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NUMERICAL PREDICTION: AN EXPOSITION AND REVIEW FOR THE NONSPECIALIST

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18 April 1957

National Oceanic and Atmospheric Administration

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April 13, 2004

Mr. Chairman, Ladies and Gentlemen:

I have been asked to give a brief survey or review of numerical prediction-that is, to describe what it is, how far it has gone, and how far one can reasonably expect it to go in the future. I have also been asked to do so in simple and completely nonmathematical terms. To the extent that it isn't really necessary to write down any equations, this is certainly possible. It would be extremely difficult, however, to avoid mentioning any mathematical ideas. In a way, this would be rather like trying to describe what a language is without using any words. Accordingly, I shall try to describe some of the physical concepts and principles that underlie numerical or dynamical methods of prediction, without going into the mathematical procedures by which those methods are applied.

After thinking about this job for a while, I was much struck with the many ramifications of numerical prediction, and with the difficulty of giving a coherent and comprehensive account of all the important points in anything less than a small book. I should warn you that I have picked out what seemed to me the most salient features, and must apologize in advance if my remarks sound a little disjointed. You must realize that the topics I shall discuss here are not, in fact, isolated, but are connected by many lesser ideas that can't be elaborated on in 25 minutes.

I should also warn you that I shall say nothing new-a considerable departure from the usual aims of a scientific meeting. But, in a broad sense, there is nothing new about numerical prediction. The general approach is exactly the same one that has become established and accepted in other (and perhaps better developed) branches of physical science over the past few centuries. This is simply the methodology of predicting the behaviour of a physical system by developing the mathematical consequences of a quantitative theory. Whether the purpose is to predict an event that has not yet happened - or whether it is to "predict" a body of facts that have been observed, but not yet explained - is totally irrelevant. In either case, the method is the same.

One analogy that is frequently cited is that of astronomy, a large part of which is concerned with predicting the positions of the planets. In this case, the physical basis of prediction is Newton's Law of Gravitation, which is expressed quantitatively by equating the change in the momentum of each body to the resultant of the gravitational forces exerted on it by all the other bodies. The process of prediction is one of solving a set of differential equations, frequently by approximate numerical methods, starting with the positions and velocities of the planets as observed at isolated moments in time. Although extremely simple in basic conception, predictions computed by this method are very accurate, and are generally regarded as one of the crowning achievements of astronomical science.

But why, in the first place, should one have thought that the Law of Gravitation, originally suggested and verified by the falling of apples and stones, would apply to the motions of the planets? No man has set foot on them, let alone test the validity of <u>our</u> physics in <u>another</u> world. It is simply that one intuitively suspects that general physical laws apply everywhere and under all circumstances; moreover, astronomical calculation and observations have substantiated that this deeply ingrained principle of universality in probably right.

It is also interesting to note certain similarities between the dynamical approaches to astronomical and meteorological prediction. In astronomy, as in numerical weather prediction, one deals with mathematical idealizations or abstractions of the real physical system - as, for instance, bodies whose mass is concentrated in infinitesmal points, which carry no electrostatic charge, and which possess no magnetic polarity. Another noteworthy point is that even the equations which apply to such idealized solar systems are not solved exactly, but are solved by approximate numerical methods or by perturbation methods, frequently borrowed by the meteorologist but originally devised by the astronomer to circumvent the famous N-body problem.* But these criticisms - which are of the same type as those most frequently levelled at numerical weather prediction - certainly do not cause the astronomer to discard the dynamical approach and to revert to Ptolemy's circles-within-circles or other forms of rank empiricism. In this case, the situation is quite clear: The validity of the method of prediction is not a question of total right or total wrong, nor of black and white. Astronomical predictions are not absolutely accurate, nor is the underlying theory wholly correct. The real question is one of degree of validity, and one that can be resolved only by quantitative test, rather than by qualitative judgements of complete correctness and generality. In the last analysis, the validity of approach is judged by the relative accuracy of the predictions and the essential unity and economy of the theory. The point of this remark is that no more should be expected of the dynamical approach to meteorological prediction.

