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UNITED STATES
DEPARTMENT OF THE INTERIOR
HAROLD L. ICKES, Secretary

FISH AND WILDLIFE SERVICE
IRA N. GABRIELSON, Director

Special Scientific Report No. 24

STUDIES ON THE PACIFIC PILCHARD OR SARDINE

(SARDINOPS CAERULEA)

5.—A method of Computing Mortalities and Replacements

By

Ralph P. Silliman

R. F. SHUMAN

Chicago
1943

Explanatory Note

The series, Special Scientific Reports, embodies results of investigations, usually of restricted scope, intended to aid or direct management or utilization practices and as guides for administrative or legislative action. It is issued in limited quantities for the official use of Federal, State, or cooperating agencies and in processed form for economy and to avoid delay in publication.

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INTRODUCTION

The place of estimates of mortality and replacement in a management program, and their position in the investigation of the pilchard resource of the Pacific coast, are pointed out by Sette (1943). Derivation of such statistics for fish populations has been investigated by Baranov (1918), Thompson and Bell (1934), and Ricker (1940), whose theories underlie the computations described herein. The last named student has developed concepts not included in the work of the earlier authors and gives mathematical formulations in a form more readily adaptable to the data available for the pilchard population than do the others. This report describes the application of Ricker's formulations to the pilchard data, and also develops new equations where necessary. To avoid adding to the symbols in the already confused set employed in fisheries statistics, those used by Ricker are adapted for this report.

This analysis was conceived and carried out under the inspiration and guidance of Oscar Elton Sette, in charge, South Pacific Investigations of the United States Fish and Wildlife Service. For critical reading of the manuscript and suggestions for improvement, the author is grateful to Drs. W. E. Ricker, M. B. Schaefer, J. L. Hart, and Frances N. Clark.

BASIC ASSUMPTIONS

In treating fisheries statistics by the method used herein it is necessary to make certain simplifying assumptions. These have been set forth by Ricker (p. 44) and may be condensed as follows:

1. The amount of effort expended toward catching fish is distributed uniformly over the geographical range of the species during the fishing season.

2. There is no competition between units of fishing gear for fish to be caught during one and the same instant.

Of the two types of fisheries postulated by Ricker, his "Type II" most nearly conforms to that of the pilchard fishery. It differs from his "Type I" in that natural mortality and recruitment take place during the fishing season. The listed assumptions are given by Ricker as applying to his "Type I" fishery, but are later implied to be inherent in "Type II" also (Ricker, 1940, p. 59).

They do not hold strictly true for the pilchard fishery, especially with regard to the uniform distribution of fishing effort. This does not preclude the use of the involved formulations, however, if it is kept in mind that they apply only to the extent to which the basic assumptions hold true.

KNOWN QUANTITIES AND RELATIONSHIPS

Use of the series of formulations given in this report is based on the assumption that the fishery has been stabilized at two different levels of exploitation during its development. Although the pilchard fishery has never reached complete stability, there were two periods during its history within which there was little shift in the total catch, and apparently little trend in the total fishing effort expended. The first period occurred when the expansion of the industry was temporarily halted by the economic depression of the early thirties and the second included the recent years of the fishery, during which the total catch, after having reached a maximum in the 1936-37 season, has fluctuated around an annual take of about half a million tons per year.

Uniform recruitment, as well as uniform exploitation, are requisites of a stabilized fishery. During the early years of the pilchard fishery, most of the recruitment was furnished by unusually successful age groups or broods, which occurred at intervals of about four years. Four "superabundant groups" have been recognized by Clark (1936) during the 15 seasons, 1919-20 to 1932-33. To average out these fluctuations in recruitment, the first period used in the study was chosen to include the passage through the fishery of two of the successful broods (designated groups "C" and "D" by Clark, 1936) and included the fishing seasons 1925-26 through 1932-33, thus representing a time somewhat prior to the depression. During the more recent period (1937-38 through 1941-42) the dominance of unusually successful groups has largely disappeared, (Clark, 1939 p. 21) indicating more uniform recruitment.

