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DANIEL C. ROPER, Secretary

COAST AND GEODETIC SURVEY
R. S. PATTON, Director

MANUAL OF SECOND AND THIRD ORDER TRIANGULATION AND TRAVERSE

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PREFACE

The methods described in this publication are the result of gradual changes extending over many years and are due to the suggestions and investigations of many persons. No attempt has been made to indicate the sources of all these changes or the reasons for them.

Assistance in the preparation of this manual has been given by a large number of the engineers and mathematicians of the Coast and Geodetic Survey. Particular mention should be made of W. F. Reynolds, senior mathematician, and W. D. Sutcliffe, associate mathematician, who prepared the rough draft of the sections on the field computations for triangulation and traverse, respectively. C. H. Swick, senior mathematician, rendered invaluable assistance in arranging and correcting the text and supplying general criticisms of the material.

CONTENTS

	Page
Preface	II
General statement	1
Classification of control surveys	2
Requirements for horizontal control	3
CHAPTER 1.—RECONNAISSANCE	
Specifications for reconnaissance for second-order triangulation	6
Strength of figure	7
Factors for determining strength of figure	8
Computation of strength of figure	9
Some values of the quantity $\frac{D-C}{D}$	9
Examples of various triangulation figures	11
Determination of C and D in strength of figure formula	16
Overlapping of figures	17
Other considerations in reconnaissance	18
Length of lines of triangulation	18
Base sites and base nets	18
Supplementary stations	19
Modifications of specifications	19
Public value of control surveys	20
Connections to existing triangulation	20
Intervisibility of stations	21
Correction for earth's curvature and refraction	21
Computation for the determination of the intervisibility of stations	22
Selecting sites for triangulation stations	26
Signals and signal building	27
Pole signal	28
Tripod signal	29
Other types of signals	31
Reconnaissance for third-order triangulation	31
CHAPTER 2.—SECOND AND THIRD ORDER TRIANGULATION	
Specifications, second-order triangulation	33
Initial settings for 2-micrometer theodolite	34
Marking of stations	37
Triangulation and traverse marks of other organizations	37
Specifications for marking triangulation stations	38
Horizontal angle measurement	41
Instrumental procedure, repetition method	41
Instrumental procedure, direction method	42
Program for observing vertical angles	43
Correction of records	43
Instruments	44
Theodolites	44
Theodolite adjustments	46
Discussion of adjustments	60
Wild theodolite	60
Care of the theodolite	61
Tests to determine the quality of a theodolite	65
Vertical collimator	69
Instruments and apparatus for light keepers	70
Principal sources of error in horizontal-angle measurements	72
Instrumental errors	75
Phase and eccentricity	76
Horizontal refraction	77
Other conditions affecting observations	78
Naming of stations	79
Courtesy to property owners and others	80
Determination of value of one division of a level bubble	80
Determination of height of station by observing sea horizon	81
Determination of distance to breaker by observing angle of depression	82
Specifications, third-order triangulation	83
Field computations, second and third order triangulation	83
Record, horizontal angles (repeating theodolite)	84
Record, horizontal directions (direction theodolite)	86
Abstract of horizontal directions (direction theodolite)	87
List of directions	89
Reduction to center	91
Triangle-side computation	94
Mathematical solution of the three-point problem	98
Position computation	100
List of geographic positions	100
Record, double zenith distances	104
Abstract of zenith distances	106
Computation of elevations from observations of zenith distances	107
Description of stations	110
Season's report	115
Records of fourth-order control	116
CHAPTER 3.—SECOND AND THIRD ORDER BASE MEASUREMENT	
Specifications, second-order bases	117
Preparation of base	118
Alignment	118
Staking	119
Procedure in staking	120
Measurement of base	122
Instruments and appliances	122
Handling and testing of base apparatus	122
Corrections to measured lengths and precautions against errors	125
Grade correction	125
Alignment correction	126
Corrections due to sag of tape and to stretching	126
Corrections due to erroneous tension	127
Corrections for method of support of tape and for change in weight	128
Standardization data	129
Friction over supports	130
Temperature correction	130
Parallax in marking	131
Wind effect	131
Blunders	131

	Page
Measuring with tapes	132
Personnel and duties	132
Forms for records and computations	139
Probable error of measurement	144
Reduction to sea level	145
Third-order base measurement	146

CHAPTER 4.—SECOND AND THIRD ORDER TRAVERSE

Specifications, second-order traverse	148
Signal building	151
Organization of party	151
Subparties	151
Measurement with invar tapes	152
Field comparisons	153
Details of measurement	153
Support of tape	154
Tension	155
Temperature	156
Care of tapes	156
Set-ups, setbacks, and offset distances	156
Invar-tape record	157
Check measurement of distances	158
Measurements along highways and city streets	159
Levels	159
Angle measurements	160
Main traverse stations	160
Observations with micrometer direction theodolite	161
Observations with repeating theodolite	161
Conditions affecting the accuracy of observations	162
Intersection stations	162
Eccentric stations	162
Subsidiary stations	163
Azimuth observations on second-order traverse	164
Third-order traverse	164
Specifications, third-order traverse	164
Field records and computations, second and third-order traverse	165
Traverse sketches on computation sheets	166
Closure and projection of loops	168
Computation of geographic positions	170
Distribution of discrepancies	171

CHAPTER 5.—ASTRONOMIC AZIMUTHS

	Page
Classification of azimuths	173
Method of observing a second-order azimuth	174
Azimuth observations with direction theodolite	175
Azimuth observations with repeating vernier theodolite	177
Azimuth mark	177
Recorder's duties	177
When to observe Polaris	179
Chronometer correction	180
Errors in time observations	182
Third-order azimuth	183

CHAPTER 6.—CONSTANTS, FORMULAS, AND TABLES

Constants and formulas	184
Tables	186
Differences of elevation and inclination corrections for varying angles of inclination	186
Inclination corrections for 50-meter tape lengths	187
Inclination corrections for 25-meter lengths	190
Inclination corrections for 5, 10, 15, 20, 25, 30, 35, 40, and 45 meter lengths	193
Factors for computing catenary correction	196
Catenary corrections for various lengths and weights of tape	196
Temperature corrections for steel tapes	198
Table of log m	199
Arc-sin corrections for inverse position computations	200
Proportional change in a number corresponding to a change in its logarithm	201
Logarithms of radii of curvature of the earth's surface (in meters)	202
Factors used in the computation of elevations from reciprocal and nonreciprocal observations	205
Length of 1 degree of the meridian at different latitudes	206
Length of 1 degree of the parallel at different latitudes	207
Conversion tables (feet to meters and meters to feet)	208
Publications of the Coast and Geodetic Survey relating to field methods	216
Index	218

ILLUSTRATIONS

Fig.	Page
1 to 14. Triangulation figures	11–16
15. Quadrilateral to illustrate C	16
16. Overlapping of figures	17
17. Excess of observed lines	17
18. Intervisibility of stations across water, case (*a*)	22
19. Intervisibility of stations across water, case (*b*)	22
20. Intervisibility of stations across wooded plain, case (*a*)	23
21. Intervisibility of stations across wooded plain, case (*b*)	23
22. Intervisibility of stations in mountain country	24
23. Intervisibility of stations in mountain country, solution (*a*)	25
24. Intervisibility of stations in mountain country, solution (*b*)	25
25. Intervisibility of stations in mountain country, solution (*c*)	26
25a. Determining the approximate height of a tree on reconnaissance	27
26. Signal 235 feet high built of poles	28
27. Tree signal 143 feet high	28
28. Diagram of pole signal, with target	28
29. Tripod stand for theodolite	28
30. Small signal lamp with target centered above	29

Fig.		Page
31.	Tripod-and-pole signal	30
32.	Diagram showing construction of tripod-and-pole signal	30
33.	Steel triangulation tower, 77 feet high to *top of inner tripod*	31
34.	Standard marks of the United States Coast and Geodetic Survey	38
35.	Triangulation station mark	39
36.	Repeating vernier theodolite, 7-inch circle	44
37.	A 4½-inch theodolite with vernier microscopes	44
38.	Micrometer theodolite, 6¼-inch	44
39.	Sectional view of Parkhurst 9-inch direction theodolite	45
40.	Wild prism-microscope direction theodolite, 5½-inch	45
41.	Device for holding theodolite firmly to stand	46
42.	Diagram of pressure clamp for theodolite	47
43.	Micrometer box, compression spring type	54
44.	Apparent arrangement of comb, wires, and graduation marks	54
45.	Micrometer box, tension spring type	54
46.	Graph of readings, *A* micrometer minus *B* micrometer	67
47.	Vertical collimator, old type	70
48.	Vertical collimator, new type	71
49.	Heliotrope, box type	72
50.	Emergency heliotrope	72
51.	Automatic signal lamp	73
52.	Specimen, record horizontal angles	85
53.	Specimen, record horizontal angles, intersection stations	85
54.	Specimen, record horizontal directions	86
55.	Specimen, abstract of directions	87
56.	Specimen, list of directions	90
57.	Specimen, reduction to center	92
58.	Nomogram for reduction to center	93
59.	Diagram, quadrilateral	94
60.	List of directions for two stations of quadrilateral	95
61.	Specimen, computation of triangles	97
62.	Specimen, computation of three-point problem	99
63.	Specimen, position computation (2 pages)	101
64.	Specimen, inverse position computation	103
65.	Specimen, list of geographic positions	103
66.	Specimen, record double zenith distances	105
67.	Specimen, abstract of zenith distances	106
68.	Specimen, computation of elevations from reciprocal observations	107
69.	Specimen, computation of elevations from nonreciprocal observations	109
70.	Specimen, description of station	111
71.	Specimen, recovery note	114
72.	Tape stretcher and spring balance	124
73.	Sample certificate of tape standardization	129
74.	Making rear contact with tape	134
75.	Making forward contact with tape	134
76.	Portable iron tripod for tape support, single joint	135
77.	Portable iron tripod for tape support, double joint	135
78.	Nomogram of errors due to incorrect reductions for grade	136
79.	Microphotograph of defaced mark on tape	138
80.	Specimen, record of tape measurements	140
81.	Specimen, record of wye leveling	141
82.	Specimen, abstract of wye levels	142
83.	Specimen, computation of base line	143
84.	Traverse station at intersection of two tangents	148
85.	Traverse station on extension of one tangent	148
86.	Traverse loop	149
87.	Connection between triangulation and traverse by quadrilateral	149
88.	Connection between triangulation and traverse by two triangles	150
89.	Type of tape stretcher used on traverse along railroad	156
90.	Combination of traverse and triangulation	163
91.	Traverse sketch, direct measurement between stations	167
92.	Specimen, projection computation, one offset station	167
93.	Specimen, projection computation, stations on same side of track	168
94.	Specimen, projection computation, stations on opposite sides of track	168
95.	Specimen, loop closure and projection computation	169
96.	Specimen, observations on stars for time	174
97.	Specimen, observations on Polaris for azimuth, direction theodolite	176
98.	Specimen, observations on Polaris for azimuth, repetition method	178

MANUAL OF SECOND AND THIRD ORDER TRIANGULATION AND TRAVERSE

By C. V. Hodgson, *Hydrographic and Geodetic Engineer, United States Coast and Geodetic Survey*

GENERAL STATEMENT

Instructions and specifications for second-order triangulation and traverse have heretofore been available in abbreviated form only. Since most of the coastal control surveys in the future will be of second-order accuracy, this manual will supersede the instructions for third-order triangulation contained in Special Publication No. 26, General Instructions for Field Work. Condensed specifications for third-order control surveys will be found in this publication at the end of the chapters on reconnaissance, triangulation, base measurement, and traverse.

The purpose of this publication is to provide a manual on second-order horizontal control surveys which will conform closely to the modern requirements and approved practices of the Coast and Geodetic Survey. The only discussions of theoretical principles which have been included are those necessary to a proper understanding of field methods. The manual is not designed to take the place of textbooks teaching the basic principles of control surveys but is intended to supplement them.

Until recent years the terms "primary," "secondary," and "tertiary" were used, in order of decreasing accuracy, to designate the principal grades of triangulation and traverse used on Government surveys. This led to considerable confusion, for on any extended piece of triangulation the more accurate class was usually called "primary," irrespective of the degree of accuracy obtained, while the subsidiary schemes were usually classed as "secondary."

In order to secure more uniform specifications and nomenclature, representatives of the various Federal map-making and map-using organizations in Washington in 1921 agreed upon a uniform classification. Under this agreement triangulation of the highest accuracy was termed "precise," the next order "primary," and the third grade was called "secondary," corresponding, respectively, to what had previously been known in the Coast and Geodetic Survey as primary, secondary, and tertiary.

This terminology did not find wide favor or acceptance, and in May, 1925, the Federal Board of Surveys and Maps, after referring the matter to various Federal map-making bureaus represented on the board, recommended that the four grades of vertical and horizontal control ordinarily used be designated as first order, second order, third order, and fourth order, respectively, the first named being the most accurate. These terms have been authorized by the Director of the Coast and Geodetic Survey and made applicable to the various grades of control executed by this bureau. These terms have also been adopted by the committee on triangulation of the Section of Geodesy of the International Geodetic and Geophysical Union, so that no doubt there will soon be international concurrence in this terminology.

The national scheme of control surveys approved by the Federal Board of Surveys and Maps will, when completed, consist of a framework of intersecting belts of first-order triangulation or traverse about 100 miles apart. Second-order triangulation or traverse will subdivide the intermediate spaces, so that no considerable area will be farther than about 25 miles from a horizontal control point of the first or second order. Horizontal control of the third order would then be established over the entire area to be surveyed, with a density of distribution of points depending upon the requirements of the detailed surveys and the nature of the terrain.

CLASSIFICATION OF CONTROL SURVEYS

The basis of classification of control surveys is the accuracy with which the length and azimuth of a line of the triangulation or traverse are determined. Since it is impossible to ascertain the absolute error in the determination of the length or azimuth of each line of triangulation or traverse, indirect gauges must be used. On triangulation the principal criterion is that the discrepancy between the measured length of a base line and its length as computed through the scheme from the next preceding base shall not, after the side and angle equations have been satisfied, be greater than 1 part in 25,000 of the length of the base for first-order work, 1 part in 10,000 for second order, and 1 part in 5,000 for third order. Similar ratios are prescribed for the error of closure in position of traverse of the corresponding order of accuracy. Coupled with this gauge of the length agreement between bases and almost coordinate in importance are the requirements limiting the error of angle measurements, for the limits imposed on angular errors serve to maintain a uniform accuracy along the chain of triangles.

The specifications for procuring a required accuracy make use of other criteria, such as the number and strength of the geometrical figures between adjacent bases, the observation of an astronomic azimuth at specified intervals, and the accuracy of measurement of base lines. All these tests are subsidiary to the controlling tests of the agreement between the measured and computed length of a base and the limits specified for angle errors, even though they are essential in securing a sustained accuracy for the control lines.

A comparison of the three grades of horizontal control ordinarily used by this bureau is readily obtained from the following table, which shows the limits for the principal items of the specifications for these grades:

Requirements for horizontal control

TRIANGULATION

	First order	Second order	Third order
Strength of figures:			
Desirable limit, ΣR_1 between bases	80	100	125
Maximum limit, ΣR_1 between bases	110	130	175
Desirable limit, R_1, single figure	15	25	25
Maximum limit, R_1, single figure	25	40	50
Discrepancy between computed length and measured length of base or adjusted length of check line, not to exceed	1 in 25,000	1 in 10,000	1 in 5,000
Triangle closure:			
Average, not to exceed	1 sec.	3 sec.	6 sec.
Maximum, not to exceed	3 sec.	8 sec.	12 sec.
Usual number of observations:			
Positions with 1-second direction theodolite	16	4	2
Positions with 2-second direction theodolite	20 to 24	4 to 8	2 to 4
Sets with 10-second repeating theodolite	5 to 6	2 to 3	1 to 2
Base measurement:			
Actual error of base not to exceed	1 in 300,000	1 in 150,000	1 in 75,000
Probable error of base not to exceed	1 in 1,000,000	1 in 500,000	1 in 250,000
Discrepancy between 2 measures of a section, not to exceed	10 mm. $\sqrt{k}$	20 mm. $\sqrt{k}$	25 mm. $\sqrt{k}$
Astronomic azimuth probable error of result	0.5 sec.	2.0 sec.	5.0 sec.

TRAVERSE

	First order	Second order	Third order
Closing error in position, not to exceed	1 in 25,000	1 in 10,000	1 in 5,000
Probable error of main scheme angles	1.5 sec.	3.0 sec.	6.0 sec.
Number of stations between astronomical azimuths	10 to 15	15 to 25	20 to 35
Astronomic azimuth, discrepancy per main angle station, not to exceed	1.0 sec.	2.0 sec.	5.0 sec.
Astronomic azimuth, probable error of result	0.5 sec.	2.0 sec.	5.0 sec.

For many years the Coast and Geodetic Survey controlled its coastal hydrographic and topographic surveys by triangulation and traverse of third-order (formerly called tertiary) accuracy. Horizontal control along the coast is indispensable for hydrographic surveys, because the method of determining the position of a vessel while making soundings is a form of triangulation, and unless the signals on shore are accurately located it is impossible to determine such positions at a distance from shore with any degree of accuracy.

In addition, the horizontal control along the coast is of great importance from a national standpoint, for the zone between the storm-water line and the low-water line is the field of numerous legal controversies regarding jurisdiction and property rights, and all surveys of this marginal belt must be accurately made. In recognition of this fact and of the increasing economic importance of the coastal area this bureau has within recent years gradually increased the accuracy of the control surveys of those regions, though they are still specified as of third-order accuracy for most of the coastal control.

The fact that with modern instruments and methods second-order accuracy can be secured almost as cheaply as third order led the Director of the Coast and Geodetic Survey in the spring of 1928 to decide that in the future all main-scheme coastal triangulation and traverse should be of second-order accuracy, except in those regions where first-order control is available. Third-order triangulation and traverse will be used for subsidiary schemes where connections to first or second order control are made every 40 to 50 miles.

For second-order control the standards of accuracy stated previously need some further explanation. A base line may mean either a measured base or a line of adjusted first-order triangulation. The check on the base of 1 part in 10,000 or better means that the length of the base line, as computed through the triangulation from the preceding base, must agree with the measured or adjusted length within 1 part in 10,000 *after the side and angle equations have been satisfied* by the method of least squares. Ordinarily, the field computation of the length discrepancy through the triangles is a sufficiently accurate guide, since an adjustment of the lengths of the sides and angles by the least-squares method usually reduces the discrepancy between the computed and measured lengths of the forward base line.

Second-order traverse will usually begin and end upon adjusted triangulation of second or higher order or upon adjusted traverse of first or second order. The distance by which the traverse fails to close on the adjusted control position should not exceed one ten-thousandth part of the total length of the unadjusted traverse unless there is reason to believe that there may be considerable error in the adjusted work to which connection is being made.

It will be noticed that the standards of accuracy prescribed on page 3 apply only to the field observations. Other standards are used for the adjusted work. The process of adjusting observations by the method of least squares makes the results consistent throughout but does not remove all errors. If the observational errors are small and are indiscriminately plus and minus, then the adjustment will distribute them so that there will be but a slight accumulation of errors; or, if the accumulation of error in length between bases, or in azimuth between Laplace[1] stations, is fairly constant in amount and direction, then the adjustment will distribute the errors approximately where they belong. Blunders, large accidental errors, and systematic errors of varying signs are not distributed correctly by the adjustment process.

Under certain conditions the specified allowable error in the length of a line may be found to be exceeded even when the triangulation meets the other specifications for that particular grade of control. Where two points are close together, as compared with the size of the triangulation figure of which they are a part, the distance between those points may be in error in excess of that indicated by the class of triangulation of the scheme. The accuracy of the computed length of any line can be estimated by computing the ΣR_1 from the base to that line in accordance with the formula for the strength of figures as given on page 7.

Triangle closure and agreement in length are not the only standards for triangulation which should be applied. It is possible by a lucky balancing of errors to secure small triangle closures in a short scheme of triangulation even when the observations are below standard. It is also possible by omitting from the computations observations which differ greatly from the mean to reduce triangle closures greatly. It may also happen that a balancing of errors in computing a chain of triangles will result in a very small discrepancy in length on the next fixed line. The accuracy of triangulation is perhaps best indicated by the probable error of a direction, but since this gauge of the work is not available until after the adjustment has been made, the triangle closure and the agreement in length, as given by the preliminary computations, are the best available field criteria. To insure that the requisite accuracy is maintained throughout the triangulation, it is essential to give careful considerations to the instrumental equipment and the methods of observing in order that the systematic and accidental errors may be kept within the prescribed limits and that no part of the triangulation will exhibit undue weakness.

Since the methods employed in triangulation differ essentially from those used in traverse, the two operations will be treated separately. (See p. 147 for the chapter on traverse.)

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[1] See p. 173.

CHAPTER 1.—RECONNAISSANCE

The general term "triangulation" properly includes, in addition to the observation of horizontal angles, the operations of reconnaissance and base measurement, since the specifications for each must be decided upon with due regard for the other two. Starting with a base of specified accuracy, the computed lengths of the successive triangle sides will gradually become less accurate until finally a new base will be necessary if the required accuracy is to be maintained. The stronger the geometrical figures through which the lengths are carried by the triangulation process, and the more accurately the angles are measured, the farther apart may be the measured bases.

SPECIFICATIONS FOR RECONNAISSANCE FOR SECOND-ORDER TRIANGULATION

The subject of reconnaissance and signal building has been covered in great detail in United States Coast and Geodetic Survey Special Publication No. 93, Reconnaissance and Signal Building. The following specifications, approved by the director on December 10, 1928, are for second-order schemes of triangulation and must be followed strictly, except perhaps in a few unusual cases where particularly difficult country is to be covered. In such cases special instructions will be issued to the chief of party.

1. **Character of figures.**—The main scheme of the triangulation shall be made up of figures of from four to seven points each, in which certain stations may be left unoccupied as indicated under paragraph 2, "Strength of figures." It may be reduced in exceptional cases to a single chain of triangles with all angles observed where otherwise the cost and time would be excessive. On the other hand, there should be no overlapping of figures, except that in a four-sided, central-point figure one of the diagonals of the figure may be observed, and with the above exception there should be no excess of observed lines beyond those necessary to secure a double determination of every length. It is permissible, however, to observe between stations which are not in the same figure in order to avoid a back computation in locating supplementary or intersection stations. Observations over lines which will make the main scheme any more complicated than that defined above will practically be wasted. If it is necessary to occupy other stations than those in the main scheme in order to locate by intersection certain stations which must be fixed to control hydrographic or topographic operations, connect these additional occupied stations (which will be called supplementary stations) with the main scheme by the simplest figures possible in which there is a check. Single triangles with all the angles measured will, in general, be sufficient for the purpose. Supplementary stations are distinguished from intersection stations by being occupied and by being determined with at least third-order accuracy. An intersection station

is not occupied and, although it may be of second or third order accuracy if determined with a check and with a sufficient number of observations, it is frequently located with fourth-order accuracy.

2. **Strength of figures.**—In the main scheme of triangulation the value of the quantity $R=\frac{D-C}{D}\Sigma[\delta_A^2+\delta_A\delta_B+\delta_B^2]$* for any one figure should not, in the selected best chain of triangles (call it R_1), exceed 40, nor in the second best (call it R_2) exceed 120 in units of the sixth place of logarithms. These are outside limits never to be exceeded except when it is extremely difficult under existing conditions to keep within them. The quantities R_1 and R_2 should be kept down to the limits 25 and 80 for the best and second-best chains, respectively, whenever the estimated total cost does not exceed that for the chain barely within the extreme limits by more than 25 per cent. One station in each quadrilateral or central-point figure may be left unoccupied or certain lines in a figure may be observed over in one direction only if the values of R_1 and R_2 do not exceed the specified limits and if a considerable saving of time can be effected thereby. When a supplementary station is connected to the main scheme by a single triangle, the angle at the supplementary station should not be less than 30° if possible to avoid it.[1]

3. **Frequency of bases.**—If the character of the country is such that a base site can be found near any desired location, ΣR_1 between base lines, whether these are actually measured base lines or lines of first-order triangulation used as bases, should be made about 100. This value will be found to correspond to a chain of from 10 to 30 triangles, according to the strength of the figures involved. With strong figures, but few measured base lines will be needed, and a corresponding saving will be made on this part of the work. If topographic conditions make it difficult to secure a base site at the desired location, ΣR_1 may be allowed to approach but not to exceed 130. There will be danger, when this is done, that an intervening base will be necessary to meet the requirements stated in the next sentence. If in any case the discrepancy between adjacent bases is found to exceed 1 part in 10,000, after the side and angle equations have been satisfied, an intermediate base must be measured or the angle observations made more accurate.

STRENGTH OF FIGURE

The square of the probable error of the logarithm of a side of a figure is $\frac{4}{3}(d^2)\frac{D-C}{D}\Sigma[\delta_A^2+\delta_A\delta_B+\delta_B^2]$, in which d is the probable error of an observed direction, D is the number of directions observed in the figure, C is the number of conditions to be satisfied in the figure (see p. 16), and δ_A, δ_B are the respective logarithmic differences of the sines, expressed in units of the sixth decimal place, corresponding to a change of one second in the distance angles A and B of a triangle. (See p. 9, "Computation of strength of figure.") The summation, indicated by Σ, is to be taken for the triangles used in computing the value of the side in question from the side supposed to be absolutely known.

* See p. 9.

[1] The computation of R_1 and R_2 is described on pp. 9 to 17.

In the preceding formula the two terms $\frac{D-C}{D}$ and $\Sigma[\delta_A{}^2+\delta_A\delta_B+\delta_B{}^2]$ depend entirely upon the figures chosen and are independent of the accuracy with which the angles are measured. The product of these two terms is therefore a measure of the strength of the figure with respect to length, in so far as the strength depends upon the selections of stations and of lines to be observed over.

In the following table the values tabulated are $[\delta_A{}^2+\delta_A\delta_B+\delta_B{}^2]$. The two arguments of the table are the distance angles in degrees, the smaller distance angle being given at the top of the table. The distance angles are the angles in each triangle opposite the known side and the side required.

Factors for determining strength of figure

°	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°
10	428	359																					
12	359	295	253																				
14	315	253	214	187																			
16	284	225	187	162	143																		
18	262	204	168	143	126	113																	
20	245	189	153	130	113	100	91																
22	232	177	142	119	103	91	81	74															
24	221	167	134	111	95	83	74	67	61														
26	213	160	126	104	89	77	68	61	56	51													
28	206	153	120	99	83	72	63	57	51	47	43												
30	199	148	115	94	79	68	59	53	48	43	40	33											
35	188	137	106	85	71	60	52	46	41	37	33	27	23										
40	179	129	99	79	65	54	47	41	36	32	29	23	19	16									
45	172	124	93	74	60	50	43	37	32	28	25	20	16	15	11								
50	167	119	89	70	57	47	39	34	29	26	23	18	14	11	9	8							
55	162	115	86	67	54	44	37	32	27	24	21	16	12	10	8	7	5						
60	159	112	83	64	51	42	35	30	25	22	19	14	11	9	7	5	4	4					
65	155	109	80	62	49	40	33	28	24	21	18	13	10	7	6	5	4	3	2				
70	152	106	78	60	48	38	32	27	23	19	17	12	9	7	5	4	3	2	2	1			
75	150	104	76	58	46	37	30	25	21	18	16	11	8	6	4	3	2	2	1	1	1		
80	147	102	74	57	45	36	29	24	20	17	15	10	7	5	4	3	2	1	1	1	0	0	
85	145	100	73	55	43	34	28	23	19	16	14	10	7	5	3	2	2	1	1	0	0	0	0
90	143	98	71	54	42	33	27	22	19	16	13	9	6	4	3	2	1	1	1	0	0	0	0
95	140	96	70	53	41	32	26	22	18	15	13	9	6	4	3	2	1	1	0	0	0	0	
100	138	95	68	51	40	31	25	21	17	14	12	8	6	4	3	2	1	1	0	0	0		
105	136	93	67	50	39	30	25	20	17	14	12	8	5	4	2	2	1	1	0	0			
110	134	91	65	49	38	30	24	19	16	13	11	7	5	3	2	2	1	1	1				
115	132	89	64	48	37	29	23	19	15	13	11	7	5	3	2	2	1	1					
120	129	88	62	46	36	28	22	18	15	12	10	7	5	3	2	2	1						
125	127	86	61	45	35	27	22	18	14	12	10	7	5	4	3	2							
130	125	84	59	44	34	26	21	17	14	12	10	7	5	4	3								
135	122	82	58	43	33	26	21	17	14	12	10	7	5	4									
140	119	80	56	42	32	25	20	17	14	12	10	8	6										
145	116	77	55	41	32	25	21	17	15	13	11	9											
150	112	75	54	40	32	26	21	18	16	15	13												
152	111	75	53	40	32	26	22	19	17	16													
154	110	74	53	41	33	27	23	21	19														
156	108	74	54	42	34	28	25	22															
158	107	74	54	43	35	30	27																
160	107	74	56	45	38	33																	
162	107	76	59	48	42																		
164	109	79	63	54																			
166	113	86	71																				
168	122	98																					
170	143																						

COMPUTATION OF STRENGTH OF FIGURE

To compare with each other two alternative figures, whether triangles, quadrilaterals, or central-point figures, in so far as the strength with which the length is carried is concerned, proceed as follows:

(a) For each figure take out the distance angles, to the nearest degree if possible, for the best and second best chains of triangles through the figure. These chains are to be selected at first by estimation, and the estimate is to be checked later by the results of comparison.

(b) For each triangle in each chain enter the table with the distance angles as the two arguments and take out the tabular value.

(c) For each chain, the best and second best, through each figure, take the sum of the tabular values.

(d) Multiply each sum by the factor $\frac{D-C}{D}$ for that figure, where D is the number of directions observed and C is the number of conditions to be satisfied in the figure. (See p. 16.) The quantities so obtained, namely, $\frac{D-C}{D}\Sigma[\delta_A{}^2+\delta_A\delta_B+\delta_B{}^2]$, will for convenience be called R_1 and R_2 for the best and second best chains, respectively.

(e) The strength of the figure is dependent mainly upon the strength of the best chain through it, hence the smaller the R_1 the greater the strength of the figure. The second best chain contributes somewhat to the total strength, and the other weaker and progressively less independent chains contribute still smaller amounts. In deciding between alternative figures in reconnaissance they should be selected according to their best chains, unless said best chains are very nearly of equal strength and their second best chains differ greatly.

SOME VALUES OF THE QUANTITY $\frac{D-C}{D}$

The starting line is supposed to be completely fixed.

The directions observed along the fixed line are not included in computing D but *are* included in computing C.

For a single triangle, $\frac{4-1}{4}=0.75$.

For a completed quadrilateral, $\frac{10-4}{10}=0.60$.

For a quadrilateral with one station on the fixed line unoccupied, $\frac{8-2}{8}=0.75$.

For a quadrilateral with one station not on the fixed line unoccupied, $\frac{7-2}{7}=0.71$.

For a three-sided, central-point figure, $\frac{10-4}{10}=0.60$.

For a three-sided, central-point figure with one station on the fixed line unoccupied, $\frac{8-2}{8}=0.75$.

For a three-sided, central-point figure with one station not on the fixed line unoccupied, $\frac{7-2}{7}=0.71$.

For a four-sided, central-point figure, $\frac{14-5}{14}=0.64$.

For a four-sided, central-point figure with one corner station on the fixed line unoccupied, $\frac{12-3}{12}=0.75$.

For a four-sided, central-point figure with one corner station not on the fixed line unoccupied, $\frac{11-3}{11}=0.73$.

For a four-sided, central-point figure with the central station not on the fixed line unoccupied, $\frac{10-2}{10}=0.80$.

For a four-sided, central-point figure with one diagonal also observed, $\frac{16-7}{16}=0.56$.

For a four-sided, central-point figure with the central station not on the fixed line unoccupied and one diagonal observed, $\frac{12-4}{12}=0.67$.

For a five-sided, central-point figure, $\frac{18-6}{18}=0.67$.

For a five-sided, central-point figure with a station on a fixed outside line unoccupied, $\frac{16-4}{16}=0.75$.

For a five-sided, central-point figure with an outside station not on the fixed line unoccupied, $\frac{15-4}{15}=0.73$.

For a five-sided, central-point figure with the central station not on the fixed line unoccupied, $\frac{13-2}{13}=0.85$.

For a six-sided, central-point figure, $\frac{22-7}{22}=0.68$.

For a six-sided, central-point figure with one outside station on the fixed line unoccupied, $\frac{20-5}{20}=0.75$.

For a six-sided, central-point figure with one outside station not on the fixed line unoccupied, $\frac{19-5}{19}=0.74$.

For a six-sided, central-point figure with the central station not on the fixed line unoccupied, $\frac{16-2}{16}=0.88$.

To illustrate the application of the preceding strength table the R_2 for Figure 14 will be considered. Let it be assumed that the direction of progress is from the bottom line toward the top line. It will be found that the smallest R, called R_1, for this figure will be obtained by computing through the three best-shaped triangles around the central point. The next best R, called R_2, will be obtained by computing through the two triangles formed by the diagonal. The R_2 is easily computed as follows: From the known side to the diagonal the distance angles are 89 and 27°. Using these angles as arguments in the preceding strength table, the factor 17.5 is obtained. Similarly, from the diagonal to the top line the distance angles are 91 and 26° and the corresponding factor is 18.8. The sum of the two factors is 36.3. If the central point of the figure is an occupied station, $\frac{D-C}{D}=0.56$ (see p. 10) and $R_2=36.3\times0.56=20$. If the central point is unoccupied, as shown in Figure 14, $\frac{D-C}{D}=0.67$ (see p. 10) and $R_2=36.3\times0.67=24$, as given opposite the figure.

The R_1 may be computed in a similar manner by using the distance angles in the three best-shaped triangles around the central point.

EXAMPLES OF VARIOUS TRIANGULATION FIGURES

The following 14 figures are given to illustrate some of the principles involved in the selection of strong figures and to illustrate the use of the preceding strength table:

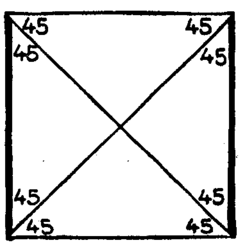

FIG. 1.—All stations occupied. $R_1=5$, $R_2=5$

Same, any one station not occupied. $R_1=6$, $R_2=6$

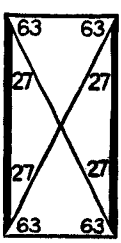

FIG. 2.—All stations occupied. $R_1=1$, $R_2=1$

Same, any one station not occupied. $R_1=2$, $R_2=2$

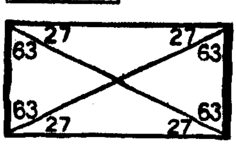

FIG. 3.—All stations occupied. $R_1=22$, $R_2=22$

Same, one station on fixed line not occupied. $R_1=27$, $R_2=27$

690689°—46——2

In every figure the line which is supposed to be fixed in length and the line of which the length is required are represented by heavy lines. Either of these two heavy lines may be considered to be the fixed line and the other the required line. Opposite each figure R_1 and R_2, as computed by the table on page 8 are shown. The

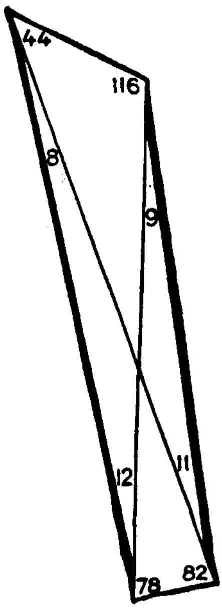

FIG. 4.—All stations occupied $R_1=1$ $R_2=2$

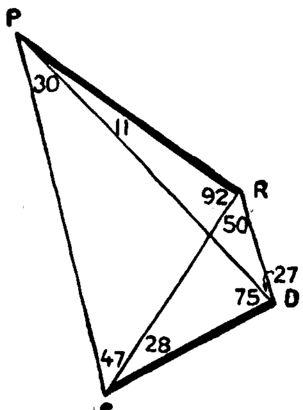

FIG. 5.—All stations occupied $R_1=10$ $R_2=12$

smaller the value of R_1 the greater the strength of the figure. R_2 need not be considered in comparing two figures unless the two values of R_1 are equal, or nearly so.

Compare Figures 1, 2, and 3. Figure 1 is a square quadrilateral; Figure 2 is a rectangular quadrilateral which is one-half as long in the direction of progress as it is wide; Figure 3 is a rectangular quadrilateral twice as long in the direction of progress as it is wide. The comparison of the values of R_1 in Figures 1 and 2 shows that shortening a rectangular quadrilateral in the direction of progress increases its strength. A comparison of Figures 1 and 3 shows that extending a rectangular quadrilateral in the direction of progress weakens it.

Figure 4, like Figure 2, is short in the direction of progress. Such short quadrilaterals are in general very strong, even though badly distorted from the rectangular shape, but they are not economical, as progress with them is slow.

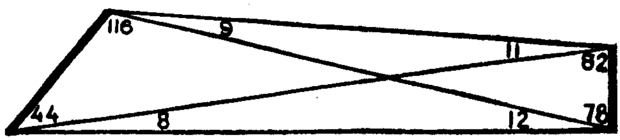

FIG. 6.—All stations occupied R_1=164 (approx.) R_2=176 (approx.)

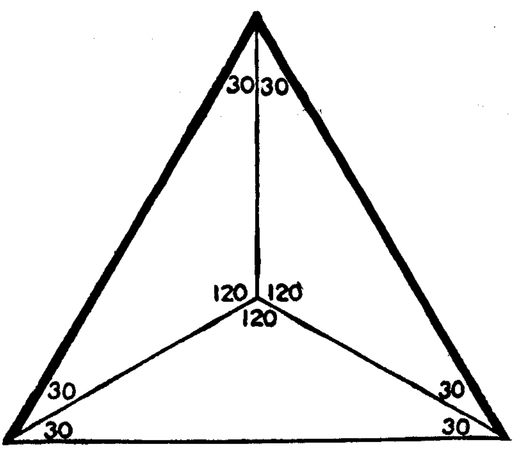

FIG. 7.—All stations occupied R_1= 2 R_2=12

One outside station, on fixed line, not occupied R_1= 3 R_2=15

Figure 5 is badly distorted from a rectangular shape but is still a moderately strong figure. The best pair of triangles for carrying the length through this figure are *D S R* and *R S P*. As a rule, one diagonal of the quadrilateral is common to the two triangles forming the best pair, and the other diagonal is common to the second-best pair. In the unusual case illustrated in Figure 5 a side line of the quadrilateral is common to the second-best pair of triangles.

Figure 6 is an example of a quadrilateral so much elongated, and therefore so weak, that it is not allowable in any class of triangulation.

Figure 7 is the regular three-sided, central-point figure. It is extremely strong.

Figure 8 is the regular four-sided, central-point figure. It is very much weaker than Figure 1, the corresponding quadrilateral.

Figure 9 is the regular five-sided, central-point figure. Note that it is much weaker than any of the quadrilaterals shown in Figures 1, 2, or 4.

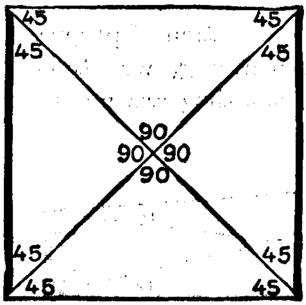

FIG. 8.—All stations occupied $R_1=13$ $R_2=13$

Same, one corner station not occupied $R_1=16$ $R_2=16$

Same, central station not occupied $R_1=17$ $R_2=17$

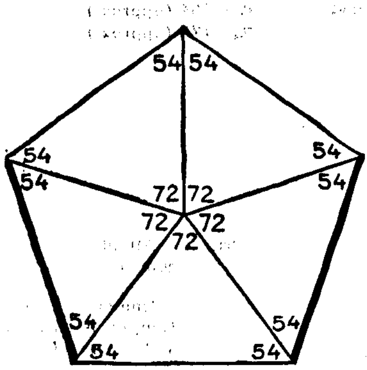

FIG. 9.—All stations occupied $R_1=10$ $R_2=15$

Same, any one outside station not occupied $R_1=11$ $R_2=16$

Same, central station not occupied $R_1=13$ $R_2=19$

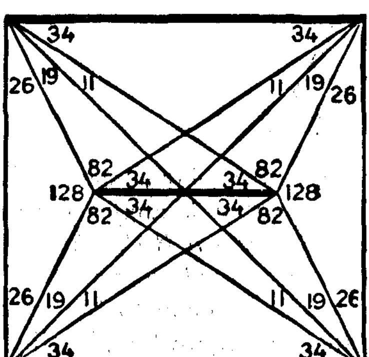

FIG. 10.—All stations occupied $R_1=5$ $R_2=5$

$$\frac{D-C}{D}=\frac{28-16}{28}=0.43$$

Figure 10 is a good example of a strong, quick expansion from a base. The expansion is in the ratio of 1 to 2.

Figures 11 and 12 are given as a suggestion of the manner in which, in second and third order triangulation, a point A, difficult or impossible to occupy, may be used as a concluded point common to several figures.

Figure 13 shows a figure which is frequently used in expanding from a base, the shorter, heavy line representing the measured length. length.

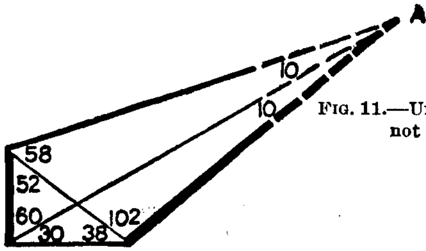

FIG. 11.—Unoccupied station not on fixed line $R_1 = 36$ $R_2 = 102$

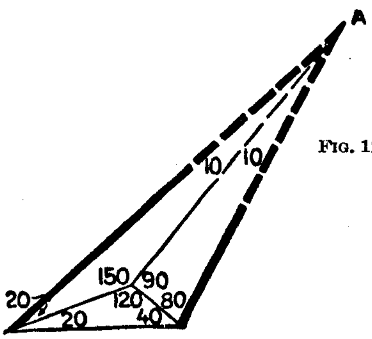

FIG. 12.—Unoccupied station at intersection of fixed line and line to be determined $R_1 = 4$ $R_2 = 20$

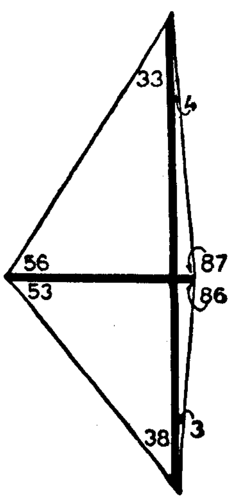

FIG. 13.—All stations occupied (A strong and quick expansion figure.) $R_1 = 9$ $R_2 = 9$

Figure 14 is one which can frequently be used to advantage on second and third order coastal triangulation. By placing the unoccupied point near the coast the number of control stations near the coast is increased.

Some of the figures given on the preceding pages are too weak to be used in the main scheme of triangulation, but for convenience

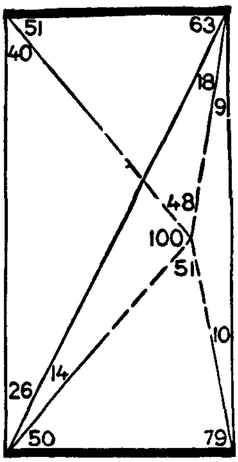

FIG. 14.—Central station not occupied $R_1=18$ $R_2=24$

of reference and to illustrate the principles involved, they are included with the figures which it is permissible to use.

DETERMINATION OF C AND D IN STRENGTH OF FIGURE FORMULA

Referring to page 9, where some values are given for the quantity $\frac{D-C}{D}$, the number of conditions to be satisfied in any figure may be computed from the following formula:

$$C=(n'-s'+1)+(n-2s+3)$$

in which

n=total number of lines,
n'=number of lines observed in both directions,
s=total number of stations,
s'=number of occupied stations.

Thus, in a quadrilateral with one station unoccupied and one unobserved line at one of the occupied stations (see fig. 15)

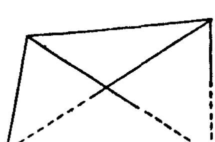

FIG. 15.—Quadrilateral to illustrate C

$$C=(2-3+1)+(6-8+3)=1$$

(In using these formulas allowance must be made for lines or triangles previously fixed.)

The number of conditions to be satisfied in any given figure may also be determined in another way. Starting with the fixed line or the line fixed by the preceding figure, build up the figure, station by station, computing the number of conditions at each new station. To obtain the number of conditions for the entire figure, simply add the number of conditions at all the stations. At each station the number of angle conditions is one less than the total number of full lines to the station from previously considered stations. The number of side conditions at each station is two less than the total number of lines, full and broken, to the station from previously considered stations.

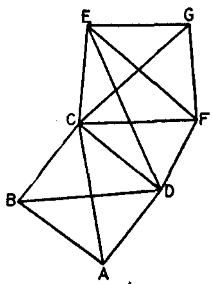

FIG. 16.—Overlapping of figures

As previously stated, D is the number of observed directions in a figure. It must be remembered, however, that the directions over the fixed line are not counted in obtaining D.

OVERLAPPING OF FIGURES

To illustrate what is meant by "overlapping of figures" and an "excess of observed lines" (see p. 6), reference is made to Figure 16, where the figures overlap in space but where there is no excess of observed lines. Assume the direction of progress to be from AB to EG. After AB the next line to go ahead on, or the known line of

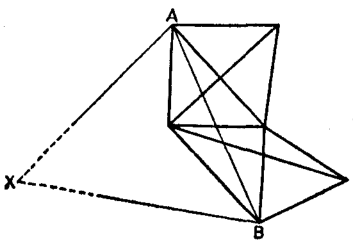

FIG. 17.—Excess of observed lines

the second figure, is CD. In the figure $CDEF$ the diagonal CF is determined as the known line of the succeeding figure $FCEG$. The last two figures are thus seen to overlap in space but to be simple quadrilaterals.

In Figure 17 the line AB connects stations in separate figures and should not be observed as a main-scheme line. It may, however, be observed in locating a point X, in order to avoid a back-position computation or the computation of a triangle by the two-sides-and-included-angle method.

OTHER CONSIDERATIONS IN RECONNAISSANCE

LENGTH OF LINES OF TRIANGULATION

The lower limit of length of line is fixed by two considerations. On very short lines it is difficult to get observations of the degree of accuracy necessary to close the triangles within the required limit. Extreme caution is required in centering and plumbing the signals and the theodolite to avoid the errors due to eccentricity. Very short lines are apt to be accompanied, though not necessarily so, by poor geometric conditions as expressed by large values of R. The extreme lower limit fixed by these two considerations should be avoided. On the other hand, there is no advantage in so far as accuracy is concerned in using very long lines. Long lines are apt to introduce delays due to signals not being visible. With long lines supplementary stations to reach required points in all portions of the area covered are much more apt to be needed than with short lines. Therefore, in any project, endeavor in laying out the main scheme to use the economical length of line—that is, endeavor to use in each region lines of such lengths as to make the total cost of reconnaissance, signal building, triangulation, and base measurement a minimum, subject to the requirements stated in these specifications.

BASE SITES AND BASE NETS

A base may be measured with the degree of accuracy specified over fairly rough ground and steep slopes with steel or invar tapes. Smooth, level ground is a convenience but not a necessity for base measurement of this grade of accuracy. There should be no hesitancy in placing the base on rough ground if by so doing the geometric conditions in the base net are improved; that is, values of R made smaller. The length of a base is to be determined primarily by the desirability of securing small values of R in the base net. Ordinarily the longer the base the easier it will be found to secure small values of R, and the smaller the values of R the longer the chain of triangles through which the lengths may be carried before another base becomes necessary. The base net shall consist of a figure or figures of the same character and subject to the same conditions as to strength as the main scheme previously described. If the net is made up of two or more figures, they may overlap in space but there should be no overlapping of figures in the sense of the existence of observed lines which tie together the separate figures.

Broken bases are permissible when found advantageous. They will frequently be found necessary along rugged coasts. No large section of a broken base should make an angle of more than 30° with the projected base line. The two terminal stations of a broken base must be intervisible; the directions at each intermediate angle station on the broken base must also be observed in order to form a closed loop. It is permissible to include, in the main scheme observations of the base net, one of the intermediate angle stations on a broken base, in order to provide an additional check on the projected length of the measured sections.

SUPPLEMENTARY STATIONS

The distribution of supplementary stations can not be definitely specified, for the number and location of extra second-order, third-order, and intersection stations will be governed entirely by the topography of the region, the accuracy required for the detailed hydrographic or topographic surveys, and the scale of the working sheets. Each hydrographic or topographic sheet should have at least two and preferably three triangulation stations within its limits, and such greater number as special conditions may require. A little extra time spent in locating additional triangulation stations will frequently save much time and trouble in making the detailed surveys check within the specified limits of accuracy.

MODIFICATIONS OF SPECIFICATIONS

To interpret correctly the foregoing reconnaissance specifications it must be borne in mind that they were drawn to meet particularly one requirement, namely, to enable the triangulator to determine the lengths of successive lines of the triangulation as economically as possible consistent with the maintenance of a specified accuracy. In some respects they meet the requirements for long belts of triangulation better than those for the development of an area. It is very easy, however, to modify the specifications to meet other conditions. For instance, on short arcs where a check in length can be obtained in a few figures it will be permissible to use a chain of triangles instead of figures which afford two or more ways of computing lengths through the scheme; or, where an area is to be covered with a network of triangulation, belts of triangulation figures can be located which will form a checkerboard system over the area to be controlled, and the location of the bases required can be determined by computing the R_1 through the strongest chain of figures. The intervening areas can then be covered by triangles extending from the most convenient stations.

PUBLIC VALUE OF CONTROL SURVEYS

Second-order triangulation is normally executed for the purpose of furnishing main-scheme control for some hydrographic or topographic survey which is to follow immediately, and the control is, necessarily, planned to meet the requirements of that particular survey. The officer in charge of the reconnaissance should bear in mind, however, that any control survey of third, or higher, order of accuracy should be so established as to fulfill all reasonable demands for control in that area for many years to come. Not only may this bureau need the data for additional surveys, but other organizations will almost certainly use the stations established. An effort should therefore be made to leave lines of the triangulation of proper strength to face up rivers or river valleys which lie outside the limits of the work assigned. It costs little to establish supplementary stations of second and third order accuracy while work on the main scheme is in progress. Similarly, a station placed on the shoulder of a hill or mountain may answer every requirement for the work in hand, but if it is placed on the summit the chances are that it will be of much greater future value. The summit should, therefore, be selected if it does not involve too much additional expense.

The mistake most frequently made in extending control from existing triangulation for a topographic or hydrographic survey of limited extent is to locate intersection stations only for the new control needed, when but little more time and expense would be required to locate some of the new stations with second and third order accuracy and to mark them permanently. The aim should always be to leave an area adequately controlled with stations of third-order, or higher, accuracy and to have intersection stations in sufficient number to provide connections to the detailed surveys.

CONNECTIONS TO EXISTING TRIANGULATION

In starting from or connecting with existing triangulation it is essential that the connection be made to a line of proper strength, and also that observations be made from the two ends of that line upon a third point of the existing triangulation. If the new and old angles upon the third point agree closely, the exact recovery of the old stations is then assured. Connection in position alone, namely, to a single point, or in position and azimuth, namely, to a single point but with a direction observed from that point to another old station, may sometimes be advantageously made at intervals between the connections in length just described.

INTERVISIBILITY OF STATIONS

Reconnaissance for triangulation can be executed by either of two general methods, or by a combination of them. In the first method, which can be used in hilly or mountainous country, the intervisibility of the stations is tested by visiting each station. In the second method reliance is placed upon obtaining the elevations of the stations and of the intervening country from maps or other sources and determining the intervisibility of points and the required heights of towers from those data. This second method is necessary in flat country. In actual practice a combination of the two methods is generally used.

The difference between the apparent and true difference in elevation of two points is affected by two factors—the curvature of the earth's surface and the refraction of light by the earth's atmosphere. These factors are of opposite sign and of an approximately fixed relation to each other, so that the combined effect can be applied as a single factor. The effect of refraction is about one-seventh as much as the curvature. The formulas for the separate effect of each can be found in various works on geodetic surveying, but the formulas below give the approximate resultant:

$$h \text{ (in feet)} = K^2 \text{ (in miles) times } 0.574,$$

or

$$K \text{ (in miles)} = \sqrt{h \text{ (in feet)}} \text{ times } 1.32.$$

Below is a table, condensed from the one given in Appendix 9, Report for 1882, which gives the distance K (in statute miles) at which a line from the height h (in feet) will touch the horizon, taking into account terrestrial refraction with a mean assumed coefficient of refraction of 0.070.

Correction for earth's curvature and refraction

Distance	Correction	Distance	Correction	Distance	Correction	Distance	Correction
Miles	*Feet*	*Miles*	*Feet*	*Miles*	*Feet*	*Miles*	*Feet*
1	0.6	16	146.9	31	551.4	46	1,214.2
2	2.3	17	165.8	32	587.6	47	1,267.7
3	5.2	18	185.9	33	624.9	48	1,322.1
4	9.2	19	207.2	34	663.3	49	1,377.7
5	14.4	20	229.5	35	703.0	50	1,434.6
6	20.6	21	253.1	36	743.7	51	1,492.5
7	28.1	22	277.7	37	785.6	52	1,551.6
8	36.7	23	303.6	38	828.6	53	1,611.9
9	46.4	24	330.5	39	872.8	54	1,673.3
10	57.4	25	358.6	40	918.1	55	1,735.8
11	69.4	26	388.0	41	964.7	56	1,799.6
12	82.7	27	418.3	42	1,012.2	57	1,864.4
13	97.0	28	449.9	43	1,061.0	58	1,930.4
14	112.5	29	482.6	44	1,111.0	59	1,997.5
15	129.1	30	516.4	45	1,162.0	60	2,065.8

To determine how much the line of sight between two stations will clear or fail to clear an intervening hill, either the table above may be used or the following formula employed:

$$h = h_1 + (h_2 - h_1)\frac{d_1}{d_1 + d_2} - 0.574 d_1 d_2,$$

where

h = height of line at obstruction, in feet.
h_1 = height of lower station, in feet.
h_2 = height of higher station, in feet.
d_1 = distance from lower station to intervening obstruction, in miles.
d_2 = distance from intervening obstruction to higher station, in miles.

This formula is also based on a mean assumed coefficient of refraction of 0.070.

COMPUTATION FOR THE DETERMINATION OF THE INTERVISIBILITY OF STATIONS

A few examples will be given to illustrate the application of the formulas relating to curvature and refraction.

Example.—Two stations are at the water's level on opposite shores of a bay 18 miles wide. In this problem the line of sight is not supposed to approach the water nearer than 10 feet.

(*a*) How high above the water must the instrument be at station *A* with the instrument at station *B* 10 feet above the surface?

(*b*) How high must the towers be, supposing them to be of equal height at the two stations?

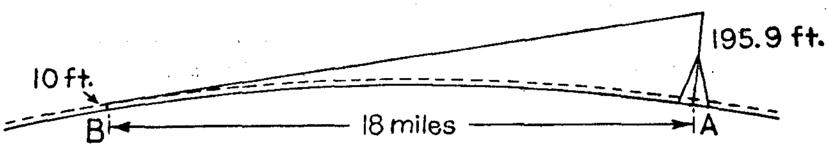

FIG. 18.—Intervisibility of stations across water, case (*a*)

Solution of (*a*).—From the table for curvature and refraction, page 21, the instrument must be at an elevation of 185.9 feet for the line of sight, to be tangent to the earth's surface at a distance of 18 miles. Since it must not approach the water surface nearer than 10 feet at station *B*, the instrument at *A* must be 195.9 feet above the water surface. (See fig. 18.)

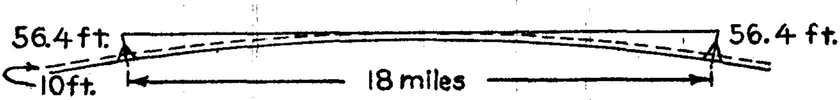

FIG. 19.—Intervisibility of stations across water, case (*b*)

Solution of (*b*).—Since the towers are to be of equal height, the line of sight will approach the water the nearest at a point midway between the two stations. From the table the instrument must be elevated 46.4 feet to see the water surface 9 miles distant. Since the line of sight must clear the surface by 10 feet, the instrument must be elevated 56.4 feet at each station. (See fig. 19.) Or, by the formula

$$h \text{ (in feet)} = K^2 \text{ (in miles)} \times 0.574,$$
$$h = 81 \times 0.574 = 46.49 \text{ feet},$$
$$46.49 + 10 = 56.49 \text{ feet}.$$

Example.—A level plain is wooded with trees 35 feet high. It is desirable that the line of sight clear the trees by 10 feet at least.

(*a*) With towers 70 feet high, without superstructure—that is, with lamp and theodolite mounted at same height—what is the maximum length of line?

(*b*) Under the same conditions of terrain, what is the maximum length of line when the theodolite is at a height of 70 feet and the lamp at a height of 90 feet above the ground?

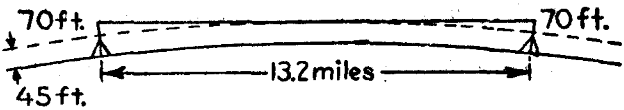

FIG. 20.—Intervisibility of stations across wooded plain, case (*a*)

Solution of (*a*).—Since the line of sight must not approach nearer than 45 feet to the surface of the plain or 10 feet above the tree tops, and the towers are 70 feet high, the problem is the same as if the towers were 70–45 feet high and the line of sight could be tangent to the surface of a level plain. From the table it is seen that the line of sight from a tower 25 feet high would be tangent to the surface of the sphere at a distance between 6 and 7 miles.

Applying the formula

$$h \text{ (in feet)} = K^2 \text{ (in miles)} \times 0.574$$

$$K^2 = \frac{25}{0.574}$$

or

$$K = 6.6 \text{ miles}.$$

The stations could, therefore, be 13.2 miles apart. (See fig. 20.)

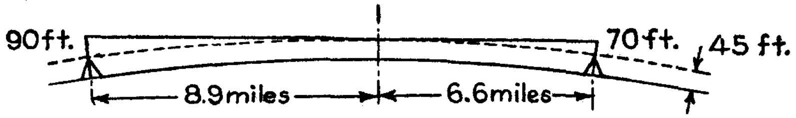

FIG. 21.—Intervisibility of stations across wooded plain, case (*b*)

Solution of (*b*).—From the previous example the line of sight from a tower 70 feet high is tangent to the spherical surface 45 feet above the station at a distance of 6.6 miles. The distance at which the line of sight from the 90-foot tower will be tangent to the 45-foot surface is found from the formula in a similar manner, as follows:

$$(90-45) = h = K^2 \times 0.574$$

$$K^2 = \frac{45}{0.574}$$

or

$$K = 8.9 \text{ miles}.$$

Therefore, the maximum distance between stations would be 6.6+8.9=15.5 miles. (See fig. 21.)

When contoured maps of a hilly or mountainous country are available they are of great assistance in reconnaissance. When laying out a scheme from a map the most frequent problem is to determine if a certain ridge or mountain between two stations will cut the line of sight, and if so, if it will be practicable to build at either station and thus make the stations intervisible. Such a problem may be solved either by the formula on page 22 or by the table for curvature and refraction. A solution of such a problem by each method is given below.

Example.—Two stations, A and B, are 54 miles apart and at an elevation above sea level of 1,050 and 4,500 feet, respectively. At X on the line between A and B and at a distance of 18 miles from A is a ridge 1,840 feet high. (See fig. 22.)

(*a*) How much below the crest of the ridge does the line between stations strike?

(*b*) Supposing a tower is to be built at only one station, what height would be necessary at each station for the line of sight to barely clear the ridge?

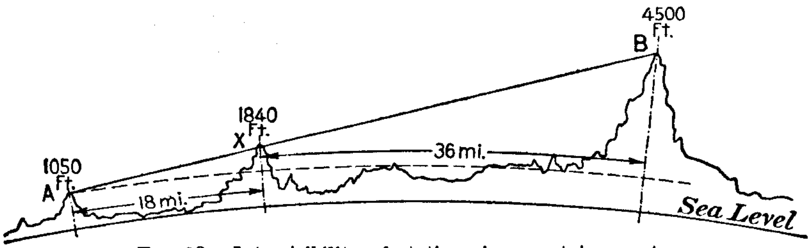

FIG. 22.—Intervisibility of stations in mountain country

Solution of (*a*).—At 18 miles, the distance from A to X (fig. 22), the correction for curvature and refraction is 185.9 feet and at 54 miles, the distance from A to B, it is 1,673.3 feet. Subtracting these amounts from the elevations of X and B, respectively, will have the effect of reducing the sea-level bases of A, X, and B to a plane surface.

Therefore, in Figure 23,

$$bc = 4{,}500 \text{ feet} - 1{,}050 \text{ feet} - 1{,}673.3 \text{ feet},$$
$$= 1{,}776.7 \text{ feet}.$$

and

$$ed : bc :: 18 : 54$$

Therefore $ed = 592.2$ feet.

Correcting elevation of X for curvature and refraction and reducing it to the plane of peak A

$$X = 1{,}840 - 1{,}050 - 185.9$$
$$= 604.1 \text{ feet}.$$

Since the line of sight from A to B, at a point 18 miles from A, is only 592.2 feet above A, therefore the line of sight fails to clear peak X by $604.1 - 592.2 = 11.9$ feet.

Another method for obtaining the elevation at X of the straight line between the two stations is by the formula given on page 22,

$$h=h_1+(h_2-h_1)\frac{d_1}{d_1+d_2}-0.574\ d_1\ d_2.$$

In which for the example given above

$$h_1=1{,}050,\ h_2=4{,}500,\ d_1=18,\ d_2=36.$$

Substituting these values

$$h=1{,}050+(4{,}500-1{,}050)\frac{18}{18+36}-0.574\times18\times36=1{,}828.$$

Therefore, the line between A and B is 12 feet below the top of the ridge.

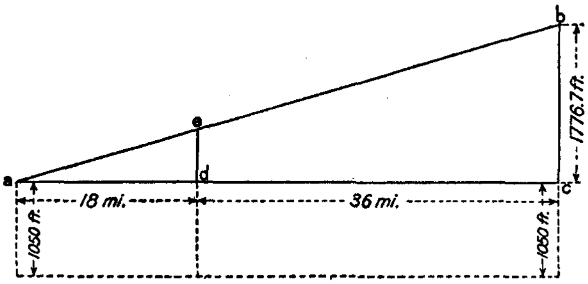

FIG. 23.—Intervisibility of stations in mountain country, solution (*a*)

There may be times in the field when the observer will not have a book at hand from which to obtain the formula for the second method and he may not be able to recall the formula. He may use the first method, however, if he merely remembers that the correction in feet for curvature and refraction is the square of the distance in miles times 0.574. The second method may be a little easier as regards the numerical operations.

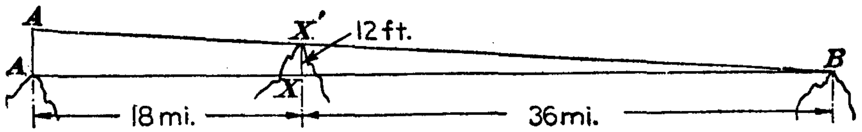

FIG. 24.—Intervisibility of stations in mountain country, solution (*b*)

Solution of (b).—Accepting 1,828 feet as the elevation at X of the line from A to B, the line strikes 12 feet below the crest of the ridge. If the station at A is to be elevated, the necessary height can be computed by means of a triangle, as shown in Figure 24. The line from A to B has already been corrected for curvature and refraction.

Let AA' be the height by which A is to be increased. From similar triangles,

$$AA' : XX' :: AB : XB,$$

or $$AA' : 12 :: 54 : 36.$$

Therefore, $$AA'=18\text{ feet.}$$

If the station at B is to be elevated, a similar method may be used as shown in Figure 25.

$$BB' : 12 :: 54 : 18.$$

Therefore, $BB' = 36$ feet.

These two solutions show that, other things being equal, it is always more economical to build at the station nearest the obstruction, for the height necessary to clear the obstruction increases in direct proportion to the distance from it.

SELECTING SITES FOR TRIANGULATION STATIONS

Triangulation stations, as far as practicable, should be placed on the crests of ridges and on the highest points of hills and mountains. In a mountainous country it is not necessary to place the stations on the highest peaks, but each one should be on the highest point of the peak selected, and this peak should be the highest one in the immediate vicinity in order that there may be an unobstructed view in all directions.

In making a choice between a high peak and a lower one many factors must be taken into account, part of which relate to the cost

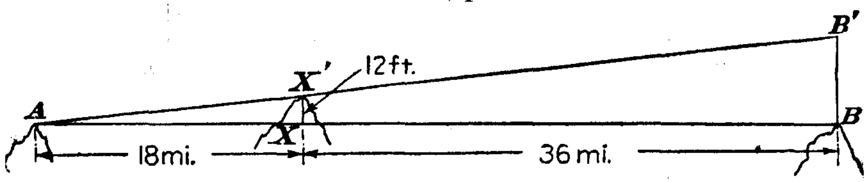

Fig. 25.—Intervisibility of stations in mountain country, solution (c)

of the immediate project in hand and others to the usefulness of the triangulation itself.

The *cost of the reconnaissance* per mile of progress measured along the axis of the scheme of triangulation will be practically the same in any region where two schemes are possible, whether a large or a small scheme is used, and whether the points selected are on the highest peaks or on lower ones. The use of the highest points usually results in large figures in the scheme of triangulation, though not necessarily so. It is usually easier to secure strong figures when the highest peaks are used as main-scheme points.

Where all stations are convenient of access on both large and small alternative schemes, the cost of observing will be almost in proportion to the number of stations to be occupied on the two schemes. Usually, however, the stations of the larger scheme will be so much more difficult to reach that the cost of making the observations through the smaller scheme will not be much more than through the larger one with its smaller number of stations. Weather conditions affecting the visibility of stations and the accuracy of the observations will often be the controlling factor in deciding upon the economical length of lines.

SIGNALS AND SIGNAL BUILDING

When observations are to be made in daylight upon targets the signals must be constructed not only to meet the demands of visi-

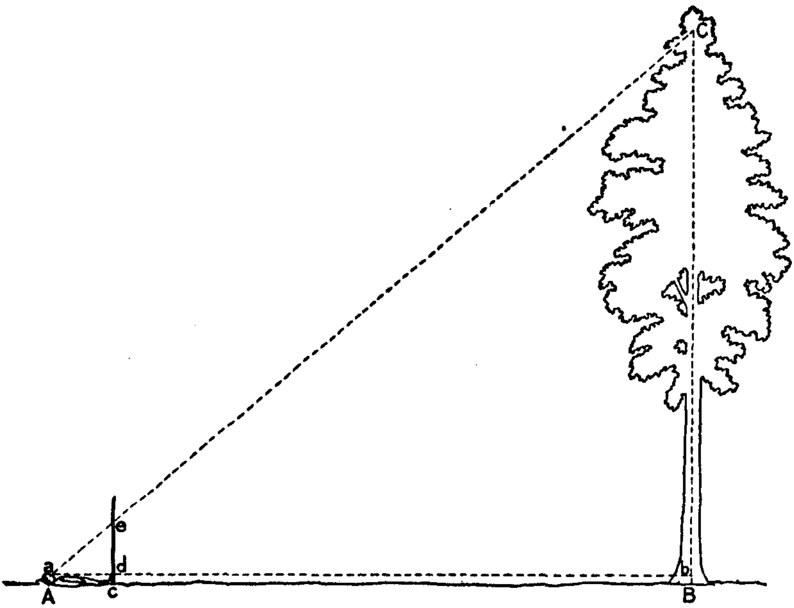

FIG. 25a.—Determining the approximate height of a tree on reconnaissance

A distance AB approximately equal to the height of the tree should be paced or measured out from the base of the tree and the point A marked. Also a point c should be located toward the tree from A such that Ac equals the height of the observer's eye from the ground when standing upright. At c a straight pole about 2 feet higher than the observer should be set perpendicularly in the ground. A point e should then be marked on the pole at such a height that $ce=cd+de$, where cd equals the distance from the observer's eye to the back of his head and de equals the height of his eye from the ground when standing upright. Then if the observer lies on the ground with his feet against the pole and his eye above the point A the line of sight from his eye past the point e on the pole will strike the perpendicular from B at the point C, and $bC=ab=AB$. Approximate allowance can easily be made for slope of ground along the line AB, and an estimate can be made of the distance the point C falls above or below the top of the tree.

bility but must also be of a type which will reduce the effects of phase and eccentricity to a minimum. (See p. 76.) The lengths of the lines of sight and the materials available for constructing the signal will also impose their requirements. In many places, such as the Philippines, sawed lumber can not be readily obtained, and round timbers cut near the signal, or bamboo poles, must be used. (See fig. 26.) Where tall signals are required the trunks of standing trees are frequently of great assistance. (See fig. 27.) In building tree signals ingenuity in the utilization of the materials available and the assistance of one or two experienced men are of much greater value than written instructions.

Occasionally it is necessary to build tall towers of sawed timbers of the type frequently used on first-order triangulation. The construction of these towers is described in Special Publication No. 93, Reconnaissance and Signal Building. Only the more ordinary types of signals will be described here.

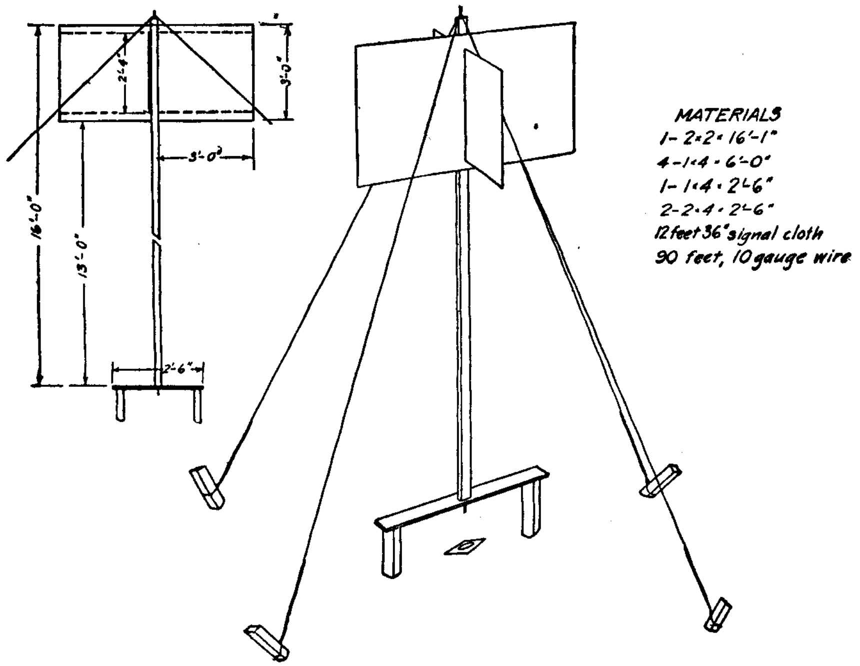

FIG. 28.—Diagram of pole signal, with target

POLE SIGNAL

Figure 28 shows a pole signal held in a vertical position by wire guys with the foot of the pole resting on a low bench. The bench may be made of two stakes driven in the ground on either side of the station mark, with a piece of scantling placed across on top and nailed to them. The foot of the pole should have a spike driven at its center projecting about an inch, and when the pole is erected this spike should be placed in a hole bored in the crosspiece of the bench directly over the station. Each set of guys should consist of four guys of No. 12 smooth galvanized wire. The number of sets depends

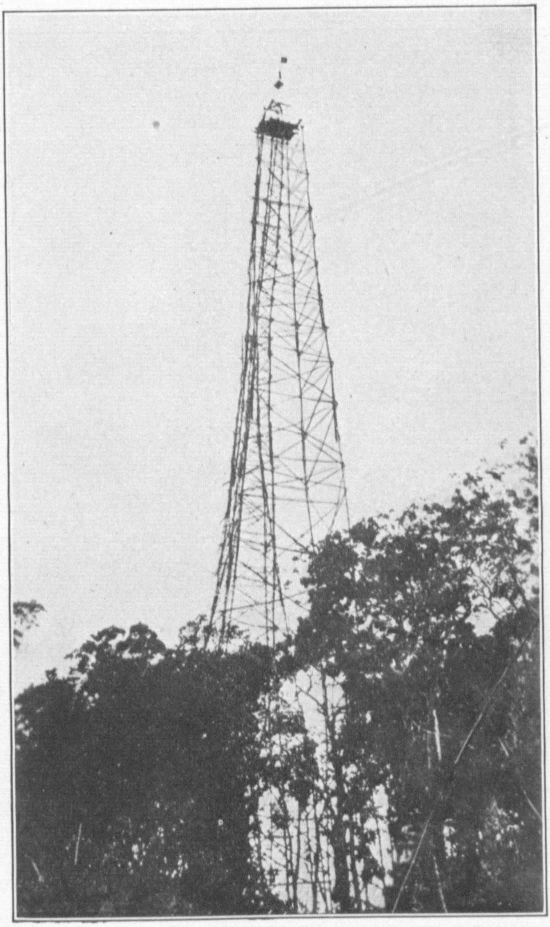

FIG. 26.—SIGNAL 235 FEET HIGH BUILT OF POLES

Inner instrument tripod is entirely separate from outer structure

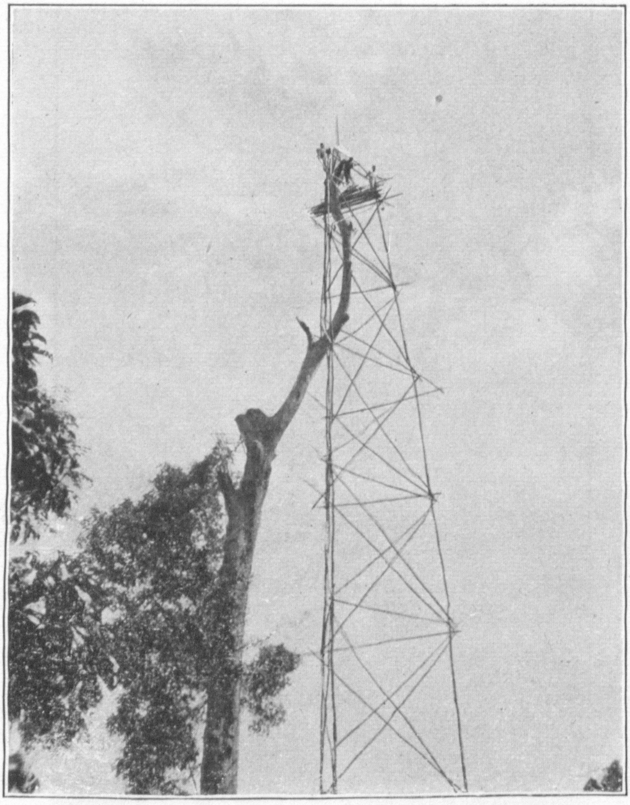

FIG. 27.—TREE SIGNAL 143 FEET HIGH

The theodolite was mounted on the top of the tree trunk and the observer was supported by the scaffold built of poles

FIG. 29.—TRIPOD STAND FOR THEODOLITE

FIG. 30.—SMALL SIGNAL LAMP WITH TARGET CENTERED ABOVE

An aluminum cup-shaped plate is made for this type of lamp, which may be attached to the top of the lamp and which automatically centers the base of the target over the center of the reflector of the lamp. The target is held in place by guy wires upon the height of the pole. The pole is easily lowered, when the station is occupied, by loosening the guy or guys on only one side. The guys on the other three sides are not loosened from their anchors. To replace the pole it is only necessary to stand it up on the bench and fasten the loosened guy or guys on the one side. The centering of the pole or that part on which observations are made should be tested after the pole has thus been replaced. The ease with which a pole signal can be erected, removed, and replaced is a strong recommendation for it when the station is to be occupied with a theodolite mounted on its own tripod. It also makes a satisfactory object to point on provided the diameter of the center pole is regulated for the length of line and consideration is given to the effect of phase. A serious objection to this type of signal is the fact that it can very easily get out of plumb. Unequal expansion and contraction of the guys, displacement of the lower part of the pole, people tripping over the guys or cattle rubbing against them—these and other causes may displace the signal enough to cause considerable errors in the observations.

When signal lights are used to observe upon it will be found more convenient to use the type of stand shown in Figure 29 as a support for the light and theodolite. The horizontal board on the top of the stand has a gimlet hole, centered over the mark, by means of which the lamp or heliotrope is centered and fastened and the theodolite centered. The combined use of target and lamp is shown in Figure 30.

TRIPOD SIGNAL

When the signal must be seen from a greater distance, and especially when it is to be used also for hydrographic or topographic purposes, a tripod signal is used of the type shown in Figure 31. With this type of signal the theodolite may be set up over the mark without disturbing the signal. One objection to this type of signal is that it is difficult to center over a mark already in place. Tripod signals along the coast will usually be erected with one leg toward the water and with the two adjacent sides of the tripod covered with cloth or painted boards. The part of the center pole or target to be observed upon must be carefully centered over the station mark. Each leg should be secured to the ground by a stake or other means. Wire guys to hold the center pole are necessary if the pole has considerable length.

The largest tripod signals usually have a height of about 20 feet to the apex of the tripod and from 30 to 35 feet to the top of the center pole. When cut lumber is available, timbers 4 inches square

are generally used for the legs and center pole of large signals and 2 by 4 inch lumber for those of smaller tripods. Ordinarily the center pole should be large enough in diameter to be visible through the theodolite from the adjacent stations, as otherwise observations will be made upon the tripod or banners and errors due to phase and eccentricity will develop. One-inch boards are ordinarily used for dressing signals and should be rough or with only one side planed, as a rough surface is superior for holding whitewash.

