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OBJECTIVE NUMERICAL PREDICTION OUT TO SIX DAYS  
USING THE PRIMITIVE EQUATION MODEL-A TEST CASE

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## **U.S. Joint Numerical Weather Prediction Unit**

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## Abstract

This report is a brief summary of the results of a test case study using the primitive equations to make numerical predictions out to six days after the initial time. Daily and selected 5-day mean charts of 500-, 700-, and 1000-mb. height and 1000 to 500- and 1000 to 700-mb. thickness, along with their departures from normal, were produced directly from the model. For comparison, the barotropic 500-mb. forecast heights were also obtained, and both primitive and barotropic forecasts were compared with the observed heights and thicknesses.

Although the 500-mb. forecasts made from the primitive equation model were not noticeably better or worse than the barotropic forecasts on the average over the hemisphere, the 1000-mb. and thickness forecasts were of useful quality even at the 6th day in the test case. The 5-day mean forecast patterns of height and thickness correctly indicated a temperature reversal over the United States and are thus expected to be useful tools in the preparation of the Weather Bureau's 5-day mean extended forecasts.

### I. Introduction:

On November 4, 1966 the primitive equation model was run out to 144 hours, or 6 days, using the 00Z data as input. The run, which was partially sponsored by NASA in support of the Gemini XII space shot, afforded an excellent opportunity to study the behavior of the primitive equation model on a semi-operational basis. The primitive equation model is currently operationally run out to only 36 hours, after which the barotropic model is used.

For details of the theory of the primitive equation model the reader is referred to a forthcoming paper by Shuman and Hovermale [1]. In addition to the hydrodynamic terms, the model used in the test run has friction at the earth's surface (including orography) and diabatic heating due to sensible heat transfer from the ocean to the atmosphere. Radiational cooling at the rate of  $0.1^{\circ}\text{C}$  per hour is assumed to operate in the boundary layer over snow-covered ground in areas with descending motion (assumed to be clear) when the sun is less than  $10^{\circ}$  above the horizon. The model thus has a source of potential energy and a sink for kinetic energy, similar to the real atmosphere. Other diabatic effects are being studied and may be added later.

## II. Daily Predictions:

The initial weather situation at the beginning of the period studied was one of generally cold in the East and mild in the West, following a record early snowstorm throughout much of the Midwest. The surface low which had produced the snow was located near James Bay (Fig. 4A) and its associated 500-mb. low was just northeast of Lake Superior (Fig. 5A). These two figures show the subsequent evolution of the atmosphere at 48-hour intervals out to 6 days from initial time.

The outstanding feature of the development was the discontinuous retrogression of the trough over North America, brought about by the motion of a short wave from the Gulf of Alaska. As a consequence, mid-tropospheric heights increased over eastern North America and temperatures rose rapidly. The trough initially over the Mediterranean, which was associated with disastrous floods in Italy, moved northeastward and weakened as a deepening Low plunged rapidly southward from Iceland to the coast of Portugal, thereby effectively retrograding the mean trough in this area also. The strong ridge in the Atlantic moved eastward, while in the Pacific a series of short waves progressed at mid-latitudes and a cut-off Low which formed in the trough north of Hawaii retrograded slowly.

For convenience of comparison, the 500-mb. forecasts by the primitive equation model and the barotropic model are displayed in Figs. 2 and 3 respectively. The most obvious difference between the two is the gradually increasing energy appearing in short wavelengths in the primitive equation (P.E.) model. This is due to the growth of gravity waves, which are allowed by the theory of the model. These short waves do not materially affect the 5-day mean pattern (Fig. 6A) of basic interest to the Extended Forecast Division, but do complicate preparation of the daily prognostic surface maps when the gravity waves approach the magnitude of the desired synoptic short waves, as they do by the 5th and 6th days. Gravity waves may be filtered internally within the model as it runs, or they may be filtered in the output stage, at some risk to masking the desired short-range features. In any case the P.E. model does appear to be stable out to 6 days.

The barotropic model, on the other hand, has the disadvantage of being perhaps a little too "smooth," such that it is sometimes difficult to follow the weaker short synoptic waves beyond 72 or 96 hours. This shortcoming could also result in difficulty in preparing the daily maps.

Now let us consider the individual 500-mb. progs by the two models and compare them with each other and with the corresponding observed maps (See Figs. 2, 3, and 5). Both models more or less correctly moved a strong short wave trough from the Gulf of Alaska

to the vicinity of Vancouver Island by 48 hours. The actual trough movement was a little faster and a small closed center was observed just inland. Another short wave trough which was observed over Indiana was also predicted somewhat too weak and too slow. Both models predicted the southward plunge of the Low originally near Iceland, although the forecast movement was too slow and too much deepening was indicated, with the barotropic model having the larger errors on both counts.

At 72 hours both models brought the Pacific trough in over the northern Rockies, although the P.E. model suggested a separate vorticity maximum remaining off the Oregon Coast. The observed map (not shown) also indicated two vorticity maxima, with the stronger one over northern California. The motion of the eastern trough predicted by both models was only to eastern Ohio, whereas the observed position was off the New England coast. A high-latitude short wave trough component predicted in a NE-SW position over the Northwest Territory of Canada had actually moved to western Hudson Bay with a N-S orientation, and was completely sheared from the Rockies trough.

The two models had diverged more noticeably from each other by 96 hours. The P. E. prog indicated a broad long-wave trough in the Rockies with distinct short waves in the Basin, the Central Plains, and off the Oregon coast. The barotropic model had the trough off Oregon but suggested only one trough in the Western Plains. Both models indicated a weak short wave over western New England and New York, with the P.E. slightly more intense. The observed map had the separate short waves over the northern Basin area and the eastern Dakotas, but a small closed low was also found off southern California. A ridge was also building noticeably over the southern Mississippi Valley, and the eastern short wave had moved well out to sea east of Nova Scotia. The intense Low in the eastern Atlantic was again too deep and too far north on the progs, with worse errors in the barotropic model. It should be noted at this stage that neither model handled the low latitude cut-off Low in the west central Atlantic well. It was incorrectly predicted to move northward, whereas it remained south of 25°N.

On the sixth day (144 hours) the P.E. model indicated short waves near James Bay, the western Plains, and a relatively vigorous one over Utah and Arizona. The barotropic had only one, broad trough, not much changed from 96 hours. The observed map had the principal trough in the western Plains and the James Bay short wave already in the Davis Straits, indicating that the P.E. model was slow. The P.E. gave a better indication of the observed ridging in the East than did the barotropic, and also had a somewhat better indication than did the barotropic of short wave troughs approaching Vancouver Island and the western Aleutians.

The P. E. model, however, indicated the formation of a weak closed Low in the Gulf of Maine, which was completely in error. Errors in prediction of the closed Low in the Eastern Atlantic were of similar relative magnitudes as in the previous progs, with neither model indicating enough weakening or the eastward motion to the Gibraltar area at the end of the forecast period.

