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The Error in Numerical Forecasts Due To Retrogression of Ultra-Long Waves

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ABSTRACT

Error charts for the numerical barotropic forecasts prepared at the Joint Numerical Weather Prediction Unit since October, 1957 have revealed retrogressive patterns of very long wave-length. These errors are shown to be due to changes in the large scale components predicted by the numerical model. These components are actually quasi-stationary in the atmosphere. Forecasts prepared with an approximation to these components held unchanged show significantly increased accuracy. Finally, some of the difficulties in developing a more acceptable physical approach to this problem are outlined.

Any opinions expressed by the author is his own and do not necessarily reflect the views of the Navy Department.

1. INTRODUCTION

Beginning in October, 1957 500 millibar barotropic forecasts at the Joint Numerical Weather Prediction Unit were prepared on a hemispheric basis with boundaries in the tropics. The smaller area of previous forecasts had boundaries in meteorologically active locations, and it was widely anticipated that this expansion would be accompanied by a large reduction in gross error. Instead, the new error patterns were of very long wave-length and showed westward motion during the forecast.

Figure (1) is a 48-hour error chart in which the type of error treated here is prominent. The scale of the pattern is much greater than that of the corresponding 48-hour observed height changes. The largest scale error is positive over the Central Pacific Ocean and negative over the Eastern Atlantic and Western Europe. The persistence in location of these errors from day to day has been so high that the variations in shape and intensity due to other causes is not large enough to cause a reversal of sign in these areas. That is; while the magnitude of the error over the Pacific varies it always has a positive sign in the 48 hour forecasts. Similarly the sign of the error on the west coast of Northern Europe has always been negative.

Some other persistent errors of smaller scale and slightly less persistence should also be noted in fig. (1). These patterns are continental in scale and have positive centers off the East Coasts of North America and Asia and negative centers inland from the West Coast of North America and Europe.

These observations led to the hypothesis that the errors were due to improper treatment in the prediction model of waves of very long wave length and low wave number. To test this hypothesis a numerical method for computing the large scale component was devised and applied to many 500 millibar initial charts and forecasts from the Winter of 1957-58. This paper will give evidence in support of the following observations and conclusions derived from this investigation:

a. There is a large amount of energy present in low wave numbers in the atmosphere and these waves are quasi-stationary.

b. In current numerical models these ultra-long waves are moved westward at very great speeds and altered in shape and intensity.

c. Numerical forecasts made with these components held stationary are consistently superior to those prepared without this modification.

2. Method of Computation

The sinusoidal form has been frequently assumed for atmospheric wave disturbances and the Fourier representation was chosen in this investigation for ease in machine computation. The form used was

$$\Phi(\lambda) = A_o + \sum_{n=1}^{n=\infty} A_N \cos n\lambda + B_n \sin n\lambda$$

where Φ = height of 500 mb surface

λ = Longitude in radions

n = wave number

Computations of A_n and B_n were made for wave numbers one through five for each initial chart and 48 hour forecast for January and February, 1958. Due to an insufficient number of grid points, the computations for wave numbers higher than one were terminated at Latitude 70 degrees North.

3. Description of Wave One in the Atmosphere

Of the components computed, wave number one had the greatest intensity. For these sixty winter days there were two pairs of centers on each initial chart of wave number one. Since the Fourier computation enforces symmetry in each pair, it is sufficient to describe the negative center——the positive center completing the pair being the mirror image of the negative one.

The lower latitude centers were fixed in phase with the negative center over the Western Pacific Ocean. These centers varied in intensity from 300 to 850 ft.

There was another pair of centers in higher latitudes. These centers were almost randomly distributed in phase. They varied in intensity from 150 to 1000 ft. Figure (2) shows wave one for 00Z 6 Jan. 1958. This was the weakest wave number one pattern observed in the two month period.

4. Distortion of Wave Number One in the Numerical Forecasts

The basic equation of the forecast model requires the individual conservation of vorticity. From this same consideration Rossby (1) derived his famous equation governing the motion of long waves on a zonal current. The phase-speed is given by the relation:

$$C = U - \frac{\beta L^2}{4 \pi^2}$$

where U is the mean zonal wind, β is the variation of the Coriolis parameter with latitude and L is the wave length.

From this formula it is found that the maximum value of U observed in the atmosphere will give eastward displacement only for wave number five and higher. Therefore in this numerical model wave numbers one through four will be moved rapidly westward. In his computation of "influence functions" this effect was dismissed by Charney (2) with the statement "These (very long waves) are associated with little of the total energy".

Figure (3) shows the initial and 48 hour forecast position of the Southern negative center of wave number one for January 1958. The initial positions are all clustered in the Western Pacific, illustrating the almost stationary nature of these waves. The forecast centers have been moved southward slightly and westward at high speed. Table I summarizes the treatment of both centers in the 48 hour forecasts.

Table I - Wave One Average Values

	Southern Centers			
	Jan.		Feb.	
	D00	D48	D00	D48
LAT.	48	35	44	30
LONG.	168E	014E	172W	69E
INTENSITY	570 ft	280 ft	460 ft	240 ft
	Northern Centers			
LAT.	73	73	68	69
INTENSITY	480 ft	620 ft	440 ft	600 ft

Returning to 6 January data, fig (4) and fig (5) show the retrogression of wave number one at 24 hours and 48 hours in the forecast from this initial data. This January 6 initial chart is the weakest wave number one from the 60 day investigation. This false retrogression is easily seen to be a major contributor to the 48 hour error, owing to the almost exact reversal of phase.

5. Wave Numbers 2, 3, 4, and 5

Bristor (3) has recently developed a convenient form of machine computation of kinetic energy. This analysis was made for the individual wave components for the January and February months. A summary of the distribution of energy among wave numbers is shown in Table II.

Table II Average Kinetic Energy (units proportional to knots^2)

	Total Perturbation KE	WV1	WV2	WV3	WV4	WV5	WV6 +up
Jan	426	77	53	41	59	39	157
Feb	486	66	40	45	60	30	245

Table II shows a surprising amount of energy rather uniformly distributed in these low wave numbers. The familiar long waves (wave number circa 5) have less important intensity.

