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Saturation Thickness Tables for the Dry Adiabatic, Pseudo-Adiabatic, and Standard Atmospheres

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SATURATION THICKNESS TABLES FOR THE DRY ADIABATIC, PSEUDO-ADIABATIC, AND STANDARD ATMOSPHERES

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ABSTRACT

The requirements for a complete moisture parameter include not only a measure of the degree of saturation but also a measure of the quantity of water vapor. "Saturation Thickness" meets these two requirements when the thickness of the layer is also considered. The saturation thickness is defined as the hypothetical thickness required to produce saturation, given the moisture quantity and lapse rate of the layer. Tables of saturation thickness are given for the moist and dry adiabatic, and U. S. Standard Atmosphere lapse rates over a temperature range at 1000 mb. of -40°C. to +40°C. The 1000-500-mb. layer has been divided into the three layer intervals corresponding to the standard radiosonde reporting levels. The tables should obviate the efforts of other investigators to derive this information. The significance of errors arising from assumptions and computational approximations is investigated in the Appendix.

1. INTRODUCTION

A relatively accurate forecast of the degree of saturation is an a priori condition for successful cloud and precipitation forecasts. In addition, for reliable estimates of amounts of precipitation, a quantitative measure of the available moisture is needed. At the National Meteorological Center the use of "saturation thickness" [1] has been found convenient since it fulfills both of these requirements.

It should be emphasized that neither the relative humidity nor the precipitable water alone, the two moisture parameters routinely available on the National Weather Facsimile Circuit, provides the answer to both moisture requirements. Neither does the saturation thickness alone provide the complete answer, since it is necessary to compare the saturation thickness and the actual thickness of the layer to determine the degree of saturation. However, the advantage in using saturation thickness is that the thicknesses are in the same units and of the same order of magnitude and are thus easily comparable. Further, since thickness prognoses are a product of the forecast program at the National Meteorological Center, they are routinely available. This allows a comparison of the two thicknesses with just a crude advective or continuity-type forecast of the saturation thickness. A more sophisticated numerical forecast of the saturation thickness has been developed and is in operation at the National Meteorological Center providing a complete numerical cloud, precipitation, and quantitative precipitation forecast program.

2. DEFINITIONS AND THEORY

Saturation thickness (h_s) is the thickness of a hypothetical saturated column between specified isobaric surfaces with a prescribed lapse rate and base temperature. Therefore, when using an integrated column moisture value, i.e., the precipitable water, the saturation thickness is defined by the precipitable water content [2]. The thickness of a layer is approximately,

$$h \approx 29.27 T_m \ \ln \frac{p_1}{p_2} \quad (1 + 0.61\, w_m) \qquad (1)$$

h = thickness of the stratum in geopotential meters,

T_m = mean absolute temperature of the stratum,

p_1 = pressure at the base of the stratum,

p_2 = pressure at the top of the stratum,

w_m = mean mixing ratio of the stratum (grams of water vapor per gram of dry air).

The first term on the right side of the equation gives the thickness of the layer due to the density of the dry air while the second term represents the additional thickness due to the water vapor present. For saturated conditions w_m becomes the mean saturated mixing ratio (w_{sm}) and it can be further shown that w_m is closely approximated by fw_{sm}, where f is the mean relative humidity, therefore equation (1) becomes

$$h_f \approx 29.27 T_m \ \ln \frac{p_1}{p_2} \quad (1 + 0.61\, fw_{sm}) \qquad (2)$$

where h_f is the thickness at any mean relative humidity value; at 100 percent mean relative humidity, h_f is equal to h_s.

Recently the authors had need of the saturation thickness and corresponding precipitable water values for the pseudo-adiabatic lapse rates through a wide range of temperatures. It was decided to expand this work to include the dry adiabatic and U. S. Standard Atmosphere lapse rates for the 1000-850, 850-700, and 700-500-mb. strata. The 1000-500-mb. data were arrived at by summing the three layers. The results of these detailed computations are published to obviate laborious computations on the part of others. While the method of computation does not permit accuracy to the decimal extent published in the tables, the last figures may be of help in making reasonable interpolations. Computational errors for the entire 1000-500-mb. layer at highest summertime values of moisture and temperature over the southern United States are less than 0.02 in. in precipitable water and 2 gpm. in thickness.

Although saturation of a dry adiabatic lapse rate through considerable depth is physically unlikely, the information given in table 3 is useful for obtaining thickness values for lower relative humidities. For example, assume a 1000-500-mb. stratum with dry adiabatic lapse rate, a 1000-mb. temperature of 30°C., and a relative humidity of 50 percent. The correct thickness would be half way between 5572 gpm., the dry-air thickness, and 5591 gpm., the saturated thickness, or 5582 gpm. Also, the precipitable water would be half that of the saturated layer. The same interpolation procedures may be used in tables 1 and 2 to obtain thickness and precipitable water values for relative humidities differing from 100 percent.

The mean saturation mixing ratio w_m in grams per kilogram may be obtained from the precipitable water values given in the tables. This follows from the usually accepted precipitable water equation

$$W = \frac{1}{g} \int_{p_1}^{p_2} w \, dp \text{ cm.} \tag{3}$$

$$= .0004 \int_{p_1}^{p_2} w \, dp \text{ inches} \tag{4}$$

$$= .0004 w_m (p_1 - p_2)$$

$$w_m = \frac{W}{.0004(p_1 - p_2)} \tag{5}$$

where W is the precipitable water, p_1 and p_2 are the pressures at the bottom and top of the layer, and g is the acceleration of gravity with a value of 980 cm. sec.$^{-2}$. Considering a layer of 150-mb. depth

$$w_m = \frac{W}{.0004(150)} = 16.67W$$

Considering a layer of 200-mb. depth

$$w_m = \frac{W}{.0004\ (200)} = 12.50W$$

Considering a layer of 500-mb. depth

$$w_m = \frac{W}{.0004\ (500)} = 5.00W$$

Thus, the factors for converting the precipitable water content of the layer to the mean mixing ratio are:

	1000-850 mb.	850-700 mb.	700-500 mb.	1000-500 mb.
conversion factor	16.67W	16.67W	12.50W	5.00W

3. PROCEDURE

With the 1000-mb. temperature as a base, individual temperature curves in the three theoretical atmospheres (moist and dry adiabatic, and U. S. Standard) were built up. The procedures for computing the dry-air thickness, saturation thickness, and precipitable water for the three different lapse rate atmospheres were as follows:

1. The mean dry-air temperatures to tenths of a degree Celsius were determined for each layer for 1000-mb. base temperatures at 5°C. intervals (and at intermediate points if necessary) from -40°C. to +40°C. The Smithsonian Tables [3], and the Skew T and pseudo-adiabatic diagrams were used to determine temperatures at constant pressure intersections.

