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REPORT

OF THE

SUPERINTENDENT

OF THE

U. S. COAST AND GEODETIC SURVEY

FOR THE

FISCAL YEAR ENDING JUNE 30, 1893,

43

IN TWO PARTS.

PART II.

APPENDICES RELATING TO THE METHODS, DISCUSSIONS, AND RESULTS OF THE COAST AND GEODETIC SURVEY.

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Annual Report of the Superintendent of the Coast Survey

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U. S. COAST AND GEODETIC SURVEY REPORT FOR 1893.—PART II.

PREFATORY NOTE.

The text of this Report for the fiscal year 1892 has been arranged for publication in two parts, like that for the fiscal year preceding.

Part I, in quarto form, contains the historical portion, presenting reports of progress in the field and office operations of the Survey; estimates for future progress, and statements of expenditures during the fiscal year. Maps of general progress, and sketches showing localities of field work, exhibit graphically the advance of the Survey to June 30, 1893.

Part II, it will be observed, is in octavo, and includes the professional papers relating to the methods, discussions, and results of the Survey which have been approved for publication during the year. Such illustrations as are needed accompany them.

The octavo form is more convenient and suitable for the scientific and professional papers, while the quarto form appears to be demanded for the statistical matter and the progress sketches. Since the latter are of less general interest than the former, in the future distribution of the Report, Part II only will be sent, as it is believed that this will include all that is generally desired, and in a much more compact and convenient form than that of the old quarto.

In special cases, where both parts are desirable, they will be sent.

T. C. MENDENHALL,
Superintendent.

U. S. COAST AND GEODETIC SURVEY REPORT FOR 1893.—PART II.

TITLES OF APPENDICES.

ILLUSTRATIONS.

APPENDIX No. 1—1893.

STATE LAWS AUTHORIZING OFFICERS OF THE UNITED STATES COAST AND GEODETIC SURVEY TO ENTER UPON LANDS WITHIN STATE LIMITS FOR THE PURPOSES OF THE SURVEY.

LETTER OF TRANSMISSION.

UNITED STATES COAST AND GEODETIC SURVEY,
Washington, D. C., December 29, 1893.

SIR: In accordance with your suggestion that it is desirable to have published, as an appendix to the report for 1893, the laws of several of the States enacted for the protection of Coast Survey parties working therein, I have had the accompanying typewritten copies made, for the printer's use, of all such laws that we have any record of in this office. According to this record, laws have been passed in the following nineteen States: California, Connecticut, Georgia, Illinois, Indiana, Maine, Maryland, Massachusetts, Minnesota, Missouri, New Hampshire, New Jersey, Ohio, Oregon, South Carolina, Tennessee, Vermont, Virginia, and West Virginia.

In order to insure the accuracy of the manuscript copies of the laws on file in this office, I took them over to the Law Library of Congress and compared them with the printed laws on file there.

Finding that they were incomplete and full of inaccuracies, I carefully corrected them, and from the corrected text Miss Hein then made typewritten copies. These I have carefully compared, and now submit them (55 pages) as exact copies, following strictly the punctuation and capitalization of the printed laws.

Respectfully, yours,

GEORGE A. FAIRFIELD,
Assistant in Charge of State Surveys.

DR. T. C. MENDENHALL,
Superintendent Coast and Geodetic Survey.

STATE LAWS

FOR THE

PROTECTION OF COAST SURVEY PARTIES.

CALIFORNIA.

CHAPTER LXXV.

An Act to authorize persons engaged in the United States Coast Survey, upon the Coast of California, to enter on lands within this State, for the purpose of said Survey; to protect the operations of the same from injury and molestation; to ascertain the mode of assessing damages caused to any property in the progress of the same, and to provide for the punishment of offenders against the provisions of this Act, and for other purposes.

The People of the State of California, represented in Senate and Assembly, do enact as follows:

SEC. 1. That from and after the passing of this Act, any and every person employed under and by virtue of an Act of Congress of the United States, passed the tenth day of February, one thousand eight hundred and seven, and the supplements thereto concerning the United States Coast Survey, may enter upon Lands and clear and cut the timber, within this State, upon the same, and may erect any works, buildings, or appendages requisite for the purpose of exploring, surveying, triangulation, leveling, or doing any other act requisite to effect the object of said Act of Congress, without being considered as a trespasser: *Provided,* no unnecessary injury be done thereto.

SEC. 2. That if the parties interested—namely, party or parties representing the Government of the United States Coast Survey on the Coast of California, and the owners or possessors, of the Land so entered upon, and to which damage may have been done—cannot agree together upon the amount to be paid for the damages caused by doing any of the acts aforesaid, either of them may complain, in a summary manner, to the nearest Justice of the Peace for the District of the County where the damages may have been committed, who shall associate with himself two disinterested freeholders of the said County, one to be named by each party interested, who shall, upon hearing the parties, and with or without view of the premises, as they may determine, proceed to assess and award any damages which may have accrued to the owners or possessors of the Land so entered upon: *Provided,* nevertheless, that the party complaining as, aforesaid, shall serve upon the opposite party interested, ten days notice, in writing, of the time and place where said complaint is to be heard, and the name of the freeholder by him selected.

SEC. 3. That the said magistrate and freeholders shall, without unreasonable delay, file in the Office of the Clerk of the County Court of the County where the said complaint may have been heard, a report of their proceedings, which report shall be conclusive against the parties, and be evidence of their assent to the same; unless either of them shall, within ten days after filing of the said report, file a general or special objection to the same in the office of the said Clerk, of which the other party shall have notice; whereupon an issue shall be made up and a trial had at the next term of the County Court of said County, in the same manner in which civil cases are tried; except that the judgment shall be rendered and the damages assessed at the first term.

SEC. 4. That any person so entering upon Land, as aforesaid, for the purposes aforesaid, may tender to the party injured sufficient amends for any damages done upon said Land; and if, upon examination before the Justice of the Peace and freeholders as aforesaid, or upon trial before the County Court, the damages finally assessed shall not exceed the amount so tendered, the person who has so entered and tendered the amount, shall recover his costs.

SEC. 5. That the Justices of the Peace and Freeholders aforesaid, upon complaint made to them as aforesaid, and decision given, shall receive the same costs to which, by law, Justices of the Peace are entitled in a civil case from summons to judgment; and upon the trial in the County Court the costs shall be taxed by analogy to the Bill of Costs in said Court, established by law.

SEC. 6. That if any person or persons shall wilfully or wantonly injure, deface, or remove any instrument, signal, monument, building, or any appendage thereto, used or constructed in the State of California, under and by virtue of the Act of Congress aforesaid, he and they shall be liable to indictment for the same, under this Statute, for each and every offence, and upon conviction, shall be sentenced to pay a fine of two hundred dollars, one-half of which shall go to the prosecutor, and the remainder shall be appropriated according to the Laws of this State regulating the disposal of such fines, or shall be imprisoned not more than one month, or both, at the discretion of the Court before which such conviction shall take place, and he and they shall also be liable for all damages sustained by the United States of America, by reason of any such injury, defacement, or removal; to be recovered by action on the case in any Court of competent jurisdiction.

SEC. 7. This Act shall take effect from and after its passage

Approved April 2, 1852.

CONNECTICUT.

CHAPTER XI.

An Act relating to the Survey of the Coast of Connecticut.

Be it enacted by the Senate and House of Representatives in General Assembly convened:

SEC. 1. Persons employed under an act of the Congress of the United States, passed the tenth day of February, in the year eighteen hundred and seven, and the supplements thereto, may enter upon lands within this state, for any purpose which may be necessary to effect the objects of said act, and may erect works, stations, buildings, or appendages for that purpose, doing no unnecessary injury.

SEC. 2. If the parties interested cannot agree upon the amount to be paid for damages caused thereby, either of them may petition the county commissioners of the county in which the land lies, who shall appoint a time for a hearing, as soon as may be, and order at least fourteen days' notice to all persons interested, and, with or without a view of the premises, as they may determine, hear the parties and their witnesses and assess the damages.

SEC. 3. The county commissioners shall file in the office of the clerk of the superior court of the county in which the land is situated, a report of their doings, which shall be conclusive, unless one of the parties shall, within thirty days after the filing of such report, file a petition to said court for a new hearing to be had in such superior court; in which case, after such notice of such petition to the opposite party as the said superior court, if in session, or, if in vacation, as any judge thereof or of the supreme court, or any county commissioner of the county in which such petition is pending, shall direct, a trial shall be had in said court, in the same manner as other civil actions are tried, and such hearing shall take precedence of all other civil actions.

SEC. 4. The person so entering upon land may tender to the party injured amends therefor; and if the damages finally assessed do not exceed the amount tendered, the person entering shall recover costs; otherwise, the prevailing party shall recover costs.

SEC. 5. The costs to be taxed and allowed in all such cases, either before the county commissioners or the superior court, shall be the same as are ordinarily taxed, according to the rules and practice in the superior court.

SEC. 6. Whoever wilfully injures, defaces or removes any signal, monument, building or appendage thereto, erected, used or constructed, under said acts of Congress, shall forfeit the sum of fifty dollars for each offense; and shall be liable for damages sustained by the United States, to be recovered in an action of tort.

Approved June 5th, 1861.

GEORGIA.

[From "The Code of the State of Georgia, 1882."]

Part I—Title I—Chapter I.

ARTICLE III.

COAST SURVEY.

[Act of 1847, Code, p. 155.

§ 23. (23.) (25.) *Coast surveyors.*—Any person employed under the Act of the Congress of the United States, providing for a survey of the coasts, may enter upon lands and clear or cut timber within this State upon the same, for any purpose legitimately connected with, and requisite to effect, the said object: *Provided*, no unnecessary injury be done thereby, and all damages to the owner of the land be promptly paid.

[Act of 1847, Code, p. 155.]

§ 24. (24.) (26.) *Damage to land-owners.*—If the parties representing the Government of the United States, and the owner or possessor of the land so entered upon, cannot agree upon the amount to be paid for said damages, either party may complain in a summary manner to the nearest Justice of the Peace of the county in which the land lies, who shall associate with him two disinterested freeholders of the county—one to be named by each party interested—who shall, upon hearing the parties, and with or without view of the premises, as they may determine, proceed to assess and award the damages, if any: *Provided*, the party complaining shall give the opposite party ten days' notice, in writing, of the time and place when and where said complaint is to be heard, and the name of the freeholder by him selected.

[Act of 1847, Code, p. 156.]

§ 25. (25.) (27.) *Award and objections thereto.*—The said assessors, without unreasonable delay, shall file their award in the office of the Ordinary of the county, which shall be conclusive upon both parties, unless objections are filed to the same within ten days after the filing of the award. If objections are filed, the other party shall have written notice; whereupon an issue shall be made and tried at the first term thereafter of said Court, under the same rules as other civil cases.

[Act of 1847, Code, p. 156.]

§ 26. (26.) (28.) *Damages, tender of.*—The person so entering upon lands may tender such amount as he chooses for the damage done, and if the damages finally assessed shall not exceed the sum tendered, the party complaining shall pay all costs.

[Act of 1847, Code, p. 156.]

§ 27. (27.) (29.) *Costs.*—The costs before an Ordinary shall be the same as are allowed in civil cases in said Courts.

Part IV—Title I—Division XII.

[(a) Acts of 1865-6, p. 233.]

§ 4619. (4529.) (4483.) *Injuries to coast survey fixtures.*—Any person who shall willfully or wantonly injure, deface or remove any signal, monument, building, or any other appendage thereto, erected within this State by virtue of any Act of Congress authorizing a coast survey, shall be guilty of a misdemeanor, and, on conviction, [shall be punished as prescribed in section 4310 of this Code.] (*a*)

Part IV—Title I—Division III.

§ 4310. (4245.) (4209.) *Punishment of accessories after the fact.*—Accessories after the fact, except where it is otherwise ordered in this Code, shall be punished by a fine not to exceed one thousand dollars, imprisonment not to exceed six months, to work in the chain-gang on the public works, or on such other works as the county authorities may employ the chain-gang, not to exceed twelve months, and any one or more of these punishments may be ordered in the discretion of the Judge: *Provided*, that nothing herein contained shall authorize the giving the control of convicts to private persons, or their employment by the county authorities in such mechanical pursuits as will bring the products of their labor into competition with the products of free labor.

ILLINOIS.

SURVEY.

UNITED STATES COAST AND GEODETIC SURVEY.

An Act relating to the operations of the United States coast and geodetic survey.

SECTION 1. *Be it enacted by the People of the State of Illinois, represented in the General Assembly,* That any person employed under and by virtue of an act of congress of the United States, approved the tenth day of February, one thousand eight hundred and seven, and of the supplements thereto, for the survey of the coasts of the United States, or, under the direction of congress, to form a geodetic connection between the Atlantic and Pacific coasts, and to furnish triangulation points for state surveys, may enter upon lands within this state, for the purpose of exploring, triangulating, leveling, surveying, and of doing any other act which may be necessary to carry out the object of said laws, and may erect any works, stations, buildings and appendages requisite for that purpose, doing no unnecessary injury thereby.

§ 2. If the parties interested cannot agree upon the amount to be paid for damages caused thereby the United States of America may proceed to condemn said land, as provided by "An act to provide for the exercise of the right of eminent domain," approved April 10, 1872, in force July 1, 1872.

§ 3. If any person shall wilfully deface, injure or remove any signal, monument, building, or other property of the United States coast and geodetic survey, constructed or used under or by virtue of the act of congress aforesaid, he shall forfeit a sum not exceeding fifty dollars for each offense, and shall be liable for damages sustained by the United States, in an action on the case in any court of competent jurisdiction.

Approved April 21, 1881.

INDIANA.

Chapter XCV.

An Act relating to the operations of the United States Coast and Geodetic Survey in the State of Indiana, and declaring an emergency.

Section 1. *Be it enacted by the General Assembly of the State of Indiana*, That any person employed under and by virtue of an act of Congress of the United States, passed the tenth day of February, one thousand eight hundred and seven, and of the supplements thereto, for the survey of the coasts of the United States, or under the direction of Congress, to form a geodetic connection between the Atlantic and Pacific coasts, and to furnish triangulation points for State surveys, may enter upon lands within this State for the purpose of exploring, triangulating, leveling, surveying and doing any other act which may be necessary to carry out the objects of said laws, and may erect any works, stations, buildings and appendages requisite for that purpose, doing no unnecessary injury thereby.

Sec. 2. If the parties interested can not agree upon the amount to be paid for damages caused thereby, either of them may petition the Circuit Court in the county in which the land is situated, which Court shall appoint a time for a hearing as soon as may be, and order at least fourteen days notice to be given to all parties interested and with or without a view of the premises, as the Court may determine, hear the parties and their witnesses and assess damages.

Sec. 3. The person so entering upon land, may tender to the party injured, amends therefor, and if, in case of appeal to the Circuit Court, the damages finally assessed do not exceed the amount tendered, the person entering shall recover costs, otherwise the prevailing party shall recover costs.

Sec. 4. The costs to be allowed in all such cases shall be the same as allowed according to rules by the Court.

Sec. 5. If any person shall wilfully deface, injure, or remove, any signal, monument, building, or other property of the United States coast survey, constructed, or used under or by virtue of the acts of Congress, aforesaid, he shall forfeit a sum not exceeding fifty dollars, for each offense, and shall be liable for damages sustained by the United States in consequence of such defacing, injury, or removal, to be recovered in an action on the case in any Court of competent jurisdiction.

Sec. 6. Whereas an emergency exists for the immediate taking effect of this act, therefore, the same shall take effect and be in force from and after its passage.

Approved April 9, 1891.

STATE OF MAINE.

Chapter 181.

An Act relating to the survey of the coast of Maine.

Be it enacted by the Senate and House of Representatives in Legislature assembled, as follows:

Section 1. Any person employed under and by virtue of an act of the congress of the United States, passed the tenth day of February, one thousand eight hundred and seven, and the supplements thereto, may enter upon lands within this state for the purpose of exploring, surveying, triangulating, leveling and doing any other act which may be necessary to effect the objects of said act, and may erect any works, stations, buildings or appendages, requisite for that purpose, doing no unnecessary injury thereby.

SECT. 2. If the parties interested cannot agree upon the amount to be paid for the damages caused by doing any of the acts aforesaid, either of them may petition the commissioners of the county in which the land entered upon is situated, to hear the parties and assess any damages, which in the opinion of the commissioners has accrued to the owner or possessor of the land so entered upon.

SECT. 3. The commissioners as soon as may be, shall hear the parties either with or without a view of the premises, as the commissioners shall determine, and before any hearing shall be had, shall order notice to be given to all persons interested, at least fourteen days before the time of hearing.

SECT. 4. The commissioners shall file in the office of the clerk of the district court for said county, a report of these doings, which report shall be conclusive upon the parties unless one of them shall file within thirty days after the term of said court, which shall be held next after said report shall be filed, a petition to the said court that a trial shall be had in the case in said court, and after notice to the opposite party a trial shall be had in said court in the same manner in which other civil cases are there tried.

SECT. 5. The person so entering upon land as aforesaid, may tender to the party injured sufficient amends therefor, and if the damages finally assessed shall not exceed the amount so tendered, the person so entering shall recover his costs, and in all other cases the prevailing party shall recover his costs.

SECT. 6. In the taxation and allowance of costs in the district court upon a trial of the case, the proceedings of the said court shall hold the same relation to the report of the commissioners, as proceedings of the same court hold to judgments of justices of the peace, in cases of appeal from said judgments, and the costs shall be taxed accordingly.

SECT. 7. If any person shall wilfully injure, deface, or remove any signal, monument, building, or any appendage thereto, used and constructed under and by virtue of the act of congress aforesaid, he shall forfeit a sum not exceeding fifty dollars for each offense, to be recovered by indictment for the use of the person prosecuting, and shall also be liable for all damages sustained by the United States of America, to be recovered in an action on the case in any court of competent jurisdiction.

SECT. 8. This act shall take effect from and after its approval by the Governor.

Approved June 16, 1846.

AMENDMENT.

CHAPTER 125.

An Act to amend the second chapter of the revised statutes, relating to the coast survey.

Be it enacted by the Senate and House of Representatives, in Legislature assembled, as follows:

SECTION 1. The second chapter of the revised statutes is hereby amended by striking out the eighth section thereof, and inserting instead the following, viz:

SECT. 8. The person so entering upon land, may tender to the party injured sufficient amends therefor, and if the damages finally assessed do not exceed the tender, judgment shall be rendered against the owner for costs. The costs recovered by the prevailing party shall be taxed as in case of appeal from the judgment of a justice of the peace.

SECT. 2. This act shall take effect when approved by the governor.

Approved January 27, 1860.

MARYLAND.

An Act concerning the Survey of the Coast of Maryland.

SECTION 1. *Be it enacted by the General Assembly of Maryland*, That it shall and may be lawful for any person or persons employed under and by virtue of an act of the Congress of the United States, passed the tenth of February in the year eighteen hundred and seven, and of the supplement thereto, at any time hereafter to enter upon lands within this State for the purpose of exploring, surveying, triangulating or levelling or doing any other matter or thing which may be necessary to affect the objects of said act, and to erect any works, stations, buildings or appendages requisite for that purpose, doing no unnecessary injury to private or other property.

SEC. 2. *And be it enacted*, That in case the person or persons employed under the act of congress aforesaid, cannot agree with the owners or possessors of the land so entered upon and used as to the amount of damage done thereto by reason of the removal of fences, cutting of trees or injury to the crop or crops growing on the same, it shall and may be lawful for the said parties or either of them to apply to the chief justice for the time being or one of the associate judges of the judicial district in which such land may be situated, who shall thereupon appoint three disinterested and judicious freeholders, residents of the same judicial district, to proceed with as much despatch as possible to the examination of the matter in question, and the faithful assessment of the damages sustained by the owners or possessors aforesaid, and the said freeholders or a majority of them, having first taken and subscribed an oath or affirmation before the chief or associate justice aforesaid or other person duly authorized to administer the same, that they will well and truly examine and assess as aforesaid, and having given five days notice to both parties of the time of their meeting, shall proceed to the spot, and then and there upon their own view and if required upon the evidence of witnesses, (to be by them sworn or affirmed and examined) shall assess the said damages, and shall afterwards make report thereof and of their proceedings in writing under their hands and seals and file the same within five days thereafter in the office of the clerk of the county in which the land aforesaid is situated, subject to an appeal by either party to the county court of the said county within ten days after filing as aforesaid, and the said report so made as aforesaid, if no appeal as aforesaid be taken, shall be held to be final and conclusive as between the said parties, and the amount so assessed and reported shall be paid to the said owners or possessors of the land so damaged within twenty days after the filing of said report, and the said chief or associate justice as aforesaid, shall have authority to tax and allow upon the filing of said report, such costs, fees and expenses to the said freeholders for the performance of their duty as he shall think equitable and just, which allowance shall be paid by the person or persons employed under the act of congress aforesaid, within the time last above limited, but if an appeal as aforesaid be taken, the case shall be set down for hearing at the first term of county court aforesaid, ensuing upon and after said appeal, and it shall be lawful for either party immediately after the entry of such appeal, to take out summons for such witnesses as may be necessary to be examined upon the hearing aforesaid, and the said court shall have power in its discretion to award costs against which ever the final judgment shall be entered, and such appeal at the option of either party may and shall be heard before and the damages assessed by a jury of twelve men to be taken from the regular pannel and elected as in other cases.

SEC. 3. *And be it enacted*, That if any person or persons shall wilfully injure or deface or remove any signal, monument or building or any appendage thereto, erected, used or constructed under and by virtue of the act of congress aforesaid, such person or persons so offending shall severally forfeit and pay the sum of fifty dollars, with costs of suit to be sued for and recovered by any person who shall first prosecute the same before any justice of the peace of the county where the person so offending may reside, and shall also be liable to pay the amount of damages thereby sustained, to be recovered with costs of suit in an action on the case, in the name and for the use of the United States of America in any court of competent jurisdiction.

Passed March 9, 1842.

COMMONWEALTH OF MASSACHUSETTS.

CHAPTER 192.

An Act relating to the Survey of the Coast of Massachusetts.

Be it enacted by the Senate and House of Representatives, in General Court assembled, and by the authority of the same, as follows:

SECT. 1. Any person employed under and by virtue of an act of the Congress of the United States, passed the tenth day of February, in the year eighteen hundred and seven, and the supplement thereto, may enter upon lands within this State, for the purpose of exploring, surveying, triangulating, levelling, or doing any other act which may be necessary to effect the objects of said act, and may erect any works, stations, buildings or appendages, requisite for that purpose, doing no unnecessary injury thereby.

SECT. 2. If the parties interested cannot agree upon the amount to be paid for the damages caused by doing any of the acts aforesaid, either of them may petition the commissioners of the county in which the land entered upon is situated, to hear the parties and assess any damages which, in the opinion of the commissioners, has accrued to the owner or possessor of the land so entered upon.

SECT. 3. The commissioners, as soon as may be, shall hear the parties either with or without a view of the premises, as the commissioners shall determine, and before any hearing shall be had, shall order notice to be given to all persons interested, at least fourteen days before the time of hearing.

SECT. 4. The commissioners shall file in the office of the clerk of the court of common pleas for said county, a report of their doings, which report shall be conclusive upon the parties, unless one of them shall file, within thirty days after the term of said court, which shall be held next after said report shall be filed, a petition to the said court, that a trial be had in the case in said court; and after notice to the opposite party, a trial shall be had in said court, in the same manner in which other civil cases are there tried.

SECT. 5. The person so entering upon land as aforesaid, may tender to the party injured, sufficient amends therefor, and if the damages finally assessed shall not exceed the amount so tendered, the person so entering shall recover his costs; and, in all other cases, the prevailing party shall recover his costs.

SECT. 6. In the taxation and allowance of costs in the court of common pleas, upon a trial of the case, the proceedings of the said court shall hold the same relation to the report of the commissioners, as proceedings of the same court hold to judgments of justices of the peace, in cases of appeal from said judgments, and the costs shall be taxed accordingly.

SECT. 7. If any person shall wilfully injure, deface or remove any signal, monument, building, or any appendage thereto erected, used or constructed under and by virtue of the act of Congress aforesaid, he shall forfeit the sum of fifty dollars for each offence, to be recovered by indictment, to the use of the person prosecuting; and shall also be liable for all damages sustained by the United States of America, to be recovered in an action on the case, in any court of competent jurisdiction.

SECT. 8. This act shall take effect from and after its passage.

Approved by the Governor, March 25, 1845.

MINNESOTA.

CHAPTER 60.

[S. F. No. 219.]

An Act to provide for surveys authorized by Congress of the United States in the State of Minnesota.

Be it enacted by the Legislature of the State of Minnesota:

SECTION 1. Any person employed in the execution of any survey authorized by the congress of the United States may enter upon lands within this state for the purpose of exploring, triangulating, leveling, surveying, and of doing any work which may be necessary to carry out the objects of then existing laws relative to surveys, and may establish permanent station marks, and erect the necessary signals and temporary observatories, doing no unnecessary injury thereby.

SEC. 2. If the parties interested cannot agree upon the amount to be paid for damages caused thereby, either of them may petition the district court in the county in which the land is situated, which court shall appoint a time for a hearing as soon as may be, and order at least twenty (20) days' notice to be given to all parties interested, and, with or without a view of the premises, as the court may determine, hear the parties and their witnesses and assess damages.

SEC. 3. The person so entering upon land may tender to the injured party damages therefor, and if, in case of petition or complaint to the court, the damages finally assessed do not exceed the amount tendered, the person entering shall recover costs; otherwise, the prevailing party shall recover costs.

SEC. 4. The costs to be allowed in all such cases shall be the same as allowed according to the rules of the court, and provisions of law relating thereto.

SEC. 5. If any person shall wilfully deface, injure or remove any signal, monument, building or other property of the U. S. coast and geodetic survey, constructed or used under or by virtue of the act of congress aforesaid, he shall forfeit a sum not exceeding fifty (50) dollars for each offense, and shall be liable for damages sustained by the United States in consequence of such defacing, injury or removal, to be recovered in a civil action in any court of competent jurisdiction.

SEC. 6. This act shall take effect from and after its passage.

Approved April 2, 1889.

MISSOURI.

GEODETIC SURVEY.

An Act to provide for the protection of citizens of the State of Missouri, the interests of the United States, and persons engaged in the triangulation of the State of Missouri, under an act of Congress to form a geodetic connection between the Atlantic and Pacific Coasts.

Be it enacted by the General Assembly of the State of Missouri, as follows:

SECTION 1. Persons employed under an Act of Congress of the United States, passed the tenth day of February, 1807, and the supplement thereto, may, upon making satisfactory amends, enter upon lands within this State for any purpose which may be necessary to effect the object of said act, and may erect works, stations, buildings or appendages for that purpose, doing no unnecessary injury.

SECTION 2. If the parties interested cannot agree upon the amount to be paid for damages caused thereby, either of them may petition the County Court in the county in which the land is situated, which court shall appoint a time for a hearing as soon as may be, and order at least fourteen days' notice to be given to all persons interested, and, with or without a view of the premises, as the Court may determine, hear the parties and their witnesses and assess damages.

SECTION 3. The person so entering upon land may tender to the party injured amends therefor, and if, in case of appeal to the county court, the damages finally assessed do not exceed the amount tendered, the person entering shall recover costs; otherwise the prevailing party shall recover costs.

SECTION 4. The costs to be allowed in all such cases shall be the same as allowed according to the rules by the circuit court.

SECTION 5. Whosoever wilfully injures, defaces or removes any signal, monument, building or appendage thereto, erected, used or constructed under said acts of Congress, shall forfeit a sum not exceeding fifty dollars for each such offence, and shall be liable for damages sustained by the United States in consequence of such injuring, defacing or removing, to be recovered in an action before the circuit court of the county in which such offense is committed.

SECTION 6. Any party to the proceeding under the provisions of this act, who may feel aggrieved by the decision of any county court, may take an appeal to the circuit court, in the same term, in the same manner, and with like effect, as in other proceedings in the county courts of this State; *Provided*, that no appeal herein provided for shall prevent the continuation of the work referred to in this act.

SECTION 7. This act to take effect and be in force from and after its passage.

Approved March 9, 1872.

STATE OF NEW HAMPSHIRE.

CHAPTER 337.

An Act relating to the survey of the coast of New Hampshire.

SECTION 1. *Be it enacted by the Senate and House of Representatives in General Court convened*, That any person employed under and by virtue of an act of the congress of the United States, passed the 10th day of February, one thousand eight hundred and seven, and the supplements thereto, may enter upon lands within this state for the purpose of exploring, surveying, triangulating, levelling, or doing any other act which may be necessary to effect the objects of said acts, and may erect any works, buildings, stations or appendages requisite for that purpose, doing no unnecessary damage thereby.

SEC. 2. If the parties interested cannot agree upon the amount to be paid for the damages caused by doing any of the acts aforesaid, either of them may petition the court of common pleas for the county in which the land entered upon is situated, for an assessment of said damages, who shall refer the same to the road commissioners for such county, who shall hear the parties and make report, as in the case of assessing damages for land taken for highways, upon which the court shall render judgment as in other cases: *Provided*, that either of the parties dissatisfied with the amount of damages so assessed may appeal to the court of common pleas next to be holden in said county and not afterwards, and thereupon said court shall assess the damages of such party by a jury.

SEC. 3. The person so entering upon land as aforesaid may tender to the party injured sufficient amends therefor; and if the damages finally assessed shall not exceed the amount so tendered, the person so entering shall recover his costs, and in all cases the party prevailing shall recover his costs.

SEC. 4. If any person shall wilfully deface, injure or remove any signals, monuments, buildings, or any appendage thereto, used or constructed under and by virtue of the acts of congress aforesaid, he shall forfeit the sum of fifty dollars for each offence, to be recovered by indictment to the use of the party prosecuting, and shall also be liable for all damages sustained by the United States of America, to be recovered in an action on the case in any court of competent jurisdiction.

SEC. 5. This act shall take effect from and after its passage.

Approved, June 30, 1846.

CHAPTER XXIX.

An Act in co-operation with the United States Coast Survey, in the triangulation of the State.

Be it enacted by the Senate and House of Representatives in General Court convened:

SECTION 1. The acting assistant, in charge of the triangulation now being carried on in this state by the United States coast survey, is hereby authorized to set such signals as may be necessary to render this survey complete, and of the greatest service and benefit for future use in the construction of a map of the state, at an expense not exceeding twenty dollars in any town or city of the state, and to draw upon the state treasurer for the sums so expended.

SECT. 2. The state treasurer is hereby directed to pay out of any money in the treasury such expenses as may be incurred in carrying out the object named in the preceding section, the bills for the same having been previously approved by the governor.

SECT. 3. This act shall take effect on its passage.

Approved July 3, 1872.

NEW JERSEY.

ACTS OF THE SIXTY-FIFTH GENERAL ASSEMBLY OF THE STATE OF NEW JERSEY.

An Act concerning the survey of the coast of New Jersey.

SECTION 1. *Be it enacted by the Council and General Assembly of this State, and it is hereby enacted by the authority of the same,* That it shall and may be lawful for any person or persons, employed under and by virtue of the act of the Congress of the United States entitled, "An act to provide for surveying the coasts of the United States," passed the tenth day of February, in the year of our Lord eighteen hundred and seven, at any time hereafter to enter upon any lands within this state, for the purpose of exploring, surveying, or levelling, or doing any other matter or thing which may be necessary to effect the objects of the said act, and to erect any works, stations, buildings, and appendages necessary for that purpose, doing no unnecessary injury to private or other property.

SECTION 2. *And be it enacted,* That in case the person or persons so employed under the said act cannot agree with the owners or possessors of the said land so entered upon, for the use of the same, or upon the amount of the damage done thereto, it shall and may be lawful for the person or persons so employed, or the owners or possessors of the said lands, to apply to one of the justices of the supreme court of this state, who shall thereupon appoint three disinterested and judicious freeholders resident in the county wherein the said lands do lie, which said freeholders, having first severally taken and subscribed an oath or affirmation, before some person duly authorized to administer the same, faithfully to examine the matter in question, and assess the damages sustained by the owners or possessors of the lands so occupied, by reason of such occupation thereof, according to the best of their skill and understanding; and the said freeholders, or a majority of them, having given to the owners or possessors of the said lands, and to the person or persons so employed, five days' notice of the time and place of meeting, shall proceed upon the testimony of witnesses, to be by them sworn or affirmed and examined, or upon their own view, or both, to assess the said damages; and shall make report thereof in writing, under their hands and seals, and file the same within five days thereafter in the office of the clerk of the county in which the said lands do lie; which report, as between the said parties, shall be final and conclusive, and the amount so assessed and reported be paid to the said owners or possessors of the said lands within ten days after the filing of the said report; and upon default of such payment, any person or persons so entering upon the said lands shall forfeit all his or their right of entry given by this Act, and shall be taken and considered as guilty of trespass, in like manner as if this act had not been passed; and the said justice of the said supreme court shall, on application of either party, tax and allow such costs, fees, and expenses, to any person or persons performing any of the duties prescribed in this act, as he shall think equitable and just, which shall be paid by the person or persons employed under the said act, within the time above limited.

SECTION 3. *And be it enacted,* That, if any person or persons shall wilfully injure, deface, or remove any signal, station, monument, or building, or any appendage thereto erected, used, or constructed under the said act of the Congress of the United States, or under this act, such person or persons so offending shall severally forfeit and pay the sum of one hundred dollars, with costs of suit, to be sued for and recovered by any person who shall first sue for the same in any court having cognizance thereof; one half thereof for the use of the said prosecutor, and the other half thereof to be paid to the overseers of the poor of the township in which the offence was committed, for the use of the poor of said township, and shall be also liable to pay the amount of damages thereby sustained, to be recovered, with costs of suit, in an action on the case, in the name and for the use of the United States of America, in any court of competent jurisdiction.

SECTION 4. *And be it enacted,* That this act shall go into effect immediately after the passage thereof.

Passed March 11, 1841.

OHIO.

An Act relating to surveys authorized by the congress of the United States, in the state of Ohio.

SECTION 1. *Be it enacted by the General Assembly of the State of Ohio,* That any person employed in the execution of any survey authorized by the congress of the United States, may enter upon lands within this state for the purpose of exploring, triangulating, leveling, surveying, and of doing any work which may be necessary to carry out the objects of existing laws, and may establish permanent stations, marks, and erect the necessary signals and temporary observatories, doing no unnecessary injury thereby.

SEC. 2. If the parties interested cannot agree upon the amount to be paid for damages caused thereby, either of them may petition the probate court in the county in which the land is situated, which court shall appoint a time for a hearing as soon as may be, and order at least fourteen days' notice to be given to all parties interested, and with or without a view of the premises, as the court may determine, hear the parties and their witnesses, and assess damages.

SEC. 3. The person so entering upon land may tender to the injured parties damages therefor, and if, in case of appeal to the probate court, the damages finally assessed do not exceed the amount tendered, the persons entering shall recover costs; otherwise the prevailing party shall recover costs.

SEC. 4. The costs to be allowed in all such cases shall be the same as allowed according to the rules of the court.

SEC. 5. If any person shall wilfully deface, injure, or remove any signal, monument, building, or other property of the United States coast survey constructed or used under or by virtue of the acts of congress aforesaid, he shall forfeit a sum not exceeding fifty dollars for each offense, and shall be liable for damages sustained by the United States in consequence of such defacing, injury, or removal, to be recovered in an action in the case in any court of competent jurisdiction.

SEC. 6. This act shall take effect from and after its passage.

JAMES E. NEAL,
Speaker of the House of Representatives.
JAMES W. OWENS,
President pro tem. of the Senate.

Passed April 14, 1879.

OREGON.

An Act relating to Surveys Authorized by the Congress of the United States in the State of Oregon.

Be it enacted by the Legislative Assembly of the State of Oregon:

SECTION 1. Any person employed in the execution of any survey authorized by the congress of the United States may enter upon lands within this State for the purpose of exploring, triangulating, leveling, surveying, and of doing any work which may be necessary to carry out the objects of existing laws, and may establish permanent station marks and erect the necessary signals and temporary observatories, doing no unnecessary injury thereby, having first paid or tendered to the owner thereof the compensation or damages hereinafter prescribed.

SECTION 2. If the parties interested cannot agree upon the amount to be paid for damages caused thereby, either of them may petition the county court in the county in which the land is situated, which court shall appoint a time for a hearing as soon as may be, and order at least fourteen days' notice to be given to all parties interested and, with or without a view of the premises, as the court may determine, hear the parties and their witnesses and assess damages.

SECTION 3. The person so entering upon land may tender to the injured party damages therefor, and if in case of appeal to the county court the damages finally assessed do not exceed the amount tendered, the person entering shall recover costs; otherwise the prevailing party shall recover costs.

SECTION 4. The costs to be allowed in all such cases shall be the same as allowed according to the rules of the court.

SECTION 5. If any person shall wilfully deface, injure or remove any signal monument, building or other property of the U. S. coast survey, constructed or used under or by virtue of the Acts of congress aforesaid, he shall forfeit a sum not exceeding fifty dollars for each offense, and shall be liable for damages sustained by the United States in consequence of such defacing, injury or removal, to be recovered in an action on the case in any court of competent jurisdiction.

SECTION 6. Inasmuch as there is no law on this subject, this Act shall be in force from and after its approval by the Governor.

Approved February 25, 1889.

SOUTH CAROLINA.

An Act relating to the Survey of the Coast of South Carolina under the authority of the United States. No. 3021.

I. *Be it enacted, by the Senate and House of Representatives, now met and sitting in General Assembly, and by the authority of the same,* That any person employed under and by virtue of an Act of the Congress of the United States, passed the tenth day of February, in the year of our Lord one thousand eight hundred and seven, and the supplements thereto, may enter upon lands within this State, for the purpose of exploring, surveying, triangulation, leveling, or doing any other act which may be necessary to effect the object of the said Act of Congress, doing no unnecessary injury thereby, so that the dwelling house, yard, garden, graveyard, or ornamental trees, of any person be not invaded without his consent: *And provided,* that before such entry, the person so employed as aforesaid, shall enter into bond, with sufficient security, in such sum as may be agreed upon by and between the said persons so employed as aforesaid, and the owner of the said lands, conditioned to pay whatever damages may be done after such entry; and in case of disagreement of the parties as to the amount of the penalty of the bond, the same may be determined by any Judge of the Court of Common Pleas of this State in chambers or open court, upon application to him, after ten days' notice to the opposite party, which application may be supported or answered by affidavit.

II. If the parties interested cannot agree upon the amount to be paid for the damages caused by doing any of the acts aforesaid, either of them may petition the Court of Common Pleas for the district in which the damage has been done for the appointment of five commissioners, a majority of whom shall value and fix the amount of the said damage, either upon view or upon competent testimony, as the said commissioners may deem best. And the said commissioners, before they act, shall severally take an oath before some magistrate, faithfully and impartially to discharge the duty assigned them, and shall return their proceedings, with a full description of the damage done, under the hands and seal of a majority of them, to the Court from which the commission issued, there to remain of record.

III. In case either party shall appeal from the valuation of the damage so fixed by the said commissioners, or a majority of them, to the Court at its next sitting thereafter, and give fifteen days' notice to the opposite party, of such appeal, the Court shall order a new valuation to be made by a jury, who shall be charged therewith in the same term or as soon as practicable, and their verdict shall be final and conclusive between the parties, unless a new trial shall be granted.

IV. If any person shall wilfully and maliciously destroy, or in any manner hurt, damage, or obstruct, or shall wilfully and maliciously cause, or aid, or assist, or counsel, or advise any other person or persons to destroy or in any manner to hurt, damage, injure or obstruct any signal, monument, building, or any appendage thereto, used or constructed under and by virtue of the Act of Congress aforesaid, he shall be liable to be indicted therefor, and on conviction shall be imprisoned not more than one month, or pay a fine not exceeding fifty dollars, or both, at the discretion of the Court before which such conviction shall take place, and shall be further liable to pay all expenses of repairing the same, and it shall not be competent for any person so offending, to defend himself, by pleading or giving in evidence that he was the owner, or agent, or servant of the owner of the land where such damage was done or caused at the time the same was caused or done.

In the Senate House, the seventeenth day of December, in the year of our Lord one thousand, eight hundred and forty-eight,[1] and in the seventy-second year of the Sovereignty and Independence of the United States of America.

R. F. W. ALLSTON,
President of the Senate pro. tem.
W. F. COLCOCK,
Speaker of the House of Representatives.

TENNESSEE.

CHAPTER XXIV.

An Act relating to the United States Coast Survey in the State of Tennessee.

SECTION 1. *Be it enacted by the General Assembly of the State of Tennessee,* That any person employed under and by virtue of an Act of Congress of the United States, passed the tenth day of February, one thousand eight hundred and seven and of the supplements thereto, or under the direction of Congress to form a Geodetic connection between the Atlantic and Pacific coasts, and to furnish triangulation points for State Surveys, may enter upon such lands within this State for the purpose of exploring, triangulating, leveling, surveying and of doing any other act which may be necessary to carry out the objects of said laws, and may erect any works, stations, buildings and appendages requisite for that purpose, doing no unnecessary injury thereby.

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[1] Seven.

SEC. 2. *Be it further enacted,* That if the person or persons, over whose lands the survey has been made, or upon whose lands monuments, stations or buildings have been erected, or who has in any way sustained damage by such survey, cannot agree with the officer of the Coast Survey as to the damage sustained, the amount of such damage may be ascertained in the manner provided by Chapter II, of Title 8, Code of Tennessee, providing for taking private property for public uses.

SEC. 3. *Be it further enacted,* That if any person shall wilfully deface, injure or remove any signals, monuments, buildings or other property of the United States Coast Survey, constructed or used under or by virtue of the Acts of Congress aforesaid, he shall forfeit a sum not exceeding fifty dollars for each offense, and shall be liable for damages sustained by the United States in consequence of each defacing, injury or removal, in an action on the case in any Court of competent jurisdiction.

SEC. 4. *Be it further enacted,* That this Act shall take effect from and after its passage, the public welfare requiring it.

Passed March 17, 1877.

HUGH M. MCADOO,
Speaker of the Senate.
EDWIN T. TALIAFERRO,
Speaker of the House of Representatives.

Approved March 21, 1877.

JAMES D. PORTER, *Governor.*

VERMONT.

No. 251. An Act relating to the Survey of Lake Champlain, and to the operations connected therewith, in the State of Vermont.

It is hereby enacted by the General Assembly of the State of Vermont:

SEC. 1. Any person employed under and by virtue of an act of the Congress of the United States, passed the tenth day of February, one thousand eight hundred and seven, and the supplements thereto, may enter upon lands within this state for the purpose of exploring, surveying, triangulating, levelling, and doing any other act which may be necessary to effect the object of said act, or of the act of Congress passed the fifteenth day of July, one thousand eight hundred and seventy, and may erect any works, stations, buildings, and appendages requisite for that purpose, doing no unnecessary injury thereby.

SEC. 2. If the parties interested cannot agree upon the amount to be paid for the damages caused by doing any of the acts aforesaid, either of them may petition a judge of the county court of the county where such land is situated for the appointment of commissioners to appraise such damages; and such judge shall give reasonable notice to the parties interested of the time when and place where he will hear the parties in such petition; and such judge may appoint three judicious and disinterested persons commissioners to ascertain the damages to such land-owner. And such commissioners shall notify the parties interested, and shall proceed to ascertain and appraise the damages to the land-owner, and shall make a report thereof to the county court then next to be held in the same county; and said court may, for sufficient reasons, accept or reject said report, in whole or in part, and may render judgment in favor of the person interested in the land for such damages as it shall appear he has sustained, and may tax costs as said court shall judge just and equitable, and shall issue execution therefor.

SEC. 3. The person so entering upon the land and doing any of the acts aforesaid, may tender to the parties injured sufficient amends therefor, and if the damages finally assessed shall not exceed the amount so tendered, the person so entering shall recover his costs.

SEC. 4. If any person shall wilfully deface, injure, or remove any signals, monuments, buildings, or any appendage thereto, used and constructed under and by virtue of the acts of Congress aforesaid, he shall forfeit a sum not exceeding fifty dollars for each offense, to be recovered by indictment for the use of the party prosecuting, and shall also be liable for all damages sustained by the United States of America, to be recovered in an action on the case in any court of competent jurisdiction.

SEC. 5. This act shall take effect from and after its approval by the governor.

Approved November 8, 1870.

VIRGINIA.

LAWS OF VIRGINIA PASSED IN 1843-4—PAGE 65. CHAPTER 85.

§ 1 of Title I, Chap. 2. refers to "Places purchased by the United States for forts and other buildings."

§ 2. Any person employed under the act of congress providing for a survey of the coasts of the *United States* approved the tenth of February, eighteen hundred and seven, or under any act supplemental thereto, may, for the purpose of exploring, surveying, triangulating or leveling, to effect the objects of the first mentioned act, enter upon any lands within this state, remove the fences, cut down trees, or do any other matter or thing necessary to effect those objects.

§ 3. The damages sustained by removal of the fences, cutting of trees, injury to the crops, or otherwise, if the same be not agreed upon, shall be ascertained either on the application of the person so employed, or of the owner or possessor of the land, as follows, that is to say: notice shall be given by one of them to the other for ten days that at a certain time and place he will apply to a justice to appoint persons to assess the damages. Upon its being shewn to the justice at such time and place that such notice has been given, the justice shall appoint three intelligent, disinterested and impartial freeholders to make such assessment. They shall be duly sworn, and after giving five days' notice of the time of their meeting, both to the applicant and the other party, shall go upon the premises, and then and there, upon their own view and the evidence of such witnesses as may be adduced, to be by them sworn and examined, shall assess the damages.

§ 4. They shall make a report of their proceedings, under their hands, and file the same within five days thereafter in the office of the clerk of the court of the county wherein the land is situated.

§ 5. Within ten days after the same be filed, either party may file with the clerk a written notice stating that he appeals from the assessment to the county court.

§ 6. If no such notice be filed, the county court shall at the first term thereafter confirm the report, make a reasonable allowance to the freeholders for their services, and order payment to be made of the amount so assessed, of such allowance, of the officers' fees and of what the witnesses may be entitled to for their attendance.

§ 7. If such notice be filed, either party may thereupon take out subpoenas for witnesses; and at the first term at which the same can conveniently be done, the case shall be heard. If either party desire it, a jury may be impanelled to assess the damages; but if this be not asked, the court shall itself hear the witnesses and make such assessment as may seem to it proper. And the court shall give such directions in regard to the costs as it may deem right.

§ 8. If any person shall wilfully injure, deface or remove any signal, monument or building or any appendage thereof, erected, used or constructed under the act of congress aforesaid, such person shall forfeit fifty dollars to any person who shall sue for the same, and shall also be liable to the *United States* for the damages thereby sustained.

Code of Virginia, published in 1849, pp. 60, 61.

WEST VIRGINIA.

CHAPTER LXXXIV.

An Act concerning the United States Coast and Geodetic Survey in this State.

Be it enacted by the Legislature of West Virginia:

1. That it shall and may be lawful for any person or persons employed under and by virtue of an act of the Congress of the United States, passed February the tenth, one thousand eight hundred and seven, and all acts supplemental thereto, at any time hereafter to enter upon lands within this state for the purpose of exploring, surveying, triangulating or leveling, or doing any other matter or thing which may be necessary to effect the objects of said act; and to erect any works, stations, buildings or appendages requisite for that purpose, doing no unnecessary injury to private or other property.

2. That in case the person or persons employed under the act of Congress aforesaid, or acts supplemental thereto, cannot agree with the owners or possessors of the land so entered upon and used, as to the amount of damages done thereto by reason of the removal of fences, cutting of trees, or injury to the crop or crops growing on the same, it shall and may be lawful for the said parties, or either of them, to apply to the circuit court of the county to have the same condemned, and such application shall be proceeded in, tried and determined, in all respects, as provided in chapter forty two of the code of West Virginia.

3. That if any person or persons shall wilfully injure or deface or remove any signal, monument, or building, or any appendage thereto, erected, used or constructed under and by virtue of the act of congress aforesaid, or any act or acts supplemental thereto, such persons so offending shall severally forfeit and pay the sum of fifty dollars with the costs of suit, to be sued for and recovered by any person who shall first prosecute the same before any justice of the peace of the county where the person so offending may reside, and shall also be liable to pay the amount of damages thereby sustained, to be recovered with costs of suit in an action on the case, in the name and for the use of the United States of America, in any court of competent jurisdiction.

Passed March 14, 1881.

Approved March 16, 1881.

[Note by the Clerk of the House of Delegates.]

The foregoing act takes effect from its passage, two-thirds of the members elected to each House, by a vote taken by yeas and nays, having so directed.

APPENDIX No. 2—1893.

ON THE RESULTING HEIGHTS FROM GEODETIC LEVELING ALONG THE TRANSCONTINENTAL LINE OF LEVELS BETWEEN ST. LOUIS AND JEFFERSON CITY, MO., EXECUTED IN THE YEARS 1882 AND 1888, BY ANDREW BRAID AND GERSHOM BRADFORD, ASSISTANTS, AND ISAAC WINSTON, SUBASSISTANT.

Discussion and report by CHARLES A. SCHOTT, Assistant and Chief of the Computing Division.

Submitted for publication August 29, 1893.

The report which I have the honor to submit herewith gives the resulting heights from geodetic leveling along the transcontinental line of levels between St. Louis and Jefferson City, Mo., executed in the years 1882 and 1888 by Assistants Andrew Braid and Gershom Bradford.

In Appendix No. 11, Report for 1880, Assistant Braid explains the method of leveling then in use, viz: Two parallel lines were run simultaneously and in the same direction, one using (say) Staff E, the other Staff F, the rods being placed at slightly different distances from the intrument; *alternate parts* of the double line were run in opposite directions. On level ground or where the slope did not interfere, the average distance between the staves was 220 metres, the instrument being as near as may be midway between them. This method was afterwards found unsatisfactory and was superseded in 1885 and 1886 by the better one of running two *independent lines*, one forward, the other backward. The latter method was employed in 1888 by Assistant Bradford, who usually took the forward and Subassistant Winston the backward measures.

Route of levels.—The line starts from the Coast and Geodetic Survey bench mark J_3, as marked by a bronze plate on the western land pier of the Great Bridge across the Mississippi at St. Louis, and identical in level with bench mark K_3, known as the St. Louis Directrix, which is used by city surveyors and United States engineers. (For description see Appendix No. 11, C. and G. Survey Report for 1882, p. 556.) The line of 1882 follows the Missouri Pacific Railroad track to New Haven and a few miles beyond to Etlah, at which point it was taken up and carried, in 1888, along the same road to Moreau Creek (secondary bench mark XXV), a few miles east of Jefferson City. Total development of line of levels from St. Louis mark K_3 to temporary mark XXV, 194·5 kilometres, or 120·86 statute miles. (See illustration No. 1.)

Observers and dates of leveling.—Assistant A. Braid carried the line from St. Louis to Etlah between October 15 and December 6, 1882, and Assistant G. Bradford, aided by Subassistant I. Winston, extended it to the vicinity of Jefferson City between April 19 and June 30, 1888.

Instruments and rods.—Geodetic spirit level No. 1 was used by Assistant Braid; it is described and illustrated in Appendix No. 11, Report for 1880. The metric rods E and F are of the pattern shown on plate 23, Coast and Geodetic Survey Report for 1879, Appendix No. 15. Assistant Bradford used almost exclusively spirit level No. 2 and No. 3 on only four days; the rods A (A_1), B, C, D were used at one time or another.[1] Both instruments are described in Appendix No. 15, Report for 1879. The instrumental constants are as follows:

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[1] A_1 and B from April 19 to May 18; then C and D until May 28; then B and D until June 7; after that date A_1 and B, but from June 23 to June 30 C and D were again used, their broken thermometers having been replaced by new ones.

Geodetic Micrometer Level No. 1.	*Geodetic Micrometer Level No. 2.*	*Geodetic Micrometer Level No. 3.*
Aperture of telescope, $3{\cdot}5^{cm}$	$4{\cdot}3^{cm}$	$4{\cdot}3^{cm}$
Focal length of telescope, $40{\cdot}7^{cm}$	$41{\cdot}0^{cm}$	$41{\cdot}0^{cm}$
Magnifying power of telescope, 26	Used with low-power eyepiece, 25·6	Used with low-power eyepiece, 25·6
Value of 1 div. of striding level, $5''{\cdot}29$	$3''{\cdot}37$	Value of striding level, $4''{\cdot}48$ " " chambered level,† $2''{\cdot}73$
Determined by A. Braid, Apr. 25, 1879.	Determined by J. B. Weir, Apr. 2, 1887.	Determined by { I. Winston, Apr. 2 and 17, 1888. / Office determination, 1888.
Collar inequality, object-end large,* $2''{\cdot}74$	Object-end large, $0''{\cdot}25$ and $0''{\cdot}24$.	Object-end small, $0''{\cdot}03$ and $0''{\cdot}41$.
Determined by A. Braid, Dec. 9, 1882.	Determined by I. Winston, Apr. 18, July 10, 1888.	Determ'd by I. Winston, Apr. 18, 19, July 10, 1888.
Telemeter diaphragm of 3 horizontal spider lines.	Three equidistant telemeter threads.	Three equidistant telemeter threads.
Upper to middle thread, $16'\ 52''{\cdot}7$ Lower " " " $16'\ 35''{\cdot}3$	Angular distance, $16'\ 39''{\cdot}3$	Angular distance, $14'\ 00''{\cdot}7$
Value of 1 turn=100 divisions of microm., $443''{\cdot}1$ and $442''{\cdot}9$.	Value adopted,‡ $257''{\cdot}5$	Value adopted,‡ $257''{\cdot}5$
determined by O. H. Tittmann, Aug. and Sept., 1877, and A. Braid, May 21, 23, 1879.	determined by McGrath, Winston } in 1887.	determined by Tittmann, Winston, McGrath } in 1879–'80–'87.
Weight of instrument and stand, 10·4 kg.	20·4 kg.	20·4 kg.
Increasing t's and d's of microm. correspond to depressing object-end of telescope.	Increasing turns depress object-end of telescope.	

Rods E and F are each 3^m long.
The graduation of these rods is of standard length at 62°.1 and 66°·1 Fah. or 16°·7 and 18°·9 C.

Rods A_1, B, C, D are each 3^m long (see App. No. 9, Rep. for 1887).

The graduation of A_1 is standard at	67°·0 Fah. or	19°·4 C.
B " " "	71 ·7 "	" 22 ·1 "
C " " "	68 ·4 "	" 20 ·3 "
D " " "	58 ·1 "	" 14 ·4 "

Coefficient of expansion of brass for { Fahrenheit scale, 0·000010 / Centigrade " 0·000018

Index corr. of E (Oct., 1883), $64{\cdot}0^{mm}$
F ("), $61{\cdot}0^{mm}$

Index corr. of A_1 (in Apr, and July, 1888), $77{\cdot}2^{mm}$
B (" " " " "), $77{\cdot}0^{mm}$
C (" May " July, "), $60{\cdot}4^{mm}$
D (" " " " "), $60{\cdot}0^{mm}$

* It was but 1″.01 in the period April, 1881, to June, 1882, as computed by H. Farquhar from records by A. Braid.
† Used when determining collar inequality in July, 1888, the tube of striding level broken.
‡ For particulars, see Appendix No. 14, Report for 1887, p. 276.

N. B.—This difference between the terminal point of the rods and the zero of the "brass scale" does not ordinarily come into consideration. None of these rods have undergone any change since their construction except that due to an accident to rod A in August, 1881, and that due to wear of supporting surface. Comparisons for lengths of rods C and D were made by J. J. Clark, September 21, 1880, and August 30, 1882, and computed by H. Farquhar.

