Serial No. 9

DEPARTMENT OF COMMERCE

U. S. COAST AND GEODETIC SURVEY

E. LESTER JONES, SUPERINTENDENT

GEODESY

APPLICATION OF THE THEORY OF LEAST SQUARES TO THE ADJUSTMENT OF TRIANGULATION

BY

OSCAR S. ADAMS

COMPUTER

UNITED STATES COAST AND GEODETIC SURVEY

QB
275
.U35
no. 28
1915

Special Publication No. 28

LIBRARY

APR 1 0 2006

National Oceanic &
Atmospheric Administration
U.S. Dept. of Commerce

WASHINGTON
GOVERNMENT PRINTING OFFICE
1915

National Oceanic and Atmospheric Administration

ERRATA NOTICE

One or more conditions of the original document may affect the quality of the image, such as:

Discolored pages
Faded or light ink
Binding intrudes into the text

This has been a co-operative project between the NOAA Central Library and the Climate Database Modernization Program, National Climate Data Center (NCDC). To view the original document, please contact the NOAA Central Library in Silver Spring, MD at (301) 713-2607 x124 or www.reference@nodc.noaa.gov.

LASON
Imaging Contractor
12200 Kiln Court
Beltsville, MD 20704-1387
January 1, 2006

ADDITIONAL COPIES
OF THIS PUBLICATION MAY BE PROCURED FROM
THE SUPERINTENDENT OF DOCUMENTS
GOVERNMENT PRINTING OFFICE
WASHINGTON, D. C.
AT
40 CENTS PER COPY
▽

CONTENTS.

	Page.
General statement	7
Station adjustment	7
Observed angles	8
List of directions	8
Condition equations	9
Formation of normal equations by differentiation	9
Correlate equations	11
Formation of normal equations	11
Normal equations	12
Discussion of method of solution of normal equations	12
Solution of normal equations	13
Back solution	13
Computation of corrections	13
Adjustment of a quadrilateral	14
General statement	14
Lists of directions	16
Figure	16
Angle equations	17
Side equation	17
Formation of normal equations by differentiation	17
Correlate equations	18
Normal equations	18
Solution of normal equations	19
Back solution	19
Computation of corrections	19
Adjustment of a quadrilateral by the use of two angle and two side equations	20
Angle equations	20
Side equations	20
Correlate equations	20
Normal equations	20
Solution of normal equations	21
Back solution	21
Computation of corrections	21
Solution of a set of normals including terms usually omitted	22
Discussion of the solution	22
Solution of triangles	23
Position computations, secondary triangulation	24
List of geographic positions	26
Development of condition equations for latitude and longitude closures	26
Equations in a net	32
Adjustment of a figure with latitude, longitude, azimuth, and length closure conditions	34
Figure	34
Angle equations	35
Azimuth equation	35
Side equations	35
Length equation	37

Adjustment of a figure with latitude, longitude, azimuth, and length closure conditions—Continued. Page.
Figure for latitude and longitude equations 37
Formation of azimuth equation 38
Preliminary computation of triangles 38
Preliminary computation of positions, primary form 40
Formation of latitude and longitude condition equations 50
Latitude equation 51
Longitude equation 51
Correlate equations 52
List of corrections 55
Normal equations 56
Solution of normals 58
Back solution 68
Computation of corrections 69
Final solution of triangles 71
Final computation of positions 76
List of geographic positions 91
Adjustment of triangulation by the method of variation of geographic coordinates 91
Development of formulas 91
Adjustment of a quadrilateral with two points fixed 94
Lists of observed directions 94
Preliminary computation of triangles 95
Preliminary computation of positions 96
Formation of observation equations 98
Table for formation of normals, No. 1 100
Table for formation of normals, No. 2 101
Normal equations 101
Solution of normals 101
Back solution 102
Computation of corrections 102
Adjusted computation of triangles 103
Adjustment of three new points by variation of geographic coordinates 103
General statement 103
Figure 104
First method 105
List of directions 105
Lists of fixed positions 106
Preliminary computation of triangles 107
Preliminary computation of positions 110
Formation of observation equations 114
Table for formation of normals, No. 1 118
Table for formation of normals, No. 2 119
Normal equations 119
Solution of normals 120
Back solution 121
Computation of corrections 121
Final computation of triangles 122
Second method 125
Formation of observation equations 126
Table for formation of normals, No. 1 127
Table for formation of normals, No. 2 127
Normal equations 127
Solution of normals 128

Adjustment of three new points by variation of geographic coordinates—Con.
Second method—Continued. Page.
Back solution 129
Computation of corrections 129
Final computation of triangles 130
Final computation of positions 134
Computation of probable errors 138
Adjustment of a figure with latitude and longitude, azimuth, and length conditions by variation of geographic coordinates 139
Table of fixed positions 139
Preliminary computation of triangles 140
Preliminary computation of positions 144
Figure 157
Formation of observation equations 158
Table for formation of normals, No. 1 162
Table for formation of normals, No. 2 166
Normal equations 168
Solution of normals 169
Back solution 174
Computation of corrections 175
Final computation of triangles 178
Final computation of positions 182
Adjustments by the angle method 196
Adjustment of verticals 197
General statement 197
Figure 197
Computation of elevations from reciprocal observations 198
Computation of elevations from nonreciprocal observations 199
Fixed elevations 200
Assumed and adjusted elevations 200
Formation of observation equations 200
Table of formation of equations 201
Computation of probable error 202
Formation of normal equations by differentiation 202
Table for formation of normal equations 204
Normal equations 204
Solution of normal equations 204
Back solution 205
Development of formulas for trigonometric leveling 205
General statement 205
Development of formulas 207
Examples of computation by formulas 214
Recapitulation of formulas 216
Notes on construction and use of tables 217
Tables 218
Notes on the developments 219

Blank page retained for pagination

APPLICATION OF THE THEORY OF LEAST SQUARES TO THE ADJUSTMENT OF TRIANGULATION

By Oscar S. Adams
Computer United States Coast and Geodetic Survey

GENERAL STATEMENT

In this publication the aim has not been to develop the theory of least squares, but to illustrate the application of the method to the problems arising in the adjustment of triangulation. The general idea has been to collect material in one volume that will serve as a working manual for the computer in the office and for such other members of the Survey as may desire to make these special applications. It has not been deemed necessary to insert the derivation of formulæ except in the case of a few special ones that are not usually found in the textbooks on least squares.

For the general theory reference should be made to such books as the following:

Crandall: Geodesy and Least Squares.
Helmert: Die Ausgleichungsrechnung nach der Metode der kleinsten Quadrate.
Jordan: Handbuch der Vermessungskunde, volume 1.
Merriman: Textbook of Least Squares.
Wright and Hayford: Adjustment of Observations.

Some of the simpler cases are treated first, such as the local adjustment at a station, the adjustment of a simple quadrilateral, etc. After these is given the development of the condition equations for latitude and longitude closures, followed by a sample adjustment including the condition equations for these closures, together with the equations for length and azimuth conditions.

A method of adjustment by the variation of geographic coordinates is then developed and applied first to a quadrilateral, then to a figure with a few new points connected with a number of fixed points. The same method is applied to the adjustment of a figure with latitude, longitude, length, and azimuth conditions. A sample adjustment of a vertical net is carried through and lastly there is given the development of the formulæ for the computation of vertical observations, together with examples of the method of computation.

STATION ADJUSTMENT

The general rule followed by the observers of the Coast and Geodetic Survey is to measure the angles at each station in the order of azimuth, thus giving rise to no conditions except the horizon closure. Occasionally, however, sum angles are observed and, when this is done, other conditions are introduced in addition to the horizon closure making it necessary to adjust the angles at the station by the method of least squares. If all angles were observed in the same way, the weight of each would be unity and the adjustment would be made without regard to weights. In the adjustment given below the angles were measured by the usual Coast and Geodetic Survey repetition method; that is, six measures of the angles with the telescope direct and six with it reversed for each set. A station has been chosen at which there are angles measured with one, two, and three sets in order to illustrate the method of weighting.

Observed angles, Gray Cliff

Observed stations	Angle	Weight p	v	Adjusted final seconds*
	° ′ ″ ″			″
Boulder-Tower	65 06 26.6, 30.9, 30.4 } 29.3	3	v_1	29.9
Tower-Tyonek	19 46 27.7, 25.0, 28.0 } 26.9	3	v_2	27.5
Tyonek-Round Point	8 39 14.6, 18.4, 14.8 } 15.9	3	v_3	15.9
Round Point-Boulder	266 27 47.9	1	v_4	46.7
Round Point-Birch Hill	66 23 20.6, 23.0 } 21.8	2	v_5	22.4
Birch Hill-Boulder	200 04 22.2, 22.3 } 22.2	2	v_6	24.3
Boulder-Tyonek	84 52 56.2	1	v_7	57.4
Tyonek-Birch Hill	75 02 35.0	1	v_8	38.3
Round Point-Moose Point	64 32 12.9, 09.8 } 11.4	2	v_9	11.3
Moose Point-Birch Hill	1 51 11.2	1	v_{10}	11.1

List of directions, Gray Cliff

Observed station	Direction	Adjusted final seconds*
	° ′ ″	″
Boulder	0 00 00.0	00.0
Tower	65 06 29.3+v_1	29.9
Tyonek	84 52 56.2+v_1+v_2	57.4
Round Point	93 32 12.1+v_1+v_2+v_3	13.3
Moose Point	158 04 23.5+v_1+v_2+v_3+v_9	24.6
Birch Hill	159 55 34.7+v_1+v_2+v_3+v_9+v_{10}	35.7

* These values result from the computation on p. 13.

There have been formed a complete list of directions without using five of the angles, each of which, then, gives rise to a condition, there being five conditions in all. The equations expressing these conditions are formed in the following manner:

Angle Round Point–Boulder, observed,	266	27	$47.9+v_4$
Angle Round Point–Boulder, from the list,	266	27	$47.9-v_1-v_2-v_3$
Condition No. 1,			$0=+0.0+v_1+v_2+v_3+v_4$
Angle Round Point–Birch Hill, from the list,	66	23	$22.6+v_9+v_{10}$
Angle Round Point–Birch Hill, observed,	66	23	$21.8+v_5$
Condition No. 2,			$0=+0.8-v_5+v_9+v_{10}$

In the same way the other condition equations are formed. As a result there are finally:

Condition equations

1. $0=+0.0+v_1+v_2+v_3+v_4$
2. $0=+0.8-v_5+v_9+v_{10}$
3. $0=-3.1+v_1+v_2+v_3+v_6+v_9+v_{10}$
4. $0=+0.0+v_1+v_2-v_7$
5. $0=+3.5+v_3-v_8+v_9+v_{10}$

FORMATION OF NORMAL EQUATIONS BY DIFFERENTIATION

According to the theory of least squares, the most probable values will be determined by making the $\Sigma\, p_n\, v_n^2$ a minimum, subject to the given conditions. By the method of Lagrangian multipliers the formation of the normal equations can be much simplified.

With the use of these the function u that is to be made a minimum is

$$u=3\,v_1^2+3\,v_2^2+3\,v_3^2+1\,v_4^2+2\,v_5^2+2\,v_6^2+1\,v_7^2+1\,v_8^2+2\,v_9^2+1\,v_{10}^2-2\,C_1(+v_1+v_2+v_3+v_4+0.0)-2C_2(-v_5+v_9+v_{10}+0.8)-2C_3(+v_1+v_2+v_3+v_6+v_9+v_{10}-3.1)-2C_4(+v_1+v_2-v_7+0.0)-2C_5(+v_3-v_8+v_9+v_{10}+3.5).$$

The C's are merely undetermined multipliers, the values of which will be determined by the solution. The factor 2 is included to obviate later on the use of the fraction $\frac{1}{2}$; the minus sign is used for convenience. The function will be rendered a minimum if the partial differential coefficients with respect to v_1, v_2, etc., are equated to zero. By this means ten equations will be formed, giving the ten v's expressed in terms of the C's.

Differentiating with respect to v_1, v_2, etc., in succession and equating the results to zero, the following equations are obtained:

$$\begin{aligned}
3\,v_1 &-C_1-C_3-C_4=0\\
3\,v_2 &-C_1-C_3-C_4=0\\
3\,v_3 &-C_1-C_3-C_5=0\\
v_4 &-C_1=0\\
2\,v_5 &+C_2=0\\
2\,v_6 &-C_3=0\\
v_7 &+C_4=0\\
v_8 &+C_5=0\\
2\,v_9 &-C_2-C_3-C_5=0\\
v_{10} &-C_2-C_3-C_5=0
\end{aligned}$$

Therefore

$$
\begin{aligned}
v_1 &= +\tfrac{1}{3}\,C_1 + \tfrac{1}{3}\,C_3 + \tfrac{1}{3}\,C_4 \\
v_2 &= +\tfrac{1}{3}\,C_1 + \tfrac{1}{3}\,C_3 + \tfrac{1}{3}\,C_4 \\
v_3 &= +\tfrac{1}{3}\,C_1 + \tfrac{1}{3}\,C_3 + \tfrac{1}{3}\,C_5 \\
v_4 &= +C_1 \\
v_5 &= -\tfrac{1}{2}\,C_2 \\
v_6 &= +\tfrac{1}{2}\,C_3 \\
v_7 &= -C_4 \\
v_8 &= -C_5 \\
v_9 &= +\tfrac{1}{2}\,C_2 + \tfrac{1}{2}\,C_3 + \tfrac{1}{2}\,C_5 \\
v_{10} &= +C_2 + C_3 + C_5
\end{aligned}
$$

Thus all of the v's are now expressed in terms of the C's. These can now be substituted in the condition equations forming five normal equations containing five C's and these equations may then be solved for the C's. If the normals are formed from these values, fractions will occur in practically all of the coefficients. This can be avoided by replacing C_1 by 6 C_1', C_2 by 6 C_2', etc. This is equivalent to using 12 C_1', 12 C_2', etc., in the original function instead of 2 C_1, 2 C_2, etc., which, of course, is perfectly valid.

The equations will then stand as follows:

$$
\begin{aligned}
v_1 &= +2\,C_1' + 2\,C_3' + 2\,C_4' \\
v_2 &= +2\,C_1' + 2\,C_3' + 2\,C_4' \\
v_3 &= +2\,C_1' + 2\,C_3' + 2\,C_5' \\
v_4 &= +6\,C_1' \\
v_5 &= -3\,C_2' \\
v_6 &= +3\,C_3' \\
v_7 &= -6\,C_4' \\
v_8 &= -6\,C_5' \\
v_9 &= +3\,C_2' + 3\,C_3' + 3\,C_5' \\
v_{10} &= +6\,C_2' + 6\,C_3' + 6\,C_5'
\end{aligned}
$$

Dropping the prime and substituting these values in the first condition equation the following normal equation is obtained:

$$
\begin{aligned}
&2\,C_1 + 2\,C_3 + 2\,C_4 + 2\,C_1 + 2\,C_3 + 2\,C_4 + 2\,C_1 + 2\,C_3 + 2\,C_5 + 6\,C_1 + 0.0 = 0 \\
&\quad +12\,C_1 \qquad + 6\,C_3 + 4\,C_4 + 2\,C_5 + 0.0 = 0
\end{aligned}
$$

In a similar manner the other normal equations are formed, giving in all the following five equations:

$$
\begin{aligned}
&+12\,C_1 \qquad\qquad + 6\,C_3 + 4\,C_4 + 2\,C_5 + 0.0 = 0 \\
&\qquad\quad +12\,C_2 + 9\,C_3 \qquad\qquad + 9\,C_5 + 0.8 = 0 \\
&+6\,C_1 + 9\,C_2 + 18\,C_3 + 4\,C_4 + 11\,C_5 - 3.1 = 0 \\
&+4\,C_1 \qquad\qquad + 4\,C_3 + 10\,C_4 \qquad\quad + 0.0 = 0 \\
&+2\,C_1 + 9\,C_2 + 11\,C_3 \qquad\qquad + 17\,C_5 + 3.5 = 0
\end{aligned}
$$

This manner of forming the normal equations is called the method of correlates and is most conveniently carried out by means of a table of correlates formed as on page 11.

After the determination of the C's by the solution of the normal equations, the v's may be computed from the equations of the v's

in terms of the C's. In the tabulated form below the first column is multiplied by C_1, the second by C_2, etc. The sum of the first line multiplied by the $\frac{6}{p}$ for that line gives v_1; so also for the other v's.

Correlate equations

	$\frac{6}{p}$	1	2	3	4	5	Σ	v's*	Adopted v's
1	2	+1		+1	+1		+3	+0.618	+0.6
2	2	+1		+1	+1		+3	+0.618	+0.6
3	2	+1		+1		+1	+3	−0.050	−0.0
4	6	+1					+1	−1.182	−1.2
5	3		−1				−1	+0.585	+0.6
6	3			+1			+1	+2.133	+2.1
7	6				−1		−1	+1.230	+1.2
8	6					−1	−1	+3.234	+3.3
9	3		+1	+1		+1	+3	−0.069	−0.1
10	6		+1	+1		+1	+3	−0.138	−0.1

* These values result from the computation on p. 13.

FORMATION OF THE NORMAL EQUATIONS

After the condition equations are tabulated in correlates as above, the next step is the formation of the normal equations. In forming these the various products must be multiplied by $\frac{1}{p}$ or by $\frac{a}{p}$ in which p is the weight of the given v and a is some constant. (See the direct formation on p. 9.) It is most convenient to choose a so as to make most of the values integers, if this can be done without making the quantities too large. In this case 6 is the L. C. M. of the p's, hence it is chosen for a. The normal equations are formed by taking the algebraic sums of $\frac{6}{p}$ times the products of the various columns. Normal No. 1 is, in symbols—

$$\Sigma\frac{6}{p}\cdot 1\cdot 1+\Sigma\frac{6}{p}\cdot 1\cdot 2+\Sigma\frac{6}{p}\cdot 1\cdot 3+\Sigma\frac{6}{p}\cdot 1\cdot 4+\Sigma\frac{6}{p}\cdot 1\cdot 5+\eta+\left(\Sigma\frac{6}{p}\cdot 1\cdot \Sigma+\eta^{\dagger}\right)$$

The algebraic sum of the sigma products in the formation checks or controls the formation of the normals. Each Σ line in the correlates is the algebraic sum of that line in the table. As is easily seen, the sum of the products of this column in the formation of the normals should check the algebraic sum of the coefficients of the normal. On the first normal $+12+6+4+2=+24$, which is the same as the algebraic sum of the products in the correlates. The Σ column in the normals also includes the constant term. In the third normal $+6+9+18+4+11=+48$. In the Σ columns of the normal $+48-3.1=+44.9$.

† η is the constant term of the condition equation.

Normal equations

	1	2	3	4	5	η	Σ	C's*
1	+12		+ 6	+ 4	+ 2	+0.0	+24	−0.19700
2		+12	+ 9		+ 9	+0.8	+30.8	−0.19531
3			+18	+ 4	+11	−3.1	+44.9	+0.71069
4				+10		+0.0	+18	−0.20547
5					+17	+3.5	+42.5	−0.53917

* These values result from the computation on p. 13.

DISCUSSION OF METHOD OF SOLUTION OF NORMAL EQUATIONS

In the normal equations the coefficients in each equation occurring before the diagonal term are omitted, as the equations are symmetrical with regard to the diagonal line. The set just given when written in full is as follows:

	1	2	3	4	5	η	Σ
1	+12		+ 6	+ 4	+ 2	+0.0	+24
2		+12	+ 9		+ 9	+0.8	+30.8
3	+ 6	+ 9	+18	+ 4	+11	−3.1	+44.9
4	+ 4		+ 4	+10		+0.0	+18
5	+ 2	+ 9	+11		+17	+3.5	+42.5

It can be seen that the coefficients may be omitted to the left of the diagonal line and each equation may be read from the top down to the diagonal term and then across the page.

The Doolittle method of solution is used. Equation No. 1 is copied and then divided by the diagonal term (+12 in this case), the signs being changed. Since No. 2 does not occur on No. 1, this also is divided at once by the diagonal term with a change of sign. No. 3 has +6 on No. 1 and +9 on No. 2; accordingly, the divided coefficients of No. 1 are multiplied by +6 and those of No. 2 by +9 and these give the two products on No. 3. These are then added algebraically and divided by the diagonal term with change of sign to give C_3 in terms of No. 4 and No. 5 plus a constant term. In a similar manner No. 4 and No. 5 are eliminated, the division on No. 5 giving the value of C_5. The back solution is then carried through $C_4 = +0.17778 C_5 - 0.10962$. When the value of C_5 is substituted, $C_4 = -0.09585 - 0.10962 = -0.20547$. So also for the remaining C's. For an explanation of the omission of the terms before the diagonal term, see page 22. For a full discussion of the Doolittle method of solution, see Wright & Hayford, Adjustment of Observations, page 114 et seq.

Solution of normal equations

1	2	3	4	5	η	Σ
+12 C_1		+ 6 − 0.5	+ 4 − 0.33333	+ 2 − 0.16667	+0.0 +0.0	+ 24 − 2
	+12 C_2	+ 9 − 0.75		+ 9 − 0.75	+0.8 −0.06667	+30.8 − 2.56667
	1 2	+18 − 3 − 6.75	+ 4 − 2	+11 − 1 − 6.75	−3.1 −0.6	+44.9 −12 −23.1
		+ 8.25 C_3	+ 2 − 0.24242	+ 3.25 − 0.39394	−3.7 +0.44848	+ 9.8 − 1.18788
		1 3	+10 − 1.3333 − 0.4848	− 0.6667 − 0.7879	+0.0 +0.8969	+18 − 8 − 2.3758
			+ 8.1819 C_4	− 1.4546 + 0.17778	+0.8969 −0.10962	+ 7.6242 − 0.93184
			1 2 3 4	+17 − 0.3333 − 6.75 − 1.2803 − 0.2586	+3.5 −0.6 +1.4576 +0.1595	+42.5 − 4 −23.1 − 3.8606 + 1.3554
				+ 8.3778 C_5	+4.5171 −0.53917	+12.8949 − 1.53917

Back solution

5	4	3	2	1
−0.53917	−0.10962 −0.09585	+0.44848 +0.21240 +0.04981	−0.00667 +0.40438 −0.53302	+0.08986 +0.06849 −0.35535
−0.53917	−0.20547	+0.71069	−0.19531	−0.19700

Computation of corrections

1	2	3	4	5
−0.197 +0.711 −0.205	−0.197 +0.711 −0.205	−0.197 +0.711 −0.539	−0.197	+0.195
+0.309 2 +0.618	+0.309 2 +0.618	−0.025 2 −0.050	−0.197 6 −1.182	+0.195 3 +0.585
6	**7**	**8**	**9**	**10**
+0.711	+0.205	+0.539	−0.195 +0.711 −0.539	−0.195 +0.711 −0.539
+0.711 3 +2.133	+0.205 6 +1.230	+0.539 6 +3.234	−0.023 3 −0.069	−0.023 6 −0.138

ADJUSTMENT OF A QUADRILATERAL

GENERAL STATEMENT

After the local conditions—that is, those arising from the relations of the angles to one another at each station—are satisfied there are general conditions arising from the geometrical relations necessary to form a closed figure which must be satisfied. To illustrate this, let the case of a quadrilateral be taken. The angles of each triangle should sum up to 180° plus the spherical excess of the triangle. Except in rare cases this does not happen with the observed angles; therefore condition equations are needed to bring it about. There are four triangles in a quadrilateral, but if three of them close the other will also close. There will then be three angle equations in the quadrilateral. A fourth equation must be included to insure that the lines at the pole will pass through the same point. When this condition is satisfied, and the triangles are closed, the same values will be obtained for the various sides when the computation is carried through different triangles.

In the adjustment of triangulation in the United States Coast and Geodetic Survey the method of directions is used; that is, an angle is considered as the difference of two directions.* If v_1 is the correction to the first direction in order of azimuth at a given station and v_2 the correction to the second direction, the correction to the angle will be $-v_1+v_2$, or the algebraic difference of the v's applying to the directions. To avoid the use of so many v's, the custom is to write (1) instead of v_1; thus the angle given above will have the correction symbol $-(1)+(2)$, in which 1 and 2 are not quantities but the subscripts of the corresponding v's.

An angle equation simply states that the sum of the corrections to the angles of a given triangle is to equal the failure in the closure of the triangle. In the triangle $A_3A_2A_1$ (see fig. 1 on p. 16) the angle at A_2 is to be corrected by $-(1)+(2)$, the angle at A_1 by $-(4)+(6)$, and the angle at A_3 by $-(8)+(9)$. The sum of the angles needs to be increased by 2″.3 to make up the sum of 180° plus the spherical excess. (See triangle on p. 23.) Therefore $-(1)+(2)-(4)+(6)-(8)+(9)=+2.3$, or, as it is usually written, $0=-2.3-(1)+(2)-(4)+(6)-(8)+(9)$.

Three angle equations in a quadrilateral will bring about the closure of the four triangles, but it is possible to have all of the triangles close and still the sides fail to check when computed through different triangles. To make the computation of lengths consistent a side equation must be added to the three angle equations. In figure 1 on page 16 the sides can be made consistent in the following manner:

* See lists of directions on p. 16.

In the triangle $A_1A_4A_2$

$$\frac{\text{side } A_2A_4}{\text{side } A_1A_4}=\frac{\text{sine angle } A_1}{\text{sine angle } A_2}=\frac{\text{sine } [-(5)+(6)]}{\text{sine } [-(1)+(3)]}$$

in the triangle $A_1A_3A_4$

$$\frac{\text{side } A_1A_4}{\text{side } A_3A_4}=\frac{\text{sine angle } A_3}{\text{sine angle } A_1}=\frac{\text{sine } [-(7)+(9)]}{\text{sine } [-(4)+(5)]}$$

and in the triangle $A_2A_3A_4$

$$\frac{\text{side } A_3A_4}{\text{side } A_2A_4}=\frac{\text{sine angle } A_2}{\text{sine angle } A_3}=\frac{\text{sine } [-(2)+(3)]}{\text{sine } [-(7)+(8)]}$$

If the sides are consistent, the product of these three equations gives

$$\frac{\text{sine } [-(5)+(6)] \text{ sine } [-(7)+(9)] \text{ sine } [-(2)+(3)]}{\text{sine } [-(1)+(3)] \text{ sine } [-(4)+(5)] \text{ sine } [-(7)+(8)]}=1$$

In a spherical triangle the same equation is obtained by using the sine of the side in place of the side. In the end the equation given above results, since the sines of the sides cancel out as did the sides above.

Passing to logarithms, we have

$$\log \text{sine } [-(5)+(6)]+\log \text{sine } [-(7)+(9)]+\log \text{sine } [-(2)+(3)]-\log \text{sine } [-(1)+(3)]$$
$$-\log \text{sine } [-(4)+(5)]-\log \text{sine } [-(7)+(8)]=0$$

As this will not be exactly true when the observed angles or angles adjusted only for closing errors of the triangles are used except in rare cases, a condition equation must be formed to accomplish this result. From the table of logarithms we find the amount of change of the log sine of the given angle for 1″ change in the angle, and this multiplied by the v's applying to the angle will give the change in the log sine of that angle. It is customary to consider the log sines in six places of decimals, hence the change in the log sine for 1″ will be taken as units in the sixth place of decimals.

	°	′	″	
log sine [−(5)+(6)]	20	50	56.7	=9.5513374−5.53(5)+5.53(6)
log sine [−(7)+(9)]	61	47	35.0	=9.9450972−1.13(7)+1.13(9)
log sine [−(2)+(3)]	32	09	01.2	=9.7260280−3.35(2)+3.35(3)
Total				=9.2224626−3.35(2)+3.35(3)−5.53(5)+5.53(6) −1.13(7)+1.13(9)

	°	′	″	
log sine [−(1)+(3)]	133	53	46.3	=9.8576926+2.03(1)−2.03(3)
log sine [−(4)+(5)]	26	40	23.5	=9.6521506−4.19(4)+4.19(5)
log sine [−(7)+(8)]	31	03	42.5	=9.7126180−3.50(7)+3.50(8)
Total				=9.2224612+2.03(1)−2.03(3)−4.19(4)+4.19(5) −3.50(7)+3.50(8)

9.2224626 −3.35(2)+3.35(3) −5.53(5)+5.53(6)−1.13(7) +1.13(9)

9.2224612+2.03(1) −2.03(3)−4.19(4)+4.19(5) −3.50(7)+3.50(8)

0=+1.4−2.03(1)−3.35(2)+5.38(3)+4.19(4)−9.72(5)+5.53(6)+2.37(7)−3.50(8) +1.13(9)

(See tabulated form of this equation on p. 17.)

This condition requires the lines from A_2, A_1, and A_3 to pass through the same point at A_4. If v's are found that satisfy this equation, at the same time satisfying the three angle equations given on page 17, they will render the quadrilateral consistent in all respects.

In a full quadrilateral (see figure 1) there are four conditions. These can be put in as three angle equations and one side equation, or two angle and two side, or one angle and three side equations. (See article by C. A. Schott, Appendix No. 17, United States Coast and Geodetic Survey Report of 1875, p. 280.) To illustrate the fact, a quadrilateral is adjusted using two angle equations and two side equations. (See p. 20.) In order to hold the closure of the triangles, the logarithms in the side equations must be found at least to seven places to hold the closure to tenths of seconds. Of course this method would never be used in practice, as the side equations require much more work, but the fact is interesting as an illustration of what can be done in the method of adjustment. Four side equations or four angle equations could not be used, for the fourth is functionally related to the other three, and hence they would not be independent conditions.

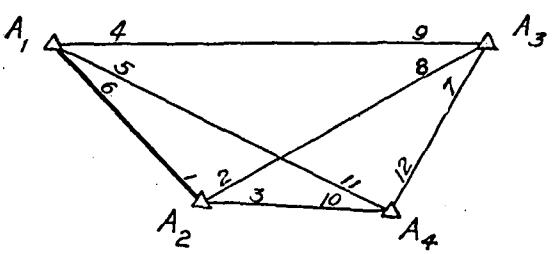

FIG. 1.

In a set of equations, if an identical one is included, the diagonal term of the reduced normal will become zero with the possible exception, of course, of a few units in the last place of solution due to accumulations. In any case, if the reduced diagonal term falls below unity, there may be danger of instability, since in this case any accumulations in the last place of the solution are increased when the normal is divided by this term.

Lists of directions

Stations observed	Directions after local adjustment	Final seconds*	Stations observed	Directions after local adjustment	Final seconds*
STATION A_1	° ′ ″	—	STATION A_3	° ′ ″	″
A_3	0 00 00.0	59.8	A_4	0 00 00.0	00.7
A_4	26 40 23.5	23.5	A_2	31 03 42.5	42.0
A_2	47 31 20.2	20.5	A_1	61 47 35.0	34.8
STATION A_2		—	STATION A_4		
A_1	0 00 00.0	59.5	A_2	0 00 00.0	00.1
A_3	101 44 45.1	46.1	A_1	25 15 16.2	16.9
A_4	133 53 46.3	45.8	A_3	116 47 20.0	19.2

* These values result from the following computation.

Angle equations *

$$0=-2.3-(1)+(2)-(4)+(6)-(8)+(9).$$
$$0=+3.6-(2)+(3)-(7)+(8)-(10)+(12).$$
$$0=+2.2-(4)+(5)-(7)+(9)-(11)+(12).$$

Side equation

Symbol	Angle	Logarithm	Tabular difference	Symbol	Angle	Logarithm	Tabular difference
	° ′ ″				° ′ ″		
−7+9	61 47 35.0	9.9450972	+1.13	−4+5	26 40 23.5	9.6521506	+4.19
−5+6	20 50 56.7	9.5513374	+5.53	−1+3	133 53 46.3	9.8576926	−2.03
−2+3	32 09 01.2	9.7260280	+3.35	−7+8	31 03 42.5	9.7126180	+3.50
		9.2224626				9.2224612	

$$0=+1.4-2.03(1)-3.35(2)+5.38(3)+4.19(4)-9.72(5)+5.53(6)+2.37(7)-3.50(8)+1.13(9).$$

FORMATION OF NORMAL EQUATIONS BY DIFFERENTIATION

The function u to be rendered a minimum is the sum of the squares of the v's, subject to the four given conditions.

$$u=v_1^2+v_2^2+v_3^2+v_4^2+v_5^2+v_6^2+v_7^2+v_8^2+v_9^2+v_{10}^2+v_{11}^2+v_{12}^2-2C_1(-2.3-v_1+v_2-v_4+v_6-v_8+v_9)-2C_2(+3.6-v_2+v_3-v_7+v_8-v_{10}+v_{12})-2C_3(+2.2-v_4+v_5-v_7+v_9-v_{11}+v_{12})-2C_4(+1.4-2.03\,v_1-3.35\,v_2+5.38\,v_3+4.19\,v_4-9.72\,v_5+5.53\,v_6+2.37\,v_7-3.50\,v_8+1.13\,v_9)$$

Differentiating with respect to the v's in succession and equating to zero, there result after transposition the following equations:

$$v_1=-C_1-2.03\,C_4$$
$$v_2=+C_1-C_2-3.35\,C_4$$
$$v_3=+C_2+5.38\,C_4$$
$$v_4=-C_1-C_3+4.19\,C_4$$
$$v_5=+C_3-9.72\,C_4$$
$$v_6=+C_1+5.53\,C_4$$
$$v_7=-C_2-C_3+2.37\,C_4$$
$$v_8=-C_1+C_2-3.50\,C_4$$
$$v_9=+C_1+C_3+1.13\,C_4$$
$$v_{10}=-C_2$$
$$v_{11}=-C_3$$
$$v_{12}=+C_2+C_3$$

By the substitution of these values in the four condition equations the following normal equations result:

$$+6\,C_1-2\,C_2+2\,C_3+4.65\,C_4-2.3=0$$
$$-2\,C_1+6\,C_2+2\,C_3+2.86\,C_4+3.6=0$$
$$+2\,C_1+2\,C_2+6\,C_3-15.15\,C_4+2.2=0$$
$$+4.65\,C_1+2.86\,C_2-15.15\,C_3+206.0470\,C_4+1.4=0$$

These normal equations are formed most easily by means of the tabular form of the correlate equations given on page 18.

* For triangles see p. 23.

The sum of the squares of each column gives the diagonal term in that equation in the normals. All coefficients before the diagonal term are omitted; each equation is read by starting at the top of the tabular form below, reading down the column to the diagonal term, and then along the horizontal line. Compare the full normals given above with the tabular form below. After the diagonal terms are determined column No. 1 in the correlates is multiplied by column No. 2 and the algebraic sum of the products taken for the coefficient of normal No. 1 on No. 2; this is also the coefficient of No. 2 on No. 1. Column No. 1 times No. 3, with the algebraic sum of the products, gives the coefficient of No. 1 on No. 3 in the normals; also No. 3 on No. 1. Finally, the algebraic sum of the products of column No. 1 by column No. 4 gives the coefficient of normal No. 1 on No. 4. The algebraic sum of the products of column No. 1 by the Σ column should check the algebraic sum of the coefficients of normal No. 1. To this should be added algebraically the constant term of normal No. 1 and the sum placed in the Σ column of normal No. 1. (See the table of normals below.)

In the same way the sum of the products of column No. 2 times column No. 3 is determined for the second normal, and by continuing the process all of the normals are formed.

After the C's are determined by the solution of the normals the v's are most conveniently computed by multiplying column No. 1 in the correlates by C_1, column No. 2 by C_2, column No. 3 by C_3, and column No. 4 by C_4. Then the algebraic sum of line No. 1 gives v_1; of No. 2, v_2, etc. (See the computation of the v's on p. 19.)

Correlate equations

	1	2	3	4	Σ	v's *	Adopted v's	v^2
1	−1			−2.03	−3.03	−0.503	−0.5	0.25
2	+1	−1		−3.35	−3.35	+1.004	+1.0	1.00
3		+1		+5.38	+6.38	−0.501	−0.5	0.25
4	−1		−1	+4.19	+2.19	−0.227	−0.2	0.04
5			+1	−9.72	−8.72	−0.015	−0.0	0.00
6	+1			+5.53	+6.53	+0.242	+0.3	0.09
7		−1	−1	+2.37	+0.37	+0.663	+0.7	0.49
8	−1	+1		−3.50	−3.50	−0.493	−0.5	0.25
9	+1		+1	+1.13	+3.13	−0.170	−0.2	0.04
10		−1			−1	+0.099	+0.1	0.01
11			−1		−1	+0.740	+0.7	0.49
12		+1	+1		+2	−0.840	−0.8	0.64
							Σv^2	3.55

Normal equations

1	2	3	4	η	Σ	C's *
+6	−2	+2	+ 4.65	−2.3	+ 8.35	+0.6547
	+6	+2	+ 2.86	+3.6	+ 12.46	−0.0994
		+6	− 15.15	+2.2	− 2.95	−0.7401
			+206.0470	+1.4	+199.8070	−0.07461

* These values result from the computation on p. 19.

$$\text{Probable error of an observed direction} = \pm 0.6745\sqrt{\frac{3.55}{4}} = \pm 0.6.$$

Solution of normal equations

1	2	3	4	η	ζ
+6	−2	+2	+ 4.65	−2.3	+ 8.35
C_1	+0.33333	−0.33333	− 0.775	+0.38333	− 1.39167
	+6	+2	+ 2.86	+3.6	+ 12.46
1	−0.6667	+0.6667	+ 1.55	−0.7667	+ 2.7833
	+5.3333	+2.6667	+ 4.41	+2.8333	+ 15.2433
	C_2	−0.50001	− 0.82688	−0.53125	− 2.85814
		+6	− 15.15	+2.2	− 2.95
	1	−0.6667	− 1.55	+0.7667	− 2.7833
	2	−1.3333	− 2.205	−1.4167	− 7.6217
		+4	− 18.905	+1.55	− 13.355
		C_3	+ 4.72625	−0.3875	+ 3.33875
			+206.0470	+1.4	+199.8070
		1	− 3.6038	+1.7825	− 6.4712
		2	− 3.6465	−2.3428	− 12.6044
		3	− 89.3498	+7.3257	− 63.1191
			+109.4469	+8.1654	+117.6123
			C_4	−0.07461	− 1.07461

Back solution

4	3	2	1
−0.07461	−0.3875	−0.5312	+0.3833
	−0.3526	+0.0617	+0.0578
−0.07461		+0.3701	+0.2467
	−0.7401		−0.0331
		−0.0994	
			+0.6547

Computation of corrections

1	2	3	4	5	6
−0.6547	+0.6547	−0.0994	−0.6547	−0.7401	+0.6547
+0.1515	+0.0994	−0.4014	+0.7401	+0.7252	−0.4126
	+0.2499		−0.3126		
−0.5032		−0.5008		−0.0149	+0.2421
−0.5	+1.0040	−0.5	−0.2272	−0.0	+0.3
	+1.0		−0.2		

7	8	9	10	11	12
+0.0994	−0.6547	+0.6547	+0.0994	+0.7401	−0.0994
+0.7401	−0.0994	−0.7401			−0.7401
−0.1768	+0.2611	−0.0843	+0.0994	+0.7401	
			+0.1	+0.7	−0.8295
+0.6627	−0.4930	−0.1697			−0.8
+0.7	−0.5	−0.2			

ADJUSTMENT OF A QUADRILATERAL BY THE USE OF TWO ANGLE AND TWO SIDE EQUATIONS *

(See fig. 1 on p. 16.)

Angle equations

$$0=-2.3-(1)+(2)-(4)+(6)-(8)+(9)$$
$$0=+3.6-(2)+(3)-(7)+(8)-(10)+(12)$$

Side equations

Symbol	Angle	Logarithm	Tabular difference	Symbol	Angle	Logarithm	Tabular difference
−7+9	61 47 35.0	9.9450972	+1.13	−4+5	26 40 23.5	9.6521506	+4.19
−5+6	20 50 56.7	9.5513374	+5.53	−1+3	133 53 46.3	9.8576926	−2.03
−2+3	32 09 01.2	9.7260280	+3.35	−7+8	31 03 42.5	9.7126180	+3.50
		9.2224626				9.2224612	

$$0=+1.4-2.03(1)-3.35(2)+5.38(3)+4.19(4)-9.72(5)+5.53(6)+2.37(7)-3.50(8)+1.13(9)$$

− 2+ 3	32 09 01.2	9.7260280	+3.35	− 7+ 8	31 03 42.5	9.7126180	+3.50
−11+12	91 32 03.8	9.9998442	−0.06	− 4+ 5	26 40 23.5	9.6521506	+4.19
− 8+ 9	30 43 52.5	9.7084309	+3.54	− 1+ 2	101 44 45.1	9.9908094	−0.44
− 5+ 6	20 50 56.7	9.5513374	+5.53	−10+11	25 15 16.2	9.6300613	+4.46
		8.9856405				8.9856393	

$$0=+1.2-0.44(1)-2.91(2)+3.35(3)+4.19(4)-9.72(5)+5.53(6)+3.50(7)-7.04(8)+3.54(9)+4.46(10)-4.40(11)-0.06(12)$$

Correlate equations

	1	2	3	4	Σ	v's†	Adopted v's
1	−1		−2.03	−0.44	− 3.47	−0.495	−0.5
2	+1	−1	−3.35	−2.91	− 6.26	+0.996	+1.0
3		+1	+5.38	+3.35	+ 9.73	−0.502	−0.5
4	−1		+4.19	+4.19	+ 7.38	−0.227	−0.2
5			−9.72	−9.72	−19.44	−0.013	−0.0
6	+1		+5.53	+5.53	+12.06	+0.240	+0.3
7		−1	+2.37	+3.50	+ 4.87	+0.659	+0.7
8	−1	+1	−3.50	−7.04	−10.54	−0.500	−0.5
9	+1		+1.13	+3.54	+ 5.67	−0.159	−0.2
10		−1		+4.46	+ 3.46	+0.113	+0.1
11				−4.40	− 4.40	+0.717	+0.7
12		+1		−0.06	+ 0.94	−0.830	−0.8

Normal equations

	1	2	3	4	η	Σ	C's*
1	+6	−2	+ 4.65	+ 9.45	−2.3	+ 15.80	+0.2328
2		+6	+ 2.86	− 8.80	+3.6	+ 1.66	−0.8398
3			+206.0470	+208.2153	+1.4	+423.1723	+0.16435
4				+276.0980	+1.2	+486.1633	−0.16302

* For triangles, see p. 23

† These values result from the computation on p. 21.

Solution of normal equations

1	2	3	4	η	Σ
+6 C_1	−2 +0.33333	+ 4.65 − 0.775	+ 9.45 − 1.575	−2.3 +0.38333	+ 15.80 − 2.63333
1	+6 −0.6667	+ 2.86 + 1.55	− 8.80 + 3.15	+3.6 −0.7667	+ 1.66 + 5.2667
	+5.3333 C_2	+ 4.41 − 0.82688	− 5.65 + 1.05938	+2.8333 −0.53125	+ 6.9266 − 1.29875
	1 2	+206.0470 − 3.6038 − 3.6465	+208.2153 − 7.3238 + 4.6719	+1.4 +1.7825 −2.3428	+423.1723 − 12.2450 − 5.7275
		+198.7967 C_3	+205.5634 − 1.034038	+0.8397 −0.004224	+405.1998 − 2.038262
		1 2 3	+276.0980 − 14.8838 − 5.9855 −212.5604	+1.2 +3.6225 +3.0015 −0.8683	+486.1633 − 24.8850 + 7.3379 −418.9920
			+ 42.6683 C_4	+6.9557 −0.16302	+ 49.6240 − 1.16302

Back solution

4	3	2	1
−0.16302	−0.00422 +0.16857	−0.5312 −0.1727 −0.1359	+0.3833 +0.2568 −0.1274 −0.2799
−0.16302	+0.16435	−0.8398	+0.2328

Computation of corrections

1	2	3	4	5	6
−0.2328 −0.3336 +0.0717	+0.2328 +0.8398 −0.5506 +0.4744	−0.8398 +0.8842 −0.5461	−0.2328 +0.6886 −0.6831	−1.5975 +1.5846 −0.0129 −0.0	+0.2328 +0.9089 −0.9015
−0.4947 −0.5	+0.9964 +1.0	−0.5017 −0.5	−0.2273 −0.2		+0.2403 +0.3
7	**8**	**9**	**10**	**11**	**12**
+0.8398 +0.3895 −0.5706	−0.2328 −0.8398 −0.5752 +1.1477	+0.2328 +0.1857 −0.5771	+0.8398 −0.7271 +0.1127 +0.1	+0.7173 +0.7173 +0.7	−0.8398 +0.0098 −0.8300 −0.8
+0.6587 +0.7	−0.5001 −0.5	−0.1586 −0.2			

SOLUTION OF A SET OF NORMALS INCLUDING TERMS USUALLY OMITTED

A set of four normal equations is solved below with inclusion of the terms omitted in the Doolittle method of solution.

Solution of normals

1	2	3	4	η	Σ
+6	−2	+ 2	+ 4.65	−2.3	+ 8.35
−1 C_1	+0.33333	− 0.33333	− 0.775	+0.38333	− 1.39167
−2	+6	+ 2	+ 2.86	+3.6	+ 12.46
+2 (1)	−0.6667	+ 0.6667	+ 1.55	−0.7667	+ 2.7833
	+5.3333	+ 2.6667	+ 4.41	+2.8333	+ 15.2433
	−1 C_2	− 0.50001	− 0.82688	−0.53125	− 2.85814
+2	+2	+ 6	− 15.15	+2.2	− 2.95
−2	+0.6667 (1)	− 0.6667	− 1.55	+0.7667	− 2.7833
	−2.6667 (2)	− 1.3333	− 2.205	−1.4167	− 7.6217
		+ 4	− 18.905	+1.55	− 13.355
		− 1 C_3	+ 4.72625	−0.3875	+ 3.33875
+4.65	+2.86	−15.15	+206.0470	+1.4	+199.8070
−4.65	+1.55	− 1.55 (1)	− 3.6038	+1.7825	− 6.4712
	−4.41	− 2.205 (2)	− 3.6465	−2.3428	− 12.6044
		+18.905 (3)	− 89.3498	+7.3257	− 63.1191
			+109.4469	+8.1654	+117.6123
			− 1 C_4	−0.07461	− 1.07461

DISCUSSION OF THE SOLUTION

The quantities in heavy type are the ones omitted in the Doolittle method of solution of normal equations. They sum up to zero with the possible variation of a few units in the last place of the solution. This shows that the method is one of curtailed substitution. It can also be seen that the quantity in the Σ column is the direct sum of all the quantities in each horizontal line including those in heavy type. All of the quantities in heavy type occur in the regular solution. This is of value in the control of the solution. If an equation fails to check the Σ column after it is added up, the error can generally be located by adding back through noting that the coefficient is changed in sign because it is multiplied by -1. Note the product of equation No. 1 on No. 4; -1.55 and $+1.55$ are the products of No. 1 on No. 3 and No. 2, respectively; -4.65 is the coefficient of No. 4 on No. 1 with sign changed. The method is the same in all cases. Care should be taken with such coefficients as No. 2 and No. 3 on No. 1. They have the same value with opposite sign. If a mistake should be made on them the Σ column control would not catch it. Care should be taken not to make a mistake in the η column and a compensating one in the Σ column. There is most danger of this in the addition. The control would not catch this and it would take much labor to correct it later.

After each equation is added, it should be added horizontally to check the Σ column. If the check fails an error has been made and it must be found before proceeding. A slight variation in the last place of the solution is of course unavoidable. After the division of each equation by the reduced diagonal term, a horizontal addition should be made (including, of course, -1) to check the correctness of the division. No time is ever lost in using care in the solution of the equations. It takes so much time and labor to rectify a mistake later that every means should be employed to detect and correct it in the solution. The larger the set, the more important it is to be on guard against errors. It is possible to carry a set through with almost absolute assurance that the solution is correct.

If, in a given equation, the solution fails to check and the check of adding back through is satisfied, a mistake has been made somewhere in the solution columns and a compensating mistake in the Σ column. This can be caught by building up the omitted columns to the left of the given equation. They should each sum up to zero. If any one does not, the mistake in addition has been made in that equation in the column of the one being eliminated.

Solution of triangles *

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
	A_2-A_1	° ′ ″	″	″	″	″	3.772745
− 8+ 9	A_3	30 43 52.5	+0.3	52.8	0.0	52.8	0.291568
− 1+ 2	A_2	101 44 45.1	+1.5	46.6	0.1	46.5	9.990809
− 4+ 6	A_1	47 31 20.2	+0.5	20.7	0.0	20.7	9.867787
			+2.3		0.1		
	A_3-A_1						4.055122
	A_3-A_2						3.932100
	A_2-A_1						3.772745
−10+11	A_4	25 15 16.2	+0.6	16.8	0.0	16.8	0.369936
− 1+ 3	A_2	133 53 46.3	+0.0	46.3	0.1	46.2	9.857693
− 5+ 6	A_1	20 50 56.7	+0.3	57.0	0.0	57.0	9.551339
			+0.9		0.1		
	A_4-A_1						4.000374
	A_4-A_2						3.694020
	A_2-A_3						3.932100
−10+12	A_4	116 47 20.0	−0.9	19.1	0.1	19.0	0.049306
− 2+ 3	A_2	32 08 61.2	−1.5	59.7	0.0	59.7	9.726023
− 7+ 8	A_3	31 03 42.5	−1.2	41.3	0.0	41.3	9.712614
			−3.6		0.1		
	A_4-A_3						3.707429
	A_4-A_2						3.694020
	A_1-A_3						4.055122
−11+12	A_4	91 32 03.8	−1.5	02.3	0.1	02.2	0.000156
− 4+ 5	A_1	26 40 23.5	+0.2	23.7	0.0	23.7	9.652151
− 7+ 9	A_3	61 47 35.0	−0.9	34.1	0.0	34.1	9.945096
			−2.2		0.1		
	A_4-A_3						3.707429
	A_4-A_1						4.000374

* For the method of solution of triangles see United States Coast and Geodetic Survey Special Publication No. 8, p. 6.

Position computation,

STATION A_2

								°	′	″
α	A_2 to A_1							156	20	26.6
Second angle	A_1 and A_2							+101	44	46.6
α	A_2 to A_3							258	05	13.2
Δα								+	8	05.9
								180	00	00.0
α′	A_3 to A_2							78	13	19.1
				First angle of triangle				30	43	52.8
	°	′	″							
φ	60	56	01.089	A_2		λ		149	34	19.237
Δφ	+		56.720			Δλ		−	9	15.880
φ′	60	56	57.809	A_3		λ′		149	25	03.357
	° ′ ″		s	3.932100	s^2	7.86420				
½(φ+φ′)	60 56 29		cos α	9.314765	sin² α	9.98109	h^2	3.5122		
			B	8.509299	C	1.65750	D	2.3224		
	″		h	1.756164		9.50279		5.8346		
1st term	−57.0380									
2d and 3d terms	+ 0.3184					0.3183				
						0.0001				
−Δφ	−56.7196									

	s	3.932100		
	sin α	9.990514		
	A′	8.508600	Δλ	2.744981
	sec φ′	0.313737	sin ½(φ+φ′)	9.941572
		2.744981		2.686553
		″		″
	Δλ	−555.8800	−Δα	−485.89

STATION A_4

								°	′	″
α	A_2 to A_3							258	05	13.2
Second angle	A_3 and A_4							+ 32	08	59.7
α	A_2 to A_4							290	14	12.9
Δα								+	4	29.0
								180	00	00.0
α′	A_4 to A_2							110	18	41.9
				First angle of triangle				116	47	19.1
	°	′	″							
φ	60	56	01.089	A_2		λ		149	34	19.237
Δφ	−		55.340			Δλ		−	5	07.795
φ′	60	55	05.749	A_4		λ′		149	29	11.442
	° ′ ″		s	3.694020	s^2	7.38804				
½(φ+φ′)	60 55 33		cos α	9.538954	sin² α	9.94466	h^2	3.484		
			B	8.509299	C	1.65750	D	2.322		
	″		h	1.742273		8.99020		5.806		
1st term	+55.2425									
2d and 3d terms	+ 0.0979					0.0978				
−Δφ	+55.3404					1				

	s	3.694020		
	sin α	9.972328		
	A′	8.508601	Δλ	2.488262
	sec φ′	0.313313	sin ½(φ+φ′)	9.941507
		2.488262		2.429769
		″		″
	Δλ	−307.7953	−Δα	−269.01

* For an explanation of the forms for computing differences of latitude, longitude, and azimuth see United States Coast and Geodetic Survey Special Publication No. 8, pp. 6–11.

secondary triangulation *

STATION A_3

										°	′	″
α	A_1 to A_3									336	18	08.4
Third angle	A_3 and A_2									− 47	31	20.7
α	A_1 to A_3									288	46	47.7
Δα										+	10	24.3
α′	A_3 to A_1									180	00	00.0
										108	57	12.0
												−.1
φ	° 60	′ 58	″ 56.416		A_1		λ	149	36	57.360
Δφ	−	1	58.608				Δλ	−	11	54.003
φ′	60	56	57.808		A_3		λ′	149	25	03.357
			+1							
½(φ+φ′)	° ′ ″ 60 57 57		s cos α B	4.055122 9.507767 8.509295	s² sin² α C	8.11024 9.95248 1.65837	h² D	4.1444 2.3221	−h s² sin² α E	2.072 8.063 6.640
	″		h	2.072184		9.72109		6.4665		6.775
1st term	+118.0821									
2d, 3d, and 4th terms	+ 0.5258					0.5261				
						0.0003				
−Δφ	+118.6079					−0.0006				

s	4.055122		
sin α	9.976241		
A′	8.508600	Δλ	2.853700
sec φ′	0.313737	sin ½(φ+φ′)	9.941676
	2.853700		2.795376
	″		″
Δλ	−714.0029	−Δα	−624.27

STATION A_4

										°	′	″
α	A_3 to A_2									78	13	19.1
Third angle	A_4 and A_2									− 31	03	41.3
α	A_3 to A_4									47	09	37.8
Δα										−	3	36.8
α′	A_4 to A_3									180	00	00.0
										227	06	01.0
φ	° 60	′ 56	″ 57.809		A_3		λ	149	25	03.357
Δφ	−	1	52.060				Δλ	+	4	08.086
φ′	60	55	05.749		A_4		λ′	149	29	11.443
										− 1
½(φ+φ′)	° ′ ″ 60 56 02		s cos α B	3.707429 9.832475 8.509298	s² sin² α C	7.41486 9.73052 1.65808	h² D	4.098 2.322		
	″		h	2.049202		8.80346		6.420		
1st term	+111.9959									
2d and 3d terms	+ 0.0639					0.0636				
−Δφ	+112.0598					3				

s	3.707429		
sin α	9.865259		
A′	8.508601	Δλ	2.394602
sec φ′	0.313313	sin ½(φ+φ′)	9.941540
	2.394602		2.336142
	″		″
Δλ	+248.0858	−Δα	+216.84

List of geographic positions, Turnagain Arm, Alaska. Valdez datum

Station	Latitude and longitude	Seconds in meters	Azimuth	Back azimuth	To station	Distance	Logarithm
	° ′ ″		° ′ ″	° ′ ″		*Meters*	
A_3	60 56 57.809	1789.4	78 13 19.1	258 05 13.2	A_2	8552.6	3.932100
	149 25 03.357	50.5	108 57 11.9	288 46 47.7	A_1	11353.3	4.055122
A_4	60 55 05.749	177.9	110 18 41.9	290 14 12.9	A_2	4943.3	3.694020
	149 29 11.442	172.4	135 33 58.7	315 27 11.4	A_1	10008.6	4.000374
			227 06 01.0	47 09 37.8	A_3	5098.3	3.707429

DEVELOPMENT OF CONDITION EQUATIONS FOR LATITUDE AND LONGITUDE CLOSURES

After the conditions arising from the closure of triangles and from the equality of sides or lengths computed by different routes have been satisfied, cases frequently arise where azimuth, latitude, and longitude conditions must be satisfied. There is given now a development of a form of condition equations that will bring about a closure in geographic position.

Discrepancies in latitude and longitude arise whenever a chain of triangulation or a traverse closes on itself. The discrepancies may be distributed throughout the whole loop or in a selected portion of it, depending upon the circumstances. Of course the most rigid adjustment would require the discrepancies to be distributed throughout the whole chain. At times, however, this would require more labor than the importance of the work would justify. Also some parts of the loop may be much better determined than other parts, in which case the more poorly determined part should be required to make up the discrepancies.

The discussion of the form of equations to be employed to effect the closure without discrepancies will be based upon the position computation formulæ employed by the United States Coast and Geodetic Survey. (See United States Coast and Geodetic Survey Special Publication No. 8, p. 8.)

The amount to be distributed being, of course, small compared with the total change in latitude and longitude, the only term of the latitude computation formula that need be considered is the first one. No appreciable changes due to the adjustment will take place in the other terms.

The formation of the equations must always start from a line fixed in length and azimuth. If a scheme of triangulation should start from a fixed line and run to two points which are fixed in position but are not the ends of a single line, then the formation of the equations for each of the two points must start from the fixed line.

There are, of course, two elements that enter into the determination of the position of any point as computed from a known point; these

are the distance from the known point and the azimuth of the line from the known to the unknown point.

In the triangle 1 2 3, let 1 and 2 be fixed in position, and let us consider what change in the position of 3 will be produced by small changes in angles A, B, and C. The length to be carried forward is 1 to 3. Starting with the length 1 to 2, we have log 1 to 3 = log 1 to 2 − log sin B + log sin A. The change in length, then, depends upon the changes in angles A and B and the change in azimuth of the line 1 to 3 depends upon the change in angle C. The angles A and B, therefore, are called the length angles and angle C the azimuth angle.

If we can derive a linear expression for the effect of each of these separately, the total effect will be the sum of the two.

Let δ_A and δ_B represent the change of the log sin for a change of one second in the angles A and B; v_A and v_B the number of seconds change in angles A and B, respectively. Then the change in log sin A will equal $\delta_A(v_A)$, and the change in log sin B will equal $\delta_B(v_B)$; therefore, the change in log 1 to 3 is $+\delta_A(v_A)-\delta_B(v_B)$. The change in the logarithm of the first term of the latitude due to the change in length is equal to $+\delta_A(v_A)-\delta_B(v_B)$. This is the change in the logarithm, but for convenience of computation it is better to determine what change in the antilogarithm will be produced by this change; or, in other words, to determine what this logarithmic change will amount to in seconds of arc. From the nature of logarithms, if we know the number to which a given logarithm corresponds, the change in the number due to any *small* change in the logarithm can be found by multiplying the logarithm change by the number and dividing by M (the modulus of the common system of logarithms). This can also be shown by differentiation.

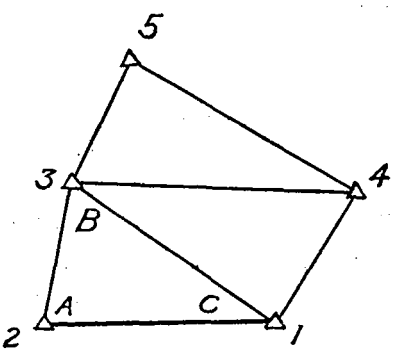

FIG. 2.

Let $y=\log_{10}x$

$$dy=M\frac{dx}{x}$$

Therefore $dx=\frac{x}{M}dy$, dy being the small change in the log and dx the corresponding small change in the number.

$+\delta_A(v_A)-\delta_B(v_B)$ must then be multiplied by $(\phi_B-\phi_C)$, (in which ϕ_B is the computed latitude of 3 and ϕ_C is the latitude of 1), and the product divided by M; this will give the change in seconds in the latitude of 3 due to the change in length of 1 to 3.

Next must be considered the change in latitude due to the change in the azimuth angle C. If s is the length in meters of 1 to 3, the length of the small arc through which 3 turns is equal to $s(v_c)$ arc $1''$ (as $ds = rd\theta$ for a circle about the origin in polar coordinates), v_c equals the number of seconds change in angle C, and arc $1''$ is included to reduce this angle to circular measure.

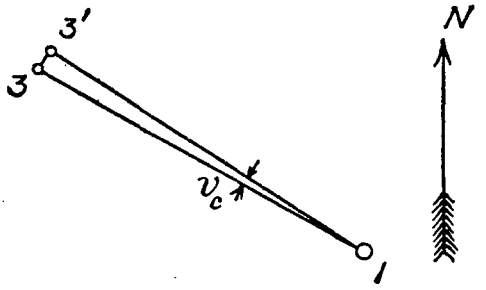

FIG. 3.

Let 3 be the original position of 3 and 3′ the position due to a small rotation of 1 to 3 about 1.

$$3 \text{ to } 3' = sv_c \text{ arc } 1''$$

The azimuth of 3 to 3′ is $90° + \alpha$. The change in latitude due to $s(v_c)$ arc $1''$ is equal to $-s(v_c)$ arc $1''$ cos $(90° + \alpha)$ times the B factor in the position computation,

$$= +s(v_c) \text{ arc } 1'' \sin \alpha B$$

$$= +(v_c) \text{ arc } 1'' Bs \sin \alpha$$

But $\lambda_B - \lambda_C = s \sin \alpha A' \sec \phi'$

Therefore $s \sin \alpha = \dfrac{\lambda_B - \lambda_C}{A' \sec \phi'}$

Therefore the change in latitude $= \dfrac{B \text{ arc } 1''}{A' \sec \phi'} (\lambda_B - \lambda_C)(v_c)$.

In a similar way the change in longitude due to a change in length is,

$$\frac{\lambda_B - \lambda_C}{M}\left[+\delta_A(v_A) - \delta_B(v_B)\right]$$

and the change in longitude due to the change in azimuth is,

$$s(v_c) \text{arc } 1'' \sin (90° + \alpha) A' \sec \phi' = +(v_c) \text{ arc } 1'' s \cos \alpha A' \sec \phi'$$

$$-s \cos \alpha \cdot B = \phi_B - \phi_C \text{ (neglecting the small terms)}$$

$$s \cos \alpha = -\frac{\phi_B - \phi_C}{B}$$

Therefore change in longitude $= -\dfrac{A' \sec \phi' \text{ arc } 1''}{B} (\phi_B - \phi_C)(v_c)$.

The usage is to point off the log change for one second of arc as an integer in the sixth place of logarithms; therefore as a number the tabular difference $= \dfrac{\delta_A}{10^6}$ and $\dfrac{\delta_B}{10^6}$.

The total change in seconds of latitude in the triangle is

$$\frac{(\phi_B - \phi_C)}{10^6 M}\left[+\delta_A(v_A) - \delta_B(v_B)\right] \pm \frac{B \text{ arc } 1''}{A' \sec \phi'} (\lambda_B - \lambda_C)(v_c).^*$$

The total change in longitude is,

$$\frac{(\lambda_B - \lambda_C)}{10^6 M}\left[+\delta_A(v_A) - \delta_B(v_B)\right] \mp \frac{A' \sec \phi' \text{ arc } 1''}{B} (\phi_B - \phi_C)(v_c).^*$$

* The upper sign being used for a right azimuth angle, the lower sign for a left.

In this way the change could be determined for each triangle in the chain and the sum placed equal to the discrepancy, but this would require a very great amount of work.

If any change takes place in the first triangle while the remaining triangles are for the moment supposed to remain fixed, this length change and azimuth change will affect not only this triangle, but will persist in each succeeding triangle. As a consequence the change of length and azimuth in the first triangle will be felt in the computation of every point after it in the chain. Let ϕ_n and λ_n be the computed ϕ and λ of the end point. The change in the first triangle will apply not merely to $\phi_B - \phi_C$, etc., but to $\phi_n - \phi_C$, etc.

Therefore the change in the final position due to the changes in the first triangle is, for latitude,

$$\frac{(\phi_n - \phi_C)}{10^6\ M}\left[+\delta_A(v_A) - \delta_B(v_B)\right] \pm \frac{B_C \text{ arc } 1''}{A_n \sec \phi_n}(\lambda_n - \lambda_C)\ (v_C),^*$$

and for longitude,

$$\frac{(\lambda_n - \lambda_C)}{10^6\ M}\left[+\delta_A(v_A) - \delta_B(v_B)\right] \mp \frac{A_n \sec \phi_n \text{ arc } 1''}{B_C}(\phi_n - \phi_C)\ (v_C).^*$$

In the same way the change in the final position due to changes in the second triangle can be determined, and so on through the whole chain. Each triangle will have an A, B, and C angle, A being the length angle next to the known side; B, the one opposite the known side; and C, the azimuth angle.

The equations will finally stand

$$0 = +(\phi_n - \phi_{n'}) + \Sigma\left[\frac{(\phi_n - \phi_C)}{10^6\ M}\delta_A(v_A) - \frac{(\phi_n - \phi_C)}{10^6\ M}\delta_B(v_B)\right]$$
$$+\Sigma \pm \frac{B_C \text{ arc } 1''}{A_n \sec \phi_n}(\lambda_n - \lambda_C)\ (v_C).^*$$

$$0 = +(\lambda_n - \lambda_{n'}) + \Sigma\left[\frac{\lambda_n - \lambda_C}{10^6\ M}\delta_A(v_A) - \frac{\lambda_n - \lambda_C}{10^6\ M}\delta_B(v_B)\right]$$
$$+\Sigma \mp \frac{A_n \sec \phi_n \text{ arc } 1''}{B_C}(\phi_n - \phi_C)(v_C).^*$$

ϕ_n is the computed latitude of the final point and $\phi_{n'}$, the fixed latitude; so also for λ_n and $\lambda_{n'}$.

It is exact enough to take $\phi_n - \phi_C$ and $\lambda_n - \lambda_C$ to minutes and tenths of a minute, so that it is advisable to divide the equations by 60 since, as they stand, $\phi_n - \phi_C$, etc., are in seconds. Also it is best to multiply through by $10^6\ M$ to remove this factor from the denominator of the first summation.

---

* Upper sign for right azimuth angle, lower for left.

Then we have

$$0 = +\frac{M}{60}\,10^6(\phi_n - \phi_{n'})'' + \Sigma[(\phi_n - \phi_c)'\ \delta_A(v_A) - (\phi_n - \phi_c)'\ \delta_B(v_B)]$$
$$+\Sigma \pm\ 10^6\ M\frac{B_c \text{ arc } 1''}{A_n \sec \phi_n}\ (\lambda_n - \lambda_c)'\ (v_c).\ *$$

$$0 = +\frac{M}{60}\,10^6\ (\lambda_n - \lambda_{n'})'' + \Sigma[(\lambda_n - \lambda_c)'\ \delta_A(v_A) - (\lambda_n - \lambda_c)'\ \delta_B(v_B)]$$
$$+\Sigma \mp\ 10^6\ M\frac{A_n \sec \phi_n \text{ arc } 1''}{B_c}\ (\phi_n - \phi_c)'(v_c).\ *$$

The A and B factors change so slowly that for any chain they can be taken for the mean ϕ and also sec ϕ_n can be used in the same way. A table can then be prepared for functions designated as a_1 and a_2 and defined as follows:

$$a_1 = +10^6\ M\frac{B \text{ arc } 1''}{A \sec \phi}$$
$$a_2 = -10^6\ M\frac{A \sec \phi \text{ arc } 1''}{B},$$

the A and B factors and the ϕ being used at a convenient interval. A table has been computed for latitudes starting at 24° for intervals of 4° up to 56°. The minus sign is used with a_2 in order that the same sign can be used on the directions of the azimuth angle for both latitude and longitude equations. If the discrepancy to be made up by the adjustment is large, or if the chain extends over a great distance of latitude, it would be best to compute the values of a_1 and a_2 using A_n and ϕ_n and the B for the mean ϕ.

If the chain to be adjusted extends principally east and west, in place of $\phi_n - \phi_c$ a summation of the first terms (h) in the position computations should be used. $\Sigma_n^c h$ would then replace $\phi_n - \phi_c$, the sign being used that would conform to $\phi_n - \phi_c$. These quantities should then be used throughout in forming the equations.

If the latitude and longitude equations are to be included in the main adjustment and the equations all solved simultaneously, the computation of the positions through the chain must be made with one length carried through the figures by means of the observed plane angles; that is, the angles as observed each diminished by $\frac{1}{3}$ of the spherical excess of the triangle. This could be done by carrying the length through a selected chain of triangles and then computing each of the various positions over a single line. Both lines of the triangle could not be used because the observed plane angles must be used in carrying the length and, under ordinary circumstances, the triangle would not be closed. To obviate this difficulty, it is best to use only the observed A and B angles and to conclude the C angle, using, of course, the concluded correction symbols on this

* Upper sign for right azimuth angle, lower for left.

angle. This method gives a much more reliable determination of the discrepancy, as it furnishes a check on each position, and thus prevents a mistake being left in the computation. If the figure adjustment is carried out first, there is no need to follow this method as the triangles would then be closed. In this case it is the general custom of the United States Coast and Geodetic Survey to choose the best chain of triangles and to form the equations through them, using the angle method in place of the direction method. Equations with absolute terms equal to zero must be included for the various triangles in order to hold them closed; also, if a length equation is included in the figure adjustment, it must be retained with zero discrepancy to hold the length. If the figure ends on a fixed line and a length equation is not put in the figure adjustment, the discrepancy must be put on the length equation used with the latitude and longitude equations. After adjustment is made for these final discrepancies the cross lines are computed by two sides and the included angle.

The best results are probably obtained by the solution of all the equations at once, but this entails so much work that the angle method is often used in chains of minor importance.

We have finally:

$$\frac{M}{60}\,10^6 = 7238.24$$

$$0 = +7238.24(\phi_n - \phi_{n'})'' + \Sigma[(\phi_n - \phi_C)'\delta_A(v_A) - (\phi_n - \phi_C)'\delta_B(v_B)] + \Sigma \pm a_1(\lambda_n - \lambda_C)'(v_C).$$

$$0 = +7238.24(\lambda_n - \lambda_{n'})'' + \Sigma[(\lambda_n - \lambda_C)'\delta_A(v_A) - (\lambda_n - \lambda_C)'\delta_B(v_B)] + \Sigma \pm a_2(\phi_n - \phi_C)'(v_C).$$

In the equations v_A, v_B, and v_C would be replaced by their correction symbols, care being taken to use $v_C = -v_A - v_B$, if the azimuth angle has been concluded in carrying the position computation through the chain.

If an azimuth equation occurs, the constant term must be corrected by $+(\lambda_n - \lambda_{n'}) \times$ sine of the mean ϕ, this being the amount that the azimuth will change from the changes in the back azimuths due to the changes in longitude.

It should be noted that whenever a discrepancy of position is adjusted into a section of a loop, an external condition is placed upon the chain, as at best only part of this discrepancy is due to errors in the chain, the rest being due to the remainder of the loop. It is necessary to hold some parts of the triangulation fixed; otherwise when a loop closure is put in it would frequently be necessary to readjust nearly all of the triangulation of the country. The result is, however, that some chains of triangulation, excellent in themselves, get some rather large corrections due to the position closure.

776318°—48——3

EQUATIONS IN A NET

In the adjustment of a quadrilateral, use is made of the two kinds of condition equations that are necessary for the adjustment of any figure that does not contain external conditions such as length, azimuth, or loop closure. In fact most figures can be broken up into successive quadrilaterals. In forming the length equation, use is made of the two length angles in the various triangles passed through. In fig. 5, on page 37, the length angles are lettered *A* and *B*. The angle omitted is the azimuth angle of the given triangle. The log sin of the *A* angle is added to the first length and the log sin of the *B* angle to the final length. So with all of the triangles through which the length is carried. The discrepancy is found and the equation formed in the same way as in the case of an ordinary side equation. See the formation of the length equation on page 37. If the spherical angles are used a correction for arc to sine must be applied to each length. (See the table of these corrections in Special Publication No. 8, p. 17.)

An azimuth equation is formed by adding algebraically to the first azimuth the various azimuth angles up to the second line fixed in azimuth. When passing from one end of a line to another, the azimuth difference due to convergence of the meridians, must be applied as determined in the computation of positions. The algebraic sum of the *v*'s upon these angles must make up the discrepancy between the computed and fixed azimuths. See the computation on page 38.

The determination of the exact number of side and angle equations in a net and the manner in which they come in, is one of the difficulties encountered by a beginner in the adjustment of triangulation. This is especially true if the net is somewhat complicated. The best method for this determination is to plot the figure point by point. By plotting the triangle Tower, Turn, and Dundas, in the figure on page 34 one angle equation is determined. Add Lazaro by the lines Lazaro to Turn and Lazaro to Tower. This gives another angle equation, making two. Another angle equation and a side equation are obtained by putting in the line Lazaro to Dundas. This makes a total of three angle equations and one side equation for the quadrilateral, just as it should be. Next plot Nichols by the lines Nichols to Lazaro and Nichols to Tower; this is a closed triangle and gives a fourth angle equation. Put in Tow Hill by the lines Tow Hill to Nichols and Tow Hill to Lazaro; this does not give an angle equation as it is not a closed triangle. Draw the line Tow Hill to Tower; this gives a second side equation. In this way one can continue through the whole figure. If a full line Nichols to Turn were in the figure, it would give another angle and another side

equation. The angle equation added would have to include the directions on this line as would also the side equation. This method shows at once where the equations come in and what new v's must appear in the equations.

Lines sighted over in only one direction have no effect on the number of angle equations. If the closed part of the figure is plotted, omitting all of the extra lines—that is, putting in each station with only two lines from those already plotted, a closed framework of the figure will be formed. The first triangle requires three lines, those after the first require two lines. The number of angle equations in the framework of the figure is thus equal to the number of lines in the figure minus the number of stations plus one. Every full line added to this framework gives another angle equation. Therefore, the whole number of angle equations in a net is equal to the whole number of full lines minus the number of occupied stations plus one.

The lines sighted over in one direction have the same effect on the number of side equations that the full lines have. If the full framework of the figure is plotted with two lines to each station from those already determined, no side equation will as yet appear in the figure. Every extra line put in gives a side equation. The first triangle fixes three stations; the stations after these require two lines to be used in plotting them. Thus the number of lines needed to plot the framework is equal to twice the number of stations minus three. The full number of side equations will then be equal to the number of all the lines minus twice the number of all the stations plus three.

Let n = total number of lines.

n' = number of lines sighted over in both directions.

S = total number of stations.

S' = number of occupied stations.

Then

The number of angle equations in a net $= n' - S' + 1$.

The number of side equations in a net $= n - 2\,S + 3$.

These formulas should be used to check the number determined by directly plotting the figure.

In figure 4 on page 34,

$$n = 41$$
$$n' = 38$$
$$S = 18$$
$$S' = 17$$

Therefore number of angle equations $= 38 - 17 + 1 = 22$.

number of side equations $= 41 - 36 + 3 = 8$.

For convenience in solution it is best to use triangles with the larger angles for the angle equations, reserving the small angles to be used in the side equations. This will keep the large coefficients

in the side equations from appearing on the same directions as are used in the angle equations and will aid in the solution of the normals. The small angles need to appear in the side equations, as their tabular differences are proportionally much less affected by the dropping of decimal places than are those of the larger angles.

ADJUSTMENT OF A FIGURE WITH LATITUDE, LONGITUDE, AZIMUTH, AND LENGTH CLOSURE CONDITIONS

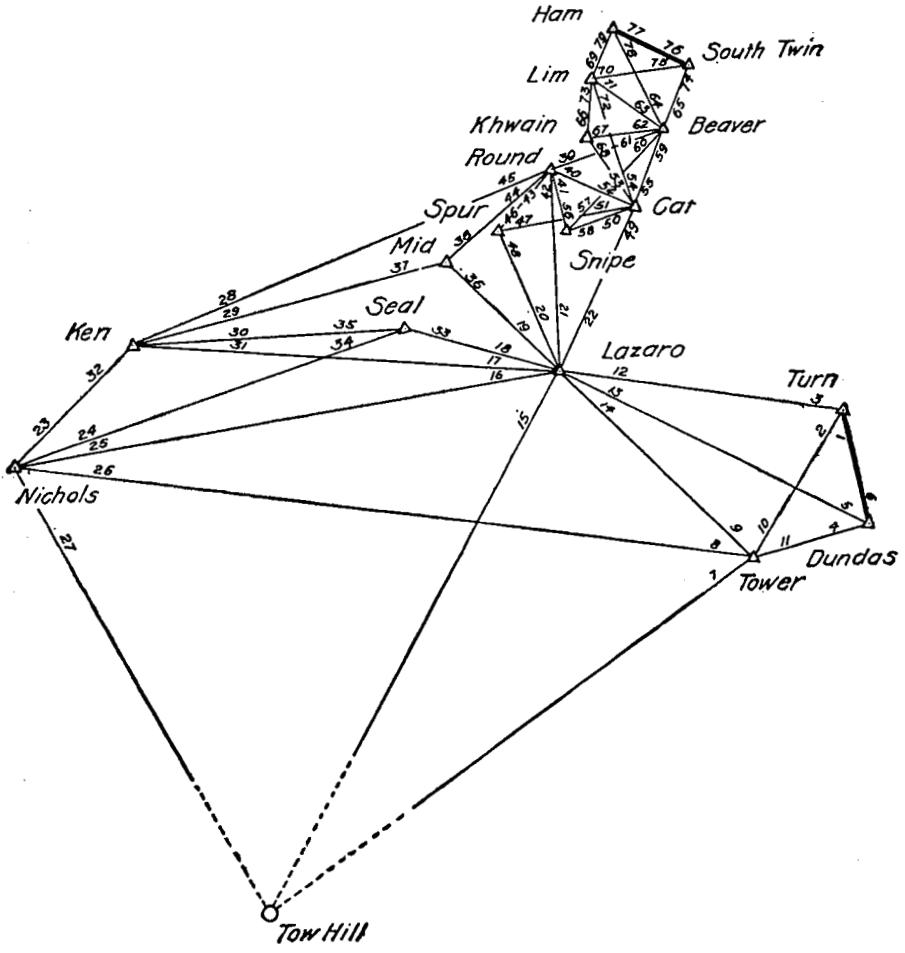

FIG. 4.

In this figure, in addition to the angle, side, and length conditions, there are included conditions for azimuth, latitude, and longitude.

Angle equations

$$0=-\ 5.5-(1)+(2)-(4)+(6)-(10)+(11)$$
$$0=+\ 4.0-(1)+(3)-(5)+(6)-(12)+(13)$$
$$0=+12.0-(2)+(3)-(9)+(10)-(12)+(14)$$
$$0=-\ 8.0-(8)+(9)-(14)+(16)-(25)+(26)$$
$$0=-\ 2.9-(16)+(17)-(23)+(25)-(31)+(32)$$
$$0=-\ 0.0-(16)+(18)-(24)+(25)-(33)+(34)$$
$$0=-\ 3.2-(23)+(24)-(30)+(32)-(34)+(35)$$
$$0=-\ 2.3-(17)+(19)-(29)+(31)-(36)+(37)$$
$$0=-\ 3.2-(17)+(21)-(28)+(31)-(42)+(45)$$
$$0=-\ 3.2-(19)+(21)+(36)-(38)-(42)+(44)$$
$$0=+\ 5.0-(21)+(22)-(40)+(42)-(49)+(52)$$
$$0=-\ 1.0-(20)+(22)-(47)+(48)-(49)+(51)$$
$$0=+\ 3.7-(40)+(43)-(46)+(47)-(51)+(52)$$
$$0=+\ 5.2-(40)+(41)-(50)+(52)-(56)+(58)$$
$$0=-\ 2.5-(39)+(40)-(52)+(55)-(59)+(61)$$
$$0=+\ 2.8-(50)+(55)-(57)+(58)-(59)+(60)$$
$$0=-\ 5.1-(53)+(55)-(59)+(62)-(67)+(68)$$
$$0=-\ 1.7-(54)+(55)-(59)+(63)-(71)+(72)$$
$$0=+\ 2.1-(62)+(63)-(66)+(67)-(71)+(73)$$
$$0=+\ 2.3-(63)+(65)-(70)+(71)-(74)+(75)$$
$$0=+\ 6.1-(64)+(65)-(74)+(76)-(77)+(78)$$
$$0=+\ 0.1-(69)+(70)-(75)+(76)-(77)+(79)$$

Azimuth equation

$$0=(-7.1^{*}-0.2)-(1)+(2)+(9)-(10)-(14)+(21)+(40)-(42)$$
$$-(52)+(55)-(59)+(63)+(70)-(71)-(75)+(76)$$

Computation of correction to azimuth constant:

log 0.277=9. 442
log sin. mean ϕ=9. 912
log correction=9. 354
correction=−0. 2

Side equations

Symbol	Angle	Logarithm	Tabular difference	Symbol	Angle	Logarithm	Tabular difference
	° ′ ″		′		° ′ ″		
− 9+11	93 11 39.1	9.9993248	−0.12	−13+14	16 37 43.4	9.4566222	+7.05
−12+13	25 52 38.1	9.6399291	+4.34	− 1+ 3	110 36 08.7	9.9712965	−0.79
− 1+ 2	24 17 25.4	9.6142236	+4.67	−10+11	42 00 30.0	9.8255810	+2.34
		9.2534775				9.2534997	

$$0=-22.2-5.46(1)+4.67(2)+0.79(3)+0.12(9)+2.34(10)-2.46(11)-4.34(12)+11.39(13)$$
$$-7.05(14)$$

− 7+ 9	111 09 20.5	9.9696969	−0.81	{ + 7− 9 +14−15 }	21 40 38.8	9.5674745	+5.30
+15−16 +25−27 }	36 08 04.3	9.7706188	+2.88	−25+27	89 23 18.6	9.9999753	+0.02
−25+26	30 04 51.8	9.7000325	+3.64	− 8+ 9	48 16 10.2	9.8729041	+1.88
		9.4403482				9.4403539	

$$0=-5.7-4.49(7)+1.88(8)+2.61(9)-5.30(14)+8.18(15)-2.88(16)-0.74(25)+3.64(26)$$
$$-2.90(27)$$

* See computation on p. 38.

