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U. S. COAST AND GEODETIC SURVEY
E. LESTER JONES, DIRECTOR

TIDAL DATUM PLANES

By

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U. S. Coast and Geodetic Survey

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PREFACE

Of datum planes that may be used as planes of reference for elevations, those based on the rise and fall of the tide have the advantages of simplicity of definition, accuracy of determination, and certainty of recovery, even though all bench-mark connection be lost. Tidal datum planes are, therefore, the basic planes of reference used in the hydrographic and geodetic work of the Coast and Geodetic Survey.

In the present publication two objects have been kept in mind. It is aimed to provide a working manual for the determination of the more important tidal datum planes and at the same time to provide a sufficient discussion of the principles involved and accuracy attainable. Since no such discussion is elsewhere available, this phase of the subject has been treated in detail.

A considerable body of observational material formed the basis of the investigations here undertaken, long-continued observations being especially important. Of these latter, however, those at hand were limited almost without exception to the observations made by this bureau on the coasts of the United States. It is for this reason, and for the further reason that the publication is to serve as a manual in the Coast and Geodetic Survey, that the examples chosen are from observations on the coasts of the United States.

TIDAL DATUM PLANES

By H. A. Marmer, *Assistant Chief, Division of Tides and Currents, Coast and Geodetic Survey*

I. INTRODUCTORY

DEFINITIONS

A tidal datum plane is a datum plane determined from the rise and fall of the tides. Various tidal planes may be derived, and each is designated by a definite name, as, for example, the plane of mean high water, the plane of half-tide level, the plane of lower low water.

The tide is the name given to the alternate rising and falling of the level of the sea, which at most places occurs twice daily. The striking feature of the tide is its intimate relation to the movement of the moon. High water and low water at any given place follow the moon's meridian passage by a very nearly constant interval; and since the moon in its apparent movement around the earth crosses the meridian at any place 50 minutes later each day on the average, the tide at most places likewise comes later each day by 50 minutes on the average.

With respect to the tide, the "moon's meridian passage" has a special significance. It refers not only to the instant when the moon is directly above the meridian but also to the instant when the moon is directly below the meridian, or 180° distant in longitude. In this sense there are two meridian passages in a tidal day, and they are distinguished by being referred to as the upper and lower meridian passages or upper and lower transits.

The interval between the moon's meridian passage (upper or lower) and the following high water is known as the "high-water lunitidal interval." Likewise, the interval between the moon's meridian passage and the following low water is known as the "low-water lunitidal interval." For short they are called, respectively, high-water interval and low-water interval, and abbreviated as follows: HWI and LWI.

The tide-producing forces are due to the differences in the attraction of sun and moon for the earth as a whole and for the water covering the earth. The tide-generating force of a celestial body varies directly as its mass and inversely as the cube of its distance from the earth. The mass of the sun is about 26,000,000 times as great as that of the moon, but, because its distance from the earth is 389 times that of the moon, its tide-producing power is to that of the moon as 26,000,000 is to $(389)^3$, or somewhat less than half.

In its rise and fall the tide is accompanied by a horizontal forward and backward movement of the water called the tidal current. The

two movements—the vertical rising and falling of the tide and the horizontal forward and backward movement of the tidal current—are intimately related, forming parts of the same phenomenon brought about by the tidal forces of sun and moon.

It is necessary, however, to distinguish clearly between tide and tidal current, for the relation between the two is not a simple one nor is it everywhere the same. At one place a strong current may accompany a tide having a very moderate rise and fall, while at another place a like rise and fall may be accompanied by a very weak current. Furthermore, the time relation between current and tide varies widely from place to place. For the sake of clearness, therefore, "tide" should be used to designate the vertical movement of the water and "tidal current" the horizontal movement.

With respect to the rise and fall of the water due to the tide, high water and low water have precise meanings. They refer not so much to the height of the water as to the phase of the tide. High water is the maximum height reached by each rising tide and low water the minimum height reached by each falling tide.

It is important to note that it is not the absolute height of the water which is in question, for it is not at all infrequent at many places to have the low water of one day higher than the high water of another day. Whatever the height of the water, when the rise of the tide ceases and the fall is about to begin, the tide is at high water, and when the fall of the tide ceases and the rise is about to begin the tide is at low water. The abbreviations HW and LW are frequently used to designate high and low water, respectively.

In its rising and falling the tide does not move at a uniform rate. From low water the tide begins rising very slowly at first, but at a constantly increasing rate for about three hours, when the rate of rise is at a maximum. The rise then continues at a constantly decreasing rate for the following three hours, when high water is reached and the rise ceases. The falling tide behaves in a similar manner, the rate of fall being least immediately after high water, but increasing constantly for about three hours, when it is at a maximum, and then decreasing for a period of three hours till low water is reached.

The rate of rise and fall and other characteristics of the tide may best be studied by representing the rise and fall of the tide graphically. This may be done by reading the height of the tide at regular intervals on a fixed vertical staff graduated to feet and tenths and plotting these heights to a suitable scale on cross-section paper and drawing a smooth curve through these points. A more convenient method is to make use of an automatic tide gauge by means of which the rise and fall of the tide is recorded on a sheet of paper as a continuous curve drawn to a suitable scale. Figure 1 shows a tide curve for Fort Hamilton, N. Y., for July 4, 1922.

In Figure 1 the consecutive figures from 0 to 24, increasing from left to right, represent the hours of the day beginning with midnight. Numbering the hours consecutively to 24 eliminates all uncertainty as to whether morning or afternoon is meant and has the further advantage of great convenience in computation. The figures on the left, increasing upward from 2.0 to 9.0, represent the height of the tide in feet as referred to a fixed vertical staff. The tide curve presents the well-known form of the sine or cosine curve.

The difference in height between a high water and a preceding or following low water is known as the "range of tide" or "range." The average difference in the heights of high and low water at any given place is called the mean range.

VARIATIONS IN RANGE

The range of the tide at any given place is not constant but varies from day to day; indeed, it is exceptional to find consecutive ranges equal. Obviously, changing meteorological conditions will find reflection in variations of range, but the principal variations are due to astronomic causes, being brought about by variations in the position of the moon relative to earth and sun.

At times of new moon and full moon the tidal forces of moon and sun are acting in the same direction. High water then rises higher and low water falls lower than usual, so that the range of the tide at such times is greater than the average. The tides at such

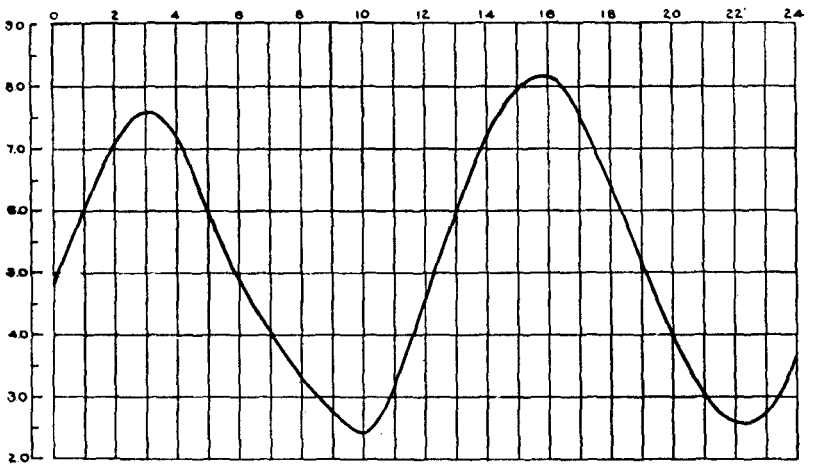

FIG. 1.—Tide curve for Fort Hamilton, N. Y., July 4, 1922

times are called "spring tides," and the range of the tide is then known as the "spring range."

When the moon is in its first and third quarters, the tidal forces of sun and moon are opposed and the tide does not rise as high nor fall as low as on the average. At such times the tides are called "neap tides," and the range of the tide then is known as the "neap range."

It is to be noted, however, that at most places there is a lag of a day or two between the occurrence of spring or neap tides and the corresponding phases of the moon—that is, spring tides do not occur on the days of full and new moon, but a day or two later; likewise, neap tides follow the moon's first and third quarters after an interval of a day or two. This lag in the response of the tide is known as the "age of phase inequality" or "phase age" and is generally ascribed to the effects of friction.

The varying distance of the moon from the earth likewise affects the range of the tide. In its movement around the earth the moon

describes an ellipse in a period of approximately 27½ days. When the moon is in perigee or nearest the earth, its tide-producing power is increased, resulting in an increased rise and fall of the tide. These tides are known as "perigean tides," and the range at such times is called the "perigean range." When the moon is farthest from the earth, its tide-producing power is diminished, the tides at such time exhibiting a decreased rise and fall. These tides are called "apogean tides" and the corresponding range the "apogean range."

In the response to the moon's change in position from perigee to apogee it is found that, like the response in the case of spring and neap tides, there is a lag in the occurrence of perigean and apogean tides. The greatest rise and fall does not come on the day when the moon is in perigee, but a day or two later; likewise, the least rise and fall does not occur on the day of the moon's apogee, but a day or two later. This interval varies somewhat from place to place, and in some regions it may have a negative value. This lag is known as the "age of parallax inequality" or "parallax age."

The moon does not move in the plane of the Equator, but in an orbit making an angle with that plane of approximately 23½°. During the month, therefore, the moon's declination is constantly changing, and this change in the position of the moon produces a variation in the consecutive ranges of the tide. When the moon is on or close to the Equator—that is, when its declination is small—consecutive ranges do not differ much, morning and afternoon tides being very much alike. As the declination increases the difference in consecutive ranges increases, morning and afternoon tides beginning to show decided differences; and at the times of the moon's maximum semimonthly declination these differences are very nearly at a maximum. But, like the response to changes in the moon's phase and parallax, there is a lag in the response to the change in declination, this lag being known as the "age of diurnal inequality" or "diurnal age." Like the phase and parallax ages, the diurnal age varies from place to place, being generally about one day, but in some places it may have a negative value.

When the moon is on or close to the Equator and the difference between morning and afternoon tides small, the tides are known as "equatorial tides." At the times of the moon's maximum semimonthly declination, when the differences between morning and afternoon tides are at a maximum, the tides are called "tropic tides," since the moon is then near one of the tropics.

The three variations in the range of the tide noted above are exhibited by the tide the world over, but not everywhere to the same degree. In many regions the variation from neaps to springs is the principal variation; in certain regions it is the variation from apogee to perigee that is the principal variation, and in other regions it is the variation from equatorial to tropic tides that is the predominant variation.

The month of the moon's phases (the synodic month) is approximately 29½ days in length; the month of the moon's distance (the anomalistic month) is approximately 27½ days in length; the month of the moon's declination (the tropic month) is approximately 27⅓ days in length. It follows, therefore, that very considerable varia-

tion in the range of tide occurs during a year due to the changing relations of the three variations to each other.

DIURNAL INEQUALITY

The difference between morning and afternoon tides due to the declination of the moon is known as diurnal inequality, and, where the diurnal inequality is considerable, the rise and fall of the tide is affected to a very marked degree both in time and in height. Figure 2 represents graphically the differences in the tide at San Francisco on October 18 and 24, 1922. On the former date the moon was over the Equator, while on the latter date the moon was at its maximum

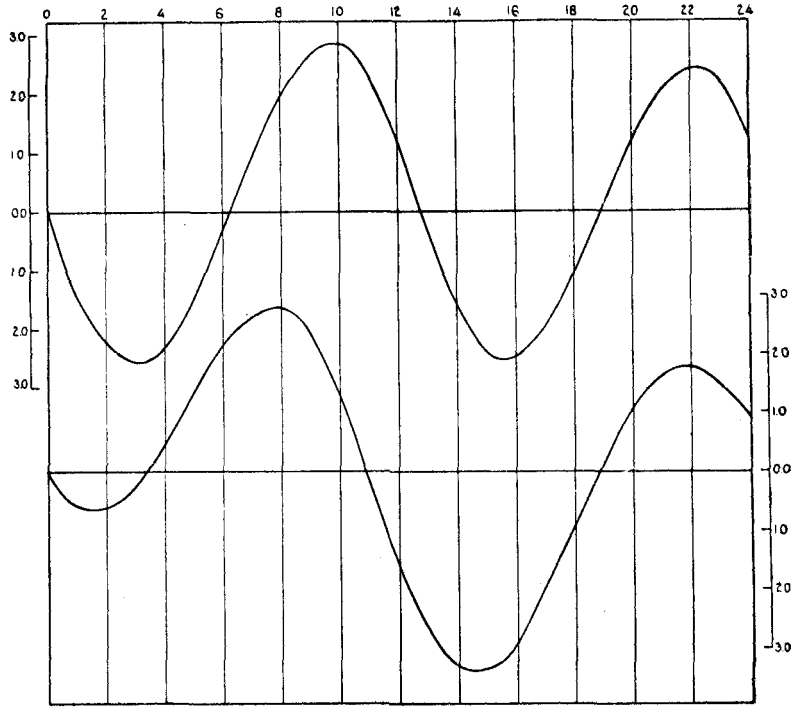

FIG. 2.—Tide curves, San Francisco, Calif., October 18 and 24, 1922

south declination for the month. The upper diagram thus represents the equatorial tide for San Francisco, while the lower diagram represents the tropic tide.

It will be noted that on October 18 the morning and afternoon tides show very close resemblance. In both cases the rise from low water to high water and the fall from high water to low water took place in approximately six hours. The heights to which the two high waters attained were very nearly the same, and likewise the depressions of the two low waters.

On October 24, when the moon attained its extreme declination for the fortnight, tropic tides occurred. The characteristics of the rise and fall of the tide on that day differ markedly from those on the 18th, when equatorial tides occurred, these differences pertaining both

to the time and the height. Instead of an approximately equal duration of rise and fall of six hours, both morning and afternoon, as was the case on the 18th, we now have the morning rise occupying less time than the afternoon rise, and the morning and evening falls likewise taking place in different periods. Even more striking are the differences in extent of rise and fall of morning and afternoon tides. The tide curve shows that there was a difference of 1 foot in the two high waters of the 24th and a difference of almost 3 feet in the low waters.

Definite names have been given to each of the two high and two low waters of a tidal day. Of the high waters, the higher is called the "higher high water" and the lower the "lower high water." Likewise, of the two low waters of any tidal day, the lower is called "lower low water" and the higher "higher low water."

The diurnal inequality may be related directly to the ratio of the tides brought about respectively by the diurnal and semidiurnal tide-producing forces. Those bodies of water which offer relatively little response to the diurnal forces will exhibit but little diurnal inequality, while those bodies which offer relatively considerable response to these diurnal forces will exhibit considerable diurnal inequality. On the Atlantic coast of the United States there is relatively little diurnal inequality, while on the Pacific coast there is considerable inequality.

It is obvious that with increasing diurnal inequality the lower high water and higher low water tend to become equal and merge. When this occurs, there is but one high and one low water in a tidal day, instead of two. This occurs frequently at Galveston, Tex., and at a number of other places.

TYPES OF TIDE

From place to place the characteristics of the rise and fall of the tide generally differ in one or more respects. But, according to the predominating features, the various kinds of tide may be grouped under three types, namely, semidiurnal, diurnal, and mixed. Instead of semidiurnal and diurnal the terms semidaily and daily are frequently used.

The semidiurnal type of tide is one in which two high and two low waters occur each tidal day with but little diurnal inequality; that is, morning and afternoon tides resemble each other closely. Figure 1 may be taken as representive of this type of tide, and this is the type found on the Atlantic coast of the United States.

In the diurnal type of tide but one high and one low water occur in a tidal day. Do-Son, French Indo-China, may be cited as a place where the tide is always of the daily type. But it is to be noted that there are not many such places. When the moon's declination is zero, the diurnal tidal forces tend to vanish, and there are generally two high and two low waters during the day at such times. Galveston, Tex., and Manila, P. I., may be mentioned as ports at which the tide is frequently diurnal, while St. Michael, Alaska, may be cited as a port at which the tide is largely diurnal.

The mixed type of tide is one in which two high and two low waters occur during the tidal day, but which exhibits marked diurnal inequality. Several forms may occur under this type. In one form the diurnal inequality is exhibited principally by the high waters;

in another form it is the low waters which exhibit the greater inequality; or the diurnal inequality may be features of both high waters and low waters.

It is to be noted that when the tide at any given place is assigned to a particular type, it refers to the characteristics of the predominating tide at that place. At the time of the moon's maximum semimonthly declination the semidiurnal type of tide exhibits more or less diurnal inequality and thus approaches the mixed type; and when the moon is on or near the Equator, the diurnal inequality in the mixed type of tide is at a minimum, resembling the semidiurnal type. It is the characteristics of the predominating tide that determine the type of tide at any given place. With the aid of harmonic constants, the type of tide may be defined by definite ratios of the semidiurnal to the diurnal constituents.

Type of tide is intimately associated with diurnal inequality and hence depends on the relation of the semidiurnal to the diurnal tides, and it is due to the variation in this relation that makes possible the various forms of the mixed type of tide.

MEAN VALUES

Since the rise and fall of the tide varies from day to day, chiefly in accordance with the changing positions of sun and moon relative to the earth, any tidal quantities determined directly from a short series of tidal observations must be corrected to a mean value. The principal variations are those connected with the moon's phase, parallax, and declination, the periods of which are approximately 29½ days, 27½ days, and 27⅓ days, respectively.

In a period of 29 days, therefore, the phase variation will have almost completed a full cycle, while the other variations will have gone through a full cycle and but very little more. Hence, for tidal quantities varying largely with the phase variation, tidal observations covering 29 days, or multiples, constitute a satisfactory period for determining approximate values of these quantities. Such are the lunitidal intervals, the mean range, mean high water, and mean low water. For quantities varying largely with the declination of the moon, as, for example, higher high water and lower low water, 27 days, or multiples, constitute the more satisfactory period.

As will be seen in the detailed discussion of the various tidal datum planes, the values determined from two different 29-day or 27-day periods may differ very considerably. This is due to the fact that these periods are not exact synodic periods for the different variations, and to the further fact that variations having periods greater than a month are not taken into account. Furthermore, meteorological conditions, which change from month to month, leave their impress on the tides. For accurate results, the direct determination of the tidal datum planes and other tidal quantities should be based on a series of observations that cover a number of years. Values derived from shorter series must be corrected to a mean value.

Two methods may be employed for correcting the results of short series to mean values. One method makes use of tabular values, determined both from theory and from observations, for correcting for the different variations. The other method makes use of

direct comparison with simultaneous observations at some near-by port for which mean values have been determined from a series of considerable length.

II. TIDE OBSERVATIONS

LOCATION OF TIDE STATION

Generally the rise and fall of the tide at any given point is characteristic of the tide over an area of greater or less extent in its vicinity. Hence a single tide station may frequently supply the tidal data for regions of considerable extent. In selecting the site for a tide station that may be representative of tidal conditions for a wide area it is important that the location be such as to provide free communication with the sea, deep water at low tides, sufficient shelter from storm waves, and comparative freedom from freshets.

In passing over areas of shoal water the tide is affected profoundly; hence sites near the heads of tidal bays and rivers are not suitable for a tide station that is to be representative for any considerable area. In tidal rivers draining large areas the effects of freshets or of the seasonal variation in amount of drainage waters are most pronounced in the upper reaches but become less pronounced further seaward. In the smaller bays and bights, seiche effects are likely to become prominent and such locations are, therefore, in general not suitable for a tide station that is to furnish the data for a region of wide extent.

TIDE STAFF

The simplest means for obtaining tidal observations consists in the use of a tide staff. This may be made from a board 5 to 6 inches wide and 1 inch thick, graduated to feet and tenths, with the numbers increasing upward. It should be of such length that the extreme fluctuation of the water in the locality in which it is to be used will be within its lowest and highest graduation, and it should be fastened securely in a vertical position to a pile or other suitable support.

Where the surface of the water is disturbed by considerable wave motion, it becomes difficult to read the height of the tide with any degree of accuracy. In such cases it is of advantage to fasten a glass tube to the face of the staff. Stock glass tubing about one-half inch in diameter, having a wall thickness of about one thirty-secondth of an inch and about 6 feet in length, has been found quite satisfactory. The tubing may be secured to the face of the staff by means of spring clips or cup hooks. The wave motion is reduced by partially closing the submerged end of the tube by a notched cork. A floating object introduced into the tube, such as a thin slice of cork having a diameter somewhat less than that of the tube, then permits the reading of the height of the water with ease to the nearest tenth or half tenth of a foot.

Where the tide staff is to be used for a considerable period of time, and therefore subject to weathering, it will be found of advantage to cut the graduations denoting the feet and tenths into the wood and to form the figures marking each foot from brass upholstery tacks.

Readings on the staff should be recorded every half hour or hour, except near the times of high and low water, when the readings should be made every 15 minutes or even more frequently. Continuous observations covering both day and night are most satisfactory, but, where this is not feasible, daylight observations over a period of 13 consecutive hours every day should be made.

BENCH MARKS

The zero of the tide staff should be connected by spirit levels with at least three good bench marks. This will make possible the replacing of the tide staff at the same elevation during the progress of the observations, should it become destroyed or should its elevation be changed by accident. The bench marks will also serve the further purpose of preserving for future use the datum planes that are determined from the tidal observations. The bench marks should

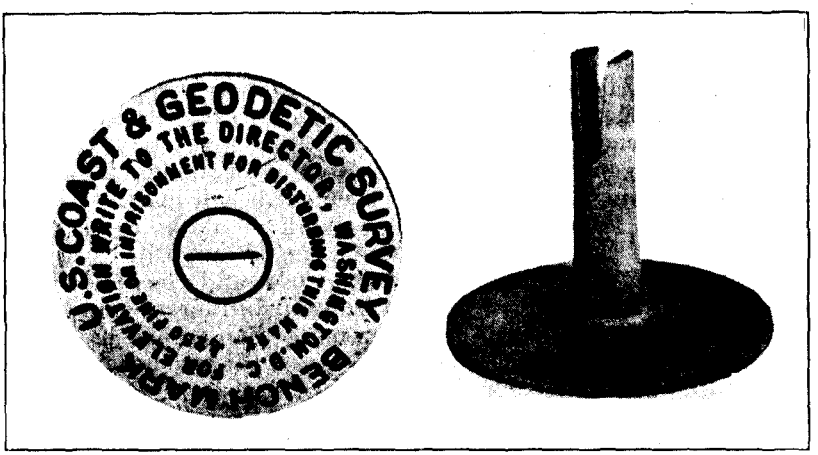

FIG. 3.—Standard tidal bench mark, U. S. Coast and Geodetic Survey

be placed at some distance from each other so that they are all not likely to be destroyed by a common cause.

It is the practice of the Coast and Geodetic Survey to establish and maintain at each tidal station not less than one standard disk bench mark for each year of observations up to 10 years, with a minimum of five such marks for a series of one year in length and a minimum of three for a series less than a year. Three of these bench marks are located within a short leveling distance of the tide staff while the remainder are more widely distributed to insure against loss from a common cause. Care is taken to avoid locating the bench marks on filled-in ground.

The qualities that distinguish a good bench mark are freedom from likelihood of change in elevation and ease of finding and identification. Disk bench marks fulfill these requirements well. The standard tidal bench mark of the Coast and Geodetic Survey consists of a brass disk about 3 inches in diameter, with a shank about 2½ inches long for insertion into a building or other substantial support, and carries the inscription shown in Figure 3.

Permanent and substantial buildings afford the best locations for setting the disk bench marks. The bench mark is countersunk, with its face flush with the surface of the part of the building into which it is set, and is securely cemented in, so that it will effectively resist extraction, rotation, or change of elevation. If the wall of a building is used, the bench mark should be set with its central line horizontal, for it is the elevation of this central line that is taken as the elevation of the bench mark. If a suitable location on a building is found which permits the disk bench mark to be set with its face horizontal, it is to be preferred, since this position is a more convenient one for placing a leveling rod.

A bowlder or a ledge of rock makes a very satisfactory support for a bench mark. A good foundation for a bench mark is also furnished by a mass of concrete with its upper surface slightly above the level of the ground, about 2½ feet deep, 2 feet square on the bottom, and 1½ feet square on top. A satisfactory mixture for this purpose consists of one part cement, two parts sand, and four parts gravel or broken stone.

If standard disk bench marks are not available, a small cross cut on a rock, building, or other structure may serve the purpose of a bench mark. A copper bolt set into rock or into a cement block makes a satisfactory bench mark. Water hydrants, curbstones, and nails in growing trees, while frequently suitable for temporary use in leveling, do not make satisfactory permanent bench marks.

The bench marks and zero of the tide staff should be connected by a double line of levels, the lines being run in opposite directions. With a wye level the regulations of the Coast and Geodetic Survey prescribe that when the forward and backward differences in the elevations of two bench marks differ by more than $0.05\sqrt{K}$ feet (K being distance in statute miles leveled between the two bench marks), both the forward and backward leveling between these two bench marks are to be repeated until the difference falls within the required limit. It is important that the leveling record and the descriptions of the bench marks be made in such form that no ambiguity will arise in the use of such records at future times by other persons.

For convenience in use the following table of maximum closing error allowed in leveling between bench marks is given. It is based on the formula given above.

TABLE 1.—*Maximum allowable errors in leveling between bench marks*

Distance between B. Ms. (*feet*)	Maximum error allowed (*feet*)
500 or less	0.015
1,000	.022
2,000	.031
3,000	.038
4,000	.044
5,000	.049

BOX GAUGE

Where conditions make readings on a plain staff difficult, a box gauge may be used. Essentially this consists of a float that rises and falls in a vertical box to which the tide has access. The box may be made of 1-inch boards 12 inches wide, the bottom being all closed,

except a hole about 1 inch in diameter through which the tide has access and which reduces the wave motion in the float box very considerably. A convenient form of float for a box gauge is a copper cylinder about 8 inches in diameter and 3 inches high, the bottom of which is weighted to give it steadiness in water.

Various means may be used for determining the rise and fall of the float in a box gauge. Where the range of the tide is moderate, a light wooden rod graduated to feet and tenths may be secured to the top of the float and at a convenient point above the top of the float box the rod made to pass through a metal ring secured in such wise that the axis of the rod is vertical. The metal ring serves the further purpose of furnishing a reference point for reading the height of the tide. It is to be noted that it is necessary to graduate the rod with the numbers increasing from top downward, in order that the heights of the tide as read on the rod may be direct and not inverted.

Where the range of the tide, or the distance from the top of the box gauge to the surface of the water, is considerable, a graduated steel or phosphor-bronze tape is more convenient than a rod. In this case the lower end of the tape is attached to the float, and the upper end is made to pass over a fixed pulley. To keep the tape in tension, a weight is attached to its upper extremity, and, for a reading point for measuring the rise and fall of the float, the tape may be made to pass a metal ring fixed at a convenient distance from the top of the float box, or a board may be fixed vertically near the tape and a reading line marked on the board.

The relation of the zero of a box gauge to fixed bench marks on shore may be determined in two different ways. In the first method simultaneous readings of the box gauge and a fixed tide staff are made and the relation of the zeros derived. The elevations of the bench marks above the zero of the fixed tide staff are then determined in the usual way, and the relation of the zero of the box gauge to the bench marks is then determined through the difference of the zeros of the tide staff and box gauge. In this method care must be taken to have the fixed tide staff near the box gauge, so that at any instant the height of the tide is the same in the box gauge as on the tide staff.

Another method of determining the relation of the zero of the box gauge to fixed bench marks on shore consists in determining the elevations of the bench marks relative to the reading point and adding the length of the float rod or tape from the zero graduation to the line of flotation of the float. The first part of this operation is accomplished in the usual manner with the spirit level. The second part is accomplished by floating the float with rod or tape attached in a pan of water, care being taken to have the density of water in the pan the same as that in the float box, and measuring the distance between the zero of the rod or tape and the line of flotation of the float.

AUTOMATIC TIDE GAUGES

Where the tidal observations are to cover a period of several months, the automatic or self-recording tide gauge is the more satisfactory. Various forms of automatic tide gauges are on the market, some of these tracing a continuous curve and others printing the

height of the tide at regular intervals. For certain purposes the printing gauges are preferred, but for general purposes the curve-tracing gauges have several advantages, among which may be mentioned the visualizing of any breaks in the record, whether due to stoppage or accidental change in adjustment. The curve-tracing gauge furthermore permits the recording and studying of rapid changes in level and of any unusual features in the rise and fall of the tide.

The Coast and Geodetic Survey makes use of two types of automatic tide gauge, a three-roller gauge at its principal tide stations and a single-roller portable gauge at its secondary stations. These two gauges are briefly described below.

THE THREE-ROLLER GAUGE

For a number of years the Coast and Geodetic Survey has made use of the three-cylinder automatic tide gauge shown in Figures 4 and 5. The essential parts of the machine consist of a clock that moves a strip of paper forward at a uniform rate and a float that is free to rise and fall with the tide and is so connected with a pencil that the latter moves perpendicularly to the motion of the paper and proportional to the rise and fall of the tide. The combined motion of paper and pencil produces a continuous curve known as the tide curve, which shows the rise and fall of the tide to a reduced scale, and from which the height of the tide for any given instant during the period of observations may be determined.

There are two clocks, as shown in Figure 4. Clock No. 1 in the figure is the motor clock which turns the main cylinder (No. 4), regulating the motion of the paper. This cylinder turns once in 12 hours and moves the paper forward at the rate of 1 inch per hour. Clock No. 2 is the time clock, the purpose of which is to mark the hours on the record. It is similar to an ordinary striking clock, but, instead of striking a bell, it operates a device that trips the recording pencil, making a short horizontal mark on the record at the beginning of each hour.

The paper used on the machine is about 13 inches wide and is furnished in rolls about 66 feet long, which is sufficient for one month of record. The roll is placed on the supply roller (No. 3, fig. 5), fed over the main cylinder where the tidal record is made, and is then wound upon the receiving roller (No. 5, fig. 4). The supply and receiving rollers have removable flanges which permit the paper roll to be put on and taken off easily.

One end of the receiving roller is fitted with a pulley around which is wound a cord (No. 22, fig. 4) which has a weight at its lower end. This pulley is provided with a pawl and ratchet for winding the weight from time to time. The action of this weight winds the tide roll on the receiving roller, keeps the paper on this side of the machine taut, and also assists the motor clock in turning the main cylinder.

The float used with the tide gauge is a copper cylinder 8½ inches in diameter, 3 inches high, and weighted so as to float with about one-third of its height above the surface of the water. It is connected with the float pulley of the machine (No. 11, figs. 4 and 5) by a phosphor-bronze wire. A set of four interchangeable float pulleys, with

circumferences of 6 inches, 9 inches, 12 inches, and 16 inches, is provided with every machine in order to adapt it to different ranges of tide. These pulleys are about 1 inch wide and have threads cut on their surfaces to prevent the float wire, one end of which is fixed near the edge of the pulley, from winding upon itself.

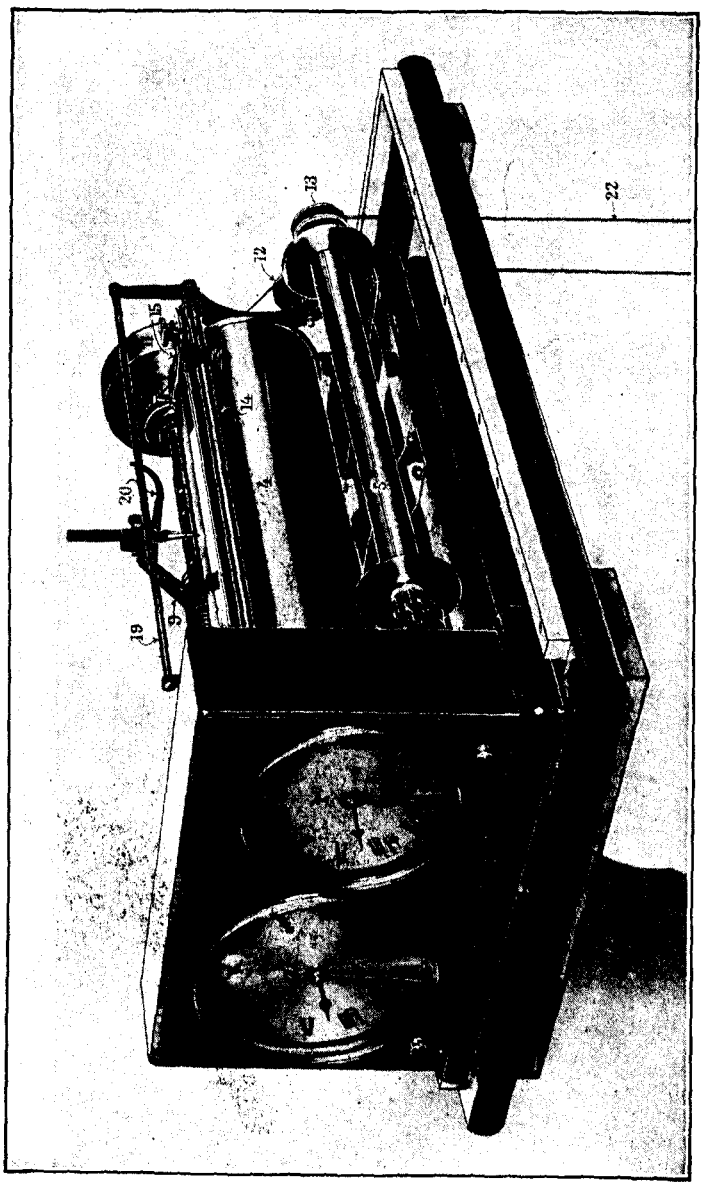

FIG. 4.—Three-roller gauge, showing arrangement of clocks

Clamped to the float pulley is another threaded pulley (No. 10, fig. 5), known as the counterpoise pulley. This carries a wire or cord to which is attached a counterpoise weight that serves to take up the slack in the float wire and rewind it as the float rises. Both float pulley and counterpoise pulley are clamped to the pencil screw

(No. 8, fig. 5), causing the latter to turn as the float wire winds on and off the float pulley. In turning, the pencil screw moves the pencil arm (No. 9, fig. 5) across the paper.

Besides the pencil carried in the pencil arm (No. 9, fig. 5) which traces the tide curve, there is another pencil carried in the datum

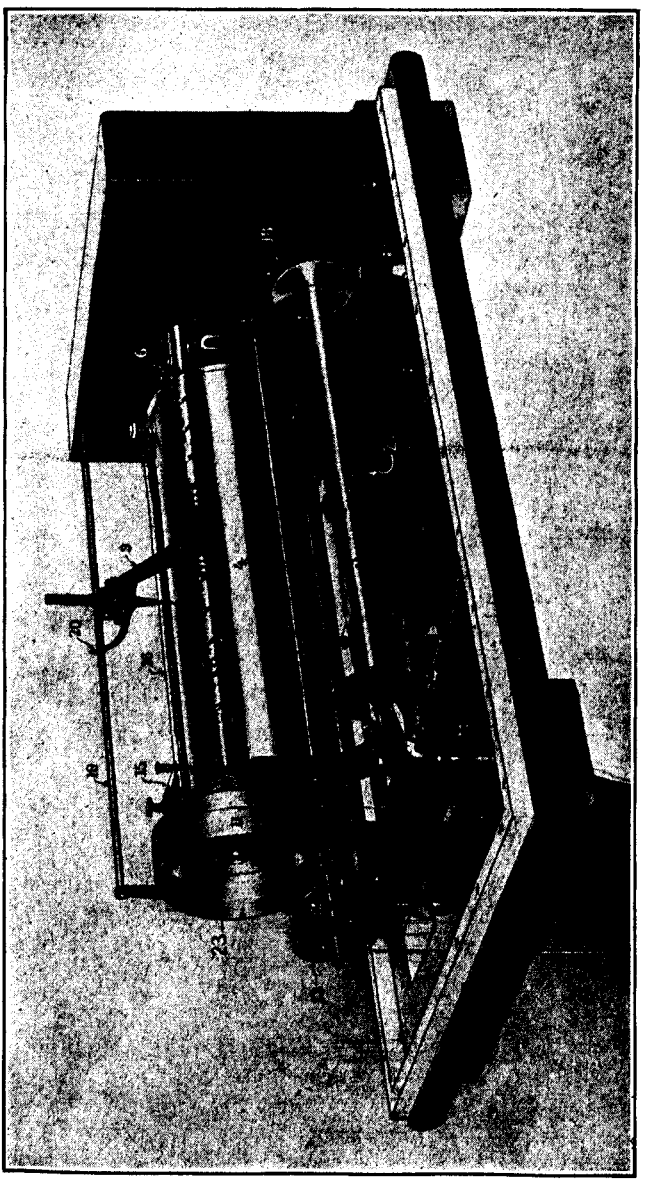

FIG. 5.—Three-roller gauge, showing float pulley and pencil screw

pencil holder (No. 15, fig. 5). This may be clamped in any desired position on the datum pencil rod (No. 14, fig. 4) but is preferably clamped near the middle of the rod. This pencil marks the datum line on the record from which the height of the tide on the curve is read.

FLOAT WELL

The float box or float well for the automatic tide gauge should be located where the water is several feet deep, even at the time of the lowest tides. The box is generally made about 1 foot square on the inside, or 1 foot in diameter if cylindrical in shape, and long enough to reach several feet below the lowest tide. It is set and securely fastened in a vertical position to prevent the float from scraping against the sides as it rises and falls with the tide. A single opening about 1 inch in diameter in the bottom of the box has been found most satisfactory. An opening in the bottom, rather than in the side of the box, has the advantage of being easily cleaned by means of a slender gas pipe.

For a station that is to be occupied but a short time a plain square wooden box made of 1-inch boards is sufficient. For a longer series a wooden box made of 2-inch boards 14 inches wide and covered on the outside with copper sheathing or yellow metal has been found very satisfactory. In waters infested with teredos the metal sheathing serves the added purpose of preventing the teredos from attacking the float box. Where the float box has a considerable length, stock wrought iron or steel pipe with a diameter of 12 inches has been found satisfactory. As an inlet for such a pipe an inverted conical-shaped casting with an opening 1 inch in diameter is used. Where such a pipe is used, it is important that the inlet be cleaned frequently, about once a month, to remove the rust scales which collect in the bottom. For this purpose gas pipe about one-half inch in diameter has been found suitable.

To prevent the formation of ice in the float well during freezing weather, petroleum or kerosene oil in the well has been found satisfactory. A column of oil in the float well 2 or 3 feet high will probably suffice for all but arctic latitudes. In a 12-inch float tube a column of oil 1 foot high will require about 6 gallons of oil. The amount of oil that can be used is limited by the depth of the opening in the float well, the maximum height of oil being the distance of the opening below the lowest tide. If a greater amount is used, it will be lost through the opening at extreme low tides.

When the tide gauge is in operation, the observer visits the gauge every day or every other day. He notes upon the tidal record the correct time and the height of the tide on the staff and also the weather conditions. The tension weight is wound up at each visit, and the clocks are wound twice a week. The tide roll is changed every month, preferably on the 1st of the month.

PORTABLE AUTOMATIC TIDE GAUGE

For use by hydrographic parties in the field, the Coast and Geodetic Survey has developed a small portable automatic tide gauge shown in Figure 6. This gauge is 10 inches square on its base and with its weatherproof metal cover in place is 10 inches high. It was designed to provide a gauge which can be easily installed in remote localities where wharves and docks are not available.

The tide curve is made on cross-section paper on a drum 7 inches long and 19.2 inches in circumference. This drum is geared to a clock movement within the drum so as to rotate once in 48 hours, giving a time coordinate of 0.4 inch to the hour. The axle carrying

the float pulley carries also at one end a gear wheel which meshes into an idler gear wheel which in turn meshes into a third gear wheel at one end of the pencil screw. The upper and lower of the three gear wheels are removable, permitting various height scales for the tide curve, and provision is made for five scales, allowing tides from less than 6 feet up to 25 feet to be recorded on the paper.

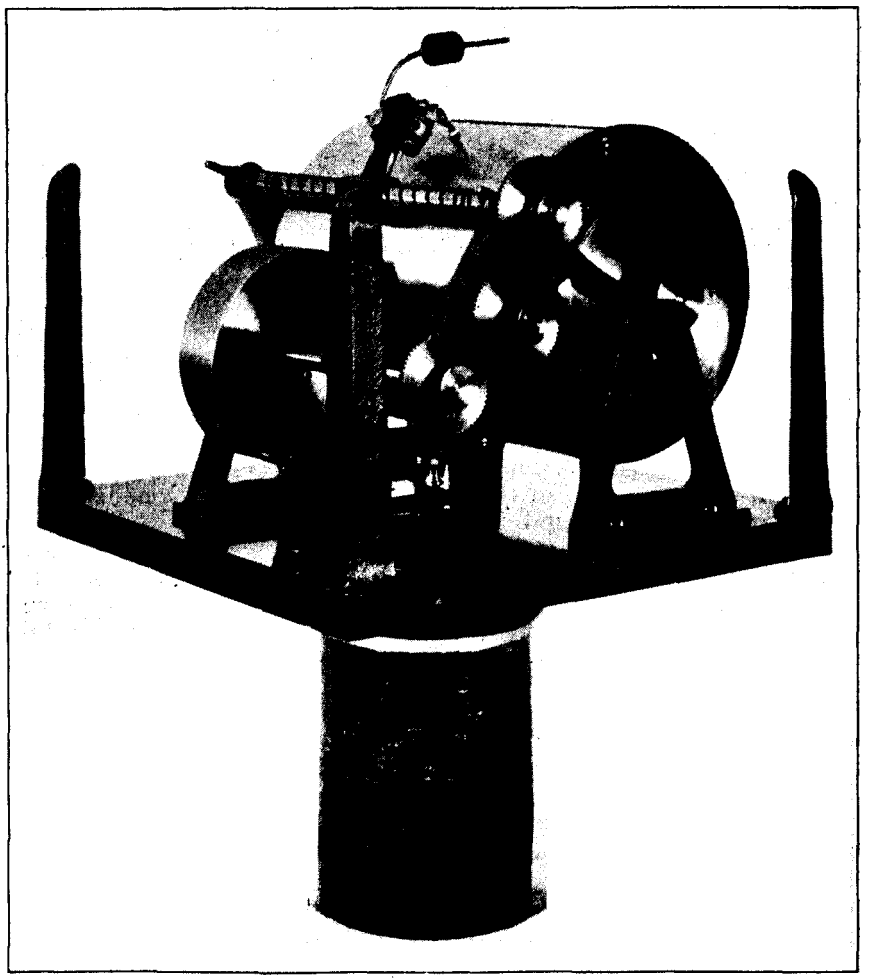

FIG. 6.—Portable automatic gauge

Instead of employing the usual counterpoise weight for taking up the slack of the float wire on a rising tide, the portable automatic tide gauge is fitted with a flat spring coiled within the case of the float-wire pulley. This spring is connected with a ratchet and pawl, so that any desired tension may be put on the spring.

The float well for this portable gauge consists of a section or sections of ordinary stock 3½-inch iron pipe. The base of the gauge has a sleeve fitted with a short piece of 3½-inch pipe, the lower end of which is threaded so that it can be screwed directly on to the float

pipe. A bronze casting with an inverted cone inside, having a ¼-inch hole, is screwed to the bottom of the float well and allows free ingress and egress to the tide. In this gauge, since it uses cross-section paper, the recording pencil is set to read the same as the tide staff with which it is used. Detailed instructions for the installation and operation of this tide gauge have been issued in Coast and Geodetic Survey Special Publication No. 113, entitled "Portable Automatic Tide Gauge."

TIDE STAFF AND STAFF SUPPORT

In connection with the automatic tide gauge a tide staff is used, the simultaneous readings of the tide curve and tide staff furnishing the necessary data for referring the height of the tide on the tide curve to the zero of the tide staff. For long series of observations a portable staff is preferable to a fixed staff, since a fixed staff will need frequent renewing. The practice of the Coast and Geodetic Survey at its principal tidal stations is to use a portable staff in connection with a fixed staff support.

Figure 7 shows one form of staff support and tide staff. The staff support consists of a plank about 1½ inches thick, 6 inches wide, and about 9 feet long. On the front face of the staff support are fitted several sets of wooden or metallic guides which permit the tide staff to slide vertically along the staff support. To the top of the staff support, as shown in Figure 7, is fitted a phosphor-bronze stop plate. To the back of the tide staff is fitted a similar stop plate set at an exact footmark. As the staff is slid through the guides the stop plate of the staff fits on the stop plate of the staff support, as shown in Figure 8.

The staff support is securely fastened in a vertical position to a pile or other suitable object. When it is desired to obtain a reading of the height of the tide, the tide staff is inserted into the guides and allowed to slide down until the two stop plates come together. After a reading has been obtained the staff is withdrawn and put into some protected place. The phosphor-bronze stop plate on the staff

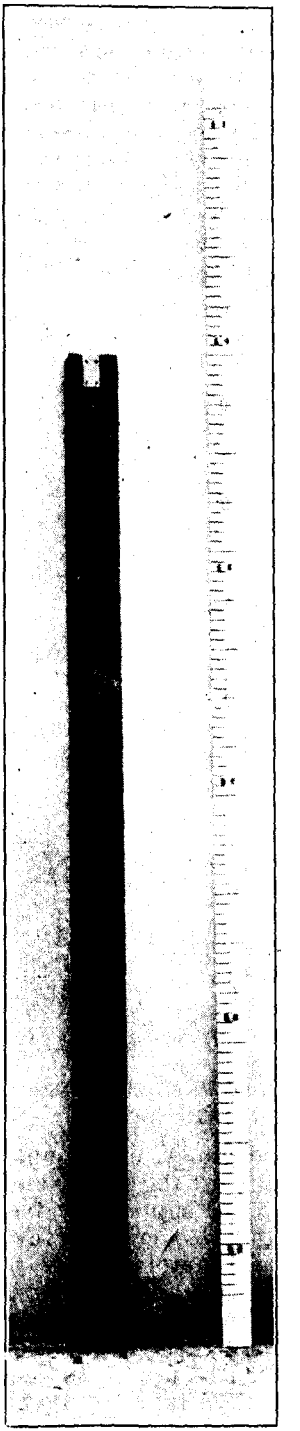

FIG. 7.—Staff support and tide staff

support also serves as a convenient bench for the leveling rod in connecting the tide staff with bench marks.

Where the water is infested with teredos, the staff support is sheathed with thin sheet copper or yellow metal. In this case, instead of wooden guides, use is made of brass castings 6 inches long, shaped somewhat like the letter Z, which, when screwed on the staff support, act as guides for the tide staff. To prevent the buoyancy of the water from floating the stop of the tide staff off the stop of the support, various means may be used. A strip of lead fixed to the bottom of the staff will give sufficient weight to counteract the

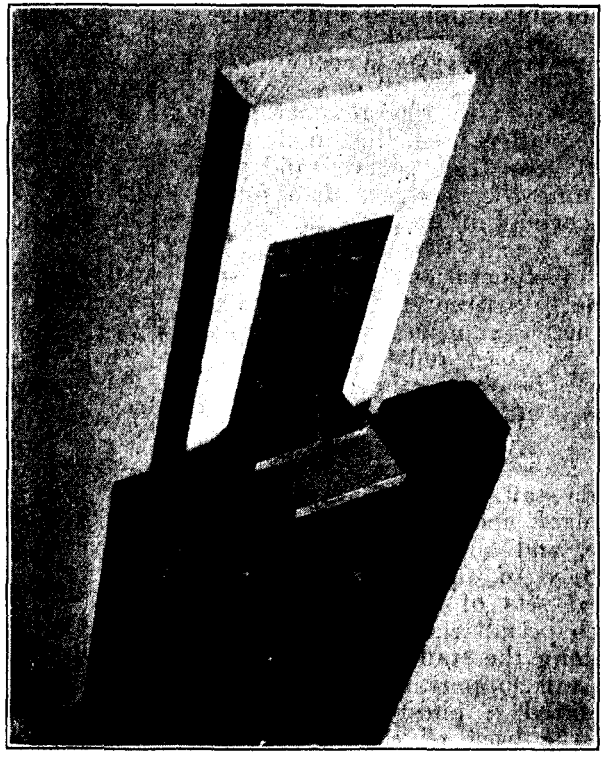

FIG. 8.—Stop plate of staff in place on stop plate of staff support

buoyancy, or springs may be fitted to the sides of the staff to slide against the guides and maintain the stop of the staff against the stop of the support by friction. In one form of staff used by the Coast and Geodetic Survey the staff is held against the stop of the support automatically by means of a locking device fitted into the stops.

In certain places, as, for example, at the ends of long piers many feet above the surface of the sea, a tide staff can not very well be used. In such cases a box gauge with a steel or phosphor-bronze tape, as described under "Box gauge" may be used instead of a tide staff for referring the tide curve made by an automatic tide gauge to a fixed zero.

III. THE TIDE RECORD

STAFF READINGS

When the tide record consists of staff readings made at intervals during the day, it is of advantage, in preparing it for tabulation, to plot these staff readings on cross-section paper to a suitable scale, plotting the time along the axis of X and the height along the axis of Y. The plotting of the staff readings permits the smoothing out of accidental irregularities and the detection of errors, thus permitting a more accurate determination of the height of the tide at any desired time and of the times and heights of high and low water.

A convenient form of plotting staff readings consists in plotting the tide curves for consecutive days under each other. Altogether apart from the economy in cross-section paper, this method brings out any unusual departures from normal conditions and aids in the interpolation of breaks. Figure 9 shows on a reduced scale the plottings of the tide curves for San Francisco, Calif., from June 1 to June 7, inclusive, made by plotting the hourly heights of the tide. The height scale is shown from 10 to 11 feet for the first tide curve, but for the others only the 10-foot line is marked.

AUTOMATIC TIDE-GAUGE RECORD

With automatic gauges employing cross-section paper the recording pencil is set to give the height referred to a tide staff. Hence such a record is ready for tabulation as soon as taken off the gauge. A comparison of the tide curve with the time and height notes made by the observer will indicate whether any time and height corrections are required. Generally no such corrections are necessary, since in tabulating tide records it is customary to tabulate times to the nearest tenth of an hour and heights to the nearest tenth of a foot, except in regions of little range of tide, in which case heights are tabulated to the nearest half tenth or even more closely.