The barotropic model occasionally held one or two of these waves stationary but spurious retrogression was the normal behavior. The slower westward motion in the forecast was somewhat balanced by the shorter wave-length so that reversal of phase was again possible in these wave numbers. Thompson (4) has shown that particular forms of zonal wind profiles are capable of holding waves of these lengths stationary.

Fig (6) is the analysis of Wave Number Two for 6 January initial data and fig (7) is Wave Three from the same day. This wave number two is somewhat more intense than average while the wave number three is about average intensity.

6. Comparative Verification Data

In order to correct the operational forecasts for these effects wave numbers one, two, and three were computed and added to form an approximate stationary component. This correction was applied to a series of forecasts and was incorporated in the operational computations on 10 April 1958. In order to avoid excessive stabilization of the flow pattern at the high latitudes, the stabilization of wave numbers 1, 2, and 3 was terminated at latitudes 75° N, 65° N, and 55° N respectively.

A series of forecasts were corrected for waves one two and three. Table III lists the RMS errors in feet for these corrected forecasts along with the original error and the error of persistence.* Petterssen (5) has recently suggested the comparison of RMSE's of forecast and persistence as a valid scheme of verification.

* Essentially similar results were obtained by L. Carstensen who obtained the stationary component by repeated smoothing and subtracted a tendency computed from this component at each time step in the forecast.

Table III Root Mean Square Errors in Feet

Day	Persistence	Operational	WV1	WV1, 2, & 3
Jan 8	325	406	278	230
Jan 10	324	377	251	201
Jan 12	296	411	293	229
Jan 14	309	414	296	252
Jan 16	352	482	319	217
Feb 8	272	339	262	181
Feb 12	310	372	339	217
Feb 16	303	479	320	227
Average	311	410	295	219

The mechanics of correcting the forecast consists of operating on the stream function periodically during the forecast with the following identity.

$$\psi_{\text{corrected}} = \psi_\gamma + S_{oo} - S_\gamma$$

where $\psi_{\text{corrected}}$ is corrected stream

ψ_γ is forecast stream at time γ

S_{oo} is initially computed stationary components

S_γ is stationary component at time γ

Figure (8) shows the same type of comparison for a forecast made from 12Z 31 March 1958. These curves show positive skill for the corrected forecast out to 72 hours compared with persistence.

7. Improvement in the Empirical Procedure

Fig (9) shows the error pattern for the same forecast as figure (1) after correction for the stationary component as defined in para 6. In addition to the marked reduction in error the scale of the error pattern is much more acceptable.

Although these results may appear impressive, further improvement is expected from a different method of defining the stationary component. The symmetry of the fourier computation must introduce local errors even while effecting an over-all improvement. The discontinuity at 70° Lat may produce gradient errors in high latitudes. Since the removal of the errors arising from spurious retrogression of the ultra-long waves, certain systematic errors of treatment of the zonal profile have become relatively more prominent. Improvements in the definition of the stationary component should be accompanied by some method of removing systematic profile errors.

8. Improvement in the Physical Model

The empirical corrections now employed should obviously be replaced by terms in the equations which describe the mechanism by which energy is transferred to and from these waves and the fields which hold them stationary in the atmosphere.

Figure (10) is the long wave component plus the zonally-averaged flow for 6 Jan 1958. Both this quasi-stationary component and the error locations suggest that the physical mechanism by which the large scale quasi-stationary components are maintained must in some way reflect differences in the surface characteristics of land and sea.

A test for any formulation of the physical mechanism is a numerical prediction model which will produce forecasts superior to those obtained by the present method.

Acknowledgments

The investigation reported here would not have been possible except for the continued encouragement and guidance of Lt. Col. Philip Thompson, Head Development Section, JNWPU.

The author is also indebted to LCdr W.E. Hubert, who noted the regular retrogression of the 12 hour error patterns in October 1957 and also read the completed manuscript. Walker, R.E. AG2, USN assisted in the many computations involved.

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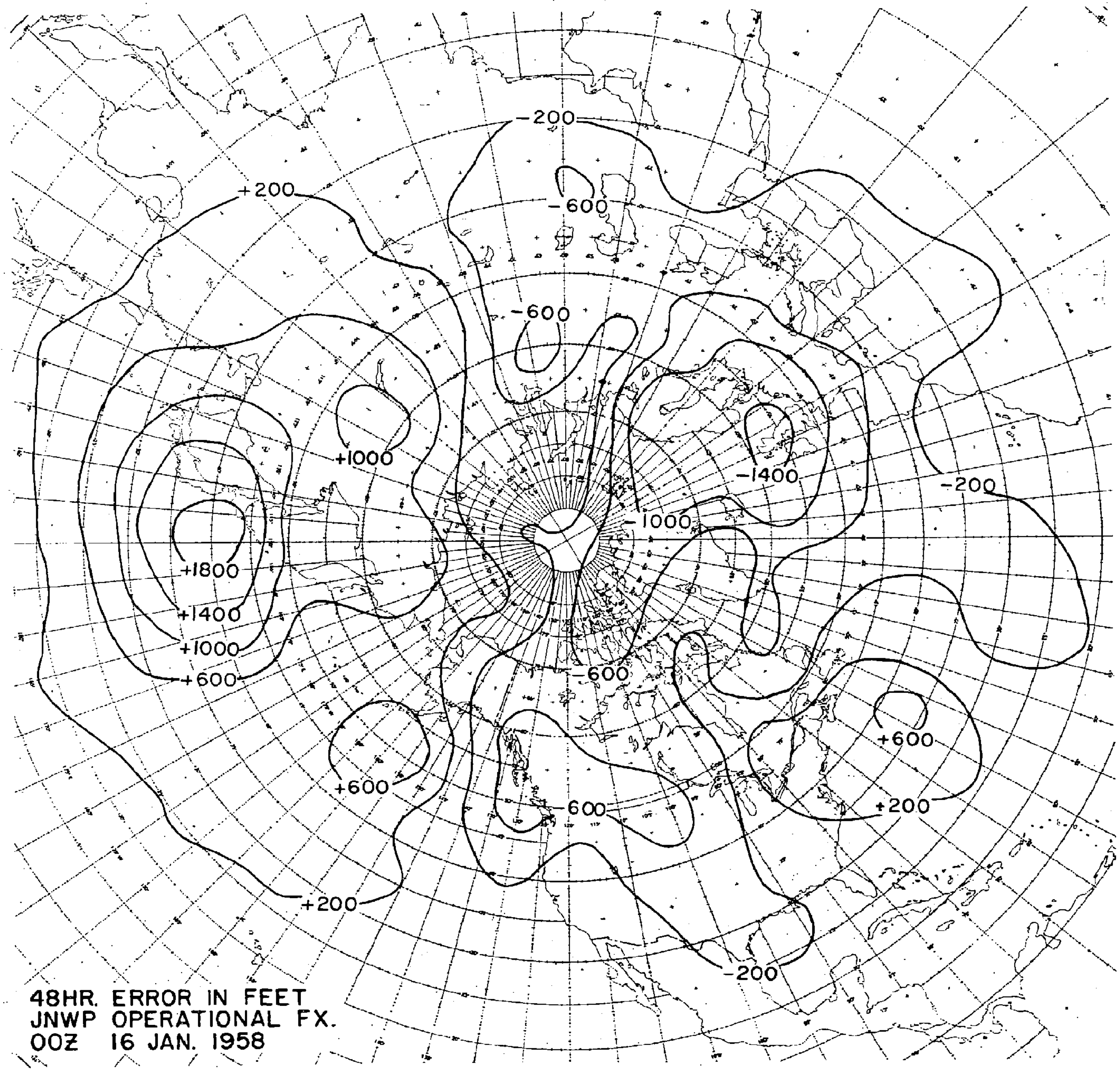
List of Illustrations with Legends



