ON THE ENERGY EXCHANGE BETWEEN THE BAROCLINIC AND BAROTROPIC COMPONENTS OF ATMOSPHERIC FLOW*

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ABSTRACT

The energy conversion between the vertical shear flow and the vertical mean flow has been computed using atmospheric data from the isobaric surfaces: 850, 700, 500, 300, and 200 mb. In comparison with earlier calculations based on a smaller vertical resolution (2 levels) and a smaller sample, it is found that the new calculations give larger numerical values in better agreement with the results of numerical experiments concerning the general circulation of the atmosphere. The energy transformation has been computed in the wave number regime, and it is found that the medium-scale waves are responsible for the major portion of the transformation.

The amounts of energy in the baroclinic component (the vertical shear flow) and the barotropic component (the vertical mean flow) have been computed as a function of wave number. It is found that the kinetic energy in the barotropic component is about 2.6 times the kinetic energy in the baroclinic component. The partitioning of the kinetic energy between the zonal flow and the eddies is such that the eddies contain more energy than the zonal flow. This result applies for the vertical shear flow as well as the vertical mean flow and is in contrast to the results obtained from numerical experiments regarding the general circulation.

The present computations include only the energy calculations which would be present in a quasi-non-divergent model. Later calculations will provide estimates of the remaining term of the energy conversion.

1. INTRODUCTION

A few years ago one of the authors (Wiin-Nielsen [10]) made a pilot calculation of the energy conversion from the kinetic energy of the vertical shear flow to the kinetic energy of the vertical mean flow. The study which contains the derivation of the basic formulas for the energy conversion was based on data with a very limited vertical resolution (2 levels) and on data from a single winter month (January 1959).

Since then Smagorinsky [8] has published his basic experiment on the numerical simulation of the general circulation of the atmosphere. He uses the same energy conversion to investigate the energetics of his model. The original idea to divide the energy conversion from available potential energy to kinetic energy into the energy conversion from available potential to the kinetic energy of the vertical shear flow and the conversion from this form of energy to the kinetic energy of the vertical mean flow was, as a matter of fact, proposed by Smagorinsky. Although there is agreement with respect to direction of the energy conversion in question between the results of Smagorinsky and those reported in [10], there are differences in the order of magnitude. The observational study [10] gave a time-averaged value of 3.8×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$, while the numerical study resulted in a value of 16.1×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$, or more than 4 times as much.

It was pointed out by Smagorinsky [8] that one of the reasons for the low value found for January 1959 from data at 850 and 500 mb., could be that only data from the lower part of the atmosphere were used in the evaluations. For this reason alone, it is worthwhile to extend the calculations to a larger vertical resolution. In addition, data from a single winter month may not be very representative for the general circulation of the atmosphere and it becomes important to extend the calculations to other months selected from different seasons and different years. It is one of the purposes of this paper to report the results of such calculations.

In [10] it was necessary to give a crude first estimate of the amounts of shear flow kinetic energy and mean flow kinetic energy in order to estimate the energy decay times. No estimate of the two forms of energy has been made from observations to the knowledge of the authors, although results from numerical experiments (Smagorinsky

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[8]) have been published. A second purpose of this paper is to describe the results of such observational studies.

It was shown in [10] that the kinetic energy conversion from the vertical shear flow to the vertical mean flow may be written as a sum of two integrals (see equation (3.12) of [10]). The evaluation of the first integral requires a knowledge of the velocity divergence in the atmosphere, while the second integral can be evaluated from the vertical component of vorticity and a reasonable approximation to the horizontal wind field. While the second integral readily is estimated from standard data, it is as difficult to evaluate the first as it is to compute vertical velocities from atmospheric data. Estimates of the first integral will not be given in this paper, but computations of vertical velocities from a quasi-balanced five-level model of the atmosphere are under way and will be reported later.

2. THE CALCULATION OF THE ENERGY CONVERSION

Since the basic derivations were given in [10], it will not be necessary to repeat them here. It suffices to point out that we have computed the integral

$$C_{ND}(K_S, K_M)=\frac{p_0}{g}\int_A (\mathbf{k}\times\mathbf{V}_M)\cdot[\zeta_S\mathbf{V}_S]_M\, dA \qquad (2.1)$$

in which K_M is the kinetic energy of the vertical mean flow and K_S the kinetic energy of the vertical shear flow. $\mathbf{V}$ is the horizontal component of the wind vector, ζ the vertical component of the vorticity vector, $\mathbf{k}$ the vertical unit vector, p_0 a standard value of the surface pressure, and g is the acceleration of gravity.

A means the total area over which the integration is carried out, while dA is the area element. In spherical coordinates we get:

$$dA=a^2\cos\varphi d\varphi d\lambda \qquad (2.2)$$

where a is the radius of the earth, φ is latitude and λ is longitude.

A subscript M means a vertical average defined by the relation

$$(\ )_M=\frac{1}{p_0}\int_0^{p_0}(\ )dp \qquad (2.3)$$

while a subscript S is defined by the relation

$$(\ )_S=(\ )-(\ )_M \qquad (2.4)$$

The integral (2.1) was evaluated using data from the isobaric surfaces: 850, 700, 500, 300, and 200 mb. The original data consisted of height data analyzed by the National Meteorological Center (NMC), U.S. Weather Bureau. A streamfunction, ψ, was computed for each level at each observation time by solving the balance equation

$$\nabla^2\psi=\frac{1}{f}\nabla^2\phi-\frac{1}{f^2}\nabla f\cdot\nabla\phi \qquad (2.5)$$

All quantities in the integrand of (2.1) can now be expressed in terms of the streamfunction derived from (2.5).

The vertical mean of the streamfunction was defined by the following weighted average:

$$\psi_M=0.25\psi(20)+0.15\psi(30)+0.20\psi(50)+0.175\psi(70)+0.225\psi(85) \qquad (2.6)$$

where the number in parenthesis refers to the pressure at the isobaric level, measured in cb.

The shear flow at each level is then defined by the relation

$$\psi_S(p)=\psi(p)-\psi_M \qquad (2.7)$$

The fields of the streamfunctions defined by (2.5), (2.6), and (2.7) were obtained on the grid used by NMC. The six fields Ψ_M and Ψ_S (p) were next obtained in a grid of spherical coordinates using a grid size of 2.5° in latitude and longitude, i.e. $\Delta\lambda=\Delta\varphi=2.5°$. The interpolation procedure was carefully checked by first interpolating to the spherical grid and next, interpolating back to the quadratic NMC grid. It was possible to regenerate the original field with a very high accuracy.

It is the purpose of this investigation to compute the spectral distribution of energies and energy transformations. We have used a technique very similar to those employed in earlier investigations (Wiin-Nielsen, Brown, and Drake, [11], [12]). Each streamfunction is written in a Fourier series of the form

$$\psi=A_0(\varphi)+\sum_{n=1}^{N}\{A_n(\varphi)\cos(n\lambda)+B_n(\varphi)\sin(n\lambda)\} \qquad (2.8)$$

The Fourier coefficients, A_0, A_n, and B_n, were computed by standard procedures for each level and latitude with $N=15$. The lowest latitude was $\varphi=17.5°$ N., while the highest latitude was $\varphi=87.5°$ N. The total number of Fourier coefficients for the 6 fields at 29 different latitudes, each characterized by 31 coefficients is therefore 5395.

The integral (2.1) may also be written in the form

$$C_{ND}(K_S,K_M)=\frac{1}{g}\int_0^{p_0}\int_0^{2\pi}\int_{\varphi_1}^{\varphi_2}\zeta_S(u_Mv_S-u_Sv_M)a^2\cos\varphi d\varphi d\lambda dp \qquad (2.9)$$

when it is expressed in spherical coordinates. The term in parentheses in the integrand may also be written in the form $J(\Psi_M, \Psi_S)$, when the wind components are expressed by the streamfunction. We may therefore write:

$$C_{ND}(K_S,K_M)=\frac{1}{g}\int_0^{p_0}\int_0^{2\pi}\int_{\varphi_1}^{\varphi_2}\zeta_S J(\Psi_M,\Psi_S)a^2\cos\varphi\, d\varphi d\lambda dp \qquad (2.10)$$

which shows that the contribution from a given level to the total energy conversion depends on the correlation between the vorticity of the shear flow and the advection of the streamfunction for the shear flow in the streamfunction for the vertical mean flow. It is the form (2.10) which was used in [10] to evaluate the energy conversion. The relative vorticity of the shear flow and the Jacobian were expanded in Fourier series, and C_{ND} can then be expressed in the Fourier coefficients in the two series. In this investigation we have preferred to work directly with the Fourier coefficients for the streamfunctions. Under these circumstances we are faced with the problem of finding the spectrum for an energy conversion which depends on an integral of a triple product, of which two factors depend on the shear flow while the third factor depends on the mean flow. The problem can be solved by forming the sum of all the terms from the shear flow which contribute to a given component of the vertical mean flow. The general derivation is given in Appendix A of this paper.

When formula (2.1) is expanded, and we write the contribution from a single level representing a layer of pressure difference Δp we get

$$C_\Delta(K_S,K_M)=\frac{\Delta p}{g}\int_0^{2\pi}\int_{\varphi_1}^{\varphi_2}\zeta_S(u_Mv_S-u_Sv_M)a^2\cos\varphi\, d\varphi d\lambda \tag{2.11}$$

The contributions from the different layers are added to form C_{ND} (K_S, K_M). The values of Δp for the levels 850, 700, 500, 300, and 200 mb. are 22.5, 17.5, 20.0, 15.0, and 25.0 respectively.