Since the barotropic model does not produce surface progs automatically, comparisons can be made only between the P. E. forecasts and the observed maps, shown in Figs. 1 and 4. Although the output is in height of the 1000-mb. surface, the maps are labelled in the more familiar millibars of surface pressure, using the approximate relationship that 60 meters of 1000-mb. height gradient is equivalent to 8 mb. of surface pressure gradient. Frontal positions were subjectively analyzed on the P. E. charts using primarily the configuration of the predicted 1000-mb. heights with the help of the associated thickness patterns (not shown), continuity, and the vertical velocity patterns (not shown) which were also used to subjectively locate expected precipitation areas. Moisture content was not explicitly considered, either subjectively or in the model. The intensity of the precipitation was subjectively estimated from the intensity of the vertical motion and whether the expected precipitation was near a frontal zone. The type (frozen or unfrozen) was specified from the value of the predicted 1000 to 500-mb. thickness [2] .

The 48-hour position of the Alberta Low was forecast quite well, being a little slow and somewhat too deep. The general movement of the Low initially near James Bay to Baffin Island was well indicated, although in this case the forecast was slow but too weak. The P. E. model predicted the formation of a Low in southern Mississippi which was not observed although a weak front did form from the Southern Plains to the Ohio Valley. Precipitation was observed not far north of the area where it was predicted over West Virginia and Virginia, but none was observed where it was predicted in the South at 48 hours.

At 72 hours, the Alberta Low was moved to the Dakotas and deepened to about 975 mb. This was felt at the time to be unrealistically deep, as indeed was the case. At least part of the error was due to the fact that the model brought most of the vorticity in from the Pacific Northwest coast to the Rockies whereas in actuality part of it was temporarily deflected southward. The model also may have a tendency to overdo leeside cyclogenesis. The Gulf coastal Low, in the South, which had been incorrectly formed in the first place, was unrealistically left stationary, while a separate center was predicted to form over eastern Virginia. Rather strong vertical motions were associated with this development, which corresponded to the eastern short wave discussed previously. Over an inch of rain was observed over southern New England the day previous, as the model moved the associated upper trough

too slowly. Although the timing was poor, the fact that the development was indicated at all in almost the exact place where it occurred should be considered a point in favor of the model.

By 96 hours, the deep Dakotas Low was rapidly weakened and moved eastward to Minnesota with a N-S trough extending toward Hudson Bay. A high center which had originally been in the Arctic Ocean was indicated north of Montana, very close to where it was actually observed, although a stronger High than forecast remained over the Yukon-Northwest Territory area. The P. E. prog suggested that another Low might be forming over the Panhandle area, with a diffuse warm front south of Tennessee, as well as one over the southern Lakes associated with the Minnesota storm. The observed map had a storm over Wisconsin which, however, did not evolve from the old Alberta Low but had rapidly developed from a weak wave in the Central Plains. The Alberta storm actually moved rapidly northeastward as an occluding wave and by 96 hours had been absorbed in the cyclone north of Hudson Bay.

The prog for the last day (144 hours) scored a remarkable success in the almost exactly correct prediction of a second Alberta Low just north of Montana. A wave was suggested over western Kentucky while one was observed over southern Wisconsin. The second Wisconsin Low also developed from a wave in the Plains, while the wave which had been there 48 hours previously moved rapidly northeastward to the Davis Strait. The Arctic High also weakened with the bulk of it moving eastward to James Bay, whereas the prog incorrectly kept the principal mass of the cold air in the Plains.

It is remarkable that even on the 6th day the surface prog from the P. E. model was still producing reasonable synoptic features in most middle and high latitude areas, and that many of these features were not only reasonable but in good phase with observed systems. The low latitudes were contaminated with numerous weak high and low centers presumably generated in the slack gradients by the spurious gravity wave developments. The experienced synoptician would probably disregard these in any case, so they may not be too serious a problem.

### III. Mean Forecasts:

The 5-day mean P. E. and barotropic 500-mb. forecasts centered at 96 hours are shown in Fig. 6, along with their departures from the seasonal normal. These charts, referred to as "D+4" in the Extended Forecast Division, since the period of the mean is centered 4 days after the initial time, are produced by summing the 48, 72, 96,

120, and 144-hr. maps. "D+2" charts were also calculated but are not shown here since the "D+4" charts show up the differences in the models more clearly. Qualitatively the charts are quite similar, and probably differ less from each other or the observed pattern than corresponding daily charts due to smoothing of errors in the short waves. The differences between the charts are perhaps brought out more clearly in the departures from normal (Fig. 6B, D) or in their errors (Fig. 7C, D).

Over the United States, the P. E. was better than the barotropic as more ridging was predicted over the Southeast, consequently reducing the size and magnitude of the negative error covering most of the country. The negative error of 780 ft. on the P. E. forecast in the eastern Atlantic, although large for a 5-day mean chart, was only a little over half the error made by the barotropic model. The P. E. model, however, had considerably greater positive errors at high latitudes with several maximum error centers over 700 ft. too high as compared with a single error of 600 ft. on the barotropic.

Surprisingly and somewhat disappointingly, the P. E. model also had slightly worse errors than the barotropic in the Pacific east of Japan. A study by Andrews [3] has shown persistent large positive errors in the barotropic progs near the Asiatic Coast during the cold months of the year. It has been felt these errors were due to strong sensible heat transfer from the ocean to the cold air spilling out from the Asian continent. The version of the P. E. model used in this test case has sensible heating, although latent heat has not yet been introduced.<sup>1</sup> Since the overall error pattern is one of generally too high heights at high latitudes and too low at low latitudes, there may also be a problem in the way momentum is transported or in the surface frictional constraints. The large-scale error pattern of the barotropic forecast was similar to that of the P. E. though not as bad over high latitudes and the Pacific sector.

Comparison of the departure from normal (DN) patterns of the P. E., barotropic, and observed D+4 maps (Figs. 6B, 6D, 7B) indicates that both models gave the right overall indication. It is the "D+4 DN" chart which is probably the most important single tool used by the Extended Forecast Division in preparing its 5-day forecasts of temperature and precipitation. Location and magnitude of DN centers and direction and strength of anomalous gradients are objectively and subjectively evaluated in preparing these forecasts. See, for example, Klein [4], and O'Connor [5].

All three DN patterns suggest cold over most of western North America and milder to the east, although the barotropic model, which had the largest negative error over the southeastern United States,

would not strongly imply above normal temperatures in that area. Both progs were almost perfect on the position and intensity of the eastern Pacific ridge and positive height anomaly center. The spurious increase of heights to above normal values at high latitudes, particularly by the P. E. model, resulted in the negative DN center being predicted over Montana by the P. E., and Saskatchewan by the barotropic, whereas the observed center was west of Hudson Bay.

