DEPARTMENT OF COMMERCE

U. S. COAST AND GEODETIC SURVEY

E. LESTER JONES, DIRECTOR

MANUAL OF TRIANGULATION COMPUTATION AND ADJUSTMENT

BY

WALTER F. REYNOLDS

Mathematician

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FOREWORD

The purpose of this publication is to explain the methods used in the United States Coast and Geodetic Survey in the computation and adjustment of triangulation. It will not only serve as a guide to the younger mathematician just learning to make triangulation computations but will tend to standardize the methods of computation and adjustment of triangulation so that greater efficiency and economy will result.

Beginning on page 215 will be found a number of suggestions and general rules which have been formulated as the result of many years of experience in the adjustment of triangulation. Those just starting work on such computations will find it helpful to study these rules and suggestions before attempting to study the volume as a whole.

Acknowledgment is gratefully made to C. H. Swick, Dr. O. S. Adams, mathematicians, and O. P. Sutherland, associate mathematician, of the division of geodesy of this bureau, and to R. N. Ashmun, mathematician of the International Boundary Commission, who have carefully reviewed the entire manuscript and offered many valuable suggestions.

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MANUAL OF TRIANGULATION COMPUTATION AND ADJUSTMENT

By WALTER F. REYNOLDS, *Mathematician, United States Coast and Geodetic Survey*

GENERAL STATEMENT

For many years most of the computations and adjustments of triangulation in the United States Coast and Geodetic Survey have been more or less standardized, but, as there have been no printed instructions for the work, the standard methods have not been used as consistently as is desirable. This publication, giving the methods of computation resulting from years of experience by the various mathematicians, will tend to more consistency and thus to greater accuracy and speed.

The theory of least squares as applied to the adjustment of triangulation is not covered in this publication, since that is fully treated in Special Publication No. 28, Application of the Theory of Least Squares to the Adjustment of Triangulation. Instead, there is shown, step by step, how the computation of the triangulation is carried on from the time the field observations are received in the office until the final results are published. Examples of each part of the computation are given. The methods used in the computation and adjustment of traverse are not shown in this publication, since they are contained in Special Publication No. 137, Manual of First-Order Traverse.

CHAPTER 1.—PRELIMINARY COMPUTATIONS

ABSTRACT OF HORIZONTAL DIRECTIONS

The instructions for field work require that the lists of directions giving the observed horizontal directions or angles shall be made out and checked in the field, so that ordinarily the office computations should begin with the checked directions, and the mathematician should not have to go back to the original field record books.

But occasionally, due to a rush of work or a shortage of personnel, the field directions are not checked in the field, and this must be done in the office. The method of computing the list of directions is therefore shown here. As the methods of forming the lists of directions for triangulation of the first and second order (formerly precise and primary) and for triangulation of the third order (formerly secondary) are somewhat different, examples of both methods are given.

Below is given a sample abstract of observed directions on triangulation of the first order, as received in the office, from which the list of directions is computed. This abstract and the instructions for making out the list of directions for first-order triangulation are taken from Special Publication No. 120, Manual of First-Order Triangulation.

It is important that this form be made out carefully, because the mean directions derived from the abstract of horizontal directions constitute the basis for all the later computations. Every position observed at a station, except observations on objects where only one or two positions are taken, should appear on the abstract, the rejected readings being indicated by the letter R. Sample forms are shown in Figures 1 and 2.

Where more than one station is used as an initial, there will frequently be different ways in which the observations can be combined to give the directions from some one initial station. Figures 1 and 2 will illustrate the proper way to form the combined direction in a number of typical cases.

At station Granite both South Base and Westedge were used as initials in the observations, but South Base was chosen as the initial for the list of directions. A supplemental abstract of directions, Figure 2, was first made out for the observations in which Westedge was used as initial, and the abstract, shown in Figure 1, was then made out for such observations as had South Base for initial. It was then necessary to transfer the observations made with Westedge as initial to equivalent values with South Base as initial, marking such transferred directions with the letter T on the abstract to show their origin. For example, in Figure 2 the direction of Floyd from Westedge, position 1, is 271° 11′ 44.″0, while the direction of Westedge from South Base, position 1, is 17° 17′ 49.″5, and the sum of the two

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Form 470

ABSTRACT OF DIRECTIONS

State Arizona

Station Granite — Computed by O. P. S. — Date July 25–Aug. 6
Observer C. V. H. — Checked by W. F. R. — Inst. No.

DO NOT WRITE IN THIS MARGIN

STATIONS OBSERVED

Position No.	South Base	Westedge	Union	Floyd	N. Base	Williams	Frisco
	(INITIAL) 0° 00′ ″	17° 17′ ″	89° 58′ ″	288° 29′ ″	318° 34′ ″	325° 54′ ″	344° 12′ ″	° ′ ″
1	0.00	49.5	01.3	33.5T	47.8 (41.6)R	49.0	57.0	
2	0.00	53.0	00.4	35.7T	46.5	53.0T	54.5	
3	0.00	49.6	59.6	32.4T	49.3	51.1T	56.7	
4	0.00	51.5	59.8	36.3T	47.3	53.3	59.6	
5	0.00	50.8	02.2	37.3T	46.2	51.3	58.3	
6	0.00	51.0	02.8	38.3T 36.6 } 37.4	48.0 (42.5)R	51.8 (57.2)R	60.7	
7	0.00	48.2	58.5	32.8	45.8	52.8	56.5	
8	0.00	50.4	03.0	36.8	46.8	52.2	60.0	
9	0.00	52.2	05.0	35.5	45.8	53.0	54.5	
10	0.00	52.0	00.5	36.2	47.7	52.8	56.7	
11	0.00	50.5	00.6	35.7	46.4	52.2	55.7	
12	0.00	51.7	00.7	35.3	48.3	53.8	59.7	
13	0.00	50.5	02.5	32.3	48.0	51.0	56.5	
14	0.00	50.7	03.8	35.4	47.2	54.0	59.2	
15	0.00	48.8	59.2	34.2	47.0	52.8	59.6	
16	0.00	51.7	04.6	33.7	47.0	52.2	57.8	
Sum,		812.1	24.5	560.5	755.1	836.3	923.0	
Mean,		50.76	01.53	35.03	47.19	52.27	57.69	
Cor. for ecc.,			−0.26					
Direction,		50.76	01.27	35.03	47.19	52.27	57.69	

Fig. 1.—Sample abstract of directions

directions, 288° 29′ 33.″5, is the direction from South Base to Floyd, as shown for position 1, Figure 1. Similarly, the values for the other positions for Floyd and Williams are transferred from the supplemental abstract to the combined one, using for each position the corresponding value of the angle between South Base and Westedge.

It is not necessary to transfer Frisco from the supplemental abstract shown in Figure 2 to the combined abstract for the reason that a complete set was observed on that station from each initial. The mean of the directions on Frisco, with Westedge as initial, viz, 326° 55′ 07″.57, plus 17° 17′ 50″.76, the mean of the directions on Westedge

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 670

ABSTRACT OF DIRECTIONS

State *Arizona*

Station *Granite* Computed by *O. P. S.* Date *July 25 – Aug. 6*

Observer *C. V. H.* Checked by *W. F. R.* Inst. No. *Wanschaff #2*

DO NOT WRITE IN THIS MARGIN

POSITION NO.	*Westedge* (INITIAL) 0° 00′ ″	*Floyd* 271° 11′ ″	*William* 301° 17′ ″	*Frisco* 326° 55′ ″
1	0.00	44.0	59.5	07.2
2	0.00	42.7	60.0	06.5
3	0.00	42.8	61.5	07.1
4	0.00	44.6 43.0		09.0
5	0.00	46.5		07.0
6	0.00	47.3		10.0
7	0.00			06.3
8	0.00	*Transferred to sheet 1.*	*Transferred to sheet 1.*	08.7
9	0.00			07.1
10	0.00			05.9
11	0.00			08.2
12	0.00			08.0
13	0.00			06.7
14	0.00			07.0
15	0.00			09.1
16	0.00			07.3
Sum,				121.1
Mean,				07.57
Co. in m.,				
Direction,				07.57

FIG. 2.—Sample abstract of directions for missing signals

with South Base as initial, gives 344° 12′ 58″.33. The mean of this value of the direction to Frisco and that obtained with South Base as initial, 344° 12′ 57″.69, is used in the list of directions shown in Figure 3. If more than 10 or 12 acceptable positions are obtained on any one night for a direction, that night should be given unit weight with any other night in determining the mean direction. In general, where 10 or more positions of a direction have been measured on each of two or more nights no one night's observations should be rejected unless it is more than one-half second from the mean of all the values for that direction. If the divergence from the mean is greater than one-half second, that night's observations should be selected which will best close the triangles, provided that at least 12 acceptable positions are available for the retained direction.

It will be noted that since the angles were measured from South Base to Westedge, from Westedge to Frisco, and from South Base to Frisco, the proper value of the direction from South Base to the other two points could be secured most accurately by a least-squares adjustment (see pp. 8–16). In most cases, however, the results obtained by this station adjustment do not justify the time required to make the computation, but a mean value for the sum angles can usually be obtained by arbitrary methods which will meet sufficiently well the final demands for accuracy. When a number of sum angles are measured, however, and especially when the means obtained by different combinations vary considerably, a station adjustment may be made.

The direction to triangulation station Floyd, Figure 1, has two acceptable values for position 6. In such cases the mean is taken of all values for a position and that mean given unit weight in the final mean, on the theory that a symmetrical distribution of the readings around the circle is essential to accuracy. With an accurately graduated circle it is probable that the variation due to the graduation is not quite so large as that due to errors in reading, but the rule of unit weight for each position is the safest to follow as a uniform procedure.

REJECTION OF OBSERVATIONS

The chief difficulty in making out the form lies in deciding what observations to reject. The usual formulæ for the rejection of observational quantities are too cumbersome to apply and are not satisfactorily applicable to a short series of observations. It is, therefore, customary to apply an arbitrary limit of rejection, determined empirically from previous experience with the instrument used or with one of similar qualities. For observations with the type of theodolite usually used on first-order triangulation the rejection limit for the angular value of a direction on any one position of the circle may ordinarily be taken as four seconds from the mean.

The following rules will be a sufficient guide to the rejection of observed directions:

1. No reading should be rejected if it falls within the limit of retention (in the sample this limit is $\pm 4''.0$ from the mean) unless rejected at the time of taking the observation. The observer's

reason for rejection should then appear in the original record. This rule will not apply to the case where one set of observations of a direction is rejected in favor of another set of 12 or more positions, as provided for on page 5.

2. If two or more readings have been taken for a single position, the mean should be used if all readings come within the limit of retention.

3. If one reading of a position falls without the limit and one within the limit, do not use a mean even though the mean be within the limit. Use instead the single reading within the limit.

4. If both readings of a position fall without the limit, reject the position entirely, using the remaining positions to compute the mean direction.

5. In case the 16 readings seem to fall in two groups, the mean of one group differing considerably from the mean of the other, extreme care is necessary in making the rejections.

6. Before computing a trial mean any observations so far from the approximate mean as to be very evidently the result of blunders should be rejected. After a trial mean is obtained and the rejection limit applied, the observations so rejected should not be again included even though the new mean would bring them within the limit of rejection.

7. The results obtained by applying rigorously the limit of rejection, even though the quantities rejected are just outside the limit, will probably be but little different from those derived after long consideration, and much time can be saved by a strict application of the rule.

LIST OF DIRECTIONS

On the list of directions, Figure 3, the mean directions of all unrejected observations are arranged in order of azimuth from some one selected initial. Not only the mean directions to the principal stations as listed and computed on the abstract of directions should be shown, but also the directions to intersection points and reference marks.

The data on this form constitute the material upon which the office computations are based, and these data should be so completely checked in the field that there will be no need in the office to resort to the record book or the abstract of directions. The only exception to this rule is where there is not sufficient time in the field to make all the eccentric reductions without delay to the progress of the party.

On the back of the form for the list of directions are instructions for its preparation. (See fig. 5.) Only two points covered by those instructions need be emphasized here, viz, the number of decimal places to be shown in the mean angle and the treatment of eccentric directions. As regards the first, on first-order triangulation the directions to main-scheme stations should be carried to hundredths of a second, directions to second-order stations and to sharply defined permanently marked intersection stations to tenths of seconds, and directions to other points, such as mountain peaks, to seconds. Directions to near-by objects, such as witness or reference marks, need be taken to the nearest 10 seconds only. In general, two uncertain figures should be given; that is, the third digit from the right in the

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 2??

LIST OF DIRECTIONS

State: Arizona

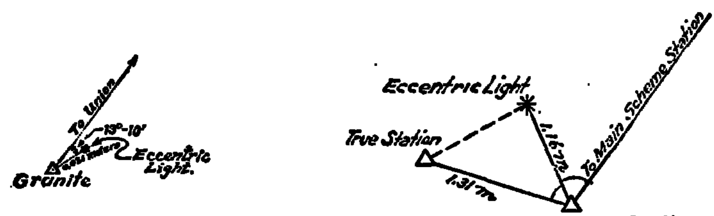
Station Granite — Computed by O.P.S. — Station ____ — Computed by ____

Observer C.V.H. — Checked by W.G.R. — Observer ____ — Checked by ____

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″	Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
South Base	00 00 00.00				
Westedge	17 17 50.76				
R.M. #1; dist. 6.738 m.	28 36 09.				
Union	89 58 01.27				
South Smoke stack sawmill	92 43 21.4				
Eccentric Light; dist. 0.021 m.	103 08				
R.M. #2; dist. 23.762 m.	200 25 08				
Floyd	288 29 35.00				
N. Base	318 34 47.19				
Williams	325 54 52.27				
Frisco	344 12 58.01				

Do not write in this margin.

(Method of recording eccentricity when theodolite is eccentric and eccentric light is more than 0.1 m from the true station)



Light eccentric as given above when shown to Union and Williams.
No eccentricity of light to other stations.
No eccentricity of theodolite.

All directions reduced for eccentricity.

(Record must state to what stations the light was shown in above position.
Measuring the distance from the true station to the eccentric light will give a check on the values for the elements of the small triangle)

FIG. 3.—Sample list of directions from horizontal directions

number denoting the direction should not be in error more than one unit.

The second point to be emphasized in the preparation of the list of directions is the computation of the eccentricity and the reduction of the observed directions to center. If a direction has not been reduced to center, the seconds pertaining to that direction should not be written on the form in ink, but in pencil. This rule should be invariably followed, for otherwise an unreduced direction may be used for a reduced one.

LIST OF DIRECTIONS FROM HORIZONTAL ANGLES

For making out the list of directions from observed horizontal angles in third-order triangulation, the method is slightly different from that used with observed horizontal directions. As shown below, the angles at the station are simply corrected for the closing of the horizon. Since each angle was observed in the same manner, its weight is unity, and the correction to each angle is obtained by dividing the difference between 360° and the sum of all the angles by the number of angles involved.

Observed angles, Vance Mt.

Observed stations	Angle			Correction	Final seconds
	°	′	″	″	″
Neal–Tomah Mt.	22	18	41.5	−0.5	41.0
Tomah Mt.–Spruce Mt.	59	51	12.0	−.5	11.5
Spruce Mt.–Mt. Henry	116	54	37.1	−.5	36.6
Mt. Henry–Brandy Hill	60	36	00.3	−.5	59.8
Brandy Hill–Oak	45	23	46.2	−.6	45.6
Oak–Neal	54	55	46.1	−.6	45.5
	360	00	03.2	−3.2	00.0

The corrections to close the horizon are usually applied in the Horizontal Angle Record Book, and the list of directions is made directly from that record on Form 24A, as shown in Figure 4. Complete instructions for making out the list of directions, which are given on the back of Form 24A, are given in Figure 5.

DEPARTMENT OF COMMERCE
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FORM 24A

State: Maine

Station Vance Mt. Computed by O.P.S.

Observer O.V.H. Checked by W.F.R.

STATIONS OBSERVED	DIRECTIONS AFTER LOCAL ADJUSTMENT ° ′ ″	FINAL SECONDS ″
Neal	0 00 00.0	
Tomah Mt.	22 18 41.0	
Spruce Mt.	82 09 52.5	
Mt. Henry	199 04 29.1	
Brandy Hill	259 40 28.9	
Oak	305 04 14.5	

FIG. 4.—Sample list of directions from horizontal angles

STATION ADJUSTMENT, DIRECTION METHOD

Under the present system of observing, no local adjustments are necessary, and all the computations, such as the taking of means and the closing of the horizon, are made in the record book in the field. But as the mathematician has to deal occasionally with observations made a number of years ago, when it was the custom to measure as many angles as possible, including sum angles, the method used in computing the list of directions at a station where local adjustment is necessary is given here.

As explained on page 8, the angles at each station must be corrected for the closing of the horizon. If no sum angles are observed, this is the only condition; but if sum angles are observed, new conditions are imposed on the station adjustment. If all angles are observed with the same accuracy, then the weight of each is unity; but when the angles are measured with different numbers of sets, they must be weighted accordingly. A set consists of six measures of the angle with the telescope direct and six measures of the explement of the angle with the telescope reversed.

State: Kansas.

Station Chase — Computed by A. T. M.

Observer A. T. M. — Checked by A. R. L.

This form, properly filled out and checked, must be furnished by field parties. *To be acceptable it must contain every direction observed.*

It is to be used for observations with repeating theodolites, as well as direction theodolites.

Start each new station at the head of a new column.

If a repeating theodolite is used, do not abstract the angles in *tertiary* triangulation. The local adjustment corrections (to close horizon only) are to be written in the Horizontal Angle Record, and the List of Directions is to be made from that record directly.

Choose as an initial for Form 24A some station involved in the local adjustment, and preferably one which has been used as an initial for a round of directions on objects not in the main scheme. Use but one initial at a station. Call the direction of the initial 0° 00′ 00.″00, and by applying the corrected angles to this, fill in opposite each station its direction reckoned *clockwise* around the whole circumference regardless of the direction of graduation of the instrument. The clockwise reckoning is necessary for uniformity and to make the directions comparable with azimuths.

Stations Observed	Directions After Local Adjustment			Final Seconds
	°	′	″	″
Central	0	00	00.00	Do not write in this column. It is for Office computation only.
White church spire, 8 miles	6	28	56.4	
Chase M. E. church, white spire	18	10	11.9	
Little River	18	20	10.78	
Lyons, salt works, center hoist	24	33	53.0	
Lyons, white spire, short	27	19	39.7	
Lyons, courthouse	27	55	34.2	
Lyons, white spire, slim	28	02	54.2	
Gilmore	53	32	33.44	
Savage	83	59	57.32	
Reference mark distant 66.65 meters.	171	34		
Section 3, T. 20, R. 10 W., NW. corner stone, distant 252.6 meters.	290	37	36	
Bossing	314	52	23.61	

If a station has been occupied eccentrically, reduce to the center and enter in this form, in ink, the resulting directions at the center. If the reduction is not made for some directions, they should be entered in pencil, with a footnote to that effect.

Directions in the main scheme should be entered to hundredths of seconds in primary triangulation; otherwise, to tenths only. Points observed upon but once, direct and reverse, should be carried to tenths in primary and secondary triangulation, and in tertiary triangulation to even seconds only. In general, but two uncertain figures should be given.

It is recommended that the following simple plan of observing be used with a repeating instrument: Measure each single angle in the scheme at each station and the outside angle necessary to close the horizon. Measure no sum angles. Follow each measurement of every angle immediately by a measurement of its explement. Six repetitions are to constitute a measurement. The local adjustment will consist simply of the distribution of the error of closure of the horizon.

In next to the last paragraph above the designation "primary" should be changed to "first-order," "secondary" to "second-order" and "tertiary" to "third-order."

FIG. 5.—Back of Form 24A, giving instructions for making list of directions

In the example below the angles were measured with different numbers of sets, and so are weighted.

Observed angles, Mag

Observed station	Angle	Weight p	v	Adjusted final seconds [1]
	° ′ ″			
Chamcook–Cooper	115 58 09.8, 10.4, 10.0, 10.2; Mean 10.1	4	v_1	10.1
Cooper–Rye	34 29 21.3, 20.5, 21.0; Mean 20.9	3	v_2	20.8
Rye–Middlemiss	46 01 29.3, 28.1; Mean 28.7	2	v_3	28.4
Middlemiss–Collins	80 07 16.5, 16.1; Mean 16.3	2	v_4	16.8
Rye–Mohannas	16 16 06.7, 05.5; Mean 06.1	2	v_5	06.3
Anderson–Mohannas	11 59 06.7, 06.1; Mean 06.4	2	v_6	05.6
Collins–Chamcook	83 23 44.3, 43.5, 44.0; Mean 43.9	3	v_7	43.9
Rye–Collins	126 08 44.8, 45.4, 45.0; Mean 45.1	3	v_8	45.2
Anderson–Middlemiss	41 44 27.0, 26.6; Mean 26.8	2	v_9	27.7
Mohannas–Collins	109 52 39.4, 39.7; Mean 39.6	2	v_{10}	38.9

[1] This column is filled out after the adjustment is completed. (See p. 16.)

List of directions, Mag

Observed station	Direction	Adjusted final seconds [1]
	° ′ ″	
Chamcook	0 00 00.0	00.0
Cooper	115 58 $10.1+v_1$	10.1
Rye	150 27 $31.0+v_1+v_2$	30.9
Anderson	154 44 $30.7+v_1+v_2+v_5-v_6$	31.6
Mohannas	166 43 $37.1+v_1+v_2+v_5$	37.2
Middlemiss	196 28 $59.7+v_1+v_2+v_3$	59.3
Collins	276 36 $16.0+v_1+v_2+v_3+v_4$	16.1

[1] This column is filled out after the adjustment is completed. (See p. 16.)

The complete list of directions has been formed using six of the angles, the remaining four not being necessary. As each of these angles not used gives rise to a condition, there will be four conditions. The equations expressing these conditions are formed as follows:

		°	′	″
Angle Collins–Chamcook, observed,	=	83	23	$43.9+v_7$
Angle Collins–Chamcook, from list,	=	83	23	$44.0-v_1-v_2-v_3-v_4$
Condition 1,				$0=-0.1+v_1+v_2+v_3+v_4+v_7$
Angle Rye–Collins, observed,	=	126	08	$45.1+v_8$
Angle Rye–Collins, from list,	=	126	08	$45.0+v_3+v_4$
Condition 2,				$0=+0.1-v_3-v_4+v_8$
Angle Anderson–Middlemiss, observed,	=	41	44	$26.8+v_9$
Angle Anderson–Middlemiss, from list,	=	41	44	$29.0+v_3-v_5+v_6$
Condition 3,				$0=-2.2-v_3+v_5-v_6+v_9$
Angle Mohannas–Collins, observed,	=	109	52	$39.6+v_{10}$
Angle Mohannas–Collins, from list,	=	109	52	$38.9+v_3+v_4-v_5$
Condition 4,				$0=+0.7-v_3-v_4+v_5+v_{10}$

After the condition equations are formed, they are tabulated in correlates as shown below.

Correlate equations

	$\frac{a}{p}$	1	2	3	4	Σ_c	v *	Adopted v †
1	3	+1				+1	−0.024	0.0
2	4	+1				+1	−.032	−.1
3	6	+1	−1	−1	−1	−2	−.342	−.3
4	6	+1	−1		−1	−1	+.510	+.5
5	6			+1	+1	+2	+.168	+.2
6	6			−1		−1	−.852	−.8
7	4	+1				+1	−.032	.0
8	4		+1			+1	+.084	+.1
9	6			+1		+1	+.852	+.9
10	6				+1	+1	−.684	−.7

* This column is filled out after the adjustment is completed. (See p. 15.)
† See explanation of this column on p. 15.

In the second column above, headed $\frac{a}{p}$, a is some constant, and p is the weight of a given v. It is best to take a as the least common multiple of all the weights, so as to make all the values in this column integers, provided this can be done without making the quantities too large. Thus, in the example above $a=12$. The values of the p's are given in the table on page 10. The quantities in the column headed Σ_c are obtained by adding across algebraically the quantities in columns 1, 2, 3, and 4, in the same horizontal line.

After forming the correlate equations, the normal equations are formed as shown in the table below:

Normal equations

	1	2	3	4	η	Σ_n	C *
1	+23	−12	−6	−12	−0.1	−7.1	−0.0075
2		+16	+6	+12	+.1	+22.1	+.0206
3			+24	+12	−2.2	+33.8	+.1417
4				+24	+.7	+36.7	−.1140

* See p. 13 for values in this column.

45861°—34——2

The normal equations are obtained by taking the algebraic sums of $\frac{a}{p}$ times the products of the various columns in the correlates. For example, normal equation No. 1 may be expressed as: $\Sigma\left(\frac{a}{p}\cdot 1\cdot 1\right)+\Sigma\left(\frac{a}{p}\cdot 1\cdot 2\right)+\Sigma\left(\frac{a}{p}\cdot 1\cdot 3\right)+\Sigma\left(\frac{a}{p}\cdot 1\cdot 4\right)+\eta_1+\left[\Sigma\left(\frac{a}{p}\cdot 1\cdot\Sigma_c\right)+\eta_1\right]$, normal equation No. 2 as $\Sigma\left(\frac{a}{p}\cdot 2\cdot 2\right)+\Sigma\left(\frac{a}{p}\cdot 2\cdot 3\right)+\Sigma\left(\frac{a}{p}\cdot 2\cdot 4\right)+\eta_2+\left[\Sigma\left(\frac{a}{p}\cdot 2\cdot\Sigma_c\right)+\eta_2\right]$, normal equation No. 3 as $\Sigma\left(\frac{a}{p}\cdot 3\cdot 3\right)+\Sigma\left(\frac{a}{p}\cdot 3\cdot 4\right)+\eta_3+\left[\Sigma\left(\frac{a}{p}\cdot 3\cdot\Sigma_c\right)+\eta_3\right]$, and normal equation No. 4 as $\Sigma\left(\frac{a}{p}\cdot 4\cdot 4\right)+\eta_4+\left[\Sigma\left(\frac{a}{p}\cdot 4\cdot\Sigma_c\right)+\eta_4\right]$, in which the Σ before each parenthesis indicates the sum of the products in the parentheses and should be distinguished from the Σ_c and the Σ_n of the preceding tables. η is the constant term for the corresponding condition equation.

In each symbolized normal equation above, the part in the square brackets should equal the corresponding Σ_n in the preceding table, the Σ_n being the sum of the values in columns 1, 2, 3, 4, and η, in the same horizontal line, including the omitted terms as explained below. This gives a check on the formation of the normals. In obtaining the Σ_n it must be remembered that due to symmetry, as explained below, certain coefficients have been omitted in the preceding table of normal equations and that these must be taken into consideration. The term in square brackets of the first normal equation is $\left[\Sigma\left(\frac{a}{p}\cdot 1\cdot\Sigma_c\right)+\eta_1\right]=-7.1$. Adding the coefficients of the terms in the first normal equation as given in the table we have $+23-12-6-12-0.1=-7.1$, which checks the formation of this equation.

In the same way the term in square brackets of the second normal equation $\left[\Sigma\left(\frac{a}{p}\cdot 2\cdot\Sigma_c\right)+\eta_2\right]=22.1$, and this checks the sum of the coefficients of the terms in this equation, $-12+16+6+12+0.1=+22.1$. The third and fourth normal equations are checked in the same way.

In the preceding table of normal equations the coefficients occuring before what is called the "diagonal term" are omitted, as the equations are symmetrical with regard to the diagonal line shown in the table below. Thus the table above, if written in full, would be as follows:

Normal equations in full

	1	2	3	4	η	Σn
1	+23	−12	−6	−12	−0.1	−7.1
2	−12	+16	+6	+12	+0.1	+22.1
3	−6	+6	+24	+12	−2.2	+33.8
4	−12	+12	+12	+24	+0.7	+36.7

It can be readily seen from this table that all the coefficients to the left of the diagonal line may be omitted and each equation may be read from the top down to the diagonal term and then across the page.

SOLUTION OF NORMAL EQUATIONS

In the solution of the normal equations the Doolittle method is used. As a full discussion of this method is given in Adjustment of Observations, by Wright and Hayford, second edition, page 114 et seq., no attempt is made here to discuss it or give any of the theory concerning it. A complete solution of the preceding normal equations is given below, followed by an explanation of the computation.

Forward solution

1	2	3	4	η	Σn
+23	−12	−6	−12	−0.1	−7.1
					6
C_1	+.5217	+.2609	+.5217	+.0043	+.3087
	+16	+6	+12	+.1	+22.1
1	−6.260	−3.131	−6.260	−.052	−3.703
	+9.740	+2.869	+5.740	+.048	+18.397
	C_2	−.2946	−.5893	−.0049	−1.8828
		+24	+12	−2.2	+33.8
	1	−1.565	−3.130	−.026	−1.852
	2	−.345	−1.691	−.014	−5.419
		+21.590	+7.179	−2.240	+26.529
		C_3	−.3325	+.1038	−1.2287
			+24	+.7	+36.7
		1	−6.260	−.052	−3.703
		2	−3.383	−.028	−10.842
		3	−2.387	+.745	−8.821
					5
			+11.970	+1.365	+13.334
			C_4	−.1140	−1.1140

Back solution (computation of C's)

4	3	2	1
−0.1140	+0.1038	−0.0049	+0.0043
	+.0379	+.0672	−.0595
		−.0417	+.0370
			+.0107
	+0.1417	+0.0206	−0.0075

EXPLANATION OF SOLUTION

The forward solution is computed as follows:

Normal equation No. 1 is written down and divided by its diagonal term, +23, all the signs being changed. C_1 is thus given in terms of C_2, C_3, C_4, and the constant term. Normal equation No. 2 is next set down, and since it has a coefficient of −12 for C_1, the divided coefficients of equation No. 1 are multiplied by −12, and the products are placed under equation No. 2. The quantities in each column are then added algebraically and are divided by the new diagonal term, +9.740, all the signs being changed. C_2 is thus given in terms of C_3, C_4, and the constant term. Normal equation No. 3 is then written down, and since it has a coefficient of −6 for C_1 and +2.869 for C_2, the divided coefficients of equation No. 1 are multiplied by −6 and those of equation No. 2 by +2.869, giving products which are set down under equation No. 3. The quantities in each column are added algebraically and these sums divided by the new diagonal term, all the signs being changed. C_3 is thus given in terms of C_4 and the constant term. Normal equation No. 4 is then set down, and since it has a coefficient of −12 for C_1, +5.740 for C_2, and +7.179 for C_3, the divided coefficients of equation No. 1 are multiplied by −12, those of equation No. 2 by +5.740, and those of equation No. 3 by +7.179. The quantities in each column are then added algebraically and these sums divided by the new diagonal term, all the signs being changed. The value of C_4 is thus obtained.

As can readily be seen from the forward solution, $C_3 = -0.3325C_4 + 0.1038$; $C_2 = -0.2946C_3 - 0.5893C_4 - 0.0049$; and $C_1 = +0.5217C_2 + 0.2609C_3 + 0.5217C_4 + 0.0043$. These C's can be obtained most conveniently by arranging the back solution in the form shown on page 13. There will be as many columns as there are C's to be determined, and they will be headed in reverse order from the forward solution. Thus, in this particular solution, the columns will be headed 4, 3, 2, and 1.

On the first line is written the constant term for each divided equation of the forward solution; that is, the quantities in the column headed η. Then the value of C_4, −0.1140, is multiplied into each of the quantities of the divided equations in the column headed 4 of the forward solution, these products being placed in the second line of the back solution beginning with the column headed 3. The quantities in the column headed 3 of the back solution are then added algebraically to give C_3. The value of C_3, +0.1417, is then multiplied into each of the quantities of the divided equations in the column headed 3 of the forward solution, these products being placed in the third line of the back solution, beginning with the column headed 2. The quantities in the column headed 2 of the back solution are then added algebraically to give C_2. The value of

C_2, +0.0206, is multiplied into the quantity of the divided equation in the column headed 2 of the forward solution, this product being placed in the fourth line of the back solution in the column headed 1. The quantities in the column headed 1 of the back solution are then added algebraically to give C_1.

COMPUTATION OF v's

After the C's are determined, the next step is to compute the v's by substituting the values of the C's in the correlate equations tabulated on page 11, taking into account the weights as shown in the column headed $\frac{a}{p}$. It can be easily seen from this table that $v_1 = 3C_1$, $v_2 = 4C_1$, $v_3 = 6\ (C_1 - C_2 - C_3, - C_4)$, $v_4 = 6\ (C_1 - C_2 - C_4)$, $v_5 = 6\ (C_3 + C_4)$, $v_6 = -6C_3$, $v_7 = 4C_1$, $v_8 = 4C_2$, $v_9 = 6C_3$, and $v_{10} = 6C_4$.

The v's are best obtained by means of a table as shown below, which has a column for each v. The values of C_1, C_3, C_3, and C_4 are placed in the different columns to correspond with the set of correlate equations on page 11. These are then added algebraically in each column, and each sum is then multiplied by the corresponding weight for that v to give the final v.

As the constant terms of the condition equations are carried to the nearest tenth of a second, it is customary to round off the v's to the nearest tenth of a second. Occasionally, when the v's are substituted in the condition equations, one or more equations may fail by a tenth of a second due to this dropping of hundredths of seconds in the v's.

For instance, if in condition equation (1) we substitute the values of the v's computed to the nearest tenth of a second, we have $0 = -0.1 + 0.0 + 0.0 - 0.3 + 0.5 + 0.0$, or $0 = +0.1$; and in equation (3) we have $0 = -2.2 + 0.3 + 0.2 + 0.9 + 0.9$, or $0 = +0.1$.

In order that the equations may check exactly and the results be consistent, it is customary to adopt a set of v's, a few of which may not be the same to the nearest tenth of a second as the computed values. These adopted v's are shown at the bottom of each column in the table of v's below. In this set of adopted v's, v_2 has been given the value −0.1 instead of 0.0, and v_6 −0.8 instead of −0.9, in order that equations (1) and (3) may be satisfied.

Computation of v's

1	2	3	4	5	6	7	8	9	10
−0.008 3	−0.008 4	−0.008 −.021 −.142 +.114	−0.008 −.021 +.114	+0.142 −.114	−0.142 6	−0,008 4	+0.021 4	+0.142 6	−0.114 6
−.024 .0	−.032 −.1	−.057 6	+.085 6	+.028 6	−.852 −.8	−.032 .0	+.084 +.1	+.852 +.9	−.684 −.7
		−.342 −.3	+.510 +.5	+.168 +.2					

The adopted values of the v's are substituted in the table of observed angles and in the list of directions on page 10 to give the values in the column headed "Adjusted final seconds" in each table.

It will be found now that if the list of directions is formed from the corrected observed angles, it will be consistent; that is, the direction at each station will be the same no matter from which angles it is computed.

The list of directions after the local adjustment should appear in the following form:

List of directions, Mag

Observed station	Direction after local adjustment	Final seconds
	° ′ ″	
Chamcook	0 00 00.0	
Cooper	115 58 10.1	
Rye	150 27 30.9	
Anderson	154 44 31.6	
Mohannas	166 43 37.2	
Middlemiss	196 28 59.3	
Collins	276 36 16.1	

STATION ADJUSTMENT, ANGLE METHOD, OBSERVATIONS OF EQUAL WEIGHT

The method of station adjustment explained on pages 8–16 is used in cases of first-order triangulation, and particularly where the observations are of unequal weight and the adjustment is involved. When the observations are of equal weight and the number of sum angles observed is not great, the adjustment can be much simplified by using the method shown in the example below:

In the record book of horizontal angles, at the end of the observations of angles for each station, there is always given an abstract of the observed angles. Where the station adjustment is made by the angle method, the corrections are applied directly to the observed angles in this abstract, and the list of directions is made directly from the abstract.

The following example illustrates the method. The abstract below is found in the record book:

Station Cora

Observed stations	Observed angle	Final seconds[1]
	° ′ ″	″
(1) Decision—Mac	16 11 53.6+v_1	53.5
(2) Decision—Nation	99 42 50.6+v_2	50.9
(3) Mac—Nation	83 30 57.6+v_3	57.4
(4) Howard—Nation	74 11 11.0+v_4	11.1
(5) Mac—Howard	9 19 46.1+v_5	46.3
(6) Nation—Decision	260 17 08.8+v_6	09.1

[1] This column is filled in after the adjustment is completed. See p. 18.

In the above table v_1, v_2, v_3, v_4, v_5, and v_6 represent corrections to be applied to the angles to make them consistent in themselves. Angles (2) and (3) are sum angles, and angle (6) gives a horizon closure with (2). There are, therefore, 3 condition equations, which are formed in the following manner:

1st	(1) − (2) + (3)	gives $0 = +0.6 + v_1 - v_2 + v_3$.
2d	− (3) + (4) + (5)	gives $0 = -0.5 - v_3 + v_4 + v_5$.
3d	(2) + (6)	gives $0 = -0.6 + v_2 + v_6$.

The correlate and normal equations below are formed in the same manner, as explained on pages 11–13. The normals are solved and the corrections computed in the manner explained on pages 13–16.

Correlate equations

	1	2	3	Σ_c	v*	Adopted v†
1	+1	--------	--------	+1	−0.062	−0.1
2	−1	--------	+1	0	+.331	+.3
3	+1	−1	--------	0	−.208	−.2
4	--------	+1	--------	+1	+.146	+.1
5	--------	+1	--------	+1	+.146	+.2
6	--------	--------	+1	+1	+.269	+.3

* This column is filled in after the adjustment is completed.
† See explanation of this column on p. 15.

Normal equations

	1	2	3	η	Σ_n	C
1	+3	−1	−1	+0.6	+1.6	−0.0616
2		+3	--------	−.5	+1.5	+.1462
3			+2	−.6	+.4	+.2692

Solution of normal equations

1	2	3	η	Σ_n
+3 C_1	−1 +.33333	−1 +.33333	+0.6 −.2	+1.6 −.53333
1	+3 −.3333	−.3333	−.5 +.2	+1.5 +.5333
	+2.6667 C_2	−.3333 +.125	−.3 +.1125	+2.03334 −.7625
	1 2	+2 −.3333 −.0417	−.6 +.2 −.0375	+.4 +.5333 +.2542
		+1.6250 C_3	−.4375 +.26923	+1.1875 −.73077

Back solution

3	2	1
+0.26923	+0.1125 +.0337	−0.2 +.0897 +.0487
	+.1462	−.0616

Computation of corrections (v's)

1	2	3	4	5	6
−0.062	+0.062 +.269	−0.062 −.146	+0.146	+0.146	+0.269
−.062 −.1	+.331 +.3	−.208 −.2	+.146 +.1	+.146 +.2	+.269 +.3

After the final seconds are placed on the observed angles in the abstract in the record book, the directions are made out on form 24A. (See fig. 6.)

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 24A

State: ____________

Station Cora ____ Computed by O. P. S.

Observer C.V.H. ____ Checked by W. F. R.

STATIONS OBSERVED	DIRECTIONS AFTER LOCAL ADJUSTMENT ° ′ ″	FINAL SECONDS ″
Decision	0 00 00.0	
Mac	16 11 53.5	
Howard	25 31 39.8	
Nation	99 42 50.9	

FIG. 6.—List of directions resulting from station adjustment by angle method

REDUCTION TO CENTER

When a station is not occupied centrally, the directions or angles observed at the eccentric point must be corrected to what they would have been if the instrument had been centered over the station mark. Also, if the signal observed upon is eccentric, the directions and angles involving this station must be corrected to what they would have been if the station itself had been observed upon. The computation for the reduction to center is made on Form 382. The instructions given on that form are repeated below, and examples of the two different cases of eccentricity are shown.

First are given a list of directions for a station occupied eccentrically (fig. 7) and the computations necessary to obtain the corrections for reducing the directions to center.

INSTRUCTIONS

The required reduction to center is, in seconds, $c=\frac{d \sin a}{s \sin 1''}$, in which d is the distance from the eccentric station to the true station, and s is the length in meters of the line between the true stations involved, and, therefore, log s is taken directly from the computation of triangle sides. a is the direction of the distant station involved, reckoned in a clockwise direction as usual but referred to the direction from the eccentric to the true station, or center, taken as zero. This definition of a is true for the case in which the object pointed upon is eccentric, as well as for the case in which the instrument is eccentric.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 24A

LIST OF DIRECTIONS

State: ____________

Station Ken Ecc. — Computed by O.P.S.
Observer C.V.H. — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment °	′	″	~~Final Seconds~~ Cor.″ for eccentr.
Chevy	0	00	00.00	
Tank west of Δ Dulce	29	03	37.0	-62.5
Ken (center) 3.469 m.	176	42		
Forest Glen Standpipe	313	24	53.0	+188.~~6~~5
Home	326	31	30.21	+39.2~~2~~4
Bu. of Stand. wireless pole	352	17	20.8	+13.0
Reno	357	28	48.63	+6.15

Station Ken — Computed by O.P.S.
Observer C.V.H. — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment °	′	″	Final Seconds ″
Chevy	0	00	00.00	
Tank west of Δ Dulce	29	02	34.5	
Forest Glen Standpipe	313	28	01.~~6~~5	
Home	326	32	09.4~~3~~5	
Bu. of Stand. wireless pole	352	17	33.8	
Reno	357	23	54.78	

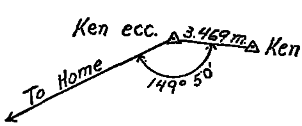


FIG. 7.—Lists of directions before and after reduction for eccentric station

Carry a to minutes only and all logarithms to five decimal places only. Do not in any case carry the derived reductions to more than two decimal places. There is no advantage in carrying them to more decimal places than the directions to which they are to be applied are carried on Form 24A.

REDUCTIONS FOR AN ECCENTRIC INSTRUMENT

If the instrument is eccentric, the first column of Form 382 should contain the names of the stations observed from that eccentric position of the instrument.

The values in the fifth column are derived by subtracting those in the fourth column from those in the third. The values in the fourth column may need to be derived by successive approximations from the triangle side computations if the eccentric reductions are large. The values in the sixth column are obtained from those in the fifth by adding log $\frac{d}{\sin 1''}$, derived as indicated in the heading of the form, if d is expressed in meters. If d is expressed in feet, to the other two logarithms add also 9.48402 to convert to meters. To obtain a direction as shown on Form 24A, subtract the reduction c for the station which is the initial on Form 24A from the reduction c for the required direction and apply the difference to the observed direction. Similarly, the correction to any angle is the difference of the reductions on this form to the two directions involved in that angle.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 382

REDUCTION TO CENTER

Eccentric Station: Ken

d = 3.469 meters

Log d = 0.54020
Colog sin 1″ = 5.3 1 4 4 3
Sum = 5.8 5 4 6 3

STATIONS	α °	′	Log sin α	Log s	Log $\frac{\sin \alpha}{s}$	Logarithm of reduction in seconds	Reduction = c ″
Center	0	00					
Chevy	183	18	8.76015	3.75~~097~~ 100	5.0091~~8~~ 5	0.863~~81~~ 78	-7.31
Tank west of Δ Dulce	212	22	9.72863	3.739~~64~~ 12	5.9895~~9~~ 1	1.844~~28~~ 14	-69.8~~6~~ 5
Forest Glen standpipe	136	43	9.83608	3.432~~40~~ 65	6.403~~68~~ 43	2.258~~51~~ 06	+181.~~26~~ 16
Home	149	50	9.70115	4.0516~~1~~ 3	5.649~~55~~ 52	1.50~~895~~ 415	+31.9~~1~~ 3
Bu. of Standards wireless pole	175	35	8.88654	3.98487	4.90167	0.75630	+5.71
Reno	180	47	8.13581	3.9263~~2~~ 5	4.2094~~5~~ 6	0.064~~18~~ 09	-1.16

FIG. 8.—Reduction to center of eccentric station

In order to compute the corrections to the directions due to eccentricity, it is necessary to know the logarithm of the distance from the station itself to each of the other stations. For this purpose preliminary triangles are computed. (See fig. 9.) The logarithms of the distances from Ken to Home, Reno, and Chevy are computed from the triangles Ken-Home-Reno and Ken-Home-Chevy. In computing these triangles it is not necessary to use the angles at Ken eccentric, as in each triangle the other two angles are known, and the concluded angles at Ken may be computed and used. This method of computing the logarithms of the lengths will give values which are more nearly the true values than if the eccentric angles were used.

However, as the other three stations observed from Ken eccentric, namely, "Tank near Dulce," "Forest Glen Standpipe," and "Bureau of Standards wireless pole," were not occupied, it is necessary to use the eccentric angles at Ken for the preliminary triangles used in computing the logarithms of the lengths.

After the logarithms of the lengths have been computed, all the data necessary to compute the eccentric corrections are known, and the computation can be made as shown in Figure 8.

The corrections thus determined are now applied to the corresponding directions observed at Ken eccentric, as shown in Figure 7. As it is desired to keep the initial direction (in this case to station Chevy) 0° 00′ 00″.0, the correction at Chevy is subtracted algebraically from each of the other corrections and these differences applied to the other directions. The following corrections are therefore applied: At Chevy, 0″.00; at Tank, west of Dulce, −69″.86−(−7″.31)=−62″.5; at Forest Glen Standpipe, +181″.26 −(−7″.31) =+188″.6; at Home, +31″.91−(−7″.31)=+39″.22; at Bureau of Standards wireless pole, +5″.71−(−7″.31)=+13″.0; at Reno, −1″.16−(−7″.31)=+6″.15. The corrected directions are shown in Figure 7.

When the eccentric corrections are large, the logarithms of the lengths computed by the use of the eccentric angles are usually not sufficiently accurate to give the exact corrections. In this case the triangles must be recomputed by using the corrected list of directions (see p. 19) and more accurate logarithms of the lengths obtained. These logarithms are then used to compute new eccentric corrections which are applied to the directions in the list of directions to give the final corrected directions. Ordinarily the first computation of the eccentric corrections is sufficiently accurate, but occasionally two and sometimes three computations are required.

REDUCTIONS FOR AN ECCENTRIC OBJECT OBSERVED

If the object observed is eccentric the heading "Eccentric station ——" on Form 382, should be changed to "Eccentric observed object at station ——," the first column should contain the names of the stations from which this eccentric object was observed, and in each case *a* is the direction from the eccentric object to the distant station involved, reckoned in a clockwise direction as usual but referred to the direction from the eccentric object to the true station, or center, taken as zero. (No distinction need be made between the direction from the eccentric object to the distant station and the direction from the true station to the distant station except when the eccentric reduction is more than one minute.) The remainder of the computation on Form 382 is made in the manner indicated above with reference to an eccentric instrument. The reductions to directions are, however, to be applied to observed directions, at the stations named in the first column, to the eccentric object at the station named in the heading. The directions to which these reduc-

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Maryland

NO.		STATIONS	OBSERVED ANGLE			CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLES AND DISTANCES	LOGARITHM
	2-3	Home-Reno								3.772214
c	1	Ken	30	56	~~55.6~~ 45.3	+3.5	48.8			0.2888~~03~~ 31
	2	Home	47	09	57.7	+3.4	61.1			9.865~~298~~ 304
	3	Reno	101	53	06.7	+3.4	10.1			9.99058~~8~~ 7
	1-3	Ken-Reno								3.9262~~30~~ 49
	1-2	Ken-Home								4.0516~~10~~ 32
					49.7					
	2-3	Home-Chevy								3.860903
c	1	Ken	33	27	~~57.2~~ 50.6	+2.3	52.9			0.2585~~01~~ 15
	2	Home	25	20	54.6	+2.2	56.8			9.6315~~69~~ 78
	3	Chevy	121	11	08.1	+2.2	10.3			9.93221~~7~~ 4
	1-3	Ken-Chevy								3.7509~~75~~ 96
	1-2	Ken-Home								4.0516~~21~~ 32
					53.3					
	2-3	Ken-Reno								3.9263~~20~~ 49
c	1	Tank near Dulce	(111	~~06~~ 07	~~46.2~~ 53.7)					0.030~~176~~ 252
	2	Ken	31	~~54~~ 33	~~47.2~~ 39.7					9.71~~9070~~ 8839
	3	Reno	37	18	26.6					9.782538
	1-3	Tank-Reno								3.675~~568~~ 420
	1-2	Tank-Ken								3.759~~036~~ 119
	2-3	Reno-Ken								3.9263~~20~~ 49
c	1	Forest Glen Standpipe	(119	~~46~~ 49	~~19.0~~ 22.6)					0.061~~476~~ 697
	2	Reno	16	09	44.2					9.444605
	3	Ken	44	~~03~~ 00	~~56.8~~ 53.2					9.84~~2237~~ 1887
	1-3	Forest Glen Standpipe - Ken								3.432~~401~~ 651
	1-2	Forest Glen Standpipe - Reno								3.83~~6098~~ 29933
	2-3	Reno-Ken								3.9263~~20~~ 49
c	1	Bu. of Stand. wireless pole	(31	22	~~06.8~~ 14.8)					0.2835~~45~~ 17
	2	Reno	143	26	24.2					9.775001
	3	Ken	5	11	~~29.0~~ 21.0					8.956~~566~~ 381
	1-3	Wireless pole-Ken								3.98486~~6~~ 7
	1-2	Wireless pole-Reno								3.166~~481~~ 247

FIG. 9.—Preliminary computation of triangles for reduction of eccentric station

tions are to be applied are therefore found in various of the lists of directions on Form 24A, not all in one list, as is the case when the instrument is eccentric.

On page 24 is given an example of a computation for reduction to center, when the observed object is eccentric as seen from several other

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 24A

LIST OF DIRECTIONS

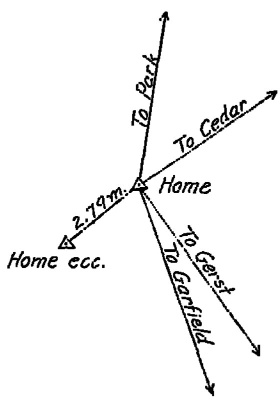
State: Maryland

Station Home — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Park	0 00 00.0	
Cedar	47 07 49.6	
Gerst	137 42 15.9	
Garfield	152 25 42.7	
Home Ecc. 2.79 m.	222 55 20.	

Station Park — Computed by O.P.S. — Observer C.L.G. — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Cedar	0 00 00.0	
Wallace	44 45 54.8	
{Home	116 24 38.6	
{Home Ecc	116 26 12.1	
Reno	173 33 28.9	



Station Cedar — Computed by O.P.S. — Observer C.L.G. — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Wallace	0 00 00.0	
{Home	43 02 10.6	
{Home Ecc	43 02 07.4	
Takoma	58 59 17.6	
Park	59 29 28.5	

Station Gerst — Computed by O.P.S. — Observer C.L.G. — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Garfield	0 00 00.0	
Stanton	12 13 22.0	
{Home	46 21 33.0	
{Home Ecc	46 20 53.2	

Station Garfield — Computed by O.P.S. — Observer C.L.G. — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Insane	0 00 00.0	
{Home	62 58 17.2	
{Home Ecc	62 57 25.9	
Stanton	88 40 09.0	
Potomac	93 11 08.1	

FIG. 10.—Lists of directions for eccentric observed object

stations. Station Home is eccentric as seen from stations Park, Cedar, Gerst, and Garfield.

In Figure 10 there is given a list of directions at station Home, showing directions to stations Park, Cedar, Gerst, Garfield, and Home eccentric, and also lists of directions at stations Park, Cedar, Gerst, and Garfield, at which station Home is seen eccentrically. The computation is made in the same way as where the station is occupied eccentrically. The directions used in the computation are obtained from the lists of directions at station Home, using Home eccentric as initial, remembering, however, that 180° must be added to these directions, since the directions are taken at the station itself and not at the eccentric station.

As in the case of an eccentric instrument, the approximate logarithms of the lengths are obtained by a preliminary computation of triangles. As the details of this computation are given on pages 20–21, it is not necessary to repeat them here.

After the eccentric corrections are determined, they are applied, not to the directions observed at station Home, but to the direction Home eccentric in each of the lists of directions for stations Park, Cedar, Gerst, and Garfield. (See fig. 10.)

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 382

REDUCTION TO CENTER

observed object at
Eccentric Station: Home

$d =$ 2.79 meters

Log d = 0.44560
Colog sin 1″ = 5.31443
Sum = 5.76003

STATIONS	α ° ′	Log sin α	Log s	Log $\frac{\sin \alpha}{s}$	Logarithm of Reduction in Seconds	Reduction = c ″
Center	0 00					
Cedar	4 12	8.86474	4.12189	4.74285	0.50288	+3.18
Gerst	94 46	9.99850	4.10683	5.89167	1.65170	+44.84
Garfield	109 30	9.97435	4.02427	5.95008	1.71011	+51.30
Park	317 06	9.83297	3.62200	6.21097	1.97100	-93.54

FIG. 11.—Reduction to center of eccentric observed object

REDUCTION OF HORIZONTAL DIRECTIONS TO SEA LEVEL

In case the elevation of a triangulation station is very great, a correction must be applied to the observations upon that station to reduce them to sea level. It is only necessary to compute this correction for triangulation of the first order, as it usually amounts to only a few hundredths of a second. The correction, expressed in seconds, is

$$\frac{e^2 h \sin 2\alpha \cos^2 \phi}{2\rho \sin 1''},$$

where $e^2 = \frac{a^2 - b^2}{a^2}$, h = the height of the station observed and α = its azimuth reckoned in a clockwise direction from south, ρ = the radius of curvature of the earth in a plane normal to the meridian at the station from which the direction is measured, and ϕ = the latitude of the station from which the direction is measured.

Below is given an example of the computations required to obtain the corrections by means of the formula. The computations are arranged in a table for convenience.

Computation of sea-level corrections using formula

[Station Bull, latitude 48° 20′]

Observed station	h	α	2α	Log sin 2α	Log h	Log $\frac{e^2 \cos^2 \phi}{2\rho \sin 1''}$	Log correction	Correction
	m.	° ′	° ′					″
Snake	813	50 45	101 30	9.991	2.910	5.684	8.585	+0.04
Gladys	761	221 52	83 44	9.997	2.881	5.684	8.562	+.04
Bonetrail	740	239 21	118 43	9.943	2.869	5.684	8.496	+.03
Williston	723	293 57	227 54	9.870	2.859	5.684	8.413	−.03
Buford	756	353 42	347 24	9.339	2.879	5.684	7.902	−.01

Since ϕ is a constant for a particular station, e^2 does not vary, and ρ does not vary enough for any given latitude to affect the result, the same value of the factor $\frac{e^2 \cos^2 \phi}{2\rho \sin 1''}$ can be used for all the directions at any given station. The values of h are obtained from the vertical angle computations, and the values of ϕ and α are obtained from the geographic position computations which, necessarily, must be made before the sea-level corrections can be determined. To obtain the corrections to the directions it is necessary to add the sum of log h and log sin 2α for each direction to log $\frac{e^2 \cos^2 \phi}{2\rho \sin 1''}$ which, as was stated, is the same for all directions at any given station. The sign of each correction is determined by the sign of sine α.

Since the sea-level correction is always small and since a large part of the above formula may be considered a constant for a given station, it is possible to make use of a table which greatly facilitates the computations. Such a table is given on page 26.

Correction to horizontal direction for elevation of mark

[Corr. = $+\frac{e^2 h}{2\rho \sin 1''}\cos^2\phi \sin 2\alpha$; $\frac{e^2}{2\rho \sin 1''}\cos^2\phi \sin 2\alpha$ is tabulated below for the *sixth decimal place*. It is to be multiplied by h in meters. The sign of the correction is + for azimuths in the first and third quadrants and − for azimuths in the second and fourth quadrants.]

	Azimuth of direction														
	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
+	0 90	5 85	10 80	12.5 77.5	15 75	17.5 72.5	20 70	22.5 67.5	25 65	27.5 62.5	30 60	32.5 57.5	35 55	40 50	45 45
−	90 180	95 175	100 170	102.5 167.5	105 165	107.5 162.5	110 160	112.5 157.5	115 155	117.5 152.5	120 150	122.5 147.5	125 145	130 140	135 135
+	180 270	185 265	190 260	192.5 257.5	195 255	197.5 252.5	200 250	202.5 247.5	205 245	207.5 242.5	210 240	212.5 237.5	215 235	220 230	225 225
−	270 360	275 355	280 350	282.5 347.5	285 345	287.5 342.5	290 340	292.5 337.5	295 335	297.5 332.5	300 330	302.5 327.5	305 325	310 330	315 315
ϕ															
20	0	17	33	41	48	55	62	68	74	79	84	88	91	95	97
21	0	17	33	40	48	55	61	67	73	78	83	86	90	94	95
22	0	16	32	40	47	54	60	67	72	77	81	85	88	93	94
23	0	16	32	39	46	53	60	66	71	76	80	84	87	91	93
24	0	16	31	39	46	52	59	65	70	75	79	83	86	90	91
25	0	16	31	38	45	52	58	64	69	74	78	81	84	89	90
26	0	15	30	37	44	51	57	63	68	72	77	80	83	87	88
27	0	15	30	37	43	50	56	61	67	71	75	79	82	86	87
28	0	15	29	36	43	49	55	60	65	70	74	77	80	84	85
29	0	15	29	35	42	48	54	59	64	69	73	76	79	82	84
30	0	14	28	35	41	47	53	58	63	67	71	74	77	81	82
31	0	14	28	34	40	46	52	57	62	66	70	73	76	79	80
32	0	14	27	33	39	45	51	56	60	64	68	71	74	78	79
33	0	13	26	33	38	44	49	54	59	63	67	70	72	76	77
34	0	13	26	32	38	43	48	53	58	62	65	68	71	74	75
35	0	13	25	31	37	42	47	52	56	60	64	67	69	72	73
36	0	12	25	30	36	41	46	51	55	59	62	65	67	71	72
37	0	12	24	30	35	40	45	49	53	57	60	63	66	69	70
38	0	12	23	29	34	39	44	48	52	56	59	62	64	67	68
39	0	12	23	28	33	38	42	47	51	54	57	60	62	65	66
40	0	11	22	27	32	37	41	45	49	53	56	58	60	63	64
41	0	11	21	26	31	36	40	44	48	51	54	56	59	61	62
42	0	10	21	26	30	35	39	43	46	50	52	55	57	60	60
43	0	10	20	24	29	34	38	41	45	48	51	53	55	58	59
44	0	10	19	24	28	32	36	40	43	46	49	51	53	56	57
45	0	10	19	23	27	31	35	39	42	45	47	50	51	54	55
46	0	9	18	22	26	30	34	37	40	43	46	48	50	52	53
47	0	9	17	22	25	29	33	36	39	42	44	46	48	50	51
48	0	9	17	21	25	28	31	35	38	40	42	44	46	48	49
49	0	8	16	20	24	27	30	33	36	39	41	43	44	46	47
50	0	8	15	19	23	26	29	32	35	37	39	41	42	45	45
51	0	8	15	18	22	25	28	31	33	36	38	39	41	43	43
52	0	7	14	18	21	24	27	29	32	34	36	38	39	41	41
53	0	7	14	17	20	23	25	28	30	32	34	36	37	39	40
54	0	7	13	16	19	22	24	27	29	31	33	34	36	37	38
55	0	6	12	15	18	21	23	25	28	29	31	33	34	35	36
56	0	6	12	14	17	20	22	24	26	28	30	31	32	34	34
57	0	6	11	14	16	19	21	23	25	27	28	29	31	32	32
58	0	5	11	13	15	18	20	22	24	25	27	28	29	30	31
59	0	5	10	12	15	17	19	21	22	24	25	26	27	29	29
60	0	5	9	12	14	16	18	19	21	22	24	25	26	27	27
61	0	4	9	11	13	15	17	18	20	21	22	23	24	25	26
62	0	4	8	10	12	14	15	17	18	20	21	22	23	23	24
63	0	4	8	10	11	13	14	16	17	18	20	20	21	22	23
64	0	4	7	9	11	12	14	15	16	17	18	19	20	21	21
65	0	3	7	8	10	11	13	14	15	16	17	18	18	19	20
66	0	3	6	8	9	10	12	13	14	15	16	16	17	18	18
67	0	3	6	7	8	10	11	12	13	14	14	15	16	16	17
68	0	3	5	6	8	9	10	11	12	13	13	14	14	15	15
69	0	2	5	6	7	8	9	10	11	12	12	13	13	14	14
70	0	2	4	5	6	7	8	9	10	10	11	12	12	13	13
71	0	2	4	5	6	7	7	8	9	10	10	11	11	11	12
72	0	2	4	4	5	6	7	7	8	9	9	9	10	10	10
73	0	2	3	4	5	5	6	7	7	8	8	8	9	9	9
74	0	1	3	4	4	5	5	6	6	7	7	8	8	8	8
75	0	1	3	3	4	4	5	5	6	6	6	7	7	7	7

The arguments used in this table are the latitude of the station at the left and the azimuth of the direction at the top. For convenience the computation should be made in tabular form as shown below.

Computation of sea-level corrections using table

[Station Bull, latitude 48° 20′]

Observed station	h	α	$\frac{e^2 \sin 2\alpha \cos^2\phi}{2\rho \sin 1''}$	Correction
	m.	° ′		″
Snake	813	50 45	0.000047	+0.04
Gladys	761	221 52	.000048	+.04
Bonetrail	740	239 21	.000042	+.03
Williston	723	293 57	.000036	−.03
Buford	756	353 42	.000011	−.01

In the first and second columns are given the name and height of the station observed, in the third column the azimuth of the observed station from the occupied station, in the fourth column the factor

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 24A

LIST OF DIRECTIONS

State: North Dakota

Station Bull — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Williston	0 00 ~~00.00~~ 59.97	
Buford	59 45 10.0~~6~~5	
Snake	116 48 35.3~~2~~6	
Gladys	287 55 21.1~~2~~6	
Bonetrail	305 24 33.8~~0~~3	

Station Bull — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Williston	0 00 00.00	
Buford	59 45 10.08	
Snake	116 48 35.39	
Gladys	287 55 21.19	
Bonetrail	305 24 33.86	

FIG. 12.—List of directions corrected for sea-level reduction

$\frac{e^2 \sin 2\alpha \cos^2 \phi}{2\rho \sin 1''}$ as taken from the table on page 26, and in the last column the sea level correction which is obtained by multiplying the factor in the fourth column by the height in the second column. The correction is plus for azimuths in the first (0° to 90°) and third (180° to 270°) quadrants and minus for azimuths in the second (90° to 180°) and fourth (270° to 360°) quadrants.

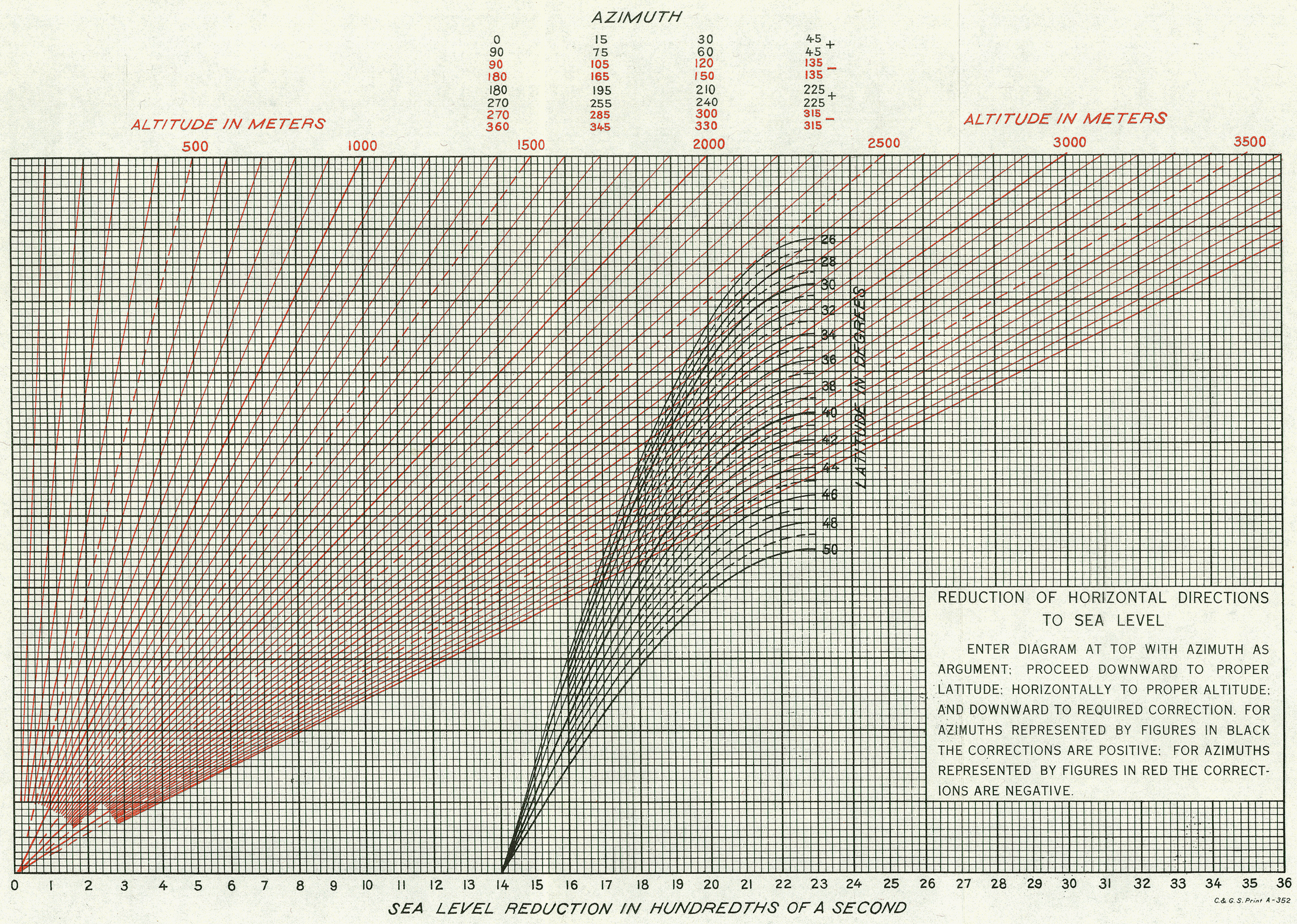
After the sea level corrections have been determined, they are applied to the corresponding directions at station Bull. The list of directions is then rewritten to make the reading of the initial station, Williston, 0° 00′ 00.″00. (See fig. 12.)

The sea-level corrections may also be determined by means of the nomogram shown in Figure 13 which was designed by H. S. Rappleye, associate mathematician of the division of geodesy of this bureau. Directions for using this nomogram are given on the figure.

LIST OF DIRECTIONS, ALL PRELIMINARY CORRECTIONS APPLIED

After all corrections have been applied to the observed directions or angles as the case may be, the list of directions is made out on form 24A, as shown in Figure 12. The directions are arranged by giving the initial direction a value of 0° 00′ 00.″00 and continuing in a clockwise direction around the horizon.

The list of directions should then be checked and initialed by the checker before it is used in taking out the angles for the triangles.



AZIMUTH
0 15 30 45 +
90 75 60 45
90 105 120 135 −
180 165 150 135
180 195 210 225 +
270 255 240 225
270 285 300 315 −
360 345 330 315
ALTITUDE IN METERS
500 1000 1500 2000 2500 3000 3500
ALTITUDE IN METERS
LATITUDE IN DEGREES
26 28 30 32 34 36 38 40 42 44 46 48 50
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36
SEA LEVEL REDUCTION IN HUNDREDTHS OF A SECOND
REDUCTION OF HORIZONTAL DIRECTIONS TO SEA LEVEL
ENTER DIAGRAM AT TOP WITH AZIMUTH AS ARGUMENT: PROCEED DOWNWARD TO PROPER LATITUDE: HORIZONTALLY TO PROPER ALTITUDE: AND DOWNWARD TO REQUIRED CORRECTION. FOR AZIMUTHS REPRESENTED BY FIGURES IN BLACK THE CORRECTIONS ARE POSITIVE: FOR AZIMUTHS REPRESENTED BY FIGURES IN RED THE CORRECTIONS ARE NEGATIVE.
C.& G. S. Print A-352

CHAPTER 2.—ADJUSTMENT OF A QUADRILATERAL

SKETCH

Before starting the adjustment of a net of triangulation the mathematician should make a good clear sketch showing all the lines over which observations were made. The unobserved directions should be shown by dotted lines. A sketch of a typical quadrilateral is shown in Figure 14. In this figure C and A can not be seen from D.

TRIANGLES

After the figure is drawn the triangles should be written out in *clockwise* order on Form 25. In the quadrilateral above with the line AB fixed the four triangles should be written as follows starting at station C: CAB, DAB, DAC, and DBC; or starting with station D, DAB, CDA, CDB, and CAB. That is, at each station not on the fixed line all the triangles formed by connecting it with stations on the fixed line or previously named stations should be written in clockwise order.

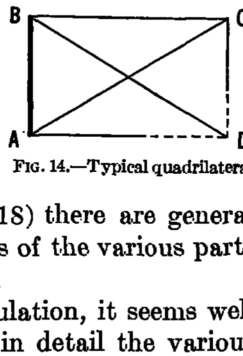


FIG. 14.—Typical quadrilateral

After the local conditions, that is, those arising from the relations of the angles at each station to one another, are satisfied (see pp. 8–18) there are general conditions arising from the geometrical relations of the various parts forming a closed figure which must be satisfied.

To illustrate the method of adjusting triangulation, it seems well to start with a simple quadrilateral, and give in detail the various steps of the adjustment. The adjustment of a larger figure or net of triangulation, involving all the various conditions which enter into such an adjustment, is shown on pages 50–109.

In the sample given below, a quadrilateral of first-order triangulation was selected for illustrating the methods. The adjustment of triangulation of the lower orders is similar except that the angles, lengths, and logarithms are not carried to as many decimal places. In the adjustment of triangulation of the first order, the angles are carried to hundredths of seconds, and the logarithms are carried to eight places in the equations and to seven places in the final lengths used in the triangles.

There are given below the lists of directions for stations Roman, Spencer, Yellow, and Fairview, the four stations making up the sample quadrilateral. These directions are assumed to have been corrected for sea-level reduction and for any local adjustment required and to have been checked. In the adjustment, the method of directions is used; that is, an angle is considered as the difference of two directions. The geographic positions (latitudes and longitudes) of stations Roman and Spencer are considered fixed and also the length and azimuth of the line joining them.

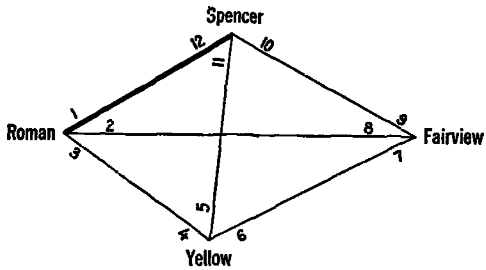


FIG. 15.—Quadrilateral used for sample adjustment

The sketch showing the relative positions of the stations is drawn and the directions numbered as shown in Figure 15. It is not necessary to spend much time in making the sketch absolutely to scale, although it should be approximately correct in order to give an idea of the relative size of the angles, the sketch being used as an aid in forming the equations.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 24A

LIST OF DIRECTIONS

State: Oregon

Station Roman — Computed by W.F.R. — Observer O.B.French — Checked by O.P.S.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Spencer	0 00 00.00	59.96
Fairview	31 04 11.58	11.86
Yellow	65 12 45.72	45.48
Mary	291 34 34.04	
Peterson	321 25 23.53	
Twin	330 41 33.42	

Station Spencer — Computed by W.F.R. — Observer O.B.French — Checked by O.P.S.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Peterson	0 00 00.00	
Twin	1 08 19.38	
Fairview	131 12 05.23	04.91
Yellow	197 25 26.30	26.80
Roman	251 46 38.49	38.31
Ranch	270 37 26.91	
Willamette South Base	311 51 09.89	
Mary	318 12 01.16	
Ridge	319 15 00.47	
Willamette North Base	328 26 41.12	

Station Yellow — Computed by W.F.R. — Observer O.B.French — Checked by O.P.S.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
White	0 00 00.00	
Roman	178 40 38.63	38.69
Spencer	239 06 47.30	47.52
Fairview	297 46 09.74	09.96
Scott	337 15 48.07	

Station Fairview — Computed by W.F.R. — Observer O.B.French — Checked by O.P.S.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
White	0 00 00.00	
Scott	23 26 03.35	
Yellow	54 53 23.69	23.22
Roman	81 39 24.54	24.90
Spencer	110 00 45.96	46.06
Black	310 44 55.62	

FIG. 16.—Lists of directions for stations of quadrilateral

The triangles are then written out in clockwise order on Form 25 as described on page 29. (See fig. 17.)

EXPLANATION OF TRIANGLES

In the first column of Form 25 is given the designation of the angle, in the second the name of the station, in the third the observed angle at the station, in the fourth the correction to the angle as determined

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Oregon

1 NO.		2 STATIONS	3 OBSERVED ANGLE			4 CORR'N	5 SPHER'L ANGLE	6 SPHER'L EXCESS	7 PLANE ANGLES AND DISTANCES	8 LOGARITHM
	2-3	Roman-Spencer								4.7176307
-4+5	1	Yellow	60	26	09.17	-0.34	08.83	1.95	06.88	0.0605813
-1+3	2	Roman	65	12	45.72	-0.20	45.52	1.96	43.56	9.9580218
-11+12	3	Spencer	54	21	12.19	-0.68	11.51	1.95	09.56	9.9098872
	1-3	Yellow-Spencer				-1.22		5.86		4.7362338
	1-2	Yellow-Roman								4.6880992
					07.08					
	2-3	Yellow-Roman								4.6880992
-7+8	1	Fairview	26	46	00.85	+0.83	01.68	2.19	59.49	0.3464441
-4+6	2	Yellow	119	05	31.11	+0.16	31.27	2.19	29.08	9.9414345
-2+3	3	Roman	34	08	34.14	-0.52	33.62	2.19	31.43	9.7491538
	1-3	Fairview-Roman				+0.47		6.57		4.9759778
	1-2	Fairview-Yellow								4.7836971
					06.10					
	2-3	Yellow-Spencer								4.7362338
-7+9	1	Fairview	55	07	22.27	+0.57	22.84	2.39	20.45	0.0859876
-5+6	2	Yellow	58	39	21.94	+0.50	22.44	2.39	20.05	9.9314862
-10+11	3	Spencer	66	13	21.07	+0.82	21.89	2.39	19.50	9.9614757
	1-3	Fairview-Spencer				+1.89		7.17		4.7537076
	1-2	Fairview-Yellow								4.7836971
					05.28					
	2-3	Roman-Spencer								4.7176307
-8+9	1	Fairview	28	21	21.42	-0.26	21.16	2.15	19.01	0.3233636
-1+2	2	Roman	31	04	11.58	+0.32	11.90	2.15	09.75	9.7127133
-10+12	3	Spencer	120	34	33.26	+0.14	33.40	2.16	31.24	9.9349835
	1-3	Fairview-Spencer				+0.20		6.46		4.7537076
	1-2	Fairview-Roman								4.9759778
					06.26					

FIG. 17.—Triangle computation for stations of quadrilateral

by the adjustment, in the fifth the adjusted spherical angle at the station, in the sixth the spherical excess or difference between the plane and spherical angle, in the seventh the plane angle, and in the eighth column the logarithms of the distances and the logarithms of the sines of the plane angles (cologarithm of the sine of the first angle).

It is apparent that to begin with, we have only the data in the first, second, and third columns. The other columns are filled in after the adjustment is completed. The first and second columns can be filled in directly from the sketch. The observed angles in the third column are obtained from the lists of directions, an angle being the difference of two directions. For example, the angle at Yellow, between Roman and Spencer, is obtained from the list of directions for station Yellow (fig. 16) by subtracting the direction to Roman from the direction to Spencer.

To obtain the angle at Roman between Peterson and Fairview, it is necessary to add 360° to the direction to Fairview before subtracting the direction to Peterson; (31° 04′ 11.″58 + 360°) − 321° 25′ 23.″53 = 69° 38′ 48.″05. In other words, if a direction is less than the one which is to be subtracted from it, then 360° must be added to it before the subtraction is made.

As an angle is obtained by subtracting the direction to the left hand station from the direction to the right hand station, it is designated by the two numbers representing these directions on the sketch, the number of the left hand direction having a minus sign because that direction is subtracted from the other. For example, the designation of the angle at Yellow between Roman and Spencer is −4 + 5, 4 and 5 being the designations of the directions from Yellow to Roman and to Spencer, respectively.

SPHERICAL EXCESS

The total spherical excess for each triangle as given on Form 25 is obtained by the formula (see Special Publication No. 8, p. 7)

$$\epsilon = \frac{a_1\ b_1\ \sin\ C_1\ (1 - e^2\ \sin^2\ \phi)^2}{2\ a^2\ (1 - e^2)\ \sin\ 1''} = a_1\ b_1 \sin\ C_1 \times m.$$

where ϵ is the spherical excess; a_1, b_1 and C_1 are the two sides and the included angle, respectively, of the corresponding triangle; e is the eccentricity and a the semimajor axis of the spheroid of reference; and ϕ is the mean latitude of the three vertices of the triangle. The letter m is used to designate that part of the expression above which depends only on the latitude and the dimensions of the spheroid and the values of log m are given with the latitude as an argument in the table on page 234.

To compute the spherical excess of a triangle it is necessary to make a preliminary computation of the triangle to obtain the logarithms of the sines of the angles and the logarithms of the lengths. In this preliminary computation, the logarithms need be carried out to only

4 places of decimals. (See fig. 17a.) As an example, the spherical excess of the triangle, Yellow–Roman–Spencer, is computed in the following manner:

In this triangle the length Roman-Spencer may be called a_1, and Yellow–Spencer b_1. Then the angle at Spencer will be C_1. Log m is taken out of the table for latitude 43° 49′, which is approximately the mean latitude of the three vertices. We then have

$$
\begin{array}{lcl}
\log a_1 & = & 4.7176 \\
\log b_1 & = & 4.7362 \\
\log \sin C_1 & = & 9.9099-10 \\
 & & {}^{1}\,9.3637 \\
\log m & = & 1.4041-10 \\
\hline
\log \epsilon & = & 0.7678 \\
\epsilon & = & 5''.859
\end{array}
$$

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 23

COMPUTATION OF TRIANGLES

State: Oregon

NO.	STATION	OBSERVED ANGLE	CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
2-3		Roman-Spencer					4.7176
1	Yellow	60 26 09.17					0.0606
2	Roman	65 12 45.72					9.9580
3	Spencer	54 21 12.19					9.9099
1-3		Yellow-Spencer					4.7362
1-2		Yellow-Roman					4.6881

FIG. 17a.—Preliminary triangle computation to obtain lengths for spherical excess computation

One third of the total spherical excess for the triangle is applied to each of the three angles as shown in the sixth column of the triangle computation in Figure 17. In like manner the spherical excess for the other three triangles of the quadrilateral can be obtained and applied.

