# Numerical Weather Prediction in the Soviet Union

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## ABSTRACT

This article gives a review of the research done in the Soviet Union through 1959 on the theory and practice of numerical weather prediction by hydrodynamical methods. Russian meteorologists have used the same geostrophic forecast system as have other meteorologists and have carried out a number of test forecasts with electronic computers. Comparatively little has been published so far in objective weather-map analysis, general-circulation experiments and the use of non-geostrophic equations.

## 1. Introduction and historical background

W. Baum and P. Thompson (1959) have reviewed the research on long-range weather prediction in the Soviet Union; L. Battan (1959) has performed a similar service for English-speaking meteorologists by summarizing Russian work in cloud physics. The present article is a survey of Soviet work on the numerical prediction of large-scale weather patterns over periods of, at most, several days in advance by means of the equations of hydro- and thermo-dynamics. Numerical methods based on statistical techniques were not considered by us.

In addition to his book on numerical weather prediction (1957b), I. A. Kibel (Kibel and Blinova, 1957) has published in English a short summary of Soviet work in this field. However, the present writers have come to the conclusion that some of the Soviet work merits a more detailed description than was given by Kibel in the latter article; this is especially true of work on the theory of quasi-geostrophic motions. Where possible, we have compared the Russian work with equivalent non-Soviet work on the same subject. For the reader familiar with the English-language literature, this has the double merit of emphasizing differences in approach where they exist and of providing a familiar background on which to view the Soviet work where it is similar to non-Soviet work. Where there are interesting or significant differences in approach, we have described the Soviet work in some detail.

Russian work in numerical weather prediction may be considered to have begun in 1940, when I. A. Kibel (1940) derived his simple formula for predicting the motion of surface pressure centers. The attempt to apply Kibel's method during the 1940's resulted in the so-called "Advective-Dynamic Analysis" or ADA. Judging from the critical essay by Monin (1952a), this method did not give satisfactory results in practice.

Kibel began by introducing the idea of the *quasi-geostrophic expansion* into the equations of motion:

$$f(v - v_g) = \frac{du}{dt} \approx \left(\frac{\partial}{\partial t} + v_g \cdot \nabla\right) u_g, \qquad (1)$$

$$f(u - u_g) = -\frac{dv}{dt} \approx -\left(\frac{\partial}{\partial t} + v_g \cdot \nabla\right) v_g. \qquad (2)$$

Here, $f$ is the Coriolis parameter, and $\mathbf{v}$ is the horizontal velocity (with components $u$ and $v$). The subscript $g$ signifies "geostrophic." $d/dt$ is the substantial derivative. (Except where specified differently, the hydrostatic $x, y, p, t$—coordinate system will be used throughout this article.) The same technique was used by Philipps (1939) and later by Eliassen (1948).[1] Experience has shown that (1) and (2) are essentially correct for large-scale motions; interpreted properly, they can be used as the basis of the presently accepted quasi-geostrophic forecast system.

The formula Kibel derived for the surface pressure tendency, $\partial p_0/\partial t$, was very simple:

$$\frac{\partial p_0}{\partial t} = J\left(\frac{p_0, \theta}{x, y}\right). \qquad (3)$$

$\theta$ is here a linear function of the surface temperature and surface pressure. (3) amounts to an

[1] (1) and (2), or simplified forms of them, are also to be found in the works of Brunt and Douglas (1928) and Ertel (1941).

advection of the existing field of surface pressure in the wind field determined by $\theta$; the maximum and minimum values of $p_0$ therefore cannot change. The relations (1) and (2) were not used in deriving (3), although they were used by Kibel in deriving later modifications of (3). An analysis of this forecasting method has been made by Charney (1951).

As part of his justification of (1) and (2), Kibel introduced the important parameter $\epsilon$;

$$\epsilon = \frac{V}{fS} \sim 0.1. \tag{4}$$

Here, $V$ and $S$ are the orders of magnitude respectively of the horizontal wind and the horizontal wave-length of synoptic disturbances. $\epsilon$ has played an important role not only in later Russian works in dynamic meteorology but the same non-dimensional ratio is also very important in the scale theory of Charney (1947). $\epsilon$ is frequently called the "Rossby number" in American work.

A basic revision in the Soviet approach to numerical weather prediction seems to have been due to a remarkable paper by Obukhov (1949). This paper treated the mutual adjustment of wind and pressure in a barotropic atmosphere and is the Russian counterpart of the well-known papers by Rossby (1938) and Cahn (1945). However, it went beyond the simple adjustment concepts treated by Rossby and Cahn, and, as will be shown below, it contained an elegant derivation of the barotropic geostrophic forecast equation. From this point, Soviet work on numerical weather prediction is quite parallel to the developments which took place almost simultaneously in the rest of the world.

## 2. Justification of the theory of quasi-geostrophic motion

This subject has two aspects:

(a) What is the detailed mechanism by which the large-scale pressure fields and wind fields in the atmosphere are kept in approximate geostrophic balance?

(b) What equations govern the relatively slow rates of change with time of the geostrophic flow patterns?

The first aspect is normally analysed by considering any motion as a small perturbation on a *resting* atmosphere—*i.e.*, the same approach that is used in tidal theory. The linearized equations of motion then have as solutions three broad types of wave motion: acoustic waves, gravity waves, and a steady-state solution which is geostrophic. This limited perturbation approach does not end up with a complete explanation of the quasi-geostrophic nature of the large-scale flow pattern, however. It gives only a *description* of the way in which an initial localized unbalanced disturbance in a stable atmosphere will approach a steady-state geostrophic motion; the acoustic and gravity waves are simply free to disperse over the entire atmosphere. The fundamental explanation of *why* the flow is mainly geostrophic involves also the *stability* of atmospheric motions, together with the length and time scales of the external energy sources affecting the atmosphere.

Both aspects (a) and (b) have been treated in the Soviet and non-Soviet literature. In the latter, the two aspects have been treated more-or-less separately. For example, Rossby (1938), Cahn (1945) and Bolin (1953) have treated question (a), while Bjerknes and Holmboe (1944), Sutcliffe (1947), Charney (1948) and Eliassen (1948) have considered question (b) directly. The Russian approach has been more methodical, with both aspects frequently being treated in the same paper. As an example of this, we will first summarize the 1949 paper by Obukhov.

Obukhov considered a hydrostatic barotropic atmosphere. His basic equations may be written

$$\frac{\partial u}{\partial t} - fv + c^2\frac{\partial \chi}{\partial x} = - \mathbf{v}\cdot\nabla u, \tag{5}$$

$$\frac{\partial v}{\partial t} + fu + c^2\frac{\partial \chi}{\partial y} = - \mathbf{v}\cdot\nabla v, \tag{6}$$

$$\frac{\partial \chi}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = - \mathbf{v}\cdot\nabla \chi. \tag{7}$$

Here, $\chi$ is the logarithm of the surface pressure, $c^2 = gH$, and $f$ is taken as constant. $g$ is the acceleration of gravity, and $H$ is the average depth of the atmosphere.

Obukhov first examined the perturbation problem obtained by neglecting the non-linear terms on the right sides of (5)–(7). Two types of perturbations are possible:

(1) *Gravity-inertia waves.* These satisfy the differential equation

$$\frac{\partial^2 \chi}{\partial t^2} = c^2\nabla^2\chi - f^2\chi \tag{8}$$

$$(\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2)$$

and have a large phase speed. They also have the property that the potential vorticity, $\Omega$, associated with these waves is zero:

$$\Omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - f\chi. \tag{9}$$

(2) *Steady geostrophic motion.* Let $\phi$ and $\psi$ represent the horizontal velocity potential and stream function:

$$\begin{aligned} u &= \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial y}, \\ v &= \frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}. \end{aligned} \tag{10}$$

In addition to the gravity inertia waves, a steady-state motion is possible with the following properties:

$$\begin{aligned} \frac{\partial}{\partial t} &= 0, \\ \phi &= 0, \\ \psi &= \frac{c^2}{f}\,x, \\ \Omega &= \frac{c^2}{f}\nabla^2\chi - fx. \end{aligned} \tag{11}$$

It is easy to show directly from the linearized forms of (5)–(7) that $\partial\Omega/\partial t = 0$. Obukhov now argues as follows. Suppose that an initial disturbance, which in general will contain both gravity waves and geostrophic motion, is limited to a finite area. The gravity waves will disperse over the rest of the atmosphere, and a final equilibrium satisfying (11) will be reached. [He demonstrates this mathematically from the influence function solution of (8).] Since the potential vorticity is independent of time,

$$\Omega(x, y, t) \equiv \Omega(x, y, 0).$$

The final equilibrium state can therefore be determined by using the last equation of (11):

$$\frac{c^2}{f}\nabla^2\chi_\infty - f\chi_\infty = \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - f\chi\right]_{t=0}. \tag{12}$$

$\chi_\infty$ is the final equilibrium distribution of $\chi$. The final wind field is geostrophic and determined by (10) and (11) from $\chi_\infty$. The results obtained by Obukhov in this analysis of the linearized problem are the same as those obtained by Rossby and by Cahn.

