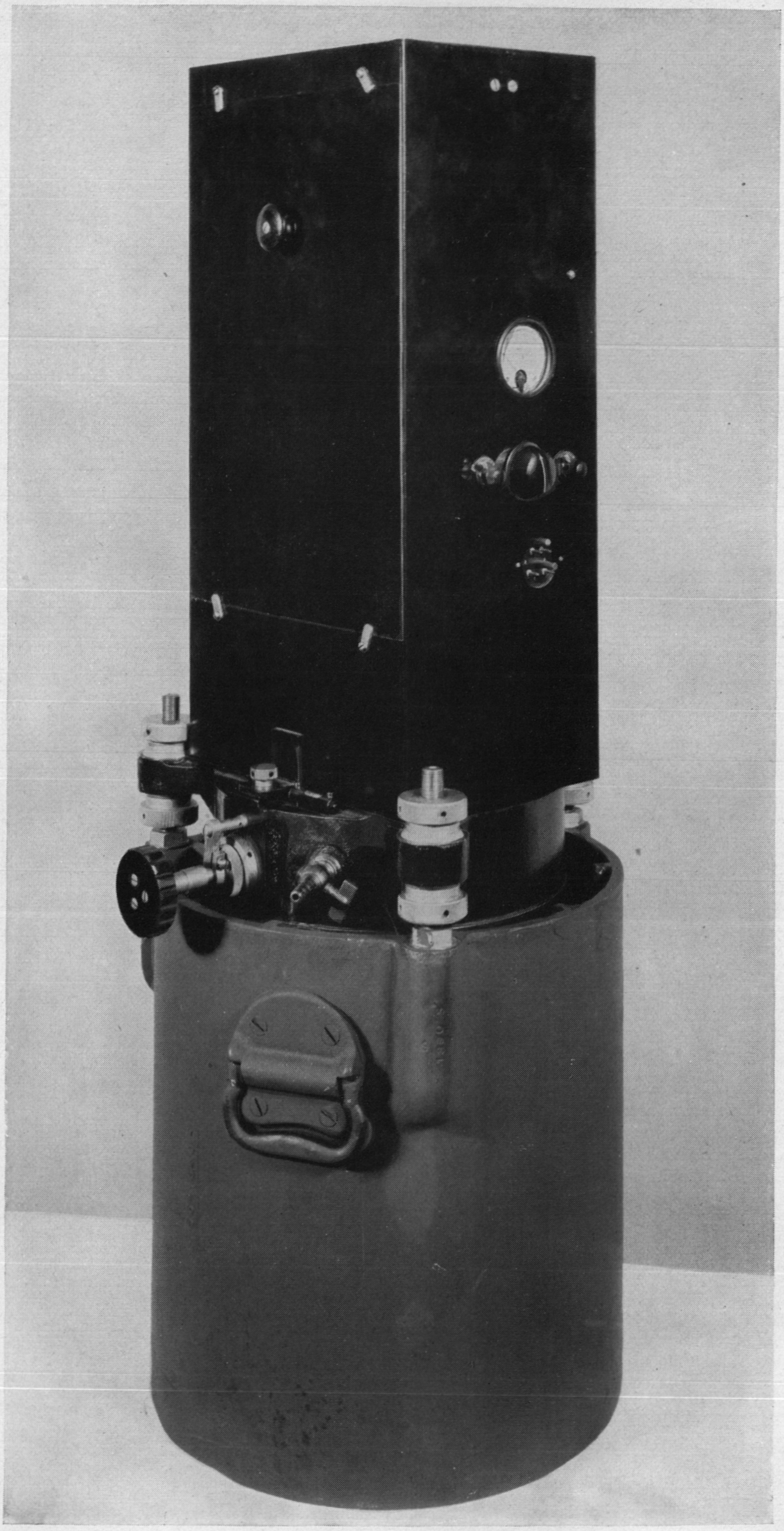

Frontispiece. Brown gravity apparatus.

U. S. DEPARTMENT OF COMMERCE
JESSE H. JONES, Secretary
COAST AND GEODETIC SURVEY
LEO OTIS COLBERT, Director

Special Publication No. 232

# PENDULUM GRAVITY MEASUREMENTS AND ISOSTATIC REDUCTIONS

By
CLARENCE H. SWICK



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# PENDULUM GRAVITY MEASUREMENTS AND ISOSTATIC REDUCTIONS

## INTRODUCTION

This publication supersedes Special Publication No. 69 "Modern methods for measuring the intensity of gravity" which was published in 1921 and which related principally to the Mendenhall type of pendulum apparatus. In 1932 this apparatus was largely superseded by the Brown gravity apparatus, designed and constructed by the late E. J. Brown, of this Bureau. The principal consideration in designing the new apparatus was to speed up the observations without any loss of accuracy.

Although many parts of the Mendenhall apparatus, including the pendulums themselves, were kept unchanged in the new apparatus two major improvements were made at this time. One was a clamping device in the receiver to hold the pendulum securely and safely during transportation from station to station. This avoided the necessity for breaking the vacuum seal and removing the pendulum each time a move was made. The second improvement was a pick-up and amplifying device for recording the pendulum oscillations on a chronograph and thus permit direct time comparison between the pendulum oscillations and radio time signals. A photoelectric cell was used as the basis for this device, as explained on page 12.

The apparatus was, of course, adapted to transportation by automobile truck. A special truck, later replaced by a trailer, was equipped with permanent mountings for much of the auxiliary apparatus such as chronograph, chronometers, radio receiver, amplifier, etc. The "overhead," or time required to prepare for the observations, was considerably decreased by this arrangement and it was also decreased by carefully designing the equipment that had to be removed from the truck at each station in order to give the maximum facility for handling.

Another improvement in the equipment should be mentioned. In order to obviate the necessity for building a special concrete pier at each station or of finding some rigid support already in place on which the pendulum apparatus could be mounted, a bucket-shaped aluminum box was designed which could be used as the support. At each station, a hole is now dug in the ground and the box is set in it in plaster of Paris which becomes sufficiently hard in about 30 minutes to permit starting the pendulum observations. Since many of the recent stations of this Bureau are located in the open country where it would be difficult to find concrete floors or other supports already in place, this substitute arrangement saves much time.

Methods for measuring gravity have changed very rapidly during the past few years and many new types of apparatus have been devised. A major part of the development work has been done by oil companies in their efforts to obtain an instrument with which comparisons of gravity over limited areas could be made rapidly and accurately. The instruments used for this purpose are mostly of the type called gravimeters. The typical gravimeter consists essentially of a spring, changes in the extension of which, under the load of a constant mass, can be measured with extreme accuracy as the instrument is moved from place to place and is thus affected by changes in gravity.

Until very recently the different types of gravimeters have not been well adapted for use on general gravimetric surveys over wide areas. The Coast and Geodetic Survey has therefore continued to use pendulum apparatus to determine the basic stations scattered over the country. These stations are now fairly well distributed over the United States and gravimeters will probably be used for most of the remaining intermediate work. It seems likely, however, that pendulum apparatus will be used to some extent for many more years for special problems such as the international comparison of national base stations and the establishment of fundamental stations in special locations.

This manual includes a brief description of the Brown pendulum apparatus and of the corresponding methods and computations. No attempt has been made to describe the construction or use of the many types of gravimeters. Most of them are of extreme sensitivity and require great care in construction, but they are, nevertheless, fairly simple to use.

A part of this manual is devoted to instructions for processing gravity results and making the isostatic reductions. This work constitutes one of the principal contributions of the Bureau to gravity investigations. The isostatic computations follow the general method outlined by John F. Hayford and William Bowie in Special Publication No. 10 but many changes have been made in the detailed methods for the purpose of increasing the accuracy or of expediting the computations. The reader will do well to study the general principles of the isostatic method as given in Special Publication No. 10 before using this manual as a guide for making isostatic computations.

In the preparation of this manual the author has received generous assistance from various members of the Division of Geodesy of the Coast and Geodetic Survey. He has used, without individual acknowledgment, several tables and other material prepared by his coworkers in the Bureau. He wishes particularly to acknowledge gratefully the help received from A. J. Hoskinson, C. I. Aslakson, W. D. Lambert, F. W. Darling, J. A. Duerksen, and C. A. Whitten in their critical reading of parts or all of the text and for other assistance they have rendered.

## Chapter 1. BROWN PENDULUM APPARATUS

From 1890 to about 1930 all gravity determinations by the Coast and Geodetic Survey were made with the Mendenhall quarter-meter pendulum apparatus. Up until 1921 bronze pendulums were used in this apparatus. Because of their large temperature coefficient, measurements with these pendulums had to be made in rooms where the temperature remained fairly constant. This limited the selection of stations to a serious extent and often made necessary the construction of special rooms or buildings for the observations. It was finally decided, therefore, to make a set of invar pendulums because of the low-temperature coefficient of this alloy. The invar pendulums had a coefficient only about one-fifteenth that of the bronze pendulums and they therefore proved quite satisfactory as far as temperature changes were concerned. They were, however, somewhat inferior to the bronze pendulums in other respects, principally because of their magnetic properties and their lack of precise stability.

Field determinations with the Mendenhall apparatus were relatively slow, although they were not excessively expensive because they were ordinarily made with a small party of one or two men. One of the causes for the slow progress was the necessity for breaking the vacuum seal of the pendulum case several times at each station in order to change pendulums or to remove them before the apparatus was shipped to a new station. The vacuum pumps were not very efficient and the necessary reduction of pressure within the case required some time and much strenuous effort. Four or five stations per month was the usual rate of progress with the Mendenhall apparatus. (A detailed description of this apparatus will be found in Special Publication No. 69, which is now out of print but still available for reference in depository libraries.)

About 1930, interest in the gravity work of this Bureau had increased to the point where only a small part of the numerous, legitimate requests for new stations could be met. There was urgent need for a number of major changes in the apparatus in order that the gravity work could be considerably expedited. The task of making these changes and of redesigning the apparatus was assigned to the late E. J. Brown of this Bureau. The resulting apparatus was later named the Brown gravity apparatus in his honor. It was first used in the field in 1932 and has been employed almost exclusively since then for the field determinations of this Bureau. During the 9 years following 1932 more than three times as many stations were determined as during the preceding 40 years.

The principal changes in the apparatus have already been described briefly on page 1. They will be covered in more detail in the following pages where the individual parts of the Brown apparatus are described.

At first only one set of the apparatus was constructed and this was used by itself for field determinations. It was not feasible to change pendulums one or more times at each station as had been the practice with the Mendenhall apparatus, but a single pendulum remained in the vacuum case for an entire season without the seal being broken except on the rare occasions when field repairs became necessary. The integrity of the pendulum was thus checked only by the standardizations at the Washington base station and by such reoccupations of previously determined stations as might be made during the field season. This was not entirely satisfactory because there was a possibility, even though a slight one, that the pendulum, or some other part of the apparatus, might undergo a change which would affect the results at one or more stations but which would become corrected before being detected.

Since 1936 two sets of the apparatus have been used at each station in order to obviate this uncertainty. They are used simultaneously but entirely independently except that the periods of both pendulums are based on the same radio time signals and are recorded on the same chronograph. The additional cost per station is relatively small although the work is considerably increased because of the extra equipment required. The planes of oscillation of the two pendulums are oriented approximately at right angles to each other at each station, one approximately north and south and the other east and west.

## QUARTER-METER PENDULUMS

The pendulum is quite simple in construction as can be seen in figure 1. It consists of a thin stem with a heavy bob attached to the lower end and with an inverted stirrup attached to the upper end to hold the agate plane (or knife edge in the newer apparatus) on which the pendulum swings. The stem is 4 by 14 mm in cross section, with rounded corners. The bob is lenticular in shape, about 9 cm in diameter and 4.4 cm thick at the center. Both it and the stirrup at the upper end are fastened securely to the stem by means of rivets of the same material (invar) as the rest of the pendulum. From the lower face of the plane (or from the knife edge) to the center of the bob is about 24.8 cm and the pendulum weighs about 1.5 kg.

The pendulums used with the Brown apparatus are made of invar. They are the same pendulums that were used in the Mendenhall apparatus. When changed over to the new apparatus the vertical mirror attached to the face of the head of the pendulum was removed, and a new mirror was attached in a horizontal position on top of the head. Just recently, on apparatus made in 1940, another change was made by reversing the positions of knife edge and plane. In the 1940 apparatus the knife edge is attached to the pendulum and the plane to the support. It is believed that this arrangement will give somewhat more reliable results, since a slight shift in the position of the pendulum with relation to its support will be less likely to affect the period of the pendulum. It has been found from long experience that any wear that occurs in the agate knife edge is extremely slow and, therefore, that any lengthening of the pendulum which might be caused by this wear when the knife edge is attached to the pendulum is evaluated with sufficient accuracy by the standardizations at the Washington base station.

## VACUUM CASE

The vacuum case or chamber consists of two main parts, a relatively massive head of cast aluminum or bronze, and a piece of brass tubing which is attached to it. (See fig. 2.) The tubing is 7 inches in inside diameter and 5/16 inch thick except where grooved out to half this thickness to form channels about 1 inch wide as shown in figure 2. The lower end of the tube is sealed by means of a spherical plate soldered in a groove on the inside of the tube. A ring about 1⅜ inches wide attached to the top of the tube is used to fasten the tube to the cast head. The temperature insulation of this chamber is described on page 15.

The head has three projecting arms by means of which the case is supported and leveled. These are at practically the same height as the knife edge on which the

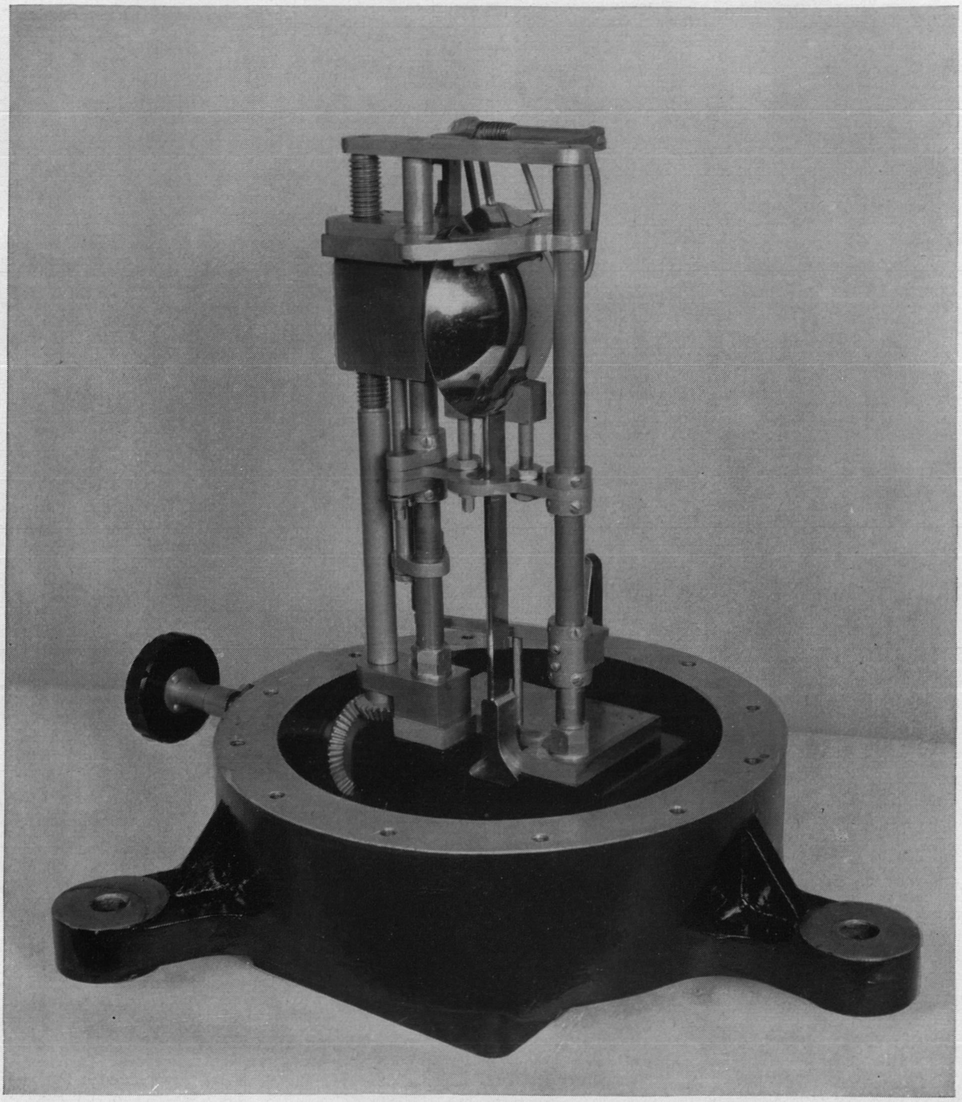

FIGURE 1.—Gravity pendulum and its support, upside-down position.

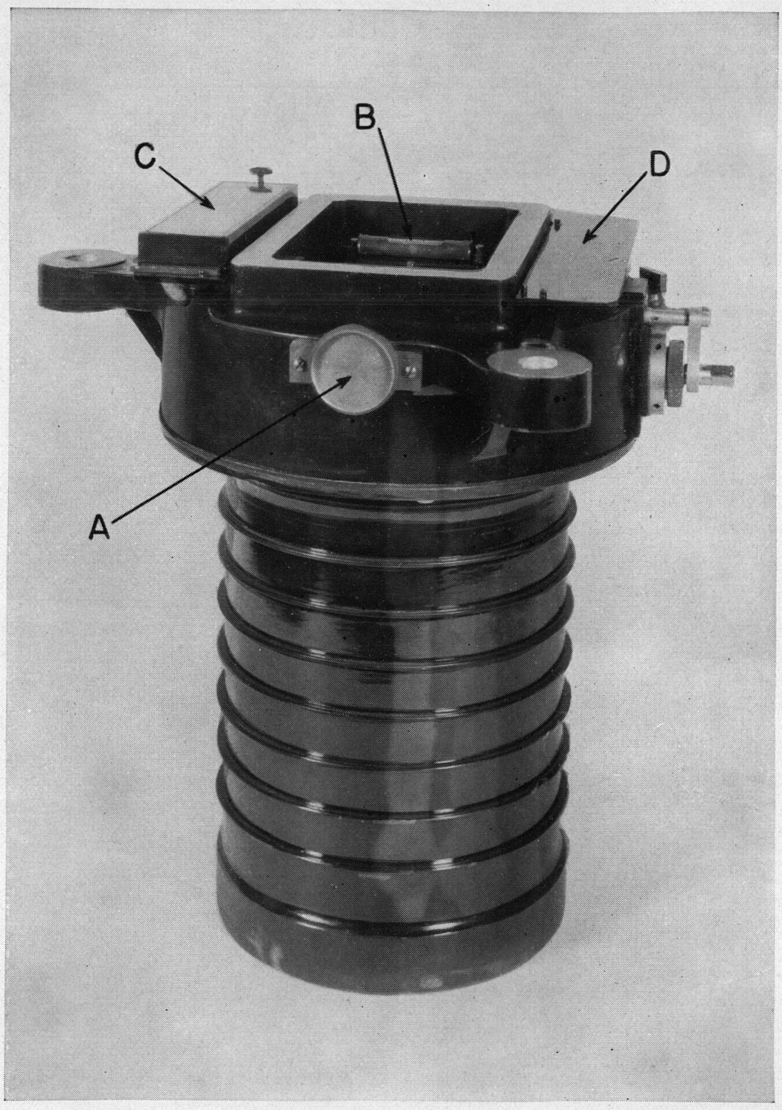


FIGURE 2.—Vacuum case in which gravity pendulum swings.

A is interferometer plate, B is sensitive level vial attached to knife-edge plate, C is compass declinometer, and D is removable plate for support of compass declinometer.

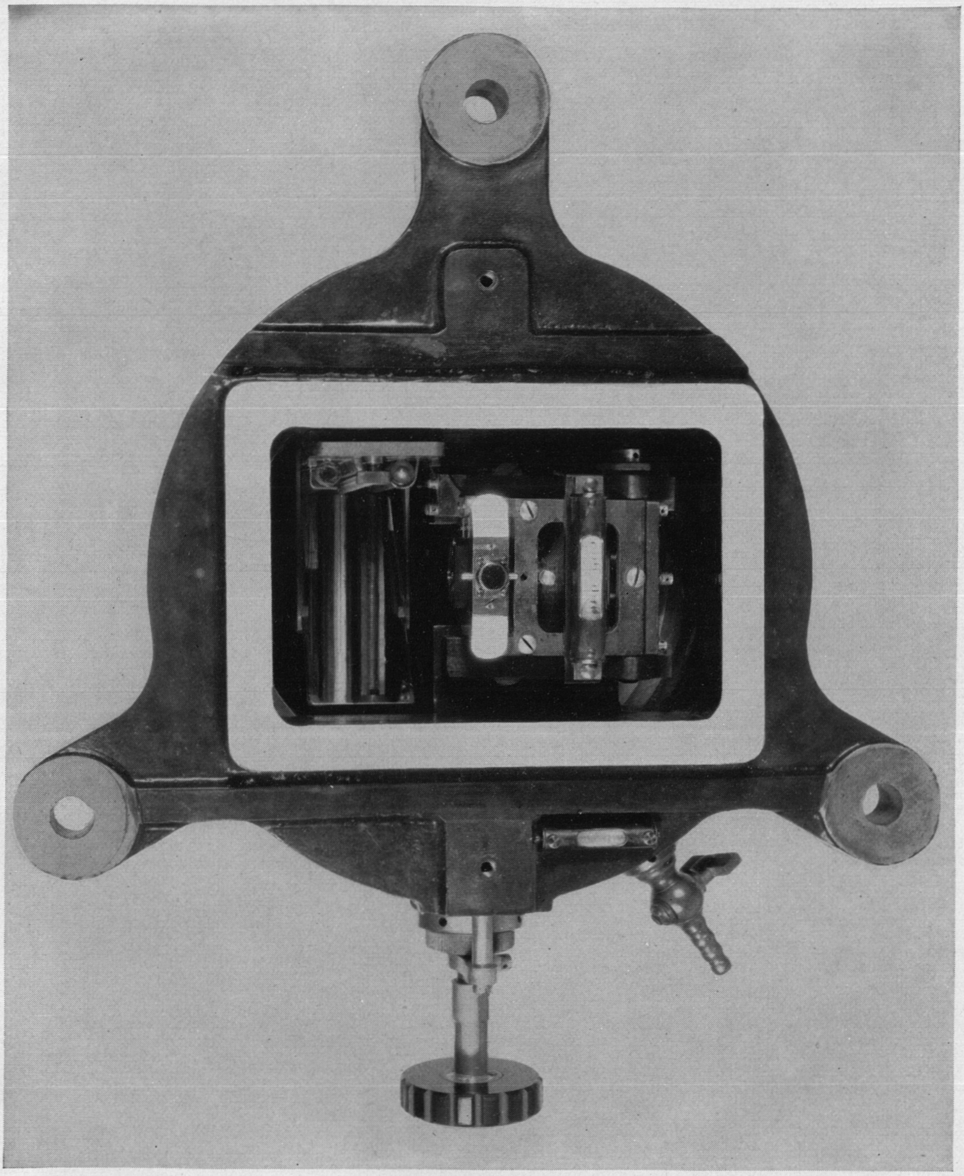

FIGURE 3.—View of pendulum apparatus from directly above.

The small circular mirror attached to top of pendulum, which is used to reflect the light beam of the recording apparatus, can be seen near center of opening.

FIGURE 4.—Base chamber, used as support for pendulum apparatus.

pendulum swings. They were placed at this height in order to decrease flexure as much as possible. The case really hangs from the support instead of resting on foot screws at the bottom as is the more common practice with instruments of this kind.

In addition to the opening to which the large tube forming the vacuum chamber is attached, the head has three other openings, a large rectangular opening in the top which is sealed with a piece of heavy plate glass, an opening in the side for the rod which is used for all manipulation of the pendulum after the case is sealed, and a stopcock opening.

There is a small, rather crude, level attached to the top of the head in a position parallel to the plane of oscillation of the pendulum, which is used for leveling the case in that direction. The more precise leveling which is required in the direction parallel to the knife edge is done with a level attached to the pendulum support inside the case. This level can be seen in figure 3.

Some difficulty was experienced in the construction of the Brown apparatus to make the case airtight. It was found, for example, that the cast head leaked badly. Several coats of Glyptal paint had to be applied on the outside while a partial vacuum was maintained on the inside to draw the paint into the small pores or interstices before the leaks could be stopped. A few of the soldered joints gave trouble also because of the corrosive action of the flux. The rod used to manipulate the pendulum has a long slender taper shaft very closely fitted to its bearing surface so that no packing is required to make an airtight seal. It is advisable that the entire length of the shaft be coated with a thin coat of good graphite stopcock grease to make it turn freely and also aid in making an airtight fit. The top opening, with the plate-glass cover, has a flange ground to a true plane and is readily sealed by the use of lubriseal or other good vacuum sealing compound. Since this cover is seldom removed, Glyptal paint is also used around the edge of the glass.

## BASE CHAMBER

As stated on page 1, the problem of finding a rigid support for the pendulum case, when the Mendenhall apparatus was used, was frequently a troublesome one and became a serious limitation on the selection of stations. For the Brown apparatus this difficulty was overcome by the design of a bucket-shaped support which is shown in figure 4. This support is known as the base chamber. It is made of cast aluminum, 12½ inches in inside diameter, 14½ inches high, and ⅝ inch thick. It has nine vertical ribs on the inside to give it greater rigidity and three projections on the outside, near the top, to which the leveling bolts are attached.

On each leveling bolt are two plano-convex washers and two nuts with concave surfaces to fit the washers. (See fig. 4.) The holes through the washers and those through the arms of the vacuum case are made large enough to give considerable latitude in leveling the case before there will be any tendency to bind. The large bearing surfaces tend to eliminate a large part of the flexure which is caused by the elastic compression of the foot screws when the ordinary type of foot screws are used. The bolts are 9/16 inch in diameter. When the case has been leveled the upper nuts on the leveling bolts are tightened and the case is thus clamped securely to the base chamber.

In order to obtain a rigid support for the base chamber itself, at each station a hole is dug in the ground somewhat larger than the chamber and about 1 foot deep. Several pounds (sometimes as much as 40 or 50) of plaster of Paris are placed in the hole to bond the base chamber to the ground. The moistening of the plaster is done in the hole in order that the wet plaster may seep into the soil and bond it together more firmly. Usually the plaster sets in about 30 minutes rigid enough for the observations to be started although this depends upon the quality of the plaster. With poor plaster it is sometimes necessary to wait an hour or more. All heat must be out of the plaster before the instrument levels will stop drifting. The base chamber is set approximately level by use of a small universal level attached to the inside of the bottom of the chamber.

At a few special stations, concrete piers are built as a support for the apparatus. These piers are sometimes constructed at universities or at stations where it may be desirable to repeat the observations at some later time for the purpose of verifying the apparatus. They are made about 15 inches square on top and extend deep enough to have a firm foundation. The top may be flush with the ground or a few inches above it. Two piers are required, one for each set of apparatus. They should be placed far enough apart (not less than 4 feet) to prevent any possible disturbance of either one by the oscillations of the pendulum on the other one. The base chamber is fastened to the top of the pier by a layer of plaster of Paris under it and extending up a few inches around the outside. It has been found that a pier furnishes a much better support for the apparatus than a concrete floor.

## KNIFE EDGE AND PLANE

The design and dimensions of the knife edge and plane which have been used for many years in the gravity apparatus of this Bureau are shown in figure 5. Because of its extreme hardness and toughness, agate is used as the material for these highly important parts of the apparatus. The two plane surfaces which meet to form the knife edge are ground at an angle of 130°. This large angle makes the edge much less likely to be chipped or otherwise injured, and tends to lessen the distortion of the edge due to the weight of the pendulum. It should be remembered that the edge itself is very narrow (less than a micron) and that therefore the unit pressure due to the load of the pendulum is enormous.

The grinding of the knife edge must be done with great care. Not only must the two planes which meet to form the edge be ground optically flat but there must be no rounding off of the edges of these planes near the line where they should meet. The following notes on the method employed for the grinding of an agate knife edge were furnished the writer many years ago by E. G. Fischer, former chief of the Instrument Division of this Bureau, who personally ground most of the edges and planes which are still in use in the gravity apparatus of this Bureau:

"The grinding of the agate is done on an Arkansas oilstone. The oilstone is first given a perfectly plane surface as follows: A thick piece of lead is planed in a planing machine and emery powder is rolled into the surface. The loose particles of emery are carefully brushed off and the oilstone is ground on this surface. When the surface of the oilstone has been made perfectly plane in this manner it is dusted with sapphirine powder and rubbed against another stone similarly ground and treated. The loose powder is then carefully brushed off.

"The agate is secured in a frame to hold it at the correct angle and is ground on the stone. At intervals of 3 or 4 strokes the stone is again brushed to remove loose particles. This is very important since the loose particles will roll and have a tendency to cut deepest near the edge of the agate where they roll under.

"The knife edge may be tested to determine its width (or the width of the 'missing triangle') by placing it under a microscope and illuminating the two planes by suitable lights placed on opposite sides. A black line in the microscope will indicate that the planes fail to meet and the width of this line is a measure of the imperfection of the knife edge. This width was found to be less than half a micron for the knife edges of the gravity apparatus."

Another method for testing the effective width of a knife edge, or for determining its radius of curvature, as it is sometimes considered, is described by Heyl and Cook.[1]

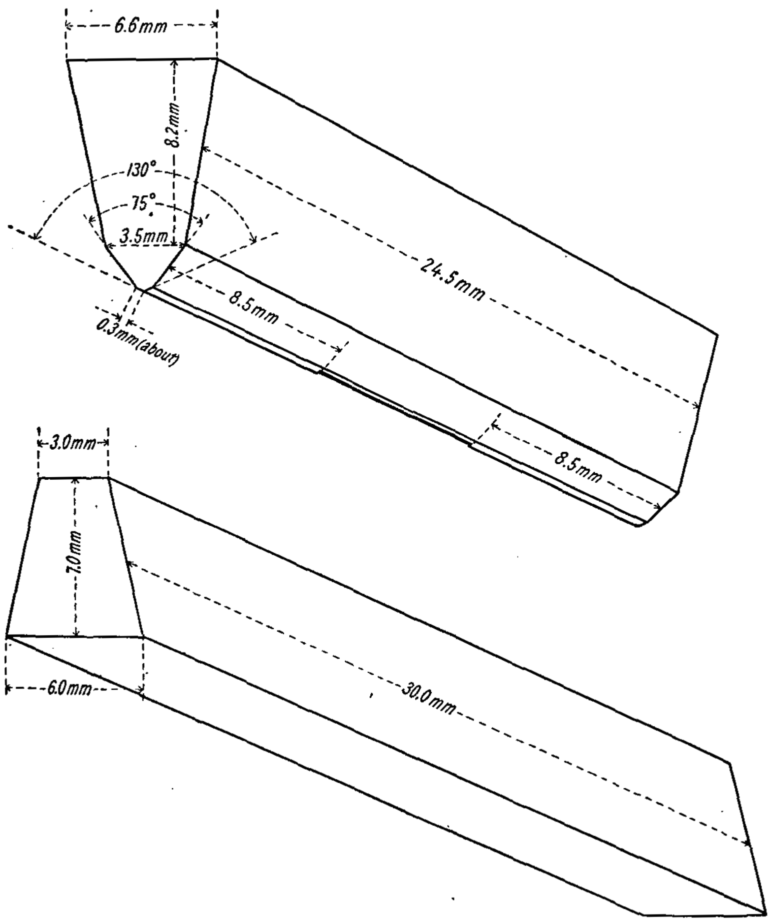


FIGURE 5.—Agate knife edge and plane, old type.

On the older apparatus, the middle of the knife edge was cut away for about one-third of the length of the edge to prevent any tendency of the pendulum to "walk" on its support. The newer knife edges, shown in figure 6, have not been cut away in the middle. They were used for about 25 stations in South America in 1941 and seemed to be entirely satisfactory. They have the advantage of the longer supporting edge and the corresponding smaller unit pressure which, of course, is still very large on such a finely ground edge.

[1] Paul R. Heyl and Guy S. Cook, The value of gravity at Washington, National Bureau of Standards Research Paper RP 946, p. 816.

As can be seen in figure 5, the bases of the knife edge and plane were ground wedge shape in cross section. They were also made slightly wedge shape in a longitudinal direction. They were attached to bronze or invar holders by being driven into slots made to fit these bases except for being slightly smaller. This method of attachment was not entirely satisfactory because the wedging of the agate in the metal holder had a tendency to distort the agate slightly unless the fitting was done very precisely.

A better method for attaching the agates is shown in figure 6. The design was made by D. L. Parkhurst, Chief of the Instrument Division of this Bureau. The bases of these new agates are ground optically flat as are also the metal surfaces to

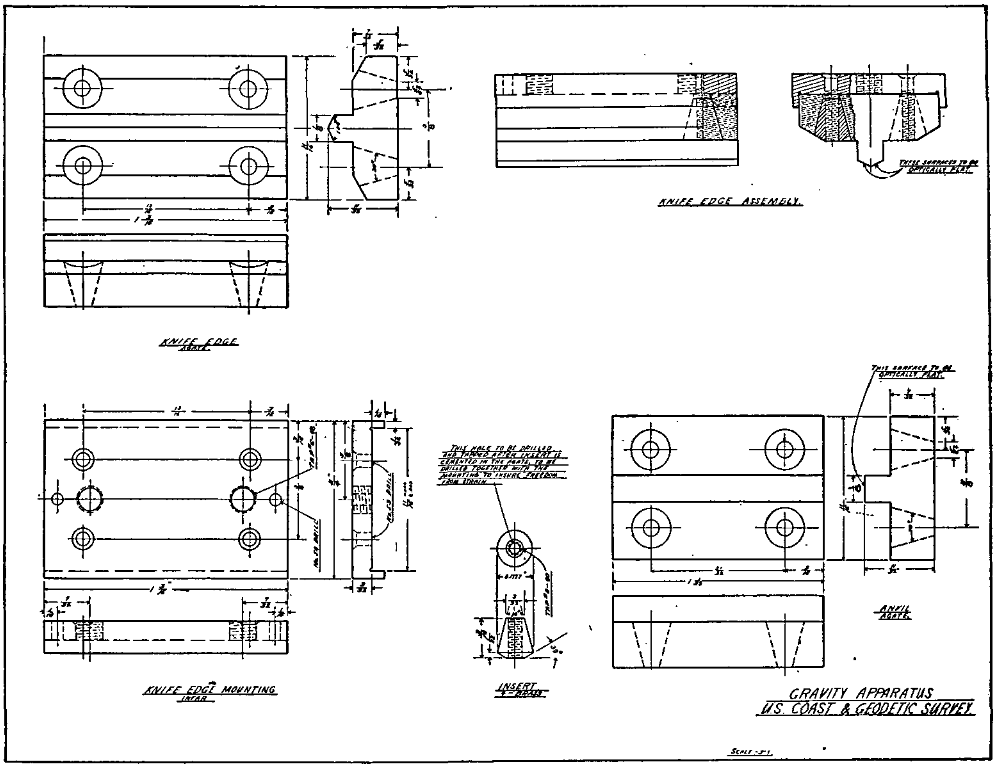


FIGURE 6.—Agate knife edge and plane, new type.

The agates, and the blocks to which they are attached, are carefully lapped together to prevent distortion of knife edge and plane when they are fastened in place.

which they are attached. This avoids any distortion of the knife edge or plane when they are fastened to the metal holders by means of screws.

## LEVEL

Immediately below the plate glass cover of the vacuum chamber is a level which is attached to the pendulum support and oriented parallel with the knife edge. (See fig. 2.) This is a sensitive level which is needed to insure that the knife edge is exactly horizontal during the observations. This level is made parallel with the knife edge by use of an auxiliary level mounted on a small pendulum which can be suspended from the agate knife edge (or from the agate plane of the more recent apparatus) in

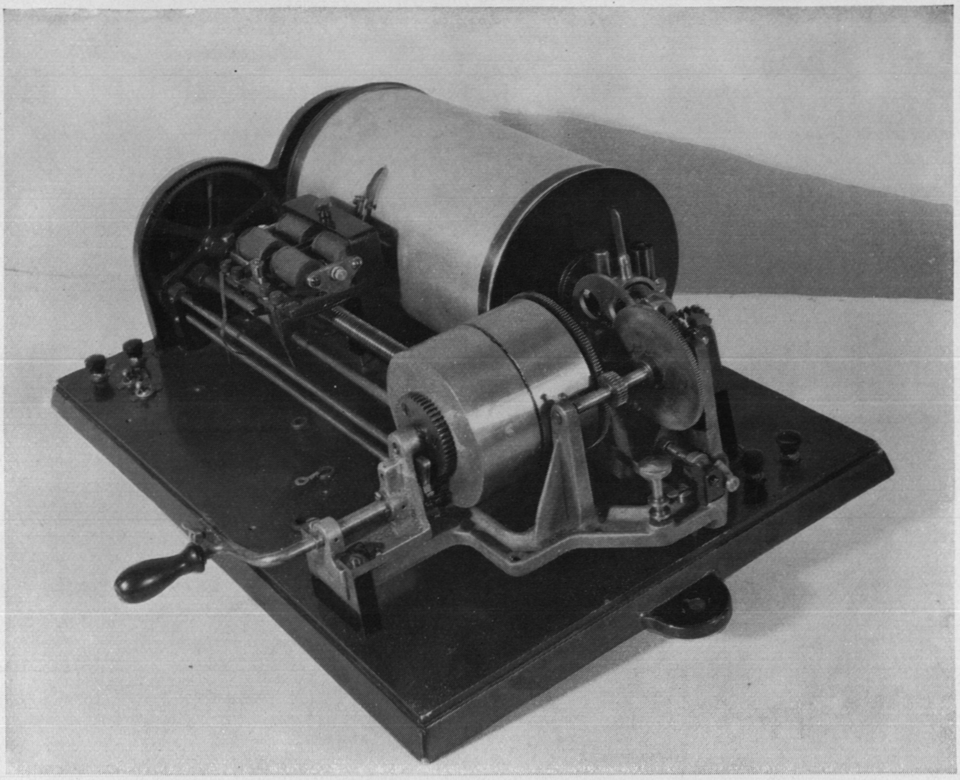

FIGURE 7.—Gravity chronograph.