A specimen sheet (reduced to about two-fifths of its original size) of the record made by a portable automatic gauge, showing the observer's time and height notes, is shown in Figure 10. This record was made on the Potomac River at Washington, D. C., from November 24–29, 1924. At each visit to the gauge the observer places the impression of the stamp above or below the curves, noting in the proper places the correct time and the height of the water on the tide staff. He then indicates the exact place on the curve to which each note refers by marking a right angle with its vertex on the curve and writing the date in the angle. Six days' observations are shown on the one sheet, but no confusion arises and no difficulty is experienced in tabulating from such a record. The "saw teeth" or short-period oscillations seen on the record about 7 a. m. and 7 p. m. are caused by the waves produced by the arrival and departure of a steamboat at those times.

The three-roller gauge does not employ cross-section paper. The record is made on a roll of plain paper on which the time scale is 1 inch to the hour, each roll containing generally the record for a calendar month. At the beginning of each hour the hour-marking device on the gauge causes the pencil to make a short horizontal stroke, thus indicating on the curve the beginning of the hour.

Every day, when the observer visits the tide station, he stamps and fills in the data of the note shown on Figure 11. This gives the necessary information for marking the hours correctly and for referring the heights measured on the curve to the tide staff. The

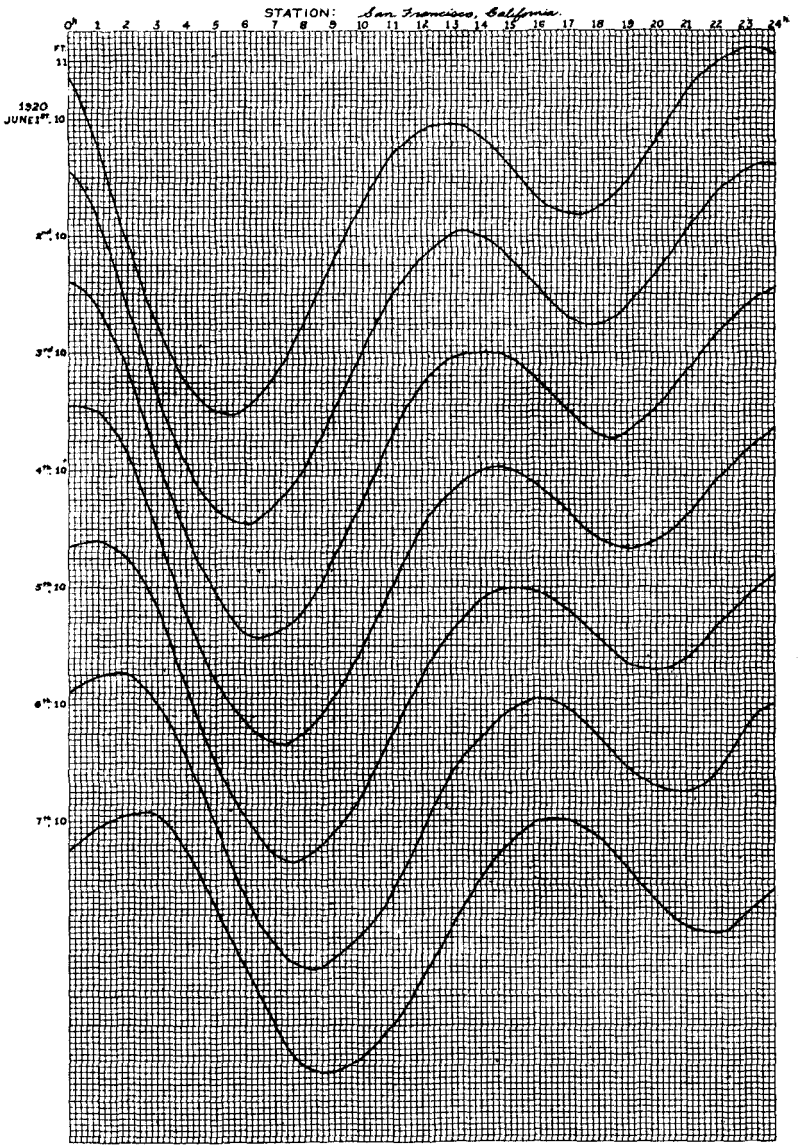

FIG. 9.—Tide curves, San Francisco, Calif., plotted from hourly heights of the tide

observer reads the tide staff generally to the nearest half tenth of a foot and indicates the place on the curve to which the time and height note pertains by a line as shown in Figure 11, which is a reproduction on a reduced scale of the tide-gauge record for Sitka, Alaska, for a part of December 9, 1924.

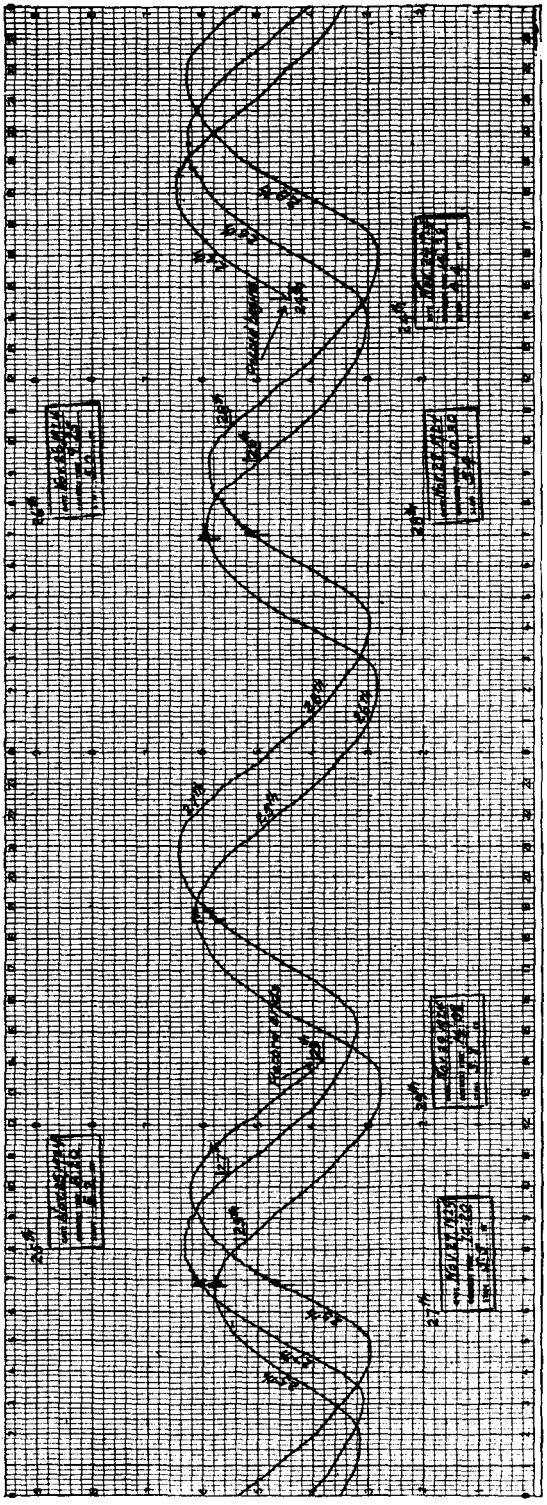

FIG. 10.—Specimen sheet (reduced) of tide record, portable automatic tide gauge. Washington, D. C., November 24–29, 1924

When the tide roll is taken off the three-roller gauge at the end of the month, the observer affixes a label on the outside of the roll on which he notes the name of the station, the number of the gauge, the scale to which the record is drawn, the dates of beginning and ending of the observations, and the kind of time used. The roll is then ready for tabulation.

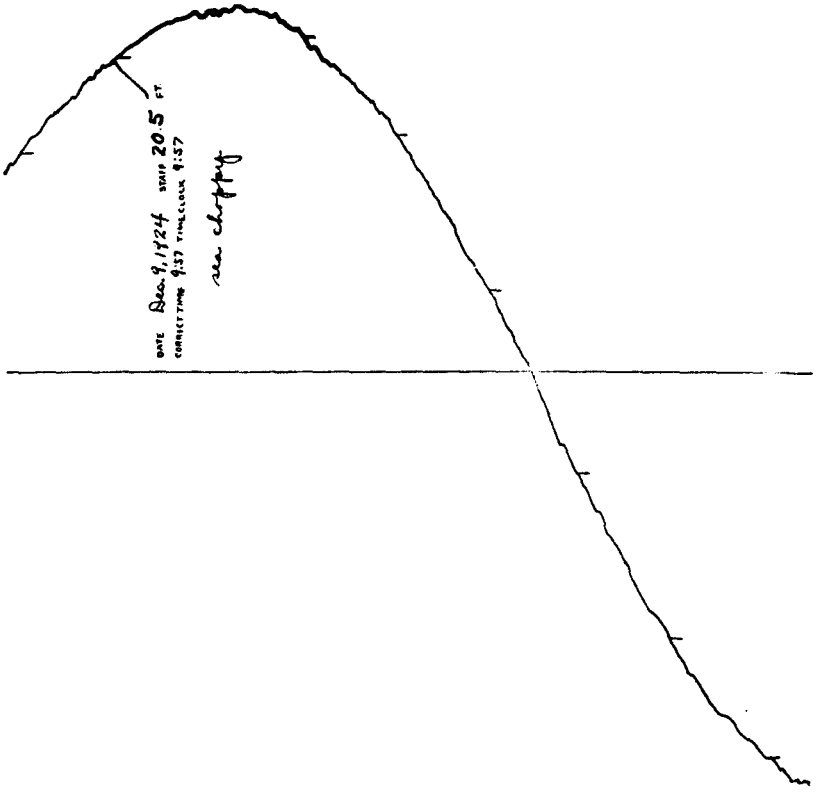

FIG. 11.—Specimen of record made by three-roller tide gauge, Sitka, Alaska, December 9, 1924

IV. TABULATION OF THE TIDE RECORD

HOURLY HEIGHTS AND HIGH AND LOW WATERS

A complete tabulation of a tide record comprises two sets of tabulations, the first giving the hourly heights of the tide and the second the times and heights of the high and low waters. In addition to constituting a full and convenient record of the tide, these two tabulations furnish the data requisite for the determination of all tidal datum planes and the characteristic features of the tide.

A complete tabulation of the tide record is, however, required only for primary tide stations; that is, stations at which the observations are to be used not only for the determination of local tidal datum planes but also for correcting to a mean value short series of observations in the general vicinity, and for other purposes in connection with tidal investigations. For the determination of tidal

datum planes at any particular place it is frequently sufficient to tabulate only the high and low waters.

For convenience in tabulating and in filing the Coast and Geodetic Survey makes use of printed forms 8 by 10½ inches for tabulating the hourly heights and the high and low waters. Both sides of the sheets are used, one sheet of high and low waters covering a month and one sheet of hourly heights covering two weeks. Specimen copies of these forms are shown on a reduced scale in Figures 12 and 13. The wide spacing of the days on the form for hourly heights is brought about by the fact that these tabulated forms are used for other purposes in connection with certain stencils which require that particular spacing.

THE RECORD ON CROSS-SECTION PAPER

If the tide record is on cross-section paper, whether made by an automatic tide gauge or plotted from staff readings, the tabulation is a relatively simple matter. Generally no time corrections are necessary, since corrections up to three minutes are ignored; but should time corrections be necessary, the tabulator indicates them on the tide curves. The height of the tide pertaining to each hour of the day is then read from the tide curve and entered into its appropriate place on the hourly height form. The tabulation of the hourly heights for each sheet of curves is completed before taking up the tabulation of the high and low waters. A specimen page of hourly heights of the tide for the week beginning June 10, 1920, for San Francisco, Calif., is shown in Figure 12.

The horizontal and vertical sums shown to the right and bottom in Figure 12 are obtained later in connection with the determination of mean sea level. The figures in the horizontal column "Day of series" give the sequence of each day with reference to the beginning of the series. When a tide station is continued for a number of years, it is most convenient to begin each series of observations on the 1st of January and continue the tabulation of the hourly heights consecutively throughout the year. Table 2 gives the day of series, the page, and the column corresponding to the 1st of every month and the last day of the year for a series beginning January 1. This table serves as a convenient check to insure against the omission or duplication of a day in the tabulation of the hourly heights of the tide.

TABLE 2.—*Day of series, page, and column for hourly height tabulations beginning January 1*

Common year				Leap year			
Month	Page	Column	Day of series	Month	Page	Column	Day of series
Jan. 1	1	1	1	Jan. 1	1	1	1
Feb. 1	5	4	32	Feb. 1	5	4	32
Mar. 1	9	4	60	Mar. 1	9	5	61
Apr. 1	13	7	91	Apr. 1	14	1	92
May 1	18	2	121	May 1	18	3	122
June 1	22	5	152	June 1	22	6	153
July 1	26	7	182	July 1	27	1	183
Aug. 1	31	3	213	Aug. 1	31	4	214
Sept. 1	35	6	244	Sept. 1	35	7	245
Oct. 1	40	1	274	Oct. 1	40	2	275
Nov. 1	44	4	305	Nov. 1	44	5	306
Dec. 1	48	6	335	Dec. 1	48	7	336
Dec. 31	53	1	365	Dec. 31	53	2	366

In tabulating the high and low waters the tabulator notes in succession the highest and lowest points of the tide curve, tabulating the times to the nearest tenth of an hour and the heights to the nearest tenth of a foot as read directly from the cross-section paper. How-

Form 362
DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY

TIDES: HOURLY HEIGHTS

Station: San Francisco, California Year: 1920
Observer: N. S. Ballard Lat. 37° 48' N. Long. 122° 27' W
Time Meridian: -120° W Tide Gauge No. 79 Scale 1: 12 Reduced to Staff.

GOVERNMENT PRINTING OFFICE 11—792

Month and Day	mo. d.	d.	d.	d.	d.	d.	d.	Horizontal Sum
Day of Series	June 10 162	11 163	12 164	13 165	14 166	15 167	16 168	
Hour	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet
0	7.2	6.9	6.8	7.3	8.3	9.7	11.0	57.2
1	7.6	6.7	6.2	6.2	6.8	7.8	9.2	50.5
2	7.9	7.0	6.1	5.5	5.7	6.3	7.3	45.8
3	8.3	7.4	6.4	5.4	5.0	5.1	5.7	43.3
4	8.7	8.0	6.9	5.8	4.9	4.4	4.7	43.4
5	9.0	8.6	7.6	6.5	5.5	4.5	4.0	45.7
6	9.1	9.0	8.4	7.4	6.3	5.2	4.2	49.6
7	8.8	9.2	9.0	8.4	7.5	6.3	5.1	54.3
8	8.4	9.1	9.3	9.2	8.6	7.6	6.3	58.5
9	7.6	8.6	9.2	9.5	9.4	8.8	7.7	60.8
10	7.0	8.0	8.9	9.6	9.8	9.5	8.9	61.7
11	6.8	7.4	8.3	9.2	9.8	10.1	9.8	61.4
Noon	6.9	7.0	7.7	8.6	9.5	9.9	10.1	59.7
13	7.4	7.2	7.4	8.0	8.9	9.6	10.0	58.5
14	8.0	7.7	7.5	7.7	8.2	9.0	9.6	57.7
15	8.9	8.4	8.1	7.8	7.9	8.3	8.8	58.2
16	9.6	9.4	9.0	8.5	8.2	8.0	8.2	60.9
17	10.4	10.2	9.8	9.4	8.9	8.4	8.0	65.1
18	10.8	10.9	10.7	10.4	9.8	9.1	8.3	70.0
19	10.8	11.2	11.4	11.2	10.8	10.2	9.1	74.7
20	10.4	11.0	11.6	11.9	11.6	11.2	10.2	77.9
21	9.6	10.4	11.3	11.9	12.2	11.9	11.2	78.5
22	8.4	9.2	10.3	11.3	12.1	12.4	11.9	75.6
23	7.5	7.8	8.7	9.9	11.2	12.1	12.2	69.4
Sum	205.1	206.3	206.6	206.6	206.9	205.4	201.5	1438.4

Sum for month of Divisor=(28d) 672; (29d) 696; (30d) 720: (31d) 744. Mean for month=

Tabulated by N. B. B. Date July 1920 Summed by N. S. B. Date July 1920

FIG. 12.—Specimen sheet, tabulation of hourly height of tide

ever, with tides of small range it is better to tabulate the heights of the high and low waters to the nearest half tenth of a foot, or even to the nearest hundredth of a foot. A specimen sheet of the tabulated high and low waters for the first half of the month of June, 1920,

for San Francisco, is shown in Figure 13. The last half of the month is tabulated on the other side of the sheet.

It is important to note that in determining the points of high and low water on the tide curve, which points give the times and heights

Form 138
DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY

TIDES: HIGH AND LOW WATERS

Station: San Francisco, California Lat. 37° 48′ N.
Acc. No. Party of H. S. Ballard Long. 122° 27′ W.
Observations begin Observations end Time meridian 120° W.

DATE Year 1920	MOON'S TRANSITS (Greenwich mean civil)	TIME OF— High Water	TIME OF— Low Water	LUNITIDAL INTERVAL High Water	LUNITIDAL INTERVAL Low Water	HEIGHT OF— High Water	HEIGHT OF— Low Water	REMARKS
mo. d.	hr. dec.	hr. dec.	hr. dec.	hr. dec.	hr. dec.	feet	feet	
June 1		12.8	5.4	.	.	9.9	5.0	
	.	23.0	17.2	.	.	11.3	8.4	
2	.	13.5	6.0	.	.	10.0	5.1	
	.	23.7	17.9	.	.	11.3	8.5	
3	.	14.0	6.6	.	.	10.0	5.2	
	.	—	18.4	.	.	—	8.6	
4	.	0.2	7.2	.	.	11.1	5.3	
	.	14.5	19.2	.	.	10.1	8.6	
5	.	0.7	7.6	.	.	10.8	5.4	
	.	15.2	20.0	.	.	10.0	8.6	
6	.	1.5	8.2	.	.	10.6	5.5	
	.	15.9	20.7	.	.	10.1	8.5	
7	.	2.0	8.9	.	.	10.1	5.8	
	.	16.6	21.9	.	.	10.0	8.1	
8	.	3.1	9.2	.	.	9.6	6.0	
	.	17.2	22.9	.	.	10.2	7.6	
9	.	4.4	10.2	.	.	9.4	6.3	
	.	18.0	23.8	.	.	10.5	7.2	
10	.	5.7	11.2	.	.	9.1	6.8	
	.	18.5	—	.	.	10.9	—	
11	.	7.2	0.8	.	.	9.2	6.8	
	.	19.2	12.2	.	.	11.2	7.0	
12	.	8.4	1.8	.	.	9.4	6.0	
	.	20.0	13.3	.	.	11.6	7.4	
13	.	9.6	2.7	.	.	9.6	5.4	
	.	20.6	14.2	.	.	11.9	7.7	
14	.	10.6	3.6	.	.	9.9	4.9	
	.	21.3	15.0	.	.	12.3	8.0	
15	.	11.5	4.5	.	.	10.1	4.4	
	.	22.2	16.0	.	.	12.4	8.0	
16	.	12.3	5.3	.	.	10.2	4.0	
	.	23.0	16.9	.	.	12.2	8.0	
17	.	13.2	6.1	.	.	10.2	3.8	
	.	23.9	17.8	.	.	12.0	7.8	
Sums; carried forward		.	.	.	.	.	.	

FIG. 13.—Specimen sheet, tabulation of high and low waters

to be tabulated, attention is to be centered on an arc of the curve that covers a time interval of an hour or more. The highest or lowest part of the smooth arc is chosen for the high or low water and not merely the highest or lowest point on the curve, which may be due to wave action or other disturbing factors. This matter will receive further

consideration in connection with the discussion of irregularities in the tide curve.

In tabulating times to the nearest tenth of an hour and heights to the nearest tenth of a foot no provision is made for the tabulation of values which lie exactly half way between tenths. For example, 8.25 hours may be tabulated either as 8.2 or 8.3 hours and, likewise, 6.75 feet may be tabulated as 6.7 or 6.8 feet. Obviously, some definite rule is desirable for such cases. A rule sometimes used is to drop the last figure, but this introduces a systematic error. A much better rule in such cases is to make the first decimal place even; for example, 8.25 would be written 8.2 while 8.35 would be written 8.4.

THREE-ROLLER GAUGE RECORD

Before a tabulation of the hourly heights or of the high and low waters can be made from the tide record furnished by the three-roller gauge, it is necessary to determine the relation of the curve to the zero of the tide staff. This is done by means of a tabulation of comparative readings of staff and curve, using a glass or celluloid reading scale graduated in feet and tenths to the same scale as that to which the tide curve is drawn by the tide gauge. A specimen sheet of the tabulation of the comparative readings for the tide record at Charleston, S. C., for the month of November, 1925, is shown in Figure 14.

In the first three columns of the comparative readings tabulation the tabulator notes, respectively, the day, the time of staff reading, and the height of staff, which items are taken from the tide roll as recorded in the observer's notes. In the fourth column the tabulator notes the height of the curve by his reading scale at the time of the staff reading. This height obviously will depend on the height assumed for the datum line on the curve. It is most convenient to assume for the datum line a height which will be somewhat less than the staff reading for that point on the curve, so that the differences between staff and scale will be positive and lie between zero and 2 feet. The scale reading for the datum line in the specimen sheet of comparative readings shown in Figure 14 was taken as 4 feet.

In the fifth column of the comparative readings tabulation the difference between staff and scale is derived, and in the sixth column the phase of the tide at the time of staff reading is noted. The letters *F*, *R*. *H*, and *L* are used to designate, respectively, the falling tide, rising tide, high water, and low water.

Any change in the adjustment of the gauge will change the relation between scale and staff. Thus, a note in the column of remarks in Figure 14 states that on November 13, 1925, a new float wire was put on the gauge at Charleston. This is reflected in a change of the relation of staff to scale from 1.1 feet to 0.2 foot. When such a change occurs, the two parts of the record must be tabulated separately.

For the period November 1–12, Figure 14 shows that the difference between staff and scale averaged 1.11 feet, the derivation of this average being shown in the column headed "Remarks." Hence, for the portion of the record from November 1–12, the datum line corresponds to 5.11 feet, and by marking this height on the scale

the hourly heights and also the heights of high and low water may be tabulated to correspond to the true staff readings.

Beginning with the time the new float wire was put on the gauge on November 13, the datum line on the tide record corresponded to a staff reading of 4.16 feet, and this setting must be used on the scale

Form 433
DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY

TIDES: COMPARATIVE READINGS.

Station: Charleston, S. C. Lat. 32° 47' N.
Party of L. C. Lockwood Time meridian 75° W. Long. 79° 55' W.
Obs. begin Obs. end Tabulated by R. C. L. Date Nov. 1925
Tide Gauge No. 97 Scale 1 : 12 The scale reading of Datum Line for this comparison is 4.00 feet.

DATE. Year. 1925 mo.	d.	TIME OF STAFF READING. h.	m.	STAFF A. feet.	SCALE B. feet.	DIFFERENCE A–B. feet.	PHASE OF TIDE.*	REMARKS.	
Nov	1	10	33	7.10	6.00	1.10	F	Nov. 1 – Nov. 12	
	2	13	09	4.30	3.15	1.15	F	Sum	12.20
	3	16	23	3.00	1.90	1.10	R	Divisor	11
	4	16	48	3.60	2.50	1.10	R	Mean	1.11
	5	16	26	4.00	2.90	1.10	F	Original Setting	4.00
	6	16	52	3.80	2.70	1.10	F	Corrected Setting	5.11
	7	16	28	4.85	3.75	1.10	F		
	9	16	02	6.95	5.85	1.10	F		
	10	16	31	7.10	6.00	1.10	F		
	11	15	48	7.55	6.45	1.10	R		
	12	16	37	7.95	6.80	1.15	R		
	13							New float wire put on	
	13	16	46	6.50	6.30	0.20	R		
	14	12	30	1.90	1.80	0.10	L		
	16	16	49	4.05	3.90	0.15	R	Nov. 13 – Nov. 30	
	17	15	49	1.60	1.50	0.10	R	Sum	2.20
	18	16	16	1.70	1.60	0.10	R	Divisor	14
	19	16	23	1.95	1.85	0.10	L	Mean	0.16
	20	16	48	2.50	2.35	0.15	F	Original Setting	4.00
	21	15	50	5.20	5.10	0.10	F	Corrected Setting	4.16
	23	16	30	6.50	6.30	0.20	F		
	24	16	13	7.50	7.30	0.20	F		
	25	16	30	7.30	7.10	0.20	H		
	27	16	18	6.90	6.70	0.20	R		
	28	12	46	2.60	2.40	0.20	L		
	30	16	30	4.90	4.70	0.20	R		

* In the column headed "Phase of Tide" write the appropriate one of the four following symbols: H, for high water; L, for low water; R, for rising tide; and F, for falling tide. Use Form 136 for tabulating high and low water. 11—4501

FIG. 14.—Specimen sheet, tabulation of comparative readings

for tabulating that portion of the roll in order that the tabulation should correspond with true staff readings.

The differences between staff and scale in the fifth column of the comparative readings tabulation will vary somewhat from day to day, primarily because of the difficulty of reading the staff to the nearest half tenth of a foot if any wave motion is present. Since the

figures on the staff increase upward an error of a foot is occasionally made by the observer in reading the staff, and this error of 1 foot will appear in the column of differences. An error of this kind, however, is easily noted and should be corrected before the differences are summed for the derivation of the mean.

In general, during periods when no change is made in the adjustment of the gauge, the differences between staff and scale will be approximately constant. Any difference which stands out strikingly from the others should be rejected from the computation for the mean difference.

It is to be observed that the differences between staff and scale will vary systematically if the inlet to the float well becomes clogged. In that case the difference will be greater than the average for the rising tide and less than the average for the falling tide. These differences thus furnish a check on the proper functioning of the float well.

With the determination of the corrected setting for the scale, the tabulation of the hourly heights of the tide and of the high and low waters is carried on as outlined in the preceding paragraphs for the tabulation of the record on cross-section paper.

In connection with the tabulation of the tide record it is assumed that throughout the period of observations the tabulations are referred to a staff the zero of which is maintained at a fixed level. If during the period of observations the staff is changed, the height relation between the two positions of the staff must be accurately determined. Whenever possible it is preferable to take account of this change in staff in connection with the tabulation, so that the whole series may be referred to the same staff. However, it frequently happens that the tabulations must be made prior to the determination of the exact relationship between the two staffs. In that case full explanation should be noted in the columns of remarks of the hourly ordinates and high and low water tabulations; and, as soon as the corrections necessary to reduce these readings to the zero of the previous staff are determined, this correction should be noted on the tabulated sheets.

IRREGULARITIES IN TIDE CURVES

To secure a correct representation of the rise and fall of the tide, the inlet to the float well of an automatic tide gauge is made sufficiently large to insure free communication with the water outside the float well. If the inlet is too small, the tide curve will show a smaller range of tide and a retardation in the times of high and low water. As a result of making it large enough to insure free communication, disturbed conditions of the sea will be reflected by irregularities in the tide curve. An example of such irregularities is shown in Figure 15, which is a representation on a reduced scale of the San Francisco tide curve for November 21, 1910.

Two kinds of irregularities in the tide curve are seen in Figure 15. The first consists of numerous small "saw teeth," which appear throughout the curve and which show up particularly well between the 13th and 15th hours of the day. These "saw teeth" represent the rise and fall of the larger waves and ocean swells which enter San Francisco Bay from the ocean. The second irregularity consists

of larger and slower fluctuations, which appear suddenly about 4.45 a. m. and continue with diminishing amplitude until about 10 o'clock. These fluctuations are brought about by stationary-wave oscillations of the water within the bay and are known as seiches.

Seiches are brought about by various agencies. Heavy winds, sudden variations in barometric pressure, and seismic waves due to seaquakes—all these bring about seiche movements of the water which cause irregularities in the tide curve. The seiches shown in Figure 15 were caused by a rapid fall and rise of atmospheric pressure at San Francisco. In Figure 16 are shown seiches due to heavy winds. The curve of Figure 16 represents the rise and fall of the tide at Atlantic City, N. J., from 5.30 a. m. to 9.30 p. m. on January 2, 1925. The Atlantic City tide gauge is located about 1,500 feet from shore on one of the piers that juts out into the open sea.

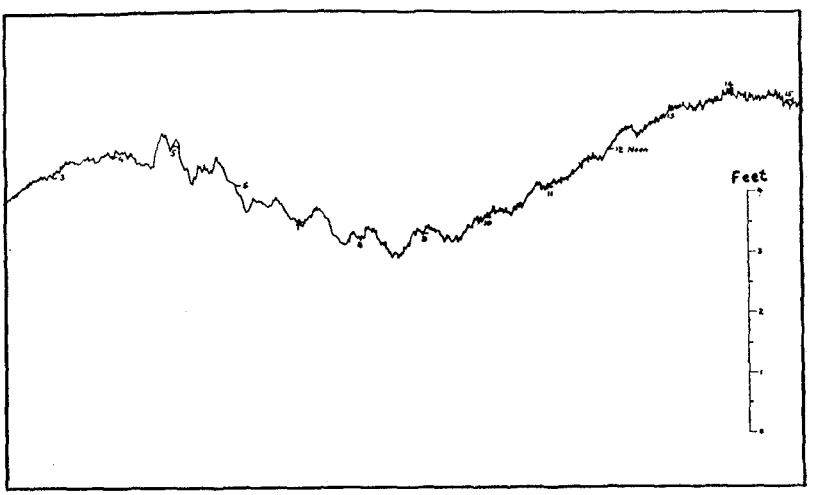

FIG. 15.—Tide curve, San Francisco, Calif., showing seiches due to changes in barometric pressure

On the day represented by Figure 16 the wind between 6 a. m. and 8 p. m. blew from the northeast with velocities varying from 54 to 78 miles per hour, the latter velocity occurring a little after noon. The short "saw teeth" are the traces of the swells and larger waves which have short periods; but the larger oscillations, with a period of about half an hour beginning about 11.30 a. m., undoubtedly represent seiches of some part of a wide embayment of the coast.

Figure 17 is an example of seiches due to a seaquake. On November 11, 1922, about 4.30 a. m. Greenwich civil time, a seaquake occurred off the coast of South America in the vicinity of Carrizal, Chile. The three curves in Figure 17 were drawn from the tide curves for San Diego, Calif., San Francisco, Calif., and Honolulu, Territory of Hawaii, for the period 4 p. m., November 11, to 4 a. m., November 12, 1922. To make these curves readily comparable, they are drawn to the same height scale and referred to the same time meridian. The height scale is shown in feet to the right. The time used is that for the meridian of Greenwich, or Greenwich civil time.

In round numbers, San Diego, San Francisco, and Honolulu are distant from Carrizal, respectively, 4,500, 5,000, and 6,000 nautical miles. Nevertheless, the seaquake off Carrizal brought about well-developed seiches at each of those places and introduced irregularities in the tide curves. It is somewhat difficult to pick out the exact instant of beginning of the fluctuation due to the arrival of the seismic sea waves, but in general it appears that this time was a little before 6 p. m. at San Diego, about 6.30 p. m. at San Francisco, and about 7.30 p. m. at Honolulu.

In the tabulation of the tidal record "saw teeth" and seiches introduce difficulties. For use in the determination of tidal datum planes it is preferable to consider a smooth curve drawn through such irregularities and tabulate the hourly heights directly from this smooth curve. The times and heights of the high and low waters should also be tabulated from the smooth curve, but note should be

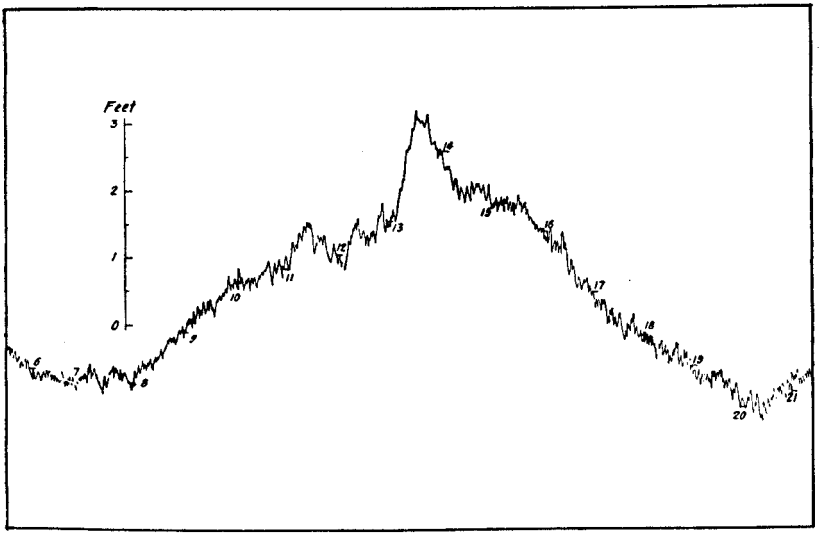

FIG. 16.—Tide curve, Atlantic City, N. J., showing seiches due to heavy winds

made in the column of remarks of the time and height of the highest (or lowest) point of the short-period oscillations.

Smoothing the tide curve must not be carried beyond the legitimate purpose of eliminating short-period oscillations. When the tide curve is disturbed in time and height by unusual weather conditions, the tabulator may be tempted to substitute for the actual tide curve a hypothetical tide curve which disregards the disturbances in time and height, on the mistaken notion that better mean values are derived through this substitution. A moment's reflection will make it evident that in such cases it is much better to tabulate the data directly from the actual tide curve and reject, if necessary, the disturbed values from the computation for mean values.

Figure 18 illustrates the disturbing effects of heavy winds on the tide. The upper curve represents the average or mean rise and fall of the tide at Fort Hamilton, N. Y., while the lower curve represents the rise and fall of the tide at Fort Hamilton for December 13 to

14, 1917, on which days heavy northerly winds prevailed. To make the tide curves comparable in time, they are referred to the time of the moon's transit over the meridian at Fort Hamilton, zero hours

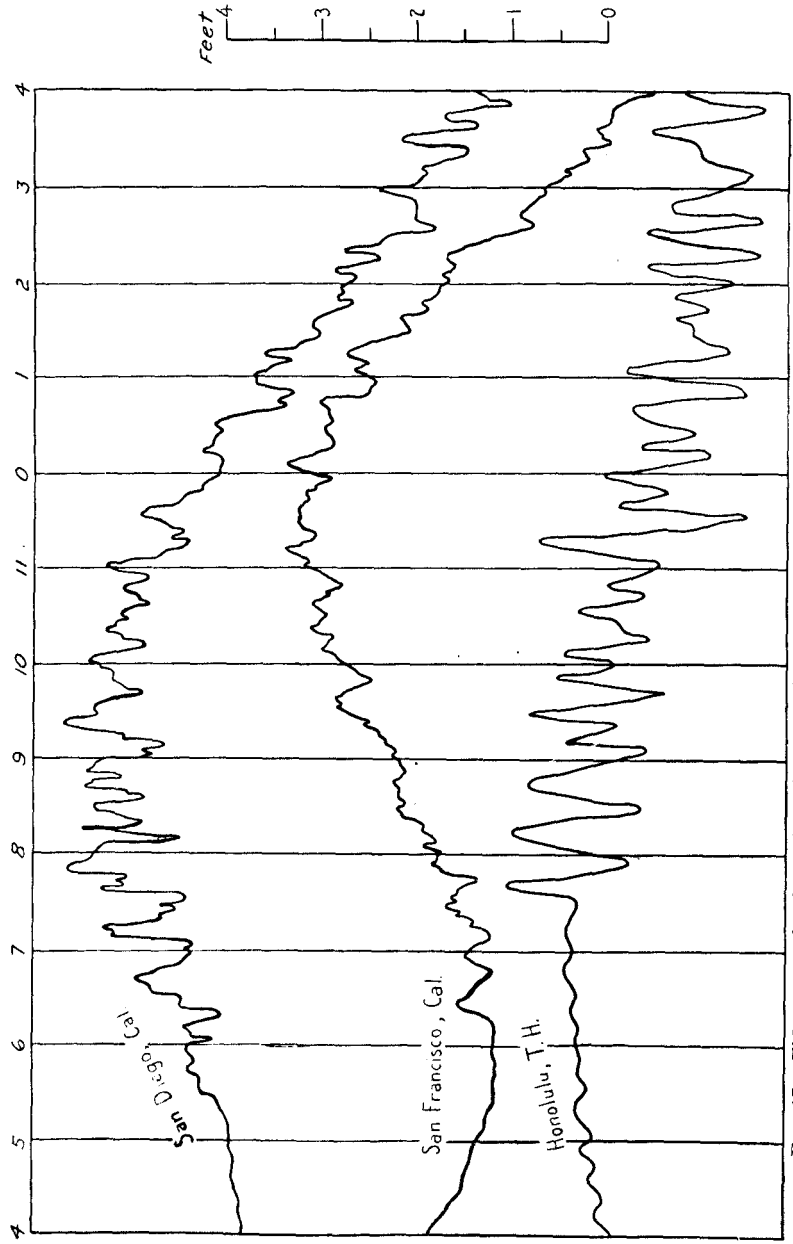

FIG. 17.—Tide curves for San Diego, San Francisco, and Honolulu showing seiches due to distant seaquake

being the instant of the moon's upper meridian passage. For the lower curve zero hours correspond to 11.10 a. m. on December 13, 1917. In reproducing the tide curve for December 13–14 the short-period oscillations were smoothed out.

Comparing the two curves, it is seen that for the first six hours they agree closely. From that time on the increasing velocity of the wind on December 13, 1917, is reflected in a disturbed condition of the tide, the high water being retarded by an hour and the following low water by four hours. In all such cases smoothing of the tide curve should be limited strictly to the elimination of short-period fluctuations, the times and heights of the high and low waters being tabulated from the actual tide curve so smoothed and note made in the column of remarks of the weather conditions prevailing.

INTERPOLATION OF BREAKS IN THE RECORD

Since the time and height of tide varies from day to day, it is desirable both for the purpose of determining mean values and for

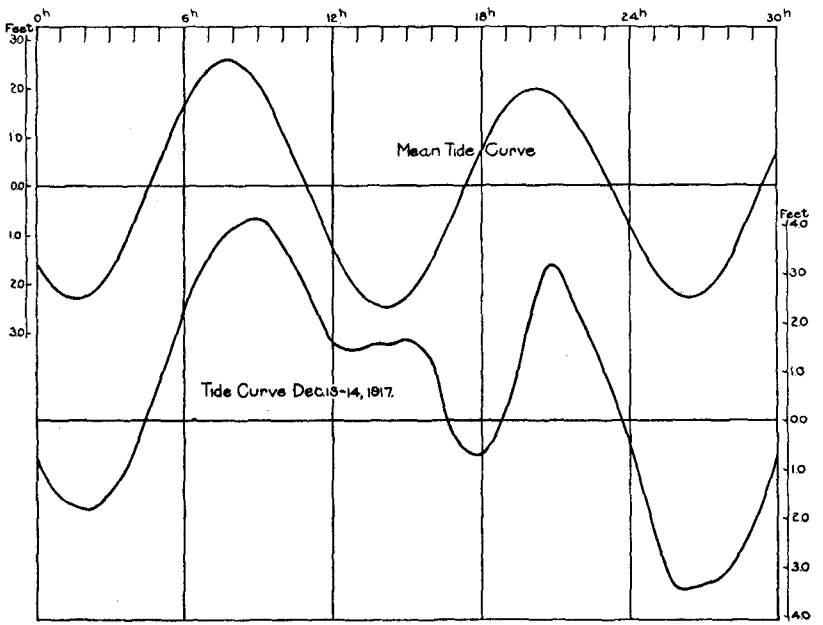

FIG. 18.—Mean and storm tide curves, Fort Hamilton, N. Y.

purposes of comparison to interpolate any breaks that may occur in the tide record at stations where the series of observations cover several months or more. Various methods may be used, depending on the location of the station and the duration of the break. In general the procedure is to tabulate the hourly ordinates and the high and low waters for such portions of the record as are complete, leaving the interpolations to be made later.

To distinguish interpolated values from those derived directly from the tide record, the interpolated values are tabulated in red ink or they are inclosed in parentheses. If the duration of the break is no more than a day or two, a convenient method is to interpolate linearly the times and heights of the high and low waters. These interpolated values are then used for constructing the tide curve on

cross-section paper, from which the hourly ordinates are tabulated. An example will make this method clear.

Suppose that on June 13, 1920, the tide gauge at San Francisco had failed to function. In tabulating the record for the month of June, 1920, the tabulator would leave that day blank in both the hourly heights and in the high and low water tabulations (figs. 12 and 13) and complete both tabulations before making the interpolations. The tabulation of the high and low waters illustrated in Figure 13 shows that on the day preceding the assumed break the morning high water came at 8.4 hours with a height of 9.4 feet, while on the day succeeding the break these values were, respectively, 10.6 hours and 9.9 feet. A direct mean of the above values gives for the time of the missing high water 9.5 hours and for the height 9.6 feet as compared with 9.6 hours and 9.6 feet, the values actually observed.

In the same way the morning low water for the 13th would be determined as 2.7 hours and 5.4 feet, while the afternoon high and low waters interpolated from the corresponding tides the day previous and the day following are, respectively, 20.6 hours and 12 feet and 14.2 hours and 7.7 feet. Of these eight interpolated values, comparison with the values given for that day on Figure 13 shows that six agree exactly and two differ by one-tenth.

To interpolate the hourly ordinates for the day in question, the values determined above for the times and heights of the high and low waters are plotted on cross-section paper and a curve drawn through these points as maxima and minima, the shape of the curve being made to conform to the curves of the days preceding and following. The hourly heights are then tabulated directly from this curve.

The linear method of interpolation obviously can be used only for relatively short breaks—rarely more than for three days. Breaks of greater duration may be interpolated by use of the observations at some other tide station, not too far away, which has a tide of the same type. The differences in the time and height of the tide at the two stations are determined from simultaneous observations and these differences applied to the observed times and heights of the high and low waters for the days in question, the hourly ordinates being interpolated as before.

Another method of interpolating a break of more than three days is to take a mean of the times and heights of the high and low waters 29 days before and after. This method is based on the fact that the three principal lunar cycles, the phase cycle, the parallax cycle, and the declinational cycle are, respectively, 29½ days, 27½ days, and 27⅓ days in length. As an example of this method, it may be used for interpolating the high and low waters at San Francisco for June 13, 1920, the day used to exemplify the method of linear interpolation.

Twenty-nine days prior to June 13 is May 15, and 29 days after June 13 is July 12. For May 15, 1920, the high waters at San Francisco occurred at 9.6 and 21.2 hours, the heights being 9.9 and 11.7 feet. For July 12 the corresponding values were 9.5 and 20.2 hours and 9.6 and 12 feet. Hence the interpolated high waters for June 13 would be 9.6 and 20.7 hours and 9.8 and 11.8 feet. These agree quite well with the observed values of 9.6 and 20.6 hours and 9.6 and 11.9 feet. A mean of the times and heights of the low waters on May 15

and July 12, 1920, gives 2.8 and 14.2 hours and 5.5 and 7.6 feet, which agree well with the observed values of 2.7 and 14.2 hours and 5.4 and 7.7 feet.

V. TIME OF TIDE

LUNITIDAL INTERVALS

In connection with time relations of the tide, it is convenient to use the term "time of tide" to denote the time of high or low water. Differences in time of tide at various places are readily determined through lunitidal intervals. These intervals also furnish the data for determining the durations of rise and fall of the tide. Hence, in connection with the determination of tidal datum planes, the computations for deriving the lunitidal intervals are generally included.

With regard to the tide at a given place, the lunitidal interval is defined as the time elapsing between the moon's local meridian passage and the following high water or low water. In other words, it is the difference between the mean local time of tide and the mean local time of the moon's transit across the local meridian. But it is found more convenient to use the Greenwich transits of the moon and the times of high and low water as tabulated with regard to some standard time meridian, the necessary corrections being applied only to the mean values.

A specimen computation of the lunitidal intervals for a month of high and low waters is shown in Figures 19 and 20. It will be noted that this computation is carried out on the sheet of tabulated high and low waters, the first 17 days on one side of the sheet and the last half of the month on the other side. In the second column the computer tabulates the Greenwich mean time of the moon's transit across the meridian of Greenwich to the nearest tenth of an hour, these times being taken from a nautical almanac. To distinguish the upper from the lower transits the latter are inclosed in parentheses.

In the fifth and sixth columns, under the appropriate headings, are entered the differences between the times of high and low water and the immediately preceding times of the moon's transits, the intervals derived from the lower transits being inclosed in parentheses to distinguish them from those derived from the upper transits. The intervals are then summed, the number of intervals and the sums of the first half of the month being brought forward and summed with the last half of the month, as shown in Figure 20. The sums for the whole month are then entered opposite "Sums," with the number of intervals immediately above and the means derived to two decimal places. For the month of June, 1920, these means, as shown in Figure 20, are 12.06 hours for the high-water interval and 5.47 hours for the low-water interval.

Two corrections must be applied to these mean values to derive the true lunitidal intervals. The first correction is to take account of the fact that the Greenwich times of the moon's transit across the meridian of Greenwich were used and not the local times across the local meridian. The second correction is to take account of the difference between local time and standard time, since the times of the high and low waters were tabulated with reference to the standard time meridian of 120° W., while the longitude of the tide station is 122° 27′ W.

To derive general formulas for these two corrections, it is to be noted that, since the moon's transit across any given meridian comes later each day by about 50 minutes, the local time of the moon's transit is earlier than the Greenwich transit for places east of Greenwich and later for places west of Greenwich. This means that for places west of Greenwich the use of Greenwich transits gives a luni-

Form 188
DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY

TIDES: HIGH AND LOW WATERS

Station: San Francisco, California Lat. 37° 48' N
Acc. No. _____ Party of H. S. Ballard Long. 122° 27' W
Observations begin _____ Observations end _____ Time meridian 120° W. (Greenwich)

DATE Year 1920	MOON'S TRANSITS (Greenwich mean civil)	TIME OF— High Water	TIME OF— Low Water	LUNITIDAL INTERVAL High Water	LUNITIDAL INTERVAL Low Water	HEIGHT OF— High Water	HEIGHT OF— Low Water	REMARKS
mo. d.	hr. dec.	hr. dec.	hr. dec.	hr. dec.	hr. dec.	feet	feet	
May 31	23.4							
June 1	(11.8)	12.8	5.4	13.4	6.0	9.9	5.0	
	—	23.0	17.2	(11.2)	(5.4)	11.3	8.4	
2	0.2	13.5	6.0	13.3	5.8	10.0	5.1	
	(12.6)	23.7	17.9	(11.1)	(5.3)	11.3	8.5	
3	1.0	14.0	6.6	13.0	5.6	10.0	5.2	
	(13.4)	—	18.4	—	(5.0)	—	8.6	
4	1.8	0.2	7.2	(10.8)	5.4	11.1	5.3	
	(14.2)	14.5	19.2	12.7	(5.0)	10.1	8.6	
5	2.6	0.7	7.6	(10.5)	5.0	10.8	5.4	
	(15.0)	15.2	20.0	12.6	(5.0)	10.0	8.6	
6	3.3	1.5	8.2	(10.5)	4.9	10.6	5.5	
	(15.7)	15.9	20.7	12.6	(5.0)	10.1	8.5	
7	4.1	2.0	8.9	(10.3)	4.8	10.1	5.8	
	(16.5)	16.6	21.9	12.5	(5.4)	10.0	8.1	
8	4.8	3.1	9.2	(10.6)	4.4	9.6	6.0	
	(17.2)	17.2	22.9	12.4	(5.7)	10.2	7.6	
9	5.6	4.4	10.2	(11.2)	4.6	9.4	6.3	
	(18.0)	18.0	23.8	12.4	(5.8)	10.5	7.2	
10	6.4	5.7	11.2	(11.7)	4.8	9.1	6.8	
	(18.7)	18.5	—	12.1	—	10.9	—	
11	7.1	7.2	0.8	(12.5)	(6.1)	9.2	6.8	
	(19.6)	19.2	13.2	12.1	5.1	11.2	7.0	
12	8.0	8.4	1.8	(12.8)	(6.2)	9.4	6.0	
	(20.4)	20.0	13.8	12.0	5.3	11.6	7.4	
13	8.9	9.6	2.7	(13.2)	(6.3)	9.6	5.4	
	(21.4)	20.6	14.2	11.7	5.3	11.9	7.7	
14	9.9	10.6	3.6	(13.2)	(6.2)	9.9	4.9	
	(22.4)	21.3	15.0	11.4	5.1	12.3	8.0	
15	10.9	11.5	4.6	(13.1)	(6.1)	10.1	4.4	
	(23.4)	22.2	16.0	11.3	5.1	12.6	8.0	
16	11.9	12.3	5.3	(12.4)	(5.9)	10.2	4.0	
	—	23.0	16.9	11.1	5.0	12.2	8.0	
17	(0.5)	13.2	6.1	(12.7)	(5.6)	10.2	3.8	
	12.0	23.9	17.8	10.9	4.8	12.0	7.8	
Sums; carried forward				395.8	177.0	347.2	219.7	
				33	33	33	33	

FIG. 19.—Specimen sheet, computations for lunitidal intervals, high water and low water

tidal interval too large, and the correction to be applied is therefore negative. Since the average daily retardation in the time of the moon's meridian passage is 50.47 minutes or 0.841 hours, the correction for local meridian passage is $0.841 \div 360 = 0.00234$ hours for each degree of longitude. Hence, if L is the west longitude in degrees of any place, the first correction to the intervals derived by use of Greenwich transits is, in hours, $-0.00234L$.

To correct the intervals for the use of standard time, it is evident that since the difference in local time between two places is 1 hour for each 15 degrees of longitude, if S is the longitude in degrees of the standard time meridian used and L the longitude of a given place, the correction to the interval, in hours, will be $(S-L) \div 15$ where S and L are taken as positive for west longitude

Form 136
DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY

TIDES: HIGH AND LOW WATERS

Station: San Francisco, California Lat. 37° 48′ N.

