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no. 237
1947

U. S. DEPARTMENT OF COMMERCE
W. AVERELL HARRIMAN, Secretary
COAST AND GEODETIC SURVEY
LEO OTIS COLBERT, Director

Special Publication No. 237

MANUAL OF GEODETIC ASTRONOMY

DETERMINATION OF LONGITUDE, LATITUDE, AND AZIMUTH

By

Commander ALBERT J HOSKINSON

and

J. A. DUERKSEN, Senior Mathematician

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UNITED STATES
GOVERNMENT PRINTING OFFICE
WASHINGTON : 1947

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National Oceanic and Atmospheric Administration

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PREFACE

The purpose of this publication is to set forth the methods currently used by this Survey in its astronomical work in the field and to describe in some detail the office methods used in computing and processing the field records. Much of the material in a previous manual, Special Publication No. 14, "Determination of Time, Longitude, Latitude and Azimuth," has become obsolete, or nearly so, by the improvement in astronomical instruments and equipment in recent years.

This manual covers only those instruments now in use by the U. S. Coast and Geodetic Survey and does not include discussions of experimental work with newer instruments or estimates of the usefulness of such instruments.

The material in this manual is the product of the accumulated experience in office and field of the members of the U. S. Coast and Geodetic Survey from its earliest days to the present.

This publication, which is the work of the Section of Gravity and Astronomy of the Division of Geodesy, has been made possible by valuable contributions of material and suggestions, both by officers of this Bureau who, at one time or another, have been assigned to astronomical work, and by various mathematicians of the Division of Geodesy who either have had assignments with this Section or else have taken a keen interest in some phase or phases of the subjects dealt with in the manual.

This publication has been prepared under the direction of Commander H. W. Hemple, Chief, Division of Geodesy, and under the general supervision of Walter D. Lambert, former Chief of the Section of Gravity and Astronomy, who is also responsible for several portions of the Appendix. The authors especially wish to acknowledge the substantial debt owed to F. W. Darling, Mathematician, for his share in the appendix and tables and for his share in writing those sections of Parts II and III dealing with office procedure, and to Lieutenant H. J. Seaborg for the use of his manuscript prepared about 1943, on "Determinations of Wireless Longitude." A large part of his material dealing with field practice in longitude determinations has been incorporated in this manual. Thanks are also due to C. N. Claire, Mathematician, who edited the manuscript and served as technical adviser on instruments and radio equipment, and to A. D. Sollins, Associate Mathematician, whose critical judgment and mathematical background were of great value.

Finally the authors wish to thank the numerous other members of the U. S. Coast and Geodetic Survey, both in the field and in the Office, who have so cheerfully contributed their services in one way or another.

The history and development of astronomical work is more or less reflected in various papers and treatises on the determination of time, latitude, and azimuth which have appeared in the Annual Reports or as Special Publications of the Coast and Geodetic Survey.

The titles of some papers, appearing in earlier Annual Reports of the Coast and Geodetic Survey, which were intended to form the basis of an astronomical manual are:

Determination of Time by the Transit Instrument, C. A. Schott. Appendix No. 9, Annual Report of the Coast Survey of 1866.

Determination of the Astronomical Latitude of a Station by Means of the Zenith Telescope, C. A. Schott. Appendix No. 10, Annual Report of the Coast Survey of 1866.

Addenda to Appendices No. 9 and No. 10 of the Coast Survey Report of 1866, C. A. Schott, Appendix No. 10, Annual Report of the Coast Survey of 1868.

Determination of Weights to be given for Observations for Determining Time with Portable Instruments Recorded by the Chronographic Method, C. A. Schott. Appendix No. 12. Annual Report of the Coast Survey of 1872.

The above group of papers is usually referred to as the first edition.

The reprint of these papers as they appear in Professional Papers: Determination of Time, Latitude and Azimuth, 1876, is considered the second edition.

What is known as the third edition by C. A. Schott appeared as Appendix No. 14 of the 1880 Report.

The fourth edition by J. F. Hayford appeared as Appendix No. 7 of the 1898 Report.

The fifth edition by William Bowie appeared as Special Publication No. 14 in 1917.

These publications deal with certain types of instruments not discussed in this publication. These older instruments, although excellent of their kind, and although still in the possession of this Bureau have been found in general less serviceable than the broken-telescope transit of the Bamberg type, which is now the main reliance of the Bureau's astronomical parties in the field. In case of necessity these older instruments could again be used.

The general theory of these instruments, especially the meridian telescope and the zenith telescope, is essentially the same as that of the Bamberg instrument when used for time or latitude but certain details of construction require at times a somewhat different manipulation and a somewhat different arrangement of the computation from that of the Bamberg instrument. For particulars regarding these older instruments the publications listed above should be consulted, especially Special Publication No. 14, Fifth Edition.

Since then, two more publications have been issued which deal particularly with the description of the Bamberg broken-telescope transit and its application to longitude and latitude observations; one of them also introduces the use of radio or wireless telegraphy for longitude determination. These publications are:

Special Publication No. 35, Determination of the Difference in Longitude between each two of the Stations, Washington, Cambridge, and Far Rockaway, by Fremont Morse and O. B. French, 1916.

Special Publication No. 109, Wireless Longitude, by G. D. Cowie and E. A. Eckhardt, 1924.

These last two publications are no longer available, for they have been out of print for more than ten years.

MANUAL OF GEODETIC ASTRONOMY

DETERMINATION OF LONGITUDE, LATITUDE, AND AZIMUTH

INTRODUCTION

The orientation of the triangulation executed by this Bureau is controlled by Laplace azimuths placed at an average interval of about 8 triangulation quadrilaterals. This orientation control is necessary because of the probability of a large accumulation of error in an azimuth carried through a long arc of triangulation.

Because of the existence of the deflection of the vertical or what is sometimes called "station error", it is not possible to use the astronomical azimuth directly in controlling the orientation of the triangulation. If, however, both an astronomical longitude and an astronomical azimuth have been observed at a triangulation station, the astronomical azimuth may be corrected for the effect of the deflection of the vertical in the following manner: η, the component of the deflection in the prime vertical (positive when the astronomical zenith is west of the geodetic zenith) is given by either of the following expressions:

$$\eta = -(\alpha_A - \alpha_G) \cot \phi, \tag{1}$$

and

$$\eta = (\lambda_A - \lambda_G) \cos \phi \tag{2}$$

where

λ_A = astronomical longitude
λ_G = geodetic longitude
α_A = astronomical azimuth
α_G = geodetic or Laplace azimuth.

We may equate these two expressions for η and solve either for λ_G in terms of λ_A α_A, and α_G or for α_G in terms of λ_A, α_A, and λ_G. Since λ_G is well determined by the triangulation and since it is desired to control the orientation of the triangulation, the second of these alternatives is chosen and α_G, the Laplace azimuth, is solved for as follows:

$$\alpha_A - \alpha_G = -(\lambda_A - \lambda_G) \sin \phi \tag{3}$$

or

$$\alpha_G = \alpha_A + (\lambda_A - \lambda_G) \sin \phi. \tag{4}$$

For a more detailed development of the deflection of the vertical and the corresponding Laplace correction to the astronomical azimuth, see "Astronomic Determinations", Special Publication No. 110, pp. 87–92.

Frequently, the triangulation party observes the astronomical azimuth at the proposed Laplace station using another triangulation station as the mark for the azimuth observations. Later, the astronomical party will observe the astronomical longitude and thus the necessary field data are available for the computation of the Laplace azimuth. In cases where the triangulation party has not observed the astronomical azimuth, it is the duty of the astronomical party to observe the azimuth also. In addition, the astronomical party makes an astronomical latitude determination at each station

whenever this does not seriously delay the progress of the party; since, for studies of the figure of the earth, both components of the deflection of the vertical are needed.

The astronomical party normally consists of 4 men, the chief of party, one recorder, one radioman and a hand. The chief of party directs and supervises all the work; usually he also does a great deal of the observing and computing. The recorder, in addition to his obvious duties makes some of the astronomical observations and computations. The radioman has immediate charge of keeping all the radio equipment functioning properly and helps with the recording, observing and computing. The hand assists wherever needed, and does much of the manual labor and the truck driving.

The most convenient and economical method of transportation is by motor truck. Two 1½-ton trucks and a combination house-trailer have been found very satisfactory. The trailer should be large enough to house 4 men and still provide sufficient room for office work. The usual procedure is to camp right at the station site in order to facilitate the work program. See figure 13.

Before starting out on an astronomical expedition, the chief of party should check all instruments and equipment carefully for completeness and for being in good working order. The instruments and equipment should be packed in boxes which will properly safeguard the delicate instruments and which can be most conveniently handled in the loading and unloading of trucks.

The Bamberg broken-telescope transit has been used extensively by this Bureau for determining latitude and longitude since 1916.[1] It is a good portable instrument, not too heavy and cumbersome and yet relatively stable for precise work.

In general, the direction theodolite is used for azimuth observations, although occasionally the repeating theodolite is used. For azimuth observations in high latitudes a method has been devised using the Bamberg broken-telescope transit.

INSTRUMENTS AND EQUIPMENT

The following is a list of the instruments and equipment necessary for astronomical observations when the Bamberg transit is used. A first-order theodolite and signal lamps are also included because the astronomical party will have to observe the azimuth at those astronomical stations at which the triangulation party has, for any reason, failed to do so.

Instruments

Item	*No.*	*Item*	*No.*
Ammeter, pocket	1	Rule, slide	1
Amplifier, astronomic, (spare if available)	1	Receiver, short-wave radio (spare if available)	1
Barometer, aneroid	1	Scales, glass, chronograph reading	3
Binoculars, ordinary	1	Tape, steel, 30-meter	1
Chronograph, astronomical	1	Test set, radio, (circuit and voltage tester)	1
Chronometers, sidereal, 2 second break-circuit	2	Theodolite, first-order	1
Compass, magnetic	1	Thermometer, centigrade	1
Dividers, hairspring	1	Transit, 4-inch, ordinary	1
Filter, audio	1	Transit, astronomic, Bamberg	1
Headset, radio	2	Tribrachs	2
Lamp, signal, flashlight type	1	Tripod, aluminum for Bamberg	1
Lamps, signal, large	2	Typewriter, portable	1
Machine, calculating	1	Watches, mean-time	2
Pens, fountain, for chronograph	2		
Pens, fountain, fine point for recording	2		

[1] Special Publication No. 35, Determination of the Difference in Longitude between each two of the Stations, Washington, Cambridge, and Far Rockaway.

Equipment

Item	*No.*
Bag, recorder's	1
Batteries, storage, 6-volt	4 to 8
Batteries, 'B', 45-volt	15 to 30
Batteries, 'C', 22½-volt	6 to 10
Batteries, 1½-volt dry cell	10 to 20
Batteries, flashlight	50
Chairs, folding, canvas	2
Heater, oil or gasoline	1
Hydrometer, battery tester	1
Lamps, hand electric	3
Lantern, gasoline	1
Level, carpenter's, small	1
Packboards	4
Plaster of paris (25 to 50 pounds per station)	
Plumb bobs, ordinary	3
Scribers	1
Table, folding for recorder	1
Table, wood, knock-down for electrical equipment	1
Tarpaulins, large	4
Tent, astronomic, with frame and fly	1
Tent, silk, hexagonal observing with frame	1
Tubes, radio (complete set for radio used)	1 set
Tubes, amplifier, (complete set for amplifier)	1 set

As stated before, the astronomical party generally camps at the station site. This necessitates a complete camping outfit in addition to the equipment needed for the astronomical work. When the party is operating in remote regions throughout the country, and especially in Alaska or on some of the smaller islands, it is well to have spare parts and enough repair material along for minor repairs. The following additional list includes various tools and miscellaneous items:

Axes.
Bits, screwdriver and boring.
Brace, carpenter's.
Buckets.
Canteens.
Cans, gasoline.
Drill, hand.
Extinguisher, fire, Pyrene.
Generator, gasoline motor.
Hacksaw.
Hammers, claw.
Hatchets.
Pliers.
Saw, hand.
Screwdrivers.
Shovels.
Soldering iron.
Tarpaulins.
Trowels.
Wire, connecting and antenna.
Wrench, adjustable end.

BOOKS, FORMS, TABLES, AND DIAGRAMS

The following is a list of books, forms, and tables needed for the work.

Books

Special Publication No. 237. Manual of Geodetic Astronomy, Determination of Longitude, Latitude, and Azimuth.
Special Publication No. 120. Manual of First-Order Triangulation.
American Ephemeris and Nautical Almanac. (For the year.)
Apparent Places of Fundamental Stars. International Astronomical Union, His Majesty's Stationery Office, London. (For the year.)
Boss, General Catalogue of Stars, Epoch 1950.
The Radio Amateur's Handbook, A. R. R. L. (For the year.)
Vega, Logarithmic Tables.
Shortrede, Logarithms of Sines and Tangents.
Olcott, A Field Book of the Stars.
Special Publication No. 231, Natural Sines and Cosines to Eight Decimal Places.

Forms

FOR LONGITUDE:

No.
256 Longitude Record and Computation.
34A Longitude Record.
Chronograph sheets.

FOR LATITUDE:

485 Observing List for Latitude.
255 Latitude Observations.
33 Latitude Computations
772 Reduction, Mean to Apparent Declinations.

FOR AZIMUTH:

251A Observations of Horizontal Directions.
605 Comparison of Chronometer and Radio Signals.
381a Computation of Time, Observations on a Star with Vertical Circle.
380 Computation of Azimuth, Direction Method.
380a Computation of Azimuth, Micrometer Method.
380b Computation of Azimuth, Micrometer Method.

OTHER FORMS:

525a Description of Astronomical Station.
526 Recovery Note Triangulation Station.
382 Reduction to Center.
470 Abstract of Directions.
24A List of Directions.

Tables and Diagrams

Factors for Bessel's Probable Error Formulas.
Reduction of Mean Solar Time to Sidereal Time.
Diagram of Astronomical Amplifier Hookup.
Star-place Charts for finding Star Pairs.
G–35. P. G. C., and G. C. Star Numbers of Identical Stars.
Table of A, B, C Factors (for azimuth, level, and collimation corrections).
Nomogram for A, B, C Factors
List of Instrumental Constants.

PART I

DETERMINATION OF LONGITUDE

Instruments. The large transit instruments made by Troughton and Simms of London were used almost exclusively for telegraphic determination of astronomical longitudes until 1888. These were replaced at that time by a slightly smaller transit made in the Instrument Division of this Bureau. In 1914, two Bamberg broken-telescope transits were purchased. Since that time these transits have been used almost exclusively for both longitude and latitude observations except in those cases where the Bamberg transits were in use elsewhere. In the hands of skillful observers, all these transits give good results, and they compare favorably with the much larger transits usually employed at astronomical observatories, where special difficulties are encountered in consequence of strains or temporary instability of the instrument due to reversal of axis, and the more serious effect of flexure.

In addition, the Bureau has some smaller transits which are known as meridian telescopes. They were used for time determinations in connection with longitude observations and for latitude observations by the Horrebow-Talcott method.

LONGITUDE DETERMINATION WITH THE BAMBERG TRANSIT

The principal instruments used for radio longitude determinations are the Bamberg broken-telescope transit with transit micrometer, sidereal break-circuit chronometer, weight-driven chronograph, two-stage radio amplifier, short-wave radio receiver, and audio filter.

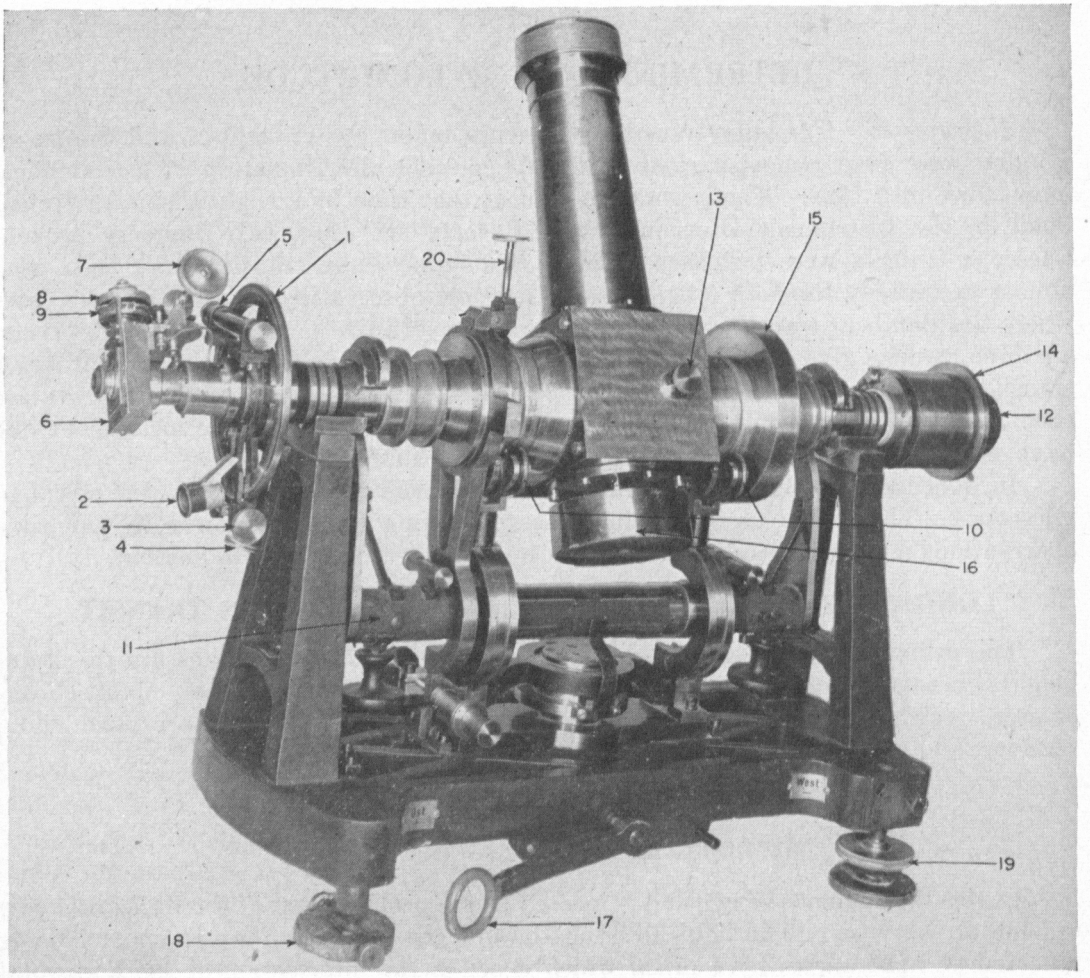
INSTRUMENTS

BAMBERG BROKEN-TELESCOPE TRANSIT

In this instrument the eye end is located in the prolongation of the horizontal axis outside of the wyes. The light entering the objective reaches the eye end by being reflected at right angles by a prism which is set in the intersection of the horizontal axis and the axis of the telescope. See Figures 1 and 2. The objective has a clear aperture of 7 centimeters and a focal length of 67 centimeters. The instrument is fitted with an ocular longitude micrometer which records electrically the times of the passage of a star through the field. See Figure 3. The micrometer screw has an equatorial value of about 10.5 seconds of time for one turn and the head of the micrometer has an agate rim in which are set 10 equidistant metallic strips, each strip being about one one-hundredth of a turn in width. In addition to these 10 strips, there are also two others, set one on each side of the zero strip and equidistant from it, to provide a means of identifying the marks of the record. A platinum point is pressed by a spring against the surface of the micrometer rim, and as the head is revolved this point makes a contact and completes a circuit each time one of the metallic strips passes under it, and a record is made on the chronograph sheet.

At the opposite end of the telescope axis from the micrometer there is arranged a very small electric light for the illumination of the cross hairs, and the light from it is

transmitted through the large prism to the eyepiece by means of a small prism not over one-third of a centimeter in size which is cemented with Canada balsam onto the diagonal face of the large prism. The faces of the two prisms must be accurately parallel to allow the passage of the light.



1. Setting circle.
2. Magnifying glass for reading setting circle.
3. Slow-motion adjustment screw for setting circle.
4. Setting circle clamping screw.
5. Setting circle level.
6. Longitude micrometer box.
7. Handwheel for operating longitude micrometer.
8. Graduated micrometer drum for longitude.
9. Agate micrometer drum with metal contact strips.
10. Friction wheels on springs supporting telescope axis.
11. Hanging level.
12. Housing for light which illuminates the crosshairs.
13. Knurled knob for adjusting light on crosshairs.
14. Counterpoise for micrometer eyepiece and setting circle.
15. Counterpoise for clamping arrangement.
16. Counterpoise for telescope tube.
17. Lever for lifting the telescope axis out of the wyes for reversing.
18. Footplate for adjusting instrument in azimuth.
19. One of the leveling footscrews.
20. Telescope clamping screw.

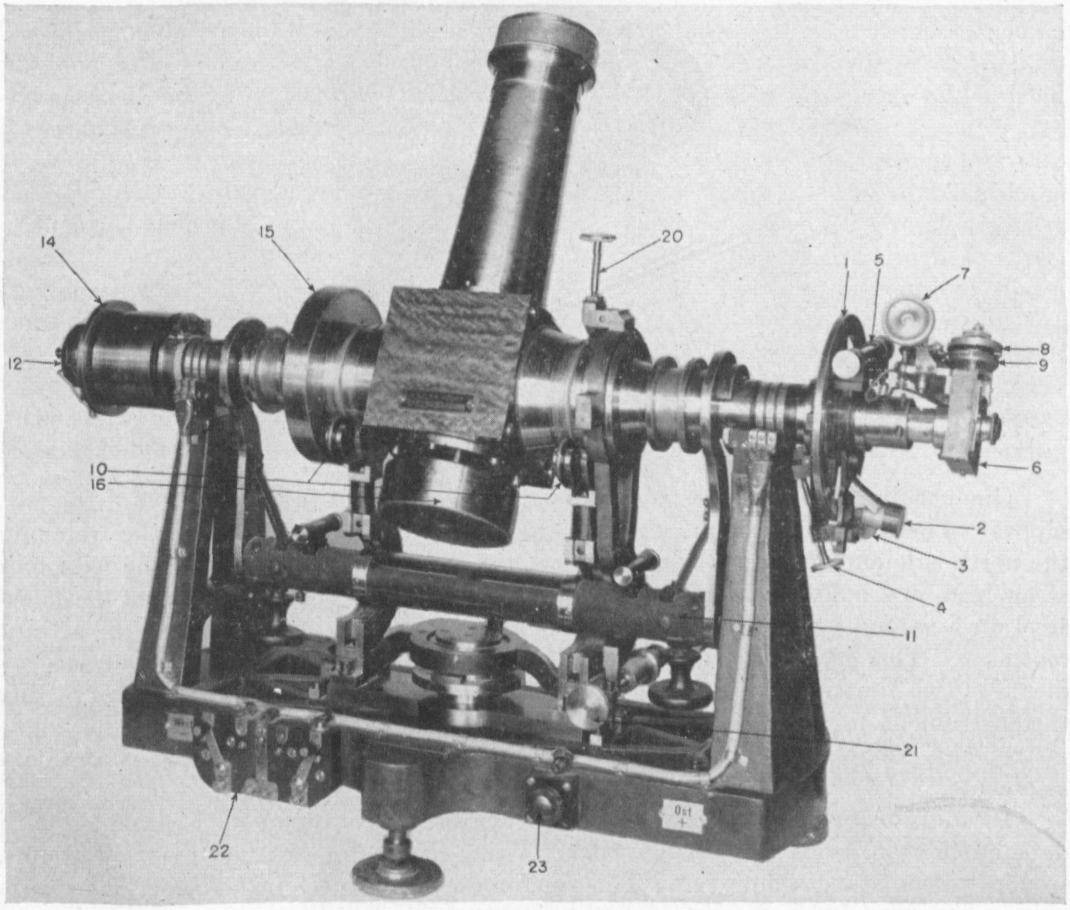
FIGURE 1.—Bamberg broken-telescope transit set up for longitude observations, north view.

A small setting circle, about 15 centimeters in diameter, is attached to the telescope axis back of the micrometer box at the eye end. This circle carries a movable vernier and level, and reads to one minute of arc. The graduations of the circle are numbered from 0 to 360, and the zero is so placed that the circle reads zenith distances for stars $\frac{\text{north}}{\text{south}}$ of the zenith when the eye end is $\frac{\text{east}}{\text{west}}$, and explements of zenith distances for

stars $\frac{\text{south}}{\text{north}}$ of the zenith when the eye end is $\frac{\text{east}}{\text{west}}$. The explement of the zenith distance is the angle that must be added to the zenith distance to make 360°.

A reversing apparatus is provided, and attached to it are four friction wheels running in two grooves cut in the horizontal axis of the telescope. These wheels, two of which form a sort of cradle, one on each side of the telescope tube, rest in turn on springs which are sufficiently strong to support the greater part of the weight of the telescope. Thus, only a small part of the weight of the telescope is borne by the pivots and wyes. This reduces flexure to a minimum and permits the telescope to be made heavier and more solid than would be desirable if all the weight were carried on the pivots.

Instead of the usual striding level the instrument is fitted with a hanging level, and, to permit its hanging directly below the axis of the telescope, each standard of the reversing gear is cast with a semicircular arc at a point about halfway of its length.



1. Setting circle.
2. Magnifying glass for reading setting circle.
3. Slow-motion adjustment screw for setting circle.
4. Setting circle clamping screw.
5. Setting circle level.
6. Longitude micrometer box.
7. Handwheel for operating longitude micrometer.
8. Graduated micrometer drum for longitude.
9. Agate micrometer drum with metal contact strips.
10. Friction wheels on springs supporting telescope axis.
11. Hanging level.
12. Housing for light which illuminates the crosshairs.
14. Counterpoise for micrometer eyepiece and setting circle.
15. Counterpoise for clamping arrangement.
16. Counterpoise for telescope tube.
20. Telescope clamping screw.
21. Telescope slow motion screw.
22. Illumination and transit circuit switches.
23. Rheostat for adjustment of brightness of illumination of crosshairs.

FIGURE 2.—Bamberg broken-telescope transit set up for longitude observations, south view.

748977°—48——2

Two small cross levels are attached to the hanging level, one near each end, to indicate when it is in proper position for reading.

The frame of the instrument is skeletonized, but strength has not been sacrificed, nor has it been made too light in weight. In size the base of the frame is 28 by 55 centimeters, with the top of the base of the frame 8 centimeters above the top of the pier.

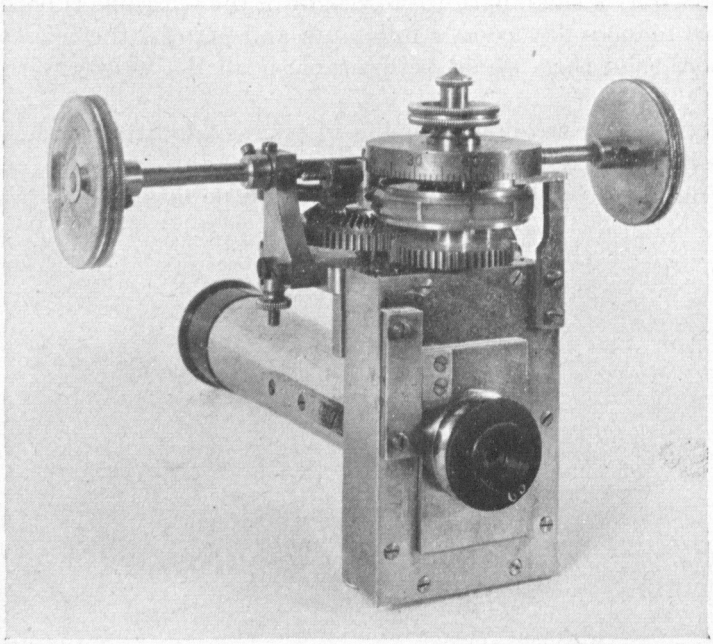


FIGURE 3.—Longitude micrometer eyepiece.

One peculiarity of the construction of the instrument is in the position of the foot screws. These are three in number, two being placed at the corners of the frame on the north side and the third being at the middle of the opposite side. The west foot screw rests in a hole in a footplate; the south one rests on a plane surface of hardened steel on a second footplate, while the east one rests in a V-shaped groove in a third footplate. This groove is cut in a steel block, which is movable in the footplate by means of two abutting screws. The object of this motion is to provide means of setting the instrument in the meridian. When the abutting screws are turned, the whole instrument, frame and all, revolves horizontally about the hole in the west footplate as a center.

In addition to what has already been described, the instrument is provided with detachable twin levels for latitude work. When in use for latitude determination a second micrometer is employed. This micrometer has no electric connections. It is arranged so that it can be used for time work, either by the key method or by eye and ear.

The care taken to avoid flexure by supporting the telescope on friction wheels has been mentioned. As another means to this end the telescope is very efficiently counterpoised. Thus, as a balance to the telescope tube, a weight is attached to the opposite side of the axis; as a counterpoise to the ocular micrometer and setting circle the mounting of the illuminating lamp is made heavy; as a balance to the clamp a counterweight

is provided on the opposite side; and finally, when the latitude levels are in use, a counterdisk is attached opposite them.

There are two separate electric circuits in the instrument, one for the illumination of the field and the other for transmitting the record of the transit micrometer, and they are so arranged as to be effective in both positions of the telescope. There are only three wires for the two circuits, one wire being common to both. Hence, in making the connections, the central, or common wire must be connected with like poles of the two batteries. The two circuits enter the instrument at the southwest corner, where there are four binding posts. Two switches are placed here, by means of which either circuit can be opened or closed. The three wires are insulated and lead along the frame of the instrument to three metal springs, which are fastened to a strip of hard rubber at the top of each standard. These springs press upward against three insulated metal bands in the axis just outside the pivots. The outer and middle bands are in the micrometer circuit and the inner and middle ones are in the illuminating circuit.

It is desirable to describe the micrometer and recording device more in detail. The micrometer screw, with its divided head and metallic strips already mentioned, is provided with a suitable train of beveled wheels, which are in turn connected with an axis located back of and above the micrometer box. This axis has a large milled head at each end for use, one with each hand, in following a star as it transits.

The back of the micrometer box is attached to a tube which fits accurately into the end of the telescope axis. A lug on the tube fits in a slot in the axis, and the slot is made larger than the lug, so as to allow the tube to be rotated through a small angle by means of two abutting screws for adjustment of the verticality of the movable wires. The focusing of the objective is done by moving the tube in or out as required, and a clamp is provided to hold the tube in place after the adjustment for focus is made.

The movable wires of the diaphragm are parallel and three in number. Two of them are placed close together, so that observations may be made by keeping the image of the star midway between them, and the third is separated from the pair by the space of about half a turn of the screw. Observations may be made with this wire by bisecting the star image.

From this point on, the phrase "follow the star" will be used for the cumbersome but more exact phraseology, "follow the image of the star."

There are five fixed wires in the reticule of the micrometer. Two of these are intended to indicate the path through which the star transits and are placed about 50 seconds of arc apart and parallel. These are of course at right angles to the movable wires. The other three fixed wires are parallel to the movable wires. One of them is in the middle and serves to define the line of collimation; the other two are placed one at each side of the field at a distance of five turns of the micrometer screw from the line of collimation and indicate where the observations on a star should begin and end. A small scale on the outside of the micrometer box numbered from 0 to 10 indicates the position of the movable wires. On the west end of the frame of the instrument there is a small plate bearing the inscription "0 R," and on the east end a similar plate is inscribed "10 R." These are intended to remind the observer that in position ocular West the micrometer should be set at 0 revolution in preparation for the approach of the star, while in position ocular East it should be set at 10 revolutions.

CHRONOGRAPH

Figure 4 shows the form of chronograph now in use in this Bureau. The train of gears seen at the right is driven by a falling weight. It drives the speed governor (seen

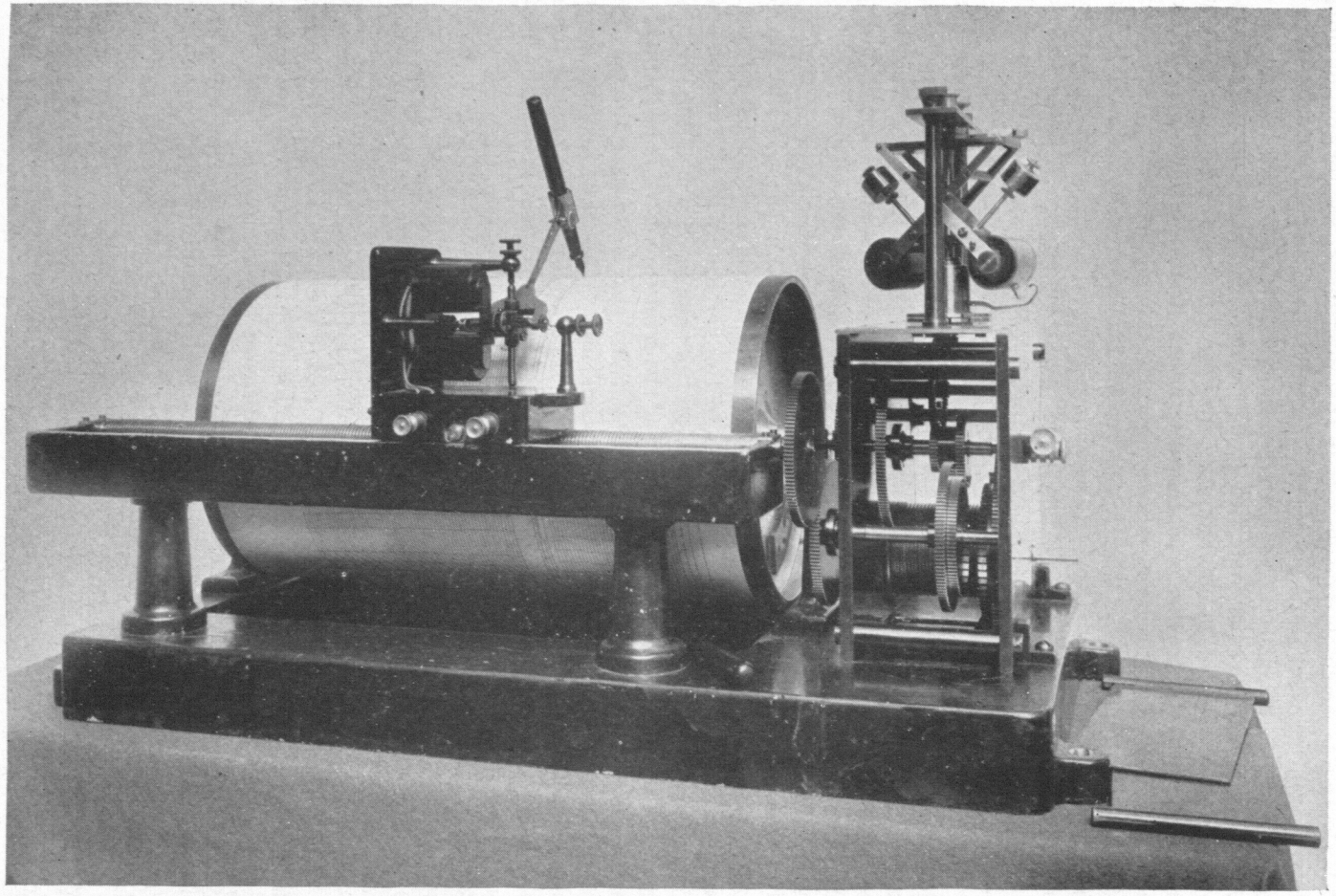


FIGURE 4.—Chronograph.

above the case containing the gears), the cylinder upon which the record sheet is wound, and the screw which gives the pen carriage a slow motion parallel to the axis of the record cylinder. When the speed governor is first released, the speed continually increases until the governor balls have moved far enough away from the axis of revolution to cause a small projection upon one of them to strike a small hook. This impact and the effect of the friction at the base of the weight attached to the hook causes the speed to decrease continually until the hook is released. The speed then increases again until the hook is engaged, decreases until it is released, and so on. The total range of variation in the speed is, however, surprisingly small, so small that in interpreting the record of the chronograph the speed is assumed to be uniform during the intervals between chronometer breaks. The speed may be regulated by screwing or unscrewing the movable weights which are above the governor balls and attached to the same arm. This moves them nearer to or farther from the axis, and thus decreases or increases the critical speed at which the hook is engaged. To get a convenient record it is desirable to adjust the speed so that the cylinder makes one revolution per minute with the ordinary arrangement of the train of gears. This is the speed used for recording transits.

The gears may be changed quickly to another combination by means of a clutch which is worked by moving the small horizontal shaft on the outer side of the gear box a short distance in or out. The center position of the shaft is neutral. In this position the cylinder is free to revolve, thus facilitating the placing of the chronograph sheet on the cylinder. When the clutch is set at the inner position the surface of the cylinder runs at a speed of about 1 centimeter per second. This position is normally used for the recording of the time of star transits. When set at the outer position, the cylinder runs at double speed, or about two centimeters per second. This speed is used for recording radio time signals. A small pin is inserted in the proper hole in the clutch shaft to maintain the proper speed position. Additional weights are required to drive the cylinder at double speed.

The cylinder is about 19 centimeters in diameter, and about 33 centimeters long, thus requiring a sheet of paper of about 33 by 60 centimeters. Actually a sheet 33 by 63 centimeters is used, thus allowing 3 centimeters for overlap. Since the circumference of the drum is 60 centimeters, one minute of time will be recorded on one line of the sheet for each revolution of the drum when the drum runs at a speed of one centimeter per second of time. This is particularly convenient for reading the chronograph sheet, in that corresponding second breaks will fall on a nearly horizontal line across the sheet. See figure 5.

The pen carriage holds a fountain pen, which traces a helix on the sheet attached to the chronograph cylinder. The pitch of the helix is about 2 millimeters. This pen is clamped in position at the outer end of the armature which normally is held away from the magnet by means of a small spring. The chronograph-pen armature is connected to the terminals of the output of the amplifier. See wiring diagram, figure 6.

CHRONOMETER

The chronometer is a two-second sidereal break-circuit chronometer. The chronometer is connected into the astronomical amplifier circuit as illustrated in figure 6. Normally, the chronometer circuit is closed. Every two seconds the break-circuit wheel moves the distance of one notch, a spring is released which trips a contact, and the circuit is thus broken for just an instant. For purpose of identification, at the end of each minute, some chronometers skip the 58th second break and others break on the 59th second. Thus, the identification of the beginning of a minute is positive. It is

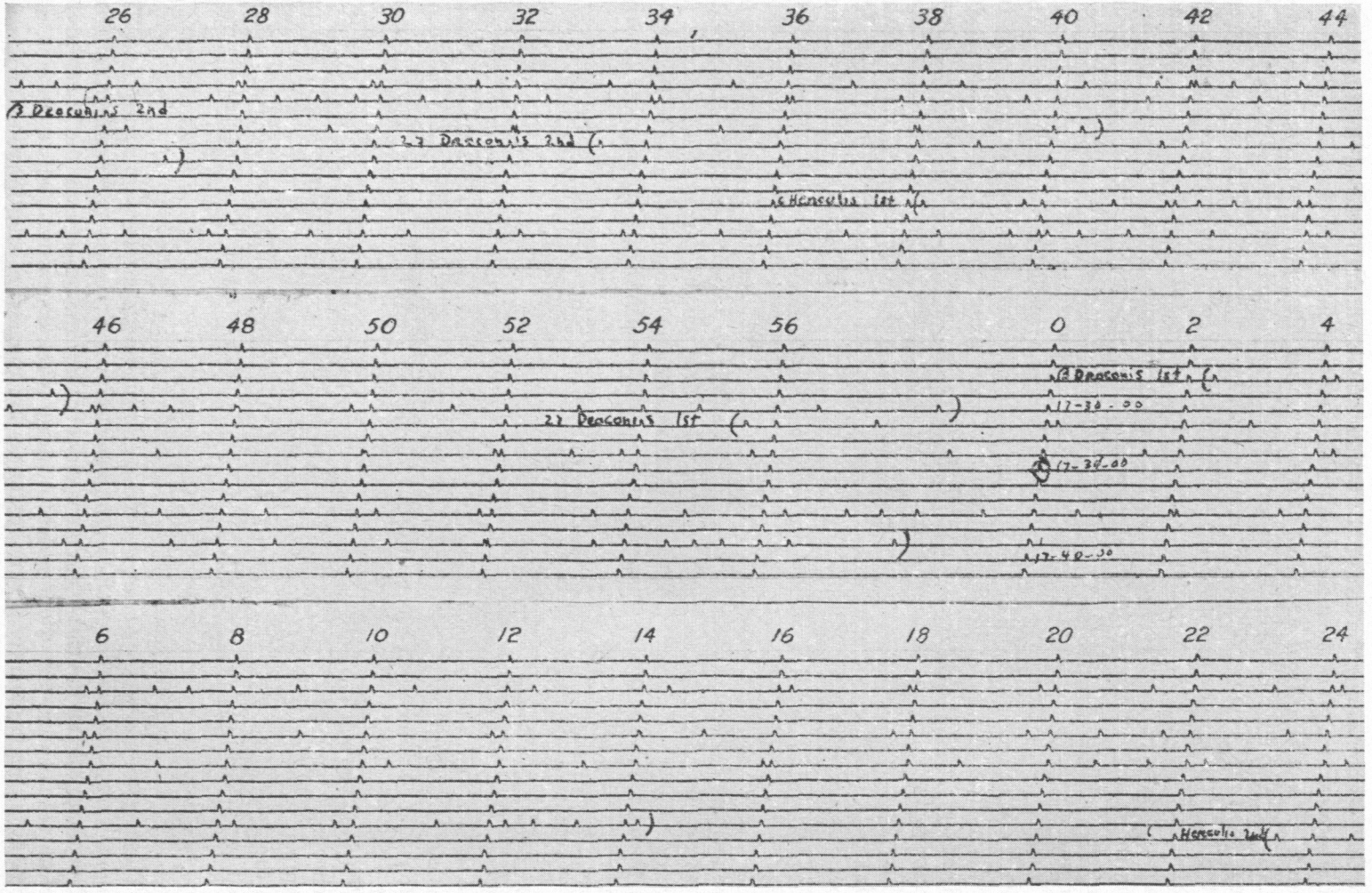


FIGURE 5.—Chronograph record.

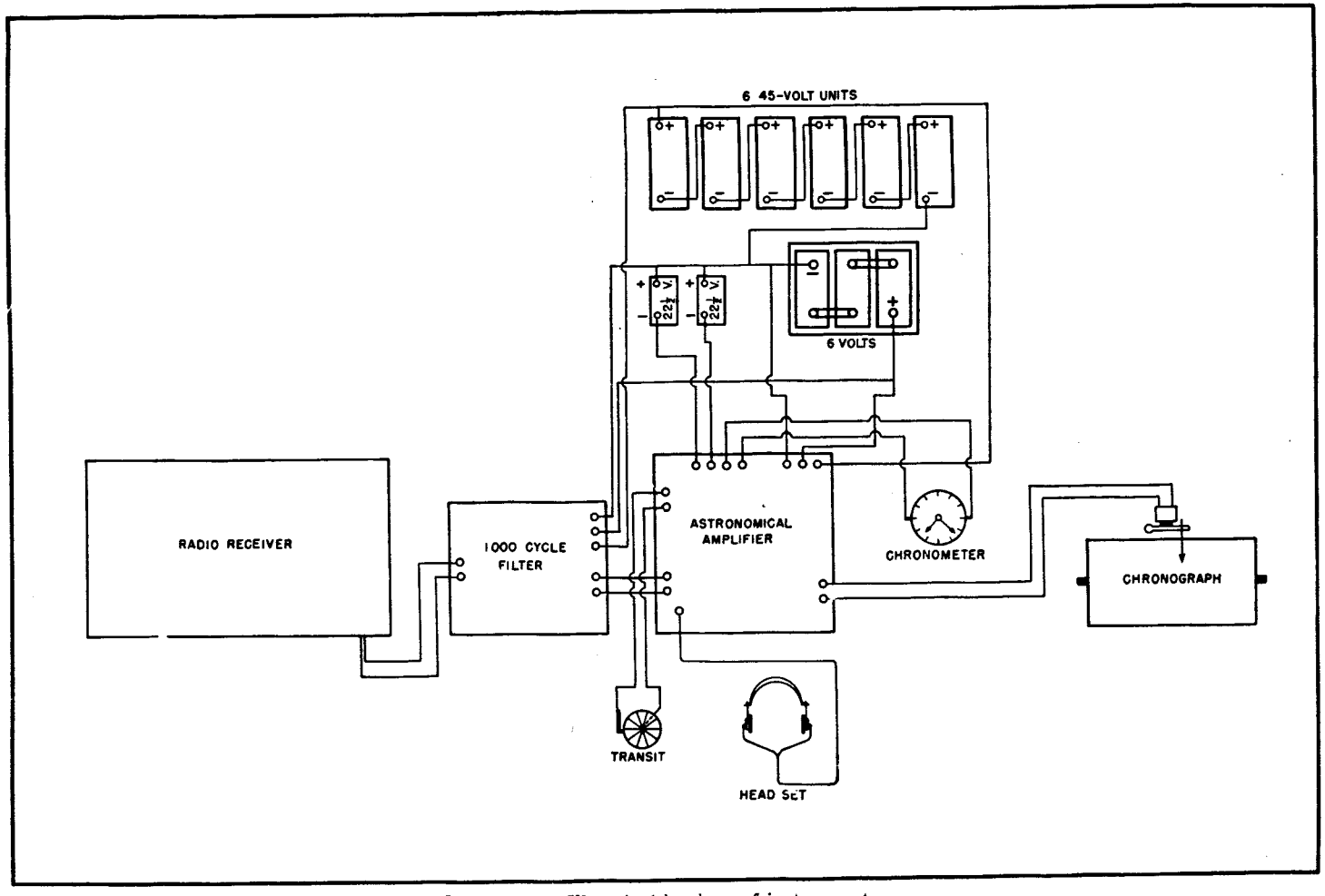


FIGURE 6.—Electrical hookup of instruments.

the first break after the long gap of 4 seconds, or it is the first break after the extra break at the 59th second, depending on the type of chronometer used. A one-second break-circuit chronometer is sometimes used for longitude work. However, it is less satisfactory because there is then more interference between chronometer breaks and transit breaks on the chronograph sheet.

The function of the chronometer is to carry the time between the star observations and the radio time signal. In brief, the local sidereal time as determined from the transit of a star is carried by the chronometer to the instant of the radio time signal, at which instant the sidereal time at Greenwich is known. The difference between the sidereal time at the station and the sidereal time at Greenwich for the same instant is the longitude of the station.

Packing of the Chronometer for Shipment

There are several precautions to be observed in the preparation of a chronometer for shipment.

1. The balance wheel should first be stopped by allowing it to strike against a small piece of writing paper until motion ceases. Never use the fingers or any rigid body for this purpose. As the balance wheel is very heavy compared to its supporting pivots, it is necessary to lock it carefully to prevent motion during shipment, thus preventing breakage of the delicate pivots. This is done by inserting soft cork wedges, as shown in Figure 7. These must be placed as nearly opposite the spokes of the wheel as possible and both should be carefully pressed in at the same time to avoid a sidewise pressure on the pivots. They should never come in contact with any of the adjusting screws in

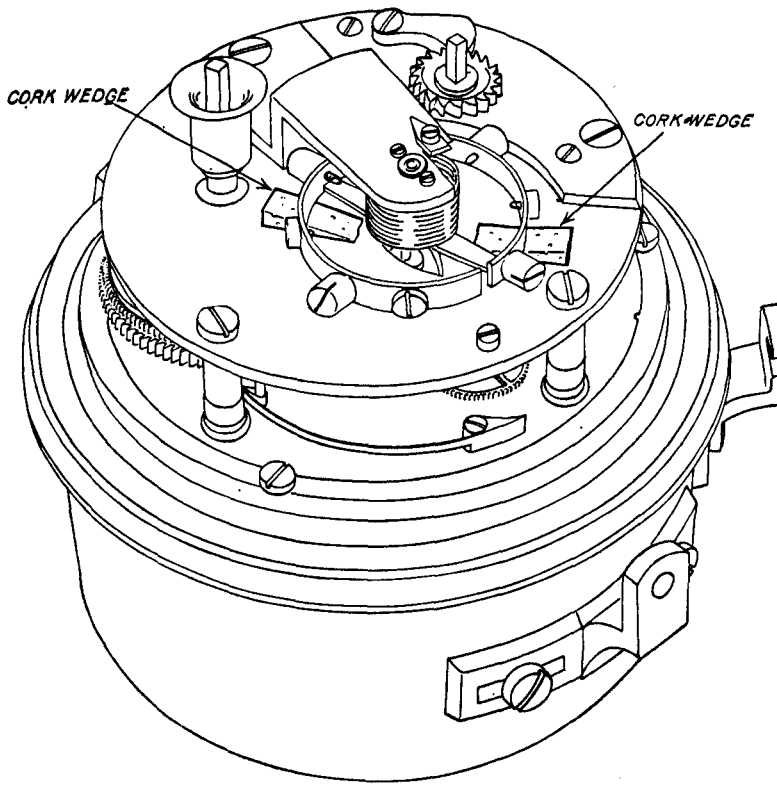


FIGURE 7.—Chronometer balance wheel blocked for transportation.

the balance wheel. The wedges should be pressed in with only sufficient firmness to insure their remaining in place. They should never be forced. Cork wedges are issued with most chronometers, but should none of these be available, care should be exercised in the selection of this material to insure that it is soft and resilient.

2. After the wedges are inserted, the chronometer should be replaced in its case and the cover screwed tightly in place. If a gimbal case is used, the chronometer should be removed from the gimbal rings and packed in the case with paper. The chronometer should be securely wedged with paper so that it cannot move.

3. In packing in an outer shipping case, it is advisable to wrap the chronometer case with paper to prevent infiltration of dust and to use paper as a cushioning material in the packing box. EXCELSIOR SHOULD NEVER BE USED, as it is dusty and the fine powder will sift through exceedingly small openings.

RADIO RECEIVER

Almost any standard commercial make of a good quality short wave receiver can be used in the astronomical setup. The frequency range should cover from about one to thirty megacycles per second. The receiver is connected to the amplifier by means of a phone jack.

Radio receivers as now built usually require only the simplest of antennas, such as the inverted L-type. Best results will probably be obtained by following the manufacturer's instructions concerning the length and type of antenna to be used. An antenna length of at least 65 feet including lead-in is desirable; a ground connection to the receiver may or may not be necessary, depending on the installation. If the poles carrying the antenna are guyed by wire, it is well to insulate these guys from the ground. At the beginning of a season's work the required antenna and guy wires can be obtained locally; poles 20 to 30 feet long can be made up of two-by-fours. A telescoping vertical pole made of aluminum has been used with success.

In cases of reception at extreme distances, consideration should be given to the use of a resonant antenna, oriented properly to obtain maximum signal strength. For best results, some form of feeder system is required to transfer energy from the antenna to the receiver. This type of antenna, although more difficult to erect and less flexible as to frequency, may allow satisfactory reception in distant locations where the conventional antenna would fail. Various directivity patterns and gains may be obtained, depending on the type selected. If the use of an antenna of this type is contemplated, reference should be made to an up-to-date radio handbook for details of construction and operation.

ASTRONOMICAL AMPLIFIER

Various types and designs of amplifiers have been used in the past for the recording of radio longitude observations. Those in use up to 1939 were designed by the late Lieutenant E. J. Brown of this Bureau and were designed to operate a sensitive relay which in turn operated the chronograph pen. A new type of amplifier was designed in 1939 by Lieutenant Albert J Hoskinson which eliminated the relay. The plate currents from the output tubes are fed directly into the high impedance coils of the chronograph pen assembly. It is a two-stage class "B" amplifier using 6N7 radio tubes. Impulses from the chronometer circuit, transit micrometer circuit and radio receiver all enter this amplifier before being recorded on the chronograph sheet through the pen-carriage assembly. As all of the electrical circuits are hooked up through the amplifier, this piece of apparatus becomes very important to the field observer. However, this amplifier has been simplified in construction, so that circuits are rather easy to follow and minor

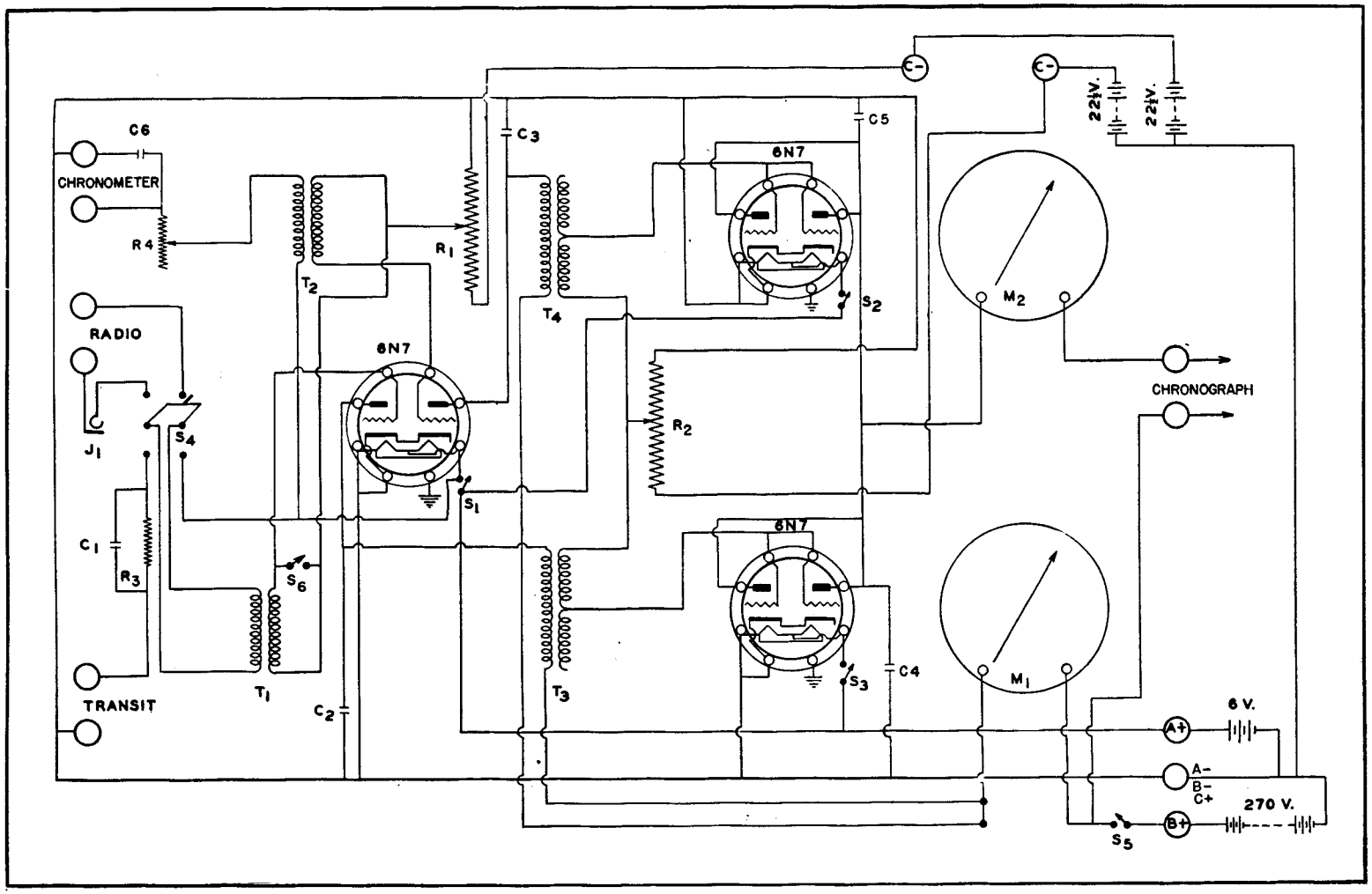


FIGURE 8.—Wiring diagram for transit, chronometer, and radio amplifier circuits.

repairs, even by an inexperienced man, are possible. The wiring diagram is shown in figure 8.

The grid circuits of the tubes are biased so that the no-signal plate current of the tubes is less than 0.5 milliampere.

Description of Parts

T_1 is an audio transformer. The primary winding should have a resistance of about 800 ohms, and the secondary winding about 8,000 ohms.

Nearly any interstage transformer will work in this circuit but the very best results will be obtained if the transformer is wound to match the particular radio used and the grid circuits of the tubes used in the amplifier.

In our amplifier we used transformers in stock (Type T 65) supplied by the Signal Corps, U. S. Army.

T_2 is the same kind of an audio transformer as T_1. The Thordarson type 23 A 57 may be used, provided a fixed resistance of at least 600 ohms is placed in the circuit of the primary, or the variable resistance R_4 is so designed that the limits are 600 ohms to 5,000 ohms.

THERE MUST BE A RESISTANCE OF AT LEAST 600 OHMS IN THE CHRONOMETER CIRCUIT TO LIMIT THE CURRENT TO 0.010 AMPERE OR LESS IN ORDER TO PROTECT THE CONTACT POINTS OF THE CHRONOMETER.

T_3 and T_4 are Thordarson transformers type 67 D 47.

R_1 and R_2 are 20,000 ohm potentiometers.

R_3 is a fixed resistor of 20,000 ohms (1 watt). The value of this resistor should be just large enough to block the current in the transit circuit so that no break of the chronograph pen will be recorded when the contact points of the micrometer head in the Bamberg transit *break*. But it must be small enough so that the condenser C_1 will be sufficiently discharged after one break to make a positive break on the next contact. These contacts may come rather close together for stars nearly on the equator and for the three identification contacts on the Bamberg head.

The R_4 resistor unit is desirable but not absolutely necessary if the fixed resistance in this circuit is 600 ohms or more.

Any variable resistance that will carry 0.010 ampere is satisfactory.

C_1 is a 10-microfarad 25-volt condenser. The capacity of this condenser must be large enough so that a positive break is obtained from the chronograph pen. There are many combinations of values of C_1 and R_3 that will work satisfactorily in this circuit, but the main consideration should be to get a good reserve of power so that the chronograph pen will work well in any kind of weather, particularly on those nights when moisture condenses on the instruments and has a tendency to partially short-circuit the electrical components.

C_2 and C_3 are 0.006-microfarad 600-volt paper by-pass condensers.

C_4 and C_5 are 0.5-microfarad 600-volt paper by-pass condensers.

C_6 is a 0.1-microfarad paper by-pass condenser.

S_1, S_2, S_3, and S_5, are single-pole single-throw snap switches.

S_4 is a double-pole double-throw switch such as General Radio rotary switch or Federal anti-capacity switch.

M_1 is a direct-current milliammeter 0 to 25 milliamperes. Any standard make.

M_2 is a direct-current milliammeter 0 to 50 milliamperes. Any standard make.

J_1 is a standard head phone jack, make-circuit type.

For binding posts shown for radio transit and other circuits, use EBY COMMANDER.

The magnetic coils of the chronograph pen should be wound to have a resistance of 2,000 to 5,000 ohms, so that the output current from the amplifier will work the pen without the use of a sensitive relay.

Any automobile storage battery will furnish the "A" current required to work the amplifier.

Any standard make of 45-volt "B" batteries hooked in series to obtain the 270 volts required will furnish the desired "B" current required to work the amplifier.

Any standard make of 22½-volt "C" batteries are satisfactory for the potentiometer units R_1 and R_2.

In operation, the chronometer-break impulses will travel through the amplifier and be recorded continuously on the chronograph sheet. By means of a two-way switch, S_4, either the radio beats or the transit micrometer breaks can be introduced and recorded on the chronograph sheet along with the chronometer-break marks.

When the transit micrometer breaks and the chronometer breaks are both being recorded on the chronograph sheet, the chronometer breaks can easily be identified by drawing a straight line from a certain chronometer second break, before the transit micrometer is operated, to corresponding second breaks after the transit micrometer has been stopped. If the chronograph is running uniformly, any breaks that line up exactly with this line are chronometer breaks. The slope of this line depends on the speed of the chronograph cylinder. Occasionally a transit break will occur simultaneously with a chronometer break. But since the spacing of the transit breaks is usually decidedly different from the spacing of the two-second chronometer breaks, there is usually no difficulty in finding enough chronometer and transit breaks for reading the chronograph sheets.

However, when recording radio time signal beats and chronometer breaks together, simultaneous breaks are more serious. The sidereal chronometer gains only 0.16 second on the radio time signal per minute. Since the radio beat is 0.4 second in duration, it is therefore entirely possible that the radio beats will cover up all chronometer breaks for about 2½ minutes, except those between 55 and 60 seconds of each minute of the time signal, that is, the records may overlap half of the time that the signal is being broadcast. Unfortunately, it sometimes happens that the greater part of the other half of the broadcasting time may be consumed in tuning in the signal. For the purpose of obtaining a satisfactory chronograph record for scaling that part of the time signal where radio beats and chronometer breaks overlap, a cut-out switch has been provided on the amplifier so connected that the radio beats may be cut out of the circuit at will. By using this hand-operated switch and by proper timing, every other radio beat may be cut out and every two-second break mark of the chronometer will be preserved: The record will have only one-half as many radio time beats as otherwise, but all these beats will be scalable against chronometer breaks.

Two milliammeters, M_1 and M_2, are provided to indicate the direct-current flow and are controlled by potentiometers R_1 and R_2, respectively. The hook-up diagram and explanation with regard to the rest of the electrical equipment is given in figure 8.

AUDIO FILTER

Two standard audio frequencies, 440 cycles per second and 4000 cycles per second, are superimposed on a good share of the *WWV* radio time signal broadcasts. An audio filter was designed by Herbert Grove Dorsey and T. J. Hickley and built by G. D.

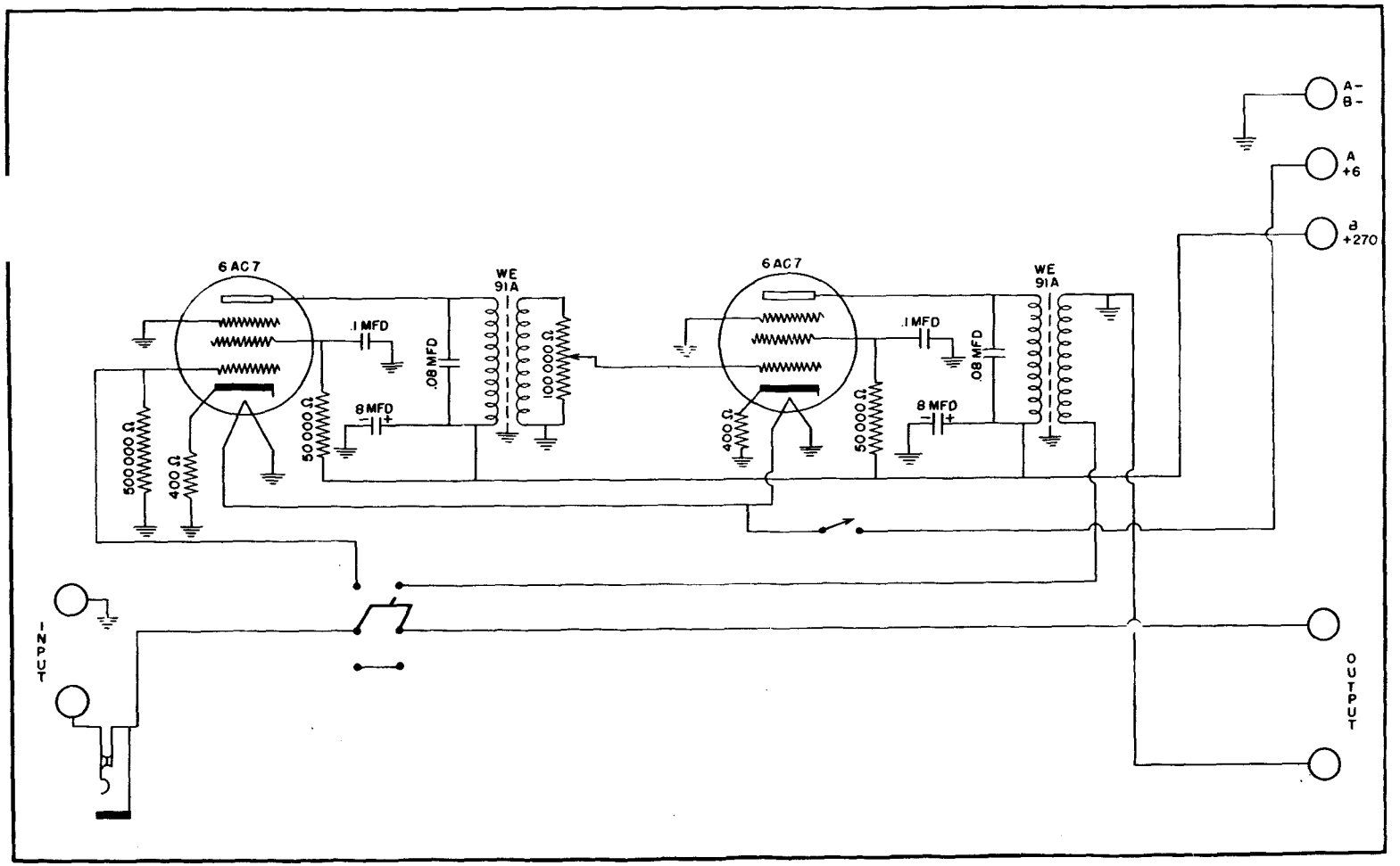


FIGURE 9.—Wiring diagram of audio filter.

Nedley, all of this Bureau. This apparatus filters out the above mentioned audio frequencies, while allowing the time signal impulses to be recorded without interference. The wiring diagram of the audio filter is illustrated in figure 9.

ASTRONOMICAL TENT

Various types of covering have been used in the past to house the instruments. Wooden observatories have been used extensively in Alaska and elsewhere. At present a 6 x 10-foot canvas tent is used. This tent is placed over a light but sturdy wooden frame bolted together, an arrangement that makes for ease in setting up and dismantling. A small galvanized iron pipe, supported at the ends of the frame by gable points, serves as a lengthwise ridgepole within the tent. A slot 3 feet wide in the tent top provides the necessary opening for the observations. There are flaps one foot wide all around the bottom of the tent, so that earth or stones may be piled on them to make a tight connection against the weather. A canvas fly large enough to cover the entire top and to overlap the sides and ends by about 8 inches is placed over the tent when the observatory is not being used. See figures 10 and 11. Various guy lines and tying lines hold the tent and fly in a secure position. At the southeast corner is a doorway, which is provided with an 18-inch overlap for closing. This tent has been used in all types of weather conditions encountered in Alaska and has proved to be a good covering for the Bamberg transit and for other instrumental equipment.

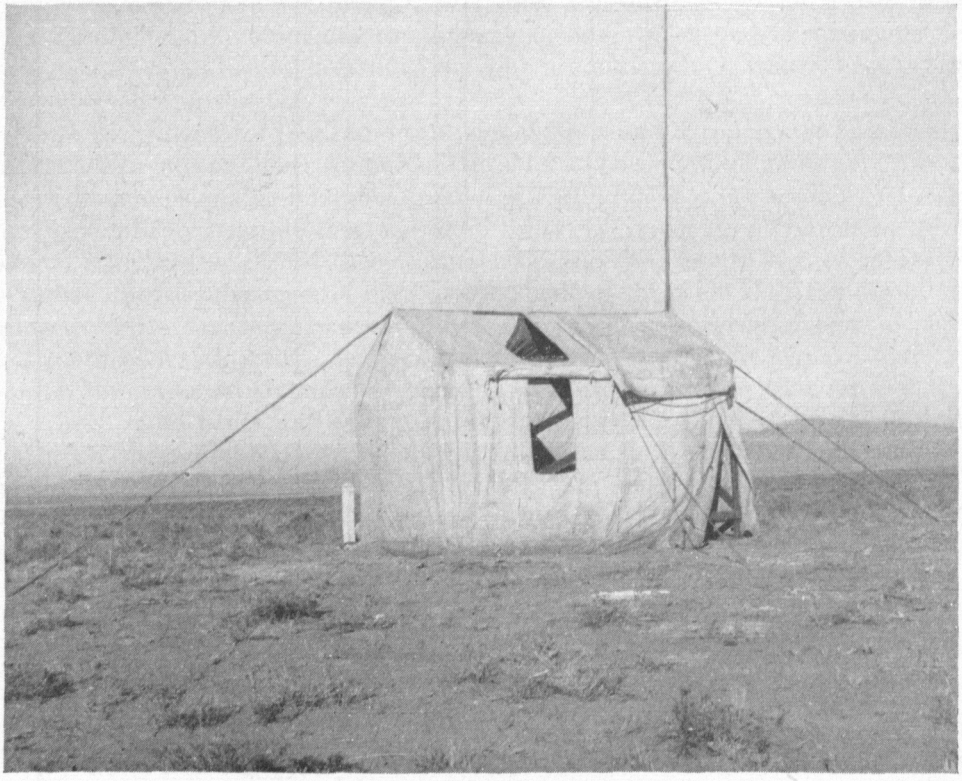


FIGURE 10.—Observing tent with slot for observing uncovered.

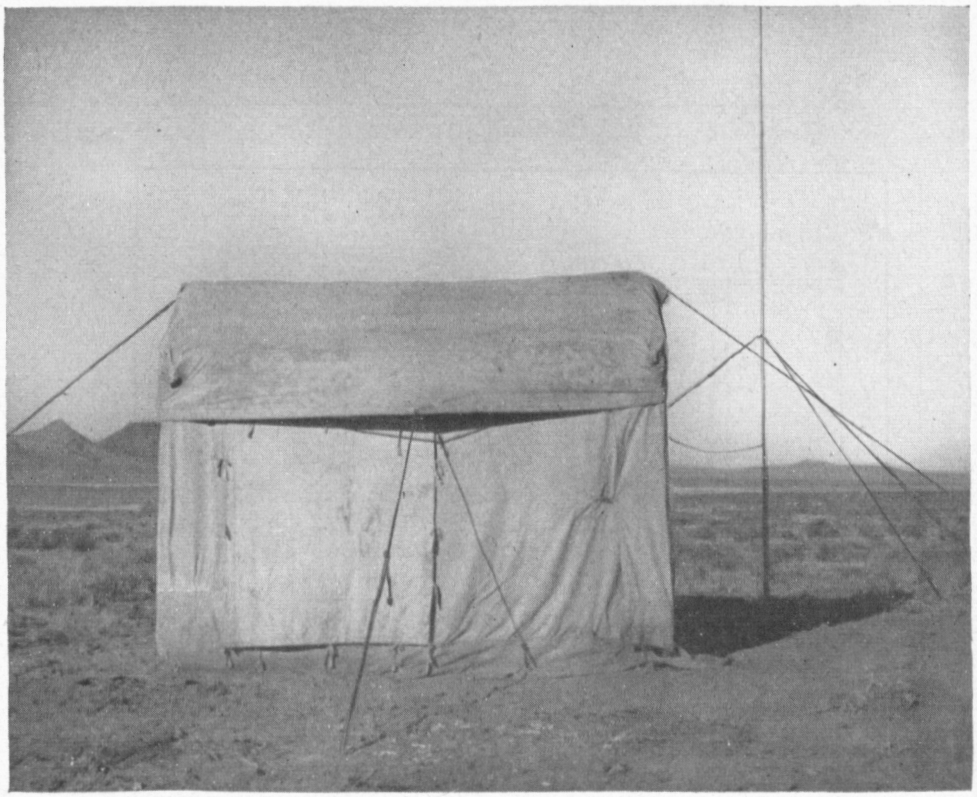


FIGURE 11.—Observing tent.

SPECIFICATIONS FOR ASTRONOMIC OBSERVING TENT AND FLY

Tent

Dimensions.—Height seven (7) feet six (6) inches; length of ridge, nine (9) feet; width, six (6) feet six (6) inches; height of wall, five (5) feet six (6) inches.

Material.—To be made of cotton duck, clear of all imperfections, and of a weight equivalent to ten (10) ounces per linear yard of material twenty-eight and one-half (28½) inches wide; or, first quality, double filled, treated material, weighing eight (8) to ten (10) ounces per square yard. A good quality of the latter type material is sold under the trade name of "Vivatex."

Workmanship.—All seams to be double sewed with lock stitch, with at least one-half (½) inch overlap. If sewed by machine, a good quality linen thread of suitable weight is to be used All work must be in accordance with the best practices and subject to inspection and approval.

Grommets.—Grommets are to be placed in all corners, three (3) inches below the eave line, to permit guy lines attached to the tent frame to be passed out through the tent. Grommets for foot-stops are to be worked in the tabling at the bottom of the tent, in each corner and at the middle of each long side. Grommets to be made with three-quarter (¾) inch malleable-iron rings, galvanized, well sewed in the holes with linen or well-waxed cotton twine of suitable weight.

Door.—One corner of the tent is to be opened from the eave line to bottom, as shown in sketch, and a door flap, one (1) foot wide, attached at this corner. The

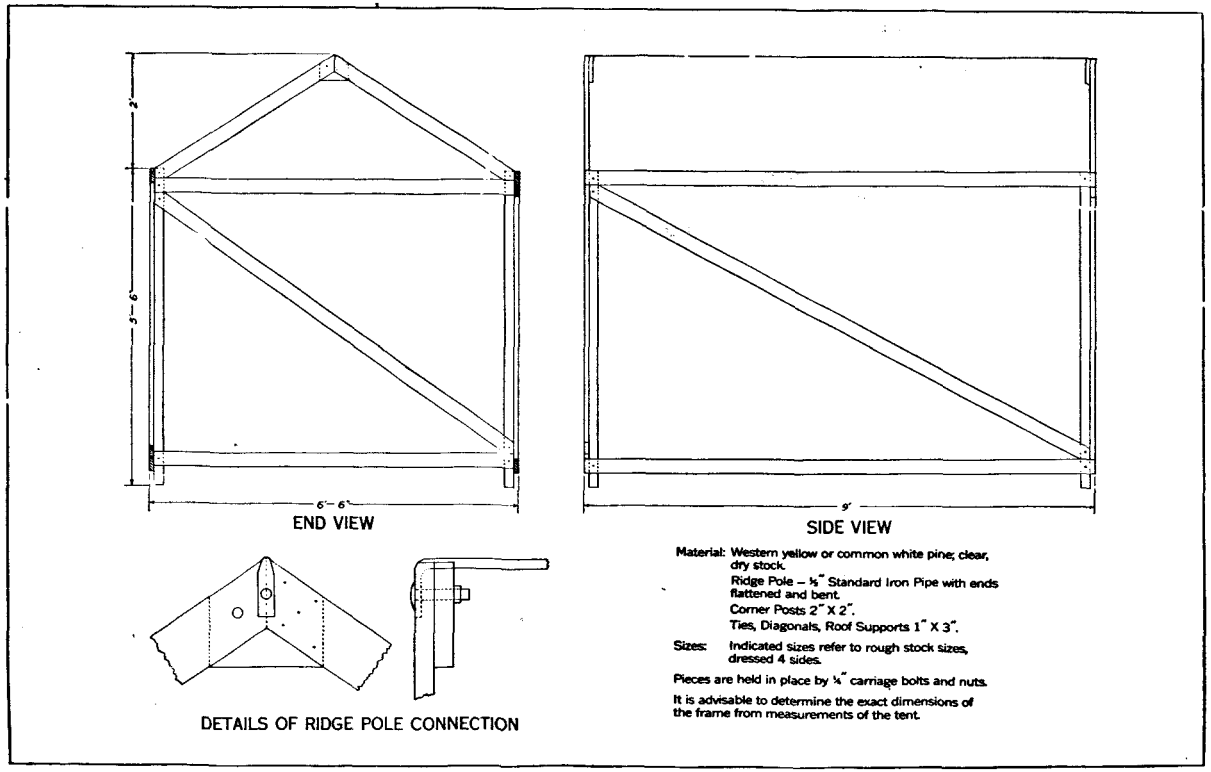


FIGURE 11a.—Specifications of demountable frame for astronomic observing tent.

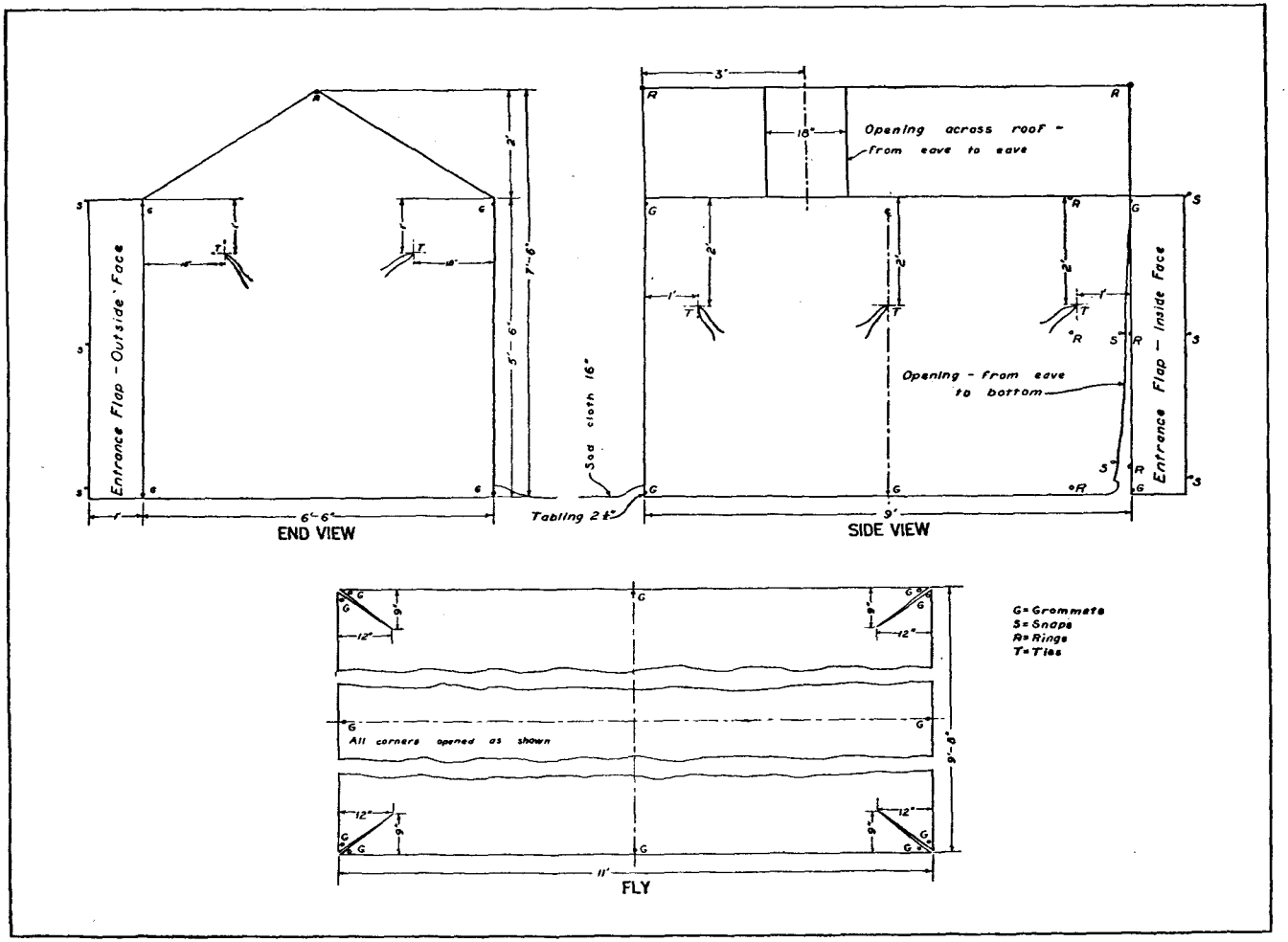


FIGURE 11b.—Specifications for astronomic observing tent and fly.

This design is for longitude and latitude work in low latitudes. For azimuth work in high latitudes an additional opening in the side wall, as shown in fig. 10, will be required.

door fastening is to consist of two (2) snaps and rings to hold the sidewall, and three (3) snaps and rings to secure the flap.

Roof.—An opening, eighteen (18) inches wide, is to be made across the top of tent from eave line to eave line, as shown on the sketch. This opening is to be bound on all sides by heavy one-half (½) inch webbing. A similar piece of webbing is to be sewed the entire length of the ridge line, inside. A three-quarter (¾) inch iron ring is to be attached to the tent at each end of the ridge, outside. The rings are to be fastened by means of one-half (½) inch webbing, sewed back along ridge line eight (8) inches.

Ties.—Double ties of one-fourth (¼) inch cotton rope, two (2) feet long on each side, are to be attached to the tent at the points indicated on the sketch; three (3) ties on each side of tent and two (2) on each end.

Reinforcing.—Reinforcing patches, of the same material as tent, shall be placed inside each corner at the eave line and at each end of the ridge. These patches to extend eight (8) inches from the corners. The tent shall also be reinforced at the points where ties are attached.

Tabling.—The tabling on the foot of the tent, when finished, to be two and one-half (2½) inches in width.

Foot-stops.—Foot-stops, six (6) in number, to be loops four (4) inches long in the clear, of nine-thread manila line, both ends passing through a single grommet worked in the tabling, and to be held by what is known as the "Mathew Walker" knot. Foot-stops to fall outside tent.

Sod Cloth.—The sod cloth to be of same material as the tent, sixteen (16) inches wide, to be sewed to top of tabling, and to fall *outside* the tent. The sod cloth is to be filled in at the corners except at the door opening.

Guy Lines.—Guy lines, six (6) in number, to be of nine-thread manila line (5/16 inch), fifteen (15) feet long in the clear, with one end well whipped, and an iron snap attached to the other end. The snaps are to be large enough to snap onto the nine-thread line, and they may be either rope snaps, or round eye snaps back spliced onto the line.

Fly

Dimensions.—When finished, length, eleven (11) feet; width, nine (9) feet and eight (8) inches.

Material.—As specified for tent.

Tabling.—A one and one-half (1½) inch tabling to be worked on sides and ends.

Corners and Stay Pieces.—Corners are to be opened as shown on the sketch, and reinforced by a stay piece sixteen (16) inches square, opened in the same manner.

Grommets.—Grommets as specified for tent; two in each corner, and one in the middle of each side and end, as shown on the sketch.

Workmanship.—The fly is to be made in a workmanlike manner in every respect, with all seams double sewed lock stitch, and having a one (1) inch overlap.

DETERMINATION OF EQUATORIAL VALUE OF LONGITUDE MICROMETER

The approximate equatorial value of one turn of the longitude micrometer screw for each of the Bamberg transits Nos. 20 and 21 is 10.5 seconds of time. For longitude observations it is usually not necessary to know this value any more closely, because it is not used in the longitude computation. However, if the Bamberg instrument is used

for measuring micrometrically the angle between the azimuth mark and Polaris, it is very important to determine accurately the equatorial value of the longitude micrometer screw.

The micrometer value should be determined by recording chronographically the passage of equatorial stars across the field of view. The instrument should be in the meridian for these observations. The movable wire of the micrometer should be used to follow the star as it moves across the portion of the field of view bounded by the two outer wires of the fixed reticule. This covers a distance of about 10 turns of the micrometer screw. Because of uncertain optical and parallactic conditions the space beyond these outer wires is not considered a usable portion of the field of view.

Since it is rather fatiguing to follow a star continuously with the transit micrometer for ten turns, the following procedure is suggested:

The observer should start to follow the star when it crosses the outer wire and should continue following it for two or three turns. The recording switch should then be thrown open; the movable wire moved across the field to the approximately corresponding part on the other side of the usable field; the switch should then be closed; when the star catches up with the movable wire, the observer should continue following it carefully until the outer wire is reached.

Let n=number of turns in the interval concerned

t_o=chronometer reading for the first break of the chosen interval

t_n=chronometer reading for the last break of the chosen interval, and

R'=value of one turn in seconds of time for δ, the declination of the star being followed.

Then $R'=\frac{t_n-t_o}{n}$ seconds of time on a parallel or small circle whose angular distance from the equator or great circle is equal to the declination of the star.

In order to reduce the micrometer value R' into seconds of time on the equator or great circle, it is necessary to multiply R' by the cosine of the declination. Thus, the equatorial micrometer value $R=R' \cos \delta$.

Now, instead of depending on just one break near the beginning and one near the end, a mean of ten breaks near the beginning should be used and a mean of ten near the end. It is still better to make a least-squares adjustment of the means of several groups of ten near the beginning with the means of several groups of ten near the end.

For the least-squares adjustment, let x be the reading in units of turns of the micrometer and t the corresponding chronometer reading in seconds of time.

Then

$$R'=\frac{[xt]-\frac{[x]\,[t]}{n}}{[xx]-\frac{[x]^2}{n}} \tag{5}$$

and as before $R=R' \cos \delta$.

Each chronometer reading, t, should be reckoned from some arbitrary initial, $t_o=0$, preferably the last whole minute before the star comparison.

If the number of turns increases uniformly by a constant d, it can readily be shown that the denominator is then independent of x and that it is always equal to $\frac{n(n^2-1)}{12}d^2$.

Determination of the equatorial value of the longitude micrometer screw for Bamberg No. 21, Sheep Astro, Alaska September 17, 1943. H. J. Seaborg, Observer

ν Piscium

Turn x	*Chron. Reading* t
0. 55	53. 57
1. 55	64. 17
2. 55	74. 64
3. 55	85. 29
4. 55	95. 80
5. 55	106. 41
6. 55	116. 90
7. 55	127. 47
8. 55	138. 04
9. 55	148. 69

$n=10$

$[x]=50.5$

$[t]=1010.98$

$[x]\,[t]=51054.49$

$[xt]=5976.829$

$$[xx]-\frac{[x]^2}{n}=n\,\frac{(n^2-1)}{12}=82.5$$

$$R'=\frac{5976.829-\frac{51054.49}{10}}{82.5}$$

$R'=10^s.5622$

$\delta=5°12'13''$

$\cos\delta=0.995879$

$R=R'\cos\delta=10^s.5187$

Each recorded chronometer reading is the mean of ten breaks.

In this example, the star has been followed continuously for ten turns of the micrometer.

Summary of observations on six stars

Star	R	v
ν Piscium	$10^s.5187$	+0. 0177
o Piscium	10. 5437	−0. 0073
ξ Piscium	10. 5373	−0. 0009
232 G Ceti	10. 5387	−0. 0023
ζ^2 Ceti	10. 5483	−0. 0119
ν Ceti	10. 5320	+0. 0044
Mean	10. 5364	

$[v^2]=0.00053365$

Probable error $=\sqrt{0.00053365}\;\frac{0.6745}{\sqrt{n(n-1)}}=0.0231\times0.1231$

Probable error $=\pm0.0028$

This second factor in the probable error formula, $\frac{0.6745}{\sqrt{n(n-1)}}$, may be taken from a table on p. 194.