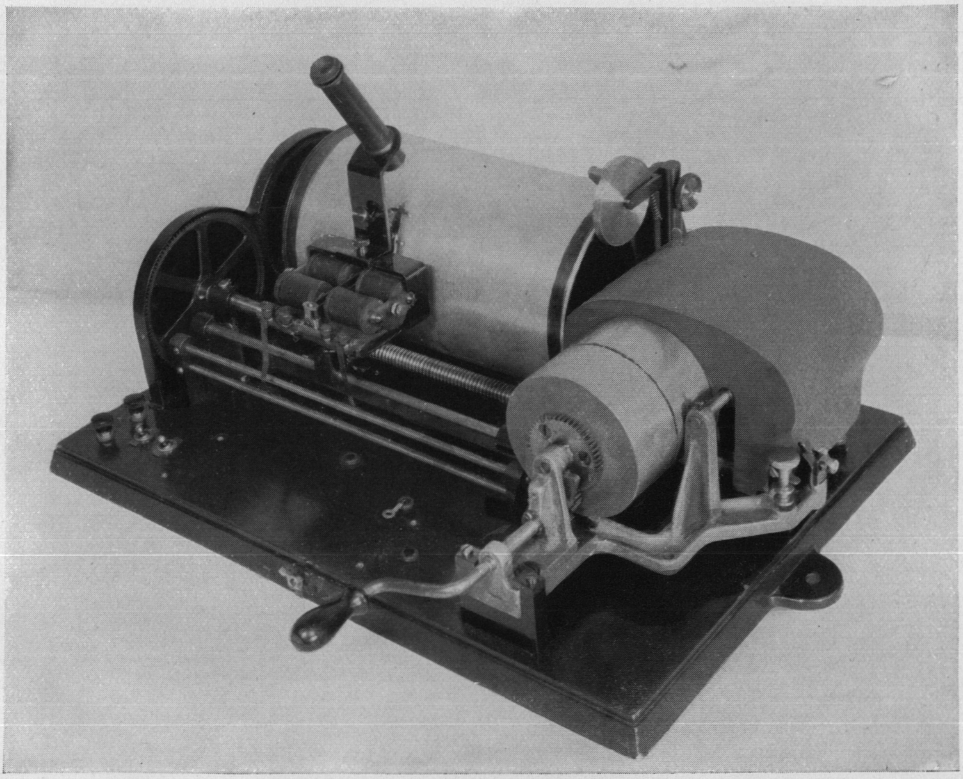

FIGURE 8.—Chronograph with scaling device attached.

a manner similar to the pendulum itself and which can be reversed on the knife edge for accurate adjustment. After the auxiliary level has thus been adjusted, and the knife edge made horizontal, the level which is mounted permanently on the support can be adjusted.

As stated on page 19, the leveling of the apparatus at right angles to the knife edge, that is, in the plane of oscillation of the pendulum, need be done only approximately. A small level on top of the case (or just inside the glass cover on some of the instruments) is used for this purpose. The leveling in this direction may be perfected, if this seems desirable, by lowering the pendulum on the knife edge and noting where the beam of light from the recording apparatus, after reflection from the mirror on the pendulum, strikes the scale used to measure the arc of oscillation of the pendulum. This will be explained in more detail in the next chapter.

## CHRONOGRAPH

A special chronograph was designed for the Brown apparatus in order to obtain sufficient amplification of the record to permit accurate scaling. This chronograph is shown in figure 7. It has a drum about 16 cm in outside diameter or 50 cm in circumference, which is driven at a speed of 12 revolutions per minute. This produces a record on which each second is 10 cm in length. This even length makes it easy to check the scaling of any part of the record by use of a metric scale after the sheet has been removed from the drum, if such checking becomes necessary.

The drum may be driven by a synchronous motor or by a spring motor, as shown in figure 7, of the type used on old style phonographs. The spring motor seems to be more satisfactory for field work than the electric motor, although the load is rather heavy for the size of spring motor available. Where a good constant 60-cycle current is available, a synchronous motor will give the most uniform record.

The paper used on the drum is a special wax-coated paper and the record is made by two styluses which press lightly on the drum. One stylus is somewhat longer than the other to prevent interference, and the two may be adjusted to follow in the same track. One is operated by a relay which is in the circuit through the pendulum recording device, or the chronometer, and the other is operated by a relay connected to the radio receiver or to the chronometer. The impulses received by the relays cause one stylus to move to the left and the other to the right, thus producing independent records of the pendulum oscillations and the time "breaks." The usual worm gear carries the relays longitudinally along the drum to give a helical record.

Each sheet is scaled before it is removed from the drum if time permits. For this purpose a scaling wheel is attached to the end of the drum and a microscope to the frame of the relays. (See fig. 8.) The styluses are held free from the paper by a small spring while the scaling is being done. As each mark is scaled, the drum is turned until the mark coincides with the cross hair in the microscope and then the scaling wheel on the end of the drum is read.

## RADIO RECEIVER AND AMPLIFIER

In order to obtain the requisite accuracy in gravity determinations with pendulum apparatus, it is necessary to have precise time control as a basis for evaluating the period of the pendulum. For a number of years this time control has been obtained from the hourly time signals sent out by the Naval Observatory and transmitted by the Naval Communications Service of the U. S. Navy through the Arlington or Annapolis radio stations.[2] This time service has been maintained on a regular schedule and at a very high standard of precision by the Navy organizations concerned and has greatly facilitated the gravity work of the Coast and Geodetic Survey.

The experience of this Bureau has shown that the short-wave, high frequency signals can in general be received more satisfactorily than the long wave signals by a field party using portable equipment. The frequencies most used for this work are 4390, 9425, and 12630 kc. Ordinarily, it is only when the gravity station is close enough to the transmitting station to be in the first or second skip distance and still too far away to receive the ground wave satisfactorily that there is any particular difficulty in recording the signals on one of these frequencies. Less equipment in the form of long antennas and high poles, etc., are required for the short-wave reception than for the long-wave.

A good battery-operated, short-wave, commercial radio receiver is used to receive the radio time signals. Best reception will usually be obtained with a single antenna of a length equal to about one-fourth to one-half of the wave length of the signals to be received and set to point towards the transmitting station and with the lead-in end nearest the transmitting station. The approximate wave length in meters may be obtained by dividing 300,000 by the frequency of the signals in kilocycles. For example, an antenna 6 to 12 meters long should be used for a frequency of 12,630 kc. The lead-in wire should be counted in the length of the antenna unless it is set approximately normal to the antenna. It should be as short as convenient.

Other important parts of the gravity radio equipment consist of two amplifiers, a small one which forms a part of the pendulum recording device (see p. 12) and a power amplifier for operating the relays on the chronograph. The designs for these amplifiers were originally made by Albert J. Hoskinson, of this Bureau, and later somewhat modified by C. I. Aslakson and other members of the Bureau. The wiring diagrams are shown in figures 9 and 10. Both of these amplifiers are special equipment and must be built from specifications. They are not available as standard commercial equipment.

Only a brief description of the power amplifier is included in this publication since the continuous and rapid development of radio equipment would probably make a detailed description largely obsolete within a comparatively short time. In addition to that of amplification, the principal requirement of the power amplifier is that the lags involved in the transmission of the different kinds of impulses through it shall be small and of equal amounts. It is so constructed that the radio time signals, the impulses caused by the pendulum through the photoelectric cell, and the chronometer "breaks" shall be transmitted through identical circuits or those having identical characteristics. The following brief description of the power amplifier was furnished by Albert J. Hoskinson, its designer.

"The power amplifier is a twin unit in which the pendulum beats are amplified through one half of the amplifier and the time signals through the other half. Two entirely separate units might be used for this work but the design is somewhat simplified by placing both units in one case. Each half of the power amplifier should be of class "B" or class "AB" construction having two or three stages of audio amplification and should deliver an undistorted output of 10 watts or more to the chronograph. The amplifier may be either resistance or transformer coupled, or a combination of both may be used to good advantage.

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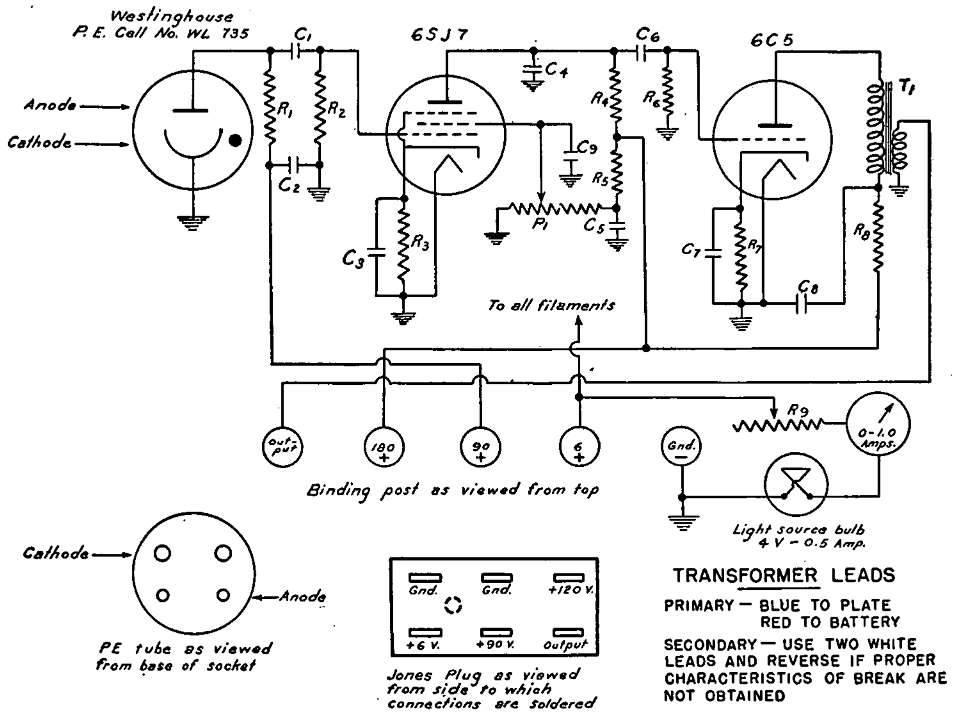
[2] The operation of the Arlington radio station was discontinued in March 1941, and the time signals are now transmitted directly from Annapolis only.




$R_1$ = 5 megohms, 1 watt
$R_2$ = 5 megohms, 1 watt
$R_3$ = 3,000 ohms, 1 watt
$R_4$ = 500,000 ohms, 1 watt
$R_5$ = 300,000 ohms, 1 watt
$R_6$ = 1 megohm, 1 watt
$R_7$ = 5,000 ohms, 1 watt
$R_8$ = 50,000 ohms, 1 watt
$R_9$ = 0 − 50 ohms, variable
$P_1$ = 50,000 ohms, potentiometer
$C_1$ = 0.01 microfarad, 450 volts
$C_2$ = 0.5 microfarad, 450 volts
$C_3$ = 5 microfarads, 25 volts
$C_4$ = 0.003 microfarad, 450 volts
$C_5$ = 0.1 microfarad, 450 volts
$C_6$ = 0.005 microfarad, 450 volts
$C_7$ = 5 microfarads 25 volts
$C_8$ = 1 microfarad, 450 volts
$C_9$ = 0.1 microfarad, 450 volts
$T_1$ = Thordarson 62 A 26 or T 14 A 91

**PHOTO ELECTRIC CELL AMPLIFIER**

BROWN GRAVITY APPARATUS
1941 AND 1942 MODELS

FIGURE 9.—Photoelectric amplifier, wiring diagram.

"The beats of the pendulum picked up by the movement of the light beam across the photo cell are fed into one half of the amplifier and recorded on one of the styluses of the chronograph. The time signals received by short-wave radio are fed into the other half of the amplifier and recorded on the other stylus of the chronograph simultaneously with the pendulum beats. One of the important considerations in the design of the amplifier is that there must be complete and effective shielding between the two halves of the amplifier so that there will be no coupling between the radio and the pendulum styluses of the chronograph.

"A suitable switching system must be provided so that the seconds breaks from the chronometer may be quickly fed into either half of the amplifier thus making it possible to record radio-pendulum, radio-chronometer, or chronometer-pendulum breaks on the two styluses of the chronograph. The switching system must also be arranged so that either of the two pendulum instruments can be used to furnish the pendulum beats for the amplifier by simply throwing a switch in the amplifier."

## RECORDING DEVICE FOR PENDULUMS

The accuracy of gravity determinations with pendulums depends to a great extent upon the accuracy with which the oscillations of the pendulum can be compared with some primary time standard at the beginning and end of the interval during which the pendulum swings. With the Mendenhall apparatus, about 48 hours were required for the determinations at each station in order that the unavoidable errors of making the comparisons would be distributed over a long enough interval to have a small effect on the resulting average period of the pendulum. At that time the pendulums were first compared with an accurate chronometer, and then the rate of the chronometer was obtained from telegraphic or radio time signals or, in the early days of gravity determinations in this country, from astronomical time determinations.

From the standpoint of accuracy there were two principal objections to this method: First, was the necessity for using the chronometer as an intermediate time control, thus requiring twice as many comparisons as would otherwise be needed; and, second, was the possibility that the rate of the chronometer during the interval required for the comparisons differed from its average rate. One of the main considerations in designing the Brown apparatus was to overcome both of these objections by making it possible to compare the oscillations of the pendulum directly with time signals.

In order to accomplish this result satisfactorily, it was necessary to devise some method for recording simultaneously the pendulum oscillations and the time signals. This could not be done by any kind of direct mechanical connection to the pendulum as such a connection would affect the period of the pendulum. It was therefore done by reflecting a beam of light from a mirror on the head of the pendulum to a photoelectric cell and amplifying the resulting impulses until they could be recorded on the chronograph.

The device used for this purpose is shown in figure 11. A straight-filament galvanometer lamp is mounted in the upper end of the vertical tube shown in the picture. The beam of light from this lamp is directed through the tube onto a small mirror attached to the head of the pendulum and is there reflected back to a mirror mounted beside the upper end of the tube. From there the beam is reflected downward to a photoelectric cell which is housed in the horizontal tube shown in figure 11 (barely visible over the lower edge of the box). There is an adjustable slit in the upper part of the housing of this tube, and the image of the filament of the lamp crossing this slit

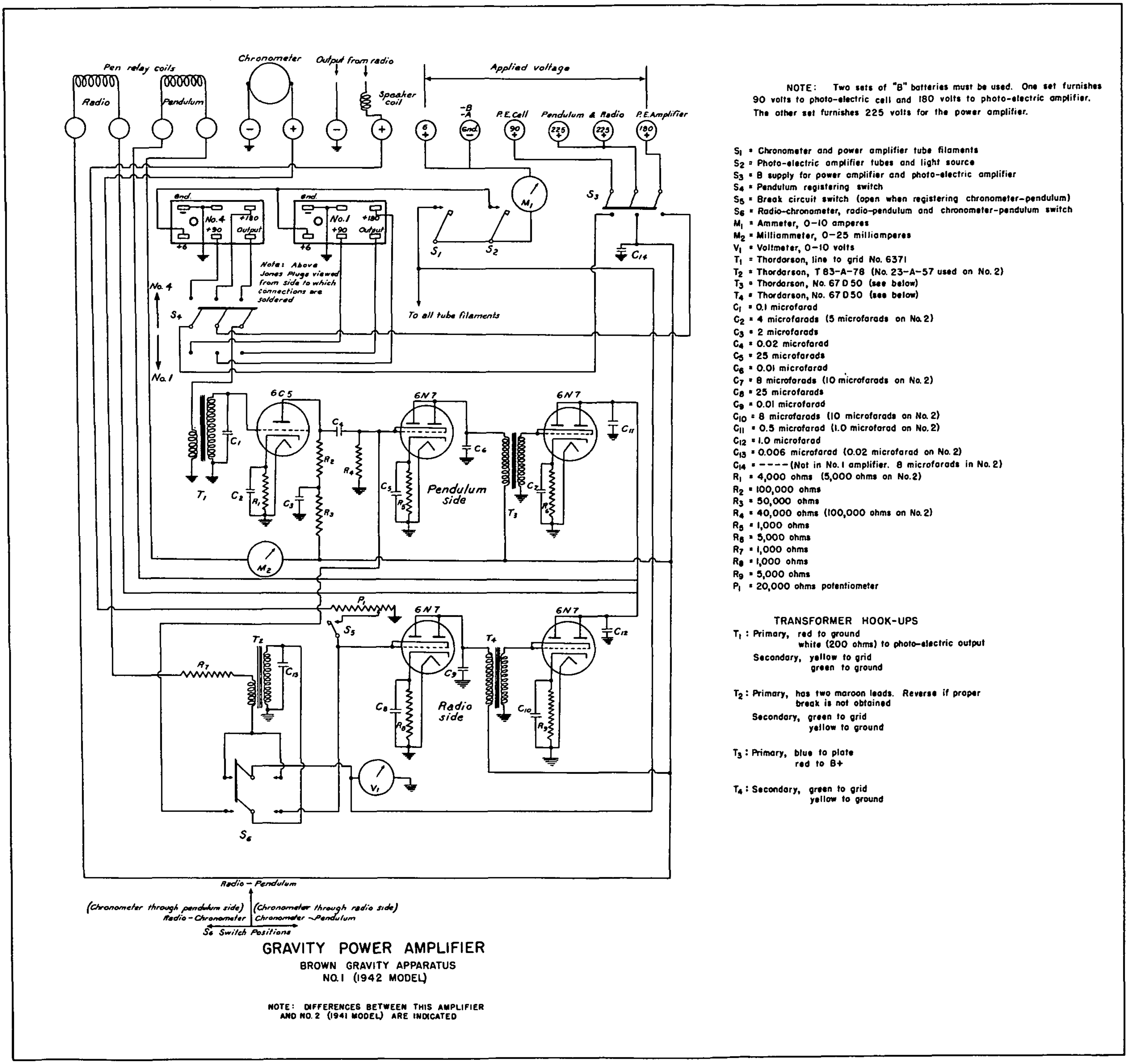



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FIGURE 10.—Power amplifier, wiring diagram.

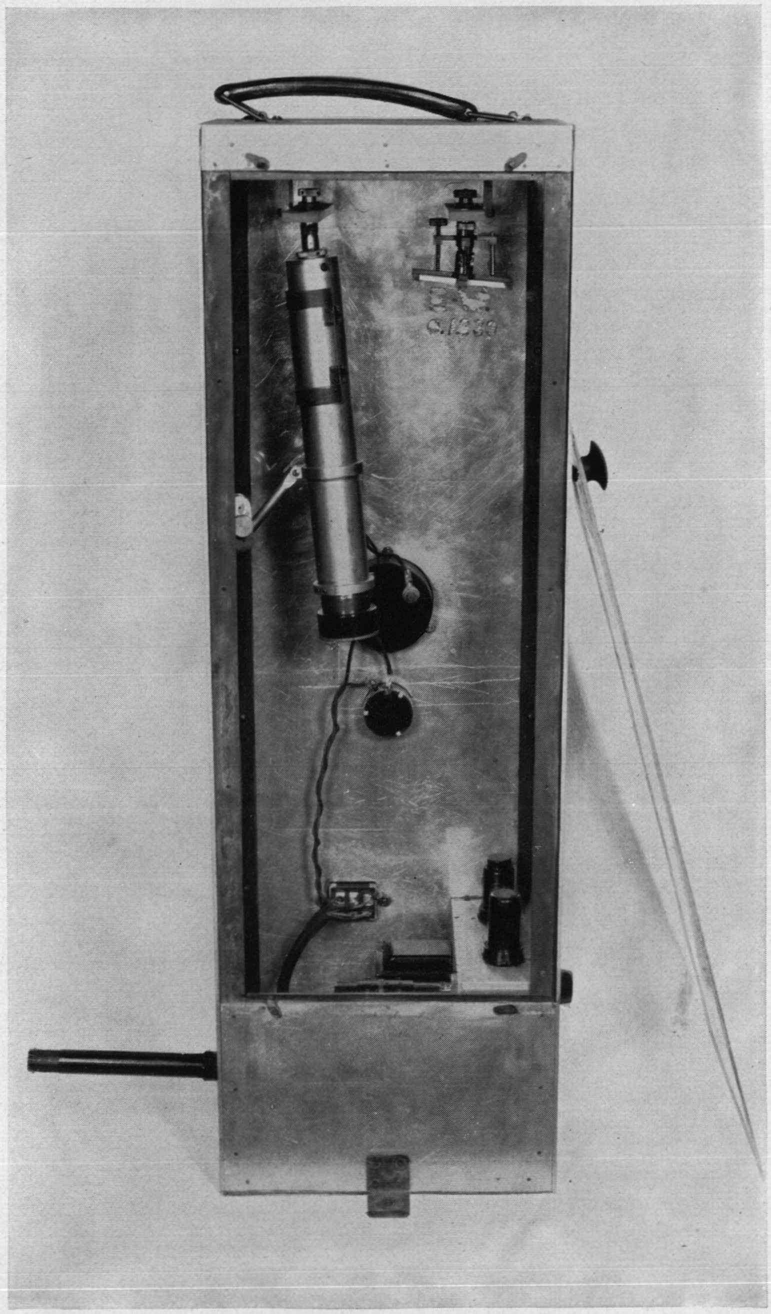

Figure 11.—Photoelectric recording device.

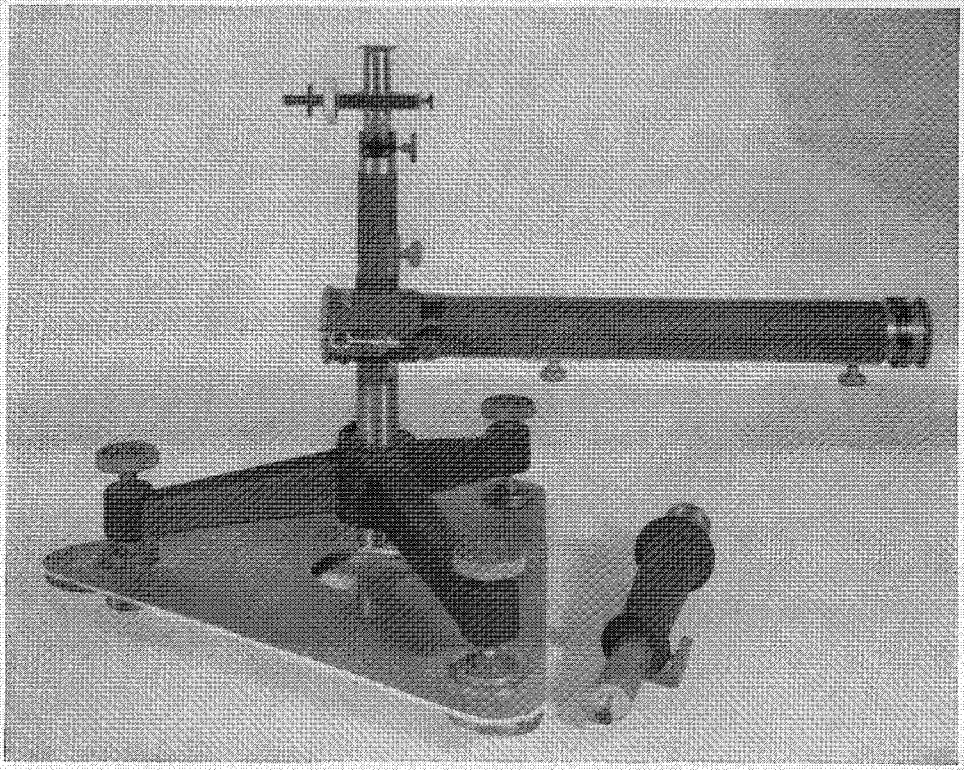

FIGURE 12.—Interferometer.

causes two impulses in the photoelectric cell, one when it reaches the slit and the other when it leaves the slit. These can be amplified and recorded on the chronograph. The first one of the two impulses is the one used in timing the pendulum. The slit should be made as narrow as possible (0.2 mm. or less) and still permit enough light to reach the cell to give a sharp impulse.

The galvanometer lamp is so oriented in its mounting that the image of its straight filament will coincide with the slit over the photoelectric cell when the pendulum reaches a certain position in its oscillation. The beam is focused by suitable lenses in the light tube and by the mirror attached to the top of the pendulum, which is slightly concave (radius of curvature equals 1 meter). As shown in figure 11, both the light tube and the mirror attached to the top of the box are adjustable.

The amplitude of oscillation of the pendulum is determined by means of a scale attached to the upper part of the housing of the photoelectric cell. The end positions of the oscillating beam of light are noted on this scale, and the sweep of the beam which is thus determined is multiplied by a factor to obtain the movement of the tip of the pendulum which is used as the argument in the amplitude correction tables. (See p. 53.)

A special amplifier is used to build up the impulses from the photoelectric cell before they are passed through the power amplifier to the chronograph. (See p. 10.)

## CRYSTAL CHRONOMETER FOR TIME CONTROL

In the winter of 1936–37, a special type of portable crystal chronometer was used as the time control for a series of gravity observations at sea. The chronometer was designed and constructed by the Bell Telephone Laboratories of New York and was lent to the American Geophysical Union as the sponsoring organization for a cooperative scientific expedition to the West Indies for determining gravity at sea. The gravity measurements on this expedition were made with the Vening Meinesz type of pendulum apparatus, an apparatus adapted for use on a submerged submarine.

The crystal chronometer proved to be an exceptionally precise timepiece. It gained about one-third of a second in one and one-half months and could have been used without time-signal check for the duration of the expedition without affecting appreciably the corresponding gravity results. It is rugged and portable and its rate is remarkably free from temperature effects. It could undoubtedly be readily adapted for use with the Brown gravity apparatus at land stations. It would make possible the use of a shorter period of observations at a station, because of its more accurate time control, and would tend to increase the accuracy of the results, even with the shorter period.

During the fall and winter of 1937–38, the Coast and Geodetic Survey obtained the loan of the crystal chronometer for use in redetermining certain constants of the Brown gravity apparatus at the Washington base station. The continuous and accurate time control provided by the chronometer made it possibe to complete the experiments in a much shorter period than would otherwise have been possible and also with appreciably greater accuracy. The many independent determinations of the period of the pendulum required in these experiments, were started and stopped at will, without reference to the hourly radio time signals which would otherwise have had to be used, and each determination was completed in a relatively short time. It was also possible to make the comparisons between pendulum and timepiece more accurately than can be done when the pendulum is compared with time signals.

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The Coast and Geodetic Survey has not as yet obtained a chronometer of this type for its own use. The cost of the instrument itself, plus the cost of making the necessary changes in the recording apparatus, would hardly be justified in view of the limited number of pendulum stations now needed to complete the basic control net of the country. It is anticipated that gravimeters will soon be used to a large extent to fill in the intermediate stations in the net and these instruments, of course, are not dependent upon accurate time control.

## INTERFEROMETER

One of the supplementary measurements which must be made when single pendulums are used for gravity determinations, as in the Brown apparatus,[3] is the flexure or sway of the support caused by the oscillation of the pendulum. The movement is extremely small and of course has a half-second period. It is therefore difficult to measure. The most satisfactory way of measuring it is with an interferometer.

An interferometer of the type used by this Bureau is shown in figure 12. In this instrument the monochromatic light from a helium lamp is reflected from adjacent surfaces of two glass plates, one attached to the interferometer itself and the other to the front of the case in which the pendulum swings (A, fig. 2). When the plates are so adjusted that the two reflecting surfaces are close together but at a very slight angle to each other, interference bands are formed. As the pendulum swings back and forth (at right angles to the two plates) it produces a slight displacement of the plate attached to the case relative to the plate on the interferometer. This causes a small transverse movement of the bands which can be observed and measured.

In figure 12 the helium lamp is shown at the left-hand end of the horizontal tube. A beam of light from this lamp enters the horizontal tube through a small opening and is reflected from the plates at the other end. One of the reflecting plates is shown at the extreme right-hand end of the horizontal tube. The outside surface of this plate is the one which is used as the reflecting surface, the reflection being from the inside. The plate which is attached to the case is not shown in the illustration. A prism at the intersection of the horizontal and vertical tubes reflects the light into the small telescope at the upper end of the latter, through which the movements of the interference bands may be observed. There is a collimating lens in the horizontal tube.

The reflecting surfaces of the two plates which are used to produce the bands must be optically flat. To prevent interference by the reflection from the other surface of each plate, they are made slightly wedge shape in thickness and this deflects the unwanted reflections to one side of the line of sight.

Movements of the case of the order of one-twentieth of the wave length of helium light (or about 0.03 micron) may be measured with this instrument with a fair degree of accuracy. (A movement of one-twentieth of a wave length corresponds to a movement of one-tenth of a fringe width as seen in the interferometer, because the light reflected from the second plate must traverse the distance between the plates in both directions.) (See p. 38.) As stated on page 5, the Brown apparatus was designed to make the flexure or sway as small as possible. The size of the flexure depends upon the type of soil in which the support is set and upon the care used in setting the support. Under

[3] Other types of pendulum apparatus, such as the Vening Meinesz apparatus for gravity determinations at sea or the Lenox-Conyngham apparatus for determinations on land, have two or more pendulums which are practically identical and are used simultaneously. They swing out of phase from the same support. In this way the flexure or sway effect is either eliminated (if the two pendulums are in opposite phase) or may be evaluated from the records of the pendulum oscillations themselves.

normal conditions the flexure is quite small with the Brown apparatus, seldom exceeding two- or three-tenths of a wave length of helium light.

## TEMPERATURE INSULATION AND CONTROL

Although invar pendulums have a very low coefficient of expansion, it is important that they be kept at a fairly uniform temperature while they are being used for gravity measurements. If they change rapidly in temperature, uncertainties are introduced into the results which cannot be entirely eliminated. The correction for temperature which is applied to the observations consists of two parts, one the standard correction to take care of the thermal expansion of the pendulums, and the other an additional, secondary, empirical correction, called the dynamic temperature correction,[4] which depends upon the rapidity of change of the temperature and which is probably due to lag between the temperature of the pendulum and that indicated by the thermometer. A part of the remaining uncertainties caused by temperature variations is probably due to small inaccuracies in the temperature coefficients of the pendulums and part to variations in the lag between the temperature indicated by the thermometer and the actual effective temperature of the pendulum itself.

The pendulum is normally fairly well protected from variations in outside temperatures both because it is in a partial vacuum and also because it has no direct connection to the case while it is oscillating except through the thin line of the agate knife edge. Unless special care is used, however, it will vary quite rapidly in temperature when outside variations are rapid. When this occurs, the temperature of the pendulum does not change uniformly. The bob of the pendulum serves as a reservoir of heat, either positively or negatively depending on the direction of change, and this makes it practically impossible to determine with sufficient accuracy the actual effective temperature of the stem of the pendulum which of course is what is needed.

In order to keep the temperature of the pendulum at a more uniform level, an insulating cover and a thermostated heating element were designed and constructed for the vacuum case in 1940 by Albert J. Hoskinson of this Bureau. The lower part of the cover consists of a layer of rock wool protected on the outside by a thin sheet of aluminum. The upper part of the cover is of bakelite. It has an opening in the top corresponding with the observing window in the case. The leveling screws are provided with bakelite washers in order to prevent as far as possible any metallic contact between the case and the base chamber which serves as the support.

The heating element consists of about 15 turns of small copper wire, having a resistance of about 1 ohm, wound around the vacuum case inside the insulating cover. It is looped back on itself to form a pair of wires twisted together. This insures that the current in adjacent wires will always be flowing in opposite directions and the magnetic effect will thus be neutralized. The necessary current is provided by 6-volt storage batteries. Under normal operating conditions and with moderate outside temperatures the current will be on about half the time, in periods of approximately 1 minute. The temperature of the vacuum chamber is controlled by a mercury thermoregulator in close contact with the chamber. A variation of about 0.1° C. in temperature is enough to trip the regulator and cut on or cut off the heating current.

Two sets of the Brown gravity apparatus, equipped with the insulating covers and the thermostatic devices, were used in the field in the fall and winter of 1940–41. This

[4] The first published reference to the "dynamical" correction is probably that given on p. 20 of Professional Paper No. 10 of the Survey of India. It is contained in a report entitled "The Pendulum Operations in India, 1903 to 1907" by G. P. Lenox-Conyngham.

was the first field test of the temperature-control equipment. It was hoped at the beginning of the season that the pendulums could be kept continuously within 2° or 3° of 35° C. It was found, however, that this could not be done. Severe cold waves were encountered during which the temperature inside the case dropped several times this amount. Even then the drain on the batteries was found to be a serious problem in cold weather. Although the temperature of the pendulums was maintained at a much more uniform level than would otherwise have been possible, a more adequate type of insulation must be provided before the temperature can be held within the desired limits. The insulation is made rather difficult by the large size and awkward shape of the gravity apparatus and by the restricted space available for the cover.

## MISCELLANEOUS INSTRUMENTS

### THERMOMETER

The thermometer used to indicate the temperature inside the vacuum case is a standard precise thermometer except that the stem has an S-bend immediately above the bulb to permit a better contact between it and the piece of metal in which it is mounted. Because of the small available space in the vacuum case of the Brown apparatus the thermometer is not mounted in the stem of a dummy pendulum as in the Mendenhall apparatus, but instead is mounted in a small piece of invar of about the same dimensions as the stem of the pendulum. This invar strip is fastened inside the vacuum case in such a position that the reflection of the thermometer in a diagonal mirror which is directly in front of it can be observed through the plate-glass window in the top of the case. (See fig. 3.) In order to make the lag between the outside temperature and that of the invar strip about the same as for the pendulum the strip should be thermally insulated from the case to about the same extent as the pendulum. The bulb of the thermometer is packed in invar filings.

### MANOMETER

The manometer is a simple U-tube of Pyrex glass with one end closed. It is graduated to millimeters and will indicate pressures up to about 90 millimeters of mercury. It is attached to the inside of the case by means of a suitable holder in such a position that the readings can be made in the same diagonal mirror as that used for reading the thermometer. (See fig. 3.)

Since the Brown apparatus is, if possible, used for an entire season without the case being opened, the manometer cannot be taken out and sealed to prevent the spilling of mercury during transportation. The gravity apparatus is always kept in an upright position and therefore the only danger of spilling is that due to splashing of the mercury in the manometer, which may be caused by the jars received during transportation. Mercury will not amalgamate with invar, and the possibility of loose droplets destroying the constancy of the pendulum is not so great as when bronze pendulums are used. There is the danger, however, that small particles of any mercury that may be spilled inside the vacuum case will stick to the invar or perhaps lodge in small crevices of the pendulum.

It can be seen, therefore, that it is very important that no mercury be spilled or jarred out of the manometer during the moves between stations. In order to guard against this, the open end of the manometer tube is partly closed with a small funnel which is inserted into the opening with pointed end down and fastened in place with wax. Frequently, two of these funnels are used, one above the other. The small end of each one has a small opening to let the air in and equalize the pressure, but any mercury that splashes against the funnel either will hit the sides and fall back or its velocity will be so much decreased by passing through the small opening that it will fall back into the funnel and drain back into the manometer tube. This little device has been used for a number of years and has proved quite satisfactory. It was designed by Albert J. Hoskinson.

The filling of a manometer tube is rather difficult and should not be attempted by anyone lacking experience. The tube and the mercury must be very carefully cleaned and for best results the mercury should be freshly redistilled or taken from a bottle which has been filled with distilled mercury and immediately sealed from the air. There must be no air bubbles in the mercury after the manometer tube has been filled. The older practice was to fill the tube with a long slender glass funnel as carefully as possible and then to remove any air bubbles by boiling them out or by fishing them out with a small steel wire. Either of these methods requires a great deal of skill and patience to fill the manometer properly, and both have been abandoned in recent years by the gravity observers.

The present practice is to fill the manometer while the air is pumped out of the tube so that it is impossible to get an air bubble in the mercury. This is readily done by attaching one end of a T-shaped tube to the open end of the manometer. The lower end of the upright part of the tube is sealed and enough mercury poured into it to fill the manometer to the desired level. With one end of the horizontal tube attached to the manometer and the other to the power vacuum pump, the system is then completely evacuated. This will generally require about 20 minutes with the power pump. The manometer can then be filled by simply upending the T tube and allowing the mercury to flow into the manometer. Care must be taken to see that the end of the T tube and the manometer tube have a very close fit so that none of the mercury touches the rubber tube used to hold them together, as the mercury will pick up grease and dirt if it touches the rubber. With the mercury in the manometer hold it in an upright position and slowly break the vacuum seal. This must be done very slowly or the inrushing air will force the mercury against the closed end of the manometer so hard that it will be broken. A good method is to puncture the rubber tube with a small pin.

The amount of mercury in the tube should be such that the readings on the two sides of the tube will be approximately equal. To be properly graduated the zero lines on the two sides should be in the same horizontal line when the tube is in its correct vertical position. Pieces of white paper pasted on the back of the tube directly behind the graduations will greatly facilitate the readings.

## CHRONOMETER

With the present methods of comparing the pendulum oscillations directly with the radio time signals, the chronometer is used only as a counting device to make sure that there is no error in deriving the number of pendulum oscillations between time signals, especially if these are from 2 to 6 hours apart. Over a 1-hour interval there is very little chance of an undetected error in the count as the resulting value of gravity would be obviously in error. The chronometer, therefore, need not have an extremely uniform rate. It should be regulated to sidereal time to prevent possible continuous interference with the mean time radio breaks and should have an electrical break-circuit device, either 1-second or 2-second (preferably the former) in order that its time may be recorded on the chronograph sheet.

## COMPASS DECLINOMETER

A compass needle, mounted in a small oblong box with glass cover, is used to test the pendulums for magnetization. It is shown at C in figure 2, at the left-hand side of the opening of the pendulum case. It is supported on a special plate, a part of which can be seen at D, on the right-hand side of the opening, in figure 2. The plate is provided with three small lugs on each side which make it possible to place the compass box in exactly corresponding and parallel positions on opposite sides of the head of the pendulum.

# Chapter 2. GRAVITY DETERMINATIONS WITH PENDULUMS

The various parts of the Brown gravity apparatus and of the supplementary equipment are described in Chapter 1. In Chapter 2 are given the general instructions which govern the making of gravity determinations with this apparatus and a brief explanation of the methods employed. The gravity work of this Bureau must be done according to the following instructions except as these are modified by the specific instructions for any particular project. These instructions have been made as concise as possible. Detailed explanations will be found in the text immediately following them.

## GENERAL INSTRUCTIONS

1. Immediately preceding and following each field season, the gravity apparatus must be standardized at the Washington base station. (See p. 23.)