I won't belabor this analogy much further, but will simply review the situation, as regards weather prediction, in the light of astronomical prediction and general scientific methodology. To begin with, we already possess a perfectly good quantitative theory of the behavior of fluids, expressed in the familiar principles of conservation of momentum, mass, and energy.

*Never solved for N greater than 2.

These principles have been known since 1850, and have been successfully applied to the explanation and prediction of a great variety of hydrodynamical phenomena - as, for example, the propagation of sound in air, the motion of waves on the sea surface, and the flow of air around a wing. So far as is known, these principles are completely general - or, at least, general enough to account for the behavior of all fluids at normal pressures and temperatures, and more then a few mean free paths away from a rigid boundary. Thus, by the principle of universality of physical law, it is quite reasonable to suppose that the general laws of fluid dynamics apply to the meteorological behavior of the atmosphere, and that they would provide a rational approach to the problem of weather prediction. This, at any rate, is the principal article of faith.

As in astronomy, the underlying theory of numerical weather prediction can be expressed quantitatively in a set of differential equations. In this case, they are:

1) The Newtonian equations of motion, relating the acceleration of a unit mass of air to the resultant of all the forces acting on it,

2) The continuity equation, which simply states that any local increase or decrease of density must be exactly accounted for by a net import or export of mass in that region,

3) The thermodynamic energy equation, requiring that the heat energy added to any mass of air be equal to the change in its internal or molecular energy, plus the work done by it in expanding against pressure forces, and finally

4) The Boyle —Charles equation of state.

This set contains exactly as many equations as there are unknowns. Thus, the state of the atmosphere at all times in the future is determined (in principle) by its state at a single instant. The fundamental problem of numerical weather prediction is to solve this set of general equations, beginning with observations at some arbitrarily chosen initial time, and numerical prediction itself is the process of solving this or some more specialized set of equations.

Put in such general terms, the problem of numerical weather prediction is easily, but rather deceptively stated. In the first place, the only known methods for solving the general hydrodynamical equations (as they are called) are purely numerical in character, and require an enormous volume of calculations - more than can be handled by even the fastest of existing high-speed automatic computers. Second, there is still some prejudice for understanding the evolution of a prediction from the observed initial state, even if the forecast were absolutely correct - a feat of interpretation that is made all but impossible by the complexity of the general equations. In short, on grounds of both economy and scientific advancement, it is desirable not to deal with the general form of the hydrodynamical equations.

This rather paradoxical fact has led to the study of a series of "models" or mathematical idealizations of the atmosphere, which preserve the most essential aspects of the atmosphere's meteorological behavior, but which do not exhibit all aspects of the behavior of fluids under the most general conditions. In fact, it is fair to say that the main theoretical problem in numerical weather prediction over the past ten years has been to synthesize the most general model that is incapable of maintaining certain nonmeteorological types of motion, such as sound and gravity waves.

To many, this approach may appear artificial and negligent of the obvious reality. I have pointed out earlier, however, that the study of models is not peculiar to meteorology,and has acquired an odor of sanctity from the methods of astronomy and other branches of theoretical mechanics. The real distinction between dynamical meteorology and its sister sciences - if there is such a distinction - lies in the much greater complexity of the physical and mathematical problems that confront the meteorologist. But, for the very reason that the meteorologist's problems are enormously complicated, it would be foolhardy to abandon the most powerful tools that have yet been developed to deal with such problems. One cannot expect hard questions to have easy answers.

The simplest and one of the most successful models that has been studied so far is the familiar barotropic model. Loosely described, this idealized atmosphere is one in which the winds are the same at all levels and in which the percentage changes of surface pressure are not abnormally large - of the order of a few percent per day. Under these restrictions, the motions of the model are governed by a single dynamical principle, namely, the socalled principle of vorticity conservation.