Since surface temperature is more directly related to lower tropospheric thickness than to a mid-tropospheric height, it might be worth while to examine the errors in the D+4 1000-500 mb. thickness pattern, shown in Fig. 7A for ease of comparison with the errors of the height forecasts. It can be seen that the errors in the P. E. thickness forecast are less than the P. E. height errors in nearly all areas, most notably near the eastern Atlantic Low, over the United States, and at low latitudes in the Pacific, where they are also less than the barotropic errors. This reduction of error in the thickness is due largely to compensating errors at 500 mb. and 1000 mb. Least improvement seemed to occur at high latitudes, where, perhaps due to the greater airmass stability, there is less coupling between the surface and mid-troposphere. Perhaps more cooling than is now suspected occurs in the mid-troposphere during the Arctic night as well. The model allowed for radiational cooling to occur in only the lowest layer.<sup>2</sup>

The D+2 and D+4 P. E. and observed thickness DN's are shown in Fig. 8. The temperature reversal which actually occurred over the United States (Fig. 9) was well indicated by the change in the departure from normal of the 5-day mean thickness patterns. The error in position of the negative D+4 thickness DN center over Canada was less than for the corresponding height center. Warm and cold areas were generally well indicated on both the D+2 and D+4 charts. There is some suggestion from the D+4 thickness DN pattern that the high latitudes are too warm and low latitudes too cold. This error might be reduced by applying a latitudinal heating gradient, in addition to the effects now incorporated in the model, although probably it would be better to incorporate into the model all known diabatic effects in as realistic a manner as possible before making empirical corrections.

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<sup>1</sup>Beginning on February 20, 1967, latent heat feedback was incorporated into the P. E. model.

<sup>2</sup>Subsequent to the time of the P. E. forecast run reported in this case study, radiational cooling at the rate of 1/4 that in the lowest layer has been added throughout the depth of the atmosphere.

## SUMMARY

The primitive equation model has been demonstrated to give reasonably stable and accurate predictions out as far as 6 days after the initial time. Output is available at several levels, and the 500-mb. and 1000-mb. daily maps studied in this report appear to be useful. (Although not shown or discussed in detail in this report, daily thickness forecasts are also available.) The overall accuracy of the primitive equation 500-mb. progs does not appear to be materially different from that of the barotropic model; however, the added utility of reasonably accurate completely objective 1000-mb. progs appears to be advantageous. Results of this initial case study indicate that principal errors appeared to be in timing of short waves and in the latitudinal pressure profile, with the primitive equation model handling the short waves a little better but the barotropic model having less spurious height rises at high latitudes. Long wave positions were predicted similarly and reasonably well by both models.

The forecast 5-day mean charts of height and thickness and their departures from normal centered 4 days after the initial time were good enough to quite accurately indicate a marked temperature reversal over the United States. The D+4 thickness forecasts, which could not be produced from the barotropic model, appear to hold much promise as a new and useful tool for extended-range forecasting.

## REFERENCES

1. F. G. Shuman and J. B. Hovermale, "An Operational Primitive Equation Model," to be published in 1967 in the Journal of Applied Meteorology.
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3. J. F. Andrews (unpublished research and personal communication).
4. W. H. Klein, "Application of Synoptic Climatology and Short-Range Numerical Prediction to Five-Day Forecasting," Weather Bureau Research Paper No. 46, Washington, D. C., 1965, 109 pp.
5. J. F. O'Connor, "Catalog of 5-Day Mean 700-mb. Height Anomaly Centers 1947-1963 and Suggested Applications," NMC Technical Memorandum No. 37, Washington, D. C., 1966, 63 pp.

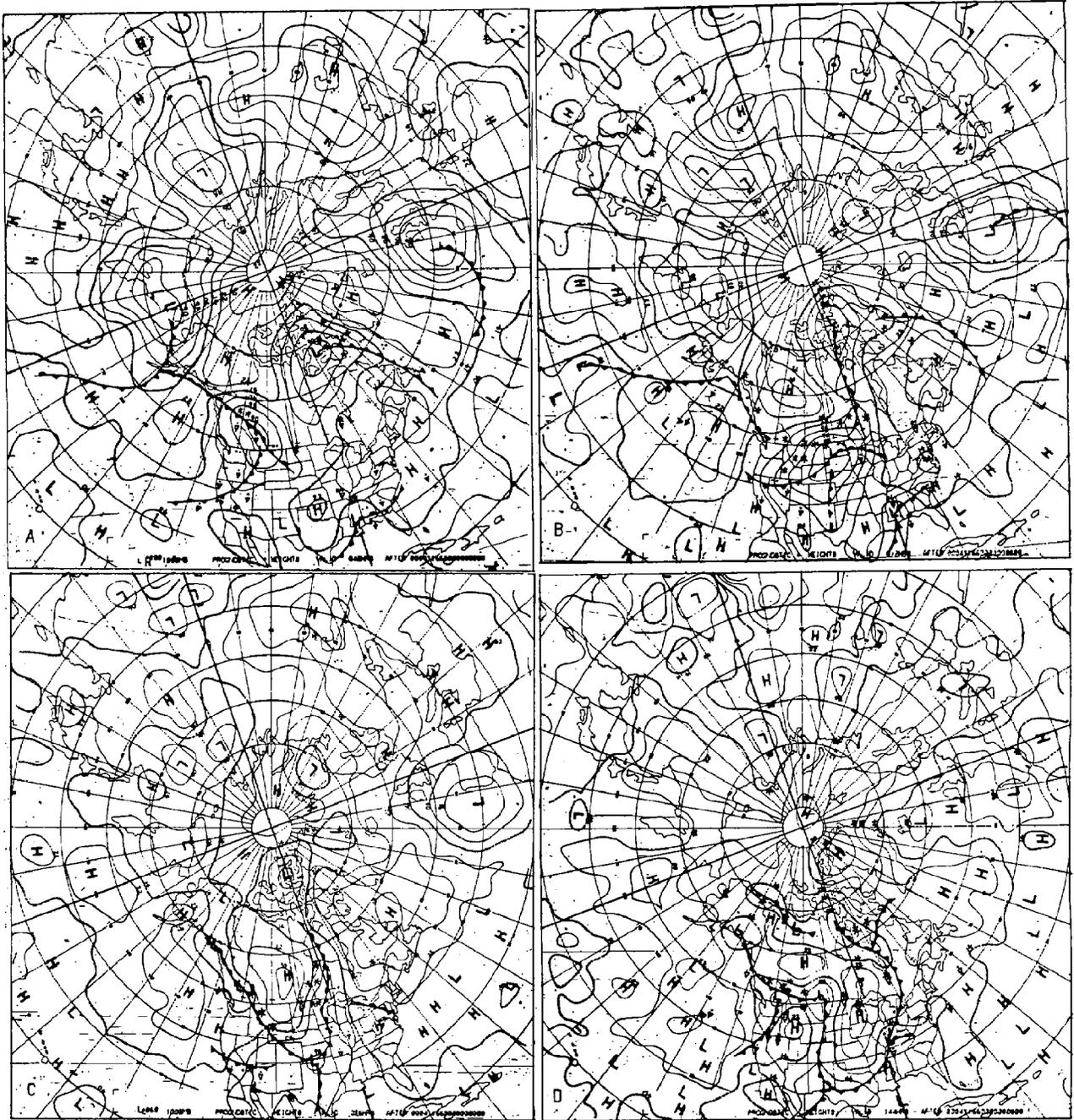


Figure 1 1000-mb. prognostic charts from the Primitive Equation Model verifying: (A) 48 hrs., (B) 72 hrs., (C) 96 hrs., and (D) 144 hrs. after the initial time. Contours are for intervals of 60 m. (approx. 200 ft.) and are labeled in mb. (approx. 8 mb. to 200 ft.).