Side equations—Continued

Symbol	Angle	Logarithm	Tabular difference	Symbol	Angle	Logarithm	Tabular difference
−16+18	38 29 18.8	9.7940405	+ 2.65	−24+25	12 35 33.3	9.3384902	+9.43
−23+24	27 51 39.2	9.6696203	+ 3.98	−30+32	126 31 06.3	9.9050754	−1.56
−30+31	9 31 16.8	9.2185744	+12.55	−17+18	15 56 21.5	9.4387305	+7.37
		8.6822352				8.6822961	

0=−60.9−2.65(16)+7.37(17)−4.72(18)−3.98(23)+13.41(24)−9.43(25)−14.11(30)
+12.55(31)+1.56(32)

−17+19	35 17 43.3	9.7617712	+ 2.98	−29+31	16 21 23.9	9.4496564	+7.17
−28+29	9 58 42.9	9.2387486	+11.97	−44+45	23 37 23.0	9.6028386	+4.81
−42+44	51 05 11.2	9.8910323	+ 1.70	−19+21	43 39 34.4	9.8390831	+2.21
		8.8915521				8.8915781	

0=−26.0−2.98(17)+5.19(19)−2.21(21)−11.97(28)+19.14(29)−7.17(31)−1.70(42)
+6.51(44)−4.81(45)

−49+52	89 47 52.8	9.9999973	+0.01	−21+22	26 22 55.0	9.6477280	+4.25
−20+21	26 37 18.2	9.6513730	+4.20	−46+48	105 41 48.0	9.9834943	−0.59
−46+47	33 20 40.5	9.7401044	+3.20	−51+52	35 09 14.0	9.7602524	+2.99
		9.3914747				9.3914747	

0=+0.0−4.20(20)+8.45(21)−4.25(22)−3.79(46)+3.20(47)+0.59(48)−0.01(49)
+2.99(51)−2.98(52)

−39+41	105 37 20.7	9.9836521	−0.59	−60+61	22 08 21.3	9.5761788	+5.17
−59+60	27 49 50.7	9.6691879	+3.99	−50+55	125 14 18.4	9.9120934	−1.49
−50+52	37 39 24.5	9.7859918	+2.73	−40+41	63 10 28.9	9.9505530	+1.07
		9.4388318				9.4388252	

0=+6.6+0.59(39)+1.07(40)−1.66(41)−4.22(50)+2.73(52)+1.49(55)−3.99(59)
+9.16(60)−5.17(61)

−71+73	59 25 24.7	9.9349784	+1.24	−62+63	43 56 28.3	9.8413093	+2.19
−59+62	58 43 17.2	9.9317900	+1.28	−53+55	59 03 08.4	9.9333037	+1.26
−53+54	18 17 51.0	9.4968619	+6.37	−72+73	22 50 29.2	9.5890357	+5.00
		9.3636303				9.3636487	

0=−18.4−5.11(53)+6.37(54)−1.26(55)−1.28(59)+3.47(62)−2.19(63)−1.24(71)
+5.00(72)−3.76(73)

−74+76	94 10 29.2	9.9988462	−0.15	−64+65	50 11 06.3	9.8854273	+1.76
−63+64	28 13 32.1	9.6748099	+3.92	−69+71	102 20 31.6	9.9898451	−0.46
−69+70	61 25 13.8	9.9435708	+1.15	−75+76	33 30 23.0	9.7419627	+3.18
		9.6172269				9.6172351	

0=−8.2−3.92(63)+5.68(64)−1.76(65)−1.61(69)+1.15(70)+0.46(71)+0.15(74)
+3.18(75)−3.33(76)

Length equation

Symbol	Angle	Logarithm	Tabular difference	Symbol	Angle	Logarithm	Tabular difference
		−6				−1	
Turn-Dundas		4.266771		Ham-South Twin		3.898371	
− 4+ 6	113 41 59.6	9.9617359	−0.92	−10+11	40 00 30.0	9.8255810	+2.34
− 2+ 3	86 18 43.3	9.9990997	+0.14	−12+14	42 30 21.5	9.8297327	+2.30
− 7+ 9	111 09 20.5	9.9696969	−0.81	{ + 7− 9 +14−15 }	21 40 38.8	9.5674745	+5.30
+15−16 +25−27	36 08 04.3	9.7706188	+2.88	−25+27	89 23 18.6	9.9999753	+0.02
−23+25	40 27 12.5	9.8121311	+2.47	−31+32	116 59 49.5	9.9498922	−1.07
−28+31	26 20 06.8	9.6470132	+4.25	−42+45	74 42 34.2	9.9843478	+0.58
−21+22	26 22 55.0	9.6477280	+4.25	−49+52	89 47 52.8	9.9999973	+0.01
−39+40	42 26 51.8	9.8292505	+2.30	−59+61	49 58 12.0	9.8840631	+1.77
−54+55	40 45 17.4	9.8147959	+2.44	−71+72	36 34 55.5	9.7752272	+2.84
−63+65	78 24 38.4	9.9910544	+0.43	−74+75	60 40 06.2	9.9404164	+1.18
−69+70	61 25 13.8	9.9435708	+1.15	−77+79	85 04 23.4	9.9983924	+0.18
		2.6534656				2.6534708	

$$0=-5.2-0.14(2)+0.14(3)+0.92(4)-0.92(6)-4.49(7)+4.49(9)+2.34(10)-2.34(11)$$
$$+2.30\,(12)-7.60\,(14)+8.18\,(15)-2.88(16)-4.25(21)+4.25(22)-2.47(23)+5.37(25)$$
$$-2.90\,(27)-4.25\,(28)+3.18\,(31)+1.07(32)-2.30(39)+2.30(40)+0.58(42)-0.58(45)$$
$$+0.01\,(49)-0.01\,(52)-2.44\,(54)+2.44(55)+1.77(59)-1.77(61)-0.43(63)+0.43(65)$$
$$-1.15\,(69)+1.15\,(70)+2.84\,(71)-2.84(72)+1.18(74)-1.18(75)+0.18(77)-0.18(79)$$

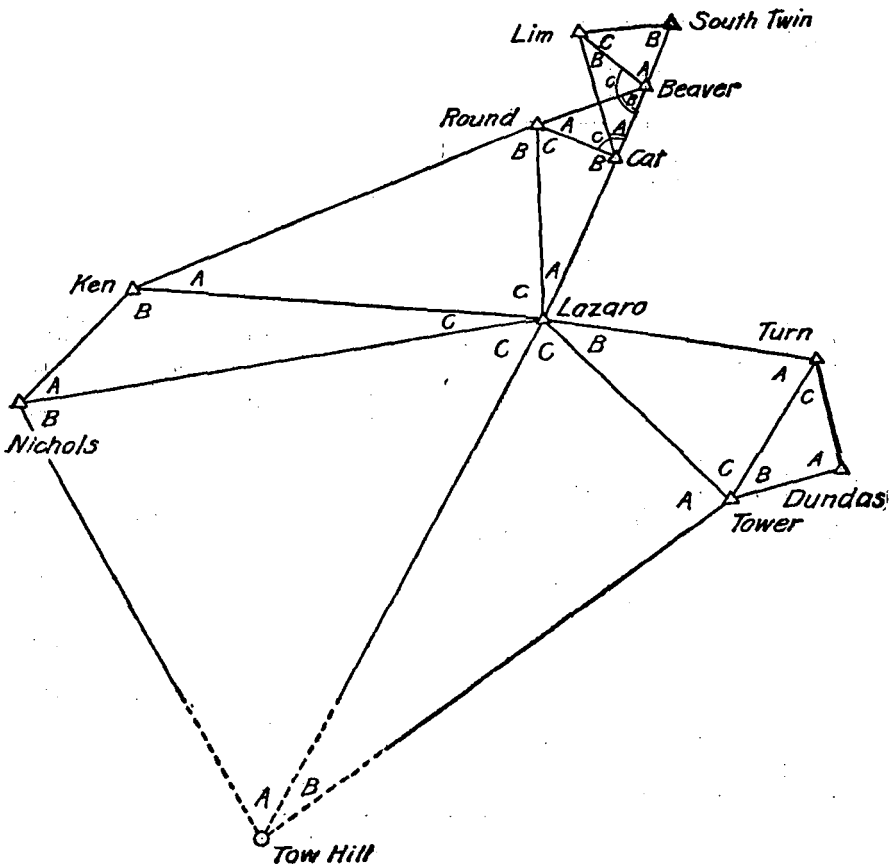

FIG. 5.

Formation of azimuth equation

	°	′	″
Turn–Dundas	357	33	07.7
−1+2	24	17	25.4
Turn–Tower	21	50	33.1
Azimuth difference*	−180	07	07.6
Tower–Turn	201	43	25.5
+9−10	51	11	09.1
Tower–Lazaro	150	32	16.4
Azimuth difference	−180	14	01.2
Lazaro–Tower	330	18	15.2
−14+21	101	38	55.1
	22	32	57.3
	78	57	17.7
Lazaro–Round	173	27	25.3
Azimuth difference	−180	01	37.8
Round–Lazaro	353	25	47.5
+40−42	63	49	17.6
Round–Cat	289	36	29.9
Azimuth difference	+180	05	59.6
Cat–Round	109	42	29.5

	°	′	″
Cat–Round	109	42	29.5
−52+55	87	34	53.9
Cat–Beaver	197	17	23.4
Azimuth difference	+180	01	40.2
Beaver–Cat	17	19	03.6
−59+63	102	39	45.5
Beaver–Lim	119	58	49.1
Azimuth difference	−180	05	20.3
Lim–Beaver	299	53	28.8
+70−71	40	55	17.8
Lim–South Twin	258	58	11.0
Azimuth difference	+180	06	48.1
South Twin–Lim	79	04	59.1
−75+76	33	30	23.0
South Twin–Ham	112	35	22.1
Fixed azimuth	112	35	29.2
Discrepancy			−7.1

Preliminary computation of triangles

Designation of angle	Symbol	Station	Observed angle			Correction	Spherical angle	Spherical excess	Plane angle			Logarithm
			°	′	″			″	°	′	″	
		Turn–Dundas										4.266771
B	−10+11	Tower	42	00	30.0			0.2			29.8	0.1744195
C		Turn			30.9			0.2	24	17	30.7	9.6142483
A	− 4+ 6	Dundas	113	41	59.6			0.1			59.5	9.9617359
								0.5				
		Tower–Dundas										4.0554388
		Tower–Turn										4.4029264
		Turn–Tower										4.4029264
B	−12+14	Lazaro	42	30	21.5			0.6			20.9	0.1702686
A	− 2+ 3	Turn	86	18	43.3			0.7			42.6	9.9990996
C		Tower			57.1			0.6	51	10	56.5	9.8916183
								1.9				
		Lazaro–Tower										4.5722946
		Lazaro–Turn										4.4648133
		Lazaro–Tower										4.5722946
B		Tow Hill			38.8			2.2	21	40	36.6	0.4325371
C	−14+15	Lazaro	47	10	07.2			2.2			05.0	9.8653119
A	− 7+ 9	Tower	111	09	20.5			2.1			18.4	9.9696986
								6.5				
		Tow Hill–Tower										4.8701436
		Tow Hill–Lazaro										4.9745303
		Lazaro–Tow Hill										4.9745303
B	−25+27	Nichols	89	23	18.6			3.6			15.0	0.0000248
C	−15+16	Lazaro	54	28	47.9			3.6			44.3	9.9105723
A		Tow Hill			04.3			3.6	36	08	00.7	9.7706083
								10.8				
		Nichols–Tow Hill										4.8851274
		Nichols–Lazaro										4.7451634

* See position computation, p. 40.

Preliminary computation of triangles—Continued

Designation of angle	Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
			° ′ ″			″	° ′ ″	
		Lazaro-Nichols						4.7451634
B	−31+32	Ken	116 59 49.5			0.8	48.7	0.0501070
C		Lazaro	60.2			0.7	22 32 59.5	9.5837509
A	−23+25	Nichols	40 27 12.5			0.7	11.8	9.8121294
						2.2		
		Ken-Nichols						4.3790213
		Ken-Lazaro						4.6073998
		Lazaro-Ken						4.6073998
B	−42+45	Round	74 42 34.2			0.6	33.6	0.0156525
C		Lazaro	20.9			0.7	78 57 20.2	9.9918811
A	−28+31	Ken	26 20 06.8			0.6	06.2	9.6470108
						1.9		
		Round-Ken						4.6149334
		Round-Lazaro						4.2700631
		Lazaro-Round						4.2700631
B	−49+52	Cat	89 47 52.8			0.2	52.6	0.0000027
A	−21+22	Lazaro	26 22 55.0			0.1	54.9	9.6477276
C		Round	12.6			0.1	63 49 12.5	9.9529926
						0.4		
		Cat-Round						3.9177934
		Cat-Lazaro						4.2230584
		Cat-Round						3.9177934
B	−59+61	Beaver	49 58 12.0			0.1	11.9	0.1159371
C		Cat	56.4			0.0	87 34 56.4	9.9996132
A	−39+40	Round	42 26 51.8			0.1	51.7	9.8292503
						0.2		
		Beaver-Round						4.0333437
		Beaver-Cat						3.8629808
		Beaver-Cat						3.8629808
B	−71+72	Lim	36 34 55.5			0.0	55.5	0.2247728
C		Beaver	47.2			0.1	102 39 47.1	9.9893057
A	−54+55	Cat	40 45 17.4			0.0	17.4	9.8147959
						0.1		
		Lim-Cat						4.0770593
		Lim-Beaver						3.9025495
		Beaver-Lim						3.9025495
B	−74+75	South Twin	60 40 06.2			0.0	06.2	0.0595836
A	−63+65	Beaver	78 24 38.4			0.1	38.3	9.9910543
C		Lim	15.5			0.0	40 55 15.5	9.8162528
						0.1		
		South Twin-Lim						3.9531874
		South Twin-Beaver						3.7783859

Preliminary position computation,

STATION TOWER

						°	′	″
α Second angle	Turn to Dundas					357	33	07.7
	Dundas and Tower					+ 24	17	30.9
α	Turn to Tower					21	50	38.6
$\Delta\alpha$						−	7	07.6
						180		
α'	Tower to Turn					201	43	31.0
				First angle of triangle		42	00	30.0
	°	′	″					
ϕ	54	48	06.742	Turn	λ	130	56	04.052
$\Delta\phi$	−	12	39.419		$\Delta\lambda$	+	8	43.993
ϕ'	54	35	27.323	Tower	λ'	131	04	48.045
s	4.4029264	s^2	8.80585			−h		2.8803
$\cos\alpha$	9.9676416	$\sin^2\alpha$	9.14128	$(\partial\phi)^2$	5.7609	$s^2\sin^2\alpha$		7.9471
B	8.5097251	C	1.55459	D	2.3672	E		6.4574
h	2.8802931		9.50172		8.1281			7.2848
	″		″					
1st term	+759.0896	3d term	+0.0134					
2d term	+ 0.3175	4th term	−0.0019					
	+759.4071							
			−6					
3d and 4th terms	+ 0.0115	s	4.4029264					
		$\sin\alpha$	9.5706383	Arg.		$\Delta\lambda$		2.7193259
$-\Delta\phi$	+759.4186	A′	8.5087480	s	−11	$\sin\frac{1}{2}(\phi+\phi')$		9.9117440
		$\sec\phi'$	0.2370138	$\Delta\lambda$	+ 5	$\sec\frac{1}{2}(\Delta\phi)$		7
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	54 41 47		2.7193259	Corr.	− 6			2.6310706
			″					″
		$\Delta\lambda$	+523.9935			$-\Delta\alpha$		+427.63

STATION LAZARO

						°	′	″
α Second angle	Turn to Tower					21	50	38.6
	Tower and Lazaro					+ 86	18	43.3
α	Turn to Lazaro					108	09	21.9
$\Delta\alpha$						−	21	10.7
						180		
α'	Lazaro to Turn					287	48	11.2
				First angle of triangle		42	30	21.5
	°	′	″					
ϕ	54	48	06.742	Turn	λ	130	56	04.052
$\Delta\phi$	+	4	51.101		$\Delta\lambda$	+	25	54.244
ϕ'	54	52	57.843	Lazaro	λ'	131	21	58.296
								−1
s	4.4648133	s^2	8.92963			−h		2.4681
$\cos\alpha$	9.4938068	$\sin^2\alpha$	9.95564	$(\partial\phi)^2$	4.9281	$s^2\sin^2\alpha$		8.8853
B	8.5097251	C	1.55459	D	2.3672	E		6.4574
h	2.4681452		0.43986		7.2953			7.8108
	″		″					
1st term	−293.8632	3d term	+0.0020					
2d term	+ 2.7534	4th term	+0.0065					
	−291.1098							
			+27					
3d and 4th terms	+ 0.0085	s	4.4648133					
		$\sin\alpha$	9.9778202	Arg.		$\Delta\lambda$		3.1915191
$-\Delta\phi$	−291.1013	A′	8.5087409	s	−15	$\sin\frac{1}{2}(\phi+\phi')$		9.9125251
		$\sec\phi'$	0.2401420	$\Delta\lambda$	+42	$\sec\frac{1}{2}(\Delta\phi)$		
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	54 50 32.3		3.1915191	Corr.	+27			3.1040442
			″					″
		$\Delta\lambda$	+1554.2441			$-\Delta\alpha$		+1270.70

primary triangulation

STATION TOWER

						°	′	″
α	Dundas to Turn					177	33	43.6
Third angle	Tower and Turn					−113	41	59.6
α	Dundas to Tower					63	51	44.0
$\Delta\alpha$						−	7	43.1
						180		
α'	Tower to Dundas					243	44	00.9
								+.1
	°	′	″					
ϕ	54	38	09.559	Dundas	λ	130	55	20.042
$\Delta\phi$	−	2	42.236		$\Delta\lambda$	+	9	28.003
ϕ'	54	35	27.323	Tower	λ'	131	04	48.045
s	4.0554388	s^2	8.11087			−h		2.2091
cos α	9.6439767	$\sin^2\alpha$	9.90630	$(\delta\phi)^2$	4.4203	$s^2\sin^2\alpha$		8.0172
B	8.5097372	C	1.55194	D	2.3681	E		6.4528
h	2.2091527		9.56911		6.7884			6.6791
	″		″					
1st term	+161.8649	3d term	+0.0006					
2d term	+ 0.3708	4th term	−0.0005					
	+162.2357							
3d and 4th terms	+0.0001		+3 4.0554388					
		s	9.9531493	Arg.		$\Delta\lambda$		2.7543502
$-\Delta\phi$	+162.2358	$\sin\alpha$	8.5087480	s	−2	$\sin\frac{1}{2}(\phi+\phi')$		9.9112981
		A′	0.2370138	$\Delta\lambda$	+5	$\sec\frac{1}{2}(\Delta\phi)$		
	° ′ ″	$\sec\phi'$						
$\frac{1}{2}(\phi+\phi')$	54 36 48.4		2.7543502	Corr.	+3			2.6656483
			″					″
		$\Delta\lambda$	+568.0025			$-\Delta\alpha$		+463.07

STATION LAZARO

						°	′	″
α	Tower to Turn					201	43	31.0
Third angle	Lazaro and Turn					− 51	10	57.1
α	Tower to Lazaro					150	32	33.9
$\Delta\alpha$						−	14	01.2
						180		
α'	Lazaro to Tower					330	18	32.7
	°	′	″					
ϕ	54	35	27.323	Tower	λ	131	04	48.045
$\Delta\phi$	+	17	30.520		$\Delta\lambda$	+	17	10.250
ϕ'	54	52	57.843	Lazaro	λ'	131	21	58.295
s	4.5722946	s^2	9.14459			−h		3.0219
cos α	9.9398800	$\sin^2\alpha$	9.38353	$(\delta\phi)^2$	6.0428	$s^2\sin^2\alpha$		8.5281
B	8.5097404	C	1.55122	D	2.3683	E		6.4516
h	3.0219150		0.07934		8.4111			8.0016
	″		″					
1st term	−1051.7559	3d term	+0.0258					
2d term	+ 1.2004	4th term	+0.0100					
	−1050.5555							
3d and 4th terms	+ 0.0358		−7 4.5722946					
		s	9.6917656	Arg.		$\Delta\lambda$		3.0129424
$-\Delta\phi$	−1050.5197	$\sin\alpha$	8.5087409	s	−25	$\sin\frac{1}{2}(\phi+\phi')$		9.9119609
		A′	0.2401420	$\Delta\lambda$	+18	$\sec\frac{1}{2}(\Delta\phi)$		
	° ′ ″	$\sec\phi'$						
$\frac{1}{2}(\phi+\phi')$	54 44 12.6		3.0129424	Corr.	− 7			2.9249033
			″					″
		$\Delta\lambda$	+1030.2498			$-\Delta\alpha$		+841.20

Preliminary position computation,

STATION TOW HILL

						°	′	″
α	Lazaro to Tower					330	18	32.7
Second angle	Tower and Tow Hill					+ 47	10	07.2
α	Lazaro to Tow Hill					17	28	39.9
$\Delta\alpha$						−	21	07.7
						180		
α	Tow Hill to Lazaro					197	07	32.2
					First angle of triangle	21	40	38.8
	°	′	″					
ϕ	54	52	57.843	Lazaro	λ	131	21	58.295
$\Delta\phi$	−	48	31.870		$\Delta\lambda$	+	25	57.539
ϕ'	54	04	25.973	Tow Hill	λ'	131	47	55.834
			−1					
s	4.9745303	s^2	9.94906			−h		3.4637
cos α	9.9794727	$\sin^2\alpha$	8.95521	$(\partial\phi)^2$	6.9283	$s^2\sin^2\alpha$		8.9043
B	8.5097191	C	1.55589	D	2.3667	E		6.4597
h	3.4637221		0.46016		9.2950			8.8277
	″		″					
1st term	+2908.8550	3d term	+0.1972					
2d term	+ 2.8851	4th term	−0.0673					
	+2911.7401							
			−117					
3d and 4th terms	+ 0.1299		4.9745303					
		s	9.4776065	Arg.		$\Delta\lambda$		3.1924389
$-\Delta\phi$	+2911.8700	$\sin\alpha$	8.5087606	s	−159	$\sin\frac{1}{2}(\phi+\phi')$		9.9105687
		A′	0.2315532	$\Delta\lambda$	+ 42	$\sec\frac{1}{2}(\Delta\phi)$		108
	° ′ ″	$\sec\phi'$						
$\frac{1}{2}(\phi+\phi')$	54 28 41.9		3.1924389	Corr.	−117			3.1030184
			″					″
		$\Delta\lambda$	+1557.5388			$-\Delta\alpha$		+1267.70

STATION NICHOLS

						°	′	″
α	Lazaro to Tow Hill					17	28	39.9
Second angle	Tow Hill and Nichols					+ 54	28	47.9
α	Lazaro to Nichols					71	57	27.8
$\Delta\alpha$						−	40	14.3
						180		
α'	Nichols to Lazaro					251	17	13.5
					First angle of triangle	89	23	18.6
	°	′	″					
ϕ	54	52	57.843	Lazaro	λ	131	21	58.295
$\Delta\phi$	−	9	27.012		$\Delta\lambda$	+	49	14.397
ϕ'	54	43	30.831	Nichols	λ'	132	11	12.692
								+1
s	4.7451634	s^2	9.49033			−h		2.7458
cos α	9.4909674	$\sin^2\alpha$	8.95620	$(\partial\phi)^2$	5.5072	$s^2\sin^2\alpha$		9.4465
B	8.5097191	C	1.55589	D	2.3667	E		6.4597
h	2.7458499		1.00242		7.8739			8.6520
	″		″					
1st term	+556.9933	3d term	+0.0075					
2d term	+ 10.0559	4th term	−0.0449					
	+567.0492							
			+92					
3d and 4th terms	− 0.0374	s	4.7451634					
		$\sin\alpha$	9.9781021	Arg.		$\Delta\lambda$		3.4704688
$-\Delta\phi$	+567.0118	A′	8.5087447	s	− 56	$\sin\frac{1}{2}(\phi+\phi')$		9.9123203
		$\sec\phi'$	0.2384494	$\Delta\lambda$	+148	$\sec\frac{1}{2}(\Delta\phi)$		
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	54 48 14.3		3.4704688	Corr.	+ 92			3.3827891
			″					″
		$\Delta\lambda$	+2954.3966			$-\Delta\alpha$		+2414.3

primary triangulation—Continued

STATION TOW HILL

						°	′	″
α	Tower to Lazaro					150	32	33.9
Third angle	Tow Hill and Lazaro					−111	09	20.5
α	Tower to Tow Hill					39	23	13.4
$\Delta\alpha$						−	35	02.4
						180		
α'	Tow Hill to Tower					218	48	11.0
	°	′	″					
ϕ	54	35	27.323	Tower	λ	131	04	48.045
$\Delta\phi$	−	31	01.351		$\Delta\lambda$	+	43	07.789
ϕ'	54	04	25.972	Tow Hill	λ'	131	47	55.834
s	4.8701436	s^2	9.74029			−h		3.2680
$\cos\alpha$	9.8881103	$\sin^2\alpha$	9.60494	$(\delta\phi)^2$	6.5397	$s^2\sin^2\alpha$		9.3452
B	8.5097404	C	1.55122	D	2.3683	E		6.4516
h	3.2679943		0.89645		8.9080			9.0648
	″		″					
1st term	+1853.5073	3d term	+0.0809			$(\Delta\lambda)^3$		0.239
2d term	+ 7.8786	4th term	−0.1160			F		7.733
	+1861.3859							7.972
			+16					
3d and 4th terms	− 0.0351		4.8701436					
		s	9.8024699	Arg.		$\Delta\lambda$		3.4129289
$-\Delta\phi$	+1861.3508	$\sin\alpha$	8.5087606	s	− 98	$\sin\frac{1}{2}(\phi+\phi')$		9.9097770
		A′	0.2315532	$\Delta\lambda$	+114	$\sec\frac{1}{2}(\Delta\phi)$		44
	° ′ ″	$\sec\phi'$						
$\frac{1}{2}(\phi+\phi')$	54 19 56.6		3.4129289	Corr.	+ 16			3.3227103
			″					″
		$\Delta\lambda$	+2587.7893			$-\Delta\alpha$		+2102.38

STATION NICHOLS

						°	′	″
α	Tow Hill to Lazaro					197	07	32.2
Third angle	Nichols and Lazaro					− 36	08	04.3
α	Tow Hill to Nichols					160	59	27.9
$\Delta\alpha$						−	18	55.8
						180		
α'	Nichols to Tow Hill					340	40	32.1
	°	′	″					
ϕ	54	04	25.972	Tow Hill	λ	131	47	55.834
$\Delta\phi$	+	39	04.860	′	$\Delta\lambda$	+	23	16.859
ϕ'	54	43	30.832	Nichols	λ'	132	11	12.693
			−1					
s	4.8851274	s^2	9.77025			−h		3.3706
$\cos\alpha$	9.9756468	$\sin^2\alpha$	9.02568	$(\delta\phi)^2$	6.7403	$s^2\sin^2\alpha$		8.7959
B	8.5097780	C	1.54301	D	2.3709	E		6.4373
h	3.3705522		0.33894		9.1112			8.6038
	″		″					
1st term	−2347.2114	3d term	+0.1292					
2d term	+ 2.1824	4th term	+0.0402					
	−2345.0290							
			−70					
3d and 4th terms	+ 0.1694	s	4.8851274					
		$\sin\alpha$	9.5128381	Arg.		$\Delta\lambda$		3.1451536
$-\Delta\phi$	−2344.8506	A′	8.5087447	s	−104	$\sin\frac{1}{2}(\phi+\phi')$		9.9101427
		$\sec\phi'$	0.2384494	$\Delta\lambda$	+ 34	$\sec\frac{1}{2}(\Delta\phi)$		70
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	54 23 58.9		3.1451526	Corr.	− 70			3.0553033
			″					″
		$\Delta\lambda$	+1396.8592			$-\Delta\alpha$		+1135.80

Preliminary position computation,

STATION KEN

						°	′	″
α	Lazaro to Nichols					71	57	27.8
Second angle	Nichols and Ken					+ 22	33	00.2
α	Lazaro to Ken					94	30	28.0
$\Delta\alpha$						−	30	53.7
						180		
α'	Ken to Lazaro					273	59	34.3
				First angle of triangle		116	59	49.5
	°	′	″					
ϕ	54	52	57.843	Lazaro	λ	131	21	58.295
$\Delta\phi$	+	1	37.056		$\Delta\lambda$	+	37	45.861
ϕ'	54	54	34.899	Ken	λ'	131	59	44.156
s	4.6073998	s^2	9.21480			−h		2.0125
cos α	8.8953917	$\sin^2\alpha$	9.99731	$(\delta\phi)^2$	4.0250	$s^2\sin^2\alpha$		9.2121
B	8.5097191	C	1.55589	D	2.3667	E		6.4597
h	2.0125106		0.76800		6.3917			7.6843
	″		″					
1st term	− 102.9226	3d term	+0.0002					
2d term	+ 5.8614	4th term	+0.0048					
	− 97.0612							
3d and 4th terms	+ 0.0050		+58					
		s	4.6073998					
		sin α	9.9986545	Arg.		$\Delta\lambda$		3.355234
$-\Delta\phi$	− 97.0562	A′	8.5087403	s	−29	sin ½($\phi+\phi'$)		9.912798
		sec ϕ'	0.2404328	$\Delta\lambda$	+87	sec ½($\Delta\phi$)		
	° ′ ″							
½($\phi+\phi'$)	54 53 46.4		3.3552332	Corr.	+58			3.268032
			″					″
		$\Delta\lambda$	+2265.8607			$-\Delta\alpha$		+1853.67

STATION ROUND

						°	′	″
α	Lazaro to Ken					94	30	28.0
Second angle	Ken and Round					+ 78	57	20.9
α	Lazaro to Round					173	27	48.9
$\Delta\alpha$						−	1	37.8
						180		
α'	Round to Lazaro					353	26	11.1
				First angle of triangle		74	42	34.2
	°	′	″					
ϕ	54	52	57.843	Lazaro	λ	131	21	58.295
$\Delta\phi$	+	9	58.318		$\Delta\lambda$	+	1	59.402
ϕ'	55	02	56.161	Round	λ'	131	23	57.697
s	4.2700631	s^2	8.54013			−h		2.7769
cos α	9.9971678	$\sin^2\alpha$	8.11255	$(\delta\phi)^2$	5.5538	$s^2\sin^2\alpha$		6.6527
B	8.5097191	C	1.55589	D	2.3667	E		6.4597
h	2.7769500		8.20857		7.9205			5.8893
	″		″					
1st term	−598.3427	3d term	+0.0083					
2d term	+ 0.0161	4th term	+0.0001					
	−598.3266							
3d and 4th terms	+ 0.0084		−6					
		s	4.2700631					
		sin α	9.0562746	Arg.		$\Delta\lambda$		2.077013
$-\Delta\phi$	−598.3182	A′	8.5087369	s	−6	sin ½($\phi+\phi'$)		9.913183
		sec ϕ'	0.2419389	$\Delta\lambda$	0	sec ½($\Delta\phi$)		
	° ′ ″							
½($\phi+\phi'$)	54 57 57		2.0770129	Corr.	−6			1.990196
			″					″
		$\Delta\lambda$	+119.4024			$-\Delta\alpha$		+97.77

primary triangulation—Continued

STATION KEN

						°	′	″
α	Nichols to Lazaro					251	17	13.5
Third angle	Ken and Lazaro					− 40	27	12.5
α	Nichols to Ken					210	50	01.0
$\Delta\alpha$						+	9	22.8
						180		
α'	Ken to Nichols					30	59	23.8
	°	′	″					
ϕ	54	43	30.831	Nichols	λ	132	11	12.693
$\Delta\phi$	+	11	04.068		$\Delta\lambda$	−	11	28.537
ϕ'	54	54	34.899	Ken	λ'	131	59	44.156
s	4.3790213	s^2	8.75804			−h		2.8226
$\cos\alpha$	9.9338209	$\sin^2\alpha$	9.41947	$(\delta\phi)^2$	5.6450	$s^2\sin^2\alpha$		8.1775
B	8.5097307	C	1.55337	D	2.3676	E		6.4553
h	2.8225729		9.73088		8.0126			7.4554
	″		″					
1st term	−664.6192	3d term	+0.0103					
2d term	+ 0.5381	4th term	+0.0029					
	−664.0811							
3d and 4th terms	+ 0.0132		−2					
		s	4.3790213					
		$\sin\alpha$	9.7097334	Arg.		$\Delta\lambda$		2.837928
$-\Delta\phi$	−664.0679	A′	8.5087403	s	−10	$\sin\frac{1}{2}(\phi+\phi')$		9.912392
	° ′ ″	$\sec\phi'$	0.2404328	$\Delta\lambda$	+ 8	$\sec\frac{1}{2}(\Delta\phi)$		
$\frac{1}{2}(\phi+\phi')$	54 49 02.9		2.8379276	Corr.	− 2			2.750320
			″					″
		$\Delta\lambda$	−688.5375			$-\Delta\alpha$		−562.76

STATION ROUND

						°	′	″
α	Ken to Lazaro					273	59	34.3
Third angle	Round and Lazaro					− 26	20	06.8
α	Ken to Round					247	39	27.5
$\Delta\alpha$						+	29	17.8
						180		
α'	Round to Ken					68	08	45.3
	°	′	″					
ϕ	54	54	34.899	Ken	λ	131	59	44.156
$\Delta\phi$	+	8	21.261		$\Delta\lambda$	−	35	46.458
ϕ'	55	02	56.160	Round	λ'	131	23	57.698
			+1					−1
s	4.6149334	s^2	9.22987			−h		2.7046
$\cos\alpha$	9.5799436	$\sin^2\alpha$	9.93222	$(\delta\phi)^2$	5.4091	$s^2\sin^2\alpha$		9.1621
B	8.5097172	C	1.55631	D	2.3666	E		6.4604
h	2.7045942		0.71840		7.7757			8.3271
	″		″					
1st term	−506.5172	3d term	+0.0058					
2d term	+ 5.2288	4th term	+0.0212					
	−501.2884							
3d and 4th terms	+ 0.0270		+49					
		s	4.6149334					
		$\sin\alpha$	9.9661083	Arg.		$\Delta\lambda$		3.331722
$-\Delta\phi$	−501.2614	A′	8.5087369	s	−30	$\sin\frac{1}{2}(\phi+\phi')$		9.913254
	° ′ ″	$\sec\phi'$	0.2419389	$\Delta\lambda$	+79	$\sec\frac{1}{2}(\Delta\phi)$		
$\frac{1}{2}(\phi+\phi')$	54 58 46.0		3.3317224	Corr.	+49			3.244976
			″					″
		$\Delta\lambda$	−2146.458			$-\Delta\alpha$		−1757.83

Preliminary position computation,

STATION CAT

						°	′	″
α Second angle	Lazaro to Round					173	27	48.9
	Round and Cat					+ 26	22	55.0
α	Lazaro to Cat					199	50	43.9
$\Delta\alpha$						+	4	21.5
						180		
α'	Cat to Lazaro					19	55	05.4
								−.1
				First angle of triangle		89	47	52.8
	°	′	″					
ϕ	54	52	57.843	Lazaro	λ	131	21	58.295
$\Delta\phi$	+	8	28.257		$\Delta\lambda$	−	5	19.363
ϕ'	55	01	26.100	Cat	λ'	131	16	38.932
s	4.2230584	s^2	8.44612			−h		2.7062
$\cos\alpha$	9.9734102	$\sin^2\alpha$	9.06164	$(\delta\phi)^2$	5.4124	$s^2\sin^2\alpha$		7.5078
B	8.5097191	C	1.55589	D	2.3667	E		6.4597
h	2.7061877		9.06365		7.7791			6.6737
	″		″					
1st term	−508.3791	3d term	+0.0060					
2d term	+ 0.1158	4th term	+0.0005					
	−508.2633							
			−3					
3d and 4th terms	+ 0.0065	s	4.2230584					
		$\sin\alpha$	9.5308211	Arg.		$\Delta\lambda$		2.504284
$-\Delta\phi$	−508.2568	A′	8.5087375	s	−5	$\sin\frac{1}{2}(\phi-\phi')$		9.913117
		$\sec\phi'$	0.2416677	$\Delta\lambda$	+2	$\sec\frac{1}{2}(\Delta\phi)$		
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	54 57 42.0		2.5042844	Corr.	−3			2.417401
			″					″
		$\Delta\lambda$	−319.3629			$-\Delta\alpha$		−261.46

STATION BEAVER

						°	′	″
α Second angle	Cat to Round					109	42	58.1
	Round and Beaver					+ 87	34	56.4
α	Cat to Beaver					197	17	54.5
$\Delta\alpha$						+	1	40.2
						180		
α'	Beaver to Cat					17	19	34.7
				First angle of triangle		49	58	12.0
	°	′	″					
ϕ	55	01	26.100	Cat	λ	131	16	38.932
$\Delta\phi$	+	3	45.192		$\Delta\lambda$	−	2	02.273
ϕ'	55	05	11.292	Beaver	λ'	131	14	36.659
s	3.8629808	s^2	7.7260			−h		2.353
$\cos\alpha$	9.9798982	$\sin^2\alpha$	8.9465	$(\delta\phi)^2$	4.705	$s^2\sin^2\alpha$		6.672
B	8.5097090	C	1.5584	D	2.366	E		6.464
h	2.3525880		8.2309		7.071			5.489
	″		″					
1st term	−225.2102	3d term	+0.0012					
2d term	+ 0.0170	4th term						
	−225.1932							
			−1					
3d and 4th terms	+0.0012	s	3.8629808					
		$\sin\alpha$	9.4732671	Arg.		$\Delta\lambda$		2.087330
$-\Delta\phi$	−225.1920	A′	8.5087360	s	−1	$\sin\frac{1}{2}(\phi+\phi')$		9.913657
		$\sec\phi'$	0.2423463	$\Delta\lambda$	0	$\sec\frac{1}{2}(\Delta\phi)$		
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	55 03 18.7		2.0873301	Corr.	−1			2.000987
			″					″
		$\Delta\lambda$	−122.2728			$-\Delta\alpha$		−100.23

primary triangulation—Continued

STATION CAT

						°	′	″
α	Round to Lazaro					353	26	11.1
Third angle	Cat and Lazaro					− 63	49	12.6
α	Round to Cat					289	36	58.5
$\Delta\alpha$						+	5	59.6
						180		
α'	Cat to Round					109	42	58.1
	°	′	″					
ϕ	55	02	56.161	Round	λ	131	23	57.697
$\Delta\phi$	−	1	30.061		$\Delta\lambda$	−	7	18.765
ϕ'	55	01	26.100	Cat	λ'	131	16	38.932
s	3.9177934	s^2	7.8356			−h		1.9535
$\cos\alpha$	9.5259756	$\sin^2\alpha$	9.9481	$(\delta\phi)^2$	3.9069	$s^2\sin^2\alpha$		7.7837
B	8.5097071	C	1.5586	D	2.3658	E		6.4643
h	1.9534761		9.3423		6.2727			6.2015
	″		″					
1st term	+89.8413	3d term	+0.0002					
2d term	+ 0.2199	4th term	−0.0002					
	+90.0612							
3d and 4th terms			+2					
		s	3.9177934					
		$\sin\alpha$	9.9740335	Arg.		$\Delta\lambda$		2.642232
$-\Delta\phi$	+90.0612	A′	8.5087375	s	−1	$\sin\frac{1}{2}(\phi+\phi')$		9.913558
	° ′ ″	$\sec\phi'$	0.2416677	$\Delta\lambda$	+3	$\sec\frac{1}{2}(\Delta\phi)$		
$\frac{1}{2}(\phi+\phi')$	55 02 11.1		2.6422323	Corr.	+2			2.555790
			″					″
		$\Delta\lambda$	−438.7654			$-\Delta\alpha$		−359.58

STATION BEAVER

						°	′	″
α	Round to Cat					289	36	58.5
Third angle	Beaver and Cat					− 42	26	51.8
α	Round to Beaver					247	10	06.7
$\Delta\alpha$						+	7	40.0
						180		
α'	Beaver to Round					67	17	46.7
	°	′	″					
ϕ	55	02	56.161	Round	λ	131	23	57.697
$\Delta\phi$	+	2	15.131		$\Delta\lambda$	−	9	21.038
ϕ'	55	05	11.292	Beaver	λ'	131	14	36.659
s	4.0333437	s^2	8.0667			−h		2.132
$\cos\alpha$	9.5888562	$\sin^2\alpha$	9.9291	$(\delta\phi)^2$	4.264	$s^2\sin^2\alpha$		7.996
B	8.5097071	C	1.5586	D	2.366	E		6.464
h	2.1319070		9.5544		6.630			6.592
	″		″					
1st term	−135.4899	3d term	+0.0004					
2d term	+ 0.3584	4th term	+0.0004					
	−135.1315							
3d and 4th terms			+3					
	+ 0.0008	s	4.0333437					
		$\sin\alpha$	9.9645661	Arg.		$\Delta\lambda$		2.748992
$-\Delta\phi$	−135.1307	A′	8.5087360	s	−2	$\sin\frac{1}{2}(\phi+\phi')$		9.913723
	° ′ ″	$\sec\phi'$	0.2423463	$\Delta\lambda$	+5	$\sec\frac{1}{2}(\Delta\phi)$		
$\frac{1}{2}(\phi+\phi')$	55 04 03.7		2.7489924	Corr.	+3			2.662715
			″					″
		$\Delta\lambda$	−561.0382			$-\Delta\alpha$		−459.95

Preliminary position computation,

STATION LIM

						°	′	″
α	Beaver to Cat					17	19	34.7
Second angle	Cat and Lim					+102	39	47.2
α	Beaver to Linn					119	59	21.9
$\Delta\alpha$						−	5	20.3
						180		
α'	Lim to Beaver					299	54	01.6
				First angle of triangle		36	34	55.5
	°	′	″					
ϕ	55	05	11.292	Beaver	λ	131	14	36.659
$\Delta\phi$	+	2	08.973		$\Delta\lambda$	+	6	30.479
ϕ'	55	07	20.265	Lim	λ'	131	21	07.138
s	3.9025495	s^2	7.8051			−h		2.111
$\cos\alpha$	9.6988310	$\sin^2\alpha$	9.8751	$(\delta\phi)^2$	4.222	$s^2\sin^2\alpha$		7.680
B	8.5097044	C	1.5591	D	2.366	E		6.465
h	2.1110849		9.2393		6.588			6.256
	″		″					
1st term	−129.1471	3d term	+0.0004					
2d term	+ 0.1735	4th term	+0.0002					
	−128.9736							
			+2					
3d and 4th terms	+ 0.0006	s	3.9025495					
		$\sin\alpha$	9.9375769	Arg.		$\Delta\lambda$		2.591597
$-\Delta\phi$	−128.9730	A′	8.5087351	s	−1	$\sin\frac{1}{2}(\phi+\phi')$		9.913918
	° ′ ″	$\sec\phi'$	0.2427356	$\Delta\lambda$	+3	$\sec\frac{1}{2}(\Delta\phi)$		
$\frac{1}{2}(\phi+\phi')$	55 06 15.8		2.5915973	Corr.	+2			2.505515
			″					″
		$\Delta\lambda$	+390.4787			$-\Delta\alpha$		+320.27

STATION SOUTH TWIN

						°	′	″
α	Beaver to Lim					119	59	21.9
Second angle	Lim and South Twin					+ 78	24	38.4
α	Beaver to South Twin					198	24	00.3
$\Delta\alpha$						+	1	27.7
						180		
α'	South Twin to Beaver					18	25	28.0
				First angle of triangle		60	40	06.2
	°	′	″					
ϕ	55	05	11.292	Beaver	λ	131	14	36.659
$\Delta\phi$	+	3	04.190		$\Delta\lambda$	−	1	46.962
ϕ'	55	08	15.482	South Twin	λ'	131	12	49.697
	Fixed latitude, 15.517					Fixed longitude, 49.974		
s	3.7783859	s^2	7.5568			−h		2.265
$\cos\alpha$	9.9772093	$\sin^2\alpha$	8.9984	$(\delta\phi)^2$	4.530	$s^2\sin^2\alpha$		6.555
B	8.5097044	C	1.5591	D	2.366	E		6.465
h	2.2652996		8.1143		6.896			5.285
	″		″					
1st term	−184.2042	3d term	+0.0008					
2d term	+ 0.0130	4th term						
	−184.1912							
			−1					
3d and 4th terms	+ 0.0008	s	3.7783859					
		$\sin\alpha$	9.4992064	Arg.		$\Delta\lambda$		2.029229
$-\Delta\phi$	−184.1904	A′	8.5087347	s	−1	$\sin\frac{1}{2}(\phi+\phi')$		9.913958
	° ′ ″	$\sec\phi'$	0.2429024	$\Delta\lambda$		$\sec\frac{1}{2}(\Delta\phi)$		
$\frac{1}{2}(\phi+\phi')$	55 06 43.4		2.0292293	Corr.	−1			1.943187
			″					″
		$\Delta\lambda$	−106.9619			$-\Delta\alpha$		−87.74

Discrepancy in latitude:
−0.035
×7238.24÷100=
−2.5334

Discrepancy in longitude:
−0.277
×7238.24÷100=
−20.0499

primary triangulation—Continued

STATION LIM

						°	′	″
α	Cat to Beaver					197	17	54.5
Third angle	Lim and Beaver					− 40	45	17.4
α	Cat to Lim					156	32	37.1
$\Delta\alpha$						−	3	39.9
						186		
α'	Lim to Cat					336	28	57.2
								−.1
	°	′	″					
ϕ	55	01	26.100	Cat	λ	131	16	38.932
$\Delta\phi$	+	5	54.165		$\Delta\lambda$	+	4	28.206
ϕ'	55	07	20.265	Lim	λ'	131	21	07.138
s	4.0770593	s^2	8.1541			−h		2.549
cos α	9.9625415	sin² α	9.1999	$(\delta\phi)^2$	5.099	s^2 sin² α		7.354
B	8.5097090	C	1.5584	D	2.366	E		6.464
h	2.5493098		8.9124		7.465			6.367
	″		″					
1st term	−354.2499	3d term	+0.0029					
2d term	+ 0.0817	4th term	+0.0002					
	−354.1682							
3d and 4th terms	+ 0.0031	s	4.0770593					
		sin α	9.5999381	Arg.		$\Delta\lambda$		2.428468
$-\Delta\phi$	−354.1651	A′	8.5087351	s	−2	sin ½($\phi+\phi'$)		9.913752
		sec ϕ'	0.2427356	$\Delta\lambda$	+2	sec ½($\Delta\phi$)		
	° ′ ″							
½($\phi+\phi'$)	55 04 23.2		2.4284681	Corr.	0			2.342220
			″					″
		$\Delta\lambda$	+268.2057			$-\Delta\alpha$		+219.90

STATION SOUTH TWIN

						°	′	″
α	Lim to Beaver					299	54	01.6
Third angle	South Twin and Beaver					− 40	55	15.5
α	Lim to South Twin					258	58	46.1
$\Delta\alpha$						+	6	48.1
						180		
α'	South Twin to Lim					79	05	34.2
	°	′	″					
ϕ	55	07	20.265	Lim	λ	131	21	07.138
$\Delta\phi$	+		55.217		$\Delta\lambda$	−	8	17.440
ϕ'	55	08	15.482	South Twin	λ'	131	12	49.698
								−1
s	3.9531874	s^2	7.9064			−h		1.744
cos α	9.2813974	sin² α	9.9838	$(\delta\phi)^2$	3.488	s^2 sin² α		7.890
B	8.5097018	C	1.5597	D	2.365	E		6.467
h	1.7442866		9.4499		5.853			6.101
	″		″					
1st term	−55.4992	3d term	+0.0001					
2d term	+ 0.2818	4th term	+0.0001					
	−55.2174							
			+2					
3d and 4th terms	+ 0.0002	s	3.9531874					
		sin α	9.9919164	Arg.		$\Delta\lambda$		2.696741
$-\Delta\phi$	−55.2172	A′	8.5087347	s	−2	sin ½($\phi+\phi'$)		9.914053
		sec ϕ'	0.2429024	$\Delta\lambda$	+4	sec ½($\Delta\phi$)		
	° ′ ″							
½($\phi+\phi'$)	55 07 47.9		2.6967411	Corr.	+2			2.610794
			″					″
		$\Delta\lambda$	−497.4404			$-\Delta\alpha$		−408.12

Formation of latitude and longitude condition equations

Station	ϕ	λ	$\phi_n-\phi_C$	$\lambda_n-\lambda_C$	$+\delta_A$	$-\delta_B$	$\frac{\phi}{(\phi_n-\phi_C)\delta_A}$	A	$\frac{\lambda}{(\lambda_n-\lambda_C)\delta_A}$	$\frac{\phi}{(\phi_n-\phi_C)\delta_B}$	B	$\frac{\lambda}{(\lambda_n-\lambda_C)\delta_B}$	$\frac{\phi}{(\lambda_n-\lambda_C)a_1}$	$+R$ or $-L$ C	$\frac{\lambda}{(\phi_n-\phi_C)a_2}$
	° ′	° ′					÷ 100		÷ 100	÷ 100		÷ 100	÷ 100		÷ 100
Turn	54 48.11	130 56.07	+20.18	+16.76	−0.92	−2.34	−18.6	− 4+ 6	−15.4	−47.2	−10+11	−39.2	+20.2	+ 4− 6 +10−11	− 74.2
Tower	54 35.46	131 04.80	+32.84	+ 8.03	+0.14	−2.30	+ 4.6	− 2+ 3	+ 1.1	−75.5	−12+14	−18.5	+ 9.7	− 2+ 3 −12+14	−120.7
Lazaro	54 52.96	131 21.97	+15.31	− 9.14	−0.81	−5.30	−12.4	− 7+ 9	+ 7.4	−81.1	+ 7− 9 +14−15	+48.4	−11.0	−14+15	− 56.3
Lazaro	54 52.96	131 21.97	+15.31	− 9.14	+2.88	−0.02	+44.1	+15−16 +25−27	−26.3	− 0.3	−25+27	+ 0.2	−11.0	−15+16	− 56.3
Lazaro	54 52.96	131 21.97	+15.31	− 9.14	+2.47	+1.07	+37.8	−23+25	−22.6	+16.4	−31+32	− 9.8	−11.0	+23−25 +31−32	− 56.3
Lazaro	54 52.96	131 21.97	+15.31	− 9.14	+4.25	−0.58	+65.1	−28+31	−38.8	− 8.9	−42+45	+ 5.3	−11.0	+28−31 +42−45	− 56.3
Round	55 02.94	131 23.96	+ 5.33	−11.13	+4.25	−0.01	+22.7	−21+22	−47.3	− 0.1	−49+52	+ 0.1	−13.4	−21+22 −49+52	− 19.6
Cat	55 01.43	131 16.65	+ 6.83	− 3.82	+2.30	−1.77	+15.7	−39+40	− 8.8	−12.1	−59+61	+ 6.8	− 4.6	+39−40 +59−61	− 25.1
Beaver	55 05.19	131 14.61	+ 3.08	− 1.78	+2.44	−2.84	+ 7.5	−54+55	− 4.3	− 8.7	−71+72	+ 5.1	− 2.1	+54−55 +71−72	− 11.3
Lim	55 07.34	131 21.12	+ 0.92	− 8.29	+0.43	−1.18	+ 0.4	−63+65	− 3.6	− 1.1	−74+75	+ 9.8	−10.0	−63+65 −74+75	− 3.4
South Twin	55 08.26	131 12.83													

ϕ	a_1	a_2
°	+	−
24	1.93	2.29
28	1.87	2.37
32	1.79	2.47
36	1.71	2.59
40	1.62	2.74
44	1.52	2.92
48	1.41	3.14
52	1.30	3.41
56	1.18	3.76

$a_1 = +1.206$
$a_2 = -3.675$

$\frac{M}{60}10^8 = 7238.24$

Latitude equation

$$
\begin{aligned}
0 = & -2.5334 - 0.14(2) + 0.14(3) + 0.39(4) - 0.39(6) - 0.69(7) + 0.69(9) + 0.67(10) - 0.67(11) \\
& - 0.66(12) - 1.36(14) + 1.25(15) - 0.55(16) - 0.09(21) + 0.09(22) - 0.49(23) + 0.93(25) \\
& - 0.44(27) - 0.76(28) + 0.49(31) + 0.27(32) - 0.20(39) + 0.20(40) - 0.02(42) + 0.02(45) \\
& + 0.14(49) - 0.14(52) - 0.10(54) + 0.10(55) + 0.08(59) - 0.08(61) + 0.10(63) - 0.10(65) \\
& + 0.07(71) - 0.07(72) + 0.11(74) - 0.11(75)
\end{aligned}
$$

Longitude equation

$$
\begin{aligned}
0 = & -20.0499 + 1.20(2) - 1.20(3) - 0.59(4) + 0.59(6) + 0.41(7) - 0.41(9) - 0.35(10) + 0.35(11) \\
& + 1.39(12) - 0.34(14) - 0.75(15) - 0.30(16) + 0.67(21) - 0.67(22) - 0.34(23) + 0.07(25) \\
& + 0.27(27) - 0.17(28) - 0.29(31) + 0.46(32) - 0.16(39) + 0.16(40) - 0.62(42) + 0.62(45) \\
& + 0.20(49) - 0.20(52) - 0.07(54) + 0.07(55) - 0.32(59) + 0.32(61) + 0.07(63) - 0.07(65) \\
& - 0.16(71) + 0.16(72) - 0.06(74) + 0.06(75)
\end{aligned}
$$

Correlate.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	−1	−1																				
2	+1		−1																			
3		+1	+1																			
4	−1																					
5		−1																				
6	+1	+1																				
7																						
8				−1																		
9			−1	+1																		
10	−1		+1																			
11	+1																					
12		−1	−1																			
13		+1																				
14			+1	−1																		
15																						
16				+1	−1	−1																
17					+1			−1	−1													
18						+1																
19								+1		−1												
20												−1										
21									+1	+1	−1											
22											+1	+1										
23					−1		−1															
24						−1	+1															
25				−1	+1	+1																
26				+1																		
27																						
28									−1													
29								−1														
30							−1															
31					−1			+1	+1													
32					+1		+1															
33						−1																
34						+1	−1															
35							+1															
36								−1		+1												
37								+1														
38										−1												
39															−1							
40											−1		−1	−1	+1							
41														+1								
42									−1	−1	+1											
43													+1									
44										+1												
45									+1													
46													−1									
47												−1	+1									
48												+1										
49											−1	−1										
50														−1		−1						
51												+1	−1									
52											+1		+1	+1	−1							
53																	−1					
54																		−1				
55															+1	+1	+1	+1				
56														−1								
57																−1						
58														+1		+1						
59															−1	−1	−1	−1				
60																+1						
61															+1							
62																	+1		−1			
63																		+1	+1	−1		
64																					−1	
65																				+1	+1	

equations

23	24	25	26	27	28	29	30	x 31	l 32	ϕ 33	λ 34	Σ	
−0.55	.....	.....	.....	.....	.....	.....	.....	−1	.....	.....	.....	− 3.55	1
+0.47	.....	.....	.....	.....	.....	.....	.....	+1	−0.14	−0.14	+1.20	+ 2.39	2
+0.08	.....	.....	.....	.....	.....	.....	.....	.....	+0.14	+0.14	−1.20	+ 1.16	3
.....	.....	.....	.....	.....	.....	.....	.....	.....	+0.92	+0.39	−0.59	− 0.28	4
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	− 1.00	5
.....	.....	.....	.....	.....	.....	.....	.....	.....	−0.92	−0.39	+0.59	+ 1.28	6
.....	−4.49	.....	.....	.....	.....	.....	.....	.....	−4.49	−0.69	+0.41	− 9.26	7
.....	+1.88	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	+ 0.88	8
+0.01	+2.61	.....	.....	.....	.....	.....	.....	+1	+4.49	+0.69	−0.41	+ 8.39	9
+0.24	.....	.....	.....	.....	.....	.....	.....	−1	+2.34	+0.67	−0.35	+ 1.90	10
−0.25	.....	.....	.....	.....	.....	.....	.....	.....	−2.34	−0.67	+0.35	− 1.91	11
−0.43	.....	.....	.....	.....	.....	.....	.....	.....	+2.30	+0.66	+1.39	+ 1.92	12
+1.14	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	+ 2.14	13
−0.71	−5.30	.....	.....	.....	.....	.....	.....	−1	−7.60	−1.36	−0.34	−16.31	14
.....	+8.18	.....	.....	.....	.....	.....	.....	.....	+8.18	+1.25	−0.75	+16.86	15
.....	−2.88	−0.27	.....	.....	.....	.....	.....	.....	−2.88	−0.55	−0.30	− 7.88	16
.....	.....	+0.74	−0.30	.....	.....	.....	.....	.....	.....	.....	.....	− 0.56	17
.....	.....	−0.47	.....	.....	.....	.....	.....	.....	.....	.....	.....	+ 0.53	18
.....	.....	.....	+0.52	.....	.....	.....	.....	.....	.....	.....	.....	+ 0.52	19
.....	.....	.....	.....	−4.20	.....	.....	.....	.....	.....	.....	.....	− 5.20	20
.....	.....	.....	−0.22	+8.45	.....	.....	.....	+1	−4.25	−0.09	+0.67	+ 6.56	21
.....	.....	.....	.....	−4.25	.....	.....	.....	.....	+4.25	+0.09	−0.67	+ 1.42	22
.....	.....	−0.40	.....	.....	.....	.....	.....	.....	−2.47	−0.49	−0.34	− 5.70	23
.....	.....	+1.34	.....	.....	.....	.....	.....	.....	.....	.....	.....	+ 1.34	24
.....	−0.74	−0.94	.....	.....	.....	.....	.....	.....	+5.37	+0.93	+0.07	+ 5.69	25
.....	+3.64	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	+ 4.64	26
.....	−2.90	.....	.....	.....	.....	.....	.....	.....	−2.90	−0.44	+0.27	− 5.97	27
.....	.....	.....	−1.20	.....	.....	.....	.....	.....	−4.25	−0.76	−0.17	− 7.38	28
.....	.....	.....	+1.92	.....	.....	.....	.....	.....	.....	.....	.....	+ 0.92	29
.....	.....	−1.41	.....	.....	.....	.....	.....	.....	.....	.....	.....	− 2.41	30
.....	.....	+1.25	−0.72	.....	.....	.....	.....	.....	+3.18	+0.49	−0.29	+ 4.91	31
.....	.....	+0.16	.....	.....	.....	.....	.....	.....	+1.07	+0.27	+0.46	+ 3.96	32
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	− 1.00	33
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.00	34
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	+ 1.00	35
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.00	36
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	+ 1.00	37
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	− 1.00	38
.....	.....	.....	.....	.....	+0.59	.....	.....	.....	−2.30	−0.20	−0.16	− 3.07	39
.....	.....	.....	.....	.....	+1.07	.....	.....	+1	+2.30	+0.20	+0.16	+ 2.73	40
.....	.....	.....	.....	.....	−1.66	.....	.....	.....	.....	.....	.....	− 0.66	41
.....	.....	.....	−0.17	.....	.....	.....	.....	−1	+0.58	−0.02	−0.62	− 2.23	42
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	+ 1.00	43
.....	.....	.....	+0.65	.....	.....	.....	.....	.....	.....	.....	.....	+ 1.65	44
.....	.....	.....	−0.48	.....	.....	.....	.....	.....	−0.58	+0.02	+0.62	+ 0.58	45
.....	.....	.....	.....	−3.79	.....	.....	.....	.....	.....	.....	.....	− 4.79	46
.....	.....	.....	.....	+3.20	.....	.....	.....	.....	.....	.....	.....	+ 3.20	47
.....	.....	.....	.....	+0.59	.....	.....	.....	.....	.....	.....	.....	+ 1.59	48
.....	.....	.....	.....	−0.01	.....	.....	.....	.....	+0.01	+0.14	+0.20	− 1.66	49
.....	.....	.....	.....	.....	−4.22	.....	.....	.....	.....	.....	.....	− 6.22	50
.....	.....	.....	.....	+2.99	.....	.....	.....	.....	.....	.....	.....	+ 2.99	51
.....	.....	.....	.....	−2.98	+2.73	.....	.....	−1	−0.01	−0.14	−0.20	+ 0.40	52
.....	.....	.....	.....	.....	.....	−5.11	.....	.....	.....	.....	.....	− 6.11	53
.....	.....	.....	.....	.....	.....	+6.37	.....	.....	−2.44	−0.10	−0.07	+ 2.76	54
.....	.....	.....	.....	.....	+1.49	−1.26	.....	+1	+2.44	+0.10	+0.07	+ 7.84	55
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	− 1.00	56
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	− 1.00	57
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	+ 2.00	58
.....	.....	.....	.....	.....	−3.99	−1.28	.....	−1	+1.77	+0.08	−0.32	− 8.74	59
.....	.....	.....	.....	.....	+9.16	.....	.....	.....	.....	.....	.....	+10.16	60
.....	.....	.....	.....	.....	−5.17	.....	.....	.....	−1.77	−0.08	+0.32	− 5.70	61
.....	.....	.....	.....	.....	.....	+3.47	.....	.....	.....	.....	.....	+ 3.47	62
.....	.....	.....	.....	.....	.....	−2.19	−3.92	+1	−0.43	+0.10	+0.07	− 4.37	63
.....	.....	.....	.....	.....	.....	.....	+5.68	.....	.....	.....	.....	+ 4.68	64
.....	.....	.....	.....	.....	.....	.....	−1.76	.....	+0.43	−0.10	−0.07	+ 0.50	65

Correlate

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
66	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	−1	....	....	....
67	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	−1	....	+1	....	....	....
68	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	+1	....	....	....	....	....
69	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	−1
70	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	−1	....	+1
71	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	−1	−1	+1	....	....
72	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	+1	....	....	....	....
73	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	+1	....	....	....
74	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	−1	−1	....
75	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	+1	....	−1
76	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	+1	+1
77	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	−1	−1
78	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	+1	....
79	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	+1

equations—Continued

23	24	25	26	27	28	29	30	α 31	ι 32	ϕ 33	λ 34	Σ	
												− 1.00	66
												0.00	67
												+ 1.00	68
							−1.61		−1.15			− 3.76	69
							+1.15	+1	+1.15			+ 3.30	70
						−1.24	+0.46	−1	+2.84	+0.07	−0.16	− 0.03	71
						+5.00			−2.84	−0.07	+0.16	+ 3.25	72
						−3.76						− 2.76	73
							+0.15		+1.18	+0.11	−0.06	− 0.62	74
							+3.18	−1	−1.18	−0.11	+0.06	+ 0.95	75
							−3.33	+1				− 0.33	76
									+0.18			− 1.82	77
												+ 1.00	78
									−0.18			+ 0.82	79

List of corrections

	v's.*	Adopted v's.	v^2.		v's.*	Adopted v's.	v^2.
1	+0.699	+0.7	0.49	41	−0.789	−0.8	0.64
2	+2.448	+2.4	5.76	42	−1.997	−2.0	4.00
3	−3.146	−3.2	10.24	43	+0.028	+0.0	0.00
4	−1.369	−1.4	1.96	44	+1.554	+1.6	2.56
5	−0.207	−0.2	0.04	45	+0.581	+0.6	0.36
6	+1.576	+1.6	2.56	46	+0.697	+0.7	0.49
7	+0.806	+0.7	0.49	47	−1.478	−1.5	2.25
8	−1.876	−1.9	3.61	48	+0.781	+0.8	0.64
9	+0.498	+0.5	0.25	49	+0.735	+0.8	0.64
10	−0.117	−0.1	0.01	50	+1.145	+1.2	1.44
11	+0.688	+0.7	0.49	51	+0.294	+0.3	0.09
12	+3.097	+3.1	9.61	52	−0.317	−0.3	0.09
13	+1.159	+1.2	1.44	53	−1.522	−1.5	2.25
14	−2.691	−2.7	7.29	54	−0.138	−0.1	0.01
15	−1.472	−1.4	1.96	55	−0.197	−0.2	0.04
16	−0.728	−0.7	0.49	56	+0.741	+0.7	0.49
17	+0.755	+0.8	0.64	57	+0.525	+0.5	0.25
18	−0.168	−0.1	0.01	58	−1.266	−1.3	1.69
19	+0.945	+1.0	1.00	59	−0.592	−0.6	0.36
20	−0.090	−0.1	0.01	60	−0.262	−0.2	0.04
21	+0.102	+0.1	0.01	61	+0.524	+0.6	0.36
22	−0.910	−0.9	0.81	62	+1.193	+1.2	1.44
23	−1.665	−1.6	2.56	63	+0.065	+0.1	0.01
24	+0.614	+0.7	0.49	64	+0.364	+0.4	0.16
25	−1.570	−1.5	2.25	65	−1.294	−1.3	1.69
26	+2.090	+2.1	4.41	66	−0.190	−0.2	0.04
27	+0.530	+0.5	0.25	67	−0.898	−0.9	0.81
28	−0.966	−1.0	1.00	68	+1.088	+1.1	1.21
29	+0.183	+0.2	0.04	69	−0.748	−0.8	0.64
30	−1.164	−1.2	1.44	70	−0.490	−0.5	0.25
31	+0.311	+0.3	0.09	71	+0.134	+0.1	0.01
32	+1.636	+1.6	2.56	72	+1.234	+1.2	1.44
33	−0.457	−0.4	0.16	73	−0.130	−0.2	0.04
34	+1.167	+1.2	1.44	74	+1.360	+1.3	1.69
35	−0.710	−0.7	0.49	75	−0.203	−0.2	0.04
36	−0.494	−0.5	0.25	76	−1.158	−1.2	1.44
37	+1.484	+1.5	2.25	77	+0.445	+0.4	0.16
38	−0.990	−1.0	1.00	78	−1.482	−1.5	2.25
39	−0.319	−0.3	0.09	79	+1.037	+1.0	1.00
40	+0.941	+0.9	0.81		Total.....	.....	103.76

* These values result from the computation on p. 69.

Normal equations

	1	2	3	4	5	6	7	8	9	10	11	23	24	25	26	27	31	32	33	34	η	Σ	C's*
1	+6	+2	−2									+0.53					+3	− 6.66	−2.26	+3.08	− 5.5	− 1.81	+0.0070
2		+6	+2									+2.20					+1	− 3.08	−0.91	−2.00	+ 4.0	+11.21	+0.2070
3			+6	−2								−0.44	−7.91				−4	−11.77	−1.76	−4.07	+12.0	−13.95	−0.7922
4				+6	−2	−2						+0.72	+7.53	+0.67			+2	+ 3.84	+0.57	−0.44	− 8.0	+ 6.89	+1.9492
5					+6	+2	+2	−2	−2				+2.14	−0.62	+0.42			+ 8.61	+1.75	+1.46	− 2.9	+14.86	+1.1901
6						+6	−2						+2.14	−2.48				+ 8.25	+1.48	+0.37	− 0.0	+13.76	+0.4571
7							+6							+3.31				+ 3.54	+0.76	+0.80	− 3.2	+11.21	−0.7099
8								+6	+2	−2				+0.51	−1.82			+ 3.18	+0.49	−0.29	− 2.3	+ 3.77	+1.4844
9									+6	+2	−2			+0.51	+0.25	+8.45	+2	+ 2.02	+1.20	+1.79	− 3.2	+19.02	−0.3254
10										+6	−2				+0.08	+8.45	+2	− 4.83	−0.07	+1.29	− 3.2	+ 7.72	+0.9897

	11	12	13	14	15	16	17	18	19	20	21	22	26	27	28	29	30	31	32	33	34	η	Σ	C's
11	+6	+2	+2	+2	−2								+0.05	−15.67	+ 1.66			−4	+6.76	−0.32	−2.52	+5.0	− 3.04	−1.2334
12		+6	−2											+ 0.34					+4.24	−0.05	−0.87	−1.0	+ 8.66	+0.8942
13			+6	+2	−2									+ 1.02	+ 1.66			−2	−2.31	−0.34	−0.36	+3.7	+ 7.37	+0.0285
14				+6	−2	+2								− 2.98	+ 4.22			−2	−2.31	−0.34	−0.36	+5.2	+11.43	−0.7413
15					+6	+2	+2	+2						+ 2.98	− 1.94	+0.02		+4	+3.51	+0.48	+1.23	−2.5	+13.78	−0.0027
16						+6	+2	+2							+18.86	+0.02		+2	+0.67	+0.02	+0.39	+2.8	+38.76	−0.5247
17							+6	+2	−2						+ 5.48	+8.60		+2	+0.67	+0.02	+0.39	−5.1	+22.06	+1.0884
18								+6	+2	−2					+ 5.48	−2.30	−4.38	+4	−3.00	+0.08	+0.85	−1.7	+11.03	+0.5038
19									+6	−2						−8.18	−4.38	+2	−3.27	+0.03	+0.23	+2.1	− 7.47	+0.1899
20										+6	+2	−2				+0.95	+4.50	−4	+0.19	−0.35	−0.18	+2.3	+ 5.41	−0.0433

	21	22	23	24	25	26	27	28	29	30	31	32	33	34	η	Σ	C's
21	+6	+2								−10.92	+ 1	− 0.93	− 0.21	− 0.01	+ 6.1	+ 5.03	−1.4817
22		+6								− 3.75	+ 3	+ 3.12	+ 0.11	− 0.06	+ 0.1	+ 8.52	+1.0414
23			+2.6386	+ 3.7891							+ 1.50	+ 5.5439	+ 0.9624	− 0.0639	− 2.22	+ 15.1601	+0.83524
24				+156.0106	+1.4732						+ 7.91	+151.8020	+24.5038	− 7.2148	− 5.7	+336.4739	+0.03872
25					+7.2568	−1.1220						+ 0.8640	+ 0.1260	− 0.1377	− 6.09	+ 4.2703	+1.32912
26						+6.7354	− 1.8590				− 0.05	+ 3.9252	+ 0.5728	+ 0.0732	− 2.60	+ 4.6556	+0.86820
27							+149.8778	− 8.1354			+11.43	− 53.9453	− 0.7272	+ 9.1030	+ 0.0	+108.3339	−0.19135
28								+158.2846	+ 3.2298		+ 3.82	+ 6.8009	− 0.0428	− 0.7425	+ 6.6	+205.2346	+0.02875
29									+127.4272	+ 8.0144	− 0.93	− 37.6627	− 1.5212	+ 0.7206	−18.4	+ 79.9881	+0.08500
30										+76.0764	− 9.74	+ 1.8338	− 0.5171	− 0.0430	− 8.2	+ 48.4945	−0.19677
31											+16	+ 6.82	+ 1.67	+ 3.69	− 7.3	+ 46.82	−1.37166
32												+351.4744	+49.3161	−13.0822	− 5.2	+478.9301	+0.02103
33													+ 8.7558	− 1.0341	− 2.5334	+ 79.9111	−0.27282
34														+ 9.4562	−20.0499	− 18.6051	+2.16065

* These values result from the computation on p. 68.

