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Spurious Deepening in Baroclinic Prediction Models.

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Introduction

It has been noted in JNWP Unit operations during 1955 that the baroclinic model in use has certain systematic errors. Some of these errors can be traced to errors in the lateral boundary conditions, others to terrain effects which are omitted from the prediction model. Although the evidence is not clear, still others are probably due to effects of non-adiabatic heating and to latent heat of vaporization —— neither of which are incorporated into the model.

This paper will not deal with these, but will take up the problem of spurious anticyclogenesis, which sometimes is predominant, particularly in cold, statically stable situations, and particularly in the lower levels of the forecasts. A good example of this type of error appeared in the forecast of the Thanksgiving Day storm of 1950 (Charney, 1954).

A Descriptive Analysis of the Problem

It is becoming clear to dynamical meteorologists that errors of spurious anticyclogenesis are due to faults of the model itself. Specifically, they are due to linearizations of the dynamical equations (Charney et al., 1955).

In the explanation immediately following, we will omit all terms extraneous to the gross behavior of the atmosphere, as set forth by Rossby (1940). Consider the vorticity in isobaric coordinates.

$$\frac{D\eta}{Dt} = \eta \frac{\partial \omega}{\partial p} \qquad (1)$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}$

$\omega = \frac{dp}{dt}$

$\frac{d}{dt}$ is the substantial derivative

η is absolute vorticity.

If the thermodynamic equation

$$\omega = -\frac{1}{S}\frac{D\theta}{Dt}$$

is differentiated by p, viz.,

$$\frac{\partial \omega}{\partial p} = -\frac{1}{S}\frac{DS}{Dt} \qquad (2)$$

and ω is eliminated from equations (1) and (2), we have

$$\frac{D\eta}{Dt} + \frac{\eta}{S}\frac{DS}{Dt} = 0 \qquad (3)$$

Here θ is potential temperature and $S = -\frac{\partial\theta}{\partial p}$

In equation (2) a term has been omitted on the assumption that it has to do with only modifications of the principal atmospheric processes. In so doing, it is not suggested that it is unimportant in accurate forecasting. In fact, it can be shown that it contains Charney's (1949) mechanism for finite speed of propagation of influences in the vertical. Without it influences affect all levels simultaneously. In justification of its omission from this discussion, it may be noted that equation (3) contains the interaction of vorticity and stability discussed by Rossby (1940). This interaction represents the principal machinery of the large-scale atmospheric disturbances, and the forecast errors under discussion pertain to this interaction.

If, as in the case of the JNWP Unit baroclinic model, mean values are substituted for η and S where they appear undifferentiated in equation (3), then in anticyclones, where vorticity is small and stability large, the effects of divergence (increase of stability) are over-estimated. Conversely, the effects of divergence are underestimated in cyclones. Thus in JNWP Unit predictions, cyclones are stable, whereas any anticyclone with a tendency toward anticyclogenesis deepens too much.

The correction of spurious anticyclogenesis would seem to lie in not performing the linearization of coefficients in equation (3). Computations carrying the full non-linear effects of equation (3) have been done by Charney et al. (1955). Indeed, spurious anticyclogenesis did not appear in these computations, but spurious cyclogenesis predominated to such an extent as to render the computations useless for practical purposes.

The explanation for the spurious cyclogenesis lies in the quasi-geostrophic assumption itself. Winds, and therefore vorticity, are considerably smaller in cyclones than the estimate obtained from the geostrophic. In cyclonic regions the effects of divergence is over-estimated if vorticity is replaced by geostrophic vorticity in equation (3), and deepening of cyclones is over-forecast.

At this date the foregoing will be familiar ground to meteorologists working in the field of numerical weather prediction (Charney et al., 1955). Although all aspects of the problem are not yet explained, we have a basis for a working hypothesis, viz.,

> the correction of spurious anticyclogenesis lies in the non-linear effects of vorticity and stability, but the non-linear effects must be coupled with an accounting of the ageostrophic vorticity in cyclones.

A Corrected Non-linear Quasi-geostrophic Model.