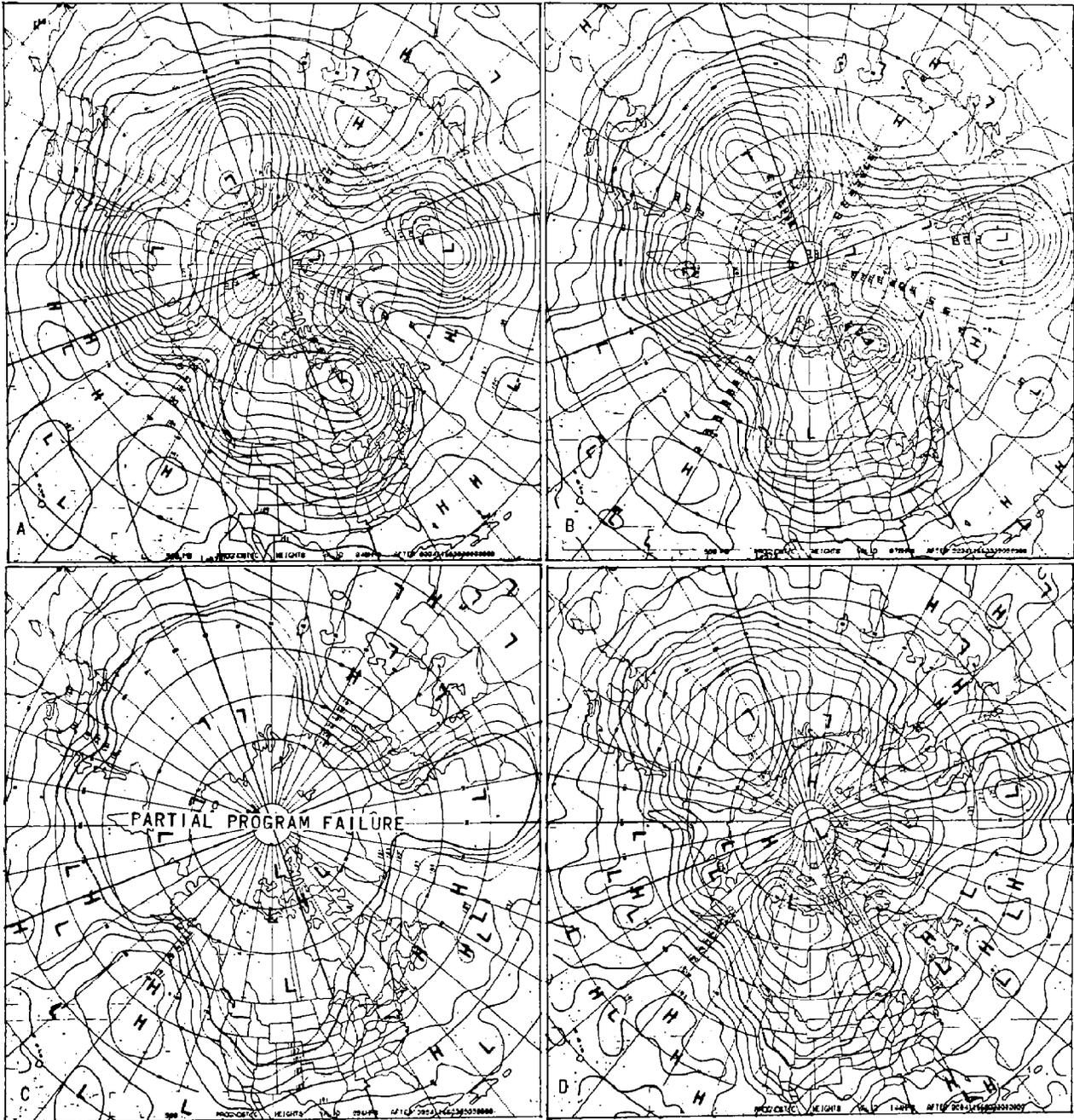


Figure 2 500-mb. prognostic charts from the Primitive Equation Model verifying: (A) 48 hrs., (B) 72 hrs., (C) 96 hrs., and (D) 144 hrs. after the initial time. Contours are for intervals of 60 m. (approx. 200 ft.) and are labeled in hundreds of ft.