DETERMINATION OF LEVEL VALUE

The level trier, a laboratory instrument in the Instrument Shop of this Bureau, should be used for determining the value of one division of the level vial of the hanging level. The level trier is protected by a glass case with an opening at one end that gives access to the micrometer head which is graduated in seconds of arc.

The level vial, with the chambered end to the right, should first be securely fastened to the level trier but without imposing any strain on the vial. The length of the bubble should be made about one half of the total length of vial. Then the micrometer head should be turned until the left end of the bubble is near the left end of the graduations of the vial, and the micrometer set at an exact second. After the bubble has been allowed to come to rest, both ends of the bubble are to be read and recorded. The readings should be estimated to the nearest tenth of a division. The micrometer is then moved a convenient whole number of seconds causing the bubble to move to the right. After the bubble has come to rest, both ends of the bubble are again read and recorded. This process of successively moving the micrometer the same number of seconds and reading and recording the ends of the bubble is repeated until the right end of the bubble has moved close to the right end of the scale. Similar observations are then made starting the readings with the bubble at the right end of the vial scale, and successively moving the bubble across the vial to the left end.

The level vial is then reversed, so that the chambered end is to the left. The same process as before is repeated, first by moving the bubble across the vial from left to right and then from right to left. This makes a total of four level determinations; two with chambered end left, and two with chambered end right. The set of observations for each of these level determinations is then adjusted by the method of least squares, as shown in the following example.

Then the level value, d, becomes:

$$d=\frac{[xx]-\frac{[x]^2}{n}}{[xy]-\frac{[x]\,[y]}{n}} \text{ seconds of arc per division of the vial.} \tag{6}$$

Where x is the micrometer setting on the level trier, y is the mean of the readings of the ends of the bubble, and n is the number of settings.

Determination of value of one division of hanging level, Bamberg No. 21, March 17, 1943, H. J. Seaborg, Observer

Micrometer Setting	*Level Reading*		*Bubble*	*Mean Level*
	Left	*Right*	*Movement*	*Reading*
x				y
80	69.0	48.1	--------	58.55
84	66.4	45.4	2.65	55.90
88	63.3	42.2	3.15	52.75
92	60.9	39.7	2.45	50.30
96	57.1	36.2	3.65	46.65
100	54.5	33.6	2.60	44.05
104	51.8	30.7	2.80	41.25
108	48.4	27.2	3.45	37.80
112	45.6	24.4	2.80	35.00
116	43.1	22.0	2.45	32.55

Determination of value of one division of hanging level, Bamberg No. 21, March 17, 1943, H. J. Seaborg, Observer—Continued

Micrometer Setting	Level Reading Left	Level Reading Right	Bubble Movement	Mean Level Reading
x ″				y
120	40.2	19.0	2.95	29.60
124	37.7	16.3	2.60	27.00
128	35.0	13.9	2.55	24.45
132	32.6	11.2	2.55	21.90
136	29.6	8.4	2.90	19.00
140	26.9	5.8	2.65	16.35
144	24.4	3.1	2.60	13.75
148	22.1	0.8	2.30	11.45

$$n=18$$

$$[x]=2052$$

$$[y]=618.3$$

$$[xy]=65068.8$$

$$[x]\,[y]=1268751.6$$

$$[xx]-\frac{[x]^2}{n}=\frac{n(n^2-1)}{12}D^2=7752$$

$$d=\frac{7752}{65068.8-\frac{1268751.6}{18}}$$

$$d=1''.431 \text{ per division}$$

Summary of 4 sets of observations

Set	d ″
1	1.431
2	1.414
3	1.383
4	1.352
Mean	1″.395

DETERMINATION OF MEAN WIDTH OF CONTACT STRIPS

In following a star with the micrometer eyepiece, the micrometer screw is turned, thus bringing the metallic strips on the micrometer head into contact with the platinum point. As soon as the front edge of each strip comes into contact with the platinum tip, the electric circuit is closed causing a break to be recorded on the chronograph sheet. After the telescope is reversed, the star is followed with an opposite turning motion of the micrometer screw, and consequently the other edge of each metallic strip makes contact first with the platinum point. Hence it is evident that the chronographic recordings are made too early by an amount corresponding to one-half of the time required for the whole width of the contact strip to pass the platinum point.

The width of the contact strips can be determined at any time. It is not even necessary that the transit instrument be set up and mounted. The chronograph, amplifier and longitude micrometer should be hooked up electrically, similarly as for

observing. The observations for the determination of the width of contact strips are made as follows. The micrometer screw is turned very slowly and is stopped at the instant that the click of the chronograph relay is heard. The calibrated micrometer head is then read and recorded. The turning of the micrometer screw is continued for 2 or 3 divisions of the micrometer head after the contact is broken. Then the micrometer screw is turned in the opposite direction and is stopped at the instant the click is heard, which is the instant when the contact is made on the other edge of the contact strip. The micrometer head is again read and recorded. The difference between these two readings is the effective width of the contact strip in terms of divisions of the micrometer head. For one determination of the width of the contact strip, at least 5 and preferably 10, observations should be made on each contact strip except the 2 contact strips flanking the zero strip, thus making a minimum of 50 observations.

Continued use of the micrometer with frequent sparking between the strips and the platinum point causes a wearing of the edges. Therefore, the width of contact strip determination should be made at least once during each season's work.

The correction to the observed time of transit of a star due to the average width of the contact strips is equal to $\frac{1}{2}\frac{Rs}{100}\sec\delta$, where R is the equatorial value of one turn of the longitude micrometer screw, δ is the declination of the star, and s is the average width of the contact strips in terms of divisions on the micrometer head.

DETERMINATION OF LOST MOTION OF THE MICROMETER SCREW

In any screw working against a spring there is probably always a certain amount of lost motion. This is true with regard to the two compression coil springs attached to the slide in the transit micrometer box. When the micrometer screw is turned so that the slide moves upward, the coil springs are being compressed and when the motion is reversed, they are partially released.

It is desired to obtain the actual lost motion during the time the star is being followed, that is, at the time the micrometer screw is in actual motion. To determine this dynamic lost motion rather elaborate equipment is required. The simpler method of determining the static lost motion is therefore used and here described. It is probably a fair approximation to the dynamic lost motion, actually desired.

The lost motion is determined at different parts of the screw by making a set of observations on each of the fixed wires as follows:

Pointings on a fixed wire are made by turning the screw so as to bring the movable wire *down* into coincidence with the fixed wire. It is understood that the movable wire is actually moving down. At coincidence, the reading of the micrometer head is recorded, and then the same screw motion is continued for a short distance. The screw motion is then reversed, and when the movable wire has been brought *up* into coincidence with the fixed wire, the reading of the micrometer head is again recorded. The difference between these two micrometer-head readings in the sense "down" minus "up" is the lost motion in terms of divisions of the micrometer head.

To show how the correction to the time of transit for this lost motion is deduced and how it should be applied to the observed time of transit, let us suppose for eyepiece East, that 1, 2, 3, and 4 in figure 12 represent four successive recording positions of the movable wire while following a star going in the direction indicated by the arrow. The movable wire is stopped after the fourth recording. Now let 4, 3, 2, and 1 represent the same positions of the movable wire after reversal of the telescope on the assumption that there is no lost motion in the screw.

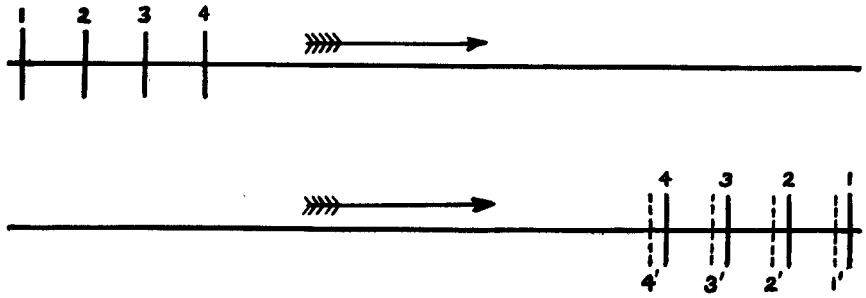


FIGURE 12.—Correction for lost motion.

However, since there is lost motion, as the observer follows the star (still going in the same direction), the movable wire will lag behind its proper position and instead of taking the positions 4, 3, 2, and 1, it will be at the points indicated by the dotted lines 4′, 3′, 2′, and 1′. It is evident that all the star observations after reversal will be recorded too soon by the time required for the star image to traverse each of the spaces 4′–4, 3′–3, 2′–2 and 1′–1, and the correction to be applied to the observed time of transit of a star will be one-half of the time interval represented by the mean of these spaces. Similar reasoning shows that if the observations on a star are begun with the telescope reversed from the above position, the same correction must be applied, the first half of the positions having been made too soon. Hence, the correction for lost motion is $\frac{1}{2}\frac{R}{100}m \sec \delta$, where m is the lost motion expressed in terms of one division on the micrometer head.

Since the corrections for lost motion and width of contact strips each involve $\frac{1}{2}\frac{R}{100}$ and $\sec \delta$, it is convenient to combine these two corrections into one correction l, which would then become:

$$l=\frac{1}{2}\frac{R}{100}(m+s) \sec \delta \tag{7}$$

or since $\sec \delta$ is by definition the collimation factor C, see page 41,

$$l=\frac{1}{2}\frac{R}{100}C(m+s)\cdot \tag{8}$$

The constant $\frac{1}{2}\frac{R}{100}(m+s)$ should be determined at least once every season.

Warning: The method described above for determining lost motion measures this effect at three widely separated points on the screw; furthermore, it measures the lost motion at these points when the screw is practically stationary, since a careful pointing or bisection requires a very slow motion of the screw. Experiment indicates that this "stationary" lost motion may vary quite unpredictably between these three points; also, it is possible that the "stationary" lost motion differs from what might be called the "dynamic" lost motion, this being the lost motion at a point on the screw when the screw is rotating rapidly enough for a star to be tracked across the field.

It is believed that if the stationary lost motion is close to zero, the effect of the dynamic lost motion can be ignored. As this publication goes to press, instrumental changes are being made in order to reduce the stationary lost motion, but if these fail, it will be necessary to resort to a more elaborate procedure capable of measuring the dynamic lost motion for the speeds encountered in actual observations.

Determination of mean width of contact strips, Bamberg Transit No. 21, Range, Tex., May 12, 1945, F. R. Gossett, Observer

Up	*Down*	*Diff.*	*Up*	*Down*	*Diff.*	*Up*	*Down*	*Diff.*
28.9	29.6	+0.7	28.8	29.6	+0.8	28.8	29.6	+0.8
38.8	39.6	+0.8	38.7	39.7	+1.0	38.7	39.7	+1.0
48.9	49.3	+0.4	48.8	49.5	+0.7	48.9	49.5	+0.6
58.9	59.4	+0.5	58.8	59.4	+0.6	58.8	59.4	+0.6
68.9	69.4	+0.5	68.9	69.4	+0.5	68.9	69.5	+0.6
78.8	79.8	+1.0	78.7	79.8	+1.1	78.8	79.7	+0.9
88.9	89.7	+0.8	88.8	89.6	+0.8	88.8	89.5	+0.7
98.9	99.4	+0.5	98.8	99.4	+0.6	98.8	99.3	+0.5
08.8	09.6	+0.8	08.8	09.6	+0.8	08.8	09.5	+0.7
18.8	19.5	+0.7	18.8	19.6	+0.8	18.8	19.6	+0.8
Mean		+0.670±0.040			+0.770±0.039			+0.720±0.033

Mean width of contact strips, s, from these three sets and seven additional sets is +0.693 division on micrometer head.

Determination of lost motion, Bamberg Transit No. 21, Range Tex., May 19, 1945, F. R. Gossett, Observer

Lower Wire			*Middle Wire*			*Upper Wire*		
Up	*Down*	*Diff.*	*Up*	*Down*	*Diff.*	*Up*	*Down*	*Diff.*
1.2	1.9	+0.7	1.1	0.9	−0.2	97.7	97.8	+0.1
1.9	2.1	+0.2	0.9	1.0	+0.1	97.7	97.8	+0.1
2.2	1.9	−0.3	1.1	0.8	−0.3	97.6	97.8	+0.2
1.9	2.1	+0.2	1.2	0.9	−0.3	97.1	97.6	+0.5
2.1	1.6	−0.5	0.6	0.9	+0.3	97.6	97.3	−0.3
2.1	1.5	−0.6	0.9	0.8	−0.1	97.2	97.2	0.0
2.1	2.2	+0.1	0.7	0.7	0.0	97.1	97.5	+0.4
1.9	2.2	+0.3	0.9	0.7	−0.2	97.2	97.3	+0.1
1.8	1.9	+0.1	0.8	0.9	+0.1	97.1	97.1	0.0
1.8	2.2	+0.4	0.8	1.1	+0.3	97.0	97.3	+0.3
Mean		+0.06±0.072			−0.03±0.048			+0.14±0.050

Mean lost motion, m,=+0.057 division on micrometer head. Combining the corrections for mean width of contact strips and lost motion and reducing the fraction of a division to seconds of time, we have

$$l=+\frac{1}{2}\frac{10.54}{100}(m+s)C$$

$$l=+0.0527(0.057+0.693)C=+0.0395C \text{ in seconds of time.}$$

LAG IN THE AUDIO FILTER

The lag in the audio filter described on page 14 was determined by H. G. Dorsey with a cathode ray oscilloscope and a special sweep circuit. The lag in the filter circuit was found to be 0.008 second which agrees satisfactorily with the computed lag of 0.0073 second.

CAMP LOCATION, STATION SITE AND INSTRUMENTAL SETUP

It is most desirable for the party to be camped at the station site whenever possible. This practice facilitates the observations and is the most economical one for the members of the party. Tent camps may be used; or in the case of a house trailer, it is an easy matter to unhitch and block up the trailer in any desired location. At the present time, with the existence of good roads in most sections of the country and with careful selection of sites for astronomical stations, trucks and trailers usually may be driven directly to the station sites. In Alaska and other less developed territories, various conditions and difficulties may be encountered in transportation, and other methods of transportation such as launches and airplanes may be used.

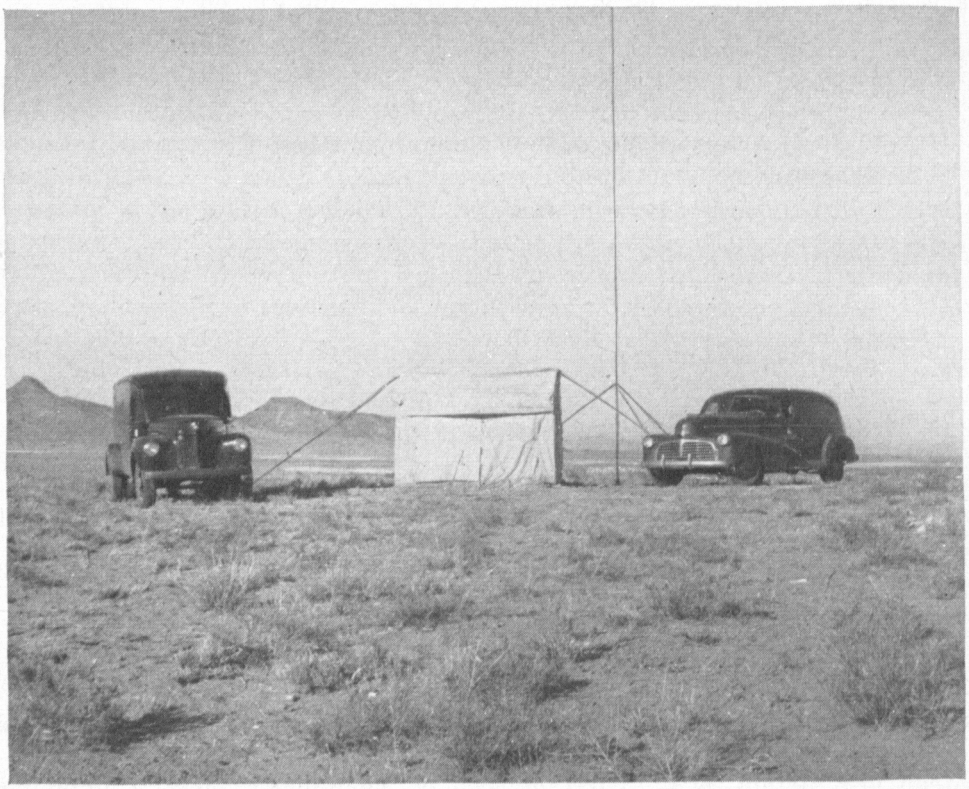


FIGURE 13.—Observing tent and trucks at station site.

(a) Selection of transit location. A level area free of any large boulders or other obstructions should be chosen for the transit location. It is not necessary to place the astronomical transit at the triangulation station. In this case, the eccentric station must be determined with respect to the triangulation station. This may be done in several ways. One way is to measure the distance between the triangulation station and the eccentric station and turn the angle at the triangulation station between the eccentric station and the azimuth mark. One may also measure the angle at the eccentric station between the north or south point and the triangulation station. Whenever the astronomical station is eccentric to the triangulation station, a sketch should be entered in the record book, showing all measured distances and angles. The horizontal distances and angles should be measured accurately enough so that the eccentric reduction can be made with an accuracy of about an inch.

The observatory should be placed so as to have an unobstructed view of at least 50 degrees north and south of the zenith. A good way to ascertain whether the tentatively selected location for the observatory is satisfactory is to bolt the tent frame together and place it approximately in position, the long side in an East-West direction. After shifting the frame about so that the opening in the tent top comes above the instrument it can readily be seen whether the place chosen will be satisfactory.

(b) Methods of Orientation. The first field operation in setting up an astronomical observatory is the establishment of a true North-South line through the instrument position for the purpose of placing the instrument roughly in the local meridian. A stake placed some 500 feet to the north or south of the station site will suffice for this orientation.

There are several methods by which the stake can be set in the approximate meridian of the transit location. It has been found through practice that if all precautions are taken to insure as accurate an orientation as possible with the means at hand, the final placing of the transit in the true meridian will be simplified.

If the station occupied is a previously determined triangulation station, the true azimuth of the reference marks, of the station azimuth mark, or of some distant triangulation station visible from the ground may be used and a North-South line established through the instrument position by means of a small engineer's transit. Using the reference or azimuth marks affords the quickest and easiest method of orientation. If the triangulation data are not available, a magnetic compass may be used for orientation. Observations on the sun or Polaris will also give the desired azimuth. When using Polaris, the hour angle is ascertained from Table VII of the American Ephemeris and Nautical Almanac and then Table IV of the same publication is used to find the azimuth of Polaris corresponding to this hour angle.

(c) Placing tripod, and instrument setup. A very rigid support should be provided for the Bamberg transit. Concrete piers are undoubtedly the best type of support but considerable time is required for their construction and seasoning. It has been the custom in this Bureau for several years to support the instrument on a special metal tripod constructed so that it may be made very rigid by tightening the cross braces with turnbuckles. The tripod consists of a triangular head plate supported by three adjustable legs. Horizontal members and diagonal rods with turnbuckles in each panel keep the tripod rigidly braced. Care should be exercised to make sure that the tripod head rests snugly on the tops of the legs at all points. The head may be drawn down into proper position by means of the turnbuckles on the diagonal rods.

The tripod head has one pointed corner and two rounded ones. When placing the tripod over the mark, the pointed corner should be toward the south in the Northern Hemisphere. After the Bamberg transit has been centered, leveled and lined up with the orientation stake, the ground is marked at the base of each tripod leg and a small circular hole is later dug at each mark. These holes should be dug from 18″ to 24″ deep depending upon the nature of the ground. Six to eight inches of very wet plaster of paris or concrete should be poured into the bottom of each hole and allowed to set firmly. If the mix is very wet it will make much better bond with the soil and provide a more stable support. The top of the plaster or concrete in each hole should be at least 10″ to 12″ below the surface of the ground so that the observer walking about the instrument will not disturb it. Upon setting, this plaster or concrete provides a smooth surface over which the tripod legs may slide in making the necessary shifts to place the instrument in the meridian.

While the plaster is setting, the canvas tent may be placed over the frame in the

position such that the slot in the tent top is in a North-South line directly over the instrument location. See figures 10 and 14. As soon as the plaster has hardened, the tripod is placed in position and again lined up with the orientation stake. The transit is set up on the tripod. It is not necessary that the transit be exactly centered over the mark for latitude and longitude observations. In fact, there is no direct way of plumbing the Bamberg transit over the mark. It will be sufficient to have the tripod plumbed over the mark within one inch and the transit centered on the tripod head.

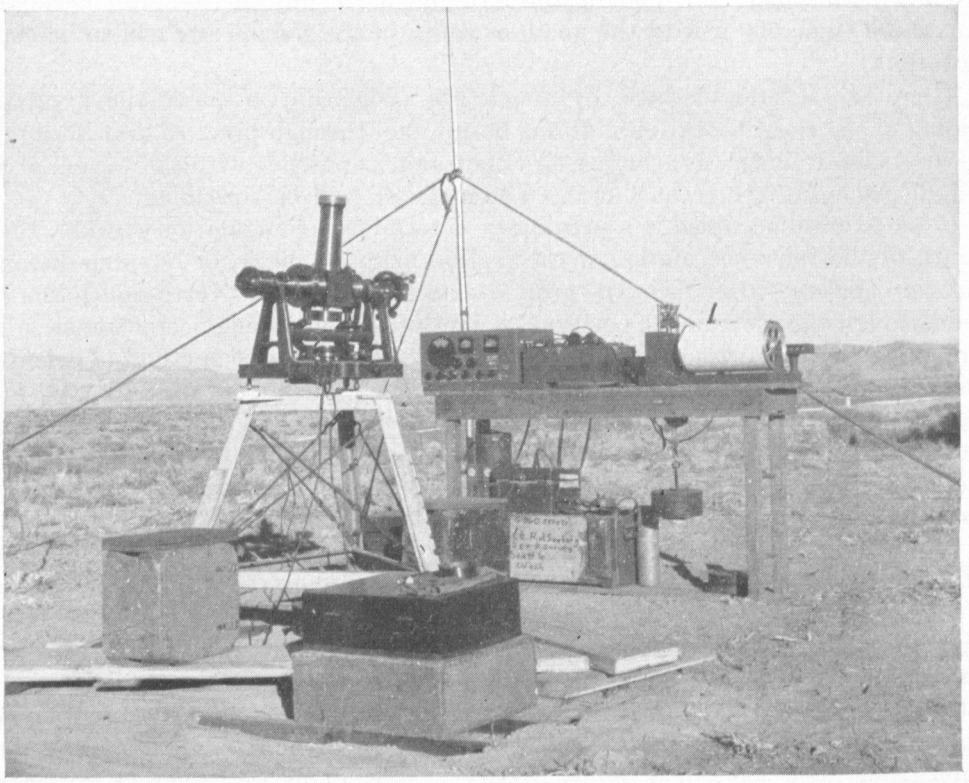


FIGURE 14.—Catwalk and arrangement of instruments.

The three footplates should be set on the tripod-head first, with the smooth surfaced footplate at the south end, the plate with the small hole in it at the west end, and the plate with the V-shaped groove at the east end. These plates are placed thus so that the instrument can be moved through a small arc about the west foot screw as a center. After the instrument has been set up and leveled, the telescope should be lined up with the orientation stake and the instrument shifted if necessary. The line of sight will then lie very nearly in the true meridian. Wet plaster of paris should be poured around the bases of the tripod legs in order to provide rigid instrument support. The three footplates are also fastened rigidly to the tripod head with plaster of paris before the final adjustment into the meridian is made.

It will be found that, if the foregoing procedure has been followed closely, very little further shifting of the instrument will be required to bring it finally into the meridian. Sometimes, especially if the ground is springy, a 12-inch wide platform should be built

around the transit, so that the weight of the observer will not bear directly upon the ground near the tripod. See Figure 14. In fact, some observers use such a ready-built platform at all stations. This platform may rest on 2 by 4 inch pieces of lumber whose support points are well away from the instrument, preferably at least 3 feet from any tripod leg. The boxes or seats upon which the observer sits during observation, one at the east end and one at the west end, should be placed on this platform. When the tripod is being set up, the legs should be adjusted for length with respect to the depth of the holes and height of platform so as to leave the observer in a comfortable position for observing. For complete details for placing the instrument in the meridian see page 34.

(d) Electrical Instrument Table and Hookup. A table with detachable legs should be set up along the east wall of the tent for holding electrical equipment. A 1 by 5 foot table is quite satisfactory. A 12-inch board set under this table will be useful for giving the table a steady support and for providing a place for the "B" and "C" batteries, tools and other paraphernalia. From north to south, the instruments are lined up in this order on the table: receiver, amplifier, chronometer and chronograph. This arrangement is the most convenient for the electrical hookup. See fig. 8. Some observers find a different arrangement more convenient.

The "B" batteries are usually blocked into place in wooden boxes specially built for carrying the required number of batteries. They are transported in these boxes from station to station. Separate "B" battery connections are necessary for the receiver and amplifier. The "C" batteries for the amplifier may be placed in this box also. Some type of plug and socket may be assembled and fitted to the end of the battery box for convenience in making and breaking connections. The "A" storage batteries are placed in any convenient location, usually along the north wall.

A 3-inch square hole is cut near one end of the table through which a small pulley line runs down from the chronograph gear box. The weights for driving the chronograph mechanism are attached to the lower end of the line.

The orientation for and the method of erecting the antennas have been discussed on page 11. The observer should check the guying of the poles carefully, so that they will not be blown over and fall upon the observatory tent and thus cause damage.

During the summer months, in temperate or warm climates, the recorder may sit outside of the tent for his recording. However, in winter in the temperate climates or in summer in the arctic regions, some kind of tent should be erected for the recorder. A silk triangulation observing tent has been found quite satisfactory for this purpose. It should be set east or west of the observing tent, not in the meridian of the instrument. When observations are made in cold weather, the electrical instrument table may be set up in the recorder's tent. However, care should be used if this tent is heated since the heat waves may interfere with the astronomical observations. A heater should never be placed in the observer's tent. The multiple wire connection between the transit and the instruments in the recorder's tent is strung between the two tents.

ADJUSTMENTS OF THE INSTRUMENTS

(a) Adjustments of the Transit. After the transit is set up, it should be inspected. The pivots and wyes of both instrument and level should be cleaned with watch oil, which must afterwards be wiped off to keep it from accumulating dust. The pivots and wyes should be carefully inspected to insure that there is no dirt gummed to them. The lens should be examined occasionally to see that it is tight in its cell. It may be

dusted off with a camel's-hair brush, and when necessary may be cleaned by rubbing gently with soft, clean tissue paper, first moistening the glass slightly by breathing on it.

Focusing the Eyepiece. The eyepiece should be focused by turning the telescope to a light surface such as the sky and the eyepiece moved in and out until that position is found in which the most distinct vision of the micrometer wire is obtained.

Focusing the Objective. The objective should then be focused by directing the telescope to some well-defined object, not less than a mile away, and moving the transit-micrometer draw tube in and out, thus changing the distance from the objective to the plane in which the micrometer wire moves, until there is no apparent change of relative position (or parallax) of the micrometer wire and the image of the object when the eye is shifted sideways or up and down in front of the eyepiece. The purpose of the adjustment, namely, bringing the image formed by the objective into coincidence with the micrometer wire, has then been accomplished. The focus of the objective will need to be checked at night, using a star as the object, and corrected if necessary. Unless the focus is made nearly right by daylight, none but the brightest stars will be seen at all at night, and the observer may lose time trying to learn the cause of the trouble. If the objective is focused at night, a preliminary adjustment should be made on a bright star and the final adjustment on a faint star, as it is almost impossible to get a very sharp image of a bright star. A planet or the moon is an ideal object on which to make a preliminary focusing of the objective. A scratch upon the drawtube to indicate its approximate position for sidereal focus will be found convenient. After a satisfactory focus has been found, the drawtube is clamped in position with screws provided for that purpose.

The illumination of the cross hairs is controlled by a screen grid in the telescope axis between the light and the prism. By turning the knurled knob, which is on one of the square faces of the telescope, more or less light may be cut off from passing through the grid, thus decreasing or increasing the illumination of the cross hairs at will. The final adjustment of the illumination of the cross hairs is made by turning the rheostat knob on the base of the frame.

Hanging-level Adjustment. The hanging level is first adjusted for *wind* (pronounced to rhyme with *find*). The bubble is brought to the center by the leveling foot screws. The level is tested for wind by slowly rocking the level forward and backward on its supporting arms. If the bubble does not remain centered, then the bubble axis and the horizontal axis of the instrument lie at an angle to each other when projected upon a horizontal plane, and the level is said to *have wind*. The adjustment for wind is by means of the screws which permit a lateral adjustment of one end of the tube. The adjustment for wind is very important, because any wind introduces a serious error into the result.

After the adjustment for wind is perfected, the bubble is brought to the center again by the leveling footscrews. Then the hanging level is reversed. If the bubble does not return to the center, half of the discrepancy is corrected for by the foot screws and half by the vertical adjusting screws on the level. This process of adjustment is repeated until the lack of adjustment does not exceed one or two divisions of the level.

Adjustment of Cross Hairs. The two wires which define the path of the star should be parallel to the telescope tube. This adjustment is made by means of the small screws which butt against the lug on the side of the drawtube. These wires are correctly adjusted if an equatorial star appears to move centrally between them as it crosses the field of view.

Adjustment for Collimation. The middle wire of the fixed field of the transit is set in the line of collimation by the customary method of direct and reverse pointings on a distant object and correcting for half of the difference by adjusting the two small collimation screws on the collar of the drawtube. An alternate method is to record the readings of the micrometer head when the movable wire is set on an object near the center of the field with the instrument in both the direct and reverse positions. Setting the micrometer head at the mean reading puts the movable wire in the line of collimation. The fixed middle wire should then be made to coincide with the movable wire by adjusting the collimation screws.

Wherever practicable, the adjustment for collimation should be made at sidereal focus on a terrestrial object at least one mile distant, or on the cross hairs of a theodolite or collimator which has previously been adjusted to sidereal focus. The theodolite or collimator is set up just in front of the telescope of the transit. If necessary the cross hairs of the theodolite are artificially illuminated. Occasionally, if neither a distant object nor a theodolite is available for making the collimation adjustment, a near object may be used for the purpose. In this case, however, collimation error may exist when the telescope is in sidereal focus. A rapid and careful observer may sometimes be able to make this collimation adjustment on a slow-moving close circumpolar star. In so doing he will have to estimate the amount the star moves while he is reversing his instrument and securing the second pointing. No attempt should be made to reduce the collimation error to zero. If it is already less than 0.2 second of time it should not be changed, for experience has shown that frequent adjustment of an instrument causes looseness in the screws and the moving parts.

Adjustment of the Setting Circle. To test the setting circle which reads zenith distances, point upon some object, bringing the image of the object midway between the two horizontal lines (guide lines); bring the bubble of the setting circle level to the center and read the circle. Next reverse the telescope and point again on the same object; bring the bubble to the center and read the setting circle as before. The mean of the two readings is the true zenith distance of the object, and their half difference is the index error of the circle. The index error may be made zero by setting the circle to read the true zenith distance, pointing on the object, and bringing the bubble to the center with the setting circle level-adjusting screw. At night if the latitude is known, this adjustment may be made by keeping a known star between the horizontal lines as it transits. While the telescope remains clamped in this position the setting circle is set to read the known zenith distance of the star and the bubble is brought to the middle position of the tube as before by means of the setting circle level-adjusting screw.

The preceding adjustments cannot always be made in the order named, as, for instance, when no distant mark can be seen in the meridian. Not all of these adjustments need be made at every station. The observer must examine and correct them often enough to make certain that the errors due to them are always within allowable limits.

(b) Setting the Chronometer. To set the chronometer on local sidereal time or very nearly so, local sidereal time must be determined.

If the approximate longitude is known within three minutes of arc, either from a map or otherwise, a very practical way for determining local sidereal time is based upon receiving a scientific radio time signal some time before star observations are begun.

The local sidereal time for the instant of the time signal to be used for setting the chronometer, is computed beforehand in the following manner:

Example

Given: Longitude of station is 119°15′10″ W.
Time signal at 3 P. M. Pacific standard time, June 5, 1944

To Find: The local sidereal time for the instant of the time signal

Pacific standard time, June 5, 1944	15–00–00
Difference between P. S. T. and G. C. T	8–00–00
Greenwich civil time	23–00–00
Greenwich sidereal time of 0^h G. C. T. (RAMS+12 hr.), June 5, 1944	16–53–10. 4
Cor., mean solar to sidereal time (for 23 hrs.)	03–46. 7
Greenwich sidereal time of the time signal	15–56–57. 1
Longitude of station (if west, subtract; if east, add)	7–57–00. 7
Local sidereal time of time signal	7–59–56. 4

Abbreviations:

P. S. T., Pacific standard time.
G. C. T., Greenwich civil time.
RAMS+12, Right ascension of mean sun+12 hrs.

The hands of the chronometer are set beforehand at this predetermined local sidereal time for the instant of the time signal preferably for one whole minute or two whole minutes before the final time signal. The setting is done by turning the hands clockwise by means of a key which fits the end of the axis on which the hands rotate. The minute hand should be set so as to agree with the position of the second hand, that is if the second hand is on thirty, the minute hand should be set midway between two minute divisions. When the time for the predetermined minute of the signal approaches, the signal is tuned in, and at the instant of the minute break, the chronometer, already wound, is started with a gentle rotary movement. As the second hand cannot be set, allowance is usually made for this by starting the chronometer after the signal, either by counting seconds or using a hack watch. With care the chronometer can be started within a few seconds of local sidereal time. The chronometer is then checked at the next minute break.

If, for the purpose of setting the chronometer, the radio time signal cannot be obtained at that time, star observations may be used for that purpose. With the instrument leveled and properly adjusted for collimation, the telescope is set for a zenith star which is about to transit. The chronometer time of the instant when the image of the star crosses the line of collimation is recorded. The instant of transit is approximately local sidereal time, differing from it only by the effects of the collimation and azimuth error. Since the star is nearly in the zenith, its time of transit is affected but very little by the azimuth error of the instrument. Hence the chronometer correction is approximately equal to the right ascension of the star minus the chronometer time of transit. To avoid waiting for stars very close to the zenith the chronometer correction may also be estimated closely by comparing observations of two stars not very distant from the zenith, one north and one south; the difference of the chronometer correction obtained from these two will give some idea of the amount and direction of the azimuth error.

(c) Putting Transit in Meridian. The transit must be put in the meridian with an accuracy of one second of time or better before the longitude observations are started. Proceed as follows: Level the instrument accurately. Select a slow moving north star

that is about to transit and compute the time of transit. Pick up the star in the telescope one or two minutes before the time of transit. Keep the star on the cross hairs until the instant of transit by shifting the instrument on the tripod or, if necessary, shifting the tripod on the plaster base. Repeat with another north star if necessary to place the instrument in the meridian within an accuracy of ten to twenty seconds of time. Then fasten the tripod legs securely in position by pouring a small amount of wet plaster of paris about the foot of each leg, being sure that it makes good bond with the metal leg and with the plaster or concrete base upon which it rests. The three foot plates should also, at this time, be fastened securely to the tripod by placing a small amount of wet plaster under each plate. As soon as this plaster has set (10 to 20 minutes) the final adjustment of placing the instrument into the meridian can be made. Proceed as follows: Level the instrument very accurately, reversing as necessary. Select two stars, one north and one south of the zenith that will transit one to five minutes apart. Record the time of transit for each star. Write an equation for each star as follows:

$$\Delta T + Aa - (\alpha - t) = 0. \tag{9}$$

To be exact and complete this equation should be written

$$\Delta T + Aa + Bb + Cc - (\alpha - t) = 0.$$

However, if the instrument is leveled very accurately the Bb term, the inclination correction, may be neglected and if the instrument has been accurately adjusted for collimation the Cc term, the collimation correction, may also be neglected.

The symbols in the above equations are:

ΔT is the chronometer correction,

A is the azimuth factor, $\sin \zeta \sec \delta$,

a is the azimuth of the line of collimation in seconds of time,

α is the right ascension of the star,

t is the chronometer time of transit,

δ is the declination of the star,

ζ is the zenith distance of the star at culmination, that is, latitude of station minus declination of star, $\phi - \delta$, where ϕ is the latitude. In north latitudes ζ is positive for stars south of the zenith, negative for stars north of it.

ΔT and a are the two unknowns in these equations and by subtracting one equation from the other, ΔT cancels out and the azimuth a may be computed directly. The value of a may then be substituted in either equation and ΔT computed. If stars are selected with A factors of 0.5 or larger and if t is known to ½ second, a will be determined with an accuracy of one second or better. If a is positive, the telescope pointing north is too far to the west and if negative too far to the east. Let r equal the value of one turn of the east foot screw in seconds of time, and n equal the number of turns to be applied to the foot screw to bring the instrument to the meridian.

$$\text{Then } n = \frac{a}{r}. \tag{10}$$

The value of one turn of the east foot screw is approximately ten seconds of time for Bamberg transits Nos. 20 and 21. The instrument should then be turned through an azimuth a, by turning the east foot screw n turns in the proper direction.

This process should be repeated until the azimuth, a, of the instrument is less than 1.0 second of time. After having placed the instrument in the meridian, it is desirable to reverse the instrument and check with one pair of stars to verify that the collimation

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error is still within the desired limits. The instrument should always be tested for azimuth before beginning work on any evening, and if necessary it should be adjusted to make a less than 1 second of time.

LONGITUDE DETERMINATION [2]

Basically, a longitude determination consists of measuring the difference between the local sidereal time at the base station and the local sidereal time at the field station at the same instant of absolute time. The local sidereal time at the field station is determined by observations of transits of stars across the local meridian. At the instant of transit of a star across the meridian, the local sidereal time at the field station is equal to the right ascension of the star. Since the longitude of the base station is known, the local sidereal time at the base station can be computed for any given instant.

In practice, a radio time signal is received and from the Greenwich civil time of the broadcast of that signal, the local sidereal time at the base station is computed for the instant of the reception of the time signal. The time interval between the transit of the star and the reception of the time signal is carried by a good sidereal chronometer and thus the observed local sidereal time for the field station and for the base station are reduced to a common instant.

The determination of time in any latitude is based on the observation equation already given on page 35, known as Mayer's Formula,

$$\Delta T + Aa + Bb - (\alpha - t) = v. \tag{11}$$

There should be also a term, $+Cc$, on the left to take care of the collimation unless its effect is eliminated by reversal of the instrument as is done with the Bamberg transit. In this equation A, B, $\alpha - t$, and b are known. It is therefore necessary to determine the two unknown quantities, ΔT and a, from the observation equations.

A and B are constants for the star depending on its declination and zenith distance,
α is the right ascension of the star,
t is the observed chronometer time of transit of the star,
b is the observed inclination of the horizontal axis,
ΔT is the chronometer correction and
a is the azimuth of the line of collimation.

The field computation to determine these two quantities is made by giving each star unit weight even when the station is located in high latitudes, say above 50°. A more elaborate and accurate process is preferable for the office computations in all latitudes, although the field method has sometimes been used in the low latitudes.

GENERAL INSTRUCTIONS FOR WIRELESS LONGITUDE DETERMINATIONS WITH BAMBERG TRANSIT IN LOW LATITUDES

1. A determination of a difference of longitude normally will consist of at least three separate determinations, not all of which are made on the same night. If observations are being made in an area where very unfavorable weather conditions exist, and if one separate determination has been made on each of two nights, and it is found that the two separate determinations differ by less than 0.05 second, the observer may proceed to the next station if the weather reports indicate that several days of unfavorable weather are expected. Three nights of observations with at least one determination each night shall constitute a complete longitude determination, even if the com-

[2] Since this section was written some of the procedures have been altered. For a discussion of this, see the "Appendix to Longitude Determination", p. 121.

putations have not been completed before leaving the station. A single determination of longitude should normally consist of observations of two time sets. Each time set should consist of observations on from five to eight stars, preferably about equally divided between north and south stars. All of these are to be time stars; no azimuth stars are to be observed. For the purpose of this paragraph an azimuth star is defined as one for which the azimuth factor, A, is greater than 0.75, while a time star is one whose azimuth factor is less than 0.75. Preferably the azimuth factor for time stars should be held to less than 0.60.

2. The observer should select the stars for each time set with great care. He should endeavor to secure the maximum number of stars per hour subject to the following requirements. The algebraic sum of the A factors for any time set should be less than unity and the sum for the two sets that constitute a longitude determination should also be less than unity. It is desirable to have this sum as small as possible. Each time set should have about an equal number of north and south stars. A three-three or a three-four combination is the ideal one. A two-five combination is undesirable even though the A factors may balance satisfactorily. Sufficient time must be allowed between successive stars for setting the instrument, reading the level, reversing the instrument etc. The ideal time is about 5 minutes but some intervals may be 4 minutes or even slightly less. The observer must remember that two equatorial stars may be observed upon if their times of transit are 3½ minutes apart but two north stars with that same interval between their times of transit cannot be observed upon.

3. The detailed observing program for any night will depend principally upon the weather conditions and the number of time signals available. If only one time signal is available it should be near the center of the two time sets. If time signals are available at hourly intervals, or even more frequently, then a night's program should consist of time signal, time set, time signal, time set, etc., ending with a time signal. In this manner two or more longitude determinations may be made in one night under favorable weather conditions. There is, however, little to be gained in making more than two complete longitude determinations in any one night, for at least one more night's work is required under any circumstances.

4. *Observing List for Longitude.* In making an observing list, it is necessary to know the approximate latitude of the station. If the latitude is not known from the triangulation data, it may be obtained by scaling from a good map or obtained from astronomical observations. It should be known within one minute of arc. The observing list should be made up beforehand according to the general instructions in paragraphs 1, 2, and 3. The stars should be selected from "Apparent Places of Fundamental Stars" produced under the auspices of the International Astronomical Union and published annually by His Majesty's Stationery Office in London.

5. The position of the ocular of the transit (east or west) should alternate for the beginning of observation on each star. It may be in either position for the first star, but most observers prefer to start with ocular east. The position of the ocular should be given in the records for the first star at least.

6. When a longitude station is referred to a triangulation station, the measurements of angle and distance should be made and checked with such an accuracy that the position of the longitude station may at any time be reproduced within one inch.

7. The field computations should be kept as closely up-to-date as practicable and the method shown on pages 48, 51–53 of this publication should be followed.

8. *Radio Time Signals.* If possible, the scientific short-wave radio time signals transmitted by the Naval Communications Service or the continuous short-wave

standard-frequency signals by the National Bureau of Standards should be used. In any case, only signals monitored by the Naval Observatory may be used.

If other than the Annapolis or Bureau of Standards time signals are used, the observer should verify from this Office, before leaving the station, whether the Naval Observatory has received the particular signals.

About 20 second breaks of each radio time signal should be scaled from the chronograph sheet to the nearest hundredth of a second, rejecting any whose residual is 0.05 second or more from the mean.

9. *Chronograph Record.* The observations upon each star are given unit weight regardless of the declination of the star and regardless of whether or not the observation of the transit is complete. Ten consecutive transit breaks (identification breaks excluded) corresponding to one complete revolution of the micrometer contact wheel before reversal of the instrument and the ten corresponding breaks after reversal constitute a complete observation. (For identification of the zero mark there are two extra marks one on each side of the zero mark.) In scaling the chronograph sheet for star observations, each star-transit break should be scaled from the chronograph sheet to the nearest tenth of a second. See figure 5.

The limit of rejection for an observation on a star is a residual of 0.20 second. No observation corresponding to a residual smaller than this should be rejected unless rejection is made at the time of observation for reasons stated in the record.

DIRECTIONS FOR OBSERVING

Before starting to observe, the electrical hookup and recording apparatus should be carefully checked to insure proper recording. All outside binding-post connections on the amplifier should be tightened and the switch placed in the transit position. With the transit recording switch on, the two handwheels of the micrometer eyepiece should be given a few turns in order to test the action of the recording pen. The hanging level should be inspected for proper bubble length, wind and parallelism with horizontal axis. See page 32. The chronograph should be started and the chronometer breaks recorded for a few turns to check the speed of the chronograph cylinder and the marking action of the armature pen. The speed of revolution of the cylinder should be adjusted either by increasing or decreasing the number of weights or by adjusting the governor balls. It is very desirable that the cylinder should make very nearly one revolution per minute so that the corresponding chronometer second breaks will line up approximately horizontally across the paper. If the chronograph operates appreciably too fast, or too slow, the scaling of the sheets becomes more difficult because of the diagonal slant of the line of chronometer breaks.

In setting for the first star it is immaterial whether the eyepiece is in position East or position West. It is general practice to start with eyepiece East. The first position in every time set should be noted on the record. The proper zenith distance angle is set on the vertical setting circle for stars $\frac{\text{north}}{\text{south}}$ of the zenith when the eyepiece is $\frac{\text{east}}{\text{west}}$; and the explement of the zenith distance angle for stars $\frac{\text{south}}{\text{north}}$ of the zenith when the eyepiece is $\frac{\text{east}}{\text{west}}$. The bubble of the setting circle level is then brought to the center by tilting the telescope, making the final adjustment by tapping the telescope with the finger tips. Depending on whether the zenith distance or explement of the zenith distance angle has been used in the first case, the opposite one for that same star is

then set on the setting circle, so as to have the setting ready for the reversed position of the instrument. By using this procedure the instrument will be ready for centering the setting circle level immediately after reversal of the telescope in the wyes.

By turning the micrometer wheels, with the transit switch off, the movable slide of the transit micrometer is brought into proper position with respect to the entrance of the star's image into the field of view. With the transit micrometer eyepiece in the position East, all stars above the pole will come into the field of view at the apparent top. Subpolar stars will come in at the apparent bottom. With eyepiece West, exactly the opposite is true, stars above the pole coming in at the apparent bottom and subpolar stars coming in at the top.

The usable field of view is taken as that between the outer wires of the fixed reticule. The time of entrance of the image of the star into the telescope field of view depends upon its declination, circumpolar stars of course traveling much slower than equatorial stars. When the star enters the usable field of view, tap the telescope to bring the star between the two guide wires. It should be followed with the cross wire of the movable slide, bisecting the star at all times and attempting to make the motion of the hand-wheels as uniform as possible. The star image should be followed well into the field of view, 20 to 30 breaks, but enough time should be allowed between corresponding breaks before and after reversal to read the hanging level and to reverse the telescope. Upon reversal, if the circle setting has been made beforehand the telescope can be brought quickly into the proper position by tilting the telescope until the bubble of the setting circle level is centered. After the star has been followed across the field of view and the hanging level read, the proper setting for the next star should be made. From time to time the chronograph sheet should be inspected to see that all of the transit breaks are recording properly. Occasionally, a time identification should be marked on the sheet at the beginning of some whole minute and the information given to the recorder, who should record it on Form 34A. About two or three identifications of the chronometer minute should be made during each time set and at least one during the time when the chronograph is running at double speed for the reception of the radio signal.

RECEPTION AND RECORDING OF TIME SIGNALS

About ten minutes before the start of the time signal the radio receiver should be turned on to allow the tube filaments to warm up properly.

After the last star before the signal has been observed the chronograph should be changed over to double speed and the amplifier switch turned to "RADIO". Some time shortly before or after the time signal broadcast a chronometer time identification should be recorded at double chronograph speed. If during the radio time signal recording there is interference between the chronometer breaks and the radio time-signal breaks, the cut-out switch should be used, as previously described in this publication. After the record of the time signal has been completed, the second set of stars should be observed and recorded in the same way as the first set.

METHOD OF AND FORMS FOR RECORDING

At the field station the star observations and time signals are recorded on the chronograph sheets and other pertinent data are entered on Form 34A. As each star is observed, the readings of the hanging level are recorded on Form 34A opposite the name of the star. A glance at the difference between the west and east readings of the level will show the amount of inclination of the horizontal axis and whether the bubble needs adjusting. A west reading is considered positive. Under "Remarks" on this Form the

chronometer time identifications are recorded as given by the observer from time to time. They are placed in consecutive order and numbered consecutively to correspond with the proper identification on the chronograph sheet. Any remarks pertinent to some particular star, such as "very dim," "brighter of two," "thin clouds," should also be entered in the "Remarks" column, or if space permits, under the star name itself. It is especially important to note any irregularities in following the star, so that these may be kept in mind when scaling the chronograph sheet. It is also helpful to indicate the type of star such as north, south or azimuth. A sample is shown on page 42. At the bottom of this Form, information is entered with regard to the time, frequency and call letters of the time signal or signals received.

The chronograph sheet, 33 by 63 centimeters, covers the record for about two hours of observing. Normally one longitude determination will be recorded on one chronograph sheet. The following information should be placed on the chronograph sheet before leaving the station: Name of station, State, local Standard Time of signal, both hour and date, transmitting station and frequency of signal. Since the date changes at midnight, it is very important that the date is carefully noted and the dates of evening and the following morning distinguished.

As soon as possible after the longitude is observed, the chronograph record should be "marked up." The proper hour and minute are entered for each identification mark. The name of each star is entered just before the transit record of that star. All of these data have been recorded on Form 34A as the longitude observations were made, and so they may be copied directly onto the chronograph sheet.

RADIO TIME SIGNALS

The Naval Observatory Time Signals broadcast by the Naval Communications stations, *NSS* at Annapolis, *NPG* at Mare Island, and some others are monitored by the Naval Observatory. These signals are sent out at one second intervals during the five minutes preceding the hour for specified hours of the day. Each signal or beat is 0.4 second in duration. The beginning of the beat is the beginning of the second. For identification every 29th second is omitted; at the end of the first minute the 51st and 56th to 59th are omitted, in other words, there are four successive beats before the last five second gap of that minute, signifying that there are still 4 minutes left before the full hour; at the end of the second minute, there are three successive beats before the last five second gap; at the end of the third two, and at the end of the fourth one. At the end of the last minute the 51st to 59th beats are omitted. The last beat on the full hour is a decidedly longer beat.

The National Bureau of Standards furnishes continuous time signal service throughout the 24 hours of a day. These signals are broadcast from the Bureau of Standards station *WWV* by high-frequency radio waves. The Naval Observatory monitors these signals at twelve hour intervals.

The *WWV* signal is transmitted by impulses of 0.005 second duration at precise second intervals. The 59th second of each minute is omitted for identification purposes. The standard audio frequencies which are transmitted throughout the day are interrupted precisely on the hour and each five minutes thereafter; after an interval of precisely one minute the audio frequencies are resumed. At the hour and the half hour, a detailed announcement is given by voice.

Several weeks after the time signals, the Naval Observatory furnishes upon request the corrections to the monitored time signals.

LONGITUDE RECORD AND FIELD COMPUTATION

As an example for longitude determination, station Wills, Arkansas, has been chosen. The observations were made by Lt. J. P. Lushene on Dec. 18 and 20, 1935.

On Form 34A, the readings of the hanging level have been recorded for the different stars. At the bottom of the sheet, the hour, frequency and station are shown for the time signal received. Under "Remarks", the hours and minutes are given for the corresponding identification marks on the chronograph sheet. These data under "Remarks" and the corresponding identification marks on the chronograph are all recorded at the time of observation. Thus the time by the chronometer and the chronograph record are positively linked with each other.

The entire longitude computation is in one book, Form 256. This book contains the various forms necessary for the computation of the longitude.

On pages 4 and 5, Form 256, are shown the magnitude, right ascension, declination, meridian zenith distance of the star, the star factors (A, B and C) and the corrections l and κ, for width of contact strips combined with lost motion, and for diurnal aberration respectively.

The factors A, B, and C are taken directly from the table on pages 156-171.

The factors are defined by the following formulas:

$$\text{Azimuth factor}=A=\sin\zeta\sec\delta=\sin\phi-\tan\delta\cos\phi \tag{12}$$

$$\text{Level factor}=B=\cos\zeta\sec\delta=\cos\phi+\tan\delta\sin\phi \tag{13}$$

$$\text{Collimation factor}=C=\sec\delta \tag{14}$$

where $\zeta=(\phi-\delta)$ is the zenith distance of the star at culmination, δ is the declination of the star, and ϕ is the latitude of the station. In the northern hemisphere ζ is positive for stars south of the zenith, negative for stars north of it. For lower transits $180°-\delta$ is used instead of δ.

For north latitudes, the signs of the factors are as follows:

A is plus except for stars between the zenith and the pole,

B is plus except for stars observed at lower culmination.

For observations with the Bamberg transit, the factor C is used for computing the correction l for lost motion and average width of the contact strips. See page 26. The correction l is positive for all stars.

For observations with an instrument, such as the meridian telescope, where the telescope is not reversed between transit observations on the same star, the factor C is necessary for computing the collimation error of the instrument. In this case: C is $\frac{\text{plus}}{\text{minus}}$ for stars at upper culmination and $\frac{\text{minus}}{\text{plus}}$ for stars at lower culmination when observations are made with the instrument, band (clamp or illumination) $\frac{\text{west}}{\text{east}}$.

The correction for diurnal aberration, κ, is explained on page 46.

The data relating to the station (date, observer, etc.) are copied directly from Form 34A onto pages 6–29, Form 256. The names of the stars and the level data are also copied from Form 34A.

The chronograph sheets are usually read in the field before leaving the station. A set of 10 transit breaks before transit of a star and the corresponding set of 10 breaks after transit are selected and the chronometer time of each break is read to the nearest tenth of a second and entered in the proper columns on pages 6–29, Form 256. The reading is done by the use of a glass scale with uniformly spaced converging lines.

DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
Form 34A
Rev. Feb. 1935

LONGITUDE RECORD

ORIGINAL TRANSIT LEVEL READINGS

Station: WILLS ; Instrument: BAMBERG No. 21

State: ARKANSAS ; Chief of party: J. P. LUSHENE

Local date: DEC. 18, 1935 ; Observer: J. P. LUSHENE

PASTE THIS SHEET ON TO THE CHRONOGRAPH SHEET

SET I				SET II			
STARS		LEVELS W.	LEVELS E.	STARS		LEVELS W.	LEVELS E.
ν ANDROMEDAE	N	57.9	18.5	ν ARIETIS	S	20.8	59.9
		18.4	57.2			58.9	19.8
		39.5	38.7			38.1	40.1
φ PERSEI	N	18.2	57.4	θ PERSEI	N	58.9	19.1
		58.8	19.4			20.8	59.8
		40.6	38.0			38.1	40.7
α TRIANGULI	S	58.1	18.6	τ PERSEI	N	19.8	58.8
		19.0	58.2			59.5	20.2
		39.1	39.6			39.7	38.6
γ' ANDROMEDAE	N	18.5	57.6	ε ARIETIS	S	59.1	20.0
TEMPERATURE:		59.4	20.1	TEMPERATURE:		20.3	59.2
		40.9	37.5			38.8	39.2
α ARIETIS	S	59.4	20.1	β PERSEI	N	20.1	59.1
		19.2	58.5			59.0	20.0
		40.2	38.4			38.9	39.1
6 PERSEI	N	19.8	59.0	δ ARIETIS	S	58.9	19.8
		59.0	19.6			20.9	59.8
		39.2	39.4			38.0	40.0
γ TRIANGULI	S	58.5	19.1				
		19.8	58.9				
		38.7	39.8				

Greenwich date and G. C. T. of signal: 03 00 00 Chronometer: 3479

Frequency of signal: NAA 9250 KC

Remarks: (1) 1 29 00 (6) 2 30 00
(2) 1 42 00 (7) 2 31 00
(3) 1 56 00 (8) 2 44 00
(4) 2 15 00
(5) 2 22 00 (SIGNAL)

U. S. GOVERNMENT PRINTING OFFICE 16—7901

FIGURE 15.—Longitude record.

FIELD COMPUTATION

Form 256 STATION: WILLS, ARK.
$\phi = 36° 26' 45.''30$ sin ϕ= cos ϕ=

CAT.	STAR	MAG.	α	δ	Z. D.	A	C	B	k (−)	l (+)
	ν ANDROMEDAE	4.2	$1^h 33^m 04^s$	41° 05'	N 4° 38'	− .10	1.32	1.32	.02	.07
	φ PERSEI	4.2	1 39 40	50 22	N 13 55	− .38	1.57	1.53	.03	.08
	α TRIANGULI	3.6	1 49 27	29 16	S 7 11	+ .14	1.15	1.14	.02	.06
	γ' ANDROMEDAE	2.3	2 00 00	42 02	N 5 35	− .13	1.35	1.34	.02	.07
	α ARIETIS	2.2	2 03 35	23 10	S 13 17	+ .25	1.09	1.06	.02	.05
	6 PERSEI	5.4	2 09 22	50 46	N 14 19	− .39	1.59	1.53	.03	.08
	γ TRIANGULI	4.1	2 13 32	33 33	S 2 54	+ .06	1.20	1.20	.02	.06
	ν ARIETIS	5.4	2 35 13	21 41	S 14 46	+ .28	1.08	1.04	.02	.05
	θ PERSEI	4.2	2 39 51	48 58	N 12 31	− .33	1.52	1.48	.03	.08
	τ PERSEI	4.1	2 49 45	52 30	N 16 03	− .46	1.64	1.58	.03	.08
	ε ARIETIS	4.6	2 55 35	21 05	S 15 22	+ .29	1.07	1.03	.02	.05
	β PERSEI	VAR.	3 04 02	40 43	N 4 16	− .10	1.32	1.31	.02	.07
	δ ARIETIS	4.5	3 08 00	19 29	S 16 58	+ .31	1.06	1.02	.02	.05

FIGURE 16.—Star factors and corrections for diurnal aberration and width of contact strip, field computation.

There are 21 converging lines, uniformly spaced, on this glass scale; and the spacing is such that at some point of the scale, the two outer lines can be made to coincide with the beginnings of successive chronometer breaks, thus dividing the two second chronometer interval into twenty equal parts or tenths of seconds. The transit breaks can then be read directly to the nearest tenth of a second.

The chronometer times of ten breaks before transit (before reversing the telescope in the wyes) are recorded in the first column successively from the top down, and the chronometer times for the corresponding ten breaks after transit are recorded in the second column from the bottom up. In that way the first scaled break of the set before transit and the last scaled break of the set after transit are recorded on the same line. The mean of the chronometer times for each such pair of breaks will then yield a chronometer time of transit of the star. In practice, for each pair the sum of the seconds in the first and second columns is entered in the third column, a mean is taken of these sums and this mean is divided by two. The result will be the mean observed chronometer time of transit of the star based on ten observations.

Correction for Inclination of Axis

If the horizontal axis of the telescope is slightly inclined to the horizon and the telescope is otherwise in perfect adjustment, the line of collimation describes a plane which passes through the north and south points of the horizon, when the telescope is rotated about its horizontal axis. This plane will make an angle with the meridian plane equal to the inclination of the horizontal axis with respect to the horizon. If the east end of the axis is too high, the transits of all stars above the pole (apparently moving westward) will be observed too late, and the transits of all subpolars will be too early; it is therefore necessary to correct the observed times of transit by means of the readings of the hanging level.

Form 256

Station, WILLS, ARKANSAS — Observer, J. P. LUSHENE

Date, DEC 18, 1935 — Recorder, J. P. LUSHENE

Instrument, BAMBERG NO. 21 — Chronometer, 3479 — Temp., 13°C

Level value, 1."200 — GREENWICH CIVIL DAY 19.1

Star:	υ ANDROMEDAE			φ PERSEI			α TRIANGULI			γ' ANDROMEDAE		
Clamp.												
Level:	W	E		W	E		W	E		W	E	
	57.9	18.5		18.2	57.4		58.1	18.6		18.5	57.6	
	18.4	57.2		58.8	19.4		19.0	58.2		59.4	20.1	
	+39.5	−38.7		+40.6	−38.0		+39.1	−39.6		+40.9	−37.5	
	+ .8			+ 2.6			− .5			+3.4		
	+ .016			+ .052			− .010			+ .068		
B	1.32			1.52			1.14			1.34		
	1 h. 33 m.			1 h. 39 m.			1 h. 49 m.			2 h. 00 m.		
	32	33		38	40		48	50		59	60	
	s	s	Sums	s	s	Sums	s	s	Sums	s	s	Sums
	17.1	69.8	86.9	47.9	52.6	100.5	48.1	25.5	73.6	15.8	62.9	78.7
	18.5	68.3	86.8	49.7	50.8	100.5	49.2	24.3	73.5	17.2	61.5	78.7
	19.8	67.0	86.8	51.2	49.3	100.5	50.3	23.1	73.4	18.8	60.2	79.0
	21.2	65.6	86.8	52.9	47.7	100.6	51.8	21.8	73.6	20.2	58.8	79.0
	22.8	64.2	87.0	54.5	46.0	100.5	53.0	20.5	73.5	21.6	57.3	78.9
	24.0	62.8	86.8	56.1	44.3	100.4	54.1	19.5	73.6	23.0	55.8	78.8
	25.4	61.3	86.7	57.8	42.8	100.6	55.3	18.4	73.7	24.2	54.4	78.6
	26.8	60.1	86.9	59.3	41.0	100.3	56.6	17.0	73.6	25.8	53.1	78.9
	28.2	58.7	86.9	61.0	39.3	100.3	57.8	15.9	73.7	27.2	51.7	78.9
	29.6	57.2	86.8	62.8	37.8	100.6	59.0	14.8	73.8	28.8	50.2	79.0
	Mean		86.84	Mean		100.48	Mean		73.60	Mean		78.85
	Mean/2		13.42	Mean/2		50.24	Mean/2		36.80	Mean/2		09.42
l			+ .07			+ .08			+ .06			+ .07
k			− .02			− .03			− .02			− .02
Bb			+ .02			+ .08			− .01			+ .09
t	1	33	13.49	1	39	50.37	1	49	36.83	2	00	09.56
α	1	33	03.617	1	39	40.113	1	49	27.293	1	59	59.632
Δα (short-period term)			− .012			− .014			− .010			− .012
R (RATE = +.00112)			− .025			− .018			− .007			+ .005
α+Δα−(t+R)			−9.86			−10.25			−9.54			−9.94
C (FIELD COMP.)			−9.87			−10.26			−9.54			−9.93

11—3205

FIGURE 17.—Computation of the chronometer correction for each star.

Form 256

Station, WILLS, ARKANSAS — Observer, J. P. LUSHENE

Date, DEC. 18, 1935 — Recorder, J. P. LUSHENE

Instrument, BAMBERG NO. 21 — Chronometer, 3479 — Temp., 13°C

Level value, 1".200

Star:	α ARIETIS		6 PERSEI			γ TRIANGULI					
Clamp.											
Level: W		E	W		E	W		E	W		E
59.4		20.1	19.8		59.0	58.5		19.1			
19.2		58.5	59.0		19.6	19.8		58.9			
+ 40.2		− 38.4	+ 39.2		− 39.4	+ 38.7		− 39.8	+		−
	+1.8			−.2			−1.1		$d\psi$ = −.11		
	+.036			−.004			−.022		$d\omega$ = +.08		
B	1.06			1.53			1.20				
2 h. 03 m.			2 h. 09 m.			2 h. 13 m.			h.		m.
02	04		08	10		12	14				
s	s	Sums	s	s	Sums	s	s	Sums	s	s	Sums
58.5	30.8	89.3	29.7	35.5	65.2	50.8	32.6	83.4			
59.5	29.6	89.1	31.3	33.8	65.1	52.0	31.4	83.4			
60.8	28.5	89.3	33.0	32.4	65.4	53.3	30.0	83.3			
61.8	27.4	89.2	34.6	30.5	65.1	54.6	28.8	83.4			
63.1	26.2	89.3	36.2	28.9	65.1	55.8	27.4	83.2			
64.2	25.1	89.3	38.0	27.1	65.1	57.2	26.3	83.5			
65.3	24.0	89.3	39.5	25.5	65.0	58.4	25.0	83.4			
66.3	22.9	89.2	41.2	23.8	65.0	59.6	23.8	83.4			
67.6	21.8	89.4	43.0	22.3	65.3	60.8	22.6	83.4			
68.8	20.6	89.4	44.5	20.6	65.1	62.0	21.3	83.3			
	Mean	89.28		Mean	65.14		Mean	83.37		Mean	
Mean/2		44.64	Mean/2		32.57	Mean/2		41.68	Mean/2		
l		+.05			+.08			+.06			
k		−.02			−.03			−.02			
Bb		+.04			−.01			−.03			
t	2 03	44.71	2 09		32.61	2 13		41.69			
α	2 03	35.338	2 09		22.337	2 13		32.025			
Δα (short-period term)		−.009			−.014			−.011			
R		+.009			+.016			+.020	MEAN EPOCH		
α+Δα−(t+R)		−9.39			−10.30			−9.70	= $1^h 55^m 41^s.32$		
11—9206		−9.37			−10.27			−9.66			

The divisions of the hanging level are numbered consecutively from one end of the level to the other. Let w and e be the readings of the west and east ends of the bubble, respectively, when the lower readings of the level are toward the east; and let w' and e' be the corresponding west and east readings when the telescope is reversed in the wyes, that is, when the lower readings are toward the west. Let d be the value of one division of the level in seconds of arc, then $\frac{d}{15}$ will be the value of one division in seconds of time. Then the apparent inclination of the telescope axis, b, expressed in seconds of time will be:

$$b=[(w-w')+(e-e')]\frac{d}{60}$$

and the inclination correction becomes

$$b(\cos\phi+\tan\delta\sin\phi)=bB. \tag{15}$$

Correction for Diurnal Aberration

The effect of the annual aberration due to the motion of the earth in its orbit is taken account of in the apparent positions of the stars as published in any star catalogue giving apparent places, such as the American Ephemeris or Apparent Places of Fundamental Stars. Therefore, a correction for this effect is not needed.

The correction for diurnal aberration to be applied to an observed time of transit of a star across the meridian is:

$$\kappa=0^s.021\cos\phi\sec\delta \tag{16}$$

where ϕ is the latitude of the station and δ is the declination of the star.[3] This correction is *minus* for all stars observed at upper culmination and *plus* for all stars observed at lower culmination. Table I for diurnal aberration is given on page 154.

The three corrections, l for lost motion and average width of contact strips, κ for diurnal aberration, and bB for inclination of the horizontal axis are applied to the observed chronometer time of transit to obtain a partially corrected chronometer time of transit, t.

The apparent right ascension of the star, α, is obtained directly from a star catalogue giving the apparent positions of the stars for every ten days. This Office used the American Ephemeris for the right ascensions of the longitude stars until the year 1940, inclusive. Since that date the Apparent Places of Fundamental Stars, prepared under the auspices of the International Astronomical Union, has been used for obtaining the apparent right ascensions. The apparent right ascension, α, is computed or rather interpolated for the mean epoch of the observations between two successive published values. The short-period terms in right ascension need not be considered in the field computation. However, they must be included in the final office computation. See page 54.

The chronometer corrections are now computed for each individual star by taking the difference:

$$\text{Chronometer correction}=\alpha-t \tag{17}$$

where t has been corrected for lost motion and average width of contact strips, diurnal aberration and inclination of the axis.

[3] See William Chauvenet, Spherical and Practical Astronomy, Vol. I, p. 640.

Collimation Correction

No correction is needed for collimation in observations with the Bamberg instrument. The collimation error is eliminated by making observations on each star in both positions of the telescope, eyepiece East and eyepiece West.

Azimuth and Chronometer Corrections

In general, there is still one more error left in the chronometer correction, $\alpha-t$, as defined above and that is the error due to the fact that the instrument is not exactly in the meridian. If the instrument is otherwise in adjustment, but has a small error in azimuth, the micrometer wire in its mean position will describe a vertical circle on the celestial sphere at a small angle with the meridian. The correction in seconds to an observed time of transit for this azimuth error is:

$$\begin{aligned}\text{Azimuth correction} &= a \sin \zeta \sec \delta = a(\sin\phi - \cos\phi \tan\delta) \\ &= aA \end{aligned} \tag{18}$$

in which a is the angle expressed in seconds of time between the meridian and the vertical circle described by the mean position of the micrometer wire.[4] It is considered positive when the collimation axis is too far to the east with the telescope pointed south. In north latitudes, the factor A is positive for all stars except those between the zenith and the pole.

The computations of the azimuth errors and the most probable chronometer corrections are made on pages 34–41, Form 256. One star set is computed on each page. For each star, the chronometer correction, $\alpha-t$, is listed as computed on pages 6–29, Form 256, and the factor A as computed on pages 4 and 5, Form 256. If the chronometer correction, $\alpha-t$, is large numerically, a constant (the approximate mean) is subtracted from all the $(\alpha-t)$'s, so that most of the residuals will be near zero. This is only a matter of convenience, so that one may work with smaller numbers; as the final results will be the same whether the constant is subtracted or not. In the sample given on page 48, -10 seconds are subtracted from each $\alpha-t$, and the results are entered in the column, headed d.

Each observation equation may now be written:

$$\delta t + Aa - d = v. \tag{19}$$

The following field method of computation was devised shortly after 1905, the year when the transit micrometer was adopted by this Bureau for time determination. It consists of combining all observation equations with *positive* A factors into one equation whose coefficients are the sums of the corresponding coefficients of the observation equations, and whose constant term is the sum of the constant terms of the observation equations. Similarly all equations with *negative* A factors are combined into one equation. In each set of equations, the sum of the v's on the right hand side is assumed to be zero.

The two equations will be:

$$n\delta t + [+A]a - [+d] = 0 \tag{20}$$

$$n'\delta t + [-A]a - [-d] = 0 \tag{21}$$

[4] In practice there always exists an error of collimation, so in general a is the angle between the meridian and the axis of collimation.

Form 256

FIELD COMPUTATION

Station, WILLS, ARKANSAS Observer, J. P. LUSHENE

Date, DEC. 18, 1935 Set, 1 Computer, J. P. LUSHENE

STAR	Clamp	$\alpha-t$	δt	C	A	Co	Aa	ΔT	v
			−10.00						
υ ANDROMEDAE		− 9.87	+ .13		− .10		− .14	− 9.73	.00
Φ PERSEI		− 10.26	− .26		− .38		− .53	− 9.73	.00
α TRIANGULI		− 9.54	+ .46		+ .14		+ .20	− 9.74	+ .01
γ' ANDROMEDAE		− 9.93	+ .07		− .13		− .18	− 9.75	+ .02
α ARIETIS		− 9.37	+ .63		+ .25		+ .35	− 9.72	− .01
6 PERSEI		− 10.27	− .27		− .39		− .55	− 9.72	− .01
γ TRIANGULI		− 9.66	+ .34		+ .06		+ .08	− 9.74	+ .01
			a =	+ 1.^{s}398					
			δt =	+ 0.^{s}267					
			ΔT =	− 9.^{s}737					

$3\delta t + 0.45a - 1.43 = 0$

$4\delta t - 1.00a + .33 = 0$

$12\delta t + 1.80a - 5.72 = 0$

$12\delta t - 3.00a + .99 = 0$

$4.80a - 6.71 = 0$

$a = +1.^s398$

$3\delta t + .629 - 1.43 = 0$

$\delta t = +0.^s267$

$\Delta T = -10.00 + .267 = -9.^s733$

FIGURE 18.—Field computation of the chronometer correction for a set.

where n is the number of equations with positive A factors,

n' is the number of equations with negative A factors,

$[+A]$ is the sum of the positive A factors,

$[-A]$ is the sum of the negative A factors,

$[+d]$ is the sum of the chronometer corrections, $(\alpha-t)$, for the equations whose A factors are positive,

$[-d]$ is the sum of the chronometer corrections, $(\alpha-t)$, for the equations whose A factors are negative.

In other words, equation (20) is the resultant of all observation equations whose A factors are positive, that is, for south stars; and equation (21) is the resultant of all the observation equations whose A factors are negative, that is, for stars between the zenith and the pole.

In the example, equation (20) becomes:

$$3\delta t+(+0.14+0.25+0.06)a-(+0.46+0.63+0.34)=0$$
$$3\delta t+0.45a-1.43=0. \tag{22}$$

Equation (21) becomes:

$$4\delta t+(-0.10-0.38-0.13-0.39)a-(+0.13-0.26+0.07-0.27)=0$$
$$4\delta t-1.00a+0.33=0. \tag{23}$$

By multiplying equation (22) by 4 and equation (23) by 3, the coefficients of δt become equal, and by subtracting one of the resulting equations from the other, δt is eliminated.

$$12\delta t+1.80a-5.72=0$$
$$12\delta t-3.00a+0.99=0$$
$$4.80a-6.71=0$$
$$a=+1^s.398.$$

By substituting $+1.398$ for a in equation (22), we solve for δt as follows:

$$3\delta t=+0.629-1.43=0$$
$$\delta t=+0^s.267$$
$$\Delta T=-10.000+0.267=-9^s.733$$

which is the chronometer correction, $\alpha-t$, for the mean epoch of the star set.

The azimuth correction $A_i a$ is now computed for each star and added, with the sign reversed, to its (α_i-t_i) in order to obtain the chronometer correction, ΔT_i, for each star. The difference between the mean ΔT and the individual ΔT_i, in the sense, $\Delta T-\Delta T_i$, gives the residual, v_i, for the individual star.

Remainder of the Computations

The scalings of the radio time signals are recorded on pages 30–33, Form 256. The first column is the radio time of the break counted from the beginning of the signal. For example, for a five-minute Naval Observatory time signal, the first column gives the number of minutes and seconds elapsed since the beginning of the signal. For the 10 P. M. signal, the designation $1^m\ 35^s$ in the first column corresponds to one minute and 35 seconds after 9:55 P. M. or $9^h\ 56^m\ 35^s$. In the second column, only the scaled fraction of the second is recorded. For a first approximation, it is assumed that the chronometer is set on sidereal time. The correction of mean time to sidereal time for the mean-time interval between each scaled break and the final second is entered in the third column. The idea is to reduce each scaled break to the final break at the full hour. At $1^m\ 35^s$, the correction for 3 minutes and 25 seconds of mean time, the interval between the break at $1^m\ 35^s$ and 10 P. M., is $0^s.561$.

This correction added to the scaled chronometer time will give the fractional second of the final signal for that particular break. The mean of all the reduced fractional seconds is the fractional second of the chronometer time for the particular time signal.

The mean epoch for each star set is computed. If the stars in a set are fairly uniformly spaced with respect to time, it is sufficient to take the mean of the chronometer times of transit of the first and last star in the set as the mean epoch of the set, T_o. However, if that is not the case, the mean of the chronometer times of transit of all the stars will have to be computed.

Station, WILLS, ARKANSAS Observer, J.P. LUSHENE Recorder,

Date, DEC 18, 1935 E. S. time of sig., 10 PM. Freq. of sig., NAA 9250

* RADIO TIME SIGNALS

Sending time of signal ——— h	Local chronometer time of signal ——— h	Correction to reduce to final signal	Local chronometer time of final signal ——— h. ——— m.	Sending time of signal ——— h	Local chronometer time of signal ——— h	Correction to reduce to final signal	Local chronometer time of final signal ——— h. ——— m.
m. s.	m. s.	s.	s.	m. s.	m. s.	s.	s.
0 18	.09	.772	.862				
0 20	.10	.766	.866				
0 28	.10	.744	.844				
0 30	.10	.739	.839				
0 35	.13	.726	.856				
0 39	.16	.715	.875				
0 43	.16	.704	.864				
0 45	.18	.698	.878				
0 48	.18	.690	.870				
0 50	.18	.684	.864				
1 00	.20	.657	.857				
1 05	.21	.643	.853				
1 09	.22	.632	.852				
1 13	.25	.621	.871				
1 15	.24	.616	.856				
1 18	.26	.608	.868				
1 20	.26	.602	.862				
1 30	.27	.575	.845				
1 35	.30	.561	.861				
1 39	.31	.550	.860				
1 43	.31	.539	.849				
1 45	.32	.534	.854				
1 48	.33	.526	.856				
1 50	.33	.520	.850				
2 05	.40	.479	.879				
2 09	.40	.468	.868				
2 13	.40	.457	.857				
2 15	.41	.452	.862				
2 18	.42	.444	.864				
2 28	.45	.416	.866				
2 30	.44	.411	.851				
2 39	.48	.386	.866				
2 45	.50	.370	.870				
2 48	.50	.361	.861				
3 00	.54	.329	.869				
3 15	.57	.287	.857				
3 30	.60	.246	.846				
4 00	.70	.164	.864				
4 30	.77	.082	.852				
5 00	.85	.000	.850				
CHRONOMETER TIME AT 10 PM. = 2h			29m 23.860s				
CHRONOMETER RATE CORRECTION =			— .003				
		2	29 23.857				

FIGURE 19.—Computation of the chronometer time of a radio time signal.

The chronometer rates with respect to sidereal time and the chronometer corrections for the radio time signals are computed on pages 42 and 43, Form 256.

In the first column are given the mean epochs of the first and second time sets and the epoch of the time signal. In the second column, the chronometer corrections, ΔT, for the first and second time sets are entered on the corresponding lines. The rate of the chronometer is usually expressed in seconds per minute; it is obtained by dividing the difference in the chronometer corrections by the difference in the mean

epochs of the two time sets. The chronometer rate is $\frac{\text{negative}}{\text{positive}}$ if the chronometer is $\frac{\text{gaining}}{\text{losing}}$ on sidereal time. The chronometer correction is now interpolated for the epoch (chronometer time) of the time signal.

Form 256

CHRONOMETER CORRECTIONS AND RATES

FIELD COMPUTATION Station, WILLS, ARKANSAS

Date	Chronometer No.		Rate per Minute	Chronometer No.		Rate per Minute
	T_0 t_0	ΔT		T_0	ΔT	
1935 DEC 18	1 55.69	-9.733				
	2 52.23	-9.688	+ .00080			
10 P.M.	2 29.40	-9.706				
DEC 18	4 06.23	-9.615				
	4 56.38	-9.609	+ .00812			
12 P.M.	4 29.72	-9.612				
DEC 20	2 06.42	+21.852				
	2 57.09	+21.995	+ .00282			
10 P.M.	2 36.76	+21.938				

FIGURE 20.—Field computation of chronometer corrections and rates.

The longitude of the station is computed on pages 44 and 45, Form 256. The first two columns are self-explanatory. In the third column, the local chronometer time of the final signal is entered as computed on pages 30–32, Form 256. The chronometer correction for the epoch of the time signal as computed on pages 42 and 43 is entered in the fourth column. The sum of columns 3 and 4 is the corrected local chronometer time of the final signal as given in the fifth column. In the sixth column, the Greenwich sidereal time of the time signal is entered for field computation. A sample of the computation of the Greenwich sidereal time for any given Standard Time is given on page 34.

The difference between the fifth and sixth columns is the difference in longitude between Greenwich and the station. Hence it is the longitude of the station.

If the triangulation station and the station occupied for astronomical observations are not identical, the necessary data for the geodetic connection between these two stations should be given on page 44, Form 256, under that heading. If the stations are identical, that fact should be stated.

LONGITUDE DETERMINATION AND FIELD COMPUTATION FOR HIGH LATITUDES

For high latitudes such as over 50° the selection of a good observing list of stars is less simple than for lower latitudes. This is because of the convergence of the meridians toward the pole and the consequent inherent uncertainty of all longitudes near the pole. In fact, at the pole itself longitude has no meaning. All meridians pass through the pole.

The rule of using approximately the same number of north and south stars and of balancing the A factors, as is done in low latitudes, becomes infeasible as one approaches

748977°—48——5

Form 256

FIELD COMPUTATION COMPUTATION OF RADIO

Station, WILLS, ARKANSAS

Date	Eastern standard time of signal	Local chronometer time of final signal			Correction to chronometer at time of final signal		Corrected local chronometer time of final signal		
1935		h.	m.	s.	m.	s.	h.	m.	s.
DEC 18	10 P.M.	2	29	23.860		− 9.706	2	29	14.154
DEC 18	12 P.M.	4	29	43.484		− 9.612	4	29	33.872
DEC 20	10 P.M.	2	36	45.301		+ 21.938	2	37	07.239

Geodetic connection:

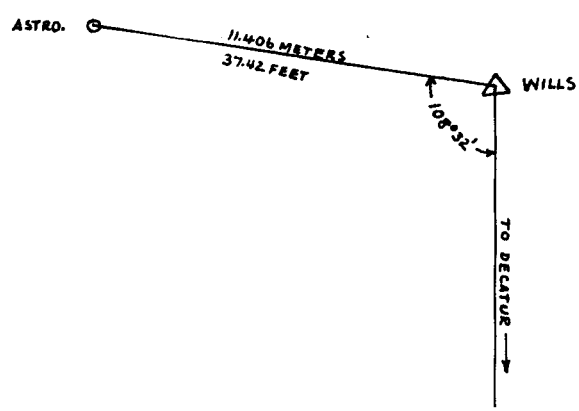


FIGURE 21.—Field computation of difference of longitude.

the pole. There are only a few stars between the zenith and the pole. Moreover, the numerical values of the A factors increase very rapidly as the stars approach the pole.

This Office has found the following system quite satisfactory for latitudes 50° or 55° to 70°.

Each time set should consist of 5 or 6 time stars and of one or two azimuth stars. The time stars should fall between 5° north of the zenith and about 20° or possibly 25° south of the zenith. The azimuth stars should be either above or below the pole with A factors of about 3 or 4 units. It is inadvisable to have the azimuth stars much closer to the pole, because of their slow movement.

The field adjustment is made by setting up two equations. One equation should have as its coefficients, term for term, the sum of the corresponding coefficients of all equations for time stars and the other equation should have as its coefficients, term for term, the sum of the corresponding coefficients of all equations involving azimuth stars.

Warning. Any equation derived from the coefficients of azimuth stars should involve only azimuth stars whose A factors are all of the same sign. If this precaution is not observed and the azimuth equation is derived from the sum of the coefficients of an azimuth star with a negative A factor and an azimuth star with a positive A factor, the

DIFFERENCE OF LONGITUDE

SIGNALS

Sidereal time at ~~Naval Observatory (center of clock room)~~ GREENWICH of final signal			Difference of time ~~Naval Obsy.~~ GREENWICH—station			Correction for lag	Difference of longitude Δλ			v	v²
h.	m.	s.	h.	m.	s.	s.	h.	m.	s.		
8	47	09.355	6	17	55.201						
10	47	29.068	6	17	55.196						
8	55	02.463	6	17	55.224						
						Mean					

Transmission time for ..... miles,					
Longitude of Naval Observatory (center of clock room),	5	08	15.784		
Longitude of astronomical station in time,					
Longitude of astronomical station in arc,	°	′	″		
Reduction to geodetic station,					
Astronomical longitude of geodetic station,					
Geodetic longitude of geodetic station,					
A − G (astronomical − geodetic),					
Sin ϕ, (ϕ = ° ′ ″)				Cos ϕ	
(A − G) sin ϕ,				Cot ϕ	

Laplace azimuth computation:

A factors will tend to cancel each other and the azimuth will be poorly determined by the resulting equation. If a time set includes both an azimuth star above the pole and one below the pole, that is, one azimuth star with a minus A factor and one with a plus A factor, the equation involving the time stars and the equation involving either one of the azimuth stars may be used for solving for ΔT and a. There is, however, no objection to using both azimuth equations in the final least-squares adjustment in the Office. In fact, there is an advantage in using both azimuth equations, for the additional azimuth equation will strengthen the least-squares adjustment whether the signs of the A factors are the same or different.

The solution of these two equations is identical with that shown on page 48. The remainder of the longitude computation is identical with that for low latitudes.

OFFICE COMPUTATION OF LONGITUDES AT LOW LATITUDES

In general, the office computation is identical with the field computations, except that various refinements of computation and additional small corrections are included. These points will be stressed in this section. Where the procedure is the same as in the field computation, it will not be repeated.

Computation of A Factors. If the azimuth error is greater than 1½ seconds of time, the A factors should be recomputed to three decimal places by the formula:

$$A = \sin\phi - \cos\phi \tan\delta. \tag{24}$$

For convenience, the factor B may be computed at the same time by the formula:

$$B = \cos\phi + \sin\phi \tan\delta. \tag{25}$$

It should be noted, that ordinarily the accuracy obtained for the factor B from the table on pages 156 to 171 is sufficient.

Short-Period Term in Nutation. The apparent right ascension of each star, as interpolated for the particular fraction of the day for the date of observation, should be corrected by the short-period term in nutation according to the formula:

$$\Delta\alpha = d\alpha(\psi)d\psi + d\alpha(\epsilon)d\epsilon. \tag{26}$$

OFFICE COMPUTATION

Form 256 STATION: WILLS, ARKANSAS

ϕ = 36° 26′ 45.″30 sin ϕ = .5941 cos ϕ = .8044

CAT.	STAR	MAG.	α	δ	Z. D.	A	C	B	k (−)	l (+)
	υ ANDROMEDAE	4.2	1h 33m 04s	41° 05′ 29″	N 4 39	−.107	1.33	1.32	.02	.07
	φ PERSEI	4.2	1 39 40	50 22 22	N 13 56	−.377	1.57	1.52	.03	.08
	α TRIANGULI	3.6	1 49 27	29 16 19	S 7 10	+.144	1.15	1.14	.02	.06
	γ′ ANDROMEDAE	2.3	2 00 00	42 01 43	N 5 35	−.131	1.35	1.34	.02	.07
	α ARIETIS	2.2	2 03 35	23 09 52	S 13 17	+.251	1.09	1.06	.02	.05
	6 PERSEI	5.4	2 09 22	50 46 30	N 14 20	−.390	1.58	1.53	.03	.08
	γ TRIANGULI	4.1	2 13 32	33 33 23	S 2 53	+.061	1.20	1.20	.02	.06
	ν ARIETIS	5.4	2 35 13	21 41 21	S 14 45	+.275	1.08	1.04	.02	.05
	θ PERSEI	4.2	2 39 51	48 57 50	N 12 31	−.329	1.52	1.49	.03	.08
	τ PERSEI	4.1	2 49 45	52 30 24	N 16 04	−.454	1.64	1.58	.03	.08
	ε ARIETIS	4.6	2 55 35	21 05 19	S 15 21	+.284	1.07	1.03	.02	.05
	β PERSEI	VAR.	3 04 02	40 42 52	N 4 16	−.098	1.32	1.32	.02	.07
	δ ARIETIS	4.5	3 08 00	19 29 19	S 16 57	+.309	1.06	1.01	.02	.05

FIGURE 22.—Star factors and corrections for diurnal aberration and width of contact strip, office computation.

The coefficients $d\psi$ and $d\epsilon$ are taken from the table of Short-period Terms of Nutation, in the "Apparent Places of Fundamental Stars" for the year in question. The quantities $d\alpha(\psi)$ and $d\alpha(\epsilon)$, which depend on the position of the star, are given at the bottom of the page for the particular star. The notation for these quantities is different in the "American Ephemeris" than that used here, which follows the "Apparent Places of Fundamental Stars." The "American Ephemeris" uses $D_\psi\alpha$ and $D_\epsilon\alpha$ instead of $d\alpha(\psi)$ and $d\alpha(\epsilon)$.