Consider the balance equation

$$f\eta - f\eta_g + 2\frac{\partial(u, v)}{\partial(x, y)} = 0$$

where η_g is geostrophic vorticity and f is Coriolis parameter. The balance equation may be derived by taking the divergence of the horizontal equations of motion, and omitting terms involving vertical motion and divergence. As pointed out by Shuman (1955), by a mere re-arrangement of terms, the balance equation may be written

$$\eta^2 - 2f(\eta_g - \frac{1}{2} f) - r^2 = 0 \qquad (4)$$

where $r^2 = (\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y})^2 + (\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y})^2$

In natural coordinates r^2 may be written

$$r^2 = (\frac{\partial V}{\partial n} + V\frac{\partial \alpha}{\partial s})^2 + 4(V\frac{\partial \alpha}{\partial n})^2 \qquad (5)$$

where V is the magnitude of the velocity vector, and α, n, and s are as indicated in Figure 1. The form of r^2 shows it to be a combination of deformation terms.

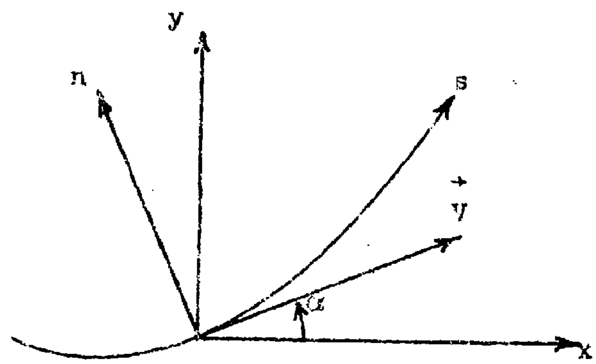

Figure 1. Plan of natural coordinates.

The last term in equation (5) depends on difluence, i.e., spreading of the streamlines. The regions of principal interest and significant activity are anticyclones and cyclones. Geometry of the streamlines in such regions of organized flow puts a limit on that term.

Likewise, the two parts of the first term are of opposite sign in both cyclones and anticyclones within the band of maximum wind around them. Thus, for the regions of principal meteorological interest, one arrives at a rough estimate of the relationship between vorticity and geostrophic vorticity by ignoring r^2 in the balance equation (4). Then,

$$\eta^2 = 2f\ (\eta_g - \frac{1}{2} f) \qquad (6)$$

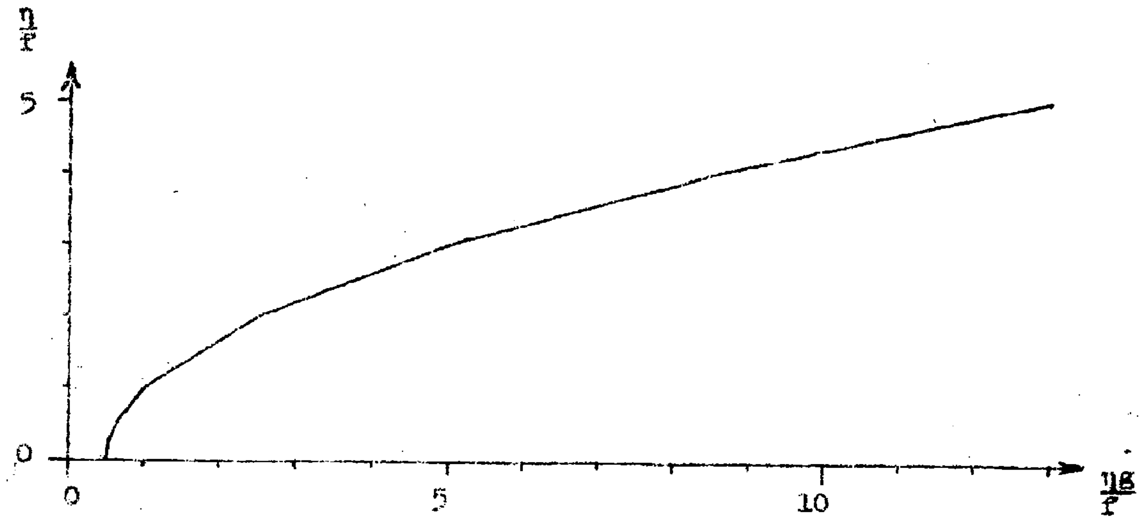

Figure 2. A graph of equation (6), an abridged form of the balance equation.