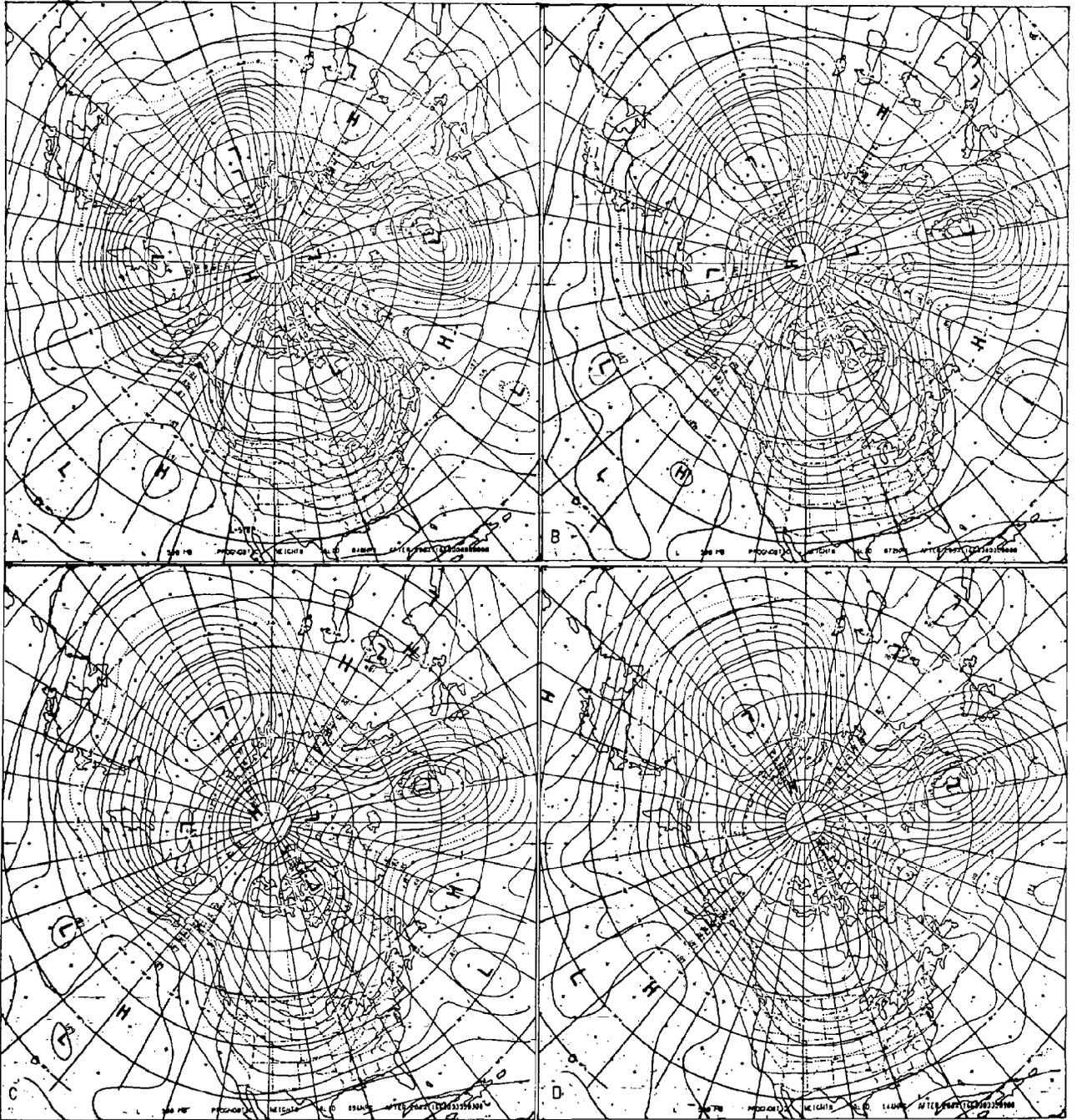


Figure 3 Same as Fig. 2, using output from the Barotropic Model.

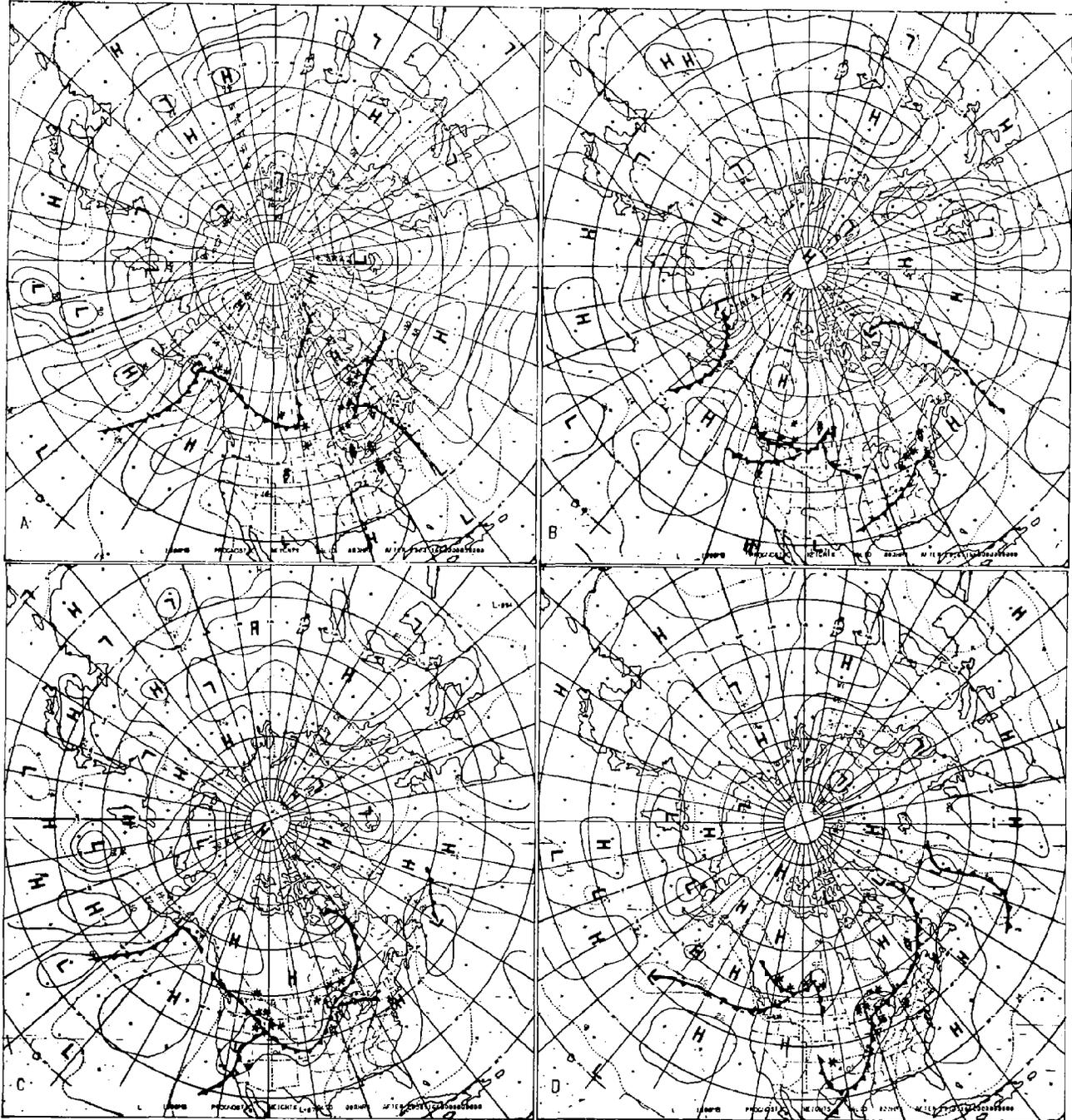


Figure 4 1000-mb. observed charts for (A) initial, (B) 48 hrs., (C) 96 hrs., and (D) 144 hrs. Labeling as in Fig. 1