Method of observing.—As already stated, the method employed for the part of the line between St. Louis and Etlah was that of running simultaneously two parallel lines, but this was changed for the remainder of the line to the better practice of running two independent lines—one forward, the other backward. In the latter work, before taking the micrometer reading for "horizon," the bubble of the level was always brought to the center of the scale.

Computations.—The field computation was made by the observer, and the office computation of the 1882 work by Subassistant J. F. Pratt and Mr. H. Farquhar, with results drawn up by Mr. A. S. Christie.[1] The observations of 1888 were reduced by the observers, and the office computation was made by Mr. F. M. Little in November, 1888, and completed by Subassistant J. Nelson, in April, 1893. The usual corrections were made for micrometric difference when pointing to horizon and to target of staff; for effect of collar inequality; for curvature and refraction; for length of staff at various temperatures, and for index error where necessary.

Results.—They are given here in the usual tabular form, but instead of starting from the sea level the results are given differentially with respect to the St. Louis bench mark K_3. Its height above the ocean is at present not known with precision, but the value given in the Annual Report for 1882, page 554, appears too high, to judge from the two independent lines of levels now extending to the Gulf. Until the fieldwork is completed, and if temporarily approximate results of the bench marks west of St. Louis be required, we may take for the height of this mark 126 metres, or 413·4 feet, nearly.

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[1] Results reported by me, August 25, 1883.

Results from geodetic spirit leveling in Missouri—First part from St. Louis to New Haven (and Etlah), 1882.

Date, 1882.		Bench mark. From—	Bench mark. To—	Distance between successive bench marks.	Distance from initial mark K_3.	Difference of height between bench marks. E or first line.	Difference of height between bench marks. F or second line.	Difference of height between bench marks. Mean.	Discrepancy. Partial E—F.	Discrepancy. Total accumulated.	Height of mark above St. Louis bench mark K_3.
				km.	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
			K_3	-----	0.000	-------	-------	-------	---	0.0	0.0000
Oct.	15	K_3	181	0.372	0.372	+13.5064	+13.5061	+13.5062	+0.3	+ 0.3	+13.5062
	22	181	187	2.059	2.431	— 3.1293	— 3.1224	— 3.1259	—6.9	— 6.6	10.3803
	22	187	188	0.885	3.316	+18.9172	+18.9179	+18.9175	—0.7	— 7.3	29.2978
	25	188	189	1.339	4.655	—15.7988	—15.7945	—15.7966	—4.3	—11.6	13.5012
	25	189	190	2.562	7.217	+ 9.7236	+ 9.7250	+ 9.7243	—1.4	—13.0	23.2255
Nov.	2	190	191	2.849	10.066	—10.9293	—10.9311	—10.9302	+1.8	—11.2	12.2953
	2	191	192	3.954	14.020	+ 7.1345	+ 7.1342	+ 7.1344	+0.3	—10.9	19.4297
	3	192	193	0.952	14.972	— 5.7505	— 5.7473	— 5.7489	—3.2	—14.1	13.6808
	3	193	194	2.101	17.073	+17.1256	+17.1265	+17.1260	—0.9	—15.0	30.8068
	3	194	195	2.012	19.085	+15.2657	+15.2582	+15.2620	+7.5	— 7.5	46.0688
	3	195	196	1.153	20.238	+10.8348	+10.8342	+10.8345	+0.6	— 6.9	56.9033
	4	196	197	2.336	22.574	+10.5820	+10.5790	+10.5805	+3.0	— 3.9	67.4838
	4	197	198	0.928	23.502	— 0.4905	— 0.4892	— 0.4898	—1.3	— 5.2	66.9940
	9	198	204	2.196	25.698	—17.8612	—17.8662	—17.8637	+5.0	— 0.2	49.1303
	9	204	203	3.634	29.332	—29.1377	—29.1404	—29.1390	+2.7	+ 2.5	19.9913
	7	203	202	1.102	30.434	— 9.1403	— 9.1363	— 9.1383	—4.0	— 1.5	10.8530
	7	202	201	1.680	32.114	— 6.8468	— 6.8444	— 6.8456	—2.4	— 3.9	4.0074
	7	201	200	2.134	34.248	+ 0.1142	+ 0.1181	+ 0.1162	—3.9	— 7.8	4.1236
	7	200	199	2.217	36.465	— 0.1627	— 0.1669	— 0.1648	+4.2	— 3.6	3.9588
	13	199	205	1.907	38.372	+ 1.1323	+ 1.1364	+ 1.1344	—4.1	— 7.7	5.0932
	13	205	206	2.063	40.435	+ 3.1798	+ 3.1756	+ 3.1777	+4.2	— 3.5	8.2709
	13	206	X	1.979	42.414	— 2.4040	— 2.3983	— 2.4012	—5.7	— 9.2	5.8697
	13	X	207	2.141	44.555	+ 1.2115	+ 1.2148	+ 1.2132	—3.3	—12.5	7.0829
	14	207	208	1.827	46.382	+ 0.9690	+ 0.9705	+ 0.9698	—1.5	—14.0	8.0527

Results from geodetic spirit leveling in Missouri—First part from St. Louis to New Haven (and Etlah), 1882—Continued.

Date, 1882.		Bench mark. From—	Bench mark. To—	Distance between successive bench marks.	Distance from initial mark K_3.	Difference of height between bench marks. E or first line.	Difference of height between bench marks. F or second line.	Difference of height between bench marks. Mean.	Discrepancy. Partial E—F.	Discrepancy. Total accumulated.	Height of mark above St. Louis bench mark K_3.
				km.	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
Nov.	14	208	209	1·830	48·212	+ 3·9695	+ 3·9629	+ 3·9662	+6·6	— 7·4	12·0189
	14	209	210	1·712	49·924	+ 3·4154	+ 3·4143	+ 3·4148	+1·1	— 6·3	15·4337
	18	210	215	2·177	52·101	+ 5·4122	+ 5·4089	+ 5·4105	+3·3	— 3·0	20·8442
	18	215	216	2·325	54·426	+ 6·0574	+ 6·0593	+ 6·0583	—1·9	— 4·9	26·9025
	18	216	XI	0·925	55·351	— 6·7483	— 6·7480	— 6·7482	—0·3	— 5·2	20·1543
	18	XI	217	2·655	58·006	— 4·7312	— 4·7287	— 4·7299	—2·5	— 7·7	15·4244
	18	217	218	0·640	58·646	— 0·8689	— 0·8680	— 0·8685	—0·9	— 8·6	14·5559
	21	218	224	2·428	61·074	+ 1·1732	+ 1·1719	+ 1·1725	+1·3	— 7·3	15·7284
	21	224	223	1·573	62·647	+11·7666	+11·7637	+11·7652	+2·9	— 4·4	27·4936
	21	223	222	2·045	64·692	+14·6175	+14·6112	+14.6144	+6·3	+ 1·9	42·1080
	20	222	221	0·217	64·909	+ 1·5192	+ 1·5185	+ 1·5188	+0·7	+ 2·6	43·6268
	20	221	220	3·051	67·960	+23·9667	+23·9673	+23·9670	—0·6	+ 2·0	67·5938
	20	220	219	1·890	69·850	—14·3620	—14·3641	—14·3631	+2·1	+ 4·1	53·2307
	20	219	214	1·928	71·778	—15·4343	—15·4353	—15·4348	+1·0	+ 5·1	37·7959
	16	214	213	1·569	73·347	—12·9388	—12·9412	—12·9400	—2·4	+ 7·5	24·8559
	15	213	212	2·476	75·823	— 3·2351	— 3·2341	— 3·2346	—1·0	+ 6·5	21·6213
	15	212	211	3·309	79·132	— 0·4045	— 0·3990	— 0·4017	—5·5	+ 1·0	21·2196
	22	211	225	2·329	81·461	+ 1·5481	+ 1·5503	+ 1·5492	—2·2	— 1·2	22·7688
	22	225	226	2·450	83·911	— 0·5850	— 0·5792	— 0·5821	—5·8	— 7·0	22·1867
	22	226	227	1·943	85·854	+ 0·8756	+ 0·8736	+ 0·8746	+2·0	— 5·0	23·0613
	23	227	XII	0·861	86·715	+ 0·2358	+ 0·2395	+ 0·2377	—3·7	— 8·7	23·2990
	23	XII	228	2·026	88·741	— 0·1652	— 0·1704	— 0·1678	+5·2	— 3·5	23·1312
	23	228	229	1·305	90·046	+ 1·1348	+ 1·1383	+ 1·1365	—3·5	— 7·0	24·2677
	25	229	L_3	0·451	90·497	+15·9322	+15·9332	+15·9327	—1·0	— 8·0	40·2004

	29	229	237	1·265	91·311	− 1·2030	− 1·2052	− 1·2041	+2·2	− 4·8	23·0636
	29	237	236	2·416	93·727	+ 1·4947	+ 1·4987	+ 1·4967	−4·0	− 8·8	24·5603
	29	236	235	2·057	95·784	− 0·4924	− 0·4891	− 0·4908	−3·3	−12·1	24·0695
	28	235	234	1·869	97·653	+ 1·4992	+ 1·4960	+ 1·4976	+3·2	− 8·9	25·5671
	28	234	233	2·079	99·732	+ 1·0356	+ 1·0373	+ 1·0365	−1·7	−10·6	26·6036
	28	233	232	2·025	101·757	− 0·4750	− 0·4763	− 0·4756	+1·3	− 9·3	26·1280
	27	232	231	2·485	104·242	+ 0·9905	+ 0.9834	+ 0·9869	+7·1	− 2·2	27·1149
	27	231	230	3·095	107·337	+ 0·4680	+ 0·4674	+ 0·4677	+0·6	− 1·6	27·5826
Dec.	4	230	238	2·040	109·377	− 0·3351	− 0·3330	− 0·3341	−2·1	− 3·7	27·2485
	4	238	M_3	1·103	110·480	+ 1·9052	+ 1·9015	+ 1·9034	+3·7	0·0	29·1519
	4	M_3	XIII	0·129	110·609	− 0·8890	− 0·8887	− 0·8889	−0·3	− 0·3	28·2630
	4	XIII	239	1·080	111·689	+ 0·6239	+ 0·6260	+ 0·6250	−2·1	− 2·4	28·8880
	6	239	240	2·126	113·815	+ 0·1278	+ 0·1268	+ 0·1273	+1·0	− 1·4	29·0153
	6	240	XIV	2·347	116·162	+ 0·0659	+ 0·0674	+ 0·0666	−1·5	− 2·9	29·0819

Description of primary and secondary bench marks between St. Louis, Mo., and Etlah, Mo.

K_3.—This mark is known at St. Louis as the "City Directrix." It has been in use for many years in connection with the levels of the city. It was originally the top surface of a pedestal of a monument which stood on Front street, near Market. The monument shaft was destroyed at the time of the great fire in that locality, but the pedestal remained. It is now (1882) level with the curbstone and forms a part thereof. A T mark has since been cut to indicate the point used for a bench mark. The large bronze-plate bench marks I_3, on south face of the eastern land pier of the Great Bridge at East St. Louis, Ill., and J_3, on the western land pier of the bridge, were placed, as near as possible, on the same level with the City Directrix mark K_3. (See C. and G. Survey Report for 1882, p. 554; also report of the Miss. River Commission for 1883.)

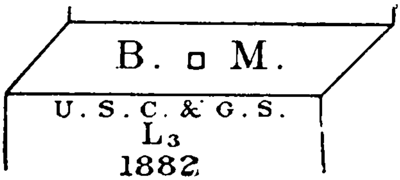


FIG. 1.

Secondary B. M. X.—Is cut on the upper surface of the middle top stone of the south side of the east abutment of railroad bridge (Missouri Pacific) at St. Paul, Mo. It is marked thus: B. □ M.

Secondary B. M. XI.—Is cut on top of the south side of the west abutment of the Missouri Pacific Railroad bridge at Allenton, Mo. It is marked thus: B □ M.

U. S. C. & G. S.
B. ✣ M.
M_3
1882

FIG. 2.

Secondary B. M. XII.—Is a cross on the head of a copper bolt inserted in the face of a perpendicular rocky bluff about three-eighths of a mile west of South Point Station (Mo. Pac. R. R.). The bolt was inserted by the United States engineers at work on improvement of Missouri River.

Primary B. M. L_3.—Is cut on the horizontal surface of the stone ledge under the windows of the east face of the German Catholic church at Washington, Mo. It is marked as shown in fig. 1.

Primary B. M. M_3.—Is cut on the northeast corner of the building occupied by the "New Haven Merchandise Company," at New Haven, Mo. The building stands a short distance south of the Missouri Pacific Railroad track and west of the railroad station. The B. M. is marked as shown in fig. 2.

Secondary B. M. XIII.—Is cut on the north side of the east abutment of railroad culvert (Mo. Pac. R. R.) about one-eighth mile west of New Haven, Mo. It is marked thus: B. ▫ M.

Secondary B. M. XIV.—Is cut on the top surface of the north end of the east abutment of a small railroad bridge or culvert (Mo. Pac. R. R.) about one-fourth mile east of Etlah Station, Mo. It is

U. S.
marked thus: B. ▫ M.
XIV.

(See route diagram.)

Results from geodetic spirit leveling in Missouri—Second part from New Haven to vicinity of Jefferson City, 1888.

Date, 1888.		Bench mark. From—	Bench mark. To—	Distance between successive bench marks.	Distance from initial mark K_3.	Difference of height between bench marks. Forward measure.	Difference of height between bench marks. Backward measure.	Difference of height between bench marks. Mean.	Discrepancy. Partial F—B.	Discrepancy. Total accumulated.	Height of mark above St. Louis mark K_3.
				km.	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
1882.			M_3	-----	110·480				---	---	+29·1519
Dec. 4	Dec. 4	M_3	XIII	0·129	110·609	—0·8890	—0·8887	—0·8889	---	---	28·2630
4	4	XIII	239	1·080	111·689	+0·6239	+0·6260	+0·6250	---	---	28·8880
6	6	239	240	2·126	113·815	+0·1278	+0·1268	+0·1273	---	---	29·0153
6	6	240	XIV	2·347	116·162	+0·0659	+0·0674	+0·0666	---	---	29·0819
1888.			M_3	-----	110·480				---	0·0	+29·1519
Apr. 19	May 2	M_3	1	0·521	111·001	—0·6548	—0·6541	—0·6544	—0·7	— 0·7	28·4975
19	1	1	2	0·956	111·957	+0·3198	+0·3169	+0·3184	+2·9	+ 2·2	28·8159
19	1	2	3	0·831	112·788	+0·2576	+0·2548	+0·2562	+2·8	+ 5·0	29·0721
20	1	3	4	0·505	113·293	—0·0502	—0·0528	—0·0515	+2·6	+ 7·6	29·0206
20	1	4	5	0·724	114·017	—0·8910	—0·8885	—0·8898	—2·5	+ 5·1	28·1308
20	1	5	6	0·830	114·847	+0·4366	+0·4375	+0·4370	—0·9	+ 4·2	28·5678
Apr. 21	Apr. 30	6	7	0·759	115·609	+0·0287	+0·0227	+0·0268	+2·6	+ 6·8	28·5946
May 2	May 2			0·766		+0·0275	+0·0283				
Apr. 21	Apr. 30	7	XIV	0·490	116·099	+0·4901	+0·4891	+0.4896	+1·0	+ 7·8	29·0842
	Mean		XIV	-----	116·131					+ 7·8	29·0830
Apr. 21	Apr. 30	XIV	8	0·744	116·875	—0·0784	—0·0746	—0·0765	—3·8	+ 4·0	29·0065
23	30	8	9	0·662	117·537	—0·1600	—0·1607	—0·1604	+0·7	+ 4·7	28·8461
23	30	9	10	0·858	118·395	+0·1742	+0·1779	+0·1760	—3·7	+ 1·0	29·0221
24	28	10	11	0·782	119·177	—0·6261	—0·6286	—0·6274	+2·5	+ 3·5	28·3947
24	28	11	12	0·911	120·088	+0·4952	+0·4950	+0·4951	+0·2	+ 3·7	28·8898
25	28	12	13	0·818	120·906	+0·4874	+0·4837	+0·4856	+3·7	+ 7·4	29·3754
25	27	13	14	0·936	121·842	—0·3931	—0·3926	—0·3928	—0·5	+ 6·9	28·9826
26	27	14	15	0·782	122·624	—0·0657	—0·0664	—0·0660	+0·7	+ 7·6	28·9166

May 26 3 10	*May 27 9 10*	*15*	*16*	*0·834 0·831 0·835*	*123 457*	*—0·1302 —0·1293 —0·1232*	*—0·1247 —0·1217 —0·1229*	—0·1254	—4·5	+ 3·1	*28·7912*
Apr. 26	Apr. 26	16	XV	0·114	123·571	+1·0344	+1·0339	+1·0342	+0·5	+ 3·6	29·8254
May 4	May 9	16	17	0·927	124·384	+0·3828	+0·3825	+0·3826	+0·3	+ 3·4	29·1738
4	9	17	18	0·985	125·369	+0·3205	+0·3186	+0·3196	+1·9	+ 5·3	29·4934
4	9	18	19	0·982	126·351	—0·2762	—0·2740	—0·2751	—2·2	+ 3·1	29·2183
5 10	9 10	19	20	0·693 0·692	127·043	+0·7521 +0·7564	+0·7561 +0·7549	+0·7548	—1·3	+ 1·8	29·9731
5	9	20	21	0·805	127·848	—0·1655	—0·1630	—0·1642	—2·5	— 0·7	29·8089
5	8	21	22	1·028	128·876	+0·4504	+0·4515	+0·4510	—1·1	— 1·8	30·2599
5	8	22	23	1·106	129·982	—0·0178	—0·0196	—0·0187	+1·8	0·0	30·2412
7	8	23	24	1·038	131·020	+0·1218	+0·1210	+0·1214	+0·8	+ 0·8	30·3626
7	8	24	25	1·062	132·082	+0·4491	+0·4536	+0·4513	—4·5	— 3·7	30·8139
7	6	25	26	0·852	132·934	—0·0794	—0·0802	—0·0798	+0·8	— 2·9	30·7341
7	7	26	N_3	0·052	132·986	+1·4628	+1·4632	+1·4630	—0·4	— 3·3	32·1971
7	12	26	27	0·677	133·611	—0·1193	—0·1209	—0·1201	+1·6	— 1·3	30·6140
11	12	27	28	0·968	134·579	+0·9055	+0·9081	+0·9068	—2·6	— 3·9	31·5208
11	12	28	29	0·896	135·475	+0·3169	+0·3180	+0·3174	—1·1	— 5·0	31·8382
11	12	29	30	0·934	136·409	+0·1725	+0·1769	+0·1747	—4·4	— 9·4	32·0129
11 14	12 14	30	31	1·066 1·066	137·475	+0·5562 +0·5644	+0·5663 +0·5644	+0·5628	—5·1	—14·5	32·5757
11, 14	15	31	XVI	1·052	138·527	+0·6489	+0·6450	+0·6470	+3·9	—10·6	33·2227
14	15	XVI	32	0·998	139·525	+0·3731	+0·3766	+0·3748	—3·5	—14·1	33·5975
14 June 8	25 June 8	32	33	0·944 0·944	140·469	—0·2549 —0·2593	—0·2633 —0·2581	—0·2589	+3·6	—10·5	33·3386
May 16 25	May 23 25	33	34	1·184 1·184	141·653	+0·8881 +0·8815	+0·8744 +0·8849	+0·8822	+5·2	— 5·3	34·2208
16	23	34	35	1·210	142·863	—0·6930	—0·6888	—0·6909	—4·2	— 9·5	33·5299
16 25	23 25	35	36	0·995 1·004	143·863	+0·0598 +0·0670	+0·0693 +0·0661	+0·0656	—4·3	—13·8	33·5955
16	23	36	XVII	0·656	144·519	—0·1385	—0·1373	—0·1379	—1·2	—15·0	33·4576

Results from geodetic spirit leveling in Missouri—Second part from New Haven to vicinity of Jefferson City, 1888—Continued.

Date, 1888.		Bench mark. From—	Bench mark. To—	Distance between successive bench marks.	Distance from initial mark K_3.	Difference of height between bench marks. Forward measure.	Backward measure.	Mean.	Discrepancy. Partial F—B.	Total accumulated.	Height of mark above St. Louis mark K_2.
1888.				*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
May 16, 18	May 22, 23	XVII	XVIII	0·583	145·102	+0·9390	+0·9389	+0·9390	+0·1	−14·9	34·3966
18	22, 23	XVIII	37	1·144	146·246	−0·8794	−0·8799	−0·8796	+0·5	−14·4	33·5170
18	22	37	38	1·492	147·740	−0·1412	−0·1331	−0·1362	−0·9	−15·3	33·3808
31	June 2			1·495		−0·1321	−0·1383				
May 18	May 22	38	39	1·566	149·305	+0·5681	+0·5816	+0·5774	−2·8	−18·1	33·9582
31	May 31, June 2			1·564		+0·5775	+0·5786				
June 4	June 4			1·564		+0·5824	+0·5763				
May 18	May 21	39	40	1·553	150·858	+0·2375	+0·2436	+0·2406	−6·1	−24·2	34·1988
18, 19	21	40	XIX	1·278	152·136	+0·3278	+0·3275	+0·3276	+0·3	−23·9	34·5264
19	21	XIX	41	0·870	153·006	−0·2010	−0·1991	−0·2000	−1·9	−25·8	34·3264
19	21	41	42	0·914	153·919	−0·2624	−0·2522	−0·2564	−4·9	−30·7	34·0700
24	June 1			0·914		−0·2595	−0·2565				
June 4	4			0·911		−0·2547	−0·2534				
May 19	May 21	42	43	0·974	154·893	+1·0060	+1·0090	+1·0075	−3·0	−33·7	35·0775
19, 24	June 1	43	44	1·211	156·104	+1·1416	+1·1437	+1·1426	−2·1	−35·8	36·2201
May 24	1	44	45	1·364	157·468	+0·0470	+0·0455	+0·0462	+1·5	−34·3	36·2663
24	May 29, June 1	45	46	1·082	158·550	−1·0914	−1·0948	−1·0931	+3·4	−30·9	35·1732
26	May 29	46	47	0·960	159·511	+0·4225	+0·4284	+0·4257	−0·2	−31·1	35·5989
June 1	June 1			0·962		+0·4287	+0·4233				
May 26	May 29	47	48	0·848	160·359	−0·2077	−0·2035	−0·2056	−4·2	−35·3	35·3933
26	28	48	49	0·900	161·259	+1·4371	+1·4345	+1·4358	+2·6	−32·7	36·8291
26	28	49	50	1·039	162·298	−0·3258	−0·3272	−0·3265	+1·4	−31·3	36·5026
26	28	50	51	1·061	163·359	−0·0518	−0·0538	−0·0528	+2·0	−29·3	36·4498

June 5, 28	28	51	52	0·726 0·812	164·128	+0·5028 +0·5046	+0·5017	+0·5027	+2·0	—27·3	36·9525
May 28	May 28	52	O3	0·092	164·220	+0·2664	+0·2661	+0·2662	+0·3	—27·0	37·2187
June 5	June 11	52	53	0·820	164·948	—0·0465	—0·0464	—0·0464	—0·1	—27·4	36·9061
5 22	11 22	53	54	0·832 0·834	165·781	—0·3289 —0·3304	—0·3239 —0·3301	—0·3283	—2·6	—30·0	36·5778
5	11	54	55	0·928	166·709	—0·2724	—0·2698	—0·2711	—2·6	—32·6	36·3067
5	11	55	56	0·986	167·695	+0·8934	+0·8934	+0·8934	0·0	—32·6	37·2001
6	11	56	57	0·905	168·600	—0·3543	—0·3498	—0·3520	—4·5	—37·1	36·8481
6	11	57	58	0·820	169·420	+0·7511	+0·7553	+0·7532	—4·2	—41·3	37·6013
6	9	58	59	0·817	170·237	—0·1016	—0·1022	—0·1019	+0·6	—40·7	37·4994
6	9	59	60	0·942	171·179	+0·0600	+0·0610	+0·0605	—1·0	—41·7	37·5599
7	9	60	61	0·870	172·049	—0·0732	—0·0748	—0·0740	+1·6	—40·1	37·4859
7	9	61	XX	0·314	172·363	+0·4994	+0·4985	+0·4990	+0·9	—39·2	37·9849
7	9	XX	62	0·739	173·102	—0·0079	—0·0080	—0·0080	+0·1	—39·1	37·9769
7	9	62	XXI	1·223	174·325	—0·1606	—0·1593	—0·1600	—1·3	—40·4	37·8169
7, 12	18	XXI	63	1·002	175·327	+0·8696	+0·8746	+0·8721	—5·0	—45·4	38·6890
12	18	63	64	1·084	176·411	+0·4163	+0·4159	+0·4161	+0·4	—45·0	39·1051
12	18	64	65	0·896	177·307	—0·0178	—0·0220	—0·0199	+4·2	—40·8	39·0852
12	18	65	XXII	0·786	178·093	—0·7713	—0·7691	—0·7702	—2·2	—43·0	38·3150
12	18	XXII	66	0·844	178·937	+1·7270	+1·7287	+1·7278	—1·7	—44·7	40·0428
12, 13	16, 18	66	67	1·039	179·976	+0·4371	+0·4389	+0·4380	—1·8	—46·5	40·4808
13	16	67	68	1·078	181·054	—0·8449	—0·8461	—0·8455	+1·2	—45·3	39·6353
13	16	68	69	0·806	181·860	—0·7478	—0·7454	—0·7466	—2·4	—47·7	38·8887
13	16	69	70	0·842	182·702	+0·2703	+0·2710	+0·2706	—0·7	—48·4	39·1593
13 21 21	16 21 21	70	71	0·836 0·838 0·837	183·539	+0·3512 +0·3518 +0·3493	+0·3590 +0·3556 +0·3495	+0·3528	—3·9	—52·3	39·5121
14	16	71	72	1·192	184·731	+0·1626	+0·1612	+0·1619	+1·4	—50·9	39·6740
14	15	72	P3	0·097	184·828	+2·3964	+2·3966	+2·3965	—0·2	—51·1	42·0705
14	15	P3	73	0·656	185·484	—2·5319	—2·5302	—2·5310	—1·7	—52·8	39·5395
14	15	73	74	0·778	186·262	+0·2088	+0·2078	+0·2083	+1·0	—51·8	39·7478
14	15	74	75	0·838	187·100	+0·1475	+0·1513	+0·1494	—3·8	—55·6	39·8972

Results from geodetic spirit leveling in Missouri—Second part from New Haven to vicinity of Jefferson City, 1888—Continued.

Date, 1888.		Bench mark. From—	Bench mark. To—	Distance between successive bench marks.	Distance from initial mark K_3.	Difference of height between bench marks. Forward measure.	Backward measure.	Mean.	Discrepancy. Partial F−B.	Total accumulated.	Height of mark above St. Louis mark K_3.
1888.				*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
June 14, 19	June 20	75	76	0·911	188·011	+0·1597	+0·1572	+0·1584	+2·5	—53·1	40·0556
19	20	76	77	0·906	188·917	+0·5809	+0·5782	+0·5796	+2·7	—50·4	40·6352
19	20	77	78	0·832	189·749	+0·5384	+0·5410	+0·5397	—2·6	—53·0	41·1749
19	20	78	79	0·789	190·538	—0·0681	—0·0668	—0·0674	—1·3	—54·3	41·1075
25	20, 23	79	XXIII	0·640	191·178	—0·3141	—0·3080	—0·3094	—2·3	—56·6	40·7981
26	26			0·638		—0·3101	—0·3081				
28	28			0·643		—0·3072	—0·3084				
25	23	XXIII	XXIV	0·421	191·599	—0·3978	—0·3964	—0·3971	—1·4	—58·0	40·4010
25	20, 23	79	80	0·534	191·072	—4·8475	—4·8501	—4·8488	+2·6	—51·7	36·2587
26	26	80	81	0·381	191·453	—0·2260	—0·2348	—0·2292	+8·4	—43·3	36·0295
26	26			0·381		—0·2239	—0·2320				
25	23	81	XXIV	0·301	191·754	+4·3750	+4·3750	+4·3750	0·0	—43·3	40·4045
	Mean		XXIV		191·676					—50·6	40·4028
27	28	XXIV	82	0·938	192·619	+4·0272	+4·0195	+4·0222	+2·1	—48·5	44·4250
30	30			0·948		+4·0194	+4·0230				
27	28	82	83	0·882	193·501	—0·0624	—0·0602	—0·0613	—2·2	—50·7	44·3637
27	30	83	84	1·214	194·715	—1·5940	—1·5944	—1·5942	+0·4	—50·3	42·7695
29	30	84	XXV	0·191	194·906	+0·8160	+0·8143	+0·8152	+1·7	—48·6	43·5847

Description of primary and secondary bench marks between Etlah, Mo., and vicinity of Jefferson City, Mo.

Secondary B. M. XV.—Berger, Franklin County, Mo. A limestone post 1·7 feet long, rough at the bottom and dressed to 6 by 6 inches at the top to a depth of 6 inches, was used as this B. M. It is buried 1·5 feet in the ground. It is situated on the west side of Mrs. M. M. Schaub's house, close to the wall of the foundation and 3·3 feet from the southwest corner of the house. This house is quite close to the track of the Missouri Pacific Railroad (50 feet), just north of the point where the main street of the village crosses it. Both corners of the stone on the south side are chipped off, and the stone appears to be rather soft.

Primary B. M. N_3.—Hermann, Gasconade County, Mo. A cross cut on the northeast corner (east side) of the stone foundation of the "White House" hotel, A. C. Leisner, proprietor, at Hermann, Gasconade County, Mo., and the center of this cross was used as the bench mark. The cross is 1·24 feet south of the corner and 1·26 feet above the surface of the ground. This bench was marked as follows:

U. S.
+
B. M.
N_3.
1888.

Secondary B. M. XVI.—Gasconade County, Mo. This bench is the bottom of a square hole cut in the top of the stone abutment to the iron bridge on the Missouri Pacific Railroad across Coles Creek. The bridge rests on a portion of the abutment which is about 4 feet lower than that portion where the bench is cut. Near the corner of the stone a cross is cut with the letters B. + M.

B. M. XVI is 0·750 metre east of this, on the same stone. It is on the east side of the creek and is north of the railroad. Mr. Eaffner lives near the creek, on the west side. The bench is marked as follows:

U. S.
□
B. M.

Secondary B. M. XVII.—Gasconade County, Mo. This bench is the bottom of a square hole cut in the top of the middle stone pier of the Missouri Pacific Railroad bridge over the Gasconade River. It is south of the track and marked as follows:

U. S.
□
B. M.

Secondary B. M. XVIII.—Gasconade County, Mo. This bench is the bottom of a square hole cut in the center of the top of a limestone post which was set in the ground in the yard of Mr. J. Wolter's dwelling and storehouse at Gasconade Station. The post is about 0·7 metre from the southeast corner of the house, which is situated about 100 feet south of the Missouri Pacific Railroad and about

200 feet west of the railroad station house. The post is dressed to 6 by 6 inches at the top and is 18 inches long, buried 15 inches in the ground. It is marked on top:

U. S.
□
B. M.

Secondary B. M. XIX.—Gasconade County, Mo. This bench is the bottom of a small square hole cut in the top of the stone foundation to H. Binkholter & Co.'s grain elevator at Morrison Station, Missouri Pacific Railroad. The building is about 6 inches inside the outer face of the foundation, and the bench is on this ledge, near the northeast corner of the building, which is situated quite near the track, on the south side. The stone is soft and the letters are roughly cut; the bottom of the hole is smooth. This bench is marked as follows:

U. S.
□
B. M.

Primary B. M. O_3.—Chamois, Osage County, Mo. This bench is the bottom of a square hole cut in the top of the stone across the bottom of the side door to the saloon on the northwest corner of Main and Pacific streets. This door is on the Pacific street side and faces the railroad. The building is a two-story brick, with imitation stone foundation. The bench is near the west side of the door and is marked as follows:

U. S.
□
B. M.
O_3.
1888.

Secondary B. M. XX.—St. Aubert, Osage County, Mo. This bench is the bottom of a square hole cut in the top of the stone abutment of the Missouri Pacific Railroad bridge across ——— Creek, opposite the village of St. Aubert. The bench is on the east abutment, and is south of the track. The letters are very roughly cut. The bridge is about one-fourth mile west of the depot. The bench is marked as follows:

U. S.
□
B. M.

Secondary B. M. XXI.—Near St. Aubert, Osage County, Mo. Is on the north side of east abutment of the first trestle west of mile post 106 on the Missouri Pacific Railroad, and is about 1 mile west of St. Aubert Station and between it and Isbell Station. The B. M. is a spot surrounded by a square trench (about an inch square), with the letters

U S
□
B M

rudely and slightly cut.

Secondary B. M. XXII.—Isbell, Osage County, Mo. This bench is the bottom of a square hole cut in the top of the stone abutment of the Missouri Pacific Railroad bridge over Loose Creek. It is on the east abutment, and is north of the track. It is situated on the step in the abutment on which the bridge rests, and is about 5 feet below the track. Near the corner of the stone a cross is cut, with the letters B. M., thus: B. + M. This bench is about one-half a mile east of Isbell Station. Marked as follows:

U S
□
B M

The stone is a soft sandstone and the letters are roughly cut.

Primary B. M. P_3.—At Bonnot's Mill, Osage County, Mo. It is on the northwest corner of a brick building used as a store and owned by Mrs. L. Bonnot, and is on the limestone block forming the corner stone, which is about 8 inches square at the end and projects some 4 inches. The mark is a square cavity in center of projection, and has on upper surface U. S. □ B. M. and on western face P_3. 1888. The stone is 35 paces south of railroad. The exact B. M. is the bottom of the square cavity.

Secondary B. M. XXIII.—This B. M. is the surface of the stone inside a square (□) cut on top of the fourth pier (from east bank) of the Missouri Pacific Railroad bridge over the Osage River at Osage, Mo. The B. M. is under the center of the track and about the center of the top of the pier.

Secondary B. M. XXIV.—This B. M. is the bottom of a square cavity cut in the top of a stone post set in the southwest corner of Mrs. Rassler's boarding house yard at Osage, Mo. The stone post (limestone) is dressed to 6 by 6 inches at the top and 6 inches below; it is about 2 feet long and is set 22 inches in the ground. The top of the post is lettered as follows:

U S
□
B M

Secondary B. M. XXV.—This is the bottom of a square cavity cut in the capstone on south end of west abutment of first trestle west of mile post 119 on the Missouri Pacific Railroad, between the Osage River and the Moreau Creek. The letters

U S
□
B M

are placed thus, and roughly cut.

(See route diagram, illustration No. 1.)

Accuracy of the preceding results for heights.

The temporary marks of the line between St. Louis and New Haven are fairly regularly distributed, with an average distance apart of 1·9 kilometres; hence we may assume the weights for these partial lines to be equal and the probable error of a difference of height of 1 kilometre from a double measure (here two simultaneous measures) becomes

$$r_{\prime\prime} = 0{\cdot}675\sqrt{\frac{[dd]}{4\,[s]}}$$

and the probable error for height of a terminal point at the distance $S=[s]$ will be

$$r=0\cdot675\sqrt{\frac{[dd]}{4}}$$

These expressions suppose the two measures to be independent of one another; this, however, is not the case with simultaneous lines, the condition of the atmosphere at the time being the same for both, and this is also partially true of the condition of the instrument, so that the weight of results from two simultaneous lines is but little better than that for one line. Experience showed that in case of two simultaneous lines the above probable error should be increased by its one-fourth part in order to approximate to a more correct value.

We have $[dd]=695\cdot1$ and $[s]=116\cdot2$;
hence $r_{,,}=\pm 0\cdot83^{mm}$ and adding one-fourth, the corrected value $=\pm 1\cdot04^{mm}$, also $r=\pm 11\cdot2^{mm}$.

In the line between New Haven and vicinity of Jefferson the temporary marks are also regularly distributed, but only 0·9 kilometre apart on the average; here we have $[dd]=719\cdot9$ and $[s]=84\cdot4$; hence $r_{,,}=\pm 0\cdot98^{mm}$ and $r=\pm 9\cdot06^{mm}$.

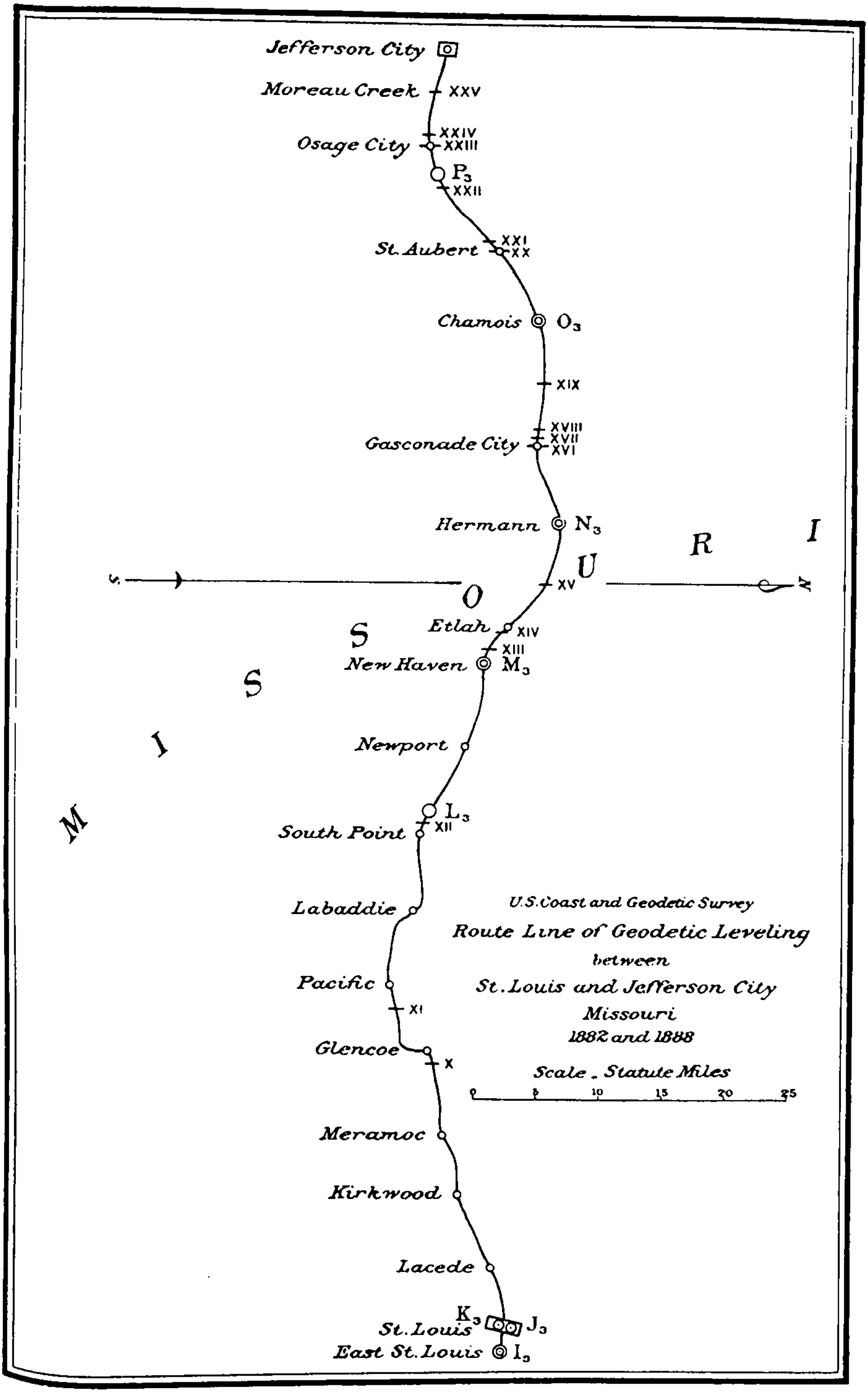
Also for first part of line $m_{,}=\sqrt{\frac{[dd]}{2\,[s]}}$, or the mean error of a single leveling of one kilometre, after increasing the r by its fourth part

$$m_{,}=\pm 2\cdot16^{mm}$$

and for second part $\quad m_{,}=\pm 2\cdot06$

The probable error of the difference of height between St. Louis (K_3) and vicinity of Jefferson (XXV)

$$\pm 11\cdot2 \pm 9\cdot1 = \pm 14\cdot4^{mm}$$


Jefferson City
Moreau Creek
XXV
XXIV
Osage City
XXIII
P3
XXII
St. Aubert
XXI
XX
Chamois
O3
XIX
XVIII
XVII
Gasconade City
XVI
Hermann
N3
XV
Etlah
XIV
XIII
New Haven
M3
Newport
L3
South Point
XII
Labaddie
Pacific
XI
Glencoe
X
Meramoc
Kirkwood
Lacede
K3
St. Louis
J3
East St. Louis
I3
S.
N.
M I S S O U R I
U.S. Coast and Geodetic Survey
Route Line of Geodetic Leveling
between
St. Louis and Jefferson City
Missouri
1882 and 1888
Scale - Statute Miles
0
5
10
15
20
25

APPENDIX No. 3—1893.

PHOTOTOPOGRAPHY AS PRACTICED IN ITALY UNDER THE AUSPICES OF THE ROYAL MILITARY GEOGRAPHICAL INSTITUTE, AND AS PRACTICED IN THE DOMINION OF CANADA UNDER THE AUSPICES OF THE DEPARTMENT OF THE INTERIOR. ALSO A SHORT HISTORICAL REVIEW OF OTHER PHOTOGRAPHIC SURVEYS AND PUBLICATIONS ON THE SUBJECT.

Submitted for publication December 9, 1893, by J. A. FLEMER, Assistant.

PREFACE.

A topographic survey of a large area or of an entire country has been and still is a very laborious, time-absorbing, and expensive undertaking. Nearly all the European countries have such surveys of a more or less elaborate and detailed nature, which are the fruits of ceaseless work, begun many years ago, and in most instances the topographic work is continued to this day, in order to maintain the value of the maps, particularly for military purposes, by making frequent resurveys, covering all changes subsequent to the time at which the original surveys had been completed.

The completion of a topographic survey of the United States, executed on a scale to be useful for general purposes, if undertaken now, could not be witnessed by many of the present generation. With a practical people like the Americans such an undertaking would probably be looked upon with more favor if the generation that begins this work would also reap some of the benefits thereof.

The topography of this country is so diversified and the population is so unevenly distributed over the same that the methods to be employed for such a survey should also be diversified; the character and value of the different sections should govern the accuracy and amounts of detail of the survey, in order to reach the quickest yet practically useful and valuable results.

Minute and detailed methods, with ensuing accurate results, should be applied to cities and all closely settled regions, to the coast, larger rivers, and lakes, and the work should be platted on a large scale. Arid, barren, and mountainous regions, as well as prairies and swamp lands, should be more generalized in their cartographic representations and platted on a small scale.

The new survey of Italy demonstrates this fully, and it is there that phototopography, the subject to be considered in this paper, has reached a high state of perfection under the auspices of the Military Geographical Institute of that country.

Photogrammetry proper (or *metrophotography*) should be applied to the art of taking perspective views of buildings with a photographic camera for the purpose of constructing therefrom the elevations and ground plans of buildings. It is used chiefly for architectural purposes (remodeling, illustrating, copying, etc.).

The term *phototopography* should be generally adopted for all topographic surveys based on perspective views of the terrene obtained by means of the camera.

Photographic survey, finally, could then be applied to all surveys based on photographic data which do not include the delineation of the terrene (nonhypsometric surveys).

We have endeavored to give in the following pages a short review of the more important photographic surveys, and of some of the publications on photogrammetry and phototopography, as well as a concise description of the general methods and principles of phototopography as practiced in Europe and in the Dominion of Canada, in order that this branch of surveying may become more generally known, tested, and amplified also in this country.

SHORT REVIEW OF PHOTOGRAPHIC SURVEYS AND PUBLICATIONS.

In Europe the possibility of applying photography for constructive and surveying purposes was recognized many years ago.

Photographs obtained by aid of lenses ground specially with a view toward reducing astigmatic aberration as much as possible and giving a uniform extension of definition and depth over a strictly flat field will represent geometrically true perspectives.

Photogrammetry, or metrophotography, is the art of ascertaining graphically the true dimensions of objects from their perspectives, in which the relative dimensions of the objects are changed and distorted (chiefly foreshortened) and can not be ascertained by direct linear measurements in consequence of being represented in perspective view on a plane surface.

The study of constructing geometrical views and ground plans of objects represented in perspective can be divided into two groups or chapters.

1. To construct geometrical plans from perspectives, composed of regular figures and taken from points of view close to the objects thus represented, for instance, to construct the elevations and ground plans of buildings, machines, and the like from photographs taken from stations sufficiently close to the same to delineate all details. This art

may properly be termed photogrammetry or metrophotography; it is of interest only to constructors, architects, paleologists, artists, etc.

2. The objects represented in perspective are of irregular shape and at various distances from the stations or points of view, like distant landscapes, and it is desired to construct therefrom, graphically, a topographic map of the terrene, projected in horizontal plan. This art may be termed phototopography and it interests topographers, geographers, geologists, explorers, hydrographers, etc.

Descriptive geometry teaches the laws which are to be followed when representing objects by drawings on plane surfaces. The eye receives the natural image or the view of an object by aid of the rays of light—termed visual rays—which emanate from the illuminated parts of an object facing the spectator.

If we regard the eye as a fixed point and imagine the rays of light, emanating from different points of the object in view, intercepted by a vertical plane, we will obtain a central projection or a perspective view of the object in the vertical plane.

The greater the distance of the object from the eye, the less great will be the deviation of the extreme visual rays from the direction of the central ray; for an infinite length of the central ray all the rays will become parallel.

If the picture or image of the object is given us as a true perspective in a plane, we can, inversely, construct therefrom a geometrical projection of the object in a plane placed at right angles to the picture plane, if we know the distance and relative position of the point of view with reference to the picture plane, and if we have views taken from a sufficient number of stations in space to envelop the irregularly formed object in question.

Regarding a photograph as a geometrically true perspective, photogrammetry will be the art of reconstructing geometrical horizontal projections from given perspective views.

The theoretical fundamental ~~principles upon~~ which such reconstructions rest were known to Lambert in 1759, but the first practical application of the same ~~was made by the~~ celebrated French savant and hydrographer, Beautemps-Beaupré, while on a scientific expedition during the years 1791 to 1793. Although the camera had not yet been invented, it is ~~said that~~ Beautemps-Beaupré was an expert sketcher, and he made perspective drawings and sketches of coast regions while on that expedition, from which, at a later period, he constructed topographic maps of a part of Van Diemen's Land (now Tasmania) and of the island of Santa Cruz. Notwithstanding Beautemps-Beaupré's frequent allusions to the feasibility of this method of making reconnaissance surveys and topographic maps, nothing more was accomplished until Laussedat, major in the French army, took the study of this subject up in 1850, using, however, the camera to obtain the perspectives.

In 1839, shortly after Daguerre had presented his memorial upon photography to the Academy of Sciences in Paris through Arago, the latter called attention to the possibilities of photography in the Chamber of Deputies, where he said:

* * * Nous pourrions, par exemple, parler de quelques idées qu'on a eu sur les moyens rapides d'investigation, que le topographe pourra emprunter à la photographie.

Gay-Lussac similarly called attention to the probable adaptability of photography to topographic surveys.

In 1858 Chevallier had an instrument patented which he called a "planchette photographique." This photographic plane table is mentioned and described by Alophe (1861), d'Abbadie, Baté (1862), Jouart (1866), Tronquoy, etc.

Jouart, Wiganowski, Baté, and others also made practical tests and topographic surveys with Chevallier's photographic plane table.

Captain Cannette used the sextant and photographic camera to make topographic surveys, chiefly of fortifications.

In 1851 Laussedat constructed a "camera clara," which in 1858 was superseded by the "camera obscura" with additional improvements for surveying purposes, by Reynault. Laussedat, as "chef du génie corps," made numerous experimental surveys and studies with Reynault's improved "camera obscura," partly on his own behalf and partly under the direction of the French ministry of war. He also was the first to make topographical surveys with the aid of balloon photography. During the years 1863 to 1870 he had the assistance of Captain, now Commandant, Javary, who improved the French phototheodolite and made experimental surveys in the mountains of the Dauphiné and Savoie, in the Vosges, and in Alsatia. The first practical survey of a more extended character made with the aid of photography in France was made by Laussedat in 1861, when he mapped a portion of Paris and also the town of Grenoble under the auspices of the ministry of war. The area covered by this survey was 0·4 square mile; the field-work consumed sixty hours, and the office work was accomplished in two months.

Pujo and Fourcade published an article in Les Mondes, 1865, on "Goniométrie photographique."

Other publications in French are:

Comptes Rendus de l'Académie des Sciences, Paris, XLIX. 1859; L, 1860; LI, 1860; LIX, 1864; 1885 and III, p. 729-732, 1890.

Magasin Pittoresque, XXIX, 1861.

Annales du Conservatoire National des Arts et Métiers, 2e série, IV, 1892.

Comptes Rendus du Congrès de Pau et Revue Scientifique de 1892.

"Éléments de Photogrammétrie," in Bulletin de la Soc. d'Éditions Scient., Paris, 1891, by V. Legros.

Application de la Photographie à la Topographie Militaire, par E. Paté. 1862.

Mémorial de l'Officier du Génie, No. 16, 1854; No. 17, 1864; No. 22, 1874.

Bulletin de la Société de Géographie de Paris, Déc. 1862.

Application de la Photographie aux Levers Militaires, par A. Jouart, 1866.

La Photographie Appliquée aux Études Géographiques, par Jules Girard, 1872.

De la Photographie et ses Applications aux Besoins de l'Armée, par Fl. Dumas, 1872.

La Photographie Appliquée au Lever des Plans, par J. Bornecque, 1886.

La Photographie dans les Armées, par Alfred Hanot, 1875.

La Photographie sans Objectif, par R. Colson, 1887.

La Nature (Paris).

La Revue d'Artillerie (Paris).

Bulletin de la Société Française de Photographie (Paris).

Les Levers Photographiques et la Photographie en Voyage, par le Dr. Gustave Le Bon, 1889.

Annales du Conservatoire des Arts et Métiers. Édouard Monet: Principes Fondamentaux de la Photogrammétrie, published by La Société d'Éditions Scientifiques, No. 4 Rue Antoine-Dubois, Paris.

In recent years the French ministry of war has had numerous experiments made with balloon surveying (using both the captive and free balloon), balloon photography being better adapted for military and secret surveys.

France had an exhibit at the World's Columbian Exposition in Chicago, 1893, showing photographic instruments and specimens in illustration of topographic and astronomical results, gained chiefly under the direction of Col. A. Laussedat and taken from the collection of the Conservatoire National des Arts et Métiers, in Paris, of which Bureau Col. A. Laussedat is the director.

The first German publication bearing on this subject ~~is probably~~ the article in Horn's Photographische Mittheilungen, April, 1863, being a German translation of A. Laussedat's explanations ~~and~~ descriptions, as given by him on January 9, 1863, in a meeting of the French Photographic Society.

Dr. A. Meydenbaur's first publication on this subject is in the June edition of the Photographische Mittheilungen of 1863, where he uses the term "photometrography," which was subsequently changed into "photogrammetry."

Vogel published, in the March number of the same magazine for 1866, an article on the use of Johnson's photographic instrument for making topographic surveys, showing the method of obtaining horizontal and vertical angles from the perspectives.

Ever since Dr. Meydenbaur first became interested in this method he endeavored to interest German private and Government surveyors in photogrammetry. He has been recently appointed director of the Photogrammetrical Institute in Berlin, founded by the Prussian Government as a branch bureau of the ministry of culture. May 4, 1893, Dr. Meydenbaur (royal counsilor), gave a lecture on metrophotography, or photogrammetry, in the ministerial building, under the auspices of Mr. Bosse, minister of culture.

Although this photogrammetrical institute was founded several years ago, no official publications or reports have been issued yet to the public.

Professor Jordan, Dr. Doergens, Dr. Stolze, Dr. Vogel, and Dr. Hauck have done much toward popularizing the photographic methods of surveying in Germany and Austria.

Professor Jordan published a treatise upon "The application of photography for geometrical representations" in the Zeitschrift für Vermessungswesen, 1876, Heft 2, Bd. V, and he points out the future importance of photogrammetry in his closing remark: "Photogrammetry can be applied with the greatest advantage in certain cases, e. g., for the survey of inaccessible mountain groups and ranges, on scientific expeditions," etc.

The first attempt at a photogrammetric survey in Germany was made under the direction of the Prussian ministry of war and commerce in 1867, when a survey of the town of Freiburg and also an architectural survey of the cathedral in Freiburg were made. The fieldwork was continued through four days and the area surveyed comprised about 0·04 square mile. The office work for the construction of the map consumed three weeks, while it took one week to draw the ground plan, one side, and one front elevation of the cathedral.

During the Franco-Prussian war phototopography was called into service by the German army, and a detachment of the engineer corps, under Dr. Doergens (later professor of geodesy in Berlin), was formed to obtain certain distances about the city of Strasburg with the aid of a camera, during the siege of that city. This detachment made a map on the scale of 1 to 25,000 of the besieged front of the city. However, the result was not utilized by the army, the city having capitulated before the map was fully platted.

Professor Jordan, as member of Rohlf's African exploring expedition in 1873–74, made a phototopographic survey of the Oasis Gassr Dachel in the Libyan desert.

In 1874 Dr. Stolze used a Meydenbaur camera-theodolite to make a survey of the ruins of Persepolis, and also an architectural survey of the mosque of Djumâht, in Shiraz, Persia.

In 1885 the students of the technical high school in Berlin, under direction of Professor Pietsch, used two instruments specially constructed to obtain views also under an inclined position of the optical axis of the camera. They obtained satisfactory results from various ascensions made in a free balloon, as well as from views taken on the ground for architectural purposes.

Photography has also not only found practical application in topographical surveys of Austria, principally in Steiermark and Kärnten, but the art of phototopography has made rapid strides in gaining public favor in that country, owing to the treatises and works published on this subject by Pollack, Hofferl, Steiner, and others, as well as to recent improvements in the instruments.

The following are the principal publications in German on photographic surveying:

Photographisches Archiv, Septr., 1865.

Zeitschr. für Bauwesen, 1867.

Archiv für die Offiziere des k. preuss. Artillerie- und Ingenieur-Corps, Bd. 63, 1868.

Deutsche Bauzeitung, 1872.

Zeitschr. f. Vermessungswesen, Heft 23 and 24, 1887.

Journal für die reine und angewandte Mathematik, Bd. 95.

Das Licht. S. G. Stein. Heft 5, 1887. (Photogrammetrie, von V. Stolze.)

Lechner's Mittheilungen aus dem Gebiete der Photographie und Kartographie. R. Lechner, Graben 31, Wien.

Dr. C. Koppe: Die Photogrammetrie, oder Bildmesskunst. Weimar, 1889.

V. Pollock: Die photographischen Terrainaufnahmen mit Berücksichtigung der Arbeit in Steiermark. R. Lechner, Wien, 1891.

V. Pollock: Photogrammetrie und Phototopographie. Mittheilungen der k. k. geogr. Gesellsch., 1891 (pages 175–195), Wien.

Fr. Steiner: Das Problem der fünf Punkte, eine Aufgabe der Photogrammetrie, 1891. Wochenschr. d. östr. Ing.- und Archt.-Vereins (pages 214–217).

