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OBJECTIVE ANALYSIS OF THE TROPOPAUSE

Prepared by

Arthur F. Gustafson
Statistical Techniques and Analysis Branch
Development Division

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This memorandum describes the method and the theoretical basis for an objective tropopause analysis program. This IBM 7094 program is used operationally at NMC to provide input to Dr. Frederick G. Shuman's primitive equations, "PE," forecast model. Two fields for the 1977-point hemispheric grid are produced: one for the tropopause pressure; the other for the tropopause temperature.

Neither of these fields is an "analysis" in the sense that the grid values represent a weighted average of reported tropopause data from surrounding stations. Instead, they are derived from modeling assumptions using a selected set of objective isentropic temperature analyses. The basic model used conforms to the ideas of Newton and Persson [1], in that it subdivides the tropopause into three major types or "leafs"; i.e.:

1. A "polar" tropopause north of the polar jet stream.

2. A "middle" tropopause between the polar and subtropical jet streams.

3. A "subtropical" tropopause south of the subtropical jet stream.

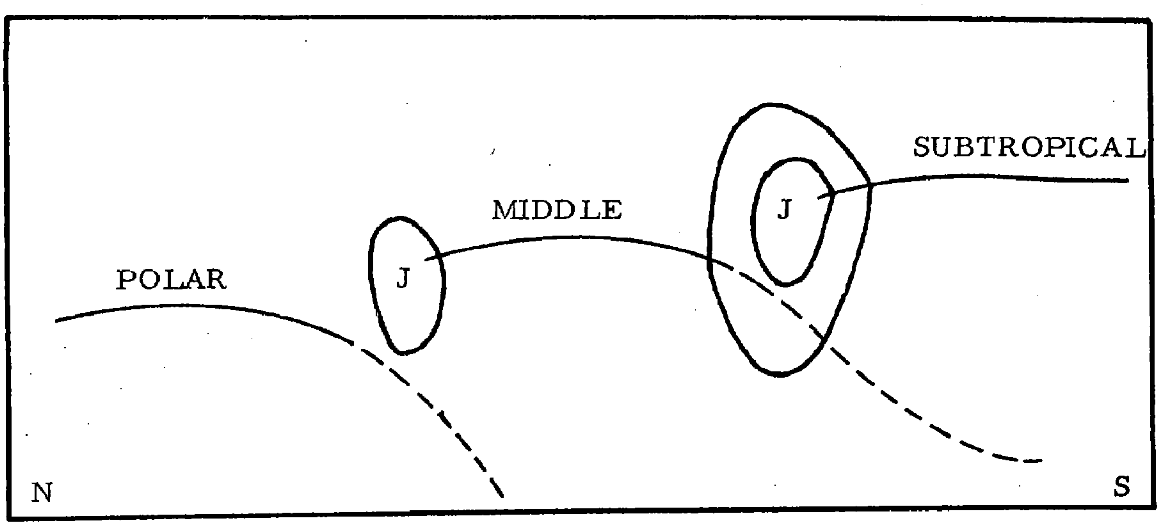

FIGURE 1: Schematic Illustrating Polar, Middle and Subtropical Jet Streams (Medium) and Associated Fronts (Dashed)

In addition to the above classification scheme, some further modeling assumptions are made, i.e.:

1. The subtropical tropopause coincides with a selected isentropic surface.

2. The lapse rate in the lower stratosphere above the polar and middle tropopauses is zero (isothermal).

3. The lapse rate for 100 mbs or less below the polar tropopause is approximately standard ($.7\gamma_d$) except under arctic conditions when it is half this rate.

4. The lapse rate below the middle tropopause is approximately standard, i.e., $\gamma = .7\gamma_d = 6.83$ °C/km.

The method is based on the observed fact that a tropopause leaf tends either to be associated with a certain isentropic surface or to be in-phase with the pressure variations of an isentropic surface above it. The latter case is typical of the polar tropopause whose pressure variation generally has a larger amplitude than that of the in-phase isentropes above it. The former is more often the case for middle and subtropical tropopauses.