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48HR. ERROR IN FEET
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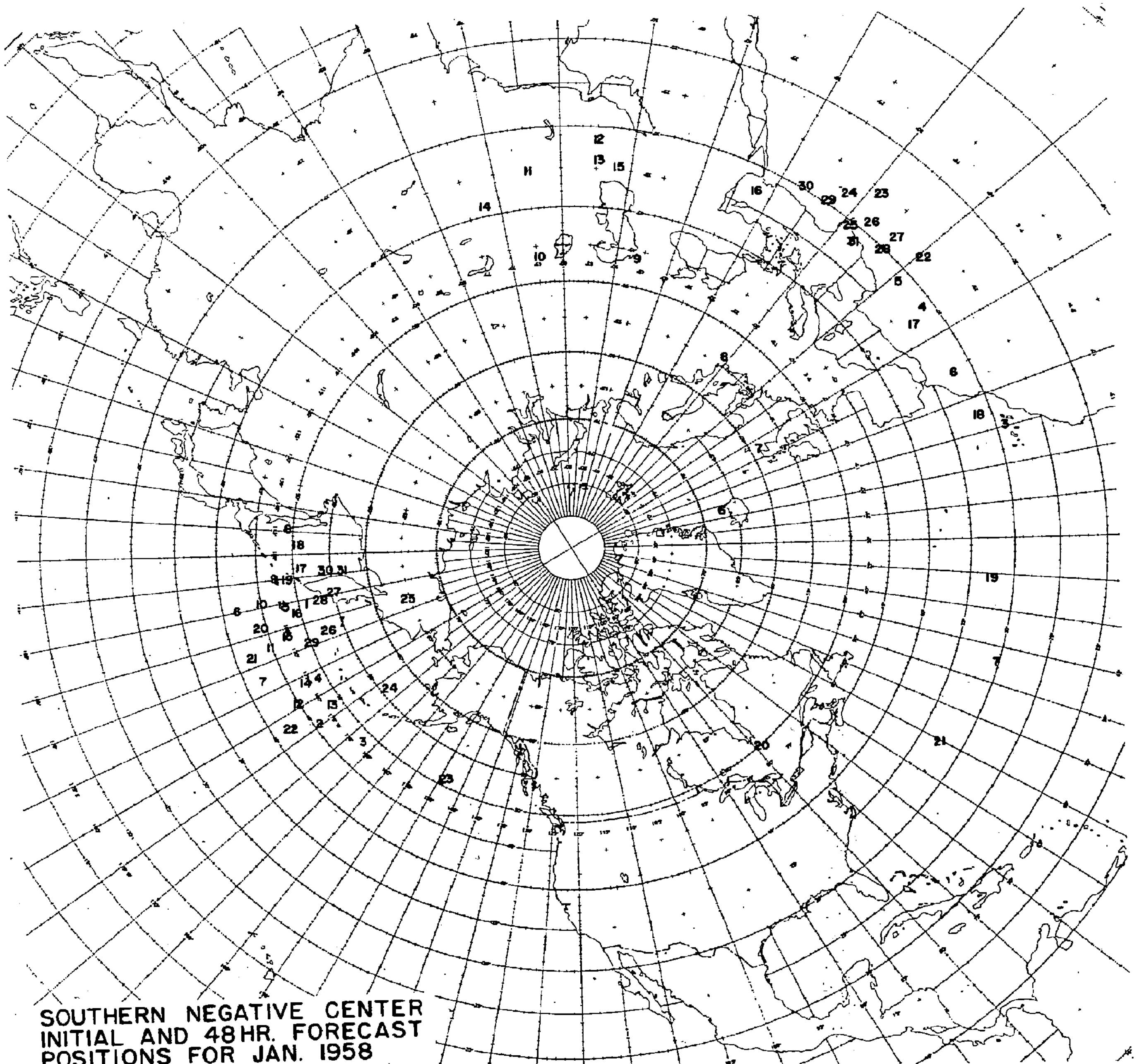
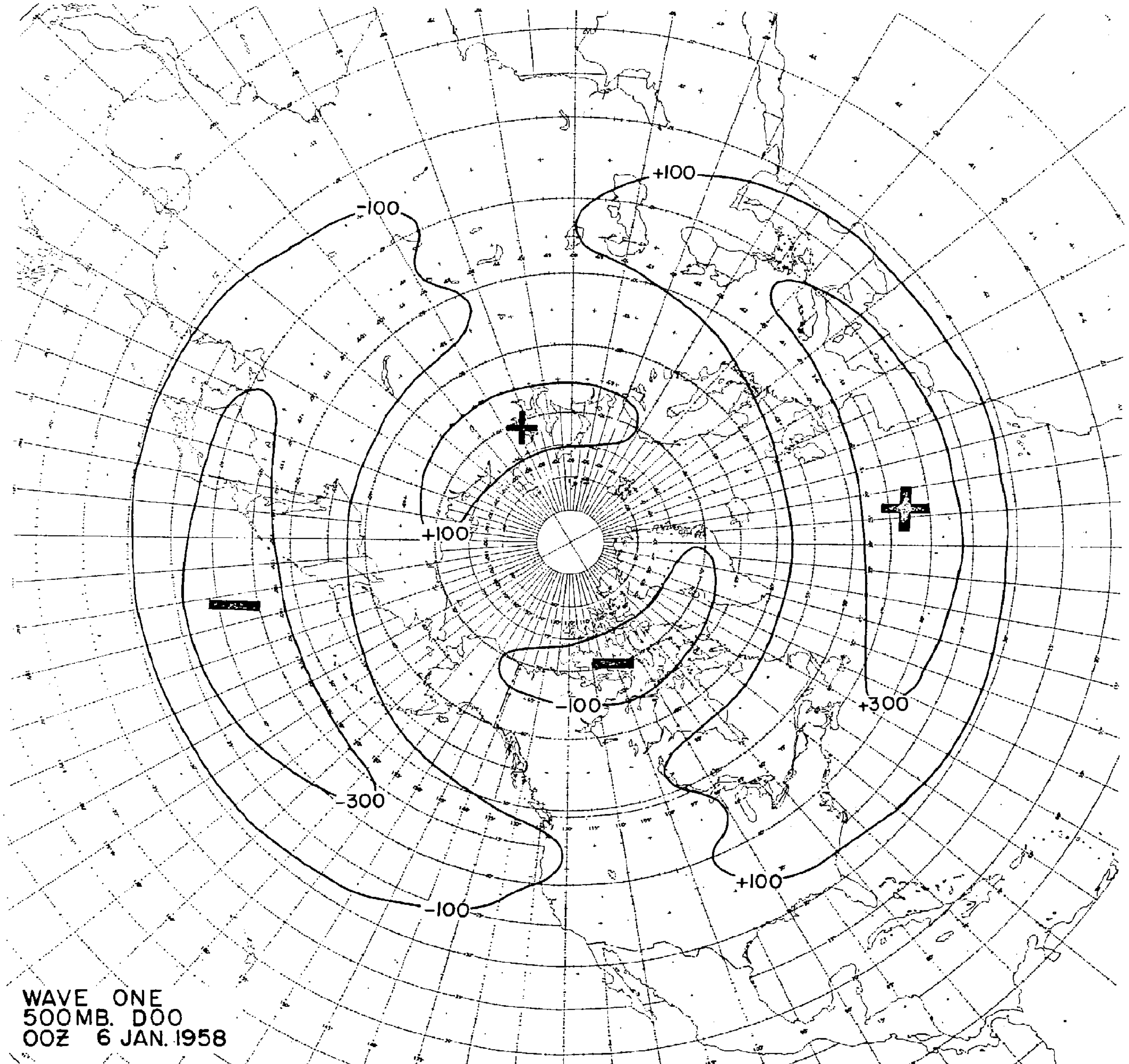
00Z 16 JAN. 1958