2. These mean temperatures were then converted to dry-air thicknesses in accordance with the Smithsonian Tables.

3. Computed dry-air thickness values were plotted against 1000-mb. base temperatures; smooth curves were drawn; and dry-air thickness values were read at intervals of 1°C.

4. The foregoing steps provide a solution to the first term on the right side of equation (1). The second term on the right is a function of the preceding term and its solution requires additionally only the mean mixing ratio of the strata under consideration. The previously mentioned conversion factors were used to change precipitable water values (Step 6) into mean mixing ratios. Mathematical solutions for the additional thickness due to column saturation then were obtained at 5°C. temperature intervals.

5. The thicknesses due to the moisture in the saturated columns were added to the appropriate thickness values obtained in Step 3 to obtain saturated thickness values for each stratum and lapse rate. Values of saturation thickness were then plotted and read for the 1000-mb. base temperatures at 1°C. intervals.

6. Values of precipitable water for the pseudo-adiabatic atmosphere for base temperatures of -6.7° to +26.7°C. were obtained from the Tables of Precipitable Water and Other Factors for a Saturated Pseudo-Adiabatic Atmosphere [4]. Precipitable water values for other than these temperatures in the pseudo-adiabatic atmosphere, and for the entire 80°C. range for the dry adiabatic and the U. S. Standard atmospheres were computed by use of the Skew T and pseudo-adiabatic diagrams in a manner described by Solot [5] and Showalter [6]. Computations were made by layers for the appropriate indentifying temperatures. These were plotted against saturated thickness values, again smooth curves were drawn, and precipitable water values were read at intervals of 1°C.

4. ACKNOWLEDGMENT

The authors wish to express their indebtedness to Mr. Vernon Bohl for his assistance in preparing these tables.

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TABLE I PSEUDO-ADIABATIC ATMOSPHERE

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T° C 1000 mb	1000-850 mb Dry h	Saturated h_s	W	850-700 mb Dry h	Saturated h_s	W	700-500 mb Dry h	Saturated h_s	W	1000-500 mb Dry h	Saturated h_s	W
-40	1086	1086	.0044	1235	1235	.0014	1972	1972	.0003	4293	4293	.0061
-39	1091	1091	.0049	1240	1240	.0015	1981	1981	.0003	4312	4312	.0067
-38	1096	1096	.0055	1245	1245	.0017	1990	1990	.0004	4331	4331	.0076
-37	1100	1100	.0062	1250	1250	.0019	1999	1999	.0004	4350	4350	.0085
-36	1105	1105	.0069	1256	1256	.0022	2008	2008	.0005	4369	4369	.0096
-35	1110	1110	.0077	1261	1261	.0025	2018	2018	.0006	4388	4388	.0108
-34	1114	1114	.0086	1266	1266	.0028	2027	2027	.0007	4407	4407	.0121
-33	1119	1119	.0096	1272	1272	.0032	2036	2036	.0008	4426	4426	.0136
-32	1123	1123	.0107	1277	1277	.0037	2045	2045	.0009	4445	4445	.0153
-31	1128	1128	.0119	1282	1282	.0041	2054	2054	.0010	4464	4464	.0170
-30	1133	1133	.0131	1288	1288	.0046	2063	2063	.0011	4484	4484	.0188
-29	1137	1137	.0144	1293	1293	.0052	2073	2073	.0012	4503	4503	.0208
-28	1142	1142	.0158	1299	1299	.0058	2082	2082	.0014	4523	4523	.0230
-27	1147	1147	.0174	1304	1304	.0065	2092	2092	.0016	4543	4543	.0255
-26	1152	1152	.0191	1310	1310	.0072	2101	2101	.0018	4563	5563	.0281
-25	1156	1156	.0210	1315	1315	.0081	2111	2111	.0021	4583	4583	.0312
-24	1161	1161	.0231	1321	1321	.0091	2121	2121	.0024	4603	4603	.0346
-23	1166	1166	.0254	1326	1326	.0102	2130	2130	.0028	4623	4624	.0384
-22	1171	1171	.0279	1332	1332	.0114	2140	2140	.0031	4643	4644	.0424
-21	1176	1176	.0306	1338	1338	.0127	2150	2150	.0035	4663	4664	.0468
-20	1180	1180	.0335	1343	1343	.0140	2160	2160	.0040	4683	4684	.0515
-19	1185	1185	.0367	1349	1349	.0155	2170	2170	.0046	4704	4705	.0568
-18	1190	1190	.0402	1355	1355	.0171	2180	2180	.0052	4725	4726	.0625
-17	1195	1195	.0439	1361	1361	.0189	2190	2190	.0059	4746	4747	.0686
-16	1200	1200	.0479	1367	1367	.0209	2201	2201	.0067	4768	4769	.0755
-15	1205	1205	.0522	1373	1373	.0231	2211	2211	.0075	4789	4790	.0828
-14	1209	1210	.057	1380	1380	.026	2222	2222	.008	4811	4812	.091
-13	1214	1215	.062	1386	1386	.028	2233	2233	.009	4833	4834	.099
-12	1219	1220	.068	1392	1392	.031	2244	2244	.010	4855	4856	.109
-11	1224	1225	.074	1398	1398	.034	2254	2254	.012	4877	4879	.120
-10	1229	1230	.081	1405	1405	.038	2265	2265	.014	4899	4901	.133
-9	1234	1235	.089	1411	1411	.042	2276	2276	.016	4921	4923	.147
-8	1238	1240	.097	1417	1418	.047	2288	2288	.018	4944	4946	.162
-7	1243	1245	.105	1424	1425	.052	2300	2300	.020	4967	4969	.177
-6	1248	1250	.114	1430	1431	.057	2312	2312	.023	4990	4993	.194
-5	1253	1255	.123	1437	1438	.063	2324	2324	.026	5014	5017	.212
-4	1258	1260	.133	1444	1445	.070	2336	2337	.029	5038	5041	.232
-3	1263	1265	.144	1450	1451	.077	2349	2350	.033	5062	5066	.254
-2	1268	1270	.156	1457	1458	.084	2361	2362	.037	5086	5090	.277
-1	1273	1275	.169	1464	1465	.092	2374	2375	.041	5111	5115	.302
0	1279	1281	.182	1471	1472	.100	2387	2388	.046	5136	5141	.328
1	1284	1286	.197	1477	1479	.110	2400	2401	.052	5161	5166	.359
2	1289	1292	.213	1484	1486	.122	2413	2414	.060	5186	5192	.393
3	1294	1297	.230	1491	1493	.135	2426	2427	.068	5212	5218	.433
4	1299	1303	.249	1498	1500	.149	2440	2441	.077	5238	5245	.475
5	1305	1308	.269	1505	1507	.164	2455	2456	.086	5264	5272	.519

