NOAA Technical Memorandum NWS SR-129

Q-VECTORS: A NEW APPROACH

TO AN OLD PROBLEM

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May 1990

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Q-Vectors: A New Approach to an Old Problem

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Abstract:
Many forecasters infer synoptic-scale vertical motion from the traditional form of the quasi-geostrophic (QG) omega (ω) equation, which relates vertical motion to fields of thickness and vorticity advection. This brief paper discusses the weaknesses typically involved in real-time interpretation of this form of the equation and acquaints the reader with an alternative form which greatly simplifies the task of inferring synoptic-scale vertical motion.

1. Introduction

Inferring synoptic-scale vertical motion is an old and formidable problem in meteorology. From work in the 1930s and 1940s (Sutcliffe 1939, 1947) ultimately came the QG ω-equation which relates synoptic-scale vertical motion to fields of vorticity and thickness advection. For reference the QG ω-equation is given as

$$\underbrace{(\sigma\nabla^2 + f_0^2 \frac{\partial^2}{\partial p^2})\omega}_{\text{Term A}} = \underbrace{f_0 \frac{\partial}{\partial p} (Vg\cdot\nabla(\frac{1}{f_0}\nabla^2\Phi + f))}_{\text{Term B}} + \underbrace{\nabla^2 (Vg\cdot\nabla(-\frac{\partial\Phi}{\partial p}))}_{\text{Term C}} \qquad (1)$$

where σ is static stability (which is related to the change of potential temperature with height), ∇^2, is the horizontal Laplacian, f is the Coriolis parameter at a given latitude, p is pressure, Vg^0, the geostrophic wind, Φ is geopotential, f, is the Coriolis parameter, and all derivatives are evaluated on constant pressure surfaces. In order to distinguish this form of the QG ω-equation from another which will be discussed shortly, this paper will adopt the terminology of Hoskins et al. (1978) by referring to eq. 1 as the conventional form of the QG ω-equation. Readers interested in a derivation and very thorough discussion of the equation should consult a dynamics textbook (e.g., Holton, 1979).

Although expressions very similar to eq. 1 formed the basis of numerical weather prediction (NWP) in the 1950s and 1960s (Thompson, 1961), NWP has progressed to the point where the primitive equations are integrated, and ω fields (700 mb) are available as part of the NGM and LFM product suites. However, during integration of the primitive equations, predicted changes emerge as small residuals from a great number of additions and subtractions of much larger numbers. Thus, the integration procedure cannot be understood in the sense that it can be quantitatively followed in our minds. To the meteorologist who wants as far as possible to understand what is occurring, and wants to check the numerical guidance, this is not a very satisfactory situation. Therefore, the meteorologist must have some understanding of basic atmospheric processes in a way not possible by reasoning based on the primitive equations alone. Eq. 1 allows the meteorologist to have such an understanding as applied to processes governing synoptic-scale weather systems. However, its mathematical form forces him or her to make many assumptions and approximations, the integrated effect of which is to make the equation difficult to accurately use in real time. Fortunately, alternative forms of the equation have been developed (Hoskins et al., 1978; Trenberth, 1978) which lend themselves more easily and profitably to real-time use.

This paper will acquaint the operational meteorologist with one of these alternative forms, the Q-vector form, which was introduced by Hoskins et al. (1978). This will be accomplished by (1) illustrating the difficulties normally encountered with real-time interpretation of eq. 1, (2) showing how the Q-vector form circumvents many of the difficulties, (3) discussing some of the potential problems associated with using the Q-vector form, and (4) offering suggestions for future research.

2. The Typical Approach to the Old Problem: Using the Conventional Form of the QG ω-Equation.

Before proceeding with a term-by-term discussion of the approximations commonly used to evaluate eq. 1, a few general comments are necessary. First, as with any equation, the meteorologist must always consider the assumptions used to derive it. Specifically, horizontal velocities are assumed to be geostrophic, the atmosphere is assumed to be hydrostatic and frictionless, and stability, as expressed in terms of the static stability parameter, σ, is assumed to be constant. Thus, situations wherein some or all of these assumptions are not likely to be valid (e.g., MCCs) will likely compromise the validity of the equation. Second, vertical motions, in the context of eq. 1, are seen as being forced by two mechanisms: differential vorticity advection (term B) and the Laplacian of thickness advection (term C). The vertical motions are part of the secondary circulations which maintain geostrophic and hydrostatic balance in the atmosphere (Holton, 1979).

Since vertical motions, in the context of eq. 1, are forced by two mechanisms, it is conceivable, if not highly probable, that contributions toward ω from each mechanism might offset the other. In fact, as will be discussed in Section 3, this occurs quite often, and therefore will be a motivation for deriving alternative forms of eq. 1. For the time being, however, we will concern ourselves with a term-by-term discussion of eq. 1, concentrating on how each term is commonly evaluated.