The Fourier series are written:

$$\psi_M=A_0^M(\varphi)+\sum_{n=1}^{N}\left\{A_n^M(\varphi)\cos(n\lambda)+B_n^M(\varphi)\sin(n\lambda)\right\} \tag{2.12}$$

and

$$\psi_S=A_0^S(\varphi,p)+\sum_{n=1}^{N}\left\{A_n^S(\varphi,p)\cos(n\lambda)+B_n^S(\varphi,\ )\sin(n\lambda)\right\} \tag{2.13}$$

The series expansions for the shear vorticity and the horizontal wind components are easily derived from (2.12) and (2.13) by using the relations

$$u=-\frac{1}{a}\frac{\partial\psi}{\partial\varphi},\qquad v=\frac{1}{a\cos\varphi}\frac{\partial\psi}{\partial\lambda} \tag{2.14}$$

and

$$\zeta=\frac{1}{a^2}\left[\frac{1}{\cos^2\varphi}\frac{\partial^2\psi}{\partial\lambda^2}+\frac{\partial^2\psi}{\partial\psi^2}-\tan\varphi\frac{\partial\psi}{\partial\varphi}\right] \tag{2.15}$$

The notations used in these expansions are summarized below:

$$u_M=U_0^M(\varphi)+\sum_{n=1}^{N}[UC_n^M(\varphi)\cos(n\lambda)+US_n^M(\varphi)\sin(n\lambda)] \tag{2.16}$$

$$v_M=V_0^M(\varphi)+\sum_{n=1}^{N}[VC_n^M(\varphi)\cos(n\lambda)+VS_n^M(\varphi)\sin(n\lambda)] \tag{2.17}$$

and similar expressions for u_S and v_S in which M is replaced by S. The expansion for shear vorticity is written:

$$\zeta_S=Z_0^S(\varphi)+\sum_{n=1}^{N}[ZC_n^S(\varphi)\cos(n\lambda)+ZS_n^S(\varphi)\sin(n\lambda)] \tag{2.18}$$

The contribution from the layer given in (2.11) is now written in the form

$$C_\Delta(K_S,K_M)=C_\Delta^0+\sum_{n=1}^{n}C_\Delta^n \tag{2.19}$$

By applying the results from Appendix A, it is then possible to find the expressions for C_Δ^0 and C_Δ^n. These formulas are given below, converted to energy conversions per unit area:

$$C_\Delta^0=\frac{\Delta p}{g}\frac{1}{2(\sin\varphi_2-\sin\varphi_1)}\int_{\varphi_1}^{\varphi_2}T_0(\varphi)\cos\varphi\, d\varphi \tag{2.20}$$

in which φ_1 and φ_2 are the southern and northern latitudes determining the boundaries of the region ($\varphi_1=18.75°$ and $\varphi_2=88.75°$) and

$$T_0(\varphi)=U_0^M\sum_{n=1}^{N}\{VC_n^S\cdot ZC_n^S+VS_n^S\cdot ZS_n^S\} \tag{2.21}$$

In the calculations we replace the integral in (2.20) by a finite sum.

The expression for C_Δ^n, $n=1, 2, \ldots, N$ may be written in the form

$$C_\Delta^n=\frac{\Delta p}{g}\frac{1}{2(\sin\varphi_2-\sin\varphi_1)}\int_{\varphi_1}^{\varphi_2}(-T_n^{(1)}+T_n^{(2)}-T_n^{(3)})\cos\varphi\, d\varphi$$
$$+\frac{\Delta p}{g}\frac{1}{4(\sin\varphi_2-\sin\varphi_1)}\int_{\varphi_1}^{\varphi_2}(T_n^{(4)}-T_n^{(5)})\cos\varphi\, d\varphi \tag{2.22}$$

in which

$$T_n^{(1)}=U_0^S(VC_n^M\cdot ZC_n^S+VS_n^M\cdot ZS_n^M) \tag{2.23}$$

$$T_n^{(2)}=Z_0^S(UC_n^M\cdot VC_n^S+US_n^M\cdot VS_n^S) \tag{2.24}$$

$$T_n^{(3)}=Z_0^S(VC_n^M\cdot UC_n^S+VS_n^M\cdot US_n^S) \tag{2.25}$$

The three expressions (2.23) to (2.25) represent the contribution from the waves of wave number n in the shear flow and the mean flow in their interaction with the zonal flow. The contributions from these terms are included in the analysis given in section 5 of [10], in which the energy conversion was analyzed for the quasi-non-divergent, two-parameter model. The remaining two terms, $T_n^{(4)}$ and $T_n^{(5)}$, represent the non-linear interaction between components which combine to contribute to the kinetic energy of wave number n of the vertical mean flow.

The expressions are:

$$T_n^{(4)}=\sum_{m=1}^{n}\{UC_n^M[VC_m^S(ZC_{n+m}^S+ZC_{n-m}^S)+VS_m^S(ZS_{n+m}^S-ZS_{n-m}^S)]+US_n^M[VC_m^S(ZS_{n+m}^S+ZS_{n-m}^S)-VS_m^S(ZC_{n+m}^S-ZC_{n-m}^S)]\}+\sum_{m=n+1}^{N}\{UC_n^M[VC_m^S(ZC_{m+n}^S+ZC_{m-n}^S)+VS_m^S(ZS_{m+n}^S+ZS_{m-n}^S)]+US_n^M[VC_m^S(ZS_{m+n}^S-ZS_{m-n}^S)-VS_m^S(ZC_{m+n}^S-ZC_{m-n}^S)]\} \quad (2.26)$$

and

$$T_n^{(5)}=\sum_{m=1}^{n}\{VC_n^M[UC_m^S(ZC_{n+m}^S+ZC_{n-m}^S)+US_m^S(ZS_{n+m}^S-ZS_{n-m}^S)]+VS_n^M[UC_m^S(ZS_{n+m}^S+ZS_{n-m}^S)-US_m^S(ZC_{n+m}^S-ZC_{n-m}^S)]\}+\sum_{m=n+1}^{N}\{VC_n^M[UC_m^S(ZC_{m+n}^S+ZC_{m-n}^S)+US_m^S(ZS_{m+n}^S+ZS_{m-n}^S)]+VS_n^M[UC_m^S(ZS_{m+n}^S-ZS_{m-n}^S)-US_m^S(ZC_{m+n}^S-ZC_{m-n}^S)]\} \quad (2.27)$$

The following conventions have been incorporated in the formulas for $T_n^{(4)}$, and $T_n^{(5)}$:

1. If $(m+n)>N$, the corresponding Fourier coefficient is zero.
2. If $n-m=0$, the corresponding coefficient is zero.

The first point is equivalent to the fact that the Fourier expansions are truncated at $n=N$, while the second point is necessary because these components are accounted for in formulas (2.23) to (2.25).

Formulas which in many respects are similar to those just developed have been used by Saltzman and Fleisher [4].

The formulas described in this section were used to compute the energy conversion C_{ND} (K_S, K_M) in the wave number regime with $N=15$. For each calculation we have also obtained the contribution from each level to the total energy conversion. The results of the calculations will be described in section 4 of this paper.

3. CALCULATION OF THE KINETIC ENERGY SPECTRA

In order to estimate the decay times for the different forms of energy in the atmosphere, it is necessary to know the amounts of energy in the different reservoirs. In our case we must know the amounts of kinetic energy in the vertical shear flow and the vertical mean flow. Some preliminary estimates of decay times were made in [10], but the partition of kinetic energy between the shear flow and the mean flow was estimated from an extremely simple assumption about the vertical structure of the atmosphere. The estimate was, as a matter of fact, based on an integrated two-parameter model.

It is possible to compute the amounts of shear flow and mean flow kinetic energies from the same basic data which are used in the energy conversion calculations. The procedures used in these calculations will be described in the following paragraphs.

The kinetic energy of the horizontal motion in the atmosphere per unit area may be written.

$$K=\frac{1}{g}\int_0^{p_0}\int_A \frac{1}{2}(u^2+v^2)dAdp \quad (3.1)$$

When we write $u=u_M+u_S$, $v=v_M+v_S$ we may also divide the integral in (3.1) in the following way:

$$K=\frac{p_0}{2g}\int_A(u_M^2+v_M^2)dA+\frac{1}{2g}\int_0^{p_0}\int_A(u_S^2+v_S^2)dAdp \quad (3.2)$$

The first term in (3.2) is the kinetic energy of the vertical mean flow while the second is the kinetic energy of the vertical shear flow. They will be denoted K_M and K_S, respectively, and (3.2) may therefore be written:

$$K=K_M+K_S \quad (3.3)$$

K_M and K_S have been computed by the assumption that the wind components are non-divergent (see equation (2.14)). The streamfunction for the vertical mean flow and the vertical shear flow were computed using (2.6) and (2.7). The Fourier series are unchanged; see (2.12) and (2.13). The spectra for K_M and K_S are written:

$$K_M=K_0^M+\sum_{n=1}^{N}K_n^M,\quad K_S=K_0^S+\sum_{n=1}^{N}K_n^S \quad (3.4)$$