WB 413-50 **TABLE I (continued)**

6	1310	1314	.290	1512	1515	.180	2469	2471	.097	5291	5299	.567
7	1315	1319	.313	1519	1522	.197	2484	2486	.110	5318	5327	.620
8	1320	1324	.337	1526	1530	.216	2499	2501	.124	5345	5355	.677
9	1325	1330	.363	1534	1537	.237	2514	2516	.140	5372	5383	.740
10	1330	1335	.392	1541	1545	.260	2528	2531	.157	5399	5411	.809
11	1336	1341	.422	1548	1552	.285	2543	2546	.177	5427	5440	.884
12	1341	1346	.454	1555	1560	.311	2558	2562	.200	5454	5469	.965
13	1346	1352	.488	1562	1567	.339	2573	2577	.225	5481	5497	1.052
14	1351	1358	.523	1569	1575	.368	2588	2593	.253	5508	5526	1.144
15	1356	1364	.560	1576	1582	.400	2603	2609	.284	5535	5555	1.244
16	1361	1369	.600	1583	1590	.434	2619	2625	.318	5563	5584	1.352
17	1366	1375	.644	1590	1598	.472	2634	2641	.356	5590	5613	1.472
18	1371	1380	.690	1597	1605	.512	2649	2657	.398	5617	5643	1.600
19	1376	1386	.740	1604	1613	.556	2664	2673	.444	5644	5672	1.740
20	1381	1392	.792	1610	1620	.604	2679	2689	.494	5670	5701	1.890
21	1386	1398	.848	1617	1628	.656	2694	2705	.549	5697	5731	2.053
22	1391	1404	.908	1624	1636	.712	2709	2721	.609	5723	5760	2.229
23	1396	1410	.971	1630	1643	.773	2723	2737	.674	5749	5789	2.418
24	1401	1416	1.038	1637	1651	.837	2737	2753	.744	5775	5819	2.619
25	1406	1422	1.110	1644	1659	.905	2751	2768	.820	5800	5848	2.835
26	1411	1428	1.188	1650	1666	.977	2764	2783	.902	5825	5877	3.067
27	1416	1434	1.270	1657	1674	1.053	2778	2799	.991	5851	5907	3.314
28	1421	1440	1.357	1663	1682	1.133	2791	2814	1.087	5875	5937	3.577
29	1426	1447	1.450	1670	1690	1.217	2804	2829	1.191	5900	5967	3.858
30	1431	1453	1.553	1677	1698	1.306	2817	2845	1.302	5925	5997	4.161
31	1436	1460	1.657	1683	1706	1.400	2830	2861	1.422	5949	6027	4.479
32	1441	1466	1.764	1689	1714	1.500	2842	2876	1.550	5973	6057	4.814
33	1446	1473	1.874	1696	1723	1.608	2855	2892	1.686	5997	6088	5.168
34	1451	1480	1.989	1702	1731	1.724	2867	2907	1.831	6020	6118	5.544
35	1456	1487	2.110	1708	1740	1.848	2879	2922	1.985	6043	6148	5.943
36	1461	1494	2.238	1714	1748	1.980	2891	2938	2.147	6066	6180	6.365
37	1466	1501	2.374	1720	1757	2.120	2904	2955	2.318	6090	6213	6.812
38	1471	1508	2.518	1726	1766	2.269	2916	2971	2.497	6113	6245	7.284
39	1475	1515	2.672	1733	1775	2.427	2928	2988	2.685	6136	6278	7.784
40	1480	1522	2.838	1739	1784	2.595	2940	3005	2.883	6159	6311	8.316

SATURATION IS WITH RESPECT TO WATER

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Table 2.