Fr. Steiner: Die Photographie im Dienste des Ingenieurs. Ein Lehrbuch der Photogrammetrie. R. Lechner, Wien, 1891.

Fr. Schiffner: Die photographische Messkunst, oder Photogrammetrie, Bildmesskunst und Phototopographie. Wilhelm Knapp, Halle a. S., 1892.

Dr. A. Meydenbaur: Das photographische Aufnehmen zu wissenschaftlichen Zwecken, ins besondere das Messbildverfahren. Unte's Verlags-Anstalt, Berlin, 1892.

Gustav Fritsch, in Dr. G. Neumayer's Anleitung zu wissenschaftlichen Beobachtungen auf Reisen. Robert Oppenheim, Berlin, 1888.

Fr. Schiffner: Ueber die photogrammetrische Aufnahme einer Küste im Vorbeifahren. Mittheilungen aus dem Gebiete des Seewesens, 1890, pages 412–417.

F. Hafferl: Ueber Photogrammetrie. Vortrag. Wochenschrift des östr. Ing.- u. Archit.-Vereins, 1890, pages 199–203.

V. Pollock: Ueber Anwendung der Photogrammetrie im Hochgebirge. Vortrag. Wochenschrift d. östr. Ing.- u. Archit.-Vereins, 1890, pages 207–209.

Volkmer: Das Wesen der Photogrammetrie. Wochenschrift des östr. Ing.- u. Archit.-Vereins, Vol. XIV, page 157.

Jordan: Vermessungskunde. II. Feld-Landmessen. Metzler'sche Verlagsbuchhandlung, Stuttgart, 1893.

In Italy we find, as previously mentioned, that phototopography has been brought to a high state of perfection in recent years.

Porro spent much time, labor, and energy in perfecting photography as applied to tachymetry and topography. The results of his labors were published in Il Politecnico, Vols. X and XI, under "Applicazione della Fotografia alla Geodesia," 1853, Saldini, Milano. Porro's instruments have all been preserved by Salmairaghi, director of the Polytechnic Institute at Milan, of which Porro was a member.

In 1875 Manzi Michele, officer of the Military Geographical Institute of Italy, utilized some photographic views of the "Abruzzi" to supplement his plane-table survey of the "Gran Sasso." In 1876 the same officer continued the practical application of photography for the topographical survey of "Mont Cenis" (Bart Glacier).

The Military Geographical Institute then decided to suspend all photographic work indefinitely, as many maintained that photographical data for topographic purposes were unreliable.

In 1878 General Ferrero, chief of the geodetic department of the institute, called the attention of the directory of the institute to the desirability of resuming the studies in photogrammetry; and in the same year L. P. Paganini, engineer geographer of the institute, was commissioned to proceed to the Alps, near Apua, to resume the studies in photography applied to topographic surveys, with a view to ascertain whether phototopography was economical and expedient for practical work.

During Paganini's first season he obtained 17 cycloramic views, composed of 110 perspectives. A number of these perspectives were used to construct a map, in Florence, on a scale of 1 to 25,000, of the marble quarries at Colonnata (Carrara), with hypsometric contours in intervals of 5 metres.

In 1879 Paganini (using bromo-gelatin plates instead of wet plates, as heretofore, also having improved the camera theodolite) surveyed the Serra dell' Argentera, which was platted on a scale of 1 to 25,000, with contours in 10 metres intervals. This survey was based upon panoramic views obtained from fifteen stations, on elevated points, comprised 113 perspectives, and was the result of a field season of two months and a half. Also, this map was constructed in Florence during the following winter, and it represents an area of 28 square miles. The contours were controlled by 490 points, the elevations of which had been ascertained.

In 1880 the same officer commenced the survey of the area bounded by the valleys of the Orco, the Valsoana, the Cogne, and the Valsavaranche, representing an area of about 386 square miles. The survey of this area was finished in 1885.

However, since 1884 Paganini used an improved instrument, made by Galileo for the institute after plans submitted by Paganini. He also invented three instruments which greatly facilitate and accelerate the otherwise tedious graphic operations of the map construction from the perspectives, and which will be described later on.

Paganini's results proved the efficiency of phototopography for Alpine work, to be platted on a scale of 1 to 25,000 or 1 to 50,000, and the technical solution of the problem has been fully established. Owing to the untiring efforts of the officers of the Military Geographical Institute, and the good results which they obtained, phototopography has been adopted as an auxiliary to the plane table for the new survey of Italy.

In a more recent report on phototopographic work by Paganini to the first geographical congress in Italy, he described his latest improvements to the camera theodolite. An extract from this report, made by Fenner, can be found in the Zeitschr. f. Verm., 1893.

A German translation, by A. Schepp, of Paganini's "La fototopografia in Italia" can be found in the same periodical for 1891 and 1892.

C. W. Verner, "Notes on military topography," 1891. Also, Mechanics, Vol. II, p. 168, "Application of photography to surveying."

A short article on photogrammetry has also been published by Henry A. Reed, lieutenant, United States Army, "Topographical drawing and photography applied to surveying." Another work in English has been issued by Allen, in London.

Civil and Military Photogrammetry (read before the American Philosophical Society, May 6, 1892), by R. Meade Bache, assistant, United States Coast and Geodetic Survey.

The following works are very explicit and full of details:

Photography Applied to Surveying, by Lieut. Henry A. Reed, United States Army. John Wiley & Sons, 15 Astor place, 1889.

Photographic Surveying, by E. Deville, surveyor-general, Canada. Ottawa, 1889. This work, unfortunately, is out of print. Only a limited number of copies had been printed, chiefly to supply the Dominion land surveyors.

a copy on file in US C & G S

INTRODUCTION.

Topography (description of locality) serves to represent and describe, in horizontal plan, limited areas of the earth's surface, showing vertical and horizontal distances (the relief) between points of the area thus represented. On every topographic map, therefore, the characteristic lines and forms of the terrene, including natural objects which appear on the earth's surface, must be recognizable with more or less minute detail, according to the scale and purpose of the map.

Generally speaking, the scales of topographic maps vary from 1 to 10,000 to 1 to 200,000. On a larger scale than 1 to 10,000 such maps are chiefly used for constructive (engineering) purposes; they show all artificial as well as natural objects on the earth's surface, and they are the result of special surveys for special purposes.

Maps on a smaller scale than 1 to 200,000 are designated as geographic maps. Their topography is generalized to show only prominent and characteristic features (mountain ranges, valleys, plateaus, seas, lakes, rivers, etc.).

The topographer inspects the area to be mapped and intuitively separates characteristic features from minor detail. He immediately, on inspection, recognizes what is to be shown on the map, and consequently embodies only such topographic features of the terrene as will harmonize with the power of representation of the adopted scale for the work. It is useless to locate features during the instrumental survey which are too small to be represented on the scale of the map. An expert topographer will make a topographic map of a given area in the least time necessary, yet show all characteristic features that can be drawn and represented in harmony with the scale of the map.

After a system of triangulation has been extended over a country and the topographic details of the same area have been gathered by a series of photographic panorama views, taken from these trigonometric stations, and supplemented, where needed, by views taken from other points (the latter to be subsequently connected with the triangulation by resecting or otherwise), an experienced topographic draftsman can select all characteristic topographic features from such photographs, for mapping purposes, with the same good result as the topographer would select them in nature from the same area, as shown in the photographs, were he to draw the map in the field (plane-table method).

If by a simple graphic method such selected characteristic points on the photographs can be platted upon the map, a great deal would be gained in the survey of certain regions and under certain conditions toward a saving of time and money, compared with the instrumental methods of topographic surveying as generally practiced. At present phototopography can not replace instrumental topography; still, experience in Italy, France, Canada, Austria, and Germany has proven it to be a very valuable adjunct to the plane table and transit for topographic surveys of rugged mountainous regions, if they are not too wooded.

The following description of photographic instruments and methods is chiefly a condensed extract from the previously mentioned article printed in the Rivista di Topografia e Catasto, 1889, by L. P. Paganini, engineer of the Royal Italian Military Geographical Institute, followed by a description of instruments as used and methods as practiced in Canada, with a short reference to German, French, and Austrian instruments.

Chapter I.

THE PHOTOGRAMMETRIC APPARATUS—PRINCIPAL COMPONENT PARTS OF THE PERSPECTIVES.

Figure A shows Paganini's Italian photogrammetric apparatus. It has a tripod which can be dismembered into three "alpenstocks" A, A, and A, a theodolite, and a camera C. All three parts can be firmly united by means of spiral springs and screws.

The three screws S_1 of the tripod head H (but two shown in drawing) support the theodolite, and the camera C is supported by another set of three screws S_1', S_2', and S_3', which are connected with the horizontal limb of the theodolite in such a manner that the camera can be revolved about the vertical axis of the theodolite.

A spirit level L and telescope T are supported by an upright piece U, placed at right angles to the horizontal limb of the theodolite and at one side of, but close to, the camera.

The telescope T is provided with two cross hairs (one vertical and the other horizontal), in the usual adjustable manner. The camera box is made of hardened pasteboards, which are stiffened by a metal skeleton casing B. The camera is provided with an aplanatic objective (antiplanet), made by Steinheil, the focal length of which is 244·5^{mm} and the aperture in the diaphragm has a diameter of 5^{mm}.

Regarding the general arrangement of the camera, it may be said that—

1. The optical axis of the photographic lens (objective) is vertical to the image plate I.

2. The intersection O of the optical axis and image plate I is marked on the latter by the point of intersection of two very fine platinum wires p_1 and p_2, placed at right angles to each other and very near the image plate I; when in adjustment one p_1 of these fine wires is horizontal and the other p_2 vertical.

The optical axis of the camera can be adjusted in horizontal plane by means of the three milled head screws S_1', S_2', and S_3' which support the camera. The horizontal wire p_1 is adjusted (after the instrument has been carefully leveled up) in horizontal plane by finding some easily identified point on the ground-glass plate I, which is bisected by this wire, and by gently revolving the camera about the vertical axis of the instrument; if the wire p_1 is in horizontal plane, the observed point will be seen to move along p_1 during the revolving motion of the camera. Should the bisected point appear above or

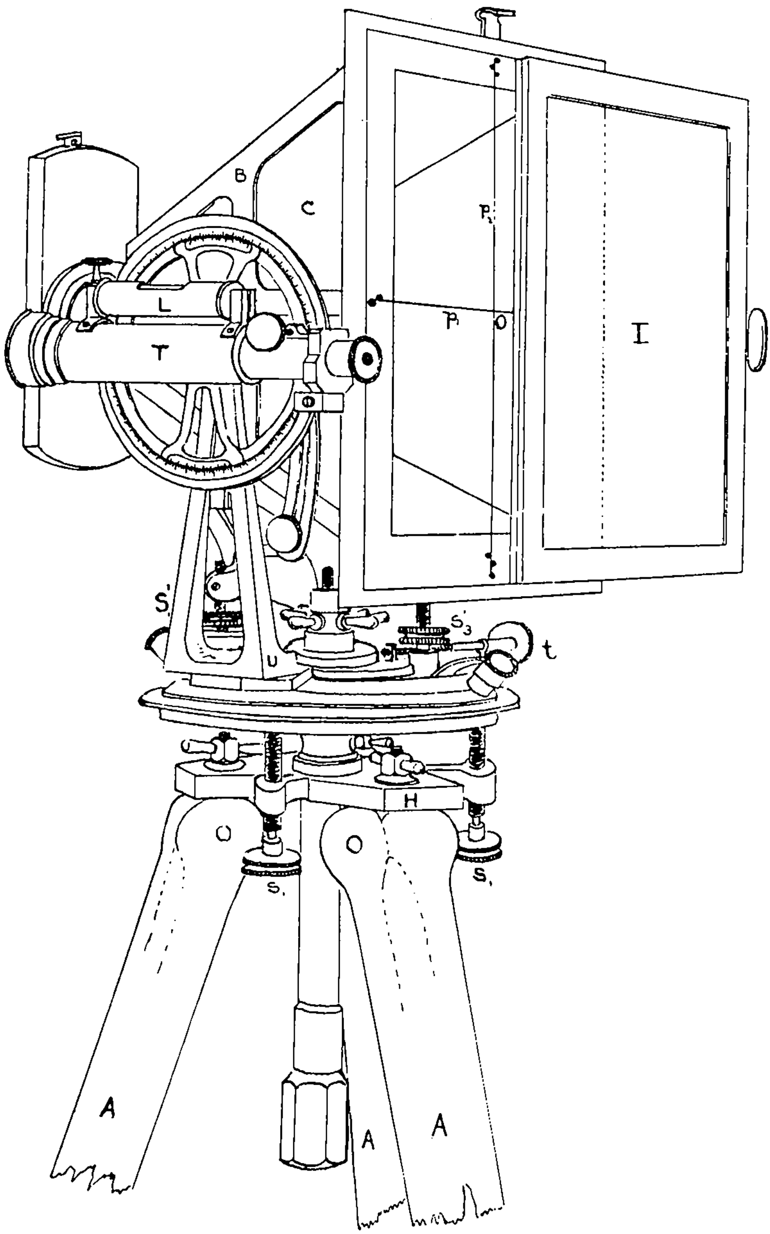


Fig A

ITALIAN PHOTOGRAMMETRIC APPARATUS.

below the wire p_1 at any time during the revolution of the camera, the same must be adjusted in horizontal plane by aid of the two forward screws S_1' and S_2' which support the camera.

The camera is provided with a short tangent screw t, by means of which the same can be slightly moved in azimuth, while the telescope and horizontal limb of the theodolite remain stationary. This will enable the observer to place the optical axis of the camera parallel to that of the telescope T, provided both are adjusted in horizontal plane. This correction is made by observing some distant point in the intersection of the cross wires of the telescope and then clamping the theodolite. The camera is now moved by means of the tangent screw t to the right or left until the same point appears in the intersection O of the two wires p_1 and p_2, it being already bisected by the camera wire p_1, as described in the preceding adjustment.

The points of the camera cross wires p_1 and p_2 appear upon every plate taken with the camera, and as these plates are vertical to the optical axis of the camera, the perspectives obtained after the camera had been adjusted, as described in the preceding paragraphs,* are in vertical plan and each shows the principal point of view O, as well as the two axes, p_1 and p_2, intersecting each other in O at right angles. The line p_1 represents the horizon of the station whence the picture was taken.

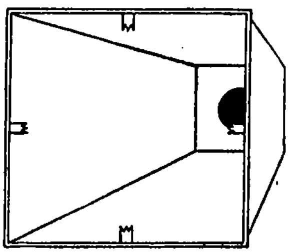


FIG. 1.

Instead of the fixed platinum wires p_1 and p_2, some recent instruments (among others those used in Canada) have a set of teeth attached to the camera, close to the plate I, as shown in figure 1. If solar prints are used for the map construction instead of the plates, this arrangement is more desirable than that of the fixed wires, as the prints will unavoidably be a little distorted. The lines p_1 and p_2 are preferably drawn in red ink on the prints after their positions in regard to the teeth have been experimentally ascertained or checked. Great care must be exercised in locating these lines properly, as they form a rectangular system of coordinates to which every point in the picture is referred during the process of the subsequent map construction. They also aid in ascertaining the value of the constant focal length of the camera.

Figure 2 shows the longitudinal section of a camera with the diaphragm AB in position, the aperture of which is 5^{mm} in diameter. Only such rays of light, emanating from a point, N, in nature, will reach the point n on plate I which form a cone around the central ray nON

*After a perspective has been taken and it appears desirable to obtain a view of the terrene immediately below (or above) the same, the construction of the instrument will permit a depression (or an elevation) of the optical axis of 30° below (or above) the horizon.

with apex in n and base in O of 5^{mm} diameter. (In our diagram, figure 2, this base will be an ellipse with 5^{mm} length for the short axis; the cone of rays emanating from a point, C, would be intercepted by the plane of diaphragm AB in a circle of 5^{mm} diameter.) The camera lenses are so focused that even for the largest diaphragm aperture used all

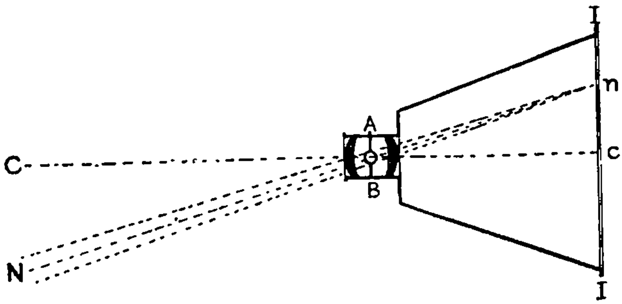


FIG. 2.

points from 10 metres to infinite distance from O, will be clearly photographed with a maximum error of 6^{mm}, as will readily be seen from the following discussion:

a=distance of object from O (10 metres to infinite distance).
f=principal focal distance (240^{mm}).
b=focal distance, variable for different lengths of a.

From the well-known relation:

$$\frac{1}{f}=\frac{1}{a}+\frac{1}{b}$$

we find

$$b=\frac{af}{a-f} \tag{1}$$

By adopting 240^{mm} as value for f, and substituting different values, from 1 metre to 300 metres, for a, in formula (1), we obtain the following values for b:

a (in m.)	1	10	20	30	40	50	75	100	200	300
b (in mm.)	315·8	245·9	242.9	241·9	241·4	241·1	240·7	240·5	240·2	240·02

The error, therefore, in maintaining the focal distance constant is 6^{mm}, if the object is 10^{m} distant from the image point; is 1^{mm} if the object is 50–100^{m} distant from the image point; is inappreciable if the object is 300^{m} or more distant from the image point.

The value $\frac{x}{2}$ of the error (distortion), maintaining a constant focal distance $= 240^{mm}$ in the photograph for points at different distances, can be seen from the following:

Assuming again that the plate I is held in a fixed position and 200^{mm} distant from the "image point" (principal focus), it is evident that the plane of image I will intersect some of the cones of rays (passing through aperture of diaphragm) in a circle (or in an ellipse) instead of intercepting their apex. From the foregoing table we see that this circle of diffused rays will increase in size with the decreasing

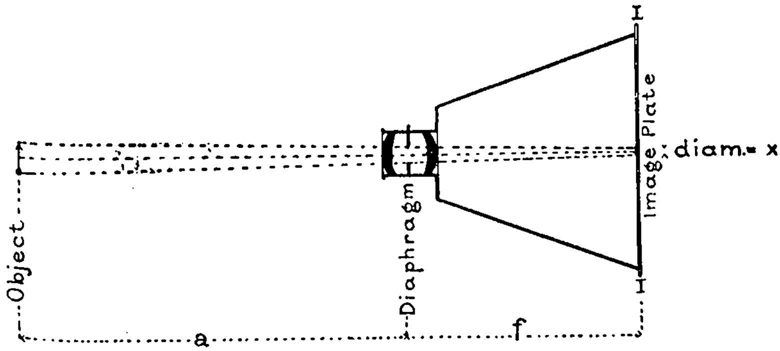


FIG. 3.

distance of the object photographed. The length of the diameter, x, of this circle (or ellipse) can be ascertained from the following relation (fig. 3):

$$x : O = f : a$$

$$x = \frac{f\,O}{a}.$$

If we assume this aperture, O, in the diaphragm, to be 5^{mm} in diameter, and assume the same values for f and a as in the preceding, we find the respective values for x as follows:

a (in *m.*) =	10	20	30	40	50	75	100	200	300	400	500	700	1000
x (in *mm.*) =	0·12	0·06	0·04	0·03	0·025	0·016	0·012	0·006	0·004	0·003	0·003	0·002	0·001

The diameter, x, of the circle (or ellipse) is evidently quite small, and the maintaining of a constant focal distance can be well carried out for all practical purposes without appreciable error.*

* The apparatus described in this paper is provided with a metal graduation plate, extending in the direction of the camera axis, and bearing a scratch to mark the focal length of the camera when focused upon objects at infinite distance. From this scratch, toward the sensitive plate of the camera, a millimetre graduation is engraved upon the plate, by means of which the observer can directly measure the focal length of the camera if the same is changed at any time for any pictures. The objective cylinder can be moved in the direction of the camera axis by means of a spiral groove cut into a second cylinder which is firmly attached to the camera, and as one turn of this screw is equal to an axial motion of 1 millimetre, the change of focus can readily be ascertained to one-tenth of a millimetre, as will be described in the following:

By revolving the external cylinder, the metal strip or plate attached to the same, glides around the outer surface of the inner cylinder (firmly attached to the camera), which bears a circular scratch lying in a plane vertical to the camera axis and passing through the constant focus (or through a point whose distance from the image plate equals the principal focal length). This circular scratch is divided into ten

The distance of the point of view from the perspective plate, the principal point of view on the perspective, and the line of horizon can always be ascertained or rectified by instrumental observations and computations, or graphically, as has been indicated, and as will be shown more fully.

We have described how the optical axes of the telescope and of the camera can be brought into two vertical parallel planes. Both can be kept in this position and yet be revolved about the vertical axis of the instrument. The horizontal limb of the theodolite is divided into 360°, with subdivisions of 20′, and by means of two verniers 30″ can be read. The vertical circle is provided with the same graduation. Thus the means are given to ascertain the azimuthal positions of the optical axis for each perspective; or, in other words, the means for orientation are thus provided for. The magnetic azimuth of the principal ray of the perspectives (i. e., direction of optical axis for each exposure), or the horizontal angle made with any other line passing through the station and some known point (e. g., trigonometrical point), can readily be ascertained.

All the perspectives which are to be used for mapping must be obtained from stations with known geographical positions. Generally trigonometrical points are selected for photographic stations, but if other points have to be occupied the elements needed (horizontal and vertical angles) to determine their positions with respect to surrounding triangulation points, can readily be observed by aid of the theodolite before leaving the camera station.

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equal parts, and the metal scale, passing over this graduated ring when the objective tube is moved toward the sensitive plate, will indicate by its position on this circular scale the number of tenths of millimetres it has moved beyond the number of millimetres which are read off on the metal scale first mentioned.

The focal length is a very important factor in all phototopographic work, and it is advisable to verify at the beginning of operations the reading of the metal scale, and if the principal focal length is changed, the difference must be entered into the notebook, so that the proper correction can be applied later on.

Chapter II.

FUNDAMENTAL PRINCIPLES OF PHOTOTOPOGRAPHY.

It was comparatively easy to obtain a close connection between the phototopographic stations and the new triangulation of Italy, as the committee who had charge of this triangulation has provided Italy with a generous and harmonious disposition of triangulation points, which have been very carefully located, their exact positions computed and permanently marked in the field, irrespective of the character of the surrounding topography or of the order of triangulation to which they belong.

This great number of triangulation points not only facilitates the application of the camera and assures the accurate determination of the panorama stations, but it also simplifies the subsequent map construction, as the greater part of the perspective contains one, two, or more triangulation points, although the instrument commands a field view of but 42° horizontally.

Thus all cardinal points of the perspectives can readily be adjusted, the pictures are easily orientated for the map construction, and the salient topographical features, deduced from the perspectives, can be frequently checked.

The camera used for this work gave pictures of 18·5 x 14^{cm} (the plates were 19 x 24·5^{cm}); the lens controlled a field of 42° horizontally and 52° vertically (26° above and 26° below the horizon). Recently, however, the lens of the camera has been exchanged for one with a principal focal length of 240^{mm} in order to use plates of 18 x 24^{cm}, which size is readily supplied by photographic dealers.

For an exact determination of the primary points of the perspectives it is necessary to measure the "coordinates" of the points in question upon the perspectives as accurately as possible. To insure good results these measurements should be made upon the negatives with a pair of dividers made especially for this purpose, by means of which the desired distances are obtained in millimetres and tenths.

Secondary and tertiary points need not be obtained with the same degree of accuracy as the primary points (needed for the control principally), and as the plates are not well adapted for the making of direct measurements thereon, nor for the marking of identical points with numerals or other characters, it will be better to take all measurements for secondary and tertiary points from the prints, as the permanent changes which such prints undergo, compared with the negatives, are, when some care is exercised, hardly ever greater than the irregularities which can be discovered in any drawing on paper, no matter how carefully made.

The perspectives (panoramic views) which are to subserve the map making, and some of which are also to be reproduced as illustrations to accompany certain maps of the Alpine ranges, are obtained by ten successive exposures. After the camera has been adjusted at a panorama station, it will have to be revolved about the vertical axis through an angle of 36° after each exposure, and, as each plate subtends a horizontal angle of 42°, the two ends of adjoining plates will lap over by a vertical strip of the horizon of 3°, or each one of two adjoining plates will have a vertical margin of 15mm width reproduced on the neighboring plate.

Figure 4 shows the horizontal projection of the positions of ten plates ($P^1 P^2$ . . . . P^{10}) at a panorama station. $VP^1 = VP^2 =$ . . . . $VP^{10} = f =$ principal focal length of camera $= 244 \cdot 5$mm.

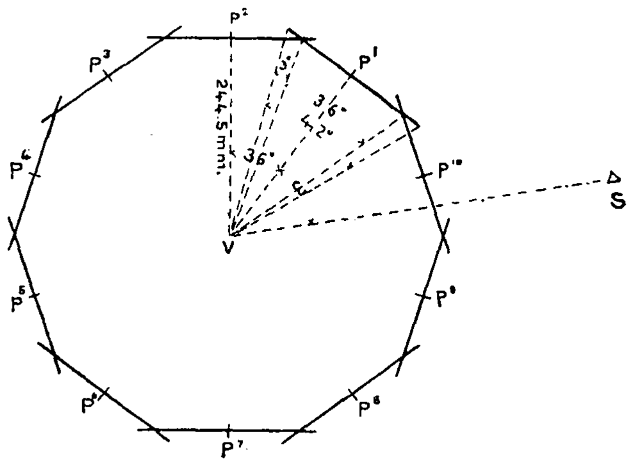


FIG. 4.

The common margins of two adjoining plates serve principally to ascertain whether the adjustments of the instrument have been disturbed during the occupancy of a station. Should, for instance, the position of the horizontal thread have been disturbed by some cause, this would be shown by different results when measuring the ordinates of identical points of two adjoining margins. These margins also serve admirably for the correct trimming of the edges of adjoining pictures if they are to be fitted together for the panorama.

The horizontal projection of the ten plates exposed from one panorama station is a regular decagon (fig. 4), with a radius of the inscribed circle equal to the principal focal length of the camera.

After the position of one panoramic view has been found on the map (i. e., after the angle ω, figure 4, has been plotted from the station V to the triangulation point, shown in the figure), the orientation of all the other plates is accomplished by adding 36°, 72°, 108° . . . . to the angle ω; thus the entire decagon can be plotted after the horizontal projection of plate P^1 has been oriented with reference to the station V and the triangulation point plotted on the map.

The orientation in horizontal plan having been made, it will be an easy matter to draw lines of direction from the platted camera station to points photographed in the panorama.

The plate $m\ m'\ n'\ n$, figure 5, may represent a vertical and oriented perspective and OO' be the line of the horizon of said plate. V is the point of view; ω' represents the angle of orientation for this plate in

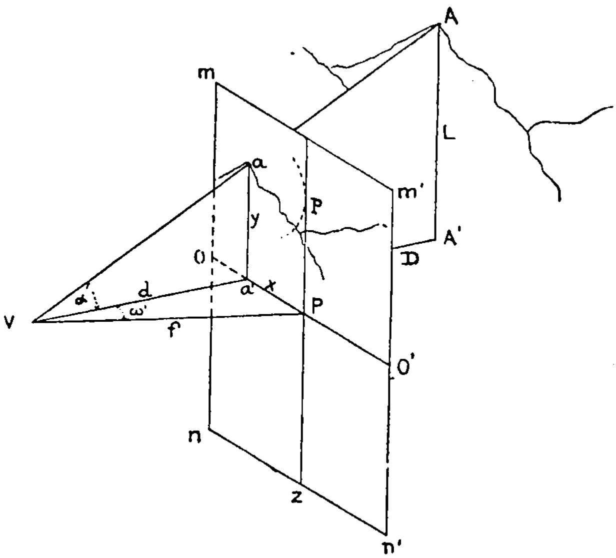


FIG. 5.

reference to the secondary point A (i. e., the horizontal angle between the direction from V to a' and the vertical ray $V\,P$ to principal point of view P of the perspective).

$VP=f$ is the principal focal length. If A represents a point of the terrene in space and a is the representation of the same on the plate, and if the vertical $a\ a'$ has been drawn from a upon the line of horizon $O\,O'$, then $P\,a'=x$ will be the abscissa of the point a.

From the rectangular triangle $V\ Pa'$ we find:

$$x = f \tan \omega'$$

Figure 6 shows this triangle as $V'\ P'\ a'$ in the plane of the drawing; P'= horizontal projection of the principal point of view (P in fig. 5) of the horizontal projection $O\ O'$ of the plate ($m\ m'\ n'\ n$ in fig. 5) which had been oriented in the plane of the map by the direction $V\cdot S$ (or rather by the horizontal angle ω, fig. 6) to the triangulation point S.

In order to draw the horizontal projection of the ray from V to the point A (fig. 5) the distance $P' a'$, (fig. 6) $= P a'$ (fig. 5) $= x$ is laid off upon $O O'$ (fig. 6) from P', in the sense of direction to A (whether to the right or left of the vertical thread). This abscissa x is taken from the picture by means of a pair of dividers as the radius $a p$ (fig. 5). After drawing a line from V' through a' (fig. 6), this line $V' a'$ will be the horizontal direction of the ray $V A$ (fig. 5).

The position of A' (horizontal projection of A plotted on the map) will be in the intersection of two or more lines of direction, obtained, in a similar manner, from other pictures containing a and taken from other stations. The same refers to all other points of the terrene, if their pictures can be identified upon plates taken from different panorama stations.

Every perspective containing the picture of a triangulation point will give evidence of the grade of accuracy for the relative plotted positions of the horizontal projections of the point of view and of the perspective, as well as for the orientation of the plate, by laying off upon $O O'$ (fig. 6) the abscissa x' of the triangulation point S and drawing the line $V' S'$, which should bisect the plotted point S.

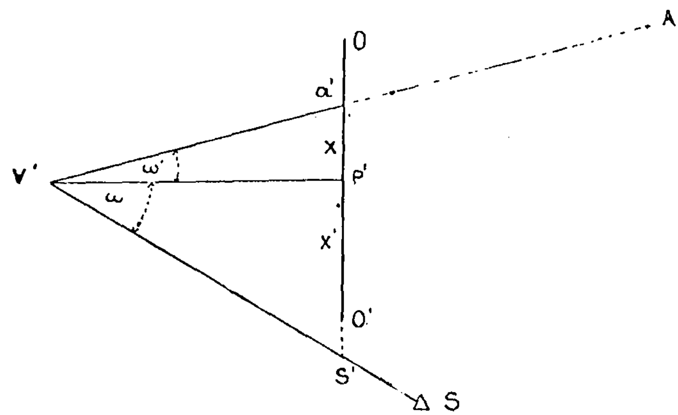


FIG. 6.

If the horizontal projection of a point, A, has been determined by the intersection of two rays from two stations, V^1 and V^2, a check is obtained if a third plate, taken from another camera station, V^3, shows the same point, as all rays to the same object, A, must intersect each other in the same point on the map.

The hypsometric determinations of the terrene can readily be accomplished after the selected points have been determined and plotted in horizontal plan in the foregoing manner. If the elevation of the station V (fig. 5) is known, the elevation of the line of horizon $O O'$ on the plate $m m' n' n$ can be easily obtained by adding the height of the instrument to the elevation of the station V. The elevations of all the points on the plate which are bisected by the line $O\ O'$ have the same elevation as the optical axis of the instrument at the station V, disregarding the effects of curvature and refraction. The apparent elevations of other secondary points, selected from the plate for the construction of the map, are obtained by determining their elevations above or their depression below the line $O\ O'$.

If D = horizontal distance of a point A from the station V;
= VA' (to be measured on the map) (fig. 5);
L = difference of elevation between point A and station V;
= $A\ A'$ (A' is the projection of A upon the horizontal plane through optical axis of instrument at station V);
d = horizontal distance of the picture of A from V;

Then we have from the similar triangles $Va'a$ and $VA'A$ the relation:

$$L : D = y : d \tag{1}$$

$$L = \frac{Dy}{d} \tag{2}$$

From the rectangular triangle VPa' we find:

$$d = \frac{f}{\cos\omega'} = f\sec\omega' \tag{2^a}$$

whence:

$$L = \frac{Dy}{f\sec\omega'} \tag{3}$$

Should the point A be bisected by the vertical thread then $\omega' = O$ and $\sec\omega' = 1$, or:

$$L = \frac{Dy}{f} \tag{3^a}$$

(Formula 3ª would answer for all points of the perspective if the image plate were a cylindrical surface of radius $= f$ instead of being a plane, if the decagon were a circle.)

The differences of elevation, taken from the perspectives, are positive or negative according to the relative positions of their points in respect to the line of horizon $O\ O'$, whether above or below the same. In order to obtain the apparent elevations of these points above mean sea level their ordinates must be added to or subtracted from the elevation of the camera station. Two graphical instruments have been constructed based upon the formulas (1) and (2). The one serves to locate the points in horizontal plan by intersections after their abscissæ (x) have been measured upon the perspectives. The other serves to ascertain the differences of their elevations compared with the height of instrument after the abscissæ (x) and the ordinates (y) of the points have been measured upon the perspectives.

The use of these auxiliary instruments enables the draftsman to dispense with the construction of the decagons (representing the horizontal projections of the panoramas about the plotted camera stations), and as these polygons must be very carefully drawn, the dispensing with them altogether, especially if the area to be plotted is extensive and the stations numerous, will save much time and labor. It is particularly desirable to avoid the drawing of these polygons if the plotting is to be done on 1 to 25,000 or 1 to 50,000 scales, as the numerous decagons will intermingle in such a manner that great care and painstaking must be exercised to avoid confusion in selecting the proper lines from the intricate network of lines upon the face of the drawing. An attempt has been made to overcome this difficulty by drawing the different polygons in lines of different colors, but even this expedient failed when plotting on small scales.

Also, the hypsometric determinations of secondary points can be checked by comparing the elevations obtained in the preceding manner with the results obtained in the same way from other stations and on other plates.

Furthermore, any perspective containing one or more triangulation points (the elevations of which are determined by double zenith distances, or by any other instrumental method) will serve to check the horizontal adjustments of the instrument during the exposure of such plate, by comparing the elevation obtained from the perspective with the elevation of the trigonometrical point obtained by former instrumental measurements. Should the elevations of the triangulation points be unknown, this check can still be made by using the vertical circle of the instrument and measuring the vertical angles (α, fig. 5) from the panorama station to the surrounding triangulation points. By observing vertical angles from every camera station, a check on the measured values of the ordinates on the perspectives is obtained, inasmuch as then these ordinates can be computed and compared with those obtained by direct measurements on the perspectives.

From the triangles $VA'A$ and $Va'a$ we find (fig. 5):

$$\tan AVA' = \tan \alpha = \frac{L}{D} = \frac{y}{d}$$

According to formula (2″) we had

$$d = \frac{f}{\cos \omega'}$$

therefore:

$$\tan \alpha = \frac{y}{f} \cos \omega' \tag{4}$$

where ω' is the horizontal angle formed at V by the vertical plane containing the line of direction from the camera station V to the triangulation point and the vertical plane containing the line drawn from V to the principal point P of the perspective, which horizontal angle can also be measured directly on the horizontal limb of the camera theodolite in the field before leaving the station. If these different values are not in accord, the horizontal line on the perspective must be adjusted by determining the value of the ordinate y by aid of the following formula (5) derived from (4):

$$y = \frac{f}{\cos \omega'} \tan \alpha \tag{5}$$

From the preceding the necessity of the precise determination of the value for f is evident, and this value can readily be found if the area to be surveyed is provided with a number of triangulation points, marked by signals, and if the secondary points are of such a character (for instance, when surveying mountain ranges) that the differences of their elevations, compared with the elevation of the panorama station, are sufficiently great to give their ordinates on the perspectives, lengths sufficient to be readily measured. This will permit the determination of f by means of the line of horizon $O\,O'$, the latter being an element obtainable from the perspectives with a great degree of accuracy.

The instrument is placed over any well-determined point and adjusted, then it is turned in azimuth until the vertical thread bisects a geodetic point, which can readily be identified upon the image plate (desirable also that the ordinate y of such point be sufficiently long to assure a correct measurement).

Then will be given: the difference of elevation of bisected point and panorama station $= L$; the horizontal distance between these two $= D$, and $y =$ ordinate of bisected point; and from equation (3^a) we find for

$$f = \frac{Dy}{L}$$

a fairly accurate value, if the adjustments for securing the horizontal position of the instrument were carefully made and if the ordinate y was measured upon the negative plate correct within 0.1^{mm}.

The value for f can also be found, if the perspective contains several other points (besides the one bisected by the vertical thread) which can readily be identified, by measuring both the vertical and horizontal angles (by observing all the points, including the one bisected). The values for α and ω thus obtained, including the values of the abscissæ (x) and ordinates (y), measured upon the image or negative plate, will enable us to compute f by means of equations (1) and (4). By using the mean of these different determinations for f the computation, based upon the new values for x and y, can be repeated until an agreement is found.

Although the Italian pictures commanded an angle of but 42°, the greater part of them contain one or more triangulation points. In all similar cases, simultaneously with the determination of the value f (principal focal length), it can be ascertained whether the picture of the intersection of the crosswires coincides with the principal point of view P upon the perspective (fig. 7):

S' and S represent the pictures of two triangulation points upon the perspective $m\,n$;
V = station point or point of view;
$S'\,O'$ and $S\,O$ = vertical lines from S' and S upon the line of horizon $O\,O'$ = ordinates y' and y of these two points S' and S.

It is desired to find $V P$ (vertical to plane $m\ n$) and the position of the point P in regard to O and O' or the abscissæ x and x' of the two triangulation points S and S'.

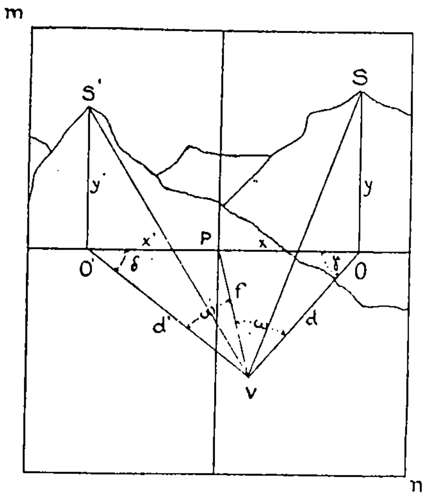


FIG. 7.

L and L' represent the differences of elevation between the camera station (V) or the horizon (OO') and the triangulation points S and S'. D and D' are the horizontal distances of S and S' from the camera station (V).

L, L', D, D', as well as y and y', are known or can be measured upon the chart projection and negative plate; therefore the horizontal distances d and d' of the pictured triangulation points S and S' from the point of view V can be computed from the following equations (see fig. 5):

$$d = \frac{D\,y}{L}$$

$$d' = \frac{D'\,y'}{L'}$$

The horizontal angle $O'\ V\ O$ ($= \omega + \omega'$) being observed in the field, the other two angles, γ and δ, of the horizontal triangle $O'\ V\ O$ (fig. 7) can be computed as follows:

$$\tan \frac{\gamma - \delta}{2} = \frac{d' - d}{d + d'} \cot \frac{O'\ V\ O}{2}$$

By substituting

$$N \text{ for } \frac{\gamma - \delta}{2} \text{ and}$$

$$M \text{ for } \frac{\gamma + \delta}{2} = 90^\circ - \frac{O'\ V\ O}{2}$$

we will have:

$$\gamma = M + N$$
$$\delta = M - N$$

From the two triangles $O'\ P\ V$ and $O\ P\ V$ (both are rectangular at P) we find (fig. 7):

$$f = d \sin \gamma = d' \sin \delta$$
$$x = f \cot \gamma\ ; \quad x' = f \cot \delta$$

also the angles of orientation

$$\omega = 90^\circ - \gamma$$
$$\omega' = 90^\circ - \delta$$

The sum $x + x'$ must be equal to the value $O\ O'$ found by careful measurement upon the plate, and it must also be equal to the value of $O\ O'$ obtained from the following formula:

$$OO' = \frac{(d + d') \sin \frac{O'VO}{2}}{\cos \frac{\delta - \gamma}{2}}$$

Should the horizontal angle OVO' not have been measured in the field, then the angles γ and δ can be computed by carefully measuring $O'O$ on the negative plate and using the following well-known formulas:

$$\tan \frac{\delta}{2} = \sqrt{\frac{(p - d')\ (p - OO')}{p\ (p - d)}} \text{ and}$$

$$\tan \frac{\gamma}{2} = \sqrt{\frac{(p - d)\ (p - OO')}{p\ (p - d')}}$$

where $p = \dfrac{d + d' + OO'}{2}$

The angles of elevation α and α', which are either obtained by direct measurement in the field or computed from the formulas:

$$\tan \alpha = \frac{L}{D}$$

$$\tan \alpha' = \frac{L'}{D'}$$

serve to obtain check values for the values of y and y', measured upon the negative plate, by using the formulas:

$$y = \frac{f}{\cos \omega} \tan \alpha \text{ and}$$

$$y' = \frac{f}{\cos \omega'} \tan \alpha'$$

The value for f in above formulas being the same as found (page 60) from the equation:

$$f = d \sin \gamma = d' \sin \delta.$$

By repeating the computation with these new values for y and y' the true value for f can be obtained very closely. For all practical purposes, however, it will suffice to take several pictures with a constant focal length, and to take the mean value of the different f determined from these pictures.*

* A comparison of the value read off on the graduated metal plate on the objective cylinder with the result obtained by computation will give the correction to be applied to the reading on said graduation.

Examples showing the application of the methods described in the foregoing pages.

I.

In the panorama obtained from the "Punta Percia," September 19, 1884 (this peak is on the divide between the valleys of the "Rhêmes" and the "Valsavaranche"), two stations, "Punta Rouletta" and "Gran Punta di Nomenon," of the new Italian geodetic triangulation appear upon the same plate (see fig. 8).

The following values are given for the computation:

Elevation of Punta Rouletta $= 3384{\cdot}10^{m}$ }
Elevation of Gran Punta di Nomenon $= 3488{\cdot}42^{m}$ } Taken from the catalogue of triangulation points.

Elevation of Punta Percia $= 3202{\cdot}3 =$ horizon of panorama station.

Distance: Percia-Rouletta $= D = 3250^{m}$ }
Distance: Percia-Nomenon $= D' = 9720^{m}$ } Measured graphically upon the projection on drawing board, scale: $\frac{1}{50000}$.

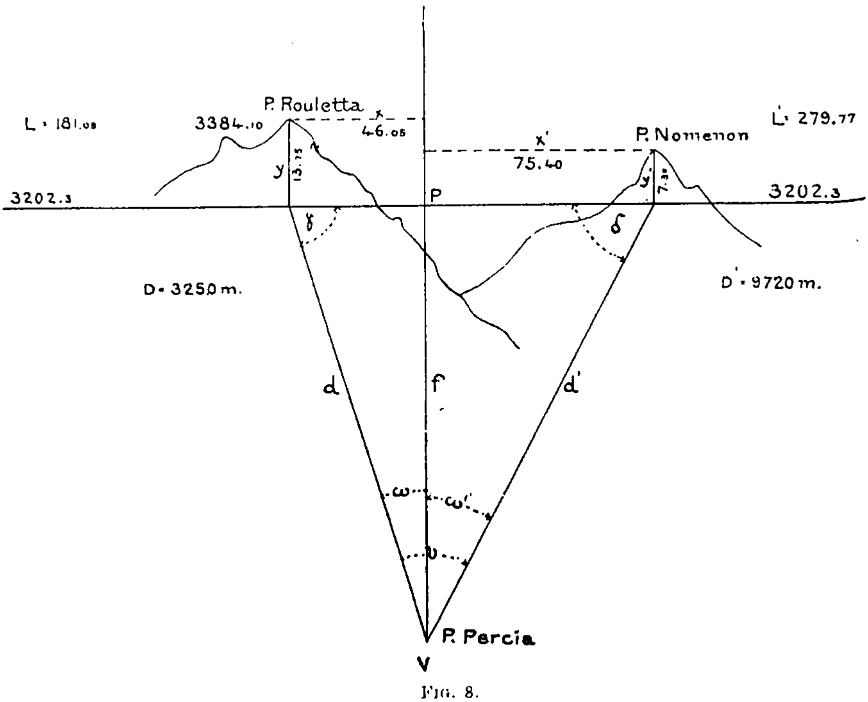


FIG. 8.

The angle V (fig. 8) at Percia included by the horizontal directions to Rouletta and Nomenon $= 28^{\circ}\ 02'\ 30''$.

It is desired to find:

(1) Focal distance $= f$, the preliminary value, read off on the graduation on plate attached to objective cylinder, is found to be about $244{\cdot}5^{mm}$.

(2) The position of the principal point of view, which is fixed by the determination of the abscissæ x and x'.

(3) The position of the line of horizon, which will be located by ascertaining the values for y and y'.

The difference in elevation of Percia and Rouletta, according to the given data, is:

$$3384{\cdot}10 - 3202{\cdot}30 = 181{\cdot}80^{m}$$

The difference in elevation between Percia and Nomenon is

$$3488{\cdot}42 - 3202{\cdot}30 = 286{\cdot}12^{m}.$$

If we consider that the distances D and D' are great, and that the above values are the apparent elevations, we will have to make a correction for curvature and refraction to obtain the true values of L and L', as follows:

$$\begin{aligned} \text{Difference of elevations: Percia–Rouletta,} &= 181{\cdot}80^{m} \\ \text{Correction for curvature and refraction,} &= -0{\cdot}70^{m} \\ L &= 181{\cdot}09^{m} \\ \text{Difference of elevations: Percia–Nomenon,} &= 286{\cdot}12^{m} \\ \text{Correction for curvature and refraction,} &= -6{\cdot}35^{m} \\ L' &= 279{\cdot}77^{m} \end{aligned}$$

By measuring the ordinates (y and y') and the abscissæ (x and x') upon the negative plate with a millimetre scale, provided with a vernier, which enables the computer to read $\frac{5}{100}$ mm (the vernier being divided to read $\frac{1}{20}$ of the graduation), we find:

The coordinates of Punta–Rouletta: $x = 46{\cdot}05^{mm}$ $y = 13{\cdot}75^{mm}$
The coordinates of Punta–Nomenon: $x' = 75{\cdot}40^{mm}$ $y' = 7{\cdot}30^{mm}$

The value of d is found from the following formula:

$$d = \frac{Dy}{L} \text{ (see page 60).}$$

$$\begin{aligned} \log D = \log 3250 &= 3{\cdot}5118834 \\ \log y = \log 0{\cdot}01375 &= 8{\cdot}1383027 \\ \text{co.log } L = \text{co.log } 181{\cdot}09 &= 7{\cdot}7421055 \\ \hline \log d &= 9{\cdot}3922916 \\ d &= 0{\cdot}24677^{m} = 246{\cdot}77^{mm} \end{aligned}$$

d is similarly found:

$$\begin{aligned} \log D' = \log 9720 &= 3{\cdot}9876663 \\ \log y' = \log 0{\cdot}00730 &= 7{\cdot}8633229 \\ \text{co.log } L' = \text{co.log } 279{\cdot}77 &= 7{\cdot}5531989 \\ \hline \log d' &= 9{\cdot}4041881 \\ d' = 0{\cdot}25362^{m} &= 253{\cdot}62^{mm} \\ d + d' &= 0{\cdot}50039^{m} \\ d' - d &= 0{\cdot}00685^{m} \end{aligned}$$

The angles γ and δ (fig. 8) are computed by aid of the formula:

$$\tan \frac{\gamma-\delta}{2} = \frac{d'-d}{d+d'} \cot. \frac{O'VO}{2} \text{ (page 60)}$$

as follows:

$$V = 28^\circ\ 02'\ 30''\ ; \qquad \frac{V}{2} = 14^\circ\ 01'\ 15''$$

$$\gamma+\delta = 180^\circ - V = 151^\circ\ 57'\ 30''\ ; \ \frac{\gamma+\delta}{2} = 75^\circ\ 58'\ 45'' = M$$

$$\log (d'-d) = \log 0{\cdot}00685 \qquad = 7{\cdot}8356906$$

$$\log.\cot \frac{V}{2} = \log \cot\ 14^\circ\ 01'\ 15'' = 0{\cdot}6025567$$

$$\text{co.log } (d+d') = \text{co.log } 0{\cdot}50039 \qquad = 0{\cdot}3006914$$

$$\log \frac{\gamma-\delta}{2} \qquad = 8{\cdot}7389387$$

$$\frac{\gamma-\delta}{2} = 3^\circ\ 08'\ 16''{\cdot}1 = N$$

Hence $M+N = \gamma = 79^\circ\ 07'\ 01''{\cdot}1$ and
$M-N = \delta = 72^\circ\ 50'\ 28''{\cdot}9$

The computation of f based on the formulas (page 60)

$$f = d \sin \gamma \text{ and } f = d' \sin \delta,$$

gives the following two values:

$$\log d = 9{\cdot}3922916 \qquad \log d' = 9{\cdot}4041881$$
$$\log \sin \gamma = 9{\cdot}9921180 \qquad \log \sin \delta = 9{\cdot}9802269$$
$$\log f = 9{\cdot}3844096 \qquad \log f = 9{\cdot}3844150$$
$$f = 0{\cdot}242331^{\text{m}} \qquad f = 0{\cdot}242334^{\text{m}}$$

mean value for $f = 242{\cdot}332^{\text{mm}}$.

The abscissæ x and x' are computed by aid of the two formulas (page 60)

$$x = f \tan \omega \qquad x' = f \tan \omega'$$
$$\omega = 90^\circ - \gamma = 10^\circ\ 52'\ 58''.9 \qquad \omega' = 90^\circ - \delta = 17^\circ\ 09'\ 31''.1$$
$$\log f = 9{\cdot}3844123 \text{ (mean log)} \qquad \log f' = 9{\cdot}3844123 \text{ (mean log)}$$
$$\log \tan \omega = 9{\cdot}2838945 \qquad \log \tan \omega' = 9{\cdot}4896221$$
$$\log x = 8{\cdot}6683068 \qquad \log x' = 8{\cdot}8740344$$
$$x = 46{\cdot}59^{\text{mm}} \qquad x' = 74{\cdot}82^{\text{mm}}$$
$$x \text{ measured on plate} = 46{\cdot}05^{\text{mm}} \qquad \text{measured } x' = 75{\cdot}40^{\text{mm}}$$
$$\text{diff.} = {\cdot}54^{\text{mm}} \qquad \text{diff.} = {\cdot}42^{\text{mm}}$$

Computation of the ordinates y and y':

$$y=\frac{f}{\cos\omega}\tan\alpha \qquad y'=\frac{f}{\cos\omega'}\tan\alpha' \quad \text{(page 61).}$$

α = angle of elevation of Punta Rouletta = 3° 11′ 30″ ⎫ from station Percia. See model in Supplement.
α' = angle of elevation of Punta Nomenon = 1° 38′ 30″ ⎭

$\log f =$	9·3844123	$\log f =$	9·3844123
$\log\tan\alpha =$	8·7463444	$\log\tan\alpha' =$	8.4572812
co.log $\cos\omega =$	0·0078820	co.log $\cos\omega' =$	0·0197731
$\log y =$	8·1386387	$\log y' =$	7·8614666
$y =$	13·761mm	$y' =$	7·269mm
y measured on plate =	13·75mm	measured $y' =$	7·30mm
diff. =	0.01mm	diff. =	0·03mm

II.

Owing to the fact that the distances D and D' in the example treated in Subject I are large, while the ordinates y and y' are quite small (due to the small difference in the elevations of the two points Rouletta and Nomenon compared with the camera station Percia), it will be preferable first to determine f by means of the abscissæ and then to compute the values for the ordinates (y and y'), based upon this value of f and the observed angles of orientation ω_1 and ω_1' (see Supplement, remarks to station Percia).

The direction to the main point of view P of the perspective containing the pictures of Rouletta and Nomenon is = 350° 00′ 00″

Direction to point Nomenon (signal) = 332° 42′ 00″

Direction to point Rouletta (signal) = 0° 44′ 30″

Direction to Rouletta = 360° 44′ 30″

Direction to point P = 350° 00′ 00″

Hence, ω_1 = 10° 44′ 30″

Direction to point P = 350° 00′ 00″

Direction to point Nomenon = 332° 42′ 00″

Hence, ω_1' = 17° 18′ 00″

(a) Computation of f:

$$f=\frac{x}{\tan\omega_1}$$

$\log x = \log 46{\cdot}05 = 1{\cdot}6632296$

co.log $\tan\omega_1$ = co.log tan 10° 44′ 30″ = 0·7219207

$\log f = 2{\cdot}3851503$

$f = 242{\cdot}745$mm

(*b*)

$$f = \frac{x}{\tan \omega_1'}$$

$$\log x' = \log 75{\cdot}40 = 1{\cdot}8773713$$
$$\text{co.log} \tan \omega_1' = \text{co.log} \tan 17^\circ\ 18'\ 00'' = 0{\cdot}5065903$$

$$\log f = 2{\cdot}3839616$$
$$f = 242{\cdot}082^{\text{mm}}$$

$$\text{Mean value for } f = 242{\cdot}41^{\text{mm}}$$
$$\text{and from computation I we had } f = 242{\cdot}33^{\text{mm}}$$

$$\text{diff.} = 0{\cdot}08^{\text{mm}}$$

Computation of the ordinates y and y':

$$y = \frac{f}{\cos \omega_1} \tan \alpha$$

$$\log f = \log 242{\cdot}41 = 2{\cdot}3845505$$
$$\log \tan \alpha = \log \tan\ 3^\circ\ 11'\ 30'' = 8{\cdot}7463444$$
$$\text{co.log} \cos \omega_1 = \text{co.log} \cos 10^\circ\ 44'\ 30'' = 0.0076774$$

$$\log y = 1{\cdot}1385723$$
$$y = 13{\cdot}758^{\text{mm}}$$
$$\text{and computation I gave } y = 13{\cdot}761^{\text{mm}}$$

$$\text{diff.} = 0{\cdot}003^{\text{mm}}$$

$$y' = \frac{f}{\cos \omega'} \tan \alpha'$$

$$\log f = \log 242{\cdot}41 = 2{\cdot}3845505$$
$$\log \tan \alpha' = \log \tan 1^\circ\ 38'\ 30'' = 8{\cdot}4572812$$
$$\text{co.log} \cos \omega' = \text{co.log} \cos 17^\circ\ 18'\ 00'' = 0{\cdot}0201054$$

$$\log y' = 0{\cdot}8619371$$
$$y' = 7{\cdot}277^{\text{mm}}$$
$$\text{and computation I gave: } y' = 7{\cdot}269^{\text{mm}}$$

$$\text{diff.} = 0{\cdot}008^{\text{mm}}$$

III.

The following computation is of greater interest, the camera station having been selected over a trigonometrical point of the Italian geodetic triangulation system, thus admitting a direct comparison between the elements of the perspective and the exact values of these same elements, taken from the data of the triangulation work.

In the round of perspectives, obtained on September 21, 1884, vertically above the trigonometrical point near the Royal Hunting Lodge of Valsavaranche, there is one plate (P^5) which contains the pictures of two triangulation points, "Punta Ruja" and "Gran Punta di Nomenon," of the new geodetic triangulation.