Solution of

1	2	3	23	4	6	24	7	5
+6	+2	−2	+0.53					
C_1	−0.33333	+0.33333	−0.08833					
	+6	+2	+2.20					
1	−0.6667	+0.6667	−0.1767					
	+5.3333	+2.6667	+2.0233					
	C_2	−0.50001	−0.37937					
		+6	−0.44	−2		− 7.91		
	1	−0.6667	+0.1767					
	2	−1.3333	−1.0117					
		+4	−1.275	−2		− 7.91		
		C_3	+0.31875	+0.5		+ 1.9775		
			+2.6386	+0.72		+ 3.7891		
		1	−0.0468					
		2	−0.7676					
		3	−0.4064	−0.6375		− 2.5213		
			+1.4178	+0.0825		+ 1.2678		
			C_{23}	−0.05819		− 0.89420		
				+6	−2	+ 7.53		−2
			3	−1		− 3.955		
			23	−0.0048		− 0.0738		
				+4.9952	−2	+ 3.5012		−2
				C_4	+0.40038	− 0.70091		+0.40038
					+6	+ 2.14	−2	+2
				4	−0.8008	+ 1.4018		−0.8008
					+5.1992	+ 3.5418	−2	+1.1992
					C_6	− 0.68122	+0.384675	−0.23065
						+156.0106		+2.14
					3	− 15.6420		
					23	− 1.1337		
					4	− 2.4540		+1.4018
					6	− 2.4127	+1.3624	−0.8169
						+134.3682	+1.3624	+2.7249
						C_{24}	−0.0101393	−0.0202794
							+6	+2
						6	−0.7693	+0.4613
						24	−0.0138	−0.0276
							+5.2169	+2.4337
							C_7	−0.46650

normals

25	31	32	33	34	η	Σ
	+3	− 6.66	− 2.26	+3.08	− 5.5	− 1.81
	−0.5	+ 1.11	+ 0.37667	−0.51333	+ 0.91667	+ 0.30167
	+1	− 3.08	− 0.91	−2.00	+ 4.0	+ 11.21
	−1	+ 2.22	+ 0.7533	−1.0267	+ 1.8333	+ 0.6033
		− 0.86	− 0.1567	−3.0267	+ 5.8333	+ 11.8132
		+ 0.16125	+ 0.02938	+0.56751	− 1.09375	− 2.21499
	−4	− 11.77	− 1.76	−4.07	+12.0	− 13.95
	+1	− 2.22	− 0.7533	+1.0267	− 1.8333	− 0.6033
		+ 0.43	+ 0.0783	+1.5133	− 2.9167	− 5.9067
	−3	− 13.56	− 2.435	−1.53	+ 7.25	− 20.4600
	+0.75	+ 3.39	+ 0.60875	+0.3825	− 1.8125	+ 5.115
	+1.50	+ 5.5439	+ 0.9624	−0.0639	− 2.22	+ 15.1601
	−0.2650	+ 0.5883	+ 0.1996	−0.2721	+ 0.4858	+ 0.1599
		+ 0.3263	+ 0.0594	+1.1482	− 2.2130	− 4.4816
	−0.9562	− 4.3222	− 0.7762	−0.4877	+ 2.3109	− 6.5216
	+0.2788	+ 2.1363	+ 0.4452	+0.3245	− 1.6363	+ 4.3166
	−0.19664	− 1.50677	− 0.31401	−0.22888	+ 1.15411	− 3.04458
+0.67	+2	+ 3.84	+ 0.57	−0.44	− 8.0	+ 6.89
	−1.5	− 6.78	− 1.2175	−0.7650	+ 3.6250	− 10.23
	−0.0162	− 0.1243	− 0.0259	−0.0189	+ 0.0952	− 0.2512
+0.67	+0.4838	− 3.0643	− 0.6734	−1.2239	− 4.2798	− 3.5912
−0.13413	−0.09685	+ 0.61345	+ 0.13481	+0.24502	+ 0.85678	+ 0.71893
−2.48		+ 8.25	+ 1.48	+0.37	− 0.0	+ 13.76
+0.2683	+0.1937	− 1.2269	− 0.2697	−0.4900	− 1.7135	− 1.4378
−2.2117	+0.1937	+ 7.0231	+ 1.2103	−0.12	− 1.7135	+ 12.3221
+0.42539	−0.03726	− 1.35080	− 0.23279	+0.02308	+ 0.32957	− 2.37000
+1.4732	+7.91	+151.8020	+24.5038	−7.2148	− 5.7	+336.4739
	−5.9325	− 26.8149	− 4.8152	−3.0256	+14.3369	− 40.4597
	−0.2493	− 1.9103	− 0.3981	−0.2902	+ 1.4632	− 3.8599
−0.4696	−0.3391	+ 2.1478	+ 0.4720	+0.8578	+ 2.9998	+ 2.5171
+1.5067	−0.1320	− 4.7843	− 0.8245	+0.0817	+ 1.1673	− 8.3941
+2.5103	+1.2571	+120.4403	+18.9380	−9.5911	+14.2672	+286.2773
−0.0186822	−0.0093556	− 0.8963453	− 0.1409411	+0.0713792	− 0.1061799	− 2.1305435
+3.31		+ 3.54	+ 0.76	+0.80	− 3.2	+ 11.21
−0.8508	+0.0745	+ 2.7016	+ 0.4656	−0.0462	− 0.6591	+ 4.7400
−0.0255	−0.0127	− 1.2212	− 0.1920	+0.0972	− 0.1447	− 2.9027
+2.4337	+0.0618	+ 5.0204	+ 1.0336	+0.8510	− 4.0038	+ 13.0473
−0.46650	−0.01185	− 0.96233	− 0.19813	−0.16312	+ 0.76747	− 2.50097

Solution of

	5	25	8	9	26	10	12	27
	+6	−0.62	−2	−2	+0.42			
4	−0.8008	+0.2683						
6	−0.2766	+0.5101						
24	−0.0553	−0.0509						
7	−1.1353	−1.1353						
	+3.7320	−1.0278	−2	−2	+0.42			
	C_5	+0.27540	+0.53591	+0.53591	−0.11254			
		+7.2568	+0.51	+0.51	−1.1220			
	4	−0.0899						
	6	−0.9408						
	24	−0.0469						
	7	−1.1353						
	5	−0.2831	−0.5508	−0.5508	+0.1157			
		+4.7608	−0.0408	−0.0408	−1.0063			
		C_{25}	+0.00857	+0.00857	+0.21137			
			+6	+2	−1.82	−2		
		5	−1.0718	−1.0718	+0.2251			
		25	−0.0003	−0.0003	−0.0086			
			+4.9279	+0.9279	−1.6035	−2		
			C_8	−0.18830	+0.32539	+0.40585		
				+6	+0.25	+2		+8.45
			5	−1.0718	+0.2251			
			25	−0.0003	−0.0086			
			8	−0.1747	+0.3019	+0.3766		
				+4.7532	+0.7684	+2.3766		+8.45
				C_9	−0.16166	−0.5		−1.77775
					+6.7354	+0.08		−1.8590
				5	−0.0473			
				25	−0.2127			
				8	−0.5218	−0.6508		
				9	−0.1242	−0.3842		−1.3660
					+5.8294	−0.9550		−3.2250
					C_{26}	+0.16382		+0.55323
						+6		+8.45
					8	−0.8117		
					9	−1.1883		−4.2250
					26	−0.1564		−0.5283
						+3.8436		+3.6967
						C_{10}		−0.96178
							+6	+0.34
							C_{12}	−0.05667

normals—Continued

11	13	31	32	33	34	η	Σ
			+8.61	+1.75	+1.46	−2.9	+14.86
		+0.1937	−1.2269	−0.2697	−0.4900	−1.7135	− 1.4378
		−0.0447	−1.6199	−0.2792	+0.0277	+0.3952	− 2.8421
		−0.0255	−2.4425	−0.3841	+0.1945	−0.2893	− 5.8055
		−0.0288	−2.3420	−0.4822	−0.3970	+1.8678	− 6.0866
		+0.0947	+0.9787	+0.3348	+0.7952	−2.6398	− 1.3122
		−0.02538	−0.26225	−0.08971	−0.21308	+0.70734	+ 0.35161
			+0.8640	+0.1260	−0.1377	−6.09	+ 4.2703
		−0.0649	+0.4110	+0.0903	+0.1642	+0.5740	+ 0.4817
		+0.0824	+2.9876	+0.5148	−0.0510	−0.7289	+ 5.2417
		−0.0235	−2.2501	−0.3538	+0.1762	−0.2665	− 5.3483
		−0.0288	−2.3420	−0.4822	−0.3970	+1.8678	− 6.0866
		+0.0261	+0.2695	+0.0922	+0.2190	−0.7270	− 0.3614
		−0.0087	−0.0600	−0.0127	−0.0233	−5.3706	− 1.8024
		+0.00183	+0.01260	+0.00267	+0.00489	+1.12809	+ 0.37859
			+3.18	+0.49	−0.29	−2.3	+ 3.77
		+0.0508	+0.5245	+0.1794	+0.4262	−1.4147	− 0.7032
		−0.0001	−0.0005	−0.0001	−0.0002	−0.0460	− 0.0154
		+0.0507	+3.7040	+0.6693	+0.1360	−3.7607	+ 3.0516
		−0.01029	−0.75164	−0.13582	−0.02760	+0.76314	− 0.61925
−2		+2	+2.02	+1.20	+1.79	−3.2	+19.02
		+0.0508	+0.5245	+0.1794	+0.4262	−1.4147	− 0.7032
		−0.0001	−0.0005	−0.0001	−0.0002	−0.0460	− 0.0154
		−0.0095	−0.6975	−0.1260	−0.0256	+0.7081	− 0.5746
−2		+2.0412	+1.8465	+1.2533	+2.1904	−3.9526	+17.7270
+0.42077		−0.42944	−0.38848	−0.263675	−0.46083	+0.83157	− 3.72949
+0.05		−0.05	+3.9252	+0.5728	+0.0732	−2.60	+ 4.6556
		−0.0107	−0.1101	−0.0377	−0.0895	+0.2971	+ 0.1477
		−0.0018	−0.0127	−0.0027	−0.0049	−1.1352	− 0.3810
		+0.0165	+1.2052	+0.2178	+0.0443	−1.2237	+ 0.9930
+0.3233		−0.3300	−0.2985	−0.2026	−0.3541	+0.6390	− 2.8657
+0.3733		−0.3760	+4.7091	+0.5476	−0.3310	−4.0228	+ 2.5496
−0.06404		+0.06450	−0.80782	−0.09394	+0.05678	+0.69009	− 0.43737
−2		+2	−4.83	−0.07	+1.29	−3.2	+ 7.72
		+0.0206	+1.5033	+0.2716	+0.0552	−1.5263	+ 1.2385
+1		−1.0206	−0.9233	−0.6266	−1.0952	+1.9763	− 8.8635
+0.0612		−0.0616	+0.7714	+0.0897	−0.0542	−0.6590	+ 0.4177
−0.9388		+0.9384	−3.4786	−0.3353	+0.1956	−3.4090	+ 0.5126
+0.24425		−0.24415	+0.90504	+0.08724	−0.05089	+0.88693	− 0.13336
+2	−2		+4.24	−0.05	−0.87	−1.0	+ 8.66
−0.33333	+0.33333		−0.70667	+0.00833	+0.145	+0.16667	− 1.44333

Solution of

	27	11	13	14	15	28	16	17
	+149.8778	−15.67	+1.02	−2.98	+2.98	− 8.1354		
9	− 15.0220	+ 3.5555						
26	− 1.7842	+ 0.2065						
10	− 3.5554	+ 0.9029						
12	− 0.0193	− 0.1133	+0.1133					
	+129.4969	−11.1184	+1.1333	−2.98	+2.98	− 8.1354		
	C_{27}	+ 0.0858584	−0.0087516	+0.0230121	−0.0230121	+ 0.0628231		
		+ 6	+2	+2	−2	+ 1.66		
	9	− 0.8415						
	26	− 0.0239						
	10	− 0.2293						
	12	− 0.6667	+0.6667					
	27	− 0.9546	+0.0973	−0.2559	+0.2559	− 0.6985		
		+ 3.2840	+2.7640	+1.7441	−1.7441	+ 0.9615		
		C_{11}	−0.84166	−0.53109	+0.53109	− 0.29278		
			+6	+2	−2	+ 1.66		
		12	−0.6667					
		27	−0.0099	+0.0261	−0.0261	+ 0.0712		
		11	−2.3263	−1.4679	+1.4679	− 0.8093		
			+2.9971	+0.5582	−0.5582	+ 0.9219		
			C_{13}	−0.18625	+0.18625	− 0.30760		
				+6	−2	+ 4.22	+ 2	
			27	−0.0686	+0.0686	− 0.1872		
			11	−0.9263	+0.9263	− 0.5106		
			13	−0.1040	+0.1040	− 0.1717		
				+4.9011	−0.9011	+ 3.3505	+ 2	
				C	+0.18386	− 0.68362	− 0.40807	
					+6	− 1.94	+ 2	+2
				27	−0.0686	+ 0.1872		
				11	−0.9263	+ 0.5106		
				13	−0.1040	+ 0.1717		
				14	−0.1657	+ 0.6160	+ 0.3677	
					+4.7354	− 0.4545	+ 2.3677	+2
					C_{15}	+ 0.09598	− 0.5	−0.42235
						+158.2846	+18.86	+5.48
					27	− 0.5111		
					11	− 0.2815		
					13	− 0.2836		
					14	− 2.2905	− 1.3672	
					15	− 0.0436	+ 0.2273	+0.1920
						+154.8743	+17.7201	+5.6720
						C_{28}	− 0.114416	−0.036623
							+ 6	+2
						14	− 0.8161	
						15	− 1.1839	−1
						28	− 2.0275	−0.6490
							+ 1.9725	+0.3510
							C_{16}	−0.17795

normals—Continued

18	29	31	32	33	34	η	Σ
		+ 11.43	−53.9453	−0.7272	+9.1030	+0.0	+108.3339
		− 3.6287	− 3.2826	−2.2281	−3.8940	+7.0267	− 31.5142
		− 0.2080	+ 2.6052	+0.3029	−0.1831	−2.2255	+ 1.4105
		− 0.9025	+ 3.3456	+0.3225	−0.1881	+3.2787	− 0.4930
			− 0.2403	+0.0028	+0.0493	+0.0567	− 0.4908
		+ 6.6908	−51.5174	−2.3271	+4.8871	+8.1366	+ 77.2464
		− 0.0516676	+ 0.3978273	+0.0178703	−0.0377391	−0.0628324	− 0.5965116
		− 4	+ 6.76	−0.32	−2.52	+5.0	− 3.04
		+ 0.8589	+ 0.7770	+0.5274	+0.9217	−1.6631	+ 7.4590
		+ 0.0241	− 0.3016	−0.0351	+0.0212	+0.2576	− 0.1633
		+ 0.2292	− 0.8496	−0.0819	+0.0478	−0.8326	+ 0.1252
			− 1.4133	+0.0167	+0.2900	+0.3333	− 2.8866
		+ 0.5745	− 4.4232	−0.1998	+0.4196	+0.6986	+ 6.6323
		− 2.3133	+ 0.5493	−0.0927	−0.8197	+3.7938	+ 8.1269
		+ 0.70442	− 0.16727	+0.02823	+0.24960	−1.15524	− 2.47470
		− 2	− 2.31	−0.34	−0.36	+3.7	+ 7.37
			+ 1.4133	−0.0167	−0.2900	−0.3333	+ 2.8866
		− 0.0586	+ 0.4509	+0.0204	−0.0428	−0.0712	− 0.6760
		+ 1.9470	− 0.4623	+0.0780	+0.6899	−3.1931	− 6.8401
		− 0.1116	− 0.9081	−0.2583	−0.0029	+0.1024	+ 2.7405
		+ 0.03724	+ 0.30299	+0.08618	+0.00097	−0.03417	− 0.91438
		− 2	− 2.31	−0.34	−0.36	+5.2	+ 11.43
		+ 0.1540	− 1.1855	−0.0536	+0.1125	+0.1872	+ 1.7776
		+ 1.2286	− 0.2917	+0.0492	+0.4353	−2.0148	− 4.3161
		+ 0.0208	+ 0.1691	+0.0481	+0.0005	−0.0191	− 0.5104
		− 0.5966	− 3.6181	−0.2963	+0.1883	+3.3533	+ 8.3811
		+ 0.12173	+ 0.73822	+0.06046	−0.03842	−0.68419	− 1.71004
+2	+0.02	+ 4	+ 3.51	+0.48	+1.23	−2.5	+ 13.78
		− 0.1540	+ 1.1855	+0.0536	−0.1125	−0.1872	− 1.7776
		− 1.2286	+ 0.2917	−0.0492	−0.4353	+2.0148	+ 4.3161
		− 0.0208	− 0.1691	−0.0481	−0.0005	+0.0191	+ 0.5104
		− 0.1097	− 0.6652	−0.0545	+0.0346	+0.6165	+ 1.5409
+2	+0.02	+ 2.4869	+ 4.1529	+0.3818	+0.7163	−0.0368	+ 18.3697
−0.42235	−0.00422	− 0.52517	− 0.87699	−0.08063	−0.151265	+0.00777	− 3.87923
+5.48	+3.2298	+ 3.82	+ 6.8009	−0.0428	−0.7425	+6.6	+205.2346
		+ 0.4203	− 3.2365	−0.1462	+0.3070	+0.5112	+ 4.8529
		+ 0.6773	− 0.1608	+0.0271	+0.2400	−1.1107	− 2.3794
		+ 0.0343	+ 0.2793	+0.0795	+0.0009	−0.0315	− 0.8430
		+ 0.4078	+ 2.4734	+0.2026	−0.1287	−2.2924	− 5.7295
+0.1920	+0.0019	+ 0.2387	+ 0.3986	+0.0366	+0.0688	−0.0035	+ 1.7631
+5.6720	+3.2317	+ 5.5984	+ 6.5549	+0.1568	−0.2545	+3.6731	+202.8988
−0.036623	−0.020867	− 0.036148	− 0.042324	−0.001012	+0.001643	−0.023717	− 1.310087
+2	+0.02	+ 2	+ 0.67	+0.02	+0.39	+2.8	+ 38.76
		+ 0.2435	+ 1.4764	+0.1209	−0.0768	−1.3684	− 3.4201
−1	−0.01	− 1.2435	− 2.0764	−0.1909	−0.3582	+0.0184	− 9.1849
−0.6490	−0.3698	− 0.6405	− 0.7500	−0.0179	+0.0291	−0.4203	− 23.2149
+0.3510	−0.3598	+ 0.3595	− 0.6800	−0.0679	−0.0159	+1.0297	+ 2.9401
−0.17795	+0.18241	− 0.18226	+ 0.34474	+0.03442	+0.00806	−0.52203	− 1.49054

Solution of

	17	18	29	19	20	21	22
	+6	+2	+ 8.60	−2			
15	−0.8447	−0.8447	− 0.0084				
28	−0.2077	−0.2077	− 0.1184				
16	−0.0625	−0.0625	+ 0.0640				
	+4.8851	+0.8851	+ 8.5372	−2			
	C_{17}	−0.18118	− 1.74760	+0.40941			
		+6	− 2.30	+2	−2		
	15	−0.8447	− 0.0084				
	28	−0.2077	− 0.1184				
	16	−0.0625	+ 0.0640				
	17	−0.1604	− 1.5468	+0.3624			
		+4.7247	− 3.9096	+2.3624	−2		
		C_{18}	+ 0.82748	−0.50001	+0.42331		
			+127.4272	−8.18	+0.95		
		15	− 0.0001				
		28	− 0.0674				
		16	− 0.0656				
		17	− 14.9196	+3.4952			
		18	− 3.2351	+1.9548	−1.6550		
			+109.1394	−2.73	−0.7050		
			C_{29}	+0.025014	+0.006460		
				+6	−2		
			17	−0.8188			
			18	−1.1812	+1		
			29	−0.0683	−0.0176		
				+3.9317	−1.0176		
				C_{19}	+0.25882		
					+6	+2	−2
				18	−0.8466		
				29	−0.0046		
				19	−0.2634		
					+4.8854	+2	−2
					C_{20}	−0.40938	+0.40938
						+6	+2
					20	−0.8188	+0.8188
						+5.1812	+2.8188
						C_{21}	−0.54404
							+6
						20	−0.8188
						21	−1.5335
							+3.6477
							C_{22}

normals—Continued

30	31	32	33	34	η	Σ
	+2	+ 0.67	+0.02	+0.39	− 5.1	+22.06
	−1.0503	− 1.7540	−0.1613	−0.3025	+ 0.0155	− 7.7584
	−0.2050	− 0.2401	−0.0057	+0.0093	− 0.1345	− 7.4308
	−0.0640	+ 0.1210	+0.0121	+0.0028	− 0.1832	− 0.5232
	+0.6807	*− 1.2031*	*−0.1349*	*+0.0996*	*− 5.4022*	*+ 6.3475*
	−0.13934	+ 0.24628	+0.02761	−0.02039	+ 1.10585	− 1.29936
− 4.38	+4	− 3.00	+0.08	+0.85	− 1.7	+11.03
	−1.0503	− 1.7540	−0.1613	−0.3025	+ 0.0155	− 7.7584
	−0.2050	− 0.2401	−0.0057	+0.0093	− 0.1345	− 7.4308
	−0.0640	+ 0.1210	+0.0121	+0.0028	− 0.1832	− 0.5232
	−0.1233	+ 0.2180	+0.0244	−0.0180	+ 0.9788	− 1.1500
− 4.38	*+2.5574*	*− 4.6551*	*−0.0505*	*+0.5416*	*− 1.0234*	*− 5.8325*
+ 0.92704	−0.54128	+ 0.98527	+0.01069	−0.11463	+ 0.21661	+ 1.23447
+ 8.0144	−0.93	−37.6627	−1.5212	+0.7206	−18.4	+79.9881
	−0.0105	− 0.0175	−0.0016	−0.0030	+ 0.0002	− 0.0776
	−0.1168	− 0.1368	−0.0033	+0.0053	− 0.0766	− 4.2339
	+0.0656	− 0.1240	−0.0124	−0.0029	+ 0.1878	+ 0.5363
	−1.1896	+ 2.1025	+0.2358	−0.1741	+ 9.4409	−11.0929
− 3.0244	+2.1162	− 3.8520	−0.0418	+0.4482	− 0.8468	− 4.8203
+ 4.3900	−0.0651	−39.6905	−1.3445	+0.9941	− 9.6945	+60.2939
− 0.040224	+0.000596	+ 0.363668	+0.012319	−0.009109	+ 0.088827	− 0.552449
− 4.38	+2	− 3.27	+0.03	+0.23	+ 2.1	− 7.47
	+0.2787	− 0.4926	−0.0552	+0.0408	− 2.2117	+ 2.5987
+ 2.19	−1.2787	+ 2.3276	+0.0252	−0.2708	+ 0.5117	+ 2.9163
+ 0.1098	−0.0016	− 0.9926	−0.0336	*+0.0249*	− 0.2425	+ 1.5082
− 2.0802	*+0.9984*	*− 2.4278*	*−0.0336*	*+0.0249*	+ 0.1575	− 0.4467
+ 0.52908	−0.25394	+ 0.61749	+0.00855	−0.00633	− 0.04006	+ 0.11361
+ 4.50	−4	+ 0.19	−0.35	−0.18	+ 2.3	+ 5.41
− 1.8541	+1.0826	− 1.9706	−0.0214	+0.2293	− 0.4332	− 2.4690
+ 0.0284	−0.0004	− 0.2564	−0.0087	+0.0064	− 0.0626	+ 0.3895
− 0.5384	+0.2584	− 0.6284	−0.0087	+0.0064	+ 0.0408	− 0.1157
+ 2.1359	−2.6594	− 2.6654	−0.3888	+0.0621	+ 1.8450	+ 3.2148
− *0.43720*	*+0.54436*	+ *0.54558*	*+0.07958*	−0.01271	− 0.37766	− 0.65804
−10.92	+1	− 0.93	−0.21	−0.01	+ 6.1	+ 5.03
− 0.8744	+1.0887	+ 1.0912	+0.1592	−0.0254	− 0.7553	− 1.3161
−11.7944	+2.0887	+ 0.1612	−0.0508	−0.0354	+ 5.3447	+ 3.7140
+ 2.27638	−0.40313	− 0.03111	+0.00980	+0.00683	− 1.03156	− 0.71682
− 3.75	+3	+ 3.12	+0.11	−0.06	+ 0.1	+ 8.52
+ *0.8744*	−1.0887	− 1.0912	−0.1592	+0.0254	+ 0.7553	+ 1.3161
+ 6.4166	−1.1363	− 0.0877	+0.0276	+0.0193	− 2.9077	− 2.0206
+ 3.5410	+0.7750	+ 1.9411	−0.0216	−0.0153	− 2.0524	+ 7.8155
− 0.97075	−0.21246	− 0.53214	+0.00592	+0.00419	+ 0.56266	− 2.14258

Solution of normals—Continued

	30	31	32	33	34	η	Σ
	+76.0764	− 9.74	+ 1.8338	− 0.5171	− 0.0430	− 8.2	+ 48.4945
18	− 4.0604	+ 2.3708	− 4.3155	− 0.0468	+ 0.5021	− 0.9487	− 5.4070
29	− 0.1766	+ 0.0026	+ 1.5965	+ 0.0541	− 0.0400	+ 0.3900	− 2.4253
19	− 1.1006	+ 0.5282	− 1.2845	− 0.0178	+ 0.0132	+ 0.0833	− 0.2363
20	− 0.9338	+ 1.1627	+ 1.1653	+ 0.1700	− 0.0272	− 0.8066	− 1.4055
21	−26.8485	+ 4.7547	+ 0.3670	− 0.1156	− 0.0806	+12.1666	+ 8.4545
22	− 3.4374	− 0.7523	− 1.8843	+ 0.0210	+ 0.0149	+ 1.9924	− 7.5869
	+39.5191	− 1.6733	− 2.5217	− 0.4522	+ 0.3394	+ 4.6770	+ 39.8883
	C_{30}	+ 0.042342	+ 0.063810	+ 0.011443	− 0.008588	− 0.118348	− 1.009342
		+16	+ 6.82	+ 1.67	+ 3.69	− 7.3	+ 46.82
	1	− 1.5	+ 3.33	+ 1.13	− 1.54	+ 2.75	+ 0.9050
	3	− 2.25	− 10.17	− 1.8262	− 1.1475	+ 5.4375	− 15.3450
	23	− 0.0548	− 0.4201	− 0.0875	− 0.0638	+ 0.3218	− 0.8488
	4	− 0.0469	+ 0.2968	+ 0.0652	+ 0.1185	+ 0.4145	+ 0.3478
	6	− 0.0072	− 0.2617	− 0.0451	+ 0.0045	+ 0.0638	− 0.4591
	24	− 0.0118	− 1.1268	− 0.1772	+ 0.0897	− 0.1335	− 2.6783
	7	− 0.0007	− 0.0595	− 0.0122	− 0.0101	+ 0.0474	− 0.1546
	5	− 0.0024	− 0.0248	− 0.0085	− 0.0202	+ 0.0670	+ 0.0333
	25	−	− 0.0001	−	−	− 0.0008	− 0.0033
	8	− 0.0005	− 0.0381	− 0.0069	− 0.0014	+ 0.0387	− 0.0314
	9	− 0.8766	− 0.7930	− 0.5382	− 0.9406	+ 1.6974	− 7.6127
	26	− 0.0243	+ 0.3037	+ 0.0353	− 0.0213	− 0.2595	+ 0.1644
	10	− 0.2291	+ 0.8493	+ 0.0819	− 0.0478	+ 0.8323	− 0.1252
	27	− 0.3457	+ 2.6618	+ 0.1202	− 0.2525	− 0.4204	− 3.9911
	11	− 1.6295	+ 0.3869	− 0.0653	− 0.5774	+ 2.6724	+ 5.7248
	13	− 0.0042	− 0.0338	− 0.0096	− 0.0001	+ 0.0038	+ 0.1021
	14	− 0.0726	− 0.4404	− 0.0361	+ 0.0229	+ 0.4082	+ 1.0202
	15	− 1.3060	− 2.1810	− 0.2005	− 0.3762	+ 0.0193	− 9.6472
	28	− 0.2024	− 0.2369	− 0.0057	+ 0.0092	− 0.1328	− 7.3344
	16	− 0.0655	+ 0.1239	+ 0.0124	+ 0.0029	− 0.1877	− 0.5759
	17	− 0.0948	+ 0.1676	+ 0.0188	− 0.0139	+ 0.7527	− 0.8845
	18	− 1.3843	+ 2.5197	+ 0.0273	− 0.2932	+ 0.5539	+ 3.1570
	29	−	− 0.0237	− 0.0008	+ 0.0006	− 0.0058	+ 0.0359
	19	− 0.2535	+ 0.6165	+ 0.0085	− 0.0063	− 0.0400	+ 0.1134
	20	− 1.4477	− 1.4509	− 0.2116	+ 0.0338	+ 1.0043	+ 1.7500
	21	− 0.8420	− 0.0650	+ 0.0205	+ 0.0143	− 2.1546	− 1.4972
	22	− 0.1647	− 0.4124	+ 0.0046	+ 0.0033	+ 0.4361	− 1.6605
	30	− 0.0709	− 0.1068	− 0.0191	+ 0.0144	+ 0.1980	+ 1.6890
		+ 3.1119	+ 0.2312	− 0.0558	− 1.3082	+ 7.0750	+ 9.0541
		C_{31}	− 0.07430	+ 0.01793	+ 0.42039	− 2.27353	− 2.90951
			+351.4744	+49.3161	−13.0822	− 5.2	+478.9301
		1	− 7.3926	− 2.5086	+ 3.4188	− 6.1050	− 2.0091
		2	− 0.1387	− 0.0253	− 0.4881	+ 0.9406	+ 1.9049
		3	− 45.9684	− 8.2546	− 5.1867	+24.5775	− 69.3594
		23	− 3.2189	− 0.6708	− 0.4889	+ 2.4655	− 6.5041
		4	− 1.8798	− 0.4131	− 0.7508	− 2.6254	− 2.2030
		6	− 9.4868	− 1.6349	+ 0.1621	+ 2.3146	− 16.6447
		24	−107.9561	−16.9750	+ 8.5969	−12.7883	−256.6033
		7	− 4.8313	− 0.9947	− 0.8189	+ 3.8530	− 12.5558
		5	− 0.2567	− 0.0878	− 0.2085	+ 0.6923	+ 0.3441
		25	− 0.0008	− 0.0002	− 0.0003	− 0.0677	− 0.0227
		8	− 2.7841	− 0.5031	− 0.1022	+ 2.8267	− 2.2937
		9	− 0.7173	− 0.4869	− 0.8509	+ 1.5355	− 6.8866
		26	− 3.8041	− 0.4424	+ 0.2674	+ 3.2497	− 2.0596
		10	− 3.1483	− 0.3035	+ 0.1770	− 3.0853	+ 0.4639
		12	− 2.9963	+ 0.0353	+ 0.6148	+ 0.7067	− 6.1198
		27	− 20.4950	− 0.9258	+ 1.9442	+ 3.2370	+ 30.7307
		11	− 0.0919	+ 0.0155	+ 0.1371	− 0.6346	− 1.3594
		13	− 0.2751	− 0.0783	− 0.0009	+ 0.0310	+ 0.8303
		14	− 2.6710	− 0.2187	+ 0.1390	+ 2.4755	+ 6.1871
		15	− 3.6421	− 0.3348	− 0.6282	+ 0.0323	− 16.1100
		28	− 0.2774	− 0.0066	+ 0.0108	− 0.1555	− 8.5875
		16	− 0.2344	− 0.0234	− 0.0055	+ 0.3550	+ 1.0136
		17	− 0.2963	− 0.0332	+ 0.0245	− 1.3305	+ 1.5633
		18	− 4.5865	− 0.0498	+ 0.5336	− 1.0083	− 5.7466
		29	− 14.4342	− 0.4890	+ 0.3615	− 3.5256	+ 21.9270
		19	− 1.4991	− 0.0207	+ 0.0154	+ 0.0973	− 0.2758
		20	− 1.4542	− 0.2121	+ 0.0339	+ 1.0066	+ 1.7539
		21	− 0.0050	+ 0.0016	+ 0.0011	− 0.1663	− 0.1155
		22	− 1.0329	+ 0.0115	+ 0.0081	+ 1.0922	− 4.1589
		30	− 0.1609	− 0.0289	+ 0.0217	+ 0.2984	+ 2.5453
		31	− 0.0172	+ 0.0041	+ 0.0972	− 0.5257	− 0.6727
			+105.7210	+13.6619	− 6.0470	+14.5692	+127.9051
			C_{32}	− 0.129226	+ 0.057198	− 0.137808	− 1.209836

Solution of normals—Continued

	33	34	η	Σ
	+8.7558	−1.0341	− 2.5334	+79.9111
1	−0.8513	+1.1601	− 2.0717	− 0.6818
2	−0.0046	−0.0889	+ 0.1714	+ 0.3471
3	−1.4823	−0.9314	+ 4.4134	−12.4550
23	−0.1398	−0.1019	+ 0.5138	− 1.3555
4	−0.0908	−0.1650	− 0.5770	− 0.4841
6	−0.2817	+0.0279	+ 0.3989	− 2.8685
24	−2.6691	+1.3518	− 2.0108	−40.3482
7	−0.2048	−0.1686	+ 0.7933	− 2.5851
5	−0.0300	−0.0713	+ 0.2368	+ 0.1177
25	—	−0.0001	− 0.0143	− 0.0048
8	−0.0909	−0.0185	+ 0.5108	− 0.4145
9	−0.3305	−0.5776	+ 1.0422	− 4.6742
26	−0.0514	+0.0311	+ 0.3779	− 0.2395
10	−0.0293	+0.0171	− 0.2974	+ 0.0447
12	−0.0004	−0.0072	− 0.0083	+ 0.0721
27	−0.0416	+0.0873	+ 0.1454	+ 1.3804
11	−0.0026	−0.0231	+ 0.1071	+ 0.2294
13	−0.0223	−0.0002	+ 0.0088	+ 0.2362
14	−0.0179	+0.0114	+ 0.2027	+ 0.5067
15	−0.0308	−0.0578	+ 0.0030	− 1.4811
28	−0.0002	+0.0003	− 0.0037	− 0.2053
16	−0.0023	−0.0005	+ 0.0354	+ 0.1012
17	−0.0037	+0.0027	− 0.1492	+ 0.1753
18	−0.0005	+0.0058	− 0.0109	− 0.0623
29	−0.0166	+0.0122	− 0.1194	+ 0.7428
19	−0.0003	+0.0002	+ 0.0013	− 0.0038
20	−0.0309	+0.0049	+ 0.1468	+ 0.2558
21	−0.0005	−0.0003	+ 0.0524	+ 0.0364
22	−0.0001	−0.0001	− 0.0122	+ 0.0463
30	−0.0052	+0.0039	+ 0.0535	+ 0.4564
31	−0.0011	−0.0261	+ 0.1410	+ 0.1804
32	−1.7655	+0.7814	− 1.8827	−16.5287
	+0.5568	+0.2254	− 0.3351	+ 0.4471
	C_{33}	−0.40481	+ 0.60183	− 0.80298
		+9.4562	−20.0499	−18.6051
	1	−1.5811	+ 2.8233	+ 0.9291
	2	−1.7177	+ 3.3105	+ 6.7041
	3	−0.5825	+ 2.7731	− 7.8260
	23	−0.0743	+ 0.3745	− 0.9880
	4	−0.2999	− 1.0486	− 0.8799
	6	−0.0028	− 0.0395	+ 0.2844
	24	−0.6846	+ 1.0184	+20.4342
	7	−0.1388	+ 0.6531	− 2.1283
	5	−0.1694	+ 0.5625	+ 0.2796
	25	−0.0001	− 0.0263	− 0.0088
	8	−0.0038	+ 0.1038	− 0.0842
	9	−1.0094	+ 1.8215	− 8.1691
	26	−0.0188	− 0.2284	+ 0.1448
	10	−0.0100	+ 0.1735	− 0.0261
	12	−0.1262	− 0.1450	+ 1.2557
	27	−0.1844	− 0.3071	− 2.9152
	11	−0.2046	+ 0.9469	+ 2.0285
	13	—	+ 0.0001	+ 0.0027
	14	−0.0072	− 0.1288	− 0.3220
	15	−0.1084	+ 0.0056	− 2.7787
	28	−0.0004	+ 0.0060	+ 0.3334
	16	−0.0001	+ 0.0083	+ 0.0237
	17	−0.0020	+ 0.1102	− 0.1294
	18	−0.0621	+ 0.1173	+ 0.6686
	29	−0.0091	+ 0.0883	− 0.5492
	19	−0.0002	− 0.0010	+ 0.0028
	20	−0.0008	− 0.0234	− 0.0409
	21	−0.0002	+ 0.0365	+ 0.0254
	22	−0.0001	− 0.0080	+ 0.0327
	30	−0.0029	− 0.0402	− 0.3426
	31	−0.5500	+ 2.9743	+ 3.8063
	32	−0.3459	+ 0.8333	+ 7.3159
	33	−0.0912	+ 0.1357	− 0.1810
		+1.4672	− 3.1701	− 1.7029
		C_{34}	+ 2.16065	+ 1.16065

Back solution

34	33	32	31	30	22	21	20
+2.16065	+0.60183	−0.13781	−2.27353	−0.11835	+0.5627	−1.0316	−0.3777
+2.16065	−0.87465	+0.12358	+0.90832	−0.01856	+0.0091	+0.0148	−0.0275
	−0.27282	+0.03526	−0.00489	−0.00312	−0.0016	−0.0027	−0.0217
		+0.02103	−0.00156	+0.00134	−0.0112	−0.0007	+0.0115
			−1.37166	−0.05808	+0.2914	+0.5530	−0.7467
				−0.19677	+0.1910	−0.4479	+0.0860
					+1.0414	−0.5666	+0.4263
						−1.4817	+0.6065
							−0.0433

19	29	18	17	16	28	15	14
−0.0401	+0.08883	+0.2166	+1.1058	−0.5220	−0.02372	+0.0078	−0.6842
−0.0137	−0.01968	−0.2477	−0.0441	+0.0174	+0.00355	−0.3268	−0.0830
−0.0023	−0.00336	−0.0029	−0.0075	−0.0094	+0.00028	+0.0220	−0.0165
+0.0130	+0.00765	+0.0207	+0.0052	+0.0072	−0.00089	−0.0184	+0.0155
+0.3483	−0.00082	+0.7425	+0.1911	+0.2500	+0.04958	+0.7204	−0.1670
−0.1041	+0.00791	−0.1824	+0.0777	+0.0155	−0.00177	−0.0004	+0.2141
−0.0112	−0.00028	−0.0183	−0.1485	−0.0897	−0.01845	−0.2128	−0.0197
+0.1899	+0.00475	−0.0950	−0.0913	−0.1937	−0.03086	−0.4597	−0.0005
	+0.08500	+0.0703	+1.0884	−0.5247	+0.06003	+0.2624	−0.7413
		+0.5038			+0.02875	+0.0028	
						−0.0027	

13	11	27	12	10	26	9	8
−0.0342	−1.1552	−0.06283	+0.1667	+0.8869	+0.69009	+0.8316	+0.7631
+0.0021	+0.5393	−0.08154	+0.3133	−0.1100	+0.12268	−0.9957	−0.0596
−0.0235	−0.0077	−0.00488	−0.0023	−0.0238	+0.02563	+0.0719	+0.0371
+0.0064	−0.0035	+0.00837	−0.0149	+0.0190	−0.01699	−0.0082	−0.0158
−0.0511	−0.9662	+0.07087	+0.0095	+0.3349	−0.08847	+0.5890	+0.0141
−0.0088	−0.0084	+0.00181	+0.4111	−0.3013	+0.07899	−0.5190	+0.4017
−0.0005	−0.0014	+0.00006	+0.0108	+0.1840	−0.10586	+0.3402	+0.2825
+0.1381	+0.3937	−0.01706	+0.8942	+0.9897	+0.16213	−0.4948	+0.0613
+0.0285	−0.0240	−0.00025			+0.86820	−0.1404	+1.4844
	−1.2334	−0.10590				−0.3254	
		−0.19135					

25	5	7	24	6	4	23	3
+1.12809	+0.7073	+0.7675	−0.10618	+0.3296	+0.8568	+1.15411	−1.8125
+0.01057	−0.4604	−0.3524	+0.15423	+0.0499	+0.5294	−0.49453	+0.8264
−0.00073	+0.0245	+0.0541	+0.03845	+0.0635	−0.0368	+0.08567	−0.1661
+0.00026	−0.0055	−0.0202	−0.01885	−0.0284	+0.0129	−0.03169	+0.0713
−0.00251	+0.0348	+0.0163	+0.01283	+0.0511	+0.1328	+0.26972	−1.0287
+0.18351	−0.0977	−0.6200	−0.02483	+0.5654	−0.1783	−0.03462	+0.0766
−0.00279	−0.1744	−0.5552	−0.02413	−0.2745	+0.4765	−0.11342	+0.9746
+0.01272	+0.7955	−0.7099	+0.00720	−0.2731	−0.0271	+0.83524	+0.2662
+1.32912	+0.3660		+0.03872	−0.0264	+0.1830		−0.7922
	+1.1901			+0.4571	+1.9492		

2	1
−1.0938	+0.9167
+1.2262	−1.1091
−0.0080	−0.1028
+0.0034	+0.0233
−0.3169	+0.6858
+0.3961	−0.0738
+0.2070	−0.2641
	−0.0690
	+0.0070

Probable error of an observed direction $= \pm 0.6745\sqrt{\frac{103.76}{34}} = \pm 1''.2$

Computation of corrections

1	2	3	4	5	6	7	8
−0.007	+0.007	+0.207	−0.007	−0.207	+0.007	−0.174	−1.949
−0.207	+0.792	−0.792	+0.019		+0.207	−0.094	+0.073
−0.459	+0.393	+0.067	−0.106	−0.207	−0.019	+0.188	
+1.372	−1.372	+0.003	−1.275	−0.2	+0.106	+0.886	−1.876
	−0.003	−0.038			+1.275		−1.9
+0.699	+0.038	−2.593	−1.369			+0.806	
+0.7	+2.593		−1.4		+1.576	+0.7	
		−3.146			+1.6		
	+2.448	−3.2					
	+2.4						

9	10	11	12	13	14	15	16
+0.792	−0.007	+0.007	−0.207	+0.207	−0.792	+0.317	+1.949
+1.949	−0.792	−0.209	+0.792	+0.952	−1.949	+0.172	−1.190
+0.008	+0.200	−0.049	−0.359		−0.593	−0.341	−0.457
+0.101	+1.372	+0.183	+0.048	+1.159	−0.205	−1.620	−0.112
−1.372	+0.049	+0.756	−0.180	+1.2	+1.372		−0.359
+0.094	−0.183		+3.003		−0.160	−1.472	−0.061
−0.188	−0.756	+0.688			+0.371	−1.4	+0.150
−0.886		+0.7	+3.097		−0.735		−0.648
	−0.117		+3.1				
+0.498	−0.1				−2.691		−0.728
+0.5					−2.7		−0.7

17	18	19	20	21	22	23	24
+1.190	+0.457	+1.484	−0.894	−0.325	−1.233	−1.190	−0.457
−1.484	−0.625	−0.990	+0.804	+0.990	+0.894	+0.710	−0.710
+0.325		+0.451		+1.233	+0.813	−0.532	+1.781
+0.984	−0.168		−0.090	−0.191	+0.089	−0.052	
−0.260	−0.1	+0.945	−0.1	−1.617	−0.025	+0.134	+0.614
		+1.0		−1.372	−1.448	−0.735	+0.7
+0.755				−0.089			
+0.8				+0.025	−0.910	−1.665	
				+1.448	−0.9	−1.6	
				+0.102			
				+0.1			

25	26	27	28	29	30	31	32
−1.949	+1.949	−0.112	+0.325	−1.484	+0.710	−1.190	+1.190
+1.190	+0.141	−0.061	−1.042	+1.667	−1.874	+1.484	−0.710
+0.457		+0.120	−0.089			−0.325	+0.213
−0.029	+2.090	+0.583	+0.207	+0.183	−1.164	+1.661	+0.023
−1.249	+2.1		−0.367	+0.2	−1.2	−0.625	−0.074
+0.113		+0.530				+0.067	+0.994
−0.254		+0.5	−0.966			−0.134	
+0.161			−1.0			−0.627	+1.636
							+1.6
−1.570						+0.311	
−1.5						+0.3	

33	34	35	36	37	38	39	40
−0.457	+0.457	−0.710	−1.484	+1.484	−0.990	+0.003	+1.233
	+0.710		+0.990			+0.017	−0.028
−0.457		−0.710		+1.484	−0.990	−0.048	+0.741
−0.4	+1.167	−0.7	−0.494	+1.5	−1.0	+0.055	−0.003
	+1.2		−0.5			−0.346	+0.031
							−1.372
						−0.319	+0.048
						−0.3	−0.055
							+0.346
							+0.941
							+0.9

Computation of corrections—Continued

41	42	43	44	45	46	47	48
−0.741	+0.325	+0.028	+0.990	−0.325	−0.028	−0.894	+0.894
−0.048	−0.990		+0.564	−0.417	+0.725	+0.028	−0.113
	−1.233	+0.028		−0.012		−0.612	
−0.789	−0.148	+0.0	+1.554	−0.005	+0.697		+0.781
−0.8	+1.372		+1.6	+1.340	+0.7	−1.478	+0.8
	+0.012					−1.5	
	+0.005			+0.581			
	−1.340			+0.6			
	−1.997						
	−2.0						

49	50	51	52	53	54	55	56
+1.233	+0.741	+0.894	−1.233	−1.088	−0.504	−0.003	+0.741
−0.894	+0.525	−0.028	+0.028	−0.434	+0.541	−0.525	
+0.002	−0.121	−0.572	−0.741		−0.051	+1.088	+0.741
−0.038			+0.003	−1.522	+0.027	+0.504	+0.7
+0.432	+1.145	+0.294	+0.570	−1.5	−0.151	+0.043	
	+1.2	+0.3	+0.078			−0.107	
+0.735			+1.372		−0.138	−1.372	
+0.8			+0.038		−0.1	+0.051	
			−0.432			−0.027	
						+0.151	
			−0.317				
			−0.3			−0.197	
						−0.2	

57	58	59	60	61	62	63	64
+0.525	−0.741	+0.003	−0.525	−0.003	+1.088	+0.504	+1.482
	−0.525	+0.525	+0.263	−0.149	−0.190	+0.190	−1.118
+0.525		−1.088		−0.037	+0.295	+0.043	
+0.5	−1.266	−0.504	−0.262	+0.022		−0.186	+0.364
	−1.3	−0.115	−0.2	+0.691	+1.193	+0.771	+0.4
		−0.109			+1.2	−1.372	
		+1.372		+0.524		−0.009	
		+0.037		+0.6		−0.027	
		−0.022				+0.151	
		−0.691					
						+0.065	
		−0.592				+0.1	
		−0.6					

65	66	67	68	69	70	71	72
−0.043	−0.190	−1.088	+1.088	−1.041	+0.043	−0.504	+0.504
−1.482		+0.190		+0.317	+1.041	−0.190	+0.425
+0.346	−0.190		+1.088	−0.024	−0.226	−0.043	−0.060
+0.009	−0.2	−0.898	+1.1		−1.372	−0.105	+0.019
+0.027		−0.9		−0.748	+0.024	−0.091	+0.346
−0.151				−0.8		+1.372	
					−0.490	+0.060	+1.234
−1.294					−0.5	−0.019	+1.2
−1.3						−0.346	
						+0.134	
						+0.1	

73	74	75	76	77	78	79	
+0.190	+0.043	−0.043	−1.482	+1.482	−1.482	+1.041	
−0.320	+1.482	−1.041	+1.041	−1.041		−0.004	
	−0.030	−0.626	+0.655	+0.004	−1.482		
−0.130	+0.025	+1.372	−1.372		−1.5	+1.037	
−0.2	−0.030	−0.025		+0.445		+1.0	
	−0.130	+0.030	−1.158	+0.4			
		+0.130	−1.2				
	+1.360						
	+1.3	−0.203					
		−0.2					

Final solution of triangles

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″	″	° ′ ″	
	Turn-Dundas						4.266771
−10+11	Tower	42 00 30.0	+ 0.8	30.8	0.1	30.7	0.174417
− 1+ 2	Turn	24 17 25.4	+ 1.7	27.1	0.2	26.9	9.614231
− 4+ 6	Dundas	113 41 59.6	+ 3.0	62.6	0.2	42 02.4	9.961733
			+ 5.5		0.5		
	Tower-Dundas						4.055419
	Tower-Turn						4.402921
	Turn-Dundas						4.266771
−12+13	Lazaro	25 52 38.1	− 1.9	36.2	0.4	35.8	0.360081
− 1+ 3	Turn	110 36 08.7	− 3.9	04.8	0.5	04.3	9.971300
− 5+ 6	Dundas	43 31 18.5	+ 1.8	20.3	0.4	19.9	9.837989
			− 4.0		1.3		
	Lazaro-Dundas						4.598152
	Lazaro-Turn						4.464841
	Turn-Tower						4.402921
−12+14	Lazaro	42 30 21.5	− 5.8	15.7	0.6	15.1	0.170282
− 2+ 3	Turn	86 18 43.3	− 5.6	37.7	0.7	37.0	9.999099
− 9+10	Tower	51 11 09.1	− 0.6	08.5	0.6	07.9	9.891638
			−12.0		1.9		
	Lazaro-Tower						4.572302
	Lazaro-Turn						4.464841
	Dundas-Tower						4.055419
−13+14	Lazaro	16 37 43.4	− 3.9	39.5	0.4	39.1	0.543408
− 4+ 5	Dundas	70 10 41.1	+ 1.2	42.3	0.4	41.9	9.973475
− 9+11	Tower	93 11 39.1	+ 0.2	39.3	0.3	39.0	9.999325
			− 2.5		1.1		
	Lazaro-Tower						4.572302
	Lazaro-Dundas						4.598152
	Lazaro-Tower						4.572302
	Tow Hill	38.8		37.7	2.2	21 40 35.5	0.432543
−14+15	Lazaro	47 10 07.2	+ 1.3	08.5	2.2	06.3	9.865314
− 7+ 9	Tower	111 09 20.5	− 0.2	20.3	2.1	18.2	9.969699
					6.5		
	Tow Hill-Tower						4.870159
	Tow Hill-Lazaro						4.974544
	Lazaro-Tower						4.572302
−25+26	Nichols	30 04 51.8	+ 3.6	55.4	1.7	53.7	0.299961
−14+16	Lazaro	101 38 55.1	+ 2.0	57.1	1.7	55.4	9.990962
− 8+ 9	Tower	48 16 10.2	+ 2.4	12.6	1.7	10.9	9.872905
			+ 8.0		5.1		
	Nichols-Tower						4.863225
	Nichols-Lazaro						4.745168^{+1}
	Lazaro-Tow Hill						4.974544
−25+27	Nichols	89 23 18.6	+ 2.0	20.6	3.6	17.0	0.000025
−15+16	Lazaro	54 28 47.9	+ 0.7	48.6	3.6	45.0	9.910573
	Tow Hill	64.3		61.6	3.6	36 07 58.0	9.770601
					10.8		
	Nichols-Tow Hill						4.885142
	Nichols-Lazaro						4.745170^{-1}

Final solution of triangles—Continued

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″	″	° ′ ″	
	Tower-Tow Hill						4.870159
−26+27	Nichols	59 18 26.8	− 1.6	25.2	4.1	21.1	0.005550
− 7+ 8	Tower	62 53 10.3	− 2.6	07.7	4.0	03.7	9.949433
	Tow Hill	35.1		39.3	4.1	57 48 35.2	9.927516
					12.2		
	Nichols-Tow Hill						4.885142
	Nichols-Tower						4.863225
	Lazaro-Nichols						4.745169
−31+32	Ken	116 59 49.5	+ 1.3	50.8	0.8	50.0	0.050108
−16+17	Lazaro	22 32 57.3	+ 1.5	58.8	0.7	58.1	9.583744
−23+25	Nichols	40 27 12.5	+ 0.1	12.6	0.7	11.9	9.812130
			+ 2.9		2.2		
	Ken-Nichols						4.379021
	Ken-Lazaro						4.607407
	Lazaro-Nichols						4.745169
−33+34	Seal	128 55 09.3	+ 1.6	10.9	0.4	10.5	0.109005
−16+18	Lazaro	38 29 18.8	+ 0.6	19.4	0.5	18.9	9.794041
−24+25	Nichols	12 35 33.3	− 2.2	31.1	0.5	30.6	9.338465
			+ 0.0		1.4		
	Seal-Nichols						4.648215
	Seal-Lazaro						4.192639
	Lazaro-Ken						4.607407
−33+35	Seal	154 32 21.8	− 0.3	21.5	0.2	21.3	0.366640
−17+18	Lazaro	15 56 21.5	− 0.9	20.6	0.1	20.5	9.438723
−30+31	Ken	9 31 16.8	+ 1.5	18.3	0.1	18.2	9.218592
			+ 0.3		0.4		
	Seal-Ken						4.412770+1
	Seal-Lazaro						4.192639
	Nichols-Ken						4.379021
−34+35	Seal	25 37 12.5	− 1.9	10.6	0.4	10.2	0.364122
−23+24	Nichols	27 51 39.2	+ 2.3	41.5	0.4	41.1	9.669628
−30+32	Ken	126 31 06.3	+ 2.8	09.1	0.4	08.7	9.905072
			+ 3.2		1.2		
	Seal-Ken						4.412771
	Seal-Nichols						4.648215
	Lazaro-Ken						4.607407
−36+37	Mid	128 20 51.4	+ 2.0	53.4	0.3	53.1	0.105542
−17+19	Lazaro	35 17 43.3	+ 0.2	43.5	0.3	43.2	9.761771
−29+31	Ken	16 21 23.9	+ 0.1	24.0	0.3	23.7	9.449655
			+ 2.3		0.9		
	Mid-Ken						4.474720
	Mid-Lazaro						4.162604
	Lazaro-Mid						4.162604
−42+44	Round	51 05 11.2	+ 3.6	14.8	0.2	14.6	0.108962
−19+21	Lazaro	43 39 34.4	− 0.9	33.5	0.2	33.3	9.839081
+36−38	Mid	85 15 11.7	+ 0.5	12.2	0.1	12.1	9.998508
			+ 3.2		0.5		
	Round-Mid						4.110647
	Round-Lazaro						4.270074

Final solution of triangles—Continued

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″	″	° ′ ″	
	Lazaro-Ken						4.607407
−42+45	Round	74 42 34.2	+ 2.6	36.8	0.6	36.2	0.015651
−17+21	Lazaro	78 57 17.7	− 0.7	17.0	0.7	16.3	9.991879
−28+31	Ken	26 20 06.8	+ 1.3	08.1	0.6	07.5	9.647016
			+ 3.2		1.9		
	Round-Ken						4.614937
	Round-Lazaro						4.270074
	Mid-Ken						4.474720
−44+45	Round	23 37 23.0	− 1.0	22.0	0.2	21.8	0.397167
−37+38	Mid	146 23 56.9	− 2.5	54.4	0.1	54.3	9.743050
−28+29	Ken	9 58 42.9	+ 1.2	44.1	0.2	43.9	9.238760
			− 2.3		0.5		
	Round-Ken						4.614937
	Round-Mid						4.110647
	Lazaro-Round						4.270074
−49+52	Cat	89 47 52.8	− 1.1	51.7	0.2	51.5	0.000003
−21+22	Lazaro	26 22 55.0	− 1.0	54.0	0.1	53.9	9.647723
−40+42	Round	63 49 17.6	− 2.9	14.7	0.1	14.6	9.952995
			− 5.0		0.4		
	Cat-Round						3.917800
	Cat-Lazaro						4.223072
	Round-Cat						3.917800
−46+47	Spur	33 20 40.5	− 2.2	38.3	0.1	38.2	0.259903
−40+43	Round	111 30 09.4	− 0.9	08.5	0.0	08.5	9.968671
−51+52	Cat	35 09 14.0	− 0.6	13.4	0.1	13.3	9.760250
			− 3.7		0.2		
	Spur-Cat						4.146374
	Spur-Round						3.937953
	Round-Lazaro						4.270074
−46+48	Spur	105 41 48.0	+ 0.1	48.1	0.1	48.0	0.016506
−42+43	Round	47 40 51.8	+ 2.0	53.8	0.1	53.7	9.868888
−20+21	Lazaro	26 37 18.2	+ 0.2	18.4	0.1	18.3	9.651373
			+ 2.3		0.3		
	Spur-Lazaro						4.155468
	Spur-Round						3.937953
	Cat-Lazaro						4.223072
−47+48	Spur	72 21 07.5	+ 2.3	09.8	0.1	09.7	0.020934
−49+51	Cat	54 38 38.8	− 0.5	38.3	0.2	38.1	9.911462
−20+22	Lazaro	53 00 13.2	− 0.8	12.4	0.2	12.2	9.902368
			+ 1.0		0.5		
	Spur-Lazaro						4.155468
	Spur-Cat						4.146374
	Cat-Round						3.917800
−59+61	Beaver	49 58 12.0	+ 1.2	13.2	0.1	13.1	0.115935
−52+55	Cat	87 34 53.9	+ 0.1	54.0	0.0	54.0	9.999613
−39+40	Round	42 26 51.8	+ 1.2	53.0	0.1	52.9	9.829253
			+ 2.5		0.2		
	Beaver-Round						4.033348
	Beaver-Cat						3.862988

Final solution of triangles—Continued

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″	″	° ′ ″	
	Round-Beaver						4.033348
−56+57	Snipe	52 14 18.1	− 0.2	17.9	0.1	17.8	0.102063
−39+41	Round	105 37 20.7	− 0.5	20.2	0.0	20.2	9.983652
−60+61	Beaver	22 08 21.3	+ 0.8	22.1	0.1	22.0	9.576182
			+ 0.1		0.2		
	Snipe-Beaver						4.119063
	Snipe-Round						3.711593
	Round-Cat						3.917800
−56+58	Snipe	79 10 11.9	− 2.0	09.9	0.0	09.9	0.007806
−40+41	Round	63 10 28.9	− 1.7	27.2	0.0	27.2	9.950551
−50+52	Cat	37 39 24.5	− 1.5	23.0	0.1	22.9	9.785987
			− 5.2		0.1		
	Snipe-Cat						3.876157
	Snipe-Round						3.711593
	Beaver-Cat						3.862988
−57+58	Snipe	26 55 53.8	− 1.8	52.0	0.0	52.0	0.343980
−59+60	Beaver	27 49 50.7	+ 0.4	51.1	0.0	51.1	9.669189
−50+55	Cat	125 14 18.4	− 1.4	17.0	0.1	16.9	9.912095
			− 2.8		0.1		
	Snipe-Cat						3.876157
	Snipe-Beaver						4.119063
	Beaver-Cat						3.862988
−67+68	Khwain	62 13 29.4	+ 2.0	31.4	0.1	31.3	0.053161
−59+62	Beaver	58 43 17.2	+ 1.8	19.0	0.0	19.0	9.931792
−53+55	Cat	59 03 08.4	+ 1.3	09.7	0.0	09.7	9.933305
			+ 5.1		0.1		
	Khwain-Cat						3.847941
	Khwain-Beaver						3.849454
	Beaver-Cat						3.862988
−71+72	Lim	36 34 55.5	+ 1.1	56.6	0.0	56.6	0.224770
−59+63	Beaver	102 39 45.5	+ 0.7	46.2	0.0	46.2	9.989306
−54+55	Cat	40 45 17.4	− 0.1	17.3	0.1	17.2	9.814795
			+ 1.7		0.1		
	Lim-Cat						4.077064
	Lim-Beaver						3.902553
	Beaver-Khwain						3.849454
−71+73	Lim	59 25 24.7	− 0.3	24.4	0.0	24.4	0.065022
−62+63	Beaver	43 56 28.3	− 1.1	27.2	0.0	27.2	9.841307
−66+67	Khwain	76 38 09.2	− 0.7	08.5	0.1	08.4	9.988077
			− 2.1		0.1		
	Lim-Khwain						3.755783
	Lim-Beaver						3.902553
	Cat-Khwain						3.847941
−72+73	Lim	22 50 29.2	− 1.4	27.8	0.1	27.7	0.410972
−53+54	Cat	18 17 51.0	+ 1.4	52.4	0.0	52.4	9.496870
−66+68	Khwain	138 51 38.6	+ 1.3	39.9	0.0	39.9	9.818151
			+ 1.3		0.1		
	Lim-Khwain						3.755783
	Lim-Cat						4.077064

Final solution of triangles—Continued

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″	″	° ′ ″	
	Beaver-Lim						3.902553
−74+75	South Twin	60 40 06.2	− 1.5	04.7	0.0	04.7	0.059585
−63+65	Beaver	78 24 38.4	− 1.4	37.0	0.1	36.9	9.991054
−70+71	Lim	40 55 17.8	+ 0.6	18.4	0.0	18.4	9.816260
			− 2.3		0.1		
	South Twin-Lim						3.953192
	South Twin-Beaver						3.778398
	South Twin-Beaver						3.778398
−77+78	Ham	35 38 30.7	− 1.9	28.8	0.0	28.8	0.234548
−74+76	South Twin	94 10 29.2	− 2.5	26.7	0.1	26.6	9.998847
−64+65	Beaver	50 11 06.3	− 1.7	04.6	0.0	04.6	9.885424
			− 6.1		0.1		
	Ham-Beaver						4.011793
	Ham-South Twin						3.898370+1 3.898371
	South Twin-Lim						3.953192
−77+79	Ham	85 04 23.4	+ 0.6	24.0	0.1	23.9	0.001608
−75+76	South Twin	33 30 23.0	− 1.0	22.0	0.0	22.0	9.741959
−69+70	Lim	61 25 13.8	+ 0.3	14.1	0.0	14.1	9.943571
			− 0.1		0.1		
	Ham-Lim						3.696759
	Ham-South Twin						3.898371
	Beaver-Lim						3.902553
−78+79	Ham	49 25 52.7	+ 2.5	55.2	0.0	55.2	0.119395
−63+64	Beaver	28 13 32.1	+ 0.3	32.4	0.0	32.4	9.674811
−69+71	Lim	102 20 31.6	+ 0.9	32.5	0.1	32.4	9.989845
			+ 3.7		0.1		
	Ham-Lim						3.696759
	Ham-Beaver						4.011793

Final position computation,

STATION TOWER

						°	′	″
α	Turn to Dundas					357	33	07.7
Second angle	Dundas and Tower					+ 24	17	27.1
α	Turn to Tower					21	50	34.8
$\Delta\alpha$						−	7	07.6
						180		
α'	Tower to Turn					201	43	27.2
					First angle of triangle	42	00	30.8
	°	′	″					
ϕ	54	48	06.742	Turn	λ	130	56	04.052
$\Delta\phi$	−	12	39.415		$\Delta\lambda$	+	8	43.963
ϕ'	54	35	27.327	Tower	λ'	131	04	48.015
			−1					
s	4.402921	s^2	8.80585			−h		2.8803
cos α	9.9676448	$\sin^2\alpha$	9.14128	$(\delta\phi)^2$	5.7609	$s^2\sin^2\alpha$		7.9471
B	8.5097251	C	1.55459	D	2.3672	E		6.4574
h	2.8802909		9.50172		8.1281			7.2848
	″		″					
1st term	+759.0859	3d term	+0.0134					
2d term	+ 0.3175	4th term	−0.0019					
	+759.4034							
			−6					
3d and 4th terms	+ 0.0115	s	4.402921					
		sin α	9.5706188	Arg.		$\Delta\lambda$		2.7193010
$-\Delta\phi$	+759.4149	A′	8.5087480	s	−11	$\sin\frac{1}{2}(\phi+\phi')$		9.9117440
		sec ϕ'	0.2370138	$\Delta\lambda$	+ 5	$\sec\frac{1}{2}(\Delta\phi)$		7
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	54 41 47		2.7193010	Corr.	− 6			2.6310457
			″					″
		$\Delta\lambda$	+523.9635			$-\Delta\alpha$		+427.61

STATION LAZARO

						°	′	″
α	Turn to Tower					21	50	34.8
Second angle	Tower and Lazaro					+ 86	18	37.7
α	Turn to Lazaro					108	09	12.5
$\Delta\alpha$						−	21	10.8
						180		
α'	Lazaro to Turn					287	48	01.7
					First angle of triangle	42	30	15.7
	°	′	″					
ϕ	54	48	06.742	Turn	λ	130	56	04.052
$\Delta\phi$	+	4	51.079		$\Delta\lambda$	+	25	54.366
ϕ'	54	52	57.821	Lazaro	λ'	131	21	58.418
			−1					−1
s	4.464841	s^2	8.92963			−h		2.4681
cos α	9.4935464	$\sin^2\alpha$	9.95564	$(\delta\phi)^2$	4.9281	$s^2\sin^2\alpha$		8.8853
B	8.5097251	C	1.55459	D	2.3672	E		6.4574
h	2.4681125		0.43986		7.2953			7.8108
	″		″					
1st term	−293.8411	3d term	+0.0020					
2d term	+ 2.7534	4th term	+0.0065					
	−291.0877							
			+27					
3d and 4th terms	+ 0.0085	s	4.464841					
		sin α	9.9778267	Arg.		$\Delta\lambda$		3.1915533
$-\Delta\phi$	−291.0792	A′	8.5087409	s	−15	$\sin\frac{1}{2}(\phi+\phi')$		9.9125251
		sec ϕ'	0.2401420	$\Delta\lambda$	+42	$\sec\frac{1}{2}(\Delta\phi)$		
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	54 50 32.3		3.1915533	Corr.	+27			3.1040784
			″					″
		$\Delta\lambda$	+1554.3661			$-\Delta\alpha$		+1270.8

primary triangulation

STATION TOWER

						°	′	″
α	Dundas to Turn					177	33	43.6
Third angle	Tower and Turn					−113	42	02.6
α	Dundas to Tower					63	51	41.0
$\Delta\alpha$						−	7	43.0
						180		
α'	Tower to Dundas					243	43	58.0
	°	′	″					
ϕ	54	38	09.559	Dundas	λ	130	55	20.042
$\Delta\phi$	−	2	42.233		$\Delta\lambda$	+	9	27.973
ϕ'	54	35	27.326	Tower	λ'	131	04	48.015
s	4.055419	s^2	8.11087			−h		2.2091
$\cos\alpha$	9.6439895	$\sin^2\alpha$	9.90630	$(\partial\phi)^2$	4.4203	$s^2\sin^2\alpha$		8.0172
B	8.5097372	C	1.55194	D	2.3681	E		6.4528
h	2.2091457		9.56911		6.7884			6.6791
	″		″					
1st term	+161.8623	3d term	+0.0006					
2d term	+ 0.3708	4th term	−0.0005					
	+162.2331							
3d and 4th terms	+0.0001		+3					
		s	4.055419					
		$\sin\alpha$	9.9531462	Arg.		$\Delta\lambda$		2.7543283
$-\Delta\phi$	+162.2332	A′	8.5087480	s	−2	$\sin\frac{1}{2}(\phi+\phi')$		9.9112981
	° ′ ″	$\sec\phi'$	0.2370138	$\Delta\lambda$	+5	$\sec\frac{1}{2}(\Delta\phi)$		
$\frac{1}{2}(\phi+\phi')$	54 36 48.4		2.7543273	Corr.	+3			2.6656264
	″		″					″
		$\Delta\lambda$	+567.9725			$-\Delta\alpha$		+463.05

STATION LAZARO

						°	′	″
α	Tower to Turn					201	43	27.2
Third angle	Lazaro and Turn					− 51	11	08.5
α	Tower to Lazaro					150	32	18.7
$\Delta\alpha$						−	14	01.3
						180		
α'	Lazaro to Tower					330	18	17.4
	°	′	″					
ϕ	54	35	27.326	Tower	λ	131	04	48.015
$\Delta\phi$	+	17	30.494		$\Delta\lambda$	+	17	10.401
ϕ'	54	52	57.820	Lazaro	λ'	131	21	58.416
								+1
s	4.572302	s^2	9.14459			−h		3.0219
$\cos\alpha$	9.9398618	$\sin^2\alpha$	9.38353	$(\partial\phi)^2$	6.0428	$s^2\sin^2\alpha$		8.5281
B	8.5097404	C	1.55122	D	2.3683	E		6.4516
h	3.0219042		0.07934		8.4111			8.0016
	″		″					
1st term	−1051.7298	3d term	+0.0258					
2d term	+ 1.2004	4th term	+0.0100					
	−1050.5294							
3d and 4th terms	+ 0.0358		−7					
		s	4.572302					
		$\sin\alpha$	9.6918222	Arg.		$\Delta\lambda$		3.0130064
$-\Delta\phi$	−1050.4936	A′	8.5087409	s	−25	$\sin\frac{1}{2}(\phi+\phi')$		9.9119609
	° ′ ″	$\sec\phi'$	0.2401420	$\Delta\lambda$	+18	$\sec\frac{1}{2}(\Delta\phi)$		
$\frac{1}{2}(\phi+\phi')$	54 44 12.6		3.0130064	Corr.	− 7			2.9249673
			″					″
		$\Delta\lambda$	+1030.4012			$-\Delta\alpha$		+841.3

Final position computation,

STATION TOW HILL

						°	′	″
α	Lazaro to Tower					330	18	17.4
Second angle	Tower and Tow Hill					+ 47	10	08.5
α	Lazaro to Tow Hill					17	28	25.9
$\Delta\alpha$						−	21	07.5
						180		
α'	Tow Hill to Lazaro					197	07	18.4
				First angle of triangle		21	40	37.7
	°	′	″					
ϕ	54	52	57.820	Lazaro	λ	131	21	58.417
$\Delta\phi$	−	48	32.022		$\Delta\lambda$	+	25	57.250
ϕ'	54	04	25.798	Tow Hill	λ'	131	47	55.667 −2
s	4.974544	s^2	9.94909			−h		3.4637
$\cos\alpha$	9.9794818	$\sin^2\alpha$	8.95504	$(\delta\phi)^2$	6.9283	$s^2\sin^2\alpha$		8.9043
B	8.5097191	C	1.55589	D	2.3667	E		6.4597
h	3.4637449		0.46002		9.2950			8.8277
	″		″					
1st term	+2909.0080	3d term	+0.1972					
2d term	+ 2.8842	4th term	−0.0673					
	+2911.8922							
3d and 4th terms	+ 0.1299		−117					
		s	4.974544					
		$\sin\alpha$	9.4775129	Arg.		$\Delta\lambda$		3.1923584
$-\Delta\phi$	+2912.0221	A′	8.5087606	s	−159	$\sin\frac{1}{2}(\phi+\phi')$		9.9105687
		sec ϕ'	0.2315526	$\Delta\lambda$	+ 42	$\sec\frac{1}{2}(\Delta\phi)$		108
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	54 28 41.9		3.1923584	Corr.	−117			3.1029379
			″					″
		$\Delta\lambda$	+1557.2502			$-\Delta\alpha$		+1267.5

STATION NICHOLS

						°	′	″
α	Lazaro to Tow Hill					17	28	25.9
Second angle	Tow Hill and Nichols					+ 54	28	48.6
α	Lazaro to Nichols					71	57	14.5
$\Delta\alpha$						−	40	14.3
						180		
α'	Nichols to Lazaro					251	17	00.2
				First angle of triangle		89	23	20.6
	°	′	″					
ϕ	54	52	57.820	Lazaro	λ	131	21	58.417
$\Delta\phi$	−	9	27.129		$\Delta\lambda$	+	49	14.369
ϕ'	54	43	30.691	Nichols	λ'	132	11	12.786 +1
s	4.745169	s^2	9.49034			−h		2.7458
$\cos\alpha$	9.4910534	$\sin^2\alpha$	9.95019	$(\delta\phi)^2$	5.5072	$s^2\sin^2\alpha$		9.4465
B	8.5097191	C	1.55589	D	2.3667	E		6.4597
h	2.7459415		1.00242		7.8739			8.6520
	″		″					
1st term	+557.1107	3d term	+0.0075					
2d term	+ 10.0559	4th term	−0.0449					
	+567.1666							
3d and 4th terms	− 0.0374		+92					
		s	4.745169					
		$\sin\alpha$	9.9780929	Arg.		$\Delta\lambda$		3.4704648
$-\Delta\phi$	+567.1292	A′	8.5087447	s	− 56	$\sin\frac{1}{2}(\phi+\phi')$		9.9123203
		sec ϕ'	0.2384490	$\Delta\lambda$	+148	$\sec\frac{1}{2}(\Delta\phi)$		
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	54 48 14.3		3.4704648	Corr.	+ 92			3.3827851
			″					″
		$\Delta\lambda$	+2954.3694			$-\Delta\alpha$		+2414.27

primary triangulation—Continued

STATION TOW HILL

						°	′	″
α	Tower to Lazaro					150	32	18.7
Third angle	Tow Hill and Lazaro					−111	09	20.3
α	Tower to Tow Hill					39	22	58.4
$\Delta\alpha$						−	35	02.3
						180		
α'	Tow Hill to Tower					218	47	56.1
	°	′	″					
ϕ	54	35	27.326	Tower	λ	131	04	48.015
$\Delta\phi$	−	31	01.527		$\Delta\lambda$	+	43	07.648
ϕ'	54	04	25.799	Tow Hill	λ'	131	47	55.663
			−1					+2
s	4.870159	s^2	9.74032			−h		3.2680
cos α	9.8881363	$\sin^2\alpha$	9.60486	$(\delta\phi)^2$	6.5397	$s^2\sin^2\alpha$		9.3452
B	8.5097404	C	1.55122	D	2.3683	E		6.4516
h	3.2680357		0.89640		8.9080			9.0648
	″		″					
1st term	+1853.6842	3d term	+0.0809					
2d term	+ 7.8777	4th term	−0.1160					
	+1861.5619							
			+16					
3d and 4th terms	− 0.0351	s	4.870159					
		sin α	9.8024314	Arg.		$\Delta\lambda$		3.4129052
$-\Delta\phi$	+1861.5268	A′	8.5087606	s	− 98	sin ½($\phi+\phi'$)		9.9097770
		sec ϕ'	0.2315526	$\Delta\lambda$	+114	sec ½($\Delta\phi$)		44
	° ′ ″							
½($\phi+\phi'$)	54 19 56.6		3.4129052	Corr.	+ 16			3.3226866
			″					″
		$\Delta\lambda$	+2587.6482			$-\Delta\alpha$		+2102.3

STATION NICHOLS

						°	′	″
α	Tow Hill to Lazaro					197	07	18.4
Third angle	Nichols and Lazaro					− 36	08	01.6
α	Tow Hill to Nichols					160	59	16.8
$\Delta\alpha$						−	18	56.0
						180		
α'	Nichols to Tow Hill					340	40	20.8
	°	′	″					
ϕ	54	04	25.798	Tow Hill	λ	131	47	55.665
$\Delta\phi$	+	39	04.894		$\Delta\lambda$	+	23	17.123
ϕ'	54	43	30.692	Nichols	λ'	132	11	12.788
			−1					−1
s	4.885142	s^2	9.77028			−h		3.3706
cos α	9.9756388	$\sin^2\alpha$	9.02582	$(\delta\phi)^2$	6.7403	$s^2\sin^2\alpha$		8.7959
B	8.5097780	C	1.54301	D	2.3709	E		6.4373
h	3.3705588		0.33911		9.1112			8.6038
	″		″					
1st term	−2347.2470	3d term	+0.1292					
2d term	+ 2.1833	4th term	+0.0402					
	−2345.0637							
			−70					
3d and 4th terms	+ 0.1694	s	4.885142					
		sin α	9.5129060	Arg.		$\Delta\lambda$		3.1452347
$-\Delta\phi$	−2344.8943	A′	8.5087447	s	−104	sin ½($\phi+\phi'$)		9.9101427
		sec ϕ'	0.2384490	$\Delta\lambda$	+ 34	sec ½($\Delta\phi$)		70
	° ′ ″							
½($\phi+\phi'$)	54 23 58.9		3.1452347	Corr.	− 70			3.0553844
			″					″
		$\Delta\lambda$	+1397.1232			$-\Delta\alpha$		+1136.02

Final position computation,

STATION KEN

						°	′	″
α	Lazaro to Nichols					71	57	14.5
Second angle	Nichols and Ken					+ 22	32	58.8
α	Lazaro to Ken					94	30	13.3
$\Delta\alpha$						−	30	53.7
						180		
α'	Ken to Lazaro					273	59	19.6
				First angle of triangle		116	59	50.8
	°	′	″					
α	54	52	57.820	Lazaro	λ	131	21	58.417
$\Delta\phi$	+	1	36.965		$\Delta\lambda$	+	37	45.909
ϕ'	54	54	34.785	Ken	λ'	131	59	44.326

s	4.607407	s^2	9.21480			−h	2.0125
cos α	8.8949989	sin² α	9.99731	$(\delta\phi)^2$	4.0250	s^2 sin² α	9.2121
B	8.5097191	C	1.55589	D	2.3667	E	6.4597
h	2.0121250		0.76800		6.3917		7.6843
	″		″				
1st term	−102.8312	3d term	+0.0002				
2d term	+ 5.8614	4th term	+0.0048				
	− 96.9698		+58				
3d and 4th terms	+ 0.0050	s	4.607407				
		sin α	9.9986569	Arg.		$\Delta\lambda$	3.355242
$-\Delta\phi$	− 96.9648	A′	8.5087403	s	−29	sin ½($\phi+\phi'$)	9.912798
	° ′ ″	sec ϕ'	0.2404325	$\Delta\lambda$	+87	sec ½($\Delta\phi$)	
½($\phi+\phi'$)	54 53 46.4		3.3552425	Corr.	+58		3.268040
			″				″
		$\Delta\lambda$	+2265.9094			$-\Delta\alpha$	+1853.70

STATION ROUND

						°	′	″
α	Lazaro and Ken					94	30	13.3
Second angle	Ken and Round					+ 78	57	17.0
α	Lazaro to Round					173	27	30.3
$\Delta\alpha$						−	1	37.8
						180		
α'	Round to Lazaro					353	25	52.5
				First angle of triangle		74	42	36.8
	°	′	″					
ϕ	54	52	57.820	Lazaro	λ	131	21	58.417
$\Delta\phi$	+	9	58.327		$\Delta\lambda$	+	1	59.499
ϕ'	55	02	56.147	Round	λ'	131	23	57.916
								+1
s	4.270074	s^2	8.54013			−h	2.7769
cos α	9.9971633	sin² α	8.11255	$(\delta\phi)^2$	5.5538	s^2 sin² α	6.6527
B	8.5097191	C	1.55589	D	2.3667	E	6.4597
h	2.7769564		8.20857		7.9205		5.8893
	″		″				
1st term	−598.3514	3d term	+0.0083				
2d term	+ 0.0161	4th term	+0.0001				
	−598.3353		−6				
3d and 4th terms	+ 0.0084	s	4.270074				
		sin α	9.0566163	Arg.		$\Delta\lambda$	2.077365
$-\Delta\phi$	−598.3269	A′	8.5087369	s	−6	sin ½($\phi+\phi'$)	9.913183
	° ′ ″	sec ϕ'	0.2419389	$\Delta\lambda$	+0	sec ½($\Delta\phi$)	
½($\phi+\phi'$)	54 57 57		2.0773655	Corr.	−6		1.990548
			″				″
		$\Delta\lambda$	+119.4993			$-\Delta\alpha$	+97.85

primary triangulation—Continued

STATION KEN

						°	′	″
α	Nichols to Lazaro					251	17	00.2
Third angle	Ken and Lazaro					− 40	27	12.6
α	Nichols to Ken					210	49	47.6
$\Delta\alpha$						+	9	22.7
						180		
α'	Ken to Nichols					30	59	10.3
								+.1
	°	′	″					
ϕ	54	43	30.691	Nichols	λ	132	11	12.787
$\Delta\phi$	+	11	04.093		$\Delta\lambda$	−	11	28.462
ϕ'	54	54	34.784	Ken	λ'	131	59	44.325
			+1					+1
s	4.379021	s^2	8.75804			−h		2.8226
$\cos\alpha$	9.9338378	$\sin^2\alpha$	9.41937	$(\delta\phi)^2$	5.6450	$s^2\sin^2\alpha$		8.1775
B	8.5097307	C	1.55337	D	2.3676	E		6.4553
h	2.8225895		9.73078		8.0126			7.4554
	″		″					
1st term	−664.6446	3d term	+0.0103					
2d term	+ 0.5380	4th term	+0.0029					
	−664.1066							
3d and 4th terms	+ 0.0132		−2					
		s	4.379021					
		$\sin\alpha$	9.7096861	Arg.		$\Delta\lambda$		2.837880
$-\Delta\phi$	−664.0934	A′	8.5087403	s	−10	$\sin\frac{1}{2}(\phi+\phi')$		9.912392
		sec ϕ'	0.2404325	$\Delta\lambda$	+ 8	$\sec\frac{1}{2}(\Delta\phi)$		
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	54 49 02.9		2.8378797	Corr.	− 2			2.750272
			″					″
		$\Delta\lambda$	−688.4616			$-\Delta\alpha$		−562.69

STATION ROUND

						°	′	″
α	Ken to Lazaro					273	59	19.6
Third angle	Round and Lazaro					− 26	20	08.1
α	Ken to Round					247	39	11.5
$\Delta\alpha$						+	29	17.8
						180		
α'	Round to Ken					68	08	29.3
	°	′	″					
ϕ	54	54	34.785	Ken	λ	131	59	44.326
$\Delta\phi$	+	8	21.362		$\Delta\lambda$	−	35	46.407
ϕ'	55	02	56.147	Round	λ'	131	23	57.919
								−2
s	4.614937	s^2	9.22087			−h		2.7046
$\cos\alpha$	9.5800256	$\sin^2\alpha$	9.93219	$(\delta\phi)^2$	5.4091	$s^2\sin^2\alpha$		9.1621
B	8.5097172	C	1.55631	D	2.3666	E		6.4604
h	2.7046798		0.71837		7.7757			8.3271
	″		″					
1st term	−506.6171	3d term	+0.0058					
2d term	+ 5.2285	4th term	+0.0212					
	−501.3886							
3d and 4th terms	+ 0.0270		+49					
		s	4.614937					
		$\sin\alpha$	9.9660945	Arg.		$\Delta\lambda$		3.331712
$-\Delta\phi$	−501.3616	A′	8.5087369	s	−30	$\sin\frac{1}{2}(\phi+\phi')$		9.913254
		sec ϕ'	0.2419389	$\Delta\lambda$	+79	$\sec\frac{1}{2}(\Delta\phi)$		
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	54 58 46.0		3.3317122	Corr.	+49			3.244966
			″					″
		$\Delta\lambda$	−2146.4074			$-\Delta\alpha$		−1757.79

Final position computation,

STATION CAT

						°	′	″
α Second angle	Lazaro to Round Round and Cat					173 + 26	27 22	30.3 54.0
α Δα	Lazaro to Cat					199 +	50 4	24.3 21.4
α′	Cat to Lazaro					180 19 89	54 47	45.7 51.7
					First angle of triangle			
	°	′	″					
φ Δφ	54 +	52 8	57.820 28.290	Lazaro	λ Δλ	131 −	21 5	58.417 19.289
φ′	55	01	26.110	Cat	λ′	131	16	39.128 +1
s cos α B	4.223072 9.9734251 8.5097191	s² sin² α C	8.44612 9.06164 1.55589	(δφ)² D	5.4124 2.3667	−h s² sin² α E		2.7062 7.5078 6.4597
h	2.7062162		9.06365		7.7791			6.6737
	″		″					
1st term 2d term	−508.4124 + 0.1158	3d term 4th term	+0.0060 +0.0005					
	−508.2966							
3d and 4th terms	+ 0.0065		−3					
		s sin α A′ sec φ′	4.223072 9.5307069 8.5087375 0.2416677	Arg. s Δλ	−5 +2	Δλ sin ½(φ+φ′) sec ½(Δφ)		2.504184 9.913117
−Δφ	−508.2901							
	° ′ ″							
½(φ+φ′)	54 57 42.0		2.5041838	Corr.	−3			2.417301
			″					″
		Δλ	−319.2889			−Δα		−261.4

STATION BEAVER

						°	′	″
α Second angle	Cat to Round Round and Beaver					109 + 87	42 34	37.4 54.0
α Δα	Cat to Beaver					197 +	17 1	31.4 40.2
α′	Beaver to Cat					180 17 49	19 58	11.6 13.2
					First angle of triangle			
	°	′	″					
φ Δφ	55 +	01 3	26.110 45.204	Cat	λ Δλ	131 −	16 2	39.129 02.231
φ′	55	05	11.314	Beaver	λ′	131	14	36.898
s cos α B	3.862988 9.9799133 8.5097090	s² sin² α C	7.7260 8.9465 1.5584	(δφ)² D	4.705 2.366	−h s² sin² α E		2.353 6.672 6.464
h	2.3526103		8.2309		7.071			5.489
	″		″					
1st term 2d term	−225.2218 + 0.0170	3d term 4th term	+0.0012					
	−225.2048							
3d and 4th terms	+ 0.0012		−1					
		s sin α A′ sec φ′	3.862988 9.4731109 8.5087360 0.2423463	Arg. s Δλ	−1 0	Δλ sin ½(φ+φ′) sec ½(Δφ)		2.087181 9.913657
−Δφ	−225.2036							
	° ′ ″							
½(φ+φ′)	55 03 18.7		2.0871811	Corr.	−1			2.000838
			″					″
		Δλ	−122.2309			−Δα		−100.19

primary triangulation—Continued

STATION CAT

						°	′	″
α	Round to Lazaro					353	25	52.5
Third angle	Cat and Lazaro					− 63	49	14.7
α	Round to Cat					289	36	37.8
$\Delta\alpha$						+	5	59.6
						180		
α'	Cat to Round					109	42	37.4
	°	′	″					
ϕ	55	02	56.147	Round	λ	131	23	57.917
$\Delta\phi$	−	1	30.037		$\Delta\lambda$	−	7	18.788
ϕ'	55	01	26.110	Cat	λ'	131	16	39.129
s	3.917800	s^2	7.8356			−h		1.9535
$\cos\alpha$	9.5258533	$\sin^2\alpha$	9.9481	$(\partial\phi)^2$	3.9069	$s^2\sin^2\alpha$		7.7837
B	8.5097071	C	1.5586	D	2.3658	E		6.4643
h	1.9533604		9.3423		6.2727			0.2015
	″		″					
1st term	+89.8174	3d term	+0.0002					
2d term	+ 0.2199	4th term	−0.0002					
	+90.0373							
3d and 4th terms	0		+2					
		s	3.917800					
		$\sin\alpha$	9.9740491	Arg.		$\Delta\lambda$		2.642254
$-\Delta\phi$	+90.0373	A′	8.5087375	s	−1	$\sin\frac{1}{2}(\phi+\phi')$		9.913558
		$\sec\phi'$	0.2416677	$\Delta\lambda$	+3	$\sec\frac{1}{2}(\Delta\phi)$		
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	55 02 11.1		2.6422545	Corr.	+2			2.555812
			″					″
		$\Delta\lambda$	−438.7877			$-\Delta\alpha$		−359.59

STATION BEAVER

						°	′	″
α	Round to Cat					289	36	37.8
Third angle	Beaver and Cat					− 42	26	53.0
α	Round to Beaver					247	09	44.8
$\Delta\alpha$						+	7	39.9
						180		
α'	Beaver to Round					67	17	24.7
								+.1
	°	′	″					
ϕ	55	02	56.147	Round	λ	131	23	57.917
$\Delta\phi$	+	2	15.166		$\Delta\lambda$	−	9	21.019
ϕ'	55	05	11.313	Beaver	λ'	131	14	36.898
			+1					
s	4.033348	s^2	8.0667			−h		2.132
$\cos\alpha$	9.5889657	$\sin^2\alpha$	9.9291	$(\partial\phi)^2$	4.264	$s^2\sin^2\alpha$		7.996
B	8.5097071	C	1.5586	D	2.366	E		6.464
h	2.1320208		9.5544		6.630			6.592
	″		″					
1st term	−135.5254	3d term	+0.0004					
2d term	+ 0.3584	4th term	+0.0004					
	−135.1670							
3d and 4th terms	+ 0.0008		+3					
		s	4.033348					
		$\sin\alpha$	9.9645467	Arg.		$\Delta\lambda$		2.748977
$-\Delta\phi$	−135.1662	A′	8.5087360	s	−2	$\sin\frac{1}{2}(\phi+\phi')$		9.913723
		$\sec\phi'$	0.2423463	$\Delta\lambda$	+5	$\sec\frac{1}{2}(\Delta\phi)$		
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	55 04 03.7		2.7489773	Corr.	+3			2.662700
			″					″
		$\Delta\lambda$	−561.0186			$-\Delta\alpha$		−459.94