In the last part of his paper, Obukhov investigated aspect (b); *i.e.*, what equations govern the slow rate of change with time of the geostrophic flow pattern? His development here is more formal than the heuristic derivations familiar to Western meteorologists. First, the variables $x, y, t$ and $u, v, \chi$ in (5)–(7) are replaced by non-dimensional variables $\xi, \eta, t'$ and $u_1, v_1, \pi_1$:

$$\xi = \frac{x}{L_0}, \eta = \frac{y}{L_0}, t' = ft;$$

$$u_1 = \frac{u}{W_0}, v_1 = \frac{v}{W_0}, \pi_1 = \left(\frac{c^2}{fW_0L_0}\right)\chi. \tag{13}$$

$L_0$ and $W_0$ are length and velocity scales. The non-dimensional numbers

$$\lambda = \frac{W_0}{fL_0}, \beta = \frac{fL_0}{c} = \frac{L_0}{L_1}, \tag{14}$$

are thereby introduced. $L_1 = f^{-1}\sqrt{gH}$ is equal to Rossby's "radius of deformation" (1938). (5)–(7) may now be written

$$\frac{\partial u_1}{\partial t'} - v_1 + \frac{\partial \pi_1}{\partial \xi} = -\lambda \mathbf{v}_1\cdot\nabla' u_1, \tag{15}$$

$$\frac{\partial v_1}{\partial t'} + u_1 + \frac{\partial \pi_1}{\partial \eta} = -\lambda \mathbf{v}_1\cdot\nabla' v_1, \tag{16}$$

$$\frac{\partial \pi_1}{\partial t} + \frac{1}{\beta^2}\nabla'\cdot\mathbf{v}_1 = -\lambda \mathbf{v}_1\cdot\nabla'\pi_1. \tag{17}$$

($\nabla'$ is the gradient operator expressed by derivatives with respect to $\xi$ and $\eta$.) The parameter $\lambda$, which is equivalent to Kibel's $\epsilon$, has a value of about 0.1 for large-scale processes.

Obukhov's reasoning is now the following. If $\lambda$ were equal to zero, (15)–(17) would reduce to the linear problem, whose solution is given by (8)–(11). Since $\lambda$ is small, the *non-linear* solution must differ only by quantities of the order of magnitude of $\lambda$ from the *linear solution*. We are interested in the geostrophic type of motion. We may therefore assume that in the non-linear case the first three relations of (11) will have the form

$$\begin{aligned} \frac{\partial}{\partial t'} &= O(\lambda), \\ \phi_1 &= O(\lambda) = \lambda\phi_1^* + O(\lambda^2), \\ \pi_1 - \psi_1 &= O(\lambda) = \lambda r_1 + O(\lambda^2). \end{aligned} \tag{18}$$

$\phi_1$ and $\psi_1$ are non-dimensional values of the velocity potential and stream function:

$$v_1 = \frac{\partial \psi_1}{\partial \xi} + \frac{\partial \phi_1}{\partial \eta}, \quad u_1 = -\frac{\partial \psi_1}{\partial \eta} + \frac{\partial \phi_1}{\partial \xi}. \tag{19}$$

$r_1$ and $\phi_1{}^*$ are quantities of order 1. When multiplied by $\lambda$, they give the corrections to the purely geostrophic values of $\psi_1$ and $\phi_1$.

Introducing (18) and (19) into (15)–(17) then gives us

$$\frac{\partial u_1}{\partial t'} = -\lambda\left[v_1\cdot\nabla' u_1 + \frac{\partial r_1}{\partial \xi} - \frac{\partial \phi_1{}^*}{\partial \eta}\right] + O(\lambda^2), \quad (20)$$

$$\frac{\partial v_1}{\partial t'} = -\lambda\left[v_1\cdot\nabla' v_1 + \frac{\partial r_1}{\partial \eta} + \frac{\partial \phi_1{}^*}{\partial \xi}\right] + O(\lambda^2), \quad (21)$$

$$\frac{\partial \pi_1}{\partial t'} = -\lambda\left[v_1\cdot\nabla' \pi_1 + \frac{1}{\beta^2}\nabla'^2\phi_1{}^*\right] + O(\lambda^2). \quad (22)$$

The $\lambda^2$ terms are now dropped, and $t'$ is replaced by the new time variable $\tau$:

$$\tau = \lambda t' = \lambda f t = \frac{W_0}{L_0}t. \quad (23)$$

(20)–(22) then assume the form

$$\frac{\partial u_1}{\partial \tau} = -v_1\cdot\nabla' u_1 - \frac{\partial r_1}{\partial \xi} + \frac{\partial \phi_1{}^*}{\partial \eta}, \quad (24)$$

$$\frac{\partial v_1}{\partial \tau} = -v_1\cdot\nabla' v_1 - \frac{\partial r_1}{\partial \eta} - \frac{\partial \phi_1{}^*}{\partial \xi}, \quad (25)$$

$$\frac{\partial \pi_1}{\partial \tau} = -\frac{1}{\beta^2}\nabla'^2\phi_1{}^*. \quad (26)$$

[$v_1'\cdot\nabla'\pi_1$, according to (18), is at most of order $\lambda$.] In these equations, $u_1$ and $v_1$ must be of zero order in $\lambda$ since all other terms are of zero order in $\lambda$ and are therefore given by the geostrophic relations

$$u_1 = -\frac{\partial \pi_1}{\partial \eta}, \quad v_1 = \frac{\partial \pi_1}{\partial \xi}.$$

An equation for $r_1$ is readily obtained from (24) and (25):

$$\begin{aligned}\nabla'^2 r_1 &= -\frac{\partial}{\partial \xi}(v_1\cdot\nabla' u_1) - \frac{\partial}{\partial \eta}(v_1\cdot\nabla' v_1)\\ &= 2\left[\frac{\partial^2\pi_1}{\partial \xi^2}\frac{\partial^2\pi_1}{\partial \eta^2} - \left(\frac{\partial^2\pi_1}{\partial \xi\partial\eta}\right)^2\right].\end{aligned} \quad (27)$$

This, as is clear from (18), determines the first-order correction to the pure geostrophic value for the streamfunction $\psi_1$. *It is equivalent to the so-called* "balance equation" *proposed by Charney (1955) and by Bolin (1955)*, except that the nonlinear terms in (27) are evaluated geostropically.

The corresponding equation for $\phi_1{}^*$ is

$$\begin{aligned}\nabla'^2\phi_1{}^* &= \frac{\partial}{\partial \eta}\left[\frac{\partial u_1}{\partial \tau} + v_1\cdot\nabla' u_1\right]\\ &\quad - \frac{\partial \xi}{\partial}\left[\frac{\partial v_1}{\partial \tau} + v_1\cdot\nabla' v_1\right] = -\nabla'^2\frac{\partial \pi_1}{\partial \tau}\\ &\quad - J\left(\frac{\pi_1, \nabla'^2\pi_1}{\xi, \eta}\right).\end{aligned} \quad (28)$$

This is the familiar geostrophic vorticity equation for the barotropic atmosphere, since $\nabla'^2\phi_1{}^*$ is the horizontal divergence. (Obukhov is aware of the Rossby effect due to the variability of $f$, but in this simple model he prefers not to include it.)

Finally, (28) can be combined with (26):

$$\nabla'^2\frac{\partial \pi_1}{\partial \tau} - \beta^2\frac{\partial \pi_1}{\partial \tau} = -J\left(\frac{\pi_1, \nabla'^2\pi_1}{\xi, \eta}\right). \quad (29)$$

This is the geostrophic forecast equation for a barotropic atmosphere.

Obukhov's paper has since been followed by six or seven other Russian papers, each of which has extended his theory to more realistic atmospheres. Monin (1952b) reproduced Obukhov's barotropic analysis and studied the vertical velocity field associated with (24)–(26). Yaglom (1953) extended Obukhov's analysis by introducing spherical geometry. He showed that in the spherical case the two types of perturbations derived by Obukhov corresponded to the "first" and "second" class of tidal motions studied by Hough (1897).

Kibel (1955) applied Obukhov's linear perturbation analysis to a hydrostatic, resting, stably-stratified atmosphere with a constant $f$. He first simplified the analysis by eliminating "external" gravity waves through the assumption that $\partial p_0/\partial t = 0$. ($p_0$ is the surface pressure.) His "internal" gravity waves are governed by the differential equation

$$c^2\nabla^2\phi + \frac{\partial}{\partial p}\left[p^2\frac{\partial}{\partial p}\left(\frac{\partial^2\phi}{\partial t^2} + f^2\phi\right)\right] = 0, \quad (30)$$

where $\phi$ is again the horizontal velocity potential. The constant $c^2$ is here equal to $(\gamma_a - \gamma)R^2\bar{T}/g$, with $\gamma_a$ and $\gamma$ the adiabatic and actual lapse rates of temperature. $R$ is the gas constant, and $\bar{T}$ is a mean temperature. Kibel first obtained the general solution of the initial value problem (30) in the form of an influence function. From the form of the influence function, he was able to show that an initial limited disturbance would approach a steady-state geostrophic equilibrium, just as Obukhov had demonstrated for the barotropic atmosphere. [The analysis by Bolin (1953) is very similar to that of Kibel's, except that Bolin dealt with an incompressible fluid and retained the external gravity waves.] Kibel was also able to show that the initial potential vorticity field determined the details of the final steady geostrophic motion in this baroclinic atmosphere.

Later, Kibel (1957a) extended this analysis in the following interesting way. Instead of discarding the non-linear terms in the derivation of (30), he carried them along and obtained thereby an equation similar to (30) except for the presence of some non-linear terms on the right side. These latter express the effect of vorticity advection and temperature advection and are considered by Kibel in this analysis as known functions of space but independent of time. He then obtained the influence function solution for this *non-homogeneous* initial-value problem. From the form of this solution, he was able to show that, as $t \to \infty$, the velocity potential $\phi$ does not approach the zero-value characteristic of the homogeneous solution but rather approaches the same value of $\phi$ that would be given by the usual quasi-geostrophic theory for the same vorticity and temperature advections. (See also section 8.)

In 1957, Sadokov and Dobryshman (1957) studied the solution of (30) without neglecting the external gravity waves. Their results, together with the earlier work of Obukhov, Yaglom and Kibel, are contained in a thorough survey paper by Monin (1958). Monin's analysis of the adjustment problem for the hydrostatic baroclinic atmosphere follows very closely the approach used by Obukhov in 1949 for the barotropic atmosphere. Instead of the simple barotropic formula (29), Monin derived the baroclinic form of the geostrophic equation (35).[2]

As the last paper in this series, we may mention the paper by Monin and Obukhov (1958). [For an English summary, see Monin and Obukhov (1959).] In this paper, Monin and Obukhov did not make the hydrostatic assumption but limited themselves to a consideration only of the perturbations on a resting atmosphere. In addition to internal gravity waves and stationary geostrophic motion, acoustic waves are now possible. The results are very similar to those given by Eliassen (1957). The acoustic waves disperse even more rapidly than the gravity waves and therefore do not slow up the process of geostrophic adaptation.