2.1 Term A: The Three-Dimensional Laplacian of ω

In order to evaluate this term, the meteorologist must have some understanding of the meaning of the Laplacian of a data field. Roughly speaking, the Laplacian of a data field at a point is determined by whether the field is at a maximum or minimum about that point. By using crude mathematical arguments, (e.g., Holton, 1979) this term is usually taken to be proportional to $-\omega$. This approximation allows vertical velocities to be estimated from the magnitude and sign of terms B and C without resorting to complicated numerical computations as described by Barnes (1987). Despite the simplicity offered by this approximation, it does introduce some error, the magnitude of which is a function of the wavelength and altitude of the particular disturbance under consideration (Durran and Snellman, 1987).

2.2 Term B: Differential Geostrophic Vorticity Advection

In order to properly evaluate this term, the meteorologist must know the geostrophic vorticity and geostrophic winds on a number of pressure surfaces. Since this information is normally not available, the term is commonly approximated as vorticity advection at a single level in the middle troposphere (e.g., 500 mb).

However, numerous assumptions are implicit to this approximation. The first two assumptions are (1) measured 500 mb winds are approximately geostrophic, and (2) 500 mb vorticity is approximately geostrophic. Other assumptions are (3) the vorticity of a parcel is changed only by convergence or divergence, and (4) vorticity patterns at upper levels move slower than the winds.

Given these four assumptions, consider the arguments of Petterssen (1956) based on calculations from extratropical systems. Following his arguments, we further assume (5) vorticity advection is nearly zero at the surface, so that regions of increasing (decreasing) vorticity are associated with convergence (divergence), (6) at upper levels, (300 mb) local changes in vorticity are small, so that PVA (NVA) implies divergence (convergence), and (7) the only level of non-divergence (LND) is found at approximately the 500 mb level, so the amplitude of vertical motions would be maximized and vorticity would be conserved at that level.

If all of these assumptions are valid, vorticity changes implied by PVA at 500 mb produce convergence below and divergence above that level, rising motion, or in the context of eq. 1, positive forcing. Using similar arguments, NVA at 500 mb implies sinking motion, or negative forcing.

2.3 Term C: The Laplacian of Thickness Advection

From Section 2.1, we know that in order for the meteorologist to correctly evaluate this term, he or she would have to determine where thickness advection is at a maximum or minimum. Since this is usually very difficult to do in real time, many meteorologists evaluate this term by (1) approximating the Laplacian as discussed in Section 2.1, and (2) assuming that measured winds are approximately geostrophic. Many meteorologists also substitute temperature for thickness ($-\partial\Phi/\partial p$) as shown by Holton (1979 p. 128). Considering these approximations in the context of eq. 1, warm advection (positive forcing) would contribute toward rising motion ($\omega < 0$) and cold advection (negative forcing), toward sinking ($\omega > 0$).

In Section 2.2, it was noted that many meteorologists evaluate term B at 500 mb. However, we have yet to define a level (or levels) where term C should be evaluated. Since temperature advection is usually strongest near the surface, many meteorologists evaluate the term at the 850 or 700 mb. levels.

3. Potential Problems with the Conventional Form of the QG ω-Equation.

As discussed, meteorologists typically make quite a number of assumptions when evaluating eq. 1. Many of these assumptions have been addressed in recent literature. Doswell (1982) examined the assumptions commonly used to evaluate the differential vorticity advection term (term B) and noted (1) actual winds may be far from being geostrophic, (2) vorticity patterns occasionally move faster than the winds, (3) convergence may not necessarily be the only mechanism by which parcels change their vorticity, and (4) the 500 mb level may not be near the LND. Durran and Snellman (1987) compared the qualitative evaluations of the terms in eq. 1 against their calculated values and concluded that accurate determination of vertical motion can be extremely difficult to do without resorting to numerical calculations. Other work by Barnes (1987) suggested that eq. 1 should be best evaluated by calculations involving layers of the atmosphere, as opposed to calculations performed on constant-pressure surfaces.

Nevertheless, many meteorologists approximate the conventional form of the QG ω-equation, and at times the approximations appear to work very well when judged in light of ensuing weather. Applying the approximations to Figs. 1-2, overlapping 500 mb PVA and 850 mb warm advection patterns suggested strong rising motion over western Texas and western Oklahoma, which helped to provide a favorable synoptic-scale environment for convection which soon developed (Fig. 3). Thus, in this example, the approximations appeared to work surprisingly well.