For each of the two levels of exploitation under consideration, it is necessary to have information on: (1) Age composition of the stock available to the commercial fishery, (2) total catch per season, and (3) relative amounts of effort (product of gear and time) expended per season. For the more recent of the two periods, age composition was determined from scales. During the earlier period scale readings were not taken and it was necessary to deduce age composition from length composition by a method involving growth rate and other characteristics of year classes. The total catch records were obtained from the weigh-in reports of the industry and converted to numbers by means of length distributions in samples of the catch in conjunction with previously determined length-weight relationships. Relative amounts of effort were calculated by dividing the total catch by the computed catch-per-unit-of-effort.[1]

Numerical values obtained for the three items of information are given in the accompanying Table 1. The data on age composition are from samples taken in the commercial fishery throughout each fishing season, exclusive of the summer fishery carried on in the Pacific Northwest.

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[1] This computation is in process of revision (1943).

Samples for the eight seasons of the early period were combined to discount the effect of fluctuating recruitment. For the latter period, when recruitment was more uniform, age readings for the last season (1941-42) only were used.[2]

Table 1.-- Numerical values determined for age composition, relative fishery effort, and catch.

Period	Hundreds of fish in samples of age (to last birthday)													Relative effort	Catch, billions of fish
	1	2	3	4	5	6	7	8	9	10	11	12	13		
1925-33	47	186	558	338	202	120	72	43	25	13	10	5	3	1	1.66
1937-42	58	527	310	56	14	2	1							4	4.98

The numerical values given in Table 1 represent the results of a canvass of the data available at present. Many shortcuts and approximations have entered into their derivation, but they have been used in the computations to obtain a preliminary estimate of the changes which have taken place in the pilchard stock, and to show the way in which more precise data may be used when available. The analyses necessary to the derivation of such precise data are now being carried on by the United States Fish and Wildlife Service and the California State Fisheries Laboratory.

DETERMINING TOTAL MORTALITY

The rate of disappearance, or total mortality, of the year classes may be determined from the relative age composition of the stock. As pointed out by Baranov (1918), in a stabilized population, the numbers of fish of each age, starting with the youngest commercial-size, are the same as the numbers of a given year-class surviving each year since its entry into the fishery. Therefore, the effective rate of disappearance may be determined from the age composition of the stock at any time after stabilization at that rate. The simplest method is to plot the logarithms of the numbers of fish of each age, so that a constant percentage decrease

[2] A recent study of the method of taking scale samples for use in the age analysis indicated that there was a tendency to select the larger fish of each sample (scales were taken from only 10 fish out of each 50 measured) for the removal of scales. This may have caused an indicated mortality rate lower than the actual. Computations to correct for this selection are now being made.

will be represented by a straight line. Figure 1 shows in this fashion the numerical values used in this paper. It will be seen at once that in both periods the numbers of fish over age two decline at a constant rate[3/], while those of ages one and two years do not. It was assumed that the one- and two-year-olds are not fully available to the fishery, and the line was fitted to the other ages by inspection. The lines shown indicate an annual decline in logarithm of .225 for the first period and .700 for the second. These may be translated into mortality rates (a) by the following formulations:

Let S_1 and S_2 be the relative stocks of any two successive ages.

Then $S_2 = S_1 - aS_1 = S_1(1-a)$

and $1 - a = \frac{S_2}{S_1}$

taking logs $\log (1-a) = \log S_2 - \log S_1$

or $\log (1-a) = -(\log S_2 - \log S_1) = -(\text{annual decline in log})$

Substituting numerical values given above, and letting subscripts refer to the two periods, we have:

$\log (1-a_1) = -.225 \qquad 1-a_1 = .60 \qquad a_1 = 1-.60 = .40$
$\log (1-a_2) = -.700 \qquad 1-a_2 = .20 \qquad a_2 = 1-.20 = .80$

In other words, the total mortality, or rate of disappearance, as indicated by the age compositions of the stocks, was 40 percent for the first, and 80 percent for the second, period.