SOUTHERN NEGATIVE CENTER
INITIAL AND 48 HR. FORECAST
POSITIONS FOR JAN. 1958



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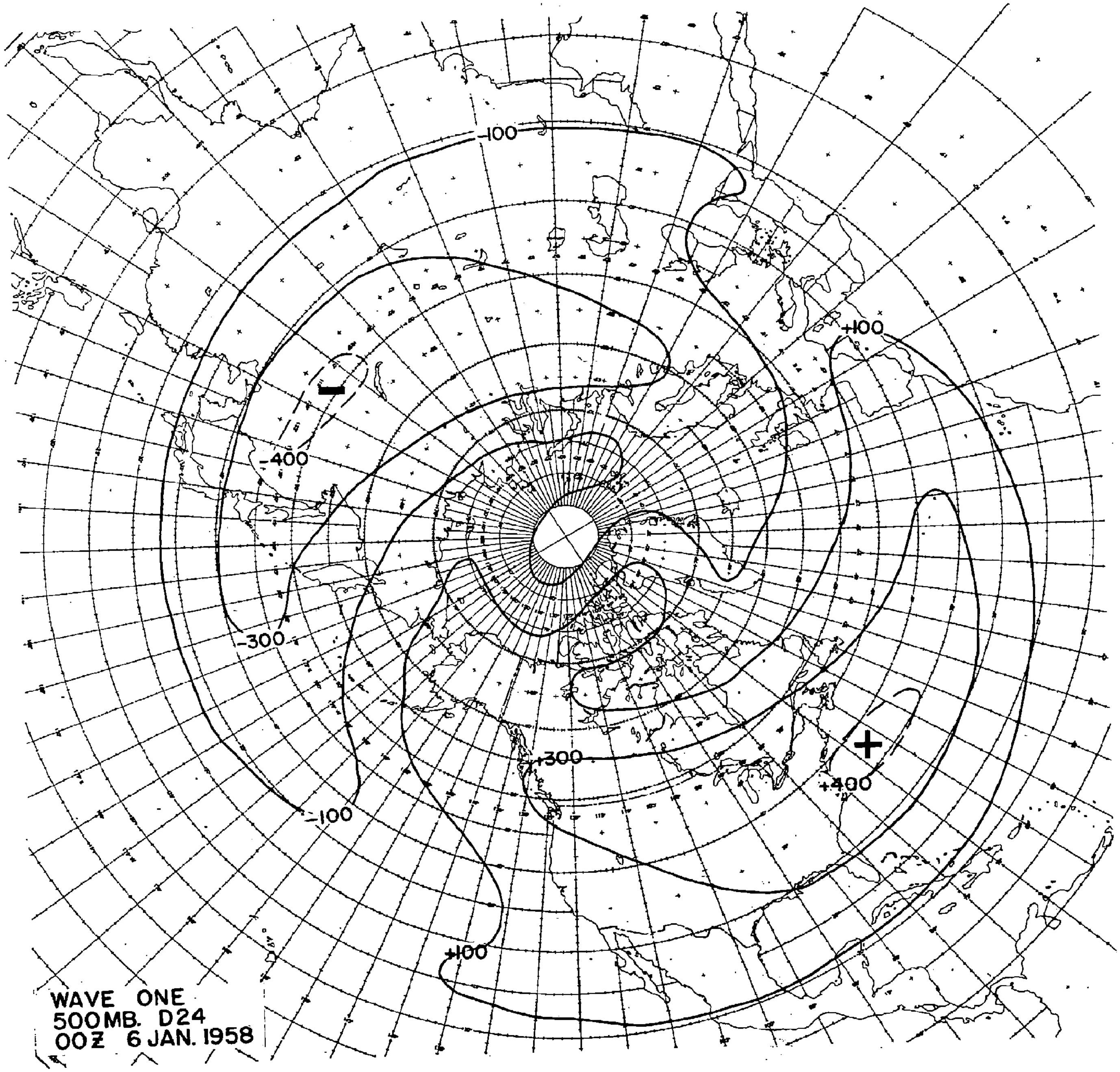