STANDARD ATMOSPHERE

T °C 1000mb	1000-850mb DRY h	SATURATED h_s	W	850-700mb DRY h	SATURATED h_s	W	700-500mb DRY h	SATURATED h_s	W	1000-500mb DRY h	SATURATED h_s	W
	gpm	gpm	inches	gpm	gpm	inches	gpm	gpm	inches	gpm	gpm	inches
-40	1088	1088	.0047	1247	1247	.0018	2027	2027	.0007	4362	4362	.0072
-39	1093	1093	.0052	1252	1252	.0021	2037	2037	.0008	4382	4382	.0081
-38	1098	1098	.0058	1258	1258	.0023	2047	2047	.0009	4403	4403	.0090
-37	1102	1102	.0065	1264	1264	.0026	2057	2057	.0010	4423	4423	.0101
-36	1107	1107	.0073	1269	1269	.0030	2067	2067	.0011	4443	4443	.0114
-35	1112	1112	.0081	1275	1275	.0034	2077	2077	.0013	4464	4464	.0128
-34	1117	1117	.0090	1281	1281	.0038	2086	2086	.0014	4484	4484	.0142
-33	1121	1121	.0100	1287	1287	.0043	2096	2096	.0016	4504	4504	.0159
-32	1126	1126	.0111	1292	1292	.0048	2106	2106	.0018	4524	4524	.0177
-31	1131	1131	.0124	1298	1298	.0054	2116	2116	.0021	4545	4545	.0199
-30	1135	1135	.0138	1304	1304	.0061	2126	2126	.0024	4565	4565	.0223
-29	1140	1140	.0152	1310	1310	.0068	2136	2136	.0027	4586	4586	.0247
-28	1145	1145	.0167	1316	1316	.0076	2145	2145	.0031	4606	4606	.0274
-27	1150	1150	.0183	1321	1321	.0084	2155	2155	.0035	4626	4626	.0302
-26	1155	1155	.0201	1327	1327	.0093	2165	2165	.0039	4647	4647	.0333
-25	1159	1159	.0222	1333	1333	.0103	2175	2175	.0044	4667	4667	.0369
-24	1164	1164	.0244	1339	1339	.0114	2185	2185	.0049	4688	4688	.0407
-23	1169	1169	.0268	1344	1344	.0126	2195	2195	.0055	4708	4708	.0449
-22	1174	1174	.0294	1350	1350	.0140	2204	2204	.0061	4728	4728	.0495
-21	1178	1178	.0322	1356	1356	.0155	2214	2214	.0068	4748	4748	.0545
-20	1183	1183	.0353	1361	1361	.0172	2224	2224	.0076	4768	4768	.0601
-19	1188	1188	.0387	1367	1367	.0190	2234	2234	.0085	4789	4789	.0662
-18	1193	1194	.0425	1372	1372	.0210	2244	2244	.0095	4809	4810	.0730
-17	1198	1199	.0466	1378	1378	.0231	2254	2254	.0106	4830	4831	.0803
-16	1202	1203	.0509	1384	1384	.0253	2264	2264	.0118	4850	4851	.0880
-15	1207	1208	.0555	1390	1390	.0278	2273	2274	.0132	4870	4871	.0965
-14	1212	1213	.060	1395	1395	.031	2283	2283	.015	4890	4891	.106
-13	1216	1217	.066	1401	1402	.034	2293	2293	.016	4910	4912	.116
-12	1221	1222	.072	1406	1407	.037	2303	2303	.018	4930	4932	.127
-11	1226	1227	.078	1412	1413	.040	2313	2313	.020	4951	4953	.138
-10	1230	1232	.085	1418	1419	.044	2323	2323	.022	4971	4974	.151
-9	1235	1237	.092	1423	1424	.048	2333	2333	.025	4991	4994	.165
-8	1240	1242	.099	1429	1430	.052	2342	2342	.027	5011	5014	.178
-7	1245	1247	.107	1435	1436	.057	2352	2352	.030	5032	5035	.194
-6	1250	1252	.115	1440	1441	.062	2362	2363	.033	5052	5056	.210
-5	1254	1256	.125	1446	1448	.068	2372	2373	.037	5072	5077	.230
-4	1259	1261	.136	1452	1454	.074	2382	2383	.040	5093	5098	.250
-3	1264	1266	.147	1457	1459	.080	2392	2393	.044	5113	5118	.271
-2	1269	1271	.159	1463	1465	.087	2401	2402	.048	5133	5138	.304
-1	1273	1276	.171	1469	1471	.094	2411	2412	.053	5153	5159	.318
0	1278	1281	.184	1475	1477	.102	2421	2422	.058	5174	5180	.344
1	1283	1286	.198	1480	1482	.111	2431	2432	.064	5194	5200	.373
2	1288	1291	.213	1485	1488	.120	2441	2442	.071	5214	5221	.404
3	1293	1296	.229	1491	1494	.130	2451	2452	.077	5235	5242	.436
4	1297	1301	.246	1497	1500	.141	2461	2462	.084	5255	5263	.471
5	1302	1306	.265	1503	1506	.152	2470	2471	.091	5275	5283	.508

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Table 2. (Continued)

6	1307	1311	.285	1508	1511	.164	2480	2481	.099	5295	5303	.548
7	1312	1316	.306	1514	1517	.177	2490	2491	.107	5316	5324	.590
8	1316	1321	.328	1520	1523	.191	2500	2502	.116	5336	5346	.625
9	1321	1326	.352	1525	1529	.206	2510	2512	.126	5356	5367	.684
10	1326	1331	.377	1531	1535	.222	2520	2523	.137	5377	5389	.736
11	1330	1336	.403	1537	1542	.239	2530	2533	.149	5397	5411	.791
12	1335	1341	.431	1543	1548	.258	2539	2543	.162	5417	5432	.851
13	1340	1346	.461	1548	1554	.278	2549	2554	.176	5437	5454	.915
14	1345	1352	.493	1554	1560	.299	2559	2564	.191	5458	5476	.983
15	1349	1357	.527	1560	1567	.322	2569	2575	.208	5478	5499	1.057
16	1354	1363	.564	1565	1573	.346	2579	2585	.226	5498	5521	1.136
17	1359	1368	.603	1571	1579	.372	2589	2596	.245	5519	5543	1.220
18	1363	1374	.645	1577	1585	.400	2599	2607	.265	5549	5566	1.310
19	1368	1379	.690	1582	1591	.430	2609	2618	.286	5559	5588	1.406
20	1373	1384	.738	1588	1597	.462	2618	2629	.309	5579	5610	1.509
21	1378	1390	.788	1594	1603	.496	2628	2639	.333	5600	5632	1.617
22	1382	1395	.841	1600	1609	.532	2638	2650	.359	5620	5654	1.732
23	1387	1401	.897	1605	1615	.570	2648	2661	.387	5640	5677	1.854
24	1392	1406	.956	1611	1621	.610	2658	2671	.417	5661	5698	1.983
25	1397	1411	1.022	1616	1627	.653	2668	2681	.449	5681	5719	2.124
26	1401	1417	1.087	1622	1634	.699	2678	2691	.484	5701	5742	2.270
27	1406	1423	1.155	1628	1640	.748	2687	2701	.522	5721	5764	2.425
28	1411	1429	1.227	1634	1646	.800	2697	2712	.563	5742	5787	2.590
29	1416	1435	1.303	1639	1653	.856	2707	2722	.607	5762	5810	2.766
30	1420	1441	1.384	1645	1660	.916	2717	2732	.654	5782	5833	2.954
31	1425	1446	1.470	1651	1667	.980	2727	2743	.704	5803	5856	3.154
32	1430	1452	1.563	1656	1674	1.048	2737	2753	.758	5823	5879	3.369
33	1435	1459	1.662	1662	1681	1.120	2746	2763	.806	5843	5903	3.588
34	1440	1465	1.768	1668	1688	1.196	2756	2773	.868	5864	5926	3.832
35	1445	1472	1.882	1673	1695	1.277	2766	2784	.934	5884	5951	4.093
36	1450	1478	2.004	1679	1702	1.352	2776	2794	1.004	5905	5974	4.360
37	1454	1485	2.135	1685	1709	1.431	2786	2805	1.079	5925	5999	4645
38	1459	1492	2.276	1690	1716	1525	2796	2816	1.159	5945	6024	4.960
39	1464	1499	2.428	1696	1723	1.624	2806	2827	1.234	5966	6049	5.286
40	1469	1506	2.592	1702	1730	1.729	2816	2838	1.324	5987	6074	5.645