Highest tide: Date June 15 Height 12.4 ft. Lowest tide: Date June 17 & 18 Height 3.8 ft. Long. 122° 27′ W

I= ____ $(K_1+O_1)+M_2$ or $2(DHQ+DLQ)+Mn$= ____ $F(Mn)$= ____ $F_1 \times 1.02$= ____ Time meridian 120° W.

DATE Year 1920	MOON'S TRANSITS (Greenwich mean civil)	TIME OF—		LUNITIDAL INTERVAL		HEIGHT OF—		REMARKS	
		HIGH WATER	LOW WATER	HIGH WATER	LOW WATER	HIGH WATER	LOW WATER		
mo. d.	hr. dec.	hr. dec.	hr. dec.	hr. dec.	hr. dec.	feet	feet		
Brought forward				33 395.8	33 177.0	33 347.2	33 218.7		
June 18	(1.5)	14.0	6.9	(12.5)	(5.4)	10.4	3.8	HHW	LLW
	14.0	—	18.7	—	4.7	—	7.8	Sums	
19	(2.5)	0.7	7.6	10.7	(5.1)	11.7	4.1	Means	
	15.0	14.7	19.7	(12.2)	4.7	10.6	7.7		
20	(3.4)	1.7	8.4	10.7	(5.0)	11.3	4.9	DHQ= DLQ=	
	15.9	15.6	20.9	(12.1)	5.0	10.9	7.8		feet
21	(4.3)	2.7	9.2	10.8	(4.9)	10.8	5.4	Corr. Mn= × =	
	16.7	16.4	22.1	(12.1)	5.4	11.0	7.4	Corr. DHQ= × =	
22	(5.1)	3.8	10.0	11.1	(4.9)	10.2	5.9	Corr. DLQ= × =	
	17.5	17.2	23.4	(12.1)	5.9	11.2	7.1		
23	(5.9)	5.1	10.9	11.6	(5.0)	9.6	6.5		
	18.3	17.9	—	(12.0)	—	11.2	—		
24	(6.7)	6.6	0.5	12.3	6.2	9.4	6.6		
	19.0	18.7	11.9	(12.0)	(5.2)	11.4	7.1		
25	(7.4)	8.1	1.4	13.1	6.4	9.5	6.2		
	19.8	19.5	13.0	(12.1)	(5.6)	11.5	7.7		
26	(8.2)	9.2	2.5	13.4	6.7	9.9	6.1		
	20.6	20.2	13.9	(12.0)	(5.7)	11.8	8.4		
27	(9.0)	10.1	3.2	13.5	6.6	10.0	5.9		
	21.4	20.9	14.7	(11.9)	(5.7)	11.8	8.5		
28	(9.8)	11.2	4.0	13.8	6.6	10.1	5.6		
	22.2	21.8	15.7	(11.7)	(5.9)	11.8	8.8		
29	(10.6)	11.7	4.6	13.5	6.4	10.2	5.6		
	23.0	22.2	16.2	(11.6)	(5.6)	11.8	8.7		
30	(11.4)	12.4	5.3	13.4	6.3	10.2	5.4		
	23.8	22.6	16.9	(11.2)	(5.5)	11.6	8.7		
31									
Sums				58 699.2	58 317.4	58 617.1	58 387.4		
Means				12.06	5.47	10.64	6.68		
Correction to intervals				−0.45	−0.45	6.68			
				11.61	5.02	3.96	Mn	Tabulated by H. L. B.	
Duration of rise						8.66	MTL	Date July 1920	
Mean rise interval								Reduced by H. L. B.	
								Date July 1920	

FIG. 20.—Specimen sheet, computation for lunitidal intervals, high water and low water

and negative for east longitude. Combining the two corrections, the total correction to intervals determined by use of Greenwich transits and standard time becomes $0.06667(S-L)-0.00234L=0.06667S-0.06901L$. In this formula it is to be noted that S and L are positive for west longitude and negative for east longitude.

For the lunitidal intervals derived for San Francisco for the month of June, 1920, since the time used is that for the meridian of

120° W. and the longitude of the station is 122° 27′ (=122.45°), the correction to the intervals becomes $0.06667 \times 120 - 0.06901 \times 122.45 = -0.45$ hour. The corrected lunitidal intervals for the month, therefore, are 11.61 hours for the high water and 5.02 hours for the low water, as shown on Figure 20.

From the formula above it is a simple matter to compute tables of corrections to lunitidal intervals for different longitudes and various time meridians. Table 3 gives these corrections for the 13 primary standard time meridians. The time meridian 157° 30′ is also included, since this is the time meridian used in the Hawaiian Islands.

TABLE 3.—*Corrections to lunitidal intervals for Greenwich transits and standard time*

[For west longitude, use sign given; for east longitude, reverse sign]

Time meridian, 0°		Time meridian, 15°		Time meridian, 30°		Time meridian, 45°		Time meridian, 60°	
Longitude	Correction	Longitude	Correction	Longitude	Correction	Longitude	Correction	Longitude	Correction
°	*Hour*	°	*Hour*	°	*Hour*	°	*Hour*	°	*Hour*
		5	+0.655	20	+0.620	35	+0.585	50	+0.550
		6	+.586	21	+.551	36	+.516	51	+.481
		7	+.517	22	+.482	37	+.447	52	+.412
		8	+.448	23	+.413	38	+.378	53	+.343
		9	+.379	24	+.344	39	+.309	54	+.274
		10	+.310	25	+.275	40	+.240	55	+.205
		11	+.241	26	+.206	41	+.171	56	+.136
		12	+.172	27	+.137	42	+.102	57	+.067
		13	+.103	28	+.068	43	+.033	58	−.002
		14	+.034	29	−.001	44	−.036	59	−.071
0	0.000	15	−.035	30	−.070	45	−.105	60	−.140
1	−.069	16	−.104	31	−.139	46	−.174	61	−.209
2	−.138	17	−.173	32	−.208	47	−.243	62	−.278
3	−.207	18	−.242	33	−.277	48	−.312	63	−.347
4	−.276	19	−.311	34	−.346	49	−.381	64	−.416
5	−.345	20	−.380	35	−.415	50	−.450	65	−.485
6	−.414	21	−.449	36	−.484	51	−.519	66	−.554
7	−.483	22	−.518	37	−.553	52	−.588	67	−.623
8	−.552	23	−.587	38	−.622	53	−.657	68	−.692
9	−.621	24	−.656	39	−.691	54	−.726	69	−.761
10	−.690	25	−.725	40	−.760	55	−.795	70	−.830

Time meridian, 75°		Time meridian, 90°		Time meridian, 105°		Time meridian, 120°		Time meridian, 135°	
Longitude	Correction	Longitude	Correction	Longitude	Correction	Longitude	Correction	Longitude	Correction
°	*Hour*	°	*Hour*	°	*Hour*	°	*Hour*	°	*Hour*
65	+0.515	80	+0.480	95	+0.445	110	+0.410	125	+0.375
66	+.446	81	+.411	96	+.376	111	+.341	126	+.306
67	+.377	82	+.342	97	+.307	112	+.272	127	+.237
68	+.308	83	+.273	98	+.238	113	+.203	128	+.168
69	+.239	84	+.204	99	+.169	114	+.134	129	+.099
70	+.170	85	+.135	100	+.100	115	+.065	130	+.030
71	+.101	86	+.066	101	+.031	116	−.004	131	−.039
72	+.032	87	−.003	102	−.038	117	−.073	132	−.108
73	−.037	88	−.072	103	−.107	118	−.142	133	−.177
74	−.106	89	−.141	104	−.176	119	−.211	134	−.246
75	−.175	90	−.210	105	−.245	120	−.280	135	−.315
76	−.244	91	−.279	106	−.314	121	−.349	136	−.384
77	−.313	92	−.348	107	−.383	122	−.418	137	−.453
78	−.382	93	−.417	108	−.452	123	−.487	138	−.522
79	−.451	94	−.486	109	−.521	124	−.556	139	−.591
80	−.520	95	−.555	110	−.590	125	−.625	140	−.660
81	−.589	96	−.624	111	−.659	126	−.694	141	−.729
82	−.658	97	−.693	112	−.728	127	−.763	142	−.798
83	−.727	98	−.762	113	−.797	128	−.832	143	−.867
84	−.796	99	−.831	114	−.866	129	−.901	144	−.936
85	−.865	100	−.900	115	−.935	130	−.970	145	−1.005

TABLE 3.—*Corrections to lunitidal intervals for Greenwich transits and standard time*—Continued

[For west longitude, use sign given; for east longitude, reverse sign]

Time meridian, 150°		Time meridian, 157° 30′		Time meridian, 165°		Time meridian, 180°	
Longitude	Correction	Longitude	Correction	Longitude	Correction	Longitude	Correction
°	*Hour*	°	*Hour*	°	*Hour*	°	*Hour*
140	+0.340	148	+0.288	155	+0.304	170	+0.269
141	+.271	149	+.219	156	+.235	171	+.200
142	+.202	150	+.149	157	+.166	172	+.131
143	+.133	151	+.080	158	+.097	173	+.062
144	+.064	152	+.011	159	+.028	174	−.007
145	−.005	153	−.058	160	−.041	175	−.076
146	−.074	154	−.127	161	−.110	176	−.145
147	−.143	155	−.196	162	−.179	177	−.214
148	−.212	156	−.265	163	−.248	178	−.283
149	−.281	157	−.334	164	−.317	179	−.352
150	−.351	158	−.403	165	−.386	180	−.421
151	−.420	159	−.472	166	−.455	181	−.490
152	−.489	160	−.541	167	−.524	182	−.559
153	−.558	161	−.610	168	−.593	183	−.628
154	−.627	162	−.679	169	−.662	184	−.697
155	−.696	163	−.748	170	−.731	185	−.766
156	−.765	164	−.817	171	−.800	186	−.835
157	−.834	165	−.886	172	−.869	187	−.904
158	−.903	166	−.955	173	−.938	188	−.973
159	−.972	167	−1.024	174	−1.007	189	−1.042
160	−1.041	168	−1.093	175	−1.076	190	−1.111

VARIATIONS IN LUNITIDAL INTERVALS

A glance down the fifth and sixth columns of Figures 19 and 20 shows that both the high-water and low-water lunitidal intervals vary from day to day. In part these variations may be brought about by disturbing effects of wind and weather; but, altogether apart from such disturbing effects, the lunitidal intervals vary periodically through the phase and declinational cycles. In general, during the phase cycle the intervals decrease from spring tides to neap tides and increase from neap tides to spring tides. During the declinational cycle, in general, the two high-water and also the two low-water lunitidal intervals will be approximately the same at the time of equatorial tides and differ most at the time of tropic tides.

In June, 1920, the moon's phases were as follows: Full on the 1st, third quarter on the 9th, new on the 16th, first quarter on the 22d. Corresponding to these phases both high and low water intervals in Figures 19 and 20 show decreasing values from the 1st to about the 8th, followed by a period of increasing values to the 13th, then a period of decreasing values to about the 20th, after which the intervals begin increasing until the 28th.

An inspection of Figures 19 and 20 shows that it is the high-water interval that exhibits the greatest difference between the two intervals of a day. The moon's declination during the month of June, 1920, was as follows: Farthest south on the 2d, on the Equator on the 9th, farthest north on the 15th, on the Equator again on the 22d, and farthest south on the 29th. Corresponding to these changes in declination we find that on the 2d, 15th, and 29th, when the moon was near the tropics, considerable difference existed between the two high-water intervals, while on the 9th and again on the 22d, when the moon was on the Equator, the two high-water intervals of the day show less variation.

In addition to the relatively short-period variations of the phase and declination cycles, lunitidal intervals also exhibit variations of a seasonal nature, especially at stations located some distance upstream on tidal waterways subject to seasonal variations in fresh-water discharge. At San Francisco, for example, the high-water lunitidal interval, on the average, has its greatest value in July or August and its least value in November, this difference, however, averaging but little over 0.1 hour. At Philadelphia, which is situated on the Delaware River, about 100 miles from the sea, the variation in fresh-water discharge is such as to give an average seasonal difference of 0.3 hour in the high-water interval, the greatest value coming in December and the least in April. Figure 21 gives in diagrammatic form the annual variation in the high and low water intervals at Philadelphia, derived from the monthly values for the 10-year period 1911–1920.

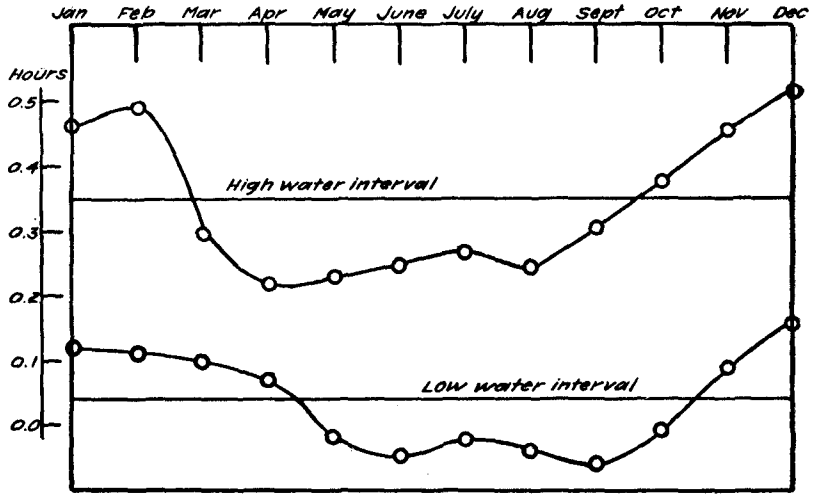

FIG. 21.—Annual variation in lunitidal intervals, Philadelphia, Pa.

The upper curve of Figure 21 shows the average variation in the monthly values of the high-water interval for the period 1911–1920, while the lower curve shows it for the low-water interval. The two horizontal lines through the middle of the curves represent, respectively, the mean high-water and the mean low-water intervals at Philadelphia as derived from the 10-year series, the scale in hours being shown to the left. It appears immediately that at Philadelphia the high-water interval is below its average value in spring and summer and above its average value in winter. The low-water intervals exhibit a somewhat similar annual variation, though it is to be noted that the curves for the two intervals are not exactly alike.

The variations in the intervals discussed thus far are average periodic variations. Superimposed upon these periodic variations are variations due to nonperiodic causes, such as unusual weather conditions. It follows, therefore, that to determine the lunitidal intervals at any given place directly from the tidal observations it is necessary to have these observations cover a period of at least one

month. For stations located on tidal waterways subject to considerable variation in level due to fresh-water discharge the observations should cover at least a year. Results derived from shorter series of observations may differ considerably from mean values, and such results must be corrected to mean values.

REDUCTION TO MEAN VALUES

The simplest and perhaps the most satisfactory method of reducing the results of short series of tidal observations to mean values consists in a comparison with the results of simultaneous observations at a near-by tide station at which mean values have been derived from a long series of observations. The method is simple in application, as the examples following will make clear. It is based on the fact that any departures from average values in the lunitidal intervals during a given period of time will be the same at all places subject to like weather conditions. This is obvious for places in the same body of water, but it is found true even for places some distance from each other during periods not disturbed by widely varying weather conditions.

As an example of the applicability of the method of comparison, the results from the tide stations at San Francisco and at La Jolla, Calif., will be taken. La Jolla is situated on the open coast more than 400 miles southeast from San Francisco. Assuming that the tide records for the first week in July, 1925, were at hand for both places, that the mean values of the lunitidal intervals at San Francisco were known from a long series of observations, and that it was desired to derive as accurate values as possible for La Jolla from that week's record, the procedure would be as follows:

The lunitidal intervals are derived from the week's records at both places as described previously. At La Jolla these intervals are, respectively, 9.46 hours for the high water and 3.23 hours for the low water, while for San Francisco they are 11.68 and 5.09 hours. But from a series of observations at San Francisco, covering a period of 20 years, the mean high-water interval has been determined as 11.67 hours and the low-water interval as 4.97 hours. Hence, to correct to mean values the lunitidal intervals at San Francisco derived from the observations during the first week in July, corrections of -0.01 and -0.12 hour, respectively, to the high and the low water intervals must be applied. Applying these same corrections to the intervals derived from the week's record at La Jolla, we have $9.46-0.01=9.45$ hours for the high-water interval and $3.23-0.12=3.11$ hours for the low-water interval.

For comparison it will be of interest to derive the mean intervals from another week of record. Taking the record for the second week of July, 1925, the high and low water lunitidal intervals at La Jolla were, respectively, 9.03 and 2.79 hours and at San Francisco 11.25 and 4.77 hours. The latter values, compared with the mean values of 11.67 and 4.97 hours, indicate corrections of $+0.42$ and $+0.20$ hour, respectively, to the high and low water intervals derived from the tide record for the second week in July. Applying these corrections to the intervals derived from the data for the second week in July at La Jolla, the corrected values are $9.03+0.42=9.45$ hours for

the high-water interval and 2.79+0.20=2.99 hours for the low-water interval.

Comparing with the results derived from the data for the first week in July, it is seen that intervals with values of 9.46 and 3.23 hours were corrected to 9.45 and 3.11 hours, while the values of 9.03 and 2.79 hours from the second week were corrected to 9.45 and 2.99 hours. The mean values of the lunitidal intervals at La Jolla derived by comparing a full year of observations with simultaneous observations at San Francisco give 9.41 hours for the high-water interval and 3.07 hours for the low-water interval. For each of the two weeks used in the above examples the mean values derived by comparison with San Francisco are correct within a tenth of an hour, even though the corrections in the first case are negative and in the second case positive.

The correction to mean values by the method of comparison is applicable to a series of any length. Care, however, must be taken to choose a station for comparison which has similar tides and which is not so far away or so situated that variations brought about by varying meteorological conditions will be different at the two places. It is also necessary to use the same number of tides at both places, even though this may make necessary the inclusion at one of the places the high or low water of another day.

In general, it may be assumed that the lunitidal intervals derived from a series one month in length, when corrected by comparison with simultaneous observations at some near-by station for which mean values based on a long series of observations are at hand, will be correct to the nearest tenth of an hour.

TIME DIFFERENCES FROM LUNITIDAL INTERVALS

Since in lunitidal intervals the time is referred to the instant of the moon's local meridian passage, the direct difference between the high-water or the low-water intervals at two places gives the difference in the time of tide at the two places only if they have the same longitude. In general, therefore, the direct difference between the lunitidal interval at two places does not give the difference in time of tide at these places. To derive from the intervals the differences in time of tide at different places, a correction must be applied for the difference in longitude. The formula for this is derived as follows:

If $L_{,}$ and $L_{,,}$ are the west longitudes in degrees of two places, $L_{,}$ being greater than $L_{,,}$, and $I_{,}$ and $I_{,,}$ the respective lunitidal intervals in hours, either for high water or low water, it follows that $I_{,}$ is referred to a time origin zero hours of which occurs as much later than at $I_{,,}$ as it takes the moon to traverse the distance represented by the difference between $L_{,}$ and $L_{,,}$. Since the average period of the moon's apparent revolution about the earth is 24.84 hours, the time required for the moon to travel 1° of longitude is $24.84 \div 360 = 0.069$ hour. Hence to make $I_{,}$ and $I_{,,}$ refer to the same zero hour the correction $0.069\ (L_{,}-L_{,,})$ must be added to $I_{,}$ (since this is the interval at the place having the greater west longitude) or subtracted from $I_{,,}$. The difference in the time of tide in hours at the two places is, therefore, given by $I_{,}-I_{,,}+0.069\ (L_{,}-L_{,,})$.

As an example, let it be required to determine the difference in time of tide at La Jolla as compared with San Francisco. For San Francisco HWI=11.67 hours and for La Jolla 9.41 hours; LWI=4.97 hours for San Francisco and 3.07 hours for La Jolla; longitude of La Jolla=117° 16′ (=117.27°), of San Francisco=122° 27′ (=122.45°). Therefore, in the formula above we have for the high-water time difference $11.67-9.41+0.069\ (122.45-117.27)=2.62$ hours, and for the low-water time difference $4.97-3.07+0.069\ (122.45-117.27)=2.26$ hours. High water at San Francisco is therefore 2.62 hours later than at La Jolla, and low water 2.26 hours later.

To make the above formula applicable when one or both places are in east longitude, it is only necessary to make L negative for the places having east longitude. Thus if $I_{,}$ is the interval for a place situated in 30° east longitude and $I_{,,}$ the interval for a place in 15° west longitude, the difference in time of tide at the two places is $I_{,}-I_{,,}+0.069\ (-30-15)=I_{,}-I_{,,}-0.069\times 45$. If both places are in east longitude, the difference is $I_{,}-I_{,,}+0.069\ (-30+15)=I_{,}-I_{,,}-0.069\times 15$.

It should be noted that time differences determined through lunitidal intervals are absolute differences of time. If time differences derived from lunitidal intervals are used for computing the times of tide at one station from the observed times at another, it will be necessary to take into consideration the standard time meridians at the two places and to make the necessary allowance if these time meridians differ.

DURATIONS OF RISE AND FALL

The lunitidal intervals at any given place permit the determination of the average periods of rise and fall of the tide at that place. The duration of rise is given by subtracting the low-water interval from the high-water interval and the duration of fall by subtracting the high-water interval from the low-water interval. In other words, in abbreviated form we have, Rise=HWI−LWI; Fall=LWI−HWI.

A tidal cycle involving a single rise and fall averages 12.42 hours. Hence in using the two formulas above, when the greater interval is to be subtracted from the lesser, 12.42 hours is to be added to the latter before subtraction. Thus, since for San Francisco HWI=11.67 hours and LWI=4.97 hours, the average duration of rise is given by $11.67-4.97=6.70$ hours and the average duration of fall by $4.97+12.42-11.67=5.72$ hours.

VI. MEAN SEA LEVEL

DEFINITION

Mean sea level is the primary tidal datum plane to which all other tidal datum planes are referred. As a datum plane, mean sea level at any point may be defined as the mean level of the sea at that point; and since mean sea level is determined through tidal observations, it may also be defined as the plane about which the tide oscillates.

Strictly, mean sea level should be determined by integrating the tidal curve. It is much more convenient, however, to derive mean sea level as the average of the tabulated hourly heights of the tide.

For a very short period of observations the difference between the two determinations may be relatively large, but for a series covering a month or more the difference, if any, would be insignificant. The hourly heights of the tide are generally tabulated to the nearest tenth of a foot, and the mean sea level derived therefrom is taken to the nearest hundredth of a foot for series up to a year in length.

Mean sea level is generally assumed to constitute an equipotential surface; but it is to be noted that, as derived from tidal observations, mean sea level must deviate somewhat from a theoretical equipotential surface in consequence of the net or resultant effects of various agencies, as, for example, variations in barometric pressure and in air movements. As a first approximation, however, mean sea level as derived from tidal observations may, for most purposes, be regarded as constituting an equipotential surface.

Within coastal bodies of water draining large areas subject to considerable fresh-water run-off the mean level of the sea obviously tends to stand somewhat higher than along an open coast. In tidal rivers in which variations in the fresh-water run-off cause relatively large fluctuations in level it is sometimes preferable to speak of mean river level rather than mean sea level, though this mean river level is determined in precisely the same manner as mean sea level, namely, by averaging the hourly heights of the tide.

The fact that mean sea level determined from tidal observations at different places does not constitute an equipotential surface may bring about discrepancies between mean sea level determined from tidal observations and that determined by geodetic leveling from some distant point. Thus, recent geodetic leveling between New York City and Portland, Me., brings out the fact that mean sea level determined from tidal observations at Portland is about half a foot higher than that determined from tidal observations in New York Harbor. To distinguish the two planes of mean sea level, that determined by means of geodetic leveling may be called geodetic mean sea level while that determined locally through tidal observations may be called local or geographic mean sea level.

It is convenient at times to use the expressions daily, weekly, monthly, and yearly sea level. These terms denote, respectively, the sea level derived by averaging the hourly heights of the tide for the period of a day, week, month, and year. With respect to weekly, monthly, or yearly sea level no ambiguity arises, but with respect to daily sea level it is necessary to define precisely how it is determined from the hourly heights of the day, for this determination is possible in three different ways.

If the hourly heights of the tide for any given day are denoted by $h_0, h_1, h_2, \ldots h_{23}, h_{24}$, in which h_0 is the height at midnight beginning the day and h_{24} the height at midnight ending the day, then, strictly, sea level for the day is given by $\frac{1}{24}(\frac{1}{2}h_0+h_1+h_2+h_3+\ldots+h_{22}+h_{23}+\frac{1}{2}h_{24})$. It is much simpler, however, to sum the hourly heights as tabulated; furthermore, no useful purpose is served by the refinement of the first and last terms of the formula. Hence daily sea level is frequently taken as $\frac{1}{25}(h_0+h_1+\ldots+h_{23}+h_{24})$. But, as shown in Figure 12, the hourly heights of the tide are tabulated with the 23d hour of the day as the last hour. It is therefore more convenient to derive daily sea level as $\frac{1}{24}$

$(h_0+h_1+h_2+ \quad . \quad . \quad . \quad +h_{22}+h_{23})$. Throughout this publication, unless otherwise specifically stated, daily sea level will be derived in accordance with the last formula.

It is to be observed that, excepting regions in which the range of the tide is very large, there is very little difference in the values of daily sea level determined by the three formulas. Thus, taking the first day shown in Figure 12 (June 10, 1920), the first formula gives as the value of sea level on the tide staff at San Francisco 8.54 feet, the second formula makes it 8.48 feet, and the third formula makes it 8.55 feet. As will be seen later, sea level derived from one day of observation may differ from mean sea level by several feet. In such cases a difference of a few hundredths of a foot is negligible, and hence, for most purposes, the third formula is the most convenient one.

HALF-TIDE LEVEL

Mean sea level must be carefully distinguished from half-tide level or, as it is frequently called, mean tide level. Half-tide level is the plane that lies exactly midway between the planes of mean high water and mean low water and is determined by averaging the heights of the high and low waters.

If the curve representing the rise and fall of the tide were that of a simple sine curve, the planes of mean sea level and of half-tide level would coincide. But the tide curve is not a simple sine curve; it is compounded of a number of simple sine curves, some of which have fixed phase relations with respect to each other. The average rise of high water above mean sea level is, therefore, generally not exactly the same as the average fall of low water below mean sea level, and hence mean sea level and half-tide level generally differ.

It will be more convenient to take up in detail the plane of half-tide level after the discussion of the plane of mean sea level. Here it will be sufficient to call attention to the fact that at any point on the open coast the planes of mean sea level and of half-tide level generally differ only by small quantities, and that over periods of a year or more the differences between these two planes are very nearly constant.

VARIATIONS IN SEA LEVEL

If the level of the sea were to fluctuate only in response to daily and semidaily tide-producing forces of unvarying periods, then mean sea level could be determined from one day of tidal observations. Averaging the hourly heights of the tide through one day would eliminate the effect of the tide, the resulting average height being the height of mean sea level. But the tide-producing forces to which the sea responds include, besides those of daily and semidaily periods, also those with periods of half a month or more. Daily sea level therefore varies from one day to another in consequence of these so-called long-period tides.

It can be shown that the variations in sea level from day to day resulting from the long-period tidal forces are relatively small. Far greater variations are brought about by the response of the waters to changes in wind and weather. It is a matter of common knowledge that a wind blowing toward the shore tends to raise the

level of the sea along the shore, while a wind blowing from the shore tends to lower it.

Variations in barometric pressure likewise bring about fluctuations in sea level. Indeed, as a first approximation, any arm of the sea may be regarded as constituting a huge inverted water barometer. When the barometric pressure over this arm of the sea rises, the level of the water will be lowered, while with a decrease in barometric pressure the level of the water will rise.

DAILY SEA LEVEL

Wind and weather vary from day to day; this, together with the variation due to the long-period tides, brings about variations in the height of sea level from day to day. Figure 22 shows in diagrammatic form the changes in sea level from day to day at Fort Hamilton in New York Harbor during the month of June, 1919, when weather conditions were relatively uniform. For each day sea level was determined as the average of the 24 hourly heights of the tide, from 0 hour to 23 hours, inclusive.

The changes in sea level from day to day in Figure 22 are seen to vary from less than 0.1 foot to more than 0.5 foot. For the month in question the difference between the highest and lowest daily sea level is 0.9 foot. Obviously, during periods of heavy winds or of considerable changes in barometric pressure, the variation in sea level from day to day will be greater than during periods of relatively uniform weather. Figure 23 shows the changes in daily sea level at Fort Hamilton during February, 1919, four months earlier than the period represented in Figure 22.

On the North Atlantic coast of the United States February is generally a month with considerable variation in wind and weather. Consequently, Figure 23 shows greater variation in daily sea level than Figure 22. As against a difference between the highest and lowest daily sea level of 0.9 foot in June, this difference in February is 2.4 feet. A rapid change in sea level is seen to have taken place between the 14th and 17th of February, a fall of somewhat more than 2 feet occurring between these dates. An examination of the weather record for these days discloses marked changes in the velocity and direction of the wind. On the 13th and 14th the wind was easterly, with velocities up to 35 miles per hour, but on the 15th the wind shifted to the northwest, the force increasing steadily until the 16th, when it attained a velocity of 62 miles per hour.

The change in sea level from day to day depends primarily on variations in meteorological conditions, hence such changes are not periodic; that is, from one day to the next sea level may be either higher or lower, depending on the weather. But, as will be shown in the discussion of monthly sea level, there is a seasonal variation in sea level or, more precisely, an annual variation. Thus, in New York Harbor sea level is, on the average, lowest during the first months of the year and highest late in summer or early in fall. Since daily sea level may differ by as much as 2½ feet during a single month, as shown in Figure 23, it follows that within a year daily sea level may differ by even greater quantities. During 1919 sea level for January 11 was 2.1 feet below the mean sea level for

the year, and on November 8 it was 1.6 feet above. For these days. therefore, the difference in sea level is 3.7 feet.

Regions subject to storms of great intensity, especially those fronting shallow bodies of water, exhibit much greater variation in daily sea level than those found in New York Harbor. For example, while in New York Harbor in 1919 the difference between the highest and lowest values of daily sea level was 3.7 feet, at Galveston, which faces the Gulf of Mexico on the coast of Texas, in the same year the difference was about 5 feet, and during periods of exceptional storms it is much more.

In tidal streams in which there is considerable variation of fresh-water discharge daily sea level exhibits greater variation than along an open coast, and this becomes especially marked in the upper reaches of the streams. Thus, in 1914 the highest and lowest values of daily sea level at the places mentioned below differed by the following amounts: Open coast near the entrance to New York Harbor, 5.4 feet; Fort Hamilton, N. Y., the entrance to New York Harbor, 6 feet; Albany, near the head of tidewater on the Hudson River, 15.2 feet. These figures may be taken as representative of the sea-level

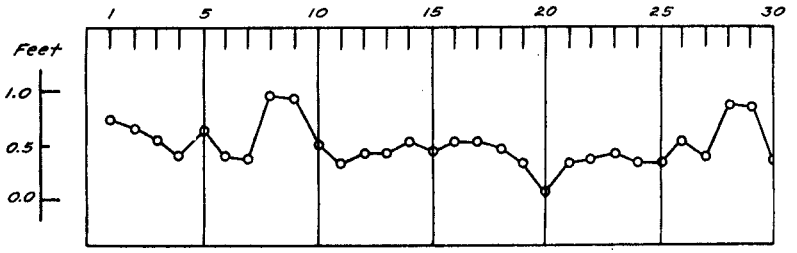

FIG. 22.—Daily sea level, Fort Hamilton, N. Y., June, 1919

changes within a long tidal river subject to considerable differences in fresh-water discharge and subject also to freshets.

The dates of occurrence of the respective highest and lowest daily sea levels at the three places mentioned above immediately give a clue to the agencies responsible. In the year in question (1914) the lowest sea level occurred on January 13 at all three places. On that day, and also the day previous, heavy northwesterly winds had been blowing, with velocities up to 76 miles per hour. This tended to drive the water out of the river and harbor and away from the coast. The highest sea level in 1914 at Fort Hamilton and on the open coast occurred on December 7, but at Albany this occurred on April 22. On December 7, and the day before, winds with velocities up to 60 miles per hour from the northeast prevailed. And even at Albany this resulted in a height of sea level above the average. But here it is the fresh-water flow in the upper reaches of the Hudson and its tributaries that is the principal factor in the variation of sea level, so that the highest sea level is reached early in spring, during freshets.

The very large variations in sea level, or more accurately in river level, in the upper reaches of tidal streams which are subject to fluctuations in fresh-water run-off introduce troublesome questions in connection with the determination of tidal datum planes. Mani-

festly, where daily river level may vary as much as 15 feet within a year, the plane of mean river level or any other tidal datum plane determined from short series of observations is subject to relatively large errors.

It is obvious that changes in sea level from day to day must, in general, be much the same at points near each other and which are subject to similar meteorological conditions. As will be seen later, advantage is taken of this fact in determining the plane of mean sea level from short series of observations by correcting the sea level derived from these observations to a mean value. Just how far two points may be separated and still exhibit similar sea level changes depends on a number of factors. Within a long tidal river subject to considerable variation in fresh-water run-off the changes in daily river level may be quite different for points relatively near each other. But on the open coast and in tidal waters not subject to large variations in fresh-water discharge the changes in daily sea level resemble each other closely over areas of considerable extent.

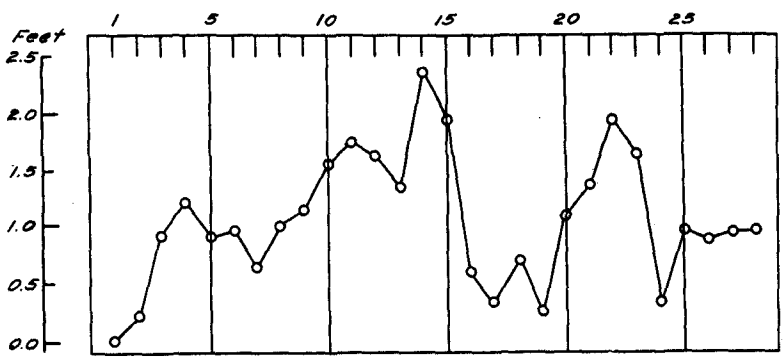

FIG. 23.—Daily sea level, Fort Hamilton, N. Y., February, 1919

Figure 24 represents the daily heights of sea level for the month of April, 1923, at five stations on the Atlantic coast from Portland, Me., to Fernandina, Fla. A glance shows that at Portland and Boston the sea-level changes resemble each other closely. At Atlantic City these changes are decidedly different from those of Portland and Boston, and, while the curves for Charleston and Fernandina resemble each other, they differ from the preceding curves.

Examining the locations of these stations it is found that, though Boston lies about 100 miles south of Portland, both harbors lie open to the same arm of the sea known as the Gulf of Maine. Atlantic City is about 250 miles south of Boston but lies in a different embayment of the coast; this, together with differences in meteorological conditions, makes the daily sea-level changes at the two places show but little similarity. Charleston and Fernandina are almost exactly the same distance from each other as Atlantic City and Boston, but, lying in the same embayment of the coast, they exhibit similar changes in sea level from day to day.

There appears to be an impression that sea-level changes are a function of the range of tide. That is, it is assumed that at stations where the range of tide is large, greater variations in sea level are

to be expected than at stations having a small range of tide. That there is no basis for this impression, Figure 24 brings out as regards daily sea level. The average ranges of the tide at the five stations are as follows: Portland, 8.9 feet; Boston, 9.4 feet; Atlantic City, 4 feet; Charleston, 5.2 feet; and Fernandina, 6 feet. At Boston, therefore, the range of tide is very nearly two and one-half times as large

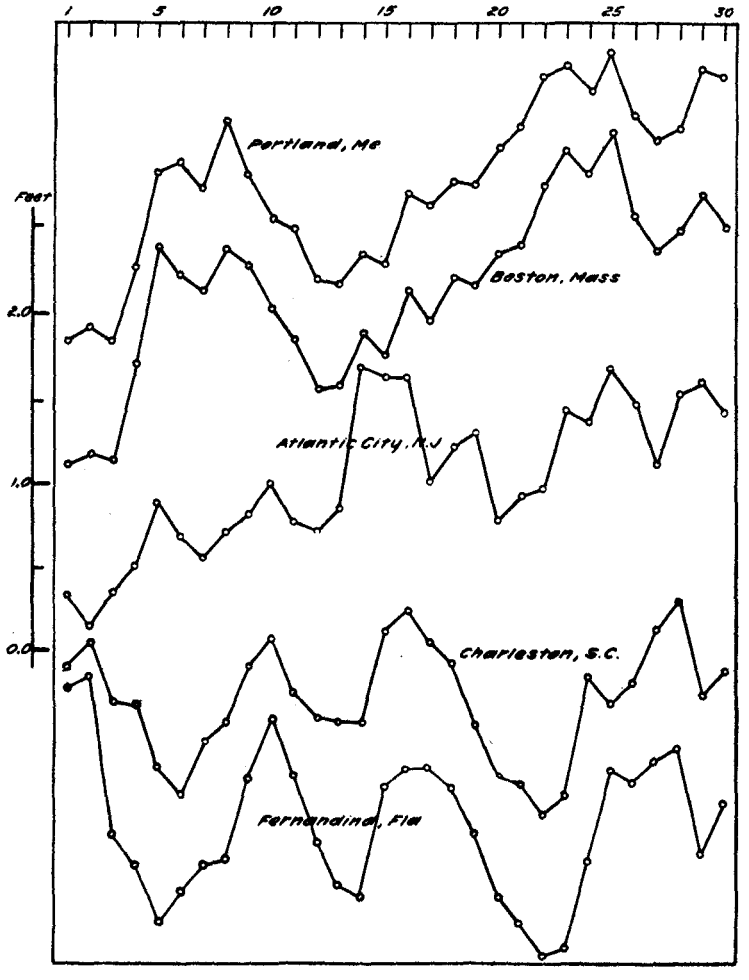

FIG. 24.—Daily sea level at five stations on the Atlantic coast, April, 1923

as at Atlantic City; nevertheless the variations in sea level at the two places for the month shown are very nearly the same.

MONTHLY SEA LEVEL

It is obvious that sea level determined for periods of a week will show smaller variations than daily sea level. There is, however, no need of discussing such variations in detail, since they will lie between the daily variations and the monthly variations. Within a

month the larger fluctuations exhibited by daily sea level will tend to balance out, so that monthly sea level shows much less variation than daily sea level. For example, as noted in the discussion of daily sea level, in New York Harbor during the year 1914 the difference between two daily values of sea level, one in January and the other in December, differed by 6 feet; during this same year the difference between the highest and lowest monthly values of sea level differed by but 0.74 foot. In Figure 25 are shown in diagrammatic form the

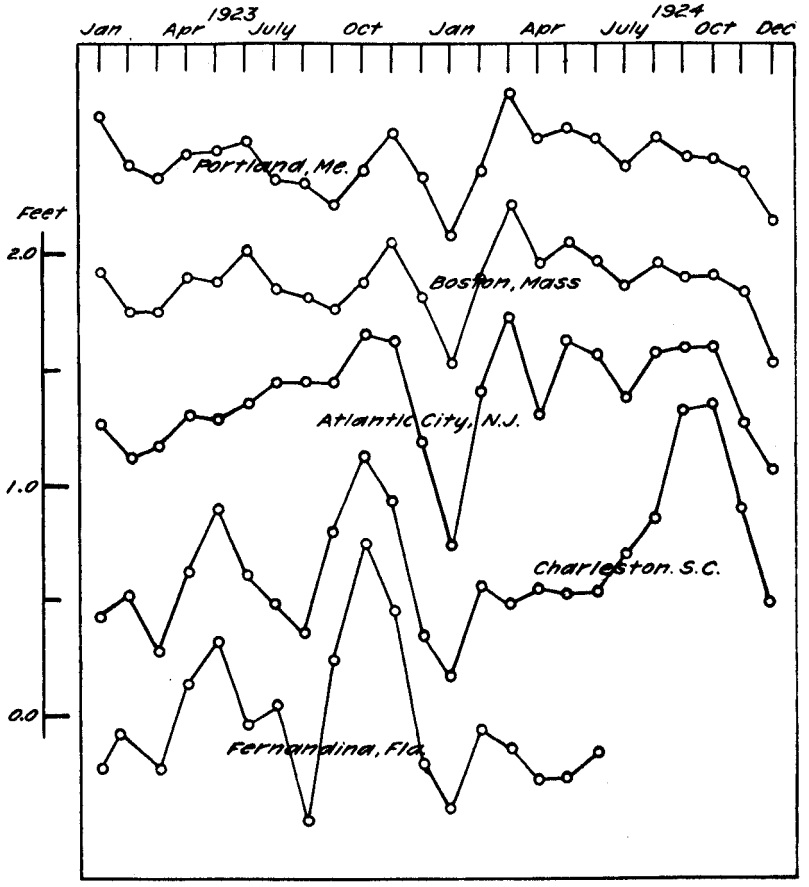

FIG. 25.—Monthly sea level at five Atlantic coast stations, 1923–24

variations in height of sea level from month to month for the years 1923 and 1924 at the five stations for which daily sea-level changes are given in Figure 24. For Fernandina the record stops at the end of June, 1924, the station at that time being discontinued.

In discussing changes in sea level from day to day attention was directed to the fact that such changes are in no way related to the range of tide. Figure 25 emphasizes this fact with regard to monthly sea level. Atlantic City with a mean range of tide of 4 feet, Charleston with a range of 5.2 feet, and Fernandina with a range

of 6 feet show much greater changes in monthly sea level than do Boston and Portland with ranges of 9.4 and 8.9 feet, respectively.

For daily sea level the variations at Atlantic City were found quite different from those at Boston and Portland. But for monthly sea level the curves of Figure 25 show similar changes at the three stations with respect to the larger variations. Indeed, with respect to these larger variations there is considerable resemblance in all the curves of Figure 25. Furthermore, they all give evidence of a seasonal or, more acccurately, of an annual variation in sea level with a minimum in the winter months and a maximum in the late summer or early fall months. Differences in wind and weather from one year to another mask somewhat the periodic annual variation in sea level. But, if the monthly heights are averaged over a number of years, the irregularities tend to balance out.

ANNUAL VARIATION

The six curves of Figure 26 represent, as indicated, the annual variation in sea level at Fort Hamilton in New York Harbor as derived from monthly heights. The five upper curves give the annual variation for each of the consecutive years 1916–1920, while the lowest curve is the mean curve of annual variation derived by averaging the corresponding monthly heights of the five-year period. For any one of the individual years shown in Figure 26 there are seen to be irregularities in the change of sea level from month to month; there appears, however, at the same time a large element of periodicity. Not only in the curve representing the average monthly heights during the five-year period, but also in the individual years, it is seen that monthly sea level is generally lowest in the winter months and highest in the summer months. And this is a characteristic feature of sea level in New York Harbor.

The annual variation of sea level at any point is characteristic for a considerable area in its vicinity, but from this statement must be excluded the upper reaches of tidal streams subject to large fluctuations in fresh-water flow. Thus, the annual variation in sea level is very much the same at Atlantic City, N. J., as at Fort Hamilton, although the two places are nearly 100 miles apart. But at Albany, N. Y., which is not much farther from Fort Hamilton than is Atlantic City, the annual variation in sea level is very considerably different from that at Fort Hamilton, as Figures 26 and 27 show, despite the fact that Fort Hamilton and Albany lie on the same tidal waterway.

For Albany there are at hand tide observations covering the years 1910, 1911, 1914, and 1920, the monthly heights of sea level or, more properly, of river level for each of these years being shown by the four upper curves of Figure 27, while the lowest curve gives the average monthly height as derived by averaging the height of river level for the corresponding months of 1910, 1911, 1914, and 1920. It will be noted that the height scale to which Figure 27 is plotted is but one-fourth that of Figure 26. This was necessary to bring Figure 27 within reasonable compass. It is at once apparent, therefore, that the change in sea level from month to month is very much greater at Albany than at Fort Hamilton, near the open sea.

For 1920 Figure 26 shows a difference between the lowest and highest values of monthly sea level of 0.9 foot at Fort Hamilton; Figure 27 for the same year at Albany shows a difference of 5.5 feet. For 1914 the contrast is even more striking; Figure 27 shows a difference in monthly height of river level at Albany of 7.8 feet as between the months of April and November, 1914. In that same year the tide record at Fort Hamilton shows the greatest difference in the monthly heights of sea level to have been 0.7 foot, the highest monthly sea level coming in October and the lowest in February.

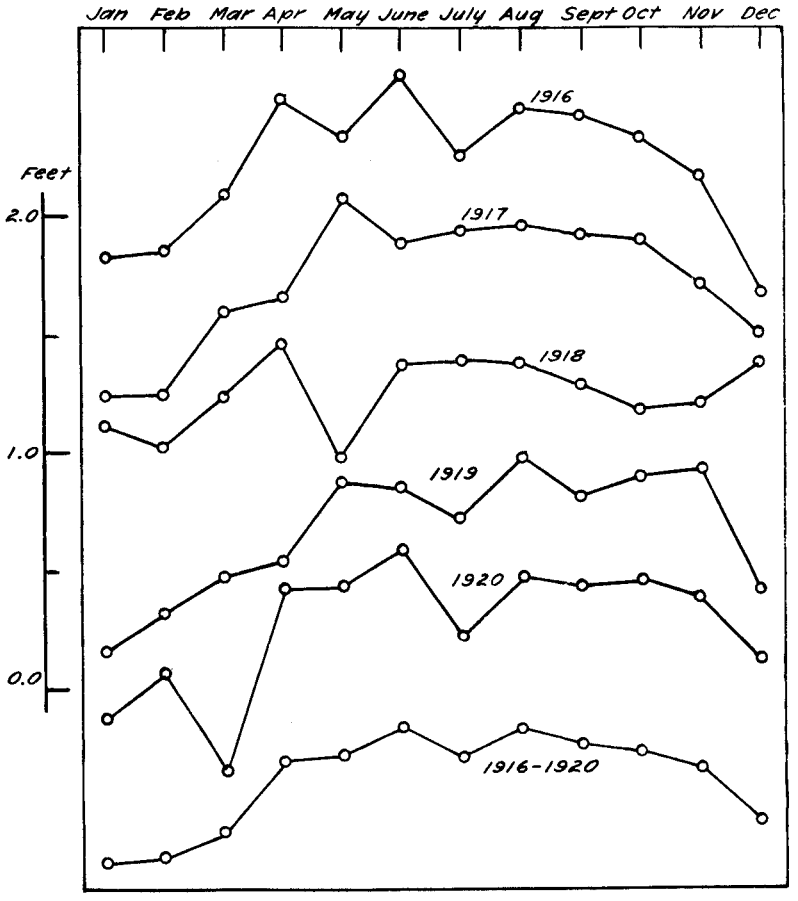

FIG. 26.—Annual variation in sea level, Fort Hamilton, N. Y., 1916–1920

Another striking difference between the curves of annual variation of sea level at Fort Hamilton and at Albany is that in phase. Figures 26 and 27 show that at Fort Hamilton the monthly height of sea level is highest in the summer months and lowest in the winter months, while at Albany the river level is highest in the spring months and lowest in the summer months. The influence of the fresh-water flow is evident in the curves of Figure 27, the sudden rise during the month of April coinciding with freshet conditions in the river and the low level during the summer months correspond-

ing with the minimum of the run-off which occurs during these months.

Atlantic coast.—The characteristics of the annual variation in sea level on the Atlantic coast of the United States for a number of stations from Maine to Florida are shown in Figure 28. Comparing the curves of this figure with those in Figures 25 and 26, it is evident

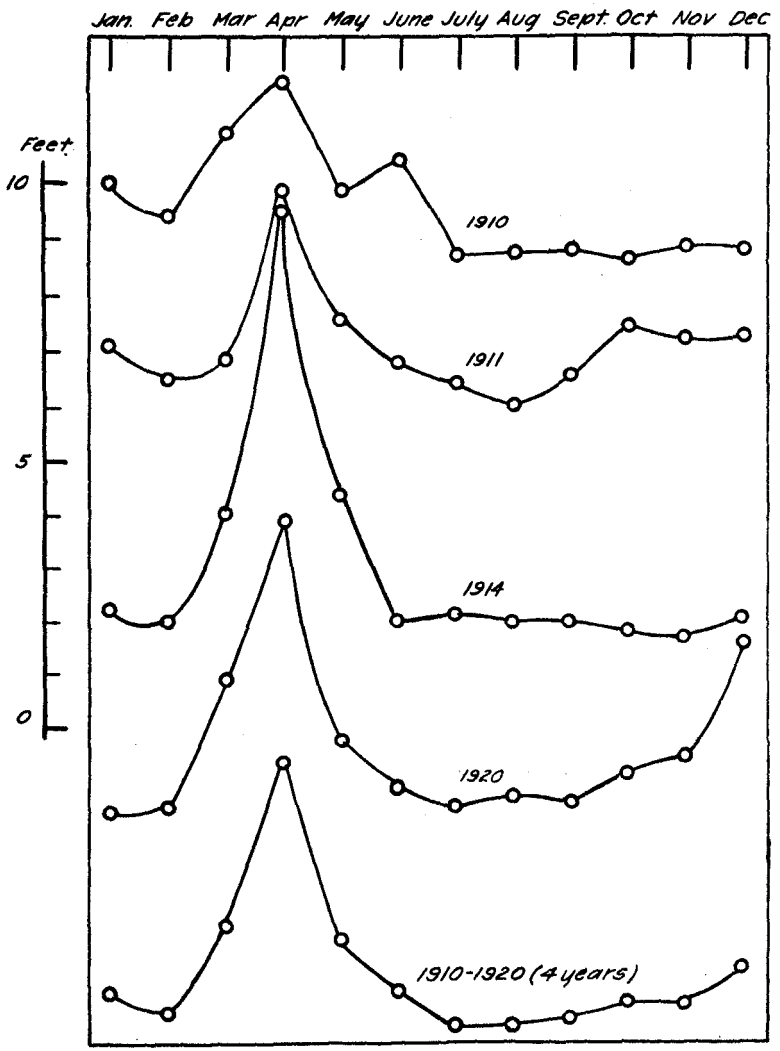

FIG. 27.—Annual variation in sea level, Albany, N. Y.

that the nonperiodic variations of monthly sea level are of greater magnitude than the strictly periodic. Notwithstanding this, a comparison of the curve of annual variation at Fort Hamilton, as derived from five years of observations (lowest curve, fig. 26), with the curve in Figure 28 derived from 28 years, shows that the characteristics of the periodic constituent in the annual variation of sea level

may be determined with a considerable degree of precision from five years of observations.