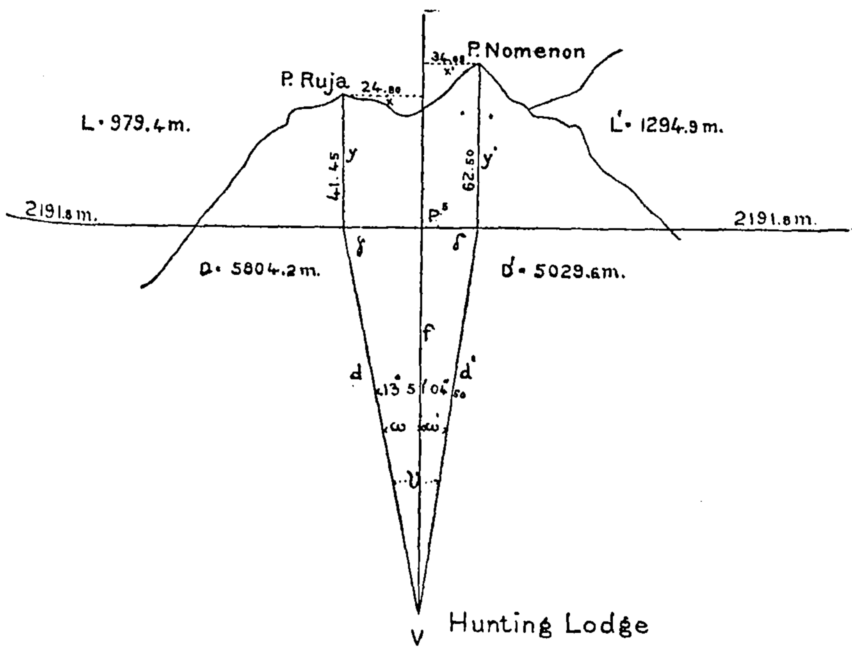


FIG. 9.

The following data are given for the computation:

Elevation of Punta Ruja (signal) $= 3173{\cdot}5^{m}$

Elevation of Gran Punta di Nomenon (signal) $= 3488{\cdot}4^{m}$

Elevation of the horizon of panorama station near the Royal Hunting Lodge of Valsavaranche $= 2191{\cdot}8^{m}$

The triangle side Hunting Lodge — Ruja $= D$ $= 5804{\cdot}2^{m}$

The triangle side Hunting Lodge — Nomenon $= D'$ $= 5029{\cdot}6^{m}$

The angle, V, between Ruja and Nomenon is $= 13^{\circ}\ 51'\ 04''{\cdot}50$. (See fig. 9.)

It is desired to find:

(1) The focal length, f, approximately found by reading the graduation on objective tube, $= 244{\cdot}5^{mm}$.

(2) The position of the principal point of view (located by the abscissæ x and x').

(3) The true position of the line of horizon (located by the ordinates y and y').

By careful measurements (made as in the preceding) we find:

The coordinates of Punta Ruja: $x = 24{\cdot}80^{mm}$; $y = 41{\cdot}45^{mm}$

The coordinates of Punta Nomenon: $x' = 34{\cdot}05^{mm}$; $y' = 63{\cdot}50^{mm}$

Elevation of Punta Ruja	$= 3173.5^{m}$
Elevation of point V	$= 2191{\cdot}8^{m}$
Apparent difference of elevation	$= 981{\cdot}7^{m}$
Correction for refraction and curvature	$= -\ 2{\cdot}3^{m}$
True difference of elevation	$= 979{\cdot}4^{m} = L$
Elevation of Punta di Nomenon	$= 3488{\cdot}4^{m}$
Elevation of point of view (V)	$= 2191{\cdot}8^{m}$
Apparent difference of elevation	$= 1296{\cdot}6^{m}$
Correction for curvature and refraction	$= -\ 1{\cdot}7^{m}$
True difference of elevation	$= 1294{\cdot}9^{m} = L'$

Computation of $d = \frac{Dy}{L}$

$$\begin{aligned} \log D &= \log\ 5804{\cdot}2 &&= 3{\cdot}7637424 \\ \log y &= \log\ 0{\cdot}04145 &&= 8{\cdot}6175245 \\ \text{co.}\log L &= \text{co.}\log\ 979{\cdot}4 &&= 7{\cdot}0090399 \\ & & \log d &= 9{\cdot}3903068 \\ & & d &= 0{\cdot}245644^{m} = 245{\cdot}644^{mm} \end{aligned}$$

Computation of $d' = \frac{D'y'}{L'}$

$$\begin{aligned} \log D' &= \log\ 5029{\cdot}6 &&= 3{\cdot}7015334 \\ \log y' &= \log\ 0{\cdot}06350 &&= 8{\cdot}8027737 \\ \text{co.}\log L' &= \text{co.}\log\ 1294{\cdot}9 &&= 6{\cdot}8877638 \\ & & \log d' &= 9{\cdot}3920709 \\ & & d' &= 0{\cdot}246644^{m} = 246{\cdot}644^{mm} \end{aligned}$$

$$d + d' = 492{\cdot}29^{mm}$$
$$d' - d = 1{\cdot}00^{mm}$$

Computations ' the angles γ and δ:

$$\tan \frac{\gamma - \delta}{2} = \frac{d' - d}{d + d'} \cot \frac{V}{2}$$

$$V = 13^{\circ}\ 51'\ 04''{\cdot}50$$

$$\frac{V}{2} = 6^{\circ}\ 55'\ 32''{\cdot}25$$

$$\gamma + \delta = 180^{\circ} - V = 166^{\circ}\ 08'\ 55''{\cdot}50$$

$$\frac{\gamma + \delta}{2} = 83^{\circ}\ 04'\ 27''{\cdot}75 = M$$

$$\log (d' - d) = \log \quad 0{\cdot}00100 \quad = 7{\cdot}0000000$$
$$\log \cot \frac{V}{2} = \log \cot 6^\circ\ 55'\ 32''{\cdot}3 = 0{\cdot}9155406$$
$$\text{co.log}\ (d + d') = \text{co.log} \quad 0{\cdot}49229 \quad = 0{\cdot}3077790$$

$$\log \tan \frac{\gamma - \delta}{2} = 8{\cdot}2233196$$

$$\frac{\gamma - \delta}{2} = 0^\circ\ 57'\ 29''{\cdot}10 = N$$

$$M + N = 84^\circ\ 01'\ 56''{\cdot}9 = \gamma$$
$$M - N = 82^\circ\ 06'\ 58''{\cdot}7 = \delta$$

Computation of $f = d \sin \gamma = d' \sin \delta$

$$\log d = \quad \log 0{\cdot}24564 \quad = 9{\cdot}3903068$$
$$\log \sin \gamma = \log \sin 84^\circ\ 01'\ 56''{\cdot}9 = 9{\cdot}9976401$$

$$\log f = 9{\cdot}3879469$$
$$f = 0{\cdot}244313$$
$$\log d' = \quad \log 0{\cdot}24664 \quad = 9{\cdot}3920709$$
$$\log \sin \delta = \log \sin 82^\circ\ 06'\ 58''{\cdot}7 = 9{\cdot}9958757$$

$$\log f = 9{\cdot}3879466$$
$$f = 0{\cdot}244313^{\text{m}}$$

Mean value for $f = 244{\cdot}31^{\text{mm}}$

Computation of the abscissæ:

$$x = f \tan \omega$$
$$\omega = 90^\circ - \gamma = 5^\circ\ 58'\ 03''{\cdot}1$$
$$\log\ (\text{mean value of}\ f) = 9{\cdot}3879468$$
$$\log \tan \omega = \log \tan 5^\circ\ 58'\ 03''{\cdot}1 = 9{\cdot}0192462$$

$$\log x = 8{\cdot}4071930$$
$$x = 0{\cdot}025538^{\text{m}} = 25{\cdot}54^{\text{mm}}$$

$$x' = f \tan \omega'$$
$$\omega' = 90^\circ - \delta = 7^\circ\ 53'\ 01''{\cdot}3$$
$$\log f = 9{\cdot}3879468$$
$$\log \tan \omega' = \log \tan 7^\circ\ 53'\ 01''{\cdot}3 = 9{\cdot}1413601$$

$$\log x' = 8{\cdot}5293069$$
$$x' = 0{\cdot}033830 = 33{\cdot}83^{\text{mm}}$$

$x = 25{\cdot}54^{\text{mm}}$	$x' = 33{\cdot}83^{\text{mm}}$
measured $x = 24{\cdot}80^{\text{mm}}$	measured $x' = 34{\cdot}05^{\text{mm}}$
diff. $= 0{\cdot}74^{\text{mm}}$	diff. $= 0{\cdot}22^{\text{mm}}$

Mean difference $= 0.48^{\text{mm}}$ = correction for principal point of view (vertical thread) on the plate.

IV.

The plate (P^5) treated of, under Subject III, is the fifth perspective of the ten plates forming the panorama. These perspectives were obtained, as previously mentioned, by moving the optical axis of the camera successively by 36° in horizontal plan, as indicated in the examples for phototopographic stations in the Supplement.

The orientation for the entire panorama (set of ten plates) was obtained by the exposure of the first plate (P') by directing the optical axis for this exposure to the trigonometrical point "Punta Chandellei" (i. e., by bisecting the signal at Punta Chandellei with the vertical thread of the camera). (See fig. 10.)

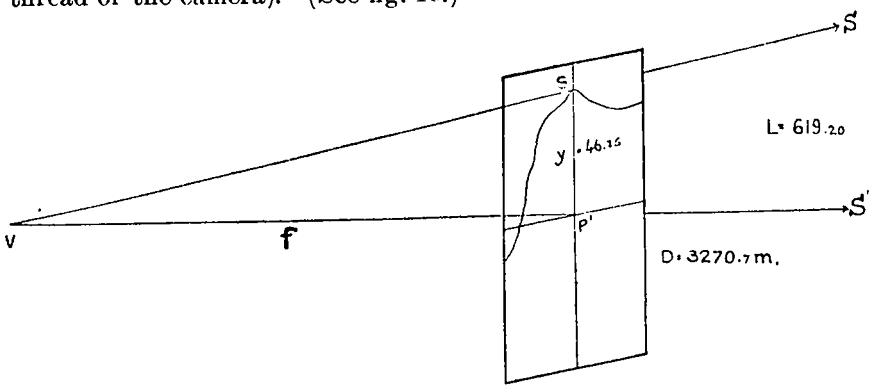


FIG. 10.

We find, therefore, on plate P' (fig. 11) the picture of the signal Punta Chandellei (S in fig. 10), bisected by the vertical thread, and this fact enables us to obtain the value for f more readily from this plate by means of the following formula (fig. 10):

$$VP' = f = \frac{VS' \times P'S}{SS'}$$

For this computation the following data are given:

D = triangle side: Royal Lodge (signal)—Punta Chandellei (signal) = VS' = 3270·7 m

The elevation of Punta Chandellei = 2811·72m

The elevation of Royal Lodge—elevation of horizon of V = 2191·80m

The ordinate y of image (S) of P. Chandellei, measured on plate P' = 46·25mm

Computation of L = SS':

Elevation of Punta Chandellei	= 2811·72m
Elevation of point V (camera horizon)	= 2191·80m
Apparent difference of elevation	= 619·92m
Correction for curvature and refraction	= —0·72m
True difference of elevation	= 619·20m = L.

Computation of $f = \frac{Dy}{L} = \frac{VS \times P'S}{SS'}$

$$\begin{aligned} \log D &= \log 3270{\cdot}7 &&= 3{\cdot}5146407 \\ \log y &= \log 0{\cdot}04625 &&= 8{\cdot}6651117 \\ \text{co.log } L &= \text{co.log } 619{\cdot}20 &&= 7{\cdot}2081691 \\ \hline \log f & &&= 9{\cdot}3879215 \end{aligned}$$

$$f = 0{\cdot}24430 = 244{\cdot}30^{\text{mm}}$$

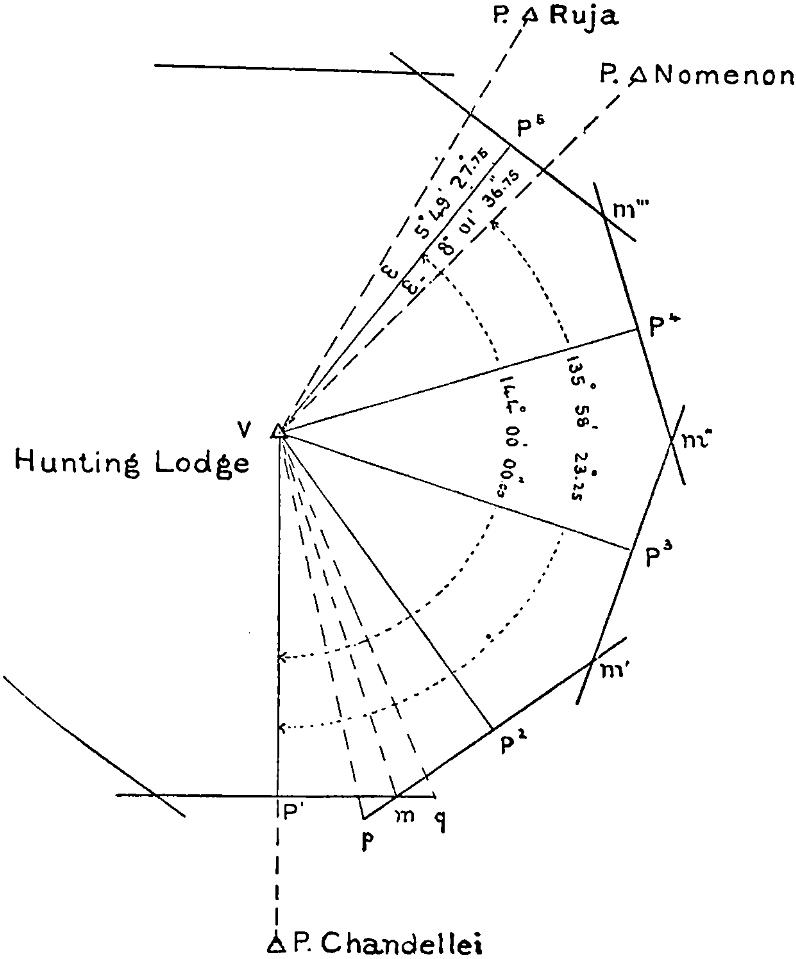


FIG. 11.

V.

Perspective P^5 (fig. 11) shows, as previously mentioned, the pictures of Punta Ruja and Punta Nomenon, while the vertical thread of plate P' bisects the picture of the signal Punta Chandellei, and as the horizontal angles $P'VP^2$, P^2VP^3, P^3VP^4 . . . . are uniformly $=36°$, the orientation of plate P^5, for instance, will be:

$$P'VP^5 = 4 \times 36° = 144°$$

From the geodetic records we take the angle: Chandellei-Royal Lodge-Nomenon $= 135° \; 58' \; 23''\cdot25$ and the angles: P^5VP'—(Nomenon-Royal Lodge-Chandellei)$= \omega'$

$$\text{(Ruja-Royal Lodge-Nomenon)} - \omega' = \omega$$

With aid of these values for ω, ω' and the value for f, see Subject III, we can compute the values for x and y very closely. (See fig. 11.)

The angle: (Chandellei-Lodge-P^5)$=P'VP^5$ $= 144° \; 00' \; 00''$
The angle: (Chandellei-Lodge Nomenon)$=P'V$ Nomenon. $= 135° \; 58' \; 23''\cdot25$

Therefore $\omega' = 8° \; 01' \; 36''\cdot75$

$$\omega = \text{(Ruja-Lodge-Nomenon)} - \omega' = 13° \; 51' \; 04''\cdot50 - 8° \; 01' \; 36''\cdot75 = 5° \; 49' \; 27''\cdot75$$

Computation of the abscissæ:

$$x = f \tan \omega$$

$\log f = \log 244.31 = 9\cdot3879468$ (see computation Subject III)
$\log\tan\omega = \log\tan 5° \; 49' \; 27''\cdot75 = 9\cdot0086263$
$\log x = 8\cdot3965731$
$x = 24\cdot92^{mm}$
by measurement: $x = 24\cdot80^{mm}$

diff. $= 0\cdot12^{mm}$

$$x' = f \tan \omega'$$

$\log f = 9\cdot3879468$
$\log \tan \omega' = \log \tan 8° \; 01' \; 36''\cdot75 = 9\cdot1492780$

$\log x' = 8\cdot5372248$
$x' = 34\cdot45^{mm}$
by measurement: $x' = 34\cdot05^{mm}$

diff. $= 0\cdot40^{mm}$

The mean difference: $\frac{0\cdot12 + 0\cdot40}{2} = 0\cdot26^{mm}$ is the correction which should be applied to the vertical axis of plate P^5.

Computation of the ordinates:

$$y = \frac{fL}{\cos \omega D}$$

$$\begin{aligned}
\log f &= \log 0{\cdot}24431 &&= 9{\cdot}3879468 \\
\log L &= \log 979{\cdot}4 &&= 2{\cdot}9909601 \\
\text{co.log} \cos \omega &= \text{co.log} \cos 5^\circ\, 49'\, 27''{\cdot}75 &&= 0{\cdot}0022478 \\
\text{co.log}\, D &= \text{co.log}\, 5804{\cdot}2 &&= 6{\cdot}2362576
\end{aligned}$$

$$\log y = 8{\cdot}6174123$$

$$y = 0{\cdot}041439$$

$$y = 41{\cdot}44^{\mathrm{mm}}$$

by measurement: $y = 41{\cdot}45^{\mathrm{mm}}$

$$y' = \frac{f\,L'}{\cos \omega'\, D'}$$

$$\begin{aligned}
& & \log f &= 9{\cdot}3879468 \\
\log L' &= & \log 1294{\cdot}9 &= 3{\cdot}1122362 \\
\text{co.log} \cos \omega' &= & \text{co.log} \cos 8^\circ\, 01'\, 36''{\cdot}75 &= 0{\cdot}0042759 \\
\text{co.log}\, D' &= & \text{co.log}\, 5029{\cdot}6 &= 6{\cdot}2984666
\end{aligned}$$

$$\log y' = 8{\cdot}8029255$$

$$y' = 0{\cdot}063522^{\mathrm{m}} = 63{\cdot}52^{\mathrm{mm}}$$

by measurement: $y' = 63{\cdot}50^{\mathrm{mm}}$

These five examples will elucidate the various relations between parts of the perspectives and the terrene, as well as give the means to judge of the degree of accuracy of phototopography.

In practical work it would become too time consuming to make such computations for all the plates, or even for all the panoramas, with the necessary minute graphical measurements.

If the camera has been carefully constructed it is generally accepted that its optical axis is vertical to the image plate, and the value for f for any, or for all, panoramas which were obtained with the same objective and with the same constant focal length (that is to say, obtained with the same reading of the scale on the objective tube) is computed in practice in the following manner:

As the horizontal shiftings

$$P'VP^2,\ P^2VP^3,\ P^3VP^4\ \ .\ \ .\ \ .\ \ .$$

are all $= 36^\circ$, the angles

$$P'Vm,\ mVP^2,\ P^2Vm'\ \ .\ \ .\ \ .\ \ .$$

will be $= 18^\circ$ each. The value of $P'm = mP^2 = P^2m' = \ .\ .\ .\ .$ is $= f \tan 18^\circ = x^{\mathrm{m}}$ = the value of the greatest abscissa of the plates; hence,

$$f = \frac{x^{\mathrm{m}}}{\tan 18^\circ}$$

In the preceding (page 54) it has been stated that two adjoining plates have a common margin, representing the terrene included by an angle $p\,V q$ (fig. 11). If the negative plates are sufficiently clear, it will be an easy matter to identify a point, m (fig. 11), on the two strips $p\ q$ of two adjoining plates, which will be on or near the line of horizon, and which will be $m\,P'$ distant from the vertical axis of plate P' and $m\,P^2$ distant from the vertical axis of plate P^2. If we select such points m', m^2, m^3, ... which can be identified upon two adjoining plates, P^2—P^3, P^3—P^4, P^4—P^5, ... we will obtain a mean value, x^{m}, for the entire panorama, by aid of which a good value for f can be obtained from the formula:

$$f = \frac{x^{\text{m}}}{\tan 18^\circ}$$

For example: By means of ten negatives of a panorama station, occupied with the latest improved Italian apparatus, it was found:

$$\left.\begin{array}{lll}
 & & mm \\
x^{\text{m}} \text{ for } P' & - P^2 & = 77{\cdot}10 \\
x^{\text{m}} \quad P^2 & - P^3 & = 77{\cdot}15 \\
x^{\text{m}} \quad P^3 & - P^4 & = 77{\cdot}00 \\
x^{\text{m}} \quad P^4 & - P^5 & = 77{\cdot}40 \\
x^{\text{m}} \quad P^5 & - P^6 & = 77{\cdot}40 \\
x^{\text{m}} \quad P^6 & - P^7 & = 77{\cdot}20 \\
x^{\text{m}} \quad P^7 & - P^8 & = -\!- \\
x^{\text{m}} \quad P^8 & - P^9 & = -\!- \\
x^{\text{m}} \quad P^9 & - P^{10} & = 77{\cdot}40. \\
x^{\text{m}} \quad P^{10} & - P' & = 76{\cdot}90
\end{array}\right\} x^{\text{m}} = 77{\cdot}194^{\text{mm}} = \text{mean value.}$$

$$\begin{aligned}
\log 77{\cdot}194 &= 1{\cdot}8875835 \\
\text{co.log} \tan 18^\circ &= 0{\cdot}4882240 \\
\hline
\log f &= 2{\cdot}3758075 \\
f &= 237{\cdot}6^{\text{mm}}
\end{aligned}$$

The above values were obtained by using the negative plates and reading the measurements, obtained by means of dividers, off the graduated rulers of the graphical instruments of the Royal Military Geographical Institute.

Using the positives (prints) of the same panorama, the following results were obtained:

$$\begin{array}{lll}
 & & mm \\
x^{\text{m}} \text{ for } P' & - P^2 & = 76{\cdot}25 \\
x^{\text{m}} \quad P^2 & - P^3 & = 76{\cdot}20 \\
x^{\text{m}} \quad P^3 & - P^4 & = 76{\cdot}10 \\
x^{\text{m}} \quad P^9 & - P^{10} & = 76{\cdot}70 \\
x^{\text{m}} \quad P^{10} & - P' & = 76{\cdot}00
\end{array}$$

From this the mean value for x^{m} is found $= 76{\cdot}25^{mm}$

$$\log 76.25 = 1{\cdot}8822398$$
$$\text{co.log} \tan 18^{\circ} = 0{\cdot}4882240$$
$$\log f = 2{\cdot}3704628$$
$$f = 234{\cdot}67^{mm} = 234{\cdot}7^{mm}$$

The negatives gave $x^{m} = 77{\cdot}19$

The positives gave $x^{m} = 76{\cdot}25$

diff. — $0{\cdot}94^{mm}$

This shortening of the greatest abscissa of half a millimetre at either side of the vertical thread on the prints is due to shrinkage of the 24×18^{cm} paper. The positive prints being extensively used in the map construction, this shrinkage must be taken into account.

Chapter III.

THE EXECUTION OF THE FIELDWORK.

By a close inspection of the various panoramas upon which the construction of the map is based, it readily becomes evident that not all of the perspectives are adapted for illustrative purposes (to be used to illustrate the Alpine character).

For cartographic purposes the panoramic views should not be taken at too great a distance from the terrene which is to be delineated, in order to preserve and show as much as possible of the topographic details and also that the selected triangulation points may appear sufficiently clear and well defined in the two or three views of the panorama set which contain their pictures.

It will be best to select the distance from which views for illustrative purposes are to be taken in such a manner that the camera station may command an extensive field of the terrene. Illustrative views should therefore, be taken from isolated prominent points and from such that can readily be recognized upon the topographic map containing the section photographed, thus assuring a rapid orientation and giving the student of the map the means to form a correct opinion of the topographic character of the terrene by comparing such illustrative views with the map.

With this object in view, a selection was made from the numerous panoramas obtained during former years and the selected perspectives were copied with pen and ink by expert draftsmen, whose drawings were reproduced by photozincography and published with the addition of all data needed to identify the camera station and to enable the student to orient each view properly upon the map.

The requirements which the camera theodolite constructed under the auspices of the Royal Military Geographical Institute was to satisfy have been previously mentioned, and as a result of the improvements suggested and made by practical experience the apparatus now in use in Italy furnishes the elements of the panorama station in such completeness that little needs to be added by extra operations and computations before the map construction is begun, and, with due reference to the rough character of the terrene, the apparatus can easily be dismembered into pieces small enough to be taken to the most inaccessible points. Three small-sized knapsacks, each weighing 7 to 8 kilogrammes, contain the theodolite, the camera, and ten negative plates. They are carried by two soldiers and one guide, each bearing one tripod leg, to be used as an alpenstock.

The fieldwork consists in the fitting up of a small laboratory, conveniently located with due regard to communication, to a central position, to facilities of transportation, accessibility of good water, etc. A sufficient number of bromo-gelatin plates are kept on hand packed in air and water tight cases. From this laboratory the camp outfit is taken to the neighborhood of the stations which are to be occupied. The observer and party take daily excursions from this camp to the surrounding mountain peaks, replacing the plates exposed during the day every evening by new ones to be used the next day.

After the camera theodolite has been put together and placed in position at a station, with favorable weather and light, precluding unforeseen accidents to the corrections and adjustments of the instrument, an experienced observer will execute the panorama and determine the camera station within an hour.

To secure the position of a camera station at least three or four directions to surrounding geodetic points should be taken, as, if so many are not visible, that number of horizontal directions must be taken to some other points which have previously been determined as phototopographic stations, and which were provided with signals before leaving them. The vertical angles at these points are recorded in a notebook. After the terrene to be photographed has been focused upon, the circle reading of the focal length on the graduated metal plate on the objective tube is also recorded in this book if the principal focal distance has not been used.

The panorama is obtained by clamping the instrument, after the direction of the optical axis of the first perspective P^1 has been secured, by bisecting a geodetic point (see fig. 11), and then revolving the camera 36° for each successive exposure in order to obtain the directions of the optical axes of the following perspectives: P^2, P^3. . . . .

In the notebook (Model No. 1, supplement) all data are recorded which may be deemed useful or necessary for the selection of subsequent camera stations, also the general incidents of the fieldwork at each station (time or duration of exposure of the different plates, according to the character of illumination, in order to gather the means for regulating the subsequent developing of the plates). Finally, a pencil outline sketch of the terrene, with valuable notes for the map (names, roads, paths, buildings, etc.), is made before leaving the station.

After all the stations around this camp have been completed, and if all required data have been gathered during the several traverses through the country, from the camp to the stations, the camp is packed and the party returns to the laboratory, where the recently obtained negatives are developed and the occupied camera stations are plotted and marked upon the chart projection. All the finished panoramas left at the laboratory are catalogued and labeled. Then the party, with a new supply of plates, proceeds to another camping ground to continue the work as before.

In order to save time and trouble, it will be advisable to regulate the general progress of the work in such a manner that the elevated points are visited in the most favorable season (i. e., when the snow has least depth, when the passes are free from snowdrifts, and when the glaciers can be passed over with the least risk). The lower regions, being nearer to civilization, require less time and can be occupied at leisure at any time during the season.

A good selection of the camera station is important, and should be well considered and be made dependent upon the elevation and distance of the points of the terrene to be surveyed, upon the scale of the chart, and upon the character of the country (a diversified and broken terrene will need more stations to control the same than an undulating and a more regular section), still, with due regard to the limited length of the working season in these elevated regions, it will also be advisable to occupy no more stations than are really needed to develop the terrene properly. The stations, finally, should be selected in such a manner that the smallest area of the represented terrene is visible from three or more stations. If any part is visible from two stations only, and if it is of minor importance, its determination by two directions only may be accepted if the points of the same are determined by good intersections (if the two lines of direction intersect each other at an angle of 60°-90°).

Regarding the most favorable hours for exposing the plates, one must be guided by local conditions; generally speaking, the trend of the valleys in comparison with the course of the sun is important; slopes totally in shadow should not be photographed; neither is it advisable to execute panoramas when the sun is low or near the horizon. In the latter case an additional source of trouble would arise from the fact that one or two perspectives, taken in the direction toward the sun, will be cloudy and have a double set of cross wires, one set being printed by rays which penetrate into the camera and the other in the usual manner by the refracted rays through the lenses of the objective. Generally speaking, it will be found that the best results are obtained from exposures made during the latter part of the forenoon, the atmosphere having a greater percentage of vapor in the afternoon, particularly in mountainous countries.

From every negative plate obtained at least two positive prints are made. One serves to determine the panorama necessary for the location of the secondary points, while the other is used to measure the abscissæ and ordinates needed for the map construction. All measurements should be made upon loose prints, as the pictures become greatly distorted by being pasted on cardboards.

CHAPTER IV.

THE HORIZONTAL PROJECTION.

In order to obtain the map, based upon the panoramas, two drawing boards are covered with paper (gummed down on the edges). One is used as a constructing board (to make all graphical determinations of points) and the other for the drawing of the finished charts; both are provided with a chart projection upon which the trigonometric points which were used during the field season are plotted by means of their coordinates.

With the aid of a specially constructed graphical protractor and tracing paper the directions obtained with the theodolite in the field can readily be plotted upon both sheets. This protractor is shown in figure 12. It consists of two concentric circles *A A* and *B B*. The former can be moved concentric within the latter about the axis *C*, secured in the center of *A A*. This rotary motion is applied to *A A* by means of two projecting ribs *S* and *S* on the plate *a a*, the latter being secured to the movable circle.

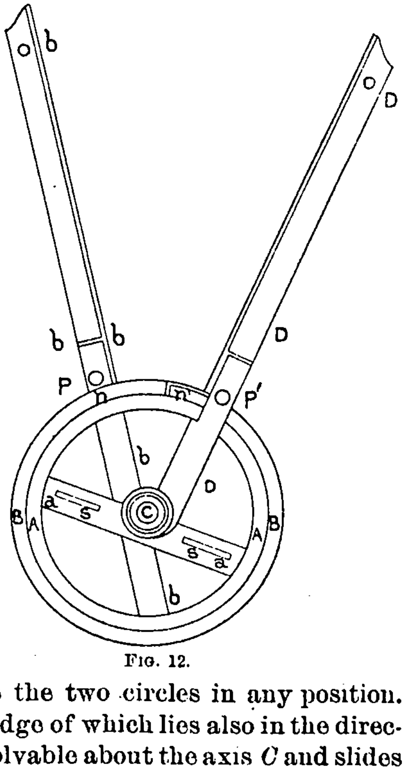


FIG. 12.

The inner circle *A A* has a graduation, divided into degrees and half degrees, while the outer circle *B B* bears a vernier *n*, the zero of which lies in the prolongation of the fiducial edge of an arm *b b*,' which is securely fastened to the outer circle and in a radial position to the same. This vernier *n* reads to half minutes. The clamp screw *P* serves to secure the two circles in any position.

An alidade ruler, *D D*, the fiducial edge of which lies also in the direction of a diameter of the circles, is revolvable about the axis *C* and slides upon the upper surfaces of the circles which are in the same plane. This alidade bears a vernier *n'*, graduated like *n*, the zero of which.

coincides with the fiducial edge of *D D*. The clamp screw *P′* serves to clamp this movable arm *D D* to the outer circle *B B*.

The axis *C* is a hollow screw bolt with conical interior, at the bottom of which a thin piece of isinglass is secured in such a manner that it can be renewed. This has a small puncture to indicate the center of the circles and revolving axis.

When an ordinary protractor is used to lay off the different directions from one camera station which were obtained by the theodolite in the field all the necessary additions and subtractions to be made in order to obtain the successive angles between these lines of directions absorb much valuable time, especially when plotting a series of panorama stations.

The protractor shown in figure 12 can be used not only as any ordinary protractor (by making the zero of the inner circle coincide with the zero of the outer circle and clamping the two circles in that position by means of the clamp screw *P*), but it can also be used to plot the directions upon the map in the same manner as they were obtained in the field, by aid of the theodolite; that is to say, they can be referred to zero or any other direction as the beginning, and then be plotted in successive order. To do this, the inner circle is revolved until the zero of the outer circle (vernier *n*) gives the same reading upon the graduation of the movable circle as the theodolite reading for the prime direction; then both circles are clamped together by the same clamp screw *P*. The line of prime direction is drawn along the fiducial edge of the fixed ruler *b b* upon the drawing (or upon the tracing paper, if the station is to be fixed or located upon the tracing of the lines), while the center of the instrument coincides with the point representing the station upon the paper.

The zero of the vernier *n′* of the alidade *D D* is then successively brought, upon the inner circle graduation, to the readings of the other directions radiating from the station point under the center of the protractor; each successive direction is plotted by drawing a pencil line along the edge of the alidade *D D*. Care must be taken not to change the primary position of the instrument as defined by the first line, during these motions of the alidade.

The tracings of the lines radiating from the stations are obtained with great accuracy by means of this instrument. If we have a sufficient number of directions to well-determined points which are evenly distributed about the station, their corresponding intersections upon both drawing boards can be located with as much rapidity and accuracy as a graphical construction will admit of.

This protractor serves also to locate points on the construction board which on account of importance or for reasons of control had been bisected from numerous stations with the theodolite, and also, as will be shown, to orient a perspective view upon the board, if such perspective contains no trigonometrical point, or if the image of such is blurred and not sufficiently clear to be identified with precision.

After all stations, including such secondary points as have been determined by theodolite directions from these camera stations, have been plotted upon the two boards, the work of determining upon the construction board such secondary points as seem needed to complete the map is taken up. For this purpose the various elements of the perspectives are corrected and adjusted in the manner previously indicated, and all secondary points are selected and marked by searching for well-defined points which are common to two or more plates, carefully selecting therefrom only such as seem to be the most useful either for drawing the contours or for tracing the general trend of mountain ranges, torrents and streams, boundary lines of glaciers, etc. The number to be selected depends chiefly upon the adopted scale and upon the accuracy to be attained. All such points are marked upon the prints (perspectives) by numerals or letters in red ink.

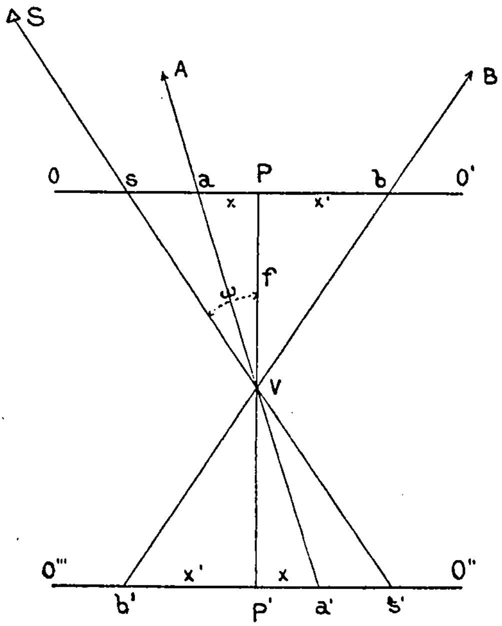


FIG. 13.

Instead of drawing the horizontal projections of all perspectives (or the polygons of the panoramas) upon the construction board, much time can be saved by using the instrument represented in figure 14. With this instrument we can draw directly upon the construction board the horizontal directions to the pairs or trios of points marked upon the prints without drawing the horizontal projections of said prints.

In figure 13, V represents the station point plotted upon the board, $O\ O'$ the horizontal projection of a perspective (which has been oriented with reference to a signal point S, a known and plotted point of the terrene).

$V\ P$ is vertical to $O\ O'$.

$f =$ focal length for the perspective $O\ O'$.

$P =$ principal point of view of the perspective.

$P\ s$ upon $O\ O'$ is the measure of orientation ($= \omega$) of the perspective.

We now prolong $V\ P$ through V by $V\ P = V\ P' = f$ and erect a perpendicular to $VP' = O'''\ O''$ in P'. Likewise, prolong $V\ B$, $V\ A$, $V\ S$ to their intersections with $O'''\ O''$, which intersections are marked b', a', s', respectively. $V\ P' = V\ P$ and $O\ O'$ parallel to $O'''\ O''$; hence the rectangular triangles $V P' a'$, $V P' b'$, and $V P' s'$ are congruent with $V P a$, $V P b$, and $V P s$, respectively; therefore:

$$P'a' = Pa = x$$
$$P'b' = Pb = x' \text{ and}$$
$$P's' = Ps, \text{ giving also the measure of orien-}$$

tation ($= \omega$) of the perspective.

The construction of the graphic sector (fig. 14) is based on the preceding consideration, and it serves to draw from the station point, in the plane of drawing, the various horizontal directions to secondary points of the perspectives.

The metal plate $V S S'$, shaped like a sector, can be revolved in the plane of drawing about the center of a strong needle, puncturing the station point in center (r) of sector. This needle passes through an oblong opening (of the same width as the needle) of a revolvable button, r, secured in V, and through a similar slot in the metal plate $V S S'$ at V. The metal ruler $R R'$ is revolvable about V, gliding with the end R' over the arc $S S'$ of the sector, and the fiducial edge of the ruler passing through the center of V.

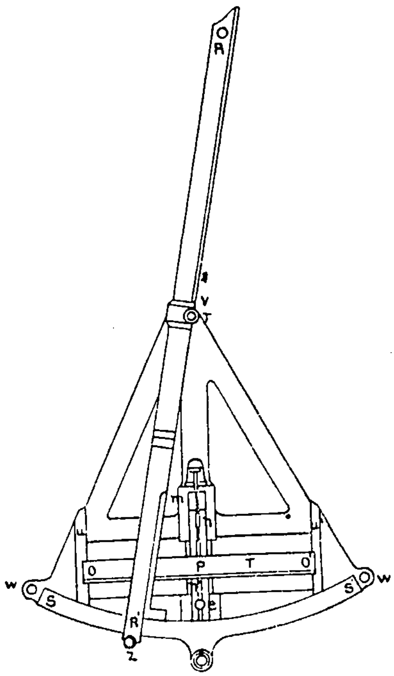


FIG. 14.

This ruler is secured to the revolvable button r by means of a cylinder, the bottom of which also bears a slot similar to those in the button and sector plate.

When the ruler and button are in a certain position the three slots in sector plate, button, and ring of ruler will coincide, and the needle can be inserted into the station point under V, the center of rotation, through the three slots. By a quarter turn of the button r the needle will become inclosed in a square, of which the needle circumference forms the inscribed circle. The entire instrument can now be revolved about the needle center in V.

The lever screw m serves to move the prolongation arms $n\,n'$ in a suitable slide in the direction of the middle line of the sector. The axis of this instrument coincides with that radius of the sector which falls together with the middle line, passing through the center of V. The steel ruler T is perpendicular to this axis, and is secured to the prolongation arms $n\,n'$; it can be moved up or down, while maintaining a position perpendicular to the axis of the instrument, by means of the screw m to positions parallel to each other. During such movements of T the ruler ends O and O' glide over two graduated metal strips $u\ u'$, which are parallel to the axis of the instrument and upon which the distance of a line coinciding with the fiducial edge $O\ O'$ of T from the center of the needle in V can be read off to $0{\cdot}1^{mm}$.

If the edge $O\ O'$ of the steel ruler T is brought to a distance $=f$ from the camera station in center of V (by means of the screw m) it will represent the line of horizon or the horizontal projection of a perspective, obtained with the focal length $=f$ (in inverse position, like the line of horizon, viewed upon the ground-glass plate of the camera).

The point P, intersection of the axis of the instrument with $O\ O'$, is the principal point of view, and it is accentuated by a small conical cavity to receive the point of one arm of the divider.

The screw c serves to give the steel ruler T a permanent position after it has been brought to the desired distance from the center of rotation V. Two thumbscrews W and W' (into which fine needles can be inserted and held in place by clamp screws) serve to secure the metal sector in any desired position upon the drawing board. The arc $S\ S'$ of the sector is graduated to ten minutes and the zero of this graduation coincides with the axis $V\ P$ of the instrument, giving readings from 0° to 25° on either side of $V\ P$ upon this arc.

The ruler or alidade $R\ R'$ bears a vernier to read fractions of the arc graduation. It is graduated to read half minutes. The thumbscrew (clamp screw) Z of the alidade has a counterplate at its lower end and it serves to secure the end R' of the alidade upon the arc of the sector and upon the steel ruler T.

In order to draw the lines of direction upon the construction board to a point of the terrene, the picture of which has been selected and marked upon the perspective, the instrument is placed with its center of rotation, V, over the needle, marking the camera station on the board and giving the button r a quarter turn (care must be taken that the side bearings of the button r of the instrument have no loose play about the needle), then $O\ O'$ is moved by turning the screw m until $O\ O'$ is distant from center of $V=f$, whereupon the orientation of the instrument is accomplished as follows:

$V\ P$ is directed to bisect a plotted triangulation point, the image of which appears on the perspective with sufficient distinctness; its abscissa is taken from the print by means of a pair of dividers and plotted in the inverse direction, upon the line $O\ O'$, from the puncture in P; the alidade $R\ R'$ is now gently brought into contact with the other point of the dividers and secured in this position by clamping the screw Z. Now the entire instrument is revolved about V until the other end R of the alidade bisects the plotted point. The instrument is held in this position by pressing the small needles W and W' into the drawing board; the end R' of the alidade is now released and the abscissæ of all the desired points of the perspective are transferred to the drawing along the line $O\ O'$ from P, by means of the dividers, in their successive order, but in inverse direction (the fiducial edge of the alidade being gently brought into contact with the divider point each time), and the lines of direction are drawn with a sharp pencil along the fiducial edge of the alidade end R.

Should the image of the triangulation point appear blurred upon the perspective, the instrument will have to be oriented upon the drawing by means of the angle of orientation $= \omega$ of the perspective, which angle is taken from the field book (Model No. 1, Supplement). The end R' of the alidade is placed and secured in such a position that the alidade $R\ R'$ forms the angle ω with the axis $V\ P$ of the instrument, which angle is read off (in the inverse direction) on the arc $S\ S'$ of the sector. The instrument is then revolved about the needle in V, the same as before, until the end R of the ruler passes through the trigonometrical point in question. The instrument is then secured upon the board in this position by means of the screws W and W' and the horizontal directions are drawn to the secondary points along R, in the manner just described.

If a plate has been exposed while the vertical wire bisected a trigonometrical point, the orientation of such perspective is accomplished by making the zero of alidade-vernier coincide with the zero of the arc graduation, $S\ S'$, clamping $R\ R'$ in this position and directing the end R to bisect the plotted trigonometrical point in question.

Should, finally, the perspective contain no images of points previously located and plotted, then the zero of alidade is again made to coincide with the zero of the arc graduation and the instrument is revolved about the center of the needle until the fiducial edge R of the alidade coincides with a line (which had been drawn by means of the previously described protractor) which forms an angle in the station-point V with the direction to a triangulation point, which is equal to the angle of orientation ($= \omega$) of the plate and which is taken from the field notebook (Model No. 1, Supplement).

After the horizontal directions have been drawn to the different points of the panorama, they are provided with numerals or symbols corresponding with the characters affixed to the points upon the panoramic views, in order to facilitate their identification when seeking for the subsequent intersections with lines to the same points from other camera stations. In this manner, shown in the preceding, the positions of the secondary points in the plan of the drawing are secured by intersections, which will serve to make up the control of the map. It is well to transfer to the fair drawing, by means of tracings, which are oriented by the plotted trigonometrical points and previously located panorama stations, all the different points obtained by intersections upon the construction board, in order to erase therefrom all lead pencil lines, which served for their determination and to obscure the subsequent constructions for the positions of other points of the terrene as little as possible.

Chapter V.

THE HYPSOMETRICAL WORK.

After the position of the most important points of the second order is well under control it remains to ascertain the elevations of the various station and secondary points of the perspectives, in order to enable the draftsman to interpolate the contours between these points.

By means of the graphical hypsometer, figure 15, the elevations of the plotted camera stations can be ascertained, by means of their graphically measured distances from triangulation points and the corresponding angles of elevation of said points (measured with the theodolite), which are recorded in the field notebook (Model No. 1, Supplement). The elevations of all secondary points are determined with the same instrument by means of their graphically measured distances from the camera station and their corresponding ordinates, y, to be measured upon the perspectives.

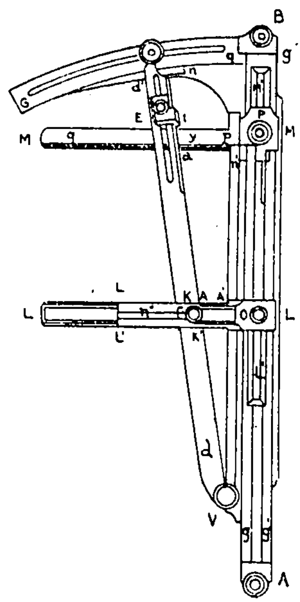


Fig. 15.

Two rulers, $L L'$ and $M M'$ (fig. 15), can be made to glide with their ends along a ruler, $A B$, but always maintaining a perpendicular position to the same, for which purpose their ends are secured to two sleighs, M' and L', which glide in two parallel grooves, g and g', along $A B$. The motion of L' is free, and is accomplished by pushing the button O up or down the ruler $A B$.

M' is provided with a ratchet and screw P. By turning the latter in one direction or the other the ruler $M M'$ is gradually moved up or down $A B$, the latter being provided with a row of fine teeth into which the ratchet wheel of M' bites while P is being revolved. The alidade ruler $d'd$ is secured with one end, d, in V, in such a manner that $d'd$ can be revolved about the axis of V as a center, while the other end, d', passes over a graduated arc, $G g g'$. The plug in V is similarly constructed to the one in V of the graphical sector, figure 14, previously described (it is provided with a revolvable button which contains a slot in such a manner that the ruler $A B$ can be revolved simultaneously with the alidade $d'd$ about a needle, marking the station point on the construction board). In this instrument the plug, the revolvable button, and the alidade have each a slot, which intersect each other in the center of rotation V, and through which the needle can be passed when they have a certain position and then be secured in place by a quarter turn of the button. The entire instrument can be revolved about the needle, the center of which lies in the directions of the fiducial edges of the ruler $A\ B$ and alidade $d'd$.

The alidade is provided with a vernier, n, graduated to read half minutes, on the graduation of the limb $G\ g\ g'$. This vernier serves to lay off angles from V between the fiducial edges of $A\ B$ and $d'd$. When $d'd$ is brought close to and in contact with $A\ B$, the zero of the vernier n and the zero of the arc graduation will coincide. The axis of this instrument is represented by that edge of $A\ B$ (lying toward $d'd$) which passes through the center of rotation V, and which passes through the zero of the graduated arc $G\ g\ g'$; it also passes through the point p of the line $p\ q$, which is marked upon the ruler $M\ M'$. This line $p\ q$ corresponds with the zero of a vernier, n', which is attached to the ruler $M\ M'$ and which glides along the groove g when $M\ M'$ is moved up or down $A\ B$. $A\ B$ has a millimetre graduation, and by means of the vernier n' the distance of the line $p\ q$ from center of V can be read to 0.1^{mm}.

When this line $p\ q$ is brought to the distance $=f$ from V, by means of the fine ratchet movement at M', the line $p\ q$ can be regarded as the axis of abscissæ drawn upon the perspective, while the point P represents the principal point of view of the perspective (see fig. 5).

In this case the line $p\ q$ can also be regarded as the axis of the ordinates of the perspective $m\ n$ (fig. 5), provided the vertical plane (containing $V\ P$ and axis of ordinates) is supposed to be rotated about $V\ P$ until it coincides with the horizontal plane $V\ P\ O$.

The point p is marked upon the line $p\ q$ (in the same way as described for the sector) by a small puncture, which serves to receive one point of the dividers, when such are used to lay off the abscissæ and ordinates, taken from the perspectives.

After $p\ q$ has been secured at a distance $=f$ from the center V and the abscissa x of a point a, taken from the perspective $m\ n$, has been transferred to the line $p\ q$ from p, the second point of the dividers upon $p\ q$ will represent the horizontal projection a' of the point a. If we now move the alidade, $d'd$, until its fiducial edge touches the second point of the dividers, the triangle formed by the edge of the alidade $d'd$, the edge of the ruler $A\ B$, and line $a'\ p$ will represent the triangle $V\ P\ a'$ of figure 5.

The end d' of the alidade is provided with a steel index mark, i which can be moved along $d'\ d$ by means of a revolvable button, E, a ratchet movement, and teeth in a groove along $d'\ d$. If this index mark is moved to a' (the intersection of fiducial edge of the alidade and line $p\ q$ on $M\ M'$), the distance $V\ a'$ cut off on $d'\ d$ will represent the horizontal distance of the point a' of the perspective from V (i. e., the value d in figure 5).

Maintaining this index mark i (fig. 15) in this position on $d'\ d$ and revolving $d'\ d$ about V until its fiducial edge coincides with the edge $p\ V$ of $A\ B$ (i. e., with the axis of the hypsometer), then moving the ruler $M\ M'$ away from V (by turning the ratchet button P) until the line $p\ q$ coincides with the index mark i, we will have transferred the distance d (fig. 5) upon the hypsometer axis; we will have brought the line $p\ q$ (engraved upon $M\ M'$) to a distance from the center of rotation (of the needle) in V equal to d, and by transferring the ordinate y (fig. 5), measured on the perspective $m\ n$ with a pair of dividers, upon the line $p\ q$ (while the latter is still in the position just described), by inserting one point of the dividers into the cavity p and bringing the fiducial edge of the alidade $d'\ d$ gently into contact with the other point of the dividers, resting on the line $p\ q$ (fig. 15), then the triangle $V\ p\ a$ of the hypsometer will also represent the vertical triangle $V\ a'\ a$ of figure 5, except that it is now turned about $V\ a'$, as axis, into horizontal plan.

The movable ruler $L\ L'$, which will always remain perpendicular to the hypsometer axis, consists of two plates joined firmly together at their ends, between which the alidade $d'\ d$ (fig. 15) can glide when revolved about V. The upper plate of $L\ L'$ is slotted like the handle of a penknife, and the edges $L\ L$ and $L\ L'$ are beveled and provided with a millimetre graduation, the numerals of which correspond with a scale of 1:50000 (50 m = 1 mm). A ratchet screw, c, serves to move a plate ($K\ O\ K'$) with two index marks K and K', which can be made to coincide with the intersections of the fiducial edge of the alidade $d'\ d$ and the two sharp graduated edges $L\ L$ and $L\ L'$. The index plate $K\ O\ K'$ also has a double vernier, n'', on the opposite side of the ratchet screw c, graduated to read $\frac{1}{50}^{\text{mm}}$ (i. e., to read metres for the $\frac{1}{50000}$ scale) in connection with the millimetre scales $L\ L$ and $L\ L'$.

When the zeros of this double vernier n'' coincide with the zeros of the graduated edges $L\ L$ and $L\ L'$, the marks of the double index K and K' will coincide with the edge $V\ p$ of $A\ B$ (i. e., with the axis of the instrument) and also with the fiducial edge of alidade $d'\ d$, the zero of the vernier n of the alidade also coinciding with the zero of the arc graduation $G\ g\ g'$ (i. e., the fiducial edge of $d\ d'$ will fall together with the axis $p\ V$ of the instrument).

In figure 5 A represents a point of the terrene, the image of which is designated by a in the perspective $m\ n$. If A' is the projection of A in the horizontal plane passing through V, then $A\ A'$ will represent the difference of elevation $= L$ between the points A and V. $V\ A'$ will be the horizontal distance $= D$ of the point A from the camera station V, which distance is represented by $\frac{D}{50000}$ for a scale of map of 1:50000.

Returning to figure 15, we imagine the hypsometer revolved about the needle center in V until the hypsometer axis $p\ V$ passes through a plotted point A' in the drawing. If the ruler $M\ M'$ had previously been secured in such a position that the distance of p from $V = d$ and if $d\ d'$

had been set to lay off the ordinate y upon $p\,q$ from p, and if we now bring the index mark K in a position to mark the intersection of the fiducial edge of the alidade with the edge $L\,L$, then the triangle $V\,A\,A'$ (fig. 15) will also represent (in the scale of 1:50000) the triangle $V\,A'\,A$ of figure 5. The index mark K indicating on the edge $L\,L$ the length $\frac{L}{50000}$, we will find the difference of elevation between the point A and camera station V by reading the corresponding vernier n''

The triangles $V\,p\,a$ and $V\,A'\,A$ (fig. 15) being similar ones, we will have:

$$\frac{A\,A'}{V\,A'} = \frac{P\,a}{V\,p} = \frac{y}{d}$$

we found (page 58).

$$\frac{y}{d} = \frac{L}{D}$$

hence

$$\frac{A\,A'}{V\,A'} = \frac{L}{D}$$

and as $V\,A' = \frac{D}{50000}$, we have

$$A\,A' = \frac{L}{50000}$$

$$L = 50000 \times A\,A'.$$

The numerals of the graduation of the edges $L\,L$ and $L\,L'$ and of the double vernier n'' give the value $A\,A'$ multiplied by 50,000, which is the difference of elevation.

It has been previously shown (page 58) that

$$\tan\,\alpha = \frac{L}{D} = \frac{y}{d}$$

and therefore

$$\tan\,\alpha = \frac{A\,A'}{V\,A'}$$

Hence, if we have the angle of elevation of a point A of the terrene we need only to lay off this angle upon the graduated arc $G\,g\,g'$ by means of the alidade vernier n, from g, and place the index mark K upon the intersection of the fiducial edge of alidade and edge $L\,L$ (the instrument having been placed upon the drawing in such a position that the hypsometer axis passes through the plotted point A'), and then read off on L and corresponding vernier n'' the difference of elevation between camera station and point A.

This case becomes very much simplified when the image A' of A is bisected by the vertical thread of perspective (axis of y), as then:

$$x = o \text{ and } d = f.$$

The alidade is placed so as to lay off the ordinate y of the point a upon pq from p, after the ruler $M\,M'$ had been secured in a position at a distance $=f$ from V; then the index mark K or K' is brought into the point of intersection of the fiducial edge of $d\ d'$, with edge $L\ L$ or $L\ L'$ of the ruler $L\ L'$ (the axis of hypsometer passing through the plotted point A'), and the difference of elevation between A and V is read off either on the vernier corresponding to the graduation $L\ L$ or to the graduation $L\ L'$. The correction for curvature and refraction to be applied to these differences of elevation is taken from the ordinary field tables.

A special list (Model No. 2, Supplement) is made for the secondary points, in which they are tabulated according to the numerals or symbols with which they were characterized on the perspectives, and they are catalogued according to the panorama and perspective to which they belong. This list also contains the differences of elevation between them and the two or more stations whence they were determined, as well as their absolute elevations, the latter being the mean of the values obtained from the different stations and corrected for curvature and refraction.

The elevations of the camera stations are the mean results of the values obtained by adding or subtracting the difference of elevation (obtained by means of the graphical hypsometer) to or from the known elevations of the triangulation points (using the vertical angles observed with the theodolite and the graphically measured horizontal distances between plotted camera station and triangulation points).

After the secondary points, including their subscribed elevations, have been transferred from the construction board to the final plan, it remains only to interpolate the contours between these points, in harmony with the affixed figures, to sketch in the details and everything that is needed, and to give the terrene its proper character, all based upon frequent reference to the perspectives of the terrene in question.

CONCLUSION.