SEPARATING FISHING MORTALITY FROM NATURAL MORTALITY

The total mortalities determined above may be broken down into fishing mortality and natural mortality if the latter is assumed to be the same for the two periods. In the absence of any evidence of a change in the rate of natural mortality, this assumption was made, and the method noted below was adopted.

This depends upon the fact that, with the same natural mortality (n) for two periods, a given pair of total mortalities (a_1, a_2) combined with a given ratio between the two fishing mortalities (m_1, m_2) is associated with a certain ratio between the amounts of effort (f_1, f_2) expended per year during the two periods. The procedure is to determine what ratio ($R = \frac{m_2}{m_1}$) between the two fishing mortalities gives the known ratio between

3/ The regular decrease in numbers of the fish in each age class for the earlier period is a result of the method of age analysis employed, rather than a natural phenomenon.

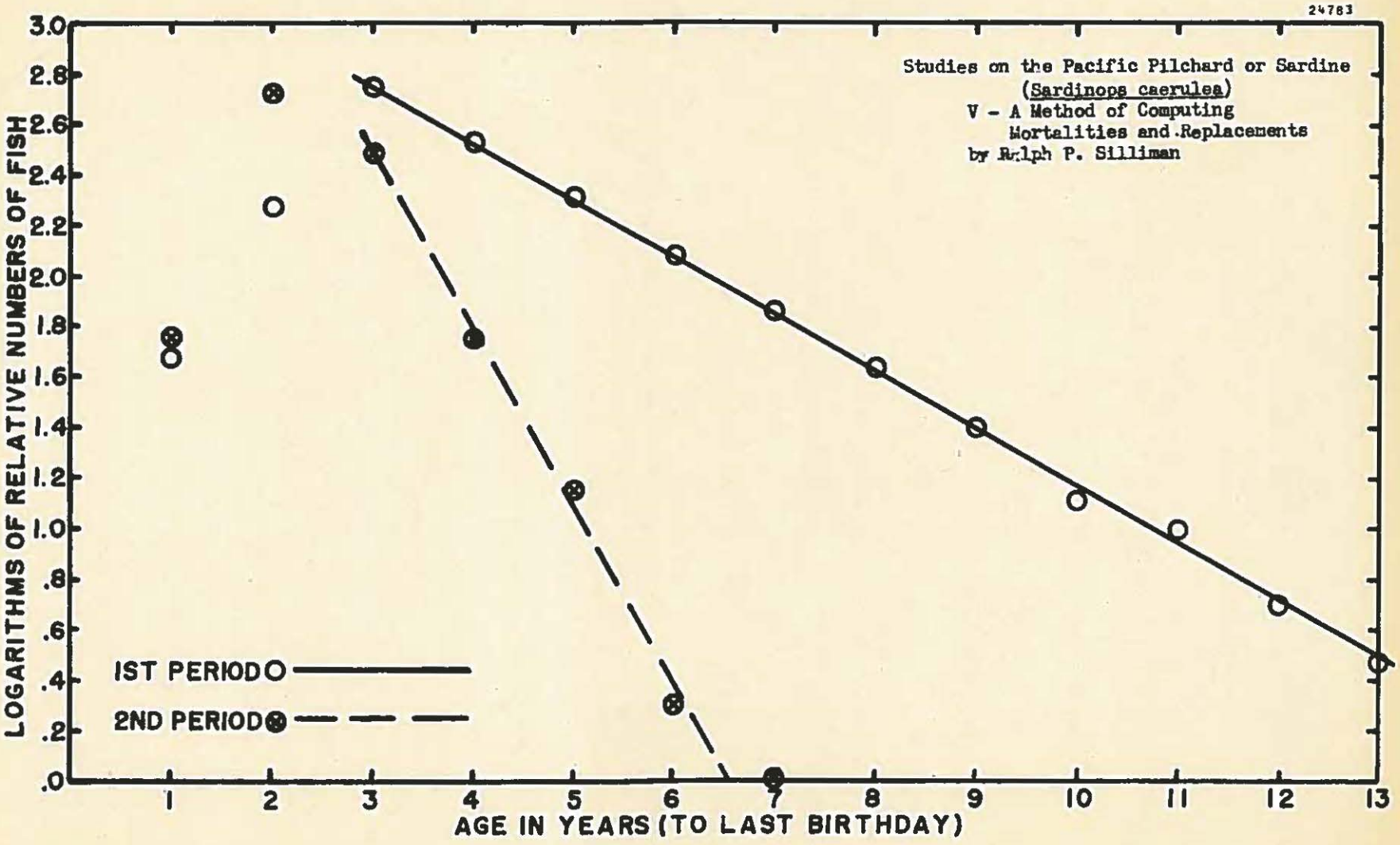

Figure 1. Relative age compositions of the stocks in the two periods of the pilchard fishery mentioned in the text.

the two amounts of effort ($\frac{f_2}{f_1} = 4$). Natural mortality (n) is put in terms of R, a_1 and a_2 as follows:

$$a_1 = m_1 + n - m_1 n$$
$$a_2 = m_2 + n - m_2 n$$

(Ricker, p. 60, 1st paragraph)

But, by definition: $R = \frac{m_2}{m_1}$ and $m_2 = Rm_1$

Substituting in the second equation above we have: $a_2 = Rm_1 + n - Rm_1 n$

Multiplying the first equation by R: $Ra_1 = Rm_1 + Rn - Rm_1 n$

Subtracting from the last equation the one above it:

$$Ra_1 - a_2 = Rn - n = n(R - 1)$$