Examples of this tendency can be found in Newton and Persson's paper or in the series of "Daily Aerological Cross-Sections" made for the IGY period, July 1957 thru December 1958. The cross-section for 29 January 1958, from this series has been reproduced in Figure 2. The heavy lines are the tropopauses as analyzed by the authors from the reported soundings. A few isotachs (meters/sec) have been copied to indicate the polar and subtropical jet stream maxima. Portions of the 300° and 320° isentropes have been dashed and the 345° isentrope added for illustrative purposes.

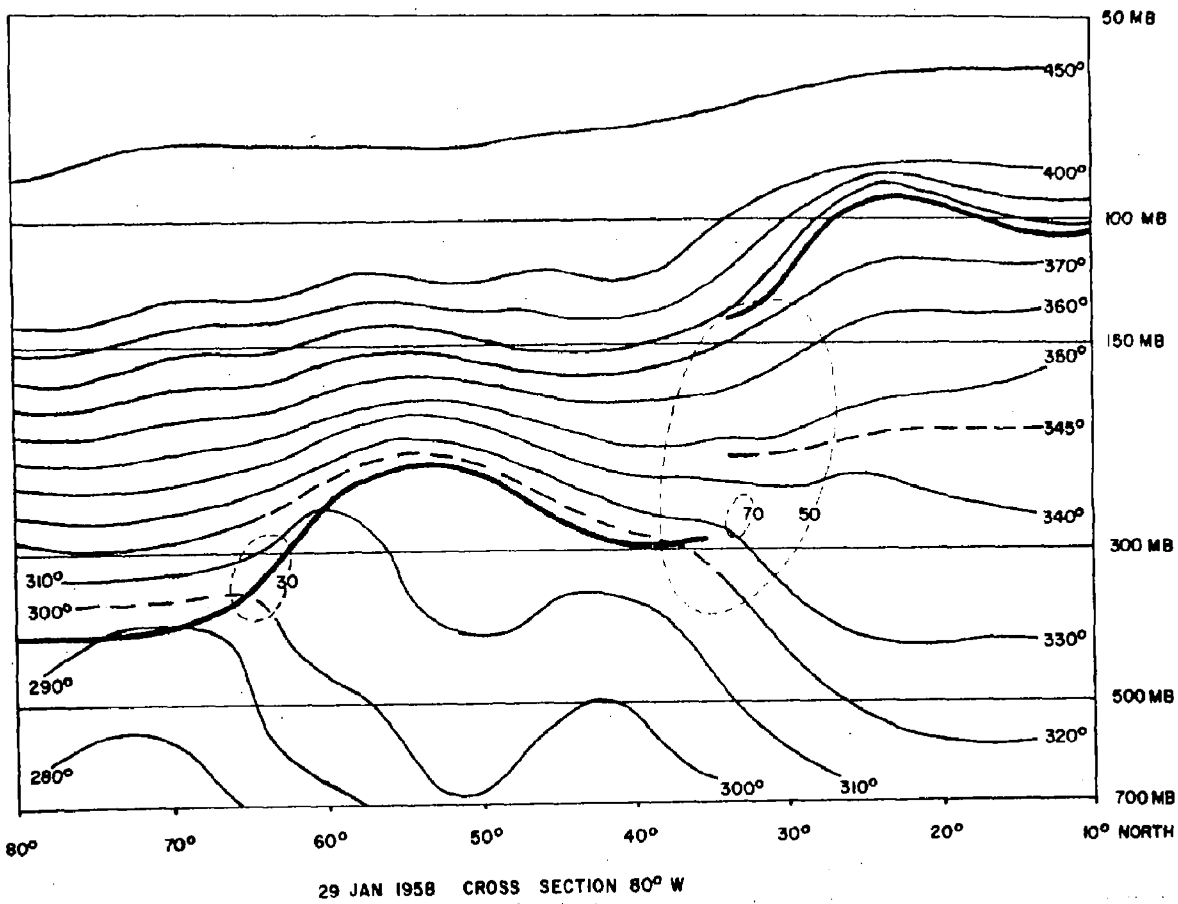

FIGURE 2: CROSS-SECTION SHOWING ISENTROPES, TROPOPAUSE LEAFS, AND JET STREAMS

Note how the 320° isentrope parallels the middle leaf in the dashed portion between the jet maxima. Note, also, the abrupt separation which occurs between the 300° and 320° isentropes as one proceeds southward through the polar jet stream. This separation, which is associated with the polar front, is used to determine the "break-line" between the polar tropopause leaf and the middle leaf. A similar separation associated with the subtropical front is used to determine the break between the middle and subtropical leafs. Note in Figure 2 that the 320° isentrope separates from 345° isentrope south of the subtropical jet maximum.

This example is a case where the subtropical tropopause is weak and does not meet WMO criteria. A stabilization does occur at about 345° K, however, and this isentrope has been added to represent the subtropical tropopause. A strong stabilization occurs at the tropical tropopause which, in this case, is seen to parallel the 380° isentrope. No attempt is made, however, to analyze the tropical tropopause in the system under consideration. The large pressure difference which occurs between it and the middle tropopause would make the analyses unsuitable for use as a "sigma" surface in the PE forecast model. Furthermore, the level of the maximum wind is generally associated with the subtropical tropopause and not the tropical tropopause.

The current tropopause analysis system is not entirely automatic in that the isentropic surfaces associated with the various leafs must be selected subjectively. To assist the monitoring analyst in making this selection, a chart, on which the reported tropopause pressure and temperature and the corresponding computed potential temperature are printed for each station, is produced automatically. This chart, for which a plastic overlay of the geography is available, covers the North American continent and surrounding ocean areas. The analyst also has available various constant pressure analyses from which the location of jet streams and other upper-air features can be determined.