Saturation is with respect to water

WB 413-50 TABLE 3 DRY ADIABATIC ATMOSPHERE

T° C 1000 mb	1000-850 mb Dry h	Saturated h_s	W	850-700 mb Dry h	Saturated h_s	W	700-500 mb Dry h	Saturated h_s	W	1000-500 mb Dry h	Saturated h_s	W
	gpm	gpm	Inches	gpm	gpm	inches	gpm	gpm	inches	gpm	gpm	inches
-40	1084	1084	.0045	1230	1230	.0014	1975	1975	.0003	4290	4290	.0062
-39	1089	1089	.0050	1236	1236	.0016	1984	1984	.0003	4309	4309	.0069
-38	1094	1094	.0055	1241	1241	.0018	1992	1992	.0003	4327	4327	.0076
-37	1098	1098	.0061	1246	1246	.0020	2000	2000	.0004	4345	4345	.0085
-36	1103	1103	.0067	1251	1251	.0022	2009	2009	.0004	4363	4363	.0093
-35	1108	1108	.0074	1257	1257	.0025	2017	2017	.0005	4382	4382	.0104
-34	1112	1112	.0081	1262	1262	.0028	2025	2025	.0006	4400	4400	.0115
-33	1117	1117	.0090	1267	1267	.0031	2034	2034	.0007	4418	4418	.0128
-32	1121	1121	.0100	1273	1273	.0035	2042	2042	.0008	4436	4436	.0143
-31	1126	1126	.0111	1278	1278	.0039	2051	2051	.0009	4455	4455	.0159
-30	1130	1130	.0122	1283	1283	.0043	2059	2059	.0010	4473	4473	.0175
-29	1135	1135	.0134	1288	1288	.0048	2068	2068	.0011	4491	4491	.0193
-28	1139	1139	.0147	1294	1294	.0053	2076	2076	.0013	4509	4509	.0213
-27	1144	1144	.0162	1299	1299	.0058	2085	2085	.0014	4528	4528	.0234
-26	1149	1149	.0178	1304	1304	.0064	2093	2093	.0016	4546	4546	.0258
-25	1153	1153	.0195	1309	1309	.0070	2102	2102	.0018	4564	4564	.0283
-24	1158	1158	.0214	1315	1315	.0077	2110	2110	.0020	4583	4583	.0311
-23	1163	1163	.0235	1320	1320	.0085	2119	2119	.0022	4601	4601	.0342
-22	1167	1167	.0258	1325	1325	.0093	2127	2127	.0024	4620	4620	.0375
-21	1172	1172	.0282	1331	1331	.0102	2136	2136	.0027	4638	4639	.0411
-20	1177	1177	.0308	1336	1336	.0112	2144	2144	.0030	4657	4658	.0450
-19	1181	1181	.0336	1341	1341	.0123	2153	2153	.0033	4675	4676	.0492
-18	1186	1186	.0366	1347	1347	.0135	2161	2161	.0037	4694	4695	.0538
-17	1191	1191	.0398	1352	1352	.0148	2170	2170	.0041	4712	4713	.0587
-16	1196	1196	.0433	1357	1357	.0163	2178	2178	.0045	4731	4732	.0641
-15	1200	1201	.0471	1363	1363	.0179	2186	2186	.0050	4749	4750	.0700
-14	1205	1206	.0513	1368	1368	.0196	2195	2195	.0055	4768	4769	.0764
-13	1210	1211	.0559	1373	1373	.0214	2203	2203	.0061	4786	4787	.0834
-12	1215	1216	.0609	1378	1378	.0234	2211	2211	.0067	4804	4805	.0910
-11	1219	1220	.0663	1384	1384	.0255	2220	2220	.0074	4823	4824	.0992
-10	1224	1225	.0721	1389	1389	.0278	2228	2228	.0081	4841	4843	.1080
-9	1229	1230	.0783	1394	1394	.0303	2237	2237	.0089	4860	4862	.1175
-8	1234	1235	.0849	1399	1400	.0330	2245	2245	.0098	4878	4880	.1277
-7	1238	1239	.0919	1405	1405	.0359	2254	2254	.0108	4897	4899	.1386
-6	1243	1244	.0994	1410	1411	.0390	2262	2262	.0119	4915	4917	.1503
-5	1248	1249	.1074	1415	1416	.0423	2271	2271	.0130	4933	4935	.1627
-4	1252	1254	.116	1420	1421	.046	2279	2279	.014	4952	4954	.176
-3	1257	1259	.125	1426	1427	.050	2288	2288	.016	4970	4973	.191
-2	1261	1263	.135	1431	1432	.054	2296	2296	.017	4988	4991	.206
-1	1266	1268	.145	1436	1437	.058	2305	2305	.019	5007	5010	.222
0	1270	1272	.156	1442	1443	.063	2313	2313	.020	5025	5028	.239
1	1275	1277	.168	1447	1448	.068	2322	2322	.022	5044	5048	.258
2	1280	1282	.181	1452	1453	.074	2330	2330	.024	5062	5066	.279
3	1284	1287	.195	1457	1458	.080	2339	2339	.026	5080	5084	.301
4	1289	1292	.210	1463	1464	.087	2347	2348	.029	5099	5103	.326
5	1294	1297	.227	1468	1469	.094	2356	2357	.031	5117	5122	.352
6	1298	1302	.245	1473	1475	.102	2364	2365	.034	5135	5140	.381
7	1303	1307	.263	1478	1480	.110	2373	2374	.037	5154	5159	.410
8	1308	1312	.282	1484	1486	.119	2381	2382	.040	5172	5178	.441
9	1312	1316	.302	1489	1491	.128	2390	2391	.044	5191	5197	.474
10	1317	1321	.324	1494	1496	.138	2398	2399	.047	5209	5216	.509

WB 413.50 **TABLE 3** (continued)