Correction for Rate of Chronometer. If the rate of the chronometer with respect to sidereal time is not zero, the chronometer correction changes during the progress of the time set. In order to reduce each observed time of transit to the same instant, such as the mean epoch of the time set, the following rate correction is applied:

$$R = (t - t_0)r \tag{27}$$

where t is the chronometer time of transit of a star, t_o is the mean epoch of the time set (mean of the chronometer times of transit of all the stars of the set), and r is the rate of the chronometer with respect to sidereal time, expressed in seconds per minute. The difference in chronometer time, $t-t_o$, should therefore be expressed in minutes.

The rate, r, is considered positive, if losing, and negative, if gaining. The above is the correction as applied to the observed time of transit, t. Ordinarily, the rate correction should be applied if the rate is numerically greater than $0^s.001$ per minute.

The observed chronometer time of the time signal should also be corrected for rate. As explained on page 49, each scaling of a radio break has been reduced to the final break of the time signal at the full hour. That reduction takes care of reducing mean time to sidereal time. If the chronometer does not keep exact sidereal time, then each break should be corrected for the chronometer rate on sidereal time for the interval between the individual break and the final break. But instead of computing the correction for each separate interval, it is considerably simpler to apply one correction to the mean of the radio times of all breaks. The rate correction to the observed chronometer time of the signals will then be:

$$R=(t-t_o)r \tag{28}$$

where t is the mean of the radio times of all breaks, t_o is the time of the final break of the signal, and r is the rate of the chronometer with respect to sidereal time per minute, plus, if losing, minus, if gaining. It is again expressed in seconds per minute.

Solution of the Time Sets. As stated before, an observation equation of this type:

$$\Delta T+Aa-(\alpha-t)=v \tag{29}$$

is set up for each star observation. The term Cc, correction for collimation, does not occur in this equation for observations made with the Bamberg transit, since collimation is eliminated by reversing the instrument between observations on each star.

If the average chronometer correction, $\alpha-t$, is large numerically, it is convenient to add a constant so that the average chronometer correction will be nearly equal to zero.

The time set at Wills, Arkansas, is again used as a sample (see figure 23). If $-10^s.000$ is added to each $\alpha-t$, the seven observation equations become:

$$\begin{aligned}
\delta t-0.107a-0.14&=v_1\\
\delta t-0.377a+0.25&=v_2\\
\delta t+0.144a-0.46&=v_3\\
\delta t-0.131a-0.06&=v_4\\
\delta t+0.251a-0.61&=v_5\\
\delta t-0.390a+0.30&=v_6\\
\delta t+0.061a-0.30&=v_7.
\end{aligned}$$

In symbols, the normal equations will be:

$$n\delta t+[A]a-[(\alpha-t)]=0 \tag{30}$$

$$[A]\delta t+[AA]a-[A(\alpha-t)]=0. \tag{31}$$

In our example, they become:

$$7\delta t-0.549a-1.02=0$$

$$-0.549\delta t+0.4103a-0.4261=0.$$

These normal equations are solved by the Doolittle method or any other method for solving simultaneous equations. The solution of these equations gives $+1^s378$ for a, $+0^s254$ for δt, and -9^s746 for ΔT.

The products Aa are computed for each star and subtracted from the individual $(\alpha-t)$, to obtain the individual chronometer corrections ΔT. The difference between the individual ΔT and the final ΔT, in the sense, -9.746 minus individual ΔT equals the residual v for the individual star.

Form 256 OFFICE COMPUTATION BY LEAST SQUARES

Station, WILLS, ARKANSAS Observer, J. P. LUSHENE

Date, DEC 18, 1935 Set, Computer, F. W. DARLING, J. A. DUERKSEN

STAR	Clamp	$\alpha-t$	$+\frac{t}{d}$	C	A	Cc	Aa	ΔT	v	v^2
			−10.00							
υ ANDROMEDAE		−9.86	+ .14		−.107		−.147	−9.713	−.033	.001089
φ PERSEI		−10.25	− .25		−.377		−.520	−9.730	−.016	.000256
α TRIANGULI		−9.54	+ .46		+.144		+.198	−9.738	−.008	.000064
γ ANDROMEDAE		−9.94	+ .06		−.131		−.181	−9.759	+.013	.000169
α ARIETIS		−9.39	+ .61		+.251		+.346	−9.736	−.010	.000100
6 PERSEI		−10.30	− .30		−.390		−.537	−9.763	+.017	.000289
γ TRIANGULI		−9.70	+ .30		+.061		+.084	−9.784	+.038	.001444
									Σv^2	.003411
			$a=$	$+1^s.378$						
			$\delta t=$	$+ .254$						
			$\Delta T=$	$-9^s.746$						

FIGURE 23.—Office computation of the chronometer correction for a set.

The chronometer correction for the instant of each time signal is computed on pages 42 and 43, Form 256, see figure 23a. The office computation of the longitude is shown on pages 44 and 45, see figure 24. The local sidereal time at Naval Observatory (center of clock room) is computed for the instant the radio time signal is received there. It is simply, the longitude of the center of clock room, Naval Observatory ($5^h\ 08^m\ 15^s\ 784$), subtracted from the algebraic sum of the sidereal time for 0^h Greenwich Civil time, the correction for the change in nutation between 0 hour and the hour in question, the Greenwich Civil Time of time signal reckoned from 0^h, its reduction to sidereal time, and the correction for signal being early or late.

Probable Errors

The probable errors are computed according to the standard formulas:

e_s, the probable error of an observation on a single star, is $\pm 0.6745\sqrt{\frac{[vv]}{n-2}}$ (32)

$e_{\Delta T}$, the probable error of the chronometer correction, ΔT, is $\pm e_s\sqrt{\frac{1}{n-\frac{[A]^2}{[AA]}}}$ (33)

e_a, the probable error of the azimuth a, is $\pm e_s\sqrt{\frac{1}{[AA]-\frac{[A]^2}{n}}}$ (34)

Form 256

CHRONOMETER CORRECTIONS AND RATES

OFFICE COMPUTATION Station, WILLS, ARKANSAS

Date	Chronometer No.			Rate per Minute	Chronometer No.			Rate per Minute
		T_0	ΔT			T_0	ΔT	
1933 DEC 18	1	55.69	-9.746					
	2	52.23	-9.688	+.00103				
10 P.M.	2	29.40	-9.711					
DEC 18	4	06.23	-9.612					
	4	56.38	-9.606	+.00012				
12 P.M.	4	29.72	-9.609					
DEC 20	2	06.42	+21.849					
	2	57.09	+21.995	+.00288				
10 P.M.	2	36.76	+21.936					

FIGURE 23a.—Office computation of chronometer corrections and rates.

Form 256

OFFICE COMPUTATION OF RADIO

Station, WILLS, ARKANSAS

Date	Eastern standard time of signal	Local chronometer time of final signal			Correction to chronometer at time of final signal		Corrected local chronometer time of final signal		
1935		h.	m.	s.	m.	s.	h.	m.	s.
DEC 18	10 P.M.	2	29	23.857		− 9.711	2	29	14.146
DEC 18	12 P.M.	4	29	43.484		− 9.609	4	29	33.875
DEC 20	10 P.M.	2	36	45.296		+21.936	2	37	07.232

Geodetic connection:

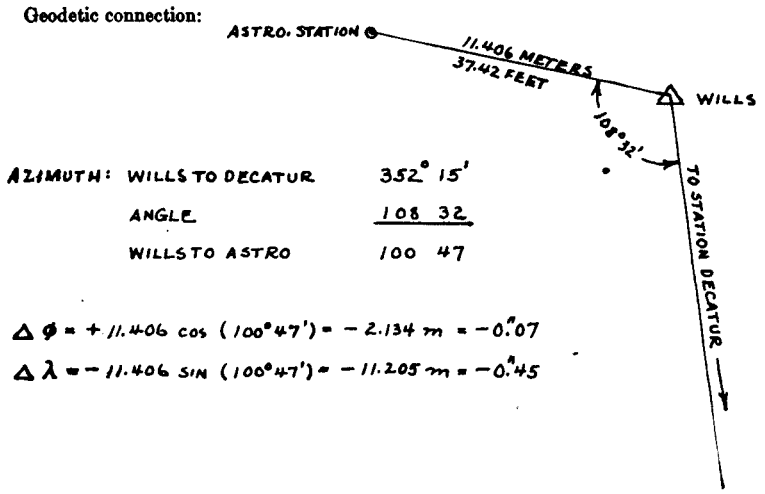


FIGURE 24.—Office computation of difference of longitude.

In our example

$$e_s = \pm 0.6745\sqrt{\frac{0.003411}{7-2}} = \pm 0^s0261$$

$$e_{\Delta T} = \pm 0.0261\sqrt{\frac{1}{7-\frac{0.3014}{0.4103}}} = \pm 0^s0104$$

$$e_a = \pm 0.0261\sqrt{\frac{1}{0.4103-\frac{0.3014}{7}}} = \pm 0^s0431.$$

Sidereal Time of Final Signal at Naval Observatory, Dec. 18, 1935, 10 p. m.

G. S. T. Dec. 19, 0^h, 1935	5^h	46^m	39^s 786
G. C. T. Dec. 19, 3^h, 1935	3	00	00. 000
Reduction, mean time to sidereal	--	--	29. 569
Correction for change in nutation	--	--	0. 000
Time signal correction	--	--	+0. 008
G. S. T. of final signal	8	47	09. 363
Longitude at Naval Observatory	5	08	15. 784
Sidereal Time of final signal at Naval Observatory	3	38	53. 579

DIFFERENCE OF LONGITUDE

SIGNALS

Sidereal time at Naval Observatory (center of clock room) of final signal	Difference of time Naval Obsy.–station	Correction for ~~lag~~ TRANSMISSION TIME	Difference of longitude Δλ	v	v^2
h. m. s.	h. m. s.	s.	h. m. s.		
3 38 53.579	+ 1 09 39.433	+ .005	+ 1 09 39.438	-.007	
5 39 13.293	+ 1 09 39.418	+ .005	39.423	+.008	
3 46 46.658	+ 1 09 39.426	+ .005	39.431	.000	
		Mean	+ 1 09 39.431	±.003	
Transmission time for ..... miles,					
Longitude of Naval Observatory (center of clock room),			5 08 15.784		
Longitude of astronomical station in time,			6 17 55.215		
Longitude of astronomical station in arc,			94° 28′ 48.22″		
Reduction to geodetic station,			− .45		
Astronomical longitude of geodetic station,			94 28 47.77		
Geodetic longitude of geodetic station,			94 28 43.603		
A−G (astronomical−geodetic),			+4.17		
Sin φ, .5940 (φ= 36° 26′ 41.236″)				Cos φ	.8044
(A−G) sin φ,			+2.48	Cot φ	1.3541
Laplace azimuth computation:					

Longitude Computation

After the difference of longitude has been computed, there is one correction to be applied and that is the correction for the difference in transmission time of the radio signal from the broadcasting station to the observer and to the Naval Observatory. For our purposes, it is sufficiently accurate to use the approximate speed of light, namely, 300,000 kilometers per second, in deriving this correction.

The correction for the difference in transmission time as applied to the difference in longitude is:

$$\frac{(A-B)}{300{,}000}. \tag{35}$$

Here A is the distance between the field station and the broadcasting station, and B is the distance between the Naval Observatory and the broadcasting station, where A and B are expressed in kilometers.

The mean of the differences of longitude is computed, and the residuals between the mean difference of longitude and the individual differences of longitude are found and entered in the column headed v. The residual is here defined as the correction to be applied to the observed difference of longitude in order to obtain the mean. The probable error of the difference in longitude is computed according to the standard formula:

$$e=\pm 0.6745\sqrt{\frac{[vv]}{n(n-1)}}. \tag{36}$$

In our example, the probable error is:

$$e=\pm 0.6745\sqrt{\frac{0.000113}{(3)(2)}}=\pm 0^s.003.$$

If the instrument has not been set up over the triangulation station, the observed astronomic longitude must be reduced to it, by applying the east-west component of the distance between the transit station and the triangulation station. The geodetic reduction in longitude in seconds of arc is equal to $-\frac{a \sin A}{b}$ where a is the distance between the stations in meters, b is the length of 1 second of arc along the parallel for the latitude of the station, and A is the azimuth of the transit station, reckoned from south, at the triangulation station. The length of one second of arc along the parallel may be taken directly from "Tables for a Polyconic Projection of Maps," Special Publication No. 5.

OFFICE COMPUTATION OF LONGITUDES AT HIGH LATITUDES

So far as the office computation is concerned, the only difference between the computations for low and high latitudes is in the weighting of the star observations. In low latitudes, unit weight is given to each star. This procedure is justifiable for low latitudes, since only time stars are used and their weights for time observation do not vary greatly. In high latitudes, azimuth stars are needed to determine the azimuth of the instrument quite accurately, but since these azimuth stars move so slowly, they should receive much less weight than time stars for determining the clock correction.

Relative Weights of Time Observations Depending on the Declinations of the Stars

The relative weights to be assigned to the time observations as obtained from the transits of stars are based on a study of the probable errors, ϵ, of the transit of a star over a single line from 1047 transits taken by G. Davidson at San Francisco in February and March 1869 with the large transit C. S. No. 3 and 875 transits taken by A. T. Mosman and F. Blake with the large transit C. S. No. 5. See Coast and Geodetic Survey Report, 1872, App. No. 12; also Astronomical Report Vol. IV, pp. 294–310 (not published).

According to Albrecht,[5] the mean square error of an observation of a transit of a star may be expressed in this manner:

$$\mu=\pm\sqrt{a^2+\left(\frac{b}{v}\right)^2 \sec^2 \delta} \tag{37}$$

where v is the magnification power of the telescope, δ is the declination of the star and a and b are constants.

This mean square error may be transformed into:

$$\mu=\pm\sqrt{a^2+\left(\frac{b}{v}\right)^2(1+\tan^2 \delta)}$$

$$=\pm\sqrt{a^2+\left(\frac{b}{v}\right)^2+\left(\frac{b}{v}\right)^2 \tan^2 \delta}. \tag{38}$$

If we now substitute m^2 for $a^2+\left(\frac{b}{v}\right)^2$ and n^2 for $\left(\frac{b}{v}\right)^2$ we have

$$\mu=\pm\sqrt{m^2+n^2 \tan^2 \delta} \tag{39}$$

[5] Th. Albrecht, Formeln und Hilfstafeln für geographische Ortsbestimmungen, p. 21, 1908 edition.

the form actually used for the study of the relative weights, except that the probable error was used instead of the mean square error, which is perfectly legitimate.

Table of Probable Errors, ϵ

Transit No. 3		*Transit No. 5*	
δ	ϵ	δ	ϵ
87°.2	±0.s74	86°.9	±0.s66
86. 6	±0. 49	80. 0	±0. 20
83. 0	±0. 38	76. 3	±0. 19
81. 0	±0. 31	72. 6	±0. 12
68. 4	±0. 12	68. 8	±0. 11
62. 9	±0. 088	3. 2	±0. 066
48. 6	±0. 075		
28. 5	±0. 058		
7. 8	±0. 060		

These tabular probable errors are fairly well represented by the expression:

$$\text{Transit No. 3, } \epsilon = \pm\sqrt{(0.060)^2 + (0.036)^2 \tan^2 \delta} \tag{40}$$

$$\text{Transit No. 5, } \epsilon = \pm\sqrt{(0.066)^2 + (0.036)^2 \tan^2 \delta}. \tag{41}$$

Or combining the probable errors for these two transits,

$$\epsilon = \pm\sqrt{(0.063)^2 + (0.036)^2 \tan^2 \delta}. \tag{42}$$

Now the weight of an observation is inversely proportional to the square of the probable error; the actual magnitude of the weight is immaterial. We may, therefore, define the weight p, as follows:

$$p = \frac{C}{(0.063)^2 + (0.036)^2 \tan^2 \delta}. \tag{43}$$

For convenience, let p=unity for equatorial stars.
That is:

$$\frac{C}{(0.063)^2 + (0.036)^2 \tan^2 0°} = 1 = \frac{C}{(0.063)^2}.$$

From which, $C = (0.063)^2$.
Hence the weight for stars for any declination will be;

$$p = \frac{(0.063)^2}{(0.063)^2 + (0.036)^2 \tan^2 \delta} = \frac{1}{1 + 0.32653 \tan^2 \delta}. \tag{44}$$

Table II, Weights of Star Transits, on page 155 is computed by formula (44).

Computation of Time Set

Each time set is now adjusted by assigning weights, p, to the observation equations depending on the declination of the stars as given in table II on page 155.

Each observation equation will now be of the type:

$$\sqrt{p}\delta t + \sqrt{p}Aa - \sqrt{p}(\alpha - t) = v. \tag{45}$$

The two normal equations will be:

$$[p]\delta t + [pA]a - [pd] = 0 \tag{46}$$

$$[pA]\delta t + [pAA]a - [pAd] = 0. \tag{47}$$

These normal equations are solved by the standard method.

A convenient arrangement of this computation is shown below. This example is based on a time set observed by Lieut. H. J. Seaborg, on Sept. 1, 1944, at Steese West Base, Alaska, with Bamberg Transit No. 21.

ADJUSTMENT OF TIME SET

STEESE WEST BASE, ALASKA
SEPTEMBER 21, 1944
H. J. SEABORG, OBSERVER

STAR	a-t	d (+17.00)	A	p	pA	pA²	pd	pAd	Aa	Δt	v	Δt
Φ Cygni	+19.53	+2.53	+.105	.447	+.0469	+.0049	+1.1309	+.1187	−.482	+20.012	+.050	+.022
α Cygni	+17.86	+.86	+.496	.753	+.3735	+.1853	+.6476	+.3212	−2.279	+20.139	−.077	−.058
η Cephei	+19.46	+2.46	+.145	.472	+.0684	+.0099	+1.1611	+.1683	−.666	+20.126	−.064	−.030
ν Cygni	+17.56	+.56	+.551	.803	+.4425	+.2438	+.4497	+.2478	−2.532	+20.092	−.030	−.024
59 Cygni	+17.96	+.96	+.462	.723	+.3340	+.1543	+.6941	+.3206	−2.123	+20.083	−.021	−.015
ζ Cygni	+17.64	+.64	+.515	.770	+.3966	+.2042	+.4928	+.2538	−2.366	+20.006	+.056	+.043
δ Cygni	+17.34	+.34	+.574	.822	+.4718	+.2708	+.2795	+.1604	−2.638	+19.978	+.084	+.069
24 Ursa Ma	+10.65	+6.35	+2.052	.287	+.5889	+1.2084	+1.8224	+3.7395	−9.429	+20.079	−.017	+.005
				5.077	+2.7226	+2.2816	+3.0333	+2.1487				+.002

δt	a	d
+5.077	+2.7226	−3.0333
	−.53626	+.59746
	+2.2816	+2.1487
	+.8216	+3.7753
	a	= −4.59506
	δt	= +3.06161
	Δt	= +20.062

FIGURE 25.—Adjustment of time set using weights.

PART II

DETERMINATION OF LATITUDE BY THE HORREBOW-TALCOTT METHOD

INTRODUCTION

HISTORICAL NOTE ON THE HORREBOW-TALCOTT METHOD

Although what is now known as the Horrebow-Talcott method of latitude determinations was first publicly announced by Peter Horrebow in his *Atrium Astronomiae*, published in 1732, nevertheless very little attention was paid to it until over a hundred years later when Captain Andrew Talcott of the United States Engineers rediscovered and developed the method independently and demonstrated its practicality with the zenith telescope.[6]

The Horrebow-Talcott method was introduced in the Coast Survey by Superintendent A. D. Bache in 1846, and after numerous trials was finally adopted in 1851 as the standard method for the determination of latitude by the Coast Survey. The first complete set of observations by this method made for the Coast Survey was obtained by T. J. Lee [7] at Thompson, Mass., in 1846, with a zenith telescope loaned by the United States Military Academy. In 1847 the Survey acquired an instrument of its own known as zenith telescope No. 1.

The Horrebow-Talcott method of measuring the difference in zenith distances for the determination of latitude superseded the methods formerly used which depended on the measurement of absolute zenith distances with the zenith sector or on observations of the transit of stars over the prime vertical with a prime vertical transit.[8]

DESCRIPTION OF THE HORREBOW-TALCOTT METHOD

The salient feature of the Horrebow-Talcott method is the measurement on the meridian of the small difference of the zenith distances of two stars which have nearly equal altitudes and which culminate on opposite sides of the zenith at about the same time. This measurement is made by an eyepiece micrometer and two sensitive latitude levels.

The latitude determined by observations on such a pair of stars is the mean of the declinations of the two stars combined algebraically with the following quantities expressed in arc, namely, one-half of the observed micrometer difference, one-half of the difference in meridional inclination of the collimation axis during the observations as measured by the levels, and one-half of the difference in refraction of the two zenith angles. If either of the stars is not on the meridian when observed, there will be an additional term to be applied in order to reduce the observations to the meridian.

Because of its simplicity, great accuracy, rapidity and convenience in making observations, the method is generally preferred to other field methods.

[6] J. E. McGrath. A Question of Priority in Originating a Very Important Astronomical Method. Journal of the Royal Astronomical Society of Canada, Vol. 8, No. 6, Jan.-Feb., 1914, pp. 36-40.

[7] Capt. T. J. Lee, U. S. Corps Topographical Engineers, Assistant on the Survey of the Coast. On the Use of the Zenith and Equal-Altitude Telescope in the Determination of Latitude. Topographical Bureau of the War Department, 1848.

[8] U. S. Coast and Geodetic Survey Special Pub. No. 14. Edition 5, p. 103, Special Pub. No. 110, pp. 47-48, and Appendix 14, Report of the Superintendent of the Coast and Geodetic Survey for 1880, p. 245. Also, Special Pub. 173, p. 16.

The high degree of accuracy attained by the Horrebow-Talcott method follows from the fact that the method depends entirely on differential measurements. The measurement of the small meridional difference with the eyepiece micrometer is capable of great precision; likewise, the measurement of the difference in meridional inclination of the collimation axis with the two sensitive latitude levels will be of high accuracy when proper precautions are taken; while the difference in refraction for two stars at nearly the same altitude is small and can be computed without appreciable error.

GENERAL INSTRUCTIONS FOR LATITUDE WORK

1. The purpose of these instructions is to insure uniformity in records and computations and in the precision of the results, and also to serve as a guide to the latitude observers of the Coast and Geodetic Survey.

2. The Horrebow-Talcott method should be followed, using the Bamberg broken-telescope transit (see description, p. 1ff). If a Bamberg instrument is not available, a zenith telescope or a meridian telescope may be used. (For descriptions of these instruments and the procedure, see Special Pub. No. 14, fifth edition, p. 103 ff.)

3. A standard first-order latitude should, in general, depend on the observations of sixteen pairs of stars with a minimum requirement of 12 accepted observations, and a probable error not greater than 0".2.

4. The stars to be observed upon should be selected from "The General Catalogue of 33342 Stars for the Epoch 1950" by Benjamin Boss, published by the Carnegie Institution of Washington in 1937.

5. The pairs to be observed upon at a station should be so selected that the algebraic sum of the measured micrometer differences in turns of the micrometer is less than the total number of pairs, that is, the algebraic sum divided by the number of pairs should be numerically less than one turn.

6. A pair of stars should be observed only once at a given station, unless some gross error is discovered, in which case the pair may be reobserved. Not more than two stars should be observed at one setting of the instrument. If a star is observed on more than one night, it must be paired with a different star each night, but it is better to have a completely different star list for each night.

7. No determination of the micrometer value should be made in the field, as this value is computed in the office from the observations themselves.

8. All observations should be entered immediately at the time of observation in the field record book, Form 255.

9. Abstracts of the latitude records on the latitude computation Form 33 should be made and checked in the field. No field computations are required.

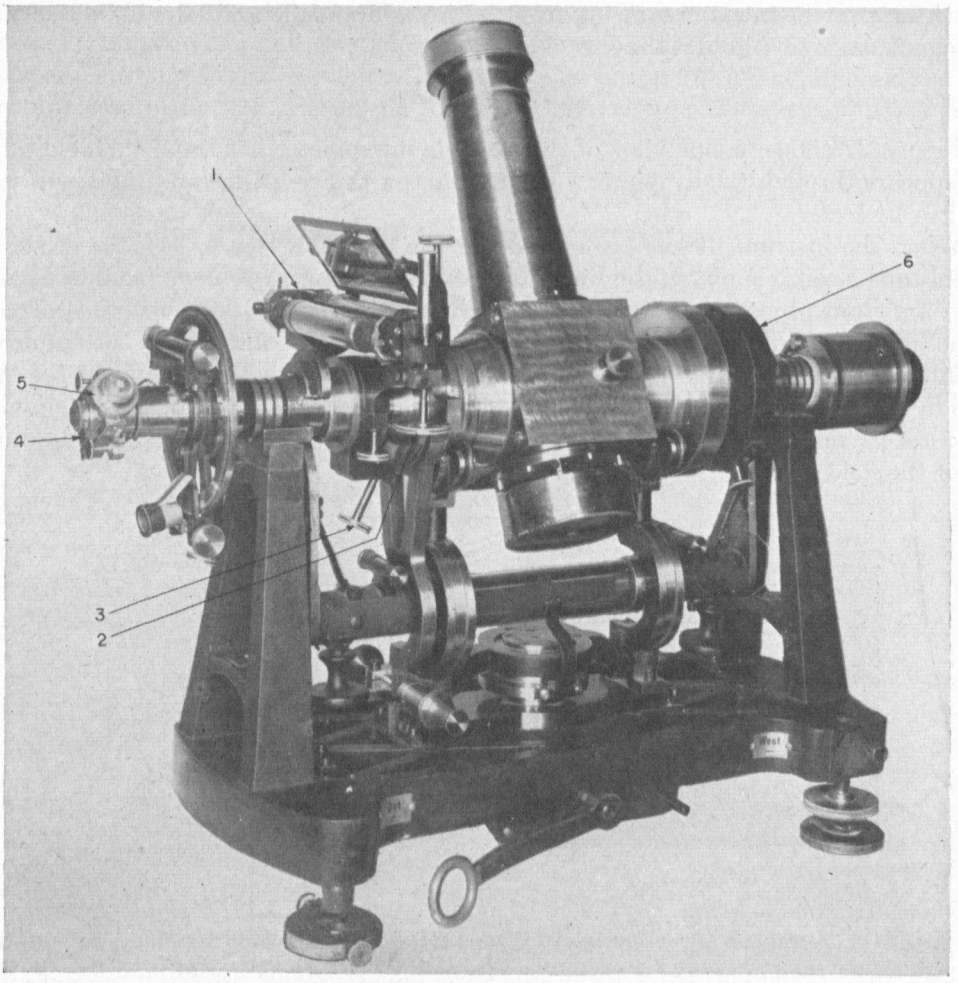
10. The abstracts of the latitude records should be sent to the office by registered mail as soon as practicable after the completion of the occupation of a station. Soon after the abstracts have been forwarded, but by a later mail, the corresponding original records on Form 255 should also be forwarded by registered mail. It is desirable that these records be sent to the office promptly, so as to avoid their possible loss.

11. Original descriptions of stations should either be inserted in the original latitude record book, Form 255, or written on an astronomical description card, Form 525a. If, however, the station has been described before, a recovery note should be made. The description or recovery note should be sent to the Office promptly after the completion of the occupation of the station.

These General Instructions will be referred to from time to time in the succeeding text.

THE BAMBERG BROKEN-TELESCOPE TRANSIT

The Bamberg transit has already been described in connection with longitude observations on page 1ff. For latitude observations the instrument has been provided with detachable twin levels and a counterpoise which can be attached to the horizontal axis of the telescope, and with a latitude micrometer which has no electrical connections. Figure 26 shows the Bamberg instrument equipped for latitude work.



1. Twin levels for latitude work.
2. Twin-levels adjustment screw.
3. Clamp for twin-levels attachment.
4. Latitude micrometer box.
5. Graduated micrometer drum for latitude.
6. Counterpoise for twin-levels attachment.

FIGURE 26.—Bamberg broken-telescope transit set up for latitude observations.

The same support for the instrument and the same observing tent are used in latitude work as are used in longitude work. (See p. 29.) The same is true for the platform on which the observer stands when observing. (See p. 30.)

ADVANTAGES IN THE USE OF THE BAMBERG BROKEN-TELESCOPE TRANSIT FOR LATITUDE

There are several advantages in using the Bamberg instrument instead of the zenith telescope for observing latitude, although the accuracy of the results is about the same. These advantages are: The Bamberg instrument is more stable in azimuth; less care is

necessary in the use of the eyepiece micrometer in preventing pressure which might cause flexure of the telescope; the observer may make observations while comfortably seated, as his eye is always at the same elevation; when latitude is done in connection with longitude work, the use of the same instrument for both results is a great saving of time, since it will require but 15 to 30 minutes to make the change for latitude work after the instrument has been set up in the meridian for longitude work. Hence, it is possible on the same night for a single observer without undue effort to make a night's observations for longitude requiring from 3 to 5 hours' time and latitude observations requiring from 2 to 4 hours' time.

ADJUSTMENTS

Figure 27 shows a side view of the latitude micrometer head and the field of view as it appears through the eyepiece. The salient parts are all lettered and will be referred to by letter.

When the instrument has been set up in the meridian (see p. 34), the crosshair *h* parallel to the comb is put in the line of collimation in the customary manner by sighting on a distant object or collimator in the direct and reverse positions and correcting by one half of the difference; and the movable wire *g* perpendicular to the comb must be made truly perpendicular by means of the small screws which butt against the lug on the side of the draw tube. This latter adjustment is very important and should be so accurately made that the star will follow exactly along the wire across the middle part of the field of view.

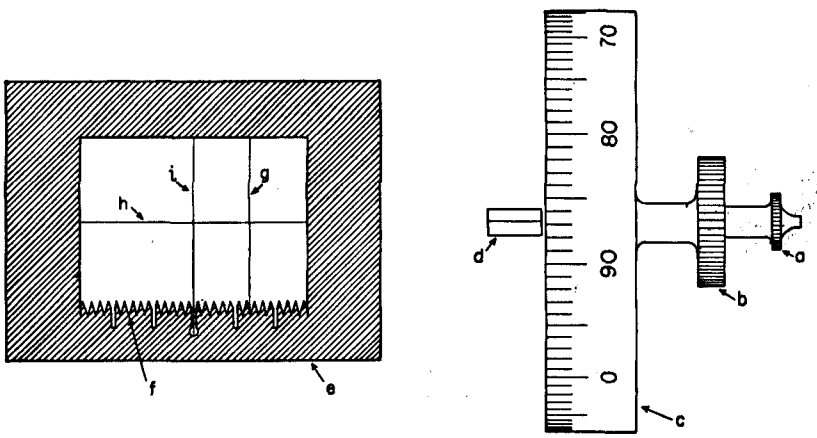


FIGURE 27.—Diagram of micrometer head and field of view.

When the wire *g* is set in a notch in the comb the micrometer head *c* should read zero. If it does not, loosen the screw *a*, hold *b* and turn *c* until it reads zero. Then tighten *a*.

Care should be taken to keep the position of the eyepiece unchanged during a set of observations, since otherwise there will be a change in focal length resulting in a change in the value of one turn of the micrometer screw, thus producing an uncertainty in the measurements.

OBSERVING LIST FOR LATITUDE

Before starting the observations, a list of all the stars to be observed should be made. See example, page 67. The stars should be taken from the "Boss General Catalogue of 33342 Stars for the Epoch 1950." (See paragraph No. 4 of the General Instructions,

p. 64.) The mean declination for the year of observation should be computed to the nearest minute and the corresponding mean right ascensions to the nearest second. The latitude of the station should be obtained to the nearest minute from such sources as a map, triangulation data, or preliminary observations on the sun or stars.

To find from the catalogue all available pairs of stars for a given latitude, one may, for each star in succession within the zone of observation, that is, 45° or preferably 30° each way from the zenith, subtract the declination from twice the latitude and then compare this difference with the declination of every star in the catalogue within the following 20 minutes of right ascension. Any star whose declination (or, in the case of a subpolar, the supplement of whose declination) is within 20′ of the above difference, will combine with the star in question to make a pair, provided other conditions stated below are fulfilled.[9]

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 485
Rev. April 1939

OBSERVING LIST FOR LATITUDE

Station: UNIVERSITY, MISS. *Date:* April 21, 1935 *Instrument:* Bomberg No. 20

ϕ: 34° 22′.0 2ϕ: 68° 44′.0 *Value of one turn:* 78.″94 *a:* .76

Star No. Boss G.C.	Mag.	Right ascension	Declination δ	Difference between δ's	$\Sigma\delta$ = sum of declinations	$2\phi-\Sigma\delta$*	$E-W = a(2\phi-\Sigma\delta)$	Star north or south, eyepiece east or west	Setting: ½ difference of δ's Circle	Setting: Micrometer Turns**
		h m s	° ′	° ′	° ′	′			° ′	
12565	4.7	9 02 24	38 43					N E		22
12593	5.4	9 04 06	29 55	8 48	68 38	+6	+5	S W	4 24	18
12722	6.1	9 09 55	21 33					S W		18
12799	5.7	9 13 10	47 05	25 32	68 38	+6	+5	N E	12 46	22
13157	3.3	9 28 31	51 58					N W		19
13277	5.9	9 33 28	16 44	35 14	68 42	+2	+2	S E	342 23	21

* This quantity should not exceed 20′.
** The algebraic sum of the number of turns should not exceed the number of pairs.

11—10538 U. S. GOVERNMENT PRINTING OFFICE

FIGURE 28.—Observing list for latitude.

Some observers in making their observing lists prefer to use the latitude star-finder chart prepared by the Coast and Geodetic Survey for the epoch 1940. It comprises four sheets, each covering six hours of right ascension and each containing stars whose declinations are between −10° and +90°. Since the catalogue used in making the chart was the Boss Preliminary General Catalogue (or P. G. C.) the chart contains the star numbers of that catalogue and it will therefore be necessary to identify these numbers with star numbers in the Boss General Catalogue (or G. C.). A concordance, G–35, entitled "P. G. C. and G. C. Star Numbers of Identical Stars" will serve this purpose. The change in declination has to be taken into account also.

[9] At stations in Alaska there are but few stars in the zone extending 45° northward from the zenith as compared with the corre sponding zone to the southward, and the above process may be improved by taking in succession only stars to the north of the zenith and comparing each with stars in both the preceding and the following 20^m. To make the search with a subpolar star, subtract 180°−δ from twice the latitude and pair the star with any star whose declination is within 20′ of this difference, provided the right ascension differs from that of the subpolar anywhere from 11^h 40^m to 12^h 20^m.

To use a sheet of the chart, it is folded along the line of the declination equal to the latitude of the station. Then by placing the chart where light will show through it, stars can be paired whose zenith distances differ by not more than 20′ and whose right ascensions differ by not more than 20 minutes.

The above chart for the epoch 1940 is now becoming obsolete.

The observing list is made on Form 485 as shown in the illustration, page 67. The first column contains the catalogue numbers of the stars; the second column their magnitudes. The third column contains the mean right ascension to the nearest second for the year of observation. This is obtained by applying to the right ascension given in the catalogue the product of the annual variation in right ascension by the number of years from 1950. If the observations are made before 1950 this number of years must be treated as negative. The mean declinations in the fourth column to the nearest minute for the year of observation are obtained from the catalogue by using the annual variation in declination in the same way. The fifth and sixth columns contain, respectively, the differences and sums of the declinations of each pair. The seventh column contains the differences in minutes of arc between twice the latitude and the sum of the declinations of the stars of each pair, namely, $2\phi-\Sigma\delta$. This expresses the zenith distance of the south star minus that of the north star. The next column headed $E-W$ expresses these differences in number of turns of the micrometer screw, that is, $a(2\phi-\Sigma\delta)$, where a is the number of turns of the micrometer screw per minute of arc. In other words, $a=\frac{60''}{R}$, where R is the value of one turn of the micrometer in seconds of arc. Column 9 states whether the star is north or south of the zenith and whether the position of the ocular is East or West. In column 10 are given the settings for the vertical circle. For ocular East and the star North or ocular West and the star South, the setting angle is one half the difference of the declinations of a pair. For ocular East and star South or ocular West and star North, the setting angle is the explement of one half the difference of the declinations of a pair.

The last column in the observing list contains the setting of the micrometer for the point at which the image of each star will appear in the field of view. To avoid negative readings, the center of the comb is called 20. Then, regardless of the position of the star with respect to the zenith, the micrometer setting for ocular East is $20+\frac{E-W}{2}$ and for ocular West $20-\frac{E-W}{2}$.

In the description of the observing list it was shown that $E-W=a(2\phi-\Sigma\delta)$, where $a(2\phi-\Sigma\delta)$ represents the zenith distance of the south star minus that of the north star of the pair in turns of the micrometer head. Consequently, $E-W$ is positive when the zenith distance of the south star is greater than that of the north star, in which case the setting of the micrometer for ocular East will be greater than that for ocular West. The reverse will be true when the zenith distance of the north star is the greater of the two zenith distances.

Furthermore, the Bamberg broken-telescope transit is so designed that for ocular East an increase in the micrometer reading, M_E, corresponds to an increase in the zenith distance of a south star or a decrease in the zenith distance of a north star, and for ocular West an increase in the micrometer reading, M_W, corresponds to an increase in the zenith distance of a north star or a decrease in the zenith distance of a south star.

It follows, therefore, from the above considerations, that $\frac{1}{2}(M_E-M_W)$ expressed in arc measurement is to be added algebraically to the mean of the apparent declinations of the pair of stars.

DIRECTIONS FOR OBSERVING

Although it is not absolutely essential, it will nevertheless be definitely to the advantage of the observer to have a recorder, for with a recorder the observer will be able, not only to have a faster observing schedule, but also to devote his entire time and attention to the observing.

After the Bamberg instrument has been properly prepared for observing latitude (see p. 66), turn to the next to last column of the observing list and set the small setting circle for the first pair of stars to be observed on.

Then put the telescope in position for the first star with the eyepiece or ocular to the East or West as indicated in the observing list. Tilt the telescope until the setting circle bubble is centered. After temporarily loosening the clamp, rotate the twin level attachment on the horizontal axis until the bubbles of the twin latitude levels are brought to their approximate centers. Reclamp and perfect the centering by the use of the tangent screw acting directly on the level holder.

Set the micrometer for the appearance of the first star in the field of view by placing the movable wire at the point on the comb indicated in the last column of the observing list. Then watch the chronometer carefully so as to be ready for the arrival of the star. When the star appears, place the movable wire on it and then when it is nearly in the center of the field, bisect it carefully as it crosses the middle wire, calling "tip" for the recorder to note the time of transit. Thereupon while still seated, read both ends of the latitude levels promptly, the number of turns of the micrometer from the comb and the number of divisions from the micrometer head, estimating the tenths of a division. The recorder enters these data in the field book on Form 255.

Warning: Always bisect the star carefully, being sure that the final motion is always in the same direction and preferably in that direction which compresses the springs in the micrometer box.

To prepare for the observation on the second star of the pair, reverse the telescope in its wyes by rotating it about its vertical axis. After reversal, if necessary, bring the latitude level bubbles back to center by means of the tangent screw, which changes the inclination of the axis of the telescope. However, *do not touch* the tangent screw that acts directly on the level holder.

Set the micrometer for the second star following the procedure described above for the first star and proceed as before. This will complete the observation for latitude on one pair of stars.

Repeat the operations described above until the required number of pairs has been observed upon.

The observer should place in the column headed "Remarks" such data as comparison of chronometer with radio signal, weather and atmospheric conditions or any unusual condition prevailing during the observations, as well as stating in the case of two or more stars very close together which one was observed on, whether it was the first or second, the brighter or fainter. Also make note, when a star is not observed on the meridian, approximately how long before or after transit it was observed.

It is very important that the position of the eyepiece micrometer in the telescope axis remain unchanged during a set of observations. A change in this adjustment will change the focus, which in turn changes the value of a turn of the micrometer in seconds of arc. If a change is made, the recorder should make a note at the time it occurred. Furthermore, if a second night's observation is made, the recorder should note at the beginning of the observations whether the eyepiece micrometer has been removed or the adjustment changed since the previous night's observations.

U. S. Coast and Geodetic Survey
Form 253

ZENITH-TELESCOPE RECORD FOR LATITUDE

Station: University Date: April 21, 1935 Chronometer: 202 Observer: J. E. Bondurant

No. of pair	Star number Cat. Boss GC.	N. or S.	E. or W.	Micrometer Turns	Micrometer Div's	Level North	Level South	Accum. microm. Diff.	Chronometer time of culmination	Chronometer time of observation	Mer'n dist.	Remarks
1	12565	N	E	22	85.3	77.9	57.9			H M S 9 02 27		Chronometer corr.:
						34.1	10.0					7.00 PM. E.S.T
	12593	S	W	18	01.6	60.7	80.7	-4		9 04 09		April 21
						13.9	37.0					E.S.T #202
												7 00 00 7 58 19.0
2	12722	S	W	17	79.1	60.1	80.3			9 09 57		
						13.9	37.0					
	12799	N	E	22	10.1	80.0	59.8	-9		9 13 12		
						36.7	13.1					
3	13157	N	W	20	21.7	59.7	80.0			9 28 35		
						12.0	35.8					
	13277	S	E	21	65.8	80.5	60.0	-10		9 33 29		
						36.0	12.2					
4	13406	S	E	25	85.1	80.0	59.3			9 39 47		
						35.0	11.1					
	13573	N	W	14	80.8	59.3	80.0	-21		9 48 28		
						10.1	34.0					
5	13836	S	W	26	19.3	60.0	81.0			10 00 49		
						11.0	35.1					
	14123	N	E	16	46.9	80.7	59.8	-11		10 13 23		
						35.9	11.6					

Figure 29.—Zenith-telescope record for latitude.

While observing avoid letting the warmth of the observer's breath or face, or that of a flashlight disturb the bubbles of the latitude levels. Heat will affect the position of the bubble ends and hence tend to cause erratic level readings. In fact, by heating one end of a level vial holder more than the other, the whole bubble may be moved several divisions.

DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
Form 605
(Rev. June 1946)

COMPARISON OF CHRONOMETER AND RADIO SIGNALS

Station: University Latitude: 34° 22′ Longitude: 5h 58m 09.8s

Chief of party: Chas. A. Whitten Observer: J. E. Bondurant

Year: 1935 Chronometer No.: 202 (~~mean~~, sidereal) (Strike out one)

Local date		April 21	April 22		
Standard time of signal, 75 Mer.		h. m. s. 19 00 00	h. m. s. 0 00 00	h. m. s.	h. m. s.
Chronometer time of signal		7 58 19.0	12 59 09.0		
Transmitting station					
Frequency of signal					
G. C. T. of signal	Date	April 22	April 22		
	Time	h. m. s. 0 00 00.0	h. m. s. 5 00 00.0	h. m. s.	h. m. s.
Sidereal time of 0h G. C. T.		13 56 29.8	13 56 29.8		
Cor., mean solar to sidereal time		00.0	49.3		
Δ Transmission time					
Correction to signal					
G. S. T. of signal		13 56 29.8	18 57 19.1		
Longitude of station		5 58 09.8	5 58 09.8		
Local sidereal time		7 58 20.0	12 59 09.3		
Chronometer time of signal		7 58 19.0	12 59 09.0		
Chronometer correction		+ 01.0	+ 00.3		
Rate per minute			-0.0023		
Rate per hour					

Remarks:

FIGURE 30.—Comparison of chronometer and radio signals.

The accumulated sum of the micrometer differences of the pairs should be recorded in the record book, Form 255, after each pair as observed. If necessary, near the end of the set of observations, those pairs should be chosen which will reduce this accumulated sum to less than the number of pairs observed.

The correction to the sidereal chronometer used in obtaining the times of observations should be known, or sufficient data should be given for the correction to be computed in the office. A comparison of the chronometer with a radio time signal immedi-

ately before or after or during the observations will suffice, or even a comparison with some timepiece accurate enough to give the correction to even seconds. Enter these data in the remarks column, Form 255. The correction should be computed on Form 605.

Upon completion of the latitude observations which have been duly entered on Form 255, the data are abstracted on Form 33 (see example, p. 73) and sent to the Office in accordance with paragraphs 8 and 9, page 64 of the General Instructions.

Examples of the record and the several steps in the office computation for latitude follow.

LATITUDE COMPUTATION

FORMULA

The formula for latitude by the Horrebow-Talcott method with the Bamberg broken-telescope transit is

$$\phi=\frac{1}{2}(\delta+\delta')+\frac{1}{2}R(M_E-M_W)+\frac{1}{4}\left(\frac{d+d_1}{2}\right)\frac{1}{2}[(n+n_1+s+s_1)_E-(n+n_1+s+s_1)_W]+$$
$$\frac{1}{2}(r-r')+\frac{1}{2}(m+m'). \qquad (1)$$

Here δ and δ' are the apparent declinations of the stars of a pair, at the time of observation; M_E and M_W are the micrometer readings with the ocular East and West, respectively; $\frac{1}{2}R$ is the value in seconds of arc of one half turn of the micrometer; d and d_1 are the values in seconds of arc of one division of the two levels, respectively; n, n_1, s, s_1 are the readings of the north and south ends of the two levels, respectively, the letters without subscript pertaining to the level, numbered from 0 to 40, whose level value is d, and those with the subscripts pertaining to the one numbered from 50 to 90 whose level value is d_1; the subscripts E and W pertain to the position of the ocular; $(r-r')$ is the difference in refraction of the two stars; and $\frac{1}{2}(m+m')$ is the sum of their meridian distance corrections. The unprimed letters in the first and the last two terms refer to the star observed with ocular East and the primed letters pertain to the star observed with ocular West. $\frac{1}{2}(r-r')$ will have the same sign as the difference of the micrometer readings.

The U. S. Coast and Geodetic Survey owns two Bamberg instruments, Nos. 20 and 21. The value R of one turn of the latitude micrometer screw is 78″.94 for No. 20 and 79″.08 for No. 21. These values for R are sufficiently accurate for the first approximation. A final value for R is obtained from the final adjustment of the results themselves in the manner explained later.

COMPUTATION OF MEAN PLACES FOR THE YEAR OF OBSERVATION

The first step in the computation of the apparent places is to obtain the mean place of each star for the year of observation from a standard star catalogue. The best available star catalogue at the present time is the "General Catalogue of 33342 Stars for the Epoch 1950," by Benjamin Boss. In the Horrebow-Talcott method the right ascensions need be accurate only to the nearest second.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 88—Rev. Jan. 1936

LATITUDE COMPUTATION

Station University Chief of party Chas A. Whitten Date April 21, 1935 Instrument Bamberg 20 Chronometer 202 Chron. correction about 0s.6

Elevation of station Approx. 450 ft. Observer J.E. Bondurant Value of half-turn of micrometer 39".47 Level value $\frac{1}{8}\left(\frac{d+d_1}{2}\right) = .167$

	Boss Star No. G.C.	N. S	E. W.	Micrometer Reading t. d	Micrometer Diff. Z. D t. d.	Level N d	Level S d.	Level Diff d.	Chronometer Time of Observation h. m s.	Meridian Distance s.	Declination ° ′ ″	Half Sum of Declinations ° ′ ″	Corrections Microm ′ ″	Corrections Level ″	Corrections Refr. ″	Corrections Merid. ″	Latitude ° ′ ″	Remarks
1.	12565	N	E	22 85.3		77.9	57.9		9 02 27	-3	38 42 48.65							Radio Comparison
					+4.837	34.1	10.0	-12.4				34 18 53.32	+3 10.92	-2.07	+.05		34 22 02.22	7 P.M. E.S.T. April 21
	12593	S	W	18 01.6		60.7	80.7		9 04 09	-3	29 54 57.99							E.S.T. Chron. 202
						13.9	37.0											7-00-00 7-58-19.0
2.	12722	S	W	17 79.1		60.1	80.3		9 09 57	-2	21 33 02.09							
					+4.310	13.9	37.0	-1.7				34 19 13.47	+2 50.12	-.28	+.05		03.36	
	12799	N	E	22 10.1		80.0	59.8		9 13 12	-2	47 05 24.85							
						36.7	13.1											
3	13157	N	W	20 21.7		59.7	80.0		9 28 35	-3	51 58 32.30							
					+1.441	12.0	35.8	+1.2				34 21 06.12	+0 56.88	+.20	+.02		03.22	
	13277	S	E	21 65.8		80.5	60.0		9 33 29	-1	16 43 39.93							
						36.0	12.2											

FIGURE 31.—Latitude computation.

This accuracy will usually be attained by adding algebraically to the catalogue right ascension the product of the annual variation by the number of years between the year of observation and the epoch of the catalog. If the year of observation is earlier than the epoch of the catalogue, then the difference in years must be considered negative.

If we denote by α_o, the mean right ascension for the beginning of the year of observation, t_o, and by α_m the catalogue right ascension, then the complete formula for use with the Boss General Catalogue, Epoch 1950, is:

$$\alpha_o=\alpha_m+(t_o-1950)\text{ An. Var.}+\frac{1}{2}(t_o-1950)^2\left(\frac{1}{100}\right)\text{Sec. Var.}+\left(\frac{t_o-1950}{100}\right)^3 3^d t. \quad (2)$$

In the case of circumpolar stars it may be necessary to use more than two terms of the above formula.

If δ_m represents the catalogue declination and δ_o the mean declination for the beginning of the year, t_o, then the complete formula for mean declination is:

$$\delta_o=\delta_m+(t_o-1950)\text{ An. Var.}+\frac{1}{2}(t_o-1950)^2\left(\frac{1}{100}\right)\text{Sec. Var.}+\left(\frac{t_o-1950}{100}\right)^3 3^d t. \quad (3)$$

All the terms of the above formula will be needed to determine the mean declinations with the accuracy required by the latitude. Usually the mean declinations are computed accurately to the nearest hundredth of a second.

The above formulas for reducing the mean right ascension and declination of a star from the epoch of the catalogue to the year of observation contain all the elements necessary for the reduction. These elements consist of first- and second-order terms of the precession and proper motion. The terms of formulas (2) and (3) are arranged according to ascending powers of the time interval from the epoch of the catalogue to the year of observation in the manner indicated in these formulas.

Specimen Computation of Mean Place

For example, consider the first star observed at University, Miss., namely, 12565 in the General Catalogue. From the catalogue we have

$$\begin{aligned} \alpha_m&=9^h03^m21^s.337 & \delta_m&=38°39'12''.08\\ \text{An. Var.}&=+3^s.8154 & \text{An. Var.}&=-14''.401\\ \text{Sec. Var.}&=-0^s.0304 & \text{Sec. Var.}&=-0''.381\\ & & 3^d t&=+0''.24 \end{aligned} \quad (4)$$

The year of observation is 1935, hence $t_o-1950=-15$ years.

Then for mean right ascension for 1935, we have by (2)

$$\begin{aligned} \alpha_o&=9^h03^m21^s.337+(-15)(+3^s.8154)+\frac{1}{2}(-15)^2\left(\frac{1}{100}\right)(-0^s.0304)=\\ &9^h03^m21^s.337-57^s.23-0^s.03=9^h02^m24^s. \end{aligned} \quad (5)$$

(The third term is inappreciable.)

And for declination

$$\begin{aligned} \delta_o&=38°39'12''.08+(-15)(-14''.401)+\frac{1}{2}(-15)^2\left(\frac{1}{100}\right)(-0''.381)+\\ &(-.15)^3(+0''.24)=38°39'12''.08+3'36''.015-0''.4286-0''.0008=\\ &38°42'47''.67. \end{aligned} \quad (6)$$

DEPARTMENT OF COMMERCE
U.S. COAST AND GEODETIC SURVEY
FORM NO. 772

REDUCTION, MEAN TO APPARENT DECLINATION, WITH BESSEL'S STAR NUMBERS

Latitude Station: University
State: Mississippi
Date: April 21, 1935
Chief of Party: Chas. A. Whitten
Observer: J.E. Bondurant

Formulas:
$$\begin{cases} \delta = \delta_0 + \tau\mu' + i\cos\delta_0 + X\sin\alpha_0 + Y\cos\alpha_0 \\ X = -(B + C\sin\delta_0) \\ Y = D\sin\delta_0 + A\dfrac{a'}{\cos a} \end{cases}$$

Cat. Boss G.C. St. No.	Mag.	α_0	δ_0	$\sin\delta_0$	$\cos\delta_0$	μ'	X	$\sin\alpha_0$	Y	$\cos\alpha_0$	δ	Remarks
12565	4.7	9h 02m 24s	38° 42' 47.67"	.6254	.7802	-.023	+14.384	+.6997	+5.098	-.7145	38° 42' 48.65"	
12593	5.4	9 04 06	29 55 00.09	.4987	.8668	-.005	+12.346	.6943	+6.436	-.7196	29 54 57.99	
12722	6.1	9 09 55	21 33 07.43	.3673	.9301	-.014	+10.233	.6758	+7.825	-.7370	21 33 02.09	
12799	5.7	9 13 10	47 05 21.84	.7324	.6808	+.008	+16.105	.6653	+3.967	-.7465	47 05 24.85	
13157	3.3	9 28 31	51 58 29.00	.7877	.6160	-.543	+16.994	.6138	+3.383	-.7894	51 58 32.30	
13277	5.9	9 33 28	16 43 48.22	.2878	.9577	-.010	+8.954	.5967	+8.665	-.8025	16 43 39.93	

	h. m.	h. m.
Mean epoch of observation	11 03	
Correction for longitude	5 58	
Greenwich sidereal time	17 01	
Sidereal time, 0 hr. G.C.T.	13 56	
Interval	3 05	
Greenwich day	April 22.128	

Star numbers and constants

A	+0.5840	
B	-4.325	
C	-16.084	
D	-10.566	
i	-6.97	
τ	0.3035	
a'/cos a	20."0439	

The mean epoch should be for a period not exceeding 4 hours. Use all sines and cosines to 4 decimal places; use X and Y to 3 decimal places.

8-756-2 m

FIGURE 32.—Reduction, mean to apparent declination, with Bessel's star numbers.

REDUCTION FROM MEAN TO APPARENT DECLINATION

After the mean places of the stars for the beginning of the year of observation have been obtained, and entered on Form 772, the apparent declination of each star is computed by means of Bessel's Star Numbers and entered on Form 33. Using the trigonometrical expressions for Bessel's star constants (*Formulas For the Reduction of Stars*, American Ephemeris and Nautical Almanac, any year) the formula for apparent declination customarily expressed in terms of Bessel's Star Constants and Bessel's Star Numbers has been transformed for the purpose of machine computation on Form 772, into

$$\delta=\delta_o+\tau\mu'+i\cos\delta_o+X\sin\alpha_o+Y\cos\alpha_o \tag{7}$$

where $X=-(B+C\sin\delta_o)$

$$Y=D\sin\delta_o+A\left(\frac{a'}{\cos\alpha_o}\right).$$

See the specimen computation, page 75. For further and more detailed discussion of the subject see pages 128 ff.

EXPLANATION OF LATITUDE COMPUTATION

If paragraph 9, page 64, of the General Instructions has been followed, the latitude computation Form 33 already contains:

At the top of the form, the names of the station, Chief of Party and observer, the date of observation, the kind and number of instrument used and the number of the chronometer.

In the designated columns for each star, the star catalogue number, the position of the star with respect to the zenith N or S, the position, E or W, of the ocular during the observation on the star, the micrometer reading in turns of the screw and in divisions and tenths of division of the micrometer head, the respective readings of the north and south ends of the two latitude levels, and the chronometer time of observation.

In the office the approximate elevation of the station is usually entered at the top of the page along with the value of one half turn of the micrometer and the level value. If the field party is able to obtain the approximate elevation of the station, it should be entered on Form 33.

The apparent declinations computed on Form 772 (see p. 75) should be entered on Form 33 in the column designated "Declination," and the half-sum of these declinations, for each pair, computed and entered in the next column.

The algebraic difference of the micrometer readings for each pair in the sense ocular East minus ocular West is then placed in the "Diff. Z. D." column, usually in decimal form. This difference is then converted to seconds of arc by multiplying by the value of one half turn of the micrometer and the result placed in the micrometer correction column.

Next, the algebraic difference of the sum of the level readings for the star with ocular East minus the sum of the level readings with ocular West is set down in the designated column. This difference multiplied by the level value, i. e., by $\frac{1}{8}\left(\frac{d+d_1}{2}\right)$, constitutes the level correction.

The approximate meridian distance is computed by the formula $\alpha_o-(t+\Delta t)$, where α_o is the mean right ascension, t the chronometer time of observation and Δt the correction to the chronometer time obtained from a radio time signal. Actually, the appar-

ent right ascension should be used, but since, with the exception of polar stars, the difference between mean and apparent right ascensions of all stars is about of the same order of magnitude, the mean of all the differences of the mean right ascensions minus the corrected chronometer time may be considered to be approximately the mean of the differences between mean and apparent right ascensions for the stars of the set. Hence, by applying the mean $\alpha_o-(t+\Delta t)$ with reverse sign to each individual $\alpha_o-(t+\Delta t)$ a better meridian distance is obtained. This distance is entered in the proper column on Form 33 and the meridian-distance correction is obtained from Table X, p. 177.

If, for any reason, the observer has not observed the star on the meridian, he should note it under "Remarks," giving his estimation of the time of observation before or after transit. The $\alpha_o-(t+\Delta t)$ of such stars then would not be used in taking the mean $\alpha_o-(t+\Delta t)$.

For explanation of meridian-distance computation, see page 78.

When the micrometer, level, refraction and meridian-distance corrections have been combined algebraically with the mean of the declinations of a pair of stars, the latitude from observations on that pair is obtained.

Correction for Differential Refraction

Although the difference in refraction of a pair of stars used in the Horrebow-Talcott method is small, it is usually not negligible and must be applied as a small correction to the latitude. The refraction for each star of a pair is very nearly proportional to the tangent of the zenith distance, so that the differential refraction will be very nearly proportional to the square of the secant of the mean zenith distance. In addition, the differential refraction depends upon the pressure and temperature of the atmosphere at the time of the observation. For a mean state of the atmosphere (pressure 29.9 inches or 76 cm. and temperature 50° F. or 10° C.), the correction to be applied to the latitude for differential refraction will be given by the formula:

$$\frac{r-r'}{2}=\frac{57''.9}{2}\sin(z-z')\sec^2 z \tag{8}$$

where r and z are the refraction and zenith distance, respectively, of the star observed with ocular East, the primed letters referring to the star observed with ocular West. Differential refraction will therefore have the same sign as half the difference of the zenith distances as measured by the micrometer. The two zenith distances of a pair of stars used in the Horrebow-Talcott method are so nearly equal that either may be used in $\sec^2 z$ in formula (8), although for accuracy the mean of the two zenith distances should be used; such refinement, however, is unnecessary.

Table VIII, page 176, has been computed by means of formula (8) with half the difference of the zenith distances as measured by the micrometer for one argument and the mean zenith distance for the other.

Inasmuch as the differential refraction obtained from the above table is only valid for the assumed mean state of the atmosphere, it will be necessary to apply to this differential refraction, factors obtained from the regular refraction Tables VI and VII, in order to reduce it to the differential refraction for the pressure and temperature at the time of observation.

Accordingly, it is important that the observer should always have a barometer and thermometer as part of his regular equipment. A reading just before beginning the observations and another at the completion of them should ordinarily be sufficient.

Only in an emergency should the rough correction based on the elevation of the station above sea level be permitted.

Reduction to the Meridian

If a star is observed off the meridian while the line of collimation of the telescope remains in the meridian, the measured zenith distance is in error on account of the curvature of the apparent path of the star. Let m be the correction to reduce the measured zenith distance to what it would have been if the star had been observed upon the meridian.

Then,

$$m=\frac{\sin^2 \frac{1}{2}\tau}{\sin 1''} \sin 2\delta \tag{9}$$

in which τ is the hour-angle of the star. The signs are such that the correction to the latitude, $\frac{m}{2}$, is always *plus* for the stars of *positive* declination and *minus* for stars of *negative* declination (south of the equator), regardless of whether the star is to the northward or to the southward of the zenith. $\frac{m}{2}$ or $\frac{m'}{2}$ is, then, always applied as a correction to the latitude, with the sign of the right-hand member of the above equation. For a subpolar, $180°-\delta$ must be substituted for δ, making the correction negative for a northern subpolar and positive for a southern subpolar. Table X, page 177, gives the corrections to the latitude computed from the above formula. If both stars of a pair are observed off the meridian, two such corrections must be applied to the computed latitude.

ADJUSTMENT OF LATITUDE RESULTS

The correction to the mean latitude and the correction to the value of one-half turn of the micrometer are computed for each night's observations separately by the method of least squares. See remarks on page 69. It is understood, unless stated otherwise by the observer, that the adjustment of the eyepiece micrometer remains untouched during the night's observations.

Form 663, Summary of Latitude Computations, should be used for the adjustment. The data in the first four columns are obtained directly from Form 33, the micrometer differences being taken to the nearest tenth.

Before proceeding further with the adjustment, it is necessary to find out which, if any, of the results are to be rejected. An absolute rejection limit of 3″ from the mean of all the latitudes in column 4, each considered to have unit weight, is first used. Then a mean of the remaining latitudes is taken, and the probable error of a single observation, e_p, is computed. Any latitude with a residual, $\Delta\phi$, equal to or greater than $5\,e_p$ is automatically rejected. Furthermore $3\frac{1}{2}\,e_p$ may be used as a doubtful limit beyond which rejection is to be made if evidence exists in favor of rejection, other than the residual itself. Such evidence may consist of positive notes indicating bad conditions during the observations of the particular pair concerned, contradictions in the record indicating a probable misreading, or a mean declination of a star with a probable error so large that it might account for the large residual. (Consult the appropriate column in the star catalogue used.)

Another criterion for rejection is discussed on page 136. For a small number of observations its use will increase slightly the number of rejections.

After all rejections have been made, the accepted observations remain to be adjusted in order to determine from the observations themselves the most probable value of a turn of the micrometer and the most probable latitude of the station. The information at the foot of the third column is now entered. Then a mean ϕ_m is taken of the

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 663
(Rev. Oct. 1934)

SUMMARY OF LATITUDE COMPUTATIONS

State: Mississippi Date: April 21, 1935 Chief of Party: C.A. Whitten

Station: University Instrument: Bamberg No. 20 Observer: J.E. Bondurant

GC	Star No. Boss catalog 1950	Mic. diff. M	ϕ 34° 22′	$\Delta\phi$	$\bar{\Delta\phi}$	Mr_1	Corrected 34° 22′	$\Delta\phi$	$\bar{\Delta\phi}$
12565	12593	+4.8	02.″22	+.46		-.12	02.″10	+.58	
12722	12799	+4.3	03.36	-.68		-.11	03.25	-.57	
13157	13277	+1.4	03.22	-.54		-.04	03.18	-.50	
13406	13573	+11.0	03.30	-.62		-.28	03.02	-.34	
* 13836	14123	-9.7	04.12	Rejected					
14170	14357	-8.9	02.39	+.29		+.23	02.62	+.06	
14389	14625	+2.4	02.68	.00		-.06	02.62	+.06	
14688	14789	-6.8	02.58	+.10		+.17	02.75	-.07	
14961	15006	-9.5	03.24	-.56		+.24	03.48	-.80	
15032	15185	+4.7	02.27	+.41		-.12	02.15	+.53	
15302	15340	+9.7	02.94	-.26		-.25	02.69	-.01	
15441	15485	-7.9	02.22	+.46		+.20	02.42	+.26	
15600	15745	+10.1	02.78	-.10		-.26	02.52	+.16	
16137	16173	+1.0	02.91	-.23		-.03	02.88	-.20	
16524	16871	-10.1	01.96	+.72		+.26	02.22	+.46	
16965	17127	-7.0	02.44	+.24		+.18	02.62	+.06	
17342	17769	-0.8	02.33	+.35		+.02	02.35	+.33	
* See appendix									
+sum		+49.4		+3.03	2.9664			+2.50	2.4173
−sum		-51.0		-2.99				-2.49	
Algebraic sum		-1.6		+.04				+.01	
Mean		-0.1	02.68				02.68		

	Uncorrected	Corrected
Value of half-turn of micrometer as used in field	39.47	39.4444
(1) Mean ϕ, 9 pairs with plus micrometer differences	02.85	02.71
(2) Mean ϕ, 7 pairs with minus micrometer differences	02.45	02.64
Difference, (2)−(1)	-.40	

Normal equations

$pc - \Sigma M r + \Sigma\Delta\phi = 0$ $\quad -\Sigma M c + \Sigma M^2 r - \Sigma M \Delta\phi = 0$

$16c + 1.6r + .04 = 0$ $\quad 1.6c + 819.20r + 20.934 = 0$

$c =$ 0.00 $\quad r =$ -.″0256

$e_p = \pm$ 0.28 $\quad e_\phi = \pm$ 0.″07 $\quad e_r = \pm$ .0098

Latitude of University latitude station	34°	22′	02.″68
Reduction to sea level (elevation of station 135 meters)			-.02
Reduction to mean position of pole			
Final latitude of University latitude station	34	22	02.66
Reduction to geodetic station University			-.10
Latitude of geodetic station University	34	22	02.56

U. S. GOVERNMENT PRINTING OFFICE

FIGURE 33.—Summary of latitude computations.

unrejected latitudes in column 4 and the difference $\Delta\phi=\phi_m-\phi$ entered in the next column and summed algebraically at the foot of the column.

If p is the number of accepted latitudes, c the amount in seconds by which the mean latitude deviates from the most probable latitude, and r the amount by which the preliminary value of one half-turn of the micrometer is to be corrected, then there will be p equations of the form

$$c-Mr+\Delta\phi=v. \tag{10}$$

Inserting the condition that $[v^2]$ or $[vv]$ must be a minimum, the normal equations to be solved for c and r are

$$pc-[M]r+[\Delta\phi]=0 \tag{11}$$

$$-[M]c+[MM]r-[M\Delta\phi]=0$$

where [ ] is the standard symbol indicating summation in operations with least squares. The details of the solution of the normal equations and the computation of the probable errors are explained on pages 141 and 142.

In the example, page 79, there are 16 equations of which the first two are

$$c-4.8\ r+0.46=v_1 \tag{12}$$

$$c-4.3\ r-0.68=v_2$$

The resulting normal equations are

$$16\ c+1.6\ r+0.04=0 \tag{13}$$

$$1.6\ c+819.20\ r+20.934=0.$$

Solving

$$c=0''.00$$

$$r=-0''.0256$$

and

$$\tfrac{1}{2}\ R=39''.47-0''.0256=39''.4444. \tag{14}$$

That is, 39″.4444 is the value of ½ turn of the micrometer determined by the observations themselves.

Each micrometer difference, M, of the accepted latitudes is now multiplied by the value of r and the results placed in the proper column. Then to each preliminary latitude ϕ there is added algebraically the corresponding Mr to produce the corrected latitude which is entered in the designated column of Form 663. A mean of these latitudes is now taken and entered at the foot of the column. In the next column the difference, mean latitude minus individual latitude, is set down for each pair of stars. The sum of the squares of these new $\Delta\phi$'s is entered in the next column.

The mean of the corrected latitude is the mean observed astronomical latitude of the latitude station, uncorrected, however, for elevation of station above sea level or variation of the pole.

As indicated by the value of $c=0.00$, the mean latitude of University was unchanged by the adjustment. It is

$$34°22'02''.68\pm0''.07. \tag{15}$$

To reduce this latitude to sea level see Table XI, page 178. The correction is found to be $-0''.02$ for an elevation of 135 meters at this latitude. See discussion below. Regarding the correction for the variation of the pole, see discussion below.

The latitude of the latitude station or point of observation is therefore

$$34°22'02''.66 \pm 0''.07. \tag{16}$$

From data furnished by the observer it was found that the instrument was set up at a point $0''.10$ in latitude north of the geodetic (triangulation) station. The astronomical latitude of the geodetic station, therefore, is

$$34°22'02''.56 \pm 0''.07. \tag{17}$$

The probable errors have been computed by means of the following formulas with data used in the adjustment.

$$e_p = \pm\sqrt{\frac{0.455[\Delta\phi^2]}{p-2}} \tag{18}$$

$$e_\phi = \pm\sqrt{\frac{0.455[\Delta\phi^2]}{(p-2)\left(p-\frac{[M]^2}{[MM]}\right)}} \tag{19}$$

$$e_r = \pm\sqrt{\frac{0.455[\Delta\phi^2]}{(p-2)\left([MM]-\frac{[M]^2}{p}\right)}}. \tag{20}$$

For detailed explanation of adjustment and probable errors, see pages 78 and 141.

GENERAL REMARKS

Correction to the Latitude for Elevation of Station

The correction to the latitude to reduce it to sea level is given by the formula

$$\Delta\phi = -0''.000171\ h \sin 2\phi \tag{21}$$

where $\Delta\phi$ is the correction in seconds of arc, h the elevation of the station in meters, and ϕ is the latitude.

This is a correction for the curvature of the vertical or, otherwise stated, for the fact that two level surfaces at different elevations are not parallel but converge slightly as the poles are approached.

Since this correction has been regularly applied in the past, for the sake of uniformity, it should continue to be applied. It is very doubtful, however, whether this correction in its present form accomplishes its purpose. It is based on two assumptions: (1) that the rate of change of gravity with latitude is normal; (2) that the space between the geoid and the place of observation is empty. Neither assumption is generally valid.

The correction to reduce the latitude to sea level according to the above formula can be obtained directly from Table XI, page 178.

Correction to the Latitude for the Motion of the Pole

It was the original intention when the variation of latitude was discovered to apply a correction for the motion of the pole. This has proved impracticable, since there are many years for which no definitive reductions of the motion of the pole are

available and since it is not always clear whether the effect of the published definitive reductions is always to reduce the latitudes to the same mean position of the pole on the surface of the earth. It is hoped, however, that the International Latitude Service will soon make available the necessary data for making definitive reductions to the same mean pole.

In that case it will be desirable to apply the correction to all latitudes observed within the period covered by the reductions.

Probable Error and Weights

The probable error of the latitude derived from a single pair of stars is the resultant of the probable errors of observation and of the mean declination of the pair, the former probable error being, in general, greater.

In previous editions of this manual much attention was given to the weighting of pairs of latitude stars under various possible arrangements of the program of observation. This was because star catalogues were then less accurate and more variable from star to star and because it was the practice to put in several nights of observation at a single station. Schemes of weighting were devised that depended on whether a given star pair was observed on more than one night.

It is the present practice of the Coast and Geodetic Survey to neglect the differences between the probable errors of the mean declinations for various star pairs and to assume that, with equal care and under equally good conditions of observation, the resultant probable errors of all star pairs taken from the Boss catalogue are equal. A pair recurring in a later observing list is treated as if it were a different and independent pair.

It is good practice, however, to note in the Boss catalogue the probable errors of the declinations in 1950 and, if the probable error of the mean declination of the pair is much larger than for the other pairs in the list, to give that pair a little less weight. The probable error of the mean declination of the pair is, of course, not the sole source of error of the latitude. As a rough estimate, the probable error of observation on a pair and the probable error of the mean declination of the pair for the less accurately determined stars of the Boss Catalogue contribute about equally to the resultant probable error.

Note: The probable error of the mean declination of a pair is given by the formula

$$\epsilon=\sqrt{\frac{\epsilon_1^2+\epsilon_2^2}{2}} \tag{22}$$

where ϵ_1 and ϵ_2 are the probable errors of the declinations of the separate stars and ϵ the probable error of the mean declination of the pair.

PART III

DETERMINATION OF THE ASTRONOMIC AZIMUTH OF A DIRECTION

GENERAL REMARKS

Various methods have been employed by the Coast and Geodetic Survey for determining astronomically the azimuth of a triangulation line or, what is the same thing, the direction of that line with respect to the meridian of the station at which observations are being made, and there is, perhaps, no other geodetic operation in which the choice of the method, the perfection of the instrument and the skill of the observer enter so directly into the value of the result. It is intended to give here in a concise form an account of the two principal methods now in use by this Bureau and to present the formulas involved as well as specimens of record and examples of computation.

These two methods are: (1) the direction method with a direction theodolite for low latitudes and (2) the micrometer method with the Bamberg broken-telescope transit for high latitudes. For other methods the reader is referred to Special Publication No. 14, fifth edition, or, in the case of observations on the sun at any hour angle, to Serial No. 166, *Directions for Magnetic Measurements*. Both are publications of the U. S. Coast and Geodetic Survey.

DEFINITIONS

The *meridian* of an observer is the great circle passing through both poles of the celestial sphere and the zenith of the observer.

The *astronomical azimuth* is the angle which the plane of the meridian through the astronomical zenith of the observer makes with the vertical plane passing through the observer and the object whose direction is to be determined. It is the practice of this Survey to reckon the azimuth from the south in a clockwise direction.

For purposes of consistency in the computation of the triangulation net of a country, it is necessary, first, to adopt a surface of reference which is assumed to fit closely the sea-level surface of the earth and, second, to choose after adequate investigation an initial station, the adopted position of which is assumed to be its true position on the adopted surface of reference.

In North America the adopted surface of reference is that of the Clarke Spheroid of 1866, and the initial point is Meade's Ranch, Kansas, the adopted position of which is assumed to be its position on the surface of the Clarke Spheroid. See Special Publication No. 110, pages 87–92.

In a triangulation net, computed latitudes, longitudes and azimuths referred to the surface of reference and to the initial station are called *geodetic latitudes*, *longitudes* and

azimuths respectively, and at the stations involved the geodetic zeniths are considered to be defined by the directions of the normals to the spheroid of reference at those stations.

On account of the variation in horizontal attraction caused by irregularities in the physical surface of the earth and the variable densities beneath the surface, the direction of the plumb line does not in general coincide with the direction of the normal to the spheroid at a station, and therefore the astronomical and geodetic azimuths are not in general identical. There is, however, a definite relation between the two kinds of azimuths at a station. The astronomical azimuth and the geodetic azimuth, consistent with the adopted surface of reference and the initial station, differ by a quantity called the *Laplace correction* which is obtained from the expression

$$(\lambda_A - \lambda_G) \sin \phi \tag{1}$$

where ϕ is the geodetic latitude of the station and where λ_A and λ_G are, respectively, the astronomical and geodetic longitudes of the station. See also p. XI.

The azimuth obtained by adding the Laplace correction to the astronomical azimuth is called the *Laplace azimuth* or *geodetic azimuth* of the station.

Since in the computation of the positions in a triangulation net, the computation of the azimuth involves a greater accumulation of errors than the longitude, it is necessary at frequent intervals, usually of from six to ten quadrangles, to observe the Laplace azimuth to control the computed geodetic azimuths.

The use of the Laplace correction assumes that all the error is contained in the computed geodetic azimuth, but that the computed or geodetic longitude is flawless. There have been some proposals to give some weight to the computed geodetic azimuth, but so far these have not been adopted by this Bureau.

With the above intervals, the accumulation of errors in azimuth between consecutive Laplace stations will not be large and when distributed properly will have a minimum effect on the azimuths of the intervening stations.

SPECIFICATIONS FOR LAPLACE AZIMUTH

It follows, therefore, that at stations selected for Laplace azimuths, astronomical observations must be made for both longitude and azimuth.

For use in triangulation a Laplace azimuth must be observed with first-order accuracy. The specifications for this are:

(1) In the northern hemisphere, except near the equator, Polaris preferably should be used, though any circumpolar will do, especially near elongation. With time determinations better than one second, Polaris may be used at any hour angle. If the point of observation is close to the equator, however, Polaris is too near the horizon for practical use. In high latitudes it is difficult to observe on Polaris with the theodolite. It is more satisfactory at high latitudes to use the micrometer method by making observations on Polaris with the broken-telescope transit.

(2) Observations should be made on at least two nights.

(3) No observation which gives a residual of 5″ or greater from the mean should be accepted.

(4) The azimuth should depend upon at least 24 acceptable observations, not less than 12 of them being on any one night.

(5) The probable error of the azimuth should not exceed 0″.30.

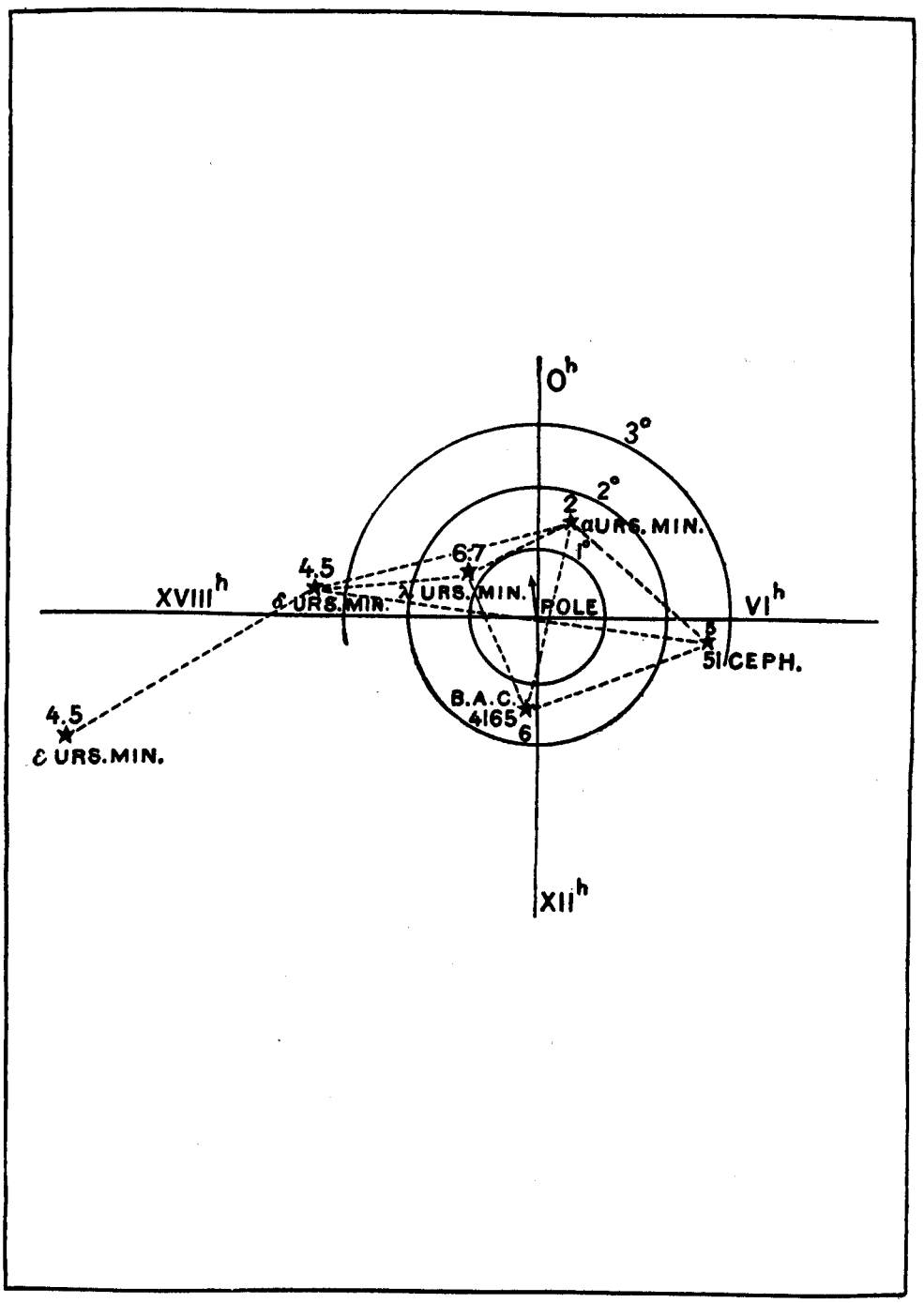


FIGURE 34.—Circumpolar stars.

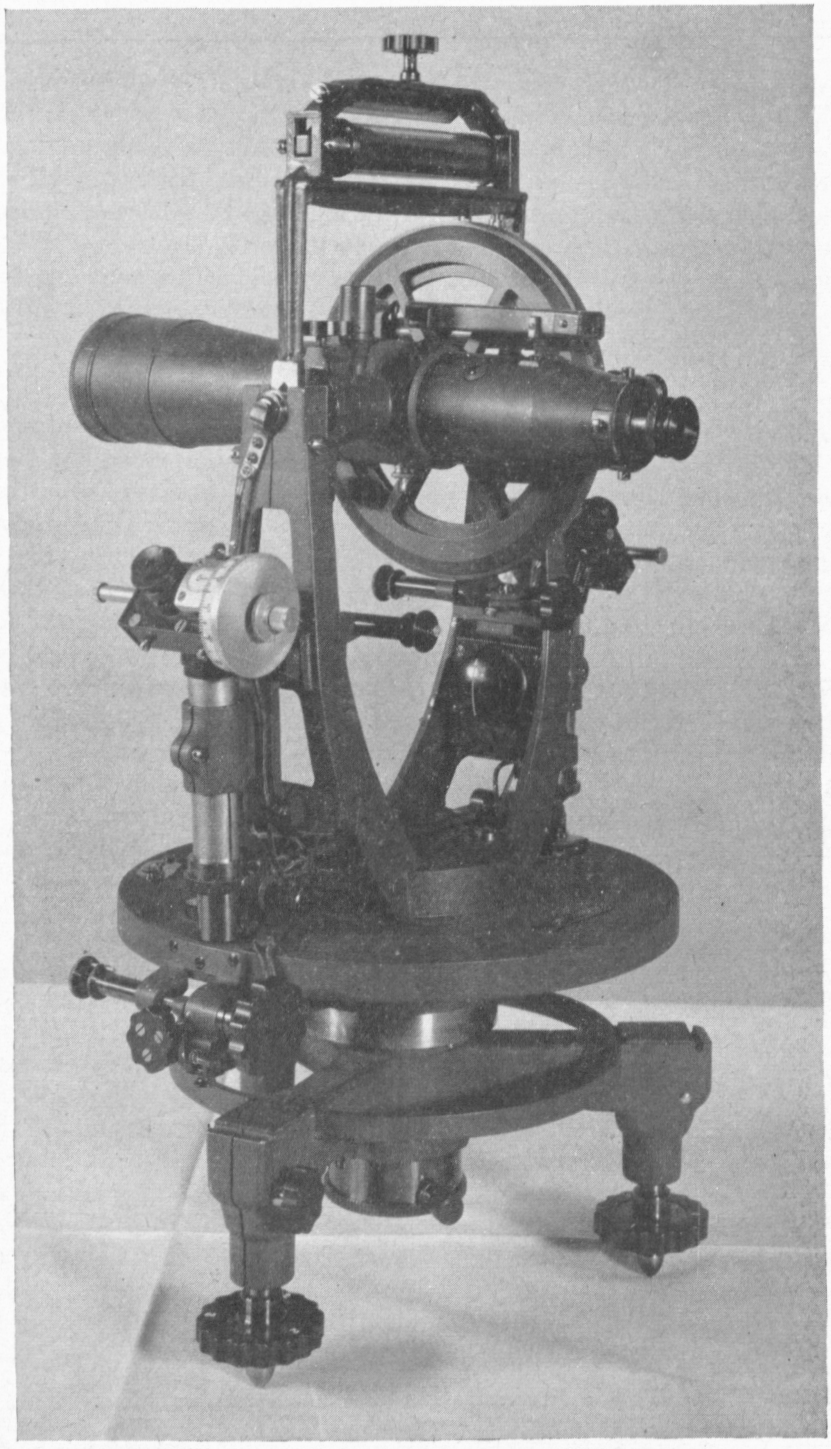


FIGURE 35.—Parkhurst theodolite.

INSTRUMENTS

The principal instruments now used by the Coast and Geodetic Survey for observing azimuth are the direction theodolite in low latitudes and the Bamberg broken-telescope transit in high latitudes such as occur in Alaska.

The theodolite in general use at present is the Parkhurst theodolite, which was designed and developed in the Coast and Geodetic Survey just prior to 1930. See illustration, page 86. This theodolite has a 9-inch horizontal circle graduated to 5 minutes. The micrometer head reads to 1″. The instrument has a 13-inch telescope with an objective of 2½ inches clear aperture and is fitted with a 6-inch vertical circle read by verniers to 10″. This theodolite has several unusual features and has proved very satisfactory.

The Bamberg broken-telescope transit has already been described in connection with longitude and latitude observations. See pages 1 to 5.

INSTRUMENT SUPPORT AND PROTECTION

If, as is customary, the triangulation party observes azimuth, the instrument, its support and its means of protection are standard equipment of the triangulation party.

On the other hand, if the astronomical party observes azimuth with a direction theodolite, support and protection must be provided both for the theodolite and for the Bamberg transit used in the observations for longitude and latitude. This is necessary because the two instruments are set up in different locations, so that observations for azimuth, longitude and latitude may be made all on the same night. The

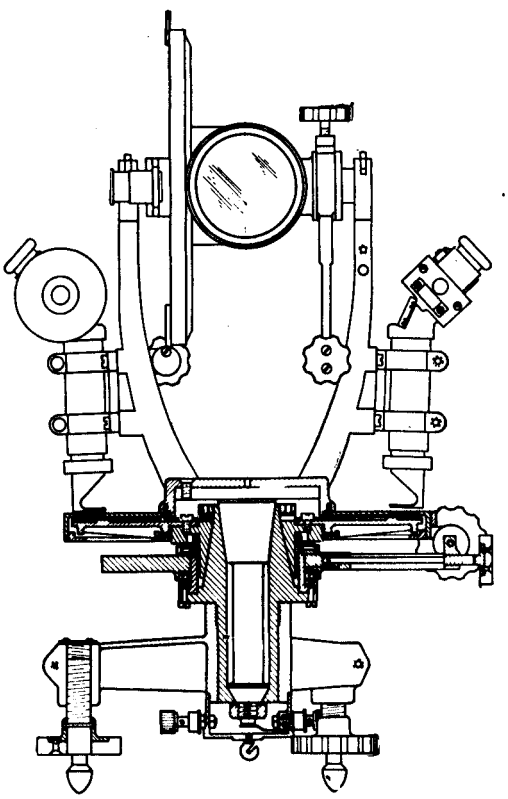


FIGURE 36.—Sectional view of Parkhurst 9-inch direction theodolite.

theodolite is set up at the station itself and the transit a short distance away. A type of tripod stand for the theodolite that is commonly used is pictured below.

When the tripod is set up on the ground a *catwalk* should be constructed for the observer to move about on. The walk should be supported far enough away from the legs of the tripod so that there is no danger of the instrument being disturbed by movements of the observer. Such a catwalk is described and illustrated on pages 30 and 31.