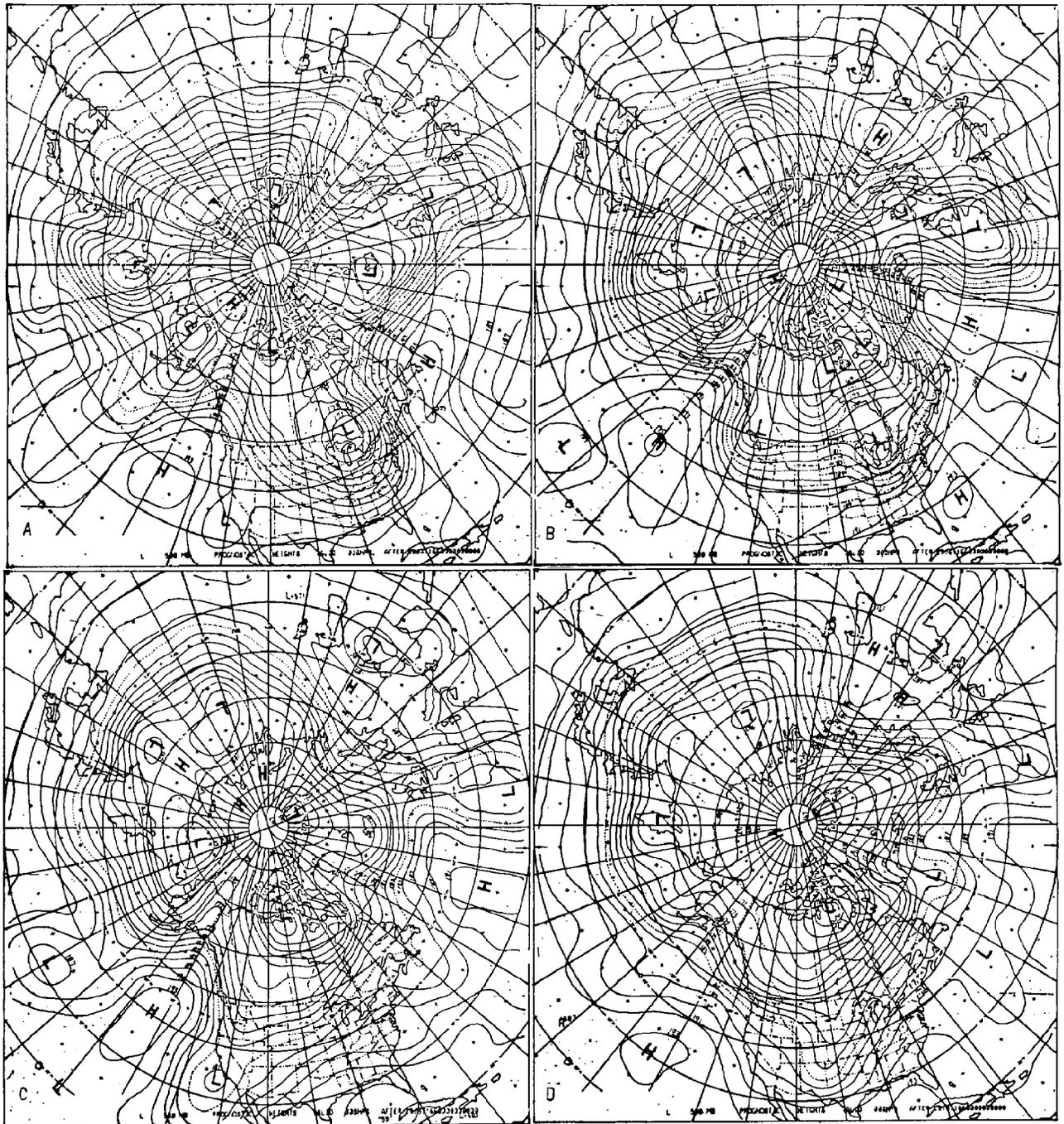


Figure 5 500-mb. observed charts for: (A) initial, (B) 48 hrs., (C) 96 hrs., and (D) 144 hrs., Labeling as in Fig. 2.

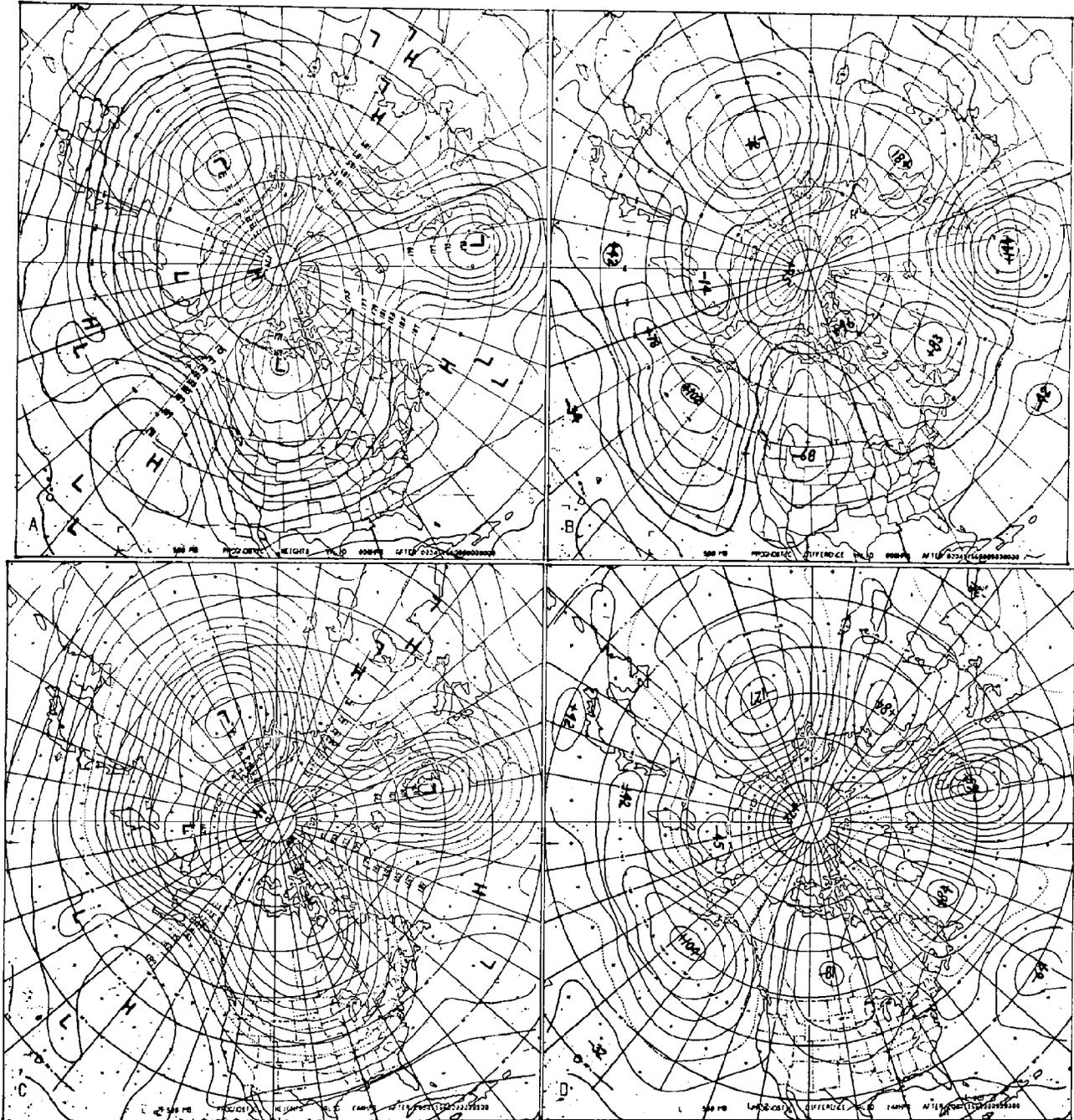


Figure 6 (A) D+4 500 mb. height from the Primitive Eq. Model  
 (B) its departure from normal  
 (C) D+4 500 mb. height from the Barotropic Model.  
 (D) its departure from normal  
 Contours are labeled as in previous figures, and central values of departure from normal are in tens of feet.