2. The pendulum for each of the two sets of apparatus must be swung at least 6 hours at each field station if radio time signals are used as the time control. It must be compared with the time signals at the beginning and end of each 6-hour swing. As a check it must also be compared at some intermediate time, at an interval of from 1 to 5 hours from the starting signal. If an intermediate time signal cannot be received, the pendulum should be kept swinging and an additional signal recorded at an interval of 7 or 8 hours from the initial signal. (If a crystal chronometer is used as the time control, an equal accuracy can probably be obtained by swinging the pendulum only 2 hours, or possibly less.)

3. Two sets of the Brown apparatus must be used at each station to provide a check on the accuracy of the work. The difference in period between the two pendulums should remain nearly constant from station to station. If it changes by as much as 5 in the seventh decimal place of period from one station to the next, the observations should be repeated at the second station. If the change exceeds 10 in the seventh place as compared with the difference for the preseason standardization, the Washington office should be notified at once.

4. At each station, the base chambers should be set with sufficient care (see p. 25) that the flexure correction will ordinarily be less than 20 in the seventh decimal place of period and preferably less than 15. Only fresh plaster of Paris or quick-setting building plaster should be used and it should be mixed in a very thin, wet mix to give a good bond between the base chamber and the ground.

5. Each pendulum case must be leveled very carefully in the direction parallel to the knife edge, that is, at right angles to the plane of oscillation, by use of the level inside the case. This level should be read at the beginning, middle, and end of each swing and if it changes by as much as three divisions, the pendulum observations must be repeated. The leveling in the plane of oscillation of the pendulum is not so important. No adjustment of the leveling should be made while the pendulum is swinging.

6. Readings of temperature, pressure, and arc must be made at the beginning and end of each swing and at the intermediate time signal. Temperature readings should be made at intermediate intervals of 1 hour or less depending upon the rapidity of change. Care should be used not to start the pendulum observations before most of the heat caused by the chemical action of the plaster of Paris has been dissipated. As stated on page 23, it is desirable that the temperature, pressure, and starting arc be kept as nearly as possible the same as they were for the preseason Washington standardization. The pressure should normally be about 5 mm. of mercury (not appreciably less than that) and the starting arc should be such that the tip of the pendulum will oscillate about 6 or 7 mm. at the beginning of the swing.

7. The frequency and strength of the radio signals should be noted in the record each time they are received. Any serious interference with radio reception should be reported to the Washington office as soon as convenient.

**8.** Flexure readings may be made just before the swing is started or just after it is finished, preferably the latter. This is a difficult observation but it should be made as accurately as possible and by both the observer and his assistant, each using two or three different arcs. The probable error of the derived flexure correction should be less than 2 in the seventh decimal place of period.

**9.** Great care must be exercised in starting the pendulum swinging to guard against injury to the knife edge and to make sure that the pendulum is correctly seated on the plane. The pendulum must always be raised and lowered carefully just before a swing is started to make sure it is in the correct position, and the starting lever must be brought in contact with the pendulum very carefully to prevent jarring. The pendulum should never be lifted while swinging as this will cause scraping and injury to the knife edge, but all motion should first be stopped by bringing up the starting lever very carefully.

**10.** In making gravity determinations it should be constantly borne in mind that the desired accuracy of one part in a million is considerably higher than that of most other types of field measurements in geodetic surveying. Very careful attention must be given to a large number of details of observation if the specified accuracy is to be obtained.

**11.** Once or twice a month, or oftener if there is any suspicion regarding the accuracy of the pressure readings, the manometers should be inspected to make sure there are no air bubbles in the closed ends of the tubes. This can be done by letting a little air into the pendulum cases to bring the mercury to the top of the closed end of the tube in each case. A more positive test of the manometers is to compare them with a standard manometer, or even with each other. The method for doing this is explained on page 26.

**12.** Extreme care must always be used in handling mercury around the gravity apparatus. Although mercury will not amalgamate with the invar pendulums there is always danger that some of it may lodge in small cavities in the pendulum, as for example where the stem enters the bob, and thus destroy the constant properties of the pendulum upon which the accuracy of the results depends. Mercury will, of course, amalgamate with the bronze pendulums.

**13.** At each station the pendulum must be tested for magnetization before the beginning of the swing. If the pendulum causes a deflection of 2° or more in the compass declinometer when that instrument is in position on top of the pendulum case, the pendulum must be demagnetized by means of the solenoid. The test for magnetization must be repeated, of course, after the solenoid has been used and the demagnetization process repeated if necessary. The aim should be to bring the deflection of the needle well under 2°, or preferably under 1°, before the demagnetization is considered satisfactory. If at several consecutive stations the pendulum is always found to be magnetized in the same direction, it is sometimes desirable to magnetize it slightly in the opposite direction by means of the solenoid, enough to cause a deflection of the compass needle of 1° or 1½°.

**14.** The magnification factor for the arc reading scale should be checked from time to time to make sure that it remains constant. It must be redetermined whenever any change is made in the adjustments of the light tube and mirrors in the recording apparatus. (See p. 22.)

**15.** The latitude and longitude of each station must be determined, either by scaling from a reliable, large-scale map or by astronomical observations. Wherever practicable, these two coordinates should be determined within 5 seconds of arc and it is very essential that they be known within half a minute. The latitude is slightly more important than the longitude because of the variation of gravity with latitude. For explanation of methods see page 31.

**16.** The elevation of the station must be determined with reference to sea level with an accuracy of a few feet. It is quite essential that the elevation be known within 5 feet because of the variation of gravity with elevation. (See p. 33 for explanation of methods.)

**17.** In rough terrain, a rough topographic sketch must be made for the area immediately around the station; that is, out to a radius of about 275 meters. If the differences of elevation between the station and different parts of the area exceed 3 meters within a radius of 75 meters or 10 meters within a radius of 275 meters, the sketch should be made. It need not be elaborate nor particularly accurate but it should be roughly to scale and the scale must be given on the sketch. (See p. 36.)

**18.** Each gravity station should be marked with the standard bronze gravity disk. This disk may be set in the top of a concrete post projecting a few inches above the ground, in a block of concrete or in rock. In locations where it is apt to be disturbed if placed at the point actually occupied, the mark (a reference mark in this case) may be placed in a more secure location at a distance from the station not exceeding 300 or 400 feet. The distance and direction from the mark to the station in such cases should be measured and noted in the description. Of even greater importance, the difference in elevation of the mark and the station should be determined and recorded. It is desirable to have the mark at about the same elevation as the station if this is practicable. If the mark is placed at the station itself, the standard bronze disk without the inscribed arrow should be used. If the mark is not at the station the disk with the inscribed arrow should be used and the arrow should be pointed toward the station.

**19.** A detailed description of each gravity station and of the marking must be furnished. This should be such as to make possible the recovery of the station both in horizontal position and in elevation for many years in the future. If the station is in a building, the difference in elevation of the knife edge and the general level of the ground just outside the building should be stated. The type of mounting of each pendulum case and the approximate direction of oscillation of each pendulum should also be given in the description.

**20.** To insure against the loss of records, the following abstracts and computations for each station should be made in the field. All chronograph sheets should be read and the essential data copied on the proper computation forms; all temperature, pressure and arc readings should be copied on the forms and also the derived flexure corrections. The periods of the pendulums should be computed in order to verify their performance by the derived difference in periods. The computations need not be carried beyond this point in the field if to do so would delay the progress of the party or place an undue burden on the personnel. The records and computations should be forwarded by registered mail on different days according to the general practice of this Bureau. Each record book should contain stations in one State only.

**21.** Great care must be exercised to prevent any variation of lag in the chronograph or radio equipment during a swing. (See p. 27.) No change should be made in the batteries, "A" or "B," no radio tubes or photo cells should be replaced and no change should be made in the relay adjustment of the pen mechanism on the chronograph. If any of the above changes become essential before a swing can be completed, a new set of observations must be made.

## INSPECTION AND ADJUSTMENT OF APPARATUS

At the beginning of each season's work and before the apparatus is standardized, each pendulum case should be opened and all parts of the apparatus which are contained in it should be carefully inspected, checked, and adjusted. Unless the observer has had previous experience in making this inspection he should not attempt it without the help of a competent gravity observer who is thoroughly familiar with the operations involved from having done them before. The centering of the knife edge on the agate plane should first be tested by coating the plane with a thin layer of Prussian blue and then lowering the pendulum. If the centering is unsatisfactory as shown by the thin line made by the knife edge, the lifting lever should be adjusted until it is satisfactory. The Prussian blue should then be carefully wiped off. The pendulum should be removed, inspected carefully, and cleaned if necessary with a clean chamois and alcohol. The pendulum should never be touched with bare hands. The agate knife edge and plane should be similarly cleaned and inspected. The manometer should be taken out and tested for a possible air bubble. A *slight* amount of light oil should be put on the screw that raises and lowers the heavy brass block near the bottom of the receiver by means of which the pendulum is clamped for transportation and otherwise controlled. No oil must be allowed to get on the pendulum. Before the pendulum is replaced, it should be examined with a reading glass and all lint and dust particles removed with a camel's hair brush.

A very important thing to be done while the pendulum case is open is to check the level attached to the knife edge block and adjust it if necessary. This testing must be done while the pendulum is out of the case. A level tube attached to a small pendulum which was constructed for use with the old Mendenhall apparatus should be used to make this test. After this pendulum level has been adjusted in the usual manner by reversals, and the knife edge has been made perfectly level, then the bubble of the level

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attached to the knife edge block should be exactly in the center. If it is not, the level must be carefully adjusted.

As explained on page 13, the scale used for reading the amplitude is directly in front of the photoelectric cell on the recording device and the amplitude is read by noting on the scale the limits of motion of the beam of light which operates the cell. Since the arc correction tables are based on the movement of the lowest tip of the pendulum, it is convenient to determine a factor to reduce the scale readings to the corresponding movements of the pendulum tip. This may be done in two ways. The actual movement of the tip may be measured and compared with the corresponding scale readings or the length of the pendulum from the tip to the knife edge may be compared with the length of the light path from the reflecting surface on top of the pendulum to the scale.

If the former method is used, the cylindrical part of the vacuum chamber must be detached and removed. The device for starting the pendulum to oscillate consists of a lever working against a spring which can be made to press against the edge of the bob and push the pendulum away from its vertical position. It has a ratchet to hold the lever in any desired position until the lever can be released to start the pendulum oscillating. The ratchet has 8 or 10 teeth, and the position of the tip of the pendulum and scale reading are ordinarily determined for each of these teeth. The starting lever, *B*, and the ratchet, *C*, may both be seen in figure 19, near the pendulum bob. Both are manipulated by means of the heavy block, *A*, to the front and left of the bob. When the block is lowered, the lever pushes the bob to one side. When the proper notch in the ratchet is reached to give the pendulum the correct oscillation, the block is raised a short distance and this releases the ratchet and permits the spring to pull the starting lever out of the way.

When all tests and adjustments of the apparatus have been completed and the pendulum has been replaced in the case, a very careful final inspection should be made before the case is sealed and evacuated to make sure that all parts are correctly placed, adjusted, and secured as it is desirable to keep the case sealed during the following pre-season standardization, the entire field season, and the post-season standardization. The case should not be opened in the field if this can be avoided.

For making the magnetization test at the Washington base station it is necessary to take the apparatus entirely outside the building, as abnormal magnetic conditions inside the building and in the court adjacent to the gravity room affect the compass declinometer to such an extent that it cannot be used there satisfactorily.

The sealing of the vacuum case should be made as perfect as possible both because it is desirable to have the seal last the entire season and because leaks are very difficult to repair satisfactorily in the field. The most common locations for leaks are in the stop cock, around the plate-glass cover, and through pores in the cast head. Leaks may also develop around the edge of the plate which forms the bottom of the case and in the joint where the tubular part of the case is attached to the head. As explained on page 5, small leaks may ordinarily be stopped by the use of Glyptal paint. There should be a partial vacuum in the case to force the paint into any unclosed pore or crack. By watching carefully while the paint is applied in thin coats, the leak may sometimes be detected by the disappearance of paint into the opening.

The plate-glass cover is put on with vacuum grease. If the flange around the opening has been carefully ground and the glass has a good plane surface, this seal gives very little difficulty. The vacuum grease should be applied as a very thin and uniform coat to the flange and when the pump is started, the glass should be shifted back and forth a few times to perfect the seating. Since the seal is seldom, if ever, broken in the field, Glyptal paint may be used around the edge of the glass to perfect the seal and make it more permanent.

As an emergency measure, when a leak cannot be found or stopped by the methods explained above, it may be necessary to fasten the glass cover with clamps and pump air into the case. If a strong soap solution is then applied to the outside of the case, the leak may usually be found by the formation of bubbles.

## STANDARDIZATION OF PENDULUMS

The relative method is, of course, used for gravity measurements with the Brown apparatus; that is, the difference in gravity between the known base station and each field station is determined by noting the change in the period of the invariable pendulum. Before field work is started, therefore, it is necessary to determine the period of the pendulum at the base station. This is known as standardizing the pendulum. It is again standardized at the end of the field season to make sure that it has maintained its constancy within certain narrow limits during the season's work.

The gravity base station for the United States is located in a specially constructed, constant-temperature room in the basement of the Commerce Department building, near the corner of Fifteenth Street and Constitution Avenue NW., Washington, D. C. The value of gravity at this point, 980.118 gals, was determined by very careful relative measurements from the world base station at Potsdam,[5] Germany.

The standardization measurements are carried out in almost exactly the same manner as a field determination except that somewhat greater care is used and a larger number of observations are made. The base chambers are fastened with plaster of Paris to the tops of the base-station piers and the flexure is therefore somewhat smaller than at the average field station where they are set in the ground. The orientation of each instrument should always be the same whether at the field station or at the base station. As a precaution against small, undetected errors of observation which would, of course, affect all the field results based on the standardization, a minimum of three independent determinations are made with each of the two sets of apparatus. So far as practicable the observing conditions are made the same as at the average field station. This tends to minimize the effect of slight inaccuracies in the constants of the apparatus. For example, each pendulum is swung for the same length of time as in the field (6 hours) for each determination and the pressure inside the case and the initial arc of oscillation are made the same as in the field.

## OTHER PREPARATIONS FOR FIELD WORK

Automobile trucks are used for the transportation of the gravity party. The instruments are carried in one of these trucks or in a trailer. Some parts of the apparatus are permanently mounted in the truck or trailer and other parts are taken out at each station. A few parts have to be carefully protected against excessive shaking or jarring on the moves between stations. It is particularly important that the pen-

[5] Recent absolute-gravity determinations by Paul R. Heyl and Guy S. Cook in Washington, D. C., and by J. S. Clark in Teddington, England, indicate that the Potsdam value is probably too high by as much as 15 or 20 milligals. The Potsdam value, however, was adopted many years ago as the international standard and practically all relative-gravity measurements in the world are based on it. It is more important that all gravity values be on a uniform datum than that the datum be the most accurate one available. A change should be made only when there can be a definite international agreement on the new value and all determinations, old and new, can be changed over to the more precise datum. See the following references: Paul R. Heyl and Guy S. Cook, The value of gravity at Washington, National Bureau of Standards Research Paper RP 946, 1936; J. S. Clark, An absolute determination of the acceleration due to gravity, Phil. Trans. R. Soc. of London, Series A, Vol. 238, 1939, pp. 65–123; B. C. Browne and E. C. Bullard, Comparison of the acceleration due to gravity at the National Laboratory, Teddington and the Bureau of Standards, Washington, D. C., Proc. R. Soc., Series A, Vol. 175, 1940, pp. 110–117.

dulum case itself be transported carefully in order to prevent any injury to the pendulum and to guard against the breaking of the thermometer or manometer and the consequent spilling of mercury. The delicate parts of the apparatus are supported on thick pieces of sponge rubber and are provided with snubbers to limit their motion.

The radio receiver and power amplifier are mounted permanently in the truck or trailer as are also the chronograph and chronometer and the battery equipment. A multiple-circuit cable is used to connect the apparatus in the truck with the pendulum case and the photoelectric amplifier which are housed in a tent a few feet away from the truck, at the point selected for the station. (See fig. 13.)

On recent work, the gravity party has consisted of the chief of party, an assistant observer, and 2 hands. Lt. R. W. Woodworth, who has had charge of the gravity party for several seasons, has recommended the addition of another hand because of the extra work involved in maintaining the temperature-control equipment, including batteries, and because of the need for additional help on some of the many other operations required at each station. It is believed that the progress of the party would be enough faster to justify the additional expense. The present normal progress of the gravity party is between 10 and 15 stations per month, depending upon weather conditions, distances between stations, condition of roads, and other factors.

Three trucks or two trucks and a trailer are minimum equipment for the economical transportation of the party. A part of this equipment is shown in figure 13. Since the work at each station requires only 1 or 2 days it is quite important that sleeping and cooking accommodations be provided for the men. The best solution of this problem seems to be to have bunks arranged in the trailer and in one of the trucks and to have sufficient cooking utensils and storage space to make possible the preparation of simple but adequate meals. The progress of the work would often be seriously delayed if the men had to find local accommodations at each station.

## SELECTION OF STATIONS

Ordinarily, only the general locations of the stations are specified in the instructions to the chief of party. These locations are usually in villages or small cities. Except in rare cases, the chief of party may select any suitable location within one or two miles of the center of the town named in the instructions. Occasionally, however, a definite location is specified.

Various requirements or conditions should be given careful consideration in selecting the exact site for a station. Damages to growing crops or to lawns should be avoided as much as possible. Whenever practicable, the consent of the property owner must be obtained before the work is started. Proximity to heavy power lines or to transformers should be avoided because of radio interference. Proximity to heavy machinery should also be avoided because of the danger of its rhythmic vibrations coupling with those of the pendulum. Other things being equal, the station should be placed where its position and elevation may be determined within the required limits (see p. 20) with a minimum amount of surveying operations. It should be placed where it can be readily recovered after several years within 50 or 100 feet in horizontal position and within 2 or 3 feet in elevation even though the mark has been destroyed. It should be where it can be marked, either at the station itself or at some nearby reference point, and where the mark will be reasonably secure from possible interference or destruction.

FIGURE 13.—Typical gravity station.

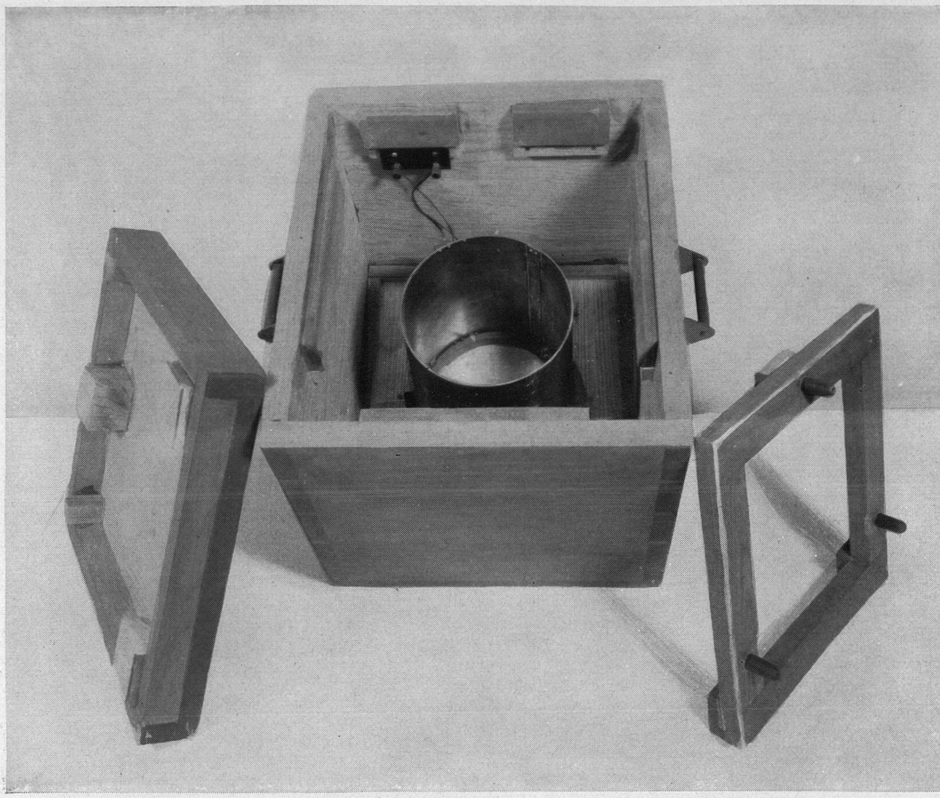

FIGURE 14.—Carrying box for pendulum case equipped with demagnetizing solenoid.

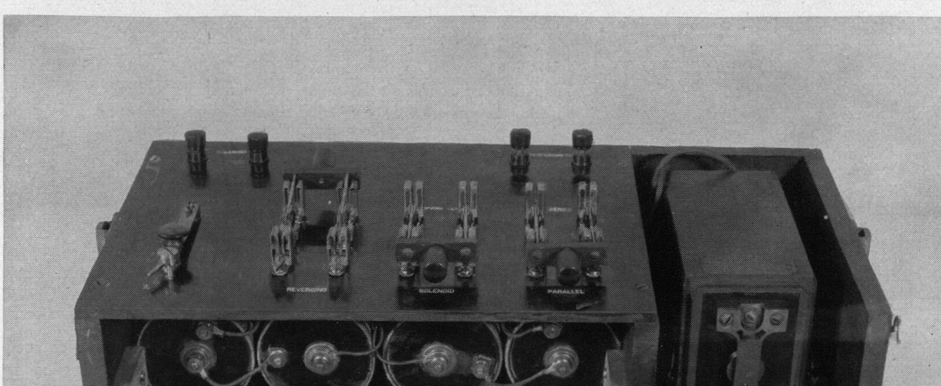


FIGURE 15.—Combined switchboard and battery box for use with solenoid and interferometer.

Colleges and universities usually make ideal locations for stations. They furnish a fairly permanent location and the results are frequently very useful to the physics departments of these institutions. The head of the physics department is ordinarily the one most interested in having a gravity determination made and, if possible, he should be consulted before the exact site for the station is selected.

### REPEAT STATIONS

Experience in the past few years has shown the desirability of reoccupying previously determined stations at fairly regular intervals during a season's work. If this can be arranged in such a way as to avoid excessive travel, the reoccupied stations should be selected from those determined earlier in the same season. Otherwise, any station, the reliability of which has not been questioned, may be redetermined. These reoccupations give approximate checks on the constancy of the pendulums at intermediate intervals of a month or 6 weeks between the preseason and post-season standardizations. In case there is any change in the pendulums, these checks make possible an adjustment of the results which will largely eliminate the effects of this change.

A few of the older stations of this Bureau do not quite meet the standards of accuracy now considered desirable. This lower accuracy has sometimes been indicated by a difference between the preseason and post-season standardizations somewhat larger than normal. Whenever it can be done without seriously delaying the party, a few of these old stations should be reoccupied each season for verification of the original results.

It is not always possible at these old stations to reoccupy the exact spot used for the first determination. New buildings or the proximity of power lines or other causes may interfere. In such cases another point in the general vicinity should be occupied although it may be a half mile or more distant and several feet different in elevation. An approximate comparison of the two determinations will still be possible if the relative positions are known within a few hundred feet and the relative elevations within 5 or 10 feet.

## SETTING OF SUPPORT FOR APPARATUS

The base chamber which serves as the support for the pendulum case is described on page 5. The setting of this support is one of the first things to be done after the station site has been selected. Unless there is some heavy concrete floor or foundation or outcropping rock to which it can be cemented, the base chamber is set in the ground. A hole is dug about 1 foot deep and about 5 or 6 inches larger in diameter than the chamber. A quantity of dry plaster of Paris, sometimes as much as 50 pounds, is then placed in the hole and is moistened and stirred up in the hole. If the soil is porous, the wet plaster tends to seep into the ground and to cement it together. The stirring should be thorough enough to insure that all of the plaster is mixed with water. No harm is done if sand or dirt gets mixed in. Plenty of water should be used so that the mix is quite thin. This insures a good bond between the plaster and the case and also between the plaster and the soil.

Before the plaster starts to harden, the base chamber is pushed down into it in an approximately level position as indicated by the small universal level attached to the inside of the bottom of the chamber. If the plaster is of good quality and fresh, it will harden sufficiently in about 30 minutes to permit setting up the rest of the apparatus, leveling the case, and making final preparations for the observations.

In ordinary soils, this method of supporting the instrument works out very satisfactorily. In loose, mucky soils, however, such as are found near the Red River in North Dakota and Minnesota, along the Mississippi River and the Gulf Coast, and in other places, it will not give sufficient rigidity to the pendulum case. Sometimes this condition can be remedied by driving three wooden stakes or iron rods, long enough to reach firm soil, in the bottom of the hole before the plaster is put in. The tops of the stakes should project an inch or two above the bottom of the hole in order to be embedded in the plaster.

The two base chambers for the two sets of apparatus should be set far enough apart, usually about 3 feet, to prevent any transmission of flexure from one to the other. They should be so oriented that one pendulum case will be at right angles to the other and one pendulum will swing approximately north and south and the other east and west. This tends to eliminate any influence of one pendulum on the other if any slight oscillation of the base chamber is transmitted through the ground.

## INSTALLING INSTRUMENTS

After the base chambers have been set, the observing tent should be set up to protect the pendulum cases against sudden changes in temperature due to sun and wind when they are unloaded from the truck. The opening of the tent should not be toward the south nor toward the direction of prevailing winds if this can be avoided.

### DEMAGNETIZATION OF PENDULUMS

Before the pendulum cases are taken out of their carrying boxes, the pendulums should be tested for magnetization. This is done with the small box compass described on page 18. Each pendulum case is first oriented approximately in the magnetic meridian. The compass box is then placed on top of the case, alternately on the east and west sides of the opening, and readings of the needle are made. Care should be taken to make sure that the compass box is in the correct position on each side as indicated by the three stops in the special plate provided for this purpose (D, fig.2).

If the pendulum causes a deflection of 2° or more in the compass needle, it must be demagnetized by means of the solenoid which is permanently installed in the carrying box. (See fig. 14.) Because of their peculiar shape, the pendulums have a tendency to become magnetized more readily with the poles oriented in one direction than with them oriented in the reverse direction. A switchboard and battery box for operating the solenoid is shown in figure 15. It has a series-parallel switch which makes it possible to have either 3 or 6 volts across the terminals of the solenoid. It also has a variable resistor in the circuit to further control the current in the solenoid. With fresh batteries, 3 volts is always sufficient to reverse the magnetization of the pendulum. Generally a few, very short, contacts of the switch will accomplish the desired purpose.

The test for magnetization must be repeated, of course, after the solenoid has been used, and the demagnetization process must be repeated if necessary. The aim should be to bring the deflection of the needle well under 2°, or preferably under 1°, before considering the demagnetization satisfactory.

### MANOMETER TESTS

A U-tube manometer will indicate low pressures accurately only if the mercury is clean and free of air bubbles. Any air that may be in the mercury will accumulate in the closed end of the tube. By letting a little air *slowly* into the vacuum chamber through the stopcock, the mercury will be pushed back into the closed end of the tube and the air will collect in small bubbles at the top of the mercury. These bubbles are not easy to see, but can be detected by a careful inspection.

A better test is to compare the manometer in the case with a standard manometer outside. A piece of rubber vacuum tube is slipped over the open end of a standard manometer and then connected to the stopcock of the vacuum case. When the stopcock is opened, the two manometers should indicate the same pressure within the accuracy of reading. If there is air in the inside manometer, it will indicate a lower pressure than the standard manometer. If it indicates a higher pressure, then the standard manometer must be at fault. In making this test, care must be taken not to let any mercury get drawn into the vacuum case either by spilling some in the vacuum tube or by opening the stopcock too quickly.

If the manometer in only one of the two sets of apparatus is suspected, it may be tested by connecting the stopcocks of the two sets with a piece of vacuum tubing and then opening both stopcocks. This should equalize the pressure in the two vacuum cases if there are no leaks, and the two manometers should register the same if both are free of air bubbles.

The manometers should be tested once or twice a month by one of the methods described above. If there is any suspicion regarding the accuracy of the pressure readings, they should be tested more frequently.

At least two spare manometers should be carried by the gravity party. To make it possible to carry them safely, they should be entirely filled with mercury and sealed with a small piece of cloth and sealing wax. If it becomes necessary to use one of these manometers, the seal must be removed and the surplus mercury poured out carefully to avoid letting any air into the closed end. The party should also carry several of the small glass funnels which are inserted in the open end to prevent splashing. (See p. 16.) The observer should obtain instructions on how to install these funnels before starting field work.

## ADJUSTMENT OF ELECTRICAL EQUIPMENT

The amplifying system used in recording the beats of the pendulum on the chronograph sheet consists of three units: the photoelectric amplifier in the recorder box; the power amplifier; and the pen relays on the chronograph. There is also the radio receiver. For accurate results, the electrical and mechanical lags in the various units and circuits must be kept constant during each pendulum determination. The units have been designed with great care to eliminate variable lags so far as possible, but it is also essential that the precautions mentioned in the following descriptions of the units be carefully noted. Although the two units are separate, they are designed to work with each other and to this extent they are not independent.

**Photoelectric Amplifier.**—This unit consists of a photo cell of the gas-filled type which picks up the beats of the pendulum as the light beam reflected from the mirror on top of the pendulum crosses a narrow slit in the cover of the cell. The current thus induced in the cell is amplified through two stages of amplification and then transmitted through a power cable to the large amplifier where it is further amplified so that it will operate the stylus of the recording chronograph. A schematic wiring diagram of the photoelectric amplifier is shown in figure 9. The values of resistors and condensers used in this amplifier are selected to give a sharp, low-frequency cut-off and thus to minimize the tendency of multiple stages to "motorboat."

There are two adjustments shown in the diagram; one ($P_1$) serves as a volume control by varying the voltage on the screen grid of the 6SJ7 tube of the amplifier. The other ($R_9$) regulates the amount of current drawn by the light-source bulb and therefore controls the intensity of the light beam that moves across the photo cell. This control should ordinarily be set so that the light-source bulb draws very nearly its rated capacity, or 0.5 ampere, as indicated on the milliammeter on the back of the case. The life of these bulbs will be considerably lengthened if they are allowed to heat up well before the full current is applied.

The volume control, $P_1$, should be set so that when the pendulum beat occurs, the plate current of the last tube of the power amplifier shown on milliammeter, $M_2$, increases from its constant value of about 0.2 milliampere to a maximum of from 4 to 6 milliamperes. This potentiometer should be adjusted (before each time signal if necessary) to give the same constant current and the same kick of the meter for all of the time comparisons of a swing. The kick of the meter can, of course, be changed by varying $R_9$, and thus changing the intensity of the light source. It should not be controlled in this manner, however, but instead the light source should be set to rated capacity and a check made to see that it is on that setting for each time comparison of the swing.

**Power Amplifier.**—This unit is so designed that there are no adjustments in either side of the amplifier. The only variable unit is the $P_1$ which controls the strength of the radio signal fed into the first tube of the amplifier. This control is a useful aid in receiving some of the weaker radio signals and in keeping the output on the radio side of the amplifier more nearly constant for all of the time comparisons of a swing. A schematic diagram of the power amplifier is shown in figure 10. The value of the resistors and condensers used in the amplifier have been selected to give a sharp, low-frequency cut-off and thus minimize the tendency of multiple stages to "motorboat." The design of the amplifier has been somewhat simplified by using two sets of "B" batteries, one for the photo-electric amplifier and another for the power amplifier. This has eliminated the use of many filter units in the "B" supply.

There should be no large change in the voltages supplied by the batteries during a swing. A fully charged "A" battery should not be substituted for a weak one after a swing has been started, nor should the "B" batteries be changed. The drain on the "B" batteries is small, and their life in the amplifier circuits is approximately their shelf life. After they have been in use for about a year, they should either be renewed or watched carefully. The "B" battery may show full voltage when idle, but the voltage may drop off very rapidly as soon as the load is applied. The radio tubes should not be changed or shifted during a swing, nor should any of the other units of the amplifier. If any major changes are required due to failure of a battery or tube or other part after a swing has been started, it is advisable to start the swing over again.

**Chronograph.**—The relays and pen mechanism of the chronograph are more likely to introduce variable lags into the record than are any other parts of the recording apparatus. The pen relay works against a small spring which returns the pen to its neutral position after it is released by the relay. This spring should be adjusted to give quick and positive action, but the tension in it should be no greater than is necessary to accomplish this result. The tension must not be changed during a swing.

As explained on page 9, the two pens of the chronograph are set to follow the same line in their neutral positions. This is done by setting one pen just far enough ahead of the other to avoid any interference between them. The distance between their points may be easily determined by tripping both pens with the same impulse when the drum is revolving or by tripping them independently when the drum is stationary. Since the total time of a swing as scaled from the chronograph sheet will not be affected by the distance apart of the two pens, so long as the distance remains constant, the only reason for measuring it is to make sure that it does not change.

The pens, or styluses, of the chronograph are made of spring steel. They should be kept well sharpened in order to function properly in cutting through the paraffine. The tips of the points should be squared off slightly as this will make them more effective in cutting through the paraffine instead of sliding over the top. The pressure of the pens on the paper should be just enough to give a good record but no more. Any necessary adjustment of the pens should be made before the first time signal of a swing and then great care taken to avoid having to readjust or disturb them until the swing is completed.

**Radio receiver.**—The radio time-signal reception and recording are the most difficult to maintain at a constant intensity on account of the difference in reception at different times of day. The observer should learn what tone of signal will operate the relay most satisfactorily. This tone will not necessarily be sharp or pleasing to the ear. After it has been determined, the same tone should be used on all signals so far as possible. The intensity of the signal should be just enough to give positive action to the pen. A strong, clear signal should be reduced somewhat in volume in order that a possible weak signal later in the day, during the same swing, may be recorded with about the same intensity.

It has been found by experience that the high-frequency (short-wave) radio time signals can be received in the field with less difficulty than the low-frequency signals. Wherever possible the signals transmitted by the Annapolis radio station (NSS) should be used. The frequency of 9425 kc is transmitted every hour except 1, 11, 12, and 18, Greenwich civil time (20, 6, 7, and 13, eastern standard time) and is usually a very satisfactory one for gravity work. Other high frequencies, namely, 4390, 12630, and 17370, are available on several of the hours and any one of these may be substituted for the 9425 if it can be recorded more satisfactorily. Except in rare instances, the U. S. Naval Observatory determines the lag of each of these signals by recording on a chronograph both the signal as sent out from the Observatory and the same signal as received by radio from Annapolis. These lags are then combined with the error of the sending clock and scientific corrections are thus determined which are furnished free of charge by the U. S. Naval Observatory to a selected list of interested organizations and individuals.

## MARKING STATION

Each gravity station must be marked as specified in paragraph 18 of the general instructions (p. 20). The bronze disks which are used for this purpose have been made hexagonal in form in order that they may be readily distinguishable from other standard marks of this Bureau. They are of two kinds, as shown in figure 16, one for use at the station and the other as a reference mark. One or the other is used at each station, but not both. If the station is in a cultivated field or in some location where the mark would be in the way, it is not marked but instead a reference mark is placed near a fence or other location where it can be readily found without becoming a nuisance. The arrow is pointed toward the station, and the distance and direction to it from the station are carefully measured and included in the description of station.

473355°—42——5

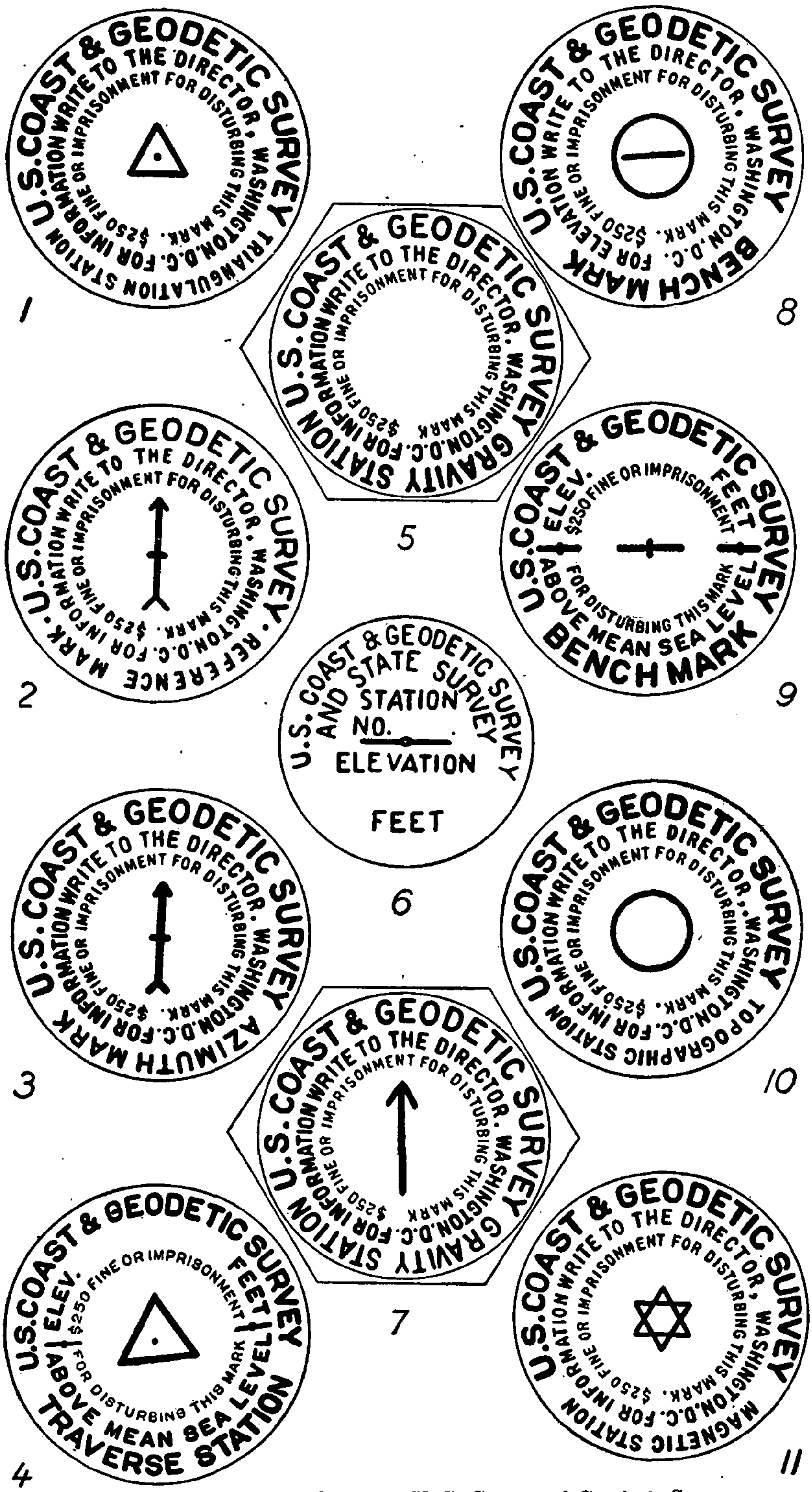


FIGURE 16.—Standard marks of the U. S. Coast and Geodetic Survey.