As the work at a phototopographic station can be finished within an hour or an hour and a half, and as two or three well-selected stations will control the horizontal and vertical representation of an extended area, the fieldwork will take no more time for a detailed survey than for a general survey, as the perspectives will be the same for both. The difference in the time needed for obtaining a more or less detailed topographic map depends upon the office work only, every panorama giving the means to construct therefrom an unlimited number of horizontal directions from the camera station to surrounding points, as the number of secondary points selected from the perspectives can be increased indefinitely to the limit of the patience and ability of the draftsman.

The panoramic perspectives, however, are not only important aids for the construction of the map, but they can also serve as subsequent checks upon the work of the draftsman, and if they are preserved, together with the catalogues containing the numerals, symbols, etc., of their secondary points, their elevations, etc., they can serve for future illustrations of the mapped terrene, giving the relief modeler important details to enhance and complete the natural character of the model, etc.

From the foregoing it is evident that phototopography is especially well adapted for topographical surveys of mountainous regions, as the ordinary topographical methods for such regions can be carried on only during a few summer months each year with advantage. Even in favorable seasons the weather will be very vacillating; clouds will obscure during the warmer hours of nearly every bright day the more elevated peaks, winds will carry misty vapors from one valley to another, etc., so that the camera can obtain in a short bright interval more topographical data than could be obtained in weeks of time with the other instrumental methods. Even if the selected camping ground is most favorably situated for the work, the ordinary topographer will have to traverse long and difficult distances before he will reach a favorable point for a topographical station. He can not leave camp before daybreak, and he can not risk a late return on account of the danger to life and limb (not mentioning his instruments) attending a tramp through rough mountain regions by night. He will arrive at the selected station in a fatigued and nervous condition, have but a short time to spend there, and consequently will hurry through his observations. As is well known, the topographer can, under general circumstances, determine prominent points by the intersections of horizontal directions from a number of stations or by telemeter readings from one station. In order to secure the details, however, he will have to traverse the country quite extensively and make numerous sketches in order to give it the proper character and to delineate the terrene by horizontal contours. Not many horizontal and vertical angles can be obtained in a single day either with the plane table, the theodolite, the tachemeter, or other instruments, as the topographer will have to spend a good deal of his time, at the station, in making sketches in order to identify the points of the lines of direction for subsequent lines to the same objects from other stations (in order to get the correct intersections).

The work will be still less encouraging if the use of the telemeter is depended upon to determine secondary and tertiary points, on account of the slow progress, the danger to life and limb of the telemeter men, etc. If we add to this the low temperature and snow, it will be impossible to work more than a few hours at one station with the plane table, theodolite, etc., to get directions and make sketches. It appears certain, at best, that, under the conditions which always prevail in mountains of an alpine character, the positions and elevations of prominent points only will be determined (and often only such as are absolutely necessary), the telemeter will be discarded, and the characteristic forms of the terrene, which are only seen at a distance, are sketchily represented. If details are mapped at all they will be unreliable, and the hours at which they can be seen to the best advantage are few. These facts render topographic surveys of such mountains not only tedious, difficult, and expensive, but also unreliable. They explain why so few maps give the true representation of such regions, and they also show the great advantage to be derived by applying photography to the surveys of all regions which are difficult of access or inaccessible, and where snow, ice, and bad weather prevail the entire year. The photographic perspectives will not only reproduce the terrene before one's eyes at any place, and at any time, but we can also construct a topographic map, based on such views, with the utmost correctness that may be demanded by science or industry. It is evident, therefore, that phototopography is to be recommended in the following cases:

1. For all mountains of an alpine character where, if the ordinary topographic methods are followed, the lack of control will give but mediocre results.

2. For extensive scientific expeditions and explorations, for reconnaissance in times of war, for topographic surveys in unhealthy localities along the frontiers of belligerent nations, etc.

3. For surveys for geological studies, for projected railroads through mountains, for hydrographical surveys for river ameliorations; in short, for surveys for all purposes where correct representation and character of the terrene, as well as full details, are desired.

4. For naval purposes or on board of vessels fitted out for explorations, to obtain coast views, topographic and hydrographic sketches of hostile or barren coasts. (Two or more shore stations are selected from the deck of the vessel and panoramic views are taken therefrom, care being had to include in these perspectives the vessel, anchored boats, buoys, moored flags and other secured objects which served to control the soundings, simultaneously carried on with the topographic survey.)

A special apparatus, for use on shipboard, has been invented by Paganini. It furnishes a vertical photographic perspective of a known focal length and at the same time gives the magnetic azimuth of the optical axis of the camera for each perspective. The azimuths of all the points along the coast shown on the perspective can be taken directly from the perspective.

Pio Paganini, engineer geographer and director of the phototopographical work in Italy, in a report recently made to the First Geographical Congress in Italy, says the following, relating to the improvements of his camera theodolite (a German translation, by Fenner, of this report has been published in the Zeitschrift für Vermessungswesen, 1892):

The principal improvement to the camera theodolite consists in dropping the eccentric telescope of the theodolite (Fig. A) and changing the instrument so that the "*photographic camera in itself will serve as a centrally located telescope.*"

Paganini accomplished this by replacing the ground-glass plate of the camera by an opaque plate which has a Ramsden ocular lens in the center. This new apparatus has all the details of a transit, with a centrally located telescope. The same instrument serves to obtain the photographic panorama as well as to measure the horizontal angles necessary to orient the panorama or needed for the determination of the camera station by resection, and to measure the vertical angles for the determination of the elevations.*

The plates Nos. 6 and 7 of the new map of Italy, comprising the terrene to the north of Chiavenna to Splügen, were obtained in 1889 by means of the former instrument (Fig. A), and they are now completed and have been published. A comparison between a recent edition (scale $\frac{1}{50000}$, with contours of 50 metres interval, excepting the lowlands, where the interval is 10 metres) and the adjoining sheet of the Swiss "Dufour Atlas" shows that the former appears to represent the terrene more true to nature, and although the Swiss map ranks higher from an artistic point of view, it also evinces a certain uniform undisputable neglect of characteristic topographic features.

During the exposition of charts and maps at Vienna (in 1891), under the auspices of the Ninth Congress of German Geographers, this Italian map was generally praised and declared by competent judges to deserve the first rank above all other exhibits.

In 1890 Paganini, assisted by the topographer Rimbotti, began the work of phototopographing the elevated parts of the terrene of plate No. 29 of the new Italian map, which comprises the difficult group of Monte Rosa, with elevations of 4,600 metres. They used two instruments, one of the older pattern and one of the latest construction. This work, however, had to be interrupted in 1891 in order to do "more important work for military purposes." Paganini also mentions that he had been engaged in the same year upon an "important military work," to accomplish which he doubtless would not have succeeded without the aid of photogrammetry.

Concluding his report, Paganini made some very interesting remarks concerning a recently invented instrument, which, however, is not yet constructively finished. It is also a photographic instrument, but to be used on shipboard, and which he terms a "photographic azimutale."

Formerly the perspectives used to illustrate portions of the coasts in order to facilitate the identification of such portions by sailors when approaching the coast from the sea, were published with and upon the charts or in the coast-pilot books. They were obtained in the following manner:

From the deck of a vessel at anchor a free-hand perspective drawing would be made of the desired part of the coast, including all prominent

* Paganini proposes to publish a detailed description of this new apparatus shortly.

features, particularly light-houses and navigation marks. (The use of an ordinary camera was precluded on account of the rocking motion of the vessel.)

The angles formed by the lines of direction from the vessel to the various prominent points shown in the perspective were measured with a sextant, and the local magnetic azimuth of one of these lines would also be determined (giving the local magnetic azimuth of all other lines of direction to points on the perspective).

These magnetic bearings were inscribed in the drawing above the points to which they referred. The place of anchorage had to be determined as accurately as possible and plotted upon the coast chart.

Such perspectives (it is said that Porro showed a remarkable skill in making such views) would naturally be obtained more readily and far more accurately if a photographic instrument could be constructed to be used on shipboard for this purpose.

Paganini (having been an officer in the Italian navy until 1877) had for several years made studies and investigations with the above object in view, particularly since the instantaneous process in photography had been developed to the present degree of perfection.

The "photographic azimutale," the construction of which is now well under way, if not already completed, is the direct result of Paganini's studies in this direction. This instrument can be called a transit, the telescope of which is replaced by a photographic camera, which can be converted into a telescope by replacing the ground-glass plate by an opaque plate with an ocular lens in the center. This instrument differs from Paganini's latest improved camera theodolite by its mounting and by the additional attachment of a dial compass.

Regarding the mounting of this "photographic azimutale," we will say that it rests upon a plate which swings in gimbals; both are connected by a central clamp screw, which has a heavy weight attached to secure a permanent horizontal position of the horizontal limb or the vertical position of image plate.

The compass resembles the Schmalkalder or azimuth compass and is placed centrally above the horizontal plate and within the ring-shaped alidade. The magnetic bearing of the optical axis for every perspective is secured by photographing directly upon the image plate simultaneously with the picture of the coast (and immediately below the vertical wire) that part of the compass graduation which lies in the direction of the view photographed.

The zero diameter of the dial compass always being in the magnetic meridian, the compass reading designated by that graduation mark which we find bisected by the prolonged vertical thread under the picture will represent the magnetic azimuth of the optical axis of the instrument at the moment of the exposure, or it will indicate the angle of orientation for the picture.

This picture of the compass graduation, caught simultaneously with that of the coast view, is obtained by means of a small secondary camera placed immediately above the compass and below the main camera. The optical axes of these two cameras are at right angles with each other. The image of the compass graduation in the secondary camera is reflected by means of a suitably placed prism upon the image plate of the main camera.

In order to obtain the pictures of both cameras simultaneously, the shutters of both are operated automatically and at the same moment.

The "photographic azimutale" is to be permanently secured to the captain's bridge, forming a part of the instrumental outfit of every naval vessel. By replacing the gimbal support by a tripod the instrument can be used for work on land. It is also tested and adjusted on shore in order to adjust the horizontal thread by means of the sea horizon.

Paganini mentions that this instrument is well adapted to photograph the illuminated sectors of light-houses and the range of visibility of navigation marks. He also believes that the same can be employed with advantage for the topographic and hydrographic surveys of harbors, wharves, seldom-frequented coasts, for military or scientific expeditions, for the determination of the geographical latitude of a vessel's position by means of the image of the sun, which can readily be obtained with sufficient sharpness, including the illuminated sea horizon, to give good results, etc.

From every picture showing the image of the sun we can find the sun's declination and azimuth, and the time being known we can compute the geographical position. Whether such semigraphical determinations are sufficiently accurate for practical use and whether sextant observations will be supplanted by these, time and experience can only teach; at present there are no comparative results to communicate.

The preceding chapters show that photographic surveying is being pushed to a high degree of perfection in Italy, and we are particularly indebted to Paganini for the numerous improvements so recently made in photographic and graphical instruments, including methods of use for topographic and hydrographic surveys.

SUPPLEMENT.

MODEL NO. 1.

Station on Punta Bivula (trig. pt.), on the ridge between the valleys of the Valsavaranche and Rêmes.

[September 18, 1884.]

Orientation of the panorama.	Perspectives belonging to the panorama.	Directions to the principal points of view.	Focal distance.	Remarks.
		° ′		
Punta Gran Paradiso: 78° 27′ 00″	P^1	78 27	244·5^{mm}	Time of exposure: 10^s, with smallest diaphragm, No. 7.
	P^2	114 27	Steinheil's objective: "Antiplanate."	10^s.
	P^3	150 27		9^s.
	P^4	186 27		12^s.
	P^5	222 27		9^s.
Punta Della Grivola: 123° 47′ 00″	P^6	258 27		10^s.
	P^7	294 27		9^s. Fine weather.
	P^8	330 27		10^s.
	P^9	6 27		10^s.
	P^{10}	42 27		10^s.

Directions and vertical angles of the trigonometrical points.	Computation of elevation of station and elevation of line of horizon.

Station upon the half-destroyed signal. Elev. of instr. = 2·30^m.
Geodetic point, elevation = 3413·69^m.
Elev. of instr. = 2·30^m.
Elevation of lines of horizon of panorama = 3415·99^m = 3416·0^m.

The adjoining page is utilized for topographic sketch from station, detailed remarks, names of roads, etc.

MODEL No. 1.

Station on Punta Percia, on ridge between the valleys of the Valsavaranche and the Rêmes.

[September 19, 1884.]

Orientation of the panorama.	Perspectives belonging to the panorama.	Directions to the principal points of view. ° ″	Focal distance.	Remarks.
Punta dell' Erbetet: 282° 04′ 00″	P1	170 00	244·5mm	Time of exposure: 6s. Shorter exposure than before on account of the great reflection of surrounding glacier.
	P2	206 00	Steinheil's objective: "Antiplanate."	7s.
	P3	242 00		8s.
	P4	278 00		9s.
	P5	314 00		10s.
	P6	350 00		8s.
	P7	26 00		9s.
	P8	62 00		9s. Fine weather.
	P9	98 00		10s. Diaphragm No. 7.
	P10	184 00		7s.

Directions and vertical angles of the trigonometrical points.	°	′	″
Cima di Breuil,	220	54	00
Elevation,	1	33	00
Punta dell' Erbetet,	282	04	10
Elevation,	3	36	30
Cima di Nomenon,	222	42	00
Elevation,	1	38	30
Cima di Rouletta,	0	44	30
Elevation,	3	11	30
Punta dell' Invergnan,	80	07	00
Elevation,	3	42	00
Cima di Toss,	34	11	30
Elevation,	0	30	30

Computation of elevation of station and elevation of line of horizon.	m.
Elevation Invergnan	= 3607·72
Diff. of elev. + corr.	= 406·15
	3201·57
Elevation Nomenon	= 3488·42
Diff. of elev. + corr.	= 284·94
	3202·48
Elevation Toss	= 3302·24
Diff. of elev. + corr.	= 99·84
	3202·40
Elevation Breuil	= 3454·62
Diff. of elev. + corr.	= 252·64
	3201·98
Elevation Rouletta	= 3384·10
Diff. of elev. + corr.	= 182·28
	3201·82
Elev. of line of horizon	= 3202·3

MODEL No. 2.

Elevations of secondary points of the panorama.

Names or numbers of points.	Stations whence they were derived.	Elevations of stations.	Diff. of elevations.	Elevation of point.	Remarks.

PHOTOGRAPHIC INSTRUMENTS AND METHODS EMPLOYED FOR TOPOGRAPHIC SURVEYS IN THE DOMINION OF CANADA.

The phototopography of the Rocky Mountain region in the Northwest Territory of the Dominion of Canada proved a success, and several of the Dominion topographic and land surveyors (J. J. McArthur, W. S. Drewry, etc.), under the direction of the surveyor general, Capt. E. Deville, have acquired skill and valuable experience in this branch of surveying, as is well proven by Deville's topographic map of the Rocky Mountains along the Canadian Pacific Railroad, based on triangulation and phototopography, plotted on 1:20000 and published on 1:40000 scale, and which was on exhibition at the Columbian World's Fair.

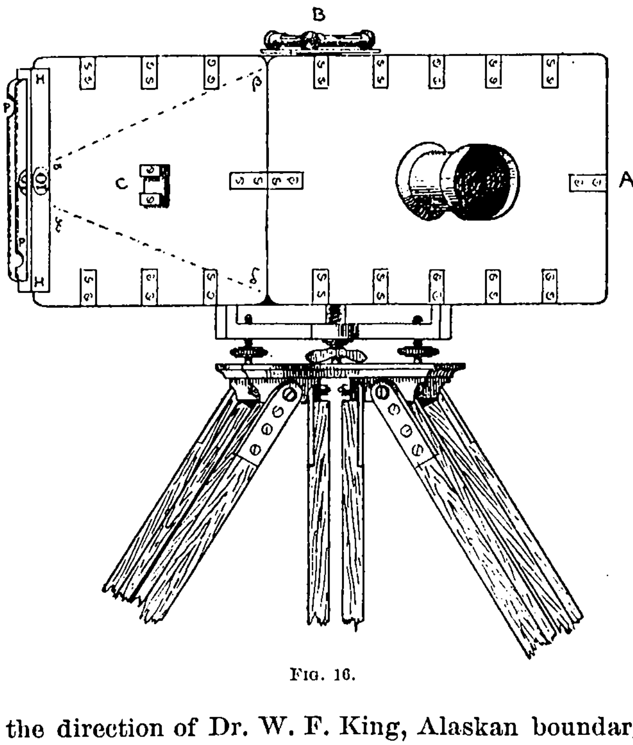


FIG. 16.

Under the direction of Dr. W. F. King, Alaskan boundary commissioner to Her Majesty, phototopography has been successfully employed for the topographic survey of southeastern Alaska, as far as this topographic reconnaissance has been executed under the Government of the Dominion of Canada.

The views taken from the camera stations of the Dominion surveys are not complete or full sets of panoramic views, and when the stations are close together, even those few plates which are exposed from one station do not always comprise adjoining pictures. According to the desired greater vertical or horizontal extension of the view, the camera can be placed with either the long or short side in an upright position upon the tripod.

The camera is a rectangular box of well-seasoned mahogany (fig. 16), strongly bound in brass and very carefully constructed, with opposite sides parallel and adjoining sides at right angles to each other. The camera has neither telescope, horizontal nor vertical circles, as it is used in conjunction with a transit, the same tripod serving for both instruments. All angles are observed with this transit, either before

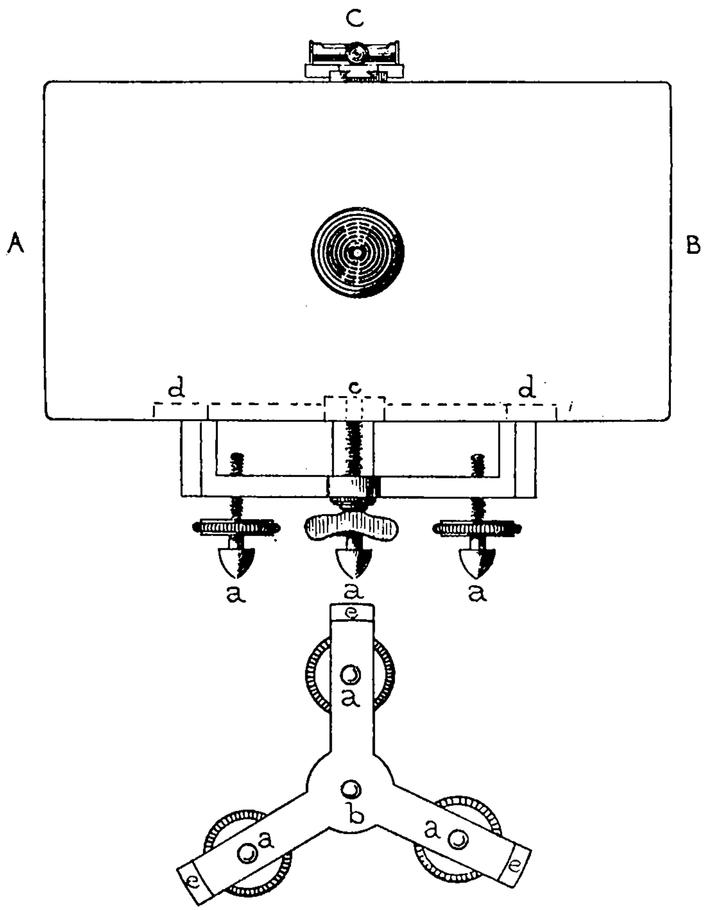


FIG. 17.

or after the exposure of the plates has been made. Care must be exercised not to disturb the tripod when changing the instruments. The camera is secured to the tripod by means of a separate triangular support (fig. 17); the three screws marked *a* serve to level up the camera before each exposure of a plate. A brass plate, with two spirit levels placed at right angles to each other, can be attached to the uppermost side of the camera, and this pair of levels is used for the leveling up. The central clamp screw, *b*, serves also as vertical axis when revolving the camera in azimuth.

The camera box *A B C* (fig. 17) is provided with two nuts inserted into and made flush with the face of the camera, one in the center of a small side and the other in the center of an adjoining long side. These nuts receive the central clamp screw, *b* (fig. 17), of the triangular camera support, and a circular brass plate inserted into these same sides, with the nuts as centers, forms the bearings for the three camera rests, *c* (fig. 17), when revolving the camera horizontally. The clamp screw, *b*, of the camera support is drawn only tight enough still to permit the camera to be rotated in horizontal plane, and after the double levels have been inserted into the slide of the uppermost side of the camera, the latter leveled up and oriented, this central screw, *b*, is tightened sufficiently to secure the fixed position of the camera when the slide is drawn and the plate exposed.

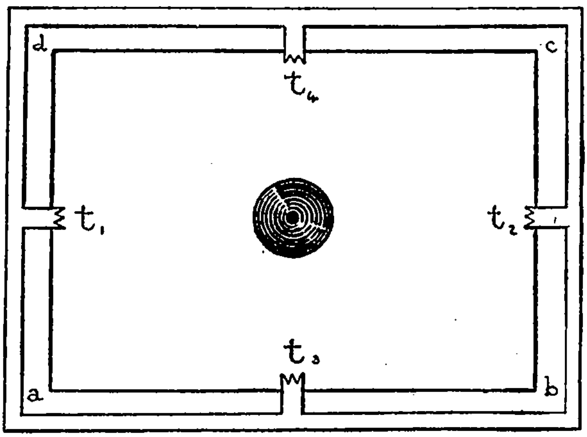


FIG. 18.

Each camera is provided with six double plate holders, *H H* (fig. 16), bearing a number on each side (from 1 to 12) to enable the operator to keep trace of the plates. The latter are made by B. J. Edwards & Co., The Grove, Hackney, London, England. They are the so-called isochromatic instantaneous plates of 4¾ by 6½ inches (old English half plate), all of one emulsion and made as uniformly in every respect as possible.

Four sets of teeth (fig. 18), each set about one-eighth of an inch wide, are securely fastened to the camera box, as close as possible to the plate-holder slides, in such a manner that the lines (horizon and principal lines) joining the middles of two opposite sets are parallel to the faces of the camera box. These metal teeth t_1 t_2 t_3 t_4 (fig. 18) are placed close enough to the plates to give sharp and well-defined prints of the same. After the camera has been leveled up the plates are vertical, the line t_1 t_2 is horizontal, t_3 t_4 is vertical, and the "principal point" (the intersection of t_1 t_2 and t_3 t_4) is in the optical axis of the camera. Capt. E. Deville has changed these teeth, as they were too long, and inasmuch as the lens, levels, sunshade, etc., are carried within the camera box during transportation, and the jarring motion to which the pack is exposed is liable to dislodge, or at least to bend, the teeth, he advocates their being placed farther back, as shown in figure 18*a*.

Some of the Canadian cameras have a revolvable plate, with lens eccentrically located, so that the width of the picture remains the same throughout, but the horizon can be elevated or lowered by turning this revolvable plate. This plate, being of wood, swells in damp weather and then can not be moved. Then, too, every movable part of an instrument is a source of uncertain errors. Capt. E. Deville's experience teaches that the best results are obtained with a camera that is perfectly rigid in all its parts, of a constant focal length, immovable lens, situated in one-third the length of one short side from either long side and midway between two short sides of the camera as indicated in figure 18*a*. This arrangement will enable the surveyor to elevate or depress the horizon by resting the camera on the face *A B* or *C D* (fig. 18*a*).

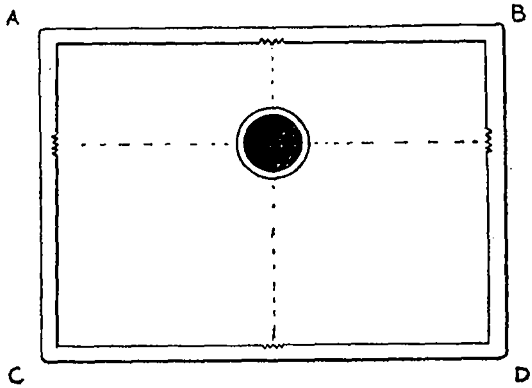


FIG. 18*a*.

A square diaphragm, *a b c d* (fig. 18), placed within the camera box admits only the light needed for the development of the negative, excluding side lights or rays which may possibly be reflected from the camera sides.

A small mahogany box, with a shutter made like a venetian blind, can readily be secured to the tube of the camera lens in case it becomes necessary to exclude the direct sunlight and shade the lens (fig. 19).

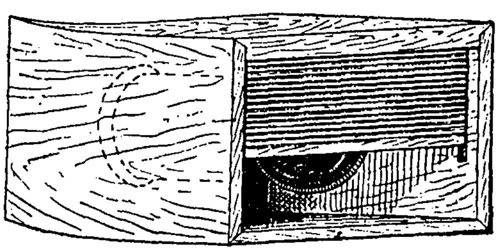


FIG. 19.

The camera faces, which are provided with the level attachment (*B* and *C*, fig. 16), also show two converging lines, $\alpha\ \beta$ and $\gamma\ \delta$ (side *C*, fig. 16), which indicate the range of the lens in horizontal and vertical plane. One set of these lines will appear on the upper face (*B*, fig. 16) of the camera when in use, while the other set ($\alpha\ \beta$ and $\gamma\ \delta$, side *C*) will appear on one vertical side. This arrangement enables the surveyor to see what part of the panorama he is taking during the exposure by sighting along the two lines of the horizontal face, and also up and down the two lines $\alpha\ \beta$ and $\gamma\ \delta$ marked on one of the vertical faces of the camera, thus dispensing with the use of the ground-glass plate and shade cloth altogether. After the camera has been oriented by means of these sight lines and leveled up, as mentioned before, the central clamp screw is tightened and the plate exposed.

The lens is a wide-angle lens, No. 1*a*, of 5⅝-inch focus, made by J. H. Dallmeyer, in London. It really is a combination of two similar lenses, between which the diaphragm is inserted. The aperture of the latter (the stop) used is always the same for all pictures. That end of the lens tube which faces the negative is closed by a planoparallel plate of a yellow or orange color to lessen the actinic action of the blue and violet rays upon the isochromatic plates, thus securing a sharp outline of distant mountain ranges and ridges.

With the plates used, this lens gives an angle of about 45° for the small side and 60° for the long side of the picture.

The camera, six double plate holders (including twelve plates), sunshade, levels, camel's-hair brush for removing dust particles from the slides, etc., are securely packed into a sole-leather case, which has straps attached to it in such a way that the whole can easily be carried on the back like a knapsack.

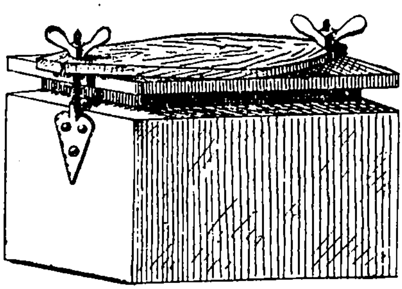


FIG. 20.

The triangular support of the camera is packed with the transit, and the case of the latter is also inclosed in a sole-leather knapsack, with straps for the extension tripod, both being carried together on the back.

The cameras are made by J. H. Dallmeyer, No. 25 Newman street, London, W.

The transits and tripods are made by Troughton & Sims, 138 Fleet street, London, E. C.

Every evening the surveyor replaces the exposed plates in his dark tent by new ones, using a ruby-colored light. He marks the exposed plates in one corner, before their removal from the holder, with his initials, the number of the dozen and of the plate, using a soft lead-pencil. e. g., III 5 means plate No. 5 of the third dozen. (The plates are packed in sets of a dozen each.)

The exposed and marked plates are placed into a double tin box (fig. 20) which can be closed hermetically and which will float when filled with two dozen plates, if by accident it should be thrown into water. These boxes are shipped to the head office, in Ottawa, where the plates are developed by a specialist (Mr. Topley).

The outline sketches of the different perspectives are designated by the same numerals as the plates to which they belong. They show the peaks, saddles, and points to which horizontal directions were taken with the transit (or altazimuth), and they also contain remarks about the weather, illumination, time of exposure, names of localities and features, and any other needed data.

The data obtained with aid of the transit for triangulation purposes are recorded in the usual manner.

The length of exposure for the plates is determined experimentally, as it may be assumed that the same length of exposure will suit a similar subject under similar conditions, with a light of equal intensity, the plates being all of one emulsion.

So-called photometers are used to measure the intensity of the light. They consist of an endless strip of sensitized paper incased in a small metal box—like a small tapeline—a short portion of the paper being exposed to the light and the time noted (in seconds) which it takes to bronze the exposed part of the paper. The nature and coloring of the subject vary but little in phototopography, and the time of exposure should be regulated with reference to the shadows or dark colors of the distant landscape; the darker these are, the longer the time of exposure should be (ten to forty seconds).

On the southeastern Alaskan boundary survey, Mr. O. J. Klotz, Dominion topographical surveyor, received the exposed plates from the different shore parties, and by way of test developed one plate out of every set of a dozen plates in a dark room fitted up for this purpose on the steamer *Thistle* to see that no bad plates had crept in.

The other plates were shipped to Ottawa, where the photographic specialist developed them and also made the enlarged prints (four times the size of the original negative) on heavy bromide paper, which enlarged prints are preferably used for the map construction, as they permit a greater precision in making direct linear measurements and the drawing of construction lines, which would become too minute and intricate if done on the small contact prints. However, if the loss of detail becomes a serious objection, larger cameras should be used or the enlargements should be made on glass. Glass transparencies (enlarged from the small negatives) show minute details in the shadows as well as in the high lights and assure more accurate results, there being no irregular expansion and contraction, as will always be more or less the case with paper prints. The only objection against enlargements on glass lies in the fact of their being less handy in manipulation during the process of the map construction than paper prints. Still the latter could be used for the location of points of detail and minor importance, while all data forming the control of the map are preferably deduced from the enlarged glass prints. Captain Deville is greatly in favor of dispensing with the use of paper prints altogether, and advocates the use of glass enlargements exclusively, the ensuing loss of time being outweighed by far by the great gain in accuracy.

The horizontal angles observed with the transit (or altazimuth) to the points of the terrene marked on the outline sketch which accompanies each negative serve not only for the orientation of the horizontal projection of the plate on the plan (the picture trace), but they also aid to counteract in a measure and to ascertain the distortion of the paper prints. The vertical angles, with the plotted distances, are used to check and verify the position of the horizon line on the different photographs.

To test whether the plate is vertical after the camera has been leveled up the following process is carried out:

Insert a piece of plate glass or a plano parallel mirror of $4\frac{3}{4}$ by $6\frac{1}{2}$ inches into the plate holder, open the rear slide of the latter, and level the camera carefully. Now set up a level (altazimuth or transit, with the vernier of the vertical circle set at 0°) near the back of the camera and revolve in azimuth until a well-defined and distant point of the landscape is covered by the intersection of the cross wires of the level telescope; the reflection of the same point must also be visible in the reflecting surface at the back of the camera from the level station. If on directing the level toward this reflected image the latter is also

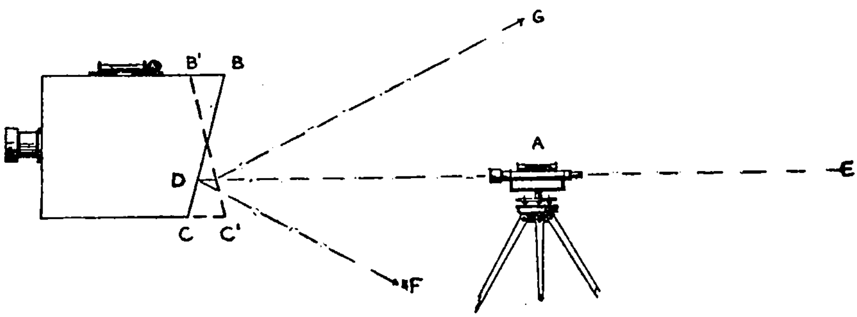


FIG. 21.

bisected by the horizontal wire of the level the plate in the camera will be vertical. (See fig. 21.)

If A is the position of the level and E the selected point of the terrene on level with the elevation of the instrument and B C the reflecting surface at the back of the camera, then the line A E will be horizontal, and if the plate B C is vertical E will be reflected in B C at D and D A and E will be in the same horizontal plane.

Should the top of B C be inclined toward A (as in fig. 21), then the ray E D will be reflected in the direction of D F, and the reflected point D will no longer be bisected by the horizontal wire of the level, but will fall below the same. Should the plate B C incline upward, as shown in B' C', the reflection D of E would appear above the horizontal thread of the level telescope, E D being now reflected in the direction D G.

Should the plate be thus found not to be vertical after the camera has been leveled up, the inclination must be changed by means of the leveling screws of the camera until D falls upon the horizontal thread of the level telescope, and the level, which is at right angles to the camera plate, must now be adjusted to conform with this corrected position of *B C*.

The plate holders must of course be well made, and all be exactly alike, so that the above conditions are fulfilled by every one of them, and that the distance of the sensitive plate from the lens be the same; i. e., the sensitive surface of every plate should fall into the focal plane of the camera lens.

The focal length of the camera, which has a constant value for every camera, must be determined directly if the negatives are to be used for plotting; but if prints are to subserve the construction of the map, this determination should be made from a print.

It has been previously mentioned that the prints rarely correspond in size with the negatives. They either expand or contract, sometimes both, and the distortion is greater in one direction of the paper than in the other. If this distortion is uniform in all directions the print will be similar to the negative and correspond to the perspective of the same landscape on a vertical plane (parallel to the plate), but nearer the lens when contracted and farther from the lens when expanded. The prints have either a shorter or a longer distance line (focal length) than the camera plate.

As (enlarged) prints are used for the map construction of the Canadian survey in southeastern Alaska, the constants required for this construction of the horizontal plan (i. e., the focal length, the horizontal and principal lines) are obtained from such a print.

This is done by taking a picture of some large building or any landscape with well-defined points from a station of which the distances to said points are known or can be ascertained by direct measurements in the field. From the same station vertical and horizontal angles are measured to the selected points, and the points as well as the station are plotted on a sheet of paper, and radials are drawn from the plotted station through the selected and plotted points marked on the print and plotted on the paper.

On a strip of paper, one edge of which is made perfectly straight, the points marked on the photograph are laid off, and this strip is moved over the plotted radials until the lines bisect the corresponding points marked off on the straight edge. A line is now drawn along this edge on the drawing sheet and a perpendicular dropped on this line from the plotted station. (See fig. 22.)

The line *H H'*, representing the paper edge, will be the picture trace, the perpendicular line *Ò P* will be the distance line, and *P* will represent the horizontal projection of the principal point.

The paper is now again laid on *H H'* in such a manner that the radials bisect the points marked on the straight edge, and *P* is marked off on the latter.

From the known distances of the reference points from the station and their vertical angles the elevations of these points, above or below the horizon of the station, are computed and laid off on the photograph. This will enable the draftsman to draw the horizon line on the photograph, and after projecting the marked reference points upon this line the strip of paper is placed on this line in such a manner that the corresponding points cover each other, and P is transferred to the picture. A vertical to $H H'$ through P will represent the principal line $V V'$ on the print. The points where the principal and horizon line bisects the comb marks are noted and will serve to draw these lines on all other similar prints without any new determinations being necessary.

In order to lay off the elevations of the selected points above or below the horizon of the station an approximate horizon line had to be drawn on the photograph. This is done by setting the vernier of the vertical circle of the transit (which replaced the camera in such a way that the

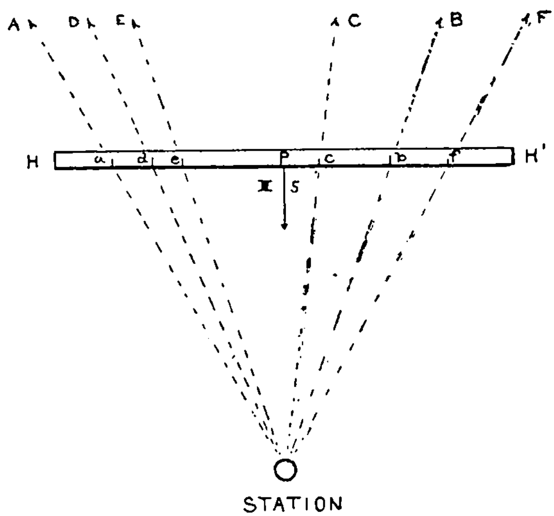


FIG. 22.

optical axes of camera and level telescope were in the same horizontal plane) at 0° and noting several points of the building or landscape which were bisected by the horizontal thread of the telescope while level and by drawing a line through the same points pictured on the photograph.

PLOTTING.

The field data of the Canadian surveys in Alaska are plotted on a scale of 1:80000, with a contour interval of 250 feet, indicating the 1,000-foot contours by heavier lines.

From the original negatives copies are made, four times enlarged on heavy bromide paper (9½ by 13 inches), which are used for the construction of the maps.

The triangulation points, obtained by means of a 4-inch transit, are plotted, and their elevations (obtained barometrically, and checked by vertical angles whenever convenient) are added in red ink.

Now the principal line and the horizon line are drawn on the prints through the proper points of the comb marks, or in proper position with reference to the position of the lens during the exposure of the original plate, if the lens was attached to a revolvable plate, an arrangement not approved by Capt. E. Deville, as a surveying camera should be rigid in all its parts in order to obtain uniform results.

The horizon line, obtained by means of the comb marks, is checked and corrected, if necessary, by means of pictured points of known elevation, each print containing two or more points which have been determined trigonometrically and which have previously been plotted on the working sheet and the elevations of which are inscribed in red ink.

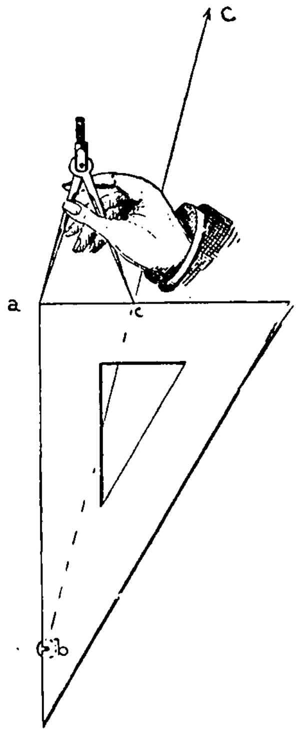


FIG. 23.

The picture traces are plotted on the working sheet as follows (fig. 23):

Take a triangle of hard rubber or wood and mark off along one side the focal distance of the print $a\ b$ from the right angle a, and carefully notch b so that the center of a needle inserted into this notch will be at a distance from a equal to this focal length. From the outline sketch select a point, C (fig. 23a), to which angles have been read, and take the abscissa of $C = C_0\ P_0$ between the points of a pair of dividers. Insert a fine needle into the plotted station (b, fig. 23) whence the picture was taken and place the triangle notch b close to this needle; now move the triangle with the left hand so that the line of direction to the plotted point c will be covered by the triangle, and with the right hand hold the dividers, moving one point over this line of direction until $a\ c$ is equal to $C_0\ P_0$ (the triangle must be moved likewise until this takes place). The triangle is securely held in this position and lines are drawn on the working sheet along $a\ b$ and $a\ c$; prolong $a\ c$ beyond a, and now check the distance $a\ c$ again with the dividers (to be $= C_0\ P_0$). Then $b\ a =$ horizontal projection of principal ray.

$a\ c =$ horiz. projection of picture plane.
$= H\ H' =$ picture trace.
$a =$ principal point.

The trace of the principal plane ($= a\ b$) is only marked by a short line bearing an arrow pointing toward the plotted station whence the picture was taken, and the point a is marked the same as the print to which it refers; e. g., III 5 (fig. 22). The prints are marked with the name of the camera station whence they were taken, with the numerical designation of their negative, and with the elevation of the horizon line above the datum plane; e. g., *N N*, III 2, 4,860, means name of camera station, second plate of third set of a dozen plates (or really the twenty-sixth plate), and elevation of camera horizon = 4,860 feet above mean sea level.

After the traces of the principal and corresponding picture planes have been plotted, the draftsman selects from the prints a pair of photographs which are represented on the plan by their traces, taken from two different camera stations and overlooking common ground. On these pairs he identifies as many points as possible, marking all corresponding points alike with numbers or letters and with a dot in red ink. Of course such points are chiefly selected which indicate characteristic features of and changes in the terrene like knolls and depressions in ridges, peaks, bends in streams, coves in the shore line of lakes and rivers, buildings, etc.

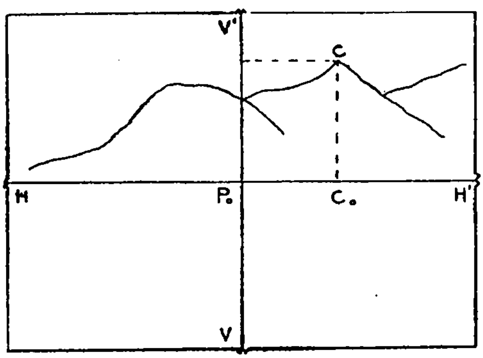


FIG. 23*a*.

After enough pictures have been selected to develop a certain area and the identification and marking of corresponding points have been completed, projections of all these points on the horizon-line are marked (their abscissas are measured) and transferred to the straight edge of a strip of paper, including the principal point of every photograph.

These strips are now placed upon the plotted traces (fig. 22) to which they belong in such a manner that the principal points of trace and paper strip coincide, and in this position they are held on the working sheet by means of small thumb tacks or paper weights.

To plot the horizontal projections of a point shown and marked on two prints we insert two fine needles into the plotted stations of these two prints and attach a fine silk thread to each needle. The other end of the thread is connected with a small paper weight by means of a thin rubber band (fig. 23*b*).

The weighted thread attached to station needle *I* is now moved over the weighted strip (indicating the picture trace) until it bisects the projection a' of the sought point *A*. The weight is now placed upon the paper, holding this thread under slight tension in this position. The second thread connected with the needle in Station *II* is placed over the projection a'' of the sought point, also under tension. The point of intersection of these two threads will be the desired point *A* plotted on the map. After this position of point A on the plan has been checked, in the same manner, by means of another photograph taken from a third station and containing the picture of this point, its plotted position is marked by a dot and its designation, as given on the prints, in red ink.

After a sufficient number of points have been plotted in this manner by intersections, and after they have been supplied with the letters or numerals (in red ink) as given on the prints, their elevations are deter-

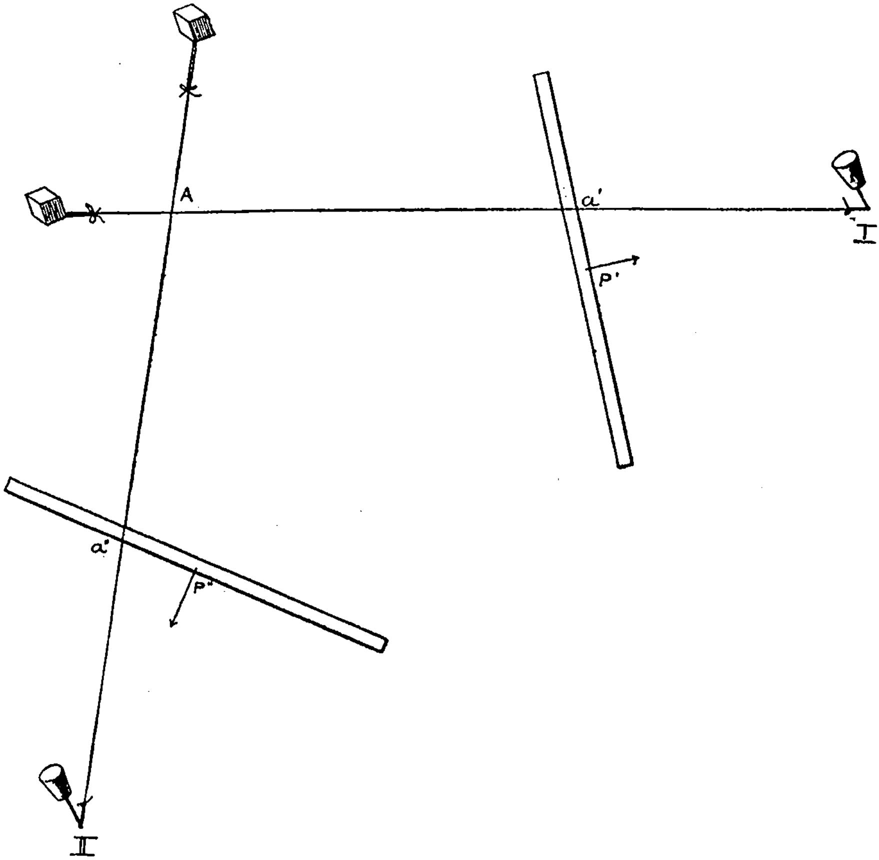


FIG. 23b.

mined and also added in red ink. Frequently the designation of the points by letters or numerals are only added in pencil, to be erased after the elevations have been added in red ink.

ELEVATIONS.

All points of the prints which are bisected by the horizon line $H\ H'$ have the same elevation as the horizon of the camera station, which fact will greatly assist in drawing in the contours on the plan. The latter can be plotted or drawn in with the same precision as is attainable in other "irregular methods" of contouring if only enough points can be identified on the prints and established by intersections and their elevations to cover the area sufficiently close to leave no place for doubt. Ridges, which appear in profile on the prints, will also facilitate contouring, inasmuch as lines of directions drawn to characteristic points of these ridges can be regarded as tangents to the contours passing through such points. The heights of the points fixed by intersections are found by means of a so-called "scale of heights" (fig. 24).

$S\,x\,P$ = straight line divided into equal parts.

$S\,P$ = focal length of prints.

$P\,P'$ perpendicular to $S\,P$ and divided into equal parts.

Erect verticals to $S\,P$ in the points of division. Join the points of division along $P\,P'$ to S.

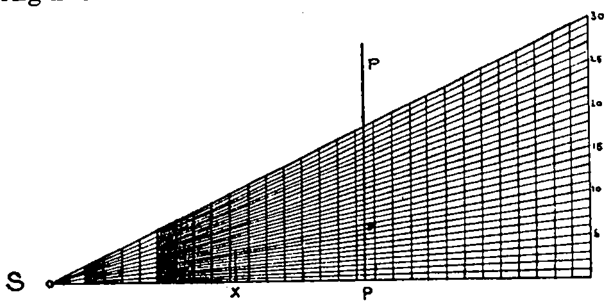


FIG. 24.

This scale is used as follows:

Take, with a pair of dividers, from the photograph the ordinate of any point bisected by the principal line of which the elevation is sought, transfer this length to $P\,P'$ from P. Suppose it corresponds to $P\pi = 11$ parts of the graduation of $P\,P'$.

Now take from the plan with the dividers the distance of the horizontal projection of the point (previously plotted by intersections) from the picture trace and lay this length off on $S\,P$ from P to the right or left of P according to the position of the point on the plan in regard to the picture trace if beyond or within the trace and station. Suppose the point was between the plotted station and picture trace and it fell on x. Then the distance from x to a point vertically above x on the ray $S\,\pi$ (= 11) and measured on the plotting scale will represent the elevation of the point above or below the camera horizon. If the point on the photograph was above $H\,H'$ this length will have to be added to the camera elevation and the sum is entered on the plan in pencil close to the plotted point. After it has been checked by a second photograph, and the discrepancy between these two heights is within the permissible limit of error, the mean is entered in red ink on the plan and the pencil marks are erased. After the elevations of all the points plotted on the working sheet have been determined and entered on the drawing in red ink, the streams, ridges, bluffs, and shore lines are drawn in, using intersections and tangents, whenever possible, to identify and locate their characteristic bends, their terminals, etc.

Now the contour lines are drawn in by estimation between the established points of known elevation ("irregular method"), having the shore lines, streams, and ridges as guides, and studying the photographs as much as possible to modify the contours so as to represent minor inequalities and accidents of the terrene.

As long as a sufficient number of points is obtained by intersections there will be little difficulty in drawing in the contour lines, but in a rapid reconnaissance it may happen that the points which can be plotted are too few and too far apart for defining the surface, when it will become necessary to resort to so-called "tricks of trade" and less accurate methods. The perspectograph and similar instruments to convert perspectives into plan drawings and vice versa are too complicated (the numerous movable parts are sources of too much lost motion) to give results sufficiently accurate for topographical maps.

THE PHOTOGRAPH BOARD.

(Fig. 25.)

So many lines are needed and drawn for the constructions on the photographs that it is advisable to prepare a special drawing board on which as many of the construction lines are drawn, once for all, as would have to be repeated for the different prints of uniform size. This so-called "photograph board" is an ordinary drawing board covered with tough drawing paper, the surface of which is to represent the picture plane, and it is used in conjunction with the photographs.

Two lines, $H H'$ and $V' V$, are drawn at right angles to each other; they represent the horizon and principal lines, while $V P_0 = H P_0 = V' P_0 = H' P_0 =$ focal length of prints. By revolving the horizontal plane about $H H'$ we obtain the upper and lower distance points V' and V in the picture plane, and by turning the principal plane about the line $V' V$ into the picture plane we obtain the left and right distance points H and H'.

The photograph is put on the middle of the board in such a manner that the principal line coincides with $V' V$ and the horizon line with $H H'$. The four scales, forming the sides of the square $T U R S$ (a little larger than the photograph which falls within $T U R S$), can be used to draw parallels to the horizon and principal lines, without obscuring the print by too many pencil lines, by placing a ruler on the corresponding graduation marks of two opposite scales. Also for marking the "ground line" for any station by joining the graduations of the vertical scales representing the height of the station.

At a suitable distance from H, outside of the photograph field, a perpendicular, $K L$ to $H H'$, is erected, on which line are marked, by means of a table of tangents, the angles formed with $H H'$ by lines drawn from the left-hand distance point H. This graduation, $K L$, serves for measuring the horizontal angles or the altitudes of points selected on the photographs, as will be explained in the following:

From V as a center describe a circular arc, $P_o C$, with $V P_o =$ focal length as radius, and divide this arc into any number of equal parts. Through the points of division, between $P_o C$ and $P_o H'$, radials are drawn from V as center.

In order to obtain the elevations of points marked on the photographs the radial distance from the station to the horizontal projection of the point in the picture trace must be known. If the sensitive plate formed

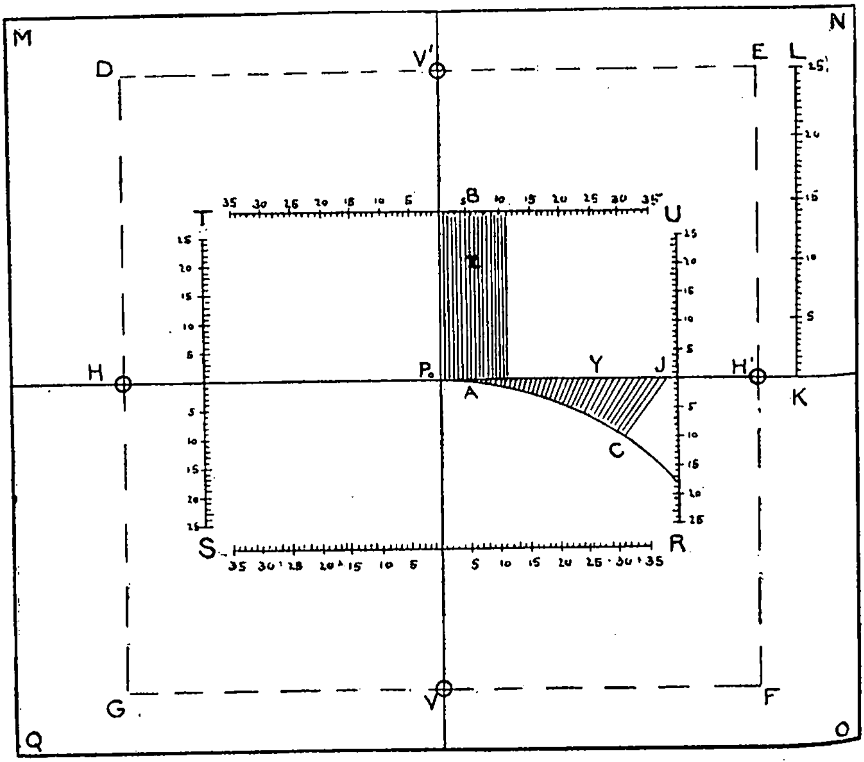


FIG. 25.

a part of a vertical cylinder with radius = focal length, then these distances would all be constant and equal to the focal length of the print. The arc $P_o C$ (divided into any number of equal parts) and the line $P_o H'$ cuts off pieces of the radials drawn from the station V to the various horizontal projections of pictured points, which must be added to the focal length $V P_o = V C$ to give the horizontal distance of the station to the projected point on the photograph.

The equidistant lines $A B$, drawn parallel to the principal line, are also drawn sufficiently close together and cover a space in width equal to the largest radial difference $C J$. All these lines ($A B$ and $A C J$)

are used in connection with the scale of degrees and minutes *L K*; e. i., on a print, *T U S R* (fig. 26), we wish to obtain the elevation $c\ c'$, the vertical and horizontal angles to *c* having been observed in the field and noted on outline sketch. From *H* (fig. 25) we draw a line through that division mark on *K L* which corresponds to the vertical angle of *c*, say $10\frac{1}{2}°$. Now the abscissa $P_0 c' = x_c$ is laid off (on the photograph board) along $P_0 H'$ from P_0; the second point of the dividers may fall upon the twentieth radial difference (counting from P_0 toward *c*). The length of the difference between the radius of the arc $P_0 c$ and the distance *V C* (from the plotted camera station *V* to the plotted point *c*) is now laid off along $P_0 H'$ from P_0, which may fall midway between the fourth and fifth line (counting from P_0 as zero) of the set *A B*. Then the distance from the line $P_0 H'$, taken midway between the fourth and fifth *A B* line to the line *H* $10\frac{1}{2}°$ (on degree scale *K L*) and measured on the plotting scale, will be the difference of elevation $c\ c'$, which, added to the elevation of the camera horizon, will give the elevation of the point *c* above the datum plane. Sometimes the angle between a point, *a*, on the print and the principal and horizon lines (altitude and azimuthal angles) may be wanted.

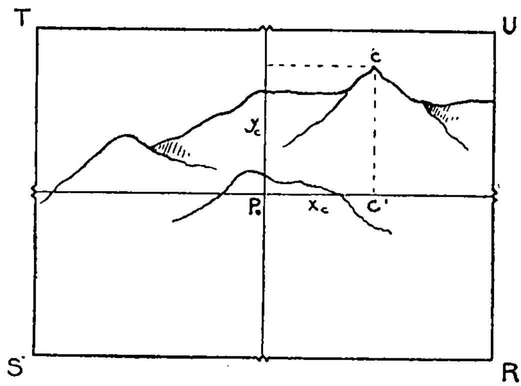


FIG. 26.

The azimuthal angle $P_0 P a_0$ (fig. 27) can be found directly by joining the plotted station *P* to the horizontal projection a_0 of the point a'. To find the same in degrees and minutes we transfer the abscissa $P_0 a_0$ of the point (fig. 27) a' to $P_0 V'$ (on the photograph board) from P_0 and draw a line from *H* through this point on $P_0 V'$. Where this line intersects the scale *K L* will be the reading indicating the value of the azimuthal angle in degrees and minutes.

The altitude of the point a' on the photograph (fig. 27) is represented by the angle $a' P a_0$, and to find its value we transfer $P_0\ a_0$ the abscissa of a' (fig. 27) to $P_0 H'$ from P_0 on the photograph board; say, equal to $P_0 Y$. With the same pair of dividers we take the radial difference at *Y* (distance of *Y* to the arc $P_0\ C$) and transfer the same to $P_0\ H'$ from P_0, and note which of the verticals *A B* falls upon the second point of the dividers. On this vertical we transfer the ordinate of $a' = a'\ a_0$ from $P_0 H'$; say, equal to *A Z*. If we draw a line through *Z* from *H*, this line will indicate on the divisions of scale *K L* the value of the angle $a'\ P\ a_0$ (fig. 27) in degrees and minutes.