Figure 2 is a graph in $\frac{\eta}{f}$, $\frac{\eta g}{f}$ - space of equation (6).

With preliminary computations as a basis, it is predicted that the product of S and η, as defined by equation (5), will display more constancy in time and space than does η_g X S, particularly within cyclones. The significance of this fact, if the prediction is borne out, may be explained as follows.

From synoptic experience, we know that temperature fields retain their form essentially unchanged for periods of a day or two, while features of the fields are translated in space. A forecasting system correct in its essentials would have this characteristic in common with the real atmosphere. If we consider two forecasting systems, specifically,

$$\frac{D\eta S}{Dt} = 0 \qquad (7)$$

$$\frac{D\eta_g S}{Dt} = 0 \qquad (8)$$

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where η is defined by equation (6), then, speaking in general terms, developments from a set of initial conditions depend on:

1. changes in the lower bounding temperature field
2. re-arrangements of the three-dimensional field of either ηS or $\eta_g S$, depending on which of the two systems (7) or (8) are being used.

In the case of an atmosphere in which potential vorticity ($\eta_g S$ or ηS as the case may be) is constant, only the first of the above listed two items comes into play——and then only through non-uniformity of lateral boundary conditions. But in view of our synoptic experience concerning developments in the lower bounding temperature field, these developments would lead to quite stable pressure systems during a forecast covering a day or two.

It may thus be reasonably expected that if our prediction concerning the constancy of ηS compared with $\eta_g S$ is borne out, a prediction system incorporating the relation (6) will forecast less deepening of pressure systems than did the non-linear systems (Charney et al., 1955) in which η_g was taken as the absolute vorticity.

In the spirit of representing only the gross machinery of the atmosphere, the prediction model which is here proposed is, grossly,

$$\frac{D}{DT}[fS(\eta_g S - \frac{1}{2}fS)] = 0 \qquad (9)$$

which results from a substitution from equation (6) into equation (3). In view of previous remarks concerning the importance of the term omitted in writing equation (2), viz.,

$$\frac{1}{S^2}\frac{\partial S}{\partial p}\frac{D\theta}{Dt}$$

it is also proposed to take this term into account in the proposed model, which can be done in a number of ways.

Contrary Indications.

If equation (9) is decomposed into a form similar to equation (3), we have, neglecting variations of f,

$$\frac{D\eta_g}{Dt} + \frac{2\eta_g - f}{S} \frac{DS}{Dt} = 0 \qquad (10)$$

Compare this with the quasi-geostrophic form of (3), which was the basis of the non-linear models of Charney et al. (1955).

$$\frac{D\eta_g}{Dt} + \frac{\eta_g}{S} \frac{DS}{Dt} = 0 \qquad (11)$$

It is seen that use of the abridged balance equation (6), in preference to a direct use of geostrophic vorticity, leads to an increase of the effects of divergence in cyclonic areas——indeed, to doubling of the effects where η_g is very large. This is contrary to the course of reasoning in a previous section of this paper, where it was assumed that using a vorticity more correct than the geostrophic would lead to a decrease of the effects of divergence in regions of cyclonic vorticity. It can only be said that the arguments based on the weight given to divergence implicitly assume that the weight it is given does not materially affect the divergence itself, both as an instantaneous field and as a field being predicted during the course of the forecast. It is admitted that this implicit assumption applies to the JNWP linearized model well enough to qualitatively determine a priori, the effects on the forecast of tampering with coefficients in the equations. But we have no experience with extending the assumption to the non-linear models. It is suggested by the author that we do so cautiously.

A subject of immediate research by the author will be to determine whether equation (6) is an accurate representation of the balance equation (4) for systems of meteorological dimensions and form. If it is, then the predictive behavior of the model (9) will carry with it indications of success or failure for forecasting systems which incorporate the balance equation as an accounting of the ageostrophic wind component.

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