Final position computation,

STATION LIM

						°	′	″
α	Beaver to Cat					17	19	11.6
Second angle	Cat and Lim					+102	39	46.2
α	Beaver to Lim					119	58	57.8
$\Delta\alpha$						−	5	20.3
						180		
α'	Lim to Beaver					299	53	37.5
				First angle of triangle		36	34	56.6
	°	′	″					
ϕ	55	05	11.314	Beaver	λ	131	14	36.898
$\Delta\phi$	+	2	08.948		$\Delta\lambda$	+	6	30.508
ϕ'	55	07	20.262	Lim	λ'	131	21	07.406
								+1
s	3.902553	s^2	7.8051			−h		2.111
cos α	9.6987431	$\sin^2\alpha$	9.8751	$(\delta\phi)^2$	4.222	$s^2\sin^2\alpha$		7.680
B	8.5097044	C	1.5591	D	2.366	E		6.465
h	2.1110005		9.2393		6.588			6.256
	″		″					
1st term	−129.1221	3d term	+0.0004					
2d term	0.1735	4th term	+0.0002					
	−128.9486							
3d and 4th terms	+ 0.0006		+2					
		s	3.902553					
		sin α	9.9376062	Arg.		$\Delta\lambda$		2.591630
$-\Delta\phi$	−128.9480	A′	8.5087351	s	−1	sin $\frac{1}{2}(\phi+\phi')$		9.913918
		sec ϕ'	0.2427356	$\Delta\lambda$	+3	sec $\frac{1}{2}(\Delta\phi)$		
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	55 06 15.8		2.5916301	Corr.	+2			2.505548
			″					″
		$\Delta\lambda$	+390.5082			$-\Delta\alpha$		+320.3

STATION SOUTH TWIN

						°	′	″
α	Beaver to Lim					119	58	57.8
Second angle	Lim and South Twin					+ 78	24	37.0
α	Beaver to South Twin					198	23	34.8
$\Delta\alpha$						+	1	27.7
						180		
α'	South Twin to Beaver					18	25	02.5
				First angle of triangle		60	40	04.7
	°	′	″					
ϕ	55	05	11.314	Beaver	λ	131	14	36.898
$\Delta\phi$	+	3	04.203		$\Delta\lambda$	−	1	46.925
ϕ'	55	08	15.517	South Twin	λ'	131	12	49.973
								+1
			15.517					49.974
s	3.778398	s^2	7.5568			−h		2.265
cos α	9.9772271	$\sin^2\alpha$	8.9984	$(\delta\phi)^2$	4.530	$s^2\sin^2\alpha$		6.555
B	8.5097044	C	1.5591	D	2.366	E		6.465
h	2.2653295		8.1143		6.896			5.285
	″		″					
1st term	−184.2169	3d term	+0.0008					
2d term	+ 0.0130	4th term						
	−184.2039							
3d and 4th terms	+ 0.0008		−1					
		s	3.778398					
		sin α	9.4990449	Arg.		$\Delta\lambda$		2.029080
$-\Delta\phi$	−184.2031	A′	8.5087347	s	−1	sin $\frac{1}{2}(\phi+\phi')$		9.913958
		sec ϕ'	0.2429024	$\Delta\lambda$	0	sec $\frac{1}{2}(\Delta\phi)$		
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	55 06 43.4		2.0290799	Corr.	−1			1.943038
			″					″
		$\Delta\lambda$	−106.9252			$-\Delta\alpha$		−87.7

primary triangulation—Continued

STATION LIM

						°	′	″
α	Cat to Beaver					197	17	31.4
Third angle	Lim and Beaver					− 40	45	17.3
α	Cat to Lim					156	32	14.1
$\Delta\alpha$						−	3	40.0
α'	Lim to Cat					180 336	28	34.1
	°	′	″					
ϕ	55	01	26.110	Cat	λ	131	16	39.129
$\Delta\phi$	+	5	54.152		$\Delta\lambda$	+	4	28.278
ϕ'	55	07	20.262	Lim	λ'	131	21	07.407
s	4.077064	s^2	8.1541			−h		2.549
cos α	9.9625205	$\sin^2\alpha$	9.1999	$(\delta\phi)^2$	5.099	$s^2\sin^2\alpha$		7.354
B	8.5097090	C	1.5584	D	2.366	E		6.464
h	2.5492935		8.9124		7.465			6.367
	″		″					
1st term	−354.2367	3d term	+0.0029					
2d term	+ 0.0817	4th term	+0.0002					
	−354.1550							
3d and 4th terms	+ 0.0031	s	4.077064					
		sin α	9.6000497	Arg.		$\Delta\lambda$		2.428584
$-\Delta\phi$	−354.1519	A′	8.5087351	s	−2	sin $\frac{1}{2}(\phi+\phi')$		9.913752
	° ′ ″	sec ϕ'	0.2427356	$\Delta\lambda$	+2	sec $\frac{1}{2}(\Delta\phi)$		
$\frac{1}{2}(\phi+\phi')$	55 04 23.2		2.4285844	Corr.	0			2.342336
			″					″
		$\Delta\lambda$	+268.2776			$-\Delta\alpha$		+219.96

STATION SOUTH TWIN

						°	′	″
α	Lim to Beaver					299	53	37.5
Third angle	South Twin and Beaver					− 40	55	18.4
α	Lim to South Twin					258	58	19.1
$\Delta\alpha$						+	6	48.1
α'	South Twin to Lim					180 79	05	07.2
	°	′	″					
ϕ	55	07	20.262	Lim	λ	131	21	07.407
$\Delta\phi$	+		55.255	,	$\Delta\lambda$	−	8	17.433
ϕ'	55	08	15.517	South Twin	λ'	131	12	49.974
s	3.953192	s^2	7.9064			−h		1.744
cos α	9.2816903	$\sin^2\alpha$	9.9838	$(\delta\phi)^2$	3.488	$s^2\sin^2\alpha$		7.890
B	8.5097018	C	1.5597	D	2.365	E		6.467
h	1.7445841		9.4499		5.853			6.101
	″		″					
1st term	−55.5372	3d term	+0.0001					
2d term	+ 0.2818	4th term	+0.0001					
	−55.2554							
3d and 4th terms	+ 0.0002		+2					
		s	3.953192					
		sin α	9.9919052	Arg.		$\Delta\lambda$		2.696734
$-\Delta\phi$	−55.2552	A′	8.5087347	s	−2	sin $\frac{1}{2}(\phi+\phi')$		9.914053
	° ′ ″	sec ϕ'	0.2429024	$\Delta\lambda$	+4	sec $\frac{1}{2}(\Delta\phi)$		
$\frac{1}{2}(\phi+\phi')$	55 07 47.9		2.6967345	Corr.	+2			2.610787
			″					″
		$\Delta\lambda$	−497.4329			$-\Delta\alpha$		−408.12

Final position computation,

STATION SEAL

						°	′	″
α	Nichols to Ken					210	49	47.6
Second angle	Ken and Seal					+ 27	51	41.5
α	Nichols to Seal					238	41	29.1
$\Delta\alpha$						+	29	04.7
						180		
α'	Seal to Nichols					59	10	33.8
								+.1
					First angle of triangle	25	37	10.6
	°	′	″					
ϕ	54	43	30.691	Nichols	λ	132	11	12.787
$\Delta\phi$	+	12	22.365		$\Delta\lambda$	−	35	34.411
ϕ'	54	55	53.056	Seal	λ'	131	35	38.376
			−1					+1
s	4.648215	s^2	9.2964			−h		2.874
$\cos\alpha$	9.7157086	$\sin^2\alpha$	9.8633	$(\delta\phi)^2$	5.747	$s^2\sin^2\alpha$		9.160
B	8.5097307	C	1.5534	D	2.368	E		6.455
h	2.8736543		0.7131		8.115			8.489
	″		″					
1st term	−747.5741	3d term	+0.0130					
2d term	+ 5.1654	4th term	+0.0308					
	−742.4087							
3d and 4th terms	+ 0.0438		+4					
		s	4.648215					
		$\sin\alpha$	9.931652	Arg.		$\Delta\lambda$		3.329278
$-\Delta\phi$	−742.3649	A′	8.508740	s	−4	$\sin\frac{1}{2}(\phi+\phi')$		9.912450
		$\sec\phi'$	0.240667	$\Delta\lambda$	+8	$\sec\frac{1}{2}(\Delta\phi)$		
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	54 49 41.9		3.329278	Corr.	+4			3.241728
			″					″
		$\Delta\lambda$	−2134.4108			$-\Delta\alpha$		−1744.72

STATION MID

						°	′	″
α	Round to Lazaro					353	25	52.5
Second angle	Lazaro and Mid					+ 51	05	14.8
α	Round to Mid					44	31	07.3
$\Delta\alpha$						−	6	56.5
						180		
α'	Mid to Round					224	24	10.8
					First angle of triangle	85	15	12.2
	°	′	″					
ϕ	55	02	56.147	Round	λ	131	23	57.917
$\Delta\phi$	−	4	57.777		$\Delta\lambda$	+	8	28.436
ϕ'	54	57	58.370	Mid	λ'	131	32	26.353
								−1
s	4.110647	s^2	8.2213			−h		2.473
$\cos\alpha$	9.853103	$\sin^2\alpha$	9.6916	$(\delta\phi)^2$	4.947	$s^2\sin^2\alpha$		7.913
B	8.509707	C	1.5586	D	2.366	E		6.464
h	2.473457		9.4715		7.313			6.850
	″		″					
1st term	+297.4795	3d term	+0.0021					
2d term	+ 0.2961	4th term	−0.0007					
	+297.7756							
3d and 4th terms	+ 0.0014							
		s	4.110647					
		$\sin\alpha$	9.845806	Arg.		$\Delta\lambda$		2.706236
$-\Delta\phi$	+297.7770	A′	8.508740	s		$\sin\frac{1}{2}(\phi+\phi')$		9.913405
		$\sec\phi'$	0.241043	$\Delta\lambda$		$\sec\frac{1}{2}(\Delta\phi)$		
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	55 00 27.3		2.706236	Corr.				2.619641
			″					″
		$\Delta\lambda$	+508.4356			$-\Delta\alpha$		+416.52

primary triangulation—Continued

STATION SEAL

						°	′	″
α	Ken to Nichols					30	59	10.4
Third angle	Seal and Nichols					−126	31	09.1
α	Ken to Seal					264	28	01.3
$\Delta\alpha$						+	19	43.3
						180		
α'	Seal to Ken					84	47	44.6
								−.1
	°	′	″					
ϕ	54	54	34.785	Ken	λ	131	59	44.326
$\Delta\phi$	+	1	18.270		$\Delta\lambda$	−	24	05.949
ϕ'	54	55	53.055	Seal	λ'	131	35	38.377
s	4.412771	s^2	8.8255			−h		1.907
cos α	8.984161	$\sin^2\alpha$	9.9960	$(\delta\phi)^2$	3.813	$s^2 \sin^2 \alpha$		8.821
B	8.509717	C	1.5563	D	2.367	E		6.460
h	1.906649		0.3778		6.180			7.188
	″		″					
1st term	−80.6583	3d term	+0.0002					
2d term	+ 2.3867	4th term	+0.0015					
	−78.2716							
3d and 4th terms	+ 0.0017		+3					
		s	4.412771					
		sin α	9.997972	Arg.		$\Delta\lambda$		3.160153
$-\Delta\phi$	−78.2699	A′	8.508740	s	−1	sin ½($\phi+\phi'$)		9.912942
	° ′ ″	sec ϕ'	0.240667	$\Delta\lambda$	+4	sec ½($\Delta\phi$)		
½($\phi+\phi'$)	54 55 13.9		3.160153	Corr.	+3			3.073095
			″					″
		$\Delta\lambda$	−1445.949			$-\Delta\alpha$		−1183.30

STATION MID

						°	′	″
α	Lazaro to Round					173	27	30.3
Third angle	Mid and Round					− 43	39	33.5
α	Lazaro to Mid					129	47	56.8
$\Delta\alpha$						−	8	33.9
						180		
α'	Mid to Lazaro					309	39	22.9
								+.1
	°	′	″					
ϕ	54	52	57.820	Lazaro	λ	131	21	58.417
$\Delta\phi$	+	5	00.550		$\Delta\lambda$	+	10	27.934
ϕ'	54	57	58.370	Mid	λ'	131	32	26.351
								+1
s	4.162604	s^2	8.3252			−h		2.479
cos α	9.806246	$\sin^2\alpha$	9.7710	$(\delta\phi)^2$	4.957	$s^2 \sin^2 \alpha$		8.096
B	8.509719	C	1.5559	D	2.367	E		6.460
h	2.478569		9.6521		7.324			7.035
	″		″					
1st term	−301.0017	3d term	+0.0021					
2d term	+ 0.4488	4th term	+0.0011					
	−300.5529							
3d and 4th terms	+ 0.0032							
		s	4.162604					
		sin α	9.885527	Arg.		$\Delta\lambda$		2.797914
$-\Delta\phi$	−300.5497	A′	8.508740	s		sin ½($\phi+\phi'$)		9.912963
	° ′ ″	sec ϕ'	0.241043	$\Delta\lambda$		sec ½($\Delta\phi$)		
½($\phi+\phi'$)	54 55 28.1		2.797914	Corr.				2.710877
			″					″
		$\Delta\lambda$	+627.9340			$-\Delta\alpha$		+513.90

Final position computation,

STATION SPUR

						°	′	″
α Second angle	Round to Cat Cat and Spur					289 +111	36 30	37.8 08.5
α $\Delta\alpha$	Round to Spur					41 −	06 4	46.3 22.7
α'	Spur to Round			First angle of triangle		180 221 33	02 20	23.6 38.3
	°	′	″					
ϕ $\Delta\phi$	55 −	02 3	56.147 31.319	Round	λ $\Delta\lambda$	131 +	23 5	57.917 20.567
ϕ'	54	59	24.828 −1	Spur	λ'	131	29	18.484
s cos α B	3.937953 9.877035 8.509707	s^2 sin² α C	7.8759 9.6358 1.5586	$(\partial\phi)^2$ D	4.649 2.366	−h s^2 sin² α E		2.325 7.512 6.464
h	2.324695		9.0703		7.015			6.301
1st term 2d term	″ +211.2006 + 0.1176	3d term 4th term	″ +0.0010 −0.0002					
	+211.3182							
3d and 4th terms	+ 0.0008	s sin α A′ sec ϕ'	3.937953 9.817925 8.508738 0.241303	Arg. s $\Delta\lambda$		$\Delta\lambda$ sin ½(ϕ+ϕ') sec ½($\Delta\phi$)		2.505919 9.913468
$-\Delta\phi$	+211.3190							
½(ϕ+ϕ')	° ′ ″ 55 01 10.5		2.505919	Corr.				2.419387
		$\Delta\lambda$	″ +320.5672			$-\Delta\alpha$		″ +262.66

STATION SNIPE

						°	′	″
α Second angle	Round to Cat Cat and Snipe					289 + 63	36 10	37.8 27.2
α $\Delta\alpha$	Round to Snipe					352 +	47	05.0 29.8
α'	Snipe to Round			First angle of triangle		180 172 79	47 10	34.8 09.9
	°	′	″					
ϕ $\Delta\phi$	55 −	02 2	56.147 45.140	Round	λ $\Delta\lambda$	131 −	23	57.917 36.371
ϕ'	55	00	11.007	Snipe	λ'	131	23	21.546
s cos α B	3.711593 9.996547 8.509707	s^2 sin² α C	7.4232 8.1980 1.5586	$(\partial\phi)^2$ D	4.436 2.366	−h s^2 sin² α E		2.218 5.021 6.464
h	2.217847		7.1798		6.802			4.303
1st term 2d term	″ +165.1380 + 0.0015	3d term 4th term	″ +0.0006					
	+165.1395							
3d and 4th terms	+ 0.0006	s sin α A′ sec ϕ'	3.711593 9.098982 8.508738 0.241442	Arg. s $\Delta\lambda$		$\Delta\lambda$ sin ½(ϕ+ϕ') sec ½($\Delta\phi$)		1.560755 9.913502
$-\Delta\phi$	+165.1401							
½(ϕ+ϕ')	° ′ ″ 55 01 33.6		1.560755	Corr.				1.474257
		$\Delta\lambda$	″ −36.3710			$-\Delta\alpha$		″ −29.80

primary triangulation—Continued

STATION SPUR

						°	′	″
α	Cat to Round					109	42	37.4
Third angle	Spur and Round					− 35	09	13.4
α	Cat to Spur					74	33	24.0
$\Delta\alpha$						−	10	22.1
α'	Spur to Cat					180 254	23	01.9
	°	′	″					
ϕ	55	01	26.110	Cat	λ	131	16	39.129
$\Delta\phi$	−	2	01.283		$\Delta\lambda$	+	12	39.355
ϕ'	54	59	24.827	Spur	λ'	131	29	18.484
s	4.146374	s^2	8.2927			−h		2.081
cos α	9.425347	$\sin^2\alpha$	9.9681	$(\partial\phi)^2$	4.163	$s^2\sin^2\alpha$		8.261
B	8.509709	C	1.5582	D	2.366	E		6.464
h	2.081430		9.8190		6.529			6.806
	″		″					
1st term	+120.6230	3d term	+0.0003					
2d term	+ 0.6592	4th term	−0.0006					
	+121.2832							
3d and 4th terms	−0.0003		+1					
		s	4.146374					
		$\sin\alpha$	9.984029	Arg.		$\Delta\lambda$		2.880445
$-\Delta\phi$	+121.2829	A′	8.508738	s		$\sin\frac{1}{2}(\phi+\phi')$		9.913402
		$\sec\phi'$	0.241303	$\Delta\lambda$	+1	$\sec\frac{1}{2}(\Delta\phi)$		
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	55 00 25.5		2.880445	Corr.	+1			2.793847
			″					″
		$\Delta\lambda$	+759.3552			$-\Delta\alpha$		+622.07

STATION SNIPE

						°	′	″
α	Cat to Round					109	42	37.4
Third angle	Snipe and Round					− 37	39	23.0
α	Cat to Snipe					72	03	14.4
$\Delta\alpha$						−	5	29.7
α'	Snipe to Cat					180 251	57	44.7
	°	′	″					
ϕ	55	01	26.110	Cat	λ	131	16	39.129
$\Delta\phi$	−	1	15.103		$\Delta\lambda$	+	6	42.417
ϕ'	55	00	11.007	Snipe	λ'	131	23	21.546
s	3.876157	s^2	7.7523			−h		1.975
cos α	9.488721	$\sin^2\alpha$	9.9567	$(\partial\phi)^2$	3.949	$s^2\sin^2\alpha$		7.709
B	8.509709	C	1.5582	D	2.366	E		6.464
h	1.874587		9.2672		6.315			6.148
	″		″					
1st term	+74.9181	3d term	+0.0002					
2d term	+ 0.1850	4th term	−0.0001					
	+75.1031							
3d and 4th terms	+ 0.0001		3.876157					
		$\sin\alpha$	9.978339	Arg.		$\Delta\lambda$		2.604676
$-\Delta\phi$	+75.1032	A′	8.508738	s		$\sin\frac{1}{2}(\phi+\phi')$		9.913436
		$\sec\phi'$	0.241442	$\Delta\lambda$		$\sec\frac{1}{2}(\Delta\phi)$		
	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	55 00 48.6		2.604676	Corr.				2.518112
			″					″
		$\Delta\lambda$	+402.4167			$-\Delta\alpha$		+329.69

Final position computation, primary triangulation—Continued

STATION KHWAIN

						°	′	″
α Second angle	Beaver to Cat Cat and Khwain					17 +58	19 43	11.6 19.0
α $\Delta\alpha$	Beaver to Khwain					76 −	02 5	30.6 17.1
α'	Khwain to Beaver			First angle of triangle		180 255 62	57 13	13.5 31.4
	°	′	″					
ϕ $\Delta\phi$	55 −	05	11.314 55.322	Beaver	λ $\Delta\lambda$	131 +	14 6	36.898 26.681
ϕ'	55	04	15.992 −1	Khwain	λ'	131	21	03.579
s $\cos\alpha$ B	3.849454 9.3824014 8.5097044	s^2 $\sin^2\alpha$ C	7.6989 9.9740 1.5591	$(\delta\phi)^2$ D	3.483 2.366	−h $s^2\sin^2\alpha$ E		1.742 7.673 6.465
h	1.7415598		9.2320		5.849			5.880
	″		″					
1st term 2d term	+55.1518 + 0.1706	3d term 4th term	+0.0001 −0.0001					
3d and 4th terms	+55.3224 0.0000	s $\sin\alpha$	3.849454 9.986983	Arg.		$\Delta\lambda$		2.587353
$-\Delta\phi$	+55.3224	A′ $\sec\phi'$	8.508736 0.242180	s $\Delta\lambda$		$\sin\frac{1}{2}(\phi+\phi')$ $\sec\frac{1}{2}(\Delta\phi)$		9.913782
$\frac{1}{2}(\phi+\phi')$	° ′ ″ 55 04 43.7		2.587353	Corr.				2.501135
			″					″
		$\Delta\lambda$	+386.6812			$-\Delta\alpha$		+317.06

STATION KHWAIN*

						°	′	″
α Third angle	Cat to Beaver Khwain and Beaver					197 − 59	17 03	31.4 09.7
α $\Delta\alpha$	Cat to Khwain					138 −	14 3	21.7 36.8
α'	Khwain to Cat					180 318	10	44.9
	°	′	″					
ϕ $\Delta\phi$	55 +	01 2	26.110 49.881	Cat	λ $\Delta\lambda$	131 +	16 4	39.129 24.450
ϕ'	55	04	15.991	Khwain	λ'	131	21	03.579
s $\cos\alpha$ B	3.847941 9.872700 8.509709	s^2 $\sin^2\alpha$ C	7.6959 9.6470 1.5584	$(\delta\phi)^2$ D	4.461 2.366	−h $s^2\sin^2\alpha$ E		2.230 7.343 6.464
h	2.230350		8.9013		6.827			6.037
	″		″					
1st term 2d term	−169.9613 + 0.0797	3d term 4th term	+0.0007 +0.0001					
3d and 4th terms	−169.8816 + 0.0008	s $\sin\alpha$	3.847941 9.823487	Arg.		$\Delta\lambda$		2.422344
$-\Delta\phi$	−169.8808	A′ $\sec\phi'$	8.508736 0.242180	s $\Delta\lambda$		$\sin\frac{1}{2}(\phi+\phi')$ $\sec\frac{1}{2}(\Delta\phi)$		9.913616
$\frac{1}{2}(\phi+\phi')$	° ′ ″ 55 02 51.1		2.422344	Corr.				2.335960
			″					″
		$\Delta\lambda$	+264.4502			$-\Delta\alpha$		+216.75

* This is right-hand portion of computation above.

List of geographic positions—Felice Strait, Alaska, southeast Alaska datum

Station	Latitude and longitude			Seconds in meters	Azimuth			Back azimuth			To station	Distance	Logarithm
	°	′	″		°	′	″	°	′	″		*Meters*	
Tower	54	35	27.326	845.0	201	43	27.2	21	50	34.8	Turn	25288.4	4.402921
1907	131	04	48.015	862.3	248	43	58.0	63	51	41.0	Dundas	11361.1	4.055419
Lazaro	54	52	57.820	1788.0	287	48	01.7	108	09	12.5	Turn	29163.6	4.464841
1907	131	21	58.417	1041.5	313	40	37.9	134	02	23.3	Dundas	39641.7	4.598152
					330	18	17.4	150	32	18.7	Tower	37351.0	4.572302
Tow Hill	54	04	25.798	797.6	197	07	18.4	17	28	25.9	Lazaro	94307.0	4.974544
1908	131	47	55.665	1012.2	218	47	56.1	39	22	58.4	Tower	74158.2	4.870159
Nichols	54	43	30.691	949.0	251	17	00.2	71	57	14.5	Lazaro	55612.1	4.745169
1907	132	11	12.787	228.9	281	21	55.6	102	16	06.1	Tower	72983.6	4.863225
					340	40	20.8	160	59	16.8	Tow Hill	76761.2	4.885142
Ken	54	54	34.785	1075.7	273	59	19.6	94	30	13.3	Lazaro	40495.5	4.607407
1907	131	59	44.326	789.7	30	59	10.4	210	49	47.6	Nichols	23934.3	4.379021
Seal	54	55	53.055	1640.6	290	15	23.0	110	26	33.9	Lazaro	15582.6	4.192639
1907	131	35	38.377	683.4	59	10	33.9	238	41	29.1	Nichols	44485.1	4.648215
					84	47	44.5	264	28	01.3	Ken	25868.5	4.412771
Mid	54	57	58.370	1805.0	309	39	23.0	129	47	56.8	Lazaro	14541.3	4.162604
1914	131	32	26.352	468.8	78	00	16.4	257	37	55.6	Ken	29834.6	4.474720
Round	55	02	56.147	1736.3	353	25	52.5	173	27	30.3	Lazaro	18624.0	4.270074
1914	131	23	57.917	1028.3	44	31	07.3	224	24	10.8	Mid	12901.7	4.110647
					68	08	29.3	247	39	11.5	Ken	41203.8	4.614937
Spur	54	59	24.827	767.7	221	02	23.6	41	06	46.3	Round	8668.7	3.937953
1914	131	29	18.484	328.7	326	44	11.7	146	50	11.9	Lazaro	14304.3	4.155468
Cat	55	01	26.110	807.4	19	54	45.7	199	50	24.3	Lazaro	16713.7	4.223072
1914	131	16	39.129	695.2	74	33	24.0	254	23	01.9	Spur	14007.9	4.146374
					109	42	37.4	289	36	37.8	Round	8275.6	3.917800
Snipe	55	00	11.007	340.4	172	47	34.8	352	47	05.0	Round	5147.5	3.711593
1914	131	23	21.546	383.0	251	57	44.7	72	03	14.4	Cat	7518.9	3.876157
Beaver	55	05	11.314	349.9	17	19	11.6	197	17	31.4	Cat	7294.4	3.862988
1914	131	14	36.898	654.5	45	09	02.7	225	01	52.7	Snipe	13154.2	4.119063
					67	17	24.8	247	09	44.8	Round	10798.1	4.033348
					148	12	30.2	328	08	19.3	Ham	10275.3	4.011793
					198	23	34.8	18	25	02.5	South Twin	6003.4	3.778398
Khwain	55	04	15.991	494.5	255	57	13.5	76	02	30.6	Beaver	7070.6	3.849454
1914	131	21	03.579	63.5	318	10	44.9	138	14	21.7	Cat	7046.0	3.847941
Lim	55	07	20.262	626.6	197	33	05.0	17	34	14.5	Ham	4974.6	3.696759
1914	131	21	07.407	131.3	258	58	19.1	79	05	07.2	South Twin	8978.3	3.953192
					299	53	37.5	119	58	57.8	Beaver	7990.1	3.902553
					336	28	34.1	156	32	14.1	Cat	11941.6	4.077064
					359	19	01.9	179	19	05.0	Khwain	5698.8	3.755783

ADJUSTMENT OF TRIANGULATION BY THE METHOD OF VARIATION OF GEOGRAPHIC COORDINATES

DEVELOPMENT OF FORMULAS

A scheme of triangulation may be adjusted not only by means of equations of condition * but also by means of observation equations in which the number of independent unknowns is just sufficient to

* There is some confusion in usage as to the term *equation of condition*, or *condition equation*. In this publication the meaning is restricted to that of an equation expressing some condition which is imposed *a priori* and independently of anything arising from the observations themselves, and which must be rigorously satisfied by the adopted results. An equation which expresses the results of an observation, and which will, in general, be satisfied only approximately by the adopted results, is not herein termed an equation of condition, but an *observation equation*.

determine the entire triangulation. These independent unknowns may very conveniently be taken as the small corrections to the assumed approximate geographic coordinates (that is, the latitudes and longitudes) of the points in the triangulation. To form the observation equations the relation must be found that connects the small change in the direction of a line with the small arbitrary changes in the geographic coordinates of its ends. The following derivation of the formulas is based on the formulas for the computation of geographic positions given in U. S. Coast and Geodetic Survey Special Publication No. 8 and on the notation there used. A "δ" before the symbol of a quantity denotes a small arbitrary change in that quantity. ϕ and λ are, respectively, the latitude and longitude of A_1, the initial point of the position computation, which may also be thought of as the occupied point, while ϕ' and λ' are the latitude and longitude of B_1, the terminal point in the position computation, which may also be thought of as the point sighted on. By definition also,

$$\Delta\phi = \phi' - \phi$$

$$\Delta\lambda = \lambda' - \lambda$$

$$h = sB \cos \alpha$$

α is the azimuth at A_1 of the line A_1B_1 reckoned from the south toward the west.

$$\Delta\lambda = sA' \sec \phi' \sin \alpha$$

$$\cos \alpha = \frac{h}{sB}$$

$$\sin \alpha = \frac{\Delta\lambda}{sA' \sec \phi'}$$

$$\cot \alpha = \frac{A' \sec \phi'}{B} \frac{h}{\Delta\lambda}$$

The meaning of A' and B is explained in Special Publication No. 8.

By differentiating the preceding equation and neglecting the effects of changes in A', B, and $\sec \phi'$ there results:

$$-\operatorname{cosec}^2 \alpha \, d\alpha = \frac{A' \sec \phi'}{B} \left[\frac{\Delta\lambda \delta h - h\delta(\Delta\lambda)}{(\Delta\lambda)^2} \right]$$

Multiplying by $-\sin^2 \alpha = -\dfrac{(\Delta\lambda)^2}{s^2 A'^2 \sec^2 \phi'}$

and dividing by arc 1″ in order to express $d\alpha$ in seconds instead of in radians gives,

$$d\alpha \text{ in seconds} = \frac{1}{s^2BA' \sec \phi' \text{ arc } 1''}[h\delta(\Delta\lambda) - \Delta\lambda\delta h]$$

$$= \frac{sB \cos \alpha}{s^2BA' \sec \phi' \text{ arc } 1''}\delta(\Delta\lambda) - \frac{sA' \sin \alpha \sec \phi'}{s^2BA' \sec \phi' \text{ arc } 1''}\delta h$$

$$= \frac{\sin \alpha \cos \alpha}{sA' \sec \phi' \sin \alpha \text{ arc } 1''}\delta(\Delta\lambda) - \frac{\sin \alpha \cos \alpha}{sB \cos \alpha \text{ arc } 1''}\delta h$$

$$= \frac{\sin \alpha \cos \alpha}{\text{arc } 1''}\left[\frac{\delta(\Delta\lambda)}{\Delta\lambda} - \frac{\delta h}{h}\right]$$

By neglecting the variations in all the terms of the expression given for $\Delta\phi$ in Special Publication No. 8 except the first or principal term, h, there results,

$$\delta(\Delta\phi) = -\delta h = \delta\phi' - \delta\phi$$

Evidently, also,

$$\delta(\Delta\lambda) = \delta\lambda' - \delta\lambda$$

It thus appears that, to the degree of approximation here adopted, it is the difference in the changes of coordinates at the ends of a line that turns the line in azimuth. The formulas for computing $d\alpha$ become,

$$d\alpha \text{ in sec.} = \frac{1}{s^2BA' \sec \phi' \text{ arc } 1''}[\Delta\lambda(\delta\phi' - \delta\phi) + h(\delta\lambda' - \delta\lambda)]$$

$$= \frac{\sin \alpha \cos \alpha}{\text{arc } 1''}\left[\frac{\delta\phi' - \delta\phi}{h} + \frac{\delta\lambda' - \delta\lambda}{\Delta\lambda}\right]$$

In practice $-\Delta\phi$ *may be used for* h, but if a position computation has been made over the line, log h will be immediately available. The change in the azimuth α' at B_1 of the line $B_1 A_1$ for given changes in the coordinates of A_1 and B_1 may usually be taken the same as the change in α, the azimuth at A_1 of the line $A_1 B_1$.* If the point A_1 is fixed $\delta\phi$ and $\delta\lambda$ are zero, and if B_1 is fixed $\delta\phi'$ and $\delta\lambda'$ are zero.

This formula will now be applied to three examples, first, the adjustment of a quadrilateral, next the adjustment of three new points connected with a number of fixed points, and, lastly, to a figure involving a closure in geographic position. The steps to be taken and the precautions to be observed will be explained as they arise in the course of the examples.

* For more exact formula to be used with longer lines, see Dr. F. R. Helmert's Höhere Geodäsie, vol. 1, pp. 495 and 496. For such lines some of the approximations made in the derivation here given are no longer permissible.

In all cases treated by this method, however complicated they may be, a start is made with the assumed positions of the points to be determined and the assumed azimuths and lengths of the lines sighted over. These positions, azimuths, and lengths must be consistent with each other and not too far from the final result so that the corrections to the assumed quantities are in fact small, as is implied in the development of the formulas. Otherwise it is not important how these preliminary quantities are found.

ADJUSTMENT OF A QUADRILATERAL WITH TWO POINTS FIXED

As a simple example a quadrilateral, A_1, A_2, A_3, A_4, with two points, A_1 and A_2, fixed is adjusted. The coordinates of A_1 and A_2 and the length and direction of the line A_1-A_2 are fixed as shown in the first lines of the position computation that follows. The angles of the preliminary computation of the triangles are obtained from the list of directions. To obtain the preliminary positions, directions and lengths, the triangles A_1, A_2, A_3, and A_2, A_3, A_4 were made to close by correcting each angle by approximately one-third of the error of closure as indicated in the triangle computation. This determined the entire quadrilateral. In each of the other triangles two sides and an included angle became known and thus their remaining parts were computed.

List of observed directions *

AT A_2

Station	Direction †
Initial	0 00 00.0+z_1
A_1	0 00 00.0+v_1
A_3	101 44 45.1+v_2
A_4	133 53 46.3+v_3

AT A_3

Station	Direction †
Initial	0 00 00.0+z_3
A_4	0 00 00.0+v_7
A_2	31 03 42.5+v_8
A_1	61 47 35.0+v_9

AT A_1

Initial	0 00 00.0+z_2
A_3	0 00 00.0+v_4
A_4	26 40 23.5+v_5
A_2	47 31 20.2+v_6

AT A_4

Initial	0 00 00.0+z_4
A_2	0 00 00.0+v_{10}
A_1	25 15 16.2+v_{11}
A_3	116 47 20.0+v_{12}

* See fig. 1 on p. 16.

† Each observed value has its symbolic correction affixed.

Preliminary computation of triangles

Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
	° ′ ″	″	″	″	″	
A_7–A_1						3.772745
A_3	30 43 52.5	+0.8	53.3			0.291566
A_2	101 44 45.1	+0.7	45.8	0.1	45.7	9.990809
A_1	47 31 20.2	+0.8	21.0			9.867787
		+2.3		0.1		
A_7–A_1						4.055120
A_7–A_2						3.932098
A_7–A_4*						3.772745
A_4	25 15 16.2		17.3			0.369934
A_3	133 53 46.3		45.8	0.1	45.7	9.857694
A_7	20 50 56.7		57.0			9.551339
		+0.9		0.1		
A_4–A_1						4.000373
A_4–A_3						3.694018
A_7–A_3						3.932098
A_4	116 47 20.0	−1.2	18.8	0.1	18.7	0.049306
A_2	32 09 01.2	−1.2	00.0			9.726024
A_3	31 03 42.5	−1.2	41.3			9.712614
		−3.6		0.1		
A_4–A_3						3.707428
A_4–A_2						3.694018
A_1–A_3*						4.055120
A_4	91 32 03.8		01.5	0.1	01.4	0.000156
A_1	26 40 23.5		24.0			9.652153
A_3	61 47 35.0		34.6			9.945097
		−2.2		0.1		
A_4–A_3						3.707429^{-1}
A_4–A_1						4.000373

* This triangle is computed from two sides and the included angle.

Preliminary position computation,

STATION A_3

α Second angle	A_2 to A_1 A_1 and A_3						° 156 +101	′ 20 44	″ 26.6 45.8
α $\Delta\alpha$	A_2 to A_3						258 +	05 8	12.4 05.9
α'	A_3 to A_2			First angle of triangle			180 78 30	00 13 43	00.00 18.3 53.3
ϕ $\Delta\phi$	° 60 +	′ 56	″ 01.089 56.720	A_3		λ $\Delta\lambda$	149 −	34 9	19.237 15.877
ϕ'	60	56	57.809	A_3		λ'	149	25	03.360
$\frac{1}{2}(\phi+\phi')$	° ′ ″ 60 56 29	s cos α B	3.932098 9.314773 8.509299	s^2 sin² α C		7.86420 9.98109 1.65750	h² D		3.512 2.322
1st term	″ −57.0388	h	1.756170			9.05279			5.834
2d and 3d terms	+ 0.3184					+0.3183 +0.0001			
$-\Delta\phi$	−56.7204								
	s sin α A′ sec ϕ'	3.932098 9.990544 8.508600 0.313737		$\Delta\lambda$ sin $\frac{1}{2}(\phi+\phi')$		2.744979 9.941572			
		2.744979				2.686551			
	$\Delta\lambda$	″ −555.8774		$-\Delta\alpha$		″ −485.90			

STATION A_4

α Second angle	A_3 to A_2 A_2 and A_4						° 258 + 32	′ 05 09	″ 12.4 00.0
α $\Delta\alpha$	A_3 to A_4						290 +	14 4	12.4 29.0
α'	A_4 to A_3			First angle of triangle			180 110 116	00 18 47	00.00 41.4 18.8
ϕ $\Delta\phi$	° 60 −	′ 56	″ 01.089 55.340	A_3		λ $\Delta\lambda$	149 −	34 5	19.237 07.794
ϕ'	60	55	05.749	A_4		λ'	149	29	11.443
$\frac{1}{2}(\phi+\phi')$	° ′ ″ 60 55 33	s cos α B	3.694018 9.538951 8.509299	s^2 sin² α C		7.38804 9.94466 1.65750	h² D		3.484 2.322
1st term	″ +55.2418	h	1.742268			8.99020			5.806
2d and 3d terms	+ 0.0979					+0.0978 +0.0001			
$-\Delta\phi$	+55.3397								
	s sin α A′ sec ϕ'	3.694018 9.972328 8.508601 0.313313		$\Delta\lambda$ sin $\frac{1}{2}(\phi+\phi')$		2.488260 9.941507			
		2.488260				2.429767			
	$\Delta\lambda$	″ −307.7939		$-\Delta\alpha$		″ −269.0			

secondary triangulation.

STATION A_3

									°	′	″
α	A_1 to A_3								336	18	08.4
Third angle	A_3 and A_2								−47	31	21.0
α	A_1 to A_3								288	46	47.4
$\Delta\alpha$									+	10	24.3
									180	00	00.00
α'	A_3 to A_1								108	57	11.7
											−.1
	°	′	″								
ϕ	60	58	56.416	A_1			λ		149	36	57.360
$\Delta\phi$	−	1	58.607				$\Delta\lambda$		−	11	54.000
ϕ'	60	56	57.809	A_3			λ'		149	25	03.360
	° ′ ″	s	4.055120	s^2	8.11024					−h	2.072
$\frac{1}{2}(\phi+\phi')$	60 57 57	$\cos\alpha$	9.507765	$\sin^2\alpha$	9.95248	h^2	4.144		$s^2\sin^2\alpha$	8.063
		B	8.509295	C	1.65837	D	2.322		E	6.640
	″									
1st term	+118.0810	h	2.072180		9.72109		6.466			6.775
2d, 3d, and 4th terms	+ 0.5258				+0.5261					
					+0.0003					
$-\Delta\phi$	+118.6068				−0.0006					
	s	4.055120								
	$\sin\alpha$	9.976241								
	A′	8.508600	$\Delta\lambda$		2.853698					
	$\sec\phi'$	0.313737	$\sin\frac{1}{2}(\phi+\phi')$		9.941676					
		2.853698			2.795374					
		″			″					
	$\Delta\lambda$	−713.9996	$-\Delta\alpha$		−624.27					

STATION A_4

									°	′	″
α	A_3 to A_2								78	13	18.3
Third angle	A_4 and A_3								− 31	03	41.3
α	A_3 to A_4								47	09	37.0
$\Delta\alpha$									−	3	36.8
									180	00	00.00
α'	A_4 to A_3								227	06	00.2
	°	′	″								
ϕ	60	56	57.809	A			λ		149	25	03.360
$\Delta\phi$	−	1	52.060				$\Delta\lambda$		+	4	08.084
ϕ'	60	55	05.749	A_4′			λ'		149	29	11.444
											−1
	° ′ ″	s	3.707428	s^2	7.41486						
$\frac{1}{2}(\phi+\phi')$	60 56 02	$\cos\alpha$	9.832477	$\sin^2\alpha$	9.73052	h^2			4.098		
		B	8.509298	C	1.65808	D			2.322		
	″										
1st term	+111.9962	h	2.049203		8.80346				6.420		
2d and 3d terms	+ 0.0639				+0.0636						
					+0.0003						
$-\Delta\phi$	+112.0601										
	s	3.707428									
	$\sin\alpha$	9.865257									
	A′	8.508601	$\Delta\lambda$		2.394599						
	$\sec\phi'$	0.313313	$\sin\frac{1}{2}(\phi+\phi')$		9.941540						
		2.394599			2.336139						
		″			″						
	$\Delta\lambda$	+248.0841	$-\Delta\alpha$		+216.8						

FORMATION OF OBSERVATION EQUATIONS

The observation equations used in making the adjustments are formed on the assumptions of the direction method.* Each pointing of the telescope is treated as an independent observation and the sum of the squares of the corrections to the separate pointings is to be made a minimum. A single pointing, however, taken by itself determines nothing, for if each of the pointings at a station be changed by the same amount the set of pointings has the same significance as before. The effect is simply a change in the zero direction, which is a purely arbitrary matter. If a set of corrections to directions at a point has been determined by any method and the mean of these corrections is not zero, the sum of the squares of these corrections can always be diminished by subtracting from each correction the mean of all of the corrections so that the algebraic sum of the reduced corrections is zero. Hence in any set of directions adjusted by the method of least squares the algebraic sum of the corrections at a point is zero.† To allow for this change of zero direction, or for the constant correction to all directions at a point, an unknown constant correction, "z," is introduced into all equations expressing the results of observations at a point, a different "z" for each point where observations are taken.

The observation equation may be written,

$$\text{Assumed azimuth} + d\alpha - \text{observed azimuth} + z - v = 0.$$

The coefficients of the $\delta\phi$'s and $\delta\lambda$'s come from the last equation on page 93. As a sample, take those in the expression for v_9. A_1 corresponds to the A_1 and A_3 to the B_1 of the explanation of the formulas. Sin α, cos α, h, and $\Delta\lambda$ come from the position computation on page 97.

log sin α	9.9762n		4.7984n		4.7984n
log cos α	9.5078	log h	2.0722	log $\Delta\lambda$	2.8537n
colog arc 1″	5.3144				
			2.7262n		1.9447
	4.7984n	Number	−532	Number	+88

The observed angles in the following formation of equations come from the list of directions on page 94.

Azimuth A_2 to A_1 (initial direction).....	156	20	26.6
Observed angle initial direction to A_1.....	0	00	$00.0-z_1+v_1$
Observed azimuth A_2 to A_1.....	156	20	$26.6-z_1+v_1$
Assumed azimuth A_2 to A_1.....	156	20	$26.6+d\alpha$
Assumed azimuth−observed azimuth.....			$0=0.0+d\alpha+z_1-v_1$

* See Wright and Hayford, Adjustment of Observations, Chap. VII.

† This does not necessarily hold good when a line whose direction has already been fixed enters into the set.

Azimuth A_2 to A_1 is fixed. Therefore $d\alpha=0$ and $v_1=z_1$

Azimuth A_2 to A_1 (initial direction).....	156	20	26.6
Observed angle initial direction to A_3.....	101	44	$45.1-z_1+v_2$
Observed azimuth A_2 to A_3.....	258	05	$11.7-z_1+v_2$
Assumed azimuth A_2 to A_3.....	258	05	$12.4+d\alpha$
Assumed azimuth−observed azimuth.....			$0=+0.7+d\alpha+z_1-v_2$

$d\alpha=-730\delta\phi_3-75\delta\lambda_3$. Therefore $v_2=z_1-730\ \delta\phi_3-75\delta\lambda_3+0.7$.

Azimuth A_2 to A_1 (initial direction).....	156	20	26.6
Observed angle initial direction to A_4.....	133	53	$46.3-z_1+v_3$
Observed azimuth A_2 to A_4.....	290	14	$12.9-z_1+v_3$
Assumed azimuth A_2 to A_4.....	290	14	$12.4+d\alpha$
Assumed azimuth−observed azimuth.....			$0=-0.5+d\alpha+z_1-v_3$

$d\alpha=-1212\ \delta\phi_4+218\ \delta\lambda_4$. Therefore $v_3=z_1-1212\ \delta\phi_4+218\ \delta\lambda_4-0.5$

In the same way at A_1 $v_4=z_2-532\ \delta\phi_3+88\ \delta\lambda_3-0.8$

$v_5=z_2-447\ \delta\phi_4+221\ \delta\lambda_4-0.3$

$v_6=z_2$

Azimuth A_3 to A_4 (initial direction).....	47	09	37.0
Observed angle initial direction to A_4.....	0	00	$00.0-z_3+v_7$
Observed azimuth A_3 to A_4.....	47	09	$37.0-z_3+v_7$
Assumed azimuth A_3 to A_4.....	47	09	$37.0+d\alpha$
Assumed azimuth−observed azimuth.....			$0=0.0+d\alpha+z_3-v_7$

$d\alpha=+918\ (\delta\phi_4-\delta\phi_3)+414\ (\delta\lambda_4-\delta\lambda_3)$. Therefore $v_7=z_3-918\ \delta\phi_3-414\ \delta\lambda_3+918\ \delta\phi_4+414\ \delta\lambda_4+0.0$

Similarly

$v_8=z_3-730\ \delta\phi_3-75\ \delta\lambda_3-1.2$

$v_9=z_3-532\ \delta\phi_3+88\ \delta\lambda_3-0.4$

$v_{10}=z_4-1211\ \delta\phi_4+218\ \delta\lambda_4+0.0$

$v_{11}=z_4-447\ \delta\phi_4+221\ \delta\lambda_4+1.1$

$v_{12}=z_4-918\ \delta\phi_3-414\ \delta\lambda_3+918\ \delta\phi_4+414\ \delta\lambda_4-1.2$

We have then the set of observation equations:

$v_1=z_1$

$v_2=z_1-730\ \delta\phi_3-75\ \delta\lambda_3+0.7$

$v_3=z_1-1212\ \delta\phi_4+218\ \delta\lambda_4-0.5$

$v_4=z_2-532\ \delta\phi_3+88\ \delta\lambda_3-0.8$

$v_5=z_2-447\ \delta\phi_4+221\ \delta\lambda_4-0.3$

$v_6=z_2$

$v_7=z_3-918\ \delta\phi_3-414\ \delta\lambda_3+918\ \delta\phi_4+414\ \delta\lambda_4+0.0$

$v_8=z_3-730\ \delta\phi_3-75\ \delta\lambda_3-1.2$

$v_9=z_3-532\ \delta\phi_3+88\ \delta\lambda_3-0.4$

$v_{10}=z_4-1211\ \delta\phi_4+218\ \delta\lambda_4+0.0$

$v_{11}=z_4-447\ \delta\phi_4+221\delta\ \lambda_4+1.1$

$v_{12}=z_4-918\ \delta\phi_3-414\ \delta\lambda_3+918\ \delta\phi_4+414\ \delta\lambda_4-1.2$

These equations contain z's which are of no particular interest in themselves. The normal equations might be formed and the z's eliminated in the regular way, but this work is made easier by the following mechanical rule, the effect of which is to form at once the

reduced normal equations with the z's eliminated. For the proof of the rule and further particulars see Jordan's Handbuch der Vermessungskunde, Vol. I, pages 151–171, of the third edition. Each direction is assumed to have equal weight. Write the observation equations dropping the z's and giving each unit weight. Add together as they stand all observation equations containing the z for any particular point. Drop the z term out and treat the resulting equation as a new observation equation with a negative weight equal to $-1/r$, where r is the total number of directions, both fixed and to be determined, that have been observed at the point in question. To reduce the new fictitious observation to unit weight it must be multiplied through by $\sqrt{\frac{-1}{r}}=\frac{1}{r}\sqrt{r}\,i$ where $i=\sqrt{-1}$.

Table 1 below shows the coefficients of the unknowns, the coefficients formed by adding the equations containing any particular z, and the weights. Table 2 shows these equations divided through by 100 for convenience. This has no effect on the relative weights. The table also contains the fictitious observation equations obtained by multiplying the sum equation by $\sqrt{\frac{-1}{r}}$. From Table 2 the normal equations which do not contain the z's are formed in the ordinary way for observation equations of equal weight, using the i's strictly according to algebraic laws. Thus in the first line of Table 2 $(-4.23i)^2$ contributes to the first diagonal coefficient not $+17.8929$ but $+17.8929i^2$, or -17.8929, and $(-4.23i)\times+(1.26i)$ contributes toward the side coefficient not -5.3298, but $-5.3298i^2$ or $+5.3298$.

Table for formation of normals, No. 1

		$\delta\phi_2$	$\delta\lambda_2$	$\delta\phi_4$	$\delta\lambda_4$	l	p	$\sqrt{p}$
	z_1							
	1						1	1
	1	− 730	− 75			+0.7	1	1
	1			−1212	+218	−0.5	1	1
Sum	3	− 730	− 75	−1212	+218	+0.2	$-\frac{1}{3}$	$0.58i$
	z_2							
	1	− 532	+ 88			−0.8	1	1
	1			− 447	+221	−0.3	1	1
	1						1	1
Sum	3	− 532	+ 88	− 447	+221	−1.1	$-\frac{1}{3}$	$0.58i$
	z_3							
	1	− 918	−414	+ 918	+414	+0.0	1	1
	1	− 730	− 75			−1.2	1	1
	1	− 532	+ 88			−0.4	1	1
Sum	3	−2180	−401	+ 918	+414	−1.6	$-\frac{1}{3}$	$0.58i$
	z_4							
	1			−1211	+218	+0.0	1	1
	1			− 447	+221	+1.1	1	1
	1	− 918	−414	+ 918	+414	−1.2	1	1
Sum	3	− 918	−414	− 740	+853	−0.1	$-\frac{1}{3}$	$0.58i$

Table for formation of normals, No. 2

	$\delta\phi_2$	$\delta\lambda_2$	$\delta\phi_4$	$\delta\lambda_4$	l	Σ
2	− 7.30	−0.75			+0.007	− 8.043
3			−12.12	+2.18	−0.005	− 9.945
z_1	− 4.23i	−0.44i	− 7.03i	+1.26i	+0.00116i	−10.43884i
4	− 5.32	+0.88			−0.008	− 4.448
5			− 4.47	+2.21	−0.003	− 2.263
z_2	− 3.09i	+0.51i	− 2.59i	+1.28i	−0.00638i	− 3.89638i
7	− 9.18	−4.14	+ 9.18	+4.14	+0.0	0.0
8	− 7.30	−0.75			−0.012	− 8.062
9	− 5.32	+0.88			−0.004	− 4.444
z_3	−12.64i	−2.33i	+ 5.32i	+2.40i	−0.00928i	− 7.25928i
10			−12.11	+2.18	+0.0	− 9.93
11			− 4.47	+2.21	+0.011	− 2.249
12	− 9.18	−4.14	+ 9.18	+4.14	−0.012	− 0.012
z_4	− 5.32i	−2.40i	− 4.29i	+4.95i	−0.00058i	− 7.06058i

Normal equations

	$\delta\phi_2$	$\delta\lambda_2$	$\delta\phi_4$	$\delta\lambda_4$	η	Σ
1	+116.2166	+35.0927	−161.8628	−10.0554	+0.0753	− 20.5336
2		+25.3104	− 75.6831	−16.9056	+0.0236	− 32.1620
3			+399.2176	+24.0721	−0.0468	+185.6970
4				+20.0637	−0.01105	+ 17.16375

The forward and back solution of the normals, conducted according to the Doolittle method, is next shown.

To compute the v's from the observation equations a knowledge of the z's is required. Substitute the $\delta\phi$'s and $\delta\lambda$'s in the right-hand side of the sum equation formed from the observation equations that contain the z in question as if the z were not there and divide the result of the substitution by the weight $-r$. As a check the sum of the v's about a point should equal zero. The computation of the v's is shown in the table on page 102. Below each v as computed to 3 decimals is given its value as adopted and reduced to 1 decimal.

Following the computation of the v's there is given a computation of the triangles using the adjusted directions.

Solution of normals

$\delta\phi_2$	$\delta\lambda_2$	$\delta\phi_4$	$\delta\lambda_4$	η	Σ
+116.2166	+35.0927	−161.8628	−10.0544	+0.0753	− 20.5336
$\delta\phi_2$	− 0.301959	+ 1.392768	+ 0.086523	−0.000648	+ 0.176684
	+25.3104	− 75.6831	−16.9056	+0.0236	− 32.1620
1	−10.5966	+ 48.8759	+ 3.0363	−0.0227	+ 6.2003
	+14.7138	− 26.8072	−13.8693	+0.0009	− 25.9618
	$\delta\lambda_2$	+ 1.821909	+ 0.942605	−0.000061	+ 1.764452
		+399.2176	+24.0721	−0.0468	+185.6970
	1	−225.4373	−14.0048	+0.1049	− 28.5985
	2	− 48.8403	−25.2686	+0.0016	− 47.3000
		+124.9400	−15.2013	+0.0597	+109.7984
		$\delta\phi_4$	+ 0.121669	−0.000478	− 0.878809
			+20.0637	−0.01105	+ 17.16375
		1	− 0.8700	+0.00652	− 1.77663
		2	−13.0733	+0.00085	− 24.47172
		3	− 1.8495	+0.00726	+ 13.35904
			+ 4.2709	+0.00358	+ 4.27448
			$\delta\lambda_4$	−0.000838	− 1.000838

Back solution

$\delta\lambda_4$	$\delta\phi_4$	$\delta\lambda_3$	$\delta\phi_3$
−0.00084	−0.00048	−0.00006	−0.00065
	−0.00010	−0.00079	−0.00007
−0.00084		−0.00106	−0.00081
	−0.00058		+0.00058
		−0.00191	
			−0.00095

Computation of corrections

$1=z_1$	2	3	z_1
−0.519	+0.6935	+0.7030	+0.6935
−0.5	+0.1432	−0.1831	+0.1432
	+0.7	−0.5	+0.7030
	−0.519	−0.519	−0.1831
			+0.2
	+1.018	−0.499	+1.5566+−3
	+1.0	−0.5	−0.519

4	5	$6=z_2$	z_2
+0.5054	+0.2593	+0.230	+0.5054
−0.1681	−0.1856	+0.3	−0.1681
−0.8	−0.3		+0.2593
+0.230	+0.230		−0.1856
			−1.1
−0.233	+0.004		−0.689 +−3
−0.2	0.0		+0.230

7	8	9	z_3
+0.8721	+0.6935	+0.5054	+2.0710
+0.7907	+0.1432	−0.1681	+0.7659
−0.5324	−1.2	−0.4	−0.5324
−0.3478	−0.119	−0.119	−0.3478
−0.119			−1.6
	−0.482	−0.182	
+0.664	−0.5	−0.2	+0.3567+−3
+0.7			−0.119

10	11	12	z_4
+0.7024	+0.2593	+0.8721	+0.8721
−0.1831	−0.1856	+0.7907	+0.7907
−0.425	+1.1	−0.5324	+0.4292
	−0.425	−0.3478	−0.7165
+0.094		−1.2	−0.1
+0.1	+0.749	−0.425	
	+0.7		+1.2755+−3
		−0.842	−0.425
		−0.8	

Adjusted computation of triangles

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″	″	″	
	A_3-A_1						3.772745
− 8+ 9	A_3	30 43 52.5	+0.3	52.8	0.0	52.8	0.291568
− 1+ 2	A_2	101 44 45.1	+1.5	46.6	0.1	46.5	9.990809
− 4+ 6	A_1	47 31 20.2	+0.5	20.7	0.0	20.7	9.867787
			+2.3		0.1		
	A_3-A_1						4.055122
	A_3-A_2						3.932100
	A_3-A_1						3.772745
−10+11	A_4	25 15 16.2	+0.6	16.8	0.0	16.8	0.369936
− 1+ 3	A_2	133 53 46.3	0.0	46.3	0.1	46.2	9.857693
− 5+ 6	A_1	20 50 56.7	+0.3	57.0	0.0	57.0	9.551339
			+0.9		0.1		
	A_4-A_1						4.000374
	A_4-A_2						3.694020
	A_3-A_2						3.932100
−10+12	A_4	116 47 20.0	−0.9	19.1	0.1	19.0	0.049306
− 2+ 3	A_2	32 08 61.2	−1.5	59.7	0.0	59.7	9.726023
− 7+ 8	A_3	31 03 42.5	−1.2	41.3	0.0	41.3	9.712614
			−3.6		0.1		
	A_4-A_3						3.707429
	A_4-A_2						3.694020
	A_1-A_2						4.055122
−11+12	A_4	91 32 03.8	−1.5	02.3	0.1	02.2	0.000156
− 4+ 5	A_1	26 40 23.5	+0.2	23.7	0.0	23.7	9.652151
− 7+ 9	A_3	61 47 35.0	−0.9	34.1	0.0	34.1	9.945096
			−2.2		0.1		
	A_4-A_3						3.707429
	A_4-A_1						4.000374

ADJUSTMENT OF THREE NEW POINTS CONNECTED WITH SEVERAL FIXED POINTS BY VARIATION OF GEOGRAPHIC COORDINATES

GENERAL STATEMENT

The method of adjustment by geographic coordinates seems to be especially suitable for the adjustment of a few new points depending upon a number of fixed points. The number of normal equations in such case is $2n$, n being the number of new points. In the figure used the number of condition equations would be 15, which would form a very intricate set of normals. By the method of coordinates the number of normal equations is only six.

The adjustment of figure 6 is carried out in two different ways, the first one being more rigorous but a trifle longer than the second. The first method corresponds in its treatment of observed directions to the method developed in Jordan's Vermessungskunde, volume 1, pages 144–173, of the third edition. The second method resembles somewhat the method given by Jordan on pages 173–179 for the approximate treatment of the z's and corresponds in its treatment of fixed directions to the ordinary practice of the Coast and Geodetic Survey for subsidiary triangulation as treated by the method of condition equations.

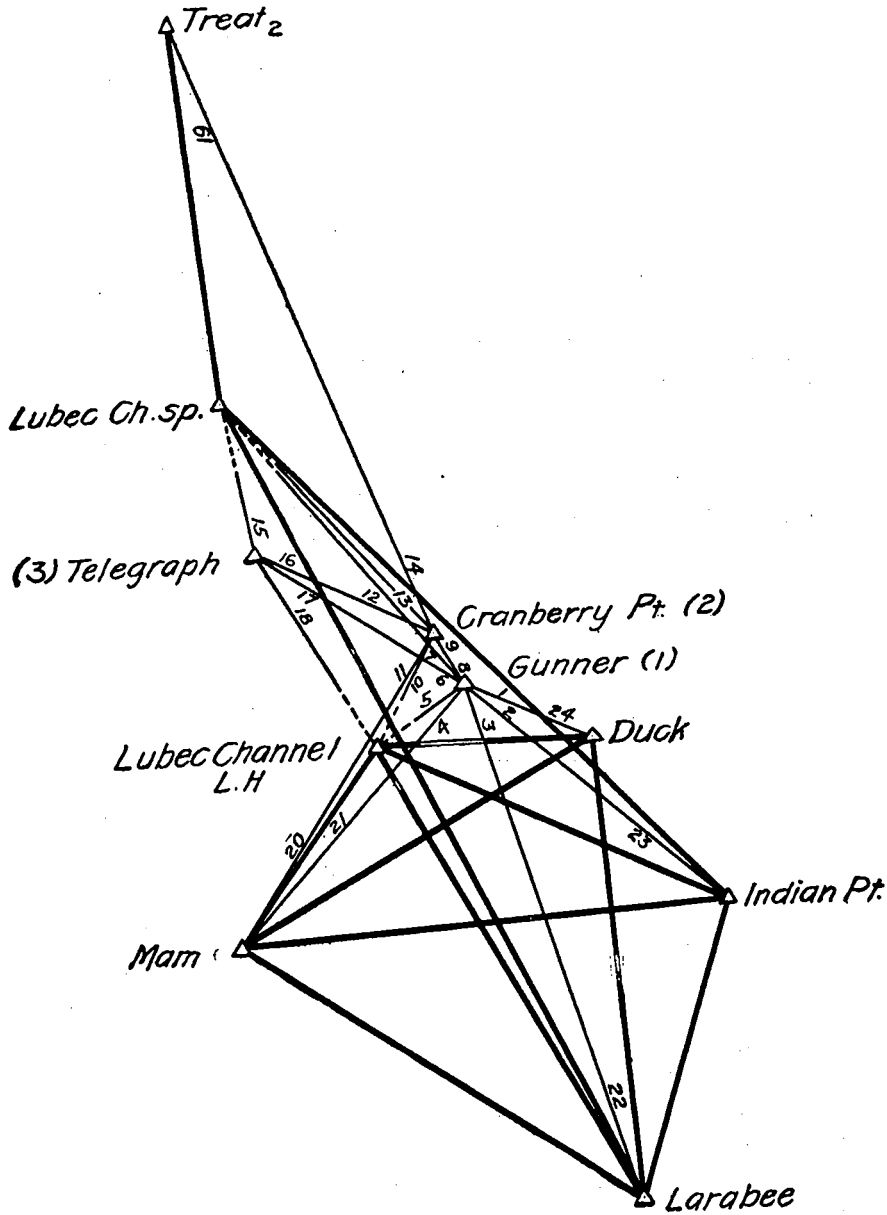
Treat 2
Lubec Ch.sp.
(3) Telegraph
Cranberry Pt. (2)
Gunner (1)
Duck
Lubec Channel
L.H
Indian Pt.
Mam
Larabee

FIG. 6.

The solution by the method of condition equations was carried out for figure 6 and gave almost the same results, the greatest difference in the correction to a direction being 0.2″. This difference was quite to be expected in view of the different formulas and the fact that the fixed positions, distances, and azimuths may not be strictly consistent with each other to the last figure given.

FIRST METHOD

The first method is fundamentally the same as the method used in the adjustment of the quadrilateral previously given, but the greater complication of the figure, particularly the great number of fixed lines, brings to light points that need mention. The groundwork of the adjustment by either method is shown in the tables of observed directions and of fixed positions, azimuths, and lengths which follow. In the list of directions the names of stations that are sighted on over fixed lines are shown in heavy type. For these stations the directions corrected from a previous adjustment are also shown. In forming the table of triangles for the preliminary computation these corrected directions were taken with the directly observed directions of new points in order to obtain such of the angles in the column "Observed angle" as have a fixed line for one of its sides.* No particular procedure to obtain the consistent set of positions, azimuths, and lengths necessary to form the observation equations is essential to the method. In this particular case the corrections to the angles of the triangles Gunner-Larrabee-Mam, Cranberry Point-Gunner-Lubec Channel Lighthonse, and Telegraph-Cranberry Point-Gunner were arbitrarily assumed as shown in the table of preliminary computation of triangles. These assumptions, with the lines already fixed, determined enough parts in every one of the other triangles to make possible its solution with results as shown in the table.

Following the table of triangles the necessary preliminary computation of positions is included.

* This corresponds to the idea followed out in the second method of solution, but in the preliminary computation this is of no consequence, as is shown in the next sentence.

Lists of directions

AT INDIAN POINT

Stations observed	Observed directions	Seconds after adjustment*
	° ′ ″	″
Larrabee	0 00 00.0	01.5
Mam	66 55 01.0	01.2
Lubec Channel Lighthouse	95 24 38.4	41.2
Gunner	114 00 37.5	
Lubec Church Spire	117 33 55.5	71.9

AT LARRABEE

Stations observed	Observed directions	Seconds after adjustment*
	° ′ ″	″
Indian Point	0 00 00.0	56.7
Mam	281 47 48.4	47.9
Lubec Channel Lighthouse	312 58 58.5	55.0
Lubec Church Spire	315 12 10.0	24.9
Gunner	325 58 24.9	
Duck	336 12 51.0	50.6

AT MAM

Indian Point	0 00 00.0	55.4
Larrabee	34 52 43.6	46.7
Cranberry Point	317 37 23.5	
Lubec Channel Lighthouse	318 56 54.1	55.2
Gunner	322 01 44.8	
Duck	336 18 52.1	52.5

AT DUCK

Lubec Channel Lighthouse	0 00 00.0	59.9
Gunner	33 11 41.8	
Larrabee	269 09 40.7	40.9
Mam (computed)	336 10	44.0

AT GUNNER

Indian Point	0 00 00.0	
Larrabee	31 57 41.4	
Mam	94 56 05.9	
Lubec Channel Lighthouse	101 48 54.9	
Telegraph	168 28 59.0	
Lubec Church Spire	186 30 29.8	
Cranberry Point	191 54 53.6	
Duck	346 14 12.0	

AT CRANBERRY POINT

Gunner	0 00 00.0	
Lubec Channel Lighthouse	75 45 12.8	
Mam	78 37 05.9	
Telegraph	153 39 19.7	
Lubec Church Spire	174 08 32.3	
Treat$_2$	191 52 35.2	

AT TREAT$_2$

Cranberry Point	0 00 00.0	
Lubec Church Spire	17 34 38.1	45.4

AT TELEGRAPH

Cranberry Point	0 00 00.0	
Gunner	2 54 53.2	
Lubec Channel Lighthouse	29 52 44.5	
Lubec Church Spire	231 21 00.1	

* Refers to final values of heavy lines in Fig. 6, p. 104, obtained from a previous adjustment.

List of fixed positions

Station	Latitude and longitude	Azimuth	Back azimuth	To station	Logarithm of distance
	° ′ ″	° ′ ″	° ′ ″		
Lubec Church Spire	44 51 38.470 66 59 17.418				
Lubec Channel Lighthouse	44 50 31.652 66 58 38.299				
Treat$_2$	44 52 44.333 66 59 25.919	354 45 17.5	174 45 23.5	Lubec Church Spire	3.309982
Indian Point	44 50 03.537 66 57 12.788	114 48 33.8	294 47 33.5	Lubec Channel Lighthouse	3.315762
		136 58 04.6	316 56 36.7	Lubec Church Spire	3.603123
Larrabee	44 49 10.841 66 57 38.857	152 22 33.8	332 21 51.9	Lubec Channel Lighthouse	3.449573
		154 36 03.7	334 34 54.3	Lubec Church Spire	3.702872
		199 23 35.7	19 23 54.1	Indian Point	3.236668
Duck	44 50 33.886 66 57 47.822	355 36 23.1	175 36 29.4	Larrabee	3.410111
		86 26 42.1	266 26 06.5	Lubec Channel Lighthouse	3.045619
Mam	44 49 57.369 66 59 26.892	225 14 19.0	45 14 53.3	Lubec Channel Lighthouse	3.176968
		242 36 16.3	62 37 26.2	Duck	3.389282
		266 17 19.2	86 18 53.8	Indian Point	3.470097
		301 10 10.5	121 11 26.7	Larrabee	3.443126

Preliminary computation of triangles

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″		° ′ ″	
	Duck-Larrabee						3.410111
−1+ 3	Gunner	45 43 29.4	+ 4.8	34.2			0.145080
+24	Duck	124 01 60.9	− 1.9	59.0			9.918405
−22	Larrabee	10 14 25.7	+ 1.1	26.8			9.249896
			+ 4.0				
	Gunner-Larabee						3.473596
	Gunner-Duck						2.805087
	Duck-Mam						3.389282
−1+ 4	Gunner	108 41 53.9	+ 3.6	57.5			0.023552
+24	Duck	57 00 57.8	− 1.9	55.9			9.923668
−21	Mam	14 17 07.7	− 1.1	06.6			9.392254
			+ 0.6				
	Gunner-Mam						3.336502
	Gunner-Duck						2.805088^{-1}
	Duck-Lubec Channel Lighthouse						3.045619
−1+ 5	Gunner	115 34 42.9	+ 7.1	50.0			0.044803
+24	Duck	33 11 41.9	− 1.9	40.0			9.738370
	Lubec Channel Lighthouse	35.2	− 5.2	30.0		31 13 30.0	9.714665
	Gunner-Lubec Channel Lighthouse						2.828792+1
	Gunner-Duck						2 805087
	Indian Point-Larrabee						3.236668
−2+ 3	Gunner	31 57 41.4	+ 7.0	48.4			0.276234
+23	Indian Point	114 00 36.0	+ 2.5	38.5			9.960694
−22	Larrabee	34 01 32.0	+ 1.1	33.1			9.747852
			+10.6				
	Gunner-Larrabee						3.473596
	Gunner-Indian Point						3.260754
	Indian Point-Mam						3.470097
−2+ 4	Gunner	94 56 05.9	+ 5.8	11.7			0.001614
+23	Indian Point	47 05 36.3	+ 2.5	38.8			9.864792
−21	Mam	37 58 10.6	− 1.1	09.5			9.789044
			+ 7.2				
	Gunner-Mam						3.336503^{-1}
	Gunner-Indian Point						3.260755^{-1}
	Indian Point-Lubec Channel Lighthouse						3.815762
−2+ 5	Gunner	101 48 54.9	+ 9.3	64.2			0.009304
+23	Indian Point	18 35 56.3	+ 2.5	58.8			9.503728
	Lubec Channel Lighthouse	68.8		57.0		59 34 57.0	9.935688
	Gunner-Lubec Channel Lighthouse						2.828794^{-1}
	Gunner-Indian Point						3.260754
	Larrabee-Mam						3.443126
−3+ 4	Gunner	62 58 24.5	− 1.2	23.3			0.050223
+22	Larrabee	44 10 37.0	− 1.1	35.9			9.843153
−21	Mam	72 51 01.9	− 1.1	00.8			9.980247
			− 3.4				
	Gunner-Mam						3.336502
	Gunner-Larrabee						3.473596

Preliminary computation of triangles—Continued

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″		° ′ ″	
	Larrabee-Lubec Channel Lighthouse						3.449573
−3+ 5	Gunner	69 51 13.5	+ 2.3	15.8			0.027417
+22	Larrabee	12 59 29.9	− 1.1	28.8			9.351803
	Lubec Channel Lighthouse	16.6		15.4		97 09 15.4	9.996606
	Gunner-Lubec Channel Lighthouse						2.828793
	Gunner-Larrabee						3.473596
	Larrabee-Lubec church spire						3.702872
− 3+ 7	Gunner	154 32 48.4	+20.7	69.1		33 09.1	0.366851
+22	Larrabee	10 45 60.0	− 1.1	58.9			9.271388
	Lubec church spire	71.6		52.0		14 40 52.0	9.403873
	Gunner-Lubec church spire						3.341111
	Gunner-Larrabee						3.473596
	Mam-Lubec Channel Lighthouse						3.176968
− 4+ 5	Gunner	6 52 49.0	+ 3.5	52.5			0.921500
+21	Mam	3 04 49.6	+ 1.1	50.7			8.730324
	Lubec Channel Lighthouse	21.4		16.8		170 02 16.8	9.238033
	Gunner-Lubec Channel Lighthouse						2.828792^{+1}
	Gunner-Mam						3.336501^{+1}
	Lubec church spire-Indian Point						3.603122
+ 2− 7	Gunner	173 29 30.2	−27.7	02.5			0.945080
	Lubec church spire	56 55.4		85.6		2 57 25.6	8.712552
−23	Indian Point	3 33 34.4	− 2.5	31.9			8.792909
	Gunner-Indian Point						3.260754
	Gunner-Lubec church spire						3.341111
	Gunner-Lubec Channel Lighthouse						2.828793
− 9+10	Cranberry Point	75 45 12.8	0.0	12.8			0.013566
− 5+ 8	Gunner	90 05 58.7	0.0	58.7			9.999999
	Lubec Channel Lighthouse	48.5		48.5		14 08 48.5	9.388114
	Cranberry Point-Lubec Channel Lighthouse						2.842358
	Cranberry Point-Gunner						2.230473
	Gunner-Mam						3.336502
− 9+11	Cranberry Point	78 36 65.9	−13.1	52.8			0.008631
− 4+ 8	Gunner	96 58 47.7	+ 3.5	51.2			9.996768
−20+21	Mam	4 24 21.3	− 5.3	16.0			8.885341
			−14.9				
	Cranberry Point-Mam						3.341901
	Cranberry Point-Gunner						2.230474^{-1}
	Gunner-Lubec church spire						3.341111
− 9+13	Cranberry Point	174 08 32.3	+11.5	43.8			0.991389
− 7+ 8	Gunner	5 24 23.8	−18.4	05.4			8.973748
	Lubec church spire	03.9		10.8		0 27 10.8	7.897971
	Cranberry Point-Lubec church spire						3.306248
	Cranberry Point-Gunner						2.230471^{+2}

Preliminary computation of triangles—Continued

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″		° ′ ″	
	Lubec Channel Lighthouse-Mam						3.176968
−10+11	Cranberry Point	2 51 53.1	−13.1	40.0			1.301768
	Lubec Channel Lighthouse			54.7		175 48 54.7	8.863166
−20	Mam	1 19 31.7	− 6.4	25.3			8.363626
	Cranberry Point-Mam						3.341902−1
	Cranberry Point-Lubec Channel Lighthouse						2.842362−1
	Lubec church spire-Treat$_2$						3.309982
−13+14	Cranberry Point	17 43 62.9	− 4.7	58.2			0.516300
	Lubec church spire		19.0	28.0		144 41 28.0	9.761916
−19	Treat$_2$	17 34 38.1	− 4.3	33.8			9.479966
	Cranberry Point-Treat$_2$						3.588198
	Cranberry Point-Lubec church spire						3.306248
	Lubec church spire-Cranberry Point						3.306248
−15+16	Telegraph	128 38 59.9	−52.6	07.3			0.107274
	Lubec church spire	51 47.5		85.9		30 52 25.9	9.710244
−12+13	Cranberry Point	20 29 12.6	+14.2	26.8			9.544138
	Telegraph-Cranberry Point						3.123766
	Telegraph-Lubec church spire						2.957660
	Lubec church spire-Gunner						3.341111
−15+17	Telegraph	131 33 53.1	−54.8	58.3		32 58.3	0.125876
	Lubec church spire	24 36.1		75.1		30 25 15.1	9.704449
− 6+ 7	Gunner	18 01 30.8	+15.8	46.6			9.490673
	Telegraph-Gunner						3.171436
	Telegraph-Lubec church spire						2.957660
	Cranberry Point-Gunner						2.230473
−16+17	Telegraph	2 54 53.2	− 2.2	51.0			1.293796
− 9+12	Cranberry Point	153 39 19.7	− 2.7	17.0			9.647167
− 6+ 8	Gunner	23 25 54.6	− 2.6	52.0			9.599497
			− 7.5				
	Telegraph-Gunner						3.171436
	Telegraph-Cranberry Point						3.123766
	Cranberry Point-Lubec Channel Lighthouse						2.842358
−16+18	Telegraph	29 52 44.5	− 9.6	34.9			0.302657
−10+12	Cranberry Point	77 54 06.9	− 2.7	04.2			9.990244
	Lubec Channel Lighthouse	08.6		20.9		72 13 20.9	9.978751
	Telegraph-Lubec Channel Lighthouse						3.135259+1
	Telegraph-Cranberry Point						3.123766
	Gunner-Lubec Channel Lighthouse						2.828793
−17+18	Telegraph	20 57 51.3	− 7.4	43.9			0.343516
− 5+ 6	Gunner	66 40 04.1	+ 2.6	06.7			9.962951
	Lubec Channel Lighthouse	04.6		09.4		86 22 09.4	9.999127
	Telegraph-Lubec Channel Lighthouse						3.135260
	Telegraph-Gunner						3.171436

Preliminary position computation,

STATION GUNNER

		°	′	″
α	Duck to Lubec Channel Lighthouse	86	26	42.1
Second angle	Lubec Channel Lighthouse and Gunner	+ 33	11	40.0
α	Duck to Gunner	119	38	22.1
$\Delta\alpha$		−		17.8
		180	00	00.00
α'	Gunner to Duck	299	38	04.3
	First angle of triangle	115	34	50.0

	°	′	″			°	′	″
ϕ	44	50	33.886	Duck	λ	66	57	47.822
$\Delta\phi$	+		10.227		$\Delta\lambda$	+		25.265
ϕ'	44	50	44.113	Gunner	λ'	66	58	13.087

	° ′ ″	s	2.805087	s^2	5.6102
$\frac{1}{2}(\phi+\phi')$	44 50 39	$\cos\alpha$	9.694202	$\sin^2\alpha$	9.8782
	″	B	8.510480	C	1.4016
1st term	−10.2275	h	1.009769		6.8900
2d term	+ 0.0008				
$-\Delta\phi$	−10.2267				

	s	2.805087		
	$\sin\alpha$	9.939097		
	A′	8.508994	$\Delta\lambda$	1.402526
	$\sec\phi'$	0.149348	$\sin\frac{1}{2}(\phi+\phi')$	9.848301
		1.402526		1.250827
		″		″
	$\Delta\lambda$	+25.2654	$-\Delta\alpha$	+17.82

STATION CRANBERRY POINT

		°	′	″
α	Gunner to Lubec Channel Lighthouse	55	12	54.3
Second angle	Lubec Channel Lighthouse and Cranberry Point	+ 90	05	58.7
α	Gunner to Cranberry Point	145	18	53.0
$\Delta\alpha$		−		03.1
		180	00	00.00
α'	Cranberry Point to Gunner	325	18	49.9
	First angle of triangle	75	45	12.8

	°	′	″			°	′	″
ϕ	44	50	44.113	Gunner	λ	66	58	13.087
$\Delta\phi$	+		04.529		$\Delta\lambda$	+		04.405
ϕ'	44	50	48.642	Cranberry Point	λ'	66	58	17.492

	° ′ ″	s	2.230473	s^2	4.4609
$\frac{1}{2}(\phi+\phi')$	44 50 46	$\cos\alpha$	9.915025	$\sin^2\alpha$	9.5103
	″	B	8.510480	C	1.4016
1st term	−4.5288	h	0.655978		5.3728
2d term	0.0000				
$-\Delta\phi$	−4.5288				

	s	2.230473		
	$\sin\alpha$	9.755164		
	A′	8.508994	$\Delta\lambda$	0.643988
	$\sec\phi'$	0.149357	$\sin\frac{1}{2}(\phi+\phi')$	9.848315
		0.643988		0.492303
		″		″
	$\Delta\lambda$	+4.4054	$-\Delta\alpha$	+3.11

secondary triangulation

STATION GUNNER

		°	′	″
α	Lubec Channel Lighthouse to Duck	266	26	06.5
Third angle	Gunner and Duck	− 31	13	30.0
α	Lubec Channel Lighthouse to Gunner	235	12	36.5
$\Delta\alpha$		+		17.8
		180	00	00.00
α'	Gunner to Lubec Channel Lighthouse	55	12	54.3

	°	′	″					
ϕ	44	50	31.652	Lubec Channel Lighthouse	λ	66	58	38.299
$\Delta\phi$	+		12.461		$\Delta\lambda$	−		25.212
ϕ'	44	50	44.113	Gunner	λ'	66	58	13.087

	° ′ ″	s	2.828793	s^2	5.6576
$\frac{1}{2}(\phi+\phi')$	44 50 36	cos α	9.756308	sin² α	9.8289
	″	B	8.510480	C	1.4016
1st term	−12.4618	h	1.095581		6.8881
2d term	+ 0.0008				
$-\Delta\phi$	−12.4610				

	s	2.828793		
	sin α	9.914474		
	A′	8.508994	$\Delta\lambda$	1.401609
	sec ϕ'	0.149348	sin $\frac{1}{2}(\phi+\phi')$	9.848294
		1.401609		1.249903
		″		″
	$\Delta\lambda$	−25.2121	$-\Delta\alpha$	−17.78

STATION CRANBERRY POINT

		°	′	″
α	Lubec Channel Lighthouse to Gunner	235	12	36.5
Third angle	Cranberry Point and Gunner	− 14	08	48.5
α	Lubec Channel Lighthouse to Cranberry Point	221	03	48.0
$\Delta\alpha$		+		14.7
		180	00	00.00
α'	Cranberry Point to Lubec Channel Lighthouse	41	04	02.7

	°	′	″					
ϕ	44	50	31.652	Lubec Channel Lighthouse	λ	66	58	38.299
$\Delta\phi$	+		16.990		$\Delta\lambda$	−		20.807
ϕ'	44	50	48.642	Cranberry Point	λ'	66	58	17.492

	° ′ ″	s	2.842358	s^2	5.6847
$\frac{1}{2}(\phi+\phi')$	44 50 40	cos α	9.877362	sin² α	9.6350
	″	B	8.510480	C	1.4016
1st term	−16.9903	h	1.230200		6.7213
2d term	+ 0.0005				
$-\Delta\phi$	−16.9898				

	s	2.842358		
	sin α	9.817494		
	A′	8.508994	$\Delta\lambda$	1.318203
	sec ϕ'	0.149357	sin $\frac{1}{2}(\phi+\phi')$	9.848303
		1.318203		1.166506
		″		″
	$\Delta\lambda$	−20.8067	$-\Delta\alpha$	−14.67

Preliminary position computation,

STATION TELEGRAPH

		°	′	″
α	Cranberry Point to Lubec Channel Lighthouse	41	04	02.7
Second angle	Lubec Channel Lighthouse and Telegraph	+ 77	54	04.2
α	Cranberry Point to Telegraph	118	58	06.9
$\Delta\alpha$		−		37.4
		180	00	00.00
α'	Telegraph to Cranberry Point	298	57	29.5
	First angle of triangle	29	52	34.9

	°	′	″			°	′	″
ϕ	44	50	48.642	Cranberry Point	λ	66	58	17.492
$\Delta\phi$	+		20.860		$\Delta\lambda$	+		52.980
ϕ'	44	51	09.502	Telegraph	λ'	66	59	10.472

	° ′ ″	s	3.123766	s^2	6.2475
$\frac{1}{2}(\phi+\phi')$	44 50 59	cos α	9.685141	$\sin^2\alpha$	9.8839
		B	8.510480	C	1.4016
	″				
1st term	−20.8635	h	1.319387		7.5330
2d term	+ 0.0034				
$-\Delta\phi$	−20.8601				

	s	3.123766		
	sin α	9.941951		
	A′	8.508994	$\Delta\lambda$	1.724112
	sec ϕ'	0.149401	sin $\frac{1}{2}(\phi+\phi')$	9.848343
		1.724112		1.572455
		″		″
	$\Delta\lambda$	+52.9800	$-\Delta\alpha$	+37.36

secondary triangulation—Continued.