[2] (35), or an equation similar to it, was evidently derived earlier by Kibel (1951).

The Soviet work described above represents a very thorough and methodical mathematical investigation of the theory of quasi-geostrophic adjustment. Non-Soviet work has in general treated the same questions with similar results. However, the elegant type of formal mathematical development introduced by Obukhov [eq. (13)–(29) above] has not been used by non-Soviet meteorologists[3]. The latter have instead concerned themselves from the beginning with the effects of variable $f$ and a basic zonal current on the gravitational and geostrophic wave motions. These effects confuse the simple relations (8)–(11) enough so that a development similar to (13)–(29) is too complicated, and a more heuristic justification of the quasi-geostrophic forecast equations becomes necessary.

The advantage of the Soviet approach would seem to be primarily the equal emphasis given to (27) and (28). According to this view, the "balance equation" is a logical *result* of the quasi-geostrophic development, whereas Charney (1955) and Bolin (1955) have instead presented the balance equation as an assumed generalization of the geostrophic wind formula.[4]

### 3. The geostrophic forecast equations

The well-known geostrophic forecast system may be expressed in terms of the vorticity equation and the thermodynamic equation:

$$\frac{\partial \zeta_g}{\partial t} + v_g \cdot \nabla (f + \zeta_g) = f_0 \frac{\partial \omega}{\partial p}, \tag{31}$$

$$\frac{\partial^2 \Phi}{\partial t \partial p} + v_g \cdot \nabla \left(\frac{\partial \Phi}{\partial p}\right) = -\sigma\omega. \tag{32}$$

In these equations, $\Phi$ is the geopotential of an isobaric surface, $\omega = dp/dt$, and $f_0$ is a mean value of $f$. The static stability parameter, $\sigma$, is positive and may be a function of $p$ only. It is equal to $(\partial \ln \theta/\partial p)(\partial \Phi/\partial p)$. $\zeta_g$ and $v_g$ are evaluated geostrophically:

$$v_g = k \times \nabla \left(\frac{\Phi}{f_0}\right), \tag{33}$$

$$\zeta_g = \nabla^2 \left(\frac{\Phi}{f_0}\right). \tag{34}$$

Friction and non-adiabatic heating have been neglected in (31) and (32).

[3] The article by Morikawa (1960) appeared shortly after this survey was completed.

[4] The interpretation of the balance equation as a second-order geostrophic approximation is, however, implicit in an iterative sequence of higher-order geostrophic approximations proposed by Charney, Gilchrist and Shuman (1956).

(31) and (32) may also be combined to give an equation for $\partial\Phi/\partial t$ alone:

$$f_0^{-1}\left[\nabla^2 + \frac{\partial}{\partial p}\left(\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\right)\right]\frac{\partial \Phi}{\partial t} = -v_g\cdot\nabla\left[f + \zeta_g + \frac{\partial}{\partial p}\left(\frac{f_0}{\sigma}\frac{\partial \Phi}{\partial p}\right)\right]. \quad (35)$$

Alternatively, an equation for $\omega$ may be obtained from (31) and (32):

$$\left[\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right]\omega = f_0\frac{\partial}{p\partial}\left[v_g\cdot\nabla(f+\zeta_g)\right] - \frac{f_0^2}{\sigma}\nabla^2\left(v_g\cdot\nabla\frac{\partial\Phi}{\partial p}\right). \quad (36)$$

This system (or equations which are equivalent to it) was formulated in 1948 by Charney[5] and a few years later by Kibel (1951).[6]

The formulations of (32), (35) or (36) which have been used by Soviet meteorologists have usually contained a special form of the stability factor $\sigma$. If $\gamma_a$ and $\gamma$ are the adiabatic and actual lapse rates of temperature, and $R$ is the gas constant, $\sigma$ can be written as

$$\sigma = \frac{\partial \ln\theta}{\partial p}\frac{\partial\Phi}{\partial p} = \frac{R^2T(\gamma_a - \gamma)}{gp^2}.$$

In the Soviet literature, the factor $R^2T(\gamma_a - \gamma)g^{-1} = c^2$ is normally taken as a constant ($c^2 \approx 10^4$ m$^2$ sec$^{-2}$). $\sigma$ in (32), (35) and (36) is then replaced by $c^2p^{-2}$.

In 1951, Buleyev and Marchuk (1958) constructed the three-dimensional Green's functions which are needed to "invert" the elliptic operators on the left sides of (35) and (36) and obtain the distributions of $\partial\Phi/\partial t$ and $\omega$. They assumed a constant value of $c^2$. From the form of these Green's functions, they could infer the influence of the "dynamic factor" $v_g\cdot\nabla(f+\zeta_g)$ and the "thermal factor" $v_g\cdot\nabla(\partial\Phi/\partial p)$ on the resulting distributions of $\partial\Phi/\partial t$ and $\omega$. Kuo (1956) has also computed the influence function for $\partial\Phi/\partial t$, using a slightly different assumption about $\sigma$.

The same type of procedure can be used to compute the non-geostrophic component of the wind:

$$u' = u - u_g; \quad v' = v - v_g. \quad (37)$$

This was done in 1953 by Marchuk and Kireeva (1958). The divergence of the fields of $u$ and $v$ is equal to $-\partial\omega/\partial p$, where $\omega$ is given by (36). The vorticity of $u'$ and $v'$ is given by an equation similar to Obukhov's form of the balance equation (27).

In non-Soviet work, the system (31)–(36) has normally not been solved by a Green's function method but by *relaxation*. Hinkelmann (1953) has in fact even obtained the Green's functions themselves by a relaxation computation. In this way, he could conveniently allow for variation of the stability factor $\sigma$ with pressure and more faithfully reproduce the effect of the stratosphere. This difference in approach will be brought out more fully in sections 5 and 6, when we discuss the Soviet numerical work with electronic computers. At this point, however, it is interesting to note that it becomes impractical to solve (35) or (36) by a Green's-function method if the coefficients on the left side of the equations are allowed to vary with $x$, $y$ or $t$. By relying on this method of solution, one is therefore not tempted to "generalize" the geostrophic equations by introducing such artificial variability in the coefficients. This may be one reason why the atmospheric "models" used by Soviet meteorologists have been relatively free of inconsistencies that are sometimes found in non-Soviet numerical-prediction experiments. Charney, Gilchrist and Shuman (1956) have analysed the effect of non-geostrophic terms in the geostrophic equations and have shown that their inclusion gives no gain in accuracy.

## 4. Graphical and linearized forecasting methods

Graphical methods of solving (31)–(36) were developed around 1951 by Buleyev and Yudin at the Main Geophysical Observatory in Leningrad (in collaboration with the Northwest Hydrometeorological Service in that city). The techniques used are very similar to the scheme devised by Fjörtoft (1952). However, the models developed by Yudin (1957) are more complicated than those which have been customarily used for graphical prediction in non-Soviet work. Yudin's computation scheme uses geopotential data from the 1000-, 850-, 700-, 500- and 300-mb levels. The approximate method used to invert the elliptic operator in this model is also similar to the techniques used by Fjörtoft, but with the added complication of more pressure levels.

Dubov and Orlova (1957) have described the results of these experimental 24-hr graphical

[5] Charney's first formulation (1948) was not quite as simple as (35). $\sigma$ and $f_0$ were allowed to vary, and the term $f_0\partial\omega/\partial p$ in (31) was replaced by $(f+\zeta)\partial\omega/\partial p$. A more consistent application of his scale theory, however, leads to the simpler forms given above.

[6] Unfortunately, this paper of Kibel's was not available to the writers. Our knowledge of it is based on material in Kibel's book (1957b) and on the 1951 paper by Chistyakov (1958).

forecasts and compared them with other types of forecasts. Predictions were made of the surface pressure and the 700-mb or 500-mb height field over Europe. As measures of forecast skill, the following parameters were used:

$n$ = the per cent of grid points at which the error in the forecast isobaric height (or sea-level pressure) was less than 40 m (or 5 mb),

$r$ = correlation coefficient between actual and forecast changes of isobaric height or sea-level pressure,

$\mu$ = the average of the absolute value of the error, and

$\sigma$ = root-mean-square error.

During July–October 1954, forty-eight graphical forecasts were made of sea-level pressure and 500-mb height. $n$ was 0.73 for sea-level pressure and 0.66 for 500-mb height. A comparison during August–September with the subjectively-prepared forecast charts issued by the Central Institute of Forecasting in Moscow showed the latter to be better, with an $n$-score of 0.77 and 0.78. During January–November 1955 an average of twelve forecasts of surface pressure and 700-mb height was made each month. These forecasts included a correction for surface friction similar to the type used by Charney and Eliassen (1949). The grid network was also changed to a triangular instead of a square lattice. $n$ was now 0.72 for both surface pressure and 700-mb height, while $\mu$ was 4.2 mb and 35 m. The conventional Moscow forecasts during the spring were still slightly better.

On 14 days of January-February 1955, the graphical forecasts were made for the same days as were used by the Institute of Meteorology in Stockholm to make barotropic forecasts on the BESK (Bergthorsson, *et al.*, 1955). The graphical method had an average $r$ of 0.52 for those days (700-mb height), while the Swedish machine forecasts had $r = 0.68$ (500-mb height).