$$\text{and } n = \frac{Ra_1 - a_2}{R - 1} \quad \text{(I)}$$

Now $\frac{f_2}{f_1}$ must be gotten in terms of R, n, a_1 and a_2 as follows:

Starting again with: $a_1 = m_1 + n - m_1 n$
and transposing: $a_1 - n = m_1 - m_1 n = m_1(1 - n)$

Dividing by $(1 - n)$: $m_1 = \frac{a_1 - n}{1 - n}$ (II)

Also, by definition: $m_2 = Rm_1$ (III)

Finally: $\frac{f_2}{f_1} = \frac{\log (1 - m_2)}{\log (1 - m_1)}$ (IV)

This last formula was developed by Ricker for fisheries of Type I. However, as shown by Schaefer (1943), it may be used equally well for fisheries of Type II.

Starting with a value of R of 2.4 and substituting our previously determined values of a_1 (.40) and a_2 (.80), $\frac{f_2}{f_1}$ may be computed from the above equations as follows:

$$\text{(I)} \quad n = \frac{2.4(.40) - .80}{2.4 - 1} = .1143$$

$$\text{(II)} \quad m_1 = \frac{.40 - .1143}{1 - .1143} = .3226$$

$$\text{(III)} \quad m_2 = 2.4(.3226) = .7742$$

$$\text{(IV)} \quad \frac{f_2}{f} = \frac{\log (1 - .7742)}{\log (1 - .3226)} = 3.82$$

Table 2.--Substituting successive values of R at intervals of .1, we have the following:

R	n	m_1	m_2	$\frac{f_2}{f_1}$
2.4	.1143	.3226	.7742	3.82
2.5	.1333	.3077	.7692	3.99
2.6	.1500	.2941	.7647	4.15

Obviously, the value of R most closely approaching the true value is 2.5, since the value of $\frac{f_2}{f_1}$ of 3.99 is closest to the empirical value of 4.0. Considering the variability of our data, any interpolation of values of R would be superfluous. Therefore, referring to the center line of the table, we now have natural mortality and fishing mortality for the two periods.