11	1322	1326	.347	1499	1501	.148	2401	2408	.051	5228	5235	.546
12	1326	1331	.372	1505	1507	.159	2415	2416	.055	5246	5254	.586
13	1331	1336	.399	1510	1512	.171	2424	2425	.060	5265	5274	.630
14	1336	1342	.427	1515	1518	.183	2432	2433	.065	5283	5293	.675
15	1340	1347	.457	1520	1523	.196	2441	2442	.070	5302	5312	.723
16	1345	1352	.489	1526	1529	.210	2449	2450	.075	5320	5331	.774
17	1350	1357	.523	1531	1534	.225	2458	2459	.081	5339	5351	.829
18	1354	1362	.559	1536	1540	.241	2466	2468	.087	5357	5370	.887
19	1359	1367	.597	1541	1545	.259	2475	2477	.094	5375	5389	.950
20	1363	1372	.637	1547	1551	.278	2483	2485	.102	5394	5409	1.017
21	1368	1377	.679	1552	1557	.298	2492	2494	.109	5412	5428	1.086
22	1373	1383	.723	1557	1562	.319	2500	2502	.118	5430	5447	1.160
23	1377	1388	.770	1563	1568	.342	2509	2511	.127	5449	5467	1.239
24	1382	1393	.820	1568	1574	.366	2517	2520	.137	5467	5487	1.323
25	1387	1399	.872	1573	1579	.391	2526	2529	.148	5485	5506	1.411
26	1391	1404	.927	1578	1585	.418	2534	2537	.159	5503	5526	1.504
27	1396	1410	.985	1584	1591	.447	2543	2546	.171	5522	5546	1.603
28	1401	1416	1.046	1589	1597	.477	2551	2555	.183	5540	5566	1.706
29	1405	1421	1.110	1594	1602	.509	2560	2564	.196	5559	5587	1.815
30	1410	1427	1.178	1600	1609	.543	2568	2572	.209	5578	5607	1.930
31	1415	1433	1.250	1605	1614	.579	2577	2581	.223	5596	5627	2.052
32	1419	1438	1.326	1610	1620	.617	2585	2590	.239	5614	5648	2.182
33	1424	1444	1.407	1616	1626	.657	2593	2598	.256	5633	5669	2.320
34	1429	1450	1.493	1621	1632	.699	2602	2607	.274	5651	5689	2.466
35	1434	1456	1.585	1626	1638	.743	2610	2616	.293	5670	5710	2.621
36	1438	1462	1.683	1631	1644	.790	2618	2625	.313	5688	5731	2.786
37	1443	1469	1.788	1637	1651	.840	2627	2634	.334	5706	5752	2.962
38	1448	1475	1.900	1642	1657	.893	2635	2642	.356	5725	5774	3.149
39	1453	1482	2.020	1647	1663	.949	2644	2652	.380	5743	5796	3.349
40	1457	1488	2.148	1652	1669	1.008	2652	2660	.405	5761	5817	3.561

SATURATION IS WITH RESPECT TO WATER

APPENDIX

MATHEMATICAL DERIVATIONS AND A DISCUSSION OF ERRORS

A number of errors have been introduced through approximations used in equation (1), in the formula for the precipitable water equation (3), and in the theoretical treatment of the subject. While all of these errors are minor and the approximations made are consistent with those usually made in investigations of this nature, an analysis of their value may prove worthwhile.

Equation (1) is derived in the following steps. The well known hypsometric equation is

$$h = \frac{1}{g} \int_{p_2}^{p_1} R_a T_v d(\ln p) \qquad (6)$$

Where h is the thickness, g is the acceleration of gravity, R_a is the gas constant for dry air, T_v is the virtual temperature, and p_1 and p_2 are the pressures at the bottom and top of the layer.

The virtual temperature T_v is defined as RT/R_a (7)
where R is the gas constant of the mixture.

$$R = \frac{M_a R_a + M_v R_v}{M_a + M_v} \qquad (8)$$

where R_v is the gas constant for water vapor and M_a and M_v are the mass of dry air and water vapor respectively.

By definition the mixing ratio, w, is

$$w = \frac{M_v}{M_a}, \text{ so that } R = \frac{R_a + wR_v}{1 + w} \qquad (9)$$

Substitution of R into (7) gives

$$T_v = T \left[\frac{1 + w \frac{R_v}{R_a}}{1 + w} \right] \qquad (10)$$

$R_v/R_a = m_a/m_v = 28.97/18.02$ where m_a and m_v are the molecular weights of dry air and water vapor respectively. Then

$$T_v = T \left[\frac{1+1.61w}{1+w} \right] \qquad (11)$$

This may be written as

$$T_v = T\left[\frac{(1+w)+.61w}{1+w}\right] = T\left[1+\frac{.61w}{1+w}\right]. \tag{12}$$

Binomial expansion of (12) with squared and higher powers of w considered insignificant, since in the atmosphere w seldom exceeds 0.03 gm. per gm., gives

$$T_v \approx T(1+.61w). \tag{13}$$

Substituting this approximation into (6) gives

$$h \approx \frac{R_a}{g}\int_{p_2}^{p_1} T(1+.61w)\, d(\ln p) \tag{14}$$

Integration of (14) gives

$$h \approx \frac{R_a}{g}\left[T_m \ln\frac{p_1}{p_2} + (T\cdot .61w)_m \ln\frac{p_1}{p_2}\right] \tag{15}$$

Assuming a linear decrease in temperature and mixing ratio with pressure, which is particularly appropriate in saturated soundings with the specified lapse rates used in this article, the second term on the right can be closely approximated by

$$T_m \cdot .61w_m \ln\frac{p_1}{p_2}. \tag{16}$$

This reduces (15) to

$$h \approx \frac{R_a}{g} T_m \ln\frac{p_1}{p_2}(1+.61w_m) \tag{17}$$

Evaluating constants, where $R_a = .287$ joules gm. cm.$^{-1}$ and $g = 980.62$ cm. sec.$^{-2}$ gives

$$h \approx 29.27T_m \ln\frac{p_1}{p_2}(1+.61w_m). \tag{18}$$

The error in (18) evolves from the approximation of T_v. Returning to (6) and using the exact equation for T_v yields, after integration

$$h = \frac{R_a}{g}\left[T_m \ln\frac{p_1}{p_2} + T\left(\frac{1+.61w}{1+w}\right)_m \ln\frac{p_1}{p_2}\right] \tag{19}$$

The second term on the right side is very closely approximated by

$$T_m \ \ln \frac{p_1}{p_2} \left(\frac{1 + .61w_m}{1 + w_m} \right) \tag{20}$$

Investigation of this latter approximation in integration using 10-mb. pressure intervals through the 1000-850-mb. stratum for the pseudo-adiabatic lapse rate with base 1000-mb. temperature of 4.44°C. showed an error of only 0.01 percent in the saturated case. Even with a highly variable vertical profile of moisture in the layer, the maximum error was less than 0.03 percent.