The horizontal lines in Figure 28 represent, respectively, mean sea level at each of the stations based on the years of observations

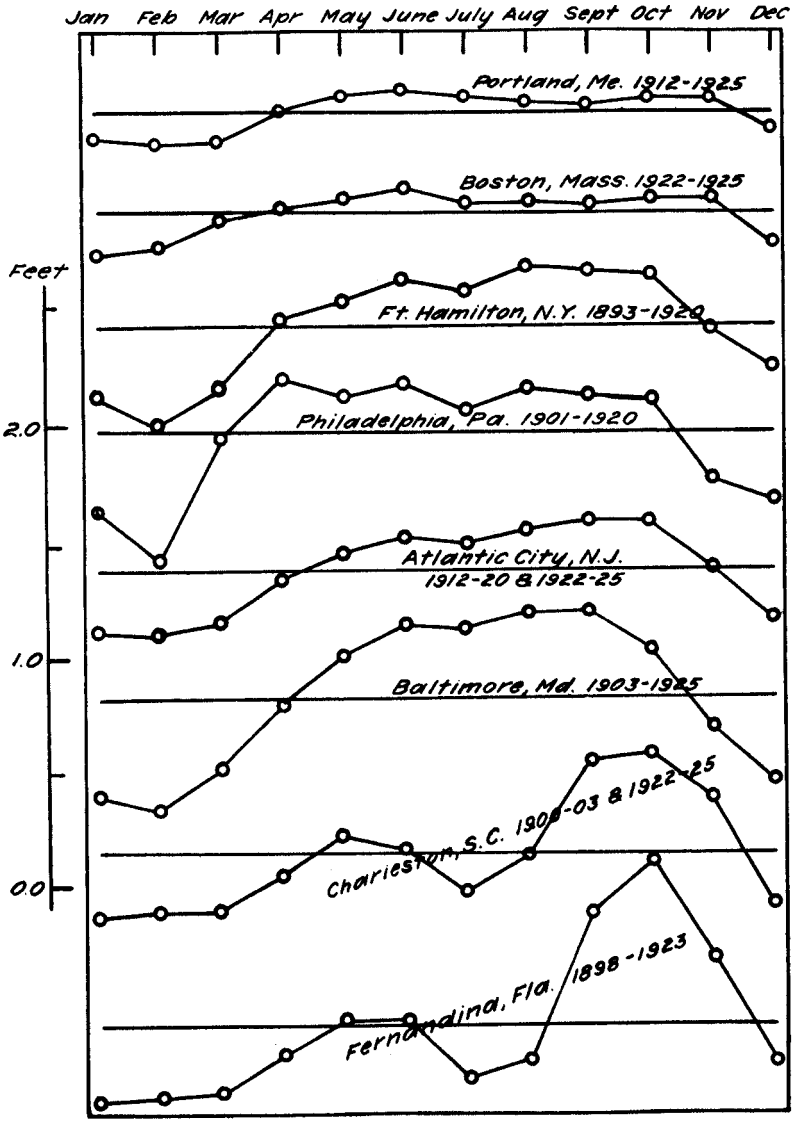

FIG. 28.—Annual variation in sea level, Atlantic coast

following the name of the station. In general, it is seen that along the Atlantic coast of the United States sea level is below its mean level in the winter and early spring months and above its mean level in the summer and fall months, though exception must be made of the southern embayment of the coast from North Carolina to Flor-

ida. Here during the months of July and August sea level is below its mean level.

Figure 28 shows that the minimum level all along the Atlantic coast is reached between January and March, with the maximum coming between August and September. The curve for Philadelphia stands out somewhat from the other curves, the highest level being

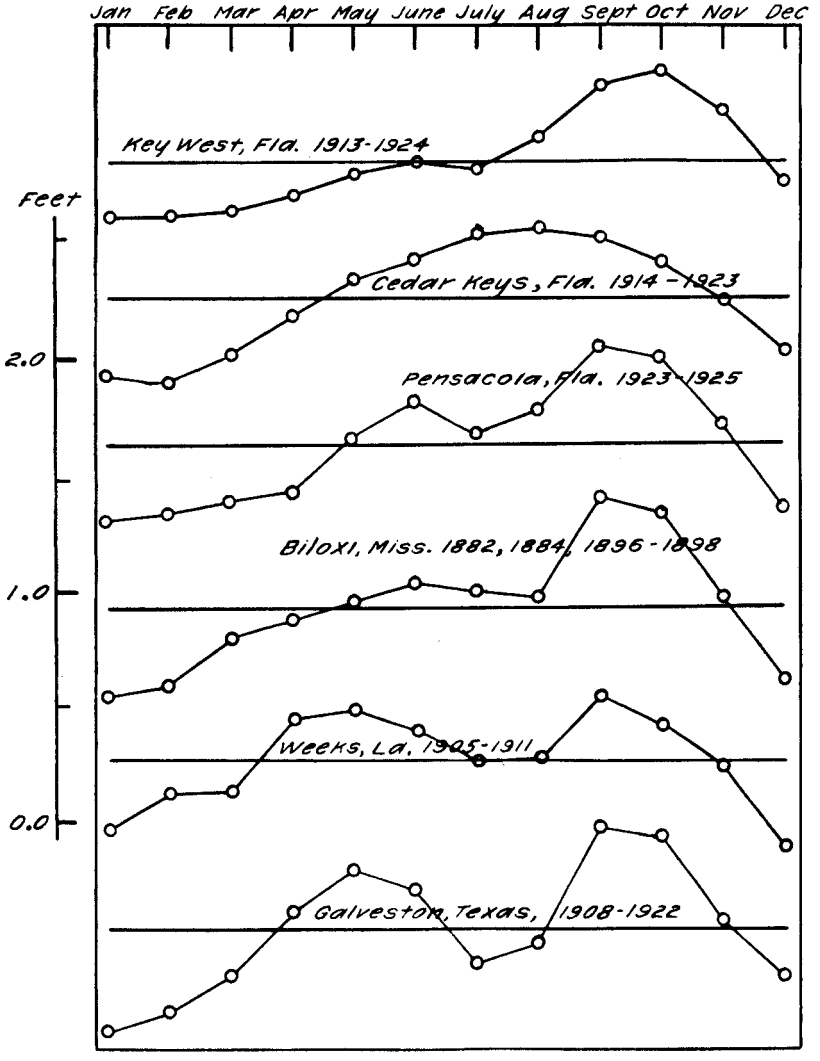

FIG. 29.—Annual variation in sea level, Gulf coast

reached in April. This, however, is due to the freshet conditions obtaining in the upper reaches of the Delaware and tributaries during that month. In general, too, it may be said that the range of the annual variation in sea level increases from north to south. At Portland it is just over 0.2 foot; at Fort Hamilton it is 0.6 foot; Baltimore, 0.8 foot; Fernandina, 1 foot. Philadelphia can not be

considered as invalidating this general conclusion, because there the annual variation is dependent to a large extent on the fresh-water flow.

Gulf coast.—In the Gulf of Mexico the characteristics of the annual variation in sea level differ somewhat from those on the Atlantic coast. In Figure 29 are shown the curves of annual variation at six stations from Key West, Fla., to Galveston, Tex. Here, too, the figures following the names of the stations give the years of observations on which the curves are based, and the horizontal lines represent, respectively, the mean sea level derived for each of the stations from those years of observations.

From Figure 29 it follows that along the Gulf coast of the United

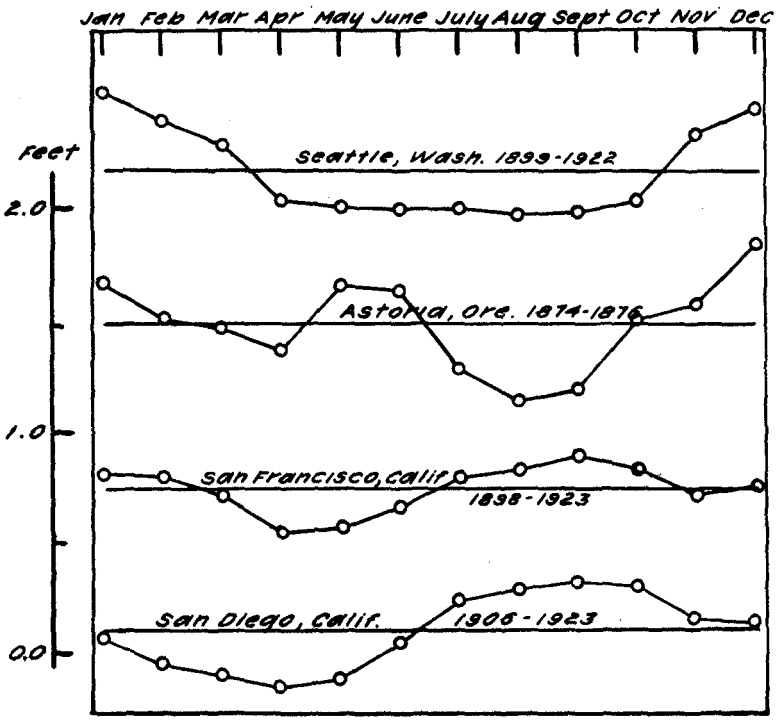

FIG. 30.—Annual variation in sea level, Pacific coast

States, as on the Atlantic coast, sea level is lowest in January or February and highest in September or October. A secondary minimum in midsummer, which is apparent at Key West, becomes well developed toward the western end of the Gulf. At Cedar Keys, however, this secondary minimum is missing. In fact, the curve of annual variation for Cedar Keys differs considerably from the other curves in Figure 29. For all six stations shown the range of the annual variation is approximately the same, being about three-quarters of a foot.

Pacific coast.—For the Pacific coast of the United States Figure 30 gives the curves of annual variation at four stations from San Diego, Calif., to Seattle, Wash. For the southern portion of this

coast, as exemplified by the curves for San Diego and San Francisco, the annual variation is much the same, with a minimum in April and a maximum in September, the range being slightly less than half a foot. For the northern stretch Seattle may be taken as representative. There sea level is below its mean level from April to October and above for the rest of the year.

The annual variation at Astoria is not so well determined as at the other stations shown in Figure 30, since it is based on but three years of observations. There is also present another factor, arising from the fact that Astoria is situated on the fresh-water harbor at the mouth of the Columbia River and is thus subject to the seasonal variation in the fresh-water run-off from the large territory that drains into the Columbia. The maximum in May and June

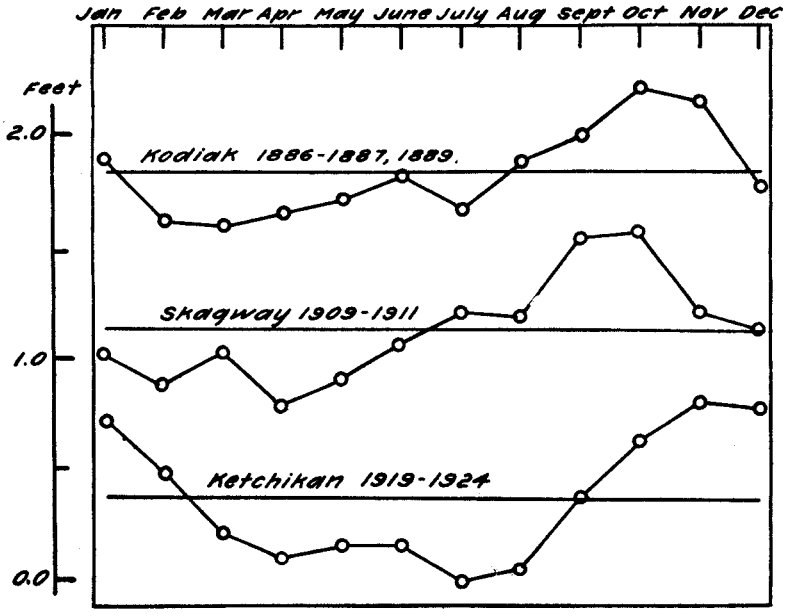

FIG. 31.—Annual variation in sea level, Gulf of Alaska

and the minimum in August are due primarily to the effect of the fresh-water flow..

All along the Pacific coast of the United States the range of the annual variation in sea level is practically the same, being about half a foot, but the phase of the variation is different. Along the Atlantic coast the phase was found much the same, but the range was found to vary, increasing from north to south.

Alaska.—For the coast of Alaska fronting the Gulf of Alaska there are three stations at which there are observations covering a period of three years or more, which permits a determination of the annual variation in sea level. These stations are Ketchikan, on Tongass Narrows; Skagway, near the head of Lynn Canal; and Kodiak, on Kodiak Island. The curves of annual variation at these stations are shown in Figure 31.

The curve for Ketchikan, being based on six years of observations, is regular in outline, but the curves for Skagway and Kodiak, being

based on but three years, are somewhat irregular. In general, it appears that along this coast sea level is below its mean level during the spring and summer months and above mean level in the fall months. The maximum appears to come in October or November and the minimum some time between March and July. The range of the annual variation here is about three-quarters of a foot.

YEARLY SEA LEVEL

In the previous section monthly sea level was found to be subject to an annual variation with a range up to a foot. This means that, due to this cause alone, mean sea level determined directly from one month of observations may be in error by half a foot. This error may be further augmented very considerably by the nonperiodic variation from month to month arising from variations in wind and weather. Within a year, however, the annual variation balances out, and it now remains to consider whether there are any variations in sea level from year to year.

Atlantic coast.—In Figure 32 are plotted the heights of sea level as derived for each of a number of years at six stations on the Atlantic coast. The horizontal lines associated with the curves represent for each station the mean sea level for the period of observations at the station. It appears at once that sea level does vary from year to year, although it is to be noted that this variation is considerably less than the variation from month to month. For example, Figure 25 shows that sea level in February, 1924, at Atlantic City was 0.7 foot higher than the preceding month. Figure 32 shows that at Atlantic City the greatest difference in yearly sea level for two consecutive years, from 1912 to 1925, was 0.14 foot.

The difference in sea level from year to year at any point along the Atlantic coast is generally less than a tenth of a foot. Occasionally, however, it may be as much as a quarter of a foot or even more, as exemplified by the variation between the years 1900 and 1901 at Fort Hamilton and between 1918 and 1919 at Fernandina. It is to be noted, too, that notwithstanding the fact that Philadelphia is situated about 100 miles from the sea on a tidal river subject to considerable variation in fresh-water flow the variations in sea level from year to year are of about the same magnitude as at the stations which are situated on the coast.

Figure 32 shows that the principal changes in sea level from year to year on the Atlantic coast are of a cyclic and not of a progressive character. For several years sea level will be rising, and for the period of several years following sea level will be falling. Unfortunately the observations at no station have been continued for a sufficient number of years to determine accurately the periods of these cycles. In a general way the existence of a cycle with a period of four to five years is indicated. There appears also to be a cycle with a period of eight to nine years.

What stands out strikingly in Figure 32 is the fact that sea level changes from year to year in much the same way all along the Atlantic coast. It will be recalled that for daily sea level the area in which like sea-level changes took place was relatively restricted.

For the change in sea level from month to month the area was considerably larger, and from year to year it appears to be much the same all the way from Maine to Florida. At Philadelphia and at Atlantic City repairs to the wharves on which the tide gauges were located necessitated the interruption of the observations during the

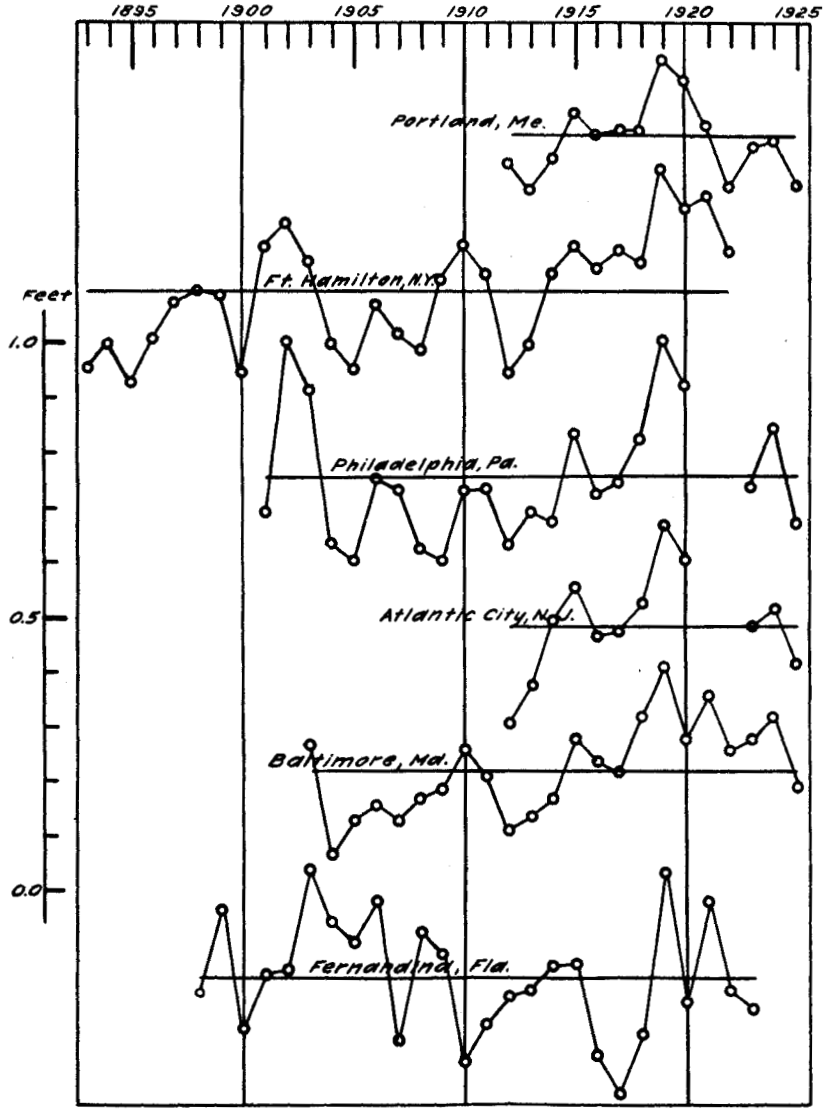

FIG. 32.—Yearly sea level, Atlantic coast

years 1921 and 1922; but, when the observations were resumed, the yearly sea levels gave values which fit in with the observations at the other stations.

Generally, when sea level during any one year is high (or low) at any of the stations shown in Figure 32, it is also high (or low)

at the other stations. Thus, in 1900 sea level was in a low-level phase at Fort Hamilton, being about 0.15 foot below its mean level. At Fernandina, very nearly a thousand miles away, sea level likewise is seen to have been in a low-level phase, and the record shows it to have been about 0.1 foot below its mean value. Similarly, 1919 appears to have been a year of unusually high sea level on the Atlantic coast, and from Portland to Fernandina Figure 32 shows the yearly sea level to have been higher than for any of the years following or for a number of years preceding.

While the changes in sea level from year to year along the Atlantic coast are generally similar, it must be emphasized that they are not exactly the same. But it is to be noted that, when the changes at any two stations are different, these changes are of small magnitude, rarely as much as 0.1 foot. In this connection attention is to be directed to the curve for Fernandina for the period from 1909 to 1912, during which time the changes in sea level differ from those at the other stations. During this period, however, difficulties were encountered in the operation of this tidal station, and the results for this period are not as satisfactory as at the other stations.

Gulf coast.—Along the Gulf coast of the United States there are three stations at which tide observations have been continued for a number of consecutive years. Various difficulties, however, were experienced in the operation of these stations, especially during the earlier years. These difficulties conspire to make the sea-level results for certain years not as free from question as might be desired. In Figure 33 the yearly heights at these three Gulf-coast stations are shown in graphic form. As in the previous figures, the horizontal lines represent, respectively, for each station the mean sea level derived from the years of observations shown.

At none of these stations have the observations been continued for so long a period as on the Atlantic coast; but here, too, the cyclic character of the variations in yearly sea level is clearly shown. The periods of the cycles are not as apparent as in the variation of yearly sea level on the Atlantic coast. In general, however, it appears that there is a cycle with a period of something like four to five years. The Galveston observations also indicate the existence of a cycle with a period of about nine years.

In a general way it may be said that there is considerable similarity in the changes of sea level from year to year at the stations along the Gulf coast, though this similarity is not so close as on the Atlantic coast. This means, in other words, that along the Gulf coast like changes in yearly sea level are confined to smaller areas than on the Atlantic coast. This is undoubtedly to be ascribed to the shallower depths along the Gulf coast and to the greater frequency of local tropical storms. For Key West and Cedar Keys the changes in sea level from year to year are of about the same magnitude as on the Atlantic coast. At Galveston, however, these changes are somewhat greater.

Pacific coast.—For illustrating the changes in sea level along the Pacific coast of continental United States there are at hand observations covering periods of 20 years or more at San Diego, San Francisco, and Seattle. These yearly heights of sea level are shown in Figure 34, the horizontal lines, as before, representing, respectively,

mean sea level at each of the stations, derived from the years of observations plotted for each station.

The changes in sea level from year to year on the Pacific coast are of about the same magnitude as on the Atlantic and Gulf coasts, being only infrequently greater than 0.1 foot. A variation, with a period of between two and three years, is indicated by Figure 34, and a secondary variation, with a period of about nine years, is likewise indicated. A comparison of the curves shows that differences

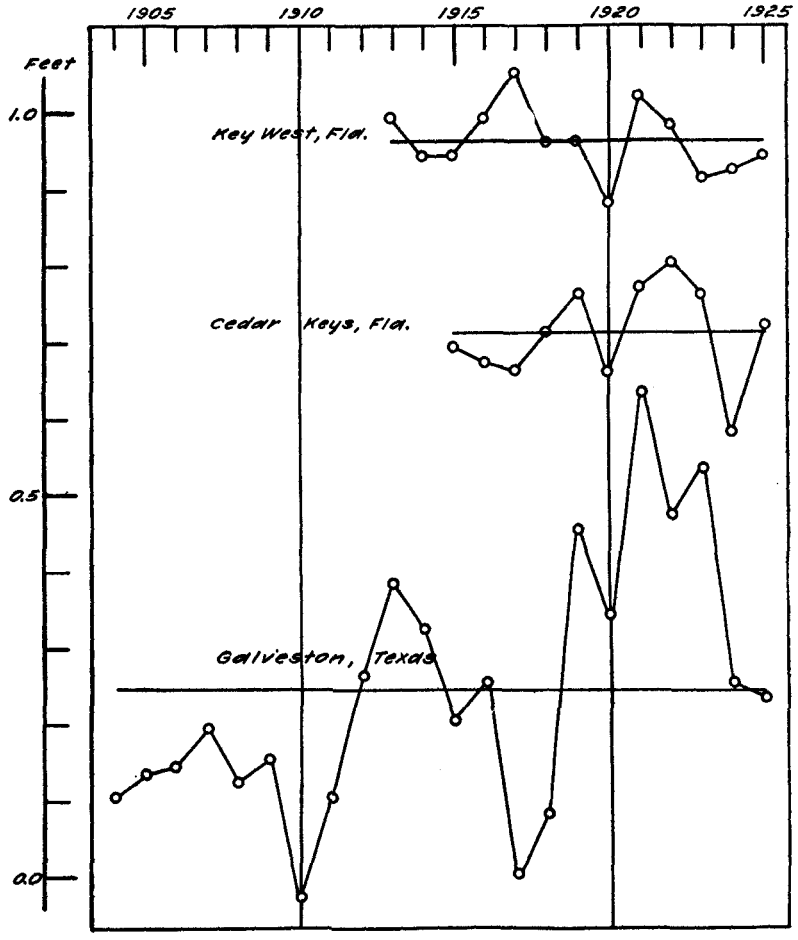

FIG. 33.—Yearly sea level, Gulf coast

in the variation of sea level from year to year occur; but in general it appears that when sea level is for any one year high (or low) at one point on the Pacific coast of the United States, it is also high (or low) all along the coast.

Alaska.—With the exception of Ketchikan, there is no station in Alaska at which tide observations have been continued for more than three consecutive calendar years. For Ketchikan there are at hand yearly heights of sea level for the period 1919–1925. On the tide

staff the height of sea level averaged from the seven years of observation is 14.25 feet. For each year the height of sea level on the tide staff was as follows: 1919, 14.31 feet; 1920, 14.25; 1921, 14.29; 1922, 14.12; 1923, 14.26; 1924, 14.21; 1925, 14.29. Comparing these heights with the yearly heights of sea level at Seattle for the same years, shown in Figure 34, it is seen that the sea level at Ketchikan from year to year changes in much the same way as at Seattle.

PRIMARY DETERMINATION

The variations in sea level discussed in the preceding sections may be summarized as follows: At any point on the coast sea level

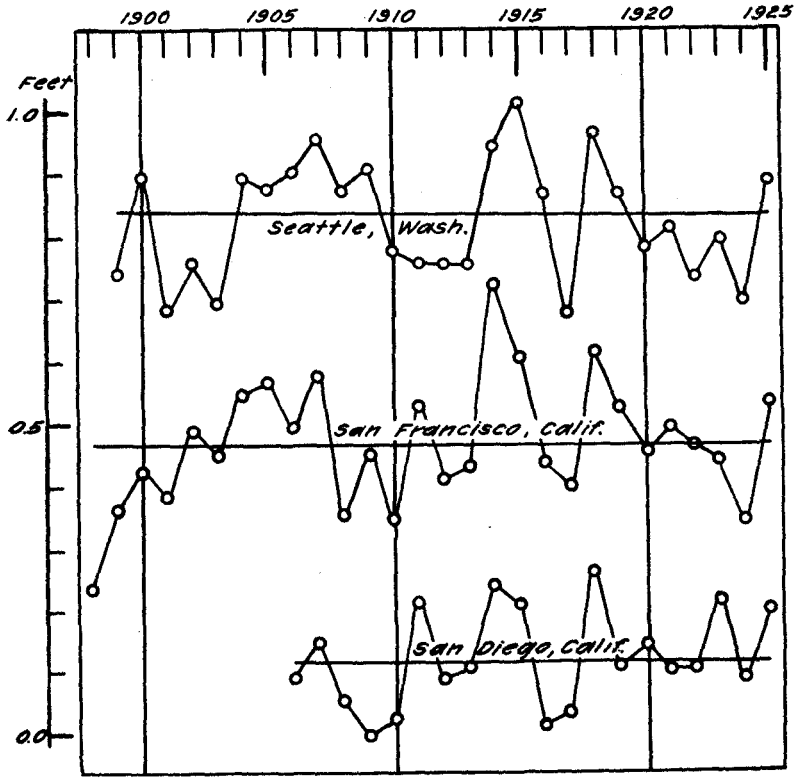

FIG. 34.—Yearly sea level, Pacific coast

varies from day to day, from month to month, and from year to year. From one day to the next sea level may vary by a foot or more, and within the same year two values of daily sea level may differ by 5 feet or more. Monthly sea level is subject to variations of both periodic and nonperiodic character, so that within a year sea level for two different months may differ by as much as a foot. Yearly values of sea level may show differences of a quarter of a foot or even more.

The determination of mean sea level therefore involves two different problems. The first is, how long a series of tide observations is required to give an accurate determination of mean sea level? The

second problem is, how can the sea level derived from a short series of observations be corrected to mean value?

A period of 19 years is generally considered as constituting a full tidal cycle, for during this period of time the more important of the tidal variations will have gone through complete cycles. It is therefore customary to regard results derived from 19 years of tide observations as constituting mean values. Hence sea level derived from 19 years of observations has frequently been taken to constitute a primary determination of mean sea level and as giving accurately the datum of mean sea level.

For the practical purposes of datum plane determination 19 years is a considerable period of time. Moreover, an examination of the yearly heights of sea level shown in Figures 32, 33, and 34 discloses the existence of a prominent variation in sea level with a period of about nine years. Therefore for practical purposes sea level based on nine years or more of tidal observations may be taken as constituting a primary determination of mean sea level and as giving with sufficient accuracy for all practical purposes the datum of mean sea level.

In this connection, however, it is not to be overlooked that in speaking of an accurate determination of mean sea level at any given place two assumptions are tacitly implied. It is assumed that the mean level of the sea remains constant, and it is further assumed that the relative elevation of land to sea at the given place remains constant. For practical purposes these tacit assumptions are justified, for it is obvious that, in general, any changes either in the volume of the ocean waters or in the relative elevation of land to sea must be very small even over a period of 19 years.

While the question of change of relative elevation of land to sea may be disregarded as of little practical concern in determining the plane of mean sea level at present, it is obvious that, if such changes of a progressive nature are taking place, a primary determination of mean sea level at one time may differ appreciably from that made 50 or 100 years later. No confusion, however, will arise if a given determination of mean sea level is specified by the years of observations on which it is based. This will make possible the correlation of mean sea levels determined many years apart.

The differences in consecutive values of mean sea level from nine-year groups are illustrated in Figure 35 for two stations, one on the Atlantic coast and the other on the Pacific coast. For Fort Hamilton observations covering the period 1893–1925 were used, the data for the years 1923–1925 being interpolated; for San Francisco observations covering the years 1898–1925 were used. For Fort Hamilton, therefore, the first nine-year group centers in 1897, while for San Francisco it centers in 1902. The horizontal lines represent, respectively, mean sea level for each station derived from the whole series of observations.

Since two consecutive nine-year groups include eight years of the same data, the difference between such consecutive groups must be small. Figure 35 shows that this difference is generally 0.01 or 0.02 foot. For the 33-year period represented by the Fort Hamilton data the lowest sea level is given by the first nine-year group and the highest by the last, the difference between the two being 0.18

foot. For San Francisco the lowest sea level is given by the first nine-year group, while the highest is given by the group centering in 1918, the difference being 0.09 foot. It is to be observed, however, that in part these differences are to be ascribed to errors inherent in observing tides over long periods. At the same time these differences bring out the necessity for specifying the years of observations used in deriving a primary determination of mean sea level.

SECONDARY DETERMINATION

Observations covering a period of nine years or more for primary determinations of mean sea level are required at but few places on the coast. At all other places a satisfactory secondary determination of this datum plane can be made by means of observations covering much shorter periods if the results are corrected to a mean value by comparison with the primary determination at some suitably located tide station. The precision with which mean sea level can be derived by a secondary determination from various periods of tide observations can best be illustrated by examples.

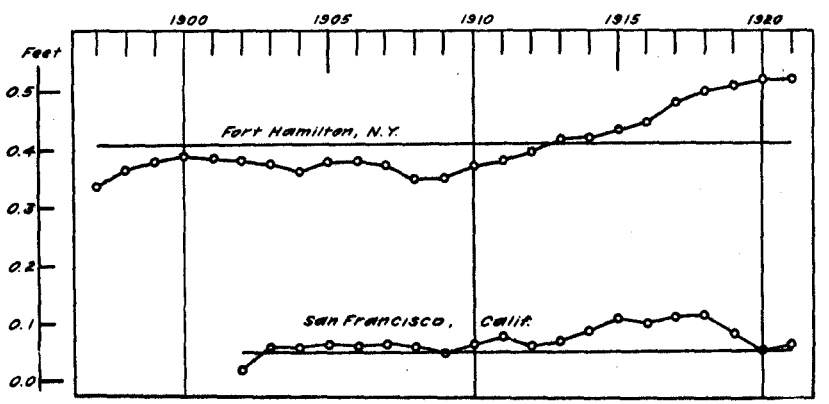

FIG. 35.—Nine-yearly sea level, Fort Hamilton, N. Y., and San Francisco, Calif.

Day.—Since weather conditions at widely separated places may be markedly different on the same day, it is obvious that, in deriving mean sea level at any point from one day of tide observations, comparison must be made with a near-by primary station at which the changes in sea level will be similar.

Figure 24 shows that daily values of sea level fluctuated about 1¾ feet at Boston during the month of April, 1923, with a minimum value on the 1st and a maximum value on the 25th, the tide-staff values being, respectively, 7.12 and 8.96 feet. Suppose that from the observations of each of these two days it is desired to derive a value of mean sea level for Boston. It is obvious that Portland, Me., is a suitably located primary tide station with which to compare the Boston observations. At Portland, from 14 years of observation, mean sea level reads 13.13 feet on the tide staff. For April 1 and 25, 1923, daily sea level on the staff, determined by averaging the 24 hourly heights of the day, was, respectively, 12.30 and 13.85 feet.

On April 1, therefore, sea level at Portland was 0.83 foot below its mean value. And since it is reasonable to assume that the fluc-

tuation in sea level at Boston is closely similar to that at Portland, it follows that mean sea level on the staff at Boston would read 7.12+0.83=7.95 feet. Similarly, on the 25th of April sea level at Portland was 0.72 foot above its mean value, giving for that day at Boston a mean sea-level value on the staff of 8.96—0.72=8.24 feet, which compares with 7.95 feet derived from the observations on April 1. From four years of observations, compared with simultaneous observations at Portland, mean sea level at Boston corresponds to a reading of 8.13 feet on the tide staff. From one day of observations, therefore, by comparison with simultaneous observations at a primary tide station, mean sea level can be determined within a quarter of a foot.

Month.—As an example of the determination of mean sea level from a month of observations we may again take Boston and Portland. From Figure 25 it is seen that for the two-year period 1923–1924 monthly sea level at Boston reached its lowest stage in January, 1924, and its highest stage in March, 1924, the values on the tide staff being, respectively, 7.75 and 8.44 feet. For the same months sea level on the tide staff at Portland read 12.80 and 13.41 feet.

Since 13.13 feet represents the primary determination of sea level on the staff at Portland, sea level in January, 1924, was 0.33 foot below its mean value and in March 0.28 foot above. Applying these corrections to the corresponding sea-level values at Boston, we derive for mean sea level 7.75+0.33=8.08 feet from the January observations and 8.44—0.28=8.16 feet from the March observations, which values agree with each other within 0.08 foot and which differ by 0.05 and 0.03 foot from the mean sea-level value of 8.13 feet, found in the preceding paragraph, from four years of observations. From a month of tide observations, therefore, it appears that mean sea level can be determined within 0.1 foot by comparison with simultaneous observations at a suitably located primary tide station.

In the discussion of the variations of monthly sea level it was shown that this variation is much the same over a larger area than the variation in daily sea level. To illustrate the determination of mean sea level by comparison with a station which is at a greater distance from Boston than is Portland, we may take Atlantic City and derive mean sea level at Boston for the same months by comparison with simultaneous observations at Atlantic City. It should be noted in passing that Atlantic City is more than 200 miles from Boston and lies in a different embayment of the coast. Furthermore, January and March are months when the effects of meteorological conditions are more likely to be different at the two places than during the summer.

From 12 years of tide observations the primary determination of mean sea level on the staff at Atlantic City is 6.59 feet. For the months of January and March, 1924, sea level on the staff read 5.95 and 6.94 feet, or 0.64 foot below and 0.35 foot above mean sea level. Applying these corrections respectively to the January and March sea-level values at Boston, we derive for mean sea level at Boston 7.75+0.64=8.39 feet and 8.44—0.35=8.09 feet.

The best determined value of mean sea level on the staff at Boston is 8.13 feet. Hence the value determined from the month of January, 1924, when compared with simultaneous observations at Atlantic

City, differs by 0.26 foot from the best determined mean sea level, while the same month of observations compared with Portland gave a mean sea-level value which differed by only 0.05 foot. The importance of a suitably located tide station for correcting the results from short series of observations is brought out by this example.

One year.—To exemplify the secondary determination of mean sea level from one year of observations, we may take stations farther apart than those used in connection with one month of observations. Portland and Baltimore are approximately 400 miles apart by air line and considerably farther as measured along the coast line. Moreover, Baltimore lies on an arm of Chesapeake Bay 140 miles from the open sea, which feature tends to make the variations at the two places more unlike than if both were situated on the open coast.

Since the series at Baltimore is the longer, that station will be taken as the primary tide station, mean sea level from 23 years of observations reading 4.11 feet on the tide staff. From Figure 32 yearly sea level at Portland is seen to have been at its lowest level in 1913 and at its highest in 1919, the heights on the tide staff being 13.03 and 13.27 feet, respectively. These two years will therefore be used in deriving mean sea-level values by comparison with simultaneous observations at Baltimore.

Sea level on the tide staff at Baltimore read 4.03 feet in 1913 and 4.30 feet in 1919. Hence in 1913 sea level was 0.08 foot below, and in 1919 it was 0.19 foot above its mean level. Applying these corrections to the sea-level values for the corresponding years at Portland, mean sea level from the 1913 observations is $13.03+0.08=13.11$ feet and from the 1919 observations $13.27-0.19=13.08$ feet. The primary determination of mean sea level at Portland from 14 years of observations is 13.13 feet. Therefore the values derived from the two years of observations differ from the mean value by 0.02 and 0.05 foot, respectively.

We may test the applicability of the method of correction by comparison to stations still farther apart. As an example we may turn to the Pacific coast and take Seattle and San Francisco, which are over 600 miles apart. And since San Francisco has the longer series of tide observations, we may use it as the primary station for correcting two yearly values of sea level at Seattle. Figure 34 shows that at Seattle yearly sea level was at its highest in 1915 and at its lowest in 1917, the heights on the staff being, respectively, 14.23 and 13.88 feet. These two years will therefore be used.

From 28 years of observations the primary determination of mean sea level at San Francisco is 8.55 feet on staff. The yearly sea level was 8.69 feet in 1915 and 8.48 in 1917, giving corrections to mean sea level for those years of -0.14 and $+0.07$ foot, respectively. Applying these corrections to the figures for sea level at Seattle, we derive mean sea-level values of $14.23-0.14=14.09$ feet and $13.88+0.07=13.95$ feet. The primary determination of mean sea level at Seattle from 27 years of observations is 14.04 feet.

The two determinations of mean sea level at Seattle from a year of observations, corrected by comparison with San Franciso, are seen to differ by 0.05 and 0.09 foot from a primary mean sea-level determination. But it is important to note that, notwithstanding the fact of the very considerable distance between Seattle and San Francisco and the further fact of differences in meteorological conditions,

the two yearly values of sea level used in the example were both brought nearer to a mean sea-level value by comparison with San Francisco. In general it may be said that one year of tide observations, when corrected by comparison with simultaneous observations at a suitably located primary tide station, will give a mean sea-level determination correct to within 0.05 foot.

Four years.—In discussing yearly sea level attention was directed to the fact that on the Atlantic, Gulf, and Pacific coasts of the United States there appears to be a variation in sea level with a period of about four years. This period may therefore be regarded as giving very nearly a primary determination of sea level and, if corrected by comparison with simultaneous observations at a suitable primary tide station, should give a very good secondary determination of mean sea level. As examples, we may use the same stations as in the preceding section, namely, Portland and Baltimore on the Atlantic coast and Seattle and San Francisco on the Pacific coast.

Since the years 1912 and 1919 were used in deriving mean sea-level determinations from one year of observations, we may, for the sake of securing comparable results, use four-year groups including those years. Taking 1912–1915 and 1919–1922 we find sea level at Portland to have read 13.09 feet for the first period and 13.17 feet for the second period. At Baltimore for these same periods the heights of sea level on the tide staff were, respectively, 4.06 and 4.22 feet. Compared with the primary determination of mean sea level at Baltimore of 4.11 feet derived from 23 years of observations, a correction of +0.05 foot is indicated for the 1912–1915 sea level and of −0.11 foot for the 1919–1922 sea level. Applying these corrections to the corresponding sea levels at Portland, mean sea-level values of 13.14 and 13.06 feet are derived. These compare with a primary determination at Portland of 13.13 feet derived from 14 years of observations.

In the Seattle-San Francisco comparison of a year of observations the years 1915 and 1917 were used. For the four-year comparison we may therefore take the two periods 1912–1915 and 1917–1920. At Seattle sea level for the period 1912–1915 read 14.08 feet on the tide staff, and for the period 1917–1920, 14.03 feet. At San Francisco the staff readings of sea level for the same periods were, respectively, 8.62 and 8.58 feet. Since the primary determination of mean sea level at San Francisco for the 28-year period ending 1925 is 8.55 feet, the corrections to the sea-level values for the periods 1912–1915 and 1917–1920 are, respectively, −0.07 and −0.03 foot. Applying these corrections to the corresponding sea-level values at Seattle, mean sea-level values of 14.01 and 14.00 feet are derived. The primary determination of mean sea level at Seattle from 27 years of observations is 14.04 feet.

It is obvious that, if stations closer together than those in the examples above had been used, the secondary determinations of mean sea level from the four years of observations would have been closer to the primary determination. If corrected by comparison with simultaneous observations at a primary tide station lying within the same embayment of the coast, four years of tide observations should give, in general, a mean sea-level value correct to within 0.02 foot.

PRIMARY TIDE STATIONS

The possibility of determining the plane of mean sea level from short series of observations is thus seen to depend on the existence of tide stations at which long series of observations are being made. Such tide stations are designated as primary tide stations. At the present time the Coast and Geodetic Survey is operating eight such stations on the Atlantic coast, three on the Gulf coast, four on the Pacific coast, and two in Alaska.

The primary tide stations serve a number of purposes. They furnish primary determinations of mean sea level at these stations, which are then used as the starting and "tie-in" points of the precise level net which is being spread over the country. The data from these stations make possible the correction to mean values of the results of short series of tide observations. In addition, they also furnish the basic data for the study and advancement of the subject of tides.

REFERENCE TO TIDE STAFF AND BENCH MARKS

From the tide observations at any point the plane of mean sea level is determined as corresponding to a certain height on the fixed tide staff used in the tide observations at that point. In other words, mean sea level at that point may be said to be so many feet and hundredths above the zero of a given tide staff. And if it were a simple matter to maintain that tide staff for many years without change in elevation, it would serve very well for preserving the determination of the plane of mean sea level.

But unfortunately it is only rarely that a tide staff can be maintained without change for a number of years. Deterioration of the material used, changes in wharves and piling, and accidents of one kind or another make it necessary to replace a tide staff at intervals more or less frequent. To make certain that the new tide staff will be replaced at the same elevation as the preceding one, so as to make succeeding observations comparable with those preceding, it is necessary to refer the zero of the tide staff to bench marks.

As soon as the elevation of the tide staff with reference to one or more bench marks is known it becomes possible to refer the determination of the plane of mean sea level to these bench marks. These bench marks, established in rock or concrete, or on some substantial structure, thus preserve the determination of mean sea level much better than would the tide staff; and generally this plane at any point is given as so many feet and hundredths below one or more bench marks.

VII. HALF-TIDE LEVEL

DEFINITION

The plane of half-tide level, or mean tide level, as it is sometimes called, is defined as lying exactly halfway between the planes of mean high water and mean low water. It is thus a plane lying close to mean sea level, and frequently the two are taken as synonomous. As accurate datum planes, however, the two must be carefully distinguished.

Strictly, the plane should be designated as that of "mean half-tide level," rather than "half-tide level" in consonance with the distinc-

tion between sea level and mean sea level. No confusion, however, results from the dropping of "mean," since the context clearly indicates the sense in which the term is used, whether to designate the half-tide level for a short period of time, as a day, week, or month, or as a datum plane.

Prior to the invention of the automatic tide gauge the recording of the tide throughout the 24 hours of the day was a matter of considerable expense. It was therefore customary to observe the tide only near the times of high and low water. This permitted a tabulation of the high and low waters but not of the hourly heights. Half-tide level could be determined from such tabulations, but not mean sea level; and as a rule the earlier determinations were those of the plane of half-tide level.

While the tabulation of the hourly ordinates is necessary in the harmonic analysis of the tide, the only datum plane derived from such tabulation is the plane of mean sea level. From the high and low water tabulation, however, not only is the plane of half-tide level determined, but also the various high-water and low-water planes. Moreover, since mean high water and mean low water are symmetrical with respect to half-tide level, a determination of the one is also a determination of the other. It is therefore customary to derive these high-water and low-water datum planes with regard to half-tide level.

VARIATIONS IN HALF-TIDE LEVEL

The tide oscillates about sea level, high water and low water being the maximum and minimum of the oscillation. And, on the average, the rise of high water above sea level is approximately the same as the fall of low water below sea level. Since half-tide level lies halfway between high water and low water, it follows that it must vary in much the same way as sea level.

This conclusion is borne out by an examination of daily, monthly, and yearly values of half-tide level. The variations in sea level discussed in the previous section may be taken to represent also the corresponding variations in half-tide level. It is unnecessary, therefore, to go into a detailed discussion of the variations in half-tide level. It will be sufficient to note in summary that, like sea level, half-tide level at any point on the coast varies from day to day, from month to month, and from year to year. From one day to the next half-tide level may vary by a foot or more, and within the same year two values of daily half-tide level may differ by 5 feet or more. Monthly half-tide level is subject to variations of both periodic and nonperiodic character, so that within a year half-tide level for two different months may differ by as much as a foot. Yearly values of half-tide level may show differences of a quarter of a foot or even more.

RELATION TO MEAN SEA LEVEL

If the curve representing the rise and fall of the tide were that of a simple sine curve, the planes of mean sea level and of half-tide level would coincide. But the rise and fall of the tide does not take place in accordance with the ordinates of a simple sine curve. The movement of the tide is compounded of the movements of a number

of simple sine curves, some of which have fixed phase relations with respect to each other. The rise of high water above sea level is therefore generally not exactly the same as the fall of low water below sea level, and hence mean sea level and half-tide level generally differ.

Obviously, any cause that tends to disturb the regularity of the tide-curve tends to change the relation between sea level and half-tide level. Decided changes in wind and weather may therefore change that relationship somewhat. In general, however, the relation is very nearly constant. Figure 36 shows in diagrammatic form, for each day of the month of April, 1923, the relation of sea level to half-tide level at Boston, Mass. Half-tide level for each day was derived as the average of the four high and low waters of the day. On days when but one high water or one low water occurred, the other one occurring nearest to the day in question was used to make

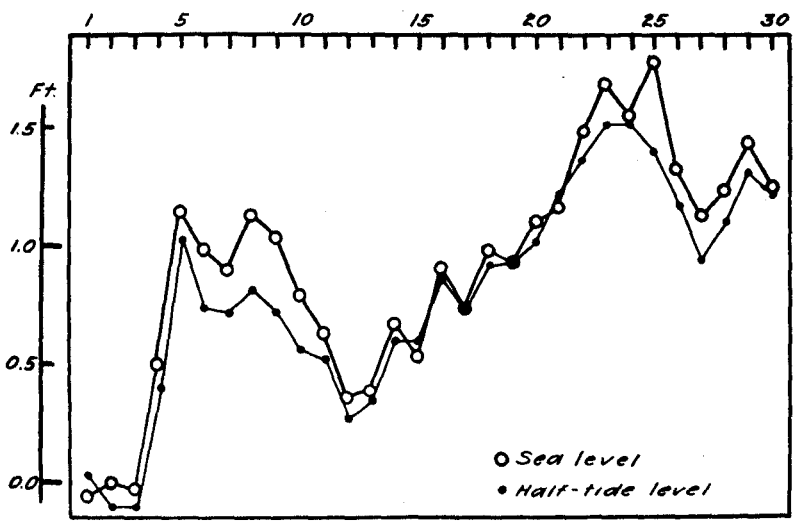

FIG. 36.—Daily sea level and half-tide level, Boston, Mass., April, 1923

up the group of four high and low waters. Sea level for each day was, as heretofore, derived as the average of the 24 hourly heights of the tide.

From Figure 36 it is seen that, despite the relatively large changes in sea level from day to day, the relation of sea level to half-tide at Boston remains very nearly constant. To be sure, this relation changes somewhat from day to day, but these changes are relatively small. Sea level here is, almost without exception, above half-tide level, on the average, by 0.1 foot. The greatest difference between the two shown in the figure—for the 25th—is but 0.2 foot greater than the average.

It is not difficult to see why the relation of sea level to half-tide level is not constant from day to day. In the first place the fact that the tidal cycle has a period of very nearly 25 hours, and not 24 hours, introduces slight variations; and in the second place it is obvious that changes in wind and weather must vary that relationship. For

example, suppose that at any given place we take two days during which the high waters, and likewise the low waters were exactly similar. Half-tide level for the two days would therefore be exactly the same. And if the weather conditions during the two days were similar, sea level likewise would be the same for the two days.

Suppose, however, that weather conditions on the second of the two days were the same as on the first day only until the occurrence of the last high or low water of the day (which, for the sake of illustration, we may assume to have occurred about 6 p. m.). Suppose that from that time to the end of the day the direction or velocity of the wind was different. Obviously, the half-tide level for that day would not be changed, since the last high or low water used in deriving it has already occurred. But the hourly heights of the tide for the remainder of the day would differ from the corresponding heights on the first day, and hence, although half-tide level for the two days would still be the same, the sea levels would differ.

If monthly heights of sea level and half-tide level are compared, the relation between the two is found to be less variable than in

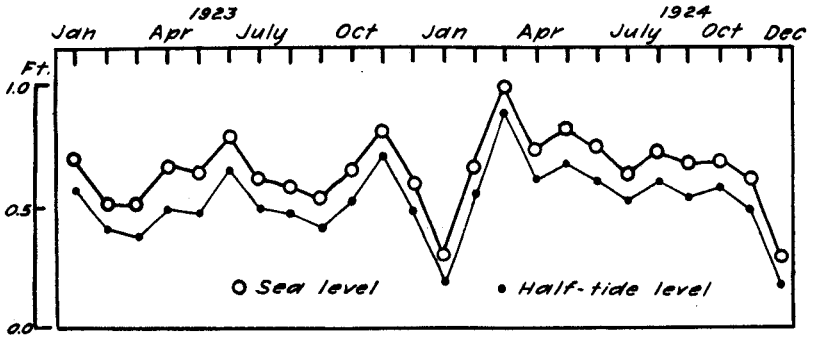

FIG. 37.—Monthly sea level and half-tide level, Boston, Mass.

the case of the daily levels. Figure 37 gives the monthly heights of sea level and half-tide level at Boston for the years 1923 and 1924. Without exception, monthly sea level is seen to be above half-tide level. For the years shown in the figure sea level averaged higher than half-tide by 0.12 foot, the least difference between monthly values being 0.10 foot and the greatest difference 0.17 foot.