The theodolite should be protected by a tent, as in triangulation. On towers it is well to have a tarpaulin about 5 feet high fitted with ties and of a proper length to fasten around the observing platform.

FIGURE 37.—Tripod stand for theodolite.

Where the observing is done on the ground, a hexagonal silk tent of standard type is used. This tent is supported by a frame of thin steel tubing and is very light. Whatever type of tent is used, it should be pitched before any observing is done. If heavy winds are anticipated, the guy ropes should lead straight out from the stand and poles, with two extra ropes leading from the top of one of the poles to the two adjacent tent pegs in order to prevent the tent from swinging in azimuth. In moderate or light winds the pegs may be driven on lines passing out from the station mark midway between each pair of poles and a single long rope used for the guys, passing alternately behind a peg and then in a hitch over the top of the next pole.

In windy regions it will be advisable to carry as a part of the observing outfit, usually as a wrapping for the poles of the tent, a piece of canvas about 6 by 16 feet, which can be used as a windbreak. When the soil around the stand is soft and dry, the dust is often bothersome, and a piece of light canvas used as a floor in the observing tent will prove helpful. Another piece of light canvas laid outside the tent to windward, close to the bottom of the wall, will also decrease the annoyance from dust.

FIGURE 38.—Bilby steel tower.

If wind of considerable strength is permitted to strike the theodolite, it delays the observing and may also cause errors by deflecting the telescope from its proper position in the wyes. For that reason the flaps of the observing tent are usually kept tied down on the windward side when the wind is strong, except for small openings on line to the stations to be observed. If the wind is especially constant and bothersome, a strip of muslin of the proper size to cover the windward openings of the observing tent may be tied or pinned in place and a hole 4 or 5 inches in diameter cut on line to the station through which the mark may be observed.

When the Bamberg transit is also used for azimuth observations by the micrometer method, the instrument should be accurately plumbed over the triangulation station

as described on p. 109. All three types of astronomical observations are made on the same support and with the same protection. See pages 29 to 31.

Electric signal lamps, standard equipment for a triangulation party, are used at the azimuth marks.

DETERMINATION OF AZIMUTH

GENERAL FORMULAS

Two methods of determining azimuth will be treated in detail in this manual, namely (1) the direction method of measuring horizontal angles using a direction theodolite; (2) the micrometer method, using the Bamberg broken-telescope transit. Certain formulas which are common to these methods will be stated here for convenient reference.

The computation of the azimuth of a terrestrial line of sight from a set of azimuth observations consists essentially of the computation of the azimuth of a star at the instant of observation, the computation of the horizontal angle between the star and the mark, and the combination of these two results by addition or subtraction. The general practice of the Coast and Geodetic Survey is to prefer the use of Polaris, though any other close circumpolar star would serve.

The astronomical triangle is defined by the pole, the zenith of the observer and a star. The side zenith-pole is the co-latitude, the side star-pole is the polar distance or complement of the declination of the star, and the angle at the pole is the hour angle or its explement. Starting from these three as known parts, the astronomical triangle may be solved by the ordinary trigonometrical formulas. The solution to obtain the azimuth of a star, which is the angle of this triangle at the zenith, may without any approximation, be put in the form

$$\tan A = -\frac{\sin t}{\cos \phi \tan \delta - \sin \phi \cos t} \tag{2}$$

in which A is the azimuth of the star reckoned from the north in a clockwise direction, and t is the hour angle reckoned westward from upper culmination continuously to 24^h, at the next upper culmination. This is the most convenient formula for use with either of the two methods. The first term of the denominator changes very slowly on account of the very small change in the declinations during the period of observation and hence may be assumed constant during a period of observation not exceeding four hours. The second term, for a close circumpolar star, may be computed with sufficient accuracy by five-place logarithms.

The computation of the azimuth from this formula may be considerably shortened by transforming it, as indicated below, and using the table given on pages 183 to 191.

$$\tan A = -\frac{\cot \delta \sec \phi \sin t}{1 - \cot \delta \tan \phi \cos t} \tag{3}$$

$$= -\cot \delta \sec \phi \sin t \left(\frac{1}{1-a}\right) \tag{4}$$

in which $a = \cot \delta \tan \phi \cos t$.

The second form of this formula, equation (3), is about as convenient as the first. It involves the same number of logarithms as the first and one less reduction from logarithms to numbers.

The third form, equation (4), in connection with the table given on pages 183 to 184 gives a much quicker computation process than either of the other two. In using

this form and the tables, log cot δ sec ϕ sin t must be carried to six decimal places and log cot δ tan ϕ cos t to five decimal places. A convenient arrangement of the computation is shown on page 104. The formula and tables involve no approximations, and the only errors resulting from their use are those arising from the cast-off decimal places (logarithms limited to six decimal places). These errors are of the accidental class, and will seldom exceed 0".04 for any case covered by the table, and for most observations made below latitude 50°, the error will not exceed 0".01. These quantities are so small in comparison with the errors of observation as to be negligible. A few observations made in Alaska may be beyond the range of the table on pages 183 to 191, and when that is found to be the case, one may easily substitute formula (2) for formula (4), page 90.

Various other formulas for computing the azimuth of circumpolar stars have been proposed and used, but this Office prefers the use of the above formulas, and the tables and forms in this manual have been prepared for that purpose. Furthermore, these formulas are accurate and applicable to any star at any hour angle.

CURVATURE CORRECTION

In the direction method, an observation on a star consists of two pointings, one with the instrument direct and the other with the instrument reversed. The resulting azimuth is computed by means of formulas (4) or (3) using for hour angle t the mean of the hour angles of the two pointings. The azimuth so computed will be the azimuth corresponding to the mean hour angle of the observation, but will not be equal to the required mean of the azimuths corresponding to the hour angles of the separate pointings of the observation, since the rate of change of the azimuth is constantly varying on account of the curvature of the apparent path of the star. The difference is small but not always negligible.

The general formula for computing the correction for curvature is:

$$(\tan A)\,\frac{1}{n}\sum\left(\frac{2\sin^2\frac{1}{2}\tau}{\sin 1''}\right) \tag{5}$$

where A is the azimuth of the star; n is the number of separate pointings; and τ is the interval between the mean hour angle of all pointings and the hour angle of a single pointing. The general formula is used in computing the azimuth by the method of repetitions which is discussed on pages 155 to 157 of Special Publication No. 14, Fifth Edition.

In the direction method, $n=2$ and 2τ is the interval between the direct and reverse pointings on the star.

For Polaris and other circumpolar stars the correction for curvature may be taken directly from Table XIV, which has been computed by means of formula (5) for $n=2$ with the help of Table XII, in which are tabulated the values of $\left(\frac{2\sin^2\frac{1}{2}\tau}{\sin 1''}\right)$.

The sign of the curvature correction is always such as to decrease numerically the azimuth reckoned from the north, that is, if the azimuth is reckoned clockwise, the algebraic sign of the correction will be positive when the star is west of north and negative when the star is east of north.

The above formula (5) is approximate, but for circumpolar stars and for the interval of time usually covered by a complete observation, the errors involved are negligible.

If the star observed is Polaris, a convenient check on the computation for azimuth may be obtained from Table IV of the American Ephemeris and Nautical Almanac, entitled Azimuth of Polaris at all Hour Angles.

DIURNAL ABERRATION

Because of the rapid motion of the observer, due to the rotation of the earth on its axis, a star is seen slightly displaced from its real position. The required correction for diurnal aberration is:

$$0''.32 \frac{\cos A \cos \phi}{\cos h}. \tag{6}$$

The sign of the correction is always positive when applied to azimuths reckoned clockwise. For Polaris and for latitudes not greater than 74°, the greatest variation of the correction from its mean value of 0″.32, is 0″.02. The correction for diurnal aberration need not be applied to the separate results but simply to the mean result for a station.

LEVEL CORRECTION

If the horizontal axis is inclined when the pointings are made upon either the star or the mark, the corrections indicated below must be applied where d is the value of one division of the level and h is the altitude of the star:

(a) If the graduations on the striding level are numbered continuously in one direction, the level correction is

$$\frac{d}{4}[(w-w')+(e-e')] \tan h \tag{7}$$

where w, e, w', e' represent the west and east readings of the striding level in the two positions of the level, the primed letters referring to readings taken in the position in which the numbering increases toward the east.

Practically all of the striding levels now in use are numbered continuously in one direction.

(b) If the graduations are numbered in both directions from the middle, the level correction will be

$$\frac{d}{4}[(w+w')-(e+e')] \tan h \tag{8}$$

where w, e and w', e' are the readings of the west and east ends of the striding level before and after reversing it, respectively.

If the azimuth station and mark are at different elevations, the angle of elevation or depression of the mark from the station should be measured twice.

It is necessary to know h only approximately. An occasional reading of the vertical circle will give it with abundant accuracy.

If Polaris is the star, h may be obtained with sufficient accuracy by means of Table I of the American Ephemeris and Nautical Almanac, using the declination and hour angle for arguments and applying to the latitude the tabulated correction with sign reversed.

If the mark is not in the horizontal plane of the instrument, a similar correction, if appreciable, must be applied to readings on the mark. The same formula is used

as for the star except that tan h is replaced by tan H, where H is the angle of elevation or depression of the mark. Ordinarily the mark is so nearly in the horizon of the instrument, that tan H is nearly zero and the corrections required to pointings on the mark are negligible. If the mark is elevated or depressed more than 1° from the horizon, the striding level readings should be taken on the mark in the same way as on the star.

The formula as written gives the sign of the correction to be applied to the readings of the horizontal circle of which the numbering increases in a clockwise direction. This is also the sign of the correction to the computed azimuth (reckoned clockwise) for level readings in connection with pointings upon the mark, but in connection with pointings upon Polaris the sign must be reversed to give corrections to the computed azimuth of the mark.

DIRECTION METHOD

ADJUSTMENTS

The measurement of an azimuth by this method is essentially similar to the process of measuring a difference of two horizontal directions with a direction theodolite. The quantity measured in this case is the difference in horizontal direction between a circumpolar star and a mark instead of that between two triangulation signals. The fact that the azimuth of the star is continually changing adds new features to the computation, and makes it necessary to know the time of each pointing upon the star. The fact that the star is at a considerable altitude makes reading of the striding level a necessity and decreases the accuracy of the measurement because errors of inclination of the horizontal axis have a marked influence as contrasted with their comparatively unimportant effects upon the measurements of horizontal angles in triangulation.

It is important that the level adjustment of the instrument be carefully watched throughout the observations. The inclinations should be kept small and preferably should vary in sign so that they will tend to cancel out. When inclinations are large, any lack of uniformity in the graduations of the level vial will produce sizable errors in the azimuth.

The adjustments required are identical with those which are necessary when the instrument is to be used for the measurement of horizontal directions. The adjustments, of the focus of the telescope to eliminate parallax between image and wires, of the line of collimation for bringing the vertical lines of the reticule into vertical planes, of the setting circle (if used), and of the striding level may be made as described in connection with a transit on pages 31 to 33. The vertical axis of the instrument must be made to point as nearly as is feasible to the zenith by bringing the striding level to the proper reading in each of two positions at right angles to each other.

The microscopes with which the horizontal circle is read must be kept in adjustment. Ordinarily it will be found necessary to adjust the eyepiece alone by pushing it in or pulling it out until the most distinct vision is obtained of the micrometer lines and of the circle graduation. If the micrometer lines are not apparently parallel to the graduation upon which the pointing is to be made, they should be made so by rotating the micrometer box about the axis of figure of the microscope. If, to do this, it is necessary to loosen the microscope in its supporting clamp, great caution is necessary to insure that the distance of the objective from the graduated circle is not changed. The error of run of the reading micrometers should be kept small. In other words, the value of one turn of the micrometer in terms of the circle graduation should not be allowed to differ much from its nominal value. The value of the micrometer may be adjusted by changing the distance of the objective from the graduation. The

nearer the objective is to the graduation the smaller is the value of one turn. A change in this distance also necessitates a change in the distance from the objective to the micrometer lines, these lines and the graduation being necessarily at conjugate foci of the objective. This adjustment of the micrometer value is a difficult one to make, but when once well made, it usually remains sufficiently good for a long period.

SELECTION OF THE LAPLACE STATION AND THE AZIMUTH MARK

Inasmuch as the Coast and Geodetic Survey at the present time observes precise azimuth only for use in triangulation and traverse, the observations are usually entrusted to a triangulation party since the two triangulation stations to be used for observing station and mark are often not intervisible from the ground and therefore the towers in use by the triangulation party are needed for the azimuth observations. Furthermore, the instruments and equipment required for these observations are already part of the standard equipment of the triangulation party.

As a first consideration, a triangulation station selected for a Laplace station should be readily accessible, if possible by truck, because of the heavy instruments and the numerous pieces of equipment that the astronomical party requires for longitude observations. In the case where the stations of the main scheme are too inaccessible, a supplemental point may be chosen. This point must then be connected to the main scheme with first-order accuracy. This may be done by means of a single well-shaped triangle whose angles are measured with first-order accuracy.

If the astronomical party is to observe azimuth as well as longitude, the triangulation stations selected for Laplace station and azimuth mark should be, if possible, intervisible from the ground, or, if not, the mark should be so chosen that a minimum of tower construction is necessary.

When the triangulation party observes azimuth any convenient station of the scheme may be chosen for the azimuth mark. If the observer intends to observe on Polaris during the course of the observations of the horizontal angles, he must arrange his program so that Polaris will be observed last in order that the time interval between the direct and reverse pointings on it will be a minimum. It is advisable also that the last station observed on before Polaris should be the one chosen for the mark, in order to keep the instrumental changes between the pointings on the mark and Polaris as small as possible.

DIRECTIONS FOR OBSERVING AZIMUTH

Point on the mark and read the micrometers; then point on Polaris, bringing the star to within a half minute or so of arc of the vertical wires in the middle of the field; clamp the horizontal motion; place the striding level in position on the standard, at the same time calling "stand by" to the recorder; perfect the pointing, calling "tip" sharply at the moment of bisection; read striding level, the west end always first. Now reverse the striding level; read the micrometers in order and finally read the reversed striding level, again west end first.

Next loosen the horizontal clamp, turn the instrument 180 degrees, plunge the telescope and point again upon Polaris, going through the same procedure as for the first pointing. This routine permits the striding level bubble to come fully to rest without delay to the observing. With an extra man to read one micrometer, 16 positions can be observed on Polaris and the mark in a little more than an hour.

The principal precautions to take in observing upon Polaris are to train the recorder to note accurately the chronometer time corresponding to the call of "tip," and then

to give him sufficient time to make a record of it before he is confused by other readings. The recorder should be trained to carry mentally a staccato count of the seconds of the chronometer, as, "twenty′ half, twenty-one′ half, twenty-two′ half,"—the accented word or syllable and the word "half" synchronizing with the half-second beat. It is then easy for him to note within a quarter of a second the time of the observer's "tip."

If a sidereal watch is used instead of a chronometer, its fifth-second beat can not readily be followed, but with practice the mental staccato half-second count can be regulated so that the count of the whole second can be made to coincide with the passage of the second hand over each successive second division of the dial as registered by the eye and the time of the "tip" noted with reference to the count. This is more accurate than noting the time by the eye alone.

To avoid errors due to the instrument being out of level, the azimuth mark should be as nearly as possible in the horizon. If it is elevated or depressed more than 1 degree out of the horizon, striding-level readings should be taken when observing upon the mark in the same manner as when observing upon Polaris.

In northern latitudes, such as Alaska, particular care must be taken that the instrument stand is stable and not disturbed by the movements of the observer about it. Because of the high altitude of Polaris the striding level used should be very sensitive and care taken that the bubble has come to rest before the readings are made. A 2-millimeter division on the vial should have a value of not to exceed 4 seconds of arc for azimuth work in high latitudes.

The record book in which are recorded the observations upon Polaris should always contain a record of the eccentricity of both the light and the theodolite. If there is no eccentricity of either it should be so stated. Often there is uncertainty as to whether the eccentricity recorded for the regular angle observations should be applied to the azimuth observations also. This applies in particular on the occasions when the azimuth observations are made on a different night than the other horizontal directions.

As stated on page 84, the observations for a first-order azimuth should be made on at least two nights and must consist of 24 acceptable observations, not less than 12 of them being on any one night. Usually a night's azimuth program consists of one or two sets of observations, each set for 16 positions of the circle.

TIME DETERMINATION

Generally, the timepiece used in the azimuth observations is a sidereal chronometer. It is usually set approximately at local sidereal time, though this is not absolutely necessary. The observed chronometer time must be corrected to obtain local sidereal time. Two principal methods of obtaining the chronometer corrections and rate will be discussed.

1. Time by Radio. The simpler and more accurate method of obtaining the chronometer corrections and rate is by means of radio time signals. A radio-chronometer comparison should be made just before the azimuth observations begin and for every two hours thereafter, including a comparison immediately after the conclusion of the observations.

Any scientific time signals, such as the Navy time signals, *NSS* or *NPG*, or the National Bureau of Standards' signal, *WWV*, will be satisfactory. The latter is particularly convenient because it is transmitted continuously throughout the 24 hours of the day. It is sufficient to obtain the chronometer times of several identifiable breaks by ear to the nearest quarter of a second. This method requires that the astronomical longitude of the place of observation be accurately known.

The chronometer correction should be computed on Form 605. The astronomical longitude should be used. A sample computation is given on page 114.

2. Time by Zenith Distances of Stars Near the Prime Vertical. This method is recommended when the party observing azimuth does not have the use of a radio.

The method consists in the observation of the zenith distances of an east and a west star, each of which is within 30° of the prime vertical and not less than 15° above the visible horizon. Four observations should be made on each star. The above combination of stars is commonly known as a *time set.*

For the reduction of observed chronometer time to local sidereal time in the azimuth observations, two time sets must be observed, one immediately preceding the azimuth observations and the other immediately following them. For each set the difference between chronometer time and local sidereal time is then computed. By means of the rate of change of this difference from the first set to the second, it will be a simple matter to compute the local sidereal time of each azimuth observation.

The observations may be made with any instrument having a good vertical circle and a fairly sensitive level.

Since the observed zenith distances must be corrected for refraction, it is essential that barometer and temperature readings be taken for each time set.

DIRECTIONS FOR OBSERVING ZENITH DISTANCES

The method of observing depends upon whether the level vial is attached to the vernier or to the frame of the graduated circle. With either method the star is brought near the intersection of the middle vertical and horizontal wires, the horizontal wire brought just ahead of the star in the direction in which the star is moving and the star allowed to make contact, thus eliminating the error due to the thrust upon the instrument when the horizontal wire is moved into contact with the star.

Level Vial Attached to Vernier. With circle right bring the star near the intersection of the middle wires, as described above; call "tip" to the recorder for the noting of the chronometer time to one-half second as the star makes contact with the horizontal wire; bring the bubble to the center of the tube with the vernier screw, watching it three or four seconds to see that it remains centered; read the verniers; loosen the vernier clamp; plunge the telescope and with circle left bring the star into approximate position with the vernier screw, calling "tip" to the recorder as the star makes contact; bring the bubble to the center and read the verniers. This constitutes one measure of zenith distance. The next measure should be made beginning circle left and ending with circle right.

Level Vial Attached to Frame or to Graduated Circle. With circle right read the verniers; bring the star near the horizontal wire with the tangent screw, which operation does not change the vernier readings; call "tip" to the recorder as the star makes contact; *then read the level,* objective end first. Loosen the vernier clamp; reverse instrument; plunge the telescope; and with circle left bring the star into position with the vernier slow-motion screw; call the contact; read the level and then the verniers.

If one end of the bubble falls beyond the graduations after reversal, it can be brought back by moving one or more of the foot screws of the instrument before pointing upon the star. During a single measure of the zenith distance, the relation between the line tangent to the axis of the bubble and the line passing through both the zero and the 180° mark on the graduated circle must not be changed in the period between two star contacts.

DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
Form 252
Rev. Feb. 1935

DOUBLE ZENITH DISTANCES

Station Doyle State Ohio Instrument 368 Date Dec. 3, 1943

Observer R.L. Tucker County Wayne

Object Observed	Time	Level O.	Level E.	Circle Right or Left	Circle Reading		Verniers A	Verniers B	Verniers Mean	Zenith Distance	Remarks
Star East	h m	s			° ′		″	″	″	° ′ ″	∡ from pole 96°05′
α Arietis	18 55	22.5		R	314 47		40	50	45.0		
										45 00 20.0	Barometer = 28.7
	18 58	34		L	44 48		20	30	25.0		Temperature = 3.5 C
mean	18 56	58.2									
	19 00	27.5		L	44 27		40	25	32.5		"B" Micro: Ed. Hartley
										44 10 28.8	Recorder: A.L. Powell
	19 02	26.5		R	316 06		30	40	35.0		Checker: M.Y. Poling
mean	19 01	27.0									
	19 04	19.5		R	316 28		00	55	57.5		
										43 27 08.8	
	19 06	17.5		L	43 22		25	05	15.0		
mean	19 05	18.5									
	19 08	07.0		L	43 01		30	30	30.0		
										42 46 20.0	
	19 09	47.5		R	317 28		40	60	50.0		
mean	19 08	57.2									

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FIGURE 39.—Double zenith distances.

If a repeating circle is being used, the reading of the circle should be changed by at least a degree between each two measures; otherwise, an error in a single reading would cause the rejection of two measures of the zenith distance. See U. S. Coast and Geodetic Survey Special Publication No. 120, Manual of First-Order Triangulation, page 152.

As far as possible, only the brighter stars which can be easily identified should be chosen, whose apparent places appear in the American Ephemeris and Nautical Almanac or the Apparent Places of Fundamental Stars.

In every instance for the purpose of checking the identification of the star, the approximate azimuth of the star from Polaris should be observed and recorded along with the time of such observation.

It is recommended that the observer compute in the field the results of at least one observation on each star.

Errors in Time Observations

Some of the more common sources of error in time observations with the vertical circle are mentioned below, with the remedy for each indicated:

1. *Incorrect noting of time.* An inexperienced recorder should be trained in the way explained in the paragraphs relating to the observations on Polaris. Do not confuse him by calling out the readings of the verniers or levels before he has finished recording the time.

2. *Incorrect circle readings.* The difficulty of securing an adequate illumination of the verniers by flashlight increases the chances of incorrect readings. Check carefully the minutes of each vernier reading, for the mistakes are more apt to occur in the minutes than in either the degrees or seconds.

3. *Wrong star.* The approximate azimuth from Polaris and the time of its measurement will serve to correct the identification of the star.

4. *Refraction errors.* The zenith distances must be corrected for refraction by means of Tables V–VII, pages 173 to 175, in accordance with the barometric pressure and temperature at the time of observation.

The differential effect of using one temperature and barometer reading for the whole time set instead of separate temperature and barometer readings for each observation will be lessened by having the zenith distances of the east and west stars as nearly equal as possible. Usually, however, the error from this source is negligible if no star is used whose altitude is less than 15°.

5. *Poor selection of stars.* More serious errors will be introduced by selecting stars too far from the prime vertical. For instance, at 30° from the prime vertical the effect of an error in zenith distance on time is 15 percent more than it would be exactly in the prime vertical; at 20° from the prime vertical the effect on time is 6 percent more than exactly in the prime vertical. In the early evening there is always the temptation to use the first stars visible in order to begin the night's work. A delay of a quarter of an hour is usually not serious and will often result in securing time stars that will give a much more accurate chronometer correction.

6. *Parallax.* The effect of parallax is almost invariably opposite in sign for east and west stars. Hence, to eliminate this effect, an east star and a west star should always be observed, as the mean of the computed times for the two stars will then be measurably free from the effect of parallax, unless there is a great difference in their zenith distances.

All of the foregoing observation data are entered in the field book for Double

Zenith Distances, Form 252. A sample for station *Doyle* in Ohio is given on page 97. The mean of the two chronometer readings corresponding to one observation, namely, for circle left and for circle right, is computed. Then the zenith distance is computed by subtracting vertical circle reading for circle right, from the vertical circle reading for circle left, and dividing the difference by two. This procedure of computation is based on the assumption that the circle is graduated continuously from 0° to 360°; the circle reading 0° for the telescope pointing to the zenith and reading 90° for the telescope pointing to the horizon, when the vertical circle is on the left.

These mean chronometer readings and the corresponding zenith distance observations for the observations on the stars are then entered on Form 381a and computed as explained in the following section.

Computation of Time From Observations on a Star With Vertical Circle

This computation is made on Form 381a. Specimens of the record and the computation will be found on pages 100 and 101. The formula used is:

$$\sin^2 \frac{1}{2} t = \frac{\sin \frac{1}{2}[z+(\phi-\delta)] \sin \frac{1}{2}[z-(\phi-\delta)]}{\cos \phi \cos \delta} \tag{9}$$

where t is the hour angle of the star, z is the zenith distance of the star corrected for refraction, ϕ is the astronomical latitude of the station and δ is the declination of the star. If the star is a west star, sin $\frac{1}{2}t$ will be positive, and $\frac{1}{2}t$ will be in the first quadrant. If the star is an east star, sin $\frac{1}{2}t$ will be negative and $\frac{1}{2}t$ will be in the fourth quadrant.

Knowing the mean barometric pressure and temperature for each star, the refraction is computed from Tables V, VI, and VII, in the manner explained below.

To obtain the refraction from Tables V, VI, and VII, first enter Table VII with the temperature reading as argument to obtain C_T. Then enter Table VI with the barometer reading as argument to obtain C_B. Take the product of C_T by C_B.

Next using the zenith distance, z, as argument enter Table V for r_m (the mean refraction for 29.9 inches pressure and temperature 10° C.).

The refraction for the temperature and barometer readings at the time of observation will now be the product:

$$C_B \text{ times } C_T \text{ times } r_m. \tag{10}$$

After the hour angle has been found from formula (9) the local sidereal time is found by the formula:

$$L.S.T. = \alpha + t \tag{11}$$

where α is the right ascension of the star.

Note: When the star is east and the hour angle, as defined on page 100, is in the third or fourth quadrant, it is often convenient to use, instead, the explement of the hour angle as a negative small angle. For instance, in the sample computation on page 100, for the first observation on α Arietis, $\frac{1}{2}t = -(24°51'57''.7)$ would be used in place of $\frac{1}{2}t = (335°08'02''.3)$. The sidereal time would then be $(2^h04^m00^s.7) + [-(3^h18^m55^s.7)] + 24^h = 22^h45^m05^s.0$, as in the sample computation.

The chronometer correction will now be local sidereal time minus chronometer time.

If it is desired to keep the chronometer correction always positive, 24 hours should be added to the local sidereal time before making the subtraction when the sidereal time is less than the chronometer time.

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DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 881 a
Rev. Jan. 1946

COMPUTATION OF TIME, OBSERVATIONS ON A STAR WITH VERTICAL CIRCLE

STATE Ohio STATION Doyle BAROMETER READ. 28.7" C_B 0.959

DATE Dec. 3, 1943 CHRONOMETER No. 235 TEMPERATURE 3.5°C C_T 1.023

INSTRUMENT 368 LEVEL VALUE ..... $C_B \times C_T$ 0.981

CHIEF OF PARTY A.P. Ratti | APPROX. ANGLE BETWEEN STAR AND POLARIS 96° 05'
CHRO. TIME OF ANGLE READING 18h 56m

OBSERVER R.L. Tucker CHRO.–WATCH COMPARISON | CHRONOMETER .....
WATCH (..... MER. TIME)

	STAR {East / West}; α Arietis		STAR {East / West}; α Arietis	
Chron. Reading. Zenith Dist.	18h 56m 58.2s	45° 00' 20.0"	19h 01m 27.0s	44° 10' 28.8"
Refraction		+ 56.8		+ 55.4
Corrected Z. D. = ζ		45 01 17		44 11 24
log cos φ . φ	9.8779372	40 58 34		
log cos δ . δ	9.9633858	23 11 53		
log cos φ + log cos δ = log D, φ − δ	9.8413230	17 46 41	9.8413230	17 46 41
log sin ½ [ζ + (φ − δ)], ½ [ζ + (φ − δ)]	9.7168423	31 23 59	9.7116378	30 59 02.5
log sin ½ [ζ − (φ − δ)], ½ [ζ − (φ − δ)]	9.3720087	13 37 18	9.3587956	13 12 21.5
Sum two log sines = log N	9.0888510		9.0704334	
log N − log D = log sin² ½ t	9.2475280		9.2291104	
log sin ½ t . ½ t (arc)	9.6237640	335 08 02.3	9.6145552	335 41 23.5
t (time) . t (arc)	20h 41m 04.3s	310 16 04.6	20h 45m 31.1s	311 22 47.0
Right ascension of star	2 04 00.7		2 04 00.7	
Sidereal time	22 45 05.0		22 49 31.8	
Chronometer reading	18 56 58.2		19 01 27.0	
Chronometer correction	+3 48 06.8		+3 48 04.8	

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The correction is plus if the chronometer is slow, and minus if fast.
Carry all angles to seconds only, all times to tenths of seconds, and all logarithms to seven decimal places.
In space below, compute rate of chronometer, etc.

FIGURE 40.—Computation of time, observations on a star with vertical circle. East star.

The mean chronometer time and the mean chronometer correction of each set are obtained by first taking the means for each star and then the means of these respective means. The rate of the chronometer then will be the difference in the mean corrections for the two sets divided by the difference in the mean chronometer times for the sets, or expressed symbolically, the rate of the chronometer per hour with respect to sidereal time will be:

$$R = \frac{C_t - C_o}{T_t - T_o}$$

where T_o and T_t are the mean chronometer times expressed in hours for the first and last sets, respectively, and C_o and C_t are the mean chronometer corrections at T_o and T_t, respectively.

If $C_t - C_o$ is expressed in seconds of time, R is the rate in seconds per hour. If R is positive, the chronometer is losing. If negative, the chronometer is gaining.

In the sample computation, page 100, the east star of the first set is α Arietis. The first observed zenith distance is 45°00′20″.0. The barometer reading is 28.7 inches and

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 381 a
Rev. Jan. 1946

COMPUTATION OF TIME, OBSERVATIONS ON A STAR WITH VERTICAL CIRCLE

STATE Ohio STATION Doyle BAROMETER READ. 28″8 C_B 0.963

DATE Dec. 3, 1943 CHRONOMETER No. 235 TEMPERATURE 2°5 C C_T 1.027

INSTRUMENT 368 LEVEL VALUE ..... $C_B \times C_T$ 0.989

CHIEF OF PARTY A. P. Ratti | APPROX. ANGLE BETWEEN STAR AND POLARIS 90°25′ | CHRO. TIME OF ANGLE READING 19h 14m

OBSERVER R. L. Tucker CHRO.-WATCH COMPARISON | CHRONOMETER ..... | WATCH (..... MER. TIME)

	STAR {~~East~~ West}; β′ Cygni		STAR {~~East~~ West}; β′ Cygni	
Chron. Reading, Zenith Dist.	19h 15m 10.0s	45° 26′ 28.8″	19h 18m 45.8s	46° 06′ 57.5″
Refraction		+ 58.4		59.8
Corrected Z. D. = r		45 27 27		46 07 57
~~log cos φ~~ φ	9.8779372	40 58 34		
log cos δ δ	9.9465609	27 50 39		
log cos φ + log cos δ = log D, φ − δ	9.8244981	13 07 55	9.8244981	13 07 55
log sin ½ [r + (φ − δ)], ½ [r + (φ − δ)]	9.6895771	29 17 41	9.6941055	29 37 56
log sin ½ [r − (φ − δ)], ½ [r − (φ − δ)]	9.4446180	16 09 46	9.4533489	16 30 01
Sum two log sines = log N	9.1341951		9.1474544	
log N − log D = log sin² ½ t	9.3096970		9.3229563	
log sin ½ t ½ t (arc)	9.6548485	26 51 09.7	9.6614782	27 17 59.3
t (time) t (arc)	3h 34m 49.3s	53 42 19.4	3h 38m 23.9s	54 35 58.6
Right ascension of star	19 28 25.7		19 28 25.7	
Sidereal time	23 03 15.0		23 06 49.6	
Chronometer reading	19 15 10.0		19 18 45.8	
Chronometer correction	+ 3 48 05.0		+ 3 48 03.8	

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The correction is plus if the chronometer is slow, and minus if fast.
Carry all angles to seconds only, all times to tenths of seconds, and all logarithms to seven decimal places.
In space below, compute rate of chronometer, etc.

FIGURE 41.—Computation of time, observations on a star with vertical circle. West star.

temperature is 3°5 Centigrade. From Table VII, page 175, we find $C_T = 1.023$ and from Table VI, page 174, we obtain $C_B = 0.959$. The product of C_B by $C_T = 0.981$. The altitude is $90° - (45°00'20''.0) = 44°59'40''.0$. Consulting Table V, page 174, r_m is found to be 57″.9. Hence the refraction correction is (0.981) (57″.9) = 56″.8.

Summary of Time Computation

Doyle, Ohio

Star	*Mean chronometer time*	*Mean chronometer correction*
	First Set	
East (4 obs.)	19^h 03^m17	$+3^h$ 48^m 05^s90
West (4 obs.)	19 20.74	+3 48 04.28
Mean, 1st set	19 11.96	+3 48 05.09
	Second Set	
East (4 obs.)	22 02.61	+3 48 04.38
West (3 obs.)	22 23.12	+3 48 03.40
Mean, 2nd set	22 12.86	+3 48 03.89
Difference	3 00.90	−1.20

Rate = -0^s3980 per hour or -0^s00663 per minute

COMPUTATION OF AZIMUTH, DIRECTION METHOD

For the purpose of explaining the computation of an azimuth observed by the direction method, the azimuth from *Doyle* to *Barberton* was chosen. The observations were made on two nights, Dec. 3, 4, 1943, sixteen observations being made on each night. The correction for rate from the observations with the theodolite has already been explained. (See p. 100.)

There follow samples of the record of the observations on Polaris and the mark on Form 251a; computation of the azimuth on Form 380; and the summary of azimuth computations.

Explanation of Computation

The first step in the computation for azimuth is to abstract on Form 380, the necessary data from the horizontal angle record book Form 251a. Then, with the chronometer correction and rate computed on Form 381a, correct the chronometer time for each observation on Polaris to obtain the local sidereal time of the observation. Next, compute the Greenwich civil time of the mean epoch of the observations expressed in a fraction of a day, as follows.

Usually, it is sufficient to take the mean of the local sidereal times of the first and last observations of the night's observations, provided the period is not greater than 4 hours. To this mean, add the longitude of the station if west, subtract if east of Greenwich. The result is now the Greenwich sidereal time of the approximate mean epoch of the observations. Subtract from this the Greenwich sidereal time of the nearest preceding 0^h civil time and express this difference as a fractional part of a civil day, taking into account that there are 24.066 sidereal hours or 1444 sidereal minutes in a civil day. In the example this computation is as follows:

Mean epoch of observations (L. S. T.)	0^h	00^m	39^s7	
Astro. Longitude (W)	+5	26	45.9	
G. S. T. of mean epoch	5	27	25.6	
0^h Dec. 4, 1943, G. S. T	4	47	44.4	
Difference	0	39	41.2	$=39^m687$

$39^m687 \div 1444 = 0.027$ day after 0^h Dec. 4.

Hence, the mean Greenwich civil time of the mean epoch of the observations is Dec. 4.027, 1943.

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FORM 261a

Horizontal Directions

Station: Doyle Observer: R.L. Tucker Instrument: 368 Date: Dec. 3, 1943

Position	Objects Observed	Time h. m.	Tel. D or R	Mic.	°	′		Back'w'd ″	Forward ″	Mean	Mean D and R	Direction ′	Remarks
1	Barberton		D	A	00	00		41	41				1 division of striding
				B				50	50	45.5			level = 6.507
			R	A	180	00		40	40				
				B				32	33	36.2	40.8		
													Level readings
	Polaris		D	A	308	11*		24	24				W E
	7h 38m 47.0s			B				38	38	31.0			06.9 20.2
	7 41 40.0												23.4 10.0
	Mean 7 40 13.5		R	A	128	10		55	55				16.5 +6.3 10.2
	Diff 2 53.0			B				37	38	46.2	08.6		08.6 21.9
													21.6 08.1
				A									13.0 -0.8 13.8
				B									+2.8
2	Barberton		R	A	191	01		28	28				
				B				23	23	25.5			
			D	A	11	01		29	30				
				B				32	32	30.8	28.2		
	Polaris												W E
	7 45 10.0		R	A	139	10*		42	43				08.0 21.1
	7 46 46.5			B				27	27	34.8			22.1 08.9
	Mean 7 45 58.2												14.1 +1.9 12.2
	Diff. 1 36.5		D	A	319	09		55	55				06.8 20.0
				B				66	67	60.8	17.8		23.5 10.1
													16.7 +6.8 09.9
				A									+4.4
				B									
	* Correct minute for mean of direct and reverse												

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FIGURE 42.—Horizontal directions (Azimuth observations, direction method).

DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY

FORM 289
Ed. Jan. 1942

COMPUTATION OF AZIMUTH, DIRECTION METHOD

STATE: Ohio STATION: Doyle ECCENTRIC* INSTR.: None
MARK: Barberton INSTR. No.: 368 LIGHT: None
CHRONOMETER NO.: 235 LEVEL VALUE (D): 6.507 OBSERVER: R. L. Tucker
Ast. POSITION OF STATION, ϕ: 40° 58′ 33.″668 λ: 81° 41′ 29.″138 GREENWICH CIVIL DAY: 4.03

Date, 1943 Dec. 3, position,	1	2	3	4
Chronometer reading,	19h 40m 13.5s	19h 45m 58.2s	19h 50m 17.0s	19h 54m 13.2s
" correction,	+ 3 48 04.9	+ 3 48 04.9	+ 3 48 04.8	+ 3 48 04.8
Sidereal time,	23 28 18.4	23 34 03.1	23 38 21.8	23 42 18.0
α of Polaris,	1 46 01.6			
t of Polaris (time),	21 42 16.8	21 48 01.5	21 52 20.2	21 56 16.4
t of Polaris (arc),				
δ of Polaris,	89° 00′ 01.″77			
log cot δ,	8.24171 } =	8.18051		
log tan ϕ,	9.93880 }			
log cos t,	9.91636	9.92362	9.92882	9.93338
log a (to 5 places),	8.09687	8.10413	8.10933	8.11389
log cot δ,	8.241708 } =	8.363770		
log sec ϕ,	0.122062 }			
log sin t,	9.752355 n	9.736036 n	9.723187 n	9.710976 n
log $\frac{1}{1-a}$ = colog(1-a)	0.005462	0.005555	0.005622	0.005682
log (−tan A) (to 6 places),	8.121587 n	8.105361 n	8.092579 n	8.080428 n
A = Azimuth of Polaris, from north,†	+ 0° 45′ 28.″9	0° 43′ 48.″8	0° 42′ 32.″6	0° 41′ 22.″2
Difference in time between D. and R.,	2m 53.0s	1m 36.5s	1m 30.0s	1m 39.5s
Curvature correction,	− 0.″1	0.″0	0.″0	0.″0
Altitude of Polaris=h,	41° 48′	° ′	° ′	41° 50′
$\frac{4}{3}$tan h=level factor,	1.″454			
Inclination,	+ 2.8	+ 4.4	+ 4.4	+ 4.6
Level correction,	+ 04.1	+ 06.4	+ 06.4	+ 06.7
Circle reads on Polaris,	308 11 08.6	139 10 17.8	330 10 54.9	161 10 45.8
Corrected reading on Polaris,	308 11 12.7	139 10 24.2	330 11 01.3	161 10 52.5
Circle reads on Mark,	0 00 40.8	191 01 28.2	22 03 25.1	213 04 25.0
Difference, Mark—Polaris,	51 49 28.1	51 51 04.0	51 52 23.8	51 53 32.5
Corrected azimuth of Polaris, from north,†	0 45 28.8	0 43 48.8	0 42 32.6	0 41 22.2
	180 00 00.0	180 00 00.0	180 00 00.0	180 00 00.0
Azimuth of (Clockwise from south)	232 34 56.9	232 34 52.8	232 34 56.4	232 34 54.7

To the mean result from the above computation must be applied corrections for diurnal aberration, elevation of mark, and eccentricity (if any) of station and mark. Carry times and angles to tenths of seconds only.

* Give volume and page of record for eccentricity, if any. † Minus, if west of north.

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FIGURE 43.—Computation of azimuth. Direction method.

Although theoretically the longitude used in the above computation should be the astronomical one, the use of the geodetic longitude in this computation will not essentially change the result.

The next step is to obtain from the table in the American Ephemeris the right ascension α and declination δ of Polaris for the Greenwich civil time just computed, being careful not to overlook the corrections for the short-period terms in the table.

The right ascension is taken out accurate to one-tenth second to conform with the practice for the local sidereal time. The declination, however, is taken out to hundredths of a second.

In the designated places, the logarithms of the tangent and secant of the astronomical latitude, ϕ, and the logarithm of the cotangent of the declination δ of Polaris can now be entered on the form.

The right ascension α of Polaris is subtracted from the local sidereal time, the result being t the hour angle of Polaris in units of time. If there is available a 6- or 7-place logarithm table of trigonometric functions with time as an argument, log cos t and log sin t can be taken directly from the table without first going through the tedious process of converting time to arc. Such a table is that of Shortrede, which is standard equipment in this Office. It is important that the trigonometric functions be used with the proper signs.

The expression for a will have the same sign as cos t. Colog $(1-a)$ is taken from Table XIII, page 183 ff., using log a as argument. There are two parts to this table, one for positive a, the other for negative a.

Next, we obtain log tan $(-A)$. The sign of tan $(-A)$ will depend on the sign of sin t. If sin t is positive, tan $(-A)$ will be positive; hence, the azimuth A will be negative, when Polaris is west of the meridian. If sin t is negative, tan $(-A)$ will be negative; hence, the azimuth A will be positive, when Polaris is east of the meridian.

The azimuth of Polaris from the north, that is, azimuth A, must now be corrected for curvature. The sign of the curvature correction is always such that it diminishes the size of the angle of Polaris from the north. The correction is found in Table XIV, page 192. On page 91 there is a discussion of the curvature correction.

Ordinarily, the altitude of Polaris is not observed in the field, since for the purpose of obtaining $\frac{d}{4}$ tan h, the value of the altitude, h, as taken from Table I in the American Ephemeris to the nearest minute will be sufficiently accurate. The arguments are the hour angle and declination of Polaris. It will usually suffice also to compute this factor for one in every four observations and to interpolate between the computed factors. To obtain the altitude of Polaris the quantity found in Table I is applied to the latitude with its sign reversed.

Later (p. 115) in connection with the micrometer method of determining azimuth in high latitudes, it will be shown how the altitude can be obtained with greater accuracy.

Each circle reading on Polaris must be corrected for the inclination of the horizontal axis. This correction is obtained by multiplying the inclination, expressed in divisions of the level vial, by $\frac{d}{4}$ tan h seconds of arc.

If H the angle of elevation or depression of the mark from the station is greater than 1°, level readings should be made when observing on the mark, and the correction in seconds to be applied to the circle reading on the mark is obtained by multiplying the inclination expressed in divisions of the level vial by $\frac{d}{4}$ tan H. The angle of elevation, H, is positive; the angle of depression, H, is negative. This factor is constant for all the observations.

The corrected circle reading on Polaris is now subtracted from the corrected circle reading on the mark, the result being the direction of the mark measured from Polaris. To this direction, add algebraically the corrected azimuth of Polaris from the north. The result is the azimuth of the mark from the north. To reduce the azimuth from the

north to an azimuth from the south, add 180° if the former is less than 180° and subtract 180° if it is greater than 180°.

On the summary sheet the mean of the 32 results (all observations being considered of equal weight) is 232°34′55″.75.

There were no rejections, as the largest residual was −4″.9, which is less than the rejection limit of 5″ from the mean.

The probable error of the result is ±0″.29, so that the azimuth conforms to the criterion of first order accuracy.

The mean observed azimuth has now to be corrected for diurnal aberration, eccentricity of instrument or light, if any, elevation of mark and variation of pole.

The correction for diurnal aberration is explained on page 92. The correction for eccentricity when it occurs is computed on Form 382 as in triangulation. The correction for variation of pole and the reasons for not applying it at present are discussed on page 136.

The correction for elevation of mark. When the mark is at a considerable elevation, it is necessary to apply a correction to obtain the projection of the direction of the mark on the surface of the spheroid of reference. This correction in seconds of arc is:

$$+\frac{e^2h}{2a \sin 1''} \cos^2 \phi \sin 2A \tag{12}$$

where e is the eccentricity and a the semi-major axis of the spheroid of reference; ϕ is the latitude of the station; h is the elevation of the mark and A the mean observed azimuth of the mark.

Summary of Azimuth Computation

Date: Dec. 3, 4, 1943
Observer: R. L. Tucker
Inst.: Theod. No. 368

Doyle to Barberton, Ohio

Position	*Azimuth* 232°34′	*v*	*v*²	*Position*	*Azimuth* 232°34′	*v*	*v*²
Dec. 3, 1	56″.9	−1″.1	1.21	Dec. 4, 1	50″.9	+4″.9	24.01
2	52.8	+3.0	9.00	2	57.7	−1.9	3.61
3	56.4	−0.6	0.36	3	53.0	+2.8	7.84
4	54.7	+1.1	1.21	4	51.3	+4.5	20.25
5	59.5	−3.7	13.69	5	53.7	+2.1	4.41
6	56.0	−0.2	0.04	6	58.1	−2.3	5.29
7	55.5	+0.3	0.09	7	54.4	+1.4	1.96
8	54.0	+1.8	3.24	8	55.1	+0.7	0.49
9	58.0	−2.2	4.84	9	54.1	+1.7	2.89
10	55.9	−0.1	0.01	10	60.5	−4.7	22.09
11	58.7	−2.9	8.41	11	56.8	−1.0	1.00
12	55.1	+0.7	0.49	12	54.2	+1.6	2.56
13	57.4	−1.6	2.56	13	51.2	+4.6	21.16
14	58.3	−2.5	6.25	14	54.8	+1.0	1.00
15	57.9	−2.1	4.41	15	55.9	−0.1	0.01
16	58.7	−2.9	8.41	16	56.4	−0.6	0.36
							Σv^2=183.15

Mean observed azimuth	232°34′55″.75	±0″.29
Corrections for:		
Diurnal aberration	+0.32	
Elevation of mark (300 m.)	+0.02	
Eccentricities	0.00	
Variation of pole: [10]		
Final astronomical azimuth:	232°34′56″.09	±0″.29

32 observations
No rejections
Probable error of a single observation = ±1″.64.

For h in meters and the dimensions in meters of the Clarke spheroid of 1866, the formula (12) becomes

$$+0.000109\ h \cos^2 \phi \sin 2A \tag{13}$$

$$\text{or } +[6.0392-10]h \cos^2 \phi \sin 2A,$$

where the expression in brackets is a logarithm.

The sign of formula (13) shows that the correction will be positive when the azimuth of the mark is in the first or third quadrants and negative when the azimuth is in the second or fourth quadrants. In other words, the sign will be positive when the mark is northeast or southwest of the station and negative when the mark is northwest or southeast of the station. The correction may be obtained by use of the formula or by means of the nomogram, Table XVIII, facing page 196.

The corrections to the mean observed azimuth at *Doyle* are indicated on the summary sheet, page 106. The final astronomical azimuth is 232°34′56″.09±0″.29.

Note: For the International Spheroid of reference the formula with numerical coefficient is $+0''.0001087\ h \cos^2 \phi \sin 2A$ or when the coefficient is expressed as a logarithm $[6.03623-10]\ h \cos^2 \phi \sin 2A$. The elevation h is expressed in meters.

The nomogram, Table XVIII, is based on the formula for the Clarke spheroid of 1866. By means of the nomogram the correction may be found quickly and with sufficient accuracy. For futher explanation, see page 133.

MICROMETER METHOD

Azimuth Determinations in High Latitudes With the Bamberg Broken-Telescope Transit

GENERAL STATEMENT

Precise azimuth determinations become increasingly more difficult with increasing latitude, especially beyond latitude 50°. In Alaska, for instance, there are only limited areas (in Southeast Alaska and along the Aleutian Islands) where the standard method of Polaris observations with a theodolite can be used at all satisfactorily.

The method which has been used in the past until recently for azimuth determinations in Alaska is preferable to the standard theodolite method, but is not entirely satisfactory. It involves the use of an astronomical transit, as does the method about to be described, but does not lend itself to convenient repetition of observations or to high accuracy. In this older method a pointing is made on a mark set north or south of the station, in the collimation axis of the instrument, after each longitude deter-

[10] No correction has been made for variation of pole. See p. 136.

mination at a station. The azimuth of this pointing is then derived by taking into account the azimuth and collimation errors of the instrument as obtained from the longitude computations. One azimuth observation is thus obtained for each longitude determination or, under normal conditions, from three to five observations at each station.

In the method here described, the azimuth observations are entirely independent of the longitude observations. They can be made quite rapidly and with any desired number of repetitions. About 40 repetitions can be made in about three hours. The method consists essentially of measuring with the eyepiece micrometer used in time determinations, the small angle, never greater than about 25′, between the horizontal direction of Polaris and that of a mark set nearly in the same azimuth (or 180° different). It is desirable to make the observations when Polaris is near elongation in order that its movement in a horizontal direction will be slow and that many observations may be made before Polaris moves beyond the safe range of the instrument, that is, moves too close to the edge of the field of view.

With the exception that the angle measured is necessarily small, the method is almost the same as the standard method using a theodolite. For high latitudes, however, the astronomical transit, particularly the "broken-telescope" type, has several advantages over the theodolite. It has greater stability due to its size and weight. It has a longer horizontal axis and a longer and more accurate level, a very important consideration in bringing the direction of Polaris down to the horizon from an altitude of 60° or more. The eyepiece is conveniently located for the observer and is always in the same location whether the instrument is pointed on Polaris or on the mark.

One technical detail should be mentioned as a caution to anyone wishing to use the method. The micrometer reading on Polaris must be multiplied by the secant of the altitude to reduce it to the corresponding angle on the horizon. Only the collimation axis projected moves along a great circle of the celestial sphere when the telescope is turned on its horizontal axis. The pointing wire, if away from the collimation axis, will move on a small circle parallel to the great circle described by the collimation axis and a correction must therefore be applied. Also, if the mark is at an elevation different from that of the instrument a similar correction may have to be applied to its micrometer reading. In practice the reduction is made to the difference of the micrometer readings on Polaris or on the mark, as the case may be, rather than to each micrometer reading separately.

The method has one requirement which is sometimes difficult to meet. A mark must be set about 2° away from the meridian and at a distance of two to five miles from the station, if possible. This mark may be either north or south of the station occupied but must have nearly the same azimuth as Polaris at elongation (or 180° different). It should be placed, if possible, where it can be made a station of the triangulation net or, at least, where it can be accurately connected to the net, by a closed triangle, if possible. Except in flat, wooded terrain, this requirement can be met without too much difficulty by a little care in the selection of the Laplace station.

Two different orientations of the instrument are required at each station, one in the meridian for the longitude and latitude observations and the other about 2° out of the meridian for the azimuth observations. In orienting the transit for the azimuth observations it is desirable to have it point from 5′ to 10′ to the westward or eastward of Polaris at elongation, as the case may be, in order that the observations may extend over a fairly long period and have the mean position of Polaris near the collimation axis. The mark is placed near this desired azimuth of the collimation axis.

SETTING THE AZIMUTH MARK

If the triangulation party precedes the astronomical party, the triangulation party should preferably establish the mark, since it has better facilities for tying the mark into the triangulation with the accuracy required. However, to establish the mark properly it is necessary for the triangulation observer to know in advance the time of year at which the astronomical observations are to be made in order to be able to select a site appropriate for the particular elongation of Polaris that will occur at night.

With this knowledge, Table VII of the American Ephemeris and Nautical Almanac will furnish the observer with the desired information concerning the particular elongation of Polaris that can be observed. With declination obtained from Table VII and knowing the latitude from the triangulation, the observer may then find in Table V the azimuth of Polaris at elongation for that latitude.

In order to have the observations extend over a fairly long period, the azimuth of the proposed mark (or its supplement if the mark is to the south of the station) should be from 5′ to 8′ less than the azimuth of Polaris at elongation. By means of this azimuth of the proposed mark and the geodetic azimuth of some convenient triangulation station in the scheme, the angle between that station and the proposed mark can be determined. Then when this angle is turned off from the above triangulation station by means of an engineer's transit or a small theodolite, the direction of the proposed mark from the observing station is found.

The mark may now be set in the direction just found, either to the northward or to the southward of the observing station. The mark may be set in the daytime by signalling with flags for proper alignment.

If the astronomical party has to set the mark and if it has received the triangulation data from the triangulation party, the manner of selecting the site of the mark is the same as just described above. If there are no triangulation data available, the astronomical party will have to rely almost entirely upon the results of the astronomical observations. The latitude could be obtained accurately enough by scaling from a good map or by observations on two or three pairs of stars. Using Tables VII and V as before and turning the proposed azimuth of the mark from north in the direction of the elongation of Polaris, the alignment of the mark with the observing station is found and the mark can be set up as described above.

It is assumed in the above that the Bamberg broken-telescope transit has been plumbed over the observing station. (It is very important that the transit be accurately centered over the station mark.) Since the Bamberg transit is not designed as an azimuth instrument, it has no special means for plumbing it over the observing station mark.

The plumbing may be done with a small engineer's transit from two points about 100 feet away from the instrument, the lines from which make an approximate right angle at the station. The telescope of the transit is pointed to the zenith during this operation and a small rod at the base of the cubical part of the telescope tube is taken to be identical with the collimation axis. The error in this assumption is undoubtedly very small.

DETAILED INSTRUCTIONS FOR OBSERVING

Local sidereal time should be determined preferably by the reception of radio time signals, but may be determined also by some method of star observations.

In what follows, by collimation axis is meant the line of collimation passing through the object pointed on and extending in opposite directions from the station.

In these instructions for the sake of simplicity it will be assumed that the azimuth mark is northward of the observing station.

First read and record the temperature and barometric pressure. Then measure the angle of elevation or depression of the mark at the station to the nearest minute of arc. These three details need be recorded only twice during a night's observations, namely, near the beginning and the end of the work, but must not be omitted.

Plunge telescope to the mark. Record whether mark is north or south of station. Record position of instrument, whether the eyepiece is east or west. Set micrometer wire on mark by turning the hand wheels of the longitude micrometer; read and record, both the index which shows the number of turns and the micrometer head which records the fraction of a turn. Without changing the instrument in azimuth, raise the telescope until it is in line with Polaris. Set micrometer wire on Polaris. At the instant the pointing is perfected and the wire bisects Polaris exactly, call "Tip". The recorder should then note and record the chronometer time to the nearest half-second. Read and record the hanging level.

Next reverse the telescope in the wyes, and again point on Polaris. Read and record micrometer and level readings and record the position of the instrument. Point on the mark and repeat the observations as before. This will complete one observation of azimuth.

Ordinarily 32 such observations should be made for the determination of a Laplace azimuth. A minimum of 25 observations with a probable error of 0".30 will be acceptable.

The observations may be made on a single night if terrain and weather conditions are such as to indicate that horizontal refraction effects are apt to be negligible. Otherwise, they should be made on two different nights with not less than 8 observations on either night.

The angle at the azimuth station between the mark and a triangulation station, preferably one of the main scheme, should be measured with a theodolite with first-order accuracy. Great care should be used to guard against any possibility of blunders in this measurement.

If it can be done without too much difficulty, the mark used for the azimuth observations (or a station a few feet away) should be observed with first-order accuracy from one or more additional triangulation stations, besides the azimuth station itself. This will make it possible to use the Laplace azimuth directly in the adjustment of the triangulation and thus avoid the loss of accuracy likely to be involved in turning through an angle between the mark and a triangulation station. The triangulation party should, if possible, arrange for this connection.

Earlier in these instructions it was stated that the value of one turn of the micrometer must be known or determined. It is desirable to have it determined immediately before or after the azimuth observations at each station. It can be done very simply by connecting the instrument to the chronograph and following equatorial stars with the micrometer. For this determination, of course, the instrument should not be reversed in the middle of the observations, as for time work, but instead the star should be followed continuously across the central part of the field, including about 8 or 10 turns of the micrometer. Observations on at least 4 different fast-moving stars should be sufficient. The stars must be identified, of course, for use in the computations. See page 20 ff.

If there is much change in temperature during the azimuth observations, it will be desirable to determine the micrometer value at two different temperatures in order that the effect of temperature on the micrometer value may be eliminated. This

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY

Horizontal Directions

Station: Redoubt Astro, Alaska Observer: H. J. Seaborg Instrument: Bamberg Transit #21 Date: Aug. 31, 1944

Position	Objects Observed	Time h. m.	Ocular E W	Mic. reading turns	Mic. Diff	Levels W	Levels E	Level Diff.	Remarks
1	Muth	P.M. 11:10	E	5.274					Radio-Chron comparison
			W	6.005	-0.731				NPG 9255 kc.
									135 Meridian = 11 P.M.
	Polaris $20^h\ 45^m\ 21^s.0$		E	5.468		49.0	22.1		Chron. #3479 = $20^h\ 30^m\ 07^s.5$
	20 47 14.5		W	5.856	-0.388	22.5	49.5		
	mean = 20 46 17.8					26.5	27.4	-0.9	Recorder: J. D. Harrow
	Diff. = 1 53.5								
									Sky overcast
2	Muth		W	6.004					Wind moderate, southerly
			E	5.318	-0.686				
									Double crosshairs
	Polaris 20 49 28.0		W	5.921		21.9	48.9		
	20 50 27.0		E	5.340	-0.581	49.9	22.9		Z.D. Polaris = 29° 18'
	mean = 20 49 57.5					28.0	26.0	+2.0	330 42
	Diff. = 0 59.0								
									The mark, Muth, is
3	Muth		E	5.295					to the southward of the
			W	6.011	-0.716				station.
	Polaris 20 54 22.5		E	5.254		49.2	22.2		
	20 55 43.0		W	6.080	-0.826	22.7	49.8		
	mean = 20 55 02.8					26.5	27.6	-1.1	
	Diff. = 1 20.5								

Do not write in this margin

Do not write in this margin

FIGURE 44.—Horizontal directions (Azimuth observations, micrometer method).

need not be done at every station. The focusing of the transit must not be changed between the determination of the micrometer value and the azimuth observations, since this will affect the micrometer value.

In the azimuth observations considerable care must be used to guard against variable personal equation between the pointings on the star and on the mark. Since Polaris is at a high elevation, the movable wire during the pointing on it will appear to be inclined to the vertical at an angle equal to the latitude. For the pointing on the mark the wire will appear nearly vertical. Personal equation is apt to differ considerably for these two conditions. The observer should try to keep it as constant as possible by turning his head sidewise for the pointing on the star, if he can do so without assuming a position so awkward as to introduce other errors of pointing.

The Bamberg transits are equipped with two sets of pointing wires, one consisting of a single wire and the other of two closely spaced parallel wires. The observer may use either the single or double wires, as he prefers. However, he should not use one for the star and the other for the mark, as the distance between the two sets of wires is not well enough determined.

Since the small angle between the collimation axis and Polaris, measured by the pointings on Polaris in the two positions of the instrument, is in a line that is at an angle with the plane of the horizon, the measured angle has to be multiplied by sec h, in order to bring it into the plane of the horizon. Similarly, if the mark is elevated or depressed 1° or more with respect to the horizontal plane, the measured angle between the mark and collimation axis, resulting from the pointings on the mark in both positions of the instrument, must be divided by the cosine of the angle of elevation or depression, so that the measurement will be brought into the plane of the horizon.

The difference between the above measurements in the horizontal plane combined with the azimuth of Polaris from the north, computed in the usual way on Form 380A, will give the azimuth of the mark.

Let M_E and M_W be the readings of the longitude micrometer with ocular East and West, respectively, when pointing on the mark. Now, since the mark is stationary, the mean of the above readings on the mark is the micrometer reading of the collimation axis. That is, if S represents the micrometer reading on the collimation axis, then

$$S=\frac{M_E+M_W}{2}. \tag{14}$$

Then M_E-S represents the angle between the collimation axis and the mark in turns of the micrometer for ocular East and $S-M_W$ for ocular West. Then the mean angle in turns of the micrometer between the mark and collimation axis is

$$\frac{(M_E-S)+(S-M_W)}{2}=\frac{M_E-M_W}{2}. \tag{15}$$

If this difference is positive, then the mark is east of the collimation axis and if negative, the mark is west.

The correction for the inclination of the axis is

$$\frac{d}{4}\tan H[(w+e)-(w'+e')]=\frac{d}{4}\tan H[(w-w')-(e'-e)] \tag{16}$$

where d is the value of one division of the level in seconds, H is the altitude of the mark, w and e are the west and east readings of the bubble of the hanging level in the

position in which the numbering increases from east to west, and the primed letters refer to the corresponding readings when the position is reversed.

Hence the total angle in the horizontal plane between the collimation axis and the mark is

$$\frac{R}{2}(M_E-M_W)\sec H+\frac{d}{4}\tan H[(w-w')-(e'-e)]. \tag{17}$$

If the mark is to the northward of the station, this angle, when positive, will represent the direction of the mark measured in a clockwise direction from the north with respect to the collimation axis. If negative, the explement of this angle will represent the direction of the mark.

If the mark, however, is to the southward of the station, the above angle will represent the direction of the mark from the south with respect to the collimation axis. Therefore the direction of the mark in a clockwise direction from the north with respect to the collimation axis will be the supplement of the angle expressed by (17). The direction will then be expressed by

$$180°-\left\{\frac{R}{2}(M_E-M_W)\sec H+\frac{d}{4}\tan H[(w-w')-(e'-e)]\right\}. \tag{18}$$

Formula (18) may be rewritten

$$180°+\left\{\frac{R}{2}(M_W-M_E)\sec H+\frac{d}{4}\tan H[(w'+e')-(w+e)]\right\}. \tag{19}$$

The expression in brackets is the direction from the north with respect to the collimation axis, of the line of sight on the mark extended backward through the station. Hence to obtain the direction of the mark from the north with respect to the collimation axis 180° must be added, just as is indicated in formula (19).

If M represents the difference of the micrometer readings in (17) and (19); and $O_E=w+e$, $O_W=w'+e'$, while i represents the level difference, then it follows that:

$$\text{When the mark is } \begin{Bmatrix}\text{North}\\ \text{South}\end{Bmatrix},\ M=\begin{Bmatrix}M_E-M_W\\ M_W-M_E\end{Bmatrix},\ \text{and } i \text{ equals } \begin{Bmatrix}O_E-O_W\\ O_W-O_E\end{Bmatrix}.$$

The above procedure is followed on Form 380 B. See the specimen computation on this form, page 116.

If P_E and P_W are the micrometer readings on Polaris with ocular East and ocular West, respectively, h is the altitude of Polaris, and w, w', e and e' have the same meaning as before, then

$$\frac{R}{2}(P_E-P_W)\sec h+\frac{d}{4}\tan h\ [(w-w')-(e'-e)] \tag{20}$$

if positive, or, if negative, its explement will represent the direction of Polaris measured in a clockwise direction from the north with respect to the collimation axis.

A sample azimuth computation is shown on pages 116 and 117.

COMPUTATION OF AZIMUTH, MICROMETER METHOD, FIELD AND OFFICE

The computation of the directions of the mark and Polaris from the northward extension of the collimation axis are made on Form 380B as shown on page 116. The computation of the azimuth of Polaris from the north and of the azimuth of the mark from the north and south are made on Form 380A, as shown on page 117.

As stated previously on page 108, the directions measured by the longitude micrometer must be reduced to directions in the plane of the horizon; in the case of Polaris, by multiplying by the secant of its apparent altitude, h; and in the case of the mark, by multiplying by the secant of the angle of elevation or depression of the mark with respect to the station, this angle being designated H. Since H remains constant and is comparatively small, it need be determined only twice, the second determination for the purpose of checking the first.

On the other hand, since the apparent altitude of Polaris is constantly varying and since the effect of multiplying by its secant will have a very appreciable effect, it is necessary to obtain it for each observation to within a very few seconds of arc.

DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
Form 605
(Rev. June 1946)

COMPARISON OF CHRONOMETER AND RADIO SIGNALS

Station: Redoubt Astro Latitude: 60° 29′ 02.″57 Longitude: 152° 17′ 53.″64

Chief of party: H.J. Seaborg Observer: H.J. Seaborg

Year: 1944 Chronometer No.: 3479 (~~mean~~, sidereal) (Strike out one)

Local date	Aug. 31	Sept. 1		
Standard time of signal, 135 Mer.	h. m. s. 23 00 00	h. m. s. 1 00 00	h. m. s.	h. m. s.
Chronometer time of signal	20 32 07.5	22 32 27.5		
Transmitting station	NPG	NPG		
Frequency of signal	9255 kc.	9255 kc.		
G. C. T. of signal — Date	Sept. 1	Sept. 1		
G. C. T. of signal — Time	h. m. s. 8 00 00.0	h. m. s. 10 00 00.0	h. m. s.	h. m. s.
Sidereal time of 0h G. C. T.	22 40 07.4	22 40 07.4		
Cor., mean solar to sidereal time	1 18.9	1 38.6		
Δ Transmission time				
Correction to signal				
G. S. T. of signal	30 41 26.3	32 41 46.0		
Longitude of station	10 09 11.6	10 09 11.6		
Local sidereal time	20 32 14.7	22 32 34.4		
Chronometer time of signal	20 32 07.5	22 32 27.5		
Chronometer correction	+ 7.2	+ 6.9		
Rate per minute		- 0.0025		
Rate per hour		- 0.15		

Remarks:

FIGURE 45.—Comparison of chronometer and radio signals.

The observation of the apparent altitude, h, of Polaris is not required, in the field, since the true altitude, h', can always be computed directly from the declination, hour angle and the azimuth, and when refraction is added, the result is the apparent altitude. Although the argument in the refraction tables is the apparent altitude, the difference between the true and apparent altitude is so small, except when the star is close to the horizon, that sufficient accuracy is obtained by using the true altitude as argument.

The method of obtaining the true altitude of Polaris by means of Tables I and IA of the American Ephemeris with the hour angle as argument is not considered quite accurate enough, principally because the quantities in these tables are given to whole seconds only and because in addition, double interpolation is necessary.

The Office, therefore, has adopted the method which makes use of the following formula:

$$\cos h' = \frac{\cos \delta \sin t}{-\sin A} = \frac{\cos \delta \sin t}{-\tan A \cos A}. \tag{21}$$

Formula (21) for the true altitude of Polaris will give all the accuracy required and is simple to compute. The following methods of computing formula (21) in connection with the computations on Form 380A have been found expeditious and to give the necessary accuracy.

First, formula (21) should be expressed in logarithmic form, as follows:

$$\log \cos h' = \log \cos \delta + \log \sin t - \log (-\tan A) - \log \cos A. \tag{22}$$

It will be noted that log sin t and log (—tan A) are parts of the computation on Form 380A; that log cos δ may be taken from the table at the same time as log cot δ; and that log cos A may be taken from the table at the same time as the antilogarithm of log (—tan A).

With a little practice it will be found simple to perform the subtraction, log sin t —log (—tan A), mentally and write down the result directly on the line reserved for the purpose on Form 380A.

Formula (22) gives the true altitude, h', but, since the telescope is pointing at the apparent altitude, h, it is necessary to convert from the true to the apparent altitude by adding to the true altitude the refraction at the observed temperature and barometric pressure. The computation of h' and h is shown on Form 380A page 116.

The refraction is computed by means of Tables V, VI and VII, pages 174 and 175. A sample computation of refraction follows, with the first observation at *Redoubt Astro*, being chosen for the purpose.

The barometer reading is 29.26 inches; the temperature is 46° F and the true altitude is 60° 43′ 48″.5.

For the barometer reading 29.26 inches, Table VI, page 174, gives

$$C_B = 0.978. \tag{23}$$

For the temperature 46° F, Table VII, page 175, gives

$$C_T = 1.008. \tag{24}$$

For the true altitude, 60° 43′.8, Table V, page 174, gives

$$r_m = 0'32''.6. \tag{25}$$

The refraction is then

$$\begin{aligned}(C_B)(C_T)(r_m) &= (0.978)(1.008)(32''.6)\\ &= (0.986)(32''.6) = 32''.1.\end{aligned} \tag{26}$$

Hence the apparent altitude is

$$h = 60°43'48''.5 + 32''.1 = 60°44'21''. \tag{27}$$

DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
Form 380 B

COMPUTATION OF AZIMUTH, MICROMETER METHOD
(Directions to Polaris and Mark)

Station: Redoubt Astro State: Alaska Chief of party: H.J. Seaborg

Mark: Muth Instrument: Bamberg No. 21 Observer: H.J. Seaborg

Chronometer No.: 3479 Level value, d: 1."366

Date: August 31, 1944 Value of half turn of micrometer, ½ R: 79."0515

Position	1	2	3	4
POLARIS				
Micrometer dif., M	-0.388	-0.581	-0.826	-0.961
Level difference, i	-0.9	+2.0	-1.1	+2.6
h	60° 44′ 21″	60° 45′ 17″	60° 46′ 35″	60° 47′ 09″
tan h	1.7848	1.7859	1.7876	1.7883
$\frac{d}{4}$ tan h	0.610	0.610	0.610	0.611
cos h	0.488786	0.488549	0.488219	0.488075
½ RM	-30."6720	-45."9289	-65."2965	-75."9685
½ RM ÷ cos h	-62."75	-94."01	-133."74	-155."65
$i \frac{d}{4}$ tan h	- 0.55	+ 1.22	- 0.67	+ 1.59
Direction in seconds	-63.30	-92.79	-134.41	-154.06
Direction	359° 58′ 56."70	359° 58′ 27."21	359° 57′ 45."59	359° 57′ 25."94
MARK (~~north~~—south) (strike out one)				
Micrometer dif., M*	-0.731	-0.686	-0.716	-0.724
Level difference, i*				
H	0° 21′.5			
tan H	0.0063			
$\frac{d}{4}$ tan H	.0022			
cos H	0.999980			
½ RM	+57."79	+54."23	+56."60	+57."23
½ RM ÷ cos H	+57.79	+54.23	+56.60	+57.23
$i \frac{d}{4}$ tan H				
Direction in seconds	+57.79	+54.23	+56.60	+57.23
Direction†	180° 00′ 57."79	180° 00′ 54."23	180° 00′ 56."60	180° 00′ 57."23

REMARKS: Sky overcast. Wind moderate, southerly Temp. 46°F.
Barom # 240 = 29."26

Recorder: J.D. Harrow

Double crosshair.

* When Mark is {north / south}, M equals {$M_S - M_W$ / $M_W - M_S$} and i equals {$O_S - O_W$ / $O_W - O_S$}

† When Mark is south, add 180°.

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FIGURE 46.—Computation of azimuth, micrometer method. (Directions to Polaris and mark)

DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
Form 380A

COMPUTATION OF AZIMUTH, MICROMETER METHOD

STATION: Redoubt Astro. STATE: Alaska VOL. AND PAGE OF ECCENTRIC — INSTR: LIGHT: —
MARK: Muth INSTR. NO.: Bamberg No. 21
CHRONOMETER NO.: 3479 CHIEF OF PARTY: H.J. Seaborg OBSERVER: H.J. Seaborg
POSITION OF STATION, ϕ: 60° 29′ 02″.57 λ: 152° 17′ 53″.64 GREENWICH CIVIL DAY: Sept. 1.38

Date, 1944 Aug. 31, position,	1	2	3	4
Chronometer reading,	20h 46m 17s.8	20h 49m 57s.5	20h 55m 02s.8	20h 57m 21s.0
" correction,	+07.2	+07.2	+07.1	+07.1
Sidereal time,	20 46 25.0	20 50 04.7	20 55 09.9	20 57 28.1
α of Polaris,	1 46 12.6	1 46 12.6	1 46 12.6	1 46 12.6
t of Polaris (time),	19 00 12.4	19 03 52.1	19 08 57.3	19 11 15.5
t of Polaris (arc),	285° 03′ 06″.0	285° 58′ 01″.5	287° 14′ 19″.5	287° 48′ 52″.5
δ of Polaris,	88° 59′ 47″.21			
log cot δ,	8.24346			
log tan ϕ,	0.24708	8.49054	8.49054	8.49054
log cos t,	9.41446	9.43947	9.47181	9.48563
log a (to 5 places),	7.90500	7.93001	7.96235	7.97617
log cot δ,	8.243462			
log sec ϕ,	0.307448	8.550910	8.550910	8.550910
log sin t,	9.984839 n	9.982913 n	9.980039 n	9.978660 n
colog (1—a),	0.003504	0.003712	0.004000	0.004131
log (—tan A) (to 6 places),	8.539253 n	8.537535 n	8.534949 n	8.533701 n
A = Azimuth of Polaris, from north,	1° 58′ 56″.8	1° 58′ 28″.7	1° 57′ 46″.5	1° 57′ 26″.2
Difference in time between D. and R.,	1m 53s.5	0m 59s.0	1m 20s.5	0m 43s.0
Curvature correction,	-0″.1	0″.0	-0″.1	0″.0
log sin t—log (—tan A),	1.445586	1.445378	1.445090	1.444959
log cos δ,	8.243395	8.243395	8.243395	8.243395
colog cos A,	0.000260	0.000258	0.000255	0.000253
log cos h′,	9.689241	9.689031	9.688740	9.688607
True altitude, h′,	60° 43′ 49″	60° 44′ 44″	60° 46′ 02″	60° 46′ 37″
Refraction,	+ 32	+ 32	+ 32	+ 32
Apparent altitude, h,	60 44 21	60 45 16	60 46 34	60 47 09
Direction of Polaris,	359° 58′ 56″.70	359° 58′ 27″.21	359° 57′ 45″.59	359° 57′ 25″.94
Direction of Mark,	180 00 57.79	180 00 54.23	180 00 56.60	180 00 57.23
Difference, Mark—Polaris,	180 02 01.1	180 02 27.0	180 03 11.0	180 03 31.3
Corrected azimuth of Polaris, from north,	1 58 56.7	1 58 28.7	1 57 46.4	1 57 26.2
(Minus if west of north)	180 00 00.0	180 00 00.0	180 00 00.0	180 00 00.0
Azimuth of Muth (Clockwise from south)	2° 00′ 57″.8	2° 00′ 55″.7	2° 00′ 57″.4	2° 00′ 57″.5

To the mean result from the above computation must be applied corrections for diurnal aberration, elevation of mark, and eccentricity (if any) of station and mark.

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FIGURE 47.—Computation of azimuth, micrometer method.

After the apparent altitudes of Polaris for all the observations have been computed, the remainder of the computation for azimuth follows in the manner illustrated below for the first observation on August 31, 1944 at *Redoubt.*

The instrumental constants for this night's observation were

$$\tfrac{1}{2}R=79''.0515$$

$$\tfrac{1}{4}d=0''.3415 \tag{28}$$

where R is the value of one turn of the longitude micrometer screw and d is the value of one division of the level.

The longitude micrometer value is determined for each continuous set of azimuth observations, but the level value need not in general be determined more often than once a year.

For Polaris the data will now be:

$$h=60°\ 44'\ 21'' \qquad P_E-P_W=-0.388 \text{ of a micrometer turn}$$

$$\cos\ h=0.488786 \qquad i=(w-w')-(e'-e)=-0.9 \text{ of a level division.} \tag{29}$$

$$\tan\ h=1.7848$$

Then

$$\tfrac{1}{2}R(P_E-P_W)=-30''.6720$$

$$\frac{d}{4}\tan h=0''.610. \tag{30}$$

Hence it follows that

$$\tfrac{1}{2}R(P_E-P_W)\sec h=-62''.75$$

$$\frac{d}{4}\,i\tan h=-0''.55. \tag{31}$$

The direction of Polaris from the north with respect to the collimation axis, will therefore be

$$-62''.75-''.55=-63''.30$$

or

$$359°58'56''.70. \tag{32}$$

For the *mark* the data are:

$$M_E-M_W=-0.731 \text{ of a micrometer turn}$$

$$H=\text{small enough to be ignored.} \tag{33}$$

Then

$$\frac{1}{2}R(M_E-M_W)=-57''.79$$

or

$$\frac{1}{2}R(M_W-M_E)=+57''.79. \tag{34}$$

Since the mark is to the south of the station, the direction of the mark from the north with respect to the collimation axis is

$$180°+57''.79=180°00'57''.79 \text{ by (19).} \tag{35}$$

The direction from Polaris to the mark will then be (35)−(32). This and the remainder of the computation are tabulated on the next page.

Direction of mark	180°00′57″.79
Direction of Polaris	359 58 56.70
Direction from Polaris to Mark	180 02 01.09
Azimuth of Polaris East of North	1 58 56.7
Azimuth of Mark East of North	182 00 57.8
	−180
Azimuth of Mark from South	2°00′57″.8 (36)

Summary of Azimuth Computation—Micrometer Method
Redoubt Astro to Muth, Alaska

Date: Aug. 31, 1944
Observer: H. J. Seaborg
Instrument: Bamberg No. 21

Position	*Azimuth* 2° 00′	*v*	*v²*	*Position*	*Azimuth* 2° 00′	*v*	*v²*
1	57″.8	+1″.4	1.96	21	60″.8	−1″.6	2.56
2	55.7	+3.5	12.25	22	59.4	−0.2	0.04
3	57.4	+1.8	3.24	23	(49.1)	Reject	----
4	57.5	+1.7	2.89	24	61.0	−1.8	3.24
5	59.6	−0.4	0.16	25	59.9	−0.7	0.49
6	62.3	−3.1	9.61	26	60.5	−1.3	1.69
7	59.9	−0.7	0.49	27	58.8	+0.4	0.16
8	58.5	+0.7	0.49	28	59.4	−0.2	0.04
9	58.4	+0.8	0.64	29	59.9	−0.7	0.49
10	56.0	+3.2	10.24	30	57.5	+1.7	2.89
11	56.3	+2.9	8.41	31	58.0	+1.2	1.44
12	58.8	+0.4	0.16	32	58.4	+0.8	0.64
13	58.2	+1.0	1.00	33	62.2	−3.0	9.00
14	61.0	−1.8	3.24	34	57.5	+1.7	2.89
15	58.4	+0.8	0.64	35	58.6	+0.6	0.36
16	59.4	−0.2	0.04	36	59.3	−0.1	0.01
17	62.2	−3.0	9.00	37	58.5	+0.7	0.49
18	61.6	−2.4	5.76	38	61.3	−2.1	4.41
19	59.5	−0.3	0.09	39	59.3	−0.1	0.01
20	59.5	−0.3	0.09	40	61.1	−1.9	3.61
							$\Sigma v^2 = 104.86$

Mean observed azimuth	2°00′59″.22±0″.18
Corrections for:	
Diurnal aberration	+0.32
Elevation of mark (120 ft.)	0.00
Eccentricities	0.00
Variation of pole[11]	----
Final astronomical azimuth	2°00′59″.54±0″.18

39 observations
1 rejection
Probable error of a single observation = ±1″.12

[11] No correction has been made for variation of pole. See p. 136.

REMINDERS

(1) Micrometer value should be determined from observations in the field at each station as explained on page 20.

(2) Barometer and temperature readings should be made before and after each set of observations.

(3) The position of the mark whether to the north or to the south of station should be noted in the record.

(4) The date and the local time of the beginning of the observations should be recorded. The 0–24-hour system of recording time should be used so that there will be no confusion about date and time of day.

APPENDIX

LATEST PROCEDURE IN LONGITUDE DETERMINATION

The introduction of continuous time-signal service has made it possible to modify to advantage the procedure for comparing the chronometer with the time signal. Because this continuous time signal may not always be available and because of the delay involved in recasting completely the pages dealing with this subject, they have been left unchanged and the following explanation, applicable to continuous time signal, has been incorporated in this appendix. The procedure here given, when applicable, supersedes that explained on pages 36 to 59.

Since December, 1944, the U. S. Naval Observatory has furnished corrections for the radio time signals transmitted by *WWV*, the National Bureau of Standards station. This has made available throughout the continental United States and usually in Alaska a practically continuous time service of sufficient accuracy to be used by this Bureau for its longitude determinations. With this time service, it is possible to obtain radio-chronometer comparisons as frequently as desired, and this in turn makes it possible to determine chronometer rates directly from the radio-chronometer comparisons. The rates so determined are unquestionably more reliable than those obtained, as in the past, from the star sets themselves.

Radio-chronometer comparisons are now to be made immediately before and after each set of six stars. This results in comparisons at intervals of 45 to 60 minutes. It is sufficient to make all radio-chronometer comparisons at the normal speed for the chronograph of one centimeter per second of time. Considerable time will be saved by this method in that it will not be necessary to change the speed of the chronograph between star observations and radio time signal recordings or to make extra identifications of minute breaks either for the radio or for the chronometer.

It would be better yet to have even more frequent radio-chronometer comparisons. This could be accomplished by making a radio-chronometer comparison between every two consecutive stars which are far enough apart to allow a one- or two-minute radio-chronometer comparison. If an extra man is available, he could switch the amplifier to "Radio" between star observations and then switch back to "Transit" again for the star observation. This method would give excellent time control for all star observations.

A set of six stars bracketed within two radio-chronometer comparisons shall constitute a single longitude observation previously called a time set. Additional radio-chronometer comparisons within the set shall be obtained when possible. The composition of each set remains as stated in the General Instructions. (See p. 37.) A longitude determination shall consist of at least six longitude observations as defined above, of which at least two shall be observed on a night separate from the others.

This revision in observational procedure is accompanied by a natural revision in computational procedure. Other slight changes have been made as well.

Only about 20 radio breaks are scaled from the chronograph sheet for each comparison. By correcting for the difference between mean and sidereal time, the breaks are reduced to the whole second nearest the mean epoch of the breaks. For this,

Table XV, p. 193, is convenient. Previously when the Navy signals were being used, the reduction was made to the final break which was sometimes two or three minutes distant from the mean epoch of the breaks. The new procedure eliminates the effect of the chronometer rate over such a period.

The chronometer corrections are computed on Form 605 for each comparison, using an assumed longitude of the station. See page 124. Almost any longitude may be assumed. However, for convenience of working with small numbers, it is desirable to use the best longitude available, such as the geodetic longitude or the longitude scaled from a map. The transmission time, the Naval Observatory correction to the signal, and the change in nutation between midnight and the time of the signal should all be included in this computation. The chronometer rates are determined from the chronometer corrections for each of the intervals between successive radio-chronometer comparisons and with these data, the chronometer correction Δt is computed for each star. See page 125.

For field computation, two equations are to be formed from the six star observations of the type:

$$3\Delta\lambda - a[+A] + [\alpha - t - \Delta t] = 0 \tag{1}$$

$$3\Delta\lambda - a[-A] + [\alpha - t - \Delta t] = 0 \tag{2}$$

These equations are very similar to those discussed on pages 47 to 49 and they are solved in the same manner as illustrated there. In these equations, $\Delta\lambda$ represents the correction to the assumed longitude. The value of $\Delta\lambda$ for each set is then applied to the assumed longitude used on Form 605 p. 124, to obtain the individual observed longitudes of the station. In the field computation of Form 605, the Naval Observatory correction to the signal and the corrections for nutation and transmission time need not be applied.

For office computation, an observation equation of the type:

$$\Delta\lambda - Aa + (\alpha + \Delta\alpha - t - \Delta t) = v \tag{3}$$

is formed for each star, and a least-squares adjustment made for each set. The values, for each set, of $\Delta\lambda$, the correction to the assumed longitude, and of a, the azimuth of the collimation axis of the instrument, are then used to compute the residuals for each star

Form 256 STATION: FENN WEST BASE, IDAHO
$\phi = 45°57'41''$ $\sin\phi = .71887$ $\cos\phi = .69514$

CAT.	STAR	MAG.	α	δ	Z.D.	A	C	B	k (−)	l* (+)
	72 HERCULES	5.4	17 18 38	32 32	S 13 25	+.275	1.19	1.15	.02	.05
	77 HERCULIS	5.8	17 25 18	45 18	N 02 21	−.062	1.50	1.50	.02	.06
	β DRACONIS	3.0	17 29 12	52 20	N 06 23	−.182	1.64	1.63	.02	.07
	ι HERCULIS	3.8	17 37 56	46 02	N 00 05	−.002	1.44	1.44	.02	.06
	μ HERCULIS	3.5	17 44 20	27 45	S 18 13	+.353	1.13	1.07	.02	.05
	ξ HERCULIS	3.8	17 55 39	29 15	S 16 42	+.330	1.15	1.10	.02	.05
	* l = CORRECTION FOR MEAN WIDTH OF CONTACT STRIPS (= ˢ.041) ONLY									

FIGURE 48.—Star factors and corrections for diurnal aberration and width of contact strip.

Form 256

Station, FENN WEST BASE Observer, C.R. REED Recorder, W.G. RAMSAY

Date, 27 JULY, 1946 E. S. time of sig., Freq. of sig., WWV 10mc

RADIO TIME SIGNALS

Sending time of signal 22 h P.S.T.		Local chronometer time of signal 18 h		Correction to reduce to final signal MEAN EPOCH	Local chronometer time of final signal 18 h. 55 m. S MEAN EPOCH	Sending time of signal ---- h		Local chronometer time of signal ---- h		Correction to reduce to final signal	Local chronometer time of final signal ---- h. ---- m.
m.	s.	m.	s.	s.	s.	m.	s.	m.	s.	s.	s.
19	30	54	45.23	+.077	13.307						
	31		.21	+.074	.284						
	32		.21	+.071	.281						
	33		.21	+.068	.278						
	34		.21	+.066	.276						
	35		.22	+.063	.283						
	36		.23	+.060	.290						
	37		.23	+.057	.287						
	38		.24	+.055	.295						
	39		.24	+.052	.292						
	40	54	55.26	+.049	.309						
	49	55	04.27	+.025	.295						
	50		.28	+.022	.302						
	52		.28	+.016	.296						
	53		.27	+.014	.284						
	55		.27	+.008	.278						
	56		.28	+.005	.285						
	57		.29	+.003	.293						
19	58	55	13.29	+.000	.290						
20	00		.30	−.005	.295						
	01		.30	−.008	.292						
	02		.31	−.011	.299						
	03		.31	−.014	.296						
	04		.31	−.016	.294						
	05		.30	−.019	.281						
	07		.31	−.025	.285						
	08		.32	−.027	.293						
	09		.32	−.030	.290						
	10		.33	−.033	.297						
	11		.33	−.036	.294						
	12	55	27.34	−.038	.302						
	49	56	04.42	−.140	.280						
	50		.43	−.142	.288						
	51		.43	−.145	.285						
20	52	56	07.43	−.148	.282						
					MEAN .290^s						
MEAN EPOCH = 22^h 19^m 58^s.5 P.S.T.											
MEAN SCALED READINGS =					.291^s						
CORRECTION, MEAN TO SID. TIME (.5 SEC) =					−.001^s						
					.290^s						
ADOPTED MEAN EPOCH = 22^h 19^m 58^s P.S.T.											
CORRESPONDING CHRONOMETER TIME = 18^h 55^m 13^s.290											

FIGURE 49.—Computation of the chronometer time of a radio time signal.

in the set. As before, if the residual for a star exceeds 0^s.20, that star is to be rejected. A sample adjustment is given on page 127.