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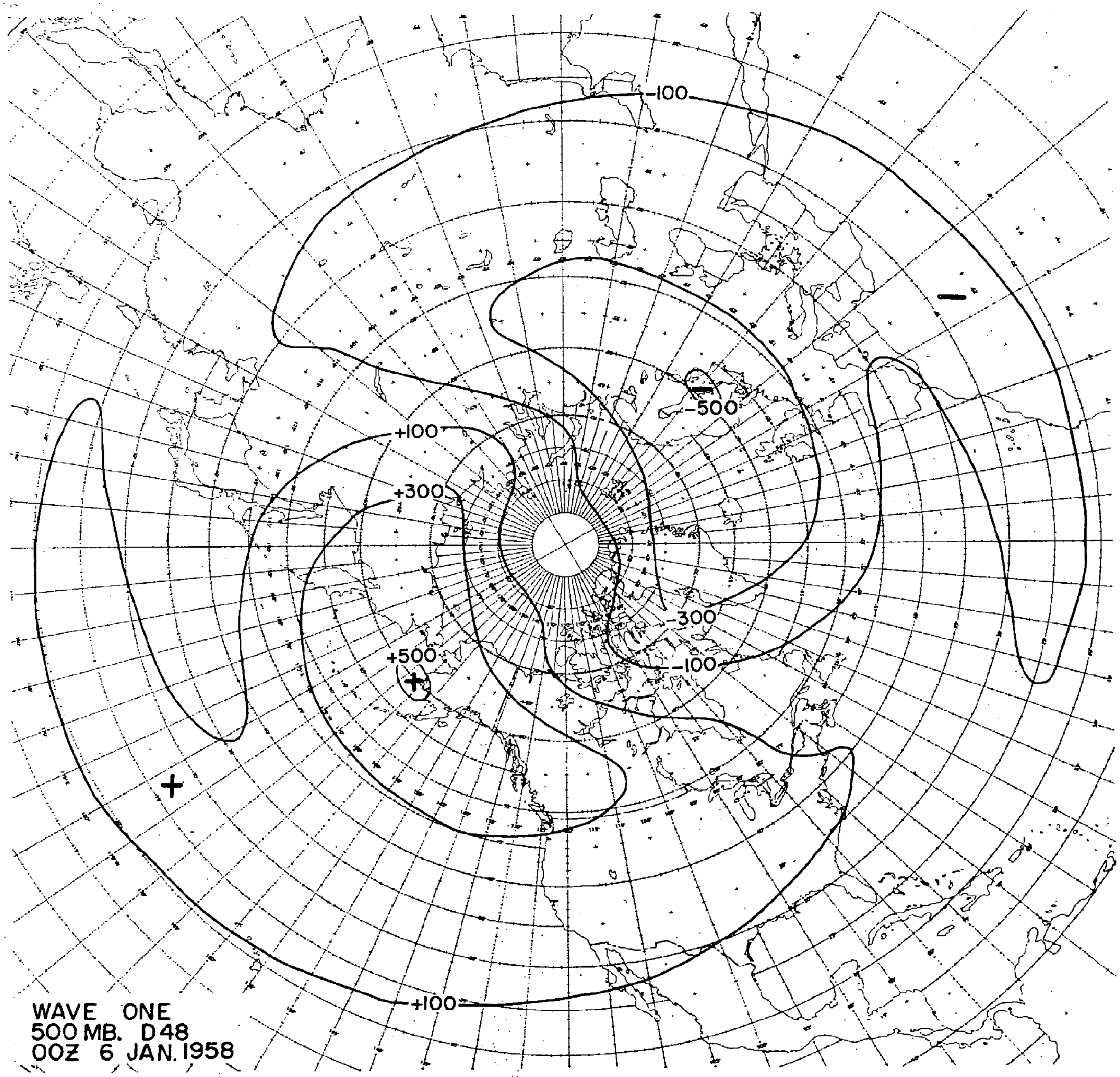
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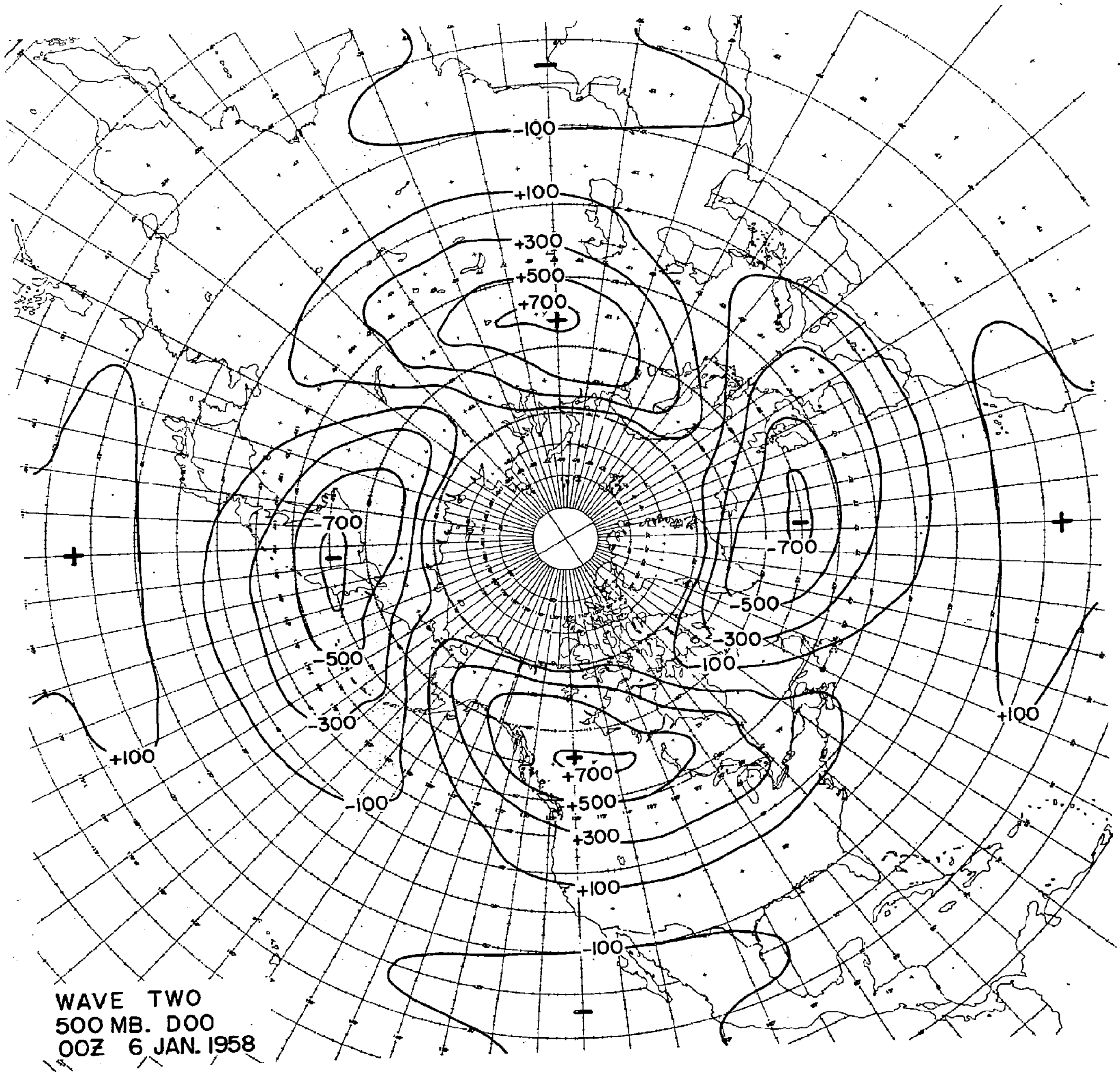