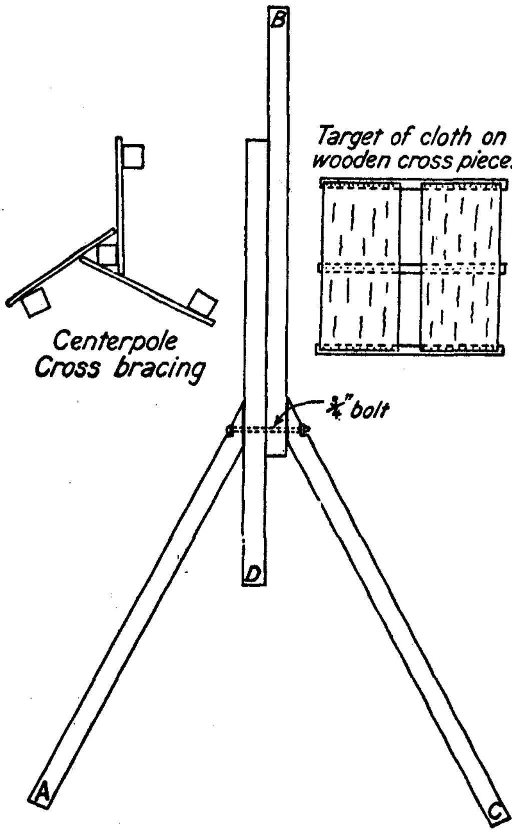

FIG. 32.—Diagram showing construction of tripod-and-pole signal

A convenient method of constructing a large tripod signal is indicated in Figure 32. The drawing shows the legs and center pole as assembled on the ground. The identifying letter is placed at the lower end of each part. This assembly is arranged so that the lower ends of the legs *A* and *C* are in the approximate positions they will occupy when the signal is completed. The tripod is then erected by using leg *B* as a lifting pole and prop. When the tripod is erected and secured in the proper location the lower end *D* of the center

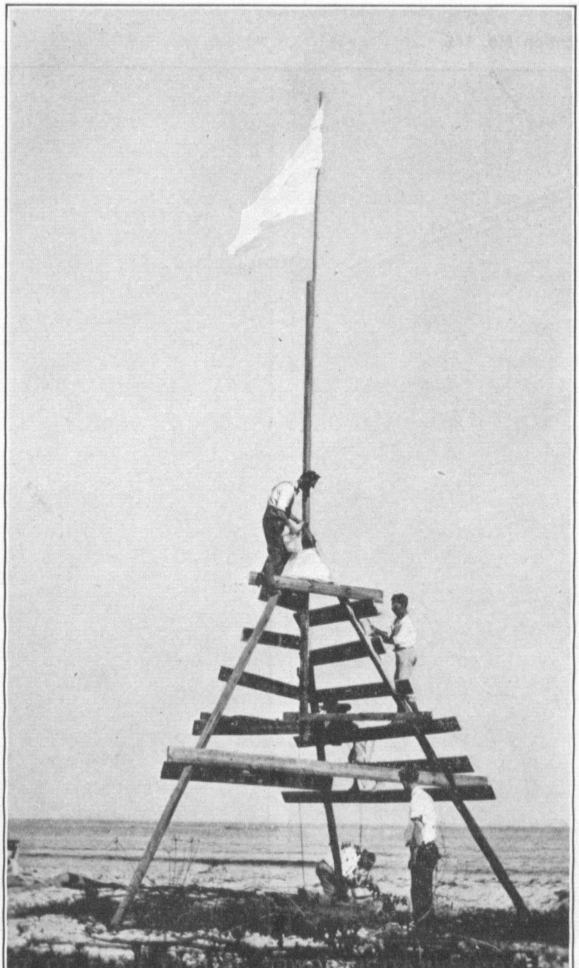

FIG. 31.—TRIPOD-AND-POLE SIGNAL

Signals of this type should be used with caution because of the danger of errors due to phase and eccentricity. The portion of the pole just above the tripod should be the part observed upon when it is visible

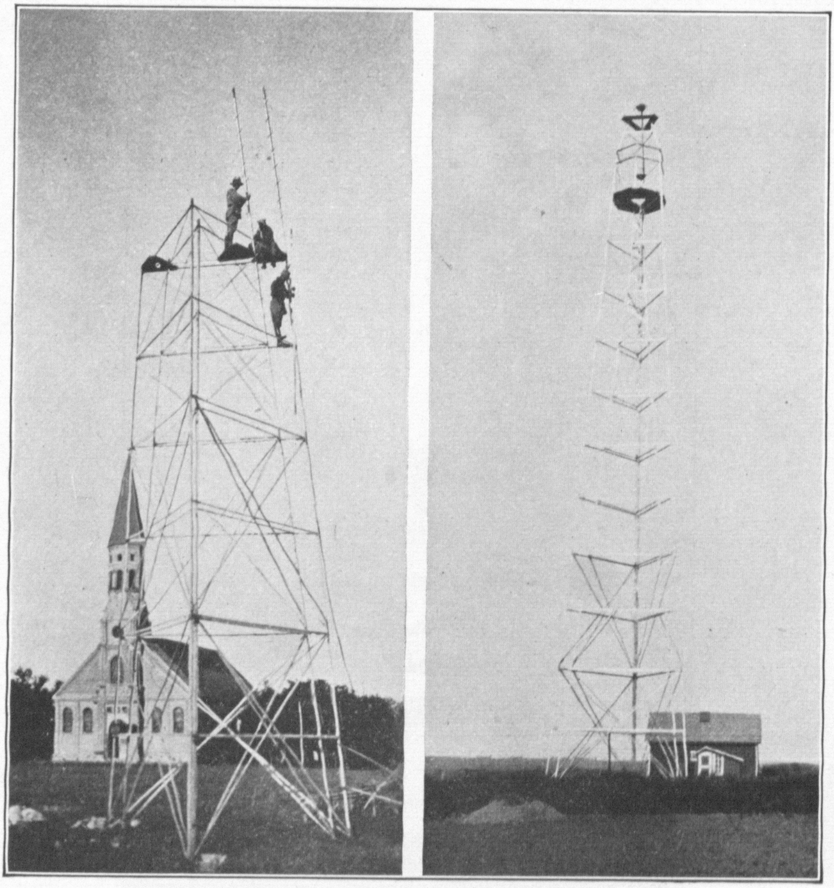

FIG. 33.—STEEL TRIANGULATION TOWER, 77 FEET HIGH TO TOP OF INNER TRIPOD

A tower of this type can be erected in half a day by five men. Similar towers 25 feet higher have been used successfully pole, which pivots on the bolt, is brought within reach by means of a line attached at D, and the pole is then adjusted to a vertical position and secured by cross braces to each leg of the tripod. The targets on the center pole may be attached before or after this operation. The sides of the tripods are then boarded up as desired. The spaces between boards need not be less than the width of the boards, and wider spacing may be desirable in order to cover as much surface as possible with a limited amount of material.

OTHER TYPES OF SIGNALS

Occasionally the type of steel tower used on first-order triangulation can be employed to advantage. (See fig. 33.) These towers weigh from 2 to 3 tons each and have a maximum height of 103 feet. They can be erected without the heavy tackle needed for tall wooden towers and in much less time.

Where lumber is not readily available a portable wooden signal has been used to advantage on coastal triangulation. It consists of inner and outer tripods, the inner one having a height of 15 feet. The members of both the inner and outer tripods are fastened together by bolts. The diagonals are of $\frac{3}{16}$-inch wire and are provided with turnbuckles. The entire signal weighs about 175 pounds and can be dismantled by two men in half an hour. These signals have been used with a direction theodolite on very swampy, unstable ground and second-order accuracy secured, but care must be taken under such conditions not to have the footings for the inner tripod legs too close to the footings for the corresponding legs of the outer tripod.

When the requirements of the hydrographic or topographic surveys, carried along in conjunction with the triangulation, necessitate a special kind of signal, the types described in the Hydrographic Manual (Spec. Pub. No. 143) or the Topographic Manual (Spec. Pub. No. 144) should be used, so long as they permit the observer to obtain the specified accuracy on the triangulation.

RECONNAISSANCE FOR THIRD-ORDER TRIANGULATION

The same principles and methods are applied in reconnaissance for third-order triangulation as for second-order, though the allowable limits of some of the controlling factors are different, as noted below.

Strength of figures.—In the main scheme the value of R_1 (see p. 9) for any one figure must not exceed 50, and that of R_2 for the figure must not exceed 150, except when conditions are such that it is very difficult to keep within these limits. The quantities R_1 and R_2 for a single figure should be kept down to 35 and 120 for the best and second-best chains, respectively, whenever the total estimated cost for the reconnaissance and observing does not exceed that for the chain barely within the extreme limits by more than 25 per cent.

Frequency of bases.—When a base site can be found near any desired location, ΣR_1 between base lines, whether these are actual measured bases or lines of first or second order triangulation used as bases, should be made about 125. Where topographic conditions make it difficult to measure a base near the desired location, ΣR_1 between bases may be increased but should never greatly exceed 175. When this upper limit is approached an intervening base may be necessary, for if the discrepancy in length between bases exceeds 1 part in 5,000, after the side and angle equations have been satisfied, either an intermediate base must be measured or the angle observations made more accurate.

Marking of stations.—Stations of third-order triangulation must be marked and referenced by marks of the same size and character as stations of second-order triangulation. (See p. 38.)

CHAPTER 2.—SECOND AND THIRD ORDER TRIANGULATION

SPECIFICATIONS, SECOND-ORDER TRIANGULATION

The following specifications for angle measurements on second-order triangulation were approved by the Director of the Coast and Geodetic Survey on December 10, 1928, and supersede all previous instructions for work of this character. Specifications for third-order triangulation will be found on page 83.

1. **Accuracy.**—Either a direction or a repeating instrument may be used in triangulation of this class, though the required results can usually be obtained more quickly and economically with a direction theodolite with micrometers. The immediate requirements which the angle measurements must meet are that the average closure of the triangles in the main scheme shall not exceed 3 seconds, that an effort be made to keep the average closure down to 2.5 seconds, and that the maximum closure shall but seldom exceed 6 seconds.

2. **Observations with repeating theodolite.**—A set of observations should consist of six repetitions of the angle with the telescope in the direct (or reversed) position, followed immediately by six repetitions of the explement of the angle with the telescope in the reversed (or direct) position.[1] With the ordinary type of 7-inch repeating theodolite equipped with verniers reading to 10 seconds, the accuracy specified for second-order triangulation will usually be obtained by making from two to three sets of observations on each angle.

3. **Circle settings.**—When two or more sets of observations with either a direction or a repeating theodolite are made on the same angle the initial setting for each set should differ by an amount depending upon the number of positions to be observed and the number of verniers or micrometers on the theodolite. The interval in degrees between successive settings with a 2-micrometer or a 2-vernier theodolite is given by the formula $I=\frac{360°}{mn}$, where I is the interval in degrees, m the number of verniers or micrometers, and n the number of positions or sets. In addition an increment represented by the value of one division of the circle divided by the number of sets to be observed should be added to the difference in degrees between settings in order to eliminate the error of graduation of the verniers or the run of the micrometers. For instance, with a circle graduated to 10 minutes and with two sets observed on an angle, the settings would be 0° 00′ 00″ and 90° 05′ 00″.

4. **Program of observing.**—With a repeating theodolite, measure only the single angles between adjacent lines of the main scheme, including the angle necessary to close the horizon. In the comparatively rare cases in which the failure of adjacent signals to show at the same time prevents carrying out this program make as near an approach to it as possible and then take

[1] The abbreviation "6 D/R" is ordinarily used to indicate a set of six observations with the telescope direct and six with it reversed. Similarly, "3 D/R" means a set of three observations direct and reversed.

the remaining signals in another series together with some one, and only one, of the signals observed in the first series and measure in the new series only the angles between the signals which have not been observed upon and the angle necessary to close the horizon. In other words, the measurement of an angle which is the sum of two or more observed angles should be avoided. With this scheme of observing no local adjustment is necessary, except to distribute each horizon closure uniformly among the angles measured in that series.

5. Observations with direction instrument.—One complete measure of the horizontal direction to each station from some one selected initial station, with telescope both direct and reversed, is called a position. With the type of micrometer direction instrument which is read to single seconds four to six positions are usually sufficient to secure second-order accuracy, and with a theodolite reading to 2 seconds, six to eight positions will ordinarily be adequate.

6. Circle settings.—In order that the readings of the micrometer microscopes may be uniformly distributed around the graduated circle during the measurement of any set of directions the circle is shifted between any two successive positions by an amount determined in the same manner as described for repeating theodolites on page 33.

Initial settings for 2-micrometer theodolite

TWO POSITIONS OF CIRCLE

Position No.	1 division of circle = 4 minutes	1 division of circle = 5 minutes	1 division of circle = 10 minutes
	° ′ ″	° ′ ″	° ′ ″
1	0 01 00	0 01 00	0 02 30
2	90 03 00	90 03 40	90 07 30

FOUR POSITIONS OF CIRCLE

1	0 00 30	0 00 40	0 01 20
2	45 01 30	45 01 50	45 03 50
3	90 02 30	90 03 10	90 06 20
4	135 03 30	135 04 20	135 08 50

SIX POSITIONS OF CIRCLE

1	0 00 40	0 00 50	0 01 40
2	30 02 00	30 02 30	30 05 00
3	60 03 20	60 04 10	60 08 20
4	90 00 40	90 00 50	90 01 40
5	120 02 00	120 02 30	120 05 00
6	150 03 20	150 04 10	150 08 20

EIGHT POSITIONS OF CIRCLE

1	0 00 30	0 00 40	0 01 20
2	22 01 30	22 01 50	22 03 50
3	45 02 30	45 03 10	45 06 20
4	67 03 30	67 04 20	67 08 50
5	90 00 30	90 00 40	90 01 20
6	112 01 30	112 01 50	112 03 50
7	135 02 30	135 03 10	135 06 20
8	157 03 30	157 04 20	157 08 50

A 3-micrometer theodolite will ordinarily not be used on second-order triangulation. Settings for either a 3-micrometer or a 2-micrometer instrument for 8, 12, and 16 positions of the circle may be found in Special Publication No. 137, Manual of First-Order Traverse, pages 43 and 44. Settings for any other number of positions may be easily computed by the formula already given.

The settings given in the tables are those actually to be made with the *A*-micrometer of a 2-micrometer theodolite. Since the telescope is reversed between the settings for any two consecutive positions, the given settings require the changing of the circle almost 180° in orientation for each new position. This tends to maintain the circle at a uniform temperature, even though temperature conditions may not be the same on all sides of the instrument. It is possible to arrange other tables of settings which will give an equally good distribution of the readings around the circle, but those given above are as easy to memorize as any. It is not necessary to set the circle nearer than about one-half minute to the setting given.

On second-order triangulation the settings given may be used even when a new initial must be used to complete a series of observations at a station. On first-order triangulation, especially with a poorly graduated circle, the angle between the old and new initial is usually added to the prescribed setting for each position when using the new initial.

7. Incomplete series, direction theodolite.—It frequently happens that one or more signals are not visible during all or a part of the time that observations are being made upon the other stations. Little time should be spent in waiting for a signal or a light to show. The positions missing from the first series can be observed later, using the same initial as was used during the first series, or some other main-scheme station observed upon during that series. Not more than two initials should be used at any one station. With this system of observing, no local adjustment is necessary aside from that arising from observing back upon the initial at the middle of each position and using the mean of all readings upon the initial. (See fig. 54, p. 86.)

8. Observations on intersection stations.—An intersection station is one which is not occupied, the position of which is determined by observations upon it from stations of the main scheme or from supplementary stations. It may be a signal over a marked point or it may be a well-defined natural or artificial object, such as a tank, church spire, or sharp mountain peak. In these specifications the term "intersection station" is used in a restricted sense to mean a station located by intersections with fewer observations than are specified for second-order triangulation. A line to such a station must not be used as a base from which to start second-order triangulation.

The direction method of observation should be used in observations upon intersection stations even if the theodolite is a repeater. Each series of observations on intersection stations should contain some one line, and only one, of the main scheme. Two positions should be observed upon each intersection station with a vernier instrument or with a direction instrument reading to two seconds. One position is adequate with a direction instrument reading to one second, but in this case a second round of pointings should be made with the telescope either direct or reversed in order to provide a check on the readings of degrees and minutes on the first position. It is advisable to observe upon each intersection station from at least three stations in order to obtain a check upon the position, and the directions from at least two of the stations should, if practicable, form a good angle of intersection at the object to be located. A possible intersection station should not be disregarded if only two directions to it can be secured. Even one direction may sometimes be of value when used with another direction to the same object taken during a different season's work.

9. Value of intersection stations.—In selecting intersection stations it should be kept in mind that the geographic value of triangulation depends upon the number of points determined, the size of the area over which they are distributed, and the permanence with which they are marked. The geographic value of a triangulation is lost for a given area when stations can not be recovered within that area. The chance of permanency is made greater by increasing the number of stations as well as by thorough marking. For the reasons stated there should be determined as intersection stations many artificial objects of a permanent character, such as lighthouses, church spires, cupolas, towers, chimneys, and standpipes. Make the description definite whenever practicable. Instead of describing the object as "church spire" with the name of the town, make its identity certain by giving street location and denomination of church. Descriptions of intersection stations should be submitted on Form 525, even though the name itself may be a description. (See p. 112.)

10. Distance between points.—On all triangulaton near the coast, recoverable points suitable for the control of topography shall be located not more than 5 miles apart, measured along the general trend of the coast line. This distance should be reduced to 2 miles when the nature of the country makes it practicable without considerable additional expense. If objects suitable for the purpose do not exist, then marked stations must be established. A station to be suitable for the control of topography must be located on the shore or be easily accessible from the shore. In addition, on work of this character all prominent objects visible from the sea and suitable for charting as landmarks or as hydrographic signals should be located. In a mountainous country, such as Alaska, the location and elevation of prominent mountain peaks visible from the sea and beyond the limit of the fringe along the coast usually covered by the topographer shall be determined if visible from triangulation stations. Information indicating the relation of such elevations to the surrounding country is of value, and it is suggested that a sketch be made on a small-scale chart showing the trend of the ridges and location of valleys.

11. Report on "landmarks for charts."—All objects located by a triangulation party which should be shown on the charts shall be listed on Form 567 and shall be forwarded under separate cover from other records at the end of the season.

12. Trigonometric leveling.—No measurements of vertical angles for the determination of the elevation of stations need be made on second or third order triangulation unless specifically provided for in the instructions for a particular project. When such observations are authorized a sufficient number of lines should be observed over in both directions to provide a check value of the elevation of each station. Nonreciprocal observations of vertical angles are of little value in carrying elevations through a scheme of triangulation but are necessarily used in determining the elevations of intersection stations. Vertical angles should always be measured on prominent peaks lying outside the limits of plane-table sheets. When practicable, connections should be made at frequent intervals to bench marks established by spirit levels or to the water level of the sea or to that of lakes and tidal rivers. In connecting to water level it is sufficient to record an observed vertical angle to the water's edge and an approximate measured or scaled distance to the point sighted upon, with a note as to the time of the observation or to the height of the tide when the water's edge is sighted upon. (See p. 81.) Record and computation forms are shown on pages 104–110.

13. Plane of reference.—All heights will be referred to mean sea level.

MARKING OF STATIONS

Each triangulation station which is not in itself a permanent mark, such as a lighthouse, a spire, or a tank, but which is in a location where it can be permanently marked or referenced, shall be marked in accordance with the specifications which follow. When an old triangulation station, established by this bureau, is recovered and found to be inadequately marked it shall be re-marked and referenced. Care shall be taken that the descriptions of stations established or recovered shall fit existing conditions.

Certain classes of objects used frequently as intersection stations, such as flagpoles, signal masts, beacons, and even church spires, are frequently moved or destroyed and rebuilt near their former location. The inadvertent use of the old geographical position for the new location of the object often leads to serious errors and delays. Care should be taken to make the original description as definite as possible to assist in later identification. When such points are recovered they should not be used without adequate checking to make certain that no change in location has taken place.

TRIANGULATION AND TRAVERSE MARKS OF OTHER ORGANIZATIONS

On April 10, 1928, the Board of Surveys and Maps adopted the following report:

Whenever an adequately equipped field party of the United States Geological Survey, the United States Coast and Geodetic Survey, the Corps of Engineers of the United States Army, the Forest Service, the Mississippi River Commission, the General Land Office, or other member organization of the Board of Surveys and Maps is engaged in triangulation or traverse it should make connections with any control marks in the vicinity of the work established by other member organizations, provided such connections can be made without undue expenditure of time. The connection should be made with third-order accuracy and should be made either by direct measurement of a distance and direction from a triangulation or a traverse station by the three-point method with check, or by the intersection method. Recovery notes for such stations shall be sent to the main office of the organization to which the field party belongs, where a duplicate will be made and sent to the organization which originally established the station.

Where a station of third-order or higher accuracy, originally established by another organization, is definitely recovered, but is found to be inadequately marked, it should be re-marked in a permanent manner, and the character of the new mark described in the recovery note. If the station was originally marked by a tablet, the tablet should be reset in the new mark. If the station was not marked by a tablet, then a tablet such as is now used by the organization which established the station should be secured by the field party from its own headquarters and used as the new station mark. If there is not time to secure the tablet, a copper bolt may be stamped with the initials of the organization which first established the mark, and this bolt should be used as the station mark. If the original station can not be exactly recovered or if, for any reason, it is not desirable to reoccupy it, a new station may be established near by and marked, and the old station should be connected with the new one and used as a reference mark for it. In general, no survey monument established by one organization of the Government should be changed or replaced by another organization unless there is an agreement between the two organizations regarding such replacement.

However, a station established by another organization should never be re-marked unless it had a subsurface mark and this was definitely recovered or unless the tablet was established in rock and its original location can be definitely and accurately determined. The recovery note should describe the present condition of the surface, subsurface, and reference marks in sufficient detail to make the record clear as to the certainty of recovery of the station.

Triangulation parties of this bureau will follow the procedure outlined in the above report.

SPECIFICATIONS FOR MARKING TRIANGULATION STATIONS

1. Metal tablets.—Each station which has been located with first, second, or third order accuracy (see p. 3) should be marked by a standard tablet of copper alloy, so fastened in the rock or concrete as to effectively resist extraction, change of elevation, or rotation. (See figs. 34 and 35.) The name of the station and the year established should be stamped upon the mark, preferably before it is set in the rock or concrete.[2]

2. Setting of tablets.—Stations for horizontal control must often be located where the permanent marking of them is difficult, and for that reason a great variety of settings for the tablets must be permitted. The location of the station, depth of soil, or presence of rock ledges, and the availability of materials will usually control the choice of the mark to be used. The precautions to be taken in establishing each kind of mark are briefly stated below.

(1) *In rock outcrop.*—Care should be taken that the rock in which a mark is set is hard and is part of the main ledge, not a detached fragment. The tablet should be countersunk and well cemented in.

(2) *In bowlders.*—When a tablet is set in a bowlder the latter should be of durable material and of cross section, area, and depth below the surface not less than the standard concrete mark as described below.

(3) *In rock ledges below surface.*—When the ledge is only slightly below the surface a tablet set in the usual manner in the ledge will be sufficient, provided two reference marks are established. Where the ledge is so far below the surface that a surface mark is required a tablet or copper bolt should be set in the ledge, the ledge carefully brushed or washed off for a space at least 18 inches in diameter, and a concrete surface mark placed above the subsurface mark. A tablet should be set in the surface mark directly over the subsurface tablet or bolt. If the rock ledge in which the subsurface mark is set is very

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[2] The authority for the warning concerning punishment for disturbing the mark is contained in an act of Congress, approved March 4, 1909, entitled "An act to codify, revise, and amend the penal laws of the United States," and reads as follows: "Whoever * * * shall willfully deface, change, or remove any monument or bench mark of any Government survey shall be fined not more than $250, or imprisoned not more than six months, or both." (35 Stat. 1088, sec. 57.) Many States have also enacted additional laws on the same subject, among them being California, Connecticut, Georgia, Illinois, Indiana, Maine, Maryland, Massachusetts, Minnesota, Michigan, Missouri, Mississippi, New Hampshire, New Jersey, Ohio, Oregon, South Carolina, Tennessee, Vermont, Virginia, Washington, and West Virginia.

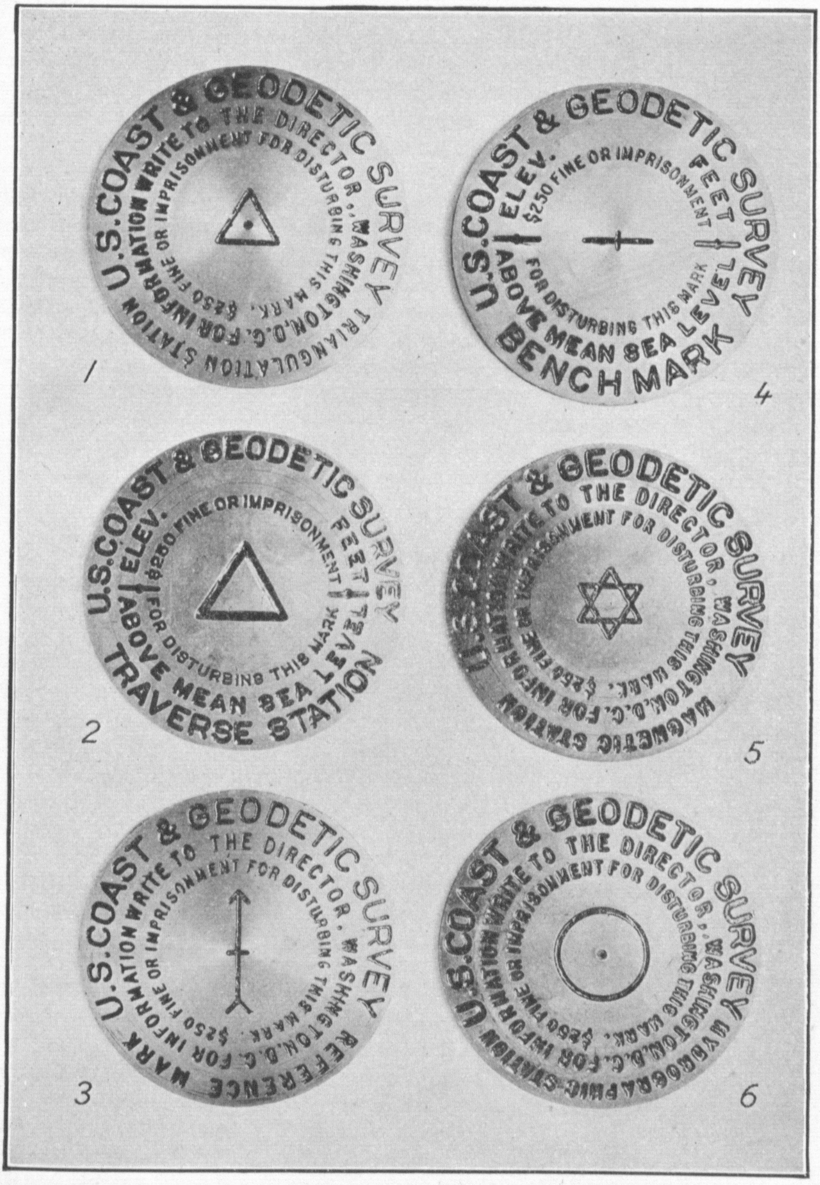

FIG. 34.—STANDARD MARKS OF THE U. S. COAST AND GEODETIC SURVEY

1. Triangulation station mark
2. Traverse station mark
3. Reference mark
4. Bench mark
5. Magnetic station mark
6. Hydrographic station mark

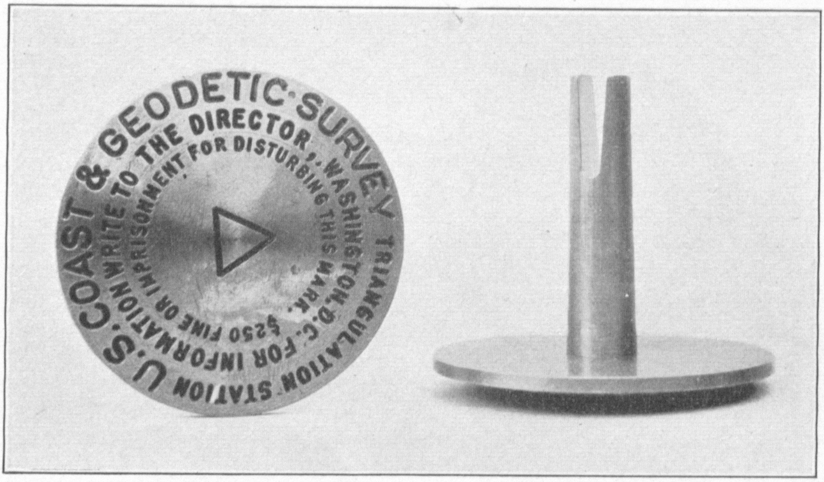

FIG. 35.—TRIANGULATION STATION MARK

smooth, it should be furrowed with a chisel to afford better anchorage for the concrete.

(4) *In concrete.*—(*a*) *Shape.*—The mark should be either a frustum of a cone or of a pyramid, or have the form of a post with an enlarged base. If of pyramidal or conical form, the sides should have a batter of at least 1 inch to 1 foot. When a post with an enlarged base is used the bottom of the base should be 4 inches larger in least horizontal dimension than the post proper and should have a vertical thickness of at least 6 inches. If the concrete is cast in place, the enlarged base can easily be provided for by enlarging the bottom of the hole at the sides with the digger. Extreme care should be used to avoid making the mark with a mushroom top or with projecting corners near the surface, which would provide leverage points for frost action and would make easier the malicious destruction of the mark.

(*b*) *Size and depth.*—The concrete post should extend to a depth of from 30 to 36 inches, depending upon the kind of soil. It should be not less than 14 inches in diameter, except that the upper 12 inches may be in the shape of a frustum of a cone or pyramid with the upper surface not less than 12 inches in diameter. Where the mark is not in the path of traffic or in soil subject to cultivation it should extend from 2 to 4 inches above the surface. When located where traffic passes over it the top of the mark should be slightly below the surface.

(*c*) *Underground mark.*—A standard station-mark tablet, set in a mass of concrete at least 10 inches in diameter and accurately centered under the station mark, should always be established for an underground mark where soil conditions permit. The tablet should be stamped with the same designation as the tablet in the surface mark.

The procedure in making the standard concrete mark is as follows: A hole is dug to a depth of 3½ feet or more. It should be 16 inches in diameter for the top 2½ feet and 10 inches in diameter at the lower end. Concrete made of good cement, sand, and gravel or broken rock is placed in the lower part of the hole to a depth of 6 inches. A standard tablet station mark (fig. 35) is then set in the concrete, with the top of the tablet slightly depressed. This completes the underground mark. A layer of from 4 to 6 inches of sand or dirt is then put into the hole. The hole is then enlarged about 2 inches in radius near the bottom in order that the lower end of the block of concrete for the surface mark will be mushroomed, and then the hole is filled with concrete to within 9 inches of the surface of the ground. Next a mold or frame 12 inches on a side at the top, 13 inches at the bottom, and 12 inches in depth is set in the hole on top of the concrete and filled in around the outside with dirt tamped firmly. The frame is then filled with concrete level with its top and a standard tablet station mark is set in the center of the concrete, with the top of the tablet slightly depressed. The tablet must be centered exactly over the underground mark. The top of the concrete should be smoothed with a trowel and the frame should be left in place to protect the concrete until it becomes firmly set.

Care must be taken not to disturb the position of the tablet in the underground mark when placing the layer of sand or dirt and when pouring the concrete for the surface mark. A piece of thin board should be placed over the lower mark or other suitable means used to insure against any horizontal movement of the tablet due to the impact or pressure of the material above.

3. Special conditions.—Under certain conditions special marks will often be required, and these should conform in size and durability to the marks described above.

(1) *Sand.*—In sand, which if used as a mold would spoil the concrete by absorbing the water from it, sewer tiles 8 inches in diameter and 30 inches long may be used, set with the bell end down, filled with concrete, and with the base end set in concrete. A sheet-iron mold of the same dimensions filled with concrete may also be used. A metal tablet should be set in the center of the top.

(2) *Marsh.*—Where the surface of the ground is too soft to hold a mark of the usual type a post of durable wood should be forced down vertically as far as it will go, its top cut off flush with the surface, and a sewer tile at least 6 inches in diameter set into the marsh around the top of the post. The tile should then be filled with concrete and a tablet set in the top. The post should always be water-soaked before being used, as otherwise it will swell and break the concrete which incases it. Where the marsh is very soft but dries out at certain seasons of the year successive tiles can be forced down around the post, the post then can be withdrawn and the mud worked out from within the tiles, and the tiles then filled with a hydraulic cement mixture.

(3) *Land subject to cultivation.*—The subsurface or lower mark should be a tablet in a block of concrete 10 inches square or 10 inches in diameter and 6 inches thick, set with its top 3 feet below the surface. The upper mark should be a tablet set in a block of concrete 15 inches in least horizontal cross-section dimension and 20 inches high, with its top at least 12 inches below the surface of the ground. About 3 inches of dirt should be placed between the concrete blocks bearing the upper and lower marks.

All stations so marked should be referenced by two standard reference marks preferably placed on property boundary lines, in a location where there is little likelihood of their being disturbed. The directions to the reference marks should be such as to give a good angle of intersection at the station. The reference marks may be as much as a half mile from the station, if necessary, provided they can be seen from the station. The distance to each reference mark should be carefully measured. Other distances, such as those to the center of a highway, the corner of a building, or the center of a well, should be measured if feasible. Two or more such measurements will intersect so near the station that the concrete block will be easily found with a small prodding rod. When measurements are made to buildings or other objects the directions must be given also. If measurements of this kind are made the station may usually be easily recovered, though the reference marks may both be destroyed. The measurements to a road should always be to the center of the road and not to the fence line. All distances must be carefully measured and not estimated, but those to distant objects need not be reduced to the horizontal if the fact that the measurement is inclined is noted in the description. Care should be taken in placing reference marks along highways, for nearly all States are widening the highways.

4. Reference marks.—Each reference mark should consist of a metal tablet similar in material and shape to the station mark but bearing an arrow which points to the station. A reference mark should be stamped with the same designation as its station mark, and where there is more than one reference mark they should be numbered serially in a clockwise direction, the number to be stamped upon each one. Each should be set under the same conditions as specified for the station mark, except that the concrete post in which it is set may be 2 inches smaller in diameter and 6 inches shorter than for the station mark. No subsurface marks need be used with reference marks.

Each station mark must have at least one reference mark and should preferably have two. If the station mark, due to surface conditions, is entirely beneath the surface, there should be two reference marks, unless there are permanent witness marks, such as road crossings, etc., which will serve to locate the station without an excessive amount of digging. If the station mark is on ground liable to be disturbed or washed away, two reference marks should invariably be established. These should be so located as to avoid the probability of both being disturbed by the same cause. They should also preferably be so located as to give a good angle of intersection at the station, or else be placed in range with the station. The distance and direction to each reference mark must be carefully measured.

5. **Azimuth mark.**—At each station where a tall signal tower is needed to enable the observer to see the adjacent stations of the scheme an azimuth mark should be established. This mark should be placed several hundred feet away, at some point visible from an instrument mounted on an ordinary tripod set on the ground over the mark. An engineer recovering the station will then not only have a geographic position but will be able to obtain an azimuth as well. The azimuth mark should be of the same size and character as a reference mark.

6. **Material for marks.**—The main considerations in making concrete are to have clean materials, mix them well before adding water, have the mixture not too wet, and tamp well into the form. Each streak of dirt in concrete means a line of cleavage. Where rough aggregate is available the proportions should be about 1–2–3, with the top 12 inches of the mark of slightly richer mixture. Where only cement and sand are available the lower part of the mark should be proportioned 1 part of cement to 3 parts of sand, and the upper part should be 1 part of cement to 2 parts of sand. With a mark of the proper size it will not be necessary to reinforce the concrete with metal rods or wire. To avoid cracking of the concrete, due to rapid drying, it should be covered with paper or cloth and then with earth or other material for a period of at least 48 hours.

HORIZONTAL ANGLE MEASUREMENT

INSTRUMENTAL PROCEDURE, REPETITION METHOD

The procedure used in making a set of observations is as follows: Set the circle approximately at one of the readings given in the appropriate table of circle settings on page 34 and record the exact reading. Point on the left-hand object by means of the lower clamp and tangent screw, which does not change the reading. Then unclamp the upper motion and point upon the right-hand object, perfecting the pointing with the upper clamp and tangent screw. Record the approximate reading of the circle. This completes one measure of the angle. The lower clamp is then released and the operation repeated, except that the circle is not read. The circle reading, if made, would equal the original setting plus twice the angle measured. The process of adding the angle to itself is continued until several measures, usually six, are accumulated on the circle. A reading of the circle is made and recorded after the third repetition as a check on the value of a single angle, and a reading is also made at the completion of the sixth repetition. Next revolve the telescope about its horizontal axis, keeping the upper clamp tight and point upon the right-hand object by means of the lower clamp and screw. Then loosen the upper clamp, move the telescope clockwise, and point upon the left-hand object. This completes one measure of the explement of the angle. Make the same number of measures of the explement as was made of the angle, when the circle reading should be nearly the same as on the original setting. The circle should then be read. (For sample of record, see fig. 52.) Before beginning another set, the circle reading should be changed in order that an error in reading may not affect two angles.

INSTRUMENTAL PROCEDURE, DIRECTION METHOD

The direction method of measuring angles consists of measuring the direction to each station from some one station taken as an initial. The directions are the angles measured clockwise from the initial station to each of the other stations. The angle at a station between any two observed stations is the difference of their directions.

In observing, a pointing is made on the initial station and then upon each station around the horizon in a clockwise direction; the telescope is then reversed and the readings made back in a reverse direction. A direction theodolite does not usually have a slow-motion screw for the lower motion, though the direction method of observing may be used with a theodolite arranged for repetitions by keeping the lower motion clamped. A direction theodolite is almost invariably read by means of micrometer microscopes. On first-order triangulation the horizon is not closed on each position, though an occasional reading is taken back on the initial just before and immediately after reversing to ascertain if any indications of drag are present. On second-order triangulation, however, where the number of positions taken is small, the horizon should be closed on each half position. The form of record is shown in Figure 54.

A theodolite having a graduated circle 9 inches or more in diameter is usually read by micrometers to single seconds. Eight-inch direction theodolites are usually read to the nearest even second. The least division on the micrometer drum of a 6-inch direction theodolite is usually equal to 10 seconds and the readings should be estimated to the nearest even second. The range of the readings secured with any theodolite will determine the refinement with which it should be read. For instance, if a number of readings of a single setting with the circle clamped shows a range of 2 or 3 seconds all readings should be taken to single seconds, but if the range is 4 or 5 seconds readings to the nearest even second are all that are warranted. When a comparatively small number of readings are taken, as with the Wild theodolite (see p. 60 and fig. 40, p. 45), readings may be made to a fraction of a second.

PROGRAM FOR OBSERVING VERTICAL ANGLES

On practically all theodolites used in the Coast and Geodetic Survey for second-order and third-order triangulation the vertical circle is of the type having the graduated circle rigidly attached to the horizontal axis of the telescope and the level bubble attached to the vernier circle. With this mechanical arrangement either one of two observing programs may be used. When using the first method level the theodolite, then with the circle right and the object near the vertical wire (*a*) bisect the object with the horizontal wire, using the telescope-clamp slow-motion screw; (*b*) bring the bubble to the center of the level vial; (*c*) read both verniers; (*d*) turn the instrument 180° in azimuth and transit the telescope, then repeat (*a*), (*b*), and (*c*) in the same order. Do not change the adjustment of the level on the vertical circle between the two pointings of a set. With the second method level the theodolite and with circle right perform operations (*a*), (*b*), and (*c*) as described above on each object around the horizon, then transit the telescope and again perform the same operations upon each object. This is a more rapid method than the one first described.

If the vertical circle is graduated clockwise from zero to 360°, as is usually the case, the reading with circle right minus the reading with the circle left is equal to twice the zenith distance. When the graduation is counterclockwise the subtraction is made in the inverse order. If the system of graduation is different from that described above, a statement and diagram showing the kind of graduation should be made in the record book when the first observations with the instrument are recorded.

When observing upon an object at a considerable elevation, using a straight eyepiece, care should be taken to eliminate the parallax as completely as possible, for otherwise a constant error may enter into the observations. When observing upon stars near the prime vertical for the determination of local time the error in the derived times due to parallax is of opposite sign for east and west stars.

CORRECTION OF RECORDS

All observations should be recorded clearly and distinctly on the proper forms. The numbers must be written so plainly that there is no chance of misunderstanding them. If for any reason a temporary record must be made and the observations transferred later to the record book, the temporary record should be sent to the office with the record book.

Erasures must never be made in the original records; if it is necessary to change a figure, it should be lightly crossed out and the

696689°—46——4

correct figure placed above it or to one side. If a figure is changed after the complete set of observations have been recorded and no chance exists of checking the reading by a reobservation, the initials of the person making the change must be placed near the alteration.

More uncertainty in the results is caused by hurried and ill-considered changes in the recorded figures to correct supposed discrepancies than from any other one source. The doubt is greatly increased if the original figures are illegible. The observer must make certain before beginning work that the recorder understands the importance of observing the requirements of this section.

INSTRUMENTS

THEODOLITES

In the past by far the greater percentage of second and third order triangulation has been done with repeating theodolites. This probably has been due to the fact that the older direction instruments were large and heavy, could not be used on light tripods, and were thus unsuited to the class of work where the stay at a station is short and the number of stations occupied relatively large. However, recent advances in the construction of direction theodolites have resulted in reduced size and weight of these instruments, and it is probable that an increasing amount of second-order and third-order triangulation will be done in the future with theodolites of this type.

In the matter of speed the direction instrument with micrometer microscopes is superior, as a given accuracy in the angle measurements can be attained more quickly with it than with the repeating type of instrument. On the other hand, the fact that with the usual type of direction theodolite the observer must constantly move about the instrument while observing makes the direction theodolite unsuitable for work in cramped quarters.

With the method of repetitions, some multiple of the angle between any two stations is accumulated on the graduated circle between successive readings. Theoretically the method of repetitions is an ideal one, since the value of the single angle should be determined very accurately from the accumulated multiple of the angle, even though the circle can be read to only the nearest 5 or 10 seconds. Experience has proved, however, that there are certain sources of error in the instrument which prevent the securing of extreme accuracy, the principal one being the necessary play in the arrangement of the double centers of the vertical axis.

It should be remembered that of more importance than the size of an instrument is its workmanship, as shown particularly in the accuracy with which the circle is graduated and the centers fitted

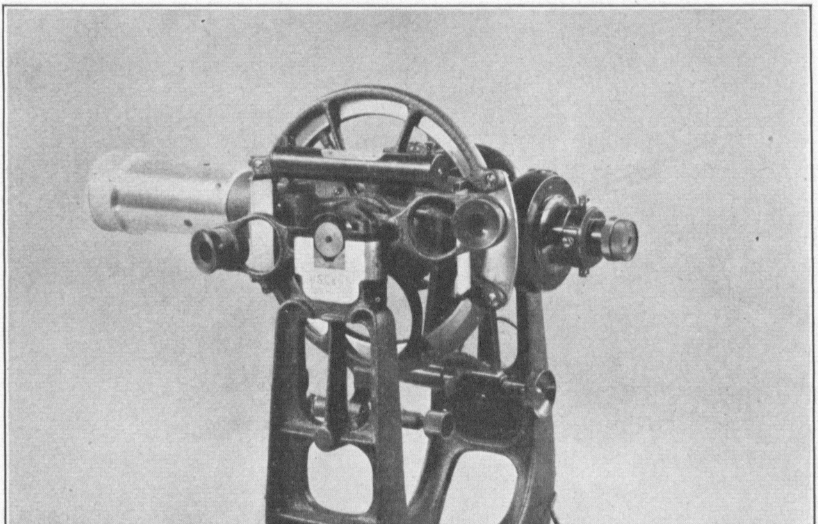

FIG. 36.—REPEATING VERNIER THEODOLITE, 7-INCH CIRCLE

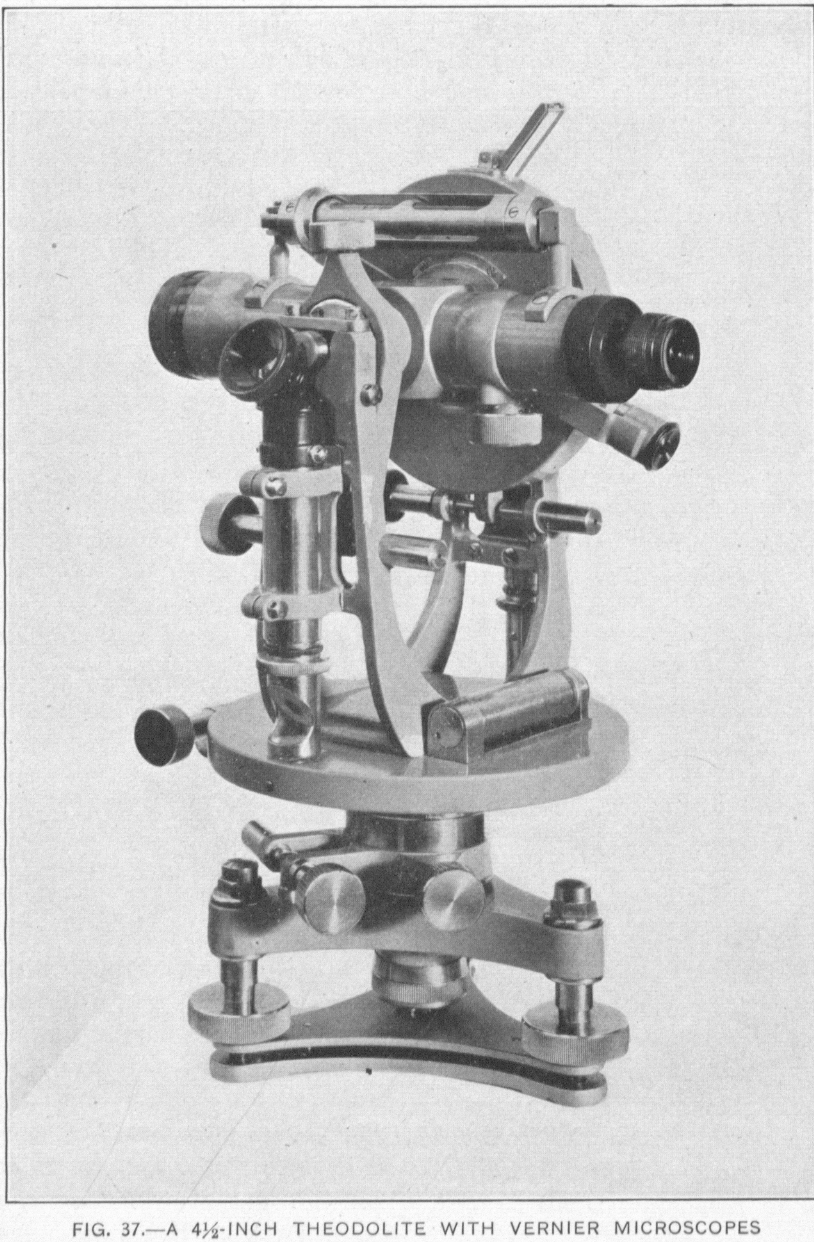

FIG. 37.—A 4½-INCH THEODOLITE WITH VERNIER MICROSCOPES

(For description see p. 60)

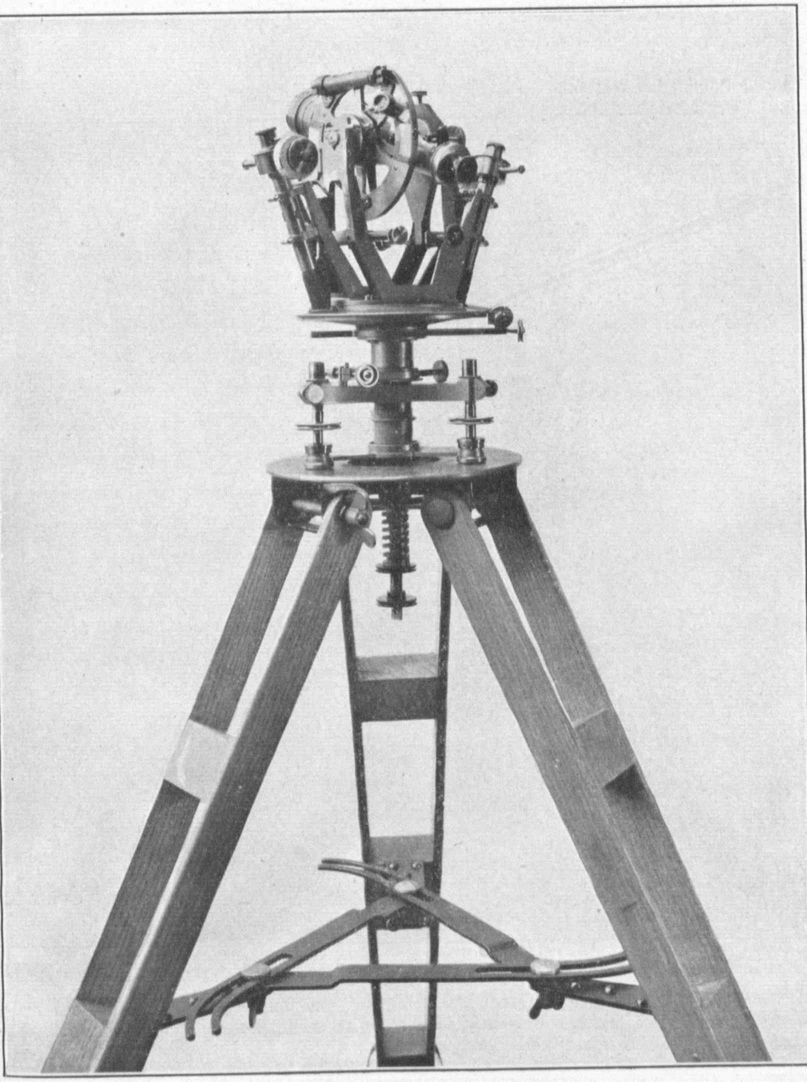

FIG. 38.—MICROMETER THEODOLITE, 6¼-INCH

This instrument may be used with either the direction or repetition method. The braced tripod greatly increases its stability

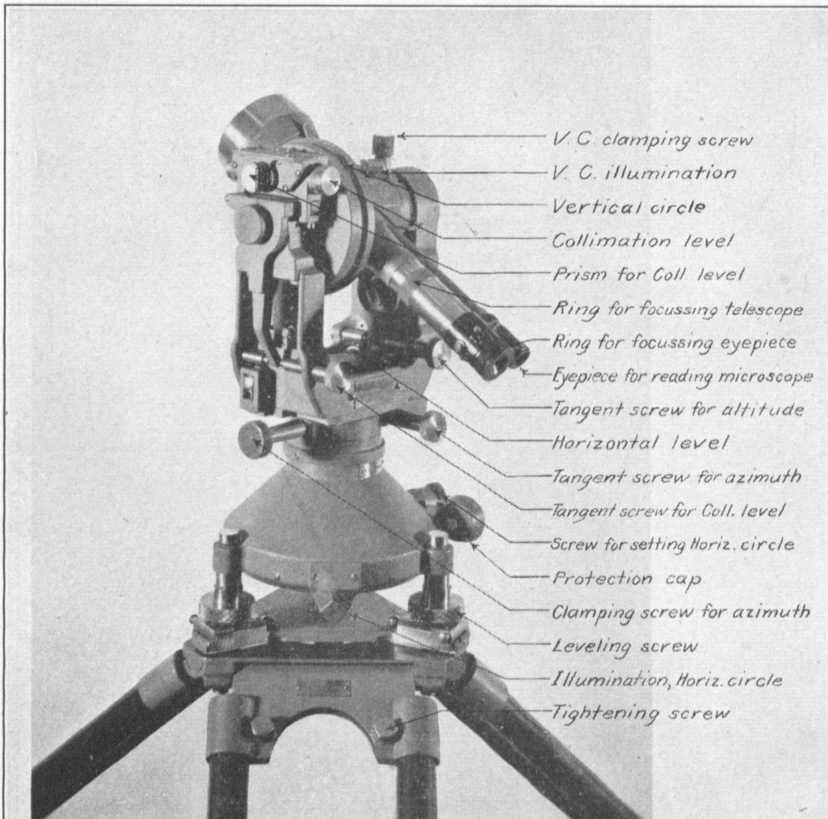

FIG. 40.—WILD PRISM-MICROSCOPE DIRECTION THEODOLITE, 5½-INCH

This theodolite of novel design has graduations on glass. The horizontal circle can be read to $\frac{1}{10}$ second. Opposite readings of both circles can be taken through the ocular alongside the main telescope without changing from the observing position

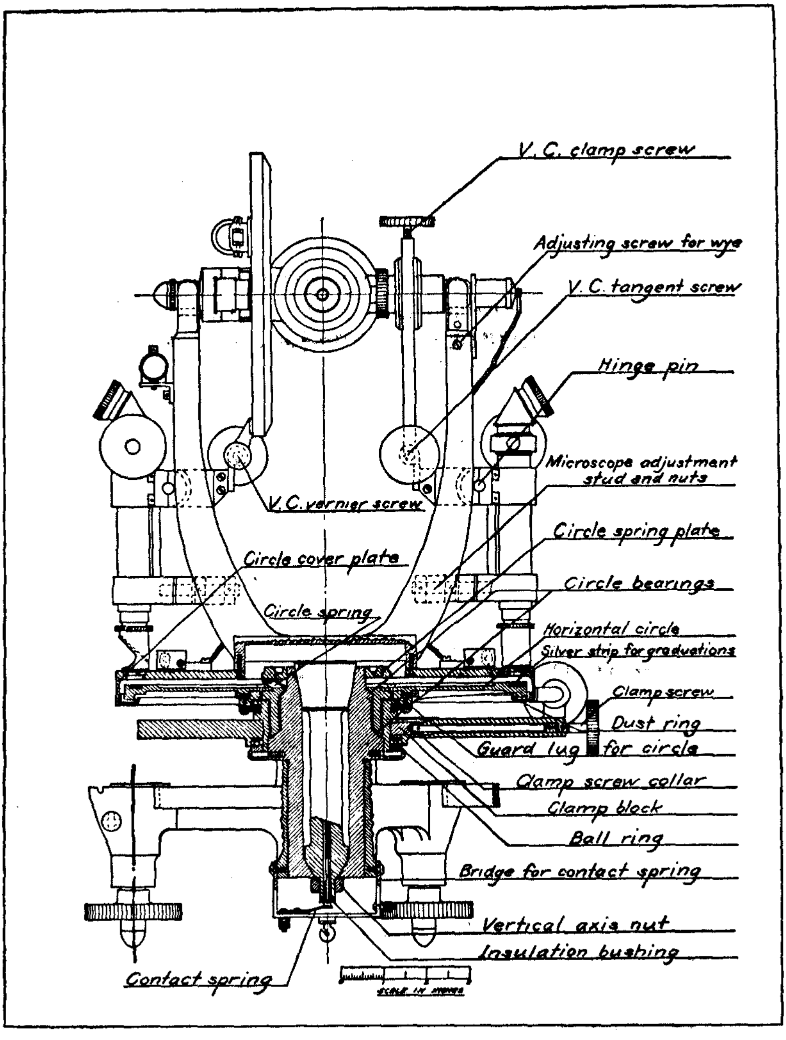

FIG. 39.—Sectional view of Parkhurst 9-inch direction theodolite

The distinctive features of this theodolite are its nonbinding center, ball-bearing clamp ring, illumination through central axis, discontinuous conical bearings for horizontal circle, illuminated glass micrometer drums, and improved designs for tangent-screw assembly, clamp-block assembly, and micrometer mountings.

To remove alidade: (1) Remove screws holding dust ring and drop it clear from alidade cover plate; (2) remove bridge for contact spring; (3) remove vertical axis nut; (4) lift alidade carefully to avoid scraping axis.

To remove circle: (1) Remove threaded circle spring plate; (2) remove guard lugs for circle; (3) lift circle off carefully.

To remove clamp: (1) Pull back and turn spring plunger opposite tangent screw until plunger is in a position to remain disengaged; (2) remove two screws holding clamp together; (3) remove halves of clamp; upper ball race and ball ring can be lifted up and cleaned if desired, using alcohol or high-grade gasoline, then *light oil*; (4) *never* try to remove clamp screw, for the clamp-screw collar prevents its extraction.

together and in the care used in the construction of all micrometer and tangent screws. The proper adaptation of the magnification of the reading micrometers to the pitch of the micrometer screw is also an important factor.

A theodolite is frequently supplied with two or more eyepieces of varying magnifications. The lower-powered eyepieces should be used in pointing upon objects to be located by intersections, which in a hazy atmosphere would not be clearly visible when a high-powered eyepiece is used. In general, the highest-powered eyepiece should be used which will give a fair definition of the object. When observing upon signal lights the highest-powered eyepiece available should ordinarily be used.

On work in mountainous regions a lightweight theodolite is desirable, but if a very light instrument is used, it is not sufficiently stable in azimuth. To increase the pressure upon the support a system of springs is sometimes used in connection with an aluminum base which is screwed fast to the top of the stand supporting the instrument. Such an arrangement is shown in Figure 41. The same result is effected by using the device shown in Figure 42, which has mechanical means for exerting a downward pressure upon the leveling screws.

Many factors enter into the design of a theodolite to make it acceptable to the engineer using it, such as simplicity of construction, convenience in manipulation, compactness, and ease of adjustment; but the observer must usually direct his energies to making the best use of the instrument furnished him instead of deciding upon the theoretically best instrument for his particular purpose. This *should not deter him, however, from informing himself* as fully as possible regarding the principles underlying the design and construction of theodolites, for such knowledge will enable him to better estimate the capabilities and weaknesses of any instrument supplied to him.

A theodolite for use on second-order triangulation should be equipped with a vertical circle. The vertical circles built into modern theodolites are sufficiently accurate for the trigonometrical leveling and for the time observations occasionally required on second-order triangulation. The extra cost of transporting and mounting a separate vertical circle for making zenith-distance observations is not warranted on this class of work.

THEODOLITE ADJUSTMENTS

When measuring angles with the accuracy required on any geodetic triangulation it is best to keep the theodolite in good adjustment, even though the program of observing tends to eliminate most of the errors due to lack of perfect adjustment. The mechan-

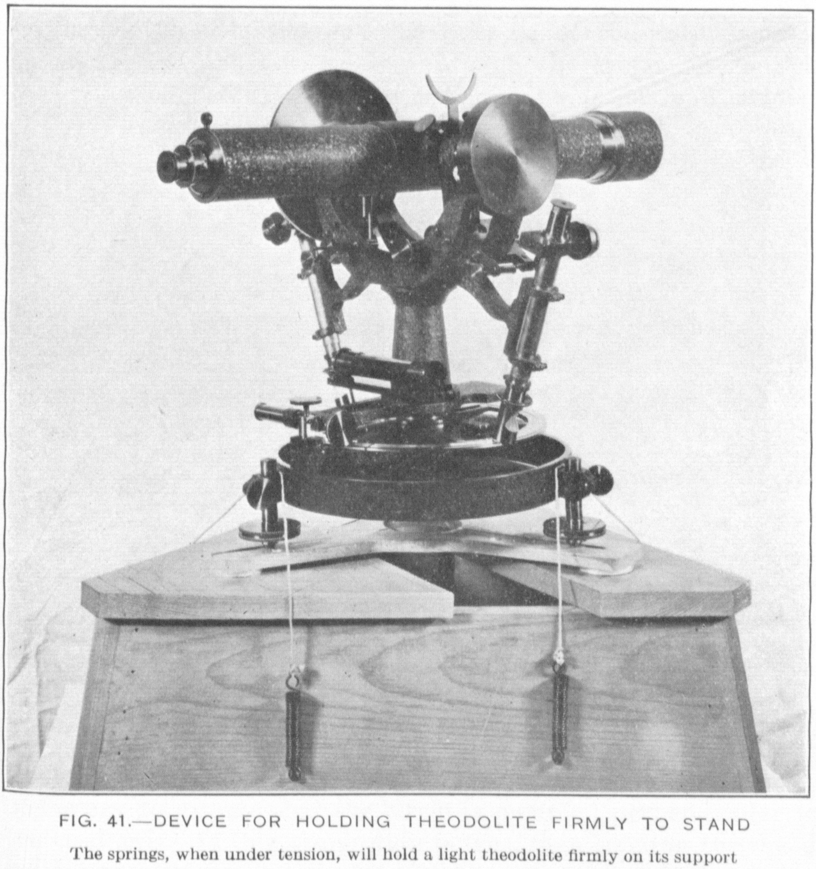

FIG. 41.—DEVICE FOR HOLDING THEODOLITE FIRMLY TO STAND

The springs, when under tension, will hold a light theodolite firmly on its support ical devices provided for making the adjustments vary somewhat on different instruments, but an inspection will usually quickly disclose the method of operation. A general knowledge of the structural features of a theodolite is assumed in the description of adjustments which follows. If a new type of theodolite is to be used and the mechanical means of adjusting it are not readily seen, proceed carefully, for a strained and weakened joint or a stripped screw thread may necessitate the return of the instrument to the shop.

Plate-level adjustment.—The purpose of this adjustment is to make vertical the line passing through the center of the spindle, which is designed to be the vertical axis of rotation, and incidentally

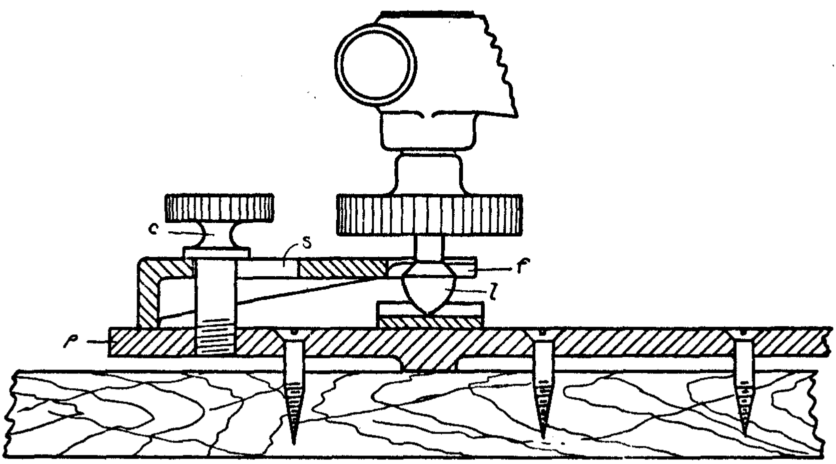

FIG. 42.—Diagram of pressure clamp for theodolite

The slot *s* and the fork *f* in the clamp enables it to be slipped over the enlarged portion *l* of the leveling screw of the theodolite. By tightening the screw *c* the leveling screw is pressed firmly against the plate *p*, which in turn is fastened to the tripod head of the tower. This device increases the steadiness of a theodolite and does not set up strains in the tribrach.

to make the plane of the graduated circle horizontal, since it is necessary to assume that the circle is mounted perpendicular to the vertical axis. If this assumption is true, then leveling the theodolite with a properly adjusted plate bubble will achieve both results.

With the upper motion loose, bring a plate level parallel to the line joining two of the leveling screws and bring the bubble to center with the leveling screws. Turn the alidade bearing the level 180°, checking the angle by sighting over the wyes or telescope or by reading the circle. Correct half the bubble error by the leveling screws and half by the adjusting screws on the bubble. If the theodolite is considerably out of level, turn the alidade 90° and bring the bubble to center by the third leveling screw. Then place the bubble in its original position and repeat the operation until the bubble is in adjustment within less than one division, always checking the final movement of the adjusting screws by an additional testing of the bubble by reversal. After the instrument is properly leveled the bubble of the second plate level, if the theodolite is equipped with two, can at once be brought to the center by the adjusting screws.

Lack of verticality of the vertical axis introduces an error in the measured angle which can not be eliminated by the method of observing, and it is therefore advisable to test this adjustment and relevel the instrument at comparatively frequent intervals. Since on an inclined circle one diametral line can be drawn which is horizontal, it follows that the directions observed will be unequally affected. The error on any pointing is given by the formula,

$$\text{error} = i \tan h,$$

where i is the angle of inclination in seconds as given by the bubble at right angles to the line of sight and h is the angle above or below the horizon of the object sighted upon. The error of any angle is obtained by combining the mean errors of the two directions involved. The magnitude of the errors due to an inclined circle is indicated by the following table:

Errors in horizontal directions due to inclination of vertical axis of instrument

Inclination of the vertical axis (i)	Angle of elevation or depression of the line of collimation (h)	Correction to horizontal direction ($i \tan h$)
"	'	"
10	20	0.06
20	20	0.12
30	20	0.17
10	40	0.12
20	40	0.23
30	40	0.35
10	60	0.17
20	60	0.35
30	60	0.52

Striding-level adjustment.—In describing the adjustments of the level the term "bubble axis" denotes that horizontal line, tangent to the surface of the centered bubble, which lies in the vertical plane through the axis of the bubble tube.

The purpose of the striding-level adjustment is to make the bubble axis parallel to the horizontal axis of the telescope. Strictly speaking, the bubble axis is brought into parallelism with a line which approximates more closely the true horizontal axis the more nearly the telescope pivots assume the form of true cylinders having equal diameters and with their major axes in alignment.

To make the adjustment, place the level in position on the pivots of the horizontal axis of the telescope. Bring the bubble to the center with the leveling screws. Test for "*wind*" by rocking the level slowly forward and back on its supports. If the bubble does not remain centered, then the bubble axis and the horizontal axis of the telescope lie at an angle to each other when projected upon a horizontal plane, and the level is said to have "*wind*." Correct the defect by the screws at one end of the tube, which permit a lateral adjustment.

After the adjustment for "wind" is perfected, bring the bubble to the center again by the leveling screws and then reverse the level. If the bubble does not return to the center, adjust half the discrepancy by the foot screws and half by the adjusting screws on the level. Repeat the test and adjustment until the lack of adjustment does not exceed one or two divisions of the level.

Adjustment of standard.—The purpose of this adjustment is to make the horizontal axis of the telescope perpendicular to the vertical axis of rotation of the alidade.

Adjust the striding level and place it in position on the pivots of the horizontal axis. Bring the bubble to the center by the leveling screws, then rotate the alidade 180° around its vertical axis. The amount by which the bubble is displaced must be corrected, half by adjustment of a standard and half by the leveling screws of the instrument. This process must be repeated until the striding-level bubble shows no displacement when the alidade is rotated through 180°.

In a few instruments no mechanical arrangement is made for the standard adjustment, and the only way it can be made is by carefully filing down and polishing the higher standard. This should be done in the field only when the lack of adjustment is large and will usually not be necessary unless one of the standards has been knocked out of its true position. If the pivots of the telescope are appreciably unequal in diameter, allowance must be made for that inequality in adjusting the standard.

When the axis of rotation of the alidade is vertical the error introduced in the measured horizontal angles by the horizontal axis of the telescope not being at right angles to the vertical axis of the alidade is completely corrected by the reversing of the instrument in the middle of the observation.

Inequality of pivots.—If the pivots are unequal in diameter, the defect can be detected and the amount of the inequality determined by placing a carefully adjusted striding level on the pivots, bringing the bubble to center, reversing the telescope in the wyes so that each pivot lies in a different wye than at first, and then reading the level. With the method of observing in use in the Coast and Geodetic Survey, whereby there is no change of the pivots in the wyes during the occupation of a station, no error in the angle measures results from the pivots being unequal in diameter.

Irregularity of pivots.—If the pivots are not truly cylindrical, a striding level placed in position on the pivots will change its reading as the telescope is slowly rotated on its horizontal axis, unless the pivots are irregular in exactly the same amount and phase. If the irregularity is large enough to cause an appreciable movement of the bubble of the level, it will cause errors in the measured horizontal angles.

If either irregularity or inequality of pivots exists, that fact should be called to the attention of the office when the instrument is returned at the end of the season, in order that the pivots may be reground.

Focusing adjustment.—The error due to change of focus is eliminated by the usual method of reversal of the telescope during the observations.

Adjustment for parallax.—Point the telescope toward a light surface, such as the sky. Screw the eyepiece of the telescope in or out until the wires show the sharpest and blackest. Next focus the telescope on a distant object, and then test the adjustment by moving the eye slowly across the front of the eyepiece. If the wires appear to move over the image of the object sighted upon, parallax is present. The focus of the object glass of the telescope should be changed until the objective is at the proper distance to cause the image of the object sighted upon to fall exactly on the plane of the cross wires; in this position no movement of the wires over the field of view will be apparent when the eye is moved across the eyepiece. The adjustment for parallax must be closely watched, for the error due to lack of proper adjustment is not eliminated by the method of observing. It is especially noticeable if the eye of the observer, because of a strained position, is not in front of the exact center of the eyepiece.

The adjustment of the eyepiece must also be tested frequently, for as the eyes of the observer tire the focal distance of the lenses of the eye changes, causing a blurring of the wires and an increased eye effort in centering the image of the light between the sighting wires.

Adjustment for verticality of sighting wires.—To test if this adjustment is necessary, point upon a well-defined object with the instrument leveled. Swing the telescope slowly in elevation while watching the position of the object in the field of view. If the object changes its position with relation to the vertical wires as the telescope changes in elevation, the diaphragm must be rotated around the longitudinal axis of the telescope. An examination of the telescope will usually quickly disclose the mechanical means for accomplishing this purpose.

Collimation adjustment.—The purpose of this adjustment is to make the line of collimation of the telescope perpendicular to its horizontal axis. To make the adjustment, point the telescope upon some sharply defined object, and, with the alidade clamped, lift the telescope from the wyes, rotate it 180° around its longitudinal axis, and replace it in the wyes, the pivots being in different wyes than in the original position. If the object is not bisected after reversal, correct half the discrepancy by shifting the reticule by means of the screws provided and again bisect the object by using the tangent screw. Repeat the test to check the adjustment.

If the instrument can not be reversed in the wyes, set a stake, *A*, several hundred feet distant and bisect it with the wires. Plunge the telescope with the alidade clamped and set a second stake, *B*, in the opposite direction at almost the same distance to avoid having to change the focus. Both points should be as nearly as possible in the plane of the horizon of the instrument to prevent errors due to imperfect leveling. Rotate the alidade about its vertical axis and then bisect the stake *A*. Plunge the telescope with the alidade clamped, and if the wires do not bisect stake, *B*, set another stake, *C*, in the line of sight close to *B*. Set a fourth stake, *D*, one-fourth the distance from *C* toward *B* and adjust the wires by means of the reticule screws to bisect *D*. Check the adjustment.

A procedure which will give an approximately correct adjustment for collimation is to point on some sharply defined distant object with both motions clamped, read both verniers or micrometers, plunge or reverse the telescope, loosen the upper motion, and set the micrometers so the mean reading will be exactly 180° from the mean of the first readings. If the object is not bisected, correct half the discrepancy by the reticule screws and half by the tangent screw. Repeat the test as a check.

If the collimation adjustment is perfect, the line of sight defined by the center point of the sighting wires describes a plane perpendicular to the horizontal axis when the telescope is rotated on the horizontal axis. When the adjustment is not perfect the line of sight describes a cone. The correction for the error of collimation is equal to $c \sec h$, where c is the angle of error in the horizon and h is the altitude of the object sighted upon. The error of collimation is eliminated from the result by taking the mean of the direct and reversed readings.

Adjustment of level on vertical circle.—Such theodolites of this survey as are equipped with vertical circles have that circle rigidly attached to the horizontal axis of the telescope, with the level bubble mounted on the vernier frame, the verniers not being adjustable. Since the method of observing entails the reversal of the telescope during each measure of a vertical angle, thus eliminating the error of the level, it is customary to leave the level bubble on the vertical circle unadjusted.

If it is desired to adjust the bubble, use the following method: Establish a truly horizontal line by means of the "peg" method. This is done by setting up the theodolite and leveling it. Two pegs, *A* and *B*, are driven into the ground for rod supports at equal distances from the theodolite and about 180° apart in azimuth. A level rod is read successively on the two pegs, with the vertical movement of the telescope clamped, the bubble of the level on the vernier frame in the center of the tube, and the vertical circle set at the reading corresponding to the horizontal position of the telescope. The difference in the rod readings is the true difference of elevation of the two pegs. Next set up the theodolite very near peg *A*. With the bubble centered on the vertical circle level and the vertical circle reading the same as before, read the rod on that peg by looking through the objective end of the telescope. If this reading is greater than the first reading on peg *A*, add the difference of the two readings to the first reading on peg *B* and, if it is smaller, subtract the difference from the first reading on peg *B*. Set the horizontal wire of the telescope on the rod at peg *B* at the reading so obtained. The telescope is now horizontal. Adjust the vernier to read zero or whatever corresponds to the horizontal position of the telescope and then adjust the bubble to the middle of the tube.

The error due to an inclined horizontal axis of the vertical circle is usually of no consequence if ordinary care is used in leveling the instrument. The formula for obtaining the true altitude is,

$$\sin h = \sin h' \cdot \cos i$$

where h is the true altitude h' the observed altitude, and i the inclination of the horizontal axis.

For method of observing vertical angles, see page 43. Sample records of observations are given in Figures 66 and 96.

Adjustment of centers.—In precision theodolites so little tolerance is permitted in the fit of the centers that an adjustment is often necessary to regulate the variations of friction caused by wear and by changes of temperature. This usually takes the form of a nut or screw at the lower end of the vertical axis which, by pressing upward upon the lower end of the alidade axis, lessens the weight of the alidade and telescope upon the conical bearing surfaces. This adjustment must be made with caution, for if too much of the weight is removed from the bearing surfaces, there is play in the centers, with a resultant loss of accuracy. To make the adjustment, raise the vertical axis with the adjusting nut or screw until the alidade appears to move just freely enough on its vertical axis. Test for play in the conical bearings by pointing on an object and noticing if a slight pressure on the alidade will move the telescope off its pointing. A better test is to point the telescope on some well-defined object, read the circle, swing the telescope around clockwise, and again point and read. Repeat the process, except that the alidade is swung in the opposite direction. If a series of three or four sets of such alternating pointings shows the effect of drag, the pressure of the screw upward on the vertical axis should be increased; if the readings are erratic and cover a considerable range, the pressure should be lessened.

Eccentricity of centers.—Since it is almost impossible to secure an exact coincidence between the center of the graduated circle and the axis of rotation of the alidade bearing the verniers or micrometers, the readings of the verniers or micrometers around the circle will differ by varying amounts. No error is caused by this eccentricity if two (or more) equally spaced verniers or micrometers are read at each pointing. The mean of the direct and reversed readings of a single vernier or micrometer is likewise free from error due to an eccentric center.

The observer should not take a mental mean of two vernier or micrometer readings and record the mean as the reading of each as if both read the same, for the judgment is apt to be swayed by following that method.

Micrometer adjustment.—The micrometer microscope is a most satisfactory device for measuring accurately the angular value of any part of the interval between adjacent marks of a graduated circle. It consists essentially of a compound microscope with a micrometer box mounted between the objective and eyepiece, at such a distance from each that the movable wires in the micrometer box can be brought into the focal plane of each lens system. The principle of operation will be more readily understood after the mechanical details of the micrometer box have been described.

The mechanical arrangements of the box vary somewhat on different instruments, but two typical arrangements, illustrated in Figures 43, 44, and 45, will be described. The micrometer shown in Figure 43 will be described first. An outer case *c*, into which are screwed from opposite sides the objective tube and the eyepiece, contains a slide *d*, carrying the comb *e*, the center notch of which, taken in conjunction with the zero of the micrometer head *f*, furnishes a fiducial point for all readings. The slide for the comb is adjustable transversely by the screw *g*, acting against the spring *h*, movement in other directions being prevented by the machined surfaces of the slot in which the slide moves.

A movable slide bearing two wires is fastened rigidly to a finely machined screw, *k*, on which the micrometer head, *f*, and the attached spindle works, a washer on the micrometer head bearing against the end, *l*, of the micrometer case. In reality there are usually two pairs of these parallel wires, but for simplicity the function and adjustment of one pair will be first explained and the use of the second pair will be explained later. Rigidly attached to the case end, *l*, on its inner side are two rods, *m*, around each of which is a helical spring, *n*.

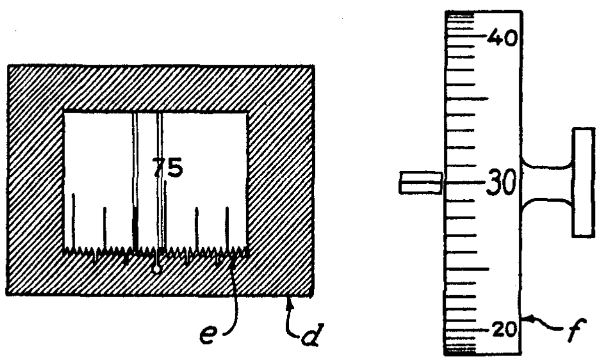

FIG. 44.—Apparent arrangement of comb, wires, and graduation marks

The diagram on the left shows the apparent arrangement in the field of view of the comb, wires, and circle graduations; on the right, the graduated drum of the micrometer. The reading as shown by the movable wires, comb, and drum is 74° 58′ 30″, the circle being graduated in 5-minute spaces.

The rods and springs pass through holes in the upright partition, *o*, of the slide, and the rods fit into depressions in the other upright partition of the slide, while the springs bear against its inner surface, thus maintaining a pressure of the slide and its screw against the threaded bearing in the head, preventing slackness and play.

With this construction the theory of operation is readily seen. The objective forms a magnified image of a small portion of the graduated circle in the plane of the parallel wires. This image in turn is magnified by the eyepiece. The angular value of the portion of the circle between the zero point and the next preceding graduation of the circle can then be measured in terms of whole turns and fractional turns of the micrometer screw. The magnifying power of the objective and the pitch of the micrometer screw are so

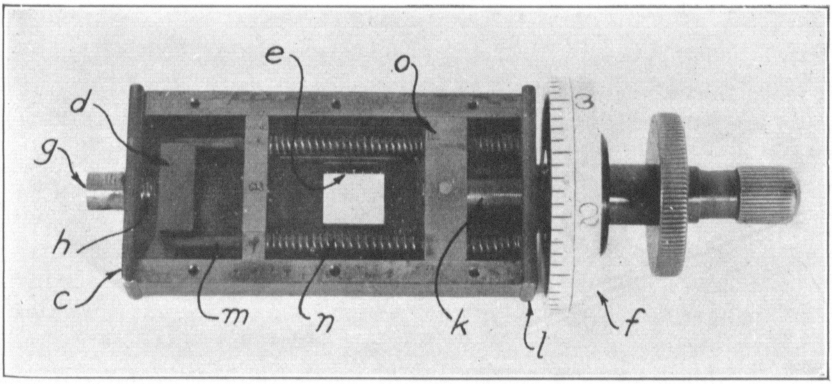

FIG. 43.—MICROMETER BOX, COMPRESSION SPRING TYPE

View of lower side of box with plate removed

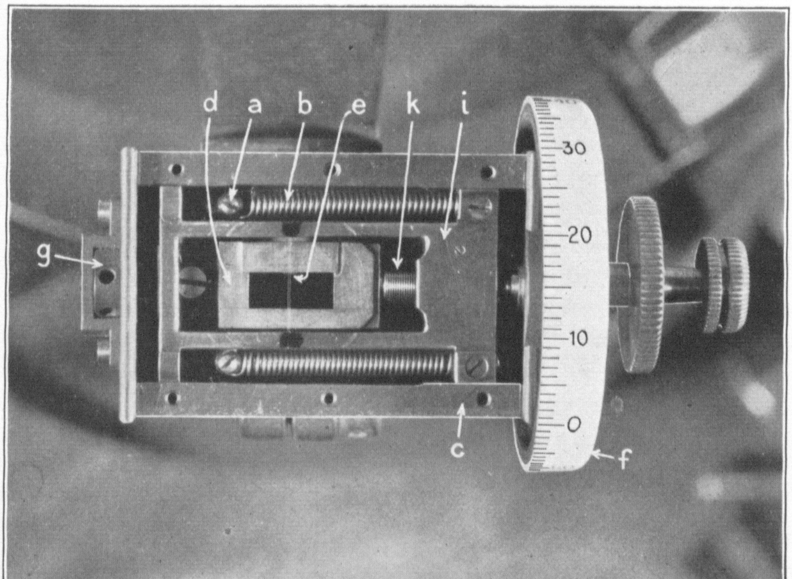

FIG. 45.—MICROMETER BOX, TENSION SPRING TYPE

The block *d* carries the comb and the slide *i* the wires. The screw *k* is threaded into *i* and bears against *d* related that the adjustment can make one full turn of the micrometer screw equal to a minute of arc, or to some multiple thereof. The micrometer drum is graduated to single seconds, or to some integral multiple of one second. The whole turns or minutes are read off from the comb, each notch of which corresponds usually to one minute of arc, while the fractional part of a minute is read from the micrometer drum in seconds.