The algebraic mean of the six or more $\Delta\lambda$'s is applied to the assumed longitude, yielding the mean observed longitude of the station. The probable error of this mean is obtained by treating each $\Delta\lambda$ as a single observation for longitude. The same weight is assigned to each set regardless of its apparent internal probable error. If there are only a very few sets, the probable error computed in this way is quite uncertain but it is believed that this procedure is preferable to a system of weighting according to apparent internal probable error because it takes account of possible night error and even

of "set" error, a possible systematic difference between observations at different hours of the night.

The following example for station Fenn West Base in Idaho illustrates the method. Attention should be called to the seventh column (headed $\Delta\lambda$) in the adjustment of the set of six star observations, see page 127. These quantities are the corrections to the assumed longitude, one for each star observation. Then the residual, v, for any given star will be equal to the adjusted value of $\Delta\lambda$ for the given set, namely, $+0^s.004$ minus the value of $\Delta\lambda$ for that star. The residuals can also be obtained directly from the observation equation given at the top of this adjustment, without computing the individual values of $\Delta\lambda$ for each star.

The $\Delta\lambda$'s in the "Summary of Results" are the adjusted $\Delta\lambda$'s obtained from the adjustments of the six sets of six stars each. The first one, $+0^s.004$, is the value of the $\Delta\lambda$ obtained from the set illustrated in this example.

DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
Form 605
(Rev. June 1946)

COMPARISON OF CHRONOMETER AND RADIO SIGNALS

Station: FENN WEST BASE Latitude: 45° 57′41″ Longitude: 116°17′57″

Chief of party: F.R. GOSSETT Observer: F.R. GOSSETT, C.R. REED

Year: 1946 Chronometer No.: 177 (~~mean~~, sidereal) (Strike out one)

Local date		JULY 27	JULY 27	JULY 27	JULY 27
Standard time of signal, 120TH Mer.		h. m. s. 20 29 23	h. m. s. 21 25 07	h. m. s. 22 19 58	h. m. s. 23 40 27
Chronometer time of signal		17 04 19.935	18 00 13.203	18 55 13.290	20 15 55.610
Transmitting station		WWV	WWV	WWV	WWV
Frequency of signal		10 mc	10 mc	10 mc	10 mc
G. C. T. of signal	Date	JULY 28	JULY 28	JULY 28	JULY 28
	Time	h. m. s. 4 29 23	h. m. s. 5 25 07	h. m. s. 6 19 58	h. m. s. 7 40 27
Sidereal time of 0h G. C. T.		20 20 13.313	20 20 13.313	20 20 13.313	20 20 13.313
Cor., mean solar to sidereal time		44.253	53.408	01 02.419	01 15.640
Δ Transmission time		+ .011	+ .011	+ .011	+ .011
~~CORR. FOR CHANGE IN NUTATION~~		~~+ .001~~	~~+ .001~~	~~+ .002~~	~~+ .002~~
Correction to signal		− .013	− .013	− .013	− .013
G. S. T. of signal		24 50 20.565	25 46 13.720	26 41 13.732	28 01 55.953
Longitude of station (ASSUMED)		7 45 11.800	7 45 11.800	7 45 11.800	7 45 11.800
Local sidereal time		17 05 08.765	18 01 01.920	18 56 01.932	20 16 44.153
Chronometer time of signal		17 04 19.935	18 00 13.203	18 55 13.290	20 15 55.610
Chronometer correction		+48.830	+48.717	+48.642	+48.543
Rate per minute			−.002022	−.001364	−.001227
Rate per hour					

Remarks:

FIGURE 50.—Comparison of chronometer and radio signals.

Form 256 G.C.T. = JULY 28.2 $dV = +.21$ $d\delta = -.06$

Station, FENN WEST BASE, IDAHO Observer, F. R. GOSSETT

Date, JULY 27, 1946 Recorder, W. G. RAMSAY

Instrument, BAMBERG #21 , Chronometer, #177 Temp., 64°F

Level value, 1".353

Star:	72 HERCULIS		77 HERCULIS		β DRACONIS		ι HERCULIS	
Clamp.								
Level: W	E		W	E	W	E	W	E
50.0	23.0		23.0	50.0	50.1	23.0	21.3	48.8
24.2	51.1		50.2	23.1	22.1	49.2	51.2	23.8
+25.8	−28.1		+27.2	−26.9	+28.0	−26.2	+29.9	−25.0
	− 2.3			+ .3		+ 1.8		+ 4.9
	− .052			+ .007		+ .041		+ .110
B	1.15			1.50		1.63		1.44
17 h. 17 m.			17 h. 24 m.		17 h. 28 m.		17 h. 37 m.	
16m 18m			23m 25m		27m 29m		36m 37m	
s	s	Sums	s	s	Sums	s	s	Sums	s	s	Sums
56.3	47.5	103.8	27.6	34.6	62.2	15.0	35.4	50.4	17.7	61.3	79.0
57.5	46.2	103.7	29.1	33.1	62.2	16.6	34.1	50.7	19.1	59.3	78.4
59.0	45.0	104.0	30.5	31.6	62.1	18.3	32.4	50.7	21.1	58.0	79.1
61.6	42.4	104.0	32.4	29.9	62.3	19.8	30.5	50.3	22.3	56.8	79.1
62.7	41.1	103.8	33.9	28.4	62.3	21.5	28.8	50.3	23.5	55.2	78.7
63.9	39.9	103.8	35.5	26.6	62.1	23.0	27.0	50.0	25.1	53.8	78.9
65.0	38.8	103.8	37.0	25.0	62.0	25.3	25.4	50.7	28.2	50.8	79.0
66.1	37.6	103.7	38.9	23.3	62.2	27.0	23.5	50.5	29.7	49.0	78.7
67.4	36.3	103.7	40.4	21.9	62.3	28.8	21.7	50.5	31.2	47.6	78.8
68.7	35.1	103.8	41.9	20.5	62.4	30.6	19.7	50.3	32.7	46.0	78.7
	Mean	103.81		Mean	62.21		Mean	50.44		Mean	78.84
		51.90			31.10			25.22			39.42
Mean/2		51.90	Mean/2		31.10	Mean/2		25.22	Mean/2		09.42
l		+ .05			+ .06			+ .07			+ .06
k		− .02			− .02			− .02			− .02
Bb		− .06			+ .01			+ .07			+ .16
t	$17^h 17^m$	51.87	$17^h 24^m$		31.15	$17^h 28^m$		25.34	$17^h 37^m$		09.62
α	$17^h 18^m$	39.930	$17^h 25^m$		20.142	$17^h 29^m$		14.637	$17^h 37^m$		58.255
Δα (short-period term)		+ .009			+ .006			+ .005			+ .007
ΔC CHRONOMETER CORRECTION		+48.803			+48.789			+48.781			+48.764
α+Δα−(t+ΔC)		− .734			+ .209			+ .521			− .122

11—9906

FIGURE 51.—Computation of the chronometer correction for each star.

Form 256

Station, FENN WEST BASE, IDAHO — Observer, F. R. GOSSETT

Date, JULY 27, 1946 — Recorder, W. G. RAMSAY

Instrument, BAMBERG #21 — Chronometer, #177 — Temp., 64° F

Level value, 1".353

Star:	κ HERCULIS		ξ HERCULIS								
Clamp.											
Level: W	E		W	E		W	E		W	E	
49.3	21.8		21.9	49.8							
22.9	50.2		50.8	22.8							
+ 26.4	− 28.4		+ 28.9	− 27.0		+	−		+	−	
	− 2.0			+ 1.9							
	− .045			+ .043							
B	1.07			1.10							
17 h. 43 m.			17 h. 54 m.			h.	m.		h.	m.	
42m 44m			54m 55m								
s	s	Sums	s	s	Sums	s	s	Sums	s	s	Sums
46.3	23.0	69.3	10.2	37.4	47.6						
47.3	22.1	69.4	11.5	36.2	47.7						
48.6	20.8	69.4	12.5	35.0	47.5						
49.8	19.7	69.5	13.8	33.8	47.6						
50.8	18.2	69.0	14.9	32.6	47.5						
51.9	17.5	69.4	16.1	31.4	47.5						
53.1	16.1	69.2	17.2	30.3	47.5						
54.4	14.5	68.9	18.4	29.3	47.7						
55.7	13.3	69.0	21.0	26.6	47.6						
58.1	11.1	69.2	22.3	25.4	47.7						
Mean		69.23	Mean		47.59	Mean			Mean		
		34.62			23.80						
Mean/2		34.62	Mean/2		53.80	Mean/2			Mean/2		
l		+ .05			+ .05						
k		− .02			− .02						
Bb		− .05			+ .05						
t 17h43m		34.60	17h54m		53.88						
α 17h44m		22.391	17h55m		41.760						
Δα (short-period term)		+ .010			+ .010						
Δt chronometer correction		+ 48.750			+48.728						
α+Δα−(t+Δt)		− .949			− .838						

11—9206

LONGITUDE COMPUTATION

STATION: FENN WEST BASE IDAHO
CHIEF OF PARTY: F. R. GOSSETT
OBSERVER: F. R. GOSSETT & C. R. REED
DATE: JULY 27, 1946

OBSERVATION EQUATION: $\Delta\lambda - Aa + (\alpha + \Delta\alpha - t - \Delta t) = v$

STAR	A	$\alpha+\Delta\alpha-t-\Delta t$	AA	$A(\alpha+\Delta\alpha-t-\Delta t)$	Aa	$\Delta\lambda$	v
72 HERC	+.275	−.734	.0756	−.2018	−.730	+.004	.000
77 HERC	−.062	+.209	.0038	−.0130	+.165	−.044	+.048
β DRAC	−.182	+.521	.0331	−.0948	+.483	−.038	+.042
ι HERC	−.002	−.122	.0000	+.0002	+.005	+.127	−.123
μ HERC	+.353	−.949	.1246	−.3350	−.937	+.012	−.008
ξ HERC	+.330	−.838	.1089	−.2765	−.876	−.038	+.042
SUM	+.712	−1.913	.3460	−.9209			+.001

NORMAL EQUATIONS

$$\begin{cases} n\,\Delta\lambda - [A]\,a + [(\alpha+\Delta\alpha-t-\Delta t)] = 0 \\ -[A]\,\Delta\lambda + [AA]\,a - [A(\alpha+\Delta\alpha-t+\Delta t)] = 0 \end{cases}$$

SOLUTION OF NORMAL EQUATIONS

$\Delta\lambda$	a	η
6.0000	−.7120	−1.9130
$\Delta\lambda$ =	+.11867	+.31883
	+.3460	+.9209
	−.0845	−.2270
	+.2615	+.6939
	a =	−2.65354
	$\Delta\lambda$ =	+.00393

SUMMARY OF RESULTS

GREENWICH DATE 1946	OBSERVER	a	$\Delta\lambda$	v
JULY 28.2	F.R.G.	−2ˢ.654	+0ˢ.004	+0ˢ.034
28.2	F.R.G.	−2.351	+.072	−.034
28.3	C.R.R.	−2.193	+.029	+.009
28.3	C.R.R.	−2.301	+.085	−.047
30.3	F.R.G.	−2.146	−.012	+.050
30.4	F.R.G.	−2.266	+.053	−.015
		MEAN $\Delta\lambda$ =	+ˢ.038 ± ˢ.011	
			Σv^2 = .007327	

MEAN $\Delta\lambda$	+ˢ.038
CORRECTION FOR LAG IN AUDIO FILTER	+ˢ.008
ASSUMED λ	$7^h 45^m 11^s.800$ FROM FORM 606
MEAN OBSERVED LONGITUDE (TIME)	$7^h 45^m 11^s.846 \pm {}^s.011$
MEAN OBSERVED LONGITUDE (ARC)	116° 17′ 57″.69
REDUCTION TO GEODETIC STATION	−″.20
ASTRONOMIC LONGITUDE GEODETIC STATION	116° 17′ 57″.49

FIGURE 52.—Longitude computation.

REDUCTION OF MEAN TO APPARENT PLACE

Each year the American Ephemeris and Nautical Almanac publishes in Part II Stars, two sets of formulas for reducing the mean place of a star to the apparent place. One set uses the Besselian Star Numbers and Bessel's Star Constants and the other uses the Independent Star Numbers. There are two tables for the Besselian Star Numbers, one giving the numbers themselves, the other their logarithms. There is also a table of Independent Star Numbers for every Greenwich civil day.

FORMULAS

With Besselian Star Numbers and Bessel's Star Constants. If α and δ represent the apparent right ascension and declination, respectively, and α_0 and δ_0 the corresponding mean places, then the set using the Besselian Star Numbers and Bessel's Constants is:

$$\alpha=\alpha_o+\tau\mu+Aa+Bb+Cc+Dd+E \text{ (in time)} \quad (1)$$

$$\delta=\delta_o+\tau\mu'+Aa'+Bb'+Cc'+Dd' \text{ (in arc)}. \quad (2)$$

The Besselian Star Numbers are A, B, C, D and E and the Bessel's Star Constants are a, b, c, d, a', b', c', and d' which are quantities depending on trigonometric functions of α_0 and δ_0 and are defined in the Ephemeris. The table for Independent Star Numbers contains τ, the fractional part of a year elapsed from the beginning of the Besselian Fictitious year.[12]

The annual proper motions, μ and μ', in right ascension and declination respectively, are in the star catalogue used, namely, the Boss General Catalogue of 33342 Stars for the Epoch 1950.

With Independent Star Numbers. The set of formulas involving the Independent Star Numbers is

$$\alpha=\alpha_o+f+f'+\tau\mu+\frac{1}{15}g \sin (G+\alpha_o) \tan \delta_o+\frac{1}{15}h \sin (H+\alpha_o) \sec \delta_o \text{ (in time)} \quad (3)$$

$$\delta=\delta_o+\tau\mu'+g \cos (G+\alpha_o)+h \cos (H+\alpha_o) \sin \delta_o+i \cos \delta_o \text{ (in arc)}. \quad (4)$$

The quantities f, f', τ, log g, log h, G, H, and i are all found in the table for Independent Star Numbers.

In order to make use of the four formulas, it is necessary to know the fractional part of the Greenwich civil day [13] accurately, so that the values of the Star Numbers can be taken from the tables by interpolation.

An example using a modification of formula (2) for reduction of mean declination to apparent declination will be shown before illustrating the use of formulas (1) to (4). This modified formula is the one now used by this Bureau.

REDUCTION USING BESSELIAN STAR NUMBERS AND BESSEL'S STAR CONSTANTS

Mean to Apparent Declination

Modified Formula. It is now the standard practice in this Office to use the Besselian Star Numbers and Bessel's Constants in reducing the mean to the apparent

[12] The Besselian fictitious year is the solar year which begins at the instant the right ascension of the mean sun is 280° or $18^h\ 40^m$. The beginning of this year coincides very nearly with that of the Gregorian calendar year. The Besselian fictitious year is used in computations relating to fixed stars.

[13] Prior to 1925, the astronomical day was reckoned from noon of the civil day of the same name. One purpose of this practice was to avoid a change of date during a night's work. Beginning with the year 1925, however, the astronomical day has coincided exactly with the civil day. For instance, May 5.0, G. C. T. is also May 5.0, astronomical time. From 1925 to 1938 in the American Ephemeris astronomical time was also called Greenwich civil time. In 1939, however, the American Ephemeris, introduced the expression "Universal Time or Greenwich civil time."

Since in the system now used the date often changes during a set of observations, this change of date should be carefully noted in the records of observations.

declination. For the purpose of machine computation formula (2) above has been modified to become

$$\delta=\delta_o+\tau\mu'+i\cos\delta_o+X\sin\alpha_o+Y\cos\alpha_o \tag{5}$$

where

$$X=-(B+C\sin\delta_o)$$

and (6)

$$Y=D\sin\delta_o+A\left(\frac{a'}{\cos\alpha_o}\right).$$

Form 772 has been arranged for this computation.

α_o and δ_o are computed for the year of observation, as explained on page 74, and entered on Form 772, along with their natural sines and cosines, each in its designated column.

Four place tables will give sufficient accuracy for the trigonometric functions. Watch the signs of these functions.

The proper motion in declination, μ', is taken from the star catalogue (G. C.). (If the change in proper motion per hundred years, namely, 100 $\Delta\mu'$, is large enough it must be taken into account in taking the value of μ' from the catalogue.)

Usually when the period of the observations does not exceed four hours, the values of A, B, C, D, τ, and i can be obtained with sufficient accuracy by using the mean Greenwich civil time, commonly abbreviated, G. C. T. of the observations. If the mean right ascensions of the set of stars are spaced with approximate regularity, the mean of the first and last right ascensions will give a sufficiently accurate mean local sidereal time of the observations. Sometimes when there is a pronounced break in the regularity, the set should be divided and the mean epoch computed for each part.

To the mean local sidereal time add the longitude if west of Greenwich, subtract if east, thus obtaining the corresponding Greenwich sidereal time. From this Greenwich sidereal time or as abbreviated, G. S. T., subtract the Greenwich sidereal time of the nearest preceding 0^h, G. C. T. Divide this interval if expressed in hours by 24.066 or if in minutes by 1444, and the result will be the fractional part of a civil day from the 0^h, G. C. T. chosen.

In the example, page 75,

Mean of the first and last right ascensions	11^h	03^m	
West longitude	5	58	
Greenwich sidereal time	17	01	
Greenwich sidereal time of 0^h, April 22	13	56	
Sidereal interval after 0^h	3	05	$= 3^h.083$

$$3^h.083 \div 24^h.066 = 0^d.128$$

Hence mean G. C. T. of observations=April 22.128.

After quantities A, B, C, D, τ, and i have been found, it is a simple matter to compute X and Y for all the stars by machine. When these have been entered in the proper places on the form, all the necessary information for computing the apparent declination of a star is on the same horizontal line with the exception of τ and i which are at the bottom of the form.

By machine all the steps of the computation from mean to apparent declination may be carried through without setting down anything but the result. The signs of the products must be watched.

In the example page 75 for star No. 12593, we have

$$\delta = 29^\circ\ 54'\ 60''.09 + (0.3035)(-0.005) + (-6.97)(0.8668)$$
$$+ (12.346)(0.6943) + (6.436)(-0.7196)$$
$$= 29^\circ\ 54'\ 57''.99.$$

Regular Formula:

$$\delta = \delta_o + \tau\mu' + Aa' + Bb' + Cc' + Dd' \text{ (in arc).} \tag{2}$$

The Bessel's Star Constants in declination for 1935 are defined by

$$a' = 20''.0439 \cos \alpha_o \qquad c' = \tan \omega \cos \delta_o - \sin \alpha_o \sin \delta_o$$
$$b' = -\sin \alpha_o \qquad d' = \cos \alpha_o \sin \delta_o.$$

The formula for the mean obliquity of the ecliptic is given under Astronomical Constants, American Ephemeris, as

$$\omega = 23^\circ 27' 08''.26 - (0''.4684)(t - 1900).$$

Hence in 1935

$$\omega = 23^\circ 27' 08''.26 - 16''.39 = 23^\circ 26' 51''.87.$$

Note: The true obliquity is given in the American Ephemeris as part of the ephemeris of the sun. (See page 18, et seq. of the 1945 American Ephemeris.)

In the computation of the Bessel's Star Constants, either the true or the mean obliquity may be used.

$\log 20''.0439 =$	1.30198	$\log \tan \omega =$	$9.63722 - 10$
$\log \cos \alpha_o =$	$9.85712 - 10n$	$\log \cos \delta_o =$	$9.93789 - 10$
$\log a' =$	$1.15910n$	$\log (1)$	$9.57511 - 10$
$a' =$	$-14''.424$	$(1) =$	0.37593
$\log (-\sin \alpha_o) =$	$9.84158 - 10n = \log b'$	$\log \sin \alpha_o =$	$9.84158 - 10$
$b' =$	-0.69435	$\log \sin \delta_o =$	$9.69787 - 10$
		$\log (2) =$	$9.53945 - 10$
$\log \cos \alpha_o =$	$9.85712 - 10n$	$(2) =$	0.34630
$\log \sin \delta_o =$	$9.69787 - 10$	$(1) =$	0.37593
$\log d' =$	$9.55499 - 10n$	$c' = (1) - (2) =$	$+0.02963$
$d' =$	-0.35891		

Example: Computation af Apparent Declination

The quantities A, B, C, D, τ and μ' are taken from Form 772, page 75.

$\delta_o =$	$29^\circ\ 54'\ 60''.09$
$Aa' =$	-08.424
$Bb' =$	$+03.003$
$Cc' =$	-00.477
$Dd' =$	$+03.792$
$\tau\mu' =$	-00.002
$\delta =$	$29^\circ\ 54'\ 57''.982$

The same computation by means of logarithms. The logarithms of A, B, C, and D, are taken from the first part of the table for Besselian star numbers in the American Ephemeris.

$$\begin{array}{ll} \log A & =9.76637-10 \\ \log a' & =1.15910n \\ \hline \log Aa' & =0.92547n \end{array} \qquad \begin{array}{ll} \log B & =0.6360n \\ \log b' & =9.8416-10n \\ \hline \log Bb' & =0.4776 \end{array} \qquad \begin{array}{ll} \log C & =1.20642n \\ \log c' & =8.47173-10 \\ \hline \log Cc' & =9.67815-10n \end{array}$$

$$\begin{array}{ll} \log D & =1.02384n \\ \log d' & =9.55499-10n \\ \hline \log Dd' & =0.57883 \end{array}$$

$$\begin{array}{rr} \delta_o=29°54' & 60''.09 \\ Aa'= & -08.423 \\ Bb'= & +03.003 \\ Cc'= & -00.477 \\ Dd'= & +03.792 \\ \tau\mu'= & -00.002 \\ \hline \delta=29°54' & 57''.983 \end{array}$$

The value of δ obtained on Form 772 is 29°54′57″.987 before being rounded off to two decimals.

Mean to Apparent Right Ascension

Bessel's Star Constants are, as the name implies, practically constant for any given star for a period of at least a year. If the apparent place of a star is needed many times during the year, the formulas based on the Besselian Star Numbers and Bessel's Star Constants are, in general, more convenient than the formulas involving the Independent Star Numbers. The latter, on the other hand, are more convenient if the computations for the same star are to be made only once or twice.

To illustrate the computation of the Bessel's Star Constants, let us take Star No. 12593 for 1935 with

$$\alpha_o=9^h04^m06^s$$

$$\delta_o=29°55'00''.09.$$

The formula for apparent right ascension is:

$$\alpha=\alpha_o+\tau\mu+Aa+Bb+Cc+Dd+E \text{ (in time).} \tag{1}$$

The Bessel's Star Constants in right ascension for 1935 are defined by

$$a=3^s.07299+1^s.33626 \sin \alpha_o \tan \delta_o \qquad c=\frac{1}{15} \cos \alpha_o \sec \delta_o$$

$$b=\frac{1}{15} \cos \alpha_o \tan \delta_o \qquad d=\frac{1}{15} \sin \alpha_o \sec \delta_o$$

$\log 1.33626 = 0.1259$	$\log \frac{1}{15} = 8.8239 - 10$
$\log \sin \alpha_o = 9.8416 - 10$	$\log \cos \alpha_o = 9.8571 - 10n$
$\log \tan \delta_o = 9.7600 - 10$	$\log \tan \delta_o = 9.7600 - 10$
$9.7275 - 10$	$\log b = 8.4410 - 10n$
Number $= 0^{s}.5339$	$b = -0.02761$

$$a = 3^{s}.07299 + 0^{s}.5339 = 3^{s}.6069$$

$\log \frac{1}{15} = 8.8239 - 10$	$\log \frac{1}{15} = 8.8239 - 10$
$\log \cos \alpha_o = 9.8571 - 10n$	$\log \sin \alpha_o = 9.8416 - 10$
$\log \sec \delta_o = 0.0621$	$\log \sec \delta_o = 0.0621$
$\log c \quad 8.7431 - 10n$	$\log d \quad 8.7276 - 10$
$c = -0.05535$	$d = +0.05341$

Example: Computation of Apparent Right Ascension

For April 22.128, 1946:

$A = +\ 0.5840$		$\alpha_o = 9^h04^m$	$06^{s}.184$
$B = -\ 4.325$		$\tau\mu =$	$-\ 0.001$
$C = -16.084$		$aA =$	$+\ 2.106$
$D = -10.566$		$bB =$	$+\ 0.119$
$E = +\ 0^{s}.002$		$cC =$	$+\ 0.890$
$\tau = +\ 0.3034$		$dD =$	$-\ 0.564$
$\mu = -\ 0.0022$		$E =$	$+\ 0.002$
		$\alpha = 9^h04^m$	$08^{s}.736$

REDUCTION USING INDEPENDENT STAR NUMBERS

Example: Computation of Apparent Declination

$$\delta = \delta_o + \tau\mu' + g \cos (G + \alpha_o) + h \cos (H + \alpha_o) \sin \delta_o + i \cos \delta_o \tag{4}$$

$\log \cos (G + \alpha_o)$	$= 9.6379 - 10n$	$\delta_o = 29°54'$	$60''.09$
$\log g$	$= 1.0961$	$\tau\mu' =$	-0.002
$\log (1)$	$= 0.7340n$	$(1) =$	-5.420
$\log \cos (H + \alpha_o)$	$= 9.9892 - 10$	$(2) =$	$+9.363$
$\log h$	$= 1.2843$	$i \cos \delta_o =$	-6.046
$\log \sin \delta_o$	$= 9.6979 - 10$		
$\log (2)$	0.9714	$\delta = 29°54'$	$57''.985$

If the computation is to be made in the field where multiplying machines are not available, the computation with logarithms as shown above would be most convenient.

Summary of results of the different computations for apparent declination.

By Besselian star numbers and star constants

	δ
Form 772	29°54′57″.987
Regular formula (Numbers)	57.982
Regular formula (Logarithms)	57.983
By Independent Star Numbers	57.985

Example: Computation of Apparent Right Ascension

$$\alpha=\alpha_o+f+f'+\tau\mu+\frac{1}{15}\,g\sin(G+\alpha_o)\tan\delta_o+\frac{1}{15}\,h\sin(H+\alpha_o)\sec\delta_o \qquad (3)$$

$\alpha_o=$	$9^h\ 04^m.1$	colog 15	$=8.8239-10$	colog 15	$=8.8239-10$
$G=$	22 38.9	log g	$=1.0961$	log h	$=1.2843$
$H=$	15 46.8	log sin $(G+\alpha_o)$	$=9.9546-10$	log sin $(H+\alpha_o)$	$=9.3430-10$
$G+\alpha_o=$	7 43.0	log tan δ_o	$=9.7600-10$	log sec δ_o	$=0.0621$
$H+\alpha_o=$	0 50.9	log (1)	$9.6346-10$	log (2)	$9.5133-10$

$\alpha_o=$	$9^h04^m06^s.184$
$f=$	$+1.807$
$f'=$	-0.010
$\tau\mu=$	-0.001
(1)=	$+0.431$
(2)=	$+0.326$
$\alpha=$	$9^h04^m08^s.737$

CORRECTION TO AZIMUTH FOR THE ELEVATION OF THE MARK

Since the verticals of an ellipsoid of revolution do not meet at its center but intersect the axis at points whose position varies with the latitude, the verticals of the point of observation and of the mark will not in general lie in the same plane; the exceptions occur when both the point of observation and the mark lie on the same meridian or the same parallel of latitude. Since the two verticals are not in the same plane, a plane containing the vertical of the point of observation and the mark will have to be rotated slightly to pass through the point where the vertical of the mark intersects the ellipsoid.

This means a correction in seconds to the azimuth dependent on the elevation of the mark. To a first approximation the correction is:

$$+\frac{e^2h}{2a\sin 1''}\cos^2\phi\,\sin 2A \qquad (1)$$

where e is the eccentricity and a the semimajor axis of the spheroid of reference; ϕ is the latitude of the station and A the azimuth from station to mark; h is the elevation of the mark expressed in terms of the same unit as a.

For the Clarke Spheroid of 1866, and with h expressed in meters this expression becomes

$$+0''.000109\,h\cos^2\phi\,\sin 2A. \qquad (2)$$

To a first approximation it is thus independent of the distance of the mark from the point of observation.

CORRECTION FOR THE DISPLACEMENT OF THE POLE

The direction of the earth's axis of rotation is subject to changes of various sorts. The *direction of the axis* is conveniently specified by the coordinates of the pole of the axis; accordingly when in the following brief discussion the position of the pole is referred to, it should be understood that it is really direction of the axis of rotation that is involved.

The direction of the axis or position of the pole shifts *in space*. This shift in space causes changes in the position of the coordinate axes to which the stars are referred

and these changes in the coordinate axes cause changes in the coordinates, that is, in the right ascension and the declination of the stars. These changes in the coordinates are taken care of by the corrections for precession and nutation, which are elsewhere discussed. In connection with these corrections for precession and nutation it is convenient to apply corrections for proper motion and for aberration, although these two latter corrections are corrections for displacements, real or apparent, of the stars themselves, not of the coordinate axes to which the stars are referred.

In addition to the shifts of the axis or the pole *in space*, the axis or pole is subject to a small shift in the *body of the earth*. This shift leaves the coordinates of the stars practically unaffected but changes astronomical latitudes and longitudes of points on the earth by small amounts. It also affects astronomical azimuths.

These shifts of the pole are small. If the position of the terrestrial pole, north or south, could be laid down on the ground and followed as it changed, it would be found to wander only perhaps about thirty or forty feet from some mean position. The laws of this displacement of the pole are known only in the most general way. This motion can be represented by an annual term and by a term with a period of about 14 months but these terms are subject to unexplained changes of amplitude and phase.

The annual term represents the aggregate effect of all the various seasonal meteorological effects having a period of one year, annual changes in barometric pressure, load of ice, etc. The 14-month motion, the so-called Chandlerian motion, named from its discoverer, S. C. Chandler, represents the motion in the natural free period of a body having the size, shape, physical properties and angular velocity of rotation of the earth, when such a body is set rotating about an axis not quite coincident with its axis of figure, that is, with the axis about which the moment of inertia is a maximum.

The study of this shift of the pole within the body of the earth is the special task of the International Latitude Service, a cooperative international organization. Special observatories are maintained for this purpose. They have been in operation since the autumn of 1899. Since this shifting of the pole is so unpredictable, the only way of correcting for it is to make use of the published results of the International Latitude Service. These are:

Resultate des Internationalen Breitendienstes. Th. Albrecht and B. Wanach.

Band I	Fall, 1899 to Jan. 4, 1902	1903
Band II	Jan. 5, 1902 to Jan. 4, 1905	1906
Band III	Fall, 1899 to Jan. 1, 1906	1909
Band IV	1906–1908	1911

Resultate des Internationalen Breitendienstes. B. Wanach.

Band V	1899–1912.0	1916

The definitive results in Band V are to be taken as superseding those published in earlier volumes of the series.

Ergebnisse des Internationalen Breitendienstes. B. Wanach and H. Mahnkopf. 1912.0–1922.7 1932

This volume is really Vol. VI of the series although not so designated on its title page.

Results of the International Latitude Service, Hisashi Kimura.

Vol. VII	1922.7–1931.0	1935
Vol. VIII	1922.7–1935.0	1940

Provisional Results of the Work of the International Latitude Service in the North Parallel +39° 08′ for the year 1935 by Hisashi Kimura are published in Vol. XII, No. 5, 1936 of the Proceedings of the Imperial Academy, Tokyo, Japan.

Provisional results of the work of the International Latitude Service by Luigi Carnera are published under the following titles:

Il movimento del Polo di rotazione terrestre in the Astronomische Nachrichten: For 1936 in Vol. 263, pp. 17–32; for 1937 in Vol. 266, pp. 181–196.

Le variazioni della latitudine osservate durante il 1937. Risultati provvisorii dedotti dalle Stazioni boreali ed australi. Rendiconto della R. Accademia delle Scienze Fisiche e Matematiche della Società Reale di Napoli. Serie 4, Vol. VIII, 1937–1938–XVI.

Risultati tratti dallo studio delle variazioni di latitudine negli anni 1936 e 1937. Rendiconti della R. Accademia nazionale dei Lincei. Vol. XXVII, Serie 6a, May 1938–XVI.

Il movimento del Polo durante il 1938 dalle osservazioni delle Stazioni Internazionali. Rendiconto della R. Accademia delle Scienze Fisiche e Matematiche della Società Reale di Napoli, Serie 4a, Vol. IX, 1938–39–XVII.

Le variazioni di latitudine osservate nel 1939 dalle Stazioni Internazionali. Rendiconto della R. Accademia delle Scienze Fisiche e Matematiche della Società Reale di Napoli, Serie 4a, Vol. X, 1939–1940–XVIII.

Le variazioni della latitudine negli anni 1935 e 1940 dedotte dalle osservazioni delle Stazioni Internazionali. Rendiconto della R. Accademia delle Scienze Fisiche e Matematiche della Società Reale di Napoli, Serie 4a, Vol. XI, 1940–41–XIX.

Because of two world wars these published results are far from being up to date or even approximately so. There is the further difficulty that, though these published results afford corrections to be applied to reduce latitudes, longitudes and azimuths to *some* mean position of the pole, it is not always to the *same* mean position. Occasionally corrections are published to reduce from one assumed mean position of the pole to another but ordinarily it is difficult to determine to just what mean position of the pole the published figures refer.

The problem of defining and determining a mean position of the pole and of referring all current positions of the pole to that assumed mean position is a problem of considerable technical difficulty and the results, when obtained, will be somewhat uncertain at best.

The present Director of the International Latitude Service intends to publish a systematic discussion which will provide the means of reducing astronomical latitudes, longitudes and azimuths to the same mean pole, as nearly as may be. It seems therefore best to await the appearance of this discussion before actually applying the corrections.

The results, when available, will give means for computing (at least with the aid of previously published results) the rectangular coordinates of the north pole from 1900.0 on. The coordinates of the instantaneous north pole, x and y, expressed in seconds of arc will be referred to the assumed position of the mean pole as origin. The positive direction of the axis of x is southward along the meridian of Greenwich. The positive direction of the axis of y is southward along the meridian of 90° west longitude.

When the x and y of the instantaneous north pole are known for a given date, the reduction to be applied to an astronomical latitude observed in west longitude λ to reduce it to the mean pole is

$$\Delta\phi = -(x \cos \lambda + y \sin \lambda),$$

x and y are in seconds of arc.

In some of the discussions published by the International Latitude Service an additional term, z, independent of the longitude has been introduced. Presumably the z-term represents a combination of seasonal effects in refraction, with periodic and other more or less systematic errors in the declinations of the stars.

Whether to include this z-term at all, and if so, how to apply it, is a question that may be left open for the present. Some sort of answer may be given in the publication referred to.

The correction to an astronomical azimuth in latitude ϕ and west longitude λ is given by

$$\Delta\alpha = (x \sin \lambda - y \cos \lambda) \sec \phi.$$

An observed difference of longitude involves two stations. Formerly, both were often field stations. At present one of the stations is more commonly the fixed observatory that furnishes the time signals. Let ϕ_1, λ_1 be the latitude and longitude of the observatory furnishing the time signals and ϕ_2, λ_2 the latitude and longitude of the field station. The correction to the difference of longitudes, $\lambda_2 - \lambda_1$, for the displacement of the pole is

$$\Delta(\lambda_2 - \lambda_1) = (x \sin \lambda_1 - y \cos \lambda_1) \tan \phi_1 - (x \sin \lambda_2 - y \cos \lambda_2) \tan \phi_2.$$

Ordinarily no correction for the shifting of the pole is applied to the corrections to the radio time signals published from time to time by various observatories. In particular no correction for the shift of the pole has been applied to the time signals of the U. S. Naval Observatory. An obvious reason is that the necessary data for such a correction are usually not available until long afterwards. However, the International Time Service (Bureau international de l'Heure) has studied the effect of applying a correction for the shifting of the poles and found that the corrections for polar shift tend to smooth out the irregularities of the recorded signals. The effect of the polar shift on the longitudes of various observatories is published in the Bulletin Horaire of the International Time Service.

REJECTION OF DOUBTFUL OBSERVATIONS

In general the process of the rejection of doubtful observations involves two stages. If the observations are of a familiar kind and are presumed to be of a certain average quality, a rejection limit for observations of that kind and quality or order of accuracy will usually have been established. This rejection limit frequently forms part of the definition of the quality or order of accuracy. Any residual exceeding the rejection limit is to be rejected without further question. This is the first stage.

For instance, in the case of latitude, those results having residuals of 3″ or more are rejected without question. This is the first stage for latitude and is an important part of the definition of the order of accuracy.

For azimuth, a 5″ rejection limit for a residual is the first stage. It also defines the order of accuracy.

Even after the residuals are rejected in the first stage, there may still be residuals whose size suggests a blunder or an error of a kind not contemplated by the theory of probability. We have no satisfactory means of evaluating the probability, large or small, of the occurrence of these blunders or special errors. Hence any criterion for further rejection must be somewhat arbitrary. The following process which represents a possible second stage is given by Chauvenet (Manual of Spherical and Practical Astronomy, Vol. II, page 564, et seq.) and allows for the fact that, even if only the errors contemplated by the theory of probability are considered, the probability of the occurrence of a large resultant error increases with the number of observations.

Chauvenet derives a rejection limit for a single doubtful observation based on the number of the observations and the fundamental theorem of the theory of least squares, namely,

$$\Theta(\rho t')=\frac{2}{\sqrt{\pi}}\int_0^{\rho t'} e^{-t^2}dt=\frac{2}{\sqrt{\pi}}\int_0^{\rho t'} e^{-t^2}dt \tag{1}$$

where ρ is found from the solution of the equation,

$$\frac{2}{\sqrt{\pi}}\int_0^{\rho} e^{-t^2}dt=\frac{1}{2}. \tag{2}$$

It follows that

$$\rho=\frac{0.6745}{\sqrt{2}}=0.4769. \tag{3}$$

The quantity t' is the ratio of the limit of doubtful observations to the probable error of a single observation.

The above formula (1) represents in general the number of errors less than $a=rt'$ which may be expected to occur in any extended series of observations when the whole number of observations is taken as unity, r being the probable error of a single observation. If this is multiplied by the number of observations, n, we shall have the actual number of errors less than rt'. Hence it follows that

$$n-n\Theta(\rho t')=n[1-\Theta(\rho t')] \tag{4}$$

expresses the number of errors to be expected greater than the limit rt'. But if this quantity is less than ½, it will follow that an error of the magnitude rt' will have a greater probability against it than for it and may therefore be rejected. The limit of rejection of a single doubtful observation which is rt' according to this rule is obtained from

$$\frac{1}{2}=n[1-\Theta(\rho t')] \tag{5}$$

or

$$\Theta(\rho t')=\frac{2n-1}{2n} \tag{6}$$

and Table IX. A (Chauvenet's Practical Astronomy, Vol. II) which gives the value of t' for values of $\Theta(\rho t')$.

To facilitate the use of this criterion, a table on page 138 has been prepared for this publication based on formula (4) and the above-mentioned Table IX.A, whereby t' may be found directly for values of n.

Evidently, it would be unreasonable to apply this process to a very small number of observations, since these would give no adequate determination of the probable error.

Example of Application. At **University,** page 79, the latitude determined from the pair of stars Nos. 13836 and 14123 has an outstanding residual of $-1''.36$. Here $n=17$ and $r=\pm 0''.37$. The residual is less than $5e_p$ or $5r$ and slightly larger than $3\frac{1}{2}\ e_p$. The record gives no special ground for rejection but from the table on page 138 the value of t' when $n=17$ is 3.229.

Hence

$$rt'=e_pt'=\pm 0''.37\times 3.229=\pm 1''.19. \tag{7}$$

Thus by Chauvenet's criterion, the rejection of the above pair of stars is justified.

Table for t'

Based on Table IX.A for $\Theta(\rho t')$ (Chauvenet, Practical Astronomy, Vol. II) and

$$\Theta(\rho t')=\frac{2n-1}{2n}$$

n	t'	n	t'	n	t'	n	t'
8	2.76	14	3.11	20	3.32	26	3.47
9	2.84	15	3.15	21	3.35	27	3.49
10	2.91	16	3.19	22	3.38	28	3.51
11	2.97	17	3.23	23	3.40	29	3.53
12	3.02	18	3.26	24	3.43	30	3.55
13	3.07	19	3.29	25	3.45		

This table shows that as a rough working rule, if the number of observations is moderately large, 20 or over, a residual greater than $3\frac{1}{2}$ times the probable error should be rejected.

The probable error to be used in applying the test should be the probable error computed by including the questioned observation. This practice will diminish the number of rejections, especially when the number of observations is small.

PROBABLE ERRORS OF ADJUSTED VALUES

In the great majority of cases, the observational procedures used to obtain precise physical data lead to more or less complicated functional relations amongst the quantities which are desired and those which it is possible to measure. The observation equations obtained from the determinations of latitude and longitude according to the methods discussed elsewhere in this publication are rather simple examples of this type of relation.

If the functional relation is non-linear, it is usually reduced to linear form by an expansion in Taylor series in which terms of the second and higher orders are neglected. We may therefore consider observation equations of the form

$$ax+by+cz-l=v, \tag{1}$$

from which it is desired to obtain the most probable or adjusted values of x, y and z as well as the probable errors of these values.

The coefficients a, b, and c are presumed to be known to the degree of accuracy required, and l is either a directly observed quantity, or one which is derived from an observed quantity but which is not sensibly affected by the as yet unknown quantities x, y, z.

The method to be employed is a general one which will hold for any number of unknowns, but the procedure will be apparent from its application to the problem with three unknowns. The theory underlying this method is given in Wright and Hayford, "The Adjustment of Observations," pages 121 to 137.

From the observation equations of the form (1), we obtain three symmetric normal equations, namely,

$$\begin{array}{ll} \text{I} & [aa]x+[ab]y+[ac]z-[al]=0, \\ \text{II} & [ab]x+[bb]y+[bc]z-[bl]=0, \\ \text{III} & [ac]x+[bc]y+[cc]z-[cl]=0. \end{array} \tag{2}$$

As is customary, the square brackets indicate a summation of all the products of the type indicated. At the same time we obtain the quantity $[ll]$, which will be used to

obtain $[vv]$, the summation of the squares of the residuals, by an extension of the solution of the normal equations. Since $[vv]$ can also ba obtained after substitution of the unknowns into the observation equations, this affords a valuable check on the solution.

The normal equations are now treated in the usual schematic way known as the "forward solution" of the Doolittle process, with the inclusion of three additional columns headed R, S, and T. The numerical or absolute values of certain quantities later to be obtained from these columns are designated u_x, u_y, and u_z, and are the so-called weight coefficients, which together with ϵ, the probable error of a single observation, will enable us to determine the probable errors ϵ_x, ϵ_y, ϵ_z of the adjusted values x, y and z.

Forward Solution (3)

	x	y	z	l	R	S	T
1	$+[aa]$	$+[ab]$	$+[ac]$	$-[al]$	$+1$	0	0
2	$x=$	$-\frac{[ab]}{[aa]}$	$-\frac{[ac]}{[aa]}$	$+\frac{[al]}{[aa]}$	$-\frac{1}{[aa]}$	0	0
3		$+[bb]$	$+[bc]$	$-[bl]$	0	$+1$	0
4		$-\frac{[ab]}{[aa]}[ab]$	$-\frac{[ab]}{[aa]}[ac]$	$+\frac{[ab]}{[aa]}[al]$	$-\frac{[ab]}{[aa]}$	0	0
5		$+[bb_1]$	$+[bc_1]$	$-[bl_1]$	$-[bR_1]$	$+1$	0
6		$y=$	$-\frac{[bc_1]}{[bb_1]}$	$+\frac{[bl_1]}{[bb_1]}$	$+\frac{[bR_1]}{[bb_1]}$	$-\frac{1}{[bb_1]}$	0
7			$+[cc]$	$-[cl]$	0	0	$+1$
8			$-\frac{[ac]}{[aa]}[ac]$	$+\frac{[ac]}{[aa]}[al]$	$-\frac{[ac]}{[aa]}$	0	0
9			$-\frac{[bc_1]}{[bb_1]}[bc_1]$	$+\frac{[bc_1]}{[bb_1]}[bl_1]$	$+\frac{[bc_1]}{[bb_1]}[bR_1]$	$-\frac{[bc_1]}{[bb_1]}$	0
10			$+[cc_2]$	$-[cl_2]$	$-[cR_2]$	$-[cS_2]$	$+1$
11			$z=$	$+\frac{[cl_2]}{[cc_2]}$	$+\frac{[cR_2]}{[cc_2]}$	$+\frac{[cS_2]}{[cc_2]}$	$-\frac{1}{[cc_2]}$
12				$[ll]$	0	0	0
13				$-\frac{[al]}{[aa]}[al]$	$-\frac{1}{[aa]}$	0	0
14				$-\frac{[bl_1]}{[bb_1]}[bl_1]$	$-\frac{[bR_1]}{[bb_1]}[bR_1]$	$-\frac{1}{[bb_1]}$	0
15				$-\frac{[cl_2]}{[cc_2]}[cl_2]$	$-\frac{[cR_2]}{[cc_2]}[cR_2]$	$-\frac{[cS_2]}{[cc_2]}[cS_2]$	$-\frac{1}{[cc_2]}$
16				$[vv]$	u_x	u_y	u_z

Lines 1, 3, and 7 through column l are simply the coefficients in the normal equations I, II, and III, respectively, the terms to the left of the main diagonal being omitted because of the symmetry of the normal equations. The three additional columns contain, for line 1, the numbers $+1$, 0, and 0, respectively; for line 3, the numbers 0, $+1$, and 0, respectively; and for line 7, the numbers 0, 0, and $+1$, respectively. Line 12 consists of the quantity $[ll]$, which is entered in column l.

Line 2 is obtained by dividing each term of line 1 by the negative of $[aa]$. Note that -1 which would have been the leading term of line 2 is replaced by x. This process is of course equivalent to dividing by the leading term, and then transposing all but the first term of line 2 considered as an equation. Note also that the three additional columns are treated exactly as are the preceding columns.

Line 4 is the product of each term of line 1 by the leading term of line 2. Again, a term at the beginning of line 4 is omitted. The sum of lines 3 and 4 is line 5, in which, for compactness, this summation is indicated by the subscript, as $[bb_1]$, $[bc_1]$, etc. Examination will show that the terms omitted in lines 3 and 4 sum to zero.

The above process, resulting in line 5, is one complete cycle which is repeated as many times as necessary, the number of times being determined by the number of normal equations to be solved.

Line 6 is obtained from line 5 just as line 2 was obtained from line 1, that is, by dividing line 5 by the negative of its first term, $-[bb_1]$. Lines 8 and 9 correspond to the single line 4, line 8 being the result of multiplying each term of line 1 by the second term of line 2, and line 9 the result of multiplying each term of line 5 by the first term of line 6. Note that these multipliers lie in the same column and that they are in corresponding rows of the two cycles. (This correspondence is emphasized by the practice of underlining such rows, although the underlining serves a further purpose in that these are the only rows required for the "back solution".) Just as line 5 was the sum of lines 3 and 4, so line 10 is the sum of lines 7, 8, and 9, the summation being indicated by the subscript 2, as $[cc_2]$, etc. This completes the second cycle. Again, the terms omitted in 7, 8, and 9 sum to zero.

We start the third cycle by dividing each term of line 10 by the negative of its first term, thus obtaining line 11. This, however, completes the forward solution, since the quantity $\frac{[cl_2]}{[cc_2]}$ in line 11 is obviously the value of z.

Lines 12 through 16 represent the additional computation required to obtain $[vv]$, u_x, u_y, and u_z. This portion of the computation is carried out vertically, column by column.

Note that $[vv]$ is the result of carrying out one more cycle, or rather, completing the cycle just started at the completion of the forward solution. The summation of lines 12 through 15 is indicated by $[vv]$ rather than by a subscript.

The weight coefficients are now obtained in the following manner. Imagine each term of column l through line 12 replaced by those of column R. With these terms, repeat the process for obtaining $[vv]$. The absolute value of the result is u_x. The weight coefficients u_y and u_z are obtained analogously from columns S and T.

We can now, even before obtaining the values of y and z obtain the required probable errors. For ϵ, the probable error of a single observation, is given by

$$\epsilon = \pm\sqrt{\frac{0.455\ [vv]}{n-k}} \tag{4}$$

where n is the number of observations and k is the number of unknowns, in this case, three. The probable errors of the adjusted values are then given by

$$\begin{aligned} \epsilon_x &= \epsilon\sqrt{u_x}, \\ \epsilon_y &= \epsilon\sqrt{u_y}, \\ \epsilon_z &= \epsilon\sqrt{u_z}. \end{aligned} \tag{5}$$

The "back solution", to obtain the values of y and x may now be carried out in the usual fashion, the additional columns and lines being ignored. For completeness, we outline this briefly.

The value of z obtained in line 11 is multiplied by the quantity in line 6, column z, and this product is added to the quantity in column l, line 6. This results in the value of y. With the values of y and z, x is obtained from line 2 in a similar way, two products being added to the quantity in column l, line 2.

The method outlined above for obtaining the weight coefficients and the summation of the squares of the residuals can be expressed explicitly in terms of the quantities indicated in the forward solution. We now give these expressions of the weight coefficients for the case with two unknowns x and y:

$$u_x=\frac{1}{[aa]}+\frac{[bR_1]^2}{[bb_1]}, \text{ where } [bR_1]=\frac{[ab]}{[aa]} \text{ and}$$

$$[bb_1]=[bb]-\frac{[ab][ab]}{[aa]} \tag{6}$$

$$u_y=\frac{1}{[bb_1]}.$$

If more than two unknowns are involved, the expressions become more complicated, although the weight coefficient of the unknown found first by the forward solution (i. e., the unknown written last in the observation equations) will always be the reciprocal of the quantity analogous to $[bb_1]$.

The expression for $[vv]$, due originally to Gauss, is

$$[vv]=[ll]-\frac{[al]^2}{[aa]}-\frac{[bl_1]^2}{[bb_1]}-\frac{[cl_2]^2}{[cc_2]}-\ \cdots, \tag{7}$$

which is valid for any number of unknowns.

As a practical example of this method, the details of the computation for determining the adjusted latitude at Station University are given.

EXAMPLE OF LATITUDE ADJUSTMENT FOR STATION UNIVERSITY

Normal equations

$$\begin{aligned} &\text{I.} \quad 16c+1.6r+0.04 &&=0 \\ &\text{II.} \quad 1.6c+819.2r+20.934 &&=0 \end{aligned} \tag{8}$$

Forward Solution (9)

	c	r	$\Delta\overline{\phi}$	R	S
	+16.0000	+ 1.6000	+ 0.0400	+1.0000	0
	$c=$	− 0.10000	− 0.00250	−0.06250	0
		+819.2000	+20.9340	0	+1.0000
		− 0.1600	− 0.0040	−0.1000	0
		+819.0400	+20.9300	−0.1000	+1.0000
		$r=$	− 0.02555	+0.00012	−0.00122
Computation of $[vv]$ and the weight coefficients			+ 2.9664	0	0
			− 0.0001	−0.0625	0
			− 0.5348	−0.0000	−0.00122
			+ 2.4315	0.0625	0.00122
			$[vv]$	u_c	u_r

Back Solution

$$\begin{aligned} r &= -0.02555 \\ c &= (-0.10000)(-0.02555) - 0.00250 \\ &= +0.00006 \end{aligned} \tag{10}$$

Probable Errors

$$\begin{aligned} \epsilon = e_p &= \pm\sqrt{\frac{0.455(2.4315)}{16-2}} = \pm 0''.28 \\ \epsilon_c = e_\phi &= \pm e_p\sqrt{0.0625} = \pm 0''.07 \\ \epsilon_r = e_r &= \pm e_p\sqrt{0.00122} = \pm 0''.0098 \end{aligned} \tag{11}$$

It is frequently required to calculate the probable error of a *function* of the adjusted values. This value can in practice be reduced to the probable error of a linear function of the adjusted values. The actual calculation is very similar to the calculation, just explained, of the probable errors of the adjusted values themselves, the numerical values of certain partial derivatives being substituted for the zeros and ones in the "R" and "S" columns. For the details see comprehensive treatises on the method of least squares, in particular Hayford.

One caution, however, may be given here in regard to a not uncommon fallacy in the computation of such probable errors. Suppose we have a linear function of x, y and z, namely

$$px+qy+rz$$

and that we have the probable errors of x, y, and z, themselves. Denote by ϵ_x, ϵ_y, and ϵ_z, the probable errors of x, y, and z, and by ϵ_f the probable error of the function

$$px+qy+rz.$$

The following simple relation then holds *under certain conditions*

$$\epsilon_f^2 = p^2\epsilon_x^2 + q^2\epsilon_y^2 + r^2\epsilon_z^2. \tag{12}$$

The fundamental condition is that x, y, and z, are completely independent of one another. This condition is not fulfilled in the problem under discussion. Our x, y, and z, are not independent but are connected by observation equations of the form

$$ax+by+cz-l=0. \tag{13}$$

The simple formula for ϵ_f does not apply and the more complete calculation given in Hayford and other treatises must be made in order to determine ϵ_f. The fallacious application of formula (12) to cases where it does not apply is not the fault of the authors of treatises on least squares but of their readers who do not give sufficient attention to the underlying assumptions.

ASTRONOMICAL REFRACTION

This article of the Appendix is intended to explain the use of the elaborate tables of refraction prepared by astronomers, and more especially the assumptions on which these tables are based and the limitations on their accuracy imposed by these underlying assumptions. Many elaborate mathematical developments of the theory of

refraction have been given but it has not been found necessary to reproduce them. The general characteristics and the limitations of the tables can be explained in a general way without going beyond a treatment of refraction given by Laplace, who was to a certain extent anticipated by Oriani. Laplace's discussion showed that the refraction may be expanded into a series, the first two terms of which are practically independent of the assumed law of density of the atmosphere.

RELATION BETWEEN THE INDEX OF REFRACTION AND THE DENSITY OF AIR

The index of refraction for air, relative to a vacuum, is a quantity slightly greater than unity, say $1+\epsilon$, where ϵ is a small quantity. The quantity ϵ depends on the composition of air, and the amount of water vapor present, but above all on the density. It is not the same for all wave-lengths of light but within the visible spectrum it does not vary greatly. For air at 0° C and a pressure of one atmosphere, ϵ is about 0.0003, and the index of refraction is 1.0003. Various laws connecting the index of refraction with the density of the air have been suggested but within the limits of experimental error it has been found that ϵ varies directly as the density of the air, regardless of the particular combination of pressure and temperature that has given rise to that particular density. We may also assume that pressure, temperature and density are connected by the equation for a perfect gas

$$\rho=\frac{kp}{T}$$

where ρ is the density, p is the pressure, T the absolute temperature, and k a constant depending on the units used and on the gas. With another value of k we may therefore write

$$\epsilon=\frac{kp}{T}.$$

We may call ϵ the approximate *refractive power* of the atmosphere with reference to a vacuum.

The treatment follows in general that of Simon Newcomb, Compendium of Spherical Astronomy, Chapter VIII, 1906, and John E. Willis, Transactions of the American Geophysical Union, 22nd meeting, Part II, 1941, pages 324–336. The tables here reproduced by permission of the author are those of Willis. Neither Newcomb nor Willis, however, confines himself to two terms of an expansion of the refraction in series. Both of them make the necessary assumptions about the atmosphere and carry the series out to many terms.

All tables of refraction are based on the assumption that the atmosphere is stratified horizontally, strata of equal density being level surfaces. In most theories the ellipticity of these level surfaces is neglected. If the strata of equal density are not horizontal, then anomalous refractions in any direction may occur in any part of the sky including the zenith, where all tables assign to the refraction a zero value.

Up to a zenith distance of 60° or so the refraction R may be expressed to a high degree of approximation in the form

$$R=\frac{KB}{T}\tan z, \tag{1}$$

where B is the barometric pressure at the point of observation, T the absolute temperature there, z the zenith distance and K a constant, the value of which depends on the units in which B and T are expressed. Some refinements having to do with the accu-

rate specification of the barometric pressure will be mentioned later. The refraction thus computed is independent of any hypothesis about the constitution of the atmosphere, except that it is horizontally stratified. The factors for reducing from standard pressure and temperature to the actual pressure and temperature are thus very simple.

A second term may be added to the refraction formula, thus extending its applicability to a zenith distance of 70° or somewhat beyond. This second term involves the constitution of the atmosphere only to a very slight extent. In the notation of Newcomb the formula for refraction extended to two terms may be written

$$R=\frac{\alpha}{1-\alpha}(\tan z)\{1-(\nu-\tfrac{1}{2}\alpha)\}\sec^2 z. \tag{2}$$

This formula is of the type assumed by Willis but his formula includes further terms in ascending even powers of sec z. The quantity $\left(\frac{\alpha}{1-\alpha}\right)$ is very nearly equal to the refractive power of air under the assumed conditions. Willis and Newcomb start from different hypotheses, both equally valid as far as experimental verification is concerned, regarding the connection between density and index of refraction. In both, α is approximately proportional to the density. It can be shown, however, that Newcomb's $\frac{\alpha}{1-\alpha}$ and Willis' log n_0 are the same up to and including quantities of the order of α^2.

The quantity α or $\frac{\alpha}{1-\alpha}$ may be considered to vary, as in equation (1), directly as the barometric pressure and inversely as the temperature.

The quantity ν is equal to $\frac{H}{r}$, where r is the radius of curvature of the level surface in the direction of observation and H is the so-called "height of the homogeneous atmosphere," that is, the height to which the atmosphere would extend if its density were the same as at the point of observation and the pressure (or total mass per unit area above the point of observation) were equal to the observed pressure (or mass). The first alternative, based on pressure, assumes gravity to be constant throughout the atmosphere; if the second alternative definition, based on total mass per unit area, is adopted, some knowledge of the constitution of the atmosphere and of the variation of gravity with elevation must be assumed.

Let σ be the density of mercury, let B be the height of the mercurial barometer and ρ_1 the density of the atmosphere at the place of observation. Then according to the first definition (pressure)

$$H=\frac{\sigma}{\rho_1}B. \tag{3}$$

According to the second definition (mass)

$$H'=\frac{\sigma}{\rho_1}B\frac{g_1}{g_m} \tag{3*}$$

where g_1 is the acceleration of gravity at the place of observation and g_m is some average value of gravity between the point of observation and the upper limit of the atmosphere. Since $\sigma=13.6$ gm/cm^3, approximately, and in the same units $\rho_1=1/800$, we have approximately by either definition for $B=0.76$ meter, $H=8$ km$=5$ miles; and since r is roughly 6400 km, $\nu=\frac{H}{r}=\frac{1}{800}$.

The full formula for refraction requires an infinite series of the form

$$R=\frac{\alpha}{1-\alpha}(\tan z)\ (1-c_2 \sec^2 z+c_4 \sec^4 z-c_6 \sec^6 z\text{----}) \tag{4}$$

The coefficient $c_2=\nu-\frac{1}{2}\alpha$, as we have seen, is almost independent of atmospheric conditions, except those at the place of observation, always assuming horizontality of the strata, but the remaining coefficients depend, and to an increasing degree as the subscript increases, on the constitution of the atmosphere. Even after these coefficients have been determined for some assumed state of things, the series ceases to converge long before the horizon is reached and special transformations are needed. Willis' numerical series ceases to converge for practical computations at a zenith distance of about 83° or 84°.

Some idea of the peculiarities of the more elaborate tables of refraction may be derived simply from a study of the two-term formula that stops with the term in c_2.

Since the discussion that follows deals mainly with the peculiarities of the coefficient $\left(\frac{\alpha}{1-\alpha}\right)c_2 \sec^2 z$, it may as well be said at once that Willis' value of this coefficient is $0''.06692 \sec^2 z$ for conditions not quite identical with those on which Table XXa is based. At $z=75°$ this amounts to less than one second of arc. If we deal with zenith distances less than 75°, we are dealing with the *variations* (due to departure from standard atmospheric conditions) of a quantity that is less than one second.

The tables for the practical computation of refraction used by astronomers, such as Bessel's tables and the Pulkovo tables, also those of Willis here reproduced, are all based on formulas of the same general type, though with differences of notation.

Willis writes

$$\log_{10} R=\log_{10} \tan z+\alpha_w+\beta+\gamma+(\lambda-1)\left(\gamma+\frac{\beta}{10}\right). \tag{5}$$

The terms in β and γ represent corrections to the refraction for departures from the assumed standard conditions of pressure and temperature. We pass these over for the present in order to discuss the quantity α_w.

The quantity α_w of Willis' formula is obviously not the α of equation (4) but is related to it. We have temporarily applied a subscript w (for *Willis*) to the α of Willis' formula. If we limit our refraction formula to the term in tan z, α_w is constant and we have approximately

$$\log_{10}\left(\frac{\alpha}{1-\alpha}\right)=\alpha_w. \tag{6}$$

The quantity α_w is not, of course, strictly constant but an inspection of Willis' principal table on p. 332 or our Table XXa shows that it varies slowly for moderate zenith distances. It is easy to show that this variation can be approximately accounted for by equation (4). Taking logarithms of both sides of the equation, this gives

$$\begin{aligned}\log_{10} R&=\log_{10} \tan z+\log_{10}\left(\frac{\alpha}{1-\alpha}\right)+\log_{10}(1-c_2 \sec^2 z)\\ &=\log_{10} \tan z+\log_{10}\left(\frac{\alpha}{1-\alpha}\right)-Mc_2 \sec^2 z \qquad (7)\\ &=\log_{10} \tan z+\log_{10}\left(\frac{\alpha}{1-\alpha}\right)-Mc_2-Mc_2 \tan^2 z\end{aligned}$$

by expanding the logarithm to one term only, where M is the modulus of common logarithms. Thus we would have for the zenith to a slightly greater degree of accuracy than by (6)

$$\alpha_w = \log_{10}\left(\frac{\alpha}{1-\alpha}\right) - Mc_2 \tag{8}$$

and for the variation in α_w from the zenith to zenith distance z

$$Mc_2 \tan^2 z. \tag{9}$$

On p. 329 equation (19) Willis gives c_2 as 0.00115062, which will serve for illustration, although his equations (18) and (19) do not correspond to the mean state of the atmosphere assumed in his Table 8, which is our Table XXa. The following shows that the simple expression (9) represents the change in α_w satisfactorily up to $z=70°$ and not too badly at $z=75°$.

(α_w for zenith) − (α_w for given z)

z	Table XXa	Expression (9)
30°	0. 00017	0. 00017
45°	0. 00050	0. 00050
60°	0. 00150	0. 00150
65°	0. 00229	0. 00230
70°	0. 00373	0. 00377
75°	0. 00679	0. 00696

The presence of the second term in the expression for the refraction shows why the whole refraction may not be taken as directly proportional to the barometric pressure and inversely proportional to the absolute temperature. The quantity $\frac{\alpha}{1-\alpha}$ may be so taken with sufficient accuracy but the quantity ν which is dominant in the expression $\nu - \frac{1}{2}\alpha$, since $\frac{1}{2}\alpha$ is only about one-eighth of ν, depends only on the pressure and not on the temperature. Thus the expression $\frac{\alpha}{1-\alpha}\left(\nu - \frac{1}{2}\alpha\right)$ depends on pressure and temperature in a rather complicated way. This is taken account of by the term $(\lambda - 1)\left(\gamma + \frac{\beta}{10}\right)$. The proof is omitted but the fact may be verified in a numerical example. Newcomb gives the proof for an analogous formula for the correction for departure from standard conditions.

The same term shows why in strictness the refraction at a mountain station, where barometric pressure is low because of altitude, can not be computed simply by reducing the refraction in proportion to the lowered pressure. If the ratio of the pressures is f, the factor $\frac{\alpha}{1-\alpha}$ is reduced in the ratio f but the term $\frac{\alpha}{1-\alpha}\left(\nu - \frac{1}{2}\alpha\right)$ is reduced in the ratio f^2, apart from considerations of temperature, so that no single reduction ratio applies to the entire refraction. What has been said about the corrections for departure from standard conditions and about the unequal reduction-factors for elevation in the two terms of the formula applies with even greater force to the next term $\frac{\alpha}{1-\alpha} c_4 \sec^4 z \tan z$. For c_4 Newcomb gives the expression $c_4 = \frac{18\nu^2}{7} - \frac{24}{11}\nu\alpha + \frac{1}{2}\alpha^2$.

Willis' value of $\left(\frac{\alpha}{1-\alpha}\right) c_4$ is 0".000215 which is to be multiplied by $\sec^4 z$. This amounts to only 0".05 for $z=75°$ but to 0".24 for $z=80°$.

The reduction factor for this term would be f^3 instead of f^2 or f. Of course, the value of c_4 corresponds to special assumptions regarding the atmosphere but any reasonable set of assumptions will lead to the same general conclusion.

If our place of observation is elevated above the sea to a height that is a considerable fraction of H, the "height of the homogeneous atmosphere," not only does the value of H change because the new "height of the homogeneous atmosphere" depends only on the mass of air above the level of the observer, but the coefficients of ν^2, $\nu\alpha$ and α^2 in the formula may change also, because we are dealing with only a part of the atmosphere, not with the whole of it.

For aviators the Nautical Almanac Office of the U. S. Naval Observatory has worked out a table of refraction for various zenith distances and various large elevations above sea level. This table is Table XXI on p. 200, and is the same as that given in the *American Air Almanac*. This refraction table is valid only under average atmospheric conditions. At a particular time and place actual atmospheric conditions may vary widely from the average, and thus errors of several minutes of arc may occur at low altitudes. Similar computations were made on a somewhat different set of assumptions regarding the atmosphere; some of the results, especially for large zenith distances, differed from those in the table by several tenths of a minute of arc.

The definition of ν involves r, the radius of curvature of the level surface of the atmosphere in the direction of observation and at the point of observation. At the surface of the earth this radius would be the radius of curvature of the earth itself. Here there is the possibility of a variation of about one percent between pole and equator and between meridian and prime vertical. As may be seen by Willis' Table 5 on page 331, a variation of one percent might affect the value of α_w by a few units in the fifth decimal. The effect of elevations attainable on the surface of the earth would ordinarily be negligible.

Explanation of the Use of Willis' Tables of Refraction

The formula for computing the refraction for pressures and temperatures not too far from standard conditions is

$$\log_{10} R=\log_{10} \tan z+\alpha+\beta+\gamma+(\lambda-1)\left(\gamma+\frac{\beta}{10}\right). \tag{10}$$

The subscript w has been dropped from the α as now unnecessary, since the law of variation of α has been sufficiently discussed. We simply take it from Table XXa for the appropriate zenith distance, correcting it in extreme cases for the radius of curvature in the direction of observation.

The quantity β represents the effect of the departure of the barometric pressure from assumed standard conditions. To pass from the observed height B of the mercurial barometer to the quantity β in formula (10) requires several steps. The observed barometer reading B is corrected for humidity by adding to it the term eC, where the factor C is obtained from Table XXd using the wave-length of light as argument. The quantity e is the vapor pressure of water in millimeters of mercury.

Then $B'=B+eC$ is the equivalent height of the barometer after allowance has been made for the humidity or pressure of water vapor.

748977°—48——11

In order not to interrupt the discussion of refraction some brief notes on wave-length and color of light and on the relation of vapor pressure to humidity have been placed at the end of this discussion on p. 149.

Having corrected B to obtain B' we use B' as argument to obtain β_0. What is really important in the computation of refraction is the atmospheric pressure in absolute units, such as dynes per square centimeter. Before the observed height of the barometer can be considered as proportional to the pressure in dynes per square centimeter it must be reduced to some standard temperature to allow both for the expansion of the mercury itself and for the expansion of the scale. In a fixed observatory the barometer may be in a warm room, whereas the observation is made at nearly the temperature of the open air. Table XXc is computed on the assumption that the scale is made of brass; the temperature of the brass scale is denoted by t_B in this table and it should be used as argument for taking out $\Delta\beta_t$; this is the correction $\Delta\beta_t$ needed for the argument, Table XXb. If the scale is of another material, proper allowance must be made, and if an aneroid barometer is used the corrections appropriate to the particular instrument must be applied in order to allow for the temperature.

The quantity β_0 in Table XXb is based on the theoretical value of gravity at the elevation and latitude of the U. S. Naval Observatory, Washington, D. C. An additional correction to refraction is therefore needed for the effect of the ratio of the gravity at the station to that at the Naval Observatory. Table XXe gives the correction, $\Delta\beta_g$, to the logarithm of the refraction, for this change in gravity which in turn is due to change in elevation and latitude. The arguments for $\Delta\beta_g$ in Table XXe are elevation and latitude.

The effect of the elevation of the station on gravity is computed on the assumption that the decrease in gravity is due simply to the normal vertical gradient of gravity with no allowance for the intervening masses of the earth's crust; this is the so-called "free-air" hypothesis. It will probably give better results on the whole than an allowance for the intervening masses in the crust at full face value, which is the so-called "Bouguer hypothesis," although the Bouguer hypothesis has hitherto been preferred by astronomers. If gravity at the place of observation has been observed, the observed value should, of course, be used in preference to any computed theoretical value.

The readings of the ordinary aneroid barometer should need no reduction to standard gravity. If there is such a reduction, the instrument maker should supply it.

There is also a correction for the wave-length of light. For convenience Willis treats it as a correction to β_0, although it would be quite as logical to treat it as a correction to α. The correction $\Delta\beta_w$ is given in table XXd where w refers to wave-length of light. We have finally $\beta=\beta_0+\Delta\beta_t+\Delta\beta_g+\Delta\beta_w$.

The quantity γ is simple. It is the logarithm of the ratio of the observed absolute temperature to the standard absolute temperature. It is obtained from Table XXc using t, the temperature of the local outside air, as argument. If the formula (10) stopped at this point, that is, if it were written simply

$$\log_{10} R=\log_{10} \tan z+\alpha+\beta+\gamma,$$

we would have the refraction directly proportional to the barometric pressure and inversely proportional to the absolute temperature. The additional term $(\lambda-1)\left(\gamma+\frac{\beta}{10}\right)$ represents the departure from this simple proportionality. This term in $\lambda-1$ does not

become important except at the larger zenith distances. It takes account principally of the fact that the quantities $\frac{\alpha}{1-\alpha}, \left(\frac{\alpha}{1-\alpha}\right)c_2, \left(\frac{\alpha}{1-\alpha}\right)c_4$ in equation (4) do not change in the same ratio when actual conditions depart from the assumed standard condition of the table.

Color and Wave-length

In Table XXd the wave-lengths are given in microns (1 micron$=10^{-3}$ millimeter$=10^{-6}$ meter). To convert wave-length in microns to wave-length in Ångstrom units multiply the length in microns by 10,000. To the large majority of human eyes light is readily visible between wave-lengths of 0.33 micron (violet) to 0.77 micron (red). The maximum sensitiveness for low intensities is near 0.50 micron (green). The following wave-lengths are typical of the colors ordinarily recognized.

red	0.693 micron
yellow	0.575
green	0.505
blue	0.475

Willis' tables are computed for yellow light, 0.578 micron.

Vapor Pressure

The following table gives the vapor pressure corresponding to *saturation* at various temperatures.

Pressure of Saturated Water Vapor

Temp. °*C*	*mm. of mercury*
−5 (over ice)	3.02
0	4.58
+5	6.54
10	9.21
15	12.79
20	17.55
25	23.78
30	31.86
35	42.23
40	55.40

For the determination of vapor pressure by readings of wet and dry bulb thermometers see treatises on practical meteorology or the Smithsonian Meteorological Tables.

Example of Computation of Vertical Refraction, Using Willis' Formulas and Tables

Formulas: $\log R = \log \tan z + \alpha + \beta + \gamma + (\lambda - 1)\left(\gamma + \frac{\beta}{10}\right)$

$$\beta = \beta_0 + \Delta\beta_t + \Delta\beta_w + \Delta\beta_g$$

$$B' = B + eC$$

Where R is astronomical refraction in seconds of arc

z is apparent zenith distance

B is observed brass scale barometer reading in millimeters of mercury

e is vapor pressure in millimeters of mercury

t is local air temperature, °C

t_B is temperature of barometer, °C.

Given:

$z=81°$
$B=720$ mm.
$t=-5°\ C$
$t_B=20°\ C$
$e=3$ mm.
$\phi=20°$
$h=400$ meters
$w=0.60$ micron

Computation

$$\log \tan z \quad =0.80029$$
$$\text{From table XXd, } C \quad =-0.1566$$
$$B'=720+3(-0.1566)=719.53 \text{ mm.}$$
$$\text{From table XXb, } \beta_0 \quad =-0.01801$$
$$\text{From table XXc, } \Delta\beta_t \quad =-0.00141$$
$$\text{From Table XXd, } \Delta\beta_w=-0.00056$$
$$\text{From Table XXe, } \Delta\beta_g=-0.00093$$
$$\text{Hence } \beta=-0.01801-0.00141-0.00056-0.00093=-0.02091$$
$$\text{From Table XXa, } \alpha=+1.73993 \text{ and } (\lambda-1)=+0.0488$$
$$\text{From Table XXc, } \gamma=+0.02372$$

Hence $\log R=0.80029+1.73993-0.02091+0.02372+(0.0488)(+0.02372-0.00209)=2.54409$

$$R=350''.02.$$

The refraction as computed by the simplified method, using Tables V, VI, and VII is $351''.0$.

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EXPLANATION OF TABLES

Table I. Since the diurnal aberration is computed by means of the formula

$$\kappa = -0''.021 \cos\phi \sec\delta \tag{1}$$

the sign of κ will always be negative for stars at upper culmination, and positive for stars at lower culmination. In the latter case δ is replaced by $180°-\delta$ in formula (1).

Consult page 46 for further discussion.

Table II. For explanation and discussion, see pages 59 to 61.

Table III. The following formulas were used in computing this table:

$$A = \sec\delta \sin z = \sec\delta \sin(\phi-\delta) = \sin\phi - \cos\phi \tan\delta \tag{2}$$

$$B = \sec\delta \cos z = \sec\delta \cos(\phi-\delta) = \cos\phi + \sin\phi \tan\delta \tag{3}$$

$$C = \sec\delta \tag{4}$$

$$A^2 + B^2 = C^2 \tag{5}$$

where δ=declination of the star

ϕ=latitude of the station

$z=\phi-\delta$=zenith distance at time of transit at upper culmination.

The second expression for A in (2) shows that for upper culmination, A is negative for a star transiting north of the zenith and positive for a star transiting south of the

zenith. The second expression for B in (3) shows that at upper culmination B is always positive.

In high latitudes, where there are fewer stars culminating between zenith and pole, it is sometimes found necessary to observe the transit of a star at lower culmination, in which case $180°-\delta$ must be used in the above formulas. For such a star, then, A will be positive and B negative.

However, although sec δ is a factor of the expression for l, the correction for the lost motion and width of contact strip, l, is always positive.

Table IV. This table replaces the table on page 60 of Special Publication No. 14, Fifth Edition. The new table is based on the solar parallax of 8″.80 which was the value adopted at the Paris Conference. The new table gives the horizontal parallax to two decimal figures and is computed for altitudes from 0° to 42° by 3° intervals and from 42° to 90° by 2° intervals.

The parallax for $h=0°$ for the first day of each month was taken from the American Ephemeris and Nautical Almanac. Then to find the parallax for the first day of each month at different altitudes, the following formula was used:

$$\text{Parallax}=H \cos h \tag{6}$$

where h is the altitude, and H is the parallax at $h=0°$ for the first day of the month under consideration.

Table V. Mean refraction, r_m.—The table for mean refraction in this publication supersedes the one in Special Publication No. 14, Fifth edition, page 58.

The mean refraction in this table is computed for an assumed standard condition of the atmosphere for the latitude and elevation of the U. S. Naval Observatory in Washington, D. C. The assumed standard condition is: barometric pressure 760 mm., temperature 10° C., and relative humidity 60%. This is equivalent to computing the table for a station at sea level in latitude 38° 37′ 20″.

The new table presents some innovations. For instance the argument given is zenith distance, z, instead of altitude; then too, the new table is computed for every ten minutes of arc for zenith distances from 0° to 90°, whereas in the previous table the interval of the argument was different in different parts of the table.

For zenith distances from 0° to 85° the new table is computed by means of the data and formulas furnished by Mr. John E. Willis,[14] who kindly permitted their use for this purpose. The formula used is

$$\text{Log}_{10}\, r_m=\log_{10} \tan z+\alpha+\beta+\gamma+(\lambda-1)\left(\gamma+\frac{\beta}{10}\right) \tag{7}$$

in which r_m is mean refraction, z is the zenith distance of a star, and $\alpha, \beta, \gamma, (\lambda-1)$ are quantities taken from John E. Willis' article mentioned above.

As Mr. Willis in his article did not furnish data for any zenith distance greater than 85°, it was decided to compute the remainder of the table from 85° to 90° by means of the data and formula given in the "Pulkovo Observatory Refraction Tables, Third edition, 1930". A comparison between values computed with the data and formula of Willis and those computed with the data and formula of the Pulkovo tables for the same zenith distances from 83° to 85° show an almost complete agreement.

Table VI. See page 99.

Table VII. See page 99.

[14] John E. Willis, A Determination of Astronomical Refraction from Physical Data. American Geophysical Union Transactions of 1941. Part II, pp. 324-336.

Table VIII. See page 77.

Table IX. These factors should be applied only as a last resort when there are no barometer or temperature readings at the station.

Table X. See page 78.

Table XI. See page 81.

Table XII. See page 91.

Table XIII. See pages 90 and 105.

Table XIV. See pages 91 and 105.

Table XV. See page 49.

Table XVI. These factors are coefficients of $\sqrt{[vv]}$.

Table XVII. These factors are coefficients of $\sqrt{[v]}$.

Table XVIII. By using the latitude, azimuth, and elevation of the mark, and then following the directions placed on the nomogram, the correction to the azimuth for elevation of mark may be readily obtained.

For formula and discussion of this correction see pages 106 and 107.

Table XIX. This nomogram was devised many years ago by Mr. C. R. Duvall, a computer in the U. S. Coast and Geodetic Survey, for the purpose of obtaining graphically the star factors A, B, and C and the correction for diurnal aberration. It has two advantages over Table III: it is more expeditious and it eliminates double interpolation.

At present this nomogram is not being used in the Office as more precision is required in the computations for longitude than is possible with either the nomogram or Table III. However, for field computations either of these will give sufficient accuracy.

The nomogram is shown reduced in size. It consists of two systems of equidistant parallel lines perpendicular to each other, a system of arcs of equidistant concentric circles, and a transparent arm, carrying a graduated straight line which revolves about the common center of the circles. The decimeter has been the unit of length in the nomograms used. The three systems of lines are drawn at a common distance apart of 1 centimeter. The estimated tenth of this centimeter space gives the second decimal place in the required factors.

The graduated line on the under surface of the transparent arm passes through the center of the axis about which the arm revolves. A secant graduation is made upon this line, measured from the center of the axis of revolution. That is, the graduation corresponding to any angle is at a distance from the center equal to the secant of the angle in question. This center of the axis of revolution is the common center of the concentric circles and also the origin of the two systems of parallel lines.

The graduations on the arm are for the declinations. In the nomograms used the graduations have not been carried beyond three decimeters from the center, which limits the use of the instrument to declinations from 0° to slightly over 70°.

The zenith distances are graduated on one of the concentric circles at a convenient distance from the center. In the instrument shown in the illustration the distance is 25 centimeters. Since stars are never observed at zenith distances approaching 90°, the upper part of the quadrant is not used.

To determine the factors A, B, and C of a given star, revolve the transparent arm until the graduated line of the arm coincides with the star's zenith distance on the graduated arc. Holding the arm in this position, place a needle point at that point of the graduated line which corresponds to the star's declination. The position of this point in the three systems of equidistant lines gives the three factors, A being the ordinate, B the abscissa, and C the radius vector.

The nomogram shown is of thin bristol board pasted smoothly on thick cardboard. The transparent arm is of celluloid one-sixteenth of an inch thick. The axis of the arm is a solid metal cylinder with a head which fits against the back of the cardboard. The axis is made long so that the arm can be placed on it and revolved without being made fast.

The correction for aberration may be taken from the same nomogram, as follows: Set the revolving arm at that angle on the graduated circle which is equal to the latitude of the given station. From the graduated line of the arm read off the declination at each intersection with a broken-line ordinate. These declinations are the limits between which κ has the values $0^s.00$, $0^s.01$, $0^s.02$, etc., for the latitude of the station in question. By means of these limits the κ of any star can be immediately written down from its declination. The broken-line ordinates are drawn at distances from the origin equal to $\frac{.005}{.021}$, $\frac{.015}{.021}$, $\frac{.025}{.021}$, etc.---decimeters.

Tables XXa–XXe. The use of these tables is explained in the text on pages 147 to 150.

Table XXI. This table is a table of refraction for high elevations, such as attainable by an airplane. It is published by the U. S. Naval Observatory on the back outside cover of its 1947 Air Almanac.

Table I.—Correction to longitude for diurnal aberration, κ

($\kappa = 0^s.021 \cos \phi \sec \delta$)

(Negative for stars at upper culmination, positive for stars at lower culmination)

C = sec δ \ ϕ	0°	10°	20°	30°	40°	50°	60°	70°	80°
	s	s	s	s	s	s	s	s	s
1.00	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.00
1.20	.03	.02	.02	.02	.02	.02	.01	.01	.00
1.40	.03	.03	.03	.03	.02	.02	.01	.01	.01
1.60	.03	.03	.03	.03	.03	.02	.02	.01	.01
1.80	.04	.04	.04	.03	.03	.02	.02	.01	.01
2.00	0.04	0.04	0.04	0.04	0.03	0.03	0.02	0.01	0.01
2.20	.05	.05	.04	.04	.04	.03	.02	.02	.01
2.40	.05	.05	.05	.04	.04	.03	.03	.02	.01
2.60	.05	.05	.05	.05	.04	.04	.03	.02	.01
2.80	.06	.06	.06	.05	.05	.04	.03	.02	.01
3.00	0.06	0.06	0.06	0.05	0.05	0.04	0.03	0.02	0.01
3.20	.07	.07	.06	.06	.05	.04	.03	.02	.01
3.40	.07	.07	.07	.06	.05	.05	.04	.02	.01
3.60	.08	.07	.07	.07	.06	.05	.04	.03	.01
3.80	.08	.08	.08	.07	.06	.05	.04	.03	.01
4.00	0.08	0.08	0.08	0.07	0.06	0.05	0.04	0.03	0.01
4.20	.09	.09	.08	.08	.07	.06	.04	.03	.02
4.40	.09	.09	.09	.08	.07	.06	.05	.03	.02
4.60	.10	.10	.09	.08	.07	.06	.05	.03	.02
4.80	.10	.10	.09	.09	.08	.06	.05	.03	.02
5.00	0.10	0.10	0.10	0.09	0.08	0.07	0.05	0.04	0.02
5.20	.11	.11	.10	.09	.08	.07	.05	.04	.02
5.40	.11	.11	.11	.10	.09	.07	.06	.04	.02
5.60	.12	.12	.11	.10	.09	.08	.06	.04	.02
5.80	.12	.12	.11	.11	.09	.08	.06	.04	.02
6.00	0.13	0.12	0.12	0.11	0.10	0.08	0.06	0.04	0.02
6.50	.14	.13	.13	.12	.10	.09	.07	.05	.02
7.00	.15	.14	.14	.13	.11	.09	.07	.05	.03
7.50	.16	.16	.15	.14	.12	.10	.08	.05	.03
8.00	.17	.17	.16	.15	.13	.11	.08	.06	.03
8.50	0.18	0.18	0.17	0.15	0.14	0.11	0.09	0.06	0.03
9.00	.19	.19	.18	.16	.14	.12	.09	.06	.03
9.50	.20	.20	.19	.17	.15	.13	.10	.07	.03
10.00	.21	.21	.20	.18	.16	.14	.10	.07	.04
11.00	.23	.23	.22	.20	.18	.15	.12	.08	.04

Table II.—Weights of star transits, p

$$p=\frac{(.063)^2}{(.063)^2+(.036)^2\tan^2\delta}=\frac{1}{1+.32653\tan^2\delta}$$

δ	p	Tab. Diff.	δ	p	Tab. Diff.	δ	p	Tab. Diff.
°			°			°		
0	1.0000		30	0.9018		60	0.5052	
1	.9999	1	31	.8945	73	61	.4848	204
2	.9996	3	32	.8869	76	62	.4640	208
3	.9991	5	33	.8790	79	63	.4429	211
4	.9984	7	34	.8707	83	64	.4215	214
5	0.9975	9	35	0.8620	87	65	0.3997	218
6	.9964	11	36	.8530	90	66	.3778	219
7	.9951	13	37	.8436	94	67	.3556	222
8	.9936	15	38	.8338	98	68	.3333	223
9	.9919	17	39	.8236	102	69	.3109	224
10	0.9899	20	40	0.8131	105	70	0.2886	223
11	.9878	21	41	.8021	110	71	.2664	222
12	.9855	23	42	.7907	114	72	.2443	221
13	.9829	26	43	.7788	119	73	.2225	218
14	.9801	28	44	.7666	122	74	.2012	213
15	0.9771	30	45	0.7538	128	75	0.1802	210
16	.9739	32	46	.7407	131	76	.1599	203
17	.9704	35	47	.7270	137	77	.1403	196
18	.9667	37	48	.7129	141	78	.1215	188
19	.9627	40	49	.6983	146	79	.1037	178
20	0.9585	42	50	0.6832	151	80	0.0869	168
21	.9541	44	51	.6676	156	81	.0713	156
22	.9494	47	52	.6515	161	82	.0570	143
23	.9444	50	53	.6349	166	83	.0441	129
24	.9392	52	54	.6178	171	84	.0327	114
25	0.9337	55	55	0.6002	176	85	0.0229	98
26	.9279	58	56	.5822	180	86	.0148	81
27	.9219	60	57	.5636	186	87	.00834	646
28	.9155	64	58	.5446	190	88	.00372	462
29	.9088	67	59	.5251	195	89	.000932	2788
30	0.9018	70	60	0.5052	199	90	0.000000	932

See Special Publication No. 14, 5th edition, pp. 38, 39. Coast Survey Annual Report, 1872.