A comparison of yearly heights of sea level and half-tide level shows a more nearly constant relation than between monthly values. Thus, at Boston, for the four consecutive years 1922–1925, yearly sea level was above the corresponding yearly half-tide level, in feet, by 0.13, 0.13, 0.12, and 0.12.

The relation between half-tide level and sea level at any point depends upon the amplitude and phase relations between the various constituents of the tide at that point. In the harmonic notation the relation is given by the formula [1]

$$\mathrm{HTL} = \mathrm{SL} + \mathrm{M}_4 \cos(2\mathrm{M}_2^\circ - \mathrm{M}_4^\circ) - \frac{0.04(\mathrm{K}_1 + \mathrm{O}_1)^2}{\mathrm{M}_2} \cos(\mathrm{M}_2^\circ - \mathrm{K}_1^\circ - \mathrm{O}_1^\circ).$$

[1] See R. A. Harris, Manual of Tides, Pt. III, p. 148.

Here HTL stands for half-tide level, SL for sea level, and the other terms have their usual significance in the harmonic notation. Since the amplitudes of the various components vary somewhat from year to year, it follows that the relation between sea level and half-tide level may differ from year to year. Furthermore, the cosine of $(2M_2^\circ - M_4^\circ)$ and also of $(M_2^\circ - K_1^\circ - O_1^\circ)$ may be either positive or negative. Hence sea level may be either above or below half-tide level, depending upon the phase and amplitude relations.

Along the Atlantic coast of the United States cos $(2M_2^\circ - M_4^\circ)$ is generally negative, while cos $(M_2^\circ - K_1^\circ - O_1^\circ)$ is generally positive. Hence along this coast half-tide level is below sea level, with but few exceptions. Along the Gulf coast both cosine terms in the formula are generally positive, so that here half-tide level may be either above or below sea level, depending upon which term has the greater value. On the Pacific coast the first cosine term is positive at some places, while at others it is negative; the second cosine term, however, is generally negative. Here, however, $\frac{(K_1+O_1)^2}{M_2}$ is, as a rule, more than 25 times as great as M_4, and therefore at most places along this coast half-tide level is above sea level.

The periodic variation in K_1 and O_1 from year to year is much greater than in M_4. Hence, where the ratio of $(K_1+O_1)^2$ to M_2 is large, appreciable variations in the relation of half-tide level to sea level may be expected from year to year. On the Atlantic coast this ratio is small, being at most places about 0.1, but on the Gulf and Pacific coasts it is relatively large, being at most places greater than unity. It is therefore to be expected that the relation of half-tide level to sea level will differ but little from year to year on the Atlantic coast, while on the Gulf and Pacific coasts larger differences will appear. This is brought out by the tide observations.

In Figure 38 the variation in the relation of yearly half-tide level to sea level is shown for Fort Hamilton, N. Y., Galveston, Tex., and Seattle, Wash. These stations may be taken to represent, respectively, this variation on the Atlantic, Gulf, and Pacific coasts of the United States. For each station the horizontal line represents the best-determined difference between half-tide level and sea level from the whole series of observations, this difference in feet being indicated by the figures to the right or left of the horizontal line. The difference given is half-tide level minus sea level. Hence, when a negative sign is prefixed, it indicates that half-tide level is below sea level.

The curve for Fort Hamilton shows that the relation of yearly half-tide level to sea level is very nearly constant, the extreme deviation from the mean value being but 0.01 foot. For Galveston the variation is somewhat greater, attaining in some years a value differing by 0.04 foot from the mean value. At both of these stations the variation is so small that any periodicity is masked. At Seattle, however, the periodicity is unmistakable, the minimum coming about 1904 and 1923 and the maximum about 1913, giving 19 years as the period of this variation. At times of maximum or minimum the difference from the mean value may be as much as 0.05 foot.

Table 4 gives the relation of half-tide level to mean sea level as determined directly from observations at a number of stations on the Atlantic, Gulf, Pacific, and Alaskan coasts of the United States. The table gives the values for half-tide level minus mean sea level. Negative values therefore indicate that mean sea level lies above half-

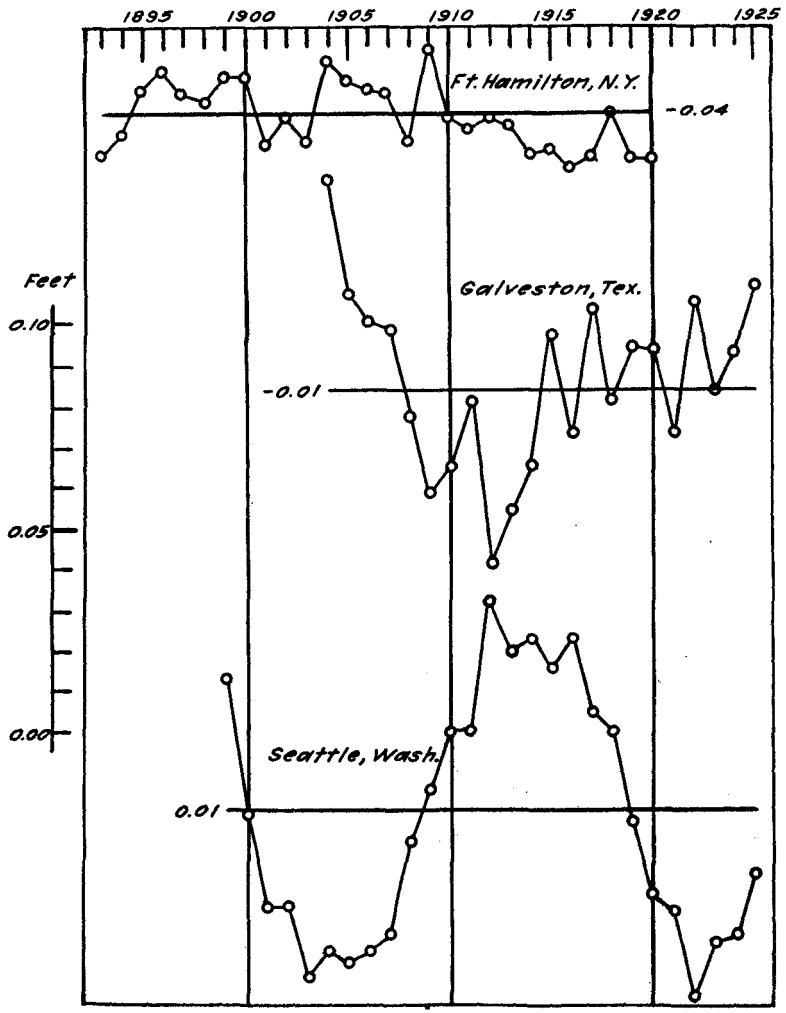

FIG. 38.—Variation in relation of yearly half-tide level to sea level

tide level, while positive values indicate that mean sea level lies below half-tide level by the amounts shown. For each station the value given is based on at least two years of observations, and it may therefore be taken to approximate the mean value of this relation within 0.01 or 0.02 foot.

TABLE 4.—*Half-tide level minus mean sea level*

	Feet
Atlantic coast:	
Portland, Me	−0.02
Boston, Mass	−.12
Providence, R. I	−.10
New London, Conn	−.05
Fort Hamilton, N. Y	−.04
Atlantic City, N. J	−.03
Philadelphia, Pa	−.16
Baltimore, Md	−.02
Washington, D. C	−.01
Wilmington, N. C	−.02
Charleston, S. C	−.12
Tybee Island, Ga	−.11
Fernandina, Fla	−.11
Gulf coast:	
Key West, Fla	−.01
Cedar Keys, Fla	−.02
Gulf coast—Continued.	
Pensacola, Fla	0.02
Weeks, La	.01
Galveston, Tex	−.01
Pacific coast:	
San Diego, Calif	.05
La Jolla, Calif	.02
Los Angeles, Calif	.03
San Francisco, Calif	.06
Astoria, Oreg	.04
Seattle, Wash	.01
Olympia, Wash	−.05
Anacortes, Wash	.22
Alaskan coast:	
Ketchikan	−.02
Skagway	−.13
Valdez	−.09

Table 4 shows that, in general, mean sea level and half-tide level differ by about 0.1 foot. On the Atlantic coast the plane of mean sea level generally lies above that of half-tide level, while on the Pacific coast it is the plane of half-tide level that is the higher. The five stations given for the Gulf coast make it appear that there mean sea level and half-tide level may for all practical purposes be taken as coinciding.

DETERMINATION OF HALF-TIDE LEVEL

Since the variations in half-tide level are very nearly the same as those in sea level, the procedure employed in determining the plane of mean sea level becomes applicable also to the determination of the plane of half-tide level; and, as in the case of mean sea level, a determination based on nine years of tide observations may be taken as constituting a primary determination of the datum of half-tide level.

For deriving the datum of half-tide level from a short series of observations the direct determination is corrected by means of simultaneous observations at some near-by station for which a primary determination is at hand. The procedure is similar in all respects to that employed in correcting to a mean value the determinations of sea level from short series of observations. As an example, we may derive the datum of half-tide level at San Francisco, Calif., from two different months of observations, using San Diego, Calif., as the primary station.

The tabulation of the high and low waters at San Francisco for the month of June, 1920, is shown in Figures 19 and 20. Taking this month we derive half-tide level—half the sum of the average heights of the high and low waters—as 8.66 feet on the staff. At San Diego for the same month half-tide level read 6.59 feet, while a primary determination based on 18 years of observations read 6.58 feet. Compared with this primary determination, the half-tide level for the month of June, 1920, is 0.01 foot too high, so that the correction is −0.01 foot. Hence the corrected half-tide level at San Francisco, based on this month of observations, reads 8.66−0.01= 8.65 feet.

Taking as the other month December, 1914, half-tide level at San Francisco is found to read 9.26 feet, while at San Diego it read 7.10 feet. Compared with the primary determination of 6.58 at San Diego, a correction of −0.52 feet is indicated for December, 1914. Hence the corrected half-tide level at San Francisco will be given by 9.26−0.52=8.74 feet. The two determinations of half-tide level, each based on a month of observations, which differed by 0.6 foot, after correction agree within 0.09 foot. These two corrected values, moreover, differ from the primary determination of 8.61 feet for the half-tide level at San Francisco only by 0.04 and 0.13 foot, respectively.

The precision with which half-tide level can be determined from a short series of tide observations may be taken to be the same as for mean sea level; that is, in general, when corrected by comparison with a suitably located primary station, a month of observations will determine the datum of half-tide level within 0.1 foot, a year of observations will determine it within 0.05 foot, while four years of observations will determine it within 0.02 foot.

VIII. MEAN HIGH WATER

VARIATIONS IN HEIGHT OF HIGH WATER

The height to which high water rises varies from day to day. Indeed, it is exceptional that the two high waters of the same day are alike. Primarily these variations are related to the varying positions of the moon relative to earth and sun. Thus, near the times of new moon and full moon high water rises higher than usual, while at the times of the moon's first and third quarters the rise of high water is less than usual. Likewise, when the moon is near perigee, the height to which high water rises is above the average, while near the time of the moon's apogee the height of high water is below the average.

The periodic semimonthly change in the declination of the moon also brings about variations in the height of high water, causing the two high waters of a day to differ. This gives rise to the feature known as diurnal inequality, necessitating the distinction between higher high water and lower high water. When the moon is near its semimonthly maximum declination, the two high waters of a day show the greatest difference in height; when the declination of the moon is small—that is, when the moon is near the Equator—the difference in height of the two high waters is least.

The seventh column of the specimen sheet of the tabulation shown in Figures 19 and 20 gives the height of each high water at San Francisco, Calif., for the month of June, 1920, as referred to the tide staff. In Figure 39 these heights are shown graphically, together with the various positions of the moon during the month. The small circles give the height of each high water above mean sea level, which is represented by the horizontal line in the figure. To indicate clearly the succession each high water is joined by a straight line to the preceding and succeeding high waters.

Figure 39 brings out the relation of the height of high water to the position of the moon relative to earth and sun. When the declination of the moon is small—that is, when the moon is close to the Equator (in the figure about the 9th and 22d of the month)—suc-

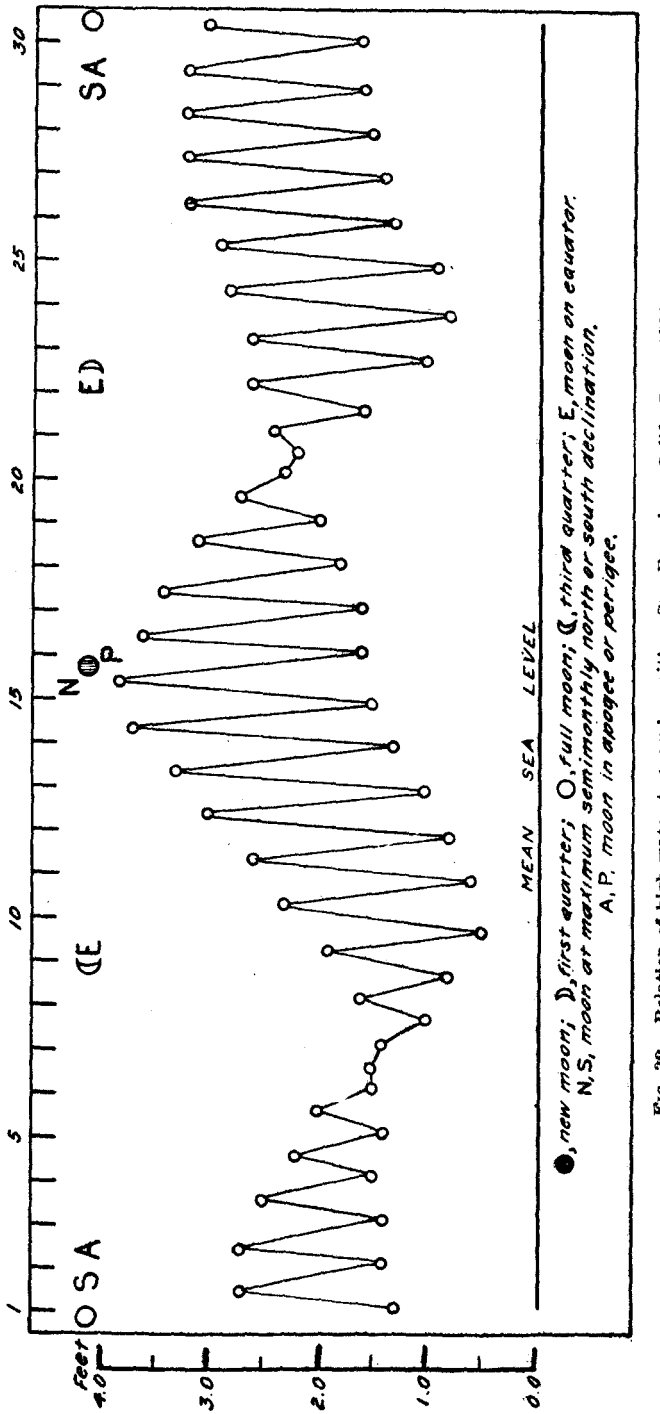

FIG. 39.—Relation of high water to moon's position, San Francisco, Calif., June, 1920

cessive high waters do not differ greatly in height; near the times of the moon's maximum north and south declination (the 1st, 15th, and 29th of the month) the difference between the two high waters is greatest. On the 14th the two high waters differed by 2.4 feet.

Because of the difference in the periods of the various motions of the moon various combinations occur. Thus, on the 1st of June, 1920, the increased height of high water due to full moon was offset by the decrease due to the moon being close to apogee. Likewise, on the 16th the high waters bear the impress of the new moon coinciding with perigee and very nearly coinciding with the moon's maximum semimonthly declination.

In addition to the variations in the height of high water enumerated above, which are due to astronomic causes, still further variations are brought about by wind and weather. These are especially marked on coasts fronting shallow bodies of water and in tidal rivers subject to wide variations in fresh-water flow. It is obvious, therefore, that the term "plane of high water" is ambiguous, for there are various planes of high water—mean high water, spring high water, neap high water, perigean high water, apogean high water, higher high water, lower high water, storm high water. This does not exhaust the list; for, since the months of the moon's phase, distance, and declination are not of the same length, various combinations occur which define such datum planes as perigean spring high water, tropic perigean spring higher high water, etc.

In different places and for different purposes different high-water planes have been used. Of the great variety of such high-water planes possible, however, it is found that four cover the practical needs for datum planes. These are the planes of mean high water, higher high water, spring high water, and monthly highest high water.

DEFINITION OF MEAN HIGH WATER

Mean high water at any place is the average height of all the high waters at that place over a considerable period of time. To determine just how long a series of tide observations is required to derive a satisfactory value of mean high water it is necessary to consider in detail the variations to which high water is subject.

The variations in the heights of successive high waters during the period of a month were discussed in the previous section and illustrated diagrammatically in Figure 39. These variations, which are of a periodic character, are due to changes in the position of the moon with respect to phase, distance, and declination. There are also variations of a nonperiodic character due to changes in wind and weather. Indeed, since the tide is primarily an oscillation above and below sea level, it is obvious that changes in sea level due to wind and weather will be reflected in the height to which high water rises. This is brought out in Figure 40, which gives the daily heights of high water and of sea level at Atlantic City for the month of February, 1925.

For the purpose of exhibiting the relation of high water to sea level it is sufficient to take the average of the two high waters for each day, and this was done in constructing Figure 40. In this figure the upper diagram gives the average height of the two high waters each day, while the lower diagram gives the daily height of

sea level. A glance suffices to show that, notwithstanding the variations in high water from day to day due to changes in the position of the moon, there is a variation also due to the daily variation of sea level.

The average rise of high water above sea level at Atlantic City is, to the nearest tenth of a foot, 2.0 feet. For the month shown in Figure 40 the rise of high water above sea level is seen to vary from 1 foot to 3 feet, this variation reflecting the periodic changes due to the changes in the position of the moon. On the 1st of February, 1925, the moon was in her first quarter and only three days before apogee; hence a much reduced rise of high water. On the 21st the moon was in perigee, and on the 23d new moon occurred; hence on

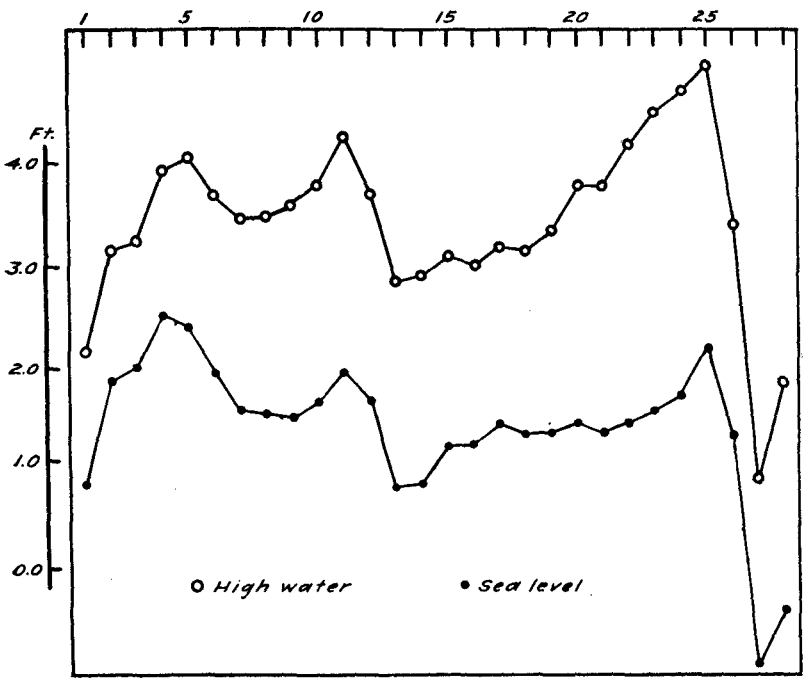

FIG. 40.—Relation of daily high water to sea level, Atlantic City, February, 1915

the 23d and 24th the rise of high water was considerably above the average, being then about 3 feet above sea level.

MONTHLY HIGH WATER

The daily height of high water is subject to relatively large variations both periodic and nonperiodic in character. The periodic variations depend primarily on the phase, distance, and declination of the moon, the periods of these being approximately 29½, 27½, and 27⅓ days, respectively. Such variations are therefore largely eliminated within a month. And within a month, too, the large variations in sea level due to wind and weather tend to balance out. It follows, therefore, that a much closer approximation to the plane of mean high water can be secured from a month than from one day of

observations. It remains now to determine what variations monthly high water is subject to.

In Figure 41 are shown the relative heights of monthly high water at five stations on the Atlantic coast for the years 1923 and 1924. In general, consecutive values of monthly high water at any point on the coast vary about 0.1 or 0.2 foot, though occasionally the difference may be more than half a foot. Within the same year the monthly heights of high water may differ by as much as a foot.

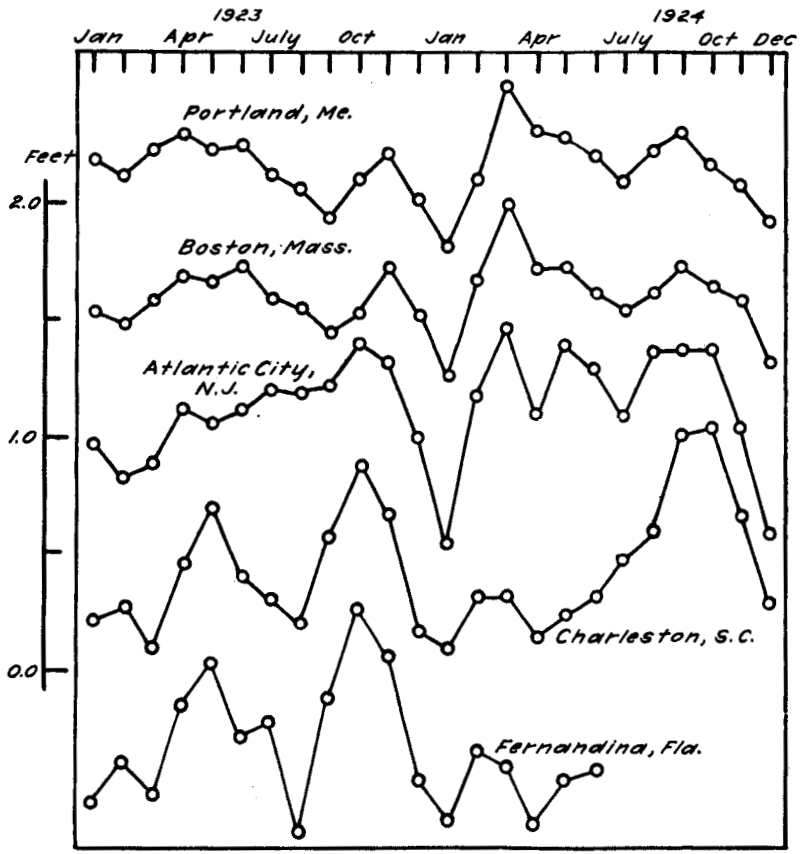

FIG. 41.—Monthly heights of high water, Atlantic coast tide stations

A comparison of the diagrams of Figure 41 with the corresponding diagrams of Figure 25, which illustrate the variations of monthly sea level, brings out the fact that monthly high water follows the variations in sea level much more closely than does daily high water; and, obviously, this is to be ascribed to the fact that, within a month, the periodic variations due to changes in the position of the moon very largely balance out.

Since the height of monthly high water varies approximately as the height of sea level, it follows that monthly high water must be subject to an annual variation such as was found characteristic of monthly sea level. This is borne out by observations and for three

stations—one on each of the coasts of continental United States—is shown graphically in Figure 42. The corresponding curves of annual variation in sea level for these three stations are shown in Figures 28, 29, and 30. Comparing corresponding curves, it is seen that the annual variation of high water follows that in sea level closely.

Monthly high water at any point is thus subject to variations both periodic and nonperiodic in character; and in both of these it follows closely the like variations in sea level at that point.

YEARLY HIGH WATER

Turning now to the variations in the height of yearly high water, it is found, as was to be expected, that these are much smaller than

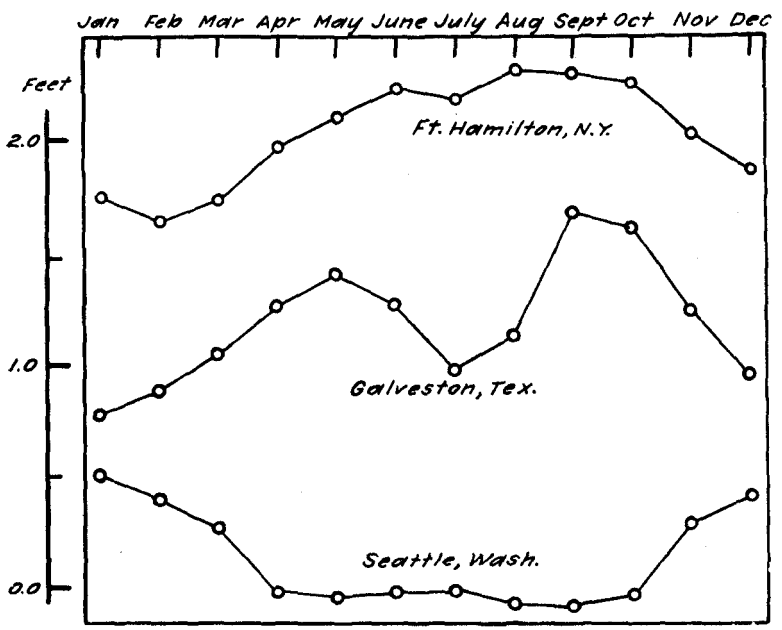

FIG. 42.—Annual variation in high water

the variations from month to month. In Figure 43 are shown the yearly heights of high water at six stations on the Atlantic coast. Generally, consecutive yearly heights of high water at a station are seen to differ by not more than a few hundredths of a foot, though at times this difference may be as much as a quarter of a foot. Within a period of 20 years yearly values of high water may differ by as much as half a foot, even at stations free from the disturbing effects brought about by large fluctuations in river discharge.

A comparison of the curves of Figure 43 with the corresponding curves of Figure 32, which give for the same stations the variations in sea level from year to year, shows that, like daily and monthly high water, yearly high water varies in much the same way as yearly sea level. In fact, yearly high water follows the variation in sea level much more closely than monthly sea level.

If yearly high water varied exactly as sea level, the rise of high water above sea level at any point would, from year to year, be constant. This, however, is not the case, for observations show that the rise of high water above sea level differs from year to year. In part, such differences must be ascribed to difficulties inherent in the operation of the tide stations over considerable periods of time and

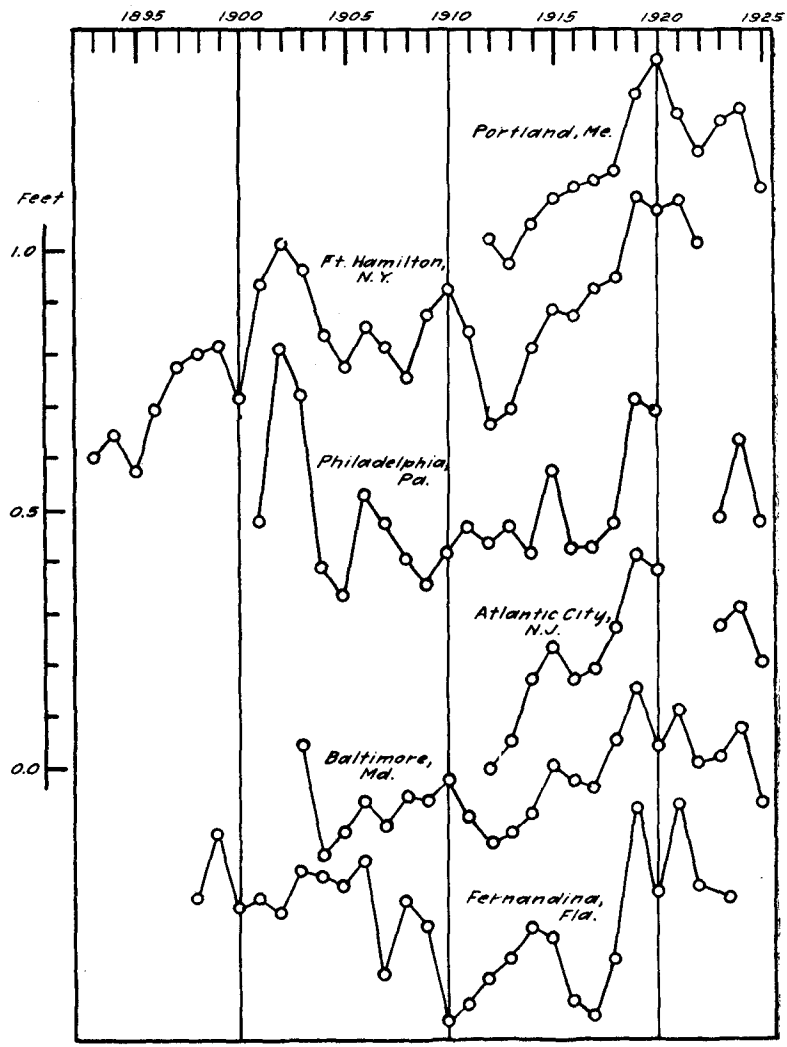

FIG. 43.—Yearly high water, Atlantic coast tide stations

also to the disturbing effects of unusual weather conditions. Changes in hydrographic features, whether natural or artificial, likewise may change the relation of high water to sea level, for such changes generally do not affect sea level but do affect the range of the tide.

The causes enumerated above for the variation in the relation of yearly high water to sea level are not of a periodic character. If

the height of high water above sea level from year to year is plotted for a number of years, a distinct periodic variation comes to light, the period being approximately 19 years. In Figure 44 the variation of yearly high water above sea level is shown graphically for three stations, one each on the Atlantic, Gulf, and Pacific coasts.

The horizontal lines associated with the diagrams represent for each of the stations the average height of high water above sea level derived from the whole series of observations at that station, this height in feet being given by the figures at the right or left of the horizontal lines. For Fort Hamilton the periodic variation is very definitely brought out with minima about 1894 and 1914 and maxima about 1904 and 1922. For Galveston variations of nonperiodic character mask the periodic variation, but a curve drawn to fit the

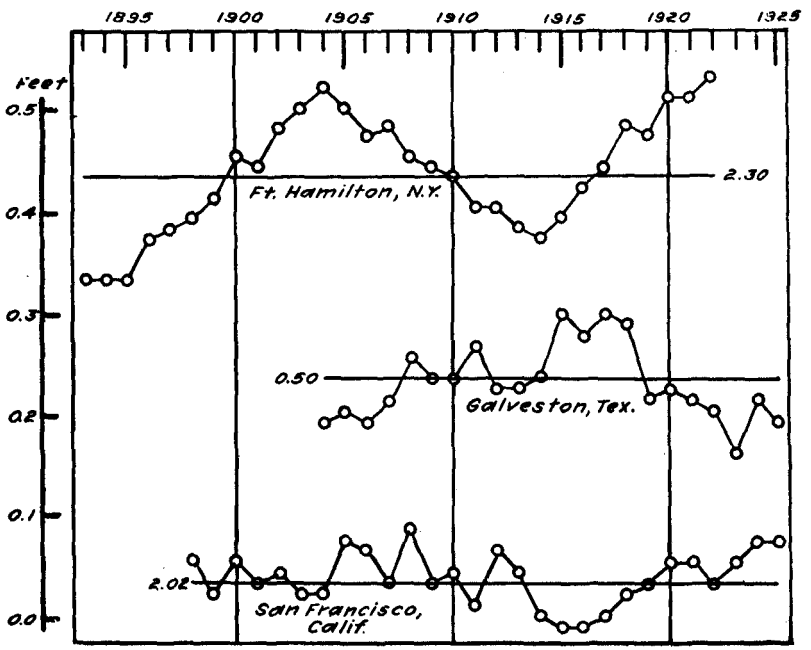

FIG. 44.—Variation of yearly high water in relation to sea level

yearly heights would show a maximum about 1916 and minima about 1905 and 1923. For San Francisco a like curve indicates minima about 1899 and 1916 and maxima about 1907 and 1925.

From theoretical considerations of an astronomical character it follows that there should be a periodic variation in the rise of high water above sea level having a period of 18.6 years. This is brought about by the change in longitude of the moon's node, which introduces a variation in the inclination of the lunar orbit to the plane of the earth's Equator. The effect of this, however, is different on the daily and the semidaily constituents of the tide. This means that in computing factors to correct the rise of high water for the longitude of the moon's node account must be taken of the relative magnitudes of the daily and semidaily constituents of the tide.

The ratio of the daily to the semidaily constituents of the tide at any place can be computed very readily from the harmonic constants of the tide at that place. For the purpose of deriving factors to reduce the rise of high water to a mean value the ratio of the daily to the semidaily constituents is taken as $(K_1+O_1)\div M_2$. Where harmonic constants are not at hand this ratio may be derived, approximately, from the formula, $2(DHQ+DLQ)\div Mn$, in which DHQ is the mean high-water diurnal inequality, DLQ is the mean low-water diural inequality, and Mn is the mean range of the tide. The derivation of the diurnal inequalities will be taken up in connection with the higher high-water and lower low-water datums. In Table 5 the ratio of K_1+O_1 to M_2 at a number of points along the coasts of the United States are given.

TABLE 5.—*Ratio of* K_1+O_1 *to* M_2

	Feet
Atlantic coast:	
Eastport, Me	0.10
Portland, Me	.19
Boston, Mass	.18
Newport, R. I	.22
Providence, R. I	.20
New London, Conn	.37
New York, N. Y	.24
Albany, N. Y	.34
Sandy Hook, N. J	.23
Atlantic City, N. J	.31
Lewes, Del	.31
Philadelphia, Pa	.24
Old Point Comfort, Va	.27
Richmond, Va	.21
Washington, D. C	.20
Baltimore, Md	.83
Wilmington, N. C	.32
Charleston, S. C	.24
Savannah, Ga	.18
Fernandina, Fla	.21
Gulf coast:	
Key West, Fla	1.02
Cedar Keys, Fla	.97
Pensacola, Fla	13.21
Fort Morgan, Ala	10.46
Biloxi, Miss	9.66
Cat Island, Miss	8.66
Port Eads, La	14.34
Galveston, Tex	2.71
Pacific coast:	
San Diego, Calif	1.04
Morro, Calif	1.31
San Francisco, Calif	1.14
Humboldt, Bay, Calif	.98
Astoria, Oreg	.71
Port Townsend, Wash	1.79
Seattle, Wash	1.21
Alaskan coast:	
Craig	.63
Ketchikan	.43
Sitka	.67
Sergius Narrows	.53
Cross Sound	.59
Juneau	.41
Skagway	.41
Kokinhenic Island	.68
Orca Inlet	.58
Seldovia	.41
Anchorage	.29
Kodiak	.69
Dutch Harbor	2.07
St. Paul Island	1.88
Nushagak Bay	.71
Apokak	.85
St. Michael	3.82
Point Barrow	.65

It will be noted from Table 5 that the ratio of the daily constituent of the tide to the semidaily is approximately the same for large regions, so that at any desired point this ratio may be taken to be the same as at a station in the general vicinity where it has been determined. Thus, for the Atlantic coast of the United States this ratio is about 0.2, while for the Pacific coast it is about 1.1. On the Gulf coast this ratio shows wide variations, but even here the value at any given point may be taken as characteristic of the tide over a considerable area.

Where the ratio of the daily constituent of the tide to the semidaily, defined by $(K_1+O_1)\div M_2$, is less than 0.5, the tide is of the semidaily type—that is, there is but little difference between morning and afternoon tides; where the ratio is between 0.5 and 2, the tide is

of the mixed type; and where greater than 2, the tide begins to be of the daily type. Strictly, the plane of mean high water should be confined to the semidaily and the mixed types of tide. It is convenient, however, in regions of small range, to adopt the plane of mean high water even when the tide is partly of the daily type.

Instead of giving the tabular values of the percentage increase or decrease in the height of high water brought about by the change in longitude of the moon's node, it will be more useful to give the reciprocals of such values, for these reciprocals constitute the factors for correcting the rise of high water over a given period to a mean value. Table 6 gives these factors for each year of the 60-year period 1891–1950 as computed from Tables 6 and 14 of Harris's Manual of Tides, Part III. As will be seen later, these are also the factors for correcting the fall of low water to a mean value.

TABLE 6.—*Factors for correcting high water and low water to mean values*

Year \ $\frac{K_1+O_1}{M_2}$	0.0 to 0.2	0.3 to 0.4	0.5 to 0.6	0.7 to 0.8	0.9 to 1.0	1.1 to 1.2	1.3 to 1.4	1.5 to 1.6	1.7 to 1.8	1.9 to 2.0
1891	1.013	1.013	1.012	1.011	1.009	1.008	1.006	1.004	1.001	0.998
1892	1.022	1.021	1.020	1.017	1.015	1.012	1.008	1.005	1.001	0.996
1893	1.027	1.026	1.025	1.022	1.019	1.016	1.012	1.007	1.001	0.995
1894	1.029	1.028	1.026	1.023	1.020	1.017	1.013	1.008	1.001	0.994
1895	1.029	1.028	1.026	1.023	1.020	1.017	1.013	1.007	1.001	0.995
1896	1.025	1.024	1.023	1.020	1.017	1.014	1.011	1.006	1.001	0.996
1897	1.018	1.018	1.017	1.015	1.013	1.010	1.007	1.005	1.001	0.996
1898	1.009	1.009	1.009	1.008	1.006	1.005	1.004	1.002	1.001	0.998
1899	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
1900	0.989	0.989	0.990	0.991	0.992	0.994	0.996	0.998	1.000	1.002
1901	0.980	0.981	0.982	0.984	0.986	0.989	0.992	0.995	0.999	1.004
1902	0.974	0.975	0.976	0.979	0.982	0.985	0.989	0.993	0.999	1.005
1903	0.971	0.972	0.974	0.976	0.979	0.983	0.987	0.992	0.999	1.006
1904	0.971	0.972	0.974	0.976	0.979	0.983	0.987	0.992	0.999	1.006
1905	0.973	0.974	0.976	0.978	0.981	0.984	0.988	0.993	0.999	1.005
1906	0.979	0.980	0.981	0.982	0.985	0.988	0.991	0.995	0.999	1.004
1907	0.987	0.987	0.988	0.989	0.991	0.993	0.995	0.997	0.999	1.002
1908	0.997	0.997	0.997	0.998	0.998	0.999	0.999	0.999	1.000	1.000
1909	1.007	1.007	1.007	1.006	1.005	1.004	1.003	1.002	1.000	0.998
1910	1.017	1.016	1.015	1.014	1.012	1.010	1.007	1.004	1.001	0.997
1911	1.024	1.023	1.022	1.019	1.017	1.014	1.010	1.006	1.001	0.996
1912	1.028	1.028	1.026	1.023	1.019	1.016	1.013	1.007	1.001	0.995
1913	1.029	1.028	1.026	1.023	1.020	1.017	1.013	1.008	1.001	0.994
1914	1.028	1.027	1.025	1.022	1.019	1.016	1.013	1.007	1.001	0.995
1915	1.023	1.022	1.021	1.018	1.016	1.013	1.010	1.006	1.001	0.996
1916	1.015	1.014	1.014	1.012	1.010	1.009	1.006	1.004	1.001	0.997
1917	1.005	1.005	1.005	1.005	1.004	1.003	1.002	1.001	1.000	0.999
1918	0.995	0.995	0.995	0.996	0.997	0.997	0.998	0.999	1.000	1.001
1919	0.985	0.985	0.986	0.988	0.990	0.992	0.994	0.997	0.999	1.003
1920	0.978	0.979	0.980	0.981	0.984	0.987	0.990	0.994	0.999	1.005
1921	0.972	0.973	0.975	0.978	0.981	0.984	0.988	0.993	0.999	1.006
1922	0.971	0.972	0.974	0.976	0.979	0.983	0.987	0.992	0.999	1.006
1923	0.971	0.972	0.974	0.976	0.979	0.983	0.987	0.993	0.999	1.006
1924	0.975	0.976	0.977	0.980	0.983	0.986	0.989	0.993	0.999	1.005
1925	0.981	0.982	0.983	0.985	0.988	0.990	0.993	0.996	0.999	1.003
1926	0.991	0.991	0.991	0.992	0.994	0.995	0.997	0.998	1.000	1.002
1927	1.001	1.001	1.001	1.001	1.001	1.001	1.000	1.000	1.000	1.000
1928	1.011	1.011	1.010	1.009	1.008	1.006	1.005	1.003	1.001	0.998
1929	1.020	1.019	1.018	1.016	1.014	1.011	1.008	1.005	1.001	0.996
1930	1.026	1.026	1.024	1.021	1.018	1.015	1.011	1.006	1.001	0.995
1931	1.029	1.028	1.026	1.023	1.020	1.017	1.013	1.008	1.001	0.994
1932	1.029	1.028	1.026	1.023	1.020	1.017	1.013	1.008	1.001	0.994
1933	1.026	1.026	1.024	1.021	1.018	1.015	1.011	1.006	1.001	0.995
1934	1.020	1.019	1.018	1.016	1.014	1.011	1.008	1.005	1.001	0.996
1935	1.011	1.011	1.010	1.010	1.008	1.007	1.005	1.003	1.001	0.998
1936	1.002	1.002	1.002	1.001	1.001	1.001	1.001	1.000	1.000	1.000
1937	0.991	0.991	0.992	0.992	0.994	0.995	0.997	0.998	1.000	1.001
1938	0.981	0.982	0.983	0.985	0.988	0.990	0.993	0.996	0.999	1.003
1939	0.975	0.976	0.977	0.980	0.983	0.986	0.989	0.993	0.999	1.005
1940	0.971	0.972	0.974	0.976	0.979	0.983	0.987	0.993	0.999	1.006

TABLE 6.—*Factors for correcting high water and low water to mean values*—Con.

Year \ $\frac{K_1+O_1}{M_2}$	0.0 to 0.2	0.3 to 0.4	0.5 to 0.6	0.7 to 0.8	0.9 to 1.0	1.1 to 1.2	1.3 to 1.4	1.5 to 1.6	1.7 to 1.8	1.9 to 2.0
1941	0.971	0.972	0.974	0.976	0.979	0.983	0.987	0.992	0.999	1.006
1942	0.972	0.973	0.975	0.977	0.980	0.984	0.988	0.993	0.999	1.006
1943	0.977	0.978	0.979	0.981	0.984	0.987	0.991	0.994	0.999	1.005
1944	0.985	0.985	0.986	0.987	0.990	0.992	0.994	0.997	0.999	1.003
1945	0.995	0.995	0.995	0.996	0.996	0.997	0.998	0.999	1.000	1.001
1946	1.005	1.005	1.005	1.004	1.004	1.003	1.002	1.001	1.000	0.999
1947	1.014	1.014	1.013	1.012	1.010	1.008	1.006	1.004	1.001	0.997
1948	1.023	1.022	1.021	1.018	1.016	1.013	1.009	1.006	1.001	0.996
1949	1.028	1.027	1.025	1.022	1.019	1.016	1.012	1.007	1.001	0.995
1950	1.029	1.028	1.026	1.023	1.020	1.017	1.013	1.008	1.001	0.994

An examination of Table 6 shows that the factor for correcting yearly high water to mean high water decreases with increasing values of the ratio K_1+O_1 to M_2. Thus, for a ratio of K_1+O_1 to M_2 of 0.0 to 0.2 it appears that the rise of high water determined from one year of observations may require a correction as great as 3 per cent to derive a mean value, while, when the ratio is 1.5, this correction is at most barely 1 per cent. In other words, this correction is relatively large for tides of the semidaily type and small for tides of the mixed type.

PRIMARY DETERMINATION

The variations to which the height of high water is subject were, in the preceding sections, found to be of two kinds, namely, those due to changes in the height of sea level and those due to changes in the position of the moon relative to earth and sun. Within a period of approximately 19 years the more important of the moon's movements, in so far as the tide is concerned, will have gone through one or more complete cycles. And 19 years has been found to give a primary determination of the plane of mean sea level. It follows, therefore, that the direct average of all the high waters during a period of 19 years will constitute a primary determination of the plane of mean high water.

In the discussion of mean sea level it was shown that nine years of tide observations may be taken, for the practical purposes of datum plane determination, as giving a primary determination of the plane of mean sea level. This means that changes in the height of high water due to changes in the height of sea level may be considered as eliminated within a period of nine years. In a period of nine years the only periodic change in high water that has not gone through a complete cycle is that due to the change in the longitude of the moon's node, and this can be corrected by means of the factor in Table 6. It is obvious, therefore, that the plane of mean high water from nine years of observations, when corrected by Table 6, may be taken to constitute a primary determination of that plane.

The procedure and also the accuracy of the determination of the plane of mean high water from nine years of observations may be exemplified by such determinations from the observations at Fort Hamilton, N. Y., on the Atlantic coast, and at San Francisco, Calif., on the Pacific coast. For the former place the series of observations

at hand covers the years 1893–1922, and for the latter the years 1898–1925. Three nine-year groups can therefore be used at each station. From Table 5 the ratio of K_1+O_1 to M_2 is found to be 0.2 for New York Harbor and 1.1 for San Francisco. With these values the correction factors for each nine-year group are derived from Table 6.

Attention must here be directed to the fact that the heights of high water as tabulated from the tide record refer to some fixed tide staff, the zero of which is at an arbitrary elevation. In themselves, therefore, the heights of high water as they appear in a tabulation are in a sense arbitrary, for they have meaning only in reference to the particular tide staff to which they refer. For example, two independent observers establishing tide staffs near each other would record the same high water differently. One might find it reading 10 feet on his staff, while the other, near by, might find the same high water reading 15 feet on his staff, depending on the difference in elevation of the zeros of the two staffs.

But if the height of high water, instead of being given with reference to a tide staff, is given with reference to sea level or half-tide level, all arbitrariness is eliminated. For, no matter what the relative elevations of the staffs be, all observers at any one place would, within the limits of error, derive the same height of high water above sea level; or, if half-tide level instead of sea level is used, they would derive the same height of high water above half-tide level. And in this connection it should be noted that, while sea level and half-tide level can not be used as synonomous with each other, either plane may be used as reference for high water. In the illustrative examples given below it will obviously be most convenient to use but one of these planes, and half-tide level will be used, since this is derived directly from the tabulation of the high and low waters.

It is clear that in correcting the height of high water for the longitude of the moon's node the factors of Table 6 are to be applied to the height of high water above half-tide level and not to the height of high water as given with reference to a tide staff. Below are given in tabular form the data used in deriving the planes of mean high water, from groups of nine years, at Fort Hamilton and at San Francisco.

Mean high water from nine years of observations

Nine-year group	Fort Hamilton, N. Y.					Nine-year group	San Francisco, Calif.				
	High water on staff	Half-tide level on staff	High water above half-tide level	Factor from Table 6	Mean high water above half-tide level		High water on staff	Half-tide level on staff	High water above half-tide level	Factor from Table 6	Mean high water above half-tide level
	Feet	*Feet*	*Feet*		*Feet*		*Feet*	*Feet*	*Feet*		*Feet*
1893–1901	8.18	5.89	2.29	1.012	2.32	1898–1906	10.55	8.57	1.98	0.990	1.96
1902–1910	8.32	5.94	2.38	.986	2.35	1907–1915	10.59	8.64	1.95	1.009	1.97
1911–1919	8.32	5.98	2.34	1.015	2.38	1916–1924	10.57	8.58	1.99	.992	1.97

The second and eighth columns of the table give the average heights of all the high waters referred to the staff at each of the places for the nine-year periods in question and are taken from

the tabulations. The third and ninth columns give the heights of half-tide level on staff for the corresponding periods, while the fourth and tenth columns give the heights of high water above half-tide level for these periods. It will be noted that the values in the columns giving high water above half-tide level are, for each of the stations, more nearly equal than are the values for high water on staff. This, obviously, is due to the elimination of the variations in high water due to changes in sea level.

The direct average of the observed heights of high water over a period of 19 years constitutes the best determination of the plane of mean high water. For Fort Hamilton this gives 2.34 feet above half-tide level for the 19-year period 1893–1911 and 2.37 feet for the 19-year period 1904–1922. For San Francisco high water above half-tide level is 1.97 feet for each of the two overlapping 19-year periods 1898–1916 and 1907–1925. Comparing the values derived for the plane of mean high water from the nine-year periods in the table above, it is seen that these are within 0.01 or 0.02 foot of the 19-year values for both Fort Hamilton and San Francisco. It appears, therefore, that nine years of observations, corrected by the factor for the longitude of the moon's node, give the average rise of high water above half-tide level correct within one or two hundredths of a foot; or, expressed as a percentage of the rise of high water above half-tide level, correct within 1 per cent. Hence the plane of mean high water based on nine years of observations and corrected to a mean value may be taken to constitute a primary determination of this plane.

SECONDARY DETERMINATION

For a series of observations varying in length from less than a month to one or more years two different methods may be used to derive a mean value of the plane of mean high water. The results from the tabulation may be corrected either by Table 6 or by comparison with the results of simultaneous observations at a near-by station for which a primary determination is at hand. The procedure and accuracy attainable will be illustrated below for periods of various lengths.