The foregoing is a general description of the plotting methods employed by the topographers under Dr. W. F. King, boundary commissioner for the southeastern Alaskan survey. There remain, of course, some minor details which serve in a measure to facilitate the work of plotting and which every draftsman acquires by practical application, and which are not touched upon in the preceding pages.

The enlarged prints are made upon positive bromide paper, using a good copying lens to secure as much detail as possible. The prints are developed and dried in the usual manner, and classified. The essential requirement for a true copy or correct enlargement is that the sensitized paper be parallel to the negative. Both the printing camera and the easel upon which the paper rests are provided with graduations on their slides to facilitate giving the correct distances both for enlarging and reducing. An inclined position of the easel and negative is the

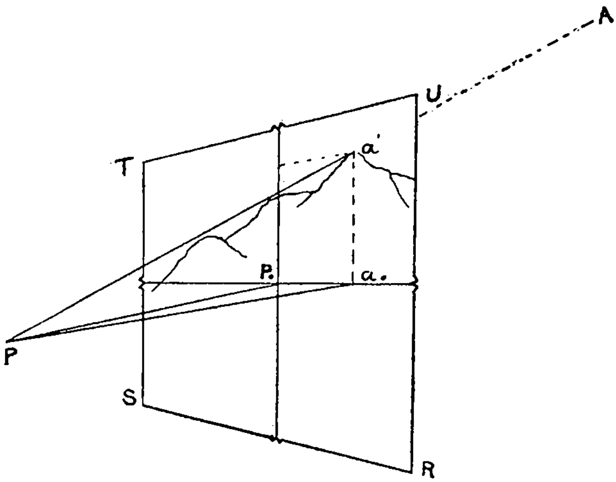


FIG. 27.

essential feature of the Canadian printing apparatus in order to give the negative the full skylight and not have a part of the plate illuminated by reflected rays from the earth, thus giving the entire plate a uniform light during the time of printing.

In France, Germany, and Italy the tendency has been toward combining the transit and camera into one instrument, while in Canada these instruments have been kept separate. The reason, probably, is that in the European countries mentioned, photographic surveys were made on a large scale and the means for safe transportation were comparatively within easy reach, while in the Canadian surveys the work was done on a small scale and the instruments had to be carried over rough country, frequently on the human back, thus making it essential to reduce the weight to a minimum (which could be best done by keeping the instruments separate) and to make them rigid and easy of adjustment.

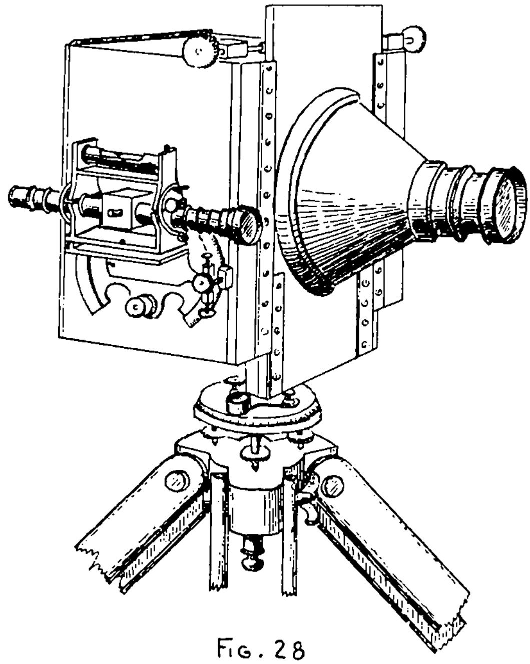


FIG. 28

FRENCH PHOTO-THEODOLITE.

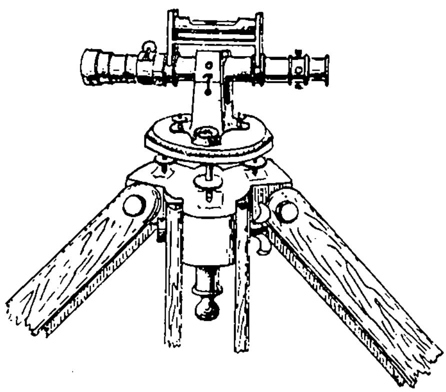


FIG. 29

THE SAME WITH CAMERA DETACHED.

Figures 28 and 29 show the latest model, in its general form, of the French "phototheodolite," which was on exhibition at the World's Columbian Fair, in Chicago. Figure 28 shows the complete phototheodolite, and figure 29 shows the theodolite without the camera, in which form it is used for trigonometrical purposes, the triangulation being made before the phototopography is begun.

This phototheodolite has a "declinatoire" (compass) and a pair of sights, which will enable the observer to direct the optical axis of the camera quickly toward any desired object or point of the panorama. The "declinatoire" is principally used for traverse work, to note the magnetic meridian from time to time. This compass and the sights are not visible in the figure; they are on the side opposite to the one with the telescope.

The near side of the instrument (fig. 28) shows the telescope, level and vertical circle for making angular measurements, in addition to the previously executed triangulation, in order to determine the position of the camera station (by the three-point problem) with reference to surrounding triangulation points or for running traverse lines between triangulation stations. The horizontal circle is under the camera proper and carries a box level. The optical axis can be elevated or depressed (maintaining a horizontal position) by means of a slide or shutter to which the camera lens is attached.

Dr. Meydenbaur's camera theodolite is a camera of constant focal length, constructed in metal throughout, with neither telescope nor vertical circle, but it is provided with a horizontal limb and mounted on a tripod.

After the instrument is leveled up, the panorama surrounding the station can be surveyed (photographed) by making six consecutive exposures, revolving the camera 60° in azimuth after each exposure by means of the horizontal circle, which is graduated to read to single minutes.

The lens is a pantoscopic one, made by E. Busch, in Rathenow, Prussia. It commands an angle of about 100°, but by excluding the external ring of this lens by means of a small stop in the diaphragm, pictures are obtained subtending a horizontal angle of about 66°, thus giving two consecutive plates a common margin of 3° width, horizontally.

The horizontal projections of the six picture planes, representing the panorama surrounding one station, form a regular hexagon, and after one picture trace has been plotted and oriented on the plan, the remaining five are readily plotted by constructing this hexagon of which one side is given.

To use this instrument it is necessary that the country to be sursurveyed photographically be well supplied with a generous number of carefully selected triangulation points of a recently made triangulation, as it is necessary that the signals shall be still intact and standing during the photographic operations. The triangulation must also include the hypsometrical determinations of all the stations, as no direct measurements of vertical angles can be made with this camera theodolite. The elevations of all the other points needed for the topographic development of terrene will have to be obtained by constructions from the negatives or photographic prints by means of the elevations of the horizon line, obtained from the given elevation of the station (triangulation point) and the height of the instrument above the ground. The survey will have to be plotted on a large scale and numerous camera stations will have to be occupied.

The recently made small traveling camera theodolite of Dr. Meydenbaur dispenses with plate holders, inasmuch as the plates are placed directly against the rear frame of the camera by an ingenious arrangement with which the plates can be exchanged under exclusion of light.

One of the more recent productions of photographic surveying instruments in Austria is Captain Hübl's plane table photogrammeter, made by R. Lechner in Vienna, and described in "Lechner's Mittheilungen aus dem Gebiete der Photographie und Kartographie." Wien, Graben No. 31.

The camera proper has also been modified like the latest Meydenbaur camera by discarding the plate holders. Captain Hübl places the sensitive plate directly against the camera casing, where it is held in position by means of springs, thus securing a constant permanent focal length. The camera box is cube shaped and has sides of 21^{cm} length. The plates are 12 by 16^{cm}, but the pictures are only 10 by 14^{cm}. The camera alone weighs $3\frac{1}{2}$ kilogrammes; with knapsack, including entire outfit for one day's work and stout tripod, the weight is $11\frac{1}{2}$ kilogrammes and the cost in Vienna is 400 florins.

This instrument is the result of Captain Hübl's efforts to reduce the weight and cost of the camera theodolite and to simplify the adjustments and manipulation of the same.

For this reason the theodolite has been replaced by a plane table and small alidade.

The upper surface (horizontal) of the camera, 21 by 21^{cm}, serves as plane table; it is provided with a pivot with which the alidade is connected. By means of special appliances the picture trace, principal line, and point, as well as horizontal directions to known (triangulation) points, for the orientation of the picture trace, are drawn directly on the paper, resting on the upper horizontal surface of the camera.

Each negative can thus be accompanied by a small plane-table sheet showing a bunch of rays radiating from the station point to a number of known points (triangulation points) in correct relation and position to the picture trace and principal line, besides numerous data which can be sketched and inscribed upon the paper.

The results obtained with this photographic plane table are easily transferred to the working sheet containing a plot of the triangulation. The adjustments of this instrument are few and simple.

APPENDIX No. 4—1893.

ON PHOTOGRAPHY AS APPLIED TO OBTAIN AN INSTANTANEOUS RECORD OF LUNAR DISTANCES FOR DETERMINATIONS OF LONGITUDE.

By C. Runge.

Translated and submitted for publication December 9, 1893, by J. A. Flemer, Assistant.

Since Dr. F. Stolze's treatise on photographical determination of geographical positions without the use of chronometers was published, Mr. C. Runge, in Hanover, Prussia, has made experimental observations with an ordinary camera, such as travelers and explorers generally carry in their outfits, to develop a photographic method for obtaining the geographical longitude and latitude of a place, as well as the local time by means of photography.

Photographic determinations of the latitude and the local time of a place, however, do not offer great advantages, compared with the general methods heretofore in use for obtaining these values, as nearly every explorer will carry in his outfit instruments which can readily be used for this purpose, the ordinary methods for astronomical latitude and time observations being comparatively simple and easily applied. We will, therefore, in the following pages, consider only Mr. Runge's method for determining the geographical longitude photographically.

The desirability of developing a method for finding the longitude without the use of chronometers, which, when the geographical longitude of a place had been determined by means of chronometer readings, could also be used to check the latter, had not only been recognized by Mr. Runge, but he also felt convinced that if the method should find favor with explorers, the necessity of making astronomical observations—such as would be necessary, for instance, for longitude determinations based on lunar distances—could be avoided.

A full description of Mr. C. Runge's first application of photography, made June 17, 1893, in Hanover, for this purpose, will be found in the Zeitschr. f. Verm., Heft 15, 1893, of which the following is a free translation.

The camera, placed upon a window sill and its position secured, as well as possible, against accidents, was directed upon the new moon at 10 p. m. (June 17, 1893) and eight successive short exposures of the same plate were made; at 10^{h}, $10^{h}\ 2^{m}$, $10^{h}\ 4^{m}$, $10^{h}\ 8^{m}$, $10^{h}\ 10^{m}$, $10^{h}\ 12^{m}$, $10^{h}\ 14^{m}$, and at $10^{h}\ 23^{m}$ p. m., an ordinary watch being used for timing the latter.

The camera, with objective closed, was left undisturbed in the same position until the constellation Leonis appeared in the same part of the firmament where the moon had been photographed. At $10^{h}\ 51^{m}$ (by the same watch) the objective of the camera was uncovered and the plate remained exposed, short interruptions excepted, until $12^{h}\ 45^{m}$.

These interruptions, of five seconds' duration each, were effected by means of a dark cloth, with which the objective was covered, without having been brought into contact with the same.

Twenty such interruptions of the exposure were made in toto, and their times of occurrence were carefully noted by the same watch and recorded. Between $10^{h}\ 51^{m}$ and $11^{h}\ 00^{m}$ two such interruptions took place. From 11^{h} till 12 p. m. interruptions were made every five minutes, beginning at the full minute and lasting five seconds (12 breaks); from 12 p. m. till $12^{h}\ 40^{m}$ the breaks occurred in the same manner, but at intervals of ten minutes (4 interruptions), and two breaks were made at odd times, one at $11^{h}\ 37^{m}$ and the other at $11^{h}\ 54^{m}$.

After the plate had been developed the moon's crescent appeared eight times, as was to be expected, in the central portion of the plate. Above and below this row of moon pictures the star traces were plainly visible in the shape of smooth curves of a regular curvature.

From the relative positions of these curves and the positions of their beginnings and end points it could readily be conjectured which star of the constellation Leonis belonged to each curve. The star traces of α, β, γ, δ, ε, ζ, η, and ϑ Leonis, besides some other faint star traces, were plainly distinguishable upon the developed plate. The trace made by δ Leonis was the most distinct of all, this star having been farther north (in a darker portion of the firmament) than the other bright stars of the constellation.

The traces of β and δ Leonis, scrutinized under a microscope, distinctly showed the gaps, corresponding with the recorded interruptions, made during the exposure. These breaks were less clearly shown in the other star traces, partly because they appeared less bright, having been nearer the horizon, and partly because their light was dimmed by the illuminated western horizon.

The positions of the two breaks, corresponding with the two interruptions of the exposure, made at random (at $11^{h}\ 37^{m}$ and $11^{h}\ 54^{m}$) among the regular series, formed characteristic pointers toward identifying the breaks with their corresponding recorded time observations (by the watch).

On the lower part of the plate the upper outlines of two buildings were shown, one with a lightning rod and the other with a flag pole.

In order to ascertain the Greenwich time by means of this plate, Mr. Runge employed different methods of mensuration. The mensuration was done with an instrument used heretofore for making measurements on photographic plates in analytical investigations of spectra (with the spectroscope). Its principal parts are a frame over which a sleigh can be moved horizontally by means of a horizontal screw having a very fine thread.

The plate is placed upon the movable holder (sleigh) and illuminated from below by means of an inclined mirror. A microscope with cross wires is secured to the stationary frame in such a manner that the plate can be studied through the same while the plate is passed underneath in a horizontal plane by turning the screw passing through the plate holder (sleigh). The horizontal linear change in position of the plate when thus moved is measured by the number of turns of the screw. This screw has two threads per 1^{mm} length, and is supplied with a micrometer at one end, divided into 100 parts, and a vernier. Thus, $\frac{1}{200}^{mm}$ can be read with the index mark, and by using the vernier $\frac{1}{2000}^{mm}$ can be measured. A registering apparatus marks the full turns of the screw.

With the aid of this micrometer the following measurements on the plate were taken:

1. *To determine the right ascensions of the pictured crescents*, we find and mark on the trace of δ Leonis that point which corresponds with the meridian of one of the crescents—for instance, the first one of the eight shown on the plate—and by means of the gaps shown in this star trace the time of transit (the time as shown by the watch when δ Leonis had reached that particular spot on the trace) of δ Leonis at this marked point is ascertained.

The time which had elapsed from the moment of photographing the first position of the moon (10^h) until δ Leonis had reached the point in question on its trace gives in watch time the difference of the hour angles of the moon and δ Leonis.

This civil time interval converted into sidereal time will represent the difference of the moon's and δ Leonis's right ascensions; and after the value for the right ascension of δ Leonis has been taken from the fixed star catalogue of the Ephemeris, we will thus have found the right ascension of the moon at the time of the first exposure.

As only short time intervals enter into consideration, the quality of the watch is immaterial.

In detail the mensuration was made as follows:

By means of the difference between the right ascensions of δ and β Leonis, as taken from the Ephemeris, two points were located on the corresponding two star traces, situated as nearly as possible upon the same hour circle, which two points were joined by a fine line scratched into the film of gelatin on the plate by means of a fine needle and straight edge.

The photograph being a true perspective, all meridians are represented by straight lines on the same, and this scratch, if carefully made, should represent a meridian line. If the direction of this scratch does not appear perfectly correct, its deviation from the true position can be ascertained by measuring the distances from two corresponding breaks in the two star traces (of δ and β Leonis) to the intersections of the scratch with these two curves, and the necessary correction can be applied.

After this has been done, the distance between the crescents and the scratch are measured; and as the distance between the breaks—shown in the star traces of δ and β Leonis—between which the crescents are situated are known, we can compute the time interval corresponding with a certain length of the moon's circle of declination.

If the scratch passes close to the picture of the moon, a small error in the value with which the distance is to be multiplied in order to obtain the time interal will barely affect the result.

As only the edge of the moon could be measured in our case, a correction for the moon's semidiameter had to be applied in order to obtain the right ascension of the moon's center, which was done by measuring a chord and corresponding height of the crescent's arc. The resulting right ascension is free from atmospheric refraction, as it has been determined from the relative position of the moon with reference to the pictured stars on the plate, which were also subjected to the same atmospherical influences (and are likewise affected by refraction). In order to reduce this right ascension to the center of the earth the declination and the local time must be known or will have to be found.

2. *Mensuration of the declination of the lunar pictures.*—For this purpose the plate was placed upon the movable holder in such a manner that the direction of its course under the microscope (when moved in a horizontal plane by turning the micrometer) was vertical to the star traces.

Pointings were now made to as many of the star traces as possible, as well as to the edge of the crescent; for instance, the traces of ζ, δ, γ, β, and α Leonis were bisected, and also the edge of the crescent falling between γ and β.

If we now write opposite the micrometer values for the bisected star traces the declinations of the corresponding stars, we can regard the latter as a linear function of the micrometer readings. The two unknown values of this linear function are computed by means of the method of least squares, and after substituting the micrometer readings for the edge of the moon we find her declination.

The following example will show the degree of precision obtained in this manner:

	Micrometer reading.	Declination, taken from the Ephemeris.			Declination computed.			Diff.
		°	′	″	°	′	″	″
α Leonis.	249·0	12	29	26	12	29	15	+11
β "	2515·1	15	10	12	15	10	31	—19
Edge of moon.	6441·1				19	49	53	
δ Leonis.	7515·9	21	6	38	21	6	23	+15
ζ "	9916·7	23	57	07	23	57	13	— 6

We believe that the declination of the moon's edge can be found by this method within a limit of error of 20″, and if the star curves are all close to the lunar picture even a more close value may be obtained.

The semidiameter of the moon having been determined as mentioned above, we can now compute the declination of the moon's center. The declinations having yet to be reduced to the center of the earth, we will need for this purpose, besides the right ascensions, also the local time, which is found as follows:

3. *Determination of the local time.*—We could assume the local time to be known, as the explorer will generally determine the same astronomically, especially if he intends to determine the longitude of the place of observation by means of chronometer readings. He can readily find the local time by observing sun, moon, or star altitudes with a sufficient degree of accuracy for practical purposes.

Whenever the photographic plate contains the image of a fixed terrestrial point—for instance, the lightning rod, gable, or chimney of a distant building, the peak of a distant mountain, etc.—such point can be utilized in the same manner as a star. As mentioned before, the exposed plate contained a picture of a conspicuous lightning rod. The circle of declination and hour circle of this point were determined in the same manner as shown for the lunar pictures (by means of civil time as indicated by the watch). The following day (June 18) the elevation of this point (of the lightning rod) was determined from the place occupied by the camera in the preceding night. From this elevation and the declination we can determine the hour angle, if the geographical latitude be known, and we are thus enabled to compute the difference between the time indicated by the watch and the local time.

After the local time has been found in this manner, the determined values for the right ascension and declination can be reduced to the center of the earth, the altitude of the lunar pictures being known if the local time and the declination are given.

4. *Measuring a lunar distance.*—We measured the distance between the first lunar picture and that break in the stellar curve belonging to δ Leonis which was nearest the crescent without being intersected by the same hour circle.

The angle corresponding to a measured length on the plate can be computed if we know the distance between the traces of two stars. It is true, the plate represents the area of the firmament on a variable scale (it being a perspective representation of the photographed area of the firmament), yet for a small portion in the center of the plate we can assume that there is no distortion. In the present case the distance amounted to only $2\frac{1}{2}^{\circ}$. A star, the image of which would have appeared at the time of exposure for the first lunar picture on that part of the plate indicated by the break referred to, would have to have had the same declination as δ Leonis, and its right ascension would be found from the time which would have elapsed until δ Leonis had reached the same place.

By means of this imaginary star point we can compute in the same manner as for a true star its lunar distance for any given Greenwich time, and also from the measured lunar distance, reduced to the center of the earth, we can interpolate the Greenwich time.

The following tabulated results were obtained by these three methods:

Mean of the R. A.'s of the first three lunar picture centers reduced to the center of the earth.	Mean time for Greenwich.	Local time.	Difference of time.
h. m. s.	*h. m.*	*h. m.*	*m.*
9 23 40·7	8 56·3	9 35·4	39·1

Declination of the eight lunar picture centers reduced to the center of the earth.	Mean Greenwich time.	Local time.	Difference of time.
° ′ ″	*h. m.*	*h. m.*	*m.*
20 19 33	8 54·1	9 33·4	39·3
18 58	8 57·1	35·4	38·3
18 28	8 59·6	37·4	37·8
18 1	9 1·8	41·4	39·6
17 27	9 4·7	43·4	38·7
17 22	9 5·1	45·4	40·3
16 50	9 7·8	47·4	39·6
15 9	9 17·2	56·4	39·2
		Mean	39·1(±0·2)

Lunar distances measured and reduced.	Lunar distances computed.	Mean Greenwich time.	Interpolated mean Greenwich time.	Local time.	Diff. of time.
° ′ ″	° ′ ″	*h. m.*	*h. m.*	*h. m.*	*m.*
2 35 19	2 39 34 2 38 20	8 45 9 0	8 54·8	9 33·4	38·6

According to these three methods we find the three corresponding differences of time:

m.
39·1
39·1
38·6
Mean 38·93

The true difference of time for the "market tower" in Hanover City is, according to Gauss:

m.
38·943

And as the place of observation was 650 metres west from this tower, the difference of time for the camera station would be:

m.
38·90

This close result seems to partake of the nature of a coincidence. Still, we believe that an error of more than $0{\cdot}2^{m}$ is precluded if the measurements on the plate are made as carefully as the one just described. In order to obtain an equally good result by the ordinary astronomical methods, the observations would have to be made within 6 seconds. We believe, however, that this photographic method can be raised to a still higher degree of precision without having to add many mechanical devices.

In this first practical attempt the stellar pictures in reference to the pictured crescents were not very favorably situated, more favorable positions being of frequent occurrence. For instance, if we photograph the moon at the moment when she has the same apparent declination as the star, and after the moon has passed this point allow the star to trace its path (making suitable breaks in the star trace by means of short interruptions of the exposure) over the plate until this moon picture is bisected by the star trace, it will be possible to obtain the right ascension with the same degree of precision with which the breaks in the star trace can be measured (bisected with the cross wires in the microscope). If we assume that the tenth part of such a gap or break can be bisected in the microscope (which is feasible), then a single reading will be correct within 0·5 seconds of time and the right ascension equally as close; consequently the Greenwich time can be ascertained in this case within 15 seconds.

If we wonder how it is possible to reach such close results with the crude means employed, we find that this crudeness is only an apparent one. The work has simply been divided in a happy manner. The entire work of mensuration has been separated from that part by means of which the observations are gained and recorded. Photography simply records instantaneously the relative positions of the stars and moon at fixed-time intervals, which positions can afterwards be studied and measured at leisure.

The described measuring apparatus takes the place of the theodolite or sextant, with its graduated limb, arc, verniers, and micrometers.

This division of work is of great value for geographical exploration parties, as the actual measuring can be done by experts at any subsequent time after the plates have been shipped to the mother country.

In regard to the camera used for the foregoing described experimental observations, it may be said that the objective was a so-called antiplanetic group lens, of a focal length of about 24^{cm}, made by Steinheil, in Munich, Bavaria. The stop used had a diameter of 17^{mm}. This objective really consists of four lenses—i. e., two cemented pairs—which are placed as closely together as the interposition of the diaphragm (with the 17^{mm} stop) will admit of. The peculiarities of this lens combination have been utilized in increasing the depth and the field of the objective without sacrificing uniformity of definition and an even distribution of light.

The constants of the camera need not be known, as they do not enter into the work. All we need to know is the geographical latitude of the station and at what time periods (given in civil time) the breaks in the star traces were made and when the lunar pictures were obtained. All remaining data can be culled from the plate.

APPENDIX No. 5—1893.

ON THE MEASUREMENT OF BASE LINES WITH STEEL TAPES AND WITH STEEL AND BRASS WIRES.

By EDV. JÄDERIN.

Translated and submitted for publication by Prof. J. HOWARD GORE, November 27, 1893.

In the report on a method for measuring geodetic base lines with steel tapes, which was published in Öfversigt af. K. Vetenskapsakaemiens förhandlingar, No. 9, 1879, only the first attempts at such measuring were described. Since these experiments were not sufficiently general to give an accurate and detailed account of the practical application of the method, and as the plans pursued were not satisfactory in every respect, especially as they could not be carried out under certain atmospheric conditions—conditions which may always be expected during any extensive measurement—with any hope of materially diminishing the error, I found it necessary within the past years to continue my experiments on a larger scale.

In the report which here follows I have, for the sake of continuity, deemed it advisable to repeat in outline the principles of this method.

The steel tapes which can be purchased in the stores are of various kinds and grades. Those which are best suited for geodetic measuring, in accordance with the method hereinafter described, are about 13^{mm} broad and 20^{m} or more long. They are divided into centimetres, with the first decimetre further divided into millimetres.

If it should be desired to make use of a steel tape in the field, where it is not possible to provide a smooth support, it is necessary to support only the two end points, allowing the tape to hang freely throughout the rest of its length. In such a case the exact distance between the two ends can be mathematically determined. For this a tension is needed at both ends, exerted either by means of a spring balance or by weights. This use of the tape requires two corrections—a negative correction, increasing the length of the tape, which arises from the stretching of the tape due to the applied tension, and a positive correction, shortening the straight-line distance between the ends, which amounts to the difference between the length of the curve formed by the freely hanging tape and its chord. Should it be desired to have as the normal length of the tape—that is, the straight-line distance from the zero mark on one end to the similar mark on the other—a length exactly equal to the distance between these marks while the tape is lying on a smooth support throughout its entire length, it is only necessary that the sum of the two corrections named be equal to zero or that a tension be found sufficient to give to the hanging tape its normal length. From this it can be seen that this tension takes the place of the support.

It is better to provide the line which is to be measured with tripods than with stakes, since the latter are less stable and not so convenient as the former. If the tripod should be placed on rocks or on ground too hard for the feet to secure a firm hold, the requisite stability can be obtained by placing a stone in a sling attached to the under part of the tripod. In measuring, the tripods are placed so that the distance between the fine needles which are fixed in their upper surfaces is some centimetres less than the full length of the tape. The zero mark on the rear end of the tape is brought into coincidence with the needle on the rear tripod. The spring balance is attached to the forward end and a tension predetermined upon is applied. The reading on the tape is then made to the tenth of a millimetre by approximation, and recorded. (The forward tripod now becomes the rear one, and the operation just described is repeated.)

The length of the line is computed by considering, besides the readings just referred to, the following corrections: The constant correction to the length of the tape, obtained by measuring with it a line of known length—the comparator; the reduction to the horizontal projection, determined from the difference in the elevation of the tripod heads for each tape length, and the correction for temperature, that of the tape being taken as the temperature of the air.

When the measurement is prosecuted in this manner the weather occasionally interferes. The steel tapes, which are usually 13^{mm} in breadth, offer in a length of 20^{m}, when turned flat side to the wind, a surface of one-fourth square metre, approximately. A strong wind, therefore, causes a waving motion and moves the millimetre scale along the needle so rapidly as to make a reading inaccurate. Again, it is easy to see that in the sunshine the temperature of the air is not the same as that of the tape or even that of the thermometer, which makes the thermometric reading very unreliable. For this reason the application of this method of obtaining the temperature of the tape should be made use of only when the sky is clouded and the air perfectly still or in a gentle wind.

To obviate these difficulties was the purpose of the following experiments:

The effect of the wind is diminished partly by selecting a tape as small in cross section as possible and partly by increasing the tension.

In the matter of cross section there is a minimum which must not be exceeded in order to avoid an appreciable stretching in applying the tension. Therefore, as the width is diminished the thickness must be increased, which leads to the fact that a circular cross section is the best, and instead of tapes wires should be used. When the tension is increased it is necessary to know the length correction for this increased tension; to find this the comparator is measured with that tension. Whenever the entire length of the tape is not employed, it is best to determine the tension which will make the correction to the length proportional to the length utilized.

In order to be able to safely determine the temperature under all conditions, there appears to be no better way than to employ two wires—for example, one of steel and one of brass—whose coefficients of expansion are different. If these wires are of the same size and have a similar surface—nickel plated, for instance—and are handled in the same manner, there is no apparent reason why, on the average, they should not have for the entire measurement the same temperature; and this temperature can be determined from the differences in the readings of the two wires stretched successively between the same two fixed points.

After making these suggestions and before passing on to a description of the instruments and a report of the results, I shall give the development of the requisite formulas.

The following notation is employed:

L_0 = the normal length of the tape or wire, or the length which it indicates when supported and without tension.

L = the straight-line distance between the two zero marks when the tape or wire is supported at both ends, hangs freely throughout its length, and is subjected to a tension I.

g = gravity, force of.

m = mass of a unit's length of the tape.

w = weight of a unit's length of the tape.

s = the extension of a unit's length due to a unit tension.

c = correction to the length due to a curvature of the tape.

c_1 = correction to the length due to tension.

One has then

$$w = m\,g$$

The curve line which the tape forms when it hangs suspended from its two end points has, according to Sturm's Cours de Mécanique, vol. 2, p. 48, the following equations:

$$\frac{y}{k} = \frac{e^{\frac{x}{k}} + e^{-\frac{x}{k}}}{2} \qquad (a)$$

$$\frac{l}{k} = \frac{e^{\frac{x}{k}} - e^{-\frac{x}{k}}}{2} \qquad (b)$$

$$\frac{\rho}{k} = \left(\frac{y}{k}\right)^2 \qquad (c)$$

$$y^2 = k^2 + l^2 \qquad (d)$$

$$T = w\,y \qquad (e)$$

in which the y axis is vertical and the x axis horizontal, the former passing through the curve at its lowest point and the latter at a distance, k, below it; l the length of the curve reckoned from the point where the y axis cuts the same, ρ the radius of curvature, and T the tension exerted in the direction of its length.

When the two ends are at the same height the ordinates will be equal, while the abscissas will be equal but with opposite signs. If L be the length of the tape and a the straight-line distance between the end points, then

$$L = 2\,k,\ \frac{e^{\frac{a}{2\,k}} - e^{-\frac{a}{2\,k}}}{2}$$

or expressed in a series

$$L = 2\,k\left(\frac{a}{2\,k} + \frac{1}{1.\,2.\,3}\left(\frac{a}{2\,k}\right)^3 + \frac{1}{1.\,2.\,3.\,4.\,5}\left(\frac{a}{2\,k}\right)^5 \cdots\cdots\right)$$

and
$$L - a = \frac{a^3}{24.\,k^2} + \frac{a^5}{1920.\,k^4} + \cdots\cdots \qquad (f)$$

In the experiments it was found that all the terms after the first would fall between $0{\cdot}0001^{mm}$ and $0{\cdot}001^{mm}$; hence it can be assumed that

$$L - a = \frac{a^3}{24\,k^2}$$

From (e)
$$1 = \frac{T}{w\,y}$$

or
$$k = \frac{T\,k}{w\,y}$$

From a series of tabulated values it was found that $\frac{k}{y}$ changed value with k, but only to the extent of $\frac{1}{2000}$ of the whole, or that the factor $\frac{k}{y} = 1$. This gives:

$$k = \frac{T}{w}$$

and
$$L - a = \frac{a^3 w^2}{24\,T^2}$$

or
$$c = a - L = -\frac{a^3 w^2}{24\,T^2}$$

or placing
$$L_0 = a^3$$

and
$$w = m\,g$$

we have $$c = -\frac{L_0^3 m^2 g^2}{24\, T^2}$$

Likewise $$c_1 = s\, L_0\, T$$

from which we have

$$L = L_0 - \frac{L_0^3\, m^2\, g^2}{24\, T^2} + s\, L_0\, T \; \ldots\ldots \tag{1}$$

For the force T_0 (the normal tension), which makes L equal to L_0, it would give

$$T_0 = \sqrt[3]{\frac{L_0^2\, m^2\, g^2}{24 s}} = \sqrt[3]{\frac{V^2}{24 s}} \; \ldots\ldots \tag{2}$$

in which V (or $w\, L_0$) is the weight of the entire tape.

If it should be necessary to employ a fractional part—say, the nth part of the tape—and we wished to know the corresponding part of the normal length, we would have to introduce a tension, $T^{0\prime}$, which is obtained by substituting in (2) $n\, L_0$ for L_0, from which we obtain

$$T^{0\prime} = \sqrt[3]{\frac{n^2\, L_0^2\, m^2\, g^2}{24 s}} = T_0 \sqrt[3]{n^2} \; \ldots\ldots \tag{3}$$

If, on the other hand, a greater tension, T, is made use of, in order to diminish the effect of the wind, the distance between the zero points will be greater than L_0. We therefore place

$$L = L_0\, (1 + f)$$

or the factor $$f = \frac{L - L_0}{L_0}$$

From (1) we obtain

$$f = -\frac{L_0^2\, m^2\, g^2}{24\, T^2} + sT = -\frac{L_0^2\, w^2}{24\, T^2} + sT$$

or

$$f = -\frac{V^2}{24\, T^2} + sT \; \ldots\ldots \tag{4}$$

When only a part of the tape is utilized, the corresponding or nth part of f is obtained by first finding from T the value T'. From (1) we have

$$nL_0\, (1 + f) = nL_0 - \frac{n^3\, L_0^3\, m^2\, g^2}{24\, T'^2} + nsL_0T',$$

from which

$$f = -\frac{n^2\, V^2}{24\, T'^2} + sT'$$

which combined with (4) gives

$$T'^3 + \left(\frac{V^2}{24\, sT^2} - T\right) T'^2 = \frac{V^2}{24\, s}\, n^2,$$

or

$$T' = T - \frac{V^2}{24\, sT^2} + \frac{V^2}{24\, s}\, n^2, \frac{1}{T'^2} \; \ldots\ldots \tag{5}$$

where the value of T' is given by approximation.

If p is the sag of the tape at its lowest point, or the height of the segment of the circle formed by the tape, we have

$$p = \tfrac{1}{8}\frac{L_0^2 w}{T} = \tfrac{1}{8}\frac{L_0 V}{T}$$

For three steel tapes = A and B (both 20^m long, the latter nickel plated) and C, nearly 30^m in length, but used as a 20^m tape, I found, using the metre as a unit of length and a kilogramme as the unit of weight:

A,	$w = 0{\cdot}01977^k$,	$s = 0{\cdot}0000214 \pm 6$,	$sw = 0{\cdot}000000424 \pm 11$
B,	$0{\cdot}01898$	$0{\cdot}0000226 \pm 6$,	$0{\cdot}000000429 \pm 12$
C,	$0{\cdot}01653$,	$0{\cdot}0000234 \pm 5$,	$0{\cdot}000000387 \pm 8$

The determination of s and sw were made while subjecting the tapes when horizontal to a tension by means of a dynamometer; also when vertical by suspending them from a high tower and attaching weights to the lower ends. The mean temperature was 6° (C.).

From these values, by means of (2), we obtain

for A,	$T_0 = 6{\cdot}72^k$
B,	$6{\cdot}43$
C,	$5{\cdot}80$.

For the elucidation of the formulas given above, the following table is given for A, which was used in the experimental determinations of length:

n	nL	T_0; $T_0=6{\cdot}72^k$ $f=0$	T: $T=10^k$ $f=0{\cdot}000149$	T: $T=15^k$ $f=0{\cdot}000293$
	$m.$	$k.$	$k.$	$k.$
0·00	0	0·00	6·96	13·65
0·05	1	0·91	6·98	13·65
0·10	2	1·45	7·02	13·67
0·15	3	1·88	7·10	13·69
0·20	4	2·30	7·20	13·71
0·25	5	2·67	7·32	13·75
0·30	6	3·01	7·45	13·79
0·35	7	3·34	7·60	13·84
0·40	8	3·65	7·77	13·90
0·45	9	3·95	7·94	13·96
0·50	10	4·24	8·11	14·03
0·55	11	4·51	8·30	14·11
0·60	12	4·78	8·48	14·19
0·65	13	5·05	8·67	14·28
0·70	14	5·30	8·86	14·37
0·75	15	5·55	9·05	14·47
0·80	16	5·79	9·24	14·57
0·85	17	6·03	9·43	14·67
0·90	18	6·27	9·62	14·78
0·95	19	6·50	9·81	14·89
1·00	20	6·72	10·00	15·00

If the length L_0 is known when the tension T_0 was used, it is possible by means of the formula already given to determine the length under a tension, T, introducing for that purpose the factor f. However, it is preferable to ascertain from the comparator this length with the application of the new tension.

The disturbing influence of the wind, as has been said, chiefly shows itself in adding to the difficulty of reading the scale on the tape. If this interference of the wind is regarded as manifesting itself in a constant thrust sidewise, or a lateral curving of the tape, it is possible by means of equation (1) to determine the length, introducing in the place of g a factor representing the strength of the wind. From this it will be seen that the distortion in question is proportional to the square of the force of the wind and inversely proportional to the square of the tension. But the force of the wind is proportional to the exposed surface of the tape, from which it is apparent that to avoid these obstacles the use of wires is well-nigh imperative.

In case the measurement takes place under a different latitude or at a greater elevation than that of the comparator, it is necessary, in order to be able to compute the effect of gravity upon the results of the measuring, to take into consideration two distinct cases:

1. When the tension is effected by means of a spring balance.
2. When the tension is effected by means of weights.

In the first instance the tension is unchanged, although the weight of the entire tape or wire is different at the two places. If the length of the wire was found from the comparator to be L under a tension of T, and at the base line the same tension was applied, then the straight-line distance between the zero marks, L^1, can be obtained from the equation

$$L^1 - L = \frac{L_0^3\, m^2\, g^2}{24\, T^2}\left(1 - \frac{g'^2}{g^2}\right)$$

in which g and g' represent the gravity at the comparator and the base line, respectively. This equation can also be written

$$L^1 - L = \frac{L_0^3\, V^2}{24\, T} \times \frac{g+g'}{g^2}\left(g - g'\right)$$

or with sufficient approximation

$$L^1 - L = \frac{L_0^3\, V^2}{12\, T^2}\left(\frac{g-g'}{g}\right) \ldots\ldots \qquad (6)$$

In the second case, where the tension is exerted by means of weights, their tensive force, as well as the weight of the wire or tape, will vary. If M represent the mass of the object used to produce the tension, then its weight (taking the place of T in formula (1)) at the two places would be $M\,g$ and $M\,g^1$; this would give

$$L^1 - L = -s L_0 M g\left(1 - \frac{g'}{g}\right).$$

or

$$L^1 - L = -s L_0 T\, \frac{g-g'}{g} \ldots\ldots \qquad (7)$$

For the numerical results, see pages 133–134.

DESCRIPTION OF THE INSTRUMENTS.

In the measurements, wires were chiefly made use of, and steel tapes employed only when the distance between two tripods happened to be less than the length of a wire. In such cases it was necessary to use a tape, since it was not possible to graduate a wire throughout its entire length.

As has already been remarked, wires were used in pairs—one of steel and one of brass. Piano wire was found to be the best size for the steel. The brass wire was repeatedly pulled through a draw plate so as to give to it its greatest possible ductility. In selecting a wire one must see to it that there are no longitudinal cracks in it, a fault which may be found by cold hammering even when invisible to the naked eye.

When it becomes necessary to roll up the wires, it is advisable to give to the rings the size which, if left to themselves, they would naturally take, so that they may not suffer a permanent change in length through bending. The winding and unwinding must be done with the greatest care, and every possible precaution taken that no kinks are formed. Should any occur, it will be necessary to redetermine its length before continuing the measurement. It may be remarked that whenever any slight unevenness occurs on the wire the error in length resulting therefrom is always the same under equal tension. The danger, therefore, arising from such misfortune is, as experience shows, considerably less than one would be inclined to imagine.

The wires are provided at both their ends with scales 1 decimetre in length, divided into millimetres. These, marked *s* in figure 2, are attached longitudinally to brass tubes. The free end of the twisted wire is inserted into the outer terminals of this tube and firmly soldered thereto. To prevent the wire from twisting when in use, a turn buckle, *r*, is inserted, which also makes it possible to bring the scale into the most favorable position with respect to the needle in the top of the tripod for reading.

Of the ten wires made by the mechanician, Fr. J. Berg, of Stockholm, eight have been repeatedly investigated on the comparator, which will be described later on. All these wires have a length of 25^{m}. This length was made necessary by the desire to employ each wire as often as possible on the 100^{m} comparator. Otherwise, wires of 30^{m} or more would have been used.

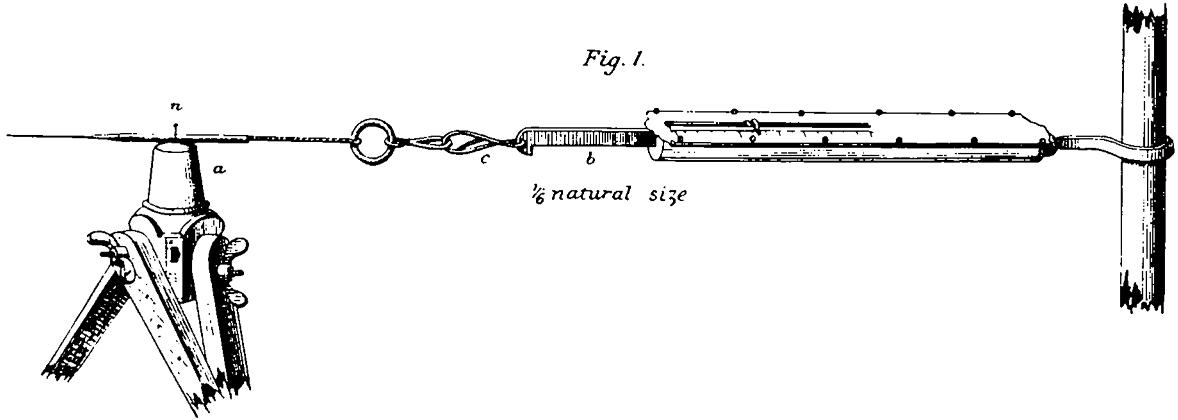
The pair *A B* was utilized since March, 1882, *C D* since April, *E F* since September, and *M N* since April, 1883. The reason why I employed so many wires was to determine from experience what diameter is the best, and also to confirm or disprove the oft-expressed fear of a continuous change in the length of tapes or wires under use.

The following wires were nickel plated:

For *A* (steel) and *B* (brass) before plating the weights were found to be

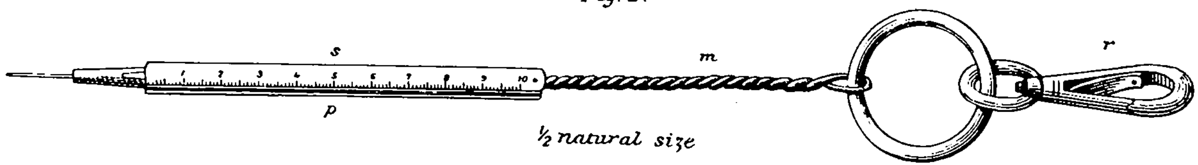
$$A \quad w = 0{\cdot}01459^{k}$$
$$B \quad\quad 0{\cdot}01624$$

Fig. 1.



1/6 natural size

Fig. 2.



1/2 natural size

After plating

$$A \quad w = 0{\cdot}01469^{k}$$
$$B \quad 0{\cdot}01629\,*$$

Likewise it was found that under tension, with a temperature of $+1^{\circ}$ (Celsius), the following wires gave:

$$A\ldots,\ s = 0{\cdot}00002799,\ sw = 0{\cdot}000000411$$
$$B\ldots,\ 0{\cdot}00005769,\ 0{\cdot}000000940$$

Since the wires are always employed throughout their entire length, which is not the case with tapes, it is of slight importance whether these quantities are known, even if approximate values are at times needed. A knowledge of such values can be obtained with sufficient accuracy if it is supposed that for the same metal w is proportional to the surface of a transverse section or to the square of the diameter ($a\ d^2$, in which a represents a constant and d the diameter); also, that $s\ w$ is a constant and s inversely proportional to the square of the diameter, or equal to $\frac{b}{d}$ For A, $d = 1{\cdot}53^{mm}$, and for B, $d = 1{\cdot}57^{mm}$. If a be expressed in millimetres, we would have:

for steel, $a = 0{\cdot}006275^{k}$; $w = ad^2$........[log., 7·7976]

$b = 0{\cdot}0000655^{m}$; $s = \frac{b}{d^2}$........[log., 5·8162]

for brass, $a = 0.006609^{k}$................[log., 7·8201]

$b = 0{\cdot}0001422^{m}$...............[log., 6·1529]

From these two values for a the specific gravity was found to be for steel, 7·99, and for brass, 8·41.

After having determined the diameters of the other wires, the remaining quantities were found to have the following approximate values:

Wire.		d	w	s	sL_0	V	T_0	T	p
		mm.	k.			k.	k.	k.	m.
Steel.	A	1·53	0·01469	0·00002799	0·000700	0·367	5·85	10	0·11
Brass.	B	1·57	0·01629	0·00005769	0·001442	0·407	4·92	10	0·13
Brass.	C	1·52	0·0153	0·0000616	0·00154	0·38	4·6	10	0·12
Steel.	D	1·51	0·0143	0·0000287	0·00072	0·36	5·7	10	0·11
Steel.	E	2·04	0·0270	0·0000157	0·00039	0·65	10·4	10	0·20
Brass.	F	2·02	0·0270	0·0000349	0·00087	0·67	8·2	10	0·21
Steel.	M	2·66	0·0444	0·0000091	0·00023	1·11	17·7	15	0·23
Brass.	N	2·66	0·0468	0·0000201	0·00050	1·17	14·2	15	0·24

* If for any reason w, and likewise V, suffered any change—for example, through the wear of the plating or removal of the plating—c took on a new value, which was determined from

$$d\,c = -\frac{L_0\,V}{12\,T^2}\,d\,V$$

Also s would change, giving

$$d\,c_1 = t\,L_0\,T\,ds.$$

From these equations it can be seen how small the dreaded changes from these sources are.

We readily obtain from (2)

$$T_0 = d^2 \sqrt[3]{\frac{L_0^2 a^2}{24b}} \cdots\cdots \tag{8}$$

and

$$T_0 = w \sqrt[3]{\frac{L_0^2}{24ab}} \cdots\cdots \tag{9}$$

With a wire it is easy to obtain d by measuring, and in a tape w can be found by weighing; therefore the formulas just given can serve in determining an approximate value for T_0. The values given in the eighth column of the above table were obtained in this way. The knowledge of T_0 is of no direct value, yet for the sake of comparing values of it with the arbitrarily taken values of T, as shown in column 9, they are here given. If it is desired to obtain T_0 more easily, it can be done, from formulas (8) and (9), expressing T_0 and w in kilogrammes, and d in millimetres.

Steel, $L = 20^m$, $T_0 = 2{\cdot}16\ d^2 = 344\ w$
Brass, $T_0 = 1{\cdot}72\ d^2 = 261\ w$
Steel, $L = 25^m$, $T_0 = 2{\cdot}50\ d^2 = 399\ w$
Brass, $T_0 = 2{\cdot}00\ d^2 = 303\ w$

If we take the values given in the above table and insert them in formulas (6) and (7), we will obtain for the influence of the varying force of gravity the following, in millimetres:

1. *Tension with spring balance.*

A	*B*	*C*	*D*	*E*	*F*	*M*	*N*
+2·8	+3·5	+3·0	+2·7	+8·8	+9·4	+11·4	+12·7

2. *Tension from weights.*

A	*B*	*C*	*D*	*E*	*F*	*M*	*N*
−7·0	−14·4	−15·4	−7·2	−3·9	−8·7	−3·5	−7·5

These values have not been multiplied by the factor $\frac{g - g'}{g}$; but as this factor never exceeds 0·0052, it would give a correction of only $0{\cdot}066^{mm}$ for N and 0·080 for C.

The wires A B and C D were found to be easily handled, while more trouble was experienced with M N. The latter were given a greater thickness that they might serve as standards for the others, it being accepted that they would be less subject to change than the lighter wires.

For the corrections to reduce a 25^m wire to the horizontal projection, the following table was computed from the formula

$$k = \frac{h^2}{2L} + \frac{h^4}{8L^3} + \frac{h^6}{16L^5} \cdots\cdots \tag{10}$$

in which k is the correction, always positive; L the length of the wire, and h the difference in elevation of the two tripod heads:

Correction for reduction to a horizontal projection.

h	*k*	*h*	*k*	*h*	*k*	*h*	*k*
m.	*mm.*	*m.*	*mm.*	*m.*	*mm.*	*m.*	*mm.*
0·00	0·0	0·66	8·7	1·26	31·8	1·86	69·3
0·04	0·0	0·67	9·0	1·27	32·3	1·87	70·0
0·05	0·1	0·68	9·2	1·28	32·8	1·88	70·8
0·08	0·1	0·69	9·5	1·29	33·3	1·89	71·5
0·09	0·2	0·70	9·8	1·30	33·8	1·90	72·3
0·11	0·2	0·71	10·1	1·31	34·3	1·91	73·1
0·12	0·3	0·72	10·4	1·32	34·9	1·92	73·8
0·13	0·3	0·73	10·7	1·33	35·4	1·93	74·6
0·14	0·3	0·74	11·0	1·34	36·0	1·94	75·4
0·15	0·4	0·75	11·3	1·35	36·5	1·95	76·2
0·16	0·5	0·76	11·6	1·36	37·0	1·96	76·9
0·17	0·5	0·77	11·9	1·37	37·6	1·97	77·7
0·18	0·6	0·78	12·2	1·38	38·1	1·98	78·5
0·19	0·7	0·79	12·5	1·39	38·7	1·99	79·3
0·20	0·8	0·80	12·8	1·40	39·2	2·00	80·1
0·21	0·9	0·81	13·1	1·41	39·8	2·01	80·9
0·22	1·0	0·82	13·5	1·42	40·4	2·02	81·7
0·23	1·1	0·83	13·8	1·43	40·9	2·03	82·6
0·24	1·2	0·84	14·1	1·44	41·5	2·04	83·4
0·25	1·3	0·85	14·5	1·45	42·1	2·05	84·2
0·26	1·4	0·86	14·8	1·46	42·7	2·06	85·0
0·27	1·5	0·87	15·1	1·47	43·3	2·07	86·7
0·28	1·6	0·88	15·5	1·48	43·8	2·08	87·5
0·29	1·7	0·89	15·8	1·49	44·4	2·09	88·4
0·30	1·8	0·90	16·2	1·50	45·0	2·10	89·2
0·31	1·9	0·91	16·6	1·51	45·6	2·11	90·0
0·32	2·0	0·92	16·9	1·52	46·3	2·12	90·9
0·33	2·2	0·93	17·3	1·53	46·9	2·13	91·8
0·34	2·3	0·94	17·7	1·54	47·5	2·14	92·6
0·35	2·5	0·95	18·1	1·55	48·1	2·15	93·5
0·36	2·6	0·96	18·4	1·56	48·7	2·16	94·4
0·37	2·7	0·97	18·8	1·57	49·3	2·17	95·2
0·38	2·9	0·98	19·2	1·58	50·0	2·18	96·1
0·39	3·0	0·99	19·6	1·59	50·6	2·19	97·0
0·40	3·2	1·00	20·0	1·60	51·3	2·20	97·9
0·41	3·4	1·01	20·4	1·61	51·9	2·21	98·8
0·42	3·5	1·02	20·8	1·62	52·5	2·22	99·7
0·43	3·7	1·03	21·2	1·63	53·2	2·23	100·6
0·44	3·9	1·04	21·6	1·64	53·8	2·24	101·5
0·45	4·1	1·05	22·1	1·65	54·5	2·25	102·4
0·46	4·2	1·06	22·5	1·66	55·2	2·26	103·3
0·47	4·4	1·07	22·9	1·67	55·8	2·27	103·3
0·48	4·6	1·08	23·3	1·68	56·5	2·28	104·2
0·49	4·8	1·09	23·8	1·69	57·2	2·29	105·1
0·50	5·0	1·10	24·2	1·70	57·9	2·30	106·0
0·51	5·2	1·11	24·7	1·71	58·6	2·31	106·9
0·52	5·4	1·12	25·1	1·72	59·2	2·32	107·9
0·53	5·6	1·13	25·5	1·73	59·9	2·33	108·8
0·54	5·8	1·14	26·0	1·74	60·6	2·34	109·8
0·55	6·1	1·15	26·5	1·75	61·3	2·35	110·7
0·56	6·3	1·16	26·9	1·76	62·0	2·36	111·6
0·57	6·5	1·17	27·4	1·77	62·7	2·37	112·6
0·58	6·7	1·18	27·9	1·78	63·4	2·38	113·5
0·59	7·0	1·19	28·3	1·79	64·2	2·39	114·5
0·60	7·2	1·20	28·8	1·80	64·9	2·40	115·5
0·61	7·4	1·21	29·3	1·81	65·6	2·41	116·4
0·62	7·7	1·22	29·8	1·82	66·3	2·42	117·4
0·63	7·9	1·23	30·3	1·83	67·1	2·43	118·4
0·64	8·2	1·24	30·8	1·84	67·8	2·44	119·4
0·65	8·5	1·25	31·3	1·85	68·5	2·45	120·3

Correction for reduction to a horizontal projection—Continued.

h	k	h	k	h	k	h	k
m.	*mm.*	*m.*	*mm.*	*m.*	*mm.*	*m.*	*mm.*
2·46	121·3	2·54	129·4	2·62	137·7	2·70	146·2
2·47	122·3	2·55	130·4	2·63	138·6	2·71	147·3
2·48	123·3	2·56	131·4	2·64	139·8	2·72	148·4
2·49	124·3	2·57	132·4	2·65	140·8	2·73	149·5
2·50	125·3	2·58	133·5	2·66	141·9	2·74	150·6
2·51	126·3	2·59	134·4	2·67	143·0	2·75	151·7
2·52	127·3	2·60	135·6	2·68	144·1	2·76	152·8
2·53	128·3	2·61	136·5	2·69	145·1		

The table extends only to a difference of $2{\cdot}76^{m}$ in the height of the two tripods, because a greater inclination than this—1 in 9—should be avoided. In such extreme cases an error in elevation of 1^{mm} would produce an error of $0{\cdot}1^{mm}$ in the length.

From equation (10) we obtain by differentiating

$$dk = -\frac{h^2}{2L^2}\,dL \ldots\ldots \tag{11}$$

from which it appears that when L is not exactly 25^{m}, but, as is usually the case, varies a few centimetres, owing to the scale readings, the corrections taken from the table will not be correct. If $h=3^{m}$ and $d\,L=+50^{mm}$, then the error referred to would amount to $-0{\cdot}36^{mm}$.