The total spherical excess of a quadrilateral should have the same value when computed by adding together the sperical excesses of either pair of triangles which cover its area. In the preceding example, the sum of the spherical excesses of the two triangles, Yellow–Roman–Spencer and Fairview–Yellow–Spencer, should equal the sum of the spherical excesses of the two triangles Fairview–Yellow–Roman and Fairview–Roman–Spencer, that is, $5.86+7.17=6.57+6.46$.

In computing the spherical excesses, it is sometimes found that these sums fail to check by 0″.01, due to the dropping of the third decimal place. They should always be made to check by arbitrarily changing the spherical excess of one of the triangles by 0″.01.

[1] This sum can be obtained directly from the triangle computation without recopying the individual values.

CLOSURES OF TRIANGLES

The sum of the three angles of a triangle should equal 180° plus the spherical excess of the triangle. This rarely happens when the observed angles are used, and consequently a triangle closure arises. The triangle closure (closing correction) is obtained by subtracting the sum of the three angles of the triangle from 180° plus the spherical excess. For example, the sum of the three angles of the triangle Yellow–Roman–Spencer is 180° 00′ 07″.08, and 180° plus the spherical excess is 180° 00′ 05″.86, so the triangle closure is 180° 00′ 05″.86 − 180° 00′ 07″.08 = −1″.22.

In a similar manner the closures are obtained for the other three triangles of the quadrilateral. The closure for the whole quadrilateral should be the same when computed by adding together the closures of either pair of triangles which cover its area. For example, the algebraic sum of the closing corrections of the two triangles, Yellow–Roman–Spencer and Fairview–Yellow–Spencer should equal the algebraic sum of the closing corrections of the two triangles Fairview–Yellow–Roman and Fairview–Roman–Spencer, that is, $-1.22+1.89=+0.47+0.20$. This check should always be applied before beginning the adjustment.

NUMBER AND SELECTION OF EQUATIONS

After the first, second, third, and sixth columns of the triangle computation (see fig. 17) are filled in as already explained, the quadrilateral is ready to be adjusted. The first thing to be done is to determine the number of equations in the adjustment. In a simple quadrilateral where the length and azimuth of only one line are fixed, we have two kinds of equations, angle and side equations.

Condition equations must be included in the adjustment to eliminate the closing errors of the triangles, that is, to make the sum of the angles of each triangle exactly 180° plus the spherical excess of the triangle. These are called angle equations. A condition equation must also be included to insure that the lines at the pole (the point around which the equation is formed) pass through the same point (see Special Publication No. 28, p. 14). This is called a side equation.

The formulas to be used in computing the number of equations in the adjustment of a triangulation net are

$$\text{number of angle equations} = n' - S' + 1,$$
$$\text{number of side equations} = n - 2S + 3,$$

in which n is the total number of lines, n' is the number of lines sighted over in both directions, S is the total number of stations, and S' is the number of occupied stations. In using these formulas allowance must be made for lines or triangles fixed by previous adjustments. (See p. 59 for another method of determining the number of equations.)

METHOD FOR DETERMINING NUMBER OF EQUATIONS IN AN ADJUSTMENT

by B. K. Meade

In the adjustment of a triangulation net the total number of equations involved is given by the formula,

$$\text{number of equations} = v - 3S_n + S_u$$

in which v is the total number of v's excluding those on lines fixed by previous adjustment, S_n is the total number of new stations, and S_u is the total number of unoccupied new stations.

This formula takes care of azimuth, length, latitude and longitude equations. With any Laplace azimuths or base lines in the net, one equation is added for each condition introduced thereby. In case of a fixed station, that is, position tie only, this should be considered a new station, then two equations are added, one for latitude and one for longitude. When a fixed station is connected to any other fixed station, with directions observed to new stations only, then the fixed station should be considered a new station and two equations should be added to the number obtained by the above formula.

This method of determining the total number of equations in a net will serve as a check against the number derived by the usual procedure of building up the figure point by point.

In the quadrilateral Roman-Spencer-Fairview-Yellow (see fig. 15 $n=6$, $n'=6$, $S=4$, $S'=4$. Therefore

$$\text{number of angle equations} = 6-4+1=3,$$
$$\text{number of side equations} = 6-8+3=1.$$

The number of equations in a net having been determined, it is necessary to carefully select the equations so that all discrepancies will be eliminated by the adjustment. In forming the side equations it is necessary to select the pole so that the small angles will be used in the equation, as their tabular differences are proportionately much less affected by the dropping of decimal places than those of the larger angles. On the other hand, the triangles with the large angles should be used in forming the angle equations. (This rule in regard to angle equations need not be followed for a simple quadrilateral.)

The corrections to directions are designated by $v_1, v_2, \ldots\ldots v_n$, but for convenience it is customary to drop the v's, and simply write (1), (2), (3). . . . etc., in which the numbers are not quantities but subscripts of the corresponding v's.

ANGLE EQUATIONS

The angle equations then for the quadrilateral shown in Figure 15 are formed as follows:

Angle equation 1, $-(1)+(3)-(4)+(5)-(11)+(12)=-1.22$, or
as usually written $0=+1.22-(1)+(3)-(4)+(5)-(11)+(12)$,
angle equation 2, $0=-0.47-(2)+(3)-(4)+(6)-(7)+(8)$,
angle equation 3, $0=-1.89-(5)+(6)-(7)+(9)-(10)+(11)$.

Since there are four closed triangles (see fig. 17), one might suppose that there could be four angle equations. However, by studying the formation of the angle equations, it can be seen that the fourth equation would not be independent but would be a combination of the other three. For example, the fourth angle equation of the quadrilateral would read

$$0=-0.20-(1)+(2)-(8)+(9)-(10)+(12).$$

But equation 1 is, $0=+1.22-(1)+(3)-(4)+(5)-(11)+(12)$,
and equation 2 is $0=-0.47-(2)+(3)-(4)+(6)-(7)+(8)$.

Therefore equation 1 − equation 2 is,

$$0=+1.69-(1)+(2)+(5)-(6)+(7)-(8)-(11)+(12).$$

But equation 3 is,

$$0=-1.89-(5)+(6)-(7)+(9)-(10)+(11),$$

therefore equation 1 − equation 2 + equation 3 is,

$$0=-0.20-(1)+(2)-(8)+(9)-(10)+(12)$$

which is identical with the fourth equation above. This shows that the fourth equation is simply a combination of the first three and so will necessarily be satisfied by any values of the v's that satisfy the other three.

SIDE EQUATIONS

In order to include in the side equation the small angles of the quadrilateral, designated by $-7+8$ and $-8+9$, respectively, the pole must be taken at Roman. The equation is formed by expressing the condition that the lines Roman-Spencer, Roman-Fairview, and Roman-Yellow meet in a point; that is,

$$\frac{\text{Roman-Spencer}}{\text{Roman-Fairview}} \times \frac{\text{Roman-Fairview}}{\text{Roman-Yellow}} \times \frac{\text{Roman-Yellow}}{\text{Roman-Spencer}} = 1.$$

For the sides of the triangles the sines of the opposite angles may be substituted, and the equation becomes

$$\frac{\sin[-8+9]\ \sin[-4+6]\ \sin[-11+12]}{\sin[-10+12]\ \sin[-7+8]\ \sin[-4+5]} = 1.$$

Or for computation by logarithms [1] we have

$$\log\sin[-8+9] + \log\sin[-4+6] + \log\sin[-11+12] = \log\sin[-10+12] + \log\sin[-7+8] + \log\sin[-4+5]$$

For convenience in computing, the equation is arranged in tabular form as shown on page 37. The designations of the angles are placed in the first and fifth columns, the angles themselves in the second and sixth columns, the logarithms of the sines of the angles in the third and seventh columns and the tabular differences of the logarithms of the sines for 1 second of the angles in the fourth and eighth columns. The sums of the logarithms in the third and seventh columns are then taken, and the constant term of the side equation is obtained by subtracting the sum in column 7 from the sum in column 3 and pointing off this difference in units of the sixth decimal place. The quantities in columns 4 and 8 are the coefficients of the quantities in columns 1 and 5, respectively, and the rest of the equation is formed by multiplying together the quantities in columns 4 and 1 and those in columns 8 and 5, and changing the signs of the latter products.

The designations of the angles in the first and fifth columns are taken directly from the sketch (see fig. 15), and the angles themselves in the second and sixth columns are obtained from the triangle computation (fig. 17). Each value in the fourth and eighth columns is the amount of change in the logarithm of the sine of the angle corresponding to a change of 1″ in the angle and this multiplied by

[1] It is customary in all the computations to designate logarithm by *log*, sine by *sin* and cosine by *cos*.

the v applying to the angle gives the change in the logarithm of the sine of the angle produced by the v. In the work of the United States Coast and Geodetic Survey, the tabular differences of the logarithms are taken in units of the sixth place of decimals.

Side equation

1	2	3	4	5	6	7	8
	° ′ ″				° ′ ″		
−8+9	28 21 21.42	9.67664586	+3.90	−10+12	120 34 33.26	9.93498097	−1.24
−4+6	119 05 31.11	9.94143208	−1.18	−7+8	26 46 00.85	9.65356160	+4.18
−11+12	54 21 12.19	9.90989120	+1.51	−4+5	60 26 09.17	9.93942144	+1.19
		9.52796914				9.52796401	

4. $$0=+5.13+2.37(4)-1.19(5)-1.18(6)+4.18(7)-8.08(8)+3.90(9)$$
$$-1.24(10)-1.51(11)+2.75(12)$$

EXPLANATION OF COMPUTATION

The equations can now be entered in the table of correlates as shown on page 38. This table is arranged like that shown on page 11, except that there is no column for weights as all the directions were considered as observed in the same manner and therefore of equal weight. The formation and solution of the normals and the computation of the v's are also made in the same manner as shown on pages 11–16. After the v's are determined, the extra decimal places are dropped to give the adopted values. In first-order work the adopted v's are taken to hundredths of a second.

As a check to insure that the C's were properly substituted in the correlates in computing the v's, the v's on all directions around a point are added together. The sum should equal zero, unless there is at the point a fixed direction to which no correction is applied. For example, at Roman the corrections on directions (1), (2), and (3) are (see p. 38) $-0.039+0.284-0.245=0$; at Yellow on (4), (5), and (6) they are $+0.056-0.282+0.226=0$; at Fairview on (7), (8), and (9) they are $-0.466+0.362+0.104=0$; and at Spencer on (10), (11), and (12) they are $-0.317+0.498-0.181=0$.

As was the case for the station adjustment (see p. 15) the adopted v's are not simply the computed v's taken to the nearest hundredth of a second. In order to satisfy all the angle equations exactly, it is necessary on some of the directions to adopt the hundredth above or below the computed value. In doing this it is well, if possible, to change those values which involve the smallest change in the thousandth decimal place. (See the adopted value of v_6 on p. 38.)

The full solution, both forward and backward, of the four normal equations, is given on page 38. The computation of the v's is given on page 39. As a solution similar to this was fully explained on page 14, it is not necessary to explain this one.

Correlate equations

	1	2	3	4	Σ_a	v	Adopted v
1	−1	-------	-------	----------	−1.00	−0.039	−0.04
2	-------	−1	-------	----------	−1.00	+.284	+.28
3	+1	+1	-------	----------	+2.00	−.245	−.24
4	−1	−1	-------	+2.37	+.37	+.056	+.06
5	+1	-------	−1	−1.19	−1.19	−.282	−.28
6	-------	+1	+1	−1.18	+.82	+.226	+.22
7	-------	−1	−1	+4.18	+2.18	−.466	−.47
8	-------	+1	-------	−8.08	−7.08	+.362	+.36
9	-------	-------	+1	+3.90	+4.90	+.104	+.10
10	-------	-------	−1	−1.24	−2.24	−.317	−.32
11	−1	-------	+1	−1.51	−1.51	+.498	+.50
12	+1	-------	-------	+2.75	+3.75	−.181	−.18

Normal equations

	1	2	3	4	η	Σ_n	C
1	+6	+2	−2	+0.70	+1.22	+7.92	+0.03928
2		+6	+2	−15.81	−.47	−6.28	−.28422
3			+6	−.54	−1.89	+3.57	+.41561
4				+117.7744	+5.13	+107.2544	−.08003

Solution of normal equations

1	2	3	4	η	Σ_n
+6	+2	−2	+0.70	+1.22	+7.92
C_1	−.33333	+.33333	−.11667	−.20333	−1.32
	+6	+2	−15.81	−.47	−6.28
1	−.6667	+.6667	−.2333	−.4067	−2.64
	+5.3333	+2.6667	−16.0433	−.8767	−8.92
	C_2	−.50001	+3.00814	+.16438	+1.67251
		+6	−.54	−1.89	+3.57
	1	−.6667	+.2333	+.4067	+2.64
	2	−1.3333	+8.0217	+.4383	+4.46
		+4.0000	+7.7150	−1.0450	+10.67
		C_3	−1.92875	+.26125	−2.6675
			+117.7744	+5.13	+107.2544
		1	−.0817	−.1423	−.9240
		2	−48.2605	−2.6372	−26.8326
		3	−14.8803	+2.0155	−20.5798
					79
			+54.5519	+4.3660	+58.918Ø
			C_4	−.08003	−1.08003

Back solution

4	3	2	1
−0.08003	+0.26125	+0.16438	−0.20333
	+.15436	−.24079	+.00933
		−.20781	+.13854
	+.41561		+.09474
		−.28422	
			+.03928

In the forward and back solutions of the normal equations for a simple quadrilateral it is not necessary ordinarily to use as many decimal places as are used in the example above. Three decimal places for the multiplied terms, four for the division terms, and four for the back solution are usually sufficient. In some cases two, three, and three decimal places, respectively, may be used and the desired accuracy still be obtained.

Computation of corrections (v's)

1	2	3	4	5	6	7	8	9	10	11	12
−0.039 −.04	+0.284 +.28	+0.039 −.284	−0.039 +.284 −.189	+0.039 −.416 +.095	−0.284 +.416 +.094	+0.284 −.416 −.334	−0.284 +.646	+0.416 −.312	−0.416 +.099	−0.039 +.416 +.121	+0.039 −.220
		−.245 −.24	+.056 +.06	−.282 −.28	+.226 +.22	−.466 −.47	+.362 +.36	+.104 +.10	−.317 −.32	+.498 +.50	−.181 −.18

COMPUTATION OF TRIANGLES

After adopting values for the *v*'s the next step is to substitute these values in column 4 of the triangle computation. (See fig. 17.) For instance, in the triangle Yellow-Roman-Spencer, the correction to angle Yellow is $-4+5=-(+0.06)+(-0.28)=-0.34$; the correction to angle Roman is $-1+3=-(-0.04)+(-0.24)=-0.20$; and the correction to angle Spencer is $-11+12=-(+0.50)+(-0.18)=-0.68$. These three corrections should sum up to the closure of the triangle, $-1''.22$, and we find this to be the case for $-0.34-0.20-0.68=-1.22$. Likewise, the corrections to the angles in each of the other three triangles should sum up to the closure of that triangle. The corrections should always be written in the triangles in *pencil* until it is certain that all the results will check.

DISTRIBUTION OF SPHERICAL EXCESS

The spherical angles in column 5 (see fig. 17) are next computed. The spherical excess of each triangle is then distributed among the three angles, one-third of it being placed on each angle. If it is not exactly divisible by 3, the spherical excess is so distributed that the small angles will have their correct share as nearly as possible, since changes in the small angles affect the lengths to a greater degree than changes in the large angles. That is, if the spherical excess of the triangle were only $0''.02$, then $0''.01$ should be placed on each of the two smaller angles and $0''.00$ on the largest. If the total spherical excess were $0''.04$, then $0''.02$ should be placed on the largest angle, and $0''.01$ on each of the two smaller angles; or if the spherical excess were $0''.05$, then $0''.02$ should be placed on each of the smaller angles and $0''.01$ on the largest angle.

COMPUTATION OF LOGARITHMS OF LENGTHS

The plane angles in column 7 (see fig. 17) can now be computed and finally the logarithms (or co-logarithms) of the sines of these angles are placed in column 8. (In the case of the first angle of each triangle the co-logarithm of the sine of the angle is used.) In each triangle, the logarithm of the length 2–3 is added to the co-logarithm of the sine of angle 1 and the logarithm of the sine of angle 2 to give the logarithm of the length 1–3; and the logarithm of the length 2–3 is added to the co-logarithm of the sine of angle 1 and the logarithm of the sine of angle 3 to give the logarithm of the length 1–2.

After the logarithms of the lengths for each of the four triangles are computed, the logarithm of each length will appear in two different triangles. These should be the same except possibly for a difference of 1 in the last place of decimals, which may be due to accumulation. In a flat triangle, however, having one or two very small angles the discrepancies in lengths may amount to several units of the last place of decimals used.

Where there is a difference of one or more in the last place of decimals in the adjusted logarithms of the lengths, the logarithms should be made consistent before going ahead with the work. The question naturally arises: To which logarithm should the correction be applied? Other things being equal, the correction should be applied to that logarithm which was computed through the smallest angles. However, in taking out the logarithms of the sines, one more decimal place than is necessary should be taken out and placed in small figures to the right of each logarithm. It can then readily be seen where the adding or dropping of units in this decimal place has accumulated enough to change any particular logarithm of a length one in the last decimal place used, and the correction can be applied accordingly.

CORRECTIONS TO DIRECTIONS

After the correctness of the *v*'s has been checked by the closures of the triangles and the agreement of the lengths, the corrections should be applied to the directions and the final values placed in the list of directions. (See fig. 16.)

The first step is to put the computed and adopted values of the *v*'s in the columns intended for them in the table of correlates. (See p. 38.) Then using the sketch, Figure 15, for the designations of the directions, the corresponding corrections should be applied to the directions in the list of directions.

If the *v* to be applied to a direction is negative it sometimes happens that the minutes of the final direction will be one less than the minutes of the observed direction. Where this occurs a bar should be placed over the value in the final seconds column. For example, in Figure 16, in the list of directions at Roman, the observed direction at

Spencer is 0° 00′ 00″.00 and the final seconds are $\overline{59.96}$, that is, the final direction is 359° 59′ 59″.96.

At station Yellow the directions to Roman, Spencer, and Fairview are numbered 4, 5, and 6, respectively. In the table of correlates, page 38, we find the values of the adopted *v*'s for 4, 5, and 6 are +0.06, −0.28, and +0.22, respectively. Then in the list of directions at Yellow, Figure 16, we apply these corrections to the directions Roman, Spencer, and Fairview as follows: To Roman, 38″.63 + 0″.06 = 38″.69; to Spencer, 47″.80 − 0″.28 = 47″.52; to Fairview, 09″.74 + 0″.22 = 09″.96.

The values of the final seconds in the list of directions should be checked by using them to verify the corrected angles in the triangle computation. The directions at Yellow should be checked, therefore, by the adjusted angles at Yellow in the various triangles. In the triangle Yellow–Roman–Spencer, Figure 17, the spherical angle at Yellow is 60° 26′ 08″.83. From the list of directions at Yellow the final angle between Roman and Spencer is 239° 06′ 47″.52 − 178° 40′ 38″.69 = 60° 26′ 08″.83 which checks the adjusted angle in the triangle. In the triangle Fairview–Yellow–Roman, the adjusted spherical angle at Yellow is 119° 05′ 31″.27. From the list of directions at Yellow the final angle between Roman and Fairview is 297° 46′ 09″.96 − 178° 40′ 38″.69 = 119° 05′ 31″.27 which also checks the angle from the triangle. In the triangle Fairview-Yellow-Spencer the adjusted angle at Yellow is 58° 39′ 22″.44. From the list of directions at Yellow the final angle between Spencer and Fairview is 297° 46′ 09″.96 − 239° 06′ 47″.52 = 58° 39′ 22″.44, which again checks the adjusted angle from the triangle.

In the same manner all the adjusted angles at Roman, Spencer, and Fairview in the triangles will be found to be checked by the angles from the lists of directions, and this shows that all the angles are consistent.

It is important not to omit the placing of the corrections in the list of directions after an adjustment is completed. The final values in the list show that the adjustment has been made, and they are the values of the directions that must be used if other adjustments are made depending on this one.

COMPUTATION OF GEOGRAPHIC POSITIONS

After the adjustment is completed and all the triangles are computed and made consistent, the geographic positions of the two new points, Yellow and Fairview, can be computed by starting from the fixed positions of Roman and Spencer and the fixed azimuth, Roman-Spencer.

Since all the angles and lengths in the quadrilateral are now consistent due to the adjustment, any triangle may be used in the com-

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Roman	to	Spencer	261	04	53,49
∠		Spencer	&	Yellow	+65	12	45,52
α	2.	Roman	to 1.	Yellow	326	17	39,01
Δα					+	13	53,26
					180		
α′	1.	Yellow	to 2.	Roman	146	21	32,27

First Angle of Triangle 60 26 08,83

	°	′	″				°	′	″
φ	43	54	45,041	2.	Roman	λ	123	44	14,987
Δφ	-	21	56,195	s=		Δλ	-	20	05.419
φ′	43	32	48,846	1.	Yellow	λ′	123	24	09.568

s	4,688 0992	s²	9,37620			−h	3,1187
cos α	9,920 0699	sin² α	9,48848	(δφ)²	6,2384	s² sin² α	8,8647
B	8,510 5516	C	1,38763	D	2,3922	E	6,1886
h	3,118 7207		0,26231		8,6306		8,1720

1st term.	+1314,3792	3d term.	+0,0427			(Δλ)²	9,245
2d term.	+ 1,7878	4th term.	−0,0149			F	7,845
	+1316,1670						7,092
3d and 4th terms.	+0,0278	s	4,688 0992				
−Δφ	+1316,1948	sin α	9,744 2375	Arg.		Δλ	3,081 1382
½(φ+φ′)	43 43 46,94	A′	8,509 0278	s	−42	sin ½(φ+φ′)	9,839 6397
		sec φ′	0,139 7754	Δλ	+25	sec ½(Δφ)	22
			82 3,081,1399	Corr.	−17		2,920,7601
		Δλ	−,1205,4194			−Δα	−833,259
							− 0,001
							−833,260

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DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY.
FORM 26

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Spencer	to	Roman	81	31	39,26
∠		Yellow	&	Roman	−54	21	11,61
α	3.	Spencer	to 1.	Yellow	27	10	27,74
Δα					-	12	46,64
					180		
α′	1.	Yellow	to 3.	Spencer	206	57	41,10

Do not write in this margin.

	°	′	″				°	′	″
φ	43	59	00,715	3.	Spencer	λ	123	05	41,248
Δφ	-	26	11,869	s=		Δλ	+	18	28,320
φ′	43	32	48,846	1.	Yellow	λ′	123	24	09,568

s	4,736 2338	s²	9,47247			−h	3,1960
cos α	9,949 2049	sin² α	9,31926	(δφ)²	6,3928	s² sin² α	8,7917
B	8,510 5462	C	1,38869	D	2,3923	E	6,1902
h	3,195 9849		0,18042		8,7851		8,1779

1st term.	+1570,3083	3d term.	+0,0610			(Δλ)²	9,134
2d term.	+ 1,5150	4th term.	−0,0151			F	7,848
	+1571,8233						6,982
3d and 4th terms.	+ 0,0459	s	4,736 2338				
−Δφ	+1571,8692	sin α	9,659 6311	Arg.		Δλ	3,044 6650
½(φ+φ′)	43 45 54,78	A′	8,509 0278	s	−52	sin ½(φ+φ′)	9,839 9209
		sec φ′	0,139 7754	Δλ	+21	sec ½(Δφ)	32
			50 3,044 6681	Corr.	−31		2,884 5891
		Δλ	+ 1108,3196			−Δα	+766,656
							+ 001
							+766,657

11—239

FIG. 18.—Position computation for new stations of quadrilateral

45861°—34——4

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Yellow	to	Spencer	206	57	41.10
∠		Spencer	&	Fairview	+58	39	22.44
α	2.	Yellow	to 1.	Fairview	265	37	03.54
Δα					+	31	01.50
					180		
α′	1.	Fairview	to 2.	Yellow	86	08	05.04 +1
				First Angle of Triangle	55	07	22.84

	°	′	″				°	′	″
φ	43	32	48.846	2.	Yellow	λ	123	24	09.568
Δφ	+	2	21.607	s=		Δλ	-	45	00.954
φ′	43	35	10.453	1.	Fairview	λ′	122	39	08.614

s	4.783 6971	s²	9.56739			−h	2.1774
cos α	8.883 1608	sin² α	9.99746	(δφ)²	4.3022	s² sin² α	9.5648
B	8.510 5798	C	1.38211	D	2.3920	E	6.1805
h	2.177 4377		0.94696		6.6942		7.9227
1st term.	″ -150.4658	3d term.	″ +0.0005			(Δλ)²	0.295
2d term.	+8.8503	4th term.	+0.0034			F	7.351
	-141.6155						8.146
3d and 4th terms.	+0.[illegible]89	s	4.783 6971				
−Δφ	-141.6066	sin α	9.998 7284	Arg.		Δλ	3.431 5172
½(φ+φ′)	43 33 59.65	A′	8.509 0268	s	-65	sin ½(φ+φ′)	9.838 3433
		sec φ′	0.140 0590	Δλ	+124	sec ½(Δφ)	0
			72 3.431 5118	Corr.	+59		3.2698 605
		Δλ	″ -2700.9540			−Δα	-1861.489
							- 0.014
							-1861.503

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 26

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Spencer	to	Yellow	27	10	27.74
∠		Fairview	&	Yellow	− 66	13	21.89
α	3.	Spencer	to 1.	Fairview	320	57	05.85
Δα					+	18	22.04
					180		
α′	1.	Fairview	to 3.	Spencer	141	15	27.89

	°	′	″				°	′	″
φ	43	59	00.715	3.	Spencer	λ	123	05	41.248
Δφ	-	23	50.262	s=		Δλ	-	26	32.635
φ′	43	35	10.453	1.	Fairview	λ′	122	39	08.613 +1

s	4.753 7076	s²	9.50742			−h	3.1545
cos α	9.890 2054	sin² α	9.59865	(δφ)²	6.3108	s² sin² α	9.1061
B	8.510 546°	C	1.38869	D	2.3923	E	6.1902
h	3.154 4692		0.49476		8.7031		8.4508
1st term.	″ +1427.1158	3d term.	″ +0.0505			(Δλ)²	9.606
2d term.	+ 3.1244	4th term.	-0.0282			F	7.848
	+1430.2402						7.454
3d and 4th terms.	+ 0.0223	s	4.753 7076				
−Δφ	+1430.2625	sin α	9.799 3242	Arg.		Δλ	3.202 1162
½(φ+φ′)	43 47 05.58	A′	8.509 0268	s	-57	sin ½(φ+φ′)	9.840 0764
		sec φ′	0.140 0530	Δλ	+43	sec ½(Δφ)	26
			62 3.202 1176	Corr.	-14		3.0421 952
		Δλ	-1592.6348			−Δα	-1102.035
							- 0.003
							-1102.038

Do not write in this margin.

FIG. 18.—Position computation for new stations of quadrilateral—Continued

putation and the position of the point computed should be the same. It is best, however, to use the triangle with the shorter lengths, as there will be less trouble in making the computation over the two lines to the new point agree.

The positions for first-order triangulation are computed on form 26. The formulas used in the computation of geographic positions, the development of these formulas, and the tables used in the position computation, are found in United States Coast and Geodetic Survey Special Publication No. 8, and are not repeated here.

The computation of the geographic positions of stations Yellow and Fairview are given on pages 42 and 43.[2] (See fig. 18.)

It should be noted that the angles used in the computation of the geographic positions are the spherical angles, the seconds of which appear in column 5 of the table of triangles (fig. 17). The first angle of the triangle, although not used in the actual computation of the position, should always be written down in its proper place on the form since it is used as a check on the computed azimuth.

For example, in computing the geographic position of station "Yellow," the angles used are 60° 26′ 08.″83, 65° 12′ 45.″52, and 54° 21′ 11.″51. After the computation is finished the azimuth 1 to 2 plus the first angle of the triangle should equal the azimuth 1 to 3; that is, 146° 31′ 32.″27 + 60° 26′ 08.″83 should equal 206° 57′ 41.″10, which they do.

Care should be taken to make sure that the latitude and longitude of the new station as computed from the two different stations check and that the azimuths as computed from the two stations differ exactly by the amount of the first angle of the triangle. Occasionally these values will fail to check by one unit in the last place of decimals used, due to accumulation in the next decimal place, but if the discrepancy is greater than one the computation should be checked over to see whether a mistake has been made.

If no error has been made in the computation and the latitudes (or longitudes) fail to check by one unit in the last place of decimals used, then the two values should be made to agree by adding one unit of the last place of decimals to one value or by subtracting one unit from the other value. Which value should be corrected depends upon which one should be changed to make the next position computation check. (See p. 77.)

In the same manner if the azimuth of station 1 to station 2 plus the first angle of the triangle fails to check the azimuth of station 1

[2] After the manuscript of this publication was prepared new forms with the same numbers (26 and 27) as the old forms were prepared for the computation of geographic positions of the first and third orders. Although there has been no change in the formulas used for the computation, the different terms of the formulas have been rearranged in a more compact form, which, it is believed, will expedite the computations. These new forms have not yet been finally adopted by this bureau, but are at present being tried out in the division of geodesy A copy of each of these forms with sample computations of geographic positions are shown in Figures 19 and 20.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 27

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

					°	′	″
α	2	Outer	to 3	Hard	144	43	42.5
2d ∠		Hard	&	Parson	+ 23	51	16.1
α	2	Outer	to 1	Parson	168	34	58.6
$\Delta\alpha$					−		20.7
					180	00	00.0
α'	1	Parson	to 2	Outer	348	34	37.9
				First Angle of Triangle.	103	12	18.8

α	3	Hard	to 2	Outer	324	42	28.9
3d ∠		Parson	&	Outer	− 52	56	25.1
α	3	Hard	to 1	Parson	271	46	03.8
$\Delta\alpha$					+		52.9
					180	00	00.0
α'	1	Parson	to 3	Hard	91	46	56.7

	°	′	″				°	′	″
ϕ	40	35	18.742	2	Outer	λ	73	36	33.964
$\Delta\phi$	+	1	59.853			$\Delta\lambda$	+		31.763
ϕ'	40	37	18.595	1	Parson	λ'	73	37	05.727

	°	′	″				°	′	″
ϕ	40	37	20.514	3	Hard	λ	73	38	27.008
$\Delta\phi$	−		01.919			$\Delta\lambda$	−	1	21.281
ϕ'	40	37	18.595	1	Parson	λ'	73	37	05.727

	Logarithms	Values in seconds	
s	3.576526		
Cos α	9.991320		
B	8.510807		
h	2.078653	1st term	−119.8541″
s^2	7.15305		
$\sin^2\alpha$	8.59311		
C	1.33731		
	7.08347	2d term	+0.0012
h^2	4.1573		
D	2.3872		
	6.5445	3d term	+0.0004
		$-\Delta\phi$	−119.8525

	° ′ ″	
$\frac{1}{2}(\phi+\phi')$	40 36 18.7	
	Logarithms	Values in seconds
s	3.576526	
Sin α	9.296554	
A′	8.509103	
sec ϕ'	0.119745	
$\Delta\lambda$	1.501928	+31.7635″
Sin $\frac{1}{2}(\phi+\phi')$	9.813476	
$-\Delta\alpha$	1.315404	+20.67

	Logarithms	Values in seconds	
s	3.281346		
Cos α	8.489222		
B	8.510804		
h	0.281372	1st term	+1.9115″
s^2	6.56269		
$\sin^2\alpha$	9.99959		
C	1.33783		
	7.90011	2d term	+0.0079
h^2	0.5627		
D	2.3873		
	2.9500	3d term	+ 0
		$-\Delta\phi$	+1.9194

	° ′ ″	
$\frac{1}{2}(\phi+\phi')$	40 37 19.6	
	Logarithms	Values in seconds
s	3.281346	
Sin α	9.999793	
A′	8.509103	
sec ϕ'	0.119745	
$\Delta\lambda$	1.909987	−81.2806″
Sin $\frac{1}{2}(\phi+\phi')$	9.813626	
$-\Delta\alpha$	1.723613	−52.92

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FIG. 19.—Sample position computation, third-order triangulation, new form

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 26

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

	Station		°	′	″
α	2 Mt. Nebo	to 3 Tushar	20	05	36.08
2d $\angle$	Tushar	& Wheeler Peak	+48	04	05.50
α	2 Mt. Nebo	to 1 Wheeler Peak	68	09	41.58
$\Delta\alpha$			−1	37	01.48
			180	00	00.00
α'	1 Wheeler Peak	to 2 Mt. Nebo	246	32	40.10
	First Angle of Triangle		43	40	37.34

	°	′	″	Station		°	′	″
ϕ	39	48	38.316	2 Mt. Nebo	λ	111	45	56.235
$\Delta\phi$	−	49	29.300		$\Delta\lambda$	+2	32	50.783
ϕ'	38	59	09.016	1 Wheeler Peak	λ'	114	18	47.018

	Logarithms	+1	″		Logs		° ′ ″
s	5.3761505	(1)	+2867.8000	½	9.699	½$(\phi+\phi')$	39 23 53.67
cos α	9.5705323	(2)	+ 103.0505	s^2	10.752	Logarithms	
B	8.5108661	Sum	+2970.8505	K	2.013	s	5.3761505
(1)=h	3.4575489	(3)	+ 0.2140	E	6.130	sin α	9.9676586
s^2	10.75230	(4)	− 1.7599	(5)	8.564	A′	8.5091440
sin² α	9.93532	(5)	− 0.0366	3	0.477	sec ϕ'	0.1094105
C	1.32543	(6)	+ 0.0152	cos² α	9.141	Arc-sin corr.	+ 428
(2)=K	2.01305	(7)	+ 0.0167	(6)	8.182	$\Delta\lambda$	3.9624064
$(\delta\phi)^2$	6.9452	$-\Delta\phi$	+2969.2999	(colog) E	13.900	sin ½$(\phi+\phi')$	9.8025731
D	2.3852	$\frac{\Delta\phi}{2}$	−1484.64	$\frac{A^2 \sec^2 1''}{3}$	$\bar{5}$.912	sec $\frac{\Delta\phi}{2}$	113
(3)	9.3304			sec² ϕ	0.229	(approx.) $-\Delta\alpha$	3.7649908
−h	3.4575			(7)	$\bar{8}$.223	do	″ +5820.909
s^2 sin² α	10.6876					(8)	″ + 0.571
E	6.1004	Arc-sin corr.				$-\Delta\alpha$	″ +5821.480
(4)	0.2455	for s	−1003	$(\Delta\lambda)^2$	11.887		
		for $\Delta\lambda$	+1431	F	$\bar{\bar{7}}$.870	$\Delta\lambda$	″ +9170.7830
		Total	+ 428	(8)	9.757		

	Station		°	′	″
α	3 Tushar	to 2 Mt. Nebo	199	41	08.58
3d $\angle$	Wheeler Peak	& Mt. Nebo	−88	16	30.92
α	3 Tushar	to 1 Wheeler Peak	111	24	37.66
$\Delta\alpha$			−1	11	20.22
			180	00	00.00
α'	1 Wheeler Peak	to 3 Tushar	290	13	17.44

	°	′	″	Station		°	′	″
ϕ	38	25	09.551	3 Tushar	λ	112	24	42.168
$\Delta\phi$	+	33	59.467		$\Delta\lambda$	+1	54	04.851
ϕ'	38	59	09.018	1 Wheeler Peak	λ'	114	18	47.019

	Logarithms	−1	″		Logs		° ′ ″
s	5.2478407	(1)	−2094.8860	½	9.699	½$(\phi+\phi')$	38 42 09.28
cos α	9.5623485	(2)	+ 54.6500	s^2	10.496	Logarithms	
B	8.5109712	Sum	−2040.2360	K	1.739	s	5.2478407
(1)=h	3.3211604	(3)	+ 0.1000	E	6.072	sin α	9.9689446
s^2	10.49568	(4)	+ 0.6708	(5)	8.005	A′	8.5091440
sin² α	9.93789	(5)	− 0.0101	3	0.477	sec ϕ'	0.1094105
C	1.30402	(6)	+ 0.0040	cos² α	9.125	Arc-sin corr.	+242
(2)=K	1.73759	(7)	+ 0.0046	(6)	7607	$\Delta\lambda$	3.8353640
$(\delta\phi)^2$	6.6191	$-\Delta\phi$	−2039.4667	(colog) E	13.928	sin ½$(\phi+\phi')$	9.7960730
D	2.3807	$\frac{\Delta\phi}{2}$	+1019.73	$\frac{A^2 \sec^2 1''}{3}$	$\bar{5}$.912	sec $\frac{\Delta\phi}{2}$	53
(3)	8.9998			sec² ϕ	0.212	(approx.) $-\Delta\alpha$	3.6314423
−h	3.3212			(7)	$\bar{7}$.659	do	″ +4279.985
s^2 sin² α	10.4336					(8)	″ 0.240
E	6.0718	Arc-sin corr.				$-\Delta\alpha$	″ +4280.225
(4)	9.8266	for s	−555	$(\Delta\lambda)^2$	11.506		
		for $\Delta\lambda$	+797	F	$\bar{\bar{7}}$.874	$\Delta\lambda$	″ +6844.8508
		Total	+242	(8)	9.330		

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FIG. 20.—Sample position computation, first-order triangulation, new form

to station 3 by one unit in the last decimal place, a correction must be applied to one of the azimuths to make the result consistent before proceeding with the next position. (See p. 43.)

LIST OF GEOGRAPHIC POSITIONS

After the geographic positions of the stations are computed a list of these geographic positions, together with the azimuths and logarithms of the distances to the other stations, is made on Form 28B. (See fig. 21.) The azimuths from each station to the other stations should be arranged in clockwise order. The names of the stations in the first and sixth columns of the list of positions can be filled in most easily in proper order from the sketch, since this shows the arrangement of the stations in regard to azimuth. For each station the azimuth and logarithm of the distance to each of two stations can be obtained directly from the position computation and should be written in the list before the other azimuths and logarithms of distances which are obtained from the tables of triangles are entered on the form. In case the quadrilaterals are complete, that is, all the lines are included, it is possible to get a check on the computation of all extra azimuths in the list.

At station Fairview, for example, the azimuths and back azimuths to Yellow and Spencer are obtained directly from the position computation, but the azimuth and back azimuth to Roman must be computed from the triangles by using the two triangles Fairview–Yellow–Roman and Fairview–Roman–Spencer. They are computed in the following manner: The azimuth, Fairview to Yellow already in the list (fig. 21) is 86° 08′ 05″.05 and the angle at Fairview from Yellow to Roman in the triangle Fairview–Yellow–Roman is 26° 46′ 01″.68, so the azimuth of Fairview to Roman as derived from the first triangle is 86° 08′ 05″.05 + 26° 46′ 01″.68 = 112° 54′ 06″.73. In a similar manner the azimuth Fairview to Spencer already in the list is 141° 15′ 27″.89, and the angle at Fairview from Roman to Spencer in the triangle Fairview–Roman–Spencer is 28° 21′ 21″.16, so the azimuth of Fairview to Roman as derived from the second triangle is 141° 15′ 27″.89 − 28° 21′ 21″.16 = 112° 54′ 06″.73, which checks the value above.

The back azimuth Roman to Fairview is computed as follows: The azimuth Roman to Yellow already in the list (fig. 21) is 326° 17′ 39″.01, and the angle at Roman from Fairview to Yellow in the triangle Fairview–Yellow–Roman is 34° 08′ 33″.62, so the azimuth of Roman to Fairview as derived from the first triangle above is 326° 17′ 39″.01 − 34° 08′ 33″.62 = 292° 09′ 05″.39. Again, the azimuth Roman–Spencer already in the list is 261° 04′ 53″.49, and the angle at Roman from Spencer to Fairview in the triangle Fairview–Roman–Spencer is 31° 04′ 11″.90, so the azimuth Roman to Fairview as derived from the second triangle is 261° 04′ 53″.49 + 31° 04′ 11″.90 = 292° 09′ 05″.39, which checks the other value.

The extra azimuths, that is, those computed from the triangles, should *always* be checked in every possible triangle, as this will show whether or not the adjustment has made all the angles consistent. It should be borne in mind that the angles used in the computation of the azimuths are the *spherical angles*. One can readily tell whether to add or subtract the angles in the computation of the azimuths by referring to the triangulation sketch on which the relative positions of the stations are shown.

After all the azimuths and logarithms of distances have been entered in the list of positions, the other columns are filled out. The quantities in the column headed "Seconds in meters" are obtained by means of the tables in Special Publication No. 5, entitled "Tables for a polyconic projection of maps," in which the value in meters corresponding to 1 second of either latitude or longitude is given for all latitudes from 0° to 90°. Next, under the general heading "Distance," the column headed "Meters" is filled out by taking the antilogarithms of the values in the preceding column, and finally the one headed "Feet" is obtained by conversion of the meter values.

In the first column of the list of positions, in addition to the name of the station, there should be given the year of the first establishment of the station, the date when the station was last visited, and letters to indicate whether the station is described and marked or not. If the station is described but not marked the letter "d" is used; if it is marked and not described the letters "m. n. d." are used; and if it is described and marked the letters "d. m." are used. Other letters used in this column are: "n. d." which means that the station is not described, but whether or not it is marked is not known; "r" which means that the station was recovered but no description was furnished of its condition; "r. d." which means that the station was recovered and described; and "r. d. m." which means that the station was recovered, described, and marked.

The column headed "Seconds in meters" is ordinarily not filled out for first-order triangulation. The values in this column are computed for the convenience of the draftsman in constructing charts along the coast but as first-order triangulation is mostly in the interior of the country it is not used in making the charts.

The adjustment of the quadrilateral which has been given in detail on pages 29–41, fully illustrates the various steps that are necessary in an adjustment when only the geographic positions (latitudes and longitudes) of two initial stations, and the distance and azimuth between these stations are fixed. Where a number of connected triangles and quadrilaterals are to be adjusted, but the only fixed conditions are those given above, the steps in the adjustment are exactly the same as shown for the single quadrilateral, the only difference being in the number of equations, both angle and side.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 26 B

GEOGRAPHIC POSITIONS

Accession No. of Computation: 35,607

Locality Eugene. *Datum* North American *State* Oregon

STATION.		LATITUDE AND LONGITUDE. °	′	″	SECONDS IN METERS.	AZIMUTH °	′	″	BACK AZIMUTH. °	′	″	TO STATION.	DISTANCE. LOGARITHM (METERS).	METERS.	FEET.
Yellow		43	32	48.846	1507.5	146	31	32.27	326	17	39.01	Roman	4.688 0992	48763.99	159986.5
1904	d.m.	123	24	09.568	214.8	206	57	41.10	27	10	27.74	Spencer	4.736 2338	54479.59	178738.5
Fairview		43	35	10.453	322.6	86	08	05.05	265	37	03.54	Yellow	4.783 6971	60771.10	199379.9
1904	d.m.	122	39	08.614	193.2	112	54	06.73	292	09	05.39	Roman	4.975 9773	94618.87	310428.7
						141	15	27.89	320	57	05.85	Spencer	4.753 7076	56716.26	186076.6

FIG. 21.—List of geographic positions for new stations of quadrilateral

CHAPTER 3.—ADJUSTMENT OF A NET OF TRIANGULATION BY THE DIRECTION METHOD

CONDITIONS INVOLVED

In the adjustment of the single quadrilateral just considered, only angle and side equations were required. The adjustment of a net of triangulation is given on the following pages for which are required not only angle and side equations, but condition equations to eliminate the closing errors in length, azimuth, latitude, and longitude. The direction method will be used, as first-order triangulation is always adjusted by that method in this bureau. On pages 110–146 is shown a net of third-order triangulation adjusted by the angle method.

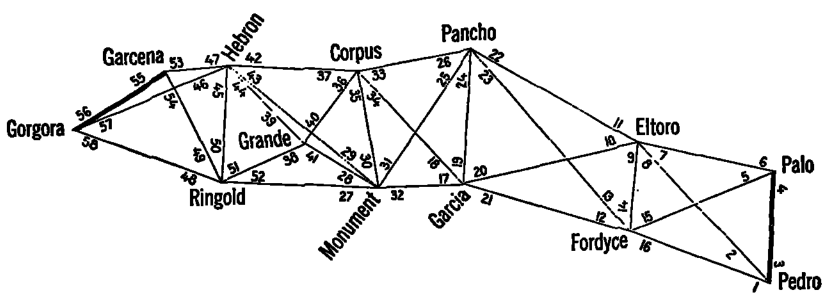

FIG. 22.—Triangulation net used in sample adjustment

As shown in the sketch of this net (fig. 22), the lines Garcena-Gorgora and Palo-Pedro are fixed in length and azimuth and the stations Garcena, Gorgora, Palo, and Pedro are fixed in position (latitude and longitude). The angle and side condition equations for this net are formed in the same manner as explained on pages 35–37 for the single quadrilateral.

The length equation is formed by starting with the fixed length Palo-Pedro and computing through the observed angles of a single chain of triangles the length of the line Garcena-Gorgora. This computed length will usually differ from the fixed length of the line and the length condition is thus obtained.

The azimuth equation is formed by starting with the fixed azimuth of the line Palo-Pedro and computing through the observed angles of the triangles an azimuth for the line Garcena-Gorgora. This computed azimuth will usually differ from the fixed azimuth of the line and the azimuth condition is thus obtained.

The latitude and longitude equations are formed by starting with the fixed position (latitude and longitude) of either Palo or Pedro and computing through the observed angles of a single chain of triangles the geographic position (latitude and longitude) of Garcena. This computed position of Garcena will usually differ from the fixed position and the latitude and longitude conditions are thus obtained.

All the steps in the adjustment of this net of triangulation are given in detail on the following pages. Although the net is only a small one all the conditions that would appear in a large net are present, so that an understanding of this small adjustment will enable one to adjust a large net.

The formation of the equations representing all the conditions considered above are shown below in detail. If the lists of directions at the various stations have not been made out in the field they are made in the form shown on page 7, with the column "Final seconds" left blank.

A sketch (fig. 22) is then drawn showing the relative positions of the stations and all the observed directions between the stations, full lines representing directions observed from both stations at the ends of the line and lines dotted at one end representing directions observed from the station at the full end of the line but not from the other station. The directions are numbered in clockwise order on the sketch as shown in Figure 22.. The triangles (fig. 24) are next written out and the angles filled in from the list of directions in Figure 23. Particular attention should be paid to the order in which the triangles are written. (See p. 29.)

As full instructions for determining the spherical excess and triangle closures, and for checking•the triangle closures are given on pages 32–34 for the single quadrilateral, they will not be repeated here.

NUMBER OF EQUATIONS

After the closures of the triangles have been determined, the next step is to determine the number of equations in the net. The formulas for computing the number of equations to be used in the adjustment of a triangulation net are given on page 34. In the net considered here, $n=29$, $n'=29$, $s=13$, and $s'=13$. The number of angle equations is therefore $n'-s'+1=29-13+1=17$, and the number of side equations is $n-2s+3=29-26+3=6$. Since there are fixed azimuths and lengths at the two ends of the net, there will also be one azimuth and one length equation. In addition to these, due to the fixed position (latitude and longitude) at each end of the net, there will be one latitude and one longitude equation. Altogether then there are 17 angle, 6 side, 1 azimuth, 1 length, 1 latitude, and 1 longitude equations, or a total of 27 equations.

Another way of determining the number of angle and side equations in a net is to build up the figure point by point, starting from the fixed line at one end, and count the number of equations at each

(Text continued on p. 59)

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 24A

LIST OF DIRECTIONS

State: Texas

Station: Palo. Computed by: A. C. D. Observer: C. V. H. 1917. Checked by: W. F. R.

Stations observed	Directions after local adjustment ° ′ ″	Final seconds ″
Pedro	0 00 00.00	59.95
Fordyce	61 27 31.53	31.71
Eltoro	99 37 25.05	24.93

Station: Pedro. Computed by: A. C. D. Observer: C. V. H. 1917. Checked by: W. F. R.

Stations observed	Directions after local adjustment ° ′ ″	Final seconds ″
Fordyce	0 00 00.00	00.29
Eltoro	26 37 19.81	19.61
Palo	70 32 18.94	18.86

Station: Fordyce. Computed by: A. C. D. Observer: C. V. H. 1917. Checked by: W. F. R.

Stations observed	Directions after local adjustment ° ′ ″	Final seconds ″
Garcia	0 00 00.00	59.99
Pancho	33 42 51.34	51.75
Eltoro	76 47 59.67	60.01
Palo	140 50 53.52	53.15
Pedro	188 51 03.46	03.08

Station: Eltoro. Computed by: A. C. D. Observer: C. V. H. 1917. Checked by: W. F. R.

Stations observed	Directions after local adjustment ° ′ ″	Final seconds ″
Palo	0 00 00.00	00.43
Pedro	36 27 36.43	36.47
Fordyce	77 47 14.58	14.28
Garcia	149 39 01.51	01.57
Pancho	194 18 20.42	20.19

Station: Garcia. Computed by: A. C. D. Observer: C. V. H. 1917. Checked by: W. F. R.

Stations observed	Directions after local adjustment ° ′ ″	Final seconds ″
Monument	0 00 00.00	00.46
Corpus	51 15 33.50	33.51
Pancho	89 53 49.28	49.12
Eltoro	160 46 14.32	14.39
Fordyce	192 06 27.73	27.35

Station: Pancho. Computed by: A. C. D. Observer: C. V. H. 1917. Checked by: W. F. R.

Stations observed	Directions after local adjustment ° ′ ″	Final seconds ″
Eltoro	0 00 00.00	00.43
Fordyce	20 23 46.79	46.52
Garcia	64 28 16.62	16.92
Monument	92 56 19.84	19.79
Corpus	142 14 59.23	58.83

Station: Monument. Computed by: A. C. D. Observer: C. V. H. 1917. Checked by: W. F. R.

Stations observed	Directions after local adjustment ° ′ ″	Final seconds ″
Ringold	0 00 00.00	59.64
Grande	25 00 27.23	27.85
Hebron	31 47 59.55	59.97
Corpus	80 32 48.42	48.61
Pancho	118 19 55.00	54.61
Garcia	179 58 03.72	03.25

Station: Corpus. Computed by: A. C. D. Observer: C. V. H. 1917. Checked by: W. F. R.

Stations observed	Directions after local adjustment ° ′ ″	Final seconds ″
Pancho	0 00 00.00	00.68
Garcia	63 35 03.71	03.37
Monument	92 54 15.91	15.83
Grande	139 12 15.04	15.26
Hebron	186 42 58.48	58.00

FIG. 23.—Lists of directions for stations of net

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 24A

LIST OF DIRECTIONS

State: Texas

Station Grande — Computed by A.C.D.
Observer C.V.H. 1917 — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Ringold	0 00 00.00	00.09
Hebron	63 08 11.31	11.49
Corpus	152 41 08.92	08.87
Monument	230 50 49.05	48.82

Station Hebron — Computed by A.C.D.
Observer C.V.H. 1917 — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Corpus	0 00 00.00	00.55
Monument	37 26 30.55	30.03
Grande	42 56 20.48	20.61
Ringold	90 19 02.33	02.02
Gorgora	161 52 16.28	16.58
Garcena	173 35 47.89	47.73

Station Ringold — Computed by A.C.D.
Observer C.V.H. 1917 — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Gorgora	0 00 00.00	00.03
Garcena	41 19 43.99	44.21
Hebron	70 44 38.44	38.50
Grande	140 13 45.67	45.83
Monument	166 04 06.90	06.45

Station Garcena — Computed by A.C.D.
Observer C.L.G. 1917 — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Hebron	0 00 00.00	00.16
Ringold	67 18 20.19	20.25
Gorgora	162 14 05.66	05.43

Station Gorgora — Computed by A.C.D.
Observer C.L.G. 1917 — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Garcena	0 00 00.00	00.57
Hebron	6 02 24.09	24.18
Ringold	43 44 32.07	31.41

FIG. 23.—Lists of directions for stations of net—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Texas

[illegible]:

	NO.	STATION	OBSERVED ANGLE	CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
	2-3	Palo - Pedro						3.9781520
-15+16	1	Fordyce	48 00 09.94	-0.01	09.93	0.08	09.85	0.1289079
-4+5	2	Palo	61 27 31.53	+0.23	31.76	0.09	31.67	9.9437288
-1+3	3	Pedro	70 32 18.94	-0.37	18.57	0.09	18.48	9.9744497
	1-3	Fordyce - Pedro		-0.15		0.26		4.0507887
	1-2	Fordyce - Palo						4.0815096
			00.41					
	2-3	Palo - Pedro						3.9781520
-7+8	1	Eltoro	36 27 36.43	-0.39	36.04	0.09	35.95	0.2260226
-4+6	2	Palo	99 37 25.05	-0.07	24.98	0.09	24.89	9.9938449
-2+3	3	Pedro	43 54 59.13	+0.12	59.25	0.09	59.16	9.8411144
	1-3	Eltoro - Pedro		-0.34		0.27		4.1980195
	1-2	Eltoro - Palo						4.0452890
			00.61					
	2-3	Palo - Fordyce						4.0815096
-7+9	1	Eltoro	77 47 14.58	-0.73	13.85	0.07	13.78	0.0099416
-5+6	2	Palo	38 09 53.52	-0.30	53.22	0.07	53.15	9.7909358
-14+15	3	Fordyce	64 02 53.85	-0.71	53.14	0.07	53.07	9.9538377
	1-3	Eltoro - Fordyce		-1.74		0.21		3.8823870
	1-2	Eltoro - Palo						4.0452889 +1
			01.95					
	2-3	Pedro - Fordyce						4.0507887
-8+9	1	Elton	41 19 38.15	-0.34	37.81	0.07	37.74	0.1302208
-1+2	2	Pedro	26 37 19.81	-0.49	19.32	0.06	19.26	9.6513775
-14+16	3	Fordyce	112 03 03.79	-0.72	03.07	0.07	03.00	9.9670100
	1-3	Eltoro - Fordyce		-1.55		0.20		3.8823870
	1-2	Eltoro - Pedro						4.1980195
			01.75					
	2-3	Eltoro - Fordyce						3.8823870
-20+21	1	Garcia	31 20 13.41	-0.45	12.96	0.09	12.87	0.2839387
-9+10	2	Eltoro	71 51 46.93	+0.36	47.29	0.09	47.20	9.9778678
-12+14	3	Fordyce	76 47 59.67	+0.35	60.02	0.09	59.93	9.9883711
	1-3	Garcia - Fordyce		+0.26		0.27		4.1441935
	1-2	Garcia - Eltoro						4.1546963
			00.01					

FIG. 24.—Triangle computation for stations of net

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 26

COMPUTATION OF TRIANGLES

State: Texas

	No.	Station	Observed angle	Corr'n	Spher'l angle	Spher'l excess	Plane angle and distance	Logarithm
	2-3	Eltoro - Fordyce						3.8823870
-22+23	1	Pancho	20 23 46.79	-0.70	46.09	0.08	46.01	0.4577866
-9+11	2	Eltoro	116 31 05.84	+0.07	05.91	0.09	05.82	9.9517221
-13+14	3	Fordyce	43 05 08.33	-0.07	08.26	0.09	08.17	9.8344781
	1-3	Pancho - Fordyce		-0.70		0.26		4.2918957
	1-2	Pancho - Eltoro						4.1746517 +1
			00.96					
	2-3	Eltoro - Garcia						4.1546968
-22+24	1	Pancho	64 28 16.62	-0.13	16.49	0.13	16.36	0.0446159
-10+11	2	Eltoro	44 39 18.91	-0.29	18.62	0.12	18.50	9.8468552
-19+20	3	Garcia	70 52 25.04	+0.23	25.27	0.13	25.14	9.9753391
	1-3	Pancho - Garcia		-0.19		0.38		4.0461679
	1-2	Pancho - Eltoro						4.1746518
			00.57					
	2-3	Fordyce - Garcia						4.1441935
-23+24	1	Pancho	44 04 29.83	+0.57	30.40	0.13	30.27	0.1576402
-12+13	2	Fordyce	33 42 51.34	+0.42	51.76	0.13	51.63	9.7443342
-19+21	3	Garcia	102 12 38.45	-0.22	38.23	0.13	38.10	9.9900620
	1-3	Pancho - Garcia		+0.77		0.39		4.0461679
	1-2	Pancho - Fordyce						4.2918957
			59.62					
	2-3	Pancho - Garcia						4.0461679
-31+32	1	Monument	61 38 08.72	-0.08	08.64	0.06	08.58	0.0555446
-24+25	2	Pancho	28 28 03.22	-0.35	02.87	0.05	02.82	9.6782083
-17+19	3	Garcia	89 53 49.28	-0.62	48.66	0.06	48.60	9.9999993
	1-3	Monument - Garcia		-1.05		0.17		3.7799207
	1-2	Monument - Pancho						4.1017118
			01.22					
	2-3	Pancho - Garcia						4.0461679
-33+34	1	Corpus	63 35 03.71	-1.02	02.69	0.07	02.62	0.0478917
-24+26	2	Pancho	77 46 42.61	-0.70	41.91	0.07	41.84	9.9900438
-18+19	3	Garcia	38 38 15.78	-0.17	15.61	0.07	15.54	9.7954581
	1-3	Corpus - Garcia		-1.89		0.21		4.0841034
	1-2	Corpus - Pancho						3.8895177
			02.10					

FIG. 24.—Triangle computation for stations of net—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 26

COMPUTATION OF TRIANGLES

State: Texas

	NO.	STATION	OBSERVED ANGLE	CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
	2-3	Pancho - Monument						4.1017118
-33+25	1	Corpus	92 54 15.91	-0.76	15.15	0.07	15.08	0.0005531
-25+26	2	Pancho	49 18 39.39	-0.35	39.04	0.06	38.98	9.8798168
-30+31	3	Monument	37 47 06.58	-0.58	06.00	0.06	05.94	9.7872478
	1-3	Corpus - Monument		-1.69		0.19		3.9820867
	1-2	Corpus - Pancho						3.8895177
			01.88					
	2-3	Garcia - Monument						3.7799207
-34+35	1	Corpus	29 19 12.20	+0.26	12.46	0.05	12.41	0.3100800
-17+18	2	Garcia	51 15 33.50	-0.45	33.05	0.05	33.00	9.8920860
-30+32	3	Monument	99 25 15.30	-0.66	14.64	0.05	14.59	9.9941028
	1-3	Corpus - Monument		-0.85		0.15		3.9820867
	1-2	Corpus - Garcia						4.0641035
			01.00					
	2-3	Corpus - Monument						3.9820867
-40+41	1	Grande	78 09 40.13	-0.18	39.95	0.05	39.90	0.0093379
-35+36	2	Corpus	46 17 59.13	+0.30	59.43	0.04	59.39	9.8591173
-38+30	3	Monument	55 32 21.19	-0.43	20.76	0.05	20.71	9.9161972
	1-3	Grande - Monument		-0.31		0.14		3.8505419
	1-2	Grande - Corpus						3.9076218
			00.45					
	2-3	Corpus - Monument						3.9820867
-42+43	1	Hebron	37 26 30.55	-1.07	29.48	0.09	29.39	0.2161314
-35+37	2	Corpus	93 48 42.57	-0.40	42.17	0.10	42.07	9.9990382
-29+30	3	Monument	48 44 48.87	-0.23	48.64	0.10	48.54	9.8761042
	1-3	Hebron - Monument		-1.70		0.29		4.1972563
	1-2	Hebron - Corpus						4.0743223
			01.99					
	2-3	Corpus - Grande						3.9076218
-43+44	1	Hebron	42 56 20.48	-0.42	20.06	0.06	20.00	0.1667139
-36+37	2	Corpus	47 30 43.44	-0.70	42.74	0.06	42.68	9.8677132
-39+40	3	Grande	89 32 57.61	-0.23	57.38	0.06	57.32	9.9999866
	1-3	Hebron - Grande		-1.35		0.18		3.9420489
	1-2	Hebron - Corpus						4.0743223
			01.53					

FIG. 24.—Triangle computation for stations of net—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Texas

	No.	Station	Observed angle	Corr'n	Spher'l angle	Spher'l excess	Plane angle and distance	Logarithm
	2-3	Monument - Grande						3.8505419
-43+44	1	Hebron	5 29 49.93	+0.65	50.58	0.01	50.57	1.0136334
-28+29	2	Monument	6 47 32.32	-0.20	32.12	0.01	32.11	9.0728735
-39+41	3	Grande	167 42 37.74	-0.41	37.33	0.01	37.32	9.3280810
	1-3	Hebron - Grande		+0.04		0.03		3.9420488 +1
	1-2	Hebron - Monument						4.1972563
			59.99					
	2-3	Hebron - Grande						3.9420489
-50+51	1	Ringold	69 29 07.23	+0.10	07.33	0.05	07.28	0.0284539
-44+45	2	Hebron	47 22 41.85	-0.44	41.41	0.04	41.37	9.8667328
-38+39	3	Grande	63 08 11.31	+0.09	11.40	0.05	11.35	9.9504064
	1-3	Ringold - Grande		-0.25		0.14		3.8372856
	1-2	Ringold - Hebron						3.9209092
			00.39					
	2-3	Hebron - Monument						4.1972563
-50+52	1	Ringold	95 19 28.46	-0.51	27.95	0.09	27.86	0.0018779
-43+45	2	Hebron	52 52 31.78	+0.21	31.99	0.09	31.90	9.9016361
-27+29	3	Monument	31 47 59.55	+0.78	60.33	0.09	60.24	9.7217750
	1-3	Ringold - Monument		+0.48		0.27		4.1007703
	1-2	Ringold - Hebron						3.9209092
			59.79					
	2-3	Grande - Monument						3.8505419
-51+52	1	Ringold	25 50 21.23	-0.61	20.62	0.03	20.59	0.3606682
+38-41	2	Grande	129 09 10.95	+0.32	11.27	0.04	11.23	9.8895602
-27+28	3	Monument	25 00 27.23	+0.98	28.21	0.03	28.18	9.6260755
	1-3	Ringold - Monument		+0.69		0.10		4.1007703
	1-2	Ringold - Grande						3.8372856
			59.41					
	2-3	Hebron - Ringold						3.9209092
-53+54	1	Garcena	67 18 20.19	-0.10	20.09	0.03	20.06	0.0349980
-45+47	2	Hebron	83 16 45.56	+0.15	45.71	0.03	45.68	9.9970055
-49+50	3	Ringold	29 24 54.45	-0.16	54.29	0.03	54.26	9.6911991
	1-3	Garcia - Ringold		-0.11		0.09		3.9529127
	1-2	Garcia - Hebron						3.6471063
			00.20					

FIG. 24.—Triangle computation for stations of net—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Texas

	NO.	STATION	OBSERVED ANGLE	CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
	2-3	Garcena - Hebron						3.6471063
-56+57	1	Gorgora	6 02 24.09	-0.48	23.61	0.01	23.60	0.9778983
-53+55	2	Garcena	162 14 05.66	-0.39	05.27	0.01	05.26	9.4844665
-46+47	3	Hebron	11 43 31.61	-0.46	31.15	0.01	31.14	9.3079664
	1-3	Gorgora - Hebron		-1.33		0.03		4.1094711
	1-2	Gorgora - Garcena						3.9329710
			01.36					
	2-3	Garcena - Ringold						3.9529127
-56+58	1	Gorgora	43 44 32.07	-1.23	30.84	0.07	30.77	0.1602639
-54+55	2	Garcena	94 55 45.47	-0.29	45.18	0.07	45.11	9.9983908
-43+49	3	Ringold	41 19 43.99	+0.19	44.18	0.06	44.12	9.8197944
	1-3	Gorgora - Ringold		-1.33		0.20		4.1115674
	1-2	Gorgora - Garcena						3.9329710
			01.53					
	2-3	Hebron - Ringold						3.9209092
-57+58	1	Gorgora	37 42 07.98	-0.75	07.23	0.08	07.15	0.2135649
-45+46	2	Hebron	71 33 13.95	+0.61	14.56	0.09	14.47	9.9770934
-48+50	3	Ringold	70 44 38.44	+0.03	38.47	0.09	38.38	9.9749970
	1-3	Gorgora - Ringold		-0.11		0.26		4.1115675 -1
	1-2	Gorgora - Hebron						4.1094711
			00.37					

FIG. 24.—Triangle computation for stations of net—Continued

point. The rules to follow in this case are: At each point the number of angle equations is one less than the number of full lines (observed at both ends) and the number of side equations is two less than the total number of lines, counting both the full lines and those dotted at one end.

For example, in the net to be adjusted, start at the fixed line Palo-Pedro. Then at station Fordyce there are 2 full lines to stations already considered and so there are one angle and no side equations; at station Eltoro there are 3 full lines to stations already used and so there are 2 angle and 1 side equations; at Garcia there are 2 full lines and therefore 1 angle and no side equations; at Pancho there are 3 full lines and therefore 2 angle and 1 side equations. Continuing this process of building up the figure point by point until the final point, Gorgora, is reached, we have the following number of angle and side equations:

Number of angle and side equations in net

Station	Equations	
	Angle	Side
Fordyce	1	0
Eltoro	2	1
Garcia	1	0
Pancho	2	1
Monument	1	0
Corpus	2	1
Grande	1	0
Hebron	2	1
Ringold	2	1
Garcena	1	0
Gorgora	2	1
Total	17	6

A total of 17 angle and 6 side equations is thus obtained and this agrees with the number of equations determined by means of the formulas. In a complicated figure for which it is difficult to determine the correct number of equations the number should be checked by using both methods.

FORMATION OF CONDITION EQUATIONS

The angle equations, which are shown on page 60, are obtained in the same manner as in the adjustment of the single quadrilateral. (See p. 35.) The side equations are also formed in the same manner as the one used in the adjustment of the single quadrilateral. One should be careful at all times to choose the pole in each side equation so that the two smallest angles will be included for, otherwise, the final adjusted logarithms of the distances in the triangle computation may not check.

The angle and side equations should be formed by building up the figure point by point in the same manner as for counting the equations and writing out the equations at each new point as the figure is built up. In this way not only will the correct number of equations be obtained, but the introduction of "identical" equations will be avoided. "Identical" equations are sometimes introduced accidentally and not discovered until the solution of the normals is considerably advanced. (An illustration of an "identical" equation is given on p. 181.)

Angle equations

1. 0=+0.15−(1)+(3)−(4)+(5)−(15)+(16)
2. 0=+0.34−(2)+(3)−(4)+(6)−(7)+(8)
3. 0=+1.74−(5)+(6)−(7)+(9)−(14)+(15)
4. 0=−0.26−(9)+(10)−(12)+(14)−(20)+(21)
5. 0=+0.19−(10)+(11)−(19)+(20)−(22)+(24)
6. 0=−0.77−(12)+(13)−(19)+(21)−(23)+(24)
7. 0=+1.05−(17)+(19)−(24)+(25)−(31)+(32)
8. 0=+1.89−(18)+(19)−(24)+(26)−(33)+(34)
9. 0=+1.69−(25)+(26)−(30)+(31)−(33)+(35)
10. 0=+0.31−(28)+(30)−(35)+(36)−(40)+(41)
11. 0=−0.69−(27)+(28)+(38)−(41)−(51)+(52)
12. 0=+1.70−(29)+(30)−(35)+(37)−(42)+(43)
13. 0=+1.35−(36)+(37)−(39)+(40)−(42)+(44)
14. 0=+0.25−(38)+(39)−(44)+(45)−(50)+(51)
15. 0=+0.11−(49)+(50)−(45)+(47)−(53)+(54)
16. 0=+1.33−(48)+(49)−(54)+(55)−(56)+(58)
17. 0=+0.11−(45)+(46)−(48)+(50)−(57)+(58)

Side equations

Symbol	Angle	Logarithm	Tabular difference	Symbol	Angle	Logarithm	Tabular difference
	° ′ ″				° ′ ″		
−15+16	48 00 09.94	9.87109231	+1.90	−4+5	61 27 31.53	9.94372863	+1.15
−8+9	41 19 38.15	9.81978015	+2.40	−1+2	26 37 19.81	9.65137982	+4.20
−5+6	38 09 53.52	9.79093675	+2.67	−14+15	64 02 53.85	9.95383853	+1.02
−2+3	43 54 59.13	9.84111431	+2.19	−7+8	36 27 36.43	9.77397878	+2.85
		9.32292352				9.32292576	

[1] 18. 0=−2.24+4.20 (1)−6.39 (2)+2.19 (3)+1.15 (4)−3.82 (5)+2.67 (6)+2.85 (7)−5.25 (8)+2.40 (9)+1.02 (14)−2.92 (15)+1.90 (16)

−20+21	31 20 13.41	9.71606319	+3.46	−12+14	76 47 59.67	9.98837099	+0.49
−22+24	64 28 16.62	9.95538436	+1.00	−19+20	70 52 25.04	9.97533904	+.73
−13+14	43 05 08.33	9.83447846	+2.25	−22+23	20 23 46.79	9.54221776	+5.66
		9.50592601				9.50592779	

19. 0=−1.78+0.49 (12)−2.25 (13)+1.76 (14)+0.73 (19)−4.19 (20)+3.46 (21)+4.66 (22)−5.66 (23)+1.00 (24)

[1] This equation is formed with the pole at the intersection of the two diagonals. (See explanation on p. 185.)

Side equations—Continued

Symbol	Angle ° ′ ″	Logarithm	Tabular difference	Symbol	Angle ° ′ ″	Logarithm	Tabular difference
−34+35	29 19 12.20	9.63991918	+3.74	−30+32	99 25 15.30	9.99410258	−0.34
−24+26	77 46 42.61	9.99004413	+.45	−33+34	63 35 03.71	9.95210943	+1.05
−31+32	61 38 08.72	9.94445563	+1.13	−24+25	28 28 03.22	9.67820974	+3.89
		9.62441894				9.62442175	

20. $$0 = -2.81 + 3.44\,(24) - 3.89\,(25) + 0.45\,(26) - 0.34\,(30) - 1.13\,(31) + 1.47\,(32) + 1.05\,(33) - 4.79\,(34) + 3.74\,(35)$$

−28+29	6 47 32.32	9.07287722	+17.68	−43+44	5 29 49.93	8.98135262	+21.88
−51+52	25 50 21.23	9.63933451	+4.34	−27+28	25 00 27.23	9.62607119	+4.52
−44+45	47 22 41.85	9.86678373	+1.93	−50+51	69 29 07.23	9.97154607	+.78
		8.57899546				8.57896988	

[1] 21. $$0 = +25.58 + 4.52\,(27) - 22.20\,(28) + 17.68\,(29) + 21.88\,(43) - 23.81\,(44) + 1.93\,(45) + 0.78\,(50) - 5.12\,(51) + 4.34\,(52).$$ (This equation should be divided by 5 before entering it in the correlates.)

−44+45	47 22 41.85	9.86678373	+1.93	−50+51	69 29 07.23	9.97154607	+0.78
−36+37	47 30 43.44	9.86771468	+1.93	−42+44	42 56 20.48	9.83328715	+2.27
−28+30	55 32 21.19	9.91619788	+1.45	−35+36	46 17 59.13	9.85911681	+2.01
−51+52	25 50 21.23	9.63933451	+4.34	−27+28	25 00 27.23	9.62607119	+4.52
		9.29003080				9.29002122	

[2] 22. $$0 = +9.58 + 4.52(27) - 5.97(28) + 1.45(30) + 2.01(35) - 3.94(36) + 1.93(37) + 2.27(42) - 4.20(44) + 1.93(45) + 0.78(50) - 5.12(51) + 4.34(52).$$

−48+49	41 19 43.99	9.81979414	+2.39	−56+58	43 44 32.07	9.83973894	+2.20
−45+47	83 16 45.56	9.99700544	+.25	−49+50	29 24 54.45	9.69119975	+3.74
−56+57	6 02 24.09	9.02211142	+19.90	−46+47	11 43 31.61	9.30797112	+10.14
		8.83891100				8.83890981	

[1] 23. $$0 = +1.19 - 0.25(45) + 10.14(46) - 9.89(47) - 2.39(48) + 6.13(49) - 3.74(50) - 17.70(56) + 19.90(57) - 2.20(58).$$ (This equation should be divided by 5 before entering it in the correlates.)

[1] The coefficients of the different equations of a net should be approximately the same size. If the coefficients of an equation are too large, they should be divided by a factor to bring them to the proper size.
[2] In a central point figure with one diagonal observed, which requires 2 side equations, one of the side equations should be written with the pole at the center and should be carried entirely around the figure.

EXPLANATION OF AZIMUTH EQUATION

In the formation of the azimuth equations differences between forward and back azimuths are used and it is necessary, therefore, to make a preliminary computation of the geographic positions of the various stations to determine these azimuths. This same computation is used in determining the latitude and longitude closures and an explanation of it will be found on pages 66 and 69. We shall consider here that these positions have already been computed.

Starting with the fixed azimuth, Palo-Pedro, and using the observed directions and the differences between the azimuths and back azimuths through the shortest route, the azimuth of the line Garcena-Gorgora is determined. It is well to go through the angles in a clockwise direction, wherever possible. The angles will then be added instead of subtracted.

Computation of azimuth equation

	°	′	″	Final seconds*
Palo-Pedro (fixed azimuth)	12	02	25. 00	25. 00
−4+6	+99	37	25. 05	24. 98
Palo-Eltoro	111	39	50. 05	49. 98
	−	2	45. 12	45. 11
Eltoro-Palo	291	37	04. 93	04. 87
−7+11	+194	18	20. 42	19. 76
Eltoro-Pancho	125	55	25. 35	24. 63
	−	3	14. 32	14. 32
Pancho-Eltoro	305	52	11. 03	10. 31
−22+26	+142	14	59. 23	58. 40
Pancho-Corpus	88	07	10. 26	08. 71
	−	2	04. 55	04. 55
Corpus-Pancho	268	05	05. 71	04. 16
−33+37	+186	42	58. 48	57. 32
Corpus-Hebron	94	48	04. 19	01. 48
	−	3	10. 09	10. 09
Hebron-Corpus	274	44	54. 10	51. 39
−42+47	+173	35	47. 89	47. 18
Hebron-Garcena	88	20	41. 99	38. 57
	−	1	11. 31	11. 31
Garcena-Hebron	268	19	30. 68	27. 26
−53+55	+162	14	05. 66	05. 27
Garcena-Gorgora (computed azimuth)	70	33	36. 34	32. 53
Garcena-Gorgora (fixed azimuth)	70	33	32. 53	32. 53
Closing error			+3. 81	0. 00

24. $0=+3.81\dagger-(4)+(6)-(7)+(11)-(22)+(26)-(33)+(37)-(42)+(47)-(53)+(55)$

The azimuth equation above is made up as follows (see p. 50): Starting with the azimuth Palo-Pedro, 12° 02′ 25.″00, and adding the angle at Palo from Pedro to Eltoro (designated by −4+6), 99° 37′ 25.″05, we get 111° 39′ 50.″05, which is the azimuth of the line Palo-Eltoro. Now in order to carry the azimuth ahead, it is necessary to obtain the back azimuth of this line or the azimuth

* This column is filled out after the adjustment is completed.

† The azimuth constant should be corrected by the amount $\sin \phi_m (\lambda_n - \lambda_n')$, where ϕ_m is the mean latitude and $\lambda_n - \lambda_n'$ is the discrepancy in longitude.

$\phi_m = 26° 20' 49''$ and $\lambda_n - \lambda_n' = -0.''006$

log 0.006 = 7. 7782–10 (negative).
log sin 26° 20′ 49″ = 9. 6472–10

log correction = 7. 4254–10 (negative).
correction = −0.″003.

This correction is not large enough to affect the azimuth constant.

Eltoro-Palo. This is obtained by adding 180° and (algebraically) the difference of azimuth due to convergence of meridians as computed on page 71, to the azimuth Palo-Eltoro. That is,

	°	′	″
Azimuth, Palo-Eltoro =	111	39	50. 05
Difference of azimuth and back azimuth =	180	00	00. 00
	−	2	45. 12
Azimuth, Eltoro-Palo =	291	37	04. 93

Then using the azimuth Eltoro-Palo as given above, and adding the angle at Eltoro from Palo to Pancho, we obtain the azimuth Eltoro-Pancho.

In this manner, as shown on page 62, the azimuth is carried along until the azimuth Garcena-Gorgora is determined. It must be remembered that the observed angles are used in forming the azimuth equation. They can be obtained from either the list of directions or the triangle computation. The designation of each angle used should be placed to the left of the corresponding angle as shown on page 62. At the end of the equation the fixed azimuth of Garcena-Gorgora should be placed directly under the computed azimuth. The azimuth equation may then be written by placing the fixed azimuth equal to the computed azimuth plus all of the v's denoted by the designations of the angles in the computation of the azimuth equation. For convenience all terms are transferred to the right side of the equation, and the difference of the two azimuths (computed minus fixed) is used as the constant term of the equation.

EXPLANATION OF LENGTH EQUATION

The specifications for horizontal control for first-order triangulation (see Special Publication No. 120, p. 2) require that the closure in length upon a measured base or a line of adjusted triangulation must not exceed that represented by an error of $\frac{1}{25000}$ after the angle and side equations have been satisfied in the adjustment. The closure in length can easily be expressed as a ratio by dividing the discrepancy in the logarithm by 0.4343, which is the modulus of the common system of logarithms. For instance, in the following length equation the discrepancy in length is 2.27 in the sixth place of logarithms and therefore the closure in length before adjustment is $\frac{0.00000227}{0.4343}$ or approximately $\frac{1}{191000}$. After the angle and side equations are satisfied in the adjustment the discrepancy in length is 7.25 in the sixth place of logarithms (see p. 91), that is, the closure in length after adjustment is $\frac{0.00000725}{0.4343}$ or approximately $\frac{1}{60000}$.