The design of the micrometer shown in Figure 45 is in many respects superior to the compression-spring type. The two tension springs, *b*, anchored to the outer case by the screws, *a*, hold the sliding frame, *i*, tight against the block, *d*, which carries the comb, *e*. The slide, *i*, carries the micrometer wires which are moved over the comb by the action of the micrometer screw, *k*, which is threaded into the slide. A spherical tip on the end of the micrometer screw fits into a depression of similar shape in the block, *d*. The comb is adjusted by the capstan-headed screw, *g*.

A single-micrometer microscope is in correct adjustment when the following conditions are fulfilled: First, when the micrometer lines and image of the graduated circle are so closely in the same plane that no parallactic movement can be detected by shifting the eye laterally; second, when five revolutions of the screw will exactly traverse the interval between adjacent graduations on the circle (when the graduations on the circle are 5 minutes apart one complete turn of the instrument screw will usually equal one minute; when the graduations are 10 minutes apart one turn of the screw will usually equal two minutes); and third, when the micrometer reads zero seconds with the pair of parallel wires coinciding with the zero point on the comb. After these three conditions are fulfilled the micrometers must be spaced around the circle at equal intervals.

Eyepiece adjustment.—Adjust the eyepiece by sliding or screwing it in or out until the parallel wires are in the most distinct focus. When this is accomplished the comb will be sufficiently visible.

Adjustment for focus.—Loosen the screws which hold the microscope tube in the bracket clamps and move the tube up or down until the graduation marks are as distinct as possible, then tighten the screws.

Radial adjustment of microscope.—There is usually some sort of hinged joint provided for adjusting the objective of the microscope radially. This adjustment should be made so as to bring the outer edge of the graduations near the center of the field of view, yet still leave the degree numerals visible.

Adjustment for parallelism.—If the parallel wires of the diaphragm are not parallel to the graduation marks on the circle, either turn the micrometer box slightly on the tube or loosen the binding screws and turn the tube slightly in its supports.

Adjustment of comb.—If the zero notch of the comb, usually marked by a deeper cut than the other notches or by a hole beneath it, is not in the center of the field of view, adjust it by the screw *g*.

Adjustment of graduated drum.—Center the pair of parallel wires on the zero notch of the comb, then hold the spindle of the micrometer firmly and turn the graduated drum on its friction mounting until the zero of the drum coincides with the index line, *p*, mounted on the micrometer case adjacent to the drum. Check to see that the parallel wires have not moved off the zero of the comb. If the drum is held in position by a screw as well as by a friction mounting, the screw must, of course, be loosened before making the adjustment.

If the micrometer screw is threaded to have one revolution equal to two minutes of arc, then the graduated drum will have two graduations on it marked zero seconds, the numerals of the second minute usually having some distinguishing characteristic. Care must be taken always to adjust and read to the proper zero mark.

Adjustment for run.—By "run" of the micrometer is meant the difference in seconds of arc between the intended value of one turn of the micrometer screw and its actual value as determined by measuring with the micrometer the space between two adjacent graduation marks of the circle. This quantity has sometimes been called the "error of run," but the former term seems preferable and is more generally used.

Assuming that adjacent graduations of the circle are five minutes apart and that one revolution of the micrometer screw will cause the parallel wires to traverse approximately one-fifth of the interval between adjacent graduations, the purpose of the adjustment for run is to bring about, as closely as may be, the condition that exactly five revolutions of the screw will move the parallel wires from one graduation to the next. The theory of the adjustment may be seen from the fact that the portion of the graduated circle and its magnified image are at conjugate foci of the objective, and that the magnification of the image is represented by f'/f, where f and f' are the distances of the circle and its image, respectively, from the center of the objective. The magnification of the image may be increased by decreasing the distance of the objective from the circle, and decreased by moving it in the opposite direction. Therefore, if fewer than five complete turns of the micrometer screw are necessary to move the parallel wires from one graduation to the next, the image needs to have its magnification increased, and the objective should be moved nearer the circle by protruding it from the tube by its screw adjustment.

When the objective is protruded from the tube to correct for run the image is thrown farther up in the tube. Without changing the relation of the eyepiece to the micrometer wires, move the whole micrometer tube upward to bring the image of the circle into the plane of the micrometer wires. This process may have to be repeated several times to get the run down to the required limit.

If more than five complete turns of the screw are required to make the parallel wires traverse the space between adjacent graduations, the adjustment should be made in the direction opposite to that described above.

In actual practice the preliminary tests for run should be made on five or six equidistant parts of the circle, since the error of run will vary somewhat due to eccentricity of the circle, and the adjustment should be made on a portion of the circle where the error is near the mean. Also, the final adjustment should be tested by taking the mean of at least 10 readings of the screw value of the space between graduations, and these readings should be made a part of the record of horizontal directions.

The mean run of a single micrometer should not exceed four seconds on second-order triangulation and the algebraic sum of the runs of all micrometers should not exceed two seconds. If the magnification of the micrometer is small and the required correction is very slight, the adjustment may be made by raising or lowering the entire tube instead of changing the objective in the tube, since the change in distance affects the run at a much more rapid rate than it does the parallax or distinctness of vision.

The accuracy of micrometric readings depends largely upon the proper mounting of the pairs of wires used for reading. They should be at such distance apart that a narrow strip of the bright surface of the arc is visible on either side of the graduation when the wires are accurately centered astride it. They should be parallel as closely as the eye can judge and should be adjusted to be parallel to the graduations. If the wires are not parallel or are slack, they should be remounted, as described on page 63. They should also be heavy, black, and smooth.

The formula for the correction for run may be found in various texts in different forms. That given below from Crandall's Geodesy is perhaps as convenient as any.

With a single pair of wires and the graduations on the circle five minutes apart:

Let

a=backward reading of micrometer.
b=forward reading of micrometer.
r=average run of micrometer, plus when $a>b$.

Correction to $a=\frac{-r}{300''}a$.

Correction to $b=\frac{r}{300''}(300''-b)$.

The mean, $m=\frac{a+b}{2}$.

Correction to $m=\frac{r}{300''}\,\frac{300''-(a+b)}{2}$.

$$=\frac{r}{2}-m\frac{r}{300''}. \tag{1}$$

For convenience and speed in making the readings it is the usual practice in the Coast and Geodetic Survey to have two pairs of wires mounted at such an interval apart that, when the left-hand pair of wires is centered on a graduation mark, one revolution of the micrometer screw will approximately center the right-hand pair of wires on the next adjacent graduation mark to the right. The reading on the graduation next preceding the zero of the comb (in the direction of decreasing graduations), called the forward reading, is made with the left-hand pair of wires, and the reading on the mark next following the zero of the comb (in the direction of increasing graduations), called the backward reading, with the right-hand pair.

A consideration of formula (1) above shows the following:

(a) When reading a single pair of wires successively on the two graduation marks adjacent to the zero point of the micrometer, the total correction for run is the same whether applied separately to the two readings or to their mean.

(b) With a single pair of wires the correction for run on second-order triangulation may be disregarded, provided (1) that the run for any one micrometer is less than four seconds and the algebraic mean of the runs for all micrometers is less than two seconds, and (2) that the initial settings are distributed approximately uniformly throughout the space between adjacent divisions. It is preferable with a single pair of wires to take both forward and backward readings at each pointing, but if the two conditions mentioned above are fulfilled and if the readings are always taken to the nearest graduation mark, half the corrections for run would be plus and half minus and the algebraic sum of the corrections for run on any direction would be very small if an even number of positions were observed.

(*c*) Where two pairs of wires are used mounted at an arbitrary distance apart (less than the interval between adjacent graduation marks), the correction for run can be applied to the reading of each pair of wires separately, as when a single pair of wires is read on one graduation only at a setting. The readings of the two pairs do not tend to eliminate the run.

(*d*) When conditions (1) and (2) in (*b*) above are fully met, and when the two pairs of wires are always kept each on its own side of the center mark of the comb when reading, to insure a symmetrical distribution of the readings of each pair of wires throughout the graduation space, the errors due to run of the micrometer may be neglected. The principal benefit derived from using two pairs of wires lies in the comparison obtained by sighting on two graduation marks instead of one.

The error due to run of micrometers may be very considerable unless the run is kept small and the settings on the circle for each position are kept close to the proper reading to give a symmetrical distribution of each group of readings throughout a single graduation space.

Adjustment for equidistance of microscopes.—After the individual adjustments have been made set microscope *A* exactly on an even degree graduation mark with micrometer comb and drum reading zero. To set microscope *B* at 120 or 180° distance around the circle, depending on whether it is a three-micrometer or a two-micrometer theodolite, proceed as follows: Note if the zero of the comb on *B*, when near the center of the field of view, is very distant from the proper degree mark. If it is, move microscope *B* in the proper direction by whatever mechanical means for such adjustment is provided until the proper degree mark is near the zero of the comb. (Microscope *A* is not usually adjustable circumferentially.) Then move the comb by the screw, *g*, until the zero mark coincides with the proper degree mark and adjust the graduated drum to read zero when the parallel wires read zero on the comb, as already described. If the theodolite has a third micrometer microscope, it should be adjusted to microscope *A* in the same way.

Because of eccentricity of the circle the micrometers will not maintain a constant difference when read on different portions of the circle. A sufficient number of readings should be taken around the circle to determine the approximate amount of this eccentricity and the adjustment for equidistance should be made at a point where

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the eccentricity can be closely estimated. It facilitates the taking of means of micrometer readings if microscope *B* is set sufficiently ahead of microscope *A* that *B* will seldom read less than *A*.

Adjustment of reading microscope.—On three-micrometer theodolites it is usual to have a small reading microscope with a single wire on its diaphragm by which the degrees and the next preceding 5-minute graduation of the circle is read. Such a microscope should be adjusted to read in agreement with micrometer microscope *A* by whatever means are provided for such adjustment.

Illumination of circle.—For making micrometer readings it is important that the circle be evenly illuminated from above, or else that the light be admitted normal to the circle and directly opposite the graduation to be read, as otherwise there will be an appreciable error in the readings due to phase.

DISCUSSION OF ADJUSTMENTS

The adjustments given above, with the exception of those referring to the micrometers, apply equally well to either direction or repeating types of theodolites. The verniers on a repeating theodolite are not usually adjustable in the field, and for this reason the complete adjustment of a repeating theodolite is a less tedious and difficult task than that of adjusting a direction instrument. On the other hand, the micrometers of a direction theodolite rarely need adjustment, except possibly the adjustment for equidistance around the circle, which can be made very quickly.

Some theodolites have optical verniers instead of the usual contact verniers. An optical vernier, shown in Figure 37, is a microscope with the vernier lines ruled on a glass slide placed in the focal plane common to the objective and the eyepiece. The microscope is adjusted in a manner similar to that described for the micrometer microscope until the image of the graduated circle falls in the plane of the vernier lines and until the vernier scale, superimposed on the graduated lines on the plate, subtends the proper number of divisions of the graduated circle. The optical vernier is superior to the contact vernier in that there is no attrition between contact edges with consequent parallactic errors in the readings after those edges have become worn. The optical verniers, however, are more liable to injury and to loss of adjustment than are contact verniers.

WILD THEODOLITE

A Wild theodolite (see fig. 40) recently purchased by the Coast and Geodetic Survey for first-order triangulation is constructed on novel principles and seems well adapted to triangulation of first, second, or third order. The horizontal and vertical graduated circles are of glass. By an ingenious arrangement of prisms the images of opposite portions of either the horizontal or vertical circle may be brought into the field of view of a reading microscope mounted alongside the telescope. Mechanical means are provided for bringing the graduation marks of the two images into coincidence, and the angle is read by means of an auxiliary seconds scale to the nearest second or decimal of a second without moving from the eyepiece of the telescope. The single reading so obtained is the mean of two readings at diametrically opposite points of the circle and is therefore free from errors due to the eccentricity of the graduated circle. The theodolite in its steel case weighs only 38 pounds, and the tripod only 18 pounds. When an instrument of unusual design such as this is issued special instructions will be given for its use.

CARE OF THE THEODOLITE

The most important rules to observe in caring for a delicate surveying instrument are to handle it carefully and keep it clean. When an instrument is received from the office unpack it carefully and slowly, noting the exact manner in which it is fitted into its packing case, and when replacing it in its case always avoid forcing any part into place. Avoid knocks and jars as far as possible, for many of the parts of the instrument are delicate and easily damaged. When handling a theodolite lift it entirely by the tribrach or by the lifting ring, and never by the standards or micrometer arms. Avoid setting up screws too tightly, especially capstan screws operated by adjusting pins, for the fine threads are easily stripped.

A necessary antecedent to caring for, adjusting, and repairing an instrument is for the observer to familiarize himself with the principles and details of its construction. This does not necessarily mean that before observing with a new instrument it should be dismantled and reassembled, but the observer should seize the first opportunity to see any part with which he is not familiar dismantled in the instrument shop of the survey, or by a more experienced observer. He should also study out any detail of construction which is not at once evident, in order to recognize more quickly the cause of any trouble which might develop with the instrument.

The conditions encountered on second-order and third-order coastal triangulations are particularly hard on theodolites. Not only must they be frequently landed through surf, but they are used continually in a moist, salty atmosphere. All exposed metal parts, such as screws, wyes, and pivots, must be kept oiled to prevent corrosion. If, because of weather conditions, a theodolite must be put into its box while wet, it should be thoroughly dried and oiled at the first opportunity. A damp packing box or storage space will cause an instrument to rust very quickly, and a storage place aboard ship which is moist and hot is also very hard on instruments and is especially hard on packing boxes. The chief of party should hold the officer using an instrument responsible for its care and should cause a periodical inspection and cleaning to be made of instruments not in frequent use.

When moving a theodolite by truck a large sack containing excelsior or similar material, or a pad made of blankets placed under the theodolite box, will lessen the effect of vibration and jars. Always make sure that the theodolite is securely fastened in its case. When packing for a long shipment fill vacant spaces around the instrument with paper, as this will prevent any object which may become loose from injuring the instrument; but do not use loose excelsior for that purpose, for the dust from it is very penetrating and is injurious to the working surfaces of the instrument.

A good observer almost invariably keeps a clean instrument. The atmospheric conditions met with in field work are very severe on metal surfaces, which will rust if not oiled but will collect dust and grit if an excess of oil is used. The best thing to do is to go over all exposed surfaces each day the instrument is used. When the instrument is first set up brush off the dust from the enameled or painted surfaces first, then go over the working surfaces, such as wyes, pivots, and exposed screws, with a soft rag very lightly oiled with a light oil. Next rub a soft, dry rag over the surfaces which have been oiled to remove all oil except the film adhering to the metal. If the air is dusty during the observing clean the pivots and the wyes frequently to avoid error and wear. A clean finger will free the wyes of dust and will usually leave the right amount of oil on the metal. When packing an instrument for a long shipment, especially by sea, or when putting it in the storeroom aboard ship at the end of a field season, all exposed polished surfaces should be coated with a fairly heavy oil to prevent rusting.

The centers and the micrometer slides require special treatment. It frequently happens that a large amount of friction develops in them when low temperatures are encountered. This often indicates that there is an excess of oil on the bearing surfaces, which are fitted together with a very small tolerance. The parts affected should be taken down, all the old oil wiped off, porpoise oil applied, and again wiped off with a dry, soft rag free from lint, and the parts again assembled. The film of oil left on the metal surfaces will afford sufficient lubrication. On some theodolites there is an adjustment provided for the centers for change of temperature, but even in such cases there should be no excess of oil in cold weather, for if it exists there is either excessive friction or undue play.

The outer surfaces of the lenses require frequent cleaning but should be rubbed as little as possible. First brush off the dust with a camel's-hair brush, then take soft paper or an old linen rag and lightly flick the surface to remove what dirt may remain. If further cleaning is necessary soft paper free from silicious particles, which are found in most paper, may be moistened and rubbed very lightly over the surface. A greasy film may be removed with paper or rag moistened in alcohol, but if an excess of alcohol is used it may penetrate between the component parts of the lens and affect the balsam which is sometimes used to cement the lenses together. A lens which still remains cloudy after the above treatment can not be cleared by field methods.

The component parts of a compound lens should not be taken apart in the field except in an emergency. When it is necessary to take apart a compound lens the component lenses must be so marked that they can be reassembled exactly in their former relative positions.

Emergency repairs.—Even though extreme care be observed, the conditions of transportation incident to field work frequently make necessary a certain amount of repairs to instruments during the field season. The delay to the party which would result from awaiting a relief theodolite makes it advisable for the observer to make emergency repairs where possible.

The repair job most frequently encountered in the field is replacing one or more sighting "wires," either in the telescope or the reading microscopes. This requires care and patience but is not difficult if the proper materials and appliances are at hand. In anticipation of such a contingency each chief of party should obtain from the office a spider's cocoon, a solution of pure shellac in alcohol, and a small piece of beeswax. A watchmaker's magnifying glass is a convenience, though an ordinary magnifying glass may be used, or a binocular objective, or the reading glass for the verniers of a theodolite may be mounted in a position to answer the purpose very well. A description of the method of installing micrometer wires follows:

Take the micrometer apart carefully, in order not to break any wires which do not need replacing. Clean off with alcohol all dirt and shellac from the slide where the wires are to be mounted. If only one wire of a pair is broken, it is often impossible to properly clean the slide without removing the other wire; but if this is necessary it is of little consequence, for a pair of wires can be installed almost as easily as one.

After cleaning the slide place it in a stable position on a good background, so the wires will be easily seen. Attach a bit of beeswax to each of the points of a pair of dividers, or to each end of a piece of wire bent into the form of a V with the points turned down, and to one point attach one end of a thread of the cocoon. With the cocoon suspended from the point wrap the thread two or three times around the point and then catch the thread on the other point, wrapping it two or three times around that point before cutting it. Stretch the thread until the kinks disappear, then hold it for a few seconds in warm (not hot) water; stretch it a bit more, again immerse it in water, and repeat the operation until two or three threads are broken and you can judge when to stop the stretching just short of the breaking point. Then with a thread fully stretched between the points of the dividers, place the points astride the slide so the thread will be in approximately the correct position, as shown by the scratches on the slide. Block up the points of the dividers so the thread will not be stretched too much.

If a pair of wires is to be mounted, use another pair of dividers and place the second thread in position also, the points of one pair of dividers falling outside the points of the other pair. With the aid of a magnifying glass adjust the threads with a needle until they are exactly parallel and properly spaced, as shown by the scratches on the slide. Finally, with the eye end of a needle place a very small drop of shellac on each end of each wire to cement it to the slide, and after allowing it to dry for a few minutes the dividers may be cut loose. The shellac must be of such quality that it will spread immediately upon application into a thin film over the metal surface, otherwise the thread will not be held taut. A web so mounted will rarely slacken in wet weather.

Instead of cocoon threads, fine tungsten or platinum wire may be used for diaphragm wires, but the wire must be thoroughly cleaned with alcohol, stretched in place under considerable tension, and fastened by several coats of a very thin solution of collodion. The cocoon threads are much easier to mount in the field than the wire.

Micrometer wires should be mounted at such a distance apart as will allow a narrow strip of the silver to be visible on either side of the graduation mark on which the pair of wires is centered. Scratches on the slide usually indicate the proper location for the wires.

Parallel vertical wires are used as the sighting wires in the telescopes of all large direction theodolites and may be used to advantage in any telescope employed on second or third order triangulation. In telescopes having a magnification of 35 to 50 diameters the sighting wires should be about 30 to 35 seconds apart. In telescopes having a lower magnification they should be about 50 seconds apart. With parallel wires a faint or very small signal is not blotted out, and the image of the signal may be quickly and accurately centered between the wires.

The repair of broken parts is largely a matter of ingenuity, combined with a knowledge of what is essential to the proper working of an instrument. Stripped threads on screws may sometimes be made to hold temporarily with gum or sealing wax, provided they are not such as require moving in adjusting, or a pin of hardwood may be used in place of the broken screw. Broken plate-level mountings have been temporarily replaced with sealing wax, and even a broken micrometer microscope bracket has been made sufficiently rigid with properly shaped pieces of wood wrapped with cord or wire which was then stretched with wedges. Resourcefulness is a necessary quality for a triangulator in unfrequented regions.

If erratic results are being obtained in the observations which can not be otherwise accounted for, the entire structure of the theodolite should be scrutinized in detail. See that the lenses of the objective are tight in the case, and tighten the inner screw ring if they are not. Next examine the eyepiece end of the telescope to see that the eyepiece tube fits tightly into the telescope barrel and that no screws are loose. Examine the foot screws to see that the clamping screws hold them firmly. Inspect the horizontal axis, standards, tangent-screw assembly, and microscope brackets for fractures, and see that the graduated circle is screwed firmly to its seat. Also test the junction of the barrel of the telescope and its horizontal axis, and the junction of the seating of the object glass and the telescope.

If the agreement of the separate measures of a direction is satisfactory but the closing errors of triangles large, the cause is probably not in the instrument but in those atmospheric conditions which cause lateral refraction, or else is due to instability in the stand or in the mounting of the theodolite. The trouble may also be due to the eccentricity of the lamps or other objects sighted on. There is always a reason for poor results, and the observer should not rest satisfied until he has found it.

TESTS TO DETERMINE THE QUALITY OF A THEODOLITE

In deciding what observing program will give most economically the accuracy desired for any class of work it is necessary to know the quality of the instrument employed, which, however, is not measured by the size of the circle or by the minuteness of the least reading of the verniers or micrometers. The best measure of the excellence of a theodolite is its performance in actual field work, but it is necessary to apply other tests to a new theodolite.

A preliminary examination will show a great deal about the workmanship and precision of a new instrument. The four structural features which must be scrutinized to form an estimate of the quality of a theodolite are the graduation of the circle; the design and workmanship of the micrometers, or the efficiency of the verniers as regards the ease and accuracy with which the circle may be read; the fit of the centers and the tangent screws; and, lastly, the optical properties of the telescope. Each of these features must be entirely satisfactory if the best results are to be secured.

The circle graduations of a vernier instrument must be of very poor quality to permit the errors to be detected with certainty by the engineer using the theodolite in the field. If the accuracy of graduation is suspected, a curve of *A*-vernier minus *B*-vernier readings may be constructed in the manner described for a micrometer instrument in the following paragraphs, but the comparison of vernier readings will not locate errors of smaller magnitude than the least reading of the vernier, usually 10 seconds.

With a micrometer theodolite the quality of the circle graduations as well as the efficiency of the micrometers may be gauged by making careful readings on each of two micrometers at equal intervals around the circle, say 10° apart, and constructing a curve of the means of the plotted differences of the readings, similar to that shown in Figure 46.

Such a curve will show three things: First, the algebraic mean of the differences (*A*-micrometer reading minus *B*-micrometer reading) represents the failure of the *B* micrometer to be exactly 180° from *A* and also indicates the amount the horizontal reference line of the diagram would have to be moved to make the sum of the plus ordinates equal to the sum of the minus ordinates; second, the amplitude of the mean curve drawn through the points represents the eccentricity of the graduated circle with reference to the axis of rotation of the micrometers, except as this curve is distorted by the accidental and short-period errors at the maximum and minimum points, the eccentricity curve being a sine curve; third, the variations of the plotted differences from the mean curve is a measure of the combination of local and accidental errors of graduation with those resulting from reading the micrometers. These variations from the mean curve should seldom be greater than the least reading of a single micrometer and should never exceed twice that amount; that is, with an instrument read habitually to the nearest two seconds, the plotted curve should seldom differ from the mean sine curve by more than that amount. The same general rule holds for a vernier instrument.

The fact that any one $A-B$ value falls near the mean curve does not necessarily mean that both graduations involved are in correct relation to the other graduations of the circle, for each of the two graduations on which the readings depend may be in error an approximately equal amount in the same direction. When an $A-B$ value plots far from the mean curve, however, it must necessarily mean that one or both the graduations on which the readings were made are in error by an undue amount. Additional $A-B$ readings should be taken on near-by graduations to localize the error. When a graduation mark is known to be perceptibly in error it should not be used as an initial setting with either a direction or a repeating theodolite. If, with a repeating theodolite, a set of repetitions ends on a portion of the circle where the graduations are known to be in error, an additional repetition should be made before taking the final reading, a like number of repetitions being made of the explement of the angle.

The efficiency of the micrometers and verniers can best be tested by making a number of readings. A vernier should be capable of being read quickly and accurately to its smallest graduation. If the graduation and ver-

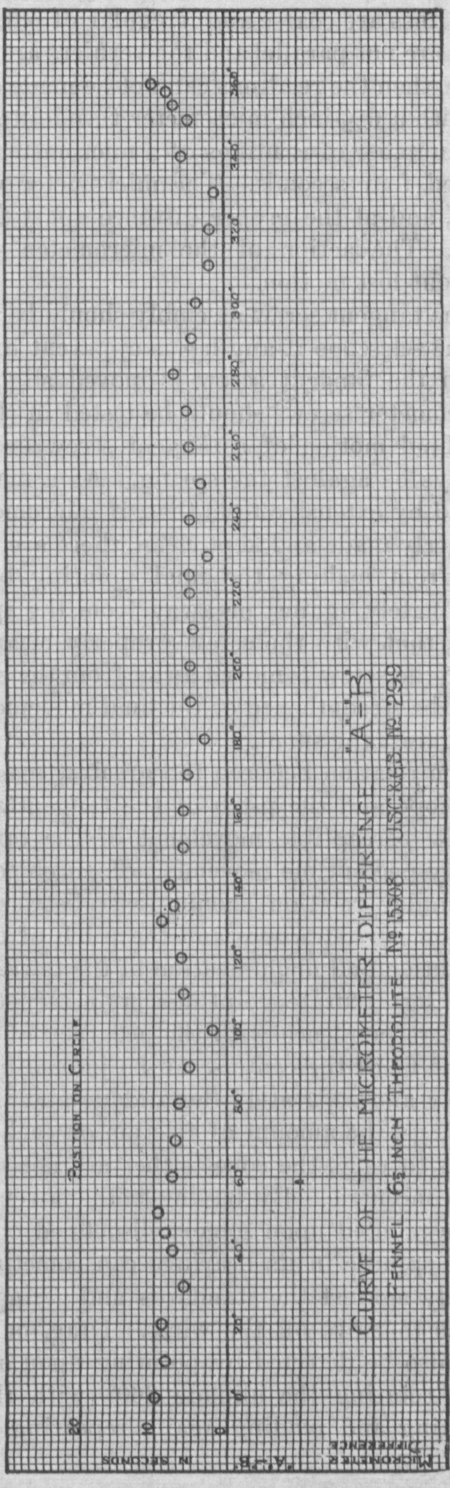

FIG. 46.—Graph of reading, A micrometer minus B micrometer (For explanation see p. 66)

nier lines are coarse and irregular of outline, or if the two systems of lines are not parallel and the verniers are not in close contact with the plate, then the readings can not be made quickly and accurately.

With a $6\frac{1}{2}$-inch micrometer theodolite of the usual type the range of a number of readings of a micrometer on a graduation should not exceed four seconds. The micrometer wires should be coarse, parallel, and at the proper distance apart for quick and accurate readings. The action of the micrometer screw must be smooth and without lost motion.

The tangent screw assembly should be tested for friction by noting in the telescope if any lag is apparent in the motion of the telescope when the tangent screw is moved slightly away from the spring. Micrometer screws should be tested in the same way. Theoretically the final motion of a tangent screw or a micrometer screw should always be against the spring. As a matter of practice it has been found by extensive tests that, if micrometers and tangent screws are properly made and kept clean, there are no appreciable errors resulting from making the final pointing by moving the screw indiscriminately against or away from the spring. If it is found, however, that with the screw properly cleaned and oiled there is a lag when the screw is moved away from the spring, then either the spring must be strengthened or the final movement of the screw must always be against the spring.

The relation between the greatest magnification obtainable by the telescope and the pitch of the tangent screw should be such that a barely perceptible movement of the tangent screw should cause a barely perceptible movement of an object across the telescope wires. A similar relation should exist between the magnification of the micrometer telescopes and the pitch of the micrometer screws. The resolution and magnification of the telescope are important factors in securing accurate pointings upon targets and lights and should be in correspondence with the accuracy of the circle readings.

The fit of the centers can be judged by various tests. By moving slowly and separately the alidade and the graduated circle on their bearings, the amount of friction on each can be felt. If the friction seems to vary, the cones either are not properly polished or else are not concentric. By having both movements clamped and the telescope pointed upon some well-defined object a lack of fit of the alidade spindle in its bearings can sometimes be detected by pressing lightly against the standards and seeing if the image of the object seems to move off the wires. Drag, or poor fitting of the centers, can also be detected by pointing upon an object, reading the circle, then rotating the alidade 360° with the upper motion loose and pointing and reading again, repeating the operation several times and reversing the direction of motion of the alidade after each reading. Any effect of drag is shown by a tendency for any reading to differ somewhat less than 360° from the preceding reading. Undue play is indicated by the readings not being closely grouped.

The stability of an instrument can be judged by the design of the standards, telescope, and tribrach. The tribrach should have a spread at least equal to the size of the circle, and the foot screws should have enough bearing surface in the tribrach arm to insure stability in azimuth. A four-point leveling head, or quadribrach, should never be used for an instrument for third or higher order of horizontal control, for the leveling process always introduces strains in the base of the instrument which tend to change the instrument in azimuth while the observations are in progress.

VERTICAL COLLIMATOR

This instrument, shown in Figure 47, is used to center the theodolite, lamp, or heliotrope on an observing tower over the center of a station mark or to set a station mark directly under a definite point on the tower.

In principle the instrument consists of a telescope fitted with a diaphragm bearing cross wires, a tribrach with three leveling screws and a long vertical collar into which the telescope is placed with the eye end uppermost. Near the eye end of the telescope and eccentrically placed is a level at right angles to the axis of the telescope.

To adjust the collimator, place the cross wires in the focus of the eyepiece by pulling out or pushing in the eyepiece until the wires are as sharply defined as possible, then focus the telescope on the object beneath so that there is no shifting of the intersection of the wires over the object as the eye is moved across the eyepiece. Next the level is adjusted in the usual manner until there is no movement of the bubble when the telescope is rotated. Finally the cross wires are adjusted by means of the diaphragm screws until the intersection of the wires remains on a point in the field of view when the telescope is rotated. When so adjusted the point covered by the intersection of the cross hairs in the field of view is in the vertical line passing through the center of the telescope.

If the collimator is slightly out of adjustment, a vertical line can be established by marking the four points in the field of view covered successively by the intersection of the wires when the telescope is rotated to four different positions approximately 90° apart. The intersection of the lines joining the diagonally opposite points will be the point sought.

After the instrument has been adjusted and it is desired to mark a point on the tower directly over the station mark, the axis of collimation of the telescope is brought directly over the center of the mark. If the center strip of the cap block has been removed to permit free vision with the collimator, the point can be marked by two threads intersecting beneath the point of the plunger. It is the usual practice, however, to bore a small hole through the cap block of such size that the vertical collimator telescope may be sighted through it and yet small enough to hold the screw which fastens the lamp or heliotrope to the tripod head. When the signal is being built it is best to center the instrument over the hole in the loose cap block and then slide the cap into a position where the hole will be directly over the station mark. The cap can then be nailed securely to the tower.

After a tower has been built for a few days or weeks the center of its cap block is likely to have been disturbed by the drying out and warping of the lumber. In such a case as this it is frequently best to find the point on the station mark directly beneath the center of the hole in the cap block, measure on the station mark the eccentric distance and direction of the projected point, and then transfer these dimensions to the cap, thus obtaining easily the point on the cap directly above the center of the station mark.

A new model of a vertical collimator has been designed to be mounted on the tripod of a Berger 7-inch theodolite and to extend the vertical line upward instead of downward. It is shown in Figure 48. It consists of a vertical telescope held in a vertical supporting collar, the telescope being supplied with a 45° reflecting mirror and a right-angle eyepiece. The telescope can be rotated in the collar through an angle of about 300° for purposes of adjustment. The level attached to the collar on the upper end of the telescope and the diaphragm wires in the eyepiece are adjusted by the usual methods. The instrument is designed to project a vertical line upward for 100 feet with a maximum error of a tenth of an inch.

INSTRUMENTS AND APPARATUS FOR LIGHT KEEPERS

Before a light keeper is placed alone on a station the chief of party should be sure that the light keeper understands thoroughly the use and adjustment of each instrument to be used. Detailed instructions should also be given each light keeper for the care of the instruments to prevent the metal parts from rusting and the leather cases from shrinking. If a light keeper thoroughly understands his apparatus, he will be able to make many of the emergency repairs which become necessary from time to time.

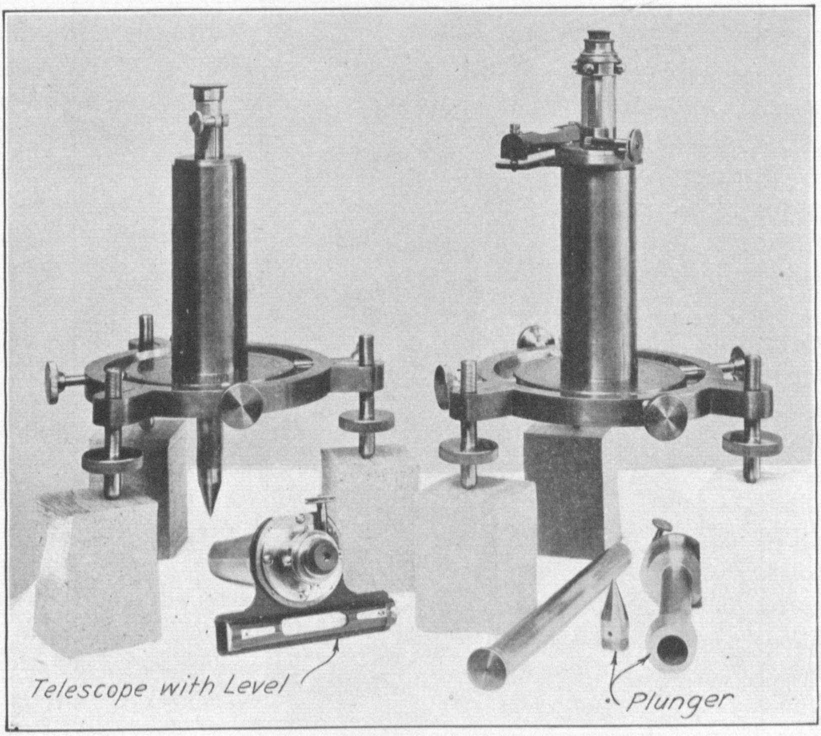

FIG. 47.—VERTICAL COLLIMATOR, OLD TYPE

(For description see p. 69)

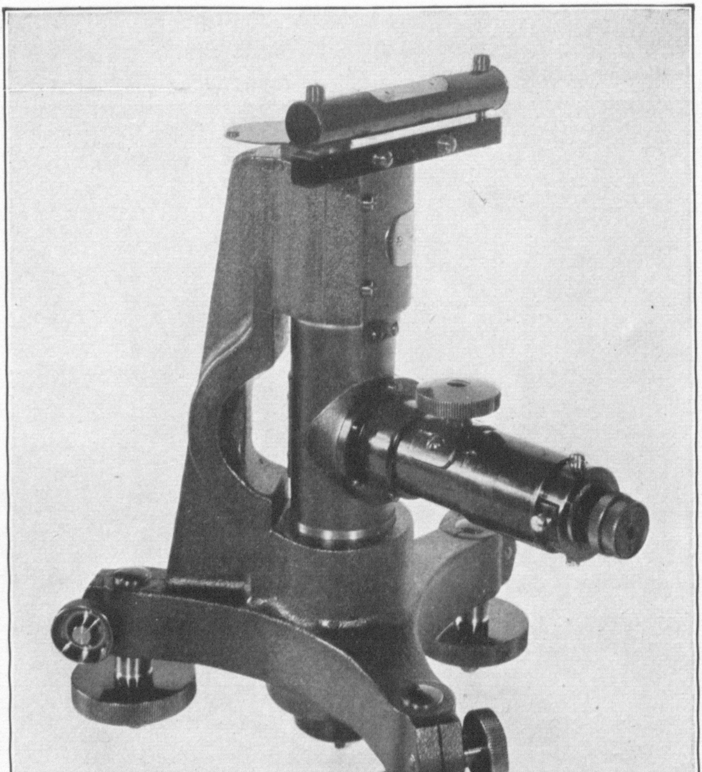

FIG. 48.—VERTICAL COLLIMATOR, NEW TYPE

(For description see p. 70)

Compass.—The compass furnished each light keeper should either be compared with other compasses or else tested on a line whose azimuth is known. The effect of magnetic objects should be explained to him, and he should be cautioned against carrying the compass with the needle resting upon the pivot point. The principles underlying the orientation of maps and sketches and the application of the magnetic declination should be fully explained, with actual tests in them, before the light keeper takes his first station.

Heliotrope.—An undamaged heliotrope (see fig. 49) needs no adjustment. The only possible lack of adjustment occurs when one of the sighting devices is bent. Should this have occurred, the line joining the sighting points is not parallel to the line passing through the centers of the alignment rings. To test, point the heliotrope at some near-by object, such as a rock or tent, and center the reflected light so the shadow of the ring next the mirror falls fair on the forward ring. The sights should point to a spot exactly above the center of the illuminated surface and just as far above as the line of the sights is above the line passing through the centers of the rings. If such is not the case, the affected part should be removed from the box, straightened, and replaced.

Each light keeper should be shown how to make an emergency heliotrope, shown in Figure 50, by driving two nails vertically about 2 feet apart into a board, the heads of the nails to be used as sighting points for the beam of reflected sunlight. Place the board on the stand and align the heads of the nails with the station of the observing party. Next fit a narrow strip of paper to the front side of the forward nail, the strip projecting slightly above the nailhead. With a common mirror a few inches in diameter throw the reflected rays of the sun along the line of the nailheads. This will be accomplished when the shadow of the head of the rear nail falls on and exactly covers the head of the forward nail. The paper strip mentioned above enables one to make this exact contact. An emergency heliotrope like the above has been satisfactorily observed upon from a station 40 miles away. The center of the mirror should be held approximately in line with the nailheads to avoid eccentricity of the light shown the observer. If the direction to the observer is nearly opposite the direction to the sun, it may be necessary to use an auxiliary mirror to reflect the sun's rays onto the mirror which is in line with the nails.

Signal lamp.—The electric signal lamp supplied with current by dry batteries has entirely superseded the acetylene lamp which was in use many years. Aside from the electric connections, only two adjustments are needed for the electric lamp, one for focus and the other for the sighting devices, and these should be tested frequently.

The focusing adjustment is made by the screw socket into which the bulb fits. Since it frequently differs for different bulbs of the same apparent size, it should be made each time a new bulb is used. It is best done by directing the light upon a flat surface, such as a tarpaulin about 100 feet or more away, and varying the adjustment until the brightest part of the disk is but little larger than the lens of the lamp. After this has been done the sighting device should be adjusted to point exactly above the center of the brightest part of the illuminated surface and as far above it as the sighting tube is above the center of the lens of the lamp. As the transportation of the lamp from station to station is apt to disturb both of these adjustments, they should be tested before the lamp is posted at a new station. If the lamp is to be operated automatically and is to be posted during the day and immediately left, the adjustments should be tested the night preceding. It is just as satisfactory to make the adjustment in daylight by changing the focus until the reflector, as viewed from a point two or three hundred feet away, is evenly illuminated. A stake can also be set with its top at a point where the light is the brighest and the sighting tube adjusted to the proper distance above that point.

A clockwork device is available for requisition which can be set to turn the signal lamp on at any desired hour each day and turn it off again after any interval not in excess of six hours. The mechanism is operated by an 8-day clock and is very satisfactory so long as visibility conditions permit the lamp to be properly pointed, and there is nothing to disturb the pointing after the lamp is set. The automatic lamp and battery box are shown in Figure 51, and full instructions for wiring and for setting the dial are contained in Instructions to Lightkeepers, Special Publication No. 65.

PRINCIPAL SOURCES OF ERROR IN HORIZONTAL-ANGLE MEASUREMENTS

A good observer is one who can consistently secure results commensurate with the possibilities of the instrument which he is using. Proficiency can be attained only by careful study of the instrument, by constant exercise of good judgment, and by making a careful study of all the factors affecting the accuracy of theodolite observations. Due regard must be had for the relative importance of the different classes of errors. It is possible to spend considerable time centering the theodolite exactly over the mark but fail to make allowance for the phase of the signal at the other end of the line of sight and point several inches to one side of the true mark. It is also possible to be very careful in perfecting all the instrumental adjustments, but omit a careful testing of the stability of the support for the theodolite,

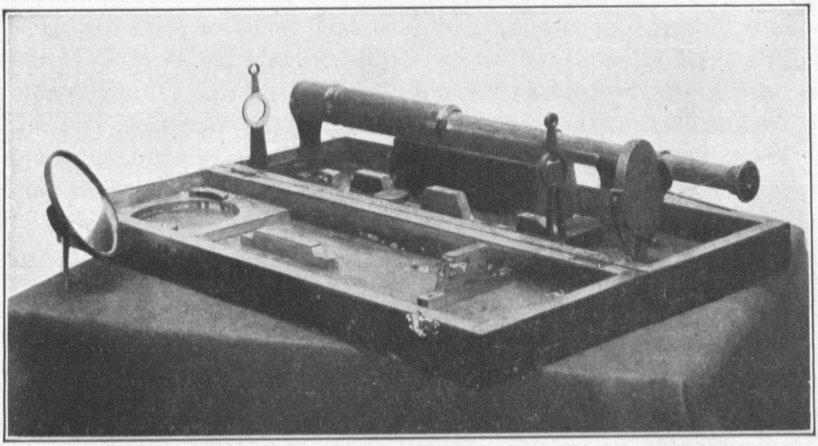

FIG. 49.—HELIOTROPE, BOX TYPE

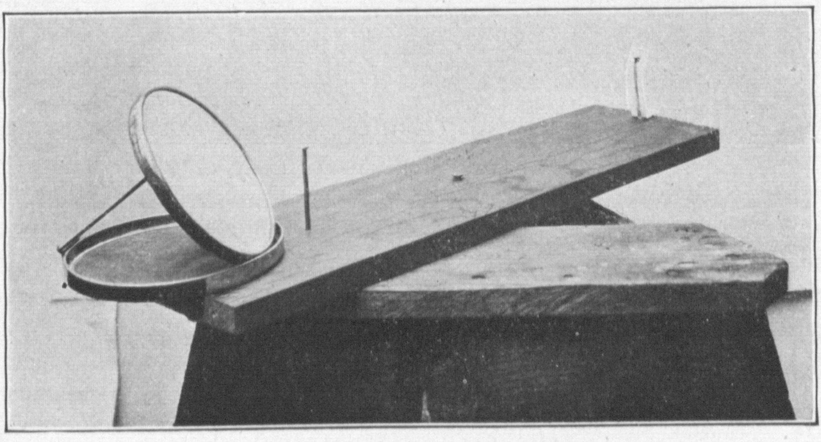

FIG. 50.—EMERGENCY HELIOTROPE

(For description see p. 71)

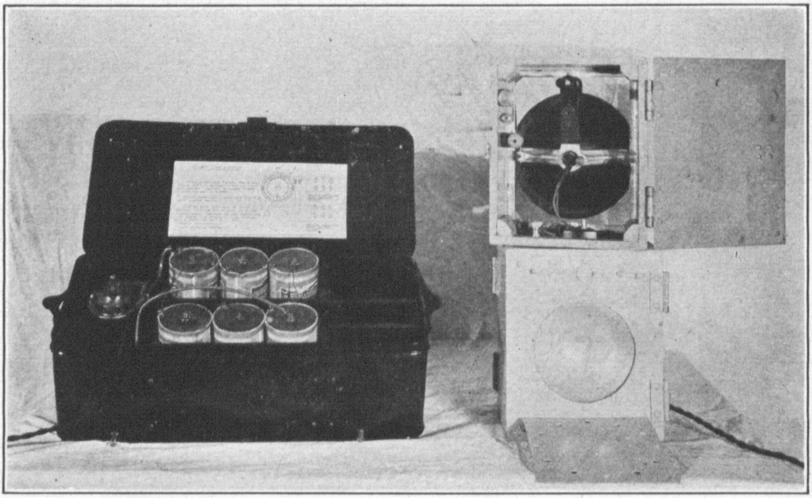

FIG. 51.—AUTOMATIC SIGNAL LAMP

The 8-day clock shown at the left end of the battery box on the left can be set to turn the light on for a definite number of hours each night. Of the two lamps at the right, the upper shows the rear view, the lower the front view though the errors from the latter source are potentially much more serious than are those due to the instrument being slightly out of adjustment.

The actual pointing of an instrument on an object is a simple operation. It is a mistake to try to perfect a pointing after the perceptions are once satisfied that the object has been centered on the wires. The most satisfactory observations are usually those which are made rapidly and methodically but not carelessly. Speed can be attained by the observer by training himself in deftness of movement in manipulating the instrument and in studying how he can perform the manipulations with the fewest movements.

It is very difficult to secure the required accuracy on second-order triangulation unless the theodolite is sheltered from the direct rays of the sun and also protected from the wind if it is of any considerable strength. Temperature changes in the instrument are particularly prejudicial to accurate results. If the temperature inside the theodolite case differs greatly from that of the outside air, it is well to give the theodolite time to assume the temperature of the air before beginning the main-scheme observations. The intervening time can be used to advantage in measuring to reference marks or in making the observations upon intersection stations.

Aside from blunders, such as reading a vernier incorrectly, and the allowable inaccuracies of pointing upon an object, there are four principal causes of error in the measurement of horizontal angles after the proper sheltering of the instrument from sun and wind has been provided for. These are instability of instrument support, instrumental errors, phase and eccentricity, and horizontal refraction. The relative importance of these four factors will vary with the field conditions encountered.

Instability of support.—It is essential that the theodolite be supported in a manner which will maintain the orientation of the bearing surfaces of the graduated circle and the alidade axis on the tribrach while the observations are being made. Everything below these bearings constitutes a part of the instrument support, and the various elements involved are here considered in succession. progressing downward.

Many theodolites, especially those of American manufacture, are held to the tripod head by a bolt with a female screw at the top which fastens to a threaded knob projecting downward from the center of the tribrach. If this bolt is set up too tight, great stress is placed upon the tribrach and strains are set up in the material which are gradually relieved by deformations of the tribrach. As these deformations may produce very sensible errors, the screw attachment should be set up only tight enough to hold the theodolite in place.

The strains would be much worse in a quadribrach, and for that reason a theodolite with four leveling screws should never be used on second or third order work if it can be avoided. The tribrach should have sufficient spread to bring the foot screws almost beneath the limb of the graduated circle.

Set screws for tightening the foot screws in the tribrach arms are always provided, and they must be set up after the leveling of the instrument is perfected. Leveling screws should have large bearing surfaces in the tribrach arm to insure that the clamping screws can make them rigid parts of the tribrach.

The tripod for the theodolite plays a very important part in the accuracy which can be secured with the instrument, and yet many types in common use are of very poor design. The usual form of tripod, with legs hinged at the top and no braces, depends for its stability in azimuth upon the friction of the bolts against the top of the legs where they are hinged and upon the torsional strength of the upper part of the legs. To make this form of tripod effective the bolts must be very tight and extreme care taken in manipulating the instrument. The form of tripod with large one-piece head and split braced legs having a wide spread at the hinges is more stable. For accurate work it is advisable to have adjustable braces, as shown in Figure 38.

A proper footing or ground support for the tripod or stand is frequently hard to secure. It must be sufficiently firm to maintain the theodolite in level and azimuth and must not transmit to the instrument any effects of the observer's movements around the instrument. Before observations are begun and after the instrument has been adjusted and leveled the telescope should be pointed upon some well-defined object and the pointing watched closely while some other person steps around the instrument as near the feet of the tripod legs as the observer would usually stand. If there is a movement of the wires with respect to the object, or if the levels of the theodolite show any change, the footings of the tripod must be made more firm or else the observer must avoid standing on the spots shown to be unstable. The observer must school himself to step wide around the points of support of the tripod or stand. If the instrument support rests upon tundra or wet sod, it is often necessary to use boards or a rough platform to distribute the weight of the observer. If the instrument must be mounted upon the same structure which supports the observer, such as a tank or building, two observers and a repeating instrument may be used to lessen the effects of unstable support. Each observer stations himself so as to face one of the stations to be observed upon and each makes the pointings upon his assigned station with as little shifting of his weight as possible.

INSTRUMENTAL ERRORS

The adjustments of the theodolite and the effects of the errors resulting from lack of perfect adjustment have been described on pages 46 to 60. These errors may be summarized as belonging to two classes: First, those which may be practically eliminated by a proper observing program, and second, those which can not be so eliminated. To the first class belong (1) the errors due to lack of proper collimation adjustment; (2) the unequal spacing of reading microscopes; (3) the eccentricity of the graduated circle with reference to the vertical axis of rotation of the reading micrometers; (4) the lack of horizontality of the horizontal axis of the telescope; and (5) the errors of graduation of the instrument, though the last error is only partly eradicated by the distribution of the readings about the circle. The methods by which these errors are eliminated are stated in the section referred to above.

Among the instrumental errors which can not be eliminated are those due to parallax and to lack of horizontality of the graduated circle. The effects of these errors are described in the section relating to the adjustments of the theodolite, pages 48 and 50. There are other errors, however, which although real are much harder to evaluate.

The changes in an instrument due to changes of temperature are such that even a small theodolite should be protected at all times from direct sunlight and wind if good results are to be obtained. The effects of these temperature changes are manifested in various ways. A graduated circle will expand on the side next the sun and there will be a differential change in the eccentricity of the circle which will not be eliminated. One side of the telescope will expand faster than the other, with a consequent change in collimation adjustment. Standards will change in their relative elevation, the collar of the diaphragm will change its adjustment, and since all of these are changing in amount from moment to moment no system of observing will eradicate them. This is one of the principal reasons why rapid pointings upon the object give more accurate results than slow pointings.

Another element which is often not appreciated is the manner of manipulation of the instrument. In an endeavor to make rapid pointings instrumental errors are sometimes introduced which are larger than those the observer is seeking to avoid. The hand should not rest heavily upon the instrument at any time. Slow-motion screws and the graduated heads on the screws of reading microscopes should be turned with a true rotary motion without lateral thrust. Slow-motion screws should always be tested to see if there is any dragging when the screw is turned in the direction which decreases the tension of the spring. If there is any sensible dragging and the cause can not be found and corrected, then the final motion of the screw must always be made against the spring. Tests for drag and for looseness of the centers have already been described.

PHASE AND ECCENTRICITY

A target or instrument is said to be eccentric when its center is not in the vertical line passing through the point to which the observations are referred. The proper correction can always be made for eccentricity if the distance and direction to the true station are recorded. Often, however, the observer desires to make an estimate of the error which would be introduced by an approximate amount of eccentricity due to phase or to some other causes. This can easily be made by remembering that "a second is a foot at 40 miles," or that an inch represents about 3⅓ seconds at a distance of 1 mile. On short lines the improper centering of either the theodolite or the target will introduce large errors. Warped or crooked signal poles must be observed upon with care, and an equal amount of care used in testing them for eccentricity. The safest plan is to point always at some certain part of the signal, say, at the bottom of the target, and to plumb that particular point over the station. Upon reaching a signal which has been observed upon it should be tested for eccentricity before its position is changed and the amount of its eccentricity recorded in such a manner that the computer can make no mistake when correcting the observations for it. (See p. 90.)

When using lamps on short lines care must be taken to point the lamps directly toward the observer, because the glass in front of the light is sufficiently illuminated to be visible to the observer and if not centered directly on the line will affect the accuracy of the observation. Occasionally a heliotrope or signal lamp must be posted at some little distance from a station in line with the distant station at which the observer is working. Errors due to eccentricity of the light are very apt to occur when this is done unless extreme care is taken to line in the light and maintain it in position. A theodolite, if available, should be used to line in the light; otherwise a plumb line should be used. The accuracy with which the light must be placed on line depends upon the distance from the observer's station, as indicated in the preceding paragraph.

The errors in horizontal-angle measures due to phase, or to the unequal illumination of a target, are often of considerable magnitude. In effect it is an eccentricity which could be corrected for if its exact amount were known. The difficulty lies in the inability of the observer to estimate its amount accurately, for it depends upon factors which change rapidly. The angle of the sun with the line of sight, the opacity of the signal, the shape of the object sighted upon, the intensity of the sunlight, will each have its effect on the appearance of the signal.

Many textbooks give trigonometric formulas for the correction of phase which are based upon the direction of the sun. These are usually not practicable to apply because other factors enter in. The apparent penumbral zone lying between the surface having a full illumination and that having no direct sunlight upon it will vary in width with the intensity of the light. The formula would also apply only to cylindrical or spherical objects, whereas many of our observations are made upon squared timbers. A target made of signal cloth will show a different phase from one made of lumber of the same shape and dimensions. For these reasons the best rule, when the outlines of the signal can be seen, is for the observer to make a close examination of the signal through the best telescope available and decide upon what part of the illuminated surface it is necessary to observe in order to eliminate errors due to phase. If the outlines of the target can not be seen, a can or other object placed at some distance away in the direction of the signal will show what part of it is illuminated and will give a rough idea as to how to point upon the object, if it is remembered that most of our telescopes are inverting. Under the conditions last stated it is practically impossible to secure second-order triangle closures, and recourse should be had to night observations on lamps.

Squared timbers should be used for signal poles on short lines, and one side should be exactly faced toward the observer. A vertical pole with a T cross section has also been used to advantage. The stem of the T is a 2 by 2 or 2 by 4 inch piece, while the bar of the T is a thin board, which is faced toward the observer. The thin edges of the board prevent any errors of phase, and the square timber, hidden from the observer's view, gives the necessary rigidity to the pole.

HORIZONTAL REFRACTION

The rays of light which pass from the object observed upon to the theodolite of the observer may curve horizontally as well as vertically. Horizontal refraction causes an error which is hard to detect and for which a correction can not be applied. It can, however, be avoided in large measure by carefulness in the reconnaissance and in the selection of observing conditions. The existence of this horizontal refraction can not always be foreseen, but certain atmospheric and topographic conditions operate strongly to cause such errors, and these will be briefly pointed out.

Air strata are generally of greater density near the ground and lie roughly parallel to it. Over a sloping terrain these strata of different densities are not horizontal, and a ray of light passing through them will be bent horizontally as well as vertically. The greater the difference in density in the air strata passed through, and the more they are inclined to the horizontal, the greater will be the horizontal bending of the light rays.

The most potent cause of this variation of density is the unequal temperature of different strata of the air. The force and direction of the wind are also determining factors, for with a strong wind the differences in the temperatures and densities of adjacent air strata are less marked, and horizontal bending of the rays of light is less apt to occur. A condition frequently met with on triangulation is that which is encountered when the line of sight passes along a bluff or mountain slope. Under these conditions it may be necessary to make observations in overcast weather or when the wind is blowing toward the bluff, if the first set of observations gives indications of horizontal refraction. If the wind blows down a slope and across the line of sight, the hotter or colder air from the slope will often cause trouble. A line passing near a building, a stone wall, or even the brace of a signal may suffer a like change in direction. The nearer the cause of the disturbance to the observer the greater will be the angular distortion.

The errors caused by horizontal refraction may be of considerable size. Night observations have occasionally been found to be in error by five or six seconds because of horizontal refraction and daylight observations by two or three times that amount. Undoubtedly, smaller errors due to this cause are frequently present but are more or less masked by errors due to other causes. If a line is suspected of having horizontal refraction, it should be reobserved, if practicable, under very different atmospheric conditions, especially with the wind in the opposite direction from that prevailing during the first observations.

OTHER CONDITIONS AFFECTING OBSERVATIONS

So long as the objects are visible there should be no hesitation in observing upon them, even though the observing conditions are apparently unfavorable. An unsteady object or a flickering or flaring light does not usually produce inaccurate observations. It is only when there is a semipermanent displacement of the image of the object sighted upon that observations must be made more slowly or stopped altogether. This creeping may be readily detected by centering the image upon the wires of the diaphragm and watching it closely for a minute or two. Sometimes the amplitude of this creeping movement may reach 10 or more seconds within a period of two or three minutes of time. Under such conditions it is not possible to secure satisfactory observations. When the period of vibration is only a few seconds of time the mean location of the image can be estimated.

The fact that the different observations upon a station exhibit a considerable range does not necessarily mean that their average value will be far from the true one, nor does a small range conclusively indicate the absence of some constant error. (See p. 88, regarding rejection of observations.) If the methods and instruments used do not give the results sought, the observer must systematically investigate the possible sources of error and locate the trouble. Above all, the observer must not try to force the observations by sighting upon a different part of the object from that which his judgment says is the proper point, for a poor triangle closure may be due to an error at any one of the three stations involved. He must cultivate an impersonal attitude toward his results and read the angles without bias, for an angle forced to give a good triangle closure will often result in large angle and side corrections when the least-squares adjustment is made.

When daylight observations are being made in an unsteady atmosphere the observer is apt to point upon the wrong object. This kind of blunder is particularly easy to make when sighting through a vista cut through timber, which frequently renders observing conditions very unfavorable. The error caused by a mistake of this kind is usually easy to detect, but it may make a reoccupation necessary. A light or signal partly obstructed from view by intervening objects may cause an error of several seconds. Occasionally changes in vertical refraction will cause a light or target to be partly obstructed that at other times is entirely clear.

NAMING OF STATIONS

The name assigned to a triangulation station is of more importance than is usually realized, since the name does not appear on the finished chart or map. It does appear, however, on the tablet which marks the station. If the name is chosen with due regard to geographic significance, the owner of the land and visitors to the station will have a greater regard for its preservation. Meaningless names should be used for triangulation stations only when names of geographical significance can not be found. Especial care should be taken to learn the proper spelling of names used for triangulation stations, whether these are the names of mountains, rivers, and points, or the name of the owner of the land on which the station is located. It is important that the same name be used for the same station throughout the records and computations.

COURTESY TO PROPERTY OWNERS AND OTHERS

The reconnaissance and signal-building party is the advance agent for the other parties. Stations must ordinarily be established on property owned by individuals or private corporations. Although many States have laws which give the right of entry upon private property to Government surveyors engaged on official work, it has not been the policy of this bureau to insist upon this right. In practically all cases the owner of the property is willing to have stations established on his land if the object of the survey is explained to him, and the chief of party should always arrange to secure permission to enter upon the premises and to establish the necessary marks. If it is necessary to damage crops, shrubs, or trees, the regulations must be closely followed in securing beforehand a written agreement which shall state the amount of damages to be paid. The good will and cooperation of property owners and their agents are very essential to the work of the parties which follow the reconnaissance party, and any source of future controversy should be avoided.

DETERMINATION OF VALUE OF ONE DIVISION OF A LEVEL BUBBLE

It is frequently necessary to determine in the field, where a level trier is not available, the value of one division of a level bubble. This is readily done by the method described below. The principle of this method consists in measuring an intercept whose length is known, at a known distance, in terms of divisions of the bubble. The angular value of the intercept is then calculated and the value of one division thus obtained.

Fasten the level to be tested, by adhesive tape or otherwise, longitudinally along the top of the telescope of a theodolite having a clamp and slow-motion screw for moving the telescope in a vertical plane. Suspend vertically an accurately graduated tape at a known distance from the theodolite, say 50 or 100 feet, or else plumb an accurately graduated rod at that distance. If the bubble has a chambered vial, adjust the length of the bubble so it will extend under about one-third to one-half the graduated portion of the vial.

Point upon some division of the tape or rod, so selected that the bubble will be near the end of the vial toward the eye end of the telescope. Read and record both ends of the bubble and repeat the pointing and readings until about 10 readings are obtained, bringing the cross wire of the telescope upon the mark half the time from above the mark and half the time from below, to neutralize the effect of any friction of the bubble against the vial. The mean of all the bubble readings will be the mean position of the center of the bubble.

Next point upon some mark on the tape or rod which will bring the bubble near the end of the vial toward the objective end of the telescope, take readings as before, and obtain the mean position of the center of the bubble. Measure carefully the distance from the rod to the horizontal axis of the instrument, compute the angular value of the intercept at that distance, and divide by the number of divisions of the bubble between the two mean positions. The value of one division of the bubble obtained by this method will be as accurate as that obtained on the average level trier.

DETERMINATION OF HEIGHT OF STATION BY OBSERVING SEA HORIZON

At times it may be difficult to connect a triangulation scheme to a bench mark. If some of the stations are within sight of the ocean, the elevations of the stations, as determined by the vertical-angle measurements carried through the chain of triangles, can be checked and made more exact by observations upon the sea horizon. Elevations determined in this manner are not as accurate as when frequent connections can be made to bench marks, for the observations are nonreciprocal and an arbitrary value must be used for the coefficient of refraction, m, which may vary for daytime observations on the sea horizon from 0.078 to 0.130. The formula for computing the height of station from the observed angle of depression is

$$h = 2\rho \frac{\sin \dfrac{\theta}{2(1-2m)} \sin \dfrac{\theta}{2}}{\cos \dfrac{(1-m)\theta}{1-2m}}$$

where h = elevation of station above sea level.

ρ = radius of curvature of the earth (in the same unit of length as h).

m = coefficient of refraction.

θ = observed angle of depression.

For ordinary purposes the following approximate formula is all that is justified, especially as the uncertainty in the coefficient of refraction is liable to cause a considerable error in the result. The approximate formula is

$$h = \frac{1}{2} \rho \sin^2 1'' \frac{\theta^2}{1-2m},$$

in which θ is expressed in seconds of arc. If we use a mean value of $\log \rho = 6.80421$ and a mean value of $\log(1-2m) = 9.92428$, the approximate formula becomes simply

$$h \text{ (in meters)} = 0.000089135\ \theta^2$$

or

$$\log h = (5.95005 - 10) + 2 \log \theta$$

This form of the formula is sufficiently exact in most cases. However, if an accurate value of m has been determined at the station by means of reciprocal observations on other triangulation stations, then it can be used to determine a better value of the constant in the approximate formula.

DETERMINATION OF DISTANCE TO BREAKER BY OBSERVING ANGLE OF DEPRESSION

In making surveys along the seacoast it quite often happens that the distance to an offshore rock or reef is desired, the position of which is indicated by breakers. If the elevation of the station from which the observations are made is known, the approximate distance may be obtained by reading the angle of depression to the breaker and computing the distance by one of the following formulas. At least two observations of the angle should be made, one with the instrument direct and one with it in the reverse position to eliminate instrumental errors.

The accuracy of the resulting distance depends principally upon the relation between the height of the station and the distance to the breaker or, in other words, upon the size of the angle of depression. If the station is high and the distance comparatively short, considerable accuracy may be obtained. Where the angle of depression is small any inaccuracy in the elevation of the station or uncertainty in the coefficient of refraction may seriously affect the accuracy of the computed distance.

The formula[8] for the distance is as follows:

$$s = h \cot \theta + \frac{1-2m+2(1-m)\tan^2\theta}{2\rho} h^2 \cot^3 \theta + \frac{[1-2m+2(1-m)\tan^2\theta]^2}{2\rho^2} h^3 \cot^5 \theta + \frac{5[1-2m+2(1-m)\tan^2\theta]^3}{8\rho^3} h^4 \cot^7 \theta + \ldots$$

in which θ is the angle of depression or the zenith distance minus 90°, ρ the radius of curvature of the earth, and m the coefficient of

[8] These formulas were derived by Dr. O. S. Adams, of the division of geodesy of this bureau.

refraction. With some approximation the above formula will take the form,

$$s=h\cot\theta+Kh^2\cot^3\theta+2K^2h^3\cot^5\theta+5K^3h^4\cot^7\theta+\ldots.$$

in which,

$$K=\frac{(1-2m)\sec^2\theta}{2\rho},$$

or, since the value of m is so uncertain, we may use simply,

$$s=h\cot\theta+Kh^2\cot^3\theta$$

SPECIFICATIONS, THIRD-ORDER TRIANGULATION

Accuracy.—The same instruments and methods will be used on the measurements of angles for third-order triangulation as were specified for second order (see p. 33), except for such minor changes in methods as result from the lower limits of accuracy imposed. The immediate requirements which the angle measurements must meet are an average triangle closure of not to exceed 6 seconds and a maximum closure of 12 seconds.

To secure this accuracy with a direction theodolite reading to one second, two positions of the circle will usually be sufficient. With a direction theodolite reading to two seconds, the type of direction instrument ordinarily used on this class of triangulation, four positions of the circle should be used. With a 10-second repeating theodolite, from one to two sets of 6 D/R (see p. 33) will suffice. Supplementary and intersection stations should be located with the same number of observations as were prescribed for second-order triangulation on page 35.

Trigonometric leveling.—Measurements of vertical angles will be made under the same conditions as were noted for second-order triangulation (see p. 36) and by the same methods.

Marking stations.—Each station located with third, or higher, order of accuracy shall be marked in accordance with the instructions on pages 38–41. In addition, enough intersection stations shall be permanently marked as will insure the distribution of permanent stations indicated on page 36.

FIELD COMPUTATIONS, SECOND AND THIRD ORDER TRIANGULATION

On both second and third order triangulation complete computations should be made in the field unless instructions to the chief of party prescribe otherwise. This includes the computation of the geographic positions of all points located by triangulation, the preparation of the "List of geographic positions," and the "Computation of elevations." All records and computations up to and including the "List of directions" and "Reduction to center" should be carefully checked in the field, for the office computation accepts the "List of directions" prepared in the field as correct. The "Computation of triangle sides" and "Computation of geographic positions" are sufficiently self-checking, if carefully computed, and do not need a field check. No least-squares adjustment of the triangulation should be made in the field. All computations should be made in a neat and orderly manner on the forms provided for the purpose.

On second and third order triangulation the angles and azimuths should be carried to the nearest tenth of a second and the latitudes and longitudes to hundredths of a second in the main scheme and for the supplementary stations. Six places in the logarithms should be used. For intersection stations the angles and azimuths should ordinarily be taken to the nearest second, latitudes and longitudes to tenths of seconds, and logarithms to five places of decimals.

Certain computations must be kept closely up to date. This includes the computing and checking of the angles in the record books and on the lists of directions, the computation of the triangle sides, and the writing of the descriptions of stations. The initials of the persons making and checking the computations in the record books and on the computation forms should be placed at the bottom of each page as the computation and checking progresses. It should be remembered that the onus of any uncaught mistake is chiefly upon the person checking. In all computations a decimal 5 which is to be dropped should be applied to make the last significant figure an even number. For instance, 1.05 would be 1.0, 1.15 would be 1.2, etc.

The record and computation forms used in the field are listed below, and a brief description of the proper preparation of each form will follow.

Record, horizontal angles (for repeating instrument), Form 250.
Record, horizontal directions (for 2-micrometer direction instrument), Form 251A.
Abstract, horizontal directions (for direction instrument), Form 470.
List of directions, Form 24A.
Reduction to center, Form 382.
Triangle-side computation, Form 25.
Position computation, Form 27.
List of geographic positions, Form 28B.
Record, double zenith distances (if observed), Form 252.
Abstract of zenith distances (if observed), Form 29.
Computation of elevations, reciprocal observations, Form 29A.
Computation of elevations, nonreciprocal observations, Form 29B.
Description of station (new stations), Form 525.
Description of station, recovery note (old stations), Form 526.

RECORD, HORIZONTAL ANGLES (REPEATING THEODOLITE)

An example of a record is given in Figure 52. From this the resulting directions should be written in the "List of directions" (fig. 56) without any other abstract. It will be noticed in the sample below that, in addition to the reading of one repetition on the first measurement of each angle, there is a reading for three repetitions in each case. The latter gives a value of the angle correct to within 10

Department of Commerce
Coast and Geodetic Survey
Form [illegible]

HORIZONTAL ANGLES

Station: Dab | State: P.I. | Island or County: Luzon | Date: Feb. 7, 1906.

Observer: E.H.P. | Instrument: B.& B. 7-inch theodolite No. 134.

Objects observed	Time h. m.	Tel. D or R	Rep's	Angle ° ′	A ″	B ″	Mean of verniers	Angle Mean D and R ° ′ ″	Remarks
	a.m.								
Pet-Dog	8.00		0	0 00	00	00	00		
			1	88 59	50				
			3	266 55	20	20			
		D	6	173 58	40	40	40	88 59 46.7	
(Dog-Pet)		R	6	0 00	10	20	15	44.2	45.5-0.7=44.8
Dog-Bat			0	0 00	15	25	20		
			1	42 30	15				
			3	127 30	35	45			
		R	6	255 01	15	25	20	42 30 10.0	
		D	6	0 00	25	25	25	09.2	09.6-0.7=08.9
Bat-Kow			0	0 00	10	10	10		
			1	27 34	10				
			3	82 43	10	20			
		D	6	165 25	20	30	25	27 34 22.5	
		R	6	0 00	$\overline{50}$	00	$\overline{55}$	25.0	23.7-0.8=22.9
Kow-Bol			0	0 00	00	10	05		
			1	37 40	40				
			3	113 02	10	20			
		R	6	226 04	20	30	25	37 40 43.3	
		D	6	0 00	10	20	15	41.7	42.5-0.8=41.7
Bol-Pet			0	0 00	20	30	25		
			1	163 15	10				
			3	129 15	30	30			
		D	6	259 30	40	40	40	163 15 02.5	
		R	6	0 00	20	30	25	02.5	02.5-0.8=01.7
								360 00	03.8 00.0

FIG. 52.—Specimen, record horizontal angles

Department of Commerce
U. S. Coast and Geodetic Survey
Form [illegible]

HORIZONTAL ANGLES

Station: Dab | State: P.I. | Island or County: Luzon | Date: Feb. 7, 1906.

Observer: E.H.P. | Instrument: 7-inch theodolite No. 134

Objects observed	Time h. m.	Tel. D or R	Rep's	Angle ° ′	A ″	B ″	Mean of verniers	Angle Mean D and R ° ′ ″	Remarks
	a.m.								
Pet	9.50	D		0 00	00	$\overline{50}$			
		R		180 00	10	00	00	0 00 00	
Bell tower, Olan		D		21 18	30	20			
		R		201 18	40	30	30	21 18 30 +5 35	
Oil		D		176 15	10	30			
		R					20	176 15 20 +10 30	
Frog		D		209 59	50	40			
		R		29 59	60	50	50	209 59 50 +5 55	
L. tang.Padang Id.		D		252 18					
		R		52 18				252 18	
Peak 17		D		241 11	00	$\overline{30}$			
		R		61 10	50	40	[illegible]	241 10 [illegible] +[illegible] 55	
Pet		D		359 59	50	40			
	10.32	R		179 59	60	50	50	0 00 $\overline{50}$ +10 00	

FIG. 53.—Specimen, record horizontal angles, intersection stations

seconds, which will check the reading of the minutes for the six repetitions. The reading of one repetition on one vernier does not give a sufficiently accurate check. Use the reading on three repetitions as a check only.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 251 B

Horizontal Directions

Station: Hester — Observer: E. O. H. — Instrument: C.& G.S. No. 302 — Date: Feb. 21, 1927.

Position	Objects observed	Time a. m. p.m.	Tel. D or R	Mic.	°	′	Back'd ″	Forward ″	Mean	Mean D and R	Direction ″	Remarks
12	Connell	3:55	D	A	123	04	15	14				
				B			25	24	19.5			
		4:08	R	A	303	04	24	25				
				B			15	14	19.5	19.5 +0.6 20.1		
	Cox		D	A	147	18	30	31				
				B			38	38	34.2			
			R	A	327	18	38	38				
				B			47	47	42.5	38.4	18.3	
	Stone Mt.		D	A	350	31	44	44				
				B			49	49	46.5			
			R	A	170	31	51	52				
				B			55	55	53.2	49.8	29.7	
	Candler		D	A	122	58	39	39				
				B			48	48	43.5			
			R	A	302	58	39	39				
				B			49	50	44.2	43.8	23.7	
	Connell		D	A	123	04	24	25				
				B			20	20	22.2			
			R	A	303	04	23	22				
				B			16	16	19.2	20.7 −.6 20.1	00.0	

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FIG. 54.—Specimen, record horizontal directions

Observations on intersection stations.—An example of a record of intersection observations is given in Figure 53. From this the resulting directions should be written in the "List of directions" without other abstract.

RECORD, HORIZONTAL DIRECTIONS (DIRECTION THEODOLITE)

A double page of the record book is shown in Figure 54. The means should be checked before the abstract of directions is made out, preferably by some one other than the original computer. Each page checked should be so indicated by a check mark at the bottom of the page, with the initials of the person checking.

ABSTRACT OF HORIZONTAL DIRECTIONS (DIRECTION THEODOLITE)

It is important that this form be made out carefully, because the mean directions derived from the abstract of horizontal directions

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 470
Ed. Jan., 1926

ABSTRACT OF DIRECTIONS

State Georgia

Station Hester Computed by B.W. Date Feb. 21, 1927.

Observer E.O.H. Checked by E.O.H. Inst. No. 302

DO NOT WRITE IN THIS MARGIN

POSITION NO.	STATIONS OBSERVED					
	Connell	Cox	Stone Mt.	Decatur E. Base	Decatur W. Base	Candler
	(INITIAL) 0° 00′	° 24 14	° ′ 227 27	° ′ 290 58	° ′ 330 20	° ′ 359 54
	″	″	″	″	″	″
1	0.00	18.3	29.7	38.7	41.7	23.7
2	0.00	15.6	29.6	39.2	44.0	21.4
3	0.00	17.4	29.5	38.4	[52.8]R 43.4	24.0
4	0.00	14.9	28.8	38.7	41.9	22.8
5	0.00	15.6	27.6	37.6	44.0	22.0
6	0.00	19.2	30.3	38.0	41.3	24.3
7	0.00	16.9	29.8	39.4	42.7	23.5
8	0.00	15.3	30.4	40.6	43.7	22.8
9	0.00					
10	0.00					
11	0.00					
12	0.00					
13	0.00					
14	0.00					
15	0.00					
16	0.00					
Sum,		133.2	235.7	310.6	342.7	184.6
Mean,		16.6	29.5	38.8	42.8	23.1
Cor. for ecc.,				-0.4		
Direction,		16.6	29.5	38.4	42.8	23.1

FIG. 55.—Specimen, abstract of directions

constitute the basis for all the later computations. Every position observed at a station on main-scheme stations should appear on the abstract, the rejected readings being indicated by the letter *R*. A sample form is shown in Figure 55.

Rejection of observations.—The chief difficulty in making out the form for abstract of directions lies in deciding what observations to reject. The usual formulas for the rejection of observational quantities are too cumbersome to apply and are not satisfactorily applicable to a short series of observations. It is customary, therefore, to apply an arbitrary limit of rejection, determined empirically from previous experience with the instrument used or with one of similar qualities. For observations with the type of direction theodolite usually used on second or third order triangulation the rejection limit for the angular value of a direction on any one position of the circle may ordinarily be taken as 5 seconds from the mean. Specific ways in which the rejection limit is to be applied are indicated below. If it is found that with such a rejection limit the number of rejections is averaging much in excess of 10 per cent of the entire number of observations, the rejection limit may be increased, though before that is done every effort should be made to determine the cause of the large range.

The following rules will be a sufficient guide to the rejection of observed directions:

1. No reading should be rejected if it falls within the limit of retention unless rejected at the time of taking the observation. The observer's reason for rejection should then appear in the original record. This rule will not apply to the case where one set of observations of a direction is rejected in favor of another set of an equal number of positions.

2. If two or more readings have been taken for a single position, the mean should be used if all readings come within the limit of retention.

3. If one reading falls without the limit and one within the limit, do not use a mean even though the mean be within the limit. Use instead the single reading within the limit.

4. If both readings fall without the limit, one being abnormally high and the other abnormally low, and the mean falls within the limit, reject both readings and try the position again. If a reading is not obtained within the required limit, reject the position entirely, using the remaining positions to compute the mean direction.

5. If the values obtained for one or more directions on the various positions of the circle show a considerable range, the safest plan, and the one which should be followed if the average closing error of the triangles involved is more than 3 seconds, is to observe all the positions a second time and use the new data to determine the value of the direction.

6. Before computing a trial mean, any observation so far from the approximate mean as to be very evidently the result of blunders should be rejected. After a trial mean is obtained and the rejection

limit applied, none of the observations rejected should be again included even though the new mean would bring them within the limit of rejection.

7. The results obtained by applying rigorously the limit of rejection, even though the quantities rejected are just outside the limit, will probably be but little different from those derived after long consideration, and much time can be saved the field party by a strict application of the rules given above.

LIST OF DIRECTIONS

On the list of directions, Figure 56, the mean directions of all unrejected observations are arranged in order of azimuth from some one selected initial. The list includes not only the mean directions to the principal stations but also the directions to intersection points and reference marks.

The data on this form constitute the material upon which the office computations are based, and these data should be so completely checked that there will be no need in the office to resort to the record book or the abstract of directions. The only exception to this rule is where there is not sufficient time in the field to make all the eccentric reductions without delay to the progress of the party, and this contingency is provided for in later paragraphs.

On the back of the form for the list of directions are instructions for its preparation. Only two points covered by those instructions need be mentioned here, viz, the number of decimal places to be shown in the mean angle and the treatment of eccentric directions. As regards the former, on second and third order triangulation the directions to main-scheme stations should be carried to tenths of seconds, and directions to other points to seconds. Directions to near-by objects, such as witness or reference marks, need be taken to the nearest 10 seconds only.

The second point to be emphasized in the preparation of the list of directions is the recording of the eccentricity and the reduction of the observed directions to center. If a direction has not been reduced to center, leave the "Eccentric reduction" column blank. If no eccentric reduction is necessary, put dashes in the column. The rule should be invariably followed, for otherwise an unreduced direction may be used for a reduced one.

On the list of directions the main-scheme stations should be easily distinguishable from the other points listed. This can be achieved by printing in heavy letters the names of the main-scheme stations, while the names of the other points are written in with ordinary width of line. The distinction between the main-scheme points and others can be further accentuated by the use of asterisks if desired.

Whether the directions shown on the form have been reduced for eccentricity or not, any eccentricity of light or instrument at a station should be recorded on the list of directions in a form that is

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 24A

LIST OF DIRECTIONS

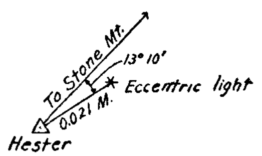
Station Hester *State* Georgia

Chief of party E.O.H. *Date* Jan. 10, 1927. *Computed by* B.W.

Observer E.O.H. *Instrument* No. 302 *Checked by* E.O.H.