Yudin has suggested a novel approach to the forecasting problem (1957b). He first showed how $u = u_g + u'$, $v = v_g + v'$, and $\omega$ could be computed from the quasi-geostrophic theory. He then proposed that, instead of computing the local derivatives $\partial\Phi/\partial t$, $\partial T/\partial t$, $\partial u/\partial t$ etc., the individual derivatives $dT/dt$, etc. should be computed directly. For example, let the position of a particle at time $t_0$ be $x_0$, $y_0$, $p_0$. At time $t_0 + \Delta t$, it will have coordinates $x$, $y$:

$$\begin{aligned} x &= x_0 + u_g(x_0, y_0, p_0, t_0)\Delta t + \tfrac{1}{2}f(\Delta t)^2 v', \\ y &= y_0 + v_g(x_0, y_0, p_0, t_0)\Delta t - \tfrac{1}{2}f(\Delta t)^2 u'. \end{aligned} \quad (38)$$

Let the temperature of this particle at time $t_0$ be $T_0$. At time $t_0 + \Delta t$, it will be $T$:

$$T = T_0 + \frac{\Delta t(\gamma_a - \gamma)RT}{gp}\,\omega(x_0, y_0, p_0, t_0). \quad (39)$$

Although this is a very interesting approach, it contains some difficuties if the scheme is to be iterated in time. The forecast values of $T$ obtained by (39) will generally not be located at the regular grid points. On the other hand, the values of $u'$, $v'$ and $\omega$ for use in (38) and (39) are presumably obtained by computations performed on a regular lattice of points, and it is not clear how this discrepancy can be overcome without re-introducing (in effect) the normal practice of computing the time derivatives at fixed points in space.

Several "linearized" forms of the geostrophic equations have also been explored as short-range forecast techniques by Russian meteorologists. (By "linearization," we here mean the assumption that the flow pattern can be treated as a basic zonal current plus small superimposed perturbations; the latter are then predictable with linear equations.) The linear system used by Blinova for long-range prediction has been described by Baum and Thompson (1959).

Marchuk (1958), in 1951, attempted to derive the instability criterion for baroclinic waves from (35). In his analysis, he neglected the effect of static stability and therefore found that all waves shorter than a certain critical wave length were unstable and that this instability increased as the wave length decreased. He assumed that a horizontal eddy viscosity effect kept these short waves from developing. Kogan (1958a, 1958b) carried out an analysis in 1952 similar to that done by Charney and Eliassen (1949), but without the one-dimensional assumption they used for space variations. He treated a non-divergent barotropic atmosphere of infinite horizontal extent and was therefore forced to arbitrarily eliminate very long waves to prevent infinite group velocities.

Nemchinov (1958, 1959a, 1959b) attempted to extend the Charney-Eliassen type of linear solution to baroclinic models. In his three-level model (700, 500, and 250 mb), he applied the *non-divergent* barotropic equation at 500 mb. This procedure is quite inconsistent since the flow pattern at 500 mb is then completely independent of that at the other two levels. Galin (1959) derived the frequency equation for a simple 2-level geostrophic model with spherical geometry. He obtains a forecast by first mak-

TABLE 1. Summary of Soviet numerical weather predictions on electronic computers. All forecasts are based on the geostrophic model. Verification statistics are given in the table wherever possible. The notation "1 +" signifies that more forecasts have been made than the single example presented in the article. The "Date of work" is taken from Kibel's article (1957) or inferred from the forecast dates.

| Date of work | Author | Machine | Levels (mb) | Area | Number of grid points | $\Delta x$ | $\Delta t$ | Jacobian | Time extrapolation | Horizontal area for Green's function | Smoothing | No. of tests | Period of forecast | Remarks |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1954 | Belousov (1957a) | BESM | 700 | Europe | 480 | 250 km | 1.5 hr | ? | ? | 9 points | Yes | 1 + | 24 hr | Non-technical description. |
| 1955 | Mashkovich (1957) | BESM | 500<br>1000 | Europe | 480 | 250 km | 22.5 min | Centered | Uncentered | $r \leq 700$ km | Yes | 1 | 24 hr | Computational instability. |
| 1956 | Belousov (1957b) | BESM | 500, 700<br>850, 1000 | Europe | 480 | 250 km | 1.5 hr | Centered | Centered | $r \leq 1000$ km | Not mentioned | 1 | 24 hr | Good 1000-mb forecast. Upper levels said to be poor. |
| 1956 | Belousov and Bykov (1957) | BESM | 700 | Europe | 480 | 250 km | 45 min | ? | Uncentered | 9 points | Every 4 time steps | 2 | 24 hr | Mean error of 27.5 m with mountains, 30.1 m without mountains. |
| 1956 | Gubin (?) | STRELA | 300<br>700 | Europe | 391 | 250 km | 45 min | Centered | Uncentered | 9 points | Yes | 2 | 24 hr | Computational instability. 300-mb forecasts said to be very poor. Included $\omega\partial\zeta/\partial p$. |
| 1957 | Birkgan and Lyubimov (1958) | STRELA | 700 | Europe | 480 | 250 km | 1.5 hr | Centered | Uncentered | 9 points | Yes | 1 | 24 hr | Same scheme as Belousov (1957a). |
| 1957 | Musayelyan and Kheifetz (1958) | POGODA | 500 | Hemisphere | 264 | 5° lat<br>15° long | — | — | — | — | — | 1 + | Month | *Linearized* Blinova temperature forecast. |
| 1957 | Belousov and Blinova (1958) | BESM | 700 | Hemisphere | 577 | 5° lat<br>10° long | 2 hr | Centered See section 6 | Uncentered (?) | Entire region | Not mentioned | 1 | Up to 10 days | Non-linear forecast of flow pattern only. |
| 1957 | Belousov (1958) | BESM | 700 | Hemisphere | 577 | 5° lat<br>10° long | ? | Centered See section 6 | Uncentered (?) | Entire region | Not mentioned | 1 + | 48 hr | 3-min machine time per time step. Other forecasts have been made to 96 hr. |
| 1958 | Dushkin Lomonosov, and Tatarskaya (1959) | STRELA | 300<br>500<br>850 | Europe | 480 | 250 km | ? | Centered | Uncentered (?) | $r \leq 900$ km | Not mentioned | 7 | 24 hr | Predicted appearance of new high and low centers. See table 2. |

Table 2. Verification statistics from the 3-level geostrophic 24-hr forecasts by Dushkin, Lomonosov and Tatarskaya (1959). $r$ is the correlation coefficient between computed and actual 24-hr changes in isobaric height, while $\sigma$ is the root-mean-square error and ($\sigma_x$, $\sigma_y$) are the root-mean-square values of the forecast and actual 24-hr changes. The latter three are given in meters.

| Forecast date (1958) | 850 mb | | | | | 500 mb | | | | | 300 mb | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $r$ | $\sigma$ | $\sigma_x$ | $\sigma_y$ | $\sigma/\sigma_y$ | $r$ | $\sigma$ | $\sigma_x$ | $\sigma_y$ | $\sigma/\sigma_y$ | $r$ | $\sigma$ | $\sigma_x$ | $\sigma_y$ | $\sigma/\sigma_y$ |
| 22 August | 0.83 | 12.9 | 21.0 | 22.3 | 0.58 | 0.93 | 12.8 | 26.0 | 25.5 | 0.50 | 0.85 | 32.3 | 40.1 | 40.0 | 0.81 |
| 29 August | 0.59 | 20.1 | 21.7 | 20.5 | 0.97 | 0.40 | 44.2 | 40.1 | 40.9 | 1.08 | 0.68 | 48.4 | 51.5 | 64.9 | 0.75 |
| 12 September | 0.78 | 20.9 | 33.1 | 25.7 | 0.81 | 0.71 | 37.4 | 50.0 | 48.2 | 0.78 | 0.60 | 65.8 | 86.9 | 106.2 | 0.62 |
| 30 September | 0.74 | 28.9 | 41.8 | 38.8 | 0.74 | 0.67 | 36.6 | 48.9 | 35.6 | 1.03 | 0.70 | 46.3 | 64.5 | 50.7 | 0.91 |
| 3 October | 0.48 | 29.3 | 26.7 | 30.7 | 0.95 | 0.84 | 26.2 | 47.5 | 46.1 | 0.57 | 0.84 | 43.7 | 76.0 | 78.0 | 0.56 |
| 11 October | 0.97 | 23.8 | 55.2 | 73.4 | 0.32 | 0.92 | 34.5 | 90.0 | 88.3 | 0.39 | — | — | — | — | — |
| 6 November | 0.77 | 25.8 | 28.4 | 40.5 | 0.63 | 0.72 | 34.7 | 39.6 | 49.6 | 0.79 | 0.85 | 51.1 | 50.0 | 86.1 | 0.59 |
| Average | 0.74 | 23.1 | 32.6 | 36.0 | 0.71 | 0.74 | 32.3 | 48.9 | 47.7 | 0.73 | 0.75 | 47.9 | 61.5 | 71.0 | 0.71 |

ing an analysis into spherical harmonics of the initial flow patterns at 700 and 300 mb and then moving each wave component with the proper frequency. (This procedure is the same as that used by Blinova for her linear barotropic model.) He gives one example of a 24-hr forecast made in this manner.

## 5. Forecasts made on electronic computers

The first numerical predictions in the Soviet Union with an electronic computer were made in 1954 by Belousov. As was the case with the first machine computations in the U. S. (Charney, *et al.*, 1950) these first Soviet calculations were made with a barotropic model. Table 1 contains a summary of all Soviet work on numerical weather prediction with electronic computers that we have been able to find described in the literature through December 1959. Most of this work has been done by meteorologists from the Central Institute of Forecasting in Moscow.

The STRELA and BESM are the two computers which have been used for this purpose.[7] They both use floating point arithmetic. The STRELA, which uses cathode-ray tubes for internal storage, has an average multiplication or addition time of 500 microsecs. The BESM, the later model of which has a magnetic-core storage, has an average operation time of only 100 microsecs. Both machines use 3-address instructions, and have about 2000 "words" of internal storage (approximately 40 "bits" per word). Both machines also have magnetic tapes, and the BESM has in addition about 10,000 words of magnetic-drum storage. As far as capabilities for numerical prediction are concerned, then, the two Russian computers would appear to be in about the same class as the IBM 701 and IBM 704 that have been used by the Joint Numerical Weather Prediction (JNWP) group in the United States. However, at the time of writing, the Soviet Union does not have an operational group making daily routine numerical weather predictions with a computer. This lack seems to be due only to the unavailability to Russian meteorologists until now of the necessary amount of computer time, and it is presumably only a temporary state of affairs. An all-union conference on the theory of pressure variations was held in Moscow in December 1958 (Mashkovich, 1959). The conference resolved that a meteorological computer center should be established in the Hydrometeorological Service.