The length equation is formed as shown in the table on page 65. It differs from a side equation in that it involves the logarithms of two lengths. Starting with the line Palo-Pedro the length is computed through the *best* chain of triangles (see fig. 22) until the computed length of the line Garcena-Gorgora is obtained. The angles turned through in carrying the length through the scheme are indicated by small arcs near the vertices of the triangles in Figure 25.

In most cases it will not be necessary to compute the ΣR (see p. 236) in order to get the best chain of triangles. An experienced mathematician will be able to tell from an inspection of the triangles which is the strongest chain. The beginner, however, when in doubt about the best triangles in any particular quadrilateral of the chain, should compute the ΣR for that quadrilateral. A good general rule to follow is to avoid the small angles in the formation of the length equation.

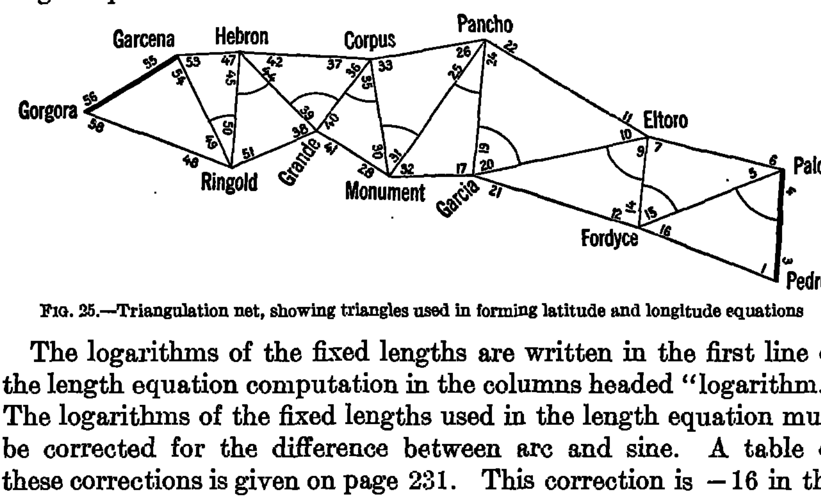

FIG. 25.—Triangulation net, showing triangles used in forming latitude and longitude equations

The logarithms of the fixed lengths are written in the first line of the length equation computation in the columns headed "logarithm." The logarithms of the fixed lengths used in the length equation must be corrected for the difference between arc and sine. A table of these corrections is given on page 231. This correction is −16 in the eighth place of logarithms for the length Palo-Pedro and −13 for the length Garcena-Gorgora. These corrections should be placed just above the logarithms of the lengths themselves. (See p. 65.)

The arc-sine correction may be explained as follows: In the formation of a length equation it is assumed that small arcs are proportional to their sines. Similarly, in the derivation of the position computation formulas it is assumed that small arcs, s and $\Delta\lambda$, are proportional to their sines. As this assumption introduces a slight error in the computations a correction must be applied to take account of the error. In the length equations we are concerned only with the corrections corresponding to log s, which are always negative. In the case of the position computation, however, the differences are taken out for both the arguments log s and log $\Delta\lambda$, the first with a negative, the second with a positive sign, and their algebraic sum is applied as a correction to log $\Delta\lambda$. A table similar to the one on page 231 is given on page 17 of Special Publication No. 8, sixth edition, but as it is only carried to seven places of decimals it can not be used for first-order work, which should be carried to eight places of decimals.

Columns 1 and 5 of the length equation, headed "symbol," can be filled out directly from Figure 22. After determining through which triangles the length is to be carried, the following rule can be used in filling out these columns: In the first column is placed the designation of the angle adjacent to the starting length but opposite the side through which the length is next carried, while in the fifth column is placed the designation of the angle opposite the starting length. For instance, in Figure 25 the line Palo-Pedro is the starting length and the line Palo-Fordyce is the line through which the length is to be carried next, so $-1+3$ is placed in the first column, and $-15+16$ in the fifth column. Palo-Fordyce now becomes the starting line and Eltoro-Fordyce becomes the next line through which the length is carried so $-5+6$ and $-7+9$ are placed in the first and fifth columns, respectively. In a similar manner, the rest of the first and fifth columns are filled out.

The remainder of the computation is handled in exactly the same manner as for a side equation. The constant term for the length equation is obtained by subtracting the sum of the quantities in the seventh column from the sum of the quantities in the third column. The quantities in the fourth and eighth columns are the coefficients of the quantities in the first and fifth columns, respectively, as in a side equation. It must be remembered that the coefficients from the eighth column change sign, the right side of the table being subtracted from the left side.

Computation of length equation

1 Symbol	2 Angle	3 Logarithm	4 Tabular difference	5 Symbol	6 Angle	7 Logarithm	8 Tabular difference
Palo-Pedro		−16* 3.9781520	----------	Garcena-Gorgora		−13* 3.9329710	----------
	° ′ ″				° ′ ″		
−1+3	70 32 18.94	9.97445004	+0.74	−15+16	48 00 09.94	9.87109231	+1.90
−5+6	38 09 53.52	9.79093675	+2.67	−7+9	77 47 14.58	9.99005871	+.45
−12+14	76 47 59.67	9.93837099	+.49	−20+21	31 20 13.41	9.71606319	+3.46
−10+11	44 39 18.91	9.84685608	+2.13	−22+24	64 28 16.62	9.95538436	+1.00
−17+19	89 53 49.28	9.99999930	.00	−31+32	61 38 08.72	9.94445563	+1.13
−25+26	49 18 39.39	9.87981749	+1.81	−33+35	92 54 15.91	9.99944177	−.11
−28+30	55 32 21.19	9.91619788	+1.45	−40+41	78 09 40.13	9.99066223	+.45
−36+37	47 30 43.44	9.86771468	+1.93	−42+44	42 56 20.48	9.83328715	+2.27
−38+39	63 08 11.31	9.95040642	+1.06	−50+51	69 29 07.23	9.97154607	+.78
−45+47	83 16 45.56	9.99700544	+.25	−53+54	67 18 20.19	9.96500209	+.88
−48+49	41 19 43.99	9.81979414	+2.39	−56+58	43 44 32.07	9.83973894	+2.20
		3.00970105				3.00970332	

* See p. 64.

25. $$0=-2.27-0.74(1)+0.74(3)-2.67(5)+2.67(6)+0.45(7)-0.45(9)$$
$$-2.13(10)+2.13(11)-0.49(12)+0.49(14)+1.90(15)-1.90(16)$$
$$+3.46(20)-3.46(21)+1.00(22)-1.00(24)-1.81(25)+1.81(26)$$
$$-1.45(28)+1.45(30)+1.13(31)-1.13(32)-0.11(33)+0.11(35)$$
$$-1.93(36)+1.93(37)-1.06(38)+1.06(39)+0.45(40)-0.45(41)$$
$$+2.27(42)-2.27(44)-0.25(45)+0.25(47)-2.39(48)+2.39(49)$$
$$+0.78(50)-0.78(51)+0.88(53)-0.88(54)+2.20(56)-2.20(58)$$

LATITUDE AND LONGITUDE EQUATIONS

The angle, side, azimuth, and length equations having been formed, the latitude and longitude equations should now be formed. The development of condition equations for latitude and longitude closures is given very fully in Special Publication No. 28. Only the actual mechanical operation of forming these equations will be given here.

Until a few years ago the usual practice was to adjust a net of triangulation for only angle, side, length, and azimuth conditions at first and then, using the adjusted angles and lengths, to compute the geographic positions and determine the latitude and longitude closures. A new adjustment was then made to eliminate the discrepancies in latitude and longitude. In this second adjustment, it was necessary to hold the constant terms of all the equations, except the latitude and longitude equations, to zero, since all closing errors except in latitude and longitude were eliminated by the first adjustment.

The present practice is to adjust the latitude and longitude equations along with the other equations. When this is done it is necessary to compute preliminary positions through a selected chain of triangles in order to determine the latitude and longitude closures.

In this chain of triangles the observed angles are used for the length angles (the angles in each triangle through which the length is carried). The azimuth angle in each triangle is concluded by subtracting the sum of the two length angles from 180° plus the spherical excess. This angle should in every case be placed in parentheses to show that it is concluded. However, if one of the length angles is not observed, then the observed azimuth angle must be used, and the unobserved length angle concluded.

The chain of triangles through which the preliminary positions are computed is shown in Figure 26. It should be noted particularly that since the azimuth angle is concluded, it should be designated by the concluded correction symbols. For instance, in the triangle Fordyce-Palo-Pedro (see fig. 26), the angle at Palo and its correction symbol are obtained as follows:

		°	′	″
−15+16	Fordyce	48	00	09.94
−1+3	Pedro	70	32	18.94
−1+3−15+16	sum of Fordyce and Pedro angles	118	32	28.88
	180°+spherical excess	180	00	00.26
+1−3+15−16	Palo	61	27	31.38

The concluded azimuth angles of all the triangles and their corresponding correction symbols are obtained in a similar manner. These triangles are given in Figure 26.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Texas

NO.	STATIONS	OBSERVED ANGLE	CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLES AND DISTANCES	LOGARITHM
	2-3 Palo-Pedro						3.978 1520
β-15+16	1 Fordyce	48 00 09.94			0.08	09.86	0.128 9079
C	2 Palo	(61 27 31.38)			0.09	31.29	9.943 7283
A -1+3	3 Pedro	70 32 18.94			0.09	18.85	9.974 4500
	1-3 Fordyce-Pedro				0.26		4.050 7882
	1-2 Fordyce-Palo						4.081 5099
	2-3 Palo-Fordyce						4.081 5099
β -7+9	1 Eltoro	77 47 14.58			0.07	14.51	0.009 9414
A -5+6	2 Palo	38 09 53.52			0.07	53.45	9.790 9366
C	3 Fordyce	(64 02 52.11)			0.07	52.04	9.953 8366
	1-3 Eltoro-Fordyce				0.21		3.882 3879
	1-2 Eltoro-Palo						4.045 2879
	2-3 Eltoro-Fordyce						3.882 3879
β -20+21	1 Garcia	31 20 13.41			0.09	13.32	0.283 9371
C	2 Eltoro	(71 51 47.19)			0.09	47.10	9.977 8678
A -12+14	3 Fordyce	76 47 59.67			0.09	59.58	9.988 3710
	1-3 Garcia-Fordyce				0.27		4.144 1928
	1-2 Garcia-Eltoro						4.154 6960
	2-3 Eltoro-Garcia						4.154 6960
β -22+24	1 Pancho	64 28 16.62			0.13	16.49	0.044 6158
A -10+11	2 Eltoro	44 39 18.91			0.12	18.79	9.846 8559
C	3 Garcia	(70 52 24.85)			0.13	24.72	9.975 3388
	1-3 Pancho-Garcia				0.38		4.046 1677
	1-2 Pancho-Eltoro						4.174 6506
	2-3 Pancho-Garcia						4.046 1677
β -31+32	1 Monument	61 38 08.72			0.06	08.66	0.055 5444
C	2 Pancho	(28 28 02.17)			0.05	02.12	9.678 2055
A -17+19	3 Garcia	89 53 49.28			0.06	49.22	9.999 9993
	1-3 Monument-Garcia				0.17		3.779 9176
	1-2 Monument-Pancho						4.101 7114

FIG. 26.—Triangle computation to obtain latitude and longitude closures of net

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Texas

NO.		STATION	OBSERVED ANGLE	CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
	2-3	Pancho-Monument						4.101 7114
β -33+35	1	Corpus	92 54 15.91			0.07	15.84	0.000 5582
A -25+26	2	Pancho	49 18 39.39			0.06	39.33	9.879 8174
C	3	Monument	(37 47 04.89)			0.06	04.83	9.787 2443
	1-3	Corpus-Monument				0.19		3.982 0870
	1-2	Corpus-Pancho						3.889 5144
	2-3	Corpus-Monument						3.982 0870
β-40+41	1	Grande	78 09 40.13			0.05	40.08	0.009 3378
C	2	Corpus	(46 17 58.82)			0.04	58.78	9.859 1161
A-28+30	3	Monument	55 32 21.19			0.05	21.14	9.916 1978
	1-3	Grande-Monument				0.14		3.850 5409
	1-2	Grande-Corpus						3.907 6226
	2-3	Corpus-Grande						3.907 6226
β-42+44	1	Hebron	42 56 20.48			0.06	20.42	0.166 7130
A-36+37	2	Corpus	47 30 43.44			0.06	43.38	9.867 7146
C	3	Grande	(89 32 56.26)			0.06	56.20	9.999 9865
	1-3	Hebron-Grande				0.18		3.942 0502
	1-2	Hebron-Corpus						4.074 3221
	2-3	Hebron-Grande						3.942 0502
β-50+51	1	Ringold	69 29 07.23			0.05	07.18	0.028 4539
C	2	Hebron	(47 22 41.60)			0.04	41.56	9.866 7831
A-38+39	3	Grande	63 08 11.31			0.05	11.26	9.950 4064
	1-3	Ringold-Grande				0.14		3.837 2872
	1-2	Ringold-Hebron						3.920 9105
	2-3	Hebron-Ringold						3.920 9105
β -53+54	1	Garcena	67 18 20.19			0.03	20.16	0.034 9979
A-45+47	2	Hebron	83 16 45.56			0.03	45.53	9.997 0054
C	3	Ringold	(29 24 54.34)			0.03	54.31	9.691 1992
	1-3	Garcena-Ringold				0.09		3.952 9138
	1-2	Garcena-Hebron						3.647 1076

FIG. 26.—Triangle computation to obtain latitude and longitude closures of net—Continued

After the preliminary triangles have been computed, the preliminary geographic positions are computed to determine the latitude and longitude closures. As the full explanation of the computation of geographic positions and the development of the formulas for computing them are given in Special Publication No. 8, it will not be necessary to repeat the explanation here. The actual computation of the positions are shown in Figure 27.

FORMATION OF LATITUDE AND LONGITUDE EQUATIONS

The formation of the latitude and longitude equations from the preceding preliminary positions is shown in Figure 28, form 496 being used for this purpose.

The formulas for the latitude and longitude equations are as follows (see Special Publication No. 28, p. 31):

Latitude: $$0=7238.24\ (\phi_n-\phi_{n'})''+\Sigma[(\phi_n-\phi_c)'\ \delta_A\ (v_A)-(\phi_n-\phi_c)'\ \delta_B\ (v_B)]+\Sigma[\pm a_1\ (\lambda_n-\lambda_c)'\ (v_C)]$$

Longitude: $$0=7238.24\ (\lambda_n-\lambda_{n'})''+\Sigma[(\lambda_n-\lambda_c)'\ \delta_A\ (v_A)-(\lambda_n-\lambda_c)'\ \delta_B\ (v_B)]+\Sigma[\pm a_2\ (\phi_n-\phi_c)'\ (v_C)]$$

In these equations v_A, v_B, and v_C are replaced by their correction symbols, care being taken to use $v_C=-v_A-v_B$, if the azimuth angle has been concluded in carrying the position computation through the chain. For convenience in computing, it is important to arrange the computation in the form of a table as shown on page 81.

EXPLANATION OF COMPUTATION, LATITUDE AND LONGITUDE EQUATIONS

In the column headed "station" is placed the name of the station corresponding to the C angle of each triangle of the preliminary set of triangles. (See p. 67.) The values in the columns headed ϕ_c and λ_c are obtained directly from the position computations, as also the values for ϕ_n and λ_n in the upper right-hand corner of the form. The columns headed $\phi_n-\phi_c$ and $\lambda_n-\lambda_c$ are self-explanatory. In the column headed δ_A is placed the tabular difference of the length angle adjacent to the starting length in each triangle, and in the column $-\delta_B$ is placed the tabular difference, with minus sign, of the length angle opposite the starting length in each triangle. If the position computations are carried through the same triangles as the length equation, as they should be, then the values in the columns headed δ_A and $-\delta_B$ can be taken directly from the columns headed tabular difference in the length equation computation (see p. 65) by simply changing the signs of the quantities placed in the column headed $-\delta_B$.

In the columns headed A, B, and C are placed the designations of the angles of the various triangles passed through in forming the lati-

(Text continued on p. 80)

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Palo	to	Pedro	12	02	25,00
∠		Pedro	&	Fordyce	+ 61	27	31,38
α	2.	Palo	to 1.	Fordyce	73	29	56,38
Δα					−	3	04,84
					180		
α′	1.	Fordyce	to 2.	Palo	253	26	51,54
				First Angle of Triangle	48	00	09,94

	°	′	″				°	′	″
φ	26	19	38,951	2.	Palo	λ	98	27	48,248
Δφ	−	1	51,518	s=		Δλ	+	6	56,990
φ′	26	17	47,433	1.	Fordyce	λ′	98	34	45,238

s	4,081 5099	s²	8,16302			−h	2,0467
cos α	9,453 3675	sin² α	9,96347	(δφ)²	4,0947	s² sin² α	8,1265
B	8,511 8083	C	1,10025	D	2,2921	E	5,8511
h	2,046 6857		9,22674		6,3868		6,0243
	″		″				
1st term.	+111,3488	3d term.	+0,0002			(Δλ)²	
2d term.	+ 0,1686	4th term.	−0,0001			F	
	+111,5174						
3d and 4th terms.	+ 0,0001	s	4,081 5099				
−Δφ	+111,5175	sin α	9,981 7347	Arg.		Δλ	2,620 1257
½(φ+φ′)	26 18 43,19	A′	8,509 4379	s	−3	sin ½(φ+φ′)	9,646 6575
		sec φ′	0,047 4432	Δλ	+3	sec ½(Δφ)	
			2,620 1257	Corr.	0		2,266 7832
		Δλ	+416,9901			−Δα	+184,835

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Pedro	to	Palo	192	01	53,35
∠		Fordyce	&	Palo	− 70	32	18,94
α	3.	Pedro	to 1.	Fordyce	121	29	34,41
Δα					−	2	32,93
					180		
α′	1.	Fordyce	to 3.	Pedro	301	27	01,48

	°	′	″				°	′	″
φ	26	14	36,740	3.	Pedro	λ	98	28	59,722
Δφ	+	3	10,693	s=		Δλ	+	5	45,516
φ′	26	17	47,433	1.	Fordyce	λ′	98	34	45,238

s	4,050 7882	s²	8,10158			−h	2,2806
cos α	9,717 9972	sin² α	9,86160	(δφ)²	4,5607	s² sin² α	7,9632
B	8,511 8134	C	1,09865	D	2,2911	E	5,8497
h	2,280 5988		9,06183		6,8518		6,0935
	″		″				
1st term.	− 190,8090	3d term.	+0,0007			(Δλ)²	
2d term.	+ 0,1153	4th term.	+0,0001			F	
	− 190,6937						
3d and 4th terms.	+ 0,0008	s	4,050 7882				
−Δφ	− 190,6929	sin α	9,930 7988	Arg.		Δλ	2,538 4681
½(φ+φ′)	26 16 12,08	A′	8,509 4379	s	−2	sin ½(φ+φ′)	9,646 0134
		sec φ′	0,047 4432	Δλ	+2	sec ½(Δφ)	
			2,538 4681	Corr.	0		2,184 4815
		Δλ	+ 345,5160			−Δα	+ 152,926

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FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

				°	′	″
α	Palo	to	Fordyce	73	29	56,38
∠	Fordyce	&	Eltoro	+ 38	09	53,52
α	2. Palo	to 1.	Eltoro	111	39	49,90
Δα				−	2	45,12
				180		
α′	1. Eltoro	to 2.	Palo	291	37	04,78

First Angle of Triangle 77 47 14,58

	°	′	″				°	′	″
φ	26	19	38,951	2.	Palo	λ	98	27	48,248
Δφ	+	2	13,006	s=		Δλ	+	6	12,057
φ′	26	21	51,957	1.	Eltoro	λ′	98	34	00,305

s	4,045 2879	s²	8,09058			−h	2,1243
cos α	9,567 2154	sin² α	9,93637	(δφ)²	4,2477	s² sin² α	8,0270
B	8,511 8083	C	1,10025	D	2,2921	E	5,8511
h	2,1243116		9,12720		6,5398		6,0024
1st term.	″ −133,1409	3d term.	″ +0,0003			(Δλ)²	
2d term.	+ 0,1340	4th term.	+0,0001			F	
	−133,0069						
3d and 4th terms.	+ 0,0004	s	4,045 2879				
−Δφ	−133,0065	sin α	9,968 1866	Arg.		Δλ	2,570 6090
½(φ+φ′)	26 20 45,46	A′	8,509 4365	s	−2	sin ½(φ+φ′)	9,647 1776
		sec φ′	0,047 6980	Δλ	+2	sec ½(Δφ)	
			2,570 6090	Corr.	0		2,217 7866
		Δλ	+ 372,0566			−Δα	+165,115

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

				°	′	″
α	Fordyce	to	Palo	253	26	51,54
∠	Eltoro	&	Palo	− 64	02	52,11
α	3. Fordyce	to 1.	Eltoro	189	23	59,43
Δα				+		19,93
				180		
α′	1. Eltoro	to 3.	Fordyce	9	24	19,36

	°	′	″				°	′	″
φ	26	17	47,433	3.	Fordyce	λ	98	34	45,238
Δφ	+	4	04,524	s=		Δλ	−		44,933
φ′	26	21	51,957	1.	Eltoro	λ′	98	34	00,305

s	3,882 3879	s²	7,76478			−h	2,3883
cos α	9,994 1291	sin² α	8,42610	(δφ)²	4,7766	s² sin² α	6,1909
B	8,511 8102	C	1,09965	D	2,2918	E	5,8506
h	2,388 3272		7,29053		7,0684		4,4298
1st term.	″ −244,5272	3d term.	″ +0,0012			(Δλ)²	
2d term.	+ 0,0020	4th term.	0,0000			F	
	−244,5252						
3d and 4th terms.	+ 0,0012	s	3,882 3879				
−Δφ	−244,5240	sin α	9,213 0480	Arg.		Δλ	1,652 5703
½(φ+φ′)	26 19 49,69	A′	8,509 4365	s	−1	sin ½(φ+φ′)	9,646 9406
		sec φ′	0,047 6980	Δλ	0	sec ½(Δφ)	
			1,652 5704 (3)	Corr.	−1		1,299 5109
		Δλ	− 44,9335			−Δα	− 19,930

FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

				°	′	″
α	Eltoro	to	Fordyce	9	24	19.36
$\angle$	Fordyce	&	Garcia	+71	51	47.19
α	2. Eltoro	to 1.	Garcia	81	16	06.55
$\Delta\alpha$				-	3	45.95
				180		
α'	1. Garcia	to 2.	Eltoro	261	12	20.60
			First Angle of Triangle	31	20	13.41

	°	′	″			°	′	″
φ	26	21	51.957	2. Eltoro	λ	98	34	00.305
$\Delta\varphi$	-	1	10.686	s=	$\Delta\lambda$	+	8	28.973
φ'	26	20	41.271	1. Garcia	λ'	98	42	29.278

s	4.154 6960	s^2	8.30939			−h	1.8478
cos α	9.181 2846	$\sin^2\alpha$	9.96987	$(\delta\varphi)^2$	3.6987	$s^2\sin^2\alpha$	8.2993
B	8.511 8060	C	1.10095	D	2.2925	E	5.8517
h	1.847 7866		9.40021		5.9912		5.9988

	″		″		
1st term.	+70.4347	3d term.	+0.0001	$(\Delta\lambda)^2$	
2d term.	+ 0.2513	4th term.	-0.0001	F	
	+70.6860				
3d and 4th terms.	0.0000				
$-\Delta\varphi$	+70.6860				
$\frac{1}{2}(\varphi+\varphi')$	26 21 16.61				

s	4.154 6960				
sin α	9.994 9374	Arg.		$\Delta\lambda$	2.706 6946
A′	8.509 4369	s	-4	$\sin\frac{1}{2}(\varphi+\varphi')$	9.647 3100
sec φ′	0.047 6243	$\Delta\lambda$	+4	$\sec\frac{1}{2}(\Delta\varphi)$	
	2.706 6946	Corr.	0		2.354 0046
$\Delta\lambda$	+ 508.9728			$-\Delta\alpha$	+225.946

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

				°	′	″
α	Fordyce	to	Eltoro	189	23	59.43
$\angle$	Garcia	&	Eltoro	- 76	47	59.67
α	3. Fordyce	to 1.	Garcia	112	35	59.76
$\Delta\alpha$				-	3	25.75
				180		
α'	1. Garcia	to 3.	Fordyce	292	32	34.01

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	°	′	″			°	′	″
φ	26	17	47.433	3. Fordyce	λ	98	34	45.238
$\Delta\varphi$	+	2	53.838	s=	$\Delta\lambda$	+	7	44.040
φ'	26	20	41.271	1. Garcia	λ'	98	42	29.278

s	4.144 1928	s^2	8.28839			−h	2.2407
cos α	9.584 6639	$\sin^2\alpha$	9.93060	$(\delta\varphi)^2$	4.4803	$s^2\sin^2\alpha$	8.2190
B	8.511 8102	C	1.09965	D	2.2918	E	5.8506
h	2.240 6669		9.31864		6.7721		6.3103

	″		″		
1st term.	-174.0471	3d term.	+0.0006	$(\Delta\lambda)^2$	
2d term.	+ 0.2083	4th term.	+0.0002	F	
	-173.8388				
3d and 4th terms.	+ 0.0008				
$-\Delta\varphi$	-173.8380				
$\frac{1}{2}(\varphi+\varphi')$	26 19 14.35				

s	4.144 1928				
sin α	9.965 3008	Arg.		$\Delta\lambda$	2.666 5549
A′	8.509 4369	s	-3	$\sin\frac{1}{2}(\varphi+\varphi')$	9.646 7901
sec φ′	0.047 6243	$\Delta\lambda$	+4	$\sec\frac{1}{2}(\Delta\varphi)$	
	2.666 5548 (9)	Corr.	+1		2.313 3450
$\Delta\lambda$	+ 464.0395			$-\Delta\alpha$	+205.752

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FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

				°	′	″
α	Eltoro	to	Garcia	81	16	06.55
∠	Garcia	&	Pancho	+ 44	39	18.91
α	2. Eltoro	to 1.	Pancho	125	55	25.46
Δα				-	3	14.32
				180		
α′	1. Pancho	to 2.	Eltoro	305	52	11.14
			First Angle of Triangle	64	28	16.62

	°	′	″				°	′	″
φ	26	21	51.957	2. Eltoro		λ	98	34	00.305
Δφ	+	4	44.835	s=		Δλ	+	7	16.977
φ′	26	26	36.792	1. Pancho		λ′	98	41	17.282

s	4.174 6506	s^2	8.34930			−h	2.4549
cos α	9.768 4220	$\sin^2 α$	9.81675	$(δφ)^2$	4.9092	$s^2 \sin^2 α$	8.1660
B	8.511 8060	C	1.10095	D	2.2925	E	5.8517
h	2.454 8786		9.26700		7.2017		6.4726

1st term.	″ −265.0222	3d term.	″ +0.0016			$(Δλ)^2$	
2d term.	+0.1849	4th term.	+0.0003			F	
	−284.8373						
3d and 4th terms.	+ 0.0019	s	4.174 6506				
−Δφ	−284.8354	sin α	9.908 3770	Arg.		Δλ	2.640 4581
½(φ+φ′)	26 24 14.37	A′	8.509 4349	s	−4	sin ½(φ+φ′)	9.648 0648
		sec φ′	0.047 9957	Δλ	+3	sec ½(Δφ)	
			2.640 4582	Corr.	−1		2.288 5229
		Δλ	″ + 436.9765			−Δα	″ + 194.322

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

				°	′	″
α	Garcia	to	Eltoro	261	12	20.60
∠	Pancho	&	Eltoro	− 70	52	24.85
α	3. Garcia	to 1.	Pancho	190	19	55.75
Δα				+		32.01
				180		
α′	1. Pancho	to 3.	Garcia	10	20	27.76

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	°	′	″				°	′	″
φ	26	20	41.271	3. Garcia		λ	98	42	29.278
Δφ	+	5	55.521	s=		Δλ	-	1	11.996
φ′	26	26	36.792	1. Pancho		λ′	98	41	17.282

s	4.046 1677	s^2	8.09233			−h	2.5509
cos α	9.992 9000	$\sin^2 α$	8.50742	$(δφ)^2$	5.1017	$s^2 \sin^2 α$	6.5998
B	8.511 8072	C	1.10057	D	2.2923	E	5.8513
h	2.550 8749		7.70032		7.3940		5.0020

1st term.	″ −355.5289	3d term.	″ +0.0025			$(Δλ)^2$	
2d term.	+ 0.0050	4th term.	0.0000			F	
	−355.5239						
3d and 4th terms.	+ 0.0025	s	4.046 1677				
−Δφ	−355.5214	sin α	9.253 7118	Arg.		Δλ	1.857 3101
½(φ+φ′)	26 23 39.03	A′	8.509 4349	s	−2	sin ½(φ+φ′)	9.647 9148
		sec φ′	0.047 9957	Δλ	0	sec ½(Δφ)	
			099 1.857 3101	Corr.	−2		1.505 2249
		Δλ	″ − 71.9963			−Δα	″ − 32.006

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FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net—Continued

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Pancho	to	Garbia	10	20	27,76
∠		Gareia	&	Monument	+28	28	02,17
α	2.	Pancho	to 1.	Monument	38	48	29,93
Δα					−	2	07,02
					180		
α′	1.	Monument	to 2.	Pancho	218	46	22,91
				First Angle of Triangle	61	38	08,72

	°	′	″			°	′	″
φ	26	26	36,792	2. Pancho	λ	98	41	17,282
Δφ	−	5	20,109	s=	Δλ	+	4	45,680
φ′	26	21	16,683	1. Monument	λ′	98	46	02,962

s	4,101 7114	s²	8,20342			−h	2,5052
cos α	9,891 6751	sin² α	9,59414	(δφ)²	5,0106	s² sin² α	7,7976
B	8,511 8012	C	1,10250	D	2,2934	E	5,8529
h	2,506 1877		8,90006		7,3040		6,1557
1st term.	″ +320,0278	3d term.	″ +0,0020			(Δλ)²	
2d term.	+ 0,0794	4th term.	−0,0001			F	
	+320,1072						
3d and 4th terms.	+ 0,0019	s	4,101 7114				
−Δφ	+320,1091	sin α	9,797 0714	Arg.		Δλ	2,455 8805
½(φ+φ′)	26 23 56,74	A′	8,509 4367	s	−3	sin ½(φ+φ′)	9,647 9901
		sec φ′	0,047 6612	Δλ	+1	sec ½(Δφ)	
			2,455 8807 (5)	Corr.	−2		2,103 8706
11—600		Δλ	″ +285,6805			−Δα	+127,020

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Garcia	to	Pancho	190	19	55,75
∠		Monument	&	Pancho	−89	53	49,28
α	3.	Garcia	to 1.	Monument	100	26	06,47
Δα					−	1	34,84
					180		
α′	1.	Monument	to 3.	Garcia	280	24	31,63

	°	′	″			°	′	″
φ	26	20	41,271	3. Garcia	λ	98	42	29,278
Δφ	+		35,412	s=	Δλ	+	3	33,684
φ′	26	21	16,683	1. Monument	λ′	98	46	02,962

s	3,779 9176	s²	7,55983			−h	1,5497
cos α	9,257 9717	sin² α	9,98551	(δφ)²	3,0983	s² sin² α	7,5453
B	8,511 8072	C	1,10057	D	2,2923	E	5,8513
h	1,549 6965		8,64591		5,3906		4,9463
1st term.	″ − 35,4566	3d term.	″ 0,0000			(Δλ)²	
2d term.	+ 0,0442	4th term.	0,0000			F	
	− 35,4124						
3d and 4th terms.	0,0000	s	3,779 9176				
−Δφ	− 35,4124	sin α	9,992 7570	Arg.		Δλ	2,329 7725
½(φ+φ′)	26 20 58,98	A′	8,509 4367	s	−1	sin ½(φ+φ′)	9,647 2352
		sec φ′	0,047 6612	Δλ	+1	sec ½(Δφ)	
			2,329 7725	Corr.	0		1,977 0077
11—[illegible]		Δλ	″ + 213,6848			−Δα	″ + 94,844

FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net—Continued

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Pancho	to	Monument	38	48	29.93
∠		Monument	&	Corpus	+49	18	39.39
α	2.	Pancho	to 1.	Corpus	88	07	09.32
Δα					-	2	04.55
					180		
α′	1.	Corpus	to 2.	Pancho	268	05	04.77
				First Angle of Triangle	92	54	15.91

	°	′	″				°	′	″
φ	26	26	36.792	2. Pancho		λ	98	41	17.282
Δφ	-		08.345	s=		Δλ	+	4	39.706
φ′	26	26	28.447	1. Corpus		λ′	98	45	56.988

s	3.889 5144	s^2	7.77903			−h	0.9174
cos α	8.516 1292	sin² α	9.99953	$(\delta\varphi)^2$	1.8428	s^2 sin² α	7.7786
B	8.511 8012	C	1.10250	D	2.2924	E	5.8529
h	0.917 4448		8.88106		4.1362		4.5489

	″		″				
1st term.	+8.2688	3d term.	0.0000			$(\Delta\lambda)^2$	
2d term.	+0.0760	4th term.	0.0000			F	
	+8.3448						
3d and 4th terms.	0.0000	s	3.889 5144				
−Δφ	+8.3448	sin α	9.999 7660	Arg.		Δλ	2.446 7024
½(φ+φ′)	26 26 32.62	A′	8.509 4350	s	-1	sin ½(φ+φ′)	9.648 6505
		sec φ′	0.047 9870	Δλ	+1	sec ½(Δφ)	
			2.446 7024	Corr.	0		2.095 3529
		Δλ	+279.7064			−Δα	+124.553

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					°	′	″
α		Monument	to	Pancho	218	46	22.91
∠		Corpus	&	Pancho	− 37	47	04.89
α	3.	Monument	to 1.	Corpus	180	59	18.02
Δα					+		02.66
					180		
α′	1.	Corpus	to 3.	Monument	0	59	20.68

	°	′	″				°	′	″
φ	26	21	16.683	3. Monument		λ	98	46	02.962
Δφ	+	5	11.764	s=		Δλ	-		05.974
φ′	26	26	28.447	1. Corpus		λ′	98	45	56.988

s	3.982 0870	s^2	7.96417			−h	2.4938
cos α	9.999 9354	sin² α	6.47352	$(\delta\varphi)^2$	4.9877	s^2 sin² α	4.4377
B	8.511 8066	C	1.10076	D	2.2924	E	5.8515
h	2.493 8290		5.53845		7.2801		2.7830

	″		″				
1st term.	-311.7662	3d term.	+0.0019			$(\Delta\lambda)^2$	
2d term.	0.0000	4th term.	0.0000			F	
	-311.7662						
3d and 4th terms.	+ 0.0019	s	3.982 0870				
−Δφ	-311.7643	sin α	8.236 7617	Arg.		Δλ	0.776 2705
½(φ+φ′)	26 23 52.56	A′	8.509 4350	s	-2	sin ½(φ+φ′)	9.647 9722
		sec φ′	0.047 9870	Δλ	0	sec ½(Δφ)	
			0.776 2709 (5)	Corr.	-2		0.424 2427
		Δλ	- 5.9741			−Δα	-2.656

11—636

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FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Corpus	to	Monument	0	59	20.68
∠		Monument	&	Grande	+ 46	17	58.82
α	2.	Corpus	to 1.	Grande	47	17	19.50
Δα					−	1	35.34
					180		
α′	1.	Grande	to 2.	Corpus	227	15	44.16
				First Angle of Triangle	78	09	40.13

	°	′	″				°	′	″
φ	26	26	28.447	2.	Corpus	λ	98	45	56.988
Δφ	−	2	58.221	s=		Δλ	+	3	34.298
φ′	26	23	30.226	1.	Grande	λ′	98	49	31.286

s	3.907 6226	s²	7.81525			−h	2.2508
cos α	9.831 4244	sin² α	9.73232	(δφ)²	4.5019	s² sin² α	7.5476
B	8.511 8013	C	1.10240	D	2.2934	E	5.8529
h	2.250 8483		8.64997		6.7953		5.6513
1st term.	″ +178.1756	3d term.	″ +0.0006			(Δλ)²	
2d term.	+ 0.0447	4th term.	0.0000			F	
	+178.2203						
3d and 4th terms.	+0.0006	s	3.907 6226				
−Δφ	+178.2209	sin α	9.866 1582	Arg.		Δλ	2.331 0173
½(φ+φ′)	26 24 59.34	A′	8.509 4359	s	−1	sin ½(φ+φ′)	9.648 2565
		sec φ′	0.047 8006	Δλ	+1	sec ½(Δφ)	
			2.331 0173	Corr.	0		1.979 2728
11—[illegible]		Δλ	″ + 214.2976			−Δα	″ + 95.339

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DEPARTMENT OF COMMERCE
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FORM 26

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Monument	to	Corpus	180	59	18.02
∠		Grande	&	Corpus	− 55	32	21.19
α	3.	Monument	to 1.	Grande	125	26	56.83
Δα					−	1	32.54
					180		
α′	1.	Grande	to 3.	Monument	305	25	24.29

	°	′	″				°	′	″
φ	26	21	16.683	3.	Monument	λ	98	46	02.962
Δφ	+	2	13.543	s=		Δλ	+	3	28.324
φ′	26	23	30.226	1.	Grande	λ′	98	49	31.286

s	3.850 5409	s²	7.70108			−h	2.1258
cos α	9.763 4128	sin² α	9.82192	(δφ)²	4.2512	s² sin² α	7.5230
B	8.511 6066	C	1.10076	D	2.2924	E	5.8515
h	2.125 7603		8.62376		6.5436		5.5003
1st term.	″ −133.5858	3d term.	″ +0.0003			(Δλ)²	
2d term.	+ 0.0421	4th term.	0.0000			F	
	−133.5437						
3d and 4th terms.	+ 0.0003	s	3.850 5409				
−Δφ	−133.5434	sin α	9.910 9608	Arg.		Δλ	2.318 7382
½(φ+φ′)	26 22 23.46	A′	8.509 4359	s	−1	sin ½(φ+φ′)	9.647 5940
		sec φ′	0.047 8006	Δλ	+1	sec ½(Δφ)	
			2.318 7382	Corr.	0		1.966 3322
11—[illegible]		Δλ	″ + 208.3235			−Δα	″ + 92.541

FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

				°	′	″
α	Corpus	to	Grande	47	17	19.50
∠	Grande	&	Hebron	+ 47	30	43.44
α	2. Corpus	to 1.	Hebron	94	48	02.94
Δα				−	3	10.09
				180		
α′	1. Hebron	to 2.	Corpus	274	44	52.85

First Angle of Triangle 42 56 20.48

	°	′	″			°	′	″
φ	26	26	28.447	2. Corpus	λ	98	45	56.988
Δφ	+		32.094	s=	Δλ	+	7	06.826
φ′	26	27	00.541	1. Hebron	λ′	98	53	03.814
			−1					+1

s	4.074 3221	s²	8.14864			−h	1.5088
cos α	8.922 6642	sin² α	9.99695	(δφ)²	3.0128	s² sin² α	8.1456
B	8.511 8013	C	1.10240	D	2.2934	E	5.8529
h	1.508 8076		9.24799		5.3062		5.5073
1st term.	−32.2706″	3d term.	0.0000″			(Δλ)²	
2d term.	+ 0.1770	4th term.	0.0000			F	
	−32.0936						
3d and 4th terms.	0.0000	s	4.074 3221				
−Δφ	−32.0936	sin α	9.998 4737	Arg.		Δλ	2.630 2512
½(φ+φ′)	26 26 44.49	A′	8.509 4348	s	−3	sin ½(φ+φ′)	9.648 7008
		sec φ′	0.048 0206	Δλ	+3	sec ½(Δφ)	
			2.630 2512	Corr.	0		2.278 9520
		Δλ	+ 426.8263″			−Δα	+ 190.087″

11—436

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FORM 26

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

				°	′	″
α	Grande	to	Corpus	227	15	44.16
∠	Hebron	&	Corpus	− 89	32	56.26
α	3. Grande	to 1.	Hebron	137	42	47.90
Δα				−	1	34.57
				180		
α′	1. Hebron	to 3.	Grande	317	41	13.33

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	°	′	″			°	′	″
φ	26	23	30.226	3. Grande	λ	98	49	31.286
Δφ	+	3	30.314	s=	Δλ	+	3	32.529
φ′	26	27	00.540	1. Hebron	λ′	98	53	03.815

s	3.942 0502	s²	7.88410			−h	2.3230
cos α	9.869 1069	sin² α	9.65582	(δφ)²	4.6458	s² sin² α	7.5399
B	8.511 8043	C	1.10146	D	2.2928	E	5.8521
h	2.322 9614		8.64138		6.9386		5.7150
1st term.	−210.3591″	3d term.	+0.0009″			(Δλ)²	
2d term.	+ 0.0438	4th term.	+0.0001			F	
	−210.3153						
3d and 4th terms.	+ 0.0010	s	3.942 0502				
−Δφ	−210.3143	sin α	9.827 9122	Arg.		Δλ	2.327 4178
½(φ+φ′)	26 25 15.38	A′	8.509 4348	s	−1	sin ½(φ+φ′)	9.648 3234
		sec φ′	0.048 0206	Δλ	+1	sec ½(Δφ)	
			2.327 4178	Corr.	0		1.975 7412
		Δλ	+ 212.5288″			−Δα	+ 94.567″

11—436

FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

				°	′	″
α	Hebron	to	Grande	317	41	13.35
∠	Grande	&	Ringold	+47	22	41.60
α	3. Hebron	to 1.	Ringold	5	03	54.93
Δα				-		11.81
				180		
α′	1. Ringold	to 2.	Hebron	185	03	43.12
			First Angle of Triangle	69	29	07.23

	°	′	″			°	′	″
φ	26	27	00.540	2. Hebron	λ	98	53	03.815
Δφ	-	4	29.783	s=	Δλ	+		26.546
φ′	26	22	30.757	1. Ringold	λ′	98	53	30.361

s	3,920 9105	s²	7.84182			−h	2.4310
cos α	9,998 3007	sin² α	7.89183	(δφ)²	4.8620	s² sin² α	5.7336
B	8.511 8008	C	1.10257	D	2.2935	E	5.8530
h	2.431 0120		6.83622		7.1555		4.0176
1st term.	″ +269.7814	3d term.	″ +0.0014			(Δλ)²	
2d term.	+ 0.0007	4th term.	0.0000			F	
	+269.7821						
3d and 4th terms.	+ 0.0014	s	3,920 9105				
−Δφ	+269.7835	sin α	8.945 9131	Arg.		Δλ	1.423 9983
½(φ+φ′)	26 24 45.65	A′	8.509 4363	s	-1	sin ½(φ+φ′)	9.648 1973
		sec φ′	0.047 7585	Δλ	0	sec ½(Δφ)	
			1.423 9984 (3)	Corr.	-1		1.072 1956
11—615		Δλ	+ 26.5459			−Δα	+ 11.809

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

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				°	′	″
α	Grande	to	Hebron	137	42	47.90
∠	Ringold	&	Hebron	− 63	08	11.31
α	3. Grande	to 1.	Ringold	74	34	36.59
Δα				-	1	46.24
				180		
α′	1. Ringold	to 3.	Grande	254	32	50.35

	°	′	″			°	′	″
φ	26	23	30.226	3. Grande	λ	98	49	31.286
Δφ	-		59.469	s=	Δλ	+	3	59.075
φ′	26	22	30.757	1. Ringold	λ′	98	53	30.361

s	3.837 2872	s²	7.67457			−h	1.7739
cos α	9.424 7935	sin² α	9.96814	(δφ)²	3.5486	s² sin² α	7.6427
B	8.511 8043	C	1.10146	D	2.2928	E	5.8521
h	1.773 8850		8.74417		5.8414		5.2687
1st term.	″ +59.4136	3d term.	″ +0.0001			(Δλ)²	
2d term.	+ 0.0555	4th term.	0.0000			F	
	+59.4690						
3d and 4th terms.	+0.0001	s	3.837 2872				
−Δφ	+59.4691	sin α	9.984 0716	Arg.		Δλ	2.378 5336
½(φ+φ′)	26 23 00.49	A′	8.509 4363	s	-1	sin ½(φ+φ′)	9.647 7515
		sec φ′	0.047 7385	Δλ	+1	sec ½(Δφ)	
			2.378 5336	Corr.	0		2.026 2851
11—436		Δλ	+ 239.0747			−Δα	+106.239

FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 26

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Hebron	to	Ringold	5	03	54.93
∠		Ringold	&	Gareena	+83	16	45.56
α	2.	Hebron	to 1.	Gareena	88	20	40.49
Δα					−	1	11.31
					180		
α′	1	Gareena	to 2.	Hebron	268	19	29.18
				First Angle of Triangle	67	18	20.19

	°	′	″			°	′	″
φ	26	27	00.540	2. Hebron	λ	98	53	03.815
Δφ	−		04.190	s=	Δλ	+	2	40.095
φ′	26	26	56.350 56.345*	1. Gareena	λ′	98	55	43.910 43.916*

s	3.647 1076	s²	7.29422			−h	0″.6196
cos α	8.460 7250	sin² α	9.99964	(δφ)²	1.2444	s² sin² α	7.2939
B	8.511 8008	C	1.10257	D	2.2936	E	5.8530
h	0.619 6334		8.39643		3.5379		3.7665
1st term.	+4″.1652	3d term.	0″.0000			(Δλ)²	
2d term.	+ 0.0249	4th term.	0.0000			F	
	+4.1901						
3d and 4th terms.	0.0000	s	3.647 1076				
−Δφ	+4.1901	sin α	9.999 8187	Arg.		Δλ	2.204 3774
½(φ+φ′)	26 26 58.44	A′	8.509 4348	s	0	sin ½(φ+φ′)	9.648 7599
		sec φ′	0.048 0163	Δλ	0	sec ½(Δφ)	
			2.204 3774	Corr.	0		1.853 1373
11—...		Δλ	+ 160.0949			−Δα	+ 71.308

* Fixed position.

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Ringold	to	Hebron	185	03	43.12
∠		Gareena	&	Hebron	− 29	24	54.34
α	3.	Ringold	to 1.	Gareena	155	38	48.78
Δα					−	−	59.41
					180		
α′	1.	Gareena	to 3.	Ringold	335	37	49.37

	°	′	″			°	′	″
φ	26	22	30.757	3. Ringold	λ	98	53	30.361
Δφ	+	4	25.593	s=	Δλ	+	2	13.549
φ′	26	26	56.350	1. Gareena	λ′	98	55	43.910

s	3.952 9138	s²	7.90583			−h	2.4242
cos α	9.959 5286	sin² α	9.23055	(δφ)²	4.8484	s² sin² α	7.1364
B	8.511 8053	C	1.10115	D	2.2926	E	5.8519
h	2.424 2477		8.23753		7.1410		5.4125
1st term.	−265″.6120	3d term.	+0″.0014			(Δλ)²	
2d term.	+ 0.0173	4th term.	0.0000			F	
	−265.5947						
3d and 4th terms.	+ 0.0014	s	3.952 9138				
−Δφ	−265.5933	sin α	9.615 2756	Arg.		Δλ	2.125 6403
½(φ+φ′)	26 24 43.55	A′	8.509 4348	s	−2	sin ½(φ+φ′)	9.648 1886
		sec φ′	0.048 0163	Δλ	0	sec ½(Δφ)	
			2.125 6405 (3)	Corr.	−2		1.773 8288
11—...		Δλ	+ 133.5489			−Δα	+ 59.406

FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net—Continued

tude and longitude equations. In columns A and B are placed the designations of the length angles, that of the adjacent angle in A and that of the opposite angle in B, and in column C is placed the designation of the azimuth angle. It should be noted particularly that the designations in column C are of the concluded angles. (See p. 67.) The symbols in columns A and B may be taken directly from the column headed symbol in the length equation computation. The signs of the symbols in column C depend upon whether the azimuth angle is turned to the right or to the left in computing through the triangles. If it is turned to the right, then the symbols in column C are the combined symbols of columns A and B but with opposite signs. If the azimuth angle is turned to the left the symbols in column C are the same as in columns A and B combined and with the same signs.

As a guide in obtaining the correct signs for the symbols in column C, it is well to indicate to the left of each name in the "station" column the direction in which the azimuth angle is turned, denoting the right or clockwise turn by +, and the left or anti-clockwise turn by −. The directions of the turns are easily obtained from Figure 25.

The quantities a_1 and a_2 are obtained from the table on the right-hand side of form 496, using as argument the computed latitude, ϕ_n, of the fixed point at the terminal of the arc, which in this case is 26°.45.

The quantities in the three columns headed "Lat. equation" and in the three columns headed "Long. equation," in Figure 28 are, as indicated in the headings, simply products of quantities in other columns previously filled out. The three columns headed Lat. equation $(\phi_n-\phi_c)\ \delta_A$, Lat. equation $(\phi_n-\phi_c)\ (-\delta_B)$ and Long. equation $(\phi_n-\phi_c)\ a_2$, respectively, should be filled out at the same time, since each of the three products contains the multiplier $(\phi_n-\phi_c)$, which can be set up on a multiplying machine and used without change for the three multiplications. In a similar manner the three columns headed Long. equation $(\lambda_n-\lambda_c)\ \delta_A$, Long. equation $(\lambda_n-\lambda_c)\ (-\delta_B)$ and Lat. equation $(\lambda_n-\lambda_c)\ a_1$, respectively, should be filled out at the same time, since they contain the multiplier $(\lambda_n-\lambda_c)$.

This completes the table on form 496 and the latitude and longitude equations themselves can now be formed.

In forming the latitude equation from the table it should be noted that the quantities in the column headed Lat. equation $(\phi_n-\phi_c)\ \delta_A$ are the coefficients of the symbols in column A, the quantities in the column headed Lat. equation $(\phi_n-\phi_c)\ (-\delta_B)$ are the coefficients of the symbols in column B, and the quantities in the column headed Lat. equation $(\lambda_n-\lambda_c)\ a_1$ are the coefficients of the symbols in the column headed C.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 496

LATITUDE AND LONGITUDE ADJUSTMENT

φ_0: 26° 26′.94 λ_0: 98° 55′.73

Station	φ_s	λ_s	$\varphi_n-\varphi_s$	$\lambda_n-\lambda_s$	$+\delta_A$	$-\delta_B$	Lat. equation $(\varphi_n-\varphi_s)\delta_A$	A	Long. equation $(\lambda_n-\lambda_s)\delta_A$	Lat. equation $(\varphi_n-\varphi_s)(-\delta_B)$	B	Long. equation $(\lambda_n-\lambda_s)(-\delta_B)$	Lat. equation $(\lambda_n-\lambda_s)a_1$	C (+B or −L)	Long. equation $(\varphi_n-\varphi_s)a_2$
◆ Palo	26° 19.65	98° 27.80	+7.29	+27.93	+0.74	−1.90	+5.39	−1 + 3	+20.67	−13.85	−15 + 16	−53.07	+52.79	+1 − 3 +15 − 16	−17.06
▬ Fordyce	26 17.79	98 34.75	+9.15	+20.98	+2.67	−0.45	+24.43	−5 + 6	+56.02	− 4.12	− 7 + 9	− 9.44	+39.65	−5 + 6 −7 + 9	−21.41
◆ Eltoro	26 21.87	98 34.01	+5.07	+21.72	+0.49	−3.46	+2.48	−12 + 14	+10.64	−17.54	−20 + 21	−75.15	+41.05	+12 − 14 +20 − 21	−11.86
▬ Garcia	26 20.69	98 42.49	+6.25	+13.24	+2.13	−1.00	+13.31	−10 + 11	+28.20	− 6.25	−22 + 24	−13.24	+25.02	−10 + 11 −22 + 24	−14.62
◆ Pancho	26 26.61	98 41.29	+0.33	+14.44	0.00	−1.13	0	−17 + 19	0	− 0.37	−31 + 32	−16.32	+27.29	+17 − 19 +31 − 32	− 0.77
▬ Monument	26 21.28	98 46.05	+5.66	+9.68	+1.81	+0.11	+10.24	−25 + 26	+17.52	+ 0.62	−33 + 35	+ 1.06	+18.30	−25 + 26 −33 + 35	−13.24
◆ Corpus	26 26.47	98 45.95	+0.47	+9.78	+1.45	−0.45	+0.68	−28 + 30	+14.18	− 0.21	−40 + 41	− 4.40	+18.48	+28 − 30 +40 − 41	−1.10
▬ Grande	26 23.50	98 49.52	+3.44	+6.21	+1.93	−2.27	+6.64	−36 + 37	+11.99	− 7.81	−42 + 44	−14.10	+11.74	−36 + 37 −42 + 44	−8.05
◆ Hebron	26 27.01	98 53.06	−0.07	+2.67	+1.06	−0.78	−0.07	−38 + 39	+2.83	+ 0.05	−50 + 51	− 2.08	+ 5.05	+38 − 39 +50 − 51	+0.16
▬ Ringold	26 22.51	98 53.51	+4.43	+2.22	+0.25	−0.88	+1.11	−45 + 47	+0.56	− 3.90	−53 + 54	− 1.95	+ 4.20	−45 + 47 −53 + 54	−10.37

Garcena

ϕ_n = 26° 26′ 56″.350 λ_n = 98° 55′ 43″.910

ϕ_n' = 26 26 56.345 λ_n' = 98 55 43.916

$(\phi_n-\phi_n')$ = +0.005 $(\lambda_n-\lambda_n')$ = −0.006

x7238.24   x7238.24

+36.19120   −43.42944

a_1 = +1.89

a_2 = −2.34

26. φ. 0 = +36.19120 + 47.40 (1) − 47.40 (3) − 64.08 (5) + 64.08 (6) − 35.53 (7) + 35.53 (9) − 38.33 (10) + 38.33 (11) + 38.57 (12)
− 38.57 (14) + 66.64 (15) − 66.64 (16) + 27.29 (17) − 27.29 (19) + 58.59 (20) − 58.59 (21) − 18.77 (22) + 18.77 (24) − 28.54 (25)
+ 28.54 (26) + 17.80 (28) − 17.80 (30) + 27.66 (31) − 27.66 (32) − 18.92 (33) + 18.92 (35) − 18.38 (36) + 18.38 (37) + 5.12 (38)
− 5.12 (39) + 18.69 (40) − 18.69 (41) − 3.93 (42) + 3.93 (44) − 5.31 (45) + 5.31 (47) + 5.00 (50) − 5.00 (51) − 0.30 (53) + 0.30 (54).
(This equation should be divided by 10 before entering it in the correlates.)

27. λ. 0 = −43.42944 − 37.73 (1) + 37.73 (3) − 34.61 (5) + 34.61 (6) + 30.85 (7) − 30.85 (9) − 13.58 (10) + 13.58 (11) − 22.50 (12) + 22.50 (14)
+36.01 (15) − 36.01 (16) − 0.77 (17) + 0.77 (19) + 63.29 (20) − 63.29 (21) + 27.86 (22) − 27.86 (24) − 4.28 (25) + 4.28 (26) − 15.28 (28)
+15.28 (30) + 15.55 (31) − 15.55 (32) + 12.18 (33) − 12.18 (35) − 3.94 (36) + 3.94 (37) − 2.67 (38) + 2.67 (39) + 3.30 (40) − 3.30 (41)
+22.15 (42) − 22.15 (44) + 9.81 (45) − 9.81 (47) + 2.24 (50) − 2.24 (51) + 12.32 (53) − 12.32 (54).
(This equation should be divided by 10 before entering it in the correlates.)

φ	a_1 (+)	a_2 (−)
0	2.12	2.08
1	2.12	2.08
2	2.12	2.09
3	2.12	2.09
4	2.11	2.10
5	2.11	2.10
6	2.11	2.10
7	2.10	2.11
8	2.10	2.11
9	2.09	2.12
10	2.09	2.12
11	2.08	2.13
12	2.07	2.14
13	2.06	2.15
14	2.06	2.16
15	2.05	2.17
16	2.04	2.18
17	2.03	2.19
18	2.02	2.20
19	2.00	2.21
20–73	[illegible]	[illegible]

$\frac{M}{\rho''}\times10^4$ = 7238.24

FIG. 28.—Formation of latitude and longitude equations of net

Correlate equations

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	α 24	l 25	φ 26	λ 27	Σc*	v*	Adopted v	v²
1	-1																	+4.20							-0.74	+4.74	-3.77	+3.43	+0.287	+0.29	0.0824
2		-1																-6.39										-7.39	-.201	-.20	.0404
3	+1	+1																+2.19							+.74	-4.74	+3.77	+3.96	-.086	-.08	.0074
4	-1	-1																+1.15						-1				-1.85	-.054	-.05	.0029
5	+1		-1															-3.82							-2.67	-6.41	-3.46	-16.36	+.176	+.18	.0310
6		+1	+1															+2.67						+1	+2.67	+6.41	+3.46	+18.21	-.121	-.12	.0146
7		-1	-1															+2.85						-1	+.45	-3.55	+3.08	-.17	+.430	+.43	.1849
8		+1																-5.25										-4.25	+.041	+.04	.0017
9			+1	-1														+2.40							-.45	+3.55	-3.08	+2.42	-.302	-.30	.0012
10				+1	-1																				-2.13	-3.83	-1.36	-7.32	+.058	+.06	.0034
11					+1																			+1	+2.13	+3.83	+1.36	+9.32	-.227	-.23	.0515
12				-1		-1													+0.49						-.49	+3.86	-2.25	-.39	-.008	-.01	.0001
13						+1													-2.25									-1.25	+.415	+.41	.1722
14			-1	+1														+1.02	+1.76						+.49	-3.86	+2.25	+1.66	+.341	+.34	.1163
15	-1		+1															-2.92							+1.90	+6.66	+3.60	+9.24	-.370	-.37	.1369
16	+1																	+1.90							-1.90	-6.66	-3.60	-9.26	-.378	-.38	.1429
17							-1																			+2.73	-.08	+1.65	+.402	+.46	.2134
18								-1																				-1.00	+.010	+.01	.0001
19					-1	-1	+1	+1											+0.73							-2.73	+.08	-1.92	-.158	-.16	.0250
20				-1	+1														-4.19						+3.46	+5.86	+6.33	+11.46	+.068	+.07	.0046
21				+1		+1													+3.46						-3.46	-5.86	-6.33	-10.19	-.381	-.38	.1452
22					-1														+4.66					-1	+1.00	-1.88	+2.79	+4.57	+.430	+.43	.1849
23						-1													-5.66									-6.66	-.271	-.27	.0734
24					+1	+1	-1	-1											+1.00	+3.44					-1.00	+1.88	-2.79	+2.53	+.300	+.30	.0900
25							+1		-1											-3.89					-1.81	-2.85	-.43	-8.98	-.054	-.05	.0020
26								+1	+1											+.45				+1	+1.81	+2.85	+.43	+8.54	-.404	-.40	.1632
27											-1										+0.90	+4.52						+4.42	-.358	-.36	.1282
28										-1	+1										-4.44	-5.97			-1.45	+1.78	-1.53	-11.61	+.615	+.62	.3782
29												-1									+3.54							+2.54	+.421	+.42	.1772
30									-1	+1		+1								-.34		+1.45			+1.45	-1.78	+1.53	+3.31	+.190	+.19	.0361
31							-1		+1											-1.13					+1.13	+2.77	+1.56	+4.33	-.396	-.39	.1568
32							+1													+1.47					-1.13	-2.77	-1.56	-2.99	-.472	-.47	.2228
33								-1	-1											+1.05				-1	-.11	-1.89	+1.22	-2.73	+.676	+.68	.4570
34								+1												-4.79								-3.79	-.342	-.34	.1170
35									+1	-1		-1								+3.74		+2.01			+.11	+1.89	-1.22	+5.53	-.077	-.08	.0059
36										+1			-1									-3.94			-1.93	-1.84	-.30	-8.10	+.219	+.22	.0480
37												+1	+1									+1.93		+1	+1.93	+1.84	+.39	+9.09	-.476	-.48	.2266
38											+1			-1											-1.06	+.51	-.27	-.82	+.096	+.09	.0092
39													-1	+1											+1.06	-.51	+.27	+.82	+.184	+.18	.0339
40										-1			+1												+.45	+1.87	+.33	+2.65	-.049	-.05	.0024
41										+1	-1														-.45	-1.87	-.33	-2.65	-.231	-.23	.0534
42												-1	-1									+2.27		-1	+2.27	-.39	+2.22	+3.37	+.554	+.55	.3069
43												+1									+4.37							+5.37	-.516	-.52	.2663
44													+1	-1							-4.76	-4.20			-2.27	+.39	-2.22	-13.06	+.133	+.13	.0177
45														+1	-1		-1				+.39	+1.93	-0.05		-.25	-.53	+.98	+1.47	-.310	-.31	.0961
46																	+1						+2.03					+3.03	+.208	+.30	.0888
47															+1								-1.98	+1	+.25	+.53	-.98	-.18	-.159	-.16	.0253
48																-1	-1						-.48		-2.39			-4.87	+.028	+.03	.0008
49															-1	+1							+1.23		+2.39			+3.62	+.219	+.22	.0480
50														-1	+1		+1				+.15	+.78	-.75		+.78	+.50	+.22	+2.68	+.056	+.06	.0031
51											-1			+1							-1.02	-5.12			-.78	-.50	-.22	-7.64	+.154	+.16	.0237
52											+1										+.87	+4.34						+6.21	-.456	-.45	.2079
53															-1									-1	+.88	-.03	+1.23	+.08	+.161	+.16	.0259
54															+1	-1									-.88	+.03	-1.23	-2.08	+.067	+.06	.0045
55																+1								+1				+2.00	-.227	-.23	.0515
56																-1							-3.54		+2.20			-2.34	+.571	+.57	.3260
57																	-1						+3.98					+2.98	+.086	+.09	.0074
58																+1	+1						-.44		-2.20			-.64	-.657	-.66	.4316

* In the Σ_c column the sum of the values for all directions around each point (except the points at which some of the directions are not numbered, due to having been fixed by a previous adjustment) is equal to zero. For example, $(1)+(2)+(3)=+3.43-7.39+3.96=0$. The same thing applies to the v column, in which the corresponding values are $+0.287-0.201-0.086=0$.

In the same way in forming the longitude equation the quantities in the column headed Long. equation $(\lambda_n - \lambda_c)\ \delta_A$ are the coefficients of the symbols in column A, the quantities in the column headed Long. equation $(\lambda_n - \lambda_c)\ (-\delta_B)$ are the coefficients of the symbols in column B, and the quantities in the column headed Long. equation $(\phi_n - \phi_c)\ a_2$ are the coefficients of the symbols in column C.

The absolute terms for the latitude and longitude equations are obtained by taking the products 7238.24 $(\phi_n - \phi_n')$ and 7238.24 $(\lambda_n - \lambda_n')$, respectively, as shown on page 69, ϕ_n and λ_n denoting the computed latitude and longitude, and ϕ_n' and λ_n' the fixed latitude and longitude of the terminal point.

If there is room on form 496 the latitude and longitude equations should be written on it directly below the table.

FORMATION OF CORRELATE AND NORMAL EQUATIONS

After all the equations have been formed they are tabulated as shown on pages 82–83. In order to have the coefficients in the different equations of about the same size, it is usually well to divide some of the equations by a constant factor before entering them in the correlates. For example, equation 21 on page 61 is divided by 5, and equations 26 and 27 on page 81 are divided by 10. When dividing a correlate equation by a constant factor, one should be careful to divide the constant term of that equation by the same factor before using it in the normal equation.

The table of correlate equations should be checked by another mathematician, especially if the set is large, before it is used for forming the normal equations. The normal equations are formed in the same manner as for the station adjustment on pages 11–13. As stated there the equations are checked automatically by the values in the Σ_n column. This does not apply, however, to the constant terms in the "η" column which should be checked by another mathematician. It should be remembered that the constant term of a divided equation (see p. 61) must be divided by the same factor before entering it in the table of normal equations. The table of normals is given on the folded page facing this page.

SOLUTION OF NORMAL EQUATIONS

The solution of the normal equations is shortened considerably if the equations are taken in the proper order. In a net of triangulation composed of simple quadrilaterals, the rule is to eliminate the three angle equations and the one side equation of each quadrilateral in order. If the net is more involved, the order of solution should be such that each succeeding equation will introduce the fewest new terms.

Normal equations

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	α 24	ζ 25	φ 26	λ 27	η	Σ_n	C
1	+6	+2	−2															−2.16						+1	−4.99	−29.21	−3.12	+0.15	−32.33	−0.33919
2		+6	+2															+2.00						+3	+2.96	+5.22	+4.15	+.34	+27.67	+.11311
3			+6	−2														+2.10	−1.76					+2	+5.85	+30.44	+2.11	+1.74	+46.48	−.73456
4				+6	−2	+2												−1.38	+8.92						−7.62	−26.82	−6.44	−.26	−29.60	−.46475
5					+6	+2	−2	−2											−8.58	+3.44				+2	+5.72	+20.01	+3.39	+.19	+28.17	−.70103
6						+6	−2	−2											+6.65	+3.44					−3.97	−5.11	−6.95	−.77	−.71	+.37423
7							+6	+2	−2										−.27	−4.73					−3.07	−15.73	−.60	+1.05	−21.35	−.50643
8								+6	+2										−.27	−8.83				+2	+2.92	+.13	+2.08	+1.89	+5.92	−.00951
9									+6	−2		−2								+6.24		+0.56		+2	+3.52	+14.03	−1.55	+1.69	+28.49	−.89215
10										+6	−2	+2	−2							−4.08	+4.44	+1.47			−.04	−11.03	+3.23	+.31	−3.70	−.26948
11											+6			−2							−3.45	−1.03			−1.28	+4.66	−1.25	−.69	−1.04	−.05744
12												+6	+2							−4.08	+0.83	−.90		+2	+1.00	−1.44	+.92	+1.70	+8.03	−.46382
13													+6	−2							−4.76	−.60		+2	−1.29	+6.84	−3.60	+1.35	+3.94	−.33785
14														+6	−2		−2				+3.98	+.23	+0.70		+2.58	−2.94	+3.30	+.25	+6.10	−.28950
15															+6	−2	+2				−.24	−1.15	−3.91	+2	−2.87	+1.62	−4.20	+.11	−4.64	−.36726
16																+6	+2						+4.81	+1	+1.26	−.03	+1.23	+1.33	+15.60	−.52368
17																	+6				−.24	−1.15	−2.61		+1.22	+1.03	−.76	+.11	+5.60	+.16843
18																		+140.9338	+1.7952					−1.33	+7.3852	+13.4924	+1.2067	−2.24	+161.8033	+.01377
19																			+93.2120	+3.4400				−4.66	−22.1867	−53.6049	−35.2972	−1.78	−19.3916	−.01822
20																				+68.7558		+7.0244		−.60	+1.2803	+17.3235	−15.5894	−2.81	+70.2246	+.03951
21																					+76.7816	+60.4347	−.1320		+18.0583	−9.3813	+18.0000	+5.116	+169.4373	−.01198
22																						+153.6336	−.6815	−.34	+41.1156	+.7959	+28.7425	+9.58	+297.7402	−.08962
23																							+38.9152	−1.98	−3.8006	−1.3979	+1.7264	+.238	+31.8776	+.06390
24																								+12	+4.30	+23.20	−5.88	+3.81	+47.52	+.29643
25																									+116.2130	+117.4082	+121.6126	−2.27	+401.0159	+.12416
26																										+451.5432	+91.8144	+3.6191	+641.4826	−.01684
27																											+235.2684	−4.3429	+429.2015	−.01628

The solution of the normal equations is given in full on pages 86–92. The method used is the same as for the adjustment of the quadrilateral on page 38, and it is therefore unnecessary to describe each step of this solution.

The check furnished by the Σ column in the forward solution make any errors comparatively easy to find. However, there is no corresponding check on the back solution, and an error is not likely to be detected until the v's are substituted in the triangles. As considerable recomputation is then required to correct for the error, it is well worth while, especially in the case of a large set of equations, to be particularly careful with the back solution. The back solution, or computation of the C's, is shown on page 93.