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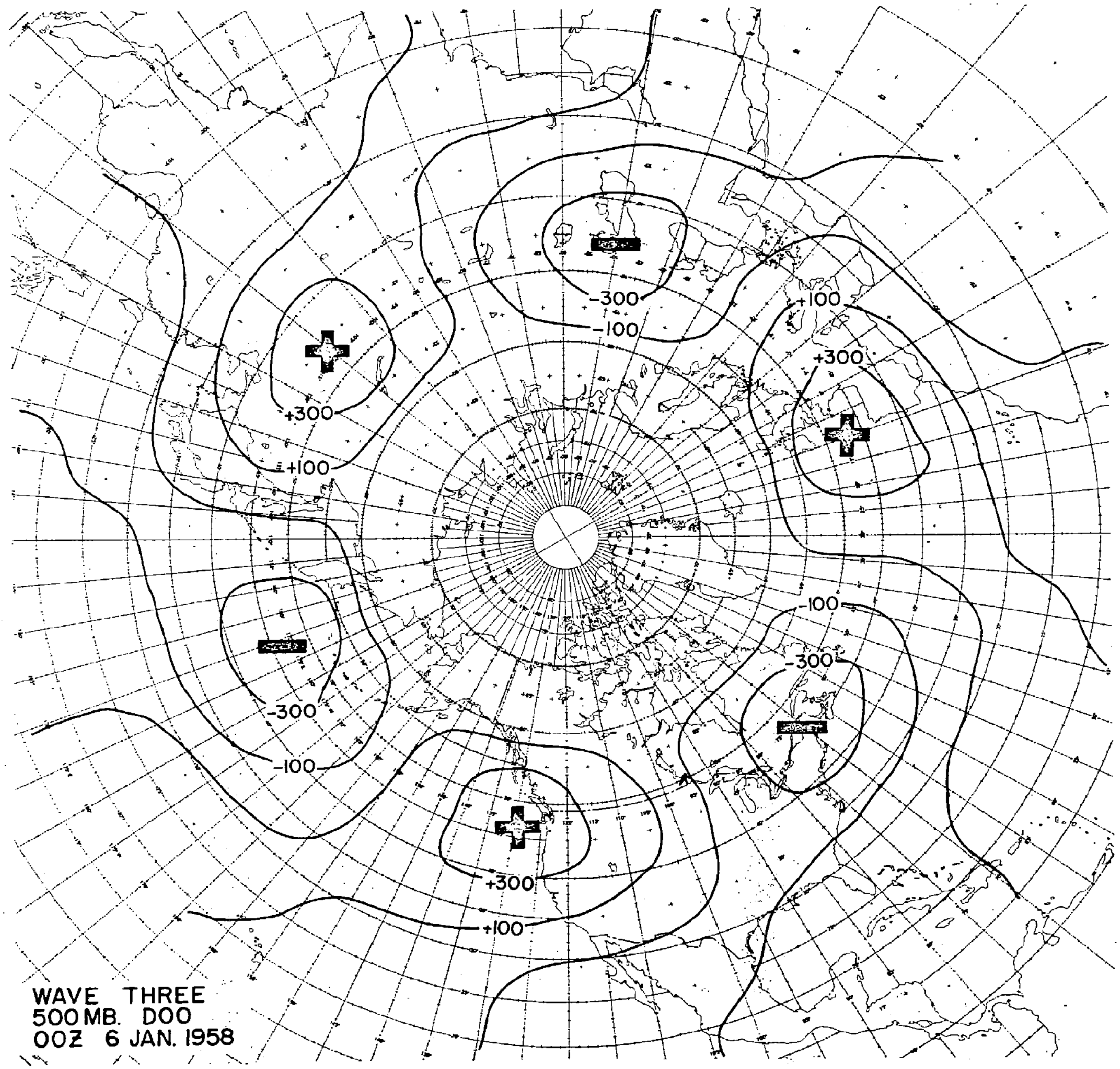