DETERMINING RATE OF RECRUITMENT

To obtain the number of recruits entering the fishery each year, we must first obtain the size of the available stocks for the two periods. These may be computed from the total catch statistics, if it is known what fraction the catch is of the available stock in each period. Ricker calls this fraction, "rate of exploitation" (μ). Its value may be determined from our previously computed values of total mortality (a), fishing mortality (m), and natural mortality (n) as follows:

$$\mu = \frac{m}{m + n}(m + n - mn) \quad \text{(Ricker, p.60, par. 2)}$$

But also: $a = m + n - mn$ (Ricker, p.60, par. 1)

Substituting: $\mu = \frac{am}{m + n}$ (V.)

Using the numerical values of a, m, and n given above, we have:

$$(V)\ \mu_1 = \frac{.40(.3077)}{.3077 + .1333} = .2791$$

$$(V)\ \mu_2 = \frac{.80(.7692)}{.7692 + .1333} = .6819$$

Since μ represents the fraction of the stock caught by the fishery, the available stock (S_A) may be computed from the catch (c) and rate of exploitation (μ) by the simple relationships:

$$c = \mu S_A \quad \text{or} \quad S_A = \frac{c}{\mu} \quad \text{(VI)}$$

Substituting the values of μ and c previously determined:

$$\text{(VI)} \quad S_{A1} = \frac{1.66}{.279} = 5.95 \text{ billions}$$

$$\text{(VI)} \quad S_{A2} = \frac{4.98}{.682} = 7.30 \text{ billions}$$

These stocks may now be broken down into their component ages in accordance with the previously determined relative age compositions.

Table 3.--Numbers of fish of each age obtained

Period	Billions of Fish of Age (to last birthday): 1	2	3	4	5	6	7	8	9	10	11	12	13
1	.18	.68	2.04	1.23	.74	.44	.26	.16	.09	.05	.04	.02	.02
2	.44	3.98	2.34	.43	.10	.02							

Since the fish of age one are not fully available to the fishery, it is necessary, in order that the recruitment to age one may be determined, to work backward from the first age class that is fully available, age three in our example. Let s_1 and s_2 be the available stocks of any two successive age classes, and S_1 and S_2 be the total stocks of these two age classes, respectively. If the stock of the second age class is considered fully available, $s_2 = S_2$. s_1 is subject to the full mortality rate a, but the balance of the age class $(S_1 - s_1)$ is subject only to natural mortality n. Formulating these statements we have:

$$s_2 = S_2 = S_1 - as_1 - n(S_1 - s_1)$$

Expanding: $s_2 = S_1 - as_1 - nS_1 + ns_1$

Transposing: $s_2 + as_1 - ns_1 = S_1 - nS_1$

Factoring: $s_2 + s_1(a - n) = S_1(1 - n)$

Dividing by $(1 - n)$: $S_1 = \dfrac{s_2 + s_1(a - n)}{1 - n}$ (VII)

Substituting the empirical values previously determined and letting subscripts refer to ages, we find for the first period:

$$\text{(VII)} \quad S_2 = \frac{2.04 + .68(.40 - .13)}{1 - .13} = 2.55 \text{ billions}$$

$$\text{(VII)} \quad S_1 = \frac{2.55 + .18(.40 - .13)}{1 - .13} = 2.99 \text{ billions}$$

And similarly for the second period:

$$\text{(VII)} \quad S_2 = \frac{2.34 + 3.98(.80 - .13)}{1 - .13} = 5.76 \text{ billions .}$$

$$\text{(VII)} \quad S_1 = \frac{5.76 + .44(.80 - .13)}{1 - .13} = 6.95 \text{ billions}$$

In other words, the estimated annual recruitment to age one during the second period was about two and a third times what it was in the first period.

EFFECT OF ERRORS IN ORIGINAL DATA

As stated earlier, the numerical values used in the foregoing computations have been obtained by approximations, and may be considerably in error. It is, therefore, desirable to know what effect errors in the various original data have on the final results.