1	2	3	18	4	5	6	19	α 24	l 25	φ 26	λ 27	η	Σ_n
+6	+2	−2	−2.16					+1	−4.99	−29.21	−3.12	+.15	−32.33
C_1	−.33333	+.33333	+.36					−.16667	+.83167	+4.86833	+.52	−.025	+5.38833
	+6	+2	+2.00					+3	+2.96	+5.22	+4.15	+.34	+27.67
1	−.6667	+.6667	+.72					−.3333	+1.6633	+9.7367	+1.04	−.05	+10.7767
	+5.3333	+2.6667	+2.72					+2.6667	+4.6233	+14.9567	+5.19	+.29	+38.4467
	C_2	−.50001	−.51000					−.50001	−.86687	−2.80440	−.97313	−.05438	−7.20880
		+6	+2.10	−2			−1.76	+2	+5.85	+30.44	+2.11	+1.74	+46.48
	1	−.6667	−.72					+.3333	−1.6633	−9.7367	−1.04	+.05	−10.7767
	2	−1.3333	−1.36					−1.3333	−2.3117	−7.4783	−2.595	−.145	−19.2233
		+4.0000	+.02	−2			−1.76	+1.0000	+1.8750	+13.2250	−1.5250	+1.645	+16.4800
		C_3	−.005	+.5			+.44	−.25	−.46875	−3.30625	+.38125	−.41125	−4.12000
			+140.9338	−1.38			+1.7952	−1.33	+7.3852	+13.4924	+1.2067	−2.24	+161.8033
		1	−.7776					+.3600	−1.7964	−10.5156	−1.1232	+.054	−11.6388
		2	−1.3872					−1.3600	−2.3579	−7.6280	−2.6469	−.1479	−19.6079
		3	−.0001	+.01			+.0088	−.0050	−.0094	−.0661	+.0076	−.0082	−.0824
			+138.7689	−1.37			+1.8040	−2.3350	+3.2215	−4.7173	−2.5558	−2.3421	+130.4742
			C_{18}	+.00987			−.01300	+.01683	−.02321	+.03399	+.01842	+.01688	−.940232
				+6	−2	+2	+8.92		−7.62	−26.82	−6.44	−.26	−29.60
			3	−1			−.88	+.50	+.9375	+6.6125	−.7625	+.8225	+8.24
			18	−.0135			+.0178	−.0231	+.0318	−.0466	−.0252	−.0231	+1.2881
				+4.9865	−2	+2	+8.0578	+.4769	−6.6507	−20.2541	−7.2277	+.5394	−20.0719
				C_4	+.40108	−.40108	−1.61592	−.09564	+1.33374	+4.06179	+1.44945	−.10817	+4.02525

	5	6	19	7	8	9	20	α 24	l 25	φ 26	λ 27	η	Σ_n
	+6	+2	−8.58	−2	−2		+3.44	+2	+5.72	+20.01	+3.39	+.19	+28.17
4	−.8022	+.8022	+3.2318					+.1913	−2.6675	−8.1236	−2.8989	+.2163	−8.0505
	+5.1978	+2.8022	−5.3482	−2	−2		+3.44	+2.1913	+3.0525	+11.8864	+.4911	+.4063	+20.11954
	C_5	−.53911	+1.02894	+.38478	+.38478		−.06182	−.42158	−.58727	−2.28681	−.09448	−.07817	−3.870754
		+6	+6.65	−2	−2		+3.44		−3.97	−5.11	−6.95	−.77	−.71
	4	−.8022	−3.2318					−.1913	+2.6675	+8.1236	+2.8989	−.2163	+8.0505
	5	−1.5107	+2.8833	+1.0782	+1.0782		−1.8546	−1.1814	−1.6456	−6.4081	−.2648	−.2190	−10.8466
		+3.6871	+6.3015	−.9218	−.9218		+1.5854	−1.3727	−2.9481	−3.3945	−4.3159	−1.2053	−3.5061
		C_6	−1.70907	+.25001	+.25001		−.42999	+.37230	+.79957	+.92004	+1.17054	+.32690	+.95091
			+93.2120	−.27	−.27		+3.4400	−4.66	−22.1867	−58.6049	−35.2972	−1.78	+19.3916
		3	−.7744					+.44	+.8250	+5.8190	−.6710	+.7238	+7.2512
		18	−.0234					+.0303	−.0419	+.0613	+.0332	+.0305	−1.6962
		4	−13.0208					−.7706	+10.7470	+32.7291	+11.0794	−.8716	+32.4347
		5	−5.5030	−2.0579	−2.0579		+3.5396	+2.2547	+3.1408	+12.2303	+.5053	+.4181	+20.7015
		6	−10.7697	+1.5754	+1.5754		−2.7096	+2.3460	+5.0385	+5.8014	+7.3762	+2.0000	+5.9922
			+63.1207	−.7525	−.7525		+4.2700	−.3596	−2.4773	−1.9638	−16.3741	+.5808	+45.20187
			C_{19}	+.01192	+.01192		−.06765	+.00570	+.03925	+.03111	+.25941	−.00919	−.717543
				+6	+2	−2	−4.73		−3.07	−15.73	−.60	+1.05	−21.35
			5	−.7696	−.7696		+1.3236	+.8432	+1.1745	+4.5736	+.1890	+.1563	+7.7415
			6	−.2305	−.2305		+.3964	−.3432	−.7370	−.8486	−1.0790	−.3013	−.8765
			19	−.0090	−.0090		+.0509	−.0043	−.0295	−.0234	−.1952	+.0069	+.5309
				+4.9909	+.9909	−2	−2.9591	+.4957	−2.6620	−12.0284	−1.6852	+.9119	−13.94573
				C_7	−.19854	+.40073	+.59290	−.00932	+.53337	+2.41007	+.33765	−.18271	+2.79415
					+6	+2	−8.83	+2	+2.92	+.13	+2.08	+1.89	+5.92
				5	−.7696		+1.3236	+.8432	+1.1745	+4.5736	+.1890	+.1563	+7.7415
				6	−.2305		+.3964	−.3432	−.7370	−.8486	−1.0790	−.3013	−.8765
				19	−.0090		+.0509	−.0043	−.0295	−.0234	−.1952	+.0069	+.5309
				7	−.1967	+.3971	+.5875	−.0984	+.5285	+2.3881	+.3346	−.1810	+2.7087
					+4.7942	+2.3971	−6.4716	+2.3973	+3.8565	+6.2197	+1.3294	+1.5709	+16.00365
					C_8	−.50000	+1.34988	−.50004	−.80441	−1.29734	−.27729	−.32767	−3.35687

Solution of normal equations—Continued

	9	20	10	12	11	13	22	21	α 24	l 25	φ 26	λ 27	η	Σn
	+6	+6.24	−2	−2			+.56		+2	+3.52	+14.03	−1.55	+1.69	+28.49
7	−.8015	−1.1858							+.1986	−1.0667	−4.8201	−.6753	+.3654	−5.5883
8	−1.1985	+3.2358							−1.1986	−1.9283	−3.1099	−.6647	−.7854	−8.0467
	+4	+8.29	−2	−2			+.56		+1	+.5250	+6.1000	−2.89	+1.27	+14.855
	C_9	−2.0725	+.5	+.5			−.14		−.25	−.13125	−1.525	+.7225	−.3175	−3.71375
		+68.7558	−4.08	−4.08			+7.0244		−.60	+1.2803	+17.3235	−15.5894	−2.81	+70.2246
5		−2.2767							−1.4502	−2.0202	−7.8666	−.3250	−.2689	−13.3153
6		−.6817							+.5902	+1.2676	+1.4596	+1.8558	+.5183	+1.5076
19		−.2889							+.0243	+.1676	+.1328	+1.1077	−.0392	−3.0639
7		−1.7545							+.2939	−1.5783	−7.1316	−.9991	+.5407	−8.2682
8		−8.7359							+3.2361	+5,2058	+8.3959	+1.7945	+2.1205	+21.7243
9		−17.1810	+4.1450	+4.1450			−1.1606		−2.0725	−1.0881	−12.6422	+5.9895	−2.6321	−30.7870
		+37.8371	+.0650	+.0650			+5.8638		+.0218	+3.2347	−.3286	−6.1660	−2.5707	+38.0221
		C_{20}	−.00172	−.00172			−.15497		−.00058	−.08549	+.00868	+.16296	+.06794	−1.0048990
			+6	+2	−2	−2	+1.47	+4.44		−.04	−11.03	+3.23	+.31	−3.70
9			−1	−1			+.28		+.5	+.2625	+3.05	−1.445	+.635	+7.4275
20			−.0001	−.0001			−.0101		.0000	−.0056	+.0006	+.0106	+.0044	−.0653
			+4.9999	+.9999	−2	−2	+1.7399	+4.44	+.5	+.2169	−7.9794	+1.7956	+.9494	+3.0622
			C_{10}	−.19998	+.40001	+.40001	−.34799	−.88802	−.10000	−.04338	+1.59591	−.35013	−.18988	−.73245
				+6		+2	−.90	+.83	+2	+1.00	−1.44	+.92	+1.70	+8.03
9				−1			+.28		+.5	+.2625	+3.05	−1.445	+.635	+7.4275
20				−.0001			−.0101		−.0000	−.0056	+.0006	+.0106	+.0044	−.0653
10				−.2000	+.4000	+.4000	−.3480	−.8879	−.1000	−.0434	+1.5958	−.3591	−.1899	−.7324
				+4.7999	+.4000	+2.4000	−.9781	−.0579	+2.4	+1.2135	+3.2064	−.8735	+2.1495	+14.0598
				C_{12}	−.08334	−.50001	+.20378	+.01206	−.50001	−.25282	−.66801	+.18198	−.44782	−3.05419

	11	13	14	22	21	15	17	23	α 24	l 25	φ 26	λ 27	η	Σn
	+6		−2	−1.03	−3.45					−1.28	+4.66	−1.25	−0.69	−1.04
10	−.8000	−0.8000		+.6960	+1.7760				+0.2000	+.0868	−3.1918	+.7183	+.3798	+1.4649
12	−.0333	−.2000		+.0815	+.0048				−.2000	−.1011	−.2672	+.0728	−.1791	−1.2217
	+5.1667	−1.0	−2	−.2525	−1.6692				.0000	−1.2943	+1.2010	−.4589	−.4893	−.79685
	C_{11}	+.19355	+.38709	+.04887	+.32307				.0000	+.25051	−.23245	+.08882	+.09470	+.15416
		+6	−2	−.60	−4.76				+2	−1.29	+6.84	−3.60	+1.35	+3.94
10		−.8		+.6960	+1.7760				+.2	+.0868	−3.1918	+.7183	+.3798	+1.4649
12		−1.2		+.4801	+.0290				−1.2	−.6068	−1.6032	+.4368	−1.0749	−7.3299
11		−.1936	−.3871	−.0489	−.3231				.0000	−.2505	+.2324	−.0888	−.0947	−.1542
		+3.8064	−2.3871	+.5362	−3.2781				+1.0	−2.0605	+2.2774	−2.5337	+.5602	−2.0792
		C_{13}	+.62713	−.14087	+.86121				−.26272	+.54133	−.59831	+.66564	−.14717	+.54624
			+6	+.23	+3.98	−2	−2	+0.70		+2.58	−2.94	+3.30	+.25	+6.10
11			−.7742	−.0977	−.6461				.0000	−.5010	+.4649	−.1776	−.1894	−.3083
13			−1.4970	+.3363	−2.0558				+.6271	−1.2922	+1.4282	−1.5889	+.3513	−1.3039
			+3.7288	+.4686	+1.2781	−2	−2	+.70	+.6271	+.7868	−1.0469	+1.5335	+.4119	+4.48789
			C_{14}	−.12567	−.34276	+.53637	+.53637	−.18773	−.16818	−.21101	+.28076	−.41126	−.11046	−1.203587
				+153.6386	+60.4347	−1.15	−1.15	−.0815	−.34	+41.1156	+.7959	+28.7425	+9.58	+297.7402
9				−.0784					−.14	−.0735	−.8540	+.4046	−.1778	−2.0797
20				−.9087					−.0034	−.5013	+.0509	+.9556	+.3984	−5.8925
10				−.6055	−1.5451				−.1740	−.0755	+2.7767	−.6247	−.3304	−1.2744
12				−.1993	−.0118				+.4891	+.2473	+.6534	−.1780	+.4380	+2.9873
11				−.0123	−.0816				.0000	−.0633	+.0587	−.0224	−.0239	−.0389
13				−.0755	+.4618				−.1409	+.2903	−.3208	+.3569	−.0789	+.2929
14				−.0589	−.1606	+.2513	+.2513	−.0880	−.0788	−.0989	+.1316	−.1927	−.0518	−.5640
				+151.7000	+59.0974	−.8987	−.8987	−.7695	−.3880	+40.8407	+3.2924	+29.4418	+9.7536	+291.170910
				C_{22}	−.38957	+.00592	+.00592	+.00507	+.00256	−.26922	−.02170	−.19408	−.06430	−1.9193940

Solution of normal equations—Continued

	21	15	17	16	23	α 24	l 25	φ 26	λ 27	η	Σ_n
	+76.7816	−0.24	−0.24	....	−0.1320	....	+18.0583	−9.3813	+18.0000	+5.116	+169.4373
10	−3.9428	....	....	....	....	−0.4440	−.1926	+7.0858	−1.5945	−.8431	−3.2521
12	−.0007	....	....	....	....	+.0290	+.0146	+.0387	−.0105	+.0259	+.1768
11	−.5393	....	....	....	....	.0000	−.4182	+.3880	−.1483	−.1581	−.2573
13	−2.8231	....	....	....	....	+.8612	−1.7745	+1.9613	−2.1820	+.4824	−1.7906
14	−.4381	+.6855	+.6855	....	−.2399	−.2150	−.2697	+.3588	−.5256	−.1412	−1.5383
22	−23.0226	+.3499	+.3499	....	+.2996	+.1513	−15.9102	−1.2824	−11.4696	−3.8000	−113.4315
	+46.0150	+.7954	+.7954	....	−.0723	+.3825	−.4923	−.8311	+2.0695	+.6819	+49.34430
	C_{21}	−.01729	−.01729	....	+.00157	−.00831	+.01070	+.01806	−.04497	−.01482	−1.07235
		+6	+2	−2	−3.91	+2	−2.87	+1.62	−4.20	+.11	−4.64
	14	−1.0727	−1.0727	....	+.3755	+.3364	+.4220	−.5615	+.8225	+.2209	+2.4071
	22	−.0053	−.0053	....	−.0046	−.0023	+.2419	+.0195	+.1744	+.0578	+1.7250
	21	−.0138	−.0138	....	+.0012	−.0066	+.0085	+.0144	−.0358	−.0118	−.8529
		+4.9082	+.9082	−2	−3.5379	+2.3275	−2.1976	+1.0924	−3.2389	+.3769	−1.360812
		C_{15}	−.18504	+.40748	+.72081	−.47421	+.44774	−.22257	+.65900	−.07679	+.277332
			+6	+2	−2.61	....	+1.22	+1.03	−.76	+.11	+5.60
		14	−1.0727	....	+.3755	+.3364	+.4220	−.5615	+.8225	+.2209	+2.4071
		22	−.0053	....	−.0046	−.0023	+.2419	+.0195	+.1744	+.0578	+1.7250
		21	−.0138	....	+.0012	−.0066	+.0085	+.0144	−.0358	−.0118	−.8529
		15	−.1681	+.3701	+.6546	−.4307	+.4036	−.2021	+.5993	−.0397	+.2519
			+4.7401	+2.3701	−1.5833	−.1032	+2.2990	+.3003	+.8004	+.3072	+9.131706
			C_{17}	−.50001	+.33402	+.02177	−.48501	−.06335	−.16886	−.06481	−1.92625
				+6	+4.81	+1	+1.26	−.03	+1.23	+1.33	+15.60
			15	−.8150	−1.4416	+.9484	−.8955	+.4451	−1.3198	+.1536	−.5546
			17	−1.1850	+.7916	+.0516	−1.1495	−.1501	−.4002	−.1536	−4.5654
				+4	+4.16	+2.00	−.7850	+.2650	−.49	+1.33	+10.48
				C_{16}	−1.04	−.5	+.19625	−.06625	+.1225	−.3325	−2.62

Solution of normal equations—Continued

	23	α 24	l 25	φ 26	λ 27	η	Σn
	+38.9152	−1.98	−3.8006	−1.3979	+1.7264	+0.238	+31.8776
14	−.1314	−.1177	−.1477	+.1965	−.2879	−.0773	−.8425
22	−.0039	−.0020	+.2072	+.0167	+.1493	+.0495	+1.4770
21	−.0001	+.0006	−.0008	−.0013	+.0033	+.0011	+.0775
15	−2.5502	+1.6777	−1.5841	+.7874	−2.3347	+.2717	−.9811
17	−.5289	−.0345	+.7679	+.1003	+.2674	+.1026	+3.0498
16	−4.3264	−2.08	+.8164	−.2756	+.5096	−1.3832	−10.8992
	+31.3743	−2.5359	−3.7417	−.5739	+.0334	−.7976	+23.759186
	C_{23}	+.08083	+.11926	+.01829	−.00106	+.02542	−.75726
		+12	+4.30	+23.20	−5.88	+5.81	+47.52
	1	−.1667	+.8317	+4.8683	+.52	−.025	+5.3883
	2	−1.3334	−2.3117	−7.4785	−2.5950	−.1450	−19.2237
	3	−.2500	−.4688	−3.3062	+.3812	−.4112	−4.1200
	18	−.0393	+.0542	−.0794	−.0430	−.0394	+2.1954
	4	−.0456	+.6361	+1.9371	+.6912	−.0516	+1.9196
	5	−.9238	−1.2869	−5.0111	−.2070	−.1713	−8.4820
	6	−.5111	−1.0976	−1.2638	−1.6068	−.4487	−1.3053
	19	−.0020	−.0141	−.0112	−.0933	+.0033	+.2580
	7	−.0492	+.2644	+1.1947	+.1674	−.0906	+1.3851
	8	−1.1987	−1.9284	−3.1101	−.6647	−.7855	−8.0474
	9	−.2500	−.1312	−1.5250	+.7225	−.3175	−3.7138
	20	−.0000	−.0019	+.0002	+.0036	+.0015	−.0219
	10	−.0500	−.0217	+.7979	−.1796	−.0949	−.3662
	12	−1.2000	−.6068	−1.6032	+.4368	−1.0748	−7.3299
	11	.0000	.0000	.0000	.0000	.0000	.0000
	13	−.2627	+.5413	−.5983	+.6656	−.1472	+.5462
	14	−.1055	−.1323	+.1761	−.2579	−.0693	−.7548
	22	−.0010	+.1045	+.0084	+.0753	+.0249	+.7447
	21	−.0032	+.0041	+.0069	−.0172	−.0057	−.4102
	15	−1.1037	+1.0421	−.5180	+1.5359	−.1787	+.6455
	17	−.0022	+.0501	+.0065	+.0174	+.0067	+.1988
	16	−1.0000	+.3925	−.1325	+.2450	−.6650	−5.2400
	23	−.2050	−.3024	−.0464	+.0027	−.0645	+1.9203
		+3.2969	−.0828	+7.5124	−6.0799	−.9395	+3.706771
		C_{24}	+.02511	−2.27863	+1.84413	+.28496	−1.124423
			+116.2130	+117.4082	+121.6126	−2.27	+401.0159
		1	−4.1500	−24.2930	−2.5948	+.1248	−26.8878
		2	−4.0078	−12.9656	−4.4991	−.2514	−33.3284
		3	−.8789	−6.1992	+.7148	−.7711	−7.7250
		18	−.0748	+.1095	+.0593	+.0544	−3.0289
		4	−8.8703	−27.0137	−9.6399	+.7194	−26.7707
		5	−1.7926	−6.9805	−.2884	−.2386	−11.8154
		6	−2.3572	−2.7141	−3.4509	−.9637	−2.8034
		19	−.0972	−.0771	−.6426	+.0228	+1.7775
		7	−1.4198	−6.4156	−.8988	+.4864	−7.4380
		8	−3.1022	−5.0032	−1.0694	−1.2637	−12.9458
		9	−.0689	−.8006	+.3793	−.1667	−1.9497
		20	−.2765	+.0281	+.5271	+.2198	−3.2506
		10	−.0094	+.3462	−.0779	−.0412	−.1589
		12	−.3068	−.8106	+.2208	−.5434	−3.7063
		11	−.3242	+.3009	−.1150	−.1226	−.1995
		13	−1.1154	+1.2328	−1.3716	+.3032	−1.1255
		14	−.1660	+.2209	−.3236	−.0869	−.9470
		22	−10.9951	−.8862	−7.9264	−2.6261	−78.3896
		21	−.0053	−.0089	+.0221	+.0073	+.5279
		15	−.9840	+.4891	−1.4502	+.1688	−.6094
		17	−1.1150	−.1456	−.3882	−.1490	−4.4284
		16	−.1541	+.0520	−.0962	+.2610	+2.0567
		23	−.4462	−.0684	+.0040	−.0951	+2.8334
		24	−.0021	+.1887	−.1527	−.0236	+.0931
			+73.4932	+25.9941	+88.5543	−7.2452	+180.79624
			C_{25}	−.35369	−1.20493	+.09858	−2.46004

Solution of normal equations—Continued

	ϕ 26	λ 27	η	Σ_n
	+451.5432	+91.8144	+3.6191	+641.4826
1	−142.2039	−15.1892	+.7302	−157.3931
2	−41.9446	−14.5548	−.8133	−107.8199
3	−43.7252	+5.0420	−5.4388	−54.4870
18	−.1603	−.0869	−.0796	+4.4353
4	−82.2679	−29.3573	+2.1909	−81.5278
5	−27.1819	−1.1230	−.9292	−46.0092
6	−3.1251	−3.9734	−1.1097	−3.2279
19	−.0611	−.5094	+.0180	+1.4091
7	−28.9893	−4.0614	+2.1977	−33.6092
8	−8.0691	−1.7247	−2.0380	−20.8787
9	−9.3025	+4.4072	−1.9368	−22.6539
20	−.0029	−.0535	−.0223	+.3302
10	−12.7344	+2.8656	+1.5151	+5.8445
12	−2.1419	+.5835	−1.4359	−9.7930
11	−.2792	+.1067	+.1137	+.1851
13	−1.3626	+1.5159	−.3352	+1.2440
14	−.2939	+.4305	+.1156	+1.2600
22	−.0714	−.6390	−.2117	−6.3194
21	−.0150	+.0374	+.0123	+.8912
15	−.2431	+.7209	−.0839	+.3029
17	−.0190	−.0507	−.0195	−.5785
16	−.0176	+.0325	−.0881	−.6943
23	−.0105	+.0006	−.0146	+.4346
24	−17.1180	+13.8538	+2.1407	−8.4472
25	−9.1939	−31.3211	+2.5625	−63.9465
	+21.0089	+18.7666	+.6592	+40.433947
	C_{26}	−.89327	−.03138	−1.92465
		+235.2684	−4.3429	+429.2015
	1	−1.6224	+.0780	−16.8116
	2	−5.0505	−.2822	−37.4137
	3	−.5814	+.6272	+6.2830
	18	−.0471	−.0431	+2.4030
	4	−10.4762	+.7818	−29.0933
	5	−.0464	−.0384	−1.9009
	6	−5.0519	−1.4109	−4.1040
	19	−4.2476	+.1505	+11.7489
	7	−.5690	+.3079	−4.7087
	8	−.3686	−.4356	−4.4626
	9	−2.0880	+.9176	+10.7327
	20	−1.0048	−.4189	+6.1962
	10	−.6449	−.3409	−1.3152
	12	−.1590	+.3912	+2.6678
	11	−.0408	−.0435	−.0707
	13	−1.6865	+.3729	−1.3840
	14	−.6307	−.1694	−1.8457
	22	−5.7141	−1.8931	−56.5106
	21	−.0931	−.0307	−2.2193
	15	−2.1374	+.2487	−.8982
	17	−.1352	−.0519	−1.5418
	16	−.0600	+.1629	+1.2838
	23	−.0000	+.0008	−.0253
	24	−11.2121	−1.7325	+6.8364
	25	−106.7017	+8.7297	−217.8471
	26	−16.7636	−.5890	−36.1191
		+58.1354	+.9462	+59.08156
		C_{27}	−.01626	−1.01628

Back solution

27	26	25	24	23	16	17
−0.01628	−0.03138 +.01454	+0.09858 +.01962 +.00596	+0.28496 −.03002 +.03837 +.00312	+0.02542 +.00002 −.00031 +.01481 +.02396	−0.3325 −.00199 +.00112 +.02437 −.14822 −.06646	−0.06481 +.00275 +.00107 −.06022 +.00645 +.02134 +.26185
	−.01684	+.12416	+.29643	+.06390	−.52368	+.16843
15	**21**	**22**	**14**	**13**	**11**	**12**
−0.07679 −.01074 +.00375 +.05559 −.14057 +.04606 −.21339 −.03117	−0.01482 +.00073 −.00030 +.00133 −.00246 +.00010 −.00291 +.00635	−0.06430 +.00316 +.00037 −.03343 +.00076 +.00032 +.00100 −.00217 +.00467	−0.11046 +.00670 −.00473 −.02620 −.04985 −.01200 +.08866 −.19699 +.00411 +.01126	−0.14717 −.01084 +.01008 +.06721 −.07788 −.01032 +.01262 −.18155	+0.09470 −.00145 +.00391 +.03110 0 −.00387 −.00438 −.11206 −.06539	−0.44782 −.00296 +.01125 −.03139 −.14822 − 00014 −.01826 +.16893 +.00479
−.36726	−.01198	−.08962	−.28950	−.33785	−.05744	−.46382
10	**20**	**9**	**8**	**7**	**19**	**6**
−0.18988 +.00585 −.02688 −.00539 −.02964 +.01064 +.03119 −.13514 −.02298 +.09275	+0.06794 −.00265 −.00015 −.01061 −.00017 +.01389 +.00080 +.00046	−0.3175 −.01176 +.02568 −.01630 −.07411 +.01255 −.23191 −.13474 −.14406	−0.32767 +.00451 +.02185 −.09988 −.14823 +.09383 +.44608	−0.18271 −.00550 −.04059 +.06622 −.02944 +.04121 −.35751 +.00189	−0.00919 −.00422 −.00052 +.00487 +.00169 −.00470 −.00011 −.00604	+0.32690 −.01906 −.01550 +.09927 +.11036 −.02989 −.00238 −.12661 +.03114
−.26948	+.06951	−.89215	−.00951	−.50643	−.01822	+.37423
5	**4**	**18**	**3**	**2**	**1**	
−0.07817 +.00154 +.03851 −.07292 −.12497 −.04600 −.00366 −.19486 −.01875 −.20175	−0.10817 −.02360 −.06840 +.16560 −.02835 +.02944 −.15010 −.28117	+0.01688 −.00030 −.00057 −.00288 +.00499 +.00024 −.00459	−0.41125 −.00621 +.05568 −.05820 −.07411 −.00802 −.23238 −.00007	−0.05438 +.01584 +.04723 −.10763 −.14822 −.00702 +.36729	−0.025 −.00847 −.08198 +.10326 −.04941 +.00496 −.24485 −.03770	
−.70103	−.46475	+.01377	−.73456	+.11311	−.33919	

After the C's are determined, the next step is the computation of the v's, that is, the corrections to the directions. These corrections are obtained, as is the case of the single quadrilateral, by substituting the values of the C's in the table of correlates. A set of v's is then adopted to make all the equations consistent. (See p. 15.)

Computation of corrections (v's)

1	2	3	4	5	6	7	8	9
+0.0614 −.0798 −.0919 +.0578 +.3392 +.2867 +.29	−0.0880 −.1131 −.2011 −.20	−0.0614 +.0798 +.0919 +.0302 +.1131 −.3392 −.0856 −.08	−0.2964 +.0158 −.1131 +.3392 −.0545 −.05	+0.0563 +.1079 −.3315 −.0526 +.7346 −.3392 +.1755 +.18	−0.0563 −.1079 +.3315 +.2964 +.0368 −.7346 +.1131 −.1210 −.12	−0.0501 +.0598 +.0559 −.2964 +.0393 +.7346 −.1131 +.4300 +.43	−0.0723 +.1131 +.0408 +.04	+0.0501 −.0598 −.0559 +.0330 +.4648 −.7346 −.3024 −.30
10	11	12	13	14	15	16	17	18
+0.0221 +.0645 −.2645 +.7010 −.4648 +.0583 +.06	−0.0221 −.0645 +.2645 +.2964 −.7010 −.2267 −.23	+0.0366 −.0650 −.0608 −.0089 −.3742 +.4648 −.0075 −.01	+0.0410 +.3742 +.4152 +.41	−0.0366 +.0650 +.0608 −.0321 +.0140 −.4648 +.7346 +.3409 +.34	−0.0586 −.1122 +.2359 −.0402 −.7346 +.3392 −.3705 −.37	+0.0586 +.1122 −.2359 +.0262 −.3392 −.3781 −.38	+0.0013 −.0460 +.5064 +.4617 +.46	+0.0095 +.01
19	20	21	22	23	24	25	26	27
−0.0013 +.0460 −.0133 −.0095 −.5064 −.3742 +.7010 −.1577 −.16	−0.1031 −.0987 +.4296 +.0763 −.7010 +.4648 +.0679 +.07	+0.1031 +.0987 −.4296 −.0630 +.3742 −.4648 −.3814 −.38	−0.0454 +.0317 +.1242 −.2964 −.0849 +.7010 +.4302 +.43	+0.1031 −.3742 −.2711 −.27	+0.0454 −.0317 −.1242 +.2391 −.0182 +.0095 +.5064 +.3742 −.7010 +.2995 +.30	+0.0070 +.0480 −.2247 −.2704 +.8922 −.5064 −.0543 −.05	−0.0070 −.0480 +.2247 +.2964 +.0313 −.8922 −.0095 −.4043 −.40	−0.4051 −.0108 +.0574 −.3585 −.36
28	29	30	31	32	33	34	35	36
+0.0249 −.0300 −.1800 +.5350 +.0532 −.0574 +.2695 +.6152 +.62	−0.0424 +.4638 +.4214 +.42	−0.0249 +.0300 +.1800 −.1299 −.0236 −.4638 −.2695 +.8922 +.1905 +.19	−0.0254 −.0466 +.1403 −.0786 −.8922 +.5064 −.3961 −.39	+0.0254 +.0466 −.1403 +.1022 −.5064 −.4725 −.47	−0.0199 +.0318 −.0137 −.2964 +.0730 +.8922 +.0095 +.6765 +.68	−0.3330 −.0095 −.3425 −.34	+0.0199 −.0318 +.0137 −.1801 +.2600 +.4638 +.2695 −.8922 −.0772 −.08	+0.0063 +.0310 −.2396 +.3531 +.3378 −.2695 +.2191 +.22
37	38	39	40	41	42	43	44	45
−0.0063 −.0310 +.2396 +.2964 −.1730 −.3378 −.4638 −.4759 −.48	+0.0044 −.0086 −.1316 +.2895 −.0574 +.0963 +.09	−0.0044 +.0086 +.1316 −.2895 +.3378 +.1841 +.18	−0.0054 −.0315 +.0559 −.3378 +.2695 −.0493 −.05	+0.0054 +.0315 −.0559 +.0574 −.2695 −.2311 −.23	−0.0361 +.0066 +.2818 −.2964 −.2034 +.3378 +.4638 +.5541 +.55	−0.0524 −.4638 −.5162 −.52	+0.0361 −.0066 −.2818 +.3764 +.0570 +.2895 −.3378 +.1328 +.13	−0.0160 +.0089 −.0310 −.0032 −.1730 −.0046 −.1684 +.3673 −.2895 −.3095 −.31

Computation of corrections (*v*'s)—Continued

46	47	48	49	50	51	52	53	54
+0.1297 +.1684 ——— +.2981 +.30	+0.0160 −.0089 +.0310 +.2964 −.1265 −.3673 ——— −.1593 −.16	−0.2967 −.0307 −.1684 +.5237 ——— +.0279 +.03	+0.2967 +.0786 −.5237 +.3673 ——— +.2189 +.22	−0.0036 −.0084 +.0968 −.0479 −.0699 −.0018 +.1684 −.3673 +.2895 ——— +.0558 +.06	+0.0036 +.0084 −.0968 +.4589 +.0122 −.2895 +.0574 ——— +.1542 +.16	−0.3890 −.0104 −.0574 ——— −.4568 −.45	−0.0200 +.0005 +.1093 −.2964 +.3673 ——— +.1607 +.16	+0.0200 −.0005 −.1093 +.5237 −.3673 ——— +.0666 +.06

55	56	57	58
+0.2964 −.5237 ——— −.2273 −.23	+0.2732 −.2262 +.5237 ——— +.5707 +.57	+0.2543 −.1684 ——— +.0859 +.09	−0.2732 −.0281 +.1684 −.5237 ——— −.6566 −.66

As a check on the computation of the *v*'s they should be summed up around each point to make sure that each sum equals zero (except where one or more directions at the point are not numbered, see p. 37).

COMPUTATION OF TRIANGLES

The adopted values of the *v*'s can now be substituted in column 4 of the triangle computation (see fig. 24) and the final triangles can be computed as explained on page 39.

In a complete quadrilateral each line is a part of two different triangles. The logarithm of each length appears therefore in two different triangles. The agreement of these two values in all cases furnishes a check on the adjustment and computation. Another check on the work is that the length as carried from the first fixed line to the second fixed line of the arc must agree with the fixed value of the latter. The elimination of the azimuth discrepancy can be checked before the positions are recomputed by substituting the adjusted angles for the corresponding observed angles on page 62.

When it is certain that all the angle, side, azimuth and length equations are satisfied, it is necessary to recompute all the geographic positions, beginning with the fixed line Palo-Pedro and ending with the fixed position Garcena. These recomputed positions are shown in Figure 29.

If the latitudes of the various stations have been only slightly changed by the adjustment, then in the recomputation of the geographic positions the second, third, and fourth terms of the $\Delta\phi$ may be taken directly from the preliminary computation of the positions and only the first term need be recomputed. In a great many cases the recomputation of positions may be made by merely correcting

(Text continued on p. 107)

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

				°	′	″
α	Palo	to	Pedro	12	02	25.00
∠	Pedro	&	Fordyce	+ 61	27	31.76
α	2. Palo	to 1.	Fordyce	73	29	56.76
Δα				−	3	04.83
				180		
α′	1. Fordyce	to 2.	Palo	253	26	51.93
			First Angle of Triangle	48	00	09.93

	°	′	″				°	′	″
φ	26	19	38.951	2.	Palo	λ	98	27	48.248
Δφ	−	1	51.517	s=		Δλ	+	6	56.990
φ′	26	17	47.434	1.	Fordyce	λ′	98	34	45.238

s	4.081 5096	s²	8.16302			−h	2.0467
cos α	9.453 3648	sin² α	9.96347	(δφ)²	4.0947	s² sin² α	8.1265
B	8.511 8083	C	1.10025	D	2.2921	E	5.8511
h	2.046 6827		9.22674		6.3868		6.0243
1st term.	″ + 111.3481	3d term.	″ +0.0002			(Δλ)²	
2d term.	+ 0.1686	4th term.	−0.0001			F	
	+ 111.5167						
3d and 4th terms.	+ 0.0001	s	4.081 5096				
−Δφ	+ 111.5168	sin α	9.981 7349	Arg.		Δλ	2.620 1256
½(φ+φ′)	26 18 43.19	A′	8.509 4379	s	−3	sin ½(φ+φ′)	9.646 6575
		sec φ′	0.047 4432	Δλ	+3	sec ½(Δφ)	
			2.620 1256	Corr.	0		2.266 7831
[illegible]		Δλ	″ + 416.9900			−Δα	″ +184.835

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

				°	′	″
α	Pedro	to	Palo	192	01	53.35
∠	Fordyce	&	Palo	− 70	32	18.57
α	3. Pedro	to 1.	Fordyce	121	29	34.78
Δα				−	2	32.92
				180		
α′	1. Fordyce	to 3.	Pedro	301	27	01.86

	°	′	″				°	′	″
φ	26	14	36.740	3.	Pedro	λ	98	28	59.722
Δφ	+	3	10.694	s=		Δλ	+	5	45.516
φ′	26	17	47.434	1.	Fordyce	λ′	98	34	45.238

s	4.050 7887	s²	8.10158			−h	2.2806
cos α	9.717 9384	sin² α	9.86160	(δφ)²	4.5607	s² sin² α	7.9632
B	8.511 8134	C	1.09865	D	2.2911	E	5.8497
h	2.280 6005		9.06183		6.8518		6.0935
1st term.	″ −190.8097	3d term.	″ +0.0007			(Δλ)²	
2d term.	+ 0.1153	4th term.	+0.0001			F	
	−190.6944						
3d and 4th terms.	+ 0.0008	s	4.050 7887				
−Δφ	−190.6936	sin α	9.930 7983	Arg.		Δλ	2.538 4681
½(φ+φ′)	26 16 12.08	A′	8.509 4379	s	−2	sin ½(φ+φ′)	9.646 0134
		sec φ′	0.047 4432	Δλ	+2	sec ½(Δφ)	
			2.538 4681	Corr.	0		2.184 4815
[illegible]		Δλ	″ + 345.5160			−Δα	″ +152.926

FIG. 29.—Final position computation for stations of net

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Palo	to	Fordyce	73	29	56.76
∠		Fordyce	&	Eltoro	+ 38	09	53.22
α	2.	Palo	to 1.	Eltoro	111	39	49.98
Δα					-	2	45.11
					180		
α′	1.	Eltoro	to 2.	Palo	291	37	04.87
				First Angle of Triangle	77	47	13.85

	°	′	″			°	′	″
φ	26	19	38.951	2. Palo	λ	98	27	48.248
Δφ	+	2	13.007	s=	Δλ	+	6	12.057
φ′	26	21	51.958	1. Eltoro	λ′	98	34	00.305

s	4.045 2890	s²	8.09058			−h	2.1243
cos α	9.567 2158	sin² α	9.93657	$(\delta\varphi)^2$	4.2477	s² sin² α	8.0269
B	8.511 8083	C	1.10025	D	2.2921	E	5.8511
h	2.124 3131		9.12780		6.5398		6.0023
1st term.	−133.″1414	3d term.	+0.″0003			$(\Delta\lambda)^2$	
2d term.	+ 0.1340	4th term.	+0.0001			F	
	−133.0074						
3d and 4th terms.	+ 0.0004	s	4.045 2890				
−Δφ	−133.0070	sin α	9.968 1865	Arg.		Δλ	2.570 6100
½(φ+φ′)	26 20 45.45	A′	8.509 4365	s	−2	sin ½(φ+φ′)	9.647 1776
		sec φ′	0.047 6980	Δλ	+2	sec ½(Δφ)	
			2.570 6100	Corr.	0		2.217 7876
		Δλ	+ 372.″0574			−Δα	+165.″115

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Fordyce	to	Palo	253	26	51.93
∠		Eltoro	&	Palo	− 64	02	53.14
α	3.	Fordyce	to 1.	Eltoro	189	23	58.79
Δα					+		19.93
					180		
α′	1.	Eltoro	to 3.	Fordyce	9	24	18.72

	°	′	″			°	′	″
φ	26	17	47.434	3. Fordyce	λ	98	34	45.238
Δφ	+	4	04.524	s=	Δλ	−		44.933
φ′	26	21	51.958	1. Eltoro	λ′	98	34	00.305

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s	3.882 3870	s²	7.76477			−h	2.3883
cos α	9.994 1295	sin² α	8.42608	$(\delta\varphi)^2$	4.7766	s² sin² α	6.1908
B	8.511 8102	C	1.09965	D	2.2918	E	5.8506
h	2.388 3267		7.29050		7.0684		4.4297
1st term.	−244.″5269	3d term.	+0.″0012			$(\Delta\lambda)^2$	
2d term.	+ 0.0020	4th term.	0000			F	
	−244.5249						
3d and 4th terms.	+ 0.0012	s	3.882 3870				
−Δφ	−244.5237	sin α	9.213 0398	Arg.		Δλ	1.652 5611
½(φ+φ′)	26 19 49.70	A′	8.509 4365	s	−1	sin ½(φ+φ′)	9.646 9406
		sec φ′	0.047 6980	Δλ	0	sec ½(Δφ)	
			1.652 5612	Corr.	−1		1.299 5017
		Δλ	− 44.″9325			−Δα	− 19.″930

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FIG. 29.—Final position computation for stations of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

				°	′	″
α	Eltoro	to	Fordyce	9	24	18.72
∠	Fordyce	&	Garcia	+71	51	47.29
α	2. Eltoro	to 1.	Garcia	81	16	06.01
Δα				-	3	45.95
				180		
α′	1. Garcia	to 2.	Eltoro	261	12	20.06
			First Angle of Triangle	31	20	12.96

	°	′	″			°	′	″
φ	26	21	51.958	2. Eltoro	λ	98	34	00.305
Δφ	-	1	10.687	s=	Δλ	+	8	28.974
φ′	26	20	41.271 -1	1. Garcia	λ′	98	42	29.279

s	4.154 6968	s^2	8.30939			−h	1.8478
cos α	9.161 2921	$\sin^2 \alpha$	9.98987	$(\delta\varphi)^2$	3.6987	$s^2 \sin^2 \alpha$	8.2993
B	8.511 8060	C	1.10095	D	2.2925	E	5.8517
h	1.847 7949		9.40021		5.9912		5.9988

1st term.	+70.4360″	3d term.	+0.0001″			$(\Delta\lambda)^2$	
2d term.	+ 0.2513	4th term.	-0.0001			F	
	+70.6873						
3d and 4th terms.	0000	s	4.154 6968				
−Δφ	+70.6873	sin α	9.994 9372	Arg.		Δλ	2.706 6952
½(φ+φ′)	26 21 16.61	A′	8.509 4369	s	-4	sin ½(φ+φ′)	9.647 3100
		sec φ′	0.047 6243	Δλ	+4	sec ½(Δφ)	
			2.706 6952	Corr.	0		2.354 0052
11-659		Δλ	+ 508.9735″			−Δα	+ 225.946″

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

				°	′	″
α	Fordyce	to	Eltoro	189	25	58.79
∠	Garcia	&	Eltoro	− 76	48	00.02
α	3. Fordyce	to 1.	Garcia	112	35	58.77
Δα				-	3	25.75
				180		
α′	1. Garcia	to 3.	Fordyce	292	32	33.02

	°	′	″			°	′	″
φ	26	17	47.434	3. Fordyce.	λ	98	34	45.238
Δφ	+	2	53.836	s=	Δλ	+	7	44.041
φ′	26	20	41.270	1. Garcia	λ′	98	42	29.279

s	4.144 1935	s^2	8.28839			−h	2.2407
cos α	9.584 6588	$\sin^2 \alpha$	9.93060	$(\delta\varphi)^2$	4.4803	$s^2 \sin^2 \alpha$	8.2190
B	8.511 8102	C	1.09965	D	2.2918	E	5.8506
h	2.240 6625		9.31864		6.7721		6.3103

1st term.	-174.0454″	3d term.	+0.0006″			$(\Delta\lambda)^2$	
2d term.	+ 0.2083	4th term.	+0.0002			F	
	-173.8371						
3d and 4th terms.	+ 0.0008	s	4.144 1935				
−Δφ	-173.8363	sin α	9.965 3017	Arg.		Δλ	2.666 5565
½(φ+φ′)	26 19 14.35	A′	8.509 4369	s	-3	sin ½(φ+φ′)	9.646 7901
		sec φ′	0.047 6243	Δλ	+4	sec ½(Δφ)	
			2.666 5564 (5)	Corr.	+1		2.313 3466
11-660		Δλ	+ 464.0412″			−Δα	+205.753″

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FIG. 29.—Final position computation for stations of net—Continued

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

				°	′	″
α	Eltoro	to	Garcia	81	16	06.01
∠	Garcia	&	Pancho	+44	39	18.62
α	2. Eltoro	to 1.	Pancho	125	55	24.63
Δα				-	3	14.32
				180		
α′	1. Pancho,	to 2.	Eltoro	305	52	10.31
			First Angle of Triangle	64	28	16.49

	°	′	″			°	′	″
φ	26	21	51.958	2. Eltoro	λ	98	34	00.305
Δφ	+	4	44.834	s=	Δλ	+	7	16.979
φ′	26	26	36.792	1. Pancho	λ′	98	41	17.284 +1

s	4.174 6518	s²	8.349[8]30			−h	2.4549
cos α	9.768 4195	sin² α	9.81676	(δφ)²	4.9092	s² sin² α	8.1661
B	8.511 8060	C	1.10095	D	2.2925	E	5.8517
h	2.454 8773		9.26701		7.2017		6.4727
1st term.	″ −285.0213	3d term.	″ +0.0016			(Δλ)²	
2d term.	+ 0.1849	4th term.	+0.0003			F	
	−284.8364						
3d and 4th terms.	+ 0.0019	s	4.174 6518				
−Δφ	−284.8345	sin α	9.908 3784	Arg.		Δλ	2.640 4607
½(φ+φ′)	26 24 14.38	A′	8.509 4349	s	−4	sin ½(φ+φ′)	9.648 0648
		sec φ′	0.047 9957	Δλ	+3	sec ½(Δφ)	
			2.640 460[7]9	Corr.	−1		2.288 5255
11—436		Δλ	″ +436.9791			−Δα	″ +194.324

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

				°	′	″
α	Garcia	to	Eltoro	261	12	20.06
∠	Pancho	&	Eltoro	− 70	52	25.27
α	3. Garcia	to 1.	Pancho	190	19	54.79
Δα				+	0	32.01
				180		
α′	1. Pancho	to 3.	Garcia	10	20	26.80

	°	′	″			°	′	″
φ	26	20	41.270	3. Garcia	λ	98	42	29.279
Δφ	+	5	55.522	s=	Δλ	−	1	11.994
φ′	26	26	36.792	1. Pancho	λ′	98	41	17.285

s	4.046 1679	s²	8.09233			−h	2.5509
cos α	9.992 9004	sin² α	8.50740	(δφ)²	5.1017	s² sin² α	6.5997
B	8.511 8072	C	1.10057	D	2.2923	E	5.8513
h	2.550 8755		7.70030		7.3940		5.0019
1st term.	″ −355.5293	3d term.	″ +0.0025			(Δλ)²	
2d term.	+ 0.0050	4th term.	0.0000			F	
	−355.5243						
3d and 4th terms.	+0.0025	s	4.046 1679				
−Δφ	−355.5218	sin α	9.253 7007	Arg.		Δλ	1.857 2989
½(φ+φ′)	26 23 39.03	A′	8.509 4349	s	−2	sin ½(φ+φ′)	9.647 9148
		sec φ′	0.047 9957	Δλ	0	sec ½(Δφ)	
			1.[8]57 29[90]89	Corr.	−2		1.505 2137
11—436		Δλ	″ − 71.9944			−Δα	″ −32.005

FIG. 29.—Final position computation for stations of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Pancho	to	Garcia	10	20	26.80
∠		Garcia	&	Monument	+28	28	02.87
α	2.	Pancho	to 1.	Monument	38	48	29.67
Δα					-	2	07.02
					180		
α′	1.	Monument	to 2.	Pancho	218	46	22.65
				First Angle of Triangle	61	38	08.64

	°	′	″				°	′	″
φ	26	26	36.792	2.	Pancho	λ	98	41	17.285
Δφ	-	5	20.110	s=		Δλ	+	4	45.680
φ′	26	21	16.682	1.	Monument	λ′	98	46	02.965

s	4.101 7116	s²	8.20342			−h	2.5052
cos α	9.891 6756	sin² α	9.59414	(δφ)²	5.0106	s² sin² α	7.7976
B	8.611 8012	C	1.10250	D	2.2934	E	5.8529
h	2.505 1886		8.90006		7.3040		6.1557
1st term.	+320.0285	3d term.	+0.0020			(Δλ)²	
2d term.	+ 0.0794	4th term.	-0.0001			F	
	+ 320.1079						
3d and 4th terms.	+ 0.0019	s	4.101 7116				
−Δφ	+320.1098	sin α	9.797 0708	Arg.		Δλ	2.455 8803
½(φ+φ′)	26 23 56.74	A′	8.509 4367	s	-3	sin ½(φ+φ′)	9.647 9901
		sec φ′	0.047 6612	Δλ	+1	sec ½(Δφ)	
			2.455 8805	Corr.	-2		2.103 8704
		Δλ	+ 285.6803			−Δα	+127.019

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Garcia	to	Pancho	190	19	54.79
∠		Monument	&	Pancho	− 89	53	48.66
α	3.	Garcia	to 1.	Monument	100	26	06.13
Δα					-	1	34.84
					180		
α′	1.	Monument	to 3.	Garcia	280	24	31.29

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	°	′	″				°	′	″
φ	26	20	41.270	3.	Garcia	λ	98	42	29.279
Δφ	+		35.412	s=		Δλ	+	3	33.686
φ′	26	21	16.682	1.	Monument	λ′	98	46	02.965

s	3.779 9207	s²	7.55984			−h	1.5497
cos α	9.257 9677	sin² α	9.98551	(δφ)²	3.0983	s² sin² α	7.5454
B	8.511 8072	C	1.10057	D	2.2923	E	5.8513
h	1.549 6956		8.64592		5.3906		4.9464
1st term.	-35.4565	3d term.	0.0000			(Δλ)²	
2d term.	+0.0443	4th term.	0.0000			F	
	-35.4122						
3d and 4th terms.	0000	s	3.779 9207				
−Δφ	-35.4122	sin α	9.992 7571	Arg.		Δλ	2.329 7757
½(φ+φ′)	26 20 58.98	A′	8.509 4367	s	-1	sin ½(φ+φ′)	9.647 2352
		sec φ′	0.047 6612	Δλ	+1	sec ½(Δφ)	
			2.329 7757	Corr.	0		1.977 0109
		Δλ	+ 213.6858			−Δα	+94.844

FIG. 29.—Final position computation for stations of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Pancho	to	Monument	38	48	29.67
∠		Monument	&	Corpus	+49	18	39.04
α	2.	Pancho	to 1.	Corpus	88	07	08.71
Δα					−	2	04.55
					180		
α′	1.	Corpus	to 2.	Pancho	268	05	04.16
				First Angle of Triangle	92	54	15.15

	°	′	″				°	′	″
φ	26	26	36.792	2.	Pancho	λ	98	41	17.285
Δφ	−		08.346	s=		Δλ	+	4	39.709
φ′	26	26	28.446	1.	Corpus	λ′	98	45	56.994

s	3,889 5177	s²	7.77904			−h	0.9175
cos α	8.516 1683	sin² α	9.99953	(Δφ)²	1.8428	s² sin² α	7.7786
B	8.511 8012	C	1.10250	D	2.2934	E	5.8529
h	0.917 4872		8.88107		4.1362		4.5490
1st term.	+8.2697	3d term.	0.0000			(Δλ)²	
2d term.	+0.0760	4th term.	0.0000			F	
	+8.3457						
3d and 4th terms.	0000	s	3,889 5177				
−Δφ	+8.3457	sin α	9.999 7660	Arg.		Δλ	2.446 7057
½(φ+φ′)	26 26 32.62	A′	8.509 4350	s	−1	sin ½(φ+φ′)	9.648 6505
		sec φ′	0.047 9870	Δλ	+1	sec ½(Δφ)	
			2.446 7057	Corr.	0		2.095 3562
		Δλ	+279.7085			−Δα	+124.554

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Monument	to	Pancho	218	46	22.65
∠		Corpus	&	Pancho	− 37	47	06.00
α	3.	Monument	to 1.	Corpus	180	59	16.65
Δα					+		02.66
					180		
α′	1.	Corpus	to 3.	Monument	0	59	19.31

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	°	′	″				°	′	″
φ	26	21	16.682	3.	Monument	λ	98	46	02.965
Δφ	+	5	11.764	s=		Δλ	−		05.972
φ′	26	26	28.446	1.	Corpus	λ′	98	45	56.993 +1

s	3,982 0867	s²	7.96417			−h	2.4938
cos α	9.999 9354	sin² α	6.47319	(Δφ)²	4.9877	s² sin² α	4.4374
B	8.511 8066	C	1.10076	D	2.2924	E	5.8515
h	2.493 8287		5.53813		7.2801		2.7827
1st term.	−311.7660	3d term.	+0.0019			(Δλ)²	
2d term.	0.0000	4th term.	0.0000			F	
	−311.7660						
3d and 4th terms.	+ 0.0019	s	3,982 0867				
−Δφ	−311.7641	sin α	8.236 5945	Arg.		Δλ	0.776 1019
½(φ+φ′)	26 23 52.56	A′	8.509 4350	s	−2	sin ½(φ+φ′)	9.647 9728
		sec φ′	0.047 9870	Δλ	+0	sec ½(Δφ)	
			0.776 1038	Corr.	−2		0.424 0741
		Δλ	− 5.9718			−Δα	−2.655

Fig. 29.—Final position computation for stations of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Corpus	to	Monument	0	59	19.31
∠		Monument	&	Grande	+ 46	17	59.43
α	2.	Corpus	to 1.	Grande	47	17	18.74
Δα					-	1	35.34
					180		
α′	1.	Grande	to 2.	Corpus	227	15	43.40
				First Angle of Triangle	78	09	39.95

	°	′	″				°	′	″
φ	26	26	28.446	2.	Corpus	λ	98	45	56.994
Δφ	-	2	58.221	s=		Δλ	+	3	34.297
φ′	26	23	30.225	1.	Grande	λ′	98	49	31.291

s	3.907 6218	s²	7.61524			−h	2.2508
cos α	9.831 4261	sin² α	9.73231	(δφ)²	4.5019	s² sin² α	7.5476
B	8.511 8013	C	1.10240	D	2.2934	E	5.8529
h	2.250 8492		8.64995		6.7953		5.6513
1st term.	+178.″1750	3d term.	+0.″0006			(Δλ)²	
2d term.	+0.0447	4th term.	0.0000			F	
	+178.2207						
3d and 4th term.	+0.0006	s	3.907 6218				
−Δφ	+178.2213	sin α	9.866 1568	Arg.		Δλ	2.331 0151
½(φ+φ′)	26 24 59.34	A′	8.509 4359	s	-1	sin ½(φ+φ′)	9.648 2555
		sec φ′	0.047 8006	Δλ	+1	sec ½(Δφ)	
			2.331 0151	Corr.	0		1.979 2706
		Δλ	+214.″2965			−Δα	+95.″339

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Monument	to	Corpus	180	59	16.65
∠		Grande	&	Corpus	− 55	32	20.76
α	3.	Monument	to 1.	Grande	125	26	55.89
Δα					-	1	32.54
					180		
α′	1.	Grande	to 3.	Monument	305	25	23.35

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	°	′	″				°	′	″
φ	26	21	16.682	3.	Monument	λ	98	46	02.965
Δφ	+	2	13.543	s=		Δλ	+	3	28.325
φ′	26	23	30.225	1.	Grande	λ′	98	49	31.290 +1

s	3.850 5419	s²	7.70108			−h	2.1258
cos α	9.763 4101	sin² α	9.82192	(δφ)²	4.2512	s² sin² α	7.5230
B	8.511 8066	C	1.10076	D	2.2924	E	5.8515
h	2.125 7586		8.62376		6.5436		5.5003
1st term.	-133.″5853	3d term.	+0.″0003			(Δλ)²	
2d term.	+0.0421	4th term.	0.0000			F	
	-133.5432						
3d and 4th term.	+ 0.0003	s	3.850 5419				
−Δφ	-133.5429	sin α	9.910 9625	Arg.		Δλ	2.318 7407
½(φ+φ′)	26 22 23.45	A′	8.509 4359	s	-1	sin ½(φ+φ′)	9.647 5940
		sec φ′	0.047 8006	Δλ	+1	sec ½(Δφ)	
			2.318 7407	Corr.	0		1.966 3347
		Δλ	+ 208.″3247			−Δα	+92.″541

FIG. 29.—Final position computation for stations of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

				°	′	″
α	Corpus	to	Grande	47	17	18.74
∠	Grande	&	Hebron	+47	30	42.74
α	2. Corpus	to 1.	Hebron	94	48	01.48
Δα				−	3	10.09
				180		
α′	1. Hebron	to 2.	Corpus	274	44	51.39

First Angle of Triangle 42 56 20.06

	°	′	″				°	′	″
φ	26	26	28.446	2.	Corpus	λ	98	45	56.994
Δφ	+		32.091	s=		Δλ	+	7	06.827
φ′	26	27	00.537	1.	Hebron	λ′	98	53	03.821

s	4.074 3223	s^2	8.14864			−h	1.5088
cos α	8.922 6476	$\sin^2 \alpha$	9.99695	$(\delta\varphi)^2$	3.0128	$s^2 \sin^2 \alpha$	8.1456
B	8.511 8013	C	1.10240	D	2.2934	E	5.8529
h	1.508 7712		9.24799		5.3062		5.5073
1st term.	−32.2679	3d term.	0.0000			$(\Delta\lambda)^2$	
2d term.	+ 0.1770	4th term.	0.0000			F	
	−32.0909						
3d and 4th terms.	0.0000	s	4.074 3223				
−Δφ	−32.0909	sin α	9.998 4739	Arg.		Δλ	2.630 2516
½(φ+φ′)	26 26 44.49	A′	8.509 4348	s	−3	sin ½(φ+φ′)	9.648 7008
		sec φ′	0.048 0206	Δλ	+3	sec ½(Δφ)	
			2.630 2516	Corr.	0		2.278 9524
		Δλ	+426.8267			−Δα	+190.087

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

				°	′	″
α	Grande	to	Corpus	227	15	43.40
∠	Hebron	&	Corpus	− 89	32	57.38
α	3. Grande	to 1	Hebron	137	42	46.02
Δα				−	1	34.57
				180		
α′	1. Hebron	to 3.	Grande	317	41	11.45

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	°	′	″				°	′	″
φ	26	23	30.225	3.	Grande	λ	98	49	31.291
Δφ	+	3	30.312	s=		Δλ	+	3	32.530
φ′	26	27	00.537	1.	Hebron	λ′	98	53	03.821

s	3.942 0489	s^2	7.88410			−h	2.3230
cos α	9.869 1033	$\sin^2 \alpha$	9.65583	$(\delta\varphi)^2$	4.6458	$s^2 \sin^2 \alpha$	7.5399
B	8.511 8043	C	1.10146	D	2.2928	E	5.8521
h	2.322 9565		8.64139		6.9386		5.7150
1st term.	−210.3568	3d term.	+0.0009			$(\Delta\lambda)^2$	
2d term.	+ 0.0438	4th term.	+0.0001			F	
	−210.3130						
3d and 4th terms.	+ 0.0010	s	3.942 0489				
−Δφ	−210.3120	sin α	9.827 9167	Arg.		Δλ	2.327 4210
½(φ+φ′)	26 25 15.38	A′	8.509 4348	s	−1	sin ½(φ+φ′)	9.648 3234
		sec φ′	0.048 0206	Δλ	+1	sec ½(Δφ)	
			2.327 4210	Corr.	0		1.975 7444
		Δλ	+ 212.5304			−Δα	+94.568

FIG. 29.—Final position computation for stations of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Hebron	to	Grande	317	41	11.46
∠		Grande	&	Ringold	+47	22	41.41
α	2.	Hebron	to 1.	Ringold	5	03	52.86
Δα					-		11.81
					180		
α′	1.	Ringold	to 2.	Hebron	185	03	41.05
				First Angle of Triangle	69	29	07.33

	°	′	″				°	′	″
φ	26	27	00.537	2. Hebron		λ	98	53	03.821
Δφ	-	4	29.783	s=		Δλ	+		26.543
φ′	26	22	30.754	1. Ringold		λ′	98	53	30.364

s	3.920 9092	s^2	7.84182			−h	2.4310
cos α	9.998 3010	$\sin^2 α$	7.89173	$(δφ)^2$	4.8620	$s^2 \sin^2 α$	5.7336
B	8.511 8008	C	1.10257	D	2.2935	E	5.8530
h	2.431 0110		6.83612		7.1555		4.0176
1st term.	+269.7808	3d term.	+0.0014			$(Δλ)^2$	
2d term.	+0.0007	4th term.	0000			F	
	+269.7815						
3d and 4th terms.	+0.0014	s	3.920 9092				
−Δφ	+269.7829	sin α	8.945 8639	Arg.		Δλ	1.423 9476
½(φ+φ′)	26 24 45.65	A′	8.509 4363	s	-1	sin ½(φ+φ′)	9.648 1973
		sec φ′	0.047 7385	Δλ	0	sec ½(Δφ)	
			1.423 9479 (8)	Corr.	-1		1.072 1449
11-686		Δλ	+26.5429			−Δα	+11.807

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Grande	to	Hebron	137	42	46.02
∠		Ringold	&	Hebron	− 63	08	11.40
α	3.	Grande	to 1.	Ringold	74	34	34.62
Δα					-	1	46.24
					180		
α′	1.	Ringold	to 3.	Grande	254	32	48.38

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	°	′	″				°	′	″
φ	26	23	30.225	3. Grande		λ	98	49	31.291
Δφ	-		59.471	s=		Δλ	+	3	59.073
φ′	26	22	30.754	1. Ringold		λ′	98	53	30.364

s	3.837 2856	s^2	7.67457			−h	1.7739
cos α	9.424 8085	$\sin^2 α$	9.96914	$(δφ)^2$	3.5486	$s^2 \sin^2 α$	7.6427
B	8.511 8043	C	1.10146	D	2.2928	E	5.8521
h	1.773 8984		8.74417		5.8414		5.2687
1st term.	+59.4153	3d term.	+0.0001			$(Δλ)^2$	
2d term.	+ 0.0555	4th term.	0.0000			F	
	+59.4708						
3d and 4th terms.	+ 0.0001	s	3.837 2856				
−Δφ	+59.4709	sin α	9.984 0704	Arg.		Δλ	2.378 5308
½(φ+φ′)	26 23 00.49	A′	8.509 4363	s	-1	sin ½(φ+φ′)	9.647 7513
		sec φ′	0.047 7385	Δλ	+1	sec ½(Δφ)	
			2.378 5308	Corr.	0		2.026 2821
11-686		Δλ	+239.0732			−Δα	−106.259

FIG. 29.—Final position computation for stations of net—Continued

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Hebron	to	Ringold	5	03	52,86
∠		Ringold	&	Garcena	+83	16	45,71
α	2.	Hebron	to 1.	Garcena	88	20	38,57
Δα					-	1	11,31
					180		
α′	1.	Garcena	to 2.	Hebron	268	19	27,26
				First Angle of Triangle	67	18	20,09

	°	′	″				°	′	″
φ	26	27	00,537	2.	Hebron	λ	98	53	03,821
Δφ	-		04,192	s=		Δλ	+	2	40.095
φ′	26	26	56,345	1.	Garcena	λ′	98	55	43.916

s	3,647 1063	s²	7.29421			−h	0,6198
cos α	8.460 8649	sin² α	9.99964	(δφ)²	1.2444	s² sin² α	7.2938
B	8.511 6008	C	1.10257	D	2.2935	E	5.8530
h	0,619 7720		8.39642		3.5379		3.7666
1st term.	+4.1665	3d term.	0.0000			(Δλ)²	
2d term.	+0.0249	4th term.	0.0000			F	
	+4.1914						
3d and 4th terms.	0.0000	s	3,647 1063				
−Δφ	+4.1914	sin α	9.999 8186	Arg.		Δλ	2,204 3760
½(φ+φ′)	26 26 58.44	A′	8.509 4348	s	0	sin ½(φ+φ′)	9,648 7599
		sec φ′	0.048 0163	Δλ	0	sec ½(Δφ)	
			2,204 3760	Corr.	0		1.853 1359
		Δλ	+160.0944			−Δα	+71,308

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Ringold	to	Hebron	185	03	41,06
∠		Garcena	&	Hebron	− 29	24	54,29
α	3.	Ringold	to 1.	Garcena	155	38	46,76
Δα					-		59,41
					180		
α′	1.	Garcena	to 3.	Ringold	335	37	47.35

	°	′	″				°	′	″
φ	26	22	30.754	3.	Ringold	λ	98	53	30.364
Δφ	+	4	25.591	s=		Δλ	+	2	13.552
φ′	26	26	56.345	1.	Garcena	λ′	98	55	43.916

s	3.952 9127	s²	7.90583			−h	2.4242
cos α	9.959 5267	sin² α	9.23057	(δφ)²	4.8434	s² sin² α	7.1364
B	8.511 6053	C	1.10115	D	2.2926	E	5.8519
h	2.424 2447		8.23755		7.1410		5.4125
1st term.	-265.6102	3d term.	+0.0014			(Δλ)²	
2d term.	+0.0173	4th term.	0000			F	
	-265.5929						
3d and 4th terms.	+ 0.0014	s	3.952 9127				
−Δφ	-265.6915	sin α	9.615 2850	Arg.		Δλ	2.125 6488
½(φ+φ′)	26 24 43.55	A′	8.509 4348	s	-2	sin ½(φ+φ′)	9.648 1685
		sec φ′	0.048 0163	Δλ	0	sec ½(Δφ)	
			2.125 6488	Corr.	-2		1.773 8573
		Δλ	+ 133.5515			−Δα	+ 59.407

FIG. 29.—Final position computation for stations of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Hebron	to	Ringold	5	03	52,86
∠		Ringold	&	Gorgora	+ 71	33	14,66
α	2.	Hebron	to 1.	Gorgora	76	37	07,42
Δα					−	3	21,11
					180		
α′	1.	Gorgora	to 2.	Hebron	256	33	46,31
				First Angle of Triangle	37	42	07,23

	°	′	″				°	′	″
φ	26	27	00,537	2.	Hebron	λ	98	53	03,821
Δφ	−	1	36,958	s=		Δλ	+	7	31,723
φ′	26	25	23,579	1.	Gorgora	λ′	99	00	35,544

s	4,109 4711	s²	8,21894			−h	1,9857
cos α	9,364 4195	sin² α	9,97609	(δφ)²	3,9732	s² sin² α	8,1950
B	8,511 8008	C	1,10257	D	2,[illegible]935	E	5,8530
h	1,985 6914		9,29760		6,2667		6,0337
1st term.	+96,7590	3d term.	+0,0002			(Δλ)²	
2d term.	+ 0,1984	4th term.	−0,0001			F	
	+96,9574						
3d and 4th terms.	+ 0,0001	s	4,109 4711				
−Δφ	+96,9575	sin α	9,988 0466	Arg.		Δλ	2,654 8722
½(φ+φ′)	26 26 12,06	A′	8,509 4353	s	−3	sin ½(φ+φ′)	9,648 5636
		sec φ′	0,047 9191	Δλ	+4	sec ½(Δφ)	
			2,654 8722	Corr.	+1		2,303 4357
		Δλ	+451 7230			−Δα	+201,111

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

					°	′	″
α		Ringold	to	Hebron	185	03	41,05
∠		Gorgora	&	Hebron	− 70	44	38,47
α	3.	Ringold	to 1.	Gorgora	114	19	02,58
Δα					−	3	09,04
					180		
α′	1.	Gorgora	to 3.	Ringold	294	15	53,54

	°	′	″				°	′	″
φ	26	22	30,754	3.	Ringold	λ	98	53	30,364
Δφ	+	2	52,825	s=		Δλ	+	7	05,180
φ′	26	25	23,579	1.	Gorgora	λ′	99	00	35,544

s	4,111 5674	s²	8,22313			−h	2,2380
cos α	9,614 6767	sin² α	9,91930	(δφ)²	4,4752	s² sin² α	8,1424
B	8,511 8053	C	1,10115	D	2,2926	E	5,8519
h	2,238 0494		9,24358		6,7678		6,2323
1st term.	−173,0013	3d term.	+0,0006			(Δλ)²	
2d term.	+ 0,1752	4th term.	+0,0002			F	
	−172,8261						
3d and 4th terms.	+ 0,0008	s	4,111 5674				
−Δφ	−172,8253	sin α	9,959 6510	Arg.		Δλ	2,628 5728
½(φ+φ′)	26 23 57,17	A′	8,509 4353	s	−3	sin ½(φ+φ′)	9,647 9918
		sec φ′	0,047 9191	Δλ	+3	sec ½(Δφ)	
			2,628 5728	Corr.	0		2,276 5646
		Δλ	+ 425,1800			−Δα	+189,045

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FIG. 29.—Final position computation for stations of net—Continued

the preliminary computation. Compare the positions in Figure 29 with those in Figure 27.

The computed position of Garcena should agree with the fixed position. Such being the case all the equations are now satisfied. The values of the C's should now be placed in the proper column in the table of normals, facing page 84. The values of the v's and the adopted v's are now completely checked, and they should be entered in the proper columns in the table of correlates, pages 82–83. Next, using the sketch for the designations of the directions, the corresponding corrections should be applied to these directions in the list of directions, Figure 23. The final seconds in the list of directions should be checked by the angles in the triangles. (See p. 41.)

PROBABLE ERROR OF A DIRECTION

In the table of correlates, pages 82–83, the last column to the right contains the squares of the adopted v's. The probable error of an observed direction is computed from the formula, p. e. $= \pm 0.6745\sqrt{\frac{\Sigma v^2}{c}}$, where $\Sigma\, v^2$ is the sum of the squares of the corrections to the directions, and c is the number of condition equations used in the adjustment. For the arc here given,

$$\Sigma v^2 = 5.9667$$
$$c = 27$$

Therefore the probable error $= \pm 0.6745\sqrt{\frac{5.9667}{27}} = \pm 0.''32$.

The last operation in the adjustment of an arc is making out the list of geographic positions with the azimuths and logarithms of the distances on form 28B. (See fig. 30.) The columns headed "Station" and "To station" are filled out first by using the sketch to pick out the proper order of the stations in regard to azimuth. The list of geographic positions for the present arc is given in Figure 30. For the method of computing the azimuths which can not be obtained directly from the position computation see page 47.

Statistics showing accuracy of triangulation

When a net of triangulation has been finally adjusted a table of statistics similar to the one below should be prepared, showing the accuracy of the observations:

Total number of triangles	23
Number of triangles with plus closures	5
Number of triangles with minus closures	18
Number of concluded triangles	0
Average closure of triangles without regard to sign	0."82
Maximum closure of a triangle	1."89
Mean error of an angle	±0."59
Probable error of an observed direction	±0."32

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 28 B

11—[illegible]

GEOGRAPHIC POSITIONS

Accession No. of Computation: ________

Locality Rio Grande River *Datum* North American *State* Texas

Station.		Latitude and longitude. ° ′ ″	Seconds in meters.	Azimuth ° ′ ″	Back azimuth. ° ′ ″	To station.	Distance. Logarithm (meters).	Distance. Meters.	Distance. Feet.
Fordyce		26 17 47.434	1459.8	253 26 51.93	73 29 56.76	Palo	4.0815096	12064.51	39581.6
1917	d.m.	98 34 45.238	1254.9	301 27 01.86	121 29 34.78	Pedro	4.0507837	11240.58	36878.5
Eltoro		26 21 51.958	1599.0	291 37 04.87	111 39 49.98	Palo	4.0452890	11099.13	35414.4
1917	d.m.	98 34 00.305	8.5	328 04 40.91	148 06 54.10	Pedro	4.1980195	15776.82	51761.1
				9 24 18.72	189 23 58.79	Fordyce	3.8823870	7627.58	25024.8
Garcia		26 20 41.270	1270.1	261 12 20.06	81 16 06.01	Eltoro	4.1546968	14278.97	46846.9
1917	d.m.	98 42 29.279	811.8	292 32 33.02	112 35 58.77	Fordyce	4.1441935	13937.78	45727.5
Pancho		26 26 36.792	1132.3	305 52 10.31	125 55 24.63	Eltoro	4.1746518	14950.37	49049.7
1917	d.m.	98 41 17.285	478.9	326 15 56.40	146 18 50.53	Fordyce	4.2918957	19583.74	64251.0
				10 20 26.30	190 19 54.79	Garcia	4.0461679	11121.62	36488.2
Monument		26 21 16.682	513.4	218 46 22.65	38 48 29.67	Pancho	4.1017118	12638.97	41466.4
1917	d.m.	98 46 02.965	82.2	280 24 31.29	100 26 06.13	Garcia	3.7799207	6024.50	19765.4
Corpus		26 26 28.446	875.4	268 05 04.16	88 07 08.71	Pancho	3.8895177	7753.85	25439.1
1917	d.m.	98 45 56.994	1579.1	331 40 06.85	151 41 39.18	Garcia	4.0841034	12136.78	39818.8
				0 59 19.31	180 59 16.65	Monument	3.9820867	9595.92	31482.6

FIG. 30.—List of geographic positions for stations of net

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 28 B

GEOGRAPHIC POSITIONS

Accession No. of Computation: ____________

Locality Rio Grande River *Datum* North American *State* Texas

STATION.		LATITUDE AND LONGITUDE.			SECONDS IN METERS.	AZIMUTH			BACK AZIMUTH.			TO STATION.	DISTANCE. LOGARITHM (METERS).	DISTANCE. METERS.	DISTANCE. FEET.
		°	′	″		°	′	″	°	′	″				
Grande		26	23	30.225	930.2	227	15	43.40	47	17	18.74	Corpus	3.9076218	8083.92	26522.0
1917	d.m.	98	49	31.291	867.3	305	25	23.35	125	26	55.89	Monument	3.8505419	7088.30	23255.5
Hebron		26	27	00.537	16.5	274	44	51.39	94	48	01.48	Corpus	4.0743223	11866.49	38932.0
1917	d.m.	98	53	03.821	105.8	312	11	20.87	132	14	28.01	Monument	4.1972563	15749.12	51670.2
						317	41	11.45	137	42	46.02	Grande	3.9420489	8750.82	28710.0
Ringold		26	22	30.754	946.5	185	03	41.05	5	03	52.86	Hebron	3.9209092	8335.07	27346.0
1917	d.m.	98	53	30.364	841.7	254	32	48.38	74	34	34.62	Grande	3.8372856	6875.20	22556.4
						280	23	09.00	100	26	27.68	Monument	4.1007703	12611.60	41376.6
Garcena		26	26	56.345	1734.0	268	19	27.26	88	20	38.57	Hebron	3.6471063	4437.17	14557.6
1917	d.m.	98	55	43.916	1216.7	335	37	47.35	155	38	46.76	Ringold	3.9529127	8972.49	29437.2
Gorgora		26	25	23.579	725.6	250	31	22.70	70	33	32.53	Garcena	3.9329710	8569.81	28116.1
1917	d.m.	99	00	35.544	984.9	256	33	46.31	76	37	07.42	Hebron	4.1094711	12866.82	42213.9
						294	15	53.54	114	19	02.58	Ringold	4.1115674	12929.07	42418.1

FIG. 30.—List of geographic positions for stations of net—Continued

CHAPTER 4.—ADJUSTMENT OF A NET OF TRIANGULATION BY THE ANGLE METHOD

EXPLANATION OF METHOD

In the adjustment of triangulation of the first and second orders it is the practice in the United States Coast and Geodetic Survey to use the direction method, and to include all the observed lines of the main scheme. As these classes of triangulation serve as control for all other surveys, it is necessary to make the adjustment of them as rigid as possible.

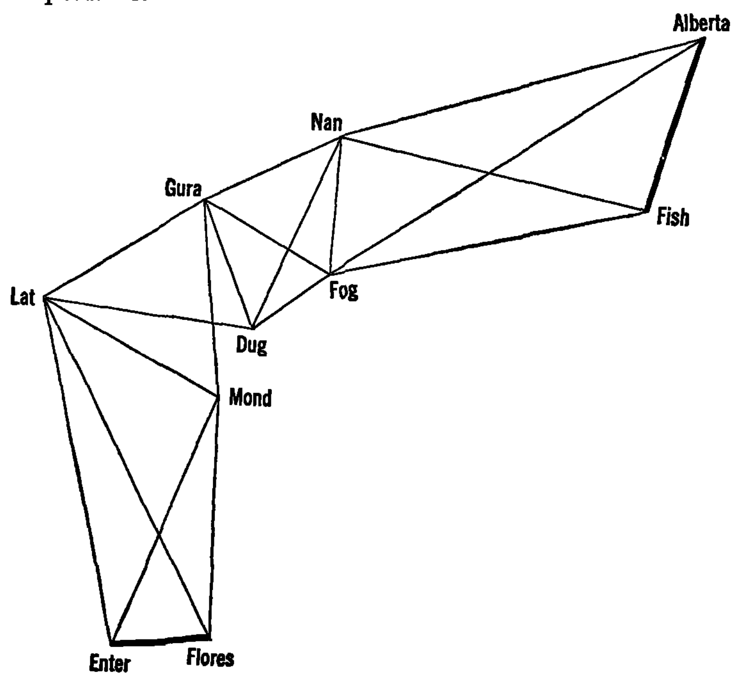

FIG. 31.—Triangulation net used in sample adjustment by angle method

However, for third-order triangulation which is fitted in between fixed points and lines of triangulation of the first and second order such a rigid adjustment is not required. Consequently in adjusting third-order triangulation the angle method should ordinarily be used.

A sample adjustment by the angle method of the arc of third-order triangulation shown in Figure 31 is given on the following pages. This small scheme requires all the different kinds of equations needed for a complete adjustment, and if these are understood, a larger scheme, which differs from this only in the number of equations, can be readily adjusted.

In Figure 31, all observed lines are shown. For the angle method of adjustment, however, it is customary to omit one diagonal in each quadrilateral and use only a chain of triangles, those of the best shape. In the example given, the diagonals Fog-Alberta, Dug-Nan, and Lat-Flores are omitted. The lines actually used in the adjustment are shown in Figure 32. After the chain of triangles has been adjusted and all the conditions are satisfied, then each omitted diagonal is computed by using the two sides and included angle of the triangle in which it occurs (see p. 139). Instructions for field work call for the observing of both diagonals of each quadrilateral in order that there may be a check on all lengths.

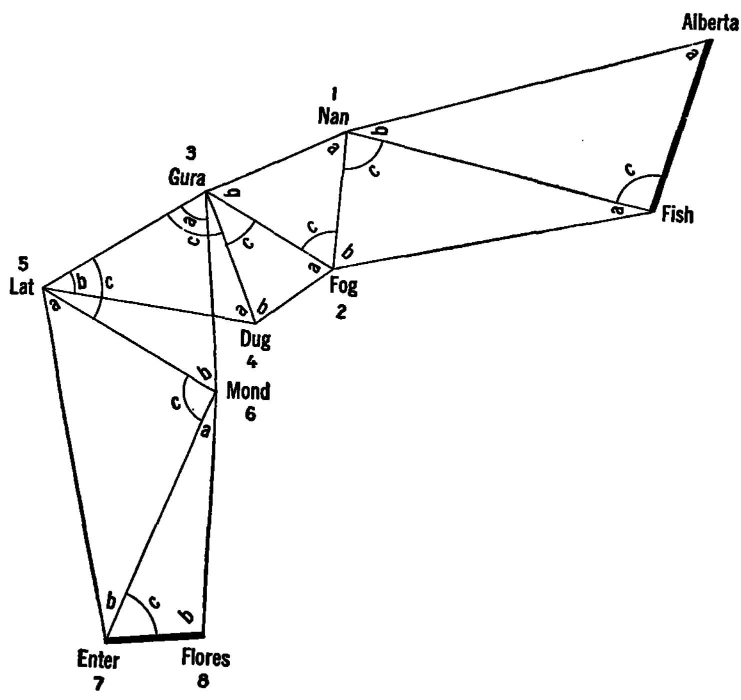

FIG. 32.—Triangulation net showing the triangles used in adjustment by angle method

The lines Alberta–Fish and Enter-Flores are fixed in length and azimuth, and the four stations at the ends of these lines are fixed in latitude and longitude.

The lists of directions for the various stations are given in Figure 33. The column headed "Final seconds" is filled out after the adjustment is completed. If the lists of directions have not been made out and checked in the field they should be computed from the horizontal angle record books in the manner shown on page 8.

The triangulation sketch (fig. 32) is numbered and lettered as follows: Starting with the fixed line Alberta-Fish and building up the sketch point by point, each point added is given a consecutive number, Nan being 1, Fog 2, Gura 3, etc. In each triangle the angles are given the letters *a*, *b*, and *c*. The angle adjacent to the starting line but opposite the line through which the length is next carried is called *a* and the angle opposite the starting line is called *b*. These are the length angles. The azimuth angle, or the angle between the two lines through which the length is carried, is called *c*.

The following triangles (fig. 34) are laid out in exactly the same manner as for adjustment by the direction method.

As was the case for the direction method, it is necessary to compute a set of preliminary triangles (fig. 35) in order to be able to compute the geographic positions (fig. 36) and determine the latitude and longitude closures. To compute the positions properly the triangles should be closed and this is done by concluding the *c* or azimuth angle in each triangle. If one of the length angles is not observed, however, then the observed azimuth angle must be used, and the unobserved length angle concluded. As the triangles and positions are computed in exactly the same manner as when the direction method is used it is not necessary to explain the computation here.

NUMBER AND FORMATION OF EQUATIONS

As the figure to be adjusted is simply a chain of triangles, it is obvious that there will be just as many angle equations as there are closed triangles, 8 in this case, and that there will be no side equations.

Angle equations

1. $0=-2.3+(1a)+(1b)+(1c)$
2. $0=-2.7+(2a)+(2b)+(2c)$
3. $0=+0.1+(3a)+(3b)+(3c)$
4. $0=+1.0+(4a)+(4b)+(4c)$
5. $0=+5.4+(5a)+(5b)+(5c)$
6. $0=+2.9+(6a)+(6b)+(6c)$
7. $0=-0.1+(7a)+(7b)+(7c)$
8. $0=+3.3+(8a)+(8b)+(8c)$

Since there are two fixed azimuths, namely, the azimuths of the lines Alberta-Fish and Enter-Flores, there will be one azimuth equation. This equation which is shown on page 125, is formed in exactly the same manner as if the direction method were used, except in the way the angles are designated.