STATION TELEGRAPH

		°	′	″
α	Lubec Channel Lighthouse to Cranberry Point	221	03	48.0
Third angle	Telegraph and Cranberry Point	− 72	13	20.9
α	Lubec Channel Lighthouse to Telegraph	148	50	27.1
$\Delta\alpha$		−		22.7
		180	00	00.00
α'	Telegraph to Lubec Channel Lighthouse	328	50	04.4

	°	′	″					
ϕ	44	50	31.652	Lubec Channel	λ	66	58	38.299
$\Delta\phi$	+		37.850	Lighthouse	$\Delta\lambda$	+		32.173
ϕ'	44	51	09.502	Telegraph	λ'	66	59	10.472

	° ′ ″	s	3.135260	s^2	0.2705
$\frac{1}{2}(\phi+\phi')$	44 50 50	cos α	9.932338	sin² α	9.4277
	″	B	8.510480	C	1.4016
1st term	−37.8511	h	1.578078		7.0998
2d term	+ 0.0012				
$-\Delta\phi$	−37.8499				

s	3.135260		
sin α	9.713841		
A′	8.508994	$\Delta\lambda$	1.507496
sec ϕ'	0.149401	sin $\frac{1}{2}(\phi+\phi')$	9.848324
	1.507496		1.355820
	″		″
$\Delta\lambda$	+32.1733	$-\Delta\alpha$	+22.69

FORMATION OF OBSERVATION EQUATIONS

In the first column of the following table is given the assumed azimuths of the various lines. Under station Gunner the azimuth to Duck, 299° 38′ 04″.3, comes from the position computation on page 110, and the other azimuths are obtained by adding to this the corrected angles from the preliminary computation of triangles. Thus in the first triangle in that list, Gunner-Duck-Larrabee, the angle at Gunner is 45° 43′ 34.″2, which, added to the azimuth of Duck, gives 345° 21′ 38″.5, as shown in the table. For any one station the assumed and observed azimuths of some one station may be taken as identical. At Gunner they are identical on station Duck. The observed azimuths in the second column of the table have their symbolic corrections affixed. These azimuths are obtained by adding the observed angles as derived from the list of observed directions on page 106 to the azimuth of the line Gunner-Duck.

At the fixed stations given in the lower part of the table the method of computing the assumed and observed azimuths is somewhat different. The assumed azimuths of the fixed lines come from the table of fixed positions on page 106. The assumed azimuths of the new lines are found by adding to one of these fixed azimuths the appropriate corrected angle from the computation of triangles on pages 107–109. In the second column of the table the observed azimuth of one fixed line used as an initial line is taken identical with its assumed azimuth, and the other observed azimuths, whether of fixed lines or of new ones, are found by adding to this azimuth the observed angles between the initial line and each of the others as derived from the list of directions.

The coefficients of the $\delta\phi$'s and $\delta\lambda$'s are found from the formulas on page 93.

GUNNER

Assumed azimuth	Observed azimuth	Equation	Station observed
° ′ ″	° ′ ″		
299 38 04.3	299 38 04.3	$v_1=z_1+8669\partial\phi_1-3509\partial\lambda_1+0.0$	Duck
313 23 50.1	313 23 52.3	$v_2=z_1+2538\partial\phi_1-1709\partial\lambda_1-2.2$	Indian Point
345 21 38.5	345 21 33.7	$v_3=z_1+541\partial\phi_1-1473\partial\lambda_1+4.8$	Larrabee
48 20 01.8	48 19 58.2	$v_4=z_1-2191\partial\phi_1-1388\partial\lambda_1+3.6$	Mam
55 12 54.3	55 12 47.2	$v_5=z_1-7756\partial\phi_1-3833\partial\lambda_1+7.1$	Lubec Channel L. H.
121 53 01.0	121 52 51.3	$v_6=z_1-3643\partial\phi_1+1612\partial\lambda_1+3643\partial\phi_3-1612\partial\lambda_3+9.7$	Telegraph
139 54 47.6	139 54 22.1	$v_7=z_1-1870\partial\phi_1+1580\partial\lambda_1+25.5$	Lubec church spire
145 18 53.0	145 18 45.9	$v_8=z_1-21313\partial\phi_1+21909\partial\lambda_1+21313\partial\phi_2-21909\partial\lambda_2+7.1$	Cranberry Point

CRANBERRY POINT

325 18 49.9	325 18 49.9	$v_9=z_2-21313\partial\phi_1+21909\partial\lambda_1+21313\partial\phi_2-21909\partial\lambda_2+0.0$	Gunner
41 04 02.7	41 04 02.7	$v_{10}=z_2-6013\partial\phi_2-4910\partial\lambda_2+0.0$	Lubec Channel L. H.
43 55 42.7	43 55 55.8	$v_{11}=z_2-2010\partial\phi_2-1485\partial\lambda_2-13.1$	Mam
118 58 06.9	118 58 09.6	$v_{12}=z_2-4189\partial\phi_2+1650\partial\lambda_2+4189\partial\phi_3-1650\partial\lambda_3-2.7$	Telegraph
139 27 33.7	139 27 22.2	$v_{13}=z_2-2045\partial\phi_2+1701\partial\lambda_2+11.5$	Lubec church spire
157 11 31.9	157 11 25.1	$v_{14}=z_2-637\partial\phi_2+1078\partial\lambda_2+6.8$	Treat_2

TELEGRAPH

170 19 22.2	170 19 22.2	$v_{15}=z_3-1181\partial\phi_3+4922\partial\lambda_3+0.0$	Lubec church spire
298 57 29.5	298 58 22.1	$v_{16}=z_3-4189\partial\phi_2+1650\partial\lambda_2+4189\partial\phi_3-1650\partial\lambda_3-52.6$	Cranberry Point
301 52 20.5	301 53 15.3	$v_{17}=z_3-3643\partial\phi_1+1612\partial\lambda_1+3643\partial\phi_3-1612\partial\lambda_3-54.8$	Gunner
328 50 04.4	328 51 06.6	$v_{18}=z_3+2413\partial\phi_3-2839\partial\lambda_3-62.2$	Lubec Channel L. H.

TREAT_2

337 10 43.7	337 10 39.4	$v_{19}=z_4-637\partial\phi_2+1078\partial\lambda_2+4.3$	Cranberry Point
354 45 17.5	354 45 17.5	$v_{19}'=z_4+0.0$	Lubec church spire

MAM

Assumed azimuth	Observed azimuth	Equation	Station observed
° ′ ″	° ′ ″		
223 54 53.7	223 54 42.7	$v_{20}=z_5-2010\partial\phi_2-1485\partial\lambda_2+11.0\ (+6.4)$	Cranberry Point
225 14 19.0	225 14 13.3	$v_{20}'=z_5+5.7\ (+1.1)$	Lubec Channel L. H.
228 19 09.7	228 19 04.0	$v_{21}=z_5-2191\partial\phi_1-1388\partial\lambda_1+5.7\ (+1.1)$	Gunner
242 36 16.3	242 36 11.3	$v_{20}''=z_5+5.0\ (+0.4)$	Duck
266 17 19.2	266 17 19.2	$v_{20}'''=z_5+0.0\ (-4.6)$	Indian Point
301 10 10.5	301 10 02.8	$v_{20}''''=z_5+7.7\ (+3.1)$	Larrabee

LARRABEE

121 11 26.7	121 11 24.1	$v_{22}'=z_6+2.6\ (-0.2)$	Mam
152 22 33.8	152 22 34.2	$v_{22}''=z_6-0.4\ (-3.2)$	Lubec Channel L. H.
154 36 03.7	154 35 54.7	$v_{22}'''=z_6+9.0\ (+6.2)$	Lubec church spire
165 22 02.6	165 22 00.6	$v_{22}=z_6+541\partial\phi_1-1473\partial\lambda_1+2.0\ (-0.8)$	Gunner
175 36 29.4	175 36 26.7	$v_{22}''''=z_6+2.7\ (-0.1)$	Duck
199 23 35.7	199 23 35.7	$v_{22}^{v}=z_6+0.0\ (-2.8)$	Indian Point

INDIAN POINT

19 23 54.1	19 23 54.1	$v_{23}'=z_7+0.0\ (-3.8)$	Larrabee
86 18 53.8	86 18 55.1	$v_{23}''=z_7-1.3\ (-5.0)$	Mam
114 48 33.8	114 48 32.5	$v_{23}'''=z_7+1.3\ (-2.4)$	Lubec Channel L. H.
133 24 32.6	133 24 31.6	$v_{23}=z_7+2538\partial\phi_1-1709\partial\lambda_1+1.0\ (-2.8)$	Gunner
136 58 04.6	136 57 49.6	$v_{23}''''=z_7+15.0\ (+11.2)$	Lubec church spire

DUCK

355 36 23.1	355 36 22.8	$v_{24}'=z_8+0.3\ (+0.2)$	Larrabee
62 37 26.2	Not observed		Mam
86 26 42.1	86 26 42.1	$v_{24}''=z_8+0.0\ (-0.1)$	Lubec Channel L. H.
119 38 22.1	119 38 23.9	$v_{24}=z_8+8669\partial\phi_1-3509\partial\lambda_1-1.8\ (-1.9)$	Gunner.

Figure 6 taken with the following two tables shows that z_1 is for directions taken at Gunner, z_2 for directions at Cranberry Point, z_3 for directions at Telegraph, and z_5 for directions at Mam. The scheme for eliminating these z's by the use of the sum equations, as fictitious additional observation equations with negative weights, is used here in the same manner as in the previous example. Each weight is the negative reciprocal of the total number of observed lines in the adjustment that radiates from the point in question. The weights are, respectively, $-1/8$, $-1/6$, $-1/4$, and $-1/6$, as shown in Table 1.

At Treat$_2$, Larrabee, Indian, and Duck where only one new line is to be determined the same process might be used, but the following method is identical in results and slightly shorter. Use is made of the fact that the directions taken at a point may each be changed by the same amount, a change equivalent to using merely a different zero point. Correct each of the directions by the averages of all the corrections necessary to reduce the observed results to the accepted results on the lines that have already been fixed. Then drop the z from the observation equation of the new line and assign the equation a positive weight equal to $\frac{s}{s+1}$, where s is the number of lines already fixed and therefore $s+1$ is the total number of lines. Thus at Larrabee the constant terms of the observation equations representing pointings on lines already fixed are $+2.6$, -0.4, $+9.0$, -0.1, and 0.0, the mean of which is $+2.8$. Subtracting this from each of the preceding numbers we have -0.2, -3.2, $+6.2$, -0.1, and -2.8 as the new constant terms, also -0.8 instead of $+2.0$ on Gunner, the new point. These new values are inclosed in parentheses and are used in forming the normal equations. There are five fixed lines, so the weight of the new equation without z that is used to replace the six equations containing z is 5/6 and the equation itself is

$$v_{22} = +541\delta\phi_1 - 1473\delta\lambda_1 - 0.8$$

which in Table 2 corresponds to the line No. 22,

$$+0.49\delta\phi_1 - 1.34\delta\lambda_1 - 0.07.$$

The z's are computed from the sum equations as in the previous example, the result of substitution in the right-hand side being divided by $-1/r$, r being the total number of lines through the point to which z applies. For fixed points where only one new line occurs, substitute $\delta\phi$ and $\delta\lambda$ in the right-hand side of the observation equation on the new line omitting the z, and divide the result by $-1/r$. Thus at Duck (see p. 116),

$$z_6 = (8669\delta\phi_1 - 3509\delta\lambda_1 - 1.9) \div (-3)$$

as shown in the computation below.

When the z's are known the v's or corrections are computed from the equations on pages 115 and 116. The details are shown in the table on page 121.

For convenience of solution in the normals it is best to divide the constant terms by 10 and the coefficients by 1000. The solution will then give $100\delta\phi_n$ and $100\delta\lambda_n$.

Table for formation of normals No. 1

		1	2	3	4	5	6			
		$\delta\phi_1$	$\delta\lambda_1$	$\delta\phi_2$	$\delta\lambda_2$	$\delta\phi_3$	$\delta\lambda_3$	η	p	$\sqrt{p}$
	z_1									
1	1	+ 8669	− 3509					+ 0.0	1	1
2	1	+ 2538	− 1709					− 2.2	1	1
3	1	+ 541	− 1473					+ 4.8	1	1
4	1	− 2191	− 1388					+ 3.6	1	1
5	1	− 7756	− 3833					+ 7.1	1	1
6	1	− 3643	+ 1612			+3643	−1612	+ 9.7	1	1
7	1	− 1870	+ 1580					+ 25.5	1	1
8	1	−21313	+21909	+21313	−21909			+ 7.1	1	1
Sum	8	−25025	+13189	+21313	−21909	+3643	−1612	+ 55.6	$-\frac{1}{8}$	0.35355i
	z_2									
9	1	−21313	+21909	+21313	−21909			+ 0.0	1	1
10	1			− 6013	− 4910			+ 0.0	1	1
11	1			− 2010	− 1485			− 13.1	1	1
12	1			− 4189	+ 1650	+4189	−1650	− 2.7	1	1
13	1			− 2045	+ 1701			+ 11.5	1	1
14	1			− 637	+ 1078			+ 6.8	1	1
Sum	6	−21313	+21909	+ 6419	−23875	+4189	−1650	+ 2.5	$-\frac{1}{6}$	0.4083i
	z_3									
15	1					−1181	+4922	+ 0.0	1	1
16	1			− 4189	+ 1650	+4189	−1650	− 52.6	1	1
17	1	− 3643	+ 1612			+3643	−1612	− 54.8	1	1
18	1					+2413	−2839	− 62.2	1	1
Sum	4	− 3643	+ 1612	− 4189	+ 1650	+9064	−1179	−169.6	$-\frac{1}{4}$	0.5i
19				− 637	+ 1078			+ 4.3	$\frac{1}{2}$	0.707
	z_5									
20	1			− 2010	− 1485			+ 6.4	1	1
21	1	− 2191	− 1388					+ 1.1	1	1
Sum	6	− 2191	− 1388	− 2010	− 1485			+ 7.5	$-\frac{1}{6}$	0.4083i
22		+ 541	− 1473					− 0.8	$\frac{4}{5}$	0.9129
23		+ 2538	− 1709					− 2.8	$\frac{4}{5}$	0.8944
24		+ 8669	− 3509					− 1.9	$\frac{2}{3}$	0.8165

Table for formation of normals No. 2

	$\delta\phi_1$	$\delta\lambda_1$	$\delta\phi_2$	$\delta\lambda_2$	$\delta\phi_3$	$\delta\lambda_3$	η	Σ
1	+ 8.67	− 3.51					+0.00	+ 5.16
2	+ 2.54	− 1.71					−0.22	+ 0.61
3	+ 0.54	− 1.47					+0.48	− 0.45
4	− 2.19	− 1.39					+0.36	− 3.22
5	− 7.76	− 3.83					+0.71	−10.88
6	− 3.64	+ 1.61			+3.64	−1.61	+0.97	+ 0.97
7	− 1.87	+ 1.58					+2.55	+ 2.26
8	−21.31	+21.91	+21.31	−21.91			+0.71	+ 0.71
z_1	− 8.85i	+ 4.66i	+ 7.54i	− 7.75i	+1.29i	−0.57i	+1.97i	− 1.71i
9	−21.31	+21.91	+21.31	−21.91			+0.00	+ 0.00
10			− 6.01	− 4.91			+0.00	−10.92
11			− 2.01	− 1.48			−1.31	− 4.80
12			− 4.19	+ 1.65	+4.19	−1.65	−0.27	− 0.27
13			− 2.04	+ 1.70			+1.15	+ 0.81
14			− 0.64	+ 1.08			+0.68	+ 1.12
z_2	− 8.70i	+ 8.95i	+ 2.62i	− 9.75i	+1.71i	−0.67i	+0.10i	− 5.74i
15					−1.18	+4.92	+0.00	+ 3.74
16			− 4.19	+ 1.65	+4.19	−1.65	−5.26	− 5.26
17	− 3.64	+ 1.61			+3.64	−1.61	−5.48	− 5.48
18					+2.41	−2.84	−6.22	− 6.65
z_3	− 1.82i	+ 0.81i	− 2.09i	+ 0.82i	+4.53i	−0.59i	−8.48i	− 6.82i
19			− 0.45	+ 0.76			+0.30	+ 0.61
20			− 2.01	− 1.48			+0.64	− 2.85
21	− 2.19	− 1.39					+0.11	− 3.47
z_5	− 0.89i	− 0.57i	− 0.82i	− 0.61i			+0.31i	− 2.58i
22	+ 0.49	− 1.34					−0.07	− 0.92
23	+ 2.27	− 1.53					−0.25	+ 0.49
24	+ 7.08	− 2.86					−0.16	+ 4.06

Normal equations

1	2	3	4	5	6	η	Σ
+987.3526	−852.5451	−823.2428	+781.3412	+ 8.0389	− 0.2265	− 8.9085	+ 91.8098
	+913.2320	+876.4443	−837.7306	−13.2644	+ 3.9464	+ 6.5262	+ 96.6088
		+923.5619	−831.4801	−39.8513	+18.6471	+ 4.1465	+128.2256
			+842.4946	+36.7824	−15.9112	+ 2.6136	− 21.8901
				+43.7028	−33.6441	−18.8752	− 17.1109
					+41.7793	+30.2371	+ 44.8281

Solution of normals

1	2	3	4	5		η	Σ
+987.3526	−852.5451	−823.2428	+781.3412	+ 8.0389	− 0.2265	− 8.9085	+ 91.8098
$\partial\phi_1$	+ 0.8634657	+ 0.8337881	− 0.7913497	− 0.0081419	+ 0.0002294	+ 0.0090226	− 0.0929858
	+913.2320	+876.4443	−837.7306	−13.2644	+ 3.9464	+ 6.5262	+ 96.6088
1	−736.1434	−710.8419	+674.6613	+ 6.9413	− 0.1956	− 7.6922	+ 79.2746
	+177.0886	+165.6024	−163.0693	− 6.3231	+ 3.7508	− 1.1660	+175.8834
	$\partial\lambda_1$	− 0.9351387	+ 0.9208345	+ 0.0357059	− 0.0211804	+ 0.0065843	− 0.9931944
		+923.5619	−831.4801	−39.8513	+18.6471	+ 4.1465	+128.2256
	1	−686.4101	+651.4730	+ 6.7027	− 0.1889	− 7.4278	+ 76.5499
	2	−154.8612	+152.4924	+ 5.9130	− 3.5075	+ 1.0904	−164.4754
		+ 82.2906	− 27.5147	−27.2356	+14.9507	− 2.1909	+ 40.3001
		$\partial\phi_2$	+ 0.334360	+ 0.330969	− 0.181682	+ 0.026624	− 0.489729
			+842.4946	+36.7824	−15.9112	+ 2.6136	− 21.8901
		1	−618.3141	− 6.3616	+ 0.1792	+ 7.0497	− 72.6536
		2	−150.1598	− 5.8225	+ 3.4539	− 1.0737	+161.9595
		3	− 9.1998	− 9.1065	+ 4.9989	− 0.7326	+ 13.4747
			+ 64.8209	+15.4918	− 7.2792	+ 7.8570	+ 80.8905
			$\partial\lambda_2$	− 0.238994	+ 0.112297	− 0.121211	− 1.247908
				+43.7028	−33.6441	−18.8752	− 17.1109
			1	− 0.0655	+ 0.0019	+ 0.0725	− 0.7475
			2	− 0.2258	+ 0.1339	− 0.0416	+ 6.2801
			3	− 9.0141	+ 4.9482	− 0.7251	+ 13.3381
			4	− 3.7024	+ 1.7397	− 1.8778	− 19.3324
				+30.6950	−26.8204	−21.4472	− 17.5726
				$\partial\phi_3$	+ 0.873771	+ 0.698720	+ 0.572491
					+41.7793	+30.2371	+ 44.8281
				1	− 0.0001	− 0.0020	+ 0.0211
				2	− 0.0794	+ 0.0247	− 3.7253
				3	− 2.7163	+ 0.3980	− 7.3218
				4	− 0.8174	+ 0.8823	+ 9.0837
				5	−23.4349	−18.7399	− 15.3544
					+14.7312	+12.8002	+ 27.5314
					$\partial\lambda_3$	− 0.86892	− 1.86892

Back solution

$\delta\lambda_3$	$\delta\phi_3$	$\delta\lambda_2$	$\delta\phi_2$	$\delta\lambda_1$	$\delta\phi_1$
−0.86892	+0.69872	−0.12121	+0.02662	+0.00658	+0.00902
	−0.75924	−0.09758	+0.15787	+0.01840	−0.00020
−0.86892		+0.01446	−0.02003	−0.00216	+0.00049
	−0.06052		−0.06832	−0.18815	+0.16170
		−0.20433		−0.08990	+0.08016
			+0.09614		−0.22038
				−0.25523	
					+0.03079

Computation of corrections

1	2	3	4	5	6	7	8
+ 2.669	+ 0.781	+ 0.167	− 0.675	− 2.388	− 1.122	− 0.576	− 6.562
+ 8.956	+ 4.362	+ 3.760	+ 3.543	+ 9.783	− 4.114	− 4.033	−55.918
−11.412	−11.412	−11.412	−11.412	− 11.412	− 2.205	−11.412	+20.490
	− 2.2	+ 4.8	+ 3.6	+ 7.1	+14.007	+25.5	+44.767
+ 0.213					−11.412		−11.412
+ 0.2	− 8.469	− 2.685	− 4.944	+ 3.083	+ 9.7	+ 9.479	+ 7.1
	− 8.5	− 2.7	− 4.9	+ 3.1		+ 9.5	
					+ 4.854		− 1.535
					+ 4.9		− 1.5
z_1	9	10	11	12	13	14	z_2
− 7.705	− 6.562	− 5.781	− 1.932	− 4.027	− 1.966	− 0.612	− 6.562
−33.662	−55.918	+10.033	+ 3.035	− 3.371	− 3.476	− 2.203	−55.918
+20.490	+20.490	− 1.129	− 1.129	− 2.535	− 1.129	− 1.129	+ 6.171
+44.767	+44.767		−13.1	+ 14.337	+11.5	+ 6.8	+48.783
− 2.205	− 1.129	+ 3.123		− 1.129			− 2.535
+14.007		+ 3.1	−13.126	− 2.7	+ 4.929	+ 2.856	+14.337
+55.6	+ 1.648		−13.1		+ 5.0	+ 2.9	+ 2.5
	+ 1.7			+ 0.575			
+91.292				+ 0.6			+ 6.776
−11.412							− 1.129
15	16	17	18	z_3	19	z_4	19′
+ 0.715	− 4.027	− 1.122	− 1.460	− 1.122	− 0.612	− 0.612	− 0.742
−42.768	− 3.371	− 4.114	+24.669	− 4.114	− 2.203	− 2.203	
+44.369	− 2.535	− 2.205	+44.369	− 4.027	− 0.742	+ 4.3	− 0.742
	+14.337	+14.007	−62.2	− 3.371	+ 4.3		− 0.7
+ 2.316	+44.369	+44.369		− 5.486		+ 1.485	
+ 2.3	−52.6	−54.8	+ 5.378	+ 10.245	+ 0.743	− 0.742	
			+ 5.4	−169.6	+ 0.8		
	− 3.827	− 3.865					
	− 3.8	− 3.8		−177.475			
				+ 44.369			
20	21	z_5	20′	20″	20‴	20⁗	22
− 1.932	− 0.675	− 0.675	− 1.912	− 1.912	− 1.912	− 1.912	+ 0.167
+ 3.035	+ 3.543	+ 3.543	+ 1.1	+ 0.4	− 4.6	+ 3.1	+ 3.760
− 1.912	− 1.912	− 1.932					− 0.521
+ 6.4	+ 1.1	+ 3.035	− 0.812	− 1.512	− 6.512	+ 1.188	− 0.8
		+ 7.5	− 0.8	− 1.5	− 6.5	+ 1.2	
+ 5.591	+ 2.056						+ 2.606
+ 5.6	+ 2.1	+11.471					+ 2.6
		− 1.912					
z_6	22′	22″	22‴	22⁗	22ᵛ	23	z_7
+ 0.167	− 0.521	− 0.521	− 0.521	− 0.521	− 0.521	+ 0.781	+ 0.781
+ 3.760	− 0.2	− 3.2	+ 6.2	− 0.1	− 2.8	+ 4.362	+ 4.362
− 0.8						− 0.469	− 2.8
	− 0.721	− 3.721	+ 5.679	− 0.621	− 3.321	− 2.8	
+ 3.127	− 0.7	− 3.7	+ 5.7	− 0.6	− 3.3		+ 2.343
− 0.521						+ 1.874	− 0.469
						+ 1.9	
23′	23″	23‴	23⁗	24	z_8	24′	24″
− 0.469	− 0.469	− 0.469	− 0.469	+ 2.669	+ 2.669	− 3.242	− 3.242
− 3.8	− 5.0	− 2.4	+11.2	+ 8.956	+ 8.956	+ 0.2	− 0.1
				− 3.242	− 1.9		
− 4.269	− 5.469	− 2.869	+10.731	− 1.9		− 3.042	− 3.342
− 4.2	− 5.5	− 2.8	+10.7		+ 9.725	− 3.0	− 3.3
				+ 6.483	− 3.242		
				+ 6.5			

Final computation of triangles

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″		° ′ ″	
	Duck-Larrabee						3.410111
−1+ 3	Gunner	45 43 29.4	− 2.9	26.5			0.145096
−24′+24	Duck	124 02 01.1	+ 9.5	10.6			9.918389
−22+22′′′′	Larrabee	10 14 26.1	− 3.2	22.9			9.249850
			+ 3.4				
	Gunner-Larrabee						3.473596
	Gunner-Duck						2.805057
	Duck-Mam						3.389282
−1+ 4	Gunner	108 41 53.9	− 5.1	48.8			0.023546
	Duck	58.8		67.5		57 00 67.5	9.923684
−21+20′′	Mam	14 17 07.3	− 3.6	03.7			9.392230
	Gunner-Mam						3.336512
	Gunner-Duck						2.805058^{-1}
	Duck-Lubec Channel Lighthouse						3.045619
−1+ 5	Gunner	115 34 42.9	+ 2.9	45.8			0.044799
−24′′+24	Duck	33 11 41.8	+ 9.8	51.6			9.738407
	Lubec Channel Lighthouse	35.3		22.6		31 13 22.6	9.714639
	Gunner-Lubec Channel Lighthouse						2.828825^{+2}
	Gunner-Duck						2.805057
	Indian Point-Larrabee						3.236668
−2+ 3	Gunner	31 57 41.4	+ 5.8	47.2			0.276238
−23′+23	Indian Point	114 00 37.5	+ 6.1	43.6			9.960689
−22+22v	Larrabee	34 01 35.1	− 5.9	29.2			9.747840
			+ 6.0				
	Gunner-Larrabee						3.473595^{+1}
	Gunner-Indian Point						3.260746
	Indian Point-Mam						3.470097
−2+ 4	Gunner	94 56 05.9	+ 3.6	09.5			0.001614
−23′′+23	Indian Point	47 05 36.5	+ 7.4	43.9			9.864802
−21+20′′′	Mam	37 58 15.2	− 8.6	06.6			9.789036
			+ 2.4				
	Gunner-Mam						3.336513^{-1}
	Gunner-Indian Point						3.260747^{-1}
	Indian Point-Lubec Channel Lighthouse						3.315762
−2+ 5	Gunner	101 48 54.9	+11.6	66.5			0.009305
−23′′′+23	Indian Point	18 35 59.1	+ 4.7	63.8			9.503759
	Lubec Channel Lighthouse	66.0		49.7		59 34 49.7	9.935679
	Gunner-Lubec Channel Lighthouse						2.828826^{+1}
	Gunner-Indian Point						3.260746
	Larrabee-Mam						3.443126
−3+ 4	Gunner	62 58 24.5	− 2.2	22.3			0.050224
−22′+22	Larrabee	44 10 36.5	+ 3.3	39.8			9.843162
−21+20′′′′	Mam	72 50 58.8	− 0.9	57.9			9.980246
			+ 0.2				
	Gunner-Mam						3.336512
	Gunner-Larrabee						3.473596
	Larrabee-Lubec Channel Lighthouse						3.449573
−3+ 5	Gunner	69 51 13.5	+ 5.8	19.3			0.027415
−22′′+22	Larrabee	12 59 26.4	+ 6.3	32.7			9.351839
	Lubec Channel Lighthouse	20.1		08.0		97 09 08.0	9.996608
	Gunner-Lubec Channel Lighthouse						2.828827
	Gunner-Larrabee						3.473596

Final computation of triangles—Continued

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″		° ′ ″	
	Larrabee-Lubec church spire						3.702872
−3+ 7	Gunner	154 32 48.4	+12.2	60.6			0.366314
−22‴+22	Larrabee	10 46 05.9	− 3.1	02.8			9.271431
	Lubec church spire	65.7		56.6		14 40 56.6	9.403910
	Gunner-Lubec church spire						3.341117
	Gunner-Larrabee						3.473596
	Mam-Lubec Channel Lighthouse						3.176968
−4+ 5	Gunner	6 52 49.0	+ 8.0	57.0			0.921421
−20′+21	Mam	3 04 50.7	+ 2.9	53.6			8.730438
	Lubec Channel Lighthouse	20.3		09.4		170 02 09.4	9.238122
	Gunner-Lubec Channel Lighthouse						2.828827
	Gunner-Mam						3.336511+1
	Lubec church spire-Indian Point						3.603122
+2− 7	Gunner	173 29 30.2	−18.0	12.2			0.945259
	Lubec church spire	11.8		21.0		2 57 21.0	8.712365
−23+23′′′′	Indian Point	3 33 18.0	+ 8.8	26.8			8.792736
	Gunner-Indian Point						3.260746
	Gunner-Lubec church spire						3.341117
	Gunner-Lubec Channel Lighthouse						2.828827
−9+10	Cranberry Point	75 45 12.8	+ 1.4	14.2			0.013565
−5+ 8	Gunner	90 05 58.7	− 4.6	54.1			9.999999
	Lubec Channel Lighthouse	48.5		51.7		14 08 51.7	9.388141
	Cranberry Point-Lubec Channel Lighthouse						2.842391
	Cranberry Point-Gunner						2.230533+1
	Gunner-Mam						3.336512
−9+11	Cranberry Point	78 30 65.9	−14.8	51.1			0.008632
−4+ 8	Gunner	96 58 47.7	+ 3.4	51.1			9.996769
−20+21	Mam	4 24 21.3	− 3.5	17.8			8.885390
			−14.9				
	Cranberry Point-Mam						3.341913
	Cranberry Point-Gunner						2.230534
	Gunner-Lubec church spire						3.341117
−9+13	Cranberry Point	174 08 32.3	+ 3.28	35.58			0.991220
−7+ 8	Gunner	5 24 23.8	−11.02	12.78			8.973913
	Lubec church spire	03.9		11.64		0 27 11.64	7.898195
	Cranberry Point-Lubec church spire						3.306250
	Cranberry Point-Gunner						2.230532+1
	Lubec Channel Lighthouse-Mam						3.176968
−10+11	Cranberry Point	2 51 53.1	−16.2	36.9			1.301899
	Lubec Channel Lighthouse	36.3		58.9		175 48 58.9	8.863046
−20+20′	Mam	1 19 30.6	− 6.4	24.2			8.363526
	Cranberry Point-Mam						3.341913
	Cranberry Point-Lubec Channel Lighthouse						2.842339−1

Final computation of triangles—Continued

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″		° ′ ″	
	Lubec church spire-Treat$_2$						3.309982
−13+14	Cranberry Point	17 44 02.9	− 2.1	00.8			0.516283
	Lubec church spire			22.6		144 41 22.6	9.761932
−19+19′	Treat$_2$	17 34 38.1	− 1.5	36.6			9.479985
	Cranberry Point-Treat$_2$						3.588197
	Cranberry Point-Lubec church spire						3.306250
	Lubec church spire-Cranberry Point						3.306250
−15+16	Telegraph	128 38 59.9	− 6.1	53.8			0.107352
	Lubec church spire			49.2		30 51 49.2	9.710115
−12+13	Cranberry Point	20 29 12.6	+ 4.4	17.0			9.544083
	Telegraph-Cranberry Point						3.123717
	Telegraph-Lubec church spire						2.957685^{-1}
	Lubec church spire-Gunner						3.341117
−15+17	Telegraph	131 33 53.1	− 6.1	47.0			0.125967
	Lubec church spire			37.6		30 24 37.6	9.704315
− 6+ 7	Gunner	18 01 30.8	+ 4.6	35.4			9.490600
	Telegraph-Gunner						3.171399
	Telegraph-Lubec church spire						2.957684
	Cranberry Point-Gunner						2.230534
−16+17	Telegraph	2 54 53.2	0.0	53.2			1.293705
− 9+12	Cranberry Point	153 39 19.7	− 1.1	18.6			9.647160
− 6+ 8	Gunner	23 25 54.6	− 6.4	48.2			9.599478
			− 7.5				
	Telegraph-Gunner						3.171399
	Telegraph-Cranberry Point						3.123717
	Cranberry Point-Lubec Channel Lighthouse						2.842391
−16+18	Telegraph	29 52 44.5	+ 9.3	53.8			0.302588
−10+12	Cranberry Point	77 54 06.9	− 2.5	04.4			9.990245
	Lubec Channel Lighthouse	08.6		01.8		72 13 01.8	9.978738
	Telegraph-Lubec Channel Lighthouse						3.135224
	Telegraph-Cranberry Point						3.123717
	Gunner-Lubec Channel Lighthouse						2.828827
−17+18	Telegraph	26 57 51.3	+ 9.2	60.5			0.343447
− 5+ 6	Gunner	66 40 04.1	+ 1.8	05.9			9.962950
	Lubec Channel Lighthouse	64.6		53.6		86 21 53.6	9.999125
	Telegraph-Lubec Channel Lighthouse						3.135224
	Telegraph-Gunner						3.171399

SECOND METHOD.

The only difference between the second method of adjustment and the first is in the treatment of the directions taken at the fixed points. At these points observation equations are written for the directions of new points only and the z's are omitted. The observations taken over the fixed lines are not used, but the observed directions of the new lines are taken in connection with the adjusted direction of a fixed line, all directions being referred to a common initial line.

The equations with z's omitted are the same as if the angle method of adjustment were used. (See p. 196.) In this treatment these equations are given unit weight. Jordan (Vermessungskunde, vol. 1, p. 179, of the third edition) suggests that on some accounts it would be better to assign the equations for observations at fixed points only half weight.

The observation equations for directions taken at Gunner, Cranberry Point, and Telegraph are the same as for the first method given on page 115 and are not repeated here. Below are given the observation equations for the remaining points, formed according to the second method. The assumed azimuths are identical with those used in the first method. As an example, to illustrate the computation of the observed azimuths, take the line Mam–Gunner. Use the observed direction for the new line and the adjusted one for the fixed line.

	°	′	″
Fixed azimuth Mam to Indian Point, page 106,	=266	17	19.2
Angle Indian Point to Gunner, page 106 (359° 59′ 55″.4 to 322° 01′ 44″.8),	=322	01	49.4
Observed azimuth Mam to Gunner,	=228	19	08.6

or by reckoning from any other fixed line through Mam the same result is reached, thus,

	°	′	″
Fixed azimuth Mam to Lubec Channel Lighthouse,	=225	14	19.0
Angle Lubec Channel Lighthouse to Gunner, page 106 (318° 56′ 55″.2 to 322° 01′ 44″.8),	= 3	04	49.6
Observed azimuth Mam to Gunner,	=228	19	08.6

Note that the coefficients of the $\delta\phi$'s and $\delta\lambda$'s are exactly the same as for the first method and that the z's are omitted.

Formation of observation equations

TREAT$_2$

Assumed azimuth	Observed azimuth	Equation	Station observed
° ′ ″	° ′ ″		
337 10 43.7	337 10 39.4+v_{19}	$v_{19}=-637\partial\phi_2+1078\partial\lambda_2+4.3$	Cranberry Point

MAM

223 44 53.7	223 44 47.3+v_{20}	$v_{20}=-2010\partial\phi_2-1485\partial\lambda_2+6.4$	Cranberry Point
228 19 09.7	228 19 08.6+v_{21}	$v_{21}=-2191\partial\phi_1-1388\partial\lambda_1+1.1$	Gunner

LARRABEE

165 22 02.6	165 22 03.7+v_{22}	$v_{22}=+541\partial\phi_1-1473\partial\lambda_1-1.1$	Gunner

INDIAN POINT

133 24 32.6	133 24 30.1+v_{23}	$v_{23}=+2538\partial\phi_1-1709\partial\lambda_1+2.5$	Gunner

DUCK

119 38 22.1	119 38 24.0+v_{24}	$v_{24}=+8669\partial\phi_1-3509\partial\lambda_1-1.9$	Gunner

The first part of each of tables 1 and 2 for the formation of normals according to the first method, pages 118 and 119, down to the line for v_{19} will serve for the second method also and is not repeated here. The remainder of the tables according to the second method is given below.

In forming the normal equations the four z's that occur are eliminated by the device of the sum equation serving as a fictitious observation equation with negative weight. The other observations that do not contain z's enter into the formation of the normal equations in the usual way. After the normal equations have been solved the four z's are found from the sum equations in the way previously explained and enter into the computation of the v's, or corrections, from 1 to 18 and 20 and 21, but not into the others.

Table for formation of normals No. 1

	1	2	3	4	5	6			
	$\delta\phi_1$	$\delta\lambda_1$	$\delta\phi_2$	$\delta\lambda_2$	$\delta\phi_3$	$\delta\lambda_3$	η	p	$\sqrt{p}$
19			− 637	+ 1078			+ 4.3	1	1
20			− 2010	− 1485			+ 6.4	1	1
21	− 2191	− 1388					+ 1.1	1	1
22	+ 541	− 1473					− 1.1	1	1
23	+ 2538	− 1709					+ 2.5	1	1
24	+ 8669	− 3509					− 1.9	1	1

Table for formation of normals No. 2

	1	2	3	4	5	6		
	$\delta\phi_1$	$\delta\lambda_1$	$\delta\phi_2$	$\delta\lambda_2$	$\delta\phi_3$	$\delta\lambda_3$	η	Σ
19			− 0.64	+ 1.08			+0.43	+ 0.87
20			− 2.01	− 1.48			+0.64	− 2.85
21	− 2.19	− 1.39					+0.11	− 3.47
22	+ 0.54	− 1.47					−0.11	− 1.04
23	+ 2.54	− 1.71					+0.25	+ 1.08
24	+ 8.67	− 3.51					−0.19	+ 4.97

Normal equations

1	2	3	4	5	6	η	Σ
+1014.5374	−863.2282	−822.5130	+781.8841	+ 8.0389	− 0.2265	− 8.5215	+109.9712
	+918.6459	+876.9117	−837.3829	−13.2644	+ 3.9464	+ 5.8167	+ 91.4452
		+924.4414	−831.3291	−39.8513	+18.6471	+ 3.7521	+130.0589
			+843.4555	+36.7824	−15.9112	+ 2.6609	− 19.8403
				+43.7028	−33.6441	−18.8752	− 17.1109
					+41.7793	+30.2371	+ 44.8281

Solution of normals

1	2	3	4	5	6	η	Σ
+1014.5374 $\partial\phi_1$	−863.2282 + 0.8508589	−822.5130 + 0.8107271	+781.8841 − 0.7706804	+ 8.0389 − 0.0079237	− 0.2265 + 0.0002233	− 8.5215 + 0.0083994	+109.9712 − 0.1083954
	+918.6459	+876.9117	−837.3829	−13.2644	+ 3.9464	+ 5.8167	+ 91.4452
1	−734.4854	−699.8425	+665.2730	+ 6.8400	− 0.1927	− 7.2506	+ 93.5700
	+184.1605 $\partial\lambda_1$	+177.0692 − 0.961494	−172.1099 + 0.934565	− 6.4244 + 0.034885	+ 3.7537 − 0.020383	− 1.4339 + 0.007786	+185.0152 − 1.004641
		+924.4414	−831.3291	−39.8513	+18.6471	+ 3.7521	+130.0589
	1	−666.8336	+633.8946	+ 6.5174	− 0.1836	− 6.9086	+ 89.1566
	2	−170.2510	+165.4826	+ 6.1770	− 3.6092	+ 1.3787	−177.8910
		+ 87.3568 $\partial\phi_2$	− 31.9519 + 0.365763	−27.1569 + 0.310873	+14.8543 − 0.170042	− 1.7778 + 0.020351	+ 41.3245 − 0.473054
			+843.4555	+36.7824	−15.9112	+ 2.6609	− 19.8403
		1	−602.5828	− 6.1954	+ 0.1746	+ 6.5674	− 84.7526
		2	−160.8479	− 6.0040	+ 3.5081	− 1.3401	+172.9087
		3	− 11.6868	− 9.9330	+ 5.4332	− 0.6503	+ 15.1150
			+ 68.3380 $\partial\lambda_2$	+14.6500 − 0.214376	− 6.7953 + 0.099437	+ 7.2379 − 0.105913	+ 83.4306 − 1.220852
				+43.7028	−33.6441	−18.8752	− 17.1109
			1	− 0.0637	+ 0.0018	+ 0.0675	− 0.8714
			2	− 0.2241	+ 0.1309	− 0.0500	+ 6.4543
			3	− 8.4423	+ 4.6178	− 0.5527	+ 12.8467
			4	− 3.1406	+ 1.4567	− 1.5516	− 17.8855
				+31.8321 $\partial\phi_3$	−27.4369 + 0.861926	−20.9620 + 0.658518	− 16.5668 + 0.520443
					+41.7793	+30.2371	+ 44.8281
				1	− 0.0001	− 0.0019	+ 0.0246
				2	− 0.0765	+ 0.0292	− 3.7712
				3	− 2.5259	+ 0.3023	− 7.0269
				4	− 0.6757	+ 0.7197	+ 8.2961
				5	−23.6486	−18.0677	− 14.2794
					+14.8525 $\partial\lambda_3$	+13.2187 − 0.89000	+ 28.0712 − 1.89000

Back solution

$\delta\lambda_3$	$\delta\phi_3$	$\delta\lambda_2$	$\delta\phi_2$	$\delta\lambda_1$	$\delta\phi_1$
−0.89000	+0.65852	−0.10591	+0.02035	+0.00779	+0.00840
−0.89000	−0.76711	−0.08850	+0.15134	+0.01814	−0.00020
	−0.10859	+0.02328	−0.03376	−0.00379	+0.00086
		−0.17113	−0.06259	−0.15993	+0.13189
			+0.07534	−0.07244	+0.06108
				−0.21023	−0.17888
					+0.02315

Computation of corrections

1	2	3	4	5	6	7	8
+ 2.007	+ 0.588	+ 0.125	− 0.507	− 1.796	− 0.843	− 0.433	− 4.934
+ 7.377	+ 3.593	+ 3.097	+ 2.918	+ 8.058	− 3.389	− 3.322	−46.059
−10.753	−10.753	−10.753	−10.753	−10.753	− 3.956	−10.753	+16.057
− 1.369	− 2.2	+ 4.8	+ 3.6	+ 7.1	+14.347	+25.5	+37.493
− 1.4	− 8.772	− 2.731	− 4.742	+ 2.609	−10.753	+10.992	−10.753
	− 8.8	− 2.8	− 4.8	+ 2.6	+ 9.7	+11.0	+ 7.1
					+ 5.106		− 1.096
					+ 5.1		− 1.1

z_1	9	10	11	12	13	14	z_3
− 5.793	− 4.934	− 4.530	− 1.514	− 3.156	− 1.541	− 0.480	− 4.934
−27.727	−46.059	+ 8.402	+ 2.541	− 2.824	− 2.911	− 1.845	−46.059
+16.057	+16.057	− 1.223	− 1.223	− 4.549	− 1.223	− 1.223	+ 4.836
+37.493	+37.493	+ 2.649	−13.1	+14.685	+11.5	+ 6.8	+40.857
− 3.956	− 1.223	+ 2.7	−13.296	− 1.223	+ 5.825	+ 3.252	− 4.549
+14.347	+ 1.334		−13.3	− 2.7	+ 5.8	+ 3.2	+14.685
+55.6	+ 1.3			+ 0.233			+ 2.5
+86.021				+ 0.2			+ 7.336
−10.753							− 1.223

15	16	17	18	z_5	19	20	21
+ 1.282	− 3.156	− 0.843	− 2.620	− 0.843	− 0.480	− 1.514	− 0.507
−43.806	− 2.824	− 3.389	+25.267	− 3.889	− 1.845	+ 2.541	+ 2.918
+44.790	− 4.549	− 3.956	+44.790	− 3.156	+ 4.3	+ 6.4	+ 1.1
+ 2.266	+14.685	+14.347	−62.2	− 2.824	+ 1.975	+ 7.427	+ 3.511
+ 2.2	+44.790	+44.790	+ 5.237	− 9.843	+ 2.0	+ 7.5	+ 3.5
	−52.6	−54.8	+ 5.3	+ 10.493			
	− 3.654	− 3.851		−169.6			
	− 3.7	− 3.9		−179.162			
				+ 44.790			

22	23	24
+ 0.125	+ 0.588	+ 2.007
+ 3.097	+ 3.593	+ 7.377
− 1.1	+ 2.5	− 1.9
+ 2.122	+ 6.681	+ 7.484
+ 2.1	+ 6.7	+ 7.5

Final computation of triangles

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″		° ′ ″	
	Duck-Larrabee						3.410111
−1+ 3	Gunner	45 43 29.4	− 1.4	28.0			0.145093
+24	Duck	124 02 00.9	+ 7.5	08.4			9.918392
−22	Larrabee	10 14 25.7	− 2.1	23.6			9.249858
			+ 4.0				
	Gunner-Larrabee						3.473596^{-1}
	Gunner-Duck						2.805062
	Duck-Mam						3.389282
−1+ 4	Gunner	108 41 53.9	− 3.4	50.5			0.023547
+24	Duck	57 00 57.8	+ 7.5	65.3			9.923681
−21	Mam	14 17 07.7	− 3.5	04.2			9.392234
			+ 0.6				
	Gunner-Mam						3.336510
	Gunner-Duck						2.805063^{-1}
	Duck-Lubec Channel Lighthouse						3.045619
−1+ 5	Gunner	115 34 42.9	+ 4.0	46.9			0.044800
+24	Duck	33 11 41.9	+ 7.5	49.4			9.738400
	Lubec Channel Lighthouse	35.2		23.7		31 13 23.7	9.714643
	Gunner-Lubec Channel Lighthouse						2.828819
	Gunner-Duck						2.805062
	Indian Point-Larrabee						3.236668
−2+ 3	Gunner	31 57 41.4	+ 6.0	47.4			0.276237
+23	Indian Point	114 00 36.0	+ 6.7	42.7			9.960690
−22	Larrabee	34 01 32.0	− 2.1	29.9			9.747842
			+10.6				
	Gunner-Larrabee						3.473595
	Gunner-Indian Point						3.260747
	Indian Point-Mam						3.470097
−2+ 4	Gunner	94 56 05.9	+ 4.0	09.9			0.001614
+23	Indian Point	47 05 36.3	+ 6.7	43.0			9.864800
−21	Mam	37 58 10.6	− 3.5	07.1			9.789037
			+ 7.2				
	Gunner-Mam						3.336511^{-1}
	Gunner-Indian Point						3.260748^{-1}
	Indian Point-Lubec Channel Lighthouse						3.315762
−2+ 5	Gunner	101 48 54.9	+11.4	66.3			0.009305
+23	Indian Point	18 35 56.3	+ 6.7	63.0			9.503754
	Lubec Channel Lighthouse	68.8		50.7		59 34 50.7	9.935680
	Gunner-Lubec Channel Lighthouse						2.828821^{-1}
	Gunner-Indian Point						3.260747

Final computation of triangles—Continued

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″		° ′ ″	
	Larrabee-Mam						3.443126
−3+ 4	Gunner	62 58 24.5	− 2.0	22.5			0.050224
+22	Larrabee	44 10 37.0	+ 2.1	39.1			9.843160
−21	Mam	72 50 61.9	− 3.5	58.4			9.980246
			− 3.4				
	Gunner-Mam						3.336510
	Gunner-Larrabee						3.473596^{-1}
	Larrabee-Lubec Channel Lighthouse						3.449573
−3+ 5	Gunner	69 51 13.5	+ 5.4	18.9			0.027415
+22	Larrabee	12 59 29.9	+ 2.1	32.0			9.351833
	Lubec Channel Lighthouse	16.6		09.1		97 09 09.1	9.996607
	Gunner-Lubec Channel Lighthouse						2.828821^{-1}
	Gunner-Larrabee						3.473595
	Larrabee-Lubec church spire						3.702872
−3+ 7	Gunner	154 32 48.4	+13.8	62.2			0.366819
+22	Larrabee	10 46 00.0	+ 2.1	02.1			9.271423
	Lubec church spire	71.6		55.7		14 40 55.7	9.403903
	Gunner-Lubec church spire						3.341114^{+1}
	Gunner-Larrabee						3.473594^{+1}
	Mam-Lubec Channel Lighthouse						3.176968
−4+ 5	Gunner	6 52 49.0	+ 7.4	56.4			0.921432
+21	Mam	3 04 49.6	+ 3.5	53.1			8.730418
	Lubec Channel Lighthouse	21.4		10.5		170 02 10.5	9.238109
	Gunner-Lubec Channel Lighthouse						2.828818^{+2}
	Gunner-Mam						3.336509^{+1}
	Lubec church spire-Indian Point						3.603122
+2− 7	Gunner	173 29 30.2	−19.8	10.4			0.945226
	Lubec church spire	56 55.4		81.9		2 57 21.9	8.712401
−23	Indian Point	3 33 34.4	− 6.7	27.7			8.792767
	Gunner-Indian Point						3.260749^{-9}
	Gunner-Lubec church spire						3.341115
	Gunner-Lubec Channel Lighthouse						2.828820
−9+10	Cranberry Point	75 45 12.8	+ 1.4	14.2			0.013565
−5+ 8	Gunner	90 05 58.7	− 3.7	55.0			9.999999
	Lubec Channel Lighthouse	48.5		50.8		14 08 50.8	9.388133
	Cranberry Point-Lubec Channel Lighthouse						2.842384
	Cranberry Point-Gunner						2.230518

Final computation of triangles—Continued

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″		° ′ ″	
	Gunner-Mam						3.336510
−9+11	Cranberry Point	78 36 65.9	−14.6	51.3			0.008632
−4+ 8	Gunner	96 58 47.7	+ 3.7	51.4			9.996768
−20+21	Mam	4 24 21.3	− 4.0	17.3			8.885376
			−14.9				
	Cranberry Point-Mam						3.341910
	Cranberry Point-Gunner						2.230518
	Gunner-Lubec church spire						3.341115
−9+13	Cranberry Point	174 08 32.3	+ 4.5	36.8			0.991245
−7+ 8	Gunner	5 24 23.8	−12.1	11.7			8.973889
	Lubec church spire	03.9		11.5		0 27 11.5	7.808156
	Cranberry Point-Lubec church spire						3.306249
	Cranberry Point-Gunner						3.230516^{+2}
	Lubec Channel Lighthouse-Mam						3.176968
−10+11	Cranberry Point	2 51 53.1	−16.0	37.1			1.301891
	Lubec Channel Lighthouse	35.2		58.7		175 48 58.7	8.863051
−20	Mam	1 19 31.7	− 7.5	24.2			8.363526
	Cranberry Point-Mam						3.241910
	Cranberry Point-Lubec Channel Lighthouse						2.842385^{-1}
	Lubec church spire-$Treat_2$						3.309982
−13+14	Cranberry Point	17 44 02.9	− 2.6	00.3			0.516286
	Lubec church spire	19.0		23.6		144 41 23.6	9.761929
−19	$Treat_2$	17 34 38.1	− 2.0	36.1			9.479981
	Cranberry Point-$Treat_2$						3.588197
	Cranberry Point-Lubec church spire						3.306249
	Lubec church spire-Cranberry Point						3.306249
−15+16	Telegraph	128 38 59.9	− 5.9	54.0			0.107352
	Lubec church spire	47.5		47.8		30 51 47.8	9.710110
−12+13	Cranberry Point	20 29 12.6	+ 5.6	18.2			9.544090
	Telegraph-Cranberry Point						3.123711
	Telegraph-Lubec church spire						2.957691
	Lubec church spire-Gunner						3.341115
−15+17	Telegraph	131 33 53.1	− 6.1	47.0			0.125967
	Lubec church spire	36.1		36.3		30 24 36.3	9.704310
−6+ 7	Gunner	18 01 30.8	+ 5.9	36.7			9.490609
	Telegraph-Gunner						3.171392
	Telegraph-Lubec church spire						2.957691
	Cranberry Point-Gunner						2.230518
−16+17	Telegraph	2 54 53.2	− 0.2	53.0			1.293713
−9+12	Cranberry Point	153 39 19.7	− 1.1	18.6			9.647161
−6+ 8	Gunner	23 25 54.6	− 6.2	48.4			9.599479
			− 7.5				
	Telegraph-Gunner						3.171392
	Telegraph-Cranberry Point						3.123710^{+1}

Final computation of triangles—Continued

Symbol	Station.	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″		° ′ ″	
	Cranberry Point-Lubec Channel Lighthouse						2.842384
−16+18	Telegraph	29 52 44.5	+ 9.0	53.5			0.302589
−10+12	Cranberry Point	77 54 06.9	− 2.5	04.4			9.990245
	Lubec Channel Lighthouse	08.6		02.1		72 13 02.1	9.978738
	Telegraph-Lubec Channel Lighthouse						3.135218
	Telegraph-Cranberry Point						3.123711
	Gunner-Lubec Channel Lighthouse						2.828820
−17+18	Telegraph	26 57 51.3	+ 9.2	60.5			0.343447
− 5+ 6	Gunner	66 40 04.1	+ 2.5	06.6			9.962951
	Lubec Channel Lighthouse	64.6		52.9		86 21 52.9	9.999125
	Telegraph-Lubec Channel Lighthouse						3.135218
	Telegraph-Gunner						3.171392

Final position computation,

STATION GUNNER

		°	′	″
α	Duck to Lubec Channel Lighthouse	86	26	42.1
Second angle	Lubec Channel Lighthouse and Gunner	+33	11	49.4
α	Duck to Gunner	119	38	31.5
$\Delta\alpha$		−		17.8
		180		
α'	Gunner to Duck	299	38	13.7
	First angle of triangle	115	34	46.9

	°	′	″			°	′	″
ϕ	44	50	33.886	Duck	λ	66	57	47.822
$\Delta\phi$	+		10.227		$\Delta\lambda$	+		25.263
ϕ'	44	50	44.113	Gunner	λ'	66	58	13.085

$\frac{1}{2}(\phi+\phi')$	° ′ ″ 44 50 39	s	2.805062	s^2	5.6101
		$\cos\alpha$	9.694237	$\sin^2\alpha$	9.8782
		B	8.510480	C	1.4016
	″				
1st term	−10.2277	h	1.009779		6.8899
2d term	+ 0.0008				
$-\Delta\phi$	−10.2269				

	s	2.805062		
	$\sin\alpha$	9.939086		
	A′	8.508994	$\Delta\lambda$	1.402490
	$\sec\phi'$	0.149348	$\sin\frac{1}{2}(\phi+\phi')$	9.848301
		1.402490		1.250791
		″		″
	$\Delta\lambda$	+25.2633	$-\Delta\alpha$	+17.82

STATION CRANBERRY POINT

		°	′	″
α	Gunner to Lubec Channel Lighthouse	55	13	00.6
Second angle	Lubec Channel Lighthouse and Cranberry Point	+ 90	05	55.0
α	Gunner to Cranberry Point	145	18	55.6
$\Delta\alpha$		−		03.1
		180		
α'	Cranberry Point to Gunner	325	18	52.5
	First angle of triangle	75	45	14.2

	°	′	″			°	′	″
ϕ	44	50	44.113	Gunner	λ	66	58	13.085
$\Delta\phi$	+		04.529		$\Delta\lambda$	+		04.406
ϕ'	44	50	48.642	Cranberry Point	λ'	66	58	17.491

$\frac{1}{2}(\phi+\phi')$	° ′ ″ 44 50 46	s	2.230518	s^2	4.4610
		$\cos\alpha$	9.915029	$\sin^2\alpha$	9.5103
		B	8.510480	C	1.4016
	″				
1st term	−4.5293	h	0.656027		5.3729
2d term	+0.0000				
$-\Delta\phi$	−4.5293				

	s	2.230518		
	$\sin\alpha$	9.755156		
	A′	8.508994	$\Delta\lambda$	0.644025
	$\sec\phi'$	0.149357	$\sin\frac{1}{2}(\phi+\phi')$	9.848315
		0.644025		0.492340
		″		″
	$\Delta\lambda$	+4.4058	$-\Delta\alpha$	+3.1

secondary triangulation

STATION GUNNER

						°	′	″
α	Lubec Channel Lighthouse to Duck					266	26	06.5
Third angle	Gunner and Duck					− 31	13	23.7
α	Lubec Channel Lighthouse to Gunner					235	12	42.8
$\Delta\alpha$						+		17.8
						180		
α'	Gunner to Lubec Channel Lighthouse					55	13	00.6
	°	′	″					
ϕ	44	50	31.652	Lubec Channel Lighthouse	λ	66	58	38.299
$\Delta\phi$	+		12.461		$\Delta\lambda$	−		25.214
ϕ'	44	50	44.113	Gunner	λ'	66	58	13.085

	° ′ ″	s	2.828819	s^2	5.6576
$\frac{1}{2}(\phi+\phi')$	44 50 36	cos α	9.756288	sin² α	9.8290
	″	B	8.510480	C	1.4016
1st term	−12.4620	h	1.095587		6.8882
2d term	+ 0.0008				
$-\Delta\phi$	−12.4612				

s	2.828819		
sin α	9.914485		
A′	8.508994	$\Delta\lambda$	1.401646
sec ϕ'	0.149348	sin $\frac{1}{2}(\phi+\phi')$	9.848294
	1.401646		1.249940
	″		″
$\Delta\lambda$	−25.2142	$-\Delta\alpha$	−17.78

STATION CRANBERRY POINT

						°	′	″
α	Lubec Channel Lighthouse to Gunner					235	12	42.8
Third angle	Cranberry Point and Gunner					− 14	08	50.8
α	Lubec Channel Lighthouse to Cranberry Point					221	03	52.0
$\Delta\alpha$						+		14.7
						180		
α'	Cranberry Point to Lubec Channel Lighthouse					41	04	06.7
	°	′	″					
ϕ	44	50	31.652	Lubec Channel Lighthouse	λ	66	58	38.299
$\Delta\phi$	+		16.990		$\Delta\lambda$	−		20.808
ϕ'	44	50	48.642	Cranberry Point	λ'	66	58	17.491

	° ′ ″	s	2.842384	s^2	5.6848
$\frac{1}{2}(\phi+\phi')$	44 50 40	cos α	9.877355	sin² α	9.6350
	″	B	8.510480	C	1.4016
1st term	−16.9910	h	1.230219		6.7214
2d term	+ 0.0005				
$-\Delta\phi$	−16.9905				

s	2.842384		
sin α	9.817504		
A′	8.508994	$\Delta\lambda$	1.318239
sec ϕ'	0.149357	sin $\frac{1}{2}(\phi+\phi')$	9.848303
	1.318239		1.166542
	″		″
$\Delta\lambda$	−20.8084	$-\Delta\alpha$	−14.67

Final position computation,

STATION TELEGRAPH

		°	′	″
α	Cranberry Point to Lubec Channel Lighthouse	41	04	06.7
Second angle	Lubec Channel Lighthouse and Telegraph	+ 77	54	04.4
α	Cranberry Point to Telegraph	118	58	11.1
$\Delta\alpha$		−		37.4
		180		
α'	Telegraph to Cranberry Point	298	57	33.7
	First angle of triangle	29	52	53.4

	°	′	″			°	′	″
ϕ	44	50	48.642	Cranberry Point	λ	66	58	17.491
$\Delta\phi$	+		20.858		$\Delta\lambda$	+		52.973
ϕ'	44	51	09.500	Telegraph	λ'	66	59	10.464
			+1					

	° ′ ″				
$\frac{1}{2}(\phi+\phi')$	44 50 59	s	3.123711	s^2	6.2474
		$\cos\alpha$	9.685157	$\sin^2\alpha$	9.8839
		B	8.510480	C	1.4016
	″				
1st term	−20.8617	h	1.319348		7.5329
2d term	+ 0.0034				
$-\Delta\phi$	−20.8583				

s	3.123711		
$\sin\alpha$	9.941946		
A′	8.508994	$\Delta\lambda$	1.724052
$\sec\phi'$	0.149401	$\sin\frac{1}{2}(\phi+\phi')$	9.848343
	1.724052		1.572395
	″		″
$\Delta\lambda$	+52.9728	$-\Delta\alpha$	+37.36

secondary triangulation—Continued.

STATION TELEGRAPH

						°	′	″
α	Lubec Channel Lighthouse to Cranberry Point					221	03	52.0
Third angle	Telegraph and Cranberry Point					− 72	13	02.2
α	Lubec Channel Lighthouse to Telegraph					148	50	49.8
$\Delta\alpha$						−		22.7
						180		
α'	Telegraph to Lubec Channel Lighthouse					328	50	27.1
	°	′	″					
ϕ	44	50	31.652	Lubec Channel Lighthouse	λ	66	58	38.299
$\Delta\phi$	+		37.849		$\Delta\lambda$	+		32.164
ϕ'	44	51	09.501	Telegraph	λ'	66	59	10.463
								+1

	° ′ ″	s	3.135218	s^2	6.2704
$\frac{1}{2}(\phi+\phi')$	44 50 50	$\cos\alpha$	9.932367	$\sin^2\alpha$	9.4275
	″	B	8.510480	C	1.4016
1st term	−37.8500	h	1.578065		7.0995
2d term	+ 0.0012				
$-\Delta\phi$	−37.8488				

s	3.135218		
$\sin\alpha$	9.713762		
A′	8.508994	$\Delta\lambda$	1.507375
$\sec\phi'$	0.149401	$\sin\frac{1}{2}(\phi+\phi')$	9.848324
	1.507375		1.355699
	″		″
$\Delta\lambda$	+32.1644	$-\Delta\alpha$	+22.69

Computation of probable errors

	Adopted v's first solution	v^2		Adopted v's second solution	v^2
1	+ 0.2	0.04	1	− 1.4	1.96
2	− 8.5	72.25	2	− 8.8	77.44
3	− 2.7	7.29	3	− 2.8	7.84
4	− 4.9	24.01	4	− 4.8	23.04
5	+ 3.1	9.61	5	+ 2.6	6.76
6	+ 4.9	24.01	6	+ 5.1	26.01
7	+ 9.5	90.25	7	+11.0	121.00
8	− 1.5	2.25	8	− 1.1	1.21
9	+ 1.7	2.89	9	+ 1.3	1.69
10	+ 3.1	9.61	10	+ 2.7	7.29
11	−13.1	171.61	11	−13.3	176.89
12	+ 0.6	0.36	12	+ 0.2	0.04
13	+ 5.0	25.00	13	+ 5.8	33.64
14	+ 2.9	8.41	14	+ 3.2	10.24
15	+ 2.3	5.29	15	+ 2.2	4.84
16	− 3.8	14.44	16	− 3.7	13.69
17	− 3.8	14.44	17	− 3.9	15.21
18	+ 5.4	29.16	18	+ 5.3	28.09
19	+ 0.8	0.64	19	+ 2.0	4.00
19′	− 0.7	0.49	20	+ 7.5	56.25
20	+ 5.6	31.36	21	+ 3.5	12.25
21	+ 2.1	4.41	22	+ 2.1	4.41
20′	− 0.8	0.64	23	+ 6.7	44.89
20″	− 1.5	2.25	24	+ 7.5	56.25
20‴	− 6.5	42.25			
20⁗	+ 1.2	1.44			734.93
22	+ 2.6	6.76			
22′	− 0.7	0.49			
22″	− 3.7	13.69			
22‴	+ 5.7	32.49			
22⁗	− 0.6	0.36			
22ᵛ	− 3.3	10.89			
23	+ 1.9	3.61			
23′	− 4.2	17.64			
23″	− 5.5	30.25			
23‴	− 2.8	7.84			
23⁗	+10.7	114.49			
24	+ 6.5	42.25			
24′	− 3.0	9.00			
24″	− 3.4	11.56			
		895.72			

In the first method of adjustment, there are 40 equations to determine 14 unknown quantities, namely, 6 $\delta\phi$'s and $\delta\lambda$'s and 8 z's. The probable error of an observed direction is therefore,

$$0.6745\sqrt{\frac{895.72}{40-14}} = \pm 4.0''$$

In the second method there are 24 equations to determine 9 unknown quantities, namely, 6 $\delta\phi$'s and $\delta\lambda$'s and 3 z's at new points. The probable error of an observed direction is therefore,

$$0.6745\sqrt{\frac{734.93}{15}} = \pm 4.7''$$

ADJUSTMENT OF A FIGURE CONTAINING LATITUDE, LONGITUDE, AZIMUTH, AND LENGTH CONDITIONS BY THE METHOD OF VARIATION OF GEOGRAPHIC COORDINATES

This example illustrates the fitting of a chain of triangulation in between fixed lines at the ends. The necessary preliminary computations of the assumed positions and directions could have been carried out in much the same way as in the preceding examples. A preliminary figure adjustment was, however, available and the results of it were used in the preliminary computations of the triangles and the geographic positions, pages 140–157. The fixed lines at the ends of the chain are shown in the following list of fixed positions. The list of observed directions is not given. The necessary data may be derived from the observed angles of the triangles, pages 140–143, taken in connection with figure 7, page 157.