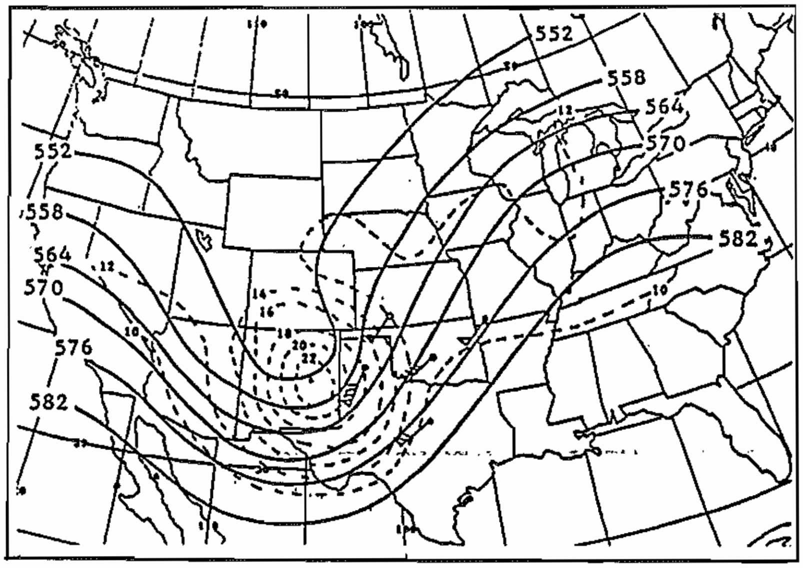

Figure 1. 500 mb height and vorticity fields 12Z November 15, 1988. Winds are plotted conventionally.

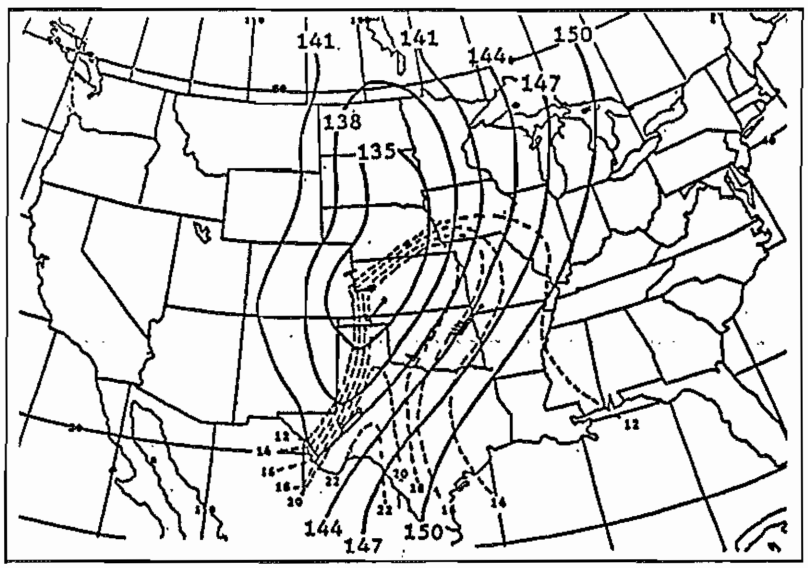

Figure 2. 850 mb height and temperature fields 12Z November 15, 1988. Winds are plotted conventionally. Temperatures are in Celsius.

Figure 3. Satellite photo, 1531Z November 15, 1988. Note squall line extending southward from western Oklahoma into western Texas.

However, there are cases when the approximations do not appear to work nearly as well; contributions toward ω from differential vorticity advection and temperature advection appear to offset each other. One such case is presented in Figs. 4-5. Fig. 4 shows strong PVA at 500 mb over northeastern Oklahoma, which alone implied rising motion, and perhaps precipitation. The corresponding 850 mb chart, Fig. 5, showed strong cold advection over that area, which alone implied sinking motion. In this and similar situations, it is not at all clear whether rising motion, implied by PVA, or sinking, implied by cold advection, dominate the vertical motion field.

Hoskins et al. (1978) noted that the terms in eq. 1 are commonly approximated, and observed that contributions toward ω from differential vorticity advection and temperature advection commonly offset each other, as they appeared to do in Figs. 4-5. These authors derived a simpler form of the QG ω-equation which allows the meteorologist to infer vertical motion without employing the numerous approximations discussed in Sections 2.2-2.3. This form of the equation, the Q-vector form, allows the meteorologist to take a new approach to the old problem of determining synoptic-scale vertical motions.

4. A New Approach to an Old Problem: The Q-vector

According to Hoskins et al. (1978), meteorologists can infer synoptic-scale vertical motion from the following equation

$$(\sigma\nabla^2 + f_0^2 \frac{\partial^2}{\partial p^2})\omega = -2\ \nabla\cdot Q + \beta \frac{\partial^2\Phi}{\partial x\partial p} \qquad (2)$$

where the Q-vector, Q, is defined as $\left(\frac{\partial V_g}{\partial x} \cdot \nabla(\frac{\partial\Phi}{\partial p})\ ,\ \frac{\partial V_g}{\partial y} \cdot \nabla(\frac{\partial\Phi}{\partial p})\right)$, $\nabla\cdot Q$ is the divergence of the Q-vector, and β is $\partial f/\partial y$.