This is, of course, a highly oversimplified version of the atmosphere. Nevertheless, it turns out that methods of prediction based on this model contain the essential physical and mathematical ingredients of more general and complicated methods. Accordingly, as an elementary introduction to the methods of numerical prediction, I shall try to define the concept of vorticity, explain the principle of vorticity conservation, and outline a method by which this principle can be used to predict the horizontal motions of the atmosphere at about the 500 mb level.

To understand the notion of vorticity, let us first consider a curved flow in which the wind speed is uniform, as shown schematically in the upper lefthand corner of the first slide. Let us also imagine that a small disk is floating in the air - a disk light enough and with a high enough drag coefficient that it follows the average motion of the air in its neighborhood - and let the ends of one of its diameters AB lie on the same streamline. Now, consider the position and orientation of this disk after a very short time, as indicated by the dashed lines. By hypothesis, the point A goes into the point A', and the point B goes into the point B'. It will be noted that the line A'B' does not have the same direction as AB, implying that the disk has been rotated slightly. It is clear that the rate of rotation in this case is proportional to the speed at which the disk is displaced along the streamlines - i.e., the wind speed - and proportional to the change of wind direction per unit length along the streamlines or, in other words, the curvature. For brevity, let us call this type of rotation "curvature rotation."

Next, let us consider a straight flow in which the wind speed is not uniform, as shown in the upper righthand corner. Again, we imagine that a small light disk is following the motion of the air. This time, however, we let the diameter CD lie perpendicular to the streamlines. As before, we now consider the position and orientation of the disk after a very short time. Since the wind speed is not uniform, the point C travels further than does the point D, with the result that the disk has been rotated. In this case, it is clear that the rate of rotation is proportional to the change in wind speed per unit length perpendicular to the streamlines. Let us call this type of rotation "shear rotation," since it depends on the lateral wind shear.

Finally, let us consider a curved flow in which the wind speed is not uniform, as shown at the bottom, and again imagine the motions of a disk following the motions of the air. The rotation of the line AB into A'B' represents the "curvature rotation," and the rotation of the line CD into C'D' represents the "shear rotation." In this case, we see that the curvature rotation is greater than the shear rotation, and that they are generally not equal unless the air is in solid rotation. At what rate, then, does the disk rotate? In general, if the disk is small enough, its net rate of rotation is the average of the curvature and shear rotations. The point that is significant for our present discussion is this: The mathematical quantity called "vorticity" is simply the sum of the curvature and shear rotations in a counterclockwise sense, as previously related to the curvature of the streamlines, wind speed, and lateral wind shear. Thus, we may think of vorticity as twice the rate of rotation of a very small disk that follows the motions of the air.

As stated earlier, the motions of the barotropic model are governed by the principle of vorticity conservation. This principle may now be visualized and interpreted in the following way: Let us first suppose that the wind field is reconstructed from observations made at some fixed initial time, and imagine that the field is covered by a vast swarm of small floating disks that are free to follow the motions of the air. This is indicated schematically on the second slide, in which the solid circles represent the initial positions of the disks. As we have shown, one can express the instantaneous rotation rate of each disk (or the initial vorticity of the air) in terms of the initial wind field. In general, disks located near the centers of cyclones are spinning in a counterclockwise sense, and the reverse in anticyclones.

Now, consider the positions of the disks after being displaced for a very short period of time in the initial wind field. These are indicated by the dashed circles. The principle of vorticity conservation simply states that the rate of rotation of each disk is the same at the end of its trajectory as it was at the beginning. This, then, provides us with a means of making a very short range forecast of the rotation rate of the disks or of the vorticity of the air, by displacing imaginary disks in the initial wind field. Now, it turns out that the wind field is mathematically determined by the vorticity pattern in barotropic flow, so that the predicted wind field can be computed from the predicted vorticity pattern. Thus, if we regard a very short range forecast of the wind field as a new initial wind field, we see that this process of vorticity displacement can be repeated again and again, to predict the wind field over periods of any length desired. It is a matter of experience that this method applies best to motions at about the 500 millibar surface. In essence, the forecasting procedure followed by the JNWP Unit in Suitland is the one outlined above, but differs in that all steps of the procedure are carried out numerically by a high-speed automatic computing machine.