For the subtropical tropopause, the mode of the potential temperatures computed from tropopause data is selected. Potential temperatures corresponding to the reported tropical tropopause are not used in this selection. When no subtropical tropopause data are reported in the area, several subtropical soundings should be examined for a stabilization point above 200 mb with a potential temperature near 355° K.

For the polar tropopause, potential temperatures to the north of the polar jet are noted. In the case of a closed low, values near the southern periphery, rather than the colder potential temperatures near the center, are considered for selection.

For the middle tropopause, tropopause data north of the subtropical jet are considered. A group of stations with potential temperatures falling within a range of about 5° K is sought. The highest value in the group is preferred since the associated isentropic surface should lie slightly above and not below the tropopause leaf in question.

The selected potential temperatures are entered through the on-line card reader. Upper-air data and isobaric objective analyses are input from the NWP "B-3" tape. The station data is used to compute isentropic temperatures in the same manner as in the objective isentropic analysis program described in NMC Technical Memorandum No. 30 [2]. Isobaric temperature analyses for eight mandatory constant pressure levels (850 to 100 mbs) are used. The completed tropopause analyses are appended to the "B-3" analysis file after the last constant pressure analysis.

The system proceeds as follows:

1. Potential temperatures are selected and entered through the card reader, one card per temperature. (This is the only non-automatic step.)

2. Objective isentropic temperature analyses are made for each potential temperature entered.

3. The stability of the layer between the two highest isentropic surfaces is tested to judge whether this layer is predominately stratospheric or not. (The difference in the isentropic temperatures for the top and bottom of the layer is used as a measure of the stability.) If it is stratospheric, the stability of the next lower isentropic layer is checked and so on. The isentropic temperature, T_θ, for the top surface of the first layer which is found not to be stratospheric, is taken as the tropopause temperature.

4. If the highest layer is not stratospheric, the pressure, p_θ, of the top isentropic surface (associated, in this case, with the subtropical tropopause) is taken as the tropopause pressure.

5. If not (4), the tropopause pressure is computed as the intersection of the isotherm $T = T_\theta$ found in (3) and an assumed constant lapse rate up from a tropospheric constant pressure surface, $p = p_S$. The nearest standard isobaric surface for which $p_S > p_\theta$ and $T_S > T_\theta + 2°C$, is used. If this level is below 400 mb, the computation is made using the 500 mb temperature, T_5, provided $T_5 > T_\theta$.