With these notations we find by substitution of (2.12) in (3.2) and (3.3):

$$K_0^M=\frac{p_0}{2ga^2(\sin\varphi_2-\sin\varphi_1)}\int_{\varphi_1}^{\varphi_2}\left(\frac{\partial A_0^M}{\partial\varphi}\right)^2\cos\varphi\, d\varphi \quad (3.5)$$

$$K_n^M=\frac{p_0}{4ga^2(\sin\varphi_2-\sin\varphi_1)}\int_{\varphi_1}^{\varphi_2}\left[\frac{n^2}{\cos\varphi}\left((A_n^M)^2+(B_n^M)^2\right)+\cos\varphi\left\{\left(\frac{\partial A_n^M}{\partial\varphi}\right)^2+\left(\frac{\partial B_n^M}{\partial\varphi}\right)^2\right\}\right]d\varphi \quad (3.6)$$

The contribution to K_S from a layer of pressure difference Δp may be written

$$K_{\Delta,S}=\frac{\Delta p}{2g}\int_A(u_S^2+v_S^2)dA \quad (3.7)$$

Substitution of (2.13) in (3.7) results in the following formulas:

$$K_{\Delta,0}^S=\frac{\Delta p}{2ga^2(\sin\varphi_2-\sin\varphi_1)}\int_{\varphi_1}^{\varphi_2}\left(\frac{\partial A_0}{\partial\varphi}\right)^2\cos\varphi\, d\varphi \quad (3.8)$$

and

$$K_{\Delta,n}^S=\frac{\Delta p}{4ga^2(\sin\varphi_2-\sin\varphi_1)}\int_{\varphi_1}^{\varphi_2}\left[\frac{n^2}{\cos\varphi}\{(A_n^S)^2+(B_n^S)^2\}+\cos\varphi\left\{\left(\frac{\partial A_n^S}{\partial\varphi}\right)^2+\left(\frac{\partial B_n^S}{\partial\varphi}\right)^2\right\}\right]d\varphi \quad (3.9)$$

The contributions from the different layers computed through application of (3.8) and (3.9) are added to obtain K_0^S and K_n^S. The derivatives in (3.5), (3.6), (3.8), and (3.9) were approximated by central finite differences, while the integrals with respect to latitude were approximated by finite sums using standard procedures. The spectra were computed with $N=15$. The results of these calculations will be described in section 5 of this paper.

4. RESULTS OF CALCULATIONS OF ENERGY CONVERSIONS

The calculations of the energy conversion $C(K_S, K_M)$ have so far been carried out for five different months: January, April, July, and October, 1962 and January 1963. The vertical resolution has been the same in all cases. In most cases we have made one calculation per day based on the data from 0000 GMT, but the calculations have been repeated using 1200 GMT data for at least one month in order to verify that one calculation per day is sufficient.

One of the main results of the calculations is summarized in table 1, which gives the mean value of the energy conversion $C(K_S, K_M)$ for each month and the contributions from each of the five levels. The unit for the numbers in table 1 is 10^{-4} kj.m.$^{-2}$ sec.$^{-1}$

By comparing the results in table 1 with those given in [10], which were based on data from January 1959 and from 850 and 500 mb. only, it is immediately apparent that the new results for the two winter months are about one order of magnitude larger.

It is naturally always possible to disregard such differences by pointing to the fact that there may be large differences between the circulations and energetics of two winter months in different years. However, it is worthwhile to try to gain further insight into these differences. We observe first of all that calculations based on data from 850 and 500 mb. indeed will give an underestimate of the energy conversion. We may verify this statement by computing the value which we would get by using the contributions from only these two levels. In that case we would have to use $\Delta p=32.5$ cb. for the 850-mb. level and $\Delta p=67.5$ cb. for the 500-mb. level. For January 1962 we would therefore get 29.4×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$, and for January 1963 we would get 26.4×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$. These values are considerably smaller than the values found in table 1. The underestimate is about 37 percent in both cases.

Second, the calculations in this paper may be too large because of the assumption that the 200-mb. flow is representative for the upper 25 percent of the mass of the atmosphere. Third, there are differences in the numerical procedures used in [10] and the present study. In [10] we computed the thermal vorticity and the temperature advection in the grid points of the quadratic grid. The values were then interpolated to the spherical grid. It is quite likely that a rather serious reduction of the maximum and minimum values was made by this procedure.

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Table 1.—*Monthly mean values of $C(K_S, K_M)$ and the contributions from the different layers. Unit: 10^{-4} kj. m.$^{-2}$ sec.$^{-1}$*

p, (mb.)	Jan. 1962	April 1962	July 1962	Oct. 1962	Jan. 1963
850	18.3	11.5	4.2	11.5	16.6
700	5.4	3.2	1.4	3.3	4.7
500	0.9	0.2	0.2	0.4	0.7
300	6.9	4.5	1.4	4.4	6.0
200	15.2	9.4	5.2	10.1	13.7
Total	46.5	28.8	12.4	29.6	41.6

Finally, the much greater complexity of the formulas in this paper increases the probability for programing mistakes, especially in the evaluation of expressions like (2.26) and (2.27). Although the programs have been checked and rechecked, it was found desirable to make a special control calculation in which only the basic formula (2.10) was used without any reference to the wave number space. The vorticity of the shear flow and the Jacobian were in this case calculated on the spherical grid in order to avoid serious interpolation errors. The results of such a calculation will not necessarily agree exactly with the previous calculation because the former contain all wave components while the latter has only the contribution from the first 15 components. However, the order of magnitude should be the same. The results of this test calculation which was carried out for only three days indicate general agreement with respect to orders of magnitude although our calculations in the wave number domain seem to be larger by about 20 percent. The only explanation for this discrepancy is that the smaller scales in the vorticity and advection fields have non-negligible amplitudes, which combine in such a way that they give negative contributions to the energy conversion $C_{ND}(K_S, K_M)$. Although it is difficult to understand why there should be a systematic negative contribution, it should be pointed out that this contribution is not necessarily real because of its small scale.

In summary, we can state that the values of the energy conversion $C_{ND}(K_S, K_M)$ are large compared with earlier estimates. Several reasons have been given for the differences, leading to the opinion that although the new estimates appear too large there are reasons to believe that the pilot calculations definitely gave underestimates.

It should also be remembered that we have only computed the part of the energy conversion which would be present in a quasi-non-divergent model. If we apply the results of the pilot calculations in [10], it is to be expected that $C(K_S, K_M)$ will be reduced by the integral depending on the divergence of the wind field.

If we accept the indication from test calculations that the contribution from the first 15 wave components overestimates $C_{ND}(K_S, K_M)$ by about 20 percent, we get the corrected total values of $C_{ND}(K_S, K_M)$ reproduced in table 2. The values given in table 2 represent the most likely values of the total energy conversion $C_{ND}(K_S, K_M)$ which we can obtain based on our present calculations.

The annual mean value obtained as an average of the

TABLE 2.—*Corrected monthly mean values of $C(K_S, K_M)$. Unit: 10^{-4} kj. m.$^{-2}$ sec.$^{-1}$*

Month	Jan. 1962	Apr. 1962	July 1962	Oct. 1962	Jan. 1963
$C(K_S, K_M)$	37	23	10	24	33

figures given in table 2 is 23×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$ This value may still be too large because of the probable negative contribution from the second integral in the total energy conversion. If we apply a 10 percent correction as indicated by the pilot calculation in [10], we obtain 21×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$, which then should be compared with the value of 16×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$ obtained by Smagorinsky [8] in his numerical experiments. It is seen that the agreement now is fair.

A further check on the magnitude of our results may be obtained by a comparison with the values obtained by observational studies of the energy conversion from available potential energy to shear flow kinetic energy, $C(A, K_S)$.

Several estimates have been made of the energy conversion $C(A,K_S)$ by different investigators. These estimates have been summarized by Oort [3]. In order to give an estimate of an annual value of $C(A,K_S)$ we shall make use of monthly values obtained by Wiin-Nielsen [9], Saltzman and Fleisher [5], and Krueger, Winston, and Haines [2] which have made use of vertical velocities from numerical prediction models. The numerical values are given in table 3. In these comparisons we do not divide the energy conversion into the contributions from the zonal flow and the eddies. This division of the energy conversion will be treated later when we consider the wave number space.

A weighted mean value which is the most likely annual value of $C(A,K_S)$ can be obtained from table 3 by weighting each value according to the number of months it represents. This value turns out to be 21×10^{-4} kj. m.$^{-2}$ sec.$^{-1}$ which is equal to the lowest value which we can obtain for $C(K_S, K_M)$. Since the estimates are based on different sets of data, we cannot expect any closer agreement although it leaves us with the impression that our values of $C(K_S, K_M)$ are too large.

If our values of $C_{ND}(K_S, K_M)$ are not reduced in any appreciable way by the contribution from the divergent component of the wind, we must conclude that the frictional dissipation $D(K_S)$ is quite small. Such a result would be in agreement with those obtained by Smagorinsky [8] who found that the contribution from the surface skin friction to $D(K_S)$ was negligible in his model.