OBSERVED STATION	Observed direction	Eccentric reduction	Direction corrected for eccentricity	Sea level reduction*	Corrected direction with zero initial*	Adjusted direction*
	° ′ ″	″	° ′ ″	″	° ′ ″	° ′ ″
Connell	0 00 00.0					
Cox	24 14 16.6					
R.M. dist. 5.254 m.	50 27 22					
Episcopal church spire	122 15 33.4					
Stone Mt.	237 27 29.5					
Eccentric light, dist. 0.021m.	240 37					
Decatur E. Base	290 58 38.4					
Decatur W. Base	330 20 42.8					
Candler	359 54 23.1					

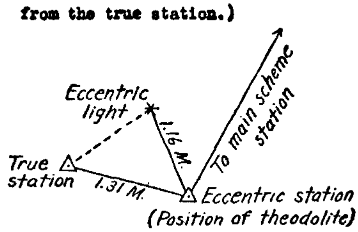
(Method of recording eccentricity when theodolite is centered over station and light is eccentric.)



Light eccentric as given above when shown to Stone Mt. No eccentricity of light to other stations. No eccentricity of theodolite.

All directions reduced for eccentricity.

(Method of recording eccentricity when theodolite is eccentric and eccentric light is more than 0.1 m. from the true station.)



(Record must state to what stations the light was shown in above position.

Measuring the distance from the true station to the eccentric light will give a check on the values for the elements of the small triangle.)

*The last three columns to the right are for office use and should be left blank in the field.

FIG. 56.—Specimen, list of directions

entirely free from any possibility of misinterpretation. The form of record shown on the sample in Figure 56 should always be used when the light is in an eccentric position and the instrument is centered. The sketch should always be included.

Wherever it is possible to mount the theodolite directly over the mark this should be done, even though the light has been shown from an eccentric point. To have an instrument mounted eccentrically when it can be avoided simply causes unnecessary computation. Directions to eccentric lights can be easily corrected by the nomogram shown in Figure 58, or by tables computed decimally for lines of various lengths and for eccentric distances of different amounts taken at right angles to the lines.

When the theodolite must be mounted eccentrically to the station, for the purposes of field computation all directions may be reduced to the eccentric station and the station should be so named—as, "Roundtop Eccentric." In this case the angle and distance to the marked station should be recorded in the manner shown in Figure 56, making sure that the points on the sketch representing the eccentric station and the marked station are distinctively marked.

When at any station the theodolite is eccentric and the light at that station has occupied a different eccentric position the measurements to the eccentric light may be made and recorded in either of two ways. When the light is not farther from the marked station than a decimeter, one edge of a straightedge of some kind can be placed in the vertical line through the marked point and sighted successively at all stations to which the eccentric light was shown, the normal distance from the eccentric light to the true lines being measured and recorded as "Light at Hester when showing to Stone Mountain 0.011 meter to the east of line to Stone Mountain." When recorded in this manner there is no chance for ambiguity.

When the instrument is eccentric and the light is shown from a different eccentric point which is more than a decimeter from the vertical line through the marked station, it is better to measure the distance to the eccentric light from the point occupied by the theodolite and the angle at the theodolite point from the eccentric light to some main-scheme station. The distance and direction from the marked point to the eccentric light can then be found by solving the small triangle shown in Figure 56.

The importance of the proper recording of the eccentricity of lights and theodolites has been emphasized for the reason that many serious mistakes and ambiguities in triangulation records are traceable to that source.

REDUCTION TO CENTER

When observations are made upon an object which is not in the vertical line through the point to which the observations are to be referred, the object sighted upon is termed an "eccentric object" and the computation necessary to correct the angles to make them

696689°—46——7

refer to the true object is called the reduction to center. The same process is used in reducing to the true station the observations made upon an eccentric object as in reducing to the true station the observations made at an eccentric station. No correction for eccentricity is necessary, of course, if the object observed upon is in the vertical

INSTRUCTIONS

The required reduction to center is, in seconds, $c = \frac{d \sin \alpha}{s \sin 1''}$, in which d is the distance from the eccentric station to the true station, and s is the length in meters of the line between the true stations involved, and, therefore, log s is taken directly from the computation of triangle sides. α is the direction of the distant station involved, reckoned in a clockwise direction as usual, but referred to the direction from the eccentric to the true station, or center, taken as zero. This definition of α is true for the case in which the object pointed upon is eccentric, as well as for the case in which the instrument is eccentric.

Carry α to minutes only and all logarithms to five decimal places only. Do not in any case carry the derived reductions to more than two decimal places. There is no advantage in carrying them to more decimal places than the directions to which they are to be applied are carried on Form 24 A.

REDUCTIONS FOR AN ECCENTRIC INSTRUMENT

If the instrument is eccentric the first column of this form should contain the names of the stations observed from that eccentric position of the instrument.

The values in the fifth column are derived by subtracting those in the fourth column from those in the third. The values in the fourth column may need to be derived by successive approximations from the triangle side computations if the eccentric reductions are large. The values in the sixth column are obtained from those in the fifth by adding log $\frac{d}{\sin 1''}$ derived as indicated in the heading of the form, if d is expressed in meters. If d is expressed in feet, to the other two logarithms add also 9.48402 to convert to meters. To obtain a direction as shown on Form 24 A, subtract the reduction c for the station which is the initial on Form 24 A from the reduction c for the required direction and apply the difference to the observed direction. Similarly, the correction to any angle is the difference of the reductions on this form to the two directions involved in that angle.

REDUCTIONS FOR AN ECCENTRIC OBJECT OBSERVED

If the object observed is eccentric the heading "Eccentric Station ——" should be changed to "Eccentric Observed Object at Station ——," the first column should contain the names of the stations from which this eccentric object was observed, and in each case α is the direction from the eccentric object to the distant station involved, reckoned in a clockwise direction as usual, but referred to the direction from the eccentric object to the true station, or center, taken as zero. (No distinction need be made between the direction from the eccentric object to the distant station and the direction from the true station to the distant station except when the eccentric reduction is more than one minute.) The remainder of the computation on this form is made in the manner indicated above with reference to an eccentric instrument. The reductions to directions are, however, to be applied to observed directions, at the stations named in the first column, to the eccentric object at the station named in the heading. The directions to which these reductions are to be applied are therefore found in various of the lists of directions on Form 24 A, not all in one list as is the case when the instrument is eccentric.

Compare the following example with that given on Form 24 A.

REDUCTION TO CENTER

Eccentric Station: Chase.

d = 10.987 *meters.*

Log d = 1.04088
Colog sin 1″ = 5.31443
Sum 6.35531

STATIONS	α ° ′	Log sin α	Log s (s in meters)	Log $\frac{\sin \alpha}{s}$	Logarithms of reduction in seconds	Reduction $= c$ ″
Center	0 00					
Bossing	179 18	8.08696	4.49198	3.59498	9.95029	+ 0.89
Central	224 27	9.84528	4.40254	5.44274	1.79805	− 62.81
Little River	242 47	9.94904	4.51928	5.42976	1.78507	− 60.96
Lyons, salt works	249 02	9.97025	4.30616	5.66409	2.01940	−104.57

FIG. 57.—Specimen, reduction to center

plane containing both the station observed upon and the station from which observations are made—in other words, if it is "in line" with the two stations.

The computation for the reduction to center is easily made on Form 382. The instructions given on that form, which include a numerical example of a reduction to center, are reproduced in Figure 57 and give ample information for making the computation. The reduction to center can also be made by the use of the nomogram shown in Figure 58.

An eccentric instrument or object should be avoided when possible because of the added computing necessary to reduce the observations

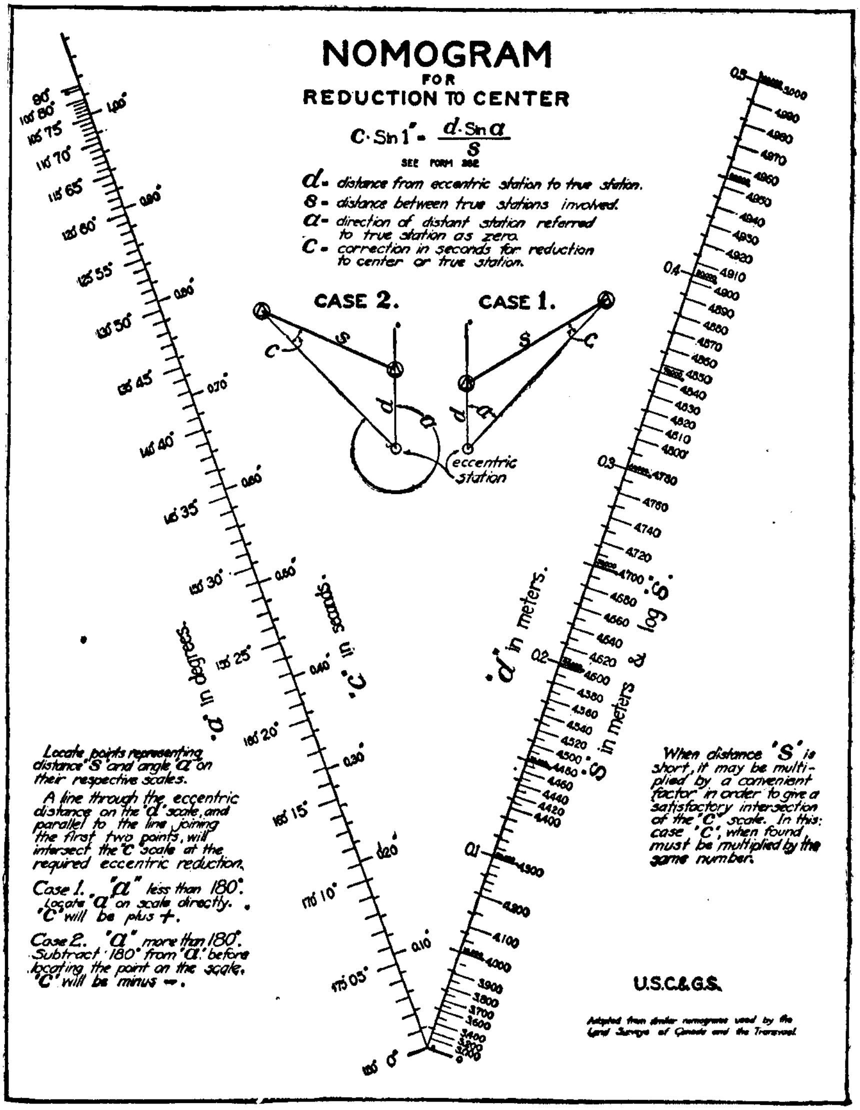


FIG. 58.—Nomogram for reduction to center

Copies of this nomogram on heavy paper can be obtained on requisition

to center and because of the greater danger of angular errors. Eccentricities of more than a meter or two are especially apt to cause poor triangle closures because of the difficulty of securing a true value of the horizontal distance to the eccentric point.

TRIANGLE-SIDE COMPUTATION

The usual arrangement of this computation is shown in Figure 61. The sketch of the quadrilateral, Figure 59, shows the relative positions of the stations, the known line being Trouble-Juan.

Every triangle should be included in this computation. The observed angles are taken from the list of directions previously compiled and the closure of the triangle obtained without regard to spherical excess. As a check on taking out the angles it may be noted that the sum of the closures of the two triangles formed by one diagonal is equal to the sum of the closures of the two triangles formed by the other diagonal. Thus, $-3''.0-6''.3=-3''.1-6''.2$.

Having checked the taking out of the observed angles in this manner, the angles are then used to compute preliminary lengths for use in the computation of spherical excess. Three places of decimals are sufficient for this computation in all cases.

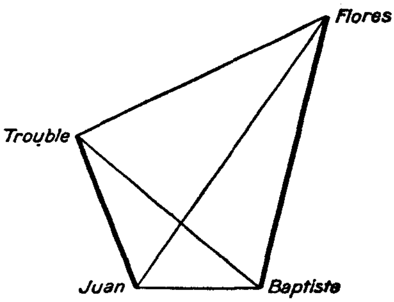


FIG. 59.—Diagram, quadrilateral

The spherical excess is computed by the formula $e=a\ b \sin C\times m$, where e is the spherical excess, a, b, and C are two sides and the included angle, respectively, of the corresponding triangle, and m a factor depending upon the latitude of the triangle and the dimensions of the spheroid. Values of log m for the different latitudes, reproduced from page 16 of Special Publication No. 8, Formulæ and Tables for the Computation of Geodetic Positions, are given on page 99. The quadrilateral of the sample computation has a mean latitude of 55° 23′.

The computation of the spherical excess of triangle 3 is given below:

log distance Baptiste to Trouble	=	3. 660
log distance Flores to Trouble	=	3. 727
log sin 66° 44′ 33″.6	=	9. 963–10
log m (lat., 55° 23′)	=	1. 403–10
log e	=	8. 753–10
e	=	0″.1

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 24A

LIST OF DIRECTIONS

Station Baptiste *State* Alaska

Chief of party H.B.O. *Date* Aug. 13, 1925 *Computed by* H.B.O.

Observer H.B.O. *Instrument* No. 287 *Checked by* O.P.S.

OBSERVED STATION	Observed direction	Eccentric reduction	Direction corrected for eccentricity	Sea level reduction*	Corrected direction with zero initial*	Adjusted direction*
	° ′ ″	″	° ″	″	″	″
Flores	0 00 00.0					
Enter	19 04 06.6					
Key (Pole on rock)	22 17 05					
Set (Whitewashed rock)	30 20 18					
Triplet	30 29 18.6					
Congrego	32 34 56.7					
Gun (Flag on bush)	34 14 50					
Ran (Whitewashed rock)	35 34 42					
Far (Whitewashed rock)	36 15 40					
Nut (Whitewashed rock)	36 57 48					
Kat (Whitewashed rock)	37 35 08					
Dik (Whitewashed rock)	38 55 38					
Dol (Whitewashed rock)	40 23 30					
R.M. No. 1 dist. 47.20 m.	41 33					
Cel (Whitewashed rock)	42 13 58					
Cabras	47 39 00.7					
Peak (Cape Bartoleme)	50 52 30					
Peak (Up)	55 18 42					
Peak (Higher twin)	64 02 42					
Peak (Nearer)	70 11 38					
Cocos	73 05 14.2					
Ignace	75 12 27.6					
Juan	256 09 42.8					
R.M. No. 2 dist. 11.82 m.	288 01					
Trouble	296 42 29.6					

Station Flores *State* Alaska

Chief of party E.F.D. *Date* July 23, 1907. *Computed by* C.V.H.

Observer C.V.H. *Instrument* No. 279 *Checked by* O.P.S.

OBSERVED STATION	Observed direction	Eccentric reduction	Direction corrected for eccentricity	Sea level reduction*	Corrected direction with zero initial*	Adjusted direction*
Enter	0 00 00.0					
Congrego	13 12 42.2					
Triplet	16 45 36.6					
Cabras	27 37 05.4					
Cocos	43 59 58.1					
Ignace	47 47 38.1					
Lat	65 28 19.2					
Mond	95 14 36.2					
Baptiste	115 19 09.7					
Juan	135 39 38.4					
R.M. No. 1 dist. 14.13 m.	140 04					
Trouble	165 17 12.0					
R.M. No. 2 dist. 5.45 m.	185 19					
Sleep	268 12 45.5					
Kelp	316 35 57.2					
Cove	338 58 58.2					

FIG. 60.—List of directions for two stations of quadrilateral
(For explanation see p. 94)

This computation may be made in small figures in abbreviated style directly upon the triangle-computation form. The same check prevails in the computation of spherical excess as was noted in the taking out of the observed angles. Thus $0''.0+0''.1=0''.0+0''.1$.

An approximate value for the spherical excess may be obtained by dividing the area of a triangle in square miles by 100 and adding one-third of itself to the quotient obtained.

The spherical excess is distributed among the three angles of the triangle, one-third to each angle.

The closing corrections of the triangles with the spherical excess considered are next obtained, being checked in the example by the equation, $-3.0-6.2= -3.1-6.1$.

This closing correction is now distributed among the three angles of the triangle, one-third to each angle, and the spherical angles obtained. If the corrections to be applied to each angle are not exactly equal, they may be applied indiscriminately regardless of the size of the angle, for the chances are equal as to whether the larger correction belongs on the large or small angle. By subtracting one-third of the spherical excess from each spherical angle the corresponding plane angles of the triangle are obtained. These plane angles are used only for the computation of the sides of the triangle.

The computation of the sides of the triangle is by the well-known law of sines. In triangle 1 of the sample 3.503741 is the log of the distance in meters from Juan to Trouble. This distance is known from previous computations; 0.187047 is the colog sine of 40° 32′ 45″.8; 9.968757 and 9.671332 are logs of the sines of 111° 28′ 24″.3 and 27° 58′ 49″.9, respectively. The sum of the first, second, and third of these logs gives 3.659545, the log distance Baptiste to Trouble, and the sum of the first, second, and fourth gives 3.362120, the log distance Baptiste to Juan. Other triangles of the quadrilateral are computed in like manner, an approximate check being obtained upon the logarithmic work by the two values obtained for each of the three lines, Flores-Juan, Flores-Baptiste, and Flores-Trouble.

The line Baptiste-Flores in the example is used to carry the triangulation forward to other quadrilaterals. The length obtained from the strongest chain of triangles through the figure should be used, and not the mean of the two or more values obtainable by using various combinations of triangles. The strongest chain of triangles should likewise be used in computing positions. By the strongest chain is meant the chain with the smallest ΣR, and not necessarily the one with the smallest triangle closures.

In triangle 1, Figure 61, the lengths of the lines Baptiste-Juan and Baptiste-Trouble are obtained. Either of those lines could have been used for the third triangle to obtain the length Baptiste-Flores, but the line Baptiste-Trouble was chosen because it gave a stronger determination of the line from which the lengths would be carried ahead through the scheme. Similarly, to go from the line Trouble-Juan to the line Flores-Juan and thence to the line Baptiste-Flores is much

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Alaska

	NO.	STATION	OBSERVED ANGLE	CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
		2-3 Juan-Trouble						3.503741
		1 Baptiste	40 32 46.8	-1.0	45.8	0.0	45.8	0.187047
1		2 Juan	111 28 25.3	-1.0	24.3	0.0	24.3	9.968757
		3 Trouble	27 58 50.9	-1.0	49.9	0.0	49.9	9.671332
			3.0	-3.0		0.0		
		1-3 Baptiste-Trouble						3.659545
		1-2 Baptiste-Juan						3.362120
		2-3 Baptiste-Juan						3.362120
		1 Flores	20 20 28.7	-1.0	27.7	0.0	27.7	0.458911
2		2 Baptiste	103 50 17.2	-1.1	16.1	0.0	16.1	9.987208
		3 Juan	55 49 17.2	-1.0	16.2	0.0	16.2	9.917657
			3.1	-3.1		0.0		
		1-3 Flores-Juan						3.808240
		1-2 Flores-Baptiste						3.738688
		2-3 Baptiste-Trouble						3.659545
		1 Flores	49 58 02.3	-2.0	00.3	0.0	00.3	0.115958
3		2 Baptiste	63 17 30.4	-2.1	28.3	0.0	28.3	9.950998
		3 Trouble	66 44 33.6	-2.1	31.5	0.1	31.4	9.963191
			6.3	-6.2		0.1		
		1-3 Flores-Trouble						3.726501
		1-2 Flores-Baptiste						3.738694
		2-3 Juan-Trouble						3.503741
		1 Flores	29 37 33.6	-2.0	31.6	0.0	31.6	0.305985
4		2 Juan	55 39 08.1	-2.0	06.1	0.0	06.1	9.916782
		3 Trouble	94 43 24.5	-2.1	22.4	0.1	22.3	9.998523
			6.2	-6.1		0.1		
		1-3 Flores-Trouble						3.726508
		1-2 Flores-Juan						3.808249

Do not write in this margin

FIG. 61.—Specimen, computation of triangles

stronger, as measured by the R's of the individual triangles, than to go by way of the line Trouble-Flores.

A word of caution.—Reference has been made repeatedly to the errors in closure of the triangles. This has been chosen as the criterion by which the observer must judge the accuracy of his work, because it is the test which can be most easily and quickly applied in the field.

It should be remembered, however, that this is but one of the tests that the season's work must pass. The lengths of the various lines of the triangulation when computed through the various chains of triangles must check to a degree of accuracy comparable to the triangle closure. When the triangles have been closed by applying one-third of the error of closure to each angle, and the triangle sides computed, the logarithms of the length of a side, as computed through the various chains, should differ by not more than four times the tabular difference corresponding to one second in the logarithm of the sine of the smallest angle entering into the computation of that logarithm side.

Finally, there must be satisfactory agreements between the azimuth as computed through the scheme and the corrected observed azimuth at each Laplace station, and also between the length of a base as computed through the triangulation from the last base and its measured length. If the observer bears in mind these additional tests which his work must meet, he will not unduly force an angle in a certain direction in order to make a better triangle closure.

MATHEMATICAL SOLUTION OF THE THREE-POINT PROBLEM

If three points, forming a triangle of which the sides and angles are known or can be computed, be visible from a fourth point *P*, it is required to determine the position of *P*. (See fig. 62.) This problem is of use in cases where, the regular triangulation having been completed, additional points are required for the topographic survey or are needed for special use.

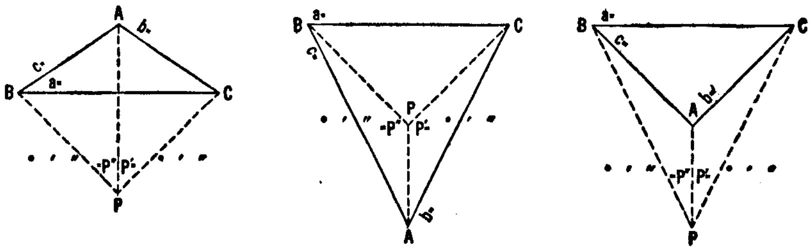
Set up the theodolite at *P* and measure the two angles subtended by any two of the given sides. The angles should be carefully measured, and in the computations the logarithms should be carried to the same number of places of decimals as in the regular triangle-side computation.

Three cases of its application are given, depending upon the location of the point *P* with reference to the sides of the triangle. If *P* falls upon the prolongation of a side of the triangle, the case resolves itself into the solution of a triangle with a side and all the angles given, while if *P* is situated on the circumference of the circle passing through the vertices of the triangle the problem is indeterminate.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 655

COMPUTATION OF THREE-POINT PROBLEM

Case 1 Case 2 Case 3



Cases 1 and 2				Case 3	
P′	21	38	06.8	P′	
P″	84	12	57.9	P″	
A	86	29	42.3		
Sum	192	20	47.0	Sum	
½ Sum	96	10	23.5	A	
				A−sum	
S = 180° − ½ sum =	83	49	36.5	S = ½ (A−sum) =	

Log c =	4.109221
Log sin P′ =	9.566668 −10
Colog b =	6.355591 −10
Colog sin P″ =	0.002217
Sum = log tan Z =	0.033697
Z =	47° 13′ 14.″0
Z + 45° =	92 13 14.0
Log cot (Z+45°) =	8.588556$_n$ 10
Log tan S =	0.965928
Sum = log tan ε =	9.554484$_n$ −10 (sign =)
ε	19° 43′ 21.″1
S	83 49 36.5
(Tan ε+)	64 06 15.4
S+ε = angle ABP	103 32 57.6
S−ε = angle ACP	

(Tan ε−)
S−ε = angle ABP
S+ε = angle ACP

BPA	84	12	57.9		APC	21	38	06.8		PCB	29	18	51.3
ABP	64	06	15.4		PCA	103	32	57.6		CBP	44	50	04.0
PAB	31	40	46.7		CAP	54	48	55.6		BPC	105	51	04.7

FIG. 62.—Specimen, computation of three-point problem

Given the sides, a, b, c, and the angle A.

Angles observed, $APC=P'$, $ABP=P''$.

To find, $ABP=x$ and $ACP=y$.

In cases I and II, let $S=180°-\frac{1}{2}\ (A+P'+P'')=\frac{1}{2}\ (x+y)$.

In case III, $S=\frac{1}{2}\ (A-P'-P'')=\frac{1}{2}\ (x+y)$.

Let $\tan Z=\frac{c \sin P'}{b \sin P''}$

then,

$$\epsilon=\tfrac{1}{2}\ (x-y)$$

$$\tan \epsilon=\cot\ (z+45°)\ \tan S.$$

If $\tan \epsilon$ be positive, $x=S+\epsilon$, $y=S-\epsilon$.

If $\tan \epsilon$ be negative, $x=S-\epsilon$, $y=S+\epsilon$.

Since all the angles and a side in each triangle are now known, the other sides, or the distances from P to the three given points, can be readily computed.

The computation is verified when both triangles give the same value for the line PA.

A tabular arrangement of the computation is shown on page 99, where Form 655 is shown with a case I problem computed.

A check may be obtained if a fourth point is observed upon and a second computation is made using two of the directions involved in the first computation.

POSITION COMPUTATION

In Figure 63 is given the computation of the geographic positions of stations Baptiste and Flores on Form 27. In this computation the azimuth and length of the line Trouble-Juan and the geographic positions of the stations Trouble and Juan are considered as fixed by a previous adjustment. No explanation of this computation is given here, as complete information for the computation of geographic positions, with tables of the factors used in the computation, is contained in Special Publication No. 8, Formulæ and Tables for the Computation of Geodetic Positions.

Inverse position computation.—This consists of the computation of the distance and azimuth between two points whose geographic positions are known. The computation can be made on Form 27, "Computation of geographic positions," but is more readily made on Form 662. (See fig. 64.)

LIST OF GEOGRAPHIC POSITIONS

The list of geographic positions (Form 28B) should always be filled out in the field on second and third order triangulation. It is little more than a tabulation of data from the position-computation sheet, arranged as shown in Figure 65. The equivalent in meters

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 27

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

				°	′	″					°	′	″
α	2 Juan	to 3	Trouble	326	41	02.7	α	3 Trouble	to 2	Juan	146	42	24.6
2d ∠	Trouble	&	Baptiste	+111	28	24.3	3d ∠	Baptiste	&	Juan	− 27	58	49.9
α	2 Juan	to 1	Baptiste	78	09	27.0	α	3 Trouble	to 1	Baptiste	118	43	34.7
Δα					-1	45.4	Δα					-3	07.3
				180	*00*	*00.0*					*180*	*00*	*00.0*
α′	1 Baptiste	to 2	Juan	258	07	41.6	α′	1 Baptiste	to 3	Trouble	298	40	27.4
			First Angle of Triangle	40	32	45.8							

	°	′	″			°	′	″		°	′	″			°	′	″
φ	55	24	27.218	2 Juan	λ	133	15	10.876	φ	55	23	01.018	3 Trouble	λ	133	13	31.373
Δφ		-	15.295		Δλ		+2	08.030	Δφ		+1	10.905		Δλ		+3	47.533
φ′	55	24	11.923	1 Baptiste	λ′	133	17	18.906	φ′	55	24	11.923	1 Baptiste	λ′	133	17	18.906

	Logarithms	Values in seconds			Logarithms	Values in seconds
				½(φ+φ′)	55 24 19.6	
s	3.362120					
Cos α	9.312224			s	3.362120	
B	8.509681					
h	1.184025	1st term	+15.2765	Sin α	9.990656	
s²	6.724			A′	8.508728	
Sin² α	9.981			Sec φ′	0.245808	
C	1.564			Δλ	2.107312	+128.0301
	8.269	2d term	+ 0.0186	Sin ½(φ+φ′)	9.915500	
h²	2.36			−Δα	2.022812	+105.39
D	2.36					
	4.72	3d term	+0.0000			
		−Δφ	+15.2951			

	Logarithms	Values in seconds			Logarithms	Values in seconds
				½(φ+φ′)	55 23 36.5	
s	3.659545					
Cos α	9.681807n			s	3.659545	
B	8.509683					
h	1.851035n	1st term	-70.9635	Sin α	9.942963	
s²	7.319			A′	8.508728	
Sin² α	9.886			Sec φ′	0.245808	
C	1.564			Δλ	2.357044	+227.5328
	8.769	2d term	+0.0588	Sin ½(φ+φ′)	9.915438	
h²	3.70			−Δα	2.272482	+187.28
D	2.36					
	6.06	3d term	+0.0001			
		−Δφ	-70.9046			

11—6363 U. S. GOVERNMENT PRINTING OFFICE

FIG. 63.—Specimen, position computation

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 27

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

		°	′	″			°	′	″
α	2 Baptiste to 3 Trouble	298	40	27.4	α	3 Trouble to 2 Baptiste	118	43	34.7
2d ∠	Trouble & Flores	+ 63	17	28.3	3d ∠	Flores & Baptiste	− 66	44	31.5
α	2 Baptiste to 1 Flores	1	57	55.7	α	3 Trouble to 1 Flores	51	59	03.2
Δα			−	08.8	Δα			−3	16.0
		180	00	00.0			180	00	00.0
α′	1 Flores to 2 Baptiste	181	57	46.9	α′	1 Flores to 3 Trouble	231	55	47.2
	FIRST ANGLE OF TRIANGLE	49	58	00.3					

	°	′	″			°	′	″
φ	55	24	11.923	2 Baptiste	λ	133	17	18.906
Δφ		−2	57.060		Δλ		+	10.665
φ′	55	21	14.863	1 Flores	λ′	133	17	29.571

	°	′	″			°	′	″
φ	55	23	01.018	3 Trouble	λ	133	13	31.373
Δφ		− 1	46.156		Δλ		+3	58.198
φ′	55	21	14.862	1 Flores	λ′	133	17	29.571

	Logarithms	Values in seconds	
s	3.738694		
Cos α	9.999744		
B	8.509681		″
h	2.248119	1st term	+177.0594
s²	7.477		
Sin²α	7.071		
C	1.564		
	6.112	2d term	+ 0.0001
h²	4.50		
D	2.36		
	6.86	3d term	+ 0.0007
		−Δφ	+177.0602

½(φ+φ′)	55 22 43.4	
	Logarithms	Values in seconds
s	3.738694	
Sin α	8.535259	
A′	8.508730	
Sec φ′	0.245268	″
Δλ	1.027951	+10.6648
Sin ½(φ+φ′)	9.915360	
−Δα	0.943311	+8.78

	Logarithms	Values in seconds	
s	3.726501		
Cos α	9.789495		
B	8.509683		″
h	2.025679	1st term	+106.0911
s²	7.453		
Sin²α	9.793		
C	1.564		
	8.810	2d term	+0.0646
h²	4.05		
D	2.36		
	6.41	3d term	+0.0003
		−Δφ	+106.1560

½(φ+φ′)	55 22 07.9	
	Logarithms	Values in seconds
s	3.726501	
Sin α	9.896439	
A′	8.508730	
Sec φ′	0.245268	″
Δλ	2.376938	+238.1979
Sin ½(φ+φ′)	9.915309	
−Δα	2.292247	+196.00

11—6363

FIG. 63.—Specimen, position computation—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 662

INVERSE POSITION COMPUTATION

$$s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{\Delta\lambda_1 \cos\phi_m}{A_m}$$

$$s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{-\Delta\phi_1 \cos\frac{\Delta\lambda}{2}}{B_m}$$

$$-\Delta\alpha=\Delta\lambda \sin\phi_m \sec\frac{\Delta\phi}{2}+F(\Delta\lambda)^3$$

in which log $\Delta\lambda_1$=log $(\lambda'-\lambda)$ − correction for arc to sin*; log $\Delta\phi_1$= log $(\phi'-\phi)$ − correction for arc to sin*; and log s=log s_1+ correction for arc to sin*.

	° ′ ″	NAME OF STATION		° ′ ″
1. ϕ	35 09 01.87	Fuller	λ	100 06 04.20
2. ϕ'	35 10 06.48	Breaks	λ'	99 59 44.57
$\Delta\phi$ $(=\phi'-\phi)$	+ 1 04.61		$\Delta\lambda$ $(=\lambda'-\lambda)$	− 6 19.63
$\frac{\Delta\phi}{2}$	+ 32.30		$\frac{\Delta\lambda}{2}$	
$\phi_m\left(=\phi+\frac{\Delta\phi}{2}\right)$	35 09 34.17			
$\Delta\phi$ (secs.)	+64.61		$\Delta\lambda$ (secs.)	−319.63

log $\Delta\phi$	1.810300			log $\Delta\lambda$	2.504648_n
cor. arc−sin	−			cor. arc−sin	−
log $\Delta\phi_1$				log $\Delta\lambda_1$	
log cos $\frac{\Delta\lambda}{2}$				log cos ϕ_m	9.912516
colog B_m	1.488788			colog A_m	1.490761
log$\left\{s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$	3.299088_n	(opposite in sign to $\Delta\phi$)		log$\left\{s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$	3.907925_n
				log$\left\{s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$	3.299088_n
log $\Delta\lambda$	2.504648_n	3 log $\Delta\lambda$		log tan $\left(\alpha+\frac{\Delta\alpha}{2}\right)$	0.608837
log sin ϕ_m	9.760313	log F		$\alpha+\frac{\Delta\alpha}{2}$	256° 10′ 21″.7
log sec $\frac{\Delta\phi}{2}$		log b		log sin $\left(\alpha+\frac{\Delta\alpha}{2}\right)$	9.987228
log a	2.264961_n			log cos $\left(\alpha+\frac{\Delta\alpha}{2}\right)$	9.378391
a				log s_1	3.920697
b				cor. arc−sin	+
$-\Delta\alpha$ (secs.)	−184″.1			log s	
$-\frac{\Delta\alpha}{2}$	−92″.0				
	° ′ ″ − 1 32.0				
$\alpha+\frac{\Delta\alpha}{2}$	256 10 21.7				
α (1 to 2)	256 08 49.7				
$\Delta\alpha$	+ 3 04.1				
	180				
α' (2 to 1)	76 11 53.8				

* Use the table on the back of this form for correction of arc to sin.

NOTE.—For log s up to 4.52 and for $\Delta\phi$ or $\Delta\lambda$ (or both) up to 10′, omit all terms below the heavy line except those printed in heavy type or those underscored, if using logarithms to 6 decimal places.

FIG. 64.—Specimen, inverse position computation

(The arc-sin correction table on the back of the form is given on p. 200)

Field Computation
GEOGRAPHIC POSITIONS

Accession No. of Computation:

Locality [illegible] Bay | Datum [illegible] Alaska | State Alaska

STATION		LATITUDE AND LONGITUDE	Meters	AZIMUTH	BACK AZIMUTH	TO STATION	Logarithm (meters)	Meters	Feet
Trouble		55 [illegible] 01.018	[illegible]						
1907	[illegible]	133 13 31.373	[illegible]						
[illegible]		55 [illegible] [illegible]	[illegible]	[illegible]	[illegible]	Trouble	[illegible]	[illegible]	[illegible]
1907	[illegible]	133 15 20.876	[illegible]						
Baptiste		55 [illegible] 11.30	[illegible]	[illegible] 07 [illegible]	[illegible] 09 [illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	133 17 16.31	[illegible]	[illegible]	[illegible]	Trouble	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible] 57 [illegible]	[illegible] 57 [illegible]	Baptiste	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	133 17 [illegible]	[illegible]	[illegible]	[illegible]	Trouble	[illegible]	[illegible]	[illegible]

FIG. 65.—Specimen, list of geographic positions

of the final seconds of latitude and longitude is computed in coastal triangulation from tables given in Special Publication No. 5, Polyconic Projection Tables, and tabulated for convenience in plotting the points on projections for hydrographic and topographic field sheets and charts.

To save time and space, any one line of the triangulation is tabulated only once. For instance, the azimuth and length of the line Baptiste—Trouble is found only under Baptiste.

Only the lengths and azimuths used in the position computation need be listed. The data for the old stations, used in computing the new triangulation, should be given as shown in Figure 65.

All stations located by triangulation should be given on the list of geographic positions, and the names should correspond to those given in the descriptions, records, and computations. Each name should be followed by the date (year) that it was established, and by an abbreviated note to show whether or not the station was marked and described. The following abbreviations are employed for this purpose: *d.*=described, *m.*=marked, *n. d.*=not described, *n. m.*=not marked, *l.*=lost, and *r.*=recovered. For example, *d. m.* after the name of a station means that it was described and marked, *d. n. m.* means that it was described but not marked, etc. Temporary stations which are not described or marked should have a parenthetical note after the name giving a brief description of the object, such as (flag in tree), (whitewashed rock), (temporary topographic signal), etc.

The list of geographic positions is one of the most important parts of the computation of triangulation and should always be carefully made out in ink, or typewritten, and should be completely checked before being sent to the office. Photostatic copies of this list are used to answer requests for information until the office adjustment of the triangulation can be made.

RECORD, DOUBLE ZENITH DISTANCES

The arrangement of the record book and a specimen computation of the zenith distances are shown in Figure 66. The program of observing is given on page 43. The form of record for observations upon a star is shown in Figure 96.

The D. Z. D. record is arranged for use with a repeating vertical circle; for observations with the instruments ordinarily used the columns headed "Rep's of DZD," "Level," "C," and "D" may be left blank.

If the bubble is not brought to the center with each reading, the value of one division of the level should be entered at the beginning of each volume of observations. If the value is not known, it can be determined by the method described on page 80.

In connection with each object observed upon it should be noted whether the ground, tripod head, heliotrope, or lamp was pointed upon, in order that the observations may be properly referred to the station mark. In the "Remarks" column on the first page of observations at a station a record should be made of the heights above the station mark of the tripod head of the tower or stand, the telescope of the vertical-angle instrument, the heliotrope, and the lamp.

In the record shown in Figure 66 some of the vernier readings have bars over them. One bar (vinculum) indicates that the seconds recorded refer to a minute on the graduated circle one less than the minute recorded for the A-vernier. For example, in the first measure

DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
FORM 566

47

DOUBLE ZENITH DISTANCES.

Station: Coon *State:* Texas *Instrument:* Hildebrand, 0-302 *Date:* Nov. 26, 1927.

Observer: W.M. *County:* Collingsworth

Object Observed.	Time.	Level. O.	Level. E.	Circle L or R.	Circle Reading °	′	Verniers. A. ″	B. ″	C. ″	D. ″	Mean. ″	Zenith Distance. ° ′ ″	Remarks.
Fuller	3.30 P.M.			L	89	56	00	00			00		
(tripod head)				R	270	04	00	30			15	89 55 52.5	Height of stand 1.07 m.
				R	270	04	00	30			15		
				L	89	56	00	30			15	89 56 00.0	
Self	3.40			R	269	57	30	00			15		
(tripod head)				L	90	03	00	00			00	90 02 52.5	
				L	90	03	00	00			00		
				R	269	57	30	30			30	90 02 45.0	
Vinson	3.52			L	90	11	00	$\overline{30}$			$\overline{45}$		
(top of O-tent, 7 ft. above mark)				R	269	49	30	30			30	90 10 37.5	
				R	269	49	30	30			30		
				L	90	11	00	00			00	90 10 45.0	
Breaks	4.05			R	269	57	30	00			15		
(top of head)				L	90	03	00	00			00	90 02 52.5	
				L	90	03	00	00			00		
				R	269	57	00	30			15	90 02 52.5	
Texola tank	4.15			R	270	00	00	$\overline{30}$			$\overline{45}$		
(top of finial)				L	90	00	30	00			15	90 00 15	

FIG. 66.—Specimen, record double zenith distances

of the zenith distance of Vinson the readings of the minutes and seconds of verniers A and B with circle right (L) are, respectively, 11′ 00″ and 10′ 30″.

A bar over a vernier or micrometer reading should refer only to that particular circle reading, and a bar should never be placed over the A-vernier reading. For instance, with a repeating instrument a zero reading might properly be either

	°	′	A ″	B ″
	0	00	00	$\overline{50}$
or	359	59	55	65

but the final reading of an angle should be recorded

	359	59	50	50
instead of	0	0	$\overline{50}$	$\overline{50}$
or as	359	59	55	65
instead of	0	0	$\overline{55}$	05

ABSTRACT OF ZENITH DISTANCES

A sample of Form 29 completely made out is shown in Figure 67. Little difficulty should be encountered in the preparation of this abstract. If the observations are taken on more than one day, give the mean result for each day the same weight, regardless of whether many or few observations were made on that day. In the record book and on Form 29 carry all angles to seconds only. The value in the column headed "Object above station" is zero if the object

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29

ABSTRACT OF ZENITH DISTANCES

Station: Coon State: Texas

Observer: W.M. Instrument: Hildebrand, G-302

11—616

DATE	HOUR	OBJECT OBSERVED	OBJECT ABOVE STATION = o METERS	TELESCOPE ABOVE STATION = t METERS	DIFF. OF HEIGHTS t − o METERS	REDUCTION TO LINE JOINING STATIONS ″	OBSERVED ZENITH DISTANCE	CORRECTED ZENITH DISTANCE
1927	P.M.						° ′ ″	
11/26	3.30	Fuller	4.12	1.47	-2.65	+25.3	89 55 52.5	
		(tripod head)					89 56 00.0	
							89 55 56.2	89 56 21.5
	3.40	Self	9.20	1.47	-7.73	+60.1	90 02 52.5	
		(tripod head)					90 02 45.0	
							90 02 48.8	90 03 48.9
	3.52	Vinson	2.13	1.47	-0.66	+10.8	90 10 37.5	
		(top of O-tent)					90 10 45.0	
							90 10 41.2	90 10 52.0
	4.05	Breaks	0.97	1.47	+0.50	-4.5	90 02 52.5	
		(tripod head)					90 02 52.5	
							90 02 52.5	90 02 48.0
	4.15	Texola tank		1.47			90 00 15	
		(top of finial)						

FIG. 67.—Specimen, abstract of zenith distances

pointed upon is the final mark for elevation, as, for example, the top of a chimney, top of a spire, etc.

Column 7, headed "Reduction to line joining stations," is essentially a vertical eccentric reduction. The formula for its computation is

$$r = -\frac{t-o}{s \sin 1''},$$

where r is the reduction in seconds, $t-o$ the difference in heights from the preceding column, and s the distance in meters between the stations involved. This reduction need be made only when the observations are reciprocal. All other parts of the form should be completely made out and checked in the field.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 A

COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS.

Station 1, occ.	Monument	Cube	Keele			
Station 2, obs.	Hastings	Hastings	Hastings			
ζ_1	90 07 59	90 05 18	90 02 19			
ζ_2	90 02 40	90 08 41	90 03 35			
$\zeta_2-\zeta_1$	- 5 19	+ 3 23	+ 1 16			
$\frac{1}{2}(\zeta_2-\zeta_1)$	- 2 40	+ 1 42	+ 0 38			
~~$\frac{1}{2}(\zeta_2-\zeta_1)$ in secs.~~						
~~log ditto~~						
~~ϕ~~ log tan ditto	6.8897	6.6942	6.2654			
log s	4.3240	4.4475	4.0684			
log $[s \tan \frac{1}{2}(\zeta_2-\zeta_1)]$	1.2137 n	1.1417	0.3338			
log A	0	0	0			
log B	0	0	0			
log C	0	0	0			
log (h_2-h_1)	1.2137 n	1.1417	0.3338			
h_2-h_1	-16.4	+13.9	+2.2			
h_1	329.9	298.8	311.0			
h_2	313.5	312.7	313.2			
2 log s						
log $p=9-2$ log s						
p of (h_2-h_1)						
α and mean ϕ						
$\zeta_1+\zeta_2-180°$						
$\zeta_1+\zeta_2-180°$ in sec.						
log ditto						
log ρ						
colog s						
log $\frac{\sin 1''}{2}=\bar{4}.38454$						
log $(0.5-m)$						
$(0.5-m)$						
p of $(0.5-m)$ *						

* Since $(0.5-m)$ varies as s^2, the weight $p=\frac{s^2}{N}$, where N is constant for a set and is preferably a power of 10.

Mean = 313.1

FIG. 68.—Specimen, computation of elevations from reciprocal observations

COMPUTATION OF ELEVATIONS FROM OBSERVATIONS OF ZENITH DISTANCES

For reciprocal observations use Form 29A (see fig. 68) in computing differences of elevation. The lower part of the form, involving the weight p and the coefficient of refraction m, is not used

696689°—46——8

in field computations. The formula for the difference of elevation between stations 1 and 2 is

$$h_2-h_1=s \tan \tfrac{1}{2} (\zeta_2-\zeta_1) \; [A \; B \; C].$$

In this formula h_1 is the elevation above mean sea level of station 1, which should be the station whose elevation is the more precisely known; h_2 is the elevation of station 2; s is the horizontal distance between the stations, reduced to sea level, log s being taken from the best available computation of triangle sides; ζ_2 is the mean corrected zenith distance of station 1, as observed from station 2; similarly, ζ_1 is the zenith distance of station 2 from station 1. The values of ζ_2 and ζ_1 are to be taken from computing Form 29. A, B, and C are correction factors whose values are nearly unity and whose logarithms may be found in the tables on page 205. A is the correction factor for the elevation of station 1; its formula is

$$A=1+\frac{h_1}{\rho},$$

in which ρ is the radius of curvature[4] of the arc between stations 1 and 2. B is the correction factor to the approximate difference of elevation, $s \tan \frac{1}{2} (\zeta_2-\zeta_1)$. Its expression is

$$B=1+\frac{s}{2\rho} \tan \tfrac{1}{2} (\zeta_2-\zeta_1).$$

C is the correction factor for the distance between stations, its expression being

$$C=1+\frac{s^2}{12\rho^2}.$$

Compute through the form by horizontal lines. In the form a brace groups those quantities which, added together, give the quantity on the line immediately below the brace. In field computations carry the angles to seconds, the logarithms to four places of decimals, and the differences of elevation to tenths of meters. In field computations the lines marked "$\frac{1}{2}$ $(\zeta_2-\zeta_1)$ in secs." and "log ditto" may be omitted and log tan $\frac{1}{2}$ $(\zeta_2-\zeta_1)$ may be taken directly from Vega's or Shortrede's tables and entered in the line marked "T." Having found log $[s \tan \frac{1}{2} (\zeta_2-\zeta_1)]$, use it to take out log B from the table on page 205. Add algebraically the logarithms of A, B, and C to log $[s \tan \frac{1}{2} (\zeta_2-\zeta_1)]$; the sum will be log (h_2-h_1), h_2-h_1 being expressed in the same unit as s, in this case the meter, which is the unit throughout the computation. To convert meters to feet

[4] For tables of radii of curvature, see p. 202.

which should be used in topographic work, multiply the number of meters by 3.28083 (log 3.28083=0.51598), or use the conversion tables on pages 208–215.

For nonreciprocal observations use Form 29B (see fig. 69) in computing differences of elevation. The computation of weights

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 B

COMPUTATION OF ELEVATIONS FROM NONRECIPROCAL OBSERVATIONS.

Station 1, occ.	Monument	Keele	Hastings	Lee		
Station 2, obs.	Cupola H.S. Hastings	Cupola H.S. Hastings	Cupola H.S. Hastings	Cupola H.S. Hastings		
Object sighted	Top	Top	Top	Top		
ζ_1	90° 04′ 54″	90° 00′ 30″	89° 52′ 57″	90° 05′ 42″		
α and mean ϕ	87 34	20 34	21 34	79 34		
log (0.5−m)	9.6325	9.6325	9.6325	9.6325		
log s	4.2520	4.1944	3.7825	4.4143		
colog ρ	3.1949	3.1966	3.1966	3.1949		
colog sin 1″	5.31443	5.31443	5.31443	5.31443	5 31443	5 31443
log (k in secs.)	2.3938	2.3379	1.9260	2.5561		
k in secs.	248	218	84	360		
(90° − ζ_1 + k) in secs.	− 46″	+ 3′ 08″	+ 8′ 27″	+ 18″		
log tan ditto	6.3483 n	6.9597	7.3906	5.9408		
log s	4.2520	4.1944	3.7825	4.4143		
log [s tan (90° − ζ_1 + k)]	0.6003 n	1.1541	1.1731	0.3551		
log A	0	0	0	0		
log B	0	0	0	0		
log C	0	0	0	0		
log ($h_2 - h_1$)	0.6003 n	1.1541	1.1731	0.3551		
$h_2 - h_1$	−4.0	+14.3	+14.9	+2.3		
h_1	329.9	310.4	313.1	319.8		
t − o	+2.1	+2.1	+1.4	+6.4		
Corrected elevation	328.0	326.8	329.4	328.5		
log p = 0 − 2 log s						
p						
Weighted mean elevation of sta. obs.	328.2					

FIG. 69.—Specimen, computation of elevations from nonreciprocal observations

provided for at the bottom of the form may be omitted in a field computation. The same rules as to the number of figures, etc., will apply here as to the computation of reciprocal observations and the braces have the same meaning as in Form 29A. The formula for difference of elevation is similar to that for reciprocal elevations, but since only one zenith distance (ζ_1) is observed, the quantity $\frac{1}{2}(\zeta_2-\zeta_1)$ must be replaced by $90°+k-\zeta_1$, the value in seconds of k being given by the equation $k=\frac{(0.5-m)\ s}{\rho \sin 1''}$. In this equation m is the coefficient of refraction, which varies with varying atmospheric conditions. In office computations the best available value of m will be used, but for field computations put

$$\log\ (0.5-m)=9.63246\text{–}10$$

which corresponds to $m=0.071$. Log ρ comes from the table on page 202, the arguments of which are the mean azimuth and mean latitude (α and ϕ) of the line. These quantities need not be known closer than the nearest degree. Having found k (to the nearest second only for field computations) the formula for the difference of elevation is given by

$$h_2-h_1=s\ \tan\ (90°+k-\zeta_1)\ [A\ B\ C].$$

The quantity ζ_1 is the mean observed zenith distance and comes from Form 29, as does also the quantity $t-o$ which is to be applied as a correction to h_2-h_1 as indicated on page 109. No vertical eccentric angular reduction is to be applied to ζ_1. This is contrary to the practice on reciprocal zenith distances.

In the field computations the lines marked "$90°+k-\zeta_1$ in secs." and "log ditto" may be omitted and log tan $(90°+k-\zeta_1)$ taken directly from Vega's or Shortrede's tables and entered in the line marked "T." Log $[s \tan\ (90°+k-\zeta_1)]$ is used as the argument for log B. The arguments of log A and log C are h_1 and log s, respectively, as in the case of reciprocal observations. The logarithms of A, B, and C added algebraically to log $[s \tan\ (90°+k-\zeta_1)]$ give log (h_2-h_1).

DESCRIPTION OF STATIONS

Specifications for the marking of stations are given on page 38. It is very important that the mark for the station be permanent. Of equal importance to the future recovery of the station is the description of the station, which should be clear, concise, and complete. The first part of the description should enable one to go with certainty to the immediate vicinity of the station, while the latter part, the detailed description, by its measured distances and directions to reference marks and its description of the station marks, must inform the searcher of the exact location of the station and make its identification certain.

The specimen description of triangulation station in Figure 70 illustrates the progressive localization referred to above, for, beginning with the naming of the State and county, the description gives the distance and direction of the station from the nearest important topographic feature, its location on a particular island, and finally the distances from reference marks and near-by topographic features. As a guide the engineer should keep in his mind the thought, "What facts should be available to engineers in order that they may recover this station 25 years hence?" Measurements should be made and recorded to prominent objects, and the location of reference marks with reference to these objects should also be noted.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 525

DESCRIPTION OF TRIANGULATION STATION

NAME OF STATION: Slide
STATE: Alaska
COUNTY: Long Island
CHIEF OF PARTY: H. B. Campbell
YEAR: 1925
LOCALITY: Kaigani Strait

Surface-station mark,	Note,*	3
Underground-station mark,	Note,*	
Reference mark, No. 1	Note,*	12 b
Reference mark, No. 2	Note,*	12 b
Witness mark,	Note,*	
Witness mark,	Note,*	
Height of signal above station mark		meters.
Height of telescope above station mark		meters.

DISTANCES AND DIRECTIONS TO REFERENCE MARKS AND PROMINENT OBJECTS

OBJECT	DISTANCE	DIRECTION	AZIMUTH
	meters	° ′ ″	° ′ ″
South Rocks		0 00 00	
Reference mark No.1	11.46	56 39 00	
Reference mark No.2	2.46	331 09 00	

Detailed description: About 10 miles southeast of the village of Howkan, on the east side of the southern entrance of Kaigani Strait, on the southernmost summit of the long north and south ridge on the southern part of Long Island. The ridge is heavily wooded, except for the summit, which is slightly higher than the tops of the surrounding trees. The station mark is set in bed rock. It was necessary to dig through a foot of moss and roots to put in the mark, and it is probable that in a short time the mark will be covered. There are two bare rocky ledges forming the northeast face of the summit and about 10 meters apart. Reference mark No. 2 is on the most northerly of these ledges and reference mark No. 1 is on the other. The station can best be reached from the southwest, landing in the bight just to the eastward of Kaigani Point. It should not be approached from any place to the eastward of the conspicuous landslide on the south end of the ridge on account of brush and fallen trees.

Described by M.O.W. Marked by M.O.W. and O.I.A.

NOTE.—The initial direction must be to a main scheme station.

FIG. 70.—Specimen, description of station

The original description should be written in the angle or direction record book or in a separate notebook carried by the recorder for that purpose. As soon as possible after leaving the station, while the topography of the vicinity is fresh in mind, the written notes should be transferred to Form 525, on a typewriter if practicable. A single copy only need be sent to the Washington office, but it is a good plan always to make one carbon copy to be retained in the field for reference until the end of the season, when the duplicates can be transmitted to the office if no longer needed. The form, after being completely filled out, should be read over carefully to see that there are no reversed directions and that no part of the description is vague, ambiguous, or erroneous.

It should be remembered that the descriptions will later be published, and they should be so written as to require as little editing as possible. If the space on the card is not large enough, continue the description on another card and not on the back of the same card. The name of the station given in the description should correspond to that given in the triangulation records and computations.

Descriptions should be furnished of all stations which may be recovered in the future. This includes all marked stations, both main scheme and supplementary, all stations of other organizations, such as the United States Geological Survey, United States Army Corps of Engineers, and General Land Office, to which connections have been made and all stations which are in themselves marks, such as lighthouses, church spires, cupolas, towers, chimneys, sharp peaks, objects valuable for future hydrographic signals, etc. The descriptions of these last-mentioned stations will usually be very brief, and several descriptions may be written on the same card. A station marked with a hydrographic station mark and located by triangulation should be described on Form 525, "Description of triangulation station." The method by which it is located, not the kind of a mark, differentiates a hydrographic station from a triangulation station.

On account of inadequate descriptions, there has been considerable confusion in identifying such objects as flagpoles, which are sometimes replaced by another pole in the immediate vicinity. The description of such objects should state the particular part of the grounds, building, or other structure where located.

Notes 1*a*, 7*a*, etc., on the specimen form refer to particular kinds of marks, as described below. The use of such a notation decreases the time required for writing the description and also appreciably reduces the cost of printing the triangulation data. These notes describe the marks in general terms only, and any essential divergence from these types should be mentioned in the description.

If a tower was required at the station described, the description should state the height of the tower. Since it frequently happens that a tall tower is required to render one or two of the adjoining stations visible while others may be visible from the ground, the description should also state the approximate heights above the ground at which the various stations observed upon become visible. Such information is of great value in deciding from what points new work should start.

Surface marks

Note 1.—A standard disk triangulation station mark set in the top of (*a*) a square block or post of concrete, (*b*) a concrete cylinder, (*c*) an irregular mass of concrete.

Note 2.—A standard disk triangulation station mark wedged in a drill hole in outcropping bedrock (*a*) and surrounded by a triangle chiseled in the rock, (*b*) and surrounded by a circle chiseled in the rock, (*c*) at the intersection of two lines chiseled in the rock.

Note 3.—A standard disk triangulation station mark set in concrete in a depression in outcropping bedrock.

Note 4.—A standard disk triangulation station mark wedged in a drill hole in a bowlder.

Note 5.—A standard disk triangulation station mark set in concrete in a depression in a bowlder.

Note 6.—A standard disk triangulation station mark set in concrete at the center of the top of a tile (*a*) which is embedded in the ground, (*b*) which is surrounded by a mass of concrete, (*c*) which is fastened by means of concrete to the upper end of a long wooden pile driven into the marsh, (*d*) which is set in a block of concrete and projects from 12 to 20 inches above the block.

Underground marks

Note 7.—A block of concrete 3 feet below the ground containing at the center of its upper surface (*a*) a standard disk triangulation station mark, (*b*) a copper bolt projecting slightly above the concrete, (*c*) an iron nail with the point projecting above the concrete, (*d*) a glass bottle with the neck projecting a little above the concrete, (*e*) an earthenware jug with the mouth projecting a little above the concrete.

Note 8.—In bedrock (*a*) a standard disk triangulation station mark wedged in a drill hole, (*b*) a standard disk triangulation station mark set in concrete in a depression, (*c*) a copper bolt set in cement in a drill hole or depression, (*d*) an iron spike set point up in cement in a drill hole or depression.

Note 9.—In a bowlder 3 feet below the ground (*a*) a standard disk triangulation station mark wedged in a drill hole, (*b*) a standard disk triangulation station mark set in concrete in a depression, (*c*) a copper bolt set with cement in a drill hole or depression, (*d*) an iron spike set with cement in a drill hole or depression.

Note 10.—Embedded in earth 3 feet below the surface of the ground (*a*) a bottle in an upright position, (*b*) an earthenware jug in an upright position, (*c*) a brick in a horizontal position with a drill hole in its upper surface.

Reference marks

Note 11.—A standard disk reference mark with the arrow pointing toward the station set at the center of the top of (*a*) a square block or post of concrete, (*b*) a concrete cylinder, (*c*) an irregular mass of concrete.

Note 12.—A standard disk reference mark with the arrow pointing toward the station (*a*) wedged in a drill hole in outcropping bedrock, (*b*) set in concrete in a depression in outcropping bedrock, (*c*) wedged in a drill hole in a bowlder, (*d*) set in concrete in a depression in a bowlder.

Note 13.—A standard disk reference mark with the arrow pointing toward the station, set in concrete at the center of the top of a tile (*a*) which is embedded in the ground, (*b*) which is surrounded by a mass of concrete, (*c*) which is fastened by means of concrete to the upper end of a long wooden pile driven into the marsh, (*d*) which is set in a block of concrete and projects from 12 to 20 inches above the block.

Witness marks

Note 14.—A conical mound of earth surrounded by a circular trench.

Note 15.—A tree marked with (*a*) a triangular blaze with a nail at the center and each apex of the triangle, (*b*) a square blaze with a nail at the center and each corner of the square, (*c*) a blaze with a standard disk reference mark set at its center into the tree.

Recovery note, triangulation station.—Whenever a station established during some previous season is recovered or searched for, Form 526, "Recovery note," Figure 71, must be made out and transmitted to the office. Any deficiency or lack of accuracy in the original description, any change in the character of the marks or of the topography near the station, or any information which will make

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 526

RECOVERY NOTE, TRIANGULATION STATION

R

NAME OF STATION: Laguna STATE: California COUNTY: Ventura
ESTABLISHED BY: W. E. Greenwell YEAR: 1857 LOCALITY: Camarillo or Hueneme
RECOVERED BY:* F. W. Hough YEAR: 1923

Detailed statement as to the fitness of the original description: The station is on top of the high peak nearest the ocean of the first range south of Hueneme, and is easily identifiable as the westernmost peak. A two mile trail from the Upper Broome Ranch leads to the station. The station was found as described and in excellent condition. All the old reference marks were recovered. A standard triangulation disk was set in the drill hole in the center of the post, thus remarking the center of the station. A new standard reference mark, note 12a, was established.

Santa Clara	0° 00′ 00″
New reference mark 34.16 meters	179 58 30

*Name of chief of party should be inserted here. The officer who actually visited the station should sign his name at the end of the recovery note.

NOTE.—One of these forms must be used for every station recovered.

FIG. 71.—Specimen, recovery note

the station more readily recovered in the future should be recorded on the recovery note. If the station is looked for and not found, the recovery note should describe in some detail the completeness of the search in order that the office may know whether or not to mark the station in the records as "lost." If the evidence seems conclusive that the station is lost, a definite recommendation to this effect should be made on the recovery form by the officer in the field.

Whenever a party is engaged in field work in an area covered by old triangulation a search should be made for as many of the old stations as practicable. This applies to parties engaged in hydrography or topography as well as to those executing triangulation. A recovery note card should be filled out and sent to the office for each station visited, whether the station was recovered or not. The name of the chief of party should be entered at the top of the form in the space after "Recovered by," and the officer who recovered or searched for the station should sign the statement about the fitness of the original description.

Shore-line reference.—Where a permanent mark is established or recovered along a shore subject to wave action and in such a position as to make the measurements feasible, the field party shall measure the distance and azimuth from the mark to one or more points on either the high-water line, top of beach slope, crest of bluff, or such other characteristic feature as, when compared with similar measurements to be made in the future, will best indicate the change in the shore line during the interim. On a straight or curving shore a single measurement normal to the shore line will suffice. At points or headlands several measurements, preferably not less than three, should be made in directions as widely separated as practicable. The resulting data should be included in the description of station or recovery note in the same manner as reference marks and in addition thereto.

SEASON'S REPORT

All triangulation records and computations should be forwarded to the Washington office as soon as possible after the field observations have been completed and should be accompanied by a progress sketch on tracing vellum and a descriptive report. The records, computations, descriptions, recovery notes, sketch, and report should all be sent within a few days of each other. The list of "Landmarks for charts" (see p. 36) should be sent under separate cover.

Each progress sketch must have a projection and a title which should include the following: Locality, scale, date of field work, and name of chief of party. If the work is along the coast, the shore line, as traced from the chart of the area covered by the triangulation, should be shown on the progress sketch. There should be a complete sketch for each piece of triangulation executed even if the observations were made in two different fiscal years. The R_1 for each figure should be shown in small numerals on the sketch. All triangulation stations, their names, and the lines between stations should be indicated as follows:

Principal triangulation schemes should be in heavy lines, and base lines should be of double width. A line observed at both ends should be full throughout. A line observed at one end should be full at the observed end and broken at the other. Reconnaissance lines should be dotted if shown on the sketch with triangulation. When the sketch contains reconnaissance only, the lines should be full if they are to be observed at both ends and broken at one end if they are to be observed in one direction only. Old stations recovered, including spires, stacks, etc., should appear thus: ⊚. New stations should appear thus: △.

All important points determined, including mountain peaks, should be shown as far as practicable. Lines to intersection stations should be drawn lighter than those of the main scheme. A confusion of lines may often be avoided by indicating, with short lines radiating from intersection points, the stations from which they were observed. All lines, letters, figures, etc., shown on the sketch should be sufficiently bold to make a good blue print.

All recovered stations should be shown on the sketch even if not connected with the new triangulation. The names of the stations on the sketch should correspond to the names used in the records and computations.

The descriptive report should contain a brief statement of the methods used in executing the triangulation, attention being called to any unusual conditions. The number of closed triangles and the average and maximum closing errors of the triangles should be given, together with the discrepancy in latitude, longitude, azimuth, and length if connection is made at more than one place to triangulation previously executed. The report should also contain the length in miles and the area in square miles of the triangulation and the number of geographic positions determined. If Form 21, "Statistics of field work," accompanies the report, the statistics given on the form need not be repeated in the written report. When the triangulation is in progress at the close of the fiscal year the above statistics covering the work up to June 30 should be forwarded to the office on June 30, or as soon after that as possible, so that this information will be available for the director's annual report, the manuscript of which is prepared soon after the close of the fiscal year.

RECORDS OF FOURTH-ORDER CONTROL

The records and computations of any fourth-order triangulation and traverse should not be sent to the office as triangulation and traverse records but should be forwarded as part of the records of the hydrographic or topographic sheet on which the positions are plotted. Fourth-order horizontal control includes sextant triangulation, or a combination of sextant and theodolite triangulation, or a theodolite and tape traverse of a lower order of accuracy than third, for which the geographic positions have been computed to prevent the accumulation of plotting errors. This paragraph does not apply to records pertaining to intersection stations located from third-order control.

CHAPTER 3.—SECOND AND THIRD ORDER BASE MEASUREMENT

SPECIFICATIONS, SECOND-ORDER BASES

On December 10, 1928, the following specifications for the measurement of second-order bases were approved by the Director of the Coast and Geodetic Survey. They supersede all previous instructions for work of this class.

1. The specifications for reconnaissance for second-order triangulation, approved also on December 10, 1928,[1] govern the character of figures and the distance between bases, but the engineer must bear in mind the close interrelation of the specifications for reconnaissance, triangulation, and base measurement. The frequency of bases, the geometrical strength of the intervening figures, and the accuracy of the measurement of angles and bases are the factors which determine the final accuracy of the adjusted lengths, azimuths, and geographic positions. If one factor is weakened, one or more of the others must be correspondingly strengthened to maintain the required accuracy.

2. A base may be measured with tapes with the required accuracy over fairly rough ground and steep slopes. It is of more importance to locate the base so as to secure in the base net as low a value for R as practicable than to locate it in the smoothest place. The longer the base the lower, as a rule, will be the value of R[2] in the base net, and therefore the longer may be the chain of triangles between base nets.

3. Where topographic conditions demand it a broken base may be used, provided the terminal stations are intervisible and the angle at each break and at each end is measured so as to form a closed polygon and with an accuracy necessary to secure the precision in length indicated below. No considerable portion of the base should be inclined at an angle of more than 30° to the final projected line of the base, and this maximum should be kept down to 20° if possible. The total error due to projecting the elements of the base upon the straight line between the base ends should not exceed 1 part in 300,000 of the length of the base.

4. A second-order base shall be measured with such accuracy that the total actual error shall not exceed 1 part in 150,000 of the length of the base, and the computed probable error shall not be greater than 1 part in 500,000. Very little increase in the average accuracy of the lengths of the lines of the triangulation will result from an increase in the accuracy indicated above, and no additional time and expense should be expended in securing an accuracy beyond the limits stated.

5. Two measurements shall be made of the base with either steel or invar tapes which have been properly standardized. If the discrepancy in millimeters between the two measurements of a section exceeds $20\sqrt{K}$ (where K is the length of the section in kilometers), additional measurements of that

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[1] See pp. 6 and 7. [2] See p. 7.

section should be made until two measures made in opposite directions are secured which agree within this limit. At the end of the season, or sooner if practicable, the tapes should be returned to the office for restandardization.

6. If steel tapes are used, the measurement should be made either at night or when the sky is heavily overcast. In the Coast and Geodetic Survey, invar tapes, because of the greater economy of operation, have so completely replaced steel tapes for all accurate measurements that in these specifications the use of invar tapes is assumed unless steel tapes are specifically mentioned.

7. Two tapes shall be used on the measurement of a base, and a third tape should be available as a field standard. The third tape should be used in case the measurements made with the other two tapes indicate injury to one or both of them. The base should be measured in sections about 1 kilometer in length with the exception of one section which may be longer or shorter than this. Each section should be measured at least twice, one measurement being made in the forward direction and the other in the backward. At the beginning and at the ending of the measurement of a base, one kilometer section should be measured with all three tapes, in order to secure a field comparison to detect any changes from the standardized lengths.

8. If the mean elevation of the base is more than 25 meters above sea level, a connection should be made between the base-line levels and some point of known elevation which will give the elevation of the base with an error not exceeding 25 meters. An error of 25 meters in elevation causes an error of about 1 part in 250,000 in the reduced length of the base; in most cases the mean elevation of the base can be easily determined with much greater accuracy than that specified.

9. The computation of the base length shall be completely made and checked in the field and all records and computations relating to the base sent to the office.

PREPARATION OF BASE

ALIGNMENT

If the flags or signals marking the terminal stations of the base are not both visible throughout the line, intermediate targets should be accurately located along the base. No part of the measured line should be more than 6 inches off the straight line between the terminal stations, or between the stations at the ends of a section in the case of a broken base, nor should any one marking strip on a stake be more than 2 inches off the line between the strips on the two adjacent stakes. When one or both of the adjacent stakes are at a less distance than the usual 50 meters the 2-inch tolerance must be proportionately reduced. If due care is used and if flags are lined in at intervals of about a kilometer along the base line, the tolerance of alignment specified above can be secured by lining in the stakes and marking strips by eye without the aid of any instrument other than a plumb bob.

If the measurement is being made over water, the tape must clear the water at all times when in place and under full tension, otherwise the shape of the catenary and the effective length of the tape will be changed. The shore parts of the line must be cleared of bushes and weeds in order that the tapes may hang free when under tension. Since those making the tape measurements work on only one side of the line, the cleared swath need not extend to an equal distance on each side of the line.

STAKING

Ordinarily the 50-meter tape is supported at the 0, 25, and 50 meter points. For second-order bases it is seldom necessary to provide a greater number of supports to overcome the effects of wind. (See p. 131.)

An old spliced invar tape or a 50-meter steel tape is usually used on the staking, since there is a possibility of injuring the tape. In either case the staking tape should be compared with a standard tape before the staking is begun, for frequently the staking tape is enough in error to cause considerable delay in the final measurements when numerous set-ups or setbacks become necessary due to the interval between stakes being different from the length of the measuring tape. If a steel tape is used, it may save time to make an approximate allowance for the change in length due to temperature. Each change of 10° C. from the temperature obtaining when the steel staking tape was compared with the standard invar tape will cause a change between the relative effective lengths of the two tapes of about half a centimeter. The 25-meter point of the tape may be marked by a strip of adhesive tape while staking to make it readily recognizable.

If the nature of the soil along the base permits of staking, a 2 by 4 inch stake is driven solidly into the ground for each tape end and a 1 by 4 inch stake is driven in on line at the 25-meter point. When the tape is in position for measuring it extends over the top of the stakes at the terminal marks, but alongside the stake at the 25-meter point, being held at the proper elevation at the mid-point by a nail driven horizontally into the stake.

Care should be used to secure lumber for the stakes which will not split easily. If the top square-cut end of the stake is beveled off slightly, the stake will not be split so readily while being driven.

The staking can best be done by a party of four men, provided the stakes have already been distributed along the line at intervals slightly less than that required in order that there may be no need of going forward for a stake and carrying it back. Extra pieces of lumber are also distributed where needed for bracing the stakes. One man carries the rear end of the tape and holds the rear mark at the stake already set while the position of the forward stake is being located. The second man at the middle of the tape supports the tape as it is being carried forward, holds the tape at the proper elevation while the trial measuring is being done, and later drives the middle stake. The third man carries the forward end of the tape, applies the tension for the trial measurement, and helps drive the forward stake. The fourth man picks up the forward stake, lines it in with the assistance of the rear man, and does most of the sledge work. If the party is short-handed, one man can do all the work at the front end of the tape.

A 16-pound sledge is best for driving the 2 by 4 inch stakes. If the ground is hard or rocky, a pointed iron bar may be used to make a hole for the stake before starting to drive it. The forward men carry a sledge, a saw, a hammer, nails, copper strips and brads for fastening the strips to the stakes, and an iron bar if necessary.

Where the ground is so rocky that stakes can not be driven a different kind of support must be provided. If the base is long and the rocky character of the soil is known beforehand, iron or wooden movable tripods may be prepared, with a marking table supported by a ball-and-socket joint as described for traverse on page 155; or the stakes may be braced by 1 by 4-inch stakes driven into crevices of the rock or held in place by rocks. The only requirement is that the stakes at the tape ends shall be sufficiently stable to hold the marks in place while the base is being measured. All stakes at kilometer ends should be braced. If the permanent mark at an angle station is low, a bench should be built over it and a scratch made on the copper strip fastened to the bench directly over the station mark to which the measurements may be made.

PROCEDURE IN STAKING

With the tape laid out along the base, the rear man holds the rear mark of the tape in contact with the mark over the base station, the forward man, approximately on line, applies to the tape with the spring balance attached to his tape stretcher the same tension which will later be used on the actual measurement, and the helper, holding the 2 by 4 inch stake opposite the forward mark on the tape, is waved into line by the rear man. While this is being done the middle man supports the middle of the tape approximately on the line between the ends and at the same time marks with the end of his stake the point on the ground under the middle mark of the tape.

With the forward stake in approximate alignment, the forward man lays aside the tape and drives the stake while the helper keeps it in position. While this is being done the rear man aligns the center stake, which is driven by the middle man, and at the same time watches the forward stake and signals if it is being driven out of line.

As soon as the forward stake has been driven and aligned, the forward man sights from the top of the forward stake to the top of the rear stake and signals the middle man who drives a nail in the center stake on this line for the support of the tape. After the nail has been driven, the rear man checks the alignment of the center stake. He then lines in a pencil or other object held on top of the forward stake to give the final alignment for that stake, a pencil line being drawn to mark it.

The tape is again brought into position with its rear mark in contact with the mark on the base station or on the bench above it, and the proper tension is applied, with the edge of the tape touching the final aligning mark on the forward stake. A copper strip is then nailed in position flush alongside the tape on the aligning side, and at the same time a pencil mark is made on the top of the stake opposite the forward mark on the tape. This mark is used as a rear contact mark in setting the stakes for the next tape length. The tape is then moved forward and the process repeated. Some time can be saved by marking with a pencil on the forward stake the position of the copper strip and letting the rear tape man nail it in position while the next stake is being driven.

When the ground is stony and a good man is available for the middle of the tape during the actual measurement no middle stake need be driven, the middle of the tape being held in position by the middleman. (See p. 133.)

From 3 to 5 kilometers a day can be staked by a 4-man party, the speed depending upon the nature of the soil and the proficiency of the party.

If for any reason the intermediate support between the tape ends can not be placed so it will be on grade with the terminal posts, it should be numbered with colored chalk and marked with a piece of cloth so it may be noticed and touched upon by the levelman. All stakes are numbered with colored crayon as they are driven, and intermediate supports not on grade should be given a fractional number. An intermediate support which is not on grade makes what is known as a broken grade. "Broken grades" should be avoided whenever possible, because the tension through the tape and the shape of the catenary may be different on the two sides of the stake at the broken grade.

At ravines or streams it is frequently necessary to place a stake on the edge of the bank and begin a tape length from that point. If it is less than half a tape length from the previous tape-end post, whose number, for instance, is 34, the fractional-length post should be numbered "34 set-up," the next terminal post being numbered 35. If the fractional-length post is more than half a tape length from the next preceding tape-end post, a 2 by 4 inch stake should be driven in at the half-tape mark and be given the number 34½, and the post on the edge of the obstruction should be numbered "34½ set-up." By following this system, which has been used in most cases for many years, there will be no uncertainty in the interpretation of records.

MEASUREMENT OF BASE

INSTRUMENTS AND APPLIANCES

The following instruments and appliances will usually be needed on the measurment of a second-order base:

- 2 awls, marking.
- 2 dividers, pairs.
- 1 level, wye, with rod.
- 2 plummets.
- 2 scales, one-tenth-meter, boxwood, reading to millimeters.
- 1 stretcher apparatus for tape, complete, consisting of two staves with loops and tape attaching clips, two balances, and an apparatus for testing balances.
- Strips, copper, for stake tops, of same thickness as tape, 20 per kilometer.
- 1 tape, steel, 30-meter (standardized or tested in field), for measuring set-ups and setbacks.
- 1 tape, steel or invar, 50-meter, unstandardized, for marking out base.
- 3 tapes, invar, 50-meter, standardized.
- 1 theodolite, 7-inch, repeating.
- 3 thermometers, backed, for tapes.

Special conditions may necessitate the use of other instruments, such as a direction theodolite on a broken base, or movable tripods.

HANDLING AND TESTING OF BASE APPARATUS

Invar tapes.—The proper handling of the tapes is most important on base measurement. When properly standardized and manipulated invar tapes are capable of giving a very high degree of accuracy, but they must be used with a full knowledge of their physical properties, some of which may introduce error. To secure the best results two general conditions must be met: First, it is necessary to avoid accidents and methods of handling which may alter their lengths; second, they must be used, so far as possible, either under the same conditions as when standardized or under only such different conditions as can be corrected for. Among the ways in which the first condition may fail to be met may be mentioned: Kinking the tape; altering its mass by abrasion against the ground when measuring; or changing its length by stretching it beyond its yielding point. The second condition involves the determination of the corrections for tension, sag, grade, alignment, and temperature within the allowable limit of error. This will be discussed in detail later.

One thing to be borne in mind is that invar is a very unstable alloy. The four invar tapes used on the measurement of all our first-order bases from about 1907 to 1916 exhibited a very satisfactory stability of length in their standardizations during that period, but later tapes have shown decided changes. Three 50-meter tapes from a particular lot of recent tapes, some of which had negative coefficients of expansion and others very low positive ones, showed changes of length while being standardized of from 0.3 to 1.0 millimeter, the largest change corresponding to 1 part in 50,000 of the length of the tape. Tapes subject to such erratic changes in length of course should not be used on base measurement.

With this potential instability it is easy to infer that care should be used in handling tapes, even when using those of fairly stable qualities. They should not be reeled or unreeled rapidly or under a heavy tension, nor wound upon a reel having a small diameter, nor dragged over the ground, nor shaken violently, nor should they be subjected to sudden large changes in temperature.

In addition to the special precautions described above, which are designed to preserve the tape from changes in length, the usual care should be taken to keep the tape free from adhering substances and from corrosion. After the tape is used it should be cleaned and then oiled with a light oil before being reeled up. If reeled up in the rain, a tape should be dried and oiled at the first opportunity.

Persons not accustomed to handling tapes will frequently kink them when unreeling or reeling them, as the tape channel on a reel is narrow and deep. Instructions on the proper way to handle the tapes should be given the members of the party when the tapes are first unreeled. Kinking a tape may render it useless as a precise measuring instrument, for, whether the tape is allowed to remain kinked or is straightened, its effective length will be different from its previous standardized length. When a tape is kinked a note should be made in the record showing the exact time the kinking took place, in order that a different standardized length may be used for later measurements if necessary. If a tape becomes badly kinked, a spare tape, if available, should be substituted for it.

As a result of the standardization of a tape at the Bureau of Standards, the following data are made available (see fig. 73):

Weight of tape in grams per meter.

Coefficient of expansion per degree centigrade.

Length at specified temperature supported at 0, 25, and 50 meter points.

Length at specified temperature supported at 0, 12.5, 25, 37.5, and 50 meter points.

Length at specified temperature supported throughout.

The last value is usually computed from the values obtained by the standardizations when the tape is supported at three and at five points. It is to be noted that the standardized length for the tape supported throughout presupposes a frictionless surface as a support for the tape. In the field, when the tape is supported throughout, a railroad rail is ordinarily used as the support. If the rail is dry and due care is taken by the middle man on the tape in lowering it to the rail, satisfactory results can be obtained on second-order base measurement with the tape supported on the rail. (See pp. 148 and 156.) With these precautions the error due to the friction of the tape on the rail will not usually exceed 1 part in 200,000.

Tape-stretching apparatus.—This is shown in Figure 72. It consists essentially of two staves of steel tubing, pointed at the bottom and with wooden tops. A loose-fitting leather loop with an attachment to receive the looped end of the tape slips over the staff used at the rear end of the tape. The leather loop can be easily slipped up and down on the staff according to the height of the rear stake. A frame for holding the spring balance is attached to the forward staff by means of a spring friction grip. The tape is fastened to the balance by the attachment shown in Figure 72. There is a finger bar by which the forward contact man carries the end of the tape when moving forward.