Before discussing eq. 2, a few general comments are necessary. First, note that the left-hand side of eq. 2 is identical to that of eq. 1, implying that eq. 2 is a restatement, or alternative form, of the QG ω-equation. This point cannot be overemphasized. Readers desiring a mathematical discussion of eq. 2. should consult Hoskins et al. (1978) and Hoskins and Pedder (1980). Second, the rightmost term in eq. 2 is usually 10 to 100 times smaller than the term involving the divergence of the Q-vector. Therefore, most investigators (e.g., Hoskins and Pedder, 1980; Barnes, 1985; Barnes, 1987; Keyser et al., 1988) approximate the equation with little loss of generality as

$$(\sigma\nabla^2 + f_0^2 \frac{\partial^2}{\partial p^2})\omega = -2\ \nabla\cdot Q \qquad (3)$$

Though we have discussed eqs. 2 and 3, we have yet to discuss the physical significance of the Q-vector. At first glance, it may appear to some that the Q-vector has very little physical

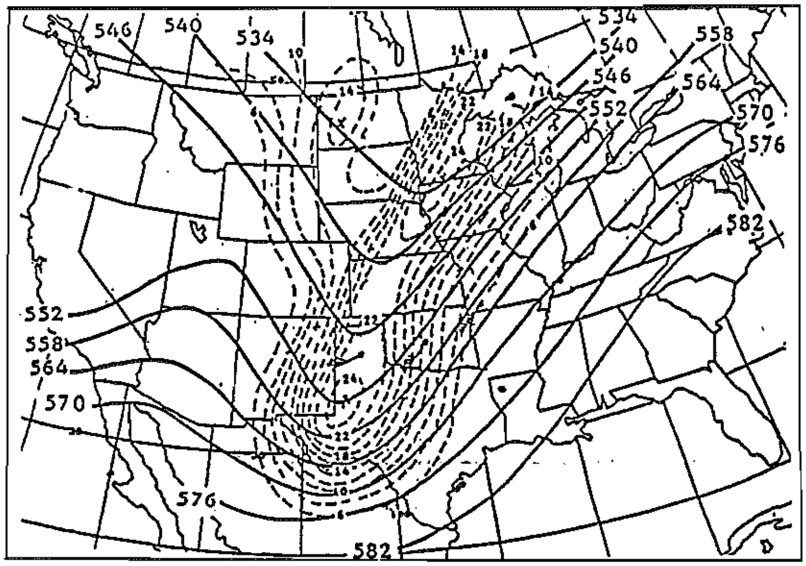

Figure 4. 500 mb height and vorticity fields 00Z December, 28 1988. Winds are plotted conventionally.

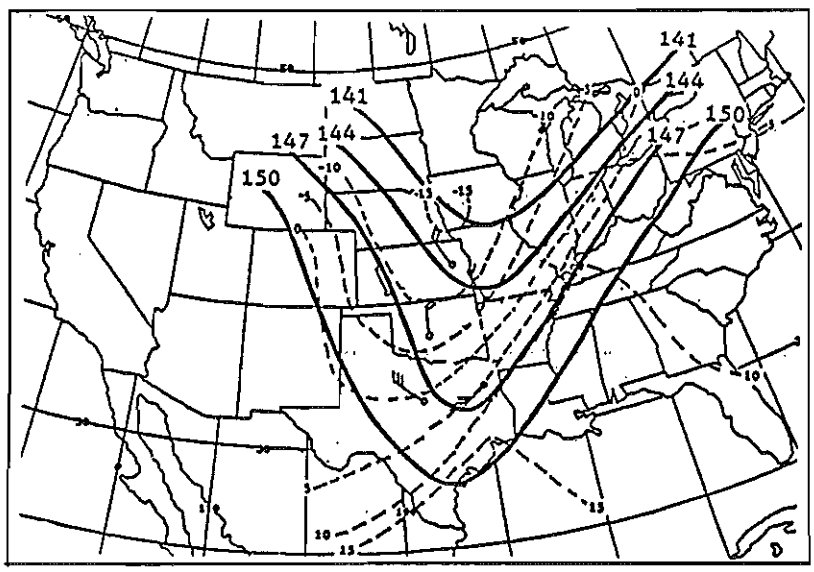

Figure 5. 850 mb height and temperature fields 00Z December 28, 1988. Winds are plotted conventionally. Temperatures are in Celsius.

significance, at least when they compare it to the old and familiar concepts of temperature and vorticity advection. However, as shown by Hoskins and Pedder (1980), the Q-vector is uniquely related to the secondary circulations briefly discussed in Section 2. As shown in Fig. 6, the Q-vector provides an approximate picture of the ageostrophic winds in the lower branch of the secondary circulation which maintains geostrophic and hydrostatic balance during an evolving synoptic situation. The Q-vector (1) points toward rising motion, (2) points away from sinking motion, and (3) lies in the same (opposite) direction as the ageostrophic wind below (above) the Q-vector's level of computation. The Q-vector is also closely related to frontogenesis, which the reader might suspect from its relationship to secondary circulations. Because the relationship between Q-vectors and frontogenesis is beyond the scope of this paper, the interested reader is strongly urged to study Keyser *et al.*, (1988) for a particularly insightful discussion.

