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An Objective Analysis Study

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Introduction

As the size of the areas included in numerical prediction increases, the successful accomplishment of automatic objective analysis becomes of greater importance. The objective analysis scheme used by the Joint Numerical Weather Prediction (JNWP) Unit for a 551 point grid centered on North America was described in {7}. This small grid was completely superseded by a 1020 point grid, the edges of which are shown in figure 1. The sucessful completion of an automatic data processing system {1} was followed by the preparation of an objective analysis program for the large grid area.

Special Problems

The problem of erroneous data was effectively handled by the checks in the automatic data processing program. These included a hydrostatic check of all soundings, a comparison of all duplicate reports with the elimination of non-comparing data, and the gross error removal procedure. These are described in {1}.

Analysis over sparse data areas such as the Pacific Area presented the most difficult problem. The chief tools of the subjective analyst for attack ~~of~~ on this problem are (a) "continuity," i.e., use of a short range forecast from the previous map, and (b) the enforcement of vertical consistency between analyses at different levels. The successful scheme of Bergthorsson and Doos {2} was used to some extent in this program. The JNWP upper air analysis program started with a preliminary map which can be obtained in either of two ways. The preliminary map may consist of either (a) a twelve-hour forecast or (b) a map obtained by adding a forecast of thickness from 500-1000 mb to the latest 1000 mb analysis. In practice, the first choice appeared to give satisfactory results, and is used in daily operations. For later purposes, one must distinguish between the 12-hour height forecast and the 12-hour wind forecast.

Another problem was introduced by the appearance of errors of large lateral scale in the 12-hour forecast. These did not appear so much in the wind forecast as in the height forecast. To complicate matters further, these errors exhibited a distinct preference for the Pacific, a sparse data area. This difficulty was overcome by use of the preliminary analysis procedure described in a later section.

Types of Data Used

The data used in the analysis are all obtained from the magnetic tape generated by the automatic data processing program. These data include reports from rawinsonde and pilot balloon stations, reports (including dropsonde data) from the USAF Air Weather Service weather reconnaissance, and selected reports from commercial and military transient aircraft, these latter reports having been edited by the monitoring analyst. Vertical extrapolations from sea level to 500 mb are occasionally inserted by the analyst.

A certain amount of difficulty was experienced in obtaining a satisfactory analysis near the boundaries where strong winds existed but from which no data were available. Two possibilities were open. The first was to use climatological data to some extent, as described by Bergthorsson and Doos. The second was to use the 12-hour old information taken from a previous subjective analysis from the National Weather Analysis Center (NAWAC). The second of these two possibilities was used, since it was believed that information 12 hours old would be more useful then climatological data. This was then accomplished by reading heights and winds from imaginary stations for these areas. Usually only the west Pacific boundary (plus the other three corners) required such data.

The total number of stations reporting after duplicate reports were removed including weather reconnaissance, other aircraft, and the imaginary statinns averages about 275 for this grid area. Most of these report both heights and winds.

The Preliminary Analysis

The 12-hour forecast would be completely suitable as a preliminary chart if its errors were small and random. Unfortunately, this is not the case. Although the 12-hour forecast winds are usually very accurate, errors of large lateral scale having magnitudes sometimes greater than 400 ft. develop in the height field. The preliminary analysis is used in order to correct these large-scale errors before the final analysis starts. This is done very quickly by the following steps.

1. A report is read in from the data tape. It is then compared with the 12-hour forecast. The height error is obtained by interpolation. The height errors at the adjacent grid points are obtained from the equation,

$$E_{i,j} = \frac{1}{D^2} (D^2 - d^2) E_o, \qquad (1)$$

where $E_{i,j}$ is the error to be applied at the point i, j in the grid, E_o is the error at the location of the actual report, d is the distance from the reporting station to the grid point at i, j, and D is the maximum distance for which it is desired to make a correction. The errors are stored in an error field which is initially set to zero.

2. Some grid points will be subject to corrections from more than one report. If an error has already been stored at a point, i, j, and another observation is received from a point closer than the distance D, the new correction at i, j is

$$E'_{i,j} = \frac{1}{D^2} \{d^2 E_{i,j} + (D^2 - d^2) E_o\}, \qquad (2)$$

where $E'_{i,j}$ is the new error to be applied at i,j. The distance D is set at a value of 4 mesh lengths.

3. When all the reports have been considered in succession, the error field is subtracted from the forecast heights. The field thus obtained is used as the preliminary height field for the analysis. The preliminary analysis is then printed for inspection.