The mark should be set in much the same manner as at a triangulation station. In bed rock or large boulders or in heavy concrete foundations, a drill hole may be made for the shank of the marker and the marker cemented securely in place. Otherwise the disk may be placed in the top of a block or post of concrete. The post should be about 10 inches square on top (or 10 inches in diameter if round) and about 12 inches square at the bottom (12 inches in diameter if round). A form should be used for the upper 10 or 12 inches to give a smooth tapered surface which will tend to minimize frost action. The mark should project a few inches above the surface of the ground and should have a 4- by 4-inch reference post driven down beside it and projecting 12 or 15 inches to make it more easily found for the first few years after it is set.

### DESCRIPTION OF STATION

The description of station should be written in the record book. It should include all information necessary to make the station readily recoverable over a period of many years. The name of the town in which it is located or the name of the nearest town with distance and direction should be given first. Next should be given directions for reaching the station from this town and then a description of the immediate locality and of the marking of the station. Distances and directions to nearby houses bridges, concrete culverts, and other fairly permanent, nearby features should be measured roughly and included in the description. The kind of support used for the pendulum cases and the direction of oscillation of the pendulum should be stated also. A sample description is given below.

**Fresno, Calif.**—About 3 miles in a northwesterly direction from the town of Fresno, on the northeast side of U. S. Route 99 and the Southern Pacific Railroad, 0.3 mile from the Clinton Avenue grade crossing and 0.2 mile from the junction of the Espee Highway with Hughes Avenue, in the NW¼ sec. 30, T. 13 S., R. 20 E. The mark, a standard gravity reference disk stamped "Fresno, 1939" and set in the top of a concrete post, is 692 feet N. 30° E. from the center line of the Southern Pacific Railroad, 228 feet N. 80° E. from the center of the railroad water tank, 28 feet S. 35° W. from the center line of the Espee Highway and 109 feet S. 60° E. from the station itself. The mark is 2.9 feet higher than the station. The apparatus was mounted in the usual manner in plaster of Paris in shallow holes in the ground. The plane of oscillation was north-south for apparatus No. 2 and east-west for No. 3.

## DETERMINATION OF POSITION

As stated on page 20, the approximate position of each gravity station must be determined. This may be done by scaling the position from a reliable map, by connecting the station to some triangulation or traverse station or other known station, or by making astronomical determinations. If a quadrangle sheet of the U. S. Geological Survey on a scale of 1 to 62,500 is available for the area around the station and if this map is recent enough to show the present positions of roads and buildings and other map features which are subject to change, the station can usually be shown on the map in its true location and the position can then be scaled with adequate accuracy.

It is possible in many cases to connect the gravity station to a triangulation station or other known position without the expenditure of an excessive amount of time or effort. If the distance between the two points is short, say a few hundred yards, a rough traverse may be run between them by use of a small transit and either a 300-foot tape or a stadia rod. For longer distances, up to 5 miles or so, a truck traverse may be run somewhat as follows: A truck is driven from the gravity station to the triangulation station and speedometer readings are made at the two stations and at each intermediate turn in the road. The differences of these readings will give the approximate lengths of the various courses. The directions of these courses are obtained with a compass.

The following precautions against large errors should be observed in running these traverses. The error of the speedometer should be determined if possible by testing it on a known distance and if the error is appreciable a correction should be applied to the speedometer distances. The inflation of the rear tires should be kept the same while the traverse is being measured as when the speedometer test was made. In reading the magnetic bearing of each course, the compass should be taken 15 or 20 paces away from the truck to have it free from the magnetic effect of the truck. On long courses the compass bearing of the course should be taken at both ends.

It is sometimes possible to determine the position of a gravity station by a "three-point fix" on three objects visible from the station, the positions of which are known. If the objects are so located with reference to each other and to the station as to give a strong fix the resulting position will be entirely adequate as regards accuracy. The observations can be made very quickly and the computations are not difficult. (See Special Publication No. 138, p. 191.)

The observer should not hesitate to determine his position by astronomical observations if none of the preceding methods can be used satisfactorily. The observations are not difficult and can be made in a comparatively short time in clear weather. They are affected by deflections of the vertical, however, and the resulting positions are therefore less reliable than those determined by direct connection to triangulation stations.

A theodolite or vertical circle instrument (see fig. 17) should be used for the longitude observations and the method employed should be that described on page 53 of Special Publication No. 14. Briefly, the method is as follows: A known star near the prime vertical is selected and the telescope pointed on it with the horizontal wire slightly ahead of the star. The time is noted accurately when the star crosses the wire and the level and vertical circle are read. The telescope is then reversed (revolved 180° about the vertical axis and rotated on its horizontal axis) and another observation made on the same star with corresponding readings of the level and circle. This gives one complete determination of the local time. Four such determinations should be made, two on east stars and two on west stars. The local time thus determined is compared with Eastern Standard Time by means of radio time signals from Annapolis and the difference in longtitude is derived.

For latitude observations it is desirable to use a vertical circle instrument (see fig. 17) but a theodolite may be substituted if necessary. The advantage of the former is that it has a large, accurately graduated, vertical circle which can be read to the nearest 5 seconds. It has also a sensitive level parallel with the circle which gives accurate control for vertical angles measured with the instrument. The latitude may be determined by measuring the altitude of Polaris, preferably when it is near upper or lower culmination, or if somewhat better accuracy is desired a modification of the Talcott method may be used.

In measuring the altitude of Polaris, a number of observations should be taken with the customary reversals of the instrument. The sensitive level should either be carefully centered or be read for each observation.

The other method mentioned above, the modified Talcott method, consists of measuring the difference in the meridian zenith distances of two stars, one north of the zenith and the other south, that transit within a few minutes of each other.

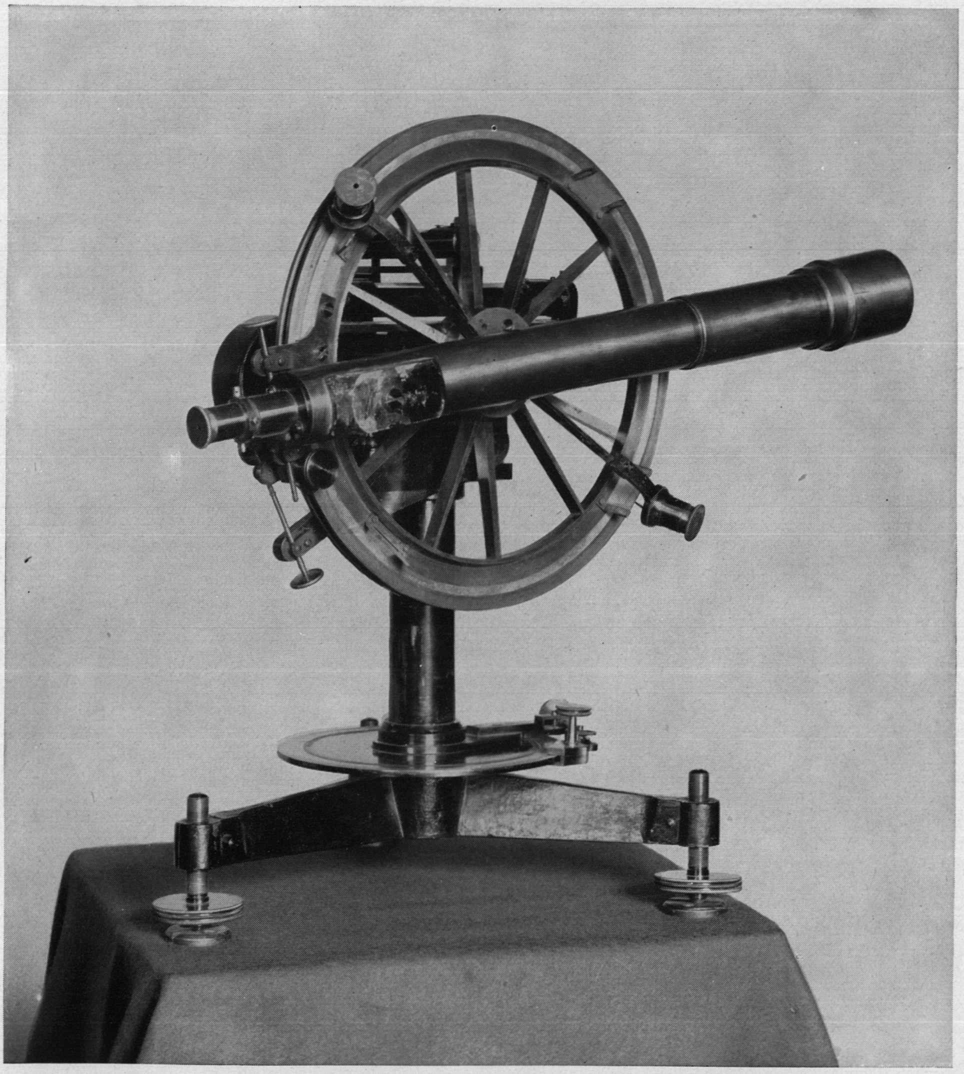

FIGURE 17.—Vertical circle.

The difference in zenith distances should not exceed 3° and the difference in right ascensions should not exceed 10 or possibly 15 minutes of time. After observing the first star of a pair the instrument should be turned 180° around its vertical axis to observe the other star. In this way only a small part of the vertical circle is used in measuring the difference of zenith distances, and instrumental errors are minimized. Refraction errors tend to be eliminated because of the nearly equal altitudes of the two stars. An instrument having a horizontal circle, of the same type as vertical circle No. 107 (fig. 17), must be used for the modified Talcott method. Observations should be made on at least three pairs of stars.

## DETERMINATION OF ELEVATION

As specified in paragraph 16 of the instructions, the elevation of each gravity station should be determined within an accuracy of 5 feet if possible. This limit may be increased to 10 feet where an excessive expenditure of time would be required to keep within the lower limit. In selecting locations for gravity stations, an effort is made to place them where accurate elevations will be available in the general vicinity. This is not always feasible, however.

The difference of elevation of the station and some point of known elevation may be determined in a number of different ways. If the distance is short, half a mile or less, and the difference of elevation not too great, the connection may be made with a hand level. For longer distances a wye level may be used. The lengths of sights need not be limited for this work and no attempt need be made to balance the fore sights and back sights. As a precaution against blunders a rod should be used which is graduated in different units (meters and feet, for example) on face and back. This rod should be turned and read on both sides at each rod point. If the distance between the gravity station and the bench mark is not too great and is accurately known, the difference in elevation may be determined with fair accuracy by measuring the vertical angle at each end of the line. A transit or plane table alidade equipped with Beaman arc scale is also sometimes used in place of a wye level.

### USE OF ALTIMETERS

For distances of 2 or 3 miles or greater, about the only practical method of determining the elevation of a gravity station is by use of aneroid barometers or altimeters. Fair accuracy can be obtained with these instruments by the use of good judgment and care. It must always be remembered, however, that the accuracy is limited fully as much by atmospheric conditions as by instrumental precision and observational procedure. If the equilibrium of the air is seriously disturbed by strong horizontal or vertical currents or if the barometer is changing rapidly, then the difference of barometric readings at points several miles apart will not give a true indication of the difference of elevation of these points. The method should be used, therefore, under favorable atmospheric conditions only.

At least two altimeters, and preferably three or four, should be used to determine a difference of elevation. One or two altimeters are kept at the starting point and are read at intervals of 5 or 10 minutes. One or two altimeters are read at the starting point, are then transported to the distant point and read, and then returned to the starting point for the final readings. It is well to arrange the timing of the observations to make the readings on the transported altimeters simultaneous with those on the stationary instruments.

The program of observations should be somewhat as follows:

1. Three sets of simultaneous readings are made on all altimeters at the starting point, which is usually the gravity station.
2. Two of the altimeters are transported to the other point which is usually a bench mark or other known elevation.
3. The altimeters at the starting point are read at regular intervals until the traveling altimeters have been returned.
4. At times which have been agreed upon beforehand, simultaneous readings are made on the altimeters at the gravity station and the bench mark.
5. The traveling altimeters are brought back to the starting point.
6. Repetition of 1.

In computing the difference of elevation it must be assumed that the variations of atmospheric pressure recorded at the starting point took place simultaneously at the distant point also. In other words, it must be assumed that for any given elevation the atmosphere has exerted the same pressure at any instant over the entire area which includes the two points involved. Under average conditions this assumption, fortunately, is a reasonably correct one. The possibility that it may not be correct during the time the observations are being made should always be kept in mind.

There are several different types of altimeters. For example the Short & Mason altimeter has both a pressure scale and an elevation scale. The former is fixed but the latter may be moved. It is thus possible to set the elevation scale to read correctly at some point of known elevation and then read other elevations off directly. This method is not recommended for accurate work. Instead the zero elevation should be set opposite the pressure reading of 31 inches and left at this setting, since this is the setting for which it is standardized.

Another type of altimeter is known as the Paulin System. It has an elevation scale only and is operated by balancing changes in atmospheric pressure by varying the tension on a spring which is attached to one side of the vacuum chamber. It has two indicators, one which shows when this balance has been attained and another which shows the corresponding elevations.

Most altimeters are approximately compensated for temperature, so far as the temperature of the instrument itself is concerned. Any inaccuracy in this compensation or in the calibration of the instrument should be determined by having each instrument standardized over its entire range and for several different temperatures at the National Bureau of Standards. Corrections can then be applied for temperature and graduation errors and for lack of uniformity in the movement of the indicator.

Regardless of whether or not an altimeter has been compensated for temperature, corrections must always be applied to altimeter readings for the temperature of the air. The instruments are ordinarily graduated for a temperature of 50° F. and are standardized for this temperature. Corrections are applied for variations from this standard temperature. The mean of the temperatures at the two points at the time the simultaneous altimeter readings are being made is considered to be the temperature of the air column. The correction in feet is approximately 0.002 times the difference of elevation in feet times the departure in degrees Fahrenheit from standard temperature. It is plus for temperatures higher than standard and minus for those lower. A table of temperature corrections will be found in Smithsonian Meteorological Tables, Table No. 52.

The following instructions for use of altimeters are taken from a report by R. W. Woodworth of this Bureau dated July 20, 1939.

Keep altimeters level at all times.

Avoid sudden jars; keep instruments cushioned against road shock on moves.

Avoid sudden temperature changes; keep instruments in shade at all times.

Set altimeters on level, stable support at height of bench mark or that of knife edge of gravity instrument.

Avoid midday observations. Best results are ordinarily obtained from 2 to 4 hours after sunrise or before sunset.

Avoid observations during thunderstorms or squally weather.

Do not exceed 2 hours for round trip to distant point, if possible.

Wait 10 minutes after setting up instruments before starting readings.

Read elevations to nearest foot (using hand glass) and temperature to nearest degree Fahrenheit.

Obtain readings simultaneously on altimeters left at starting point and those transported to other point.

On Paulin altimeters, swing middle dial counterclockwise until zero needle is against minus side before moving instrument. Failure to do this will ruin altimeter.

On Short & Mason altimeters, make sure that zero of elevation scale corresponds with 31 inches of pressure scale.

## EXAMPLE OF DETERMINATION OF ELEVATION BY ALTIMETERS

In the following example, one altimeter was kept at the gravity station and two were carried to the bench mark and back. Not all of the readings of the stationary altimeter are shown in the table but only those which are simultaneous with the readings made on the instruments transported to the bench mark and back. No standardization corrections are applied to the readings of the stationary altimeter as they would be entirely negligible for the small changes in pressure which took place at the gravity station. The standardization corrections for the other two altimeters were obtained from data furnished by the National Bureau of Standards. (See fig. 18.)

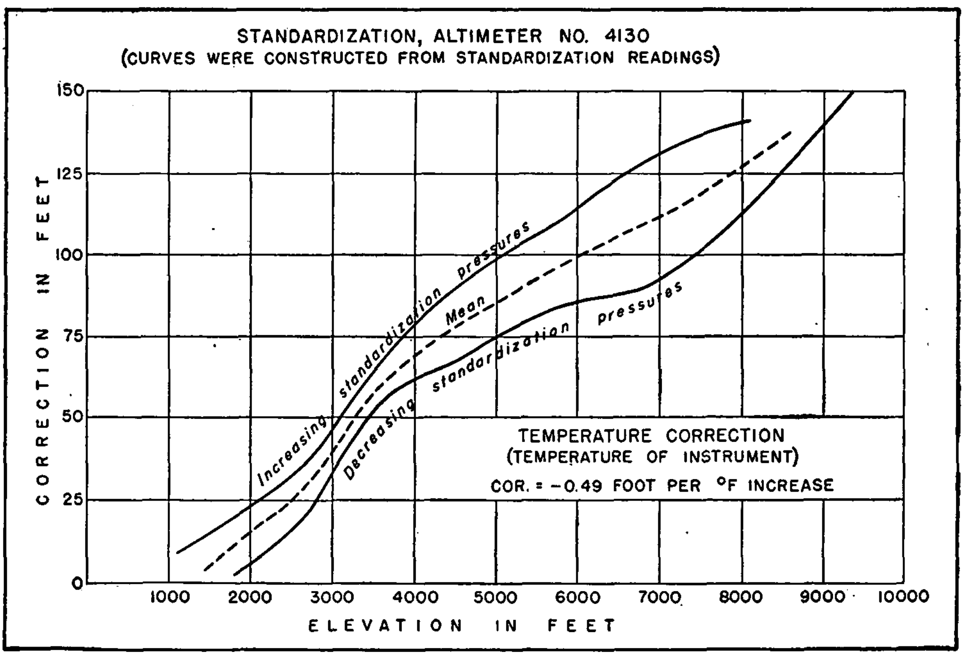


FIGURE 18.—Example of standardization curves for altimeter.

*Barometric determination of elevation*

(Gravity station No. 1028, Boulder Dam, Nevada.)

| Date and time | Temperature at station | At station, Altimeter No. 32530 | | | Traveling altimeters | | | |
|---|---|---|---|---|---|---|---|---|
| | | Reading | Correction for temperature | Corrected readings | Location | Temperature | Readings | |
| | | | | | | | No. 2 | No. 4130 |
| Apr. 26, 1939: | °F | *Feet* | *Feet* | *Feet* | | °F | *Feet* | *Feet* |
| 8:08 a. m | 70 | 2257 | 0 | 2257 | Gravity station | 70 | 2231 | 2164 |
| 8:13 a. m | 70 | 2256 | 0 | 2256 | Gravity station | 70 | 2231 | 2164 |
| 8:45 a. m | 71 | 2253 | −1 | 2252 | Bench mark | 72 | 2707 | 2650 |
| 8:55 a. m | 71 | 2252 | −1 | 2251 | Bench mark | 72 | 2706 | 2649 |
| 9:22 a. m | 73 | 2248 | −4 | 2244 | Gravity station | 73 | 2218 | 2160 |
| 9:29 a. m | 73 | 2244 | −4 | 2240 | Gravity station | 73 | 2219 | 2162 |

| Date and time | Corrections to altimeters | | | | Change in pressure at gravity station | Corrected readings [1] | | Difference of elevation | |
|---|---|---|---|---|---|---|---|---|---|
| | Temperature | | Standardization | | | | | | |
| | No. 2 | No. 4130 | No. 2 | No. 4130 | | No. 2 | No. 4130 | No. 2 | No. 4130 |
| Apr. 26, 1939: | *Feet* | *Feet* | *Feet* | *Feet* | *Feet* | *Feet* | *Feet* | *Feet* | *Feet* |
| 8:08 a. m | 0 | 0 | +28 | +19 | 0 | 2259 | 2183 | | |
| 8:13 a. m | 0 | 0 | +28 | +19 | +1 | 2260 | 2184 | 490 | 500 |
| 8:45 a. m | −2 | −1 | +39 | +29 | +5 | 2749 | 2683 | | |
| 8:55 a. m | −2 | −1 | +39 | +29 | +6 | 2749 | 2683 | 490 | 489 |
| 9:22 a. m | −3 | −1 | +28 | +19 | +13 | 2256 | 2191 | | |
| 9:29 a. m | −3 | −1 | +28 | +19 | +17 | 2261 | 2197 | | |

| | |
|---|---|
| Mean difference of elevation | 492 |
| Temperature correction | +21 |
| Corrected difference of elevation | 513 |
| Elevation of bench mark (X 167, 1935) | 1,723 |
| Elevation of gravity station | 1,210 |

[1] Includes correction for change in pressure at gravity station.

## TOPOGRAPHIC SKETCH

The topographic sketch for the area immediately around the station is required only where the terrain is fairly rough as indicated in paragraph 17 of the instructions. (See p. 20.) A quick way to determine whether or not such a sketch is necessary is to measure the vertical angle from the station to the highest or lowest points within the specified radius. If the angle of elevation or depression on any point exceeds 3°, a topographic sketch should be made.

The survey need be only a rough one and may be made by pacing the distances and by using a compass and hand level for obtaining the directions and differences of elevation. An alternate method is to use a small transit and a 300-foot tape. With the latter method the differences of elevation are computed from the distances and inclination angles.

The sketch should be drawn roughly to scale and should have the scale shown on it. An arrow pointing north should also be shown to indicate the orientation of the sketch.

In areas for which there are no published topographic maps, similar to the quadrangle sheets of the U. S. Geological Survey, it is well to make the local sketch extend somewhat beyond the 275-meter radius. The approximate distance and direction to nearby topographic features such as a small hill or stream bed or cliff should be noted on the sketch together with the approximate size or slope or elevation of these features. Notes such as these are often very useful in the isostatic computations and may be of much help in plotting the station on any detailed map of the area which may become available later.

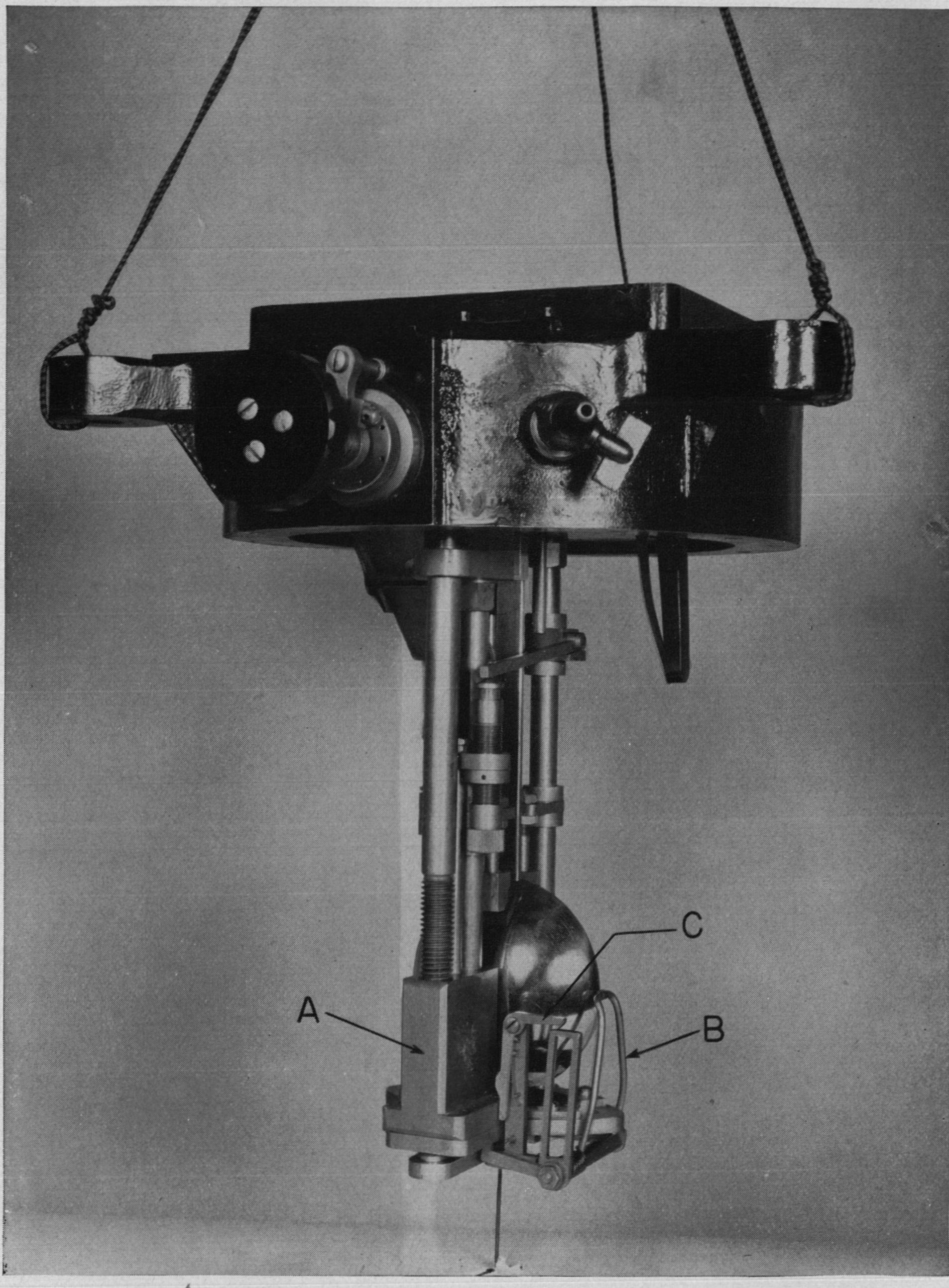


FIGURE 19.—Gravity pendulum and its support, showing lever used to start pendulum oscillating.

A is block which is raised or lowered to manipulate pendulum after lower part of vacuum case has been placed in position and sealed. B is lever which pushes pendulum away from its vertical position when starting it to oscillate. C is ratchet for holding B at any desired position until pendulum is released. C is released by raising block A a short distance.

## GRAVITY OBSERVATIONS

The observations required for a determination of gravity consist essentially of time comparisons between the swinging pendulum and the radio time signals; together with readings of temperature, pressure, flexure, and amplitude of oscillation. Time comparisons are also made between the pendulum and a chronometer, but these serve merely as a check in computing the number of pendulum oscillations in the partial swing of 1 or 2 hours included between the first and the intermediate time signals. They are essential if no intermediate time signals are recorded during a swing but may be omitted if an intermediate time signal makes it possible to compute a partial swing of 1 or 2 hours, because if an erroneous "count" is used over this short interval, the resulting value of gravity will differ so much from the theoretical value as to indicate at once that the "count" is wrong. Ordinarily, the pendulum-chronometer comparisons, even though not entirely necessary in all cases, are always made in order to serve as a precaution against error.

Before the observations are started, a final check of the two instruments is made to insure that everything is in working order and that the instruments are level and have the correct pressures in the vacuum cases. The pressure in each should be about 5 mm. of mercury. It should not be much below this because of uncertainties in the buoyancy effect at lower pressures, especially below 1 mm. of mercury.

The program of observations at a station is somewhat as follows: About 15 minutes before the time signal comes in which is to be used as the time control for the beginning of the swing, the pendulum in each apparatus should be lowered gently onto the supporting plane and started oscillating by means of the device shown in figure 19. The starting lever, *B*, is slowly brought into contact with the bob by lowering the block, *A*, and then the bob is pushed to one side by further lowering of the block. When the ratchet, *C*, which holds the starting lever in any desired position, reaches a certain notch (determined by counting the clicks) or when the ray of light which operates the photoelectric cell has reached the scale value required to give the correct amplitude to the pendulum, the ratchet is released by moving the block upward a few turns of the screw. This releases the starting lever, which is pulled back by a spring to permit the pendulum to oscillate.

Next the radio receiver is tuned and just before the beginning of the time signal, the chronograph is started. The following comparisons are then recorded on the chronograph:

Chronometer and pendulum *A* for 15 or 20 seconds.
Radio signals and pendulum *A* for about 1 minute.
Radio signals and pendulum *B* for 15 or 20 seconds (or longer if the record is poor).
Radio signals and chronometer for 15 or 20 seconds.
Radio signals and pendulum *A* for about 1 minute or less.
Radio signals and pendulum *B* for about 1 minute or until final signal is received.
Chronometer and pendulum *A* for 15 or 20 seconds immediately after final signal.

Adequate identification marks should be placed on the chronograph sheet to indicate what time piece or pendulum is shown by each section of the record, and to identify the even minutes of the time signal and the chronometer. Since the drum revolves too rapidly to permit writing on the sheet while the chronograph is running, some simple mark like a check or cross should be made at the points to be labeled and the detailed labels should be written on the sheet after the chronograph has been stopped. The coated chronograph paper is rather easily marred and must be handled carefully to avoid any obscuring of the record.

473355°—42——6

The record should include at least two identifications of the minute "break" of the chronometer, as indicated by the omission of the fifty-ninth second of each minute. The chronometer should be recorded over longer intervals than those indicated above, if necessary, to obtain these identifications.

As soon as the above comparisons have been completed, readings are made and recorded of the amplitude of oscillation (arc readings), pressure, and temperature. The frequency and strength of the radio signals are also entered in the record book as also the position of the level bubble inside the vacuum case which is recorded as "level" or as the number of divisions it is out of level and the direction ("front" or "rear"). The leveling of the instrument must not be changed while the pendulum is oscillating.

Ten minutes after the final signal "break," another comparison between the chronometer and pendulum A should be recorded on the chronograph, and again at 30 and 60 minutes after this signal.

Two hours after the first set of time signals were recorded another set of compari sons are made. These should include the following comparisons in the order indicated

Chronometer and pendulum A.
Radio signals and pendulum A.
Radio signals and pendulum B.
Radio signals and chronometer.
Radio signals and pendulum A.
Radio signals and pendulum B.
Radio signals alone until final signal.

Readings are again recorded for the amplitude, pressure, temperature, frequency and strength of radio signals, and level.

The final set of comparisons and readings are made 6 hours after the first set. They are the same as those made at the end of 2 hours except that no comparisons with the chronometer are necessary.

The program outlined above is the standard one used when the intermediate signal is 2 hours after the first signal. It must of course be modified if the intermediate signal is received at some other interval. A sample page of the record is shown below (See fig. 20.) It should be noted that temperature readings are made every hour. A longer interval may be used if the temperature variation is relatively slow and uniform. The level should also be read each hour if it is changing appreciably. The apparatus must not be releveled during the swing. The pressure should be read at the same interval as the temperature and level if the case is leaking at all.

## FLEXURE DETERMINATION

One of the corrections which must be applied to gravity observations with the Brown apparatus is that required to eliminate the effect of flexure, that is, the movement or sway of the pendulum support due to the oscillations of the pendulum itself. As already explained (see p. 14) this movement is measured with an interferometer, an instrument which can be used to measure minute distances or movements in terms of the wave length of monochromatic light. Helium light, which has an effective wave length of approximately 0.58 micron, is that ordinarily used. It is furnished by a small helium discharge tube which is connected to a 6-volt storage battery through a spark coil to give a voltage of 8,000 or 10,000.

The separate interferometer plate is attached to the left side of the vacuum chamber at the approximate height of the knife edge. The main part of the interferometer

Form 497
DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Revised April 1936

# GRAVITY OBSERVATIONS

# BROWN APPARATUS

STATION: Ludington, Mich. DATE: Sept. 10, 1940 OBSERVER: R. W. Woodworth PENDULUM: B8

SWING: 1 CHRONOMETER: 13885 GRAVITY APPARATUS: 2 KNIFE EDGE: A2

| ARC READINGS | REDUCED ARC | PRESSURE | TEMPERATURE | MEAN TIME E. S. T. | | RADIO SIGNAL FREQUENCY | RADIO SIGNAL STRENGTH | REMARKS |
|---|---|---|---|---|---|---|---|---|
| ° | mm. | mm. | °C | h. m. | | | | |
| 81.2 | | | | | | | | |
| 33.1 | | | | | | | | |
| 48.1 | 5.95 | 3.9, 2.4 | 32.0 | 11 a.m. | | 9425 | R-5 | Level |
| (114.3) | | | | | | | | |
| 78.9 | | | | | | | | |
| 35.9 | | | | | | | | |
| 43.0 | 5.31 | 3.9, 2.4 | 31.0 | 12 noon | | 9425 | R-5 | 1/4 div. rear |
| (114.8) | | | | | | | | |
| | | | 30.5 | 1 p.m. | | | | |
| | | | 30.2 | 2 p.m. | | | | |
| | | | 30.1 | 3 p.m. | | | | |
| 70.1 | | | 29.8 | 4 p.m. | | | | |
| 45.2 | | | | | | | | |
| 24.9 | 3.08 | 3.9, 2.4 | 29.6 | 5 p.m. | | 9425 | R-6 | 3/4 div. rear |
| (115.3) | | | | | | | | |

FIGURE 20.—Sample record of gravity determination.

should if possible be mounted on an independent support with the plate, which is in the end of the horizontal tube, immediately in front of the plate attached to the pendulum apparatus. The stand on which the interferometer is mounted may be of rough lumber but should if possible be supported at a distance of 2 or 3 feet from the pendulum apparatus in order to be entirely free from movements carried through the ground from the apparatus.

When the interferometer is properly adjusted, interference fringes of alternately light and dark bands will be produced which can be seen through the eyepiece of the instrument. These bands are produced when the two reflecting plates are brought as close together as practicable without touching each other and when the reflecting surfaces make a very slight angle with each other. This angle is of the order of 10″ when fringes of good visibility are obtained. If the angle is much larger than this the fringes become very narrow or invisible.

The instrument is first set with the two plates 1 or 2 millimeters apart and as nearly parallel as can be estimated by looking at them directly. The tube is then lighted and the final adjustment made by looking into the eyepiece of the instrument. Three bright spots will be seen. These are different reflections from different surfaces of the two plates of the small opening in the front end of the tube. Two of these spots must be brought very closely into the same position before fringes will be produced, but which of the spots are to be used for this purpose must be determined by tests. Fine adjustments must be made with the foot screws. When the two spots are nearly in coincidence, the instrument focus should be manipulated to be sure that the fringes are not so far out of focus as to be invisible. As soon as the fringes become visible their direction and width may be changed by very slight adjustments of the foot screws.

It must always be remembered that slight jars, either of the interferometer itself or of the gravity apparatus will cause the fringes to disappear. In making the adjustments described above, therefore, the observer must take his hand off the instrument at frequent intervals to be able to see the fringes and tell when the adjustment has been perfected. After a little experience, an observer may make the adjustment very quickly under good conditions but a great deal of patience is sometimes required when conditions are poor or the observer is inexperienced.

A brief explanation of how the fringes are formed may be of help to the new observer. Let us assume that the plates are so set that they are exactly 10 wave lengths of helium light apart along a certain line. Since the plates are set at a slight angle they will be 10½ wave lengths apart along another line parallel with the first line and close to it and 11 wave lengths apart along another line, etc. Along these lines where the plates are a certain number of wave lengths apart, or a certain number plus a half wave length, the incident and reflected light will be in phase and will produce a bright band. Halfway between these lines the incident and reflected light will be in opposite phase and will interfere to produce a black band.

Suppose the distance between the plates is increased by ½ wave length. The fringes will be displaced laterally by one fringe width. For example, the bright band where the plates were 10½ wave lengths apart will be moved laterally to the line where the plates were 10 wave lengths apart. In other words, a displacement of one of the plates of ½ wave length will cause a lateral displacement of the fringes corresponding to one wave length.

The movement of the pendulum support caused by the oscillations of the pendulum is ordinarily much less than a half wave length of helium light and therefore the movement of the fringes will be less than one fringe width. The shift of the fringes due to the pendulum can be identified from other shifts which may be due to changes in level or vibrations of the ground because of the regular half-second period of the former. Vibrations due to elevators or other machinery or to traffic may be very disturbing in some locations and may make it necessary to carry out the flexure determinations at night or when these disturbances are at a minimum.

The observations for flexure may be made by use of a scale located at the focal point of the eyepiece or they may be made by estimation relative to the fringes themselves. If the scale is used, the width of a fringe (say from the middle or edge of one dark band to the middle or edge of the next dark band) is obtained in terms of scale divisions. The half-second shift of the fringes is then read on the scale. This is a difficult observation to make because of the relatively rapid movement of the fringes.

The other method mentioned above is somewhat easier to use and is the one now commonly employed. In this second method the movement is estimated as a decimal part of a whole fringe width or more commonly as a decimal part of the black part of the fringe which is taken as one-third of the whole fringe. If the black part of the fringe is used as the unit, the observer should look at the fringes carefully to decide just what should be considered the edges of the black band to make it a third of the total fringe width. The movement is then estimated in relation to this unit.

The constant used in computing the correction to the period of the pendulum due to flexure was determined empirically in relation to an amplitude of oscillation of 5 mm. of the tip end of the half-second pendulum. In making the flexure observations, several different amplitudes of oscillation are used which are then corrected to the standard 5 mm. amplitude. This not only varies the observational conditions but also makes it possible to minimize the effect of bias or personal equation in the flexure readings provided that someone other than the observer starts the pendulum oscillating. The best method is to have two observers who will alternate in starting the pendulum oscillating and in making the flexure readings. In this way, the one making the readings can be kept free of the mental bias which would be difficult to avoid if he knew the relationship between the amplitude and that of a previous observation.

A sample set of flexure readings is given below.

*Flexure observations*

Station, Boyd, Fla.
Chief of party, C. I. Aslakson.
Assistants {W. N. Martin. / G. R. Shelton.}

Date, April 4, 1938.
Apparatus, No. 3.
Pendulum, No. A8.

| Observer | Amplitude of oscillation: Observed | Amplitude of oscillation: Reduced | Flexure reading | Flexure correction |
|---|---|---|---|---|
| | *Scale divisions* | *mm.* | *Units of ⅓ fringe width* | *Sec. × 10⁻⁷* |
| W. N. M | 123.8−18.2=105.6 | 13.6 | 0.45 | 9.5 |
| G. R. S | 116.5−26.3= 90.2 | 11.6 | .35 | 8.7 |
| C. I. A | 108.6−35.8= 72.8 | 9.4 | .25 | 7.7 |
| C. I. A | 123.8−18.2=105.6 | 13.6 | .35 | 7.4 |
| | | | | Mean... 8.3 |

The factor for reducing the amplitude as read on the scale to the amplitude of the tip of the pendulum (reduced amplitude) is 0.1288 for the apparatus used in the example above. The flexure correction in the last column is obtained as follows: The readings in the fourth column are multiplied by ⅓ to reduce them to units of a full fringe width.