TABLE 3.—*A summary of computed values for $C(A,K_S)$ by different investigators. Unit: 10^{-4} kj. m.$^{-2}$ sec.$^{-1}$*

Time periods	$C(A,K_S)$	Investigators
Jan. '59	16.0	Wiin-Nielsen [9]
Apr. '59	10.1	Wiin-Nielsen [9]
6 months '59	33.9	Saltzman and Fleisher [5]
6 winter months '62–'63	22.6	Krueger, Winston, and Haines [2]
6 summer months '62–'63	9.4	Krueger, Winston, and Haines [2]

TABLE 4.—*The energy conversion between the zonal components, $C(K_{SZ}, K_{MZ})$, and the eddies, $C(K_{SE}, K_{ME})$, in $C(K_S,K_M)$. Unit: 10^{-4} kj. m.$^{-2}$ sec.$^{-1}$*

	Jan. 1962	April 1962	July 1962	Oct. 1962	Jan. 1963
$C(K_{SZ}, K_{MZ})$	2.7	0.7	1.5	3.1	0.0
$C(K_{SE}, K_{ME})$	43.8	28.1	10.9	26.6	41.6

The next problem is to investigate the energy transformation in the wave number space. We shall first restrict ourselves to the division into the conversion between the zonal components $C(K_{SZ}, K_{MZ})$ and the eddies $C(K_{SE}, K_{ME})$. The calculation of $C(K_{SZ}, K_{MZ})$ is based on equations (2.20) and (2.21) of this paper. The contribution from a single layer was denoted C_Δ^0 in (2.19) and (2.20). The energy exchange $C(K_{SE}, K_{ME})$, on the other hand, is based on equations (2.22) to (2.25) of this paper. For a single layer we have that

$$C_\Delta \ (K_{SE}, K_{ME})=\sum_{M=1}^{N} C_\Delta^N$$

as given in equation (2.19). The results of the calculations of $C(K_{SZ}, K_{MZ})$ and $C(K_{SE}, K_{ME})$ are summarized in table 4.

It is seen from table 4 that the energy conversion in both cases goes from the shear flow to the mean flow. The energy exchange between the zonal components is small compared to the energy exchange between the eddies. There is a marked seasonal variation in $C(K_{SE}, K_{ME})$ with large values in the winter and small values in the summer.

It should be noted that a direct comparison with the results of Smagorinsky [8] is difficult because of his more detailed breakdown of the energy conversions. However, one can show that the energy conversion $C(K_{SZ}, K_{MZ})$ in this paper corresponds to the energy conversions $C(K_{SZ}, K_{MZ})$ *and* $C(K_{SE}, K_{MZ})$ in his paper, while our energy conversion $C(K_{SE}, K_{ME})$ is equal to his energy conversions $C(K_{SE}, K_{ME})$, $C(K_{SZ}, K_{ME})$ *and* $C(K_{SE}, (K_{SZ}), K_{ME})$ of which the last conversion is the catalytic energy conversion. In this interpretation our annual average value of $C(K_{SZ}, K_{MZ})$ of 2×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$ should be compared with Smagorinsky's value of 3.6×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$, computed from his tables. Our annual average of 27.1×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$ for $C(K_{SE}, K_{ME})$ corresponds to his value of 12.6×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$. While there is reasonable agreement between the two values of $C(K_{SZ}, K_{MZ})$, we find that our value of $C(K_{SE}, K_{ME})$ again appears large which, as mentioned earlier, may be due to the missing contribution from the divergent part of the wind.

We turn next to the spectral distributions of $C(K_S, K_M)$ for the different months. These are given in figures 1–5 which show the spectra for the months of January, April, July, and October 1962 and January 1963, respectively. The figures show the spectra for wave numbers 0, 1, 2, . . . , 15. All the spectra from the year 1962 show a tendency for a maximum at wave numbers 6–8. This result (figs. 1–4) is similar to the result obtained in the pilot calculation in [10] where a maximum was found for $n=7$. The maximum is apparently due to the baroclinically unstable waves in the atmosphere. Another result, consistent with those found in [10], is the rather large values found for the small wave number (1, 2, or 3). The month of January 1963 (fig. 5) turns out to be very different from the other months. It shows a marked maximum for $n=3$. The same month was included in the study of energy conversion between the zonal flow and the eddies for available potential energy and kinetic energy (Wiin-Nielsen, Brown, and Drake [12]). It was found in this investigation that wave number 3 played a dominant role in the kinetic energy conversion from the eddies to the zonal flow. There is consequently agreement between the two investigations which show that wave number 3 is dominant in the vertical mean flow of the atmosphere during January 1963.

The spectra for the different levels giving $C(K_S, K_M)$ have been investigated for each month. They are in agreement with the numbers shown in table 1. The major contributions come from the lower and higher levels with an almost negligible contribution from the 500-mb. level.

The results which have been presented so far have been monthly mean values obtained as averages of individual daily values. It is naturally possible to compute standard deviations which will give some indication of how representative the mean values are. It is perhaps even better to reproduce the daily values of $C_{ND}(K_S, K_M)$ as a function of time. This has been done in figures 6 and 7, of which the first contains the curves for January, April, and July 1962, and figure 7 shows the curves for October 1962 and January 1963. It is seen that although there are considerable variations, we find a positive value of $C_{ND}(K_S, K_M)$ on each day. There is no apparent regular behavior in the variations. An inspection of similar curves (not reproduced) for all five months giving the contributions from the five levels shows that there are positive contributions from all levels at all times.

Although the pilot calculations in [10] indicated that $C_D(K_S, K_M)$ only amounts to approximately 10 percent of $C_{ND}(K_S, K_M)$ it will nevertheless be important to compute $C_D(K_S, K_M)$. Such a calculation will be possible when vertical velocities are available, because we can estimate the divergence from them. Calculations of this nature are in preparation.

5. RESULTS OF SHEAR FLOW AND MEAN FLOW KINETIC ENERGIES

The calculations of K_S and K_M in the wave number regime were performed following the formulas developed in section 3 of this paper, in particular equations (3.5), (3.6), (3.8), and (3.9).

TABLE 5.—*Monthly mean values of the kinetic energy in the vertical mean flow, K_M. Unit: kj. m.$^{-2}$*

Month	Jan. '62	Apr. '62	July '62	Oct. '62	Jan. '63	Average
K_M	2752	1832	891	1678	2814	1796

TABLE 6.—*Monthly mean values of the kinetic energy of the vertical shear flow K_S and the contributions from the different levels. Unit: kj. m.$^{-2}$*

Month	Jan. '62	Apr. '62	July '62	Oct. '62	Jan. '63	Average
850	361	244	112	198	343	226.50
700	125	84	44	72	119	80.50
500	24	16	13	17	30	18.25
300	140	106	49	90	138	96.00
200	393	258	155	226	382	256.63
Total	1048	708	373	604	1011	678

We shall first consider the total amounts of the energies. Values of K_M are reproduced in table 5 for the five months for which we have computed $C(K_S, K_M)$. Table 5 also contains an estimated value for the annual average, given in the last column.

Table 6 contains the values of K_S for the same five months, the annual average, and the contribution from the five levels to the total value of K_S.

The values given in tables 5 and 6 for the annual average may be compared with the mean values obtained by Smagorinsky [8] in his numerical experiment. The mean values obtained by him are $K_M=2060$ kj.m.$^{-2}$ and $K_S=1016$ kj.m.$^{-2}$. One observes that the energy levels in the numerical experiments are somewhat higher than those found as annual mean values in the observational studies. The ratio K_S/K_M in the numerical experiment is 0.49. The corresponding ratio computed from the observational studies is given in table 7, which shows that the ratio is almost invariant through the year with a somewhat lower annual value, 0.38.

The fact that the energy in K_S is somewhat higher in Smagorinsky's numerical experiment might be due to the fact that we have included only the energy contained in the non-divergent motion, while his estimates naturally contain the total energy in the horizontal motion. The same argument cannot be applied to the energy K_M because the vertical mean flow is essentially non-divergent in both calculations.

We shall next turn our attention to the partitioning of the kinetic energy in the vertical shear flow and the vertical mean flow between the zonal flow and the eddies.

TABLE 7.—*The ratio of K_S/K_M for the different months and the annual average*

Month	Jan. '62	Apr. '62	July '62	Oct. '62	Jan. '63	Average
K_S/K_M	0.38	0.39	0.42	0.36	0.36	0.38

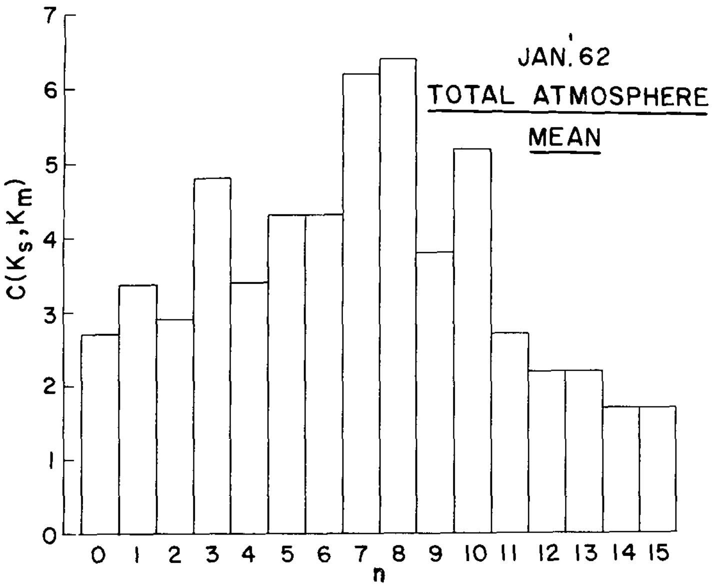

FIGURE 1.—Spectrum of the energy conversion C (K_S, K_M) in the average for January 1962 as a function of wave number n. Unit: 10^{-4} kj. m.$^{-2}$ sec^{-1}.