The barotropic model just described has certain rather obvious shortcomings that do not permit it to reproduce all of the important features of the atmosphere's meteorological behavior. For one thing, it is clear from the foregoing discussion that the number and intensity of distinct circulation centers in a barotropic flow cannot change from one day to the next, whereas in real atmospheric flow patterns new circulations develop and old ones die out. In order to simulate development and decay processes, it has been necessary to synthesize more general models. These - the socalled baroclinic models - provide for rather simple variations of wind from one level to another, and take into account the thermodynamic processes that influence changes in the vertical structure of the atmosphere. Time will not permit me to describe these models in detail, but it is perhaps sufficient to say that they are governed by dynamical principles very similar to the principle of vorticity conservation, and that the forecasting procedures are exactly analogous to the one I described earlier.

The routine of numerical forecasting is most easily described by discussing separately the two different types of information that go into the forecasts - namely, the initial data and instructions as to what to do with the data. Until recently, the JNWP Unit prepared the initial data by conventional methods. That is, the teletype data were plotted and analyzed manually, in the usual fashion. The heights of the 1000 and 500 millibar surface were then interpolated by eye at the points of a regular network and recorded on punch cards, preparatory to being fed into the computing machine.

This procedure, however, has been found unsatisfactory - not because the quality of subjective analysis is too low, but because the job of analyzing data manually over a large fraction of a hemisphere is too time-consuming. Including data transmission time, the interval between observation and analysis was already several times as long as the entire forecasting routine. To eliminate this bottleneck, Dr. Cressman and Major Bedient have developed an automatic data-processing system, whereby data are fed directly from teletype to computing machine, and there edited, processed, and analyzed automatically virtually untouched by human hand. The preedited data are first punched on a teletype tape, which is then passed through a tape-to-card converter. The resulting deck of data cards is then read into the machine's input, and the remainder of the problem is turned over to the computer. Controlled by a special set of instructions, the machine examines each message to see what kind of report it is and where it came from. If it is not relevant, the report is discarded. If it is, the machine extracts the appropriate data, converts it to standard form, and files it away in its internal memory. The data so selected by this process of editing are then "analyzed," by a scheme of quadratic interpolation and smoothing. At the end of this process, the analysis is printed out in graphical form for inspection. If the analysis looks like a weather map, the machine is instructed to proceed with the forecast. This system was put into experimental operation several months ago, and is now operating in parallel with the conventional system of analysis.

Whether analyzed manually or automatically, the initial data are ultimately stored in the machine's memory as height values at a regular network of poinms - generally covering the western half of the northern hemisphere. It then remains only to tell the machine what to do with this data.

The machine's instructions are originally derived from the differential equations for the model on which the forecasts are based. One first lays out a computing scheme by which the equations can be solved, and breaks it down into a sequence of more or less standard steps. Each of these steps, in turn, is broken down into a detailed list of elementary operations. Finally, each of these lists is translated, according to a numerical code, into a series of instructions that can be interpreted and executed by the machine. The entire list of instructions - called a "code" - is punched on cards once and for all, and is used over and over again.

When the initial data are completely processed and stored in the machine's memory, the deck of instruction cards is fed into a card reader and the machine begins its forecast. Operating at the rate of about 10, 000 numerical operations per second, it completes a 24 hour forecast in about an hour, a 48 hour forecast in two hours, and so on. At the end of each forecast period, the machine prints out its results in graphical form - 0,12,24, and 36 hour forecasts of the 1000 mb height, 500 mb height, and vertical air speed at 500 mb. It also produces 24, 48, and 72 hour forecasts of the 500 mb height. The forecasts are distributed to the three supporting services - the U. S. Weather Bureau, the Air Weather Service, and the Naval Weather Service - and to the National Weather Analysis Center. A few of the forecasts are transmitted to field offices via facsimile and teletype.