The time required for completion of the preliminary analysis, including printing the map, is four minutes. All storage is checked. It should be noted that only the heights and not the winds were used. The preliminary fields then consist of a preliminary height field, obtained as described above, and a preliminary wind field which is the 12-hour forecast wind field.

The Final Analysis

The final analysis is accomplished by fitting the polynomial

$$D = a_o + a_1 x + a_2 y + a_3 xy + a_4 x^2 + a_5 y^2 \qquad (3)$$

by the method of least squares to the data around each grid point, as described by Gilchrist and Cressman (4), where D is the height of the pressure surface and a_o to a_5 are the coefficients to be determined for each grid point from the data at various values of x and y. Actually, it is sufficient to determine the coefficient a_o, the value of D at the grid point, where x = o, and y = o. The polynomial is fit to all the data found within an octogon having a "radius" (distance from the center to a side along a perpendicular to the side) of length r.

The points of the mesh are considered successively in what can be called the scanning order. The first point to be considered is in an area of dense data. The scanning order proceeds outward from this point in a type of spiral, as illustrated in fig. 2. By scanning in this order, the octagonal area will usually include the maximum number of previously computed points. Three scans are made through the grid. On the first scan only actual data are used to fit the polynomial. Six pieces of information (counting one for each height and two for each wind) are necessary for a fit. In order to introduce some smoothing, since the data are not perfectly accurate, ten pieces of information including at least one height are actually required by the program. If insufficient data exist, no analysis is made and the program proceeds to the next point. At the completion of the first scan a second scan is begun. This time the preliminary heights and the previously computed points are counted as data, but with a smaller weight. A certain minimum of reported data is required for the analysis at a point on the second scan. The analysis is completed on the third scan. This time, the forecast winds are also included as data. Grid points farther than 4 mesh lengths from any data are not analyzed. The preliminary heights are counted as final for these points. The relevant parameters are summarized in tables 1 and 2.

Table 1. Size of Octagon

Scan	r of Octagon (in units of mesh lengths)
1	1.49
2	2.01
3	2.51

Table 2. Relative Weighting Factors

Heights		Winds	
Type	Weight	Type	Weight
Observed	1.00	Observed	1.0W
Previously computed	0.12	12-hr forecast	0.5W
Preliminary	0.50		

The weighting factor W in Table 2, which is the relative weight of winds to heights, is determined from the consideration that 35 feet height error are counted equivalent to 10 knots vector wind error. This value is designed for analyses which are to be used in the balance equation (See Charney (3) and Shuman (6)), as giving the best representation of the height field (Gilchrist and Cressman (4)). If the analyses are to be used in quasi-geostrophic forecasting models a height of 60 ft error for 10 knots wind error is more appropriate. The geostrophic equation is used in relating winds and heights. For this reason the analysis scheme described this far can be called a "geostrophic analysis" scheme.

The amount of observed data required on each scan before a value of E is computed at a point is specified by the relation that

$$\Sigma I + 5s - 15 \geqq 0, \qquad (3)$$

where I is the total amount of observed information found within the octagon (1 for each height, 2 for each wind), and s is the number of the scan (1,2, or 3).

The analysis, when completed is smoothed by an operator developed by Shuman {5} which completely eliminates perturbations having a wave length of two grid intervals and leaves those having a wave length of five or more grid intervals modified by less than one percent of their amplitude. Due to the tendency of the analysis to develop some roughness on the sparse-data boundaries there is an additional slightly heavier smoothing done on the boundary row and adjacent row on both the Atlantic and Pacific sides of the grid.

In solving the balance equation, it has been necessary to apply an ellipticity criterion to the initial data, given by

$$\zeta_g > -f/2,$$

where ζ_g is the relative vorticity of the geostrophic wind and f is the Coriolis parameter. In a study of the reality of this criterion, Mr. L. Carstensen of the JNWP Unit made a series of experiments in which the height fields of subjective analyses were altered to fit the criterion. He discovered for maps at 400 mb and lower that the only significant changes introduced by this process were in areas of little or no data. As a result of his tests the final analysis was required to satisfy the above criterion. Although relatively expensive in computing time, this tends to reduce the tendency of forecasts to develop erroneously large anticyclones. The program for this section was written by Mr. O. Fuller.

The time required for the final analysis is as follows:

Final Analysis	- 34 minutes
Smoothing	- 1
Ellipticity Criterion	- 3
Printing and Punching	- 2
Total	- 40 minutes

The 40 minutes includes the time for a complete check of all computations by duplication. Suitable recovery routines are provided in case of computer error.