Spring balances.—The balance used, shown in Figure 72, is a commercial one, altered in the instrument division of the bureau to reduce the internal friction as much as possible. The counterweight, shown in the figure, may be so adjusted as to prevent any drag of the frame of the balance on the drawbar when tension is applied.

Errors in base measurements have frequently been due to spring balances which do not read correctly. For this reason a testing apparatus for the spring balances is sent with the balances. This should be used in testing before and after each day's work, also at midday if practicable, and oftener if it is suspected that the position of the dial pointer has changed. The form of tester usually sent is a weight which is applied to the balance held vertically, after which the pointer is adjusted exactly to 15 kilograms. When this is done the balance will indicate true tensions when it is used in a horizontal position. In other words, the weight is of 15 kilograms mass, as weighed by the spring of the balance at Washington, minus the weight of the drawbar and other movable parts of balance below the spring.

The most common injury to a spring balance used on base measurement results from the tension being suddenly released, allowing the drawbar to snap back. This may change the position of the dial pointer by several hundred grams and even result in injury to the spring. For this reason the tension on the tape should always be released gradually. If through accident the drawbar of the balance is allowed to snap back, the balance should be tested before measuring is resumed.

Thermometers.—These are special and rather expensive thermometers, correct to within 0°.5 C., within the ordinary range of temperature. They are tested at the Bureau of Standards before being sent

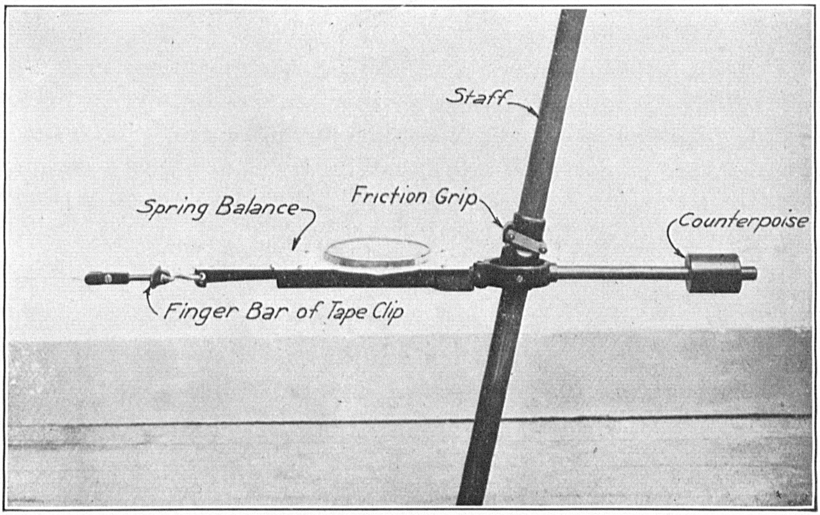

FIG. 72.—TAPE STRETCHER AND SPRING BALANCE

to the field. Field computations need not take into account the graduation errors of the thermometers found by the standardization. The glass tube is supported in a metal holder, to prevent the breaking of the tube when the tape is being handled or flexed.

During standardization a thermometer of the type described above is fastened at each end of the tape, at a point 1 meter toward the center from the terminal mark, the distance being measured from the mark to the nearer end of the thermometer. On base measurement the thermometers should always be fastened in this same position by narrow bands of adhesive tape.

CORRECTIONS TO MEASURED LENGTHS AND PRECAUTIONS AGAINST ERRORS

GRADE CORRECTION

The data for the correction for the slope of the tape are usually obtained by spirit leveling, by which the differences of elevation of the stakes supporting the two ends of the tape are obtained. If l is the inclined length and h the difference of elevation of the two ends, the correction is

$$C_G = -\left(l - \sqrt{l^2 - h^2}\right) = -\frac{h^2}{2l} - \frac{h^4}{8l^3} - \frac{h^6}{16l^5} - \cdots\cdots$$

Tables for grade corrections for various lengths of tape and differences of elevation in both meters and feet are given on pages 187–195. A table of grade corrections depending on the angular slope of the tape is given on page 186. Since for a 50-meter tape length the second term $\frac{h^4}{8l^3} < 0.1$ millimeter where $h < 3.1$ meters, on ordinary grades the correction will vary directly as the square of the difference of elevation. For this reason the leveling must be more accurately done on steep grades, and an inspection of the rate of change of the values for the correction in the tables should be the guide in deciding upon what accuracy is necessary in the leveling. The nomogram in Figure 78 also will indicate what accuracy is required in leveling for different lengths of tape and different slopes. For steep slopes and short lengths it is better to compute the grade correction by solving the triangle.

In second-order base measurement the error in C_G for a single tape length should never exceed 1 millimeter and should seldom exceed 0.5 millimeter even though the error is accidental in character. Since the correction varies inversely as the distance, fractional tape lengths are liable to a larger error in the grade correction.

The error in the grade correction which must be most closely guarded against is due to the failure to note and correct for the break in the grade of the tape at the middle support. For that reason such supports should always be flagged with cloth and given a number, such as 34½, the number signifying that the broken grade was at the half tape length between stakes 34 and 35. The tape record will then contain the note in the remarks column, "Broken grade at 34½." Before the grade corrections for any section are summed up a special check should be made to learn whether the levelman has touched upon the support at each broken grade and at the ends of fractional tape lengths, and that the leveling record has the same system of numbering of the tape supports as the tape record. By doing this confusion or doubt will be avoided when the final computation is made. In general, broken grades should be avoided whenever possible and long set-ups used instead, to avoid the errors due to the friction of the tape on the middle support.

Either the leveling should be run twice, once in each direction, or else a rod should be used which is graduated in feet on one side and in meters on the other. In the latter case the levels are run in only one direction, but both sides of the rod are read at each rod station and the differences carefully compared before the party leaves the base. If any material difference is found, the discrepancy must be checked in the field.

Special rods with metric graduations on the back may be obtained on requisition from the office if the length of the base to be measured warrants the use of the special rod.

ALIGNMENT CORRECTION

This should more properly be called the alignment error, for although the same correction formula and tables apply to differences in alignment of the tape as to differences of grade, the alignment can usually be made sufficiently exact to require no correction. It should be borne in mind, however, that alignment errors are always of the same algebraic sign, tending to make the measured length greater than the actual length, and for that reason they should be kept much smaller in magnitude than the inaccuracies in the grade corrections. The section describing the staking of the base gives details of the precautions to be taken in aligning the stakes. In addition, some member of the taping party, usually the rear contact man or the front stretcher man, should check each tape length as the measurement progresses to see that the tape does not change its horizontal direction at the middle support, and also that the forward stake has not been disturbed in alignment.

CORRECTIONS DUE TO SAG OF TAPE AND TO STRETCHING

The effective length of the tape when suspended between supports under tension is affected by the shortening due to the sag and to the stretching due to the tension. The correction due to the sag is given by the formula

$$C_S = -\frac{n}{24}\left(\frac{w}{t}\right)^2 l^3,$$

where n=number of sections into which the tape is divided by the equidistant supports.
l=length of each section in meters.
w=weight of tape in grams per meter.
t=tension in grams.

To illustrate by an example: For tape No. 922, supported at three points under a tension of 15 kilograms,

$$n=2$$
$$l=25$$
$$w=25.6$$

and

$$C_S = -\frac{1}{24} \times 2 \times 25.6^2 \times 25^3 \times \frac{1}{15000^2} = -0.00379 \text{ meter.}$$

The modulus of elasticity of invar varies greatly with the percentage of nickel in its composition and also with temperature. Five 50-meter invar tapes tested at the Bureau of Standards showed a mean change in length of 0.43 millimeter for a 500-gram change from a tension of 15 kilograms.

CORRECTIONS DUE TO ERRONEOUS TENSION

Variations in tension change the effective length of the tape in two ways: First, by changing the shape of the catenary (correction for sag), and second, by changing the length of the tape due to stretching, both changes tending to increase the effective length when the tension is increased. For invar tapes having a cross-sectional area approximately equal to that of the tapes used by the Coast and Geodetic Survey a change of 200 grams from the usual 15-kilogram tension applied to a tape will change its effective length about 0.10 millimeter, due to the change in the sag correction. The same change in tension will change the effective length by about 0.17 millimeter because of the difference due to the stretching of the tape. The total change is 0.27 millimeter, or about 1 part in 185,000. Since it is easy to keep the error in the applied tension within a 100-gram limit, half the change assumed above, the errors in length attributable to incorrect tension are negligible on second-order base measurement.

In order that a true tension of 15 kilograms may be applied the dial pointing on the spring balance should be adjusted to the proper reading whenever the tests show it to be appreciably in error. If it can not be adjusted, the proper allowance should be made in the dial reading when applying tension to the tape and a remark giving the dial reading used should be inserted in the record book immediately following the statement giving the results of the testing of the balances. In the absence of a positive statement in the record, there is always a doubt as to whether the dial was corrected or whether an allowance for the error was made.

Amount of sag.—It is sometimes desired to compute the sag of the tape; that is, the vertical distance from the lowest point of the catenary to the line joining the adjacent points of support. For all practical purposes this is given by the formula

$$\text{Amount of sag} = y = \frac{wl^2}{8t}$$

where w, l, and t represent the same quantities as in the formula for the correction to the length due to sag. (See p. 127.)

For example, taking again tape No. 922 supported at three points under a tension of 15 kilograms,

$$y = \frac{25.6 \times 25^2}{8 \times 15000} = 0.1333 \text{ meter.}$$

CORRECTIONS FOR METHOD OF SUPPORT OF TAPE AND FOR CHANGE IN WEIGHT

The change in the effective length of the tape due to a change in the number of supports can be determined by substituting in the formula for the correction due to sag different values for n and l, since a variation in the number of supports does not appreciably affect the stretching of the tape. Changes in the effective length of the tape due to a change in the weight of the tape per unit length, or in a lack of equidistance of the supports, can be determined by differentiating the same formula with respect to w and l, respectively. Moisture or grease on the tape will change its weight from 1 to 10 per cent. When thoroughly wet an invar tape is 10 per cent heavier than when dry, and after being sharply shaken to remove the drops of water it is still 1 or 2 per cent heavier than normal. An increase of 1 per cent in the weight of the tape changes its length about 1 part in 700,000 when supported at the 0, 25, and 50 meter points and an increase of 10 per cent changes its length about 1 part in 70,000. If a second-order base is measured in rain or heavy fog, the tape should be lightly shaken while the tension is being applied, to remove part of the adhering water, the shaking of course being stopped prior to the marking.

STANDARDIZATION DATA

The standardization values of the tapes are usually furnished the field party in the same form as received from the Bureau of Standards. (See fig. 73.)

LVJ:EGH
II-1

DEPARTMENT OF COMMERCE

Bureau of Standards

Certificate

FOR

50-METER INVAR TAPE

Maker:

B. S. No. 3229
U.S.C. & G. S.No.926

SUBMITTED BY

United States Coast and Geodetic Survey,
Washington, D. C.

THIS CERTIFIES that the above-described tape has been compared with the standards of the United States and found to have the length given below when under a horizontal tension of 15 kilograms and when supported at the 0, 25, and 50 meter points:

(0 to 50 meters) = 49.99595 meters, at 26.4 °C.

When supported at the 0, 12.5, 25.0, 37.5, and 50 meter points:

(0 to 50 meters) = 49.99887 meters, at 26.4 °C.

When supported on a horizontal surface, throughout its entire length. (value computed from observations taken on the tape when supported at 3 and at 5 points)

(0 to 50 meters) = 49.99989 meters, at 26.4 °C.

For the first and second of the above conditions thermometers weighing 45 grams each were attached at the points 1 meter inside the terminal marks during the test.

The above comparisons were made on the section of the lines near the end on the edge of the tape marked with a small "x" or "v" near the graduation.

The above values are correct within 1 part in 300,000 ; the probable error does not exceed 1 part in 1,000,000.

Coefficient of Expansion = 0.00000104 per degree Centigrade.
Weight per meter = 25.7 grams.

Test completed. December 18, 1926

Test No. Tw1 48928

George K. Burgess, Director.

Washington, D. C. Dec. 22, 1926

FIG. 73.—Sample certificate of tape standardization

Since the tapes are standardized under a tension of 15 kilograms when supported at three and at five points and throughout the formulas given previously for correcting the length of the tape need be used only in the cases where an unusual number of supports are used, when it is desired to apply corrections for small changes in tension, or when it is necessary to investigate the effect of small changes in one of the many factors affecting the length of the tape.

If the terminal marks on the tape extend entirely across its width, the end of the mark which should be used on standardizations and measurements is usually designated by a small X or V or by dots cut into the tape near the end of the mark. On the tapes where the terminal marks extend only about two-thirds of the way across the tape, there is no doubt as to which end of the mark to use.

FRICTION OVER SUPPORTS

With proper handling of tapes the error due to this cause is negligible. Before the tension is applied the devices attaching the tape to the stretcher staves should be so adjusted in height that the tape will be a millimeter or so above the tops of the marking posts when the tension is fully on. Frequently it is necessary to slacken off and adjust to the proper height after the tension has been partly applied. The tape must not at any time be permitted to drag over the rear post because of the danger of moving the post, which holds the mark from the previous tape length. With the tape in equilibrium under tension just above the posts a touch of the finger will depress it into position for marking.

If a nail driven into a piece of lumber is used as a middle support, the middle tape man keeps the tape vibrating on the nail by tapping it rapidly on its under side with a light stick while the tension is being applied. If the middle of the tape is supported during the measurements by a string attached to it and held by the middle man, the mid-point of the tape should be lined in vertically and horizontally between the terminal tape supports by the rear contact man and means provided for maintaining that alignment with an error of not more than an inch in either direction until the final contact and mark is made.

TEMPERATURE CORRECTION

If invar tapes are used on second-order base measurement, one may neglect the errors due to the failure of the attached thermometers to register the mean temperature of the tape. Even under adverse conditions the difference between the indicated and real temperature will probably not be more than 4° or 5° C. With a coefficient of expansion for the tape of 1 part in 1,000,000 per degree centigrade an error of temperature of 4° C. would cause an error of length of only 1 part in 250,000. With a steel tape, however, this error would be about eleven times as great, and therefore measurements with steel tapes must be made during heavily overcast days or at night.

When reading the thermometers attached to the tape they should be steadied by grasping the tape a few inches on either side of the thermometer with the hands to avoid flexing the thermometer case and breaking the tube.

PARALLAX IN MARKING

Throughout the entire measurement of the base one man should make the mark for the forward contact, for the reason that this error in marking tends to be constant in size and direction for any one person. If then the person marking remains always on the same side of the tape and stakes, the marks which tend to make the measured length too long when measuring in one direction will make it too short when measuring in the opposite direction. It is probable that there is no parallax in making the rear contact. If there is any, it is exceedingly small and need not be taken into account.

WIND EFFECT

The error caused by the bowing of the tape horizontally by the wind is always of the same sign and tends to make the measured length too long. If the tape is supported at the 0, 25, and 50 meter points and is bent by the wind 5 centimeters out of line on each section, the error due to the wind is essentially the same as the correction for grade due to a difference in elevation of 5 centimeters on a 12½-meter section, or about 0.4 millimeter for the 50-meter tape length, an error of 1 part in 125,000. If each 25-meter section of the tape is bent 1 decimeter out of line, the error would be four times as great. Measurements should not be made when the wind is strong enough to bend the tape more than 3 or 4 centimeters out of line.

It will be noticed that most of the possible systematic errors in base measurement, such as those due to wind effect, uncorrected alignment errors, and friction of the tape on its supports, tend to make the measured length too long. For this reason these sources of error should be watched closely to prevent an accumulation beyond the allowable limit.

BLUNDERS

The numbering of the stakes and the inclusion of those numbers in the record of both the tape measures and the levels practically precludes the possibility of a dropped tape length. There is a chance for error, or at least for confusion, if the record of set-ups and half tape lengths is not clear and consistent throughout all the record books. There is a chance for a compensating error in recording a small set-up as a setback, or vice versa, when measuring in one direction, with a like error of approximately the same size in the opposite running, but, with care and the usual system of having the recorder repeat clearly to the observer all data to be recorded, the probability of such an error remaining undetected is rather remote. There is one rule, however, which should invariably be observed: Any discrepancy in the records, whether of tape or level, *should be checked by field measurements*, even though the chief of party may think he has discovered the cause of the discrepancy through an examination of the record.

An added check against a dropped tape length in the record, or the recording of a half tape length as a full one, is to measure the base roughly with a 300-foot tape, checking on each kilometer section mark. Two men can do this in a short time, and the resulting certainty is worth the time spent.

Among other blunders which may occur is a tension on one tape length of 10 kilograms instead of the required 15 kilograms, since the dial pointer marks 5 kilograms at each revolution. The effect of this error is about 6 or 7 millimeters in a tape length. The possibility of making this blunder will be greatly reduced if both the forward stretcher man and the forward contact man, each time the tension is applied, will check the tension by a glance at the drawbar of the balance on which the 5, 10, and 15 kilogram lines are marked. The 15-kilogram line should be marked by white paint to distinguish it from the other lines.

If the stake holding the forward mark is jarred while the tape is being moved forward, an error may be caused. In such a case the previous tape-length measurement should be repeated before the forward progress is resumed. Any movement of the stake holding the mark designating the end of a section is usually shown by the discrepancies between the forward and backward measures of the two sections involved having opposite signs and approximately the same size.

MEASURING WITH TAPES

PERSONNEL AND DUTIES

Six men are ordinarily required for the operation of a 50-meter tape on base measurement, and their usual designations are as follows: Front contact man, rear contact man, front stretcher man, rear stretcher man, middle man, and recorder. Aside from the recorder, who must have the special qualifications usual to that position, the assignments to positions, in the order of experience and ability of the personnel, should be in the order given above. Usually the chief of party makes the forward contact, as in that position he can best supervise the manipulation of the tape and can set the pace of measurement. It is better to have experienced men for both contacts if they are available.

If any of the men are inexperienced, it is better to measure 1 or 2 practice kilometers before the recorded measurements are begun. During the preliminary work the chief of party or some other experienced officer drills each man in turn in the minutiæ of his duties. A résumé of the precautions to be observed at each position follows, beginning with the least difficult.

The middle man carries the middle of the tape high off the ground when moving forward, places the tape on the middle support when the tension is to be applied, takes the necessary precautions to see that the friction over the middle support has a negligible influence on the effective length of the tape, sees that the tape is not in contact with weeds, brush, or other obstructions, notifies the recorder of all middle supports marked "broken grade," carries and places the tape so that there is no twist in it, and each time makes sure that the middle support is not more than a decimeter distant from the middle mark on the tape. If a nail is used as a middle support for the tape, he must rapidly and lightly tap the under side of the tape near the support with a stick somewhat larger than a pencil until the front contact man calls "ready," in order to lessen the friction over the nail. The tapping must not be continued during the marking of the front contact. If no fixed support is used for the middle of the tape, he should hold it suspended by a string, after the mid-point of the tape has been lined in, using a stake or other device to steady the tape in place.

The rear stretcher man (see fig. 74) with the rear tape stretcher holds the tape in position during the time the tension is on, so that the rear terminal mark on the tape is opposite to, or slightly forward of, the mark on the copper strip on the rear stake. As he comes up to the rear stake he must place the rear staff firmly in the ground at the proper distance back of the rear stake directly in line with the stakes, and at the same time he must slip the leather loop bearing the tape link to the proper height on the staff, so that when the full tension is applied the tape will be a few millimeters above the top of the stake. The tape must not drag over the top of the stake at any time. In order to maintain a steady position of the staff, the rear stretcher man should have the top of the staff back of one of his shoulders, his body being forward of the staff. One foot should be forward of and against the base of the staff to brace it. With practice all these movements can be so coordinated that they require only a few seconds, and the man operating the rear stretcher can keep the rear mark of the tape in the proper position as the tension is applied. As soon as the front contact man calls "mark" and the thermometers are read, the tension is slackened off. The tape is then carried forward without being detached from the rear stretcher, the rear stretcher man maintaining just enough tension on the tape to keep it from touching the ground.

The front stretcher man (see fig. 75) applies the proper tension to the tape as measured by the spring balance attached to the front stretcher. In moving forward he carries the front stretcher and balance, detached from the tape. By hurrying he usually reaches the forward stake in time to check the vertical and horizontal alignment of the middle support, by sighting back over the tops of the stakes, before the front contact man reaches the stake with the front end of the tape. The checking is necessary to make sure that the stakes have not been moved since they were set or last aligned. As the tape is brought forward into position the front stretcher man with one hand holds the balance out horizontally with the hook in such position that the tape can be quickly attached. As the tape is attached he places the staff in line with the stakes at the proper distance from the front stake and applies the tension smoothly, rapidly at first, but gradually more slowly as the 15-kilogram point is neared. Jerking motions must be avoided, as they may injure the balance or tape. With the staff held in the same manner as described for the rear stretcher man, and with one hand steadying the balance so the drawbar swings free, the front stretcher man can quickly bring the tape into equilibrium under the proper tension, at which time he informs the front contact man that the tension is correct. When under tension the tape must just clear the top of the forward stake and must not drag over it, otherwise the full tension will not be transmitted throughout the tape. Care must be taken that the stretcher staff is moved in the vertical plane through the stakes, for otherwise the balance will be twisted and friction exerted on the drawbar. The tension must be kept constant at 15 kilograms and watched closely, for if the dial pointer indicates more than 100 grams from 15 kilograms when the front contact man calls "mark," the front stretcher man should immediately tell the front contact man in order that the marking may be repeated. If the tension is satisfactory at the call of "mark," the tension is quickly but smoothly slackened off while the front contact man is reading the forward thermometer, the balance is held out for the detachment of the tape by the front contact man in the same manner as for its attachment, and the advance begun to the next position.

The rear contact man (see fig. 74) makes the rear contact and reads the rear thermometer. As the tape is brought up to a new position he steadies the tape as the rear staff is being placed in

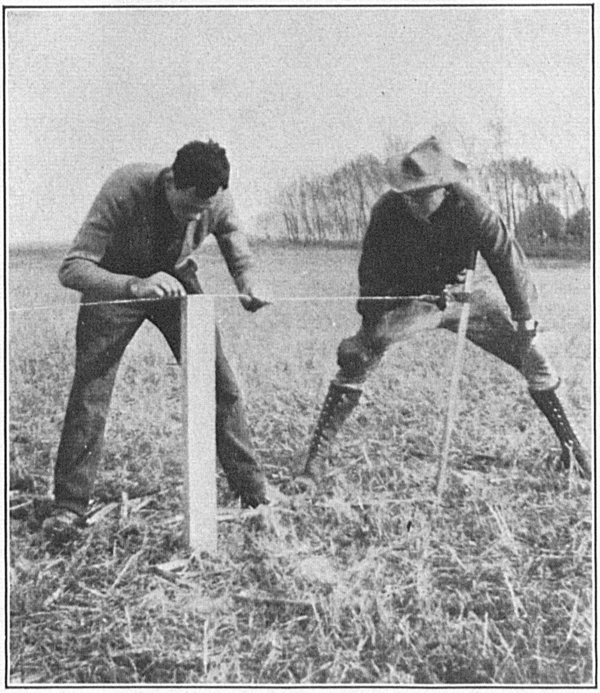

FIG. 74.—MAKING REAR CONTACT WITH TAPE

FIG. 75.—MAKING FORWARD CONTACT WITH TAPE

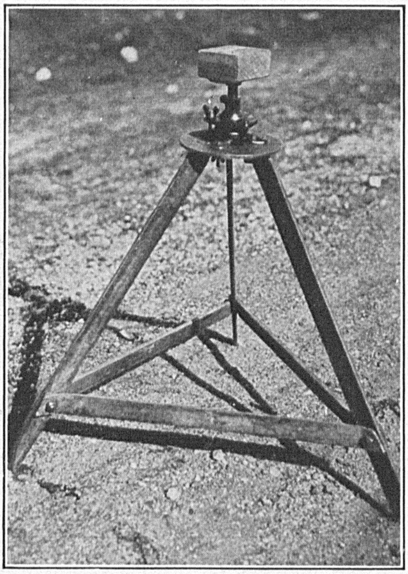

FIG. 76.—PORTABLE IRON TRIPOD FOR TAPE SUPPORT, SINGLE JOINT

The wooden marking table, which carries a strip of copper on which the mark for the tape end is made, can be placed on correct slope by the ball-and-socket base and then clamped in position

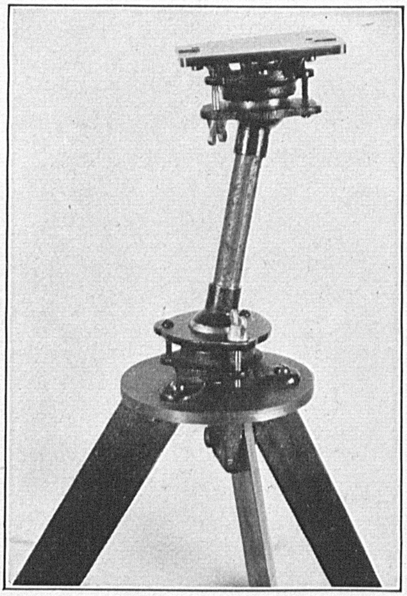

FIG. 77.—PORTABLE IRON TRIPOD FOR TAPE SUPPORT, DOUBLE JOINT

This tripod permits the marking table to be adjusted over a point as well as placed on correct slope position and the tension applied, taking care that the tape does not drag over the rear stake. As the tension is applied he advises the rear stretcher man whether to ease off or take up on the tape. Standing directly opposite the mark on the copper strip nailed to the top of the stake, with one hand he firmly grasps the tape between the staff and the mark and with the other hand he lightly touches the tape on the opposite side of the stake to steady it. He can then flex the tape with his rearward hand and bring exactly opposite each other the marks on the tape and copper strip. Before flexing the tape it is necessary, of course, that the mark on it shall be slightly forward of the mark on the strip. The marks are thus held in coincidence until the front contact man calls "ready," when the rear contact man calls "right," and the front contact man answers "mark," denoting the completion of the marking of the tape length. If more than a few seconds elapse after the calling of either "ready" or "right" before the next response can be made with accuracy, the entire process should be repeated. Immediately following the call "mark" the rear contact man reads the rear thermometer and the tape moves forward to the next position, unless some one at the forward end of the tape calls "hold" or "tension."

The front contact man (see fig. 75) has the hardest and the most important job of anyone handling the tape, for he must decide when all the conditions which affect the tape as a measuring unit are complied with, satisfying himself that the tape is in proper equilibrium under proper tension and support before he makes the forward mark. He also carries the forward end of the tape when moving forward and thus to a large extent sets the pace for the entire operation. The sequence of his movements during the measurement of a single tape length is as follows: As the forward stake is reached he lowers the front end of the tape from its position above his shoulder and attaches the link to the hook of the balance, grasping and guiding the hook to make sure that the attachment is made with a single movement. He then steps quickly back to a position alongside the forward stake, where he steadies the tape into its proper position just clear of the top of the stake, alongside the copper strip and between him and the strip. As the tension is perfected and the tape approaches equilibrium he places the point of the sharp, symmetrically pointed awl on the edge of the copper strip next to the tape and keeps it opposite the terminal mark on the tape until he is satisfied that conditions are right. After glancing at the balance to check the tension and down the tape to check the alignment he calls "ready," as above described. When he hears the response of "right" from the rear contact man he marks the copper strip with the awl, calling "mark" as the marking is completed.

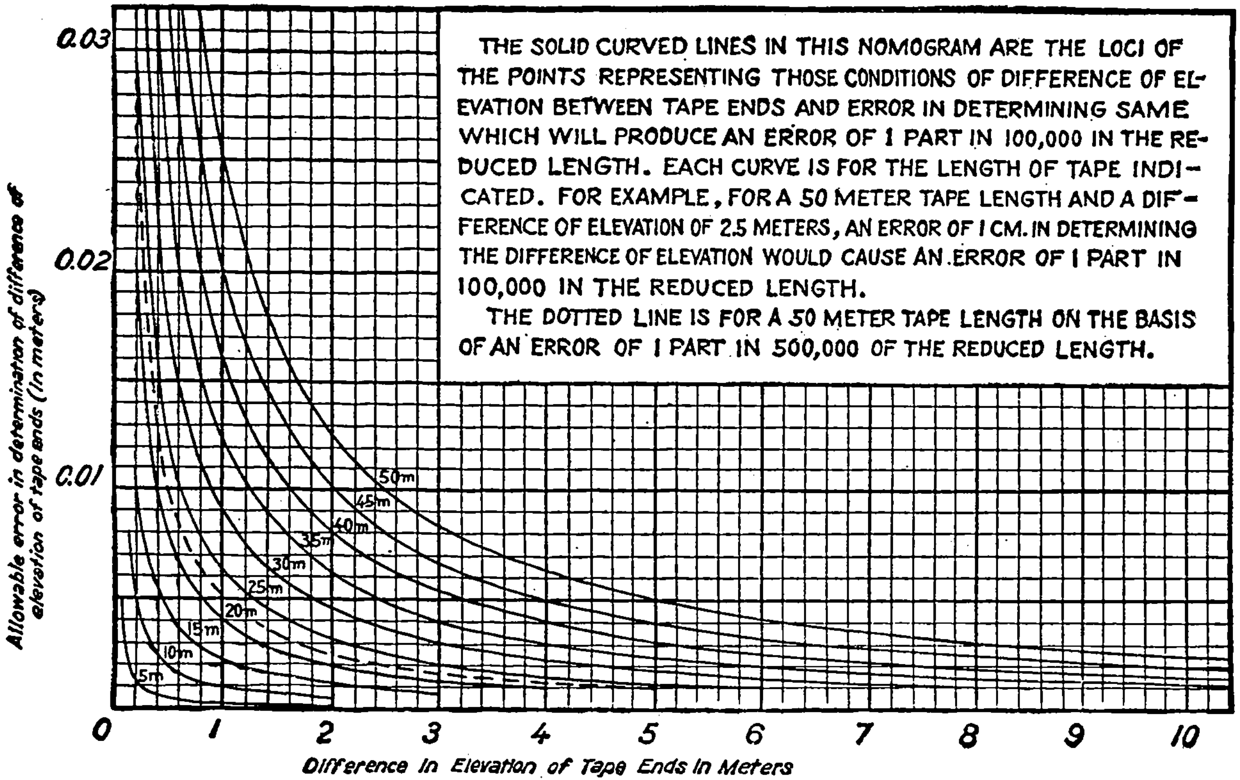

FIG. 78.—Nomogram of errors due to incorrect reductions for grade

In making the mark several precautions must be taken. The awl must be very sharp; it should *at no time* touch the tape in the region of the terminal mark,[2] the eye of the man making the forward contact, the terminal mark of the tape, and the axis of the awl should be kept in approximately the same vertical plane; and the mark should be made by the contact man moving the awl away from him in order to keep constant any error due to parallax. The mark should begin at the very edge of the copper strip in order to make it easier for the rear contact man to make contact. A straightedge is not used in making the mark, for it is believed that when the rear contact man brings the mark on the tape into coincidence with the end of the scratch on the strip and ignores the remainder of the scratch the error is less than that which would be caused by the less exact aligning of the awl point with the mark on the tape when a straightedge is used.

In order to avoid confusion, the marks placed on the copper strips during the second measurement are usually distinguished from the first markings by a bar across the scratch, while a third measurement would have a second bar. On the first measurement of each kilometer section the end mark of the section is given a distinguishing mark on the copper strip. On the second measurement, as the stake marking the end of each section is reached, a set-up or setback is taken to the original section mark on the strip, and the measurement of the next section is begun at the section mark. By this method a comparison is obtained between the forward and backward measures of a section. It is frequently better on the second measurement to make a set-up of a centimeter or so from the section mark in beginning a new kilometer, in order that the marks of the two runnings may not fall closely together

Immediately following his call of "mark" the front contact man reads the forward thermometer as the tension is released, then detaches the front tape link from the balance as he starts to pull the tape forward to the next position. If the tape is always carried in the hand which is toward the rear when making the contact, there is no danger of causing a half twist in the tape, for when detaching the tape from the balance the finger clip is caught in the rear hand, fingers pointing down, then the end of the tape is elevated above the shoulder without changing the grip, and at the next position is again attached with the hands in the same relative position.

[2] If it does, the mark soon becomes so defaced that neither measurements nor standardizations can be satisfactorily effected. Figure 79 shows a microphotograph of a tape mark almost obliterated by the careless use of the marking awl. The value of a tape increases with each successive standardization because of the evidence furnished regarding the stability of the tape, and if new marks must be made, the past history of the tape becomes to a large extent valueless.

When the tape is brought into position with its rear mark in contact with the mark on the rear stake the forward mark on the tape will sometimes fall short of or beyond the copper strip on the forward stake. If the staking was properly done and the stakes have not been moved, a second mark can be made on the strip on the rear stake, in front of or behind the first mark, at such distance as to permit of a contact mark being made on the forward strip. If the front contact man calls "set-up, 3," meaning that the forward terminal mark on the tape lacks 3 centimeters of reaching the copper strip, the rear contact man makes a mark 3 centimeters or more ahead of the mark on the rear stake, measures its distance from the previous mark with a boxwood scale, calls out the distance to the recorder, making sure that the recorder repeats it properly, measures the distance again as a check, then signals that he is ready to make contact on the new mark. Small set-ups and setbacks should be measured with an error not greater than one or two tenths of a millimeter. As an additional safeguard the recorder may check the measurement of set-ups and setbacks and have the contact man in turn check his entry in the record book.

Some base tapes have marks at approximately 5-meter intervals along the tapes, the lengths of the intermediate intervals having been determined with secondary accuracy. Where such tapes are available long set-ups are sometimes so staked as to enable the invar tape to be used for the greater part of the set-up—that is, for some multiple of 5 meters, the remainder of the set-up being measured with a standardized steel tape. For instance, if a 23-meter set-up is necessary, a stake would be set at the 20-meter point and another one at 3 meters beyond. The 20-meter interval would in such case be measured with the invar tape with the standard 15-kilogram tension. The method of supporting the tape and the tape temperature should be noted. The 3-meter interval would be measured with the steel tape, using the standardization tension, which is usually 5 kilograms, and noting the method of support. For each set-up the record should also show the part of the tape used in making the measurement. For instance, a set-up of 20 meters could be made either from the 50-meter to the 30-meter mark or from the zero to the 20-meter mark, but the standardized distance between the two pairs of marks would probably be different.

Where conditions are favorable the standardized steel tape may be used for all set-up distances up to one-half the invar tape length. Setbacks of more than a decimeter should never be used. This method will avoid in large measure the possibility of a large error in length due to an error in the sign of a partial tape-length distance on both measurements.

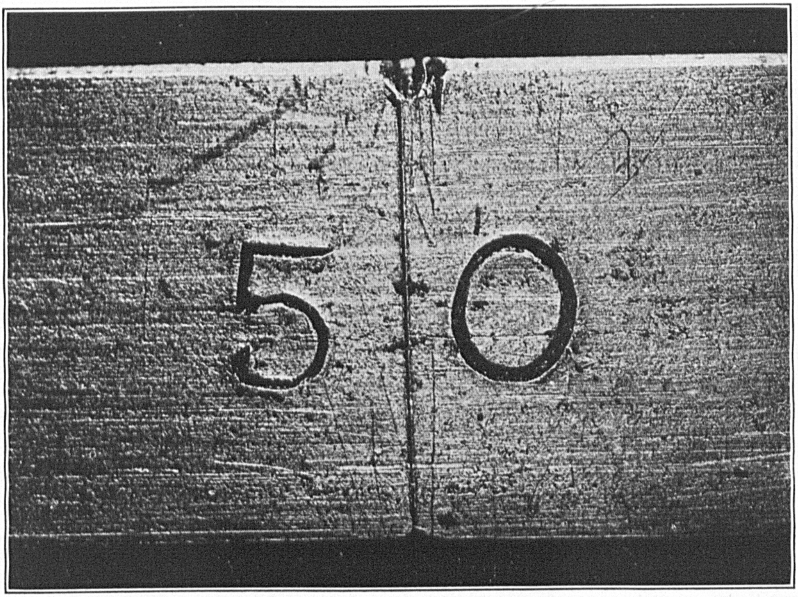

FIG. 79.—MICROPHOTOGRAPH OF DEFACED MARK ON TAPE

The contact end of mark, shown at top of illustration, has been worn away by the marking awl. The tape is useless for accurate work

The recorder should be an experienced man. If he is not, the chief of party must frequently inspect the record, especially where broken grades or set-ups and setbacks are to be recorded. It is the province of the recorder to be sure that no blunders are committed, such as the dropping or adding of a tape length, or recording a half tape length as a full one, or a set-up as a setback. He should check the chalked numbers on each stake as it is reached and make all notes necessary to a definite and correct interpretation of the record. With an experienced front contact man it is often advisable for the chief of party to assume the recorder's duties, as he can then not only record but can watch in turn the work of each man on the tape and correct such practices as need it.

FORMS FOR RECORDS AND COMPUTATIONS

Form No. 590 should be used for recording the tape measurements and Form No. 634 for recording the wye levels. The computations of the lengths of the various sections of the base should be on Form No. 589. Form No. 635 should be used for the abstract of wye levels and the computations of the grade corrections. Samples of all these forms are shown in Figures 80 to 83.

Explanation of Form No. 590 (Tape Measurements).—In recording the tape measures on Form No. 590 (fig. 80) two thermometer readings indicate a full 50-meter tape length and one thermometer reading a half tape length or a set-up. Each half tape length or large set-up should be recorded on a separate line, and not on the same line with a full tape length. The numbering of the stakes should plainly indicate the full tape lengths and the partial lengths. (See p. 121 for method of numbering.)

Notes in the "Remarks" column should clearly explain any unusual conditions. At the beginning of the day's work, and as often as changes occur, an entry should be made in the "Remarks" column giving the names and duties of the officer in charge, recorder, and the two contact men, also a statement as to the results of the comparison of the balances and the dial reading being used on the balances. All marginal notes and entries at the top of the page should be made as measurement progresses. The insertion of notes after leaving the section is very dangerous to accuracy.

Form No. 634 (Wye Leveling).—The sample form (fig. 81) shows the wye-level record when using a rod graduated in meters on one side and feet on the other. When such a rod is used the levels are run in only one direction, but both sides of the rod are read at each rod point, the reading in meters being recorded as the forward running

696680°—46——10

and the reading in feet as the backward running. If the rod used is graduated on only one side, a forward and backward running is necessary. The numbering of the stakes in this record should cor-

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 590

Base ~~TRAVERSE~~

From Station △E. Base. to Station T.E. 20

Forward measurement

Date: 7/7/24 Time: 8:10 Tape No.: 922 Bal. No.: 302 Ther. Nos.: 33260 1518

11—6328

Section.		Temp.		Set up.	Set back.	Tape support.	Remarks.
From—	To—	Forward.	Rear.				
		° C.	° C.	Meters.	Meters.		
△E.B.	△E.B set-up	20.0	—	20.0000		2	0–20
△E.B. set-up	1	21.3	21.0			3	
1	2	21.2	21.0			3	
2	3	20.6	20.0	0.0714		3	
3	4	21.2	21.0			3	
4	5	21.0	20.5			3	
5	6	20.0	19.8			3	
6	7	20.0	20.0		0.0214	3	B.G. at 6½
7	8	20.2	20.0			3	
8	9	20.4	20.5			3	
9	10	20.6	20.6			3	
10	10 set-up	—	21.5	4.7000	0.0381	2	←Steel Tape 872
10 set-up	11	20.7	21.0			2	←Crossing Gully
11	12	20.8	21.0	0.0027		3	
12	13	20.8	21.0		0.0732	3	
13	14	21.0	21.0			3	
14	15	21.0	20.0			4	Supp. 0–12½–25–50
15	16	20.8	20.5			3	
16	17	20.5	20.3			3	
17	18	20.7	20.5			3	
18	19	20.8	20.4			3	
19	20	21.0	20.5			3	

FIG. 80.—Specimen, record of tape measurements

respond to the numbering on Form No. 590. Extreme care should be taken to get readings on all broken grades and partial tape lengths, and these should be plainly indicated in the record. In the columns headed "Meters or feet" the name of the unit not used should be crossed out. This is especially important where the rod used is graduated on only one side, because the mathematician making the office computation has no other way of knowing whether the rod used was graduated in meters or feet.

89

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 684

WYE LEVELING

90

From △ E. Base To T. E. 20
FORWARD RUNNING

Date 7/7/'24 Instrument 95 Rods 118

Point.	Backsight. Meters or feet.	Foresight. Meters or feet.	Difference. Meters or feet.	Remarks.
△ E. B.	2.754			
△ E. B. set-up		2.438	+0.316	
1		2.004	+0.434	
2		1.937	+0.067	
3		1.540	+0.397	
4		1.309	+0.231	
5	2.511	1.110	+0.199	
6		2.438	+0.073	
6½ (B. G.)		2.440	−0.002	
7	0.471	2.779	−0.339	
8		0.799	−0.328	
9		0.976	−0.177	
10		1.273	−0.297	
10 set-up		1.300	−0.027	
11	1.647	1.738	−0.438	
12		1.962	−0.315	
13		1.437	+0.525	
14		0.537	+0.900	
15		1.734	−1.197	
16	1.247	1.469	+0.265	
17		1.837	−0.590	
18		2.004	−0.167	
19		2.540	−0.536	
20		2.486	+0.054	

From △ E. Base To T. E. 20
BACKWARD RUNNING

Date 7/7/'24 Instrument 95 Rods 118

Point.	Backsight. Meters or feet.	Foresight. Meters or feet.	Difference. Meters or feet.	m.	Remarks. Mean m.
△ E. B.	9.035				
△ E. B. + 20		7.999	+1.036	+0.316	+0.316
1		6.575	+1.424	+0.434	+0.434
2		6.354	+0.221	+0.067	+0.067
3		5.052	+1.302	+0.397	+0.397
4		4.290	+0.762	+0.232	+0.232
5	8.270	3.648	+0.642	+0.196	+0.198
6		8.005	+0.265	+0.081	+0.077
6½ (B. G.)		8.005	+0.000	+0.000	−0.001
7	1.550	9.121	−1.116	−0.340	−0.340
8		2.625	−1.075	−0.328	−0.328
9		3.202	−0.577	−0.176	−0.176
10		4.176	−0.974	−0.297	−0.297
10′		4.265	−0.089	−0.027	−0.027
11	5.404	5.712	−1.447	−0.441	−0.440
12		6.437	−1.033	−0.315	−0.315
13		4.714	+1.723	+0.525	+0.525
14		1.765	+2.949	+0.899	+0.900
15		5.692	−3.927	−1.197	−1.197
16	4.100	4.820	+0.872	+0.266	+0.266
17		6.031	−1.931	−0.589	−0.590
18		6.575	−0.544	−0.166	−0.166
19		8.330	−1.755	−0.535	−0.536
20		8.149	+0.181	+0.055	+0.054

FIG. 81.—Specimen, record of wye leveling

Form No. 635 (Abstract of Wye Levels).—In the first column on this form (fig. 82) are recorded the stake numbers, corresponding to the numbers on Forms No. 590 and No. 634. The second column gives the distances between stakes, each distance being that between the stake recorded on the same line as the distance and the stake on the line preceding. The third column gives the mean differences of elevation between the two stakes noted. This column is headed "Meters or feet," and here again it is important to cross out the word not applicable. The fourth column gives the grade or inclination corrections in millimeters. For 50-meter tape lengths and measurements made with the intermediate 5-meter marks these corrections can be obtained from the table on pages 193 to 195. These tables are

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 635

ABSTRACT OF WYE LEVELS
AND
COMPUTATION OF INCLINATION CORRECTIONS

POINT	DISTANCE	MEAN DIFFERENCE OF ELEVATION	INCLINATION CORRECTION	ELEVATION	MEAN ELEVATION	REMARKS
	Meters	Meters or feet	mm.	Meters	Meters	
△ E. B.				237.67		
△ E. B. set-up	20	+ 0.318	2.5			
1	50	+ 0.434	1.8			
2	50	+ 0.067	0.0			
3	50	+ 0.397	1.6			
4	50	+0.232	0.5			
5	50	+0.198	0.4			
6	50	+0.077	0.1	239.39		
6½	25	−0.001	0.0			
7	25	−0.340	2.3			
8	50	−0.328	1.1			
9	50	−0.176	0.3			
10	50	−0.297	0.9			
10 set-up	4.6619	−0.027	0.1			
11	50	−0.440	1.9			
12	50	−0.315	1.0	237.47		
13	50	+0.525	2.8			
14	50	+0.900	8.1			
15	50	−1.197	14.3			
16	50	+0.266	0.7			
17	50	−0.590	3.5			
18	50	−0.166	0.3			
19	50	−0.536	2.9			
20	50	+0.054	0.0	236.72	237.8	
			47.1			

FIG. 82.—Specimen, abstract of wye levels

made out for differences of elevation in both feet and meters, so either feet or meters may be used in the third column. The corrections for other lengths and also for differences of elevations outside the limits of the tables must be computed as explained on page 125. The sum of these corrections for the section of the base is entered on Form No. 589, "Computation of base line," in the column headed "Inclination."

It is very important that all broken grades and partial tape lengths be indicated on this form and that the grade correction be computed for the corresponding distance. The most frequent mistake made in computing grade corrections arises from using a 50-meter length instead of the real length.

Form No. 589 (Computation of Base Line).—On Form No. 589 (fig. 83) the first correction to be entered is the correction for temperature. This is computed as follows:

Temperature correction $= (T-T_s) \times$ coefficient of expansion $\times$ 50 $\times$ number of tape lengths, in which T is the mean temperature for the section and T_s is the temperature of the tape at standardization. The value of T is entered in the column headed "Temperature" and is the mean of the thermometer readings recorded on Form No. 590. The value of T_s is given in the standardization data for the tape. (See fig. 73.) The coefficient of expansion may be considered as the change in length per meter for each degree centigrade change in temperature and is also given with the standardization data. The number of tape lengths is given in the column headed "Tape lengths" and is the number of full tape lengths recorded on Form No. 590. For tapes with a positive coefficient of expansion the temperature correction is, of course, + or −, according to whether the mean

COMPUTATION OF *Exemplar* BASE LINE

Section	Date 1924	Dir. of meas.	Tape No.	Uncorrected length: Tape lengths	Uncorrected length: Meters	Temp. °C	Corrections: Temp. (Meters)	Corrections: Tape and catenary (Meters)	Corrections: Broken and ... (Meters)	Corrections: Inclination (Meters)	Corrections: Sea level (Meters)	Reduced length (Meters)	Adopted length (Meters)		
A E.B. to T.E. 20	7/7	F	922	—	—	20.0	−0.0002	−0.0007	+20.0000	−0.0471	−0.0382	1024.4704			
	"	"	922	20	1000	20.6	−0.0071	−0.0790	− 0.0586						
	"	"	872	—	—	21.5	0.0000	+0.0019	+4.7000						

FIG. 83.—Specimen, computation of base line

temperature is greater or less than the standard temperature. There are a few tapes which have negative coefficients of expansion, and for these the corrections would have the opposite signs.

The correction in the column headed "Tape and catenary" is obtained from the standardization data for the tape or by computing the catenary correction when the tape is supported in an unusual manner. The tape correction per tape length is the difference between 50 meters and the length of the tape as given for the proper method of support. For instance, in the sample of Form No. 590 (fig. 80) there are 18 tape lengths supported at three points, 1 at four points, and 1 at two points. The correction for a tape supported at three points is obtained directly from the standardization values, and this is multiplied by 18 for the 18 tape lengths. Referring again to the data on Form No. 590, the correction for the one tape length supported at four points is obtained by combining the proper fractional parts of the corrections for the three and the five point supports. The correction for the tape supported at two points may be computed by the catenary formula (p. 127), or may be taken from the table on page 196. The algebraic sum of these corrections is entered in the column headed "Tape and catenary," the sign depending on whether the length of the tape is greater or less than 50 meters.

In the column headed "Set-up and setback" is entered the algebraic sum of the set-ups and setbacks recorded on Form No. 590, the set-ups being plus and the setbacks minus. In the sample shown in Figure 83 the two large set-ups and the corrections to them are recorded separately. All set-ups, however, could have been combined, and also the temperature, tape, and catenary corrections for the large set-ups could have been combined with the same corrections for the full tape lengths, and then the entire computation of a section would have been on one line. It simplifies the checking of the computation somewhat, however, to enter each large set-up on a separate line, as shown on the sample form.

The sum of the inclination corrections is obtained from Form No. 635. Finally the algebraic sum of the uncorrected length and all corrections gives the reduced length for the section, and the mean of the reduced lengths from the forward and backward measurements gives the adopted length. The columns headed "(v)" and "(vv)" are used in computing the probable error of the measurement of the base.

PROBABLE ERROR OF MEASUREMENT

A method of computing the probable error of the measured length of the base, with separate values for the probable error due to standardization of the tapes, the determination of the coefficients of expansion, and the accidental errors of the measurement, is given in Appendix No. 4, Coast and Geodetic Survey Report for 1910, pages 160–161. The probable error is usually computed, however, by the method described in the following paragraph. This method is based on the theory that the errors of standardization and of the determination of the coefficients of expansion are either largely included in or are masked by the discrepancies in the measured lengths of the sections.

The probable error of each section is computed by the formula

$$\text{p. e.} = 0.6745\sqrt{\frac{\Sigma v^2}{n(n-1)}},$$

where v is a residual and n the number of measures of the section. Where a section is measured only twice the probable error will, of course, be 0.6745 times one-half the difference between the two measured lengths. The probable error of the entire base is the square root of the sums of the squares of the probable errors of the component sections.

REDUCTION TO SEA LEVEL

Since the lines of a scheme of triangulation are reduced to their equivalent lengths at sea level, the length of any base must be likewise reduced to sea level before it can be used in adjusting the triangulation to which it is connected. It is sometimes necessary to reduce the base to sea level in the field in order to compare the measured length with the length as computed through the triangulation from the previous base. This requires the connection of the base-line levels to a bench mark and the computation of the elevation above sea level of the tape supports in order to obtain a mean elevation for the base. Only enough elevations need be used in computing the mean elevation for a section to give a value correct to within 25 meters.

The formula used in reducing a base to sea level is

$$C = -S\frac{h}{r} + S\frac{h^2}{r^2} - S\frac{h^3}{r^3} + \ \ldots \ .$$

in which C is the correction to reduce to sea level a section of length S, of a mean elevation h, with r the radius of curvature of the earth's surface for that section. Only the first term of the formula need be used for any field reduction.

The computation of the sea level correction, shown on sample Form 589 (fig.) 83), is given below, the mean latitude of the base being 40° 30′ and its azimuth 75°, giving a value for log r (see table, p. 203) of 6.80521. The mean elevation as obtained from Form 635 is 237.8 meters.

$$\begin{aligned} \log\ 1{,}024.5 &= 3.01051 \\ \log\ 237.8 &= \dot{2}.37621 \\ \text{colog}\ r &= 3.19479 \\ \hline \log\ C &= 8.58151 \\ C &= 0.0382 \text{ meter (always negative)} \end{aligned}$$

The error per kilometer of base line for each 1-meter error in the elevation above sea level as used in computing the reduction varies from 0.000158 to 0.000156 meter, depending upon the latitude and azimuth of the base. This corresponds to a proportionate error in length of from 1 part in 6,329,000 to 1 part in 6,410,000. (See p. 118.)

THIRD-ORDER BASE MEASUREMENT

The same methods are used in measuring a third-order base as those prescribed for second order, and the precautions to be used in guarding against error are the same. The only differences are in the permissible errors, as noted in the following paragraphs.

On a third-order base the total actual error from all sources shall not exceed 1 part in 75,000 of the length of the base and the computed probable error shall not be greater than 1 part in 250,000. Two measurements shall be made of the base with two different standardized tapes, the base being divided into kilometer sections and the two measurements of each section being made in opposite directions as on second-order base measurement. If the discrepancy in millimeters between the two measurements of a section exceeds $25\sqrt{K}$ (where K is the length of a section in kilometers), additional measurements should be made until two are secured which agree within the specified limit.

If the mean elevation of the base is more than 50 meters above sea level, the mean elevation shall be determined with an error of not to exceed 50 meters in order that the measured length may be reduced to its sea-level equivalent.

Such precautions shall be taken in staking and measuring the base that the errors due to lack of alignment, wind effect, or support of the tape shall not for any one of these causes be more than 1 part in 150,000 of the length of the tape. The tension should not be in error by more than 150 grams. The error in the grade corrections and the error in projecting a broken base upon the line between the terminal stations should not exceed that specified for a second-order base.

The same forms are used for tabulating the measurements and the same computations made in determining the corrected length of the base as for a second-order base.

CHAPTER 4.—SECOND AND THIRD ORDER TRAVERSE

First-order traverse was not used extensively by the Coast and Geodetic Survey until 1916, since which time over 3,500 miles have been run with invar tapes. A description of first-order traverse methods is contained in Special Publication No. 137, Manual of First-Order Traverse. Traverse of second and third order accuracy measured with wire or tape has been used occasionally in the past by the Coast and Geodetic Survey where topographic conditions rendered control of topographic and hydrographic surveys by the triangulation method very expensive. In many respects the same methods are used for second and third order traverse as for first order, but many of the refinements necessary for first-order control are, of course, neglected or modified.

The accuracy specified for second-order traverse (see p. 3) is a position check with an error not exceeding 1 part in 10,000 of the length of the circuit when closure is made on a point of first or second order triangulation or traverse. A circuit closure is never conclusive evidence of the accuracy indicated by the error of closure, for there is always the possibility of compensating errors, or of systematic errors which will not show in the error of closure. All operations of the traverse method must, therefore, be closely scrutinized for errors and the possibility of blunders reduced to the minimum. There is no mathematical check on the accuracy of traverse such as is afforded by the closure of triangles on triangulation.

The limiting error of closure in position specified in the preceding paragraph, taken in conjunction with the requirements for angle and tape measurements which are given later, is intended to provide that each line of the traverse after adjustment will have an error not to exceed 1 part in 10,000, which will probably give an average error of all the lines of about 1 part in 30,000.

In the Coast and Geodetic Survey traverse is usually employed for the main scheme second-order control on beaches where the absence of off-lying islands and the presence of timber along shore make triangulation very expensive. These conditions are frequently accompanied by lack of transportation facilities, and this makes it necessary to reduce as much as possible the weight of the appliances used. On the other hand, it is usually possible to measure directly from one angle station to the next, and the troublesome offsets necessary when the length measurements are made along railroad tracks, as is usually the case on first-order traverse, can ordinarily be avoided. Occasionally, however, it will be economical to measure sections of second-order traverse along a road or railroad. The methods used in projecting the measured line onto the line between angle stations are described briefly on pages 168–170. A more extended discussion of projection methods may be found in Special Publication No. 137.

SPECIFICATIONS, SECOND-ORDER TRAVERSE

The following specifications for second-order traverse measurements were approved by the Director of the United States Coast and

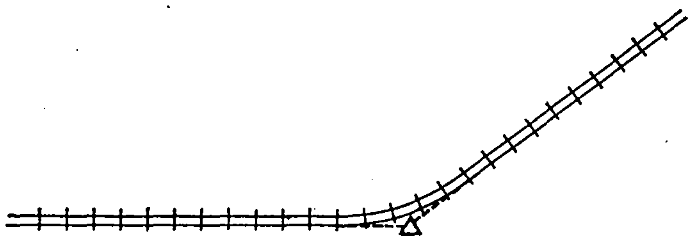

FIG. 84.—Traverse station at intersection of two tangents

Geodetic Survey on December 10, 1928, and supersede all previous instructions for work of this character.

1. **Location of stations.**—Main-scheme stations on a second-order traverse line should be located not more than 3 miles apart, and an average distance between permanently monumented main-scheme stations of not more than a mile is desirable. After the requirements of the intervisibility of stations have been satisfied other considerations must be kept in mind, such as the selection of points which can be permanently marked, which will afford the best routes for the tape measures, and which will give the observer the best opportunity for locating important objects by the intersection method. Lines of sight when opened should be inspected to see that they are not apt to be unduly affected

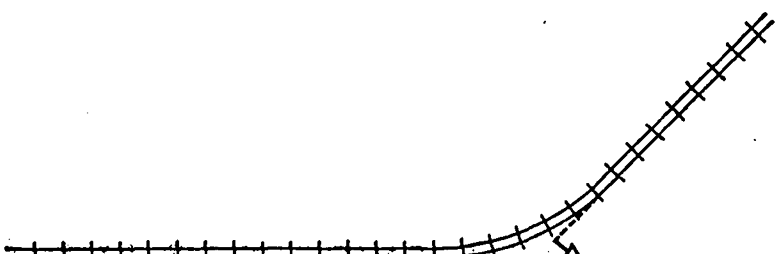

FIG. 85.—Traverse station on extension of one tangent

by lateral refraction. If practicable, a main-scheme station should be located near any highway or railroad which intersects the line of traverse. Where a line of traverse, following the ocean beach, crosses an inlet or river of any considerable width, a pair of stations should be placed in the best location for a later extension of triangulation or traverse up the inlet or river.

2. **Traverse along railroads.**—When in measuring along a railroad a station can be located at the intersection of two tangents, no projection of measured lines is necessary, and the work in both field and office is simplified. (See fig. 84.) If the presence of obstructions or the degree of curvature of the track prohibits this location, the station may be placed on the prolongation of one

tangent and at a small offset distance from the other. (See fig. 85.) In other cases it may be necessary to locate the station at a short offset distance from both tangents.

Where the tangent is longer than the maximum specified distance between stations, or where the stations at the two ends of the tangent can not be made easily intervisible, one or more intermediate stations should be placed at the side of the track. For these intermediate stations the offset point on the rail is called a "rail station." (See fig. 92.) Where the offset point is on a stake of a line of stakes it is called an "offset station." The intermediate stations along a tangent should preferably be alternated from one side of the track to the other to reduce the effect of lateral refraction. The ratio of the offset distance to the distance between stations should seldom exceed 1 part in 50.

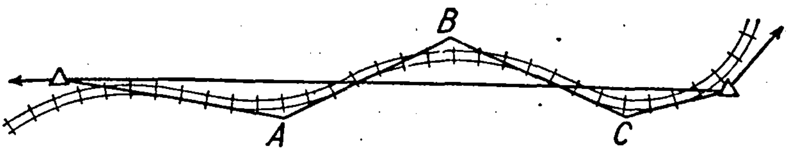

FIG. 86.—Traverse loop

3. Traverse loops.—Where the traverse follows a railroad with many curves and short tangents, or a beach where the topography necessitates carrying the taping through many short lines, an attempt should be made to provide an azimuth line extending over several curves, in order that the azimuth may be carried forward with greater accuracy. If the main traverse line can be tied in frequently to triangulation or to first-order traverse, these loops need not be provided. The subsidiary points between main traverse stations are designated by the name of the main traverse station at the beginning of the loop, followed by the letters A, B, C, etc. These subsidiary stations need not be permanently marked. Two methods of forming these loops are shown

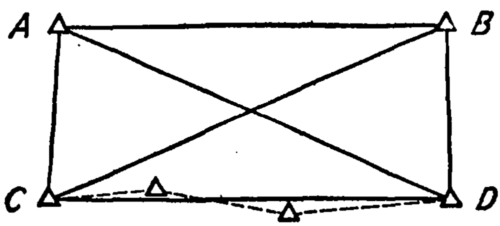

FIG. 87.—Connection between triangulation and traverse by quadrilateral

in Figures 86 and 87. No considerable portion of the measured part of a loop should be inclined at an angle of more than 45° to the line through which the azimuth is carried. The larger the angle of inclination the more accurately should the loop angles be measured.

4. Combination of triangulation and traverse.—In many situations traverse and triangulation can be advantageously combined. Sometimes triangulation can be brought down to a coast at widely separated points but triangulation along the coast is prevented by swamps or forests. Under these conditions traverse run along the beach and tied into the triangulation stations directly or by small triangles can be used for the coastal control.

5. Azimuth stations.—A second-order azimuth should be observed at intervals of 15 to 25 main-scheme stations. If the observing conditions are favorable and small angular errors probable, the upper limit of 25 may be approached, but if errors from refraction, phase, or eccentricity are apt to be large the smaller limit should be adhered to. The discrepancy between the observed astronomic azimuth of a traverse line and its azimuth as computed through the traverse from the preceding observed azimuth should not exceed 2''.0 times the number of intervening main-scheme stations.

6. Connections with existing control.—In connecting traverse with triangulation the desirable requirements are a connection in azimuth and length with a check. This means that the connecting figure must be such that there are two routes by which the length of a line of the triangulation may be carried through to a line of the traverse. In addition, if the triangulation connected with is several years old, observations should be made upon a third old station from the two stations of the triangulation involved in the connection, in order that there may be conclusive evidence of the recovery of the proper points. If the reobserved values of the angles do not agree with the old values within reasonable limits, enough of the old scheme of triangulation should be reobserved to locate the trouble.

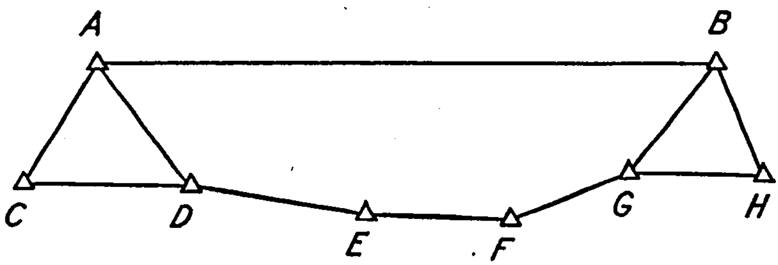

FIG. 88.—Connection between triangulation and traverse by two triangles

The connection in length is desirable for the sake of strengthening the triangulation by connecting a triangulation line to a measured section of the traverse. If the triangulation has been carried on by the same party that is executing the traverse, and there is therefore no doubt as to the exact recovery of stations, the traverse may be started from a position given by a single station of the triangulation and an azimuth obtained by measuring a single deflection angle from a line of the triangulation.

The simplest connection between triangulation and traverse is by means of a quadrilateral as shown in Figure 87. The line *CD* may be one of the measured lines of the traverse or its length may be determined by a loop projection. If the line can not be observed over, then the intermediate traverse stations between *C* and *D* must be main-angle stations. The connection may also be made by two triangles, each formed by two adjacent traverse stations and a triangulation station, with perhaps several traverse stations intervening between the two triangles, as shown in Figure 88. It is necessary to measure all interior angles of the connecting figures shown in both Figures 87 and 88.

Where a traverse joins an existing traverse line there should be a connection in position and azimuth and, in addition, one of the angles of the old traverse should be reobserved to check the recovery of the old stations. If there is any doubt of the exact recovery of the stations, one line of the old traverse should be remeasured.

7. **Marking stations.**—All main-scheme stations of second-order traverse shall be permanently marked with the kind of marks specified for second-order triangulation stations (see pp. 38–41) except where conditions are such that a permanent mark can not be established, or where the main-scheme stations are so close together that no particular advantage would be gained by marking each one. Reference marks should be established as provided for in the following paragraph. The distribution of permanently marked stations should be such that the average distance between them will be from 1 to 2 miles and the maximum distance 5 miles. Where a station is permanently marked, at least one of the adjacent main-scheme stations should also be permanently marked in order to make available a distance and an azimuth for future control.

8. **Reference marks.**—Reference marks should always be established where a station which should be preserved can not be permanently marked or where both of the adjacent stations are more than 2 miles distant. Whenever a station is referenced two reference marks should be established, placed in locations where they are not apt to be disturbed, and made to correspond in character and permanency to those prescribed for second-order triangulation. (See p. 40.)

SIGNAL BUILDING

The type of theodolite ordinarily used on second-order traverse can be mounted on its own tripod except where it is necessary to elevate it to make the stations intervisible. In the latter case the type of signal usually employed on triangulation will suffice. Where the traverse line is readily approached by boat or truck and a considerable number of low towers are needed portable tripods and scaffolds may be built which can be readily taken down and moved forward to a new location. (See p. 31.)

ORGANIZATION OF PARTY

The conditions under which second-order traverse will be run vary so greatly that no recommendations can be made regarding party organization except to point out that it requires at least one officer and five men to operate the tape, which sets the lower limit to the size of party, unless conditions are such that the tape measurements can be made with the tape lying on the ground without being supported on stakes or tripods, in which case 1 officer and 3 or 4 men will suffice.

SUBPARTIES

Building party.—This party does the detailed reconnaissance, marks the stations, clears lines, and builds whatever stands and signals are required. The reconnaissance should include the location and temporary marking of the A, B, C stations. (See p. 163.) The party will consist of one officer and from one to three men, depending upon the amount of building and clearing to be done.

Taping party.—All measurements with the invar tape are made by this party. If the measurement is being made over stakes or movable tripods, two stretcher men, two contact men, a middle man, and a recorder are required to operate the tape. In addition, one man to drive the stakes or two men to move the tripods forward into position are usually required. The officer in charge of the party will ordinarily act either as recorder or as front contact man.

Leveling party.—When grade corrections need to be applied to reduce inclined measured lengths to the horizontal (see p. 125) wye-level readings are taken on the support used at each tape end or on some point at a known distance beneath the support. The party usually consists of one officer, who acts as both observer and recorder, and one rodman.

Check-taping party.—This party makes a check measurement of the line with a 300-foot tape. It need consist of only two men, one to make contact at each end of the tape. The recording is done by one of these men.

Angle party.—This party consists of one officer as observer and a recorder. If signal lamps are used, one or more light keepers will be needed. This party measures the angles at all stations, including subsidiary loop stations and offset stations.

Ordinarily the traverse party will be of such size that practically all the personnel will be needed when the invar tapes are being used. At other times two or three subparties can be employed on operations which require only two or three men to a party. If the chief of party keeps in close touch with the progress of all the subparties, and keeps himself informed of the conditions ahead, there need be little, if any, lost motion.

MEASUREMENT WITH INVAR TAPES

The final computed lengths of the traverse will depend upon measurements with 50-meter invar tapes. Each tape will be standardized at the Bureau of Standards before being sent to the field and should be returned to the office for restandardization every two or three years, or oftener if the field comparisons show the need for it. Most of these tapes have 5-meter graduations to permit the measurement of fractional tape lengths, or set-ups, to the nearest 5 meters. Only a single measurement is made of each section of the traverse with an invar tape, unless the check measurement with the 300-foot steel tape shows that a blunder was made in the measurement with the invar tape, when a second measurement of the section with the invar tape must be made.

FIELD COMPARISONS

For purposes of making field comparisons the party should be provided with three invar tapes, one of which should be kept as a standard and not used except for comparison purposes. An invar tape should not be used on more than 20 to 25 miles of line without being compared with the other tapes. A careful measurement of a single tape length over stakes will give a comparison accurate to within one-tenth of a millimeter. In case a tape becomes kinked in use, a note should be made in the traverse record of the exact time of kinking and a field comparison made of its length before it is used on another day's measurements.

The comparisons should be entered in the record book, in which should be shown the temperature, method of support, and the set-up or setback to the forward mark of the first tape used in the comparison. If the measuring tape is shown by comparison with the standard tape to be in error by more than 1 part in 50,000, it should be sent to the office for restandardization, provided another tape is available for field measurements. Otherwise it may be used until another tape can be procured. The length of the injured tape obtained by the field comparisons will then be used in the computations involved.

DETAILS OF MEASUREMENT

For convenience in measuring fractional tape lengths the zero end of the tape should always be to the rear. Great care must be exercised in recording the number of full tape lengths and the lengths of set-ups. The temperature of a single thermometer, attached to the rear end of the tape, should be recorded for each full tape length. Fractional tape lengths including set-ups of more than 1 decimeter should be recorded on a separate line, but no temperature should be recorded for these, in order that the number of recorded temperatures may be the same as the number of full tape lengths. No temperature correction is needed for fractional tape lengths. If the measures are made over stakes or on a pavement or railroad rail, the number of each tape length should be marked with chalk or keel on the stake, pavement, or rail. The forward contact man should call out the number as he marks it, the rear contact man checking it from the mark at his end of the tape and the recorder checking it from his record. If the measurement is made along the ground without stakes, metal pins may be used as in ordinary chaining, the pins being collected and counted at the end of each section. Each pin should have a diamond-shaped cross-section or have one sharp edge to furnish a more definite point for marking the tape end. A chalk mark may be used to designate the point on the pin at which contact was made if the pin is inclined considerably. If preferred, the position of the forward end of each successive tape length may be marked by an ordinary pin stuck into a soft-pine block. The block is held in place by three or more bolts projecting several inches below the block held firmly by nuts and washers and of suitable length to hold the block firmly in place in the kind of soil encountered. The lower ends of the bolts may be sharpened if necessary.

As the measurement with the invar tape progresses, any points suitable for hydrographic or topographic signals should be connected with the traverse line and marked temporarily by stakes. The positions of these points can either be computed later or they can be plotted by distance and direction from the main-scheme points. The distances should be recorded as follows: "Stake on point, 24+4," meaning that the stake is 4 meters beyond the end of the twenty-fourth tape length from the preceding station; or "stake, 9+6, north 5 meters," meaning that the stake is 5 meters north of the traverse line at a point 6 meters beyond the end of the ninth tape length.

SUPPORT OF TAPE

A judicious selection of the methods of tape support on second-order traverse may reduce the cost of the work materially. Wherever conditions permit the tape should be stretched along the ground, but this should be done only after rather definite information has been obtained of the amount of error caused by the inequalities of the supporting surface. This information can best be obtained by selecting a few sections, 4 or 5 tape lengths long, where the ground seems to be approaching the maximum in roughness, and measuring the sections with the tape supported on stakes and also stretched along the ground. Inequalities of the supporting surface when the tape is on the ground always make the measured length too long. If the error introduced by these bumps and depressions exceeds 1 part in 20,000 (1 centimeter in four tape lengths) for the greater number of the tape lengths, the tape should be supported on stakes or light movable tripods. The error can usually be closely estimated by the officer in charge by keeping in mind the corrections for inclination. For instance, the grade correction (see p. 125) varies directly as the square of the difference of the elevation of the two ends of the tape and inversely as the length of the tape. If a tape lying on an even grade has a difference of elevation of the two ends of 0.3 meter, the correction for grade will be only 0.9 millimeter. If a tape length only 10 meters long has the same difference of elevation of the ends, the grade correction will be five times as great, or 4.5 millimeters.

The effect of sag (see p. 126) when the tape hangs unsupported between higher points of the ground surface may be computed by the formula or by the table on page 196. Since the correction for sag varies as the cube of the length of the tape, it is easy to keep the correction in mind. For a tape weighing 24 grams per meter under a tension of 15 kilograms the catenary correction for a 10-meter unsupported length would be only 0.1 millimeter, for a 20-meter unsupported length 0.8 millimeter, etc. In other words, the corrections for sag for unsupported sections of the tape when lying on the ground can usually be disregarded, but the corrections for grade must be closely noted. It will frequently happen that a small amount of time spent in clearing bushes and weeds from the line and leveling off occasional mounds will permit measurements with the tape supported on the ground.

When the terrain is such that supports must be provided for the tape the chief of party can choose between driving stakes for the end supports of the tapes or using light movable tripods. In either case the middle of the tape is usually held on grade and in alignment by the man at the middle of the tape, guided by the rear contact man. The stakes, usually of 1 by 4 inch lumber, are driven by an extra man of the party as the taping progresses.

If tripods are used, they should be light wooden ones with a wooden marking table about 4 by 6 inches in size on top, which can be easily replaced when the marks on it become so numerous that there is danger of the rear contact man not being able to detect the last mark made by the forward contact man. The marks can be made with a knife and crossed out by the rear man after the contact, or pins stuck into the marking table may be used. If the tripods are light, two or three can be carried at one time by the man whose duty it is to move them forward.

A sufficient number of signal poles with banners should be placed on the line to enable the taping party to align the stakes or tripods as the measurement proceeds, thus avoiding the need for a theodolite. The errors due to lack of alignment are the same as those due to an equal uncorrected inclination of the tape, and ordinary care will keep the alignment errors negligible in size.

TENSION

All invar-tape measurements should be made with the tape under a tension of 15 kilograms (about 33 pounds). The tension should be applied by a spring balance attached between the forward end of the tape and the front stretcher. The balances should be tested occasionally with a standard weight which is furnished on requisition by the

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office. For second-order traverse the tension applied to the tape may be in error by 200 or 300 grams without appreciable error in the measured lengths.

The tape stretchers used in taping over stakes are of the same type as those used on base measurements, described on page 124 and shown in Figure 72. For taping along a rail a different type of stretcher is used. It consists of a shoe of galvanized iron about 18 inches long and just wide enough to fit easily over the top of the rail, with uprights of the same material projecting about 6 inches above the rail. These uprights furnish the fulcrum for a lever, to the lower end of which the tape is attached by means of a hook. (See fig. 89.) This device makes it possible to apply the tension to the tape and at the same time holds the tape close to the rail. By attaching a wooden block to the under part of the shoe this same stretcher can be used for taping over a sidewalk or over the ground, since the block prevents the sides of the shoe from being crushed down.

TEMPERATURE

A single thermometer inclosed in a special protective casing is used on the tape. It is supplied by the office on requisition. It should be attached by adhesive tape to the rear end of the tape, about 1 meter forward of the rear mark, and the temperature should be read by the rear contact man for each full tape length. (See p. 134.) The thermometer used on the tape should be closely watched to detect any separation of the mercury column and should be compared each day with another kept as a standard.

CARE OF TAPES

Great care should be exercised in the use of tapes. Kinks are usually made by catching the tape under ties or spikes along the railroad track or on roots and bushes, by dragging the tape along the ground, or by careless reeling and unreeling. The tape should never be allowed to come in contact with the ground while being carried forward. Tapes should be cleaned and oiled often enough to prevent corrosion.

SET-UPS, SETBACKS, AND OFFSET DISTANCES

To avoid gross errors, due to the erroneous recording of a set-up as a setback, or vice versa, setbacks should not be greater than 1 decimeter, the amount that may be measured with a pocket decimeter scale. Any larger variation from a full tape length should be measured as a set-up, or plus correction to the measured length.

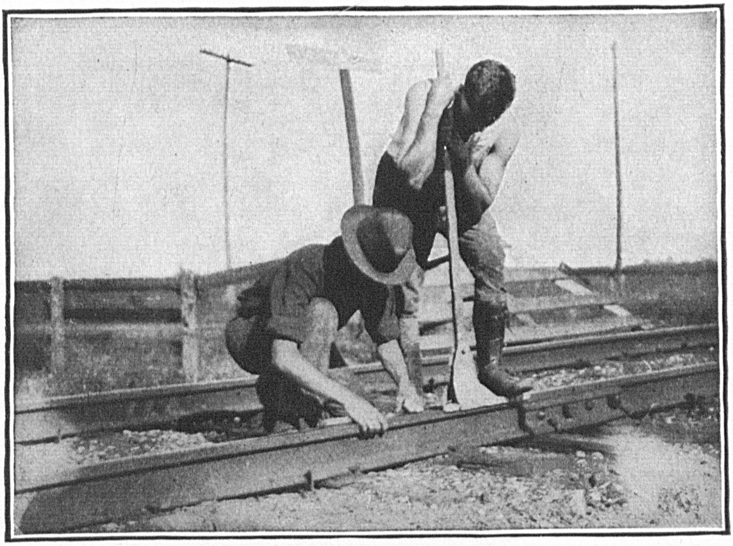


FIG. 89.—TYPE OF TAPE STRETCHER USED ON TRAVERSE ALONG RAILROAD

To measure a fraction of a 50-meter tape length, the best procedure, unless the distance is so short as to be more easily measured with a decimeter scale or a steel tape, is to make use of the 5-meter graduations on the invar tape and measure as large a part of the distance as possible in this way. The remainder should then be measured with a pocket steel tape. The horizontal distance should be measured whenever practicable by using a plumb bob when necessary.

Distances from offset and rail stations to the traverse stations should be measured with a steel tape in the same manner as set-ups and recorded in the "Remarks" column of the record. They should be measured horizontally whenever possible in order to eliminate corrections for inclination. The record should state clearly whether the horizontal or inclined distance was measured. If the inclined distance is measured, the difference of elevation should be determined.

All offset distances and set-ups measured with a steel tape should be measured in both meters and feet by using a tape graduated in meters on one side and in feet on the other. At the end of the day's work the measurements in meters should be compared with those in feet to disclose any discrepancy. If any appreciable discrepancy is found, the measurement should be repeated and corrected in the field.

INVAR-TAPE RECORD

The invar-tape measurements should be recorded on Form 590, "Traverse measurements," in exactly the same manner as indicated for base measurements, Figure 80, except that on traverse only one temperature is recorded for each tape length, no temperature for a fractional tape length, and a sketch is placed in the "Remarks" column showing the section of traverse to which the measurements relate. This sketch not only serves as a guide in projecting measured lengths onto projected lines but often helps in detecting blunders. The sketch should be made in the field and not from memory and should show the mutual relations between the measured line, the projected line (if any), and the railroad, road, or beach along which the measurement is being made.

The following data should also be recorded in the "Remarks" column:

Names of officer in charge and recorder. (At beginning of each day's work.)

Weather. (At beginning of each day and when change occurs.)

Note as to which rail of tangent was used, if measurement is along a railroad. (On each page.)

Distances to all rail and offset stations.

Statement as to whether offset distances are horizontal or inclined measurements.

Distances from beginning of section to road crossings, stream crossings, etc.
Record of broken grades.
Statement of route followed by traverse, with names of streets or highways if not shown adequately by sketches.

CHECK MEASUREMENT OF DISTANCES

A single check measurement will be made of each section of the traverse with a 300-foot tape to detect blunders. Small errors will not be detected by this check. The measurement may be made under any condition of the weather and should be made from traverse station to traverse station along the route used by the invar tape, with no reference to kilometer sections. Each section from a traverse station to an A station, from an A station to a B station, etc., should be considered as a separate section of the traverse in this measurement and should have a separate page in the record.

A moderate tension only should be used with the 300-foot tape. It is believed that about 5 kilograms is sufficient, and with this light tension two men can do the measuring. A heavy tension would require a tape stretcher at each end of the tape. No corrections for temperature need be made and ordinarily no corrections for grade. The tape should be stretched along the ground without special supports. To mark the position of the forward end of the tape when measuring on the ground long iron or heavy wire pins pointed at the end should be used. If the measurement is being made over a paved highway or a railroad, the successive tape lengths may be numbered, instead of using pins.

A comparison between the check measurements of the sections and those made with the invar tape should be made immediately after the completion of a day's work, so that, if any appreciable discrepancy is disclosed, the necessary remeasuring may be done before the party has left the vicinity. After the measurement with the 300-foot tape has been reduced to meters, the length of the section as obtained by measurement with the invar tape should be entered alongside or beneath it for convenient reference.

The party making the check measurement should have no knowledge of the length obtained with the invar tape. If the length obtained with the 300-foot tape differs from the length obtained with the 50-meter invar tape by more than 1 part in 1,000 in any section over 500 meters in length or by more than 1 part in 500 for sections under 500 meters in length, a second measurement with the 300-foot tape should be made unless the 300-foot measurement has been made over very rough ground, in which case the lack of agreement between the two measures may be allowed to exceed somewhat the amounts stated. If this second measurement agrees closely with the first one made with the 300-foot tape, then a second measurement should be made with the 50-meter invar tape. If necessary to make a second measurement with the invar tape, kilometer sections of 20 full-tape lengths supported on the rail need be remeasured only if the error can not be found elsewhere.