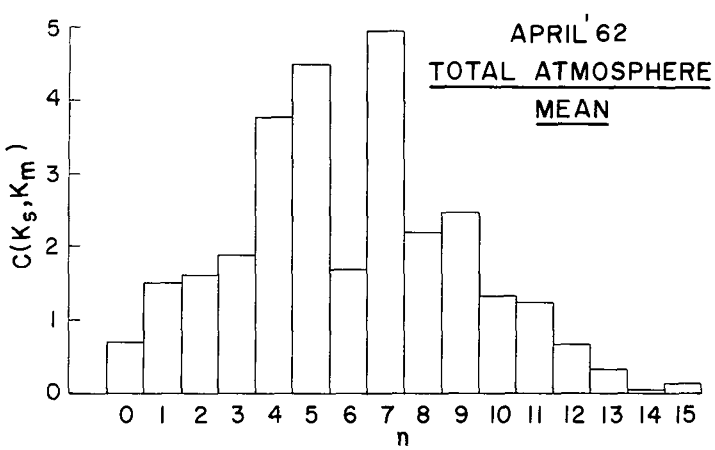

FIGURE 2.—As figure 1. April 1962.

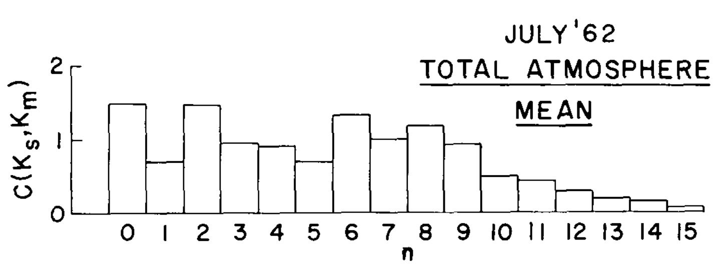

FIGURE 3.—As figure 1. July 1962.

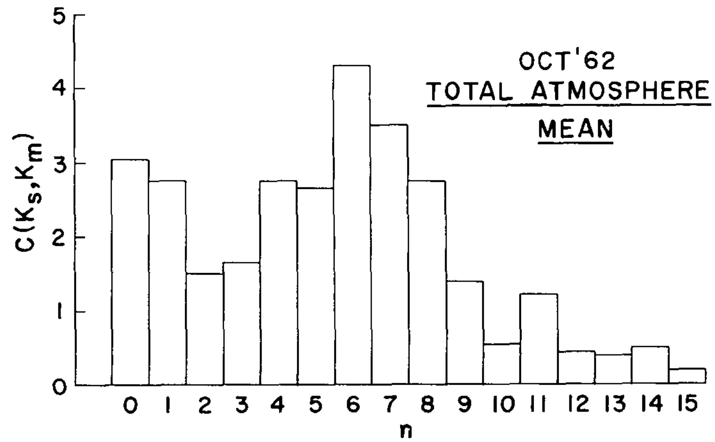

FIGURE 4.—As figure 1. October 1962.

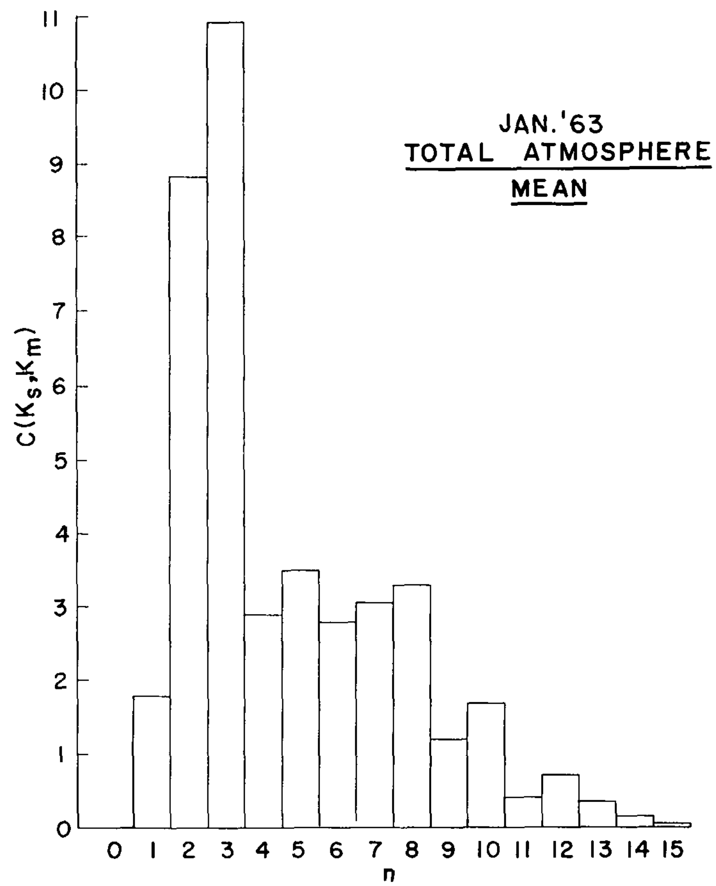

FIGURE 5.—As figure 1. January 1963.

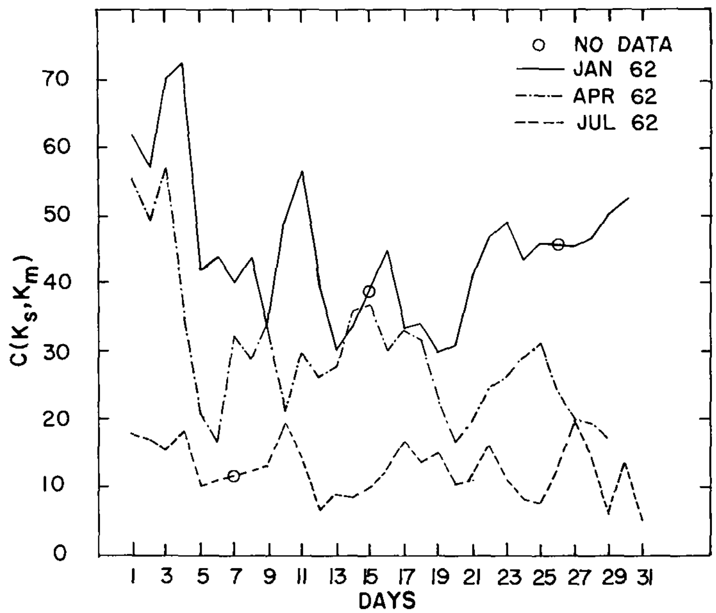

FIGURE 6.—The energy conversion $C(K_S, K_M)$ as a function of time for the months of January 1962 (solid curve), April 1962 (dashed-dotted curve) and July 1962 (dashed curve). A circle indicates missing data. Unit: 10^{-4} kj. m.$^{-2}$ sec^{-1}.

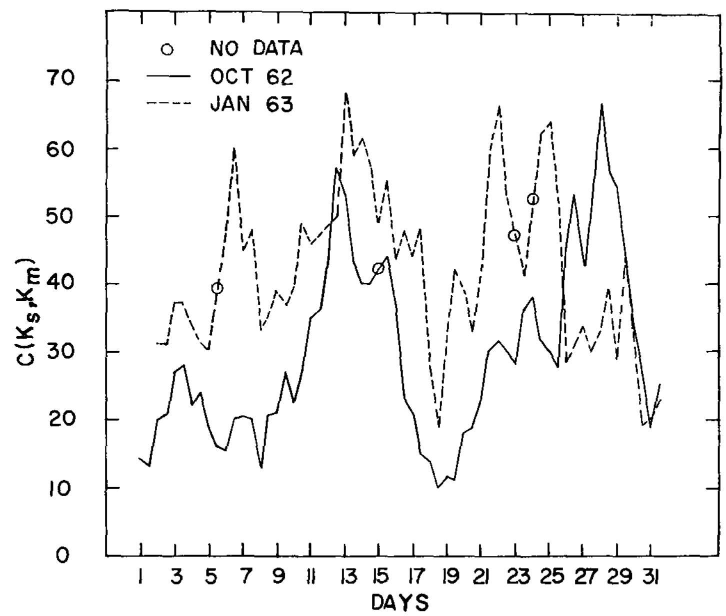

FIGURE 7.—The energy conversion $C(K_S, K_M)$ as a function of time for the months of October 1962 (solid curve) and January 1963 (dashed curve). A circle indicates missing data. Unit: 10^{-4} kj. m.$^{-2}$ sec^{-1}.

The notations for these quantities will be: $K_{MZ}=K_0^M$, $K_{ME}=\sum_{n=1}^{N} K_n^M$, $K_{SZ}=K_0^S$, and $K_{SE}=\sum_{n=1}^{N} K_n^S$. Table 8 summarizes the results of the observational studies, with an arrangement similar to the previous tables.

It is seen from table 8 that the partitioning of energy between the zonal flow and the eddies is such that we always have more energy in the eddies of the vertical mean flows than we find in the zonal flow. The same result holds in the average for the vertical shear flow although there are exceptions as seen in the results for January 1963, where K_{SZ} and K_{SE} are about equal. The partitioning of the energy between the zonal flow and the eddies found in the observational studies is in sharp contrast to the results obtained in the numerical experiment. These results have also been included in table 8, where it is seen that the kinetic energy in the eddies for both the vertical shear flow and the vertical mean flow is considerably smaller than the kinetic energy in the zonal flow. A similar result was found by a comparison of the partitioning of available potential energy between the zonal average and the eddies in Smagorinsky's experiment [8] and an observational study of available potential energy made by Winston and Krueger [13]. One therefore arrives at the conclusion that Smagorinsky's experiment has been designed in such a way that the available potential and the kinetic energy in the eddies is too small.