Year.—To exemplify the determination of the plane of mean high water from one year of observations, the observations at Fort Hamilton and at San Francisco for each of the three years 1913, 1918, and 1922 may be used. These three years are chosen because the factor in Table 6 has for 1913 a maximum value, for 1918 approximately a mean value, and for 1922 a minimum value. The plane of mean high water will be determined first by means of Table 6 and then by means of comparison with simultaneous observations at some near-by station. As before, the ratios of K_1+O_1 to M_2 for use in Table 6 are derived from Table 5.

It is clear that the correction factors are to be applied to the rise of high water above sea level or half-tide level and not to the reading of high water on the staff. Therefore in the examples following it will be unnecessary to give the height of high water on staff and the corresponding height of half-tide level on staff. The height of high water above half-tide level will be given instead. These heights, the correction factors, and the derived values of mean high water for

Fort Hamilton and San Francisco for the years 1913, 1918, and 1922 are given below in tabular form.

Mean high water from one year of observations

Year	Fort Hamilton, N. Y.			San Francisco, Calif.		
	High water above half-tide level	Factor from Table 6	Mean high water above half-tide level	High water above half-tide level	Factor from Table 6	Mean high water above half-tide level
	Feet		*Feet*	*Feet*		*Feet*
1913	2.29	1.029	2.36	1.96	1.017	1.99
1918	2.39	.995	2.38	1.98	.997	1.97
1922	2.45	.971	2.38	1.99	.983	1.96

For Fort Hamilton the values of mean high water derived from the three years of observations are in very good agreement with each other and with the primary value of 2.37 feet. The primary determination of mean high water at San Francisco makes it 1.97 feet above half-tide level. The greatest difference between the values derived from each of the three years and this primary value is 0.02 foot. It may be taken, therefore, that with respect to half-tide level a secondary determination of mean high water from one year of observations, when corrected by Table 6, gives results correct to within several hundredths of a foot or within 2 per cent of the rise.

The method of deriving mean high water by comparison of simultaneous observations may be illustrated by the observations at the two stations used above. It is important to note that, in choosing a station for comparison, care must be taken to choose one which has a like ratio of K_1+O_1 to M_2. Since along the Atlantic coast of the United States this ratio is very nearly the same, any station along that coast may be used as comparison station for Fort Hamilton, though, obviously, the nearer the station the better. Atlantic City, N. J., would undoubtedly be the best comparison station, but, to test the applicability of a station a considerable distance away, Fernandina, Fla., will be taken. For San Francisco, San Diego, Calif., will be taken as comparison station.

The determination of mean high water by the method of comparison of simultaneous observations is based on the fact that for similar types of tide the percentage increase or decrease in the rise of high water is the same for a given period of time. Thus, a primary determination of mean high water at Fernandina for the 19-year period 1905–1923 gives it as 3.00 feet above half-tide level. For the year 1913 high water at Fernandina was 2.94 feet above half-tide level. It follows, therefore, that the 1913 value must be increased by the ratio $3.00 \div 2.94$ or 1.020 to bring it to the mean value. And hence for 1913 the rise of high water at other stations on the Atlantic coast having a similar type of tide must be multiplied by 1.020 to give mean high water, if Fernandina is used as a comparison station.

Below, in tabular form, are given the results for the three years at both Fort Hamilton and San Francisco. For San Diego a primary determination of mean high water above half-tide level is 1.96 feet.

In the table mean high water at the comparison station is given in abbreviated form as MHW.

Mean high water from one year of observations

Year	Fort Hamilton, N. Y. (Comparison station, Fernandina, MHW=3.00 feet)				San Francisco, Calif. (Comparison station, San Diego, MHW=1.96 feet)			
	High water above half-tide level	High water at comparison station	Factor for correction	Mean high water above half-tide level	High water above half-tide level	High water at comparison station	Factor for correction	Mean high water above half-tide level
	Feet	*Feet*		*Feet*	*Feet*	*Feet*		*Feet*
1913	2.29	2.94	1.020	2.34	1.96	1.93	1.016	1.99
1918	2.39	3.01	.997	2.38	1.98	1.96	1.000	1.98
1922	2.45	3.09	.971	2.38	1.99	1.97	.995	1.98

The primary determination of mean high water above half-tide level at Fort Hamilton for the 19-year period 1904–1922 being 2.37 feet, it is seen that the correction to mean value of one year of observations by the method of comparison of simultaneous observations gives results correct within 0.03 foot. For San Francisco the primary value of mean high water is 1.97 feet, so that the method of comparison gives for that station results correct to within 0.02 foot. This is also the precision with which mean high water was derived through correction by means of Table 6. It may be taken, therefore, that high water derived from one year of observations can be corrected to a mean value by either method within several hundredths of a foot of a primary value, or correct to within 2 per cent of the rise of high water.

Month.—The determination of the plane of mean high water from a month of observations is in all respects similar to that for a year. The values in Table 6 are computed for July 1 of each year, so that values for each month of the year can be determined by interpolation. As an example of the accuracy attainable from one month of observations, by use of Table 6, the observations for the years 1913 and 1918 at Fort Hamilton and San Francisco may be used. For purpose of illustration it will be sufficient to take every other month of each year.

Since the months of the moon's phase, distance, and declination are respectively 29½, 27½, and 27⅓ days, the variations in the height of high water from day to day due to changes in the position of the moon will be more nearly eliminated in 29 days than in a month of 30 or 31 days. For that reason it is better to use a group of 29 days in deriving mean high water by use of Table 6. In the table below the values for each month are for a group of 29 days, beginning with the 1st of the month.

Mean high water from one month of observations, Fort Hamilton, N. Y.

Month	1913			1918		
	High water above half-tide level	Factor from Table 6	Mean high water above half-tide level	High water above half-tide level	Factor from Table 6	Mean high water above half-tide level
	Feet		*Feet*	*Feet*		*Feet*
January	2.35	1.029	2.42	2.35	1.000	2.35
March	2.33	1.029	2.40	2.37	.998	2.37
May	2.28	1.029	2.35	2.42	.996	2.41
July	2.29	1.029	2.36	2.42	.995	2.41
September	2.27	1.029	2.34	2.39	.993	2.37
November	2.31	1.029	2.38	2.36	.991	2.34

For Fort Hamilton the primary determination of the plane of mean high water is 2.37 feet above half-tide level. As derived in the table above from a month of observations, the greatest difference from the primary value is 0.05 foot. It appears, therefore, that one month of observations, when corrected by Table 6, gives the plane of mean high water, with respect to half-tide level, correct to within 0.1 foot. The results for the same years at San Francisco follow:

Mean high water from one month of observations, San Francisco, Calif.

Month	1913			1918		
	High water above half-tide level	Factor from Table 6	Mean high water above half-tide level	High water above half-tide level	Factor from Table 6	Mean high water above half-tide level
	Feet		*Feet*	*Feet*		*Feet*
January	2.01	1.017	2.04	1.99	1.000	1.99
March	1.97	1.017	2.00	1.94	.999	1.94
May	1.92	1.017	1.95	1.94	.998	1.94
July	1.92	1.017	1.95	1.98	.997	1.97
September	1.99	1.017	2.02	1.99	.996	1.98
November	1.99	1.017	2.02	2.01	.995	2.00

For San Francisco a primary determination of mean high water above half-tide level is 1.97 feet. The greatest difference in the table above from this mean value is that for January, 1913, which is 0.07 foot. It may therefore be taken that one month of observations, corrected by Table 6, will determine the plane of mean high water, with respect to half-tide level, within 0.1 foot.

From a month of observations the plane of mean high water may also be determined by comparison with simultaneous observations at some station having a similar type of tide. As an example, we may take the same stations used for deriving the plane by means of Table 6. It will be better, however, to choose a comparison station for Fort Hamilton not so far away as Fernandina, which is about 800 miles distant, since within a month meteorological conditions may be quite different at the two places. Therefore Atlantic City will be chosen as comparison station for Fort Hamilton, but San Diego will again be used as comparison station for San Francisco.

At Atlantic City the primary determination of mean high water is 2.02 feet above half-tide level. For the month of January, 1913,

high water was 1.96 feet above the corresponding monthly value of half-tide level, and therefore the factor 2.02÷1.96=1.031 is indicated as the correction factor to bring the monthly value for January to a mean value. At Fort Hamilton for that same month high water was 2.35 feet above half-tide level. Hence mean high water would be 2.35×1.031=2.42 above half-tide level. The results for every other month of the two years 1913 and 1918 at Fort Hamilton are shown below.

Mean high water from one month of observations, Fort Hamilton, N. Y.

Month	1913				1918			
	High water above half-tide level	High water at comparison station	Factor for correction	Mean high water above half-tide level	High water above half-tide level	High water at comparison station	Factor for correction	Mean high water above half-tide level
	Feet	*Feet*		*Feet*	*Feet*	*Feet*		*Feet*
January	2.35	1.96	1.031	2.42	2.35	1.98	1.020	2.40
March	2.33	1.99	1.015	2.36	2.37	2.05	.985	2.33
May	2.28	1.95	1.036	2.36	2.42	2.03	.995	2.41
July	2.29	1.95	1.036	2.37	2.42	2.06	.981	2.37
September	2.27	1.97	1.025	2.33	2.39	2.04	.990	2.37
November	2.31	1.97	1.025	2.37	2.36	1.99	1.015	2.40

Comparing the values derived for the plane of mean high water in the above table with the primary determination of 2.37 feet for Fort Hamilton, it is seen that from a month of observations this plane, with respect to half-tide level, can be derived within 0.1 foot of a primary determination, by comparison with simultaneous observations.

For San Francisco the results derived for every other month of the years 1913 and 1918 are given in tabular form below. The comparison station is San Diego, at which the plane of mean high water is 1.96 feet above half-tide level.

Mean high water from one month of observations, San Francisco, Calif.

Month	1913				1918			
	High water above half-tide level	High water at comparison station	Factor for correction	Mean high water above half-tide level	High water above half-tide level	High water at comparison station	Factor for correction	Mean high water above half-tide level
	Feet	*Feet*		*Feet*	*Feet*	*Feet*		*Feet*
January	2.01	1.94	1.010	2.03	1.99	1.95	1.005	2.00
March	1.97	1.98	.990	1.95	1.94	1.98	.990	1.92
May	1.92	1.92	1.021	1.96	1.94	1.95	1.005	1.95
July	1.92	1.86	1.054	2.02	1.98	1.99	.985	1.95
September	1.99	1.99	.985	1.96	1.99	1.97	.995	1.98
November	1.99	1.91	1.026	2.04	2.01	1.91	1.026	2.06

The primary determination of mean high water above half-tide level at San Francisco is 1.97 feet. From the values in the above table it is seen that the plane of mean high water, determined from one month of observations by comparison with simultaneous observations at a suitable station, will be correct within about 0.1 foot.

This, likewise, was the degree of accuracy obtained from a month of observations when corrected by means of Table 6.

Day.—Where the plane of mean high water is to be determined from one or several days of observations, the most practical method is by means of comparison with simultaneous observations, or with the predicted tides, at some suitable station; that is, at some station having a similar type of tide. The accuracy with which this plane may be determined from a day of observations may be illustrated by several such determinations for Fort Hamilton and San Francisco during the month of May, 1913, making use, as before, of Atlantic City and San Diego as comparison stations. The data for every fifth day of the month, in tabular form, are given below.

Mean high water from one day of observations

Date	Fort Hamilton, N. Y. (Comparison station, Atlantic City, MHW = 2.02 feet)				San Francisco, Calif. (Comparison station, San Diego, MHW = 1.96 feet)			
	High water above half-tide level	High water at comparison station	Factor for correction	Mean high water above half-tide level	High water above half-tide level	High water at comparison station	Factor for correction	Mean high water above half-tide level
1913	*Feet*	*Feet*		*Feet*	*Feet*	*Feet*		*Feet*
May 1	1.85	1.50	1.35	2.50	1.65	1.80	1.09	1.80
May 6	2.53	2.18	.93	2.35	2.05	2.20	.89	1.82
May 11	2.13	1.78	1.13	2.41	1.80	1.48	1.32	2.38
May 16	2.58	2.27	.89	2.30	2.20	2.25	.87	1.91
May 21	2.82	2.45	.82	2.31	2.20	2.35	.83	1.83
May 26	1.72	1.45	1.39	2.39	1.55	1.25	1.57	2.43
May 31	1.90	1.57	1.29	2.45	1.58	1.65	1.19	1.88

It is to be noted that the heights given in the second and third columns of the table for Fort Hamilton and in the first and second columns for San Francisco are not the heights of high water measured on the staff but the heights of high water above the half-tide levels for the respective dates; that is, from the height of high water for any day in question the height of half-tide level for that day was subtracted.

For Fort Hamilton the primary determination of mean high water is 2.37 feet above half-tide level. For the seven days of May, 1913, shown in the table above, the mean high-water values derived in no case differ by more than 0.1 foot from the primary determination. For San Francisco the primary determination of mean high water is 1.97 feet above half-tide level, and here the values of mean high water from one day of observations are in every case within 0.4 foot or less of the primary value. In general, it may be taken that one day of observations, after correction by comparison with a station even several hundred miles away, will determine the plane of mean high water with respect to half-tide level within half a foot.

At times it becomes necessary to determine the plane of mean high water from one or several days of observations when simultaneous observations at some other station are not at hand for comparison. In such cases the predicted tides for the days in question at some suitable station, as given in the tide tables, may be used for comparison. It is to be recalled that the variation from day to

day in height of high water above sea level is very largely of a periodic character; hence this can be taken account of in predicting the tides. As examples in the use of predictions for comparison, we may take the same days in May, 1913, at San Francisco and Fort Hamilton as used above. To illustrate the various steps involved, it will be of advantage to give as a part of the data also the heights of high water on the staff for each day in question.

The tide tables are published annually and are available about six months prior to the year for which the predictions are made. For the year 1913 the tide tables give predictions for two stations but few miles from Fort Hamilton, namely, Governors Island, N. Y., and Sandy Hook, N. J. Either of these would be suitable for use as comparison station for Fort Hamilton. But to emphasize the fact that, if the type of tide is about the same, a station at a considerable distance may be taken for comparison, we may use the predictions for Portland, Me., some 300 miles northeast of Fort Hamilton. That station is suitable, since Table 5 shows approximately equal ratios for K_1+O_1 to M_2 at the two stations. From the table of tidal data in the tide tables we find the mean range of tide at Portland to be 8.9 feet. And since the range is the distance between low water and high water, it follows that the plane of mean high water lies 4.45 feet above the plane of half-tide level. The data used and the results derived for Fort Hamilton are given in tabular form below. The half-tide level for each day was derived as the half sum of the heights of the average high water and the average low water of the day.

Mean high water from one day of observations, Fort Hamilton, N. Y.

[Comparison station, Portland (predictions), MHW = 4.45 feet above half-tide level]

Date	High water on staff	Half-tide level on staff	High water above half-tide level	High water at comparison station	Factor for correction	Mean high water above half-tide level
1913	*Feet*	*Feet*	*Feet*	*Feet*		*Feet*
May 1	7.60	5.75	1.85	3.40	1.31	2.42
May 6	8.35	5.82	2.53	4.32	1.03	2.61
May 11	7.85	5.72	2.13	4.13	1.08	2.30
May 16	8.70	6.12	2.58	4.62	.96	2.49
May 21	8.70	5.88	2.82	4.95	.90	2.54
May 26	7.80	6.08	1.72	3.45	1.29	2.22
May 31	7.80	5.90	1.90	3.48	1.28	2.43

The values derived for mean high water at Fort Hamilton by comparison with predictions at Portland agree well with the values derived for the same days by comparison with simultaneous observations at Atlantic City (p. 91), the greatest difference being 0.3 foot Comparing these values with the primary determination of mean high water at Fort Hamilton, 2.37 feet, it is seen that in no case is the difference greater than 0.2 foot.

The tide tables give predictions for San Diego, Calif., and we may therefore use these for deriving a mean value of high water from the observations at San Francisco for every fifth day of May, 1913. The procedure is in all respects similar to that used in deriving this plane for Fort Hamilton in the example just preceding. Since the mean range of the tide at San Diego is 3.9 feet, the plane of mean high water is 1.95 feet above half-tide level.

Mean high water from one day of observations, San Francisco, Calif.

[Comparison station, San Diego (predictions), MHW = 1.95 feet above half-tide level]

Date	High water on staff	Half-tide level on staff	High water above half-tide level	High water at comparison station	Factor for correction	Mean high water above half-tide level
1913	*Feet*	*Feet*	*Feet*	*Feet*		*Feet*
May 1	9.75	8.10	1.65	1.63	1.20	1.98
May 6	10.60	8.55	2.05	2.05	.95	1.95
May 11	10.30	8.50	1.80	1.50	1.30	2.34
May 16	10.45	8.25	2.20	2.15	.91	2.00
May 21	10.75	8.55	2.20	2.15	.91	2.00
May 26	10.25	8.70	1.55	1.13	1.73	2.68
May 31	10.20	8.62	1.58	1.50	1.30	2.05

The values derived for mean high water at San Francisco for the seven days above are, with one exception, within half a foot of the primary value, which is 1.97 feet. For May 26 the value is 0.7 foot greater than the primary value, and this is due to the very large correction factor for that day, namely, 1.73. In general, it appears that from one day of observations the plane of mean high water, with respect to half-tide level, can be determined within half a foot of its primary value by comparing with predicted tides at a suitable station.

EXAMPLES FROM A STATION HAVING A LARGE RANGE OF TIDE

In the examples illustrating the determination of the plane of mean high water from observations covering various periods of time, Fort Hamilton and San Francisco were chosen because of the long series of continuous observations at these places. It happens, however, that the range of the tide at these places is moderate, averaging 4.7 feet at Fort Hamilton and 3.9 feet at San Francisco. This is reflected in the moderate rise of high water above half-tide level, the primary determination of which was found to be 2.37 feet for Fort Hamilton and 1.97 feet for San Francisco. The question may therefore arise whether at places having large ranges of tide, mean high water can be determined with as much precision as at places having moderate ranges.

There are but few regions in the United States where the range of tide averages more than 10 feet, and in these regions there are unfortunately no long-continued series of tide observations. Thus, at Eastport, Me., the range of the tide is somewhat more than 18 feet, but the observations are not of sufficient extent to give a primary determination of mean high water. At Anchorage, Alaska, the tide has a very considerable range, averaging more than 25 feet. But here, likewise, the observations do not cover a sufficient period of time. At Ketchikan, Alaska, however, there are continuous observations since 1919 and scattered observations prior to that time. Here the tide has a range of 13 feet, and we may therefore use these observations to illustrate the determination of the plane of mean high water at a place having a relatively large range of tide.

Taking first the yearly values of high water, the table below gives in the successive columns for each year the reading of high water on staff, the corresponding reading of half-tide level on staff, high water

above half-tide level, correction factor from Table 6, and the derived value for mean high water above sea level. From Table 5 (p. 82), the value of K_1+O_1 to M_2 for Ketchikan is 0.4, and it is with this value that the correction factors are derived from Table 6.

Mean high water from one year of observations, Ketchikan, Alaska

Year	High water on staff	Half-tide level on staff	High water above half-tide level	Factor from Table 6	Mean high water above half-tide level
	Feet	*Feet*	*Feet*		*Feet*
1919	20.88	14.30	6.58	0.985	6.48
1920	20.87	14.23	6.64	.979	6.50
1921	20.96	14.28	6.68	.973	6.50
1922	20.75	14.09	6.66	.972	6.47
1923	20.92	14.25	6.67	.972	6.48
1924	20.83	14.19	6.64	.976	6.48
1925	20.85	14.26	6.59	.982	6.47

The average of the mean high waters for the seven years from 1919 to 1925 is 6.48 feet above half-tide level; and since this, obviously, must be very close to a primary determination, it will be taken as a primary value. For each of the seven years it is seen that, after correction by Table 6, the mean high water derived is within 0.02 foot of a primary determination. This agrees very well with the results derived for Fort Hamilton and San Francisco from a year of observations. Expressed as a percentage of the rise, one year of observations at Ketchikan, after correction, determines the plane of mean high water with reference to half tide within one-half of 1 per cent of the rise of high water.

Taking up now the determination of mean high water from a month of observations at Ketchikan, we may illustrate the precision by taking every other month of three years, namely, the first year, the middle year, and the last year, which are, respectively, 1919, 1922, and 1925. It will be unnecessary in this case to give the details of high-water and half-tide level on staff. It will be recalled that for correction, by means of Table 6, 29 days of observations are better than a calendar month. In the table below the data are for 29-day groups, beginning the 1st of the month.

Mean high water from one month of observations, Ketchikan, Alaska

Month	1919			1922			1925		
	High water above half-tide level	Factor from Table 6	Mean high water above half-tide level	High water above half-tide level	Factor from Table 6	Mean high water above half-tide level	High water above half-tide level	Factor from Table 6	Mean high water above half-tide level
	Feet		*Feet*	*Feet*		*Feet*	*Feet*		*Feet*
January	6.45	0.990	6.39	6.55	0.972	6.37	6.51	0.979	6.37
March	6.64	.988	6.56	6.60	.972	6.42	6.40	.980	6.27
May	6.54	.986	6.45	6.60	.972	6.42	6.36	.981	6.24
July	6.59	.985	6.49	6.77	.972	6.58	6.56	.982	6.44
September	6.65	.984	6.54	6.81	.972	6.62	6.82	.984	6.71
November	6.58	.983	6.47	6.60	.972	6.42	6.62	.985	6.52

Accepting 6.48 feet above half-tide level, derived from the seven years of observations, as giving a very close approximation to the primary value of mean high water at Ketchikan, it is seen that the values derived for the different months of the three years differ by as much as 0.24 foot from this primary value. Of those 18 monthly values, six differ by more than 0.1 foot from the primary value. In part, however, these relatively large differences are due to the fact that at Ketchikan there is an annual variation in the rise of high water above sea level which is characteristic of tidal streams subject to considerable variation in the fresh-water flow.

Prior to 1919 tide observations were made at Ketchikan for a month or more during the summers of 1911, 1914, and 1915. It will be of interest to derive mean high water from one month of observations for each of those years for comparison with the results obtained above. In the table following, the data for September, 1911, June and September, 1914, and June and September, 1915, are given. As before, the data refer to groups of 29 days, beginning with the 1st of the month.

Mean high water from one month of observations, Ketchikan, Alaska

Month	High water on staff	Half-tide level on staff	High water above half-tide level	Factor from Table 6	Mean high water above half-tide level
	Feet	*Feet*	*Feet*		*Feet*
September, 1911	20.31	13.90	6.41	1.024	6.56
June, 1914	18.41	12.14	6.27	1.027	6.44
September, 1914	18.90	12.42	6.48	1.026	6.65
June, 1915	18.27	12.02	6.25	1.022	6.39
September, 1915	18.56	12.18	6.38	1.020	6.51

Since the primary value of mean high water at Ketchikan is 6.48 feet above half-tide level, it is seen that the values derived from the five months in 1911, 1914, and 1915 are approximately within 0.1 foot of the primary value. Attention is again directed to the fact that the value derived for the month of September in each of the three years is above the mean value. This, it will be recalled, was found to be the case for the value derived for September, 1919, 1922, and 1925, when mention was made of the existence in the Ketchikan tide of an annual variation in the rise of high water above sea level.

From the foregoing it appears that, in general, a month of observations at Ketchikan will determine mean high water in relation to sea level approximately within 0.1 foot of a primary value, despite the fact of a range of 13 feet and of an annual variation in the rise of high water above sea level. And it was within 0.1 foot that a month of observations was found to give mean high water at San Francisco and Fort Hamilton.

To illustrate the accuracy with which mean high water may be derived from a day of observations at Ketchikan, it will be of advantage to test both the method of comparison with simultaneous observations and the method of comparison with predictions. From Table 5 (p. 82) it is seen that Seldovia has a ratio of K_1+O_1 to M_2 of 0.41, which is very close to the Ketchikan ratio of 0.43. Seldovia

is therefore suitable as a comparison station. Simultaneous observations are at hand for September, 1911; and since the year 1917 predictions for Seldovia have been included in the tide tables issued annually, in advance, by the Coast and Geodetic Survey.

The mean range of the tide at Seldovia is 15.4 feet; hence mean high water is 7.7 feet above half-tide level. In the table below the plane of mean high water is determined from each of four days of observations in September, 1911, and September, 1925. For 1911 the values of high water above half-tide level at Seldovia are taken from observations, while for 1925 they are taken from the tide tables.

Mean high water from a day of observations, Ketchikan, Alaska

[Comparison station, Seldovia, MHW=7.7 feet above half-tide level]

Date	High water on staff	Half-tide level on staff	High water above half-tide level	High water at comparison station	Factor for correction	Mean high water above half-tide level
1911	*Feet*	*Feet*	*Feet*	*Feet*		*Feet*
September 1	16.8	13.7	3.1	3.1	2.48	7.7
September 10	22.2	13.6	8.6	10.6	.73	6.3
September 20	22.1	14.3	7.8	9.6	.80	6.2
September 30	16.9	14.0	2.9	2.9	2.66	7.7
1925						
September 1	22.9	14.0	8.9	10.7	.72	6.4
September 10	18.0	14.2	3.8	4.0	1.92	7.3
September 20	21.8	13.6	8.2	9.9	.78	6.4
September 30	22.5	13.7	8.8	10.6	.73	6.4

The primary value of mean high water at Ketchikan is, to the nearest tenth of a foot, 6.5 feet above half-tide level. Comparing with this primary value the values of mean high water from each of the eight days of observations above, it is seen that of these eight determinations five differ by less than 0.5 foot from the primary value, one differs by 0.8 foot, and two by 1.2 feet. It will be noted, however, that the larger differences occur on days when the factor for correction has a value considerably greater than 1; that is, on days when the rise of high water was much less than the average.

SUMMARY

In the preceding sections it was found that the height to which high water rises varies considerably from day to day and from month to month and, in lesser degree, from year to year. This variation is of two kinds—the first, in response to changes in the phase, distance, and declination of the moon; and the second, in response to the variation in sea level. With the exception of the variation due to change in longitude of the moon's node, the variations due to changes in the position of the moon balance out very largely within a month, so that in a sense the variation due to changes in sea level is the primary variation.

It was found, too, that the rise of high water above half-tide level, from observations covering periods of a month or more, may be corrected to a mean value either by factors derived from theoretical considerations or by comparison with simultaneous observations at some suitable primary station. But in this comparison type

of tide, and not nearness, determined suitability. In this regard the correction of high water to a mean value differs from the like correction of sea level, in which the suitability of a station for comparison purposes depends on the existence of like meteorological conditions. Furthermore, in correcting sea level to a mean value the changes in height at two near-by stations were taken as the same, whereas the changes in the rise of high water are taken as proportional.

In general it may be taken that, when corrected to a mean value, a year of tide observations will determine the rise of high water above half-tide level correct within 0.05 foot, a month within 0.1 foot, and a day within 0.5 foot. However, in regions of large range of tide, and of considerable variation in the rise of high water, a day of observations, especially when the factor for correction differs considerably from 1.00, may give a value differing by a foot or more from a primary value.

It should be noted that mean high water is determined with respect to half-tide level which is itself subject to variations. Hence the accuracy with which the plane of mean high water can be determined depends also on the accuracy with which the plane of half tide level is determined. The degree of accuracy in deriving mean high water, noted above for observations covering various periods of time, refers only to the rise of high water above half-tide level.

To determine the plane of mean high water from any given series of observations the plane of half-tide level must first be determined; then the rise of high water above half-tide level is corrected to a mean value, and this gives the plane of mean high water above the plane of half-tide level.

IX. MEAN LOW WATER

VARIATIONS IN FALL OF LOW WATER

The variations in the fall of low water resemble closely those in the rise of high water, especially in regard to those depending on the moon's position. Not only does the tide rise higher than usual at the times of full and new moon, but it also falls lower, while at the times of the moon's first and third quarters the less-than-average rise of high water is accompanied by a fall in low water also less than average. Similarly, when the moon is in perigee the fall, like the rise, is greater than usual, while at the time of the moon's apogee the rise and also the fall are less than usual.

The periodic semimonthly change in the declination of the moon brings about variations in the fall of low water, causing consecutive low waters to differ. This diurnal inequality in the low waters necessitates the distinction between higher low water and lower low water. When the moon is near its semimonthly maximum declination, the two low waters of a day show the greatest difference in fall, and when the declination of the moon is small—that is, when the moon is near the Equator—the difference between the two low waters is least.

For the sake of clearness it will be of advantage to distinguish between "height of low water" and "fall of low water." The former term will be used when it is the height of low water as

of tide, and not nearness, determined suitability. In this regard the correction of high water to a mean value differs from the like correction of sea level, in which the suitability of a station for comparison purposes depends on the existence of like meteorological conditions. Furthermore, in correcting sea level to a mean value the changes in height at two near-by stations were taken as the same, whereas the changes in the rise of high water are taken as proportional.

In general it may be taken that, when corrected to a mean value, a year of tide observations will determine the rise of high water above half-tide level correct within 0.05 foot, a month within 0.1 foot, and a day within 0.5 foot. However, in regions of large range of tide, and of considerable variation in the rise of high water, a day of observations, especially when the factor for correction differs considerably from 1.00, may give a value differing by a foot or more from a primary value.

It should be noted that mean high water is determined with respect to half-tide level which is itself subject to variations. Hence the accuracy with which the plane of mean high water can be determined depends also on the accuracy with which the plane of half tide level is determined. The degree of accuracy in deriving mean high water, noted above for observations covering various periods of time, refers only to the rise of high water above half-tide level.

To determine the plane of mean high water from any given series of observations the plane of half-tide level must first be determined; then the rise of high water above half-tide level is corrected to a mean value, and this gives the plane of mean high water above the plane of half-tide level.

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The periodic semimonthly change in the declination of the moon brings about variations in the fall of low water, causing consecutive low waters to differ. This diurnal inequality in the low waters necessitates the distinction between higher low water and lower low water. When the moon is near its semimonthly maximum declination, the two low waters of a day show the greatest difference in fall, and when the declination of the moon is small—that is, when the moon is near the Equator—the difference between the two low waters is least.

For the sake of clearness it will be of advantage to distinguish between "height of low water" and "fall of low water." The former term will be used when it is the height of low water as

measured on some fixed staff that is in question. By fall of low water will be understood the depth or distance of low water below sea level. As tabulated from the tide record the height of low water is referred to a staff. For example, the specimen sheet of the tabulation shown in Figures 19 and 20 gives the height of each low water at San Francisco, Calif., for the month of June, 1920, referred to the tide staff at that place.

Glancing down the column of low waters of the tabulation shown on Figures 19 and 20 it will be seen that during the month the height of low water varied considerably. Differences of as much as 4 feet between consecutive low waters occur several times during the month, while the difference between the highest low water and lowest low water for that month was as much as 5 feet. Figure 45, which represents a plotting of those heights, brings out the variations in the successive low waters very clearly.

In addition to illustrating the variations in the height of successive low waters, Figure 45 also brings out the relation of the height of low water to the position of the moon with respect to earth and sun. When the declination of the moon is small—that is, when the moon is close to the Equator (in Figure 45 about the 9th and 22d of the month)—successive low waters do not differ much; near the times of the moon's maximum north or south declination (the 1st, 15th, and 29th of the month) the difference between successive low waters is greatest. The low water on the morning of the 17th was 4.2 feet lower than the preceding low water and 4.0 feet lower than the succeeding one. Figure 45 also brings out the fact that it is not at all unusual for a low water to be above mean sea level, as happened on the 3d, 4th, 5th, 28th, 29th, and 30th of the month.

The various cycles of the moon's motions have different periods. The phase cycle has a period of 29½ days, while that of the moon's parallax or distance is 27½ days, and that of the moon's declination is 27⅓ days. It follows, therefore, that various combinations of the different cycles can occur, introducing different variations in the height of low water at any given place. For example, on June 1, 1920, the augmented fall of low water that should have occurred because of full moon was reduced because the moon was then close to apogee. Similarly, on the 16th and 17th there are not only great differences in the heights of successive low waters due to the moon being close to its maximum fortnightly declination, but the average fall of the low waters on those days is greater than usual because of new moon and perigee.

The variations in low water discussed above are in response to astronomic causes. There are also variations due to changes in wind and weather. Obviously, therefore, the term "plane of low water" is ambiguous, for there are various planes of low water—mean low water, lower low water, higher low water, spring low water, neap low water, perigean low water, apogean low water, etc. The more important of these low-water planes are mean low water, lower low water, spring low water, and monthly lowest low water.

DEFINITION OF MEAN LOW WATER

Mean low water at any place is the average height of all the low waters at that place over a considerable period of time. To deter-

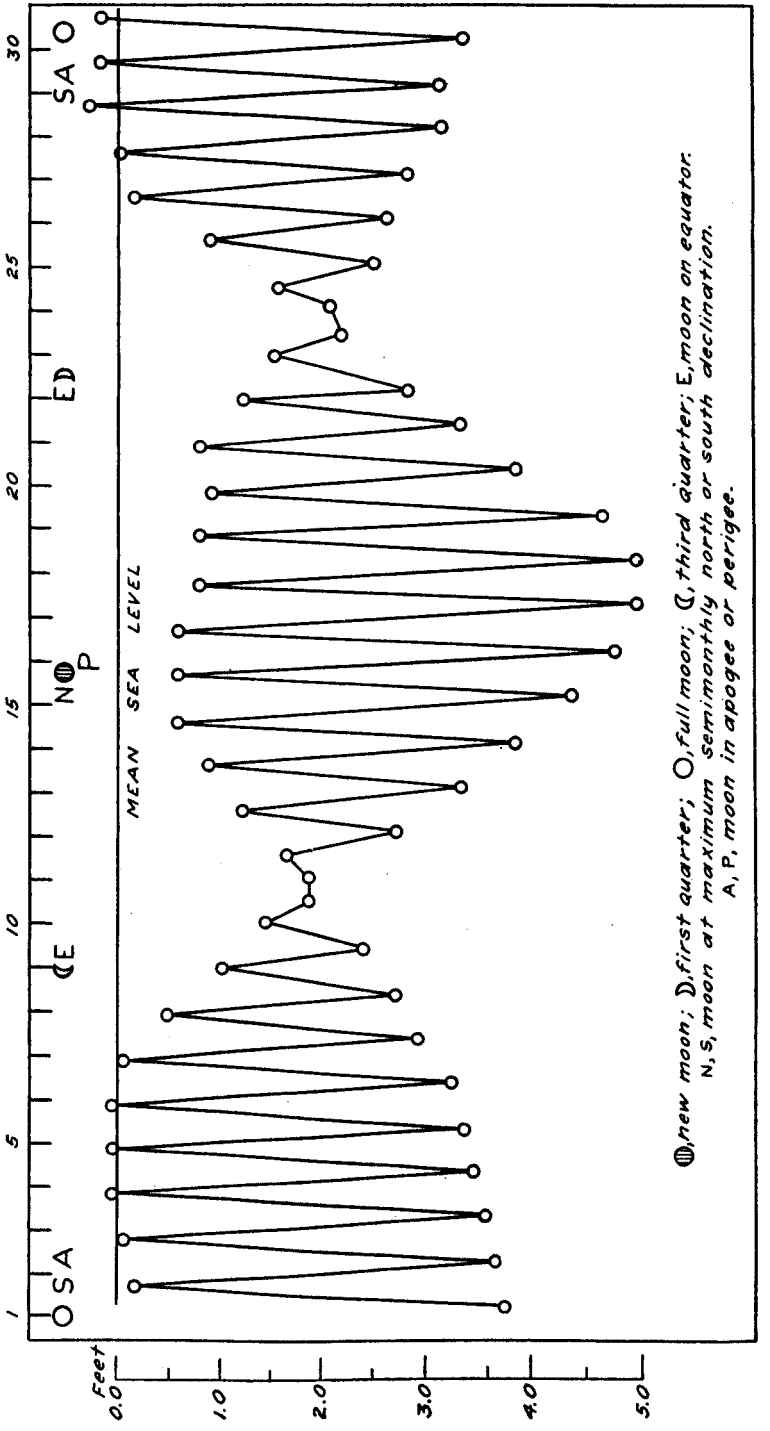

FIG. 45.—Relation of low water to moon's position, San Francisco, Calif., June, 1920

mine how satisfactorily mean low water can be derived from observations covering various periods of time it will be necessary to examine in detail the variations to which low water is subject.

From day to day the variations in successive low waters during a month appear from the heights given in the last column of the tabulation illustrated in Figures 19 and 20, and shown graphically in Figure 45. In part, such variations are due to periodic changes in the position of the moon with regard to earth and sun. But since the tide is an oscillation above and below sea level, it follows that in part, too, the variations in low water from day to day are due to changes in sea level. The relation of daily low water to sea level is brought out in Figure 46, which gives in graphic form the daily heights of low water and of sea level at Atlantic City, N. J., for the month of February, 1925.

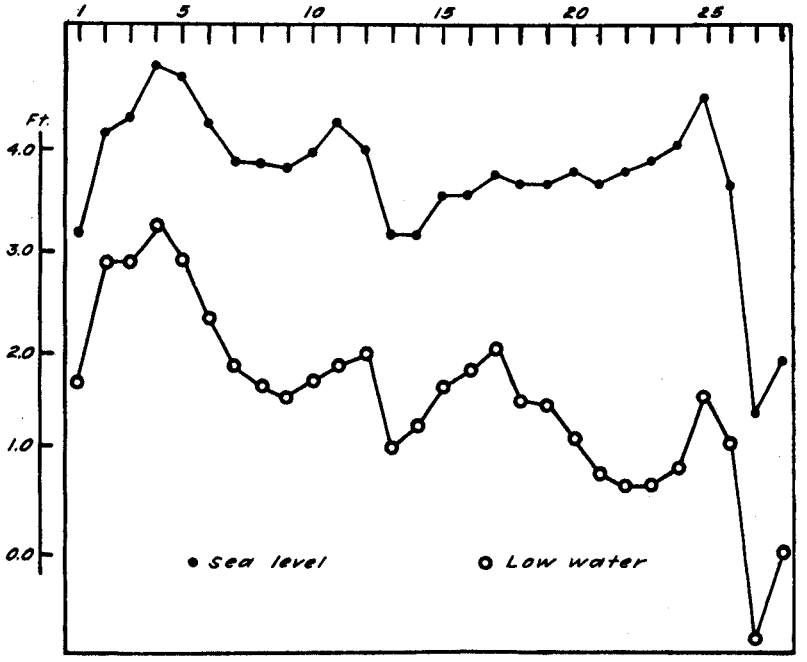

FIG. 46.—Relation of daily low water to sea level, Atlantic City, February 1–25

For the purpose of exhibiting the relationship between low water and sea level it will be sufficient to take the average height of the two low waters of each day, and this was done in Figure 46. The upper diagram of this figure gives the change in the height of sea level from day to day during the month, and the lower diagram gives the average height of the two low waters of each day with reference to sea level.

A glance at Figure 46 is sufficient to show that, notwithstanding the changes in the height of low water from day to day due to changes in the position of the moon, there is a variation also that corresponds to the daily variation of sea level. To the nearest tenth of a foot the average fall of low water at Atlantic City is 2 feet below sea level. For the month shown in Figure 46 the fall of

daily low water varied from less than 1½ feet on the 2d to more than 3 feet on the 25th, this variation reflecting primarily the periodic changes due to the changes in the position of the moon. On the 2d of February, 1925, the moon was one day past her first quarter and only two days before apogee; hence a much reduced fall of low water. On the 23d new moon occurred, and at the same time the moon was only two days past perigee; hence a much increased fall of low water.

MONTHLY LOW WATER

The daily height of low water is found to be subject to relatively large variations both periodic and nonperiodic in character. The periodic variations are principally those related to the phase, distance, and declination of the moon, the periods of which are approximately 29½, 27½, and 27⅓ days, respectively. Within a month, therefore, such variations will be very largely eliminated. And within a month, too, the large variations in sea level tend to balance out. It follows, therefore, that a much closer approximation to the plane of mean low water can be derived from a month of tide observations than from one day.

An examination of the monthly heights of low water reveals changes very much like those found in the case of monthly high waters and discussed in the section devoted to "Monthly high water." In general, consecutive values of monthly low water at any point on the coast are found to vary about 0.1 or 0.2 foot, though occasionally the difference may be more than half a foot; while within the same year two monthly heights of low water may differ by as much as a foot. The periodic variation due to the moon's changing position being very largely eliminated within a month, it is obvious that monthly low water follows the variations in sea level much more closely than does daily low water. In consequence, monthly low water is subject to the annual variation characteristic of monthly sea level. This is shown diagrammatically in Figure 47 for three stations, one on each of the coasts of continental United States.

For each of the three stations of Figure 47 the curve of annual variation of low water resembles closely the corresponding curve of annual variation in sea level shown, respectively, in Figures 28, 29, and 30. This variation of monthly low water, like the corresponding variation in sea level, has definite regional characteristics. Monthly low water at any point is thus subject to variations both periodic and nonperiodic in character, but in both of these it follows the like variations in sea level at that point.

YEARLY LOW WATER

Within a year the annual variation in low water is eliminated, and hence yearly values of low water differ much less than monthly values. Figure 43, showing the variations in the yearly heights of high water, may also be taken to represent the corresponding changes in the yearly heights of low water at the same stations. Generally, consecutive yearly heights of low water at any point differ by not more than several hundredths of a foot, though at times this dif-

ference may be as much as a quarter of a foot. Within a period of 20 years, however, yearly values of low water may differ by as much as half a foot, even at stations on the open coast free from the disturbing effects due to large fluctuations in river discharge.

A comparison of yearly values of low water at any place with the corresponding values of sea level shows that yearly low water follows yearly sea level more closely than monthly low water follows monthly sea level. But the correspondence is not exact; the fall of low water below sea level is found to vary somewhat from year to year. In Figure 48, the fall of yearly low water below sea level is shown for three stations, one each on the Atlantic, Gulf, and Pacific coasts.

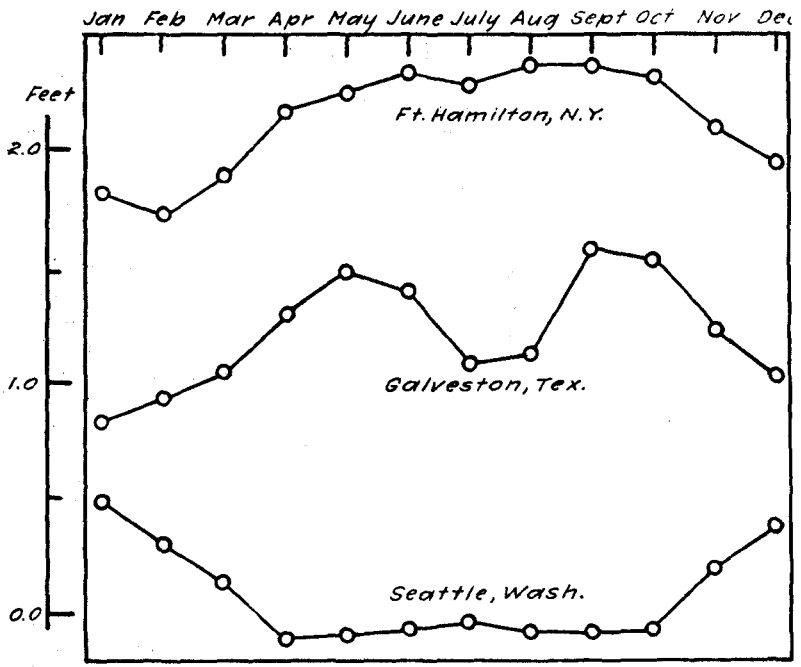

FIG. 47.—Annual variation in low water

The horizontal lines associated with the diagrams represent for each of the stations the average distance of low water below sea level derived from the whole series of observations at that station, this distance in feet being given by the figures at the right or left of the horizontal lines. For Fort Hamilton the periodic variation in the fall of low water, with a period of about 19 years, is very definitely brought out with minima in 1894 and 1913 and a maximum in 1903. For Galveston a smoothing of the curve indicates minima about 1904 and 1923 and a maximum about 1915. For San Francisco a smoothing of the curve indicates minima in the fall of low water below sea level in 1899 and 1915 and maxima in 1904 and 1924.

Comparing the curves of Figure 48 with the corresponding curves of Figure 44, which represent the variation in the rise of yearly high water above sea level, it will be noted that the two sets of curves are

complementary. In other words, when the rise of high water from sea level is above the average, the fall of low water below sea level will be greater than the average, and vice versa. This follows from theoretical considerations of an astronomic character, which indicate, with respect to sea level, a periodic variation in the rise and fall of the tide, with a period of 18.6 years depending on the longitude of the moon's node. And, as mentioned in connection with the rise of high water, the effect of this periodic variation is different on the daily and semidaily constituents of the tide. Hence, in computing factors to correct the fall of low water for the longitude of the moon's node, account must be taken of the relative magnitudes of the daily

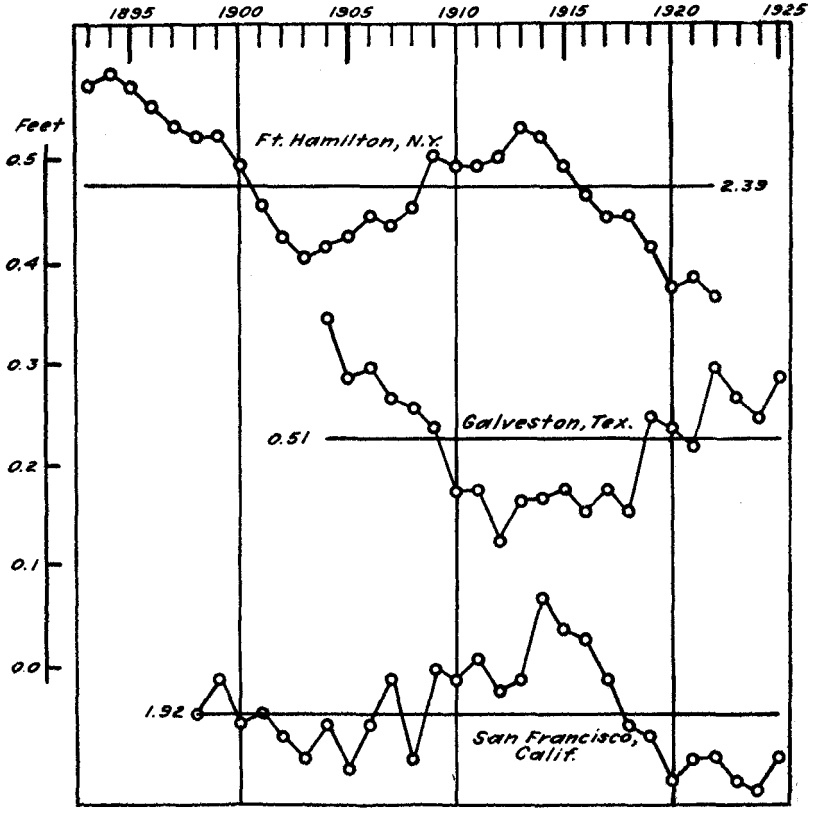

FIG. 48.—Yearly low water below sea level

and semidaily constituents of the tide. Table 5 (p. 82) gives the ratio of these constituents at a number of places on the coasts of continental United States and Alaska. With these ratios Table 6 (p. 83) may be used for correcting the fall of low water derived from short series of observations to mean values.

PRIMARY DETERMINATION

Since the variations to which low water is subject are precisely of the same nature as those of high water, it follows that a direct primary determination of the plane of mean low water requires 19 years

of observations. It follows also that nine years of observations, when corrected for the longitude of the moon's node, may be taken to give a primary determination of the plane of mean low water.

To exemplify the accuracy with which the plane of mean low water may be derived from nine years of observations we may take the stations used in the determination of the plane of mean high water, namely, Fort Hamilton, on the Atlantic coast, and San Francisco, on the Pacific coast, for each of which three nine-year groups of observations are at hand. For New York Harbor Table 5 gives a ratio of K_1+O_1 to M_2 of 0.2, and for San Francisco this ratio is 1.1. With these values the correction factors for each nine-year group are derived from Table 6. As in the case of mean high water, mean low water will be determined with reference to half-tide level. The data for low water below half-tide level, factor from Table 6, and the derived mean low water below half-tide level for each of three nine-year groups at Fort Hamilton and at San Francisco follow.

Mean low water from nine years of observations

Nine-year group	Fort Hamilton, N. Y.			Nine-year group	San Francisco, Calif.		
	Low water below half-tide level	Factor from Table 6	Mean low water below half-tide level		Low water below half-tide level	Factor from Table 6	Mean low water-below half-tide level.
	Feet		*Feet*		*Feet*		*Feet*
1893–1901	2.29	1.012	2.32	1899–1907	1.98	0.989	1.96
1902–1910	2.38	.986	2.35	1908–1916	1.95	1.011	1.97
1911–1919	2.34	1.015	2.38	1917–1925	1.99	.989	1.97

The direct average of the observed heights of low water over a period of 19 years constitutes the best determination of the plane of mean low water. For Fort Hamilton this gives 2.34 feet below half-tide level for the 19-year period 1893–1911 and 2.37 feet for the 19-year period 1904–1922. For San Francisco low water below half-tide level is 1.97 feet for each of the two overlapping 19-year periods 1898–1916 and 1907–1925. Comparing the determinations from the nine-year periods in the table above, it is seen that these are within 0.01 or 0.02 foot of the 19-year values at both Fort Hamilton and San Francisco. It appears, therefore, that nine years of observations, corrected by the factor for the longitude of the moon's node, gives the average fall of low water below sea level correct within less than 1 per cent. Hence it may be taken that the plane of mean low water based on nine years of observations and corrected to a mean value as shown above constitutes a primary determination of that plane.

In passing it may be noted that at Fort Hamilton the two 19-year periods, 1893–1911 and 1904–1922, give values for mean low water which differ by 0.03 foot. While a difference of 0.01 or even 0.02 foot in two 19-year determinations may be considered as due to unusual weather conditions during one of these periods or to difficulties inherent in making tide observations, 0.03 foot appears somewhat large to be ascribed to such causes. In this connection

attention must be directed to the fact that changes in local hydrographic features, whether natural or artificial, tend to change the rise and fall of the tide and thus tend to change the elevations of the tidal datum planes. It is important to note, however, that such changes are not the same for low-water planes as for high-water planes. This matter will receive further consideration in the section devoted to the consideration of "Changes in tidal datum planes."