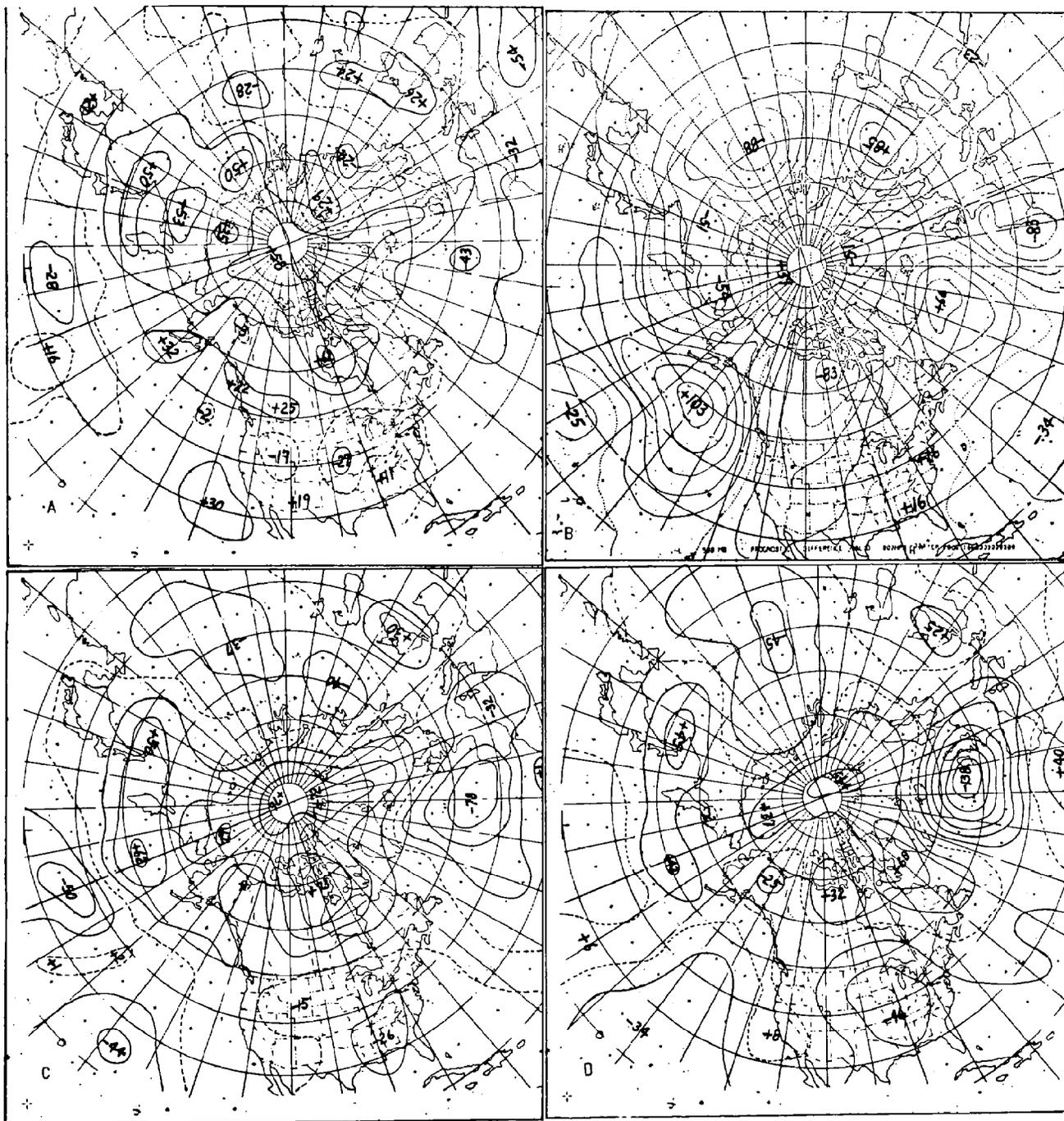


Figure 7 (A) D+4 1000-500 mb. thickness error  
 (B) D+4 500-mb. observed departure from normal  
 (C) D+4 500 mb. Primitive Eq. ht. error  
 (D) D+4 500 mb. Barotropic ht. error  
 Central values of error and departure from normal are in tens of feet.

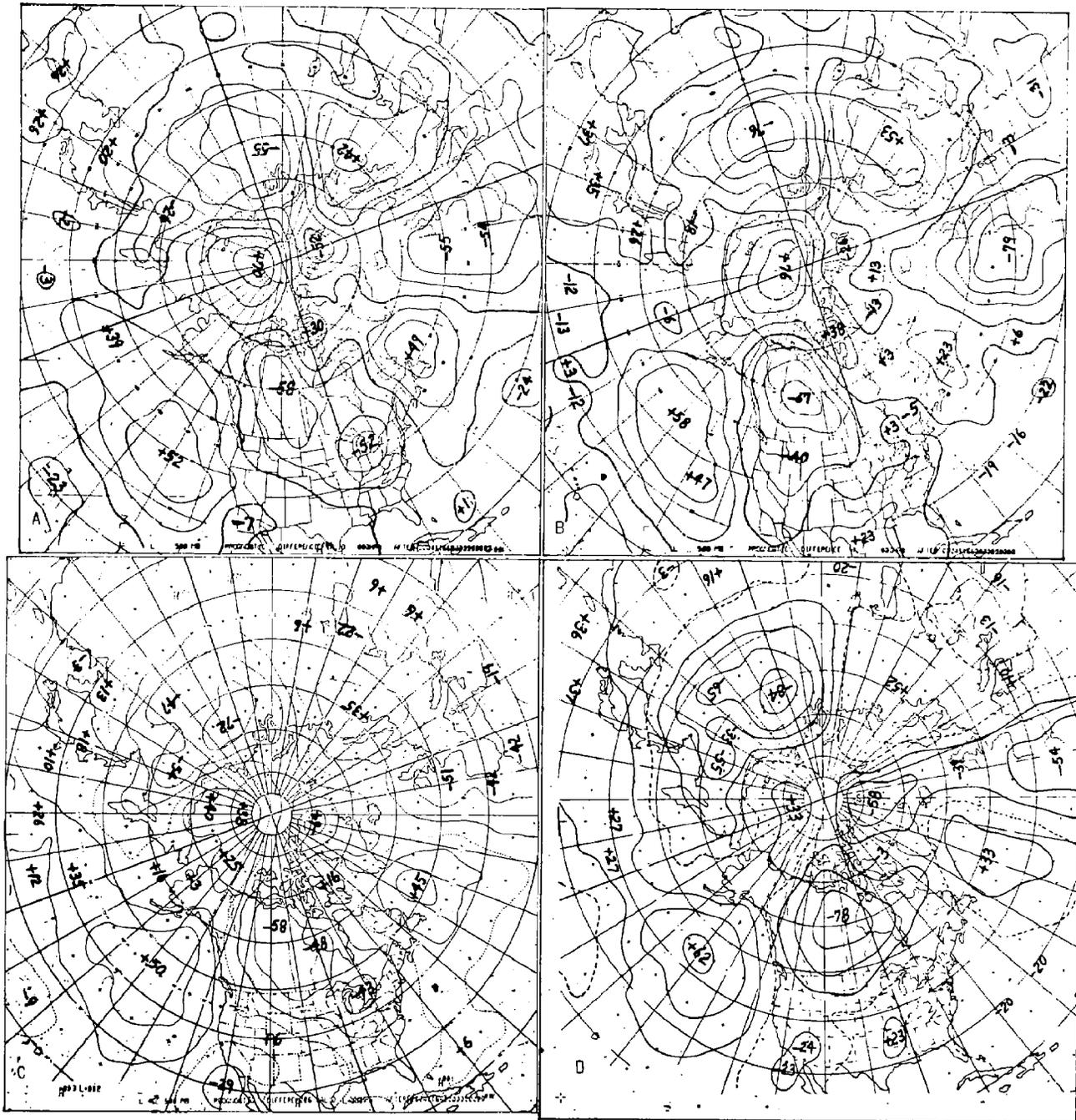


Figure 8 (A) D+2 1000-500 mb. thickness departure from normal from the Primitive Equation Model  
 (B) Same as in (A) for D+4  
 (C) D+2 1000-500 mb. observed thickness departure from normal.  
 (D) Same as in (C) for D+4