TABLE 8.—*The monthly mean values, the annual average and the mean values from Smagorinsky [8] of K_{MZ}, K_{ME}, K_{SZ}, and K_{SE}. Unit: kj. m.$^{-2}$*

	Jan '62	April '62	July '62	Oct. '62	Jan. '63	Aver.	Smag.
K_{MZ}	1240	732	310	648	1338	745	1842
K_{ME}	1512	1100	580	1030	1476	1051	208
K_{SZ}	515	313	110	218	515	289	884
K_{SE}	532	395	263	386	496	390	132

The spectral distributions of the kinetic energy in the vertical mean flow are shown in figures 8–12 representing the mean spectra for the months: January, April, July, and October 1962 and January 1963. It is seen from these figures that the amount of energy in the zonal flow (wave number 0) always is larger than the kinetic energy in any other component although, as seen from table 8, the total amount of energy in all eddies is larger than the energy in the zonal flow. During the winter (figs. 8 and 12) we find comparatively larger amounts of energy in the low wave numbers. The waves with wave numbers 2 and 3 are especially well developed during January 1963 (fig. 12). During the other seasons (figs. 9, 10, and 11) the low wave numbers are developed to a smaller extent, and there is a tendency for a maximum at higher wave numbers, but this tendency is not very marked. Only small amounts of energy are found in the waves with wave numbers larger than 10.

The spectra for the kinetic energy of the vertical shear flow are shown in figures 13–17. Everything, which has been said about the spectra for the kinetic energy of the vertical mean flow in the preceding paragraph, can also be said about the spectra for the kinetic energy of the vertical shear flow.

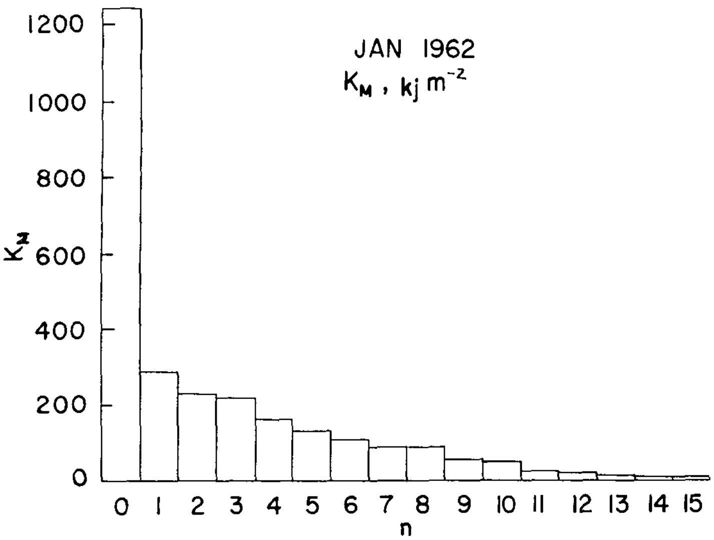

FIGURE 8.—Spectrum of the kinetic energy, K_M, of the vertical mean flow in the average for January 1962 as a function of wave number. Unit: kj. m.$^{-2}$.

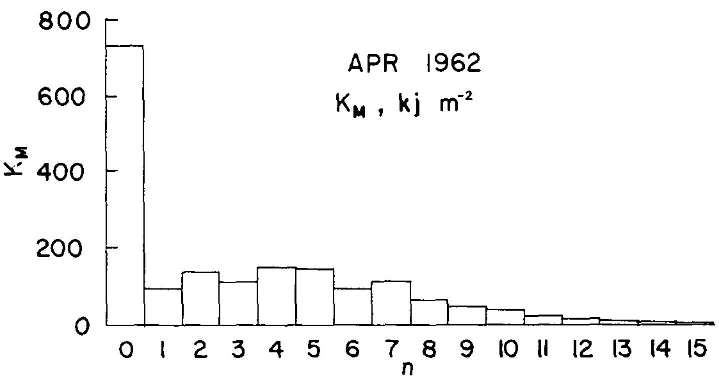

FIGURE 9.—As figure 8. April 1962.

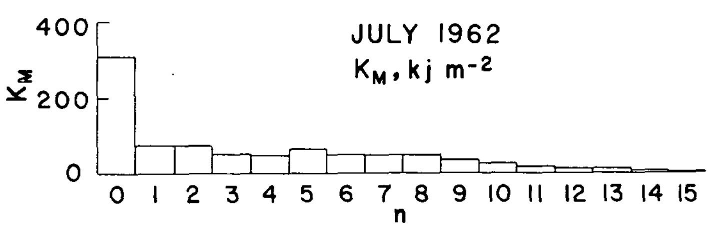

FIGURE 10.—As figure 8. July 1962.

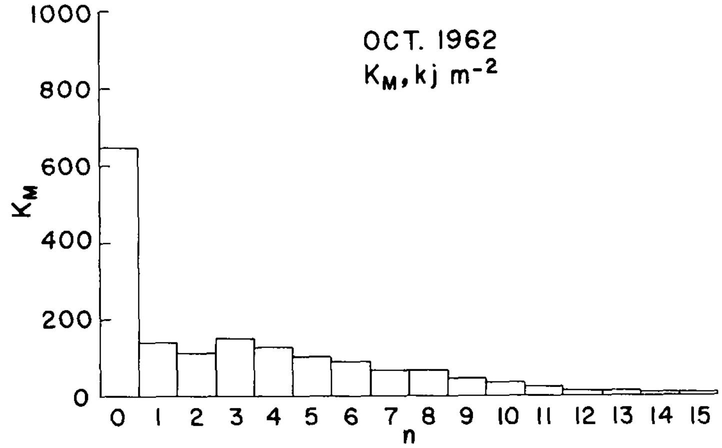

FIGURE 11.—As figure 8. October 1962.

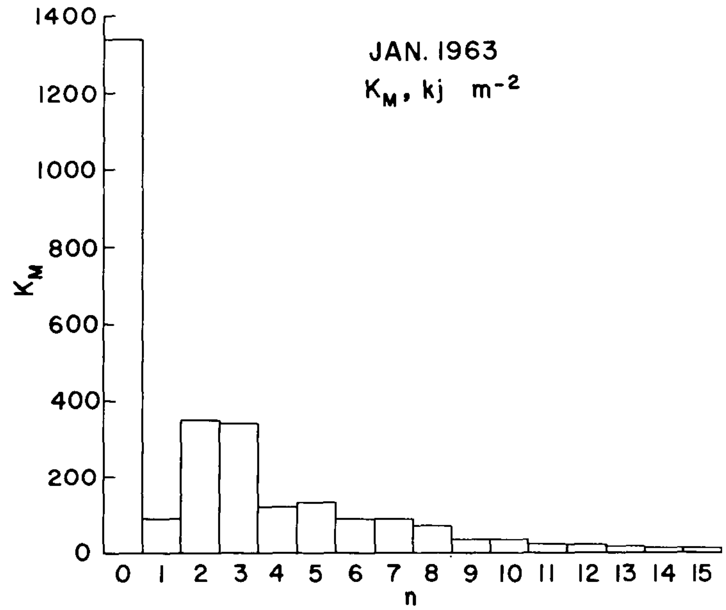

FIGURE 12.—As figure 8. January 1963.

Numerous studies have been made of the kinetic energy of different components of the atmospheric flow. Most studies have been restricted to the meridional component of the wind. The most extensive study of this nature has been made by Shapiro and Ward [7] who also give a table of previous studies. Another very interesting study of the kinetic energy of the meridional component of the wind has been made by Horn and Bryson [1]. In most of these studies, the spectra show a maximum kinetic energy around wave number 5–6 depending on the latitudes at which the calculations are made. That we do not find this maximum in our calculations is apparently due to the fact that we have included the kinetic energy of the zonal component of the wind. It has been shown by Saltzman and Fleisher [6] that the kinetic energy of the zonal component of the wind has a maximum at wave number 1 with the energy decreasing with increasing wave number. The sum of the kinetic energy of the zonal and meridional component will then result in spectra as shown in our study. Our spectra for the vertical mean flow agree, as a matter of fact, quite well with those obtained by Saltzman and Fleisher [6] for the 500-mb. level.