From table 1, it is clear that Soviet and non-Soviet meteorologists have in general both made the same type of numerical forecasts. That is to say, they have both exploited the geostrophic forecast equations, have used approximately the same size of space and time increments, and have experimented with models containing from 1 to 4 levels. The areas over which forecasts have been made have varied in both cases from a small area (of the size of Europe or of the United States) to a hemispheric-sized grid. In the few cases where numerical verification statistics are presented in the Russian articles, they are comparable with the results obtained by non-Soviet meteorologists from similar models. The detailed statistics from the work by Dushkin, Lomonosov and Tatarskaya (1959) are reproduced in table 2. Having pointed out these

[7] Several modifications (*e.g.*, BESM I and BESM II) of both machines are found. The POGODA is a small special-purpose machine with a very small internal memory. It is very useful for computing sums of products, a procedure important in Blinova's linearized scheme (Musayelyan and Kheifetz, 1958).

broad areas of agreement, we turn now to a discussion of the differences which exist.

(1) Non-Soviet work on the barotropic model has almost exclusively used the 500-mb level, this choice being based on the "equivalent-barotropic" description of the atmosphere by Charney (1949). Russian computations with the barotropic model on the other hand have used the 700-mb flow pattern.[8] It is not clear what rationale has been used by Soviet meteorologists to select the 700-mb level for this purpose. Judging by the experience of JNWP, one would expect that barotropic forecasts using the 700-mb flow pattern instead of the 500-mb flow pattern would result in a consistent underestimate of the generally eastward motion of isobaric patterns.

(2) The baroclinic 2-, 3- and 4-level models used in the Russian computations are normally formulated in a different manner than those used in non-Soviet work. In the latter, such models have been derived either by the intuitive type of approach used by Sutcliffe-Sawyer-Bushby (1953), by a process of averaging (Eliassen, 1952), or by a straightforward application of vertical finite-differences (in $p$) to eq (35) (Charney and Phillips, 1953). The models used by Mashkovich (1957), Belousov (1957b) and Dushkin, *et al.* (1959) are formulated by using the Green's functions of Buleyev and Marchuk (1958) to express the relation between the $\Phi$-fields at the levels selected for prognosis. (No reason is given for the selection of the particular levels.) Gubin's 2-level model (– – –), however, was formulated from a straightforward application of finite-differences in $p$.

(3) There seems to have been no attempt to coordinate the 10 series of computations in table 1 with one another by choosing the same data for tests with different models. Yudin (1959) has recently criticized this procedure.

(4) There are differences between the finite-difference methods used by Soviet and other meteorologists. This subject is treated more fully in the following section.

(5) The articles describing the hemispheric barotropic forecasts (Belousov and Blinova, 1958; and Belousov, 1958) do not mention the phenomenon of retrograde motion of the ultra-long waves. This persistent error was very prominent in the JNWP hemispheric forecasts until the middle of 1958, when a semi-empirical correction for divergence was added to the barotropic model to partially eliminate it (Cressman, 1958). None of the Soviet barotropic forecasts include a divergence term, although this effect was brought out in Obukhov's work (1949). [See eq (29).] Presumably this difficulty will be recognized as soon as more hemispheric forecasts have been made in Moscow.[9]

(6) With the exception of the computations by Gubin (– – –), the Soviet machine calculations have stuck very closely to the strict geostrophic model given by (35). Attempts to include the vertical advection of vorticity $\omega\partial\zeta/\partial p$, the product of $\zeta$ and divergence $\zeta\nabla\cdot\boldsymbol{v}$, or the "twisting term" $\boldsymbol{k}\cdot\partial\boldsymbol{v}/\partial p \times \nabla\omega$ in (31) have frequently been made in non-Soviet work. According to Charney's scale theory, the orders of magnitude of these extra terms are all about 10 per cent of the magnitude of the terms which have been retained in (31). Since there is an inevitable error of about 10 per cent in evaluating $\zeta$ by $\zeta_g$, it clearly is of no value to include the extra terms unless $\zeta_g$ (and $\boldsymbol{v}_g$) are at the same time replaced by a more accurate approximation to $\zeta$ and $\boldsymbol{v}$. In this respect, the Soviet machine computations have been quite faithful to the spirit of the quasi-geostrophic theory.

(7) We have found no references to any computer work by Soviet meteorologists in the areas of precipitation forecasting, general-circulation numerical experiments, objective analysis of initial data, or any computations with non-geostrophic models. However, the basic principles of objective analysis for numerical weather prediction have been described by Kalmykova (1956), and Novikov (1959) has recently improved Thompson's (1957) theoretical analysis of initial data and predictability. Bykov (1956, 1958) has suggested a method to solve the "balance equation" system of Charney (1955) and Bolin (1955). Kibel has also described a possible non-geostrophic forecast technique. (See section 7.) Such work will undoubtedly be soon taken up on computing machines.

## 6. Finite-difference techniques

It is convenient to divide this subject into two parts: (*a*) the methods used to invert the elliptic operator appearing on the left side of (35) or its simplified variants, and (*b*) the methods used to evaluate the horizontal space derivatives in the operator $\boldsymbol{v}_g\cdot\nabla$ and the time derivative, $\partial/\partial t$. Part (*b*) is intimately associated with the phenomenon of *computational stability*.

[8] Musayelyan and Kheifetz (1958) have used the 500-mb level in making linearized long-range forecasts by the Blinova method.

[9] This false retrogression should also be present in the linearized Blinova forecast technique (Kibel and Blinova, 1957).

As mentioned in sections 3 and 5, Soviet meteorologists have used a Green's function or influence-function method to solve (35) for $\partial\Phi/\partial t$. In the four baroclinic forecasts shown in table 1, the influence functions derived by Buleyev and Marchuk were used (1958):

$$\left(\frac{\partial\Phi}{\partial t}\right)_{x,y,p} = \iiint G(x, y, p; x', y', p') \times \rho(x', y', p')dx'\,dy'\,dp'. \quad (40)$$

$\rho$ represents the (known) right-hand side of (35) for the particular model employed, and $G$ is the influence or Green's function. However, instead of carrying out this integration over the entire domain of $x'$ and $y'$, the integration—which in practice is approximated by a weighted sum of values of $\rho$ at grid points—is carried out only over a small region in the neighborhood of each point. The size of this region is indicated in the eleventh column of table 1.

In the hemispheric barotropic computations by Belousov and Blinova (1958) and by Belousov (1958), the barotropic non-divergent vorticity equation,

$$\nabla^2\frac{\partial\psi}{\partial t} = -v\cdot\nabla(f + \nabla^2\psi) = -F(\theta, \lambda), \quad (41)$$

was solved for $\partial\psi/\partial t$ with a 2-dimensional Green's function for the sphere:

$$\left(\frac{\partial\psi}{\partial t}\right)_{\theta'\lambda} = -\frac{a^2}{4\pi}\int_0^{2\pi}d\lambda'\int_0^{\pi/2}\ln\left(\frac{1-\cos\gamma}{1-\cos\bar{\gamma}}\right) \times F(\theta', \lambda')\sin\theta' d\theta'. \quad (42)$$

$\theta$ is the co-latitude, $\lambda$ is longitude, and $\gamma$ and $\bar{\gamma}$ are functions of $\theta$, $\theta'$ and $(\lambda - \lambda')$. Anti-symmetry of the streamfunction ($\psi$) was assumed across the equator, and the integration (summation) involved in (42) was extended over the entire hemisphere. In the actual computations, with data given at 577 grid points, this means that 577 multiplications and 577 additions are required to compute $\partial\psi/\partial t$ at one grid point. Thus, about 300,000 multiplications and 300,000 additions are required to get a complete set of 577 values of $\partial\psi/\partial t$ at each time step. This undoubtedly is the reason for the surprisingly large time of 3 min on the BESM reported by Belousov to compute each time step in these computations. (The barotropic computations made by JNWP, on a hemispheric grid of almost 2000 points, require somewhat less than 1 min of machine time for each time step. Relaxation is used by JNWP to get $\partial\psi/\partial t$.)

A more approximate method of inverting the Laplace operator has also been used by Soviet meteorologists. Suppose the finite-difference form of the Poisson equation for a barotropic model is written in the following way:

$$-4\tau_{jk} + \tau_{j+1k} + \tau_{j-1k} + \tau_{jk+1} + \tau_{jk-1} = A_{jk}. \quad (43)$$

In a barotropic model, like (41), $\tau_{jk}$ can be thought of as representing $\partial\psi/\partial t$, with $A$ being proportional to the advection of vorticity. Consider now the solution for $\tau$ at the point 0 in fig. 1, with the 9 equations of the form (43)

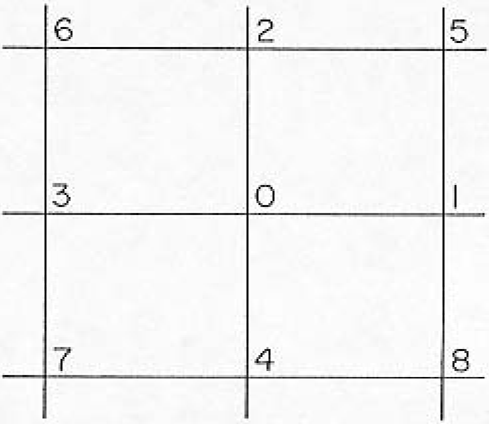


FIG. 1. The nine neighboring points in a square lattice used to compute the tendency at the central point by formula (44).

written down for point 0 and its 8 closest neighbors. If we assume that $\tau$ is zero at grid points surrounding those shown in fig. 1, then it can easily be shown from the 9 equations that $\tau_0$ is given by

$$\tau_0 = -\tfrac{3}{8}A_0 - \tfrac{1}{8}(A_1 + A_2 + A_3 + A_4) - \tfrac{1}{16}(A_5 + A_6 + A_7 + A_8). \quad (44)$$