(Text continued on p. 125)

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 24A

LIST OF DIRECTIONS

State: Alaska

Station Alberta — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Fish	0 00 00.0	00.0
Fog	40 09 09.1	09.1
Nan	57 22 26.5	26.4

Station Fish — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Fog	0 00 00.0	$\overline{59.7}$
Nan	24 54 02.9	02.2
Alberta	118 48 46.6	47.5

Station Nan — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Alberta	0 00 00.0	$\overline{59.0}$
Fish	28 42 47.6	47.4
Fog	109 26 24.4	25.5
Dug	129 52 16.6	17.0
Gura	171 17 11.1	11.1

Station Fog — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Dug	0 00 00.0	00.1
Gura	65 49 45.3	44.5
Nan	129 24 59.9	59.4
Alberta	182 45 14.9	15.7
Fish	203 47 17.6	18.9

Station Gura — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Nan	0 00 00.0	$\overline{59.9}$
Fog	54 33 58.8	59.4
Dug	93 03 00.7	01.1
Mond	109 08 46.2	46.9
Lat	172 30 45.7	44.0

Station Dug — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Lat	0 00 00.0	02.0
Gura	61 35 02.7	01.7
Nan	107 07 08.4	06.4
Fog	137 16 16.5	15.6

Station Lat — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Gura	0 00 00.0	01.1
Dug	38 57 17.8	18.6
Mond	60 10 27.4	27.7
Flores	94 59 06.6	09.3
Enter	109 18 30.8	28.7

Station Mond — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Flores	0 00 00.0	04.7
Enter	20 54 53.5	50.3
Lat	115 25 02.3	00.6
Gura	171 52 38.3	36.9

Station Enter — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Lat	0 00 00.0	$\overline{57.9}$
Mond	36 21 46.8	47.2
Flores	100 12 19.9	21.6

Station Flores — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Enter	0 00 00.0	$\overline{59.1}$
Lat	65 28 20.7	16.0
Mond	95 14 37.7	38.6

FIG. 33.—Lists of directions, angle method of adjustment

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: .. Alaska

NO.			STATION	OBSERVED ANGLE			CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHI
		2-3	Alberta-Fish								3.669907
1	b	1	Nan	28	42	47.6	+0.8	48.4		48.4	0.318371
	a	2	Alberta	57	22	26.5	-0.1	26.4		26.4	9.925419
	c	3	Fish	93	54	43.7	+1.6	45.3	-0.1	45.2	9.998986
		1-3	Nan-Fish				+2.3		0.1		3.913697
		1-2	Nan-Alberta								3.937264
						57.8					
		2-3	Nan-Fish								3.913697
2	b	1	Fog	74	22	17.7	+1.8	19.5		19.5	0.016360
	c	2	Nan	80	43	36.8	+1.3	38.1	0.1	38.0	9.994287
	a	3	Fish	24	54	02.9	-0.4	02.5		02.5	9.624330
		1-3	Fog-Fish				+2.7		0.1		3.924344
		1-2	Fog-Nan								3.554387
						57.4					
		2-3	Nan-Fog								3.554387
3	b	1	Gura	54	33	58.8	+0.7	59.5		59.5	0.088965
	a	2	Nan	61	50	46.7	-1.1	45.6		45.6	9.945312
	c	3	Fog	63	35	14.6	+0.3	14.9		14.9	9.952121
		1-3	Gura-Fog				-0.1		0.0		3.588654
		1-2	Gura-Nan								3.595463
						00.1					
		2-3	Gura-Fog								3.588654
4	b	1	Dug	75	41	13.8	+0.1	13.9		13.9	0.013694
	c	2	Gura	38	29	01.9	-0.2	01.7		01.7	9.793995
	a	3	Fog	65	49	45.3	-0.9	44.4		44.4	9.960151
		1-3	Dug-Fog				-1.0		0.0		3.396343
		1-2	Dug-Gura								3.562499
						01.0					

FIG. 34.—Triangle computation, angle method of adjustment

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Alaska

NO.			STATION	OBSERVED ANGLE			CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
		2-3	Gura-Dug								3.562499
5	b	1	Lat	38	57	17.8	-0.3	17.5		17.5	0.301551
	c	2	Gura	79	27	45.0	-2.1	42.9	0.1	42.8	9.993613
	a	3	Dug	61	35	02.7	-3.0	59.7		59.7	9.944240
		1-3	Lat-Dug				-5.4		0.1		3.756663
		1-2	Lat-Gura								3.708290
						05.5					
		2-3	Lat-Gura								3.708290
6	b	1	Mond	56	27	36.0	+0.3	36.3		36.3	0.079094
	c	2	Lat	60	10	27.4	-0.3	26.6		26.6	9.938290
	a	3	Gura	63	21	59.5	-2.4	57.1		57.1	9.951283
		1-3	Mond-Gura				-2.9		0.0		3.725674
		1-2	Mond-Lat								3.738667
						02.9					
		2-3	Lat-Mond								3.738667
7	b	1	Enter	36	21	46.3	+2.5	49.3		49.3	0.227012
	a	2	Lat	49	08	03.4	-2.4	01.0		01.0	9.878658
	c	3	Mond	94	30	09.8	0.0	09.8	0.1	09.7	9.998658
		1-3	Enter-Mond				+0.1		0.1		3.844337
		1-2	Enter-Lat								3.964337
						00.0					
		2-3	Enter-Mond								3.844337
8	b	1	Flores	95	14	37.7	+1.8	39.5		39.5	0.001822
	c	2	Enter	63	50	33.1	+1.3	34.4		34.4	9.953077
	a	3	Mond	20	54	52.5	-6.4	46.1		46.1	9.552603
		1-3	Flores-Mond				-3.3		0.0		3.799236
		1-2	Flores-Enter								3.398762
						03.3					

FIG. 34.—Triangle computation, angle method of adjustment—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

Preliminary triangles COMPUTATION OF TRIANGLES

State: Alaska

No.			Station	Observed angle			Corr'n	Spher'l angle	Spher'l excess	Plane angle and distance	Logarithm
		2-3	Alberta-Fish								3.669907
1	b	1	Nan	28	42	47.6					0.318374
	a	2	Alberta	57	22	26.5					9.925419
		3	Fish	(93	54	46.0)			0.1	45.9	9.998987
		1-3	Nan-Fish						0.1		3.913700
		1-2	Nan-Alberta								3.987268
		2-3	Nan-Fish								3.913700
2	b	1	Fog	74	22	17.7					0.016361
		2	Nan	(80	43	39.5)			0.1	39.4	9.994288
	a	3	Fish	24	54	02.9					9.624332
		1-3	Fog-Fish						0.1		3.924349
		1-2	Fog-Nan								3.554393
		2-3	Nan-Fog								3.554393
3	b	1	Gura	54	33	58.8					0.088956
	a	2	Nan	61	50	46.7					9.945314
		3	Fog	(63	35	14.5)					9.952121
		1-3	Gura-Fog								3.588663
		1-2	Gura-Nan								3.595470
		2-3	Gura-Fog								3.588663
4	b	1	Dug	75	41	13.8					0.013694
		2	Gura	(38	29	00.9)					9.793993
	a	3	Fog	65	49	45.3					9.960152
		1-3	Dug-Fog								3.396350
		1-2	Dug-Gura								3.562509

FIG. 35.—Triangle computation to obtain latitude and longitude closures of net, angle method of adjustment

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 25

COMPUTATION OF TRIANGLES

State: Alaska

NO.		STATION		OBSERVED ANGLE	CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
		2-3	Gura-Dug						3.562509
5	b	1	Lat	38 57 17.8					0.201550
		2	Gura	(79 27 39.6)			0.1	39.5	9.992611
	a	3	Dug	61 35 02.7					9.944244
		1-3	Lat-Dug				0.1		3.756670
		1-2	Lat-Gura						3.708303
		2-3	Lat-Gura						3.708303
6	b	1	Mond	56 27 36.0					0.079094
		2	Lat	(60 10 24.5)					9.938287
	a	3	Gura	63 21 59.5					9.951285
		1-3	Mond-Gura						3.725684
		1-2	Mond-Lat						3.738682
		2-3	Lat-Mond						3.738682
7	b	1	Enter	36 21 46.8					0.227019
	a	2	Lat	49 08 03.4					9.878662
		3	Mond	(94 30 09.9)			0.1	09.8	9.998658
		1-3	Enter-Mond				0.1		3.844363
		1-2	Enter-Lat						3.964359

FIG. 35.—Triangle computation to obtain latitude and longitude closures of net, angle method of adjustment—Continued

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

						°	′	″
α	2	Alberta	to 3	Fish		356	02	14.8
2d ∠		Fish	&	Nan		+ 57	22	26.5
α	2	Alberta	to 1	Nan		53	24	41.3
Δα						−	6	05.8
						180	00	00.00
α′	1	Nan	to 2	Alberta		233	18	35.5-1
					First Angle of Triangle	28	42	47.6

	°	′	″				°	′	″
φ	55	31	40.715	2	Alberta	λ	133	11	33.444
Δφ	−	3	07.393	s=		Δλ	+	7	23.898
φ′	55	28	33.322	1	Nan	λ′	133	18	57.342 −1

½(φ+φ′)	° ′ ″ 55 30 07.0	s	3.987268	s²	7.97454		
		Cos α	9.775293	Sin² α	9.80936	h²	4.544
		B	8.509672	C	1.566:6	D	2.363
		h	2.272233		9.35016		6.907
					0.2240		0.0008

	″
1st term	+187.1686
2d and 3d terms	+ 0.2:48
−Δφ	+187.3934

s	3.987268		
Sin α	9.904681		
A′	8.508727	Δλ	2.647283
sec φ′	0.246607	Sin ½ (φ+φ′)	9.916004
	2.647:33		2.563287
Δλ	+ 443.8978″	−Δα	+ 365.84″

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					°	′	″
α	3	Fish	to 2	Alberta	176	02	29.9
3d ∠		Nan	&	Alberta	− 93	54	46.0
α	3	Fish	to 1	Nan	82	07	43.9
Δα					−	6	20.9
					180	00	00.00
α′	1	Nan	to 3	Fish	262	01	23.0

	°	′	″				°	′	″
φ	55	29	09.866	3	Fish	λ	133	11	15.042
Δφ	−		36.544	s=		Δλ	+	7	42.299
φ′	55	28	33.322	1	Nan	λ′	133	18	57.341

½(φ+φ′)	° ′ ″ 55 28 51.6	s	3.913700	s²	7.82740		
		Cos α	9.136548	Sin² α	9.99178	h²	3.120
		B	8.509676	C	1.56558	D	2.363
		h	1.559924		9.38476		5.483
					0.2425		0.0000

	″
1st term	+ 36.3014
2d and 3d terms	+ 0.2425
−Δφ	+ 36.5439

s	3.913700		
Sin α	9.995889		
A′	8.508727	Δλ	2.664923
sec φ′	0.246607	Sin ½ (φ+φ′)	9.915895
	2.664923		2.580818
Δλ	+ 462.2990″	−Δα	+ 380.91″

FIG. 36.—Preliminary position computation to obtain latitude and longitude closures, angle method of adjustment

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

			°	′	″
α	2 Nan	to 3 Fish	262	01	23.0
2d ∠	Fish	& Fog	+ 80	43	39.5
α	2 Nan	to 1 Fog	342	45	02.5
Δα			+		49.8
			180	00	00.00
α′	1 Fog	to 2 Nan	162	45	52.3
	First Angle of Triangle		74	22	17.7

	°	′	″			°	′	″
φ	55	28	33.322	2 Nan	λ	133	18	57.341
Δφ	−	1	50.688	s=	Δλ	−	1	00.458
φ′	55	26	42.634	1 Fog	λ′	133	17	56.883 −1

½(φ+φ′)	55° 27′ 38.0″	s	3.554393	s²	7.10879	h²	4.088
		Cos α	9.980014	Sin²α	8.94414		
		B	8.509676	C	1.56544	D	2.363
1st term	+110.6835″	h	2.044083		7.61837		6.451
2d and 3d terms	+ 0.0045				0.0042		0.0003
−Δφ	+110.6880						

s	3.554393		
Sin α	9.472069		
A′	8.508727	Δλ	1.781457
sec φ′	0.246268	Sin ½ (φ+φ′)	9.915788
	1.781457		1.697245
Δλ	−60.4584″	−Δα	−49.80″

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			°	′	″
α	3 Fish	to 2 Nan	82	07	43.9
3d ∠	Fog	& Nan	− 24	54	02.9
α	3 Fish	to 1 Fog	57	13	41.0
Δα			−	5	31.0
			180	00	00.00
α′	1 Fog	to 3 Fish	237	08	10.0

	°	′	″			°	′	″
φ	55	29	09.866	3 Fish	λ	133	11	15.042
Δφ	−	2	27.232	s=	Δλ	+	6	41.840
φ′	55	26	42.634	1 Fog	λ′	133	17	56.882

½(φ+φ′)	55° 27′ 56.2″	s	3.924349	s²	7.84870	h²	4.335
		Cos α	9.733435	Sin²α	9.84942		
		B	8.509676	C	1.56558	D	2.363
1st term	+147.0483″	h	2.167460		9.26370		6.698
2d and 3d terms	+ 0.1840				0.1835		0.0005
−Δφ	+147.2323						

s	3.924349		
Sin α	9.924709		
A′	8.508727	Δλ	2.604053
sec φ′	0.246268	Sin ½ (φ+φ′)	9.915814
	2.604053		2.519867
Δλ	+401.8398″	−Δα	+331.03″

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FIG. 36.—Preliminary position computation to obtain latitude and longitude closures, angle method of adjustment—Continued

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

				°	′	″
a	2 Nan	to 3	Fog	342	45	02.5
2d ∠	Fog	&	Gura	+ 61	50	46.7
a	2 Nan	to 1	Gura	44	35	49.2
Δa				−	2	09.6
				180	00	00.00
a′	1 Gura	to 2	Nan	224	33	39.6
	First Angle of Triangle			54	33	58.8

	°	′	″				°	′	″
φ	55	28	33.322	2	Nan	λ	133	18	57.341
Δφ	−	1	30.740	s−		Δλ	+	2	37.375
φ′	55	27	02.582	1	Gura	λ′	133	21	34.716
									−1

½(φ+φ′) 55° 27′ 48.0″	s	3.595470	s²	7.19094	h²	3.915
	Cos a	9.852518	Sin² a	9.69282	D	2.363
	B	8.509676	C	1.56544		6.278
	h	1.957664		8.44920		0.0002
				0.0281		

1st term	+ 90.7119
2d and 3d terms	+ 0.0283
−Δφ	+ 90.7402

s	3.595470		
Sin a	9.846409		
A′	8.508727	Δλ	2.196935
sec φ′	0.246329	Sin ½ (φ+φ′)	9.915803
	2.196935		2.112738
Δλ	+ 157.3747″	−Δa	+ 129.64″

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				°	′	″
a	3 Fog	to 2	Nan	162	45	52.3
2d ∠	Gura	&	Nan	− 63	35	14.5
a	3 Fog	to 1	Gura	99	10	37.8
Δa				−	2	59.4
				180	00	00.00
a′	1 Gura	to 3	Fog	279	07	38.4

	°	′	″				°	′	″
φ	55	26	42.634	3	Fog	λ	133	17	56.882
Δφ	+		19.948	s−		Δλ	+	3	37.833
φ′	55	27	02.582	1	Gura	λ′	133	21	34.715

½(φ+φ′) 55° 26′ 52.6″	s	3.583663	s²	7.17733	h²	2.602
	Cos a	9.202727	Sin² a	9.98881	D	2.364
	B	8.509678	C	1.56492		4.966
	h	1.301068		8.73106		0.0000
				0.0538		

1st term	−20.0018
2d and 3d terms	+ 0.0538
−Δφ	−19.9480

s	3.583663		
Sin a	9.994405		
A′	8.508727	Δλ	2.338124
sec φ′	0.246329	Sin ½ (φ+φ′)	9.915722
	2.338124		2.253846
Δλ	+ 217.3332″	−Δa	+ 179.41″

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FIG. 36.—Preliminary position computation to obtain latitude and longitude closures, angle method of adjustment—Continued

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

					°	′	″
α	2	Gura	to 3	Fog	279	07	38.4
2d ∠		Fog	&	Dug	+ 38	29	00.9
α	2	Gura	to 1	Dug	317	36	39.3
Δα					+	1	55.3
					180	00	00.00
α′	1	Dug	to 2	Gura	137	38	34.6
					75	41	13.8

First Angle of Triangle

	°	′	″				°	′	″
φ	55	27	02.582	2	Gura	λ	133	21	34.715
Δφ	−	1	27.237	s−		Δλ	−	2	19.979
φ′	55	25	35.345+1	1	Dug	λ′	133	19	14.736

½(φ+φ′)	55° 26′ 19.0″	s	3.562509	s²	7.12502		
		Cos α	9.868400	Sin² α	9.65753	h²	3.881
		B	8.509678	C	1.56501	D	2.364
1st term	+87.2142″	h	1.940587		8.34756		6.245
2d and 3d terms	+0.0225				0.0233		0.0002
−Δφ	+87.2367						

s	3.562509		
Sin α	9.828764		
A′	8.508728	Δλ	2.146063
sec φ′	0.246062	Sin ½ (φ+φ′)	9.915674
	2.146063		2.061737
Δλ	−139.9790″	−Δα	−115.27″

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					°	′	″
α	3	Fog	to 2	Gura	99	10	37.8
3d ∠		Dug	&	Gura	−65	49	45.3
α	3	Fog	to 1	Dug	33	20	52.5
Δα					−	1	04.1
					180	00	00.00
α′	1	Dug	to 3	Fog	213	19	48.4

	°	′	″				°	′	″
φ	55	26	42.634	3	Fog	λ	133	17	56.882
Δφ	−	1	07.288	s−		Δλ	+	1	17.854
φ′	55	25	35.346	1	Dug	λ′	133	19	14.736

½(φ+φ′)	55° 26′ 09.0″	s	3.396350	s²	6.79270		
		Cos α	9.921867	Sin² α	9.48029	h²	3.656
		B	8.509678	C	1.56492	D	2.364
1st term	+67.2814″	h	1.827895		7.83791		6.020
2d and 3d terms	+0.0070				0.0069		0.0001
−Δφ	+67.2884						

s	3.396350		
Sin α	9.740143		
A′	8.508728	Δλ	1.891283
sec φ′	0.246062	Sin ½ (φ+φ′)	9.915659
	1.891283		1.806942
Δλ	+77.8544″	−Δα	+64.11″

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FIG. 36.—Preliminary position computation to obtain latitude and longitude closures, angle method of adjustment—Continued

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

						°	′	″
α	2	Gura	to 3	Dug		317	36	39.3
2d ∠		Dug	&	Lat		+ 79	27	38.6
α	2	Gura	to 1	Lat		37	04	18.9
Δα						−	2	24.1
						180	00	00.00
α′	1	Lat	to 2	Gura		217	01	54.8
						38	57	17.8

FIRST ANGLE OF TRIANGLE

	°	′	″				°	′	″
φ	55	27	02.582	2	Gura	λ	133	21	34.715
Δφ	−	2	11.836	s =		Δλ	+	2	55.042
φ′	55	24	50.746	1	Lat	λ′	133	24	29.757

½(φ+φ′)	55° 25′ 56.7″	s	3.708303	s^2	7.41661		
		Cos α	9.901937	Sin² α	9.56037	h²	4.240
		B	8.509678	C	1.56501	D	2.364
1st term	+ 131.8008″	h	2.115918		8.54199		6.604
2d and 3d terms	+ 0.0352				0.0348		0.0004
−Δφ	+ 131.8360						

s	3.708303		
Sin α	9.780186		
A′	8.508728	Δλ	2.243143
sec φ′	0.245926	Sin ½ (φ+φ′)	9.915641
	2.243143		2.158784
Δλ	+ 175.0423″	−Δα	+ 144.14″

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						°	′	″
α	3	Dug	to 2	Gura		137	38	34.6
3d ∠		Lat	&	Gura		− 61	35	02.7
α	3	Dug	to 1	Lat		76	03	31.9
Δα						−	4	19.4
						180	00	00.00
α′	1	Lat	to 3	Dug		255	59	12.5 +1

	°	′	″				°	′	″
φ	55	25	35.346	3	Dug	λ	133	19	14.736
Δφ	−		44.600	s =		Δλ	+	5	15.021
φ′	55	24	50.746	1	Lat	λ′	133	24	29.757

½(φ+φ′)	55° 25′ 13.0″	s	3.756670	s^2	7.51334		
		Cos α	9.381882	Sin² α	9.97403	h²	3.296
		B	8.509680	C	1.56463	D	2.364
1st term	+44.4869″	h	1.648232		9.05200		5.660
2d and 3d terms	+0.1127				0.1127		0.0000
−Δφ	+44.5996						

s	3.756670		
Sin α	9.987015		
A′	8.508728	Δλ	2.498339
sec φ′	0.245928	Sin ½ (φ+φ′)	9.915578
	2.498339		2.413917
Δλ	+ 315.0207″	−Δα	+ 259.27″

FIG. 36.—Preliminary position computation to obtain latitude and longitude closures, angle method of adjustment—Continued

45861°—34——9

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

						°	′	″
α	2	Lat	to 3	Gura		217	01	54.8
2d ∠		Gura	&	Mond		+ 60	10	24.5
α	2	Lat	to 1	Mond		277	12	19.3
Δα						+	4	14.3
						180	00	00.00
α′	1	Mond	to 2	Lat		97	16	33.6
					First Angle of Triangle	56	27	36.0

	°	′	″				°	′	″
φ	55	24	50.746	2	Lat	λ	133	24	29.757
Δφ	−		22.329	s−		Δλ	−	5	08.905
φ′	55	24	28.417	1	Mond	λ′	133	19	20.852

½(φ+φ′)	55° 24′ 39.6″	s	3.738582	s^2	7.47736		
		Cos α	9.093388	Sin² α	9.99311	h²	2.694
		B	8.509681	C	1.56443	D	2.364
1st term	″ +22.2204	h	1.346751		9.03490		5.058
2d and 3d terms	+0.1084				0.1084		0.0000
−Δφ	+22.3288						

s	3.738682		
Sin α	9.996557		
A′	8.508723	Δλ	2.489825
sec φ′	0.245858	Sin ½ (φ+φ′)	9.915529
	2.489825		2.405354
Δλ	″ −308.9050	−Δα	″ −254.30

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					°	′	″
α	3	Gura	to 2	Lat	37	04	18.9
3d ∠		Mond	&	Lat	− 63	21	59.5
α	3	Gura	to 1	Mond	333	42	19.4
Δα					+	1	50.2
					180	00	00.00
α′	1	Mond	to 3	Gura	153	44	09.6

	°	′	″				°	′	″
φ	55	27	02.582	3	Gura	λ	133	21	34.715
Δφ	−	2	34.165	s−		Δλ	−	2	13.863
φ′	55	24	28.417	1	Mond	λ′	133	19	20.852

½(φ+φ′)	55° 25′ 45.5″	s	3.725684	s^2	7.45137		
		Cos α	9.952564	Sin² α	9.29278	h²	4.376
		B	8.509678	C	1.56501	D	2.364
1st term	″ +154.1438	h	2.187926		8.30916		6.740
2d and 3d terms	+ 0.0209				0.0204		0.0005
−Δφ	+154.1647						

s	3.725684		
Sin α	9.646391		
A′	8.508723	Δλ	2.126661
sec φ′	0.245858	Sin ½ (φ+φ′)	9.915625
	2.126661		2.042286
Δλ	″ −133.8631	−Δα	″ −110.23

FIG. 36.—Preliminary position computation to obtain latitude and longitude closures, angle method of adjustment—Continued

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

					°	′	″
α	2 Lat	to 3	Mond		277	12	19.3
2d ∠	Mond	&	Enter		+ 49	08	03.4
α	2 Lat	to 1	Enter		326	20	22.7
Δα					+	3	58.4
					180	00	00.00
α′	1 Enter	to 2	Lat		146	24	21.1
					36	21	46.8

FIRST ANGLE OF TRIANGLE

	°	′	″			°	′	″
φ	55	24	50.746	2 Lat	λ	133	24	29.757
Δφ	-	4	08.033	s =	Δλ	-	4	49.720
φ′	55	20	42.713	1 Enter	λ′	133	19	40.037
Fixed	55	20	42.730			133	19	40.049

½(φ+φ′)	° ′ ″ 55 22 46.7	s	3.964359	s^2	7.92872		
		Cos α	9.920300	$\sin^2\alpha$	9.48744	h^2	4.789
		B	8.509681	C	1.56443	D	2.364
1st term	″ +247.9362	h	2.394340		8.98059		7.153
2d and 3d terms	+ 0.0970				0.0956		0.0014
−Δφ	+248.0332						

s	3.964359		
Sin α	9.743720		
A′	8.508730	Δλ	2.461979
sec φ′	0.245170	Sin ½ (φ+φ′)	9.915365
	2.461979		2.377344
Δλ	″ -289.7203	−Δα	″ -238.42

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					°	′	″
α	3 Mond	to 2	Lat		97	16	33.6
3d ∠	Enter	&	Lat		- 94	30	09.9
α	3 Mond	to 1	Enter		2	46	23.7
Δα					-		15.8
					180	00	00.00
α′	1 Enter	to 3	Mond		182	46	07.9

	°	′	″			°	′	″
φ	55	24	28.417	3 Mond	λ	133	19	20.852
Δφ	-	3	45.703	s =	Δλ	+		19.185
φ′	55	20	42.714 −1	1 Enter	λ′	133	19	40.037

½(φ+φ′)	° ′ ″ 55 22 35.6	s	3.844363	s^2	7.68873		
		Cos α	9.999491	$\sin^2\alpha$	7.36939	h^2	4.707
		B	8.509681	C	1.56433	D	2.364
1st term	″ +225.7018	h	2.353535		6.62245		7.071
2d and 3d terms	+ 0.0016				0.0004		0.0012
−Δφ	+225.7034						

s	3.844363		
Sin α	8.684697		
A′	8.508730	Δλ	1.282960
sec φ′	0.245170	Sin ½ (φ+φ′)	9.915349
	1.282960		1.198309
Δλ	″ + 19.1849	−Δα	″ + 15.79

FIG. 36.—Preliminary position computation to obtain latitude and longitude closures, angle method of adjustment—Continued

Azimuth equation

	°	′	″	Final seconds [1]
Fish-Alberta	176	02	29.9	29.9
−1c	−93	54	43.7	45.3
Fish-Nan	82	07	46.2	44.6
	−	6	20.9	20.9
Nan-Fish	262	01	25.3	23.7
+2c	+80	43	36.8	38.1
Nan-Fog	342	45	02.1	01.8
	+		49.8	49.8
Fog-Nan	162	45	51.9	51.6
−3c	−63	35	14.6	14.9
Fog-Gura	99	10	37.3	36.7
	−	2	59.4	59.4
Gura-Fog	279	07	37.9	37.3
+4c+5c	+117	56	46.9	44.6
Gura-Lat	37	04	24.8	21.9
	−	2	24.1	24.1
Lat-Gura	217	02	00.7	57.8
+6c	+60	10	27.4	26.6
Lat-Mond	277	12	28.1	24.4
	+	4	14.3	14.3
Mond-Lat	97	16	42.4	38.7
−7c	−94	30	09.8	09.8
Mond-Enter	2	46	32.6	28.9
	−		15.8	15.8
Enter-Mond	182	46	16.8	13.1
+8c	+63	50	33.1	34.4
Enter-Flores (computed)	246	36	49.9	47.5
Enter-Flores (fixed)	246	36	47.5	47.5
			+2.4	0.0

9. $0 = +2.4 - (1c) + (2c) - (3c) + (4c) + (5c) + (6c) - (7c) + (8c)$

Since there are two fixed lengths namely, Alberta to Fish and Enter to Flores, there will be one length equation. This equation, which is shown below, is formed in exactly the same manner as if the direction method were used, except in the way the angles are designated.

Length equation

Fish-Alberta				3.669907	----------	Enter-Flores				3.398762	----------
	°	′	″				°	′	″		
1a	57	22	26.5	9.9254194	+1.3	1b	28	42	47.6	9.6816265	+3.8
2a	24	54	02.9	9.6243321	+4.5	2b	74	22	17.7	9.9836394	+.6
3a	61	50	46.7	9.9453135	+1.1	3b	54	33	58.8	9.9110442	+1.5
4a	65	49	45.3	9.9601516	+.9	4b	75	41	13.8	9.9863060	+.5
5a	61	35	02.7	9.9442440	+1.1	5b	38	57	17.8	9.7984497	+2.6
6a	63	21	59.5	9.9512853	+1.1	6b	56	27	36.0	9.9209058	+1.4
7a	49	08	03.4	9.8786625	+1.8	7b	36	21	46.8	9.7729807	+2.9
8a	20	54	52.5	9.5526388	+5.5	8b	95	14	37.7	9.9981786	−.2
				2.4519542						2.4518929	

10. $0 = +61.3 + 1.3(1a) - 3.8\,(1b) + 4.5(2a) - 0.6(2b) + 1.1(3a) - 1.5(3b) + 0.9(4a) - 0.5(4b) + 1.1(5a) - 2.6(5b) + 1.1(6a) - 1.4(6b) + 1.8(7a) - 2.9(7b) + 5.5(8a) + 0.2(8b)$

[1] The values in this column are filled in after the adjustment is completed.

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U. S. COAST AND GEODETIC SURVEY
FORM 486

11—6101

LATITUDE AND LONGITUDE ADJUSTMENT

φ_c: 55° 20′.71 λ_c: 133° 19′.67

	Station	φ_s	λ_s	$\varphi_n-\varphi_s$	$\lambda_n-\lambda_s$	$+\delta_A$	$-\delta_B$	Lat. equation $(\varphi_n-\varphi_c)\delta_A$	A	Long. equation $(\lambda_n-\lambda_c)\delta_A$	Lat. equation $(\varphi_n-\varphi_c)(-\delta_B)$	B	Long. equation $(\lambda_n-\lambda_c)(-\delta_B)$	Lat. equation $(\lambda_n-\lambda_c)a_1$	C (+R or −L)	Long. equation $(\varphi_n-\varphi_c)a_2$
•	Fish	55° 29′.16	133° 11′.25	−8.45	+8.42	+1.3	−3.8	−11.0	1 a	+10.9	+32.1	1 b	−32.0	+10.1	+1a + 1b	+31.3
◆	Nan	55 28.56	133 18.96	−7.85	+0.71	+4.5	−0.6	−35.3	2 a	+ 3.2	+ 4.7	2 b	− 0.4	+ 0.9	+2a − 2b	+29.0
−	Fog	55 26.71	133 17.95	−6.00	+1.72	+1.1	−1.5	− 6.6	3 a	+ 1.9	+ 9.0	3 b	− 2.6	+ 2.1	+3a + 3b	+22.2
+	Gura	55 27.04	133 21.58	−6.33	−1.91	+0.9	−0.5	− 5.7	4 a	− 1.7	+ 3.2	4 b	+ 1.0	− 2.3	−4a − 4b	+23.4
◆	Gura	55 27.04	133 21.58	−6.33	−1.91	+1.1	−2.6	− 7.0	5 a	− 2.1	+16.5	5 b	+ 5.0	− 2.3	−5a − 5b	+23.4
+	Lat	55 24.85	133 24.50	−4.14	−4.83	+1.1	−1.4	− 4.6	6 a	− 5.3	+ 5.8	6 b	+ 6.8	− 5.8	−6a − 6b	+15.3
−	Mond	55 24.47	133 19.35	−3.76	+0.32	+1.8	−2.9	− 6.8	7 a	+ 0.6	+10.9	7 b	− 0.9	+ 0.4	+7a + 7b	+13.9
	Enter	55 20.71	133 19.67													

φ_n = 55° 20′ 42″.713 λ_n = 133° 19′ 40″.037

φ_n' = 55 20 42.730 λ_n' = 133 19 40.049

$(\varphi_n-\varphi_n')$ = −0.017 $(\lambda_n-\lambda_n')$ = −0.012

×7238.24 ×7238.24

−123.05008 −86.85888

a_1 = +1.20

a_2 = −3.70

11. φ. 0 = −123.0501 + 0.9 (1 a) + 42.2 (1 b) − 36.2 (2 a) + 3.8 (2 b) − 4.5 (3 a) + 11.1 (3 b) − 3.4 (4 a) + 5.5 (4 b) + 4.7 (5 a) + 18.8 (5 b) + 1.2 (6 a) + 11.6 (6 b) − 6.4 (7 a) + 11.3 (7 b).

(This equation should be divided by 10 before entering it in the correlates.)

12. λ. 0 = −86.8589 + 42.2 (1 a) − 0.7 (1 b) − 25.8 (2 a) + 29.4 (2 b) + 24.1 (3 a) + 19.6 (3 b) − 25.1 (4 a) − 22.4 (4 b) − 25.5 (5 a) − 18.4 (5 b) − 20.6 (6 a) − 8.5 (6 b) + 14.5 (7 a) + 13.0 (7 b).

(This equation should be divided by 10 before entering it in the correlates.)

φ	a_1 (+)	a_2 (−)
0	2.12	2.09
1	2.12	2.09
2	2.12	2.09
3	2.12	2.09
4	2.11	2.10
5	2.11	2.10
6	2.11	2.10
7	2.10	2.11
8	2.10	2.11
9	2.09	2.12
10	2.09	2.12
11	2.08	2.13
12	2.07	2.14
13	2.06	2.15
14	2.06	2.16
15	2.05	2.17
16	2.04	2.18
17	2.03	2.19
18	2.01	2.20
19	2.00	2.21
20	[illegible]	[illegible]
21	[illegible]	[illegible]
22	[illegible]	[illegible]
23	[illegible]	[illegible]
24	[illegible]	[illegible]
25	[illegible]	[illegible]
26	[illegible]	[illegible]
27	[illegible]	[illegible]
28	[illegible]	[illegible]
29	[illegible]	[illegible]
30	1.81	[illegible]
31	[illegible]	[illegible]
32	1.79	2.47
33	1.77	2.50
34	1.75	[illegible]
35	1.73	[illegible]
36	1.71	[illegible]
37	[illegible]	[illegible]
38	[illegible]	[illegible]
39	[illegible]	2.70
40	[illegible]	2.74
41	[illegible]	[illegible]
42	[illegible]	[illegible]
43	[illegible]	[illegible]
44	[illegible]	[illegible]
45	[illegible]	[illegible]
46	1.47	[illegible]
47	[illegible]	[illegible]
48	[illegible]	[illegible]
49	[illegible]	[illegible]
50	[illegible]	[illegible]
51	[illegible]	[illegible]
52	[illegible]	[illegible]
53	[illegible]	[illegible]
54	[illegible]	[illegible]
55	[illegible]	[illegible]
56	[illegible]	[illegible]
57	1.14	[illegible]
58	[illegible]	[illegible]
59	[illegible]	[illegible]
60	[illegible]	[illegible]
61	[illegible]	[illegible]
62	[illegible]	[illegible]
63	[illegible]	[illegible]
64	[illegible]	[illegible]
65	[illegible]	[illegible]
66	[illegible]	[illegible]
67	[illegible]	[illegible]
68	[illegible]	[illegible]
69	[illegible]	[illegible]
70	[illegible]	[illegible]
71	[illegible]	[illegible]
72	[illegible]	[illegible]

$\frac{M}{60}\times10^4$ = 7238.24

FIG. 37.—Formation of latitude and longitude equations, angle method of adjustment

Since the lines Alberta-Fish and Enter-Flores are fixed in position (latitude and longitude) there will be one latitude and one longitude equation. These equations, which are shown in Figure 37, are formed in exactly the same manner as when the direction method is used, except in the way the angles are designated.

In the adjustment of this net of triangulation there will be 8 angle, 1 azimuth, 1 length, 1 latitude, and 1 longitude equations, or a total of 12.

As the correlate and normal equations are formed exactly as in the case of the direction method (see p. 84), they are given below without explanation.

Correlate equations

	1	2	3	4	5	6	7	8	α 9	l 10	φ 11	λ 12	Σ_o	v	Adopted v	v^2
1a	+1									+1.3	−0.09	+4.22	+6.43	−0.102	−0.1	0.01
b	+1									−3.8	+4.22	−.07	+1.35	+.778	+.8	.64
c	+1								−1				0	+1.623	+1.6	2.56
2a		+1								+4.5	−3.62	−2.58	−0.70	−.425	−.4	.16
b		+1								−.6	+.38	−2.94	−2.16	+1.821	+1.8	3.24
c		+1							+1				+2.00	+1.302	+1.3	1.69
3a			+1							+1.1	−.45	+2.41	+4.06	−1.063	−1.1	1.21
b			+1							−1.5	+1.11	+1.96	+2.57	+.697	+.7	.49
c			+1						−1				0	+.266	+.3	.09
4a				+1						+.9	−.34	−2.51	−.95	−.902	−.9	.81
b				+1						−.5	+.55	−2.24	−1.19	+.096	+.1	.01
c				+1					+1				+2.00	−.192	−.2	.04
5a					+1					+1.1	−.47	−2.55	−.92	−2.964	−3.0	9.00
b					+1					−2.6	+1.88	−1.34	−1.56	−.324	−.3	.09
c					+1				+1				+2.00	−2.114	−2.1	4.41
6a						+1				+1.1	+.12	−2.06	+.16	−2.400	−2.4	5.76
b						+1				−1.4	+1.16	−.85	−.09	+.331	+.3	.09
c						+1			+1				+2.00	−.831	−.8	.64
7a							+1			+1.8	−.64	+1.45	+3.61	−2.370	−2.4	5.76
b							+1			−2.9	+1.13	+1.30	+.53	+2.455	+2.5	6.25
c							+1		−1				0	+.014	0	0
8a								+1		+5.5			+6.50	−6.470	−6.4	40.96
b								+1		+.2			+1.20	+1.835	+1.8	3.24
c								+1	+1				+2.00	+1.337	+1.3	1.69
															Total	88.84

$$\text{p.e.} = \pm 0.674\sqrt{\frac{\Sigma v^2}{c}} = \pm 0.674\sqrt{\frac{88.84}{12}} = \pm 1''.83$$

Normal equations

	1	2	3	4	5	6	7	8	α 9	l 10	φ 11	λ 12	η	Σ_n	C
1	+3								−1	−2.5	+4.13	+4.15	−2.3	+5.48	+0.8125
2		+3							+1	+3.9	−3.24	−5.52	−2.7	−3.56	+2.1131
3			+3						−1	−.4	+.66	+4.37	+.1	+6.73	−.5449
4				+3					+1	+.4	+.21	−4.75	+1.0	+.86	+.6186
5					+3				+1	−1.5	+1.41	−4.39	+5.4	+4.92	−1.3028
6						+3			+1	−.3	+1.28	−2.91	+2.9	+4.97	−.0205
7							+3		−1	−1.1	+.49	+2.75	−.1	+4.04	−.7967
8								+3	+1	+5.7			+3.3	+13.00	+2.1475
9									+8				+2.4	+12.40	−.8106
10										+94.34	−46.7380	−5.7790	+61.3	+107.3230	−1.5670
11											+39.7198	+5.3068	−12.3050	−9.0764	−1.4152
12												+72.7274	−8.6359	+57.2693	+.2360

SOLUTION OF NORMAL EQUATIONS

The solution of the normals is much simpler than when the direction method of adjustment is used. This is due to the fact that no two angle equations involve the same v's and the first 8 equations in the example below are eliminated by simply dividing by 3 in each case. There are only 4 equations which are much involved.

Solution of normal equations

1	2	3	4	5	6	7	8	9	10	11	12	η	Σ_n
+3	----	----	----	----	----	----	----	−1	−2.5	+4.13	+4.15	−2.3	+5.48
C_1	----	----	----	----	----	----	----	+.33333	+.83333	−1.37667	−1.38333	+.76667	−1.82667
	+3	----	----	----	----	----	----	+1	+3.9	−3.24	−5.52	−2.7	−3.56
	C_2	----	----	----	----	----	----	−.33333	−1.3	+1.08	+1.84	+.9	+1.18667
		+3	----	----	----	----	----	−1	−.4	+.66	+4.37	+.1	+6.73
		C_3	----	----	----	----	----	+.33333	+.13333	−.22	−1.45667	−.03333	−2.243334
			+3	----	----	----	----	+1	+.4	+.21	−4.75	+1.0	+.86
			C_4	----	----	----	----	−.33333	−.13333	−.07	+1.58333	−.33333	−.286676
				+3	----	----	----	+1	−1.5	+1.41	−4.39	+5.4	+4.92
				C_5	----	----	----	−.33333	+.5	−.47	+1.46333	−1.8	−1.64
					+3	----	----	+1	−.3	+1.28	−2.91	+2.9	+4.97
					C_6	----	----	−.33333	+.1	−.42667	+.97	−.96667	−1.65667
						+3	----	−1	−1.1	+.49	+2.75	−.1	+4.04
						C_7	----	+.33333	+.36667	−.16333	−.91667	+.03333	−1.34667
							+3	+1	+5.7	----	----	+3.3	+13.00
							C_8	−.33333	−1.9	----	----	−1.1	−4.33333
								+8	----	----	----	+2.4	+12.40
							1	−.3333	−.8333	+1.3767	+1.3833	−.7667	+1.8267
							2	−.3333	−1.3	+1.08	+1.84	+.9	+1.1867
							3	−.3333	−.1333	+.22	+1.4567	+.0333	+2.2433
							4	−.3333	−.1333	−.07	+1.5833	−.3333	−.2867
							5	−.3333	+.5	−.47	+1.4633	−1.8	−1.64
							6	−.3333	+.1	−.4267	+.97	−.9667	−1.6567
							7	−.3333	−.3667	+.1633	+.9167	−.0333	+1.3467
							8	−.3333	−1.9	----	----	−1.1	−4.3333
								+5.3336	−4.0666	+1.8733	+9.6133	−1.6667	+11.08679
								C_9	+.76245	−.35122	−1.80240	+.31249	−2.078698
									+94.34	−46.7380	−5.7790	+61.3	+107.3230
								1	−2.0833	+3.4417	+3.4583	−1.9167	+4.5667
								2	−5.07	+4.212	+7.176	+3.51	+4.6280
								3	−.0533	+.088	+.5827	+.0133	+.8973
								4	−.0533	−.028	+.6333	−.1333	−.1147
								5	−.75	+.705	−2.195	+2.7	+2.460
								6	−.03	+.1280	−.291	+.2900	+.4970
								7	−.4033	+.1797	+1.0083	−.0367	+1.4813
								8	−10.83	----	----	−6.27	−24.70
								9	−3.1006	+1.4283	+7.3296	−1.2708	+8.4532
									+71.9662	−36.5833	+11.9232	+58.1858	+105.49189
									C_{10}	+.50834	−.16568	−.80852	−1.465856
										+39.7198	+5.3068	−12.3050	−9.0764
									1	−5.6856	−5.7132	+3.1663	−7.5441
									2	−3.4992	−5.9616	−2.916	−3.8448
									3	−.1452	−.9614	−.022	−1.4806
									4	−.0147	+.3325	−.070	−.0602
									5	−.6627	+2.0633	−2.538	−2.3124
									6	−.5461	+1.2416	−1.2373	−2.1205
									7	−.0800	−.4492	+.0163	−.6599
									9	−.6579	−3.3764	+.5854	−3.8940
									10	−18.5968	+6.0611	+29.5783	+53.6260
										+9.8316	−1.4565	+14.2580	+22.6331
										C_{11}	+.14814	−1.45022	−2.30208
											+72.7274	−8.6859	+57.2693
										1	−5.7408	+3.1817	−7.5807
										2	−10.1568	−4.968	−6.5504
										3	−6.3656	−.1456	−9.8034
										4	−7.5208	+1.5833	+1.3616
										5	−6.4240	+7.902	+7.1996
										6	−2.8227	+2.813	+4.8209
										7	−2.5208	+.0917	−3.7033
										9	−17.3270	+3.0041	−19.9830
										10	−1.9754	−9.6402	−17.4777
										11	−.2158	+2.1122	+3.3530
											+11.6577	−2.7517	+8.905960
											C_{12}	+.23604	−.76396

The back solution, as well as the forward solution, is much shorter than when the direction method is used, as only 4 of the C's are carried back through all the equations.

Back solution

12	11	10	9	8	7	6	5	4	3	2	1
+0.2360	−1.4502	−0.8085	+0.3125	−1.1	+0.0333	−0.9667	−1.8	−0.3333	−0.0333	+0.9	+0.7667
	+.0350	−.0391	−.4254	--------	−.2163	+.2289	+.3454	+.3737	−.3438	+.4342	−.3265
		−.7194	+.4971	--------	+.2311	+.6038	+.6651	+.0991	+.3113	−1.5284	+1.9483
	−1.4152		−1.1949	+2.9773	−.5746	−.1567	−.7835	+.2089	−.2089	+2.0371	−1.3058
		−1.5670		+.2702	−.2702	+.2702	+.2702	+.2702	−.2702	+.2702	−.2702
			−.8106								
				+2.1475	−.7967	−.0205	−1.3028	+.6186	−.5449	+2.1131	+.8125

The C's determined by the back solution are substituted in the correlate equations to determine the v's, which in this case are corrections to the angles and not to directions. The adopted v's are obtained in much the same manner as when the direction method is used and consequently may not be the same as the computed values to the nearest tenth of a second. It is necessary to adopt v's which may differ slightly from the computed values in order to make the triangles consistent. Compare the computed and adopted values for 8a in the following table:

Computation of corrections (v's)

1a	b	c	2a	b	c	3a	b	c	4a	b	c
+0.812	+0.812	+0.812	+2.113	+2.113	+2.113	−0.545	−0.545	−0.545	+0.619	+0.619	+0.619
−2.037	+5.955	+.811	−7.052	+.940	+.811	−1.724	+2.350	+.811	−1.410	+.784	−.811
+.127	−5.972		+5.123	−.538		+.637	−1.571		+.481	−.778	
+.993	−.017	+1.623	−.609	−.694	+1.302	+.569	+.463	+.266	−.592	−.529	−.192
		+1.6			+1.3			+.3			−.2
−.102	+.778		−.425	+1.821		−1.063	+.697		−.902	+.096	
−.1	+.8		−.4	+1.8		−1.1	+.7		−.9	+.1	
5a	**b**	**c**	**6a**	**b**	**c**	**7a**	**b**	**c**	**8a**	**b**	**c**
−1.303	−1.303	−1.303	−0.020	−0.020	−0.020	−0.797	−0.797	−0.797	+2.148	+2.148	+2.148
−1.724	+4.074	−.811	−1.724	+2.194	−.811	−2.821	+4.544	+.811	−8.618	−.313	−.811
+.665	−2.661		−.170	−1.642		+.906	−1.599				
−.602	−.434	−2.114	−.486	−.201	−.831	+.342	+.307	+.014	−6.470	+1.835	+1.337
		−2.1			−.8			.0	−6.4	+1.8	+1.3
−2.964	−.324		−2.400	+.331		−2.370	+2.455				
−3.0	−.3		−2.4	+.3		−2.4	+2.5				

COMPUTATION OF TRIANGLES

The adopted v's or corrections are now substituted in the column headed "Corrections" in the triangles in Figure 34, and the triangles are computed in the manner already explained. If the computed length of the fixed line at the end of the scheme agrees with the fixed length, the length equation is satisfied.

Next the corrected angles are substituted for the observed angles in the formation of the azimuth equation (p. 125) to see if that equation is satisfied.

After making sure that the angle, length, and azimuth equations are all satisfied, it is necessary to recompute the geographic positions of the stations using the data of the corrected triangles. (See fig. 38.) This recomputation can usually be made quite easily and quickly by simply correcting the preliminary positions as explained on page 95. It is only after obtaining the corrected position of Enter and comparing it with the fixed position that one can be certain that the adjustment is correct, that is, that the latitude and longitude as well as the other equations have been satisfied.

CORRECTIONS TO DIRECTIONS

The angle, length, azimuth, latitude, and longitude equations are now satisfied and the corrections can be applied to the directions in Figure 33. Since the corrections determined in the angle method of adjustment are corrections to angles and not to directions, the manner of applying them is somewhat more complicated.

The sample below with the explanation following it shows in detail how the corrections are applied at station Nan. The list of directions before adjustment is given in the first two columns.

Station Nan

Station	Observed direction	Preliminary seconds	Final seconds
	° ′ ″	″	″
Alberta	0 00 00.0	00.0	59.0
Fish	28 42 47.6	48.4	47.4
Fog	109 26 24.4	26.5	25.5
Dug	129 52 16.6		
Gura	171 17 11.1	12.1	11.1

The corrected directions which go in the column headed "Final seconds" are determined as follows: From the adjusted triangles, Figure 34, using the angles at Nan, we have

	°	′	″
Triangle 1, angle Alberta to Fish	28	42	48.4(*A*)
Triangle 2, angle Fish to Fog	80	43	38.1
Angle Alberta to Fog	109	26	26.5(*B*)
Triangle 3, angle Fog to Gura	61	50	45.6
Angle Alberta to Gura	171	17	12.1(*C*)

The values of the seconds for the angles (*A*), (*B*), and (*C*) are placed in the column headed "Preliminary seconds" in the table above. Opposite the initial station, Alberta, is placed 00.0. As the direction "Dug" was not included in the adjustment, there is no correction to be applied to it.

In this method of applying the corrections, the direction at Alberta remains unchanged and so does not receive its share of the correction.

(Text continued on p. 138)

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

						°	′	″
α	2	Alberta	to 3	Fish		356	02	14.8
2d ∠		Fish	&	Nan		+ 57	22	26.4
α	2	Alberta	to 1	Nan		53	24	41.2
Δα						−	6	05.8
						180	00	00.00
α′	1	Nan	to 2	Alberta		233	18	35.4 −1
						23	42	48.4

First Angle of Triangle

	°	′	″				°	′	″
φ	55	31	40.715	2	Alberta	λ	133	11	23.444
Δφ	−	3	07.392	s−		Δλ	+	7	23.894
φ′	56	28	33.323	1	Nan	λ′	133	18	57.338

½(φ+φ′)	55° 30′ 07″.0	s	3.987264	s^2	7.97453		
		Cos α	9.775293	Sin^2 α	9.80936	h^2	4.544
		B	8.509673	C	1.56626	D	2.363
1st term	+187″.1669	h	2.272229		9.35015		6.907
2d and 3d terms	+ 0.2247				0.2239		0.0008
−Δφ	+187.3916						

s	3.987264		
Sin α	9.904681		
A′	8.508727	Δλ	2.647279
sec φ′	0.246607	Sin ½ (φ+φ′)	9.916004
	2.647279		2.563283
Δλ	+443″.8937	−Δα	+ 365″.63

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POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

						°	′	″
α	3	Fish	to 2	Alberta		176	02	29.9
3d ∠		Nan	&	Alberta		− 93	54	45.3
α	3	Fish	to 1	Nan		82	07	44.6
Δα						−	6	20.9
						180	00	00.00
α′	1	Nan	to 3	Fish		262	01	23.7

	°	′	″				°	′	″
φ	55	29	09.866	3	Fish	λ	133	11	15.042
Δφ	−		36.543	s−		Δλ	+	7	42.296
φ′	55	28	33.323	1	Nan	λ′	133	18	57.338

½(φ+φ′)	55° 28′ 51″.6	s	3.913697	s^2	7.82739		
		Cos α	9.136537	Sin^2 α	9.99178	h^2	3.120
		B	8.509676	C	1.56558	D	2.363
1st term	+36″.3003	h	1.559910		9.38475		5.483
2d and 3d terms	+ 0.2425				0.2425		0.0000
−Δφ	+36.5423						

s	3.913697		
Sin α	9.995889		
A′	8.508727	Δλ	2.664919
sec φ′	0.246607	Sin ½ (φ+φ′)	9.915895
	2.664920		2.580814
Δλ	+462″.2959	−Δα	+ 380″.90

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Fig. 38.—Final position computation, angle method of adjustment

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

					°	′	″
α	2	Nan	to 3	Fish	262	01	23.7
2d ∠		Fish	&	Fog	+ 80	43	38.1
α	2	Nan	to 1	Fog	342	45	01.8
Δα					+		49.8
					180	00	00.00
α′	1	Fog	to 2	Nan	162	45	51.6
					74	22	19.5

FIRST ANGLE OF TRIANGLE

	°	′	″				°	′	″
φ	55	28	33.323	2	Nan	λ	133	18	57.338
Δφ	-	1	50.687	s=		Δλ	-	1	00.458
φ′	55	26	42.636	1	Fog	λ′	133	17	56.880

½(φ+φ′)	55° 27′ 38.0″	s	3.554337	s²	7.10877		
		Cos α	9.980014	Sin² α	8.94416	h²	4.088
		B	8.509676	C	1.56544	D	2.363
1st term	+110.6820″	h	2.014077		7.61836		6.451
2d and 3d terms	+ 0.0045				0.0042		0.0003
−Δφ	+110.6865						

s	3.554387		
Sin α	9.472073		
A′	8.508727	Δλ	1.781454
sec φ′	0.246268	Sin ½ (φ+φ′)	9.915788
	1.781455		1.697242
Δλ	-60.4582″	−Δα	-49.80″

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POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

					°	′	″
α	3	Fish	to 2	Nan	82	07	44.6
3d ∠		Fog	&	Nan	- 24	54	02.5
α	3	Fish	to 1	Fog	57	13	42.1
Δα					-	5	31.0
					180	00	00.00
α′	1	Fog	to 3	Fish	237	08	11.1

	°	′	″				°	′	″
φ	55	29	09.866	3	Fish	λ	133	11	15.042
Δφ	-	2	27.230	s=		Δλ	+	6	41.837
φ′	55	26	42.636	1	Fog	λ′	133	17	56.879 +1

½(φ+φ′)	55° 27′ 56.3″	s	3.924344	s²	7.84869		
		Cos α	9.733432	Sin² α	9.84943	h²	4.335
		B	8.509676	C	1.56558	D	2.363
1st term	+147.0456″	h	2.167452		9.26369		6.698
2d and 3d terms	+ 0.1840				0.1835		0.0005
−Δφ	+147.2296						

s	3.924344		
Sin α	9.924711		
A′	8.508727	Δλ	2.604049
sec φ′	0.246268	Sin ½ (φ+φ′)	9.915815
	2.604050		2.519864
Δλ	+401.8370″	−Δα	+ 331.03″

FIG. 38.—Final position computation, angle method of adjustment—Continued

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

					°	′	″
α	2	Nan	to 3	Fog	342	45	01.9
2d ∠		Fog	&	Gura	+ 61	50	45.6
α	2	Nan	to 1	Gura	44	35	47.4
Δα					−	2	09.6
					180	00	00.00
α	1	Gura	to 2	Nan	224	33	37.8
					54	33	59.5

First Angle of Triangle

	°	′	″				°	′	″
φ	55	28	33.323	2	Nan	λ	133	18	57.338
Δφ	−	1	30.740	s−		Δλ	+	2	37.371
φ′	55	27	02.583	1	Gura	λ′	133	21	34.709

½(φ+φ′)	55° 27′ 48.0″	s	3.595463	s²	7.19092		
		Cos α	9.852522	Sin² α	9.69281	h²	3.915
		B	8.509676	C	1.56544	D	2.363
1st term	+ 90.7112″	h	1.957661		8.44917		6.278
2d and 3d terms	+ 0.0283				0.0281		0.0002
−Δφ	+ 90.7395						

s	3.595463		
Sin α	9.846405		
A′	8.508727	Δλ	2.196923
sec φ′	0.246329	Sin ½ (φ+φ′)	9.915803
	2.196924		2.112726
Δλ	+ 157.3708″	−Δα	+ 129.64″

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POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

					°	′	″
α	3	Fog	to 2	Nan	162	45	51.6
3d ∠		Gura	&	Nan	− 63	35	14.9
α	3	Fog	to 1	Gura	99	10	36.7
Δα					−	2	59.4
					180	00	00.00
α′	1	Gura	to 3	Fog	279	07	37.3

	°	′	″				°	′	″
φ	55	26	42.636	3	Fog	λ	133	17	56.880
Δφ	+		19.947	s−		Δλ	+	3	37.829
φ′	55	27	02.583	1	Gura	λ′	133	21	34.709

½(φ+φ′)	55° 26′ 52.6″	s	3.538654	s²	7.17731		
		Cos α	9.202713	Sin² α	9.98881	h²	2.602
		B	8.509678	C	1.56492	D	2.364
1st term	−20.0007″	h	1.301045		8.73104		4.966
2d and 3d terms	+ 0.0538				0.0538		0.0000
−Δφ	−19.9469						

s	3.538654		
Sin α	9.994405		
A′	8.508727	Δλ	2.338114
sec φ′	0.246329	Sin ½ (φ+φ′)	9.915722
	2.338115		2.253836
Δλ	+ 217.8236″	−Δα	+ 179.41″

FIG. 38.—Final position computation, angle method of adjustment—Continued

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

						°	′	″
α	2	Gura	to 3	Fog		279	07	37.3
2d ∠		Fog	&	Dug		+ 38	29	01.7
α	2	Gura	to 1	Dug		317	36	39.0
Δα						+	1	55.3
						180	*00*	*00.00*
α′	1	Dug	to 2	Gura		137	38	34.3
						75	41	13.9

First Angle of Triangle

	°	′	″				°	′	″
φ	55	27	02.583	2	Gura	λ	133	21	34.709
Δφ	-	1	27.234	s=		Δλ	-	2	19.976
φ′	55	25	35.349	1	Dug	λ′	133	19	14.733

½(φ+φ′)	° ′ ″ 55 26 19.0	s	3.562499	s²	7.12500		
		Cos α	9.868399	Sin² α	9.65753	h²	3.881
		B	8.509678	C	1.56501	D	2.364
1st term	+87.2120	h	1.940576		8.34754		6.245
2d and 3d terms	+0.0225				0.0223		0.0002
−Δφ	+87.2345						

s	3.562499		
Sin α	9.328765		
A′	8.508728	Δλ	2.146053
sec φ′	0.246062	Sin ½ (φ+φ′)	9.915674
	2.146054		2.061727
Δλ	″ -139.9761	−Δα	″ -115.27

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POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

						°	′	″
α	3	Fog	to 2	Gura		99	10	36.7
3d ∠		Dug	&	Gura		− 65	49	44.4
α	3	Fog	to 1	Dug		33	20	52.3
Δα						-	1	04.1
						180	*00*	*00.00*
α′	1	Dug	to 3	Fog		213	19	48.2

	°	′	″				°	′	″
φ	55	26	42.636	3	Fog	λ	133	17	56.880
Δφ	-	1	07.287	s=		Δλ	+	1	17.853
φ′	55	25	35.349	1	Dug	λ′	133	19	14.733

½(φ+φ′)	° ′ ″ 55 26 09.0	s	3.396343	s²	6.79268		
		Cos α	9.921868	Sin² α	9.48028	h²	3.656
		B	8.509678	C	1.56492	D	2.364
1st term	+67.2805	h	1.827889		7.83788		6.020
2d and 3d terms	+0.0070				0.0069		0.0001
−Δφ	+67.2875						

s	3.396343		
Sin α	9.740142		
A′	8.508723	Δλ	1.891274
sec φ′	0.246067	Sin ½ (φ+φ′)	9.915659
	1.891275		1.806933
Δλ	″ +77.8529	−Δα	″ +64.11

Fig. 38.—Final position computation, angle method of adjustment—Continued

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

					°	′	″
a	2	Gura	to 3	Dug	317	36	39.0
2d ∠		Dug	&	Lat	+ 79	27	42.9
a	2	Gura	to 1	Lat	37	04	21.9
Δa					−	2	24.1
					180	00	00.00
a′	1	Lat	to 2	Gura	217	01	57.8
		First Angle of Triangle			38	57	17.5

	°	′	″				°	′	″
φ	55	27	02.583	2	Gura	λ	133	21	34.709
Δφ	−	2	11.831	s=		Δλ	+	2	55.040
φ′	55	24	50.752	1	Lat	λ′	133	24	29.749

½(φ+φ′)	55° 25′ 56.7″	s	3.708290	s²	7.41658		
		Cos a	9.901933	Sin² a	9.56039	h²	4.240
		B	8.509678	C	1.56501	D	2.364
1st term	+131.7956″	h	2.119901		8.54198		6.604
2d and 3d terms	+ 0.0352				0.0348		0.0004
−Δφ	+ 131.8308						

s	3.708290		
Sin a	9.780194		
A′	8.508728	Δλ	2.243138
sec φ′	0.245926	Sin ½ (φ+φ′)	9.915641
	2.243138		2.158779
Δλ	+175.0403″	−Δa	+144.14″

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POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

					°	′	″
a	3	Dug	to 2	Gura	127	38	34.3
3d ∠		Lat	&	Gura	− 61	34	59.7
a	3	Dug	to 1	Lat	76	03	34.6
Δa					−	4	19.4
					180	00	00.00
a′	1	Lat	to 3	Dug	255	59	15.2+1

	°	′	″				°	′	″
φ	55	25	35.349	3	Dug	λ	133	19	14.733
Δφ	−		44.597	s=		Δλ	+	5	15.016
φ′	55	24	50.753	1	Lat	λ′	133	24	29.749

½(φ+φ′)	55° 25′ 13.1″	s	3.756663	s²	7.51332		
		Cos a	9.381859	Sin² a	9.97403	h²	3.286
		B	8.509680	C	1.56463	D	2.364
1st term	+44.4838″	h	1.643202		9.05198		5.660
2d and 3d terms	+0.1127				0.1127		0.0000
−Δφ	+44.5965						

s	3.756663		
Sin a	9.987016		
A′	8.508728	Δλ	2.498332
sec φ′	0.245926	Sin ½ (φ+φ′)	9.915578
	2.498333		2.413910
Δλ	+ 315.0163″	−Δa	+ 259.36″

FIG. 38.—Final position computation, angle method of adjustment—Continued

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

						°	′	″
α	2	Lat	to 3	Gura		217	01	57.8
2d ∠		Gura	&	Mond		+ 60	10	36.6
α	2	Lat	to 1	Mond		277	12	34.4
Δα						+	4	14.3
						180	00	00.00
α′	1	Mond	to 2	Lat		97	16	38.7
				First Angle of Triangle		56	27	36.3

	°	′	″				°	′	″
φ	55	24	50.752	2	Lat	λ	133	24	29.749
Δφ	-		22.332	s=		Δλ	-	5	08.893
φ′	55	24	28.420	1	Mond	λ′	133	19	20.856

½(φ+φ′)	55° 24′ 39.6″	s	3.738667	s²	7.47733		
		Cos α	9.098473	Sin² α	9.99311	h²	2.694
		B	8.508681	C	1.56442	D	2.364
1st term	+ 22.2239″	h	1.346821		9.03487		5.058
2d and 3d terms	+ 0.1084				0.1084		0.0000
−Δφ	+ 22.3323						

s	3.738667		
Sin α	9.996555		
A′	8.508728	Δλ	2.489808
sec φ′	0.245858	Sin ½ (φ+φ′)	9.915529
	2.489808		2.405337
Δλ	-308.8930″	−Δα	-254.29″

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POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

						°	′	″
α	3	Gura	to 2	Lat		37	04	21.9
3d ∠		Mond	&	Lat		− 63	21	57.1
α	3	Gura	to 1	Mond		333	42	24.8
Δα						+	1	50.2
						180	00	00.00
α′	1	Mond	to 3	Gura		153	44	15.0

	°	′	″				°	′	″
φ	55	27	02.583	3	Gura	λ	133	21	34.709
Δφ	-	2	34.163	s=		Δλ	-	2	13.853
φ′	55	24	28.420	1	Mond	λ′	133	19	20.856

½(φ+φ′)	55° 25′ 45.6″	s	3.725674	s²	7.45135		
		Cos α	9.952569	Sin² α	9.29274	h²	4.376
		B	8.509673	C	1.56501	D	2.364
1st term	+154.1420″	h	2.187921		8.30910		6.740
2d and 3d terms	+ 0.0209				0.0204		0.0006
−Δφ	+154.1629						

s	3.725674		
Sin α	9.646368		
A′	8.508728	Δλ	2.126628
sec φ′	0.245358	Sin ½ (φ+φ′)	9.915625
	2.126628		2.042253
Δλ	-133.8530″	−Δα	-110.22″

Fig. 38.—Final position computation, angle method of adjustment—Continued

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

						°	′	″
α	2	Lat	to 3	Mond		277	12	24.4
2d ∠		Mond	&	Enter		+ 49	08	01.0
α	2	Lat	to 1	Enter		326	20	25.4
Δα						+	3	58.4
						180	00	00.00
α′	1	Enter	to 2	Lat		146	24	23.8
						36	21	49.3

First Angle of Triangle

	°	′	″				°	′	″
φ	55	24	50.752	2	Lat	λ	133	24	29.749
Δφ	–	4	08.022	s=		Δλ	–	4	49.700
φ′	55	20	42.730	1	Enter	λ′	133	19	40.049
Fixed	55	20	42.730				133	19	40.049

½(φ+φ′)	° ′ ″	s	3.964337	s^2	7.92867		
	55 22 46.7	Cos α	9.930303	$\sin^2 α$	9.48742	h^2	4.789
		B	8.509681	C	1.56443	D	2.364
1st term	″ +247.9254	h	2.394321		8.98052		7.153
2d and 3d terms	+ 0.0970				0.0956		0.0014
−Δφ	+248.0224						

s	3.964337		
Sin α	9.743712		
A′	8.508730	Δλ	2.461949
sec φ′	0.245170	Sin ½ (φ+φ′)	9.915365
	2.461949		2.377314
Δλ	″ −289.7003	−Δα	″ −238.40

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POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

						°	′	″
α	3	Mond	to 2	Lat		97	16	38.7
3d ∠		Enter	&	Lat		− 94	30	09.8
α	3	Mond	to 1	Enter		2	46	28.9
Δα						–		15.8
		Enter		Mond		180	00	00.00
α′	1		to 3			182	46	13.1

	°	′	″				°	′	″
φ	55	24	28.420	3	Mond	λ	133	19	20.856
Δφ	–	3	45.690	s=		Δλ	+		19.194
φ′	55	20	42.730	1	Enter	λ′	133	19	40.050
									−1

½(φ+φ′)	° ′ ″	s	3.844337	s^2	7.68867		
	55 22 35.6	Cos α	9.999491	$\sin^2 α$	7.36985	h^2	4.707
		B	8.509681	C	1.56423	D	2.364
1st term	″ + 225.6883	h	2.353509		6.62275		7.071
2d and 3d terms	+ 0.0016				0.0004		0.0012
−Δφ	+ 225.6899						

s	3.844337		
Sin α	8.684923		
A′	8.508730	Δλ	1.283160
sec φ′	0.245170	Sin ½ (φ+φ′)	9.915349
	1.283160		1.198509
Δλ	″ + 19.1938	−Δα	″ + 15.79

FIG. 38.—Final position computation, angle method of adjustment—Continued

A mean correction is therefore computed by taking the differences between the directions in the column headed "Observed directions" and those in the column headed "Preliminary seconds."

At Alberta, the correction is	0.0
At Fish, the correction is	+0.8
At Fog, the correction is	+2.1
At Gura, the correction is	+1.0
Total	+3.9
Average correction	+1.0

The average correction +1″.0, with sign changed, is then applied to each of the directions in the "Preliminary seconds" column to obtain the values in the "Final seconds" column.

The rule to be followed, then, is: Keeping the seconds of the initial station 00.0, compute the seconds of the other directions by adding the adjusted spherical angles of the triangles in the proper order. Place these values in a column headed "Preliminary seconds." Take the algebraic sum of the differences between these values and the values in the preceding column and divide this sum by the number of directions involved. Add algebraically with opposite sign the mean thus obtained to each of the values in the column "Preliminary seconds," and place the resulting values in the "Final seconds" column.

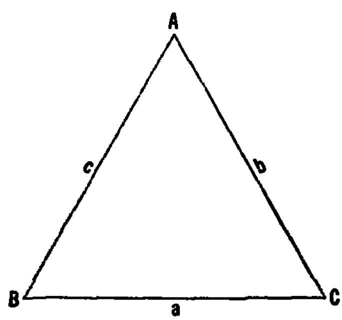

FIG. 39.—Triangle as lettered for computation using two sides and included angle

In order to see whether the corrections have been applied properly, check these final seconds from the triangles. For example, the angle at Nan between Alberta and Fish as taken from the "Final seconds" column is 28° 42′ 48″.4, which checks the angle at Nan in triangle 1. Similarly, the angle at Nan between Fish and Fog is 80° 43′ 38″.1, which checks the angle at Nan in triangle 2, and the angle at Nan between Fog and Gura is 61° 50′ 45″.6, which checks the angle at Nan in triangle 3.

CORRECTIONS TO DIRECTIONS OMITTED IN ADJUSTMENT

After the final seconds have been determined for all directions included in the adjustment, the corrections to the directions for the diagonals omitted from the adjustment are computed. The three diagonals omitted were Alberta-Fog, Nan-Dug, and Lat-Flores. These are computed by the two-sides-and-included-angle method. (See pp. 141–143.)

TRIANGLE COMPUTATION BY TWO-SIDES-AND-INCLUDED-ANGLE METHOD

The diagonal Alberta-Fog is computed as follows. The formulas used are

$$\tan (45° + \phi) = \frac{a}{b}, \text{ where } a > b$$

$$\tan 1/2\ (A - B) = \tan \phi \tan \left(90° - \frac{C}{2}\right)$$

$$1/2\ (A + B) = 90° - \frac{C}{2}$$

in which a and b, the two sides, and C, the included angle, of the triangle are known, and ϕ is an auxiliary angle.

The logarithms of the sides Fish-Alberta and Fish-Fog (see fig. 31) are given in the triangles, Figure 34, as 3.669907 and 3.924344, respectively. The angle at Fish between Fog and Alberta is the sum of the two adjusted angles between Fog and Nan, and between Nan and Alberta, namely, 24° 54′ 02″.5 plus 93° 54′ 45″.3, or 118° 48′ 47″.8.

The triangle, Alberta-Fish-Fog, is written as shown in Figure 40. The asterisk is placed opposite the fixed angle and the logarithms of the fixed lengths are placed in the last column. As there is 0″.1 spherical excess, this is applied to the large angle making the plane angle at Fish 118° 48′ 47″.8 − 0″.1, or 118° 48′ 47″.7.

The problem then is to determine A, the angle at Alberta; B, the angle at Fog; and c the line Fog-Alberta. The computation is shown in Figure 40.

These angles A and B as thus determined are placed in the triangle Alberta-Fish-Fog on the lower part of the form (fig. 40) and the triangle is computed in the usual way. The logarithm of the length, Alberta-Fog, is thus obtained. The computed logarithm of the fixed length, Alberta-Fish, should check the given value probably within one in the last place.

The value of the final seconds for the direction on Fog at station Alberta is now obtained from the triangle by simply adding the angle at Alberta between Fish and Fog, 40° 09′ 09″.1, to the fixed direction Fish, 0° 00′ 00″.0. At station Fog, the final seconds for the direction on Alberta are obtained by subtracting the angle between Alberta and Fish from the fixed direction on Fish; that is, 203° 47′ 18″.9 − 21° 02′ 03″.2 = 182° 45′ 15″.7.

45861°—34——10

In order that the length, Fog-Alberta, as determined from the two sides and included angle computation, may have a check the triangle Fog-Nan-Alberta (see fig. 40) is also computed. The three angles of this triangle are fixed by the computation of the first triangle and can be taken out directly from the lists of directions at the three stations. The logarithm of the length, Alberta-Fog, should agree in the two triangles.

In the same manner the diagonals Nan-Dug and Lat-Flores are computed, using first a triangle with two sides and the included angle known and then a second triangle, whose angles are fixed by the computation of the first triangle, to check the length of the line in question.

COMPUTATION OF THE LENGTH AND AZIMUTH OF THE OMITTED DIAGONAL BY THE METHOD OF APPROXIMATION

If the diagonals omitted in the adjustment have been observed it may be advisable or expedient at times to resort to the method of approximation in computing them after the adjustment has been completed.

An example of this method is given on page 144. (See fig. 41.) In the quadrilateral Lat-Mond-Flores-Enter (see fig. 31) the diagonal Lat-Flores was omitted in the adjustment. It may be computed from the triangle Lat-Mond-Flores in the following manner: The logarithms of the lengths Mond-Flores and Lat-Mond and the angle at Mond between Flores and Lat have been fixed by the adjustment. The observed angles at Lat and Flores are obtained from the lists of directions. (See fig. 33.) The sum of the two observed angles at Lat and Flores and the adjusted angle at Mond equals 179° 59′ 52″.0, so in order to close the triangle 4″.0 must be added to each of the angles at Lat and Flores.

Using the fixed logarithm of the length Mond-Flores, 3.799236, the logarithm of the length Lat-Mond as computed to seven decimal places through the approximate angles at Lat and Flores is 3.7386565. This value differs from the fixed logarithm of Lat-Mond, 3.738667, by 10.5 in the sixth place of logarithms. The tabular differences of the logarithmic sines of the angles at Lat and Flores for one second change in angle are +3.03 and +3.68, respectively. The discrepancy in the logarithms of the length divided by the algebraic sum of these two tabular differences, that is, $\frac{10.5}{6.71}=1''.6$, is the correction which must be applied to the angles at Lat and Flores which were used in the preliminary computation.

This correction is applied with the negative sign to the angle at Lat and with the positive sign to the angle at Flores. The triangle is then recomputed, using the corrected angles, and the final logarithm

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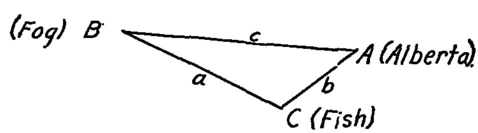
Form 665

TRIANGLE COMPUTATION USING TWO SIDES AND INCLUDED ANGLE

$\left[\frac{a}{b}=\tan(45°+\phi)\right.$ (Call longer side a) : $\tan\frac{1}{2}(A_p-B_p)=\tan\phi\ \tan\frac{1}{2}(A_p+B_p)$: $\left.c=\frac{a\sin C_p}{\sin A_p}\right]$ *

	°	′	″				
C_s	118	48	47.8	Log a	3.924344	Log m	1.403
$\frac{\text{Sph. excess}}{3}$			0.1	Log b	3.669907	Log sin C_s	9.943
C_p	118	48	47.7	Log tan $(45+\phi)$	0.254437	Log a	3.924
$\frac{1}{2}C_p$	59	24	23.85	$(45°+\phi)$	60° 53′ 54.88″	Log b	3.670
$90°-\frac{1}{2}C_p=\frac{1}{2}(A_p+B_p)$	30	35	36.15	ϕ	15 53 54.88	Log sph. ex.	8.940
$\frac{1}{2}(A_p-B_p)$	9	33	32.92	Log tan ϕ	9.4543867	Sph. excess	0.09
Sum $=A_p$	40	09	09.07	Log tan $\frac{1}{2}(A_p+B_p)$	9.7717656		
Diff. $=B_p$	21	02	03.23	Log tan $\frac{1}{2}(A_p-B_p)$	9.2263523		
C_p					(Sketch)		

Log a	3.924344
Log sin C_p	9.942601
Colog sin A_p	0.190558
Log c	4.057503



CHECK COMPUTATION

NO.	STATION	SPHERICAL ANGLE	SPHERICAL EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
2-3	Fish-Fog				3.924344
1	Alberta	40 09 09.1		09.1	0.190558
2	Fish	118 48 47.8	0.1	47.7	9.942601
3	Fog	21 02 03.2		03.2	9.555004
1-3	Alberta-Fog				4.057503
1-2	Alberta-Fish				3.669907
2-3	Nan-Alberta				3.987264
1	Fog	53 20 16.3		16.3	0.095733
2	Nan	109 26 26.5	0.1	26.4	9.974506
3	Alberta	17 13 17.3		17.3	9.471389
1-3	Fog-Alberta				4.057503
1-2	Fog-Nan				3.554387

* The subscripts s and p on this form refer to spherical and plane angles respectively.

FIG. 40.—Triangle computation using two sides and included angle

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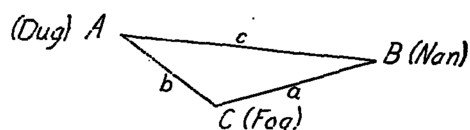
U. S. COAST AND GEODETIC SURVEY
Form 668

TRIANGLE COMPUTATION USING TWO SIDES AND INCLUDED ANGLE

$\left[\frac{a}{b} = \tan\,(45° + \phi)\right.$ (Call longer side a): $\tan \frac{1}{2}(A_p - B_p) = \tan \phi \tan \frac{1}{2}(A_p + B_p)$: $\left. c = \frac{a \sin C_p}{\sin A_p}\right]$ *

	°	′	″				
C_s	129	24	59.3	Log a	3.554387	Log m	1.403
$\frac{\text{Sph. excess}}{3}$			0.0	Log b	3.396345	Log sin C_s	9.888
C_p	129	24	59.3	Log tan $(45+\phi)$	0.158044	Log a	3.554
$\frac{1}{2} C_p$	64	42	29.65	$(45° + \phi)$	55° 12′ 08.93″	Log b	3.396
$90° - \frac{1}{2} C_p = \frac{1}{2}(A_p + B_p)$	25	17	30.35	ϕ	10 12 08.93	Log sph. ex.	8.241
$\frac{1}{2}(A_p - B_p)$	4	51	38.90	Log tan ϕ	9.2552076	Sph. excess	0.02″
Sum = A_p	30	09	09.25	Log tan $\frac{1}{2}(A_p + B_p)$	9.6744220		
Diff. = B_p	20	25	51.45	Log tan $\frac{1}{2}(A_p - B_p)$	8.9296296		
C_p					(Sketch)		

Log a	3.554387
Log sin C_p	9.887927
Colog sin A_p	0.299033
Log c	3.741347



CHECK COMPUTATION

NO.	STATION		SPHERICAL ANGLE			SPHERICAL EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
2-3		Fog-Dug						3.396343
1	Nan		20	25	51.5	0.0	51.5	0.457077
2	Fog		129	24	59.3	0.0	59.3	9.887927
3	Dug		30	09	09.2	0.0	09.2	9.700967
1-3		Nan-Dug						3.741347
1-2		Nan-Fog						3.554387
2-3		Gura-Nan						3.595463
1	Dug		45	32	04.7	0.0	04.7	0.146500
2	Gura		93	03	01.2	0.0	01.2	9.999384
3	Nan		41	24	54.1	0.0	54.1	9.820535
1-3		Dug-Nan						3.741347
1-2		Dug-Gura						3.562499

* The subscripts s and p on this form refer to spherical and plane angles respectively.

FIG. 40.—Triangle computation using two sides and included angle—Continued

DEPARTMENT OF COMMERCE
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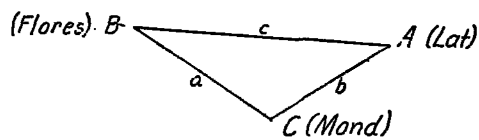
Form 665

TRIANGLE COMPUTATION USING TWO SIDES AND INCLUDED ANGLE

$\left[\frac{a}{b} = \tan(45° + \phi)\right.$ (Call longer side a) : $\tan \frac{1}{2}(A_p - B_p) = \tan\phi \tan\frac{1}{2}(A_p + B_p)$: $\left. c = \frac{a \sin C_p}{\sin A_p}\right]$ *

	°	′	″						
C_s	115	24	55.9	Log a	3.799236			Log m	1.403
$\frac{\text{Sph. excess}}{3}$			0.1	Log b	3.738667			Log sin C_s	9.956
C_p	115	24	55.8	Log tan (45+ϕ)	0.060569			Log a	3.799
$\frac{1}{2} C_p$	57	42	27.9	(45°+ϕ)	48°	58′	56.98″	Log b	3.739
$90° - \frac{1}{2}C_p = \frac{1}{2}(A_p + B_p)$	32	17	32.1	ϕ	3	58	56.98	Log sph. ex.	8.897
$\frac{1}{2}(A_p - B_p)$	2	31	09.5	Log tan ϕ	8.8427327			Sph. excess	0.08″
Sum $= A_p$	34	48	41.6	Log tan $\frac{1}{2}(A_p+B_p)$	9.8007064				
Diff. $= B_p$	29	46	22.6	Log tan $\frac{1}{2}(A_p-B_p)$	8.6434391				
C_p					(Sketch)				

Log a	3.799236
Log sin C_p	9.955793
Colog sin A_p	0.243456
Log c	3.998485



CHECK COMPUTATION

NO.	STATION		SPHERICAL ANGLE			SPHERICAL EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
2-3	Mond-Flores							3.799236
1	Lat		34	48	41.6		41.6	0.243456
2	Mond		115	24	55.9	0.1	55.8	9.955793
3	Flores		29	46	22.6		22.6	9.695975
1-3		Lat-Flores				0.1		3.998485
1-2		Lat-Mond						3.738667
2-3		Enter-Lat						3.964337
1	Flores		65	23	16.9		16.9	0.041076
2	Enter		100	12	23.7		23.7	9.993072
3	Lat		14	19	19.4		19.4	9.393350
1-3		Flores-Lat				0.0		3.998485
1-2		Flores-Enter						3.398762

* The subscripts s and p on this form refer to spherical and plane angles respectively.

FIG. 40.—Triangle computation using two sides and included angle—Continued

of the length Lat-Flores is thus obtained. The computed logarithm of the length Lat-Mond should now agree with its fixed value.

The rule to be followed in this approximation method may be stated briefly as follows: Write the triangle which includes the omitted diagonal and fill in the data fixed by the previous adjustment; that is, the logarithms of the two sides and the value of the included angle. Put in the observed values of the other two angles as obtained from the list of directions. If the sum of the three angles, after applying the spherical excess, does not equal 180°, apply one half of the triangle closure to each of the observed angles to close the triangle. Then, starting with the logarithm of one fixed length compute the logarithm of the other fixed length through the triangle, taking out the tabular differences with their proper signs for the logarithms of the sines corresponding to a change of 1 second in the two angles yet to be fixed. Next divide the discrepancy between the computed and the fixed

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: ....

Tabular dif.	NO.	STATION	OBSERVED ANGLE			CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
	2-3	Mond-Flores								3.739236
+3.03	1	Lat	34	48	39.2		41.6		41.6 ~~43.2~~	6 0.24345~~10~~
	2	Mond	115	25	02.3	-6.4	55.9	0.1	55.8	9.955793
+3.68	3	Flores	29	46	17.0		22.6		22.6 ~~21.0~~	75 9.6959~~695~~
+6.71	1-3	Lat-Flores								3.998485
	1-2	Lat-Mond								~~565~~ 3.738667

FIG. 41.—Triangle computation using two sides and included angle, method of approximation

logarithm of the second fixed length by the algebraic sum of the tabular differences, and apply this quotient with the same sign as a correction to the approximate angle which appears as the third angle in the triangle and with the opposite sign as a correction to the approximate angle which appears as the first angle in the triangle. Finally, recompute the triangle using the corrected angles.

The probable error of an observed angle is determined by the same formula as that used to obtain the probable error of a direction. (See p. 107.) In the table of correlates on page 127, $\Sigma v^2 = 88.84$, and since there are 12 equations,

$$\text{p. e.} = \pm 0.674 \sqrt{\frac{88.84}{12}} = \pm 1''.83$$

After the adjustment has been completed and the omitted diagonals have been computed, the list of geographic positions is made out on form 28B in exactly the same manner as the list on page 108. (See fig. 42.)