The curves in figure 2 indicate the values of natural and fishing mortality which will be obtained with values of effort ratio and total mortality centering about those used in the foregoing computations. It will be noted that the effect of a given deviation is always greatest on natural mortality, and least on fishing mortality, in the second period. This is favorable to the practical application of the computations, since the rate of natural mortality, in itself, is of relatively minor importance in settling questions relating to the commercial fishery. On the other hand, rate of fishing mortality under current conditions (second period) of exploitation is of the greatest interest, since it leads to the estimation of the size of the stock of pilchards from which the commercial fishery must take its catch.

Considering the relative seriousness of various types of errors, it is apparent from figure 2 that errors in computation of total mortality are more serious than those in the effort ratio. For instance, the value of the effort ratio used in our computations was 4.0. From figure 2 it may be seen that, using total mortalities of .40 the first period and .80 the second period, a change in effort ratio from 4.0 to 3.5, a decrease of 12.5 percent, would cause an increase in the computed fishing mortality for the second period of only 1.9 percent (from .77 to .785). However, retaining the effort ratio of 4.0 and changing the total mortality for the first period from .40 to .35, again a decrease of 12.5 percent, would result in an increase in fishing mortality for the second period of 2.6 percent (from .77 to .79). In other words, a given percentage error in total mortality would produce about 1.4 times the effect on computed fishing mortality as the same percentage error in effort ratio (for the example given).

In general it may be said that fairly large errors in effort ratio would not seriously affect the results of the computations, provided that