Each one is then reduced to the standard 5 mm. amplitude by multiplying it by the fraction, 5 divided by the amplitude in the third column. This value is then multiplied by the factor 173. (See p. 58.) The result is in units of the seventh decimal place of period. This operation may be indicated simply as follows:

$$\text{Cor.} = \frac{R}{3} \times \frac{5}{A_R} \times 173$$

or

$$\text{Cor.} = 288 \frac{R}{A_R}$$

where $R$ is the flexure reading in the fourth column and $A_R$ is the reduced amplitude in the third column.

Some observers find it more convenient to work directly from the observed amplitudes in scale divisions. If the factor, 0.1288, used in reducing the amplitude is included in the formula, the correction becomes,

$$\text{Cor.} = 2238 \frac{R}{A_0}$$

in which $A_0$ is the observed amplitude in the second column. It should be noted that this last formula applies only to the particular instrument used for the above example. Different instruments have different amplification factors.

## READING CHRONOGRAPH SHEETS

Each chronograph sheet is read if possible before removing it from the chronograph drum. Two auxiliary devices are attached to the chronograph for this purpose, one a microscope which is fastened to the pen (or stylus) carriage, and the other a device at the right hand end of the drum for measuring the precise amount the drum is turned to bring a mark under the cross hairs in the microscope. Both of these auxiliary devices are shown in figure 8.

Sufficient readings are taken to obtain accurate values for the different comparisons specified on page 37. For example, the comparison of the time signals with pendulum *A* is made by reading first a time-signal mark, then two half-second marks made by the pendulum, then the next time-signal mark, then the next two half-second marks made by the pendulum, etc., until 11 consecutive seconds of the radio signals and the included 20 marks of the pendulum have been read. (See the second and fourth columns of the sample radio-pendulum comparison on p. 50.) The other comparison readings are made in a similar manner as indicated on several of the computation forms given in chapter 3.

## FIELD ABSTRACT OF RECORD

The customary practice of this Bureau should be followed of sending record books (including chronograph sheets) and abstracts to the Washington office at different times and by registered mail. The abstracts should be such as to permit the computation of the final result for each station in case the corresponding records should be lost in the mail. They should therefore include all necessary readings from the chronograph sheet, and a transcription of all arc (amplitude), pressure and temperature readings, and the derived correction for flexure. The headings of each sheet used for the abstract must be carefully filled in with the name of the station, the date, the name of the observer, the identifying numbers of pendulum and receiver, etc. The elevation of the station and its latitude and longitude (or a copy of a U. S. Geological Survey quadrangle sheet on which the station has been plotted) should also be included with the abstracts. In other words, the observer should make sure that all information essential for the complete computations at each station is included in the abstracts.

Since the description of station is an essential part of the data for each station it should also be safeguarded against loss. This can be done by copying the description from the record book onto the standard description card and mailing it with the abstracts. If the observer makes a practice of writing the rough descriptions in a memorandum book and later copying them into the record book, the former may be used in place of the description card as a safeguard against loss in the mail. In this case the memorandum book should be retained and should be forwarded to the Washington office by separate mail as a part of the records for the station. This will make it possible to trace a possible error made in copying the description into the record book.

# Chapter 3—COMPUTATION OF GRAVITY RESULTS

## INTRODUCTION

Relative determinations of gravity with pendulum apparatus depend upon the simple relationship that the intensities of gravity at two points are inversely proportional to the squares of the periods of an invariable, unrestrained pendulum which is swung under identical conditions at the two points. If the value of gravity is known at one of the two points and if the periods of the pendulum have been determined at both of them, the corresponding value of gravity at the other one may be computed. For gravity determinations in this country, the known point is the fundamental or base station in the basement of the Commerce Building, Washington, D. C. The gravity at this point is 980.118 gals as determined by relative measurements from Potsdam, Germany, the world base station. (See p. 23.) Gravity determinations with pendulum apparatus in the United States may, therefore, be computed by the simple formula,

$$g_s = \frac{P_w^2}{P_s^2} g_w$$

in which $g$ is the value of gravity in gals and $P$ is the period of the pendulum in seconds, the subscripts $s$ and $w$ indicating the field station and the Washington base station, respectively. They may also be computed by the following difference formula which is somewhat more convenient to use especially when the second correction term is taken from the table on page 58:

$$g_s = g_w - \frac{2g_w}{P_w}(P_s - P_w) + \frac{3g_w}{P_w^2}(P_s - P_w)^2 - \ \ldots\ldots$$

It is of course impracticable to obtain exactly identical conditions under which to swing the invariable pendulum at two different points. The determination and computation of gravity are therefore more complex operations than the preceding paragraph might seem to indicate. Variations in temperature, pressure, and amplitude of oscillation must be taken into account as must also any differences in the rigidity of the support. The time control used in obtaining the period of the pendulum must be very precise.

An idea of the relative importance of the various observations at a gravity station may be obtained by the following comparison of errors. A change of 0.00000025 second in the period of the half-second pendulum corresponds approximately to a change of 0.001 gal in the value of gravity. An error of this magnitude may be produced by any one of the following errors of observation:

1. An error of 0.01 second in the radio time signals or in the comparison of these signals with the pendulum at either the beginning or end of a 6-hour swing.
2. An error of 0.9° C, in the determination of the mean temperature of the pendulum when invar pendulums are used or 0.06° C. when bronze pendulums are used. These values, of course, apply only to the pendulums of this Bureau as they depend upon the composition of the invar or bronze.
3. An error of 1½ mm. of mercury in the determination of the mean pressure inside the vacuum chamber for pressures of about 5mm.

4. An error of 0.7 mm. in the observed amplitude of the tip of the pendulum at the beginning of a 6-hour swing or of 0.55 mm. at the end of the swing, provided the starting amplitude is about 6 mm. These errors correspond to about 5.4 and 4.3 scale divisions, respectively, of the arc reading scale. (See p. 41.).

5. An error of 0.014 fringe width of helium (or sodium) light in the observed flexure of the pendulum support, when the amplitude of oscillation of the pendulum is 5 mm.

The period of the pendulum must be obtained by taking the mean of a large number of oscillations. The requisite accuracy, as represented by 2 or 3 ten-millionths of a second in the period of a single oscillation, cannot be obtained by timing a single oscillation or by taking the mean of 5 or 10 or 100 oscillations. With the equipment described in chapter 1, it is necessary to use an interval of about 6 hours, or about 43,000 oscillations of the pendulum, to obtain the desired accuracy in the mean period. This interval could be shortened considerably, perhaps to 1 or 2 hours, if a high grade, portable, crystal chronometer were used in timing the pendulum but this type of time piece has not as yet been used for land determinations by this Bureau, largely because of the high cost of the equipment and because of difficulties in transportation.

## DETERMINATION OF NUMBER OF OSCILLATIONS

There are two primary requirements in determining the mean period of the pendulum. First, some definite point in the oscillation of the pendulum must be selected to serve as the reference point in the timing. Second, the exact number of oscillations of the pendulum over an interval of several hours must be counted or otherwise derived in order that the unavoidable errors in the comparisons of pendulum and time signals may be largely eliminated by thus distributing them over a long interval.

The exact instant the pendulum reaches its limit of motion in one direction is taken as the reference point in timing the pendulum. This instant is determined by taking the mean of two consecutive pendulum "breaks" on the chronograph sheet. One of these "breaks" is made when the beam of light reflected from the head of the pendulum sweeps to the right across the slit of the photoelectric cell and the other when it sweeps to the left.

The beam of light of the recording apparatus is adjusted to make the middle of its motion somewhat off center in relation to the slit of the photoelectric cell. This causes the intervals between pendulum "breaks" on the chronograph sheet to be alternately long and short. The short interval is always used in deriving the reference point for timing the pendulum. This means that the same end of the pendulum motion is always used as the reference point. This may be either the right hand or left hand end of the motion, depending on the adjustment of the light beam. The advantage of this method is that the number of oscillations over any interval, the ends of which are determined in this way, will always be an even number. An error is indicated at once if the number comes out odd.

### CHRONOMETER-PENDULUM COMPARISON AND COMPUTATION OF APPROXIMATE PERIOD

The second requirement in determining the mean period of the pendulum is that of counting the oscillations. A sidereal chronometer is used as an auxiliary device for this purpose. It is convenient in determining the "count" to use as a unit a complete cycle of the pendulum, that is, from one end of its motion to the other end and back, instead of a single oscillation. The approximate double period (cycle period) of the pendulum in terms of chronometer seconds is first derived and is then corrected for the rate of the chronometer on mean time. This corrected, approximate, double period can then be used as the divisor for the interval between time signals and the number of pendulum cycles can thus be obtained.

The pendulum is slightly slower than the sidereal chronometer, that is, it takes from 1.001 to 1.003 sidereal seconds (depending on the intensity of gravity) to make a complete cycle.[6] During a short interval of from 5 to 8 minutes, therefore, an integral number of complete cycles of the pendulum will take place in an interval of the same number of sidereal seconds, plus a few tenths of a second. In the sample computation (fig. 22), for example, it is known that 464 oscillation cycles of the pendulum took place in the 464.811 sidereal seconds. If an integral number 1 or more larger or smaller than 464 were used as the number of cycles, the derived approximate double period would be so far from its expected value as to indicate the error at once.

The interval between the first and second comparisons of pendulum and chronometer is therefore made rather short, usually from 5 to 8 minutes. The double period of the pendulum in chronometer seconds computed for this interval will be roughly correct. The succeeding comparisons of pendulum and chronometer are made at progressively longer intervals. The rough double period of the pendulum derived from the first interval of 5 to 8 minutes is used as a divisor to obtain the integral number of pendulum cycles in the next longer interval. When the longer interval is then divided by this number, a better value of the rough double period is obtained, which can be used as the divisor in the next longer interval, etc. (See fig. 22.) Finally a double period is obtained which is accurate enough, after correction for the rate of the chronometer on mean time, to use as the divisor for the total mean time interval between time signals (usually 6 hours and never more than 8 hours) to obtain the integral number of pendulum cycles. The uncorrected double period of the pendulum can then be obtained by dividing the total interval by this number. Changes in temperature, pressure, amplitude, and chronometer rate may make the double period over the long interval differ appreciably from that over the first short interval, regardless of inaccuracies in the determination of the latter.

As stated on page 37, it is possible to omit the use of the chronometer and to derive the number of oscillations of the pendulum by simply using the approximate period obtained during the time signals themselves as a divisor. This can only be done satisfactorily, however, when the signals are recorded on the chronograph sheet sharply and continuously for practically the entire 5 minutes of the signals and when an intermediate set of sharp, clear time signals is received after an interval of 1 or 2 hours. Since it is impossible at the beginning of the swing to predict that the intermediate time signals required for this method can be received satisfactorily, the practice of this Bureau has been to make the chronometer-pendulum comparisons in all cases in order to avoid any difficulty in deriving the correct "count."

Sample computations of the chronometer-pendulum comparisons are shown in figure 21 and of the computations of approximate double period (in terms of chronometer seconds) in figure 22. The computations required in figure 21 are readily apparent and need no additional explanation. In figure 22, the numbers in the first column indicate what chronometer-pendulum comparisons are used in each line of the form. For example, "1–2" indicates the interval between the first and second comparisons.

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[6] As stated on page 4, the positions of knife edge and plane were reversed in apparatus made in 1940, that is, the knife edges were placed in the heads of the pendulums. This changed the periods of these pendulums to some extent and they are now slightly faster than a sidereal chronometer instead of slower. The method of obtaining the approximate period of the pendulum, however, has not been essentially affected by this change.

(These are the comparisons shown in fig. 21.) In the fourth column, in the second, third and fourth lines, two decimal places are shown. These merely indicate the accuracy of the approximate period in the preceding line which was used as the divisor.

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Form 754

## Chronometer-Pendulum Comparison

**Brown Gravity Apparatus**

State Michigan Station Ludington Date Sept. 10, 1940
Chronometer 13885 Pendulum B8 ~~B. S. T.~~
Swing No. 1 Knife edge A2 Observer R.W.W.

| | Chronometer break | Chronometer reading C. R. | Chronometer interval C. I. | Pendulum reading | Mean of pendulum readings P. M. | P. M.−C. R. | Mean (P. M.−C. R.) / Mean (C. I.) |
|---|---|---|---|---|---|---|---|
| | h. m. s. | s. | s. | s. | s. | | |
| | 17 57 10 | 1 .100 | | | | | |
| | | | | 1 .496 | | | |
| | | | | 2 .976 | .736 | .636 | |
| | 11 | 2 .108 | 1.008 | | | | |
| | | | | 3 .502 | | | |
| (1) | | | | 4 .977 | .740 | .632 | |
| | 12 | 3 .106 | .998 | | | | |
| | | | | 5 .498 | | | .632/.998 = .633 |
| | | | | 6 .968 | .733 | .627 | |
| | 13 | 4 .093 | .987 | | | | |
| | Mean | * | .998 | | | .632 | |
| | Chronometer time of comparison= | | | 17 57 | 11.633 | | |
| | 18 04 55 | 1 .100 | | | | | |
| | | | | 1 .303 | | | |
| | | | | 2 .782 | .542 | .442 | |
| | 56 | 2 .099 | .999 | | | | |
| | | | | 3 .303 | | | |
| (2) | | | | 4 .781 | .542 | .443 | |
| | 57 | 3 .095 | .996 | | | | |
| | | | | 5 .300 | | | .442/.995 = .444 |
| | | | | 6 .774 | .537 | .442 | |
| | 58 | 4 .086 | .991 | | | | |
| | Mean | * | .995 | | | .442 | |
| | Chronometer time of comparison= | | | 18 04 | 56.444 | | |

" " " " = 17 57 11.633
Difference = 7 44.811
= 464.811 s.

*Omit No. 4 in taking mean of C. R.

FIGURE 21.—Chronometer-pendulum comparison.

The number is necessarily an integer, of course, and is so taken when the division is made to obtain the corrected approximate period in the last column, which should be progressively more accurate as the progressively longer intervals are used.

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Form 751

## Computation of Approximate Period

**Brown Gravity Apparatus**

State Michigan Station Ludington Date Sept. 10, 1940

Observer R. W. W.

| Epoch | ~~(Radio time)~~* (Chronometer time) | Double Period | Beats (Complete cycles) | double New period |
|---|---|---|---|---|
| | s. | s. | | s. |
| 1-2 | 464.811 | 1.001748 | 464 | |
| 1-3 | 1004.750 | (1.001748) | 1002.99 | 1.001745 |
| 1-4 | 2024.529 | (1.001745) | 2021.00 | 1.001746 |
| 1-5 | 3615.295 | (1.001746) | 3609.00 | 1.0017443 |
| 1-6 | | | | |
| 1-7 | | | | |
| 1-8 | | | | |
| 1-9 | | | | |
| 1-10 | | | | |

* Strike out one

FIGURE 22.—Computation of approximate period.

### RADIO-CHRONOMETER COMPARISON

The rate of the chronometer on mean time must be derived in order that the period of the pendulum in chronometer seconds may be corrected to the corresponding mean time period. A sample computation of the radio-chronometer comparisons is shown in figure 23. This computation shows that an interval of 3,623 chronometer seconds corresponds to 3,615.520 mean time seconds of the Naval Observatory clock used in transmitting the time signals. These data are used in the computation of the approximate period. (See fig. 25.) Any slight error in the transmitting clock of the Naval Observatory is applied in the computation shown in figure 25. The computations shown in figure 23 are similar to those shown in figure 21 and need no detailed explanation.

### RADIO-PENDULUM COMPARISON

The radio-pendulum comparison is, of course, the most important of the time comparisons made at a station, for upon it depends directly the accuracy of the derived period of the pendulum used in the computation of the value of gravity. Eleven radio "breaks" and the 20 corresponding pendulum "breaks" are scaled from the chronograph sheet. (See fig. 24.) As stated on page 45, the index point used for timing the pendulum is the extreme end of its oscillation as obtained by taking the mean of the instant the beam of light crosses the photo-electric cell in one direction and the instant it crosses the cell in the return direction.

The radio "breaks" as read from the chronograph sheet are recorded in the second column of the form shown in figure 24 and the pendulum readings in the fourth column. The mean of the pendulum readings 1 and 2 should come within the interval limited by radio readings 1 and 2. In some cases this requires that the first pendulum reading should be earlier than the first radio reading. For example, the radio readings 1 and 2 might be 0.243 and 0.236, respectively, and the pendulum readings 1 and 2 might be 0.096 and 0.578, respectively. Although pendulum "break" 1 would come ahead of radio "break" 1, the mean (0.337) of the two pendulum readings would be inside the interval limited by the two radio readings. An inspection of the chronograph sheet quickly indicates whether the first reading from the sheet should be a radio "break" or a pendulum "break" to accomplish the desired result.

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**Radio-Chronometer Comparison**

**Brown Gravity Apparatus**

State *Michigan* Station *Ludington* Date *Sept. 10, 1940.*

Chronometer No. *13885* Radio set *Nat. — RCE* Observer *R. W. W.*

Swing No. *1* E. S. T. of signal *11 a. m.* Signal frequency *9425*

| Radio break | Radio reading R. R. | Radio interval R. I. | Chronometer break | Chronometer reading C. R.* | C. R.−R. R. | Mean (C. R.−R. R.) / Mean (R. I.) |
|---|---|---|---|---|---|---|
| h. m. s. | | | h. m. s. | | | |
| 10 59 30 | 1 .101 | | | | | |
| | | | 17 56 37 | 1 .336 | .235 | |
| 31 | 2 .094 | .993 | | | | .230 / 1.000 = .230 |
| | | | 38 | 2 .323 | .229 | |
| 32 | 3 .087 | .993 | | | | |
| | | | 39 | 3 .317 | .230 | Comparison: h. m. s. |
| 33 | 4 .089 | 1.002 | | | | Chro.= 17 56 39 |
| | | | 40 | 4 .317 | .228 | Radio= 10 59 32.230 |
| 34 | 5 .097 | 1.008 | | | | |
| | | | 41 | 5 .324 | .227 | |
| 35 | 6 .102 | 1.005 | | | | |
| Mean | ** | 1.000 | | | .230 | |
| | | | E. S. T. of signal *Noon* | | | Signal frequency *9425* |
| 11 59 45 | 1 .100 | | | | | |
| | | | 18 57 03 | 1 .858 | .758 | |
| 46 | 2 .104 | 1.004 | | | | .751 / 1.001 = .750 |
| | | | 04 | 2 .867 | .763 | |
| 47 | 3 .118 | 1.014 | | | | |
| | | | 05 | 3 .872 | .754 | Comparison: h. m. s. |
| 48 | 4 .122 | 1.004 | | | | Chro.= 18 57 05 |
| | | | 06 | 4 .865 | .743 | Radio= 11 59 47.750 |
| 49 | 5 .115 | .993 | | | | Chro. dif.= 1 00 26 |
| | | | 07 | 5 .850 | .735 | = 3626 s. |
| 50 | 6 .106 | .991 | | | | Radio dif.= 1 00 15.520 |
| Mean | ** | 1.001 | | | .751 | = 3615.520 s. |

* Reading 1 in the fifth column should be later than reading 1 in the second column.

** Omit No. 6 in taking the mean of R. R.

FIGURE 23.—Radio-chronometer comparison.

The computations required in deriving the radio time of comparison are readily apparent in the sample form shown in figure 24 and need no explanation. The numbers shown in the lower half of the last column opposite the last 5 items in the sixth column are used to detect blunders in reading the chronograph sheet or in the computations. This method is used because the items in the sixth column do not vary smoothly due probably to slight bulges in the paper on the chronograph drum or to possible irregularities in the drum itself.

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Form 763

## Radio-Pendulum Comparison

**Brown Gravity Apparatus**

State *Michigan* Station *Ludington* Date *Sept. 10, 1940*
Pendulum *B8* Knife edge *A2* E. S. T. of signal *11 a.m.*
Swing No. *1* Signal frequency *9425* Observer *R.W.W.*

| Radio break | Radio reading R. R.* | Radio interval R. I. | Pendulum reading* | Mean of pendulum readings P. M. | P. M.−R. R. | Mean (P. M.−R. R.) / Mean (R. I.) |
|---|---|---|---|---|---|---|
| 10 59 25 | 1 .099 | | | | | |
| | | | 1 .679 | | | |
| | | | 2 1.151 | .915 | .816 | |
| 26 | 2 .094 | .995 | | | | |
| | | | 3 .669 | | | |
| | | | 4 1.143 | .906 | .812 | |
| 27 | 3 .086 | .992 | | | | |
| | | | 5 .667 | | | |
| | | | 6 1.143 | .905 | .819 | |
| 28 | 4 .086 | 1.000 | | | | |
| | | | 7 .672 | | | |
| | | | 8 1.152 | .912 | .826 | |
| 29 | 5 .096 | 1.010 | | | | .8159 / 1.0003 = .816 |
| | | | 9 .677 | | | |
| | | | 10 1.154 | .916 | .820 | |
| 30 | 6 .101 | 1.005 | | | | |
| | | | 11 .674 | | | |
| | | | 12 1.146 | .910 | .809 | −7 |
| 31 | 7 .094 | .993 | | | | |
| | | | 13 .665 | | | |
| | | | 14 1.140 | .902 | .808 | −4 |
| 32 | 8 .087 | .993 | | | | |
| | | | 15 .662 | | | |
| | | | 16 1.140 | .901 | .814 | −5 |
| 33 | 9 .089 | 1.002 | | | | |
| | | | 17 .668 | | | |
| | | | 18 1.150 | .909 | .820 | −6 |
| 34 | 10 .097 | 1.008 | | | | |
| | | | 19 .674 | | | |
| | | | 20 1.151 | .912 | .815 | −5 |
| 35 | 11 .102 | 1.005 | | | | |
| Mean | ** | 1.0003 | | | .8159 | |

Radio time of comparison= *10 59 30.816*

*Note:— The above radio time of comparison is assumed to be for the pendulum oscillation which follows the sixth radio break.*

* The mean of readings 1 and 2 in the sixth column should be later than reading 1 in the third column, and the mean of 3 and 4 later than 2, etc.

** Omit No. 11 in taking the mean of R. R.

FIGURE 24.—Radio-pendulum comparison.

The drum makes a revolution in 5 seconds and therefore the 10 items in the sixth column of figure 24 represent 2 complete revolutions of the drum. The first and sixth items in the sixth column and the second and seventh items, etc., are derived from readings from the same parts of the drum and should be equally affected by irregularities. The differences of these corresponding items, which are shown in the last column, should therefore vary only slightly. The size of these differences represents the approximate gain of the pendulum on mean time over an interval of 5 seconds.

## COMPUTATION OF PERIOD

The method of computing the accurate period of the pendulum in terms of absolute sidereal seconds is shown in the sample computations in figure 25. Two computations are shown, one for a partial swing of 1 hour, from 11 a. m. to noon, and the other for the complete swing of 6 hours, from 11 a. m. to 5 p. m. The first few lines of the computation are for the purpose of deriving the approximate mean time period of the pendulum. They are filled in only for the 1-hour swing, as one value of the approximate double period is all that is required, of course. The quantities given in the third and sixth lines come directly from the computation shown in figure 23, and the quantity in the fourth line comes from the computation shown in figure 22. The number of complete cycles of the pendulum given in the fifth line is derived by dividing the chronometer time interval in the third line by the chronometer double period in the fourth line. The approximate, mean-time, double period in the seventh line is found by dividing the quantity in the sixth line by that in the fifth.

The number of complete cycles of the pendulum between time signals can now be derived. The interval involved is obtained from the radio-pendulum comparisons at the beginning and end of the interval. The comparison shown in figure 24 is for the beginning of the interval. A slight correction must be applied to the interval to take account of errors in the time signals. When the interval is divided by the approximate, mean-time, double period which has already been derived, the number of complete cycles must come out very close to an integral number, as shown in line 11, if no error has been made. When the swing interval is divided by this integral number, the precise, mean-time, double period of the pendulum is obtained, uncorrected, however, for amplitude, pressure, etc.

The factor given in the footnote is then used to obtain the corresponding sidereal period of the pendulum. The reader may wonder why this conversion is made. It is a matter of convenience only. Many years ago the time control for pendulum observations had to be obtained from star observations, and was, therefore, on a sidereal basis. The various computational methods and tables were then of course worked out for sidereal time. In the more modern methods, it is so simple to convert the period from mean time to sidereal time that the tables have not been recomputed on a mean-time basis.

The various corrections which must be applied for pressure, temperature, etc., will be explained in detail in the following sections of this chapter.

### CORRECTION FOR ERRORS OF TIME SIGNALS

The radio time signals of the U. S. Naval Observatory, which are transmitted over the air by the Naval Communications Service, are used in the gravity determinations. They are very precise. It is necessary, however, to apply corrections for any inaccuracies in these signals in obtaining the precise period of the gravity pendulum. Practically all radio signals transmitted by the Naval Radio Station at Annapolis (NSS) are recorded back at the Naval Observatory on a precision chronograph and the errors of these signals are thus derived. The signals which are checked in this manner are known as scientific signals. At intervals of two or three weeks a list of corrections to the signals is sent by the Naval Observatory to all scientific users of the signals who request this service. The corrections are given to thousandths of a second

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**COMPUTATION OF PERIOD**
**BROWN GRAVITY APPARATUS**

State Michigan | Station Ludington | Date Sept. 10, 1940
Pend. B8 | Knife edge A2 | Observer R. W. W.
Brown receiver No. 2

| | | | | |
|---|---|---|---|---|
| Swing No., | 1a | 1 | | |
| Epoch, E. S. T., | 11 a.m. - 12 m. | 11 a.m. - 5 p.m. | | |
| Chron. time interval, | 3626 | | | |
| Chron. double period, | 1.0017443 | | | |
| No. of beats, (complete cycles) | 3619.686 | | | |
| Corresponding M. T. interval, | 3615.520 | | | |
| M. T. double period (approx.), | 0.9988491 | | | |
| Uncor. swing interval, | 3395.089 | 21399.292 | | |
| Corr. for signals, | .000 | +.001 | | |
| Swing interval, | 3395.089 | 21399.293 | | |
| No. of beats, (complete cycles) | 3399 | 21424 | | |
| Uncor. M. T. double period, | 0.99884937 | 0.99884676 | | |
| Uncor. sid. ~~half~~ period *, | 0.50079206 | 0.50079076 | | |
| Corr. for ~~arc,~~ amplitude | -280 | -174 | | |
| Corr. for pressure, | -136 | -136 | | |
| Corr. for temperature, | -462 | -431 | | |
| Corr. for flexure, | -86 | -86 | | |
| Corr. for dynamic temp. | -20 | -8 | | |
| Corrected period, | 0.50078222 | 0.50078241 | | |
| Arc readings, | 5.95<br>5.31 | 5.95<br>3.08 | | |
| Pressure readings and means, | 6.30 | 6.30 | | |
| Mean temperature reading, | 31.50 | 30.40 | | |
| Rate of temp. change ~~Thermometer correction,~~ | -1.00 | -0.40 | | |
| ~~Corrected temperature reading,~~ | | | | |

* Mean time double period times the factor 0.5013680535 = sidereal ~~half~~ period.

FIGURE 25.—Computation of period.

and seldom exceed 0.01 second, plus or minus. Without the benefit of this excellent time-signal service, precise gravity determinations with pendulum apparatus would be excessively difficult and expensive in this country.

## CORRECTION FOR AMPLITUDE

The correction to the period of the pendulum for its amplitude of oscillation, or what is sometimes called the arc correction, may be taken from the table below. The table was computed by means of Borda's formula [7] which may be stated as follows:

$$\text{Correction} = -\frac{PM}{32}\,\frac{\sin(\alpha_0+\alpha_n)\,\sin(\alpha_0-\alpha_n)}{\log_{10}\sin\alpha_0-\log_{10}\sin\alpha_n}$$

where $P$ is the period of the pendulum, $M$ is the modulus of the common logarithmic system (0.43429) and $\alpha_0$ and $\alpha_n$ are the initial and final semiamplitudes of the pendulum, respectively, for the interval under consideration. $\alpha_0$ and $\alpha_n$ are expressed as angles.

As explained on page 22 the amplitude of the pendulum in the Brown apparatus is measured by reading the distance traveled by the recording light beam on a convenient scale located near the photoelectric cell. By use of a factor, the corresponding movement in millimeters of the lower tip of the pendulum is then derived. The initial and final amplitudes of the pendulum as thus obtained are then used as arguments in the table below.

Since the amplitudes in Borda's formula, above, are given as angles, it was necessary in computing the table to express these angles in terms of the movement of the tip of the pendulum. The effective length of the pendulum, from the knife edge to the tip, is 296.93 mm. and the formula may be converted to the form:

$$\text{Correction} = -0.1924\,\frac{a_0^2-a_n^2}{\log_{10}a_0-\log_{10}a_n}$$

as expressed in units of the seventh decimal place of the period, where $a_0$ and $a_n$ are the initial and final amplitudes in millimeters. In this conversion $P$ is taken as ½, which is a sufficiently close approximation.

The amplitude correction table may be readily converted to the form of a nomogram. Such a nomogram is available at the Coast and Geodetic Survey. It is too large to be reproduced satisfactorily in this publication. It has the advantage over the table of being somewhat quicker to use and of giving the correction slightly more accurately.

*Amplitude correction*

[All corrections are negative, in units of $10^{-7}$ seconds]

| Total arc at beginning of swing in millimeters | Total arc at end of swing in millimeters | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 9.5 | 9.0 | 8.5 | 8.0 | 7.5 | 7.0 | 6.5 | 6.0 | 5.5 | 5.0 | 4.5 | 4.0 | 3.5 | 3.0 | 2.5 | 2.0 | 1.5 | 1.0 |
| 2.5 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | −3 | −3 |
| 2.6 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 4 | 3 |
| 2.7 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 4 | 3 |
| 2.8 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 4 | 3 |
| 2.9 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 4 | 3 |
| 3.0 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 4 | 3 |
| 3.1 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | −6 | 4 | 3 |
| 3.2 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 6 | 5 | 4 |
| 3.3 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 6 | 5 | 4 |
| 3.4 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | −8 | 6 | 5 | 4 |
| 3.5 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 8 | 7 | 5 | 4 |
| 3.6 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 8 | 7 | 5 | 4 |
| 3.7 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 8 | 7 | 6 | 4 |
| 3.8 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 9 | 7 | 6 | 4 |
| 3.9 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 9 | 7 | 6 | 5 |

[7] See Tome III of Base du Système métrique décimal, by Mechain and Delambre, page 354 (in chapter entitled Expériences by Borda and Cassini).

## *Amplitude correction*—Continued

[All corrections are negative, in units of $10^{-7}$ seconds]

| Total arc at beginning of swing in millimeters | Total arc at end of swing in millimeters | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 9.5 | 9.0 | 8.5 | 8.0 | 7.5 | 7.0 | 6.5 | 6.0 | 5.5 | 5.0 | 4.5 | 4.0 | 3.5 | 3.0 | 2.5 | 2.0 | 1.5 | 1.0 |
| 4.0 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | −11 | −9 | −8 | −6 | −5 |
| 4.1 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 11 | 9 | 8 | 6 | 5 |
| 4.2 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 11 | 10 | 8 | 7 | 5 |
| 4.3 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 12 | 10 | 8 | 7 | 5 |
| 4.4 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 12 | 10 | 9 | 7 | 5 |
| 4.5 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 12 | 11 | 9 | 7 | 6 |
| 4.6 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | −14 | 13 | 11 | 9 | 7 | 6 |
| 4.7 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 15 | 13 | 11 | 9 | 8 | 6 |
| 4.8 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | −17 | 15 | 13 | 11 | 10 | 8 | 6 |
| 4.9 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 17 | 15 | 14 | 12 | 10 | 8 | 6 |
| 5.0 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | −20 | 18 | 16 | 14 | 12 | 10 | 8 | 7 |
| 5.1 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 20 | 18 | 16 | 14 | 12 | 10 | 9 | 7 |
| 5.2 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 21 | 19 | 17 | 15 | 13 | 11 | 9 | 7 |
| 5.3 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 21 | 19 | 17 | 15 | 13 | 11 | 9 | 7 |
| 5.4 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 22 | 19 | 17 | 15 | 13 | 11 | 9 | 7 |
| 5.5 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | −24 | 22 | 20 | 18 | 16 | 13 | 11 | 10 | 8 |
| 5.6 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 25 | 23 | 20 | 18 | 16 | 14 | 12 | 10 | 8 |
| 5.7 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 25 | 23 | 21 | 18 | 16 | 14 | 12 | 10 | 8 |
| 5.8 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 26 | 23 | 21 | 19 | 17 | 14 | 12 | 10 | 8 |
| 5.9 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 26 | 24 | 21 | 19 | 17 | 15 | 13 | 11 | 8 |
| 6.0 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | −29 | 27 | 24 | 22 | 20 | 17 | 15 | 13 | 11 | 9 |
| 6.1 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 30 | 27 | 25 | 22 | 20 | 18 | 15 | 13 | 11 | 9 |
| 6.2 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 30 | 28 | 25 | 23 | 20 | 18 | 16 | 13 | 11 | 9 |
| 6.3 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 31 | 28 | 26 | 23 | 21 | 18 | 16 | 14 | 12 | 9 |
| 6.4 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 31 | 29 | 26 | 24 | 21 | 19 | 16 | 14 | 12 | 10 |
| 6.5 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | −35 | 32 | 29 | 27 | 24 | 21 | 19 | 17 | 14 | 12 | 10 |
| 6.6 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 35 | 32 | 30 | 27 | 24 | 22 | 19 | 17 | 15 | 12 | 10 |
| 6.7 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 36 | 33 | 30 | 27 | 25 | 22 | 20 | 17 | 15 | 13 | 10 |
| 6.8 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 36 | 33 | 31 | 28 | 25 | 23 | 20 | 18 | 15 | 13 | 10 |
| 6.9 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 37 | 34 | 31 | 28 | 26 | 23 | 21 | 18 | 16 | 13 | 11 |
| 7.0 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 37 | 34 | 32 | 29 | 26 | 23 | 21 | 18 | 16 | 13 | 11 |
| 7.1 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 38 | 35 | 32 | 29 | 27 | 24 | 21 | 19 | 16 | 14 | 11 |
| 7.2 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 38 | 36 | 33 | 30 | 27 | 24 | 22 | 19 | 17 | 14 | 11 |
| 7.3 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 39 | 36 | 33 | 30 | 27 | 25 | 22 | 19 | 17 | 14 | 12 |
| 7.4 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | 40 | 37 | 34 | 31 | 28 | 25 | 22 | 20 | 17 | 15 | 12 |
| 7.5 | ----- | ----- | ----- | ----- | ----- | ----- | −43 | 40 | 37 | 34 | 31 | 28 | 26 | 23 | 20 | 18 | 15 | 12 |
| 7.6 | ----- | ----- | ----- | ----- | ----- | ----- | 44 | 41 | 38 | 35 | 32 | 29 | 26 | 23 | 21 | 18 | 15 | 12 |
| 7.7 | ----- | ----- | ----- | ----- | ----- | ----- | 45 | 41 | 38 | 35 | 32 | 29 | 26 | 24 | 21 | 18 | 15 | 13 |
| 7.8 | ----- | ----- | ----- | ----- | ----- | ----- | 45 | 42 | 39 | 36 | 33 | 30 | 27 | 24 | 21 | 19 | 16 | 13 |
| 7.9 | ----- | ----- | ----- | ----- | ----- | ----- | 46 | 43 | 39 | 36 | 33 | 30 | 27 | 24 | 22 | 19 | 16 | 13 |
| 8.0 | ----- | ----- | ----- | ----- | ----- | −50 | 46 | 43 | 40 | 37 | 34 | 31 | 28 | 25 | 22 | 19 | 16 | 13 |
| 8.1 | ----- | ----- | ----- | ----- | ----- | 50 | 47 | 44 | 40 | 37 | 34 | 31 | 28 | 25 | 22 | 20 | 17 | 14 |
| 8.2 | ----- | ----- | ----- | ----- | ----- | 51 | 48 | 44 | 41 | 38 | 35 | 32 | 29 | 26 | 23 | 20 | 17 | 14 |
| 8.3 | ----- | ----- | ----- | ----- | ----- | 52 | 48 | 45 | 42 | 38 | 35 | 32 | 29 | 26 | 23 | 20 | 17 | 14 |
| 8.4 | ----- | ----- | ----- | ----- | ----- | 52 | 49 | 46 | 42 | 39 | 36 | 33 | 30 | 26 | 24 | 21 | 18 | 14 |
| 8.5 | ----- | ----- | ----- | ----- | −57 | 53 | 50 | 46 | 43 | 39 | 36 | 33 | 30 | 27 | 24 | 21 | 18 | 15 |
| 8.6 | ----- | ----- | ----- | ----- | 57 | 54 | 50 | 47 | 43 | 40 | 37 | 34 | 30 | 27 | 24 | 21 | 18 | 15 |
| 8.7 | ----- | ----- | ----- | ----- | 58 | 54 | 51 | 47 | 44 | 41 | 37 | 34 | 31 | 28 | 25 | 22 | 19 | 15 |
| 8.8 | ----- | ----- | ----- | ----- | 59 | 55 | 51 | 48 | 44 | 41 | 38 | 35 | 31 | 28 | 25 | 22 | 19 | 16 |
| 8.9 | ----- | ----- | ----- | ----- | 59 | 56 | 52 | 49 | 45 | 42 | 38 | 35 | 32 | 29 | 25 | 22 | 19 | 16 |
| 9.0 | ----- | ----- | ----- | −64 | 60 | 56 | 53 | 49 | 46 | 42 | 39 | 36 | 32 | 29 | 26 | 23 | 19 | 16 |
| 9.1 | ----- | ----- | ----- | 65 | 61 | 57 | 53 | 50 | 46 | 43 | 39 | 36 | 33 | 29 | 26 | 23 | 20 | 16 |
| 9.2 | ----- | ----- | ----- | 65 | 62 | 58 | 54 | 50 | 47 | 43 | 40 | 37 | 33 | 30 | 27 | 23 | 20 | 17 |
| 9.3 | ----- | ----- | ----- | 66 | 62 | 58 | 55 | 51 | 47 | 44 | 40 | 37 | 34 | 30 | 27 | 24 | 20 | 17 |
| 9.4 | ----- | ----- | ----- | 67 | 63 | 59 | 55 | 52 | 48 | 44 | 41 | 38 | 34 | 31 | 27 | 24 | 21 | 17 |
| 9.5 | ----- | ----- | −72 | 68 | 64 | 60 | 56 | 52 | 49 | 45 | 42 | 38 | 35 | 31 | 28 | 25 | 21 | 18 |
| 9.6 | ----- | ----- | 72 | 68 | 64 | 61 | 57 | 53 | 49 | 46 | 42 | 39 | 35 | 32 | 28 | 25 | 21 | 18 |
| 9.7 | ----- | ----- | 73 | 69 | 65 | 61 | 57 | 54 | 50 | 46 | 43 | 39 | 36 | 32 | 29 | 25 | 22 | 18 |
| 9.8 | ----- | ----- | 74 | 70 | 66 | 62 | 58 | 54 | 50 | 47 | 43 | 40 | 36 | 33 | 29 | 26 | 22 | 18 |
| 9.9 | ----- | ----- | 75 | 71 | 67 | 63 | 59 | 55 | 51 | 47 | 44 | 40 | 37 | 33 | 30 | 26 | 22 | 19 |
| 10.0 | ----- | −80 | 76 | 71 | 67 | 63 | 59 | 56 | 52 | 48 | 44 | 41 | 37 | 33 | 30 | 26 | 23 | 19 |
| 10.1 | ----- | 81 | 76 | 72 | 68 | 64 | 60 | 56 | 52 | 49 | 45 | 41 | 38 | 34 | 30 | 27 | 23 | 19 |
| 10.2 | ----- | 82 | 77 | 73 | 69 | 65 | 61 | 57 | 53 | 49 | 45 | 42 | 38 | 34 | 31 | 27 | 24 | 20 |
| 10.3 | ----- | 82 | 78 | 74 | 70 | 65 | 61 | 57 | 54 | 50 | 46 | 42 | 39 | 35 | 31 | 28 | 24 | 20 |
| 10.4 | ----- | 83 | 79 | 75 | 70 | 66 | 62 | 58 | 54 | 50 | 46 | 43 | 39 | 35 | 32 | 28 | 24 | 20 |
| 10.5 | −89 | 84 | 80 | 75 | 71 | 67 | 63 | 59 | 55 | 51 | 47 | 43 | 40 | 36 | 32 | 28 | 25 | 21 |
| 10.6 | 89 | 85 | 80 | 76 | 72 | 68 | 64 | 59 | 55 | 52 | 48 | 44 | 40 | 36 | 33 | 29 | 25 | 21 |
| 10.7 | 90 | 86 | 81 | 77 | 73 | 68 | 64 | 60 | 56 | 52 | 48 | 44 | 41 | 37 | 33 | 29 | 25 | 21 |
| 10.8 | 91 | 87 | 82 | 78 | 73 | 69 | 65 | 61 | 57 | 53 | 49 | 45 | 41 | 37 | 33 | 30 | 26 | 22 |
| 10.9 | 92 | 87 | 83 | 78 | 74 | 70 | 66 | 61 | 57 | 53 | 49 | 45 | 42 | 38 | 34 | 30 | 26 | 22 |
| 11.0 | 93 | 88 | 84 | 79 | 75 | 71 | 66 | 62 | 58 | 54 | 50 | 46 | 42 | 38 | 34 | 30 | 26 | 22 |

## CORRECTION FOR PRESSURE

Any air in the vacuum chamber retards the pendulum oscillations and thus lengthens the period of the pendulum. This retardation is due principally to the buoyancy and the viscosity of the air. G. G. Stokes has shown that the effect of the air on the period of the pendulum may be expressed by a formula of the form

$$E=k_1\rho+k_2\sqrt{\rho}$$

where $E$ is the effect of the air density on the period, $\rho$ is the air density, and $k_1$ and $k_2$ are experimental constants.[8]

The density of air, $\rho$, can be expressed by the standard formula

$$\rho=\frac{p-0.378e}{760(1+0.00367t)}$$

where $p$ is pressure in millimeters of mercury, $e$ is the pressure of water vapor, and $t$ is the temperature in °C.