Table of fixed positions

Station	Latitude and longitude	Azimuth	Back azimuth	To station	Logarithm of distance.
	° ′ ″	° ′ ″	° ′ ″		
Fort Morgan.	30 13 42.242 88 01 23.228				
Dauphin Island east base.	30 14 56.379 88 08 14.288	281 42 17.9	101 45 44.9	Fort Morgan.	4.050203
Dauphin Island west base.	30 14 21.402 88 14 51.034	264 11 22.1	84 14 41.9	Dauphin Island east base	4.027832
Biloxi Lighthouse.	30 23 39.419 88 54 03.820				
Ship Island Lighthouse.	30 12 45.341 88 57 57.464	197 12 19.7	17 14 17.6	Biloxi Lighthouse.	4.323998

Preliminary computation of triangles

Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
	° ′ ″	″	″	″	° ′ ″	
Fort Morgan-Dauphin Island east base						4.050203
Cedar	43 54 21.8	+ 0.0	21.8	0.1	21.7	0.158968
Fort Morgan	42 33 37.7	+ 0.3	38.0	0.1	37.9	9.830183
Dauphin Island east base	93 31 59.6	+ 0.9	60.5	0.1	82 00.4	9.999174
		+ 1.2		0.3		
Cedar-Dauphin Island east base						4.039354
Cedar-Fort Morgan						4.208345
Dauphin Island east base-Dauphin Island west base						4.027832
Cedar	37 26 33.0	− 0.9	32.1	0.1	32.0	0.216124
Dauphin Island east base	103 55 36.4	− 0.9	35.5	0.1	35.4	9.987043
Dauphin Island west base	38 37 52.5	+ 0.2	52.7	0.1	52.6	9.795398
		− 1.6		0.3		
Cedar-Dauphin Island west base						4.230999
Cedar-Dauphin Island east base						4.039354
Cedar-Dauphin Island east base						4.039354
Cat	69 30 32.7	+ 0.8	33.5	0.0	33.5	0.028386
Cedar	60 11 14.2	+ 1.4	15.6	0.1	15.5	9.938349
Dauphin Island east base	50 18 11.4	− 0.3	11.1	0.1	11.0	9.886171
		+ 1.9		0.2		
Cat-Dauphin Island east base						4.006089
Cat-Cedar						3.953911
Cedar-Dauphin Island west base						4.230999
Cat	135 31 54.9	+ 2.7	57.6	0.0	57.6	0.154590
Cedar	22 44 41.2	+ 2.3	43.5	0.1	43.4	9.587303
Dauphin Island west base	21 43 18.7	+ 0.4	19.1	0.1	19.0	9.568322
		+ 5.4		0.2		
Cat-Dauphin Island west base						3.972892
Cat-Cedar						3.953911
Dauphin Island east base-Dauphin Island west base						4.027832
Cat	66 01 22.2	+ 1.9	24.1	0.1	24.0	0.039191
Dauphin Island east base	53 37 25.0	− 0.6	24.4	0.1	24.3	9.905869
Dauphin Island west base	60 21 11.2	+ 0.6	11.8	0.1	11.7	9.939066
		+ 1.9		0.3		
Cat-Dauphin Island west base						3.972892
Cat-Dauphin Island east base						4.006089
Cedar-Cat						3.953911
Pins	23 22 40.3	− 1.8	38.5	0.1	38.4	0.401445
Cedar	30 54 61.5	− 3.5	58.0	0.1	57.9	9.710779
Cat	18.4		23.7	0.0	125 42 23.7	9.909565
				0.2		
Pins-Cat						4.066135^{-1}
Pins-Cedar						4.264921
Cedar-Dauphin Island west base						4.230999
Pins	58 45 26.0	− 2.8	23.2	0.2	23.0	0.068049
Cedar	53 39 42.7	− 1.2	41.5	0.2	41.3	9.906082
Dauphin Island west base	67 34 57.9	− 2.0	55.9	0.2	55.7	9.965873
		− 6.0		0.6		
Pins-Dauphin Island west base						4.205130
Pins-Cedar						4.264921
Cat-Dauphin Island west base						3.972892
Pins	35 22 45.7	− 1.0	44.7	0.1	44.6	0.237334
Cat	35.3		38.7	0.0	98 45 38.7	9.994903
Dauphin Island west base	45 51 39.2	− 2.4	36.8	0.1	36.7	9.855908
				0.2		
Pins-Dauphin Island west base						4.205129^{+1}
Pins-Cat						4.066134

Preliminary computation of triangles—Continued

Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
	° ′ ″	″	″	″	° ′ ″	
Pins–Dauphin Island west base						4.205130
Grand	54 52 01.6	+ 1.4	00.2	0.2	00.0	0.087845
Pins	85 13 07.0	+ 1.3	08.3	0.1	08.2	9.998486
Dauphin Island west base	39 54 50.9	+ 1.1	52.0	0.2	51.8	9.807293
		+ 1.0		0.5		
Grand–Dauphin Island west base						4.290961
Grand–Pins						4.099768
Grand–Pins						4.099768
Petit	33 09 08.7	− 0.3	08.4	0.1	08.3	0.262119
Grand	114 03 53.6	− 0.6	53.0	0.1	52.9	9.960511
Pins	32 46 57.5	+ 1.4	58.9	0.1	58.8	9.733565
		+ 0.5		0.3		
Petit–Pins						4.322398
Petit–Grand						4.095452
Grand–Dauphin Island west base						4.290961
Petit	81 41 28.2	+ 0.1	28.3	0.1	28.2	0.004583
Grand	59 11 52.0	+ 0.8	52.8	0.2	52.6	9.933964
Dauphin Island west base	39 06 39.1	+ 0.3	39.4	0.2	39.2	9.799908
		+ 1.2		0.5		
Petit–Dauphin Island west base						4.229508
Petit–Grand						4.095452
Pins–Dauphin Island west base						4.205130
Petit	48 32 19.5	+ 0.4	19.9	0.2	19.7	0.125284
Pins	52 26 09.5	− 0.1	09.4	0.2	09.2	9.899093
Dauphin Island west base	79 01 30.0	+ 1.4	31.4	0.3	31.1	9.991984
		+ 1.7		0.7		
Petit–Dauphin Island west base						4.229507+1
Petit–Pins						4.322398
Grand–Petit						4.095452
Pascagoula	37 39 20.6	+ 3.0	23.6	0.1	23.5	0.214011
Grand	104 18 56.1	+ 1.5	57.6	0.2	57.4	9.986300
Petit	38 01 38.6	+ 0.6	39.2	0.1	39.1	9.789609
		+ 5.1		0.4		
Pascagoula–Petit						4.295763
Pascagoula–Grand						4.099072
Pascagoula–Grand						4.099072
Horn	38 49 39.0	+ 3.8	42.8	0.1	42.7	0.202738
Pascagoula	97 55 54.0	+ 3.5	57.5	0.2	57.3	9.995824
Grand	43 14 18.9	+ 1.2	20.1	0.1	20.0	9.835717
		+ 8.5		0.4		
Horn–Grand						4.297634+1
Horn–Pascagoula						4.137527
Pascagoula–Petit						4.295763
Horn	77 06 13.2	+ 2.9	16.1	0.2	15.9	0.011094
Pascagoula	60 16 33.4	+ 0.5	33.9	0.2	33.7	9.938732
Petit	42 37 10.3	+ 0.3	10.6	0.2	10.4	9.830670
		+ 3.7		0.6		
Horn–Petit						4.245589
Horn–Pascagoula						4.137527
Grand–Petit						4.095452
Horn	38 16 34.2	− 0.9	33.3	0.2	33.1	0.207995
Grand	61 04 37.2	+ 0.3	37.5	0.2	37.3	9.942142
Petit	80 38 48.9	+ 0.9	49.8	0.2	49.6	9.994188
		+ 0.3		0.6		
Horn–Petit						4.245589
Horn–Grand						4.297635

Preliminary computation of triangles—Continued

Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
	° ′ ″	″	″	″	° ′ ″	
Pascagoula-Horn						4.137527
Belle	48 58 49.8	− 1.8	48.0	0.2	47.8	0.123352
Pascagoula	69 43 28.0	− 3.4	24.6	0.1	24.5	9.972217
Horn	61 17 53.5	− 5.6	47.9	0.2	47.7	9.943058
		−10.8		0.5		
Belle-Horn						4.232096
Belle-Pascagoula						4.202937
Belle-Pascagoula						4.202937
Club	58 40 43.3	+ 1.1	44.4	0.1	44.3	0.068406
Belle	89 28 55.3	− 0.6	54.7	0.2	54.5	9.999982
Pascagoula	31 50 23.4	− 2.1	21.3	0.1	21.2	9.722253
		− 1.6		0.4		
Club-Pascagoula						4.271325
Club-Belle						3.993596
Belle-Horn						4.232096
Club	105 43 56.9	− 0.4	56.5	0.1	56.4	0.016582
Belle	40 30 05.5	+ 1.2	06.7	0.1	06.6	9.812561
Horn	33 45 53.7	+ 3.4	57.1	0.1	57.0	9.744918
		+ 4.2		0.3		
Club-Horn						4.061239
Club-Belle						3.993596
Pascagoula-Horn						4.137527
Club	47 03 13.6	− 1.5	12.1	0.1	12.0	0.135496
Pascagoula	37 53 04.6	− 1.3	03.3	0.1	03.2	9.788216
Horn	95 03 47.2	− 2.2	45.0	0.2	44.8	9.998302
		− 5.0		0.4		
Club-Horn						4.061239
Club-Pascagoula						4.271325
Belle-Club						3.993596
Deer	41 02 10.7	+ 0.7	11.4	0.1	11.3	0.182739
Belle	102 35 18.5	+ 1.5	20.0	0.0	20.0	9.989432
Club	36 22 26.8	+ 2.0	28.8	0.1	28.7	9.773101
		+ 4.2		0.2		
Deer-Club						4.165767
Deer-Belle						3.949436
Deer-Belle						3.949436
Ship	33 10 60.2	− 1.8	58.4	0.1	58.3	0.261764
Deer	97 49 37.9	− 1.8	36.1	0.1	36.0	9.995935
Belle	48 59 22.9	+ 2.9	25.8	0.1	25.7	9.877717
		− 0.7		0.3		
Ship-Belle						4.207135+1
Ship-Deer						4.088917
Deer-Club						4.165767
Ship	70 52 35.0	− 0.5	34.5	0.2	34.3	0.024654
Deer	56 47 27.2	− 2.5	24.7	0.1	24.6	9.922554
Club	52 20 03.5	− 2.3	01.2	0.1	01.1	9.898496
		− 5.3		0.4		
Ship-Club						4.112975
Ship-Deer						4.088917
Belle-Club						3.993596
Ship	37 41 34.8	+ 1.3	36.1	0.1	36.0	0.213650
Belle	53 35 55.6	− 1.4	54.2	0.1	54.1	9.905729
Club	88 42 30.3	− 0.3	30.0	0.1	29.9	9.999890
		− 0.4		0.3		
Ship-Club						4.112975
Ship-Belle						4.207136

Preliminary computation of triangles—Continued

Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
	° ′ ″	″	″	″	° ′ ″	
Deer–Ship						4.088917
Biloxi Lighthouse	48 11 17.4	+ 3.6	21.0	0.1	20.9	0.127640
Deer	96 30 31.2	+ 2.3	33.5	0.1	33.4	9.997191
Ship	35 18 04.4	+ 1.4	05.8	0.1	05.7	9.761838
		+ 7.3		0.3		
Biloxi Lighthouse–Ship						4.213748
Biloxi Lighthouse–Deer						3.978395
Biloxi Lighthouse–Deer						3.978395
Ship Island Lighthouse	25 30 02.1	+ 1.4	03.5	0.2	03.3	0.366001
Biloxi Lighthouse	81 55 36.0	− 1.7	34.3	0.1	34.2	9.995674
Deer	72 34 26.4	− 3.7	22.7	0.2	22.5	9.979593
		− 4.0		0.5		
Ship Island Lighthouse–Deer						4.340070
Ship Island Lighthouse–Biloxi Lighthouse						4.323989
Biloxi Lighthouse–Ship						4.213748
Ship Island Lighthouse	50 31 41.2	− 0.9	40.3	0.2	40.1	0.112420
Biloxi Lighthouse	33 44 18.6	− 5.3	13.3	0.2	13.1	9.744591
Ship	95 44 07.0	− 0.1	06.9	0.1	06.8	9.997821
		− 6.3		0.5		
Ship Island Lighthouse–Ship						4.070759
Ship Island Lighthouse–Biloxi Lighthouse						4.323989
Deer–Ship						4.088917
Ship Island Lighthouse	25 01 39.1	− 2.3	36.8	0.1	36.7	0.373615
Deer	23 56 04.8	+ 6.0	10.8	0.1	10.7	9.608227
Ship	131 02 11.4	+ 1.3	12.7	0.1	12.6	9.887537
		+ 5.0		0.3		
Ship Island Lighthouse–Ship						4.070759
Ship Island Lighthouse–Deer						4.340069+1

Preliminary position computation,

STATION CAT

		°	′	″
α	East base to west base	84	14	41.9
Second angle	West base and Cat	+53	37	24.4
α	East base to Cat	137	52	06.3
$\Delta\alpha$		−	2	08.4
		180	00	00.00
α'	Cat to east base	317	49	57.9
	First angle of triangle	66	01	24.1

		°	′	″						
ϕ		30	14	56.379	Dauphin Island east base	λ	88	08	14.288	
$\Delta\phi$		+	4	04.171		$\Delta\lambda$	+	4	14.637	
ϕ'		30	19	00.550	Cat	λ'	88	12	28.925	
$\frac{1}{2}(\phi+\phi')$	° ′ ″	30 16 58	s	4.006089	s^2	8.0122	h^2	4.776	$-h$	2.388
			$\cos\alpha$	9.870173	$\sin^2\alpha$	9.6532			$s^2\sin^2\alpha$	7.665
			B	8.511556	C	1.1712	D	2.332	E	5.917
		″								
1st term		−244.2407	h	2.387818		8.8366		7.108		5.970
2d, 3d, and 4th terms		+ 0.0700				+0.0686		+0.0013		+0.0001
$-\Delta\phi$		−244.1707								

	s	4.006089		
	$\sin\alpha$	9.826616		
	A′	8.509352	$\Delta\lambda$	2.405922
	$\sec\phi'$	0.063865	$\sin\frac{1}{2}(\phi+\phi')$	9.702663
		2.405922		2.108585
		″		″
	$\Delta\lambda$	+254.6373	$-\Delta\alpha$	+128.40

STATION CEDAR

		°	′	″
α	East base to west base	84	14	41.9
Second angle	West base and Cedar	+103	55	35.5
α	East base to Cedar	188	10	17.4
$\Delta\alpha$		+		29.4
		180	00	00.00
α'	Cedar to east base	8	10	46.8
	First angle of triangle	37	26	32.1

		°	′	″						
ϕ		30	14	56.379	Dauphin Island east base	λ	88	08	14.288	
$\Delta\phi$		+	5	51.941		$\Delta\lambda$	−		58.265	
ϕ'		30	20	48.320 +1	Cedar	λ'	88	07	16.023	
$\frac{1}{2}(\phi+\phi')$	° ′ ″	30 17 52	s	4.039354	s^2	8.0787	h^2	5.093		
			$\cos\alpha$	9.995568	$\sin^2\alpha$	8.3054				
			B	8.511556	C	1.1712	D	2.332		
		″								
1st term		−351.9476	h	2.546478		7.5553		7.425		
2d and 3d terms		+ 0.0063				+0.0036		+0.0027		
$-\Delta\phi$		−351.9413								

	s	4.039354		
	$\sin\alpha$	9.152706		
	A′	8.509351	$\Delta\lambda$	1.765408
	$\sec\phi'$	0.063997	$\sin\frac{1}{2}(\phi+\phi')$	9.702857
		1.765408		1.468265
		″		″
	$\Delta\lambda$	−58.2650	$-\Delta\alpha$	−29.39

secondary triangulation

STATION CAT

								°	′	″
α	West base to east base							264	11	22.1
Third angle	Cat and east base							− 60	21	11.8
α	West base to Cat							203	50	10.3
$\Delta\alpha$								+	1	11.7
α'	Cat to west base							180	00	00.00
								23	51	22.0
	°	′	″							
ϕ	30	14	21.492	Dauphin Island west base		λ	88	14	51.034	
$\Delta\phi$	+	4	39.058			$\Delta\lambda$	−	2	22.109	
ϕ'	30	19	00.550	Cat		λ'	88	12	28.925	
	° ′ ″	s	3.972892	s^2	7.9458			−h	2.445	
$\frac{1}{2}(\phi+\phi')$	30 16 41	cos α	9.961281	sin² α	9.2130	h^2	4.891	s^2 sin² α	7.159	
		B	8.511557	C	1.1711	D	2.331	E	5.917	
	″									
1st term	−279.0808	h	2.445730		8.3299		7.222		5.521	
2d, 3d, and 4th terms	+ 0.0231				+0.0214		+0.0017			
$-\Delta\phi$	−279.0577									
	s	3.972892								
	sin α	9.606514								
	A′	8.509352	$\Delta\lambda$		2.152623					
	sec ϕ'	0.063865	sin $\frac{1}{2}(\phi+\phi')$		9.702600					
		2.152623			1.855223					
		″			″					
	$\Delta\lambda$	−142.1095	$-\Delta\alpha$		−71.65					

STATION CEDAR

								°	′	″
α	West base to east base							264	11	22.1
Third angle	Cedar and east base							− 38	37	52.7
α	West base to Cedar							225	33	29.4
$\Delta\alpha$								+	3	49.5
α'	Cedar to west base							180	00	00.00
								45	37	18.9
	°	′	″							
ϕ	30	14	21.492	Dauphin Island west base		λ	88	14	51.034	
$\Delta\phi$	+	6	26.829			$\Delta\lambda$	−	7	35.011	
ϕ'	30	20	48.321	Cedar		λ'	88	07	16.023	
	° ′ ″	s	4.230999	s^2	8.4620			−h	2.588	
$\frac{1}{2}(\phi+\phi')$	30 17 35	cos α	9.845213	sin² α	9.7073	h^2	5.176	s^2 sin² α	8.169	
		B	8.511557	C	1.1711	D	2.331	E	5.917	
	″									
1st term	−387.0517	h	2.587769		9.3404		7.507		6.674	
2d, 3d, and 4th terms	+ 0.2227				+0.2190		+0.0032		+0.0005	
$-\Delta\phi$	−386.8290									
	s	4.230999								
	sin α	9.853675								
	A′	8.509351	$\Delta\lambda$		2.658022					
	sec ϕ'	0.063997	sin $\frac{1}{2}(\phi+\phi')$		9.702795					
		2.658022			2.360817					
		″			″					
	$\Delta\lambda$	−455.0111	$-\Delta\alpha$		−229.52					

Preliminary position computation,

STATION PINS

								°	′	″
α	Cedar to west base							45	37	18.9
Second angle	West base and Pins							+ 53	39	41.5
α	Cedar to Pins							99	17	00.4
$\Delta\alpha$								−	5	43.8
								180	00	00.00
α'	Pins to Cedar							279	11	16.6
					First angle of triangle			58	45	23.2
	°	′	″							
ϕ	30	20	48.321		Cedar		λ	88	07	16.023
$\Delta\phi$	+	1	35.926				$\Delta\lambda$	+	11	20.238
ϕ'	30	22	24.247		Pins		λ'	88	18	36.261
										+1
	° ′ ″		s	4.264921	s^2	8.5298			−h	1.984
$\frac{1}{2}(\phi+\phi')$	30 21 36		$\cos\alpha$	9.207685	$\sin^2\alpha$	9.9885	h^2	3.968	$s^2\sin^2\alpha$	8.518
			B	8.511550	C	1.1729	D	2.332	E	5.919
	″									
1st term	−96.4175		h	1.984156		9.6912		6.300		6.421
2d, 3d, and 4th terms	+ 0.4916					+0.4911		+0.0002		+0.0003
$-\Delta\phi$	−95.9259									

s	4.264921		
$\sin\alpha$	9.994274		
A′	8.509350	$\Delta\lambda$	2.832661
$\sec\phi'$	0.064116	$\sin\frac{1}{2}(\phi+\phi')$	9.703663
	2.832661		2.536324
	″		″
$\Delta\lambda$	+680.2382	$-\Delta\alpha$	+343.81

STATION GRAND

								°	′	″
α	Pins to west base							337	56	39.8
Second angle	West base and Grand							+ 85	13	08.3
α	Pins to Grand							63	09	48.1
$\Delta\alpha$								−	3	32.3
								180	00	00.00
α'	Grand to Pins							243	06	15.8
					First angle of triangle			54	52	00.2
	°	′	″							
ϕ	30	22	24.247		Pins		λ	88	18	36.262
$\Delta\phi$	−	3	04.658				$\Delta\lambda$	+	7	00.260
ϕ'	30	19	19.589		Grand		λ'	88	25	36.522
	° ′ ″		s	4.099768	s^2	8.1995			−h	2.266
$\frac{1}{2}(\phi+\phi')$	30 20 52		$\cos\alpha$	9.654608	$\sin^2\alpha$	9.9010	h^2	4.532	$s^2\sin^2\alpha$	8.100
			B	8.511548	C	1.1734	D	2.333	E	5.919
	″									
1st term	+184.4693		h	2.265924		9.2739		6.865		6.285
2d, 3d, and 4th terms	+ 0.1884					+0.1879		+0.0007		−0.0002
$-\Delta\phi$	+184.6577									

s	4.099768		
$\sin\alpha$	9.950510		
A′	8.509352	$\Delta\lambda$	2.623518
$\sec\phi'$	0.063888	$\sin\frac{1}{2}(\phi+\phi')$	9.703504
	2.623518		2.327022
	″		″
$\Delta\lambda$	+420.2600	$-\Delta\alpha$	+212.33

secondary triangulation—Continued

STATION PINS

									°	′	″
α	West base to Cedar								225	33	29.4
Third angle	Pins and Cedar								− 67	34	55.9
α	West base to Pins								157	58	33.5
$\Delta\alpha$									−	1	53.7
									180	00	00.00
α'	Pins to west base								337	56	39.8
	°	′	″								
ϕ	30	14	21.492	Dauphin Island west base			λ		88	14	51.034
$\Delta\phi$	+	8	02.754				$\Delta\lambda$		+	3	45.228
ϕ'	30	22	24.246	Pins			λ'		88	18	36.262
			+1								
	° ′ ″	s	4.205130	s^2	8.4103					−h	2.684
$\frac{1}{2}(\phi+\phi')$	30 18 23	cos α	9.967092	sin² α	9.1480		h²		5.368	s² sin² α	7.558
	″	B	8.511557	C	1.1711		D		2.331	E	5.917
1st term	−482.8131	h	2.683779		8.7294				7.699		6.159
2d, 3d, and 4th terms	+ 0.0587				+0.0534				+0.0050		+0.0001
$-\Delta\phi$	−482.7544										

s	4.205130		
sin α	9.574026		
A′	8.509350	$\Delta\lambda$	2.352622
sec ϕ'	0.064116	sin $\frac{1}{2}(\phi+\phi')$	9.702967
	2.352622		2.055589
	″		″
$\Delta\lambda$	+225.2278	$-\Delta\alpha$	+113.66

STATION GRAND

									°	′	″
α	West base to Pins								157	58	33.5
Third angle	Grand and Pins								− 39	54	52.0
α	West base to Grand								118	03	41.5
$\Delta\alpha$									−	5	25.5
									180	00	00.00
α'	Grand to west base								297	58	16.0
	°	′	″								
ϕ	30	14	21.492	Dauphin Island west base			λ		88	14	51.034
$\Delta\phi$	+	4	58.097				$\Delta\lambda$		+	10	45.488
ϕ'	30	19	19.589	Grand			λ'		88	25	36.522
	° ′ ″	s	4.290961	s^2	8.5819					−h	2.475
$\frac{1}{2}(\phi+\phi')$	30 16 50	cos α	9.672485	sin² α	9.8914		h²		4.050	s² sin² α	8.473
	″	B	8.511557	C	1.1711		D		2.331	E	5.917
1st term	−298.5403	h	2.475003		9.6444				7.281		6.865
2d, 3d, and 4th terms	+ 0.4436				+0.4410				+0.0019		+0.0007
$-\Delta\phi$	−298.0967										

s	4.290861		
sin α	9.945687		
A′	8.509352	$\Delta\lambda$	2.809888
sec ϕ'	0.063888	sin $\frac{1}{2}(\phi+\phi')$	9.702633
	2.809888		2.512521
	″		″
$\Delta\lambda$	+645.4877	$-\Delta\alpha$	+325.48

Preliminary position computation,

STATION PETIT

								°	′	″
α Second angle	Grand to west base West base and Petit							297 + 59	58 11	16.0 52.8
α $\Delta\alpha$	Grand to Petit							357 +	10	08.8 11.6
α'	Petit to Grand					First angle of triangle		180 177 81	00 10 41	00.00 20.4 28.3
ϕ $\Delta\phi$	° 30 −	′ 19 6	″ 19.589 44.089		Grand	λ $\Delta\lambda$		88 −	25	36.522 23.005
ϕ'	30	12	35.500		Petit	λ'		88	25	13.517
$\frac{1}{2}(\phi+\phi')$	° ′ ″ 30 15 58	s cos α B	4.095452 9.999470 8.511551	s^2 $\sin^2\alpha$ C	8.1909 7.3872 1.1725	h^2 D	5.213 2.332			
1st term 2d and 3d terms $-\Delta\phi$	″ +404.0853 + 0.0041 +404.0894	h	2.606473		6.7506 +0.0006		7.545 0.0035			
	s sin α A′ sec ϕ'	4.095452 8.693623 8.509354 0.063392		$\Delta\lambda$ sin $\frac{1}{2}(\phi+\phi')$	1.361821 9.702445						
		1.361821			1.064266						
	$\Delta\lambda$	″ −23.0049	$-\Delta\alpha$		″ −11.59						

STATION HORN

								°	′	″
α Second angle	Grand to Petit Petit and Horn							357 + 61	10 04	08.8 37.5
α $\Delta\alpha$	Grand to Horn							58 −.	14 5	46.3 18.1
α'	Horn to Grand					First angle of triangle		180 238 38	00 09 16	00.00 28.2 −.1 33.3
ϕ $\Delta\phi$	° 30 −	′ 19 5	″ 19.589 39.578		Grand	λ $\Delta\lambda$		88 +	25 10	36.522 31.017
ϕ'	30	13	40.011		Horn	λ'		88	36	07.539
$\frac{1}{2}(\phi+\phi')$	° ′ ″ 30 16 30	s cos α B	4.297635 9.721209 8.511551	s^2 $\sin^2\alpha$ C	8.5953 9.8592 1.1725	h^2 D	5.061 2.332	−h $s^2\sin^2\alpha$ E		2.530 8.454 5.918
1st term 2d, 3d, and 4th terms $-\Delta\phi$	″ +339.1525 + 0.4253 +339.5778	h	2.530395		9.6270 +0.4236		7.393 +0.0025			6.902 −0.0008
	s sin α A′ sec ϕ'	4.297635 9.929581 8.509354 0.063471		$\Delta\lambda$ sin $\frac{1}{2}(\phi+\phi')$	2.800041 9.702560						
		2.800041			2.502601						
	$\Delta\lambda$	″ +631.0170	$-\Delta\alpha$		″ +318.13						

secondary triangulation—Continued

STATION PETIT

								°	′	″
α	West base to Grand							118	03	41.5
Third angle	Petit and Grand							− 39	06	39.4
α	West base to Petit							78	57	02.1
$\Delta\alpha$								−	5	13.4
								180	00	00.00
α'	Petit to west base							258	51	48.7
	°	′	″							
ϕ	30	14	21.492	Dauphin Island west base		λ	88	14	51.034
$\Delta\phi$	−	1	45.992			$\Delta\lambda$	+	10	22.484
ϕ'	30	12	35.500	Petit		λ'	88	25	13.518
										−1
	° ′ ″	s	4.229508	s^2	8.4590			−h		2.024
$\frac{1}{2}(\phi+\phi')$	30 13 28	cos α	9.282521	sin² α	9.9837	h²	4.047	s^2 sin² α		8.443
	″	B	8.511557	C	1.1711	D	2.331	E		5.917
1st term	+105.5810	h	2.023586		9.6138		6.378			6.384
2d, 3d, and 4th terms	+ 0.4109				+0.4109		+0.0002			−0.0002
$-\Delta\phi$	+105.9919									
	s	4.229508								
	sin α	9.991874								
	A′	8.509354	$\Delta\lambda$	2.794128						
	sec ϕ'	0.063392	sin $\frac{1}{2}(\phi+\phi')$	9.701905						
		2.794128		2.496033						
		″		″						
	$\Delta\lambda$	+622.4837	$-\Delta\alpha$	+313.35						

STATION HORN

								°	′	″
α	Petit to Grand							177	10	20.4
Third angle	Grand and Horn							− 80	38	49.8
α	Petit to Horn							96	31	30.6
$\Delta\alpha$								−	5	29.2
								180	00	00.00
α'	Horn to Petit							276	26	01.4
	°	′	″							
ϕ	30	12	35.500	Petit		λ	88	25	13.517
$\Delta\phi$	+	1	04.511			$\Delta\lambda$	+	10	54.021
ϕ'	30	13	40.011	Horn		λ'	88	36	07.538
										−1
	° ′ ″	s	4.245589	s^2	8.4912			−h		1.813
$\frac{1}{2}(\phi+\phi')$	30 13 08	cos α	9.055530	sin² α	9.9943	h²	3.625	s^2 sin² α		8.485
	″	B	8.511559	C	1.1706	D	2.331	E		5.916
1st term	−64.9648	h	1.812678		9.6561		5.956			6.214
2d, 3d, and 4th terms	+ 0.4533				+0.4530		+0.0001			+0.0002
$-\Delta\phi$	−64.5115									
	s	4.245589								
	sin α	9.997178								
	A′	8.509354	$\Delta\lambda$	2.815592						
	sec ϕ'	0.063471	sin $\frac{1}{2}(\phi+\phi')$	9.701830						
		2.815592		2.517422						
		″		″						
	$\Delta\lambda$	+654.0214	$-\Delta\alpha$	+329.17						

Preliminary position computation,

STATION PASCAGOULA

										°	′	″
α	Grand to Horn									58	14	46.3
Second angle	Horn and Pascagoula									+ 43	14	20.1
α	Grand to Pascagoula									101	29	06.4
$\Delta\alpha$										−	3	52.8
										180	00	00.00
α'	Pascagoula to Grand									281	25	13.6
									First angle of triangle	97	55	57.5
	°	′	″									
ϕ	30	19	19.589		Grand		λ			88	25	36.522
$\Delta\phi$	+	1	21.006				$\Delta\lambda$			+	7	40.922
ϕ'	30	20	40.595		Pascagoula		λ'			88	33	17.444
	° ′ ″	s	4.099072	s^2	8.1981						−h	1.910
$\frac{1}{2}(\phi+\phi')$	30 20 00	cos α	9.299100	sin² α	9.9824		h²	3.820			s² sin² α	8.180
		B	8.511551	C	1.1725		D	2.332			E	5.918
	″											
1st term	−81.2312	h	1.909723		9.3530			6.152				6.008
2d, 3d, and 4th terms	+ 0.2256				+0.2254			+0.0001				+0.0001
$-\Delta\phi$	−81.0056											

	s	4.099072		
	sin α	9.991216		
	A′	8.509351	$\Delta\lambda$	2.663627
	sec ϕ'	0.063988	sin $\frac{1}{2}(\phi+\phi')$	9.703317
		2.663627		2.366944
		″		″
	$\Delta\lambda$	+460.9215	$-\Delta\alpha$	+232.78

STATION BELLE

										°	′	″
α	Pascagoula to Horn									19	21	11.1
Second angle	Horn and Belle									+ 69	43	24.6
α	Pascagoula to Belle									89	04	35.7
$\Delta\alpha$										−	5	01.8
										180	00	00.00
α'	Belle to Pascagoula									268	59	33.9
									First angle of triangle	48	58	48.0
	°	′	″									
ϕ	30	20	40.595		Pascagoula		λ			88	33	17.444
$\Delta\phi$	−		8.730				$\Delta\lambda$			+	9	57.323
ϕ'	30	20	31.865		Belle		λ'			88	43	14.767
			−1									
	° ′ ″	s	4.202937	s^2	8.4059							
$\frac{1}{2}(\phi+\phi')$	30 20 30	cos α	8.207256	sin² α	9.9999							
		B	8.511550	C	1.1729							
	″											
1st term	+8.3511	h	0.921743		9.5787							
2d term	+0.3790				+0.3790							
$-\Delta\phi$	+8.7301											

	s	4.202937		
	sin α	9.999944		
	A′	8.509351	$\Delta\lambda$	2.776209
	sec ϕ'	0.063977	sin $\frac{1}{2}(\phi+\phi')$	9.703447
		2.776209		2.479656
		″		″
	$\Delta\lambda$	+597.3226	$-\Delta\alpha$	+301.76

secondary triangulation—Continued

STATION PASCAGOULA

									°	′	″
α Third angle	Horn to Grand Pascagoula and Grand								238 − 38	09 49	28.1 42.8
α $\Delta\alpha$	Horn to Pascagoula								199 +	19 1	45.3 25.8
α'	Pascagoula to Horn								180 19	00 21	00.00 11.1
	°	′	″								
ϕ $\Delta\phi$	30 +	13 7	40.011 00.584	Horn			λ $\Delta\lambda$		88 −	36 2	07.539 50.094
ϕ'	30	20	40.595	Pascagoula			λ'		88	33	17.445 −1
	° ′ ″										
$\frac{1}{2}(\phi+\phi')$	30 17 10	s cos α B	4.137527 9.974803 8.511558	s^2 $\sin^2\alpha$ C	8.2750 9.0396 1.1709		h^2 D	5.248 2.331		−h $s^2\sin^2\alpha$ E	2.624 7.315 5.917
	″										
1st term 2d, 3d, and 4th terms	−420.6182 + 0.0345	h	2.623888		8.4855 +0.0306			7.579 +0.0038			5.856 +0.0001
$-\Delta\phi$	−420.5837										
	s sin α A′ sec ϕ'	4.137527 9.519824 8.509351 0.063988		$\Delta\lambda$ sin $\frac{1}{2}(\phi+\phi')$	2.230690 9.702706						
		2.230690			1.933396						
		″			″						
	$\Delta\lambda$	−170.0944		$-\Delta\alpha$	−85.78						

STATION BELLE

									°	′	″
α Third angle	Horn to Pascagoula Belle and Pascagoula								199 − 61	19 17	45.3 47.9
α $\Delta\alpha$	Horn to Belle								138 −	01 3	57.4 35.5
α'	Belle to Horn								180 317	00 58	00.00 21.9
	°	′	″								
ϕ $\Delta\phi$	30 +	13 6	40.011 51.853	Horn			λ $\Delta\lambda$		88 +	36 7	07.539 07.228
ϕ'	30	20	31.864	Belle			λ'		88	43	14.767
	° ′ ″										
$\frac{1}{2}(\phi+\phi')$	30 17 06	s cos α B	4.232096 9.871296 8.511558	s^2 $\sin^2\alpha$ C	8.4642 9.6505 1.1709		h^2 D	5.230 2.331		−h $s^2\sin^2\alpha$ E	2.615 8.115 5.917
	″										
1st term 2d, 3d, and 4th terms	−412.0501 + 0.1976	h	2.614950		9.2856 +0.1936			7.561 +0.0036			6.647 +0.0004
$-\Delta\phi$	−411.8525										
	s sin α A′ sec ϕ'	4.232096 9.825236 8.509351 0.063977		$\Delta\lambda$ sin $\frac{1}{2}(\phi+\phi')$	2.630660 9.702690						
		2.630660			2.333350						
		″			″						
	$\Delta\lambda$	+427.2283		$-\Delta\alpha$	+215.45						

Preliminary position computation,

STATION CLUB

α Second angle	Belle to Horn Horn and Club								° 317 + 40	′ 58 30	″ 21.9 06.7
α $\Delta\alpha$	Belle to Club								358 +	28	28.6 05.0
α'	Club to Belle								180 178 105	00 28 43	00.00 33.6 56.5
						First angle of triangle					
ϕ $\Delta\phi$	° 30 −	′ 20 5	″ 31.864 19.886		Belle		λ $\Delta\lambda$		88 −	43	14.767 09.812
ϕ'	30	15	11.978		Club		λ'		88	43	04.955 +1
$\frac{1}{2}(\phi+\phi')$	° ′ ″ 30 17 52 ″	s cos α B	3.993596 9.999846 8.511550	s^2 sin² α C	7.9872 6.8504 1.1729	h^2 D	5.010 2.332				
1st term 2d and 3d terms	+319.8836 + 0.0023	h	2.504992		6.0105 +0.0001		7.342 +0.0022				
$-\Delta\phi$	+319.8859										
	s sin α A′ sec ϕ'	3.993596 8.425207 8.509353 0.063584		$\Delta\lambda$ sin $\frac{1}{2}(\phi+\phi')$	0.991740 9.702856						
		0.991740			0.694596						
	$\Delta\lambda$	″ −9.8116		$-\Delta\alpha$	″ −4.05						

STATION DEER

α Second angle	Belle to Club Club and Deer								° 358 +102	′ 28 35	″ 28.6 20.0
α $\Delta\alpha$	Belle to Deer								101 −	03 2	48.6 45.3
α'	Deer to Belle								180 281	00 01	00.00 03.3 +.1
						First angle of triangle			41	02	11.4
ϕ $\Delta\phi$	° 30 +	′ 20	″ 31.864 55.356		Belle		λ $\Delta\lambda$		88 +	43 5	14.767 27.103
ϕ'	30	21	27.220		Deer		λ'		88	48	41.870
$\frac{1}{2}(\phi+\phi')$	° ′ ″ 30 21 00 ″	s cos α B	3.949436 9.283068 8.511550	s^2 sin² α C	7.8989 9.9837 1.1729	h^2 D	3.488 2.332				
1st term 2d and 3d terms	−55.4695 + 0.1137	h	1.744054		9.0555 +0.1136		5.820 +0.0001				
$-\Delta\phi$	−55.3558										
	s sin α A′ sec ϕ'	3.949436 9.991853 8.509351 0.064045		$\Delta\lambda$ sin $\frac{1}{2}(\phi+\phi')$	2.514685 9.703531						
		2.514685			2.218216						
	$\Delta\lambda$	″ +327.1033		$-\Delta\alpha$	″ +165.28						

Secondary triangulation—Continued

STATION CLUB

		°	′	″
α	Horn to Belle	138	01	57.4
Third angle	Club and Belle	− 33	45	57.1
α	Horn to Club	104	16	00.3
$\Delta\alpha$		−	3	30.2
α'	Club to Horn	180	00	00.00
		284	12	30.1

	°	′	″					
ϕ	30	13	40.011	Horn	λ	88	36	07.539
$\Delta\phi$	+	1	31.967		$\Delta\lambda$	+	6	57.418
ϕ'	30	15	11.978	Club	λ'	88	43	04.957
								−1

	° ′ ″	s	4.061239	s^2	8.1225				−h	1.964
$\frac{1}{2}(\phi+\phi')$	30 14 26	cos α	9.391705	sin² α	9.9728	h^2	3.929		s^2 sin² α	8.095
		B	8.511558	C	1.1709	D	2.331		E	5.917
	″									
1st term	−92.1514	h	1.964502		9.2662		6.260			5.976
2d, 3d, and 4th terms	+ 0.1848				+0.1845		+0.0002			+0.0001
$-\Delta\phi$	−91.9666									

	s	4.061239		
	sin α	9.986395		
	A′	8.509353	$\Delta\lambda$	2.620571
	sec ϕ'	0.063584	sin $\frac{1}{2}(\phi+\phi')$	9.702113
		2.620571		2.322684
		″		″
	$\Delta\lambda$	+417.4179	$-\Delta\alpha$	+210.22

STATION DEER

		°	′	″
α	Club to Belle	178	28	33.6
Third angle	Deer and Belle	− 36	22	28.8
α	Club to Deer	142	06	04.8
$\Delta\alpha$		−	2	50.0
α'	Deer to Club	180	00	00.00
		322	03	14.8

	°	′	″					
ϕ	30	15	11.978	Club	λ	88	43	04.956
$\Delta\phi$	+	6	15.242		$\Delta\lambda$	+	5	36.915
ϕ'	30	21	27.220	Deer	λ'	88	48	41.871
								−1

	° ′ ″	s	4.165767	s^2	8.3315				−h	2.574
$\frac{1}{2}(\phi+\phi')$	30 18 20	cos α	9.897131	sin² α	9.5767	h^2	5.149		s^2 sin² α	7.908
		B	8.511556	C	1.1713	D	2.331		E	5.918
	″									
1st term	−375.3652	h	2.574454		9.0795		7.480			6.400
2d, 3d, and 4th terms	+ 0.1233				+0.1200		+0.0030			+0.0003
$-\Delta\phi$	−375.2419									

	s	4.165767		
	sin α	9.788357		
	A′	8.509351	$\Delta\lambda$	2.527520
	sec ϕ'	0.064045	sin $\frac{1}{2}(\phi+\phi')$	9.702957
		2.527520		2.230477
		″		″
	$\Delta\lambda$	+336.9147	$-\Delta\alpha$	+170.01

Preliminary position computation,

STATION SHIP

								°	′	″
α	Deer to Club							322	03	14.8
Second angle	Club and Ship							+ 56	47	24.7
α	Deer to Ship							18	50	39.5
$\Delta\alpha$								−	1	14.8
								180	00	00.00
α'	Ship to Deer							198	49	24.7
					First angle of triangle			70	52	34.5
	°	′	″							
ϕ	30	21	27.220		Deer		λ	88	46	41.870
$\Delta\phi$	−	6	17.200				$\Delta\lambda$	+	2	28.269
ϕ'	30	15	10.020		Ship		λ'	88	51	10.139

	° ′ ″							
$\frac{1}{2}(\phi+\phi')$	30 18 19	s	4.088917	s^2	8.1778			
		$\cos\alpha$	9.976075	$\sin^2\alpha$	9.0184	h^2	5.153	
		B	8.511549	C	1.1731	D	2.332	
	″							
1st term	+377.1734	h	2.576541		8.3693		7.485	
2d and 3d terms	+ 0.0265				+0.0234		+0.0031	
$-\Delta\phi$	+377.1999							

	s	4.088917		
	$\sin\alpha$	9.509200		
	A′	8.509353	$\Delta\lambda$	2.171051
	$\sec\phi'$	0.063581	$\sin\frac{1}{2}(\phi+\phi')$	9.702952
		2.171051		1.874003
		″		″
	$\Delta\lambda$	+148.2692	$-\Delta\alpha$	+74.82

STATION BILOXI LIGHTHOUSE

								°	′	″
α	Deer to Ship							18	50	39.5
Second angle	Ship and Biloxi Lighthouse							+ 96	30	33.5
α	Deer to Biloxi Lighthouse							115	21	13.0
$\Delta\alpha$								−	2	42.9
								180	00	00.00
α'	Biloxi Lighthouse to Deer							295	18	30.1
					First angle of triangle			48	11	21.0
	°	′	″							
ϕ	30	21	27.220		Deer		λ	88	48	41.870
$\Delta\phi$	+	2	12.200				$\Delta\lambda$	+	5	22.085
ϕ'	30	23	39.420		Biloxi Lighthouse		λ'	88	54	03.955
			−1							−1

	° ′ ″								−h	2.121
$\frac{1}{2}(\phi+\phi')$	30 22 33	s	3.978395	s^2	7.9568				$s^2\sin^2\alpha$	7.869
		$\cos\alpha$	9.631650	$\sin^2\alpha$	9.9120	h^2	4.243		E	5.919
		B	8.511549	C	1.1731	D	2.332			
	″									
1st term	−132.3104	h	2.121594		9.0419		6.575			5.909
2d, 3d, and 4th terms	+ 0.1107				+0.1102		+0.0004			+0.0001
$-\Delta\phi$.	−132.1997									

	s	3.978395		
	$\sin\alpha$	9.956016		
	A′	8.509350	$\Delta\lambda$	2.507970
	$\sec\phi'$	0.064209	$\sin\frac{1}{2}(\phi+\phi')$	9.703868
		2.507970		2.211838
		″		″
	$\Delta\lambda$	+322.0846	$-\Delta\alpha$	+162.87

secondary triangulation—Continued

STATION SHIP

		°	′	″
α	Club to Deer	142	06	04.8
Third angle	Ship and Deer	− 52	20	01.2
α	Club to Ship	89	46	03.6
$\Delta\alpha$		−	4	04.4
α'	Ship to Club	180	00	00.00
		269	41	59.2

	°	′	″			°	′	″
ϕ	30	15	11.978	Club	λ	88	43	04.956
$\Delta\phi$	−		01.958		$\Delta\lambda$	+	8	05.182
ϕ'	30	15	10.020	Ship	λ'	88	51	10.138
								+1

	° ′ ″	s	4.112975	s^2	8.2260
$\frac{1}{2}(\phi+\phi')$	30 15 11	$\cos\alpha$	7.607988	$\sin^2\alpha$	0.0000
	″	B	8.511556	C	1.1713
1st term	+1.7081	h	0.232519		9.3973
2d term	+0.2496				+0.2496
$-\Delta\phi$	+1.9577				

s	4.112975		
$\sin\alpha$	9.999996		
A′	8.509353	$\Delta\lambda$	2.685905
$\sec\phi'$	0.063581	$\sin\frac{1}{2}(\phi+\phi')$	9.702275
	2.685905		2.388180
	″		″
$\Delta\lambda$	+485.1824	$-\Delta\alpha$	+244.44

STATION BILOXI LIGHTHOUSE

		°	′	″
α	Ship to Deer	198	49	24.7
Third angle	Biloxi Lighthouse and Deer	− 35	18	05.8
α	Ship to Biloxi Lighthouse	163	31	18.9
$\Delta\alpha$		−	1	27.8
α'	Biloxi Lighthouse to Ship	180	00	00.00
		343	29	51.1

	°	′	″			°	′	″
ϕ	30	15	10.020	Ship	λ	88	51	10.139
$\Delta\phi$	+	8	29.399		$\Delta\lambda$	+	2	53.815
ϕ'	30	23	39.419	Biloxi Lighthouse	λ'	88	54	03.954
	Fixed value		39.419					03.820

	° ′ ″	s	4.213748	s^2	8.4275				−h	2.707
$\frac{1}{2}(\phi+\phi')$	30 19 25	$\cos\alpha$	9.981786	$\sin^2\alpha$	8.9056	h^2	5.414	$s^2\sin^2\alpha$		7.333
	″	B	8.511556	C	1.1713	D	2.331	E		5.918
1st term	−509.4364	h	2.707090		8.5044		7.745			5.958
2d, 3d, and 4th terms	+ 0.0376				+0.0319		+0.0056			+0.0001
$-\Delta\phi$	−509.3988									

s	4.213748		
$\sin\alpha$	9.452781		
A′	8.509350	$\Delta\lambda$	2.240088
$\sec\phi'$	0.064209	$\sin\frac{1}{2}(\phi+\phi')$	9.703190
	2.240088		1.943278
	″		″
$\Delta\lambda$	+173.8153	$-\Delta\alpha$	+87.76

Preliminary position computation,

STATION SHIP ISLAND LIGHTHOUSE

		°	′	″
α	Biloxi Lighthouse to Ship	343	29	51.1
Second angle	Ship and Ship Island Lighthouse	+ 33	44	13.3
α	Biloxi Lighthouse to Ship Island Lighthouse	17	14	04.4
$\Delta\alpha$		−	1	57.9
α'	Ship Island Lighthouse to Biloxi Lighthouse	180	00	00.00
		197	12	06.5
	First angle of triangle	50	31	40.3

	°	′	″					
ϕ	30	23	39.419	Biloxi Lighthouse	λ	88	54	03.954
$\Delta\alpha$	−	10	54.077		$\Delta\lambda$	+	3	53.590
ϕ'	30	12	45.342	Ship Island Lighthouse	λ'	88	57	57.544
			+1					

	° ′ ″	s	4.323989	s^2	8.6480			−h	2.816
$\frac{1}{2}(\phi+\phi')$	30 18 12	cos α	9.980049	sin² α	8.9434	h^2	5.631	$s^2 \sin^2 \alpha$	7.592
		B	8.511546	C	1.1738	D	2.332	E	5.919
	″								
1st term	+654.0094	h	2.815584		8.7652		7.963		6.327
2d, 3d, and 4th terms	+ 0.0673				+0.0583		+0.0092		−0.0002
$-\Delta\phi$	+654.0767								

s	4.323989		
sin α	9.471708		
A′	8.509354	$\Delta\lambda$	2.368455
sec ϕ'	0.063404	sin $\frac{1}{2}(\phi+\phi')$	9.702930
	2.368455		2.071385
	″		″
$\Delta\lambda$	+233.5904	$-\Delta\alpha$	+117.87

Fixed α Biloxi Lighthouse to Ship Island Lighthouse, 17° 14′ 17.6″.

secondary triangulation—Continued

STATION SHIP ISLAND LIGHTHOUSE

									°	′	″
α	Ship to Biloxi Lighthouse								163	31	18.9
Third angle	Ship Island Lighthouse and Biloxi Lighthouse								− 95	44	06.9
α	Ship to Ship Island Lighthouse								67	47	12.6
$\Delta\alpha$									−	3	25.1
									180	00	00.00
α'	Ship Island Lighthouse to Ship								247	43	46.9
											−.1
	°	′	″								
ϕ	30	15	10.020	Ship			λ		88	51	10.139
$\Delta\phi$	−	2	24.677				$\Delta\lambda$		+	6	47.405
ϕ'	30	12	45.343	Ship Island Lighthouse			λ'		88	57	57.544
	Fixed value		45.341								57.464
	° ′ ″	s	4.070759	s^2	8.1415					−h	2.160
$\frac{1}{2}(\phi+\phi')$	30 13 58	$\cos\alpha$	9.577556	$\sin^2\alpha$	9.9330	h^2		4.320		$s^2\sin^2\alpha$	8.075
		B	8.511556	C	1.1713	D		2.331		E	5.918
	″										
1st term	+144.5010	h	2.159871		9.2458			6.651			6.153
2d, 3d, and 4th term	+ 0.1764				+0.1761			+0.0004			−0.0001
$-\Delta\phi$	+144.6774										

s	4.070759		
$\sin\alpha$	9.966509		
A′	8.509354	$\Delta\lambda$	2.610026
$\sec\phi'$	0.063404	$\sin\frac{1}{2}(\phi+\phi')$	9.702011
	2.610026		2.312037
	″		″
$\Delta\lambda$	+407.4046	$-\Delta\alpha$	+205.13

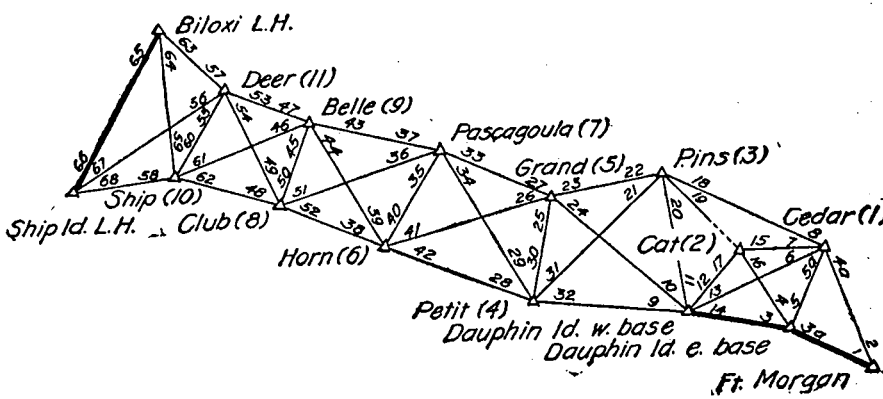

FIG. 7.

FORMATION OF OBSERVATION EQUATIONS

The position computation was carried westward from the fixed lines at the eastern end of the scheme and the observation equations were formed in the same order. The treatment of fixed lines is the same as in the first adjustment of figure 6, pages 105 et seq. No new detail arises until the points Deer and Ship are reached, which have lines connecting them with the fixed points Biloxi Lighthouse and Ship Island Lighthouse. Suppose for the moment that Biloxi Lighthouse were not fixed but that its latitude and longitude were to receive corrections of $\delta\phi_{12}$ and $\delta\lambda_{12}$ respectively. The observation equation for v_{57} would then read,

$$v_{57}=z_{13}-603\delta\phi_{11}+248\delta\lambda_{11}+603\delta\phi_{12}-248\delta\lambda_{12}+0.5$$

The latitude and longitude of Biloxi Lighthouse as developed by the preliminary position computation are 30° 23′ 39.″419 and 88° 54′ 03.″954, while the fixed values are 30° 23′ 39.″419 and 88° 54′ 03.″820, so that to reduce the preliminary to the fixed values on which the adjustment is built corrections of $\delta\phi_{12}=0.000$* and $\delta\lambda_{12}=-0.134$ are necessary. By substituting these values in the equation for v_{57} there results,

$$v_{57}=z_{13}-603\delta\phi_{11}+248\delta\lambda_{11}+603\ (0.000)-248\ (-0.134)+0.5$$

or,

$$v_{57}=z_{13}-603\delta\phi_{11}+248\delta\lambda_{11}+33.7$$

as given in the table page 16.

The equation for the reverse direction, v_{63}, if Biloxi Lighthouse were not fixed, would be,

$$v_{63}=z_{15}-603\delta\phi_{11}+248\delta\lambda_{11}+603\delta\phi_{12}-248\delta\lambda_{12}+0.0$$

which with the use of the above values of $\delta\phi_{12}$ and $\delta\lambda_{12}$ becomes,

$$v_{63}=z_{15}-603\delta\phi_{11}+248\delta\lambda_{11}+33.2$$

Similar computations must be made for v_{56}, v_{58}, v_{59}, v_{64}, v_{67}, and v_{68}.

The known terms in the expressions for v_{65}, and v_{66} may be found in a similar way by allowing for the fixity of both ends of the line. If the corrections needed to the preliminary position of Ship Island Lighthouse in order to reduce it to the fixed position are called $\delta\phi_{13}$ and $\delta\lambda_{13}$ then for the line Biloxi Lighthouse to Ship Island Lighthouse the formula gives,

$$d\alpha=89(\delta\phi_{13}-\delta\phi_{12})+250\ (\delta\lambda_{13}-\delta\lambda_{12})$$

If the line were free to be turned in azimuth, then by the adjustment,

$$v_{65}=z_{15}+89(\delta\phi_{13}-\delta\phi_{12})+250(\delta\lambda_{13}-\delta\lambda_{12})-1.7$$

* This zero discrepancy is merely accidental.

But to reduce the positions of the preliminary computation to the fixed positions, $\delta\phi_{13}=-0.002$, $\delta\phi_{12}=0.000$, $\delta\lambda_{13}=-0.080$, and $\delta\lambda_{12}=-0.134$ (see pp. 155, 157). Substituting these values gives,

$$v_{65}=z_{15}+11.6$$

as given in the table. v_{66} is found by a similar process.

The effect of using for the preliminary computation values from a previous adjustment for the figure but not for the positions appears in the constant term of the normal equations until the effect of closure in positions comes in. These constant terms should be zero except for the effect of accumulated errors in the last place of the two computations and a slight difference in the treatment of directions (3) and (3a). They are in fact almost negligible until the effect of closure appears in the equation for $\delta\phi_8$, from which point they become quite large.

Fort Morgan–Dauphin Island east base:

	°	′	″	
Assumed azimuth,	101	45	44.9	
Observed azimuth,	101	45	44.9	$-z_1+v_1$
			$0=$	$0.0+z_1-v_1$
			$v_1=$	$z_1+0.0$

Fort Morgan–Cedar:

	°	′	″	
Assumed azimuth,	144	19	22.9	$+d\alpha$
Observed azimuth,	144	19	22.6	$-z_1+v_2$
			$0=$	$+0.3+z_1-v_2+229\partial\phi_1-277\partial\lambda_1$
			$v_2=$	$z_1+229\partial\phi_1-277\partial\lambda_1+0.3$

DAUPHIN ISLAND EAST BASE

Assumed azimuth	Observed azimuth	Equation	Station observed
° ′ ″	° ′ ″		
84 14 41.9	84 14 41.9 $-z_2+v_3$	$v_3=z_2+0.0$	West base
137 52 06.3	137 52 06.9 $-z_2+v_4$	$v_4=+z_2+420\partial\phi_2-403\partial\lambda_2-0.6$	Cat
188 10 17.4	188 10 18.3 $-z_2+v_5$	$v_5=+z_2-82\partial\phi_1-498\partial\lambda_1-0.9$	Cedar
281 42 17.9	281 42 17.9 $-z_2+v_{3a}$	$v_{3a}=z_2+0.0$	Fort Morgan

(1) CEDAR

324 16 25.0	324 16 25.0 $-z_3+v_{4a}$	$v_{4a}=z_3+229\partial\phi_1-277\partial\lambda_1+0.0$	Fort Morgan
8 10 46.8	8 10 46.8 $-z_3+v_{5a}$	$v_{5a}=z_3-82\partial\phi_1-498\partial\lambda_1+0.0$	East base
45 37 18.9	45 37 19.8 $-z_3+v_6$	$v_6=z_3-266\partial\phi_1-226\partial\lambda_1-0.9$	West base
68 22 02.4	68 22 01.0 $-z_3+v_7$	$v_7=z_3-657\partial\phi_1-226\partial\lambda_1+657\partial\phi_2+226\partial\lambda_2+1.4$	Cat
99 17 00.4	99 17 02.5 $-z_3+v_8$	$v_8=z_3-341\partial\phi_1+48\partial\lambda_1+341\partial\phi_3-48\partial\lambda_3-2.1$	Pins

(2) CAT

23 51 22.0	23 51 22.0 $-z_4+v_{17}$	$v_{17}=z_4-273\partial\phi_2-537\partial\lambda_2+0.0$	West base
122 37 00.7			Pins
248 19 24.4	248 19 27.1 $-z_4+v_{15}$	$v_{15}=z_4-657\partial\phi_1-226\partial\lambda_1+657\partial\phi_2+226\partial\lambda_2-2.7$	Cedar
317 49 57.9	317 49 59.8 $-z_4+v_{16}$	$v_{16}=z_4+420\partial\phi_2-403\partial\lambda_2-1.9$	East base

DAUPHIN ISLAND WEST BASE

264 11 22.1	264 11.22.1 $-z_5+v_{14}$	$v_{14}=z_5+0.0$	East base
78 57 02.1	78 57 01.7 $-z_5+v_9$	$v_9=z_5+367\partial\phi_4+62\partial\lambda_4+0.4$	Petit
118 03 41.5	118 03 40.8 $-z_5+v_{10}$	$v_{10}=z_5+287\partial\phi_5-133\partial\lambda_5+0.7$	Grand
157 58 33.5	157 58 31.7 $-z_5+v_{11}$	$v_{11}=z_5+148\partial\phi_3-318\partial\lambda_3+1.8$	Pins
203 50 10.3	203 50 10.9 $-z_5+v_{12}$	$v_{12}=z_5-273\partial\phi_2-537\partial\lambda_2-0.6$	Cat
225 33 29.4	225 33 29.6 $-z_5+v_{13}$	$v_{13}=z_5-266\partial\phi_1-226\partial\lambda_1-0.2$	Cedar

(3) PINS

Assumed azimuth	Observed azimuth	Equation	Station observed
° ′ ″	° ′ ″		
279 11 16.6	279 11 16.6	$v_{18}=z_6-341\partial\phi_1+48\partial\lambda_1+341\partial\phi_3-48\partial\lambda_3+0.0$	Cedar
302 33 55.1	302 33 56.9	$v_{19}=z_6-459\partial\phi_2+255\partial\lambda_2+459\partial\phi_3-255\partial\lambda_3-1.8$	Cat
337 56 39.8	337 56 42.6	$v_{20}=z_6+148\partial\phi_3-318\partial\lambda_3-2.8$	West base
30 22 49.2	30 22 52.1	$v_{21}=z_6-153\partial\phi_3-228\partial\lambda_3+153\partial\phi_4+228\partial\lambda_4-2.9$	Petit
63 09 48.1	63 09 49.6	$v_{22}=z_6-450\partial\phi_3-198\partial\lambda_3+450\partial\phi_5+198\partial\lambda_5-1.5$	Grand

(4) PETIT

96 31 30.6	96 31 30.6	$v_{28}=z_7-358\partial\phi_4+36\partial\lambda_4+358\partial\phi_6-36\partial\lambda_6+0.0$	Horn
139 08 41.2	139 08 40.9	$v_{29}=z_7-209\partial\phi_4+211\partial\lambda_4+209\partial\phi_7-211\partial\lambda_7+0.3$	Pascagoula
177 10 20.4	177 10 19.5	$v_{30}=z_7-25\partial\phi_4+442\partial\lambda_4+25\partial\phi_5-442\partial\lambda_5+0.9$	Grand
210 19 28.8	210 19 28.2	$v_{31}=z_7-153\partial\phi_3-228\partial\lambda_3+153\partial\phi_4+228\partial\lambda_4+0.6$	Pins
258 51 48.7	258 51 47.7	$v_{32}=z_7+367\partial\phi_4+62\partial\lambda_4+1.0$	West base

(5) GRAND

243 06 15.8	243 06 15.8	$v_{23}=z_8-450\partial\phi_3-198\partial\lambda_3+450\partial\phi_5+198\partial\lambda_5+0.0$	Pins
297 58 16.0	297 58 17.4	$v_{24}=z_8+287\partial\phi_5-133\partial\lambda_5-1.4$	West base
357 10 08.8	357 10 09.4	$v_{25}=z_8-25\partial\phi_4+442\partial\lambda_4+25\partial\phi_5-442\partial\lambda_5-0.6$	Petit
58 14 46.3	58 14 46.6	$v_{26}=z_8-272\partial\phi_5-146\partial\lambda_5+272\partial\phi_6+146\partial\lambda_6-0.3$	Horn
101 29 06.4	101 29 05.5	$v_{27}=z_8-495\partial\phi_5+87\partial\lambda_5+495\partial\phi_7-87\partial\lambda_7+0.9$	Pascagoula

(6) HORN

104 16 00.3	104 16 00.3	$v_{38}=z_9-535\partial\phi_5+118\partial\lambda_5+535\partial\phi_8-118\partial\lambda_8+0.0$	Club
138 01 57.4	138 01 54.0	$v_{39}=z_9-249\partial\phi_5+240\partial\lambda_5+249\partial\phi_9-240\partial\lambda_9+3.4$	Belle
199 19 45.3	199 19 47.5	$v_{40}=z_9+153\partial\phi_5+379\partial\lambda_5-153\partial\phi_7-379\partial\lambda_7-2.2$	Pascagoula
238 09 28.1	238 09 26.5	$v_{41}=z_9-272\partial\phi_5-146\partial\lambda_5+272\partial\phi_6+146\partial\lambda_6+1.6$	Grand
276 26 01.4	276 26 00.7	$v_{42}=z_9-358\partial\phi_4+36\partial\lambda_4+358\partial\phi_6-36\partial\lambda_6+0.7$	Petit

(7) PASCAGOULA

281 25 13.6	281 25 13.6	$v_{33}=z_{10}-495\partial\phi_5+87\partial\lambda_5+495\partial\phi_7-87\partial\lambda_7+0.0$	Grand
319 04 37.2	319 04 34.2	$v_{34}=z_{10}-209\partial\phi_4+211\partial\lambda_4+209\partial\phi_7-211\partial\lambda_7+3.0$	Petit
19 21 11.1	19 21 07.6	$v_{35}=z_{10}+153\partial\phi_5+379\partial\lambda_5-153\partial\phi_7-379\partial\lambda_7+3.5$	Horn
57 14 14.4	57 14 12.2	$v_{36}=z_{10}-286\partial\phi_7-160\partial\lambda_7+286\partial\phi_8+160\partial\lambda_8+2.2$	Club
89 04 35.7	89 04 35.6	$v_{37}=z_{10}-398\partial\phi_7-6\partial\lambda_7+398\partial\phi_9+6\partial\lambda_9+0.1$	Belle

(9) BELLE

268 59 33.9	268 59 33.9	$v_{43}=z_{11}-398\partial\phi_7-6\partial\lambda_7+398\partial\phi_9+6\partial\lambda_9+0.0$	Pascagoula
317 58 21.9	317 58 23.7	$v_{44}=z_{11}-249\partial\phi_8+240\partial\lambda_8+249\partial\phi_9-240\partial\lambda_9-1.8$	Horn
358 28 28.6	358 28 29.2	$v_{45}=z_{11}-17\partial\phi_8+559\partial\lambda_8+17\partial\phi_9-559\partial\lambda_9-0.6$	Club
52 04 22.8	52 04 24.8	$v_{46}=z_{11}-311\partial\phi_9-211\partial\lambda_9+311\partial\phi_{10}+211\partial\lambda_{10}-2.0$	Ship
101 03 48.6	101 03 47.7	$v_{47}=z_{11}-700\partial\phi_9+119\partial\lambda_9+700\partial\phi_{11}-119\partial\lambda_{11}+0.9$	Deer

(8) CLUB

89 46 03.6	89 46 03.6	$v_{48}=z_{12}-490\partial\phi_8-2\partial\lambda_8+490\partial\phi_{10}+2\partial\lambda_{10}+0.0$	Ship
142 06 04.8	142 06 07.1	$v_{49}=z_{12}-266\partial\phi_8+297\partial\lambda_8+266\partial\phi_{11}-297\partial\lambda_{11}-2.3$	Deer
178 28 33.6	178 28 33.9	$v_{50}=z_{12}-17\partial\phi_8+559\partial\lambda_8+17\partial\phi_9-559\partial\lambda_9-0.3$	Belle
237 09 18.0	237 09 17.2	$v_{51}=z_{12}+v_{51}$ $\quad 286\partial\phi_7-160\partial\lambda_7+286\partial\phi_8+160\partial\lambda_8+0.8$	Pascagoula
284 12 30.1	284 12 30.8	$v_{52}=z_{12}-535\partial\phi_6+118\partial\lambda_6+535\partial\phi_8-118\partial\lambda_8-0.7$	Horn

(11) DEER

Assumed azimuth	Observed azimuth	Equation	Station observed
° ′ ″	° ′ ″		
281 01 03.4	281 01 03.4 $-z_{13}+v_{53}$	$v_{53}=z_{13}-700\partial\phi_9+119\partial\lambda_9+700\partial\phi_{11}-119\partial\lambda_{11}+0.0$	Belle
322 03 14.8	322 03 14.1 $-z_{13}+v_{54}$	$v_{54}=z_{13}-266\partial\phi_8+297\partial\lambda_8+266\partial\phi_{11}-297\partial\lambda_{11}+0.7$	Club
18 50 39.5	18 50 41.3 $-z_{13}+v_{55}$	$v_{55}=z_{13}+167\partial\phi_{10}+425\partial\lambda_{10}-167\partial\phi_{11}-425\partial\lambda_{11}-1.8$	Ship
42 46 50.3	42 46 46.1 $-z_{13}+v_{56}$	$v_{56}=z_{13}-197\partial\phi_{11}-185\partial\lambda_{11}-11.0$	Ship Island Lighthouse.
115 21 13.0	115 21 12.5 $-z_{13}+v_{57}$	$v_{57}=z_{13}-603\partial\phi_{11}+248\partial\lambda_{11}+33.7$	Biloxi Lighthouse

(10) SHIP

67 47 12.0	67 47 12.0 $-z_{14}+v_{58}$	$v_{58}=z_{14}-500\partial\phi_{10}-177\partial\lambda_{10}-15.2$	Ship Island Lighthouse
163 31 18.9	163 31 19.0 $-z_{14}+v_{59}$	$v_{59}=z_{14}-110\partial\phi_{10}+323\partial\lambda_{10}+43.2$	Biloxi Lighthouse
198 49 24.7	198 49 23.4 $-z_{14}+v_{60}$	$v_{60}=z_{14}+167\partial\phi_{10}+425\partial\lambda_{10}-167\partial\phi_{11}-425\partial\lambda_{11}+1.3$	Deer
232 00 23.1	232 00 23.6 $-z_{14}+v_{61}$	$v_{61}=z_{14}-311\partial\phi_9-211\partial\lambda_9+311\partial\phi_{10}+211\partial\lambda_{10}-0.5$	Belle
269 41 59.2	269 41 58.4 $-z_{14}+v_{62}$	$v_{62}=z_{14}-490\partial\phi_8-2\partial\lambda_8+490\partial\phi_{10}+2\partial\lambda_{10}+0.8$	Club

BILOXI LIGHTHOUSE

295 18 30.1	295 18 30.1 $-z_{15}+v_{63}$	$v_{63}=z_{15}-603\partial\phi_{11}+248\partial\lambda_{11}+33.2$	Deer
343 29 51.1	343 29 47.5 $-z_{15}+v_{64}$	$v_{64}=z_{15}-110\partial\phi_{10}+323\partial\lambda_{10}+46.9$	Ship
17 14 04.4	17 14 06.1 $-z_{15}+v_{65}$	$v_{65}=z_{15}+11.6$	Ship Island Lighthouse

SHIP ISLAND LIGHTHOUSE

197 12 06.5	197 12 06.5 $-z_{16}+v_{66}$	$v_{66}=z_{16}+13.3$	Biloxi Lighthouse
222 42 10.0	222 42 08.6 $-z_{16}+v_{67}$	$v_{67}=z_{16}-197\partial\phi_{11}-185\partial\lambda_{11}-13.8$	Deer
247 43 46.8	247 43 47.7 $-z_{16}+v_{68}$	$v_{68}=z_{16}-500\partial\phi_{10}-177\partial\lambda_{10}-16.1$	Ship

In order to get the quantities on a better relative basis, it is best to adopt $100\delta\phi_1$, $100\delta\lambda_1$, etc., as unknowns in the equations. The coefficients throughout will then be divided by 100, and from the solution we shall determine one hundred times the corrections in seconds to the various latitudes and longitudes

Table for formation of normals, No. 1

	$\partial\phi_1$	$\partial\lambda_1$	$\partial\phi_2$	$\partial\lambda_2$	$\partial\phi_3$	$\partial\lambda_3$	$\partial\phi_4$	$\partial\lambda_4$	$\partial\phi_5$	$\partial\lambda_5$	$\partial\phi_6$	$\partial\lambda_6$	$\partial\phi_7$	$\partial\lambda_7$	$\partial\phi_8$	$\partial\lambda_8$	η	p	$\sqrt{p}$
z_1																			
1																	0.0	1	1
1	+ 229	− 277															+0.3	1	1
2	+ 229	− 277															+0.3	$-\frac{1}{2}$	0.707i
z_2																			
1																	0.0	1	1
1			+420	−403													−0.6	1	1
1	− 82	− 498															−0.9	1	1
1																	0.0	1	1
4	− 82	− 498	+420	−403													−1.5	$-\frac{1}{4}$	0.5i
z_3																			
1	+ 229	− 277															0.0	1	1
1	− 82	− 498															0.0	1	1
1	− 266	− 226															−0.9	1	1
1	− 657	− 226	+657	+226													+1.4	1	1
1	− 341	+ 48			+341	− 48											−2.1	1	1
5	−1117	−1179	+657	+226	+341	− 48											−1.6	$-\frac{1}{5}$	0.447i
z_4																			
1			−273	−537													0.0	1	1
1	− 657	− 226	+657	+226													−2.7	1	1
1			+420	−403													−1.9	1	1
3	− 657	− 226	+804	−714													−4.6	$-\frac{1}{3}$	0.577i
z_5																			
1																	0.0	1	1
1							+367	+ 62									+0.4	1	1
1									+287	−133							+0.7	1	1
1					+148	− 318											+1.8	1	1
1			−273	−537													−0.6	1	1
1	− 266	− 226															−0.2	1	1
6	− 266	− 226	−273	−537	+148	− 318	+367	+ 62	+287	−133							+2.1	$-\frac{1}{6}$	0.408i
z_6																			
1	− 341	+ 48			+341	− 48											0.0	1	1
1			−459	+255	+459	− 255											−1.8	1	1
1					+148	− 318											−2.8	1	1
1					−153	− 228	+153	+228									−2.9	1	1
1					−450	− 198			+450	+198							−1.5	1	1
5	− 341	+ 48	−459	+255	+345	−1047	+153	+228	+450	+198							−9.0	$-\frac{1}{5}$	0.447i
z_7																			
1							−358	+ 36			+358	− 36					0.0	1	1
1							−209	+211					+209	−211			+0.3	1	1
1							− 25	+442	+ 25	−442							+0.9	1	1
1					−153	− 228	+153	+228									+0.6	1	1
1							+367	+ 62									+1.0	1	1
5					−153	− 228	− 72	+979	+ 25	−442	+358	− 36	+209	−211			+2.8	$-\frac{1}{5}$	0.447i

z_8																			
1					−450	− 198			+450	+198							0.0	1	1
1									+287	−133							−1.4	1	1
1							− 25	+442	+ 25	−442							−0.6	1	1
1									−272	−146	+272	+146					−0.3	1	1
1									−495	+ 87			+495	− 87			+0.9	1	1
5					−450	− 198	− 25	+442	− 5	−436	+272	+146	+495	− 87			−1.4	$-\frac{1}{5}$	0.447i

Table for formation of normals, No. 1—Continued

	$\partial\phi_4$	$\partial\lambda_4$	$\partial\phi_5$	$\partial\lambda_5$	$\partial\phi_6$	$\partial\lambda_6$	$\partial\phi_7$	$\partial\lambda_7$	$\partial\phi_8$	$\partial\lambda_8$	$\partial\phi_9$	$\partial\lambda_9$	$\partial\phi_{10}$	$\partial\lambda_{10}$	$\partial\phi_{11}$	$\partial\lambda_{11}$	η	p	$\sqrt{p}$
z_9																			
1					−535	+118			+535	−118							0.0	1	1
1					−249	+240					+249	−240					+ 3.4	1	1
1					+153	+379	−153	−379									− 2.2	1	1
1			−272	−146	+272	+146											+ 1.6	1	1
1	−358	+ 36			+358	− 36											+ 0.7	1	1
5	−358	+ 36	−272	−146	− 1	+847	−153	−379	+535	−118	+249	−240					+ 3.5	$-\frac{1}{5}$	0.447i
z_{10}																			
1			−495	+ 87			+495	− 87									0.0	1	1
1	−209	+211					+209	−211									+ 3.0	1	1
1					+153	+379	−153	−379									+ 3.5	1	1
1							−286	−160	+286	+160							+ 2.2	1	1
1							−398	− 6			+398	+ 6					+ 0.1	1	1
5	−209	+211	−495	+ 87	+153	+379	−133	−843	+286	+160	+398	+ 6					+ 8.8	$-\frac{1}{5}$	0.447i
z_{11}																			
1							−398	− 6			+398	+ 6					0.0	1	1
1					−249	+240					+249	−240					− 1.8	1	1
1									− 17	+559	+ 17	−559					− 0.6	1	1
1											−311	−211	+311	+211			− 2.0	1	1
1											−700	+119			+700	−119	+ 0.9	1	1
5					−249	+240	−398	− 6	− 17	+559	−347	−885	+311	+211	+700	−119	− 3.5	$-\frac{1}{5}$	0.447i
z_{12}																			
1									−490	− 2			+490	+ 2			0.0	1	1
1									−266	+297					+266	−297	− 2.3	1	1
1									− 17	+559	+ 17	−559					− 0.3	1	1
1							−286	−160	+286	+160							+ 0.8	1	1
1					−535	+118			+535	−118							− 0.7	1	1
5					−535	+118	−286	−160	+ 48	+896	+ 17	−559	+490	+ 2	+266	−297	− 2.5	$-\frac{1}{5}$	0.447i
z_{13}																			
1											−700	+119			+700	−119	0.0	1	1
1									−266	+297					+266	−297	+ 0.7	1	1
1													+167	+425	−167	−425	− 1.8	1	1
1															−197	−185	−11.0	1	1
5									−266	+297	−700	+119	+167	+425	−603	+248	+33.7	1	1
z_{14}															− 1	−778	+21.6	$-\frac{1}{5}$	0.447i
1													−500	−177			−15.2	1	1
1													−110	+323			+43.2	1	1
1													+167	+425	−167	−425	+ 1.3	1	1
1											−311	−211	+311	+211			− 0.5	1	1
1									−490	− 2			+490	+ 2			+ 0.8	1	1
5									−490	− 2	−311	−211	+358	+784	−167	−425	+29.6	$-\frac{1}{5}$	0.447i

z_{15}																			
1															−603	+248	+33.2	1	1
1													−110	+323			+46.9	1	1
3																	+11.6	1	1
z_{16}													−110	+323	−603	+248	+91.7	$-\frac{1}{3}$	0.577i
1																	+13.3	1	1
1															−197	−185	−13.8	1	1
1													−500	−177			−16.1	1	1
3													−500	−177	−197	−185	−16.6	$-\frac{1}{3}$	0.577i

Table for formation of normals, No. 2 *

1	2	3	4	5	6	7	8	9	10	11	12	13	14	η	Σ
$\partial\phi_1$	$\partial\lambda_1$	$\partial\phi_2$	$\partial\lambda_2$	$\partial\phi_3$	$\partial\lambda_3$	$\partial\phi_4$	$\partial\lambda_4$	$\partial\phi_5$	$\partial\lambda_5$	$\partial\phi_6$	$\partial\lambda_6$	$\partial\phi_7$	$\partial\lambda_7$		
+2.29	−2.77													+0.3	−0.18
+1.62i	−1.96i													+0.21i	−0.13i
		+4.20	−4.03											−0.6	−0.43
−0.82	−4.98													−0.9	−6.70
−0.41i	−2.49i	+2.10i	−2.02i											−0.75i	−3.57i
+2.29	−2.77													0.0	−0.48
−0.82	−4.98													0.0	−5.80
−2.66	−2.26													−0.9	−5.82
−6.57	−2.26	+6.57	+2.26											+1.4	+1.40
−3.41	+0.48			+3.41	−0.48									−2.1	−2.10
−4.99i	−5.27i	+2.94i	+1.01i	+1.52i	−0.21i									−0.72i	−5.72i
		−2.73	−5.37											0.0	−8.10
−6.57	−2.26	+6.57	+2.26											−2.7	−2.70
		+4.20	−4.03											−1.9	−1.73
−3.79i	−1.30i	+4.64i	−4.12i											−2.65i	−7.22i
						+3.67	+0.62							+0.4	+4.69
								+2.87	−1.33					+0.7	+2.24
				+1.48	−3.18									+1.8	+0.10
		−2.73	−5.37											−0.6	−8.70
−2.66	−2.26													−0.2	−5.12
−1.09i	−0.92i	−1.11i	−2.19i	+0.60i	−1.30i	+1.50i	+0.25i	+1.17i	−0.54i					+0.86i	−2.77i
−3.41	+0.48			+3.41	−0.48									0.0	0.0
		−4.59	+2.55	+4.59	−2.55									−1.8	−1.80
				+1.48	−3.18									−2.8	−4.50
				−1.53	−2.28	+1.53	+2.28							−2.9	−2.90
				−4.50	−1.98			+4.50	+1.98					−1.5	−1.50
−1.52i	+0.21i	−2.05i	+1.14i	+1.54i	−4.68i	+0.68i	+1.02i	+2.01i	+0.89i					−4.02i	−4.78i
						−3.58	+0.36			+3.58	−0.36			0.0	0.0
						−2.09	+2.11					+2.09	−2.11	+0.3	+0.30
						−0.25	+4.42	+0.25	−4.42					+0.9	+0.90
				−1.53	−2.28	+1.53	+2.28							+0.6	+0.60
						+3.67	+0.62							+1.0	+5.29
				−0.68i	−1.02i	−0.32i	+4.38i	+0.11i	−1.98i	+1.60i	−0.16i	+0.93i	−0.94i	+1.25i	+3.17i
				−4.50	−1.98			+4.50	+1.98					0.0	0.0
								+2.87	−1.33					−1.4	+0.14
						−0.25	+4.42	+0.25	−4.42					−0.6	−0.60
								−2.72	−1.46	+2.72	+1.46			−0.3	−0.30
								−4.95	+0.87			+4.95	−0.87	+0.9	+0.90
				−2.01i	−0.89i	−0.11i	+1.98i	−0.02i	−1.95i	+1.22i	+0.65i	+2.21i	−0.32i	−0.63i	+0.06i

* All values in this table except those in the η and Σ columns have been divided by 100.