To compute k in those instances where L is not quite 25^{m}, as in the use of parts of a steel tape, one can proceed as follows:

Place in equation (10) a for all the terms of the second member except the first, that is

$$k=\frac{h^2}{2L}+a,$$

then

$$h=\sqrt{\sqrt{8aL^3}-2aL} \ldots\ldots \tag{12}$$

If h and L are expressed in metres and k in millimetres, then

$$k=\frac{h^2}{L}\times 500^{mm}+a \ldots\ldots \tag{12a}$$

from which it is easy to compute k by taking a from the following table:

$L(m.)$	a = 0·05mm	0·15mm	0·25mm	0·35mm	0·45mm
	h				
	m.	m.	m.	m.	m.
1	0·14	0·19			
2	0·24	0·31			
3	0·32	0·42			
4	0·40	0·53			
5	0·47	0·62			
6	0·54	0·71	0·81		
7	0·61	0·80	0·91		
8	0·67	0·88	1·00		
9	0·73	0·97	1·10	1·19	
10	0·79	1·05	1·19	1·29	
11	0·85	1·12	1·28	1·39	
12	0·91	1·20	1·36	1·48	
13	0·97	1·27	1·45	1·57	1·67
14	1·02	1·35	1·53	1·66	1·77
15	1·08	1·42	1·61	1·75	1·86
16	1·13	1·49	1·69	1·84	1·96
17	1·18	1·56	1·77	1·92	2·05
18	1·24	1·62	1·85	2·01	2·14
19	1·29	1·69	1·92	2·09	2·23
20	1·34	1·76	2·00	2·17	2·31
21	1.39	1·82	2·07	2·25	2·40
22	1·44	1·89	2·15	2·33	2·48
23	1·48	1·95	2·22	2·41	2·57
24	1·53	2·02	2·29	2·49	2·65
25	1·58	2·08	2·36	2·57	2·73
26	1·63	2·14	2·43	2·65	2·82
27	1·67	2·20	2·50	2·72	2·90
28	1·72	2·26	2·57	2·80	2·98
29	1·77	2·32	2·64	2·87	3·06
30	1.81	2·38	2·71	2·95	3·14

If f represent the error in the leveling rod, then for a difference of elevation h we must write $h\,(1+f)$; therefore from (10) we will have for the reduction to the horizon

$$k'=\frac{h^2}{2L}(1+f)^2+\frac{h^4}{8L^3}(1+f)^4+\ldots\ldots$$

or with sufficient approximation

$$k'=k\,(1+2f)$$

or

$$\Sigma k'=(1+2f)\Sigma k\ldots\ldots \qquad (13)$$

Both ends of the wires are provided with balances. The one at the end where the scale on the wire is read is to bring about the desired tension, while the other is to hold the counter action at the same tension and to avoid as far as possible any drag on the tripod which carries the mark indicating the terminus of the preceding wire. For the latter an ordinary spring balance such as is found in the stores will answer, but for the former purpose a more carefully constructed balance will be required. From this it will be seen that in measuring in the field two spring balances are needed—a smaller and a larger one. These balances are employed to measure the horizontal tension and not to weigh anything in a vertical position, as is supposed to be the case when the graduation is made. Therefore it is necessary to take into consideration the difference in the readings in these two cases for actual tension. This difference can be obtained by adding to half of the weight of the spring the weight of those parts—b and c—which are attached to the lower end of the spiral spring. It is easy to find that if p represents the entire weight of the dynamometer, y the difference just mentioned, x the correction to the horizontal reading for 0 k, a the reading on the balance when it hangs vertical and free, and a' the reading when the balance is inverted and suspended by its own hook, then

$$\left.\begin{aligned} x &= \frac{p-a'-a}{2} \\ \text{and} \quad y &= \frac{p-a'+a}{2} \end{aligned}\right\} \ldots\ldots \qquad (14)$$

By way of illustration, the following quantities are added:

	Old balance.	New balance.
	$k.$	$k.$
The difference between the readings of the balance when horizontal and when vertical,	0·115	0·186
	$mm.$	$mm.$
The diameter of the steel wire which formed the spring,	2·25	2·62
The outer diameter of the spring,	20·5	26·0
The inner diameter of the spring,	16·0	20·8
The mean radius of the spring $= R$	9·1	11·4
Extension of spring for 1 $k = f$	8·34	10·14
The number of coils in the spring $= h$	42	39·5

We will have

$$f = k\frac{R^3h}{d^4} \ldots\ldots \qquad (14a)$$

where $k = 0{\cdot}0073$ for steel.

The scales on both balances were divided into tenths of a kilogramme, making it possible to read to hundredths by approximation.

If the same balances are used in measuring that were employed on the comparator, it is not necessary to determine these corrections; but if different balances are made use of, it will be absolutely necessary to investigate each balance that is employed. For this reason it is deemed well to give the correction for the two balances mentioned. In each case the vertical weighing is given with proper correction to make it the equivalent of the horizontal tension.

The old balance.

[1883, Apr. 24.]				[1884, Jan. 30.]		[1884, Feb. 3.]	
Temp.=+13°		Temp.=+6°		Temp.=+2°		Temp.=+26°	
Weight.	Corr.	Weight.	Corr.	Weight.	Corr.	Weight.	Corr.
0*k*	−0·14*k*	0*k*	−0·13*k*	0*k*	−0·115*k*	0*k*	−0·135*k*
10	−0·14	9	−0·18	2	−0·14	2	−0·195
5	−0·20	10	−0·11	4	−0·17	4	−0·245
		10	−0·10	6	−0·21	6	−0·30
		9	−0·19	9	−0·17	9	−0·27
		5	−0·19	11	−0·10	11	−0·21

The new balance.

[1883, Apr. 24.]				[1884, Jan. 30.]		[1884, Feb. 3.]	
Temp.=+13°		Temp.=+6°		Temp.=+2°		Temp.=+26°	
Weight.	Corr.	Weight.	Corr.	Weight.	Corr.	Weight.	Corr.
0*k*	−0·18*k*	0*k*	−0·18*k*	0*k*	−0·19*k*	0*k*	−0·205*k*
0	−0·18	0	−0·18	2	−0·19	2	−0·20
5	−0·18	10	−0·14	4	−0·17	4	−0·21
10	−0·18	13	−0·13	7	−0·15	6	−0·205
14	−0·14	14	−0·11	9	−0·12	9	−0·215
14	−0·14	14	−0·11	11	−0·13	11	−0·20
13	−0·17	14	−0·11	13	−0·12	13	−0·19
13	−0·16	13	−0·14	15	−0·09	15	−0·18
14	−0·13	0	−0·18				
9	−0·18	9	−0·16				
10	−0·17	10	−0·17				
0	−0·18	5	−0·17				
0	−0·19						

For both instruments the two last determinations are the mean of two series of observations.

Since the correction k can be expressed by the equation

$$k=x+my,$$

in which m is the weight suspended and x and y are constants, we have, after applying the corrections just given for reduction to horizontal readings, according to the method of least squares, the following equations:

For the old balance.

1884, Jan. 30, Temp. $=+2°$, $k=-0{\cdot}034-0{\cdot}0003m.$
1883, Apr. 24, " $=+6°$, $k=-0{\cdot}031+0{\cdot}0005m.$
" " " $=+13°$, $k=-0{\cdot}036-0{\cdot}0005m.$
1884, Feb. 3, " $=+26°$, $k=-0{\cdot}064-0{\cdot}0085m.$

For the new balance.

1884, Jan. 30, Temp. $=+2°$, $k=-0{\cdot}011+0{\cdot}0063m.$
1883, Apr. 24, " $=+6°$, $k=-0{\cdot}000+0{\cdot}0045m.$
" " " $=+13°$, $k=-0{\cdot}001+0{\cdot}0025m.$
1884, Feb. 3, " $=+26°$, $k=-0{\cdot}025+0{\cdot}0013m.$

If x changes with the temperature, it is evident that the variation is extremely slight; consequently it is regarded as practically constant.

However, the variation in y can not be made a constant function of temperature; but it is assumed that it may be expressed by the relation

$$y=u+vt,$$

in which t is the temperature and u and v are constants. In this manner it is found that for the old balance

$$y=+0{\cdot}0021-0{\cdot}000365t,$$

and $$k=-0{\cdot}041+(0{\cdot}0021-0{\cdot}000365t)m;$$

and for the new balance

$$y=+0{\cdot}0060-0{\cdot}000197t,$$

and $$k=-0{\cdot}009+(0{\cdot}0060-0{\cdot}000197t)m.$$

The temperature correction, which for the two instruments is almost the same, can not be regarded as devoid of real significance.

From what has just been given, the following table of corrections has been computed:

Temp.	The old balance tension, 10*k*.	The new balance.	
		Tension 10*k*.	Tension 15*k*.
−10°	+0·02*k*	+0·07*k*	+0·11*k*
− 5°	0·00	+0·06	+0·10
0°	−0·02	+0·05	+0·08
5°	−0·04	+0·04	+0·07
10°	−0·06	+0·03	+0·05
15°	−0·07	+0·02	+0·04
20°	−0·09	+0·01	+0·02
25°	−0·11	0·00	+0·01
30°	−0·13	−0·01	−0·01

These corrections must be applied with their proper signs to the readings of the balance in order to have the correct tension.

The correction for the length of the wire depending upon an error in tension can be found by differentiating (1), which gives

$$\frac{d\,L}{d\,T}=\frac{L_0{}^3\,m^2\,g^2}{12\,T^3}+sL_0=\frac{L_0\,V^2}{12\,T^3}+sL_0\ldots\ldots \qquad (15)$$

or if $$T=T_0$$

$$\frac{dL}{dT}=3sL_0\ldots\ldots \qquad (15a)$$

The equation (15) gives for a positive error of 1k in the balance the following positive corrections in the length of the wires in millimetres

for the wire	A	B	C	D	E	F	M	N
the correction	1·0	1·8	1·8	1·0	1·3	1·8	1·0	1·3

In order to secure reliable readings on the wire scale it is necessary to see to it that the friction on the interior of the balance is a minimum and that the observer is skilled in work of this character.

THE COMPARATOR.

In February, 1882, the earth not being in a frozen condition, two stones were set in the sidewalk extending along the Technology street, each of which weighed about 2,000 kilogrammes. Into the upper surface of each stone was inserted a brass rod 1 decimetre in length, and in the free ends of these rods fine holes were drilled, which marked the ends of the line. The line was intended to be exactly 100^m long as indicated by an old-standarded steel tape. The stones rested upon well-packed beds of gravel about $0{\cdot}5^m$ below ground and extended $0{\cdot}4^m$ above.

The line was measured three times in April, 1882, and twice in November with the base apparatus belonging to the Royal Academy of Sciences. Since the base bars are two toises long, 26 lengths give $101{\cdot}35^m$. The excess, $1{\cdot}35^m$, is such a large fractional part of the whole that it must be laid off with great care. For this purpose a 2^m bar of Bessemer steel was employed which had been tested both for length and graduation and compared with the standard by the Bureau of Weights and Measures.

At the time of the first measurement no comparison of the base bars with the standard was made, as I considered that no significant change had taken place since the last comparison; at least no change of sufficient importance to affect my results. However, it can be seen from what follows that this hypothesis was unjustifiable. A, B, C, D are the lengths of the four bars of the base apparatus and N that of the standard, all at a temperature of 13° R., or 16°·25 Celsius. The quantities are expressed in millimetres:

	1878, Oct. 10, in Stockholm.	1882, in autumn, at Jaederen.	1883, in Stockholm.	
			Nov. 9.	Nov. 15.
$A-N$	+0·0020	—0·0294	—0·0992	—0·0832
$B-N$	+0·0305	+0·1374	—0.2492	—0·2338
$C-N$	—0·1272	—0·2574	—0·2888	—0·2711
$D-N$	+0·0043	—0·0861	—0·1088	—0·0963
Mean	—0·0226	—0·0589	—0·1865	—0·1711

Before the comparisons at Jaederen were made, some flakes of rust were removed from the end surfaces of the bars, by which the lengths were somewhat shortened. Still the length of B had in the interim increased considerably and then diminished by not less than $0{\cdot}38^{mm}$, which revealed a change of a most significant character. I have here introduced the results of these comparisons in order to show why I attach so little weight to the measurements of April, 1882, although the results agree very well with those of November, 1883.

The two comparisons between the base bars and the standard were made immediately before and after the measurement of the comparator in November, 1883. In addition to this, a direct comparison was made on the 23d of November, 1883, between the traveling standard belonging to the base apparatus, which was used in the cases just mentioned, and the Pulkowa standard at Stockholm. It was here found that

$$N-P=-0{\cdot}0626^{mm},$$

a value almost identical with that obtained in a previous comparison.

The results of the measurements with the base apparatus are:

	m.
1882, Apr. 14,	99·9997
" "	99·9989
" "	99·9995
Mean	99·9994

	m.
1883, Nov. 12,	100·0003
" "	100·0000
Mean	100·0001

These determinations indicate that the length of the line had increased. Such an increase also became apparent in the discussion of the lengths obtained from measuring with the wires. This change of length presents nothing very surprising when it is understood that the line was laid out under the most unfavorable circumstances. The street along which the line extended was, in 1882, raised by a fill at the north end by as much as 2^m. To support this embankment a supporting wall was built, for which it was necessary to dig a foundation of considerable depth. Besides this, a number of large buildings were erected in the immediate neighborhood, during which it is quite likely that the stones received a jar or shock, and perhaps several, as they presented no evidence of violence and the change in the length of the line appeared gradual.

Inasmuch as a greater increase in length took place than is shown in the earlier determinations, and since the value which was obtained in November, 1883, can not be affected with any great error, the former determination must be regarded as having a value that is too large. As the result of seventeen trials, made between March 22, 1882, and November 17, 1883, it appears that the northern point was $0{\cdot}5853^m$ higher than the southern, while no change exceeding the probable error was apparent.

THE INVESTIGATION OF THE WIRES AND STEEL TAPES UPON THE COMPARATOR.

This investigation had for its object the determination of errors of length and the coefficient of expansion. All measurements were made when the sky was overcast or when the wires and tapes could be protected from the sun's rays, so that the temperature of the wires could be regarded as that of the air. The air temperature was ascertained from three thermometers, each held at the same distance from the ground, one being near each end of the wire and the third one at the middle. The thermometers were investigated in November, 1882, and the zero-point correction determined and errors of calibration by comparison with a standard thermometer at various temperatures. In January, 1884, the zero points were again examined and found to be unchanged.

In the epitomized results of the measurements of the comparator which follow, the reduction to the horizon has been made and likewise the errors from inaccuracies of the leveling rod (equation 13) and those of the balances (equation 15). The constant length correction of the wires for the 100^m is represented by $x_1, x_2, x_3 \ldots . x_8$, while $y_1, y_2, y_3 \ldots . y_8$ stand for the expansion in 100^m for 10° Celsius. The thermometer readings have been corrected and + 15° taken as the temperature at which the wires have their normal length. Therefore y will indicate the amount by which the temperature exceeds 15°; and in the majority of cases it will be negative, since the measurement of the comparator was usually made in cold weather. The length of this line is represented by x, which of course is regarded as an unknown quantity.

Wire A.

1882, Mar. 27.	$\lambda =$	$99 \cdot 9693 + x_1 - 13 \cdot 5\ y_1$,	No. of determ. = 3
Apr. 19.		$99 \cdot 9688 + x_1 - 12 \cdot 2\ y_1$,	" " = 2
" 26.		$99 \cdot 9617 + x_1 - 5 \cdot 95\ y_1$,	" " = 2
June 15.		$99 \cdot 9606 + x_1 - 4 \cdot 0\ y_1$,	" " = 2
Sept. 7.		$99 \cdot 9585 + x_1 - 0 \cdot 4\ y_1$,	" " = 2
1883, Jan. 6.		$99 \cdot 9878 + x_1 - 26 \cdot 4\ y_1$,	" " = 2
" 7.		$99 \cdot 9839 + x_1 - 22 \cdot 5\ y_1$,	" " = 2
" 31.		$99 \cdot 9753 + x_1 - 13 \cdot 4\ y_1$.	" " = 2
Apr. 20.		$99 \cdot 9717 + x_1 - 11 \cdot 7\ y_1$,	" " = 2
Sept. 25.		$99 \cdot 9640 + x_1 - 4 \cdot 9\ y_1$,	" " = 2
Oct. 16.		$99 \cdot 9630 + x_1 - 3 \cdot 4\ y_1$,	" " = 2
Nov. 17.		$99 \cdot 9764 + x_1 - 15 \cdot 9\ y_1$,	" " = 2

Wire B.

1882, Mar. 22.	$\lambda =$	$99 \cdot 9215 + x_2 - 9 \cdot 7\ y_2$,	No. of determ. = 2
" 27.		$99 \cdot 9281 + x_2 - 13 \cdot 6\ y_2$,	" " = 4
Apr. 19.		$99 \cdot 9270 + x_2 - 12 \cdot 2\ y_2$,	" " = 2
June 15.		$99 \cdot 9156 + x_2 - 4 \cdot 1\ y_2$,	" " = 2
Sept. 7.		$99 \cdot 9105 + x_2 - 0 \cdot 7\ y_2$,	" " = 2
1883, Jan. 6.		$99 \cdot 9508 + x_2 - 26 \cdot 5\ y_2$,	" " = 2
" 7.		$99 \cdot 9522 + x_2 - 22 \cdot 6\ y_2$,	" " = 2
" 31.		$99 \cdot 9377 + x_2 - 13 \cdot 4\ y_2$,	" " = 2
Apr. 20.		$99 \cdot 9327 + x_2 - 11 \cdot 6\ y_2$,	" " = 2
Sept. 25.		$99 \cdot 9197 + x_2 - 4 \cdot 9\ y_2$,	" " = 2
Nov. 17.		$99 \cdot 9405 + x_2 - 15 \cdot 9\ y_2$,	" " = 3

Wire C.

1882, Apr. 19.	$\lambda = 100{\cdot}0224 + x_3 - 11{\cdot}45 y_3$,	No. of determ. = 2
June 15.	$100{\cdot}0110 + x_3 - 4{\cdot}3\ y_3$,	" " = 2
Sept. 7.	$100{\cdot}0070 + x_3 - 1{\cdot}2\ y_3$,	" " = 2
1883, Jan. 6.	$100{\cdot}0530 + x_3 - 26{\cdot}0\ y_3$,	" " = 2
" 7.	$100{\cdot}0471 + x_3 - 21{\cdot}9\ y_3$,	" " = 2
" 31.	$100{\cdot}0330 + x_3 - 13{\cdot}4\ y_3$,	" " = 2
Apr. 26.	$100{\cdot}0224 + x_3 - 9{\cdot}2\ y_3$,	" " = 2
Sept. 25.	$100{\cdot}0124 + x_3 - 4{\cdot}5\ y_3$,	" " = 2
Nov. 17.	$100{\cdot}0342 + x_3 - 15{\cdot}75 y_3$,	" " = 2

Wire D.

1882, Apr. 19.	$\lambda = 100{\cdot}0566 + x_4 - 11{\cdot}6\ y_4$,	No. of determ. = 3
June 15.	$100{\cdot}0509 + x_4 - 4{\cdot}15 y_4$,	" " = 3
Sept. 7.	$100{\cdot}0498 + x_4 - 1{\cdot}75 y_4$,	" " = 2
1883, Jan. 6.	$100{\cdot}0792 + x_4 - 26{\cdot}95 y_4$,	" " = 2
" 7.	$100{\cdot}0752 + x_4 - 21{\cdot}8\ y_4$,	" " = 2
" 31.	$100{\cdot}0670 + x_4 - 13{\cdot}4\ y_4$,	" " = 2
Apr. 26.	$100{\cdot}0582 + x_4 - 8{\cdot}2\ y_4$,	" " = 2
Sept. 25.	$100{\cdot}0556 + x_4 - 5{\cdot}2\ y_4$,	" " = 2
Nov. 17.	$100{\cdot}0687 + x_4 - 16{\cdot}7\ y_4$,	" " = 2

Wire E.

1882, Sept. 4.	$\lambda = 100{\cdot}0107 + x_5 + 1{\cdot}9\ y_5$,	No. of determ. = 2
" 7.	$100{\cdot}0120 + x_5 + 0{\cdot}45 y_5$,	" " = 3
1883, Jan. 6.	$100{\cdot}0412 + x_5 - 26{\cdot}0\ y_5$,	" " = 2
" 7.	$100{\cdot}0384 + x_5 - 22{\cdot}25 y_5$,	" " = 2
" 31.	$100{\cdot}0307 + x_5 - 13{\cdot}4\ y_5$,	" " = 2
Apr. 20.	$100{\cdot}0262 + x_5 - 12{\cdot}3\ y_5$,	" " = 2
Sept. 25.	$100{\cdot}0198 + x_5 - 5{\cdot}5\ y_5$,	" " = 2
Nov. 17.	$100{\cdot}0278 + x_5 - 13{\cdot}1\ y_5$,	" " = 2

Wire F.

1882, Sept. 4.	$\lambda = 100{\cdot}0139 + x_6 + 0{\cdot}9\ y_6$,	No. of determ. = 2
" 7.	$100{\cdot}0157 + x_6 + 0{\cdot}2\ y_6$,	" " = 2
1883, Jan. 6.	$100{\cdot}0660 + x_6 - 26{\cdot}6\ y_6$,	" " = 3
" 7.	$100{\cdot}0584 + x_6 - 22{\cdot}5\ y_6$,	" " = 2
" 31.	$100{\cdot}0448 + x_6 - 13{\cdot}4\ y_6$,	" " = 2
Apr. 20.	$100{\cdot}0409 + x_6 - 12{\cdot}55 y_6$,	" " = 2
Sept. 25.	$100{\cdot}0284 + x_6 - 5{\cdot}9\ y_6$,	" " = 2
Nov. 17.	$100{\cdot}0431 + x_6 - 13{\cdot}4\ y_6$,	" " = 3

Wire M.

1883, Apr. 23.	$\lambda = 100{\cdot}0567 + x_7 - 13{\cdot}8\ y_7$,	No. of determ. = 3
Sept. 25.	$100{\cdot}0514 + x_7 - 6{\cdot}4\ y_7$,	" " = 2
Nov. 17.	$100{\cdot}0571 + x_7 - 13{\cdot}2\ y_7$,	" " = 2

Wire N.

1883, Apr. 23.	$\lambda = 100{\cdot}0407 + x_8 - 13{\cdot}75 y_8$,	No. of determ. = 2
Sept. 25.	$100{\cdot}0308 + x_8 - 6{\cdot}7\ y_8$,	" " = 2
Nov. 17.	$100{\cdot}0430 + x_8 - 13{\cdot}1\ y_8$.	" " = 2

The total number of simple observations made in determining the length of the comparator was 136.

The following results for the length of the line were obtained after substituting the first approximate values of x and y as given below:

Date.	Temp.	*A* (steel).	*B* (brass).	*C* (brass).	*D* (steel).	*E* (steel).	*F* (brass).	*M* (brass).	*N* (brass).	Mean.
		$x = 40·6$ mm $y = 1·04$	+87·3 mm 1·68	−7·9 mm 1·74	−52 mm 1·02	−15·0 mm 0·99	−19·0 mm 1·75	−43·6 mm 1·02	−19·0 mm 1·70	------ ------
1882.	°									
Mar. 22	+5	------	99·9925	------	------	------	------	------	------	99·9925
27	+1	99·9959	99·9926	------	------	------	------	------	------	99·9943
Apr. 19	+3	99·9967	99·9938	99·9946	99·9928	------	------	------	------	99·9945
26	+9	99·9961	------	------	------	------	------	------	------	99·9961
June 15	+11	99·9970	99·9959	99·9956	99·9947	------	------	------	------	99·9958
Sept. 4	+16	------	------	------	------	99·9976	99·9965	------	------	99·9970
7	+14	99·9987	99·9966	99·9970	99·9960	99·9974	99·9970	------	------	99·9971
1883.										
Jan. 6	−12	100·0010	100·0015	99·9999	99·9997	99·9999	100·0004	------	------	100·0004
7	−7	100·0011	100·0015	100·0011	100·0010	100·0009	100·0000	------	------	100·0009
31	+2	100·0020	100·0025	100·0018	100·0013	100·0022	100·0024	------	------	100·0020
Apr. 20	+3	100·0001	100·003	------	------	99·9988	99·9999	------	------	99·9998
23	+1	------	------	------	------	------	------	99·9990	99·9982	99·9986
26	+7	------	------	99·9985	99·9978	------	------	------	------	99·9982
Sept. 25	+10	99·9995	99·9988	99·9967	99·9983	99·9992	99·9991	100·0013	100·0003	99·9991
Oct. 16	+12	100·0001	------	------	------	------	------	------	------	100·0001
Nov. 17	+2	100·0005	100·0011	99·9989	99·9997	99·9986	100·0007	100·0000	100·0016	100·0003

Although only approximate values for the lengths of the wires and the expansion of the same were employed in the computation, still it unquestionably appears (1) that the variations are not altogether dependent upon temperature; (2) that the results obtained on any one day with different wires agree very well with one another; (3) that if the variations are made a function of the changes in the lengths of the wires, these changes would appear the same for all the wires; (4) that the line itself must be regarded as varying.

The especial difficulty of obtaining from the observations the errors of the wires is unquestionably to be regarded as an unfavorable exceptional case, caused by the grading and building operations already referred to. The errors also must be affected with a larger probable error than would otherwise have been the case. Hence the results in the longer lines depending upon this one must give a larger error than would ordinarily be expected rather than a smaller one.

So far as the change in the line is concerned, there was positive evidence of it during the early experiments, or until the beginning of the year 1883, but after that it seemed to remain quite constant. This cessation of change was expected about this time, since the work along the street was then completed. If the variation is regarded as a function of time, it may be obtained if we take terms up to the third involving time, as

$$\lambda = \lambda_0 + x\,t + y\cos t + z\sin t,$$

in which t is the time and x, y, and z are constants. It seems necessary to assume one expression for the period prior to the epoch 1883·0 and another for the time subsequent to it. I therefore place for the former

$$\lambda = \lambda_0 + m_1\,t + m_2\,t^2 \;\ldots\ldots \qquad (a)$$

and for the latter

$$\lambda = \lambda_0 + n_1\,t + n_2\,t^2 \;\ldots\ldots \qquad (b)$$

in which t is expressed in years dating from 1883·0 and λ_0 the length of the comparator at that time. The determination of the comparator by means of the base apparatus already referred to took place November 12, 1883, or 1883·86. Therefore the equation

$$100{\cdot}0001 = \lambda_0 + 0{\cdot}86\,n_1 + 0{\cdot}74\,n_2 \;\ldots\ldots \qquad (c)$$

must be satisfied.

Without introducing into the computation complications that are without any gain, I compute first of all, with the aid of the mean lengths of the line as obtained in the above approximations, the constants in the expressions for λ. Weights are ascribed to these means in proportion to the number of wires employed each day in finding this mean. For the period after 1883·0 we have from equations (b) and (c)

$$\lambda - 100{\cdot}0001 = (t - 0{\cdot}86)\,n_1 + (t^2 - 0{\cdot}74)\,n_2$$

and

$+0{\cdot}3 = -0{\cdot}85n_1$	$-0{\cdot}74n_2$,	weight	$=6$
$+0{\cdot}8 = -0{\cdot}84$	$-0{\cdot}74$	"	$=6$
$+1{\cdot}9 = -0{\cdot}78$	$-0{\cdot}73$	"	$=6$
$-0{\cdot}3 = -0{\cdot}56$	$-0{\cdot}65$	"	$=4$
$-1{\cdot}5 = -0{\cdot}55$	$-0{\cdot}64$	"	$=2$
$-1{\cdot}9 = -0{\cdot}55$	$-0{\cdot}64$	"	$=2$
$-1{\cdot}0 = -0{\cdot}13$	$-0{\cdot}21$	"	$=8$
$0{\cdot}0 = -0{\cdot}07$	$-0{\cdot}12$	"	$=1$
$+0{\cdot}2 = +0{\cdot}02$	$+0{\cdot}03$	"	$=8$

From these one finds the following most probable values:

$$n_1 = -\ 9{\cdot}67mm$$
$$n_2 = +\ 9{\cdot}59mm$$

and from equation (c)

$$\lambda_0 = 100{\cdot}0001 + 8{\cdot}31mm - 7{\cdot}09mm$$
$$= 100{\cdot}0013m$$

The equation (a) now takes the form

$$\lambda = 100{\cdot}0013 = m_1t + m_2t^2,$$

from which we have

$-8.8mm$	$=-0{\cdot}78m_1$	$+0{\cdot}61m_2$,	weight	$=1$
$-7{\cdot}0$	$=-0{\cdot}77$	$+0{\cdot}59$	"	$=2$
$-6{\cdot}8$	$=-0{\cdot}70$	$+0{\cdot}49$	"	$=4$
$-5{\cdot}2$	$=-0{\cdot}69$	$+0{\cdot}48$	"	$=1$
$-5{\cdot}5$	$=-0{\cdot}55$	$+0{\cdot}30$	"	$=4$
$-4{\cdot}3$	$=-0{\cdot}33$	$+0{\cdot}11$	"	$=2$
$-4{\cdot}2$	$=-0{\cdot}32$	$+0{\cdot}10$	"	$=6$

which gives

$$m_1 = +\ 14{\cdot}55mm$$
$$m_2 = +\ 6{\cdot}98mm$$

Hence we have

before 1883·0, $\lambda = 100{\cdot}0013m + 14{\cdot}55mm \times t + 6{\cdot}98mm \times t^2$
after 1883·0, $\lambda = 100{\cdot}0013m - 9{\cdot}67mm \times t + 9{\cdot}56mm \times t^2$

in which t is the time in years reckoning from 1883·0.

With these equations the following table has been computed:

		m.
1882, Mar. 22	$\lambda =$	99·9942
" 27		·9942
Apr. 19		·9946
" 26		·9947
June 15		·9954
Sept. 4		·9973
" 7		·9974
1883, Jan. 6		100·0012
" 7		·0011
" 31		·0006
Apr. 20		99·9993
" 23		·9993
" 26		·9993
Sept. 25		·9993
Oct. 16		·9996
Nov. 17		100·0002

Upon this determination of λ rest the computation of errors of length and the expansion of the wires, by taking the approximate values given on page 145 for x and y equal to (x) and (y). One then obtains

Wire A.

mm.

$$\begin{aligned}
-1{\cdot}7 &= (x_1) - 14(y_1)\\
-2{\cdot}1 &= (x_1) - 12(y_1)\\
-1{\cdot}4 &= (x_1) - 6(y_1)\\
-1{\cdot}6 &= (x_1) - 4(y_1)\\
-1{\cdot}3 &= (x_1) - 0(y_1)\\
+0{\cdot}2 &= (x_1) - 26(y_1)\\
0{\cdot}0 &= (x_1) - 23(y_1)\\
-1{\cdot}4 &= (x_1) - 13(y_1)\\
-0{\cdot}8 &= (x_1) - 12(y_1)\\
-0{\cdot}2 &= (x_1) - 5(y_1)\\
-0{\cdot}5 &= (x_1) - 3(y_1)\\
-0{\cdot}3 &= (x_1) - 16(y_1)
\end{aligned}$$

Wire B.

mm.

$$\begin{aligned}
+1{\cdot}7 &= (x_2) - 10(y_2)\\
+1{\cdot}6 &= (x_2) - 14(y_2)\\
+0{\cdot}8 &= (x_2) - 12(y_2)\\
-0{\cdot}5 &= (x_2) - 4(y_2)\\
+0{\cdot}8 &= (x_2) - 1(y_2)\\
-0{\cdot}3 &= (x_2) - 27(y_2)\\
-0{\cdot}4 &= (x_2) - 23(y_2)\\
-1{\cdot}9 &= (x_2) - 13(y_2)\\
-1{\cdot}0 &= (x_2) - 12(y_2)\\
+0{\cdot}5 &= (x_2) - 5(y_2)\\
-0{\cdot}9 &= (x_2) - 16(y_2)
\end{aligned}$$

Wire C.

mm.

$$\begin{aligned}
0{\cdot}0 &= (x_3) - 11(y_3)\\
-0{\cdot}2 &= (x_3) - 4(y_3)\\
+0{\cdot}4 &= (x_3) - 1(y_3)\\
+0{\cdot}3 &= (x_3) - 26(y_3)\\
0{\cdot}0 &= (x_3) - 22(y_3)\\
-1{\cdot}2 &= (x_3) - 13(y_3)\\
+0{\cdot}8 &= (x_3) - 9(y_3)\\
+2{\cdot}6 &= (x_3) - 4(y_3)\\
+1{\cdot}3 &= (x_3) - 16(y_3)
\end{aligned}$$

Wire D.

mm.

$$\begin{aligned}
+1.8 &= (x_4) - 12(y_4)\\
+0{\cdot}7 &= (x_4) - 4(y_4)\\
+1{\cdot}4 &= (x_4) - 2(y_4)\\
+1{\cdot}5 &= (x_4) - 27(y_4)\\
+0{\cdot}1 &= (x_4) - 22(y_4)\\
-0{\cdot}7 &= (x_4) - 13(y_4)\\
+1{\cdot}5 &= (x_4) - 8(y_4)\\
+1{\cdot}0 &= (x_4) - 5(y_4)\\
+0{\cdot}5 &= (x_4) - 17(y_4)
\end{aligned}$$

Wire E.

mm.

$$\begin{aligned}
-0{\cdot}3 &= (x_5) + 2(y_5)\\
0{\cdot}0 &= (x_5) + 0(y_5)\\
+1{\cdot}3 &= (x_5) - 26(y_5)\\
+0{\cdot}2 &= (x_5) - 22(y_5)\\
-1{\cdot}6 &= (x_5) - 13(y_5)\\
+0{\cdot}5 &= (x_5) - 12(y_5)\\
+0{\cdot}1 &= (x_5) - 6(y_5)\\
+0{\cdot}6 &= (x_5) - 13(y_5)
\end{aligned}$$

Wire F.

mm.

$$\begin{aligned}
+0{\cdot}8 &= (x_6) + 1(y_6)\\
+0{\cdot}4 &= (x_6) + 0(x_6)\\
+0{\cdot}8 &= (x_6) - 27(y_6)\\
+1{\cdot}1 &= (x_6) - 22(y_6)\\
-1{\cdot}8 &= (x_6) - 13(y_6)\\
-0{\cdot}6 &= (x_6) - 13(y_6)\\
+0{\cdot}2 &= (x_6) - 6(y_6)\\
-0{\cdot}5 &= (x_6) - 13(y_6)
\end{aligned}$$

Wire M.

mm.

$$\begin{aligned}
+0{\cdot}3 &= (x_7) - 14(y_7)\\
-2{\cdot}0 &= (x_7) - 6(y_7)\\
+0{\cdot}2 &= (x_7) - 13(y_7)
\end{aligned}$$

Wire N.

mm.

$$\begin{aligned}
+1{\cdot}1 &= (x_8) - 14(y_8)\\
-1{\cdot}0 &= (x_8) - 7(y_8)\\
-1{\cdot}4 &= (x_8) - 13(y_8)
\end{aligned}$$

From these we can obtain the following equations, involving the unknown quantities, the first member being in millimetres.

$$
\begin{aligned}
-11{\cdot}1 &= 12(x_1) - 134(y_1), & +0{\cdot}4 &= 11(x_2) - 137(y_2),\\
93{\cdot}7 &= -134(x_1) + 2200(y_1), & -18{\cdot}1 &= -137(x_2) + 2309(y_2),\\
+5{\cdot}0 &= 9(x_3) - 106(y_3), & +7{\cdot}8 &= 9(x_4) - 110(y_4),\\
-56{\cdot}2 &= -106(x_3) + 1820(y_3), & -86{\cdot}3 &= -110(x_4) + 1924(y_4),\\
+0{\cdot}8 &= 8(x_5) - 90(y_5), & +0{\cdot}4 &= 8(x_6) - 93(y_6),\\
-32{\cdot}4 &= -90(x_5) + 1682(y_5), & -8{\cdot}5 &= -93(x_6) + 1757(y_6),\\
-1{\cdot}5 &= 3(x_7) - 33(y_7), & -1{\cdot}3 &= 3(x_8) - 34(y_8),\\
+5{\cdot}2 &= -33(x_7) + 401(y_7), & +9{\cdot}8 &= -34(x_8) + 414(y_8),
\end{aligned}
$$

From which we obtain

$$
\begin{aligned}
(x_1) &= -1{\cdot}41, & x_1 &= 39{\cdot}19, & (x_2) &= +0{\cdot}51, & x_2 &= +87{\cdot}81,\\
(y_1) &= -0{\cdot}043, & y_1 &= 0{\cdot}997, & (y_2) &= +0{\cdot}038, & y_2 &= 1{\cdot}718,\\
(x_3) &= +0{\cdot}61, & x_3 &= -7{\cdot}29, & (x_4) &= +1{\cdot}06, & x_4 &= -50{\cdot}94,\\
(y_3) &= +0{\cdot}005, & y_3 &= 1{\cdot}745, & & 1{\cdot}016, & y_4 &= 1{\cdot}036,\\
(x_5) &= -0{\cdot}29, & x_5 &= -15{\cdot}29, & (x_6) &= -0{\cdot}02, & x_6 &= -19{\cdot}02,\\
(y_5) &= -0{\cdot}035, & y_5 &= 0{\cdot}955, & (y_6) &= -0{\cdot}006, & y_6 &= 1{\cdot}744,\\
& & x_7 &= -44{\cdot}38, & x_8 &= -19{\cdot}15,\\
& & y_7 &= 0{\cdot}996, & y_8 &= 1{\cdot}736.
\end{aligned}
$$

For the wires *M* and *N* the coefficient of expansion is taken as the mean of the preceding wires of the same metal, inasmuch as the number of determinations was too few to admit of an independent determination for each.

The wires, therefore, when hanging free and under the tension already mentioned, have the following lengths in metres from the middle of the scale at one end to the middle of the scale at the other, and also the coefficient of expansion for 1° Celsius:

Wire,	*A*	*B*	*C*	*D*
Length,	25·00980	25·02195	24·99818	24·99727
Coef. of ex.,	(0·0000)0997	1718	1745	1036
Wire,	*E*	*F*	*M*	*N*
Length,	24·99618	24·99524	24·98890	24·99521
Coef. of ex.,	(0·0000)0955	1744	0996	1736.

Since these experiments gave a noticeably smaller coefficient of expansion for steel, as well as brass, than has usually been found for these metals, I have deemed it wise to determine these coefficients independent of any geodetic operation. For this purpose I employed the proper apparatus, belonging to the school of technology, in which the wires are immersed in water of different temperatures. The changes in length were read by a telescope in a mirror, which, turned by the expansion or contraction of the wire, reflected the divisions of a large scale placed at a suitable distance from it. The result of these investigations gave for the brass wire from which *F* was taken

Coefficient of expansion = 0·00001751,

and for steel wire 1·3^{mm} in diameter

Coefficient of expansion = 0.00000979.

Some days later the experiments were repeated,

giving for the former 0·00001703

and for the latter 0·00000988;

or, taking the means we have,

Coefficient of expansion for drawn brass wire = 0·00001727
Coefficient of expansion for drawn steel wire = 0·00000984,

results which agree very closely with those found in the direct measuring.

The errors of length and coefficients of expansion for the two steel tapes A and B were also determined by measurement of the comparator; but since these tapes were only occasionally used in measuring the short overlaps at the ends of the line, a detailed account of the experiments can have no special interest. Therefore the results only will be given:

The length of the tape A at 15° Celsius = 20.00303^{m}.
The length of the tape B at 15° Celsius = 20.00493^{m}.
The coefficient of expansion for A = 0.00001046.
The coefficient of expansion for B = 0.00001030.

The length given above is for the tapes when lying on a flat surface and without tension. In addition, the errors of graduation were determined for A, it alone being used in the final measurements. They are given in the table below, combined with the errors of length for the normal tension and also the errors if 10^{k} or 15^{k} tension were employed.

Errors of graduation + error of length.

	$T_0=6{\cdot}72^k$		$T=10^k$		$T=15^k$	
m.	*k.*	*mm.*	*k.*	*mm.*	*k.*	*mm.*
0	0·00	0·0	6·96	0·0	13·65	0·0
1	0·91	+0·3	6·98	+0·4	13·65	+0·6
2	1·45	0·5	7·02	0·8	13·67	1·1
3	1·88	0·7	7·10	1·1	13·69	1·5
4	2·30	0·9	7·20	1·5	13·71	2·0
5	2·67	1·2	7·32	1·9	13·75	2·6
6	3·01	1·0	7·45	1·9	13·79	2·8
7	3·34	1·3	7·60	2·3	13·84	3·3
8	3·65	1·1	7·77	2·3	13·90	3·5
9	3·95	1·4	7·94	2·7	13·96	4·0
10	4·24	1·8	8·11	3·2	14·03	4·7
11	4·51	2·1	8·30	3·8	14·11	5·3
12	4·78	2·0	8·48	3·8	14·19	5·5
13	5·05	2·2	8·67	4·1	14·28	6·0
14	5·30	2·1	8·86	4·1	14·37	6·2
15	5·55	2·2	9·05	4·4	14·47	6·6
16	5·79	2·2	9·24	4·6	14·57	6·9
17	6·03	2·6	9·43	5·1	14·67	7·6
18	6·27	2·6	9·62	5·3	14·78	7·9
19	6·50	3·1	9·81	5·9	14·89	8·7
20	6·72	3·0	10·00	6·0	15·00	8·8

Scale $\frac{1}{15000}$

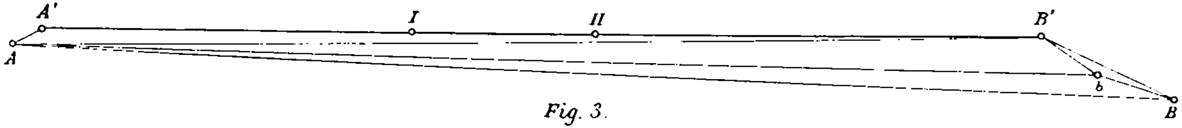


Fig. 3.

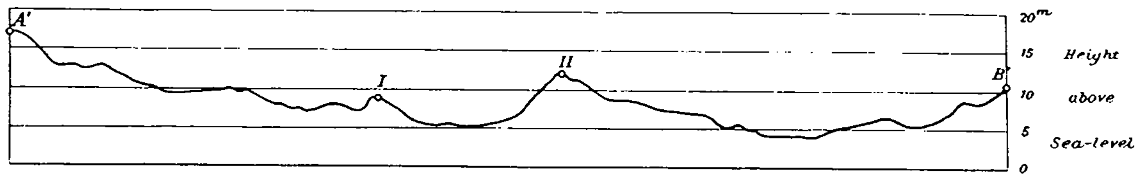


Fig. 4.

THE LONG LINE.

This line also is situated on "Ladugardsgardet," near Stockholm, but it is not the line measured in 1864 by Professor Lindhagen with the base apparatus belonging to the Academy of Sciences. One end of this line is covered by dirt, and buildings have been erected at different points along it. In figure 3 $A\ B$ is the old line and $A'\ B'$ the new one. The terminal points, as well as the two intermediate points I and II, are fixed by small holes drilled into a rod of iron firmly fastened in the upper surface of stone blocks.

The measurement in 1864 gave for $A\ B$ reduced to sea level 2320·1878^{m}.

In order to compare this result with the length of $A'\ B'$ obtained by wire measurement the two lines were connected by means of a triangulation.

At the beginning, in 1880, it was possible to determine $B\ B'$ by direct measurement, but shortly afterwards a house was built in the way, so that it was necessary to establish an intermediate point, b. Each of the values given below is the mean of at least two independent determinations.

LENGTHS.

				m.
1880, Apr. 26,	line $B\ b$,	measured with steel tape A,		161·6456
May 2,	" $B\ B'$,	" " " " "		294·4511
1882, May 17,	" $B'\ b$,	" " wires,		135·4757
May 19,	" $A\ A'$,	" " "		64·3883

When reduced to sea level the lines have the following values:

$$
\begin{aligned}
& \qquad\quad m. \\
A\ B &= 2320{\cdot}1878 \\
B\ b &= 161{\cdot}6453 \\
B\ B' &= 294{\cdot}4506 \\
B'\ b &= 135{\cdot}4757 \\
A\ A' &= 64{\cdot}3882
\end{aligned}
$$

ANGLE READINGS.

		°	′	″
1880, May 3,	$B\ B'\ A$, using a theodolite of Ertel reading to 10″ by means of verniers,	154	1	26·6
1882, May 20,	$A\ b\ B$, using a universal instrument of Wanschaff reading to 1″ by means of microscopes,	163	4	38·6
	$A\ b\ A'$,		51	38·9
	$A'\ b\ B'$,	31	29	16·6
	$B\ b\ B'$,	164	34	25·9
1882, June 12,	$B'\ A'\ b$, with the same instrument,	2	1	57·7
	$b\ A'\ A$,	149	35	42·2

In the triangle $B\,b\,B'$ the sides $B\,b$ and $b\,B'$ being known from direct measurement and the angle $B\,b\,B'$, one obtained from computation

$$B\,B'=294{\cdot}4538$$

By giving the measured value weight 2 and this computed result weight 1, the mean gives

$$B\,B'=294{\cdot}4517$$

And after making the corrections for triangle errors it was found that

$$B\,b=161{\cdot}6442$$
$$B'\,b=135{\cdot}4747$$

In the triangle $A\,B\,b$, $B\,b$, $A\,B$ and $A\,b\,B$ are known, from which one can find that

$$b\,A\,B= 1^{\circ}\ 9'\ 43{\cdot}15''$$

and

$$A\,B\,b=15\ 45\ 38.25$$

using the known equation

$$A\,b=A\,B\cos b\,A\,B+B\,b\cos A\,b\,B=A\,B-2\,A\,B\sin^2 \tfrac{1}{2}b\,A\,B+B\,b\cos A\,b\,B$$

or

$$A\,b=2320{\cdot}1878^{m}=0{\cdot}4771-154{\cdot}6448=2165{\cdot}0659.$$

In the same manner, from $A\,B'\,b$ I found

$$A\,B'=2051{\cdot}8967^{m}$$

and from $A\,b\,A$

$$A'\,b=2109{\cdot}2877^{m}$$

And, finally, from the triangle $A\,A'\,B'$

$$A'\,B'=1995{\cdot}0148$$

and from $A'\,B'\,b$

$$A'\,B'=1995{\cdot}0168$$

the mean of which was accepted as a sufficient approximation, or

$$A'\,B=1995{\cdot}0158^{m}$$

DETERMINATION OF $A'\,B'$ WITH WIRES.

In measuring, two wires were always used, one of steel and one of brass. By means of the difference in the readings of the two a check is obtained on the reading and data for the determination of temperature.

If the two wires at the normal temperature (+15° Celsius) have lengths L and L', and if we designate by c and c' the scale readings, then after applying n wire lengths the sums of the readings would be $\Sigma\, c$ and $\Sigma\, c'$. Again, if the excess of temperature above the normal temperature be indicated by t and the coefficients of expansion by α

and β, then the length of the measured line λ is for one wire ascertained from the equation

$$\lambda = n\, L + \Sigma\, e + n\, L\, \alpha\, t$$

and for the other wire

$$\lambda = n\, L' + \Sigma\, e' + n\, L'\, \beta\, t$$

therefore

$$t = \frac{n\, L - n\, L' + \Sigma\, e - \Sigma\, e'}{n\, (L'\, \beta - L\, \alpha)} .$$

By substituting this expression for t in one of the equations above, we have

$$\lambda = \frac{L'\, \beta\, (n\, L + \Sigma\, e) - L\, \alpha\, (n\, L' + \Sigma\, e')}{\beta\, L' - \alpha\, L}$$

or, if the length of the line according to the two wires be represented by s and s'—that is, without reference to the temperature—then

$$\lambda = s + \frac{\alpha}{\beta - \alpha}\, (s - s') = s' + \frac{\beta}{\beta - \alpha}\, (s - s') \ldots\ldots \qquad (16)$$

or

$$\lambda = \frac{\beta}{\beta - \alpha}\, s - \frac{\alpha}{\beta - \alpha}\, e'$$

From the latter of these equations it is evident that if at each reading of the scale there was a probable error of $\pm\, r$, the probable error for the whole line from this cause would be

$$\varphi = \pm\, \frac{2}{\beta - \alpha}\, \sqrt{n\, (\beta^2 + \alpha^2)} \ldots\ldots \qquad (17)$$

a formula which is true only when the temperature, as is here regarded, is determined by the differences in the scale readings.

Over smooth ground the errors of length resulting from errors of contact and of differences of elevation are of very slight consequence. In such a case the accidental error only is found from equation (17).

Instead of using equation (16), it would be better to compute the length of the line in such a way that the temperature may be taken from a table already prepared in which the argument is the difference in the lengths of the two wires and the tabulated values degrees. In this way, by computing the length of the line for each wire, one has a check on the work. For the wires A and B, at $+\, 15^\circ$ Celsius, the difference in lengths,

$$B - A = +\, 12{\cdot}15^{\mathrm{mm}}$$

and the relative expansion for 1°

$$25^{\mathrm{m}}\, (\beta - \alpha) = 0{\cdot}1802^{\mathrm{mm}}$$

from which the following table was prepared:

Differences of expansion.

t	$B-A$	t	$B-A$	t	$B-A$
°	*mm.*	°	*mm.*	°	*mm.*
−15	6·75	7	10·71	29	14·68
14	6·93	8	10·89	30	14·86
13	7·11	9	11·07	31	15·04
12	7·29	10	11·25	32	15·22
11	7·47	11	11·43	33	15·40
10	7·65	12	11·61		
9	7·83	13	11·79		
8	8·01	14	11·97	Differences.	
7	8·19	15	12·15		
6	8·37	16	12·34		
5	8·55	17	12·52	°	*mm.*
4	8·73	18	12·70	0·0	0·00
3	8·91	19	12·88	·1	·02
2	9·09	20	13·06	·2	·04
− 1	9·27	21	13·24	·3	·05
0	9·45	22	13·42	·4	·07
+ 1	9·63	23	13·60	·5	·09
2	9·81	24	13·78	·6	·11
3	9·99	25	13·96	·7	·13
4	10·17	26	14·14	·8	·14
5	10·35	27	14·32	·9	·16
6	10·53	28	14·50	1·0	·18

The necessary personnel for measuring is as follows: One person for the spring balance, one at each end to place the scales side by side and to make the readings, one for leveling, and one for marking—or five trusty persons. For handwork there are needed one person for the small steel balance, one to hold the leveling rod, and two for carrying instruments, pegs, etc., along the line, making four workmen. Thus it will be seen that in order to have the work move along without delays nine persons are essential. In the work on "Ladugårdsgärdet" the number of assistants was not so large as desirable. Besides this, the continuous progress of the work was often hindered by passing wagons and the drilling of soldiers. This latter interruption is the reason why the line A' B' was seldom measured as a whole, but in the sections A' I, I II, and II B' independently and irregularly.

During the first few days a leveling rod was used which was provided with two targets, the second one being on the reverse side and read downward, the zero being at the top. The advantage thus afforded was that the two readings should always have a constant sum, thereby furnishing a check.

But after some days one of the targets was injured, and thereafter the ordinary method was pursued with confidence, inasmuch as up to that time no erroneous readings had been made.

The following gives the measurements of A' B':

1882.	Line	Began	Finished	Remarks.
May 8.	A'I	10:50 a. m.	4:00 p. m.	Sunshine, scattering clouds.
" 9.	II I	7:45	10:10 a. m.	" wind = 3 to 4.*
	IIB	----------	3:15 p. m.	Thin clouds.
	II I	7:00 p. m.	8:10	Still.
" 10.	IIB	0:20	2:50	Sunshine, wind = 1.
	II I	6:45	7:25	"
" 11.	IA'	8:40 a. m.	10:50 a. m.	" still.
	A'I	0:30 p. m.	2:25 p. m.	" scattering clouds, wind = 1 to 2.
	IIB'	3:25	5:50	" wind = 2.
" 12.	A'I	8:40 a. m.	10:55 a. m.	Rain, wind = 2.
" 15.	IIB	3:10 p. m.	6:40 p. m.	Sunshine, wind at right angles to the wires.

In these measurements only wires A and B were employed and the tape A.

In the autumn of the same year the line was measured twice by a party of students, using the wires E and F, but unfortunately palpable errors were made, making the results useless. However, I mention them here merely to show how rapidly measurements of this kind can be done.

1882.	Line	Began	Finished	Remarks.
Sept. 8.	A'I	9:15 a. m.	0:00	Sunshine, scattering clouds.
	I II	2:30 p. m.	3:40 p. m.	
	IIB	3:40	6:05	
" 9.	B'II	8:30 a. m.	10:50 a. m.	" wind = 2.
	I II	0:40 p. m.	1:40 p. m.	
	IA'	1:40	3:50	

The three lines A' I, I II, and II B' have the lengths 740, 367, 888 metres, respectively. The rapidity of the measurement in metres is as follows:

	Line.	Time.		Lengths per hour.	Lengths per day.
1882.		h.	m.		
May 8	740	5	10	143	740
9	367	2	25	152	1622
	888	-----		---	
	367	1	10	315	
10	367	0	50	440	1622
	888	2	30	355	
	367	0	40	550	
11	740	2	10	342	2368
	740	1	55	386	
	888	2	25	367	
12	740	2	15	329	740
15	888	3	30	254	888
Sept. 8	740	2	45	269	1995
	367	1	10	315	
	888	2	25	367	
9	888	2	20	381	1995
	367	1	00	367	
	740	2	10	342	

* Estimated by the meteorological scale: 0=still, 6=hurricane.

The best speed was 550^{m} in an hour, and the largest day's work was on the day when operations began at 8:40 a. m. and continued until 5:50 p. m. On this day 2368^{m} were measured.

The following is the record of one measurement of a line. The numbering is from A'. Of course the record of the measurement of the line was separate from the record of the leveling, but they are united here for illustrative purposes. With the steel tape the tension was always T_0'.

For the wires A and B, $\frac{\alpha}{\beta-\alpha} = 1{\cdot}383$ and $\frac{\beta}{\beta-\alpha} = 2{\cdot}383$,[1] which values are to be employed when the computation is made according to equation (16).

[1882, May 10.]

Point.	Wire.			Leveling.				Horizontal reduction.†	Remarks.
	A	B	Diff.	Readings.		Diff.	Height.*		
	mm.	*mm.*	*mm.*	*dm.*	*dm.*	*dm.*	*dm.*	*mm.*	
I				0·38			0·00		
	72·1	60·2	11·9			− 1·65		0·5	10:30 a. m.
30				2·03			− 1·65		
	90·7	78·7	12·0			− 7·40		11·0	Sunshine.
31				9·43			− 9·05		
	86·9	75·4	11·5			−10·97		24·1	
32				20·40			−20·02		
	30·5	18·3	12·2			− 3·91		3·0	
33				24·31	2·30		−23·93		
	85·2	73·4	11·8			− 0·42		0·0	
34					2·72		−24·35		
	58·6	46·8	11·8			− 1·41		0·4	
35					4·13		−25·76		
	31·7	19·3	12·4			+ 1·54		0.5	
36					2·59		−24·22		
	91·3	79·1	12·2			− 0·70		0·1	
37				16·83	3·29		−24·92		
	54·8	42·9	11·9			+ 4·24		3·6	
38				12·59			−20·68		
	38·0	26·3	11·7			+ 3·16		2·0	
39				9·43			−17·52		
	58·5	46·4	12·1			+ 8·22		13·6	
40				1·21	20·06		− 9·30		
	56·8	45·0	11·8			+11·17		25·0	
41				12·65	8·89		+ 1·87		
	17·1	5·2	11.9			+14·65		43·0	
42				1·00	18·39		+16·52		
	98·8	57·9	11·9			+18·41		67·0	
43					−0·02		+34·93		
	17·0—66·0					− 5·24		8·1‡	
II					5·22		+29·69		11:20 a. m.
	842		$=\Sigma c$				Sum =	202·8	
		674·9	$=\Sigma c'$						

The mean of the diff. = 11·94^{mm}.
The corresponding temperature § = + 13°·85.