An example of objective analysis is shown in fig. 3. This example was chosen partly because the 12-hour forecast was unusually poor in the east Pacific, the error amounting to almost 700 ft. at the position of the weather ship at 50° N, 145° W. The forecast winds in this vicinity were also very inaccurate. The analysis is superimposed on the plotted data which were available when the analysis began. Not all the data from Europe, the USSR, Japan and the U.S., and Alaska are plotted because the high data density in these areas would destroy the legibility of the illustration. All the data used in the analysis from Canada, the Arctic and the oceans are plotted.

The satisfactory fit of the analysis to the data is evident in nearly all areas. The wind at the gulf of Alaska ship was not quite fit due to the use of the forecast winds which were unusually bad in this case. A slight tendency for smoothing in the cyclonic center over northwest California is typical of the analysis system. This can be attributed mostly to the smoothing done when fitting the quadratic surface. If a higher weight were given to winds, the presence of subgeostrophic winds in cyclonic areas would further tend to fill up cyclones in the analysis.

A map of the differences between the geostrophic winds obtained by interpolation in the grid point values of fig 3 for the positions of 54 rawinsonde stations and the observed winds is shown in figure 4. This chart was computed by Major H.A. Bedient. The mean vector difference between interpolated geostrophic winds and observed winds for this map was 13 knots.

Nongeostrophic Analysis

Several schemes have been considered for use in obtaining a nongeostrophic objective analysis, using the balance equation to represent the relation between wind and height field. The general scheme of the analysis described below was planned and tested in collaboration with Major H.A. Bedient, USAF, of the JNWP Unit. The analysis consists of the following steps:

a) Perform a preliminary height analysis using observed heights to correct a 12-hour forecast map.

b) Solve the balance equation to obtain a stream function from the preliminary height analysis. The form of the balance equation used is

$$\nabla^2\psi = \frac{1}{f}\left[\nabla^2\phi + 2\left(\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial^2\psi}{\partial x^2}\,\frac{\partial^2\psi}{\partial y^2}\right) - \nabla\psi.\nabla f\right]$$

where ψ is the stream function, ϕ is the geopotential, x and y are directions in the grid and f is the Coriolis parameter.

c) Using the stream function field as a preliminary analysis, use the final analysis scheme described above to modify the field to fit the observed winds. This is done by ignoring the heights from all observations and using only the observed winds in the analysis. In order to permit solution of the linear equations to determine a. in eq. (2), values of the stream functions from the preliminary field are used as observed heights, but with a very small weight compared to the observed winds.

Nongeostrophic analyses have been made for four different maps in May, 1957. There seems to be little difference in the goodness of the fit of observed winds by either the geostrophic or nongeostrophic analysis except in areas characterized by strong winds in curved flow, where the nongeostrophic analysis obtains a better fit. The fit of observed 500 mb winds from 50 United States stations averaged for the four maps is given in table 3.

Table 3. Vector differences between observed winds and winds interpolated from different analyses.

Type of Analysis	Mean Difference
1. Geostrophic	12.1 knots
2. Stream function obtained by solving eq.3 from final geostrophic analysis	12.0 knots
3. Nongeostrophic	10.2 knots

Factors contributing to the differences between observed and analysis winds are inaccuracies in the wind measurements and actual divergence in the wind field.

The nongeostrophic analysis of stream function can be converted to a height chart by solving the balance equation for Φ, given the field of ψ. In practice, this is not especially helpful in getting an accurate height map since currently used methods of solution of this equation do not permit recovery of the height field with an error of less than several decafeet, even though the lateral gradient of this error is very small.

An example of the success of the various analyses in fitting winds in cyclonic flow is given in figures 5 and 6. The geostrophic analysis (6a) fit the winds well with the exception of the Caribou, Me. wind, which was much slower than the geostrophic wind. The geostrophic analysis was used to supply values of geopotential to the balance equation, which was then solved for winds interpolated from this stream function map. The results are given in fig. 6b. The interpolated wind from the Caribou position was still too strong, but much better than before. However, most of the other interpolated winds, especially from Albany, N.Y. and Portland, Maine are too weak. The best over-all fit was obtained by the nongeostrophic analysis (6c).

The differences in wind verifications of the several analysis schemes are not very large. Further tests of nongeostrophic analyses at a different time of the year must be made if conclusive results are to be obtained.