7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	η	Σ
$\partial\phi_4$	$\partial\lambda_4$	$\partial\phi_5$	$\partial\lambda_5$	$\partial\phi_6$	$\partial\lambda_6$	$\partial\phi_7$	$\partial\lambda_7$	$\partial\phi_8$	$\partial\lambda_8$	$\partial\phi_9$	$\partial\lambda_9$	$\partial\phi_{10}$	$\partial\lambda_{10}$	$\partial\phi_{11}$	$\partial\lambda_{11}$		
				−5.35	+1.18			+5.35	−1.18							0.0	0.0
				−2.49	+2.40					+2.49	−2.40					+ 3.4	+ 3.40
				+1.53	+3.79	−1.53	−3.79									− 2.2	− 2.20
		−2.72	−1.46	+2.72	+1.46											+ 1.6	+ 1.60
−3.58	+0.36			+3.58	−0.36											+ 0.7	+ 0.70
−1.60i	+0.16i	−1.22i	−0.65i		+3.79i	−0.68i	−1.69i	+2.39i	−0.53i	+1.11i	−1.07i					+ 1.56i	+ 1.57i
		−4.95	+0.87			+4.95	−0.87									0.0	0.0
−2.09	+2.11					+2.09	−2.11									+ 3.0	+ 3.00
				+1.53	+3.79	−1.53	−3.79									+ 3.5	+ 3.50
						−2.86	−1.60	+2.86	+1.60							+ 2.2	+ 2.20
						−3.98	−0.06			+3.98	+0.06					+ 0.1	+ 0.10
−0.93i	+0.94i	−2.21i	+0.39i	+0.68i	+1.69i	−0.59i	−3.77i	+1.28i	+0.72i	+1.78i	+0.03i					+ 3.93i	+ 3.94i
						−3.98	−0.06			+3.98	+0.06					0.0	0.0
				−2.49	+2.40					+2.49	−2.40					− 1.8	− 1.80
								−0.17	+5.59	+0.17	−5.59					− 0.6	− 0.60
										−3.11	−2.11	+3.11	+2.11			− 2.0	− 2.00
										−7.00	+1.19			+7.00	−1.19	+ 0.9	+ 0.90
				−1.11i	+1.07i	−1.78i	−0.03	−0.08i	+2.50i	−1.55i	−3.96i	+1.39i	+0.94i	+3.13i	−0.53i	− 1.56i	− 1.57i
								−4.90	−0.02			+4.90	+0.02			0.0	0.0
								−2.66	+2.97					+2.66	−2.97	− 2.3	− 2.30
								−0.17	+5.59	+0.17	−5.59					− 0.3	− 0.30
						−2.86	−1.60	+2.86	+1.60							+ 0.8	+ 0.80
				−5.35	+1.18			+5.35	−1.18							− 0.7	− 0.70
				−2.39i	+0.53i	−1.28i	−0.72i	+0.21i	+4.01i	+0.08i	−2.50i	+2.19i	+0.01i	+1.19i	−1.33i	− 1.12i	− 1.12i
										−7.00	+1.19			+7.00	−1.19	0.0	0.0
								−2.66	+2.97					+2.66	−2.97	+ 0.7	+ 0.70
												+1.67	+4.25	−1.67	−4.25	− 1.8	− 1.80
														−1.97	−1.85	−11.0	−14.82
														−6.03	+2.48	+33.7	+30.15
								−1.19i	+1.33i	−3.13i	+0.53i	+0.75i	+1.90i		−3.48i	+9.66i	+ 6.37i
												−5.00	−1.77			−15.2	−21.97
												−1.10	+3.23			+43.2	+45.33
												+1.67	+4.25	−1.67	−4.25	+ 1.3	+ 1.30
										−3.11	−2.11	+3.11	+2.11			− 0.5	− 0.50
								−4.90	−0.02			+4.90	+0.02			+ 0.8	+ 0.80
								−2.19i	−0.01i	−1.39i	−0.94i	+1.60i	+3.50i	−0.75i	−1.90i	+13.23i	+11.15i
														−6.03	+2.48	+33.2	+29.65
												−1.10	+3.23			+46.9	+49.03
																+11.6	+11.6
												−0.63i	+1.86i	−3.48i	+1.43i	+52.91i	+52.09i
																+13.3	+13.3
														−1.97	−1.85	−13.8	−17.62
												−5.00	−1.77			−16.1	−22.87
												−2.88i	−1.02i	−1.14i	−1.07i	−9.58i	−15.69i

Normal equations

1	2	3	4	5	6	7	8	9	10	11	12
+90.0150	+ 4.1730	− 57.5385	−41.7538	− 12.6766	− 6.3049	+ 2.6686	+ 1.8229	+ 4.3305	+ 0.7642		
	+45.4427	− 3.5323	−17.5325	+ 11.5126	− 1.7807	+ 1.2372	+ 0.0158	+ 0.6543	− 0.6837		
		+117.5659	+33.7556	− 21.7139	+ 1.2849	+ 3.0590	+ 2.3685	+ 5.4192	+ 1.2251		
			+78.7027	+ 9.7277	− 3.8022	+ 2.5098	− 0.6153	+ 0.2709	− 2.1972		
				+ 84.3424	+ 6.2298	− 7.0677	− 1.7394	− 44.2628	−24.1325	+ 3.5402	+ 1.1977
					+19.9567	− 2.2687	+ 0.9316	− 6.7978	− 8.1327	+ 2.7178	+ 0.4153
						+ 59.8618	− 0.3988	− 7.2211	+ 0.8894	− 24.3542	+ 10.2336
							+34.2849	+ 1.6977	−27.5748	− 7.4852	− 3.0404
								+109.1066	+ 6.3958	− 13.4456	+ 0.4469
									+46.8472	− 2.6606	− 1.5081
										+103.3016	− 6.8476
											+ 28.4613

13	14	15	16	17	18	19	20	21	22	η	Σ
										+ 0.5960	− 13.9036
										+ 1.0071	+ 40.5135
										− 0.4396	+ 81.4539
										+ 0.5294	+ 59.5951
+ 5.0745	− 1.4231									− 0.2811	+ 8.3278
+ 2.9155	− 1.3059									− 0.1405	+ 3.9182
− 9.8322	+ 2.2660	+ 5.0144	− 0.1784	+ 3.4314	− 1.6841					+ 0.0662	+ 38.2322
+ 1.0340	− 0.2006	− 1.5856	− 0.5020	− 1.8508	+ 0.1430					− 0.1210	− 2.9055
−51.1966	− 1.6849	+ 5.7446	+ 0.9446	+ 5.2880	− 1.2391					+ 0.8374	+ 15.2886
+14.5520	− 3.7637	− 57.7023	− 0.6253	+ 0.0273	− 0.7072					+ 0.2900	+ 0.0595
−13.4998	− 8.8081	+ 1.0543	+24.4953	− 15.1399	+ 1.5610	+ 6.7770	+ 1.0673	+ 6.3184	− 3.7670	− 0.5202	− 4.4519
− 6.7278	−15.4350	− 57.7023	− 6.7932	+ 6.3530	− 1.9532	− 2.6480	− 1.0111	− 3.9798	+ 1.2720	− 0.0948	− 0.2802
+99.0967	+ 1.1818	+ 1.3790	+ 0.4952	− 32.5324	−11.4363	+ 5.2774	+ 1.6860	+ 7.0946	− 2.6458	− 0.2161	− 3.5317
	+25.6494	− 13.8524	− 0.3391	+ 8.1200	− 3.6212	+ 1.6185	+ 0.0354	+ 0.9507	− 0.9735	+ 0.0496	+ 2.1778
		− 0.1385	−19.7152	− 11.8987	+ 3.1998	− 43.9722	+ 9.8031	− 15.7932	+ 7.7351	+ 37.1447	+ 28.6369
		+122.2200	+63.1452	+ 8.9105	−43.8742	− 13.4344	− 4.8829	+ 3.1960	− 6.3741	− 10.5001	− 6.1221
				+142.9446	−21.3622	− 12.7934	− 0.8560	− 94.2862	+ 2.4115	+ 43.2741	+ 30.0408
					+61.5179	− 1.0383	− 2.8738	+ 31.3248	− 8.1976	+ 7.4331	+ 7.1924
						+106.8200	+27.9899	− 16.8102	− 7.0763	+ 34.6511	+ 85.3611
							+50.9179	− 9.2141	−26.1027	+167.6744	+214.2334
								+173.0271	−35.3120	−162.3283	−115.8122
									+54.7848	+188.1235	+163.8779

Solution of normals

1	2	3	4	5	6	7	8	9	10	11	12	13	14	η	Σ
+90.0150	+ 4.1730	− 57.5385	−41.7538	−12.6766	− 6.3049	+2.6686	+1.8229	+ 4.3305	+ 0.7642					+0.5960	−13.9036
$\partial\phi_1$	− 0.046359	+ 0.639210	+ 0.463854	+ 0.140828	+ 0.070043	−0.029646	−0.020251	− 0.048109	− 0.008490					−0.006621	+ 0.154459
	+45.4427	− 3.5323	−17.5325	+11.5126	− 1.7807	+1.2372	+0.0158	+ 0.6543	− 0.6837					+1.0071	+40.5135
1	− 0.1935	+ 2.6674	+ 1.9357	+ 0.5877	+ 0.2923	−0.1237	−0.0845	− 0.2008	− 0.0354					−0.0276	+ 0.6446
	+45.2492	− 0.8649	−15.5968	+12.1003	− 1.4884	+1.1135	−0.0687	+ 0.4535	− 0.7191					+0.9795	+41.1581
	$\partial\lambda_1$	+ 0.019114	+ 0.344687	− 0.267415	+ 0.032893	−0.024608	+0.001518	− 0.010022	+ 0.015892					−0.021647	− 0.909587
		+117.5659	+33.7556	−21.7139	+ 1.2849	+3.0590	+2.3685	+ 5.4192	+ 1.2251					−0.4396	+81.4539
	1	− 36.7792	−26.6894	− 8.1030	− 4.0302	+1.7058	+1.1652	+ 2.7681	+ 0.4885					+0.3810	− 8.8873
	2	− 0.0165	− 0.2981	+ 0.2313	− 0.0284	+0.0213	−0.0013	+ 0.0087	− 0.0137					+0.0187	+ 0.7867
		+ 80.7702	+ 6.7681	−29.5856	− 2.7737	+4.7861	+3.5324	+ 8.1960	+ 1.6999					−0.0399	+73.3535
		$\partial\phi_2$	− 0.083795	+ 0.366294	+ 0.034341	−0.059256	−0.043734	− 0.101473	− 0.021046					+0.000494	− 0.908175
			+78.7027	+ 9.7277	− 3.8022	+2.5098	−0.6153	+ 0.2709	− 2.1972					+0.5294	+59.5051
		1	−19.3677	− 5.8801	− 2.9246	+1.2378	+0.8456	+ 2.0087	+ 0.3545					+0.2765	− 6.4492
		2	− 5.3760	+ 4.1708	− 0.5130	+0.3838	−0.0237	+ 0.1563	− 0.2479					+0.3376	+14.1867
		3	− 0.5671	+ 2.4791	+ 0.2324	−0.4011	−0.2960	− 0.6868	− 0.1424					+0.0033	− 6.1467
			+53.3919	+10.4975	− 7.0074	+3.7303	−0.0894	+ 1.7491	− 2.2330					+1.1468	+61.1858
			$\partial\lambda_2$	− 0.196612	+ 0.131245	−0.069866	+0.001674	− 0.032760	+ 0.041823					−0.021479	− 1.145975
				+84.3424	+ 6.2298	−7.0677	−1.7394	−44.2628	−24.1325	+3.5402	+1.1977	+5.0745	−1.4231	−0.2811	+ 8.3278
			1	− 1.7852	− 0.8879	+0.3758	+0.2567	+ 0.6099	+ 0.1076					+0.0839	− 1.9580
			2	− 3.2358	+ 0.3980	−0.2978	+0.0184	− 0.1213	+ 0.1923					−0.2619	−11.0063
			3	−10.8370	− 1.0160	+1.7531	+1.2939	+ 3.0021	+ 0.6227					−0.0146	+26.8689
			4	− 2.0639	+ 1.3777	−0.7334	+0.0176	− 0.3439	+ 0.4390					−0.2255	−12.0299
				+66.4205	+ 6.1016	−5.9700	−0.1528	−41.1160	−22.7709	+3.5402	+1.1977	+5.0745	−1.4231	−0.6992	+10.2025
				$\partial\phi_3$	− 0.091863	+0.089882	+0.002300	+ 0.619026	+ 0.342829	−0.053300	−0.018032	−0.076400	+0.021426	+0.010527	− 0.153605
					+19.9567	−2.2687	+0.9316	− 6.7978	− 8.1327	+2.7178	+0.4153	+2.9155	−1.3059	−0.1405	+ 3.9182
				1	− 0.4416	+0.1869	+0.1277	+ 0.3033	+ 0.0535					+0.0417	− 0.9738
				2	− 0.0490	+0.0366	−0.0023	+ 0.0149	− 0.0237					+0.0322	+ 1.3538
				3	− 0.0953	+0.1644	+0.1213	+ 0.2815	+ 0.0584					−0.0014	+ 2.5190
				4	− 0.9197	+0.4896	−0.0117	+ 0.2296	− 0.2931					+0.1505	+ 8.0303
				5	− 0.5605	+0.5484	+0.0140	+ 3.7770	+ 2.0918	−0.3252	−0.1100	−0.4662	+0.1307	+0.0642	− 0.9372
					+17.8906	−0.8428	+1.1806	− 2.1915	− 6.2458	+2.3926	+0.3053	+2.4493	−1.1752	+0.1467	+13.9098
					$\partial\lambda_3$	+0.047109	−0.065990	+ 0.122494	+ 0.349111	−0.133735	−0.017065	−0.136904	+0.065688	−0.008200	− 0.777492

Solution of normals—Continued

	7	8	9	10	11	12	13	14	15	16	17	18	η	Σ
	+59.8618	− 0.3988	− 7.2211	+ 0.8894	−24.3542	+10.2336	− 9.8322	+2.2660	+5.0144	−0.1784	+3.4314	−1.6841	+0.0662	+38.2322
1	− 0.0791	− 0.0540	− 0.1284	− 0.0227									−0.0177	+0.4122
2	− 0.0274	+ 0.0017	− 0.0112	+ 0.0177									−0.0241	−1.0128
3	− 0.2836	− 0.2093	− 0.4857	− 0.1007									+0.0024	−4.3466
4	− 0.2606	+ 0.0062	− 0.1222	+ 0.1560									−0.0801	−4.2748
5	− 0.5366	− 0.0137	− 3.6956	− 2.0467	+ 0.3182	+ 0.1077	+ 0.4561	−0.1279					−0.0628	+0.9170
6	− 0.0397	+ 0.0556	− 0.1082	− 0.2942	+ 0.1127	+ 0.0144	+ 0.1154	−0.0554					+0.0069	+0.6553
	+58.6348	− 0.6123	− 11.7674	− 1.4012	−23.9233	+10.3557	− 9.2607	+2.0827	+5.0144	−0.1784	+3.4314	−1.6841	−0.1092	+30.5824
	$\partial\phi_4$	+ 0.010443	+ 0.200690	+ 0.023897	+ 0.408005	− 0.176614	+ 0.157939	−0.035520	−0.085519	+0.003043	−0.058522	+0.028722	+0.001862	−0.521574
		+34.2849	+ 1.6977	−27.5748	− 7.4852	− 3.0404	+ 1.0340	−0.2006	−1.5856	−0.5920	−1.8508	+0.1430	−0.1210	−2.9055
1		− 0.0369	− 0.0877	− 0.0155									−0.0121	+0.2816
2		− 0.0001	+ 0.0007	− 0.0011									+0.0015	+0.0625
3		− 0.1545	− 0.3584	− 0.0743									+0.0017	−3.2080
4		− 0.0001	+ 0.0029	− 0.0037									+0.0019	+0.1024
5		− 0.0004	− 0.0946	− 0.0524	+ 0.0081	+ 0.0028	+ 0.0117	−0.0033					−0.0016	+0.0235
6		− 0.0779	+ 0.1446	+ 0.4122	− 0.1579	− 0.0201	− 0.1616	+0.0776					−0.0097	−0.9179
7		− 0.0064	− 0.1229	− 0.0146	− 0.2498	+ 0.1081	− 0.0967	+0.0217	+0.0524	−0.0019	+0.0358	−0.0176	−0.0011	+0.3194
		+34.0086	+ 1.1823	−27.3242	− 7.8848	− 2.9496	+ 0.7874	−0.1046	−1.5332	−0.5939	−1.8150	+0.1254	−0.1404	−6.2420
		$\partial\lambda_4$	− 0.034765	+ 0.803450	+ 0.231847	+ 0.086731	− 0.023153	+0.003076	+0.045083	+0.017463	+0.053369	−0.003687	+0.004128	+0.183542
			+109.1066	+ 6.3958	−13.4456	+ 0.4469	−51.1966	−1.6849	+5.7446	+0.9446	+5.2880	−1.2391	+0.8374	+15.2886
		1	− 0.2083	− 0.0368									−0.0287	+0.6689
		2	− 0.0045	+ 0.0072									−0.0098	−0.4125
		3	− 0.8317	− 0.1725									+0.0040	−7.4434
		4	− 0.0573	+ 0.0732									−0.0376	−2.0044
		5	− 25.4519	−14.0958	+ 2.1915	+ 0.7414	+ 3.1412	−0.8809					−0.4328	+6.3156
		6	− 0.2684	− 0.7651	+ 0.2931	+ 0.0374	+ 0.3000	−0.1440					+0.0180	+1.7039
		7	− 2.3616	− 0.2812	− 4.8012	+ 2.0783	− 1.8585	+0.4180	+1.0063	−0.0358	+0.6886	−0.3380	−0.0219	+6.1376
		8	− 0.0411	+ 0.9499	+ 0.2741	+ 0.1025	− 0.0274	+0.0036	+0.0533	+0.0206	+0.0631	−0.0044	+0.0049	+0.2170
			+ 79.8818	− 7.9253	−15.4881	+ 3.4065	−49.6413	−2.2882	+6.8042	+0.9294	+6.0397	−1.5815	+0.3335	+20.4707
			$\partial\phi_5$	+ 0.099213	+ 0.193888	− 0.042644	+ 0.621434	+0.028645	−0.085178	−0.011635	−0.075608	+0.019798	−0.004175	−0.256262
				+46.8472	− 2.6606	− 1.5081	+14.5520	−3.7637	+1.0543	−0.6253	+0.0273	−0.7072	+0.2900	+0.0595
			1	− 0.0065									−0.0051	+0.1180
			2	− 0.0114									+0.0156	+0.6541
			3	− 0.0358									+0.0008	−1.5438
			4	− 0.0934									+0.0480	+2.5590
			5	− 7.8065	+ 1.2137	+ 0.4106	+ 1.7397	−0.4879					−0.2397	+3.4977
			6	− 2.1805	+ 0.8353	+ 0.1066	+ 0.8551	−0.4103					+0.0512	+4.8561
			7	− 0.0335	− 0.5717	+ 0.2475	− 0.2213	+0.0498	+0.1198	−0.0043	+0.0820	−0.0402	−0.0026	+0.7308
			8	−21.9536	− 6.3350	− 2.3699	+ 0.6326	−0.0840	−1.2318	−0.4772	−1.4583	+0.1008	−0.1128	−5.0151
			9	− 0.7863	− 1.5366	+ 0.3380	− 4.9251	−0.2270	+0.6751	+0.0922	+0.5992	−0.1569	+0.0331	+2.0310
				+13.9397	− 9.0549	− 2.7753	+12.6330	−4.9231	+0.6174	−1.0146	−0.7498	−0.8035	+0.0785	+7.9474
				$\partial\lambda_5$	+ 0.649576	+ 0.199093	− 0.906261	+0.353171	−0.044291	+0.072785	+0.053789	+0.057641	−0.005631	−0.570127

	11	12	13	14	15	16	17	18	19	20	21	22	η	Σ
	+103.3016	− 6.8476	−13.4998	− 8.8081	−57.7023	+24.4953	−15.1399	+ 1.5610	+6.7770	+1.0673	+6.3184	−3.7670	−0.5202	− 4.4519
5	− 0.1887	− 0.0638	− 0.2705	+ 0.0759									+0.0373	− 0.5438
6	− 0.3200	− 0.0408	− 0.3276	+ 0.1572									−0.0196	− 1.8602
7	− 9.7608	+ 4.2252	− 3.7784	+ 0.8498	+ 2.0459	− 0.0728	+ 1.4000	− 0.6871					−0.0446	+12.4778
8	− 1.8281	− 0.6839	+ 0.1826	− 0.0243	− 0.3555	− 0.1377	− 0.4208	+ 0.0291					−0.0326	− 1.4472
9	− 3.0030	+ 0.6605	− 9.6249	− 0.4437	+ 1.3193	+ 0.1802	+ 1.1710	− 0.3066					+0.0647	+ 3.9690
10	− 5.8818	− 1.8028	+ 8.2061	− 3.1979	+ 0.4010	− 0.6591	− 0.4871	− 0.5219					+0.0510	+ 5.1624
	+ 82.3192	− 4.5532	−19.1125	−11.3911	−54.2916	+23.8059	−13.4768	+ 0.0745	+6.7770	+1.0673	+6.3184	−3.7670	−0.4640	+13.3061
	$\partial\phi_6$	+ 0.055312	+ 0.232175	+ 0.138377	+ 0.659525	− 0.289190	+ 0.163714	− 0.000905	−0.082326	−0.012965	−0.076755	+0.045761	+0.005637	− 0.161640
		+28.4613	− 6.7278	−15.4350	+ 1.3790	− 6.7932	+ 6.3530	− 1.9532	−2.6480	−1.0111	−3.9798	+1.2720	−0.0948	− 0.2802
	5	− 0.0216	− 0.0915	+ 0.0257									+0.0126	− 0.1840
	6	− 0.0052	− 0.0418	+ 0.0201									−0.0025	− 0.2374
	7	− 1.8290	+ 1.6356	− 0.3678	− 0.8856	+ 0.0315	− 0.6060	+ 0.2974					+0.0193	− 5.4013
	8	− 0.2558	+ 0.0683	− 0.0091	− 0.1330	− 0.0515	− 0.1574	+ 0.0109					−0.0122	− 0.5414
	9	− 0.1453	+ 2.1169	+ 0.0976	− 0.2902	− 0.0396	− 0.2576	+ 0.0674					−0.0142	− 0.8730
	10	− 0.5525	+ 2.5151	− 0.9802	+ 0.1229	− 0.2020	− 0.1493	− 0.1600					+0.0156	+ 1.5823
	11	− 0.2518	− 1.0572	− 0.6301	− 3.0030	+ 1.3168	− 0.7454	+ 0.0041	+0.3748	+0.0590	+0.3495	−0.2084	−0.0257	+ 0.7360
		+25.4001	− 1.5824	−17.2788	− 2.8099	− 5.7380	+ 4.4373	− 1.7334	−2.2732	−0.9521	−3.6303	+1.0636	−0.1019	− 5.1990
		$\partial\lambda_6$	+ 0.062299	+ 0.680265	+ 0.110626	+ 0.225905	− 0.174696	+ 0.068244	+0.089496	+0.037484	+0.142925	−0.041874	+0.004012	+ 0.204684
			+99.0967	+ 1.1818	−13.8524	+ 0.4952	−32.5324	−11.4363	+5.2774	+1.6860	+7.0946	−2.6458	−0.2161	− 3.5317
		5	− 0.3877	+ 0.1087									+0.0534	− 0.7795
		6	− 0.3353	+ 0.1609									−0.0201	− 1.9043
		7	− 1.4626	+ 0.3289	+ 0.7920	− 0.0282	+ 0.5420	− 0.2660					−0.0172	+ 4.8302
		8	− 0.0182	+ 0.0024	+ 0.0355	+ 0.0138	+ 0.0420	− 0.0029					+0.0033	+ 0.1445
		9	−30.8488	− 1.4220	+ 4.2284	+ 0.5776	+ 3.7533	− 0.9828					+0.2072	+12.7212
		10	−11.4488	+ 4.4616	− 0.5595	+ 0.9195	+ 0.6795	+ 0.7282					−0.0711	− 7.2024
		11	− 4.4374	− 2.6447	−12.6052	+ 5.5271	− 3.1290	+ 0.0173	+1.5734	+0.2478	+1.4670	−0.8746	−0.1077	+ 3.0893
		12	− 0.0986	− 1.0765	− 0.1751	− 0.3575	+ 0.2764	− 0.1080	−0.1416	−0.0593	−0.2262	+0.0663	−0.0063	− 0.3239
			+50.0593	+ 1.1011	−22.1363	+ 7.1475	−30.3682	−12.0505	+6.7092	+1.8745	+8.3354	−3.4541	−0.1746	+ 7.0433
			$\partial\phi_7$	− 0.021995	+ 0.442202	− 0.142781	+ 0.606645	+ 0.240724	−0.134025	−0.037446	−0.166511	+0.069000	+0.003488	− 0.140699
				+25.6494	− 0.1385	− 0.3391	+ 8.1200	− 3.6212	+1.6185	+0.0354	+0.9507	−0.9735	+0.0496	+ 2.1778
			5	− 0.0305									−0.0150	+ 0.2186
			6	− 0.0772									+0.0096	+ 0.9137
			7	− 0.0740	− 0.1781	+ 0.0063	− 0.1219	+ 0.0598					+0.0039	− 1.0863
			8	− 0.0003	− 0.0047	− 0.0018	− 0.0056	+ 0.0004					−0.0004	− 0.0192
			9	− 0.0055	+ 0.1949	+ 0.0266	+ 0.1730	− 0.0453					+0.0096	+ 0.5864
			10	− 1.7387	+ 0.2180	− 0.3583	− 0.2648	− 0.2838					+0.0277	+ 2.8068
			11	− 1.5763	− 7.5127	+ 3.2942	− 1.8649	+ 0.0103	+0.9378	+0.1477	+0.8743	−0.5213	−0.0642	+ 1.8413
			12	−11.7542	− 1.9115	− 3.9034	+ 3.0185	− 1.1792	−1.5464	−0.6477	−2.4696	+0.7235	−0.0693	− 3.5367
			13	− 0.0242	+ 0.4869	− 0.1572	+ 0.6679	+ 0.2651	−0.1476	−0.0412	−0.1833	+0.0760	+0.0038	− 0.1549
				+10.3085	− 8.8457	− 1.4327	+ 9.7222	− 4.7939	+0.8623	−0.5058	−0.8279	−0.6953	−0.0447	+ 3.7470
				$\partial\lambda_7$	+ 0.858098	+ 0.138982	− 0.943125	+ 0.465043	−0.083649	+0.049066	+0.080312	+0.067449	+0.004336	− 0.363486

Solution of normals—Continued

	15	16	17	18	19	20	21	22	η	Σ
	+122.2200	−19.7152	− 11.8987	+ 3.1998	−43.9722	+ 9.8031	−15.7932	+ 7.7351	+37.1447	+28.6369
7	− 0.4288	+ 0.0153	− 0.2934	+ 0.1440					+ 0.0093	− 2.6154
8	− 0.0691	− 0.0268	− 0.0818	+ 0.0056					− 0.0063	− 0.2814
9	− 0.5796	− 0.0792	− 0.5144	+ 0.1347					− 0.0284	− 1.7437
10	− 0.0273	+ 0.0449	+ 0.0332	+ 0.0356					− 0.0035	− 0.3520
11	− 35.8067	+15.7006	− 8.8883	+ 0.0491	+ 4.4696	+ 0.7039	+ 4.1671	− 2.4844	− 0.3060	+ 8.7757
12	− 0.3108	− 0.6348	+ 0.4909	− 0.1918	− 0.2515	− 0.1053	− 0.4016	+ 0.1177	− 0.0113	− 0.5751
13	− 9.7887	+ 3.1606	− 13.4289	− 5.3288	+ 2.9668	+ 0.8289	+ 3.6859	− 1.5274	− 0.0772	+ 3.1146
14	− 7.5905	− 1.2294	+ 8.3426	− 4.1136	+ 0.7399	− 0.4340	− 0.7104	− 0.5966	− 0.0384	+ 3.2153
	+ 67.6185	− 2.7640	− 26.2388	− 6.0654	−36.0474	+10.7966	− 9.0522	+ 3.2444	+36.6829	+38.1746
	$\delta\phi_8$	+ 0.040876	+ 0.388042	+ 0.089700	+ 0.533100	− 0.159669	+ 0.133872	− 0.047981	−0.542498	− 0.564559
		+63.1452	+ 8.9105	−43.8742	−13.4344	− 4.8829	+ 3.1960	− 6.3741	−10.5001	− 6.1221
	7	− 0.0005	+ 0.0104	− 0.0051					− 0.0003	+ 0.0931
	8	− 0.0104	− 0.0317	+ 0.0022					− 0.0025	− 0.1090
	9	− 0.0108	− 0.0703	+ 0.0184					− 0.0039	− 0.2382
	10	− 0.0738	− 0.0546	− 0.0585					+ 0.0057	+ 0.5785
	11	− 6.8844	+ 3.8974	− 0.0215	− 1.9598	− 0.3087	− 1.8272	+ 1.0894	+ 0.1342	− 3.8480
	12	− 1.2962	+ 1.0024	− 0.3916	− 0.5135	− 0.2151	− 0.8201	+ 0.2403	− 0.0230	− 1.1745
	13	− 1.0205	+ 4.3360	+ 1.7206	− 0.9579	− 0.2676	− 1.1901	+ 0.4932	+ 0.0249	− 1.0056
	14	− 0.1991	+ 1.3512	− 0.6663	+ 0.1198	− 0.0703	− 0.1151	− 0.0966	− 0.0062	+ 0.5208
	15	− 0.1130	− 1.0725	− 0.2479	− 1.4735	+ 0.4413	− 0.3700	+ 0.1326	+ 1.4995	+ 1.5604
		+53.5365	+ 18.2788	−43.5239	−18.2193	− 5.3033	− 1.1265	− 4.5152	− 8.8717	− 9.7446
		$\delta\lambda_8$	− 0.341427	+ 0.812976	+ 0.340315	+ 0.099060	+ 0.021042	+ 0.084339	+ 0.165713	+ 0.182018
			+142.9446	−21.3622	−12.7934	− 0.8560	−94.2862	+ 2.4115	+43.2741	+30.0408
		7	− 0.2008	+ 0.0986					+ 0.0064	− 1.7897
		8	− 0.0969	+ 0.0067					− 0.0075	− 0.3331
		9	− 0.4566	+ 0.1196					− 0.0252	− 1.5477
		10	− 0.0403	− 0.0432					+ 0.0042	+ 0.4275
		11	− 2.2063	+ 0.0122	+ 1.1095	+ 0.1747	+ 1.0344	− 0.6167	− 0.0760	+ 2.1784
		12	− 0.7752	+ 0.3028	+ 0.3971	+ 0.1663	+ 0.6342	− 0.1858	+ 0.0178	+ 0.9082
		13	− 18.4227	− 7.3104	+ 4.0701	+ 1.1372	+ 5.0566	− 2.0954	− 0.1059	+ 4.2728
		14	− 9.1692	+ 4.5212	− 0.8133	+ 0.4770	+ 0.7808	+ 0.6558	+ 0.0422	− 3.5339
		15	− 10.1818	− 2.3536	−13.9879	+ 4.1895	− 3.5126	+ 1.2590	+14.2345	+14.8133
		16	− 6.2409	+14.8602	+ 6.2206	+ 1.8107	+ 0.3846	+ 1.5416	+ 3.0290	+ 3.3271
			+ 95.1539	−11.1481	−15.7973	+ 7.0994	−89.9082	+ 2.9700	+60.3936	+48.7633
			$\delta\phi_9$	+ 0.117159	+ 0.166018	− 0.074610	+ 0.944871	− 0.031213	− 0.634694	− 0.512468

Solution of normals—Continued

	18	19	20	21	22	η	Σ
	+ 61.5179	− 1.0383	− 2.8738	+ 31.3248	− 8.1976	+ 7.4331	+ 7.1924
7	− 0.0484					− 0.0031	+ 0.8784
8	− 0.0005					+ 0.0005	+ 0.0230
9	− 0.0313					+ 0.0066	+ 0.4053
10	− 0.0463					+ 0.0045	+ 0.4581
11	− 0.0091	− 0.0061	− 0.0010	− 0.0057	+ 0.0034	+ 0.0004	− 0.0120
12	− 0.1183	− 0.1551	− 0.0650	− 0.2477	+ 0.0726	− 0.0070	− 0.3548
13	− 2.9008	+ 1.6151	+ 0.4512	+ 2.0065	− 0.8315	− 0.0420	+ 1.6955
14	− 2.2294	+ 0.4010	− 0.2352	− 0.3850	− 0.3233	− 0.0208	+ 1.7425
15	− 0.5441	− 3.2335	+ 0.9685	− 0.8120	+ 0.2910	+ 3.2905	+ 3.4243
16	− 35.3839	− 14.8119	− 4.3115	− 0.9158	− 3.6707	− 7.2125	− 7.9221
17	− 1.3061	− 1.8508	+ 0.8318	− 10.5336	+ 0.3480	+ 7.0757	+ 5.7131
	+ 18.9087	− 19.0796	− 5.2350	+ 20.4315	− 12.3081	+ 10.5259	+ 13.2434
	$\partial\lambda_9$	+ 1.009038	+ 0.276857	− 1.080534	+ 0.650923	− 0.556670	− 0.700387
		+106.8200	+27.9899	− 16.8102	− 7.0763	+ 34.6511	+ 85.3611
	11	− 0.5579	− 0.0879	− 0.5202	+ 0.3101	+ 0.0382	− 1.0954
	12	− 0.2034	− 0.0852	− 0.3249	+ 0.0952	− 0.0091	− 0.4653
	13	− 0.8992	− 0.2512	− 1.1172	+ 0.4029	+ 0.0234	− 0.9440
	14	− 0.0721	+ 0.0423	+ 0.0693	+ 0.0582	+ 0.0037	− 0.3134
	15	− 19.2169	+ 5.7557	− 4.8257	+ 1.7296	+ 19.5557	+ 20.3509
	16	− 6.2003	− 1.8048	− 0.3834	− 1.5366	− 3.0192	− 3.3162
	17	− 2.6226	+ 1.1786	− 14.9264	+ 0.4931	+ 10.0264	+ 8.0956
	18	− 19.2520	− 5.2823	+ 20.6162	− 12.4193	+ 10.6210	+ 13.3631
		+ 57.7956	+27.4551	− 18.2225	− 17.8831	+ 71.8912	+121.0363
		$\partial\phi_{10}$	− 0.475038	+ 0.315292	+ 0.309420	− 1.243887	− 2.094213
			+50.9179	− 9.2141	− 26.1027	+167.6744	+214.2334
		11	− 0.0138	− 0.0819	+ 0.0488	+ 0.0060	− 0.1725
		12	− 0.0357	− 0.1361	+ 0.0399	− 0.0038	− 0.1949
		13	− 0.0702	− 0.3121	+ 0.1293	+ 0.0065	− 0.2637
		14	− 0.0248	− 0.0406	− 0.0341	− 0.0022	+ 0.1839
		15	− 1.7239	+ 1.4454	− 0.5180	− 5.8571	− 6.0953
		16	− 0.5253	− 0.1116	− 0.4473	− 0.8788	− 0.9653
		17	− 0.5297	+ 6.7081	− 0.2216	− 4.5060	− 3.6382
		18	− 1.4493	+ 5.6566	− 3.4076	+ 2.9142	+ 3.6665
		19	−13.0422	+ 8.6564	+ 8.4952	− 34.1511	− 57.4968
			+33.5030	+ 12.5701	− 22.0181	+125.2021	+149.2571
			$\partial\lambda_{10}$	− 0.375193	+ 0.657198	− 3.737041	− 4.455037
				+173.0271	− 35.3120	−162.3283	−115.8122
			11	− 0.4850	+ 0.2891	+ 0.0356	− 1.0213
			12	− 0.5189	+ 0.1520	− 0.0146	− 0.7431
			13	− 1.3879	+ 0.5751	+ 0.0291	− 1.1728
			14	− 0.0665	− 0.0558	− 0.0036	+ 0.3009
			15	− 1.2118	+ 0.4343	+ 4.9108	+ 5.1105
			16	− 0.0237	− 0.0950	− 0.1867	− 0.2050
			17	− 84.9517	+ 2.8063	+ 57.0642	+ 46.0750
			18	− 22.0769	+ 13.2993	− 11.3736	− 14.3099
			19	− 5.7454	− 5.6384	+ 22.6667	+ 38.1618
			20	− 4.7162	+ 8.2610	− 46.9750	− 56.0002
				+ 51.8431	− 15.2841	−136.1754	− 99.6164
				$\partial\phi_{11}$	+ 0.294815	+ 2.626683	+ 1.921498
					+ 54.7848	+188.1235	+163.8779
				11	− 0.1724	− 0.0212	+ 0.6089
				12	− 0.0445	+ 0.0043	+ 0.2177
				13	− 0.2383	− 0.0120	+ 0.4860
				14	− 0.0409	− 0.0030	+ 0.2527
				15	− 0.1557	− 1.7601	− 1.8317
				16	− 0.3808	− 0.7482	− 0.8218
				17	− 0.0927	− 1.8851	− 1.5220
				18	− 8.0116	+ 6.8516	+ 8.6204
				19	− 5.5334	+ 22.2446	+ 37.4511
				20	− 14.4703	+ 82.2826	+ 98.0915
				21	− 4.5060	− 40.1466	− 29.3684
					+ 21.1322	+254.0304	+276.0626
					$\partial\lambda_{11}$	− 12.06360	− 13.06360

Back solution

22	21	20	19	18	17	16	15
−12.0636	+2.6267	− 3.7370	−1.2439	− 0.5567	−0.6347	+0.1657	−0.5425
−12.0636	−3.5565	− 7.9282	−3.7327	− 7.8525	+0.3765	−1.0174	+0.5788
	−0.9298	+ 0.3489	−0.2932	+ 1.0047	−0.8785	−0.0196	−0.1245
		−11.3163	+5.3757	− 3.1330	+0.8443	−1.1210	+1.8069
			+0.1059	+ 0.1069	+0.0176	+0.0360	+0.0565
				−10.4306	−1.2220	−8.4798	−0.9356
					−1.4968	+0.5110	−0.5808
						−9.9251	−0.4057
							−0.1469

14	13	12	11	10	9	8	7
+0.0043	+0.0035	+0.0040	+0.0056	−0.0056	−0.0042	+0.0041	+0.0019
−0.8137	−0.8324	+0.5052	−0.5520	−0.6012	−0.2065	+0.0385	−0.2996
−0.0747	+0.1548	−0.1329	+0.0714	−0.0805	+0.1132	−0.0799	+0.0876
−0.5552	+0.4238	−0.4242	+0.1467	−0.7224	+0.1155	−0.1733	−0.0302
−0.0089	−0.0142	+0.0095	−0.0087	+0.0065	+0.0125	−0.0066	+0.0126
−4.8507	−2.5109	−0.7118	+0.0094	−2.2577	−0.1831	−0.0197	+0.2271
+1.4117	−0.9080	+0.2615	−0.2450	+1.9853	−1.3614	+0.0507	−0.3460
−1.3794	+1.4171	−2.2421	+2.8702	−1.4399	+0.3084	−0.6273	+1.2773
−0.1261	−0.0650	−0.0163	−0.0969	+0.2647	+0.0790	+0.0945	+0.1663
	+0.1406	−4.3487	−0.8846		−0.2828	−2.2905	−0.0681
−6.3927		−0.1365	−0.5086	−2.8508		+0.0490	−0.2829
	−2.1907		−0.4000		−1.4094		−0.0309
		−7.2323				−2.9605	
			+0.4075				+0.7151

6	5	4	3	2	1
−0.0082	+0.0105	−0.0215	+0.0005	−0.0216	−0.0066
−0.4199	−0.1370	−0.1192	+0.0600	−0.0453	+0.0242
+0.2999	+0.1674	+0.0462	+0.1430	+0.0141	+0.0678
+0.1234	+0.1304	−0.0050	+0.1295	−0.0045	+0.0600
−0.0545	−0 0217	−0.0500	−0.0424	−0.0176	−0.0212
−0.9952	−0.9773	−0.1310	−0.0343	−0.0328	−0.0699
−0.1726	−0.8725	+0.3049	−0.5681	+0.4148	−0.2184
+0.1954	−0.0068		−0.0020	+0.0084	+0.0113
+0.0337	+0.0643	+0.0244		−0.0060	−0.2006
	+0.0917		−0.3138		−0.0143
−0.9980				+0.3095	
	−1.5510				−0.3677

Computation of corrections.

1	2	z_1	3	4	5	3a	z_2
+0.700	−0.842	−0.842	+1.039	−1.318	+0.301	+1.039	+0.301
+0.7	−0.857	−0.857	+1.0	−0.098	−1.541	+1.0	−1.541
	+0.700	+0.3		+1.039	+1.039		−1.318
	+0.3			−0.6	−0.9		−0.098
		−1.399					−1.5
	−0.699	+0.700		−0.977	−1.101		
	−0.7			−1.0	−1.1		−4.156
							+1.039

4a	5a	6	7	8	z_3	17	15
−0.842	+0.301	+0.978	+2.416	+1.254	+4.107	+0.857	+2.416
−0.857	−1.541	−0.699	−0.699	+0.149	−3.649	−0.131	−0.699
+1.592	+1.592	+1.592	−2.062	−5.289	−2.062	+1.860	−2.062
		−0.9	+0.055	+0.479	+0.055		+0.055
−0.107	+0.352		+1.592	+1.592	−5.289	+2.586	+1.860
−0.1	+0.4	+0.971	+1.4	−2.1	+0.479	+2.6	−2.7
		+1.0			−1.6		
			+2.702	−3.915			−1.130
			+2.7	−3.9	−7.959		−1.1
					+1.592		

16	z_4	14	9	10	11	12	13
−1.318	+2.416	−0.753	+2.624	−4.045	−2.295	+0.857	+0.978
−0.098	−0.699	−0.8	−1.836	+3.792	+3.174	−0.131	−0.699
+1.860	−2.523		−0.753	−0.753	−0.753	−0.753	−0.753
−1.9	−0.174		+0.4	+0.7	+1.8	−0.6	−0.2
	−4.6						
−1.456			+0.435	−0.306	+1.926	−0.627	−0.674
−1.4	−5.580		+0.4	−0.3	+1.9	−0.7	−0.7
	+1.860						

z_5	18	19	20	21	22	z_6
+0.978	+1.254	+1.440	−2.295	+2.373	+6.980	+ 1.254
−0.699	+0.149	+0.062	+3.174	+2.275	+1.976	+ 0.149
+0.857	−5.289	−7.119	+3.728	+1.094	−6.342	+ 1.440
−0.131	+0.479	+2.545	−2.8	−6.750	−5.645	+ 0.062
−2.295	+3.728	+3.728		+3.728	+3.728	− 5.351
+3 174		−1.8	+1.807	−2.9	−1.5	+10.449
+2.624	+0.321		+1.8			+ 1.094
−1.836	+0.3	−1.144		−0.180	−0.803	− 6.750
−4.045		−1.2		−0.2	−0.8	− 6.342
+3.792						− 5.645
+2.1						− 9.0
+4.519						−18.640
−0.753						+ 3.728

28	29	30	31	32	z_7
−2.560	− 1.495	− 0.179	+2.373	+2.624	+ 2.373
−1.066	− 6.247	−13.085	+2.275	−1.836	+ 2.275
+1.454	− 4.579	− 0.352	+1.094	−0.632	− 0.515
+2.604	+13.489	+12.601	−6.750	+1.0	−28.983
−0.633	− 0.633	− 0.633	−0.633		− 0.352
	+ 0.3	+ 0.9	+0.6	+1.155	+12.601
−0.201				+1.2	+ 1.454
−0.1	+ 0.835	− 0.748	−1.041		+ 2.604
	+ 0.9	− 0.7	−1.0		− 4.579
					+13.489
					+ 2.8
					+ 3.167
					− 0.633

Computation of corrections—Continued

23	24	25	26	27	z_8
+6.980	−4.045	− 0.179	+ 3.834	+ 6.977	+ 6.980
+1.976	+3.792	−13.085	+ 4.162	− 2.480	+ 1.976
−6.342	+1.588	− 0.352	+ 1.108	−10.844	− 0.179
−5.645	−1.4	+12.601	−10.559	+ 5.562	−13.085
+1.588		+ 1.588	+ 1.588	+ 1.588	+ 0.070
	−0.065	− 0.6	− 0.3	+ 0.9	+12.429
−1.443	−0.0				+ 1.108
−1.4		− 0.027	− 0.167	+ 1.703	−10.559
		− 0.0	− 0.2	+ 1.7	−10.844
					+ 5.562
					− 1.4
					− 7.942
					+ 1.588

38	39	40	41	42	z_9
− 2.180	− 1.015	+ 0.623	+ 3.834	− 2.560	− 2.560
− 8.534	−17.358	−27.410	+ 4.162	− 1.066	− 1.066
− 0.786	− 3.727	+ 3.352	+ 1.108	+ 1.459	+ 3.834
+11.712	+25.033	+24.228	−10.559	+ 2.604	+ 4.162
− 1.284	− 1.284	− 1.284	− 1.284	− 1.284	− 0.004
	+ 3.4	− 2.2	+ 1.6	+ 0.7	−61.258
− 1.072					+ 3.352
− 1.1	+ 5.049	− 2.691	− 1.139	− 0.147	+24.228
	+ 5.0	− 2.7	− 1.2	− 0.1	− 0.786
					+11.712
					− 3.727
					+25.033
					+ 3.5
					+ 6.420
					− 1.284

33	34	35	36	37	z_{10}
+ 6.977	− 1.495	+ 0.623	+ 6.265	+8.719	− 1.495
− 2.480	− 6.247	−27.410	+10.228	+0.384	− 6.247
−10.844	− 4.579	+ 3.352	− 0.420	−5.957	+ 6.977
+ 5.562	+13.489	+24.228	−15.880	−0.626	− 2.480
− 2.538	− 2.538	− 2.538	− 2.538	−2.538	+ 0.623
	+ 3.0	+ 3.5	+ 2.2	+0.1	−27.410
− 3.323					+ 2.914
− 3.3	+ 1.630	+ 1.755	− 0.145	+0.082	+53.890
	+ 1.7	+ 1.8	− 0.1	+0.1	− 0.420
					−15.880
					− 5.957
					− 0.626
					+ 8.8
					+12.689
					− 2.538

43	44	45	46	47	z_{11}
+8.719	− 1.015	+ 0.025	+ 4.655	+10.478	− 1.015
+0.384	−17.358	−55.481	+22.009	−12.412	−17.358
−5.957	− 3.727	− 0.254	+ 0.329	− 6.509	+ 8.719
−0.626	+25.033	+58.307	−23.877	+14.356	+ 0.384
−2.716	− 2.716	− 2.716	− 2.716	− 2.716	+ 0.025
	− 1.8	− 0.6	− 2.0	+ 0.9	−55.481
−0.196					+ 5.194
−0.2	− 1.583	− 0.719	− 1.600	+ 4.097	+92.311
	− 1.6	− 0.7	− 1.6	+ 4.1	+ 0.329
					−23.877
					− 6.509
					+14.356
					− 3.5
					+13.578
					− 2.716

Computation of corrections—Continued

48	49	50	51	52	z_{12}
+0.720	+ 0.391	+ 0.025	+ 6.265	− 2.180	− 2.180
+0.199	−29.478	−55.481	+10.228	− 8.534	− 8.534
+0.519	− 2.473	− 0.254	− 0.420	− 0.786	+ 6.265
−0.226	+35.829	+58.308	−15.880	+11.712	+10.228
−1.196	− 1.196	− 1.196	− 1.196	− 1.196	− 0.071
	− 2.3	− 0.3	+ 0.8	− 0.7	−88.929
+0.016					− 0.254
+0.0	+ 0.773	+ 1.102	− 0.203	− 1.684	+58.308
	+ 0.8	+ 1.1	− 0.2	− 1.7	+ 0.519
					− 0.226
					− 2.473
					+35.829
					− 2.5
					+ 5.082
					− 1.196

53	54	55	56	57	z_{13}
+10.478	+ 0.391	+ 0.177	+ 1.832	+ 5.607	+ 0.391
−12.412	−29.478	−48.094	+22.318	−29.918	−29.478
− 6.509	− 2.473	+ 1.553	− 7.305	− 7.305	+10.478
+14.356	+35.829	+51.270	−11.0	+33.7	−12.412
− 7.305	− 7.305	− 7.305			+ 0.177
	+ 0.7	− 1.8	+ 5.845	+ 2.084	−48.094
− 1.394			+ 5.8	+ 2.1	+ 0.009
− 1.4	− 2.336	− 4.199			+93.855
	− 2.3	− 4.2			+21.6
					+36.526
					− 7.305

58	59	60	61	62	z_{14}
− 0.530	− 0.116	+ 0.177	+ 4.655	+0.720	+ 0.720
+20.030	−36.552	−48.094	+22.009	+0.199	+ 0.199
− 4.333	− 4.333	+ 1.553	+ 0.329	+0.519	+ 4.655
−15.2	+43.2	+51.270	−23.877	−0.226	+22.009
		− 4.333	− 4.333	−4.333	+ 0.379
− 0.033	+ 2.199	+ 1.3	− 0.5	+0.8	−88.720
− 0.0	+ 2.2				+ 1.553
		+ 1.873	− 1.717	−2.321	+51.270
		+ 1.9	− 1.7	−2.3	+29.6
					+21.665
					− 4.333

63	64	65	z_{15}	66	67	68	z_{16}
+ 5.607	− 0.116	−10.240	− 0.116	− 9.016	+ 1.832	− 0.530	− 0.530
−29.918	−36.552	+11.6	−36.552	+13.3	+22.318	+20.030	+20.030
−10.240	−10.240		+ 5.607		− 9.016	− 9.016	+ 1.832
+33.2	+46.9	+ 1.360	−29.918	+ 4.284	−13.8	−16.1	+22.318
		+ 1.3	+91.7	+ 4.2			−16.6
− 1.351	− 0.008				+ 1.334	− 5.616	
− 1.3	− 0.0		+30.721		+ 1.3	− 5.6	+27.050
			−10.240				− 9.016

Final computation of triangles

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″	″	° ′ ″	
	Fort Morgan-Dauphin Island east base						4.050203
−4a+5a	Cedar	43 54 21.8	+0.5	22.3	0.1	22.2	0.158967
− 1+ 2	Fort Morgan	42 33 37.7	−1.4	36.3	0.1	36.2	9.830179
+3a− 5	Dauphin Island east base	93 31 59.6	+2.1	61.7	0.1	32 01.6	9.999173
			+1.2		0.3		
	Cedar-Dauphin Island east base						4.039349
	Cedar-Fort Morgan						4.208343
	Dauphin Island east base-Dauphin Island west base						4.027832
−5a+ 6	Cedar	37 26 33.0	+0.6	33.6	0.1	33.5	0.216120
− 3+ 5	Dauphin Island east base	103 55 36.4	−2.1	34.3	0.1	34.2	9.987043
−13+14	Dauphin Island west base	38 37 52.5	−0.1	52.4	0.1	52.3	9.795397
			−1.6		0.3		
	Cedar-Dauphin Island west base						4.230995
	Cedar-Dauphin Island east base						4.039349
	Cedar-Dauphin Island east base						4.039349
−15+16	Cat	69 30 32.7	−0.3	32.4	0.0	32.4	0.028387
−5a+ 7	Cedar	60 11 14.2	+2.3	16.5	0.1	16.4	9.938350
− 4+ 5	Dauphin Island east base	50 18 11.4	−0.1	11.3	0.1	11.2	9.886171
			+1.9		0.2		
	Cat-Dauphin Island east base						4.006086
	Cat-Cedar						3.953907
	Cedar-Dauphin Island west base						4.230995
−15+17	Cat	135 31 54.9	+3.7	58.6	0.1	58.5	0.154592
− 6+ 7	Cedar	22 44 41.2	+1.7	42.9	0.0	42.9	9.587301
−12+13	Dauphin Island west base	21 43 18.7	0.0	18.7	0.1	18.6	9.568320
			+5.4		0.2		
	Cat-Dauphin Island west base						3.972888
	Cat-Cedar						3.953907
	Dauphin Island east base-Dauphin Island west base						4.027832
−16+17	Cat	66 01 22.2	+4.0	26.2	0.1	26.1	0.039189
− 3+ 4	Dauphin Island east base	53 37 25.0	−2.0	23.0	0.1	22.9	9.905867
−12+14	Dauphin Island west base	60 21 11.2	−0.1	11.1	0.1	11.0	9.939065
			+1.9		0.3		
	Cat-Dauphin Island west base						3.972888
	Cat-Dauphin Island east base						4.006086
	Cedar-Cat						3.953907
−18+19	Pins	23 22 40.3	−1.5	38.8	0.1	38.7	0.401443
− 7+ 8	Cedar	30 54 61.5	−6.6	54.9	0.1	54.8	9.710768
	Cat	18.4		26.5	0.0	125 42 26.5	9.909561
					0.2		
	Pins-Cat						4.066118
	Pins-Cedar						4.264911
	Cedar-Dauphin Island west base						4.230995
−18+20	Pins	58 45 26.0	+1.5	27.5	0.2	27.3	0.068044
− 6+ 8	Cedar	53 39 42.7	−4.9	37.8	0.2	37.6	9.906076
−11+13	Dauphin Island west base	67 34 57.9	−2.6	55.3	0.2	55.1	9.965872
			−6.0		0.6		
	Pins-Dauphin Island west base						4.205115
	Pins-Cedar						4.264911
	Cat-Dauphin Island west base						3.972888
−19+20	Pins	35 22 45.7	+3.0	48.7	0.1	48.6	0.237322
	Cat	35.3		34.9	0.0	98 45 34.9	9.994905
−11+12	Dauphin Island west base	45 51 39.2	−2.6	36.6	0.1	36.5	9.855908
					0.2		
	Pins-Dauphin Island west base						4.205115
	Pins-Cat						4.066118

Final computation of triangles—Continued

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″	″	° ′ ″	
	Pins-Dauphin Island west base						4.205115
−23+24	Grand	54 52 01.6	+1.4	03.0	0.2	02.8	0.087341
−20+22	Pins	85 13 07.0	−2.6	04.4	0.1	04.3	9.998486
−10+11	Dauphin Island west base	39 54 50.9	+2.2	53.1	0.2	52.9	9.807296
			+1.0		0.5		
	Grand-Dauphin Island west base						4.290942
	Grand-Pins						4.090752
	Grand-Pins						4.099752
−30+31	Petit	33 09 08.7	−0.3	08.4	0.1	08.3	0.262119
−23+25	Grand	114 03 53.6	+1.4	55.0	0.1	54.9	9.960510
−21+22	Pins	32 46 57.5	−0.6	56.9	0.1	56.8	9.733559
			+0.5		0.3		
	Petit-Pins						4.322381
	Petit-Grand						4.095430^{-1}
	Grand-Dauphin Island west base						4.290942
−30+32	Petit	81 41 28.2	+1.9	30.1	0.1	30.0	0.004582
−24+25	Grand	59 11 52.0	0.0	52.0	0.2	51.8	9.933962
− 9+10	Dauphin Island west base	39 06 39.1	−0.7	38.4	0.2	38.2	9.799905
			+1.2		0.5		
	Petit-Dauphin Island west base						4.229486
	Petit-Grand						4.095429
	Pins-Dauphin Island west base						4.205115
−31+32	Petit	48 32 19.5	+2.2	21.7	0.2	21.5	0.125281
−20+21	Pins	52 26 09.5	−2.0	07.5	0.2	07.3	9.899090
− 9+11	Dauphin Island west base	79 01 30.0	+1.5	31.5	0.3	31.2	9.991984
			+1.7		0.7		
	Petit-Dauphin Island west base						4.229486
	Petit-Pins						4.322380^{+1}
	Grand-Petit						4.095429
−33+34	Pascagoula	37 39 20.6	+5.0	25.6	0.1	25.5	0.214006
−25+27	Grand	104 18 56.1	+1.7	57.8	0.2	57.6	9.986300
−29+30	Petit	38 01 38.6	−1.6	37.0	0.1	36.9	9.789003
			+5.1		0.4		
	Pascagoula-Petit						4.295735
	Pascagoula-Grand						4.099038
	Pascagoula-Grand						4.099038
−40+41	Horn	38 49 39.0	+1.5	40.5	0.1	40.4	0.202744
−33+35	Pascagoula	97 55 54.0	+5.1	59.1	0.2	58.9	9.995824
−26+27	Grand	43 14 18.9	+1.9	20.8	0.1	20.7	9.835719
			+8.5		0.4		
	Horn-Grand						4.297606
	Horn-Pascagoula						4.137501
	Pascagoula-Petit						4.295735
−40+42	Horn	77 06 13.2	+2.6	15.8	0.2	15.6	0.011094
−34+35	Pascagoula	60 16 33.4	+0.1	33.5	0.2	33.3	9.938731
−28+29	Petit	42 37 10.3	+1.0	11.3	0.2	11.1	9.830672
			+3.7		0.6		
	Horn-Petit						4.245560
	Horn-Pascagoula						4.137501
	Grand-Petit						4.095429
−41+42	Horn	38 16 34.2	+1.1	35.3	0.2	35.1	0.207989
−25+26	Grand	61 04 37.2	−0.2	37.0	0.2	36.8	9.942142
−28+30	Petit	80 38 48.9	−0.6	48.3	0.2	48.1	9.994187
			+0.3		0.6		
	Horn-Petit						4.245560
	Horn-Grand						4.297605^{+1}

Final computation of triangles—Continued

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″	″	° ′ ″	
	Pascagoula-Horn						4.137501
−43+44	Belle	48 58 49.8	− 1.4	48.4	0.1	48.3	0.122351
−35+37	Pascagoula	69 43 28.0	− 1.7	26.3	0.2	26.1	9.972219
−39+40	Horn	61 17 53.5	− 7.7	45.8	0.2	45.6	9.943055
			−10.8		0.5		
	Belle-Horn						4.232071
	Belle-Pascagoula						4.202907
	Belle-Pascagoula						4.202907
−50+51	Club	58 40 43.3	− 1.3	42.0	0.1	41.9	0.068410
−43+45	Belle	89 28 55.3	− 0.5	54.8	0.2	54.6	9.999982
−36+37	Pascagoula	31 50 23.4	+ 0.2	23.6	0.1	23.5	9.722261
			− 1.6		0.4		
	Club-Pascagoula						4.271299
	Club-Belle						3.993578
	Belle-Horn						4.232071
−50+52	Club	105 43 56.9	− 2.8	54.1	0.1	54.0	0.016580
−44+45	Belle	40 30 05.5	+ 0.9	06.4	0.1	06.3	9.812560
−38+39	Horn	33 45 53.7	+ 6.1	59.8	0.1	59.7	9.744927
			+ 4.2		0.3		
	Club-Horn						4.061211
	Club-Belle						3.993578
	Pascagoula-Horn						4.137501
−51+52	Club	47 03 13.6	− 1.5	12.1	0.1	12.0	0.135496
−35+36	Pascagoula	37 53 04.6	− 1.9	02.7	0.2	02.5	9.788214
−38+40	Horn	95 03 47.2	− 1.6	45.6	0.1	45.5	9.998302
			− 5.0		0.4		
	Club-Horn						4.061211
	Club-Pascagoula						4.271299
	Belle-Club						3.993578
−53+54	Deer	41 02 10.7	− 0.9	09.8	0.1	09.7	0.182743
−45+47	Belle	102 35 18.5	+ 4.8	23.3	0.0	23.3	9.989430
−49+50	Club	36 22 26.8	+ 0.3	27.1	0.1	27.0	9.773096
			+ 4.2		0.2		
	Deer-Club						4.165751
	Deer-Belle						3.949417
	Deer-Belle						3.949417
−60+61	Ship	33 10 60.2	− 3.6	56.6	0.1	56.5	0.261770
−53+55	Deer	97 49 37.9	− 2.8	35.1	0.1	35.0	9.995936
−46+47	Belle	48 59 22.9	+ 5.7	28.6	0.1	28.5	9.877722
			− 0.7		0.3		
	Ship-Belle						4.207123
	Ship-Deer						4.088909
	Deer-Club						4.165751
−60+62	Ship	70 52 35.0	− 4.2	30.8	0.2	30.6	0.024657
−54+55	Deer	56 47 27.2	− 1.9	25.3	0.1	25.2	9.922555
−48+49	Club	52 20 03.5	+ 0.8	04.3	0.1	04.2	9.898501
			− 5.3		0.4		
	Ship-Club						4.112963
	Ship-Deer						4.088909
	Belle-Club						3.993578
−61+62	Ship	37 41 34.8	− 0.6	34.2	0.1	34.1	0.213655
−45+46	Belle	53 35 55.6	− 0.9	54.7	0.1	54.6	9.905730
−48+50	Club	88 42 30.3	+ 1.1	31.4	0.1	31.3	9.999890
			− 0.4		0.3		
	Ship-Club						4.112963
	Ship-Belle						4.207123

Final computation of triangles—Continued

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ′ ″	″	″	″	° ′ ″	
	Deer-Ship						4.088909
−63+64	Biloxi Lighthouse	48 11 17.4	+ 1.3	18.7	0.1	18.6	0.127644
−55+57	Deer	96 30 31.2	+ 6.3	37.5	0.1	37.4	9.997190
−59+60	Ship	35 18 04.4	− 0.3	04.1	0.1	04.0	9.761833
			+ 7.3		0.3		
	Biloxi Lighthouse-Ship						4.213743
	Biloxi Lighthouse-Deer						3.978386
	Ship Island Lighthouse-Biloxi Lighthouse						**4.323999**
−56+57	Deer	72 34 26.4	− 3.7	22.7	0.1	22.6	0.020407
−66+67	Ship Island Lighthouse	25 29 62.1	− 2.9	59.2	0.2	59.0	9.633980
−63+65	Biloxi Lighthouse	81 55 36.0	+ 2.6	38.6	0.2	38.4	9.995675
			− 4.0		0.5		
	Deer-Biloxi Lighthouse						3.978386
	Deer-Ship Island Lighthouse						4.340081
	Ship Island Lighthouse-Biloxi Lighthouse						**4.323999**
−58+59	Ship	95 44 07.0	+ 2.2	09.2	0.1	09.1	0.002180
−66+68	Ship Island Lighthouse	50 31 41.2	− 9.8	31.4	0.2	31.2	9.887564
−64+65	Biloxi Lighthouse	33 44 18.6	+ 1.3	19.9	0.2	19.7	9.744612
			− 6.3		0.5		
	Ship-Biloxi Lighthouse						4.213743
	Ship-Ship Island Lighthouse						4.070791
	Deer-Ship						4.088909
−67+68	Ship Island Lighthouse	25 01 39.1	− 6.9	32.2	0.1	32.1	0.373636
−55+56	Deer	23 56 04.8	+10.0	14.8	0.1	14.7	9.608246
−58+60	Ship	131 02 11.4	+ 1.9	13.3	0.1	13.2	9.877536
			+ 5.0		0.3		
	Ship Island Lighthouse-Ship						4.070791
	Ship Island Lighthouse-Deer						4.340081

Final position computation,

STATION CEDAR

								°	′	″
α	East base to west base							84	14	41.9
Second angle	West base and Cedar							+103	55	34.3
α	East base to Cedar							188	10	16.2
$\Delta\alpha$								+		29.4
								180	00	00.00
α'	Cedar to east base							8	10	45.6
							First angle of triangle	37	26	33.6
	°	′	″							
ϕ	30	14	56.379	Dauphin Island east base		λ		88	08	14.288
$\Delta\phi$	+	5	51.937			$\Delta\lambda$		−		58.262
ϕ'	30	20	48.316	Cedar		λ'		88	07	16.026
			+1							
	° ′ ″	s	4.039349	s^2	8.0787					
$\frac{1}{2}(\phi+\phi')$	30 17 52	$\cos\alpha$	9.995568	$\sin^2\alpha$	8.3054	h^2	5.093			
	″	B	8.511556	C	1.1712	D	2.332			
1st term	−351.9435	h	2.546473		7.5553		7.425			
2d and 3d terms	+ 0.0063				+0.0036		+0.0027			
$-\Delta\phi$	−351.9372									
	s	4.039349								
	$\sin\alpha$	9.152688								
	A′	8.509351	$\Delta\lambda$	1.765385						
	$\sec\phi'$	0.063997	$\sin\frac{1}{2}(\phi+\phi')$	9.702857						
		1.765385		1.468242						
		″		″						
	$\Delta\lambda$	−58.2620	$-\Delta\alpha$	−29.39						

STATION CAT

								°	′	″
α	East base to west base							84	14	41.9
Second angle	West base and Cat							+ 53	37	23.0
α	East base to Cat							137	52	04.9
$\Delta\alpha$								−	2	08.4
								180	00	00.00
α'	Cat to east base							317	49	56.5
							First angle of triangle	66	01	26.2
	°	′	″							
ϕ	30	14	56.379	Dauphin Island east base		λ		88	08	14.288
$\Delta\phi$	+	4	04.168			$\Delta\lambda$	+		6	14.637
ϕ'	30	19	00.547	Cat		λ'		88	14	28.925
	° ′ ″	s	4.006086	s^2	8.0122				−h	2.388
$\frac{1}{2}(\phi+\phi')$	30 16 58	$\cos\alpha$	9.870171	$\sin^2\alpha$	9.6532	h^2	4.776		$s^2\sin^2\alpha$	7.665
	″	B	8.511556	C	1.1712	D	2.332		E	5.917
1st term	−244.2379	h	2.387813		8.8366		7.108			5.970
2d, 3d, and 4th terms	+ 0.0700				+0.0686		+0.0013			+0.0001
$-\Delta\phi$	−244.1679									
	s	4.006086								
	$\sin\alpha$	9.826619								
	A′	8.509352	$\Delta\lambda$	2.405922						
	$\sec\phi'$	0.063865	$\sin\frac{1}{2}(\phi+\phi')$	9.702663						
		2.405922		2.108585						
		″		″						
	$\Delta\lambda$	+254.6373	$-\Delta\alpha$	+128.40						

secondary triangulation

STATION CEDAR

								°	′	″
α	West base to east base							264	11	22.1
Third angle	Cedar and east base							− 38	37	52.4
α	West base to Cedar							225	33	29.7
$\Delta\alpha$								+	3	49.5
α'	Cedar to west base							180	00	00.00
								45	37	19.2
	°	′	″							
ϕ	30	14	21.492	Dauphin Island west base		λ		88	14	51.034
$\Delta\phi$	+	6	26.825			$\Delta\lambda$		−	7	35.008
ϕ'	30	20	48.317	Cedar		λ'		88	07	16.026
	° ′ ″	s	4.230995	s^2	8.4020				−h	2.588
$\frac{1}{2}(\phi+\phi')$	30 17 35	$\cos\alpha$	9.845212	$\sin^2\alpha$	9.7073	h^2	5.176	$s^2\sin^2\alpha$		8.169
		B	8.511557	C	1.1711	D	2.331	E		5.917
	″									
1st term	−387.0473	h	2.587764		9.3404		7.507			6.674
2d, 3d, and 4th terms	+ 0.2227				+0.2196		+0.0032			+0.0005
$-\Delta\phi$	−386.8246									

s	4.230995		
$\sin\alpha$	9.853676		
A′	8.509351	$\Delta\lambda$	2.658019
$\sec\phi'$	0.063997	$\sin\frac{1}{2}(\phi+\phi')$	9.702795
	2.658019		2.360814
	″		″
$\Delta\lambda$	−455.0080	$-\Delta\alpha$	−229.52

STATION CAT

								°	′	″
α	West base to east base							264	11	22.1
Third angle	Cat and east base							− 60	21	11.1
α	West base to Cat							203	50	11.0
$\Delta\alpha$								+	1	11.7
α'	Cat to west base							180	00	00.00
								23	51	22.7
	°	′	″							
ϕ	30	14	21.492	Dauphin Island west base		λ		88	14	51.034
$\Delta\phi$	+	4	89.055			$\Delta\lambda$		−	2	22.109
ϕ'	30	19	00.547	Cat		λ'		88	12	28.925
	° ′ ″	s	3.972888	s^2	7.9458				−h	2.445
$\frac{1}{2}(\phi+\phi')$	30 16 41	$\cos\alpha$	9.961280	$\sin^2\alpha$	9.2130	h^2	4.891	$s^2\sin^2\alpha$		7.159
		B	8.511557	C	1.1711	D	2.331	E		5.917
	″									
1st term	−279.0776	h	2.445725		8.3299		7.222			5.521
2d, 3d, and 4th terms	+ 0.0231				+0.0214		+0.0017			
$-\Delta\phi$	−279.0545									

s	3.972888		
$\sin\alpha$	9.606517		
A′	8.509352	$\Delta\lambda$	2.152622
$\sec\phi'$	0.063865	$\sin\frac{1}{2}(\phi+\phi')$	9.702600
	2.152622		1.855222
	″		″
$\Delta\lambda$	−142.1092	$-\Delta\alpha$	−71.65

Final position computation,

STATION PINS

		°	′	″
α	Cedar to west base	45	37	19.2
Second angle	West base and Pins	+ 53	39	37.8
α	Cedar to Pins	99	16	57.0
$\Delta\alpha$		−	5	43.8
		180	00	00.00
α'	Pins to Cedar	279	11	13.2
	First angle of triangle	58	45	27.5

		°	′	″				°	′	″
ϕ		30	20	48.317	Cedar	λ		88	07	16.026
$\Delta\phi$		+	1	35.914		$\Delta\lambda$		+	11	20.225
ϕ'		30	22	24.231	Pins	λ'		88	18	36.251
										+1
$\frac{1}{2}(\phi+\phi')$	° ′ ″ 30 21 36	s	4.264911	s^2	8.5298	h^2	3.968	−h	1.984
		cos α	9.207641	sin² α	9.9885			s^2 sin² α	8.518
		B	8.511550	C	1.1729	D	2.332	E	5.919
	″								
1st term	−96.4055	h	1.984102		9.6912		6.300		6.421
2d, 3d, and 4th terms	+ 0.4916				+0.4911		+0.0002		+0.0003
$-\Delta\phi$	−95.9139								

	s	4.264911		
	sin α	9.9942754		
	A′	8.5093503	$\Delta\lambda$	2.832653
	sec ϕ'	0.064116	sin $\frac{1}{2}(\phi+\phi')$	9.703663
		2.8326527		2.536316
		″		″
	$\Delta\lambda$	+680.225	$-\Delta\alpha$	+343.81

STATION GRAND

		°	′	″
α	Pins to west base	337	56	40.7
Second angle	West base and Grand	+ 85	13	04.4
α	Pins to Grand	63	09	45.1
$\Delta\alpha$		−	3	32.3
		180	00	00.00
α'	Grand to Pins	243	06	12.8
	First angle of triangle	54	52	03.0

		°	′	″				°	′	″
ϕ		30	22	24.231	Pins	λ		88	18	36.252
$\Delta\phi$		−	3	04.656		$\Delta\lambda$		+	7	00.241
ϕ'		30	19	19.575	Grand	λ'		88	25	36.493
$\frac{1}{2}(\phi+\phi')$	° ′ ″ 30 20 52	s	4.099752	s^2	8.1995	h^2	4.532	−h	2.266
		cos α	9.654620	sin² α	9.9010			s^2 sin² α	8.100
		B	8.511548	C	1.1734	D	2.333	E	5.919
	″								
1st term	+184.4676	h	2.265920		9.2739		6.865		6.285
2d, 3d, and 4th terms	+ 0.1884				+0.1879		+0.0007		−0.0002
$-\Delta\phi$	+184.6560								

	s	4.099752		
	sin α	9.9505064		
	A′	8.509352	$\Delta\lambda$	2.623498
	sec ϕ'	0.063888	sin $\frac{1}{2}(\phi+\phi')$	9.703504
		2.6234984		2.327002
		″		″
	$\Delta\lambda$	+420.2409	$-\Delta\alpha$	+212.32

secondary triangulation—Continued

STATION PINS

									°	′	″
α	West base to Cedar								225	33	29.7
Third angle	Pins and Cedar								− 67	34	55.3
α	West base to Pins								157	58	34.4
$\Delta\alpha$									−	1	53.7
									180	00	00.00
α'	Pins to west base								337	56	40.7
	°	′	″								
ϕ	30	14	21.492	Dauphin Island west base			λ		88	14	51.034
$\Delta\phi$	+	8	02.739				$\Delta\lambda$		+	3	45.218
ϕ'	30	22	24.231	Pins			λ'		88	18	36.252
	° ′ ″	s	4.205115	s^2	8.4102					−h	2.684
$\frac{1}{2}(\phi+\phi')$	30 18 23	cos α	9.967093	sin² α	8.1480		h²		5.368	$s^2\sin^2\alpha$	7.558
	″	B	8.511557	C	1.1711		D		2.331	E	5.917
1st term	−482.7974	h	2.683765		8.7293				7.699		6.159
2d, 3d, and 4th terms	+ 0.0587				+0.0536				+0.0050		+0.0001
$-\Delta\phi$	−482.7387										
	s	4.205115									
	sin α	9.5740212									
	A′	8.5093503		$\Delta\lambda$	2.352602						
	sec ϕ'	0.064116		sin $\frac{1}{2}(\phi+\phi')$	9.702967						
		2.3526025			2.055569						
		″			″						
	$\Delta\lambda$	+225.2177		$-\Delta\alpha$	+113.65						

STATION GRAND

									°	′	″
α	West base to Pins								157	58	34.4
Third angle	Grand and Pins								− 39	54	53.1
α	West base to Grand								118	03	41.3
$\Delta\alpha$									−	5	25.5
									180	00	00.00
α'	Grand to west base								297	58	15.8
	°	′	″								
ϕ	30	14	21.492	Dauphin Island west base			λ		88	14	51.034
$\Delta\phi$	+	4	58.083				$\Delta\lambda$		+	10	45.459
ϕ'	30	19	19.575	Grand			λ'		88	25	36.493
	° ′ ″	s	4.290942	s^2	8.5818					−h	2.475
$\frac{1}{2}(\phi+\phi')$	30 16 50	cos α	9.672484	sin² α	9.8914		h²		4.950	$s^2\sin^2\alpha$	8.473
	″	B	8.511557	C	1.1711		D		2.331	E	5.917
1st term	−298.5266	h	2.474983		9.6443				7.281		6.865
2d, 3d, and 4th terms	+ 0.4435				+0.4409				+0.0019		+0.0007
$-\Delta\phi$	−298.0831										
	s	4.290942									
	sin α	9.945687									
	A′	8.509352		$\Delta\lambda$	2.809869						
	sec ϕ'	0.063888		sin $\frac{1}{2}(\phi+\phi')$	9.702633						
		2.809869			2.512502						
		″			″						
	$\Delta\lambda$	+645.4595		$-\Delta\alpha$	+325.46						

Final position computation,

STATION PETIT

α Second angle	Grand to west base West base and Petit							° 297 + 59	′ 58 11	″ 15.8 52.0
α $\Delta\alpha$	Grand to Petit							357 +	10	07.8 11.6
α'	Petit to Grand				First angle of triangle			180 177 81	00 10 41	00.00 19.4 30.1
ϕ $\Delta\phi$	° 30 −	′ 19 6	″ 19.575 44.068		Grand	λ $\Delta\lambda$		88 −	25	36.493 23.006
ϕ'	30	12	35.507		Petit	λ'		88	25	13.487

$\frac{1}{2}(\phi+\phi')$	° ′ ″ 30 15 58 ″	s cos α B	4.095429 9.999470 8.511551	s^2 sin² α C	8.1909 7.3873 1.1725	h² D	5.213 2.332	
1st term 2d and 3d terms	+404.0639 + 0.0041	h	2.606450		6.7507 +0.0006		7.545 +0.0035	
$-\Delta\phi$	+404.0680							

	s sin α A′ sec ϕ'	4.095429 8.693666 8.509354 0.063392	$\Delta\lambda$ sin $\frac{1}{2}(\phi+\phi')$	1.361841 9.702445
		1.361841 ″		1.064286 ″
	$\Delta\lambda$	−23.006	$-\Delta\alpha$	−11.59

STATION HORN

α Second angle	Grand to Petit Petit and Horn							° 357 + 61	′ 10 04	″ 07.8 37.0
α $\Delta\alpha$	Grand to Horn							58 −	14 5	44.8 18.1
α'	Horn to Grand				First angle of triangle			180 238 38	00 09 16	00.00 26.7 35.3
ϕ $\Delta\phi$	° 30 −	′ 19 5	″ 19.575 39.559		Grand	λ $\Delta\lambda$		88 +	25 10	36.493 30.972
ϕ'	30	13	40.016		Horn	λ'		88	36	07.465

$\frac{1}{2}(\phi+\phi')$	° ′ ″ 30 16 30 ″	s cos α B	4.297606 9.721214 8.5115514	s^2 sin² α C	8.5953 9.8592 1.1725	h² D	5.061 2.332	−h s^2 sin² α E	2.530 8.454 5.918
1st term 2d, 3d, and 4th terms	+339.1338 + 0.4253	h	2.5303714		9.6270 +0.4236		7.393 +0.0025		6.902 −0.0008
$-\Delta\phi$	+339.5591								

	s sin α A′ sec ϕ'	4.297606 9.929579 8.509354 0.063471	$\Delta\lambda$ sin $\frac{1}{2}(\phi+\phi')$	2.800010 9.702560
		2.800010 ″		2.502570 ″
	$\Delta\lambda$	+630.9719	$-\Delta\alpha$	+318.10

secondary triangulation—Continued

STATION PETIT

								°	′	″
α	West base to Grand							118	03	41.3
Third angle	Petit and Grand							− 39	06	38.4
α	West base to Petit							78	57	02.9
$\Delta\alpha$								−	5	13.3
								180	00	00.00
α'	Petit to west base							258	51	49.6
										−.1
	°	′	″							
ϕ	30	14	21.492	Dauphin Island west base		λ		88	14	51.034
$\Delta\phi$	−	1	45.985			$\Delta\lambda$		+	10	22.452
ϕ'	30	12	35.507	Petit		λ'		88	25	13.486
										+1
	° ′ ″	s	4.229486	s^2	8.4590				−h	2.024
$\frac{1}{2}(\phi+\phi')$	30 13 28	cos α	9.282513	sin² α	9.9837	h²	4.047	s^2 sin² α		8.443
	″	B	8.511557	C	1.1711	D	2.331	E		5.917
1st term	+105.5737	h	2.023556		9.6138		6.378			6.384
2d, 3d, and 4th terms	+ 0.4109				+0.4109		+0.0002			−0.0002
$-\Delta\phi$	+105.9846									

s	4.229486		
sin α	9.991874		
A′	8.509354	$\Delta\lambda$	2.794106
sec ϕ'	0.063392	sin $\frac{1}{2}(\phi+\phi')$	9.701905
	2.794106		2.496011
	″		″
$\Delta\lambda$	+622.4522	$-\Delta\alpha$	+313.34

STATION HORN

								°	′	″
α	Petit to Grand							177	10	19.4
Third angle	Horn and Grand							− 80	38	48.3
α	Petit to Horn							96	31	31.1
$\Delta\alpha$								−	5	29.1
								180	00	00.00
α'	Horn to Petit							276	26	02.0
	°	′	″							
ϕ	30	12	35.507	Petit		λ		88	25	13.487
$\Delta\phi$	+	1	04.509			$\Delta\lambda$		+	10	53.978
ϕ'	30	13	40.016	Horn		λ'		88	36	07.465
	° ′ ″	s	4.245560	s^2	8.4912				−h	1.813
$\frac{1}{2}(\phi+\phi')$	30 13 08	cos α	9.055539	sin² α	9.9943	h²	3.625	s^2 sin² α		8.485
	″	B	8.511559	C	1.1706	D	2.331	E		5.916
1st term	−64.9618	h	1.812658		9.6561		5.956			6.214
2d, 3d, and 4th terms	+ 0.4533				+0.4530		+0.0001			+0.0002
$-\Delta\phi$	−64.5085									

	+4		
s	4.245560		
sin α	9.9971774		
A′	8.5093541	$\Delta\lambda$	2.815563
sec ϕ'	0.063471	sin $\frac{1}{2}(\phi+\phi')$	9.701830
	2.815563		2.517393
	″		″
$\Delta\lambda$	+653.9778	$-\Delta\alpha$	+329.15

Final position computation,

STATION PASCAGOULA

								°	′	″
α	Grand to Horn							58	14	44.8
Second angle	Horn and Pascagoula							+ 43	14	20.8
α	Grand to Pascagoula							101	29	05.6
$\Delta\alpha$								−	3	52.8
								180	00	00.00
α'	Pascagoula to Grand							281	25	12.8
						First angle of triangle		97	55	59.1
	°	′	″							
ϕ	30	19	19.575	Grand		λ		88	25	36.493
$\Delta\phi$	+	1	20.998			$\Delta\lambda$		+	7	40.886
ϕ'	30	20	40.573	Pascagoula		λ'		88	33	17.379
	° ′ ″	s	4.099038	s^2	8.1981				−h	1.910
$\frac{1}{2}(\phi+\phi')$	30 20 00	$\cos\alpha$	9.299092	$\sin^2\alpha$	9.9824	h^2		3.820	$s^2\sin^2\alpha$	8.180
		B	8.511551	C	1.1725	D		2.332	E	5.918
	″									
1st term	−81.2232	h	1.909681		9.3530			6.152		6.008
2d, 3d, and 4th terms	+ 0.2256				+0.2254			+0.0001		+0.0001
$-\Delta\phi$	−80.9976									

s	4.099038		
$\sin\alpha$	9.991216		
A′	8.509351	$\Delta\lambda$	2.663593
$\sec\phi'$	0.063988	$\sin\frac{1}{2}(\phi+\phi')$	9.703317
	2.663593		2.366910
	″		″
$\Delta\lambda$	+460.8859	$-\Delta\alpha$	+232.76

STATION BELLE

								°	′	″
α	Pascagoula to Horn							19	21	11.9
Second angle	Horn and Belle							+ 69	43	26.3
α	Pascagoula to Belle							89	04	38.2
$\Delta\alpha$								−	5	01.7
								180	00	00.00
α'	Belle to Pascagoula							268	59	36.5
						First angle of triangle		48	58	48.4
	°	′	″							
ϕ	30	20	40.573	Pascagoula		λ		88	33	17.379
$\Delta\phi$	−		08.724			$\Delta\lambda$		+	9	57.281
ϕ'	30	20	31.849	Belle		λ'		88	43	14.660
										+2
	° ′ ″	s	4.202907	s^2	8.4059					
$\frac{1}{2}(\phi+\phi')$	30 20 36	$\cos\alpha$	8.206975	$\sin^2\alpha$	9.9999	h^2		1.844		
		B	8.511550	C	1.1729	D		2.332		
	″									
1st term	+8.3451	h	0.9214$s2$		9.5787			4.176		
2d and 3d terms	+0.3790				+0.3790					
$-\Delta\phi$	+8.7241									

s	4.202907		
$\sin\alpha$	9.999944		
A′	8.509351	$\Delta\lambda$	2.776179
$\sec\phi'$	0.063977	$\sin\frac{1}{2}(\phi+\phi')$	9.703447
	2.776179		2.479626
	″		″
$\Delta\lambda$	+597.2814	$-\Delta\alpha$	+301.73

secondary triangulation—Continued

STATION PASCAGOULA

										°	′	″
α	Horn to Grand									238	09	26.7
Third angle	Pascagoula and Grand									− 38	49	40.5
α	Horn to Pascagoula									199	19	46.2
$\Delta\alpha$										+	1	25.8
										180	00	00.00
α'	Pascagoula to Horn									19	21	12.0
												−.1
	°	′	″									
ϕ	30	13	40.016	Horn			λ			88	36	07.465
$\Delta\phi$	+	7	00.557				$\Delta\lambda$			−	2	50.086
ϕ'	30	20	40.573	Pascagoula			λ'			88	33	17.379
	° ′ ″	s	4.137501	s^2	8.2750				−h			2.624
$\frac{1}{2}(\phi+\phi')$	30 17 10	cos α	9.974802	sin² α	9.0396		h²	5.248	s^2 sin² α			7.315
	″	B	8.511558	C	1.1709		D	2.331	E			5.917
1st term	−420.5920	h	2.623861		8.4855			7.579				5.856
2d, 3d, and 4th terms	+ 0.0345				+0.0306			+0.0038				+0.0001
$-\Delta\phi$	−420.5575											
	s	4.137501										
	sin α	9.519828										
	A′	8.509351	$\Delta\lambda$	2.230667								
	sec ϕ'	0.063988	sin $\frac{1}{2}(\phi+\phi')$	9.702706								
		2.230668		1.933373								
		″		″								
	$\Delta\lambda$	−170.0858	$-\Delta\alpha$	−85.78								

STATION BELLE

										°	′	″
α	Horn to Pascagoula									199	19	46.2
Third angle	Belle and Pascagoula									− 61	17	45.8
α	Horn to Belle									138	02	00.4
$\Delta\alpha$										−	3	35.4
										180	00	00.00
α'	Belle to Horn									317	58	25.0
												−.1
	°	′	″									
ϕ	30	13	40.016	Horn			λ			88	36	07.465
$\Delta\phi$	+	6	51.834				$\Delta\lambda$			+	7	07.197
ϕ'	30	20	31.850	Belle			λ'			88	43	14.662
			−1									
	° ′ ″	s	4.232071	s^2	8.4642				−h			2.615
$\frac{1}{2}(\phi+\phi')$	30 17 06	cos α	9.871302	sin² α	9.6505		h²	5.230	s^2 sin² α			8.115
	″	B	8.511558	C	1.1709		D	2.331	E			5.917
1st term	−412.0321	h	2.614931		9.2856			7.561				6.647
2d, 3d, and 4th terms	+ 0.1976				+0.1936			+0.0036				+0.0004
$-\Delta\phi$	−411.8345											
	s	4.232071										
	sin α	9.825229										
	A′	8.509351	$\Delta\lambda$	2.630628								
	sec ϕ'	0.063977	sin $\frac{1}{2}(\phi+\phi')$	9.702690								
		2.630628		2.333318								
		″		″								
	$\Delta\lambda$	+427.1968	$-\Delta\alpha$	+215.44								

Final position computation,

STATION CLUB

									°	′	″
α	Belle to Horn								317	58	24.9
Second angle	Horn and Club								+40	30	06.4
α	Belle to Club								358	28	31.3
$\Delta\alpha$									+		05.0
									180	00	00.00
α'	Club to Belle								178	28	36.3
						First angle of triangle			105	43	54.1
	°	′	″								
ϕ	30	20	31.849		Belle		λ		88	43	14.662
$\Delta\phi$	−	5	19.873				$\Delta\lambda$		−		09.806
ϕ'	30	15	11.976		Club		λ'		88	43	04.856

	° ′ ″	s	3.993578	s^2	7.9872			
$\frac{1}{2}(\phi+\phi')$	30 17 52	cos α	9.999846	sin² α	6.8500	h^2	5.010	
		B	8.511550	C	1.1729	D	2.332	
	″							
1st term	+319.8704	h	2.504974		6.0101		7.342	
2d and 3d terms	+ 0.0023				+0.0001		+0.0022	
$-\Delta\phi$	+319.8727							

s	3.993578		
sin α	8.424985		
A′	8.509353	$\Delta\lambda$	0.991500
sec ϕ'	0.063584	sin $\frac{1}{2}(\phi+\phi')$	9.702856
	0.991500		0.694356
	″		″
$\Delta\lambda$	−9.8062	$-\Delta\alpha$	−4.95

STATION DEER

									°	′	″
α	Belle to Club								358	28	31.3
Second angle	Club and Deer								+102	35	23.3
α	Belle to Deer								101	03	54.6
$\Delta\alpha$									−	2	45.3
									180	00	00.00
α'	Deer to Belle								281	01	09.3
						First angle of triangle			41	02	09.8
	°	′	″								
ϕ	30	20	31.849		Belle		λ		88	43	14.662
$\Delta\phi$	+		55.361				$\Delta\lambda$		+	5	27.087
ϕ'	30	21	27.210		Deer		λ'		88	48	41.749

	° ′ ″	s	3.949417	s^2	7.8989			
$\frac{1}{2}(\phi+\phi')$	30 21 00	cos α	9.283133	sin² α	9.9837	h^2	3.488	
		B	8.511550	C	1.1729	D	2.332	
	″							
1st term	−55.4752	h	1.744100		9.0555		5.820	
2d and 3d terms	+ 0.1137				+0.1136		+0.0001	
$-\Delta\phi$	−55.3615							

s	3.949417		
sin α	9.991850		
A′	8.509351	$\Delta\lambda$	2.514663
sec ϕ'	0.064045	sin $\frac{1}{2}(\phi+\phi')$	9.703531
	2.514663		2.218194
	″		″
$\Delta\lambda$	+327.0872	$-\Delta\alpha$	+165.27

secondary triangulation—Continued

STATION CLUB

								°	′	″
α	Horn to Belle							138	02	00.4
Third angle	Club and Belle							−33	45	59.8
α	Horn to Club							104	16	00.6
$\Delta\alpha$								−	3	30.2
								180	00	00.00
α'	Club to Horn							284	12	30.4
	°	′	″							
ϕ	30	13	40.016	Horn		λ	88	36	07.465
$\Delta\phi$	+	1	31.961			$\Delta\lambda$	+	6	57.391
ϕ'	30	15	11.977	Club		λ'	88	43	04.856
			−1							
	° ′ ″	s	4.061211	s^2	8.1225			−h	1.964
$\frac{1}{2}(\phi+\phi')$	30 14 26	cos α	9.391708	$\sin^2\alpha$	9.9728	h^2	3.929	$s^2\sin^2\alpha$	8.095
	″	B	8.511558	C	1.1709	D	2.331	E	5.917
1st term	−92.1459	h	1.964477		9.2662		6.260		5.976
2d, 3d, and 4th terms	+ 0.1848				+0.1845		+0.0002		+0.0001
$-\Delta\phi$	−91.9611								
	s	4.061211							
	sin α	9.986395							
	A′	8.509353	$\Delta\lambda$	2.620543					
	sec ϕ'	0.063584	sin $\frac{1}{2}(\phi+\phi')$	9.702113					
		2.620543		2.322656					
		″		″					
	$\Delta\lambda$	+417.3909	$-\Delta\alpha$	+210.21					

STATION DEER

								°	′	″
α	Club to Belle							178	28	36.3
Third angle	Deer and Belle							−36	22	27.1
α	Club to Deer							142	06	09.2
$\Delta\alpha$								−	2	50.0
								180	00	00.00
α'	Deer to Club							322	03	19.2
										−.1
	°	′	″							
ϕ	30	15	11.976	Club		λ	88	43	04.856
$\Delta\phi$	+	6	15.234			$\Delta\lambda$	+	5	36.893
ϕ'	30	21	27.210	Deer		λ'	88	48	41.749
	° ′ ″	s	4.165751	s^2	8.3315			−h	2.574
$\frac{1}{2}(\phi+\phi')$	30 18 20	cos α	9.897138	$\sin^2\alpha$	9.5767	h^2	5.149	$s^2\sin^2\alpha$	7.908
	″	B	8.511556	C	1.1713	D	2.331	E	5.918
1st term	−375.3574	h	2.574445		9.0795		7.480		6.400
2d, 3d, and 4th terms	+ 0.1233				+0.1200		+0.0030		+0.0003
$-\Delta\phi$	−375.2341								
	s	4.165751							
	sin α	9.788345							
	A′	8.509351	$\Delta\lambda$	2.527492					
	sec ϕ'	0.064045	sin $\frac{1}{2}(\phi+\phi')$	9.702957					
		2.527492		2.230449					
		″		″					
	$\Delta\lambda$	+336.8930	$-\Delta\alpha$	+170.00					

Final position computation,

STATION SHIP

										°	′	″
α	Deer to Club									322	03	19.1
Second angle	Club and Ship									+ 56	47	25.3
α	Deer to Ship									18	50	44.4
$\Delta\alpha$										−	1	14.8
α'	Ship to Deer									180	00	00.00
										198	49	29.6
									First angle of triangle	70	52	30.8
	°	′	″									
ϕ	30	21	27.210			Deer		λ		88	48	41.749
$\Delta\phi$	−	6	17.189					$\Delta\lambda$		+	2	28.277
ϕ'	30	15	10.021			Ship		λ'		88	51	10.026
	° ′ ″	s	4.088909	s^2	8.1778							
$\frac{1}{2}(\phi+\phi')$	30 18 19	$\cos\alpha$	9.976071	$\sin^2\alpha$	9.0184		h^2	5.153				
		B	8.511549	C	1.1731		D	2.332				
	″	h	2.576529		8.3693			7.485				
1st term	+377.1629											
2d and 3d terms	+ 0.0265				+0.0234			+0.0031				
$-\Delta\phi$	+377.1894											
	s	4.088909										
	$\sin\alpha$	9.509230										
	A′	8.500353		$\Delta\lambda$	2.171073							
	$\sec\phi'$	0.063581		$\sin\frac{1}{2}(\phi+\phi')$	9.702952							
		2.171073			1.874025							
		″			″							
	$\Delta\lambda$	+148.2767		$-\Delta\alpha$	+74.82							

STATION BILOXI LIGHTHOUSE

										°	′	″
α	Deer to Ship									18	50	44.4
Second angle	Ship and Biloxi Lighthouse									+ 96	30	37.5
α	Deer to Biloxi Lighthouse									115	21	21.9
$\Delta\alpha$										−	2	42.9
α'	Biloxi Lighthouse to Deer									180	00	00.00
										295	18	39.0
									First angle of triangle	48	11	18.7
	°	′	″									
ϕ	30	21	27.210			Deer		λ		88	48	41.749
$\Delta\phi$	+	2	12.209					$\Delta\lambda$		+	5	22.071
ϕ'	30	23	39.419			Biloxi Lighthouse		λ'		88	54	03.820
	° ′ ″	s	3.978386	s^2	7.9568				−h			2.121
$\frac{1}{2}(\phi+\phi')$	30 22 33	$\cos\alpha$	9.631690	$\sin^2\alpha$	9.9120		h^2	4.243	$s^2\sin^2\alpha$			7.869
		B	8.511549	C	1.1731		D	2.332	E			5.919
	″	h	2.121625		9.0419			6.575				5.909
1st term	−132.3198											
2d, 3d, and 4th terms	+ 0.1107				+0.1102			+0.0004				+0.0001
$-\Delta\phi$	−132.2091											
	s	3.978386										
	$\sin\alpha$	9.956007										
	A′	8.509350		$\Delta\lambda$	2.507952							
	$\sec\phi'$	0.064209		$\sin\frac{1}{2}(\phi+\phi')$	9.703868							
		2.507952			2.211820							
		″			″							
	$\Delta\lambda$	+322.0713		$-\Delta\alpha$	+162.86							

secondary triangulation—Continued

STATION SHIP

		°	′	″
α	Club to Deer	142	06	09.2
Third angle	Ship and Deer	− 52	20	04.3
α	Club to Ship	89	46	04.9
$\Delta\alpha$		−	4	04.4
α'	Ship to Club	180	00	00.00
		269	42	00.5
				−.1

	°	′	″					
ϕ	30	15	11.976	Club	λ	88	43	04.856
$\Delta\phi$	−		01.955		$\Delta\lambda$	+	8	05.170
ϕ'	30	15	10.021	Ship	λ'	88	51	10.026

	° ′ ″	s	4.112963	s^2	8.2260
$\frac{1}{2}(\phi+\phi')$	30 15 11	cos α	7.607314	sin² α	0.0000
	″	B	8.511556	C	1.1713
1st term	+1.7054	h	0.231833		9.3973
2d term	+0.2496				+0.2496
$-\Delta\phi$	+1.9550				

s	4.112963		
sin α	9.9999964		
A′	8.5093531	$\Delta\lambda$	2.685894
sec ϕ'	0.0635812	sin $\frac{1}{2}(\phi+\phi')$	9.702275
	2.6858937		2.388169
	″		″
$\Delta\lambda$	+485.1697	$-\Delta\alpha$	+244.44

STATION BILOXI LIGHTHOUSE

		°	′	″
α	Ship to Deer	198	49	29.6
Third angle	Biloxi Lighthouse and Deer	− 35	18	04.1
α	Ship to Biloxi Lighthouse	163	31	25.5
$\Delta\alpha$		−	1	27.7
α'	Biloxi Lighthouse to Ship	180	00	00.00
		343	29	57.8
				−.1

	°	′	″					
ϕ	30	15	10.021	Ship	λ	88	51	10.026
$\Delta\phi$	+	8	29.398		$\Delta\lambda$	+	2	53.794
ϕ'	30	23	39.419	Biloxi Lighthouse	λ'	88	54	03.820
	Fixed value		39.419					03.820

	° ′ ″	s	4.213743	s^2	8.4275				−h	2.707
$\frac{1}{2}(\phi+\phi')$	30 19 25	cos α	9.981790	sin² α	8.9056	h²	5.414	s^2 sin² α		7.333
	″	B	8.511556	C	1.1713	D	2.331		E	5.918
1st term	−509.4353	h	2.707089		8.5044		7.745			5.958
2d, 3d, and 4th terms	+ 0.0376				+0.0319		+0.0056			+0.0001
$-\Delta\phi$	−509.3977									

s	4.213743		
sin α	9.452733		
A′	8.509350	$\Delta\lambda$	2.240035
sec ϕ'	0.064209	sin $\frac{1}{2}(\phi+\phi')$	9.703190
	2.240035		1.943225
	″		″
$\Delta\lambda$	+173.7941	$-\Delta\alpha$	+87.74

Final position computation,

STATION SHIP ISLAND LIGHTHOUSE

		°	′	″
α	Biloxi Lighthouse to Ship	343	29	57.7
Second angle	Ship and Ship Island Lighthouse	+ 33	44	19.9
α	Biloxi Lighthouse to Ship Island Lighthouse	17	14	17.6
$\Delta\alpha$		−	1	57.9
α'	Ship Island Lighthouse to Biloxi Lighthouse	180	00	00.00
		197	12	19.7
	First angle of triangle	50	31	31.4

	°	′	″				°	′	″
ϕ	30	23	39.419	Biloxi Lighthouse	λ		88	54	03.820
$\Delta\phi$	−	10	54.078		$\Delta\lambda$		+	3	53.644
ϕ'	30	12	45.341	Ship Island Lighthouse	λ'		88	57	57.464

	° ′ ″	s	4.323999	s^2	8.6480				−h	2.816
$\frac{1}{2}(\phi+\phi')$	30 18 12	cos α	9.980040	sin² α	8.9430	h^2	5.631	s^2 sin² α		7.592
		B	8.511546	C	1.1738	D	2.332	E		5.919
	″									
1st term	+654.0109	h	2.815585		8.7654		7.963			6.327
2d, 3d, and 4th terms	+ 0.0673				+0.0583		+0.0092			−0.0002
$-\Delta\phi$	+654.0782									

	s	4.323999		
	sin α	9.471798		
	A′	8.509354	$\Delta\lambda$	2.368555
	sec ϕ'	0.063404	sin $\frac{1}{2}(\phi+\phi')$	9.702930
		2.368555		2.071485
		″		″
	$\Delta\lambda$	+233.6442	$-\Delta\alpha$	+117.89

secondary triangulation—Continued

STATION SHIP ISLAND LIGHTHOUSE

		°	′	″
α	Ship to Biloxi Lighthouse	163	31	25.5
Third angle	Ship Island Lighthouse and Biloxi Lighthouse	− 95	44	09.2
α	Ship to Ship Island Lighthouse	67	47	16.3
$\Delta\alpha$		−	3	25.2
α'	Ship Island Lighthouse to Ship	180 247	00 43	00.00 51.1

	°	′	″			°	′	″
ϕ	30	15	10.021	Ship	λ	88	51	10.026
$\Delta\phi$	−	2	24.681		$\Delta\lambda$	+	6	47.438
ϕ'	30	12	45.340 +1	Ship Island Lighthouse	λ'	88	57	57.464
	Fixed value		45.341					57.464

	° ′ ″	s	4.070791	s^2	8.1416				−h	2.160
$\frac{1}{2}(\phi+\phi')$	30 13 58	cos α	9.577534	sin² α	9.9330	h^2	4.320		$s^2\sin^2\alpha$	8.075
	″	B	8.511556	C	1.1713	D	2.331		E	5.918
1st term	+144.5043	h	2.159881		9.2459		6.651			6.153
2d, 3d, and 4th terms	+ 0.1765				+0.1762		+0.0004			−0.0001
$-\Delta\phi$	+144.6808									

s	4.070791		
sin α	9.966513		
A′	8.509354	$\Delta\lambda$	2.610062
sec ϕ'	0.003404	sin $\frac{1}{2}(\phi+\phi')$	9.702011
	2.610062		2.312073
	″		″
$\Delta\lambda$	+407.4385	$-\Delta\alpha$	+205.15

ADJUSTMENTS BY THE ANGLE METHOD

If the adjustment be made according to the angle method* the complications due to the presence of the z's are avoided. An angle is the difference of two directions and the observation equation for an observed angle is the difference of the observation equations of its two sides, and in taking the difference the z drops out. To illustrate this suppose that at station Gunner, Figure 6, page 104, the following angles were observed: Duck to Indian Point, Indian Point to Larrabee, Larrabee to Mam, Mam to Lubec Channel Lighthouse, and Lubec Channel Lighthouse to Lubec Church spire. Call the corrections to the observed angles u_1, u_2, u_3, u_4, and u_5 respectively, and suppose the observed and assumed values to be as given on page 115. Then

$$u_1 = v_2 - v_1 = -6131\delta\phi_1 + 1800\delta\lambda_1 - 2.2$$

$$u_2 = v_3 - v_2 = -1997\delta\phi_1 + 236\delta\lambda_1 + 7.0$$

In a similar way,

$$u_3 = -2732\delta\phi_1 + \quad 85\delta\lambda_1 - 1.2$$

$$u_4 = -5565\delta\phi_1 - 2445\delta\lambda_1 + 3.5$$

$$u_5 = +5886\delta\phi_1 + 5413\delta\lambda_1 + 18.4$$

These contain no z's and the normal equations may be formed in the usual way.