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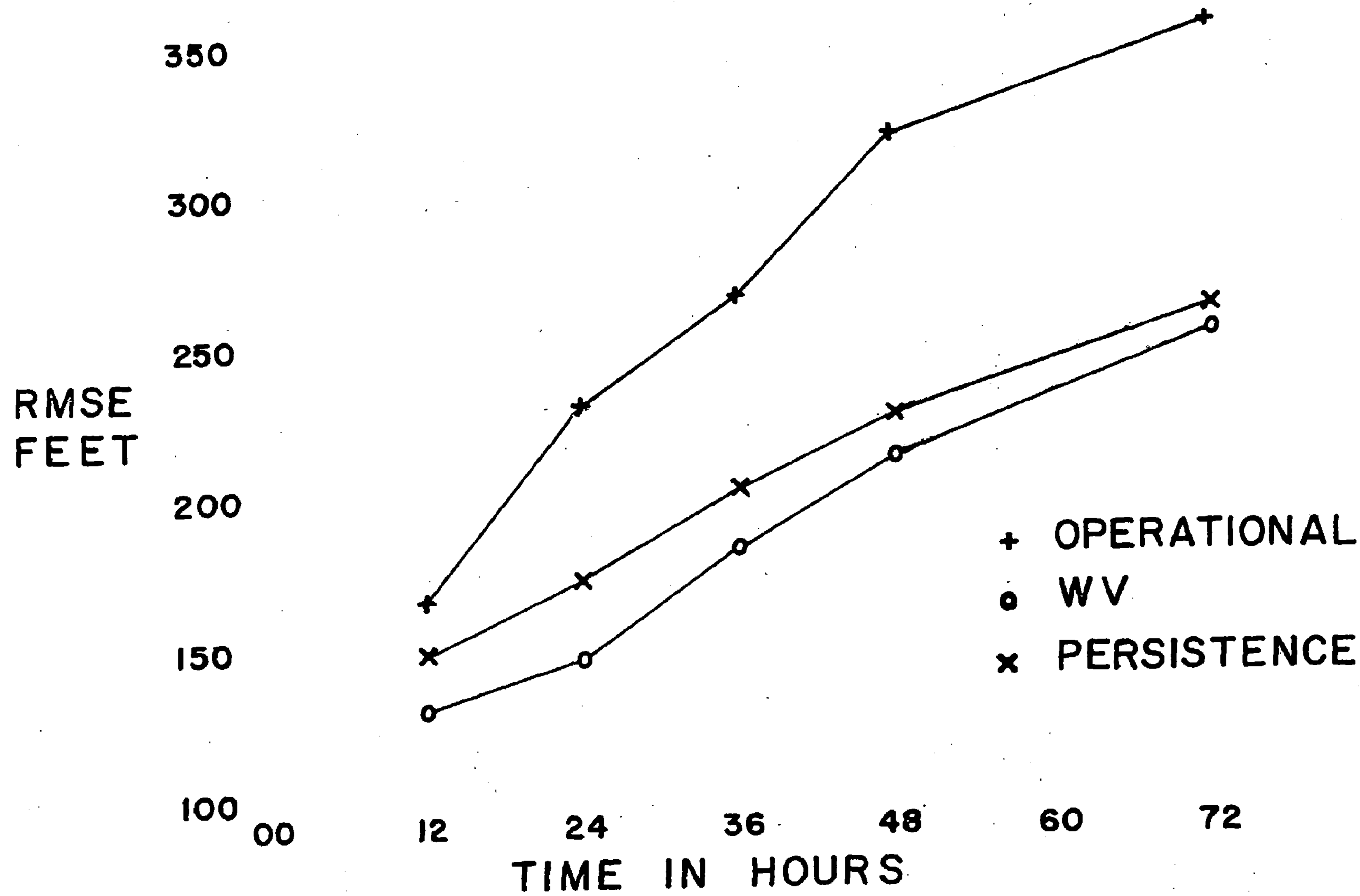
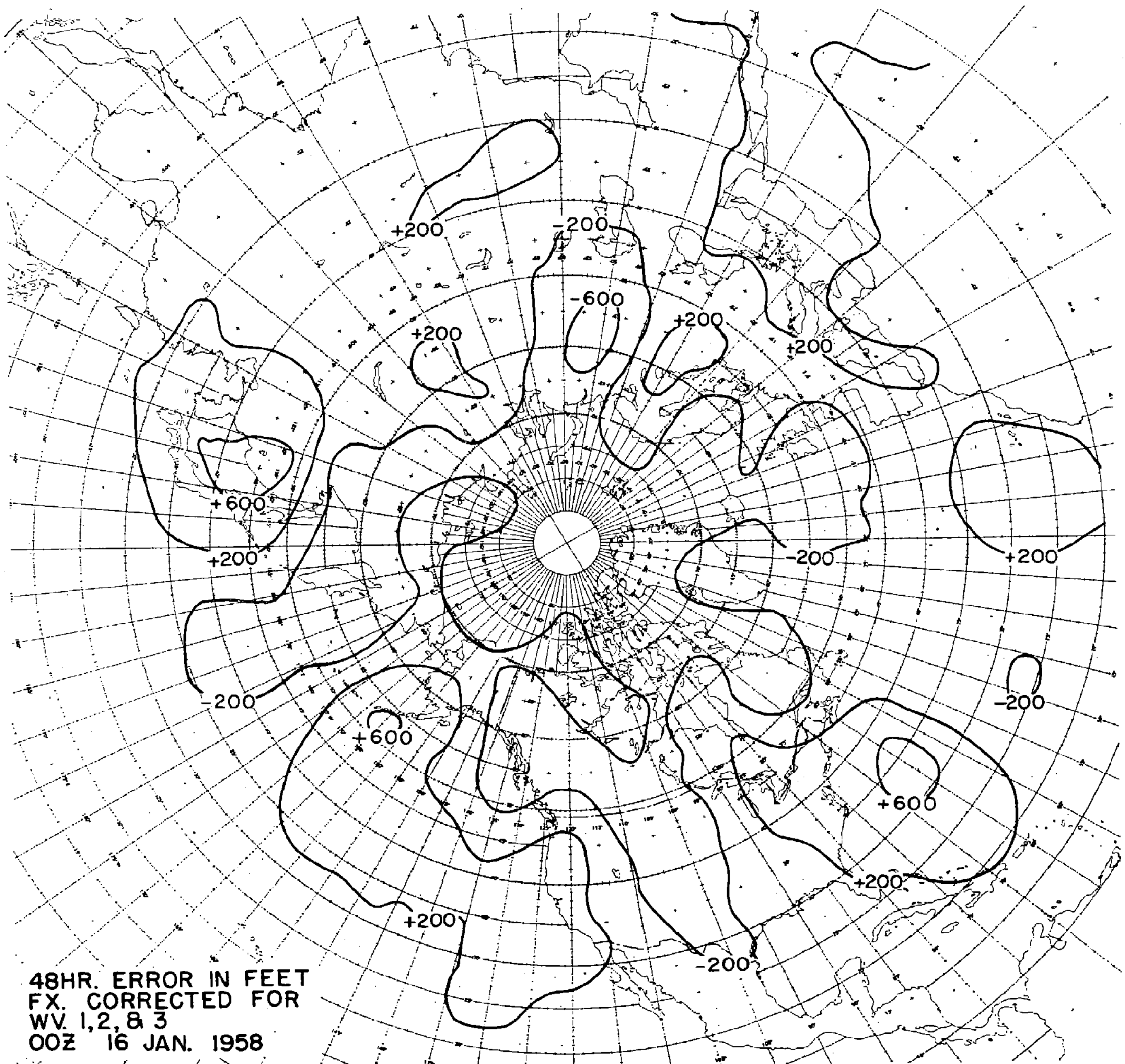
FORECAST FROM 12Z 31 MARCH 1958