Values of $\tau$ at the other points 1, 2, . . . , 8 are also implicitly defined by such a computation, but they would be influenced too much by the neglect of $\tau$ at the points not shown in fig. 1. Values of $\tau$ at the points 1, 2, . . . , 8 are therefore computed instead by merely shifting the point 0 to the location in question, so that the following formula is used at all points:

$$\tau_{jk} = -\tfrac{3}{8}A_{jk} - \tfrac{1}{8}[A_{j+1k} + A_{jk+1} + A_{j-1k} + A_{jk-1}] - \tfrac{1}{16}[A_{j+1k+1} + A_{j-1k+1} + A_{j-1k-1} + A_{j+1k-1}]. \quad (45)$$

(The subscripts indicate the different grid-point values, with $x = j\Delta$, $y = k\Delta$, where $\Delta$ is the space increment.) Such a procedure is similar to the approximate computations done in graphical solutions (Fjörtoft, 1952), and, as has been shown by Charney (Charney and Phillips, 1953), is equivalent to the carrying out of 2 or 3 iterations of a Richardson-type relaxation computation. A more complex formula of this type, suitable for a triangular lattice, has been developed by H.-Y. Tu (1959). (45) was used in the early barotropic calculations by Belousov (1957a), Birkgan and Lyubimov (1958) and Belousov and Bykov (1957). A formula such as (45) can also be considered as an approximation to a Green's function solution, where one assumes either that the Green's function itself is small beyond a certain distance from the point in question or that the contributions of $A$ beyond this distance will tend to cancel one another.

We turn now to the finite-difference techniques used by Soviet meteorologists for the computation of the Jacobian terms and the time extrapolation in (35). From table 1, we see that centered space differences have been used for evaluating Jacobians:

$$v_g \cdot \nabla\eta = \frac{1}{f} J\left(\frac{\Phi, \eta}{x, y}\right) \approx \frac{1}{4f\Delta^2} \left[(\Phi_{j+1} - \Phi_{j-1})_k(\eta_{k+1} - \eta_{k-1})_j - (\Phi_{k+1} - \Phi_{k-1})_j(\eta_{j+1} - \eta_{j-1})_k\right]. \quad (46)$$

However, this seems to have been frequently combined with an *un*centered time extrapolation:

$$\Phi_{t+\Delta t} \approx \Phi_t + \Delta t \left(\frac{\partial\Phi}{\partial t}\right)_t. \quad (47)$$

Such a combination of finite-differences is easily shown to be *computationally unstable* in the sense of the Lax-Richtmyer theory (Richtmyer, 1957). To eliminate or modify this computational instability, a considerable amount of smoothing has been introduced into many of the Russian computations.[10]

A rationale for the smoothing was developed by Obukhov (1957). He considered the one-dimensional advection equation,

$$\frac{\partial S}{\partial t} = -U\frac{\partial S}{\partial x} \quad (U = \text{constant}), \quad (48)$$

as has been done, for instance, by Platzman (1954). In his analysis, Obukhov retained the continuous time derivative, $\partial S/\partial t$, but examined the effect of finite-difference expressions for $\partial S/\partial x$ on a numerical solution. The usual centered difference approximation for $\partial S/\partial x$, when introduced into (48), gives the following system of linear differential equations:

$$\frac{\partial S_j}{\partial t} = -\frac{U}{2\Delta}(S_{j+1} - S_{j-1}). \quad (49)$$

This was found by Obukhov to give rise to "parasitic waves" when applied to a simple block-like initial distribution of $S$. The initial configuration also dispersed upstream as well as downstream when it was determined by (49).[11] Obukhov then added a smoothing term to the right side of (49):

$$\frac{\partial S_j}{\partial t} = -\frac{U}{2\Delta}(S_{j+1} - S_{j-1}) + \alpha(S_{j+1} - 2S_j + S_{j-1}). \quad (50)$$

An optimum value of $\alpha$, $\alpha_{opt} = \frac{1}{2}|U| \div \Delta$, was found to eliminate the parasitic waves and the upstream travel of the disturbance. For this choice of $\alpha$, (50) becomes

$$\frac{\partial S_j}{\partial t} = -\frac{U}{\Delta}(S_j - S_{j-1}), \quad (U > 0), \quad (51)$$
$$\frac{\partial S_j}{\partial t} = -\frac{U}{\Delta}(S_{j+1} - S_j), \quad (U < 0).$$

These are the uncentered approximations to $\partial S/\partial x$, evaluated on the *upstream* side.[12] When this is done, the smoothing operation broadens out the initial disturbance at a rate proportional to $\sqrt{t}$.

In the hemispheric barotropic computations, by Belousov (1958) and Belousov and Blinova (1958), Belousov has introduced an interesting method of computing a term like $v_g \cdot \nabla\eta$. In these computations, data are represented at the intersection of each 10 deg of longitude ($\lambda$) and each 5 deg of latitude ($\theta$). At such a grid point, an expression like $\partial\eta/\partial\theta$ is evaluated by a centered difference

$$\frac{1}{a}\frac{\partial\eta}{\partial\theta} \approx \frac{\eta(\lambda, \theta + \delta\theta) - \eta(\lambda, \theta - \delta\theta)}{2a\delta\theta}, \quad (52)$$

where $\delta\theta$ is 5 deg ($= \pi/36$), and $a$ is the radius of the earth. The partial derivative $\partial/\partial\lambda$, how-

[10] Smoothing is not mentioned in the last three calculations in table 1. Although in each of these papers there is a statement indicating that simple uncentered time extrapolation was used, this statement may not have been meant to be taken literally. In the earlier computations by Belousov (1957b), centered time extrapolation was used and smoothing was presumably unnecessary.

[11] Each individual Fourier component in (49) moves downstream, but the phase speed varies with wavelength.

[12] It can be shown that the completely uncentered computation scheme obtained from (51) by setting $\partial S/\partial t \approx [\zeta_j(t + \Delta t) - \zeta_j(t)] \div \Delta t$ is stable in the Lax-Richtmyer sense if $|u|\Delta t/\Delta \leq 1$.

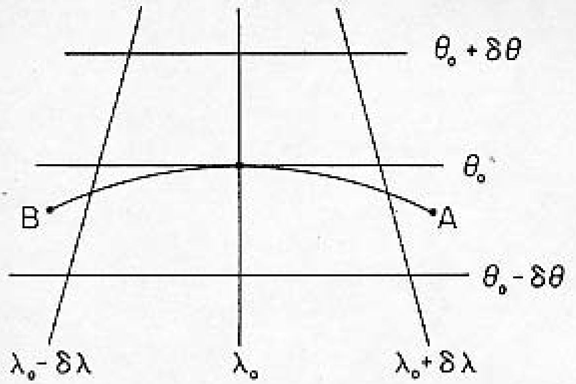


FIG. 2. Schematic diagram indicating the location of the intermediate points A and B employed by Belousov in his hemispheric computations. Straight lines in this diagram are lines of constant latitude or constant longitude. The curved line AB is a portion of a great circle.

ever, is *not* evaluated by the similar expression

$$\frac{1}{a\cos\theta}\frac{\partial\eta}{\partial\lambda}\approx\frac{\eta(\lambda+\delta\lambda,\theta)-\eta(\lambda-\delta\lambda,\theta)}{2a\delta\lambda\cos\theta} \quad (53)$$

but in the following way. Consider the great circle passing through the grid point $\theta_0$, $\lambda_0$ and tangent to the latitude circle $\theta_0$ (fig. 2). On this great circle, lay off in each direction from the point ($\theta_0$, $\lambda_0$) a line segment whose arc-length is equal to $a\delta\theta$. The ends of these segments will be at points $A$ and $B$—say, where point $A$ is to the southeast and point $B$ to the southwest of the grid point ($\theta_0$, $\lambda_0$). The $\lambda$—derivative at ($\theta_0$, $\lambda_0$) is then approximated by the expression

$$\frac{1}{a\cos\theta}\frac{\partial\eta}{\partial\lambda}\approx\frac{\eta(A)-\eta(B)}{2a\delta\theta}. \quad (54)$$

In this way, the distance on the earth over which the finite-difference approximation to $\partial/\partial\lambda$ is evaluated is the same as that over which $\partial/\partial\theta$ is approximated. Points $A$ and $B$ will not fall on the regular grid points (except when $\theta_0=0$), and therefore $\eta(A)$ and $\eta(B)$ must be obtained by interpolation between the regular grid-point values. Belousov does not describe this interpolation procedure.

In addition to the articles by Obukhov and H.-S. Tu, there are two papers by Belousov (1957c, 1958a) dealing specifically with finite-difference problems encountered in the barotropic geostrophic forecast system. The main analysis in these two papers is concerned with the computational error introduced by the combination of centered space differences and uncentered time differences in an equation similar to (48). As is to be expected, Belousov finds that his solutions are not stable, but increase exponentially with time. However, if $\Delta$ (the space increment) is 250 km and $\Delta t$ is 1.5 hr, a wavelength of $L=16\Delta=4000$ km will only amplify by 4 or 5 per cent in 24 hr. Shorter waves will amplify much more rapidly, however, although Belousov does not emphasize this point. Presumably, the smoothing operation is relied upon to keep the short waves from becoming too large in amplitude.