Remarks on the General Scheme of Analysis

The scheme of analysis used to date consists essentially of fitting the polynomial (3) by the method of least squares to the data. This method was originally designed for an area of relatively dense data, having a certain amount of redundancy. Areas such as this, however, occupy a relatively small fraction of the globe. Most of an area such as shown in fig. 1 is characterized by a few widely separated observations, which one desires to fit as exactly as possible. In analyzing the areas around such observations, lateral extrapolation is involved. A least squares polynomial is ideally suited for interpolation, but not for extrapolation. The analysis scheme above made an interpolation problem out of an extrapolation problem by using preliminary and forecast data, with a suitable choice of weights. It is felt, however, that such a scheme involves more work than should be necessary, both in the data preparation and in the computation. The next analysis experiment will therefore use a quite different scheme, not very similar to either the least squares method or to the method of Bergthorsson and Doos, but resembling the latter more than the former.

Acknowledgements:

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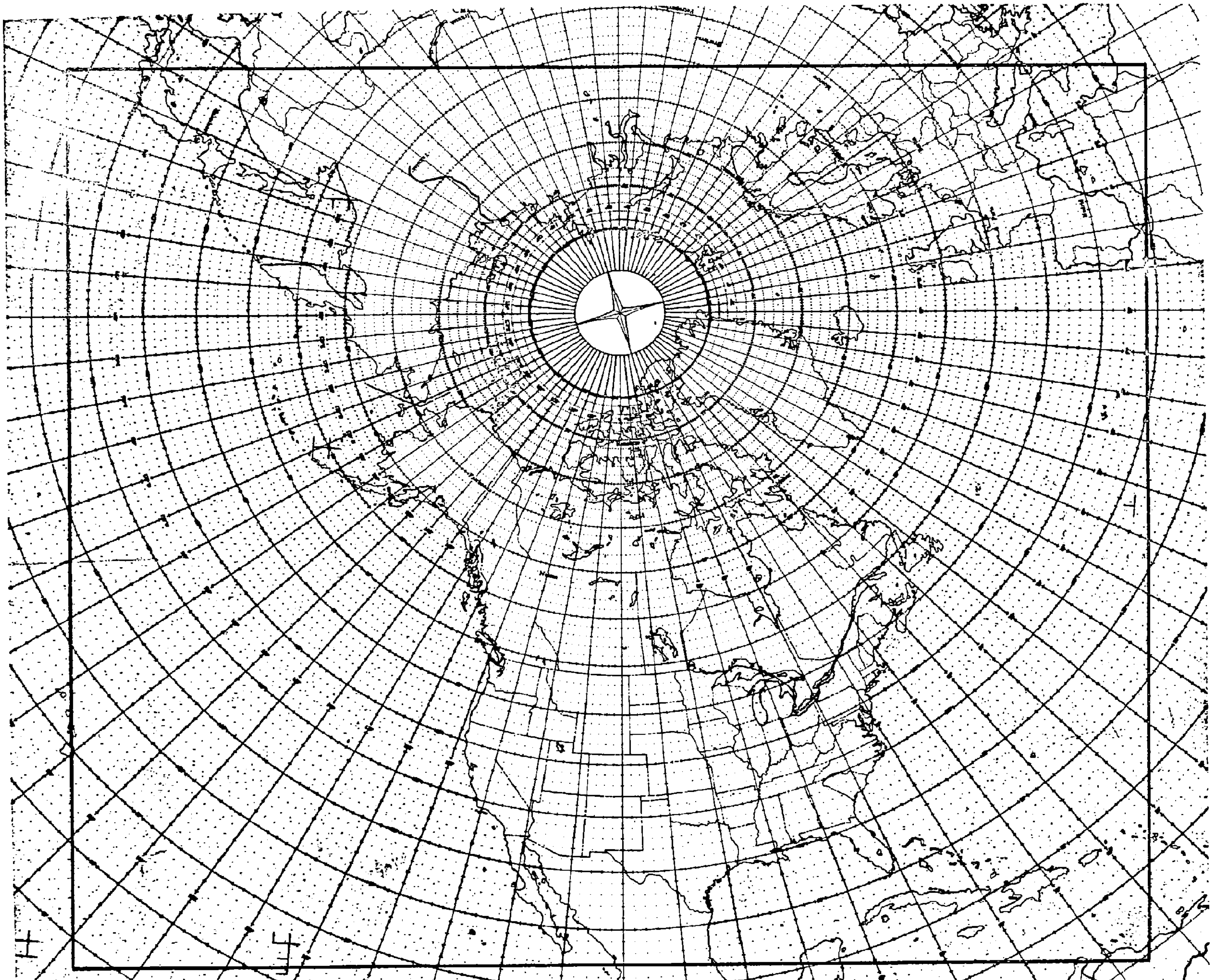