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 D

GEOGRAPHIC POSITIONS

Accession No. of Computation: ________

Locality Prince of Wales Island *Datum* North American *State* Alaska

STATION.		LATITUDE AND LONGITUDE. °	′	″	SECONDS IN METERS.	AZIMUTH °	′	″	BACK AZIMUTH. °	′	″	TO STATION.	DISTANCE. LOGARITHM (METERS).	METERS.	FEET.
Nan		55	28	33.323	1030.5	233	18	35.3	53	24	41.2	Alberta	3.987264	9711.0	31860
1907	d.m.	133	18	57.338	1007.2	262	01	23.7	82	07	44.6	Fish	3.913697	8197.8	26896
Fog		55	26	42.836	1318.6	162	45	51.6	342	45	01.8	Nan	3.554387	3584.2	11759
1907	d.m.	133	17	56.880	1000.0	216	06	07.9	36	11	23.9	Alberta	4.057503	11415.7	37453
						237	08	11.1	57	13	42.1	Fish	3.924344	8401.3	27563
Gura		55	27	02.583	79.9	224	33	37.8	44	35	47.4	Nan	3.595463	3939.7	12925
1907	d.m.	133	21	34.709	610.1	279	07	37.3	99	10	36.7	Fog	3.588654	3878.4	12724
Dug		55	25	35.349	1093.2	137	38	34.3	317	36	39.0	Gura	3.562499	3651.7	11981
1907	d.m.	133	19	14.733	259.1	183	10	39.0	3	10	53.3	Nan	3.741347	5512.6	18086
						213	19	48.2	33	20	52.3	Fog	3.396343	2490.8	8172
Lat		55	24	50.752	1569.6	217	01	57.8	37	04	21.9	Gura	3.708290	5108.6	16760
1907	d.m.	133	24	29.749	523.3	255	59	15.2	76	03	34.6	Dug	3.756663	5710.4	18735
						312	01	06.0	132	06	51.8	Flores	3.998485	9965.2	32694
						326	20	25.4	146	24	23.8	Enter	3.964337	9211.6	30222
Mond		55	24	28.420	878.9	341	51	42.8	161	53	14.4	Flores	3.799236	6298.5	20664
1907	d.m.	133	19	20.856	367.0	2	46	28.9	182	46	13.1	Enter	3.844337	6987.7	22926
						97	16	38.7	277	12	24.4	Lat	3.738667	5478.6	17974
						153	44	15.0	333	42	24.8	Gura	3.725674	5317.1	17445

FIG. 42.—List of geographic positions, angle method of adjustment

Statistics showing accuracy of triangulation

Total number of triangles	8
Number of triangles with plus closures	3
Number of triangles with minus closures	5
Number of concluded triangles	0
Average closure of triangles without regard to sign	2".2
Maximum closure of a triangle	5".4
Mean error of an angle	±1".6
Probable error of an observed angle	±1".8

COMPARISON OF DIRECTION AND ANGLE METHODS OF TRIANGULATION ADJUSTMENT

Comparing the direction and angle methods of adjustments it is seen that each has its advantages and disadvantages. The direction method gives a more rigid adjustment by making use of all the directions observed, and when the adjustment is completed the azimuths and lengths of all the lines are immediately available. There is no necessity for two-sides-and-included-angle computations. The disadvantage of the method is that the solution of the normals is more laborious, especially where the scheme of triangulation is much involved. However, for first-order and second-order triangulations, the adjustment should be as rigid as possible, and the direction method should, therefore, always be used.

The angle method of adjustment has the advantage that the number of equations is reduced considerably, there being one less angle equation in each full quadrilateral and no side equations at all. The solution of the normal equations is very simple as all the angle equations are independent of each other, and the azimuth equation does not, ordinarily, involve the same *v*'s as the length equation. The disadvantage of this method is that a number of two-sides-and-included-angle computations must be made after the adjustment is completed to obtain the azimuths and lengths of the lines omitted in the adjustment. However, the advantages of this method far outweigh the disadvantages in the adjustment of third-order triangulation and it should ordinarily be used for this class of triangulation.

CHAPTER 5.—COMPUTATION AND ADJUSTMENT OF ELEVATIONS FROM ZENITH-DISTANCE OBSERVATIONS

GENERAL STATEMENT

In connection with the observation of horizontal directions for triangulation it is customary to observe zenith distances, in order that the elevations of the stations may be determined. These elevations are needed for reducing the horizontal directions to sea level and, in some cases, if precise elevations are not available, to reduce the base lines to sea level. As there are usually two or more lines observed to each station, a rigid adjustment of the observed differences of elevation should be made in order to remove the inconsistencies and obtain the best possible elevations from the observations.

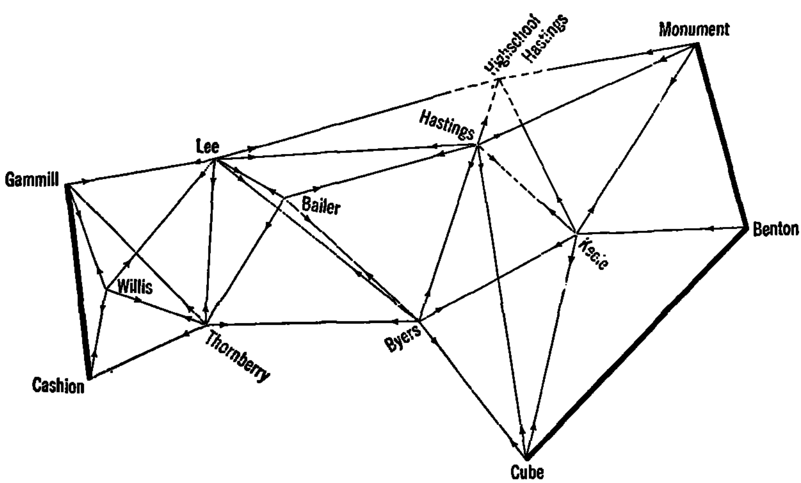


FIG. 43.—Triangulation net used in sample computation of elevations

A sample computation and least-squares adjustment of elevations based on zenith distances is given on the following pages. The triangulation stations at which the zenith distances were measured are shown in Figure 43. Arrowheads pointing away from each station denote the lines over which zenith distances were observed from that station. Arrowheads at both ends of a line indicate reciprocal observations or that zenith distances were observed in both directions over that line. Where only one arrowhead appears on a line, zenith distances were observed only in the direction in which the arrowhead points. All the stations shown in Figure 43, except "Hastings high school," are main scheme stations. The elevation of Hastings high school is computed after the elevations of the other stations are fixed.

All zenith distances are abstracted on Form 29 and checked in the field. The abstracts, Figure 44, therefore, contain the starting data for the office computation. It is sometimes necessary, however, to compute in the office the values in the column headed "Reduction to line joining stations." This column is used only when the observations are reciprocal. Each value in the column is an angle which in seconds equals $-\frac{t-o}{s \sin 1''}$, t being the height of the telescope above the station mark, o the height of the object above its station mark, and s the horizontal distance between the stations concerned. This really represents a vertical eccentric reduction which is applied as a correction to the observed zenith distance to obtain the zenith distance referred to the station marks. Only four places of logarithms should be used in computing the values.

If the observations are nonreciprocal, that is, are made in one direction only over a line, then the vertical eccentric reduction is not needed. In this case, the difference $t-o$, is applied as a correction to the computed difference of elevation, as indicated on Form 29 B (see fig. 47).

As the development of the formulas for the computation of elevations from reciprocal and nonreciprocal observations is fully shown in special publication No. 28, it is not given here.

The formula for the computation of elevations from reciprocal observations is

$$h_2-h_1=s \tan\left(\frac{\zeta_2-\zeta_1}{2}\right) A\ B\ C \qquad (1)$$

in which h_1 is the elevation above mean sea level of station 1 and h_2 that of station 2; s is the horizontal sea level distance between the two stations; ζ_1 is the zenith distance of station 2 as observed from station 1; ζ_2 is the zenith distance of station 1 from station 2; and A, B, and C are correction factors whose values are close to unity and whose logarithms are given in the tables on pages 232. The station designated 1 is the station whose elevation is already computed.

The formula for the computation of elevations from non-reciprocal observations is

$$h_2-h_1=s \cot\left[\zeta_1-(0.5-m)\frac{s}{\rho \sin 1''}\right] A\ B\ C \qquad (2)$$

in which h_2 and h_1 are elevations of the two stations, s the horizontal distance between the stations, ζ_1 the mean corrected zenith distance at the station occupied, m the coefficient of refraction, ρ the radius of curvature of the earth's surface in the mean latitude of the stations and in the azimuth of the line observed (see table on pp. 220–222), and A, B, and C the correction factors.

All but three of the main-scheme lines in Figure 43 are observed from both ends and the differences of elevations for these lines are, therefore, computed by formula (1). The three lines Benton-Keele, Bailer-Thornberry and Thornberry-Gammill are observed from one end only and these differences of elevations must be computed by formula (2). (Text continued on p. 159)

Abstract of Zenith Distances

Station Monument State Oklahoma

Observer E.O.H. Instrument V.C. 109

Date	Hour	Object Observed	Object above Station —o	Telescope above Station —t	Diff. of Heights t—o	Reduction to Line joining Stations	Observed Zenith Distance	Corrected Zenith Distance
1923			*Meters*	*Meters*	*Meters*	"	° ' "	° ' "
5/19	1:16	Benton	1.17	2.115	+ 0.945	-11.8	90 10 20.5	
	2:19						90 10 19.0	
	2:29						90 10 15.4	
							18.3	90 10 06.5
	1:24	Hastings	1.24	2.115	+0.875	-8.6	90 08 01.7	
	1:25						90 08 05.0	
	1:35						90 08 12.6	
	1:39						90 08 05.8	
	2:09						90 08 11.1	
							07.2	90 07 58.6
	1:42	Keele	1.92	2.115	+0.195	-2.0	90 08 23.4	
	1:46						90 08 23.9	
	1:49						90 08 25.2	
	1:51						90 08 32.7	
	1:54						90 08 22.3	
							25.5	90 08 23.5
	2:52	Cupola of		2.115			90 04 49.4	
	2:55	Schoolhouse					90 04 58.3	
		Hastings.					53.8	

Station Benton State Oklahoma

Observer E.O.H. Instrument V.C. 109

Date	Hour	Object Observed	Object above Station —o	Telescope above Station —t	Diff. of Heights t—o	Reduction to Line joining Stations	Observed Zenith Distance	Corrected Zenith Distance
1923			*Meters*	*Meters*	*Meters*	"	° ' "	° ' "
5/21	1:14	Keele	1.92	1.375	-0.545	+7.4	90 01 56.7	
	1:19	(Helio)					90 02 02.5	
	1:24						90 01 56.3	
	1:29						90 01 53.0	
							90 01 58.4	90 02 05.8
	1:57	Cube	9.37	1.375	-7.995	+58.2	90 06 41.0	
	2:09	(Helio)					90 06 35.6	
							38.3	90 07 36.5

FIG. 44.—Abstract of zenith distances for sample computation of elevations

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29

ABSTRACT OF ZENITH DISTANCES

Station Cube State Texas

Observer E.O.H. - A.F.B. Instrument V.C. 109

DATE	HOUR	OBJECT OBSERVED	OBJECT ABOVE STATION =o	TELESCOPE ABOVE STATION =t	DIFF. OF HEIGHTS t−o	REDUCTION TO LINE JOINING STATIONS	OBSERVED ZENITH DISTANCE	CORRECTED ZENITH DISTANCE
1923			Meters	Meters	Meters	″	° ′ ″	° ′ ″
5/28	2:06	Keele	1.92	9.575+7.655		-79.2	90 04 30.6	
	2:08	(Helio)					90 04 29.9	
	2:10						90 04 29.1	
5/29	3:33						90 04 16.2	
	3:38						90 04 08.9	
	3:44						90 04 15.2	
							21.6	90 03 03.4
5/28	2:18	Byers	3.61	9.575+5.965		-79.5	90 01 09.0	
	2:26	(Helio)					90 01 07.3	
	3:32						90 01 04.1	
	3:37						90 01 09.1	
5/29	3:07						90 01 07.0	
	3:12						90 01 02.2	
	3:20						90 00 55.2	
	3:25						90 00 51.7	
							90 01 03.2	89 59 43.7
5/28	2:56	Hastings	1:24	9.575+8.335		-61.3	90 06 37.9	
	3:21	(Helio)					90 06 34.5	
	3:33						90 06 30.1	
	3:45						90 06 37.4	
5/29	4:02						90 06 07.4	
	4:12						90 06 07.0	
	4:22						90 05 55.4	
	4:34						90 06 01.1	
							90 06 18.8	90 05 17.5

FIG. 44.—Abstract of zenith distances for sample computation of elevations—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29

ABSTRACT OF ZENITH DISTANCES

Station Keele State Oklahoma

Observer E.O.H. - A.F.B. Instrument V.C. 109

DATE	HOUR	OBJECT OBSERVED	OBJECT ABOVE STATION =o	TELESCOPE ABOVE STATION =t	DIFF. OF HEIGHTS t−o	REDUCTION TO LINE JOINING STATIONS	OBSERVED ZENITH DISTANCE	CORRECTED ZENITH DISTANCE
1923			*Meters*	*Meters*	*Meters*		° ′ ″	° ′ ″
5/22	2:06	Byers	3.61	2.125	-1.485	+20.2	90 01 33.8	
	2:09	(Helio)					90 01 36.4	
	2:12						90 01 33.0	
							34.4	90 01 54.6
	2:16	Hastings	1.24	2.125	+0.885	-15.6	90 02 37.8	
	2:20	(Helio)					90 02 30.0	
	2:28						90 02 37.0	
	2:40						90 02 32.5	
							34.3	90 02 18.7
	2:46	Cube	9.37	2.125	-7.245	+74.9	90 06 04.9	
		(Helio)					90 05 52.8	
							90 05 54.9	
							90 05 57.5	90 07 12.4
	3:02	Monument	1.91	2.125	+0.215	-2.2	90 02 03.4	
	3:10	(Helio)					90 02 04.6	
							90 01 58.6	
							02.2	90 02 00.0
	4:07	Cupola of		2.125			90 00 32.8	
		Schoolhouse					90 00 26.4	
		Hastings					29.6	

FIG. 44.—Abstract of zenith distances for sample computation of elevations—Continued

ABSTRACT OF ZENITH DISTANCES

Station Hastings State Oklahoma

Observer E.O.H. - A.F.B. Instrument V.C. 109

Date	Hour	Object observed	Object above station =o	Telescope above station =t	Diff. of heights t−o	Reduction to line joining stations	Observed zenith distance	Corrected zenith distance
1923			*Meters*	*Meters*	*Meters*	"	° ′ ″	° ′ ″
5/24	1:27	Monument	1.91	1.445	-0.465	+4.5	90 02 32.3	
	1:34	(Helio)					90 02 41.5	
	1:40						90 02 32.1	
							35.3	90 02 39.8
	1:47	Keele	1.92	1.445	-0.475	+8.4	90 03 26.1	
	1:50	(Helio)					90 03 26.8	
	1:52						90 03 27.8	
							26.9	90 03 35.3
	2:06	Bailer	1.21	1.445	+0.235	-2.7	90 04 37.9	
	2:12	(Helio)					90 04 43.9	
	2:15						90 04 44.2	
							42.0	90 04 39.3
	2:47	Byers	3.61	1.445	-2.165	+27.7	90 02 15.4	
	2:50	(Helio)					90 02 24.8	
	2:59						90 02 16.7	
	3:24						90 02 13.4	
	3:40						90 02 03.4	
	3:37						90 02 11.9	
5/26	2:40						90 02 23.2	
	2:45						90 02 15.7	
	3:52						90 02 13.8	
							15.4	90 02 43.1
5/24	2:22	Cube	9.37	1.445	-7.925	+58.3	90 07 41.3	
	2:25						90 07 39.0	
	2:27						90 07 47.6	
							42.6	90 08 40.9
5/26	2:57	Lee	1.09	1.445	+0.355	-3.1	90 04 60.1	
	3:04	(Helio)					90 04 53.6	
	3:19						90 04 55.2	
	3:30						90 04 50.3	
							54.8	90 04 51.7
5/24	4:48	Cupola		1.445			89 53 05.9	
	4:54	Schoolhouse					89 52 59.0	
5/25	3:26	Hastings					89 52 51.3	
	3:30						89 52 53.5	
							89 52 57.4	

FIG. 44.—Abstract of zenith distances for sample computation of elevations—Continued

ABSTRACT OF ZENITH DISTANCES

Station ..Byers.......... State ....Texas..........

Observer ..A.F.B. - E.O.H.......... Instrument....V.Q. 109..........

Date	Hour	Object Observed	Object above Station =o	Telescope above Station =t	Diff. of Heights t−o	Reduction to Line joining Stations	Observed Zenith Distance	Corrected Zenith Distance
			Meters	Meters	Meters	"	° ′ ″	° ′ ″
1923								
5/30	1:23	Cube	9.37	3.815	-5.555	+74.1	90 06 58.6	
	1:31	(Helio)					90 07 05.7	
	1:37						90 06 54.5	
	1:43						90 06 55.7	
							90 06 58.6	90 08 12.7
	1:53	Bailer	1.21	3.815	+2.605	-33.2	90 05 42.5	
	2:00	(Helio)					90 05 33.4	
	2:06						90 05 25.3	
	2:11						90 05 35.1	
	2:13						90 05 30.4	
	3:15						90 05 39.4	
5/31	1:35						90 05 44.4	
	1:40						90 05 47.8	
	1:45						90 05 45.6	
							38.2	90 05 05.0
	2:27	Thornberry	3.01	3.815	+0.805	-9.0	90 04 33.0	
		(Helio)					90 04 36.7	
							90 04 36.5	
6/4	1:55						90 04 20.5	
	2:00						90 04 29.8	
	2:04						90 04 35.3	
							32.0	90 04 23.0
5/31	2:43	Keele	1.92	3.815	+1.895	-25.8	90 06 02.1	
		(Helio)					90 06 06.3	
							90 06 01.5	
							03.3	90 05 37.5
	2:56	Hastings	1.24	3.815	+2.575	-32.9	90 05 47.7	
	3:03	(Helio)					90 05 48.2	
							90 05 47.8	
							47.9	90 05 15.0
6/4	2:33	Lee	6.12	3.815	-2.305	+21.0	90 04 40.7	90 05 01.7
	2:38	(Top of stand)	6.21		-2.395	+21.8	90 04 46.8	90 05 08.6
	2:44	(Helio)	6.21				90 04 51.8	90 05 13.6
								90 05 08.0

FIG. 44.—Abstract of zenith distances for sample computation of elevations—Continued

Department of Commerce
U. S. Coast and Geodetic Survey
Form 29

Abstract of Zenith Distances

Station Bailer State Oklahoma

Observer A.F.B. Instrument V.C. 109

Date	Hour	Object observed	Object above station =o	Telescope above station =t	Diff. of heights t−o	Reduction to line joining stations	Observed zenith distance	Corrected zenith distance
1923			Meters	Meters	Meters	″	° ′ ″	° ′ ″
6/2	2:20	Thornberry	3.01	1.415	-1.595	+25.7	90 01 16.8	
	2:23	(Helio)					90 01 18.1	
	2:28						90 01 19.3	
							18.1	90 01 43.8
	2:42	Byers	3.61	1.415	-2.195	+28.0	90 02 24.6	
	2:46	(Helio)					90 02 29.2	
	2:50						90 02 25.2	
							26.3	90 02 54.3
	2:55	Hastings	1.24	1.415	+0.175	-2.0	90 05 01.5	
	3:00	(Helio)					90 04 59.7	
	3:05						90 04 55.1	
							90 04 58.8	90 04 56.8
	3:12	Lee	1.09	1.415	+0.325	-10.2	89 58 56.1	
	3:17	(Helio)					89 58 51.8	
	3:21						89 58 48.9	
							52.3	89 58 42.1

Fig. 44.—Abstract of zenith distances for sample computation of elevations—Continued

Abstract of Zenith Distances

Station Lee State Oklahoma

Observer A.F.B. Instrument V.C. 109

Date	Hour	Object observed	Object above station =o	Telescope above station =t	Diff. of heights t−o	Reduction to line joining stations	Observed zenith distance	Corrected zenith distance
1923			Meters	Meters	Meters	″	° ′ ″	° ′ ″
6/5	2:10	Byers	3.61	6.415	+2.805	-25.6	90 06 15.5	
	2:15	(Helio)					90 06 21.3	
	2:18						90 06 20.7	
							19.2	90 05 53.6
	2:25	Bailer	1.21	6.415	+5.205	-164.0	90 07 53.2	
	2:29	(Helio)					90 07 55.2	
	2:34						90 07 59.4	
							55.9	90 05 11.9
	2:43	Gammill (Helio)	9.23	6.415	-2.815	+45.7	89 58 47.0	89 59 32.7
	2:48	(Top of stand)	9.14		-2.725	+44.2	89 58 43.4	89 59 27.6
	2:52		9.14				89 58 35.5	89 59 19.7
	3:00		9.14				89 58 23.5	89 59 07.7
	4:00		9.14				89 58 33.5	89 59 17.7
							33.5	89 59 21.1
	3:13	Willis	1.77	6.415	+4.645	-65.8	90 00 23.1	
	3:25	(Helio)					90 00 28.2	
	3:31						90 00 29.4	
							26.9	89 59 21.1
	3:50	Thornberry	3:01	6.415	+3.405	-49.3	90 04 29.5	
	3:52	(Helio)					90 04 31.1	
	3:56						90 04 27.6	
							29.4	90 03 40.1
	4:10	Hastings	1.24	6.415	+5.175	-45.7	90 07 25.4	
	4:13	(Helio)					90 07 33.3	
	4:28						90 07 23.2	
	4:34						90 07 35.3	
							29.3	90 06 43.6
	5:33	School House		6.415			90 05 41.4	
	5:40	top, Hastings					90 05 43.1	
							42.2	

Fig. 44.—Abstract of zenith distances for sample computation of elevations—Continued

45861°—34——11

ABSTRACT OF ZENITH DISTANCES

Station Thornberry State Texas

Observer A.F.B. - E.O.H. Instrument V.C. 109

Date	Hour	Object Observed	Object above station = o	Telescope above station = t	Diff. of heights t—o	Reduction to line joining stations	Observed Zenith Distance	Corrected Zenith Distance
1923			Meters	Meters	Meters	"	° ′ ″	° ′ ″
6/1	1:53	Byers	3.61	3.215	-0.395	+4.4	90 04 34.4	
	1:56	(Helio)					90 04 35.5	
	2:02						90 04 31.0	
							33.6	90 04 38.0
	2:07	Lee	1.09	3.215	+2.125	-30.8	90 04 47.4	
	2:11	(Helio)					90 04 44.7	
	2:19						90 04 48.3	
							46.8	90 04 16.0
	2:29	Cashion	1.16	3.215	+2.055	-37.5	90 02 28.3	
	2:34	(Helio)					90 02 34.8	
	2:37						90 02 26.6	
	2:45						90 02 35.3	
	3:23						90 02 18.2	
							28.6	90 01 51.1
	3:00	Gammill	9.23	3.215	-6.015	+71.8	90 00 01.3	
	3:08	(Helio)					90 00 05.9	
	3:12						90 00 14.1	
	3:20						90 00 04.2	
							06.4	90 01 13.2
	3:30	Willis	1.68	3.215	+1.535	-35.0	89 55 39.1	
	3:45	(Top of stand)					89 55 37.8	
	3:50						89 55 40.8	
							39.2	89 55 04.2

FIG. 44.—Abstract of zenith distances for sample computation of elevations—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29

ABSTRACT OF ZENITH DISTANCES

Station Willis State Oklahoma

Observer E.O.H. - A.F.B. Instrument V.C. 109

Date	Hour	Object observed	Object above station =o	Telescope above station =t	Diff. of heights t−o	Reduction to line joining stations	Observed zenith distance	Corrected zenith distance
1923			Meters	Meters	Meters	″	° ′ ″	° ′ ″
6/6	1:55	Gammill	9.23	1.975	-7.255	+147.6	90 01 21.7	
	2:01	(Helio)					90 01 24.0	
	2:05						90 01 22.3	
							22.7	90 03 50.3
	2:12	Lee	6.21	1.975	-4.235	+60.0	90 06 48.1	
	2:19	(Helio)					90 06 40.4	
	3:09						90 06 58.1	
							48.9	90 07 48.9
	2:55	Cashion	1.16	1.975	+0.815	-20.8	90 08 22.0	
	2:58	(Helio)					90 08 21.8	
	3:02						90 08 22.8	
							22.2	90 08 01.4
6/7	8:10	(Light)	1.26	1.975	+0.715	-18.3	90 07 56.7	
	8:20						90 07 50.9	
	8:25						90 07 46.9	
	9:15						90 07 45.9	
							50.1	90 07 31.8
								29.6
	8:40	Thornberry	3.11	1.975	-1.135	+25.9	90 07 54.1	
	8:45	(Light)					90 08 01.0	
	8:52						90 08 16.7	
	9:00						90 08 13.5	
	9:05						90 08 13.6	
	9:08						90 08 08.8	
							90 08 08.0	90 08 33.9
								29.6
								90 09 03.5

FIG. 44.—Abstract of zenith distances for sample computation of elevations—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29

ABSTRACT OF ZENITH DISTANCES

Station Gammill State Oklahoma

Observer A.F.B. - E.O.H. Instrument V.C. 109

Date	Hour	Object observed	Object above station = o	Telescope above station = t	Diff. of heights t—o	Reduction to line joining stations	Observed zenith distance	Corrected zenith distance
1923			Meters	Meters	Meters	"	° ′ ″	° ′ ″
6/18	4:49	Lee	6.12	9.435	+3.315	-53.8	90 07 26.9	
	5:12	(Top of stand)					90 07 28.3	
	5:14						90 07 31.7	
							29.0	90 06 35.2
	5:26	Willis	1.68	9.435	+7.755	-157.8	90 03 36.3	
	5:30	(Top of stand)					90 03 36.2	
	5:31						90 03 33.8	
							35.4	90 00 57.6

Station: Cashion State: Texas

Observer: A.F.B. - E.O.H. Instrument: V.C. 109

11—615

Date	Hour	Object observed	Object above station = o Meters	Telescope above station = t Meters	Diff. of heights t—o Meters	Reduction to line joining stations ″	Observed zenith distance	Corrected zenith distance
1923								
6/11	1:07	Willis	1.77	1.365	-0.405	+10.3	89 56 21.3	
	1:10	(Helio)					89 56 14.0	
	1:14						89 56 16.0	
							17.1	89 56 27.4
	1:25	Gammill	9.23	1.365	-7.865	+92.3	90 01 43.1	
	1:35	(Helio)					90 01 53.0	
	1:40						90 01 45.0	
	1:49						90 01 36.0	
							90 01 42.3	
							43.9	90 03 16.2
	4:26	Thornberry	3.01	1.365	-1.645	+30.0	90 04 01.5	
	4:36	(Helio)					90 03 59.6	
	4:41						90 04 00.7	
							90 04 00.6	90 04 30.6

FIG. 44.—Abstract of zenith distances for sample computation of elevations—Continued

COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS

Form 29A is used in computing differences of elevation from reciprocal observations. (See fig. 45.) The coefficient of refraction, *m*, which is needed in computing the nonreciprocal observations is also obtained on this form. The computations for all the reciprocal lines are given in Figure 45.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 A

COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS.

Station 1, occ.	Monument	Cube		Monument	Cube	Keele
Station 2, obs.	Keele	Keele		Hastings	Hastings	Hastings
	° ′ ″	° ′ ″		° ′ ″	° ′ ″	° ′ ″
ζ_1	90 08 23.5	90 03 02.4		90 07 58.6	90 05 17.5	90 02 18.7
ζ_2	90 02 00.0	90 07 12.4		90 02 39.8	90 08 40.9	90 03 35.3
$\zeta_2-\zeta_1$	- 6 23.5	+ 4 10.0		- 5 18.8	+ 3 23.4	+ 1 16.6
$\frac{1}{2}(\zeta_2-\zeta_1)$	- 3 11.8	+ 2 05.0		- 2 39.4	+ 1 41.7	+ 0 38.3
$\frac{1}{2}(\zeta_2-\zeta_1)$ in secs.	- 191.8	+ 125.0		- 159.4	+ 101.7	+ 38.3
log ditto	2.28285$_n$	2.09691		2.20249$_n$	2.00732	1.58320
T	4.68557	4.68557		4.68557	4.68557	4.68557
log s	4.30550	4.29977		4.32395	4.44750	4.06840
log $[s \tan \frac{1}{2}(\zeta_2-\zeta_1)]$	1.27292$_n$	1.08225		1.21201$_n$	1.14039	0.33717
log A	+ 2	+ 2		+ 2	+ 2	+ 2
log B	0	0		0	0	0
log C	0	0		0	0	0
log (h_2-h_1)	1.27394$_n$	1.08227		1.21203$_n$	1.14041	0.33719
h_2-h_1	-18.79	+12.08		-16.29	+13.82	+2.17
h_1	329.90	298.80		329.90	298.80	310.99
h_2	311.11	310.83		313.61	312.62	313.16
2 log s	8.6110	8.5995		8.6479	8.8950	8.1368
log $p=9-2 \log s$	0.3890	0.4005		0.3521	0.1050	0.8632
p of (h_2-h_1)	2.45	2.51		2.25	1.27	7.30
α and mean ϕ	38°.7 34°.2	18°.5 34°.0		71°.6 34°.2	2°.2 34°.1	39°.1 34°.1
$\zeta_1+\zeta_2-180°$	10′ 23″.5	10′ 14″.8		10′ 38″.4	13′ 58″.4	05 54″.0
$\zeta_1+\zeta_2-180°$ in sec.	623″.5	614″.8		638″.4	838″.4	354″.0
log ditto	2.79484	2.78873		2.80509	2.92345	2.54900
log ρ	6.80394	6.80334		6.80496	6.80314	6.80394
colog s	5.69450	5.70023		5.67605	5.55250	5.93160
log $\frac{\sin 1''}{2}=\bar{4}.38454$	4.38454	4.38454		4.38454	4.38454	4.38454
log $(0.5-m)$	9.67782	9.67684		9.67064	9.66363	9.66908
$(0.5-m)$	0.4762	0.4752		0.4684	0.4609	0.4667
p of $(0.5-m)$	4.08	3.98		4.45	7.85	1.37

310.99 313.19

FIG. 45.—Computation of elevations and refractions from reciprocal observations

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 39 A

COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS.

Station 1, occ.	Keele	Cube	Hastings		Byers	Hastings
Station 2, obs.	Byers	Byers	Byers		Bailer	Bailer
ζ_1	90° 01′ 54″.6	89° 59′ 43″.7	90° 02′ 43″.1		90° 06′ 06″.0	90° 04′ 39″.3
ζ_2	90 05 37.5	90 08 12.7	90 05 15.0		90 02 54.3	90 04 56.8
$\zeta_2-\zeta_1$	+3 42.9	+8 29.0	+2 31.9		−2 10.7	+0 17.5
$\frac{1}{2}(\zeta_2-\zeta_1)$	+1 51.4	+4 14.5	+1 16.0		−1 05.4	+0 08.8
$\frac{1}{2}(\zeta_2-\zeta_1)$ in secs.	+111.4	+254.5	+76.0		−65.4	+8.8
log ditto	2.04689	2.40569	1.88081		1.81558_n	0.94448
T	4.68557	4.68558	4.68557		4.68557	4.68557
log s	4.18110	4.18949	4.20738		4.20903	4.25792
log $[s \tan \frac{1}{2}(\zeta_2-\zeta_1)]$	0.91356	1.28076	0.77376		0.71018_n	9.88797
log A	+2	+2	+2		+2	+2
log B	0	0	0		0	0
log C	0	0	0		0	0
log (h_2-h_1)	0.91358	1.28078	0.77378		0.71020_n	9.88799
h_2-h_1	+8.20	+19.09	+5.94		−5.13	+0.77
h_1	310.99	298.80	313.19		318.73	313.19
h_2	319.19	317.89	319.13		313.60	313.96
2 log s	8.3622	8.3790	8.4148		8.4181	8.5158
log $p = 9-2 \log s$	0.6378	0.6210	0.5852		0.5819	0.4842
p of (h_2-h_1)	4.34	4.18	3.85		3.82	3.05
α and mean ϕ	68°.5 34°.1	30°.3 34°.0	24°.7 34°.1		43°.5 34°.1	80°.8 34°.2
$\zeta_1+\zeta_2-180°$	7′ 32″.1	07′ 56″.4	07′ 58″.1		07′ 59″.3	09′ 36″.1
$\zeta_1+\zeta_2-180°$ in sec.	452″.1	476″.4	478″.1		479″.3	576″.1
log ditto	2.65523	2.67797	2.67952		2.68061	2.76050
log ρ	6.80488	6.80365	6.80349		6.80409	6.80511
colog s	5.81890	5.81051	5.79262		5.79097	5.74208
log $\frac{\sin 1''}{2} = \bar{4}.38454$	4.38454	4.38454	4.38454		4.38454	4.38454
log $(0.5-m)$	9.66355	9.67667	9.66017		9.66021	9.69223
$(0.5-m)$	0.4608	0.4750	0.4573		0.4573	0.4923 R.
p of $(0.5-m)$	2.30	2.39	2.60		2.62	3.28
	318.73				313.76	

FIG. 45.—Computation of elevations and refractions from reciprocal observations—Continued

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U. S. COAST AND GEODETIC SURVEY
Form 29 A

COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS.

Station 1, occ.	Byers	Hastings	Bailer		Byers	Lee
Station 2, obs.	Lee	Lee	Lee		Thornberry	Thornberry
ζ_1	90° 05′ 08″.0	90° 04′ 51″.7	89° 58′ 42″.1		90° 04′ 23″.0	90° 03′ 40″.1
ζ_2	90 05 53.6	90 06 43.6	90 05 11.9		90 04 38.0	90 04 16.0
$\zeta_2-\zeta_1$	+0 45.6	+1 51.9	+6 29.8		+0 15.0	+0 35.9
$\frac{1}{2}(\zeta_2-\zeta_1)$	+0 22.8	+0 56.0	+3 14.9		+0 07.5	+0 18.0
$\frac{1}{2}(\zeta_2-\zeta_1)$ in secs.	+ 22.8	+ 56.0	+ 194.9		+ 7.5	+ 18.0
log ditto	1.35793	1.74819	2.28981		0.87506	1.25527
T	4.68557	4.68557	4.68557		4.68557	4.68557
log s	4.35434	4.36805	3.81606		4.26766	4.15328
log [s tan $\frac{1}{2}(\zeta_2-\zeta_1)$]	0.39784	0.80181	0.79144		9.82829	0.09412
log A	+ 2	+ 2	+ 2		+ 2	+ 2
log B	0	0	0		0	0
log C	0	0	0		0	0
log (h_2-h_1)	0.39786	0.80183	0.79146		9.82831	0.09414
h_2-h_1	+ 2.50	+ 6.34	+ 6.19		+0.67	+1.24
h_1	318.73	313.19	313.76		318.73	320.01
h_2	321.23	319.53	319.95		319.40	321.25
2 log s	8.7087	8.7361	7.6321		8.5353	8.3066
log $p=9-2$ log s	0.2913	0.2639	1.3679		0.4647	0.6934
p of (h_2-h_1)	1.96	1.84	23.33		2.92	4.94
α and mean ϕ	47°.2 34°.1	88°.2 34°.2	56°.5 34°.2		86°.2 34°.1	7°.5 34°.1
$\zeta_1+\zeta_2-180°$	11′ 01″.6	11′ 35″.3	03′ 54″.0		09′ 01″.0	07′ 56″.1
$\zeta_1+\zeta_2-180°$ in sec.	661″.6	695″.3	234″.0		541″.0	476″.1
log ditto	2.82060	2.84217	2.36922		2.73320	2.67770
log ρ	6.80422	6.80516	6.80455		6.80515	6.80317
colog s	5.64566	5.63195	6.18394		5.73234	5.84672
log $\frac{\sin 1''}{2}=\bar{4}.38454$	4.38454	4.38454	4.38454		4.38454	4.38454
log $(0.5-m)$	9.65502	9.66382	9.74225		9.65523	9.71213
$(0.5-m)$	0.4519	0.4611	0.5524 R		0.4521	0.5154 R
p of $(0.5-m)$	5.11	5.45	0.43		3.43	2.03

320.01

320.56

FIG. 45.—Computation of elevations and refractions from reciprocal observations—Continued

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Form 29 A

COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS.

Station 1, occ.	Lee	Thornberry		Lee	Willis	
Station 2, obs.	Willis	Willis		Gammill	Gammill	
	° ′ ″	° ′ ″		° ′ ″	° ′ ″	
ζ_1	89 59 21.1	89 55 04.2		89 59 21.1	90 03 50.3	
ζ_2	90 07 48.9	90 09 03.5		90 06 35.2	90 00 57.6	
$\zeta_2-\zeta_1$	+ 8 27.8	+13 59.3		+ 7 14.1	- 2 52.7	
$\frac{1}{2}(\zeta_2-\zeta_1)$	+ 4 13.9	+ 6 59.6		+ 3 37.0	- 1 26.4	
$\frac{1}{2}(\zeta_2-\zeta_1)$ in secs.	+ 253.9	+ 419.6		+ 217.0	- 86.4	
log ditto	2.40466	2.62284		2.33646	1.93651	
T	4.68558	4.68558		4.68558	4.68557	
log s	4.16315	3.95628		4.10401	4.00605	
log $[s \tan \frac{1}{2}(\zeta_2-\zeta_1)]$	1.25339	1.26470		1.12605	0.62813	
log A	+ 2	+ 2		+ 2	+ 2	
log B	0	0		0	0	
log C	0	0		0	0	
log (h_2-h_1)	1.25341	1.26472		1.12607	0.62815	
h_2-h_1	+17.92	+18.40		+13.37	- 4.25	
h_1	320.01	320.56		320.01	338.67	
h_2	337.93	338.96		333.38	334.42	
2 log s	8.3263	7.9126		8.2080	8.0121	
log $p = 9 - 2$ log s	0.6737	1.0874		0.7920	0.9879	
p of (h_2-h_1)	4.72	12.23		6.19	9.73	
α and mean ϕ	44.1° 34.1°	56.2° 34.1°		87.1° 34.2°	14.7° 34.1°	
$\zeta_1+\zeta_2-180°$	07′ 10.0″	04′ 07.7″		05′ 56.3″	04′ 47.9″	
$\zeta_1+\zeta_2-180°$ in sec	430.0″	247.7″		356.3″	287.9″	
log ditto	2.63347	2.39393		2.55182	2.45924	
log ρ	6.80411	6.80483		6.80515	6.80326	
colog s	5.83685	6.04372		5.89599	5.99395	
log $\frac{\sin 1''}{2} = \bar{4}.38454$	4.38454	4.38454		4.38454	4.38454	
log $(0.5-m)$	9.65897	9.62702		9.63750	9.64099	
$(0.5-m)$	0.4560	0.4237		0.4340	0.4375	
p of $(0.5-m)$	2.12	0.82		1.61	1.03	

338.67 334.02

FIG. 45.—Computation of elevations and refractions from reciprocal observations—Continued

DEPARTMENT OF COMMERCE
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Form 29 A

COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS.

Station 1, occ.	Thornberry	Willis				
Station 2, obs.	Cashion	Cashion				
ζ_1	90° 01′ 51.″1	90° 08′ 01.″4				
ζ_2	90 04 30.6	89 56 27.4				
$\zeta_2-\zeta_1$	+ 2 39.5	- 11 34.0				
$\frac{1}{2}(\zeta_2-\zeta_1)$	+ 1 19.8	- 5 47.0				
$\frac{1}{2}(\zeta_2-\zeta_1)$ in secs.	+ 79.8	347.0				
log ditto	1.90200	2.54033 n				
T	4.68557	4.68558				
log *s*	4.05335	3.90728				
log [s tan $\frac{1}{2}(\zeta_2-\zeta_1)$]	0.64092	1.13319 n				
log *A*	+ 2	+ 2				
log *B*	0	0				
log *C*	0	0				
log (h_2-h_1)	0.64094	1.13321 n				
h_2-h_1	+ 4.37	- 13.59				
h_1	320.56	338.67				
h_2	324.93	325.08				
2 log *s*	8.1067	7.8146				
log $p=9-2$ log s	0.8933	1.1854				
p of (h_2-h_1)	7.82	15.32				
α and mean ϕ	68.°7 34.°0	16.°2 34.°1				
$\zeta_1+\zeta_2-180°$	06′ 21.″7	04′ 28.″8				
$\zeta_1+\zeta_2-180°$ in sec.	381.″7	268.″8				
log ditto	2.58172	2.42943				
log ρ	6.80489	6.80329				
colog *s*	5.94665	6.09272				
log $\frac{\sin 1''}{2}=\bar{4}.38454$	4.38454	4.38454				
log $(0.5-m)$	9.71780	9.70998				
$(0.5-m)$	0.5222 R	0.5128 R				
p of $(0.5-m)$						

325.03

FIG. 45.—Computation of elevations and refractions from reciprocal observations—Continued

EXPLANATION OF COMPUTATION

The mean corrected zenith distances, ζ_1 and ζ_2, are taken from Form 29. (See fig. 44.) Log s, the logarithm of the length of the given line, α, the azimuth of the line, and "mean ϕ" the mean latitude of the two stations concerned, are obtained from the list of geographic positions. If the list of positions has not been made out, log s can be obtained directly from the triangle computations and α and "mean ϕ" from the position computations.

The following rules should be observed: Carry all angles to tenths of seconds only, and all logarithms to five decimal places only except "2 log s" and "9 − 2 log s" which should be carried to four decimal places only. If the zenith distance is to some indefinite object, such as a mountain peak for which there was no well-defined point on which to sight, the angles should be carried only to even seconds. The quantity $h_2 - h_1$ should be carried to centimeters only. Log $(0.5 - m)$ should be computed to five decimal places and $(0.5 - m)$ to four decimal places. The weights should be carried to two decimal places.

To convert $\frac{1}{2}(\zeta_2 - \zeta_1)$ from minutes and seconds, to seconds, use the tables at the bottom of pages 2–185 of the Vega Logarithmic Tables. The logarithm of the value in seconds and the corresponding value of T are found on the same page as the conversion, in each case, and should be taken out at the same time. Log $[s \tan \frac{1}{2}(\zeta_2 - \zeta_1)]$ is the sum of the three logarithms next above it. Log A, log B, and log C are obtained from the tables on page 232, which are self-explanatory. The sum of these three logarithms and log $[s \tan \frac{1}{2}(\zeta_2 - \zeta_1)]$ gives log $(h_2 - h_1)$. The elevation h_1 in each case is obtained from preceding computations.

The relative weights to be assigned to the various values of $h_2 - h_1$ in the least squares adjustment are inversely proportional to s^2, and for convenience are computed by the formula log $p = 9 - 2$ log s, as shown on the computation directly below h_2. By this formula a line 31.6 kilometers long is given unit weight.

After the value of h_2 and its weight, p, have been determined, the coefficient of refraction is computed by the formula,

$$0.5 - m = \frac{(\zeta_1 + \zeta_2 - 180^\circ)\rho \sin 1''}{2s}$$

in which m, the coefficient of refraction, is the ratio of the mean angle at the two stations, between the tangent to the line of sight and the chord joining the two stations, to the angle between the lines of gravity at the two stations. The azimuth α of the line and the mean latitude, ϕ, of the two stations are taken out to the nearest tenth of a degree only. The radius of curvature, ρ, is taken from the tables on pages 220–222 with α and "mean ϕ" as arguments. The relative weight,

p, to be assigned to each determination of $(0.5-m)$ is proportional to s^2. The logarithm of s^2 (or 2 log s) has already been taken out for use in computing the relative weights of (h_2-h_1). As the numbers corresponding to (2 log s) would usually be large quantities, it is customary to divide them by a power of 10. For example, in the computations shown above they are all divided by 10^8. The value of $(0.5-m)$ is only for use in computing elevations from nonreciprocal observations and need not be computed unless such observations are made. All values of $(0.5-m)$ which are greater than 0.5 or which are nearly 0.5 should be rejected.

After the elevation of a given station has been determined from two or more stations, the weighted mean should be taken before it is used in determining the elevation of some other station. For example, the elevation of Keele (fig. 45) as determined from Monument is 311.11 meters with a weight of 2.45, and as determined from Cube is 310.88 with a weight of 2.51. The elevation to be used for Keele is found as follows:

$$\begin{array}{lr} 2.45\times 311.11 = & 762.2195 \\ 2.51\times 310.88 = & 780.3088 \\ \hline 4.96 & 1542.5283 \end{array}$$

$$\frac{1542.5283}{4.96}=310.99 \text{ meters.}$$

This elevation is placed at the bottom of the computation for Keele. The elevations of the other stations are obtained in a similar manner.

After the elevations of the various stations connected by reciprocal observations have been determined and the value of $(0.5-m)$ obtained at each, the differences of elevation for the nonreciprocal observations are computed.

COMPUTATION OF ELEVATIONS FROM NONRECIPROCAL OBSERVATIONS

The formula for computing elevations from nonreciprocal observations is given on page 148. This formula may be rewritten in the form

$$h_2-h_1=s \cot (\zeta_1-k)\ ABC=s \tan (90°-\zeta_1+k)\ ABC$$

where $k=\frac{(0.5-m)\ s}{\rho \sin 1''}$. The quantity, ζ_1, is the mean observed zenith distance of the object sighted, no vertical eccentric reduction being made in the case of nonreciprocal observations. The other quantities in the formula have already been defined.

Form 29B is used for computing differences of elevation from nonreciprocal observations. (See fig. 46.) The quantities ζ_1 and $(t-o)$ are obtained from Form 29. (See fig. 44.) The quantities log s, α and mean ϕ are all obtained from the list of geographic positions, or from the triangle and position computations. Colog ρ is determined by subtracting the value of log ρ as taken from the tables on pages 220–222 from 10. Log A, log B, and log C are obtained from the tables on page 232. The value of $(0.5-m)$ is obtained from the computation of the reciprocal observations involving the same station. For example, for the observations on station Keele from station Benton the $(0.5-m)$ used should be a weighted mean of the $(0.5-m)$'s as determined in the computations of the reciprocal observations involving Benton. It happens in this case that the elevation of Benton was fixed to start with and as there were no reciprocal observations from this station there are no computations of $(0.5-m)$. The log value, 9.63246 − 10, given on page 63 of Special Publication No. 26, was therefore issued.

In the computation of the nonreciprocal observations on Thornberry from Bailer, the log $(0.5-m)$ is obtained as follows: In Figure 45 it is seen that $(0.5-m)$ was determined three times in the computation of the reciprocal observations involving station Bailer. In the Hastings-Bailer computation the value determined is 0.4923, and in the Bailer-Lee computation it is 0.5524. As the value is very close to 0.5 in one of these computations, and in the other it is greater than 0.5, these values are rejected. In the Byers-Bailer computation the value is 0.4573 and this is the one used, the logarithm being 9.66021.

In the computation of station Gammill from Thornberry, the value of $(0.5-m)$ is obtained as follows: The value of $(0.5-m)$ for station Thornberry is determined four times in the computation of the reciprocal observations in Figure 45. For the Lee-Thornberry line the value of $(0.5-m)$ is 0.5154 and for the Thornberry-Cashion line it is 0.5222. Both of these values are rejected since they are greater than 0.5. For the Byers-Thornberry line $(0.5-m)$ is 0.4521 with a weight of 3.43, and for the Thornberry-Willis line it is 0.4237 with a weight of 0.82. The weighted mean value is then

$$\frac{(0.4521\times 3.43)+(0.4237\times 0.82)}{3.43+0.82}=0.4466.$$

The log of this value, 9.64992 − 10, is used in the computation of the elevation of Gammill from Thornberry.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 B

COMPUTATION OF ELEVATIONS FROM NONRECIPROCAL OBSERVATIONS.

Station 1, occ.	Benton	Bailer	Thornberry			
Station 2, obs.	Keele	Thornberry	Gammill			
Object sighted	Helio	Helio	Helio			
ζ_1	90° 01′ 58″.4	90° 01′ 18″.1	90° 00′ 06″.4			
α and mean ϕ	88.1 34.1	34.9 34.1	38.9 34.1			
log (0.5−m)	9.63246	9.66021	9.64992			
log s	4.18146	4.10702	4.23756			
colog ρ	3.19485	3.19620	3.19606			
colog sin 1″	5.31443	5.31443	5.31443	5.31443	5.31443	5.31443
log (k in secs.)	2.32320	2.27786	2.39797			
k in secs.	210″.5	189″.6	250″.0			
(90°−ζ_1+k) in secs.	+92″.1	+111″.5	+243″.6			
log ditto	1.96426	2.04727	2.38668			
T	4.68557	4.68557	4.68558			
log s	4.18146	4.10702	4.23756			
log [s tan (90°−ζ_1+k)]	0.83129	0.83986	1.30982			
log A	+ 2	+ 2	+ 2			
log B	0	0	0			
log C	0	0	0			
log (h_2-h_1)	0.83131	0.83988	1.30984			
h_2-h_1	+ 6.78	+ 6.92	+ 20.41			
h_1						
$t-o$	-0.54	-1.60	-6.02			
Corrected (h_2-h_1)	+ 6.24	+ 5.32	+14.39			
log p=9−2 log s	0.6371	0.7860	0.5249			
p	4.34	6.11	3.35			
Weighted mean elevation of sta. obs.						

FIG. 46.—Computation of elevations from nonreciprocal observations

LEAST-SQUARES ADJUSTMENT OF DIFFERENCES OF ELEVATIONS

After all differences of elevations have been computed, both from the reciprocal and the nonreciprocal observations, and their weights determined, the next step is the adjustment of these differences of elevation by the method of least squares. An example of such an adjustment, from the formation of the equations to the determination of the final elevations, is given in detail on the following pages for the differences of elevation represented in Figure 43.

The adjustment of vertical observations is made by means of observation equations. Elevations in even meters approximating the final values are first assumed for the different stations. To these assumed values are added x's to be determined by the adjustment. Then observation equations are formed by comparing the differences of the assumed elevations with the differences determined by computation.

In the first one of the following tables are given the stations already fixed in elevation and their elevations. In the second table are given the names of the stations whose elevations are to be fixed by the adjustment and in the second column of this table their assumed elevations and correction symbols. The last two columns of this table are filled in after the adjustment is completed.

If any of the stations of the scheme have been determined in elevation directly from first-order leveling their elevations should be held fixed in the adjustment. The mathematician making the adjustment should be very careful at the start to ascertain whether there are any direct connections with first-order leveling.

Fixed elevations

Station	Elevation	
	Meters	Feet
Monument	329.90	1,082.3
Benton	301.40	988.8
Cube	298.80	980.3
Gammill	333.35	1,093.7
Cashion	324.65	1,065.1

Assumed and adjusted elevations

Station	Elevation		
	Assumed, plus correction	Adjusted [1]	
		Meters	Feet
	Meters		
Keele	$311+x_1$	310.41	1,018.4
Hastings	$313+x_2$	312.81	1,026.3
Byers	$319+x_3$	318.47	1,044.8
Bailer	$314+x_4$	313.60	1,028.9
Lee	$320+x_5$	319.75	1,049.0
Thornberry	$321+x_6$	319.90	1,049.5
Willis	$339+x_7$	338.05	1,109.1

[1] This column is filled in after the adjustment is completed.

FORMATION OF OBSERVATION EQUATIONS

There are just as many observation equations as there are computed differences of elevation. They are tabulated in the form shown below.

Formation of observation equations

	Station 1	Station 2	Weight p	Assumed difference of elevation h_2-h_1	Observed difference of elevation h_2-h_1	Assumed minus observed	Symbol	Adjusted difference of elevation h_2-h_1	Adjusted minus observed v	pv	v^2	pv^2
1	Monument	Keele	2.45	−18.90	−18.79	−0.11	$+x_1$	−19.49	−0.70	−1.72	0.4900	1.2005
2	Benton [1]	Keele	1.45	+9.60	+6.24	+3.36	$+x_1$	+9.01	+2.77	+4.02	7.6729	11.1257
3	Cube	Keele	2.51	+12.20	+12.08	+.12	$+x_1$	+11.61	−.47	−1.18	.2209	.5545
4	Monument	Hastings	2.25	−16.90	−16.29	−.61	$+x_2$	−17.09	−.80	−1.80	.6400	1.4400
5	Keele	Hastings	7.30	+2.00	+2.17	−.17	$-x_1+x_2$	+2.40	+.23	+1.68	.0529	.3862
6	Cube	Hastings	1.27	+14.20	+13.82	+.38	$+x_2$	+14.01	+.19	+.24	.0361	.0458
7	Hastings	Byers	3.85	+6.00	+5.94	+.06	$-x_2+x_3$	+5.66	−.28	−1.08	.0784	.3018
8	Keele	Byers	4.34	+8.00	+8.20	−.20	$-x_1+x_3$	+8.06	−.14	−.61	.0196	.0851
9	Cube	Byers	4.18	+20.20	+19.09	+1.11	$+x_3$	+19.67	+.58	+2.42	.3364	1.4062
10	Hastings	Bailer	3.05	+1.00	+.77	+.23	$-x_2+x_4$	+.79	+.02	+.06	.0004	.0012
11	Byers	Bailer	3.82	−5.00	−5.13	+.13	$-x_3+x_4$	−4.87	+.26	+.99	.0676	.2582
12	Hastings	Lee	1.84	+7.00	+6.34	+.66	$-x_2+x_5$	+6.94	+.60	+1.10	.3600	.6624
13	Bailer	Lee	23.33	+6.00	+6.19	−.19	$-x_4+x_5$	+6.15	−.04	−.93	.0016	.0373
14	Byers	Lee	1.96	+1.00	+2.50	−1.50	$-x_3+x_5$	+1.28	−1.22	−2.39	1.4884	2.9173
15	Lee	Thornberry	4.94	+1.00	+1.24	−.24	$-x_5+x_6$	+.15	−1.09	−5.38	1.1881	5.8692
16	Bailer [1]	Thornberry	2.04	+7.00	+5.32	+1.68	$-x_4+x_6$	+6.30	+.98	+2.00	.9604	1.9592
17	Byers	Thornberry	2.92	+2.00	+.67	+1.33	$-x_3+x_6$	+1.43	+.76	+2.22	.5776	1.6866
18	Lee	Willis	4.72	+19.00	+17.92	+1.08	$-x_5+x_7$	+18.30	+.38	+1.79	.1444	.6816
19	Thornberry	Willis	12.23	+18.00	+18.40	−.40	$-x_6+x_7$	+18.15	−.25	−3.06	.0625	.7644
20	Lee	Gammill	6.19	+13.35	+13.37	−.02	$-x_5$	+13.60	+.23	+1.42	.0529	.3275
21	Thornberry [1]	Gammill	1.12	+12.35	+14.39	−2.04	$-x_6$	+13.45	−.94	−1.05	.8836	.9896
22	Willis	Gammill	9.73	−5.65	−4.25	−1.40	$-x_7$	−4.70	−.45	−4.38	.2025	1.9703
23	Willis	Cashion	15.32	−14.35	−13.59	−.76	$-x_7$	−13.40	+.19	+2.91	.0361	.5531
24	Thornberry	Cashion	7.82	+3.65	+4.37	−.72	$-x_6$	+4.75	+.38	+2.97	.1444	1.1292
											Σpv^2	36.3529

[1] Computed from nonreciprocal observations. Weight used here is one-third of that determined by the computation on p. 167.

The data in the second, third, fourth, and sixth columns are obtained directly from the computations of the differences of elevations in Figures 45 and 46, except that where the difference of elevation, $h_2 - h_1$, is determined from nonreciprocal observations the weight, p, used in the adjustment is one-third of the weight determined in the computations. The quantities in the fifth column are obtained from the data given in the tables of fixed and assumed elevations on page 168.

The observation equations are formed in the following manner: First, take the equation for the difference of elevation of Monument and Keele.

(1) Monument, fixed elevation	$= 329.90$
(2) Keele, assumed elevation + correction	$= 311 \quad + x_1$
$h_2 - h_1$ (assumed)	$= -18.90 + x_1$
$h_2 - h_1$ (observed)	$= -18.79 + v_1$
assumed − observed	$= -0.11 + x_1 - v_1 = 0$
	$v_1 = -0.11 + x_1 \qquad (1)$

For the difference of elevation of Benton and Keele, we have

(1) Benton, fixed elevation	$= 301.40$
(2) Keele, assumed elevation + correction	$= 311 \quad + x_1$
$h_2 - h_1$ (assumed)	$= +9.60 + x_1$
$h_2 - h_1$ (observed)	$= +6.24 + v_2$
assumed − observed	$= +3.36 + x_1 - v_2 = 0$
	$v_2 = +3.36 + x_1 \qquad (2)$

The other 22 observation equations are formed in a similar manner. The constant terms of the equations (−0.11 in equation 1, +3.36 in equation 2, etc.) are placed in the column "Assumed minus observed" and the symbols for the unknown corrections ($+x_1$ in equation 1, $+x_1$ in equation 2, etc.) are placed in the column "Symbol."

After the first eight columns of the preceding table of observation equations are filled in, the table below in which the observation equations are written in horizontal lines with the x's in their respective columns is made out for convenience in forming the normals. In the second column of this table are given the weights, p, and in the third column the constant terms, N, of the observation equations. In the last column the products, pN, are given.

Data for formation of normal equations

	p	*N*	1	2	3	4	5	6	7	*pN*
1	2.45	−0.11	+1							−0.2695
2	1.45	+3.36	+1							+4.8720
3	2.51	+.12	+1							+.3012
4	2.25	−.61		+1						−1.3725
5	7.30	−.17	−1	+1						−1.2410
6	1.27	+.38		+1						+.4826
7	3.85	+.06		−1	+1					+.2310
8	4.34	−.20	−1		+1					−.8680
9	4.18	+1.11			+1					+4.6398
10	3.05	+.23		−1		+1				+.7015
11	3.82	+.13			−1	+1				+.4966
12	1.84	+.66		−1			+1			+1.2144
13	23.33	−.19				−1	+1			−4.4327
14	1.96	−1.50			−1		+1			−2.9400
15	4.94	−.24					−1	+1		−1.1856
16	2.04	+1.68				−1		+1		+3.4272
17	2.92	+1.33			−1			+1		+3.8836
18	4.72	+1.08					−1		+1	+5.0976
19	12.23	−.40						−1	+1	−4.8920
20	6.19	−.02					−1			−.1238
21	1.12	−2.04						−1		−2.2848
22	9.73	−1.40							−1	−13.6220
23	15.32	−.76							−1	−11.6432
24	7.82	−.72						−1		−5.6304

FORMATION OF NORMAL EQUATIONS

The normals are formed in the same manner as for condition equations (see p. 11), except that the *pN* values are multiplied by the coefficients in each numbered column in turn and the sums taken for the constant terms in the normals. For example, the constant term of the first normal equation is $(+1\times-0.2695)+(+1\times+4.8720)+(+1\times+0.3012)+(-1\times-1.2410)+(-1\times-0.8680)=-0.2695+4.8720+0.3012+1.2410+0.8680=+7.0127$.

The constant terms of the other normal equations are obtained in a similar manner. The complete set of normal equations is given below.

Normal equations

	1	2	3	4	5	6	7	η	Σ
1	+18.05	−7.30	−4.34					+7.0127	+13.4227
2		+19.56	−3.85	−3.05	−1.84			−4.2778	−.7578
3			+21.07	−3.82	−1.96	−2.92		+2.5626	+6.7426
4				+32.24	−23.33	−2.04		+2.2036	+2.2036
5					+42.98	−4.94	−4.72	−9.9465	−3.7565
6						+31.07	−12.23	+18.9324	+27.8724
7							+42.00	+25.4708	+50.5208

SOLUTION OF NORMAL EQUATIONS

These normal equations are solved in the same manner as the normal equations for the adjustment of horizontal directions. (See p. 38.) The complete forward and back solutions are given below.

Solution of normal equations

1	2	3	5	6	7	4	η	Σ
+18.05	−7.30	−4.34	----	----	----	----	+7.0127	+13.4227
x_1	+.40443	+.24044	----	----	----	----	−.38851	−.74364
	+19.56	−3.85	−1.84	----	----	−3.05	−4.2778	−.7578
1	−2.9523	−1.7552	----	----	----	----	+2.8361	+5.4286
	+16.6077	−5.6052	−1.84	----	----	−3.05	−1.4417	+4.6708
	x_2	+.33751	+.11079	----	----	+.13365	+.08681	−.28124
		+21.07	−1.96	−2.92	----	−3.82	+2.5626	+6.7426
	1	−1.0435	----		----	----	+1.6861	+3.2274
	2	−1.8918	−.6210	----	----	−1.0294	−.4866	+1.5764
		+18.1347	−2.5810	−2.92	----	−4.8494	+3.7621	+11.5464
		x_3	+.14232	+.16102	----	+.26741	−.20745	−.63670
			+42.98	−4.94	−4.72	−23.33	−9.9465	−3.7565
		2	−.2039	----	----	−.3379	−.1597	+.5175
		3	−.3673	−.4156	----	−.6902	+.5354	+1.6433
			+42.4088	−5.3556	−4.72	−24.3581	−9.5708	−1.5957
			x_5	+.12629	+.11130	+.57436	+.22568	+.03766
				+31.07	−12.23	−2.04	+18.9324	+27.8724
			3	−.4702	----	−.7808	+.6057	+1.8592
			5	−.6764	−.5961	−3.0760	−1.2086	−.2015
				+29.9234	−12.8261	−5.8968	+18.3295	+29.53010
				x_6	+.42863	+.19706	−.61255	−.98686
					+42.00	----	+25.4708	+50.5208
				5	−.5253	−2.7110	−1.0652	−.1776
				6	−5.4976	−2.5275	+7.8566	+12.6576
					+35.9771	−5.2385	+32.2622	+63.0003
					x_7	+.14561	−.89674	−1.75113
						+32.24	+2.2036	+2.2036
					2	−.5601	−.2648	+.8578
					3	−1.2968	+1.0060	+3.0876
					5	−13.9903	−5.4971	−.9165
					6	−1.1620	+3.6120	+5.8193
					7	−.7628	+4.6976	+9.1733
						+14.4680	+5.7573	+20.22513
						x_4	−.39793	−1.39793

Back solution

4	7	6	5	3	2	1
−0.39793	−0.89674	−0.61255	+0.22568	−0.20745	+0.08681	−0.38851
	−.05794	−.07842	−.22556	−.10641	−.07308	−.12655
−.39793		−.40920	−.10626	−.17715	−.02748	−.07740
−.40	−.95463		−.13894	−.03531	−.17764	
x_4	−.95	−1.10017				−.59246
	x_7	−1.10	−.24808	−.52632	−.19139	−.59
		x_6	−.25	−.53	−.19	x_1
			x_5	x_3	x_2	

The values of the x's obtained from the back solution are the quantities to be added to the assumed elevations to give the adjusted elevations which are placed in the third column of the table on page 168. These elevations which are in meters are then converted to feet for the last column to the right of the same table.

After the final elevations of the various stations have been determined, the remainder of the columns in the table on page 169 are filled out; that is, all the columns to the right of the one headed "Symbol." The column headed "Adjusted difference of elevation h_2-h_1" is filled out from the data in the tables on page 168. Each v in the next column is simply the adjusted (h_2-h_1) minus the observed (h_2-h_1) and is obtained by subtracting the quantity in column 6 from the corresponding quantity in column 9.

The column headed pv is obtained by multiplying column 10 by column 4, and the column headed pv^2 is obtained by multiplying column 12 by column 4.

As a check on the computation the sum of all the pv's for any station in columns 2 and 3 should equal zero, except for a possible discrepancy of a few hundredths due to dropping decimal places in adopting values for the x's. In computing this check the sign of the x must be taken into account. For example, for station Keele, correction symbol x_1, we have

$$\Sigma pv=(+1\times-1.72)+(+1\times+4.02)+(+1\times-1.18)+(-1\times+1.68)$$
$$+(-1\times-0.61)=-1.72+4.02-1.18-1.68+0.61=+0.05.$$

For station Hastings, correction symbol x_2, we have,

$$\Sigma pv=(+1\times-1.80)+(+1\times+1.68)+(+1\times+0.24)+(-1\times-1.08)$$
$$+(-1\times+0.06)+(-1\times+1.10)=-1.80+1.68+0.24+1.08-0.06$$
$$-1.10=+0.04.$$

All the remaining x's can be checked in a similar manner.

COMPUTATION OF PROBABLE ERROR

The probable error of an observation of unit weight derived from the adjustment is determined from the formula:

$$\text{Probable error}=\pm 0.6745\sqrt{\frac{\Sigma pv^2}{N_o-N_u}}$$

in which N_o is the number of observed zenith distances, N_u is the number of unknowns, and Σ indicates as usual "the sum of."

$$\Sigma\, pv^2=36.3529 \quad \text{(See p. .169)}$$
$$N_o-N_u=24-7=17 \quad \text{(See Fig. 43.)}$$

$$\log 36.3529=1.56054$$
$$\text{colog } 17=8.76955-10$$
$$\log\frac{\Sigma pv^2}{N_o-N_u}=0.33009$$
$$\log\sqrt{\frac{\Sigma pv^2}{N_o-N_u}}=0.16504$$
$$\log 0.6745=9.82898-10$$
$$\text{log probable error}=9.99402-10$$

Probable error (observation of unit weight) $=\pm 0.99$ m.

This means that the reciprocal observations over a line 31.6 kilometers (19⅝ miles) long, this being the length of line corresponding to unit weight, determined the difference of elevation of two points with such a degree of accuracy that it is an even chance whether the error is greater or less than 0.99 meter.

It is the general practice in the United States Coast and Geodetic Survey to compute the probable error of the elevation for that station of the net which is the farthest away from a fixed elevation and whose elevation, therefore, is least accurately determined. The probable error of this elevation can be very readily determined if the equation involving it is eliminated last in the solution of the normals. For example, in the net here considered the elevation of station Bailer is assumed to be least accurately determined. The correction to the elevation of Bailer is designated by x_4 in the adjustment, so the equation containing x_4 is eliminated last. (See p. 172.)

The formula used in computing the probable error of the elevation of any particular station is

$$(p.\ e.)_s = \pm\sqrt{\frac{(p.\ e.)_1^2}{C_w}}$$

in which $(p.\ e.)_s$ is the desired probable error of the elevation, $(p.\ e.)_1$ is the probable error for an observation of unit weight and C_w is the weight coefficient of the elevation for the station in question.

The weight coefficient for any x is the corresponding diagonal term before division in the solution of the normal equations. For x_4, the correction symbol for station Bailer, it is +14.468. (See p. 172.) The probable error of the elevation of Bailer is therefore

$$\pm\sqrt{\frac{(0.99)^2}{14.468}}$$

$$\log 0.99 = 9.99402 - 10 \quad \text{(See p. 173.)}$$

$$\log (0.99)^2 = 9.98804 - 10$$

$$\log 14.468 = \underline{1.16041}$$

$$\log \frac{(0.99)^2}{14.468} = 8.82763 - 10$$

$$\log\sqrt{\frac{(0.99)^2}{14.468}} = 9.41382 - 10$$

Probable error of elevation of Bailer = ±0.26 meter.

COMPUTATION OF ELEVATIONS OF INTERSECTION STATIONS

Figure 43 shows that Hastings High School was not occupied, but was sighted upon from four stations, Monument, Keele, Hastings, and Lee. After the final elevations of these main scheme stations have been determined, the elevation of Hastings High School may be computed from the non-reciprocal observations on it from the four stations.

(See fig. 47.) The values of $(0.5-m)$ used in this computation are obtained in the manner explained on page 166. For the final elevation of Hastings High School a weighted mean of the elevations determined from the four stations is taken.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 B

COMPUTATION OF ELEVATIONS FROM NONRECIPROCAL OBSERVATIONS.

Station 1, occ.	Momment	Keele	Hastings	Lee		
Station 2, obs.	Cupola H.S. Hastings	Cupola H.S. Hastings	Cupola H.S. Hastings	Cupola H.S. Hastings		
Object sighted	Top	Top	Top	Top		
ζ_1	90° 04′ 53″.8	90° 00′ 29″.6	89° 52′ 57″.4	90° 05′ 42″.2		
α and mean ϕ	86.8 34.2	19.5 34.2	20.8 34.2	78.9 34.2		
log (0.5−m)	9.67403	9.67367	9.66502	9.65706		
log s	4.25195	4.19443	3.78252	4.41427		
colog ρ	3.19485	3.19663	3.19660	3.19491		
colog sin 1″	5.31443	5.31443	5.31443	5.31443	5.31443	5.31443
log (k in secs.)	2.43526	2.37916	1.95857	2.58067		
k in secs.	272″.4	239″.4	90″.9	380″.8		
(90°−ζ_1+k) in secs.	−21″.4	+209″.8	+513″.5	+38″.6		
log ditto	1.33041n	2.32131	2.71054	1.58659		
T	4.68557	4.68558	4.68558	4.68557		
log s	4.25195	4.19443	3.78252	4.41427		
log [s tan (90°−ζ_1+k)]	0.26793n	1.20132	1.17864	0.68643		
log A	2	2	2	2		
log B	0	0	0	0		
log C	0	0	0	0		
log (h_2-h_1)	0.26795n	1.20134	1.17866	0.68645		
h_2-h_1	−1.85	+15.92	+15.09	+4.86		
h_1	329.90	310.41	312.81	319.75		
t−o	+2.12	+2.12	+1.44	+6.42		
Corrected elevation	330.17	328.45	329.34	331.03		
log p=9−2 log s	0.4961	0.6111	1.4350	0.1715		
p	3.13	4.08	27.23	1.48		
Weighted mean elevation of sta. etc.	329.38					

11—5048

FIG. 47.—Computation of elevation of intersection station

As the final step in the computation of elevations from zenith distances, a table is prepared giving the elevations of the stations, both in meters and feet. The elevations in feet should be given to one less place of decimals than the elevations in meters. The table should be placed at the end of the computation where it may be readily found.

Table of elevations

Station	Elevation	
	Meters	Feet
Monument	329.90	1,082.3
Benton	301.40	988.8
Cube	298.80	980.3
Gammill	333.35	1,093.7
Cashion	324.65	1,065.1
Keele	310.41	1,018.4
Hastings	312.81	1,026.3
Byers	318.47	1,044.8
Bailer	313.60	1,028.9
Lee	319.75	1,049.0
Thornberry	319.90	1,049.5
Willis	338.05	1,109.1
Hastings High School cupola	329.4	1,081

CHAPTER 6.—SPECIAL PROBLEMS CONNECTED WITH THE COMPUTATION OF TRIANGULATION

LIST OF DIRECTIONS WITH SOME DIRECTIONS PREVIOUSLY FIXED

It frequently happens that in making out a list of directions for a particular station, some of the directions from the station have already been fixed from previous adjustments. Care must be taken to correct these directions before they are used with the other directions in another adjustment, since the new adjustment will not be consistent with the old one if the observed directions are used. Below is an example of the method used in applying corrections to the observed values of these adjusted directions to make them consistent. At station "Monument" the angle from Grande to Corpus as fixed from a previous adjustment is 55° 32′ 21.″88 and the angle from Corpus to Garcia is 99° 25′ 16.″20. The observed values of the three directions involved are given in column 2 of Figure 48.

Computation of mean correction, station Monument

1 Station	2 Observed direction	3 Prelimi-nary corrected direction	4 Difference (3-2)	5 Final corrected direction
	° ′ ″	″	″	″
Grande	25 00 27.23	27.23	0.00	26.47
Corpus	80 32 48.42	49.11	+.69	48.35
Garcia	179 58 03.72	05.31	+1.59	04.55

Using Grande as an initial and adding successively the two fixed angles given above we obtain the values given in column 3. The differences between these values and the observed seconds are placed in column 4. The mean of the three differences is $\frac{+2.''28}{3} = +0.''76$. This mean is then applied with opposite sign to each of the values in column 3 to obtain the final values in column 5, which are the values to be placed in the final seconds column of the list of directions at Monument. (See fig. 48.) As a check on the computation the direction to Grande should be subtracted from the direction to Corpus and the direction to Corpus from the direction to Garcia, using the values in the final seconds column. The angles thus obtained should be identical with the fixed values given above.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 24A

State: ....

Station Monument Computed by A.C.D.

Observer C.V.H. 1917 Checked by W.F.R.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
Ringold	0 00 00.00	
Grande	25 00 27.23	26.47
Hebron	31 47 59.55	
Corpus	80 32 48.42	48.35
Pancho	118 19 55.00	
Garcia	179 58 03.72	04.55

FIG. 48.—List of directions, with some directions previously fixed

The rule to be followed then in making the directions already fixed consistent with the new directions is as follows: Take the observed direction at one of the fixed stations as an initial and add the fixed angles in order to obtain a set of preliminary corrected directions for the fixed stations. Take an algebraic mean of the differences obtained by subtracting the observed directions from these corrected directions and apply it with opposite sign to the latter to obtain the values for the final seconds column.

NUMBER OF EQUATIONS IN ADJUSTMENT OF UNUSUAL FIGURES

It frequently happens that the mathematician in forming equations for an adjustment makes a mistake in using either too many or too few. Figure 49 is an example where the number of equations necessary to adjust it is not at once apparent.

There are only two triangles in this figure and both have one concluded angle. It would appear at first as if there were only one side equation and no angle equations. There is an angle equation, however, due to the fact that the sum of the interior angles of a quadrilateral must equal 360° plus the spherical excess of the quadrilateral. This equation is formed as follows:

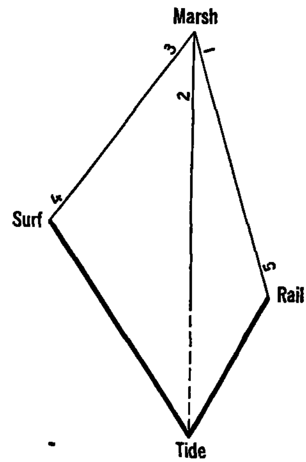


FIG. 49.—First example of unusual triangulation figure

	°	′	″
Angle at Tide, Surf to Rail (fixed)	61	09	52.0
Angle at Rail, Tide to Marsh (+5)	135	09	30.6
Angle at Marsh, Rail to Surf (−1+3)	53	44	33.5
Angle at Surf, Marsh to Tide (−4)	109	56	13.2
Sum −(1)+(3)−(4)+(5)	360	00	09.3

As there is no spherical excess in this quadrilateral, we have

$$360°\ 00'\ 00''.0 = 360°\ 00'\ 09''.3 - (1) + (3) - (4) + (5)$$

or

$$0 = +9.3 - (1) + (3) - (4) + (5)$$

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: ....

NO.	STATION	OBSERVED ANGLE	CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
2-3	Rail - Tide						3.437982
-1+2 1	Marsh	16 25 20.7					
+5 2	Rail	135 09 30.6					
+1 -2-5 3	Tide	(28 25 08.7)					
1-3	Marsh - Tide				0.0		
1-2	Marsh - Rail						
2-3	Tide - Surf						3.644409
-2+3 1	Marsh	37 19 12.3					
+2 - 3+4 2	Tide	(32 44 34.0)					
-4 3	Surf	109 56 13.2					
1-3	Marsh - Surf				0.0		
1-2	Marsh - Tide						

FIG. 50.—Triangle computation for Figure 49

Or the equation may be formed in another way. The sum of the two concluded angles should equal the fixed angle at Tide. That is

	°	′	″
Angle at Tide, Marsh to Rail (+1−2−5)	28	25	08.7
Angle at Tide, Surf to Marsh (+2−3+4)	32	44	34.0
Angle at Tide, Surf to Rail (+1−3+4−5)	61	09	42.7

Since the fixed angle at Tide, Surf to Rail, is 61° 09′ 52″.0, we have

$$61° \ 09' \ 42''.7 + (1) - (3) + (4) - (5) = 61° \ 09' \ 52''.0$$

or

$$0 = +9.3 - (1) + (3) - (4) + (5)$$

In Figure 51 is given another example, occasionally encountered, in which the proper number of equations for the adjustment is not very apparent. Ordinarily one would think that there are two angle and one side equations in the quadrilateral *C D F E* and two angle and one side equations in the quadrilateral *A B D C*, or a total of six equations. However, if the figure is built up point by point, it is seen that there are seven equations, as shown below:

Station	Number of equations: Angle	Number of equations: Side
C	1	0
D	1	1
B	1	0
A	2	1
Total	5	2

The additional angle equation is obtained by the closure of either of the two quadrilaterals *A D F C* or *A D E C*. If the latter one is used then the equation is, angle *C A D* + angle *A D E* + angle *D E C* + angle *E C A* − the closure = 360° + spherical excess.

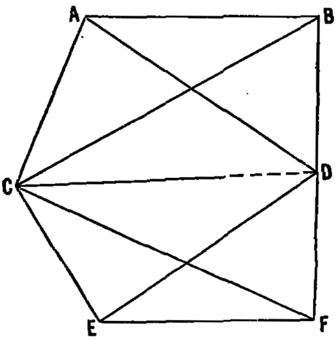


FIG. 51.—Second example of unusual triangulation figure

SIDE EQUATION IN FORM OF LENGTH EQUATION

It frequently happens in the adjustment of an intersection station from three fixed points, that the length and azimuth between two of the fixed points are not known. In this case the side equation will take the form of a length equation. An example of such an adjustment is given below. (See fig. 52.)

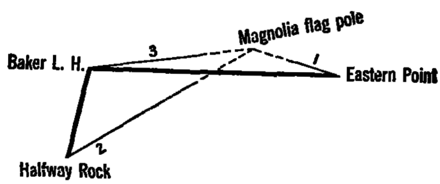


FIG. 52.—Triangulation figure requiring length equation instead of ordinary side equation

Side equation

Baker L.H.—Halfway Rock		3.588491	----------	Baker L.H.—Eastern Point		4.045264 (−2)	----------
	° ′ ″				° ′ ″		
+2	47 46 55.1	9.8695797	+1.91	−2+3	22 43 22.5	9.5868966	+5.03
−1+3	159 14 06.0	9.5496601	−5.55	+1	13 44 18.9	9.3756496	+8.61
		3.0077308				3.0078100	

$$0 = -79.2 - 3.06(1) + 6.94(2) - 10.58(3)$$

Correlate equation

		v	Adopted v
1	−3.06	−1.430	−1.4
2	+6.94	+3.244	+3.2
3	−10.58	−4.945	−4.9

Solution of normal equation

$$0 = -79.2 + 169.4636\,C$$
$$C = +0.4674$$

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: ....