4.1 Calculating Q-vectors

We see from the immediately preceding discussion that the Q-vector is quite physically meaningful. However, we still need to discuss how the Q-vector form of the QG ω-equation can be used to evaluate synoptic-scale vertical motion. We see from eq. 3 that the Laplacian of ω is related to Q-vector divergence. Therefore, if we approximate the Laplacian of ω as $-\omega$, (while realizing, of course, from Section 2.1 that this approximation is rather crude) we see that in order to infer synoptic-scale vertical motion, all the meteorologist needs to do is calculate the divergence of the Q-vector. Thus, the meteorologist would infer sinking (rising) motion where there is Q-vector divergence (convergence). Contrast the simplicity offered by the Q-vector form of the QG ω-equation to the number and the nature of the approximations used to evaluate the conventional form of the equation, as discussed in Sections 2.2 and 2.3. Thus, it should not be surprising that many in the meteorological community are adopting Q-vector analysis.

However, this simplicity comes with a penalty: Q-vectors and their divergence are very difficult to manually calculate in real time. Fortunately, a computer program is available (Foster, 1987) which computes Q-vectors and their divergence easily in real time using data extracted from the NWS AFOS. This program also calculates Q-vectors and their divergence for layers of atmosphere. Layer calculations are thought to better approximate synoptic-scale forcing than calculations on constant-pressure surfaces (Barnes, 1987). In Foster's program, Q-vectors are calculated at 700 mb using data from the 850, 700, and 500 mb levels. Examples of Q-vectors and their divergence (700 mb) fields as calculated by Foster's program will be shown in the next section.

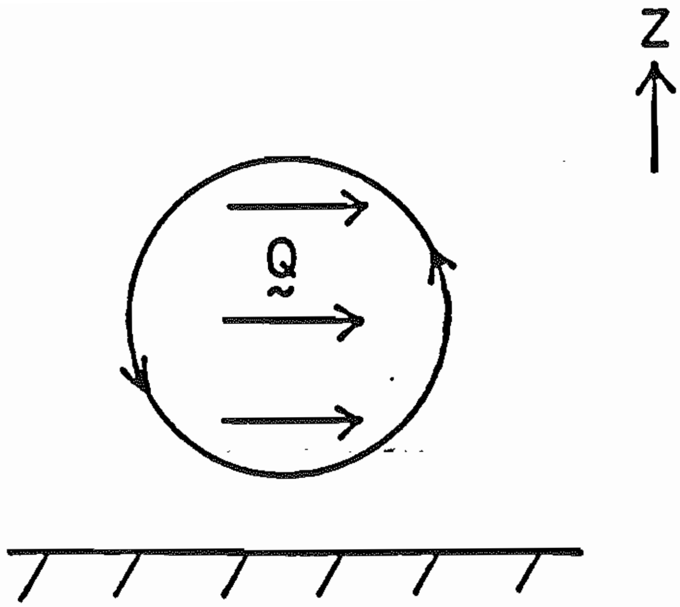

Figure 6. A representation of the relationship between a Q-vector and a secondary circulation. The vertical coordinate, z, points toward the top of the page. (adapted from Hoskins and Pedder, 1980)

4.2 Examples of Q-vectors and their Divergence Fields

Recall from Section 3 that the usual approximations of the conventional form of the QG ω-equation led to the inference from Figs. 1-2 of rising motion over western Texas and western Oklahoma. Since the Q-vector form of the ω-equation is a restatement of the conventional form of the equation, we should expect that Q-vector fields corresponding to Figs. 1-2 would also indicate rising motion over those areas. The corresponding Q-vector and Q-vector divergence fields are shown in Figs. 7-8. Since Q-vector convergence (which is negative) is associated with rising motion, rising motion is indicated over portions of the Texas Panhandle and western Oklahoma. This agrees with the conclusion reached from interpretation of the conventional form of the ω-equation given in Section 3.

The real power of Q-vector analysis will be seen in the next example. Recall from Section 3 that approximations of the conventional form of the ω-equation, as applied to Figs. 4-5, gave no definitive answer concerning the sign of vertical motion; contributions toward ω implied from PVA and cold advection appeared to offset each other. Figs. 9-10 shows the corresponding 700 mb Q-vector and Q-vector divergence fields. Note the area of Q-vector divergence (which is positive) over northeastern Oklahoma. Q-vector divergence implies sinking motion, so the atmosphere over northeastern Oklahoma was subject to sinking, drying and stabilization; hardly the large-scale conditions for precipitation. Figs. 9-10 played a prominent role in the forecast discussion between WSO Tulsa and WSFO Norman on the evening of December 28, 1988. Interpretation of the Q-vector field and other parameters culminated in the decision to delete any mention of precipitation in the forecast for the Tulsa metropolitan area on the evening of December 28. No precipitation occurred. This suggests that Q-vector analysis is a promising, real-time diagnostic technique.