## 7. Orography and surface friction

At the present time in numerical weather prediction, these two effects are introduced into the geostrophic forecast equation (35) very simply, via the boundary condition at the bottom of the atmosphere. Soviet meteorologists have done this in essentially the same manner as have other meteorologists. If we assume that we may apply the boundary condition at $p=p_0=1000$ mb, instead of at the actual value of the surface pressure, we may write

$$\omega_0=\rho_0\frac{\partial\Phi_0}{\partial t}+\rho_0(v\cdot\nabla\Phi)_0-\rho_0 g w_0. \quad (55)$$

$\rho_0$ is the density at $p=p_0$. The second term on the right of this equation is smaller than the first term for quasi-geostrophic motion and may be discarded. $w_0$ is the vertical velocity at the "bottom" of the atmosphere and is determined by both the simple upslope motion ($w_{orog}$) due to orography and the vertical motion ($w_{fr}$) caused by the frictionally-induced convergence of air in the friction layer.

$$\omega_0=\rho_0\frac{\partial\Phi_0}{\partial t}-\rho_0 g(w_{orog}+w_{fr}). \quad (56)$$

The effect of friction has been incorporated in the graphical computations by Yudin's group (1957a), although it has not been included in any of the machine computations listed in section 5. In his book (1957b), Kibel gives a thorough derivation leading up to the simple formula

$$w_{fr}=\sqrt{\frac{\nu}{2f}}\,\zeta_{g0}, \quad (57)$$

where $\nu$ is the vertical austausch coefficient and $\zeta_{g0}$ is the geostrophic vorticity at $p=p_0$. A similar formula was used by Charney and Eliassen (1949). According to Dubov (1957), the relation (57) was also formulated by Dyubyuk in 1947.

The effect of orography has been considered in the machine computations by Belousov and Bykov (1957) and also in some of the graphical computations. The kinematic boundary condition at the ground ($z = Z$) gives us the relation

$$w_{\text{orog}} = v \cdot \nabla Z. \tag{58}$$

$v$ here is the horizontal wind at the ground. Let us define $p_s$ and $\rho_s$ as the pressure and density in a standard atmosphere corresponding to the height $Z$. Then (58) may be rewritten

$$\rho_0 g w_{\text{orog}} = -\frac{\rho_0}{\rho_s} p_0 v \cdot \nabla \left(\frac{p_s}{p_0}\right) \approx -p_0 v \cdot \nabla \eta, \tag{59}$$

where $\eta = p_s/p_0$.

The computations by Belousov and Bykov (1957) were based on a barotropic model. Bykov formulated the orographic effect for this model in a somewhat different manner than the straightforward approach represented by (58) or (59). Working in the $x, y, z, t$—coordinate system, he first derived the equation

$$\frac{\partial}{\partial x}\int_Z^\infty \rho u dz + \frac{\partial}{\partial y}\int_Z^\infty \rho v dz = 0. \tag{60}$$

(This is the familiar "tendency equation" with the surface pressure-tendency discarded.) From (60), a stream function $\Psi$ may be defined

$$\begin{aligned} -\frac{\partial \Psi}{\partial y} &= \int_Z^\infty u\rho dz \approx \bar{u}\int_Z^\infty \rho dz = \bar{u}\frac{p_s}{g}, \\ \frac{\partial \Psi}{\partial x} &= \int_Z^\infty v\rho dz \approx \bar{v}\int_Z^\infty \rho dz = \bar{v}\frac{p_s}{g}. \end{aligned} \tag{61}$$

The horizontal velocity components $\bar{u}$ and $\bar{v}$ represent average values of $u$ and $v$ with respect to pressure. If a modified stream function $\psi$ is now defined by

$$\psi = \frac{g}{p_0}\Psi,$$

it is possible to rewrite (61) in the form

$$\begin{aligned} \bar{u} &= -\frac{1}{\eta}\frac{\partial \psi}{\partial y}, \\ \bar{v} &= \frac{1}{\eta}\frac{\partial \psi}{\partial x}. \end{aligned} \tag{62}$$

The vorticity equation in the form

$$\frac{\partial \zeta}{\partial t} + \bar{v}\cdot\nabla(f + \bar{\zeta}) = -f\nabla\cdot\bar{v}$$

can then be rewritten

$$\nabla^2\frac{\partial \psi}{\partial t} + \nabla \ln \eta \cdot \nabla \frac{\partial \psi}{\partial t} = -J\left(\frac{\psi, f + \bar{\zeta} + f_0 \ln \eta}{x, y}\right), \tag{63}$$

$$\bar{\zeta} = \frac{1}{\eta}\left[\nabla^2\psi + \nabla \ln \eta \cdot \nabla\psi\right]. \tag{64}$$

For actual computations, Bykov now replaces $\psi$ in (62) by $\bar{\phi}/f_0$ where $\bar{\phi}$ is taken as the geopotential of the 700-mb level and $f_0$ is an average value of $f$. (63) and (64) differ from the usual formulation of the mountain effect in a barotropic model by the presence of the $\nabla\psi\cdot\nabla \ln \eta$ term in (64) and the first-order space derivatives of $\partial\psi/\partial t$ in (63).

The system (63)–(64) was solved numerically by Belousov and Bykov (1957), using a finite-difference scheme similar to (45) to invert the operator on the left side of (63). The first-order terms in $\partial\psi/\partial t$ in (63) now make the coefficients in a formula like (45) slightly non-symmetrical and variable from point to point. However, according to Belousov and Bykov, it was found that these additional terms had negligible influence and could be disregarded. In this way, their computation of the mountain effect reduces to the more familiar scheme first used in a barotropic model by Charney and Eliassen (1949), where the only mountain effect is the $f_0 \ln \eta$ term in (63).

## 8. Non-geostrophic effects

We have not found any references to actual forecasts or experimental computations based on non-geostrophic equations. However, there are several papers which describe non-geostrophic systems. Among these is an interesting short article by Kibel (1958). Let $\psi$ and $\phi$ again represent the horizontal stream function and velocity potential:

$$u = -\frac{\partial \psi}{\partial y} + \frac{\partial \phi}{\partial x}, \quad v = \frac{\partial \psi}{\partial x} + \frac{\partial \phi}{\partial y}. \tag{65}$$

The vorticity and divergence equations may be written

$$\nabla^2\frac{\partial \psi}{\partial t} + f\nabla^2\phi = -B_\Omega, \tag{66}$$

$$\nabla^2\frac{\partial \phi}{\partial t} + \nabla^2\Phi - f\nabla^2\psi = -B_D. \tag{67}$$

In these equations, $\Phi$ is the geopotential, and $B_\Omega$ and $B_D$ represent all the non-linear terms in the complete form of the vorticity and divergence equations. The thermodynamic equation, with the assumption of hydrostatic balance, is written in a similar way:

$$\frac{\partial}{\partial p}\left(p^2 \frac{\partial^2 \Phi}{\partial p \partial t}\right) - c^2\nabla^2\phi = R\frac{\partial}{\partial p}(pB_T). \quad (68)$$

$c^2$ is the static stability parameter defined after (36), and $B_T$ is equal to $v\cdot\nabla T$. $R$ is the gas constant. The "vertical velocity" $\omega = dp/dt$ appears in $B_\Omega$ and $B_D$; it can be evaluated from $\phi$ via the continuity equation:

$$\frac{\partial \omega}{\partial p} = -\nabla\cdot v = -\nabla^2\phi. \quad (69)$$

Kibel now treats the system (66)–(68) as a *linear* initial value problem, with the non-linear terms $B_\Omega$, $B_D$ and $B_T$ as *known* functions of space only. Under these conditions, he shows how the unknown variables $\Phi$, $\phi$ and $\psi$ at a later time can be determined by an influence function type of solution if $\Phi$, $\phi$ and $\psi$ are known at the initial time. The change in value of the geopotential $\Phi$ at time $t = \Delta t$, for example, depends on the initial values of $\Phi$, $\phi$, and $\psi$ (indicated by $\Phi_0$ $\phi_0$ and $\psi_0$) and also on the values of the non-linear terms $B_\Omega$, $B_D$ and $B_T$. That part of the solution which depends on the initial values $\Phi_0$, $\phi_0$ and $\psi_0$, depends only on the values of

$$\Delta^2\phi_0 = (\nabla\cdot v)_{t=0},$$
$$\nabla^2(f\psi_0 - \Phi_0) = f(\zeta - \zeta_g)_{t=0}. \quad (70)$$

According to Kibel, this part of the solution may be discarded. [It represents the gravity-inertia waves present in the initial data at $t = 0$ and presumably disperses rapidly away; see section 2.] In this way, the solution at time $t = \Delta t$ depends only on $B_\Omega$, $B_D$ and $B_T$. Kibel proposes that $\Delta t$ be chosen small enough so that these three functions may be considered as functions only of space during the time interval $t = 0$ to $t = \Delta t$. At the end of this interval, $B_\Omega$, $B_D$ and $B_T$ would be redetermined from the new forecast values of $\Phi$, $\phi$ and $\psi$ at $t = \Delta t$, and a computation of $\Phi$, $\phi$ and $\psi$ would then be possible for time $t = 2\Delta t$, *etc.*

It is not clear what advantage this method would have over the straightforward finite-difference integration of the "primitive" equations, as has been done for example by Hinkelmann (1959) and by Smagorinsky (1958). In this latter method, the time step is limited to about 10 min to avoid computational instability. Although it might be possible to take larger time steps with the method suggested here by Kibel, the computations in his method are more complicated. For example, the value of $\Phi$ at time $t = \Delta t$ is given by the formula

$$\Phi_{\Delta t} = \Phi_0 + \frac{1}{2\pi f^2}\int_0^{\Delta t} dt' \int_0^{2\pi} d\delta \times \int_0^1 dp' \int_0^{2f(\Delta t - t')} hr'dr', \quad (71)$$

where

$$h = c^2\left(fB_\Omega G - B_D\frac{\partial G}{\partial t}\right) + p'\frac{\partial}{\partial p'}\left[\left(f^2G + \frac{\partial^2 G}{\partial t^2}\right)RB_T\right].$$

$G$ is an influence function for $\Phi$, and that part of the solution depending on $\Phi_0$, $\phi_0$ and $\psi_0$ has been discarded.[13]

An interesting variant of the geostrophic approach was proposed by Yudin (1955). Instead of the independent variables $x$, $y$, $p$ and $t$, Yudin suggests the new variables $\xi$, $\eta$, $\pi$ and $\tau$:

$$\xi = x + v/f, \quad \pi = p,$$
$$\eta = y - u/f, \quad \tau = t. \quad (72)$$

(For simplicity in describing his results here, we consider $f$ to be constant.) Consider now that form of the potential vorticity which is conserved in a frictionless, adiabatic and hydrostatic atmosphere:[14]

$$q = (f+\zeta)\frac{\partial\theta}{\partial p} + \frac{\partial u}{\partial p}\frac{\partial\theta}{\partial y} - \frac{\partial v}{\partial p}\frac{\partial\theta}{\partial x},$$
$$\frac{dq}{dt} = 0 \quad (73)$$

($\theta$ = potential temperature). According to Yudin, the usual geostrophic method of forecasting is equivalent to assuming that

$$\frac{d}{dt}\left[(f+\zeta)\frac{\partial\theta}{\partial p}\right] \approx 0. \quad (74)$$

The terms which are omitted in going from (73) to (74) are one order of magnitude smaller than those which are retained in (74).