MEASUREMENTS ALONG HIGHWAYS AND CITY STREETS

Practically the same methods should be used for traverse measurements along paved streets and improved highways as for the measurements along a railroad. The tape may be supported throughout its length on the street pavement or sidewalk or it may be supported on stakes driven along the side of the highway or on portable tripods. If the surface of the pavement is smooth and has a uniform grade, it is ordinarily advisable to use the pavement as a support for the tape.

For marking tape ends, white adhesive tape 1 inch wide may be used. A piece of this tape about 3 inches long should be stuck to the pavement and the forward mark on the tape transferred to it with a hard pencil. On a street paved with asphalt the tape ends may be marked directly on the pavement with a knife. Each tape end should be marked and numbered on the pavement with yellow lumber crayon so that it may be easily found by the rodman of the wye-level party if grade corrections are required.

The check measurement with the 300-foot tape should be made in the usual way.

LEVELS

The purpose of the leveling along a traverse is twofold—first, to provide data for reducing the measured lengths to sea level, and second, to reduce the inclined tape lengths to the horizontal.

On most second-order traverse of the Coast and Geodetic Survey levels are not needed for sea-level reduction, since the lines are usually run over terrain near the sea and at low elevation. If it is desired to reduce a measured length to sea level, or conversely to compute its length at its actual elevation above sea level after its sea-level length has been obtained, it is not necessary to know the mean elevation of the line closer than about 300 meters, since that error in elevation would affect its length less than 1 part in 20,000. The mean elevation of any line can always be estimated that closely.

Whether or not level readings are needed at each tape end to reduce the inclined measured distance to the horizontal depends entirely upon the grades. Failure to apply grade corrections will always make the measured length too long. Since any one systematic error should not exceed 1 part in 20,000 of the length between any two main-scheme stations, the chief of party must make a close estimate of the errors to be introduced by neglecting grade corrections. The tables of grade corrections on pages 187–189 will guide him in making this estimate if he bears in mind that 5 centimeters are 1/20,000 of a kilometer. This amount of error could be introduced by neglecting to correct for grade a single tape length having a difference of elevation of its two ends of about 2.2 meters, or by each tape length in the kilometer distance having a difference of elevation of 0.5 meter. If any fractional tape lengths occur, the grade corrections on them may have to be computed. (See p. 125.)

When grade corrections need to be applied a single line of wye levels should be run over the traverse line, with readings at each tape end. No restrictions as to the length of sights are specified, and no attempt need be made to keep the back sights and fore sights equal. To avoid blunders, a special rod should be used, having foot graduations on one side and meter graduations on the other. Both sides of the rod should be read at each rod station, a comparison of the two values in different units serving as a check on the readings. Differences of elevation of tape ends need be taken out for only one unit, however, after a rough comparison of the two sets of readings has been made.

On moderate grades the rod need be held at only the approximate elevation of the tape end, since an error of a few centimeters would have little effect. On steep grades more care must be exercised in holding the rod at the same elevation as the tape end and in reading the rod. For instance, where the difference in elevation of the two tape ends is 3.0 meters an error of 4 centimeters in the difference of elevation would produce an error in the reduced length of 1 part in 20,000.

When movable tripods are used to support the tape the rod may usually be held on the ground at the center of the space occupied by the tripod, the height of the tripod not being taken into account after the first one is reached. While this may be done with safety for moderate grades, the requirements for greater accuracy on steep grades must be borne in mind.

The wye levels should be recorded in Form 634 and a complete abstract of wye levels must be made on Form 635, " Abstract of wye levels," the same leveling forms being used on traverse as on base measurement. (See figs. 81 and 82.)

ANGLE MEASUREMENTS

MAIN TRAVERSE STATIONS

Either a direction instrument read by micrometers or a repeating instrument read by verniers may be used in making the angle measurements. With either type of instrument the error of the measured deflection angle should seldom exceed three seconds of arc. The station to the rear, considering the direction of progress of the traverse line, should always be used as the initial. Observations may be made upon either targets or lights, so long as the accuracy specified in subsequent paragraphs is obtained.

OBSERVATIONS WITH MICROMETER DIRECTION THEODOLITE

The class of instrument will, of course, determine the number of positions to be observed with a direction theodolite on second-order traverse. With a theodolite having a circle from 8 to 10 inches in diameter four to six positions of the circle will be sufficient. With a smaller theodolite, having a 5 to 7 inch circle which can usually be read to the nearest two seconds on each micrometer, from six to eight positions will suffice. The same observing program should be followed as described for second-order triangulation with similar instruments. The limit of rejection for the larger theodolites should ordinarily be five seconds from the mean and for the smaller theodolites six seconds from the mean.

In many respects a micrometer direction theodolite is superior to a vernier repeating theodolite for second-order traverse. Not only can the required accuracy be more quickly obtained, but there is also a decided advantage in being able to read directions to objects located by the intersection method to the nearest one or two seconds rather than to the nearest 10 seconds. If the objects to be located by intersections are at a considerable distance and the locating triangles are weak, as frequently happens, the increased accuracy of the angles is a desirable feature.

The initial settings for 2, 4, 6, and 8 positions of the circle for 2-micrometer theodolites are given on page 34.

OBSERVATIONS WITH REPEATING THEODOLITE

When the measurement of a main angle of the traverse is made with a 7-inch 10-second repeating theodolite of the type frequently used in the Coast and Geodetic Survey, two sets of 6 D/R (see p. 33) on both the exterior and interior angles will usually give the required accuracy. The initial readings for the two sets should differ by about 90°. (See p. 33 for formula for settings.) If the two values for an angle differ by more than four seconds, a third set of 6 D/R should be taken. If the horizon fails to close by more than five seconds, additional readings should be taken. The same care should be taken in instrumental adjustment and manipulation as is specified for second-order triangulation. (See p. 75.)

CONDITIONS AFFECTING THE ACCURACY OF OBSERVATIONS

On traverse the distances between main-angle stations are comparatively short, so that the accurate centering of theodolites, targets, and lights is of much greater importance than on triangulation of the same order of accuracy. For the same reason errors due to phase must be more carefully guarded against. When it is remembered that at a distance of 1 mile one-third of an inch subtends one second of arc the possible magnitude of errors of phase and centering is easily comprehended.

The atmospheric conditions attendant upon traverse angle measurements are usually more conducive to horizontal refraction than those encountered during triangulation observations. The lines of sight are apt to be near the ground, and on beach traverse the conditions are especially unfavorable. Each sand spit and inlet under the line of sight, and each bluff, or dune near it, will exert an influence. Stations should be occasionally reoccupied, under very different atmospheric conditions than were present at the first occupation, to see what changes are found in the measured angles. The best angular values can be obtained either on cloudy days or at night, provided the night observations are not made within a half hour after sundown.

INTERSECTION STATIONS

It is very important that directions should be observed to prominent objects distant from the traverse line, such as church spires, large chimneys, cupolas, etc., especially when the positions of these objects may be determined by observations from two or more stations. These observations on intersection points may be made in the daytime with a direction theodolite and using one position of the circle or with a 7-inch repeating theodolite and taking one set of three direct and three reverse. An adjacent traverse station, preferably the one to the rear to avoid the use of two different initials at one station, should be used for the initial on these observations. Horizontal directions to reference marks should be observed with two positions of the circle. The reason for the second position is not to secure greater accuracy but to avoid the possibility of a blunder in reading the angle, for the future positive recovery of the station may depend upon the accuracy of both distance and angle measurements.

ECCENTRIC STATIONS

If the instrument or the object sighted upon is eccentric, the eccentric distance and direction must be carefully measured and recorded in the manner described on pages 76 and 90.

SUBSIDIARY STATIONS

Where the loops are small, not more than a mile in length (see p. 149), the stations designated A, B, C, etc., may be occupied with a theodolite mounted on a tripod. If a 10-second repeating theodolite is used, one set of 3 D/R* on both the interior and exterior angles will be sufficient. If a small micrometer direction theodolite is used, two positions of the circle will be sufficient. Where the loops are longer the accuracy of the angle measurements at the subsidiary stations should more nearly approach in accuracy that obtained at the main-angle stations.

All loops must be closed, because a concluded angle will conceal any errors in the angle measurements that may have been made. The angles at the main-angle stations required to close loops should be measured at the same time the main angles are measured. Because of the many short lines in a loop, the chief of party must rely upon his own judgment regarding the allowable angle closure of loops, but the closing error should seldom exceed 10 seconds per angle.

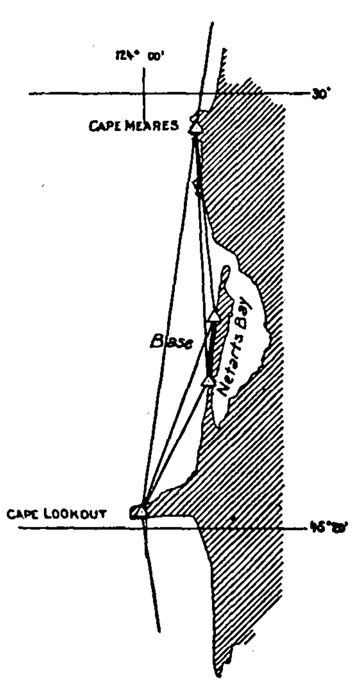


FIG. 90.—Combination of traverse and triangulation, Oregon coast

Bold headlands and wooded interior prevented the exclusive use of either triangulation or traverse. (See p. 149.)

At rail and offset stations the offset distances are so small in comparison to the distances between stations and the angles at the rail and offset stations are so nearly 90° that only the angles at the rail and offset stations themselves need be observed. Here the direction method may be used with either a repeating or direction theodolite and with two positions of the circle. If the same rail or offset station is used both in coming up to and leaving a traverse station, the angle on each side of the line to the main traverse station should be measured. Angles must always be measured in a clockwise direction. At rail or offset stations the pointing may be made upon the rail of the tangent or upon the line of stakes along which the measurement has been made, but the sights should be taken as far up the tangent or line of stakes as possible.

* See footnote on p. 88.

AZIMUTH OBSERVATIONS ON SECOND-ORDER TRAVERSE

There is a decided tendency for a traverse line to swerve in azimuth. Therefore, at intervals of 15 to 25 main-scheme stations along the traverse either a connection to first-order control must be provided or a second-order azimuth observed. (See p. 173.) Where one second-order traverse line joins onto another an azimuth should be observed at the junction station. When the horizontal-angle observations at the main-line stations are made at night or during cloudy weather it will usually be possible to space the azimuth stations about 25 main-line stations apart, but when the angle observations are made in sunshine the lower limit stated should be used. The azimuth correction per station between azimuth stations should not exceed three seconds. This extreme limit will seldom be reached except under the most unfavorable terrain conditions. Ordinarily the azimuth correction per station can be kept down to 1.5″ per station.

THIRD-ORDER TRAVERSE

The same methods are used on third-order traverse as on second order, subject to the larger permissible closure in position. Where no distinctions between second and third order traverse are noted in the following paragraphs the same methods and limits of error should be used on third order as have been specified for second-order traverse.

The accuracy to be attained on third-order traverse is a position check on an adjusted control point of a higher order than third with an error not exceeding 1 part in 5,000 of the length of the traverse line, or a circuit closure of similar accuracy when a third-order traverse is closed upon itself. The same precautions should be observed in judging a circuit closure as were mentioned on page 147. The error of closure specified will usually mean that after adjustment the length of no line of the traverse will be in error more than 1 part in 5,000.

No single systematic error, such as that due to lack of proper alignment, unapplied grade corrections, etc., should be permitted which would cause the "reduced length" (see fig. 83) of a line between angle stations to be in error more than 1 part in 10,000.

The marks for main-scheme third-order stations should be of the same character and established under the same conditions as were specified for second-order traverse.

SPECIFICATIONS, THIRD-ORDER TRAVERSE

Support of tape.—The tape may be supported on the ground whenever the effect of the inequalities of the surface does not introduce an error of more than 1 part in 10,000 for the greater number of the tape lengths. If doubt exists on this point, a test measurement on some characteristic sections of the line should be made as described on page 154. If stakes or tripods are used as tape supports, levels must be run wherever necessary to insure that the error due to unapplied grade corrections does not exceed the specified amount. If grades are not heavy, clinometer readings, if carefully made, may be used in place of levels to determine the grade corrections.

Check measurement of distances.—A check measurement with a 300-foot tape should be made in the same manner as described for second-order traverse. The permissible differences between the distances obtained with the 50-meter and with the 300-foot tape are 1 part in 500 in sections over 500 meters in length and 1 part in 300 for sections of lesser length. If the discrepancy exceeds that specified, a second and more careful measurement should be made with the 300-foot tape. If the second measurement discloses no material error in the first, the section must be measured again with the 50-meter invar tape.

Angle measurements.—A tripod theodolite, either of the micrometer direction or the vernier repeating type, will ordinarily be used and the observations made upon signal poles unless unfavorable observing conditions make the use of lights necessary. The error of the measured deflection angle should seldom exceed six seconds of arc. With a micrometer theodolite four positions will be sufficient, and with a 10-second repeating theodolite one set of 6 direct and 6 reverse (6 D/R)on both the deflection angle and its explement will be adequate. With a direction theodolite the limit of rejection for any one position may usually be taken as six seconds from the mean. With a repeating theodolite the horizon should close within six seconds.

The angle measurements on third-order traverse are subject to much greater errors from refraction than from observational errors, provided the observations are carefully made. Occasional reoccupations of a station should be made to test the changes caused by refraction, choosing atmospheric (especially wind) conditions which are very different from those present on the first occupation.

On third-order traverse extensive use of loops need not be made, and the tedious projection computations will thus be avoided. If care is taken in centering the theodolite and targets and a type of target is used which is adapted to the length of line and not conducive to phase error, short lines may be used in the main scheme without reducing the accuracy below the allowable limit.

Azimuth observations on third-order traverse.—At intervals of 20 to 35 main-angle stations along the traverse either a connection to first or second order control must be provided or else a third-order azimuth must be observed. (See p. 183.) Also, where one line of third-order traverse crosses another an azimuth should be observed at the junction station. The azimuth correction per station should never exceed five seconds and should seldom exceed three seconds. Whenever the lower limit is exceeded the observer should make an investigation to see if at some station a wrong object has not been observed upon, and he should check suspected observations.

FIELD RECORDS AND COMPUTATIONS, SECOND AND THIRD ORDER TRAVERSE

The same records and computations are made for both classes of traverse, and to a large extent they are the same as are used for base measurement, the only differences arising from the necessity of projecting the measured lengths upon the line between the main-scheme angle stations when the measurement is not made directly between them.

A complete computation of the traverse should be made in the field, including the computation of geographic positions and their tabulation on the "List of geographic positions." (See figs. 63 and 65.) Since there is no check on the accuracy of the computation of geographic positions on traverse as there is on triangulation, the computation should be repeated independently and preferably by a different person. The method of independent computation is much more apt to disclose errors than the method of checking a computation already made.

Record of tape measurements.—This record is made on Form 590 in the same manner as shown in Figure 80, except that only one temperature is recorded for each tape length and that at the bottom of each page is drawn a rough sketch showing the relation of the measured line to the beach, road, or railroad along which the traverse is being run. The record must also contain the measurement of all offset distances in both meters and feet, which must be compared at the first opportunity by converting one to the same units as the other. The mean temperature for each section must be computed and the set-ups and setbacks summed up ready for transfer to Form 589.

Level record.—If levels have been run over all or part of the traverse line, they should be recorded on Form 634 and abstracted on Form 635, as shown in Figures 81 and 82 in the section on base measurement. The sum of the inclination corrections should be taken, ready for transfer to Form 589.

Computation of traverse line.—The computation of the measured lengths of the traverse is made on Form 589 in exactly the same manner as shown in Figure 83. The "reduced length" of each section between angle stations, whether main angle stations or loop stations, is obtained separately, ready to be used in the projection computations.

TRAVERSE SKETCHES ON COMPUTATION SHEETS

A sketch should be made of each section of the traverse line, and on the sketch should be inserted in their appropriate places all the data necessary for projecting the measured lengths onto the lines between the main angle stations. The kind of sketches required is shown in Figures 84 and 92, where no projection is required, and in Figures 93 to 95, which illustrate the different conditions of projection. In Figures 93 to 95 the data above the long horizontal line are those derived from the tape and angle measurements, the projection computations being below the line.

The sketches should be made on horizontally ruled computing paper, and only one sketch should be placed on a page. The following data should be shown on each sketch:

1. Direction of progress (shown by arrow).
2. Relation of traverse line to beach, road, or railroad, taken from corresponding sketch on Form 590, "Traverse measurements."
3. Distances derived from invar-tape measurements from Form 589, "Computation of traverse line," corrected for temperature, standardization, inclination, and sea level, as given in the column headed "Reduced lengths."

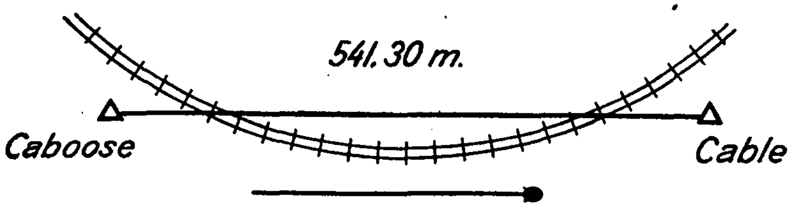


FIG. 91.—Traverse sketch, direct measurement between stations

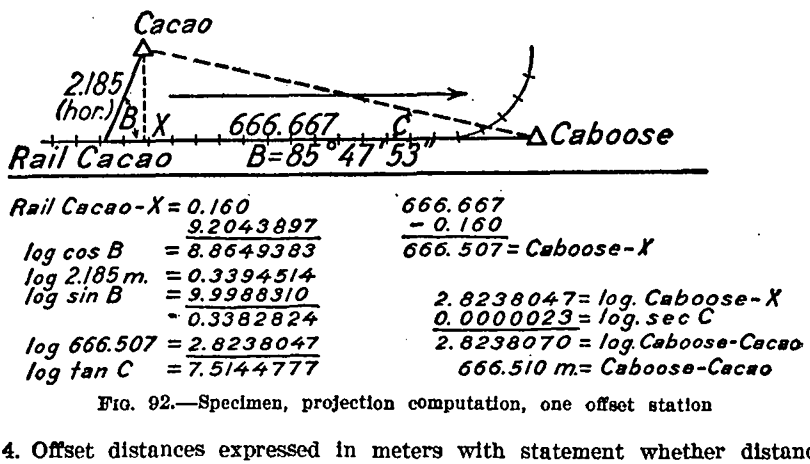


FIG. 92.—Specimen, projection computation, one offset station

4. Offset distances expressed in meters with statement whether distances were measured horizontally or inclined.
5. Angles at loop stations and at offset and rail stations.
6. Summation of loop angles with closure.

The importance of these sketches can not be overemphasized. They guide the officer in charge of field work in deciding whether all the data have been obtained and whether the necessary accuracy has been secured. These sketches and the necessary computations preceding them should be kept as nearly as possible up to date.

In Figures 92 to 95, below the horizontal line, is shown the computation necessary to make each type of projection. If the computation is made neatly and methodically, the checking, which should be done in the field before the computation of geographic positions is made, can be done more easily, quickly, and accurately.

CLOSURE AND PROJECTION OF LOOPS

As already explained (p. 149), it is often desirable to carry the azimuth through long lines extending past several intermediate sta-

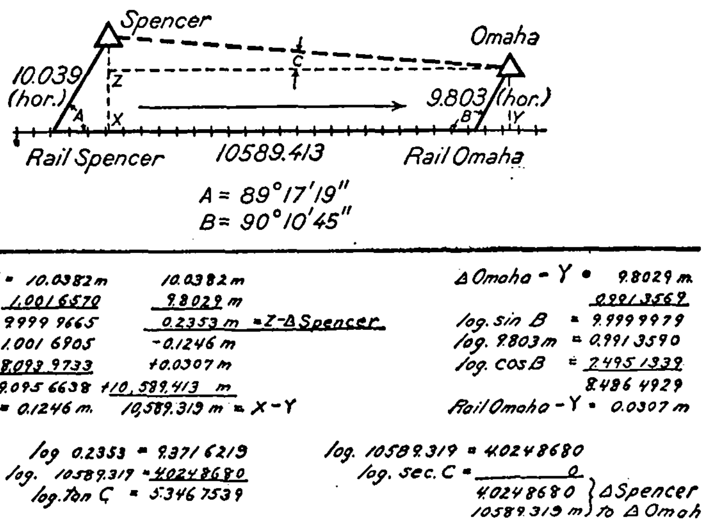


FIG. 93.—Specimen, projection computation, stations on same side of track

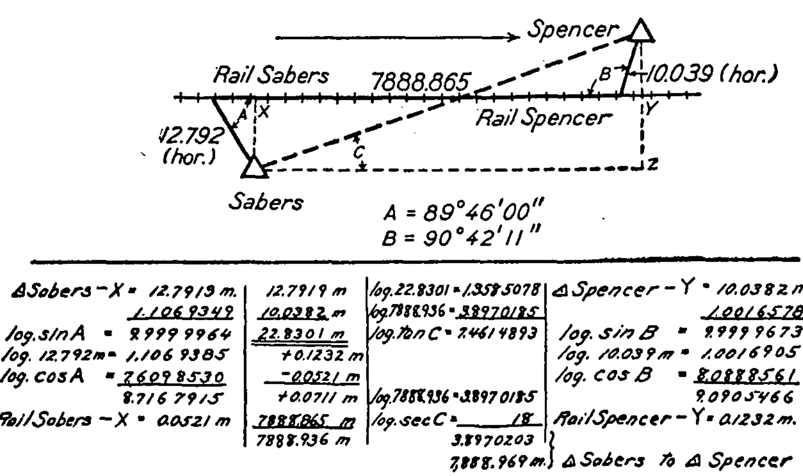
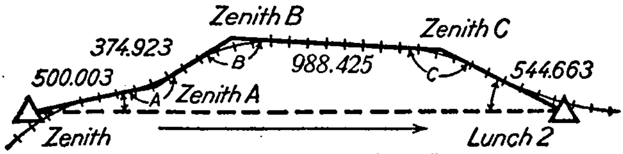


FIG. 94.—Specimen, projection computation, stations on opposite sides of track

tions and forming loops with the intermediate stations. The lengths of these long lines must be determined by projecting the measured intermediate lines upon them.

The first step in the computation is to make a diagram of the loop, as shown in Figure 95. The error of closure of the angles in the loop should be determined and distributed among the angles before making the projection. The error of closure is the difference between the sum of the interior angles in the loop and the proper number of right angles (number of sides minus 2 multiplied by 2). Distribute the error equally among the angles unless there is some good reason, such as very unequal lengths of lines, for making an arbitrary weighted distribution of the closing error. Frequently a large closing error occurs in a loop which has one or more extremely short



Zenith = 1°16′32.8″
Zenith A = 187°57′58.4″
Zenith B = 173°43′55.2″
Zenith C = 164°01′21.2″
Lunch 2 = 13°00′18.7″

Zenith–Lunch 2	0°00′00″.0		00″.0		1.046 4662	−11.129	
(1)	358 43 27.2	+1.3	28.5		8.347 4936		
Zenith–A	358 43 27.2		28.5	Zenith–A	2.698 9726		
(2)	172 02 01.6	+1.2	02.8		9.999 8924		
	180				2.698 8650		499.879
A–B	350 45 28.8		31.3		1.779 6690	−60.210	
(3)	186 16 04.8	+1.3	06.1		9.205 7260		
	180			A–B	2.573 9420		
B–C	357 01 33.6		37.4		9.994 3263		
(4)	195 58 38.8	+1.2	40.0		2.568 2683		370.057
	180				1.709 8132	−51.264	
C–Lunch 2	13 00 12.4		17.4		8.714 8693		
(5)	346 59 41.3	+1.3	42.6	B–C	2.994 9439		
	180				9.999 4151		
	179 59 53.7		60.0		2.994 3590		987.095
					2.088 3745	+122.567	
					9.352 2467		
				C–Lunch 2	2.736 1278		
					9.988 7155		
					2.724 8433		530.693
						−0.036 m	2387.724 m
							3.377 9841

FIG. 95.—Specimen, loop closure and projection computation

lines in it, where a slight eccentricity would produce a large angular error, and in such a case an unequal distribution of closing error is sometimes warranted. Whatever method is used, it should be made evident by the arrangement of the computation somewhat as shown in Figure 95. In that figure the azimuth method of distributing the error is used. To use this method, start at one end of the loop, assume the azimuth of the long line to be 0° 00′ 00″.0, and then compute the corresponding azimuth of each of the other lines with respect to the long line. The azimuth of each line is obtained by adding to the azimuth of the preceding line 180° plus the direction of the second line measured in a clockwise direction from the first. The amount that the back azimuth of the long line as thus computed through the angles differs from 180° 00′ 00.″0 will be the closing error of the angles of the loop. When this error is distributed among the angles the corrected azimuths can be obtained by merely correcting the seconds of the preliminary azimuths, the degrees and usually the minutes remaining unchanged.

The length of each measured line in the loop is next multiplied by the sine and cosine of the azimuth of that line as determined in the previous computation. The product of the length and the cosine gives the projected length of the line on the long line, and the sum of these projected lengths for all the measured lines gives the length of the long line. The product of the length and the sine gives the departure of the line or the differences between the perpendicular distances from the long line of the two ends of the measured line, the sign depending upon the azimuth. The sum of the plus perpendiculars for all the measured lines should equal the sum of the minus perpendiculars, and this gives a valuable check on the accuracy of the field measurements and the office computations. (See fig. 95.) A large discrepancy between the sums of the plus and minus perpendiculars indicates that a mistake has been made either in the computations or in the measurement of the angles or distances.

It is possible, of course, to make a least-squares adjustment of each loop and thus make the sums of the perpendiculars exactly equal as well as eliminate the closing error of the angles. This has been tried on a number of different loops and has been found to give a length for the long line which differs so slightly from the length determined as described above as not to justify the additional work, except where one of the lines is used to make a connection with triangulation.

COMPUTATION OF GEOGRAPHIC POSITIONS

After the lengths of the lines connecting the main traverse stations have been determined the geographic positions of the stations and the azimuths of the lines between them should be computed. This computation may be made either on Form 27, Figure 63, or on Form 596, which is a one-sheet form similar to Form 27 and adapted especially for traverse. The computation is described fully in Special Publication No. 8, Formulæ and Tables for the Computation of Geodetic Positions, and also in *Special Publication No.* 138, *Manual* of Triangulation Computation and Adjustment.

The geographic positions are computed by starting with the fixed position and azimuth at one end of the traverse and computing the positions and azimuths in order through the traverse until a check is obtained on a fixed position and azimuth at the other end. At each station where an astronomic azimuth has been observed the geodetic azimuth obtained by computing through the traverse is compared with the observed astronomic azimuth, and if the discrepancy is too large (see pp. 149, 164, and 165), the computations should be examined in an attempt to locate the error. If the error is not found in the computations, it will be necessary to reobserve angles at main-scheme stations until the discrepancy is reduced to allowable limits. The error may be due to a blunder at a single station, such as that due to observing upon a wrong object, or may be the result of the accumulation of errors at several stations due to unfavorable observing conditions.

The geodetic azimuths and positions derived from the fixed starting point is continued throughout the line without correction at the intervening astronomic azimuth stations. Since there is frequently a tendency for the azimuth discrepancy to accumulate in one direction along a traverse line, the discrepancy in position and azimuth at the closing point of the traverse is often rather large.

DISTRIBUTION OF DISCREPANCIES

No least-squares adjustments of traverse need be made in the field. If the geographic positions of the stations are needed immediately to control topographic and hydrographic surveys, as will frequently be the case, the discrepancies may be distributed in the following manner:

Unless the azimuth discrepancies are very large at one or more of the astronomic azimuth stations near the beginning of the traverse line, no azimuth discrepancy need be distributed. If, in the exceptional case mentioned in the preceding sentence, the azimuth discrepancy will swing the closing end of the traverse line to such an extent that it will not be sufficiently corrected for field use by the distribution of the discrepancies in latitude and longitude, then the azimuth discrepancy at the forward astronomic azimuth station of the section should be distributed equally among the main-scheme angle stations back to the preceding fixed azimuth. The positions of the stations in that section will then have to be recomputed, a comparatively easy and rapid process, since the factors remain unchanged.

The discrepancies in latitude and longitude are distributed separately back through the traverse, roughly in proportion to the lengths of the various main-scheme lines. If the lines are not so very unequal in length, if the traverse is fairly short, or if the discrepancy in position is small, the discrepancies in latitude and longitude may be distributed without weighting according to length of line. Ordinarily this will give geographic positions closely enough for field work, until the least-squares adjustment of the results can be made in the office.

The methods of making the least-squares adjustments are described in Special Publication No. 137, Manual of First-Order Traverse, and can be consulted there if needed.

CHAPTER 5.—ASTRONOMIC AZIMUTHS

CLASSIFICATION OF AZIMUTHS

The classification of azimuths given below differs somewhat from that given in Special Publication No. 120, Manual of First-Order Triangulation, but the classification below seems more logical than the one previously used and has been adopted by the United States Coast and Geodetic Survey.

A first-order astronomic azimuth is one observed with such methods as to give a probable error for the result of 0.″5 or better, obtained from observations on a single night. When it becomes necessary to secure greater accuracy the same methods are used but the number of observations is increased and the observations are usually distributed over more than one night.

To prevent the accumulation of azimuthal errors in first-order triangulation and traverse certain azimuth control points, called Laplace stations, are included in the adjustment. A Laplace station is a station of the triangulation or traverse at which both the astronomic azimuth and the astronomic longitude have been determined. A Laplace azimuth is an astronomic azimuth corrected for the deflection of the vertical in the manner described in Special Publication No. 138, Manual of Triangulation Computation and Adjustment, pages 204 to 207. Azimuths at Laplace stations are determined with a probable error of 0.″30 or less from observations made with first-order methods, usually on two separate nights.

A second-order azimuth is one observed with such methods as to give a probable error for the result of 2.″0 or less. The observed value is not corrected for the effect of the deflection of the vertical before being used in the adjustment of the triangulation or traverse, and therefore great accuracy has no advantage. In regions with no high mountains and no great differences in the density of the near-by subsurface geological structures the error in the observed azimuth due to the uncorrected effects of the deflections of the vertical will not usually exceed one or two seconds of arc, but in other regions it may amount to several times that amount. In southeast Alaska the prime vertical deflections at six sea-level stations as determined by longitude observations ranged from −7.″94 to +12.″33, with an average without regard to sign of 8.″2. This average deflection would affect the observed azimuth by more than 12 seconds of arc.

A third-order azimuth (see p. 183) is one observed with such methods as will give a probable error for the result of 5.″0 or less.

METHOD OF OBSERVING A SECOND-ORDER AZIMUTH

A detailed description of the method of making observations for azimuth and of computing the results of the observations is given

DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
Form 253

DOUBLE ZENITH DISTANCES. (On stars, for time)

Station: Finigan *State:* Mont. *Instrument:* Wanschaff No. 2 *Date:* Aug. 12, 1923

Observer: W.M. *County:* Cascade Sid. Watch No. 145, Barometer No. 300

Object Observed.	Time.	Level. O.	Level. E.	Circle right/left	Circle Reading °	Circle Reading ′	Verniers. A. ″	B. ″	C. ″	D. ″	Mean. ″	Zenith Distance. ° ′ ″	Remarks.
Alpha Aquila	9 18 57.0			R	42	28	50	60			55.0		Temp. 16.0 C
Altair	9 21 15.5			L	129	04	10	15			12.5	43 17 38.8	
(East)	9 20 06.2												Bar. 23.42
	9 22 23.5			L	128	57	40	40			40.0		Angle Δ High-wood to star = 24°
	9 24 21.5			R	43	00	00	05			02.5	42 58 48.8	
	9 23 22.5												
	9 27 44.0			R	43	18	35	30			32.5		
	9 29 57.0			L	128	16	15	30			22.5	42 28 55.0	
	9 28 50.5												
Alpha Boötis	9 35 21.5			L	144	23	10	05			07.5		Angle Δ Nelson to star = 92°
Arcturus	9 37 41.0			R	26	59	60	55			57.5	58 41 35.0	
(West)	9 36 31.2												
	9 38 55.5			R	26	47	30	30			30.0		
	9 40 45.5			L	145	17	45	60			52.5	59 15 11.2	
	9 39 50.5												
	9 41 54.0			L	145	29	30	20			25.0		
	9 44 24.5			R	25	52	05	10			07.5	59 48 38.8	
	9 43 09.2												

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FIG. 96.—Specimen, observations on stars for time

in Special Publication No. 14, Determination of Time, Longitude, Latitude, and Azimuth, with tables and specimen computations. It is assumed that this publication is available to the observer.

In the Northern Hemisphere observations upon Polaris with a theodolite provide the most convenient method for determining an azimuth. In the higher latitudes a special right-angled eyepiece must be provided for the theodolite to enable the observer to point on the north star. Either a direction or a repeating instrument may be used. A sufficient number of observations should be made to give the specified probable error.

AZIMUTH OBSERVATIONS WITH DIRECTION THEODOLITE

If a direction instrument is used, twice the number of observations on Polaris should be made as would ordinarily be taken on second-order triangulation with that class of instrument. The form of record is shown in Figure 97. The station used as the azimuth mark is observed upon next before Polaris, in order to reduce the time elapsed between the pointings upon Polaris and the mark. For the same reason the horizon is not closed when azimuth observations are being made. The observing routine for a direction instrument described in the next paragraph will reduce the time required for the operation and also minimize the chances for mistakes and omissions. The observing routine described for a direction instrument can easily be modified to apply to a repeating instrument.

After having completed the pointing and readings upon the mark point upon Polaris, bringing the star within a half minute or so of the vertical wires in the middle of the field, clamp the horizontal motion; place the stride level in position on the standards, at the same time calling " stand by " to the recorder; perfect the pointing, calling " tip " sharply at the moment of bisection; then read and make mental note of the stride-level readings, but do not call out the readings until the recorder, who is marking down the time, calls " ready." After noting the stride-level readings reverse the stride level and move to the first micrometer, calling out the stride-level readings, west end always first, then the micrometer readings in order, and lastly the readings of the reversed level, again west end first. Remove the level, turning it end for end as it is removed, and place it on the supports[1] provided for the purpose on the south side of the stand or tripod, in position to be placed later on the instrument with the same end to the west as on the first reading. This is not necessary, but will often enable the computer to detect mistakes in the recording.

Next loosen the upper horizontal clamp, turn the instrument 180°, reverse the telescope and point again upon Polaris, going through the

[1] If a wooden stand or tower has been built on which to mount the theodolite, two nails should be driven into the structure on the south side of the tripod head at the proper distance apart to serve as a rack for the stride level. If a steel tower or portable folding tripod is being used as an instrument support, a piece of stiff wire may be bent into the proper shape and fastened to the tripod for the same purpose.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 251a

Horizontal Directions

Station: Finigan Observer: W.M. Instrument: Wanschaff No. 2 Date: Aug. 12, 1923

11—738 11—738

Position	Objects Observed	Time h. m.	Tel. D or R	Mic.	°	′	Backw'd ″	Forward ″	Mean	Mean D and R	Direction ″	Remarks W.		Remarks E.
6	Goosebill		R	A	56	02	04	03						
				B			07	08	05.5					
			D	A	236	01	36	39						
				B			37	37	37.2	51.4				
	Ema		R	A	39	13	55	58						
				B			58	59	57.5					
			D	A	219	13	26	28						
				B			27	25	26.5	42.0	50.6			
	Teton		R	A	24	58	52	50						
				B			48	49	49.8					
			D	A	204	58	16	17						
				B			20	19	18.0	33.9	42.5	W.		E.
	Polaris	10 18-31.0	R	A	17	00	27	28				11.0		23.6
				B			24	24	25.8			20.5		07.9
												09.5	-06.2	15.7
		10 20-28.5	D	A	196	59	35	32				22.0		09.4
		10-19-29.8		B			35	36	34.5	00.2		09.4		22.0
												12.6	0.0	12.6
													-03.1	

FIG. 97.—Specimen, observations on Polaris for azimuth, direction theodolite

same procedure as described for the first pointing. This routine permits the stride-level bubble to come fully to rest without delay to the observing.

If the value of one division of the stride level is not known, it should be determined in the field. (See p. 80.)

AZIMUTH OBSERVATIONS WITH REPEATING VERNIER THEODOLITE

If a repeating theodolite reading by verniers to 10 seconds is used, from three to four sets will be sufficient, each set consisting of 6 D/R* between Polaris and the azimuth mark. The record of observations should include the time as marked for each pointing upon Polaris, the angle readings for the beginning and ending of each half set, and the stride-level readings, direct and reversed, for the beginning and ending of each half set. If the value of one division of the stride level is not known, it should be determined in the field. (See p. 80.) A sample form of record for a repeating instrument is shown in Figure 98. It is not necessary to record the angle readings corresponding to the times of the pointings between the first and sixth repetitions, for a curvature correction can be derived from the mean of the recorded times and the mean angle corrected accordingly. (See Special Publication No. 14, fifth edition, p. 144.)

AZIMUTH MARK

Ordinarily it will be found most convenient and desirable to use as an azimuth mark a light mounted at some adjacent triangulation station of the main scheme. If it is necessary to use some other point as an azimuth mark, it should preferably be at least a mile away from the azimuth station, and the angle between it and at least one main-scheme station should be determined by observations of the same accuracy as that used in the main scheme. Precautions must also be taken to hold the azimuth marks fixed in position and to observe upon the same point of the mark in connecting it to the main scheme as was used in the azimuth observations. A focusing flash light showing through a small slit or hole in a board will often make a satisfactory mark for azimuth work.

RECORDER'S DUTIES

The principal precautions to take in observing upon Polaris are to center the instrument and mark accurately, to see that the theodolite is firmly supported, and then to train the recorder to note accurately the chronometer time corresponding to the call of "tip,"

* See footnote on p. 33.

giving him sufficient time to make record of it before he is confused by other readings. The recorder should be trained to carry mentally a staccato count of the seconds of the chronometer, as, "twenty′ half, twenty-one′ half, twenty-two′ half"—the accented word or syllable and the word "half" synchronizing with the half-second

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 280

Azimuth Observations 27

HORIZONTAL ANGLES

STATION Cut STATE: N.C. ISLAND OR COUNTY: New Hanover DATE: June 29, 1918

OBSERVER. C.L.G. (Δ Station Isle used as mark) INSTRUMENT: Theo. No. 244 Sid. Watch No. 145

Objects observed			Time h. m.	Tel. D or R	Rep's	Angle ° ′	A ″	B ″	Mean of verniers	Angle Mean D and R ° ′ ″	Remarks
Mark - Polaris			A.M. 12:57	D	0	0 00	00	55	57.5		No eccentricity of
W. 26.4		E. 08.0									theodolite or mark.
09.6		28.0									Sidereal time h. m. s.
16.8	-3.2	20.0		D	1	355 54					6 14 55.5
				D	2						16 37.5
				D	3						19 33.5
				D	4						21 37.5
				D	5						23 24.0
W. 26.4		E. 08.0	1:07	D	6	335 25	40	40	40.0	355 54 17.1	25 13.0
11.0		29.5									
15.4	-6.1	21.5									
Polaris - Mark			1:09	R	0	335 25	40	40	40.0		
W. 24.5		E. 06.0									
11.0		29.5									h. m. s.
13.5	-10.0	23.5		R	1						6 26 28.0
				R	2						28 07.5
				R	3						29 51.0
				R	4						32 38.5
				R	5						34 55.0
W. 26.1		E. 07.8		R	6	359 51	40	35	37.5	4 04 19.6	36 43.5
09.5		28.0									
16.6	-3.6	20.2									

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FIG. 98.—Specimen, observation on Polaris for azimuth, repetition method

beat. It is then easy for him to note within a half second the time of the observer's "tip."

If a sidereal watch is used instead of a chronometer, its fifth-second beat can not readily be followed, but with practice the mental staccato half-second count can be regulated so that the count of the whole second can be made to coincide with the passage of the second hand over each successive second division of the dial as registered by the eye and the time of the "tip" noted with reference to the count. This is more accurate than noting the time by the eye alone.

The record book in which are recorded the observations upon Polaris should always contain a record of the eccentricity of both the light and the theodolite. If there is no eccentricity of either, it should be so stated. Often there is uncertainty as to whether the eccentricity recorded for the regular angle observations should be applied to the azimuth observations also, if the azimuth is measured separately from the other directions.

WHEN TO OBSERVE POLARIS

The observations upon Polaris are preferably made near the time of elongation, when an error in the chronometer correction has a relatively small effect. Observations upon Polaris may be made at any hour angle, however, if the chronometer correction is known within one or two seconds. An error of two seconds in the time would cause an error of about 0.″6 in the computed azimuth of Polaris near culmination in latitude 30° and 1.″2 in latitude 60°.

The hour angle of Polaris or its position with reference to the Meridian may be roughly determined by the fact that the line from ζ Ursæ Majoris (Mizar) to δ Cassiopeiæ passes approximately through both Polaris and the pole. Therefore when Polaris is directly above ζ Ursæ Majoris and below δ Cassiopeiæ, or vice versa, it is near the meridian, or about six hours from elongation.

The times of upper culmination and of elongation can be readily obtained from Table VII in the American Ephemeris and Nautical Almanac headed "Apparent place, time of upper culmination, and time interval between upper culmination and elongation east or west, of Polaris." As an example, suppose it is desired to find the local time of eastern or western elongation (depending on which one occurs during the hours of darkness) on the evening of November 20 or the early morning of November 21, 1928, at a station whose approximate latitude and longitude are respectively 36° 42′ and 92° 17′. In the Ephemeris table mentioned above will be found the following data:

Civil time	Upper culmination, meridian of Greenwich					Latitude	Mean time interval, elongation minus upper culmination
	Apparent right ascension	Apparent declination	Civil time	Variation per day	Variation per hour		
	h. m. s.	° ′ ″	h. m. s.	m. s.	W. E. s.	°	W. E. h. m.
November, 14.9	1 36 53	+88 55 28.0	22 01 53	−3 56.3	−9.85+	36	+5 55.9−
November, 24.9	1 36 48	+88 55 31.5	21 22 29	−3 56.5	−9.85+	38	+5 55.7−

From these data the computation is made as follows:

		h.	m.	s.
Greenwich civil time, upper culmination, November, 14.9	=	22	01	53
Variation for 6 days (6×—3 m. 56.4 s.)	=		—23	38
Greenwich civil time, upper culmination, November, 20.9	=	21	38	15
Correction for longitude (92° 17′=6.15 h.)	=		—1	01
Local civil time, upper culmination	=	21	37	14
	=	21	37.2	
Mean time interval to western elongation	=	+5	55.8	
Local civil time, western elongation (Nov. 21)		3	33.0	

It should be noted that the time of western elongation thus obtained is local civil time and not standard time. In order to obtain the corresponding standard time a correction must be applied to take account of the difference in time between the standard meridian of 90° and the meridian of 92° 17′. This correction is obtained by simply converting the difference in longitude, 2° 17′, into time and amounts in this example to 9.1 minutes. The standard time of western elongation on the morning of November 21 at the given station is therefore,

$$3^{\mathrm{h.}}\ 33.0^{\mathrm{m.}}+9.1^{\mathrm{m.}}=3^{\mathrm{h.}}\ 42.1^{\mathrm{m.}}$$

CHRONOMETER CORRECTION

Different methods may be used to determine the chronometer correction. The chronometer should have a fairly uniform rate, and its correction should be determined both before and after the observations upon Polaris, in order to determine the rate and eliminate the possibility of large errors. The most convenient method for determining the correction is by comparison of the chronometer with radio time signals. If no receiving set is available, or if the chronometer must be transported a considerable distance to the station after a comparison is made, the correction should be obtained from star observations.

If a standard chronometer aboard ship is accessible for comparison with the hack chronometer or watch both before and after the azimuth observations, no observations for time need be made, provided the standard chronometer has a satisfactory rate and has been compared with radio time signals within 24 hours of the time of the azimuth observations.

Whether a hack chronometer or a good sidereal watch is used, it should be protected as much as possible from changes in temperature. If packed in a box with cotton, with only its face exposed and shielded from the wind, the changes in rate due to temperature changes will be materially lessened. An ordinary watch left unprotected will often change several seconds in a short period of time.

The star observations for time may be made by observing the transit of stars across the middle wire of a theodolite mounted so that, with the horizontal circle clamped, its telescope swings in the meridian. The method to be followed in placing the telescope in the meridian is described on page 16 of Special Publication No. 14 (fifth edition), Determination of Time, Longitude, Latitude, and Azimuth. A star catalogue must, of course, be available when this method is used.

Probably the best method for determining the chronometer correction with the instruments usually at hand is to observe the altitudes of east and west stars near the prime vertical. No star catalogue or observing list is needed, and any star near the prime vertical of greater altitude than 25° may be used, even though it shows for only a few minutes through a broken field of clouds. A star chart can be used to identify the stars observed upon. To make identification certain, the horizontal angle should be measured from Polaris or from some adjacent triangulation station to each time star observed upon. For determining the chronometer correction four observations should be made upon an east star and four upon a west star before the azimuth observations, to constitute what is called a time set, the mean of all unrejected values being used as the chronometer correction. A time set should be observed before the azimuth observations are started and another one immediately after they have been completed, the difference in the chronometer corrections so obtained being applied as a rate distributed throughout the intervening time interval. If the probable error of a time set obtained by this method is greater than two seconds, Polaris should be observed near elongation. Any instrument having a vertical circle which reads to 30 seconds or less may be used to make the time observations. Thermometric and barometric readings should be taken at the beginning and at the end of each time set.

Since for practically all theodolites used on second-order work the level bubble on the vertical circle is attached to the vernier frame, the routine of observing with that arrangement only will be described. With circle right, bring the star near the intersection of the middle vertical and horizontal wires, with the horizontal wire just ahead of the star in the direction in which the star is moving, and allow the star to make contact with the wire, thus eliminating the error due to the thrust upon the instrument when the tangent screw is operated to make the contact. At the time of contact call "tip" to the recorder, who notes the time. Bring the bubble to the center of the tube with the vernier screw and read the verniers. Then loosen the vernier clamp, reverse the telescope, and with the circle left bring the star into approximate position with the vernier screw. Call "tip" to the recorder as the star makes contact with the horizontal wire, then bring the bubble to the center and read the verniers. This constitutes one measure of the zenith distance. The next measure should be made beginning with circle left and ending with circle right.

ERRORS IN TIME OBSERVATIONS

It is more difficult to secure good time sets than to secure good observations upon Polaris. For that reason the observer should always compute his time sets. The computation of both time and azimuth is fully explained in Special Publication No. 14, and a sample of the time record is shown in Figure 96. Some of the more common sources of error in time observations are mentioned below and the remedy for each indicated.

1. **Incorrect noting of time.**—An inexperienced recorder should be trained in the way explained in the paragraphs relating to the observations on Polaris. Do not confuse him by calling out the readings of the verniers or levels before he has finished recording the time.

2. **Incorrect circle readings.**—The difficulty of securing an even illumination of the verniers by flashlight increases the chances of incorrect readings. Check carefully the minutes of each vernier reading, for the mistakes are more apt to occur in the minutes than in either the degrees or seconds.

3. **Wrong star.**—The effect of this error can be nullified by measuring roughly the horizontal direction to each time star from either Polaris or some station of the triangulation, noting the time of the measurement. This should invariably be done, and the angle and time recorded for each time star.

4. **Refraction errors.**—The zenith distances are corrected for refraction, the correction angles being taken from tables given in Special Publication No. 14 (fifth edition), Determination of Time, Longitude, Latitude, and Azimuth. These tabulated values must themselves be corrected for temperature and barometric pressure, so thermometer and barometer readings must be recorded for each time set. The differential effect of an incorrect index of refraction being used for the tabulated values for the correction will be lessened by having the east and west stars as nearly as possible of the same zenith distance, though usually the error from this source is not serious if no star is used of an altitude less than 30°.

5. **Poor selection of stars.**—More serious errors will be introduced by selecting stars too far from the prime vertical. In the early evening there is always the temptation to use the first stars

visible in order to begin the night's work. A delay of a quarter of an hour is usually not serious and will often result in securing time stars which will give a much more accurate chronometer correction.

6. Parallax.—The effect of parallax in the telescope is very apt to be evident in the computed times, for the effect of this error is almost invariably opposite in sign for east and west stars. It is very essential that both east and west stars be observed upon for each time set, for the mean will be measurably free from this error unless there is a great difference in their zenith distances.

THIRD-ORDER AZIMUTH

A third-order azimuth is an astronomic azimuth observed with such instruments and methods as will give a probable error for the result of not to exceed five seconds.

Practically the same methods are employed in observing a third-order azimuth as those described for second-order, but the larger permissible limit of error allows greater latitude in the choice of instrument and also requires a smaller number of observations. In the Northern Hemisphere a third-order azimuth should preferably be observed upon Polaris because of the greater convenience in both the observing and in the computations. Observations upon Polaris may be made at any hour angle provided the chronometer (or watch) correction is known within four or five seconds. (See p. 179.)

If the watch correction is determined by observations upon the stars, at least two, and preferably three, observations should be made upon an east star and the same number upon a west star before the observations upon Polaris are started and a similar set after the Polaris observations are completed. Any instrument having a vertical circle reading to one minute or less may be used for the time observations.

CHAPTER 6.—CONSTANTS, FORMULAS, AND TABLES

CONSTANTS AND FORMULAS

Dimensions of the earth according to Clarke's spheroid of reference (1866):

Equatorial radius, a, $=6378206.4$ meters

log. $a=6.80469857$

Polar semi-axis, b, $=6356583.8$ meters

log. $b=6.80322378$

Eccentricity, $e, =\sqrt{\frac{a^2-b^2}{a^2}}$

$e^2=0.006768658$

log. $e^2=7.83050257-10$

Base of Naperian logarithms, ϵ, $=2.71828183$

log. $\epsilon=0.43429448$

Modulus of common logarithms, M, $=0.43429448$

log. $M=9.63778431-10$

$\pi=3.14159265$

log. $\pi=0.49714987$

log. sin $1''=4.68557487-10$

log. tan $1''=4.68557487-10$

1 kilometer = 0.621370 statute mile = 0.539593 nautical mile.

1 meter = 0.000621370 statute mile = 0.000539593 nautical mile.

1 statute mile = 1609.35 meters = 1.60935 kilometers.

1 nautical mile = 1853.25 meters = 1.85325 kilometers.

1 nautical mile = 1.151553 statute miles.

1 statute mile = 0.868393 nautical mile.

1 meter = 39.37 inches (law of July 28, 1866).

1 meter = 3.28083333 feet.

log. 3.28083333 = 0.51598417

1 foot = 0.30480061 meter.

log. 0.30480061 = 9.48401583 − 10

Probable error of an observation, $r=0.6745\sqrt{\frac{\Sigma v^2}{n-1}}$

Probable error of result, $r_0=\frac{r}{\sqrt{n}}=0.6745\sqrt{\frac{\Sigma v^2}{n(n-1)}}$

Probable error of an observation of unit weight, $\mu_1=0.6745\sqrt{\frac{\Sigma pv^2}{n-1}}$

Probable error of an observation of weight $p_1, r_1=\frac{\mu_1}{\sqrt{p_1}}=0.6745\sqrt{\frac{\Sigma pv^2}{p_1(n-1)}}$

Probable error of an observed direction, $d=0.6745\sqrt{\frac{\Sigma v^2}{c}}$ where Σv^2=sum of squares of corrections to directions, and c is the number of conditions.

Mean error of an angle, $\alpha=\sqrt{\frac{\Sigma\Delta^2}{3n}}$,
where $\Sigma\Delta^2$ is the sum of the squares of the closing errors of the triangles, and n is the number of triangles.

Differences of elevation and inclination corrections for varying angles of inclination

[Length=50 meters. Argument is inclination angle]

Angle of inclination	Difference of elevation	Inclination correction	Angle of inclination	Difference of elevation	Inclination correction	Angle of inclination	Difference of elevation	Inclination correction
° ′	*M.*	*Mm.*	° ′	*M.*	*Mm.*	° ′	*M.*	*Mm.*
0 00	0.00	0.0	0 30	0.44	1.9	1 00	0.87	7.6
01	.01	.0	31	.45	2.0	01	.89	7.9
02	.03	.0	32	.47	2.2	02	.90	8.1
03	.04	.0	33	.48	2.3	03	.92	8.4
04	.06	.0	34	.40	2.4	04	.93	8.7
0 05	0.07	0.1	0 35	0.51	2.6	1 05	0.95	8.9
06	.09	.1	36	.52	2.7	06	.96	9.2
07	.10	.1	37	.54	2.9	07	.97	9.5
08	.12	.1	38	.55	3.1	08	.99	9.8
09	.13	.2	39	.57	3.2	09	1.00	10.1
0 10	0.15	0.2	0 40	0.58	3.4	1 10	1.02	10.4
11	.16	.3	41	.60	3.6	11	.03	10.7
12	.17	.3	42	.61	3.7	12	.05	11.0
13	.19	.4	43	.63	3.9	13	.06	11.3
14	.20	.4	44	.64	4.1	14	.08	11.6
0 15	0.22	0.5	0 45	0.65	4.3	1 15	1.09	11.9
16	.23	.5	46	.67	4.5	16	.10	12.2
17	.25	.6	47	.68	4.7	17	.12	12.5
18	.26	.7	48	.70	4.9	18	.13	12.9
19	.28	.8	49	.71	5.1	19	.15	13.2
0 20	0.29	0.8	0 50	0.73	5.3	1 20	1.16	13.5
21	.31	.9	51	.74	5.5	21	.18	13.9
22	.32	1.0	52	.76	5.7	22	.19	14.2
23	.33	1.1	53	.77	5.9	23	.21	14.6
24	.35	1.2	54	.79	6.2	24	.22	14.9
0 25	0.36	1.3	0 55	0.80	6.4	1 25	1.24	15.3
26	.38	1.4	56	.81	6.6	26	.25	15.6
27	.39	1.5	57	.83	6.9	27	.27	16.0
28	.41	1.7	58	.84	7.1	28	.28	16.4
29	.42	1.8	59	.86	7.4	29	.29	16.8
0 30	0.44	1.9	1 00	0.87	7.6	1 30	1.31	17.1

Angle of inclination	Difference of elevation	Inclination correction	Angle of inclination	Difference of elevation	Inclination correction	Angle of inclination	Difference of elevation	Inclination correction
° ′	*M.*	*Mm.*	° ′	*M.*	*Mm.*	° ′	*M.*	*Mm.*
1 30	1.31	17.1	2 00	1.74	30.5	2 30	2.18	47.6
31	.32	17.5	01	.76	31.0	31	.20	48.2
32	.34	17.9	02	.77	31.5	32	.21	48.9
33	.35	18.3	03	.79	32.0	33	.22	49.5
34	.37	18.7	04	.80	32.5	34	.24	50.2
1 35	1.38	19.1	2 05	1.82	33.0	2 35	2.25	50.8
36	.40	19.5	06	.83	33.6	36	.27	51.5
37	.41	19.9	07	.85	34.1	37	.28	52.1
38	.42	20.3	08	.86	34.7	38	.30	52.8
39	.44	20.7	09	.88	35.2	39	.31	53.5
1 40	1.45	21.2	2 10	1.89	35.7	2 40	2.33	54.1
41	.47	21.6	11	.90	36.3	41	.34	54.8
42	.48	22.0	12	.92	36.9	42	.36	55.5
43	.50	22.4	13	.93	37.4	43	.37	56.2
44	.51	22.9	14	.95	38.0	44	.38	56.9
1 45	1.53	23.3	2 15	1.96	38.6	2 45	2.40	57.6
46	.54	23.8	16	.98	39.1	46	.41	58.3
47	.56	24.2	17	.99	39.7	47	.43	59.0
48	.57	24.7	18	2.01	40.3	48	.44	59.7
49	.59	25.1	19	.02	40.9	49	.46	60.4
1 50	1.60	25.6	2 20	2.04	41.5	2 50	2.47	61.1
51	.61	26.1	21	.05	42.1	51	.49	61.8
52	.63	26.5	22	.06	42.7	52	.50	62.6
53	.64	27.0	23	.08	43.3	53	.52	63.3
54	.66	27.5	24	.09	43.9	54	.53	64.0
1 55	1.67	28.0	2 25	2.11	44.5	2 55	2.54	64.8
56	.69	28.5	26	.12	45.1	56	.56	65.5
57	.70	29.0	27	.14	45.7	57	.57	66.3
58	.72	29.5	28	.15	46.3	58	.59	67.0
59	.73	30.0	29	.17	47.0	59	.60	67.8
2 00	1.74	30.5	2 30	2.18	47.6	3 00	2.62	68.5

Inclination corrections for 50-meter tape lengths

[Cor. = $-0.01h^2-0.000001h^4$ (h in meters)]

Difference of elevation		Correction	Difference of elevation		Correction	Difference of elevation		Correction	Difference of elevation		Correction
Meters	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*
0.00	0.000	0.0	.65	2.133	4.2	1.30	4.265	16.9	1.95	6.398	38.0
.01	.033	.0	.66	2.165	4.4	1.31	4.298	17.2	1.96	6.430	38.4
.02	.066	.0	.67	2.198	4.5	1.32	4.331	17.4	1.97	6.463	38.8
.03	.098	.0	.68	2.231	4.6	1.33	4.364	17.7	1.98	6.496	39.2
.04	.131	.0	.69	2.264	4.8	1.34	4.396	18.0	1.99	6.529	39.6
.05	.164	.0	.70	2.297	4.9	1.35	4.429	18.2	2.00	6.562	40.0
.06	.197	.0	.71	2.329	5.0	1.36	4.462	18.5	2.01	6.594	40.4
.07	.230	.0	.72	2.362	5.2	1.37	4.495	18.8	2.02	6.627	40.8
.08	.262	.1	.73	2.395	5.3	1.38	4.528	19.0	2.03	6.660	41.2
.09	.295	.1	.74	2.428	5.5	1.39	4.560	19.3	2.04	6.693	41.6
.10	.328	.1	.75	2.461	5.6	1.40	4.593	19.6	2.05	6.726	42.0
.11	.361	.1	.76	2.493	5.8	1.41	4.626	19.9	2.06	6.759	42.5
.12	.394	.1	.77	2.526	5.9	1.42	4.659	20.2	2.07	6.791	42.9
.13	.427	.2	.78	2.559	6.1	1.43	4.692	20.4	2.08	6.824	43.3
.14	.459	.2	.79	2.592	6.2	1.44	4.724	20.7	2.09	6.857	43.7
.15	.492	.2	.80	2.625	6.4	1.45	4.757	21.0	2.10	6.890	44.1
.16	.525	.3	.81	2.657	6.6	1.46	4.790	21.3	2.11	6.923	44.5
.17	.558	.3	.82	2.690	6.7	1.47	4.823	21.6	2.12	6.955	45.0
.18	.591	.3	.83	2.723	6.9	1.48	4.856	21.9	2.13	6.988	45.4
.19	.623	.4	.84	2.756	7.1	1.49	4.888	22.2	2.14	7.021	45.8
.20	.656	.4	.85	2.789	7.2	1.50	4.921	22.5	2.15	7.054	46.2
.21	.689	.4	.86	2.822	7.4	1.51	4.954	22.8	2.16	7.087	46.7
.22	.722	.5	.87	2.854	7.6	1.52	4.987	23.1	2.17	7.119	47.1
.23	.755	.5	.88	2.887	7.7	1.53	5.020	23.4	2.18	7.152	47.5
.24	.787	.6	.89	2.920	7.9	1.54	5.052	23.7	2.19	7.185	48.0
.25	.820	.6	.90	2.953	8.1	1.55	5.085	24.0	2.20	7.218	48.4
.26	.853	.7	.91	2.986	8.3	1.56	5.118	24.3	2.21	7.251	48.9
.27	.886	.7	.92	3.018	8.5	1.57	5.151	24.6	2.22	7.283	49.3
.28	.919	.8	.93	3.051	8.6	1.58	5.184	25.0	2.23	7.316	49.8
.29	.951	.8	.94	3.084	8.8	1.59	5.217	25.3	2.24	7.349	50.2
.30	.984	.9	.95	3.117	9.0	1.60	5.249	25.6	2.25	7.382	50.7
.31	1.017	1.0	.96	3.150	9.2	1.61	5.282	25.9	2.26	7.415	51.1
.32	1.050	1.0	.97	3.182	9.4	1.62	5.315	26.2	2.27	7.447	51.6
.33	1.083	1.1	.98	3.215	9.6	1.63	5.348	26.6	2.28	7.480	52.0
.34	1.115	1.2	.99	3.248	9.8	1.64	5.381	26.9	2.29	7.513	52.5
.35	1.148	1.2	1.00	3.281	10.0	1.65	5.413	27.2	2.30	7.546	52.9
.36	1.181	1.3	1.01	3.314	10.2	1.66	5.446	27.6	2.31	7.579	53.4
.37	1.214	1.4	1.02	3.346	10.4	1.67	5.479	27.9	2.32	7.612	53.9
.38	1.247	1.4	1.03	3.379	10.6	1.68	5.512	28.2	2.33	7.644	54.3
.39	1.280	1.5	1.04	3.412	10.8	1.69	5.545	28.6	2.34	7.677	54.8
.40	1.312	1.6	1.05	3.445	11.0	1.70	5.577	28.9	2.35	7.710	55.3
.41	1.345	1.7	1.06	3.478	11.2	1.71	5.610	29.2	2.36	7.743	55.7
.42	1.378	1.8	1.07	3.510	11.4	1.72	5.643	29.6	2.37	7.776	56.2
.43	1.411	1.8	1.08	3.543	11.7	1.73	5.676	29.9	2.38	7.808	56.7
.44	1.444	1.9	1.09	3.576	11.9	1.74	5.709	30.3	2.39	7.841	57.2
.45	1.476	2.0	1.10	3.609	12.1	1.75	5.741	30.6	2.40	7.874	57.6
.46	1.509	2.1	1.11	3.642	12.3	1.76	5.774	31.0	2.41	7.907	58.1
.47	1.542	2.2	1.12	3.675	12.5	1.77	5.807	31.3	2.42	7.940	58.6
.48	1.575	2.3	1.13	3.707	12.8	1.78	5.840	31.7	2.43	7.972	59.1
.49	1.608	2.4	1.14	3.740	13.0	1.79	5.873	32.0	2.44	8.005	59.6
.50	1.640	2.5	1.15	3.773	13.2	1.80	5.906	32.4	2.45	8.038	60.1
.51	1.673	2.6	1.16	3.806	13.5	1.81	5.938	32.8	2.46	8.071	60.6
.52	1.706	2.7	1.17	3.839	13.7	1.82	5.971	33.1	2.47	8.104	61.0
.53	1.739	2.8	1.18	3.871	13.9	1.83	6.004	33.5	2.48	8.136	61.5
.54	1.772	2.9	1.19	3.904	14.2	1.84	6.037	33.9	2.49	8.169	62.0
.55	1.804	3.0	1.20	3.937	14.4	1.85	6.070	34.2	2.50	8.202	62.5
.56	1.837	3.1	1.21	3.970	14.6	1.86	6.102	34.6	2.51	8.235	63.0
.57	1.870	3.2	1.22	4.003	14.9	1.87	6.135	35.0	2.52	8.268	63.5
.58	1.903	3.4	1.23	4.035	15.1	1.88	6.168	35.3	2.53	8.301	64.0
.59	1.936	3.5	1.24	4.068	15.4	1.89	6.201	35.7	2.54	8.333	64.6
.60	1.968	3.6	1.25	4.101	15.6	1.90	6.234	36.1	2.55	8.366	65.1
.61	2.001	3.7	1.26	4.134	15.9	1.91	6.266	36.5	2.56	8.399	65.6
.62	2.034	3.8	1.27	4.167	16.1	1.92	6.299	36.9	2.57	8.432	66.1
.63	2.067	4.0	1.28	4.199	16.4	1.93	6.332	37.2	2.58	8.465	66.6
.64	2.100	4.1	1.29	4.232	16.6	1.94	6.365	37.6	2.59	8.497	67.1

Inclination corrections for 50-meter tape lengths—Continued

Difference of elevation		Correction	Difference of elevation		Correction	Difference of elevation		Correction	Difference of elevation		Correction
Meters	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*
2.60	8.530	67.6	3.25	10.663	105.7	3.90	12.795	152.3	4.55	14.928	207.5
2.61	8.563	68.2	3.26	10.696	106.4	3.91	12.828	153.1	4.56	14.961	208.4
2.62	8.596	68.7	3.27	10.728	107.0	3.92	12.861	153.9	4.57	14.993	209.3
2.63	8.629	69.2	3.28	10.761	107.7	3.93	12.894	154.7	4.58	15.026	210.2
2.64	8.661	69.7	3.29	10.794	108.4	3.94	12.926	155.5	4.59	15.059	211.1
2.65	8.694	70.3	3.30	10.827	109.0	3.95	12.959	156.3	4.60	15.092	212.0
2.66	8.727	70.8	3.31	10.860	109.7	3.96	12.992	157.1	4.61	15.125	213.0
2.67	8.760	71.3	3.32	10.892	110.3	3.97	13.025	157.9	4.62	15.157	213.9
2.68	8.793	71.9	3.33	10.925	111.0	3.98	13.058	158.7	4.63	15.190	214.8
2.69	8.825	72.4	3.34	10.958	111.7	3.99	13.091	159.5	4.64	15.223	215.8
2.70	8.858	73.0	3.35	10.991	112.4	4.00	13.123	160.3	4.65	15.256	216.7
2.71	8.891	73.5	3.36	11.024	113.0	4.01	13.156	161.1	4.66	15.289	217.6
2.72	8.924	74.0	3.37	11.056	113.7	4.02	13.189	161.9	4.67	15.321	218.6
2.73	8.957	74.6	3.38	11.089	114.4	4.03	13.222	162.7	4.68	15.354	219.5
2.74	8.989	75.1	3.39	11.122	115.1	4.04	13.255	163.5	4.69	15.387	220.4
2.75	9.022	75.7	3.40	11.155	115.7	4.05	13.287	164.3	4.70	15.420	221.4
2.76	9.055	76.2	3.41	11.188	116.4	4.06	13.320	165.1	4.71	15.453	222.3
2.77	9.088	76.8	3.42	11.220	117.1	4.07	13.353	165.9	4.72	15.486	223.3
2.78	9.121	77.3	3.43	11.253	117.8	4.08	13.386	166.7	4.73	15.518	224.2
2.79	9.154	77.9	3.44	11.286	118.5	4.09	13.419	167.6	4.74	15.551	225.2
2.80	9.186	78.5	3.45	11.319	119.2	4.10	13.451	168.4	4.75	15.584	226.1
2.81	9.219	79.0	3.46	11.352	119.9	4.11	13.484	169.2	4.76	15.617	227.1
2.82	9.252	79.6	3.47	11.384	120.6	4.12	13.517	170.0	4.77	15.650	228.0
2.83	9.285	80.2	3.48	11.417	121.3	4.13	13.550	170.9	4.78	15.682	229.0
2.84	9.318	80.7	3.49	11.450	122.0	4.14	13.583	171.7	4.79	15.715	230.0
2.85	9.350	81.3	3.50	11.483	122.7	4.15	13.615	172.5	4.80	15.748	230.9
2.86	9.383	81.9	3.51	11.516	123.4	4.16	13.648	173.4	4.81	15.781	231.9
2.87	9.416	82.4	3.52	11.549	124.1	4.17	13.681	174.2	4.82	15.814	232.9
2.88	9.449	83.0	3.53	11.581	124.8	4.18	13.714	175.0	4.83	15.846	233.8
2.89	9.482	83.6	3.54	11.614	125.5	4.19	13.747	175.9	4.84	15.879	234.8
2.90	9.514	84.2	3.55	11.647	126.2	4.20	13.780	176.7	4.85	15.912	235.8
2.91	9.547	84.8	3.56	11.680	126.9	4.21	13.812	177.6	4.86	15.945	236.8
2.92	9.580	85.3	3.57	11.713	127.6	4.22	13.845	178.4	4.87	15.978	237.7
2.93	9.613	85.9	3.58	11.745	128.3	4.23	13.878	179.2	4.88	16.010	238.7
2.94	9.646	86.5	3.59	11.778	129.0	4.24	13.911	180.1	4.89	16.043	239.7
2.95	9.678	87.1	3.60	11.811	129.8	4.25	13.944	181.0	4.90	16.076	240.7
2.96	9.711	87.7	3.61	11.844	130.5	4.26	13.976	181.8	4.91	16.109	241.7
2.97	9.744	88.3	3.62	11.877	131.2	4.27	14.009	182.7	4.92	16.142	242.7
2.98	9.777	88.9	3.63	11.909	131.9	4.28	14.042	183.5	4.93	16.175	243.6
2.99	9.810	89.5	3.64	11.942	132.7	4.29	14.075	184.4	4.94	16.207	244.6
3.00	9.842	90.1	3.65	11.975	133.4	4.30	14.108	185.2	4.95	16.240	245.6
3.01	9.875	90.7	3.66	12.008	134.1	4.31	14.140	186.1	4.96	16.273	246.6
3.02	9.908	91.3	3.67	12.041	134.9	4.32	14.173	187.0	4.97	16.306	247.6
3.03	9.941	91.9	3.68	12.073	135.6	4.33	14.206	187.8	4.98	16.339	248.6
3.04	9.974	92.5	3.69	12.106	136.3	4.34	14.239	188.7	4.99	16.371	249.6
3.05	10.007	93.1	3.70	12.139	137.1	4.35	14.272	189.6	5.00	16.404	250.6
3.06	10.039	93.7	3.71	12.172	137.8	4.36	14.304	190.5	5.01	16.437	251.6
3.07	10.072	94.3	3.72	12.205	138.6	4.37	14.337	191.3	5.02	16.470	252.6
3.08	10.105	95.0	3.73	12.238	139.3	4.38	14.370	192.2	5.03	16.503	253.6
3.09	10.138	95.6	3.74	12.270	140.1	4.39	14.403	193.1	5.04	16.535	254.7
3.10	10.171	96.2	3.75	12.303	140.8	4.40	14.436	194.0	5.05	16.568	255.7
3.11	10.203	96.8	3.76	12.336	141.6	4.41	14.468	194.9	5.06	16.601	256.7
3.12	10.236	97.4	3.77	12.369	142.3	4.42	14.501	195.7	5.07	16.634	257.7
3.13	10.269	98.1	3.78	12.402	143.1	4.43	14.534	196.6	5.08	16.667	258.7
3.14	10.302	98.7	3.79	12.434	143.8	4.44	14.567	197.5	5.09	16.699	259.8
3.15	10.335	99.3	3.80	12.467	144.6	4.45	14.600	198.4	5.10	16.732	260.8
3.16	10.367	100.0	3.81	12.500	145.4	4.46	14.633	199.3	5.11	16.765	261.8
3.17	10.400	100.6	3.82	12.533	146.1	4.47	14.665	200.2	5.12	16.798	262.8
3.18	10.433	101.2	3.83	12.566	146.9	4.48	14.698	201.1	5.13	16.831	263.9
3.19	10.466	101.9	3.84	12.598	147.7	4.49	14.731	202.0	5.14	16.863	264.9
3.20	10.499	102.5	3.85	12.631	148.4	4.50	14.764	202.9	5.15	16.896	265.9
3.21	10.531	103.1	3.86	12.664	149.2	4.51	14.797	203.8	5.16	16.929	267.0
3.22	10.564	103.8	3.87	12.697	150.0	4.52	14.829	204.7	5.17	16.962	268.0
3.23	10.597	104.4	3.88	12.730	150.8	4.53	14.862	205.6	5.18	16.995	269.0
3.24	10.630	105.1	3.89	12.762	151.6	4.54	14.895	206.5	5.19	17.028	270.1

Inclination corrections for 50-meter tape lengths—Continued

Difference of elevation		Correction	Difference of elevation		Correction	Difference of elevation		Correction	Difference of elevation		Correction
Meters	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*
5.20	17.060	271.1	5.80	19.029	337.5	6.40	20.997	411.3	7.00	22.966	492.4
5.21	17.093	272.2	5.81	19.062	338.7	6.41	21.030	412.6	7.01	22.999	493.8
5.22	17.126	273.2	5.82	19.094	339.9	6.42	21.063	413.9	7.02	23.031	495.2
5.23	17.159	274.3	5.83	19.127	341.0	6.43	21.096	415.2	7.03	23.064	496.6
5.24	17.192	275.3	5.84	19.160	342.2	6.44	21.129	416.5	7.04	23.097	498.1
5.25	17.224	276.4	5.85	19.193	343.4	6.45	21.161	417.8	7.05	23.130	499.5
5.26	17.257	277.4	5.86	19.226	344.6	6.46	21.194	419.1	7.06	23.163	500.9
5.27	17.290	278.5	5.87	19.258	345.8	6.47	21.227	420.4	7.07	23.195	502.3
5.28	17.323	279.6	5.88	19.291	346.9	6.48	21.260	421.7	7.08	23.228	503.8
5.29	17.356	280.6	5.89	19.324	348.1	6.49	21.293	423.0	7.09	23.261	505.2
5.30	17.388	281.7	5.90	19.357	349.3	6.50	21.325	424.3	7.10	23.294	506.6
5.31	17.421	282.8	5.91	19.390	350.5	6.51	21.358	425.6	7.11	23.327	508.1
5.32	17.454	283.8	5.92	19.423	351.7	6.52	21.391	426.9	7.12	23.360	509.5
5.33	17.487	284.9	5.93	19.455	352.9	6.53	21.424	428.2	7.13	23.392	511.0
5.34	17.520	286.0	5.94	19.488	354.1	6.54	21.457	429.5	7.14	23.425	512.4
5.35	17.552	287.0	5.95	19.521	355.3	6.55	21.489	430.9	7.15	23.458	513.8
5.36	17.585	288.1	5.96	19.554	356.5	6.56	21.522	432.2	7.16	23.491	515.3
5.37	17.618	289.2	5.97	19.587	357.7	6.57	21.555	433.5	7.17	23.524	516.7
5.38	17.651	290.3	5.98	19.619	358.9	6.58	21.588	434.8	7.18	23.556	518.2
5.39	17.684	291.4	5.99	19.652	360.1	6.59	21.621	436.2	7.19	23.589	519.6
5.40	17.716	292.5	6.00	19.685	361.3	6.60	21.654	437.5	7.20	23.622	521.1
5.41	17.749	293.5	6.01	19.718	362.5	6.61	21.686	438.8	7.21	23.655	522.5
5.42	17.782	294.6	6.02	19.751	363.7	6.62	21.719	440.2	7.22	23.688	524.0
5.43	17.815	295.7	6.03	19.783	364.9	6.63	21.752	441.5	7.23	23.720	525.5
5.44	17.848	296.8	6.04	19.816	366.1	6.64	21.785	442.8	7.24	23.753	526.9
5.45	17.881	297.9	6.05	19.849	367.4	6.65	21.818	444.2	7.25	23.786	528.4
5.46	17.913	299.0	6.06	19.882	368.6	6.66	21.850	445.5	7.26	23.819	529.9
5.47	17.946	300.1	6.07	19.915	369.8	6.67	21.883	446.9	7.27	23.852	531.3
5.48	17.979	301.2	6.08	19.947	371.0	6.68	21.916	448.2	7.28	23.884	532.8
5.49	18.012	302.3	6.09	19.980	372.3	6.69	21.949	449.6	7.29	23.917	534.3
5.50	18.045	303.4	6.10	20.013	373.5	6.70	21.982	450.9	7.30	23.950	535.7
5.51	18.077	304.5	6.11	20.046	374.7	6.71	22.014	452.3	7.31	23.983	537.2
5.52	18.110	305.6	6.12	20.079	375.9	6.72	22.047	453.6	7.32	24.016	538.7
5.53	18.143	306.7	6.13	20.112	377.2	6.73	22.080	455.0	7.33	24.049	540.2
5.54	18.176	307.9	6.14	20.144	378.4	6.74	22.113	456.3	7.34	24.081	541.7
5.55	18.209	309.0	6.15	20.177	379.7	6.75	22.146	457.7	7.35	24.114	543.1
5.56	18.241	310.1	6.16	20.210	380.9	6.76	22.178	459.1	7.36	24.147	544.6
5.57	18.274	311.2	6.17	20.243	382.1	6.77	22.211	460.4	7.37	24.180	546.1
5.58	18.307	312.3	6.18	20.276	383.4	6.78	22.244	461.8	7.38	24.213	547.6
5.59	18.340	313.5	6.19	20.308	384.6	6.79	22.277	463.2	7.39	24.245	549.1
5.60	18.373	314.6	6.20	20.341	385.9	6.80	22.310	464.5	7.40	24.278	550.6
5.61	18.405	315.7	6.21	20.374	387.1	6.81	22.342	465.9	7.41	24.311	552.1
5.62	18.438	316.8	6.22	20.407	388.4	6.82	22.375	467.3	7.42	24.344	553.6
5.63	18.471	318.0	6.23	20.440	389.6	6.83	22.408	468.7	7.43	24.377	555.1
5.64	18.504	319.1	6.24	20.472	390.9	6.84	22.441	470.0	7.44	24.409	556.6
5.65	18.537	320.2	6.25	20.505	392.2	6.85	22.474	471.4	7.45	24.442	558.1
5.66	18.570	321.4	6.26	20.538	393.4	6.86	22.507	472.8	7.46	24.475	559.6
5.67	18.602	322.5	6.27	20.571	394.7	6.87	22.539	474.2	7.47	24.508	561.1
5.68	18.635	323.7	6.28	20.604	395.9	6.88	22.572	475.6	7.48	24.541	562.6
5.69	18.668	324.8	6.29	20.636	397.2	6.89	22.605	477.0	7.49	24.573	564.1
5.70	18.701	326.0	6.30	20.669	398.5	6.90	22.638	478.4	7.50	24.606	565.7
5.71	18.734	327.1	6.31	20.702	399.7	6.91	22.671	479.8			
5.72	18.766	328.3	6.32	20.735	401.0	6.92	22.703	481.2			
5.73	18.799	329.4	6.33	20.768	402.3	6.93	22.736	482.6			
5.74	18.832	330.6	6.34	20.800	403.6	6.94	22.769	484.0			
5.75	18.865	331.7	6.35	20.833	404.9	6.95	22.802	485.4			
5.76	18.898	332.9	6.36	20.866	406.1	6.96	22.835	486.8			
5.77	18.930	334.0	6.37	20.899	407.4	6.97	22.867	488.2			
5.78	18.963	335.2	6.38	20.932	408.7	6.98	22.900	489.6			
5.79	18.996	336.4	6.39	20.965	410.0	6.99	22.933	491.0			

Inclination corrections for 25-meter lengths

[Cor. $= -0.00186\,h^2 - 0.000000069\,h^4$ (h in feet)]

Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction
Foot	*Meter*	*Mm.*	*Feet*	*Meter*	*Mm.*	*Feet*	*Meter*	*Mm.*	*Feet*	*Meter*	*Mm.*
0.00	0.0000	0.0	0.70	0.2134	0.9	1.40	0.4267	3.6	2.10	0.6401	8.2
.01	.0030	.0	.71	.2164	.9	.41	.4298	3.7	.11	.6431	8.3
.02	.0061	.0	.72	.2195	1.0	.42	.4328	3.7	.12	.6462	8.4
.03	.0091	.0	.73	.2225	1.0	.43	.4359	3.8	.13	.6492	8.4
.04	.0122	.0	.74	.2256	1.0	.44	.4389	3.8	.14	.6523	8.5
.05	.0152	.0	.75	.2286	1.0	.45	.4420	3.9	.15	.6553	8.6
.06	.0183	.0	.76	.2316	1.1	.46	.4450	4.0	.16	.6584	8.7
.07	.0213	.0	.77	.2347	1.1	.47	.4481	4.0	.17	.6614	8.8
.08	.0244	.0	.78	.2377	1.1	.48	.4511	4.1	.18	.6645	8.8
.09	.0274	.0	.79	.2408	1.2	.49	.4542	4.1	.19	.6675	8.9
0.10	0.0305	0.0	0.80	0.2438	1.2	1.50	0.4572	4.2	2.20	0.6706	9.0
.11	.0335	.0	.81	.2469	1.2	.51	.4602	4.3	.21	.6736	9.1
.12	.0366	.0	.82	.2499	1.2	.52	.4633	4.3	.22	.6767	9.2
.13	.0396	.0	.83	.2530	1.3	.53	.4663	4.4	.23	.6797	9.2
.14	.0427	.0	.84	.2560	1.3	.54	.4694	4.4	.24	.6828	9.3
.15	.0457	.0	.85	.2591	1.3	.55	.4724	4.5	.25	.6858	9.4
.16	.0488	.0	.86	.2621	1.4	.56	.4755	4.6	.26	.6888	9.5
.17	.0518	.1	.87	.2652	1.4	.57	.4785	4.6	.27	.6919	9.6
.18	.0549	.1	.88	.2682	1.4	.58	.4816	4.7	.28	.6949	9.6
.19	.0579	.1	.89	.2713	1.5	.59	.4846	4.7	.29	.6980	9.7
0.20	0.0610	0.1	0.90	0.2743	1.5	1.60	0.4877	4.8	2.30	0.7010	9.8
.21	.0640	.1	.91	.2774	1.5	.61	.4907	4.8	.31	.7041	9.9
.22	.0671	.1	.92	.2804	1.6	.62	.4938	4.9	.32	.7071	10.0
.23	.0701	.1	.93	.2835	1.6	.63	.4968	5.0	.33	.7102	10.1
.24	.0732	.1	.94	.2865	1.6	.64	.4999	5.0	.34	.7132	10.2
.25	.0762	.1	.95	.2896	1.7	.65	.5029	5.1	.35	.7163	10.3
.26	.0792	.1	.96	.2926	1.7	.66	.5060	5.2	.36	.7193	10.3
.27	.0823	.1	.97	.2957	1.8	.67	.5090	5.2	.37	.7224	10.4
.28	.0853	.1	.98	.2987	1.8	.68	.5121	5.3	.38	.7254	10.5
.29	.0884	.2	.99	.3018	1.8	.69	.5151	5.3	.39	.7285	10.6
0.30	0.0914	0.2	1.00	0.3048	1.9	1.70	0.5182	5.4	2.40	0.7315	10.7
.31	.0945	.2	.01	.3078	1.9	.71	.5212	5.5	.41	.7346	10.8
.32	.0975	.2	.02	.3109	1.9	.72	.5243	5.5	.42	.7376	10.9
.33	.1006	.2	.03	.3139	2.0	.73	.5273	5.6	.43	.7407	11.0
.34	.1036	.2	.04	.3170	2.0	.74	.5304	5.6	.44	.7437	11.1
.35	.1067	.2	.05	.3200	2.0	.75	.5334	5.7	.45	.7468	11.2
.36	.1097	.2	.06	.3231	2.1	.76	.5364	5.8	.46	.7498	11.2
.37	.1128	.3	.07	.3261	2.1	.77	.5395	5.8	.47	.7520	11.3
.38	.1158	.3	.08	.3292	2.2	.78	.5425	5.9	.48	.7559	11.4
.39	.1189	.3	.09	.3322	2.2	.79	.5456	5.9	.49	.7590	11.5
0.40	0.1219	0.3	1.10	0.3353	2.2	1.80	0.5486	6.0	2.50	0.7620	11.6
.41	.1250	.3	.11	.3383	2.3	.81	.5517	6.1	.51	.7650	11.7
.42	.1280	.3	.12	.3414	2.3	.82	.5547	6.1	.52	.7681	11.8
.43	.1311	.3	.13	.3444	2.4	.83	.5578	6.2	.53	.7711	11.9
.44	.1341	.4	.14	.3475	2.4	.84	.5608	6.3	.54	.7742	12.0
.45	.1372	.4	.15	.3505	2.5	.85	.5639	6.4	.55	.7772	12.1
.46	.1402	.4	.16	.3536	2.5	.86	.5669	6.4	.56	.7803	12.2
.47	.1433	.4	.17	.3566	2.5	.87	.5700	6.5	.57	.7833	12.3
.48	.1463	.4	.18	.3597	2.6	.88	.5730	6.6	.58	.7864	12.4
.49	.1494	.4	.19	.3627	2.6	.89	.5761	6.6	.59	.7894	12.5
0.50	0.1524	0.5	1.20	0.3658	2.7	1.90	0.5791	6.7	2.60	0.7925	12.6
.51	.1554	.5	.21	.3688	2.7	.91	.5822	6.8	.61	.7955	12.7
.52	.1585	.5	.22	.3719	2.8	.92	.5852	6.8	.62	.7986	12.8
.53	.1615	.5	.23	.3749	2.8	.93	.5883	6.9	.63	.8016	12.9
.54	.1646	.5	.24	.3780	2.9	.94	.5913	7.0	.64	.8047	13.0
.55	.1676	.6	.25	.3810	2.9	.95	.5944	7.1	.65	.8077	13.1
.56	.1707	.6	.26	.3840	2.9	.96	.5974	7.1	.66	.8108	13.1
.57	.1737	.6	.27	.3871	3.0	.97	.6005	7.2	.67	.8138	13.2
.58	.1768	.6	.28	.3901	3.1	.98	.6035	7.3	.68	.8169	13.3
.59	.1798	.6	.29	.3932	3.1	.99	.6066	7.3	.69	.8199	13.4
0.60	0.1829	0.7	1.30	0.3962	3.1	2.00	0.6096	7.4	2.70	0.8230	13.5
.61	.1859	.7	.31	.3993	3.2	.01	.6126	7.5	.71	.8260	13.6
.62	.1890	.7	.32	.4023	3.2	.02	.6157	7.6	.72	.8291	13.7
.63	.1920	.7	.33	.4054	3.3	.03	.6187	7.6	.73	.8321	13.8
.64	.1951	.8	.34	.4084	3.3	.04	.6218	7.7	.74	.8352	13.9
.65	.1981	.8	.35	.4115	3.4	.05	.6248	7.8	.75	.8382	14.1
.66	.2012	.8	.36	.4145	3.4	.06	.6279	7.9	.76	.8412	14.2
.67	.2042	.8	.37	.4176	3.5	.07	.6309	8.0	.77	.8443	14.3
.68	.2073	.9	.38	.4206	3.5	.08	.6340	8.0	.78	.8473	14.4
.69	.2103	.9	.39	.4237	3.6	.09	.6370	8.1	.79	.8504	14.5

Inclination corrections for 25-meter lengths—Continued

Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction
Feet	*Meters*	*Mm.*	*Feet*	*Meters*	*Mm.*	*Feet*	*Meters*	*Mm.*	*Feet*	*Meters*	*Mm.*
2.80	0.8534	14.6	3.50	1.0668	22.8	4.20	1.2802	32.8	4.90	1.4935	44.7
.81	.8565	14.7	.51	.0699	22.9	.21	.2832	33.0	.91	.4966	44.9
.82	.8595	14.8	.52	.0729	23.1	.22	.2863	33.1	.92	.4996	45.1
.83	.8626	14.9	.53	.0759	23.2	.23	.2893	33.3	.93	.5027	45.2
.84	.8656	15.0	.54	.0790	23.3	.24	.2924	33.4	.94	.5057	45.4
.85	.8687	15.1	.55	.0820	23.5	.25	.2954	33.6	.95	.5088	45.6
.86	.8717	15.2	.56	.0851	23.6	.26	.2985	33.8	.96	.5118	45.8
.87	.8748	15.3	.57	.0881	23.7	.27	.3015	33.9	.97	.5149	46.0
.88	.8778	15.4	.58	.0912	23.8	.28	.3045	34.1	.98	.5179	46.1
.89	.8809	15.5	.59	.0942	24.0	.29	.3076	34.2	.99	.5210	46.3
2.90	0.8839	15.6	3.60	1.0973	24.1	4.30	1.3106	34.4	5.00	1.5240	46.5
.91	.8870	15.7	.61	.1003	24.2	.31	.3137	34.6	.01	.5271	46.7
.92	.8900	15.8	.62	.1034	24.4	.32	.3167	34.7	.02	.5301	46.9
.93	.8931	15.9	.63	.1064	24.5	.33	.3198	34.9	.03	.5331	47.1
.94	.8961	16.0	.64	.1095	24.6	.34	.3228	35.0	.04	.5362	47.3
.95	.8992	16.2	.65	.1125	24.8	.35	.3259	35.2	.05	.5392	47.5
.96	.9022	16.3	.66	.1156	24.9	.36	.3289	35.4	.06	.5423	47.6
.97	.9053	16.4	.67	.1186	25.0	.37	.3320	35.5	.07	.5453	47.8
.98	.9083	16.5	.68	.1217	25.1	.38	.3350	35.7	.08	.5484	48.0
.99	.9114	16.6	.69	.1247	25.3	.39	.3381	35.8	.09	.5514	48.2
3.00	0.9144	16.7	3.70	1.1278	25.4	4.40	1.3411	36.0	5.10	1.5545	48.4
.01	.9174	16.8	.71	.1308	25.5	.41	.3442	36.2	.11	.5575	48.6
.02	.9205	16.9	.72	.1339	25.7	.42	.3472	36.3	.12	.5606	48.8
.03	.9235	17.1	.73	.1369	25.8	.43	.3503	36.5	.13	.5636	49.0
.04	.9266	17.2	.74	.1400	26.0	.44	.3533	36.7	.14	.5667	49.2
.05	.9296	17.3	.75	.1430	26.1	.45	.3564	36.9	.15	.5697	49.4
.06	.9327	17.4	.76	.1461	26.2	.46	.3594	37.0	.16	.5728	49.5
.07	.9357	17.5	.77	.1491	26.4	.47	.3625	37.2	.17	.5758	49.7
.08	.9388	17.7	.78	.1521	26.5	.48	.3655	37.4	.18	.5789	49.9
.09	.9418	17.8	.79	.1552	26.7	.49	.3686	37.5	.19	.5819	50.1
3.10	0.9449	17.9	3.80	1.1582	26.8	4.50	1.3716	37.7	5.20	1.5850	50.3
.11	.9479	18.0	.81	.1613	27.0	.51	.3747	37.9	.21	.5880	50.5
.12	.9510	18.1	.82	.1643	27.1	.52	.3777	38.0	.22	.5911	50.7
.13	.9540	18.2	.83	.1674	27.3	.53	.3807	38.2	.23	.5941	50.9
.14	.9571	18.3	.84	.1704	27.4	.54	.3838	38.4	.24	.5972	51.1
.15	.9601	18.5	.85	.1735	27.6	.55	.3868	38.6	.25	.6002	51.3
.16	.9632	18.6	.86	.1765	27.7	.56	.3899	38.7	.26	.6033	51.5
.17	.9662	18.7	.87	.1796	27.9	.57	.3929	38.9	.27	.6063	51.7
.18	.9693	18.8	.88	.1826	28.0	.58	.3960	39.1	.28	.6093	51.9
.19	.9723	18.9	.89	.1857	28.2	.59	.3990	39.2	.29	.6124	52.1
3.20	0.9754	19.0	3.90	1.1887	28.3	4.60	1.4021	39.4	5.30	1.6154	52.3
.21	.9784	19.1	.91	.1918	28.4	.61	.4051	39.6	.31	.6185	52.5
.22	.9815	19.2	.92	.1948	28.6	.62	.4082	39.7	.32	.6215	52.7
.23	.9845	19.4	.93	.1979	28.7	.63	.4112	39.9	.33	.6246	52.9
.24	.9876	19.5	.94	.2009	28.9	.64	.4143	40.1	.34	.6276	53.1
.25	.9906	19.6	.95	.2040	29.0	.65	.4173	40.3	.35	.6307	53.3
.26	.9936	19.7	.96	.2070	29.1	.66	.4204	40.4	.36	.6337	53.5
.27	.9967	19.8	.97	.2101	29.3	.67	.4234	40.6	.37	.6368	53.7
.28	.9997	20.0	.98	.2131	29.4	.68	.4265	40.8	.38	.6398	53.9
.29	1.0028	20.1	.99	.2162	29.6	.69	.4295	40.9	.39	.6429	54.1
3.30	1.0058	20.2	4.00	1.2192	29.7	4.70	1.4326	41.1	5.40	1.6459	54.3
.31	.0089	20.3	.01	.2223	29.9	.71	.4356	41.3	.41	.6490	54.5
.32	.0119	20.5	.02	.2253	30.0	.72	.4387	41.5	.42	.6520	54.7
.33	.0150	20.6	.03	.2283	30.2	.73	.4417	41.6	.43	.6551	54.9
.34	.0180	20.7	.04	.2314	30.3	.74	.4448	41.8	.44	.6581	55.1
.35	.0211	20.9	.05	.2344	30.5	.75	.4478	42.0	.45	.6612	55.3
.36	.0241	21.0	.06	.2375	30.6	.76	.4509	42.2	.46	.6642	55.5
.37	.0272	21.1	.07	.2405	30.8	.77	.4539	42.4	.47	.6673	55.7
.38	.0302	21.2	.08	.2436	30.9	.78	.4569	42.5	.48	.6703	55.9
.39	.0333	21.4	.09	.2466	31.1	.79	.4600	42.7	.49	.6734	56.1
3.40	1.0363	21.5	4.10	1.2497	31.2	4.80	1.4630	42.9	5.50	1.6764	56.3
.41	.0394	21.6	.11	.2527	31.4	.81	.4661	43.1	.51	.6795	56.5
.42	.0424	21.8	.12	.2558	31.5	.82	.4691	43.3	.52	.6825	56.7
.43	.0455	21.9	.13	.2588	31.7	.83	.4722	43.4	.53	.6855	56.9
.44	.0485	22.0	.14	.2619	31.8	.84	.4752	43.6	.54	.6886	57.1
.45	.0516	22.2	.15	.2649	32.0	.85	.4783	43.8	.55	.6916	57.3
.46	.0546	22.3	.16	.2680	32.2	.86	.4813	44.0	.56	.6947	57.5
.47	.0577	22.4	.17	.2710	32.3	.87	.4844	44.2	.57	.6977	57.7
.48	.0607	22.5	.18	.2741	32.5	.88	.4874	44.3	.58	.7008	57.9
.49	.0638	22.7	.19	.2771	32.6	.89	.4905	44.5	.59	.7038	58.1

Inclination corrections for 25-meter lengths—Continued

Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction
Feet	*Meters*	*Mm.*	*Feet*	*Meters*	*Mm.*	*Feet*	*Meters*	*Mm.*	*Feet*	*Meters*	*Mm.*
5.60	1.7069	58.3	6.30	1.9202	73.9	7.00	2.1336	91.2	7.70	2.3470	110.4
.61	.7099	58.5	.31	.9233	74.1	.01	.1367	91.5	.71	.3500	110.7
.62	.7130	58.7	.32	.9263	74.4	.02	.1397	91.7	.72	.3531	111.0
.63	.7160	58.9	.33	.9294	74.6	.03	.1427	92.0	.73	.3561	111.3
.64	.7191	59.1	.34	.9324	74.8	.04	.1458	92.2	.74	.3592	111.6
.65	.7221	59.4	.35	.9355	75.1	.05	.1488	92.5	.75	.3622	111.9
.66	.7252	59.6	.36	.9385	75.3	.06	.1519	92.8	.76	.3653	112.1
.67	.7282	59.8	.37	.9416	75.5	.07	.1549	93.0	.77	.3683	112.4
.68	.7313	60.0	.38	.9446	75.7	.08	.1580	93.3	.78	.3713	112.7
.69	.7343	60.2	.39	.9477	76.0	.09	.1610	93.5	.79	.3744	113.0
5.70	1.7374	60.4	6.40	1.9507	76.2	7.10	2.1641	93.8	7.80	2.3774	113.3
.71	.7404	60.6	.41	.9538	76.4	.11	.1671	94.1	.81	.3805	113.6
.72	.7435	60.8	.42	.9568	76.7	.12	.1702	94.3	.82	.3835	113.9
.73	.7465	61.1	.43	.9599	76.9	.13	.1732	94.6	.83	.3866	114.2
.74	.7496	61.3	.44	.9629	77.2	.14	.1763	94.9	.84	.3896	114.5
.75	.7526	61.5	.45	.9660	77.4	.15	.1793	95.2	.85	.3927	114.8
.76	.7557	61.7	.46	.9690	77.6	.16	.1824	95.4	.86	.3957	115.0
.77	.7587	61.9	.47	.9721	77.9	.17	.1854	95.7	.87	.3988	115.3
.78	.7617	62.2	.48	.9751	78.1	.18	.1885	96.0	.88	.4018	115.6
.79	.7648	62.4	.49	.9782	78.4	.19	.1915	96.2	.89	.4049	115.9
5.80	1.7678	62.6	6.50	1.9812	78.6	7.20	2.1946	96.5	7.90	2.4079	116.2
.81	.7709	62.8	.51	.9843	78.9	.21	.1976	96.8	.91	.4110	116.5
.82	.7739	63.0	.52	.9873	79.1	.22	.2007	97.0	.92	.4140	116.8
.83	.7770	63.3	.53	.9903	79.4	.23	.2037	97.3	.93	.4171	117.1
.84	.7800	63.5	.54	.9934	79.6	.24	.2068	97.6	.94	.4201	117.4
.85	.7831	63.7	.55	.9964	79.9	.25	.2098	97.9	.95	.4232	117.7
.86	.7861	63.9	.56	.9995	80.1	.26	.2129	98.1	.96	.4262	118.0
.87	.7892	64.1	.57	2.0025	80.4	.27	.2159	98.4	.97	.4293	118.3
.88	.7922	64.4	.58	.0056	80.6	.28	.2189	98.7	.98	.4323	118.6
.89	.7953	64.6	.59	.0086	80.9	.29	.2220	98.9	.99	.4354	118.9
5.90	1.7983	64.8	6.60	2.0117	81.1	7.30	2.2250	99.2	8.00	2.4384	119.2
.91	.8014	65.0	.61	.0147	81.4	.31	.2281	99.5	.01	.4415	119.5
.92	.8044	65.2	.62	.0178	81.6	.32	.2311	99.8	.02	.4445	119.8
.93	.8075	65.5	.63	.0208	81.9	.33	.2342	100.0	.03	.4476	120.1
.94	.8105	65.7	.64	.0239	82.1	.34	.2372	100.3	.04	.4506	120.4
.95	.8136	65.9	.65	.0269	82.4	.35	.2403	100.6	.05	.4536	120.7
.96	.8166	66.1	.66	.0300	82.6	.36	.2433	100.9	.06	.4567	121.0
.97	.8197	66.3	.67	.0330	82.9	.37	.2464	101.2	.07	.4597	121.3
.98	.8227	66.6	.68	.0361	83.1	.38	.2494	101.4	.08	.4628	121.6
.99	.8258	66.8	.69	.0391	83.4	.39	.2525	101.7	.09	.4658	121.9
6.00	1.8288	67.0	6.70	2.0422	83.6	7.40	2.2555	102.0	8.10	2.4689	122.2
.01	.8319	67.2	.71	.0452	83.9	.41	.2586	102.3	.11	.4719	122.5
.02	.8349	67.4	.72	.0483	84.1	.42	.2616	102.6	.12	.4750	122.8
.03	.8379	67.7	.73	.0513	84.4	.43	.2647	102.8	.13	.4780	123.1
.04	.8410	67.9	.74	.0544	84.6	.44	.2677	103.1	.14	.4811	123.4
.05	.8440	68.1	.75	.0574	84.9	.45	.2708	103.4	.15	.4841	123.7
.06	.8471	68.3	.76	.0605	85.1	.46	.2738	103.7	.16	.4872	124.0
.07	.8501	68.5	.77	.0635	85.4	.47	.2769	104.0	.17	.4902	124.3
.08	.8532	68.8	.78	.0665	85.6	.48	.2799	104.2	.18	.4933	124.6
.09	.8562	69.0	.79	.0696	85.9	.49	.2830	104.5	.19	.4963	124.9
6.10	1.8593	69.2	6.80	2.0726	86.1	7.50	2.2860	104.8	8.20	2.4994	125.2
.11	.8623	69.4	.81	.0757	86.4	.51	.2891	105.1	.21	.5024	125.5
.12	.8654	69.7	.82	.0787	86.6	.52	.2921	105.4	.22	.5055	125.8
.13	.8684	69.9	.83	.0818	86.9	.53	.2951	105.6	.23	.5085	126.1
.14	.8715	70.1	.84	.0848	87.1	.54	.2982	105.9	.24	.5116	126.4
.15	.8745	70.4	.85	.0879	87.4	.55	.3012	106.2	.25	.5146	126.8
.16	.8776	70.6	.86	.0909	87.6	.56	.3043	106.5	.26	.5177	127.1
.17	.8806	70.8	.87	.0940	87.9	.57	.3073	106.8	.27	.5207	127.4
.18	.8837	71.0	.88	.0970	88.1	.58	.3104	107.0	.28	.5237	127.7
.19	.8867	71.3	.89	.1001	88.4	.59	.3134	107.3	.29	.5268	128.0
6.20	1.8898	71.5	6.90	2.1031	88.6	7.60	2.3165	107.6	8.30	2.5298	128.3
.21	.8928	71.7	.91	.1062	88.9	.61	.3195	107.8	.31	.5329	128.6
.22	.8959	72.0	.92	.1092	89.1	.62	.3226	108.2	.32	.5359	128.9
.23	.8989	72.2	.93	.1123	89.4	.63	.3256	108.4	.33	.5390	129.2
.24	.9020	72.5	.94	.1153	89.6	.64	.3287	108.7	.34	.5420	129.5
.25	.9050	72.7	.95	.1184	89.9	.65	.3317	109.0	.35	.5451	129.9
.26	.9081	72.9	.96	.1214	90.2	.66	.3348	109.3	.36	.5481	130.2
.27	.9111	73.2	.97	.1245	90.4	.67	.3378	109.6	.37	.5512	130.5
.28	.9141	73.4	.98	.1275	90.7	.68	.3409	109.8	.38	.5542	130.8
.29	.9172	73.7	.99	.1306	90.9	.69	.3439	110.1	.39	.5573	131.1
									8.40	2.5603	131.4

Inclination corrections for 5, 10, 15, 20, 25, 30, 35, 40, and 45 meter lengths

Difference in elevation		Length in meters								
		5	10	15	20	25	30	35	40	45
Meters	*Feet*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*
0.01	0.033	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.02	0.066	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.03	0.098	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.04	0.131	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	0.164	0.2	0.1	0.1	0.1	0.0	0.0	0.0	0.0	0.0
0.06	0.197	0.4	0.2	0.1	0.1	0.1	0.1	0.0	0.0	0.0
0.07	0.230	0.5	0.2	0.2	0.1	0.1	0.1	0.1	0.1	0.1
0.08	0.262	0.6	0.3	0.2	0.2	0.1	0.1	0.1	0.1	0.1
0.09	0.295	0.8	0.4	0.3	0.2	0.2	0.2	0.1	0.1	0.1
0.10	0.328	1.0	0.5	0.3	0.2	0.2	0.2	0.1	0.1	0.1
0.11	0.361	1.2	0.6	0.4	0.3	0.2	0.2	0.2	0.2	0.1
0.12	0.394	1.4	0.7	0.5	0.4	0.3	0.2	0.2	0.2	0.2
0.13	0.427	1.7	0.8	0.6	0.4	0.3	0.3	0.2	0.2	0.2
0.14	0.459	2.0	1.0	0.7	0.5	0.4	0.3	0.3	0.2	0.2
0.15	0.492	2.3	1.1	0.8	0.6	0.4	0.3	0.3	0.3	0.2
0.16	0.525	2.6	1.3	0.9	0.6	0.5	0.4	0.4	0.3	0.3
0.17	0.558	2.9	1.4	1.0	0.7	0.6	0.5	0.4	0.4	0.3
0.18	0.591	3.2	1.6	1.1	0.8	0.6	0.5	0.5	0.4	0.4
0.19	0.623	3.6	1.8	1.2	0.9	0.7	0.6	0.5	0.5	0.4
0.20	0.656	4.0	2.0	1.3	1.0	0.8	0.7	0.6	0.5	0.4
0.21	0.689	4.4	2.2	1.5	1.1	0.9	0.7	0.6	0.6	0.5
0.22	0.722	4.8	2.4	1.6	1.2	1.0	0.8	0.7	0.6	0.5
0.23	0.755	5.3	2.6	1.8	1.3	1.0	0.9	0.8	0.7	0.6
0.24	0.787	5.8	2.9	1.9	1.4	1.1	1.0	0.8	0.7	0.6
0.25	0.820	6.3	3.1	2.1	1.6	1.2	1.0	0.9	0.8	0.7
0.26	0.853	6.8	3.4	2.3	1.7	1.3	1.1	1.0	0.8	0.8
0.27	0.886	7.3	3.6	2.4	1.8	1.5	1.2	1.0	0.9	0.8
0.28	0.919	7.8	3.9	2.6	2.0	1.6	1.3	1.1	1.0	0.9
0.29	0.951	8.4	4.2	2.8	2.1	1.7	1.4	1.2	1.1	0.9
0.30	0.984	9.0	4.5	3.0	2.2	1.8	1.5	1.3	1.1	1.0
0.31	1.017	9.6	4.8	3.2	2.4	1.9	1.6	1.4	1.2	1.1
0.32	1.050	10.2	5.1	3.4	2.6	2.0	1.7	1.5	1.3	1.1
0.33	1.083	10.9	5.4	3.6	2.7	2.2	1.8	1.6	1.4	1.2
0.34	1.115	11.6	5.8	3.9	2.9	2.3	1.9	1.7	1.4	1.3
0.35	1.148	12.3	6.1	4.1	3.1	2.5	2.0	1.8	1.5	1.4
0.36	1.181	13.0	6.5	4.3	3.2	2.6	2.2	1.9	1.6	1.4
0.37	1.214	13.7	6.8	4.6	3.4	2.7	2.3	2.0	1.7	1.5
0.38	1.247	14.4	7.2	4.8	3.6	2.9	2.4	2.1	1.8	1.6
0.39	1.280	15.2	7.6	5.1	3.8	3.1	2.5	2.2	1.9	1.7
0.40	1.312	16.0	8.0	5.3	4.0	3.2	2.7	2.3	2.0	1.8
0.41	1.345	16.8	8.4	5.6	4.2	3.4	2.8	2.4	2.1	1.9
0.42	1.378	17.7	8.8	5.9	4.4	3.5	2.9	2.5	2.2	2.0
0.43	1.411	18.5	9.3	6.2	4.6	3.7	3.1	2.6	2.3	2.1
0.44	1.444	19.4	9.7	6.5	4.8	3.9	3.2	2.8	2.4	2.2
0.45	1.476	20.3	10.1	6.8	5.1	4.1	3.4	2.9	2.5	2.3
0.46	1.509	21.2	10.6	7.1	5.3	4.2	3.5	3.0	2.6	2.4
0.47	1.542	22.1	11.1	7.4	5.5	4.4	3.7	3.2	2.8	2.5
0.48	1.575	23.1	11.5	7.7	5.8	4.6	3.8	3.3	2.9	2.6
0.49	1.608	24.1	12.0	8.0	6.0	4.8	4.0	3.4	3.0	2.7
0.50	1.640	25.1	12.5	8.3	6.3	5.0	4.2	3.6	3.1	2.8
0.51	1.673	26.1	13.0	8.7	6.5	5.2	4.3	3.7	3.3	2.9
0.52	1.706	27.1	13.5	9.0	6.8	5.4	4.5	3.9	3.4	3.0
0.53	1.739	28.2	14.1	9.4	7.0	5.6	4.7	4.0	3.5	3.1
0.54	1.772	29.2	14.6	9.7	7.3	5.8	4.9	4.2	3.6	3.2
0.55	1.804	30.3	15.1	10.1	7.6	6.0	5.0	4.3	3.8	3.4
0.56	1.837	31.5	15.7	10.5	7.8	6.3	5.2	4.5	3.9	3.5
0.57	1.870	32.6	16.2	10.8	8.1	6.5	5.4	4.6	4.1	3.6
0.58	1.903	33.8	16.8	11.2	8.4	6.7	5.6	4.8	4.2	3.7
0.59	1.936	34.9	17.4	11.6	8.7	7.0	5.8	5.0	4.4	3.9
0.60	1.968	36.1	18.0	12.0	9.0	7.2	6.0	5.1	4.5	4.0
0.61	2.001	37.3	18.6	12.4	9.3	7.4	6.2	5.3	4.7	4.1
0.62	2.034	38.6	19.2	12.8	9.6	7.6	6.4	5.5	4.8	4.3
0.63	2.067	39.8	19.8	13.2	9.9	8.0	6.6	5.7	5.0	4.4
0.64	2.100	41.1	20.5	13.7	10.2	8.2	6.8	5.9	5.1	4.6
0.65	2.133	42.4	21.1	14.1	10.6	8.4	7.0	6.0	5.3	4.7

Inclination corrections for 5, 10, 15, 20, 25, 30, 35, 40, and 45 meter lengths—Continued

Difference in elevation		Length in meters								
		5	10	15	20	25	30	35	40	45
Meters	*Feet*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*
0.66	2.165	43.8	21.8	14.5	10.9	8.7	7.3	6.2	5.4	4.8
0.67	2.198	45.1	22.5	15.0	11.2	9.0	7.5	6.4	5.6	5.0
0.68	2.231	46.5	23.2	15.4	11.6	9.2	7.7	6.6	5.8	5.2
0.69	2.264	47.8	23.8	15.9	11.9	9.5	7.9	6.8	6.0	5.3
0.70	2.297	49.2	24.5	16.3	12.3	9.8	8.2	7.0	6.1	5.4
0.71	2.329	50.7	25.2	16.8	12.6	10.1	8.4	7.2	6.3	5.6
0.72	2.362	52.1	25.9	17.3	13.0	10.4	8.6	7.4	6.5	5.8
0.73	2.395	53.6	26.6	17.8	13.3	10.7	8.9	7.6	6.7	5.9
0.74	2.428	55.1	27.4	18.3	13.7	11.0	9.1	7.8	6.8	6.1
0.75	2.461	56.6	28.1	18.8	14.1	11.3	9.4	8.0	7.0	6.3
0.76	2.493	58.1	28.9	19.3	14.4	11.6	9.6	8.3	7.2	6.4
0.77	2.526	59.6	29.7	19.8	14.8	11.9	9.9	8.5	7.4	6.6
0.78	2.559	61.2	30.5	20.3	15.2	12.2	10.1	8.7	7.6	6.8
0.79	2.592	62.8	31.3	20.8	15.6	12.5	10.4	8.9	7.8	6.9
0.80	2.625	64.4	32.1	21.4	16.0	12.8	10.7	9.1	8.0	7.1
0.81	2.657	66.0	32.9	21.9	16.4	13.1	10.9	9.4	8.2	7.3
0.82	2.690	67.7	33.7	22.4	16.8	13.4	11.2	9.6	8.4	7.5
0.83	2.723	69.4	34.5	23.0	17.2	13.7	11.5	9.8	8.6	7.7
0.84	2.756	71.1	35.3	23.5	17.6	14.1	11.8	10.1	8.8	7.8
0.85	2.789	72.8	36.2	24.1	18.1	14.5	12.0	10.3	9.0	8.0
0.86	2.822	74.5	37.0	24.7	18.5	14.8	12.3	10.6	9.2	8.2
0.87	2.854	76.3	37.9	25.2	18.9	15.1	12.6	10.8	9.5	8.4
0.88	2.887	78.0	38.8	25.8	19.4	15.5	12.9	11.1	9.7	8.6
0.89	2.920	79.8	39.7	26.4	19.8	15.8	13.2	11.3	9.9	8.8
0.90	2.953	81.7	40.6	27.0	20.3	16.2	13.5	11.6	10.1	9.0
0.91	2.986	83.5	41.5	27.6	20.7	16.6	13.8	11.8	10.4	9.2
0.92	3.018	85.4	42.4	28.2	21.2	16.9	14.1	12.1	10.6	9.4
0.93	3.051	87.3	43.3	28.8	21.6	17.3	14.4	12.4	10.8	9.6
0.94	3.084	89.2	44.3	29.5	22.1	17.7	14.7	12.6	11.0	9.8
0.95	3.117	91.1	45.2	30.1	22.6	18.1	15.0	12.9	11.3	10.0
0.96	3.150	93.0	46.2	30.8	23.1	18.5	15.4	13.2	11.5	10.2
0.97	3.182	95.0	47.2	31.4	23.5	18.8	15.7	13.4	11.8	10.5
0.98	3.215	97.0	48.1	32.0	24.0	19.2	16.0	13.7	12.0	10.7
0.99	3.248	99.0	49.1	32.7	24.5	19.6	16.3	14.0	12.3	10.9
1.00	3.281	101.0	50.1	33.3	25.0	20.0	16.7	14.3	12.5	11.1
1.01	3.314	103.1	51.1	34.0	25.5	20.4	17.0	14.6	12.8	11.3
1.02	3.346	105.1	52.2	34.7	26.0	20.8	17.3	14.9	13.0	11.6
1.03	3.379	107.2	53.2	35.4	26.5	21.2	17.7	15.2	13.3	11.8
1.04	3.412	109.4	54.2	36.1	27.0	21.6	18.0	15.5	13.5	12.0
1.05	3.445	111.5	55.3	36.8	27.6	22.1	18.4	15.8	13.8	12.2
1.06	3.478	113.6	56.3	37.5	28.1	22.5	18.7	16.1	14.0	12.5
1.07	3.510	115.8	57.4	38.2	28.6	22.9	19.1	16.4	14.3	12.7
1.08	3.543	118.0	58.5	38.9	29.2	23.4	19.4	16.7	14.6	13.0
1.09	3.576	120.2	59.6	39.6	29.7	23.8	19.8	17.0	14.9	13.2
1.10	3.609	122.5	60.7	40.4	30.3	24.2	20.2	17.3	15.1	13.4
1.11	3.642	124.8	61.8	41.1	30.8	24.6	20.5	17.6	15.4	13.7
1.12	3.675	127.1	62.9	41.9	31.4	25.0	20.9	17.9	15.7	13.9
1.13	3.707	129.4	64.1	42.6	31.9	25.5	21.3	18.2	16.0	14.2
1.14	3.740	131.7	65.2	43.3	32.5	26.0	21.7	18.6	16.2	14.4
1.15	3.773	134.0	66.3	44.1	33.1	26.4	22.0	18.9	16.5	14.7
1.16	3.806	136.4	67.5	44.9	33.7	26.9	22.4	19.2	16.8	14.9
1.17	3.839	138.8	68.7	45.7	34.3	27.4	22.8	19.6	17.1	15.2
1.18	3.871	141.2	69.9	46.5	34.8	27.9	23.2	19.9	17.4	15.5
1.19	3.904	143.7	71.1	47.3	35.4	28.3	23.6	20.2	17.7	15.7
1.20	3.937	146.1	72.3	48.1	36.0	28.8	24.0	20.6	18.0	16.0
1.21	3.970	148.6	73.5	48.9	36.6	29.3	24.4	20.9	18.3	16.3
1.22	4.003	151.1	74.7	49.7	37.2	29.8	24.8	21.3	18.6	16.5
1.23	4.035	153.7	75.9	50.5	37.9	30.3	25.2	21.6	18.9	16.8
1.24	4.068	156.2	77.1	51.3	38.5	30.8	25.6	22.0	19.2	17.1
1.25	4.101	158.8	78.4	52.2	39.1	31.2	26.0	22.3	19.5	17.4
1.26	4.134	161.4	79.7	53.0	39.7	31.7	26.5	22.7	19.8	17.6
1.27	4.167	164.0	81.0	53.8	40.4	32.3	26.9	23.0	20.2	17.9
1.28	4.199	166.6	82.3	54.7	41.0	32.8	27.3	23.4	20.5	18.2
1.29	4.232	169.3	83.6	55.6	41.6	33.3	27.7	23.8	20.8	18.5
1.30	4.265	172.0	84.9	56.4	42.3	33.8	28.2	24.1	21.1	18.8
1.31	4.298	174.7	86.2	57.3	42.9	34.4	28.6	24.5	21.5	19.1
1.32	4.331	177.4	87.5	58.2	43.6	34.9	29.0	24.9	21.8	19.4
1.33	4.364	180.1	88.8	59.1	44.3	35.4	29.5	25.3	22.1	19.7
1.34	4.396	182.9	90.2	60.0	44.9	35.9	29.9	25.6	22.4	19.9
1.35	4.429	185.7	91.5	60.9	45.6	36.5	30.4	26.0	22.8	20.2

Inclination corrections for 5, 10, 15, 20, 25, 30, 35, 40, and 45 meter lengths—Continued

Difference in elevation		Length in meters								
		5	10	15	20	25	30	35	40	45
Meters	*Feet*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*
1.36	4.462	188.5	92.9	61.8	46.3	37.0	30.8	26.4	23.1	20.5
1.37	4.495	191.4	94.3	62.7	47.0	37.6	31.3	26.8	23.5	20.8
1.38	4.528	194.2	95.7	63.6	47.7	38.2	31.7	27.2	23.8	21.2
1.39	4.560	197.1	97.1	64.5	48.4	38.7	32.2	27.6	24.2	21.5
1.40	4.593	200.0	98.5	65.5	49.1	39.3	32.7	28.0	24.5	21.8
1.41	4.626	202.9	99.9	66.4	49.8	39.8	33.1	28.4	24.9	22.1
1.42	4.659	205.9	101.3	67.3	50.5	40.4	33.6	28.8	25.2	22.4
1.43	4.692	208.9	102.8	68.3	51.2	40.9	34.1	29.2	25.6	22.7
1.44	4.724	211.8	104.2	69.3	51.9	41.5	34.6	29.6	25.9	23.0
1.45	4.757	214.9	105.7	70.2	52.6	42.1	35.0	30.0	26.3	23.4
1.46	4.790	217.9	107.2	71.2	53.4	42.7	35.5	30.5	26.6	23.7
1.47	4.823	221.0	108.6	72.2	54.1	43.3	36.0	30.9	27.0	24.0
1.48	4.856	224.1	110.1	73.2	54.8	43.9	36.5	31.3	27.4	24.3
1.49	4.888	227.2	111.6	74.2	55.6	44.5	37.0	31.7	27.8	24.7
1.50	4.921	230.3	113.1	75.2	56.3	45.1	37.5	32.1	28.1	25.0
1.51	4.954	233.5	114.7	76.2	57.1	45.7	38.0	32.6	28.5	25.3
1.52	4.987	236.6	116.2	77.2	57.8	46.3	38.5	33.0	28.9	25.7
1.53	5.020	239.8	117.7	78.2	58.6	46.9	39.0	33.4	29.3	26.0
1.54	5.052	243.1	119.3	79.3	59.4	47.5	39.5	33.9	29.6	26.3
1.55	5.085	246.3	120.8	80.3	60.2	48.1	40.0	34.3	30.0	26.7
1.56	5.118	249.6	122.4	81.3	60.9	48.8	40.6	34.8	30.4	27.0
1.57	5.151	252.9	124.0	82.4	61.7	49.4	41.1	35.2	30.8	27.4
1.58	5.184	256.2	125.6	83.4	62.5	50.0	41.6	35.7	31.2	27.7
1.59	5.217	259.5	127.2	84.5	63.3	50.6	42.1	36.1	31.6	28.1
1.60	5.249	262.9	128.8	85.6	64.1	51.3	42.7	36.6	32.0	28.4
1.61	5.282	266.3	130.4	86.6	64.9	52.0	43.2	37.0	32.4	28.8
1.62	5.315	269.7	132.1	87.7	65.7	52.6	43.7	37.5	32.8	29.2
1.63	5.348	273.2	133.7	88.8	66.5	53.3	44.3	38.0	33.2	29.5
1.64	5.381	276.6	135.4	89.9	67.4	53.9	44.8	38.4	33.6	29.9
1.65	5.413	280.1	137.0	91.0	68.2	54.6	45.4	38.9	34.0	30.3
1.66	5.446	283.6	138.7	92.1	69.0	55.2	45.9	39.4	34.4	30.6
1.67	5.479	287.1	140.4	93.2	69.8	55.9	46.5	39.8	34.9	31.0
1.68	5.512	290.7	142.1	94.4	70.7	56.5	47.0	40.3	35.3	31.4
1.69	5.545	294.3	143.8	95.5	71.5	57.2	47.6	40.8	35.7	31.7
1.70	5.577	297.9	145.5	96.6	72.4	57.9	48.2	41.3	36.1	32.1
1.71	5.610	301.5	147.3	97.8	73.2	58.6	48.7	41.8	36.6	32.5
1.72	5.643	305.2	149.0	98.9	74.1	59.3	49.3	42.3	37.0	32.9
1.73	5.676	308.8	150.8	100.1	75.0	59.9	49.9	42.8	37.4	33.2
1.74	5.709	312.5	152.5	101.2	75.8	60.6	50.5	43.3	37.8	33.6
1.75	5.741	316.2	154.3	102.4	76.7	61.3	51.0	43.8	38.3	34.0
1.76	5.774	320.0	156.1	103.6	77.6	62.0	51.6	44.3	38.7	34.4
1.77	5.807	323.8	157.9	104.8	78.5	62.7	52.2	44.8	39.2	34.8
1.78	5.840	327.6	159.7	106.0	79.4	63.5	52.8	45.3	39.6	35.2
1.79	5.873	331.4	161.5	107.2	80.3	64.2	53.4	45.8	40.1	35.6
1.80	5.906	335.2	163.3	108.4	81.2	64.9	54.0	46.3	40.5	36.0
1.81	5.938	339.1	165.1	109.6	82.1	65.6	54.6	46.8	41.0	36.4
1.82	5.971	343.0	167.0	110.8	83.0	66.3	55.2	47.3	41.4	36.8
1.83	6.004	346.9	168.8	112.0	83.9	67.1	55.8	47.8	41.9	37.2
1.84	6.037	350.9	170.7	113.3	84.8	67.8	56.4	48.4	42.3	37.6
1.85	6.070	354.8	172.6	114.5	85.7	68.5	57.0	48.9	42.8	38.0
1.86	6.102	358.8	174.5	115.8	86.7	69.2	57.7	49.4	43.2	38.4
1.87	6.135	362.9	176.4	117.0	87.6	70.0	58.3	50.0	43.7	38.9
1.88	6.168	366.9	178.3	118.3	88.6	70.8	59.0	50.5	44.2	39.3
1.89	6.201	371.0	180.2	119.6	89.5	71.5	59.6	51.0	44.7	39.7
1.90	6.234	375.1	182.2	120.8	90.5	72.3	60.2	51.6	45.1	40.1
1.91	6.266	379.2	184.1	122.1	91.4	73.1	60.8	52.1	45.6	40.5
1.92	6.209	383.3	186.1	123.4	92.4	73.9	61.4	52.7	46.1	41.0
1.93	6.332	387.5	188.0	124.7	93.3	74.6	62.1	53.2	46.6	41.4
1.94	6.365	391.7	190.0	126.0	94.3	75.4	62.7	53.8	47.0	41.8
1.95	6.398	395.9	192.0	127.3	95.3	76.2	63.4	54.3	47.5	42.2
1.96	6.430	400.2	194.0	128.6	96.3	76.9	64.0	54.9	48.0	42.7
1.97	6.463	404.4	196.0	129.9	97.3	77.7	64.7	55.4	48.5	43.1
1.98	6.496	408.7	198.0	131.2	98.3	78.5	65.3	56.0	49.0	43.6
1.99	6.529	413.1	200.0	132.6	99.3	79.3	66.0	56.6	49.5	44.0
2.00	6.562	417.4	202.0	133.9	100.3	80.1	66.7	57.2	50.0	44.4

Factors for computing catenary correction

[Combined side and top arguments = weight of tape in grams per meter. Tabular values $=\frac{1}{24}\left(\frac{w}{t}\right)^2\times 10^{16}$ when $t=15{,}000$ grams. To obtain catenary correction multiply tabular value by cube of length between supports and point off 10 decimal places. The result will be in meters]

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
20	741	748	756	763	771	778	786	794	801	809
21	817	824	832	840	848	856	864	872	880	888
22	896	904	913	921	929	938	946	954	963	971
23	980	988	997	1,005	1,014	1,023	1,031	1,040	1,049	1,058
24	1,067	1,076	1,085	1,094	1,103	1,112	1,121	1,130	1,139	1,148
25	1,157	1,167	1,176	1,185	1,195	1,204	1,214	1,223	1,233	1,242
26	1,252	1,262	1,271	1,281	1,291	1,300	1,310	1,320	1,330	1,340
27	1,350	1,360	1,370	1,380	1,390	1,400	1,411	1,421	1,431	1,442
28	1,452	1,462	1,473	1,483	1,494	1,504	1,515	1,525	1,536	1,547
29	1,557	1,568	1,579	1,590	1,601	1,612	1,623	1,634	1,645	1,656
30	1,667									

Catenary corrections for various lengths and weights of tape

Length of tape (meters)	Weight of tape in grams per meter									
	20.0	20.1	20.2	20.3	20.4	20.5	20.6	20.7	20.8	20.9
	Mm.	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*
5	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
10	0.07	0.07	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
15	0.25	0.25	0.26	0.26	0.26	0.26	0.27	0.27	0.27	0.27
20	0.59	0.60	0.60	0.61	0.62	0.62	0.63	0.63	0.64	0.65
25	1.16	1.17	1.18	1.19	1.20	1.22	1.23	1.24	1.25	1.26
30	2.00	2.02	2.04	2.06	2.08	2.10	2.12	2.14	2.16	2.18
35	3.18	3.21	3.24	3.27	3.30	3.34	3.37	3.40	3.44	3.47
40	4.74	4.79	4.84	4.88	4.93	4.98	5.03	5.08	5.13	5.18
45	6.75	6.82	6.89	6.95	7.02	7.09	7.16	7.23	7.30	7.37
50	9.26	9.35	9.45	9.54	9.63	9.73	9.82	9.92	10.01	10.11
	21.0	21.1	21.2	21.3	21.4	21.5	21.6	21.7	21.8	21.9
5	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
10	0.08	0.08	0.08	0.08	0.08	0.09	0.09	0.09	0.09	0.09
15	0.28	0.28	0.28	0.28	0.29	0.29	0.29	0.29	0.30	0.30
20	0.65	0.66	0.67	0.67	0.68	0.68	0.69	0.70	0.70	0.71
25	1.28	1.29	1.30	1.31	1.33	1.34	1.35	1.36	1.38	1.30
30	2.21	2.23	2.25	2.27	2.29	2.31	2.33	2.35	2.38	2.40
35	3.50	3.53	3.57	3.60	3.64	3.67	3.70	3.74	3.77	3.81
40	5.23	5.28	5.33	5.38	5.43	5.48	5.53	5.58	5.63	5.68
45	7.44	7.51	7.58	7.66	7.73	7.80	7.87	7.95	8.02	8.09
50	10.21	10.31	10.40	10.50	10.60	10.70	10.80	10.90	11.00	11.10
	22.0	22.1	22.2	22.3	22.4	22.5	22.6	22.7	22.8	22.9
5	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
10	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.10	0.10	0.10
15	0.30	0.31	0.31	0.31	0.31	0.32	0.32	0.32	0.32	0.33
20	0.72	0.72	0.73	0.74	0.74	0.75	0.76	0.76	0.77	0.78
25	1.40	1.41	1.43	1.44	1.45	1.46	1.48	1.49	1.50	1.52
30	2.42	2.44	2.46	2.49	2.51	2.53	2.55	2.58	2.60	2.62
35	3.84	3.88	3.91	3.95	3.98	4.02	4.06	4.09	4.13	4.16
40	5.74	5.79	5.84	5.89	5.95	6.00	6.05	6.11	6.16	6.22
45	8.17	8.24	8.32	8.39	8.47	8.54	8.62	8.70	8.77	8.85
50	11.20	11.31	11.41	11.51	11.61	11.72	11.82	11.93	12.03	12.14
	23.0	23.1	23.2	23.3	23.4	23.5	23.6	23.7	23.8	23.9
5	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.11
15	0.33	0.33	0.34	0.34	0.34	0.35	0.35	0.35	0.35	0.36
20	0.78	0.79	0.80	0.80	0.81	0.82	0.83	0.83	0.84	0.85
25	1.53	1.54	1.56	1.57	1.58	1.60	1.61	1.63	1.64	1.65
30	2.65	2.67	2.69	2.71	2.74	2.76	2.78	2.81	2.83	2.86
35	4.20	4.24	4.27	4.31	4.35	4.38	4.42	4.46	4.50	4.54
40	6.27	6.32	6.38	6.43	6.49	6.55	6.60	6.66	6.71	6.77
45	8.93	9.00	9.08	9.16	9.24	9.32	9.40	9.48	9.56	9.64
50	12.25	12.35	12.46	12.57	12.68	12.78	12.89	13.00	13.11	13.22

Catenary corrections for various lengths and weights of tape—Continued

Length of tape (meters)	Weight of tape in grams per meter									
	24.0	24.1	24.2	24.3	24.4	24.5	24.6	24.7	24.8	24.9
	Mm.	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*
5	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
10	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
15	0.36	0.36	0.37	0.37	0.37	0.38	0.38	0.38	0.38	0.39
20	0.85	0.86	0.87	0.87	0.88	0.89	0.90	0.90	0.91	0.92
25	1.67	1.68	1.69	1.71	1.72	1.74	1.75	1.77	1.78	1.79
30	2.88	2.90	2.93	2.95	2.98	3.00	3.03	3.05	3.08	3.10
35	4.57	4.61	4.65	4.69	4.73	4.77	4.80	4.84	4.88	4.92
40	6.83	6.88	6.94	7.00	7.06	7.11	7.17	7.23	7.29	7.35
45	9.72	9.80	9.88	9.96	10.05	10.13	10.21	10.30	10.38	10.46
50	13.33	13.44	13.56	13.67	13.78	13.89	14.01	14.12	14.24	14.35
	25.0	25.1	25.2	25.3	25.4	25.5	25.6	25.7	25.8	25.9
5	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.02
10	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12
15	0.39	0.39	0.40	0.40	0.40	0.41	0.41	0.41	0.42	0.42
20	0.93	0.93	0.94	0.95	0.96	0.96	0.97	0.98	0.99	0.99
25	1.81	1.82	1.84	1.85	1.87	1.88	1.90	1.91	1.93	1.94
30	3.13	3.15	3.18	3.20	3.23	3.25	3.28	3.30	3.33	3.35
35	4.96	5.00	5.04	5.08	5.12	5.16	5.20	5.24	5.29	5.33
40	7.41	7.47	7.53	7.59	7.65	7.71	7.77	7.83	7.89	7.95
45	10.55	10.63	10.72	10.80	10.89	10.97	11.06	11.15	11.23	11.32
50	14.47	14.58	14.70	14.82	14.93	15.05	15.17	15.29	15.41	15.53
	26.0	26.1	26.2	26.3	26.4	26.5	26.6	26.7	26.8	26.9
5	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
10	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13
15	0.42	0.43	0.43	0.43	0.44	0.44	0.44	0.45	0.45	0.45
20	1.00	1.01	1.02	1.02	1.03	1.04	1.05	1.06	1.06	1.07
25	1.96	1.97	1.99	2.00	2.02	2.03	2.05	2.06	2.08	2.09
30	3.38	3.41	3.43	3.46	3.48	3.51	3.54	3.56	3.59	3.62
35	5.37	5.41	5.45	5.49	5.53	5.58	5.62	5.66	5.70	5.75
40	8.01	8.07	8.14	8.20	8.26	8.32	8.39	8.45	8.51	8.58
45	11.41	11.50	11.58	11.67	11.76	11.85	11.94	12.03	12.12	12.21
50	15.65	15.77	15.89	16.01	16.13	16.26	16.38	16.50	16.63	16.75
	27.0	27.1	27.2	27.3	27.4	27.5	27.6	27.7	27.8	27.9
5	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
10	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14
15	0.46	0.46	0.46	0.47	0.47	0.47	0.48	0.48	0.48	0.49
20	1.08	1.09	1.10	1.10	1.11	1.12	1.13	1.14	1.14	1.15
25	2.11	2.13	2.14	2.16	2.17	2.19	2.20	2.22	2.24	2.25
30	3.64	3.67	3.70	3.73	3.75	3.78	3.81	3.84	3.86	3.89
35	5.79	5.83	5.87	5.92	5.96	6.00	6.05	6.09	6.14	6.18
40	8.64	8.70	8.77	8.83	8.90	8.96	9.03	9.09	9.16	9.23
45	12.30	12.39	12.48	12.58	12.67	12.76	12.85	12.95	13.04	13.14
50	16.88	17.00	17.13	17.25	17.38	17.51	17.63	17.76	17.89	18.02
	28.0	28.1	28.2	28.3	28.4	28.5	28.6	28.7	28.8	28.9
5	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
10	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
15	0.49	0.49	0.50	0.50	0.50	0.51	0.51	0.51	0.52	0.52
20	1.16	1.17	1.18	1.19	1.19	1.20	1.21	1.22	1.23	1.24
25	2.27	2.28	2.30	2.32	2.33	2.35	2.37	2.38	2.40	2.42
30	3.92	3.95	3.98	4.00	4.03	4.06	4.09	4.12	4.15	4.18
35	6.22	6.27	6.31	6.36	6.40	6.45	6.49	6.54	6.59	6.63
40	9.29	9.36	9.43	9.49	9.56	9.63	9.69	9.76	9.83	9.90
45	13.23	13.32	13.42	13.52	13.61	13.71	13.80	13.90	14.00	14.09
50	18.15	18.28	18.41	18.54	18.67	18.80	18.93	19.07	19.20	19.33
	29.0	29.1	29.2	29.3	29.4	29.5	29.6	29.7	29.8	29.9
5	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
10	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.17
15	0.53	0.53	0.53	0.54	0.54	0.54	0.55	0.55	0.56	0.56
20	1.25	1.26	1.26	1.27	1.28	1.29	1.30	1.31	1.32	1.32
25	2.43	2.45	2.47	2.48	2.50	2.52	2.54	2.55	2.57	2.59
30	4.21	4.23	4.26	4.29	4.32	4.35	4.38	4.41	4.44	4.47
35	6.68	6.72	6.77	6.82	6.86	6.91	6.96	7.00	7.05	7.10
40	9.97	10.04	10.11	10.17	10.24	10.31	10.38	10.45	10.52	10.60
45	14.19	14.29	14.39	14.49	14.59	14.69	14.79	14.89	14.99	15.09
50	19.47	19.60	19.74	19.87	20.01	20.14	20.28	20.42	20.56	20.69

Temperature corrections for steel tapes

[Coefficient of expansion = 0.0000116 per degree centigrade. Standard temperature of tape = 20 degrees centigrade. For temperatures above 20 the corrections are plus, for temperatures below 20 the corrections are minus]

Length of tape (meters)	Temperature of tape in degrees centigrade																								
	19 21	18 22	17 23	16 24	15 25	14 26	13 27	12 28	11 29	10 30	9 31	8 32	7 33	6 34	5 35	4 36	3 37	2 38	1 39	0 40	−1 41	−2 42	−3 43	−4 44	−5 45
	Mm.	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*
1	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.3	0.3	0.3
2	0.0	0.0	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.4	0.4	0.4	0.4	0.5	0.5	0.5	0.5	0.6	0.6
3	0.0	0.1	0.1	0.1	0.2	0.2	0.2	0.3	0.3	0.3	0.4	0.4	0.5	0.5	0.5	0.6	0.6	0.6	0.7	0.7	0.7	0.8	0.8	0.8	0.9
4	0.0	0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.4	0.5	0.5	0.6	0.6	0.6	0.7	0.7	0.8	0.8	0.9	0.9	1.0	1.0	1.1	1.1	1.2
5	0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.5	0.5	0.6	0.6	0.7	0.8	0.8	0.9	0.9	1.0	1.0	1.1	1.2	1.2	1.3	1.3	1.4	1.4
6	0.1	0.1	0.2	0.3	0.3	0.4	0.5	0.6	0.6	0.7	0.8	0.8	0.9	1.0	1.0	1.1	1.2	1.3	1.3	1.4	1.5	1.5	1.6	1.7	1.7
7	0.1	0.2	0.2	0.3	0.4	0.5	0.6	0.6	0.7	0.8	0.9	1.0	1.1	1.1	1.2	1.3	1.4	1.5	1.5	1.6	1.7	1.8	1.9	2.0	2.0
8	0.1	0.2	0.3	0.4	0.5	0.6	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	1.9	2.0	2.1	2.2	2.3
9	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5	2.6
10	0.1	0.2	0.3	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.3	1.4	1.5	1.6	1.7	1.9	2.0	2.1	2.2	2.3	2.4	2.6	2.7	2.8	2.9
11	0.1	0.3	0.4	0.5	0.6	0.8	0.9	1.0	1.1	1.3	1.4	1.5	1.7	1.8	1.9	2.0	2.2	2.3	2.4	2.6	2.7	2.8	2.9	3.1	3.2
12	0.1	0.3	0.4	0.6	0.7	0.8	1.0	1.1	1.3	1.4	1.5	1.7	1.8	1.9	2.1	2.2	2.4	2.5	2.6	2.8	2.9	3.1	3.2	3.3	3.5
13	0.2	0.3	0.5	0.6	0.8	0.9	1.1	1.2	1.4	1.5	1.7	1.8	2.0	2.1	2.3	2.4	2.6	2.7	2.9	3.0	3.2	3.3	3.5	3.6	3.8
14	0.2	0.3	0.5	0.6	0.8	1.0	1.1	1.3	1.5	1.6	1.8	1.9	2.1	2.3	2.4	2.6	2.8	2.9	3.1	3.2	3.4	3.6	3.7	3.9	4.1
15	0.2	0.3	0.5	0.7	0.9	1.0	1.2	1.4	1.6	1.7	1.9	2.1	2.3	2.4	2.6	2.8	3.0	3.1	3.3	3.5	3.7	3.8	4.0	4.2	4.4
16	0.2	0.4	0.6	0.7	0.9	1.1	1.3	1.5	1.7	1.9	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.3	3.5	3.7	3.9	4.1	4.3	4.5	4.6
17	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.5	3.7	3.9	4.1	4.3	4.5	4.7	4.9
18	0.2	0.4	0.6	0.8	1.0	1.3	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2
19	0.2	0.4	0.7	0.9	1.1	1.3	1.5	1.8	2.0	2.2	2.4	2.6	2.9	3.1	3.3	3.5	3.7	4.0	4.2	4.4	4.6	4.8	5.1	5.3	5.5
20	0.2	0.5	0.7	0.9	1.2	1.4	1.6	1.9	2.1	2.3	2.6	2.8	3.0	3.2	3.5	3.7	3.9	4.2	4.4	4.6	4.9	5.1	5.3	5.6	5.8
21	0.2	0.5	0.7	1.0	1.2	1.5	1.7	1.9	2.2	2.4	2.7	2.9	3.2	3.4	3.7	3.9	4.1	4.4	4.6	4.9	5.1	5.4	5.6	5.8	6.1
22	0.3	0.5	0.8	1.0	1.3	1.5	1.8	2.0	2.3	2.6	2.8	3.1	3.3	3.6	3.8	4.1	4.3	4.6	4.8	5.1	5.4	5.6	5.9	6.1	6.4
23	0.3	0.5	0.8	1.1	1.3	1.6	1.9	2.1	2.4	2.7	2.9	3.2	3.5	3.7	4.0	4.3	4.5	4.8	5.1	5.3	5.6	5.9	6.1	6.4	6.7
24	0.3	0.6	0.8	1.1	1.4	1.7	1.9	2.2	2.5	2.8	3.1	3.3	3.6	3.9	4.2	4.5	4.7	5.0	5.3	5.6	5.8	6.1	6.4	6.7	7.0
25	0.3	0.6	0.9	1.2	1.4	1.7	2.0	2.3	2.6	2.9	3.2	3.5	3.8	4.1	4.4	4.6	4.9	5.2	5.5	5.8	6.1	6.4	6.7	7.0	7.2
26	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0	3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.0	6.3	6.6	6.9	7.2	7.5
27	0.3	0.6	0.9	1.3	1.6	1.9	2.2	2.5	2.8	3.1	3.4	3.8	4.1	4.4	4.7	5.0	5.3	5.6	6.0	6.3	6.6	6.9	7.2	7.5	7.8
28	0.3	0.6	1.0	1.3	1.6	1.9	2.3	2.6	2.9	3.2	3.6	3.9	4.2	4.5	4.9	5.2	5.5	5.8	6.2	6.5	6.8	7.1	7.5	7.8	8.1
29	0.3	0.7	1.0	1.3	1.7	2.0	2.4	2.7	3.0	3.4	3.7	4.0	4.4	4.7	5.0	5.4	5.7	6.1	6.4	6.7	7.1	7.4	7.7	8.1	8.4
30	0.3	0.7	1.0	1.4	1.7	2.1	2.4	2.8	3.1	3.5	3.8	4.2	4.5	4.9	5.2	5.6	5.9	6.3	6.6	7.0	7.3	7.7	8.0	8.4	8.7

Table of log m

[Computed for the Clarke spheroid of 1866 as expressed in meters]

Latitude	Log *m*	Latitude	Log *m*	Latitude	Log *m*	Latitude	Log *m*
° ′		° ′		° ′		° ′	
0 00	1.40695 −10	20 00	1.40626 −10	40 00	1.40452 −10	60 00	1.40253 −10
0 30	695 −10	20 30	623 −10	40 30	446 −10	60 30	249 −10
1 00	695 −10	21 00	619 −10	41 00	441 −10	61 00	244 −10
1 30	694 −10	21 30	616 −10	41 30	436 −10	61 30	240 −10
2 00	694 −10	22 00	612 −10	42 00	431 −10	62 00	235 −10
2 30	694 −10	22 30	608 −10	42 30	426 −10	62 30	231 −10
3 00	693 −10	23 00	605 −10	43 00	421 −10	63 00	227 −10
3 30	693 −10	23 30	601 −10	43 30	416 −10	63 30	223 −10
4 00	692 −10	24 00	597 −10	44 00	411 −10	64 00	219 −10
4 30	691 −10	24 30	594 −10	44 30	406 −10	64 30	215 −10
5 00	690 −10	25 00	590 −10	45 00	400 −10	65 00	210 −10
5 30	689 −10	25 30	586 −10	45 30	395 −10	65 30	207 −10
6 00	688 −10	26 00	582 −10	46 00	390 −10	66 00	203 −10
6 30	687 −10	26 30	578 −10	46 30	385 −10	66 30	199 −10
7 00	686 −10	27 00	573 −10	47 00	380 −10	67 00	195 −10
7 30	685 −10	27 30	569 −10	47 30	375 −10	67 30	192 −10
8 00	683 −10	28 00	565 −10	48 00	369 −10	68 00	188 −10
8 30	682 −10	28 30	560 −10	48 30	364 −10	68 30	185 −10
9 00	680 −10	29 00	556 −10	49 00	359 −10	69 00	181 −10
9 30	679 −10	29 30	552 −10	49 30	354 −10	69 30	178 −10
10 00	677 −10	30 00	548 −10	50 00	349 −10	70 00	174 −10
10 30	675 −10	30 30	544 −10	50 30	344 −10	70 30	171 −10
11 00	673 −10	31 00	539 −10	51 00	339 −10	71 00	168 −10
11 30	671 −10	31 30	534 −10	51 30	334 −10	71 30	164 −10
12 00	669 −10	32 00	530 −10	52 00	329 −10	72 00	1.40161 −10
12 30	667 −10	32 30	525 −10	52 30	324 −10		
13 00	665 −10	33 00	520 −10	53 00	319 −10		
13 30	663 −10	33 30	516 −10	53 30	314 −10		
14 00	660 −10	34 00	511 −10	54 00	309 −10		
14 30	658 −10	34 30	506 −10	54 30	304 −10		
15 00	655 −10	35 00	501 −10	55 00	299 −10		
15 30	653 −10	35 30	496 −10	55 30	295 −10		
16 00	650 −10	36 00	491 −10	56 00	290 −10		
16 30	647 −10	36 30	486 −10	56 30	285 −10		
17 00	644 −10	37 00	482 −10	57 00	280 −10		
17 30	642 −10	37 30	477 −10	57 30	276 −10		
18 00	639 −10	38 00	472 −10	58 00	271 −10		
18 30	636 −10	38 30	467 −10	58 30	266 −10		
19 00	632 −10	39 00	462 −10	59 00	262 −10		
19 30	1.40629 −10	39 30	1.40457 −10	59 30	1.40257 −10		

Arc-sin corrections for inverse position computations

Log s_1	Arc-sin correction in units of seventh decimal of logarithms	Log $\Delta\phi$ or log $\Delta\lambda$	Log s_1	Arc-sin correction in units of seventh decimal of logarithms	Log $\Delta\phi$ or log $\Delta\lambda$	Log s_1	Arc-sin correction in units of seventh decimal of logarithms	Log $\Delta\phi$ or log $\Delta\lambda$
4.177	1	2.686	5.223	124	3.732	5.525	497	4.034
4.327	2	2.836	5.234	130	3.743	5.530	508	4.039
4.415	3	2.924	5.243	136	3.752	5.534	519	4.043
4.478	4	2.987	5.253	142	3.762	5.539	530	4.048
4.526	5	3.035	5.260	147	3.769	5.543	541	4.052
4.566	6	3.075	5.269	153	3.778	5.548	553	4.057
4.599	7	3.108	5.279	160	3.788	5.553	565	4.062
4.628	8	3.137	5.287	166	3.796	5.557	577	4.066
4.654	9	3.163	5.294	172	3.803	5.561	588	4.070
4.677	10	3.186	5.303	179	3.812	5.566	600	4.075
4.697	11	3.206	5.311	186	3.820	5.570	613	4.079
4.716	12	3.225	5.318	192	3.827	5.575	625	4.084
4.734	13	3.243	5.326	199	3.835	5.579	637	4.088
4.750	14	3.259	5.334	206	3.843	5.583	650	4.092
4.765	15	3.274	5.341	213	3.850	5.587	663	4.096
4.779	16	3.288	5.349	221	3.858	5.591	674	4.100
4.792	17	3.301	5.356	228	3.865	5.595	687	4.104
4.804	18	3.313	5.363	236	3.872	5.600	702	4.109
4.827	20	3.336	5.369	243	3.878	5.604	716	4.113
4.857	23	3.366	5.376	251	3.885	5.608	729	4.117
4.876	25	3.385	5.383	259	3.892	5.612	743	4.121
4.892	27	3.401	5.390	267	3.899	5.616	757	4.125
4.915	30	3.424	5.396	275	3.905	5.620	771	4.129
4.936	33	3.445	5.403	284	3.912	5.624	785	4.133
4.955	36	3.464	5.409	292	3.918	5.628	800	4.137
4.972	39	3.481	5.415	300	3.924	5.632	814	4.141
4.988	42	3.497	5.422	309	3.931	5.636	829	4.145
5.003	45	3.512	5.428	318	3.937	5.640	845	4.149
5.017	48	3.526	5.434	327	3.943	5.644	861	4.153
5.035	52	3.544	5.440	336	3.949	5.648	877	4.157
5.051	56	3.560	5.446	345	3.955	5.652	893	4.161
5.062	59	3.571	5.451	354	3.960	5.656	909	4.165
5.076	63	3.585	5.457	364	3.966	5.660	925	4.169
5.090	67	3.599	5.462	373	3.971	5.663	941	4.172
5.102	71	3.611	5.468	383	3.977	5.667	957	4.176
5.114	75	3.623	5.473	392	3.982	5.671	973	4.180
5.128	80	3.637	5.479	402	3.988	5.674	989	4.183
5.139	84	3.648	5.484	412	3.993	5.678	1005	4.187
5.151	89	3.660	5.489	422	3.998			
5.163	94	3.672	5.495	433	4.004			
5.172	98	3.681	5.500	443	4.009			
5.183	103	3.692	5.505	453	4.014			
5.193	108	3.702	5.510	464	4.019			
5.205	114	3.714	5.515	474	4.024			
5.214	119	3.723	5.520	486	4.029			

Proportional change in a number corresponding to a change in its logarithm

[Computed from the formula $\frac{\Delta N}{N}=\frac{\Delta \log N}{\mu}$, where μ = modulus of common logarithms = 0.4343]

$\Delta \log N$ units of seventh decimal place	$\frac{\Delta N}{N}$ one part in—	$\Delta \log N$ units of seventh decimal place	$\frac{\Delta N}{N}$ one part in—	$\Delta \log N$ units of seventh decimal place	$\frac{\Delta N}{N}$ one part in—	$\Delta \log N$ units of seventh decimal place	$\frac{\Delta N}{N}$ one part in—
1	4,343,000	26	167,000	51	85,000	76	57,100
2	2,171,000	27	161,000	52	84,000	77	56,400
3	1,448,000	28	155,000	53	82,000	78	55,700
4	1,086,000	29	150,000	54	80,000	79	55,000
5	869,000	30	145,000	55	79,000	80	54,300
6	724,000	31	140,000	56	78,000	81	53,600
7	620,000	32	136,000	57	76,000	82	53,000
8	543,000	33	132,000	58	75,000	83	52,300
9	483,000	34	128,000	59	74,000	84	51,700
10	434,000	35	124,000	60	72,000	85	51,100
11	395,000	36	121,000	61	71,000	86	50,500
12	362,000	37	117,000	62	70,000	87	49,900
13	334,000	38	114,000	63	69,000	88	49,400
14	310,000	39	111,000	64	68,000	89	48,800
15	290,000	40	109,000	65	67,000	90	48,300
16	271,000	41	106,000	66	66,000	91	47,700
17	255,000	42	103,000	67	65,000	92	47,200
18	241,000	43	101,000	68	64,000	93	46,700
19	229,000	44	99,000	69	63,000	94	46,200
20	217,000	45	97,000	70	62,000	95	45,700
21	207,000	46	94,000	71	61,000	96	45,200
22	197,000	47	92,000	72	60,000	97	44,800
23	189,000	48	90,000	73	59,000	98	44,300
24	181,000	49	89,000	74	58,700	99	43,900
25	174,000	50	87,000	75	57,900	100	43,400

Logarithms of radii of curvature of the earth's surface (in meters)

[Based upon Clarke's spheroid of 1866 as expressed in meters].

Azimuth (degrees)	Latitude								
	0°	1°	2°	3°	4°	5°	6°	7°	8°
0	6.80175	6.80175	6.80175	6.80176	6.80177	6.80178	6.80180	6.80181	6.80183
5	177	177	178	178	179	180	182	184	186
10	184	184	184	185	186	187	188	190	192
15	195	195	195	196	197	198	199	201	203
20	209	209	210	210	211	212	214	215	217
25	227	228	228	228	229	230	232	233	235
30	248	249	249	250	250	251	252	254	256
35	272	272	272	273	273	274	276	277	278
40	296	297	297	297	298	299	300	301	303
45	322	322	322	323	324	324	325	326	328
50	348	348	348	348	349	350	351	352	353
55	373	373	373	373	374	374	375	376	377
60	396	396	396	396	397	398	398	399	400
65	417	417	417	418	418	418	419	420	421
70	435	435	436	436	436	437	437	438	439
75	450	450	450	450	451	451	452	452	453
80	461	461	461	461	462	462	463	463	464
85	468	468	468	468	468	469	469	470	470
90	470	470	470	470	471	471	472	472	473

Azimuth (degrees)	Latitude								
	8°	9°	10°	11°	12°	13°	14°	15°	16°
0	6.80183	6.80186	6.80188	6.80191	6.80194	6.80197	6.80201	6.80204	6.80208
5	186	188	190	193	196	199	203	206	210
10	192	194	197	200	202	206	209	213	217
15	203	205	207	210	213	216	219	223	227
20	217	219	222	224	227	230	233	236	240
25	235	237	239	242	244	247	250	254	257
30	256	257	260	262	264	267	270	273	276
35	278	280	282	284	287	289	292	295	298
40	303	304	306	308	310	313	315	318	321
45	328	329	331	333	335	337	339	342	344
50	353	354	356	358	359	361	364	366	368
55	377	379	380	382	383	385	387	389	391
60	400	401	403	404	406	407	409	411	413
65	421	422	423	424	426	427	429	430	432
70	439	440	441	442	443	444	446	447	449
75	453	454	455	456	457	458	460	461	463
80	464	465	466	467	468	469	470	471	473
85	470	471	472	473	474	475	476	478	479
90	473	474	474	475	476	477	478	480	481

Azimuth (degrees)	Latitude								
	16°	17°	18°	19°	20°	21°	22°	23°	24°
0	6.80208	6.80213	6.80217	6.80222	6.80226	6.80232	6.80237	6.80242	6.80248
5	210	215	219	224	228	234	239	244	250
10	217	221	225	230	234	239	244	250	255
15	227	231	235	239	244	249	254	259	264
20	240	244	248	252	257	262	266	271	277
25	257	261	265	269	273	277	282	287	292
30	276	280	284	287	292	296	300	305	309
35	298	301	305	308	312	316	320	324	329
40	321	324	327	330	334	338	341	345	350
45	344	347	350	353	357	360	364	367	371
50	368	371	373	376	379	382	386	389	392
55	391	394	396	398	401	404	407	410	413
60	413	415	417	419	422	424	427	430	432
65	432	434	436	438	440	443	445	448	450
70	449	451	453	454	456	459	461	463	465
75	463	464	466	468	470	472	473	476	478
80	473	474	476	478	479	481	483	485	487
85	479	480	482	483	485	487	489	490	492
90	481	482	484	485	487	489	490	492	494

Logarithms of radii of curvature of the earth's surface (in meters)—Continued

Azimuth (degrees)	Latitude								
	24°	25°	26°	27°	28°	29°	30°	31°	32°
0	6.80248	6.80254	6.80260	6.80266	6.80272	6.80279	6.80285	6.80292	6.80299
5	250	256	262	268	274	280	287	294	300
10	255	261	267	273	279	285	292	298	305
15	264	270	276	282	288	294	300	306	313
20	277	282	288	293	299	305	311	317	324
25	292	297	302	308	313	319	325	331	337
30	309	314	319	324	330	335	340	346	352
35	329	333	338	343	348	353	358	363	369
40	350	354	358	362	367	372	377	382	386
45	371	375	379	383	387	391	396	400	405
50	392	396	399	403	407	411	415	419	423
55	413	416	420	423	426	430	434	437	441
60	432	435	438	442	445	448	451	455	458
65	450	453	455	458	461	464	467	470	473
70	465	468	470	473	475	478	481	484	486
75	478	480	482	484	487	489	492	494	497
80	487	489	491	493	495	498	500	502	505
85	492	494	496	498	501	503	505	507	510
90	494	496	498	500	502	504	507	509	511

Azimuth (degrees)	Latitude								
	32°	33°	34°	35°	36°	37°	38°	39°	40°
0	6.80299	6.80306	6.80313	6.80320	6.80327	6.80335	6.80342	6.80350	6.80357
5	300	307	314	322	329	336	344	351	359
10	305	312	319	326	333	340	348	355	363
15	313	320	326	333	340	348	355	362	369
20	324	330	337	343	350	357	364	371	378
25	337	343	349	355	362	368	375	382	388
30	352	358	364	370	376	382	388	394	401
35	369	374	380	385	391	397	402	408	414
40	386	392	397	402	407	412	418	423	429
45	405	410	414	419	424	429	434	439	444
50	423	428	432	436	441	445	450	454	459
55	441	445	449	453	457	461	465	469	474
60	458	462	465	469	472	476	480	484	487
65	473	476	480	483	486	489	493	496	500
70	486	489	492	495	498	501	504	507	510
75	497	500	502	505	508	510	513	516	519
80	505	507	510	512	515	517	520	523	525
85	510	512	514	517	519	522	524	527	529
90	511	514	516	518	521	523	526	528	531

Azimuth (degrees)	Latitude								
	40°	41°	42°	43°	44°	45°	46°	47°	48°
0	6.80357	6.80365	6.80373	6.80380	6.80388	6.80396	6.80404	6.80411	6.80419
5	359	366	374	382	389	397	404	412	420
10	363	370	378	385	393	400	408	415	423
15	369	376	384	391	398	406	413	420	428
20	378	385	392	399	406	413	420	427	434
25	388	395	402	408	415	422	429	436	442
30	401	407	413	420	426	433	439	446	452
35	414	420	426	432	438	444	450	456	462
40	429	434	440	446	451	457	462	468	474
45	444	449	454	459	464	470	475	480	485
50	459	464	468	473	478	482	487	492	496
55	474	478	482	486	490	495	499	503	508
60	487	491	495	499	502	506	510	514	518
65	500	503	507	510	514	517	520	524	528
70	510	514	517	520	523	526	529	532	536
75	519	522	525	528	531	534	536	539	542
80	525	528	531	534	536	539	542	544	547
85	529	532	534	537	540	542	545	548	550
90	531	533	536	538	541	544	546	549	551

Logarithms of radii of curvature of the earth's surface (in meters)—Continued

Azimuth (degrees)	Latitude								
	48°	49°	50°	51°	52°	53°	54°	55°	56°
0	6.80419	6.80426	6.80434	6.80442	6.80449	6.80457	6.80464	6.80471	6.80479
5	420	428	435	443	450	458	465	472	479
10	423	430	438	445	453	460	467	474	481
15	428	435	442	450	457	464	471	478	485
20	434	441	448	455	462	469	476	483	489
25	442	449	456	463	469	476	482	489	495
30	452	458	465	471	477	484	490	496	502
35	462	468	474	480	486	492	498	503	509
40	474	479	485	490	496	501	506	512	517
45	485	490	495	500	505	510	515	520	525
50	496	501	506	510	515	520	524	528	533
55	508	512	516	520	524	528	533	537	541
60	518	522	526	530	533	537	541	544	548
65	528	531	534	538	541	545	548	551	555
70	536	539	542	545	548	551	554	557	560
75	542	545	548	551	554	557	559	562	565
80	547	550	553	555	558	561	563	566	568
85	550	553	555	558	560	563	566	568	570
90	551	554	556	559	561	564	566	569	571

Azimuth (degrees)	Latitude								
	56°	57°	58°	59°	60°	61°	62°	63°	64°
0	6.80479	6.80486	6.80493	6.80500	6.80506	6.80513	6.80520	6.80526	6.80532
5	479	486	493	500	507	514	520	526	532
10	481	488	495	502	509	515	522	528	534
15	485	492	498	505	511	518	524	530	536
20	489	496	502	509	515	521	527	533	539
25	495	501	508	514	520	526	531	537	542
30	502	508	514	519	525	530	536	541	546
35	509	515	520	525	531	536	541	546	551
40	517	522	527	532	537	542	546	551	556
45	525	530	534	539	543	548	552	556	560
50	533	537	542	546	550	554	558	562	565
55	541	545	548	552	556	560	563	567	570
60	548	552	555	558	562	565	568	572	575
65	555	558	561	564	567	570	573	576	579
70	560	563	566	569	572	574	577	580	582
75	565	568	570	573	575	578	580	583	585
80	568	571	573	576	578	580	583	585	587
85	570	573	575	578	580	582	584	586	588
90	571	574	576	578	580	583	585	587	589

Azimuth (degrees)	Latitude								
	64°	65°	66°	67°	68°	69°	70°	71°	72°
0	6.80532	6.80538	6.80544	6.80550	6.80555	6.80560	6.80565	6.80570	6.80575
5	532	538	544	550	555	561	566	570	575
10	534	540	545	551	556	562	566	571	576
15	536	542	547	553	558	563	568	572	577
20	539	544	550	555	560	565	570	574	578
25	542	548	553	558	562	567	572	576	580
30	546	551	556	561	565	570	574	578	582
35	551	556	560	564	569	573	577	581	584
40	556	560	564	568	572	576	580	583	587
45	560	564	568	572	576	579	583	586	589
50	565	569	573	576	579	583	586	589	592
55	570	574	577	580	583	586	589	591	594
60	575	578	581	584	586	589	591	594	596
65	579	582	584	587	589	592	594	596	598
70	582	585	587	590	592	594	596	598	600
75	585	587	590	592	594	596	598	600	601
80	587	589	591	593	595	597	599	601	602
85	588	590	592	594	596	598	600	601	603
90	589	591	593	595	597	598	600	602	603

Factors used in the computation of elevations from reciprocal and nonreciprocal observations

[The unit of length throughout these tables is the meter]

LOG A

Elevation of occupied station, h_1	Log A, units of fifth place	Elevation of occupied station, h_1	Log A, units of fifth place
Meters		*Meters*	
0	0.0	3009	20.5
73	0.5	3156	21.5
220	1.5	3303	22.5
367	2.5	3449	23.5
514	3.5	3596	24.5
661	4.5	3743	25.5
807	5.5	3890	26.5
954	6.5	4036	27.5
1101	7.5	4183	28.5
1248	8.5	4330	29.5
1394	9.5	4477	30.5
1541	10.5	4624	31.5
1688	11.5	4770	32.5
1835	12.5	4917	33.5
1982	13.5	5064	34.5
2128	14.5	5211	35.5
2275	15.5	5357	36.5
2422	16.5	5504	37.5
2569	17.5	5651	38.5
2715	18.5	5798	39.5
2862	19.5	5945	40.5

LOG B AND LOG C

[Log B has the same sign as the approximate difference of elevation; log C is always positive]

Log approximate difference of elevation $=\log s \tan\left(\frac{\zeta_2-\zeta_1}{2}\right)$*	Log B, units of 5th place	Log s	Log C
	0.0		0.0
2.167	0.5	4.875	0.5
2.644	1.5	5.113	1.5
2.866	2.5	5.224	2.5
3.011	3.5	5.297	3.5
3.121	4.5	5.352	4.5
3.208	5.5	5.395	5.5
3.281	6.5	5.432	6.5
3.343	7.5	5.463	7.5
3.397	8.5		
3.445	9.5		
3.489	10.5		
3.528	11.5		
3.565	12.5		
3.598	13.5		
3.629	14.5		
3.658	15.5		
3.685	16.5		
3.711	17.5		
3.735	18.5		
3.758	19.5		
3.779	20.5		
3.800	21.5		
3.820	22.5		
3.839	23.5		
3.857	24.5		
3.874	25.5		

*Or $\log s \cot\left[\zeta_1-(0.5-m)\frac{s}{\rho \sin 1''}\right]$ for nonreciprocal observations.