NO.	STATION	OBSERVED ANGLE	CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
	2-3 Eastern Point-Baker L.H.						4.045264
	1 Magnolia flag pole	159 14 06.0	-3.5			02.5	0.450320
+1	2 Eastern Point	13 44 18.9	-1.4			17.5	9.375637
-3	3 Baker L.H.	7 01 35.1	+4.9			40.0	9.087606
	1-3 Magnolia flag pole-Baker L.H.						3.871221
	1-2 Magnolia flag pole-Eastern Point						3.583190
	2-3 Halfway Rock-Baker L.H.						3.588491
	1 Magnolia flag pole	22 43 22.5	-8.1	14.4		14.4	0.413144
+2	2 Halfway Rock	47 46 55.1	+3.2	58.3		58.3	9.369536
-3	3 Baker L.H.	109 29 42.5	+4.9	47.4	0.1	47.3	9.974356
	1-3 Magnolia flag pole-Baker L.H.				0.1		3.871221
	1-2 Magnolia flag pole-Halfway Rock						3.975991

FIG. 53.—Triangle computation for Figure 52

IDENTICAL EQUATIONS

One must be very careful in the selection of equations for an adjustment, especially if the scheme is much involved, to avoid what is known as an "identical equation," which often is not discovered until the solution of equations is made. In the solution, the diagonal term of the identical equation will become zero, or nearly so, and the error in selecting the equation thus becomes known.

Below is given an illustration of a set of five equations, three angle and two side, in which one of the side equations is an "identical equation." The solution of the equations is carried as far as the diagonal term of the identical equation. It is seen that the diagonal term of the fifth equation is practically zero, and this means that the fifth equation is not an independent equation.

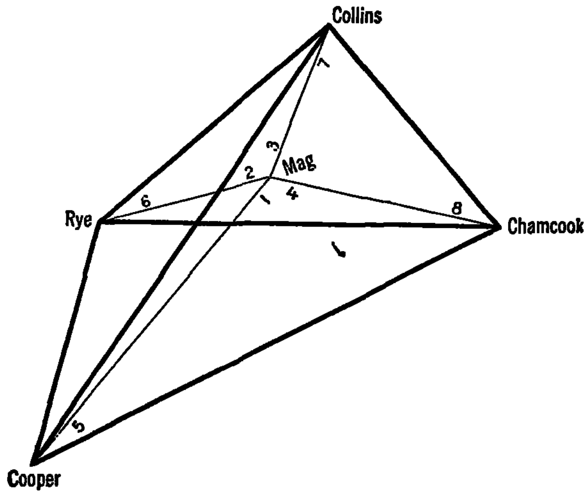


FIG. 54.—Triangulation figure for which "identical equation" was selected

Angle equations

1. $0=+3.7-(1)+(2)+(5)-(6)$
2. $0=-2.4-(2)+(3)+(6)-(7)$
3. $0=+1.1-(3)+(4)+(7)-(8)$

Side equations

	° ′ ″				° ′ ″		
−7	27 53 43.7	9.6701159	+3.98	+6	25 57 29.1	9.6411899	+4.32
−6	16 52 35.5	9.4628623	+6.94	+8	12 39 55.0	9.3409495	+9.37
−8	37 22 42.4	9.7832437	+2.76	+7	59 13 35.2	9.9340924	+1.25
		8.9162219				8.9162318	

4. $0=-9.9-11.26(6)-5.23(7)-12.13(8)$

		−13				−8	
Rye-Chamcook		4.4295047		Rye-Collins		4.3145841	
+8	12 39 55.0	9.3409495	+9.37	+2−4	150 27 31.0	9.6928929	−3.72
−2+3	126 08 45.1	9.9071522	−1.54	−7	27 53 43.7	9.6701159	+3.98
		3.6776051				3.6775921	

5. $0=+13.0+5.26(2)-1.54(3)-3.72(4)+3.98(7)+9.37(8)$

Correlate equations

	1	2	3	4	5	Σ_a
1	−1					−1.00
2	+1	−1			+5.26	+5.26
3		+1	−1		−1.54	−1.54
4			+1		−3.72	−2.72
5	+1					+1.00
6	−1	+1		−11.26		−11.26
7		−1	+1	−5.23	+3.98	−1.25
8			−1	−12.13	+9.37	−3.76

Normal equations

	1	2	3	4	5	η	Σ_n
1	+4	−2		+11.26	+5.26	+3.7	+22.22
2		+4	−2	−6.03	−10.78	−2.4	−19.21
3			+4	+6.90	−7.57	+1.1	+2.43
4				+301.2774	−134.4735	−9.9	+169.0339
5					+147.5149	+13.0	+12.9514

Solution of normal equations

1	2	3	4	5	η	Σ_n
+4	−2		+11.26	+5.26	+3.7	+22.22
C_1	+.5		−2.815	−1.315	−.925	−5.555
	+4	−2	−6.03	−10.78	−2.4	−19.21
1	−1		+5.63	+2.63	+1.85	+11.11
	+3	−2	−.40	−8.15	−.55	−8.10
	C_2	+.66667	+.13333	+2.71667	+.18333	+2.70
		+4	+6.90	−7.57	+1.1	+2.43
	2	−1.3333	−.2667	−5.4333	−.3667	−5.40
		+2.6667	+6.6333	−13.0033	+.7333	−2.97
		C_3	−2.4875	+4.87625	−.2750	+1.11375
			+301.2774	−134.4735	−9.9	+169.0339
		1	−31.6969	−14.8069	−10.4155	−62.5493
		2	−.0533	−1.0867	−.0733	−1.0800
		3	−16.5003	+32.3456	−1.8242	+7.3878
			+253.0269	−118.0215	−22.2130	+112.7924
			C_4	+.46644	+.08779	−.44577
				+147.5149	+13.0	+12.9514
			1	−6.9169	−4.8655	−29.2193
			2	−22.1409	−1.4941	−22.0050
			3	−63.4073	+3.5759	−14.4825
			4	−55.0500	−10.3611	+52.6104
				−.0002	−.1448	−.1450

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Maine

NO.	STATION	OBSERVED ANGLE	CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
	2-3 Cooper-Rye						4.1907270
-1+2	1 Mag	34 29 20.9			0.1		
+5	2 Cooper	25 52 15.7			0.1		
-6	3 Rye	119 38 27.5			0.2		
	1-3 Mag-Rye		+3.7		0.4		
	1-2 Mag-Cooper						
	2-3 Cooper-Collins						4.5386319
-1+3	1 Mag	160 38 06.0			0.1		
+5	2 Cooper	6 09 35.8					
-7	3 Collins	13 12 19.7			0.1		
	1-3 Mag-Collins		-1.3		0.2		
	1-2 Mag-Cooper						
	2-3 Rye-Collins						4.3145841
-2+3	1 Mag	126 08 45.1			0.1		
+6	2 Rye	25 57 29.1			0.1		
-7	3 Collins	27 53 43.7			0.1		
	1-3 Mag-Collins		+2.4		0.3		
	1-2 Mag-Rye						
	2-3 Collins-Chamcook						4.2624872
-3+4	1 Mag	83 23 43.9			0.2		
+7	2 Collins	59 13 35.2			0.1		
-8	3 Chamcook	37 22 42.4			0.1		
	1-3 Mag-Chamcook		-1.1		0.4		
	1-2 Mag-Collins						
	2-3 Chamcook-Cooper						4.5399291
+1-4	1 Mag	115 58 10.1			0.3		
+8	2 Chamcook	39 11 30.8			0.3		
-5	3 Cooper	24 50 17.6			0.3		
	1-3 Mag-Cooper		+2.4		0.9		
	1-2 Mag-Chamcook						
	2-3 Chamcook-Rye						4.4295047
+2-4	1 Mag	150 27 31.0			0.1		
+8	2 Chamcook	12 39 55.0					
-6	3 Rye	16 52 35.5			0.1		
	1-3 Mag-Rye		-1.3		0.2		
	1-2 Mag-Chamcook						

FIG. 55.—Triangle computation for Figure 54

A careful inspection of the side equations on page 182 will show that both equations relate to the quadrilateral Rye-Mag-Chamcook-Collins, and they differ only in that Mag is the pole in the fourth equation and Rye in the fifth. The fifth equation should have been formed for the quadrilateral Mag-Chamcook-Cooper-Rye, with the pole at Mag. Although it is easy to see in a simple figure such as the one above when an "identical equation" is selected, it is much more difficult to see in the case of more complicated figures and all equations should therefore be selected very carefully.

ADJUSTMENT OF QUADRILATERAL WITH POLE AT INTERSECTION OF DIAGONALS

In the adjustment of quadrilaterals it frequently happens that it is impossible to include the two smallest angles in the formation of the side equations by taking the pole at any one of the four vertices. In such a case the pole should be taken at the intersection of the diagonals. An example of such an adjustment is given below. (See fig. 56.)

The angle equations are formed in exactly the same manner as in the ordinary quadrilateral. (See p. 35.) The side equation is formed as follows: Call the intersection of the diagonals O. (See fig. 56.)

Then

$$\frac{\text{O}-\text{Tall}}{\text{O}-\text{Muketeo L. H.}}\times\frac{\text{O}-\text{Ridge}}{\text{O}-\text{Tall}}\times\frac{\text{O}-\text{Stump}}{\text{O}-\text{Ridge}}\times\frac{\text{O}-\text{Muketeo L. H.}}{\text{O}-\text{Stump}}=1.$$

Substituting the sines of the angles opposite the sides for the sides we have

$$\frac{\sin(-3+4)}{\sin(-5+6)}\times\frac{\sin(-6+7)}{\sin(-8+9)}\times\frac{\sin(-9+10)}{\sin(-1+2)}\times\frac{\sin(-2)}{\sin(+3)}=1.$$

The equation is then tabulated as explained on page 36 except in one particular. In the example given there the designation of the angle in the denominator appears on the same horizontal line in the tabulated equation as the designation of the angle in the numerator, and the angles corresponding to these designations are taken from the same triangle. In the example given here, however, this is not true, that is, the angle designated by $(-5+6)$ is not in the same triangle as the angle designated by $(-3+4)$, etc. It is best, however, to arrange the tabulated side equation so that the two angles from the same triangle are on the same horizontal line. This can be done as follows: With the pole at the intersection of the diagonals there will be 4 lines for the tabulated equation. Put the designation of the angles in the numerators of the first, second, third, and fourth fractions on lines 1, 2, 3, and 4, respectively, of the left side of the equation, and the designations of the angles in the denominators of the same fractions in lines 4, 1, 2, and 3, respectively, of the right side of the equation. That is, in the example given here (−3+4), (−6+7), (−9+10), and (−2) are placed in lines 1, 2, 3, and 4, respectively, of the left side of the equation, and (−5+6), (−8+9), (−1+2), and (+3) are placed in lines 4, 1, 2, and 3, respectively, of the right side of the equation. The two angles in the same horizontal line in the side equation can then be taken out of the same triangle.

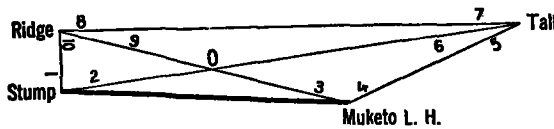


FIG. 56.—Quadrilateral with pole at intersection of diagonals

The remainder of the adjustment of the quadrilateral is similar to the one previously adjusted on pages 29–41.

Angle equations

1. 0=+4.9−(1)+(3)−(9)+(10)
2. 0=−2.0−(2)+(4)−(5)+(6)
3. 0=+3.4−(3)+(4)−(5)+(7)−(8)+(9)

Side equation

	° ′ ″				° ′ ″		
−3+4	139 26 09.8	9.8131113	−2.5	−8+9	16 03 51.3	9.4420329	+7.3
−6+7	7 26 41.4	9.1125095	+16.1	−1+2	80 24 45.5	9.9938914	+.4
−9+10	76 04 52.1	9.9870570	+.5	+3	11 53 01.8	9.3137156	+10.0
−2	11 37 25.5	9.3042404	+10.2	−5+6	17 03 20.9	9.4673165	+6.9
		8.2169182				8.2169564	

4. 0=−38.2+0.4(1)−10.6(2)−7.5(3)−2.5(4)+6.9(5)−23.0(6)+16.1(7) +7.3(8)−7.8(9)+0.5(10) (This equation should be divided by 5 before entering it in the table of correlates.)

Correlate equations

	1	2	3	4	Σ_o	v	Adopted v
1	−1	-------	-------	+0.08	−0.92	+2.202	+2.2
2	-------	−1	-------	−2.12	−3.12	−2.541	−2.6
3	+1	-------	−1	−1.50	−1.50	−.801	−.8
4	-------	+1	+1	−.50	+1.50	−.242	−.3
5	-------	−1	−1	+1.38	−.62	+.543	+.5
6	-------	+1	-------	−4.00	−3.60	+.243	+.2
7	-------	-------	+1	+3.22	+4.22	−.786	−.8
8	-------	-------	−1	+1.46	+.46	+2.386	+2.4
9	−1	-------	+1	−1.56	−1.56	−.245	−.2
10	+1	-------	-------	+.10	+1.10	−2.141	−2.1

Normal equations

	1	2	3	4	η	Σ_n	C
1	+4	--------	−2	+0.08	+4.9	+6.98	−2.1752
2		+4	+2	−4.36	−2.0	−.36	+1.8162
3			+6	−.18	+3.4	+9.22	−1.8869
4				+45.0088	−7.64	+32.9088	+.3420

Solution of normal equations

1	2	3	4	η	Σ_n
+4	--------	−2	+0.08	+4.9	+6.98
C_1		+.5	−.02	−1.225	−1.745
	+4	+2	−4.36	−2.0	−.36
	C_2	−.5	+1.09	+.5	+.09
		+6	−.18	+3.4	+9.22
	1	−1	+.04	+2.45	+3.49
	2	−1	+2.18	+1.00	+.18
		+4	+2.04	+6.85	+12.89
		C_3	−.51	−1.7125	−3.2225
			+45.0088	−7.64	+32.9088
		1	−.0016	−.098	−.1396
		2	−4.7524	−2.180	−.3924
		3	−1.0404	−3.4935	−6.5739
			+39.2144	−13.4115	+25.8029
			C_4	+0.34200	−.65800

Back solution

4	3	2	1
+0.3420	−1.7125	+0.5	−1.225
	−.1744	+.3728	−.0068
		+.9434	−.9434
	−1.8869	+1.8162	−2.1752

Computation of v's

1	2	3	4	5	6	7	8	9	10
+2.175	−1.816	−2.175	+1.816	−1.816	+1.816	−1.887	+1.887	+2.175	−2.175
+.027	−.725	+1.887	−1.887	+1.887	−1.573	+1.101	+.499	−1.887	+.034
		−.513	−.171	+.472				−.533	
+2.202	−2.541	−.801	−.242	+.543	+.243	−.786	+2.386	−.245	−2.141
+2.2	−2.6	−.8	−.3	+.5	+.2	−.8	+2.4	−.2	−2.1

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Massachusetts

		NO.	STATION	OBSERVED ANGLE			CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
		2-3	Muketeo L. H. - Stump								3.526870
	-9+10	1	Ridge	76	04	52.1	-1.9			50.2	0.012944
1	+3	2	Muketeo L.H.	11	53	01.8	-0.8			01.0	9.313708
	-1	3	Stump	92	02	11.0	-2.2			08.8	9.999726
		1-3	Ridge - Stump				-4.9				2.853522
		1-2	Ridge - Muketeo L.H.								3.539540
						04.9					
		2-3	Muketeo L.H. - Stump								3.526870
	-5+6	1	Tall	17	03	20.9	-0.3			20.6	0.532685
2	+4	2	Muketeo L.H.	151	19	11.6	-0.3			11.3	9.681169
	-2	3	Stump	11	37	25.5	+2.6			28.1	9.304267
		1-3	Tall - Stump				+2.0				3.740724
		1-2	Tall - Muketeo L.H.								3.363822
						58.0					
		2-3	Muketeo L.H. - Ridge								3.539540
	-5+7	1	Tall	24	30	02.3	-1.3			01.0	0.382268
3	-3+4	2	Muketeo L.H.	139	26	09.8	+0.5			10.3	9.813110
	-8+9	3	Ridge	16	03	51.3	-2.6			48.7	9.442014
		1-3	Tall - Ridge				-3.4				3.734918
		1-2	Tall - Muketeo L.H.								3.363822
						03.4					
		2-3	Stump - Ridge								2.853522
	-6+7	1	Tall	7	26	41.4	-1.0			40.4	0.887506
4	-1+2	2	Stump	80	24	45.5	-4.8			40.7	9.993890
	-8+10	3	Ridge	92	08	43.4	-4.5			38.9	9.999696
		1-3	Tall - Ridge				-10.3				3.734918
		1-2	Tall - Stump								3.740724
						10.3					

FIG. 57.—Triangle computation for Figure 56

ADJUSTMENT OF QUADRILATERAL HAVING ONE TRIANGLE WITH TWO CONCLUDED ANGLES

It sometimes becomes necessary to make an adjustment of a quadrilateral in which one triangle has two concluded angles. In such an adjustment the thing to be particularly careful about is the proper designation of the angles. In the following example (see fig. 58) the triangle, Ramparts flag-Bond Hill-Ten Pound Island L. H. has two concluded angles, one at Ramparts flag and the other at Ten Pound Island L. H. In triangle No. 4, Figure 59, the angle at Ten Pound Island L. H. is the sum of the two angles at Ten Pound Island L. H. between Eastern Pt. L. H. and Bond Hill, and between Ramparts flag and Eastern Pt. L. H. In triangles Nos. 1 and 3,

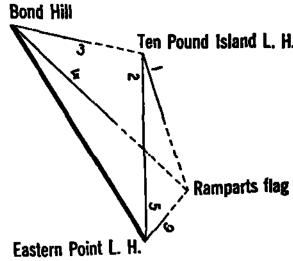


FIG. 58.—Quadrilateral having one triangle with two concluded angles

	°	′	″
+3−5 (Ten Pound Id. L. H. between Eastern Pt. L. H. and Bond Hill) =	104	53	11. 1
−1+2 (Ten Pound Id. L. H. between Ramparts flag and Eastern Pt. L. H.) =	16	09	19. 0
−1+2+3−5 (Ten Pound Id. L. H. between Ramparts flag and Bond Hill) =	121	02	30. 1

The angle at Ramparts flag in triangle No. 4 is obtained by subtracting from 180° 00′ 00″.0 plus the spherical excess (which in this case is 0″.0) the sum of the angles at Bond Hill and Ten Pound Island L. H., using the proper designations as shown below.

		°	′	″
−3+4	(Bond Hill) =	30	52	02. 6
−1+2+3−5	(Ten Pound Id. L. H.) =	121	02	30. 1
−1+2+4−5	(sum of 2 angles)	151	54	32. 7
		180	00	00. 0
+1−2−4+5	(Ramparts flag)	28	05	27. 3

After these angles have been computed and properly designated, the adjustment of the quadrilateral is exactly the same as that of any other quadrilateral, and so no further explanation is necessary here.

Side equation

	° ′ ″				° ′ ″		
−1+2+3−5	121 02 30. 1	9. 9328756	−1. 3	−3+4	30 52 02. 6	9. 7101620	+3. 5
−4	13 58 30. 0	9. 3829144	+8. 5	+6	70 59 35. 3	9. 9756521	+0. 7
−5+6	40 43 19. 0	9. 8145064	+2. 4	−1+2	16 09 19. 0	9. 4444218	+7. 3
		9. 1302964				9. 1302359	

$0 = +60.5 + 8.6(1) - 8.6(2) + 2.2(3) - 12.0(4) - 1.1(5) + 1.7(6)$

Correlate equation

	′	v	Adopted v
1	+8.6	−1.73	−1.7
2	−8.6	+1.73	+1.7
3	+2.2	−.44	−.4
4	−12.0	+2.41	+2.4
5	−1.1	+.22	+.2
6	+1.7	−.34	−.3

Solution of normal equation

$$0=+60.5+300.86C$$
$$C=-0.2011$$

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Massachusetts

NO.	STATION	OBSERVED ANGLE	CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
	2-3 Eastern Pt. L.H.- Bond Hill						3.517770
+3-5	1 Ten Pound Id.L.H.	104 53 11.1	-0.6			10.5	0.014826
1. +5	2 Eastern Pt.L.H.	30 16 16.3	+0.2			16.5	9.702512
-3	3 Bond Hill	44 50 32.6	+0.4			33.0	9.848288
	1-3 Ten Pound Id.L.H.-Bond Hill						3.235108
	1-2 Ten Pound Id.L.H.-Eastern Pt.L.H.						3.380884
	2-3 Eastern Pt.L.H. - Bond Hill						3.517770
+4-6	1 Ramparts flag	95 01 54.7	+2.7			57.4	0.001677
2. +6	2 Eastern Pt.L.H.	70 59 35.3	-0.3			35.0	9.975652
-4	3 Bond Hill	13 58 30.0	-2.4			27.6	9.382894
	1-3 Ramparts flag - Bond Hill						3.495099
	1-2 Ramparts flag - Eastern Pt.L.H.						2.902341
	2-3 Eastern Pt.L.H.-Ten Pound Id.L.H.						3.380884
+1-2 +5-6	1 Ramparts flag	(123 07 22.0)	-2.9			19.1	0.077010
3. -5+6	2 Eastern Pt.L.H.	40 43 19.0	-0.5			18.5	9.814505
-1+2	3 Ten Pound Id.L.H.	16 09 19.0	+3.4			22.4	9.444447
	1-3 Ramparts flag-Ten Pound Id.L.H.						3.272399
	1-2 Ramparts flag-Eastern Pt.L.H.						2.902341
	2-3 Bond Hill - Ten Pound Id.L.H.						3.235108
+1-2 -4+5	1 Ramparts flag	(28 05 27.3)	-5.6			21.7	0.327119
4. -3+4	2 Bond Hill	30 52 02.6	+2.8			05.4	9.710172
-1+2 +3-5	3 Ten Pound Id.L.H.	(121 02 30.1)	+2.8			32.9	9.932872
	1-3 Ramparts flag-Ten Pound Id.L.H.						3.272399
	1-2 Ramparts flag-Bond Hill						3.495099

FIG. 59.—Triangle computation for Figure 58

THREE-POINT PROBLEM

A triangulation station is sometimes determined by means of directions observed at that station to the three fixed points of a triangle, the sides and angles of which are either known or can be computed. This is called the "three-point problem."

The computations are made on Form 655 as shown in Figure 61. Three cases are illustrated on this form, depending upon the location of the point, designated P, with reference to the sides of the triangle. If P is on the circumference of the circle which passes through the vertices of the fixed triangle, the problem is indeterminate, since any point on this circumference would have the same values of the angles P' and P'' (or the supplement of one of the angles).

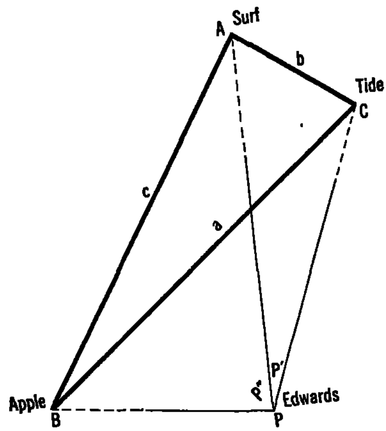


FIG. 60.—Quadrilateral, three-point problem

The formulas used in the computation are as follows: From the known sides a, b, c, and the known angle A of the triangle $A\ B\ C$, and the observed angles $A\ P\ C = P'$ and $A\ P\ B = P''$, the problem is to find the angles $A\ B\ P$ and $A\ C\ P$.

For cases 1 and 2, let $S = 180° - \frac{1}{2}(A + P' + P'')$,
and for case 3, let $S = \frac{1}{2}(A - P' - P'')$;

$$\text{Let } \tan Z = \frac{c \sin P'}{b \sin P''}$$

$$\text{and } \tan \epsilon = \cot (Z + 45°) \tan S$$

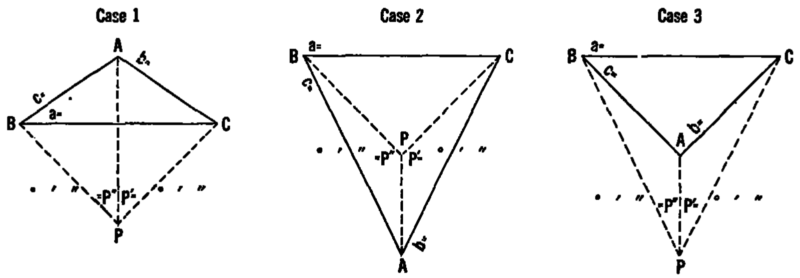
Then angle $A\ B\ P = S + \epsilon$, and angle $A\ C\ P = S - \epsilon$ if $\tan \epsilon$ is positive, and angle $A\ B\ P = S - \epsilon$, and angle $A\ C\ P = S + \epsilon$ if $\tan \epsilon$ is negative.

After the angles $A\ B\ P$ and $A\ C\ P$ are computed all the angles of the triangles can be obtained, and since the length of one side in each triangle is known all the remaining lengths can be computed. As each computed length is obtained in two different triangles, the agreement of the two values for the logarithm of each length within one, or possibly two, in the last place of decimals gives a check on the computation.

The example given below is for case 1 on Form 655. In the triangle Tide-Apple-Surf all the angles and sides are known and at Edwards the angles Apple to Surf and Surf to Tide are observed. The starting data are: $P' = 21°\ 38'\ 06''.8$, $P'' = 84°\ 12'\ 57''.9$, $A = 86°\ 29'\ 42''.3$, $\log c = 4.109221$, and $\log b = 3.644409$. The problem is to find the angle at Apple between Surf and Edwards and the angle at Tide between Edwards and Surf. After these angles have been determined, the logarithms of the lengths are computed on Form 25.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 655

COMPUTATION OF THREE-POINT PROBLEM



Cases 1 and 2	°	′	″
P′	21	38	06.8
P″	84	12	57.9
A	86	29	42.3
Sum	192	20	47.0
½ Sum	96	10	23.5
S = 180° − ½ sum =	83	49	36.5

Case 3	
P′	
P″	
Sum	
A	
A − sum	
S = ½ (A − sum) =	

Log c =	4.109221
Log sin P′ =	9.566668 −10
Colog b =	6.355591 −10
Colog sin P″ =	0.002217
Sum = log tan Z =	0.033697

	°	′	″
Z =	47	13	14.0
Z + 45° =	92	13	14.0

Log cot (Z+45°) =	8.588556$_n$ −10
Log tan S =	0.965928
Sum = log tan ε =	9.554484$_n$ −10 (sign −)

	°	′	″
ε	19	43	21.1
S	83	49	36.5

(Tan ε +)	°	′	″	(Tan ε −)
S + ε = angle ABP	64	06	15.4	S − ε = angle ABP
S − ε = angle ACP	103	32	57.6	S + ε = angle ACP

BPA	84	12	57.9	APC	21	38	06.8	PCB	29	18	51.3
ABP	64	06	15.4	PCA	103	32	57.6	CBP	44	50	04.0
PAB	31	40	46.7	CAP	54	48	55.6	BPC	105	51	04.7

(For explanation of this form see Special Publication No. 26, 2d edition, paragraph 108, page 57)

GOVERNMENT PRINTING OFFICE 11—6610

FIG. 61.—Computation of three-point problem

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Maine

NO.	STATION	OBSERVED ANGLE			CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
2-3	Apple-Surf								4.109221
1	Tide	74	14	10.9	-4.6			06.3	0.016651
2	Apple	19	16	12.3	-0.9			11.4	9.518537
3	Surf	86	29	45.9	-3.5	42.4	0.1	42.3	9.999187
1-3	Tide-Surf				-9.0		0.1		3.644409
1-2	Tide-Apple								4.125059
2-3	Apple-Surf								4.109221
1	Edwards	84	12	57.9				57.9	0.002217
2	Apple	64	06			15.5	0.1	15.4	9.954045
3	Surf	31	40			46.8	0.1	46.7	9.720299
1-3	Edwards-Surf						0.2		4.065483
1-2	Edwards-Apple								3.831737
2-3	Apple-Tide								4.125059
1	Edwards	105	51	04.7				04.7	0.016837
2	Apple	44	50			04.1	0.1	04.0	9.848226
3	Tide	29	18			51.4	0.1	51.3	9.689841
1-3	Edwards-Tide						0.2		3.990122
1-2	Edwards-Apple								3.831737
2-3	Surf-Tide								3.644409
1	Edwards	21	38	06.8				06.8	0.433331
2	Surf	54	48					55.6	9.912382
3	Tide	103	32			57.7	0.1	57.6	9.987742
1-3	Edwards-Tide						0.1		3.990122
1-2	Edwards-Surf								4.065482 +1

FIG. 62.—Triangle computation for three-point problem

ADJUSTMENT OF A FIGURE WITH AN x-CORRECTION ON ONE DIRECTION

In the adjustment of triangulation, cases sometimes arise where it is possible to obtain an approximate value for an unknown direction which is needed in the adjustment. By designating the correction to this direction as "x," it is possible to make an adjustment in which the x is eliminated and new values of the observed directions are obtained which will make the lengths consistent, as computed in the different triangles. An example of such an adjustment is given below.

In Figure 63 the triangles Tide-Apple-Surf and Rail-Tide-Surf (see triangles 1 and 2, fig. 64) are fixed and the angles and lengths are known. At station Edwards directions have been observed on stations Apple, Surf, Rail, and Tide, but no directions on Edwards have been observed at any of the four fixed stations. It is desired to make an adjustment of the figure so that all the lengths will be consistent.

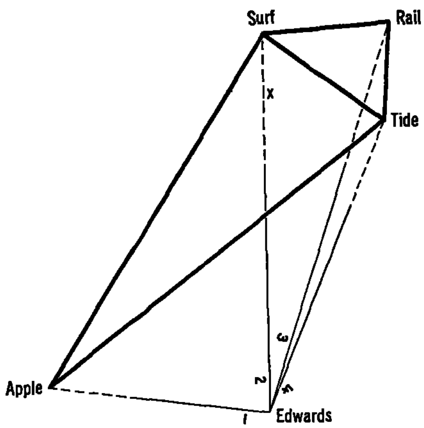


FIG. 63.—Triangulation figure for which an "x" direction is used

The triangles Edwards-Apple-Surf, Edwards-Apple-Tide, and Edwards-Surf-Tide (triangles 3, 4, and 6) are first solved by means of the three-point problem (see p. 191). Next the directions at Edwards are numbered as in an ordinary adjustment and one of the unobserved directions at one of the fixed stations is designated "x." In this example, the direction to Edwards from Surf is designated "x." By using the fixed angles of the triangles Tide-Apple-Surf and Rail-Tide-Surf and the angles in triangles 3, 4, and 6 computed by the three-point problem, it is possible to obtain the angles of the triangles Edwards-Surf-Rail and Edwards-Rail-Tide. Care must be taken in computing these angles to obtain their proper designations.

The angles of the triangle Edwards-Surf-Rail are computed as follows:

	°	′	″
Angle at Surf, Tide to Apple,	86	29	42. 4
Angle at Surf, Rail to Tide,	37	52	36. 0
Angle at Surf, Rail to Apple,	124	22	18. 4
Angle at Surf, Edwards to Apple ($-x$)	31	40	46. 8
Angle at Surf, Rail to Edwards ($+x$)	92	41	31. 6
Angle at Edwards, Surf to Rail ($-2+3$)	18	18	16. 0
Sum ($-2+3+x$)	110	59	47. 6
180° + spherical excess,	180	00	00. 1
Angle at Rail, Edwards to Surf ($+2-3-x$)	69	00	12. 5

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Maine

NO.		STATION	OBSERVED ANGLE			CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
	2-3	Apple-Surf								4.109221
	1	Tide	74	14	10.9	-4.6			06.3	0.016651
1.	2	Apple	19	16	12.3	-0.9			11.4	9.518537
	3	Surf	86	29	45.9	-3.5	42.4	0.1	42.3	9.999187
	1-3	Tide-Surf				-9.0		0.1		3.644409
	1-2	Tide-Apple								4.125059
	2-3	Tide-Surf								3.644409
	1	Rail	80	57	35.0	-3.0			32.0	0.005430
2.	2	Tide	61	09	53.5	-1.5			52.0	9.942508
	3	Surf	37	52	36.7	-0.7			36.0	9.788143
	1-3	Rail-Surf				-5.2		0.0		3.592347
	1-2	Rail-Tide								3.437982
	2-3	Apple-Surf								4.109221
-1+2	1	Edwards	84	12	57.9	+0.1	58.0		58.0	0.002217
3. +1-2+x	2	Apple	64	06	15.5	+2.9	18.4	0.1	18.3	9.954048
-x	3	Surf	31	40	46.8	-3.0	43.8	0.1	43.7	9.720289
	1-3	Edwards-Surf								4.065436-1
	1-2	Edwards-Apple						0.2		3.831727
	2-3	Apple-Tide								4.125059
-1+4	1	Edwards	105	51	04.7	-0.2	04.5		04.5	0.016837
4. +1-2+x	2	Apple	44	50	04.1	+2.9	07.0	0.1	06.9	9.848233
+2-4-x	3	Tide	29	18	51.4	-2.7	48.7	0.1	48.6	9.689831
	1-3	Edwards-Tide						0.2		3.990129-1
	1-2	Edwards-Apple								3.831727
	2-3	Surf-Rail								3.592347
-2+3	1	Edwards	18	18	16.0	0.0	16.0		16.0	0.502979
5. +x	2	Surf	92	41	31.6	+3.0	34.6	0.1	34.5	9.999520
+2-3-x	3	Rail	69	00	12.5	-3.0	09.5		09.5	9.970159
	1-3	Edwards-Rail						0.1		4.094846
	1-2	Edwards-Surf								4.065435

FIG. 64.—Triangle computation for Figure 63

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 28

COMPUTATION OF TRIANGLES

State: Maine

NO.		STATION	OBSERVED ANGLE			CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
	2-3	Surf-Tide								3.644409
-2+4	1	Edwards	21	38	06.8	-0.3	06.5		06.5	0.433333
6. +x	2	Surf	54	48	55.6	+3.0	58.6		58.6	9.912386
+2-4-x	3	Tide	103	32	57.7	-2.7	55.0	0.1	54.9	9.987743
	1-3	Edwards-Tide						0.1		3.990128
	1-2	Edwards-Surf								4.065485
	2-3	Rail-Tide								3.437982
-3+4	1	Edwards	3	19	50.8	-0.3			50.5	1.235832
7. -2+3+x	2	Rail	11	57	19.5	+3.0			22.5	9.316316
+2-4-x	3	Tide	164	42	49.7	-2.7			47.0	9.421033
	1-3	Edwards-Tide						0.0		3.990130_{-2}
	1-2	Edwards-Rail								4.094847_{-1}

FIG. 64.—Triangle computation for Figure 63—Continued

The angles of the triangle Edwards-Rail-Tide with their proper designations may be obtained in the same manner. In this triangle the angle at Edwards is an observed angle, the angle at Rail is obtained by subtracting the angle at Rail in triangle No. 5 from the fixed angle at Rail in triangle No. 2, and the angle at Tide is obtained by adding the angles at Edwards and Rail, and subtracting this sum from 180° plus the spherical excess of the triangle.

To make sure that the angles of the triangles have been correctly taken out they are checked as follows before the adjustment is made. The angles in triangles 3, 4, and 6, determined by the three-point problem, are checked by computing the lengths. (See fig. 64.) The other two triangles are checked by adding the angle at Tide in triangle No. 6 to the fixed angle at Tide in triangle No. 2 to obtain the angle at Tide in triangle No. 7.

	°	′	″
Angle at Tide, Surf to Rail,	61	09	52.0
Angle at Tide, Edwards to Surf $(+2-4-x)$	103	32	57.7
Angle at Tide, Edwards to Rail $(+2-4-x)$	164	42	49.7

This checks the angle obtained by the computation of triangle 7, and as triangle 5 was used in computing triangle 7, both triangles are verified by this check.

After the angles in the triangles have thus been checked the adjustment can be made, the "x" correction being treated just the same as the numbered corrections in forming the equations. Since there are four lines from station Edwards, there will be two side equations, one involving directions 1, 2, 4, and x, the other 2, 3, 4, and x. As both these equations contain x, it is possible to eliminate the x and combine the two equations into one equation involving directions 1, 2, 3, and 4. (See below.) From this point the adjustment is similar to that of any other quadrilateral and so no further explanation is needed.

Side equations

Surf-Tide		3.644409	....	Surf-Apple		−3 4.109221	....
	° ′ ″				° ′ ″		
$+x$	54 48 55.6	9.9123816	+1.48	−2+4	21 38 06.8	9.5666685	+5.31
$+2-4-x$	29 18 51.4	9.6898412	+3.75	$+1-2+x$	44 50 04.1	9.8482267	+2.12
−1+2	84 12 57.9	9.9977834	+.21	$-x$	31 40 46.8	9.7202996	+3.41
		3.2444152				3.2444155	

1. $0=-0.3-2.33\ (1)+11.39\ (2)-9.06\ (4)-0.98\ (x)$

Or $(x)=-0.3061-2.3775\ (1)+11.6224\ (2)-9.2449\ (4)$

Tide-Rail		3.437982	....	Tide-Surf		3.644409	....
$-2+3+x$	11 57 19.5	9.3162860	+9.94	−3+4	3 19 50.8	8.7641784	+36.18
−2+4	21 38 06.8	9.5666685	+5.31	$+x$	54 48 55.6	9.9123816	+1.48
		2.3209365				2.3209690	

2. $0=-32.5-15.25\ (2)+46.12\ (3)-30.87\ (4)+8.46\ (x)$

Multiplying equation 1 by $\frac{8.46}{0.98}$ (=8.633) and adding to equation 2 we obtain the combined equation with the x eliminated.

$0=-\ 2.59-20.11\ (1)+98.32\ (2)\qquad\qquad -78.21\ (4)-8.46(x)$

2. $0=-32.5\qquad\qquad -15.25\ (2)+46.12\ (3)-30.87\ (4)+8.46\ (x)$

$0=-35.09-20.11\ (1)+83.07\ (2)+46.12\ (3)-109.08\ (4)$. (This equation shoud be divided by 10 before entering it in the table of correlates.)

Correlate equation

	1	v	Adopted v
1	−2.01	−0.033	0.0
2	+8.31	+.136	+.1
3	+4.61	+.076	+.1
4	−10.91	−.179	−.2
x		+3.008	+3.0

Solution of normal equation

$$0=213.3764\ C-3.509$$
$$C=+0.0164$$

If an approximate value for the direction designated by "x" in Figure 63 can be obtained by inverse computation from the fixed data and the field computations it is not necessary to solve the three-point problem. For example, if a field position were available for Edwards, the approximate direction for Surf to Edwards could be obtained by an inverse position computation between Surf and Edwards instead of by the three-point problem. It is only necessary to know this direction closely enough so that the tabular difference of the logarithm of the sine will be practically the same for the approximate angle as for the final angle. If the x-correction obtained by the adjustment is large, it is no indication that the observations are poor but only that the computed direction is considerably in error. In some cases involving very small angles it may be necessary to make a second adjustment unless the changes in the small angles necessary to make the lengths check are estimated and the tabular differences are corrected accordingly before the solution of the equations.

ADJUSTMENT OF INTERSECTION STATIONS

Observations are sometimes made upon an intersection station from four or more main scheme stations. Only three of the observed lines should be used in the adjustment but these three should be selected to give the strongest intersection at the new station unless the field computation indicates that a more accurate result can be obtained by using some other combination of three lines. In forming the side equation the pole should be so selected as to include the two smallest angles in the adjustment. In Figure 65 are shown four examples of intersection stations. The way in which the pole is selected in each is explained below.

In case No. 1, the smallest two angles being *P A C* and *P B C*, the pole should be taken at *P*. In case No. 2, *A P B* and *A B P* are the smallest angles, and so the pole should be taken at *A*, the fixed lengths *A C* and *A B* being used in the formation of the equation. In case No. 3, *C P A* and *C A P* are the smallest angles and so the pole should be taken at *C*, the fixed lengths *C B* and *C A* being used in the formation of the equation, as in case No. 2. In case No. 4, the smallest angles are *P B C* and *P A B*, and so the pole should be taken at *P*.

It should be noted that in all examples like cases 3 and 4, where one of the four points is inside the triangle formed by the other three points, the pole should always be taken at the inside point, since the smallest angles will then be included in the equation.

The size of the corrections to the angles to be expected in the adjustment of intersection stations depends largely upon the instruments used, the manner in which the field work was done, and upon the size and definiteness of the point observed. For example, in the computation of mountain peaks from angles observed by sextants, large corrections must be expected. The number of decimal places used for the angles and logarithms in the adjustment will depend upon the nature of the work.

SOLUTION OF NORMAL EQUATIONS INCLUDING TERMS USUALLY OMITTED

In the forward solution of normal equations a check on the work is obtained each time an equation is eliminated. The column headed Σ is for this purpose alone. For example, in the second line of the solution on page 201, the quantity -1.32 in the Σ column should equal the algebraic sum of all the other terms in that line, that is, $-1-0.33333+0.33333-0.11667-0.20333$ should equal -1.32, which

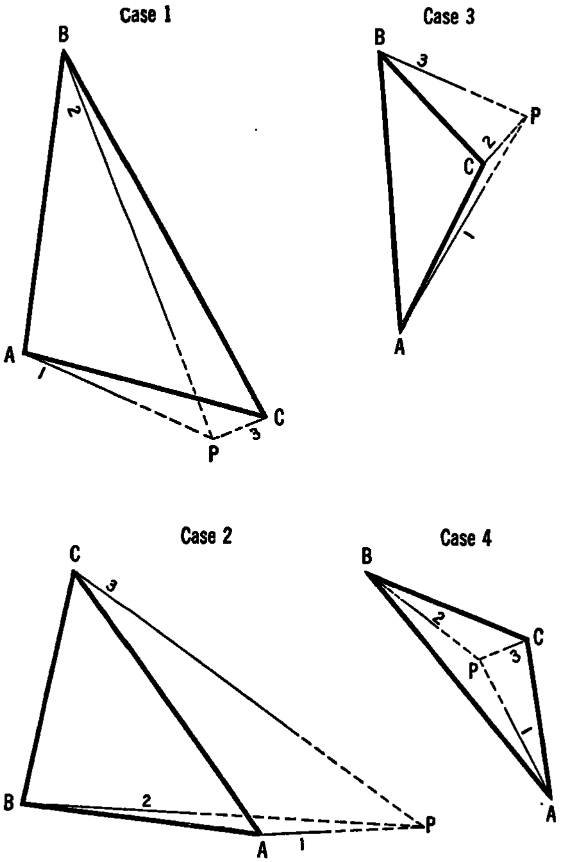


FIG. 65.—Typical figures in adjustment of intersection stations

it does, exactly. When making this check for the "divided" line, or the last line in the solution of each equation, one must always remember to include the -1 obtained by dividing the diagonal term by itself and changing the sign, as this quantity is never set down in the solution.

In the solution of the second equation of the following example the line containing the sums of the terms in the various columns should be checked before it is divided by the diagonal term. The sum of +5.3333+2.6667−16.0433−0.8767 should equal the −8.92 in the Σ column, which it does, exactly. In the next line, after division by the diagonal term, we have −1−0.50001+3.00814+0.16438, which equals +1.67251 and checks exactly the value in the Σ column.

If the line containing the sum terms does not check exactly but the discrepancy is so small as to indicate that no blunder has been made in the computation then the quantity in the Σ column should be made to agree exactly with the sum of the other terms before the line is divided by the diagonal term. For example, in the elimination of the fourth equation in the line containing the sum terms we have +54.5519+4.3660 and their sum equals +58.9179. But as the quantity in the Σ column is +50.9180 this should be changed to +58.9179 before the division is made. In the same way the quantity in the Σ column of the divided line should always be made to check the sum of the other terms in the line.

In a great many cases when a blunder has been made and the solution fails to check it is possible to find the trouble by inspection, especially if the mistake is a wrong sign on some term or the transposition of a decimal point. Sometimes, however, an error is difficult to find. In such cases, each line in the elimination of an equation can be checked. To illustrate this let us take the solution of the fourth equation on page 201 where all the usually omitted terms are shown in bold-faced type. In the second line of the solution we find that the sum of −0.70−0.2333+0.2333−0.0817−0.1423=−0.9240 which checks the value in the Σ column. Likewise, in the third line, +16.0433+8.0217−48.2605−2.6372=−26.8327 which checks the value in the Σ column within 1 in the last decimal place. In the fourth line, −7.7150−14.8803+2.0155=−20.5798 which checks exactly the quanty in the Σ column.

It should be carefully noted that all the quantities shown in bold-faced type on page 201 appear in the solution as ordinarily carried out and that it is not necessary to carry along this part of the solution to get the check on each line. For example, in the second line of the elimination of the fourth equation, the term +0.2333 appears also in column 4 in the second line of the elimination of the third equation; −0.2333 appears also in column 4 in the second line of the elimination of the second equation; and −0.70 is −1 times the term +0.70 in column 4 and equation 1. It is seen, therefore, that all the terms used in the checking of the individual lines in the elimination of equation 4 are given either in the lines themselves or in column 4. For the first line of the solution of equation 4 all the omitted terms except one are given in column 4 in the first lines of the solution of the other equations. The one exception is the quantity in column 4 in the first equation which must be multiplied by −1.

In the same way, for the second line of the solution of equation 4, all the omitted terms except one will be found in column 4 in the second line of the solution of the other equations. The exception is the quantity in the sum line of equation 2 which must be multiplied by −1. The omitted terms for the third line of the solution of equation 4 are obtained in a similar manner.

Particular care should be taken in the solution of normal equations to guard against compensating errors since they usually cause a great deal of trouble and may not be found until the triangles are computed. For example, if a mistake is made in one of the constant terms in the η column and a counterbalancing mistake in the Σ column, the solution will check and the errors will not be discovered until the triangles are solved.

Solution of normal equations, including terms usually omitted

1	2	3	4	η	Σ_n
+6	+2	−2	+0.70	+1.22	+7.92
−1 C_1	−.33333	+.33333	−.11667	−.20333	−1.32
+2	+6	+2	−15.81	−.47	−6.28
−2 (1)	−.6667	+.6667	−.2333	−.4067	−2.64
0	+5.3333	+2.6667	−16.0433	−.8767	−8.92
0	−1 C_2	−.50001	+3.00814	+.16438	+1.67251
−2	+2	+6	−.54	−1.89	+3.57
+2	+.6667 (1)	−.6667	+.2333	+.4067	+2.64
0	−2.6667 (2)	−1.3333	+8.0217	+.4383	+4.46
0	0	+4	+7.7150	−1.0450	+10.67
0	0	−1 C_3	−1.92875	+.26125	−2.6675
+.70	−15.81	−.54	+117.7744	+5.13	+107.2544
−.70	−.2333	+.2333 (1)	−.0817	−.1423	−.9240
0	+16.0433	+8.0217 (2)	−48.2605	−2.6372	−26.8326
0	0	−7.7150 (3)	−14.8803	+2.0155	−20.5798
0	0	0	+54.5519	+4.3660	+58.918079
0	0	0	−1 C_4	−.08003	−1.08003

INVERSE POSITION COMPUTATION

It sometimes becomes necessary in the adjustment of triangulation to compute the azimuths and length of a line joining two stations which are fixed in position, but which have not been directly connected by the observations. In order to compute this line an inverse or back computation must be made. This computation can be made on Form 26 or Form 27 (see Special Publication No. 8, p. 14), but it can be made more easily and simply on Form 662.

An example of an inverse position computation on Form 662 is given in Figure 66. The formulas for the computation are given at the top of the form. The table for the correction of arc to sine is given on the back of the form. (Do not confuse this table with the table given on page 17 of Special Publication No. 8, sixth edition, which is an entirely different table.) Triangulation stations Spencer and

Peterson are fixed in position (latitude and longitude) and it is desired to determine the azimuth, back-azimuth, and length of the line Spencer to Peterson. The form is self-explanatory and needs no

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 662

INVERSE POSITION COMPUTATION

$$s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{\Delta\lambda_1 \cos\phi_m}{A_m}$$

$$s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{-\Delta\phi_1 \cos\frac{\Delta\lambda}{2}}{B_m}$$

$$-\Delta\alpha=\Delta\lambda \sin\phi_m \sec\frac{\Delta\phi}{2}+F(\Delta\lambda)^3$$

in which $\log \Delta\lambda_1=\log(\lambda'-\lambda)-$correction for arc to sin*; $\log \Delta\phi_1=\log(\phi'-\phi)-$correction for arc to sin*; and $\log s=\log s_1+$ correction for arc to sin*.

	°	′	″	NAME OF STATION		°	′	″
1. ϕ	43	59	00.715	Spencer	λ	123	05	41.248
2. ϕ'	44	30	38.293	Peterson	λ'	122	58	05.537

	°	′	″
$\Delta\phi\ (=\phi'-\phi)$	+	31	37.578
$\frac{\Delta\phi}{2}$	+	15	48.789
$\phi_m\left(=\phi+\frac{\Delta\phi}{2}\right)$	44	14	49.504
$\Delta\phi$ (secs.)	+	1897.578	

	°	′	″
$\Delta\lambda\ (=\lambda'-\lambda)$		− 7	35.711
$\frac{\Delta\lambda}{2}$		− 3	47.856
$\Delta\lambda$ (secs.)		−455.711	

$\log \Delta\phi$	3.2781997	
cor. arc−sin	− 15	
$\log \Delta\phi_1$	3.2781982	
$\log \cos\frac{\Delta\lambda}{2}$	9.9999997	
colog B_m	1.4894742	
$\log\left\{s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$	4.7676721	(opposite in sign to $\Delta\phi$)
$\log \Delta\lambda$	2.6586896 n	
$\log \sin\phi_m$	9.8437024	
$\log \sec\frac{\Delta\phi}{2}$	0.0000046	
$\log a$	2.5023966 n	
a	−317.98	
b	0.00	
$-\Delta\alpha$ (secs.)	−317.98″	
$-\frac{\Delta\alpha}{2}$	−158.99″	

3 log $\Delta\lambda$	7.976
log F	7.848
log b	5.824

	°	′	″
$-\frac{\Delta\alpha}{2}$	−	2	38.99
$\alpha+\frac{\Delta\alpha}{2}$	189	47	40.69
α (1 to 2)	189	45	01.70
$\Delta\alpha$	+	5	17.98
	180		
α' (2 to 1)	9	50	19.68

$\log \Delta\lambda$	2.6586896 n		
cor. arc−sin	− 1		
$\log \Delta\lambda_1$	2.6586895 n		
$\log \cos\phi_m$	9.8551177		
colog A_m	1.4909902		
$\log\left\{s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$	4.0047974 n		
$\log\left\{s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$	4.7676721 n		
$\log \tan\left(\alpha+\frac{\Delta\alpha}{2}\right)$	9.2371253		
$\alpha+\frac{\Delta\alpha}{2}$	189°	47′	40.69″
$\log \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)$	9.2307483		
$\log \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)$	9.9936230		
$\log s_1$	4.7740491		
cor. arc−sin	+ 16		
$\log s$	4.7740507		
s	59436.15		

* Use the table on the back of this form for correction of arc to sin.

NOTE.—For log s up to 4.52 and for $\Delta\phi$ or $\Delta\lambda$ (or both) up to 10′, omit all terms below the heavy line except those printed in heavy type or those underscored, if using logarithms to 6 decimal places.

FIG. 66.—Inverse position computation, Form 662

detailed explanation. It should be noted, however, that the quadrant in which the angle $\left(\alpha+\frac{\Delta\alpha}{2}\right)$ occurs depends upon the algebraic signs of the quantities $\sin\left(\alpha+\frac{\Delta\alpha}{2}\right)$ and $\cos\left(\alpha+\frac{\Delta\alpha}{2}\right)$. In the following example the sine and cosine are both minus and therefore the angle is in the third quadrant. Log s_1 is obtained in two ways, first by subtracting $\log \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)$ from $\log\left\{s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$ and second by subtracting $\log \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)$ from $\log\left\{s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$.

Table of arc-sin corrections for inverse position computations

log s_1	Arc-sin correction in units of seventh decimal of logarithms	log $\Delta\phi$ or log $\Delta\lambda$	log s_1	Arc-sin correction in units of seventh decimal of logarithms	log $\Delta\phi$ or log $\Delta\lambda$	log s_1	Arc-sin correction in units of seventh decimal of logarithms	log $\Delta\phi$ or log $\Delta\lambda$
4.177	1	2.686	5.223	124	3.732	5.525	497	4.034
4.327	2	2.836	5.234	130	3.743	5.530	508	4.039
4.415	3	2.924	5.243	136	3.752	5.534	519	4.043
4.478	4	2.987	5.253	142	3.762	5.539	530	4.048
4.526	5	3.035	5.260	147	3.769	5.543	541	4.052
4.566	6	3.075	5.269	153	3.778	5.548	553	4.057
4.599	7	3.108	5.279	160	3.788	5.553	565	4.062
4.628	8	3.137	5.287	166	3.796	5.557	577	4.066
4.654	9	3.163	5.294	172	3.803	5.561	588	4.070
4.677	10	3.186	5.303	179	3.812	5.566	600	4.075
4.697	11	3.206	5.311	186	3.820	5.570	613	4.079
4.716	12	3.225	5.318	192	3.827	5.575	625	4.084
4.734	13	3.243	5.326	199	3.835	5.579	637	4.088
4.750	14	3.259	5.334	206	3.843	5.583	650	4.092
4.765	15	3.274	5.341	213	3.850	5.587	663	4.096
4.779	16	3.288	5.349	221	3.858	5.591	674	4.100
4.792	17	3.301	5.356	228	3.865	5.595	687	4.104
4.804	18	3.313	5.363	236	3.872	5.600	702	4.109
4.827	20	3.336	5.369	243	3.878	5.604	716	4.113
4.857	23	3.366	5.376	251	3.885	5.608	729	4.117
4.876	25	3.385	5.383	259	3.892	5.612	743	4.121
4.892	27	3.401	5.390	267	3.899	5.616	757	4.125
4.915	30	3.424	5.396	275	3.905	5.620	771	4.129
4.936	33	3.445	5.403	284	3.912	5.624	785	4.133
4.955	36	3.464	5.409	292	3.918	5.628	800	4.137
4.972	39	3.481	5.415	300	3.924	5.632	814	4.141
4.988	42	3.497	5.422	309	3.931	5.636	829	4.145
5.003	45	3.512	5.428	318	3.937	5.640	845	4.149
5.017	48	3.526	5.434	327	3.943	5.644	861	4.153
5.035	52	3.544	5.440	336	3.949	5.648	877	4.157
5.051	56	3.560	5.446	345	3.955	5.652	893	4.161
5.062	59	3.571	5.451	354	3.960	5.656	909	4.165
5.076	63	3.585	5.457	364	3.966	5.660	925	4.169
5.090	67	3.599	5.462	373	3.971	5.663	941	4.172
5.102	71	3.611	5.468	383	3.977	5.667	957	4.176
5.114	75	3.623	5.473	392	3.982	5.671	973	4.180
5.128	80	3.637	5.479	402	3.988	5.674	989	4.183
5.139	84	3.648	5.484	412	3.993	5.678	1005	4.187
5.151	89	3.660	5.489	422	3.998			
5.163	94	3.672	5.495	433	4.004			
5.172	98	3.681	5.500	443	4.009			
5.183	103	3.692	5.505	453	4.014			
5.193	108	3.702	5.510	464	4.019			
5.205	114	3.714	5.515	474	4.024			
5.214	119	3.723	5.520	486	4.029			

FIG. 67.—Arc-sine corrections for inverse position computation, back of Form 662

The values of α and log s determined by this inverse computation may be checked in the following manner. Starting with the azimuth, Peterson to Spencer, 9° 50′ 19″.68, the logarithm of the length, Peterson to Spencer, 4.7740507, and the fixed latitude and longitude of Peterson, 44° 30′ 38″.293 and 122° 58′ 05″.537, the latitude and longitude of Spencer are computed on Form 26. (See fig. 68.) The values thus obtained should check the fixed values of the latitude and longitude of Spencer within one in the last place of decimals.

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

						°	′	″
α			to					
∠			&			+		
α	2.	Peterson	to 1.	Spencer		9	50	19.68
Δα						-	5	17.98
						180		
α′	1.	Spencer	to 2.	Peterson		189	45	01.70

First Angle of Triangle

	°	′	″				°	′	″
φ	44	30	38.293	2.	Peterson	λ	122	58	05.537
Δφ	-	31	37.579	s=		Δλ	+	7	35.711
φ′	43	59	00.714 +1	1.	Spencer	λ′	123	05	41.248

s	4.7740507	s²	9.54810			−h	3.2731
cos α	9.9935651	sin² α	8.46537	(δφ)²	6.5564	s² sin² α	8.0135
B	8.5105055	C	1.39664	D	2.3925	E	6.2020
h	3.2781213		9.41011		8.9489		7.4936
	″		″				
1st term.	+1897.2358	3d term.	+0.0889			(Δλ)²	7.976
2d term.	+0.2571	4th term.	-0.0031			F	7.343
	+1897.4929						5.819
3d and 4th terms.	+0.0858	s	4.7740507				
−Δφ	+1897.5787	sin α	9.2326831	Arg.		Δλ	2.6586899
½(φ+φ′)	44 14 49.53	A′	8.5090166	s	-63	sin ½(φ+φ′)	9.8437025
		sec φ′	0.1429454	Δλ	+4	sec ½(Δφ)	46
			2.6586958 (899)	Corr.	-59		2.5023970
11—626 GOVERNMENT PRINTING OFFICE		Δλ	+ 455.7114			−Δα	+317.979

FIG. 68.—Position computation to check inverse computation

LAPLACE AZIMUTHS

A triangulation station at which both astronomic longitude and astronomic azimuth observations have been made is called a Laplace point, and the azimuth is called a Laplace azimuth.

The geodetic determinations of latitude, longitude, and azimuth at a station are referred to the point on the celestial sphere defined by the normal to the Clarke spheroid at the station; while the astronomic determinations of latitude, longitude, and azimuth at the same station are referred to the point on the celestial sphere defined by the plumb line at the station. These points of reference on the celestial sphere are called the geodetic and astronomic zeniths, respectively. By a comparison of the astronomic and geodetic determinations of latitude, longitude, and azimuth at a station, it is possible to determine the deflection of the plumb line from the normal to the Clarke spheroid. This deflection of the vertical may be expressed by the angular distance between the geodetic and the astronomic zeniths and the azimuth of the line joining them, or by the components of the deflection along the meridian and the prime vertical.

At each Laplace point the prime vertical component of the deflection of the vertical should be the same, except for errors of observation, whether derived from the observed longitude or from the observed azimuth. This relation may be expressed as follows: If ϕ_G is the geodetic latitude; λ_A and λ_G the astronomic and geodetic longitudes respectively; α_A and α_G the astronomic and geodetic azimuths, respectively, then

$$\cos \phi_G(\lambda_A - \lambda_G) = -\cot \phi_G(\alpha_A - \alpha_G),$$
$$\text{or } (\alpha_A - \alpha_G) + \sin \phi_G\ (\lambda_A - \lambda_G) = 0,$$
$$\text{or } \alpha_G = \sin \phi_G\ (\lambda_A - \lambda_G) + \alpha_A,$$

which is known as the Laplace equation, since it was first used by Laplace. (For full development of this equation see Spec. Pub. No. 110, pp. 90–91.)

The accuracy of all the data used in determining the true geodetic azimuth at a Laplace point has been thoroughly considered in the various investigations of the figure of the earth and isostasy. It is shown from these investigations that the astronomic longitudes and the astronomic azimuths are each subject to probable errors which are, upon an average, not greater than ±0.″50. The geodetic longitude anywhere in the United States is subject to a probable error of less than ±0.″50, due to all causes other than errors in geodetic azimuth. However, the geodetic azimuths as computed through the triangulation are subject to probable errors as great as ±5.″

It is clearly seen then that at each Laplace point all the quantities used in the formula for computing the true geodetic azimuth are known with a much higher degree of accuracy than the geodetic azimuth at that point is known. Therefore the true geodetic azimuth computed by the formula above is more reliable than the geodetic azimuth as computed through the triangulation, and consequently is the one held fixed in the adjustment of the triangulation.

If there were no deflections of the vertical or station errors, as they are sometimes called, the determination of the correct relative positions of different points on the surface of the earth would be a simple matter and could be done by astronomic observations alone.

However, the relation between the difference of astronomic and geodetic longitude and the difference of astronomic and geodetic azimuth, as expressed by the Laplace equation, makes it possible to correct for the deflection of the vertical and to determine the true geodetic azimuth at a Laplace station, and hence to obtain the accumulated error in the geodetic azimuth as carried through the triangulation.

The method followed by the United States Coast and Geodetic Survey in determining the true geodetic azimuth is to establish Laplace stations; that is, make astronomic observations for both longitude and azimuth at various stations along the continuous arcs of triangulation. Then at each Laplace station the observed azimuth is corrected for the deflection of the vertical by means of the Laplace equation, and the true geodetic azimuth obtained. The true geodetic azimuth is then held fixed in the adjustment of the triangulation, and in each case the discrepancy between the true geodetic azimuth and the geodetic azimuth as carried through the triangulation is distributed by means of an azimuth equation. (See p. 62.)

An example of the computation necessary to obtain the true geodetic azimuth from the observed azimuth and longitude is shown below. The astronomic longitude of triangulation station Parkersburg and the astronomic azimuth of the line Parkersburg to Denver have been determined by star observations, and the geodetic longitude of Parkersburg and the geodetic azimuth of the line Parkersburg to Denver have been computed through the triangulation. It is desired to obtain the true geodetic azimuth of the line Parkersburg to Denver, and hence determine the accumulated error developed in the geodetic azimuth as computed through the triangulation from the next preceding Laplace azimuth.

The observed astronomic azimuth of the line Parkersburg to Denver is 143° 16′ 15″.55; the observed astronomic longitude of Parkersburg is 88° 01′ 48″.30; the geodetic longitude (that computed through the triangulation) of Parkersburg is 88° 01′ 49″.00; the geodetic latitude of Parkersburg is 38° 34′ 51″.52; and the geodetic azimuth of the line Parkersburg to Denver is 143° 16′ 15″.64.

The computation necessary to obtain the true geodetic azimuth of the line Parkersburg to Denver can best be arranged as follows:

		°	′	″
Astronomic longitude of Parkersburg, λ_A	=	88	01	48. 30
Geodetic longitude of Parkersburg, λ_G	=	88	01	49. 00
$\lambda_A - \lambda_G$	=			−0. 70
Sine of geodetic latitude of Parkersburg, $\sin \phi_G$	=			+0. 624
$\sin \phi_G (\lambda_A - \lambda_G)$	=			−0. 44
Astronomic azimuth, Parkersburg to Denver, α_A	=	143	16	15. 55
Laplace azimuth, Parkersburg to Denver, α_G	=	143	16	15. 11
Geodetic azimuth, Parkersburg to Denver	=	143	16	15. 64
Correction to geodetic azimuth	=			−0. 53

The true geodetic, or Laplace, azimuth of the line Parkersburg to Denver, 143° 16′ 15″.11, is the azimuth to be held in the adjustment of the triangulation, and −0″.53 is the accumulated error developed in computing the azimuth through the triangulation, or the discrepancy which must be distributed through the triangulation by the azimuth equation in the least-squares adjustment. (See p. 63.)

COMPUTATION OF LONG DISTANCES

The formulas on Form 662 (see fig. 66) may be used in computing distances up to approximately 200 miles between points whose latitudes and longitudes are known. When it is necessary to compute distances much greater than this, such as distances between widely separated cities, the following method should be used. (See pp. 88 and 89 of the Figure of the Earth and Isostasy from Measurements in the United States.) Let ϕ, λ, and ϕ', λ' be the latitudes and longitudes of the given stations. It is required to find the distance, s, in meters between the stations (θ being the arc distance) and the azimuth α_F and the back azimuth α_B of the line joining the stations.

Let, $a = 90° - \phi$, $c = 90° - \phi'$, $x = \frac{1}{2}(a-c)\, e^2 \sin^2 \frac{1}{2}(a+c)$,

$$a' = a - x,\quad c' = c + x,\quad B = \lambda' - \lambda,$$

$$\tan \tfrac{1}{2}(A' - C') = \sin \tfrac{1}{2}(a' - c') \csc \tfrac{1}{2}(a' + c') \cot \tfrac{1}{2} B,$$

$$\tan \tfrac{1}{2}(A' + C') = \cos \tfrac{1}{2}(a' - c') \sec \tfrac{1}{2}(a' + c') \cot \tfrac{1}{2} B,$$

$$\sin b = \sin B \sin a' \csc A' = \sin B \sin c' \csc C',$$

and $b = \theta$

Then $\alpha_F = 180° - C'$,

$$\alpha_B = 180° + A',$$

and s (in meters) $=$ (θ in seconds) $\left(\frac{1}{A}\right)$

or s (in miles) $=$ (θ in seconds) $\left(\frac{1}{A}\right) \times 0.00062137$

In the formulas above e is the eccentricity of the spheroid ($\log e^2 = 7.83050$), and A is a factor whose logarithm is tabulated for each minute of latitude in Special Publication No. 8. Log A should be taken out for the mean latitude of the two stations.

In making the computation the work should be arranged in a convenient form as shown in the example below. Taking the latitude and longitude of Mexico City as 19° 27′ 20″.0 and 99° 08′ 37″.0, respectively, and the latitude and longitude of Washington, D. C., as 38° 53′ 23″.0 and 77° 00′ 34″.0, respectively, the distance and azimuths between the two cities are computed as follows.

Sample computation of long distance

	°	′	″
ϕ	19	27	20.0
λ	99	08	37.0
ϕ'	38	53	23.0
λ'	77	00	34.0
$a=90°-\phi$	70	32	40.0
$c=90°-\phi'$	51	06	37.0
$\frac{1}{2}(a+c)=\frac{1}{2}(a'+c')$	60	49	38.5
$\frac{1}{2}(a-c)$	9	43	01.5
$B=\lambda'-\lambda$	−22	08	03.0
$\frac{1}{2}B$	−11	04	01.5
$\frac{1}{2}(a-c)$ in seconds			34981.5
log $\frac{1}{2}(a-c)$ in seconds	4.54384		
log sin² $\frac{1}{2}(a+c)$	9.88218–10		
log e^2	7.83050–10		
log x	2.25652		
x			180″.5
	°	′	″
$a'=a-x$	70	29	39.5
$c'=c+x$	51	09	37.5
$\frac{1}{2}(a'-c')$	9	40	01.0
log sin $\frac{1}{2}(a'-c')$	9.22510–10		
log csc $\frac{1}{2}(a'+c')$	0.05891		
log cot $\frac{1}{2}B$	0.70864_n		
log tan $\frac{1}{2}(A'-C')$	9.99265_n–10		
log cos $\frac{1}{2}(a'-c')$	9.99379–10		
log sec $\frac{1}{2}(a'+c')$	0.31208		
log cot $\frac{1}{2}B$	0.70864_n		
log tan $\frac{1}{2}(A'+C')$	1.01451_n		

	°	′	″
$\frac{1}{2}(A'-C')$	−44	30	55
$\frac{1}{2}(A'+C')$	−84	28	33
A'	−128	59	28
C'	−39	57	38
$\alpha_F=180°-C'$	219	57	38
$\alpha_B=180°+A'$	51	00	32
log sin B	9.57608_n–10		
log sin a'	9.97433–10		
log csc A'	0.10944_n		
log sin θ	9.65985–10		
log sin B	9.57608_n–10		
log sin c'	9.89148–10		
log csc C'	0.19229_n		
log sin θ	9.65985–10		
	°	′	″
θ	27	11	21
θ (in seconds)	97881″		
log (θ in seconds)	4.99070		
colog A	1.49062		
log s (in meters)	6.48132		
log 0.00062137	6.79335–10		
log s (in miles)	3.27467		
s (in meters)	3029145		
s (in miles)	1882.2		

SIDE EQUATION TEST

Frequently in the adjustment of triangulation it is found that although the triangle closures in some particular quadrilateral are very small, the side equation for that quadrilateral has a large discrepancy. This indicates that one or more of the directions must be in error and that the small triangle closures may be due to errors that counterbalance each other. By testing the quadrilateral by means of the side equation one is usually able to find the direction or directions which should be rejected in the adjustment.

The side equation test should be made in the following manner: Add together the coefficients of the terms of the side equation, disregarding their signs. Divide the constant term of the equation by this sum. The result is the approximate average correction that must be applied to a direction to eliminate the discrepancy in the figure and should not be greater than 0″.4 for first-order work.

For the following example the quadrilateral used (fig. 69) is one taken from an arc of first-order triangulation executed in 1925, and it illustrates very clearly the value of the side equation test.

The four triangles in this quadrilateral (see fig. 70) have closures of 0″.05, 0″.15, 0″.49, and 0″.39, or an average of 0″.27, disregarding the signs. These small closures seemed to indicate accurate values for the angles. In forming the side equation for the office computation, however, it was noticed that the constant term was very large in relation to the coefficients of the various terms. In order to see just what corrections would be required to eliminate the discrepancy in the side equation, the quadrilateral was adjusted. (See p. 212.) It was found that the angles at Anarchist between Gillespie and Spur and between Spur and Oroville required corrections of −3″.28 and +3″.37, respectively, and that the angles at Spur between Anarchist and Gillespie and between Oroville and Anarchist required corrections of +3″.78 and −3″.51. This was sufficient proof that some of the angles in the quadrilateral were in error, since the small triangle closures did not justify such large corrections to the angles to eliminate the side discrepancy. Had the side equation test been made in the field the error could have been found and corrected.

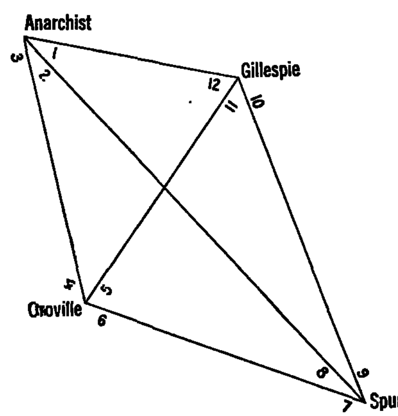


FIG. 69.—Quadrilateral used in side equation test

The smallest two angles in the quadrilateral (see fig. 69) are at Spur between Oroville and Anarchist and between Anarchist and Gillespie, so in writing the side equation the pole should be taken at Anarchist. The constant term of the side equation is then 38.3, and the sum of the coefficients without regard to sign is 30.9. Dividing 38.3 by 30′.9 we obtain 1″.24, which is the approximate average correction that must be applied to a direction in order to eliminate the side equation discrepancy. This is entirely too large considering that the average closure of the four triangles is only 0″.27, and that the maximum is only 0″.49.

Although the side equation test as made in this manner shows that some angle or angles must be in error, it does not show which ones are wrong. It is possible, however, to apply the test further by taking the pole of the side equation at some other vertex and bringing in some angles not used in the first test. If the side discrepancy is within the limit in this second test, then the angles used in it are probably correct and the error must be in some of the angles used in the first test, but not in the second test.

If necessary, the test may be further extended by using the third and fourth vertices as poles in forming other side equations. In most cases, however, especially if only one direction is in error, it can probably be located by the second test.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Washington

NO.		STATIONS	OBSERVED ANGLE			CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLES AND DISTANCES	LOGARITHM
	2-3	Gillespie - Spur								
-5+6	1	Oroville	77	58	50.51					
-10+11	2	Gillespie	54	50	54.68					
-7+9	3	Spur	47	10	15.78					
	1-3	Oroville - Spur				-0.05		0.92		
	1-2	Oroville - Gillespie								
					00.97					
	2-3	Gillespie - Spur								
-1+2	1	Anarchist	35	39	53.10					
-10+12	2	Gillespie	123	44	27.67					
-8+9	3	Spur	20	35	40.14					
	1-3	Anarchist - Spur				-0.15		0.76		
	1-2	Anarchist - Gillespie								
					00.91					
	2-3	Gillespie - Oroville								
-1+3	1	Anarchist	64	31	19.43					
-11+12	2	Gillespie	68	53	32.99					
-4+5	3	Oroville	46	35	08.71					
	1-3	Anarchist - Oroville				-0.49		0.64		
	1-2	Anarchist - Gillespie								
					01.13					
	2-3	Spur - Oroville								
-2+3	1	Anarchist	28	51	26.33					
-7+8	2	Spur	26	34	35.64					
-4+6	3	Oroville	124	33	59.22					
	1-3	Anarchist - Oroville				-0.39		0.80		
	1-2	Anarchist - Spur								
					01.19					

FIG. 70.—Observed angles and closures for triangles used in side equation test

In exceptional cases the side equation tests will show that some angles are in error, but will not indicate which ones. The example given here is such a case. As shown above the first test with the pole at Anarchist gave an approximate average correction to a direction of $\frac{38.3}{30.9}=1''.24$. The second test with the pole at Gillespie gave an approximate average correction of $\frac{33.4}{22.0}=1''.52$, the third with the pole at Spur, a correction of $\frac{25.5}{23.1}=1''.10$, and the fourth with the pole at Oroville, a correction of $\frac{30.4}{20.6}=1''.48$. In this example, therefore, all four tests show large average corrections to the directions, indicating that some of the angles are in error, but they do not disclose the erroneous angles.

It happens that this quadrilateral appears in an arc of triangulation through which a length equation was carried. It was found that if the length was carried through the triangles, Oroville-Gillespie-Spur and Anarchist-Gillespie-Oroville, the discrepancy in length was 1 part in 400,000, but if the length equation was carried through the triangles Anarchist-Gillespie-Spur and Oroville-Anarchist-Spur, the discrepancy was 1 part in 13,000. This test indicated that the line Anarchist-Spur was probably in error, and by about the same amount at both ends. Either direction 2 (at Anarchist) or direction 8 (at Spur) appears in all four side equations and consequently all four equations have large discrepancies.

This example, however, is an exceptional case as both ends of a line are seldom in error by approximately the same amount. Ordinarily the side equation test will indicate which angles are in error.

Side equation with pole at Anarchist

	°	′	″				°	′	″		
−10+12	123	44	27.67	9.9198919	−1.41	−8+9	20	35	40.14	9.5462360	+5.60
−7+8	26	34	35.64	9.6506895	+4.21	−4+6	124	33	59.22	9.9156471	−1.45
−4+5	46	35	08.71	9.8611781	+1.99	−11+12	68	53	32.99	9.9698381	+.81
				9.4317595						9.4317212	

$$0=+38.3-3.44(4)+1.99(5)+1.45(6)-4.21(7)+9.81(8)-5.60(9)+1.41(10)+0.81\,(11)-2.22\,(12)$$

$$\frac{38.3}{30.9}=1''.24$$

Side equation with pole at Gillespie

	°	′	″				°	′	″		
−7+9	47	10	15.78	9.8653329	+1.95	−5+6	77	58	50.51	9.9903732	+0.45
−4+5	46	35	08.71	9.8611781	+1.99	−1+3	64	31	19.43	9.9555679	+1.00
−1+2	35	39	53.10	9.7656995	+2.94	−8+9	20	35	40.14	9.5462360	+5.60
				9.4922105						9.4921771	

$$0=+33.4-1.94(1)+2.94(2)-1.00(3)-1.99(4)+2.44(5)-0.45(6)-1.95(7)+5.60(8)-3.65(9)$$

$$\frac{33.4}{22.0}=1''.52$$

Side equation with pole at Spur

	° ′ ″				° ′ ″		
−4+6	124 33 59.22	9.9156471	−1.45	−2+3	28 51 26.33	9.6836143	+3.82
−1+2	35 39 53.10	9.7656995	+2.94	−10+12	123 44 27.67	9.9198919	−1.41
−10+11	54 50 54.68	9.9125583	+1.48	−5+6	77 58 50.51	9.9903732	+.45
		9.5939049				9.5938794	

$$0=+25.5-2.94(1)+6.76(2)-3.82(3)+1.45(4)+0.45(5)-1.90(6)-2.89(10)+1.48(11)+1.41(12)$$

$$\frac{25.5}{23.1}=1\overset{\prime\prime}{.}10$$

Side equation with pole at Oroville

	° ′ ″				° ′ ″		
−1+3	64 31 19.43	9.9555679	+1.00	−11+12	68 53 32.99	9.9698381	+0.81
−10+11	54 50 54.68	9.9125583	+1.48	−7+9	47 10 15.78	9.8653329	+1.95
−7+8	26 34 35.64	9.6506895	+4.21	−2+3	28 51 26.33	9.6836143	+3.82
		9.5188157				9.5187853	

$$0=+30.4-1.00(1)+3.82(2)-2.82(3)-2.26(7)+4.21(8)-1.95(9)-1.48(10)+2.29(11)-0.81(12)$$

$$\frac{30.4}{20.6}=1\overset{\prime\prime}{.}48$$

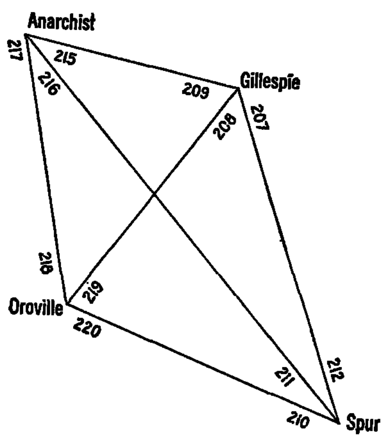


FIG. 71.—Quadrilateral used in side equation test with directions renumbered for office computation

The following adjustment of the quadrilateral shows what large corrections are required to make it consistent if all observed directions are used. The numbers on the directions are those used in the office adjustment of the arc.