4.3 Potential Limitations of Q-vector Analysis

The Q-vector form of the QG ω-equation has limitations which must be understood so that Q-vector analyses can be properly interpreted. Many of these limitations are also inherent to the conventional form of the QG ω-equation and are treated in standard dynamic meteorology texts (e.g., Holton, 1979; Pedlosky, 1979). A list of perhaps the most relevant limitations follows.

(1) The Q-vector form of the QG ω-equation strictly applies to synoptic-scale systems. Therefore, Q-vectors should not be used to determine vertical motion in mesoscale systems (e.g., MCCs).

(2) Stability is an important parameter which is modified by vertical motions (Hess, 1959). However, the Q-vector form of the QG ω-equation does not consider variations in stability. Therefore, the meteorologist will have to consider stability by other means.

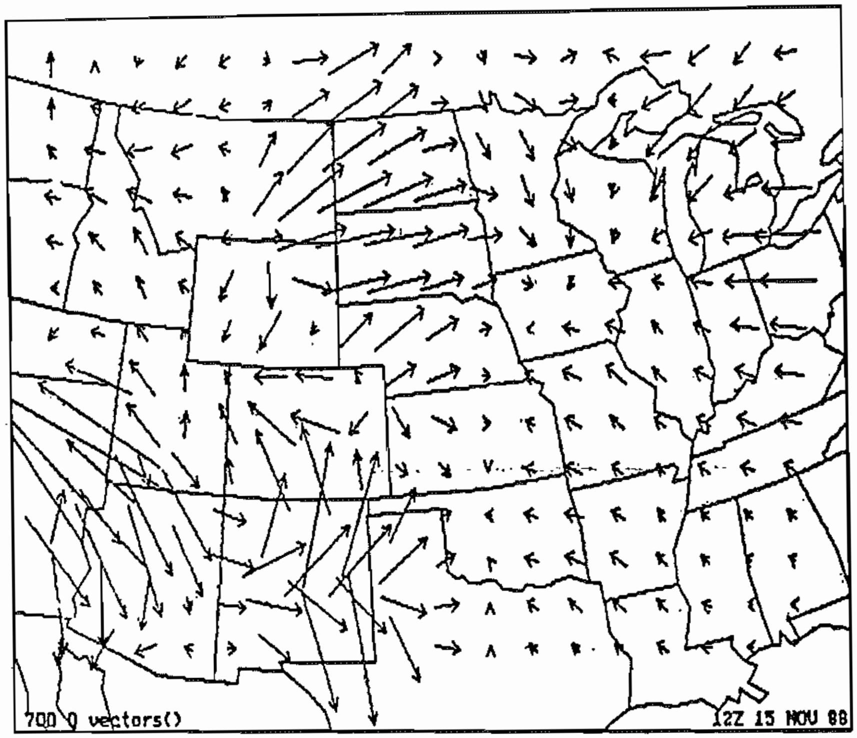

Figure 7. 700 mb Q-vector field 12Z November 15, 1988.

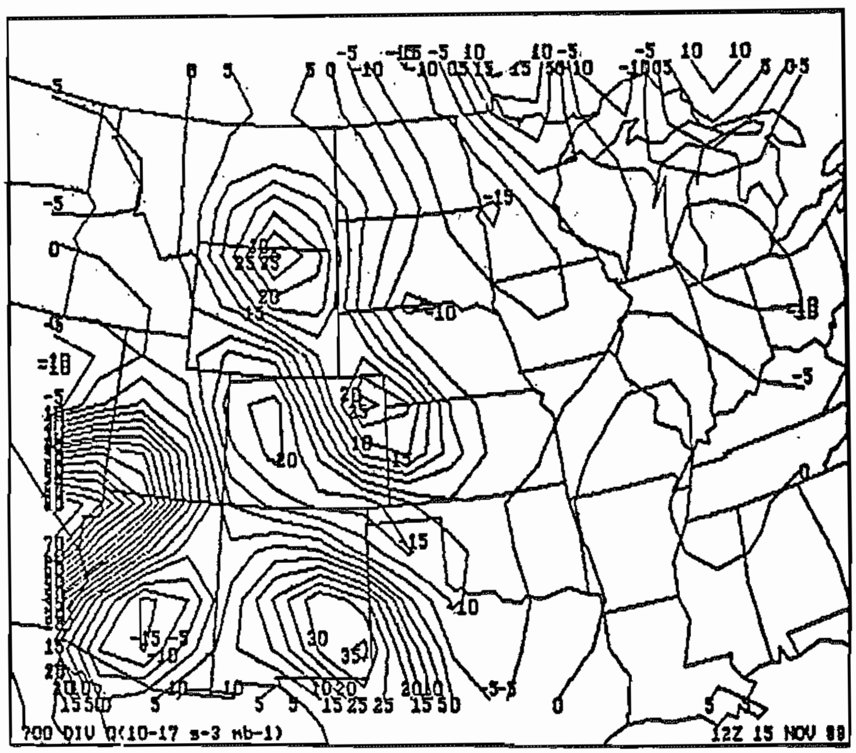

Figure 8. 700 mb Q-vector divergence field 12Z November 15, 1988. Note the area of Q-vector convergence (rising motion) over western Texas and western Oklahoma.