Length of 1 degree of the meridian at different latitudes

Latitude (degrees)	Meters	Statute miles	Nautical miles	Latitude (degrees)	Meters	Statute miles	Nautical miles
0-1	110, 567. 3	68. 703	59. 661	45-46	111, 140. 8	69. 060	59. 971
1-2	110, 568. 0	68. 704	59. 662	46-47	111, 160. 5	69. 072	59. 981
2-3	110, 569. 4	68. 705	59. 662	47-48	111, 180. 2	69. 084	59. 992
3-4	110, 571. 4	68. 706	59. 664	48-49	111, 199. 9	69. 096	60. 003
4-5	110, 574. 1	68. 707	59. 665	49-50	111, 219. 5	69. 108	60. 013
5-6	110, 577. 6	68. 710	59. 667	50-51	111, 239. 0	69. 121	60. 024
6-7	110, 581. 6	68. 712	59. 669	51-52	111, 258. 3	69. 133	60. 034
7-8	110, 586. 4	68. 715	59. 672	52-53	111, 277. 6	69. 145	60. 045
8-9	110, 591. 8	68. 718	59. 675	53-54	111, 296. 6	69. 156	60. 055
9-10	110, 597. 8	68. 722	59. 678	54-55	111, 315. 4	69. 168	60. 065
10-11	110, 604. 5	68. 726	59. 681	55-56	111, 334. 0	69. 180	60. 075
11-12	110, 611. 9	68. 731	59. 685	56-57	111, 352. 4	69. 191	60. 085
12-13	110, 619. 8	68. 736	59. 690	57-58	111, 370. 5	69. 202	60. 095
13-14	110, 628. 4	68. 741	59. 694	58-59	111, 388. 4	69. 213	60. 104
14-15	110, 637. 6	68. 747	59. 699	59-60	111, 405. 9	69. 224	60. 114
15-16	110, 647. 5	68. 753	59. 705	60-61	111, 423. 1	69. 235	60. 123
16-17	110, 657. 8	68. 759	59. 710	61-62	111, 439. 9	69. 246	60. 132
17-18	110, 668. 8	68. 766	59. 716	62-63	111, 456. 4	69. 256	60. 141
18-19	110, 680. 4	68. 773	59. 722	63-64	111, 472. 4	69. 266	60. 150
19-20	110, 692. 4	68. 781	59. 729	64-65	111, 488. 1	69. 275	60. 158
20-21	110, 705. 1	68. 789	59. 736	65-66	111, 503. 3	69. 285	60. 166
21-22	110, 718. 2	68. 797	59. 743	66-67	111, 518. 0	69. 294	60. 174
22-23	110, 731. 8	68. 805	59. 750	67-68	111, 532. 3	69. 303	60. 182
23-24	110, 746. 0	68. 814	59. 758	68-69	111, 546. 2	69. 311	60. 190
24-25	110, 760. 6	68. 823	59. 765	69-70	111, 559. 5	69. 320	60. 197
25-26	110, 775. 6	68. 833	59. 774	70-71	111, 572. 2	69. 328	60. 204
26-27	110, 791. 1	68. 842	59. 782	71-72	111, 584. 5	69. 335	60. 210
27-28	110, 807. 0	68. 852	59. 791	72-73	111, 596. 2	69. 343	60. 217
28-29	110, 823. 3	68. 862	59. 800	73-74	111, 607. 3	69. 349	60. 223
29-30	110, 840. 0	68. 873	59. 808	74-75	111, 617. 9	69. 356	60. 228
30-31	110, 857. 0	68. 883	59. 818	75-76	111, 627. 8	69. 362	60. 234
31-32	110, 874. 4	68. 894	59. 827	76-77	111, 637. 1	69. 368	60. 239
32-33	110, 892. 1	68. 905	59. 837	77-78	111, 645. 9	69. 373	60. 243
33-34	110, 910. 1	68. 916	59. 846	78-79	111, 653. 9	69. 378	60. 248
34-35	110, 928. 3	68. 928	59. 856	79-80	111, 661. 4	69. 383	60. 252
35-36	110, 946. 9	68. 939	59. 866	80-81	111, 668. 2	69. 387	60. 255
36-37	110, 965. 6	68. 951	59. 876	81-82	111, 674. 4	69. 391	60. 259
37-38	110, 984. 5	68. 962	59. 886	82-83	111, 679. 9	69. 395	60. 262
38-39	111, 003. 7	68. 974	59. 897	83-84	111, 684. 7	69. 398	60. 264
39-40	111, 023. 0	68. 986	59. 907	84-85	111, 688. 9	69. 400	60. 266
40-41	111, 042. 4	68. 998	59. 918	85-86	111, 692. 3	69. 402	60. 268
41-42	111, 061. 9	69. 011	59. 928	86-87	111, 695. 1	69. 404	60. 270
42-43	111, 081. 6	69. 023	59. 939	87-88	111, 697. 2	69. 405	60. 271
43-44	111, 101. 3	69. 035	59. 949	88-89	111, 698. 6	69. 406	60. 272
44-45	111, 121. 0	69. 047	59. 960	89-90	111, 699. 3	69. 407	60. 272

Length of 1 degree of the parallel at different latitudes

Latitude (degrees)	Meters	Statute miles	Nautical miles	Latitude (degrees)	Meters	Statute miles	Nautical miles
0	111,321	69.172	60.068	46	77,466	48.136	41.801
1	111,304	69.162	60.059	47	76,058	47.261	41.041
2	111,253	69.130	60.031	48	74,628	46.372	40.268
3	111,169	69.078	59.986	49	73,174	45.469	39.484
4	111,051	69.005	59.922	50	71,698	44.552	38.688
5	110,900	68.911	59.840	51	70,200	43.621	37.880
6	110,715	68.795	59.741	52	68,680	42.676	37.060
7	110,497	68.660	59.622	53	67,140	41.719	36.229
8	110,245	68.504	59.487	54	65,578	40.749	35.386
9	109,959	68.326	59.333	55	63,996	39.766	34.532
10	109,641	68.129	59.161	56	62,395	38.771	33.668
11	109,289	67.910	58.971	57	60,774	37.764	32.794
12	108,904	67.670	58.764	58	59,135	36.745	31.909
13	108,486	67.410	58.538	59	57,478	35.716	31.015
14	108,036	67.131	58.295	60	55,802	34.674	30.110
15	107,553	66.830	58.034	61	54,110	33.623	29.197
16	107,036	66.510	57.756	62	52,400	32.560	28.275
17	106,487	66.169	57.459	63	50,675	31.488	27.344
18	105,906	65.808	57.146	64	48,934	30.406	26.404
19	105,294	65.427	56.816	65	47,177	29.315	25.456
20	104,649	65.026	56.468	66	45,407	28.215	24.501
21	103,972	64.606	56.102	67	43,622	27.106	23.538
22	103,264	64.166	55.720	68	41,823	25.988	22.567
23	102,524	63.706	55.321	69	40,012	24.862	21.590
24	101,754	63.228	54.905	70	38,188	23.729	20.606
25	100,952	62.729	54.473	71	36,353	22.589	19.616
26	100,119	62.212	54.024	72	34,506	21.441	18.619
27	99,257	61.676	53.558	73	32,648	20.287	17.617
28	98,364	61.122	53.076	74	30,781	19.127	16.609
29	97,441	60.548	52.578	75	28,903	17.960	15.596
30	96,488	59.956	52.064	76	27,017	16.788	14.578
31	95,506	59.345	51.534	77	25,123	15.611	13.556
32	94,495	58.716	50.989	78	23,220	14.428	12.529
33	93,455	58.071	50.428	79	21,311	13.242	11.499
34	92,387	57.407	49.851	80	19,394	12.051	10.465
35	91,290	56.725	49.259	81	17,472	10.857	9.428
36	90,166	56.027	48.653	82	15,545	9.659	8.388
37	89,014	55.311	48.031	83	13,612	8.458	7.345
38	87,835	54.579	47.395	84	11,675	7.255	6.300
39	86,629	53.829	46.744	85	9,735	6.049	5.253
40	85,396	53.063	46.079	86	7,792	4.842	4.205
41	84,137	52.281	45.399	87	5,846	3.632	3.154
42	82,853	51.483	44.706	88	3,898	2.422	2.103
43	81,543	50.669	44.000	89	1,949	1.211	1.052
44	80,208	49.840	43.280	90	0	0	0
45	78,849	48.995	42.546				

CONVERSION TABLES

Lengths—Feet to meters (from 1 to 1000 units)

[Reduction factor: 1 foot=0.3048006096 meter]

Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters
0	0.0	50	15.24003	100	30.48006	150	45.72009	200	60.96012
1	0.30480	1	15.54483	1	30.78486	1	46.02489	1	61.26492
2	0.60960	2	15.84963	2	31.08966	2	46.32969	2	61.56972
3	0.91440	3	16.15443	3	31.39446	3	46.63449	3	61.87452
4	1.21920	4	16.45923	4	31.69926	4	46.93929	4	62.17932
5	1.52400	5	16.76403	5	32.00406	5	47.24409	5	62.48412
6	1.82880	6	17.06883	6	32.30886	6	47.54890	6	62.78893
7	2.13360	7	17.37363	7	32.61367	7	47.85370	7	63.09373
8	2.43840	8	17.67844	8	32.91847	8	48.15850	8	63.39853
9	2.74321	9	17.98324	9	33.22327	9	48.46330	9	63.70333
10	3.04801	60	18.28804	110	33.52807	160	48.76810	210	64.00813
1	3.35281	1	18.59284	1	33.83287	1	49.07290	1	64.31293
2	3.65761	2	18.89764	2	34.13767	2	49.37770	2	64.61773
3	3.96241	3	19.20244	3	34.44247	3	49.68250	3	64.92253
4	4.26721	4	19.50724	4	34.74727	4	49.98730	4	65.22733
5	4.57201	5	19.81204	5	35.05207	5	50.29210	5	65.53213
6	4.87681	6	20.11684	6	35.35687	6	50.59690	6	65.83693
7	5.18161	7	20.42164	7	35.66167	7	50.90170	7	66.14173
8	5.48641	8	20.72644	8	35.96647	8	51.20650	8	66.44653
9	5.79121	9	21.03124	9	36.27127	9	51.51130	9	66.75133
20	6.09601	70	21.33604	120	36.57607	170	51.81610	220	67.05613
1	6.40081	1	21.64084	1	36.88087	1	52.12090	1	67.36093
2	6.70561	2	21.94564	2	37.18567	2	52.42570	2	67.66574
3	7.01041	3	22.25044	3	37.49047	3	52.73051	3	67.97054
4	7.31521	4	22.55525	4	37.79528	4	53.03531	4	68.27534
5	7.62002	5	22.86005	5	38.10008	5	53.34011	5	68.58014
6	7.92482	6	23.16485	6	38.40488	6	53.64491	6	68.88494
7	8.22962	7	23.46965	7	38.70968	7	53.94971	7	69.18974
8	8.53442	8	23.77445	8	39.01448	8	54.25451	8	69.49454
9	8.83922	9	24.07925	9	39.31928	9	54.55931	9	69.79934
30	9.14402	80	24.38405	130	39.62408	180	54.86411	230	70.10414
1	9.44882	1	24.68885	1	39.92888	1	55.16891	1	70.40894
2	9.75362	2	24.99365	2	40.23368	2	55.47371	2	70.71374
3	10.05842	3	25.29845	3	40.53848	3	55.77851	3	71.01854
4	10.36322	4	25.60325	4	40.84328	4	56.08331	4	71.32334
5	10.66802	5	25.90805	5	41.14808	5	56.38811	5	71.62814
6	10.97282	6	26.21285	6	41.45288	6	56.69291	6	71.93294
7	11.27762	7	26.51765	7	41.75768	7	56.99771	7	72.23774
8	11.58242	8	26.82245	8	42.06248	8	57.30251	8	72.54255
9	11.88722	9	27.12725	9	42.36728	9	57.60732	9	72.84735
40	12.19202	90	27.43205	140	42.67209	190	57.91212	240	73.15215
1	12.49682	1	27.73686	1	42.97689	1	58.21692	1	73.45695
2	12.80163	2	28.04166	2	43.28169	2	58.52172	2	73.76175
3	13.10643	3	28.34646	3	43.58649	3	58.82652	3	74.06655
4	13.41123	4	28.65126	4	43.89129	4	59.13132	4	74.37135
5	13.71603	5	28.95606	5	44.19609	5	59.43612	5	74.67615
6	14.02083	6	29.26086	6	44.50089	6	59.74092	6	74.98095
7	14.32563	7	29.56566	7	44.80569	7	60.04572	7	75.28575
8	14.63043	8	29.87046	8	45.11049	8	60.35052	8	75.59055
9	14.93523	9	30.17526	9	45.41529	9	60.65532	9	75.89535

Lengths—Feet to meters (from 1 to 1000 units)—Continued

Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters
250	76.20015	300	91.44018	350	106.68021	400	121.92024	450	137.16027
1	76.50495	1	91.74498	1	106.98501	1	122.22504	1	137.46507
2	76.80975	2	92.04978	2	107.28981	2	122.52985	2	137.76988
3	77.11455	3	92.35458	3	107.59462	3	122.83465	3	138.07468
4	77.41935	4	92.65939	4	107.89942	4	123.13945	4	138.37948
5	77.72416	5	92.96419	5	108.20422	5	123.44425	5	138.68428
6	78.02896	6	93.26899	6	108.50902	6	123.74905	6	138.98908
7	78.33376	7	93.57379	7	108.81382	7	124.05385	7	139.29388
8	78.63856	8	93.87859	8	109.11862	8	124.35865	8	139.59868
9	78.94336	9	94.18339	9	109.42342	9	124.66345	9	139.90348
260	79.24816	310	94.48819	360	109.72822	410	124.96825	460	140.20828
1	79.55296	1	94.79299	1	110.03302	1	125.27305	1	140.51308
2	79.85776	2	95.09779	2	110.33782	2	125.57785	2	140.81788
3	80.16256	3	95.40259	3	110.64262	3	125.88265	3	141.12268
4	80.46736	4	95.70739	4	110.94742	4	126.18745	4	141.42748
5	80.77216	5	96.01219	5	111.25222	5	126.49225	5	141.73228
6	81.07696	6	96.31699	6	111.55702	6	126.79705	6	142.03708
7	81.38176	7	96.62179	7	111.86182	7	127.10185	7	142.34188
8	81.68656	8	96.92659	8	112.16662	8	127.40665	8	142.64669
9	81.99136	9	97.23139	9	112.47142	9	127.71146	9	142.95149
270	82.29616	320	97.53620	370	112.77623	420	128.01626	470	143.25629
1	82.60097	1	97.84100	1	113.08103	1	128.32106	1	143.56109
2	82.90577	2	98.14580	2	113.38583	2	128.62586	2	143.86589
3	83.21057	3	98.45060	3	113.69063	3	128.93066	3	144.17069
4	83.51537	4	98.75540	4	113.99543	4	129.23546	4	144.47549
5	83.82017	5	99.06020	5	114.30023	5	129.54026	5	144.78029
6	84.12497	6	99.36500	6	114.60503	6	129.84506	6	145.08509
7	84.42977	7	99.66980	7	114.90983	7	130.14986	7	145.38989
8	84.73457	8	99.97460	8	115.21463	8	130.45466	8	145.69469
9	85.03937	9	100.27940	9	115.51943	9	130.75946	9	145.99949
280	85.34417	330	100.58420	380	115.82423	430	131.06426	480	146.30429
1	85.64897	1	100.88900	1	116.12903	1	131.36906	1	146.60909
2	85.95377	2	101.19380	2	116.43383	2	131.67386	2	146.91389
3	86.25857	3	101.49860	3	116.73863	3	131.97866	3	147.21869
4	86.56337	4	101.80340	4	117.04343	4	132.28346	4	147.52350
5	86.86817	5	102.10820	5	117.34823	5	132.58827	5	147.82830
6	87.17297	6	102.41300	6	117.65303	6	132.89307	6	148.13310
7	87.47777	7	102.71781	7	117.95784	7	133.19787	7	148.43790
8	87.78258	8	103.02261	8	118.26264	8	133.50267	8	148.74270
9	88.08738	9	103.32741	9	118.56744	9	133.80747	9	149.04750
290	88.39218	340	103.63221	390	118.87224	440	134.11227	490	149.35230
1	88.69698	1	103.93701	1	119.17704	1	134.41707	1	149.65710
2	89.00178	2	104.24181	2	119.48184	2	134.72187	2	149.96190
3	89.30658	3	104.54661	3	119.78664	3	135.02667	3	150.26670
4	89.61138	4	104.85141	4	120.09144	4	135.33147	4	150.57150
5	89.91618	5	105.15621	5	120.39624	5	135.63627	5	150.87630
6	90.22098	6	105.46101	6	120.70104	6	135.94107	6	151.18110
7	90.52578	7	105.76581	7	121.00584	7	136.24587	7	151.48590
8	90.83058	8	106.07061	8	121.31064	8	136.55067	8	151.79070
9	91.13538	9	106.37541	9	121.61544	9	136.85547	9	152.09550

Lengths—Feet to meters (from 1 to 1000 units)—Continued

Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters
500	152. 40030	550	167. 64034	600	182. 88037	650	198. 12040	700	213. 36043
1	152. 70511	1	167. 94514	1	183. 18517	1	198. 42520	1	213. 66523
2	153. 00991	2	168. 24994	2	183. 48997	2	198. 73000	2	213. 97003
3	153. 31471	3	168. 55474	3	183. 79477	3	199. 03480	3	214. 27483
4	153. 61951	4	168. 85954	4	184. 09957	4	199. 33960	4	214. 57963
5	153. 92431	5	169. 16434	5	184. 40437	5	199. 64440	5	214. 88443
6	154. 22911	6	169. 46914	6	184. 70917	6	199. 94920	6	215. 18923
7	154. 53391	7	169. 77394	7	185. 01397	7	200. 25400	7	215. 49403
8	154. 83871	8	170. 07874	8	185. 31877	8	200. 55880	8	215. 79883
9	155. 14351	9	170. 38354	9	185. 62357	9	200. 86360	9	216. 10363
510	155. 44831	560	170. 68834	610	185. 92837	660	201. 16840	710	216. 40843
1	155. 75311	1	170. 99314	1	186. 23317	1	201. 47320	1	216. 71323
2	156. 05791	2	171. 29794	2	186. 53797	2	201. 77800	2	217. 01803
3	156. 36271	3	171. 60274	3	186. 84277	3	202. 08280	3	217. 32283
4	156. 66751	4	171. 90754	4	187. 14757	4	202. 38760	4	217. 62764
5	156. 97231	5	172. 21234	5	187. 45237	5	202. 69241	5	217. 93244
6	157. 27711	6	172. 51715	6	187. 75718	6	202. 99721	6	218. 23724
7	157. 58192	7	172. 82195	7	188. 06198	7	203. 30201	7	218. 54204
8	157. 88672	8	173. 12675	8	188. 36678	8	203. 60681	8	218. 84684
9	158. 19152	9	173. 43155	9	188. 67158	9	203. 91161	9	219. 15164
520	158. 49632	570	173. 73635	620	188. 97638	670	204. 21641	720	219. 45644
1	158. 80112	1	174. 04115	1	189. 28118	1	204. 52121	1	219. 76124
2	159. 10592	2	174. 34595	2	189. 58598	2	204. 82601	2	220. 06604
3	159. 41072	3	174. 65075	3	189. 89078	3	205. 13081	3	220. 37084
4	159. 71552	4	174. 95555	4	190. 19558	4	205. 43561	4	220. 67564
5	160. 02032	5	175. 26035	5	190. 50038	5	205. 74041	5	220. 98044
6	160. 32512	6	175. 56515	6	190. 80518	6	206. 04521	6	221. 28524
7	160. 62992	7	175. 86995	7	191. 10998	7	206. 35001	7	221. 59004
8	160. 93472	8	176. 17475	8	191. 41478	8	206. 65481	8	221. 89484
9	161. 23952	9	176. 47955	9	191. 71958	9	206. 95961	9	222. 19964
530	161. 54432	580	176. 78435	630	192. 02438	680	207. 26441	730	222. 50445
1	161. 84912	1	177. 08915	1	192. 32918	1	207. 56922	1	222. 80925
2	162. 15392	2	177. 39395	2	192. 63399	2	207. 87402	2	223. 11405
3	162. 45872	3	177. 69876	3	192. 93879	3	208. 17882	3	223. 41885
4	162. 76353	4	178. 00356	4	193. 24359	4	208. 48362	4	223. 72365
5	163. 06833	5	178. 30836	5	193. 54839	5	208. 78842	5	224. 02845
6	163. 37313	6	178. 61316	6	193. 85319	6	209. 09322	6	224. 33325
7	163. 67793	7	178. 91796	7	194. 15799	7	209. 39802	7	224. 63805
8	163. 98273	8	179. 22276	8	194. 46279	8	209. 70282	8	224. 94285
9	164. 28753	9	179. 52756	9	194. 76759	9	210. 00762	9	225. 24765
540	164. 59233	590	179. 83236	640	195. 07239	690	210. 31242	740	225. 55245
1	164. 89713	1	180. 13716	1	195. 37719	1	210. 61722	1	225. 85725
2	165. 20193	2	180. 44196	2	195. 68199	2	210. 92202	2	226. 16205
3	165. 50673	3	180. 74676	3	195. 98679	3	211. 22682	3	226. 46685
4	165. 81153	4	181. 05156	4	196. 29159	4	211. 53162	4	226. 77165
5	166. 11633	5	181. 35636	5	196. 59639	5	211. 83642	5	227. 07645
6	166. 42113	6	181. 66116	6	196. 90119	6	212. 14122	6	227. 38125
7	166. 72593	7	181. 96596	7	197. 20599	7	212. 44602	7	227. 68606
8	167. 03073	8	182. 27076	8	197. 51080	8	212. 75083	8	227. 99086
9	167. 33553	9	182. 57557	9	197. 81560	9	213. 05563	9	228. 29566

Lengths—Feet to meters (from 1 to 1000 units)—Continued

Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters
750	228.60046	800	243.84049	850	259.08052	900	274.32055	950	289.56058
1	228.90526	1	244.14529	1	259.38532	1	274.62535	1	289.86538
2	229.21006	2	244.45009	2	259.69012	2	274.93015	2	290.17018
3	229.51486	3	244.75489	3	259.99492	3	275.23495	3	290.47498
4	229.81966	4	245.05969	4	260.29972	4	275.53975	4	290.77978
5	230.12446	5	245.36449	5	260.60452	5	275.84455	5	291.08458
6	230.42926	6	245.66929	6	260.90932	6	276.14935	6	291.38938
7	230.73406	7	245.97409	7	261.21412	7	276.45415	7	291.69418
8	231.03886	8	246.27889	8	261.51892	8	276.75895	8	291.99898
9	231.34366	9	246.58369	9	261.82372	9	277.06375	9	292.30378
760	231.64846	810	246.88849	860	262.12852	910	277.36855	960	292.60859
1	231.95326	1	247.19329	1	262.43332	1	277.67336	1	292.91339
2	232.25806	2	247.49809	2	262.73813	2	277.97816	2	293.21819
3	232.56287	3	247.80290	3	263.04293	3	278.28296	3	293.52299
4	232.86767	4	248.10770	4	263.34773	4	278.58776	4	293.82779
5	233.17247	5	248.41250	5	263.65253	5	278.89256	5	294.13259
6	233.47727	6	248.71730	6	263.95733	6	279.19736	6	294.43739
7	233.78207	7	249.02210	7	264.26213	7	279.50216	7	294.74219
8	234.08687	8	249.32690	8	264.56693	8	279.80696	8	295.04699
9	234.39167	9	249.63170	9	264.87173	9	280.11176	9	295.35179
770	234.69647	820	249.93650	870	265.17653	920	280.41656	970	295.65659
1	235.00127	1	250.24130	1	265.48133	1	280.72136	1	295.96139
2	235.30607	2	250.54610	2	265.78613	2	281.02616	2	296.26619
3	235.61087	3	250.85090	3	266.09093	3	281.33096	3	296.57099
4	235.91567	4	251.15570	4	266.39573	4	281.63576	4	296.87579
5	236.22047	5	251.46050	5	266.70053	5	281.94056	5	297.18059
6	236.52527	6	251.76530	6	267.00533	6	282.24536	6	297.48539
7	236.83007	7	252.07010	7	267.31013	7	282.55017	7	297.79020
8	237.13487	8	252.37490	8	267.61494	8	282.85497	8	298.09500
9	237.43967	9	252.67971	9	267.91974	9	283.15977	9	298.39980
780	237.74448	830	252.98451	880	268.22454	930	283.46457	980	298.70460
1	238.04928	1	253.28931	1	268.52934	1	283.76937	1	299.00940
2	238.35408	2	253.59411	2	268.83414	2	284.07417	2	299.31420
3	238.65888	3	253.89891	3	269.13894	3	284.37897	3	299.61900
4	238.96368	4	254.20371	4	269.44374	4	284.68377	4	299.92380
5	239.26848	5	254.50851	5	269.74854	5	284.98857	5	300.22860
6	239.57328	6	254.81331	6	270.05334	6	285.29337	6	300.53340
7	239.87808	7	255.11811	7	270.35814	7	285.59817	7	300.83820
8	240.18288	8	255.42291	8	270.66294	8	285.90297	8	301.14300
9	240.48768	9	255.72771	9	270.96774	9	286.20777	9	301.44780
790	240.79248	840	256.03251	890	271.27254	940	286.51257	990	301.75260
1	241.09728	1	256.33731	1	271.57734	1	286.81737	1	302.05740
2	241.40208	2	256.64211	2	271.88214	2	287.12217	2	302.36220
3	241.70688	3	256.94691	3	272.18694	3	287.42697	3	302.66701
4	242.01168	4	257.25171	4	272.49174	4	287.73178	4	302.97181
5	242.31648	5	257.55652	5	272.79655	5	288.03658	5	303.27661
6	242.62129	6	257.86132	6	273.10135	6	288.34138	6	303.58141
7	242.92609	7	258.16612	7	273.40615	7	288.64618	7	303.88621
8	243.23089	8	258.47092	8	273.71095	8	288.95098	8	304.19101
9	243.53569	9	258.77572	9	274.01575	9	289.25578	9	304.49581

Lengths—Meters to feet (from 1 to 1000 units)

[Reduction factor: 1 meter=3.280833333 feet]

Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet
0		50	164.04167	100	328.08333	150	492.12500	200	656.16667
1	3.28083	1	167.32250	1	331.36417	1	495.40583	1	659.44750
2	6.56167	2	170.60333	2	334.64500	2	498.68667	2	662.72833
3	9.84250	3	173.88417	3	337.92583	3	501.96750	3	666.00917
4	13.12333	4	177.16500	4	341.20667	4	505.24833	4	669.29000
5	16.40417	5	180.44583	5	344.48750	5	508.52917	5	672.57083
6	19.68500	6	183.72667	6	347.76833	6	511.81000	6	675.85167
7	22.96583	7	187.00750	7	351.04917	7	515.09083	7	679.13250
8	26.24667	8	190.28833	8	354.33000	8	518.37167	8	682.41333
9	29.52750	9	193.56917	9	357.61083	9	521.65250	9	685.69417
10	32.80833	60	196.85000	110	360.89167	160	524.93333	210	688.97500
1	36.08917	1	200.13083	1	364.17250	1	528.21417	1	692.25583
2	39.37000	2	203.41167	2	367.45333	2	531.49500	2	695.53667
3	42.65083	3	206.69250	3	370.73417	3	534.77583	3	698.81750
4	45.93167	4	209.97333	4	374.01500	4	538.05667	4	702.09833
5	49.21250	5	213.25417	5	377.29583	5	541.33750	5	705.37917
6	52.49333	6	216.53500	6	380.57667	6	544.61833	6	708.66000
7	55.77417	7	219.81583	7	383.85750	7	547.89917	7	711.94083
8	59.05500	8	223.09667	8	387.13833	8	551.18000	8	715.22167
9	62.33583	9	226.37750	9	390.41917	9	554.46083	9	718.50250
20	65.61667	70	229.65833	120	393.70000	170	557.74167	220	721.78333
1	68.89750	1	232.93917	1	396.98083	1	561.02250	1	725.06417
2	72.17833	2	236.22000	2	400.26167	2	564.30333	2	728.34500
3	75.45917	3	239.50083	3	403.54250	3	567.58417	3	731.62583
4	78.74000	4	242.78167	4	406.82333	4	570.86500	4	734.90667
5	82.02083	5	246.06250	5	410.10417	5	574.14583	5	738.18750
6	85.30167	6	249.34333	6	413.38500	6	577.42667	6	741.46833
7	88.58250	7	252.62417	7	416.66583	7	580.70750	7	744.74917
8	91.86333	8	255.90500	8	419.94667	8	583.98833	8	748.03000
9	95.14417	9	259.18583	9	423.22750	9	587.26917	9	751.31083
30	98.42500	80	262.46667	130	426.50833	180	590.55000	230	754.59167
1	101.70583	1	265.74750	1	429.78917	1	593.83083	1	757.87250
2	104.98667	2	269.02833	2	433.07000	2	597.11167	2	761.15333
3	108.26750	3	272.30917	3	436.35083	3	600.39250	3	764.43417
4	111.54833	4	275.59000	4	439.63167	4	603.67333	4	767.71500
5	114.82917	5	278.87083	5	442.91250	5	606.95417	5	770.99583
6	118.11000	6	282.15167	6	446.19333	6	610.23500	6	774.27667
7	121.39083	7	285.43250	7	449.47417	7	613.51583	7	777.55750
8	124.67167	8	288.71333	8	452.75500	8	616.79667	8	780.83833
9	127.95250	9	291.99417	9	456.03583	9	620.07750	9	784.11917
40	131.23333	90	295.27500	140	459.31667	190	623.35833	240	787.40000
1	134.51417	1	298.55583	1	462.59750	1	626.63917	1	790.68083
2	137.79500	2	301.83667	2	465.87833	2	629.92000	2	793.96167
3	141.07583	3	305.11750	3	469.15917	3	633.20083	3	797.24250
4	144.35667	4	308.39833	4	472.44000	4	636.48167	4	800.52333
5	147.63750	5	311.67917	5	475.72083	5	639.76250	5	803.80417
6	150.91833	6	314.96000	6	479.00167	6	643.04333	6	807.08500
7	154.19917	7	318.24083	7	482.28250	7	646.32417	7	810.36583
8	157.48000	8	321.52167	8	485.56333	8	649.60500	8	813.64667
9	160.76083	9	324.80250	9	488.84417	9	652.88583	9	816.92750

Lengths—Meters to feet (from 1 to 1000 units)—Continued

Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet
250	820.20833	300	984.25000	350	1,148.29167	400	1,312.33333	450	1,476.37500
1	823.48917	1	987.53083	1	1,151.57250	1	1,315.61417	1	1,479.65583
2	826.77000	2	990.81167	2	1,154.85333	2	1,318.89500	2	1,482.93667
3	830.05083	3	994.09250	3	1,158.13417	3	1,322.17583	3	1,486.21750
4	833.33167	4	997.37333	4	1,161.41500	4	1,325.45667	4	1,489.49833
5	836.61250	5	1,000.65417	5	1,164.69583	5	1,328.73750	5	1,492.77917
6	839.89333	6	1,003.93500	6	1,167.97667	6	1,332.01833	6	1,496.06000
7	843.17417	7	1,007.21583	7	1,171.25750	7	1,335.29917	7	1,499.34083
8	846.45500	8	1,010.49667	8	1,174.53833	8	1,338.58000	8	1,502.62167
9	849.73583	9	1,013.77750	9	1,177.81917	9	1,341.86083	9	1,505.90250
260	853.01667	310	1,017.05833	360	1,181.10000	410	1,345.14167	460	1,509.18333
1	856.29750	1	1,020.33917	1	1,184.38083	1	1,348.42250	1	1,512.46417
2	859.57833	2	1,023.62000	2	1,187.66167	2	1,351.70333	2	1,515.74500
3	862.85917	3	1,026.90083	3	1,190.94250	3	1,354.98417	3	1,519.02583
4	866.14000	4	1,030.18167	4	1,194.22333	4	1,358.26500	4	1,522.30667
5	869.42083	5	1,033.46250	5	1,197.50417	5	1,361.54583	5	1,525.58750
6	872.70167	6	1,036.74333	6	1,200.78500	6	1,364.82667	6	1,528.86833
7	875.98250	7	1,040.02417	7	1,204.06583	7	1,368.10750	7	1,532.14917
8	879.26333	8	1,043.30500	8	1,207.34667	8	1,371.38833	8	1,535.43000
9	882.54417	9	1,046.58583	9	1,210.62750	9	1,374.66917	9	1,538.71083
270	885.82500	320	1,049.86667	370	1,213.90833	420	1,377.95000	470	1,541.99167
1	889.10583	1	1,053.14750	1	1,217.18917	1	1,381.23083	1	1,545.27250
2	892.38667	2	1,056.42833	2	1,220.47000	2	1,384.51167	2	1,548.55333
3	895.66750	3	1,059.70917	3	1,223.75083	3	1,387.79250	3	1,551.83417
4	898.94833	4	1,062.99000	4	1,227.03167	4	1,391.07333	4	1,555.11500
5	902.22917	5	1,066.27083	5	1,230.31250	5	1,394.35417	5	1,558.39583
6	905.51000	6	1,069.55167	6	1,233.59333	6	1,397.63500	6	1,561.67667
7	908.79083	7	1,072.83250	7	1,236.87417	7	1,400.91583	7	1,564.95750
8	912.07167	8	1,076.11333	8	1,240.15500	8	1,404.19667	8	1,568.23833
9	915.35250	9	1,079.39417	9	1,243.43583	9	1,407.47750	9	1,571.51917
280	918.63333	330	1,082.67500	380	1,246.71667	430	1,410.75833	480	1,574.80000
1	921.91417	1	1,085.95583	1	1,249.99750	1	1,414.03917	1	1,578.08083
2	925.19500	2	1,089.23667	2	1,253.27833	2	1,417.32000	2	1,581.36167
3	928.47583	3	1,092.51750	3	1,256.55917	3	1,420.60083	3	1,584.64250
4	931.75667	4	1,095.79833	4	1,259.84000	4	1,423.88167	4	1,587.92333
5	935.03750	5	1,099.07917	5	1,263.12083	5	1,427.16250	5	1,591.20417
6	938.31833	6	1,102.36000	6	1,266.40167	6	1,430.44333	6	1,594.48500
7	941.59917	7	1,105.64083	7	1,269.68250	7	1,433.72417	7	1,597.76583
8	944.88000	8	1,108.92167	8	1,272.96333	8	1,437.00500	8	1,601.04667
9	948.16083	9	1,112.20250	9	1,276.24417	9	1,440.28583	9	1,604.32750
290	951.44167	340	1,115.48333	390	1,279.52500	440	1,443.56667	490	1,607.60833
1	954.72250	1	1,118.76417	1	1,282.80583	1	1,446.84750	1	1,610.88917
2	958.00333	2	1,122.04500	2	1,286.08667	2	1,450.12833	2	1,614.17000
3	961.28417	3	1,125.32583	3	1,289.36750	3	1,453.40917	3	1,617.45083
4	964.56500	4	1,128.60667	4	1,292.64833	4	1,456.69000	4	1,620.73167
5	967.84583	5	1,131.88750	5	1,295.92917	5	1,459.97083	5	1,624.01250
6	971.12667	6	1,135.16833	6	1,299.21000	6	1,463.25167	6	1,627.29333
7	974.40750	7	1,138.44917	7	1,302.49083	7	1,466.53250	7	1,630.57417
8	977.68833	8	1,141.73000	8	1,305.77167	8	1,469.81333	8	1,633.85500
9	980.96917	9	1,145.01083	9	1,309.05250	9	1,473.09417	9	1,637.13583

Lengths—Meters to feet (from 1 to 1000 units)—Continued

Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet
500	1,640.41667	550	1,804.45833	600	1,968.50000	650	2,132.54167	700	2,296.58333
1	1,643.69750	1	1,807.73917	1	1,971.78083	1	2,135.82250	1	2,299.86417
2	1,646.97833	2	1,811.02000	2	1,975.06167	2	2,139.10333	2	2,303.14500
3	1,650.25917	3	1,814.30083	3	1,978.34250	3	2,142.38417	3	2,306.42583
4	1,653.54000	4	1,817.58167	4	1,981.62333	4	2,145.66500	4	2,309.70667
5	1,656.82083	5	1,820.86250	5	1,984.90417	5	2,148.94583	5	2,312.98750
6	1,660.10167	6	1,824.14333	6	1,988.18500	6	2,152.22667	6	2,316.26833
7	1,663.38250	7	1,827.42417	7	1,991.46583	7	2,155.50750	7	2,319.54917
8	1,666.66333	8	1,830.70500	8	1,994.74667	8	2,158.78833	8	2,322.83000
9	1,669.94417	9	1,833.98583	9	1,998.02750	9	2,162.06917	9	2,326.11083
510	1,673.22500	560	1,837.26667	610	2,001.30833	660	2,165.35000	710	2,329.39167
1	1,676.50583	1	1,840.54750	1	2,004.58917	1	2,168.63083	1	2,332.67250
2	1,679.78667	2	1,843.82833	2	2,007.87000	2	2,171.91167	2	2,335.95333
3	1,683.06750	3	1,847.10917	3	2,011.15083	3	2,175.19250	3	2,339.23417
4	1,686.34833	4	1,850.39000	4	2,014.43167	4	2,178.47333	4	2,342.51500
5	1,689.62917	5	1,853.67083	5	2,017.71250	5	2,181.75417	5	2,345.79583
6	1,692.91000	6	1,856.95167	6	2,020.99333	6	2,185.03500	6	2,349.07667
7	1,696.19083	7	1,860.23250	7	2,024.27417	7	2,188.31583	7	2,352.35750
8	1,699.47167	8	1,863.51333	8	2,027.55500	8	2,191.59667	8	2,355.63833
9	1,702.75250	9	1,866.79417	9	2,030.83583	9	2,194.87750	9	2,358.91917
520	1,706.03333	570	1,870.07500	620	2,034.11667	670	2,198.15833	720	2,362.20000
1	1,709.31417	1	1,873.35583	1	2,037.39750	1	2,201.43917	1	2,365.48083
2	1,712.59500	2	1,876.63667	2	2,040.67833	2	2,204.72000	2	2,368.76167
3	1,715.87583	3	1,879.91750	3	2,043.95917	3	2,208.00083	3	2,372.04250
4	1,719.15667	4	1,883.19833	4	2,047.24000	4	2,211.28167	4	2,375.32333
5	1,722.43750	5	1,886.47917	5	2,050.52083	5	2,214.56250	5	2,378.60417
6	1,725.71833	6	1,889.76000	6	2,053.80167	6	2,217.84333	6	2,381.88500
7	1,728.99917	7	1,893.04083	7	2,057.08250	7	2,221.12417	7	2,385.16583
8	1,732.28000	8	1,896.32167	8	2,060.36333	8	2,224.40500	8	2,388.44667
9	1,735.56083	9	1,899.60250	9	2,063.64417	9	2,227.68583	9	2,391.72750
530	1,738.84167	580	1,902.88333	630	2,066.92500	680	2,230.96667	730	2,395.00833
1	1,742.12250	1	1,906.16417	1	2,070.20583	1	2,234.24750	1	2,398.28917
2	1,745.40333	2	1,909.44500	2	2,073.48667	2	2,237.52833	2	2,401.57000
3	1,748.68417	3	1,912.72583	3	2,076.76750	3	2,240.80917	3	2,404.85083
4	1,751.96500	4	1,916.00667	4	2,080.04833	4	2,244.09000	4	2,408.13167
5	1,755.24583	5	1,919.28750	5	2,083.32917	5	2,247.37083	5	2,411.41250
6	1,758.52667	6	1,922.56833	6	2,086.61000	6	2,250.65167	6	2,414.69333
7	1,761.80750	7	1,925.84917	7	2,089.89083	7	2,253.93250	7	2,417.97417
8	1,765.08833	8	1,929.13000	8	2,093.17167	8	2,257.21333	8	2,421.25500
9	1,768.36917	9	1,932.41083	9	2,096.45250	9	2,260.49417	9	2,424.53583
540	1,771.65000	590	1,935.69167	640	2,099.73333	690	2,263.77500	740	2,427.81667
1	1,774.93083	1	1,938.97250	1	2,103.01417	1	2,267.05583	1	2,431.09750
2	1,778.21167	2	1,942.25333	2	2,106.29500	2	2,270.33667	2	2,434.37833
3	1,781.49250	3	1,945.53417	3	2,109.57583	3	2,273.61750	3	2,437.65917
4	1,784.77333	4	1,948.81500	4	2,112.85667	4	2,276.89833	4	2,440.94000
5	1,788.05417	5	1,952.09583	5	2,116.13750	5	2,280.17917	5	2,444.22083
6	1,791.33500	6	1,955.37667	6	2,119.41833	6	2,283.46000	6	2,447.50167
7	1,794.61583	7	1,958.65750	7	2,122.69917	7	2,286.74083	7	2,450.78250
8	1,797.89667	8	1,961.93833	8	2,125.98000	8	2,290.02167	8	2,454.06333
9	1,801.17750	9	1,965.21917	9	2,129.26083	9	2,293.30250	9	2,457.34417

Lengths—Meters to feet (from 1 to 1000 units)—Continued

Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet
750	2,460.62500	800	2,624.66667	850	2,788.70833	900	2,952.75000	950	3,116.79167
1	2,463.90583	1	2,627.94750	1	2,791.98917	1	2,956.03083	1	3,120.07250
2	2,467.18667	2	2,631.22833	2	2,795.27000	2	2,959.31167	2	3,123.35333
3	2,470.46750	3	2,634.50917	3	2,798.55083	3	2,962.59250	3	3,126.63417
4	2,473.74833	4	2,637.79000	4	2,801.83167	4	2,965.87333	4	3,129.91500
5	2,477.02917	5	2,641.07083	5	2,805.11250	5	2,969.15417	5	3,133.19583
6	2,480.31000	6	2,644.35167	6	2,808.39333	6	2,972.43500	6	3,136.47667
7	2,483.59083	7	2,647.63250	7	2,811.67417	7	2,975.71583	7	3,139.75750
8	2,486.87167	8	2,650.91333	8	2,814.95500	8	2,978.99667	8	3,143.03833
9	2,490.15250	9	2,654.19417	9	2,818.23583	9	2,982.27750	9	3,146.31917
760	2,493.43333	810	2,657.47500	860	2,821.51667	910	2,985.55833	960	3,149.60000
1	2,496.71417	1	2,660.75583	1	2,824.79750	1	2,988.83917	1	3,152.88083
2	2,499.99500	2	2,664.03667	2	2,828.07833	2	2,992.12000	2	3,156.16167
3	2,503.27583	3	2,667.31750	3	2,831.35917	3	2,995.40083	3	3,159.44250
4	2,506.55667	4	2,670.59833	4	2,834.64000	4	2,998.68167	4	3,162.72333
5	2,509.83750	5	2,673.87917	5	2,837.92083	5	3,001.96250	5	3,166.00417
6	2,513.11833	6	2,677.16000	6	2,841.20167	6	3,005.24333	6	3,169.28500
7	2,516.39917	7	2,680.44083	7	2,844.48250	7	3,008.52417	7	3,172.56583
8	2,519.68000	8	2,683.72167	8	2,847.76333	8	3,011.80500	8	3,175.84667
9	2,522.96083	9	2,687.00250	9	2,851.04417	9	3,015.08583	9	3,179.12750
770	2,526.24167	820	2,690.28333	870	2,854.32500	920	3,018.36667	970	3,182.40833
1	2,529.52250	1	2,693.56417	1	2,857.60583	1	3,021.64750	1	3,185.68917
2	2,532.80333	2	2,696.84500	2	2,860.88667	2	3,024.92833	2	2,188.97000
3	2,536.08417	3	2,700.12583	3	2,864.16750	3	3,028.20917	3	3,192.25083
4	2,539.36500	4	2,703.40667	4	2,867.44833	4	3,031.49000	4	3,195.53167
5	2,542.64583	5	2,706.68750	5	2,870.72917	5	3,034.77083	5	3,198.81250
6	2,545.92667	6	2,709.96833	6	2,874.01000	6	3,038.05167	6	3,202.09333
7	2,549.20750	7	2,713.24917	7	2,877.29083	7	3,041.33250	7	3,205.37417
8	2,552.48833	8	2,716.53000	8	2,880.57167	8	3,044.61333	8	3,208.65500
9	2,555.76917	9	2,719.81083	9	2,883.85250	9	3,047.89417	9	3,211.93583
780	2,559.05000	830	2,723.09167	880	2,887.13333	930	3,051.17500	980	3,215.21667
1	2,562.33083	1	2,726.37250	1	2,890.41417	1	3,054.45583	1	3,218.49750
2	2,565.61167	2	2,729.65333	2	2,893.69500	2	3,057.73667	2	3,221.77833
3	2,568.89250	3	2,732.93417	3	2,896.97583	3	3,061.01750	3	3,225.05917
4	2,572.17333	4	2,736.21500	4	2,900.25667	4	3,064.29833	4	3,228.34000
5	2,575.45417	5	2,739.49583	5	2,903.53750	5	3,067.57917	5	3,231.62083
6	2,578.73500	6	2,742.77667	6	2,906.81833	6	3,070.86000	6	3,234.90167
7	2,582.01583	7	2,746.05750	7	2,910.09917	7	3,074.14083	7	3,238.18250
8	2,585.29667	8	2,749.33833	8	2,913.38000	8	3,077.42167	8	3,241.46333
9	2,588.57750	9	2,752.61917	9	2,916.66083	9	3,080.70250	9	3,244.74417
790	2,591.85833	840	2,755.90000	890	2,919.94167	940	3,083.98333	990	3,248.02500
1	2,595.13917	1	2,759.18083	1	2,923.22250	1	3,087.26417	1	3,251.30583
2	2,598.42000	2	2,762.46167	2	2,926.50333	2	3,090.54500	2	3,254.58667
3	2,601.70083	3	2,765.74250	3	2,929.78417	3	3,093.82583	3	3,257.86750
4	2,604.98167	4	2,769.02333	4	2,933.06500	4	3,097.10667	4	3,261.14833
5	2,608.26250	5	2,772.30417	5	2,936.34583	5	3,100.38750	5	3,264.42917
6	2,611.54333	6	2,775.58500	6	2,939.62667	6	3,103.66833	6	3,267.71000
7	2,614.82417	7	2,778.86583	7	2,942.90750	7	3,106.94917	7	3,270.99083
8	2,618.10500	8	2,782.14667	8	2,946.18833	8	3,110.23000	8	3,274.27167
9	2,621.38583	9	2,785.42750	9	2,949.46917	9	3,113.51083	9	3,277.55250

PUBLICATIONS OF THE COAST AND GEODETIC SURVEY RELATING TO FIELD METHODS

Special Publication No. 5.—Tables for a Polyconic Projection of Maps, 20 cents. This publication contains the necessary explanation of the method employed in constructing a polyconic projection, and also gives the values in meters of degrees, minutes, and seconds of latitude and longitude for all latitudes.

Special Publication No. 8.—Formulæ and Tables for the Computation of Geodetic Positions, 25 cents. Contains the formulas, instructions, and data for computing the geographical coordinates of triangulation and traverse stations when the distances and angles are known.

Special Publication No. 14.—Determination of Time, Longitude, Latitude, and Azimuth, 35 cents. A manual describing the instruments and methods used by the Coast and Geodetic Survey on its astronomic field and office work.

Special Publication No. 28.—Application of the Theory of Least Squares to the Adjustment of Triangulation, 25 cents. Explains the principles of least-squares adjustments of triangulation, with illustrative examples.

Special Publication No. 65.—Instructions to Light Keepers on First-Order Triangulation, 10 cents. A small pamphlet issued to light keepers, containing the signaling code used and instructions for adjusting and sighting the lights.

Special Publication No. 71.—Relation between Plane Rectangular Coordinates and Geographic Positions, 10 cents. Contains formulas and tables for changing from plane to spherical coordinates, and vice versa.

Serial No. 166.—Directions for Magnetic Measurements, 15 cents. Contains discussion of the theory of magnetic measurements, directions for making magnetic observations on both land and sea, and directions for operating a magnetic observatory.

Special Publication No. 93.—Reconnaissance and Signal Building, 30 cents. A manual covering reconnaissance for triangulation and traverse, the marking of stations, and the construction of triangulation and traverse targets and towers.

Special Publication No. 109.—Wireless Longitude, 15 cents. A description of the apparatus and methods employed in the accurate measurement of differences in longitude when radio time signals are used in place of signals sent over telegraph lines.

Special Publication No. 120.—Manual of First-Order Triangulation, 40 cents. Includes detailed instructions for first-order triangulation and base measurement.

Special Publication No. 137.—Manual of First-Order Traverse, 30 cents. Contains the specifications used by this bureau for executing first-order traverse and explains how the computations, both field and office, are made.

Special Publication No. 138.—Manual of Triangulation Computation and Adjustment, 50 cents. Contains detailed instructions for computing and adjusting triangulation.

Special publication No. 139.—Instructions for Tide Observations, 20 cents. Summarizes the methods used by this bureau in obtaining tide observations and in making the reductions of the records necessary to establish planes of reference for reducing hydrographic soundings.

Special Publication No. 140.—Manual of First-Order Leveling. (In press.) Contains detailed instructions for both the field work and office computation of first-order leveling.

Special Publication No. 143.—Hydrographic Manual, 45 cents. Contains the general requirements of this bureau for the execution of hydrographic surveys and describes the equipment and methods used for hydrograhic work.

Special Publication No. 144.—Topographic Manual, 30 cents. Contains specifications for topographic surveys and complete descriptions of instruments and methods. Several useful tables are included, and samples of symbols and maps are shown.

Any of the publications listed above can be purchased by those outside the bureau from the Superintendent of Documents, Washington, D. C., at the price stated. Numerous publications of this bureau contain the results of geodetic operations in the form of geographic positions of triangulation and traverse stations and elevations of bench marks. An engineer interested in securing those data for any particular locality should address his inquiry to the Director, Coast and Geodetic Survey.

A complete list of the publications of the Coast and Geodetic Survey will be found in the List of Publications of the Department of Commerce, a copy of which may be obtained free of charge upon application to the Department of Commerce.

INDEX

	Page
Abbreviations on list of geographic positions	104
Abstract, horizontal directions (direction theodolite)	87
wye levels	141
zenith distances (vertical angles)	106
Accuracy, angle measurements	33
base measurement, specifications	117
conditions affecting, angle measurements, traverse	162
second-order traverse	147
third-order traverse	3, 164
Adjustment, centers, theodolite	52
collimation, theodolite	51
focus, theodolite	50
of micrometers	53
comb	56
drum	56
equidistance	59
eyepiece	55
focus	55
for run	56
parallelism of wires	55
radial	55
optical verniers	60
parallax, theodolite	50
plate-level, theodolite	47
reading microscope	60
standards, theodolite	49
striding level, theodolite	48
vertical circle level, theodolite	52
verticality, sighting wires, theodolite	50
Adjustments, theodolite	46
Alignment, base	118
correction, base measurement	126
A-micrometer minus B-micrometer readings, graph	66
Angle measurements, accuracy	33
circle settings, direction theodolite	34
repeating theodolite	33
direction theodolite	34
errors due to, "creeping" lights	78
horizontal refraction	77
instability of support	73
phase and eccentricity	76
errors in horizontal	72
instrumental errors	75
main stations, traverse	160
observing program, direction method	42
repeating theodolite	33
repetition method	41
repeating theodolite	33
third-order traverse	165
traverse, conditions affecting accuracy	162
micrometer theodolite	161
rail and off-set stations	163
repeating theodolite	161
Angle of depression, distance to breaker, formula	82
Arc-sin corrections for inverse position computations, table	200
A-stations, traverse	149, 163
Astronomic azimuths	173
Azimuth, compass	71
curvature correction	177
geodetic	171
Laplace	173
mark	41, 175, 177
method, distribution of error, loop closure, traverse	169
observations, chronometer correction	180
direction theodolite	175
observing routine	175
Polaris	175
record of eccentricity	179
recorder's duties	177
repeating theodolite	177
second-order traverse	164
third-order traverse	165
when to observe Polaris	179
second-order	173
discrepancy	171
method of observing	174
third-order	183
Azimuths, astronomic	173
classification	173
Balance, spring	124
Bar (vinculum), meaning	105
Base, alignment	118
apparatus, handling and testing	122
computation of length	118
Base-line, computation	143
reconnaissance, specifications	117
Base, marking broken grades	121
Base measurement	122
accuracy, specifications	117
alignment correction	126
amount of sag, formula	128
blunders	131
correction for tape support	128
corrections for tension	127
corrections to measured lengths	125
duties of front contact man	135
front stretcher man	134
middle man	133
rear contact man	134
rear stretcher man	133
recorder	139
grade correction, broken grades	126
formula	125
limit of error	125
instruments and appliances	122

Page

Base measurement—Continued.
levels 126
on railroad rail 124
personnel and duties 132
precautions against errors 125
probable error 144
records and computations 139
reduction to sea level 145
sag correction, formula 127
second-order 117
set-ups and setbacks 132, 137, 138, 144
stretching of tape 127
tapes 122
third-order 146
computations 146
methods 146
specifications 146
Base measurements, agreement, specifications 117
Base nets, reconnaissance 18
Base, numbering of broken grade stakes 121
numbering of stakes 121
preparation 118
procedure in staking 120
set-ups, setbacks 121
sites, reconnaissance 18
staking 119
Base tapes, field comparison 118
standardization 118
data 123
Bases, broken, reconnaissance 19
frequency, reconnaissance 7
third-order triangulation 32
second-order, specifications 117
Blunders, base measurement 131
Breaker, distance by angle of depression, formula 82
Broken base, specifications 117
Broken grade stakes, numbering, base 121
Broken grades, marking, base 121

Care of tapes 156
Catenary and grade errors, traverse 154
Catenary correction 126, 144
factors for computing, table 196
Catenary corrections for various lengths and weights of tape, table 196
Centers, adjustment, theodolite 52
eccentricity, theodolite 53
Check computation, sum of perpendiculars, traverse 170
traverse 166
Check measurement, base 132
divergence, traverse 158
of distances, third-order traverse 165
traverse 158
300-foot tape 132, 158
Chronometer correction, azimuth observations 180
radio time signals 180
ship's chronometer 180
star observations 181
Chronometer, ship's, chronometer correction. 180
Circle, illumination, theodolite 60
Circle settings, direction theodolite, angle measurements 84
repeating theodolite, angle measurements. 33

Page

Circuit closure, second-order traverse 147
Classification, azimuths 173
horizontal control surveys 2, 3
Closure in position, second-order traverse 147
Closure of circuit, second-order traverse 147
Coefficient of expansion, tape 129
of refraction 110
Collimation, adjustment 51
Collimator, vertical 69
new model 70
old model 69
Combination, triangulation and traverse 149
Oregon coast 163
Compass, azimuth 71
Computation, base line 143
check, sum of perpendiculars 170
elevations, nonreciprocal observations 109
reciprocal observations 107
geographic positions, traverse 170
length of base 118
projection, traverse 168
refraction, reciprocal observations 107
sheets, traverse sketches 166
traverse line 166
Computations, abstract of horizontal directions (direction theodolite) 87
and records, base measurement 139
second-order traverse 165
third-order traverse 165
field, list of forms 84
second and third order triangulation 83
geographic positions 100–102
inverse solution, geographic positions 100, 103
list of directions 89
geographic positions 100
method of recording eccentricity 90
of elevations, factors used, table 205
record, horizontal angles (repeating theodolite) 84
horizontal directions (direction theodolite) 86
observations on intersection stations 86
reduction to center 91
nomogram 93
rejection of observations, rules 88
spherical excess 94
third-order base measurement 146
three-point problem 98
triangle sides 94
Conditions, number in figure 16
Connections to existing triangulation, reconnaissance 20
Connections with existing control, traverse 150
Constants, formulas, and tables 184
Control surveys, horizontal, classification 2, 3
horizontal, tabulation, specifications 3
value 20
Conversion tables 208
Correction, curvature and refraction, table, reconnaissance 21
records 43
Courtesy to property owners 80
Curvature and refraction correction table, reconnaissance 21
Curvature and refraction formula, reconnaissance 21

Page

Curvature correction, azimuth 177
Curvature, radius 108

Deflection of vertical 173
Description of station 110
  height of signal 112
  hydrographic mark 112
  natural objects 112
  shore-line reference 115
  use of standard notes 112
Determination of height of station by observing sea horizon, formula 81
Differences of elevation and inclination corrections for varying angles of inclination, table 186
Direction, probable error 5
Direction theodolite, angle measurements 34
  circle settings, angle measurements 34
Directions, number in figure 16
Discrepancies, azimuth, distribution, traverse 171
  distribution, geographic position 171
Discrepancy in records, field check 132
Discrepancy, second-order azimuth 171
Distance to breaker by angle of depression, formula 82
Distribution, azimuth discrepancies, traverse 171
  discrepancies, geographic position 171
Distribution of error, loop closure, traverse 169
  loop closure, traverse, azimuth method 169
Double zenith distances (vertical angles), record 104

Eccentric stations, traverse 162
Eccentricity, and phase errors, angle measurements 76
  method of recording 90
  of centers, theodolite 53
  record, azimuth observations 179
Elasticity, modulus, tape 127
Elevation of base, reduction to sea level 118
Elevation of stations, difference, formula 108
Elevations, computation, nonreciprocal observations 109
  reciprocal observations 107
Elongation and culmination, time, Polaris 179
Emergency repairs, instruments 63
  sighting wires 63
Equipment and personnel, staking base 119
Erasures in records 43
Error, inclined horizontal axis, vertical circle, theodolite, formula 52
Errors due to, "creeping" lights, angle measurements 78
  horizontal refraction, angle measurements 77
  inclined vertical axis, theodolite, formula 48
  instability of support, angle measurements 73
  phase and eccentricity, angle measurements 76
Errors, in horizontal-angle measurements 72
  instrumental, angle measurements 75
  systematic, tape 131
  time observations 182
Examples, intervisibility of stations, reconnaissance 22
Expansion coefficient, tape 129

Feet to meters, conversion table 208
Field comparison, base tapes 118
  tapes, traverse 153
Field computations, list of forms 84
  second and third order triangulation 83
Figure, strength, reconnaissance 7
  triangulation, number of conditions 16
  triangulation, number of directions 16
Figures, character, reconnaissance 6
  overlapping 17
  strength, reconnaissance, third-order triangulation 31
  triangulation 11-16
Focus, adjustment, theodolite 50
Forms, list, field computations 84
Formula, amount of sag, base measurement 128
  curvature and refraction, reconnaissance 21
  determination of height of station by observing sea horizon 81
  difference of elevation of stations 108
  distance to breaker by angle of depression 82
  error due to inclined horizontal axis, vertical circle, theodolite 52
  error due to inclined vertical axis, theodolite 48
  grade correction, base measurement 125
  intervisibility of stations, reconnaissance 22
  probable error 145
  reduction of verticals to line joining stations 106
  reduction to sea level, bases 145
  run of micrometer 58
  sag correction, base measurement 127
Formulas, constants and tables 184
  probable error 184
Fourth-order control, records 116
Friction over supports, tape 130
Front contact man, duties, base measurement 135
Front stretcher man, duties, base measurement 134

Geodetic azimuth 171
Geodetic constants 184
Geographic position, distribution, discrepancies 171
Geographic positions, computation, traverse 170
  computations 100-102
  list 100
Grade and catenary errors, traverse 154
Grade correction, base measurement, broken grades 126
  formula 125
  limit of error 125
Grade corrections, traverse 159

Handling and testing of base apparatus 122
Height, of instrument above station mark 105
  of signal, description of station 112
  of station, determination by observing sea horizon, formula 81
  of trees, determination, reconnaissance 27
Heliotrope 71
Highways, traverse measurements 159
Horizontal angles, record (repeating theodolite) 84

	Page
Horizontal directions, abstract (direction theodolite)	87
record (direction theodolite)	86
Horizontal refraction errors, angle measurements	77
Hydrographic points, supplemental, traverse	154
Illumination, circle	60
Inclination corrections, 5, 10, 15, 20, 25, 30, 35, 40, and 45 meter lengths, table	193
50-meter tape lengths, table	187
25-meter tape lengths, table	190
varying angles of inclination and differences of elevation, table	186
Instability, invar tapes	122
Instruments	44
and appliances, base measurement	122
for light keepers	70
Intersection stations, observations	35
observations, record	86
shown on progress sketch	116
traverse	162
Intervisibility of stations, formula, reconnaissance	22
reconnaissance, examples	22
Intervisibility, reconnaissance stations	21
Invar tapes, handling	122
instability	122
traverse measurement	152
Inverse position computations, arc-sin corrections, table	200
Inverse solution, geographic positions	100, 103
Lamp, signal	71
automatic	72
Landmarks for charts	36
Laplace azimuth	173
Least squares adjustments, traverse	172
Length of lines, reconnaissance	18
Length of 1 degree of meridian, table	206
of parallel, table	207
Lenses, theodolite, care	63
Level bubble, determining value of one division	80
value of one division	104
Level record, traverse	166
Level, vertical circle, adjustment, theodolite	52
Leveling, trigonometric	36
wye, record	139
Levels, base measurement	126
traverse	159
wye, abstract	141
Light keepers' instruments	70
List of directions	89
List of geographic positions, temporary stations	104
abbreviations	104
Log A, table	205
Log B and log C, table	205
Log C and log B, table	205
Log m, table	199
Loop closure, traverse, distribution of error, azimuth method	169
Loop closures, traverse	168
distribution of error	169
Loops, traverse	149, 168
"Lost" station, recovery note	114
Mark, azimuth	175, 177
Marking broken grades, base	121
Marking stations	37
traverse	151
Marking tape, parallax	131
Marks, azimuth	41
material	41
other organizations	37
reference	40
station	37, 38
terminal, tape	130
underground	39
Measurement of base	122
on railroad rail	124
Measuring with tapes	132
Meridian, length of 1 degree, table	206
Meters to feet, conversion table	212
Micrometer A minus micrometer B readings, graph	66
Micrometer readings, range	68
Micrometers, adjustment	53
for equidistance	59
for run	56
parallelism of wires	55
comb adjustment	56
drum adjustment	56
eyepiece adjustment	55
focus adjustment	55
radial adjustment	55
Middle man, duties, base measurement	133
Modulus of elasticity, tape	127
Naming stations	79
Nomogram, reduction to center	93
Notes, standard, reference marks	113
surface marks	112
underground marks	113
use, description of station	112
witness marks	114
Observations on intersection stations	35
Observing program, angle measurements, repeating theodolite	83
direction method, angle measurement	42
repetition method, angle measurement	41
vertical angles	43
Offset and rail stations, angle measurement, traverse	163
Offset distances, tape measurements	166
traverse	157
Offset station, traverse	149
Optical verniers, adjustment	60
Oregon coast, combination of triangulation and traverse	149
Packing theodolite for shipment	62
Parallax, adjustment, theodolite	50
in marking tape	131
of telescope, time observations	183
vertical-angle observations	43
Parallel, length of 1 degree, table	207
Perdendiculars, sum, computation check	170
Phase and eccentricity errors, angle measurements	76
Pivot inequality, theodolite	49
Pivot irregularity, theodolite	50
Plane of reference	36

Page
Plate-level adjustment, theodolite........... 47
Polaris, azimuth observations............... 175
hour angle from Mizar—δ Cassiopeiæ..... 179
time of elongation and culmination........ 179
when to observe, azimuth observations.... 179
Preparation of base........................ 118
Pressure clamp, theodolite.................. 46
Probable error, base measurement............ 144
direction.................................. 5
formulas.................................. 145, 184
Progress sketch............................ 115
intersection stations shown................ 116
R_1 shown................................ 115
Projection computation, traverse............ 168
Property owners, courtesy.................. 80
Proportional change in a number corresponding to a change in its logarithm, table...... 201
Publications relating to field methods....... 216

R_1...................................... 3, 5, 7, 9
shown on progress sketch.................. 115
Radii of curvature of earth's surface, logarithms, table................................ 202
Radius of curvature........................ 108
Rail and offset stations, angle measurement, traverse.................................. 163
Rail station, traverse....................... 149
Range, micrometer readings................. 68
Rear contact man, duties, base measurement. 134
Rear stretcher man, duties, base measurement..................................... 133
Reconnaissance, base line, specifications..... 117
base nets................................. 18
base sites................................. 18
broken bases.............................. 19
character of figures........................ 6
connections to existing triangulation........ 20
correction for curvature and refraction, table.................................. 21
determining heights of trees............... 27
formula, curvature and refraction.......... 21
intervisibility of stations.................. 22
frequency of bases......................... 7
intervisibility of stations, examples........ 22
length of lines............................. 18
second-order triangulation, specifications.. 6
signals, triangulation...................... 27–31
sites for stations.......................... 26
specifications, modifications................ 19
stations, intervisibility.................... 21
strength of figure.......................... 7
third-order triangulation, frequency of bases.................................. 32
specifications.......................... 31
strength of figures...................... 31
Record, double zenith distances (vertical angles).................................. 104
horizontal angles (repeating theodolite).... 84
horizontal directions (direction theodolite). 86
observations on intersection stations....... 86
tape measurements, bases.................. 139
traverse............................... 166
traverse measurements..................... 157
wye leveling............................... 139
Recorder, duties, azimuth observations....... 177
base measurement........................ 139
Records and computations, base measurement.................................... 139
second-order traverse...................... 165
third-order traverse....................... 165
Records, correction......................... 43
discrepancy, field check................... 132
erasures.................................. 43
fourth-order control....................... 116
Recovery note, "lost" station................ 114
triangulation station...................... 114
Reduction to center........................ 91
Reduction to sea level, base measurement... 145
bases, formula............................ 145
elevation of base.......................... 118
traverse................................. 159
Reference marks............................ 40
standard notes............................ 113
traverse................................. 151
Refraction and curvature correction, table, reconnaissance.............................. 21
formula, reconnaissance................... 21
Refraction, coefficient...................... 110
computation, reciprocal observations....... 107
errors, time observations.................. 182
horizontal, errors, angle measurements.... 77
Rejection of observations, rules.............. 88
"Remarks" column, data, traverse........... 157
Repairs, emergency, instruments............. 63
sighting wires............................ 63
Report, season's........................... 115
Run of micrometer, effects of................ 58
formula.................................. 58
tests..................................... 57

Sag, amount, base measurement, formula..... 128
correction, base measurement, formula..... 127
Sea horizon, determination of height of station by observing, formula...................... 81
Sea-level reduction, base measurement....... 145
bases, formula............................ 145
elevation of base.......................... 118
traverse................................. 159
Season's report............................ 115
Second-order azimuth, discrepancy.......... 171
method of observing....................... 174
Second-order bases, measurement............ 117
specifications............................ 117
Second-order traverse....................... 147
accuracy................................. 147
azimuth observations...................... 164
records and computations.................. 165
Setbacks, base............................. 121
set-ups, base measurement....... 132, 137, 138, 144
traverse measurement................ 153, 156
Settings, circle, repeating theodolite, angle measurements................................ 33
Set-ups, base.............................. 121
setbacks, base measurement.... 132, 137, 138, 144
traverse measurement................ 153, 156
Sextant triangulation....................... 116
Shore-line reference, description of station... 115
Sighting wires, distance apart............... 64
Signal building, traverse.................... 151
Signal lamp................................ 71
automatic................................ 72
Signals, triangulation, reconnaissance........ 27–31

Page

Sketch, progress 115
tape measurements 166
traverse 157
computation sheets 166
Specifications, agreement of base measurements 117
base-line reconnaissance 117
base measurement, accuracy 117
broken base 117
for marking stations 38
horizontal control surveys, tabulation 3
reconnaissance, for third-order triangulation 31
modifications 19
second-order triangulation 6
second-order bases 117
second-order traverse 148
second-order triangulation 33
third-order base measurement 146
third-order traverse 164
third-order triangulation 83
Spherical excess, computations 94
Spring balance 124
care 124
testing weight 124
Stakes, numbering, base 121
Staking base 119
procedure 120
Standard notes, reference marks 113
surface marks 112
underground marks 113
use, description of station 112
witness marks 114
Standardization, base tapes 118
Standardization data, base tapes 123, 129
Standards, adjustment, theodolite 49
Star observations, chronometer correction 181
Station, description 110
marks 37, 38
height of instrument above 105
triangulation, recovery note 114
Stations, distance between 36
eccentric, traverse 162
intersection, observations 35
shown on progress sketch 116
traverse 162
intervisibility, formula, reconnaissance 22
reconnaissance, examples 22
location, second-order traverse 148
marking 37
naming 79
reconnaissance, intervisibility 21
sites, reconnaissance 26
specifications for marking 38
subsidiary, traverse 163
supplementary 19
temporary, on list of geographic positions 104
Statistics of field work, fiscal year 116
season 116
Strength of figure, reconnaissance 7
table 8
Strength of figures, reconnaissance, third-order triangulation 31
Stretcher, tape, for rail measurements, traverse 156
Stretching of tape, base measurement 127

Page

Striding level, adjustment, theodolite 48
"wind," theodolite 49
Subsidiary stations, traverse 149, 163
Supplemental hydrographic points, traverse 154
Supplementary stations 19
Support, tape, correction, base measurement 128
third-order traverse 164
Surface marks, standard notes 112
Surveys, horizontal control, classification 2, 3
tabulation, specifications 3
Systematic errors, tape 131

Table, arc-sin corrections for inverse position computations 200
catenary corrections for various lengths and weights of tape 196
conversion, feet to meters 208
meters to feet 212
correction for curvature and refraction, reconnaissance 21
differences of elevation and inclination corrections for varying angles of inclination 186
factors for computing catenary correction 196
factors used in computations of elevations 205
inclination corrections, 50-meter tape lengths 187
5, 10, 15, 20, 25, 30, 35, 40, and 45 meter lengths 193
25-meter tape lengths 190
length of 1 degree, of meridian 206
of parallel 207
log A 205
log B and log C 205
log m 199
logarithms of radii of curvature of earth's surface 202
proportional change in a number corresponding to a change in its logarithm 201
strength of figure 8
temperature corrections for steel tapes 198
Tables 186
Tabulation, specifications, horizontal control surveys 3
Tape, change in weight due to moisture 128
coefficient of expansion 129
correction for change in weight 128
for method of support 128
friction over supports 130
measurements, offset distances 166
record, bases 139
traverse 166
sketch 166
modulus of elasticity 127
parallax in marking 131
standardization data 129
stretcher for rail measurements, traverse 156
stretching apparatus 124
stretching, base measurement 127
support, third-order traverse 164
traverse 154
tripods, traverse 155
systematic errors 131
temperature correction 130
error 130
temperature, traverse 156
tension, traverse 155

Tape—Continued. Page
terminal marks 130
300-foot for check measurement 168
wind effect 131
Tapes, base measurement 122
care 156
field comparison, traverse 153
thermometers 124
Temperature correction, tape 130
error 130
Temperature corrections for steel tapes, table 198
Temperature of tape, traverse 156
Temporary stations on list of geographic positions 104
Tension apparatus, tape 124
Tension corrections, base measurement 127
Tension, tape, traverse 155
Testing quality of theodolite 65
Testing weight, spring balance 124
Theodolite, adjustments 46
care 61
direction, azimuth observations 175
lenses, care 63
micrometer, angle measurements, traverse 161
packing for shipment 62
pressure clamp 46
repeating, angle measurements 33
traverse 161
azimuth observations 177
circle settings, angle measurements 33
testing quality 65
vertical circle graduation 43
Wild 60
Theodolites 44
Thermometer, tape, traverse 156
Thermometers, attaching to tape 125
for tapes 124
Third-order, azimuth 183
base measurement 146
computations 146
methods 146
specifications 146
traverse, accuracy 3, 164
angle measurements 165
azimuth observations 165
check measurement of distances 165
records and computations 165
specifications 164
support of tape 164
triangulation, specifications 83
Three-point problem 98
Time determinations, meridian observations 181
stars near prime vertical 181
Time observations, errors 182
by recorder 182
incorrect circle readings 182
parallax of telescope 183
poor selection of stars 183
refraction errors 182
wrong star 182
Time signals, radio, chronometer correction 180
Traverse, A-stations 149, 163
and triangulation, combination 149
angle measurements, main stations 160

Traverse—Continued. Page
angle party 152
building party 151
catenary and grade errors 154
check computation 166
check measurement, distances 158
divergence 158
check-taping party 152
computation of geographic positions 170
connections with existing control 150
data in "Remarks" column 157
eccentric stations 162
field comparison, tapes 153
grade corrections 159
intersection stations 162
least squares adjustments 172
level record 166
leveling party 152
levels 159
line, computation 166
loop closures 168
loops 149, 168
marking stations 151
measurement, details 153
set-ups, setbacks 153
with invar tapes 152
measurements, along highways 159
record 157
offset distances 157
offset station 149
organization of party 151
projection computation 168
rail and offset stations, angle measurement 163
rail station 149
record, check measurements 158
tape measurements 166
reduction to sea level 159
second-order 147
accuracy 147
along railroads 148
azimuth observations 164
closure in position 147
closure of circuit 147
location of stations 148
records and computations 165
specifications 148
where used 147
set-ups and setbacks 156
signal building 151
sketch 157
sketches, computation sheets 166
subsidiary stations 149, 163
supplemental hydrographic points 154
tape stretcher for rail measurements 156
tape support 154
tripods 155
tape temperature 156
tape tension 155
taping party 152
third-order, accuracy 3, 164
angle measurements 165
azimuth observations 165
check measurement of distances 165
records and computations 165

Page

Traverse—Continued.
third-order, specifications 164
support of tape 164
Triangulation and traverse, combination 149
Oregon coast 163
Triangulation, connections to existing, reconnaissance 20
figures 11-16
second and third order, field computations 83
second-order, specifications 33
sextant 116
sides, computations 94
station, recovery note 114
third-order, specifications 83
Tribrach, design 69
Trigonometric leveling 36
Tripods, tape support, traverse 155

Underground marks 39
standard notes 113

Vertical angles 36
abstract of zenith distances 106
double zenith distances, record 104
observations, parallax 43
observing program 43

Page

Vertical circle, level, theodolite, adjustment 52
theodolite, error inclined horizontal axis, formula 52
graduation 43
Vertical collimator 69
new model 70
old model 69
Vinculum, meaning 105

Weight, tape, change due to moisture 128
correction for change 128
Wild theodolite 60
Wind effect, tape 131
"Wind" of striding level, theodolite 49
Wires, parallelism, adjustment of micrometers 55
sighting, adjustment for verticality, theodolite 50
distance apart 64
emergency repairs 63
Witness marks, standard notes 114
Wye leveling record 139
Wye levels, abstract 141

Zenith distances (vertical angles), abstract 106

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