Angle equations

1. 0=+0.05−(207)+(208)−(210)+(212)−(219)+(220)
2. 0=+0.39−(210)+(211)−(216)+(217)−(218)+(220)
3. 0=+0.49−(208)+(209)−(215)+(217)−(218)+(219)

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Washington

GOVERNMENT PRINTING OFFICE

NO.	STATIONS	OBSERVED ANGLE	CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLES AND DISTANCES	LOGARITHM
2-3	Gillespie - Spur						4.3372464
219+220[1]	Oroville	77 58 50.51	+0.94	51.45	0.31	51.14	0.0096264
-207+208[2]	Gillespie	54 50 54.68	-1.26	53.42	0.31	53.11	9.9125560
-210+212[3]	Spur	47 10 15.78	+0.27	16.05	0.30	15.75	9.8653328
1-3	Oroville - Spur		-0.05		0.92		4.3094288
1-2	Oroville - Gillespie						4.2622056
2-3	Gillespie - Spur						4.3372464
-215+216[1]	Anarchist	35 39 53.10	-3.28	49.82	0.25	49.57	0.2343109
-207+209[2]	Gillespie	123 44 27.67	-0.65	27.02	0.25	26.77	9.9198931
-211+212[3]	Spur	20 35 40.14	+3.78	43.92	0.26	43.66	9.5462557
1-3	Anarchist - Spur		-0.15		0.76		4.5414504
1-2	Anarchist - Gillespie						4.1678130
2-3	Gillespie - Oroville						4.2622056
-215+217[1]	Anarchist	64 31 19.43	+0.09	19.52	0.21	19.31	0.0444321
-208+209[2]	Gillespie	68 53 32.99	+0.61	33.60	0.21	33.39	9.9698384
-218+219[3]	Oroville	46 35 08.71	-1.19	07.52	0.22	07.30	9.8611753
1-3	Anarchist - Oroville		-0.49		0.64		4.2764761
1-2	Anarchist - Gillespie						4.1678130
2-3	Spur - Oroville						4.3094288
-216+217[1]	Anarchist	28 51 26.33	+3.37	29.70	0.27	29.43	0.3163738
-210+211[2]	Spur	26 34 35.64	-3.51	32.13	0.27	31.86	9.6506736
-218+220[3]	Oroville	124 33 59.22	-0.25	58.97	0.26	58.71	9.9156478
1-3	Anarchist - Oroville		-0.39		0.80		4.2764762
1-2	Anarchist - Spur						4.5414504

FIG. 72.—Triangle computation for quadrilateral used in side equation test

Side equation

	° ′ ″				° ′ ″		
−207+208	54 50 54.68	9.91255829	+1.48	−219+220	77 58 50.51	9.99037327	+0.45
−210+211	26 34 35.64	9.65068952	+4.21	−216+217	28 51 26.33	9.68361433	+3.82
−218+219	46 35 08.71	9.86117810	+1.99	−208+209	68 53 32.99	9.96983806	+0.81
−215+216	35 39 53.10	9.76569945	+2.94	−211+212	20 35 40.14	9.54623598	+5.60
		9.19012536				9.19006164	

4. $0=+63.72-1.48(207)+2.29(208)-0.81(209)-4.21(210)+9.81(211)-5.60(212)-2.94(215)+6.76(216)-3.82(217)-1.99(218)+2.44(219)-0.45(220)$

Correlate equations

	1	2	3	4	Σ_c	v	Adopted v
207	−1			−1.48	−2.48	+0.636	+0.64
208	+1		−1	+2.29	+2.29	−.622	−.62
209			+1	−.81	+.19	−.014	−.01
210	−1	−1		−4.21	−6.21	+1.081	+1.08
211		+1		+9.81	+10.81	−2.435	−2.43
212	+1			−5.60	−4.60	+1.354	+1.35
215			−1	−2.94	−3.94	+1.067	+1.06
216		−1		+6.76	+5.76	−2.221	−2.22
217		+1	+1	−3.82	−1.82	+1.154	+1.15
218		−1	−1	−1.99	−3.99	+.478	+.48
219	−1		+1	+2.44	+2.44	−.707	−.71
220	+1	+1		−.45	+1.55	+.229	+.23

Normal equations

1	2	3	4	η	Σ_n	C
+6	+2	−2	−0.51	+0.05	+5.54	−0.21996
	+6	+2	+4.98	+.39	+15.37	+.32198
		+6	+.45	+.49	+6.94	−.24124
			+232.4606	+63.72	+301.1006	−.28102

RECTANGULAR COORDINATES

The results of the triangulation computed by the United States Coast and Geodetic Survey are always given in geographic coordinates, since the triangulation usually covers a large area, and must necessarily be computed by means of geographic coordinates (latitudes and longitudes) on account of the curvature of the earth.

For the survey of a relatively small area, such as a city or small county, it is much more convenient to use plane coordinates, since the computations are simpler. If in the small area being surveyed, however, there are triangulation stations, these should be used to control the local survey and to connect it to the triangulation of the country.

In order to make its results of value to city and county surveyors using plane coordinates the United States Coast and Geodetic Survey has issued Special Publication No. 71, entitled "Relation between plane rectangular coordinates and geographic positions," in which tables are given that make the computations for transforming geographic to plane rectangular coordinates or vice versa very simple. This publication can be purchased from the Superintendent of Documents, Washington, D. C., for 10 cents.

CHAPTER 7.—GENERAL RULES AND SUGGESTIONS

All computations can be made much more rapidly and can be checked more easily if the work is arranged in a systematic manner. This bureau has printed forms for nearly all triangulation computations and these should be used whenever possible, as they expedite the work and lessen considerably the chances for errors.

The mathematician should bear in mind that accuracy is desired above everything else. Of course speed is desired also, but accuracy should not be sacrificed to it. Nothing is gained if a piece of work is done in an unusually short time, if as a consequence it afterwards needs considerable revision.

For a great many computations no fixed rules can be laid down, but the proper procedure depends upon the judgment of the mathematician doing the work. For instance, it is impossible to specify the number of decimal places to be used for the numbers and logarithms in all the various computations. This depends upon the particular piece of work being computed.

All work should be done neatly and all figures should be written carefully and legibly. Do not write one figure over another. Either erase entirely the one which is superseded, or draw a line through it and write the correct figure above it.

Every computation, unless self-checking, should be checked by a mathematician other than the one who made the original computation. The mathematician who does the checking should first make his changes in pencil, and then he and the mathematician making the original computation should agree on the proper values before the final corrections are made in ink.

An inexperienced mathematician should feel free at all times to consult the man under whom he is working or, in an emergency, any of the more experienced mathematicians in regard to the work. It is well to make sure you are right before carrying a computation too far. This consultation suggestion should not be abused, however. Beginners should proceed upon their own initiative in the work assigned them, unless there is doubt as to the proper method. Knowledge obtained by study and hard work is much more easily retained.

In all triangulation computations meters are used, but the final results are converted to feet, and both meters and feet are published.

In an adjustment of a large net of triangulation the selection of the equations, as well as the formation, should be checked before the work is carried ahead, as often an "identical" equation (see p. 181) will not become apparent until the forward solution of the normals is made, and considerable recomputation will then be required to substitute the proper equation.

Intersection stations, that is, unoccupied points, unless they are main scheme stations, should be adjusted by using only the three lines which give the strongest angles at the vertex.

The mathematician making the first check of a list of geographic positions should also check the angles in the triangles for all "no-check" points.

In those adjustments in which the logarithms of certain lengths each appear in only one triangle these logarithms should be carefully checked before the list is made out.

The mathematician making out a list of geographic positions and the one doing the checking should scan it carefully for errors that may easily be discovered by inspection. For example, it is easy to see whether the azimuth and back azimuth differ by approximately 180° as they should, or whether the number of figures in a length correspond to the characteristic of the logarithm from which it is taken.

All final lists of geographic positions should be checked by two mathematicians, who should put their initials at the bottom of each sheet. As soon as a list has been properly checked, a photostat copy of it should be obtained for the files of the division of geodesy. If the triangulation is along the coast another photostat copy of the list should be obtained for the files of the division of charts.

Pages of geographic positions and cards of descriptions of the stations should not be taken out of the files of the division of geodesy without leaving a memorandum of such withdrawals with the mathematician in charge of the files.

In the adjustment of a central point figure where there is more than one side equation, one of these equations should be written with the pole at the center and carried completely around the figure.

In an adjustment having several side equations, if the closure for one is large the directions used in that equation should be investigated. If the closure can be improved considerably by omitting a certain direction that should be done.

Whenever a field computation of triangles is available it is not necessary to make a preliminary office computation of triangles, but the logarithms of lengths from the field computation can be used to compute the spherical excess.

In the adjustment of triangulation by the angle method the azimuth equation should always be formed by using the *c* or azimuth angles, as then the azimuth equation will not ordinarily involve the same *v's* as the length equation, and the solution of the normal equations will be simplified.

In using the Shortrede logarithmic tables, the tables of proportional parts at the right of each page and the tabular difference for one second at the bottom of each minute column should be used except for very small angles. For small angles, where the tabular difference for one second is changing rapidly, the difference should be taken out for the particular second used.

In forming the correlate equations, the coefficients of some of the equations should, in some cases, be divided by 5, 10, or 100 to make them of approximately the same size as those in the other equations. This makes it possible to obtain a given accuracy with a smaller number of decimal places in the solution of the normal equations.

In the computation of a geodetic position for which the signs of the $\Delta\lambda$'s as computed over the two lines are the same, and the two values are approximately the same size, the resulting values of λ may check and yet the computation be wrong. This is due to the fact that the values of A' and sec ϕ' are the same for both lines used in the computation, and if there is an error in either of these terms, it will affect both $\Delta\lambda$'s by about the same amount. The error will not be apparent until this position is used in computing some other position for which the longitude will necessarily fail to check.

In the computation of a geodetic position all signs, whether plus or minus, should be indicated for all the terms. This saves much time and avoids confusion.

Eternal (true black carbon) ink should be used for making out all lists of geographic positions since this permits much better photostat copies to be made.

The last mathematician to leave a room, in which there are computing machines, at the end of a day should see that all the machines are covered. This prevents dust from entering and injuring the delicate parts of the machine.

In making out lists of geographic positions, the mathematician should always insert at the top of each sheet, in the blanks provided, the locality and State in which the triangulation is located, and also the datum on which the work is computed. This will avoid confusion if the sheet should become misplaced from the files or computation cahier.

Before starting the adjustment of a net of triangulation, the mathematician should find out just how the net is connected in position, length or azimuth to previously adjusted triangulation. If any of the points or lines of the new net are identical with points or lines of previously adjusted triangulation, they should be held fixed in the new adjustment, in order that the new triangulation may be made consistent with the existing fixed triangulation. Occasionally, however, a station already adjusted may be allowed to take a new position, if the old data on which its position was based are not considered very accurate or reliable. The final decision in a matter of this kind should rest with the more experienced judgment of the older mathematicians.

In many of the operations in the computation of triangulation more decimal places are used in the course of the computation than are necessary when the final values are reached. In dropping the extra decimal places no question arises if the figures dropped represent either more or less than one-half a unit of the last decimal place retained, for if they are less the last figure retained remains unchanged and if more it is increased one. For example, if the two numbers 0.2273 and 0.3366 are rounded off to three decimal places, then the adopted numbers should be 0.227 and 0.337, respectively.

When the figures in the dropped decimal places, however, represent exactly one-half a unit of the last decimal place retained, then the number adopted may have two values both of which are equally correct. In order to avoid confusion the United States Coast and Geodetic Survey has arbitrarily adopted the plan of using the nearest *even* figure for the last decimal place retained. For example, in rounding off to three decimal places, the numbers 0.4215 and 0.6245, the adopted numbers should be 0.422 and 0.624, respectively.

CHAPTER 8.—CONSTANTS, FORMULAS, AND TABLES

CONSTANTS AND FORMULAS

Dimensions of the earth according to Clarke's spheroid of reference (1866):

Equatorial radius, a, = 6378206.4 meters

$\log a = 6.80469857$

Polar semi-axis, b, = 6356583.8 meters

$\log b = 6.80322378$

Eccentricity, e, $= \sqrt{\frac{a^2 - b^2}{a^2}}$,

$e^2 = 0.006768658$,

$\log e^2 = 7.83050257 - 10$

Base of Naperian logarithms, ϵ, = 2.71828183

$\log \epsilon = 0.43429448$

Modulus of common logarithms, M, = 0.43429448

$\log M = 9.63778431 - 10$

$\pi = 3.14159265$

$\log \pi = 0.49714987$

$\log \sin 1'' = 4.68557487 - 10$

$\log \tan 1'' = 4.68557487 - 10$

1 kilometer = 0.621370 statute mile = 0.539593 nautical mile.

1 meter = 0.000621370 statute mile = 0.000539593 nautical mile.

1 statute mile = 1609.35 meters = 1.60935 kilometers.

1 nautical mile = 1853.25 meters = 1.85325 kilometers.

1 nautical mile = 1.151553 statute miles.

1 statute mile = 0. 868393 nautical mile.

1 meter = 39.37 inches (law of July 28, 1866).

1 meter = 3.28083333 feet.

log. 3.28083333 = 0.51598417.

1 foot = 0.30480061 meter.

log. 0.30480061 = 9.48401583 − 10.

Probable error of an observation, $r = 0.6745 \sqrt{\frac{\Sigma v^2}{n-1}}$

Probable error of result, $r_0 = \frac{r}{\sqrt{n}} = 0.6745 \sqrt{\frac{\Sigma v^2}{n(n-1)}}$

Probable error of an observation of unit weight, $\mu_1 = 0.6745 \sqrt{\frac{\Sigma pv^2}{n-1}}$

Probable error of an observation of weight p_1, $r_1 = \frac{\mu_1}{\sqrt{p_1}} = 0.6745 \sqrt{\frac{\Sigma pv^2}{p_1(n-1)}}$

Probable error of an observed direction, $d = 0.6745 \sqrt{\frac{\Sigma v^2}{c}}$ where Σv^2 = sum of squares of corrections to directions, and c is the number of conditions.

Mean error of an angle, $\alpha = \sqrt{\frac{\Sigma \Delta^2}{3n}}$,

where $\Sigma \Delta^2$ is the sum of the squares of the closing errors of the triangles, and n is the number of triangles.

Logarithms of radii of curvature of the earth's surface (in meters)

[Based upon Clarke's spheroid of 1866 as expressed in meters].

Azimuth (degrees)	Latitude								
	0°	1°	2°	3°	4°	5°	6°	7°	8°
0	6.80175	6.80175	6.80175	6.80176	6.80177	6.80178	6.80180	6.80181	6.80183
5	177	177	178	178	179	180	182	184	186
10	184	184	184	185	186	187	188	190	192
15	195	195	195	196	197	198	199	201	203
20	209	209	210	210	211	212	214	215	217
25	227	228	228	228	229	230	232	233	235
30	248	249	249	250	250	251	252	254	256
35	272	272	272	273	273	274	276	277	278
40	296	297	297	297	298	299	300	301	303
45	322	322	322	323	324	324	325	326	328
50	348	348	348	348	349	350	351	352	353
55	373	373	373	373	374	374	375	376	377
60	396	396	396	396	397	398	398	399	400
65	417	417	417	418	418	418	419	420	421
70	435	435	436	436	436	437	437	438	439
75	450	450	450	450	451	451	452	452	453
80	461	461	461	461	462	462	463	463	464
85	468	468	468	468	468	469	469	470	470
90	470	470	470	470	471	471	472	472	473

Azimuth (degrees)	Latitude								
	8°	9°	10°	11°	12°	13°	14°	15°	16°
0	6.80183	6.80186	6.80188	6.80191	6.80194	6.80197	6.80201	6.80204	6.80208
5	186	188	190	193	196	199	203	206	210
10	192	194	197	200	202	206	209	213	217
15	203	205	207	210	213	216	219	223	227
20	217	219	222	224	227	230	233	236	240
25	235	237	239	242	244	247	250	254	257
30	256	257	260	262	264	267	270	273	276
35	278	280	282	284	287	289	292	295	298
40	303	304	306	308	310	313	315	318	321
45	328	329	331	333	335	337	339	342	344
50	353	354	356	358	359	361	364	366	368
55	377	379	380	382	383	385	387	389	391
60	400	401	403	404	406	407	409	411	413
65	421	422	423	424	426	427	429	430	432
70	439	440	441	442	443	444	446	447	449
75	453	454	455	456	457	458	460	461	463
80	464	465	466	467	468	469	470	471	473
85	470	471	472	473	474	475	476	478	479
90	473	474	474	475	476	477	478	480	481

Azimuth (degrees)	Latitude								
	16°	17°	18°	19°	20°	21°	22°	23°	24°
0	6.80208	6.80213	6.80217	6.80222	6.80226	6.80232	6.80237	6.80242	6.80248
5	210	215	219	224	228	234	239	244	250
10	217	221	225	230	234	239	244	250	255
15	227	231	235	239	244	249	254	259	264
20	240	244	248	252	257	262	266	271	277
25	257	261	265	269	273	277	282	287	292
30	276	280	284	287	292	296	300	305	309
35	298	301	305	308	312	316	320	324	329
40	321	324	327	330	334	338	341	345	350
45	344	347	350	353	357	360	364	367	371
50	368	371	373	376	379	382	386	389	392
55	391	394	396	398	401	404	407	410	413
60	413	415	417	419	422	424	427	430	432
65	432	434	436	438	440	443	445	448	450
70	449	451	453	454	456	459	461	463	465
75	463	464	466	468	470	472	473	476	478
80	473	474	476	478	479	481	483	485	487
85	479	480	482	483	485	487	489	490	492
90	481	482	484	485	487	489	490	492	494

Logarithms of radii of curvature of the earth's surface (in meters)—Continued

Azimuth (degrees)	Latitude								
	24°	25°	26°	27°	28°	29°	30°	31°	32°
0	6.80248	6.80254	6.80260	6.80266	6.80272	6.80279	6.80285	6.80292	6.80299
5	250	256	262	268	274	280	287	294	300
10	255	261	267	273	279	285	292	298	305
15	264	270	276	282	288	294	300	306	313
20	277	282	288	293	299	305	311	317	324
25	292	297	302	308	313	319	325	331	337
30	309	314	319	324	330	335	340	346	352
35	329	333	338	343	348	353	358	363	369
40	350	354	358	362	367	372	377	382	386
45	371	375	379	383	387	391	396	400	405
50	392	396	399	403	407	411	415	419	423
55	413	416	420	423	426	430	434	437	441
60	432	435	438	442	445	448	451	455	458
65	450	453	455	458	461	464	467	470	473
70	465	468	470	473	475	478	481	484	486
75	478	480	482	484	487	489	492	494	497
80	487	489	491	493	495	498	500	502	505
85	492	494	496	498	501	503	505	507	510
90	494	496	498	500	502	504	507	509	511

Azimuth (degrees)	Latitude								
	32°	33°	34°	35°	36°	37°	38°	39°	40°
0	6.80299	6.80306	6.80313	6.80320	6.80327	6.80335	6.80342	6.80350	6.80357
5	300	307	314	322	329	336	344	351	359
10	305	312	319	326	333	340	348	355	363
15	313	320	326	333	340	348	355	362	369
20	324	330	337	343	350	357	364	371	378
25	337	343	349	355	362	368	375	382	388
30	352	358	364	370	376	382	388	394	401
35	369	374	380	385	391	397	402	408	414
40	386	392	397	402	407	412	418	423	429
45	405	410	414	419	424	429	434	439	444
50	423	428	432	436	441	445	450	454	459
55	441	445	449	453	457	461	465	469	474
60	458	462	465	469	472	476	480	484	487
65	473	476	480	483	486	489	493	496	500
70	486	489	492	495	498	501	504	507	510
75	497	500	502	505	508	510	513	516	519
80	505	507	510	512	515	517	520	523	525
85	510	512	514	517	519	522	524	527	529
90	511	514	516	518	521	523	526	528	531

Azimuth (degrees)	Latitude								
	40°	41°	42°	43°	44°	45°	46°	47°	48°
0	6.80357	6.80365	6.80373	6.80380	6.80388	6.80396	6.80404	6.80411	6.80419
5	359	366	374	382	389	397	404	412	420
10	363	370	378	385	393	400	408	415	423
15	369	376	384	391	398	406	413	420	428
20	378	385	392	399	406	413	420	427	434
25	388	395	402	408	415	422	429	436	442
30	401	407	413	420	426	433	439	446	452
35	414	420	426	432	438	444	450	456	462
40	429	434	440	446	451	457	462	468	474
45	444	449	454	459	464	470	475	480	485
50	459	464	468	473	478	482	487	492	496
55	474	478	482	486	490	495	499	503	508
60	487	491	495	499	502	506	510	514	518
65	500	503	507	510	514	517	520	524	528
70	510	514	517	520	523	526	529	532	536
75	519	522	525	528	531	534	536	539	542
80	525	528	531	534	536	539	542	544	547
85	529	532	534	537	540	542	545	548	550
90	531	533	536	538	541	544	546	549	551

Logarithms of radii of curvature of the earth's surface (in meters)—Continued

Azimuth (degrees)	Latitude								
	48°	49°	50°	51°	52°	53°	54°	55°	56°
0	6.80419	6.80426	6.80434	6.80442	6.80449	6.80457	6.80464	6.80471	6.80479
5	420	423	435	443	450	458	465	472	479
10	423	430	438	445	453	460	467	474	481
15	428	435	442	450	457	464	471	478	485
20	434	441	448	455	462	469	476	483	489
25	442	449	456	463	469	476	482	489	495
30	452	458	465	471	477	484	490	496	502
35	462	468	474	480	486	492	498	503	509
40	474	479	485	490	496	501	506	512	517
45	485	490	495	500	505	510	515	520	525
50	496	501	506	510	515	520	524	528	533
55	508	512	516	520	524	528	533	537	541
60	518	522	526	530	533	537	541	544	548
65	528	531	534	538	541	545	548	551	555
70	536	539	542	545	548	551	554	557	560
75	542	545	548	551	554	557	559	562	565
80	547	550	553	555	558	561	563	566	568
85	550	553	555	558	560	563	566	568	570
90	551	554	556	559	561	564	566	569	571

Azimuth (degrees)	Latitude								
	56°	57°	58°	59°	60°	61°	62°	63°	64°
0	6.80479	6.80486	6.80493	6.80500	6.80506	6.80513	6.80520	6.80526	6.80532
5	479	486	493	500	507	514	520	526	532
10	481	488	495	502	509	515	522	528	534
15	485	492	498	505	511	518	524	530	536
20	489	496	502	509	515	521	527	533	539
25	495	501	508	514	520	526	531	537	542
30	502	508	514	519	525	530	536	541	546
35	509	515	520	525	531	536	541	546	551
40	517	522	527	532	537	542	546	551	556
45	525	530	534	539	543	548	552	556	560
50	533	537	542	546	550	554	558	562	565
55	541	545	548	552	556	560	563	567	570
60	548	552	555	558	562	565	568	572	575
65	555	558	561	564	567	570	573	576	579
70	560	563	566	569	572	574	577	580	582
75	565	568	570	573	575	578	580	583	585
80	568	571	573	576	578	580	583	585	587
85	570	573	575	578	580	582	584	586	588
90	571	574	576	578	580	583	585	587	589

Azimuth (degrees)	Latitude								
	64°	65°	66°	67°	68°	69°	70°	71°	72°
0	6.80532	6.80538	6.80544	6.80550	6.80555	6.80560	6.80565	6.80570	6.80575
5	532	538	544	550	555	561	566	570	575
10	534	540	545	551	556	562	566	571	576
15	536	542	547	553	558	563	568	572	577
20	539	544	550	555	560	565	570	574	578
25	542	548	553	558	562	567	572	576	580
30	546	551	556	561	565	570	574	578	582
35	551	556	560	564	569	573	577	581	584
40	556	560	564	568	572	576	580	583	587
45	560	564	568	572	576	579	583	586	589
50	565	569	573	576	579	583	586	589	592
55	570	574	577	580	583	586	589	591	594
60	575	578	581	584	586	589	591	594	596
65	579	582	584	587	589	592	594	596	6.80598
70	582	585	587	590	592	594	596	6.80598	6.80600
75	585	587	590	592	594	596	598	6.80600	601
80	587	589	591	593	595	597	6.80599	601	602
85	588	590	592	594	596	598	6.80600	601	603
90	589	591	593	595	597	598	600	602	603

CONVERSION TABLES

Lengths—Feet to meters (from 1 to 1000 units)

[Reduction factor: 1 foot=0.3048006096 meter]

Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters
0	0.0	50	15.24003	100	30.48006	150	45.72009	200	60.96012
1	0.30480	1	15.54483	1	30.78486	1	46.02489	1	61.26492
2	0.60960	2	15.84963	2	31.08966	2	46.32969	2	61.56972
3	0.91440	3	16.15443	3	31.39446	3	46.63449	3	61.87452
4	1.21920	4	16.45923	4	31.69926	4	46.93929	4	62.17932
5	1.52400	5	16.76403	5	32.00406	5	47.24409	5	62.48412
6	1.82880	6	17.06883	6	32.30886	6	47.54890	6	62.78893
7	2.13360	7	17.37363	7	32.61367	7	47.85370	7	63.09373
8	2.43840	8	17.67844	8	32.91847	8	48.15850	8	63.39853
9	2.74321	9	17.98324	9	33.22327	9	48.46330	9	63.70333
10	3.04801	60	18.28804	110	33.52807	160	48.76810	210	64.00813
1	3.35281	1	18.59284	1	33.83287	1	49.07290	1	64.31293
2	3.65761	2	18.89764	2	34.13767	2	49.37770	2	64.61773
3	3.96241	3	19.20244	3	34.44247	3	49.68250	3	64.92253
4	4.26721	4	19.50724	4	34.74727	4	49.98730	4	65.22733
5	4.57201	5	19.81204	5	35.05207	5	50.29210	5	65.53213
6	4.87681	6	20.11684	6	35.35687	6	50.59690	6	65.83693
7	5.18161	7	20.42164	7	35.66167	7	50.90170	7	66.14173
8	5.48641	8	20.72644	8	35.96647	8	51.20650	8	66.44653
9	5.79121	9	21.03124	9	36.27127	9	51.51130	9	66.75133
20	6.09601	70	21.33604	120	36.57607	170	51.81610	220	67.05613
1	6.40081	1	21.64084	1	36.88087	1	52.12090	1	67.36093
2	6.70561	2	21.94564	2	37.18567	2	52.42570	2	67.66574
3	7.01041	3	22.25044	3	37.49047	3	52.73051	3	67.97054
4	7.31521	4	22.55525	4	37.79528	4	53.03531	4	68.27534
5	7.62002	5	22.86005	5	38.10008	5	53.34011	5	68.58014
6	7.92482	6	23.16485	6	38.40488	6	53.64491	6	68.88494
7	8.22962	7	23.46965	7	38.70968	7	53.94971	7	69.18974
8	8.53442	8	23.77445	8	39.01448	8	54.25451	8	69.49454
9	8.83922	9	24.07925	9	39.31928	9	54.55931	9	69.79934
30	9.14402	80	24.38405	130	39.62408	180	54.86411	230	70.10414
1	9.44882	1	24.68885	1	39.92888	1	55.16891	1	70.40894
2	9.75362	2	24.99365	2	40.23368	2	55.47371	2	70.71374
3	10.05842	3	25.29845	3	40.53848	3	55.77851	3	71.01854
4	10.36322	4	25.60325	4	40.84328	4	56.08331	4	71.32334
5	10.66802	5	25.90805	5	41.14808	5	56.38811	5	71.62814
6	10.97282	6	26.21285	6	41.45288	6	56.69291	6	71.93294
7	11.27762	7	26.51765	7	41.75768	7	56.99771	7	72.23774
8	11.58242	8	26.82245	8	42.06248	8	57.30251	8	72.54255
9	11.88722	9	27.12725	9	42.36728	9	57.60732	9	72.84735
40	12.19202	90	27.43205	140	42.67209	190	57.91212	240	73.15215
1	12.49682	1	27.73686	1	42.97689	1	58.21692	1	73.45695
2	12.80163	2	28.04166	2	43.28169	2	58.52172	2	73.76175
3	13.10643	3	28.34646	3	43.58649	3	58.82652	3	74.06655
4	13.41123	4	28.65126	4	43.89129	4	59.13132	4	74.37135
5	13.71603	5	28.95606	5	44.19609	5	59.43612	5	74.67615
6	14.02083	6	29.26086	6	44.50089	6	59.74092	6	74.98095
7	14.32563	7	29.56566	7	44.80569	7	60.04572	7	75.28575
8	14.63043	8	29.87046	8	45.11049	8	60.35052	8	75.59055
9	14.93523	9	30.17526	9	45.41529	9	60.65532	9	75.89535

Lengths—Feet to meters (from 1 to 1000 units)—Continued

Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters
250	76.20015	300	91.44018	350	106.68021	400	121.92024	450	137.16027
1	76.50495	1	91.74498	1	106.98501	1	122.22504	1	137.46507
2	76.80975	2	92.04978	2	107.28981	2	122.52985	2	137.76988
3	77.11455	3	92.35458	3	107.59462	3	122.83465	3	138.07468
4	77.41935	4	92.65939	4	107.89942	4	123.13945	4	138.37948
5	77.72416	5	92.96419	5	108.20422	5	123.44425	5	138.68428
6	78.02896	6	93.26899	6	108.50902	6	123.74905	6	138.98908
7	78.33376	7	93.57379	7	108.81382	7	124.05385	7	139.29388
8	78.63856	8	93.87859	8	109.11862	8	124.35865	8	139.59868
9	78.94336	9	94.18339	9	109.42342	9	124.66345	9	139.90348
260	79.24816	310	94.48819	360	109.72822	410	124.96825	460	140.20828
1	79.55296	1	94.79299	1	110.03302	1	125.27305	1	140.51308
2	79.85776	2	95.09779	2	110.33782	2	125.57785	2	140.81788
3	80.16256	3	95.40259	3	110.64262	3	125.88265	3	141.12268
4	80.46736	4	95.70739	4	110.94742	4	126.18745	4	141.42748
5	80.77216	5	96.01219	5	111.25222	5	126.49225	5	141.73228
6	81.07696	6	96.31699	6	111.55702	6	126.79705	6	142.03708
7	81.38176	7	96.62179	7	111.86182	7	127.10185	7	142.34188
8	81.68656	8	96.92659	8	112.16662	8	127.40665	8	142.64669
9	81.99136	9	97.23139	9	112.47142	9	127.71146	9	142.95149
270	82.29616	320	97.53620	370	112.77623	420	128.01626	470	143.25629
1	82.60097	1	97.84100	1	113.08103	1	128.32106	1	143.56109
2	82.90577	2	98.14580	2	113.38583	2	128.62586	2	143.86589
3	83.21057	3	98.45060	3	113.69063	3	128.93066	3	144.17069
4	83.51537	4	98.75540	4	113.99543	4	129.23546	4	144.47549
5	83.82017	5	99.06020	5	114.30023	5	129.54026	5	144.78029
6	84.12497	6	99.36500	6	114.60503	6	129.84506	6	145.08509
7	84.42977	7	99.66980	7	114.90983	7	130.14986	7	145.38989
8	84.73457	8	99.97460	8	115.21463	8	130.45466	8	145.69469
9	85.03937	9	100.27940	9	115.51943	9	130.75946	9	145.99949
280	85.34417	330	100.58420	380	115.82423	430	131.06426	480	146.30429
1	85.64897	1	100.88900	1	116.12903	1	131.36906	1	146.60909
2	85.95377	2	101.19380	2	116.43383	2	131.67386	2	146.91389
3	86.25857	3	101.49860	3	116.73863	3	131.97866	3	147.21869
4	86.56337	4	101.80340	4	117.04343	4	132.28346	4	147.52350
5	86.86817	5	102.10820	5	117.34823	5	132.58827	5	147.82830
6	87.17297	6	102.41300	6	117.65304	6	132.89307	6	148.13310
7	87.47777	7	102.71781	7	117.95784	7	133.19787	7	148.43790
8	87.78258	8	103.02261	8	118.26264	8	133.50267	8	148.74270
9	88.08738	9	103.32741	9	118.56744	9	133.80747	9	149.04750
290	88.39218	340	103.63221	390	118.87224	440	134.11227	490	149.35230
1	88.69698	1	103.93701	1	119.17704	1	134.41707	1	149.65710
2	89.00178	2	104.24181	2	119.48184	2	134.72187	2	149.96190
3	89.30658	3	104.54661	3	119.78664	3	135.02667	3	150.26670
4	89.61138	4	104.85141	4	120.09144	4	135.33147	4	150.57150
5	89.91618	5	105.15621	5	120.39624	5	135.63627	5	150.87630
6	90.22098	6	105.46101	6	120.70104	6	135.94107	6	151.18110
7	90.52578	7	105.76581	7	121.00584	7	136.24587	7	151.48590
8	90.83058	8	106.07061	8	121.31064	8	136.55067	8	151.79070
9	91.13538	9	106.37541	9	121.61544	9	136.85547	9	152.09550

Lengths—Feet to meters (from 1 to 1000 units)—Continued

Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters
500	152.40030	550	167.64034	600	182.88037	650	198.12040	700	213.36043
1	152.70511	1	167.94514	1	183.18517	1	198.42520	1	213.66523
2	153.00991	2	168.24994	2	183.48997	2	198.73000	2	213.97003
3	153.31471	3	168.55474	3	183.79477	3	199.03480	3	214.27483
4	153.61951	4	168.85954	4	184.09957	4	199.33960	4	214.57963
5	153.92431	5	169.16434	5	184.40437	5	199.64440	5	214.88443
6	154.22911	6	169.46914	6	184.70917	6	199.94920	6	215.18923
7	154.53391	7	169.77394	7	185.01397	7	200.25400	7	215.49403
8	154.83871	8	170.07874	8	185.31877	8	200.55880	8	215.79883
9	155.14351	9	170.38354	9	185.62357	9	200.86360	9	216.10363
510	155.44831	560	170.68834	610	185.92837	660	201.16840	710	216.40843
1	155.75311	1	170.99314	1	186.23317	1	201.47320	1	216.71323
2	156.05791	2	171.29794	2	186.53797	2	201.77800	2	217.01803
3	156.36271	3	171.60274	3	186.84277	3	202.08280	3	217.32283
4	156.66751	4	171.90754	4	187.14757	4	202.38760	4	217.62764
5	156.97231	5	172.21234	5	187.45237	5	202.69241	5	217.93244
6	157.27711	6	172.51715	6	187.75718	6	202.99721	6	218.23724
7	157.58192	7	172.82195	7	188.06198	7	203.30201	7	218.54204
8	157.88672	8	173.12675	8	188.36678	8	203.60681	8	218.84684
9	158.19152	9	173.43155	9	188.67158	9	203.91161	9	219.15164
520	158.49632	570	173.73635	620	188.97638	670	204.21641	720	219.45644
1	158.80112	1	174.04115	1	189.28118	1	204.52121	1	219.76124
2	159.10592	2	174.34595	2	189.58598	2	204.82601	2	220.06604
3	159.41072	3	174.65075	3	189.89078	3	205.13081	3	220.37084
4	159.71552	4	174.95555	4	190.19558	4	205.43561	4	220.67564
5	160.02032	5	175.26035	5	190.50038	5	205.74041	5	220.98044
6	160.32512	6	175.56515	6	190.80518	6	206.04521	6	221.28524
7	160.62992	7	175.86995	7	191.10998	7	206.35001	7	221.59004
8	160.93472	8	176.17475	8	191.41478	8	206.65481	8	221.89484
9	161.23952	9	176.47955	9	191.71958	9	206.95961	9	222.19964
530	161.54432	580	176.78435	630	192.02438	680	207.26441	730	222.50445
1	161.84912	1	177.08915	1	192.32918	1	207.56922	1	222.80925
2	162.15392	2	177.39395	2	192.63399	2	207.87402	2	223.11405
3	162.45872	3	177.69876	3	192.93879	3	208.17882	3	223.41885
4	162.76353	4	178.00356	4	193.24359	4	208.48362	4	223.72365
5	163.06833	5	178.30836	5	193.54839	5	208.78842	5	224.02845
6	163.37313	6	178.61316	6	193.85319	6	209.09322	6	224.33325
7	163.67793	7	178.91796	7	194.15799	7	209.39802	7	224.63805
8	163.98273	8	179.22276	8	194.46279	8	209.70282	8	224.94285
9	164.28753	9	179.52756	9	194.76759	9	210.00762	9	225.24765
540	164.59233	590	179.83236	640	195.07239	690	210.31242	740	225.55245
1	164.89713	1	180.13716	1	195.37719	1	210.61722	1	225.85725
2	165.20193	2	180.44196	2	195.68199	2	210.92202	2	226.16205
3	165.50673	3	180.74676	3	195.98679	3	211.22682	3	226.46685
4	165.81153	4	181.05156	4	196.29159	4	211.53162	4	226.77165
5	166.11633	5	181.35636	5	196.59639	5	211.83642	5	227.07645
6	166.42113	6	181.66116	6	196.90119	6	212.14122	6	227.38125
7	166.72593	7	181.96596	7	197.20599	7	212.44602	7	227.68606
8	167.03073	8	182.27076	8	197.51080	8	212.75083	8	227.99086
9	167.33553	9	182.57557	9	197.81560	9	213.05563	9	228.29566

Lengths—Feet to meters (from 1 to 1000 units)—Continued

Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters
750	228.60046	800	243.84049	850	259.08052	900	274.32055	950	289.56058
1	228.90526	1	244.14529	1	259.38532	1	274.62535	1	289.86538
2	229.21006	2	244.45009	2	259.69012	2	274.93015	2	290.17018
3	229.51486	3	244.75489	3	259.99492	3	275.23495	3	290.47498
4	229.81966	4	245.05969	4	260.29972	4	275.53975	4	290.77978
5	230.12446	5	245.36449	5	260.60452	5	275.84455	5	291.08458
6	230.42926	6	245.66929	6	260.90932	6	276.14935	6	291.38938
7	230.73406	7	245.97409	7	261.21412	7	276.45415	7	291.69418
8	231.03886	8	246.27889	8	261.51892	8	276.75895	8	291.99898
9	231.34366	9	246.58369	9	261.82372	9	277.06375	9	292.30378
760	231.64846	810	246.88849	860	262.12852	910	277.36855	960	292.60859
1	231.95326	1	247.19329	1	262.43332	1	277.67336	1	292.91339
2	232.25806	2	247.49809	2	262.73813	2	277.97816	2	293.21819
3	232.56287	3	247.80290	3	263.04293	3	278.28296	3	293.52299
4	232.86767	4	248.10770	4	263.34773	4	278.58776	4	293.82779
5	233.17247	5	248.41250	5	263.65253	5	278.89256	5	294.13259
6	233.47727	6	248.71730	6	263.95733	6	279.19736	6	294.43739
7	233.78207	7	249.02210	7	264.26213	7	279.50216	7	294.74219
8	234.08687	8	249.32690	8	264.56693	8	279.80696	8	295.04699
9	234.39167	9	249.63170	9	264.87173	9	280.11176	9	295.35179
770	234.69647	820	249.93650	870	265.17653	920	280.41656	970	295.65659
1	235.00127	1	250.24130	1	265.48133	1	280.72136	1	295.96139
2	235.30607	2	250.54610	2	265.78613	2	281.02616	2	296.26619
3	235.61087	3	250.85090	3	266.09093	3	281.33096	3	296.57099
4	235.91567	4	251.15570	4	266.39573	4	281.63576	4	296.87579
5	236.22047	5	251.46050	5	266.70053	5	281.94056	5	297.18059
6	236.52527	6	251.76530	6	267.00533	6	282.24536	6	297.48539
7	236.83007	7	252.07010	7	267.31013	7	282.55017	7	297.79020
8	237.13487	8	252.37490	8	267.61494	8	282.85497	8	298.09500
9	237.43967	9	252.67971	9	267.91974	9	283.15977	9	298.39980
780	237.74448	830	252.98451	880	268.22454	930	283.46457	980	298.70460
1	238.04928	1	253.28931	1	268.52934	1	283.76937	1	299.00940
2	238.35408	2	253.59411	2	268.83414	2	284.07417	2	299.31420
3	238.65888	3	253.89891	3	269.13894	3	284.37897	3	299.61900
4	238.96368	4	254.20371	4	269.44374	4	284.68377	4	299.92380
5	239.26848	5	254.50851	5	269.74854	5	284.98857	5	300.22860
6	239.57328	6	254.81331	6	270.05334	6	285.29337	6	300.53340
7	239.87808	7	255.11811	7	270.35814	7	285.59817	7	300.83820
8	240.18288	8	255.42291	8	270.66294	8	285.90297	8	301.14300
9	240.48768	9	255.72771	9	270.96774	9	286.20777	9	301.44780
790	240.79248	840	256.03251	890	271.27254	940	286.51257	990	301.75260
1	241.09728	1	256.33731	1	271.57734	1	286.81737	1	302.05740
2	241.40208	2	256.64211	2	271.88214	2	287.12217	2	302.36220
3	241.70688	3	256.94691	3	272.18694	3	287.42697	3	302.66701
4	242.01168	4	257.25171	4	272.49174	4	287.73178	4	302.97181
5	242.31648	5	257.55652	5	272.79655	5	288.03658	5	303.27661
6	242.62129	6	257.86132	6	273.10135	6	288.34138	6	303.58141
7	242.92609	7	258.16612	7	273.40615	7	288.64618	7	303.88621
8	243.23089	8	258.47092	8	273.71095	8	288.95098	8	304.19101
9	243.53569	9	258.77572	9	274.01575	9	289.25578	9	304.49581

Lengths—Meters to feet (from 1 to 1000 units)

[Reduction factor: 1 meter=3.280833333 feet]

Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet
0		50	164.04167	100	328.08333	150	492.12500	200	656.16667
1	3.28083	1	167.32250	1	331.36417	1	495.40583	1	659.44750
2	6.56167	2	170.60333	2	334.64500	2	498.68667	2	662.72833
3	9.84250	3	173.88417	3	337.92583	3	501.96750	3	666.00917
4	13.12333	4	177.16500	4	341.20667	4	505.24833	4	669.29000
5	16.40417	5	180.44583	5	344.48750	5	508.52917	5	672.57083
6	19.68500	6	183.72667	6	347.76833	6	511.81000	6	675.85167
7	22.96583	7	187.00750	7	351.04917	7	515.09083	7	679.13250
8	26.24667	8	190.28833	8	354.33000	8	518.37167	8	682.41333
9	29.52750	9	193.56917	9	357.61083	9	521.65250	9	685.69417
10	32.80833	60	196.85000	110	360.89167	160	524.93333	210	688.97500
1	36.08917	1	200.13083	1	364.17250	1	528.21417	1	692.25583
2	39.37000	2	203.41167	2	367.45333	2	531.49500	2	695.53667
3	42.65083	3	206.69250	3	370.73417	3	534.77583	3	698.81750
4	45.93167	4	209.97333	4	374.01500	4	538.05667	4	702.09833
5	49.21250	5	213.25417	5	377.29583	5	541.33750	5	705.37917
6	52.49333	6	216.53500	6	380.57667	6	544.61833	6	708.66000
7	55.77417	7	219.81583	7	383.85750	7	547.89917	7	711.94083
8	59.05500	8	223.09667	8	387.13833	8	551.18000	8	715.22167
9	62.33583	9	226.37750	9	390.41917	9	554.46083	9	718.50250
20	65.61667	70	229.65833	120	393.70000	170	557.74167	220	721.78333
1	68.89750	1	232.93917	1	396.98083	1	561.02250	1	725.06417
2	72.17833	2	236.22000	2	400.26167	2	564.30333	2	728.34500
3	75.45917	3	239.50083	3	403.54250	3	567.58417	3	731.62583
4	78.74000	4	242.78167	4	406.82333	4	570.86500	4	734.90667
5	82.02083	5	246.06250	5	410.10417	5	574.14583	5	738.18750
6	85.30167	6	249.34333	6	413.38500	6	577.42667	6	741.46833
7	88.58250	7	252.62417	7	416.66583	7	580.70750	7	744.74917
8	91.86333	8	255.90500	8	419.94667	8	583.98833	8	748.03000
9	95.14417	9	259.18583	9	423.22750	9	587.26917	9	751.31083
30	98.42500	80	262.46667	130	426.50833	180	590.55000	230	754.59167
1	101.70583	1	265.74750	1	429.78917	1	593.83083	1	757.87250
2	104.98667	2	269.02833	2	433.07000	2	597.11167	2	761.15333
3	108.26750	3	272.30917	3	436.35083	3	600.39250	3	764.43417
4	111.54833	4	275.59000	4	439.63167	4	603.67333	4	767.71500
5	114.82917	5	278.87083	5	442.91250	5	606.95417	5	770.99583
6	118.11000	6	282.15167	6	446.19333	6	610.23500	6	774.27667
7	121.39083	7	285.43250	7	449.47417	7	613.51583	7	777.55750
8	124.67167	8	288.71333	8	452.75500	8	616.79667	8	780.83833
9	127.95250	9	291.99417	9	456.03583	9	620.07750	9	784.11917
40	131.23333	90	295.27500	140	459.31667	190	623.35833	240	787.40000
1	134.51417	1	298.55583	1	462.59750	1	626.63917	1	790.68083
2	137.79500	2	301.83667	2	465.87833	2	629.92000	2	793.96167
3	141.07583	3	305.11750	3	469.15917	3	633.20083	3	797.24250
4	144.35667	4	308.39833	4	472.44000	4	636.48167	4	800.52333
5	147.63750	5	311.67917	5	475.72083	5	639.76250	5	803.80417
6	150.91833	6	314.96000	6	479.00167	6	643.04333	6	807.08500
7	154.19917	7	318.24083	7	482.28250	7	646.32417	7	810.36583
8	157.48000	8	321.52167	8	485.56333	8	649.60500	8	813.64667
9	160.76083	9	324.80250	9	488.84417	9	652.88583	9	816.92750

Lengths—Meters to feet (from 1 to 1000 units)—Continued

Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet
250	820.20833	300	984.25000	350	1,148.29167	400	1,312.33333	450	1,476.37500
1	823.48917	1	987.53083	1	1,151.57250	1	1,315.61417	1	1,479.65583
2	826.77000	2	990.81167	2	1,154.85333	2	1,318.89500	2	1,482.93667
3	830.05083	3	994.09250	3	1,158.13417	3	1,322.17583	3	1,486.21750
4	833.33167	4	997.37333	4	1,161.41500	4	1,325.45667	4	1,489.49833
5	836.61250	5	1,000.65417	5	1,164.69583	5	1,328.73750	5	1,492.77917
6	839.89333	6	1,003.93500	6	1,167.97667	6	1,332.01833	6	1,496.06000
7	843.17417	7	1,007.21583	7	1,171.25750	7	1,335.29917	7	1,499.34083
8	846.45500	8	1,010.49667	8	1,174.53833	8	1,338.58000	8	1,502.62167
9	849.73583	9	1,013.77750	9	1,177.81917	9	1,341.86083	9	1,505.90250
260	853.01667	310	1,017.05833	360	1,181.10000	410	1,345.14167	460	1,509.18333
1	856.29750	1	1,020.33917	1	1,184.38083	1	1,348.42250	1	1,512.46417
2	859.57833	2	1,023.62000	2	1,187.66167	2	1,351.70333	2	1,515.74500
3	862.85917	3	1,026.90083	3	1,190.94250	3	1,354.98417	3	1,519.02583
4	866.14000	4	1,030.18167	4	1,194.22333	4	1,358.26500	4	1,522.30667
5	869.42083	5	1,033.46250	5	1,197.50417	5	1,361.54583	5	1,525.58750
6	872.70167	6	1,036.74333	6	1,200.78500	6	1,364.82667	6	1,528.86833
7	875.98250	7	1,040.02417	7	1,204.06583	7	1,368.10750	7	1,532.14917
8	879.26333	8	1,043.30500	8	1,207.34667	8	1,371.38833	8	1,535.43000
9	882.54417	9	1,046.58583	9	1,210.62750	9	1,374.66917	9	1,538.71083
270	885.82500	320	1,049.86667	370	1,213.90833	420	1,377.95000	470	1,541.99167
1	889.10583	1	1,053.14750	1	1,217.18917	1	1,381.23083	1	1,545.27250
2	892.38667	2	1,056.42833	2	1,220.47000	2	1,384.51167	2	1,548.55333
3	895.66750	3	1,059.70917	3	1,223.75083	3	1,387.79250	3	1,551.83417
4	898.94833	4	1,062.99000	4	1,227.03167	4	1,391.07333	4	1,555.11500
5	902.22917	5	1,066.27083	5	1,230.31250	5	1,394.35417	5	1,558.39583
6	905.51000	6	1,069.55167	6	1,233.59333	6	1,397.63500	6	1,561.67667
7	908.79083	7	1,072.83250	7	1,236.87417	7	1,400.91583	7	1,564.95750
8	912.07167	8	1,076.11333	8	1,240.15500	8	1,404.19667	8	1,568.23833
9	915.35250	9	1,079.39417	9	1,243.43583	9	1,407.47750	9	1,571.51917
280	918.63333	330	1,082.67500	380	1,246.71667	430	1,410.75833	480	1,574.80000
1	921.91417	1	1,085.95583	1	1,249.99750	1	1,414.03917	1	1,578.08083
2	925.19500	2	1,089.23667	2	1,253.27833	2	1,417.32000	2	1,581.36167
3	928.47583	3	1,092.51750	3	1,256.55917	3	1,420.60083	3	1,584.64250
4	931.75667	4	1,095.79833	4	1,259.84000	4	1,423.88167	4	1,587.92333
5	935.03750	5	1,099.07917	5	1,263.12083	5	1,427.16250	5	1,591.20417
6	938.31833	6	1,102.36000	6	1,266.40167	6	1,430.44333	6	1,594.48500
7	941.59917	7	1,105.64083	7	1,269.68250	7	1,433.72417	7	1,597.76583
8	944.88000	8	1,108.92167	8	1,272.96333	8	1,437.00500	8	1,601.04667
9	948.16083	9	1,112.20250	9	1,276.24417	9	1,440.28583	9	1,604.32750
290	951.44167	340	1,115.48333	390	1,279.52500	440	1,443.56667	490	1,607.60833
1	954.72250	1	1,118.76417	1	1,282.80583	1	1,446.84750	1	1,610.88917
2	958.00333	2	1,122.04500	2	1,286.08667	2	1,450.12833	2	1,614.17000
3	961.28417	3	1,125.32583	3	1,289.36750	3	1,453.40917	3	1,617.45083
4	964.56500	4	1,128.60667	4	1,292.64833	4	1,456.69000	4	1,620.73167
5	967.84583	5	1,131.88750	5	1,295.92917	5	1,459.97083	5	1,624.01250
6	971.12667	6	1,135.16833	6	1,299.21000	6	1,463.25167	6	1,627.29333
7	974.40750	7	1,138.44917	7	1,302.49083	7	1,466.53250	7	1,630.57417
8	977.68833	8	1,141.73000	8	1,305.77167	8	1,469.81333	8	1,633.85500
9	980.96917	9	1,145.01083	9	1,309.05250	9	1,473.09417	9	1,637.13583

Lengths—Meters to feet (from 1 to 1000 units)—Continued

Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet
500	1,640.41667	550	1,804.45833	600	1,968.50000	650	2,132.54167	700	2,296.58333
1	1,643.69750	1	1,807.73917	1	1,971.78083	1	2,135.82250	1	2,299.86417
2	1,646.97833	2	1,811.02000	2	1,975.06167	2	2,139.10333	2	2,303.14500
3	1,650.25917	3	1,814.30083	3	1,978.34250	3	2,142.38417	3	2,306.42583
4	1,653.54000	4	1,817.58167	4	1,981.62333	4	2,145.66500	4	2,309.70667
5	1,656.82083	5	1,820.86250	5	1,984.90417	5	2,148.94583	5	2,312.98750
6	1,660.10167	6	1,824.14333	6	1,988.18500	6	2,152.22667	6	2,316.26833
7	1,663.38250	7	1,827.42417	7	1,991.46583	7	2,155.50750	7	2,319.54917
8	1,666.66333	8	1,830.70500	8	1,994.74667	8	2,158.78833	8	2,322.83000
9	1,669.94417	9	1,833.98583	9	1,998.02750	9	2,162.06917	9	2,326.11083
510	1,673.22500	560	1,837.26667	610	2,001.30833	660	2,165.35000	710	2,329.39167
1	1,676.50583	1	1,840.54750	1	2,004.58917	1	2,168.63083	1	2,332.67250
2	1,679.78667	2	1,843.82833	2	2,007.87000	2	2,171.91167	2	2,335.95333
3	1,683.06750	3	1,847.10917	3	2,011.15083	3	2,175.19250	3	2,339.23417
4	1,686.34833	4	1,850.39000	4	2,014.43167	4	2,178.47333	4	2,342.51500
5	1,689.62917	5	1,853.67083	5	2,017.71250	5	2,181.75417	5	2,345.79583
6	1,692.91000	6	1,856.95167	6	2,020.99333	6	2,185.03500	6	2,349.07667
7	1,696.19083	7	1,860.23250	7	2,024.27417	7	2,188.31583	7	2,352.35750
8	1,699.47167	8	1,863.51333	8	2,027.55500	8	2,191.59667	8	2,355.63833
9	1,702.75250	9	1,866.79417	9	2,030.83583	9	2,194.87750	9	2,358.91917
520	1,706.03333	570	1,870.07500	620	2,034.11667	670	2,198.15833	720	2,362.20000
1	1,709.31417	1	1,873.35583	1	2,037.39750	1	2,201.43917	1	2,365.48083
2	1,712.59500	2	1,876.63667	2	2,040.67833	2	2,204.72000	2	2,368.76167
3	1,715.87583	3	1,879.91750	3	2,043.95917	3	2,208.00083	3	2,372.04250
4	1,719.15667	4	1,883.19833	4	2,047.24000	4	2,211.28167	4	2,375.32333
5	1,722.43750	5	1,886.47917	5	2,050.52083	5	2,214.56250	5	2,378.60417
6	1,725.71833	6	1,889.76000	6	2,053.80167	6	2,217.84333	6	2,381.88500
7	1,728.99917	7	1,893.04083	7	2,057.08250	7	2,221.12417	7	2,385.16583
8	1,732.28000	8	1,896.32167	8	2,060.36333	8	2,224.40500	8	2,388.44667
9	1,735.56083	9	1,899.60250	9	2,063.64417	9	2,227.68583	9	2,391.72750
530	1,738.84167	580	1,902.88333	630	2,066.92500	680	2,230.96667	730	2,395.00833
1	1,742.12250	1	1,906.16417	1	2,070.20583	1	2,234.24750	1	2,398.28917
2	1,745.40333	2	1,909.44500	2	2,073.48667	2	2,237.52833	2	2,401.57000
3	1,748.68417	3	1,912.72583	3	2,076.76750	3	2,240.80917	3	2,404.85083
4	1,751.96500	4	1,916.00667	4	2,080.04833	4	2,244.09000	4	2,408.13167
5	1,755.24583	5	1,919.28750	5	2,083.32917	5	2,247.37083	5	2,411.41250
6	1,758.52667	6	1,922.56833	6	2,086.61000	6	2,250.65167	6	2,414.69333
7	1,761.80750	7	1,925.84917	7	2,089.89083	7	2,253.93250	7	2,417.97417
8	1,765.08833	8	1,929.13000	8	2,093.17167	8	2,257.21333	8	2,421.25500
9	1,768.36917	9	1,932.41083	9	2,096.45250	9	2,260.49417	9	2,424.53583
540	1,771.65000	590	1,935.69167	640	2,099.73333	690	2,263.77500	740	2,427.81667
1	1,774.93083	1	1,938.97250	1	2,103 01417	1	2,267.05583	1	2,431.09750
2	1,778.21167	2	1,942.25333	2	2,106.29500	2	2,270.33667	2	2,434.37833
3	1,781.49250	3	1,945.53417	3	2,109.57583	3	2,273.61750	3	2,437.65917
4	1,784.77333	4	1,948.81500	4	2,112.85667	4	2,276.89833	4	2,440.94000
5	1,788.05417	5	1,952.09583	5	2,116.13750	5	2,280.17917	5	2,444.22083
6	1,791.33500	6	1,955.37667	6	2,119.41833	6	2,283.46000	6	2,447.50167
7	1,794.61583	7	1,958.65750	7	2,122.69917	7	2,286.74083	7	2,450.78250
8	1,797.89667	8	1,961.93833	8	2,125.98000	8	2,290.02167	8	2,454.06333
9	1,801.17750	9	1,965.21917	9	2,129.26083	9	2,293.30250	9	2,457.34417

Lengths—Meters to feet (from 1 to 1000 units)—Continued

Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet
750	2,460.62500	800	2,624.66667	850	2,788.70833	900	2,952.75000	950	3,116.79167
1	2,463.90583	1	2,627.94750	1	2,791.98917	1	2,956.03083	1	3,120.07250
2	2,467.18667	2	2,631.22833	2	2,795.27000	2	2,959.31167	2	3,123.35333
3	2,470.46750	3	2,634.50917	3	2,798.55083	3	2,962.59250	3	3,126.63417
4	2,473.74833	4	2,637.79000	4	2,801.83167	4	2,965.87333	4	3,129.91500
5	2,477.02917	5	2,641.07083	5	2,805.11250	5	2,969.15417	5	3,133.19583
6	2,480.31000	6	2,644.35167	6	2,808.39333	6	2,972.43500	6	3,136.47667
7	2,483.59083	7	2,647.63250	7	2,811.67417	7	2,975.71583	7	3,139.75750
8	2,486.87167	8	2,650.91333	8	2,814.95500	8	2,978.99667	8	3,143.03833
9	2,490.15250	9	2,654.19417	9	2,818.23583	9	2,982.27750	9	3,146.31917
760	2,493.43333	810	2,657.47500	860	2,821.51667	910	2,985.55833	960	3,149.60000
1	2,496.71417	1	2,660.75583	1	2,824.79750	1	2,988.83917	1	3,152.88083
2	2,499.99500	2	2,664.03667	2	2,828.07833	2	2,992.12000	2	3,156.16167
3	2,503.27583	3	2,667.31750	3	2,831.35917	3	2,995.40083	3	3,159.44250
4	2,506.55667	4	2,670.59833	4	2,834.64000	4	2,998.68167	4	3,162.72333
5	2,509.83750	5	2,673.87917	5	2,837.92083	5	3,001.96250	5	3,166.00417
6	2,513.11833	6	2,677.16000	6	2,841.20167	6	3,005.24333	6	3,169.28500
7	2,516.39917	7	2,680.44083	7	2,844.48250	7	3,008.52417	7	3,172.56583
8	2,519.68000	8	2,683.72167	8	2,847.76333	8	3,011.80500	8	3,175.84667
9	2,522.96083	9	2,687.00250	9	2,851.04417	9	3,015.08583	9	3,179.12750
770	2,526.24167	820	2,690.28333	870	2,854.32500	920	3,018.36667	970	3,182.40833
1	2,529.52250	1	2,693.56417	1	2,857.60583	1	3,021.64750	1	3,185.68917
2	2,532.80333	2	2,696.84500	2	2,860.88667	2	3,024.92833	2	3,188.97000
3	2,536.08417	3	2,700.12583	3	2,864.16750	3	3,028.20917	3	3,192.25083
4	2,539.36500	4	2,703.40667	4	2,867.44833	4	3,031.49000	4	3,195.53167
5	2,542.64583	5	2,706.68750	5	2,870.72917	5	3,034.77083	5	3,198.81250
6	2,545.92667	6	2,709.96833	6	2,874.01000	6	3,038.05167	6	3,202.09333
7	2,549.20750	7	2,713.24917	7	2,877.29083	7	3,041.33250	7	3,205.37417
8	2,552.48833	8	2,716.53000	8	2,880.57167	8	3,044.61333	8	3,208.65500
9	2,555.76917	9	2,719.81083	9	2,883.85250	9	3,047.89417	9	3,211.93583
780	2,559.05000	830	2,723.09167	880	2,887.13333	930	3,051.17500	980	3,215.21667
1	2,562.33083	1	2,726.37250	1	2,890.41417	1	3,054.45583	1	3,218.49750
2	2,565.61167	2	2,729.65333	2	2,893.69500	2	3,057.73667	2	3,221.77833
3	2,568.89250	3	2,732.93417	3	2,896.97583	3	3,061.01750	3	3,225.05917
4	2,572.17333	4	2,736.21500	4	2,900.25667	4	3,064.29833	4	3,228.34000
5	2,575.45417	5	2,739.49583	5	2,903.53750	5	3,067.57917	5	3,231.62083
6	2,578.73500	6	2,742.77667	6	2,906.81833	6	3,070.86000	6	3,234.90167
7	2,582.01583	7	2,746.05750	7	2,910.09917	7	3,074.14083	7	3,238.18250
8	2,585.29667	8	2,749.33833	8	2,913.38000	8	3,077.42167	8	3,241.46333
9	2,588.57750	9	2,752.61917	9	2,916.66083	9	3,080.70250	9	3,244.74417
790	2,591.85833	840	2,755.90000	890	2,919.94167	940	3,083.98333	990	3,248.02500
1	2,595.13917	1	2,759.18083	1	2,923.22250	1	3,087.26417	1	3,251.30583
2	2,598.42000	2	2,762.46167	2	2,926.50333	2	3,090.54500	2	3,254.58667
3	2,601.70083	3	2,765.74250	3	2,929.78417	3	3,093.82583	3	3,257.86750
4	2,604.98167	4	2,769.02333	4	2,933.06500	4	3,097.10667	4	3,261.14833
5	2,608.26250	5	2,772.30417	5	2,936.34583	5	3,100.38750	5	3,264.42917
6	2,611.54333	6	2,775.58500	6	2,939.62667	6	3,103.66833	6	3,267.71000
7	2,614.82417	7	2,778.86583	7	2,942.90750	7	3,106.94917	7	3,270.99083
8	2,618.10500	8	2,782.14667	8	2,946.18833	8	3,110.23000	8	3,274.27167
9	2,621.38583	9	2,785.42750	9	2,949.46917	9	3,113.51083	9	3,277.55250

Corrections to log s and log Δλ for difference in arc and sine

Log s (−)	Log difference (units of eighth decimal place)	Log Δλ (+)	Log s (−)	Log difference (units of eighth decimal place)	Log Δλ (+)	Log s (−)	Log difference (units of eighth decimal place)	Log Δλ (+)
3.3760	1	1.8850	4.8270	799	3.3360	5.1780	4025	3.6870
3.5260	2	2.0350	4.8380	841	3.3470	5.1830	4119	3.6920
3.6140	3	2.1230	4.8500	889	3.3590	5.1880	4215	3.6970
3.6770	4	2.1860	4.8620	939	3.3710	5.1940	4333	3.7030
3.7250	5	2.2340	4.8710	979	3.3800	5.1990	4434	3.7080
3.7650	6	2.2740	4.8820	1030	3.3910	5.2040	4537	3.7130
3.7980	7	2.3070	4.8920	1078	3.4010	5.2090	4643	3.7180
3.8270	8	2.3360	4.9040	1140	3.4130	5.2140	4751	3.7230
3.8530	9	2.3620	4.9130	1188	3.4220	5.2190	4862	3.7280
3.8760	10	2.3850	4.9220	1238	3.4310	5.2240	4975	3.7330
4.0260	20	2.5350	4.9330	1303	3.4420	5.2290	5091	3.7380
4.1140	30	2.6230	4.9420	1358	3.4510	5.2330	5186	3.7420
4.1770	40	2.6860	4.9520	1422	3.4610	5.2380	5306	3.7470
4.2250	50	2.7340	4.9590	1468	3.4680	5.2420	5405	3.7510
4.2650	60	2.7740	4.9680	1530	3.4770	5.2470	5531	3.7560
4.2980	70	2.8070	4.9780	1603	3.4870	5.2520	5660	3.7610
4.3270	80	2.8360	4.9860	1663	3.4950	5.2560	5765	3.7650
4.3530	90	2.8620	4.9930	1717	3.5020	5.2600	5872	3.7690
4.3760	100	2.8850	5.0020	1790	3.5110	5.2650	6009	3.7740
4.3960	110	2.9050	5.0100	1857	3.5190	5.2690	6121	3.7780
4.4150	120	2.9240	5.0170	1918	3.5260	5.2740	6263	3.7830
4.4330	130	2.9420	5.0250	1990	3.5340	5.2780	6380	3.7870
4.4490	140	2.9580	5.0330	2064	3.5420	5.2820	6498	3.7910
4.4640	150	2.9730	5.0400	2132	3.5490	5.2860	6619	3.7950
4.4780	160	2.9870	5.0480	2212	3.5570	5.2900	6742	3.7990
4.4910	170	3.0000	5.0550	2285	3.5640	5.2940	6868	3.8030
4.5030	180	3.0120	5.0620	2359	3.5710	5.2990	7027	3.8080
4.5260	200	3.0350	5.0680	2425	3.5770	5.3030	7158	3.8120
4.5560	230	3.0650	5.0750	2505	3.5840	5.3070	7291	3.8160
4.5750	250	3.0840	5.0820	2587	3.5910	5.3110	7427	3.8200
4.5910	270	3.1000	5.0890	2672	3.5980	5.3150	7565	3.8240
4.6140	300	3.1230	5.0950	2747	3.6040	5.3190	7705	3.8280
4.6350	330	3.1440	5.1020	2837	3.6110	5.3230	7849	3.8320
4.6540	360	3.1630	5.1080	2916	3.6170	5.3270	7995	3.8360
4.6710	390	3.1800	5.1140	2998	3.6230	5.3310	8143	3.8400
4.6870	420	3.1960	5.1210	3096	3.6300	5.3350	8295	3.8440
4.7020	450	3.2110	5.1270	3183	3.6360	5.3390	8449	3.8480
4.7160	480	3.2250	5.1330	3272	3.6420	5.3430	8606	3.8520
4.7340	521	3.2430	5.1390	3364	3.6480	5.3470	8766	3.8560
4.7500	561	3.2590	5.1450	3458	3.6540			
4.7610	590	3.2700	5.1500	3538	3.6590			
4.7750	629	3.2840	5.1560	3637	3.6650			
4.7890	671	3.2980	5.1610	3722	3.6700			
4.8010	709	3.3100	5.1670	3826	3.6760			
4.8130	750	3.3220	5.1720	3916	3.6810			

FACTORS USED IN THE COMPUTATION OF ELEVATIONS FROM RECIPROCAL AND NONRECIPROCAL OBSERVATIONS

The unit of length throughout these tables is the meter.

Log A

Elevation of occupied station h_1	Log *A* units of fifth place	Elevation of occupied station h_1	Log *A* units of fifth place
Meters		*Meters*	
0	0.0	3009	20.5
73	.5	3156	21.5
220	1.5	3303	22.5
367	2.5	3449	23.5
514	3.5	3596	24.5
661	4.5	3743	25.5
807	5.5	3890	26.5
954	6.5	4036	27.5
1101	7.5	4183	28.5
1248	8.5	4330	29.5
1394	9.5	4477	30.5
1541	10.5	4624	31.5
1688	11.5	4770	32.5
1835	12.5	4917	33.5
1982	13.5	5064	34.5
2128	14.5	5211	35.5
2275	15.5	5357	36.5
2422	16.5	5504	37.5
2569	17.5	5651	38.5
2715	18.5	5798	39.5
2862	19.5	5945	40.5

Log B and log C

Log approximate difference of elevation $=\log s \tan\left(\frac{\zeta_2-\zeta_1}{2}\right)$*	Log *B* units of 5th place	Log *s*	Log *C*
	0.0		0.0
2.167	.5	4.875	.5
2.644	1.5	5.113	1.5
2.866	2.5	5.224	2.5
3.011	3.5	5.297	3.5
3.121	4.5	5.352	4.5
3.208	5.5	5.395	5.5
3.281	6.5	5.432	6.5
3.343	7.5	5.463	7.5
3.397	8.5		
3.445	9.5		
3.489	10.5		
3.528	11.5		
3.565	12.5		
3.598	13.5		
3.629	14.5		
3.658	15.5		
3.685	16.5		
3.711	17.5		
3.735	18.5		
3.758	19.5		
3.779	20.5		
3.800	21.5		
3.820	22.5		
3.839	23.5		
3.857	24.5		
3.874	25.5		

*Or $\log s \cot\left[\zeta_1-(0.5-m)\frac{s}{\rho \sin 1''}\right]$ for nonreciprocal observations.

Log *B* has the same sign as the approximate difference of elevation. Log *C* is always positive.

COMPUTATION OF SPHERICAL EXCESS

The spherical excess of a triangle is computed by the formula,

$$\epsilon=\frac{a_1 b_1 \sin C_1(1-e^2 \sin^2 \phi)^2}{2a^2(1-e^2)\sin 1''}=a_1 b_1 \sin C_1 \times m.$$

In this formula ϵ is the spherical excess; a_1 b_1 and C_1 are two sides and the included angle, respectively, of the corresponding triangle; e^2 is the square of the eccentricity, and a the major semiaxis of the spheriod of reference; and ϕ is the mean latitude of the three vertices of the triangle. That part of the above expression which depends only on the latitude and the dimensions of the spheroid may be designated by a single letter, m, as shown. In the following table the logarithms of m are given with the latitude as an argument.

The above formula gives the spherical excess too small by one one-hundredth of a second for an equilateral triangle with 200-kilometer sides, or for a nonequilateral triangle of the same area. For an equilateral triangle of 100-kilometer sides, or an equivalent nonequilateral triangle, the excess as given by this formula is too small by less than one one-thousandth of a second.

In cases where a more accurate value of the spherical excess is required the formulas given on page 51 of Special Publication No. 4, The Transcontinental Triangulation, may be used. These formulas give a slightly unequal distribution of the spherical excess among the three angles of the triangle.

Table of log m

[Computed for the Clarke spheroid of 1866 as expressed in meters]

Latitude	Log *m*	Latitude	Log *m*	Latitude	Log *m*	Latitude	Log *m*
° ′		° ′		° ′		° ′	
0 00	1.40695 −10	20 00	1.40626 −10	40 00	1.40452 −10	60 00	1.40253 −10
0 30	695 −10	20 30	623 −10	40 30	446 −10	60 30	249 −10
1 00	695 −10	21 00	619 −10	41 00	441 −10	61 00	244 −10
1 30	694 −10	21 30	616 −10	41 30	436 −10	61 30	240 −10
2 00	694 −10	22 00	612 −10	42 00	431 −10	62 00	235 −10
2 30	694 −10	22 30	608 −10	42 30	426 −10	62 30	231 −10
3 00	693 −10	23 00	605 −10	43 00	421 −10	63 00	227 −10
3 30	693 −10	23 30	601 −10	43 30	416 −10	63 30	223 −10
4 00	692 −10	24 00	597 −10	44 00	411 −10	64 00	219 −10
4 30	691 −10	24 30	594 −10	44 30	406 −10	64 30	215 −10
5 00	690 −10	25 00	590 −10	45 00	400 −10	65 00	210 −10
5 30	689 −10	25 30	586 −10	45 30	395 −10	65 30	207 −10
6 00	688 −10	26 00	582 −10	46 00	390 −10	66 00	203 −10
6 30	687 −10	26 30	578 −10	46 30	385 −10	66 30	199 −10
7 00	686 −10	27 00	573 −10	47 00	380 −10	67 00	195 −10
7 30	685 −10	27 30	569 −10	47 30	375 −10	67 30	192 −10
8 00	683 −10	28 00	565 −10	48 00	369 −10	68 00	188 −10
8 30	682 −10	28 30	560 −10	48 30	364 −10	68 30	185 −10
9 00	680 −10	29 00	556 −10	49 00	359 −10	69 00	181 −10
9 30	679 −10	29 30	552 −10	49 30	354 −10	69 30	178 −10
10 00	677 −10	30 00	548 −10	50 00	349 −10	70 00	174 −10
10 30	675 −10	30 30	544 −10	50 30	344 −10	70 30	171 −10
11 00	673 −10	31 00	539 −10	51 00	339 −10	71 00	168 −10
11 30	671 −10	31 30	534 −10	51 30	334 −10	71 30	164 −10
12 00	669 −10	32 00	530 −10	52 00	329 −10	72 00	1.40161 −10
12 30	667 −10	32 30	525 −10	52 30	324 −10		
13 00	665 −10	33 00	520 −10	53 00	319 −10		
13 30	663 −10	33 30	516 −10	53 30	314 −10		
14 00	660 −10	34 00	511 −10	54 00	309 −10		
14 30	658 −10	34 30	506 −10	54 30	304 −10		
15 00	655 −10	35 00	501 −10	55 00	299 −10		
15 30	653 −10	35 30	496 −10	55 30	295 −10		
16 00	650 −10	36 00	491 −10	56 00	290 −10		
16 30	647 −10	36 30	486 −10	56 30	285 −10		
17 00	644 −10	37 00	482 −10	57 00	280 −10		
17 30	642 −10	37 30	477 −10	57 30	276 −10		
18 00	639 −10	38 00	472 −10	58 00	271 −10		
18 30	636 −10	38 30	467 −10	58 30	266 −10		
19 00	632 −10	39 00	462 −10	59 00	262 −10		
19 30	1.40629 −10	39 30	1.40457 −10	59 30	1.40257 −10		

COMPUTATION OF STRENGTH OF FIGURE

In the following table the values tabulated are $[\delta_A^2+\delta_A\delta_B+\delta_B^2]$. The unit is one in the sixth place of logarithms. The two arguments of the table are the length angles in degrees, the smaller length angle being given at the top of the table. The length angles are the angles in each triangle opposite the known side and the side required. δ_A and δ_B are the logarithmic differences corresponding to 1 second for the length angles A and B of a triangle.

Table for determining relative strength of figures in triangulation

°	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°
10	428	359																					
12	359	295	253																				
14	315	253	214	187																			
16	284	225	187	162	143																		
18	262	204	168	143	126	113																	
20	245	189	153	130	113	100	91																
22	232	177	142	119	103	91	81	74															
24	221	167	134	111	95	83	74	67	61														
26	213	160	126	104	89	77	68	61	56	51													
28	206	153	120	99	83	72	63	57	51	47	43												
30	199	148	115	94	79	68	59	53	48	43	40	33											
35	188	137	106	85	71	60	52	46	41	37	33	27	23										
40	179	129	99	79	65	54	47	41	36	32	29	23	19	16									
45	172	124	93	74	60	50	43	37	32	28	25	20	16	13	11								
50	167	119	89	70	57	47	39	34	29	26	23	18	14	11	9	8							
55	162	115	86	67	54	44	37	32	27	24	21	16	12	10	8	7	5						
60	159	112	83	64	51	42	35	30	25	22	19	14	11	9	7	5	4	4					
65	155	109	80	62	49	40	33	28	24	21	18	13	10	7	6	5	4	3	2				
70	152	106	78	60	48	38	32	27	23	19	17	12	9	7	5	4	3	2	2	1			
75	150	104	76	58	46	37	30	25	21	18	16	11	8	6	4	3	2	2	1	1	1		
80	147	102	74	57	45	36	29	24	20	17	15	10	7	5	4	3	2	1	1	1	0	0	
85	145	100	73	55	43	34	28	23	19	16	14	10	7	5	3	2	2	1	1	0	0	0	0
90	143	98	71	54	42	33	27	22	19	16	13	9	6	4	3	2	1	1	1	0	0	0	0
95	140	96	70	53	41	32	26	22	18	15	13	9	6	4	3	2	1	1	0	0	0	0	
100	138	95	68	51	40	31	25	21	17	14	12	8	6	4	3	2	1	1	0	0	0		
105	136	93	67	50	39	30	25	20	17	14	12	8	5	4	2	2	1	1	0	0			
110	134	91	65	49	38	30	24	19	16	13	11	7	5	3	2	2	1	1	1				
115	132	89	64	48	37	29	23	19	15	13	11	7	5	3	2	2	1	1					
120	129	88	62	46	36	28	22	18	15	12	10	7	5	3	2	2	1						
125	127	86	61	45	35	27	22	18	14	12	10	7	5	4	3	2							
130	125	84	59	44	34	26	21	17	14	12	10	7	5	4	3								
135	122	82	58	43	33	26	21	17	14	12	10	7	5	4									
140	119	80	56	42	32	25	20	17	14	12	10	8	6										
145	116	77	55	41	32	25	21	17	15	13	11	9											
150	112	75	54	40	32	26	21	18	16	15	13												
152	111	75	53	40	32	26	22	19	17	16													
154	110	74	53	41	33	27	23	21	19														
156	108	74	54	42	34	28	25	22															
158	107	74	54	43	35	30	27																
160	107	74	56	45	38	33																	
162	107	76	59	48	42																		
164	109	79	63	54																			
166	113	86	71																				
168	122	98																					
170	143																						

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HOW TO USE THE TABLE

To compare with each other two alternative figures, either quadrilaterals or central-point figures, in so far as the strength with which the length is carried is concerned, proceed as follows:

(*a*) For each figure take out the length angles, to the nearest degree, for the best and second-best chains of triangles through the figure. These chains are to be selected at first by estimation, and the estimate is to be checked later by the results of comparison.

(*b*) For each triangle in each chain enter the table with the length angles as the two arguments and take out the tabular value.

(*c*) For each chain, the best and second best, through each figure, take the sum of the tabular values.

(*d*) Multiply each sum by the factor $\frac{D-C}{D}$ for that figure, where D is the number of directions observed and C is the number of conditions to be satisfied in the figure. The quantities so obtained, namely, $\frac{D-C}{D} \Sigma\ [\delta_A^2+\delta_A\delta_B+\delta_B^2]$, will for convenience be called R_1 and R_2 for the best and second-best chains, respectively. (Examples of various triangulation figures with the corresponding values of R_1 and R_2 may be found in Special Publication No. 93, pp. 8–12.)

(*e*) The strength of the figure is dependent mainly upon the strength of the best chain through it, hence the smaller the R_1 the greater the strength of the figure. The second-best chain contributes somewhat to the total strength, and the other weaker and progressively less independent chains contribute still smaller amounts. In deciding between figures they should be classed according to their best chains, unless said best chains are very nearly of equal strength and their second-best chains differ greatly.

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