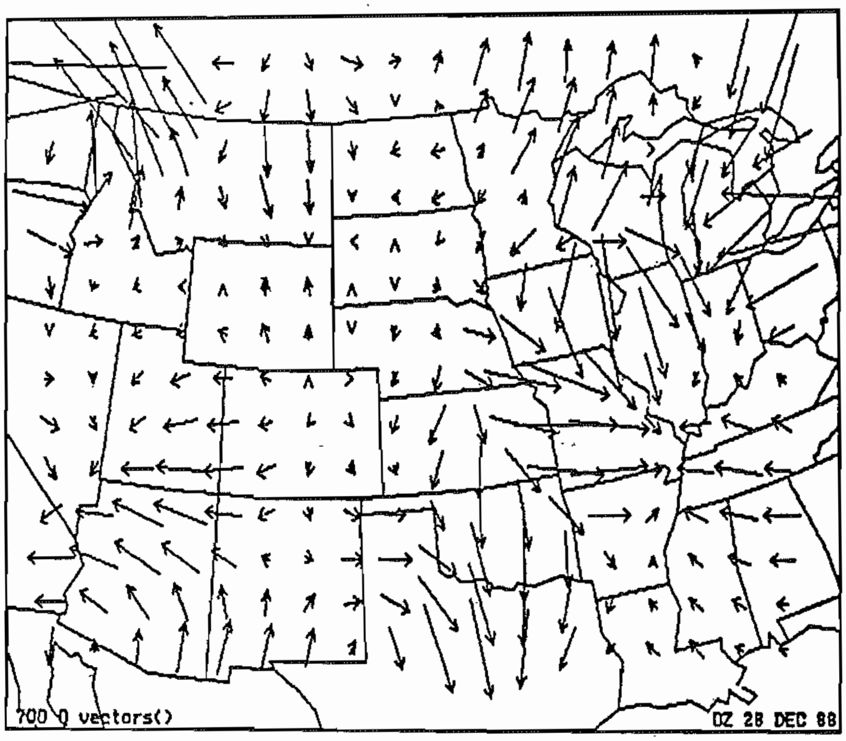

Figure 9. 700 mb Q-vector field 00Z December 28, 1988.

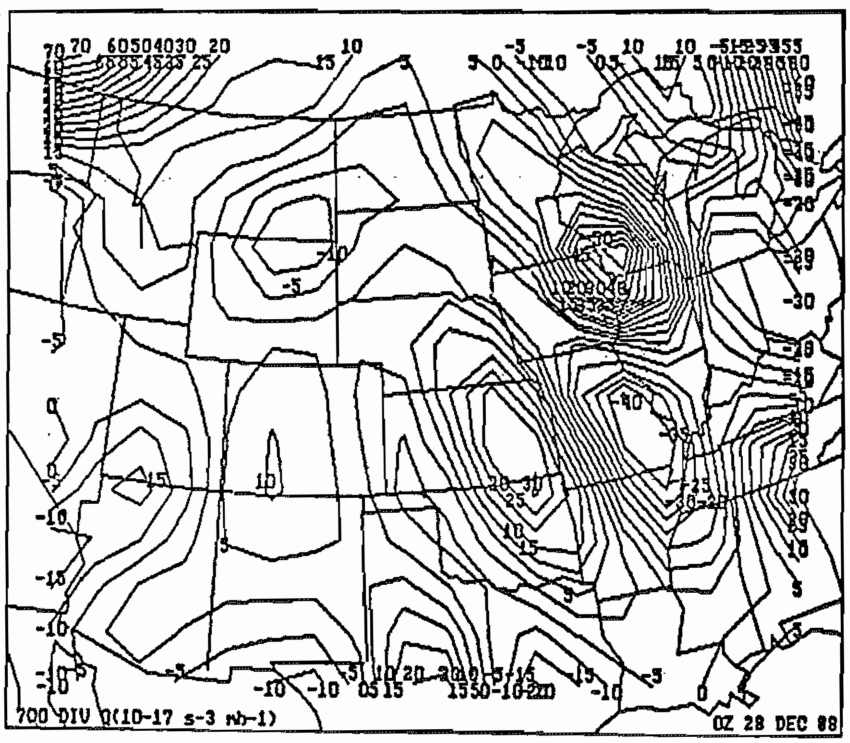

Figure 10. 700 mb Q-vector divergence field 00Z December 28, 1988. Note the area of Q-vector divergence (sinking motion) over northeastern Oklahoma.