Observation equations of this kind would arise when at an unknown point angles are taken on known points, as for example when angles are taken with a sextant from a point off-shore to determine its position, and for such observations the angle method is both easier and more logical than the direction method.

* Wright and Hayford, Adjustment of observations, p. 180.

ADJUSTMENT OF VERTICAL OBSERVATIONS

GENERAL STATEMENT

When reciprocal vertical observations are made over the lines of a triangulation scheme a computation of the differences of elevations is made by the usual Coast and Geodetic Survey formula. For an account of these observations and of the method of computation, see United States Coast and Geodetic Survey Special Publication No. 19, page 140 et seq. As there are always several lines from each station, rigid conditions are present in the figure. Thus it becomes necessary to make an adjustment of the observed values by the method of least squares. In the following figure the differences of elevations as observed are first computed and then the results are adjusted by the method employed in the United States Coast and Geodetic Survey.

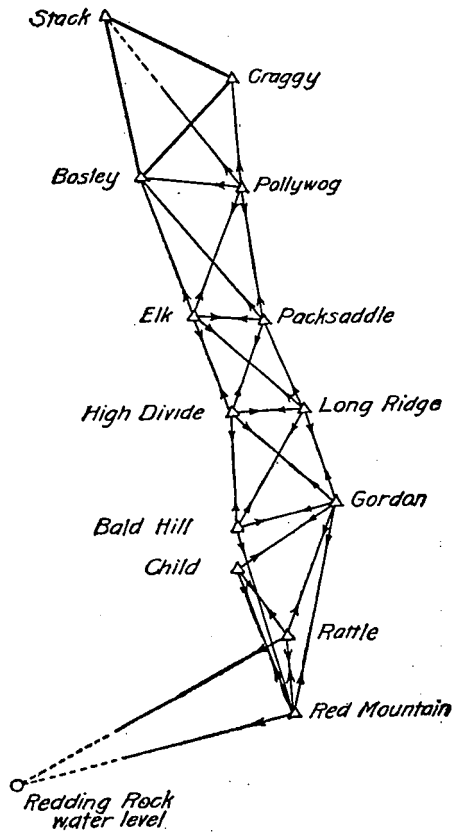

FIG. 8.

The formula used in the following computations is the one given in Special Publication No. 19, mentioned above. On pages 205 et seq. there is given a new development of the formula that takes into account some of the small terms that are needed in computation over longer and higher lines. The final form of the new formula differs slightly from the one used in this computation.

Computation of elevations from reciprocal observations

Station 1	Pollywog	Pollywog	Pack Saddle	Pack Saddle	Elk	Elk
Station 2	Pack Saddle	Elk	High Divide	Elk	Long Ridge	High Divide
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
ζ_1	90 04 13	90 57 44	90 30 00	91 53 50	88 33 16	89 15 25
ζ_2	90 04 59	89 11 15	89 37 33	88 10 59	91 36 54	90 51 04
$\zeta_2-\zeta_1$	+ 46	− 1 46 29	− 52 27	− 3 42 51	+ 3 03 38	+1 35 39
$\frac{1}{2}(\zeta_2-\zeta_1)$	+ 23	− 53 14	− 26 14	− 1 51 26	+ 1 31 49	+ 47 50
$\tan \frac{1}{2}(\zeta_2-\zeta_1)$	6.04732	8.18994	7.88258	8.51090	8.42675	8.14348
$\log s$	4.27444	4.29253	4.15543	3.98144	4.31524	4.17150
$\log s \tan \frac{1}{2}(\zeta_2-\zeta_1)$	0.32176	2.48247	2.03801	2.49234	3.74199	2.31498
$s \tan \frac{1}{2}(\zeta_2-\zeta_1)$	+2.10	−303.72	−109.15	−310.70	+552.06	+206.53
Second term	0.00	0.00	0.00	0.00	0.00	0.00
Third term	0.00	0.03	0.01	0.03	0.06	0.02
h_2-h_1	+2.10	−303.75	−109.16	−310.73	+552.12	+206.55
$2 \log s$	8.549	8.585	8.311	7.963	8.630	8.343
$\log p = 9-2 \log s$	0.451	0.415	0.689	1.037	0.370	0.657
p of h_2-h_1	2.82	2.60	4.89	10.89	2.34	4.54

Station 1	High Divide	High Divide	High Divide	Long Ridge	Long Ridge	Gordon
Station 2	Bald Hill	Gordon	Long Ridge	Gordon	Bald Hill	Bald Hill
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
ζ_1	90 29 43	88 28 23	88 12 34	89 14 01	91 27 22	92 45 09
ζ_2	89 37 50	91 40 22	91 53 03	90 52 54	88 41 46	87 21 10
$\zeta_2-\zeta_1$	− 51 53	+ 3 11 59	+ 3 40 31	+ 1 38 53	− 2 45 36	−5 23 59
$\frac{1}{2}(\zeta_2-\zeta_1)$	− 25 56	+ 1 36 00	+ 1 50 16	+ 49 26	− 1 22 48	−2 42 00
$\tan \frac{1}{2}(\zeta_2-\zeta_1)$	7.87759	8.44611	8.50632	8.15777	8.38184	8.67357
$\log s$	4.21546	4.29195	4.03443	4.14252	4.29013	4.15348
$\log s \tan \frac{1}{2}(\zeta_2-\zeta_1)$	2.09305	2.73806	2.54075	2.30029	2.67197	2.82705
$s \tan \frac{1}{2}(\zeta_2-\zeta_1)$	−123.89	+547.09	+347.34	+199.66	−469.86	−671.51
Second term	0.00	0.00	0.00	0.00	0.00	0.00
Third term	0.01	0.08	0.05	0.03	0.06	0.07
h_2-h_1	−123.90	+547.17	+347.39	+199.69	−469.92	−671.58
$2 \log s$	8.431	8.584	8.069	8.285	8.580	8.307
$\log p = 9-2 \log s$	0.569	0.416	0.931	0.715	0.420	0.693
p of h_2-h_1	3.71	2.61	8.53	5.19	2.63	4.93

Station 1	Gordon	Gordon	Gordon	Child	Child	Rattle
Station 2	Red Mountain	Rattle	Child	Rattle	Red Mountain	Red Mountain
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
ζ_1	90 03 28	90 31 05	91 56 00	87 56 00	88 33 47	89 07 26
ζ_2	90 10 44	89 38 48	88 11 27	92 09 03	91 36 13	90 57 56
$\zeta_2-\zeta_1$	+ 07 16	− 52 17	− 3 44 33	+ 4 13 03	+ 3 02 26	+ 1 50 30
$\frac{1}{2}(\zeta_2-\zeta_1)$	+ 03 38	− 26 08.5	− 1 52 16.5	+ 2 06 31.5	+ 1 31 13	+ 55 15
$\tan \frac{1}{2}(\zeta_2-\zeta_1)$	7.02404	7.88106	8.51416	8.56610	8.42390	8.20610
$\log s$	4.48786	4.31274	4.23223	4.03839	4.34662	4.06606
$\log s \tan \frac{1}{2}(\zeta_2-\zeta_1)$	1.51190	2.19380	2.74639	2.60449	2.77052	2.27216
$s \tan \frac{1}{2}(\zeta_2-\zeta_1)$	+32.50	−156.24	−557.69	+402.24	+589.55	+187.14
Second term	0.00	0.00	0.00	0.00	0.00	0.00
Third term	0.01	0.03	0.09	0.06	0.09	0.04
h_2-h_1	+32.51	−156.27	−557.78	+402.30	+589.64	+187.18
$2 \log s$	8.976	8.625	8.464	8.077	8.693	8.132
$\log p = 9-2 \log s$	0.024	0.375	0.536	0.923	0.307	0.868
p of h_2-h_1	1.06	2.37	3.43	8.38	2.03	7.38

Computation of elevations from nonreciprocal observations

Station Occ. 1.	Pollywog	Pollywog	Pollywog	Elk	Pack Saddle	Long Ridge
Station Obs. 2.	Bosley	Stack	Craggy	Bosley	Bosley	Pack Saddle
Obj. sighted	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
ζ	89 11 00	89 40 00	88 04 00	88 40 30	89 37 38	91 01 00
$90°-\zeta$	49 00	20 00	1 56 00	1 19 30	22 22	−1 01 00
$90°-\zeta$ in secs.	2940	1200	6960	4770	1342	3660
log ditto	3.46835	3.07918	3.84261	3.67852	3.12775	3.56348
T	4.68560	4.68558	4.68574	4.68565	4.68558	4.68562
log s	4.17163	4.49539	4.20117	4.33291	4.42745	4.15591
log s cot ζ	2.32558	2.26015	2.72952	2.69708	2.24078	2.40501
α and mean ϕ	95 42.2	141 42.3	175.9 42.3	157.0 42.1	137.8 42.1	151.8 42.0
log (0.5−m)	9.64167	9.64167	9.64167	9.63990	9.66260	9.66001
2 log s	8.34326	8.99078	8.40234	8.66582	8.85490	8.31182
log (0.5−m) s^2	7.98493	8.63245	8.04401	8.30572	8.51750	7.97183
log ρ	6.80535	6.80439	6.80376	6.80399	6.80446	6.80409
log (2d term)	1.17958	1.82806	1.24025	1.50173	1.71304	1.16774
s cot ζ	+211.63	+182.03	+536.44	+497.83	+174.09	−254.10
Second term	15.12	67.31	17.39	31.75	51.65	14.71
Third term	0.01	0.01	0.04	0.04	0.01	0.01
t−o	1.51	1.51	1.51	1.45	1.50	1.40
h_2-h_1	+228.27	+250.86	+555.38	+531.07	+227.25	−237.98
log p=9−2 log s	0.657	0.009	0.598	0.334	0.145	0.688
p	4.54	1.02	3.96	2.16	1.40	4.88

Station Occ. 1.	Bald Hill	Rattle	Red Mountain
Station Obs. 2.	Red Mountain	Redding Rock Water level	Redding Rock Water level
Obj. sighted	° ′ ″	° ′ ″	° ′ ″
ζ	88 41 22.5	91 51 30	92 32 45
$90°-\zeta$	+ 1 18 37.5	− 1 51 30	− 2 32 45
tan $90°-\zeta$	8.35936	8.51115	8.64799
log s	4.44942	4.56765	4.48393
log s cot ζ	2.80878	3.07880	3.13192
α and mean ϕ	21.4 41.6	31.3 41.5	48.0 41.4
log (0.5−m)	9.62941	9.64207	9.62747
2 log s	8.89884	9.13530	8.96786
log (0.5−m) s^2	8.52825	8.77737	8.59533
log ρ	6.80387	6.80410	6.80466
log (2d term)	1.72438	1.97327	1.79067
s cot ζ	+643.84	−1198.95	−1354.94
Second term	+ 53.01	+ 94.03	+ 61.75
Third term	0.06	0.21	0.27
t−o	1.36	1.45	1.43
h_2-h_1	+698.27	−1103.26	−1291.40
Cor. for reduction to mean sea level		+ 0.15	+ 0.82
h_2-h_1 (corrected)		−1103.11	−1290.67
log p=9−2 log s	0.101	9.865	0.032
p	1.26	0.73	1.07

The adjustment of vertical observations as practiced in the United States Coast and Geodetic Survey is made by means of observation equations and differs somewhat from the method of conditions. Of course condition equations could be employed if it were desired, just as triangulation can be adjusted by observation equations. (See the adjustment by the Variation of Geographic Coordinates, p. 91 et seq.)

Elevations for the various stations are assumed somewhat near what the final values will be. To these are added x's to be determined by the adjustment. (See table of assumed elevations on p. 200.) By means of these, observation equations are formed by the comparison of the assumed h_2-h_1 with that determined by computation.

The method of formation is shown below and a tabulated form of all of the computation on page 201.

Fixed elevations.

	Meters
Bosley	1037.35
Stack	1062.69
Craggy	1368.31
Redding Rock Mean sea level	0.00

Assumed and adjusted elevations

Station	Elevation	
	Assumed + correction	Adjusted
	Meters	*Meters*
Pollywog	$811+x_1$	811.06
Elk	$507+x_2$	504.61
Pack Saddle	$817+x_3$	815.74
High Divide	$710+x_4$	708.77
Long Ridge	$1059+x_5$	1055.96
Bald Hill	$589+x_6$	585.16
Gordon	$1259+x_7$	1256.12
Child	$701+x_8$	698.19
Rattle	$1103+x_9$	1100.46
Red Mountain	$1290+x_{10}$	1287.70

FORMATION OF OBSERVATION EQUATIONS

The observation equations are formed as follows:

(1) Pollywog, assumed elevation* $= 811+x_1$

(2) Craggy, fixed elevation $= 1368.31$

h_2-h_1 (assumed)* $= +557.31-x_1$

h_2-h_1 (observed)* $= +555.38+v_1$

Observed − assumed* $= -1.93+x_1+v_1=0$

$-v_1 = -1.93+x_1$

$p=\frac{1}{3}$ of $3.96=1.32$

(1) Elk, assumed elevation* $= 507+x_2$

(2) Pollywog, assumed elevation* $= 811+x_1$

h_2-h_1 (assumed)* $= +304+x_1-x_2$

h_2-h_1 (observed)* $= +303.75+v_5$

Observed − assumed* $= -0.25-x_1+x_2+v_5=0$

$-v_5 = -0.25-x_1+x_2$

$p=2.60$

In a similar manner the remaining equations are formed. These are usually formed as in the following table. The constant term is found in the column "Observed minus assumed," and the remainder of the equation in the column "Symbol."

*Including symbolic correction.

Table of formation of observation equations

Station 1	Station 2	Weight p	Assumed difference of elevation h_2-h_1	Observed difference of elevation h_2-h_1	Observed minus assumed	Symbol	Adjusted difference of elevation h_2-h_1	Adjusted minus observed v	pv	Minus † pv	pv^2
*Pollywog	Craggy	1.32	+ 557.31	+ 555.38	−1.93	$+x_1$	+ 557.25	+1.87	+ 2.47		4.6159
*Pollywog	Stack	0.34	+ 251.69	+ 250.86	−0.83	$+x_1$	+ 251.63	+0.77	+ 0.26		0.2016
*Pollywog	Bosley	1.51	+ 226.35	+ 228.27	+1.92	$+x_1$	+ 226.29	−1.98	− 2.99		5.9198
*Elk	Bosley	0.72	+ 530.35	+ 531.07	+0.72	$+x_2$	+ 532.74	+1.67	+ 1.20		2.0080
Elk	Pollywog	2.60	+ 304	+ 303.75	−0.25	$-x_1+x_2$	+ 306.45	+2.70	+ 7.02	− 7.02	18.9540
Pack Saddle	Elk	10.89	− 310	− 310.73	−0.73	$-x_2+x_3$	− 311.13	−0.40	− 4.36	+ 4.36	1.7424
*Pack Saddle	Bosley	0.47	+ 220.35	+ 227.25	+6.90	$+x_3$	+ 221.61	−5.64	− 2.65		14.9505
Pack Saddle	Pollywog	2.82	− 6	− 2.10	+3.90	$-x_1+x_3$	− 4.68	−2.58	− 7.28	+ 7.28	18.7710
High Divide	Elk	4.54	− 203	− 206.55	−3.55	$-x_2+x_4$	− 204.16	+2.39	+10.85	−10.85	25.9329
High Divide	Pack Saddle	4.89	+ 107	+ 109.16	+2.16	$-x_3+x_4$	+ 106.97	−2.19	−10.71	+10.71	23.4529
Long Ridge	High Divide	8.53	− 349	− 347.39	+1.61	$-x_4+x_6$	− 347.19	+0.20	+ 1.71	− 1.71	0.3412
Long Ridge	Elk	2.34	− 552	− 552.12	−0.12	$-x_2+x_5$	− 551.35	+0.77	+ 1.80	− 1.80	1.3874
*Long Ridge	Pack Saddle	1.63	− 242	− 237.98	+4.02	$-x_3+x_5$	− 240.22	−2.24	− 3.65	+ 3.65	8.1787
Bald Hill	High Divide	3.71	+ 121	+ 123.90	+2.90	$-x_4+x_6$	+ 123.61	−0.29	− 1.08	+ 1.08	0.3120
Bald Hill	Long Ridge	2.63	+ 470	+ 469.92	−0.08	$-x_5+x_6$	+ 470.80	+0.88	+ 2.31	− 2.31	2.0367
Gordon	Bald Hill	4.93	− 670	− 671.58	−1.58	$-x_6+x_7$	− 670.96	+0.62	+ 3.06	− 3.06	1.8951
Gordon	High Divide	2.61	− 549	− 547.17	+1.83	$-x_4+x_7$	− 547.35	−0.18	− 0.47	+ 0.47	0.0846
Gordon	Long Ridge	5.19	− 200	− 199.69	+0.31	$-x_5+x_7$	− 200.16	−0.47	− 2.44	+ 2.44	1.1405
*Red Mountain	Redding Rock	0.36	−1290	−1290.67	−0.67	$+x_{10}$	−1287.70	+2.97	+ 1.07	− 1.07	3.1755
*Red Mountain	Bald Hill	0.42	− 701	− 698.27	+2.73	$-x_6+x_{10}$	− 702.54	−4.27	− 1.79	+ 1.79	7.6578
Red Mountain	Gordon	1.06	− 31	− 32.51	−1.51	$-x_7+x_{10}$	− 31.58	+0.93	+ 0.99	− 0.99	0.9168
Child	Gordon	3.43	+ 558	+ 557.78	−0.22	$-x_7+x_8$	+ 557.93	+0.15	+ 0.51	− 0.51	0.0772
Child	Red Mountain	2.03	+ 589	+ 589.64	+0.64	$+x_8-x_{10}$	+ 589.51	−0.13	− 0.26	+ 0.26	0.0343
*Rattle	Redding Rock	0.24	−1103	−1103.11	−0.11	$+x_9$	−1100.46	+2.65	+ 0.64		1.6854
Rattle	Child	8.38	− 402	− 402.30	−0.30	$-x_8+x_9$	− 402.27	+0.03	+ 0.25	− 0.25	0.0075
Rattle	Gordon	2.37	+ 156	+ 156.27	+0.27	$-x_7+x_9$	+ 155.66	−0.61	− 1.45	+ 1.45	0.8819
Rattle	Red Mountain	7.38	+ 187	+ 187.18	+0.18	$+x_9-x_{10}$	+ 187.24	+0.06	+ 0.44	− 0.44	0.0266
										Σpv^2	146.3942

* Computed from nonreciprocal observations. Weight used $= \frac{1}{2}\,p$ from the computation.
† This column added for convenience in computing.

COMPUTATION OF PROBABLE ERROR

$$\text{Probable error} = \pm 0.6745\sqrt{\frac{\Sigma pv^2}{\text{No. observations} - \text{No. unknowns}}}$$

$$= \pm\sqrt{\frac{0.455\ \Sigma pv^2}{n_o - n_u}}$$

$\Sigma pv^2 = 146.394$, log $= 2.16552$

No. observations − No. unknowns $= 27 - 10 = 17$, colog $= 8.76955$

Constant $= 0.455$, log $= 9.65801$

log (probable error)$^2 = 0.59308$

log probable error $= 0.29654$

Probable error of unit weight $= \pm 1.98$ * m.

log (probable error, unit weight)$^2 = 0.59308$

Weight coefficient for Long Ridge $= 2.843$, log $= 0.45378$

log (probable error)$^2 = 0.13930$

log probable error $= 0.06965$

Probable error for Long Ridge $= \pm 1.17$ m.

FORMATION OF NORMAL EQUATIONS BY DIFFERENTIATION

The following equations are formed from the table just given:

$$-v_1 = -1.93 + x_1$$
$$-v_2 = -0.83 + x_1$$
$$-v_3 = +1.92 + x_1$$
$$-v_4 = +0.72 + x_2$$
$$-v_5 = -0.25 - x_1 + x_2$$

and so on for the rest of the 27 v's.

The function u to be made a minimum is $\Sigma p_n v_n^2$, or

$$\begin{aligned} u = {} & +1.32(-1.93+x_1)^2+0.34(-0.83+x_1)^2+1.51(+1.92+x_1)^2+0.72(+0.72+x_2)^2 \\ & +2.60(-0.25-x_1+x_2)^2+10.89(-0.73-x_2+x_3)^2+0.47(+6.90+x_3)^2+2.82(+3.90 \\ & -x_1+x_3)^2+4.54(-3.55-x_2+x_4)^2+4.89(+2.16-x_3+x_4)^2+8.53(+1.61-x_4+x_5)^2 \\ & +2.34(-0.12-x_2+x_5)^2+1.63(+4.02-x_3+x_5)^2+3.71(+2.90-x_4+x_6)^2+2.63(-0.08 \\ & -x_5+x_6)^2+4.93(-1.58-x_6+x_7)^2+2.61(+1.83-x_4+x_7)^2+5.19(+0.31-x_5+x_7)^2 \\ & +0.36(-0.67+x_{10})^2+0.42(+2.73-x_6+x_{10})^2+1.06(-1.51-x_7+x_{10})^2+3.43 \\ & (-0.22-x_7+x_8)^2+2.03(+0.64+x_8-x_{10})^2+0.24(-0.11+x_9)^2+8.38(-0.30-x_8+x_9)^2 \\ & +2.37(+0.27-x_7+x_9)^2+7.38(+0.18+x_9-x_{10})^2. \end{aligned}$$

The function will be rendered a minimum by equating to zero the partial differential coefficients with respect to x_1, x_2, etc. By this means the following equations are derived:

$$\begin{aligned} & +1.32(-1.93+x_1)+0.34(-0.83+x_1)+1.51(+1.92+x_1)-2.60(-0.25-x_1+x_2)-2.82 \\ & \quad (+3.90-x_1+x_3)=0 \\ & +0.72(+0.72+x_2)+2.60(-0.25-x_1+x_2)-10.89(-0.73-x_2+x_3)-4.54(-3.55-x_2+x_4) \\ & \quad -2.34(-0.12-x_2+x_5)=0 \end{aligned}$$

* This vertical net is not of a high degree of accuracy, it being a small spur of secondary triangulation that was executed in some haste with slight attention to vertical observations. It was selected on account of its small size. The more accurate work is usually in larger nets. See list of probable errors ranging from ±0.23 m. to ±1.83 m. in United States Coast and Geodetic Survey Special Publication No. 13.

$$+10.89(-0.73-x_2+x_3)+0.47(+6.90+x_3)+2.82(+3.90-x_1+x_3)-4.89(+2.16-x_3+x_4)$$
$$-1.63(+4.02-x_3+x_5)=0$$
$$+4.54(-3.55-x_2+x_4)+4.89(+2.16-x_3+x_4)-8.53(+1.61-x_4+x_5)-3.71(+2.90$$
$$-x_4+x_6)-2.61(+1.83-x_4+x_7)=0$$
$$+8.53(+1.61-x_4+x_5)+2.34(-0.12-x_2+x_5)+1.63(+4.02-x_3+x_5)-2.63(-0.08$$
$$-x_5+x_6)-5.19(+0.31-x_5+x_7)=0$$
$$+3.71(+2.90-x_4+x_6)+2.63(-0.08-x_5+x_6)-4.93(-1.58-x_6+x_7)-0.42(+2.73$$
$$-x_6+x_{10})=0$$
$$+4.93(-1.58-x_6+x_7)+2.61(+1.83-x_4+x_7)+5.19(+0.31-x_5+x_7)-1.06(-1.51$$
$$-x_7+x_{10})-3.43(-0.22-x_7+x_8)-2.37(+0.27-x_7+x_9)=0$$
$$+3.43(-0.22-x_7+x_8)+2.03(+0.64+x_8-x_{10})-8.38(-0.30-x_8+x_9)=0$$
$$+0.24(-0.11+x_9)+8.38(-0.30-x_8+x_9)+2.37(+0.27-x_7+x_9)+7.38(+0.18+x_9$$
$$-x_{10})=0$$
$$+0.36(-0.67+x_{10})+0.42(+2.37-x_6+x_{10})+1.06(-1.51-x_7+x_{10})-2.03(+0.64$$
$$+x_8-x_{10})-7.38(+0.18+x_9-x_{10})=0$$

By multiplying and collecting, we obtain the following normals:

$$+8.59x_1-2.60x_2-2.82x_3\ldots\ldots-10.2786=0$$
$$-2.60x_1+21.09x_2-10.89x_3-4.54x_4-2.34x_5\ldots\ldots+24.2159=0$$
$$-2.82x_1-10.89x_2+20.70x_3-4.89x_4-1.63x_5\ldots\ldots-10.8237=0$$
$$-4.54x_2-4.89x_3+24.28x_4-8.53x_5-3.71x_6-2.61x_7\ldots\ldots-34.8232=0$$
$$-2.34x_2-1.63x_3-8.53x_4+20.32x_5-2.63x_6-5.19x_7\ldots\ldots+18.6066=0$$
$$-3.71x_4-2.63x_5+11.69x_6-4.93x_7-\ldots\ldots0.42x_{10}\ldots\ldots+17.1914=0$$
$$-2.61x_4-5.19x_5-4.93x_6+19.59x_7-3.43x_8-2.37x_9-1.06x_{10}\ldots\ldots+0.3111=0$$
$$-3.43x_7+13.84x_8-8.38x_9-2.03x_{10}\ldots\ldots+3.0586=0$$
$$-2.37x_7-8.38x_8+18.37x_9-7.38x_{10}\ldots\ldots-0.5721=0$$
$$-0.42x_6-1.06x_7-2.03x_8-7.38x_9+11.25x_{10}\ldots\ldots-3.3228=0$$

(See the table of normals on p. 204.)

The normals are most conveniently formed from the table given on page 204. The various observation equations are written along the horizontal lines in the columns of their respective x's. The normals are then formed as in condition equations, except that the constant terms must also be multiplied by each column and the sums taken for the constant terms in the normals, as may be seen from the direct computation of the normals above.

After the x's are determined from the solution of the normals, they are added to the assumed elevations, giving the adjusted final elevations. The v's are most easily determined by computing h_2-h_1 from the adjusted values; if the observed h_2-h_1 is subtracted from the adjusted value the respective v results. They could, of course, be computed by substituting the x's in the observation equations, but this would require more work.

For a check the $\Sigma\ pv$ at any station should equal zero, with the possible exception of a small amount due to dropping the decimals on the x's. In the table on page 201, use pv from the first column if the x is positive and from the second column if the x is negative.

Table for formation of normal equations

	p	N	1	2	3	4	5	6	7	8	9	10	pN
1	1.32	−1.93	+1										− 2.5476
2	0.34	−0.83	+1										− 0.2822
3	1.51	+1.92	+1										+ 2.8992
4	0.72	+0.72		+1									+ 0.5184
5	2.60	−0.25	−1	+1									− 0.6500
6	10.89	−0.73		−1	+1								− 7.9497
7	0.47	+6.90			+1								+ 3.2430
8	2.82	+3.90	−1		+1								+10.9980
9	4.54	−3.55		−1		+1							−16.1170
10	4.89	+2.16			−1	+1							+10.5624
11	8.53	+1.61				−1	+1						+13.7333
12	2.34	−0.12		−1			+1						− 0.2808
13	1.63	+4.02			−1		+1						+ 6.5526
14	3.71	+2.90				−1		+1					+10.7590
15	2.63	−0.08					−1	+1					− 0.2104
16	4.93	−1.58						−1	+1				− 7.7894
17	2.61	+1.83				−1			+1				+ 4.7763
18	5.19	+0.31					−1		+1				+ 1.6089
19	0.36	−0.67										+1	− 0.2412
20	0.42	+2.73						−1				+1	+ 1.1466
21	1.06	−1.51							−1			+1	− 1.6006
22	3.43	−0.22							−1	+1			− 0.7546
23	2.03	+0.64								+1		−1	+ 1.2992
24	0.24	−0.11									+1		− 0.0264
25	8.38	−0.30								−1	+1		− 2.5140
26	2.37	+0.27							−1		+1		+ 0.6399
27	7.38	+0.18									+1	−1	+ 1.3284

	x's
1	+0.0595
2	−2.3925
3	−1.2577
4	−1.2304
5	−3.0402
6	−3.8403
7	−2.8757
8	−2.8107
9	−2.5441
10	−2.2951

Normal equations

	1	2	3	4	5	6	7	8	9	10	η	Σ
1	+8.59	− 2.60	− 2.82								−10.2786	− 7.1086
2		+21.09	−10.89	− 4.54	− 2.34						+24.2159	+24.9359
3			+20.70	− 4.89	− 1.63						−10.8237	−10.3537
4				+24.28	− 8.53	− 3.71	− 2.61				−34.8232	−34.8232
5					+20.32	− 2.63	− 5.19				+18.6066	+18.6066
6						+11.69	− 4.93			− 0.42	+17.1914	+17.1914
7							+19.59	− 3.43	− 2.37	− 1.06	+ 0.3111	+ 0.3111
8								+13.84	− 8.38	− 2.03	+ 3.0586	+ 3.0586
9									+18.37	− 7.38	− 0.5721	− 0.3321
10										+11.25	− 3.3228	− 2.9628

Solution of normal equations

1	2	3	4	5	6	7	η	Σ
+8.59	− 2.60	− 2.82					−10.2786	− 7.1086
x_1	+ 0.30268	+ 0.32829					+ 1.19658	+ 0.82754
	+21.09	−10.89	− 4.54	− 2.34			+24.2159	+24.9359
1	− 0.7870	− 0.8536					− 3.1111	− 2.1516
	+20.3030	−11.7436	− 4.54	− 2.34			+21.1048	+22.7842
	x_2	+ 0.57842	+ 0.22361	+ 0.11525			− 1.03949	− 1.12221
		+20.70	− 4.89	− 1.63			−10.8237	−10.3537
	1	− 0.9258					− 3.3744	− 2.3337
	2	− 6.7927	− 2.6260	− 1.3535			+12.2074	+13.1788
		+12.9815	− 7.5160	− 2.9835			− 1.9907	+ 0.4913
		x_3	+ 0.57898	+ 0.22983			+ 0.15335	− 0.03785
			+24.28	− 8.53	−3.71	−2.61	−34.8232	−34.8232
		2	− 1.0152	− 0.5232			+ 4.7192	+ 5.0948
		3	− 4.3516	− 1.7274			− 1.1526	+ 0.2845
			+18.9132	−10.7806	−3.71	−2.61	−31.2566	−29.4440
			x_4	+ 0.570004	+0.196159	+0.137999	+ 1.652634	+ 1.556796

Solution of normal equations—Continued

9	8	10	7	6	5	η	Σ
+18.37 x_9	− 8.38 + 0.45618	− 7.38 + 0.40174	− 2.37 + 0.129015			− 0.5721 + 0.03114	− 0.3321 + 0.01808
	+13.84 9 − 3.8228	− 2.03 − 3.3666	− 3.43 − 1.0811			+ 3.0586 − 0.2610	+ 3.0586 − 0.1515
	+10.0172 x_8	− 5.3966 + 0.53873	− 4.5111 + 0.45034			+ 2.7 16 − 0.27928	+ 2.9071 − 0.29021
		+11.25 9 − 2.9648 8 − 2.9073	− 1.06 − 0.9521 − 2.4303	− 0.42		− 3.3228 − 0.2298 + 1.5072	− 2.9628 − 0.1334 + 1.5661
		+ 5.3779 x_{10}	− 4.4424 + 0.82605	− 0.42 + 0.07810		− 2.0454 + 0.38033	− 1.5299 + 0.28448
			+19.59 4 − 0.3602 9 − 0.3058 8 − 2.0315 10 − 3.6696	− 4.93 − 0.5120 − 0.3469	− 5.19 − 1.4877	+ 0.3111 − 4.3134 − 0.0738 + 1.2599 − 1.6896	+ 0.3111 − 4.0632 − 0.0428 + 1.3092 − 1.2638
			+13.2229 x_7	− 5.7889 + 0.43779	− 6.6777 + 0.50501	− 4.5058 + 0.34076	− 3.7495 + 0.28356
				+11.69 4 − 0.7277 10 − 0.0328 7 − 2.5343	− 2.63 − 2.1148 − 2.9234	+17.1914 − 6.1313 − 0.1597 − 1.9726	+17.1914 − 5.7757 − 0.1195 − 1.6415
				+ 8.3952 x_6	− 7.6681 + 0.91339	+ 8.9279 − 1.06345	+ 9.6550 − 1.15006
					+20.32 2 − 0.2697 3 − 0.0857 4 − 6.1450 7 − 3.3723 6 − 7.0040	+18.6066 + 2.4323 − 0.4575 −17.8164 − 2.2755 + 8.1547	+18.6066 + 2.6259 + 0.1129 −16.7832 − 1.8935 + 8.8188
					+ 2.8433 x_5	+ 8.6442 − 3.04020	+11.4875 − 4.04020

Back solution

5	6	7	10	8	9	4	3	2	1
−3.0402	−1.0634 −2.7769	+0.3408 −1.5353 −1.6812	+0.3803 −0.2999 −2.3755	−0.2793 −1.2950 −1.2364	+0.0311 −0.3710 −0.9220 −1.2822	+1.6526 −1.7329 −0.7533 −0.3968	+0.1534 −0.6987 −0.7124	−1.0395 −0.3504 −0.2751 −0.7275	+1.1966 −0.4129 −0.7242
−3.0402	−3.8403	−2.8757	−2.2951	−2.8107	−2.5441	−1.2304	−1.2577	−2.3925	+0.0595

DEVELOPMENT OF FORMULAS FOR TRIGONOMETRIC LEVELING

GENERAL STATEMENT

The formulas used on pages 198 and 199 in the computation of vertical observations were found to be lacking in some of the quantities that were appreciable when the lines were very long and high. Accordingly, a new derivation is now given that takes into account some of these quantities. As a result, the formulas derived in this development differ slightly from those used in the computation cited above, but they ought to give practically the same result in computing over lines of such length as occur therein.

The following derivation of the formulas for trigonometric leveling is based on certain approximate assumptions which fall under four general heads:

1. *Geometric approximations.*—The verticals at the two points (P_1 the point occupied and P_2 the point sighted on) are treated as if they lay in one plane and the intersection of this plane with the ellipsoid that represents the surface of the earth is treated as the arc of a circle whose radius is the mean radius of curvature of a vertical section through P_1 and P_2. Helmert (in his Höhere Geodäsie, Vol. I, p. 520, and Vol. II, p. 563) investigates the error arising from these assumptions and finds it to be about 1/40 meter at a maximum when the distance P_1 P_2 is about 100 kilometers.

2. *Geodetic approximations.*—The difference between the geodetic zenith and the astronomic zenith, i. e., the deflection of the plumb line, is ignored. If these deflections are known, corrections may be applied to the measured zenith distances (which, of course, are referred to the astronomical zenith) to reduce them to the geodetic zenith. Furthermore, the elevations obtained by trigonometric leveling between two points are referred to an assumed ellipsoid, while spirit leveling gives elevations referred to the geoid, so that the distances between geoid and ellipsoid must be known to make the two kinds of leveling comparable. If trigonometric leveling could be carried out with great precision, its use in connection with spirit leveling would give just this information as to the distance of the ellipsoid from the geoid. The change in the distance from geoid to ellipsoid occurring between P_1 and P_2 may be found from the deflections of the vertical at those points, *provided* it is assumed that the deflections vary uniformly between P_1 and P_2, an assumption which may be considerably in error.

3. *Optical approximations.*—The path of the ray of light between P_1 and P_2 is assumed to be the arc of a circle in a vertical plane through P_1 and P_2. The angle between the chord P_1 P_2 and the tangent to the circle at either point is the refraction in zenith distance and it is evidently implied that this refraction is equal at P_1 and P_2. If we call O (see figure 9) the center of the circle referred to in approximation 1, and call the angle $P_1OP_2 = \theta$, the refraction in zenith distance of the angle TP_1P_2 ($= \angle TP_2P_1$) is written as $m\theta$ and m is termed the coefficient of refraction. The course of a ray of light through the atmosphere depends on the variations in pressure, temperature and humidity of the medium through which it passes and may be far from circular. Our lack of knowledge of the conditions which govern the refraction is the greatest obstacle to precision in trigonometric leveling.

4. *Algebraic approximations.*—After the approximations mentioned above have been made, there is the further approximation arising from the dropping of small terms after an expansion in series. In the following developments it will be seen that only extremely small terms are dropped, and that in cases arising in practice their effect even on the sixth place of logarithms is unimportant, while in fact logarithms of only five places are commonly used for this sort of computation. The accuracy of the developments is confirmed by the numerical agreement between the approximate and the exact formulas in the examples given. (*Exact* is used in the sense of dispensing with the use of series. The formula is *inexact*, owing to the first three sets of approximations.) The examples represent rather extreme cases of those arising in practice, and other numerical examples of extreme cases give a similar agreement.

DEVELOPMENT OF THE FORMULAS

Figure 9 represents the vertical plane of approximation 1 common to P_1 and P_2, being in fact the plane parallel to both verticals (see Helmert, Höhere Geodäsie, Vol. I, p. 519) on which the several points are projected.

The measured zenith distances are assumed equal to

$$\angle V_1P_1T = \zeta_1$$

and $$\angle V_2P_2T = \zeta_2.$$

The measurements are not made exactly in this plane, but the error, which is part of that involved in approximation 1, is negligible.

The refraction in zenith distance is, according to approximation 3,

$$\Delta\zeta = \angle TP_1P_2 = \angle TP_2P_1 = m\theta.$$

S_1 and S_2 are points on the earth's surface in the verticals of P_1 and P_2, so that the respective elevations of P_1 and P_2 above the surface are

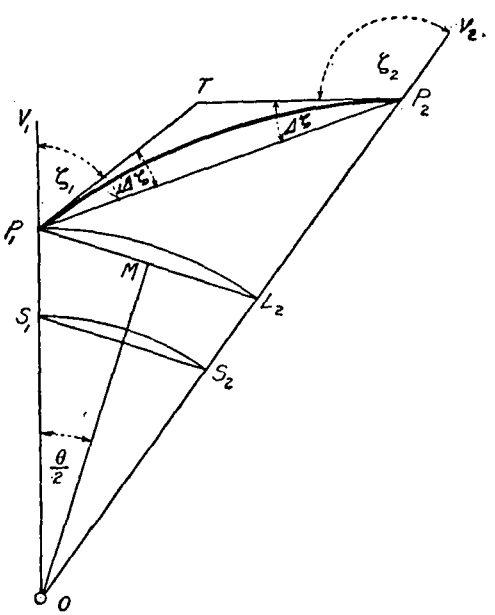

FIG. 9.

$$h_1 = S_1P_1$$

and $$h_2 = S_2P_2.$$

The mean radius of curvature ρ of approximation 1 is given by

$$\rho = OS_1 = OS_2.$$

If s denotes the distance P_1P_2 measured along the arc and if θ be expressed in radians,

$$s=\rho\theta$$

or if θ be in seconds,

$$\theta''=\frac{s}{\rho \sin 1''}.$$

There are two cases to be considered according as to whether both or only one of the zenith distances have been measured.

Case I. Reciprocal zenith distances

In the triangle P_1OP_2

$$\angle P_2P_1O=180^\circ-\zeta_1-\Delta\zeta=180^\circ-\zeta_1-m\theta$$
$$\angle P_1P_2O=180^\circ-\zeta_2-\Delta\zeta=180^\circ-\zeta_2-m\theta$$

also $\quad OP_1=\rho+h_1$

and $\quad OP_2=\rho+h_2.$

Therefore by the law of sines

$$\frac{\rho+h_1}{\rho+h_2}=\frac{\sin(\zeta_2+m\theta)}{\sin(\zeta_1+m\theta)}.$$

Treating this as a proportion and taking by division,

$$\frac{(\rho+h_2)-(\rho+h_1)}{\rho+h_1}=\frac{\sin(\zeta_1+m\theta)-\sin(\zeta_2+m\theta)}{\sin(\zeta_2+m\theta)}$$

or

$$h_2-h_1=\frac{2(\rho+h_1)\sin\left(\frac{\zeta_1-\zeta_2}{2}\right)\cos\left(\frac{\zeta_1+\zeta_2}{2}+m\theta\right)}{\sin(\zeta_2+m\theta)}. \qquad \text{(A)}$$

Since the sum of the angles of a triangle is 180°,

$$180^\circ-\zeta_1-m\theta+180^\circ-\zeta_2-m\theta+\theta=180^\circ$$

which gives

$$\frac{\zeta_1+\zeta_2}{2}+m\theta=90^\circ+\frac{\theta}{2}$$

also

$$\zeta_2+m\theta=\frac{\zeta_2+\zeta_1}{2}+m\theta+\frac{\zeta_2-\zeta_1}{2}=90^\circ+\frac{\theta}{2}+\frac{\zeta_2-\zeta_1}{2}$$

whence (A) becomes

$$h_2-h_1=\frac{2(\rho+h_1)\sin\left(\frac{\zeta_2-\zeta_1}{2}\right)\sin\frac{\theta}{2}}{\cos\left(\frac{\zeta_2-\zeta_1}{2}+\frac{\theta}{2}\right)}. \qquad (1)$$

The quantity $2(\rho+h_1)\sin\frac{\theta}{2}$ has a simple geometrical interpretation In the figure make $OL_2=OP_1$ and draw $OM\perp P_1L_2$. Then

$$P_1M=L_2M=OP_1\sin P_1OM=(\rho+h_1)\sin\frac{\theta}{2}.$$

Then $2(\rho+h_1)\sin\frac{\theta}{2}$ is the chord P_1L_2 or the chord S_1S_2 increased to allow for the elevation of P_1 above the earth's surface. In fact, the relation (1) might have been obtained by applying the law of sines directly to the triangle $P_1P_2L_2$, which makes it evident why P_1L_2 appears.

For convenient computation* (1) may be transformed as follows: By the sine series

$$2\sin\frac{\theta}{2}=2\left[\frac{\theta}{2}-\frac{1}{6}\left(\frac{\theta}{2}\right)^3+\cdots\right]$$

$$2(\rho+h_1)\sin\frac{\theta}{2}=\rho\left(1+\frac{h_1}{\rho}\right)\left(\theta-\frac{\theta^3}{24}+\cdots\right)$$

$$=\rho\theta\left(1+\frac{h_1}{\rho}\right)\left(1-\frac{\theta^2}{24}\right)=s\left(1+\frac{h_1}{\rho}\right)\left(1-\frac{s^2}{24\rho^2}\right). \qquad (2)$$

The remaining factors of the right-hand side of (1) may be written,

$$\frac{\sin\left(\frac{\zeta_2-\zeta_1}{2}\right)}{\cos\left(\frac{\zeta_2-\zeta_1}{2}+\frac{\theta}{2}\right)}=\frac{\sin\left(\frac{\zeta_2-\zeta_1}{2}\right)}{\cos\left(\frac{\zeta_2-\zeta_1}{2}\right)\cos\frac{\theta}{2}-\sin\left(\frac{\zeta_2-\zeta_1}{2}\right)\sin\frac{\theta}{2}}$$

$$=\frac{\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)\sec\frac{\theta}{2}}{1-\tan\frac{\theta}{2}\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)}$$

$$=\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)\left(1+\frac{\theta^2}{8}\right)\left[1+\frac{\theta}{2}\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)\right]. \qquad (3)$$

The last transformation comes by expanding $\sec\frac{\theta}{2}$ in powers of θ and noting that $\tan\frac{\theta}{2}=\frac{\theta}{2}$ nearly, and that the product $\frac{\theta}{2}\tan\frac{\zeta_2-\zeta_1}{2}$ is small, so that,

$$\frac{1}{1-\frac{\theta}{2}\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)}=1+\frac{\theta}{2}\tan\left(\frac{\zeta_2-\zeta_1}{2}\right) \text{ very nearly.}$$

By combining (2) and (3) and using $\theta=\frac{s}{\rho}$, equation (1) becomes

$$\left.\begin{aligned}&h_2-h_1=s\left(1+\frac{h_1}{\rho}\right)\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)\left[1+\frac{s}{2\rho}\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)\right]\left[1+\frac{s^2}{12\rho^2}\right]\\ &\text{or } h_2-h_1=s\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)ABC\end{aligned}\right\} \qquad (4)$$

* See also note 1, p. 219.

where $A=1+\frac{h_1}{\rho}$ = correction for elevation of station whose elevation is known,

$B=1+\frac{s}{2\rho}\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)$ = correction for approximate difference of elevation,

$C=1+\frac{s^2}{12\rho^2}$ = correction for distance.

The logarithms of A, B, and C are given in the tables on pages 218 and 219 with the respective arguments h_1, $\log\left[s\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)\right]$, and $\log s$. The tables show the limiting values of the respective arguments for which logarithms of A, B, and C become 1, 2, 3, etc., units of the fifth place of decimals.

Equation (4) may be compared with the expression more commonly given for h_2-h_1,

$$h_2-h_1=s\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)\left[1+\frac{h_1+h_2}{2\rho}+\frac{s^2}{12\rho^2}\right] \tag{5}$$

With the tables here given (4) will probably be found slightly more convenient for logarithmic computation than (5). The two forms are equally accurate.

Case II. Only one zenith distance (ζ_1) observed

Where two zenith distances are known, the formula, either (4) or (5), does not involve the coefficient of refraction (m) explicitly. Where only one zenith distance is known, a value of m must be assumed from the best sources of information available.

In the triangle $P_1\,L_2\,P_2$

$$\angle P_1\,L_2\,P_2=90^\circ+\frac{\theta}{2}=\angle V_1\,P_1\,L_2$$

$$\angle P_2\,P_1\,L_2=\angle V_1\,P_1\,L_2-\angle V_1\,P_1\,P_2$$

$$=90^\circ+\frac{\theta}{2}-(\zeta_1+\Delta\zeta)=90^\circ-\zeta_1+(\tfrac{1}{2}-m)\theta$$

For the third angle we find, by subtracting the sum of the other angles from 180°

$$\angle P_1\,P_2\,L=\zeta_1-(1-m)\theta.$$

By the law of sines

$$\frac{L_2P_2}{P_1L_2}=\frac{\sin P_2P_1L_2}{\sin P_1P_2L_2}$$

or

$$h_2-h_1=P_1L_2\,\frac{\cos\,[\zeta_1-(\tfrac{1}{2}-m)\theta]}{\sin\,[\zeta_1-(1-m)\theta]}. \tag{6}$$

The chord P_1L_2 = chord $S_1S_2\times\frac{\rho+h_1}{\rho}$ = chord $S_1S_2\times A$, A having the meaning previously given; chord S_1S_2 = arc s very nearly; or, if greater precision is desired, $P_1L_2=sAR$, where R is the reduction factor from arc to chord.

The logarithm of the reduction factor from arc to sine is given in the Coast and Geodetic Survey Special Publication No. 8 (Formulæ and Tables for the Computation of Geodetic Positions), page 17. The logarithm of the reduction to chord is very nearly one-fourth of the reduction from arc to sine. Granting approximations 1, 2, and 3, equation (6) may be rewritten as the so-called exact formula in the following form:

$$h_2-h_1=sAR\frac{\cos\left(\zeta_1-(\tfrac{1}{2}-m)\dfrac{s}{\rho\sin 1''}\right)}{\sin\left[\zeta_1-(\tfrac{1}{2}-m)\dfrac{s}{\rho\sin 1''}-\tfrac{1}{2}\dfrac{s}{\rho\sin 1''}\right]}. \tag{7}$$

Sin $1''$ is introduced to convert the angle from radians to seconds of arc. A and R have the meanings previously indicated. The quantity $(\frac{1}{2}-m)$ appears in the computation of the refraction from reciprocal zenith distances on the Coast and Geodetic Survey forms. A mean of the determinations of $(\frac{1}{2}-m)$ from the reciprocal zenith distances should be used in computing the nonreciprocal observations. Having found the angle $\zeta_1-(\frac{1}{2}-m)\dfrac{s}{\rho\sin 1''}$ for the numerator, the computer should subtract $\frac{1}{2}\dfrac{s}{\rho\sin 1''}$ from it to get the angle for the denominator. The angle in the denominator need not be carried out very accurately, as it is always near 90° where the sine varies slowly.

The former Coast and Geodetic Survey formula was

$$h_2-h_1=s\cot\zeta_1+\frac{(\frac{1}{2}-m)\,s^2}{\rho}+\frac{(1-m)\,s^2\cot^2\zeta_1}{\rho}. \tag{8}$$

It is obtained from (6) or (7) by expanding in series and dropping certain small quantities. On some of the longer lines the quantities dropped are appreciable in computations with five-place logarithms. The development hereafter given will show that the general form of (8) may be retained by the introduction of correction-factors D_1 and D_2, which are nearly unity, and by the further factor A, the correction-factor for elevation of the occupied station. The full formula will then be,

$$h_2-h_1=A\,D_1\,s\cot\zeta_1+\frac{(\frac{1}{2}-m)\,A\,D_2\,s^2}{\rho}+\frac{(1-m)\,s^2\cot^2\zeta_1}{\rho}.$$

This form may be obtained from (6) as follows:
As before

$$P_1L_2=2(\rho+h_1)\sin\frac{\theta}{2}$$

or expanding by the sine series

$$P_1L_2=\rho\left(1+\frac{h_1}{\rho}\right)\left[\theta-\frac{\theta^3}{24}+\cdots\right]=A\rho\left(\theta-\frac{\theta^3}{24}+\cdots\right). \tag{9}$$

The factor $\cos[\zeta_1-(\frac{1}{2}-m)\,\theta]$ in (6) may be written

$$\cos[\zeta_1-(\tfrac{1}{2}-m)\,\theta]=\cos\zeta_1\cos[(\tfrac{1}{2}-m)\,\theta]+\sin\zeta_1\sin[(\tfrac{1}{2}-m)\,\theta]$$

Since $(\frac{1}{2}-m)\,\theta$ is a small quantity, the series forms for its sine and cosine may be used, giving

$$\begin{aligned}\cos[\zeta_1-(\tfrac{1}{2}-m)\,\theta]=\cos\zeta_1&\left[1-(\tfrac{1}{2}-m)^2\frac{\theta^2}{2}+\cdots\right]\\&+\sin\zeta_1\left[(\tfrac{1}{2}-m)\,\theta-(\tfrac{1}{2}-m)^3\frac{\theta^3}{6}+\cdots\right]\end{aligned}\tag{10}$$

The third factor on the right-hand side of (6), namely,

$$\frac{1}{\sin[\zeta_1-(1-m)\,\theta]}=\operatorname{cosec}[\zeta_1-(1-m)\theta]$$

may be expanded in powers of $(1-m)\theta$ by Taylor's theorem.

$$\begin{aligned}f(\zeta_1)&=\operatorname{cosec}\zeta_1\\f'(\zeta_1)&=-\cot\zeta_1\operatorname{cosec}\zeta_1\\f''(\zeta_1)&=\operatorname{cosec}\zeta_1\,(1+2\cot^2\zeta_1)\\f'''(\zeta_1)&=-6\operatorname{cosec}^3\zeta_1\cot\zeta_1+\cot\zeta_1\operatorname{cosec}\zeta_1.\end{aligned}$$

This gives,

$$\begin{aligned}\frac{1}{\sin[\zeta_1-(1-m)\,\theta]}=\operatorname{cosec}\zeta_1&+\operatorname{cosec}\zeta_1\cot\zeta_1(1-m)\,\theta\\&+\operatorname{cosec}\zeta_1\,(1+2\cot^2\zeta_1)(1-m)^2\frac{\theta^2}{2}\\&+\operatorname{cosec}\zeta_1\cot\zeta_1\,(6\operatorname{cosec}^2\zeta_1-1)(1-m)^3\frac{\theta^3}{6}+\cdots\end{aligned}\tag{11}$$

The expressions (9), (10), and (11) for the factors on the right-hand side of (6) are now to be multiplied together.

In cases that actually occur, θ and $\cot\zeta_1$ are small quantities of about the same order of magnitude. If we call $\cot\zeta_1$ a quantity of the first order, it is evident that $\operatorname{cosec}\zeta_1$ differs from unity by a quantity of the second order. In forming the product from (9), (10), and (11) it is seen that the product is of the second order, and will moreover contain only terms of even order, so that if terms of the fourth order are retained the error will be of the sixth order, or the proportional error (the error as compared with the quantity itself) will be of the fourth order or of the order of $\frac{1}{30^4}$ part of the difference of elevation, if we suppose a quantity of the first order may be as large as $\frac{1}{30}$, a liberal allowance. The error, then, of the omitted terms should not affect the fifth place of logarithms and probably not the sixth. It will be seen that the expansions (9), (10), and (11) have been carried out sufficiently far for the purpose in hand, and if these expressions

be multiplied together, retaining in the product no terms of higher order than the fourth, the result may be written:

$$h_2-h_1=(\rho+h_1)\left\{\theta\cot\zeta_1\left[1+\frac{6\,(1-m)^2-1}{6}\cdot\theta^2\right]\right.$$
$$\left.+(\tfrac{1}{2}-m)\,\theta^2\left[1+\frac{5-10\,m+4\,m^2}{12}\cdot\theta^2\right]+(1-m)\,\theta^2\cot^2\zeta_1\right\} \tag{12}$$

Since $\theta=\frac{s}{\rho}$, we may write

$$h_2-h_1=AD_1s\cot\zeta_1+\frac{AD_2(\frac{1}{2}-m)s^2}{\rho}+\frac{(1-m)s^2\cot^2\zeta_1}{\rho} \tag{13}$$

$$\left.\begin{aligned}\text{where } D_1&=1+\frac{6\,(1-m)^2-1}{6}\cdot\frac{s^2}{\rho^2}\\ D_2&=1+\frac{5-10\,m+4\,m^2}{12}\cdot\frac{s^2}{\rho^2}\end{aligned}\right\} \tag{13a}$$

The factor A has been omitted from the last term as being unnecessary, the latter being small and A near unity. D_1 and D_2 are also near unity. Their logarithms are tabulated in the same manner as the other quantities, the tables showing the limiting values of the argument between which log D_1 or log D_2 may be taken as 1, 2, 3, etc., units of the fifth decimal.

It may be noted that in some European surveys the term $(1-m)\frac{s^2\cot^2\zeta_1}{\rho}$ is dropped and the formula for difference of elevation written as

$$h_2-h_1=s\cot\zeta_1+(\tfrac{1}{2}-m)\frac{s^2}{\rho} \tag{14}$$

The dropped terms or factors all represent quantities of the fourth order in our expansion. The term $\frac{(1-m)s^2\cot^2\zeta_1}{\rho}$ is, however, the largest of such quantities as a rule, and might be noticeable where D_1 and D_2 would not be.

Probably for short lines and small differences of elevation the most convenient formula would be

$$h_2-h_1=As\cot\zeta_1+A(\tfrac{1}{2}-m)\frac{s^2}{\rho} \tag{15}$$

and for other lines formula (7).

EXAMPLES

The data for the following examples, which illustrate the use of the formulas, come from The Transcontinental Triangulation, Special Publication No. 4, page 273, et. seq.

	At Snow Mountain West.	At Ross Mountain
	° ′ ″	° ′ ″
ζ	91 13 39.1	89 34 04.8
ϕ	39 22 38	38 30 20
α	18 56 18	197 49 29
Approx. elev.	2146 *meters*	672 *meters*

$\log s = 5.007341$.

For mean α and ϕ on the Clarke spheroid of 1866, $\log \rho = 6.80369$.

Example 1. Difference of elevation for reciprocal zenith distances, assuming Snow Mountain West as the known elevation from formula (5).

Example 2. Same data as Example 1 worked by formula (4).

Example 3. Assuming Ross Mountain as known elevation, solve by (4).

Example 4. With refraction from reciprocal zenith distances, but with only zenith distance at Snow Mountain West appearing explicitly, find difference of elevation by (7).

Example 5. Same data as Example 4, worked by (13).

Example 6. Like example 4, except zenith distance at Ross Mountain is used.

Example 7. Like example 5, except zenith distance at Ross Mountain is used.

The agreement of the differences of elevation as computed by the various combinations of data and formulas will give an idea of the accuracy of the latter.

Example 1			Example 2	Example 3
Station 1	Snow Mountain West			
Station 2	Ross Mountain			
	° ′ ″			
ζ_1	91 13 39.1			
ζ_2	89 34 04.8			
$\zeta_2-\zeta_1$	− 1 39 34.3			
$\frac{1}{2}(\zeta_2-\zeta_1)$	− 49 47.15			
$\frac{1}{2}(\zeta_2-\zeta_1)$ in secs.	− 2987.15			
log ditto	3.475257n			
T	4.685605			
log s	5.007341			
log s tan $\frac{1}{2}(\zeta_2-\zeta_1)$	3.168203n	log s tan $\frac{1}{2}(\zeta_2-\zeta_1)$	3.168203n	3.168203
s tan $\frac{1}{2}(\zeta_2-\zeta_1)$	−1473.00	log A	+146	+46
Second term*	− .03	log B	− 50	+50
Third term*	− .33	log C	+ 11	+11
h_2-h_1	−1473.36	log (h_2-h_1)	3.168310n	3.168310
		h_2-h_1	−1473.36	+1473.36
h_1	2145.66			
h_2	672.30			
	′ ″			
$\zeta_1+\zeta_2-180°$	47 43.9			
$\zeta_1+\zeta_2-180$ in secs.	2863.9			
log ditto	3.456958			
log ρ	6.803690			
log $\frac{1}{s}$	4.992659			
log $\frac{\sin 1''}{2}$ = 4.38454	4.384545			
log $(0.5-m)$	9.637852			
$(0.5-m)$	0.43436			

Example 4		Example 6
log s.	5.007341	(Same as example 4)
colog ρ	3.196310	
colog sin 1″	5.314425	
log θ†	3.518076	
log $(0.5-m)$	9.637852	
log $(0.5-m)\theta$	3.155928	
$(0.5-m)\theta$	1431.″95	
ζ_1	91° 13′ 39.″1	89° 34′ 04.″8
$(0.5-m)\theta$	23 51. 95	23 51. 95
$\zeta_1-(0.5-m)\theta$	90 49 47. 15	89 10 12. 85
$\frac{\theta}{2}$	27 28	27 28
$\zeta_1-(1-m)\theta$	90 22 19	88 42 45
log s	5.007341	5.007341
log A	+ 146	+ 46
log R	− 5	− 5
log cos $[\zeta_1-(0.5-m)\theta]$	8.160817n	8.160817
colog sin $[\zeta_1-(1-m)\theta]$	+9	+ 110
log (h_2-h_1)	3.168308n	3.168309
h_2-h_1	−1473.36	+1473.36

*Second and third terms in example 1 computed by aid of table in General Instructions for Field Work, Coast and Geodetic Survey, pp. 36–37 (edition of 1908).

† θ is used for $\frac{s}{\rho \sin 1''}$, as in the text.

Example 5					
log s	5.007341	log $(0.5-m)$	9.637852	log s^2	10.015
log A	+146	log s^2	10.014682	colog ρ	3.196
log D_1	+ 79	colog ρ	3.196310	cot^2 ζ_1	6.662
cot ζ_1	8.330975n	log A	146	log $(1-m)$	9.971
log first	3.338541n	log D_2	39	log third	9.844
		log second	2.849029		
First	−2180.424				
Second	+ 706.365				
Third	+ .698				
h_2-h_1	−1473.361				

Example 7					
log s	5.007341	log $(0.5-m)$	9.637852	log s^2	10.015
log A	+46	log s^2	10.014682	colog ρ	3.196
log D_1	+79	colog ρ	3.196310	cot^2 ζ_1	5.755
cot ζ_1	7.877539	log A	+46	log $(1-m)$	9.971
log first	2.884835	log D_2	+39	log third	8.937
		log second	2.848929		
First	+ 767.070				
Second	+ 706.202				
Third	+ .087				
h_2-h_1	+1473.359				

RECAPITULATION OF FORMULAS

(Numbered as in foregoing discussion)

Case I. Reciprocal observations

Former Coast and Geodetic Survey form,

$$h_2-h_1=s\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)\left[1+\frac{h_1+h_2}{2\rho}+\frac{s^2}{12\rho^2}\right]. \tag{5}$$

Reference: Page 210 and General Instructions for Field Work Coast and Geodetic Survey, pages 34–37 (edition of 1908).

Logarithmic form,

$$h_2-h_1=s\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)ABC. \tag{4}$$

Reference: Page 209 and tables.

Case II. Nonreciprocal observations

Former Coast and Geodetic Survey form,

$$h_2-h_1=s\cot\zeta_1+\frac{0.5-m}{\rho}s^2+\frac{(1-m)\,s^2\cot^2\zeta_1}{\rho}. \tag{8}$$

Reference: Page 211 and General Instructions for Field Work Coast and Geodetic Survey, pages 34–37 (edition of 1908).

Corrected form,

$$h_2-h_1=AD_1\,s\cot\zeta_1+\frac{0.5-m}{\rho}AD_2\,s^2+\frac{(1-m)\,s^2\cot^2\zeta_1}{\rho}\quad. \tag{13}$$

Reference: Page 213 and tables.

"Exact" form,

$$h_2 - h_1 = sAR \frac{\cos\left[\zeta_1 - (\tfrac{1}{2} - m)\dfrac{s}{\rho \sin 1''}\right]}{\sin\left[\zeta_1 - (\tfrac{1}{2} - m)\dfrac{s}{\rho \sin 1''} - \dfrac{s}{2\,\rho \sin 1''}\right]} \quad . \qquad (7)$$

Reference: Page 211 and tables; also Formulæ and Tables for Position Computation, Coast and Geodetic Survey Special Publication No. 8, for R.

See also additional note, page 220.

NOTES ON CONSTRUCTION AND USE OF TABLES

The tables are constructed with mean values of ρ and m .

$\log \rho = 6.80444$ corresponding to mean radius of curvature in latitude 40° for Clarke's spheroid of 1866.

$m = 0.06$. m varies between 0.05 and 0.10 in the great majority of cases. This value near the smaller limit was taken as probably nearer the truth for the high lines, in which the correction terms tabulated are most likely to appear, than an intermediate value of 0.07 or 0.08

A, B, C, D_1 and D_2 are all very near unity. To compute their logarithms the approximate expression $\log\ (1+x) = Mx$ was used, M being the modulus of common logarithms $= 0.43429$.

Formulas for constructing tables:

$$A = 1 + \frac{h}{\rho} \qquad \log A = \frac{Mh}{\rho} \qquad h = \frac{\rho}{M} \log A = 146.78 \log A$$

$\log A$ being in units of fifth place.

$$B = 1 + \frac{s}{2\rho} \tan\left(\frac{\zeta_2 - \zeta_1}{2}\right) \qquad \log B = \frac{Ms}{2\rho} \tan\left(\frac{\zeta_2 - \zeta_1}{2}\right)$$

$$\log\left[s \tan\left(\frac{\zeta_2 - \zeta_1}{2}\right)\right] = \log \frac{2\rho}{M} + \log\ (\log B)$$
$$= 7.4677 + \log\ (\log B)$$

$$C = 1 + \frac{s^2}{12\,\rho^2} \qquad \log C = \frac{Ms^2}{12\,\rho^2} \qquad \log s = \log\left(\rho\sqrt{\frac{12}{M}}\right) + \tfrac{1}{2} \log\ (\log C)$$
$$= 7.5251 + \tfrac{1}{2} \log\ (\log C)$$

$$D_1 = 1 + \frac{6\,(1-m)^2 - 1}{6}\,\frac{s^2}{\rho^2} \qquad \log D_1 = M\ \frac{6\,(1-m)^2 - 1}{6}\,\frac{s^2}{\rho^2}$$
$$\log s = 7.0578 + \tfrac{1}{2} \log\ (\log D_1)$$

$$D_2 = 1 + \frac{10 - 20\,m + 8\,m^2}{24}\,\frac{s^2}{\rho^2} \qquad \log D_2 = M\left(\frac{5 - 10m + 4\ m^2}{12}\right)\frac{s^2}{\rho^2}$$
$$\log s = 7.2027 + \tfrac{1}{2} \log\ (\log D_2)$$

The values of $\log A$, $\log B$, etc., were taken successively at 0.5, 1.5, 2.5, etc., units of the fifth place, namely, at the point where the value

of log A, log B, as rounded off to 5 decimals would change by one in the fifth place. The corresponding values of h, $\log\left[s \tan\left(\frac{\zeta_2-\zeta_1}{2}\right)\right]$ and log s were then computed by the formulas above. These values are carried out far enough so that the values of log A, log B, etc., may be obtained by interpolation to *six* decimals. In the numerical examples here given the values of log A, log B, etc., were computed independently for the actual values of ρ and m. These results as used in the example all agree within a unit of the sixth decimal place with those found by interpolating in the tables.

The unit of length throughout the tables and formulas is the meter.

Tables

Elevation of occupied station h_1	log A units of fifth place	Elevation of occupied station h_1	log A units of fifth place
Meters		*Meters*	
0	0.0	3009	20.5
73	0.5	3156	21.5
220	1.5	3303	22.5
367	2.5	3449	23.5
514	3.5	3596	24.5
661	4.5	3743	25.5
807	5.5	3890	26.5
954	6.5	4036	27.5
1101	7.5	4183	28.5
1248	8.5	4330	29.5
1394	9.5	4477	30.5
1541	10.5	4624	31.5
1688	11.5	4770	32.5
1835	12.5	4917	33.5
1982	13.5	5064	34.5
2128	14.5	5211	35.5
2275	15.5	5357	36.5
2422	16.5	5504	37.5
2569	17.5	5651	38.5
2715	18.5	5798	39.5
2862	19.5	5945	40.5

log A is *positive* except in the rare case when h_1 indicates a depression below mean sea level.

A is used for both reciprocal and nonreciprocal observations.

For reciprocal observations only (unless formula, p. 220, is used)				For nonreciprocal observations			
log approximate difference elevation $=\log s \tan\left(\frac{\zeta_2-\zeta_1}{2}\right)$*	log B units of 5th place	log s	log C	log s	log D_1 units of 5th place	log s	log D_2 units of 5th place
	0.0		0.0		0.0		0.0
2.167	0.5	4.875	0.5	4.407	0.5	4.552	0.5
2.644	1.5	5.113	1.5	4.646	1.5	4.791	1.5
2.866	2.5	5.224	2.5	4.757	2.5	4.902	2.5
3.011	3.5	5.297	3.5	4.830	3.5	4.975	3.5
3.121	4.5	5.352	4.5	4.884	4.5	5.029	4.5
3.208	5.5	5.395	5.5	4.928	5.5	5.073	5.5
3.281	6.5	5.432	6.5	4.964	6.5	5.109	6.5
3.343	7.5	5.463	7.5	4.995	7.5	5.140	7.5
3.397	8.5			5.023	8.5	5.167	8.5
3.445	9.5			5.047	9.5	5.192	9.5
3.489	10.5			5.068	10.5	5.213	10.5
3.528	11.5			5.088	11.5	5.233	11.5
3.565	12.5			5.106	12.5	5.251	12.5
3.598	13.5			5.123	13.5		
3.629	14.5			5.138	14.5		
3.658	15.5			5.153	15.5		
3.685	16.5			5.167	16.5		
3.711	17.5			5.179	17.5		
3.735	18.5			5.191	18.5		
3.758	19.5			5.203	19.5		
3.779	20.5			5.214	20.5		
3.800	21.5			5.224	21.5		
3.820	22.5			5.234	22.5		
3.839	23.5			5.243	23.5		
3.857	24.5			5.252	24.5		
3.874	25.5			5.261	25.5		

* Or $\log s \cot\left[\zeta_1-(0.5-m)\frac{s}{\rho \sin 1''}\right]$ for nonreciprocal observations. (See note 2, p. 220.)

log B has the same sign as the approximate difference of elevation.

log C is always positive.

log D_1 and log D_2 are always positive.

NOTES ON THE DEVELOPMENTS

NOTE 1.—The transformation of (1), page 208, may be conducted rather more simply than is there given.

$$h_2-h_1=\frac{2\,(\rho+h_1)\sin\left(\frac{\zeta_2-\zeta_1}{2}\right)\sin\frac{\theta}{2}}{\cos\left[\left(\frac{\zeta_2-\zeta_1}{2}\right)+\frac{\theta}{2}\right]} \tag{1}$$

or

$$h_2-h_1=\frac{2\,(\rho+h_1)\sin\left(\frac{\zeta_2-\zeta_1}{2}\right)\sin\frac{\theta}{2}}{\cos\left(\frac{\zeta_2-\zeta_1}{2}\right)\cos\frac{\theta}{2}-\sin\left(\frac{\zeta_2-\zeta_1}{2}\right)\sin\frac{\theta}{2}}$$

Divide numerator and denominator by $\cos\left(\frac{\zeta_2-\zeta_1}{2}\right)\cos\frac{\theta}{2}$,

$$h_2-h_1=\frac{2\,(\rho+h_1)\tan\frac{\theta}{2}\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)}{1-\tan\frac{\theta}{2}\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)}$$

or expanding $\tan\frac{\theta}{2}$ in series and using $\theta=\frac{s}{\rho}$,

$$h_2-h_1=\left(1+\frac{h}{\rho}\right)s\left(1+\frac{s^2}{12\rho^2}\right)\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)\left[1+\frac{s\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)}{2\rho}\right]$$

$$=s\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)ABC,$$

which is equation (4).

NOTE 2.—The formula for nonreciprocal observations may be put in the same form as that for reciprocal observations.

From the equation on page 208

$$\zeta_2=180^\circ-\zeta_1-2m\,\theta+\theta$$

$$\frac{\zeta_2-\zeta_1}{2}=\ 90-\zeta_1+(0.5-m)\,\theta$$

$$\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)=\cot\left[\zeta_1-(0.5-m)\,\theta\right]=\cot\left[\zeta_1-(0.5-m)\frac{s}{\rho\sin 1''}\right]$$

Substitute in (4)

$$h_2-h_1=s\cot\left[\zeta_1-(0.5-m)\frac{s}{\rho\sin 1''}\right]ABC$$

for nonreciprocal observations analogous to

$$h_2-h_1=s\tan\left(\frac{\zeta_2-\zeta_1}{2}\right)ABC$$

for reciprocal observations. B should be taken from table with argument

$$\log s\cot\left[\zeta_1-(0.5-m)\frac{s}{\rho\sin 1''}\right].$$

This is the present Coast and Geodetic Survey formula for nonreciprocal observations.

○