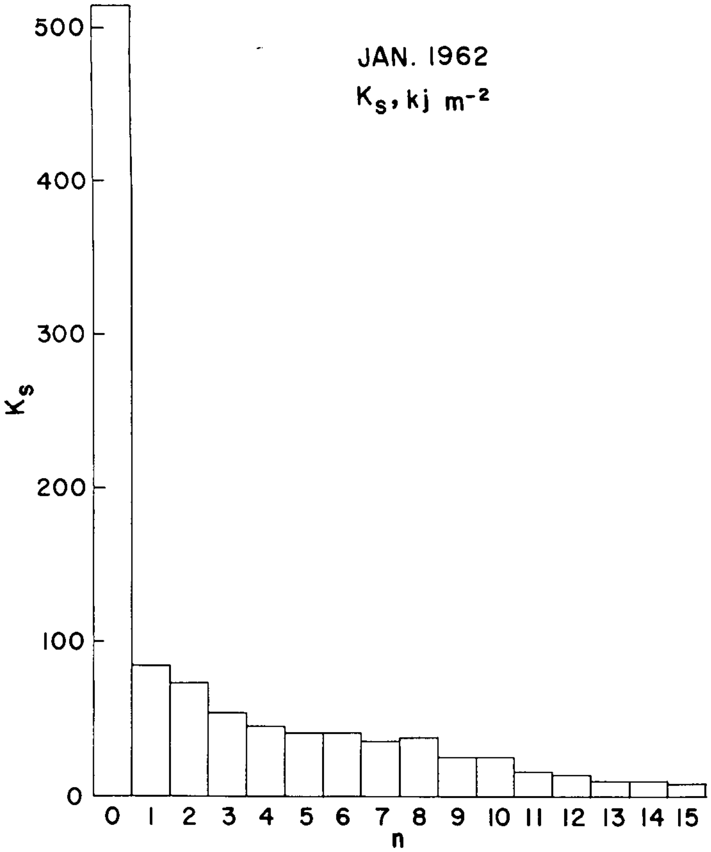

FIGURE 13.—Spectrum of the kinetic energy, K_S, of the vertical shear flow in the average for January 1962 as a function of wave number. Unit: kj. m.$^{-2}$.

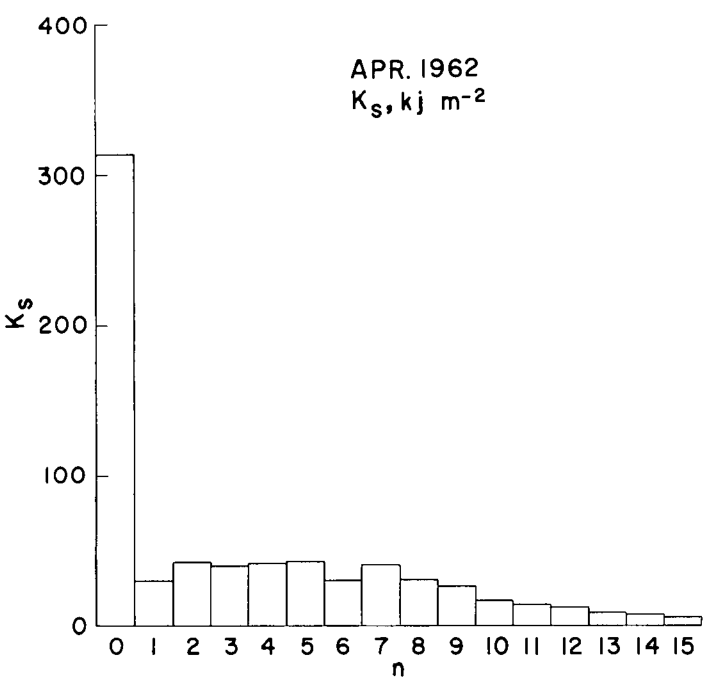

FIGURE 14.—As figure 13. April 1962.

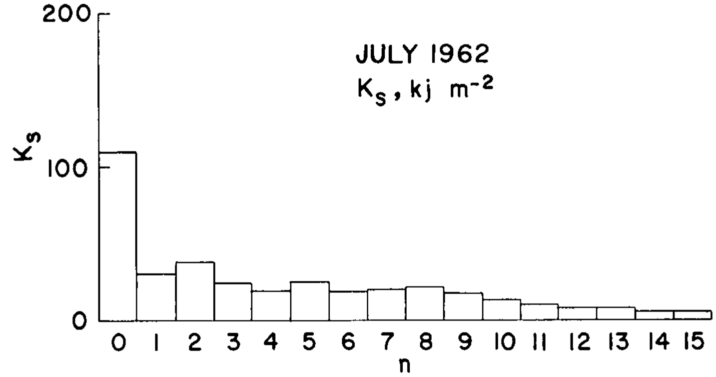

FIGURE 15.—As figure 13. July 1962.

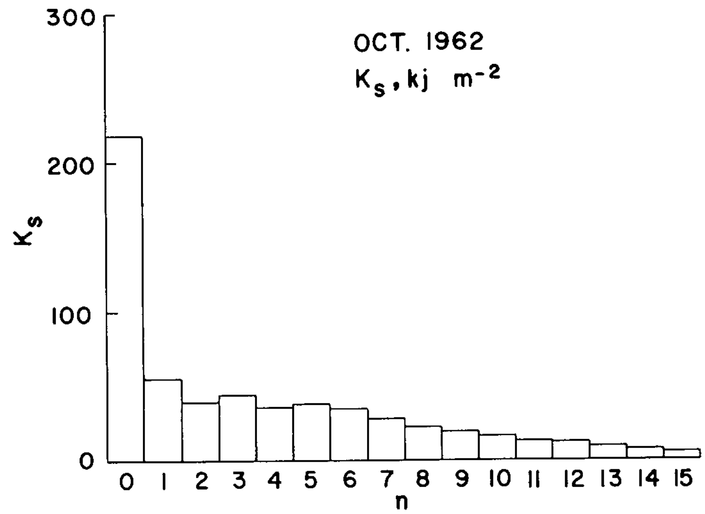

FIGURE 16.—As figure 13. October 1962.

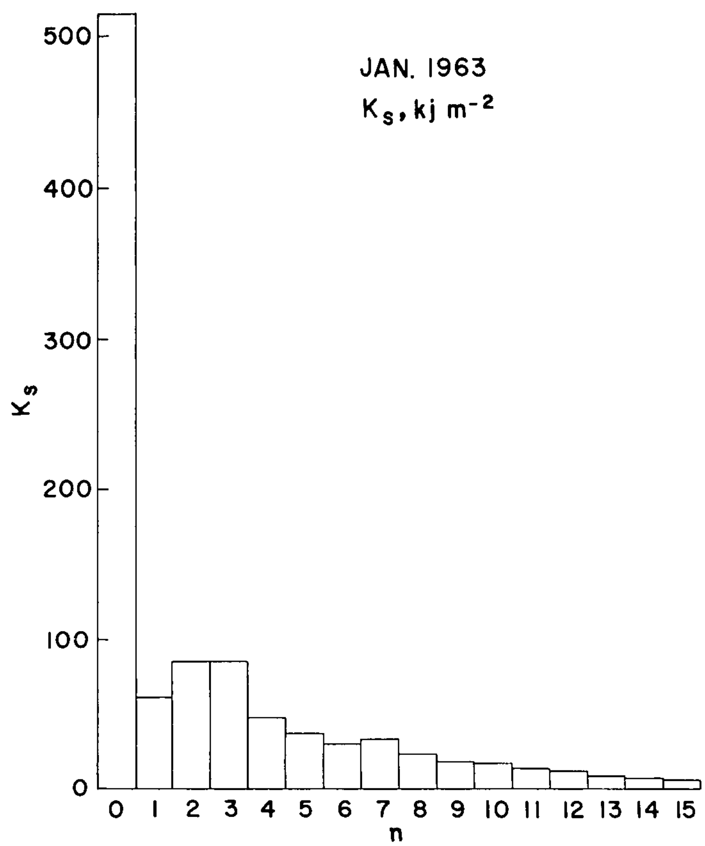

FIGURE 17.—As figure 13. January 1963.

6. DISCUSSION

The relatively large values which have been computed for the energy conversion $C_{ND}(K_S,K_M)$ indicate that the earlier results obtained in [10] were too small, partly because only two levels were used, and partly because of the computational procedures. The new results based on a greater vertical resolution and on a larger sample of atmospheric data show a better agreement with the numerical experiment performed by Smagorinsky [8]. Although our new results are larger than his, we have computed only the conversion due to the quasi-non-divergent motion. Adopting the results from the pilot calculations in [10], we would expect that $C(K_S,K_M)$ should be reduced when we add the contribution from $C_D(K_S,K_M)$. The new spectra show reasonable agreement with those obtained in [10] with a strong tendency for a maximum at intermediate wave numbers except in January 1963 which is known to be a most unusual month (Wiin-Nielsen, Brown, and Drake [12]).

Although the calculation of $C_{ND}(K_S, K_M)$ only is incomplete as far as the energetics of the atmosphere are concerned, it is nevertheless believed to be a major fraction of $C(K_S, K_M)$. Furthermore, $C_{ND}(K_S, K_M)$ is the energy conversion which would be present in a quasi-non-divergent atmospheric prediction model. According to our present results, we would therefore expect a comparatively large amount of energy to be converted into energy of the vertical mean flow which in turn seems to be closely approximated by the 500-mb. flow. The fact that the quasi-non-divergent models, frequently used in the past for short-range numerical predictions, over-predict cyclogenesis might be explained by the large values of $C_{ND}(K_S, K_M)$.

The amounts of the vertical shear flow and vertical mean flow kinetic energies have been computed in the wave number regime. The energies of both the zonal and the meridional components have been included in the calculations. The total amounts of energy found in the two components of the atmospheric flow agree reasonably well with the results of Smagorinsky's [8] numerical study, although his energy levels are slightly larger. The partitioning of the energy between the zonal flow and the eddies in the observational study is in sharp contrast to the numerical experiment. Our study shows a larger amount of energy in the eddies than in the zonal flow for both the vertical mean flow and the vertical shear flow while the opposite is the case for the numerical experiment performed by Smagorinsky [8].