(3) The effects of friction and diabatic heating (e.g., latent heat release, insolation) are neglected. Although these effects can be significant, their consideration greatly complicates both the Q-vector and conventional forms of the QG ω-equation.

(4) Q-vector analyses are not substitutes for re-analyzed NMC constant-pressure charts; they only simplify the task of inferring synoptic-scale vertical motion. The meteorologist should always re-analyze the charts to check the consistency of the Q-vector fields. Re-analyzing constant-pressure charts also occasionally highlights erroneous data, which if not corrected, causes unrepresentative Q-vector fields. Foster's upper-air program allows erroneous data to be easily deleted or corrected, and the Q-vector analyses can be re-run.

(5) The Q-vector form of the QG ω-equation does not consider the effects of terrain, which, of course, can be quite important.

(6) As discussed in Section 2.1, the Laplacian of ω is approximated as $-\omega$. This approximation is not always perfectly valid and could sometimes weaken the relationship between vertical motion and Q-vector divergence. However, as with many other approximations used in meteorology, we recognize that this approximation may not always be valid, but we usually cannot quantitatively assess its validity in a given situation.

5. Suggestions for Future Research

The Q-vector is being increasingly used by both the operational and research communities. Its ease of computation (with Foster's program) and relationship to secondary circulations suggest a number of potential topics for future research.

(1) From Hoskins and Pedder (1980) we know that the Q-vector is related to secondary circulations and ageostrophic winds above and below the Q-vector's level of computation. To the author's knowledge, no one has hitherto determined, in the context of Q-vector analysis, the vertical extent and the strength of the secondary circulations. (In other words, in a given situation, how far above and below the level of computation do the circulations go, and how strong are they?) These would be important questions, say, when a forecaster is trying to assess the effects of rising motion in the exit region of a jet on a potentially unstable atmosphere.

(2) Relatedly, Q-vectors might be used to study the relationship between upper- and low-level jets. Uccellini and Johnson (1979) suggested that vertical circulations associated with an upper-level jet could be coupled to the flow at low levels, suggesting that the intersection of an upper- and low-level jet is not merely a chance event. Assuming that the low-level jet also corresponds to a moist axis, a synoptic situation

long associated with tornadoes (Fawbush *et al.*, 1951; Beebe and Bates, 1955) would be created.

(3) Although many authors have suggested that Q-vector analyses are useful tools, no one has yet quantitatively evaluated the ability of Q-vectors to improve forecasts. If such an evaluation could be rigorously done, which would be difficult, the results would be of interest to many in the meteorological community.

(4) Currently, Foster's program calculates Q-vectors in a very reasonable amount of time. However, other means of graphically representing Q-vectors on the NGM and LFM prognoses should be investigated. Such representations would greatly increase the utility of Q-vector analysis by giving the meteorologist another way of diagnosing synoptic-scale vertical motions without relying on calculations of ω at 700 mb. Keyser *et al.*, (1988) might be a logical starting point for such work.

6. Conclusion

The Q-vector form of the ω-equation, which has existed for approximately 10 years, allows the meteorologist to infer synoptic-scale vertical motions without employing numerous and, at times, questionable assumptions concerning the distribution of thickness and vorticity fields. However, this simplicity comes with a penalty: Q-vectors and their divergence are very difficult to manually calculate in real time. Fortunately, computational power has increased by such a large degree in the past 10 years that Q-vector fields are easily calculated with a personal computer in real time.

The potential of Q-vector analysis for use in real time has been indicated in this paper and the more extensive work of others (e.g., Heflick and Fors, 1979; Hoskins and Pedder, 1980; Barnes 1985; Durran and Snellman, 1987; Doswell 1987). The author hopes that this paper will encourage other meteorologists to use Q-vector analyses in real time.

Acknowledgments

This paper originated as a presentation I gave at a WSO-RFC conference in Tulsa in March, 1989. An earlier version of this paper was used as a training paper for the staff at WSO Tulsa and WSFO Norman. The same version was also submitted to the NWSTC in Kansas City. I thank Mr. Don Devore, MIC, and the staff at WSO Tulsa for their comments and encouragement. I also thank Dr. Kenneth Crawford and Mr. Michael Foster of the WSFO Norman for being extremely helpful in the earliest stages of this endeavor. I am grateful to Dr. Charles Doswell of the National Severe Storms Laboratory, Mr. Peter Chaston of the NWSTC, and Mr. Daniel Smith, Chief, Scientific Services Division for reviewing an early version of the manuscript. I also thank Mr. Gregory Patrick, WSO Tulsa, for valuable suggestions. Last, and certainly not least, I thank

my wife, Cheryl, for tolerating my odd hours while I was writing this paper.

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