All of the thicknesses for dry conditions were obtained from existing tables rather than from equation (18). The second term of (18), which is a function of the first term, gives the additional thickness through the stratum produced by the water vapor. The percent error ensuing from approximating T_v in this term is

$$\left(\frac{1 + 1.61w}{1 + w} \right) / \ 1 + 0.61w \tag{21}$$

This error is presented in figure 1. A mean mixing ratio of 0.047 gm. gm.$^{-1}$ (the largest value given in table 1 for the 1000-850-mb. stratum) gives the largest error which is a little less than 0.13 percent. A mean mixing ratio this large even through a small layer of the atmosphere is not possible, but even when this error is applied as a correction to the difference between the dry and saturated thicknesses given in table 1, the change amounts to only +0.05 gpm. This is miniscule compared to the accuracy intended in the tables and which present observational equipment is capable of providing.

Errors which result from employing a constant gravity both latitudinally and vertically affect the precipitable water values. The equation for precipitable water W, is

$$W = \int_{p_2}^{p_1} \frac{q}{g} \ dp \ (\text{cm.}) \tag{22}$$

or

$$W \approx \frac{1}{2.54 \ g_{45}} \int_{p_2}^{p_1} q \ dp \ (\text{inches}) \tag{23}$$

g_{45} being the sea level gravity at 45° latitude and equal to 980.616 cm. sec.$^{-2}$ Evaluating constants,

$$W_{45} = .0004017 \int_{p_2}^{p_1} q \ dp \ (\text{inches}) \tag{24}$$

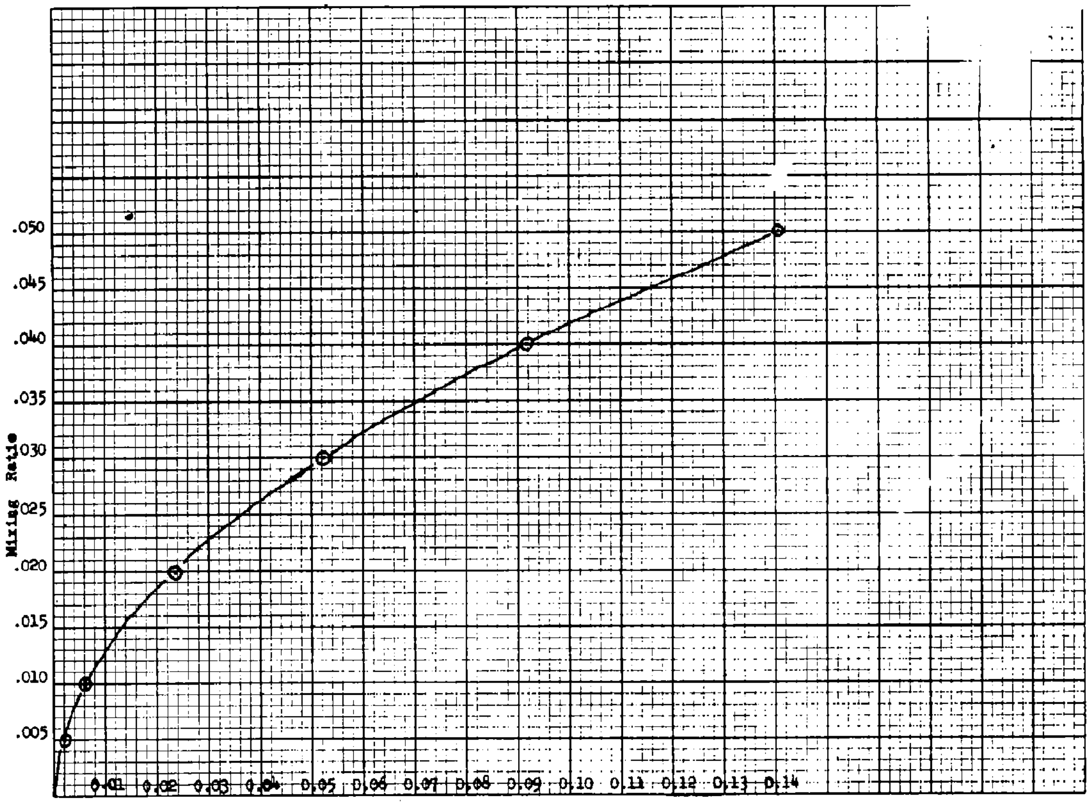

Figure 1. Error in percent due to approximating the virtual temperature in equation (6). The approximated values are larger than those using the exact equation, so the percent errors are negative.

Sea level gravity is 978.381 cm. sec.$^{-2}$ at 15° latitude and 981.911 cm. sec.$^{-2}$ at 60° according to the Smithsonian Tables. Using these figures for g_{45} in (23) gives 0.0004024 and 0.0004010 respectively for the constant in the W equation. Since in practice only the figure to the 10,000th place is used, the error due to latitudinal variation of gravity can be ignored.

Helmert's equation for the decrease of gravity with height assuming a constant vertical gradient is $g = g_\phi - 0.0003086z$ where g_ϕ is the gravity at sea level at a specified latitude and z is the height in meters above sea level. The mean height of the greatest thickness between 700 and 500 mb. is no greater than 5000 m. above m.s.l., even with high 1000-mb. heights as might occur with very high surface pressures. This gives a decrease in gravity of 1.543 cm. sec.$^{-2}$ A summation of the decrease of gravity through the 1000-500 mb. layer using equally high values for the mean 1000-850-mb. and the 850-700-mb. strata gives a decrease of 2.778 cm. sec.$^{-2}$ Applying these values at 15° latitude, where the latitudinal effect is also greatest, results in a change in the constant usually used in the precipitable water to 0.0004035. Again, since only the 10,000th place is used in practical application the error is miniscule as, for example, in the case when the precipitable water is an unattainable 8.316 in. in table 1, the error at 15° latitude is 0.058 in. and at 60° 0.026 in.

Equation (23) gives the specific humidity, q, as the moisture parameter; however, in practice and in this paper, the mixing ratio, w, is substituted as a close approximation. Mathematically

$$q = \frac{w}{1 + w}.$$

The resulting error in the precipitable water varies from a very small percentage to a maximum of nearly 5 percent at highest precipitable water values computed, but actually never observed in the atmosphere. Usual high values of precipitable water observed primarily in the 1000-850-mb. stratum in the southeastern United States are in error by only about 2 percent. Since operationally w is substituted for q in computing the precipitable water, the tables are correct for this information. If the precipitable water is computed using q then the tables will have to be revised.