Since all gravity observations are made at low pressures, however, it can be safely assumed that the vapor pressure effect is negligible. Hence the term $0.378e$ can be neglected in determining the coefficients $k_1$ and $k_2$ for the low pressures used in the Brown apparatus. The formula for the density of air thus reduces to:

$$\rho=\frac{p}{760(1+0.00367t)}$$

and the correction to the period of the pendulum for air density becomes

$$\text{cor.}=-k_1\frac{p}{760(1+0.00367t)}-k_2\frac{\sqrt{p}}{\sqrt{760}\sqrt{1+0.00367t}}$$

or,

$$\text{cor.}=k_1'\frac{p}{1+0.00367t}+k_2'\frac{\sqrt{p}}{\sqrt{1+0.00367t}}$$

where $k_1'=-\frac{k_1}{760}$

and $k_2'=-\frac{k_2}{\sqrt{760}}$

These constants, $k_1'$ and $k_2'$, were determined in 1938 by A. J. Hoskinson for invar pendulum B9 by swinging it in Brown case No. 1, at various pressures between 1 and 66 mm. of mercury. The resulting experimental constants are:

$$k_1'=-7.375\times10^{-8}$$
$$k_2'=-39.56\times10^{-8}$$

On the basis of the above formula, and using the experimental constants $k_1'$ and $k_2'$, thus determined, the air density or pressure correction table on page 56 was computed. This table may be used for any one of the invar pendulums, A7, A8, A9, B7, B8, and B9 when swung in any of the present Brown cases. All of these invar pendulums are identical in shape and made from the same batch of invar.

[8] G. G. Stokes: Mathematical and Physical Papers, Vol. III, page 141.

*Pressure corrections, invar pendulums*

[All corrections are negative, in units of $10^{-8}$ seconds, and reduce period to zero pressure]

| Pressure (millimeters of mercury) | Temperature in °C. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0° | 5° | 10° | 15° | 20° | 25° | 30° | 35° | 40° |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.10 | −13 | −13 | −13 | −13 | −13 | −13 | −13 | −12 | −12 |
| 0.25 | 22 | 21 | 21 | 21 | 21 | 21 | 20 | 20 | 20 |
| 0.50 | 32 | 31 | 31 | 31 | 30 | 30 | 30 | 30 | 29 |
| 0.75 | 40 | 39 | 39 | 39 | 38 | 38 | 37 | 37 | 37 |
| 1.00 | 47 | 46 | 46 | 46 | 45 | 45 | 44 | 44 | 43 |
| 1.5 | 59 | 59 | 58 | 57 | 57 | 56 | 56 | 55 | 55 |
| 2 | 71 | 70 | 69 | 68 | 68 | 67 | 66 | 66 | 65 |
| 3 | 91 | 90 | 89 | 88 | 87 | 86 | 85 | 84 | 83 |
| 4 | 109 | 107 | 106 | 105 | 104 | 103 | 102 | 101 | 100 |
| 5 | 125 | 124 | 123 | 121 | 120 | 119 | 117 | 116 | 115 |
| 6 | 141 | 139 | 138 | 136 | 135 | 134 | 132 | 130 | 129 |
| 7 | 156 | 154 | 153 | 151 | 149 | 148 | 146 | 144 | 143 |
| 8 | 171 | 169 | 167 | 165 | 163 | 161 | 159 | 158 | 156 |
| 9 | 185 | 183 | 181 | 178 | 176 | 174 | 172 | 171 | 169 |
| 10 | 199 | 197 | 194 | 192 | 189 | 187 | 185 | 183 | 181 |
| 11 | 212 | 210 | 207 | 205 | 202 | 200 | 198 | 195 | 193 |
| 12 | 225 | 223 | 220 | 218 | 215 | 212 | 210 | 207 | 205 |
| 13 | 239 | 236 | 233 | 230 | 227 | 224 | 222 | 219 | 217 |
| 14 | 251 | 248 | 245 | 242 | 239 | 236 | 233 | 231 | 228 |
| 15 | 264 | 260 | 257 | 254 | 251 | 248 | 245 | 242 | 239 |
| 16 | 276 | 273 | 269 | 266 | 263 | 260 | 256 | 254 | 251 |
| 17 | 288 | 285 | 281 | 277 | 274 | 271 | 268 | 265 | 262 |
| 18 | 300 | 297 | 293 | 289 | 286 | 282 | 279 | 276 | 272 |
| 19 | 313 | 309 | 305 | 301 | 297 | 293 | 290 | 287 | 283 |
| 20 | 324 | 320 | 316 | 312 | 308 | 304 | 301 | 297 | 294 |
| 21 | 336 | 332 | 327 | 323 | 319 | 315 | 311 | 308 | 304 |
| 22 | 348 | 343 | 339 | 334 | 330 | 326 | 322 | 318 | 315 |
| 23 | 359 | 355 | 350 | 345 | 341 | 337 | 333 | 329 | 325 |
| 24 | 371 | 366 | 361 | 356 | 352 | 348 | 343 | 339 | 335 |
| 25 | 382 | 377 | 372 | 367 | 363 | 358 | 354 | 350 | 345 |
| 26 | 393 | 388 | 383 | 378 | 373 | 369 | 364 | 360 | 356 |
| 27 | 405 | 399 | 394 | 389 | 384 | 379 | 375 | 370 | 366 |
| 28 | 416 | 410 | 405 | 399 | 394 | 389 | 385 | 380 | 376 |
| 29 | 427 | 421 | 416 | 410 | 405 | 400 | 395 | 390 | 385 |
| 30 | 438 | 432 | 426 | 421 | 415 | 410 | 405 | 400 | 395 |
| 31 | 449 | 443 | 437 | 431 | 426 | 420 | 415 | 410 | 405 |
| 32 | 460 | 454 | 448 | 442 | 436 | 430 | 425 | 420 | 415 |
| 33 | 470 | 464 | 458 | 452 | 446 | 440 | 435 | 429 | 424 |
| 34 | 481 | 475 | 468 | 462 | 456 | 450 | 445 | 439 | 434 |
| 35 | 492 | 486 | 479 | 473 | 467 | 461 | 455 | 449 | 444 |

## CORRECTION FOR TEMPERATURE AND FOR RATE OF CHANGE OF TEMPERATURE

The temperature correction, even when invar pendulums are used, may be relatively large and it should be determined with the greatest possible accuracy. The standard temperature to which the period of the pendulum is reduced is 15° C., and the correction is computed as follows:

$$\text{Temperature correction} = K\,(15° - T)$$

where $T$ is the temperature of the pendulum in degrees Centigrade and $K$ is a constant depending upon the material of the pendulum. The value of $K$ has been determined experimentally by swinging the pendulums at various temperatures, with other conditions maintained as nearly constant as possible and with corrections applied for all variations except that of the temperature. For invar pendulums B7, B8, and B9,

$$K = 0.000\ 00028$$

It has been assumed that invar pendulums A7, A8, and A9, since they were made from the same batch of invar, would have the same temperature constant as the B pendulums. More recent tests, which included the A pendulums but which, unfortunately, were not completed, seemed to indicate a slightly higher constant than that given above for the B pendulums. A more rigid determination of the temperature constants will be made as soon as the necessary time can be devoted to it.

For the bronze pendulums which were used in the Mendenhall apparatus the temperature constants are as follows:

$K$=0.000 00413 for pendulum A4
=0.000 00418 for pendulums A5, A6, and B5
=0.000 00419 for pendulum B4
=0.000 00415 for pendulum B6

The thermometer used for obtaining the temperature of the pendulum should be precisely graduated and should be standardized at the National Bureau of Standards. Corrections for any inaccuracies of graduation must, of course, be applied to the temperatures as read before the temperature corrections are computed.

In addition to the temperature correction described above, a small supplementary correction is required to eliminate the effects of changes in the temperature of the pendulum. This is sometimes called the dynamic temperature correction.[9] The necessity for this correction is probably due to the fact that the temperature of the pendulum, or more particularly that of the stem of the pendulum, differs from the temperature shown by the thermometer during fairly rapid changes of temperature. The pendulum seems to be affected by changes much more slowly than is the thermometer. This may be caused by the heavy bob serving as a heat reservoir.

Several years ago a study was made of the field observations which indicated that when the temperature was falling the period of the pendulum, after the ordinary temperature correction had been applied, tended to be greater than when the temperature was rising. A series of tests was therefore made at the Washington base station to determine the amount of this effect. The results of these tests were very carefully analyzed by J. A. Duerksen and C. A. Whitten and a report was prepared by them which is on file in manuscript form at the Coast and Geodetic Survey. The value adopted as a result of this study is as follows for invar pendulums A7, A8, A9, B7, B8, and B9:

Dynamic temperature correction= 0.000 00020 times the hourly increase in temperature in ° C.

The sign of this correction is minus when the temperature is decreasing.

It must be remembered that the dynamic temperature correction is very definitely related to the type of pendulum apparatus used. The mounting of the thermometer and the shape of the pendulum are probably the two principal factors involved. In another type of apparatus the factor for the dynamic temperature correction might be quite different from that given above, and it might even be of the opposite sign.

## CORRECTION FOR FLEXURE

The correction for flexure is fully explained on page 38 in connection with the instructions for its determination. The factor used in computing the correction is

[9] Early references to this correction will be found in the following:

K. R. Koch, Relative Schweremessungen, III, Veröffentlichung der Kgl. Württembergischen Kommission für die internationale Erdmessung. Stuttgart, 1903, pp. 8, 9, 20, and 21.

G. P. Lenox-Conyngham, Extracts from Narrative Reports of Officers of the Survey of India, Season of 1904–05, Calcutta, 1907, pp. 75–77.

0.0000173 as determined experimentally. For a standard amplitude of 5 mm., the movement of the pendulum support, in terms of the wave length of helium light, times the factor 0.0000173 gives the correction to the period.

## COMPUTATION OF GRAVITY VALUE

After the final corrected period of the pendulum has been derived as explained in the preceding pages, the corresponding value of gravity is readily computed by either one of the simple formulas given on page 44. The second one is the more convenient one to use. It is repeated here for convenience of reference.

$$g_s=g_w-\frac{2g_w}{P_w}(P_s-P_w)+\frac{3g_w}{P_w^2}(P_s-P_w)^2-\ \ldots$$

$g$ is the value of gravity in gals and $P$ is the period of the pendulum in seconds, the subscripts $s$ and $w$ indicating the field station and the Washington base station, respectively.

Since $g_w=980.118$ (Commerce Building base station), the principal correction term is $-\frac{1960.236}{P_w}(P_s-P_w)$.

$P_w$ is the mean of the preseason and postseason standardization periods of the pendulum, or the adopted value of the Washington period. The second correction term in the above formula is small. It can be taken directly from the following table.

*Correction term for computing $g_s$*

[Table is computed for $P_w$ equal to 0ˢ.5005. The corrections taken from the table are accurate to 0.05 milligal for all pendulums the periods of which do not differ more than 0ˢ.0010 from 0ˢ.5005]

| Approximate gravity $g_s$ | $\frac{3g_w}{P_w^2}(P_s-P_w)^2$ | Approximate gravity $g_s$ | $\frac{3g_w}{P_w^2}(P_s-P_w)^2$ | Approximate gravity $g_s$ | $\frac{3g_w}{P_w^2}(P_s-P_w)^2$ | Approximate gravity $g_s$ | $\frac{3g_w}{P_w^2}(P_s-P_w)^2$ |
|---|---|---|---|---|---|---|---|
| *Gals* | *Mgals* | *Gals* | *Mgals* | *Gals* | *Mgals* | *Gals* | *Mgals* |
| 982.0 | +2.71 | 980.5 | +.11 | 979.0 | +.96 | 977.5 | +5.24 |
| 981.9 | +2.43 | 980.4 | +.06 | 978.9 | +1.13 | 977.4 | +5.65 |
| 981.8 | +2.16 | 980.3 | +.03 | 978.8 | +1.33 | 977.3 | +6.08 |
| 981.7 | +1.91 | 980.2 | .00 | 978.7 | +1.54 | 977.2 | +6.51 |
| 981.6 | +1.68 | 980.1 | .00 | 978.6 | +1.76 | 977.1 | +6.97 |
| 981.5 | +1.46 | 980.0 | +.01 | 978.5 | +2.00 | 977.0 | +7.44 |
| 981.4 | +1.26 | 979.9 | +.04 | 978.4 | +2.26 | | |
| 981.3 | +1.07 | 979.8 | +.08 | 978.3 | +2.53 | | |
| 981.2 | +.90 | 979.7 | +.14 | 978.2 | +2.82 | | |
| 981.1 | +.74 | 979.6 | +.20 | 978.1 | +3.12 | | |
| 981.0 | +.59 | 979.5 | +.29 | 978.0 | +3.44 | | |
| 980.9 | +.47 | 979.4 | +.39 | 977.9 | +3.76 | | |
| 980.8 | +.36 | 979.3 | +.51 | 977.8 | +4.11 | | |
| 980.7 | +.26 | 979.2 | +.64 | 977.7 | +4.47 | | |
| 980.6 | +.18 | 979.1 | +.79 | 977.6 | +4.85 | | |

In order that the preliminary values of gravity at the field stations may be computed immediately after the observations are completed and before the postseason standardization has been made, the value of $P_w$ derived from the preseason standardization is used for the preliminary computations. The final values of gravity are later obtained by applying a small correction for the difference between the preseason standardization value and the final adopted value of $P_w$.

A sample gravity computation is shown in figure 26. In the upper half of the form, the computation is for apparatus No. 2 and in the lower half, for apparatus No. 3. The adopted gravity at the bottom of the form is the mean of the values obtained with the two sets of apparatus. For the sake of clearness, the details of the computation are shown for apparatus No. 2.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 638a

## GRAVITY COMPUTATION

### BROWN GRAVITY APPARATUS

*Station No.* 1089 *Name* Ludington *Chief of party* R. W. W.

*State* Mich. *Washington base value*=980.118. *Observer* R. W. W.

Apparatus 2 Pendulum B8 Knife edge A2 Wash. period: 0.50 086837

| Number of Swing | Date | Epoch, E. S. T. | Period, P, | Gravity | v | v² |
|---|---|---|---|---|---|---|
| | 1940 | | sec. | gals. | | |
| 1 | Sept. 10 | 11 a.m. – 5 p.m. | 0.50078241 | 980.4545 | | |

Details of computation

$\frac{2g_W}{P_W} = \frac{1960.236}{0.50086837} = 3913.68$

$P_S - P_W = -0.00008596$

$-\frac{2g_W}{P_W}(P_S - P_W) = 0.33642$

Second cor. term $= 0.00005$

$g_W = 980.118$

$g_S = 980.4545$

$P_W$ (prelim.) $= 0.50086837$

$P_W$ (adopt.) $= 0.50086844$

Difference $= 0.00000007$

Dif. $\times$ 3913.67 $= 0.0003$

| | |
|---|---|
| Mean gravity | |
| (Adopted $P_W = 0.50086844$) Standardization cor. | +0.0003 |
| Corrected gravity | 980.4548 |

Apparatus 3 Pendulum A8 Knife edge B1 Wash. period: 0.50 101173

| Number of Swing | Date | Epoch, E. S. T. | Period, P, | Gravity | v | v² |
|---|---|---|---|---|---|---|
| 1 | Sept. 10 | 11 a.m. – 5 p.m. | 0.50092599 | 980.4535 | | |
| $\frac{2g_W}{P_W}$ | = 3912.56 | | | | | |

| | |
|---|---|
| Mean gravity | |
| (Adopted $P_W = 0.50101241$) Standardization cor. | + 0.0027 |
| Corrected gravity | 980.4562 |
| Adopted gravity | 980.456 |

FIGURE 26.—Gravity computation.

# Chapter 4. TOPOGRAPHIC AND ISOSTATIC COMPUTATIONS AND REDUCTIONS

## INTRODUCTION

In analyzing its gravity data, the Coast and Geodetic Survey compares each observed value of gravity with theoretical values computed for the same point by several different methods and in this way obtains what are known as gravity anomalies. The ideal method of computation, of course, would be one for which the anomalies would all be zero, that is, for which the theoretical values would be identical with the observed values. If such a method were possible it would not be necessary to measure gravity although even then it might be found more economical to measure it than to use the very complicated method which would necessarily be required to compute gravity so precisely.

Investigations by Airy, Pratt, Hayford, Bowie, and others have indicated quite definitely that to obtain a reasonably accurate theoretical value of gravity, consideration must be given to the now familiar principle of isostasy which holds that the outer part of the earth adjusts itself to certain conditions of equilibrium of masses. The methods developed by Hayford and Bowie for evaluating the effects of isostasy, as well as topographic effects, have been used for many years in this and other countries, and many modified methods, more recently developed, have been based on their fundamental investigations and tables.

The Hayford-Bowie method for making isostatic reductions of gravity data is explained in detail in Special Publication No. 10 of the U. S. Coast and Geodetic Survey entitled "The effect of gravity and isostatic compensation upon the intensity of gravity." Fundamental formulas and tables are given in that publication together with complete explanations of various steps in the computations. They are not repeated here. A number of changes have been made from time to time, however, in the methods given in Special Publication No. 10. These modifications of methods will be explained briefly in the following pages, and various auxiliary tables will be included which are of assistance in expediting the computations.

Notwithstanding the various aids and short cuts that have been worked out in the past few years, the isostatic reduction of a gravity station is still a rather tedious and time-consuming task. Where the topography around the station is not too rugged and where there are other stations nearby that have already been computed, interpolation can be used in place of actual map readings for a large proportion of the distant Hayford zones. Several of the zones near the station, however, must always have their elevations determined by map readings and this takes time. Notwithstanding the effort required, it is believed that the isostatic reduction of gravity data is unquestionably worthwhile especially until an adequate distribution of stations, thus computed, has been obtained over an area. The spacing of stations to provide a good distribution cannot be definitely stated but will depend upon local conditions, especially the ruggedness of the topography, and upon the rate of variation of the anomalies.

## INTERNATIONAL GRAVITY FORMULA

At any point on the earth, the intensity of gravity depends upon the latitude and elevation of the point, upon the distribution of topographic masses with relation to the given point and upon the distribution of isostatic variations of density in the so-called crust of the earth, that is, in the outer part of the earth to a depth of 50 to 100 kilometers.

Many formulas have been derived to give the variation of gravity with latitude along a sea-level surface. Some of the more important of these are the Helmert, Hayford, Bowie, Heiskanen, and International gravity formulas. They are all of the same general form except that Helmert and Heiskanen each derived formulas containing a small longitude term in addition to the latitude terms. Not all geodesists are agreed on the validity of a longitude term and such a term is not included in the International formula which is the one in general use at the present time in most countries.

The International formula was adopted by the International Geodetic Association at its Stockholm meeting in 1930. It is as follows:

$$\gamma_0=978.0490(1+0.0052884 \sin^2 \phi-0.0000059 \sin^2 2\phi) \text{ gals}$$

in which $\gamma_0$ is the theoretical value at sea level and $\phi$ is the latitude of the point. A table of theoretical values of gravity at sea level based on the above formula is given in the following table.

The values in the following table differ slightly from those given in the table published by Lambert and Darling in *Bulletin géodésique* No. 32 (October–November–December 1931, pp. 327–340). The latter table was intended to represent gravity on the International Ellipsoid, which has a flattening of 1/297 exactly, and on which gravity at the Equator was taken as 978.049 gals exactly. Gravity on an exact ellipsoid may be expressed in closed form, but for the present purpose it is most conveniently stated in terms of an infinite series

$$g=978.049[1+\beta \sin^2 \phi-\beta_1 \sin^2 2\phi -\beta_2 \sin^2 \phi \sin^2 2\phi-\beta_3 \sin^4 \phi \sin^2 2\phi- \ . \ . \ .]$$

The conditions imposed give to eight decimal places

$$\beta=0.0052\ 8838$$
$$\beta_1=0.0000\ 0587$$
$$\beta_2=0.0000\ 0003$$

When the International Formula was adopted these numbers were rounded off to seven decimals. The term in $\beta_2$ and all subsequent terms dropped out and the formula became

$$g=978.049[1+0.0052884 \sin^2 \phi-0.0000059 \sin^2 2\phi]$$

This is the *international gravity formula* according to the understanding of those who adopted it. The table in this publication has been computed from it and the values here given, therefore, differ systematically from those of Lambert and Darling by a unit or two in the fifth decimal of a gal over a large part of the table.

Various corrections must be applied to the theoretical value at sea level, as computed by the International or similar formula, to obtain the theoretical value which approximates as closely as possible the true value of gravity at the given station or location. The manner of computing these corrections is explained in the following pages.

*Theoretical gravity, International Formula*

| Latitude φ | Gravity | Latitude φ | Gravity | Latitude φ | Gravity | Latitude φ | Gravity | Latitude φ | Gravity |
|---|---|---|---|---|---|---|---|---|---|
| ° ′ | *Gals* 978. | ° ′ | *Gals* 978. | ° ′ | *Gals* 978. | ° ′ | *Gals* 979. | ° ′ | *Gals* 980. |
| 0 0 | 04900 | 10 0 | 20429 | 20 0 | 65166 | 30 0 | 33775 | 40 0 | 18048 |
| 10 | 04904 | 10 | 20945 | 10 | 66133 | 10 | 35077 | 10 | 19529 |
| 20 | 04917 | 20 | 21470 | 20 | 67107 | 20 | 36384 | 20 | 21012 |
| 30 | 04939 | 30 | 22003 | 30 | 68088 | 30 | 37695 | 30 | 22496 |
| 40 | 04970 | 40 | 22544 | 40 | 69075 | 40 | 39010 | 40 | 23982 |
| 50 | 05009 | 50 | 23093 | 50 | 70068 | 50 | 40330 | 50 | 25469 |
| 1 0 | 05057 | 11 0 | 23650 | 21 0 | 71068 | 31 0 | 41653 | 41 0 | 26957 |
| 10 | 05113 | 10 | 24216 | 10 | 72075 | 10 | 42981 | 10 | 28447 |
| 20 | 05179 | 20 | 24789 | 20 | 73088 | 20 | 44313 | 20 | 29938 |
| 30 | 05253 | 30 | 25371 | 30 | 74108 | 30 | 45649 | 30 | 31430 |
| 40 | 05336 | 40 | 25960 | 40 | 75134 | 40 | 46988 | 40 | 32923 |
| 50 | 05427 | 50 | 26558 | 50 | 76166 | 50 | 48332 | 50 | 34417 |
| 2 0 | 05527 | 12 0 | 27163 | 22 0 | 77205 | 32 0 | 49680 | 42 0 | 35912 |
| 10 | 05636 | 10 | 27776 | 10 | 78250 | 10 | 51031 | 10 | 37408 |
| 20 | 05754 | 20 | 28398 | 20 | 79301 | 20 | 52387 | 20 | 38905 |
| 30 | 05880 | 30 | 29027 | 30 | 80358 | 30 | 53746 | 30 | 40403 |
| 40 | 06015 | 40 | 29664 | 40 | 81422 | 40 | 55109 | 40 | 41902 |
| 50 | 06158 | 50 | 30310 | 50 | 82492 | 50 | 56475 | 50 | 43401 |
| 3 0 | 06310 | 13 0 | 30963 | 23 0 | 83568 | 33 0 | 57846 | 43 0 | 44901 |
| 10 | 06471 | 10 | 31623 | 10 | 84650 | 10 | 59219 | 10 | 46402 |
| 20 | 06641 | 20 | 32292 | 20 | 85738 | 20 | 60597 | 20 | 47903 |
| 30 | 06819 | 30 | 32968 | 30 | 86832 | 30 | 61978 | 30 | 49405 |
| 40 | 07006 | 40 | 33653 | 40 | 87932 | 40 | 63362 | 40 | 50908 |
| 50 | 07202 | 50 | 34345 | 50 | 89038 | 50 | 64749 | 50 | 52411 |
| 4 0 | 07406 | 14 0 | 35044 | 24 0 | 90149 | 34 0 | 66140 | 44 0 | 53914 |
| 10 | 07618 | 10 | 35752 | 10 | 91267 | 10 | 67535 | 10 | 55417 |
| 20 | 07840 | 20 | 36467 | 20 | 92391 | 20 | 68932 | 20 | 56921 |
| 30 | 08070 | 30 | 37190 | 30 | 93520 | 30 | 70333 | 30 | 58425 |
| 40 | 08308 | 40 | 37920 | 40 | 94655 | 40 | 71737 | 40 | 59930 |
| 50 | 08556 | 50 | 38658 | 50 | 95796 | 50 | 73144 | 50 | 61434 |
| 5 0 | 08812 | 15 0 | 39404 | 25 0 | 96942 | 35 0 | 74554 | 45 0 | 62939 |
| 10 | 09076 | 10 | 40157 | 10 | 98094 | 10 | 75968 | 10 | 64443 |
| 20 | 09349 | 20 | 40918 | 20 | 99252 | 20 | 77384 | 20 | 65948 |
| 30 | 09630 | 30 | 41686 | 30 | ▲0415 | 30 | 78803 | 30 | 67452 |
| 40 | 09921 | 40 | 42461 | 40 | 01584 | 40 | 80225 | 40 | 68957 |
| 50 | 10219 | 50 | 43245 | 50 | 02758 | 50 | 81650 | 50 | 70461 |
| 6 0 | 10526 | 16 0 | 44035 | 26 0 | 03938 | 36 0 | 83077 | 46 0 | 71965 |
| 10 | 10842 | 10 | 44833 | 10 | 05123 | 10 | 84507 | 10 | 73469 |
| 20 | 11166 | 20 | 45638 | 20 | 06313 | 20 | 85940 | 20 | 74972 |
| 30 | 11499 | 30 | 46451 | 30 | 07509 | 30 | 87376 | 30 | 76475 |
| 40 | 11840 | 40 | 47271 | 40 | 08710 | 40 | 88814 | 40 | 77978 |
| 50 | 12190 | 50 | 48099 | 50 | 09916 | 50 | 90255 | 50 | 79480 |
| 7 0 | 12548 | 17 0 | 48933 | 27 0 | 11128 | 37 0 | 91698 | 47 0 | 80982 |
| 10 | 12915 | 10 | 49775 | 10 | 12344 | 10 | 93144 | 10 | 82483 |
| 20 | 13290 | 20 | 50624 | 20 | 13566 | 20 | 94592 | 20 | 83983 |
| 30 | 13673 | 30 | 51480 | 30 | 14793 | 30 | 96043 | 30 | 85483 |
| 40 | 14065 | 40 | 52344 | 40 | 16024 | 40 | 97495 | 40 | 86982 |
| 50 | 14466 | 50 | 53214 | 50 | 17261 | 50 | 98950 | 50 | 88480 |
| 8 0 | 14874 | 18 0 | 54092 | 28 0 | 18503 | 38 0 | ▲0408 | 48 0 | 89978 |
| 10 | 15292 | 10 | 54977 | 10 | 19750 | 10 | 01867 | 10 | 91474 |
| 20 | 15717 | 20 | 55868 | 20 | 21001 | 20 | 03328 | 20 | 92970 |
| 30 | 16151 | 30 | 56767 | 30 | 22258 | 30 | 04792 | 30 | 94465 |
| 40 | 16593 | 40 | 57673 | 40 | 23519 | 40 | 06258 | 40 | 95958 |
| 50 | 17044 | 50 | 58585 | 50 | 24785 | 50 | 07725 | 50 | 97451 |
| 9 0 | 17502 | 19 0 | 59505 | 29 0 | 26055 | 39 0 | 09194 | 49 0 | 98942 |
| 10 | 17970 | 10 | 60432 | 10 | 27331 | 10 | 10666 | 10 | ▲0432 |
| 20 | 18445 | 20 | 61365 | 20 | 28610 | 20 | 12139 | 20 | 01921 |
| 30 | 18929 | 30 | 62305 | 30 | 29895 | 30 | 13613 | 30 | 03409 |
| 40 | 19420 | 40 | 63252 | 40 | 31184 | 40 | 15090 | 40 | 04896 |
| 50 | 19921 | 50 | 64206 | 50 | 32477 | 50 | 16568 | 50 | 06381 |
| 10 0 | 20429 | 20 0 | 65166 | 30 0 | 33775 | 40 0 | 18048 | 50 0 | 07864 |
| | 978. | | 978. | | 979. | | 980. | | 981. |

*Theoretical gravity, International Formula*—Continued

| Latitude $\phi$ | Gravity | Latitude $\phi$ | Gravity | Latitude $\phi$ | Gravity | Latitude $\phi$ | Gravity |
|---|---|---|---|---|---|---|---|
| ° ′ | *Gals* 981. | ° ′ | *Gals* 981. | ° ′ | *Gals* 982. | ° ′ | *Gals* 983. |
| 50 0 | 07864 | 60 0 | 92391 | 70 0 | 61388 | 80 0 | 06467 |
| 10 | 09346 | 10 | 93695 | 10 | 62355 | 10 | 06980 |
| 20 | 10827 | 20 | 94994 | 20 | 63316 | 20 | 07484 |
| 30 | 12306 | 30 | 96289 | 30 | 64269 | 30 | 07981 |
| 40 | 13783 | 40 | 97579 | 40 | 65216 | 40 | 08468 |
| 50 | 15259 | 50 | 98865 | 50 | 66156 | 50 | 08948 |
| 51 0 | 16733 | 61 0 | ▲0146 | 71 0 | 67089 | 81 0 | 09419 |
| 10 | 18205 | 10 | 01423 | 10 | 68015 | 10 | 09882 |
| 20 | 19675 | 20 | 02695 | 20 | 68934 | 20 | 10336 |
| 30 | 21144 | 30 | 03962 | 30 | 69846 | 30 | 10782 |
| 40 | 22610 | 40 | 05225 | 40 | 70752 | 40 | 11219 |
| 50 | 24075 | 50 | 06482 | 50 | 71650 | 50 | 11649 |
| 52 0 | 25537 | 62 0 | 07735 | 72 0 | 72541 | 82 0 | 12069 |
| 10 | 26998 | 10 | 08983 | 10 | 73425 | 10 | 12482 |
| 20 | 28456 | 20 | 10226 | 20 | 74302 | 20 | 12885 |
| 30 | 29912 | 30 | 11464 | 30 | 75171 | 30 | 13281 |
| 40 | 31366 | 40 | 12698 | 40 | 76034 | 40 | 13668 |
| 50 | 32817 | 50 | 13926 | 50 | 76889 | 50 | 14046 |
| 53 0 | 34267 | 63 0 | 15149 | 73 0 | 77737 | 83 0 | 14416 |
| 10 | 35714 | 10 | 16366 | 10 | 78578 | 10 | 14777 |
| 20 | 37158 | 20 | 17579 | 20 | 79412 | 20 | 15130 |
| 30 | 38600 | 30 | 18786 | 30 | 80238 | 30 | 15474 |
| 40 | 40039 | 40 | 19989 | 40 | 81057 | 40 | 15810 |
| 50 | 41476 | 50 | 21186 | 50 | 81868 | 50 | 16137 |
| 54 0 | 42910 | 64 0 | 22377 | 74 0 | 82672 | 84 0 | 16455 |
| 10 | 44342 | 10 | 23563 | 10 | 83469 | 10 | 16765 |
| 20 | 45771 | 20 | 24744 | 20 | 84258 | 20 | 17066 |
| 30 | 47197 | 30 | 25919 | 30 | 85040 | 30 | 17359 |
| 40 | 48620 | 40 | 27089 | 40 | 85814 | 40 | 17643 |
| 50 | 50040 | 50 | 28253 | 50 | 86580 | 50 | 17918 |
| 55 0 | 51458 | 65 0 | 29412 | 75 0 | 87339 | 85 0 | 18185 |
| 10 | 52872 | 10 | 30565 | 10 | 88091 | 10 | 18443 |
| 20 | 54284 | 20 | 31713 | 20 | 88834 | 20 | 18693 |
| 30 | 55692 | 30 | 32854 | 30 | 89570 | 30 | 18933 |
| 40 | 57098 | 40 | 33990 | 40 | 90299 | 40 | 19165 |
| 50 | 58500 | 50 | 35120 | 50 | 91020 | 50 | 19389 |
| 56 0 | 59899 | 66 0 | 36245 | 76 0 | 91733 | 86 0 | 19603 |
| 10 | 61295 | 10 | 37363 | 10 | 92438 | 10 | 19809 |
| 20 | 62687 | 20 | 38476 | 20 | 93135 | 20 | 20007 |
| 30 | 64076 | 30 | 39583 | 30 | 93825 | 30 | 20195 |
| 40 | 65462 | 40 | 40683 | 40 | 94507 | 40 | 20375 |
| 50 | 66844 | 50 | 41778 | 50 | 95181 | 50 | 20546 |
| 57 0 | 68223 | 67 0 | 42867 | 77 0 | 95847 | 87 0 | 20708 |
| 10 | 69598 | 10 | 43949 | 10 | 96505 | 10 | 20862 |
| 20 | 70969 | 20 | 45026 | 20 | 97156 | 20 | 21007 |
| 30 | 72337 | 30 | 46096 | 30 | 97798 | 30 | 21143 |
| 40 | 73702 | 40 | 47160 | 40 | 98433 | 40 | 21270 |
| 50 | 75062 | 50 | 48218 | 50 | 99059 | 50 | 21389 |
| 58 0 | 76419 | 68 0 | 49270 | 78 0 | 99677 | 88 0 | 21499 |
| 10 | 77772 | 10 | 50315 | 10 | ▲0288 | 10 | 21600 |
| 20 | 79121 | 20 | 51354 | 20 | 00890 | 20 | 21692 |
| 30 | 80467 | 30 | 52387 | 30 | 01485 | 30 | 21775 |
| 40 | 81808 | 40 | 53413 | 40 | 02071 | 40 | 21850 |
| 50 | 83145 | 50 | 54433 | 50 | 02649 | 50 | 21916 |
| 59 0 | 84479 | 69 0 | 55446 | 79 0 | 03219 | 89 0 | 21973 |
| 10 | 85808 | 10 | 56453 | 10 | 03781 | 10 | 22022 |
| 20 | 87133 | 20 | 57453 | 20 | 04335 | 20 | 22061 |
| 30 | 88454 | 30 | 58447 | 30 | 04880 | 30 | 22092 |
| 40 | 89770 | 40 | 59434 | 40 | 05418 | 40 | 22114 |
| 50 | 91083 | 50 | 60415 | 50 | 05947 | 50 | 22127 |
| 60 0 | 92391 | 70 0 | 61388 | 80 0 | 06467 | 90 0 | 22131 |
| | 981. | | 982. | | 983. | | 983. |

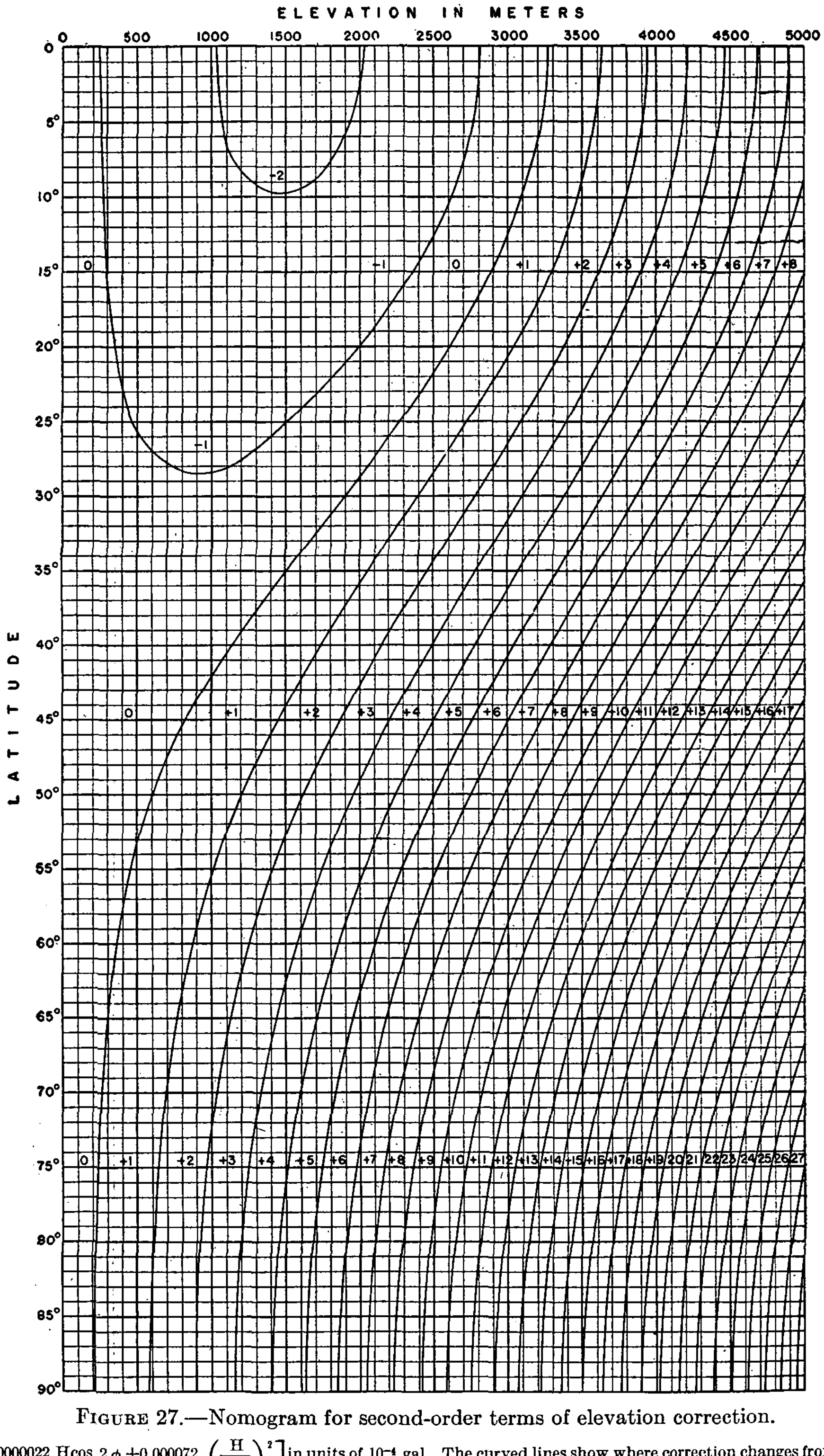


FIGURE 27.—Nomogram for second-order terms of elevation correction.