6. If $p_\theta \geq 500$ mb or if $T_\theta \geq T_5$, T_5 and 500 mb are used as the tropopause temperature and pressure.

7. The computed tropopause pressure is checked to insure that it is greater than the pressure, p_θ, of the isentropic surface selected in (3). If not, it is replaced by the isentropic pressure.

8. If the pressure of the subtropical tropopause found in (4) is greater than 200 mbs, the 200 mb temperature and pressure are used as the tropopause values. This is done to compensate for the effects of some extremely warm high-level

temperatures reported from certain countries known to use outmoded radiosonde equipment.

It should be noted that the above procedures are applied grid point by grid point using objectively analyzed isentropic and isobaric temperatures. The isentropic pressures, when needed, are derived directly from the analyzed isentropic temperature using the relation.

$$(1) \qquad p_\theta = 1000\,(T_\theta / \theta)^{7/2} .$$

For any given north-south cross-section of the atmosphere, three tropopause leafs are usually sufficient. If, however, one is making a hemispheric analysis, a subdivision into four leaves has been found to yield better over-all results. Since the system tests the stability of each layer, beginning with the highest and proceeding downward, it might appear that more layers, giving better resolution, could do no harm.

The problem with using more than three layers, however, is that a shallow non-stable layer imbedded in the stratosphere might be misinterpreted as being tropospheric. With three layers (four isentropic surfaces) the potential temperatures can easily be selected so as to be at least 10°K apart and this problem does not arise.

In any event, the current operational program makes and uses four isentropic temperature analyses: one corresponding to the "subtropical" tropopause as previously described; another for the "polar" tropopause; a third for a "low-middle" tropopause; and a fourth for a "high-middle" tropopause.

The isentropic levels are selected based on data from the North American area only. This allows for world-wide seasonal variations but not for local differences. Fortunately, the system is flexible enough to yield good large-scale analyses over most of the grid. In the case of the subtropical tropopause the system is less flexible than for the polar and middle tropopauses in that the same isentropic surface selected for the North American area must serve all tropical and subtropical regions. Fortunately, the non-seasonal variations in potential temperature for the subtropical tropopause are small. In winter, for example, it is generally confined to values between 345° and 355°K. Thus, the model values of pressure are within about ∓25 mbs of any reported subtropical tropopauses. Nevertheless, they represent nothing more than an isentropic pressure analysis except in those areas where the tropopause coincides with the selected isentropic surface either by accient or design.

As previously mentioned, the stability of each isentropic layer is tested (grid point by grid point) and a decision is made as to whether the layer is predominately stratospheric or not. This involves comparing some measure of the layer stability with an established critical value.

For the lowest layer (polar/lower-middle) the stability is measured by the pressure difference, Δp, per degree of potential temperature difference, $\Delta\theta$. A critical stability ($\Delta p/\Delta\theta$) was originally estimated from measurements scaled off the IGY daily cross-sections referenced above. The value selected, 110 mb per 25 degrees, proved to be an excellent criterion for breaking the polar tropopause where it begins to merge with the polar front. At 300 mb and for $\theta = 300°K$, this $\Delta p/\Delta\theta$ corresponds to a lapse rate of 1.95°C/km. North of the polar jet the lapse rates decrease to near zero, which is quite typical of most of the polar stratosphere.

For the separation of the subtropical and high-middle tropopause, charts of the computed temperature differences for the associated isentropic surfaces were studied to determine an optimum critical stability. A discontinuity in the gradient of stability was found which essentially divided the reported tropopause into middle types to the north and tropical or subtropical to the south (strong stability gradient). On this basis, a critical value of 12.75°C per 25°C difference in potential temperature was established. This corresponds to a lapse rate of about 4.3°C/km under typical conditions; more than twice the minimum lapse rate of 2°C/km specified by WMO. To the north of this selected break-line, the lapse rates continue to decrease so that lapse rates as stable as 2°C/km are found a few degrees latitude to the north of the break-line. The assumption of isothermal conditions above the high-middle tropopause is nevertheless somewhat questionable especially near the subtropical jet stream. This tropopause leaf, however, generally conforms closely to an isentropic surface. If it conforms nearly enough to the selected surface, isentropic and tropopause temperatures will be essentially the same regardless of the lapse rate. The problem can thus be minimized for the North American area but not for the entire hemisphere.