Another error in the precipitable water computation arises from ignoring the density of the water which is hypothetically precipitated. Since this aspect has not been previously investigated, no guide lines are available for assessing this error. The error evolves from the difference in temperatures which might be used as the proper temperature of the precipitated water. The maximum error possible would be obtained by assuming the precipitated water would have the same temperature as the base temperature. The error ranges from zero to nearly 0.8 percent at 40°C. with the maximum probable error of less than 0.5 percent at normally high temperatures encountered in the atmosphere. Under any conditions the error is insignificant considering the instrumental errors in humidity measurements.

One error ensues from the manner in which the corresponding dry and moist soundings are compared. All of the tables are based on a specified lapse rate with the family of temperature profiles defined by the base temperature. The method outlined is correct in producing that portion of the tables relating to dry thicknesses. However, the addition of moisture increases the thickness which either calls for raising the height at the top of each stratum or changing the lapse rate. Neither of these alternatives was used in the derivation of the tables and discussion of the error involved is necessary.

First, since the value of the paper lies in defining the saturation thicknesses for well-known temperature profiles, it is appropriate that the error be determined for these lapse rates and the height of the top of each stratum be decreased with the decrease in mean temperature caused by the increased thickness of the layer. The pseudo-adiabatic soundings are the most complex since they vary most in °C. change with elevation. For very cold temperatures the lapse rate is nearly 1°C. $(100\ m.)^{-1}$ while for warm temperatures the rate is less than 1/3°C. $(100\ m.)^{-1}$. High moisture values are necessarily associated with high temperatures which tend to minimize the temperature change at the top of the stratum. Thus the change at the top of the 1000-850-mb. stratum with the warmest base temperature in table 1 amounts to slightly more than 0.1°C. Upward along this pseudo-adiabat, the cumulative change in temperature at the 700-mb. level is less than 0.3°C. and at the 500-mb. level less than 0.5°C. Considering the maximum pseudo-adiabatic sounding likely to be saturated in the atmospheric range of conditions (25°C. base temperature), the cumulative negative temperature differences at the top of each stratum is, at the 850-mb. level, only .03°C., at the 700-mb. level 0.13°C., and at the 500-mb. level 0.22°C. These values represent very small decreases in the saturation thicknesses in table 1, amounting to less than 0.1 gpm. in the 1000-850-mb. layer, 0.4 gpm. in the 850-700-mb. layer, and 1.7 gpm. in the 700-500-mb. layer. Thus the total decrease from this effect in extremely warm moist conditions is a little over 2 gpm. in the 1000-500-mb. layer. In more normal soundings, even over the United States in the summer, the total decrease will seldom exceed 1 gpm.

The dry adiabatic lapse rates used in table 3 are attended by a larger error from this effect in the lower half of the 1000-500-mb. layer and smaller error above. Using a very warm saturated dry adiabat with base temperature of 25°C., the accumulative negative temperature error at the top of each stratum is 0.12°C., 0.18°C., and 0.21°C. at the 850-mb., 700-mb., and 500-mb. levels respectively. The thickness decreases due to the difference in mean temperatures in each layer amount to 0.6 gpm. in the 1000-850-mb. stratum and 0.3 gpm. in the 850-700-mb. and 700-500-mb. layers giving a total of 1.2 gpm. for the entire 1000-500-mb. layer. It is doubtful if the decrease in saturation thickness amounts to as much as 1 gpm. under normally observed conditions.

The figures given in table A-1 show, for 0°C. to 40°C. base temperatures, temperature errors at the top of each layer for the moist and dry adiabatic soundings. In all cases the temperature errors are negative. The lapse rates at these levels are also given to allow a computation of the resultant difference in lapse rates from those of the moist and dry adiabatic for those who would desire this type of correction over that given in this paper.

It should be noted that in the case of the pseudo-adiabatic lapse rate the errors are not strictly accurate since the temperature error is cumulative through each stratum while the entire error was computed for that lapse rate at the top of the layer. However, the lapse rates at the top of each layer are larger than those at the bottom, which should counterbalance in part the greater moisture, and hence, larger thickness increases contributed by the lower portion of the layer.

Summarizing the thickness errors, the approximation made for the virtual temperature is negligible while the error due to the decreased thickness in adding moisture to a previously specified lapse rate may on occasion be around 1 gpm. for the entire 1000-500-mb. layer.

Errors in the precipitable water computations are negligible except for the substitution of the mixing ratio for the specific humidity. However, since this is done universally in operational computations, no change can be made since the tables would then be inconsistent with the available precipitable water information.

Table A-I. Accumulative negative temperature errors computed for the top of each stratum due to the increase of thickness of the stratum caused by the injection of moisture sufficient to saturate the layer. Figures for both moist and dry adiabatic soundings are given with the lapse rates at the top of each layer.

1000-mb. Base Temp. (°C.)	Lapse Rate to 850 mb. (°C./100 m.)	Error in T_{850} (°C.)	Lapse Rate to 700 mb. (°C./100 m.)	Error in T_{700} (°C.)	Lapse Rate to 500 mb. (°C./100 m.)	Error in T_{500} (°C.)
			MOIST ADIABATIC			
0	.720	.0144	.814	.0225	.930	.0318
5	.665	.0200	.742	.0348	.885	.0437
10	.580	.0290	.656	.0552	.828	.0800
15	.512	.0410	.573	.0733	.732	.1172
20	.455	.0501	.496	.0997	.618	.1615
25	.404	.0646	.428	.1288	.510	.2155
30	.362	.0796	.378	.1590	.424	.2977
35	.349	.1082	.336	.2157	.360	.3704
40	.303	.1273	.307	.2654	.313	.4689
			DRY ADIABATIC			
0	1.00	.0200	1.00	.0300	1.00	.0300
5	1.00	.0300	1.00	.0400	1.00	.0500
10	1.00	.0400	1.00	.0600	1.00	.0700
15	1.00	.0700	1.00	.1000	1.00	.1100
20	1.00	.0900	1.00	.1300	1.00	.1500
25	1.00	.1200	1.00	.1800	1.00	.2100
30	1.00	.1700	1.00	.2600	1.00	.3000
35	1.00	.2200	1.00	.3400	1.00	.4000
40	1.00	.3100	1.00	.4800	1.00	.5600

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