$\left[-0.00000022\ H\cos 2\phi + 0.000072\left(\frac{H}{1000}\right)^2\right]$ in units of $10^{-4}$ gal. The curved lines show where correction changes from one value to the next.

## ELEVATION CORRECTION

Regardless of what method is used to obtain the theoretical value of gravity for a particular location, the elevation of the point must be taken into consideration. In Europe the usual procedure is to apply a correction for elevation to the observed value of gravity to obtain the corresponding value at sea level and then compare that with the theoretical value at sea level. In this country, the opposite procedure is used and the elevation correction is applied to the theoretical value to reduce it to the actual elevation of the ground. Either method will give the same anomalies, of course. The chief justification for the latter method is that comparison is made between the actual observed value and a theoretical one, while by the European method the observed value is changed to a theoretical one at a point somewhere inside of the earth. The method used in this country seems more logical to many geodesists.

If there were no matter projecting above the geoid (sea-level surface) and the geoid were a smooth ellipsoid of revolution, then the value ($g_H$) of the acceleration of gravity at a height of $H$ meters above the surface would be related to that ($\gamma_0$) at the surface as indicated by the following equation in which $\phi$ is the latitude.

$$g_H=\gamma_0-(0.000\ 30855+0.000\ 00022\cos 2\phi)H+0.000\ 072\left(\frac{H}{1000}\right)^2$$

This is known as the free-air correction and is used in deriving what are called free air anomalies. (See p. 77.)

For most purposes, unless the station is at a very high elevation, the following approximate formula is sufficiently accurate.

$$g_H=\gamma_0-0.000\ 3086H$$

in which $H$ is again the elevation of the point in meters.

The second-order terms of the more precise formula above may be conveniently evaluated with sufficient accuracy by means of the nomogram shown in figure 27.

Since the density of the outer part of the earth, to a depth of a few hundred kilometers, is less than two-thirds the mean density of the earth, the acceleration of gravity increases with depth to a considerable distance below the surface. The mean rate of increase near the surface is 0.000 0851 gal per meter of depth. The actual rate at any place depends upon the density of the crustal material in that locality and is given approximately by the following formula:

$$g_d=g_0+(0.000\ 3086-0.000\ 0837\rho)d$$

where $g_d$ is the acceleration of gravity at the depth of $d$ in meters, $\rho$ is the density, and $g_0$ is gravity at the surface of the ground.

## MAP READINGS

As explained in Special Publication No. 10 (see p. 60) the gravitational effects of topographic masses and of the isostatic compensation of these masses are computed by dividing the surface of the earth into zones and compartments with the station as a center. In actual practice this is done conveniently by the use of transparent templates constructed to the same scales as the maps on which they are to be used. Thin sheets of celluloid or cellulose acetate are most frequently used for this purpose as they permit about the maximum visibility for reading elevations from the maps over which they are placed.

The designations of the various zones, the number of compartments in each and the radii of the zones are given in the following table:

*Hayford-Bowie zones for gravity reductions*

| Designation of zone | Number of compartments | Outer radius of zone | Designation of zone | Number of compartments | Outer radius of zone |
|---|---|---|---|---|---|
| | | *Meters* | | | ° ′ ″ |
| A | 1 | 2 | 18 | [2] 1 | 1 41 13 |
| B | 4 | 68 | 17 | [2] 1 | 1 54 52 |
| C [1] | 4 | 230 | 16 | [2] 1 | 2 11 53 |
| D [1] | 6 | 590 | 15 | [2] 1 | 2 33 46 |
| E [1] | 8 | 1,280 | 14 | [2] 1 | 3 03 05 |
| F [1] | 10 | 2,290 | 13 | 16 | 4 19 13 |
| G | 12 | 3,520 | 12 | 10 | 5 46 34 |
| H | 16 | 5,240 | 11 | 8 | 7 51 30 |
| I | 20 | 8,440 | 10 | 6 | 10 44 |
| J | 16 | 12,400 | 9 | 4 | 14 09 |
| K | 20 | 18,800 | 8 | 4 | 20 41 |
| L | 24 | 28,800 | 7 | 2 | 26 41 |
| M | 14 | 58,800 | 6 | 18 | 35 58 |
| N | 16 | 99,000 | 5 | 16 | 51 04 |
| O [1] | 28 | 166,700 = 1° 29′ 58″ | 4 | 12 | 72 13 |
| | | | 3 | 10 | 105 48 |
| | | | 2 | 6 | 150 56 |
| | | | 1 | 1 | 180 |

[1] This zone is sometimes divided into 2 zones to permit greater accuracy in rough topography.
[2] To aid in estimating the average elevation for this zone, it is divided into 10 subcompartments.

The first step in the isostatic reduction of a gravity station is to estimate the elevation of each compartment of each zone beginning at the station and extending to such a distance that the remaining zones to the antipodes may be interpolated with sufficient accuracy from the values for the corresponding zones at nearby stations. It is now seldom necessary in the United States to carry the map readings beyond the lettered zones, that is, beyond a radius of about 100 miles. After the elevations have been read, the corrections for topography and compensation are taken from tables or otherwise conveniently derived, as explained later, by using the estimated elevations as a basis.

The accuracy desirable in estimating the elevations in each zone varies for the different zones. Until one has learned by experience how much care should be used in estimating for any particular zone, the tables should be watched rather closely to see what change in the elevation of the topography corresponds to a certain minimum change in the correction for either the topography or the compensation. Ten feet in a near zone may give as large a correction as 1,000 feet in a distant zone. Large scale maps, usually 1 to 62,500, are used for the near zones, from zone D to about zone L, and progressively smaller scale maps for the more distant zones. The first 3 zones, A, B, and C, are usually read from the detailed sketch prepared by the observer at the time the gravity observations are made. (See page 36.)

Proficiency and accuracy in estimating elevations depend upon experience. An inexperienced man should always be checked by one with experience until a certain amount of proficiency has been attained. At first there is usually a tendency to estimate too high, especially in rough topography. For example, a compartment nearly filled by a cone-shaped mountain should, of course, be estimated as the height of the contour which encloses about two-thirds of the area rather than that enclosing about one-half. It is sometimes helpful, where close estimating is necessary, to mentally divide the compartment into two parts if the topography is much different for one part of the compartment from that of the remainder. For example, about one-third of a compartment may contain a hill of average elevation of 1,020 feet and the other two-thirds may cover a fairly flat plain at about 900 feet elevation. By simple mental weighting, 940 feet is obtained as the average elevation of the compartment.

## TOPOGRAPHIC CORRECTIONS, LETTERED ZONES

In the Hayford-Bowie tables in Special Publication No. 10, the corrections for topography and for isostatic compensation are listed separately for the lettered zones. For zones A to I the topographic effect is in general the predominant one and from zones L to O the isostatic effect predominates. For the numbered zones Hayford and Bowie found it expedient to combine the two effects, for although both are large for these zones, the resultant is relatively small and can be evaluated more precisely than either effect by itself.

The Hayford-Bowie tables were designed to be used for obtaining the effect of topography and compensation for each individual compartment independently. Since there are 199 compartments in the lettered zones alone, this use of the tables involves considerable work and sometimes results in a small accumulation of error through the rounding off of decimal places. As explained on page 69, the isostatic effect for each zone is, for all practical purposes, proportional to the average elevation of the topography in the zone and can be derived by the use of factors. This is not the case for the topographic effect, however, at least for the lettered zones, and therefore similar factors cannot be used for it. A modification of the Hayford-Bowie method which shortens the process of deriving the topographic correction and at the same time slightly improves the accuracy of the result was derived a few years ago by E. C. Bullard of Cambridge, England. The essential details of Bullard's method of computing the topographic effect for the lettered zones is explained in the following section.

### BULLARD MODIFICATION OF HAYFORD-BOWIE METHOD [10]

In computing the effect of topography as a whole from the station to the outer limits of zone O, Bullard divides the effect into three parts, $A$, $B$, and $C$, and makes the topographic effect equal to $A+B-C$. He defines these three parts as follows:

$$A=1.118\ H \times 10^{-4} \text{ gals},$$

where $H$ is the elevation of the station in meters. $A$ is very nearly equal to the attraction of an infinite plane slab having a thickness equal to the elevation of the station and a density of 2.67. The station is on the upper surface of this slab.

$$B = \text{correction for curvature.}$$

When applied to $A$, $B$ reduces the effect of the infinite plane slab to that of a plateau or cap extending to the outer limits of zone O and curved to the mathematical surface of the earth. The $B$ correction may be obtained directly from the following table:

*Curvature correction to outer limit of zone O*

| Elevation of station, $H$ | Correction | Elevation of station, $H$ | Correction | Elevation of station, $H$ | Correction | Elevation of station, $H$ | Correction |
|---|---|---|---|---|---|---|---|
| *Meters* | *Gals* $\times 10^{-4}$ | *Meters* | *Gals* $\times 10^{-4}$ | *Meters* | *Gals* $\times 10^{-4}$ | *Meters* | *Gals* $\times 10^{-4}$ |
| 0 | 0 | 500 | +7 | 1,000 | +12 | 3,000 | +15 |
| 100 | +2 | 600 | +8 | | | 3,500 | +11 |
| 200 | +3 | 700 | +9 | 1,500 | +15 | 4,000 | +6 |
| 300 | +4 | 800 | +10 | 2,000 | +17 | 4,500 | −1 |
| 400 | +6 | 900 | +11 | 2,500 | +17 | 5,000 | −10 |

[10] See gravity measurements in East Africa by E. C. Bullard. Phil. Trans. R. Soc. London, Vol. 235, No. 757, pp. 486–497.

*C*=correction for variation of actual topography from a plateau.

The *C* correction is obtained by use of the following special table. It must be taken out for individual compartments of individual zones. The table has been carried out to the fifth decimal place of gals to prevent accumulation of error.

*C Correction, topographic effect*

[Tabular values in units of $10^{-5}$ gal. Based on density of 2.67]

| Difference of elevation, station and compartment (feet) | Zones B | C | D | E | F | G | H | I | J Station Above | J Station Below | K Station Above | K Station Below | L Station Above | L Station Below | M Station Above | M Station Below | N Station Above | N Station Below | O Station Above | O Station Below |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 1 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- |
| 10 | 4 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- |
| 15 | 8 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- |
| 20 | 11 | 0 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- |
| 40 | 25 | 1 | 0 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- |
| 60 | 39 | 5 | 1 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- |
| 80 | 50 | 9 | 1 | 0 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- |
| 100 | 61 | 12 | 2 | 1 | 0 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- |
| 150 | 84 | 27 | 5 | 1 | 1 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- |
| 200 | 99 | 43 | 9 | 2 | 1 | 0 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- |
| 300 | 121 | 79 | 20 | 5 | 2 | 1 | 0 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- |
| 400 | 136 | 115 | 33 | 9 | 3 | 1 | 1 | 0 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- |
| 500 | 144 | 148 | 49 | 14 | 4 | 2 | 1 | 1 | ---- | ---- | ---- | ---- | ---- | ---- | 0 | ---- | ---- | ---- | ---- | ---- |
| 600 | ---- | 177 | 67 | 20 | 6 | 2 | 1 | 1 | 0 | 0 | 0 | ---- | ---- | ---- | 1 | ---- | ---- | ---- | ---- | ---- |
| 800 | ---- | 224 | 106 | 36 | 11 | 4 | 2 | 1 | 1 | 1 | 1 | 0 | ---- | ---- | 1 | ---- | 0 | ---- | 0 | 0 |
| 1,000 | ---- | 259 | 145 | 54 | 17 | 7 | 3 | 2 | 1 | 1 | 1 | 1 | 0 | ---- | 1 | ---- | 1 | 0 | 1 | −1 |
| 1,200 | ---- | ---- | 182 | 75 | 25 | 10 | .4 | 3 | 2 | 2 | 1 | 1 | 1 | 0 | 2 | 0 | 1 | −1 | 1 | −1 |
| 1,400 | ---- | ---- | 218 | 96 | 34 | 13 | 6 | 4 | 3 | 2 | 1 | 1 | 1 | 1 | 2 | 1 | 1 | −1 | 1 | −1 |
| 1,600 | ---- | ---- | 250 | 120 | 43 | 17 | 8 | 5 | 3 | 3 | 2 | 2 | 1 | 1 | 3 | 1 | 2 | −1 | 1 | −1 |
| 1,800 | ---- | ---- | 278 | 144 | 54 | 21 | 10 | 6 | 4 | 4 | 2 | 2 | 1 | 1 | 3 | 1 | 2 | −1 | 1 | −1 |
| 2,000 | ---- | ---- | 304 | 169 | 65 | 26 | 12 | 8 | 5 | 5 | 3 | 3 | 2 | 1 | 4 | 1 | 2 | −1 | 2 | −1 |
| 2,200 | ---- | ---- | 328 | 193 | 77 | 31 | 15 | 9 | 6 | 6 | 4 | 3 | 2 | 2 | 4 | 2 | 3 | 0 | 2 | −1 |
| 2,400 | ---- | ---- | 349 | 218 | 90 | 36 | 17 | 11 | 7 | 7 | 4 | 4 | 3 | 2 | 5 | 2 | 3 | 0 | 2 | −1 |
| 2,600 | ---- | ---- | 367 | 241 | 103 | 42 | 20 | 13 | 8 | 8 | 5 | 5 | 3 | 2 | 6 | 3 | 3 | 0 | 2 | −1 |
| 2,800 | ---- | ---- | 384 | 264 | 117 | 48 | 23 | 15 | 10 | 9 | 6 | 5 | 3 | 3 | 7 | 4 | 4 | 0 | 2 | −1 |
| 3,000 | ---- | ---- | 400 | 287 | 131 | 54 | 26 | 17 | 11 | 11 | 7 | 6 | 4 | 3 | 8 | 4 | 4 | 0 | 3 | −1 |
| 3,500 | ---- | ---- | ---- | 339 | 168 | 73 | 35 | 23 | 15 | 15 | 9 | 8 | 5 | 4 | 10 | 6 | 5 | 0 | 3 | −1 |
| 4,000 | ---- | ---- | ---- | 387 | 205 | 93 | 46 | 30 | 20 | 19 | 12 | 11 | 7 | 6 | 13 | 8 | 6 | 1 | 4 | −1 |
| 4,500 | ---- | ---- | ---- | 430 | 242 | 113 | 56 | 37 | 25 | 24 | 15 | 14 | 9 | 8 | 16 | 11 | 8 | 1 | 4 | −1 |
| 5,000 | ---- | ---- | ---- | 467 | 280 | 135 | 69 | 45 | 31 | 30 | 18 | 17 | 11 | 9 | 19 | 14 | 9 | 2 | 5 | −1 |
| 5,500 | ---- | ---- | ---- | ---- | 315 | 158 | 82 | 54 | 37 | 36 | 22 | 21 | 13 | 11 | 23 | 17 | 11 | 3 | 6 | −1 |
| 6,000 | ---- | ---- | ---- | ---- | 349 | 181 | 96 | 64 | 44 | 43 | 26 | 25 | 15 | 14 | 27 | 20 | 12 | 4 | 7 | −1 |
| 6,500 | ---- | ---- | ---- | ---- | 381 | 204 | 110 | 74 | 51 | 50 | 30 | 29 | 17 | 16 | 31 | 24 | 14 | 5 | 7 | −1 |
| 7,000 | ---- | ---- | ---- | ---- | 412 | 227 | 125 | 85 | 59 | 58 | 35 | 34 | 20 | 19 | 36 | 28 | 16 | 6 | 8 | −1 |
| 8,000 | ---- | ---- | ---- | ---- | 469 | 274 | 156 | 109 | 76 | 75 | 46 | 44 | 26 | 24 | 47 | 38 | 20 | 9 | 10 | 0 |
| 9,000 | ---- | ---- | ---- | ---- | ---- | 318 | 188 | 135 | 95 | 94 | 57 | 56 | 33 | 31 | 58 | 48 | 24 | 12 | 12 | 0 |
| 10,000 | ---- | ---- | ---- | ---- | ---- | 360 | 220 | 162 | 116 | 114 | 70 | 68 | 41 | 38 | 71 | 60 | 29 | 16 | 14 | 1 |
| 11,000 | ---- | ---- | ---- | ---- | ---- | ---- | 252 | 190 | 138 | 137 | 84 | 82 | 49 | 46 | 85 | 73 | 35 | 20 | 16 | 2 |
| 12,000 | ---- | ---- | ---- | ---- | ---- | ---- | 284 | 220 | 162 | ---- | 99 | ---- | 58 | ---- | 101 | 87 | 40 | 24 | 19 | 3 |
| 13,000 | ---- | ---- | ---- | ---- | ---- | ---- | 315 | 250 | 187 | ---- | 115 | ---- | 67 | ---- | 118 | ---- | 46 | 29 | 21 | 4 |
| 14,000 | ---- | ---- | ---- | ---- | ---- | ---- | 346 | 280 | 213 | ---- | 133 | ---- | 78 | ---- | 136 | ---- | 53 | 34 | 24 | 6 |
| 15,000 | ---- | ---- | ---- | ---- | ---- | ---- | 375 | 311 | 241 | ---- | 151 | ---- | 89 | ---- | 155 | ---- | 60 | 40 | 27 | 7 |
| 16,000 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | 342 | 269 | ---- | 170 | ---- | 101 | ---- | 175 | ---- | 68 | ---- | 30 | 9 |
| 17,000 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | 373 | 298 | ---- | 191 | ---- | 113 | ---- | 197 | ---- | 76 | ---- | 33 | ---- |
| 18,000 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | 403 | 328 | ---- | 211 | ---- | 126 | ---- | 220 | ---- | 84 | ---- | 36 | ---- |
| 19,000 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | 433 | 358 | ---- | 233 | ---- | 140 | ---- | 244 | ---- | 93 | ---- | 40 | ---- |
| 20,000 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | 463 | 388 | ---- | 255 | ---- | 154 | ---- | 269 | ---- | 103 | ---- | 43 | ---- |
| 22,000 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | 450 | ---- | 302 | ---- | 184 | ---- | 323 | ---- | 123 | ---- | 51 | ---- |
| 24,000 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | 513 | ---- | 351 | ---- | 216 | ---- | 382 | ---- | 144 | ---- | 59 | ---- |
| 26,000 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | 575 | ---- | 401 | ---- | 250 | ---- | 445 | ---- | 168 | ---- | 68 | ---- |
| 28,000 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | 637 | ---- | 453 | ---- | 286 | ---- | 512 | ---- | 193 | ---- | 78 | ---- |
| 30,000 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | 698 | ---- | 506 | ---- | 324 | ---- | 584 | ---- | 219 | ---- | 88 | ---- |
| 32,000 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | 758 | ---- | 560 | ---- | 363 | ---- | 660 | ---- | 248 | ---- | 98 | ---- |

## ISOSTATIC COMPENSATION CORRECTIONS, LETTERED ZONES

The fundamental assumptions regarding isostatic compensation which were used by Hayford and Bowie in deriving the basic tables given in Special Publication No. 10 of this Bureau are based on Pratt's hypothesis and are as follows:

(1) In each land compartment of each zone the negative mass of the compensation is exactly equal to the positive mass of the topography included between the surface of the ground and sea level. In each water (oceanic) compartment, the negative topography is the deficiency of mass of the sea water as compared to average surface rock of density 2.67. The density of this deficiency is therefore 2.67−1.03 or 1.64. The mass of the compensation is of course a positive one for an oceanic compartment.

(2) The compensation is distributed uniformly from the surface to a depth of 113.7 km.[11]. No definite density was assumed for the crustal material below a depth of about 5 miles from the sea-level surface. It was simply assumed that this density is "normal" except as modified by the compensation of the overlying topography. The compensation was, therefore, computed as a deficiency or excess of density or of mass.

(3) The compensation is directly below the corresponding topography. In other words, each compartment is independently compensated. For the small compartments and zones near the station, this assumption appears somewhat illogical at first, since the outer part of the earth is undoubtedly too rigid to respond to such small differences of pressure. It can be readily shown, however, that larger compartments or zones near the station would make almost no difference in the computed compensation effect. The compartments were made small near the station in order to evaluate the topographic effect with sufficient precision. Larger compartments and zones would have been used if the effect of the compensation only was to be computed.

Since the isostatic compensation is assumed to be uniformly distributed over the specified depth, its effect for any compartment or zone is directly proportional to its density which in turn is proportional to the height of the topography in the compartment or zone. For land stations the effect of compensation may therefore be computed by multiplying the average height of the topography in a zone by a factor. This is a much quicker method than interpolating the corrections for individual compartments from the tables in Special Publication No. 10, and it also gives somewhat better accuracy because when the tables are used there is apt to be a slight accumulation of error from rounded off decimal places. For sea stations the compensation effect must be taken from the special sea table on page 73.

A table of factors for use in computing the compensation of the lettered zones is given below.

*Factors for computing compensation for land stations, lettered zones*

[The average elevation of each zone in feet multiplied by the factor for the zone will give the compensation correction in units of $10^{-3}$ gal.]

| Zone | Factor | Zone | Factor | Zone | Factor | Zone | Factor |
|---|---|---|---|---|---|---|---|
| A | ---------- | E | −.0209 | I | −.0914 | M | −.5848 |
| B | −0.0020 | F | −.0303 | J | −.1092 | N | −.5324 |
| C | −.0049 | G | −.0365 | K | −.1677 | O | −.5178 |
| D | −.0109 | H | −.0503 | L | −.2409 | | |

In very high topography and when considerable precision is desired there is a small additional secondary correction if the station is several thousand feet above or below

[11] The effect of the compensation for other depths of compensation are readily derived by the use of the factors on p. 76.

the individual compartment. Tables for evaluating this correction are available at the office of the Coast and Geodetic Survey. They have not been included in this publication as they are seldom needed.

In deriving the average elevation of the zones for use with the above factors, the depths in the water compartments must either be reduced to feet and multiplied by the factor 0.615 or else the depth in fathoms must be multiplied by the factor 3.69. This gives the deficiency of mass in terms of average surface rock. It must be considered as negative topography, that is, must be given a negative sign in deriving the average elevation of the zone. When a compartment is part land and part water, a similar computation must be made for it. For example, if a compartment is one-fourth land of average elevation of 60 feet and three-fourths water of average depth of 24 fathoms, the average elevation of the compartment would be −51 feet. It is sometimes convenient, when a compartment is part land and part water, to consider each part spread over the entire compartment as if the remainder of the compartment were at zero elevation. In the preceding example the elevations under this system would be recorded as 15 feet and 18 fathoms. (See zones 17 to 14 of the sample readings in figure 28.)

## CORRECTIONS FOR TOPOGRAPHY AND COMPENSATION, NUMBERED ZONES

The numbered zones extend from about 100 miles from the station to the antipodes. (See table on p. 66.) For these zones, the topography and compensation effects are not separated in the tables. Both effects are quite large, as the area covered by each compartment is large, much larger than for the lettered zones. The two effects are nearly equal and of opposite sign, however, and the algebraic sum of the two is, therefore, relatively small in each case.

The radii of the numbered zones were so selected by Hayford and Bowie and the division into compartments so made that the correction for topography and compensation of each compartment in units of the fourth decimal place of gals is a decimal part of the elevation of the compartment in feet. For zones 18 to 14 the elevation is divided by 100 to obtain the correction in units of the fourth decimal place of gals, for zones 13 to 7 it is divided by 1,000, and for zones 6 to 1 by 10,000. For depths in fathoms the corresponding divisors are 27.1, 271, and 2710. The actual elevations or depths, as read from the maps, are not recorded for the numbered zones but the corrections themselves are written directly on the computation sheet. The corrections for land compartments are negative and for oceanic compartments positive for the numbered zones.

For zones 18 to 14 the corrections are not exactly proportional to the elevations and there is, therefore, a small secondary correction for departure from proportionality. This secondary correction may be taken from the following table.

*Correction for departure from proportionality Zones 18 to 14*

ZONE 18

| Correction as read from map | Correction for departure from proportionality | Correction for elevation of station at— | | | Correction as read from map | Correction for departure from proportionality | Correction for elevation of station at— | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 5,000 feet | 10,000 feet | 15,000 feet | | | 5,000 feet | 10,000 feet | 15,000 feet |
| +150 | +1 | −2 | −5 | −7 | 0 | 0 | 0 | 0 | 0 |
| +125 | +1 | −2 | −4 | −6 | −25 | 0 | 0 | +1 | +1 |
| +100 | +1 | −2 | −3 | −5 | −50 | 0 | +1 | +2 | +2 |
| +75 | 0 | −1 | −2 | −3 | −75 | 0 | +1 | +2 | +3 |
| +50 | 0 | −1 | −2 | −2 | −100 | 0 | +2 | +3 | +5 |
| +25 | 0 | 0 | −1 | −1 | | | | | |

ZONE 17

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| +150 | +3 | −2 | −4 | −6 | 0 | 0 | 0 | 0 | 0 |
| +125 | +2 | −2 | −3 | −5 | −25 | 0 | +1 | +1 | +1 |
| +100 | +1 | −1 | −3 | −4 | −50 | 0 | +1 | +1 | +2 |
| +75 | +1 | −1 | −2 | −3 | −75 | +1 | +1 | +2 | +3 |
| +50 | 0 | −1 | −1 | −2 | −100 | +1 | +1 | +3 | +4 |
| +25 | 0 | −1 | −1 | −1 | | | | | |

ZONE 16

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| +150 | +4 | −2 | −3 | −5 | 0 | 0 | 0 | 0 | 0 |
| +125 | +3 | −1 | −3 | −4 | −25 | 0 | 0 | +1 | +1 |
| +100 | +2 | −1 | −2 | −3 | −50 | 0 | +1 | +1 | +2 |
| +75 | +1 | −1 | −1 | −2 | −75 | +1 | +1 | +1 | +2 |
| +50 | 0 | −1 | −1 | −2 | −100 | +1 | +1 | +2 | +3 |
| +25 | 0 | 0 | −1 | −1 | | | | | |

ZONE 15

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| +150 | +5 | −1 | −3 | −4 | 0 | 0 | 0 | 0 | 0 |
| +125 | +4 | −1 | −2 | −3 | −25 | 0 | 0 | 0 | +1 |
| +100 | +2 | −1 | −2 | −3 | −50 | 0 | 0 | +1 | +1 |
| +75 | +1 | −1 | −1 | −2 | −75 | +1 | +1 | +1 | +2 |
| +50 | +1 | 0 | −1 | −1 | −100 | +1 | +1 | +2 | +3 |
| +25 | 0 | 0 | 0 | −1 | | | | | |

ZONE 14

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| +150 | +6 | −1 | −2 | −3 | 0 | 0 | 0 | 0 | 0 |
| +125 | +4 | −1 | −2 | −3 | −25 | 0 | 0 | 0 | 0 |
| +100 | +3 | −1 | −1 | −2 | −50 | 0 | 0 | +1 | +1 |
| +75 | +2 | −1 | −1 | −2 | −75 | +1 | 0 | +1 | +1 |
| +50 | +1 | 0 | −1 | −1 | −100 | +2 | +1 | +1 | +2 |
| +25 | 0 | 0 | 0 | 0 | | | | | |

## INTERPOLATION FOR DISTANT ZONES

As already stated, it is seldom necessary to make map readings for more than a few of the numbered zones as the effects may be interpolated from nearby stations. The method used in making interpolations is explained on pages 58 to 65 of Special Publication No. 10. Briefly it consists of selecting three nearby, previously reduced stations which form a triangle around the new station and interpolating according to distances and directions.

Two rules are used in deciding how far from a station the map readings must be carried before interpolations can be started.

Rule 1. Interpolated values must not be accepted for any zone inside the third zone beyond that containing the nearest one of the three stations from which interpolation was made.

Rule 2. Subject to Rule 1, interpolation may be accepted for a given zone and all more distant zones if for each of the three zones next inside the given zone the interpolated and computed values agree within 0.0005 gal.

A more convenient method for interpolating some of the more distant of the numbered zones is described in the following section.

### HEISKANEN'S INTERPOLATION MAPS

In 1938, the Isostatic Reduction Office of the International Association of Geodesy, under the direction of Dr. W. Heiskanen at Helsinki, Finland, published a set of four small scale world maps which show lines of equal correction for topography and compensation for zones 10 to 1. There are separate maps for zones 10, 9, and 8 and for combined zones 7 to 1. By plotting the station on each of these maps, which need be done only very roughly, the corrections for these zones may be read directly from the maps to an accuracy of about 0.1 milligal. The accuracy is fully as good as that obtained by the more laborious interpolation process described in the preceding section but the maps do not cover the zones from *11* to *18* and, therefore, it still becomes necessary to compute the interpolation for some of these nearer zones. As a consequence, the saving of time from the use of the maps is rather small. It is hardly feasible to construct similar maps for zones 11 to 18 or even for zones 11 to 13 as the corrections for these zones are considerably larger and change much faster than for zones 10 to 1, and the equal correction lines would form an intricate pattern of closely spaced lines.

## SPECIAL REDUCTION TABLE FOR SEA STATIONS, LETTERED ZONES

A large number of gravity-at-sea stations have been isostatically reduced by this Bureau since the inauguration of gravity determinations at sea with the Vening Meinez apparatus in 1923. For these isostatic computations, the correction tables for land stations are ordinarily unsuitable because of the great depths which may occur close to the station and which are beyond the range of the land tables. The following special table was, therefore, devised for the lettered zones at sea stations.

The first part of this special table, covering zones A to J, gives the corrections by zones rather than by compartments and must be used with some caution. If the range of depth for the different compartments of a zone is small, relative to the total depth, the correction for the zone as taken from this part of the table, using the average depth of the zone as the argument, will be sufficiently accurate. This is true for the great majority of sea stations, especially those located in deep water. If the station is in shallow water or if the variation of depth is relatively large, however, the corrections should be taken out by compartments. This can be done by the use of the land tables in Special Publication No. 10 if these cover a sufficient range of depths for the case in hand or it can be done by dividing the tabular values in this part of the sea table by the number of compartments in the zones to give the corrections by compartments. The second part of the sea table, covering zones K to O, gives the corrections by both zones and compartments. Experience will soon indicate when it is desirable to take out the corrections by compartments.

### *Special reduction table for sea stations, lettered zones*

[Station at sea level. Tabular values in units of $10^{-4}$ gal. PER ZONE.][1]

| Depth in fathoms | Zone | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | | B | | C | | D | | E | | F | | G | | H | | I | | J | |
| | Comp. | Top. and comp. | Comp. | Top. and comp. | Comp. | Top. and comp. | Comp. | Top. and comp. | Comp. | Top. and comp. | Comp. | Top. and comp. | Comp. | Top. and comp. | Comp. | Top. and comp. | Comp. | Top. and comp. | Comp. | Top. and comp. |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | −1 | 0 | −5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | −1 | 0 | −10 | 0 | −1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 25 | 0 | −1 | 0 | −21 | 0 | −7 | 0 | −2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | +1 | +1 | +1 | +1 |
| 50 | 0 | −1 | 0 | −30 | 0 | −20 | 0 | −7 | 0 | −2 | +1 | 0 | +1 | 0 | +1 | +1 | +2 | +1 | +2 | +2 |
| 75 | 0 | −1 | 0 | −34 | 0 | −32 | 0 | −15 | +1 | −5 | +1 | −1 | +1 | 0 | +1 | +1 | +3 | +2 | +3 | +3 |
| 100 | 0 | −1 | 0 | −37 | 0 | −44 | 0 | −24 | +1 | −9 | +1 | −3 | +1 | 0 | +2 | +1 | +3 | +3 | +4 | +4 |
| 150 | 0 | −1 | 0 | −40 | 0 | −60 | 0 | −46 | +1 | −21 | +2 | −7 | +2 | −2 | +3 | 0 | +5 | +3 | +6 | +5 |
| 200 | 0 | −1 | 0 | −41 | 0 | −70 | +1 | −67 | +1 | −35 | +2 | −13 | +3 | −4 | +4 | −1 | +7 | +3 | +8 | +6 |
| 250 | 0 | −1 | 0 | −42 | 0 | −77 | +1 | −86 | +2 | −51 | +3 | −21 | +3 | −7 | +5 | −2 | +8 | +3 | +10 | +7 |
| 300 | 0 | −1 | 0 | −43 | 0 | −82 | +1 | −102 | +2 | −69 | +3 | −30 | +4 | −11 | +6 | −4 | +10 | +2 | +12 | +8 |
| 350 | 0 | −1 | 0 | −43 | 0 | −86 | +1 | −116 | +2 | −87 | +4 | −40 | +5 | −16 | +6 | −7 | +12 | +2 | +14 | +9 |
| 400 | 0 | −1 | 0 | −43 | 0 | −89 | +1 | −128 | +2 | −105 | +4 | −51 | +5 | −22 | +7 | −10 | +13 | 0 | +16 | +9 |
| 450 | 0 | −1 | 0 | −44 | 0 | −92 | +1 | −138 | +3 | −122 | +4 | −63 | +6 | −28 | +8 | −13 | +15 | −2 | +18 | +9 |
| 500 | 0 | −1 | 0 | −44 | 0 | −94 | +1 | −147 | +3 | −138 | +5 | −76 | +6 | −34 | +9 | −17 | +17 | −4 | +20 | +9 |
| 550 | 0 | −1 | 0 | −44 | 0 | −95 | +1 | −155 | +3 | −154 | +5 | −89 | +7 | −41 | +10 | −21 | +18 | −7 | +22 | +9 |
| 600 | 0 | −1 | 0 | −44 | 0 | −96 | +1 | −161 | +3 | −169 | +6 | −102 | +8 | −49 | +11 | −26 | +20 | −10 | +24 | +8 |
| 700 | 0 | −1 | 0 | −44 | 0 | −98 | +1 | −172 | +3 | −196 | +6 | −129 | +9 | −66 | +12 | −37 | +23 | −17 | +28 | +7 |
| 800 | 0 | −1 | 0 | −44 | 0 | −100 | +1 | −180 | +3 | −219 | +7 | −156 | +10 | −84 | +14 | −49 | +26 | −25 | +32 | +4 |
| 900 | 0 | −1 | 0 | −44 | 0 | −101 | +1 | −187 | +3 | −240 | +7 | −182 | +10 | −103 | +16 | −63 | +29 | −35 | +36 | +1 |
| 1,000 | 0 | −1 | 0 | −44 | 0 | −102 | +1 | −193 | +3 | −258 | +8 | −207 | +11 | −122 | +17 | −77 | +32 | −46 | +40 | −3 |
| 1,200 | 0 | −1 | 0 | −45 | 0 | −104 | +1 | −202 | +4 | −287 | +8 | −252 | +13 | −162 | +20 | −109 | +38 | −72 | +47 | −14 |
| 1,400 | 0 | −1 | 0 | −45 | 0 | −105 | +1 | −208 | +4 | −310 | +9 | −292 | +14 | −201 | +22 | −144 | +44 | −102 | +55 | −27 |
| 1,600 | 0 | −1 | 0 | −45 | 0 | −106 | +1 | −212 | +4 | −328 | +9 | −327 | +15 | −239 | +24 | −180 | +49 | −136 | +62 | −44 |
| 1,800 | 0 | −1 | 0 | −45 | 0 | −106 | +1 | −216 | +4 | −342 | +9 | −357 | +16 | −274 | +26 | −216 | +54 | −173 | +69 | −62 |
| 2,000 | 0 | −1 | 0 | −45 | 0 | −107 | +1 | −219 | +4 | −354 | +10 | −383 | +16 | −307 | +28 | −252 | +59 | −212 | +76 | −84 |
| 2,200 | 0 | −1 | 0 | −45 | 0 | −107 | +1 | −222 | +4 | −364 | +10 | −405 | +17 | −337 | +30 | −287 | +63 | −252 | +82 | −107 |
| 2,400 | 0 | −1 | 0 | −45 | 0 | −108 | +1 | −224 | +4 | −372 | +10 | −425 | +17 | −364 | +31 | −321 | +67 | −293 | +89 | −132 |
| 2,600 | 0 | −1 | 0 | −45 | 0 | −108 | +1 | −226 | +4 | −380 | +10 | −442 | +18 | −390 | +32 | −354 | +71 | −335 | +95 | −159 |
| 2,800 | 0 | −1 | 0 | −45 | 0 | −108 | +1 | −227 | +4 | −386 | +10 | −457 | +18 | −413 | +33 | −385 | +74 | −377 | +100 | −187 |
| 3,000 | 0 | −1 | 0 | −45 | 0 | −108 | +1 | −228 | +4 | −392 | +10 | −471 | +18 | −434 | +34 | −415 | +77 | −419 | +106 | −217 |
| 3,200 | 0 | −1 | 0 | −45 | 0 | −109 | +1 | −230 | +4 | −397 | +10 | −483 | +18 | −453 | +35 | −443 | +80 | −461 | +111 | −247 |
| 3,400 | 0 | −1 | 0 | −45 | 0 | −109 | +1 | −231 | +4 | −401 | +10 | −494 | +19 | −471 | +35 | −470 | +83 | −502 | +116 | −278 |
| 3,600 | 0 | −1 | 0 | −45 | 0 | −109 | +1 | −231 | +4 | −404 | +10 | −504 | +19 | −487 | +36 | −495 | +85 | −542 | +121 | −310 |
| 3,800 | 0 | −1 | 0 | −45 | 0 | −109 | +1 | −232 | +4 | −408 | +10 | −513 | +19 | −502 | +36 | −519 | +87 | −582 | +125 | −343 |
| 4,000 | 0 | −1 | 0 | −45 | 0 | −109 | +1 | −233 | +4 | −411 | +10 | −521 | +19 | −516 | +37 | −542 | +89 | −620 | +129 | −376 |
| 4,200 | 0 | −1 | 0 | −45 | 0 | −109 | +1 | −234 | +4 | −414 | +10 | −528 | +19 | −529 | +37 | −563 | +91 | −657 | +133 | −409 |
| 4,400 | 0 | −1 | 0 | −45 | 0 | −109 | +1 | −234 | +4 | −416 | +10 | −535 | +19 | −541 | +37 | −582 | +93 | −694 | +137 | −442 |
| 4,600 | 0 | −1 | 0 | −45 | 0 | −110 | +1 | −235 | +4 | −419 | +10 | −541 | +19 | −552 | +38 | −601 | +94 | −729 | +141 | −474 |
| 4,800 | 0 | −1 | 0 | −45 | 0 | −110 | +1 | −236 | +4 | −421 | +10 | −547 | +19 | −562 | +38 | −619 | +96 | −762 | +144 | −507 |
| 5,000 | 0 | −1 | 0 | −45 | 0 | −110 | +1 | −236 | +4 | −423 | +10 | −552 | +19 | −572 | +38 | −636 | +97 | −795 | +147 | −540 |
| 5,000 | 0 | −1 | 0 | −45 | 0 | −110 | +1 | −238 | +4 | −431 | +10 | −574 | +19 | −611 | +39 | −707 | +101 | −943 | +160 | −698 |

[1] This table should be used with caution. See instructions on p. 72.