BOUNDARIES OF 1020 POINT GRID

FIG. 1

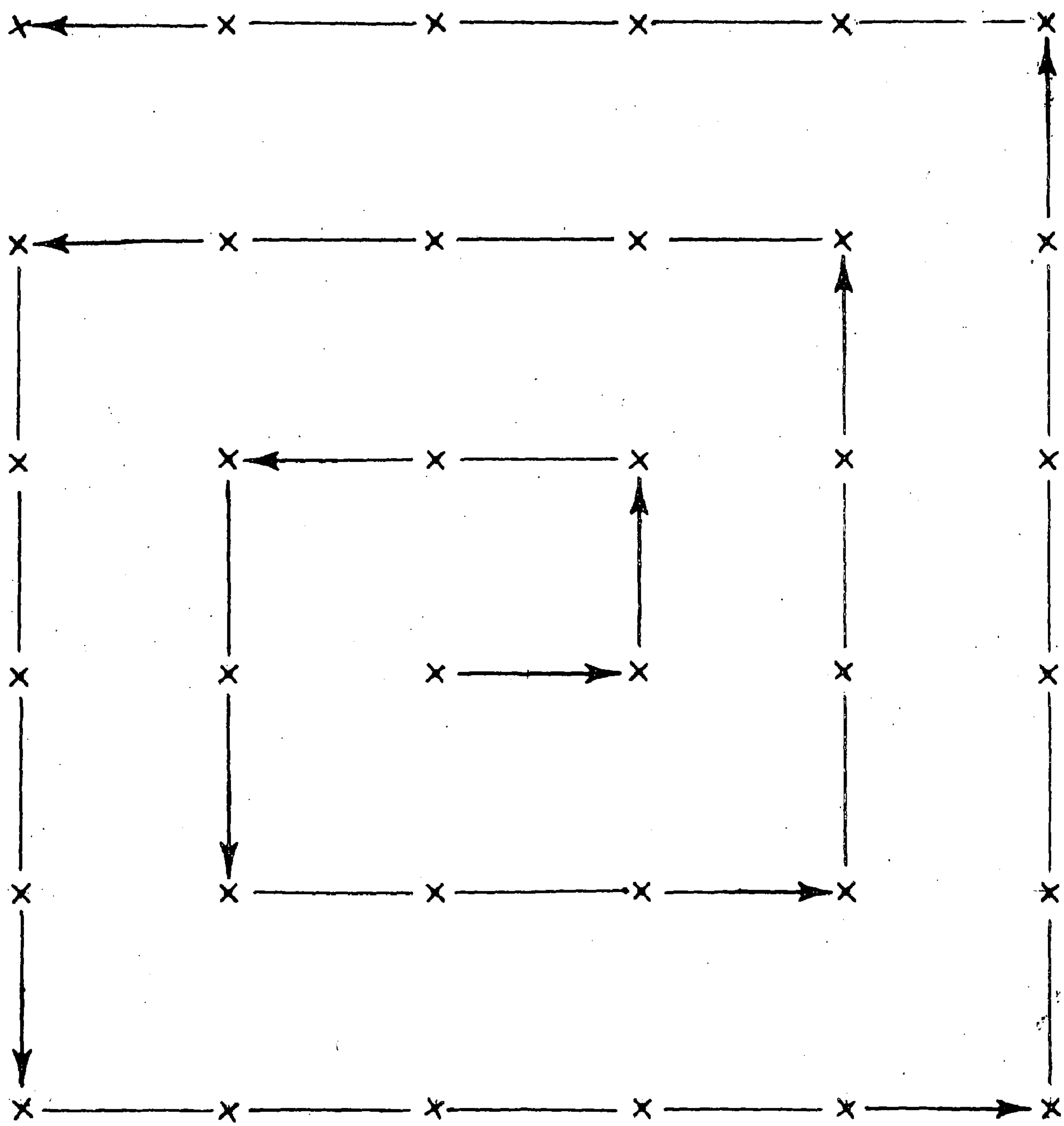
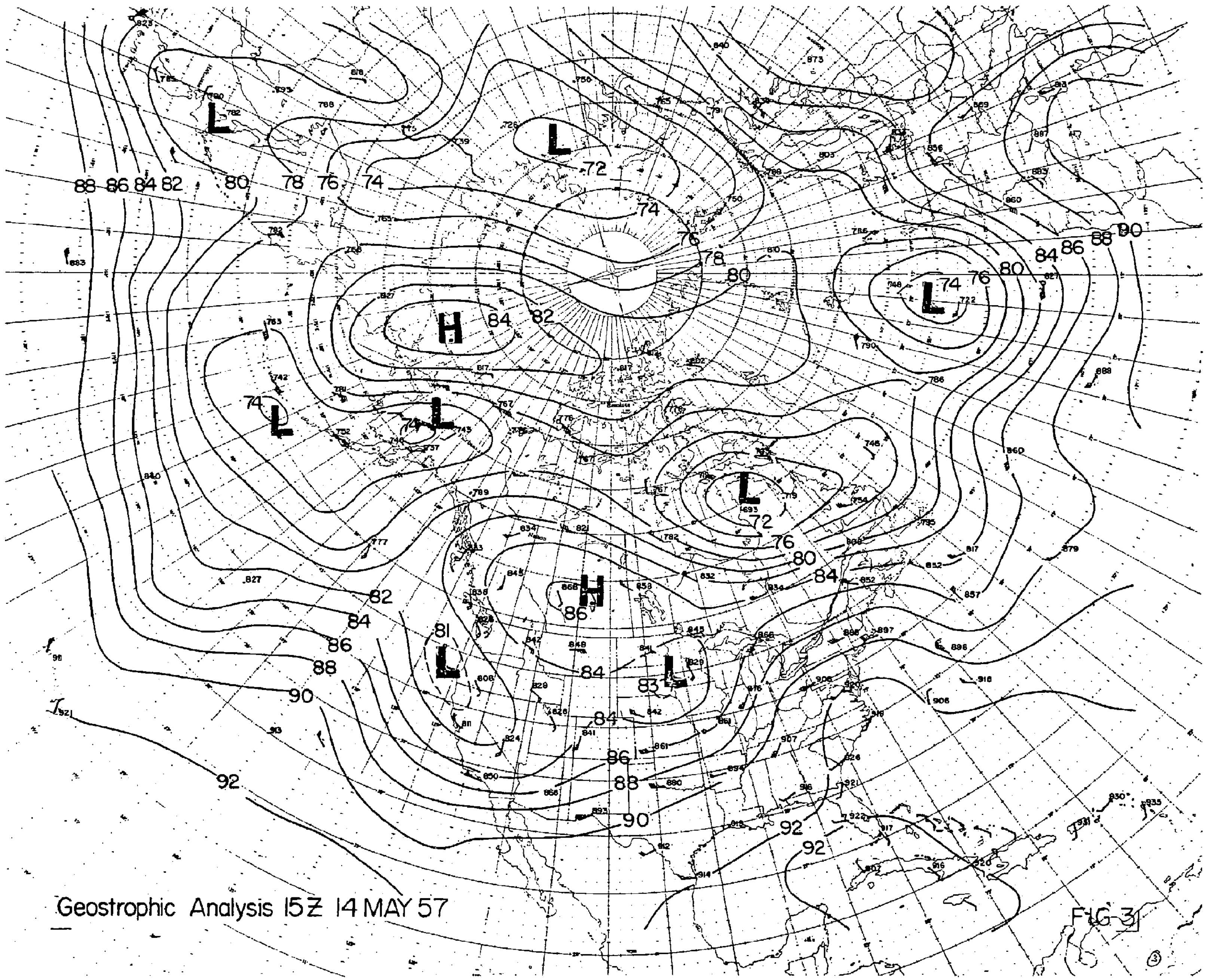