SECONDARY DETERMINATION

From a series of observations varying in length from a month to one or more years the value of the plane of mean low water may be derived in two different ways. The results from the tabulations may be corrected either by Table 6 or by comparisons with the results of simultaneous observations at a near-by station for which a primary determination is at hand.

For exemplifying the determination of the plane of mean low water from one year of observations the observations at Fort Hamilton and at San Francisco for each of the three years 1913, 1918, and 1922 may be used. As in the case of the like determination of the plane of mean high water, these years are chosen because the factor in Table 6 has for 1913 a maximum value, for 1918 approximately a mean value, and for 1922 a minimum value. The plane of mean low water will be determined first by means of Table 6 and then by comparison with simultaneous observations at some near-by station. As before, the ratios of K_1+O_1 to M_2 for use in Table 6 will be taken from Table 5.

The heights of the individual low waters as they appear in a tabulation, as, for example, in the last columns of Figures 13 and 19, are given with reference to a particular tide staff. In themselves these heights are in a sense arbitrary, for they depend on the height at which the zero of the tide staff is set. But the fall of low water below half-tide level is altogether independent of the setting of the staff. And it is this fall that is corrected to a mean value. This means that the height of low water as taken directly from the tabulations must first be referred to the half-tide level for the corresponding period before correcting to a mean value. The fall of low water below half-tide level, the correction factors, and the derived values of mean low water for Fort Hamilton and San Francisco for each of the years 1913, 1918, and 1922 are given below in tabular form.

Mean low water from one year of observations

Year	Fort Hamilton, N. Y.			San Francisco, Calif.		
	Low water below half-tide level	Factor from Table 6	Mean low water below half-tide level	Low water below half-tide level	Factor from Table 6	Mean low water below half-tide level
	Feet		*Feet*	*Feet*		*Feet*
1913	2.29	1.029	2.36	1.96	1.017	1.99
1918	2.39	.995	2.38	1.97	.997	1.96
1922	2.45	.971	2.38	1.99	.983	1.96

For both stations the values derived for mean low water from the three years are in good agreement. A primary determination of mean low water makes it 2.37 feet below half-tide level at Fort Hamilton and 1.97 feet at San Francisco. For both stations, therefore, a year of observations corrected by means of Table 6 gives the plane of mean low water correct within two hundredths of a foot, or within less than 2 per cent of the fall of low water below sea level.

For illustrating the method of deriving mean low water by comparison of simultaneous observations, we may use the same observations, as above. In choosing the station for comparison it is to be noted that type of tide should be similar, which means that the ratio of K_1+O_1 to M_2 should be approximately equal. Other things being equal, it is obviously of advantage to choose a near-by station. For Fort Hamilton, Atlantic City would be best; but to bring out the fact that good results can be derived by comparison with a suitable station many miles distant, Fernandina, Fla., will be taken. For San Francisco, San Diego, Calif., will be suitable, as a glance at Table 5 shows an approximately equal ratio of K_1+O_1 to M_2.

The determination of mean low water by the method of comparison of simultaneous observations is based on the fact that for similar types of tide the percentage increase or decrease in the fall of low water below sea level is the same over a given period of time. A primary determination of mean low water at Fernandina for the 19-year period 1905–1923 gives it as 3.00 feet below half-tide level. For the year 1913 low water at Fernandina was 2.94 feet below half-tide level for that year, and to bring this to a mean value it must be increased in the ratio $3.00 \div 2.94$ or 1.020. Hence, using Fernandina as a comparison station for the year 1913, the fall of low water below half-tide level at other stations on the Atlantic coast having a similar type of tide must be multiplied by the factor 1.020.

The data used and results derived for the three years at both Fort Hamilton and San Francisco are shown below in tabular form. For San Diego a primary determination of mean low water below half-tide level is 1.96 feet. In the table mean low water at the comparison station is given in abbreviated form as MLW.

Mean low water from one year of observations

Year	Fort Hamilton, N. Y. (Comparison station, Fernandina, MLW=3.00 feet)				San Francisco, Calif. (Comparison station, San Diego, MLW=1.96 feet)			
	Low water below half-tide level	Low water at comparison station	Factor for correction	Mean low water below half-tide level	Low water below half-tide level	Low water at comparison station	Factor for correction	Mean low water below half-tide level
	Feet	*Feet*		*Feet*	*Feet*	*Feet*		*Feet*
1913	2.29	2.94	1.020	2.34	1.96	1.93	1.016	1.99
1918	2.39	3.01	.997	2.38	1.97	1.96	1.000	1.97
1922	2.45	3.09	.971	2.38	1.99	1.97	.995	1.98

Comparing the values derived for mean low water in the table above with the primary determination of 2.37 feet for Fort Hamilton

and 1.97 feet for San Francisco, it appears that from one year of observations, by comparing with simultaneous observations, the plane of mean low water can be determined within two or three hundredths of a foot, or within less than 2 per cent of the fall of low water below half-tide level.

In the primary determination of mean low water and also in the secondary determination from one year of observations it is seen that the methods of procedure and accuracy attainable are precisely the same as in the corresponding determinations of mean high water. In fact, since mean low water is exactly the same distance below half-tide level as mean high water is above, a determination of the one plane with respect to half-tide level is also a determination of the other. The procedure and accuracy attainable in the determination of mean low water from tide observations covering various periods of time are therefore in all respects exactly the same as in the corresponding determinations of mean high water. For deriving the plane of mean low water from a month or a day of obsevations reference may be made to the discussion of the corresponding determinations of mean high water in the previous sections.

SUMMARY

The discussion of low water has brought out the fact that the depth to which low water falls varies considerably from day to day and from month to month and, in lesser degree, from year to year. This variation is of two kinds—the first, in response to changes in the phase, distance, and declination of the moon; and the second, in response to the variation in sea level. With the exception of the variation due to the change in longitude of the moon's node, the variations due to changes in the position of the moon balance out very largely within a month, so that in a sense the variation due to changes in sea level is the primary variation.

From observations covering a period of one or more months the plane of mean low water with respect to half-tide level can be corrected to a mean value, either by factors derived from theoretical considerations or by comparison with simultaneous observations at some suitable primary station. The suitability of a station for comparison purposes depends on similarity of type of tide and not merely on nearness. In this regard the correction of low water to a mean value differs from the like correction of sea level, in which the suitability of a station for comparison purposes depends on the existence of similar meteorological conditions. Furthermore, in correcting sea level to a mean value the changes in height at two near-by stations were taken as the same, whereas the changes in the fall of low water are taken as proportional.

Since the accuracy attainable in the determination of mean low water is precisely the same as that of mean high water, it may be taken, in general, that, when corrected to a mean value, nine years of tide observations will give a primary determination of mean low water; a year of observations will determine mean low water with respect to half-tide level correct to within 0.05 foot, a month within 0.1 foot, and a day within 0.5 foot. However, in regions of large range of tide and of considerable variation in the fall of low water, a day of observations, especially when the factor for correction

differs considerably from 1.00, may give a value differing by a foot or even more from a primary value.

It is to be carefully noted that mean low water is determined with respect to half-tide level, which is itself subject to variations. Hence the accuracy with which the plane of mean low water can be determined depends also on the accuracy with which the plane of half-tide level is determined. The degree of accuracy given above with which mean low water can be determined refers only to its fall below half-tide level.

To determine the plane of mean low water from any given series of observations the plane of half-tide level must first be determined; then the fall of low water below half-tide level is corrected to a mean value, and this gives the plane of mean low water below the plane of half-tide level.

X. LOWER LOW WATER

DEFINITION

The apparent daily movements of sun and moon about the earth take place in planes inclined to that of the Equator, and this gives rise to two different constituents in the tide, one having a period of half a day and the other a period of a day. The actual tide, resulting from the interaction of the semidaily and daily constituents, is therefore characterized by differences as between morning and afternoon tides, or, more precisely, by diurnal inequality. In general, consecutive low waters differ in height, necessitating the distinction between lower low water and higher low water. Of the two low waters of a day the lower is designated as the lower low water and the higher as the higher low water. The plane of lower low water at any point is the average height of all the lower low waters at that point over a considerable period of time.

Since the length of the tidal day is 24 hours and 50 minutes, there will be calendar days when but one low water occurs, the second one coming after midnight of that day and therefore on the next day. In such cases the question arises as to the designation to be applied to that single low water. Various rules may be formulated for this purpose, but for the practical purposes of datum plane determination a satisfactory rule is to give such a single low water the opposite name from the immediately preceding low water; that is, if the immediately preceding low water was the lower low water for the day, then the single low water in question will be designated as a higher low water, and vice versa. Thus, as shown in the column of low waters in the tabulation of Figure 19, but one low water occurred at San Francisco on June 10, 1920. The immediately preceding low water, which occurred at 23.8 hours on the 9th, was a higher low water; hence the single low water of the 10th would be designated as a lower low water.

At a number of places, however, only one low water may occur during a day because the tide becomes diurnal; that is, of the daily type, in which but one high water and one low water occur during a day. Where the daily constituent is more than twice as large as the semidaily constituent, the tide tends to become of the daily type. In such cases the single low water of a day is designated as the lower low water. There is usually no difficulty in deciding whether the

single low water of a day is due to the tide becoming diurnal or to the failure of the second low water to occur before midnight of that day. Diurnal tides occur only in regions having very considerable diurnal inequality, so that the characteristics of the tide for the day in question readily determine whether or not the single low water is due to the occurrence of a diurnal tide.

VARIATIONS

The depth to which lower low water falls varies from day to day. For San Francisco during the month of June, 1920, this variation appears on comparing the heights of lower low water for successive days, shown in Figures 19 and 20. For that month the difference between the highest and lowest of the lower low waters was 3.0 feet. Graphically the daily variation in lower low water is brought out in Figure 45, which is a plotting of all the low waters for that month. Not only do the successive low waters differ, but the lower low waters likewise are seen to vary in height from day to day.

An examination of successive lower low waters at any point brings out the fact that the variation from day to day is partly of a periodic nature, due to the change in position of the moon relative to earth and sun, and partly nonperiodic, due to secular variations in sea level. Referring to Figure 45, it is seen that, with regard to the moon's declination, lower low water goes through a fortnightly cycle, being lowest about the time of maximum north or south declination and highest about the time when the moon is on the Equator. The effects of the phase and parallax cycles of the moon are also reflected in lower low water, but the declinational effect is the principal one.

The difference in the morning and afternoon tides of a day, which is known as diurnal inequality, arises from the existence of daily and semidaily constituents in the tide. The greater the daily constituent in relation to the semidaily, the greater the diurnal inequality. Hence Table 5, which gives the ratios of the diurnal to the semidiurnal constituents at a number of stations on the coasts of the United States, gives also a measure of the inequality at those stations.

It is to be noted, however, that the diurnal inequality in the tides may be of three kinds. It may exist principally in the high waters, principally in the low waters, or equally in the high and low waters, depending on the phase relations of the daily and semidaily constituents. It happens that on the Atlantic coast the inequality is principally in the high waters. This, in conjunction with the small ratios of the diurnal to the semidiurnal constituents, makes the use of the plane of lower low water of little advantage on the Atlantic coast, and the plane of mean low water is used on that coast almost without exception. For this reason the examples given in connection with lower low water will be confined to the Pacific coast, where the diurnal inequality in the low waters is considerable and where the datum of lower low water is of practical importance.

MONTHLY LOWER LOW WATER

The declinational cycle of the moon has a period of 27⅓ days. Hence in a period of about a month lower low water should approximate to a mean value, and the variations from month to month should be very much less than from day to day. In Figure 49 are given

the relative monthly heights of lower low water at three stations on the Pacific coast for the years 1923 and 1924. Since 27 days is more nearly the period of the declinational cycle than is the calendar month, the monthly values plotted are for groups of 27 days, beginning with the 2d of the month and ending with the 28th.

From one month to the next Figure 49 shows that lower low water may vary by as much as half a foot at San Diego and at San Francisco and by as much as a foot at Seattle. Within a year two monthly values of lower low water may differ by as much as 1½ feet.

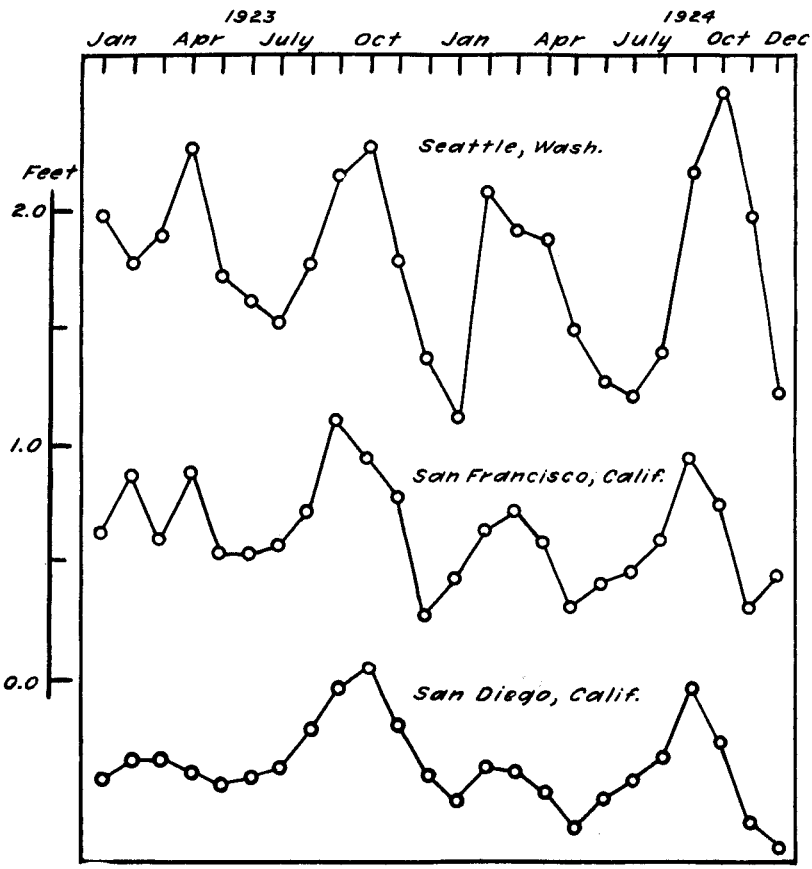

FIG. 49.—Monthly heights of lower low water

In part, these variations are obviously to be ascribed to variations in sea level.

In the consideration of monthly sea level it was found that sea level is subject to an annual variation, which is characteristic for any given region. In high water and in low water, likewise, annual variations are found which at any station resemble closely the annual variation in sea level at that station. It would therefore appear that lower low water should exhibit a similar annual variation, unless this is masked by an annual variation of a different kind. In Figure 50 are shown the monthly heights of lower low water at three Pacific

coast stations and one Alaskan station. For the Pacific coast stations the curves are based on 19 years of observations, from 1907 to 1925, while for Ketchikan the curve is based on 7 years of observations, from 1919 to 1925.

Since the annual variation in low water at any point resembles that in sea level, it appears from a comparison of Figures 30 and 31 that while the annual variation in low water at San Diego is similar to that at San Francisco it differs markedly from that at Seattle or at Ketchikan. For lower low water, however, Figure 50 shows a like annual variation from San Diego to Ketchikan. Lower low water is seen to have two maxima and two minima within a year,

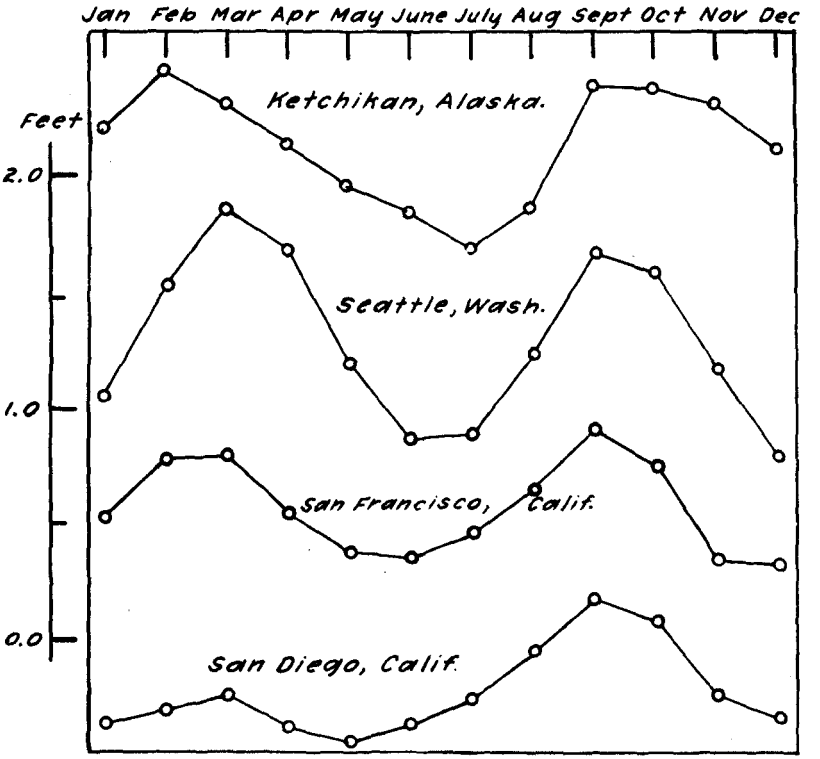

FIG. 50.—Annual variation in lower low water

being high in March and in September and low in June and December.

It is obvious that changes in sea level will be reflected by corresponding changes in lower low water. But from Figure 50 it is evident that, as regards monthly heights of lower low water, the response to changes in sea level is masked by some other more dominant variation. The characteristics of this variation may be determined by eliminating from the curves of Figure 50 the effects of the annual variation of sea level. However, from the fact that the maxima occur in June and in December, it follows that this variation is to be ascribed to the periodic change in the declination of the sun, which is over the Equator in March and in September and at its maximum declination in June and in December.

Two constituents therefore enter into the annual variation of lower low water. There is, first, the variation due to changes in sea level; and this variation, as has been shown, depends not on the range of the tide but upon its location. Second, there is the variation depending on the sun's declination; and this variation, as will be seen later, varies with the diurnal inequality.

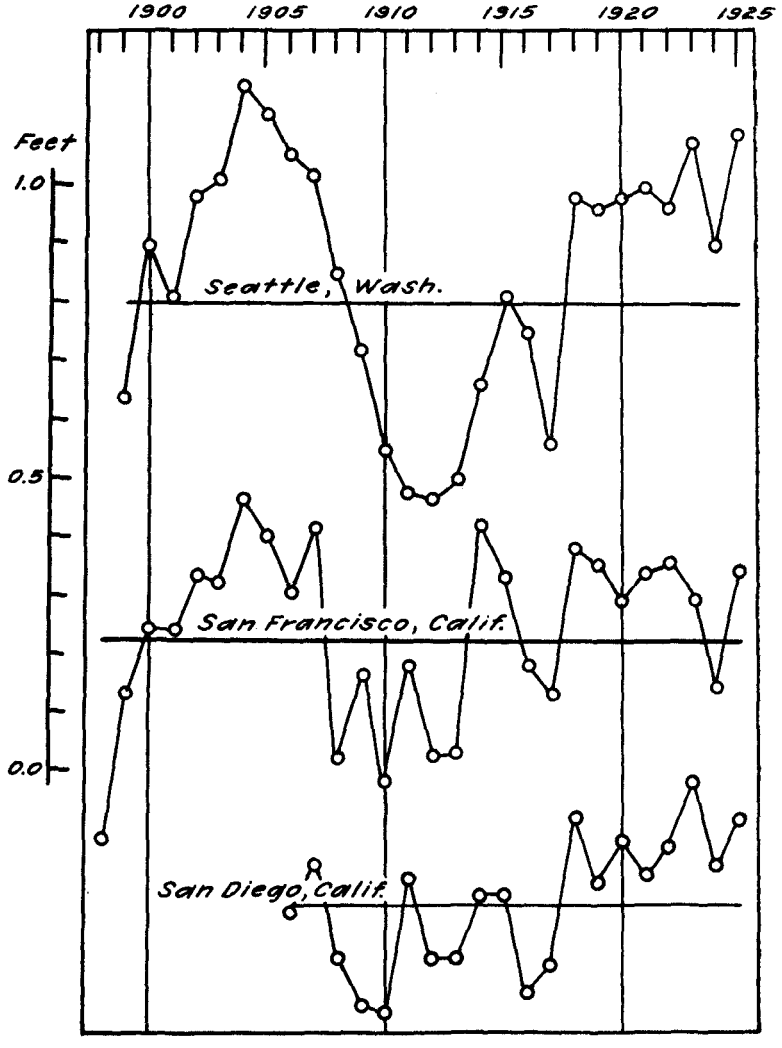

FIG. 51.—Yearly lower low water

YEARLY LOWER LOW WATER

The annual variation balances out within a year, and therefore yearly values of lower low water may be expected to show much smaller differences than monthly values. This is borne out by Figure 51, which is a plotting of the yearly heights of lower low water at three Pacific coast stations. Generally, lower low water from one

year to the next differs by one or two tenths of a foot, although occasionally this difference may be nearly half a foot. Within a 20-year period two yearly values of lower low water may differ by as much as three-quarters of a foot. The horizontal line associated with each of the diagrams represents mean lower low water derived from the whole series of observations at the station.

A comparison of the curves in Figure 51 with the corresponding curves of yearly sea level in Figure 34 shows some resemblance between them, indicating that in part the variation in yearly lower low

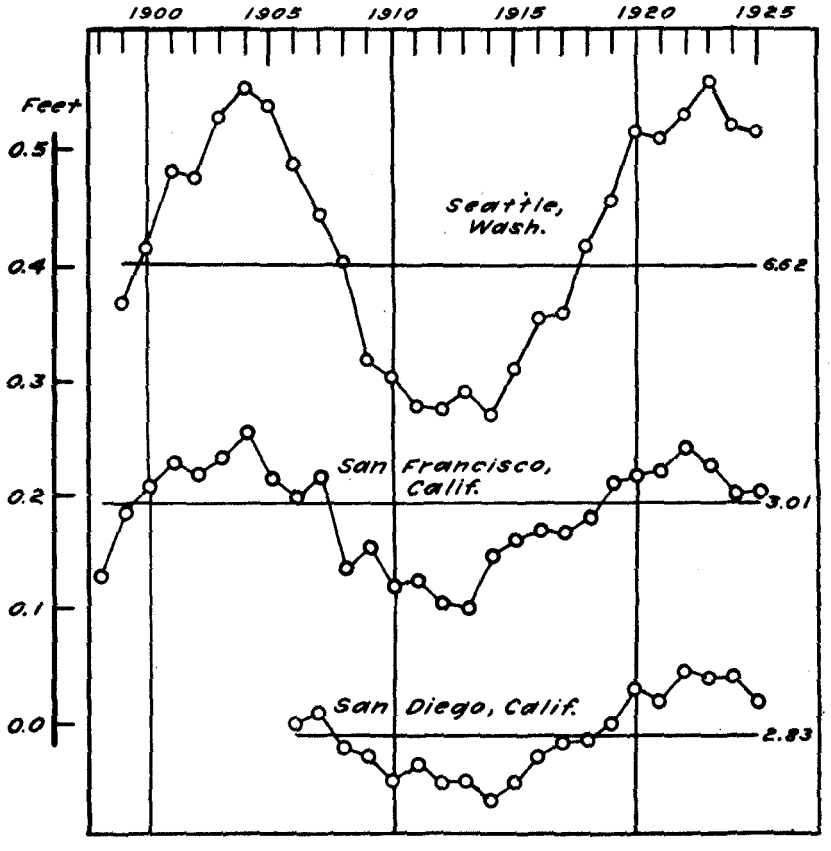

FIG. 52.—Variation of yearly lower low water in relation to sea level

water is due to changes in sea level. To determine the nature of any other constituents in the variation of yearly lower low water, the variation due to change in sea level is eliminated by subtracting each yearly height of lower low water from the corresponding yearly heights of sea level. Figure 52 shows the result for the three Pacific coast stations used above.

The horizontal line associated with each of the diagrams of Figure 52 represents the mean fall of lower low water below mean sea level, the figures to the right giving this fall in feet. The curves indicate, despite small irregularities, a regular variation with a

period of about 19 years. This variation, it can be shown from theoretical considerations, depends on the longitude of the moon's node. However, instead of deriving factors for correcting the fall of yearly lower low water below sea level or half-tide level, as was done for the rise of high water and fall of low water, it is more convenient to derive factors for correcting the diurnal inequality to a mean value. In other words, mean lower low water is derived through mean low water by correcting the diurnal low-water inequality to a mean value.

RELATION TO LOW WATER

It is necessary to distinguish not only between higher low water and lower low water, but also between the latter and low water. In general the term "low water" embraces both higher low water and lower low water; but, when used in contradistinction to lower low water, it refers to the average of the two low waters. For any day the difference between low water and lower low water is known as the diurnal low-water inequality, which in abbreviated form is written DLQ.

The diurnal low-water inequality varies from day to day throughout a fortnight, being greatest a little after the time of the moon's maximum north or south declination and least a little after the time when the moon is over the Equator. The mean value of the diurnal low-water inequality gives the difference between the planes of mean low water and lower low water. If, therefore, the plane of mean low water at any place is determined, the plane of lower low water becomes determined as soon as the mean value of the diurnal inequality is derived.

The heights of low water and lower low water vary in accordance with changes in sea level. Hence the low-water inequality, which is the difference between the heights of low water and lower low water, is independent of variations in sea level. This means that only the annual variation and the 19-year variation need be considered with regard to the diurnal inequality. These are of the same nature as the corresponding variations in lower low water, which were found to be due to changes in the relative positions of earth, moon, and sun. Factors for correcting monthly and yearly values of the diurnal inequality may therefore be derived from astronomical considerations, and in Table 7 such factors are given for each month and year of the 60-year period 1891–1950, as computed from Tables 6 and 14 of Harris's Manual of Tides, Part III.

TABLE 7.—*Factors for correcting diurnal inequality to mean value*

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Mean
1891	0.84	0.97	1.12	1.04	0.87	0.80	0.82	0.95	1.10	1.01	0.85	0.78	0.929
1892	0.81	0.94	1.07	1.01	0.85	0.78	0.80	0.93	1.06	0.98	0.83	0.77	0.902
1893	0.79	0.92	1.04	0.98	0.83	0.76	0.79	0.91	1.04	0.97	0.82	0.76	0.884
1894	0.79	0.91	1.03	0.97	0.82	0.76	0.78	0.91	1.04	0.96	0.82	0.76	0.879
1895	0.79	0.91	1.03	0.97	0.82	0.76	0.78	0.91	1.04	0.97	0.82	0.76	0.880
1896	0.79	0.92	1.04	0.98	0.83	0.77	0.79	0.92	1.06	0.98	0.83	0.77	0.890
1897	0.80	0.93	1.07	1.00	0.85	0.78	0.81	0.95	1.09	1.01	0.86	0.79	0.912
1898	0.83	0.96	1.11	1.04	0.88	0.80	0.83	0.98	1.14	1.06	0.89	0.82	0.945
1899	0.86	1.01	1.16	1.09	0.91	0.83	0.87	1.03	1.20	1.11	0.93	0.85	0.988
1900	0.89	1.06	1.23	1.15	0.96	0.87	0.91	1.08	1.29	1.18	0.97	0.89	1.040

TABLE 7.—*Factors for correcting diurnal inequality to mean value*—Continued

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Mean
1901	0.93	1.11	1.31	1.22	1.00	0.91	0.95	1.14	1.36	1.25	1.02	0.92	1.093
1902	0.97	1.16	1.39	1.29	1.04	0.94	0.98	1.19	1.43	1.30	1.06	0.95	1.142
1903	1.01	1.21	1.45	1.34	1.07	0.96	1.00	1.22	1.47	1.33	1.07	0.96	1.174
1904	1.01	1.23	1.46	1.34	1.07	0.96	1.00	1.22	1.47	1.33	1.07	0.96	1.177
1905	1.01	1.21	1.44	1.32	1.06	0.95	0.99	1.19	1.42	1.28	1.04	0.93	1.153
1906	0.98	1.17	1.38	1.26	1.02	0.92	0.95	1.14	1.34	1.22	0.99	0.90	1.106
1907	0.94	1.11	1.30	1.20	0.98	0.88	0.91	1.08	1.26	1.15	0.95	0.86	1.052
1908	0.90	1.06	1.22	1.13	0.93	0.84	0.87	1.02	1.19	1.09	0.90	0.82	0.998
1909	0.86	1.00	1.15	1.07	0.89	0.81	0.84	0.98	1.13	1.04	0.87	0.80	0.953
1910	0.83	0.96	1.10	1.02	0.86	0.79	0.81	0.94	1.08	1.00	0.84	0.78	0.918
1911	0.80	0.93	1.06	0.99	0.84	0.77	0.79	0.92	1.05	0.98	0.83	0.76	0.893
1912	0.79	0.92	1.04	0.98	0.83	0.76	0.78	0.91	1.04	0.96	0.82	0.76	0.882
1913	0.79	0.90	1.03	0.97	0.82	0.76	0.78	0.91	1.04	0.96	0.82	0.76	0.878
1914	0.79	0.90	1.03	0.97	0.83	0.76	0.79	0.91	1.04	0.97	0.83	0.76	0.882
1915	0.79	0.92	1.05	0.99	0.84	0.77	0.80	0.93	1.07	0.99	0.84	0.78	0.898
1916	0.81	0.95	1.08	1.02	0.86	0.79	0.82	0.96	1.11	1.03	0.87	0.80	0.925
1917	0.84	0.97	1.12	1.06	0.89	0.82	0.85	1.00	1.16	1.08	0.90	0.83	0.960
1918	0.87	1.02	1.19	1.11	0.93	0.85	0.88	1.05	1.23	1.14	0.94	0.86	1.006
1919	0.91	1.07	1.26	1.18	0.97	0.89	0.92	1.10	1.31	1.20	0.99	0.90	1.058
1920	0.95	1.14	1.35	1.25	1.02	0.92	0.96	1.16	1.39	1.27	1.03	0.94	1.115
1921	0.99	1.18	1.42	1.31	1.06	0.95	0.99	1.21	1.46	1.32	1.07	0.96	1.160
1922	1.01	1.22	1.46	1.34	1.08	0.96	1.00	1.22	1.47	1.33	1.07	0.96	1.177
1923	1.01	1.22	1.46	1.34	1.08	0.96	1.00	1.20	1.46	1.31	1.06	0.95	1.171
1924	1.00	1.20	1.42	1.30	1.05	0.94	0.98	1.17	1.39	1.26	1.02	0.92	1.138
1925	0.97	1.14	1.35	1.24	1.00	0.90	0.94	1.11	1.31	1.19	0.98	0.88	1.084
1926	0.92	1.08	1.26	1.17	0.96	0.87	0.89	1.06	1.23	1.13	0.93	0.85	1.029
1927	0.88	1.03	1.19	1.10	0.91	0.83	0.86	1.00	1.16	1.07	0.89	0.81	0.978
1928	0.85	0.99	1.12	1.05	0.88	0.80	0.83	0.96	1.11	1.02	0.86	0.79	0.938
1929	0.82	0.95	1.08	1.01	0.85	0.78	0.80	0.93	1.07	0.99	0.84	0.77	0.908
1930	0.80	0.92	1.05	0.99	0.83	0.76	0.79	0.91	1.04	0.97	0.82	0.76	0.887
1931	0.79	0.91	1.03	0.97	0.82	0.76	0.78	0.91	1.04	0.96	0.82	0.76	0.879
1932	0.79	0.91	1.03	0.97	0.82	0.76	0.78	0.91	1.04	0.96	0.82	0.76	0.879
1933	0.79	0.91	1.04	0.98	0.83	0.76	0.79	0.92	1.05	0.98	0.83	0.77	0.888
1934	0.80	0.92	1.06	1.00	0.84	0.78	0.80	0.94	1.08	1.01	0.85	0.78	0.905
1935	0.82	0.95	1.10	1.03	0.87	0.80	0.83	0.97	1.13	1.05	0.88	0.81	0.937
1936	0.85	1.00	1.15	1.08	0.90	0.83	0.86	1.01	1.19	1.10	0.92	0.84	0.978
1937	0.88	1.04	1.22	1.14	0.94	0.86	0.90	1.07	1.26	1.16	0.96	0.88	1.026
1938	0.92	1.09	1.30	1.21	0.99	0.90	0.94	1.13	1.34	1.23	1.01	0.92	1.082
1939	0.97	1.15	1.38	1.28	1.04	0.94	0.98	1.18	1.42	1.29	1.05	0.95	1.136
1940	1.00	1.21	1.44	1.33	1.07	0.96	1.00	1.22	1.47	1.33	1.07	0.96	1.172
1941	1.01	1.22	1.46	1.34	1.07	0.96	1.00	1.22	1.47	1.33	1.07	0.96	1.176
1942	1.01	1.21	1.45	1.33	1.07	0.96	0.99	1.20	1.43	1.30	1.04	0.94	1.161
1943	0.98	1.18	1.39	1.28	1.03	0.93	0.96	1.15	1.36	1.24	1.00	0.91	1.118
1944	0.95	1.13	1.31	1.21	0.99	0.89	0.92	1.09	1.29	1.17	0.96	0.87	1.065
1945	0.90	1.06	1.23	1.14	0.94	0.85	0.88	1.04	1.20	1.10	0.91	0.83	1.007
1946	0.87	1.01	1.16	1.08	0.90	0.82	0.84	0.99	1.14	1.05	0.88	0.80	0.962
1947	0.83	0.97	1.11	1.03	0.87	0.79	0.82	0.95	1.09	1.01	0.85	0.78	0.925
1948	0.81	0.94	1.07	1.00	0.84	0.77	0.80	0.93	1.06	0.98	0.83	0.76	0.899
1949	0.79	0.91	1.04	0.98	0.83	0.76	0.78	0.91	1.04	0.96	0.82	0.76	0.882
1950	0.79	0.90	1.03	0.97	0.82	0.76	0.78	0.91	1.04	0.96	0.82	0.76	0.878

The factors for each month in Table 7 were computed for the middle of the month. In the last column the mean of the monthly factors for each year is given, and this may be taken as the factor for correcting the yearly inequality to a mean value.

PRIMARY DETERMINATION

The variations to which lower low water is subject make it evident that a direct primary determination of the plane of lower low water requires 19 years of observations; that is, if the heights of lower low water over a period of 19 years are averaged, this average height will give a primary determination of lower low water. But, as in the case of the planes of mean high water and mean low water, lower low water observations covering a period of nine years, when corrected to a mean value, may be taken to constitute a primary determination of the plane of lower low water.

It should be noted, however, that in connection with lower low water it is the diurnal low-water inequality that is reduced to a mean value, and this, applied to the plane of mean low water, gives the plane of lower low water. To determine the plane of lower low water therefore requires the determination of the plane of mean low water.

As examples of primary determinations of lower low water from nine years of observations the observations at San Francisco, Calif., and at Seattle, Wash., may be used. At San Francisco the 28 years of observations from 1898 to 1925 give two overlapping 19-year series, 1898–1916 and 1907–1925. The first 19-year series gives the plane of lower low water as 3.11 feet below half-tide level, and the second group likewise gives it as 3.11 feet. The determination of mean lower low water from three groups of nine years each is given below. It will be convenient to use abbreviations for a number of terms, as follows: LW=low water, HTL=half-tide level, LLW=lower low water, DLQ=diurnal low-water inequality, MLW=mean low water, MLLW=mean lower low water.

Mean lower low water from nine years of observations, San Francisco, Calif.

Nine-year group	LW below HTL	LLW below HTL	DLQ	Factor from Table 7	Mean DLQ	Factor from Table 6	MLW below HTL	MLLW below HTL
	Feet	*Feet*	*Feet*		*Feet*		*Feet*	*Feet*
1899–1907	1.98	3.02	1.04	1.103	1.15	0.989	1.96	3.11
1908–1916	1.95	3.20	1.25	.914	1.14	1.011	1.97	3.11
1917–1925	1.99	3.03	1.04	1.097	1.14	.989	1.97	3.11

The fourth column in the table above, which is the difference between the second and third columns, gives the diurnal low-water inequality for each of the nine-year groups. The sixth column, which is the product of the fourth and fifth columns, gives this inequality as corrected to a mean value by the factors of Table 7. The eighth column is the product of the second and seventh columns and gives the value of low water below half-tide level reduced to a mean value. The last column, which gives the value of mean lower low water below half-tide level, is the result of adding the mean inequality to the mean low water.

The value derived for mean lower low water from each of the three nine-year groups is 3.11 feet below half-tide level, and this likewise was the value derived as the primary determination from 19 years of observations. For San Francisco, therefore, nine years of observations, corrected to a mean value by Table 7, may be taken to give a primary determination of the plane of lower low water.

At Seattle the low-water diurnal inequality is more than twice as large as at San Francisco. Greater differences in lower low water from one nine-year group to another may therefore be expected. The third column in the table below, which gives the data for Seattle, shows that two nine-year values of lower low water may differ by 0.4 foot, while for San Francisco the corresponding difference was only 0.2 foot. The data used in deriving the value of mean lower

low water from three nine-year groups, and the mean values derived, are given below, the abbreviations being the same as used for San Francisco in the preceding example.

Mean lower low water from nine years of observations, Seattle, Wash.

Nine-year group	LW below HTL	LLW below HTL	DLQ	Factor from Table 7	Mean DLQ	Factor from Table 6	MLW below HTL	MLLW below HTL
	Feet	*Feet*	*Feet*		*Feet*		*Feet*	*Feet*
1899–1907	3.87	6.44	2.57	1.103	2.83	0.989	3.83	6.66
1908–1916	3.78	6.86	3.08	.914	2.82	1.011	3.82	6.64
1917–1925	3.87	6.44	2.57	1.097	2.82	.989	3.83	6.65

The observations at Seattle begin in 1899, permitting two overlapping 19-year series, namely, 1899–1917 and 1907–1925. The first of these 19-year series gives lower low water as 6.65 feet below half-tide level, and the second series gives it as 6.64 feet. The values derived for mean lower low water from nine years of observations in the last column of the table above agree with the primary determinations within 0.01 foot. For San Francisco the agreement was even better. It may, therefore, be taken that nine years of observations, when corrected to a mean value by Table 7, give a primary determination of the plane of lower low water.

Attention was directed to the fact that, in connection with the determination of lower low water, it is the diurnal low-water inequality which is reduced to a mean value. The fact that nine years of observations, when corrected by Table 7, give a primary determination of the plane of lower low water implies that the mean value of the diurnal low-water inequality can be determined very closely from nine years of observations. That this is true is obvious from a comparison of the mean values of the diurnal inequality derived from the three groups of nine years of observations for San Francisco and Seattle in the preceding examples with the mean value determined from 19 years of observations, which is 1.14 feet for San Francisco and 2.83 feet for Seattle. It is seen that, after correction by Table 7, the value of the mean diurnal inequality from nine years of observations is correct within 0.01 foot.

In connection with the example for Seattle attention may also be directed to the next to the last column, which gives the value of mean low water before half-tide level. The direct primary determination of mean low water from 19 years of observations gives for Seattle 3.82 feet below half-tide level. The three values determined from nine years of observations agree with the primary value within 0.01 foot. Seattle therefore furnishes another example of the precision with which mean low water may be determined from nine years of observations.

SECONDARY DETERMINATION

The secondary determination of the plane of lower low water follows the procedure used in the primary determination in that mean values of the diurnal inequality and of low water are derived. The derivation of mean low water was discussed in Section IX. It

therefore remains to discuss the derivation of the mean value of the diurnal low-water inequality from observations less than nine years in length.

As in the case of mean high water and also mean low water, two different methods may be used to derive the mean low-water diurnal inequality from a series of observations varying in length from less than a month to a year or more. The result from the tabulation of the high and low waters may be corrected, either by the factors of Table 7 or by comparison with the results of simultaneous observations at a near-by station for which a primary determination is at hand. The procedure and the accuracy attainable will be exemplified below for observations covering periods of various lengths.

Year.—The determination of the plane of lower low water from a year of observations may be exemplified by deriving this plane from the observations at Seattle for the years 1913, 1918, and 1922. These years are chosen because the factor in Table 7 has for 1913 a minimum value, for 1918 approximately a mean value, and for 1922 a maximum value. The plane of lower low water will be determined first by means of Table 7 and then by comparison with simultaneous observations at a near-by station. In the table following the abbreviations are the same as used with the two preceding examples.

Mean lower low water from one year of observations, Seattle, Wash.

Year	LW below HTL	LLW below HTL	DLQ	Factor from Table 7	Mean DLQ	Factor from Table 6	MLW below HTL	MLLW below HTL
	Feet	*Feet*	*Feet*		*Feet*		*Feet*	*Feet*
1913	3.74	6.95	3.21	0.878	2.82	1.017	3.80	6.62
1918	3.84	6.64	2.80	1.006	2.82	.997	3.83	6.65
1922	3.90	6.29	2.39	1.177	2.81	.983	3.83	6.64

The primary value of mean lower low water at Seattle from the 19-year series 1907–1925 is 6.64 feet below half-tide level. The value derived for each of the three years above is thus within 0 02 foot of this primary value. It appears, therefore, that, like the planes of mean high water and mean low water, the plane of lower low water from one year of observations can be derived correct to within 0.05 foot of a primary value when corrected by the factors of Tables 6 and 7.

To determine mean lower low water from a year of observations by comparison with simultaneous observations at some near-by station we may take the same three years at Seattle, as above. For comparison station San Francisco will be used, though a station nearer Seattle would obviously be better. The procedure is as follows: First, mean low water at Seattle is derived by comparison with San Francisco, as explained in Section IX; then the diurnal low-water inequality at Seattle is reduced to a mean value by comparison with San Francisco; finally, mean lower low water is derived as the sum of mean low water plus the mean diurnal low-water inequality. The data used and results obtained are shown below.

Mean lower low water from one year of observations, Seattle, Wash.

[Comparison station, San Francisco, MLW=1.97 feet, MDLQ=1.14 feet]

Year	Seattle, Wash.		San Francisco, Calif.		Factors for correction		Seattle, Wash.		
	LW below HTL	DLQ	LW below HTL	DLQ	LW	DLQ	MLW below HTL	Mean DLQ	MLLW below HTL
	Feet	*Feet*	*Feet*	*Feet*			*Feet*	*Feet*	*Feet*
1913	3.74	3.21	1.96	1.33	1.005	0.857	3.76	2.75	6.51
1918	3.84	2.80	1.97	1.10	1.000	1.036	3.84	2.90	6.74
1922	3.90	2.39	1.99	.96	.990	1.188	3.86	2.84	6.70

The primary values of mean low water and mean diurnal low-water inequality for San Francisco are given in the brackets at the top of the table. In the sixth and seventh columns are given the factors derived by dividing these mean values by the yearly values in the fourth and fifth columns. These correction factors are then applied to the corresponding yearly values of low water and diurnal inequality at Seattle, giving the mean values of columns eight and nine, the sums of which for each year give the values of mean lower low water at Seattle in the tenth column.

In the previous example mean lower low water at Seattle for the years 1913, 1918, and 1922 was derived by means of Tables 6 and 7, and it was found that for each year the value derived agreed with the primary value of 6.64 feet within 0.02 foot. Comparing the figures in the last column derived by comparison with San Francisco, it is seen that the agreement is not nearly so close, there being one difference of as much as 0.13. foot. In part, however, this is due to the very considerable distance between the two stations. A station nearer Seattle would undoubtedly give better results than San Francisco, but the requisite data for such a station for the years in question are not at hand. To determine whether better results can be obtained from a comparison with a near-by station we may derive mean lower low water at San Francisco by comparison with simultaneous observations at San Diego, about 300 miles nearer to San Francisco than is Seattle.

Mean lower low water from one year of observations, San Francisco, Calif.

[Comparison station, San Diego, MLW=1.96 feet, MDLQ=0.92 feet]

Year	San Francisco, Calif.		San Diego, Calif.		Factors for correction		San Francisco, Calif.		
	LW below HTL	DLQ	LW below HTL	DLQ	LW	DLQ	MLW below HTL	Mean DLQ	MLLW below HTL
	Feet	*Feet*	*Feet*	*Feet*			*Feet*	*Feet*	*Feet*
1913	1.96	1.33	1.93	1.05	1.016	0.876	1.99	1.17	3.16
1918	1.97	1.10	1.96	.89	1.000	1.034	1.97	1.14	3.11
1922	1.99	.96	1.97	.78	.995	1.179	1.98	1.13	3.11

The primary value of mean lower low water at San Francisco is 3.11 feet below half-tide level. The values derived above by comparison with San Diego thus differ from the primary value by 0.05, 0.00,

and 0.00 foot, respectively. We may therefore conclude that by comparison with simultaneous observations at a suitable, not too distant, station one year of observations will give the datum of lower low water correct to within 0.1 foot.

Month.—From a month of observations mean lower low water may be determined either by means of Tables 6 and 7 or by comparison of simultaneous observations. Both methods will be exemplified for every other month of the years 1913 and 1922. The table following gives the data used and results derived for Seattle by the use of the factors of Tables 6 and 7.

Mean lower low water from one month of observations, Seattle, Wash.

Month	1913							1922						
	LW below HTL	DLQ	Factor from Table 6	Factor from Table 7	MLW below HTL	Mean DLQ	MLLW below HTL	LW below HTL	DLQ	Factor from Table 6	Factor from Table 7	MLW below HTL	Mean DLQ	MLLW below HTL
	Feet	*Feet*			*Feet*	*Feet*	*Feet*	*Feet*	*Feet*			*Feet*	*Feet*	*Feet*
Jan.	3.83	3.79	1.017	0.79	3.90	2.99	6.89	3.89	3.00	0.983	1.01	3.82	3.03	6.85
Mar.	3.76	2.84	1.017	1.03	3.82	2.93	6.75	3.84	1.76	.983	1.46	3.77	2.57	6.34
May	3.74	3.07	1.017	.82	3.80	2.52	6.32	3.88	2.42	.983	1.08	3.81	2.61	6.42
July	3.72	3.50	1.017	.78	3.78	2.73	6.51	3.97	2.64	.983	1.00	3.90	2.64	6.54
Sept.	3.79	2.56	1.017	1.04	3.85	2.66	6.51	3.93	1.95	.983	1.47	3.86	2.87	6.73
Nov.	3.72	3.45	1.017	.82	3.78	2.83	6.61	3.91	2.52	.983	1.07	3.84	2.70	6.54

A glance down the columns of the derived values of mean lower low water discloses considerable variation. The primary value of mean lower low water at Seattle is 6.64 feet below half-tide level. The extreme deviation from this mean value is 0.32 foot for the month of May, 1913. The average deviation, without regard to sign, of the values for the 12 months in the table is 0.17 foot.

Taking up, now, the determination of mean lower low water from one month of observations by the method of comparison of simultaneous observations, the datum will be determined for Seattle by comparison with San Francisco. The data and results follow:

Mean lower low water from one month of observations, Seattle, Wash.

[Comparison station, San Francisco, MLW=1.97 feet, MDLQ=1.14 feet]

Month	Seattle, Wash.		San Francisco, Calif.		Factors for correction		Seattle, Wash.		
	LW below HTL	DLQ	LW below HTL	DLQ	LW	DLQ	MLW below HTL	MDLQ	MLLW below HTL
1913	*Feet*	*Feet*	*Feet*	*Feet*			*Feet*	*Feet*	*Feet*
January	3.83	3.79	2.01	1.37	0.980	0.83	3.75	3.15	6.90
March	3.76	2.84	1.97	1.16	1.000	.98	3.76	2.78	6.54
May	3.74	3.07	1.92	1.33	1.025	.86	3.83	2.64	6.47
July	3.72	3.50	1.92	1.54	1.025	.74	3.81	2.59	6.40
September	3.79	2.56	1.99	1.04	.990	1.10	3.75	2.82	6.37
November	3.72	3.45	1.99	1.48	.990	.77	3.68	2.66	6.34
1922									
January	3.89	3.00	2.01	1.05	.980	1.09	3.81	3.27	7.08
March	3.84	1.76	1.96	.66	1.005	1.73	3.86	3.04	6.90
May	3.88	2.42	1.95	1.05	1.010	1.09	3.92	2.64	6.56
July	3.97	2.64	2.01	1.08	.980	1.06	3.89	2.79	6.68
September	3.93	1.95	2.03	.81	.970	1.41	3.81	2.75	6.56
November	3.91	2.52	2.02	1.11	.975	1.03	3.81	2.60	6.41

The values for mean lower low water derived by the method of comparison show greater differences from the primary value of 6.64 feet than the values derived by means of Tables 6 and 7. The greatest difference by the latter method was 0.32 foot, while by the comparison method the greatest difference is 0.44 foot. It may be noted in passing that the relatively large deviations from the mean value in the preceding example arise very largely from inaccuracies in the determination of MDLQ rather than in that of MLW.

Since Seattle is approximately 800 miles away from San Francisco, it is not unreasonable to assume that, in part at least, the relatively large differences from the mean value found in the preceding example are due to this distance between the two stations. In the derivation of mean low water from one year of observations it was found that a comparison of San Francisco with San Diego—500 miles apart—gave much better results than a similar comparison of Seattle with San Francisco. It will therefore be of advantage to test the results derived from a month of observations by the method of comparison, using observations at San Francisco and San Diego. As in the previous example, every other month of the years 1913 and 1922 will be taken, San Diego being used as the comparison station.

Mean lower low water from one month of observations, San Francisco, Calif.