Numerical prediction methods based on the barotropic model and two simple baroclinic models have been applied on a daily routine basis by the JNWP Unit since May 1955. The barotropic model, in particular, has been tested continuously for a year and a half. For each month, the JNWP Unit has computed the RMS errors in 36 hour 500 millibar wind forecasts over the U. S. and N. Atlantic. These errors have been compared with the RMS errors in corresponding wind forecasts prepared in the National Weather Analysis Center (NWAC) by conventional subjective methods. In a very few cases, the numerical 500 millibar forecasts were equalled or slightly excelled by NWAC's subjective forecasts. In the great majority of months, however, the barotropic 500 millibar forecasts have maintained a slight but statistically significant advantage - amounting to something like a 5% reduction in the RMS error of 36 hour wind forecasts.

But the significant fact is not merely that numerical methods may have gained some slight edge over conventional methods. Viewed historically, the important point is that, in a few short years, numerical methods have matched a human skill based on twenty years of experience with upper air forecasting. It should also be borne in mind that this standard was attained with the crudest of models, and that progress along a rational approach to prediction is directable. In short, when something goes wrong, we are not at a complete loss as to which way to turn next.

Forecasts based on the baroclinic models have been, on the whole, rather disappointing. At 500 millibars, they have been no better or slightly worse than the barotropic forecasts - not through any basic inferiority of the model itself, but because of the crudeness of initially geostrophic wind fields. Although these models exhibit the type of instability associated with cyclogenesis, they do not quantitatively predict the correct rate of development. As a result, the 1000 millibar forecasts are frequently quite poor. The forecasts of vertical motion, on the other hand, correspond surprisingly well with the observed patterns of continuous overcast and widespread precipitation. Thus, since the large-scale patterns of cloudiness and precipitation are probably more closely connected with the vertical motions than with the surface pressure pattern, the baroclinic forecasts may prove an extremely valuable aid to weather forecasting in the common or garden-variety sense.

By now, I expect that the questions foremost in your minds are: How far can numerical methods go, and what do they mean for me? And how are they to be absorbed into routine forecasting procedure? In the first place, it should be emphasized that the potentialities of existing theory are by no means exhausted. Our present difficulties in dealing with complicated types of baroclinic flow are very largely symptomatic of the necessity for a more sophisticated and realistic treatment of the problem, and the inclusion of effects that have previously been omitted, mainly for the sake of mathematical convenience - as, for instance, radiation, eddy heat conduction, and viscosity. For this reason alone, one should expect a continuous, but rather unspectacular improvement in the accuracy of upper air forecasts as numerical methods gradually progress toward greater generality.

At the same time, it should be pointed out that numerical methods are most effective in predicting phenomena for which there is a well-developed mathematical theory. Thus, although there is considerable hope of predicting condensation or formation of cloud, there is little immediate hope of accurately predicting precipitation - simply because this process involves several rather complicated physical mechanisms, none of which is particularly well understood nor for which there is an adequate mathematical theory. Much the same can be said of the predictability of phenomena that depend upon eddy diffusion, eddy heat conduction and eddy viscosity. Still another limitation is imposed by the lack of certain types of measurements - as, for example, radiation. Finally, it is appropriate to point out that nothing can be predicted in more detail than that with which it can be observed. Whether one predicts by numerical methods or by any other method it is impossible to forecast down to the last minute and mile.

To summarize the situation, we may expect that numerical methods will gradually lead to a significant improvement in general one, two and three day forecasts of the upper air flow, large-scale patterns of vertical motion and cloudiness, and the occurance of widespread precipitation. Within the foreseeable future, they will probably not succeed in predicting events on a small scale. In particular, our current state of ignorance does not permit us to apply numerical mthods to the prediction of highly localized precipation patterns.