Table III.—A, B, C Factors for reduction of transit observations

TOP ARGUMENT=STAR'S DECLINATION, δ

SIDE ARGUMENT=STAR'S ZENITH DISTANCE, ζ

[For factor *A* use left-hand argument. For factor *B* use right-hand argument. Factor *C* is on bottom line of next page]

ζ \ δ	0°	10°	15°	20°	22°	24°	26°	28°	30°	32°	34°	36°	38°	40°	41°	42°	δ / ζ
°																	°
1	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	89
2	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.05	.05	.05	88
3	.05	.05	.05	.06	.06	.06	.06	.06	.06	.06	.06	.06	.07	.07	.07	.07	87
4	.07	.07	.07	.07	.08	.08	.08	.08	.08	.08	.08	.09	.09	.09	.09	.09	86
5	.09	.09	.09	.09	.09	.10	.10	.10	.10	.10	.10	.11	.11	.11	.11	.12	85
6	.11	.11	.11	.11	.11	.11	.12	.12	.12	.12	.13	.13	.13	.14	.14	.14	84
7	.12	.12	.13	.13	.13	.13	.14	.14	.14	.14	.15	.15	.15	.16	.16	.16	83
8	.14	.14	.14	.15	.15	.15	.16	.16	.16	.16	.17	.17	.18	.18	.18	.19	82
9	.16	.16	.16	.17	.17	.17	.17	.18	.18	.18	.19	.19	.20	.20	.21	.21	81
10	.17	.18	.18	.19	.19	.19	.19	.20	.20	.21	.21	.21	.22	.23	.23	.23	80
11	.19	.19	.20	.20	.21	.21	.21	.22	.22	.23	.23	.24	.24	.25	.25	.26	79
12	.21	.21	.22	.22	.22	.23	.23	.24	.24	.25	.25	.26	.26	.27	.27	.28	78
13	.22	.23	.23	.24	.24	.25	.25	.26	.26	.27	.27	.28	.29	.29	.30	.30	77
14	.24	.25	.25	.26	.26	.27	.27	.27	.28	.29	.29	.30	.31	.32	.32	.33	76
15	.26	.26	.27	.28	.28	.28	.29	.29	.30	.31	.31	.32	.33	.34	.34	.35	75
16	.28	.28	.29	.29	.30	.30	.31	.31	.32	.33	.33	.34	.35	.36	.37	.37	74
17	.29	.30	.30	.31	.31	.32	.33	.33	.34	.34	.35	.36	.37	.38	.39	.39	73
18	.31	.31	.32	.33	.33	.33	.34	.35	.36	.36	.37	.38	.39	.40	.41	.42	72
19	.33	.33	.34	.35	.35	.36	.36	.37	.38	.38	.39	.40	.41	.42	.43	.44	71
20	.34	.35	.35	.36	.37	.37	.38	.39	.40	.40	.41	.42	.43	.45	.45	.46	70
21	.36	.36	.37	.38	.39	.39	.40	.41	.41	.42	.43	.44	.45	.47	.47	.48	69
22	.37	.38	.39	.40	.40	.41	.42	.42	.43	.44	.45	.46	.48	.49	.50	.50	68
23	.39	.40	.41	.42	.42	.43	.44	.44	.45	.46	.47	.48	.50	.51	.52	.53	67
24	.41	.41	.42	.43	.44	.45	.45	.46	.47	.48	.49	.50	.52	.53	.54	.55	66
25	.42	.43	.44	.45	.46	.46	.47	.48	.49	.50	.51	.52	.54	.55	.56	.57	65
26	.44	.45	.45	.47	.47	.48	.49	.50	.51	.52	.53	.54	.56	.57	.58	.59	64
27	.45	.46	.47	.48	.49	.50	.51	.51	.52	.54	.55	.56	.58	.59	.60	.61	63
28	.47	.48	.49	.50	.51	.51	.52	.53	.54	.55	.57	.58	.60	.61	.62	.63	62
29	.48	.49	.50	.52	.52	.53	.54	.55	.56	.57	.58	.60	.61	.63	.64	.65	61
30	.50	.51	.52	.53	.54	.55	.56	.57	.58	.59	.60	.62	.63	.65	.66	.67	60
31	.52	.52	.53	.55	.56	.56	.57	.58	.59	.61	.62	.64	.65	.67	.68	.69	59
32	.53	.54	.55	.56	.57	.58	.59	.60	.61	.63	.64	.65	.67	.69	.70	.71	58
33	.55	.55	.56	.58	.59	.60	.61	.62	.63	.64	.66	.67	.69	.71	.72	.73	57
34	.56	.57	.58	.59	.60	.61	.62	.63	.65	.66	.67	.69	.71	.73	.74	.75	56
35	.57	.58	.59	.61	.62	.63	.64	.65	.66	.68	.69	.71	.73	.75	.76	.77	55
36	.59	.60	.61	.63	.63	.64	.65	.67	.68	.69	.71	.73	.75	.77	.78	.79	54
37	.60	.61	.62	.64	.65	.66	.67	.68	.70	.71	.73	.74	.76	.79	.80	.81	53
38	.62	.63	.64	.66	.66	.67	.69	.70	.71	.73	.74	.76	.78	.80	.82	.83	52
39	.63	.64	.65	.67	.68	.69	.70	.71	.73	.74	.76	.78	.80	.82	.83	.85	51
40	.64	.65	.67	.68	.69	.70	.72	.73	.74	.76	.77	.79	.82	.84	.85	.86	50
41	.66	.67	.68	.70	.71	.72	.73	.74	.76	.77	.79	.81	.83	.86	.87	.88	49
42	.67	.68	.69	.71	.72	.73	.74	.76	.77	.79	.81	.83	.85	.87	.89	.90	48
43	.68	.69	.71	.73	.74	.75	.76	.77	.79	.80	.82	.84	.86	.89	.90	.92	47
44	.69	.71	.72	.74	.75	.76	.77	.79	.80	.82	.84	.86	.88	.91	.92	.93	46
45	.71	.72	.73	.75	.76	.77	.79	.80	.82	.83	.85	.87	.90	.92	.94	.95	45
	0°	10°	15°	20°	22°	24°	26°	28°	30°	32°	34°	36°	38°	40°	41°	42°	

Table III.—A, B, C Factors for reduction of transit observations

TOP ARGUMENT=STAR'S DECLINATION, δ

SIDE ARGUMENT=STAR'S ZENITH DISTANCE, ζ

[For factor *A* use left-hand argument. For factor *B* use right-hand argument. Factor *C* is on line opposite ζ=90°]

ζ \ δ	0°	10°	15°	20°	22°	24°	26°	28°	30°	32°	34°	36°	38°	40°	41°	42°	δ / ζ
°																	°
46	0.72	0.73	0.74	0.77	0.78	0.79	0.80	0.82	0.83	0.85	0.87	0.89	0.91	0.94	0.95	0.97	44
47	.73	.74	.76	.78	.79	.80	.81	.83	.84	.86	.88	.90	.93	.95	.97	.98	43
48	.74	.76	.77	.79	.80	.81	.83	.84	.86	.88	.90	.92	.94	.97	.98	1.00	42
49	.75	.77	.78	.80	.81	.83	.84	.86	.87	.89	.91	.93	.96	.99	1.00	1.02	41
50	.77	.78	.79	.82	.83	.84	.85	.87	.89	.90	.92	.95	.97	1.00	1.01	1.03	40
51	.78	.79	.80	.83	.84	.85	.87	.88	.90	.92	.94	.96	.99	1.01	1.03	1.05	39
52	.79	.80	.82	.84	.85	.86	.88	.89	.91	.93	.95	.97	1.00	1.03	1.04	1.06	38
53	.80	.81	.83	.85	.86	.87	.89	.91	.92	.94	.96	.99	1.01	1.04	1.06	1.07	37
54	.81	.82	.84	.86	.87	.89	.90	.92	.93	.95	.98	1.00	1.03	1.06	1.07	1.09	36
55	.82	.83	.85	.87	.88	.90	.91	.93	.95	.97	.99	1.01	1.04	1.07	1.08	1.10	35
56	.83	.84	.86	.88	.89	.91	.92	.94	.96	.98	1.00	1.02	1.05	1.08	1.10	1.12	34
57	.84	.85	.87	.89	.90	.92	.93	.95	.97	.99	1.01	1.04	1.06	1.09	1.11	1.13	33
58	.85	.86	.88	.90	.91	.93	.94	.96	.98	1.00	1.02	1.05	1.08	1.11	1.12	1.14	32
59	.86	.87	.89	.91	.92	.94	.95	.97	.99	1.01	1.03	1.06	1.09	1.12	1.14	1.15	31
60	.87	.88	.90	.92	.93	.95	.96	.98	1.00	1.02	1.04	1.07	1.10	1.13	1.15	1.17	30
61	.87	.89	.91	.93	.94	.96	.97	.99	1.01	1.03	1.05	1.08	1.11	1.14	1.16	1.18	29
62	.88	.90	.91	.94	.95	.97	.98	1.00	1.02	1.04	1.06	1.09	1.12	1.15	1.17	1.19	28
63	.89	.91	.92	.95	.96	.98	.99	1.01	1.03	1.05	1.07	1.10	1.13	1.16	1.18	1.20	27
64	.90	.91	.93	.96	.97	.98	1.00	1.02	1.04	1.06	1.08	1.11	1.14	1.17	1.19	1.21	26
65	.91	.92	.94	.96	.98	.99	1.01	1.03	1.05	1.07	1.09	1.12	1.15	1.18	1.20	1.22	25
66	.91	.93	.95	.97	.99	1.00	1.02	1.04	1.06	1.08	1.10	1.13	1.16	1.19	1.21	1.23	24
67	.92	.94	.95	.98	.99	1.01	1.02	1.04	1.06	1.09	1.11	1.14	1.17	1.20	1.22	1.24	23
68	.93	.94	.96	.99	1.00	1.02	1.03	1.05	1.07	1.09	1.12	1.15	1.18	1.21	1.23	1.25	22
69	.93	.95	.97	.99	1.01	1.02	1.04	1.06	1.08	1.10	1.13	1.15	1.18	1.22	1.24	1.26	21
70	.94	.95	.97	1.00	1.01	1.03	1.05	1.06	1.09	1.11	1.13	1.16	1.19	1.23	1.25	1.26	20
71	.95	.96	.98	1.01	1.02	1.04	1.05	1.07	1.09	1.12	1.14	1.17	1.20	1.23	1.25	1.27	19
72	.95	.97	.98	1.01	1.03	1.04	1.06	1.08	1.10	1.12	1.15	1.17	1.21	1.24	1.26	1.28	18
73	.96	.97	.99	1.02	1.03	1.05	1.06	1.08	1.10	1.13	1.15	1.18	1.21	1.25	1.27	1.29	17
74	.96	.98	1.00	1.02	1.04	1.05	1.07	1.09	1.11	1.13	1.16	1.19	1.22	1.25	1.27	1.29	16
75	.97	.98	1.00	1.03	1.04	1.06	1.08	1.09	1.12	1.14	1.16	1.19	1.23	1.26	1.28	1.30	15
76	.97	.99	1.00	1.03	1.05	1.06	1.08	1.10	1.12	1.14	1.17	1.20	1.23	1.27	1.29	1.31	14
77	.97	.99	1.01	1.04	1.05	1.07	1.08	1.10	1.13	1.15	1.17	1.20	1.24	1.27	1.29	1.31	13
78	.98	.99	1.01	1.04	1.05	1.07	1.09	1.11	1.13	1.15	1.18	1.21	1.24	1.28	1.30	1.32	12
79	.98	1.00	1.02	1.04	1.06	1.08	1.09	1.11	1.13	1.16	1.18	1.21	1.25	1.28	1.30	1.32	11
80	.98	1.00	1.02	1.05	1.06	1.08	1.10	1.12	1.14	1.16	1.19	1.22	1.25	1.29	1.30	1.33	10
81	.99	1.00	1.02	1.05	1.07	1.08	1.10	1.12	1.14	1.17	1.19	1.22	1.25	1.29	1.31	1.33	9
82	.99	1.01	1.03	1.05	1.07	1.08	1.10	1.12	1.14	1.17	1.19	1.22	1.26	1.29	1.31	1.33	8
83	.99	1.01	1.03	1.06	1.07	1.09	1.10	1.12	1.15	1.17	1.20	1.23	1.26	1.30	1.32	1.34	7
84	.99	1.01	1.03	1.06	1.07	1.09	1.11	1.13	1.15	1.17	1.20	1.23	1.26	1.30	1.32	1.34	6
85	1.00	1.01	1.03	1.06	1.07	1.09	1.11	1.13	1.15	1.17	1.20	1.23	1.26	1.30	1.32	1.34	5
86	1.00	1.01	1.03	1.06	1.08	1.09	1.11	1.13	1.15	1.18	1.20	1.23	1.27	1.30	1.32	1.34	4
87	1.00	1.01	1.03	1.06	1.08	1.09	1.11	1.13	1.15	1.18	1.20	1.23	1.27	1.30	1.32	1.34	3
88	1.00	1.01	1.03	1.06	1.08	1.09	1.11	1.13	1.15	1.18	1.20	1.23	1.27	1.30	1.32	1.34	2
89	1.00	1.02	1.04	1.06	1.08	1.09	1.11	1.13	1.15	1.18	1.21	1.24	1.27	1.31	1.32	1.35	1
90	1.00	1.02	1.04	1.06	1.08	1.09	1.11	1.13	1.15	1.18	1.21	1.24	1.27	1.31	1.32	1.35	0
	0°	10°	15°	20°	22°	24°	26°	28°	30°	32°	34°	36°	38°	40°	41°	42°	

Table III.—A, B, C Factors for reduction of transit observations

TOP ARGUMENT=STAR'S DECLINATION, δ

SIDE ARGUMENT=STAR'S ZENITH DISTANCE, ζ

[For factor *A* use left-hand argument. For factor *B* use right-hand argument. Factor *C* is on bottom line of next page]

δ / ζ	42°	43°	44°	45°	46°	47°	48°	49°	50°	51°	52°	53°	54°	55°	56°	57°	δ / ζ
°																	
1	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	89
2	.05	.05	.05	.05	.05	.05	.05	.05	.05	.06	.06	.06	.06	.06	.06	.06	88
3	.07	.07	.07	.07	.07	.08	.08	.08	.08	.08	.08	.09	.09	.09	.09	.10	87
4	.09	.10	.10	.10	.10	.10	.10	.11	.11	.11	.11	.12	.12	.12	.12	.13	86
5	.12	.12	.12	.12	.13	.13	.13	.13	.13	.14	.14	.14	.15	.15	.16	.16	85
6	.14	.14	.15	.15	.15	.15	.16	.16	.16	.17	.17	.17	.18	.18	.19	.19	84
7	.16	.17	.17	.17	.18	.18	.18	.19	.19	.19	.20	.20	.21	.21	.22	.22	83
8	.19	.19	.19	.20	.20	.20	.21	.21	.22	.22	.23	.23	.24	.24	.25	.26	82
9	.21	.21	.22	.22	.22	.23	.23	.24	.24	.25	.25	.26	.27	.27	.28	.29	81
10	.23	.24	.24	.25	.25	.25	.26	.26	.27	.28	.28	.29	.30	.30	.31	.32	80
11	.26	.26	.27	.27	.28	.28	.28	.29	.30	.30	.31	.32	.32	.33	.34	.35	79
12	.28	.28	.29	.29	.30	.30	.31	.32	.32	.33	.34	.35	.35	.36	.37	.38	78
13	.30	.31	.31	.32	.32	.33	.34	.34	.35	.36	.36	.37	.38	.39	.40	.41	77
14	.33	.33	.34	.34	.35	.35	.36	.37	.38	.38	.39	.40	.41	.42	.43	.44	76
15	.35	.35	.36	.37	.37	.38	.39	.39	.40	.41	.42	.43	.44	.45	.46	.48	75
16	.37	.38	.38	.39	.40	.40	.41	.42	.43	.44	.45	.46	.47	.48	.49	.51	74
17	.39	.40	.41	.41	.42	.43	.44	.45	.45	.46	.47	.49	.50	.51	.52	.54	73
18	.42	.42	.43	.44	.44	.45	.46	.47	.48	.49	.50	.51	.53	.54	.55	.57	72
19	.44	.45	.45	.46	.47	.48	.49	.50	.51	.52	.53	.54	.55	.57	.58	.60	71
20	.46	.47	.48	.48	.49	.50	.51	.52	.53	.54	.56	.57	.58	.60	.61	.63	70
21	.48	.49	.50	.51	.52	.52	.54	.55	.56	.57	.58	.59	.61	.62	.64	.66	69
22	.50	.51	.52	.53	.54	.55	.56	.57	.58	.60	.61	.62	.64	.65	.67	.69	68
23	.53	.53	.54	.55	.56	.57	.58	.60	.61	.62	.63	.65	.66	.68	.70	.72	67
24	.55	.56	.57	.58	.59	.60	.61	.62	.63	.65	.66	.68	.69	.71	.73	.75	66
25	.57	.58	.59	.60	.61	.62	.63	.64	.66	.67	.69	.70	.72	.74	.76	.78	65
26	.59	.60	.61	.62	.63	.64	.65	.67	.68	.70	.71	.73	.75	.76	.78	.80	64
27	.61	.62	.63	.64	.65	.67	.68	.69	.71	.72	.74	.75	.77	.79	.81	.83	63
28	.63	.64	.65	.66	.68	.69	.70	.72	.73	.75	.76	.78	.80	.82	.84	.86	62
29	.65	.66	.67	.69	.70	.71	.72	.74	.75	.77	.79	.81	.82	.84	.87	.89	61
30	.67	.68	.69	.71	.72	.73	.75	.76	.78	.79	.81	.83	.85	.87	.89	.92	60
31	.69	.70	.72	.73	.74	.75	.77	.78	.80	.82	.84	.86	.88	.90	.92	.95	59
32	.71	.72	.74	.75	.76	.78	.79	.81	.82	.84	.86	.88	.90	.92	.95	.97	58
33	.73	.74	.76	.77	.78	.80	.81	.83	.85	.87	.88	.91	.93	.95	.97	1.00	57
34	.75	.76	.78	.79	.80	.82	.84	.85	.87	.89	.91	.93	.95	.97	1.00	1.03	56
35	.77	.78	.80	.81	.83	.84	.86	.87	.89	.91	.93	.95	.98	1.00	1.03	1.05	55
36	.79	.80	.82	.83	.85	.86	.88	.90	.91	.93	.95	.98	1.00	1.03	1.05	1.08	54
37	.81	.82	.84	.85	.87	.88	.90	.92	.94	.96	.98	1.00	1.02	1.05	1.08	1.10	53
38	.83	.84	.86	.87	.89	.90	.92	.94	.96	.98	1.00	1.02	1.05	1.07	1.10	1.13	52
39	.85	.86	.87	.89	.91	.92	.94	.96	.98	1.00	1.02	1.05	1.07	1.10	1.12	1.15	51
40	.86	.88	.89	.91	.93	.94	.96	.98	1.00	1.02	1.04	1.07	1.09	1.12	1.15	1.18	50
41	.88	.90	.91	.93	.94	.96	.98	1.00	1.02	1.04	1.07	1.09	1.12	1.14	1.17	1.20	49
42	.90	.91	.93	.95	.96	.98	1.00	1.02	1.04	1.06	1.09	1.11	1.14	1.17	1.20	1.23	48
43	.92	.93	.95	.96	.98	1.00	1.02	1.04	1.06	1.08	1.11	1.13	1.16	1.19	1.22	1.25	47
44	.93	.95	.97	.98	1.00	1.02	1.04	1.06	1.08	1.10	1.13	1.15	1.18	1.21	1.24	1.28	46
45	.95	.97	.98	1.00	1.02	1.04	1.06	1.08	1.10	1.12	1.15	1.17	1.20	1.23	1.26	1.30	45
	42°	43°	44°	45°	46°	47°	48°	49°	50°	51°	52°	53°	54°	55°	56°	57°	

Table III.—A, B, C Factors for reduction of transit observations

TOP ARGUMENT=STAR'S DECLINATION, δ

SIDE ARGUMENT=STAR'S ZENITH DISTANCE, ζ

[For factor *A* use left-hand argument. For factor *B* use right-hand argument. Factor *C* is on line opposite ζ=90°]

ζ \ δ	42°	43°	44°	45°	46°	47°	48°	49°	50°	51°	52°	53°	54°	55°	56°	57°	δ / ζ
°																	°
46	0.97	0.98	1.00	1.02	1.04	1.05	1.07	1.10	1.12	1.14	1.17	1.19	1.22	1.25	1.29	1.32	44
47	.98	1.00	1.02	1.03	1.05	1.07	1.09	1.11	1.14	1.16	1.19	1.21	1.24	1.27	1.31	1.34	43
48	1.00	1.02	1.03	1.05	1.07	1.09	1.11	1.13	1.16	1.18	1.21	1.23	1.26	1.30	1.33	1.36	42
49	1.02	1.03	1.05	1.07	1.09	1.11	1.13	1.15	1.17	1.20	1.23	1.25	1.28	1.32	1.35	1.39	41
50	1.03	1.05	1.06	1.08	1.10	1.12	1.14	1.17	1.19	1.22	1.24	1.27	1.30	1.34	1.37	1.41	40
51	1.05	1.06	1.08	1.10	1.12	1.14	1.16	1.18	1.21	1.23	1.26	1.29	1.32	1.35	1.39	1.43	39
52	1.06	1.08	1.10	1.11	1.13	1.15	1.18	1.20	1.23	1.25	1.28	1.31	1.34	1.37	1.41	1.45	38
53	1.07	1.09	1.11	1.13	1.15	1.17	1.19	1.22	1.24	1.27	1.30	1.33	1.36	1.39	1.43	1.47	37
54	1.09	1.11	1.12	1.14	1.16	1.19	1.21	1.23	1.26	1.29	1.31	1.34	1.38	1.41	1.45	1.49	36
55	1.10	1.12	1.14	1.16	1.18	1.20	1.22	1.25	1.27	1.30	1.33	1.36	1.39	1.43	1.46	1.50	35
56	1.12	1.13	1.15	1.17	1.19	1.22	1.24	1.26	1.29	1.32	1.35	1.38	1.41	1.45	1.48	1.52	34
57	1.13	1.15	1.17	1.19	1.21	1.23	1.25	1.28	1.31	1.33	1.36	1.39	1.43	1.46	1.50	1.54	33
58	1.14	1.16	1.18	1.20	1.22	1.24	1.27	1.29	1.32	1.35	1.38	1.41	1.44	1.48	1.52	1.56	32
59	1.15	1.17	1.19	1.21	1.23	1.26	1.28	1.31	1.33	1.36	1.39	1.42	1.46	1.49	1.53	1.57	31
60	1.17	1.18	1.20	1.22	1.25	1.27	1.29	1.32	1.35	1.38	1.41	1.44	1.47	1.51	1.55	1.59	30
61	1.18	1.20	1.22	1.24	1.26	1.28	1.31	1.33	1.36	1.39	1.42	1.45	1.49	1.53	1.56	1.61	29
62	1.19	1.21	1.23	1.25	1.27	1.29	1.32	1.35	1.37	1.40	1.43	1.47	1.50	1.54	1.58	1.62	28
63	1.20	1.22	1.24	1.26	1.28	1.31	1.33	1.36	1.39	1.42	1.45	1.48	1.52	1.55	1.59	1.64	27
64	1.21	1.23	1.25	1.27	1.29	1.32	1.34	1.37	1.40	1.43	1.46	1.49	1.53	1.57	1.61	1.65	26
65	1.22	1.24	1.26	1.28	1.30	1.33	1.35	1.38	1.41	1.44	1.47	1.51	1.54	1.58	1.62	1.66	25
66	1.23	1.25	1.27	1.29	1.32	1.34	1.37	1.39	1.42	1.45	1.48	1.52	1.55	1.59	1.63	1.68	24
67	1.24	1.26	1.28	1.30	1.33	1.35	1.38	1.40	1.43	1.46	1.50	1.53	1.57	1.60	1.65	1.69	23
68	1.25	1.27	1.29	1.31	1.33	1.36	1.39	1.41	1.44	1.47	1.51	1.54	1.58	1.62	1.66	1.70	22
69	1.26	1.28	1.30	1.32	1.34	1.37	1.40	1.42	1.45	1.48	1.52	1.55	1.59	1.63	1.67	1.71	21
70	1.26	1.28	1.31	1.33	1.35	1.38	1.40	1.43	1.46	1.49	1.53	1.56	1.60	1.64	1.68	1.73	20
71	1.27	1.29	1.31	1.34	1.36	1.39	1.41	1.44	1.47	1.50	1.54	1.57	1.61	1.65	1.69	1.74	19
72	1.28	1.30	1.32	1.34	1.37	1.39	1.42	1.45	1.48	1.51	1.54	1.58	1.62	1.66	1.70	1.75	18
73	1.29	1.31	1.33	1.35	1.38	1.40	1.43	1.46	1.49	1.52	1.55	1.59	1.63	1.67	1.71	1.76	17
74	1.29	1.31	1.34	1.36	1.38	1.41	1.44	1.46	1.49	1.53	1.56	1.60	1.63	1.68	1.72	1.76	16
75	1.30	1.32	1.34	1.37	1.39	1.42	1.44	1.47	1.50	1.53	1.57	1.60	1.64	1.68	1.73	1.77	15
76	1.31	1.33	1.35	1.37	1.40	1.42	1.45	1.48	1.51	1.54	1.58	1.61	1.65	1.69	1.73	1.78	14
77	1.31	1.33	1.35	1.38	1.40	1.43	1.46	1.48	1.52	1.55	1.58	1.62	1.66	1.70	1.74	1.79	13
78	1.32	1.34	1.36	1.38	1.41	1.43	1.46	1.49	1.52	1.55	1.59	1.62	1.66	1.70	1.75	1.80	12
79	1.32	1.34	1.36	1.39	1.41	1.44	1.47	1.50	1.53	1.56	1.59	1.63	1.67	1.71	1.76	1.80	11
80	1.33	1.35	1.37	1.39	1.42	1.44	1.47	1.50	1.53	1.56	1.60	1.64	1.67	1.72	1.76	1.81	10
81	1.33	1.35	1.37	1.40	1.42	1.45	1.48	1.51	1.54	1.57	1.60	1.64	1.68	1.72	1.77	1.81	9
82	1.33	1.35	1.38	1.40	1.43	1.45	1.48	1.51	1.54	1.57	1.61	1.64	1.68	1.73	1.77	1.82	8
83	1.34	1.36	1.38	1.40	1.43	1.46	1.48	1.51	1.54	1.58	1.61	1.65	1.69	1.73	1.77	1.82	7
84	1.34	1.36	1.38	1.41	1.43	1.46	1.49	1.52	1.55	1.58	1.62	1.65	1.69	1.73	1.78	1.83	6
85	1.34	1.36	1.38	1.41	1.43	1.46	1.49	1.52	1.55	1.58	1.62	1.65	1.69	1.74	1.78	1.83	5
86	1.34	1.36	1.39	1.41	1.44	1.46	1.49	1.52	1.55	1.59	1.62	1.66	1.70	1.74	1.78	1.83	4
87	1.34	1.37	1.39	1.41	1.44	1.46	1.49	1.52	1.55	1.59	1.62	1.66	1.70	1.74	1.79	1.83	3
88	1.34	1.37	1.39	1.41	1.44	1.46	1.49	1.52	1.55	1.59	1.62	1.66	1.70	1.74	1.79	1.83	2
89	1.35	1.37	1.39	1.41	1.44	1.47	1.49	1.52	1.56	1.59	1.62	1.66	1.70	1.74	1.79	1.84	1
90	1.35	1.37	1.39	1.41	1.44	1.47	1.49	1.52	1.56	1.59	1.62	1.66	1.70	1.74	1.79	1.84	0
	42°	43°	44°	45°	46°	47°	48°	49°	50°	51°	52°	53°	54°	55°	56°	57°	

Table III.—A, B, C Factors for reduction of transit observations

TOP ARGUMENT=STAR'S DECLINATION, δ

SIDE ARGUMENT=STAR'S ZENITH DISTANCE, ζ

[For factor *A* use left-hand argument. For factor *B* use right-hand argument. Factor *C* is on bottom line of next page]

ζ \ δ	57°	58°	58½°	59°	59½°	60°	60½°	61°	61½°	62°	62½°	63°	63½°	64°	64½°	65°	δ / ζ
°																	°
1	0.03	0.03	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	89
2	.06	.07	.07	.07	.07	.07	.07	.07	.07	.07	.08	.08	.08	.08	.08	.08	88
3	.10	.10	.10	.10	.10	.10	.11	.11	.11	.11	.11	.12	.12	.12	.12	.12	87
4	.13	.13	.13	.14	.14	.14	.14	.14	.15	.15	.15	.15	.16	.16	.16	.17	86
5	.16	.16	.17	.17	.17	.17	.18	.18	.18	.19	.19	.19	.19	.20	.20	.21	85
6	.19	.20	.20	.20	.21	.21	.21	.22	.22	.22	.23	.23	.23	.24	.24	.25	84
7	.22	.23	.23	.24	.24	.24	.25	.25	.26	.26	.26	.27	.27	.28	.28	.29	83
8	.26	.26	.27	.27	.27	.28	.28	.29	.29	.30	.30	.31	.31	.32	.32	.33	82
9	.29	.29	.30	.30	.31	.31	.32	.32	.33	.33	.34	.35	.35	.36	.36	.37	81
10	.32	.33	.33	.34	.34	.35	.35	.36	.36	.37	.38	.38	.39	.40	.40	.41	80
11	.35	.36	.36	.37	.38	.38	.39	.39	.40	.41	.41	.42	.43	.44	.44	.45	79
12	.38	.39	.40	.40	.41	.42	.42	.43	.44	.44	.45	.46	.47	.47	.48	.49	78
13	.41	.42	.43	.44	.44	.45	.46	.46	.47	.48	.49	.50	.50	.51	.52	.53	77
14	.44	.46	.46	.47	.48	.48	.49	.50	.51	.52	.52	.53	.54	.55	.56	.57	76
15	.48	.49	.50	.50	.51	.52	.53	.53	.54	.55	.56	.57	.58	.59	.60	.61	75
16	.51	.52	.53	.54	.54	.55	.56	.57	.58	.59	.60	.61	.62	.63	.64	.65	74
17	.54	.55	.56	.57	.58	.58	.59	.60	.61	.62	.63	.64	.66	.67	.68	.69	73
18	.57	.58	.59	.60	.61	.62	.63	.64	.65	.66	.67	.68	.69	.70	.72	.73	72
19	.60	.61	.62	.63	.64	.65	.66	.67	.68	.69	.70	.72	.73	.74	.76	.77	71
20	.63	.64	.65	.66	.67	.68	.69	.70	.72	.73	.74	.75	.77	.78	.79	.81	70
21	.66	.68	.69	.70	.71	.72	.73	.74	.75	.76	.78	.79	.80	.82	.83	.85	69
22	.69	.71	.72	.73	.74	.75	.76	.77	.78	.80	.81	.82	.84	.85	.87	.89	68
23	.72	.74	.75	.76	.77	.78	.79	.81	.82	.83	.85	.86	.88	.89	.91	.92	67
24	.75	.77	.78	.79	.80	.81	.83	.84	.85	.87	.88	.90	.91	.93	.94	.96	66
25	.78	.80	.81	.82	.83	.85	.86	.87	.89	.90	.92	.93	.95	.96	.98	1.00	65
26	.80	.83	.84	.85	.86	.88	.89	.90	.92	.93	.95	.97	.98	1.00	1.02	1.04	64
27	.83	.86	.87	.88	.89	.91	.92	.94	.95	.97	.98	1.00	1.02	1.04	1.05	1.07	63
28	.86	.89	.90	.91	.93	.94	.95	.97	.98	1.00	1.02	1.03	1.05	1.07	1.09	1.11	62
29	.89	.91	.93	.94	.96	.97	.98	1.00	1.02	1.03	1.05	1.07	1.09	1.11	1.13	1.15	61
30	.92	.94	.96	.97	.99	1.00	1.01	1.03	1.05	1.07	1.08	1.10	1.12	1.14	1.16	1.18	60
31	.95	.97	.99	1.00	1.01	1.03	1.05	1.06	1.08	1.10	1.11	1.13	1.15	1.17	1.20	1.22	59
32	.97	1.00	1.01	1.03	1.04	1.06	1.08	1.09	1.11	1.13	1.15	1.17	1.19	1.21	1.23	1.25	58
33	1.00	1.03	1.04	1.06	1.07	1.09	1.11	1.12	1.14	1.16	1.18	1.20	1.22	1.24	1.26	1.29	57
34	1.03	1.05	1.07	1.09	1.10	1.12	1.14	1.15	1.17	1.19	1.21	1.23	1.25	1.27	1.30	1.32	56
35	1.05	1.08	1.10	1.11	1.13	1.15	1.16	1.18	1.20	1.22	1.24	1.26	1.29	1.31	1.33	1.36	55
36	1.08	1.11	1.12	1.14	1.16	1.18	1.19	1.21	1.23	1.25	1.27	1.30	1.32	1.34	1.37	1.39	54
37	1.10	1.14	1.15	1.17	1.19	1.20	1.22	1.24	1.26	1.28	1.30	1.33	1.35	1.37	1.40	1.42	53
38	1.13	1.16	1.18	1.20	1.21	1.23	1.25	1.27	1.29	1.31	1.33	1.36	1.38	1.40	1.43	1.46	52
39	1.15	1.19	1.20	1.22	1.24	1.26	1.28	1.30	1.32	1.34	1.36	1.39	1.41	1.43	1.46	1.49	51
40	1.18	1.21	1.23	1.25	1.27	1.29	1.31	1.33	1.35	1.37	1.39	1.42	1.44	1.47	1.49	1.52	50
41	1.20	1.24	1.26	1.27	1.29	1.31	1.33	1.35	1.37	1.40	1.42	1.45	1.47	1.50	1.52	1.55	49
42	1.23	1.26	1.28	1.30	1.32	1.34	1.36	1.38	1.40	1.42	1.45	1.47	1.50	1.53	1.55	1.58	48
43	1.25	1.29	1.30	1.32	1.34	1.36	1.39	1.41	1.43	1.45	1.48	1.50	1.53	1.56	1.58	1.61	47
44	1.28	1.31	1.33	1.35	1.37	1.39	1.41	1.43	1.46	1.48	1.50	1.53	1.56	1.58	1.61	1.64	46
45	1.30	1.33	1.35	1.37	1.39	1.41	1.44	1.46	1.48	1.51	1.53	1.56	1.58	1.61	1.64	1.67	45
	57°	58°	58½°	59°	59½°	60°	60½°	61°	61½°	62°	62½°	63°	63½°	64°	64½°	65°	

Table III.—A, B, C Factors for reduction of transit observations

TOP ARGUMENT=STAR'S DECLINATION, δ

SIDE ARGUMENT=STAR'S ZENITH DISTANCE, ζ

[For factor *A* use left-hand argument. For factor *B* use right-hand argument. Factor *C* is on line opposite ζ=90°]

ζ \ δ	57°	58°	58½°	59°	59½°	60°	60½°	61°	61½°	62°	62½°	63°	63½°	64°	64½°	65°	δ / ζ
°																	°
46	1.32	1.36	1.38	1.40	1.42	1.44	1.46	1.48	1.51	1.53	1.56	1.58	1.61	1.64	1.67	1.70	44
47	1.34	1.38	1.40	1.42	1.44	1.46	1.49	1.51	1.53	1.56	1.58	1.61	1.64	1.67	1.70	1.73	43
48	1.36	1.40	1.42	1.44	1.46	1.48	1.51	1.53	1.55	1.58	1.60	1.63	1.66	1.69	1.72	1.76	42
49	1.39	1.42	1.44	1.47	1.49	1.51	1.53	1.56	1.58	1.61	1.63	1.66	1.69	1.72	1.75	1.79	41
50	1.41	1.44	1.47	1.49	1.51	1.53	1.56	1.58	1.60	1.63	1.66	1.69	1.72	1.75	1.78	1.81	40
51	1.43	1.47	1.49	1.51	1.53	1.55	1.58	1.60	1.63	1.66	1.68	1.71	1.74	1.77	1.80	1.84	39
52	1.45	1.49	1.51	1.53	1.55	1.58	1.60	1.63	1.65	1.68	1.71	1.74	1.77	1.80	1.83	1.86	38
53	1.47	1.51	1.53	1.55	1.57	1.60	1.62	1.65	1.67	1.70	1.73	1.76	1.79	1.82	1.85	1.89	37
54	1.49	1.53	1.55	1.57	1.59	1.62	1.64	1.67	1.69	1.72	1.75	1.78	1.81	1.85	1.88	1.91	36
55	1.50	1.55	1.57	1.59	1.61	1.64	1.66	1.69	1.72	1.74	1.77	1.80	1.84	1.87	1.90	1.94	35
56	1.52	1.56	1.59	1.61	1.63	1.66	1.68	1.71	1.74	1.77	1.80	1.83	1.86	1.89	1.93	1.96	34
57	1.54	1.58	1.61	1.63	1.65	1.68	1.70	1.73	1.76	1.79	1.82	1.85	1.88	1.91	1.95	1.98	33
58	1.56	1.60	1.62	1.65	1.67	1.70	1.72	1.75	1.78	1.81	1.84	1.87	1.90	1.93	1.97	2.01	32
59	1.57	1.62	1.64	1.66	1.69	1.71	1.74	1.77	1.80	1.83	1.86	1.89	1.92	1.96	1.99	2.03	31
60	1.59	1.63	1.66	1.68	1.71	1.73	1.76	1.79	1.81	1.84	1.88	1.91	1.94	1.98	2.01	2.05	30
61	1.61	1.65	1.67	1.70	1.72	1.75	1.78	1.80	1.83	1.86	1.89	1.93	1.96	2.00	2.03	2.07	29
62	1.62	1.67	1.69	1.71	1.74	1.77	1.79	1.82	1.85	1.88	1.91	1.94	1.98	2.01	2.05	2.09	28
63	1.64	1.68	1.70	1.73	1.76	1.78	1.81	1.84	1.87	1.90	1.93	1.96	2.00	2.03	2.07	2.11	27
64	1.65	1.70	1.72	1.75	1.77	1.80	1.83	1.85	1.88	1.91	1.95	1.98	2.02	2.05	2.09	2.13	26
65	1.66	1.71	1.73	1.76	1.79	1.81	1.84	1.87	1.90	1.93	1.96	2.00	2.03	2.07	2.11	2.14	25
66	1.68	1.72	1.75	1.77	1.80	1.83	1.85	1.88	1.91	1.95	1.98	2.01	2.05	2.08	2.12	2.16	24
67	1.69	1.74	1.76	1.79	1.81	1.84	1.87	1.90	1.93	1.96	1.99	2.03	2.06	2.10	2.14	2.18	23
68	1.70	1.75	1.77	1.80	1.83	1.85	1.88	1.91	1.94	1.97	2.01	2.04	2.08	2.11	2.15	2.19	22
69	1.71	1.76	1.79	1.81	1.84	1.87	1.90	1.93	1.96	1.99	2.02	2.06	2.09	2.13	2.17	2.21	21
70	1.73	1.77	1.80	1.82	1.85	1.88	1.91	1.94	1.97	2.00	2.03	2.07	2.11	2.14	2.18	2.22	20
71	1.74	1.78	1.81	1.84	1.86	1.89	1.92	1.95	1.98	2.01	2.05	2.08	2.12	2.16	2.20	2.24	19
72	1.75	1.79	1.82	1.85	1.87	1.90	1.93	1.96	1.99	2.03	2.06	2.09	2.13	2.17	2.21	2.25	18
73	1.76	1.80	1.83	1.86	1.88	1.91	1.94	1.97	2.00	2.04	2.07	2.11	2.14	2.18	2.22	2.26	17
74	1.76	1.81	1.84	1.87	1.89	1.92	1.95	1.98	2.01	2.05	2.08	2.12	2.15	2.19	2.23	2.27	16
75	1.77	1.82	1.85	1.88	1.90	1.93	1.96	1.99	2.02	2.06	2.09	2.13	2.16	2.20	2.24	2.29	15
76	1.78	1.83	1.86	1.88	1.91	1.94	1.97	2.00	2.03	2.07	2.10	2.14	2.17	2.21	2.25	2.30	14
77	1.79	1.84	1.86	1.89	1.92	1.95	1.98	2.01	2.04	2.07	2.11	2.15	2.18	2.22	2.26	2.31	13
78	1.80	1.85	1.87	1.90	1.93	1.96	1.99	2.02	2.05	2.08	2.12	2.15	2.19	2.23	2.27	2.31	12
79	1.80	1.85	1.88	1.91	1.93	1.96	1.99	2.02	2.06	2.09	2.13	2.16	2.20	2.24	2.28	2.32	11
80	1.81	1.86	1.88	1.91	1.94	1.97	2.00	2.03	2.06	2.10	2.13	2.17	2.21	2.25	2.29	2.33	10
81	1.81	1.86	1.89	1.92	1.95	1.98	2.01	2.04	2.07	2.10	2.14	2.18	2.21	2.25	2.29	2.34	9
82	1.82	1.87	1.90	1.92	1.95	1.98	2.01	2.04	2.08	2.11	2.15	2.18	2.22	2.26	2.30	2.34	8
83	1.82	1.87	1.90	1.93	1.96	1.99	2.02	2.05	2.08	2.12	2.15	2.19	2.22	2.26	2.31	2.35	7
84	1.83	1.88	1.90	1.93	1.96	1.99	2.02	2.05	2.08	2.12	2.15	2.19	2.23	2.27	2.31	2.35	6
85	1.83	1.88	1.91	1.93	1.96	1.99	2.02	2.05	2.09	2.12	2.16	2.19	2.23	2.27	2.31	2.36	5
86	1.83	1.88	1.91	1.94	1.97	2.00	2.03	2.06	2.09	2.13	2.16	2.20	2.24	2.28	2.32	2.36	4
87	1.83	1.88	1.91	1.94	1.97	2.00	2.03	2.06	2.09	2.13	2.16	2.20	2.24	2.28	2.32	2.36	3
88	1.83	1.89	1.91	1.94	1.97	2.00	2.03	2.06	2.09	2.13	2.16	2.20	2.24	2.28	2.32	2.36	2
89	1.84	1.89	1.91	1.94	1.97	2.00	2.03	2.06	2.10	2.13	2.17	2.20	2.24	2.28	2.32	2.37	1
90	1.84	1.89	1.91	1.94	1.97	2.00	2.03	2.06	2.10	2.13	2.17	2.20	2.24	2.28	2.32	2.37	0
	57°	58°	58½°	59°	59½°	60°	60½°	61°	61½°	62°	62½°	63°	63½°	64°	64½°	65°	

Table III.—A, B, C Factors for reduction of transit observations

TOP ARGUMENT=STAR'S DECLINATION, δ

SIDE ARGUMENT=STAR'S ZENITH DISTANCE, ζ

[For factor *A* use left-hand argument. For factor *B* use right-hand argument. Factor *C* is on bottom line of next page]

ζ \ δ	65°	65½°	66°	66½°	67°	67½°	68°	68½°	69°	69° 10′	69° 20′	69° 30′	69° 40′	69° 50′	70°	70° 10′	ζ
°																	°
1	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	89
2	.08	.08	.09	.09	.09	.09	.09	.10	.10	.10	.10	.10	.10	.10	.10	.10	88
3	.12	.13	.13	.13	.13	.14	.14	.14	.15	.15	.15	.15	.15	.15	.15	.15	87
4	.17	.17	.17	.18	.18	.18	.19	.19	.20	.20	.20	.20	.20	.20	.20	.20	86
5	.21	.21	.21	.22	.22	.23	.23	.24	.24	.24	.25	.25	.25	.25	.25	.26	85
6	.25	.25	.26	.26	.27	.27	.28	.28	.29	.29	.30	.30	.30	.30	.31	.31	84
7	.29	.29	.30	.31	.31	.32	.33	.33	.34	.34	.34	.35	.35	.35	.36	.36	83
8	.33	.34	.34	.35	.36	.36	.37	.38	.39	.39	.39	.40	.40	.40	.41	.41	82
9	.37	.38	.39	.39	.40	.41	.42	.43	.44	.44	.44	.45	.45	.45	.46	.46	81
10	.41	.42	.43	.43	.44	.45	.46	.47	.48	.49	.49	.50	.50	.50	.51	.51	80
11	.45	.46	.47	.48	.49	.50	.51	.52	.53	.54	.54	.54	.55	.55	.56	.56	79
12	.49	.50	.51	.52	.53	.54	.56	.57	.58	.58	.59	.59	.60	.60	.61	.61	78
13	.53	.54	.55	.56	.58	.59	.60	.61	.63	.63	.64	.64	.65	.65	.66	.66	77
14	.57	.58	.59	.61	.62	.63	.65	.66	.67	.68	.68	.69	.70	.70	.71	.71	76
15	.61	.62	.64	.65	.66	.68	.69	.71	.72	.73	.73	.74	.74	.75	.76	.76	75
16	.65	.66	.68	.69	.71	.72	.74	.75	.77	.78	.78	.79	.79	.80	.81	.81	74
17	.69	.70	.72	.73	.75	.76	.78	.80	.81	.82	.83	.83	.84	.85	.85	.86	73
18	.73	.74	.76	.77	.79	.81	.83	.84	.86	.87	.88	.88	.89	.90	.90	.91	72
19	.77	.78	.80	.82	.83	.85	.87	.89	.91	.92	.92	.93	.94	.94	.95	.96	71
20	.81	.82	.84	.86	.88	.89	.91	.93	.95	.96	.97	.98	.98	.99	1.00	1.01	70
21	.85	.86	.88	.90	.92	.94	.96	.98	1.00	1.01	1.02	1.02	1.03	1.04	1.05	1.06	69
22	.89	.90	.92	.94	.96	.98	1.00	1.02	1.05	1.05	1.06	1.07	1.08	1.09	1.09	1.10	68
23	.92	.94	.96	.98	1.00	1.02	1.04	1.07	1.09	1.10	1.11	1.12	1.12	1.13	1.14	1.15	67
24	.96	.98	1.00	1.02	1.04	1.06	1.09	1.11	1.14	1.14	1.15	1.16	1.17	1.18	1.19	1.20	66
25	1.00	1.02	1.04	1.06	1.08	1.10	1.13	1.15	1.18	1.19	1.20	1.21	1.22	1.23	1.24	1.25	65
26	1.04	1.06	1.08	1.10	1.12	1.15	1.17	1.20	1.22	1.23	1.24	1.25	1.26	1.27	1.28	1.29	64
27	1.07	1.09	1.12	1.14	1.16	1.19	1.21	1.24	1.27	1.28	1.29	1.30	1.31	1.32	1.33	1.34	63
28	1.11	1.13	1.15	1.18	1.20	1.23	1.25	1.28	1.31	1.32	1.33	1.34	1.35	1.36	1.37	1.38	62
29	1.15	1.17	1.19	1.22	1.24	1.27	1.29	1.32	1.35	1.36	1.37	1.38	1.40	1.41	1.42	1.43	61
30	1.18	1.21	1.23	1.25	1.28	1.31	1.33	1.36	1.39	1.41	1.42	1.43	1.44	1.45	1.46	1.47	60
31	1.22	1.24	1.27	1.29	1.32	1.35	1.38	1.40	1.44	1.45	1.46	1.47	1.48	1.49	1.51	1.52	59
32	1.25	1.28	1.30	1.33	1.36	1.39	1.42	1.45	1.48	1.49	1.50	1.51	1.52	1.54	1.55	1.56	58
33	1.29	1.31	1.34	1.37	1.39	1.42	1.45	1.49	1.52	1.53	1.54	1.55	1.57	1.58	1.59	1.60	57
34	1.32	1.35	1.37	1.40	1.43	1.46	1.49	1.53	1.56	1.57	1.58	1.60	1.61	1.62	1.63	1.65	56
35	1.36	1.38	1.41	1.44	1.47	1.50	1.53	1.56	1.60	1.61	1.62	1.64	1.65	1.66	1.68	1.69	55
36	1.39	1.42	1.45	1.47	1.51	1.54	1.57	1.60	1.64	1.65	1.66	1.68	1.69	1.70	1.72	1.73	54
37	1.42	1.45	1.48	1.51	1.54	1.57	1.61	1.64	1.68	1.69	1.70	1.72	1.73	1.74	1.76	1.77	53
38	1.46	1.48	1.51	1.54	1.58	1.61	1.64	1.68	1.72	1.73	1.74	1.76	1.77	1.79	1.80	1.82	52
39	1.49	1.52	1.55	1.58	1.61	1.65	1.68	1.72	1.75	1.77	1.78	1.80	1.81	1.82	1.84	1.86	51
40	1.52	1.55	1.58	1.61	1.65	1.68	1.72	1.75	1.79	1.81	1.82	1.84	1.85	1.86	1.88	1.89	50
41	1.55	1.58	1.61	1.64	1.68	1.71	1.75	1.79	1.83	1.84	1.86	1.87	1.89	1.90	1.92	1.93	49
42	1.58	1.61	1.64	1.68	1.71	1.75	1.79	1.83	1.87	1.88	1.90	1.91	1.93	1.94	1.96	1.97	48
43	1.61	1.64	1.68	1.71	1.75	1.78	1.82	1.86	1.90	1.92	1.93	1.95	1.96	1.98	1.99	2.01	47
44	1.64	1.67	1.71	1.74	1.78	1.82	1.85	1.90	1.94	1.95	1.97	1.98	2.00	2.02	2.03	2.05	46
45	1.67	1.70	1.74	1.77	1.81	1.85	1.89	1.93	1.97	1.99	2.00	2.02	2.04	2.05	2.07	2.08	45
	65°	65½°	66°	66½°	67°	67½°	68°	68½°	69°	69° 10′	69° 20′	69° 30′	69° 40′	69° 50′	70°	70° 10′	

Table III.—A, B, C Factors for reduction of transit observations

TOP ARGUMENT=STAR'S DECLINATION, δ

SIDE ARGUMENT=STAR'S ZENITH DISTANCE, ζ

[For factor A use left-hand argument. For factor B use right-hand argument. Factor C is on line opposite $\zeta=90°$]

ζ \ δ	65°	65½°	66°	66½°	67°	67½°	68°	68½°	69°	69° 10′	69° 20′	69° 30′	69° 40′	69° 50′	70°	70° 10′	δ / ζ
°																	°
46	1.70	1.74	1.77	1.80	1.84	1.88	1.92	1.96	2.01	2.02	2.04	2.05	2.07	2.09	2.10	2.12	44
47	1.73	1.76	1.80	1.83	1.87	1.91	1.95	2.00	2.04	2.06	2.07	2.09	2.10	2.12	2.14	2.16	43
48	1.76	1.79	1.83	1.86	1.90	1.94	1.98	2.03	2.07	2.09	2.11	2.12	2.14	2.16	2.17	2.19	42
49	1.79	1.82	1.86	1.89	1.93	1.97	2.01	2.06	2.11	2.12	2.14	2.16	2.17	2.19	2.21	2.22	41
50	1.81	1.85	1.88	1.92	1.96	2.00	2.04	2.09	2.14	2.15	2.17	2.19	2.20	2.22	2.24	2.26	40
51	1.84	1.87	1.91	1.95	1.99	2.03	2.07	2.12	2.17	2.18	2.20	2.22	2.24	2.25	2.27	2.29	39
52	1.86	1.90	1.94	1.98	2.02	2.06	2.10	2.15	2.20	2.22	2.23	2.25	2.27	2.29	2.30	2.32	38
53	1.89	1.93	1.96	2.00	2.04	2.09	2.13	2.18	2.23	2.25	2.26	2.28	2.30	2.32	2.33	2.35	37
54	1.91	1.95	1.99	2.03	2.07	2.11	2.16	2.21	2.26	2.28	2.29	2.31	2.33	2.35	2.37	2.38	36
55	1.94	1.98	2.01	2.05	2.10	2.14	2.19	2.23	2.29	2.30	2.32	2.34	2.36	2.38	2.40	2.41	35
56	1.96	2.00	2.04	2.08	2.12	2.17	2.21	2.26	2.31	2.33	2.35	2.37	2.39	2.40	2.42	2.44	34
57	1.98	2.02	2.06	2.10	2.15	2.19	2.24	2.29	2.34	2.36	2.38	2.39	2.41	2.43	2.45	2.47	33
58	2.01	2.05	2.08	2.13	2.17	2.22	2.26	2.31	2.37	2.38	2.40	2.42	2.44	2.46	2.48	2.50	32
59	2.03	2.07	2.11	2.15	2.19	2.24	2.29	2.34	2.39	2.41	2.43	2.45	2.47	2.49	2.51	2.53	31
60	2.05	2.09	2.13	2.17	2.22	2.26	2.31	2.36	2.42	2.44	2.45	2.47	2.49	2.51	2.53	2.55	30
61	2.07	2.11	2.15	2.19	2.24	2.29	2.33	2.39	2.44	2.46	2.48	2.50	2.52	2.54	2.56	2.58	29
62	2.09	2.13	2.17	2.21	2.26	2.31	2.36	2.41	2.46	2.48	2.50	2.52	2.54	2.56	2.58	2.60	28
63	2.11	2.15	2.19	2.23	2.28	2.33	2.38	2.43	2.49	2.50	2.52	2.54	2.56	2.58	2.60	2.63	27
64	2.13	2.17	2.21	2.25	2.30	2.35	2.40	2.45	2.51	2.53	2.55	2.57	2.59	2.61	2.63	2.65	26
65	2.14	2.19	2.23	2.27	2.32	2.37	2.42	2.47	2.53	2.55	2.57	2.59	2.61	2.63	2.65	2.67	25
66	2.16	2.20	2.25	2.29	2.34	2.39	2.44	2.49	2.55	2.57	2.59	2.61	2.63	2.65	2.67	2.69	24
67	2.18	2.22	2.26	2.31	2.36	2.41	2.46	2.51	2.57	2.59	2.61	2.63	2.65	2.67	2.69	2.71	23
68	2.19	2.24	2.28	2.32	2.37	2.42	2.47	2.53	2.59	2.61	2.63	2.65	2.67	2.69	2.71	2.73	22
69	2.21	2.25	2.30	2.34	2.39	2.44	2.49	2.55	2.61	2.62	2.64	2.67	2.69	2.71	2.73	2.75	21
70	2.22	2.27	2.31	2.36	2.40	2.46	2.51	2.56	2.62	2.64	2.66	2.68	2.70	2.73	2.75	2.77	20
71	2.24	2.28	2.32	2.37	2.42	2.47	2.52	2.58	2.64	2.66	2.68	2.70	2.72	2.74	2.77	2.79	19
72	2.25	2.29	2.34	2.38	2.43	2.49	2.54	2.59	2.65	2.67	2.70	2.72	2.74	2.76	2.78	2.80	18
73	2.26	2.31	2.35	2.40	2.45	2.50	2.55	2.61	2.67	2.69	2.71	2.73	2.75	2.77	2.80	2.82	17
74	2.27	2.32	2.36	2.41	2.46	2.51	2.57	2.62	2.68	2.70	2.72	2.74	2.77	2.79	2.81	2.83	16
75	2.29	2.33	2.37.	2.42	2.47	2.52	2.58	2.64	2.70	2.72	2.74	2.76	2.78	2.80	2.82	2.85	15
76	2.30	2.34	2.39	2.43	2.48	2.54	2.59	2.65	2.71	2.73	2.75	2.77	2.79	2.81	2.84	2.86	14
77	2.31	2.35	2.40	2.44	2.49	2.55	2.60	2.66	2.72	2.74	2.76	2.78	2.80	2.83	2.85	2.87	13
78	2.31	2.36	2.40	2.45	2.50	2.56	2.61	2.67	2.73	2.75	2.77	2.79	2.81	2.84	2.86	2.88	12
79	2.32	2.37	2.41	2.46	2.51	2.57	2.62	2.68	2.74	2.76	2.78	2.80	2.82	2.85	2.87	2.89	11
80	2.33	2.38	2.42	2.47	2.52	2.57	2.63	2.69	2.75	2.77	2.79	2.81	2.83	2.86	2.88	2.90	10
81	2.34	2.38	2.43	2.48	2.53	2.58	2.64	2.69	2.76	2.78	2.80	2.82	2.84	2.86	2.89	2.91	9
82	2.34	2.39	2.43	2.48	2.53	2.59	2.64	2.70	2.76	2.78	2.81	2.83	2.85	2.87	2.90	2.92	8
83	2.35	2.39	2.44	2.49	2.54	2.59	2.65	2.71	2.77	2.79	2.81	2.83	2.86	2.88	2.90	2.92	7
84	2.35	2.40	2.45	2.49	2.55	2.60	2.66	2.71	2.78	2.80	2.82	2.84	2.86	2.88	2.91	2.93	6
85	2.36	2.40	2.45	2.50	2.55	2.60	2.66	2.72	2.78	2.80	2.82	2.84	2.87	2.89	2.91	2.94	5
86	2.36	2.41	2.45	2.50	2.55	2.61	2.66	2.72	2.78	2.80	2.83	2.85	2.87	2.89	2.92	2.94	4
87	2.36	2.41	2.46	2.50	2.56	2.61	2.67	2.72	2.79	2.81	2.83	2.85	2.87	2.90	2.92	2.94	3
88	2.36	2.41	2.46	2.51	2.56	2.61	2.67	2.73	2.79	2.81	2.83	2.85	2.88	2.90	2.92	2.95	2
89	2.37	2.41	2.46	2.51	2.56	2.61	2.67	2.73	2.79	2.81	2.83	2.86	2.88	2.90	2.92	2.95	1
90	2.37	2.41	2.46	2.51	2.56	2.61	2.67	2.73	2.79	2.81	2.83	2.86	2.88	2.90	2.92	2.95	0
	65°	65½°	66°	66½°	67°	67½°	68°	68½°	69°	69° 10′	69° 20′	69° 30′	69° 40′	69° 50′	70°	70° 10′	

748977°—48——12

Table III.—A, B, C Factors for reduction of transit observations

TOP ARGUMENT = STAR'S DECLINATION, δ

SIDE ARGUMENT = STAR'S ZENITH DISTANCE, ζ

[For factor *A* use left-hand argument. For factor *B* use right-hand argument. Factor *C* is on bottom line of next page]

ζ \ δ	70° 10′	70° 20′	70° 30′	70° 40′	70° 50′	71°	71° 10′	71° 20′	71° 30′	71° 40′	71° 50′	72°	72° 10′	72° 20′	72° 30′	72° 40′	ζ
°																	°
1	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	89
2	.10	.10	.10	.10	.11	.11	.11	.11	.11	.11	.11	.11	.11	.12	.12	.12	88
3	.15	.16	.16	.16	.16	.16	.16	.16	.16	.17	.17	.17	.17	.17	.17	.18	87
4	.20	.21	.21	.21	.21	.21	.22	.22	.22	.22	.22	.23	.23	.23	.23	.23	86
5	.26	.26	.26	.26	.26	.27	.27	.27	.27	.28	.28	.28	.28	.29	.29	.29	85
6	.31	.31	.31	.32	.32	.32	.32	.33	.33	.33	.34	.34	.34	.34	.35	.35	84
7	.36	.36	.37	.37	.37	.37	.38	.38	.38	.39	.39	.39	.40	.40	.41	.41	83
8	.41	.41	.42	.42	.42	.43	.43	.44	.44	.44	.45	.45	.45	.46	.46	.47	82
9	.46	.46	.47	.47	.48	.48	.48	.49	.49	.50	.50	.51	.51	.52	.52	.52	81
10	.51	.52	.52	.52	.53	.53	.54	.54	.55	.55	.56	.56	.57	.57	.58	.58	80
11	.56	.57	.57	.58	.58	.59	.59	.60	.60	.61	.61	.62	.62	.63	.63	.64	79
12	.61	.62	.62	.63	.63	.64	.64	.65	.66	.66	.67	.67	.68	.68	.69	.70	78
13	.66	.67	.67	.68	.68	.69	.70	.70	.71	.72	.72	.73	.74	.74	.75	.76	77
14	.71	.72	.72	.73	.74	.74	.75	.76	.76	.77	.78	.78	.79	.80	.80	.81	76
15	.76	.77	.78	.78	.79	.79	.80	.81	.81	.82	.83	.84	.84	.85	.86	.87	75
16	.81	.82	.83	.83	.84	.85	.85	.86	.87	.88	.88	.89	.90	.91	.92	.92	74
17	.86	.87	.88	.88	.89	.90	.90	.91	.92	.93	.94	.95	.96	.96	.97	.98	73
18	.91	.92	.93	.93	.94	.95	.96	.96	.97	.98	.99	1.00	1.01	1.02	1.03	1.04	72
19	.96	.97	.98	.98	.99	1.00	1.01	1.02	1.03	1.04	1.04	1.05	1.06	1.07	1.08	1.09	71
20	1.01	1.02	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.13	1.14	1.15	70
21	1.06	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.13	1.14	1.15	1.16	1.17	1.18	1.19	1.20	69
22	1.10	1.11	1.12	1.13	1.14	1.15	1.16	1.17	1.18	1.19	1.20	1.21	1.22	1.24	1.25	1.26	68
23	1.15	1.16	1.17	1.18	1.19	1.20	1.21	1.22	1.23	1.24	1.25	1.26	1.28	1.29	1.30	1.31	67
24	1.20	1.21	1.22	1.23	1.24	1.25	1.26	1.27	1.28	1.29	1.30	1.32	1.33	1.34	1.35	1.36	66
25	1.25	1.26	1.27	1.28	1.29	1.30	1.31	1.32	1.33	1.34	1.36	1.37	1.38	1.39	1.41	1.42	65
26	1.29	1.30	1.31	1.32	1.34	1.35	1.36	1.37	1.38	1.39	1.41	1.42	1.43	1.44	1.46	1.47	64
27	1.34	1.35	1.36	1.37	1.38	1.39	1.41	1.42	1.43	1.44	1.46	1.47	1.48	1.50	1.51	1.52	63
28	1.38	1.40	1.41	1.42	1.43	1.44	1.45	1.47	1.48	1.49	1.51	1.52	1.53	1.55	1.56	1.58	62
29	1.43	1.44	1.45	1.46	1.48	1.49	1.50	1.52	1.53	1.54	1.56	1.57	1.58	1.60	1.61	1.63	61
30	1.47	1.49	1.50	1.51	1.52	1.54	1.55	1.56	1.58	1.59	1.60	1.62	1.63	1.65	1.66	1.68	60
31	1.52	1.53	1.54	1.56	1.57	1.58	1.60	1.61	1.62	1.64	1.65	1.67	1.68	1.70	1.71	1.73	59
32	1.56	1.57	1.59	1.60	1.61	1.63	1.64	1.66	1.67	1.68	1.70	1.71	1.73	1.75	1.76	1.78	58
33	1.60	1.62	1.63	1.64	1.66	1.67	1.69	1.70	1.72	1.73	1.75	1.76	1.78	1.80	1.81	1.83	57
34	1.65	1.66	1.68	1.69	1.70	1.72	1.73	1.75	1.76	1.78	1.79	1.81	1.83	1.84	1.86	1.88	56
35	1.69	1.70	1.72	1.73	1.75	1.76	1.78	1.79	1.81	1.82	1.84	1.86	1.87	1.89	1.91	1.92	55
36	1.73	1.75	1.76	1.78	1.79	1.80	1.82	1.84	1.85	1.87	1.88	1.90	1.92	1.94	1.95	1.97	54
37	1.77	1.79	1.80	1.82	1.83	1.85	1.86	1.88	1.90	1.91	1.93	1.95	1.96	1.98	2.00	2.02	53
38	1.82	1.83	1.84	1.86	1.88	1.89	1.91	1.92	1.94	1.96	1.98	1.99	2.01	2.03	2.05	2.07	52
39	1.86	1.87	1.89	1.90	1.92	1.93	1.95	1.97	1.98	2.00	2.02	2.04	2.06	2.07	2.09	2.11	51
40	1.89	1.91	1.93	1.94	1.96	1.97	1.99	2.01	2.03	2.04	2.06	2.08	2.10	2.12	2.14	2.16	50
41	1.93	1.95	1.96	1.98	2.00	2.01	2.03	2.05	2.07	2.09	2.10	2.12	2.14	2.16	2.18	2.20	49
42	1.97	1.99	2.00	2.02	2.04	2.05	2.07	2.09	2.11	2.13	2.15	2.16	2.18	2.20	2.22	2.25	48
43	2.01	2.03	2.04	2.06	2.08	2.09	2.11	2.13	2.15	2.17	2.19	2.21	2.23	2.25	2.27	2.29	47
44	2.05	2.06	2.08	2.10	2.12	2.13	2.15	2.17	2.19	2.21	2.23	2.25	2.27	2.29	2.31	2.33	46
45	2.08	2.10	2.12	2.14	2.15	2.17	2.19	2.21	2.23	2.25	2.27	2.29	2.31	2.33	2.35	2.37	45
	70° 10′	70° 20′	70° 30′	70° 40′	70° 50′	71°	71° 10′	71° 20′	71° 30′	71° 40′	71° 50′	72°	72° 10′	72° 20′	72° 30′	72° 40′	

Table III.—A, B, C Factors for reduction of transit observations

TOP ARGUMENT=STAR'S DECLINATION, δ

SIDE ARGUMENT=STAR'S ZENITH DISTANCE, ζ

[For factor *A* use left-hand argument. For factor *B* use right-hand argument. Factor *C* is on line opposite ζ=90°]

ζ \ δ	70° 10′	70° 20′	70° 30′	70° 40′	70° 50′	71°.	71° 10′	71° 20′	71° 30′	71° 40′	71° 50′	72°	72° 10′	72° 20′	72° 30′	72° 40′	δ / ζ
°																	
46	2.12	2.14	2.15	2.17	2.19	2.21	2.23	2.25	2.27	2.29	2.31	2.33	2.35	2.37	2.39	2.41	44
47	2.16	2.17	2.19	2.21	2.23	2.25	2.27	2.28	2.30	2.32	2.35	2.37	2.39	2.41	2.43	2.45	43
48	2.19	2.21	2.22	2.24	2.26	2.28	2.30	2.32	2.34	2.36	2.38	2.40	2.43	2.45	2.47	2.49	42
49	2.22	2.24	2.26	2.28	2.30	2.32	2.34	2.36	2.38	2.40	2.42	2.44	2.46	2.49	2.51	2.53	41
50	2.26	2.28	2.29	2.31	2.33	2.35	2.37	2.39	2.41	2.44	2.46	2.48	2.50	2.52	2.55	2.57	40
51	2.29	2.31	2.33	2.35	2.37	2.39	2.41	2.43	2.45	2.47	2.49	2.51	2.54	2.56	2.58	2.61	39
52	2.32	2.34	2.36	2.38	2.40	2.42	2.44	2.46	2.48	2.50	2.53	2.55	2.57	2.60	2.62	2.64	38
53	2.35	2.37	2.39	2.41	2.43	2.45	2.47	2.50	2.52	2.54	2.56	2.58	2.61	2.63	2.66	2.68	37
54	2.38	2.40	2.42	2.44	2.46	2.48	2.51	2.53	2.55	2.57	2.60	2.62	2.64	2.67	2.69	2.72	36
55	2.41	2.43	2.45	2.47	2.50	2.52	2.54	2.56	2.58	2.60	2.63	2.65	2.68	2.70	2.72	2.75	35
56	2.44	2.46	2.48	2.50	2.52	2.55	2.57	2.59	2.61	2.64	2.66	2.68	2.71	2.73	2.76	2.78	34
57	2.47	2.49	2.51	2.53	2.55	2.58	2.60	2.62	2.64	2.67	2.69	2.71	2.74	2.76	2.79	2.82	33
58	2.50	2.52	2.54	2.56	2.58	2.61	2.63	2.65	2.67	2.70	2.72	2.74	2.77	2.79	2.82	2.85	32
59	2.53	2.55	2.57	2.59	2.61	2.63	2.66	2.68	2.70	2.72	2.75	2.77	2.80	2.82	2.85	2.88	31
60	2.55	2.57	2.59	2.62	2.64	2.66	2.68	2.71	2.73	2.75	2.78	2.80	2.83	2.85	2.88	2.91	30
61	2.58	2.60	2.62	2.64	2.66	2.69	2.71	2.73	2.76	2.78	2.80	2.83	2.86	2.88	2.91	2.94	29
62	2.60	2.62	2.64	2.67	2.69	2.71	2.74	2.76	2.78	2.81	2.83	2.86	2.88	2.91	2.94	2.96	28
63	2.63	2.65	2.67	2.69	2.71	2.74	2.76	2.78	2.81	2.83	2.86	2.88	2.91	2.94	2.96	2.99	27
64	2.65	2.67	2.69	2.72	2.74	2.76	2.78	2.81	2.83	2.86	2.88	2.91	2.94	2.96	2.99	3.02	26
65	2.67	2.69	2.71	2.74	2.76	2.78	2.81	2.83	2.86	2.88	2.91	2.93	2.96	2.99	3.01	3.04	25
66	2.69	2.71	2.74	2.76	2.78	2.81	2.83	2.85	2.88	2.90	2.93	2.96	2.98	3.01	3.04	3.07	24
67	2.71	2.74	2.76	2.78	2.80	2.83	2.85	2.88	2.90	2.93	2.95	2.98	3.01	3.03	3.06	3.09	23
68	2.73	2.76	2.78	2.80	2.82	2.85	2.87	2.90	2.92	2.95	2.97	3.00	3.03	3.06	3.08	3.11	22
69	2.75	2.77	2.80	2.82	2.84	2.87	2.89	2.92	2.94	2.97	2.99	3.02	3.05	3.08	3.10	3.13	21
70	2.77	2.79	2.81	2.84	2.86	2.89	2.91	2.94	2.96	2.99	3.01	3.04	3.07	3.10	3.12	3.15	20
71	2.79	2.81	2.83	2.86	2.88	2.90	2.93	2.95	2.98	3.01	3.03	3.06	3.09	3.12	3.14	3.17	19
72	2.80	2.83	2.85	2.87	2.90	2.92	2.95	2.97	3.00	3.02	3.05	3.08	3.10	3.13	3.16	3.19	18
73	2.82	2.84	2.86	2.89	2.91	2.94	2.96	2.99	3.01	3.04	3.07	3.09	3.12	3.15	3.18	3.21	17
74	2.83	2.86	2.88	2.90	2.93	2.95	2.98	3.00	3.03	3.06	3.08	3.11	3.14	3.17	3.20	3.23	16
75	2.85	2.87	2.89	2.92	2.94	2.97	2.99	3.02	3.04	3.07	3.10	3.13	3.15	3.18	3.21	3.24	15
76	2.86	2.88	2.91	2.93	2.96	2.98	3.01	3.03	3.06	3.08	3.11	3.14	3.17	3.20	3.23	3.26	14
77	2.87	2.90	2.92	2.94	2.97	2.99	3.02	3.04	3.07	3.10	3.12	3.15	3.18	3.21	3.24	3.27	13
78	2.88	2.91	2.93	2.95	2.98	3.00	3.03	3.06	3.08	3.11	3.14	3.16	3.19	3.22	3.25	3.28	12
79	2.89	2.92	2.94	2.96	2.99	3.02	3.04	3.07	3.09	3.12	3.15	3.18	3.20	3.23	3.26	3.29	11
80	2.90	2.93	2.95	2.97	3.00	3.02	3.05	3.08	3.10	3.13	3.16	3.19	3.22	3.24	3.27	3.31	10
81	2.91	2.94	2.96	2.98	3.01	3.03	3.06	3.09	3.11	3.14	3.17	3.20	3.23	3.25	3.28	3.32	9
82	2.92	2.94	2.97	2.99	3.02	3.04	3.07	3.09	3.12	3.15	3.18	3.20	3.23	3.26	3.29	3.32	8
83	2.92	2.95	2.97	3.00	3.02	3.05	3.08	3.10	3.13	3.16	3.18	3.21	3.24	3.27	3.30	3.33	7
84	2.93	2.96	2.98	3.00	3.03	3.06	3.08	3.11	3.13	3.16	3.19	3.22	3.25	3.28	3.31	3.34	6
85	2.94	2.96	2.98	3.01	3.03	3.06	3.09	3.11	3.14	3.17	3.20	3.22	3.25	3.28	3.31	3.34	5
86	2.94	2.96	2.99	3.01	3.04	3.06	3.09	3.12	3.14	3.17	3.20	3.23	3.26	3.29	3.32	3.35	4
87	2.94	2.97	2.99	3.02	3.04	3.07	3.09	3.12	3.15	3.18	3.20	3.23	3.26	3.29	3.32	3.35	3
88	2.95	2.97	2.99	3.02	3.04	3.07	3.10	3.12	3.15	3.18	3.20	3.23	3.26	3.29	3.32	3.35	2
89	2.95	2.97	3.00	3.02	3.04	3.07	3.10	3.12	3.15	3.18	3.21	3.24	3.27	3.30	3.33	3.36	1
90	2.95	2.97	3.00	3.02	3.05	3.07	3.10	3.12	3.15	3.18	3.21	3.24	3.27	3.30	3.33	3.36	0
	70° 10′	70° 20′	70° 30′	70° 40′	70° 50′	71°	71° 10′	71° 20′	71° 30′	71° 40′	71° 50′	72°	72° 10′	72° 20′	72° 30′	72° 40′	

Table III.—A, B, C Factors for reduction of transit observations

TOP ARGUMENT=STAR'S DECLINATION, δ

SIDE ARGUMENT=STAR'S ZENITH DISTANCE, ζ

[For factor *A* use left-hand argument. For factor *B* use right-hand argument. Factor *C* is on bottom line of next page]

ζ \ δ	72° 40′	72° 50′	73°	73° 10′	73° 20′	73° 30′	73° 40′	73° 50′	74°	74° 10′	74° 20′	74° 30′	74° 40′	74° 50′	75°	75° 10′	δ / ζ
°																	°
1	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.07	0.07	0.07	0.07	89
2	.12	.12	.12	.12	.12	.12	.12	.12	.13	.13	.13	.13	.13	.13	.13	.14	88
3	.18	.18	.18	.18	.18	.18	.19	.19	.19	.19	.19	.20	.20	.20	.20	.20	87
4	.23	.24	.24	.24	.24	.24	.25	.25	.25	.26	.26	.26	.26	.27	.27	.27	86
5	.29	.30	.30	.30	.30	.31	.31	.31	.32	.32	.32	.33	.33	.33	.34	.34	85
6	.35	.35	.36	.36	.36	.37	.37	.38	.38	.38	.39	.39	.40	.40	.40	.41	84
7	.41	.41	.42	.42	.42	.43	.43	.44	.44	.45	.45	.46	.46	.47	.47	.48	83
8	.47	.47	.48	.48	.48	.49	.50	.50	.50	.51	.52	.52	.53	.53	.54	.54	82
9	.52	.53	.53	.54	.54	.55	.56	.56	.57	.57	.58	.58	.59	.60	.60	.61	81
10	.58	.59	.59	.60	.60	.61	.62	.62	.63	.64	.64	.65	.66	.66	.67	.68	80
11	.64	.65	.65	.66	.66	.67	.68	.68	.69	.70	.71	.71	.72	.73	.74	.74	79
12	.70	.70	.71	.72	.72	.73	.74	.75	.75	.76	.77	.78	.79	.79	.80	.81	78
13	.76	.76	.77	.78	.78	.79	.80	.81	.82	.82	.83	.84	.85	.86	.87	.88	77
14	.81	.82	.83	.84	.84	.85	.86	.87	.88	.89	.90	.91	.92	.93	.94	.95	76
15	.87	.88	.89	.89	.90	.91	.92	.93	.94	.95	.96	.97	.98	.99	1.00	1.01	75
16	.92	.93	.94	.95	.96	.97	.98	.99	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.08	74
17	.98	.99	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.11	1.12	1.13	1.14	73
18	1.04	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.13	1.14	1.16	1.17	1.18	1.19	1.21	72
19	1.09	1.10	1.11	1.12	1.14	1.15	1.16	1.17	1.18	1.19	1.21	1.22	1.23	1.24	1.26	1.27	71
20	1.15	1.16	1.17	1.18	1.19	1.20	1.22	1.23	1.24	1.25	1.27	1.28	1.29	1.31	1.32	1.34	70
21	1.20	1.21	1.22	1.24	1.25	1.26	1.27	1.29	1.30	1.31	1.33	1.34	1.36	1.37	1.38	1.40	69
22	1.26	1.27	1.28	1.29	1.31	1.32	1.33	1.34	1.36	1.37	1.39	1.40	1.42	1.43	1.45	1.46	68
23	1.31	1.32	1.34	1.35	1.36	1.38	1.39	1.40	1.42	1.43	1.45	1.46	1.48	1.49	1.51	1.53	67
24	1.36	1.38	1.39	1.40	1.42	1.43	1.45	1.46	1.48	1.49	1.51	1.52	1.54	1.55	1.57	1.59	66
25	1.42	1.43	1.45	1.46	1.47	1.49	1.50	1.52	1.53	1.55	1.56	1.58	1.60	1.62	1.63	1.65	65
26	1.47	1.48	1.50	1.51	1.53	1.54	1.56	1.58	1.59	1.61	1.62	1.64	1.66	1.68	1.69	1.71	64
27	1.52	1.54	1.55	1.57	1.58	1.60	1.61	1.63	1.65	1.66	1.68	1.70	1.72	1.74	1.75	1.77	63
28	1.58	1.59	1.60	1.62	1.64	1.65	1.67	1.69	1.70	1.72	1.74	1.76	1.78	1.79	1.81	1.83	62
29	1.63	1.64	1.66	1.67	1.69	1.71	1.72	1.74	1.76	1.78	1.80	1.81	1.83	1.85	1.87	1.89	61
30	1.68	1.69	1.71	1.73	1.74	1.76	1.78	1.80	1.81	1.83	1.85	1.87	1.89	1.91	1.93	1.95	60
31	1.73	1.74	1.76	1.78	1.80	1.81	1.83	1.85	1.87	1.89	1.91	1.93	1.95	1.97	1.99	2.01	59
32	1.78	1.80	1.81	1.83	1.85	1.87	1.88	1.90	1.92	1.94	1.96	1.98	2.00	2.02	2.05	2.07	58
33	1.83	1.85	1.86	1.88	1.90	1.92	1.94	1.96	1.98	2.00	2.02	2.04	2.06	2.08	2.10	2.13	57
34	1.88	1.89	1.91	1.93	1.95	1.97	1.99	2.01	2.03	2.05	2.07	2.09	2.12	2.14	2.16	2.18	56
35	1.92	1.94	1.96	1.98	2.00	2.02	2.04	2.06	2.08	2.10	2.12	2.15	2.17	2.19	2.22	2.24	55
36	1.97	1.99	2.01	2.03	2.05	2.07	2.09	2.11	2.13	2.15	2.18	2.20	2.22	2.25	2.27	2.30	54
37	2.02	2.04	2.06	2.08	2.10	2.12	2.14	2.16	2.18	2.21	2.23	2.25	2.28	2.30	2.33	2.35	53
38	2.07	2.09	2.11	2.13	2.15	2.17	2.19	2.21	2.23	2.26	2.28	2.30	2.33	2.35	2.38	2.40	52
39	2.11	2.13	2.15	2.17	2.19	2.22	2.24	2.26	2.28	2.31	2.33	2.35	2.38	2.40	2.43	2.46	51
40	2.16	2.18	2.20	2.22	2.24	2.26	2.29	2.31	2.33	2.36	2.38	2.40	2.43	2.46	2.48	2.51	50
41	2.20	2.22	2.24	2.26	2.29	2.31	2.33	2.36	2.38	2.40	2.43	2.45	2.48	2.51	2.53	2.56	49
42	2.25	2.27	2.29	2.31	2.33	2.36	2.38	2.40	2.43	2.45	2.48	2.50	2.53	2.56	2.58	2.61	48
43	2.29	2.31	2.33	2.36	2.38	2.40	2.42	2.45	2.47	2.50	2.53	2.55	2.58	2.61	2.63	2.66	47
44	2.33	2.35	2.38	2.40	2.42	2.45	2.47	2.50	2.52	2.55	2.57	2.60	2.63	2.66	2.68	2.71	46
45	2.37	2.40	2.42	2.44	2.46	2.49	2.51	2.54	2.56	2.59	2.62	2.65	2.67	2.70	2.73	2.76	45
	72° 40′	72° 50′	73°	73° 10′	73° 20′	73° 30′	73° 40′	73° 50′	74°	74° 10′	74° 20′	74° 30′	74° 40′	74° 50′	75°	75° 10′	

Table 111.—A, B, C Factors for reduction of transit observations

TOP ARGUMENT=STAR'S DECLINATION, δ

SIDE ARGUMENT=STAR'S ZENITH DISTANCE, ζ

[For factor *A* use left-hand argument. For factor *B* use right-hand argument. Factor *C* is on line opposite ζ=90°]

ζ \ δ	72° 40′	72° 50′	73°	73° 10′	73° 20′	73° 30′	73° 40′	73° 50′	74°	74° 10′	74° 20′	74° 30′	74° 40′	74° 50′	75°	75° 10′	δ / ζ
°																	°
46	2.41	2.44	2.46	2.48	2.51	2.53	2.56	2.58	2.61	2.64	2.66	2.69	2.72	2.75	2.78	2.81	44
47	2.45	2.48	2.50	2.52	2.55	2.57	2.60	2.63	2.65	2.68	2.71	2.74	2.77	2.80	2.83	2.86	43
48	2.49	2.52	2.54	2.57	2.59	2.62	2.64	2.67	2.70	2.72	2.75	2.78	2.81	2.84	2.87	2.90	42
49	2.53	2.56	2.58	2.61	2.63	2.66	2.68	2.71	2.74	2.77	2.80	2.82	2.85	2.88	2.92	2.95	41
50	2.57	2.60	2.62	2.64	2.67	2.70	2.72	2.75	2.78	2.81	2.84	2.87	2.90	2.93	2.96	2.99	40
51	2.61	2.63	2.66	2.68	2.71	2.74	2.76	2.79	2.82	2.85	2.88	2.91	2.94	2.97	3.00	3.04	39
52	2.64	2.67	2.69	2.72	2.75	2.77	2.80	2.83	2.86	2.89	2.92	2.95	2.98	3.01	3.04	3.08	38
53	2.68	2.71	2.73	2.76	2.78	2.81	2.84	2.87	2.90	2.93	2.96	2.99	3.02	3.05	3.09	3.12	37
54	2.72	2.74	2.77	2.79	2.82	2.85	2.88	2.91	2.94	2.97	3.00	3.03	3.06	3.09	3.13	3.16	36
55	2.75	2.78	2.80	2.83	2.86	2.88	2.91	2.94	2.97	3.00	3.03	3.07	3.10	3.13	3.16	3.20	35
56	2.78	2.81	2.84	2.86	2.89	2.92	2.95	2.98	3.01	3.04	3.07	3.10	3.14	3.17	3.20	3.24	34
57	2.82	2.84	2.87	2.90	2.92	2.95	2.98	3.01	3.04	3.07	3.11	3.14	3.17	3.21	3.24	3.28	33
58	2.85	2.87	2.90	2.93	2.96	2.99	3.02	3.05	3.08	3.11	3.14	3.17	3.21	3.24	3.28	3.31	32
59	2.88	2.90	2.93	2.96	2.99	3.02	3.05	3.08	3.11	3.14	3.17	3.21	3.24	3.28	3.31	3.35	31
60	2.91	2.93	2.96	2.99	3.02	3.05	3.08	3.11	3.14	3.17	3.21	3.24	3.28	3.31	3.35	3.38	30
61	2.94	2.96	2.99	3.02	3.05	3.08	3.11	3.14	3.17	3.21	3.24	3.27	3.31	3.34	3.38	3.42	29
62	2.96	2.99	3.02	3.05	3.08	3.11	3.14	3.17	3.20	3.24	3.27	3.30	3.34	3.38	3.41	3.45	28
63	2.99	3.02	3.05	3.08	3.11	3.14	3.17	3.20	3.23	3.27	3.30	3.33	3.37	3.41	3.44	3.48	27
64	3.02	3.04	3.07	3.10	3.13	3.16	3.20	3.23	3.26	3.29	3.33	3.36	3.40	3.44	3.47	3.51	26
65	3.04	3.07	3.10	3.13	3.16	3.19	3.22	3.26	3.29	3.32	3.36	3.39	3.43	3.46	3.50	3.54	25
66	3.07	3.10	3.13	3.16	3.18	3.22	3.25	3.28	3.31	3.35	3.38	3.42	3.46	3.49	3.53	3.57	24
67	3.09	3.12	3.15	3.18	3.21	3.24	3.27	3.31	3.34	3.37	3.41	3.44	3.48	3.52	3.56	3.60	23
68	3.11	3.14	3.17	3.20	3.23	3.26	3.30	3.33	3.36	3.40	3.43	3.47	3.51	3.54	3.58	3.62	22
69	3.13	3.16	3.19	3.22	3.26	3.29	3.32	3.35	3.39	3.42	3.46	3.49	3.53	3.57	3.61	3.65	21
70	3.15	3.18	3.21	3.24	3.28	3.31	3.34	3.38	3.41	3.44	3.48	3.52	3.55	3.59	3.63	3.67	20
71	3.17	3.20	3.23	3.26	3.30	3.33	3.36	3.40	3.43	3.47	3.50	3.54	3.58	3.61	3.65	3.69	19
72	3.19	3.22	3.25	3.28	3.32	3.35	3.38	3.42	3.45	3.49	3.52	3.56	3.60	3.63	3.67	3.72	18
73	3.21	3.24	3.27	3.30	3.33	3.37	3.40	3.44	3.47	3.50	3.54	3.58	3.62	3.65	3.69	3.74	17
74	3.23	3.26	3.29	3.32	3.35	3.38	3.42	3.45	3.49	3.52	3.56	3.60	3.64	3.67	3.71	3.76	16
75	3.24	3.27	3.30	3.34	3.37	3.40	3.44	3.47	3.50	3.54	3.58	3.61	3.65	3.69	3.73	3.77	15
76	3.26	3.29	3.32	3.35	3.38	3.42	3.45	3.48	3.52	3.56	3.59	3.63	3.67	3.71	3.75	3.79	14
77	3.27	3.30	3.33	3.36	3.40	3.43	3.46	3.50	3.54	3.57	3.61	3.65	3.68	3.72	3.76	3.81	13
78	3.28	3.31	3.34	3.38	3.41	3.44	3.48	3.51	3.55	3.58	3.62	3.66	3.70	3.74	3.78	3.82	12
79	3.29	3.33	3.36	3.39	3.42	3.46	3.49	3.53	3.56	3.60	3.64	3.67	3.71	3.75	3.79	3.83	11
80	3.31	3.34	3.37	3.40	3.43	3.47	3.50	3.54	3.57	3.61	3.65	3.68	3.72	3.76	3.81	3.85	10
81	3.32	3.35	3.38	3.41	3.44	3.48	3.51	3.55	3.58	3.62	3.66	3.70	3.74	3.78	3.82	3.86	9
82	3.32	3.36	3.39	3.42	3.45	3.49	3.52	3.56	3.59	3.63	3.67	3.71	3.75	3.79	3.83	3.87	8
83	3.33	3.36	3.40	3.43	3.46	3.49	3.53	3.56	3.60	3.64	3.68	3.72	3.75	3.79	3.84	3.88	7
84	3.34	3.37	3.40	3.43	3.47	3.50	3.54	3.57	3.61	3.64	3.68	3.72	3.76	3.80	3.84	3.88	6
85	3.34	3.38	3.41	3.44	3.47	3.51	3.54	3.58	3.61	3.65	3.69	3.73	3.77	3.81	3.85	3.89	5
86	3.35	3.38	3.41	3.44	3.48	3.51	3.55	3.58	3.62	3.66	3.69	3.73	3.77	3.81	3.85	3.90	4
87	3.35	3.38	3.42	3.45	3.48	3.52	3.55	3.59	3.62	3.66	3.70	3.74	3.78	3.82	3.86	3.90	3
88	3.35	3.39	3.42	3.45	3.48	3.52	3.55	3.59	3.62	3.66	3.70	3.74	3.78	3.82	3.86	3.90	2
89	3.36	3.39	3.42	3.45	3.49	3.52	3.56	3.59	3.63	3.66	3.70	3.74	3.78	3.82	3.86	3.91	1
90	3.36	3.39	3.42	3.45	3.49	3.52	3.56	3.59	3.63	3.66	3.70	3.74	3.78	3.82	3.86	3.91	0
	72° 40′	72° 50′	73°	73° 10′	73° 20′	73° 30′	73° 40′	73° 50′	74°	74° 10′	74° 20′	74° 30′	74° 40′	74° 50′	75°	75° 10′	