[Comparison station, San Diego, MLW=1.96 feet, MDLQ=0.92 foot]

Month	San Francisco, Calif.		San Diego, Calif.		Factors for correction		San Francisco, Calif.		
	LW below HTL	DLQ	LW below HTL	DLQ	LW	DLQ	MLW below HTL	MDLQ	MLLW below HTL
1913	*Feet*	*Feet*	*Feet*	*Feet*			*Feet*	*Feet*	*Feet*
January	2.01	1.37	1.95	1.09	1.005	0.84	2.02	1.15	3.17
March	1.97	1.16	1.97	.92	.995	1.00	1.96	1.16	3.12
May	1.92	1.33	1.92	1.05	1.021	.88	1.96	1.17	3.13
July	1.92	1.54	1.87	1.22	1.048	.75	2.01	1.16	3.17
September	1.99	1.04	1.99	.81	.985	1.14	1.96	1.19	3.15
November	1.99	1.48	1.91	1.17	1.026	.79	2.04	1.17	3.21
1922									
January	2.01	1.05	1.97	.88	.995	1.05	2.00	1.10	3.10
March	1.96	.66	1.97	.54	.995	1.70	1.95	1.12	3.07
May	1.95	1.05	1.96	.86	1.000	1.07	1.95	1.12	3.07
July	2.01	1.08	1.98	.92	.990	1.00	1.98	1.08	3.06
September	2.03	.81	1.99	.64	.985	1.44	2.00	1.17	3.17
November	2.02	1.11	1.94	.87	1.010	1.06	2.04	1.19	3.23

The results derived for mean lower low water for San Francisco by comparison with San Diego are in much better agreement with each other and with the primary value of 3.11 feet than was the case for Seattle when compared with San Francisco. The greatest difference from the mean value in the present example is 0.12 foot against 0.44 foot in the case of Seattle. The average difference from the mean value in the present example for the 12 months is 0.05 foot.

Day.—To determine mean lower low water from one or several days of observation the most practical method is by comparison with simultaneous observations or with the predicted tides at some station

having a similar type of tide. The procedure may be illustrated by such determinations for San Francisco for every fifth day of May, 1913, using first the observations at San Diego and then the predictions for comparison.

Mean lower low water from one day of observations, San Francisco, Calif.

[Comparison station, San Diego, MLW=1.96 feet, MDLQ=0.92 foot]

Date	San Francisco, Calif.		San Diego, Calif.		Factors for correction		San Francisco, Calif.		
	LW below HTL	DLQ	LW below HTL	DLQ	LW	DLQ	MLW below HTL	MDLQ	MLLW below HTL
1913	*Feet*	*Feet*	*Feet*	*Feet*			*Feet*	*Feet*	*Feet*
May 1	1.65	0.45	1.80	0.55	1.09	1.67	1.80	0.75	2.55
May 6	2.05	1.60	2.20	1.15	.89	.80	1.82	1.28	3.10
May 11	1.80	2.10	1.47	1.65	1.33	.56	2.39	1.18	3.57
May 16	2.20	.05	2.25	.10	.87	9.20	1.91	.46	2.37
May 21	2.20	2.15	2.35	1.70	.83	.54	1.83	1.16	2.99
May 26	1.55	1.55	1.25	1.35	1.57	.68	2.43	1.05	3.48
May 31	1.57	.65	1.65	.30	1.19	3.07	1.87	2.00	3.87

Of the seven determinations of mean lower low water from one day of observations five differ from the primary value of 3.11 feet by less than half a foot, but for the 16th and 31st the differences are three-quarters of a foot. For these latter dates it will be noted that the large differences arise from the values derived for MDLQ.

In the tabulation from the tidal record the individual low waters are tabulated to the nearest tenth of a foot. It is obvious, therefore, that for a day when the diurnal inequality is small, as, for example, on the 16th, only rough approximations to the value of the mean diurnal inequality can be expected. Furthermore, any differences in the weather conditions at the comparison station from those prevailing at the other station will tend to bring about differences in the diurnal inequality at the two stations, which will be reflected in values for mean diurnal inequality differing considerably from primary values. In general, from one day of observations, mean lower low water derived by comparison with simultaneous observations at a suitable station may be taken as correct within three-quarters of a foot.

Frequently, when mean lower low water is to be derived from a short series of observations, simultaneous observations at another station for purpose of comparison are not at hand. In such cases the predicted tides at some suitable station may be used. The procedure is in all respects similar to that in the last example. In the following example mean lower low water is derived for the same seven days in May, 1913, as above, the predictions for San Diego in the tide tables for 1913 being used for comparison.

The tide tables give mean values for such tidal quantities as are important in connection with the purposes for which these tables are primarily issued. Thus, mean values of the lunitidal intervals and of the mean range are given for all stations listed. But the mean value of the low-water diurnal inequality is not given. If this value is

not known for the station that is used as comparison station, it may be derived from one or more months of predictions, by correcting monthly values by means of Table 7. In the example following the mean values of MLW and MDLQ for San Diego will be taken the same as in the preceding example, although, if derived from one or even several months of predictions, they would undoubtedly differ somewhat.

Mean lower low water from one day of observations, San Francisco, Calif.

[Comparison station, San Diego (predictions), MLW=1.96 feet, MDLQ=0.92 foot]

Date	San Francisco, Calif.		San Diego, Calif.		Factors for correction		San Francisco, Calif.		
	LW below HTL	DLQ	LW below HTL	DLQ	LW	DLQ	MLW below HTL	MDLQ	MLLW below HTL
1913	*Feet*	*Feet*	*Feet*	*Feet*			*Feet*	*Feet*	*Feet*
May 1	1.65	0.45	1.63	0.40	1.20	2.30	1.98	1.04	3.02
May 6	2.05	1.60	2.05	1.10	.96	.84	1.97	1.34	3.31
May 11	1.80	2.10	1.50	1.60	1.30	.58	2.34	1.22	3.56
May 16	2.20	.05	2.15	.05	.91	18.40	2.00	.92	2.92
May 21	2.20	2.15	2.15	1.70	.91	.54	2.00	1.16	3.16
May 26	1.55	1.55	1.13	1.40	1.73	.66	2.68	1.02	3.70
May 31	1.57	.65	1.50	.15	1.30	6.13	2.05	3.98	6.03

As in the preceding example, five of the seven values derived for mean lower low water differ from the primary value of 3.11 feet by less than half a foot. Of the remaining two values one differs by 0.6 foot, while the other, for the 31st, differs by 3 feet. Here again it is in the values derived for MDLQ that the trouble lies.

From the examples illustrating the derivation of mean lower low water from one day of observations it would appear that a value based on less than three days of observations can not be taken as determined correct within half a foot. If three days are used, it is better to discard the value of MDLQ for any one day which differs widely from the other two.

It is important to note that in the use of tide observations or predictions at comparison stations care must be taken to choose such as are comparable with those at the station for which mean lower low water is desired. This is necessary, since, as a general rule, there is a difference in time of tide at the two stations. For this reason it may happen that for comparison with the tides on a given day one or even two tides at the comparison station may have to be taken the day preceding or the day following. The proper tides to use are easily determined both by the times of the tide and by the order of occurrence. The following example will make this clear.

Suppose that it was required to determine mean lower low water at San Francisco from the observations on May 14, 1913, San Diego being used as comparison station. The tides observed at the two places on that day are as follows:

Observed tides, May 14, 1913

San Francisco, Calif.				San Diego, Calif.			
High water		Low water		High water		Low water	
Time	Height	Time	Height	Time	Height	Time	Height
Hours	*Feet*	*Hours*	*Feet*	*Hours*	*Feet*	*Hours*	*Feet*
5.8	9.7	0.0	7.5	3.7	8.1	10.6	3.4
19.7	10.3	12.2	5.0	17.7	8.3	23.7	5.0

A first glance at the two sets of observed tides might make them appear quite comparable. As a matter of fact, while the high waters are comparable the low waters are not. Examining the high waters it is seen that at San Diego they come, both morning and afternoon, about two hours earlier than at San Francisco. Furthermore, the order of occurrence is the same at both places, lower high water being the first high water and higher high water the second.

Examining now the low waters, it is found that instead of coming earlier than at San Francisco, as was the case of high water, low water comes about 11 hours later, if the two low waters of the day at San Diego be used as they stand. A moment's consideration, however, shows that for the day in question the first low water at San Diego is comparable, not with the first low water at San Francisco, but with the second. This means that for comparison with the first low water the one preceding 10.6 hours on the 14th is to be used, and that low water the observations give as occurring at 22.5 hours on the 13th, with a height of 5.8 feet. Hence, in deriving ratios for correcting low water and low-water diurnal inequality at San Francisco to mean values, the last low water of the 13th and the first low water of the 14th at San Diego would be used for comparison.

A consideration of the order of occurrence of the two low waters of the day leads to the same conclusion. At San Francisco the first low water on the 14th is seen to have been the higher low water, while at San Diego the first low water was the lower low. By taking the last low water of the previous day the low waters at San Diego become comparable with those at San Francisco.

SUMMARY

Lower low water is distinguished from higher low water and also from low water, the latter term in this connection referring to the average of the two low waters of the day. Lower low water, like low water, varies from day to day, from month to month, and from year to year, these variations being in part due to variations in sea level and in part to astronomic causes. Unlike low water, however, the variations in lower low water are primarily in response to the astronomic causes, and are therefore periodic.

A direct primary determination of mean lower low water requires 19 years of observations. It is found, however, that nine years of observations can be corrected to a mean value which approximates so closely to the direct primary determination that, for the purpose of datum-plane determination, such a value may be considered as a

primary value. For shorter series of observations mean lower low water can be corrected to a mean value, either by means of the factors in Tables 6 and 7 or by comparison with simultaneous observations at some suitable, not too distant, tide station. In general it may be taken that from a year of observations mean lower low water can be determined, with reference to half-tide level, correct to within 0.1 foot; from a month, correct to within a quarter of a foot; and from a day's observations, correct to within about 1 foot. It is to be noted, however, that occasionally the value derived from one day of observations may be considerably in error, so that at least three days of observations should be used if it is desired to determine mean lower low water correct within a foot.

The secondary determination of mean lower low water involves the determination of half-tide level, mean low water, and mean diurnal low-water inequality; the distance of mean lower low water below half-tide level being the distance of mean low water below half-tide level plus mean low-water diurnal inequality.

XI. HIGHER HIGH WATER

DEFINITION

The existence of daily and semidaily constituents in the tide gives rise to differences in consecutive high waters as well as to differences in consecutive low waters. As a rule, the two high waters of a day differ in height, the higher being designated the higher high water and the lower the lower high water. The plane of higher high water at any point is the average height of all the higher high waters at that point over a considerable period of time.

On days when but one high water occurs, the rule for determining whether it should be designated as the higher high or lower high is framed in the same way as for the similar case of low water. The single high water is given the name opposite that of the preceding high water; that is, if the preceding high water was the higher high water of the day, then the single high water in question is designated as the lower high water, and vice versa. Thus, in the column of high waters in the tabulation of Figures 19 and 20, the single high waters on the 10th and 18th will be designated as lower high waters.

Where the tide becomes diurnal—that is, where but one high and one low water occur in a day—the single high water obviously is a higher high water, for it is the merging of the lower high water and higher low water that gives rise to the diurnal tide.

RELATION TO LOWER LOW WATER

Manifestly the relation of higher high water to the rise of the tide is of a similar nature to that which lower low water bears to the fall of the tide. Corresponding to low-water diurnal inequality is high-water diurnal inequality, which is the difference between high water and higher high water. As distinguished from higher high water, high water refers to the average high water, whether for the day, month, or year.

Diurnal inequality depends on the relative amplitudes of the daily and semidaily tidal constituents and also on their phase relations. With given amplitudes of the two constituents, the diurnal inequality

may exist principally in the high waters, principally in the low waters, or equally in the high and low waters, depending on the phase relations of the daily and semidaily constituents. At most places the high-water and low-water diurnal inequalities differ. As mentioned in the section on lower low water, on the Atlantic coast of the United States the high-water inequality is the greater, while on the Pacific coast it is the low-water inequality that is the greater. However, the daily constituent of the tide has a small amplitude on the Atlantic coast, so that, notwithstanding the fact that on this coast the high-water inequality is the greater, it is relatively small.

VARIATIONS

Since the relation of higher high water to the rise of the tide is similar to that of lower low water to the fall, it follows that the variations in higher high water will be much the same as those in lower low water. These variations may be summarized as follows:

The height of higher high water varies from day to day, this variation being partly of a periodic nature, due to the change in position of the moon relative to earth and sun, and partly nonperiodic, due to secular variation in sea level. Referring to Figure 39, it is seen that with regard to the moon's declination higher high water goes through a fortnightly cycle, being highest about the time of maximum north or south declination and lowest about the time when the moon is on the Equator. The height of higher high water varies also with the phase and parallax of the moon, but the declinational effect is the principal one. On the Pacific coast successive higher high waters may differ by as much as a foot or more, while within a month higher high water may differ by several feet.

Monthly values of higher high water generally differ by several tenths of a foot from month to month, while within a year two such monthly values may differ by as much as a foot. The variation in monthly higher high water is partly in response to variations in sea level, but primarily in response to the change in declination of the sun, so that this latter variation has a period of a year.

From year to year higher high water varies about a tenth of a foot, although at times two consecutive yearly values of higher high water may differ by as much as three-tenths of a foot. The variation of yearly higher high water is in part due to secular variations in sea level and in part to a regular variation with a period of 19 years, depending on the longitude of the moon's node.

PRIMARY DETERMINATION

Since the variations in higher high water are brought about by the same agencies as those in lower low water, the procedure in the determination of mean higher high water is in all respects similar to that used in deriving mean lower low water. A direct primary determination requires 19 years of observations, but the value of higher high water derived from 9 years of observations, when corrected to a mean value by the factors of Table 7, may be taken to constitute a primary determination of the plane of higher high water.

As an example, we may derive mean higher high water from nine years of observations at San Francisco. Three nine-year groups may be formed from the observations covering the period 1899–1925. The data used and results derived are shown below in tabular form. The abbreviations used correspond to those used in deriving mean lower low water from nine years of observations: HW=high water, HTL=half-tide level, HHW=higher high water, DHQ=diurnal high-water inequality, MHW=mean high water, MHHW=mean higher high water.

Mean higher high water from nine years of observations, San Francisco, Calif.

Nine-year group	HW above HTL	HHW above HTL	DHQ	Factor from Table 7	Mean DHQ	Factor from Table 6	MHW above HTL	MHHW above HTL
	Feet	*Feet*	*Feet*		*Feet*		*Feet*	*Feet*
1899–1907	1.98	2.53	0.55	1.103	0.61	0.989	1.96	2.57
1908–1916	1.95	2.57	.62	.914	.57	1.011	1.97	2.54
1917–1925	1.99	2.55	.56	1.097	.61	.989	1.97	2.58

A direct primary determination from 19 years of observations at San Francisco gives mean higher high water as 2.56 feet above half-tide level. The value derived from each of the three-nine-year groups is therefore within 0.02 foot of the direct primary value, and for purposes of datum-plane determination may be taken to constitute a primary determination.

SECONDARY DETERMINATION

In deriving mean higher high water from a short series of observations the procedure and accuracy attainable are exactly the same as in the like determination of lower low water. For a series varying in length from a month to several years two methods are available. One method makes use of factors derived from theoretical considerations, while the other method consists in comparing with simultaneous observations at some suitable tide station. The procedure is detailed in the section on lower low water, to which reference is here made.

For a series of observations covering less than a month the most practicable method of deriving mean higher high water is by comparison with simultaneous observations at a suitable tide station or with the predicted tides at such a station, precisely as in the like case of determining mean lower low water, for which the detailed procedure is given.

In summary it may be stated that from a year of observations mean higher high water can be derived, with reference to half-tide level, correct within 0.1 foot; from a month, correct to within a quarter of a foot; and from a day's observations, correct to within about a foot. It must be noted, however, that occasionally the value derived from one day of observations may be considerably in error, so that at least three days of observations should be used if it is desired to determine mean higher high water correct within a foot.

XII. OTHER TIDAL DATUMS

PRINCIPAL DATUM PLANES

The six datum planes discussed in the preceding pages, namely, mean sea level, half-tide level, mean high water, mean low water, lower low water, and higher high water, constitute the principal tidal datum planes. They are more easily determined than other tidal datum planes, and from a given series of tide observations they can be derived with a greater degree of precision.

Other tidal datum planes have at times been used. Thus, the planes of monthly lowest low water and spring low water and the Indian tide plane have been used in hydrographic surveying and in tide predictions. To determine accurately such datum planes directly from observations requires a much longer series of observations than is necessary for any of the principal planes, for spring tides or tropic tides occur but twice a month and monthly lowest low water but once a month. As a rule, however, approximate determinations of such planes are quite satisfactory, especially if their relation to mean sea level or half-tide level is stated.

When the use of some datum plane other than one of the principal datums is found of advantage, it is desirable that it be defined with reference to one of these principal datums. Thus, if a plane below mean low water or mean lower low water is to be used, it is best to define it by its distance below either of these datums or mean sea level rather than seek some secondary tidal datum which approximates it. Several datum planes have, however, been used heretofore, and it is proposed here to discuss them briefly.

MONTHLY LOWEST LOW WATER

When a datum plane is desired which will be so low that most low waters will be above it, the plane of monthly lowest low water has sometimes been used. As its name signifies, it is the plane determined by the average height of the lowest low waters of each month over a considerable period of time.

This plane has sometimes been called the plane of extreme low water or of storm low water, but objections may be urged against both of those designations. Calling the lowest low water of each month an extreme low water is obviously arbitrary, while calling it a storm low water is even more arbitrary, for the lowest low water of a month is frequently not due to storms. The term monthly lowest low water is self-explanatory and definitely refers to the low water which, during the month in question, falls to the lowest level.

Obviously, the heights of consecutive monthly lowest tides will vary considerably; but within a period of two or three years the average value will not differ from a mean value based on many years of observations by more than a quarter of a foot. On the Atlantic coast of the United States the plane of monthly lowest low water is about 1.5 feet below mean low water, while on the Pacific coast it is about 1.5 feet below mean lower low water.

It is to be noted that while the plane of mean low water at different places varies in exact accordance with the range of the tide the plane of monthly lowest low water only in part varies as the range. This is evidenced by the fact that from Maine to Florida the plane of monthly lowest low water lies about the same distance below mean

low water, notwithstanding the fact that the range of tide over this region varies from 1 to 9 feet.

DATUM PLANES FROM HARMONIC CONSTANTS

The harmonic constants comprise the simple constituent tides which are derived from the harmonic analysis of the tide observations. The basis of the harmonic analysis lies in the conception of the tide as the sum of a number of simple tides, each of which has a definite period that is determined by some motion of the moon or sun relative to the earth. The number of simple constituent tides is theoretically very large, but most of them are of such small magnitude that for practical purposes they may be disregarded. The harmonic constants for many places are given in Harris's Manual of Tides, Parts IVA and IVB (Washington, D. C., 1901 and 1904); Schureman's A Manual of the Harmonic Analysis and Prediction of Tides (Washington, D. C., 1924); and in Tables for the Calculation of Tides by Means of Harmonic Constants, issued by the International Hydrographic Bureau (Monaco, 1926).

Formulas have been developed by Harris,[1] by means of which the various datum planes may be derived through the harmonic constants. These formulas are somewhat involved if it is desired to derive the datum planes accurately, but for approximate determinations the formulas may be simplified considerably.

As examples, it may be noted that the plane of mean high water is given approximately by the formula $HTL+1.1M_2$, in which HTL is half-tide level and M_2 is the principal lunar semidiurnal constituent. In the same way mean low water is given approximately by $HTL-1.1M_2$. To test the degree of approximation of these formulas we may derive the plane of mean high water for Fort Hamilton, N. Y., and for San Francisco, Calif.

From Schureman's Manual the value of M_2 is, to the nearest tenth, 2.2 feet for Fort Hamilton and 1.8 feet for San Francisco (Presidio). From the above approximate formula mean high water at Fort Hamilton is derived as 2.4 feet above half-tide level and at San Francisco as 2.0 feet above half-tide level. These values approximate closely the accurate primary values of 2.37 feet for Fort Hamilton and 1.97 feet for San Francisco. It should be stated, however, that the agreement at other places may not be quite so good and that the formula is intended to give only approximate results.

An approximate value for the datum of lower low water on the Pacific coast of the United States is given by the formula $MLW-0.6(K_1+O_1)$, in which MLW is mean low water and K_1 and O_1, respectively, the principal lunisolar diurnal and principal lunar diurnal constituents. Since mean lower low water is given by subtracting the mean low-water diurnal inequality from mean low water, the formula amounts to taking MDLQ as equal to $0.6\ (K_1+O_1)$, which is obviously but a rough approximation. Thus, if we derive the values of $0.6\ (K_1+O_1)$ for San Francisco and Seattle by using the values of K_1 and O_1 for these places given in Schureman's Manual, we get 1.2 feet for San Francisco and 2.5 feet for Seattle, which compare with the primary values of MDLQ of 1.14 and 2.83 feet, respectively.

[1] R. A. Harris, Manual of Tides, Pt. III (Washington, D. C., 1895).

The datum of higher high water on the Pacific coast is given approximately by $MHW+0.3(K_1+O_1)$, in which MHW is mean high water and K_1 and O_1 as above. It is to be emphasized, however, that the simple formulas given above for the planes of mean high water, mean low water, lower low water, and higher high water are approximations, and where accurate determinations are desired these must be derived from the observations as outlined in the previous sections. For the plane of spring low water and for the Indian tide plane, however, the determination by means of the harmonic constants is to be preferred.

SPRING LOW WATER

Spring low water has been used as a datum for hydrographic charts and for the prediction of tides. This datum may be defined as the average of the low waters that come at the time of spring tides. Spring tides are those that occur about the times of new and full moon, when the tide-producing forces of sun and moon conspire and bring about a greater rise and fall than usual. At most places there is a lag between full or new moon and the greatest rise and fall of the tide, this lag being known as the phase age of the tide. On the Atlantic and Pacific coasts of the United States the phase age of the tide is about one day; that is, spring tides come about one day after full and new moon.

It is obvious that there must be considerable variation in the height of spring low water from one fortnight to another. In the first place, it will vary in response to changes in sea level; and, in the second place, it will vary in response to changes in the positions of sun and moon as regards parallax and declination. It should be noted in passing that the two low waters coming nearest, one before and the other after, the time given by adding the phase age to the time of new and full moon are taken as constituting the spring low waters of any given new or full moon.

For all practical purposes it is sufficient to determine the plane of spring low water approximately, especially if its relation to mean sea level or half-tide level is given. This relation is given when spring low water is determined through harmonic constants, and this method therefore furnishes a convenient means for deriving the datum. As an approximate formula for the plane of spring low water, we may take it to be $MLW-S_2$ in which MLW is mean low water and S_2 the principal solar semidiurnal constituent.

As an example, we may derive spring low water from harmonic constants at Fort Hamilton and at San Francisco. From Schureman's Manual the value of S_2 is 0.44 foot for Fort Hamilton and 0.40 foot for San Francisco (Presidio). Mean low water at both these places has already been determined in the section devoted to low water, and we may therefore put the results in tabular form, as follows:

Spring low water

	Fort Hamilton, N. Y.	San Francisco, Calif.
	Feet	*Feet*
MLW below HTL	2.37	1.97
S_2	.44	.40
Sp LW below HTL	2.81	2.37

The best determined value of spring low water, based on a number of years of observations, is 2.79 feet below half-tide level at Fort Hamilton and 2.36 feet below half-tide level at San Francisco. The values derived from the approximate formula above are thus in good agreement with the more accurate values. In general it may be taken that this approximate formula will give the datum of spring low water correct to within one or two tenths of a foot. The plane of spring low water is also known as the plane of mean low-water springs.

The distance of spring low water below half-tide level is approximately the same as the distance of spring high water above half-tide level. Hence the same formula may be used for deriving approximately the datum of spring high water.

When it is necessary to determine spring low water and harmonic constants are not at hand, it may be derived by comparison with the spring low water at some other place in the same general region. If R and SpLW represent, respectively, the mean range and spring low water at the comparison station and the same abbreviations, with subscript 1, the like quantities at the station for which spring low water is desired, then we have $SpLW_1 = \frac{R_1}{R} SpLW$. In both cases SpLW represents the distance of spring low water below half-tide level.

INDIAN TIDE PLANE

In predicting the heights of high and low water for tide tables it is obviously desirable to refer these heights to a plane such that no negative heights will be necessary; that is, the datum with regard to which the predictions are given should be so low that no low water will fall below it. But it is manifestly even more desirable that the plane used in the tide predictions for any given port should be the same as that used on the hydrographic charts of that port. This consideration limits the practical datums for such purposes to some low-water datum like mean low water, lower low water, or spring low water.

The Indian tide plane, or the harmonic tide plane, as it is sometimes called, has been used for a number of ports in India. It is defined as the datum plane that lies below mean sea level a distance given by adding the amplitudes of the principal lunar semidiurnal, principal solar semidiurnal, the principal lunisolar diurnal, and the principal lunar diurnal components. In the accepted notation it may be written as follows: $MSL-(M_2+S_2+K_1+O_1)$. The Indian tide plane is sometimes called also the plane of Indian spring low water. From its definition in terms of the harmonic constants it obviously corresponds approximately to tropic spring lower low water.

XIII. CHANGES IN TIDAL DATUM PLANES

IMPLICATIONS IN ASSUMPTION OF CONSTANCY OF TIDAL DATUMS

In the use of tidal datums as planes of reference for elevations it is implied that such datums at any given place remain constant over relatively long periods of time. Underlying this implied constancy are the tacit assumptions of coastal stability and constancy of

hydrographic features. If changes take place in the relative elevation of land to sea or in the hydrographic features of the body of water on which the given place is situated, changes will also take place in the tidal datum planes, which are fixed by reference to local bench marks.

With regard to periods of time measured in thousands of years, local changes in relative elevation of land to sea of considerable magnitude have been fully demonstrated. But for the lesser periods of time involved in everyday affairs any such changes, as a general rule, are so small that with respect to tidal datum planes they may be disregarded and coastal stability taken for granted.

The changes in hydrographic features that bring in their train changes in datum planes are those that affect the local tidal régime. With regard to such changes in hydrographic features, distinction must be made between the open coast and inland bodies of tidal water. While the open coast is at all times under attack by wave and current and thus subject to change, such changes are relatively slight and only rarely bring about changes in the rise and fall of the tide, even over a period of a number of years. Hence, along the open coast, it may be assumed that tidal datum planes remain constant for periods covering many years.

But in inland bodies of tidal water changes in hydrographic features are as a rule followed by changes in the tidal régime, which are reflected by changes in the tidal datum planes. The changes that may be expected under different conditions will be discussed briefly in this section. But it will be of advantage to consider in this connection the changes in tidal datums that result from changes in relative elevation of land to sea.

CHANGES DUE TO CHANGE IN RELATIVE ELEVATION OF LAND TO SEA

If a coast is undergoing a slow gradual subsidence the first effect would obviously be an apparent elevation of all the tidal datum planes with respect to local bench marks by the same amount, this being the amount of the subsidence. That is, if after a number of years the subsidence of the coast in question amounted to one-tenth of a foot, mean sea level, half-tide and also the low-water and the high-water planes would, with respect to the local bench marks, stand one-tenth of a foot higher than at the beginning of the period.

But if the subsidence becomes sufficient to alter materially the hydrographic features of the coast, changes would ensue in the tidal régime along the coast, and as a consequence the different datums would change differently, as the following considerations will make clear.

Whatever the changes in the rise and fall of the tide along an open ocean coast due to gradual subsidence, it is obvious that the mean sea level, fixed with respect to local bench marks, will show an apparent change of the same magnitude as the subsidence, but in the opposite direction. That is, if S is the subsidence in feet, from the beginning of the period, the mean sea level would appear to have risen S feet. But, if because of the alteration of the hydrographic features consequent on this subsidence, the range of the tide is increased by A feet, mean high water will appear to have risen $S+\frac{1}{2}A$ feet, while for mean low water the apparent rise will be $S-\frac{1}{2}A$ feet. Thus the

datums of mean high water, mean sea level, and mean low water, with respect to local bench marks, will be changed by different amounts.

On a rising coast the changes that would take place in the tidal datums are similar but in the opposite direction to those just discussed. The first effect would be a lowering of all the tidal datum planes with respect to local bench marks by the amount of emergence. If the emergence becomes sufficient to alter materially the hydrographic features, and so bring about a change in the rise and fall of the tide, the changes ensuing from this latter cause would, as in the case of subsidence, be different for the different datums.

CHANGES DUE TO ALTERATION OF HYDROGRAPHIC FEATURES

It is with regard to inland tidal waters that changes in datum planes due to alteration of hydrographic features become important. On the open coast it is reasonable to assume that only profound changes in the hydrographic features can bring about changes in the range of the tide. But in inland tidal waters, because of the relatively limited areas and depths involved, changes in the features of considerably lesser magnitude are sufficient to change the range of the tide and thus bring about changes in datums. While the quantitative relations subsisting between changes in the body of water and changes in datums are difficult to establish from general considerations, qualitatively we may determine the changes in the datums that will follow proposed changes in the hydrographic features.

Tidal rivers are good examples of such inland bodies of water. These rivers serve as highways to the sea for numerous ports, some of which are situated many miles from the coast. With the increased draft and size of modern vessels, changes in depth or other alterations are frequently found necessary; and such improvements, if of sufficient magnitude, result in changes in the local tidal datum planes.

The tides in rivers are due to the tides sweeping into them from the seas into which they open. Normally the tide travels upstream until stopped by falls or rapids. If the mouth of the river is widened or deepened, this makes for a freer entry of the tide from the open sea and thus for a greater rise and fall of the tide. As a first effect, therefore, of widening or deepening a tidal river at its mouth, we may expect a rise in the high-water datums and a fall in the low-water datums. This effect, it is reasonable to expect, will generally be greatest near the mouth of the river, becoming gradually less going upstream.

Tidal rivers serve, however, not only as highways for the tide, but also as channels for carrying to the sea the drainage waters from large territories. Normally the cross-sectional area of a river increases seaward, due to the seaward slope of the river bed and the increasing width between banks. As a consequence, the mean river level in a tidal river becomes higher in going upstream. Thus, precise leveling by the Coast and Geodetic Survey shows that at Philadelphia, about 100 miles up the Delaware, the mean sea level (or mean river level) is about three-quarters of a foot higher than mean sea level on the coast, notwithstanding the fact that the range of the tide at Philadelphia is somewhat larger than at the mouth of the river.

Widening the mouth of a tidal river increases the cross-sectional area of the channel through which the drainage waters flow into the sea. As a consequence of the enlarged channel the drainage waters have freer outlet, which results in a lowering of the mean sea level some distance upstream. Deepening the mouth has a like effect; in addition, by reducing the friction per unit volume of water it brings about a further lowering of the mean level of the water.

If as a result of widening and deepening the mouth of a tidal stream the mean sea level at a given point of the river is lowered D feet and the range increased A feet, the changes in the different datums would be: Mean sea level, lowered D feet; mean high water, lowered $D-\frac{1}{2}A$ feet; mean low water, lowered $D+\frac{1}{2}A$ feet. The slope of mean sea level up a tidal stream is relatively slight, but the increase in range of tide consequent upon improvement is relatively large. For example, at Glasgow, on the Clyde, the range was increased 8 feet by river improvements. It follows, therefore, that as a rule in the above formulas D is less than $\frac{1}{2}A$. This means that $D-\frac{1}{2}A$ is negative and that high water, instead of being lowered, is raised somewhat. Low water, however, is lowered by the full amount of the increase in half range plus the depression of mean sea level.

These considerations are important in connection with the improvement of tidal rivers, since the depths in these are generally referred to mean low water. When improvements are contemplated the wording is generally to the effect that a certain depth at mean low water is to be attained. At first thought it would appear that if the desired depth below mean low water is a feet and the present depth is b feet the channel is to be deepened $a-b$ feet. But as the considerations outlined above show this is not the case, for the datum plane of mean low water is different in the two cases.

XIV. FORMS FOR TABULATIONS AND COMPUTATIONS

STANDARD FORMS

In the tabulation of the tide record and in the computation of datum planes the work is facilitated by the adoption of standard forms. This permits of a uniform procedure which, having been learned by the tabulator, reduces the time required for the tabulation and also lessens the chance of error.

Both in the tabulation and in the computation of the tidal data the Coast and Geodetic Survey makes use of printed forms of uniform size, 8 inches wide and 10½ inches long. This size is a very convenient one for handling, and it is also convenient for filing, since no folding is necessary.

TABULATION FORMS

Comparative readings.—The form used for tabulating the comparative readings, necessary with the three-roller gauge, was discussed in Section IV, "Tabulation of the tide record," and is illustrated, somewhat reduced, in Figure 14 (p. 27). The form is printed alike on both sides, one month being generally tabulated on each side.

Hourly heights.—The hourly heights of the tide are tabulated on a form printed on both sides, each side accommodating a week of

observations. This form likewise was discussed in Section IV and is illustrated in reduced size in Figure 12. The sums at the bottoms of the vertical columns permit the daily sea level to be determined, while the horizontal sums are used in the harmonic analysis. A check on the correctness of the vertical and horizontal sums is given by the final sum in the lower right-hand corner, which must check from the 7 vertical sums and from the 24 horizontal sums.

The sum of the hourly heights for each month is entered on the last sheet of hourly heights of the month in the space provided for it at the bottom of the sheet, and the mean derived by dividing by the proper divisor for the month in question. The divisors for the various months are given on the sheet.

High and low waters.—The form used for tabulating the high and low waters is illustrated in Figures 19 and 20. The form is designed for one month of observations, the first 17 days being tabulated on one side of the sheet and the remaining days of the month on the other side. The lunitidal intervals and the heights of high and low water are summed and the means derived as indicated, from which follows the range of the tide (Mn) and half-tide or mean tide level (MTL).

On the back of the sheet, illustrated in Figure 20, provision is made for the sums and means of higher high waters, lower low waters, and the inequalities of the month. A convenient method of summing the higher high and lower low waters directly from the sheet consists in checking each higher high and lower low with a small check mark. These checked figures may then be readily summed directly from the sheet and the sums and means entered in their appropriate places. The DHQ and DLQ are then derived by subtracting the mean values of the high and low waters of the month from the corresponding values of higher high and lower low waters, respectively. Provision is also made on the back of the high and low water form for deriving mean values of the range of the tide and of the inequalities by use of the factors in Tables 6 and 7.

COMPUTATION FORMS

Monthly means and extremes.—A useful form in connection with observations covering several months or more is illustrated in Figures 53 and 54, "Tides: Monthly means and extremes." One side (fig. 53) is used for monthly means and the other side (fig. 54) is used for the highest and lowest monthly tides.

One sheet of this form is generally used for the monthly means of sea level and half-tide level, the upper half for sea level and the lower half for half-tide level. The form will accommodate 10 years of monthly means. On another sheet of this same form the difference between the monthly values of sea level and half-tide level is derived. This gives a check on the monthly values of both sea level and half-tide level, for the difference from month to month should be very nearly constant. Any outstanding difference invites scrutiny, which aids in the detection of errors.

Other sheets of this form may be used for high water, low water, higher high water, etc. In each case the upper half (A) is used for the value as derived from tabulation while the lower half (B) is used for the same datum corrected to a mean value. The discussion

Form 472
DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY

TIDES: MONTHLY MEANS AND EXTREMES

Station: .....

Observations begin ..... *Observations end* .....

A

MONTH										
Jan.										
Feb.										
Mar.										
Apr.										
May										
June										
July										
Aug.										
Sept.										
Oct.										
Nov.										
Dec.										
Sum.										
Mean										

REMARKS: .....

B

MONTH										
Jan.										
Feb.										
Mar.										
Apr.										
May										
June										
July										
Aug.										
Sept.										
Oct.										
Nov.										
Dec.										
Sum.										
Mean										

REMARKS: .....

NOTE.—The following headings are to be written in lines "A" and "B" on successive forms, and in the order here given: Corrected HWI, Corrected LWI, Mean HW, Mean LW, Mean Tide Level, Mean Sea Level, Mean Range, Corrected Mean Range, Mean DHQ, Mean DLQ, Corrected DHQ, Corrected DLQ. The letters DHQ, DLQ, signify Mean HHW—Mean HW and Mean LW—Mean LLW, respectively. For a series of 4 years or less, put two headings in line "A," and two in line "B." For a series longer than 4 years, put only one heading in line "A," and one in line "B." Write the year, and the word "hour" or "feet" at the head of each column. Give Intervals in hours and hundredths, and Heights in feet and hundredths.

FIG. 53.—Form for monthly means. (Front of Form 472)

Form 472
Department of Commerce
Coast and Geodetic Survey

TIDES: MONTHLY MEANS AND EXTREMES

Station: .....

Observations begin ..... *Observations end* .....

HIGHEST TIDE OBSERVED

Month	Date	Feet	Date	Feet	Date	Feet	Date	Feet	Date	Feet
Jan.										
Feb.										
Mar.										
Apr.										
May										
June										
July										
Aug.										
Sept.										
Oct.										
Nov.										
Dec.										
Year										

Remarks: .....

LOWEST TIDE OBSERVED

Month	Date	Feet	Date	Feet	Date	Feet	Date	Feet	Date	Feet
Jan.										
Feb.										
Mar.										
Apr.										
May										
June										
July										
Aug.										
Sept.										
Oct.										
Nov.										
Dec.										
Year										

Remarks: .....

Note.—Write the year at the head of each double column. When the extreme height for any month occurs on more than one day, give the different dates, separated by commas. The heights are to be given in feet and tenths.

Fig. 54.—Form for monthly extremes. (Back of Form 472)

of the variations to which the various phases of the tide are subject makes it evident that the monthly values derived directly from the tabulation will show relatively large differences. When reduced to mean values, however, the differences are much smaller, and if any

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 248

TIDES: Comparison of Simultaneous Observations

(A) Subordinate station: Aqua Cooled, Wash. Lat. 48° 31′ N. Long. 122° 36′ W.
(B) Standard station: Seattle, Wash. Lat. 47° 37′ N. Long. 122° 20′ W.
Chief of party: ..... Time Meridian: (A) 120° W. (B) 120° W.

DATE. Year.	(A) STATION. Time of—		(B) STATION. Time of—		(A)−(B) Time difference.		(A) STATION. Height of—		(B) STATION. Height of—		(A)−(B) Height difference.	
	HW.	LW.	HW.	LW.	HW.	LW.	HW.	LW.	HW.	LW.	HW.	LW.
1922 Mo. D.	Hours.	Hours.	Hours.	Hours.	Hours.	Hours.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
Sept 24	7.4	0.1	7.2	0.6	0.2	−0.5	22.7	15.4	18.3	7.3	4.4	8.1
	18.5	12.3	18.8	13.0	−0.3	−0.7	23.2	18.7	19.2	11.0	4.0	7.7
25	8.6	1.0	8.0	1.5	0.6	−0.5	22.6	15.0	18.2	7.0	4.4	8.0
	18.9	13.0	19.4	13.8	−0.5	−0.8	22.9	19.8	18.7	12.3	4.2	7.5
26	9.9	1.9	9.4	2.3	0.5	−0.4	22.9	15.2	18.1	7.3	4.8	7.9
	20.0	14.4	20.5	15.0	−0.5	−0.6	22.9	20.7	18.4	13.5	4.5	7.2
27	11.0	2.9	10.4	3.2	0.6	−0.3	23.0	15.7	18.1	8.0	4.9	7.7
	20.5	16.0	21.5	16.4	−1.0	−0.4	22.1	20.9	17.1	13.8	5.0	7.1
28	12.5	4.0	12.0	4.3	0.5	−0.3	22.7	15.6	18.0	8.2	4.7	7.4
	21.6	18.0	22.9	17.8	−1.3	0.2	21.0	20.4	16.1	13.6	4.9	6.8
29	13.5	5.0	—	5.5	0.6	−0.5	22.7	15.8	—	8.3	4.7	7.5
	23.2	19.0	12.9	19.0	−0.9	0.0	20.7	20.0	18.0	13.0	4.9	7.0
30	—	6.0	0.1	6.6	—	−0.6	—	16.3	15.8	8.9	—	7.4
	14.3	20.4	14.0	20.0	0.3	0.4	23.2	19.8	18.4	12.7	4.8	7.1
Sums					.	.	HHW. 160.6	HLW. 140.3	HHW. 128.8	HLW. 89.9	HHW. 31.8	HLW. 50.4
Means					.	.	22.94	20.04	18.40	12.84	4.54	7.20
Sums					13 −1.2	14 −5.0	LHW. 137.0	LLW. 109.0	LHW. 129.6	LLW. 55.0	LHW. 28.4	LLW. 54.0
Means					−0.09	−0.36	22.00	15.57	17.27	7.86	4.73	7.71

(1) = −0.09 (HW.) −0.36 (LW.) = Mean difference in time of high and low water respectively.
(2) = −0.02 (HW.) −0.02 (LW.) = Correction for difference in longitude. (Table on back of form.)
(3) = −0.11 (HW.) −0.38 (LW.) = (1)+(2) = Mean difference in high and low water intervals, respectively.

(4) = 22.94 Feet. = Mean HHW height at (A).
(5) = 20.04 Feet. = Mean HLW height at (A).
(6) = 22.00 Feet. = Mean LHW height at (A).
(7) = 15.57 Feet. = Mean LLW height at (A).
(8) = 0.94 = (4)−(6) = 2DHQ at (A).
(9) = 4.47 = (5)−(7) = 2DLQ at (A).
(10) = 22.47 = ½[(4)+(6)] = Mean HW height at (A).
(11) = 17.80 = ½[(5)+(7)] = Mean LW height at (A).
(12) = 4.67 = (10)−(11) = Mn at (A).
(13) = 20.14 = ½[(10)+(11)] = MTL at (A).

(14) = 4.54 = Mean HHW difference.
(15) = 7.20 = Mean HLW difference.
(16) = 4.78 = Mean LHW difference.
(17) = 7.71 = Mean LLW difference.
(18) = −0.19 = (14)−(16) = 2DHQ difference.
(19) = −0.51 = (15)−(17) = 2DLQ difference.
(20) = 4.64 = ½[(14)+(16)] = Mean HW difference.
(21) = 7.46 = ½[(15)+(17)] = Mean LW difference.
(22) = −2.82 = (20)−(21) = Mn difference.
(23) = 6.05 = ½[(20)+(21)] = MTL difference.

(24) = 0.623 = (12)+[(12)−(22)] = Mn ratio.
(25) = 0.837 = (8)÷[(8)−(18)] = DHQ ratio.
(26) = 0.898 = (9)÷[(9)−(19)] = DLQ ratio.

Results from comparison of Stations A and B.	HWI.	LWI.	MTL.	Mn.	DHQ.	DLQ.
	Hours.	Hours.	Feet.	Feet.	Feet.	Feet.
Accepted values for standard station, from Obs. (Length of Series.) for 19 years	4.48	10.70	14.06	7.64	0.86	2.83
Differences and ratios: (3), (23), (24), (25), (26)	−0.11	−0.38	6.05	×0.623	×0.837	×0.898
Corrected values for subordinate station	4.37	10.32	20.11	4.76	0.72	2.54

Mean LW on staff at subordinate station = MTL − ½Mn = ... feet.
Mean LLW on staff at subordinate station = MTL − ½Mn − DLQ = 15.19 feet.

Computed by E. D. A. (11—4944), 5-12-27 (Date.) Verified by F. J. H., 5-12-27 (Date.)

FIG. 55.—Form for comparison of simultaneous observations. (Front of Form 248)

outstanding values occur they are subjected to examination to make sure that no errors have been made.

Comparison of simultaneous observations.—In connection with a short series of observations the form illustrated in Figure 55 is useful, as all the results desired for datum planes are derived on a single

sheet. It is designed to accommodate seven days of observations, but it is to be noted that it is not necessary that these be consecutive days. The explanation of the form is given on the back and is reproduced in Figure 56. The principles underlying the various steps have been discussed in the preceding sections, so that it is unnecessary to enter into the details here. In Figure 55 a week of observations at Anacortes, Wash., is compared with simultaneous observations at Seattle.

EXPLANATION OF FORM 248.

This form is designed for the comparison of tides at a subordinate station for which tidal results are sought, with the tides observed simultaneously at a standard station for which tidal constants are known.

For short series of observations the high and low waters observed at the subordinate station may be tabulated immediately in this form, in which case it will be unnecessary to tabulate them also in Form 138.

The time and height differences are to be obtained by subtracting the values at the standard station from the values at the subordinate station, and the results entered with proper signs in the columns indicated.

Find the sums and means of columns of time difference, height of tide at both stations, and height difference. For stations on the Pacific coast, where the plane of reference is mean lower low water, the heights of the higher high, lower high, higher low, and lower low waters are to be summed separately, the higher highs and lower lows being indicated by pencil check marks. For stations on the Atlantic coast, where the plane of reference is mean low water, the heights of the high waters may be all combined into a single sum, and similarly the low water heights; the headings of their sums being made to read HW and LW, respectively, by striking out the extra letters. All mean results should be given to two decimals of its unit, whether hour or foot. If any individual difference varies greatly from the apparent average, and an examination of the original record fails to show an error, that difference should not be included in the sum; and such a value should be encircled to show that it has been rejected.

For stations on the Atlantic coast omit (4) to (9), (14) to (19), (25), (26), and the computation of DHQ and DLQ at the bottom of the form. Take (10)=mean high water height at the subordinate station, (11)=mean low water height at subordinate station, (20)=mean high water difference, and (21)=mean low water difference.

For stations on the Pacific coast, the lower part of the form should be filled out completely as indicated.

The correction for difference in longitude (2) may be obtained from the following table. Find the difference in longitude by subtracting the longitude of the subordinate station from the longitude of the standard station, considering west as positive and east as negative. The correction has the same sign as the resulting difference of longitudes.

If the kind of time used at the two stations is different, apply this difference, expressed in hours, to the difference in the time of tide as directly obtained, adding if the time meridian of the subordinate station is west of the time meridian of the standard station, and subtracting if the time meridian of the subordinate station is east of that of the standard station.

Correction for difference in longitude.

Difference.	Correction.	Difference.	Correction.	Difference.	Correction.	Difference.	Correction.	Difference.	Correction.	Difference.	Correction.	Difference.	Correction.	Difference.	Correction.
	Hour.	°	*Hour.*	°	*Hour.*	°	*Hour.*	°	*Hour.*	°	*Hour.*	°	*Hour.*	°	*Hour.*
1	0.001	31	0.036	1	0.069	31	2.139	61	4.209	91	6.279	121	8.349	151	10.420
2	0.002	32	0.037	2	0.138	32	2.208	62	4.278	92	6.348	122	8.418	152	10.489
3	0.003	33	0.038	3	0.207	33	2.277	63	4.347	93	6.417	123	8.487	153	10.558
4	0.005	34	0.039	4	0.276	34	2.346	64	4.416	94	6.486	124	8.556	154	10.627
5	0.006	35	0.040	5	0.345	35	2.415	65	4.485	95	6.555	125	8.625	155	10.696
6	0.007	36	0.041	6	0.414	36	2.484	66	4.554	96	6.624	126	8.694	156	10.765
7	0.008	37	0.043	7	0.483	37	2.553	67	4.623	97	6.693	127	8.763	157	10.834
8	0.009	38	0.044	8	0.552	38	2.622	68	4.692	98	6.762	128	8.832	158	10.903
9	0.010	39	0.045	9	0.621	39	2.691	69	4.761	99	6.831	129	8.901	159	10.972
10	0.012	40	0.046	10	0.690	40	2.760	70	4.830	100	6.900	130	8.970	160	11.041
11	0.013	41	0.047	11	0.759	41	2.829	71	4.899	101	6.969	131	9.039	161	11.110
12	0.014	42	0.048	12	0.828	42	2.898	72	4.968	102	7.038	132	9.108	162	11.179
13	0.015	43	0.049	13	0.897	43	2.967	73	5.037	103	7.107	133	9.177	163	11.248
14	0.016	44	0.051	14	0.966	44	3.036	74	5.106	104	7.176	134	9.246	164	11.317
15	0.017	45	0.052	15	1.035	45	3.105	75	5.175	105	7.245	135	9.315	165	11.386
16	0.018	46	0.053	16	1.104	46	3.174	76	5.244	106	7.314	136	9.384	166	11.455
17	0.020	47	0.054	17	1.173	47	3.243	77	5.313	107	7.383	137	9.453	167	11.524
18	0.021	48	0.055	18	1.242	48	3.312	78	5.382	108	7.452	138	9.522	168	11.593
19	0.022	49	0.056	19	1.311	49	3.381	79	5.451	109	7.521	139	9.591	169	11.662
20	0.023	50	0.058	20	1.380	50	3.450	80	5.520	110	7.590	140	9.660	170	11.731
21	0.024	51	0.059	21	1.449	51	3.519	81	5.589	111	7.659	141	9.729	171	11,800
22	0.025	52	0.060	22	1.518	52	3.588	82	5.658	112	7.728	142	9.798	172	11.869
23	0.026	53	0.061	23	1.587	53	3.657	83	5.727	113	7.797	143	9.867	173	11.938
24	0.028	54	0.062	24	1.656	54	3.726	84	5.796	114	7.866	144	9.936	174	12.007
25	0.029	55	0.063	25	1.725	55	3.795	85	5.865	115	7.935	145	10.005	175	12.076
26	0.030	56	0.064	26	1.794	56	3.864	86	5.934	116	8.004	146	10.074	176	12.145
27	0.031	57	0.066	27	1.863	57	3.933	87	6.003	117	8.073	147	10.143	177	12.214
28	0.032	58	0.067	28	1.932	58	4.002	88	6.072	118	8.142	148	10.212	178	12.283
29	0.033	59	0.068	29	2.001	59	4.071	89	6.141	119	8.211	149	10.281	179	12.352
30	0.035	60	0.069	30	2.070	60	4.140	90	6.210	120	8.280	150	10.351	180	12.421

11—8266

FIG. 56.—Explanation of form for comparison of simultaneous observations. (Back of Form 248)

While generally confined to use with short series of observations, covering several days only, it is obvious that this form may be used for correcting to mean values the results derived from longer series of observations. In such cases the values derived from the observations are entered in the appropriate places at the bottom of the columns of "Time difference" and "Height difference" and the computation carried through as indicated.

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