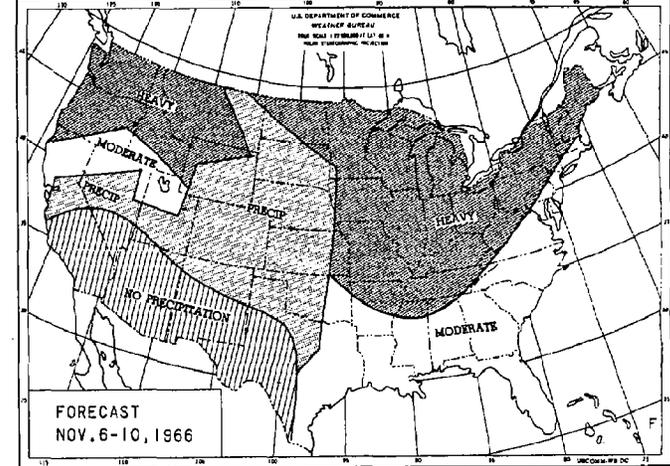
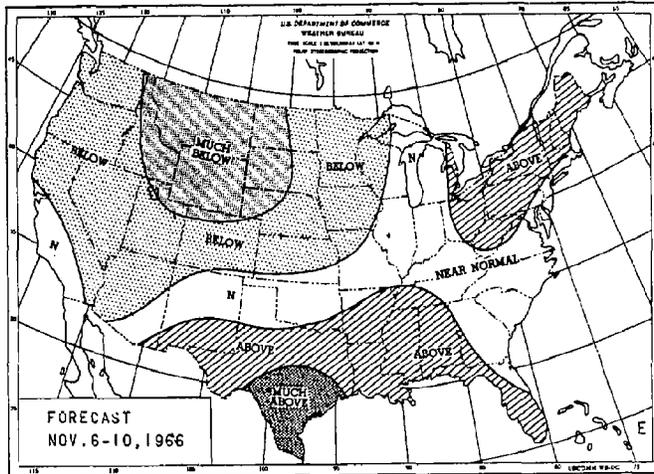
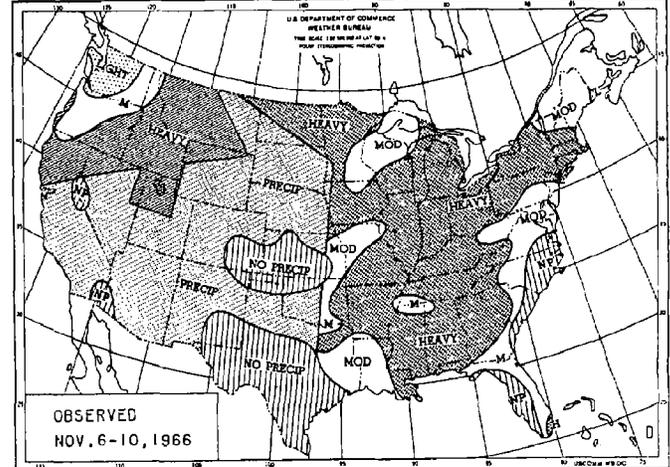
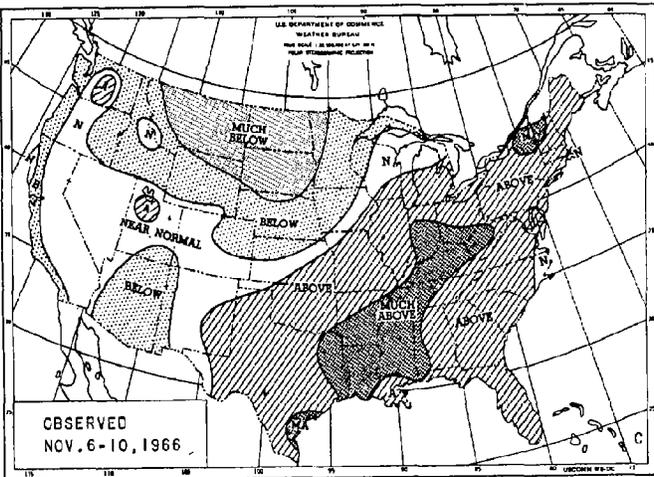
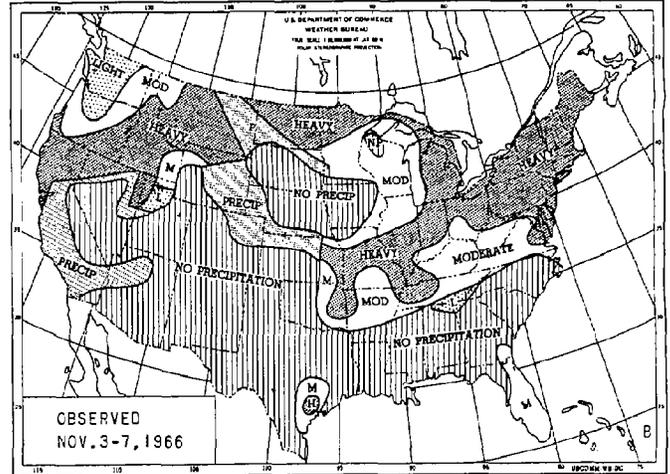
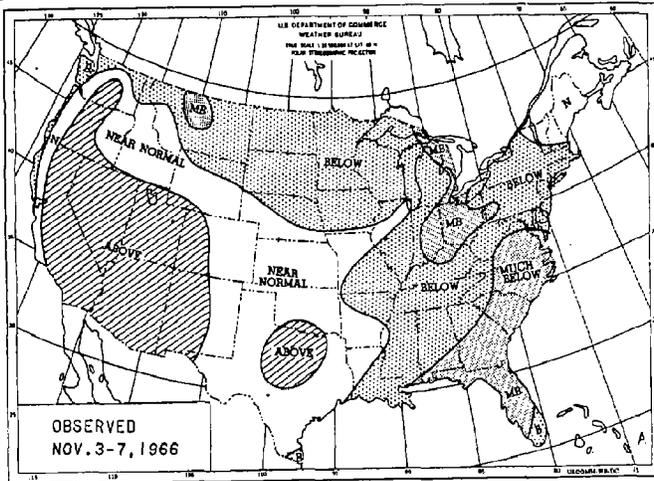


Figure 9 Observed (A) temperature and (B) precipitation categories for Nov. 3-7, 1966. (C) and (D) show observed temperature and precipitation categories for Nov. 6-10, 1966. (E) and (F) show predicted categories of temperature and precipitation for the D+4 period, which is Nov. 6-10, 1966.