Table III.—A, B, C Factors for reduction of transit observations

TOP ARGUMENT=STAR'S DECLINATION, δ

SIDE ARGUMENT=STAR'S ZENITH DISTANCE, ζ

[For factor *A* use left-hand argument. For factor *B* use right-hand argument. Factor *C* is on bottom line of next page]

ζ \ δ	75° 10′	75° 20′	75° 30′	75° 40′	75° 50′	76°	76° 10′	76° 20′	76° 30′	76° 40′	76° 50′	77°	77° 10′	77° 20′	77° 30′	77° 40′	δ / ζ
°																	°
1	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.08	0.08	0.08	0.08	0.08	0.08	0.08	89
2	.14	.14	.14	.14	.14	.14	.15	.15	.15	.15	.15	.16	.16	.16	.16	.16	88
3	.20	.21	.21	.21	.21	.22	.22	.22	.22	.23	.23	.23	.24	.24	.24	.24	87
4	.27	.28	.28	.28	.28	.29	.29	.30	.30	.30	.31	.31	.31	.32	.32	.33	86
5	.34	.34	.35	.35	.36	.36	.36	.37	.37	.38	.38	.39	.39	.40	.40	.41	85
6	.41	.41	.42	.42	.43	.43	.44	.44	.45	.45	.46	.46	.47	.48	.48	.49	84
7	.48	.48	.49	.49	.50	.50	.51	.52	.52	.53	.54	.54	.55	.56	.56	.57	83
8	.54	.55	.56	.56	.57	.58	.58	.59	.60	.60	.61	.62	.63	.64	.64	.65	82
9	.61	.62	.62	.63	.64	.65	.65	.66	.67	.68	.69	.70	.70	.71	.72	.73	81
10	.68	.69	.69	.70	.71	.72	.73	.74	.74	.75	.76	.77	.78	.79	.80	.81	80
11	.74	.75	.76	.77	.78	.79	.80	.81	.82	.83	.84	.85	.86	.87	.88	.89	79
12	.81	.82	.83	.84	.85	.86	.87	.88	.89	.90	.91	.92	.94	.95	.96	.97	78
13	.88	.89	.90	.91	.92	.93	.94	.95	.96	.98	.99	1.00	1.01	1.03	1.04	1.05	77
14	.95	.96	.97	.98	.99	1.00	1.01	1.02	1.04	1.05	1.06	1.08	1.09	1.10	1.12	1.13	76
15	1.01	1.02	1.03	1.04	1.06	1.07	1.08	1.10	1.11	1.12	1.14	1.15	1.16	1.18	1.20	1.21	75
16	1.08	1.09	1.10	1.11	1.13	1.14	1.15	1.17	1.18	1.20	1.21	1.23	1.24	1.26	1.28	1.29	74
17	1.14	1.16	1.17	1.18	1.20	1.21	1.22	1.24	1.25	1.27	1.28	1.30	1.32	1.33	1.35	1.37	73
18	1.21	1.22	1.23	1.25	1.26	1.28	1.29	1.31	1.32	1.34	1.36	1.37	1.39	1.41	1.43	1.45	72
19	1.27	1.29	1.30	1.32	1.33	1.35	1.36	1.38	1.39	1.41	1.43	1.45	1.47	1.48	1.50	1.52	71
20	1.34	1.35	1.37	1.38	1.40	1.41	1.43	1.45	1.47	1.48	1.50	1.52	1.54	1.56	1.58	1.60	70
21	1.40	1.42	1.43	1.45	1.46	1.48	1.50	1.52	1.54	1.55	1.57	1.59	1.61	1.63	1.65	1.68	69
22	1.46	1.48	1.50	1.51	1.53	1.55	1.57	1.58	1.60	1.62	1.64	1.66	1.69	1.71	1.73	1.75	68
23	1.53	1.54	1.56	1.58	1.60	1.62	1.63	1.65	1.67	1.69	1.72	1.74	1.76	1.78	1.81	1.83	67
24	1.59	1.61	1.63	1.64	1.66	1.68	1.70	1.72	1.74	1.76	1.79	1.81	1.83	1.86	1.88	1.90	66
25	1.65	1.67	1.69	1.71	1.73	1.75	1.77	1.79	1.81	1.83	1.86	1.88	1.90	1.93	1.95	1.98	65
26	1.71	1.73	1.75	1.77	1.79	1.81	1.83	1.86	1.88	1.90	1.92	1.95	1.97	2.00	2.02	2.05	64
27	1.77	1.79	1.81	1.83	1.86	1.88	1.90	1.92	1.95	1.97	1.99	2.02	2.04	2.07	2.10	2.12	63
28	1.83	1.85	1.87	1.90	1.92	1.94	1.96	1.99	2.01	2.04	2.06	2.09	2.11	2.14	2.17	2.20	62
29	1.89	1.92	1.94	1.96	1.98	2.00	2.03	2.05	2.08	2.10	2.13	2.15	2.18	2.21	2.24	2.27	61
30	1.95	1.98	2.00	2.02	2.04	2.07	2.09	2.12	2.14	2.17	2.20	2.22	2.25	2.28	2.31	2.34	60
31	2.01	2.03	2.06	2.08	2.10	2.13	2.15	2.18	2.21	2.23	2.26	2.29	2.32	2.35	2.38	2.41	59
32	2.07	2.09	2.12	2.14	2.16	2.19	2.22	2.24	2.27	2.30	2.33	2.36	2.39	2.42	2.45	2.48	58
33	2.13	2.15	2.18	2.20	2.22	2.25	2.28	2.30	2.33	2.36	2.39	2.42	2.45	2.48	2.52	2.55	57
34	2.18	2.21	2.23	2.26	2.28	2.31	2.34	2.37	2.40	2.42	2.46	2.49	2.52	2.55	2.58	2.62	56
35	2.24	2.26	2.29	2.32	2.34	2.37	2.40	2.43	2.46	2.49	2.52	2.55	2.58	2.62	2.65	2.68	55
36	2.30	2.32	2.35	2.37	2.40	2.43	2.46	2.49	2.52	2.55	2.58	2.61	2.65	2.68	2.72	2.75	54
37	2.35	2.38	2.40	2.43	2.46	2.49	2.52	2.55	2.58	2.61	2.64	2.67	2.71	2.74	2.78	2.82	53
38	2.40	2.43	2.46	2.49	2.52	2.55	2.58	2.61	2.64	2.67	2.70	2.74	2.77	2.81	2.85	2.88	52
39	2.46	2.49	2.51	2.54	2.57	2.60	2.63	2.66	2.70	2.73	2.76	2.80	2.83	2.87	2.91	2.95	51
40	2.51	2.54	2.57	2.60	2.63	2.66	2.69	2.72	2.75	2.79	2.82	2.86	2.89	2.93	2.97	3.01	50
41	2.56	2.59	2.62	2.65	2.68	2.71	2.74	2.78	2.81	2.84	2.88	2.92	2.95	2.99	3.03	3.07	49
42	2.61	2.64	2.67	2.70	2.73	2.77	2.80	2.83	2.87	2.90	2.94	2.97	3.01	3.05	3.09	3.13	48
43	2.66	2.69	2.72	2.76	2.79	2.82	2.85	2.89	2.92	2.96	2.99	3.03	3.07	3.11	3.15	3.19	47
44	2.71	2.74	2.77	2.81	2.84	2.87	2.90	2.94	2.98	3.01	3.05	3.09	3.13	3.17	3.21	3.25	46
45	2.76	2.79	2.82	2.86	2.89	2.92	2.96	2.99	3.03	3.07	3.10	3.14	3.18	3.22	3.27	3.31	45
	75° 10′	75° 20′	75° 30′	75° 40′	75° 50′	76°	76° 10′	76° 20′	76° 30′	76° 40′	76° 50′	77°	77° 10′	77° 20′	77° 30′	77° 40′	

Table III.—A, B, C Factors for reduction of transit observations

TOP ARGUMENT=STAR'S DECLINATION, δ

SIDE ARGUMENT=STAR'S ZENITH DISTANCE, ζ

[For factor *A* use left-hand argument. For factor *B* use right-hand argument. Factor *C* is on line opposite ζ=90°]

ζ \ δ	75° 10′	75° 20′	75° 30′	75° 40′	75° 50′	76°	76° 10′	76° 20′	76° 30′	76° 40′	76° 50′	77°	77° 10′	77° 20′	77° 30′	77° 40′	δ / ζ
°																	°
46	2.81	2.84	2.87	2.91	2.94	2.97	3.01	3.04	3.08	3.12	3.16	3.20	3.24	3.28	3.32	3.37	44
47	2.86	2.89	2.92	2.95	2.99	3.02	3.06	3.10	3.13	3.17	3.21	3.25	3.29	3.34	3.38	3.42	43
48	2.90	2.94	2.97	3.00	3.04	3.07	3.11	3.15	3.18	3.22	3.26	3.30	3.35	3.39	3.43	3.48	42
49	2.95	2.98	3.01	3.05	3.08	3.12	3.16	3.19	3.23	3.27	3.31	3.36	3.40	3.44	3.49	3.53	41
50	2.99	3.02	3.06	3.09	3.13	3.17	3.20	3.24	3.28	3.32	3.36	3.41	3.45	3.49	3.54	3.59	40
51	3.04	3.07	3.10	3.14	3.18	3.21	3.25	3.29	3.33	3.37	3.41	3.45	3.50	3.54	3.59	3.64	39
52	3.08	3.11	3.15	3.18	3.22	3.26	3.30	3.34	3.38	3.42	3.46	3.50	3.55	3.59	3.64	3.69	38
53	3.12	3.15	3.19	3.23	3.26	3.30	3.34	3.38	3.42	3.46	3.51	3.55	3.60	3.64	3.69	3.74	37
54	3.16	3.20	3.23	3.27	3.31	3.34	3.38	3.42	3.47	3.51	3.55	3.60	3.64	3.69	3.74	3.79	36
55	3.20	3.24	3.27	3.31	3.35	3.39	3.43	3.47	3.51	3.55	3.60	3.64	3.69	3.74	3.78	3.83	35
56	3.24	3.27	3.31	3.35	3.39	3.43	3.47	3.51	3.55	3.60	3.64	3.68	3.73	3.78	3.83	3.88	34
57	3.28	3.31	3.35	3.39	3.43	3.47	3.51	3.55	3.59	3.64	3.68	3.73	3.78	3.83	3.88	3.93	33
58	3.31	3.35	3.39	3.43	3.47	3.51	3.55	3.59	3.63	3.68	3.72	3.77	3.82	3.87	3.92	3.97	32
59	3.35	3.38	3.42	3.46	3.50	3.54	3.58	3.63	3.67	3.72	3.76	3.81	3.86	3.91	3.96	4.01	31
60	3.38	3.42	3.46	3.50	3.54	3.58	3.62	3.66	3.71	3.76	3.80	3.85	3.90	3.95	4.00	4.05	30
61	3.42	3.45	3.49	3.53	3.57	3.62	3.66	3.70	3.75	3.79	3.84	3.89	3.94	3.99	4.04	4.09	29
62	3.45	3.49	3.53	3.57	3.61	3.65	3.69	3.74	3.78	3.83	3.88	3.93	3.98	4.03	4.08	4.13	28
63	3.48	3.52	3.56	3.60	3.64	3.68	3.73	3.77	3.82	3.86	3.91	3.96	4.01	4.06	4.12	4.17	27
64	3.51	3.55	3.59	3.63	3.67	3.72	3.76	3.80	3.85	3.90	3.95	4.00	4.05	4.10	4.15	4.21	26
65	3.54	3.58	3.62	3.66	3.70	3.75	3.79	3.84	3.88	3.93	3.98	4.03	4.08	4.13	4.19	4.24	25
66	3.57	3.61	3.65	3.69	3.73	3.78	3.82	3.87	3.91	3.96	4.01	4.06	4.11	4.17	4.22	4.28	24
67	3.60	3.64	3.68	3.72	3.76	3.81	3.85	3.90	3.94	3.99	4.04	4.09	4.14	4.20	4.25	4.31	23
68	3.62	3.66	3.70	3.74	3.79	3.83	3.88	3.92	3.97	4.02	4.07	4.12	4.17	4.23	4.28	4.34	22
69	3.65	3.69	3.73	3.77	3.82	3.86	3.90	3.95	4.00	4.05	4.10	4.15	4.20	4.26	4.31	4.37	21
70	3.67	3.71	3.75	3.80	3.84	3.89	3.93	3.98	4.03	4.08	4.12	4.18	4.23	4.28	4.34	4.40	20
71	3.69	3.73	3.78	3.82	3.86	3.91	3.96	4.00	4.05	4.10	4.15	4.20	4.26	4.31	4.37	4.43	19
72	3.72	3.76	3.80	3.84	3.89	3.93	3.98	4.02	4.07	4.12	4.18	4.23	4.28	4.34	4.39	4.45	18
73	3.74	3.78	3.82	3.86	3.91	3.95	4.00	4.05	4.10	4.15	4.20	4.25	4.30	4.36	4.42	4.48	17
74	3.76	3.80	3.84	3.88	3.93	3.97	4.02	4.07	4.12	4.17	4.22	4.27	4.33	4.38	4.44	4.50	16
75	3.77	3.82	3.86	3.90	3.95	3.99	4.04	4.09	4.14	4.19	4.24	4.29	4.35	4.40	4.46	4.52	15
76	3.79	3.83	3.88	3.92	3.96	4.01	4.06	4.11	4.16	4.21	4.26	4.31	4.37	4.42	4.48	4.54	14
77	3.81	3.85	3.89	3.94	3.98	4.03	4.08	4.12	4.17	4.22	4.28	4.33	4.39	4.44	4.50	4.56	13
78	3.82	3.86	3.91	3.95	4.00	4.04	4.09	4.14	4.19	4.24	4.29	4.35	4.40	4.46	4.52	4.58	12
79	3.83	3.88	3.92	3.96	4.01	4.06	4.11	4.16	4.21	4.26	4.31	4.36	4.42	4.48	4.54	4.60	11
80	3.85	3.89	3.93	3.98	4.02	4.07	4.12	4.17	4.22	4.27	4.32	4.38	4.43	4.49	4.55	4.61	10
81	3.86	3.90	3.94	3.99	4.04	4.08	4.13	4.18	4.23	4.28	4.34	4.39	4.45	4.50	4.56	4.62	9
82	3.87	3.91	3.96	4.00	4.05	4.09	4.14	4.19	4.24	4.29	4.35	4.40	4.46	4.52	4.58	4.64	8
83	3.88	3.92	3.96	4.01	4.06	4.10	4.15	4.20	4.25	4.30	4.36	4.41	4.47	4.53	4.59	4.65	7
84	3.88	3.93	3.97	4.02	4.06	4.11	4.16	4.21	4.26	4.31	4.37	4.42	4.48	4.54	4.60	4.66	6
85	3.89	3.93	3.98	4.02	4.07	4.12	4.17	4.22	4.27	4.32	4.37	4.43	4.48	4.54	4.60	4.66	5
86	3.90	3.94	3.98	4.03	4.08	4.12	4.17	4.22	4.27	4.33	4.38	4.43	4.49	4.55	4.61	4.67	4
87	3.90	3.94	3.99	4.03	4.08	4.13	4.18	4.23	4.28	4.33	4.38	4.44	4.50	4.55	4.61	4.68	3
88	3.90	3.95	3.99	4.04	4.08	4.13	4.18	4.23	4.28	4.33	4.39	4.44	4.50	4.56	4.62	4.68	2
89	3.91	3.95	3.99	4.04	4.08	4.13	4.18	4.23	4.28	4.34	4.39	4.44	4.50	4.56	4.62	4.68	1
90	3.91	3.95	3.99	4.04	4.09	4.13	4.18	4.23	4.28	4.34	4.39	4.44	4.50	4.56	4.62	4.68	0
	75° 10′	75° 20′	75° 30′	75° 40′	75° 50′	76°	76° 10′	76° 20′	76° 30′	76° 40′	76° 50′	77°	77° 10′	77° 20′	77° 30′	77° 40′	

Table III.—A, B, C Factors for reduction of transit observations

TOP ARGUMENT=STAR'S DECLINATION, δ

SIDE ARGUMENT=STAR'S ZENITH DISTANCE, ζ

[For factor *A* use left-hand argument. For factor *B* use right-hand argument. Factor *C* is on bottom line of next page]

ζ \ δ	77°40′	77°50′	78°	78°10′	78°20′	78°30′	78°40′	78°50′	79°	79°10′	79°20′	79°30′	79°40′	79°50′	80°	δ / ζ
°																°
1	0.08	0.08	0.08	0.08	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.10	0.10	0.10	0.10	89
2	.16	.17	.17	.17	.17	.18	.18	.18	.18	.19	.19	.19	.20	.20	.20	88
3	.24	.25	.25	.26	.26	.26	.27	.27	.27	.28	.28	.29	.29	.30	.30	87
4	.33	.33	.34	.34	.34	.35	.36	.36	.37	.37	.38	.38	.39	.40	.40	86
5	.41	.41	.42	.42	.43	.44	.44	.45	.46	.46	.47	.48	.49	.49	.50	85
6	.49	.50	.51	.51	.52	.52	.53	.54	.55	.56	.56	.57	.58	.59	.60	84
7	.57	.58	.59	.59	.60	.61	.62	.63	.64	.65	.66	.67	.68	.69	.70	83
8	.65	.66	.67	.68	.69	.70	.71	.72	.73	.74	.75	.76	.78	.79	.80	82
9	.73	.74	.75	.76	.77	.78	.80	.81	.82	.83	.84	.86	.87	.89	.90	81
10	.81	.82	.84	.85	.86	.87	.88	.90	.91	.92	.94	.95	.97	.98	1.00	80
11	.89	.90	.92	.93	.94	.96	.97	.98	1.00	1.02	1.03	1.05	1.06	1.08	1.10	79
12	.97	.99	1.00	1.01	1.03	1.04	1.06	1.07	1.09	1.11	1.12	1.14	1.16	1.18	1.20	78
13	1.05	1.07	1.08	1.10	1.11	1.13	1.14	1.16	1.18	1.20	1.22	1.23	1.25	1.27	1.30	77
14	1.13	1.15	1.16	1.18	1.20	1.21	1.23	1.25	1.27	1.29	1.31	1.33	1.35	1.37	1.39	76
15	1.21	1.23	1.25	1.26	1.28	1.30	1.32	1.34	1.36	1.38	1.40	1.42	1.44	1.47	1.49	75
16	1.29	1.31	1.33	1.34	1.36	1.38	1.40	1.42	1.44	1.47	1.49	1.51	1.54	1.56	1.59	74
17	1.37	1.39	1.41	1.43	1.45	1.47	1.49	1.51	1.53	1.56	1.58	1.60	1.63	1.66	1.68	73
18	1.45	1.47	1.49	1.51	1.53	1.55	1.57	1.60	1.62	1.64	1.67	1.70	1.72	1.75	1.78	72
19	1.52	1.54	1.57	1.59	1.61	1.63	1.66	1.68	1.71	1.73	1.76	1.79	1.82	1.84	1.87	71
20	1.60	1.62	1.65	1.67	1.69	1.72	1.74	1.77	1.79	1.82	1.85	1.88	1.91	1.94	1.97	70
21	1.68	1.70	1.72	1.75	1.77	1.80	1.82	1.85	1.88	1.91	1.94	1.97	2.00	2.03	2.06	69
22	1.75	1.78	1.80	1.83	1.85	1.88	1.91	1.93	1.96	1.99	2.02	2.06	2.09	2.12	2.16	68
23	1.83	1.85	1.88	1.90	1.93	1.96	1.99	2.02	2.05	2.08	2.11	2.14	2.18	2.21	2.25	67
24	1.90	1.93	1.96	1.98	2.01	2.04	2.07	2.10	2.13	2.16	2.20	2.23	2.27	2.30	2.34	66
25	1.98	2.00	2.03	2.06	2.09	2.12	2.15	2.18	2.22	2.25	2.28	2.32	2.36	2.39	2.43	65
26	2.05	2.08	2.11	2.14	2.17	2.20	2.23	2.26	2.30	2.33	2.37	2.41	2.44	2.48	2.52	64
27	2.12	2.15	2.18	2.21	2.24	2.28	2.31	2.34	2.38	2.42	2.45	2.49	2.53	2.57	2.61	63
28	2.20	2.23	2.26	2.29	2.32	2.36	2.39	2.42	2.46	2.50	2.54	2.58	2.62	2.66	2.70	62
29	2.27	2.30	2.33	2.36	2.40	2.43	2.47	2.50	2.54	2.58	2.62	2.66	2.70	2.75	2.79	61
30	2.34	2.37	2.40	2.44	2.47	2.51	2.54	2.58	2.62	2.66	2.70	2.74	2.79	2.83	2.88	60
31	2.41	2.44	2.48	2.51	2.55	2.58	2.62	2.66	2.70	2.74	2.78	2.83	2.87	2.92	2.97	59
32	2.48	2.51	2.55	2.58	2.62	2.66	2.70	2.74	2.78	2.82	2.86	2.91	2.95	3.00	3.05	58
33	2.55	2.58	2.62	2.66	2.69	2.73	2.77	2.81	2.85	2.90	2.94	2.99	3.04	3.09	3.14	57
34	2.62	2.65	2.69	2.73	2.76	2.80	2.84	2.89	2.93	2.98	3.02	3.07	3.12	3.17	3.22	56
35	2.68	2.72	2.76	2.80	2.84	2.88	2.92	2.96	3.01	3.05	3.10	3.15	3.20	3.25	3.30	55
36	2.75	2.79	2.83	2.87	2.91	2.95	2.99	3.04	3.08	3.13	3.18	3.23	3.28	3.33	3.38	54
37	2.82	2.86	2.90	2.94	2.98	3.02	3.06	3.11	3.15	3.20	3.25	3.30	3.36	3.41	3.47	53
38	2.88	2.92	2.96	3.00	3.04	3.09	3.13	3.18	3.23	3.28	3.33	3.38	3.43	3.49	3.55	52
39	2.95	2.99	3.03	3.07	3.11	3.16	3.20	3.25	3.30	3.35	3.40	3.45	3.51	3.56	3.62	51
40	3.01	3.05	3.09	3.14	3.18	3.22	3.27	3.32	3.37	3.42	3.47	3.53	3.58	3.64	3.70	50
41	3.07	3.11	3.16	3.20	3.24	3.29	3.34	3.39	3.44	3.49	3.54	3.60	3.66	3.72	3.78	49
42	3.13	3.18	3.22	3.26	3.31	3.36	3.41	3.46	3.51	3.56	3.61	3.67	3.73	3.79	3.85	48
43	3.19	3.24	3.28	3.33	3.37	3.42	3.47	3.52	3.57	3.63	3.68	3.74	3.80	3.86	3.93	47
44	3.25	3.30	3.34	3.39	3.43	3.48	3.54	3.59	3.64	3.70	3.75	3.81	3.87	3.94	4.00	46
45	3.31	3.36	3.40	3.45	3.50	3.55	3.60	3.65	3.71	3.76	3.82	3.88	3.94	4.01	4.07	45
	77°40′	77°50′	78°	78°10′	78°20′	78°30′	78°40′	78°50′	79°	79°10′	79°20′	79°30′	79°40′	79°50′	80°	

Table III.—A, B, C Factors for reduction of transit observations

TOP ARGUMENT=STAR'S DECLINATION, δ

SIDE ARGUMENT=STAR'S ZENITH DISTANCE, ζ

[For factor *A* use left-hand argument. For factor *B* use right-hand argument. Factor *C* is on line opposite $\zeta=90°$

ζ \ δ	77° 40′	77° 50′	78°	78° 10′	78° 20′	78° 30′	78° 40′	78° 50′	79°	79° 10′	79° 20′	79° 30′	79° 40′	79° 50′	80°	δ / ζ
°																°
46	3.37	3.41	3.46	3.51	3.56	3.61	3.66	3.71	3.77	3.83	3.89	3.95	4.01	4.08	4.14	44
47	3.42	3.47	3.52	3.57	3.62	3.67	3.72	3.78	3.83	3.89	3.95	4.01	4.08	4.14	4.21	43
48	3.48	3.53	3.57	3.62	3.68	3.73	3.78	3.84	3.89	3.95	4.02	4.08	4.14	4.21	4.28	42
49	3.53	3.58	3.63	3.68	3.73	3.79	3.84	3.90	3.96	4.02	4.08	4.14	4.21	4.28	4.35	41
50	3.59	3.63	3.68	3.74	3.79	3.84	3.90	3.96	4.02	4.08	4.14	4.20	4.27	4.34	4.41	40
51	3.64	3.69	3.74	3.79	3.84	3.90	3.96	4.01	4.07	4.14	4.20	4.26	4.33	4.40	4.48	39
52	3.69	3.74	3.79	3.84	3.90	3.95	4.01	4.07	4.13	4.19	4.26	4.32	4.39	4.46	4.54	38
53	3.74	3.79	3.84	3.89	3.95	4.01	4.06	4.12	4.19	4.25	4.32	4.38	4.45	4.52	4.60	37
54	3.79	3.84	3.89	3.94	4.00	4.06	4.12	4.18	4.24	4.30	4.37	4.44	4.51	4.58	4.66	36
55	3.83	3.89	3.94	3.99	4.05	4.11	4.17	4.23	4.29	4.36	4.43	4.50	4.57	4.64	4.72	35
56	3.88	3.93	3.99	4.04	4.10	4.16	4.22	4.28	4.34	4.41	4.48	4.55	4.62	4.70	4.77	34
57	3.93	3.98	4.04	4.09	4.15	4.21	4.27	4.33	4.39	4.46	4.53	4.60	4.68	4.75	4.83	33
58	3.97	4.02	4.08	4.14	4.19	4.25	4.32	4.38	4.44	4.51	4.58	4.65	4.73	4.80	4.88	32
59	4.01	4.07	4.12	4.18	4.24	4.30	4.36	4.43	4.49	4.56	4.63	4.70	4.78	4.86	4.94	31
60	4.05	4.11	4.17	4.22	4.28	4.34	4.41	4.47	4.54	4.61	4.68	4.75	4.83	4.91	4.99	30
61	4.09	4.15	4.21	4.26	4.32	4.39	4.45	4.52	4.58	4.65	4.72	4.80	4.88	4.96	5.04	29
62	4.13	4.19	4.25	4.31	4.37	4.43	4.49	4.56	4.63	4.70	4.77	4.85	4.92	5.00	5.08	28
63	4.17	4.23	4.29	4.35	4.41	4.47	4.53	4.60	4.67	4.74	4.81	4.89	4.97	5.05	5.13	27
64	4.21	4.26	4.32	4.38	4.44	4.51	4.57	4.64	4.71	4.78	4.86	4.93	5.01	5.09	5.18	26
65	4.24	4.30	4.36	4.42	4.48	4.55	4.61	4.68	4.75	4.82	4.90	4.97	5.05	5.14	5.22	25
66	4.28	4.34	4.40	4.46	4.52	4.58	4.65	4.72	4.79	4.86	4.94	5.01	5.09	5.18	5.26	24
67	4.31	4.37	4.43	4.49	4.55	4.62	4.68	4.75	4.82	4.90	4.97	5.05	5.13	5.22	5.30	23
68	4.34	4.40	4.46	4.52	4.58	4.65	4.72	4.79	4.86	4.93	5.01	5.09	5.17	5.25	5.34	22
69	4.37	4.43	4.49	4.55	4.62	4.68	4.75	4.82	4.89	4.97	5.04	5.12	5.20	5.29	5.38	21
70	4.40	4.46	4.52	4.58	4.65	4.71	4.78	4.85	4.93	5.00	5.08	5.16	5.24	5.32	5.41	20
71	4.43	4.49	4.55	4.61	4.68	4.74	4.81	4.88	4.96	5.03	5.11	5.19	5.27	5.36	5.45	19
72	4.45	4.51	4.57	4.64	4.70	4.77	4.84	4.91	4.98	5.06	5.14	5.22	5.30	5.39	5.48	18
73	4.48	4.54	4.60	4.66	4.73	4.80	4.87	4.94	5.01	5.09	5.17	5.25	5.33	5.42	5.51	17
74	4.50	4.56	4.62	4.69	4.75	4.82	4.89	4.96	5.04	5.11	5.19	5.27	5.36	5.45	5.53	16
75	4.52	4.58	4.65	4.71	4.78	4.85	4.92	4.99	5.06	5.14	5.22	5.30	5.38	5.47	5.56	15
76	4.54	4.60	4.67	4.73	4.80	4.87	4.94	5.01	5.09	5.16	5.24	5.32	5.41	5.50	5.59	14
77	4.56	4.62	4.68	4.75	4.82	4.89	4.96	5.03	5.11	5.18	5.26	5.35	5.43	5.52	5.61	13
78	4.58	4.64	4.70	4.77	4.84	4.91	4.98	5.05	5.13	5.20	5.28	5.37	5.45	5.54	5.63	12
79	4.60	4.66	4.72	4.79	4.85	4.92	5.00	5.07	5.14	5.22	5.30	5.39	5.47	5.56	5.65	11
80	4.61	4.67	4.74	4.80	4.87	4.94	5.01	5.08	5.16	5.24	5.32	5.40	5.49	5.58	5.67	10
81	4.62	4.69	4.75	4.82	4.88	4.95	5.03	5.10	5.18	5.26	5.34	5.42	5.51	5.60	5.69	9
82	4.64	4.70	4.76	4.83	4.90	4.97	5.04	5.11	5.19	5.27	5.35	5.43	5.52	5.61	5.70	8
83	4.65	4.71	4.78	4.84	4.91	4.98	5.05	5.13	5.20	5.28	5.36	5.45	5.53	5.62	5.72	7
84	4.66	4.72	4.79	4.85	4.92	4.99	5.06	5.14	5.21	5.29	5.37	5.46	5.54	5.63	5.73	6
85	4.66	4.73	4.79	4.86	4.93	5.00	5.07	5.14	5.22	5.30	5.38	5.47	5.55	5.64	5.74	5
86	4.67	4.73	4.80	4.86	4.93	5.00	5.08	5.15	5.23	5.31	5.39	5.47	5.56	5.65	5.74	4
87	4.68	4.74	4.81	4.87	4.94	5.01	5.08	5.16	5.23	5.31	5.40	5.48	5.57	5.66	5.75	3
88	4.68	4.74	4.81	4.87	4.94	5.01	5.09	5.16	5.24	5.32	5.40	5.48	5.57	5.66	5.75	2
89	4.68	4.74	4.81	4.88	4.94	5.01	5.09	5.16	5.24	5.32	5.40	5.49	5.57	5.66	5.76	1
90	4.68	4.74	4.81	4.88	4.94	5.02	5.09	5.16	5.24	5.32	5.40	5.49	5.58	5.67	5.76	0
	77° 40′	77° 50′	78°	78° 10′	78° 20′	78° 30′	78° 40′	78° 50′	79°	79° 10′	79° 20′	79° 30′	79° 40′	79° 50′	80°	

Table IV.—Parallax of the sun for the first day of each month

(Based on solar parallax=8″.80)

Altitude	Jan. 1	Feb. 1	Mar. 1	Apr. 1	May 1	June 1	July 1	Aug. 1	Sept. 1	Oct. 1	Nov. 1	Dec. 1
0°	8″.95	8″.93	8″.88	8″.81	8″.73	8″.68	8″.66	8″.67	8″.72	8″.79	8″.87	8″.92
3	8.94	8.92	8.87	8.80	8.72	8.67	8.65	8.66	8.71	8.78	8.86	8.91
6	8.90	8.88	8.83	8.76	8.68	8.63	8.61	8.62	8.67	8.74	8.82	8.87
9	8.84	8.82	8.77	8.70	8.62	8.57	8.55	8.56	8.61	8.68	8.76	8.81
12	8.75	8.73	8.69	8.62	8.54	8.49	8.47	8.48	8.53	8.60	8.68	8.72
15	8.64	8.63	8.58	8.51	8.43	8.38	8.36	8.37	8.42	8.49	8.57	8.62
18	8.51	8.49	8.45	8.38	8.30	8.26	8.24	8.25	8.29	8.36	8.44	8.48
21	8.36	8.34	8.29	8.23	8.15	8.10	8.08	8.09	8.14	8.21	8.28	8.33
24	8.18	8.16	8.11	8.05	7.97	7.93	7.91	7.92	7.97	8.03	8.10	8.15
27	7.97	7.96	7.91	7.85	7.78	7.73	7.72	7.72	7.77	7.83	7.90	7.95
30	7.75	7.73	7.69	7.63	7.56	7.52	7.50	7.51	7.55	7.61	7.68	7.72
33	7.51	7.49	7.45	7.39	7.32	7.28	7.26	7.27	7.31	7.37	7.44	7.48
36	7.24	7.22	7.18	7.13	7.06	7.02	7.01	7.01	7.05	7.11	7.18	7.22
39	6.96	6.94	6.90	6.85	6.78	6.75	6.73	6.74	6.78	6.83	6.89	6.93
42	6.65	6.64	6.60	6.55	6.49	6.45	6.44	6.44	6.48	6.53	6.59	6.63
44	6.44	6.42	6.39	6.34	6.28	6.24	6.23	6.24	6.27	6.32	6.38	6.42
46	6.22	6.20	6.17	6.12	6.06	6.03	6.02	6.02	6.06	6.11	6.16	6.20
48	5.99	5.98	5.94	5.89	5.84	5.81	5.79	5.80	5.83	5.88	5.93	5.97
50	5.75	5.74	5.71	5.66	5.61	5.58	5.57	5.57	5.61	5.65	5.70	5.73
52	5.51	5.50	5.47	5.42	5.38	5.34	5.33	5.34	5.37	5.41	5.46	5.49
54	5.26	5.25	5.22	5.18	5.13	5.10	5.09	5.10	5.13	5.17	5.21	5.24
56	5.00	4.99	4.97	4.93	4.88	4.85	4.84	4.85	4.88	4.92	4.96	4.99
58	4.74	4.73	4.71	4.67	4.63	4.60	4.59	4.59	4.62	4.66	4.70	4.73
60	4.48	4.46	4.44	4.40	4.36	4.34	4.33	4.34	4.36	4.40	4.44	4.46
62	4.20	4.19	4.17	4.14	4.10	4.08	4.07	4.07	4.09	4.13	4.16	4.19
64	3.92	3.91	3.89	3.86	3.83	3.81	3.80	3.80	3.82	3.85	3.89	3.91
66	3.64	3.63	3.61	3.58	3.55	3.53	3.52	3.53	3.55	3.57	3.61	3.63
68	3.35	3.35	3.33	3.30	3.27	3.25	3.24	3.25	3.27	3.29	3.32	3.34
70	3.06	3.05	3.04	3.01	2.99	2.97	2.96	2.97	2.98	3.01	3.03	3.05
72	2.77	2.76	2.74	2.72	2.70	2.68	2.68	2.68	2.69	2.72	2.74	2.76
74	2.47	2.46	2.45	2.43	2.41	2.39	2.39	2.39	2.40	2.42	2.44	2.46
76	2.17	2.16	2.15	2.13	2.11	2.10	2.09	2.10	2.11	2.13	2.15	2.16
78	1.86	1.86	1.85	1.83	1.81	1.80	1.80	1.80	1.81	1.83	1.84	1.85
80	1.55	1.55	1.54	1.53	1.52	1.51	1.50	1.51	1.51	1.53	1.54	1.55
82	1.25	1.24	1.24	1.23	1.22	1.21	1.21	1.21	1.21	1.22	1.23	1.24
84	.94	.93	.93	.92	.91	.91	.90	.91	.91	.92	.93	.93
86	.62	.62	.62	.61	.61	.61	.60	.61	.61	.61	.62	.62
88	.31	.31	.31	.31	.30	.30	.30	.30	.30	.31	.31	.31
90	0	0	0	0	0	0	0	0	0	0	0	0

Table V.—Mean refraction, r_m

[Pressure=760 *mm*; Temperature=10° *C*; Relative humidity=60%]

z	00′	10′	20′	30′	40′	50′	60′
°	″	″	″	″	″	″	″
0	0.0	0.2	0.3	0.5	0.7	0.8	1.0
1	1.0	1.2	1.3	1.5	1.7	1.9	2.0
2	2.0	2.2	2.4	2.5	2.7	2.9	3.0
3	3.0	3.2	3.4	3.5	3.7	3.9	4.0
4	4.0	4.2	4.4	4.6	4.7	4.9	5.1
5	5.1	5.2	5.4	5.6	5.7	5.9	6.1
6	6.1	6.3	6.4	6.6	6.8	6.9	7.1
7	7.1	7.3	7.5	7.6	7.8	8.0	8.1
8	8.1	8.3	8.5	8.7	8.8	9.0	9.2
9	9.2	9.3	9.5	9.7	9.9	10.0	10.2
10	10.2	10.4	10.6	10.7	10.9	11.1	11.3
11	11.3	11.4	11.6	11.8	12.0	12.1	12.3
12	12.3	12.5	12.7	12.8	13.0	13.2	13.4
13	13.4	13.5	13.7	13.9	14.1	14.3	14.4
14	14.4	14.6	14.8	15.0	15.2	15.3	15.5
15	15.5	15.7	15.9	16.1	16.2	16.4	16.6
16	16.6	16.8	17.0	17.2	17.3	17.5	17.7
17	17.7	17.9	18.1	18.3	18.4	18.6	18.8
18	18.8	19.0	19.2	19.4	19.6	19.8	19.9
19	19.9	20.1	20.3	20.5	20.7	20.9	21.1
20	21.1	21.3	21.5	21.7	21.8	22.0	22.2
21	22.2	22.4	22.6	22.8	23.0	23.2	23.4
22	23.4	23.6	23.8	24.0	24.2	24.4	24.6
23	24.6	24.8	25.0	25.2	25.4	25.6	25.8
24	25.8	26.0	26.2	26.4	26.6	26.8	27.0
25	27.0	27.2	27.4	27.6	27.8	28.0	28.2
26	28.2	28.4	28.7	28.9	29.1	29.3	29.5
27	29.5	29.7	29.9	30.1	30.4	30.6	30.8
28	30.8	31.0	31.2	31.4	31.7	31.9	32.1
29	32.1	32.3	32.5	32.8	33.0	33.2	33.4
30	33.4	33.6	33.9	34.1	34.3	34.6	34.8
31	34.8	35.0	35.2	35.5	35.7	35.9	36.2
32	36.2	36.4	36.6	36.9	37.1	37.4	37.6
33	37.6	37.8	38.1	38.3	38.6	38.8	39.0
34	39.0	39.3	39.5	39.8	40.0	40.3	40.5
35	40.5	40.8	41.0	41.3	41.5	41.8	42.1
36	42.1	42.3	42.6	42.8	43.1	43.3	43.6
37	43.6	43.9	44.1	44.4	44.7	44.9	45.2
38	45.2	45.5	45.8	46.0	46.3	46.6	46.9
39	46.9	47.1	47.4	47.7	48.0	48.3	48.6
40	48.6	48.8	49.1	49.4	49.7	50.0	50.3
41	50.3	50.6	50.9	51.2	51.5	51.8	52.1
42	52.1	52.4	52.7	53.0	53.3	53.6	54.0
43	54.0	54.3	54.6	54.9	55.2	55.5	55.9
44	55.9	56.2	56.5	56.9	57.2	57.5	57.9
45	57.9	58.2	58.5	58.9	59.2	59.6	59.9
46	59.9	60.2	60.6	61.0	61.3	61.7	62.0
47	62.0	62.4	62.8	63.1	63.5	63.9	64.2
48	64.2	64.6	65.0	65.4	65.7	66.1	66.5
49	66.5	66.9	67.3	67.7	68.1	68.5	68.9
50	68.9	69.3	69.7	70.1	70.6	71.0	71.4
51	71.4	71.8	72.2	72.7	73.1	73.6	74.0
52	74.0	74.4	74.9	75.3	75.8	76.2	76.7
53	76.7	77.2	77.6	78.1	78.6	79.1	79.5
54	79.5	80.0	80.5	81.0	81.5	82.0	82.5
55	82.5	83.0	83.5	84.1	84.6	85.1	85.6
56	85.6	86.2	86.7	87.3	87.8	88.4	88.9
57	88.9	89.5	90.1	90.7	91.2	91.8	92.4
58	92.4	93.0	93.6	94.2	94.8	95.5	96.1
59	96.1	96.7	97.4	98.0	98.6	99.3	100.0

Table V.—Mean refraction, r_m

[Pressure=760 *mm*; Temperature=10°*C*; Relative humidity=60%]

z	00′	10′	20′	30′	40′	50′	60′
°	″	″	″	″	″	″	″
60	100.0	100.6	101.3	102.0	102.7	103.4	104.1
61	104.1	104.8	105.5	106.3	107.0	107.7	108.5
62	108.5	109.2	110.0	110.8	111.6	112.4	113.2
63	113.2	114.0	114.8	115.6	116.5	117.3	118.2
64	118.2	119.0	119.9	120.8	121.7	122.6	123.5
65	123.5	124.5	125.4	126.4	127.4	128.3	129.3
66	129.3	130.3	131.4	132.4	133.4	134.5	135.6
67	135.6	136.7	137.8	138.9	140.0	141.2	142.3
68	142.3	143.5	144.7	145.9	147.2	148.4	149.7
69	149.7	151.0	152.3	153.6	155.0	156.4	157.8
70	157.8	159.2	160.6	162.1	163.6	165.1	166.6
71	166.6	168.2	169.7	171.4	173.0	174.7	176.3
72	176.3	178.1	179.8	181.6	183.4	185.3	187.2
73	187.2	189.1	191.0	193.0	195.1	197.1	199.2
74	199.2	201.4	203.6	205.8	208.1	210.4	212.8
75	212.8	215.2	217.7	220.2	222.8	225.5	228.2
76	228.2	230.9	233.7	236.6	239.6	242.6	245.7
77	245.7	248.9	252.1	255.4	258.9	262.3	265.9
78	265.9	269.6	273.4	277.2	281.2	285.3	289.5
79	289.5	293.8	298.2	302.8	307.5	312.3	317.3
80	317.3	322.4	327.7	333.2	338.8	344.6	350.6
81	350.6	356.8	363.2	369.8	376.6	383.7	391.1
82	391.1	398.7	406.6	414.8	423.3	432.1	441.3
83	441.3	450.9	460.9	471.2	482.0	493.3	505.1
84	505.1	517.4	530.3	543.8	558.0	572.8	588.4
85	588.4	604.4	621.6	639.7	658.8	678.9	700.2
86	700.2	722.7	746.6	771.8	798.7	827.2	857.6
87	857.6	890.0	924.7	961.6	1,001.3	1,043.9	1,089.7
88	1,089.7	1,138.9	1,192.0	1,249.2	1,311.4	1,378.6	1,452.0
89	1,452.0	1,531.7	1,618.8	1,714.0	1,818.4	1,933.1	2,059.5

Table VI.—Pressure Correction Factor, C_B

(Apply to mean refraction in Table V)

[$r=(r_m)\ (C_B)\ (C_T)$]

Barometer		C_B	Barometer		C_B	Barometer		C_B	Barometer		C_B	Barometer		C_B
Inches	*mm*		*Inches*	*mm*		*Inches*	*mm*		*Inches*	*mm*		*Inches*	*mm*	
20.0	508	0.670	22.4	569	0.749	24.8	630	0.829	27.2	691	0.909	29.6	752	0.989
20.1	511	0.673	22.5	572	0.752	24.9	632	0.832	27.3	693	0.912	29.7	754	0.992
20.2	513	0.676	22.6	574	0.755	25.0	635	0.835	27.4	696	0.916	29.8	757	0.996
20.3	516	0.679	22.7	576	0.759	25.1	637	0.838	27.5	699	0.920	29.9	759	0.999
20.4	518	0.682	22.8	579	0.762	25.2	640	0.842	27.6	701	0.923	30.0	762	1.003
20.5	521	0.685	22.9	582	0.766	25.3	643	0.846	27.7	704	0.926	30.1	765	1.007
20.6	523	0.688	23.0	584	0.770	25.4	645	0.849	27.8	706	0.929	30.2	767	1.010
20.7	526	0.692	23.1	587	0.773	25.5	648	0.853	27.9	709	0.933	30.3	770	1.013
20.8	528	0.696	23.2	589	0.776	25.6	650	0.856	28.0	711	0.936	30.4	772	1.016
20.9	531	0.699	23.3	592	0.779	25.7	653	0.859	28.1	714	0.939	30.5	775	1.020
21.0	533	0.703	23.4	594	0.783	25.8	655	0.862	28.2	716	0.942	30.6	777	1.023
21.1	536	0.706	23.5	597	0.786	25.9	658	0.866	28.3	719	0.946	30.7	780	1.026
21.2	538	0.709	23.6	599	0.789	26.0	660	0.869	28.4	721	0.949	30.8	782	1.029
21.3	541	0.712	23.7	602	0.792	26.1	663	0.872	28.5	724	0.953	30.9	785	1.033
21.4	544	0.716	23.8	605	0.796	26.2	665	0.875	28.6	726	0.956	31.0	787	1.036
21.5	546	0.719	23.9	607	0.799	26.3	668	0.879	28.7	729	0.959			
21.6	549	0.722	24.0	610	0.803	26.4	671	0.882	28.8	732	0.963			
21.7	551	0.725	24.1	612	0.806	26.5	673	0.885	28.9	734	0.966			
21.8	554	0.729	24.2	615	0.809	26.6	676	0.889	29.0	737	0.970			
21.9	556	0.732	24.3	617	0.813	26.7	678	0.892	29.1	739	0.973			
22.0	559	0.735	24.4	620	0.816	26.8	681	0.896	29.2	742	0.976			
22.1	561	0.739	24.5	622	0.820	26.9	683	0.899	29.3	744	0.979			
22.2	564	0.742	24.6	625	0.823	27.0	686	0.902	29.4	747	0.983			
22.3	566	0.746	24.7	627	0.826	27.1	688	0.905	29.5	749	0.986			

Table VII.—Temperature Correction Factor, C_T

(Apply to mean refraction in Table V)

$[r=(r_m)\ (C_B)\ (C_T)]$

Temperature		C_T	Temperature		C_T	Temperature		C_T	Temperature		C_T	Temperature		C_T
Fahrenheit	Centigrade		Fahrenheit	Centigrade		Fahrenheit	Centigrade		Fahrenheit	Centigrade		Fahrenheit	Centigrade	
°	°		°	°		°	°		°	°		°	°	
−25	−31.7	1.172	8	−13.3	1.089	41	5.0	1.018	74	23.3	0.955	107	41.7	0.900
−24	−31.1	1.169	9	−12.8	1.087	42	5.6	1.016	75	23.9	0.953	108	42.2	0.899
−23	−30.6	1.166	10	−12.2	1.085	43	6.1	1.014	76	24.4	0.952	109	42.8	0.897
−22	−30.0	1.164	11	−11.7	1.082	44	6.7	1.012	77	25.0	0.950	110	43.3	0.895
−21	−29.4	1.161	12	−11.1	1.080	45	7.2	1.010	78	25.6	0.948	111	43.9	0.894
−20	−28.9	1.158	13	−10.6	1.078	46	7.8	1.008	79	26.1	0.946	112	44.4	0.892
−19	−28.3	1.156	14	−10.0	1.076	47	8.3	1.006	80	26.7	0.945	113	45.0	0.891
−18	−27.8	1.153	15	− 9.4	1.073	48	8.9	1.004	81	27.2	0.943	114	45.6	0.890
−17	−27.2	1.151	16	− 8.9	1.071	49	9.4	1.002	82	27.8	0.941	115	46.1	0.888
−16	−26.7	1.148	17	− 8.3	1.069	50	10.0	1.000	83	28.3	0.939	116	46.7	0.886
−15	−26.1	1.145	18	− 7.8	1.067	51	10.6	0.998	84	28.9	0.938	117	47.2	0.885
−14	−25.6	1.143	19	− 7.2	1.064	52	11.1	0.996	85	29.4	0.936	118	47.8	0.884
−13	−25.0	1.140	20	− 6.7	1.062	53	11.7	0.994	86	30.0	0.934	119	48.3	0.882
−12	−24.4	1.138	21	− 6.1	1.060	54	12.2	0.992	87	30.6	0.933	120	48.9	0.881
−11	−23.9	1.135	22	− 5.6	1.058	55	12.8	0.990	88	31.1	0.931	121	49.4	0.880
−10	−23.3	1.133	23	− 5.0	1.056	56	13.3	0.988	89	31.7	0.929	122	50.0	0.878
− 9	−22.8	1.130	24	− 4.4	1.054	57	13.9	0.986	90	32.2	0 928	123	50.6	0.877
− 8	−22.2	1.128	25	− 3.9	1.051	58	14.4	0.985	91	32.8	0.926	124	51.1	0.876
− 7	−21.7	1.125	26	− 3.3	1.049	59	15.0	0.983	92	33.3	0.924	125	51.7	0.874
− 6	−21.1	1.123	27	− 2.8	1.047	60	15.6	0.981	93	33.9	0.923	126	52.2	0.873
− 5	−20.6	1.120	28	− 2.2	1.045	61	16.1	0.979	94	34.4	0.921	127	52.8	0.871
− 4	−20.0	1.118	29	− 1.7	1.043	62	16.7	0.977	95	35.0	0.919	128	53.3	0.870
− 3	−19.4	1.115	30	− 1.1	1.041	63	17.2	0.975	96	35.6	0.917	129	53.9	0.868
− 2	−18.9	1.113	31	− 0.6	1.039	64	17.8	0.973	97	36.1	0.916	130	54.4	0.867
− 1	−18.3	1.111	32	0.0	1.036	65	18.3	0.972	98	36.7	0.914			
0	−17.8	1.108	33	+ 0.6	1.034	66	18.9	0.970	99	37.2	0.912			
+ 1	−17.2	1.106	34	1.1	1.032	67	19.4	0.968	100	37.8	0.911			
2	−16.7	1.103	35	1.7	1.030	68	20.0	0.966	101	38.3	0.909			
3	−16.1	1.101	36	2.2	1.028	69	20.6	0.964	102	38.9	0.908			
4	−15.6	1.099	37	2.8	1.026	70	21.1	0.962	103	39.4	0.906			
5	−15.0	1.096	38	3.3	1.024	71	21.7	0.961	104	40.0	0.905			
6	−14.4	1.094	39	3.9	1.022	72	22.2	0.959	105	40.6	0.903			
7	−13.9	1.092	40	4.4	1.020	73	22.8	0.957	106	41.1	0.902			

Table VIII.—Correction to latitude for differential refraction, ½(r−r′)

[The sign of the correction is the same as that of the micrometer difference]

One-half diff. of zenith distances	Zenith distance							
	0°	10°	20°	25°	30°	35°	40°	45°
′	″	″	″	″	″	″	″	″
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.5	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02
1.0	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.03
1.5	0.03	0.03	0.03	0.03	0.03	0.04	0.04	0.05
2.0	0.03	0.03	0.04	0.04	0.04	0.05	0.06	0.07
2.5	0.04	0.04	0.05	0.05	0.06	0.06	0.07	0.08
3.0	0.05	0.05	0.06	0.06	0.07	0.08	0.09	0.10
3.5	0.06	0.06	0.07	0.07	0.08	0.09	0.10	0.12
4.0	0.07	0.07	0.08	0.08	0.09	0.10	0.11	0.13
4.5	0.08	0.08	0.09	0.09	0.10	0.11	0.13	0.15
5.0	0.08	0.09	0.10	0.10	0.11	0.13	0.14	0.17
5.5	0.09	0.10	0.10	0.11	0.12	0.14	0.16	0.18
6.0	0.10	0.10	0.11	0.12	0.13	0.15	0.17	0.20
6.5	0.11	0.11	0.12	0.13	0.14	0.16	0.19	0.22
7.0	0.12	0.12	0.13	0.14	0.16	0.18	0.20	0.23
7.5	0.13	0.13	0.14	0.15	0.17	0.19	0.21	0.25
8.0	0.13	0.14	0.15	0.16	0.18	0.20	0.23	0.27
8.5	0.14	0.15	0.16	0.17	0.19	0.21	0.24	0.29
9.0	0.15	0.16	0.17	0.18	0.20	0.23	0.26	0.30
9.5	0.16	0.16	0.18	0.19	0.21	0.24	0.27	0.32
10.0	0.17	0.17	0.19	0.20	0.22	0.25	0.29	0.34
10.5	0.18	0.18	0.20	0.21	0.23	0.26	0.30	0.35
11.0	0.18	0.19	0.21	0.22	0.25	0.28	0.31	0.37
11.5	0.19	0.20	0.22	0.23	0.26	0.29	0.33	0.39
12.0	0.20	0.21	0.23	0.25	0.27	0.30	0.34	0.40
12.5	0.21	0.22	0.24	0.26	0.28	0.31	0.36	0.42
13.0	0.22	0.22	0.25	0.27	0.29	0.33	0.37	0.44
13.5	0.23	0.23	0.26	0.28	0.30	0.34	0.39	0.45
14.0	0.23	0.24	0.27	0.29	0.31	0.35	0.40	0.47
14.5	0.24	0.25	0.28	0.30	0.32	0.36	0.41	0.49
15.0	0.25	0.26	0.29	0.31	0.34	0.38	0.43	0.50
15.5	0.26	0.27	0.29	0.32	0.35	0.39	0.44	0.52
16.0	0.27	0.28	0.30	0.33	0.36	0.40	0.46	0.54
16.5	0.28	0.29	0.31	0.34	0.37	0.41	0.47	0.55
17.0	0.29	0.29	0.32	0.35	0.38	0.43	0.49	0.57
17.5	0.29	0.30	0.33	0.36	0.39	0.44	0.50	0.59
18.0	0.30	0.31	0.34	0.37	0.40	0.45	0.51	0.60
18.5	0.31	0.32	0.35	0.38	0.41	0.46	0.53	0.62
19.0	0.32	0.33	0.36	0.39	0.43	0.48	0.54	0.64
19.5	0.33	0.34	0.37	0.40	0.44	0.49	0.56	0.65
20.0	0.34	0.35	0.38	0.41	0.45	0.50	0.57	0.67

Table IX.—Elevation Factor for Differential Refraction

For elevation	Multiply by	For elevation	Multiply by	For elevation	Multiply by
feet		*meters*		*meters*	
1,000	0.96	200	0.98	3,200	.68
2,000	.93	400	.95	3,400	.66
3,000	.90	600	.93	3,600	.65
4,000	.86	800	.91	3,800	.63
5,000	.83	1,000	.89	4,000	.62
6,000	.80	1,200	.87	4,200	.60
7,000	.77	1,400	.85	4,400	.59
8,000	.75	1,600	.83	4,600	.58
9,000	.72	1,800	.81	4,800	.56
10,000	.69	2,000	.79	5,000	.55
11,000	.67	2,200	.77		
12,000	.64	2,400	.75		
13,000	.62	2,600	.73		
14,000	.60	2,800	.71		
15,000	.58	3,000	.70		

Table X.—Correction to latitude for reduction to meridian

[Star off the meridian but instrument in the meridian. The sign of the correction to the latitude is positive except for stars south of the equator and subpolars]

δ	6ᵐ	7ᵐ	8ᵐ	9ᵐ	10ᵐ	15ᵐ	20ᵐ	22ᵐ	24ᵐ	26ᵐ	28ᵐ	30ᵐ	32ᵐ	34ᵐ	36ᵐ	38ᵐ	40ᵐ	42ᵐ	44ᵐ	46ᵐ	48ᵐ	50ᵐ	52ᵐ	54ᵐ	56ᵐ	58ᵐ	60ᵐ	δ
°	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	°
1														.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.02	.02	89
2									.01	.01	.01	.01	.01	.01	.01	.01	.02	.02	.02	.02	.02	.02	.03	.03	.03	.03	.03	88
3							.01	.01	.01	.01	.01	.01	.01	.02	.02	.02	.02	.03	.03	.03	.03	.04	.04	.04	.04	.05	.05	87
4							.01	.01	.01	.01	.01	.02	.02	.02	.02	.03	.03	.03	.04	.04	.04	.05	.05	.06	.06	.06	.07	86
5						.01	.01	.01	.01	.02	.02	.02	.02	.03	.03	.03	.04	.04	.05	.05	.05	.06	.06	.07	.07	.08	.09	85
6						.01	.01	.01	.02	.02	.02	.03	.03	.03	.04	.04	.05	.05	.06	.06	.07	.07	.08	.08	.09	.10	.10	84
7						.01	.01	.02	.02	.02	.03	.03	.03	.04	.04	.05	.05	.06	.06	.07	.08	.08	.09	.10	.10	.11	.12	83
8						.01	.02	.02	.02	.03	.03	.03	.04	.04	.05	.05	.06	.07	.07	.08	.09	.09	.10	.11	.12	.13	.14	82
9						.01	.02	.02	.02	.03	.03	.04	.04	.05	.05	.06	.07	.07	.08	.09	.10	.11	.11	.12	.13	.14	.15	81
10						.01	.02	.02	.03	.03	.04	.04	.05	.05	.06	.07	.07	.08	.09	.10	.11	.12	.13	.14	.15	.16	.17	80
12					.01	.01	.02	.03	.03	.04	.05	.05	.06	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15	.16	.17	.19	.20	78
14				.01	.01	.01	.03	.03	.04	.04	.05	.06	.07	.07	.08	.09	.10	.11	.12	.14	.15	.16	.17	.19	.20	.22	.23	76
16				.01	.01	.02	.03	.03	.04	.05	.06	.07	.07	.08	.09	.10	.12	.13	.14	.15	.17	.18	.20	.21	.23	.24	.26	74
18			.01	.01	.01	.02	.03	.04	.05	.05	.06	.07	.08	.09	.10	.12	.13	.14	.16	.17	.18	.20	.22	.23	.25	.27	.29	72
20			.01	.01	.01	.02	.04	.04	.05	.06	.07	.08	.09	.10	.11	.13	.14	.15	.17	.19	.20	.22	.24	.26	.28	.29	.32	70
22			.01	.01	.01	.02	.04	.05	.05	.06	.07	.09	.10	.11	.12	.14	.15	.17	.18	.20	.22	.24	.26	.28	.30	.32	.34	68
24			.01	.01	.01	.02	.04	.05	.06	.07	.08	.09	.10	.12	.13	.15	.16	.18	.20	.21	.23	.25	.27	.29	.32	.34	.36	66
26			.01	.01	.01	.02	.04	.05	.06	.07	.08	.10	.11	.12	.14	.15	.17	.19	.21	.23	.25	.27	.29	.31	.34	.36	.39	64
28		.01	.01	.01	.01	.03	.05	.05	.07	.08	.09	.10	.12	.13	.15	.16	.18	.20	.22	.24	.26	.28	.31	.33	.35	.38	.41	62
30		.01	.01	.01	.01	.03	.05	.06	.07	.08	.09	.11	.12	.14	.15	.17	.19	.21	.23	.25	.27	.30	.32	.34	.37	.40	.42	60
32		.01	.01	.01	.01	.03	.05	.06	.07	.08	.10	.11	.13	.14	.16	.18	.20	.22	.24	.26	.28	.31	.33	.36	.39	.41	.44	58
34		.01	.01	.01	.01	.03	.05	.06	.07	.09	.10	.11	.13	.15	.16	.18	.20	.22	.24	.27	.29	.32	.34	.37	.40	.42	.45	56
36		.01	.01	.01	.01	.03	.05	.06	.07	.09	.10	.12	.13	.15	.17	.19	.21	.23	.25	.28	.30	.32	.35	.38	.41	.44	.47	54
38	.01	.01	.01	.01	.01	.03	.05	.06	.08	.09	.10	.12	.13	.15	.17	.19	.21	.23	.26	.28	.30	.33	.36	.39	.41	.44	.48	52
40	.01	.01	.01	.01	.01	.03	.05	.07	.08	.09	.11	.12	.14	.16	.17	.19	.21	.24	.26	.28	.31	.34	.36	.39	.42	.45	.48	50
45	.01	.01	.01	.01	.01	.03	.05	.07	.08	.09	.11	.12	.14	.16	.18	.20	.22	.24	.26	.29	.31	.34	.37	.40	.43	.46	.49	45

Table X.—Continued

δ	60ᵐ	65ᵐ	70ᵐ	75ᵐ	80ᵐ	85ᵐ	90ᵐ	95ᵐ	100ᵐ	105ᵐ	110ᵐ	115ᵐ	120ᵐ	δ
°	″	″	″	″	″	″	″	″	″	″	″	″	″	°
0	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	90
2	.03	.04	.05	.05	.06	.07	.08	.09	.10	.10	.12	.13	.14	88
4	.07	.08	.09	.11	.12	.14	.15	.17	.19	.21	.23	.25	.27	86
6	.10	.12	.14	.16	.18	.20	.23	.26	.28	.31	.34	.37	.41	84
8	.14	.16	.18	.21	.24	.27	.30	.34	.38	.41	.45	.50	.54	82
10	.17	.20	.23	.26	.30	.34	.38	.42	.47	.51	.56	.62	.67	80
12	.20	.24	.27	.31	.36	.40	.45	.50	.55	.61	.67	.73	.80	78
14	.23	.27	.31	.36	.41	.46	.52	.58	.64	.71	.77	.85	.92	76
16	.26	.31	.35	.41	.46	.52	.59	.65	.72	.80	.87	.96	1.04	74
18	.29	.34	.39	.45	.51	.58	.65	.72	.80	.88	.97	1.06	1.15	72
20	.32	.37	.43	.49	.56	.63	.71	.79	.88	.97	1.06	1.16	1.26	70
22	.34	.40	.46	.53	.61	.68	.77	.85	.95	1.04	1.15	1.25	1.36	68
24	.36	.43	.50	.57	.65	.73	.82	.91	1.01	1.12	1.23	1.34	1.46	66
26	.39	.46	.53	.60	.69	.78	.87	.97	1.07	1.18	1.30	1.42	1.55	64
28	.41	.48	.55	.64	.72	.82	.92	1.02	1.13	1.25	1.37	1.49	1.63	62
30	.42	.50	.58	.66	.76	.85	.96	1.07	1.18	1.30	1.43	1.56	1.70	60
32	.44	.52	.60	.69	.78	.89	.99	1.11	1.22	1.35	1.48	1.62	1.76	58
34	.45	.54	.62	.71	.81	.91	1.02	1.14	1.26	1.39	1.53	1.67	1.82	56
36	.47	.55	.63	.73	.83	.94	1.05	1.17	1.30	1.43	1.57	1.71	1.87	54
38	.48	.56	.65	.74	.85	.96	1.07	1.19	1.32	1.46	1.60	1.75	1.90	52
40	.48	.57	.66	.76	.86	.97	1.09	1.21	1.34	1.48	1.62	1.78	1.93	50
42	.49	.57	.66	.76	.87	.98	1.10	1.22	1.36	1.49	1.64	1.79	1.95	48
44	.49	.58	.67	.77	.87	.98	1.10	1.23	1.36	1.50	1.65	1.80	1.96	46
45	.49	.58	.67	.77	.87	.98	1.10	1.23	1.36	1.50	1.65	1.80	1.96	45

Table XI.—Reduction of latitude to sea level

[The correction is negative in every case]

h \ φ		5°	10°	15°	20°	25°	30°	35°	40°	45°
		85°	80°	75°	70°	65°	60°	55°	50°	
Feet	*Meters*	″	″	″	″	″	″	″	″	″
100	30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
200	61	.00	.00	.01	.01	.01	.01	.01	.01	.01
300	91	.00	.01	.01	.01	.01	.01	.01	.02	.02
400	122	.00	.01	.01	.01	.02	.02	.02	.02	.02
500	152	.00	.01	.01	.02	.02	.02	.02	.03	.03
600	183	.01	.01	.02	.02	.02	.03	.03	.03	.03
700	213	.01	.01	.02	.02	.03	.03	.03	.04	.04
800	244	.01	.01	.02	.03	.03	.04	.04	.04	.04
900	274	.01	.02	.02	.03	.04	.04	.04	.05	.05
1000	305	.01	.02	.03	.03	.04	.05	.05	.05	.05
1100	335	.01	.02	.03	.04	.04	.05	.05	.06	.06
1200	366	.01	.02	.03	.04	.05	.05	.06	.06	.06
1300	396	.01	.02	.03	.04	.05	.06	.06	.07	.07
1400	427	.01	.02	.04	.05	.06	.06	.07	.07	.07
1500	457	.01	.03	.04	.05	.06	.07	.07	.08	.08
1600	488	.01	.03	.04	.05	.06	.07	.08	.08	.08
1700	518	.02	.03	.04	.06	.07	.08	.08	.09	.09
1800	549	.02	.03	.05	.06	.07	.08	.09	.09	.09
1900	579	.02	.03	.05	.06	.08	.09	.09	.10	.10
2000	610	.02	.04	.05	.07	.08	.09	.10	.10	.10
2100	640	.02	.04	.05	.07	.08	.09	.10	.11	.11
2200	671	.02	.04	.06	.07	.09	.10	.11	.11	.11
2300	701	.02	.04	.06	.08	.09	.10	.11	.12	.12
2400	732	.02	.04	.06	.08	.10	.11	.12	.12	.13
2500	762	.02	.04	.07	.08	.10	.11	.12	.13	.13
2600	792	.02	.05	.07	.09	.10	.12	.13	.13	.14
2700	823	.02	.05	.07	.09	.11	.12	.13	.14	.14
2800	853	.03	.05	.07	.09	.11	.13	.14	.14	.15
2900	884	.03	.05	.08	.10	.12	.13	.14	.15	.15
3000	914	.03	.05	.08	.10	.12	.14	.15	.15	.16
3100	945	.03	.06	.08	.10	.12	.14	.15	.16	.16
3200	975	.03	.06	.08	.11	.13	.14	.16	.16	.17
3300	1006	.03	.06	.09	.11	.13	.15	.16	.17	.17
3400	1036	.03	.06	.09	.11	.14	.15	.17	.17	.18
3500	1067	.03	.06	.09	.12	.14	.16	.17	.18	.18
3600	1097	.03	.06	.09	.12	.14	.16	.18	.18	.19
3700	1128	.03	.07	.10	.12	.15	.17	.18	.19	.19
3800	1158	.03	.07	.10	.13	.15	.17	.19	.20	.20
3900	1189	.04	.07	.10	.13	.16	.18	.19	.20	.20
4000	1219	.04	.07	.10	.13	.16	.18	.20	.21	.21
4100	1250	.04	.07	.11	.14	.16	.19	.20	.21	.21
4200	1280	.04	.07	.11	.14	.17	.19	.21	.22	.22
4300	1311	.04	.08	.11	.14	.17	.19	.21	.22	.22
4400	1341	.04	.08	.11	.15	.18	.20	.22	.23	.23
4500	1372	.04	.08	.12	.15	.18	.20	.22	.23	.23
4600	1402	.04	.08	.12	.15	.18	.21	.23	.24	.24
4700	1433	.04	.08	.12	.16	.19	.21	.23	.24	.24
4800	1463	.04	.09	.13	.16	.19	.22	.24	.25	.25
4900	1494	.04	.09	.13	.16	.20	.22	.24	.25	.26
5000	1524	.05	.09	.13	.17	.20	.23	.24	.26	.26
5100	1554	.05	.09	.13	.17	.20	.23	.25	.26	.27
5200	1585	.05	.09	.14	.17	.21	.23	.25	.27	.27
5300	1615	.05	.09	.14	.18	.21	.24	.26	.27	.28
5400	1646	.05	.10	.14	.18	.22	.24	.26	.28	.28
5500	1676	.05	.10	.14	.18	.22	.25	.27	.28	.29

Table XI.—Reduction of latitude to sea level

h \ ϕ		5°	10°	15°	20°	25°	30°	35°	40°	45°
		85°	80°	75°	70°	65°	60°	55°	50°	
Feet	*Meters*	″	″	″	″	″	″	″	″	″
5600	1707	0.05	0.10	0.15	0.19	0.22	0.25	0.27	0.29	0.29
5700	1737	.05	.10	.15	.19	.23	.26	.28	.29	.30
5800	1768	.05	.10	.15	.19	.23	.26	.28	.30	.30
5900	1798	.05	.11	.15	.20	.24	.27	.29	.30	.31
6000	1829	.05	.11	.16	.20	.24	.27	.29	.31	.31
6100	1859	.06	.11	.16	.20	.24	.28	.30	.31	.32
6200	1890	.06	.11	.16	.21	.25	.28	.30	.32	.32
6300	1920	.06	.11	.16	.21	.25	.28	.31	.32	.33
6400	1951	.06	.11	.17	.21	.26	.29	.31	.33	.33
6500	1981	.06	.12	.17	.22	.26	.29	.32	.33	.34
6600	2012	.06	.12	.17	.22	.26	.30	.32	.34	.34
6700	2042	.06	.12	.17	.22	.27	.30	.33	.34	.35
6800	2073	.06	.12	.18	.23	.27	31	.33	.35	.35
6900	2103	.06	.12	.18	.23	.28	.31	.34	.35	.36
7000	2134	.06	.12	.18	.23	.28	.32	.34	.36	.36
7100	2164	.06	.13	.19	.24	.28	.32	.35	.36	.37
7200	2195	.07	.13	.19	.24	.29	.33	.35	.37	.38
7300	2225	.07	.13	.19	.24	.29	.33	.36	.37	.38
7400	2256	.07	.13	.19	.25	.30	.33	.36	.38	.39
7500	2286	.07	.13	.20	.25	.30	.34	.37	.38	.39
7600	2316	.07	.14	.20	.25	.30	.34	.37	.39	.40
7700	2347	.07	.14	.20	.26	.31	.35	.38	.40	.40
7800	2377	.07	.14	.20	.26	.31	.35	.38	.40	.41
7900	2408	.07	.14	.21	.26	.32	.36	.39	.41	.41
8000	2438	.07	.14	.21	.27	.32	.36	.39	.41	.42
8100	2469	.07	.14	.21	.27	.32	.37	.40	.42	.42
8200	2499	.07	.15	.21	.27	.33	.37	.40	.42	.43
8300	2530	.08	.15	.22	.28	.33	.37	.41	.43	.43
8400	2560	.08	.15	.22	.28	.34	.38	.41	.43	.44
8500	2591	.08	.15	.22	.28	.34	.38	.42	.44	.44
8600	2621	.08	.15	.22	.29	.34	.39	.42	.44	.45
8700	2652	.08	.16	.23	.29	.35	.39	.43	.45	.45
8800	2682	.08	.16	.23	.29	.35	.40	.43	.45	.46
8900	2713	.08	.16	.23	.30	.36	.40	.44	.46	.46
9000	2743	.08	.16	.23	.30	.36	.41	.44	.46	.47
9100	2774	.08	.16	.24	.30	.36	.41	.45	.47	.47
9200	2804	.08	.16	.24	.31	.37	.42	.45	.47	.48
9300	2835	.08	.17	.24	.31	.37	.42	.46	.48	.48
9400	2865	.09	.17	.24	.31	.38	.42	.46	.48	.49
9500	2896	.09	.17	.25	.32	.38	.43	.47	.49	.50
9600	2926	.09	.17	.25	.32	.38	.43	.47	.49	.50
9700	2957	.09	.17	.25	.32	.39	.44	.48	.50	.51
9800	2987	.09	.17	.26	.33	.39	.44	.48	.50	.51
9900	3018	.09	.18	.26	.33	.40	.45	.48	.51	.52
10000	3048	.09	.18	.26	.33	.40	.45	.49	.51	.52

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Table XII.—Values of $\left(\frac{2\ sin^2\ \frac{1}{2}\tau}{sin\ 1''}\right)$

τ	0 m	1 m	2 m	3 m	4 m	5 m	6 m	7 m	8 m
s	″	″	″	″	″	″	″	″	″
0	0	1.96	7.85	17.67	31.42	49.09	70.68	96.20	125.65
1	0	2.03	7.98	17.87	31.68	49.41	71.07	96.66	126.17
2	0	2.10	8.12	18.07	31.94	49.74	71.47	97.12	126.70
3	0	2.16	8.25	18.27	32.20	50.07	71.86	97.58	127.22
4	.01	2.23	8.39	18.47	32.47	50.40	72.26	98.04	127.75
5	.01	2.31	8.52	18.67	32.74	50.73	72.66	98.50	128.28
6	.02	2.38	8.66	18.87	33.01	51.07	73.06	98.97	128.81
7	.02	2.45	8.80	19.07	33.27	51.40	73.46	99.43	129.34
8	.03	2.52	8.94	19.28	33.54	51.74	73.86	99.90	129.87
9	.04	2.60	9.08	19.48	33.81	52.07	74.26	100.37	130.40
10	.05	2.67	9.22	19.69	34.09	52.41	74.66	100.84	130.94
11	.06	2.75	9.36	19.90	34.36	52.75	75.06	101.31	131.47
12	.08	2.83	9.50	20.11	34.64	53.09	75.47	101.78	132.01
13	.09	2.91	9.64	20.32	34.91	53.43	75.88	102.25	132.55
14	.11	2.99	9.79	20.53	35.19	53.77	76.29	102.72	133.09
15	.12	3.07	9.94	20.74	35.46	54.11	76.69	103.20	133.63
16	.14	3.15	10.09	20.95	35.74	54.46	77.10	103.67	134.17
17	.16	3.23	10.24	21.16	36.02	54.80	77.51	104.15	134.71
18	.18	3.32	10.39	21.38	36.30	55.15	77.93	104.63	135.25
19	.20	3.40	10.54	21.60	36.58	55.50	78.34	105.10	135.80
20	.22	3.49	10.69	21.82	36.87	55.84	78.75	105.58	136.34
21	.24	3.58	10.84	22.03	37.15	56.19	79.16	106.06	136.88
22	.26	3.67	11.00	22.25	37.44	56.55	79.58	106.55	137.43
23	.28	3.76	11.15	22.47	37.72	56.90	80.00	107.03	137.98
24	.31	3.85	11.31	22.70	38.01	57.25	80.42	107.51	138.53
25	.34	3.94	11.47	22.92	38.30	57.60	80.84	107.99	139.08
26	.37	4.03	11.63	23.14	38.59	57.96	81.26	108.48	139.63
27	.40	4.12	11.79	23.37	38.88	58.32	81.68	108.97	140.18
28	.43	4.22	11.95	23.60	39.17	58.68	82.10	109.46	140.74
29	.46	4.32	12.11	23.82	39.46	59.03	82.52	109.95	141.29
30	.49	4.42	12.27	24.05	39.76	59.40	82.95	110.44	141.85
31	.52	4.52	12.43	24.28	40.05	59.75	83.38	110.93	142.40
32	.56	4.62	12.60	24.51	40.35	60.11	83.81	111.43	142.96
33	.59	4.72	12.76	24.74	40.65	60.47	84.23	111.92	143.52
34	.63	4.82	12.93	24.98	40.95	60.84	84.66	112.41	144.08
35	.67	4.92	13.10	25.21	41.25	61.20	85.09	112.90	144.64
36	.71	5.03	13.27	25.45	41.55	61.57	85.52	113.40	145.20
37	.75	5.13	13.44	25.68	41.85	61.94	85.95	113.90	145.76
38	.79	5.24	13.62	25.92	42.15	62.31	86.39	114.40	146.33
39	.83	5.34	13.79	26.16	42.45	62.68	86.82	114.90	146.89
40	.87	5.45	13.96	26.40	42.76	63.05	87.26	115.40	147.46
41	.91	5.56	14.13	26.64	43.06	63.42	87.70	115.90	148.03
42	0.96	5.67	14.31	26.88	43.37	63.79	88.14	116.40	148.60
43	1.01	5.78	14.49	27.12	43.68	64.16	88.57	116.90	149.17
44	1.06	5.90	14.67	27.37	43.99	64.54	89.01	117.41	149.74
45	1.10	6.01	14.85	27.61	44.30	64.91	89.45	117.92	150.31
46	1.15	6.13	15.03	27.86	44.61	65.29	89.89	118.43	150.88
47	1.20	6.24	15.21	28.10	44.92	65.67	90.33	118.94	151.45
48	1.26	6.36	15.39	28.35	45.24	66.05	90.78	119.45	152.03
49	1.31	6.48	15.57	28.60	45.55	66.43	91.23	119.96	152.61
50	1.36	6.60	15.76	28.85	45.87	66.81	91.68	120.47	153.19
51	1.42	6.72	15.95	29.10	46.18	67.19	92.12	120.98	153.77
52	1.48	6.84	16.14	29.36	46.50	67.58	92.57	121.49	154.35
53	1.53	6.96	16.32	29.61	46.82	67.96	93.02	122.01	154.93
54	1.59	7.09	16.51	29.86	47.14	68.35	93.47	122.53	155.51
55	1.65	7.21	16.70	30.12	47.46	68.73	93.92	123.05	156.09
56	1.71	7.34	16.89	30.38	47.79	69.12	94.38	123.57	156.67
57	1.77	7.46	17.08	30.64	48.11	69.51	94.83	124.09	157.25
58	1.83	7.60	17.28	30.90	48.43	69.90	95.29	124.61	157.84
59	1.89	7.72	17.47	31.16	48.76	70.29	95.74	125.13	158.43

Table XII.—Values of $\left(\frac{2 \sin^2 \frac{1}{2}\tau}{\sin 1''}\right)$

τ	9 m	10 m	11 m	12 m	13 m	14 m	15 m	16 m
s	″	″	″	″	″	″	″	″
0	159.02	196.32	237.54	282.68	331.74	384.74	441.63	502.46
1	159.61	196.97	238.26	283.47	332.59	385.65	442.62	503.50
2	160.20	197.63	238.98	284.26	333.44	386.56	443.60	504.55
3	160.80	198.28	239.70	285.04	334.29	387.48	444.58	505.60
4	161.39	198.94	240.42	285.83	335.15	388.40	445.56	506.65
5	161.98	199.60	241.14	286.62	336.00	389.32	446.55	507.70
6	162.58	200.26	241.87	287.41	336.86	390.24	447.54	508.76
7	163.17	200.92	242.60	288.20	337.72	391.16	448.53	509.81
8	163.77	201.59	243.33	289.00	338.58	392.09	449.51	510.86
9	164.37	202.25	244.06	289.79	339.44	393.01	450.50	511.92
10	164.97	202.92	244.79	290.58	340.30	393.94	451.50	512.98
11	165.57	203.58	245.52	291.38	341.16	394.86	452.49	514.03
12	166.17	204.25	246.25	292.18	342.02	395.79	453.48	515.09
13	166.77	204.92	246.98	292.98	342.88	396.72	454.48	516.15
14	167.37	205.59	247.72	293.78	343.75	397.65	455.47	517.21
15	167.97	206.26	248.45	294.58	344.62	398.58	456.47	518.27
16	168.58	206.93	249.19	295.38	345.49	399.52	457.47	519.34
17	169.19	207.60	249.93	296.18	346.36	400.45	458.47	520.40
18	169.80	208.27	250.67	296.99	347.23	401.38	459.47	521.47
19	170.41	208.94	251.41	297.79	348.10	402.32	460.47	522.53
20	171.02	209.62	252.15	298.60	348.97	403.26	461.47	523.60
21	171.63	210.30	252.89	299.40	349.84	404.20	462.48	524.67
22	172.24	210.98	253.63	300.21	350.71	405.14	463.48	525.74
23	172.85	211.66	254.37	301.02	351.58	406.08	464.48	526.81
24	173.47	212.34	255.12	301.83	352.46	407.02	465.49	527.89
25	174.08	213.02	255.87	302.64	353.34	407.96	466.50	528.96
26	174.70	213.70	256.62	303.46	354.22	408.90	467.51	530.03
27	175.32	214.38	257.37	304.27	355.10	409.84	468.52	531.11
28	175.94	215.07	258.12	305.09	355.98	410.79	469.53	532.18
29	176.56	215.75	258.87	305.90	356.86	411.73	470.54	533.26
30	177.18	216.44	259.62	306.72	357.74	412.68	471.55	534.33
31	177.80	217.12	260.37	307.54	358.62	413.63	472.57	535.41
32	178.43	217.81	261.12	308.36	359.51	414.59	473.58	536.50
33	179.05	218.50	261.88	309.18	360.39	415.54	474.60	537.58
34	179.68	219.19	262.64	310.00	361.28	416.49	475.62	538.67
35	180.30	219.88	263.39	310.82	362.17	417.44	476.64	539.75
36	180.93	220.58	264.15	311.65	363.07	418.40	477.65	540.83
37	181.56	221.27	264.91	312.47	363.96	419.35	478.67	541.91
38	182.19	221.97	265.68	313.30	364.85	420.31	479.70	543.00
39	182.82	222.66	266.44	314.12	365.75	421.27	480.72	544.09
40	183.46	223.36	267.20	314.95	366.64	422.23	481.74	545.18
41	184.09	224.06	267.96	315.78	367.53	423.19	482.77	546.27
42	184.72	224.76	268.73	316.61	368.42	424.15	483.79	547.36
43	185.35	225.46	269.49	317.44	369.31	425.11	484.82	548.45
44	185.99	226.16	270.26	318.27	370.21	426.07	485.85	549.55
45	186.63	226.86	271.02	319.10	371.11	427.04	486.88	550.64
46	187.27	227.57	271.79	319.94	372.01	428.01	487.91	551.73
47	187.91	228.27	272.56	320.78	372.91	428.97	488.94	552.83
48	188.55	228.98	273.34	321.62	373.82	429.93	489.97	553.93
49	189.19	229.68	274.11	322.45	374.72	430.90	491.01	555.03
50	189.83	230.39	274.88	323.29	375.62	431.87	492.05	556.13
51	190.47	231.10	275.65	324.13	376.52	432.84	493.08	557.24
52	191.12	231.81	276.43	324.97	377.43	433.82	494.12	558.34
53	191.76	232.52	277.20	325.81	378.34	434.79	495.15	559.44
54	192.41	233.24	277.98	326.66	379.26	435.76	496.19	560.55
55	193.06	233.95	278.76	327.50	380.17	436.73	497.23	561.65
56	193.71	234.67	279.55	328.35	381.08	437.71	498.28	562.76
57	194.36	235.38	280.33	329.19	381.99	438.69	499.32	563.87
58	195.01	236.10	281.12	330.04	382.90	439.67	500.37	564.98
59	195.66	236.82	281.90	330.89	383.82	440.65	501.41	566.08