*Special reduction table for sea stations, lettered zones*—Continued

[Station at sea level. Tabular values in units of $10^{-4}$ gal PER ZONE]

| Depth in fathoms | Zone K Comp. | Zone K Top. and comp. | Zone L Comp. | Zone L Top. and comp. | Zone M Comp. | Zone M Top. and comp. | Zone N Comp. | Zone N Top. and comp. | Zone O Comp. | Zone O Top. and comp. |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | +1 | +1 | +1 | +1 | +1 | +1 |
| 10 | +1 | +1 | +1 | +1 | +2 | +2 | +2 | +2 | +2 | +2 |
| 25 | +2 | +2 | +2 | +2 | +5 | +5 | +5 | +5 | +5 | +5 |
| 50 | +3 | +3 | +4 | +4 | +11 | +11 | +10 | +10 | +10 | +9 |
| 75 | +5 | +4 | +7 | +6 | +16 | +16 | +15 | +14 | +14 | +14 |
| 100 | +6 | +6 | +9 | +9 | +22 | +21 | +20 | +19 | +19 | +18 |
| 150 | +9 | +8 | +13 | +13 | +32 | +32 | +30 | +29 | +29 | +28 |
| 200 | +12 | +11 | +18 | +17 | +43 | +42 | +39 | +38 | +38 | +37 |
| 250 | +16 | +13 | +22 | +21 | +54 | +52 | +49 | +48 | +48 | +46 |
| 300 | +19 | +16 | +27 | +25 | +65 | +62 | +59 | +57 | +58 | +55 |
| 350 | +22 | +18 | +31 | +28 | +76 | +72 | +69 | +67 | +67 | +64 |
| 400 | +25 | +20 | +36 | +32 | +87 | +82 | +79 | +76 | +77 | +74 |
| 450 | +28 | +21 | +40 | +35 | +98 | +92 | +89 | +86 | +87 | +83 |
| 500 | +31 | +23 | +44 | +39 | +108 | +102 | +99 | +95 | +97 | +92 |
| 550 | +34 | +24 | +49 | +42 | +119 | +112 | +109 | +104 | +106 | +101 |
| 600 | +37 | +25 | +53 | +45 | +130 | +121 | +119 | +114 | +116 | +110 |
| 700 | +43 | +27 | +62 | +51 | +152 | +140 | +139 | +132 | +136 | +129 |
| 800 | +49 | +29 | +71 | +57 | +174 | +158 | +159 | +151 | +155 | +147 |
| 900 | +56 | +30 | +80 | +62 | +196 | +176 | +179 | +169 | +175 | +165 |
| 1,000 | +62 | +30 | +89 | +67 | +217 | +194 | +199 | +187 | +195 | +184 |
| 1,200 | +74 | +28 | +107 | +75 | +261 | +228 | +240 | +223 | +235 | +220 |
| 1,400 | +86 | +24 | +124 | +82 | +305 | +261 | +280 | +259 | +275 | +256 |
| 1,600 | +97 | +18 | +142 | +87 | +348 | +292 | +321 | +294 | +316 | +293 |
| 1,800 | +109 | +9 | +159 | +90 | +392 | +321 | +362 | +329 | +356 | +329 |
| 2,000 | +120 | −2 | +177 | +91 | +436 | +349 | +403 | +363 | +397 | +365 |
| 2,200 | +132 | −15 | +194 | +91 | +480 | +375 | +444 | +397 | +438 | +401 |
| 2,400 | +143 | −30 | +211 | +90 | +523 | +400 | +485 | +430 | +480 | +437 |
| 2,600 | +154 | −46 | +228 | +86 | +566 | +423 | +527 | +463 | +522 | +473 |
| 2,800 | +164 | −65 | +244 | +81 | +610 | +444 | +568 | +495 | +564 | +508 |
| 3,000 | +174 | −86 | +261 | +75 | +653 | +464 | +610 | +527 | +606 | +544 |
| 3,200 | +184 | −108 | +277 | +67 | +697 | +482 | +652 | +558 | +649 | +579 |
| 3,400 | +194 | −131 | +293 | +58 | +740 | +499 | +694 | +589 | +691 | +614 |
| 3,600 | +204 | −156 | +309 | +47 | +783 | +514 | +736 | +619 | +734 | +649 |
| 3,800 | +213 | −182 | +325 | +35 | +826 | +527 | +778 | +649 | +778 | +684 |
| 4,000 | +222 | −210 | +341 | +21 | +868 | +539 | +820 | +678 | +821 | +719 |
| 4,200 | +231 | −238 | +356 | +6 | +911 | +549 | +862 | +707 | +865 | +754 |
| 4,400 | +239 | −268 | +371 | −10 | +953 | +558 | +905 | +735 | +909 | +788 |
| 4,600 | +247 | −298 | +386 | −27 | +995 | +565 | +947 | +762 | +953 | +822 |
| 4,800 | +255 | −329 | +400 | −45 | +1,037 | +571 | +989 | +789 | +997 | +856 |
| 5,000 | +262 | −360 | +414 | −64 | +1,079 | +576 | +1,032 | +816 | +1,042 | +890 |
| 6,000 | +296 | −425 | +482 | −176 | +1,283 | +575 | +1,245 | +940 | +1,267 | +1,058 |

[Tabular values in units of $10^{-4}$ gal PER COMPARTMENT]

| Depth in fathoms | Zone K Top. and comp. | Zone L Top. and comp. | Zone M Top. and comp. | Zone N Top. and comp. | Zone O Top. and comp. |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 |
| 25 | 0 | 0 | 0 | 0 | 0 |
| 50 | 0 | 0 | +1 | +1 | 0 |
| 75 | 0 | 0 | +1 | +1 | 0 |
| 100 | 0 | 0 | +2 | +1 | +1 |
| 150 | 0 | 0 | +2 | +2 | +1 |
| 200 | 0 | +1 | +3 | +2 | +1 |
| 250 | +1 | +1 | +4 | +3 | +2 |
| 300 | +1 | +1 | +4 | +4 | +2 |
| 350 | +1 | +1 | +5 | +4 | +2 |
| 400 | +1 | +1 | +6 | +5 | +3 |
| 450 | +1 | +2 | +7 | +5 | +3 |
| 500 | +1 | +2 | +7 | +6 | +3 |
| 550 | +1 | +2 | +8 | +6 | +4 |
| 600 | +1 | +2 | +9 | +7 | +4 |
| 700 | +1 | +2 | +10 | +8 | +5 |
| 800 | +1 | +2 | +11 | +9 | +5 |
| 900 | +1 | +3 | +13 | +11 | +6 |
| 1,000 | +2 | +3 | +14 | +12 | +7 |
| 1,200 | +1 | +3 | +16 | +14 | +8 |
| 1,400 | +1 | +3 | +19 | +16 | +9 |
| 1,600 | +1 | +4 | +21 | +18 | +10 |
| 1,800 | 0 | +4 | +23 | +21 | +12 |
| 2,000 | 0 | +4 | +25 | +23 | +13 |
| 2,200 | −1 | +4 | +27 | +25 | +14 |
| 2,400 | −2 | +4 | +29 | +27 | +16 |
| 2,600 | −2 | +4 | +30 | +29 | +17 |
| 2,800 | −3 | +3 | +32 | +31 | +18 |
| 3,000 | −4 | +3 | +33 | +33 | +19 |
| 3,200 | −5 | +3 | +34 | +35 | +21 |
| 3,400 | −7 | +2 | +36 | +37 | +22 |
| 3,600 | −8 | +2 | +37 | +39 | +23 |
| 3,800 | −9 | +2 | +38 | +41 | +24 |
| 4,000 | −10 | +1 | +38 | +42 | +26 |
| 4,200 | −12 | 0 | +39 | +44 | +27 |
| 4,400 | −13 | 0 | +40 | +46 | +28 |
| 4,600 | −15 | −1 | +40 | +48 | +29 |
| 4,800 | −16 | −2 | +41 | +49 | +31 |
| 5,000 | −18 | −3 | +41 | +51 | +32 |
| 6,000 | −21 | −7 | +41 | +59 | +38 |

## EXPLANATION OF COMPUTATION FORM FOR EFFECT OF TOPOGRAPHY AND COMPENSATION

In making the isostatic reductions for a gravity station, the elevations of the different compartments and zones are read from maps by use of transparent templates as already explained. The elevations are recorded on a special form which is shown as a sample computation in figure 28. The topography in the vicinity of the station selected for the example is rather low and of uniform elevation. Not until zone L is reached is there much variation in the elevations. This results in very small corrections for the inner zones and therefore only approximate map readings were needed for the first 10 or 12 zones. If the topography had been higher and more rugged near the station, the various corrections would have been larger and more careful map readings would have been necessary.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 653
Rev. April 1935

# ISOSTATIC REDUCTION OF GRAVITY STATIONS

**Station No.:** 1036 **Name:** Roseville **State:** Calif. **Latitude:** 38° 43.7′ **Longitude:** 121° 18.3′ **Elevation:** 140 ft. = 42.8 m.

| Zone | Readings (Ft.; Fms.) | Compensation corrections | Mean |
|---|---|---|---|
| A (1 comp.) | 140 | = (0) | 140 |
| B (4 comp.) | 140, 140, 140, 140 | (0) | 140 |
| C (4 comp.) | 150, 145, 145, 140 | (0) | 145 |
| D (6 comp.) | 145, 155, 150, 150, 145, 140 | (0) | 148 |
| E (8 comp.) | 135, 155, 160, 150, 150, 145, 140, 135 | (0) | 146 |
| F (10 comp.) | 145, 145, 160, 150, 155, 160, 140, 125, 120, 130 | (0) | 143 |
| G (12 comp.) | 160, 150, 160, 160, 160, 150, 140, 160, 150, 125, 130, 150 | (−1) | 150 |
| H (16 comp.) | 150, 175, 175, 175, 175, 175, 175, 150, 125, 140, 150, 125, 110, 125, 125, 130 | (−1) | 149 |
| I (20 comp.) | 150, 225, 225, 225, 300, 300, 300, 200, 200, 175, 125, 125, 120, 100, 100, 100, 100, 120, 125, 125 | (−2) | 172 |
| J (16 comp.) | 250, 300, 350, 350, 300, 250, 200, 125, 125, 75, 60, 60, 75, 100, 100, 125 | (−2) | 178 |
| K (20 comp.) | 250, 500, 500, 550, 600, 400, 300, 250, 200, 150, 110, 100, 50, 75, 100, 100, 100, 100, 100, 150 | (−4) | 234 |
| L (24 comp.) | 300, 600, 750, 1000, 1300, 1100, 1200, 950, 450, 300, 200, 100, 100, 100, 50, 50, 50, 50, 50, 50, 50, 50, 100, 150 | (−9) | 379 |
| M (14 comp.) | 1500 +1, 1700 +1, 2100 +1, 2000 +1, 1500 +1, 800, 300, 500, 100, 100, 100, 100, 100, 500 | (−48) | 814 |
| N (16 comp.) | 3000, 4000 +1, 5200 +2, 5300 +2, 5000 +2, 2600, 1200, 300, 100, 100, 500, 1000, 1100, 300, 200, 1000 | (−103) | 1931 |
| O (28 comp.) | 5500 −1, 5700 −1, 6300 −1, 6500 −1, 7000 −1, 6500 −1, 7000 −1, 7800, 8000, 5500 −1, 3500 −1, 1000, 500, 900, 2000 −1, 1000, 700, 400, 300, 600, 1100 −1, 2000 −1, 2500 −1, 2000 −1, 950, 400, 1100 −1, 4500 −1 | (−169) | 3259 |

$\Sigma C = -4\ (\times 10^{-5})$

Compartment divisions:

| Zone | Readings (Ft.; Fms.) | Mean | Cor. |
|---|---|---|---|
| 18 (1 comp.) | 6000, 5200, 5800, 7500, 800, 1200, 25, 900 15, 4000, 3000 | 3425 | −34 |
| 17 (1 comp.) | 5500, 5500, 6000, 7500, 800, 600 50, 100, 500 100, 4000, 3000 | 3248 | −32 |
| 16 (1 comp.) | 5500, 5000, 7000, 7000, 900, 200 70, 1000, 700, 4000, 3500 | 2657 | −27 |
| 15 (1 comp.) | 5600, 4800, 5000, 6500, 1500, 300 750, 1700, 1500, 2000 75, 4500 | 1535 | −15 |
| 14 (1 comp.) | 5800, 5000, 6000, 7000, 1000, 1100, 2100, 1900, 600 250, 6000 | 1166 | −12 |

| Zone | Entries | Total |
|---|---|---|
| 13 (16 comp.) | −5, −6, −5, −7, −6, −5, −3, −1, +6, +8, +9, +8, +8, +5, −1, −5 | −2 |
| 12 (10 comp.) | | +9 |
| 11 (8 comp.) | | +17 |
| 10 (6 comp.) | | +11 |
| 9 (4 comp.) | | +10 |
| 8 (4 comp.) | | +13 |
| 7 (2 comp.) | | +10 |
| 6 (18 comp.) | | +9 |
| 5 (16 comp.) | | +9 |
| 4 (12 comp.) | | +8 |
| 3 (10 comp.) | | +5 |
| 2 (6 comp.) | | +4 |
| 1 (1 comp.) | | +1 |

(The corrections for zones 12 to 1 were interpolated)

Bullard terms:
- A correction (1.118 × 42.8) = +48 (× 10⁻⁴)
- B correction (curvature) = + 1
- C correction = 0

| | |
|---|---|
| Topog. correction, letter zones | = +49 |
| Comp. correction, letter zones | = −339 |
| Topog. and comp., numbered zones | = −14 |
| Total | = −304 |

**Maps used:** U.S.G.S.: Antelope, Roseville, Arcade, Sacramento, Placerville, Davisville, Smartsville
Aeronautical charts
Nautical chart No. 5002

**Total correction** = −304

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FIGURE 28.—Map readings and computations of effects of topography and isostatic compensation.

The compensation corrections for the individual lettered zones are computed by means of the factors on page 69 after the mean elevations of the zones have been computed. The resulting corrections are encircled on the form to make them more easily distinguished from the other data. The secondary compensation corrections mentioned on page 69, which take account of the difference of elevation of the station and each compartment, have not been considered in the example, as they are quite small.

The topography corrections for the lettered zones are not taken out for individual compartments but the topography for the entire cap extending to the outer limits of zone O is computed, as a whole, by the Bullard method explained on page 67. The computation of the A and B Bullard corrections are shown in the lower right hand corner of figure 28. The C correction is taken out for individual compartments, where appreciable, by use of the table on page 68. It is shown at the right hand edge of the column for a few of the compartments in zones M, N, and O. It should be noted that these individual C corrections are given in the fifth decimal place of gals. The remainder of the computation on the form is in the fourth decimal place.

As previously explained, zones 18 to 14 are each divided into 10 subcompartments for convenience in reading the elevations and to permit greater accuracy of estimation. The average elevation of each zone as thus derived is then divided by 100 to obtain the principal part of the correction for each zone. If this part of the correction is large, there is a small, additional, secondary correction for lack of proportionality as explained on page 70.

In the example shown in figure 28, the corrections for zone 13 were taken out directly by dividing the elevations (in feet) for the land compartments by 1,000 and by dividing the depths (in fathoms) for the water compartments by 271. Zones 12 to 1 were interpolated. All interpolations were computed, that is, none were taken from the Heiskanen maps described on page 72.

In all the numbered zones, the corrections for land compartments are always negative and for water compartments, positive. A list of the maps and charts used in making the reductions is shown at the bottom of the form.

## PRINCIPAL FACTS FOR GRAVITY STATIONS

The various steps required in the observation and computation of a gravity determination, including the reductions for topography and isostatic compensation, have been described in more or less detail in the preceding pages. The final step in completing the data for a gravity determination and in making these data available in convenient form for whatever purpose they may be needed is the listing of all the observed and computed quantities in tabular form as shown in figure 29. The first nine columns of the table in figure 29 contain the basic data for the station and require no special explanation. The computed data in the remaining columns of the table are discussed briefly in the following pages.

The theoretical value of gravity at sea level, in the tenth column, is based on the International formula and may be interpolated directly from the table on page 62, with the latitude of the station as the argument. The various corrections which are applied to the theoretical value to obtain computed values of gravity at the location and elevation of the station, according to several of the more common methods in use for this purpose, are listed in the next six columns.

The correction for elevation in the eleventh column of figure 29 is computed by the formula given on page 65. The correction for topography to zone O, in the twelfth column, is the sum of the Bullard A, B, and −C corrections for the lettered zones. (See explanation on p. 67 and the example in fig. 28.) This latter correction is used in computing the Bouguer anomalies. (See p. 77.)

The correction for indirect effect, in the thirteenth column, is a small correction for the gravitational effect of the relatively thin mass which is included between the spheroid and the geoid. It is required for the precise comparison of gravity values over wide areas in order to eliminate the effects of undulations of the geoid. It would not be needed if the computed value of gravity at a station were based on the geoid instead of on a mathematical spheroid. The International formula and similar formulas for computing gravity, however, are necessarily based on the spheroid, while the correction for elevation of station is necessarily based on its elevation above the geoid since its elevation above the spheroid is not known. This tends to produce a systematic error, especially as between continental and oceanic areas, because, in general, the geoid is above the spheroid over the continents and below it over the oceans. A part of the systematic difference between continental and oceanic gravity anomalies, which has been apparent for many years, has undoubtedly been due to the neglect of this correction. It is also suspected that a part of the longitude term in the gravity formulas derived by Helmert and Heiskanen may have been due to this cause. (See p. 61.)

A mathematical discussion of the indirect effect and tables for computing it will be found in Special Publication No. 199 of this Bureau entitled, "Tables for determining the form of the geoid and its indirect effect on gravity." It should be noted that the tables for indirect effect are based on a depth of compensation of 96 kilometers. This depth was determined by Bowie as the most probable one for the area of the United States.[12] The corrections for other depths may readily be computed by means of fundamental tables included in Special Publication No. 199.

The next three columns of the table (fig. 29) contain the corrections for topography and compensation for three different depths of compensation. The correction for depth 113.7 km. is the first one computed since this is the depth on which the Hayford-Bowie tables are based. The corrections for the other two depths are then obtained by the use of the factors in the following table. It should be noted that the quantities obtained by the use of the factors must be added algebraically to the corrections for depth 113.7 km.

*Factors for changing depth of compensation from 113.7 km. to other depths*

LETTERED ZONES

[Multiply *compensation* correction for depth 113.7 km. by given factor and add algebraically]

| Zone | Factors for depths of— | | | Zone | Factors for depths of— | | |
|---|---|---|---|---|---|---|---|
| | 56.9 km. | 96 km. | 127.9 km. | | 56.9 km. | 96 km. | 127.9 km. |
| A | +1.00 | +0.18 | −0.11 | I | +0.87 | +0.17 | −0.10 |
| B | +1.00 | +.18 | −.11 | J | +.80 | +.16 | −.10 |
| C | +1.00 | +.18 | −.11 | K | +.70 | +.15 | −.10 |
| D | +1.00 | +.18 | −.11 | L | +.55 | +.13 | −.09 |
| E | +.98 | +.18 | −.11 | M | +.24 | +.08 | −.06 |
| F | +.97 | +.18 | −.11 | N | −.10 | +.01 | −.02 |
| G | +.95 | +.18 | −.11 | O | −.31 | −.06 | +.03 |
| H | +.92 | +.18 | −.11 | | | | |

[12] Investigations of gravity and isostasy, by William Bowie, U. S. Coast and Geodetic Survey Special Publication No. 40, p. 133.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 669
Rev. April 1938

# PRINCIPAL FACTS AT GRAVITY STATIONS

[POTSDAM SYSTEM]

[*Base value at Washington, Commerce Bldg., 1933 determination, is 980.118*]

International formula, $\gamma_0 = 978.049\ [1 + 0.0052884 \sin^2\phi - 0.0000059 \sin^2 2\phi]$ gals.

| Station No. | Station Name | State | Chief of party | Year | Latitude $\phi$ (° ') | Longitude $\lambda$ (° ') | Elevation (Meters) | Observed gravity $g$ (gals) | Theoretical gravity $\gamma_0$ (gals) | Correction: Elevation (gal) | Correction: Topography to zone O (gal) | Correction: Indirect effect* (gal) | Topography and compensation: Depth 56.9 km. (gal) | Topography and compensation: Depth 96 km. (gal) | Topography and compensation: Depth 113.7 km. (gal) | Anomaly: Free-air $g-\gamma$ (gal) | Anomaly: Bouguer $g-\gamma''$ (gal) | Isostatic (Pratt-Hayford): Indirect* $g-\gamma_{ci}$ (gal) | Isostatic: Depth 56.9 km. $g-\gamma_c$ (gal) | Isostatic: Depth 96 km. $g-\gamma_c$ (gal) | Isostatic: Depth 113.7 km. $g-\gamma_c$ (gal) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1082 | Pohick | Va. | R.W.Woodworth | 1940 | 38 43.2 | 77 12.7 | 23.0 | 980.087 | 980.0673 | -.0071 | +.0026 | -.0026 | -.0018 | +.0013 | +.0025 | +.027 | +.024 | +.028 | +.029 | +.026 | +.024 |
| 1083 | Leesburg | Va. | " " | 1940 | 39 05.8 | 77 34.8 | 105.8 | 980.091 | 980.1005 | -.0326 | +.0121 | -.0027 | -.0012 | +.0013 | +.0027 | +.023 | +.011 | +.024 | +.024 | +.022 | +.020 |
| 1084 | Celina | Ohio | " " | 1940 | 40 32.8 | 84 32.8 | 269.5 | 980.153 | 980.2291 | -.0832 | +.0305 | -.0028 | +.0022 | +.0046 | +.0053 | +.007 | -.023 | +.005 | +.005 | +.002 | +.002 |
| 1085 | Wright Field | Ohio | " " | 1940 | 39 46.6 | 84 05.9 | 247.8 | 980.094 | 980.1607 | -.0765 | +.0280 | -.0028 | -.0011 | +.0011 | +.0020 | +.010 | -.018 | +.012 | +.011 | +.009 | +.008 |
| 1086 | Mason | Mich. | " " | 1940 | 42 34.6 | 84 25.8 | 274.4 | 980.320 | 980.4109 | -.0847 | +.0311 | -.0027 | +.0041 | +.0068 | +.0077 | -.006 | -.037 | -.010 | -.010 | -.013 | -.014 |
| 1087 | Bay City | Mich. | " " | 1940 | 43 39.9 | 83 54.2 | 178.1 | 980.444 | 980.5090 | -.0550 | +.0202 | -.0026 | -.0006 | +.0002 | +.0006 | -.010 | -.030 | -.008 | -.009 | -.010 | -.011 |
| 1088 | West Branch | Mich. | " " | 1940 | 44 16.1 | 84 13.1 | 279.8 | 980.465 | 980.5634 | -.0863 | +.0317 | -.0027 | +.0043 | +.0072 | +.0083 | -.012 | -.044 | -.017 | -.016 | -.019 | -.020 |
| 1089 | Ludington | Mich. | " " | 1940 | 43 56.9 | 86 24.8 | 191.6 | 980.456 | 980.5344 | -.0591 | +.0217 | -.0026 | +.0024 | +.0029 | +.0032 | -.019 | -.041 | -.020 | -.022 | -.022 | -.022 |
| 1090 | Three Rivers | Mich. | " " | 1940 | 41 56.5 | 85 38.1 | 240.7 | 980.291 | 980.3539 | -.0743 | +.0272 | -.0027 | +.0018 | +.0042 | +.0049 | +.011 | -.016 | +.010 | +.010 | +.007 | +.006 |
| 1091 | Wabash | Ind. | R.W. Woodworth | 1940 | 40 47.9 | 85 49.6 | 208.2 | 980.166 | 980.2516 | -.0642 | +.0236 | -.0027 | -.0014 | +.0005 | +.0009 | -.021 | -.045 | -.019 | -.020 | -.022 | -.022 |
| 1092 | West Lafayette | Ind. | " " | 1940 | 40 25.0 | 86 55.6 | 185.8 | 980.149 | 980.2176 | -.0573 | +.0211 | -.0026 | -.0019 | -.0008 | -.0004 | -.011 | -.032 | -.008 | -.009 | -.010 | -.011 |
| 1093 | Streator | Ill. | " " | 1940 | 41 09.1 | 88 49.5 | 190.0 | 980.226 | 980.2832 | -.0586 | +.0214 | -.0026 | -.0003 | +.0006 | +.0008 | +.001 | -.020 | +.003 | +.002 | +.001 | +.001 |
| 1094 | Franksville | Wis. | " " | 1940 | 42 44.2 | 87 54.3 | 228.4 | 980.338 | 980.4253 | -.0705 | +.0258 | -.0026 | +.0038 | +.0050 | +.0053 | -.017 | -.043 | -.019 | -.021 | -.022 | -.022 |
| 1095 | Oshkosh | Wis. | " " | 1940 | 44 01.4 | 88 31.2 | 228.0 | 980.438 | 980.5412 | -.0703 | +.0258 | -.0026 | -.0008 | +.0004 | +.0007 | -.033 | -.059 | -.031 | -.032 | -.033 | -.034 |
| 1096 | Merrill | Wis. | " " | 1940 | 45 11.8 | 89 41.5 | 408.2 | 980.517 | 980.6471 | -.1260 | +.0462 | -.0029 | +.0042 | +.0075 | +.0089 | -.004 | -.050 | -.009 | -.008 | -.012 | -.013 |
| 1097 | Grand Rapids | Minn. | " " | 1940 | 47 14.9 | 93 31.3 | 395.5 | 980.701 | 980.8322 | -.1220 | +.0448 | -.0028 | +.0016 | +.0034 | +.0039 | -.009 | -.054 | -.010 | -.011 | -.013 | -.013 |
| 1098 | Bemidji | Minn. | " " | 1940 | 47 28.5 | 94 52.9 | 418.2 | 980.712 | 980.8526 | -.1290 | +.0474 | -.0028 | +.0034 | +.0054 | +.0059 | -.012 | -.059 | -.014 | -.015 | -.017 | -.018 |
| 1099 | Baudette | Minn. | " " | 1940 | 48 42.5 | 94 36.3 | 332.3 | 980.872 | 980.9634 | -.1025 | +.0377 | -.0028 | -.0015 | -.0005 | -.0001 | +.011 | -.027 | +.014 | +.013 | +.012 | +.011 |
| 1100 | Grand Forks | N.Dak. | " " | 1940 | 47 55.3 | 97 04.4 | 253.8 | 980.809 | 980.8928 | -.0783 | +.0288 | -.0027 | -.0052 | -.0067 | -.0069 | -.006 | -.034 | +.004 | .000 | +.001 | +.001 |
| 1101 | Devils Lake | N.Dak. | R.W. Woodworth | 1940 | 48 06.3 | 98 51.9 | 438.3 | 980.784 | 980.9092 | -.1352 | +.0496 | -.0029 | -.0001 | +.0009 | +.0014 | +.010 | -.040 | +.012 | +.010 | +.009 | +.009 |
| 1102 | Rugby | N.Dak. | " " | 1940 | 48 21.7 | 100 00.0 | 468.9 | 980.810 | 980.9322 | -.1447 | +.0531 | -.0030 | -.0011 | -.0003 | .0000 | +.022 | -.031 | +.026 | +.024 | +.023 | +.022 |
| 1103 | Minot | N.Dak. | " " | 1940 | 48 14.0 | 101 15.6 | 473.0 | 980.782 | 980.9207 | -.1459 | +.0536 | -.0031 | -.0068 | -.0066 | -.0063 | +.007 | -.046 | +.017 | +.014 | +.014 | +.014 |
| 1104 | Ray | N.Dak | " " | 1940 | 48 20.4 | 103 09.8 | 691.5 | 980.743 | 980.9303 | -.2133 | +.0782 | -.0033 | +.0040 | +.0059 | +.0063 | +.026 | -.052 | +.023 | +.022 | +.020 | +.020 |
| 1105 | Dickinson | N.Dak | " " | 1940 | 46 53.1 | 102 47.3 | 743.1 | 980.606 | 980.7994 | -.2292 | +.0840 | -.0034 | +.0004 | +.0020 | +.0028 | +.036 | -.048 | +.037 | +.035 | +.034 | +.033 |
| 1106 | Beulah | N.Dak. | " " | 1940 | 47 15.8 | 101 46.7 | 542.6 | 980.676 | 980.8335 | -.1674 | +.0614 | -.0032 | -.0683 | -.0074 | -.0070 | +.010 | -.052 | +.020 | +.018 | +.017 | +.017 |
| 1107 | Jamestown | N.Dak. | " " | 1940 | 46 55.3 | 98 41.0 | 455.3 | 980.656 | 980.8028 | -.1405 | +.0516 | -.0029 | +.0002 | +.0012 | +.0016 | -.006 | -.058 | -.005 | -.006 | -.008 | -.008 |
| 1108 | Lidgerwood | N.Dak. | " " | 1940 | 46 04.9 | 97 08.8 | 330.7 | 980.608 | 980.7270 | -.1020 | +.0375 | -.0028 | -.0037 | -.0044 | -.0043 | -.017 | -.054 | -.010 | -.013 | -.013 | -.013 |
| 1109 | Fargo | N.Dak | " " | 1940 | 46 53.4 | 96 48.0 | 274.4 | 980.715 | 980.7999 | -.0847 | +.0311 | -.0027 | -.0045 | -.0058 | -.0060 | .000 | -.031 | +.008 | +.004 | +.006 | +.006 |
| 1110 | Murphysboro | Ill. | " " | 1941 | 37 46.0 | 89 20.7 | 127.3 | 979.947 | 979.9837 | -.0393 | +.0144 | -.0025 | -.0019 | -.0012 | -.0013 | +.003 | -.012 | +.006 | +.004 | +.004 | +.004 |

* The indirect effect and anomaly are based on a depth of 96 km. (See explanation on separate sheet)

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FIGURE 29.—Table of principal facts at gravity stations.

*Factors for changing depth of compensation from 113.7 km. to other depths*—Con.

NUMBERED ZONES

[Multiply correction for *topography and compensation* for depth 113.7 km. by given factor and add algebraically]

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 18 | −0.41 | −0.10 | +0.06 | 9 | −0.50 | −0.15 | +0.13 |
| 17 | −.42 | −.10 | +.07 | 8 | −.50 | −.16 | +.13 |
| 16 | −.43 | −.11 | +.08 | 7 | −.50 | −.16 | +.13 |
| 15 | −.45 | −.13 | +.09 | 6 | −.50 | −.16 | +.13 |
| 14 | −.46 | −.14 | +.10 | 5 | −.50 | −.16 | +.13 |
| 13 | −.47 | −.14 | +.11 | 4 | −.50 | −.16 | +.13 |
| 12 | −.49 | −.15 | +.12 | 3 | −.50 | −.16 | +.13 |
| 11 | −.49 | −.15 | +.12 | 2 | −.50 | −.16 | +.13 |
| 10 | −.50 | −.15 | +.13 | 1 | −.50 | −.16 | +.13 |

## FREE AIR ANOMALIES

The last six columns of the table in figure 29 contain the gravity anomalies, or differences between the observed and computed values of gravity, according to several different systems of calculation. The free air anomalies are very easily derived. The theoretical value of gravity at sea level is simply corrected for the elevation of the station and then the difference is taken between the observed value of gravity and this corrected theoretical value. The free air method may therefore be considered as based on the assumption that the effects of topography and compensation, which are always of opposite sign, exactly balance each other. Except in mountainous areas, this assumption is a reasonably accurate one.

In low, flat topography the free air method gives anomalies which are about of the same magnitude as those obtained by the isostatic method. In high, rugged topography, however, the free air anomalies are larger in general than the isostatic and they are very much larger at occasional stations. They tend to be positive if the stations are above the general level of the region in which they are located, and they tend to be negative if the stations are below the general level.

## BOUGUER ANOMALIES

The Bouguer anomalies are given in the fifth column from the right of figure 29. In the Bouguer method two corrections are applied to the theoretical value of gravity at sea level before it is compared with the observed value to obtain the anomaly. One of these is the elevation correction, the same correction as that used in the free-air method, and the other is a correction for the effect of the topography out to and including zone O. The latter is the correction given in the twelfth column of the table. The Bouguer method makes no allowance for the compensation effect of the topography to zone O and neglects the effect of all distant topography and compensation. Bouguer anomalies can be computed quite easily, especially in areas where the topography is not too rugged. They are used quite extensively for geophysical prospecting to find variations in underground structure over comparatively small local areas where the topographic relief is not too great. Over extended areas the Bouguer anomalies are quite closely related to the height of the topography and are ordinarily strongly negative for mountain stations.

## ISOSTATIC ANOMALIES

The last four columns of the table in figure 29 all contain isostatic anomalies. The last three differ only in regard to the assumed depth of compensation. The anomalies in the column headed "Indirect" are the same as those in the column headed "96 km" with the exception that the former include the correction for indirect effect given in the thirteenth column.

In computing the isostatic anomalies, the theoretical value of gravity at sea level is not only corrected for the elevation of the station but it is also corrected for the effect of the topography of the whole world and for the effect of the isostatic compensation of that topography according to the Pratt-Hayford theory as to the nature of this compensation. The methods employed in computing the effects of the topography and compensation have been outlined in the earlier parts of this chapter.

If the isostatic adjustment over the surface of the earth were everywhere complete according to the Pratt-Hayford hypothesis and if there were no large masses of rock in the outer part of the earth with densities differing too much from those assumed by Hayford and Bowie in computing their fundamental tables, then the isostatic anomalies would presumably be very small. The large number of isostatically reduced gravity stations which have been accumulated over extensive areas of the earth's surface indicates quite definitely that the so-called crust of the earth is very closely in isostatic equilibrium except in a few disturbed areas such as the East and West Indies where orogenic forces are now in operation.

Except in these disturbed areas, it is believed that the isostatic anomalies give a very good indication of the location of concealed masses of rock having densities greater or less than normal. Stations over light sedimentary formations, for example, usually have negative anomalies and over heavy pre-Cambrian formations positive anomalies. The reader may wonder why the effects of these abnormal masses are not computed and included with the effects of topography and compensation. The principal reason they are not is that exact information regarding the density and the horizontal and vertical extent of the abnormal masses is often not known or available. Even if it were, however, the difficulty of computing the precise effects would be very great.

Instead of attempting to eliminate the effect of abnormal masses from the anomaalies, it is more useful to compute the anomalies according to the standard method based on the average density of surface rock and assume that the anomalies give some indication of the existence and location of abnormal masses of large extent. The pendulum method of determining gravity is too slow and cumbersome to be used in detecting small abnormalities below the earth's surface. Gravimeters are much more useful for local investigations of this sort. The pendulum method, however, may be used advantageously to detect large structural trends and to furnish basic points to which local gravimeter surveys may be connected.

# INDEX

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