It is by now obvious that numerical forecasts will probably differ from conventional area forecasts in only two important respects:

1) Increased information content, from the standpoints of geographical and vertical coverage, and number of quantities predicted. As examples of the latter, one can cite the vertical motion forecasts and related forecasts of widespread cloudiness and precipitation, and

2) Greater average reliability. Aside from these differences, however, numerical forecasts will be of the same type as the area forecasts now prepared by NWAC, and should be used in much the same way. It is our hope and conviction that numerical methods - by providing the general meteorological setting in which local influences operate to produce weather in the usual sense - will free the forecaster from routine chores of analysis and broad-scale forecasting, and will enable him to exploit more fully his experience with conditions and problems that are peculiar to his own area.

Finally, it should be apparent by now that the future evolution of numerical prediction is very closely bound to the continued advancement of computing technology. At the same time, this realization has opened our minds to several possibilities that are not directly connected with forecasting - in particular, the ever-expanding possibilities of high-speed data-processing and of providing a variety of specialized services. As examples of specialized forecasts that have already been computed from numerical forecasts, one might mention the sea height predictions that have been issued regularly by the JNWP Unit, and the machine-computed minimum flight paths that are being tested by the Air Weather Service. More and more, in fact, we are coming to think of numerical forecasting not merely as the application of modern computing techniques to short-range forecasting, but as the first step toward the automation of routine functions and a whole new way of meteorological life.