Fig. 2. Spiral Scan

FIG. 2



Geostrophic Analysis 15Z 14 MAY 57

FIG 3

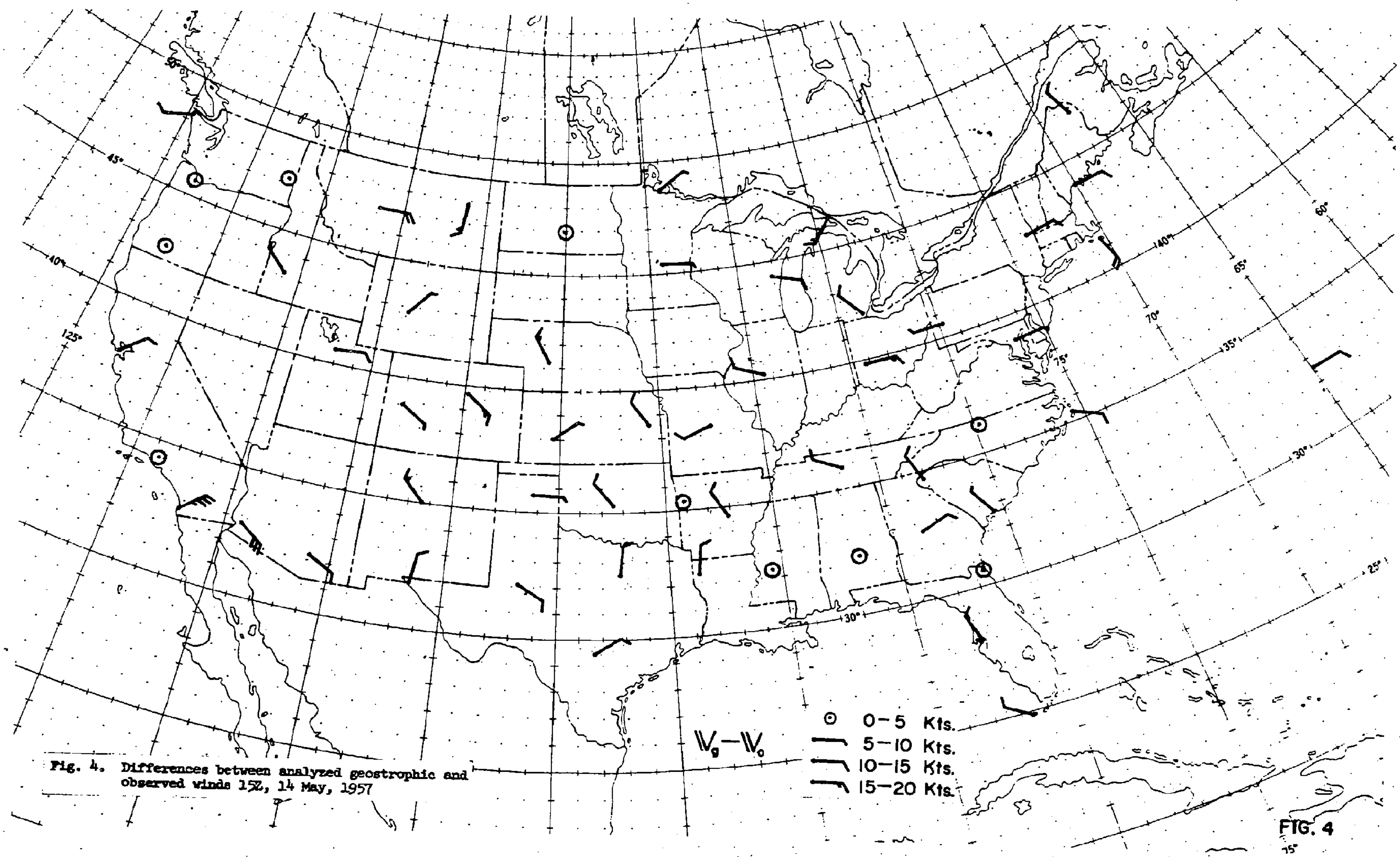