Similar empirical methods were used to determine a critical isentropic temperature difference in the layer between the isentropic surfaces corresponding to the "high-" and the "low-middle" tropopauses. Here 10°C per 25°C difference in potential temperatures was found to give the desired separation. This corresponds to a lapse rate of about 3.8°C/km under typical conditions. The same comments concerning the discrepancy between the critical lapse rate used to separate tropopauses and the assumption of an isothermal lapse rate used to compute their pressure apply here. It should be mentioned, however, that the isothermal conditions applies in the top portion of the layer below the layer for which the stability is greater than the critical value. An example may clarify this point.

Example: Suppose the stability in the high-middle/low-middle layer was greater than the critical value for that layer. Then if the stability in the low-middle polar layer is less than the critical value, the temperature of the low-middle isentropic surface is extrapolated downward with no change.

It is not intended to imply that the stability immediately above the tropopause is generally greater than in a higher layer. This is, however, quite often the case and may have some bearing on the empirical determinations of the critical values.

As discussed above, a decision is first made as to which isentropic temperature, T_θ, should apply as the tropopause temperature. Then, for polar and middle cases, a tropospheric temperature, T_s, from a constant pressure level, $p = p_s$ is found. The tropopause pressure p is then computed from:

$$p = p_s (T_\theta / T_s)^5 \quad \text{for} \quad T_5 > -40°C \quad . \tag{2a}$$

Equation (2a) is derived by assuming a constant lapse rate $\gamma = .7\gamma d$ below the tropopause, by using the well-known relation:

$$T_1 = T_2 \, (p_1/p_2) \, R\gamma/g \; ,$$

and by substituting:

$$\gamma_d = g/C_p \quad \text{and} \quad R/C_p = 2/7 \; .$$

For arctic soundings, it was found that a tropopause lapse rate of $.7\gamma_d$ was too large. Half of this rate is, therefore, used when the 500 mb temperature is - 40° C or less. The tropopause pressure is then computed from:

$$p = p_s (T_\theta / T_s)^{10} \; . \tag{2b}$$

The idea of determining the tropopause pressure using a representative stratospheric temperature as one parameter and a tropospheric temperature as another is due to Amos Eddy [3]. Eddy used mean temperatures for fixed isobaric layers (100/150 mb and 300/500 mb) as his statistical parameters.

Having determined the tropopause temperature and pressure for each grid point, the two 1977 fields are smoothed using F. G. Shuman's [4] smoothing operator with $\nu_0 = .5$ and $\nu_1 = \nu_2 = 0$. This operator is also used to smooth the fields of Δp_θ and ΔT_θ used in testing the stabilities of the lowest (polar/low-middle) and highest (high-middle/subtropical) layers. The temperature differences for the middle layer are computed point for point only as required. No smoothed field is stored for this auxiliary parameter. The isentropic temperature fields themselves are smoothed as part of the analysis program [2]. Further smoothing of their difference fields is therefore not essential.

It should be noted again that there is no attempt to fit the reported tropopause pressures and temperatures in this modeled analysis. The reports are used, however, in the selection of the isentropic surfaces to be analyzed. They are also used as significant points in determining the isentropic temperatures at the stations. Where other significant data is missing from the soundings as received at NMC, the inserted tropopause data is especially important.

If a further refinement of the analysis were desired, the analysis from the present program could serve as a first guess to which modifications could be made using the reported values in the usual way. Small initial "toss-out" criteria would have to be used, however, in order to preserve the essentially correct large-scale features of the model.

REFERENCES

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3. Eddy, A., "A Statistical Model for the Mid-Latitude Tropopause and Jet Stream Layer," Journal of Applied Meteorology, April 1963.

4. Shuman, F. G., Numerical Methods in Weather Prediction II: Smoothing and Filtering, Monthly Weather Review, Vol. 35, November 1957.