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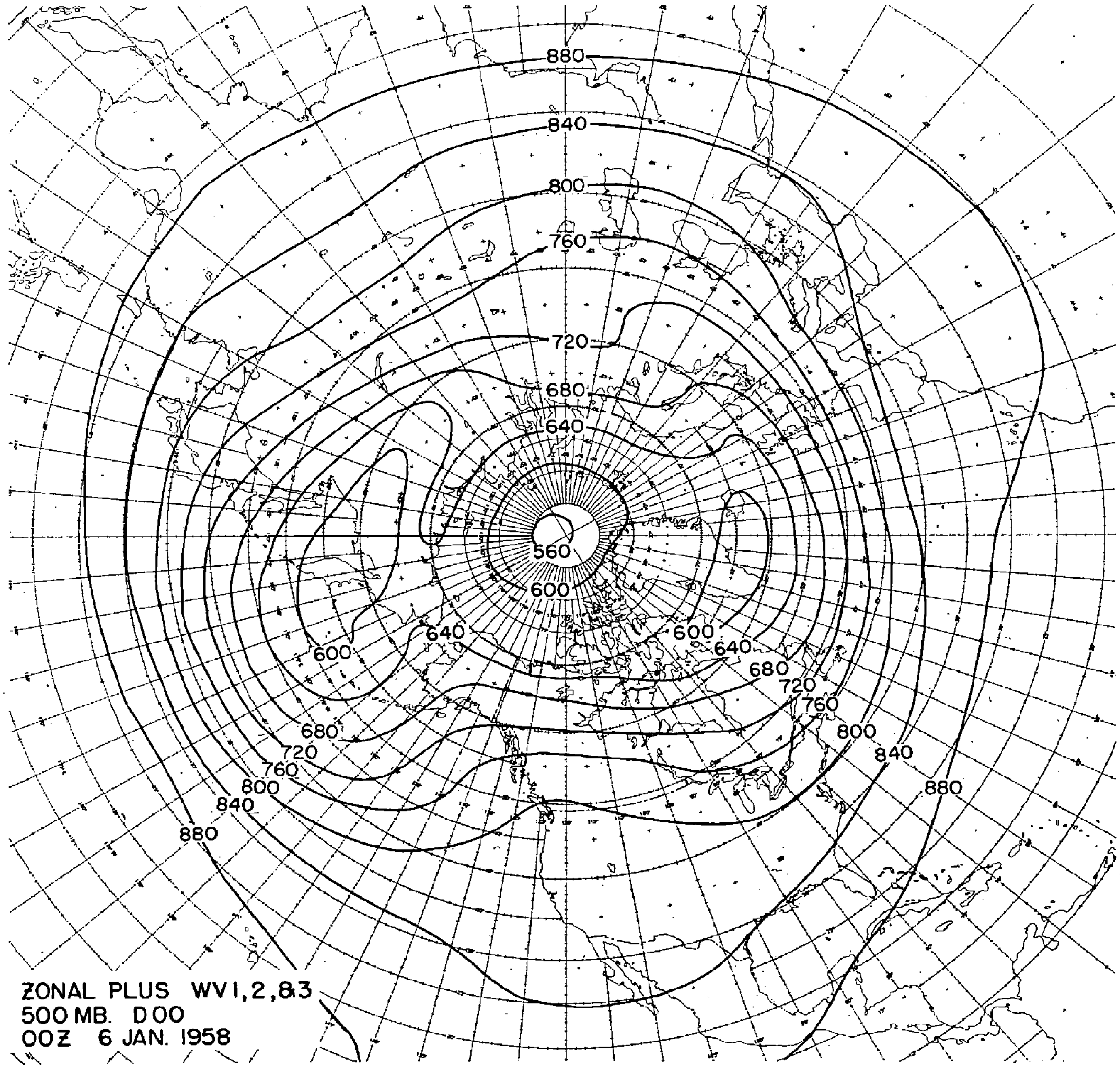
48HR. ERROR IN FEET
FX. CORRECTED FOR
WV. 1,2,& 3
00Z 16 JAN. 1958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