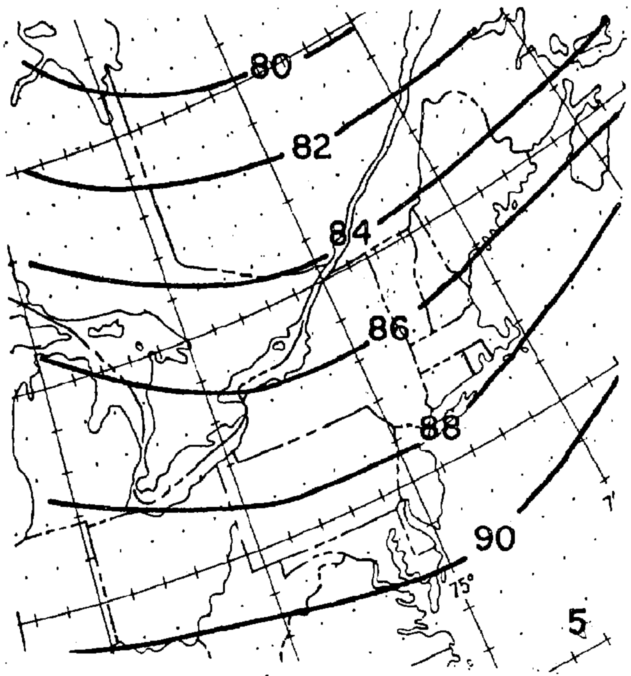
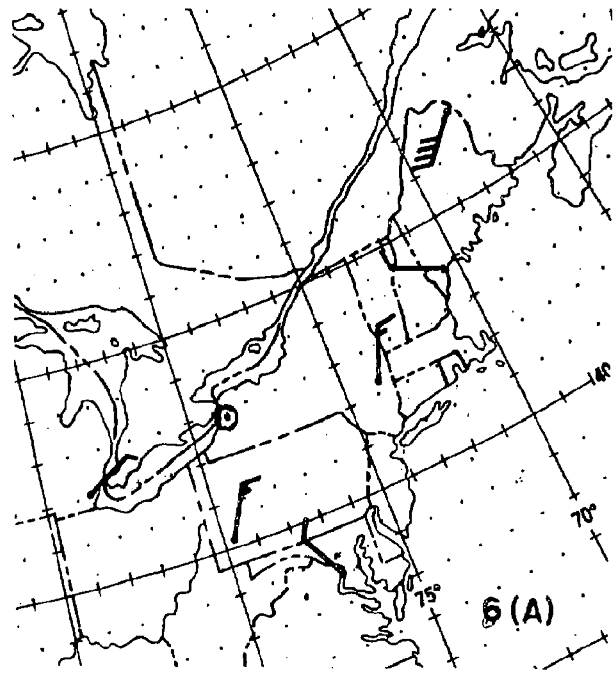
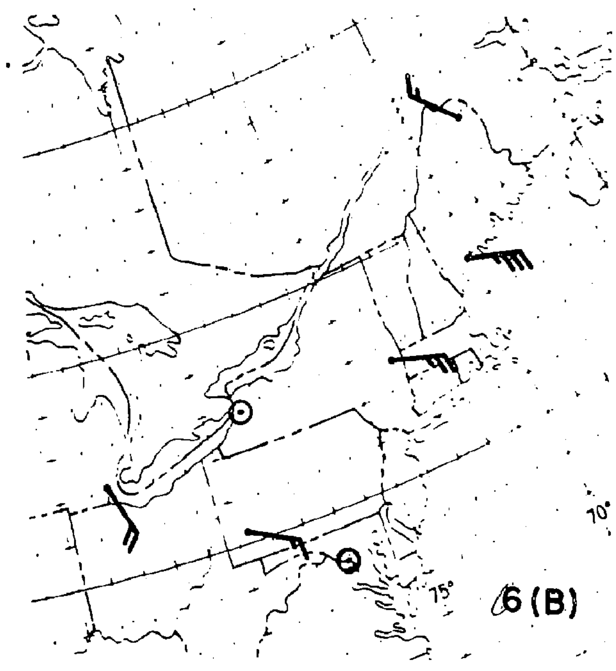