By introducing the new coordinates from (72), $q$ can be rewritten as

$$q = (f+\zeta)\frac{\partial\theta}{\partial\pi} - \frac{1}{f}\left[\frac{\partial v}{\partial x}\frac{\partial u}{\partial p}\frac{\partial\theta}{\partial\eta} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial p}\frac{\partial\theta}{\partial\xi} - \frac{\partial v}{\partial p}\frac{\partial u}{\partial x}\frac{\partial\theta}{\partial\eta} - \frac{\partial u}{\partial p}\frac{\partial v}{\partial y}\frac{\partial\theta}{\partial\xi}\right]. \quad (75)$$

[13] For $\Delta t \to \infty$, the solution (71) approaches the Green's-function solution for the geostrophic system (35), as shown in an earlier paper by Kibel (1957a).

[14] The above derivation is slightly different from the one given by Yudin. The end result is the same, however.

The terms in the bracket here are *two* orders of magnitude smaller than $(f+\zeta)\partial\theta/\partial\pi$. Therefore, the statement

$$\frac{d}{dt}\left[(f+\zeta)\frac{\partial\theta}{\partial\pi}\right]=0 \qquad (76)$$

is more accurate than (74). In order to preserve this accuracy, the operator $d/dt$ must be evaluated more precisely than by the mere substitution of the geostrophic wind Here the new variables come into play again, since it is easy to show from (72) that

$$\begin{aligned}\frac{d\xi}{dt}&=u+f^{-1}\frac{dv}{dt}=u_g,\\ \frac{d\eta}{dt}&=v-f^{-1}\frac{du}{dt}=v_g.\end{aligned} \qquad (77)$$

Therefore, with no additional approximation, we may write (76) in the form

$$\left[\frac{\partial}{\partial\tau}+u_g\frac{\partial}{\partial\xi}+v_g\frac{\partial}{\partial\eta}+\omega\frac{\partial}{\partial\pi}\right] \times\left[(f+\zeta)\frac{\partial\theta}{\partial\pi}\right]=0. \qquad (78)$$

We do not know of any papers dealing with an application of (78). In order to retain the accuracy of (78) relative to (74), an accurate determination of $v$ and $\zeta$ is required; for example, substitution of $\zeta_g$ for $\zeta$ in (78) would merely reintroduce errors of the first order It would therefore seem that any practical application of (78) would involve rather complicated computations.

V. Bykov has written two articles on the use of the "balance equation" for numerical weather prediction. The balance equation has the form

$$f\nabla^2\psi+\nabla f\cdot\nabla\psi +2\left[\frac{\partial^2\psi}{\partial x^2}\frac{\partial^2\psi}{\partial y^2}-\left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2\right]=\nabla^2\Phi. \qquad (79)$$

($\psi$ is the stream function, $\Phi$ is the geopotential.) In his first paper (1956), Bykov uses (79) to arrive at the following system of equations for a baroclinic forecast model. Differentiating (79) partially with respect to $t$, he gets

$$\nabla^2 q+f^{-1}\nabla f\cdot\nabla q+\frac{2}{f}\left[\frac{\partial^2\psi}{\partial x^2}\frac{\partial^2 q}{\partial y^2}+\frac{\partial\psi^2}{\partial y^2}\frac{\partial^2 q}{\partial x^2} -2\frac{\partial^2\psi}{\partial x\partial y}\frac{\partial^2 q}{\partial x\partial y}\right]=\frac{1}{f}\nabla^2 l, \qquad (80)$$

where $q=\partial\psi/\partial t$ and $l=\partial\Phi/\partial t$. The simple forms of the vorticity and thermodynamic equations are then combined to give another equation for $q$ and $l$:

$$\frac{\partial}{\partial p}\left(p^2\frac{\partial l}{\partial p}\right)+\frac{c^2}{f}\nabla^2 q=-\frac{c^2}{f}J\left(\frac{\psi,f+\nabla^2\psi}{x,y}\right) -\frac{\partial}{\partial p}\left[p^2 J\left(\frac{\psi,\partial\phi/\partial p}{x,y}\right)\right]. \qquad (81)$$

(80) and (81) constitute two simultaneous partial differential equations for the unknowns $q$ and $l$. Bykov does not go into any detail concerning the numerical solution of this system and does not present any computations based on (80)–(81). [He does measure the terms in (79) on a 700-mb map, however.]

In his second paper (1958), Bykov interprets this scheme in terms of a 2-level model of the atmosphere. At the 500-mb level, he assumes that $\partial\omega/\partial p=0$.[15] By defining $q_1=\partial\psi/\partial t$ at the 500-mb level, he is then able to write

$$\nabla^2 q_1=-J\left(\frac{\psi_1,f+\nabla^2\psi_1}{x,y}\right). \qquad (82)$$

An equation for $\partial\psi/\partial t=q_2$ at the 1000-mb level is then obtained by evaluating the vorticity equation at 1000 mb and eq (80) at 750 mb and combining them. The final result is a rather complicated equation for $q_2$:

$$\nabla^2(\nabla^2 q_2)+\left(K_1\frac{\partial^2\psi_2}{\partial y^2}-K_2\right)\frac{\partial^2 q_2}{\partial x^2} +\left(2K_1\frac{\partial^2\psi_2}{\partial x\partial y}\right)\frac{\partial^2 q_2}{\partial x\partial y}+\left(K_1\frac{\partial^2\psi_2}{\partial x^2}-K_2\right) \times\frac{\partial^2 q_2}{\partial y^2}-K_3\frac{\partial q_2}{\partial y}=F(x,y). \qquad (83)$$

$K_1$, $K_2$ and $K_3$ are numerical coefficients. $F(x,y)$ depends on $\psi_1$ and $\psi_2$ and the value of $q_1$ previously computed from (82). Bykov suggests that both (82) and (83) be solved for $q_1$ and $q_2$ by relaxation, although he presents no proof that such a method of solving (83) will converge. No numerical results are presented.

## 9. Remarks on the literature

The following remarks are intended to make it easier for interested meteorologists to gain access to the Russian literature on numerical weather prediction. Practically all of the published Soviet work in this field has appeared in the periodicals and serials listed below.

[15] This procedure is of course subject to the same criticism as is Nemchinov's model. (See section 4.)

I. Monthly periodicals.

a. *Meteorologiya i Gidrologiya*. Published by the Hydrometeorological Service.
b. *Izvestia Akad. Nauk SSSR, Ser. Geofizicheskaya*. One of a series of similar journals published by the Academy of Sciences. This particular journal contains papers on geophysics, oceanography and meteorology.
c. *Doklady Akad. Nauk SSSR*. The Soviet equivalent of *Comptes Rendus*.

A translated (English) table of contents appears in each issue of *a* and *c*. Regular subscriptions for all three periodicals may be placed with foreign book suppliers. The 1957 and 1958 issues of *b* have been translated in full and published by Pergamon Press (1957) and the American Geophysical Union (1958). This practice will evidently continue into 1959. Translated tables of contents of *b* are published periodically in *Transactions of the American Geophysical Union*, together with a listing of English translations of many Russian articles of geophysical and meteorological interest. Tables of contents of *a* and *b* also appear in *Meteorological Abstracts and Bibliography*.

II. Serial publications of various research institutes.

1. *Trudy Tsentralnogo Instituta Prognozov*. The Central Institute of Forecasting in Moscow is part of the Hydrometeorological Service.
b. *Trudy Glavnoi Geofizicheskaya Observ*. The Main Geophysical Observatory in Leningrad is also part of the Hydrometeorological Service.
c. *Trudy Instituta Fizika Atmosfery*. The Institute of Atmospheric Physics in Moscow is part of the Academy of Sciences.

These *Trudy* are serially numbered publications which appear at irregular intervals. Each issue normally contains a group of papers on one topic —*e.g.*, synoptic meteorology, dynamic meteorology, cloud physics *etc.* As far as is known, it is not possible to subscribe to the *Trudy* in the same way as one can for the monthly periodicals listed under I. Advance notice of their appearance is given in the Russian publication *Soviet Knigi*, and they may sometimes be obtained through foreign book agencies. Editions of the *Trudy* are not large, and it is normally impossible to obtain back copies. The library at Massachusetts Institute of Technology has recently begun an exchange program with some of these institutes, in the hope that the *Trudy* may thereby be received automatically as they appear. Tables of contents of the *Trudy* are occasionally listed in *Meteorological and Geoastrophysical Abstracts*.

Most of the Russian articles discussed in this report are available (for a nominal charge) in translated form. This has been indicated after the individual reference in the bibliography by a series of initials, according to the following scheme:

LIB Used for references to the 1957 or 1958 issues of *Izv. Akad. Nauk SSSR, Ser. Geofiz.* These issues have been translated in full, as noted above, and should be available in most well-stocked libraries.

DRB Defense Research Board,
Directorate of Scientific Information,
"A" Building, Cartier Square,
Ottawa, Ontario

OTS Office of Technical Services,
U. S. Department of Commerce,
Washington 25, D. C.

SLA SLA Translation Center,
John Crerar Library,
86 E. Randolph Street,
Chicago 1, Illinois.

Several of the articles have appeared in the serial, *Meteorological Translations*, published by the Department of Transport, Meteorological Branch, 315 Bloor Street West, Toronto.

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