ACKNOWLEDGMENTS

The authors would like to thank Mr. John A. Brown, Jr. for numerous discussions of the subjects covered in this paper. We are furthermore grateful to Mr. Placido Jordan who has processed a large amount of the results obtained in the calculations and prepared a large number of diagrams.

APPENDIX A

It is the purpose of this appendix to derive the general expression for the integral

$$I=\int_0^{2\pi} f(\lambda)g(\lambda)h(\lambda)d\lambda \tag{1}$$

when each of the three functions of f, g, and h are expressed as Fourier series in the following forms

$$f(\lambda)=a_0+\sum_{n=1}^{\infty}[a_n\cos(n\lambda)+b_n\sin(n\lambda)]$$

$$g(\lambda)=c_0+\sum_{n=1}^{\infty}[c_n\cos(n\lambda)+d_n\sin(n\lambda)]$$

$$h(\lambda)=r_0+\sum_{n=1}^{\infty}[r_n\cos(n\lambda)+s_n\sin(n\lambda)] \tag{2}$$

The following derivations are carried out in a more convenient way if we express the Fourier series in a complex form as follows:

$$f(\lambda)=a_0+\sum_{\substack{n\neq 0\\ n=-\infty}}^{n=+\infty} A_n e^{in\lambda};\ A_n=\tfrac{1}{2}(a_n-ib_n);\ A_{-n}=\tfrac{1}{2}(a_n+ib_n)$$

$$g(\lambda)=c_0+\sum_{\substack{n\neq 0\\ n=-\infty}}^{n=+\infty} C_n e^{in\lambda};\ C_n=\tfrac{1}{2}(c_n-id_n);\ C_{-n}=\tfrac{1}{2}(c_n+id_n)$$

$$h(\lambda)=r_0+\sum_{\substack{n\neq 0\\ n=-\infty}}^{n=+\infty} R_n e^{in\lambda};\ R_n=\tfrac{1}{2}(r_n-is_n);\ R_{-n}=\tfrac{1}{2}(r_n+is_n) \tag{3}$$

The integral (1) can be evaluated by introducing the series (3) in (1), multiplying term by term, and integrating. Collecting the terms which contain at least one zero-component, we get:

$$\begin{aligned}I=2\pi a_0c_0r_0&+a_0\int_0^{2\pi}\{\sum C_{n'}e^{in'\lambda}\}\{\sum R_{n''}e^{in''\lambda}\}d\lambda\\&+C_0\int_0^{2\pi}\{\sum A_{n'}e^{in'\lambda}\}\cdot\{\sum R_{n''}e^{in''\lambda}\}d\lambda\\&+r_0\int_0^{2\pi}\{\sum A_{n'}e^{in'\lambda}\}\cdot\{\sum C_{n''}e^{in''\lambda}\}d\lambda\\&+\int_0^{2\pi}\{\sum A_{n'}e^{in'\lambda}\}\cdot\{\sum C_{n''}e^{in''\lambda}\}\cdot\{R_{n'''}e^{in'''\lambda}\}d\lambda\end{aligned} \tag{4}$$

in which n', n'', and n''' are dummy indices. The three first integrals, containing a product of two series in the integrand, are straightforward to evaluate. In general we get:

$$\int_0^{2\pi}\left\{\sum_p P_pe^{ip\lambda}\right\}\cdot\left\{\sum_q Q_qe^{iq\lambda}\right\}d\lambda=2\pi\sum_p\{P_p\cdot Q_{-p}\} \tag{5}$$

The product $P_p\cdot Q_{-p}$ can easily be expressed in the real

Fourier components. When this is done, we can write (4) in the form:

$$I=2\pi a_0c_0r_0+\pi a_0\sum_{n=1}^{\infty}(c_nr_n+d_ns_n)$$
$$+\pi c_0\sum_{n=1}^{\infty}(a_nr_n+b_ns_n)+\pi r_0\sum_{n=1}^{\infty}(a_nc_n+b_nd_n)+I' \quad (6)$$

where I' is a notation for the last (triple) integral in (4). The general term in I' is evaluated to be:

$$\int_0^{2\pi}A_nC_mR_pe^{i(n+m+p)\lambda}d\lambda=\begin{cases}2\pi A_nC_mR_p,\ n+m+p=0\\ 0,\ n+m+p\neq0\end{cases} \quad (7)$$

By using the integral (7) of the general term, it is possible to express I'. In so doing, it is mathematically necessary to select one of the indices n, m, and p as the primary index, and the next as the secondary index, while the third index is determined by the relation $n+m+p=0$. We shall select n as the primary index, m as the secondary index, giving $p=-m-n$.

The physical quantity which we try to evaluate is the energy conversion as expressed in (2.11) of the present paper. Two of the three factors in the integrand are related to the vertical shear flow, while the third factor is determined by the vertical mean flow. We shall identify the primary index with the vertical mean flow and the other two with the shear flow. This means that we evaluate the amount of energy which appears in the component of wave number n in the vertical mean flow due to nonlinear interactions between different components in the shear flow.

With this convention we may write I in the form

$$I'=2\pi\sum_{\substack{n=-\infty\\ n\neq0}}^{n=+\infty}\left[A_n\sum_{\substack{m=-\infty\\ m\neq0}}^{m=+\infty}C_mR_{-m-n}\right] \quad (8)$$

In the terms appearing in the first sum in (8), we shall consider the contribution from the two specific terms with indices $n>0$ and $-n$. We may write this contribution as follows:

$$2\pi A_n\sum_{\substack{m=-\infty\\ m\neq0}}^{m=+\infty}C_mR_{-m-n}+2\pi A_{-n}\sum_{\substack{m=-\infty\\ m\neq0}}^{m=+\infty}C_mR_{n-m} \quad (9)$$

In each of the two sums in (9), we consider the contribution from the specific term with indices $m>0$ and $-m$ and obtain

$$T=2\pi A_n[c_mR_{-m-n}+C_{-m}R_{m-n}]$$
$$+2\pi A_{-n}[C_mR_{n-m}+C_{-m}R_{n+m}] \quad (10)$$

The expression (10) may now be expressed in terms of the real Fourier coefficients. The evaluation depends on the relative magnitude of m and n. If we first assume that $m<n$, we obtain the following expression for (10):

$$T=\frac{\pi}{2}[a_n(c_mr_{n+m}+d_ms_{n+m})+a_n(c_mr_{n-m}-d_ms_{n-m})$$
$$+b_n(c_ms_{n+m}-d_mr_{n+m})+b_n(c_ms_{n-m}+d_mr_{n-m})],\ m<n \quad (11)$$

In the other case, where $m>n$, we obtain:

$$T=\frac{\pi}{2}[a_n(C_mr_{m+n}+d_ms_{m+n})+a_m(c_mr_{m-n}+d_ms_{m-n})$$
$$+b_n(c_ms_{m+n}-d_mr_{m+n})-b_n(c_ms_{m-n}-d_mr_{m-n})],\ m>n \quad (12)$$

It is easy to show that the case $m=n$ is included in (11) if it is understood that $r_{n-m}=s_{n-m}=0$ for $n=m$.

Making use of the expressions (9) to (12), we may now rewrite the expression (8) for I' in the following form:

$$I'=\frac{\pi}{2}\sum_{n=1}^{\infty}\Bigg\{\sum_{m=1}^{n}[a_n\{c_m(r_{n+m}+r_{n-m})+d_m(s_{n+m}-s_{n-m})\}$$
$$+b_n\{c_m(s_{n+m}+s_{n-m})-d_m(r_{n+m}-r_{n-m})\}]$$
$$+\sum_{m=n+1}^{\infty}[a_n\{c_m(r_{m+n}+r_{m-n})+d_m(s_{m+n}+s_{m-n})\}$$
$$+b_n\{c_m(s_{m+n}-s_{m-n})-d_m(r_{m+n}-r_{m-n})\}]\Bigg\} \quad (13)$$

The only step left in the derivation is now to write the original integral (1) in the form

$$I=\sum_{n=0}^{\infty}I_n \quad (14)$$

where we have:

$$I_0=2\pi a_0c_0r_0+\pi a_0\sum_{m=1}^{\infty}(c_mr_m+d_ms_m) \quad (15)$$

and

$$I_n=\pi c_0(a_nr_n+b_ns_n)+\pi r_0(a_nc_n+b_nd_n)$$
$$+\frac{\pi}{2}\sum_{m=1}^{n}\{a_n[c_m(r_{n+m}+r_{n-m})+d_m(s_{n+m}-s_{n-m})$$
$$+b_n[c_m(s_{n+m}-s_{n-m})-d_m(r_{n+m}-r_{n-m})]\}$$
$$+\frac{\pi}{2}\sum_{m=n+1}^{\infty}\{a_n[c_m(r_{m+n}+r_{m-n})+d_m(s_{m+n}+s_{m-n})]$$
$$+b_n[c_m(s_{m+n}-s_{m-n})-d_m(r_{m+n}-r_{m-n})]\} \quad (16)$$

When the formulas (15) and (16) are used in the calculations, we have truncated Fourier series at $n=N$. We take care of this fact by setting the coefficients equal to zero if $(m+n)>N$, and also if $n=m$ in (16).

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CORRECTION

Vol. 91, No. 6, June 1963, p. 299: In table 1, in column headed "Actual day, Total", and row "Autumn, Dry", change "518" to "647".

Vol. 93, No. 1, January 1965, p. 49, col. 2, 16 lines from bottom: In the wind shear term V should be in boldface type to indicate a vector.