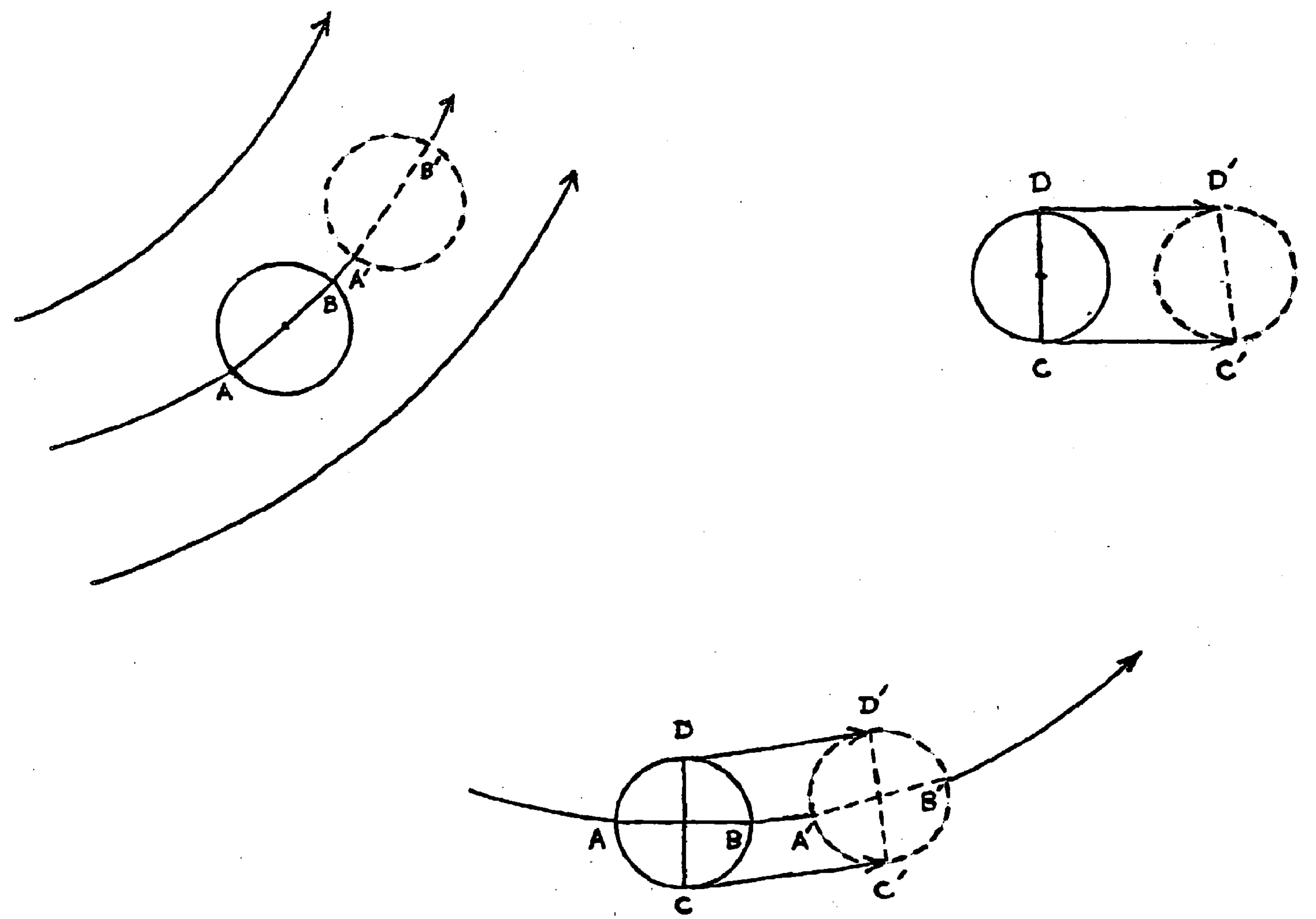

Figure 1

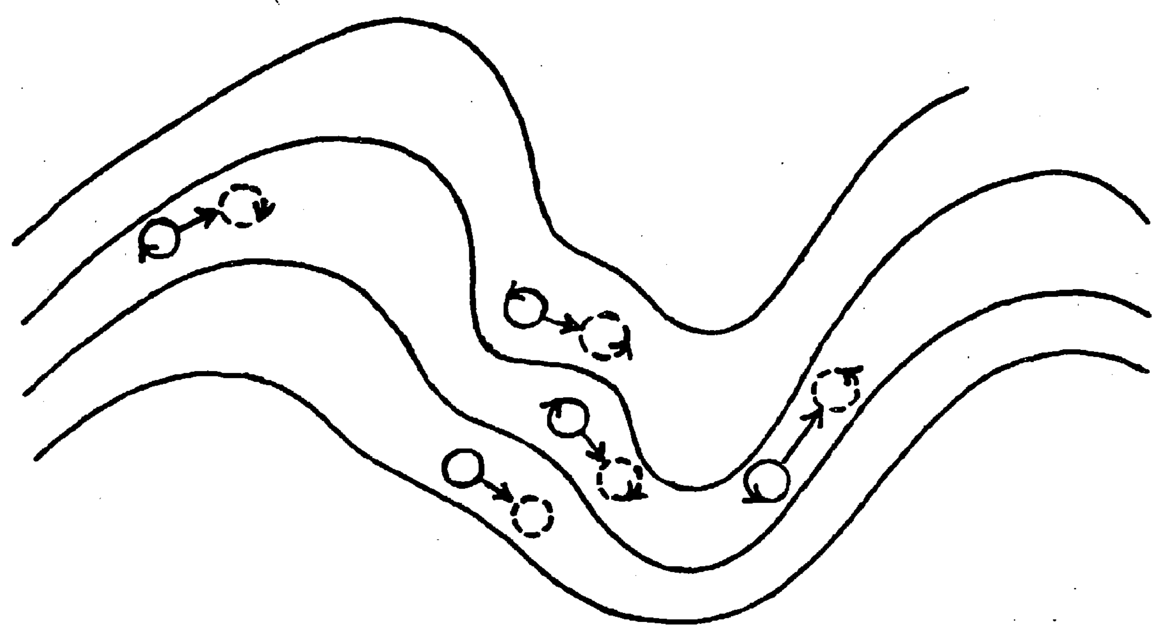

Figure 2.