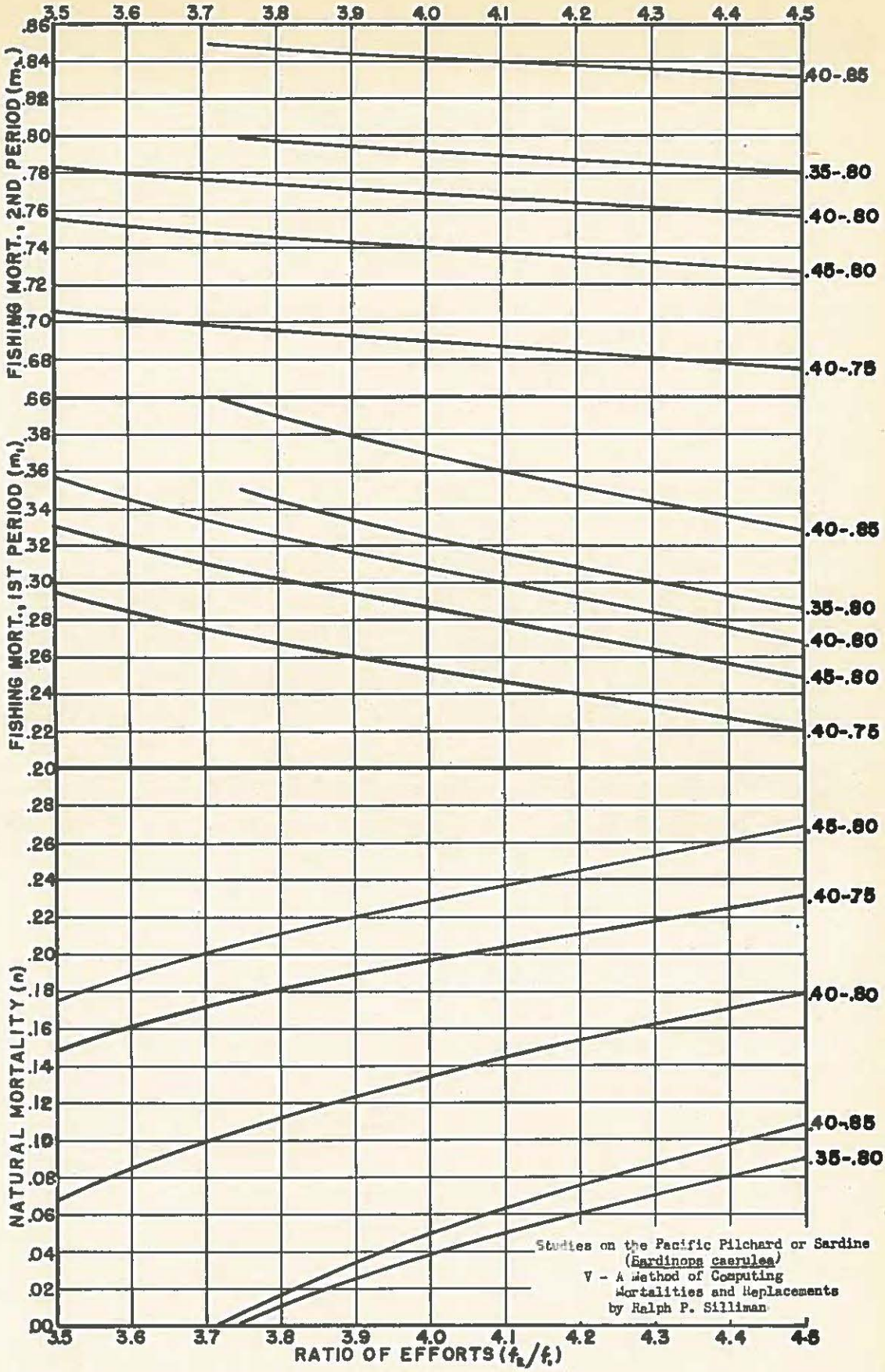

Figure 2. Natural and fishing mortalities for various combinations of effort ratio and total mortalities. Total mortalities for the first and second period respectively are given at the right-hand end of each curve.

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the estimates of total mortality were accurate. On the other hand, a relatively small error in computed total mortality might cause errors great enough seriously to affect the final interpretation of the results. Thus it will be desirable, in the more precise determinations to be made in the future, to place special emphasis on accuracy in computing total mortality.

SUMMARY

1. If it is assumed for a given population that fishing effort is uniformly distributed, and that instantaneous competition between units of gear is absent, it is possible to estimate natural mortality, fishing mortality, and rate of recruitment.

2. The source data required are: (1) Age composition of the commercial stock, (2) total catch per season, and (3) relative amount of effort expended per season. Each of these must be known for at least two periods during which the fishery was relatively stabilized.

3. Total mortality may be estimated directly from the age composition of the stock at any time after stabilization.

4. Natural mortality may be separated from fishing mortality by assuming various ratios between the fishing mortalities for the two periods, and making successive trials until a combination giving the known ratios between the amounts of effort for the two periods is found.

5. Rate of recruitment may be estimated by extrapolating the natural and fishing mortality rates from the first fully available age class to the entering age class.

6. Errors in the source data will affect the estimate of natural mortality the most seriously, and the estimate of fishing mortality during the second period the least seriously.

LITERATURE CITED

Baranov, F. I.
1918. On the question of the biological basis of fisheries. Nauchnyi issledovatelskii iktiologisheskii institut, Izvestia I (1), pp. 81-128, 12 figs.

Clark, Frances N.
1936. Interseasonal and intraseasonal changes in size of the California sardine (Sardinops caerulea). California Div. of Fish and Game, Bul. No. 47, 28 pp. 4 tables, 10 figs.

1939. Measures of abundance of the sardine (Sardinops caerulea) in California waters. California Div. Fish and Game, Bul. No. 53, 45 pp., 19 figs., 7 tables.

Ricker, W. E.
1940. Relation of "Catch per unit effort" to abundance and rate of exploitation. Journ. Fish. Res. Bd. of Canada, Vol. 5, No. 1, pp. 43-70, 4 tables, 3 figs.

Schaefer, M. B.
1943. The theoretical relationship between fishing effort and mortality. Copeia, 1943, No. 2.

Sette, O. E.
1943. Studies on the pacific pilchard or sardine (Sardinops caerulea): I - Structure of a research program to determine how fishing affects the resource. Special Scientific Report No. 19. U. S. Fish and Wildlife Service.

Thompson, W. F. and F. H. Bell
1934. Biological statistics of the Pacific halibut fishery. Int. Fish. Comm., Report No. 8; 49 pp., 15 tables, 18 figs.