Fig. 4. Differences between analyzed geostrophic and observed winds 15Z, 14 May, 1957

FIG. 4







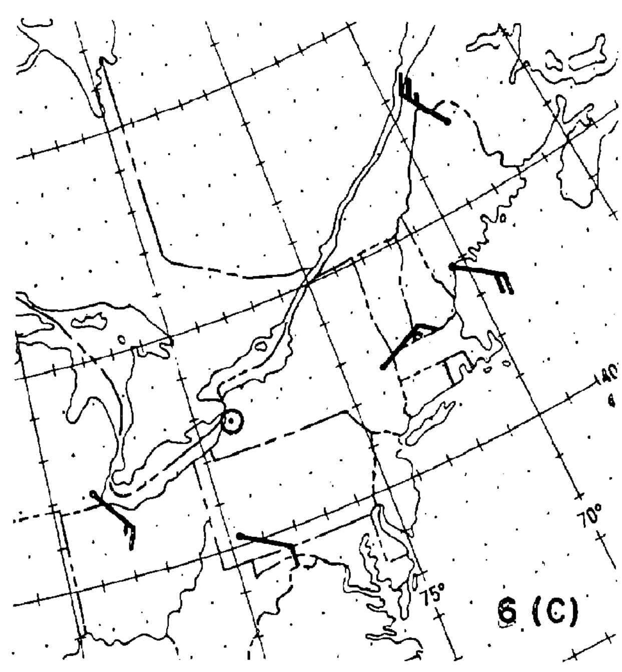


Fig. 5. Stream function from nongeostrophic analysis for 500 mb, 15Z, 16 May 1957

Fig. 6. Maps of $V_i - V_o$, where V_i is the interpolated wind from the analysis and V_o is the observed wind. Same plotting model as in fig. 5..

(a) V_i from geostrophic analysis

(b) V_i from balanced geostrophic analysis

(c) V_i from nongeostrophic analysis