Table XII.—Values of $\left(\frac{2\,sin^2\,\frac{1}{2}\tau}{sin\ 1''}\right)$

τ	17m	18m	19m	20m	21m	22m	23m	24m	25m
s	″	″	″	″	″	″	″	″	″
0	567.2	635.9	708.4	784.9	865.3	949.6	1037.8	1129.9	1225.9
1	568.3	637.0	709.7	786.2	866.6	951.0	1039.3	1131.4	1227.5
2	569.4	638.2	710.9	787.5	868.0	952.4	1040.8	1133.0	1229.2
3	570.5	639.4	712.1	788.8	869.4	953.8	1042.3	1134.6	1230.8
4	571.6	640.6	713.4	790.1	870.8	955.3	1043.8	1136.2	1232.5
5	572.8	641.7	714.6	791.4	872.1	956.7	1045.3	1137.8	1234.1
6	573.9	642.9	715.9	792.7	873.5	958.2	1046.8	1139.3	1235.7
7	575.0	644.1	717.1	794.0	874.9	959.6	1048.3	1140.9	1237.3
8	576.1	645.3	718.4	795.4	876.3	961.1	1049.8	1142.5	1239.0
9	577.2	646.5	719.6	796.7	877.6	962.5	1051.3	1144.0	1240.6
10	578.4	647.7	720.9	798.0	879.0	963.9	1052.8	1145.6	1242.3
11	579.5	648.9	722.1	799.3	880.4	965.4	1054.3	1147.2	1243.9
12	580.6	650.0	723.4	800.7	881.8	966.9	1055.9	1148.8	1245.6
13	581.7	651.2	724.6	802.0	883.2	968.3	1057.4	1150.4	1247.2
14	582.9	652.4	725.9	803.3	884.6	969.8	1058.9	1152.0	1248.9
15	584.0	653.6	727.2	804.6	886.0	971.2	1060.4	1153.6	1250.5
16	585.1	654.8	728.4	806.0	887.4	972.7	1062.0	1155.2	1252.2
17	586.2	656.0	729.7	807.3	888.8	974.1	1063.5	1156.8	1253.8
18	587.4	657.2	730.9	808.6	890.2	975.5	1065.0	1158.3	1255.5
19	588.5	658.4	732.2	809.9	891.6	977.0	1066.5	1159.9	1257.1
20	589.6	659.6	733.5	811.3	893.0	978.5	1068.1	1161.5	1258.8
21	590.8	660.8	734.7	812.6	894.4	979.9	1069.6	1163.1	1260.5
22	591.9	662.0	736.0	813.9	895.8	981.4	1071.1	1164.7	1262.2
23	593.0	663.2	737.3	815.2	897.2	982.9	1072.6	1166.3	1263.8
24	594.2	664.4	738.5	816.6	898.6	984.4	1074.2	1167.9	1265.5
25	595.3	665.6	739.8	817.9	900.0	985.8	1075.7	1169.5	1267.1
26	596.5	666.8	741.1	819.2	901.4	987.3	1077.2	1171.1	1268.8
27	597.6	668.0	742.3	820.5	902.8	988.8	1078.7	1172.7	1270.5
28	598.7	669.2	743.6	821.9	904.2	990.3	1080.3	1174.3	1272.1
29	599.9	670.4	744.9	823.2	905.6	991.8	1081.8	1175.9	1273.7
30	601.0	671.6	746.2	824.6	907.0	993.2	1083.3	1177.5	1275.4
31	602.2	672.8	747.4	825.9	908.4	994.7	1084.8	1179.1	1277.1
32	603.3	674.1	748.7	827.3	909.8	996.2	1086.4	1180.7	1278.8
33	604.5	675.3	750.0	828.6	911.2	997.6	1087.9	1182.3	1280.4
34	605.6	676.5	751.3	829.9	912.6	999.1	1089.5	1183.9	1282.1
35	606.8	677.7	752.6	831.2	914.0	1000.6	1091.0	1185.5	1283.8
36	607.9	678.9	753.8	832.6	915.5	1002.1	1092.6	1187.1	1285.5
37	609.1	680.1	755.1	833.9	916.9	1003.5	1094.1	1188.7	1287.1
38	610.2	681.3	756.4	835.3	918.3	1005.0	1095.7	1190.3	1288.8
39	611.4	682.6	757.7	836.6	919.7	1006.5	1097.2	1191.9	1290.5
40	612.5	683.8	759.0	838.0	921.1	1008.0	1098.8	1193.5	1292.2
41	613.7	685.0	760.2	839.3	922.5	1009.4	1100.3	1195.1	1293.8
42	614.8	686.2	761.5	840.7	923.9	1010.9	1101.9	1196.7	1295.5
43	616.0	687.4	762.8	842.0	925.3	1012.4	1103.4	1198.3	1297.2
44	617.2	688.7	764.1	843.4	926.8	1013.9	1105.0	1199.9	1298.9
45	618.3	689.9	765.4	844.7	928.2	1015.4	1106.5	1201.5	1300.5
46	619.5	691.1	766.7	846.1	929.6	1016.9	1108.1	1203.1	1302.2
47	620.6	692.4	768.0	847.5	931.0	1018.4	1109.6	1204.7	1303.9
48	621.8	693.6	769.3	848.9	932.4	1019.9	1111.2	1206.4	1305.6
49	623.0	694.8	770.6	850.2	933.8	1021.4	1112.7	1208.0	1307.3
50	624.1	696.0	771.9	851.6	935.2	1022.8	1114.3	1209.6	1309.0
51	625.3	697.3	773.1	852.9	936.6	1024.3	1115.8	1211.2	1310.7
52	626.5	698.5	774.5	854.3	938.1	1025.8	1117.4	1212.9	1312.4
53	627.6	699.7	775.7	855.7	939.5	1027.3	1118.9	1214.5	1314.1
54	628.8	701.0	777.1	857.1	940.9	1028.8	1120.5	1216.1	1315.7
55	630.0	702.2	778.4	858.4	942.3	1030.3	1122.0	1217.7	1317.4
56	631.2	703.5	779.7	859.8	943.8	1031.8	1123.6	1219.4	1319.1
57	632.3	704.7	781.0	861.1	945.2	1033.3	1125.1	1221.0	1320.8
58	633.5	705.9	782.3	862.5	946.6	1034.8	1126.7	1222.6	1322.5
59	634.7	707.1	783.6	863.9	948.1	1036.3	1128.3	1224.2	1324.2

Table XIII.—$Log\frac{1}{1-a}$ [$=colog\ (1-a)$]

(1) When a is positive:

Log a	0	1	2	3	4	5	6	7	8	9	10
9.00	0.045758	5869	5980	6092	6204	6317	6429	6542	6656	6769	6883
8.99	0.044660	4769	4878	4987	5096	5205	5315	5425	5536	5647	5758
98	3591	3697	3803	3909	4016	4122	4229	4337	4444	4552	4660
97	2549	2652	2755	2858	2962	3066	3171	3275	3380	3486	3591
96	1532	1633	1733	1834	1936	2037	2139	2241	2343	2446	2549
95	0.040541	0639	0737	0836	0935	1034	1133	1232	1332	1432	1532
94	0.039575	9670	9766	9862	9959	*0055	*0152	*0249	*0346	*0443	*0541
93	8633	8726	8819	8913	9007	9101	9195	9290	9385	9480	9575
92	7714	7805	7896	7987	8079	8171	8263	8355	8447	8540	8633
91	6818	6907	6996	7085	7174	7263	7353	7443	7533	7624	7714
8.90	0.035944	6031	6118	6204	6291	6379	6466	6554	6642	6730	6818
89	5092	5177	5261	5346	5431	5516	5601	5687	5772	5858	5944
88	4261	4343	4426	4508	4591	4674	4757	4841	4924	5008	5092
87	3451	3531	3611	3692	3772	3853	3934	4016	4097	4179	4261
86	2660	2738	2816	2895	2974	3053	3132	3211	3291	3371	3451
85	0.031888	1965	2041	2118	2195	2272	2349	2426	2504	2582	2660
84	1136	1210	1285	1360	1435	1510	1585	1660	1736	1812	1888
83	0.030402	0474	0547	0620	0693	0766	0840	0914	0987	1061	1136
82	0.029685	9756	9827	9898	9970	*0041	*0113	*0185	*0257	*0329	*0402
81	8987	9056	9125	9194	9264	9334	9404	9474	9544	9615	9685
8.80	0.028305	8372	8440	8508	8576	8644	8712	8780	8849	8918	8987
79	7640	7705	7771	7838	7904	7970	8037	8103	8170	8237	8305
78	6990	7055	7119	7183	7248	7313	7378	7443	7509	7574	7640
77	6357	6420	6482	6545	6608	6672	6735	6799	6862	6926	6990
76	5739	5800	5861	5923	5984	6046	6108	6170	6232	6294	6357
75	0.025136	5195	5255	5315	5375	5435	5496	5556	5617	5678	5739
74	4547	4605	4664	4722	4781	4840	4899	4958	5017	5076	5136
73	3973	4029	4086	4143	4201	4258	4316	4373	4431	4489	4547
72	3412	3467	3523	3579	3635	3691	3747	3803	3859	3916	3973
71	2865	2919	2973	3027	3082	3137	3191	3246	3301	3357	3412
8.70	0.022331	2383	2436	2489	2543	2596	2649	2703	2757	2811	2865
69	1809	1861	1913	1964	2016	2068	2121	2173	2225	2278	2331
68	1301	1351	1401	1452	1503	1553	1604	1655	1707	1758	1809
67	0804	0853	0902	0952	1001	1051	1100	1150	1200	1250	1301
.66	0.020319	0367	0415	0463	0512	0560	0609	0657	0706	0755	0804
65	0.019846	9893	9940	9987	*0034	*0081	*0128	*0176	*0223	*0271	*0319
64	9384	9430	9475	9521	9567	9613	9660	9706	9752	9799	9846
63	8933	8978	9022	9067	9112	9157	9202	9247	9293	9338	9384
62	8493	8536	8580	8624	8667	8711	8755	8800	8844	8888	8933
61	8063	8105	8148	8191	8233	8276	8319	8363	8406	8449	8493
8.60	0.017643	7685	7726	7768	7810	7852	7894	7936	7978	8020	8063
59	7233	7274	7315	7355	7396	7437	7478	7519	7560	7602	7643
58	6833	6873	6913	6952	6992	7032	7072	7112	7153	7193	7233
57	6443	6482	6520	6559	6598	6637	6676	6715	6755	6794	6833
56	6062	6099	6137	6175	6213	6251	6289	6328	6366	6404	6443
55	0.015689	5726	5763	5800	5837	5874	5912	5949	5986	6024	6062
54	5326	5362	5398	5434	5470	5507	5543	5579	5616	5653	5689
53	4971	5006	5041	5077	5112	5147	5183	5218	5254	5290	5326
52	4624	4659	4693	4727	4762	4797	4831	4866	4901	4936	4971
51	4286	4319	4353	4387	4420	4454	4488	4522	4556	4590	4624
8.50	0.013955	3988	4021	4054	4087	4120	4153	4186	4219	4253	4286

Proportional parts

	111	108	105	102	99
1	11.1	10.8	10.5	10.2	9.9
2	22.2	21.6	21.0	20.4	19.8
3	33.3	32.4	31.5	30.6	29.7
4	44.4	43.2	42.0	40.8	39.6
5	55.5	54.0	52.5	51.0	49.5
6	66.6	64.8	63.0	61.2	59.4
7	77.7	75.6	73.5	71.4	69.3
8	88.8	86.4	84.0	81.6	79.2
9	99.9	97.2	94.5	91.8	89.1

	96	93	90	87	84
1	9.6	9.3	9.0	8.7	8.4
2	19.2	18.6	18.0	17.4	16.8
3	28.8	27.9	27.0	26.1	25.2
4	38.4	37.2	36.0	34.8	33.6
5	48.0	46.5	45.0	43.5	42.0
6	57.6	55.8	54.0	52.2	50.4
7	67.2	65.1	63.0	60.9	58.8
8	76.8	74.4	72.0	69.6	67.2
9	86.4	83.7	81.0	78.3	75.6

	81	78	75	72	69
1	8.1	7.8	7.5	7.2	6.9
2	16.2	15.6	15.0	14.4	13.8
3	24.3	23.4	22.5	21.6	20.7
4	32.4	31.2	30.0	28.8	27.6
5	40.5	39.0	37.5	36.0	34.5
6	48.6	46.8	45.0	43.2	41.4
7	56.7	54.6	52.5	50.4	48.3
8	64.8	62.4	60.0	57.6	55.2
9	72.9	70.2	67.5	64.8	62.1

	66	63	60	57	55
1	6.6	6.3	6.0	5.7	5.5
2	13.2	12.6	12.0	11.4	11.0
3	19.8	18.9	18.0	17.1	16.5
4	26.4	25.2	24.0	22.8	22.0
5	33.0	31.5	30.0	28.5	27.5
6	39.6	37.8	36.0	34.2	33.0
7	46.2	44.1	42.0	39.9	38.5
8	52.8	50.4	48.0	45.6	44.0
9	59.4	56.7	54.0	51.3	49.5

	53	51	49	47	45
1	5.3	5.1	4.9	4.7	4.5
2	10.6	10.2	9.8	9.4	9.0
3	15.9	15.3	14.7	14.1	13.5
4	21.2	20.4	19.6	18.8	18.0
5	26.5	25.5	24.5	23.5	22.5
6	31.8	30.6	29.4	28.2	27.0
7	37.1	35.7	34.3	32.9	31.5
8	42.4	40.8	39.2	37.6	36.0
9	47.7	45.9	44.1	42.3	40.5

	43	41	39	37	35
1	4.3	4.1	3.9	3.7	3.5
2	8.6	8.2	7.8	7.4	7.0
3	12.9	12.3	11.7	11.1	10.5
4	17.2	16.4	15.6	14.8	14.0
5	21.5	20.5	19.5	18.5	17.5
6	25.8	24.6	23.4	22.2	21.0
7	30.1	28.7	27.3	25.9	24.5
8	34.4	32.8	31.2	29.6	28.0
9	38.7	36.9	35.1	33.3	31.5

Table XIII.—Log $\frac{1}{1-a}$ [$=colog\ (1-a)$]

When a is positive:

Log a	0	1	2	3	4	5	6	7	8	9	10
8.50	0.013955	3988	4021	4054	4087	4120	4153	4186	4219	4253	4286
49	3633	3665	3697	3729	3761	3793	3825	3858	3890	3923	3955
48	3318	3349	3380	3411	3443	3474	3506	3537	3569	3601	3633
47	3010	3040	3071	3101	3132	3163	3194	3225	3256	3287	3318
46	2709	2739	2769	2799	2829	2859	2889	2919	2949	2979	3010
45	0.012416	2445	2474	2503	2532	2562	2591	2621	2650	2680	2709
44	2129	2158	2186	2215	2243	2272	2300	2329	2358	2387	2416
43	1849	1877	1905	1933	1961	1989	2017	2045	2073	2101	2129
42	1576	1603	1630	1657	1685	1712	1739	1767	1794	1822	1849
41	1309	1335	1362	1388	1415	1442	1468	1495	1522	1549	1576
8.40	0.011048	1074	1100	1126	1152	1178	1204	1230	1256	1283	1309
39	0794	0819	0844	0869	0895	0920	0946	0971	0997	1023	1048
38	0545	0570	0594	0619	0644	0669	0694	0718	0743	0769	0794
37	0302	0326	0350	0374	0399	0423	0447	0472	0496	0520	0545
36	0.010065	0088	0112	0135	0159	0183	0207	0230	0254	0278	0302
35	0.009833	9856	9879	9902	9925	9948	9972	9995	*0018	*0041	*0065
34	9607	9629	9652	9674	9697	9719	9742	9765	9787	9810	9833
33	9386	9408	9430	9452	9474	9496	9518	9540	9562	9584	9607
32	9170	9191	9213	9234	9256	9277	9299	9320	9342	9364	9386
31	8959	8980	9001	9022	9043	9064	9085	9106	9127	9149	9170
8.30	0.008753	8773	8794	8814	8835	8855	8876	8897	8917	8938	8959
29	8552	8572	8592	8612	8632	8652	8672	8692	8712	8733	8753
28	8355	8375	8394	8414	8433	8453	8473	8492	8512	8532	8552
27	8163	8182	8201	8220	8239	8259	8278	8297	8316	8336	8355
26	7976	7994	8013	8031	8050	8069	8088	8106	8125	8144	8163
25	0.007792	7811	7829	7847	7865	7884	7902	7920	7939	7957	7976
24	7614	7631	7649	7667	7685	7702	7720	7738	7756	7774	7792
23	7439	7456	7473	7491	7508	7526	7543	7561	7578	7596	7614
22	7268	7285	7302	7319	7336	7353	7370	7387	7404	7421	7439
21	7101	7118	7134	7151	7167	7184	7201	7218	7234	7251	7268
8.20	0.006938	6954	6971	6987	7003	7019	7036	7052	7068	7085	7101
19	6779	6795	6811	6826	6842	6858	6874	6890	6906	6922	6938
18	6624	6639	6654	6670	6685	6701	6716	6732	6748	6763	6779
17	6472	6487	6502	6517	6532	6547	6562	6578	6593	6608	6624
16	6323	6338	6353	6367	6382	6397	6412	6427	6442	6457	6472
15	0.006178	6193	6207	6221	6236	6250	6265	6279	6294	6309	6323
14	6037	6051	6065	6079	6093	6107	6121	6135	6150	6164	6178
13	5898	5912	5926	5940	5953	5967	5981	5995	6009	6023	6037
12	5763	5777	5790	5803	5817	5830	5844	5857	5871	5885	5898
11	5631	5644	5657	5670	5684	5697	5710	5723	5737	5750	5763
8.10	0.005502	5515	5528	5541	5553	5566	5579	5592	5605	5618	5631
09	5376	5389	5401	5414	5426	5439	5451	5464	5477	5489	5502
08	5253	5265	5277	5290	5302	5314	5327	5339	5351	5364	5376
07	5133	5145	5157	5169	5181	5193	5205	5217	5229	5241	5253
06	5015	5027	5038	5050	5062	5074	5085	5097	5109	5121	5133
05	4900	4912	4923	4935	4946	4957	4969	4980	4992	5004	5015
04	4788	4799	4810	4822	4833	4844	4855	4866	4878	4889	4900
03	4679	4690	4700	4711	4722	4733	4744	4755	4766	4777	4788
02	4572	4582	4593	4603	4614	4625	4636	4646	4657	4668	4679
01	4467	4477	4488	4498	4509	4519	4529	4540	4550	4561	4572
8.00	0.004365	4375	4385	4395	4405	4416	4426	4436	4446	4457	4467

Proportional parts

	34	33	32	31	30
1	3.4	3.3	3.2	3.1	3.0
2	6.8	6.6	6.4	6.2	6.0
3	10.2	9.9	9.6	9.3	9.0
4	13.6	13.2	12.8	12.4	12.0
5	17.0	16.5	16.0	15.5	15.0
6	20.4	19.8	19.2	18.6	18.0
7	23.8	23.1	22.4	21.7	21.0
8	27.2	26.4	25.6	24.8	24.0
9	30.6	29.7	28.8	27.9	27.0

	29	28	27	26	25
1	2.9	2.8	2.7	2.6	2.5
2	5.8	5.6	5.4	5.2	5.0
3	8.7	8.4	8.1	7.8	7.5
4	11.6	11.2	10.8	10.4	10.0
5	14.5	14.0	13.5	13.0	12.5
6	17.4	16.8	16.2	15.6	15.0
7	20.3	19.6	18.9	18.2	17.5
8	23.2	22.4	21.6	20.8	20.0
9	26.1	25.2	24.3	23.4	22.5

	24	23	22	21	20
1	2.4	2.3	2.2	2.1	2.0
2	4.8	4.6	4.4	4.2	4.0
3	7.2	6.9	6.6	6.3	6.0
4	9.6	9.2	8.8	8.4	8.0
5	12.0	11.5	11.0	10.5	10.0
6	14.4	13.8	13.2	12.6	12.0
7	16.8	16.1	15.4	14.7	14.0
8	19.2	18.4	17.6	16.8	16.0
9	21.6	20.7	19.8	18.9	18.0

	19	18	17	16	15
1	1.9	1.8	1.7	1.6	1.5
2	3.8	3.6	3.4	3.2	3.0
3	5.7	5.4	5.1	4.8	4.5
4	7.6	7.2	6.8	6.4	6.0
5	9.5	9.0	8.5	8.0	7.5
6	11.4	10.8	10.2	9.6	9.0
7	13.3	12.6	11.9	11.2	10.5
8	15.2	14.4	13.6	12.8	12.0
9	17.1	16.2	15.3	14.4	13.5

	14	13	12	11	10
1	1.4	1.3	1.2	1.1	1.0
2	2.8	2.6	2.4	2.2	2.0
3	4.2	3.9	3.6	3.3	3.0
4	5.6	5.2	4.8	4.4	4.0
5	7.0	6.5	6.0	5.5	5.0
6	8.4	7.8	7.2	6.6	6.0
7	9.8	9.1	8.4	7.7	7.0
8	11.2	10.4	9.6	8.8	8.0
9	12.6	11.7	10.8	9.9	9.0

Table XIII.—*Log* $\frac{1}{1-a}$ [$=colog\ (1-a)$]

When a is positive:

Log a	0	1	2	3	4	5	6	7	8	9	10
8.00	0.004365	4375	4385	4395	4405	4416	4426	4436	4446	4457	4467
7.99	4265	4275	4285	4295	4305	4315	4325	4335	4345	4355	4365
98	4167	4177	4187	4196	4206	4216	4226	4235	4245	4255	4265
97	4072	4082	4091	4100	4110	4119	4129	4139	4148	4158	4167
96	3979	3988	3997	4007	4016	4025	4035	4044	4053	4063	4072
95	0.003888	3897	3906	3915	3924	3933	3942	3951	3961	3970	3979
94	3799	3808	3817	3826	3834	3843	3852	3861	3870	3879	3888
93	3712	3721	3729	3738	3747	3755	3764	3773	3782	3790	3799
92	3627	3636	3644	3653	3661	3670	3678	3687	3695	3704	3712
91	3545	3553	3561	3569	3577	3586	3594	3602	3611	3619	3627
7.90	0.003463	3472	3480	3488	3496	3504	3512	3520	3528	3536	3545
89	3384	3392	3400	3408	3416	3424	3432	3440	3448	3456	3463
88	3307	3315	3322	3330	3338	3345	3353	3361	3369	3377	3384
87	3231	3239	3246	3254	3261	3269	3277	3284	3292	3299	3307
86	3158	3165	3172	3180	3187	3194	3202	3209	3217	3224	3231
85	0.003086	3093	3100	3107	3114	3121	3129	3136	3143	3150	3158
84	3015	3022	3029	3036	3043	3050	3057	3064	3071	3078	3086
83	2946	2953	2960	2967	2974	2980	2987	2994	3001	3008	3015
82	2879	2886	2892	2899	2906	2912	2919	2926	2933	2939	2946
81	2813	2820	2826	2833	2839	2846	2852	2859	2866	2872	2879
7.80	0.002749	2755	2762	2768	2774	2781	2787	2794	2800	2807	2813
79	2686	2692	2699	2705	2711	2717	2724	2730	2736	2743	2749
78	2625	2631	2637	2643	2649	2655	2661	2668	2674	2680	2686
77	2565	2571	2577	2583	2589	2595	2601	2607	2613	2619	2625
76	2506	2512	2518	2524	2530	2535	2541	2547	2553	2559	2565
75	0.002449	2455	2460	2466	2472	2478	2483	2489	2495	2501	2506
74	2393	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449
73	2339	2344	2349	2355	2360	2366	2371	2377	2382	2388	2393
72	2285	2290	2296	2301	2306	2312	2317	2322	2328	2333	2339
71	2233	2238	2243	2249	2254	2259	2264	2269	2275	2280	2285
7.70	0.002182	2187	2192	2197	2202	2207	2213	2218	2223	2228	2233
69	2132	2137	2142	2147	2152	2157	2162	2167	2172	2177	2182
68	2084	2088	2093	2098	2103	2108	2113	2118	2122	2127	2132
67	2036	2041	2046	2050	2055	2060	2065	2069	2074	2079	2084
66	1990	1994	1999	2003	2008	2013	2017	2022	2027	2031	2036
65	0.001944	1949	1953	1958	1962	1967	1971	1976	1980	1985	1990
64	1900	1904	1909	1913	1918	1922	1926	1931	1935	1940	1944
63	1857	1861	1865	1869	1874	1878	1882	1887	1891	1896	1900
62	1814	1818	1823	1827	1831	1835	1840	1844	1848	1852	1857
61	1773	1777	1781	1785	1789	1793	1798	1802	1806	1810	1814
7.60	0.001732	1736	1740	1744	1748	1753	1757	1761	1765	1769	1773
59	1693	1697	1701	1705	1709	1713	1716	1720	1724	1728	1732
58	1654	1658	1662	1666	1670	1673	1677	1681	1685	1689	1693
57	1617	1620	1624	1628	1632	1635	1639	1643	1647	1650	1654
56	1580	1583	1587	1591	1594	1598	1602	1605	1609	1613	1617
55	0.001544	1547	1551	1554	1558	1562	1565	1569	1572	1576	1580
54	1508	1512	1515	1519	1522	1526	1529	1533	1537	1540	1544
53	1474	1477	1481	1484	1488	1491	1495	1498	1502	1505	1508
52	1440	1444	1447	1450	1454	1457	1461	1464	1467	1471	1474
51	1408	1411	1414	1417	1421	1424	1427	1431	1434	1437	1440
7.50	0.001376	1379	1382	1385	1388	1391	1395	1398	1401	1404	1408

Proportional parts

	11	10
1	1.1	1.0
2	2.2	2.0
3	3.3	3.0
4	4.4	4.0
5	5.5	5.0
6	6.6	6.0
7	7.7	7.0
8	8.8	8.0
9	9.9	9.0

	9	8
1	0.9	0.8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

	5	4
1	0.5	0.4
2	1.0	0.8
3	1.5	1.2
4	2.0	1.6
5	2.5	2.0
6	3.0	2.4
7	3.5	2.8
8	4.0	3.2
9	4.5	3.6

Table XIII.—Log $\frac{1}{1-a}$ [$=colog\ (1-a)$]

When a is positive:

Log a	0	1	2	3	4	5	6	7	8	9	10
7.50	0.001376	1379	1382	1385	1388	1391	1395	1398	1401	1404	1408
49	1344	1347	1350	1354	1357	1360	1363	1366	1369	1372	1376
48	1314	1317	1320	1323	1326	1329	1332	1335	1338	1341	1344
47	1284	1287	1290	1292	1295	1298	1301	1304	1307	1311	1314
46	1254	1257	1260	1263	1266	1269	1272	1275	1278	1281	1284
45	0.001226	1229	1231	1234	1237	1240	1243	1246	1249	1251	1254
44	1198	1201	1203	1206	1209	1212	1214	1217	1220	1223	1226
43	1170	1173	1176	1179	1181	1184	1187	1190	1192	1195	1198
42	1144	1146	1149	1152	1154	1157	1160	1162	1165	1168	1170
41	1118	1120	1123	1126	1128	1131	1133	1136	1139	1141	1144
7.40	0.001092	1095	1097	1100	1102	1105	1107	1110	1113	1115	1118
39	1067	1070	1072	1075	1077	1080	1082	1085	1087	1090	1092
38	1043	1045	1048	1050	1053	1055	1058	1060	1062	1065	1067
37	1019	1022	1024	1026	1029	1031	1033	1036	1038	1041	1043
36	0996	0998	1001	1003	1005	1008	1010	1012	1015	1017	1019
35	0.000973	0976	0978	0980	0982	0985	0987	0989	0991	0994	0996
34	951	953	956	958	960	962	964	967	969	971	973
33	929	932	934	936	938	940	942	945	947	949	951
32	908	910	913	915	917	919	921	923	925	927	929
31	888	890	892	894	896	898	900	902	904	906	908
7.30	0.000867	869	871	873	875	877	879	882	884	886	888
29	848	850	852	854	855	857	859	861	863	865	867
28	828	830	832	834	836	838	840	842	844	846	848
27	809	811	813	815	817	819	821	823	825	826	828
26	791	793	795	796	798	800	802	804	806	808	809
25	0.000773	775	777	778	780	782	784	786	787	789	791
24	755	757	759	761	762	764	766	768	769	771	773
23	738	740	742	743	745	747	748	750	752	754	755
22	721	723	725	726	728	730	731	733	735	736	738
21	705	707	708	710	711	713	715	716	718	720	721
7.20	0.000689	690	692	694	695	697	698	700	702	703	705
19	673	675	676	678	679	681	683	684	686	687	689
18	658	659	661	662	664	665	667	669	670	672	673
17	643	644	646	647	649	650	652	653	655	656	658
16	628	630	631	633	634	635	637	638	640	641	643
15	0.000614	615	617	618	620	621	622	624	625	627	628
14	600	601	603	604	605	607	608	610	611	612	614
13	586	588	589	590	592	593	594	596	597	599	600
12	573	574	576	577	578	580	581	582	584	585	586
11	560	561	562	564	565	566	568	569	570	572	573
7.10	0.000547	548	550	551	552	553	555	556	557	559	560
09	535	536	537	538	540	541	542	543	545	546	547
08	522	524	525	526	527	529	530	531	532	533	535
07	511	512	513	514	515	516	518	519	520	521	522
06	499	500	501	502	504	505	506	507	508	509	511
05	0.000488	489	490	491	492	493	494	495	497	498	499
04	476	478	479	480	481	482	483	484	485	486	488
03	466	467	468	469	470	471	472	473	474	475	476
02	455	456	457	458	459	460	461	462	463	465	466
01	445	446	447	448	449	450	451	452	453	454	455
7.00	0.000435	436	437	438	439	440	441	442	443	444	445

Proportional parts

	4	3
1	0.4	0.3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

	2	1
1	0.2	0.1
2	0.4	0.2
3	0.6	0.3
4	0.8	0.4
5	1.0	0.5
6	1.2	0.6
7	1.4	0.7
8	1.6	0.8
9	1.8	0.9

Table XIII.—Log $\frac{1}{1-a}$ [=colog $(1-a)$]

When a is positive:

Log a	0	1	2	3	4	5	6	7	8	9	10
7.00	0.000435	436	437	438	439	440	441	442	443	444	445
6.9	345	353	361	370	378	387	396	405	415	425	435
8	274	280	287	294	301	308	315	322	330	337	345
7	218	223	228	233	239	244	250	256	262	268	274
6	173	177	181	185	190	194	199	203	208	213	218
5	0.000137	141	144	147	151	154	158	161	165	169	173
4	109	112	114	117	120	122	125	128	131	134	137
3	087	089	091	093	095	097	100	102	104	107	109
2	69	70	72	74	75	77	79	81	83	85	87
1	55	56	57	59	60	61	63	64	66	67	69
6.0	0.000043	44	45	47	48	49	50	51	52	53	55
5.9	34	35	36	37	38	39	40	41	41	42	43
8	27	28	29	29	30	31	31	32	33	34	34
7	22	22	23	23	24	24	25	26	26	27	27
6	17	18	18	19	19	19	20	20	21	21	22
5	0.000014	14	14	15	15	15	16	16	17	17	17
4	11	11	11	12	12	12	13	13	13	13	14
3	09	09	09	09	10	10	10	10	10	11	11
2	7	7	7	7	8	8	8	8	8	8	9
1	5	6	6	6	6	6	6	6	7	7	7
5.0	0.000004	4	5	5	5	5	5	5	5	5	5
4	0.000000	1	1	1	1	1	2	2	3	3	4

(2) When a is negative:

4 n	0.000000	*9999	*9999	*9999	*9999	*9999	*9998	*9998	*9997	*9997	*9996
5.0 n	9.999996	96	95	95	95	95	95	95	95	95	95
1 n	95	94	94	94	94	94	94	94	93	93	93
2 n	93	93	93	93	92	92	92	92	92	92	91
3 n	91	91	91	91	90	90	90	90	90	89	89
4 n	89	89	89	88	88	88	87	87	87	87	86
5 n	9.999986	86	86	85	85	85	84	84	83	83	83
6 n	83	82	82	81	81	81	80	80	79	79	78
7 n	78	78	77	77	76	76	75	74	74	73	73
8 n	73	72	71	71	70	69	69	68	67	66	66
9 n	66	65	64	63	62	61	60	59	59	58	57
6.0 n	9.999957	56	55	53	52	51	50	49	48	47	45
1 n	45	44	43	41	40	39	37	36	34	33	31
2 n	31	30	28	26	25	23	21	19	17	15	13
3 n	913	911	909	907	905	903	901	898	896	893	891
4 n	891	888	886	883	880	878	875	872	869	866	863
5 n	9.999863	859	856	853	849	846	842	839	835	831	827
6 n	827	823	819	815	810	806	802	797	792	787	782
7 n	782	777	772	767	761	756	750	744	738	732	726
8 n	726	720	713	706	700	693	685	678	671	663	655
9 n	655	647	639	631	622	613	604	595	585	576	566
7.00 n	9.999566	565	564	563	562	561	560	559	558	557	556

Proportional parts

	10	9
1	1.0	0.9
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

	8	7
1	0.8	0.7
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

	4	3
1	0.4	0.3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

	2	1
1	0.2	0.1
2	0.4	0.2
3	0.6	0.3
4	0.8	0.4
5	1.0	0.5
6	1.2	0.6
7	1.4	0.7
8	1.6	0.8
9	1.8	0.9

Table XIII.—Log $\frac{1}{1-a}$ *[=colog (1—a)]*

When a is negative:

Log a	0	1	2	3	4	5	6	7	8	9	10
7.00 n	9.999566	565	564	563	562	561	560	559	558	557	556
01 n	556	555	554	553	552	551	550	549	548	547	545
02 n	545	544	543	542	541	540	539	538	537	536	535
03 n	535	534	533	532	531	530	528	527	526	525	524
04 n	524	523	522	521	520	519	517	516	515	514	513
05 n	9.999513	512	511	510	508	507	506	505	504	503	502
06 n	502	501	499	498	497	496	495	494	492	491	490
07 n	490	489	488	487	485	484	483	482	481	479	478
08 n	478	477	476	475	473	472	471	470	469	467	466
09 n	466	465	464	462	461	460	459	457	456	455	454
7.10 n	9.999454	452	451	450	449	447	446	445	443	442	441
11 n	441	440	438	437	436	434	433	432	430	429	428
12 n	428	427	425	424	423	421	420	419	417	416	415
13 n	415	413	412	410	409	408	406	405	404	402	401
14 n	401	400	398	397	395	394	393	391	390	388	387
15 n	9.999387	386	384	383	381	380	378	377	376	374	373
16 n	373	371	370	368	367	365	364	363	361	360	358
17 n	358	357	355	354	352	351	349	348	346	345	343
18 n	343	342	340	339	337	336	334	333	331	329	328
19 n	328	326	325	323	322	320	319	317	315	314	312
7.20 n	9.999312	311	309	307	306	304	303	301	299	298	296
21 n	296	295	293	291	290	288	286	285	283	282	280
22 n	280	278	277	275	273	272	270	268	266	265	263
23 n	263	261	260	258	256	255	253	251	249	247	246
24 n	246	244	242	241	239	237	235	234	232	230	228
25 n	9.999228	227	225	223	221	219	218	216	214	212	210
26 n	210	209	207	205	203	201	199	198	196	194	192
27 n	192	190	188	186	185	183	181	179	177	175	173
28 n	173	171	169	168	166	164	162	160	158	156	154
29 n	154	152	150	148	146	144	142	140	138	136	134
7.30 n	9.999134	132	130	128	126	124	122	120	118	116	114
31 n	114	112	110	108	106	104	102	100	098	096	094
32 n	094	091	089	087	085	083	081	079	077	075	072
33 n	072	070	068	066	064	062	060	057	055	053	051
34 n	051	049	047	044	042	040	038	036	033	031	029
35 n	9.999029	9027	9024	9022	9020	9018	9015	9013	9011	9009	9006
36 n	9006	9004	9002	8999	8997	8995	8992	8990	8988	8985	8983
37 n	8983	8981	8978	8976	8974	8971	8969	8967	8964	8962	8959
38 n	8959	8957	8955	8952	8950	8947	8945	8943	8940	8938	8935
39 n	8935	8933	8930	8928	8925	8923	8920	8918	8915	8913	8910
7.40 n	9.998910	8908	8905	8903	8900	8898	8895	8893	8890	8888	8885
41 n	8885	8883	8880	8877	8875	8872	8870	8867	8864	8862	8859
42 n	8859	8857	8854	8851	8849	8846	8843	8841	8838	8835	8833
43 n	8833	8830	8827	8825	8822	8819	8816	8814	8811	8808	8805
44 n	8805	8803	8800	8797	8794	8792	8789	8786	8783	8781	8778
45 n	9.998778	8775	8772	8769	8766	8764	8761	8758	8755	8752	8749
46 n	8749	8746	8744	8741	8738	8735	8732	8729	8726	8723	8720
47 n	8720	8717	8714	8711	8708	8705	8702	8699	8696	8693	8690
48 n	8690	8687	8684	8681	8678	8675	8672	8669	8666	8663	8660
49 n	8660	8657	8654	8651	8648	8644	8641	8638	8635	8632	8629
7.50 n	9.998629	8626	8622	8619	8616	8613	8610	8607	8603	8600	8597

Proportional parts

	1	2
1	0.1	0.2
2	0.2	0.4
3	0.3	0.6
4	0.4	0.8
5	0.5	1.0
6	0.6	1.2
7	0.7	1.4
8	0.8	1.6
9	0.9	1.8

	3	4
1	0.3	0.4
2	0.6	0.8
3	0.9	1.2
4	1.2	1.6
5	1.5	2.0
6	1.8	2.4
7	2.1	2.8
8	2.4	3.2
9	2.7	3.6

Table XIII.—Log $\frac{1}{1-a}$ [$=colog\ (1-a)$]

When a is negative:

Log a	0	1	2	3	4	5	6	7	8	9	10
7.50 n	9.998629	8626	8622	8619	8616	8613	8610	8607	8603	8600	8597
51 n	8597	8594	8590	8587	8584	8581	8577	8574	8571	8568	8564
52 n	8564	8561	8558	8554	8551	8548	8544	8541	8538	8534	8531
53 n	8531	8528	8524	8521	8517	8514	8511	8507	8504	8500	8497
54 n	8497	8493	8490	8486	8483	8479	8476	8472	8469	8465	8462
55 n	9.998462	8458	8455	8451	8448	8444	8440	8437	8433	8430	8426
56 n	8426	8422	8419	8415	8411	8408	8404	8400	8397	8393	8389
57 n	8389	8386	8382	8378	8375	8371	8367	8363	8360	8356	8352
58 n	8352	8348	8344	8341	8337	8333	8329	8325	8321	8318	8314
59 n	8314	8310	8306	8302	8298	8294	8290	8286	8282	8278	8274
7.60 n	9.998274	8271	8267	8263	8259	8255	8251	8246	8242	8238	8234
61 n	8234	8230	8226	8222	8218	8214	8210	8206	8202	8197	8193
62 n	8193	8189	8185	8181	8177	8172	8168	8164	8160	8156	8151
63 n	8151	8147	8143	8139	8134	8130	8126	8121	8117	8113	8108
64 n	8108	8104	8100	8095	8091	8087	8082	8078	8073	8069	8064
65 n	9.998064	8060	8055	8051	8047	8042	8038	8033	8028	8024	8019
66 n	8019	8015	8010	8006	8001	7997	7992	7987	7983	7978	7973
67 n	7973	7969	7964	7959	7955	7950	7945	7941	7936	7931	7926
68 n	7926	7922	7917	7912	7907	7902	7898	7893	7888	7883	7878
69 n	7878	7873	7868	7863	7859	7854	7849	7844	7839	7834	7829
7.70 n	9.997829	7824	7819	7814	7809	7804	7799	7794	7789	7783	7778
71 n	7778	7773	7768	7763	7758	7753	7748	7742	7737	7732	7727
72 n	7727	7722	7716	7711	7706	7700	7695	7690	7685	7679	7674
73 n	7674	7669	7663	7658	7652	7647	7642	7636	7631	7625	7620
74 n	7620	7614	7609	7603	7598	7592	7587	7581	7576	7570	7565
75 n	9.997565	7559	7553	7548	7542	7537	7531	7525	7519	7514	7508
76 n	7508	7502	7497	7491	7485	7479	7473	7468	7462	7456	7450
77 n	7450	7444	7438	7433	7427	7421	7415	7409	7403	7397	7391
78 n	7391	7385	7379	7373	7367	7361	7355	7349	7343	7337	7330
79 n	7330	7324	7318	7312	7306	7300	7293	7287	7281	7275	7268
7.80 n	9.997268	7262	7256	7250	7243	7237	7231	7224	7218	7211	7205
81 n	7205	7199	7192	7186	7179	7173	7166	7160	7153	7147	7140
82 n	7140	7134	7127	7120	7114	7107	7100	7094	7087	7080	7074
83 n	7074	7067	7060	7053	7047	7040	7033	7026	7019	7013	7006
84 n	7006	6999	6992	6985	6978	6971	6964	6957	6950	6943	6936
85 n	9.996936	6929	6922	6915	6908	6901	6894	6887	6880	6872	6865
86 n	6865	6858	6851	6844	6836	6829	6822	6814	6807	6800	6792
87 n	6792	6785	6778	6770	6763	6755	6748	6740	6733	6725	6718
88 n	6718	6710	6703	6695	6688	6680	6672	6665	6657	6650	6642
89 n	6642	6634	6626	6619	6611	6603	6595	6587	6580	6572	6564
7.90 n	9.996564	6556	6548	6540	6532	6524	6516	6508	6500	6492	6484
91 n	6484	6476	6468	6460	6452	6444	6435	6427	6419	6411	6403
92 n	6403	6394	6386	6378	6369	6361	6353	6344	6336	6328	6319
93 n	6319	6311	6302	6294	6285	6277	6268	6260	6251	6242	6234
94 n	6234	6225	6217	6208	6199	6190	6182	6173	6164	6155	6146
95 n	9.996146	6138	6129	6120	6111	6102	6093	6084	6075	6066	6057
96 n	6057	6048	6039	6030	6021	6012	6003	5993	5984	5975	5966
97 n	5966	5956	5947	5938	5929	5919	5910	5900	5891	5882	5872
98 n	5872	5863	5853	5844	5834	5825	5815	5805	5796	5786	5777
99 n	5777	5767	5757	5747	5738	5728	5718	5708	5698	5689	5679
8.00 n	9.995679	5669	5659	5649	5639	5629	5619	5609	5599	5589	5578

Proportional parts

	4	5
1	0.4	0.5
2	0.8	1.0
3	1.2	1.5
4	1.6	2.0
5	2.0	2.5
6	2.4	3.0
7	2.8	3.5
8	3.2	4.0
9	3.6	4.5

	6	7
1	0.6	0.7
2	1.2	1.4
3	1.8	2.1
4	2.4	2.8
5	3.0	3.5
6	3.6	4.2
7	4.2	4.9
8	4.8	5.6
9	5.4	6.3

	8	9
1	0.8	0.9
2	1.6	1.8
3	2.4	2.7
4	3.2	3.6
5	4.0	4.5
6	4.8	5.4
7	5.6	6.3
8	6.4	7.2
9	7.2	8.1

	10	11
1	1.0	1.1
2	2.0	2.2
3	3.0	3.3
4	4.0	4.4
5	5.0	5.5
6	6.0	6.6
7	7.0	7.7
8	8.0	8.8
9	9.0	9.9

Table XIII.—Log $\frac{1}{1-a}$ [$=colog\ (1-a)$]

When a is negative:

Log a	0	1	2	3	4	5	6	7	8	9	10
8.00 n	9.995679	5669	5659	5649	5639	5629	5619	5609	5599	5589	5578
01 n	5578	5568	5558	5548	5538	5528	5517	5507	5497	5486	5476
02 n	5476	5466	5455	5445	5434	5424	5413	5403	5392	5382	5371
03 n	5371	5361	5350	5339	5329	5318	5307	5296	5286	5275	5264
04 n	5264	5253	5242	5231	5220	5209	5198	5187	5176	5165	5154
05 n	9.995154	5143	5132	5121	5110	5098	5087	5076	5065	5053	5042
06 n	5042	5031	5019	5008	4996	4985	4973	4962	4950	4939	4927
07 n	4927	4916	4904	4892	4881	4869	4857	4845	4833	4822	4810
08 n	4810	4798	4786	4774	4762	4750	4738	4726	4714	4702	4690
09 n	4690	4677	4665	4653	4641	4628	4616	4604	4591	4579	4567
8.10 n	9.994567	4554	4542	4529	4517	4504	4492	4479	4466	4454	4441
11 n	4441	4428	4415	4403	4390	4377	4364	4351	4338	4325	4312
12 n	4312	4299	4286	4273	4260	4247	4234	4220	4207	4194	4181
13 n	4181	4167	4154	4141	4127	4114	4100	4087	4073	4060	4046
14 n	4046	4032	4019	4005	3991	3978	3964	3950	3936	3922	3908
15 n	9.993908	3894	3880	3866	3852	3838	3824	3810	3796	3782	3767
16 n	3767	3753	3739	3725	3710	3696	3681	3667	3652	3638	3623
17 n	3623	3609	3594	3579	3565	3550	3535	3521	3506	3491	3476
18 n	3476	3461	3446	3431	3416	3401	3386	3371	3356	3340	3325
19 n	3325	3310	3295	3279	3264	3248	3233	3218	3202	3186	3171
8.20 n	9.993171	3155	3140	3124	3108	3092	3077	3061	3045	3029	3013
21 n	3013	2997	2981	2965	2949	2933	2917	2900	2884	2868	2852
22 n	2852	2835	2819	2803	2786	2770	2753	2736	2720	2703	2687
23 n	2687	2670	2653	2636	2619	2603	2586	2569	2552	2535	2518
24 n	2518	2501	2483	2466	2449	2432	2414	2397	2380	2362	2345
25 n	9.992345	2327	2310	2292	2275	2257	2239	2222	2204	2186	2168
26 n	2168	2150	2132	2114	2096	2078	2060	2042	2024	2006	1987
27 n	1987	1969	1951	1932	1914	1896	1877	1858	1840	1821	1803
28 n	1803	1784	1765	1746	1727	1709	1690	1671	1652	1633	1613
29 n	1613	1594	1575	1556	1537	1517	1498	1478	1459	1440	1420
8.30 n	9.991420	1400	1381	1361	1341	1322	1302	1282	1262	1242	1222
31 n	1222	1202	1182	1162	1142	1122	1101	1081	1061	1040	1020
32 n	1020	0999	0979	0958	0938	0917	0896	0875	0855	0834	0813
33 n	0813	0792	0771	0750	0729	0708	0686	0665	0644	0622	0601
34 n	0601	0580	0558	0537	0515	0493	0472	0450	0428	0406	0385
35 n	9.990385	0363	0341	0319	0297	0274	0252	0230	0208	0186	0163
36 n	9.990163	0141	0118	0096	0073	0051	0028	0005	*9982	*9960	*9937
37 n	9.989937	9914	9891	9868	9845	9821	9798	9775	9752	9728	9705
38 n	9705	9682	9658	9634	9611	9587	9563	9540	9516	9492	9468
39 n	9468	9444	9420	9396	9372	9348	9323	9299	9275	9250	9226
8.40 n	9.989226	9201	9177	9152	9127	9103	9078	9053	9028	9003	8978
41 n	8978	8953	8928	8903	8877	8852	8827	8801	8776	8750	8725
42 n	8725	8699	8673	8647	8622	8596	8570	8544	8518	8492	8465
43 n	8465	8439	8413	8386	8360	8334	8307	8280	8254	8227	8200
44 n	8200	8173	8147	8120	8093	8066	8038	8011	7984	7957	7929
45 n	9.987929	7902	7874	7847	7819	7791	7764	7736	7708	7680	7652
46 n	7652	7624	7596	7568	7539	7511	7483	7454	7426	7397	7369
47 n	7369	7340	7311	7282	7253	7224	7195	7166	7137	7108	7079
48 n	7079	7049	7020	6990	6961	6931	6902	6872	6842	6812	6782
49 n	6782	6752	6722	6692	6662	6631	6601	6571	6540	6510	6479
8.50 n	9.986479	6448	6418	6387	6356	6325	6294	6263	6232	6200	6169

Proportional parts

	10	11	12	13	14
1	1.0	1.1	1.2	1.3	1.4
2	2.0	2.2	2.4	2.6	2.8
3	3.0	3.3	3.6	3.9	4.2
4	4.0	4.4	4.8	5.2	5.6
5	5.0	5.5	6.0	6.5	7.0
6	6.0	6.6	7.2	7.8	8.4
7	7.0	7.7	8.4	9.1	9.8
8	8.0	8.8	9.6	10.4	11.2
9	9.0	9.9	10.8	11.7	12.6

	15	16	17	18	19
1	1.5	1.6	1.7	1.8	1.9
2	3.0	3.2	3.4	3.6	3.8
3	4.5	4.8	5.1	5.4	5.7
4	6.0	6.4	6.8	7.2	7.6
5	7.5	8.0	8.5	9.0	9.5
6	9.0	9.6	10.2	10.8	11.4
7	10.5	11.2	11.9	12.6	13.3
8	12.0	12.8	13.6	14.4	15.2
9	13.5	14.4	15.3	16.2	17.1

	20	21	22	23	24
1	2.0	2.1	2.2	2.3	2.4
2	4.0	4.2	4.4	4.6	4.8
3	6.0	6.3	6.6	6.9	7.2
4	8.0	8.4	8.8	9.2	9.6
5	10.0	10.5	11.0	11.5	12.0
6	12.0	12.6	13.2	13.8	14.4
7	14.0	14.7	15.4	16.1	16.8
8	16.0	16.8	17.6	18.4	19.2
9	18.0	18.9	19.8	20.7	21.6

	25	26	27	28	29
1	2.5	2.6	2.7	2.8	2.9
2	5.0	5.2	5.4	5.6	5.8
3	7.5	7.8	8.1	8.4	8.7
4	10.0	10.4	10.8	11.2	11.6
5	12.5	13.0	13.5	14.0	14.5
6	15.0	15.6	16.2	16.8	17.4
7	17.5	18.2	18.9	19.6	20.3
8	20.0	20.8	21.6	22.4	23.2
9	22.5	23.4	24.3	25.2	26.1

	30	31	32
1	3.0	3.1	3.2
2	6.0	6.2	6.4
3	9.0	9.3	9.6
4	12.0	12.4	12.8
5	15.0	15.5	16.0
6	18.0	18.6	19.2
7	21.0	21.7	22.4
8	24.0	24.8	25.6
9	27.0	27.9	28.8

Table XIII.—Log $\frac{1}{1-a}$ [=*colog* $(1-a)$]

When a is negative:

Log a	0	1	2	3	4	5	6	7	8	9	10
8.50 n	9.986479	6448	6418	6387	6356	6325	6294	6263	6232	6200	6169
51 n	6169	6138	6106	6075	6043	6011	5980	5948	5916	5884	5852
52 n	5852	5820	5788	5756	5723	5691	5659	5626	5593	5561	5528
53 n	5528	5495	5462	5429	5396	5363	5330	5297	5263	5230	5197
54 n	5197	5163	5129	5096	5062	5028	4994	4960	4926	4892	4858
55 n	9.984858	4823	4789	4755	4720	4685	4651	4616	4581	4546	4511
56 n	4511	4476	4441	4406	4370	4335	4300	4264	4228	4193	4157
57 n	4157	4121	4085	4049	4013	3977	3941	3904	3868	3831	3795
58 n	3795	3758	3721	3684	3648	3611	3573	3536	3499	3462	3424
59 n	3424	3387	3349	3312	3274	3236	3198	3160	3122	3084	3046
8.60 n	9.983046	3007	2969	2930	2892	2853	2814	2776	2737	2698	2658
61 n	2658	2619	2580	2541	2501	2462	2422	2382	2343	2303	2263
62 n	2263	2223	2183	2142	2102	2062	2021	1981	1940	1899	1858
63 n	1858	1817	1776	1735	1694	1653	1611	1570	1528	1486	1444
64 n	1444	1403	1361	1319	1276	1234	1192	1149	1107	1064	1022
65 n	9.981022	0979	0936	0893	0850	0807	0763	0720	0677	0633	0589
66 n	0589	0546	0502	0458	0414	0370	0325	0281	0237	0192	0147
67 n	9.980147	0103	0058	0013	*9968	*9923	*9878	*9832	*9787	*9741	*9695
68 n	9.979695	9650	9604	9558	9512	9466	9420	9373	9327	9280	9234
69 n	9234	9187	9140	9093	9046	8999	8952	8904	8857	8809	8762
8.70 n	9.978762	8714	8666	8618	8570	8522	8473	8425	8376	8328	8279
71 n	8279	8230	8181	8132	8083	8034	7985	7935	7885	7836	7786
72 n	7786	7736	7686	7636	7586	7535	7485	7434	7384	7333	7282
73 n	7282	7231	7180	7128	7077	7026	6974	6922	6870	6818	6766
74 n	6766	6714	6662	6610	6557	6505	6452	6399	6346	6293	6240
75 n	9.976240	6187	6133	6080	6026	5972	5918	5864	5810	5756	5702
76 n	5702	5647	5593	5538	5483	5428	5373	5318	5262	5207	5152
77 n	5152	5096	5040	4984	4928	4872	4816	4759	4703	4646	4589
78 n	4589	4532	4475	4418	4361	4304	4246	4188	4131	4073	4015
79 n	4015	3957	3898	3840	3781	3723	3664	3605	3546	3487	3428
8.80 n	9.973428	3368	3309	3249	3189	3129	3069	3009	2949	2888	2828
81 n	2828	2767	2706	2645	2584	2523	2461	2400	2338	2276	2215
82 n	2215	2153	2090	2028	1966	1903	1840	1777	1714	1651	1588
83 n	1588	1525	1461	1398	1334	1270	1206	1141	1077	1013	0948
84 n	0948	0883	0818	0753	0688	0623	0557	0492	0426	0360	0294
85 n	9.970294	0228	0161	0095	0028	*9962	*9895	*9828	*9760	*9693	*9626
86 n	9.969626	9558	9490	9422	9354	9286	9218	9149	9081	9012	8943
87 n	8943	8874	8804	8735	8666	8596	8526	8456	8386	8316	8245
88 n	8245	8175	8104	8033	7962	7891	7819	7748	7676	7604	7532
89 n	7532	7460	7388	7316	7243	7170	7097	7024	6951	6878	6804
8.90 n	9.966804	6731	6657	6583	6509	6435	6360	6285	6211	6136	6061
91 n	6061	5985	5910	5834	5759	5683	5607	5531	5454	5378	5301
92 n	5301	5224	5147	5070	4992	4915	4837	4759	4681	4603	4525
93 n	4525	4446	4368	4289	4210	4130	4051	3972	3892	3812	3732
94 n	3732	3652	3571	3491	3410	3329	3248	3167	3086	3004	2922
95 n	9.962922	2840	2758	2676	2594	2511	2428	2345	2262	2179	2095
96 n	2095	2012	1928	1844	1760	1675	1591	1506	1421	1336	1251
97 n	1251	1165	1080	0994	0908	0822	0735	0649	0562	0475	0388
98 n	9.960388	0301	0213	0126	0038	*9950	*9862	*9773	*9685	*9596	*9507
99 n	9.959507	9418	9329	9239	9149	9059	8969	8879	8789	8698	8607
9.00 n	9.958607	8516	8425	8334	8242	8150	8058	7966	7874	7781	7689

Proportional parts

	32	34	36	38	40
1	3.2	3.4	3.6	3.8	4.0
2	6.4	6.8	7.2	7.6	8.0
3	9.6	10.2	10.8	11.4	12.0
4	12.8	13.6	14.4	15.2	16.0
5	16.0	17.0	18.0	19.0	20.0
6	19.2	20.4	21.6	22.8	24.0
7	22.4	23.8	25.2	26.6	28.0
8	25.6	27.2	28.8	30.4	32.0
9	28.8	30.6	32.4	34.2	36.0

	42	44	46	48	50
1	4.2	4.4	4.6	4.8	5.0
2	8.4	8.8	9.2	9.6	10.0
3	12.6	13.2	13.8	14.4	15.0
4	16.8	17.6	18.4	19.2	20.0
5	21.0	22.0	23.0	24.0	25.0
6	25.2	26.4	27.6	28.8	30.0
7	29.4	30.8	32.2	33.6	35.0
8	33.6	35.2	36.8	38.4	40.0
9	37.8	39.6	41.4	43.2	45.0

	52	54	56	58	60
1	5.2	5.4	5.6	5.8	6.0
2	10.4	10.8	11.2	11.6	12.0
3	15.6	16.2	16.8	17.4	18.0
4	20.8	21.6	22.4	23.2	24.0
5	26.0	27.0	28.0	29.0	30.0
6	31.2	32.4	33.6	34.8	36.0
7	36.4	37.8	39.2	40.6	42.0
8	41.6	43.2	44.8	46.4	48.0
9	46.8	48.6	50.4	52.2	54.0

	62	64	66	68	70
1	6.2	6.4	6.6	6.8	7.0
2	12.4	12.8	13.2	13.6	14.0
3	18.6	19.2	19.8	20.4	21.0
4	24.8	25.6	26.4	27.2	28.0
5	31.0	32.0	33.0	34.0	35.0
6	37.2	38.4	39.6	40.8	42.0
7	43.4	44.8	46.2	47.6	49.0
8	49.6	51.2	52.8	54.4	56.0
9	55.8	57.6	59.4	61.2	63.0

	72	74	76	78	80
1	7.2	7.4	7.6	7.8	8.0
2	14.4	14.8	15.2	15.6	16.0
3	21.6	22.2	22.8	23.4	24.0
4	28.8	29.6	30.4	31.2	32.0
5	36.0	37.0	38.0	39.0	40.0
6	43.2	44.4	45.6	46.8	48.0
7	50.4	51.8	53.2	54.6	56.0
8	57.6	59.2	60.8	62.4	64.0
9	64.8	66.6	68.4	70.2	72.0

	82	84	86	88	90
1	8.2	8.4	8.6	8.8	9.0
2	16.4	16.8	17.2	17.6	18.0
3	24.6	25.2	25.8	26.4	27.0
4	32.8	33.6	34.4	35.2	36.0
5	41.0	42.0	43.0	44.0	45.0
6	49.2	50.4	51.6	52.8	54.0
7	57.4	58.8	60.2	61.6	63.0
8	65.6	67.2	68.8	70.4	72.0
9	73.8	75.6	77.4	79.2	81.0

Table XIV.—Curvature correction

For azimuth observations on Polaris

Azimuth of Polaris \ 2τ	1 m	2 m	3 m	4 m	5 m	6 m	7 m	8 m	9 m	10 m
° ′	″	″	″	″	″	″	″	″	″	″
0 00	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
10	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1
20	.0	.0	.0	.0	.1	.1	.1	.2	.2	.3
30	.0	.0	.0	.1	.1	.2	.2	.3	.3	.4
40	.0	.0	.1	.1	.1	.2	.3	.4	.5	.6
50	.0	.0	.1	.1	.2	.3	.3	.5	.6	.7
1 00	.0	.0	.1	.1	.2	.3	.4	.5	.7	.9
10	.0	.0	.1	.2	.2	.4	.5	.6	.8	1.0
20	.0	.0	.1	.2	.3	.4	.6	.7	.9	1.1
30	.0	.0	.1	.2	.3	.5	.6	.8	1.0	1.3
40	.0	.1	.1	.2	.4	.5	.7	.9	1.2	1.4
50	.0	.1	.1	.3	.4	.6	.8	1.0	1.3	1.6
2 00	.0	.1	.2	.3	.4	.6	.8	1.1	1.4	1.7
10	.0	.1	.2	.3	.5	.7	.9	1.2	1.5	1.9
20	.0	.1	.2	.3	.5	.7	1.0	1.3	1.6	2.0
30	.0	.1	.2	.3	.5	.8	1.1	1.4	1.7	2.1
40	.0	.1	.2	.4	.6	.8	1.1	1.5	1.9	2.3
50	.0	.1	.2	.4	.6	.9	1.2	1.6	2.0	2.4
3 00	.0	.1	.2	.4	.6	.9	1.3	1.6	2.1	2.6
10	.0	.1	.2	.4	.7	1.0	1.3	1.7	2.2	2.7

NOTE.—2τ = Difference in time between direct and reverse pointings on Polaris.

Table XV.—Mean solar into sidereal time

[To be added to mean time interval]

1 mean time second=1.00273791 sidereal seconds

seconds \ minutes	0	1	2	3	4
0	0.000	0.164	0.329	0.493	0.657
1	.003	.167	.331	.496	.660
2	.005	.170	.334	.498	.663
3	.008	.172	.337	.501	.665
4	.011	.175	.340	.504	.668
5	0.014	0.178	0.342	0.507	0.671
6	.016	.181	.345	.509	.674
7	.019	.183	.348	.512	.676
8	.022	.186	.350	.515	.679
9	.025	.189	.353	.517	.682
10	0.027	0.192	0.356	0.520	0.684
11	.030	.194	.359	.523	.687
12	.033	.197	.361	.526	.690
13	.036	.200	.364	.528	.693
14	.038	.203	.367	.531	.695
15	0.041	0.205	0.370	0.534	0.698
16	.044	.208	.372	.537	.701
17	.047	.211	.375	.539	.704
18	.049	.214	.378	.542	.706
19	.052	.216	.381	.545	.709
20	0.055	0.219	0.383	0.548	0.712
21	.057	.222	.386	.550	.715
22	.060	.225	.389	.553	.717
23	.063	.227	.392	.556	.720
24	.066	.230	.394	.559	.723
25	0.068	0.233	0.397	0.561	0.726
26	.071	.235	.400	.564	.728
27	.074	.238	.402	.567	.731
28	.077	.241	.405	.569	.734
29	.079	.244	.408	.572	.736
30	0.082	0.246	0.411	0.575	0.739
31	.085	.249	.413	.578	.742
32	.088	.252	.416	.580	.745
33	.090	.255	.419	.583	.747
34	.093	.257	.422	.586	.750
35	0.096	0.260	0.424	0.589	0.753
36	.099	.263	.427	.591	.756
37	.101	.266	.430	.594	.758
38	.104	.268	.433	.597	.761
39	.107	.271	.435	.600	.764
40	0.110	0.274	0.438	0.602	0.767
41	.112	.277	.441	.605	.769
42	.115	.279	.444	.608	.772
43	.118	.282	.446	.611	.775
44	.120	.285	.449	.613	.778
45	0.123	0.287	0.452	0.616	0.780
46	.126	.290	.454	.619	.783
47	.129	.293	.457	.622	.786
48	.131	.296	.460	.624	.789
49	.134	.298	.463	.627	.791
50	0.137	0.301	0.465	0.630	0.794
51	.140	.304	.468	.632	.797
52	.142	.307	.471	.635	.799
53	.145	.309	.474	.638	.802
54	.148	.312	.476	.641	.805
55	0.151	0.315	0.479	0.643	0.808
56	.153	.318	.482	.646	.810
57	.156	.320	.485	.649	.813
58	.159	.323	.487	.652	.816
59	.162	.326	.490	.654	.819
60	0.164	0.329	0.493	0.657	0.821

For reducing a series of scaled radio breaks to mean epoch; before mean epoch, add; after mean epoch, subtract.

Table XVI.—Factors for Bessel's probable error formulas

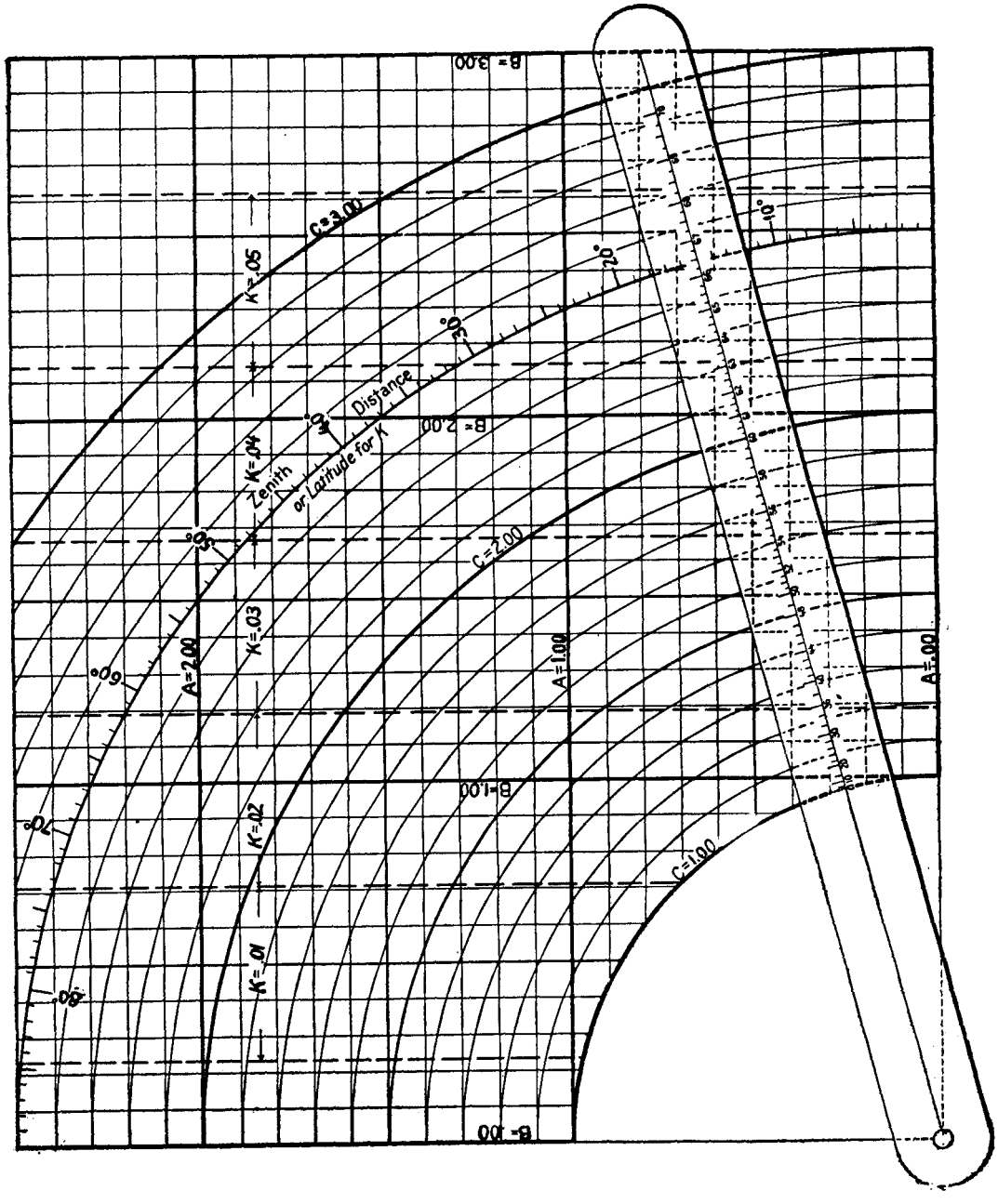
n	$\frac{.6745}{\sqrt{n-1}}$	$\frac{.6745}{\sqrt{n(n-1)}}$	n	$\frac{.6745}{\sqrt{n-1}}$	$\frac{.6745}{\sqrt{n(n-1)}}$
------	----------	----------	40	0.1080	0.0171
------	----------	----------	41	.1066	.0167
2	0.6745	0.4769	42	.1053	.0163
3	.4769	.2754	43	.1041	.0159
4	.3894	.1947	44	.1029	.0155
5	0.3372	0.1508	45	0.1017	0.0152
6	.3016	.1231	46	.1005	.0148
7	.2754	.1041	47	.0994	.0145
8	.2549	.0901	48	.0984	.0142
9	.2385	.0795	49	.0974	.0139
10	0.2248	0.0711	50	0.0964	0.0136
11	.2133	.0643	51	.0954	.0134
12	.2029	.0587	52	.0944	.0131
13	.1947	.0540	53	.0935	.0128
14	.1871	.0500	54	.0926	.0126
15	0.1803	0.0465	55	0.0918	0.0124
16	.1742	.0435	56	.0909	.0122
17	.1686	.0409	57	.0901	.0119
18	.1636	.0386	58	.0893	.0117
19	.1590	.0365	59	.0886	.0115
20	0.1547	0.0346	60	0.0878	0.0113
21	.1508	.0329	61	.0871	.0111
22	.1472	.0314	62	.0864	.0110
23	.1438	.0300	63	.0857	.0108
24	.1406	.0287	64	.0850	.0106
25	0.1377	0.0275	65	0.0843	0.0105
26	.1349	.0265	66	.0837	.0103
27	.1323	.0255	67	.0830	.0101
28	.1298	.0245	68	.0824	.0100
29	.1275	.0237	69	.0818	.0098
30	0.1252	0.0229	70	0.0812	0.0097
31	.1231	.0221	71	.0806	.0096
32	.1211	.0214	72	.0800	.0094
33	.1192	.0208	73	.0795	.0093
34	.1174	.0201	74	.0789	.0092
35	0.1157	0.0196	75	0.0784	0.0091
36	.1140	.0190	80	.0759	.0085
37	.1124	.0185	85	.0736	.0080
38	.1109	.0180	90	.0713	.0075
39	.1094	.0175	100	.0678	.0068

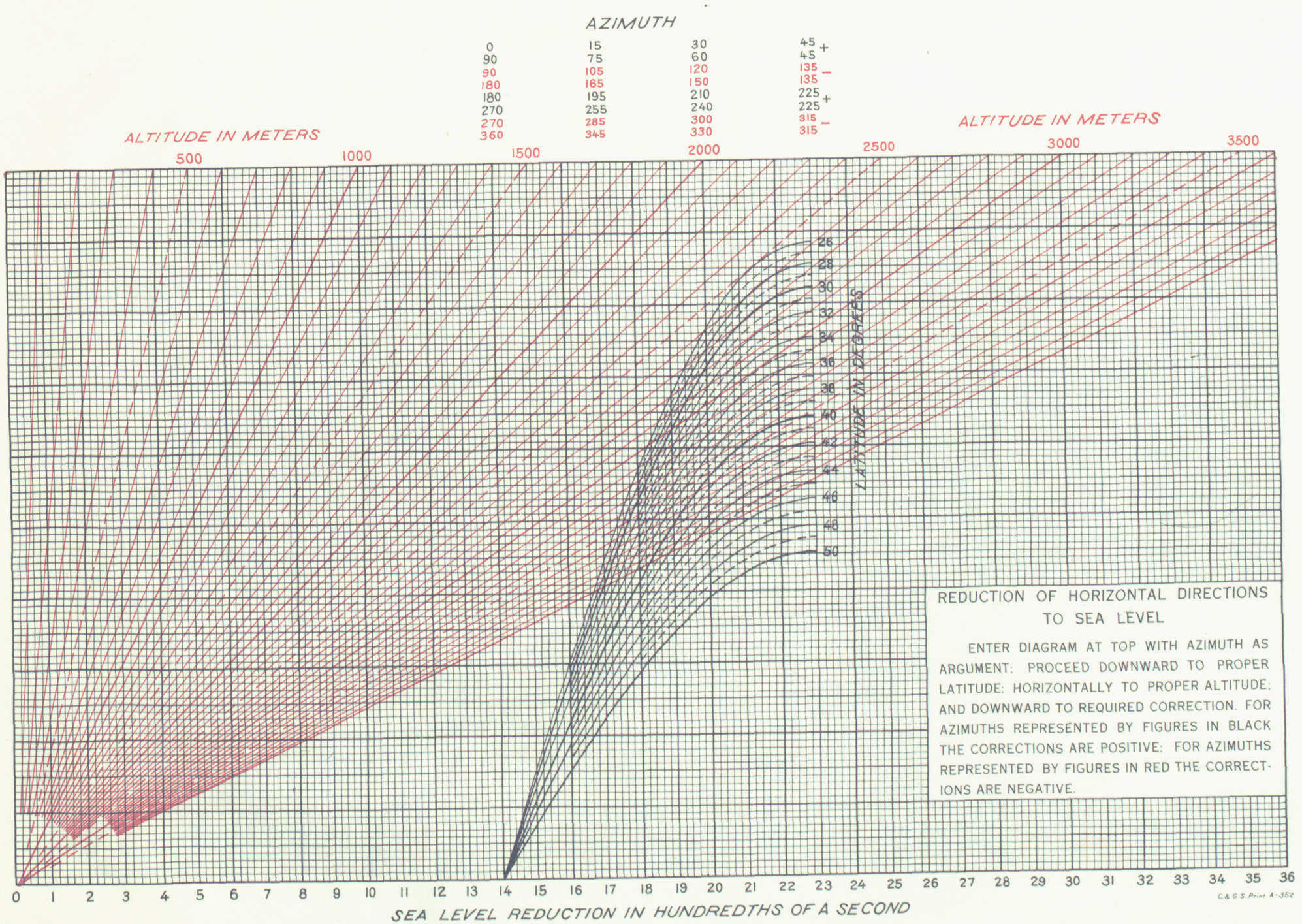
Table XVII.—Factors for Peters' probable error formulas

n	$\frac{.8453}{\sqrt{n(n-1)}}$	$\frac{.8453}{n\sqrt{n-1}}$	n	$\frac{.8453}{\sqrt{n(n-1)}}$	$\frac{.8453}{n\sqrt{n-1}}$
			40	0.0214	0.0034
			41	.0209	.0033
2	0.5978	0.4227	42	.0204	.0031
3	.3451	.1993	43	.0199	.0030
4	.2440	.1220	44	.0194	.0029
5	0.1890	0.0845	45	0.0190	0.0028
6	.1543	.0630	46	.0186	.0027
7	.1304	.0493	47	.0182	.0027
8	.1130	.0399	48	.0178	.0026
9	.0996	.0332	49	.0174	.0025
10	0.0891	0.0282	50	0.0171	0.0024
11	.0806	.0243	51	.0167	.0023
12	.0736	.0212	52	.0164	.0023
13	.0677	.0188	53	.0161	.0022
14	.0627	.0167	54	.0158	.0022
15	0.0583	0.0151	55	0.0155	0.0021
16	.0546	.0136	56	.0152	.0020
17	.0513	.0124	57	.0150	.0020
18	.0483	.0114	58	.0147	.0019
19	.0457	.0105	59	.0145	.0019
20	0.0434	0.0097	60	0.0142	0.0018
21	.0412	.0090	61	.0140	.0018
22	.0393	.0084	62	.0137	.0017
23	.0376	.0078	63	.0135	.0017
24	.0360	.0073	64	.0133	.0017
25	0.0345	0.0069	65	0.0131	0.0016
26	.0332	.0065	66	.0129	.0016
27	.0319	.0061	67	.0127	.0016
28	.0307	.0058	68	.0125	.0015
29	.0297	.0055	69	.0123	.0015
30	0.0287	0.0052	70	0.0122	0.0015
31	.0277	.0050	71	.0120	.0014
32	.0268	.0047	72	.0118	.0014
33	.0260	.0045	73	.0117	.0014
34	.0252	.0043	74	.0115	.0013
35	0.0245	0.0041	75	0.0113	0.0013
36	.0238	.0040	80	.0106	.0012
37	.0232	.0038	85	.0100	.0011
38	.0225	.0037	90	.0095	.0010
39	.0220	.0035	100	.0085	.0008

748977°—48——14

Table XIX.—Nomogram for obtaining star factors





REDUCTION OF HORIZONTAL DIRECTIONS TO SEA LEVEL

ENTER DIAGRAM AT TOP WITH AZIMUTH AS ARGUMENT; PROCEED DOWNWARD TO PROPER LATITUDE; HORIZONTALLY TO PROPER ALTITUDE; AND DOWNWARD TO REQUIRED CORRECTION. FOR AZIMUTHS REPRESENTED BY FIGURES IN BLACK THE CORRECTIONS ARE POSITIVE; FOR AZIMUTHS REPRESENTED BY FIGURES IN RED THE CORRECTIONS ARE NEGATIVE.

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Table XXa.—Principal table for computing refraction

$$\text{Log } R = \log \tan z + \alpha + \beta + \gamma + (\lambda - 1)\left(\gamma + \frac{\beta}{10}\right)$$

z	α	λ-1	z	α	λ-1
° ′			° ′		
0 00	1.75825	0.0013	74 10	1.75217	0.0180
5 00	1.75825	.0013	74 20	1.75204	.0184
10 00	1.75824	.0013	74 30	1.75190	.0187
15 00	1.75822	.0014	74 40	1.75176	.0191
20 00	1.75819	.0015	74 50	1.75161	.0195
			75 00	1.75146	.0199
25 00	1.75814	.0016			
30 00	1.75808	.0018	75 10	1.75131	.0203
35 00	1.75800	.0020	75 20	1.75115	.0207
40 00	1.75790	.0023	75 30	1.75099	.0211
			75 40	1.75082	.0216
41 00	1.75787	.0024	75 50	1.75064	.0220
42 00	1.75784	.0025	76 00	1.75046	.0225
43 00	1.75781	.0026			
44 00	1.75778	.0026	76 10	1.75027	.0230
45 00	1.75775	.0027	76 20	1.75007	.0235
			76 30	1.74987	.0240
46 00	1.75771	.0028	76 40	1.74966	.0246
47 00	1.75767	.0029	76 50	1.74944	.0251
48 00	1.75763	.0030	77 00	1.74922	.0257
49 00	1.75759	.0031			
50 00	1.75754	.0032	77 10	1.74898	.0263
			77 20	1.74874	.0270
51 00	1.75749	.0034	77 30	1.74849	.0276
52 00	1.75743	.0036	77 40	1.74823	.0283
53 00	1.75737	.0038	77 50	1.74796	.0290
54 00	1.75730	.0040	78 00	1.74768	.0297
55 00	1.75723	.0042			
			78 10	1.74739	.0304
56 00	1.75715	.0044	78 20	1.74708	.0312
57 00	1.75706	.0046	78 30	1.74676	.0320
58 00	1.75697	.0049	78 40	1.74643	.0329
59 00	1.75686	.0052	78 50	1.74609	.0337
60 00	1.75675	.0055	79 00	1.74573	.0346
61 00	1.75662	.0059	79 10	1.74536	.0356
62 00	1.75648	.0063	79 20	1.74497	.0366
63 00	1.75633	.0067	79 30	1.74456	.0376
64 00	1.75616	.0072	79 40	1.74413	.0386
65 00	1.75596	.0077	79 50	1.74369	.0397
			80 00	1.74323	.0408
66 00	1.75574	.0083			
67 00	1.75549	.0090	80 10	1.74274	.0420
68 00	1.75521	.0098	80 20	1.74223	.0433
69 00	1.75489	.0106	80 30	1.74170	.0446
70 00	1.75452	.0117	80 40	1.74114	.0460
			80 50	1.74055	.0473
70 10	1.75445	.0118	81 00	1.73993	.0488
70 20	1.75438	.0120			
70 30	1.75431	.0122	81 10	1.73928	.0504
70 40	1.75424	.0124	81 20	1.73860	.0520
70 50	1.75416	.0126	81 30	1.73788	.0537
71 00	1.75409	.0129	81 40	1.73712	.0554
			81 50	1.73633	.0573
71 10	1.75401	.0131	82 00	1.73549	.0592
71 20	1.75393	.0133			
71 30	1.75385	.0135	82 10	1.73460	.0613
71 40	1.75376	.0137	82 20	1.73366	.0635
71 50	1.75367	.0139	82 30	1.73267	.0657
72 00	1.75358	.0141	82 40	1.73161	.0680
			82 50	1.73050	.0705
72 10	1.75349	.0144	83 00	1.72931	.0732
72 20	1.75340	.0146			
72 30	1.75330	.0149	83 10	1.72805	.0760
72 40	1.75320	.0152	83 20	1.72672	.0788
72 50	1.75310	.0155	83 30	1.72529	.0819
73 00	1.75300	.0158	83 40	1.72377	.0851
			83 50	1.72215	.0885
73 10	1.75289	.0161	84 00	1.72041	.0921
73 20	1.75278	.0164			
73 30	1.75266	.0167	84 10	1.71856	.0959
73 40	1.75255	.0170	84 20	1.71657	.0999
73 50	1.75243	.0173	84 30	1.71443	.1041
74 00	1.75230	.0177	84 40	1.71213	.1086
			84 50	1.70966	.1132
			85 00	1.70700	.1179

Table XXb.—Correction for pressure

$$B' = B + eC$$

$$\beta = \beta_0 + \Delta\beta_t + \Delta\beta_w + \Delta\beta_g$$

B'	β_0	B'	β_0
mm.		*mm.*	
710	−0.02380	751	+0.00058
711	−.02319	752	+.00116
712	−.02258	753	+.00173
713	−.02197	754	+.00231
714	−.02136	755	+.00289
715	−.02076	756	+.00346
716	−.02015	757	+.00403
717	−.01954	758	+.00461
718	−.01894	759	+.00518
719	−.01833	760	+.00575
720	−.01773	761	+.00632
721	−.01713	762	+.00689
722	−.01652	763	+.00746
723	−.01592	764	+.00803
724	−.01532	765	+.00860
725	−.01472	766	+.00917
726	−.01412	767	+.00973
727	−.01353	768	+.01030
728	−.01293	769	+.01086
729	−.01233	770	+.01143
730	−.01174	771	+.01199
731	−.01114	772	+.01256
732	−.01055	773	+.01312
733	−.00996	774	+.01368
734	−.00937	775	+.01424
735	−.00877	776	+.01480
736	−.00818	777	+.01536
737	−.00759	778	+.01592
738	−.00700	779	+.01648
739	−.00642	780	+.01703
740	−.00583	781	+.01759
741	−.00524	782	+.01815
742	−.00466	783	+.01870
743	−.00407	784	+.01925
744	−.00349	785	+.01981
745	−.00290	786	+.02036
746	−.00232	787	+.02091
747	−.00174	788	+.02146
748	−.00116	789	+.02202
749	−.00058	790	+.02257
750	.00000		

Table XXc.—Correction for temperature

t_B	$\Delta\beta_t$	--------	t_B	$\Delta\beta_t$	--------
t_C	--------	γ	t_C	--------	γ
°			°		
−30	+0.00211	+0.06637	+8	−0.00057	+0.00309
−29	+.00204	+.06458	+9	−.00064	+.00154
−28	+.00197	+.06280	+10	−.00071	.00000
−27	+.00190	+.06102			
−26	+.00183	+.05926	+11	−.00078	−.00154
−25	+.00176	+.05750	+12	−.00085	−.00307
			+13	−.00092	−.00459
−24	+.00169	+.05574	+14	−.00099	−.00611
−23	+.00162	+.05400	+15	−.00106	−.00763
−22	+.00155	+.05226			
−21	+.00148	+.05053	+16	−.00113	−.00914
−20	+.00141	+.04880	+17	−.00120	−.01064
			+18	−.00127	−.01214
−19	+.00134	+.04708	+19	−.00134	−.01363
−18	+.00127	+.04537	+20	−.00141	−.01512
−17	+.00120	+.04367			
−16	+.00113	+.04197	+21	−.00148	−.01660
−15	+.00106	+.04028	+22	−.00156	−.01808
			+23	−.00163	−.01956
−14	+.00099	+.03859	+24	−.00170	−.02102
−13	+.00092	+.03691	+25	−.00177	−.02249
−12	+.00085	+.03524			
−11	+.00078	+.03358	+26	−.00184	−.02395
−10	+.00071	+.03192	+27	−.00191	−.02540
			+28	−.00198	−.02685
−9	+.00063	+.03026	+29	−.00205	−.02829
−8	+.00056	+.02862	+30	−.00212	−.02973
−7	+.00049	+.02698			
−6	+.00042	+.02534	+31	−.00219	−.03117
−5	+.00035	+.02372	+32	−.00226	−.03260
			+33	−.00234	−.03402
−4	+.00028	+.02209	+34	−.00241	−.03544
−3	+.00021	+.02048	+35	−.00248	−.03686
−2	+.00014	+.01887			
−1	+.00007	+.01726	+36	−.00255	−.03827
0	.00000	+.01567	+37	−.00262	−.03967
			+38	−.00269	−.04108
+1	−.00007	+.01407	+39	−.00276	−.04247
+2	−.00014	+.01249	+40	−.00283	−.04387
+3	−.00021	+.01091			
+4	−.00028	+.00933	+41	−.00290	−.04526
+5	−.00035	+.00776	+42	−.00298	−.04664
			+43	−.00305	−.04802
+6	−.00042	+.00620	+44	−.00312	−.04940
+7	−.00049	+.00464			

Table XXd.—Correction for wave-length of light

w	C	$\Delta\beta_w$	w	C	$\Delta\beta_w$
micron			*micron*		
0.30	−0.1326	+0.02117	0.70	−0.1586	−0.00243
.35	−.1415	+.01336	.75	−.1593	−.00309
.40	−.1471	+.00836	.80	−.1598	−.00364
.45	−.1508	+.00497	.85	−.1603	−.00409
.50	−.1533	+.00256	.90	−.1607	−.00446
.55	−.1552	+.00079	.95	−.1610	−.00478
.60	−.1566	−.00056	1.00	−.1613	−.00506
.65	−.1577	−.00160			

Table XXe.—Correction to Log R for gravity, $\Delta\beta_g$

ϕ \ h	0 m	200 m	400 m	600 m	800 m	1000 m
0°	−0.00090	−0.00093	−0.00096	−0.00098	−0.00101	−0.00104
5	−.00088	−.00091	−.00094	−.00097	−.00099	−.00102
10	−.00083	−.00086	−.00089	−.00091	−.00094	−.00097
15	−.00075	−.00077	−.00080	−.00083	−.00086	−.00088
20	−.00063	−.00066	−.00068	−.00071	−.00074	−.00077
25	−.00049	−.00052	−.00054	−.00057	−.00060	−.00062
30	−.00032	−.00035	−.00038	−.00040	−.00043	−.00046
35	−.00014	−.00017	−.00019	−.00022	−.00025	−.00028
40	+.00005	+.00003	.00000	−.00003	−.00005	−.00008
45	+.00026	+.00023	+.00020	+.00017	+.00015	+.00012
50	+.00046	+.00043	+.00040	+.00037	+.00035	+.00032
55	+.00065	+.00062	+.00060	+.00057	+.00054	+.00051
60	+.00083	+.00080	+.00078	+.00075	+.00072	+.00070
65	+.00100	+.00097	+.00094	+.00092	+.00089	+.00086
70	+.00114	+.00111	+.00109	+.00106	+.00103	+.00100
75	+.00126	+.00123	+.00120	+.00117	+.00115	+.00112
80	+.00134	+.00131	+.00129	+.00126	+.00123	+.00120
85	+.00139	+.00137	+.00134	+.00131	+.00128	+.00126
90	+.00141	+.00138	+.00136	+.00133	+.00130	+.00127

ϕ \ h	1200 m	1400 m	1600 m	1800 m	2000 m
0°	−0.00106	−0.00109	−0.00112	−0.00115	−0.00117
5	−.00105	−.00107	−.00110	−.00113	−.00116
10	−.00099	−.00102	−.00105	−.00108	−.00110
15	−.00091	−.00094	−.00096	−.00099	−.00102
20	−.00079	−.00082	−.00085	−.00088	−.00090
25	−.00065	−.00068	−.00071	−.00073	−.00076
30	−.00049	−.00051	−.00054	−.00057	−.00060
35	−.00030	−.00033	−.00036	−.00039	−.00041
40	−.00011	−.00014	−.00016	−.00019	−.00022
45	+.00009	+.00006	+.00004	+.00001	−.00002
50	+.00029	+.00026	+.00024	+.00021	+.00018
55	+.00049	+.00046	+.00043	+.00040	+.00038
60	+.00067	+.00064	+.00061	+.00059	+.00056
65	+.00083	+.00081	+.00078	+.00075	+.00073
70	+.00098	+.00095	+.00092	+.00090	+.00087
75	+.00109	+.00107	+.00104	+.00101	+.00098
80	+.00118	+.00115	+.00112	+.00110	+.00107
85	+.00123	+.00120	+.00118	+.00115	+.00112
90	+.00125	+.00122	+.00119	+.00117	+.00114

Table XXI.—Refraction table for high elevations

Elev. \ Zen. Dist.	89°	87°	85°	80°	75°	70°	60°	45°	30°
Feet	′	′	′	′	′	′	′	′	′
0	24	14	10	5	4	3	2	1	1
5,000	20	12	8	5	3	2	1	1	0
10,000	17	10	7	4	3	2	1	1	0
15,000	15	9	6	3	2	2	1	1	0
20,000	13	7	5	3	2	1	1	1	0
25,000	11	6	4	2	2	1	1	0	0
30,000	9	5	3	2	1	1	1	0	0
35,000	8	4	3	2	1	1	1	0	0
40,000	6	4	2	1	1	1	0	0	0

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