* The check is in the differences between the first and last number in each row of readings.
† This is taken from the table on pp. 135, 136. In the record the readings on the reverse scale is omitted.
‡ Based on equation (12a) and the table following.
§ Taken from table on p. 154.
[1] The coefficients of expansion, which were determined geodetically, have been applied here.

[1882, May 10.]

Point.	Wire. A	Wire. B	Wire. Diff.	Leveling. Readings.	Leveling. Readings.	Leveling. Diff.	Leveling. Height.	Horizontal reduction.	Remarks.
	mm.	*mm.*	*mm.*	*dm.*	*dm.*	*dm.*	*dm.*	*mm.*	
II				5·22			0·00		
	65·3	53·1	12·2			+ 3·54		2·5	
44				1·68			+ 3·54		0:20 p. m.
	77·2	65·4	11·8			— 6·52		8·5	
45				8·20			— 2·68		
	56·4	44·1	12·3			— 9·58		18·3	
46				17·78	0·28		—12·56		
	36·0	23·7	12·3			— 5·43		5·9	
47					5·71		—17·99		
	33·6	21·5	12·1			— 2·67		1·5	
48					8·38		—20·66		
	98·7	86·4	12·3			— 3·31		2·2	
49					11·69		—23·97		
	95·6	83·7	11·9			— 3·13		1·9	
50					14·82		—27·10		
	82·6	70·8	11·8			— 3·18		2·0	
51					18·00		—30·28		
	63·2	51·6	11·6			— 2·60		1·4	
52				1·83	20·60		—32·88		
	42·0	30·0	12·0			— 1·16		0·3	
53				2·99			—34·04		
	93·3	81·2	12·1			— 6·63		8·8	
54				9·62			—40·67		
	50·0	38·0	12·0			— 2·43		1·2	
55				12·05			—43·10		
	79·0	66·8	12·2			—11·24		25·3	
56				23·29			—54·34		
	83·7	71·6	12·1			+ 3·97		3·1	
57				19·32			—50·37		
	85·4	73·5	11·9			— 6·80		9·2	
58				26·12			—57·17		
	64·8	53·0	11·8			— 3·00		1·8	
59				29·12	2·97		—60·17		
	48·4	36·5	11·9			— 3·74		2·8	
60					6·71		—63·91		
	67·7	55·6	12·1			— 1·68		0·6	
61					8·39		—65·59		Wind = 1.
	91·4	79·2	12·2			— 3·49		2·5	
62					11·88		—69·08		
	97·2	85·1	12·1			+ 4·42		3·9	
63					7·46		—64·66		Sunshine.
	42·9	30·6	12·3			+ 7·17		10·3	
64					0·29		—57·49		
	93·8	82·1	11.7			+ 0·31		0·0	
65				14·37	—0·02		—57·18		
	95·2	83·2	12·0			+ 5·70		6·5	
66				8·67			—51·48		
	45·7	33·6	12·1			+ 3·58		2·6	
67				5·09			—47·90		
	101·5	89·2	12.3			+ 3·56		2·6	
68				1·53			—44·34		
	84·4	72·7	12·1			+ 0·70		0·1	
69				0·83			—43·64		
	40·3	28·8	11·5			— 3·79		2·9	
70				4·62			—47·43		
	91·2	79·7	11·5			— 0·61		1·3	

[1882, May 10—Continued].

Point.	Wire.			Leveling.				Horizontal reduction.	Remarks.
	A	*B*	Diff.	Readings.		Diff.	Height.		
	mm.	*mm.*	*mm.*	*dm.*	*dm.*	*dm.*	*dm.*	*mm.*	
71				5·23			—48·04		
	71·0	59·3	11·7			— 2·48		2·0	
72				7·71			—50·52		
	61·5	49·5	12·0			+ 3·20		8·6	
73				4·51	23·99		—47·32		
	91·8	80·0	11·8			+ 6·57		23·3	
74					17·42		—40·75		
	51·2	39·1	12·1			+10·79		12·9	
75					6·63		—29·96		
	26·2	14·0	12·2			+ 8·03		0·1	
76				12·38	—1·40		—21·93		
	79·1	66·7	12·4			— 0·53		23·0	
77				12·91			—22·46		
	68·1	56·0	12·1			+10·72		1·1	
78	12·4—5·9			2·19			—11·74		
B^1				0·52		—10·07	—10·07		
	2455·8		$=\Sigma e$				Sum =	201·1	
		2035·3	$=\Sigma e'$						

The mean of the diff. $=12{\cdot}01^{mm}$.
The temperature $=+14^{\circ}{\cdot}2$.

[May 11.]

Point.	Wire.			Leveling.				Horizontal reduction.	Remarks.
	A	*B*	Diff.	Readings.		Diff.	Height.		
	mm.	*mm.*	*mm.*	*dm.*	*dm.*	*dm.*	*dm.*	*mm.*	
1				1·55			0·00		
	62·6	50·5	12·1			— 4·25		3·6	
29				5·80			— 4·25		8:40 a. m.
	55·0	42·8	12·2			— 1·61		0.5	
28				7·41			— 5·86		Sunshine.
	22·6	11·0	11·6			— 0·52		0·1	
27				7·93			— 6·38		No wind.
	58·2	46·3	11·9			+ 0·16		0·0	
26				7·77			— 6·22		
	95·8	84·1	11·7			— 2·72		1·5	
25				10·49			— 8·94		
	59·2	47·3	11·9			+ 3·18		2·0	
24				7·31			— 5·76		
	35·8	24·2	11·6			+ 4·81		4·6	
23				2·50	21·48		— 0·95		
	75·2	63·1	12·1			+ 2·30		1·1	
22					19·18		+ 1·35		
	79·9	68·3	11·6			+ 4·80		4·6	
21					14·38		+ 6·15		
	58·0	45·9	12·1			+ 9·94		16·4	
20					5·24		+15·19		
	71·7	59·5	12·2			+ 1·73		0·6	
19					3·61		+16·09		
	49·1	37·0	12·1			+ 2·73		1·5	

[May 11—Continued.]

Point.	Wire.			Leveling.				Horizontal reduction.	Remarks.
	A	*B*	Diff.	Readings.		Diff.	Height.		
	mm.	*mm.*	*mm.*	*dm.*	*dm.*	*dm.*	*dm.*	*mm.*	
18					0·88		+19·65		
	44·2	32·0	12·2			+ 0·88		0·2	
17					0·00		+20·53		
	48·6	36·8	11·8			− 4·17		3·5	
16					4·17		+16·36		
	67·4	55·2	12·2			− 4·55		4·1	
15					8·72		+11·81		
	51·3	38·8	12·5			+ 3·38		2·3	
14				19·47	5·34		+15·19		
	55·3	43·2	12·1			+ 0·48		0·1	
13				18·99			+15·67		
	87·4	69·7	11·7			+ 7·10		10·2	Cloudy.
12				11·89			+22·77		
	42·3	30·6	11·7			+ 2·61		1·4	
11				9·28			+25·38		
	78·2	66·7	11·5			+ 8·21		13·5	
10				1·07	19·28		+33·59		
	73·5	61·8	11·7			+ 7·48		11·2	
9					11·80		+41·07		
	83·9	71·9	12·0			+ 6·86		9·4	
8					4·94		+47·95		
	66·2	54·7	11·5			− 0·75		0·1	
7					5·69		+47·18		
	52·7	41·1	11·6			+ 0·60		0·1	
6					5·09		+47·78		
	70·0	57·8	12·2			+ 5·09		5·2	
5					0·00		+52·87		
	99·7	87·7	12·0			− 0·51		0·1	
4				12·08	0·51		+52·36		
	54·2	42·3	11·9			+ 7·31		10·7	
3				11·77	28·59		+59·97		
	44·2	32·3	11·9					45·8	
2				13·46		+74·80	+74·80		
	64·1	52·0	12·1		+15·13			12·7	
1				5·49		+82·77	+82·77		
					+ 7·97			12·9	
0	7·6 —63·9			1·09		+87.17	+87·17		
					+ 4·40			26·0	
A	6·9 —85·2			7·04		+81.22	81·22		10:50
	1880·3 = Σe	1454·6 = Σe′			− 5·95		Sum =	205·9	

The mean of the diff. = 11·92^{mm}.
The mean temperature = 13°·7.

COMPUTATION OF THE LINE.

In determining the length of the line it is necessary to observe the following precaution: In placing each wire put the scale so that the reading increases toward the rear end of the wire. The lengths just given refer in each case to the middle mark of the scale, the scale covering 5^{cm}, except on *M*, where it begins with 5 and stops at 15. For this reason, since these 5^{cm} enter into each reading, it is necessary to subtract 50^{mm} from the reading, or, what is better, from the length of the wire, before multiplying it by the number of wires. Before taking from the table the temperature, by using the mean of the differences of readings as an argument, it is well to make the correction for the erroneous readings of the spring balance from the table on page 140. The rest of the computation will explain itself.

The leveling rod is too long by $\frac{1}{555}$ of its length; therefore each reading must be corrected by $+\frac{1}{555}$.

1882, MAY 10, LINE I II.

Approximate temperature $= +14^\circ$. The correction for the old balance at 10^k and $+14^\circ$ is (p. 140) $-0{\cdot}07^k$. The resulting error in the length of the wire $A = -0{\cdot}07^{mm}$, in $B = -0{\cdot}13^{mm}$. These values multiplied by 14, the number of wire lengths, give the correction to Σe and $\Sigma e'$, or

$\Sigma e =$	$842{\cdot}0^{mm}$	$\Sigma e' =$	$674{\cdot}9^{mm}$
error $=$	$-1{\cdot}0$	error $=$	$-1{\cdot}8$
corrected $\Sigma e =$	$841{\cdot}0$	corrected $\Sigma e' =$	$673{\cdot}1$

The difference between these values, $167{\cdot}9^{mm}$, divided by 14 gives the argument for finding the temperature.

The corrected mean of differences $= \frac{167{\cdot}9}{14} = 11{\cdot}99^{mm}$; therefore (p. 154) temp. $= 14^\circ.1$.

Wire A.	*m.*	*Wire B.*	*m.*
$14 \times 24{\cdot}95980 =$	$349{\cdot}4372$	$14 \times 24{\cdot}97195 =$	$349{\cdot}6073$
Σe (corrected) $=$	$\cdot8410$	$\Sigma e'$ (corrected) $=$	$\cdot6731$
	$350{\cdot}2782$		$350{\cdot}2804$
$(14^\circ{\cdot}1 - 15^\circ) \times 3{\cdot}5 \times 0{\cdot}997 =$	$-{\cdot}0031$	$(14^\circ{\cdot}1 - 15^\circ) \times 3{\cdot}5 \times 1{\cdot}718 =$	$-{\cdot}0054$
	$350{\cdot}2751$		$350{\cdot}2750$

		m.	
The tape A		$16{\cdot}9340$	$16{\cdot}9364$
Errors of division + errors of length	$=$	$\cdot0026$	
Temp. corr. $(-0^\circ{\cdot}9 \times 0{\cdot}17 \times 1{\cdot}046)$	$=$	$-{\cdot}0002$	$367{\cdot}2114$
		mm.	
Reduction to the horizon		$202{\cdot}8$	$\cdot2035$
Corr. for error in leveling rod $+ 202{\cdot}8 \times \frac{2}{555}$	$=$	$+{\cdot}07$	
		Line I II	$= 367{\cdot}0079$

The results are as follows:

Line A′ I.

		mm.	°	
1882, May 8.	A' I	$739{\cdot}7708$	$T = +13{\cdot}3$	Sunshine, scattering clouds.
11.	I A'	$\cdot7723$	$14{\cdot}1$	Sunshine, still.
11.	A' I	$\cdot7695$	$13{\cdot}7$	Wind $= 1$ to 2.
12.	A' I	$\cdot7725$	$8{\cdot}0$	Heavy rain, wind $= 2$.
Mean		$= 739{\cdot}7713 \pm 0{\cdot}5$		

Probable error of a measurement = $\pm 0{\cdot}9^{mm}$.

Line I II.

		mm.		°	
1882, May 9.	I II	367·0127	$T = +$	10·7	Sunshine, wind = 3 to 4.
9.	II I	·0120		8·1	Still.
10.	I II	·0079		14·1	Sunshine.
10.	II I	·0100		8·5	Sunshine.
Mean		= 367·0106 ± 0·7			

Probable error of a measurement = $\pm 1{\cdot}4^{mm}$.

Line II B′.

		mm.		°	
1882, May 9.	II B'	888·2372	$T =$	11·5	Thin clouds.
10.	II B'	·2371		14·6	Sunshine, wind = 1.
11.	II B'	·2287		14·1	Sunshine, wind = 2.
15.	II B'	·2380		10·3	Sunshine, wind vertical to wire.
Mean		= 888·2352 ± 1·5			

$$\text{Mean} = 888{\cdot}2352 \pm 1{\cdot}5^{mm}.$$

Probable error of a measurement = $\pm 2{\cdot}9^{mm}$.

$$\text{Line } A'\,B' = 1995{\cdot}0171^{m} \pm \sqrt{\overline{0{\cdot}5}^2 + \overline{0{\cdot}7}^2 + \overline{1{\cdot}5}^2}$$

$$= 1995{\cdot}0171^{m} \pm 1{\cdot}7^{mm}.$$

Probable error of a measurement = $\pm 3{\cdot}4^{mm}$, or 1: 600 000.

If the measurement of May 11 of II B' were excluded, for which unfortunately there is no good reason, we would have

$$A'\,B' = 1995{\cdot}0193 \pm 0{\cdot}9^{mm}.$$

Probable error of a measurement = $\pm 1{\cdot}7^{mm}$, or 1: 1 200 000.

The height of the three sections above sea level is

	m.
A' I	11·6
I II	7·8
II B'	7·8

which gives the following corrections for the reduction to sea level:

for A' I $- 1{\cdot}3^{mm}$, for I II $- 0{\cdot}5^{mm}$, for II B' $- 1{\cdot}1^{mm}$.

This makes the lines at sea level to have the following lengths:

	m.
A' I	739 ·7700
I II	367 ·0101
II B'	888 ·2341
A' B	1995 ·0142

The result of the computation from $A\ B$ was 1995·0158, which agrees with the measurement within $1·6^{mm}$, or 1 : 1 250 000.

By omitting the one result for II B already referred to, or taking for $A'\ B$ $1995·0164^{m}$, the difference would be only $0·6^{mm}$, or 1 : 3 300 000.

The probable error committed in making a scale reading can be obtained from the record by supposing that the temperature was constant during an entire measurement. Since the temperature was changing, the variation of differences is effected by these temperature changes, and the error obtained in this manner will be somewhat too large. The record gives

		mm.
	for I II	± 0·156
	II B'	± 0·159
	I A'	± 0·164
or the mean		± 0·164

The probable error for the difference of the two readings must therefore be less than $\pm\ 0·164^{mm}$, and each individual reading must be affected by an error which is smaller than

$$\pm\frac{0·164}{\sqrt{2}} \text{ or } \pm\ 0·116^{mm}$$

From equation (17) the probable error in a line 2000^{m} in length, when $n = 80$, is

$$\varphi < \pm\ 2·86 \text{ mm.}$$

or

$$\varphi < 1 : 700\ 000$$

The greatest accidental error is evidently that which arises in reading the scales. For in the alignment, although made with the unaided eye, the average error can not exceed 2^{cm}, an inaccuracy which, if uniformly committed, could cause an error for each wire length of only $0·008^{mm}$, or for 2000^{m}, $0·64^{mm}$; and this error would always have the same sign. The error in leveling from which result errors in determining the difference in the heights of the two ends of the wire could not well make this difference erroneous by more than 3^{mm}, which, if each inclination were as much as 1 : 100, would cause an error of only $0·03^{mm}$; and since the sign of this may be either plus or minus, for 80 lengths it would not exceed $0·03\sqrt{80} = 0·27^{mm}$. If the inclination were always 1 : 20 the errors would be 0·15 and $1·3^{mm}$, and this is less than the errors resulting from the error of measuring or from faulty scale reading.

The discussion just completed of the various sources of error which arise in the measurement of lines with wires may appear to detract from the confidence which this method appeared to beget.

The resulting probable error for the measurement finds its confirmation partly in the agreement of the measures one with another and partly in the coincidence of the final results for the whole line with the value obtained when measured with another form of apparatus.

With reference to the former of the above remarks, it is necessary to call attention to the unfavorable conditions under which the comparator was measured, for it was during this period that because of external conditions the line was very unstable. This occasioned a less accurate determination of the lengths of the wires, as well as a less reliable value for the coefficients of expansion, than under ordinary circumstances could be expected.

From the error in the determination of the lengths arises a constant error in the nature of a correction to the length as well as an error in the temperature coming from the scale readings—themselves dependent upon the lengths.

Likewise, an erroneous value for the coefficient of expansion occasions an error in the temperature reduction of the line and also a varying error in the determination of the temperature itself.

The error in the actual length of the wire has for the same line the same value, and the nearer the temperature remains constant during the measuring the nearer constant will be the error resulting from the uncertainties in the determination of the coefficient of expansion.

Since in the experiments here described the latter was the case, the total error arising from these sources may be approximately regarded as a constant correction factor, or for the line $A' B'$ it is of the same size for all the measurements.

The interagreement in the various measurements of the line $A' B'$ can not be sensibly affected by these errors.

On the other hand, in comparing the results of the measurements with wires with the results obtained with the apparatus of the Royal Academy of Sciences, the following is observed:

1. In the connection of $A' B'$ with $A B$ errors may exist which could affect our knowledge of the length of the former.

2. The point B was covered with dirt, and the stone in which it was marked might have been disarranged in the dumping.

3. The point A is in a somewhat small stone on the surface of the ground, and has for that reason an insecure position. It has been observed that this stone was raised from its position by a heavy wagon passing over it and that it afterwards fell back into its place.

4. The heads of the iron bolts in the top of the terminal stones are rusted, so that it is difficult to see distinctly the drill holes which mark A and B.

The close agreement between the measured and computed values of $A' B'$ must be regarded in a certain degree as a happy accident, but, on the other hand, it must be said that the coincidence would have been equally close if the obstacles mentioned had not been met.

Since the above communication was made to the Royal Academy of Sciences, in May of last year, the experiments have been continued and certain changes in the apparatus have been made. Therefore the description of the apparatus just given does not apply to the form now in use.

Among the experiences which have resulted from the recent experiments one at least is of importance; that is in regard to the question of a change in the lengths of wires during a long period of time.

The mean time in which the measurements of the comparator here referred to were made was 1883·0. The line was measured anew with the base apparatus of the Academy on September 3 and 4, 1884, at which time complete comparisons were made with the standard. The length of the line showed since November 12, 1882, a decrease of $0{\cdot}1^{mm}$. The wires *A*, *B*, *C*, *D*, *E*, and *F* were again compared on September 4 and 5 (temp. = + 16° Celsius) and once on November 12 (temp. = + 3°). The results are given below, except for *A*, which was injured in the interim. In the period from 1883·0 to 1884·8 all the wires were repeatedly unrolled and rolled.

Wire.	Change in length during 1·8 years.
	mm.
B (brass)	— 0·05
C "	+ 0·12
D (steel)	— 0·02
E "	+ 0·17
F (brass)	+ 0·01

On the average, without and with regard to the signs, the changes in the lengths were $0{\cdot}07^{mm}$ and $+ 0{\cdot}046^{mm}$. The comparisons of September, 1884, were made by persons who had had no experience in work of this kind. The deviations in length or the apparent changes in length of the different wires fall unquestionably within or very near the probable error of the last-made comparisons.

TRANSLATOR'S NOTE.—For later experience in the use of steel tapes see an exhaustive report by Assistant R. S. Woodward, Appendix 8, Coast and Geodetic Survey Report 1892, Part II.

APPENDIX No. 6—1893.

FUNDAMENTAL STANDARDS OF LENGTH AND MASS.*

While the Constitution of the United States authorizes Congress to "fix the standard of weights and measures," this power has never been definitely exercised, and but little legislation has been enacted upon the subject. Washington regarded the matter of sufficient importance to justify a special reference to it in his first annual message to Congress (January, 1790), and Jefferson, while Secretary of State, prepared a report, at the request of the House of Representatives, in which he proposed (July, 1790) "to reduce every branch to the decimal ratio already established for coins, and thus bring the calculation of the principal affairs of life within the arithmetic of every man who can multiply and divide." The consideration of the subject being again urged by Washington, a committee of Congress reported in favor of Jefferson's plan, but no legislation followed. In the meantime the executive branch of the Government found it necessary to procure standards for use in the collection of revenue and other operations in which weights and measures were required, and the Troughton 82-inch brass scale was obtained for the Coast and Geodetic Survey in 1814, a platinum kilogramme and metre, by Gallatin, in 1821, and a troy pound from London in 1827, also by Gallatin. In 1828 the latter was, by act of Congress, made the standard of mass for the Mint of the United States, and, although totally unfit for such purpose, it has since remained the standard for coinage purposes.

In 1830 the Secretary of the Treasury was directed to cause a comparison to be made of the standards of weight and measure used at the principal custom-houses, as a result of which large discrepancies were disclosed in the weights and measures in use. The Treasury Department, being obliged to execute the constitutional provision that all duties, imposts, and excises shall be uniform throughout the United States, adopted the Troughton scale as the standard of length; the avoirdupois pound, to be derived from the troy pound of the Mint, as the unit of mass. At the same time the Department adopted the wine

* This paper was first published as Bulletin No. 26, and is republished here to give it a more permanent form. Appended to it will be found a third edition of the Tables for converting Customary and Metric Weights and Measures.

gallon of 231 cubic inches for liquid measure and the Winchester bushel of 2150·42 cubic inches for dry measure. In 1836 the Secretary of the Treasury was authorized to cause a complete set of all weights and measures adopted as standards by the Department for the use of custom-houses and for other purposes to be delivered to the governor of each State in the Union for the use of the States, respectively, the object being to encourage uniformity of weights and measures throughout the Union. At this time several States had adopted standards differing from those used in the Treasury Department, but after a time these were rejected, and finally nearly all the States formally adopted, by act of legislature, the standards which had been put in their hands by the National Government. Thus a good degree of uniformity was secured, although Congress had not adopted a standard of mass or of length, other than for coinage purposes, as already described.

The next and in many respects the most important legislation upon the subject was the act of July 28, 1866, making the use of the metric system lawful throughout the United States and defining the weights and measures in common use in terms of the units of this system. This was the first *general* legislation upon the subject, and the metric system was thus the first, and thus far the only, system made generally legal throughout the country.

In 1875 an international metric convention was agreed upon by seventeen Governments, including the United States, at which it was undertaken to establish and maintain at common expense a permanent international bureau of weights and measures, the first object of which should be the preparation of a new international standard metre and a new international standard kilogramme, copies of which should be made for distribution among the contributing Governments. Since the organization of the Bureau, the United States has regularly contributed to its support, and in 1889 the copies of the new international prototypes were ready for distribution. This was effected by lot, and the United States received metres Nos. 21 and 27 and kilogrammes Nos. 4 and 20. The metres and kilogrammes are made from the same material, which is an alloy of platinum with 10 per cent of iridium.

On January 2, 1890, the seals which had been placed on metre No. 27 and kilogramme No. 20 at the International Bureau of Weights and Measures, near Paris, were broken in the Cabinet room of the Executive Mansion by the President of the United States in the presence of the Secretary of State and the Secretary of the Treasury, together with a number of invited guests. They were thus adopted as the national prototype metre and kilogramme.

The Troughton scale, which in the early part of the century had been tentatively adopted as a standard of length, has long been recognized as quite unsuitable for such use, owing to its faulty construction and the inferiority of its graduation. For many years, in standardizing length measures, recourse to copies of the imperial yard of Great Britain had been necessary, and to the copies of the metre of the archives in the office of weights and measures. The standard of mass originally selected was likewise unfit for use for similar reasons, and had been practically ignored.

The recent receipt of the very accurate copies of the International Metric Standards, which are constructed in accord with the most advanced conceptions of modern metrology, enables comparisons to be made directly with those standards, as the equations of the national prototypes are accurately known. It has seemed, therefore, that greater stability in weights and measures, as well as much higher accuracy in their comparison, can be secured by accepting the international prototypes as the fundamental standards of length and mass. It was doubtless the intention of Congress that this should be done when the international metric convention was entered into in 1875; otherwise there would be nothing gained from the annual contributions to its support which the Government has constantly made. Such action will also have the great advantage of putting us in direct relation in our weights and measures with all civilized nations, most of which have adopted the metric system for exclusive use. The practical effect upon our customary weights and measures is, of course, nothing. The most careful study of the relation of the yard and the metre has failed thus far to show that the relation as defined by Congress in the act of 1866 is in error. The pound as there defined, in its relation to the kilogramme, differs from the imperial pound of Great Britain by not more than one part in one hundred thousand, an error, if it be so called, which utterly vanishes in comparison with the allowances in all ordinary transactions. Only the most refined scientific research will demand a closer approximation, and in scientific work the kilogramme itself is now universally used, both in this country and in England.*

In view of these facts, and the absence of any material normal standards of customary weights and measures, the Office of Weights and Measures, with the approval of the Secretary of the Treasury, will in the future regard the International Prototype Metre and Kilogramme as fundamental standards, and the customary units—the yard and the pound—will be derived therefrom in accordance with the Act of July 28,

* NOTE.—Reference to the act of 1866 results in the establishment of the following:

Equations.

$$1 \text{ yard} = \frac{3600}{3937} \text{ metre.}$$

$$1 \text{ pound avoirdupois} = \frac{1}{2{\cdot}2046} \text{ kg.}$$

A more precise value of the English pound avoirdupois is $\frac{1}{2{\cdot}20462}$ kg., differing from the above by about one part in one hundred thousand, but the equation established by law is sufficiently accurate for all ordinary conversions.

As already stated, in work of high precision the kilogramme is now all but universally used and no conversion is required.

1866. Indeed, this course has been practically forced upon this Office for several years, but it is considered desirable to make this formal announcement for the information of all interested in the science of metrology or in measurements of precision.

T. C. MENDENHALL,
Superintendent of Standard Weights and Measures.

Approved:
J. G. CARLISLE,
Secretary of the Treasury.
APRIL 5, 1893.

[United States Coast and Geodetic Survey.—Office of Standard Weights and Measures—T. C. Mendenhall, Superintendent.]

TABLES FOR CONVERTING CUSTOMARY AND METRIC WEIGHTS AND MEASURES.

OFFICE OF STANDARD WEIGHTS AND MEASURES,
Washington, D. C., March 21, 1894.

The yard in use in the United States is equal to $\frac{3600}{3937}$ of the metre.

The troy pound of the mint is the United States standard weight for coinage. It is of brass of unknown density, and therefore not suitable for a standard of mass. It was derived from the British standard troy pound of 1758 by direct comparison. The British avoirdupois pound was also derived from the latter and contains 7,000 grains troy. The grain troy is therefore the same as the grain avoirdupois, and the pound avoirdupois in use in the United States is equal to the British pound.

2·20462234 pounds avoirdupois = 1 kilogramme.

In Great Britain the legal metric equivalent of the imperial gallon is 4·54346 litres, and of the imperial bushel 36·3477 litres.

The length of a nautical mile, as given below, is that adopted by the United States Coast Survey many years ago, and defined as the length of a minute of arc of a great circle of a sphere whose surface is equal to the surface of the earth (the Clarke spheroid of 1866).

1 foot	=	0·304801 metre, 9·4840158 log.
1 fathom	=	1·829 metres.
1 Gunter's chain	=	20·1168 metres.
1 square statute mile	=	259·000 hectares.
1 nautical mile	=	1853·25 metres.
1 avoirdupois pound	=	453·5924277 grammes.
15432·35639 grains	=	1 kilogramme.

Tables for converting United States weights and measures.

[Customary to metric.]

LINEAR.

Inches.	Millimetres.	Feet.	Metres.	Yards.	Metres.	Miles.	Kilometres.
1	25·4001	1	0·304801	1	0·914402	1	1·60935
2	50·8001	2	0·609601	2	1·828804	2	3·21869
3	76·2002	3	0·914402	3	2·743205	3	4·82804
4	101·6002	4	1·219202	4	3·657607	4	6·43739
5	127·0003	5	1·524003	5	4·572009	5	8·04674
6	152·4003	6	1·828804	6	5·486411	6	9·65608
7	177·8004	7	2·133604	7	6·400813	7	11·26543
8	203·2004	8	2·438405	8	7·315215	8	12·87478
9	228·6005	9	2·743205	9	8·229616	9	14·48412

SQUARE.

Square inches.	Square centimetres.	Square feet.	Square decimetres.	Square yards.	Square metres.	Acres.	Hectares.
1	6·452	1	9·290	1	0·836	1	0·4047
2	12·903	2	18·581	2	1·672	2	0·8094
3	19·355	3	27·871	3	2·508	3	1·2141
4	25·807	4	37·161	4	3·344	4	1·6187
5	32·258	5	46·452	5	4·181	5	2·0234
6	38·710	6	55·742	6	5·017	6	2·4281
7	45·161	7	65·032	7	5·853	7	2·8328
8	51·613	8	74·323	8	6·689	8	3·2375
9	58·065	9	83·613	9	7·525	9	3·6422

CUBIC.

Cubic inches.	Cubic centimetres.	Cubic feet.	Cubic metres.	Cubic yards.	Cubic metres.	Bushels.	Hectolitres.
1	16·387	1	0·02832	1	0·765	1	0·35239
2	32·774	2	0·05663	2	1·529	2	0·70479
3	49·161	3	0·08495	3	2·294	3	1·05718
4	65·549	4	0·11327	4	3·058	4	1·40957
5	81·936	5	0·14158	5	3·823	5	1·76196
6	98·323	6	0·16990	6	4·587	6	2·11436
7	114.710	7	0·19822	7	5·352	7	2·46675
8	131·097	8	0·22654	8	6·116	8	2·81914
9	147·484	9	0·25485	9	6·881	9	3·17154

CAPACITY.

Fluid drams.	Millilitres or cubic centimetres.	Fluid ounces.	Millilitres.	Quarts.	Litres.	Gallons.	Litres.
1	3·70	1	29·57	1	0·94636	1	3·78543
2	7·39	2	59·15	2	1·89272	2	7·57087
3	11·09	3	88·72	3	2·83908	3	11·35630
4	14·79	4	118·29	4	3·78543	4	15·14174
5	18·48	5	147·87	5	4·73179	5	18·92717
6	22·18	6	177·44	6	5·67815	6	22·71261
7	25·88	7	207·02	7	6·62451	7	26·49804
8	29·57	8	236·59	8	7·57087	8	30·28348
9	33·27	9	266·16	9	8·51723	9	34·06891

Tables for converting United States weights and measures—Continued.

[Customary to metric.]

WEIGHT.

Grains.	Milli-grammes.	Avoir-dupois ounces.	Grammes.	Avoir-dupois pounds.	Kilo-grammes.	Troy ounces.	Grammes.
1	64·7989	1	28·3495	1	0·45359	1	31·10348
2	129·5978	2	56·6991	2	0·90719	2	62·20696
3	194·3968	3	85·0486	3	1·36078	3	93·31044
4	259·1957	4	113·3981	4	1·81437	4	124·41392
5	323·9946	5	141·7476	5	2·26796	5	155·51740
6	388·7935	6	170·0972	6	2·72156	6	186·62088
7	453·5924	7	198·4467	7	3·17515	7	217·72437
8	518·3914	8	226·7962	8	3·62874	8	248·82785
9	583·1903	9	255·1457	9	4·08233	9	279·93133

By the concurrent action of the principal Governments of the world, an International Bureau of Weights and Measures has been established near Paris. Under the direction of the International Committee, two ingots were cast of pure platinum-iridium in the proportion of 9 parts of the former to 1 of the latter metal. From one of these a certain number of kilogrammes were prepared; from the other a definite number of metre bars. These standards of weight and length were intercompared without preference, and certain ones were selected as international prototype standards. The others were distributed by lot, in September, 1889, to the different Governments, and are called national prototype standards. Those apportioned to the United States were received in 1890 and are in the keeping of this office.

The metric system was legalized in the United States in 1866.

The International Standard Metre is derived from the Metre des Archives, and its length is defined by the distance between two lines at 0° centigrade on a platinum-iridium bar deposited at the International Bureau of Weights and Measures.

The International Standard Kilogramme is a mass of platinum-iridium deposited at the same place, and its weight in vacuo is the same as that of the Kilogramme des Archives.

The litre is equal to a cubic decimetre, and it is measured by the quantity of distilled water which, at its maximum density, will counterpoise the standard kilogramme in a vacuum, the volume of such a quantity of water being, as nearly as has been ascertained, equal to a cubic decimetre.

Tables for converting United States weights and measures.

[Metric to customary.]

LINEAR.

Metres.	Inches.	Metres.	Feet.	Metres.	Yards.	Kilo-metres.	Miles.
1	39·3700	1	3·28083	1	1·093611	1	0·62137
2	78·7400	2	6·56167	2	2·187222	2	1·24274
3	118·1100	3	9·84250	3	3·280833	3	1·86411
4	157·4800	4	13·12333	4	4·374444	4	2·48548
5	196·8500	5	16·40417	5	5·468056	5	3·10685
6	236·2200	6	19·68500	6	6·561667	6	3·72822
7	275·5900	7	22·96583	7	7·655278	7	4·34959
8	314·9600	8	26·24667	8	8·748889	8	4·97096
9	354·3300	9	29·52750	9	9·842500	9	5·59233

SQUARE.

Square centi-metres.	Square inches.	Square metres.	Square feet.	Square metres.	Square yards.	Hec-tares.	Acres.
1	0·1550	1	10·764	1	1·196	1	2·471
2	0·3100	2	21·528	2	2·392	2	4·942
3	0·4650	3	32·292	3	3·588	3	7·413
4	0·6200	4	43·055	4	4·784	4	9·884
5	0·7750	5	53·819	5	5·980	5	12·355
6	0·9300	6	64·583	6	7·176	6	14·826
7	1·0850	7	75·347	7	8·372	7	17·297
8	1·2400	8	86·111	8	9·568	8	19·768
9	1·3950	9	96·875	9	10·764	9	22·239

CUBIC.

Cubic centi-metres.	Cubic inches.	Cubic deci-metres.	Cubic inches.	Cubic metres.	Cubic feet.	Cubic metres.	Cubic yards.
1	0·0610	1	61·023	1	35·314	1	1·308
2	0·1220	2	122·047	2	70·629	2	2·616
3	0·1831	3	183·070	3	105·943	3	3·924
4	0·2441	4	244·094	4	141·258	4	5·232
5	0·3051	5	305·117	5	176·572	5	6·540
6	0·3661	6	366·140	6	211·887	6	7·848
7	0·4272	7	427·164	7	247·201	7	9·156
8	0·4882	8	488·187	8	282·516	8	10·464
9	0·5492	9	549·210	9	317·830	9	11·771

Tables for converting United States weights and measures—Continued.

[Metric to customary.]

CAPACITY.

Millilitres or cubic centimetres.	Fluid drams.	Centilitres.	Fluid ounces.	Litres.	Quarts.	Decalitres.	Gallons.	Hectolitres.	Bushels.
1	0·27	1	0·338	1	1·0567	1	2·6417	1	2·8377
2	0·54	2	0·676	2	2·1134	2	5·2834	2	5·6755
3	0·81	3	1·014	3	3·1700	3	7·9251	3	8·5132
4	1·08	4	1·353	4	4·2267	4	10·5668	4	11·3510
5	1·35	5	1·691	5	5·2834	5	13·2085	5	14·1887
6	1·62	6	2·029	6	6·3401	6	15·8502	6	17·0265
7	1·89	7	2·367	7	7·3968	7	18·4919	7	19·8642
8	2·16	8	2·705	8	8·4535	8	21·1336	8	22·7019
9	2·43	9	3·043	9	9·5101	9	23·7753	9	25·5397

WEIGHT.

Milligrammes.	Grains.	Kilogrammes.	Grains.	Hectogrammes.	Ounces avoirdupois.	Kilogrammes.	Pounds avoirdupois.
1	0·01543	1	15432·36	1	3·5274	1	2·20462
2	0·03086	2	30864·71	2	7·0548	2	4·40924
3	0·04630	3	46297·07	3	10·5822	3	6·61387
4	0·06173	4	61729·43	4	14·1096	4	8·81849
5	0·07716	5	77161·78	5	17·6370	5	11·02311
6	0·09259	6	92594·14	6	21·1644	6	13·22773
7	0·10803	7	108026·49	7	24·6918	7	15·43236
8	0·12346	8	123458·85	8	28·2192	8	17·63698
9	0·13889	9	138891·21	9	31·7466	9	19·84160

Quintals.	Pounds avoirdupois.	Milliers or tonnes.	Pounds avoirdupois.	Kilogrammes.	Ounces troy.
1	220·46	1	2204·6	1	32·1507
2	440·92	2	4409·2	2	64·3015
3	661·39	3	6613·9	3	96·4522
4	881·85	4	8818·5	4	128·6030
5	1102·31	5	11023·1	5	160·7537
6	1322·77	6	13227·7	6	192·9044
7	1543·24	7	15432·4	7	225·0552
8	1763·70	8	17637·0	8	257·2059
9	1984·16	9	19841·6	9	289·3567

APPENDIX No. 7—1893.

UNITS OF ELECTRICAL MEASURE.

Within but little more than a decade practical applications of electricity have developed with a rapidity unparalleled in the history of modern industries. Many millions of dollars of capital are now invested in the manufacture of machinery and various devices for the production and consumption of electricity. As it has now become a commodity of trade, its measurement is a question of the highest importance both to the producer and consumer. Both the nomenclature of electrotechnics and the methods and instruments of measure are exceptionally precise and satisfactory, but there has been lacking, up to the present time, the very important and essential element of fixed and invariable units of measure authoritatively adopted. Such units have long been in use among scientific men, but the necessity for the establishment and legalization of practical units for commercial purposes became evident in the beginning of the recent enormous development of the applications of electricity.

To meet this universally recognized want, conferences and congresses of the leading electricians of the world have been held at occasional intervals, the first being the Paris Congress of 1881. These assemblages have been international in their character, for it was wisely determined in the beginning that the new units of measure should be international and, indeed, universal in their application. It was convenient to make them so, and it was important to thus facilitate international interchange of machinery, instruments, etc. The United States was represented by official delegates in the Congress of 1881, and also in subsequent Congresses in 1884.

The difficulty of the material representation of some of the units of measure was so great at the time of holding these Congresses that no satisfactory agreement as to all of them could be arrived at. Some recommendations were made, but they at no time received the unanimous support of those interested, and were admitted by all to be tentative in their character. During the past few years the advance of knowledge and experience among electricians was such as to indicate that the time was ripe for the general adoption of the principal units of electrical measure. An International Congress of Electricians was arranged for, to meet in Chicago during the World's Columbian Exposition of 1893. In this congress the business of defining and naming units of measure was left to what was known as the "Chamber of Delegates," a body composed of those only who had been officially commissioned by their respective Governments to act as members of said chamber. The United States, Great Britain, Germany, and France were each allowed five delegates in the chamber. Other nations were represented by three, two, and in some cases one. The principal nations of the world were represented by their leading electricians, and the chamber embraced many of the most distinguished living representatives of physical science.

The delegates representing the United States have reported to the Honorable the Secretary of State, under date of November 6, 1893, giving the names and definitions of the units of electrical measure as unanimously recommended by the chamber in a resolution as follows:

Resolved, That the several Governments represented by the delegates of this International Congress of Electricians be, and they are hereby, recommended to formally adopt as legal units of electrical measure the following:

As a unit of resistance, the *international ohm*, which is based upon the ohm equal to 10^9 units of resistance of the centimetre-gramme-second system of electro-magnetic units, and is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice 14·4521 grammes in mass, of a constant cross-sectional area and of the lenght of $106{\cdot}3^{cm}$.

As a unit of current, the *international ampère*, which is one-tenth of the unit of current of the C. G. S. system of electro-magnetic units, and which is represented sufficiently well for practical use by the unvarying current which, when passed through a solution of nitrate of silver in water, and in accordance with accompanying specifications,* deposits silver at the rate of 0·001118 of a gramme per second.

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* In the following specification, the term silver voltameter means the arrangement of apparatus by means of which an electric current is passed through a solution of nitrate of silver in water. The silver voltameter measures the total electrical quantity which has passed during the time of the experiment, and by noting this time the time average of the current, or if the current has been kept constant the current itself, can be deduced.

In employing the silver voltameter to measure currents of about one ampère, the following arrangements should be adopted:

The kathode on which the silver is to be deposited should take the form of a platinum bowl not less than 10^{cm} in diameter and from 4^{cm} to 5^{cm} in depth.

The anode should be a plate of pure silver some 30 square centimetres in area and 2^{mm} or 3^{mm} in thickness.

This is supported horizontally in the liquid near the top of the solution by a platinum wire passed through holes in the plate at opposite corners. To prevent the disintegrated silver which is formed on the anode from falling onto the kathode the anode should be wrapped round with pure filter paper, secured at the back with sealing wax.

The liquid should consist of a neutral solution of pure silver nitrate, containing about 15 parts by weight of the nitrate to 85 parts of water.

The resistance of the voltameter changes somewhat as the current passes. To prevent these changes having too great an effect on the current some resistance besides that of the voltameter should be inserted in the circuit. The total metallic resistance of the circuit should not be less than 10 ohms.

As a unit of electro-motive force, the *international volt*, which is the electro-motive force that, steadily applied to a conductor whose resistance is one international ohm, will produce a current of one international ampère, and which is represented sufficiently well for practical use by $\frac{1000}{1434}$ of the electro-motive force between the poles or electrodes of the voltaic cell known as Clark's cell, at a temperature of 15° C., and prepared in the manner described in the accompanying specification.*

As a unit of quantity, the *international coulomb*, which is the quantity of electricity transferred by a current of one international ampère in one second.

As a unit of capacity, the *international farad*, which is the capacity of a condenser charged to a potential of one international volt by one international coulomb of electricity.

As a unit of work, the *joule*, which is equal to 10^7 units of work in the C. G. S. system, and which is represented sufficiently well for practical use by the energy expended in one second by an international ampère in an international ohm.

As a unit of power, the *watt*, which is equal to 10^7 units of power in the C. G. S. system, and which is represented sufficiently well for practical use by the work done at the rate of one joule per second.

As the unit of induction, the *henry*, which is the induction in a circuit when the electro-motive force induced in this circuit is one international volt, while the inducing current varies at the rate of one ampère per second.

Besides the fact that the Congress in which this important and far-reaching action was taken was held in the United States, our country has been honored by the action of the Chamber of Delegates in placing in the list of the illustrious names which are to be perpetuated in the nomenclature of electricity that of our countryman, Joseph Henry, whose splendid contributions to science, made about sixty years ago, have only in recent years met with full recognition. For these and other reasons it is extremely desirable that our Government should be among the first, if not the first, to adopt the recommendations of the Chamber. To make the use of these units obligatory in all parts of the country will require an act of Congress, but in the absence of that it is within the power of the Secretary of the Treasury to approve their adoption for use in all Departments of the Government. This, indeed, is precisely the course long ago followed in reference to the ordinary weights and measures of commerce and trade. Congress has never enacted a law fixing the value of their units, but the Secretary of the Treasury was authorized to establish and construct standards for use in the various Departments of the Government. Uniformity has followed on account of the universal adoption of these standards by the several States.

The Government is itself a large consumer of electricity and electrical machinery, and for its own protection it is important that units of measure be adopted. With the approval, therefore, of the Honorable the Secretary of the Treasury, the formal adoption by the Office of Standard Weights and Measures of the names and values of units of

*A committee, consisting of Messrs. Helmholtz, Ayrton, and Carhart, was appointed to prepare specifications for the Clark cell. Their report has not yet been received.

electrical measure as given above, the same being in accord with the recommendations of the International Congress of Electricians of 1893, is hereby announced.

T. C. MENDENHALL,
Superintendent U. S. Coast and Geodetic Survey and of Standard Weights and Measures.

Approved:

J. G. CARLISLE,
Secretary of the Treasury.

APPENDIX No. 8—1893.

[IN TWO PARTS.]

PART I.—A HISTORICAL ACCOUNT OF THE BOUNDARY LINE BETWEEN THE STATES OF PENNSYLVANIA AND DELAWARE.

By W. C. HODGKINS, Assistant.

Submitted for publication December 1, 1893.

The history of the boundary line between the States of Delaware and Pennsylvania possesses a peculiar interest to the antiquarian, the historian, and the engineer; and the consideration of its origin carried to its ultimate sources leads us far back into the colonial history of our country, to a period antedating not only this particular boundary line but even the existence and very name of the great State of Pennsylvania itself, of which province, under the proprietary government, Delaware for nearly a century formed a part.

A brief outline of the more important historical events which have left their impress in one way or another upon the long-continued controversy over the boundaries of Delaware, a controversy which has extended over more than two centuries and of which the last sounds have hardly yet been heard, may not be uninteresting nor out of place in this connection as serving to explain the somewhat intricate and frequently obscure causes which have resulted in the conditions under which this circular boundary has at last been marked by permanent monuments.

The earliest settlement by Europeans within the limits of the present State of Delaware appears to have been made by a Hollander named De Vries and a party of his countrymen, who landed in 1631 upon a tract of land near Cape Henlopen which had been purchased from the Indians two years before by another Hollander named Godyn.

De Vries called his settlement Swaanendael, and the creek on which it was situated he called the Hoornkill, after the city of Hoorn, in Holland. Swaanendael was not far from the present town of Lewes, in Sussex County, and the Hoornkill was, no doubt, the present Lewes Creek.

The name Hoornkill, it may be remarked, seems to have become more widely known than that of Swaanendael, and subsequently gave a name to the county under various corrupted forms, as Hoarkill, Horekill, or Whorekills (afterwards called New Dale, and then Sussex).

After establishing his colony De Vries returned to Holland to secure more settlers. Upon his return the following year he was horrified to find that the Indians had attacked the settlers, about thirty in number, had put them all to death, and had destroyed the village.

This tragic event, according to the best information attainable, seems to have been due to the killing by a white man of an Indian who had torn down a piece of metal ornamented with the Dutch arms from a post on which it had been placed.

There are various versions of this story, none of which are perhaps quite reliable, as they depend on the accounts of the Indians handed down by the Dutch.

It appears, however, that De Vries thought the killing not without some good reason, for he made no attempt to punish the Indians, though he parleyed with them and induced one of them to visit his ship and to give an account of the massacre.

This unfortunate affair, however, discouraged him and his party, and after a brief trip up the river they sailed to Virginia and then to New Amsterdam (New York), preferring to relinquish their dreams of wealth from a new colony on the Delaware rather than face the probability of encountering such savage foes; and for many years the Dutch made no further attempt to settle on the Delaware shore.

It might, therefore, appear that the incident thus abruptly closed could have no bearing upon the course of future settlements nor enter as a factor into the question of a boundary line between colonies subsequently established by the English.

As will be seen later, however, it proved to be an event of much moment in the conflict between the opposing claims of the proprietary governments of Pennsylvania and Maryland.

In the year 1632 King Charles I of England granted by royal letters patent to Cæcilius Calvert, second Baron Baltimore, a great tract of country on the Atlantic coast between the parallels of 38° and 40° of north latitude. This grant, which the King named Terra Mariæ, or Maryland, in honor of the Queen, Henrietta Maria, embraced not only the present area of Maryland, but the whole of Delaware and a considerable portion of the present State of Pennsylvania.

The whole of this grant, however, was included within the tract already granted by King James I for the Colony of Virginia under three charters, of 1606, 1609, and 1612, respectively. In the first charter the boundaries of Virginia are thus described:

* * * Situate, lying, or being all along the sea coasts, between four and thirty degrees of northerly latitude from the equinoctial line, and five and forty degrees of the same latitude, and in the main land between the same four and thirty and five and forty degrees and the islands thereunto adjacent, or within one hundred miles of the coast thereof. * * *

But the territory thus granted to the London and Bristol companies was materially reduced by King James himself in 1620, when he granted a charter to the Plymouth Company for all the territory between the fortieth and forty-eighth parallels, under the name of New England. In this charter we read:

We therefore * * * do grant, ordain, and establish that all that Circuit, Continent, Precincts and Limitts in America lying and being in Breadth from Fourty Degrees of Northerly Latitude from the Equinoctial Line, to Fourty eight Degrees of the said Northerly Latitude and in length by all the Breadth aforesaid throughout the Maine Land from Sea to Sea—with all the Seas, Rivers, Islands, Creekes, Inletts, Ports and Havens within the Degrees, Precincts and Limitts of the said Latitude and Longitude shall be the Limitts, and Bounds, and Precincts of the second collony—and to the end that the said Territoryes may hereafter be more particularly and certainly known and distinguished, our Will and Pleasure is, that the same shall from henceforth be nominated, termed and called by the name of New England in America.

This brought the charter limits of Virginia down to the fortieth parallel. But there was much dissatisfaction with the management of the Virginia colony by the chartered company, and in 1624 a writ of quo warranto was issued against it and the charter was forfeited.

So Virginia became a Crown Colony, and its lands were subject to the royal authority.

Sir George Calvert, the first Lord Baltimore, had received a grant of land in Newfoundland called Avalon, but finding the climate unfavorable he visited Virginia. Finding that the part of Virginia north and east of the Potomac and Chesapeake was still unsettled, he returned to England and induced the King to grant him this territory in place of the undesirable Avalon.

Before the charter was issued the first Lord Baltimore died, but the charter was confirmed and issued to his son, as already noted.

In this charter the boundaries of Maryland are, in part, described as follows:

* * * All that part of the Peninsula or Chersonese, lying in parts of America, between the ocean on the east and the Bay of Chesapeake on the west; divided from the residue thereof by a right line drawn from the promontory or headland called Watkins Point, situate upon the bay aforesaid, near the river Wighco on the west unto the main ocean on the east, and between that boundary on the south unto that part of the Bay of Delaware on the north, which lieth under the fortieth degree of north latitude from the equinoctial, where New England is terminated; and all the tract of that land within the metes underwritten (that is to say), passing from the said bay, called Delaware Bay, in a right line, by the degree aforesaid, unto the true meridian of the first fountain of the River Pattowmack; * * *

In all of these successive grants we notice a gradual reduction of the areas originally so lavishly distributed by English royalty.

By the Plymouth charter of 1620 Virginia lost a vast territory, which included the present New England States, New York, New Jersey, and the greater part of Pennsylvania, not to mention Canada and the great west. Next, by the charter of 1632, Cæcilius Calvert obtained another considerable portion of the imperial domain once included within the confines of Virginia.

So, when we find, as we shall a little later, that the barons of Baltimore themselves suffered a half century afterwards from this same trimming process, applied for the benefit of newer colonists on the Delaware, it will seem that perhaps they had little reason to complain on grounds of equity, however seriously the subsequent grants to James, Duke of York, and to William Penn may have infringed upon the letter of the Maryland charter, though legally the King might divide Virginia, as being a royal colony at the time.

The Calverts do not seem to have appreciated, until it was too late, the possible importance of the Delaware coast of the peninsula, and made no attempt to plant settlements there. Had they done so, it is not at all likely that there would be any State of Delaware to-day. They preferred to plant their new colony on the shores of Chesapeake Bay.

But the English were not alone in their attempts at colonization on this coast. As we have already seen, the Dutch had made an abortive attempt at colonizing the Lower Delaware and were firmly seated on the Hudson or North River. They had also a trading post called Fort Nassau on the eastern bank of the Delaware, near the present site of Gloucester, N. J.

The Swedish nation, then in the height of its power under the great King Gustavus Adolphus, also felt the influence of the fever for colonial aggrandizement which in the first half of the seventeenth century seems to have swept over northern Europe. Considerable preparations were made in 1627 for a Swedish colony on the Delaware, but for some reason the project was not then carried into execution.

A few years later, however, in the reign of the infant queen, Christina, the plan was again taken up by the celebrated Chancellor Oxenstiern, and in the year 1638 a Swedish expedition sailed into Delaware Bay. The commander was Peter Minuit (or Menewe), a Hollander, who had previously served the Dutch West India Company in America. The Swedes at once purchased from the Indians the whole west bank of the Delaware from Cape Henlopen to Trenton Falls. A part of their territory was included within the former purchase by Godyn, so swiftly abrogated by the massacre at Swaanendael. The whole of it was also included in the territory claimed by the English by right of discovery and disposed of by the grants to the London and Plymouth colonies and to Lord Baltimore. The English, however, had tolerated the Dutch settlements within the charter limits of New England, and the Swedes claimed that King Charles I had renounced in their favor, in 1634, any claims that England might have to that country by right of discovery. Even if this be so, it can hardly be supposed that King Charles meant to include in any such renunciation the territory of Maryland granted only two years before to his faithful servant, Lord Baltimore. This claim on the part of the Swedes affords another proof that neither they nor the English then recognized any right of the Dutch to the western bank of Delaware River and Bay.

The newcomers selected a site about where Wilmington now stands and built themselves a fort, which they named Christina, after the little Queen of Sweden. They also named the little river before them the Christina Kihl, a name still retained in the slightly altered Christiana Creek.

The Dutch governor of New Amsterdam, who claimed the country between the Connecticut and the Delaware, immediately protested against the Swedish settlement as an interference with the rights of the Dutch West India Company. The Dutch were too weak, however, to offer forcible resistance, and for nearly twenty years the Swedish settlements grew and prospered. But as their numbers and trade increased the friction between the Swedes on the western and the Dutch on the eastern bank of the Delaware became constantly greater, each colony trying to monopolize the navigation of the river. In retaliation for the Swedish pretensions, the Dutch, in 1651, repurchased from some of the Indians part of the Swedish territory below Wilmington and built a post, which they called Fort Casimir, where Newcastle now stands. The Swedes, in their turn, crossed the Jersey shore, then part of New Netherlands, and built Fort Elfsborg, about 7 miles below Fort Casimir. The Swedes were driven out of this place, however, by the swarms of mosquitoes which made life almost unendurable.

In 1654 a new Swedish governor, John Claudius Rising, arrived in the Delaware with a considerable number of colonists. One of his first acts was to compel the surrender of the Dutch at Fort Casimir, which he renamed the Fort of the Holy Trinity, on account of its having been captured on Trinity Sunday. The Dutch made no immediate reprisals, but having made thorough preparations they appeared in the Delaware at the end of August, 1655, under the redoubtable Governor Peter Stuyvesant.

Their force of seven vessels and more than 600 armed men was more than a match for the Swedes, and landing between the Fort of the Holy Trinity and Fort Christina they blocked communication between the two posts and reduced them in succession without any bloodshed. Thus the Swedish claims, such as they were, passed by conquest to the Dutch, who also claimed that the old purchase of the Hoornkill tract gave them title to the whole west bank, though that purchase was for only 30 miles of coast at Cape Henlopen.

But this conquest of the Swedes and the subsequent increase of the Dutch power on the Delaware alarmed the Lord Proprietor of Maryland, who seems to have paid little heed to the Swedish settlers, probably deeming them easy to subject to his dominion in due course of time.

In 1659 Lord Baltimore sent instructions to Maryland calling for an inquiry into the proceedings of the Dutch. A deputation was accordingly sent from Maryland to New Amstel, as Newcastle was then called, to notify the Dutch that they were unlawfully seated within the province of his lordship. The Dutch officials paid little heed to the complaints of Lord Baltimore's envoys, unbacked as they were by military force, though the spokesman of the party, Col. Nathaniel Utie, is said to have delivered his message "in a pretty harsh and bitter manner;" and the embassy came to nothing. Still the Dutch seem to have been somewhat alarmed at Utie's proceedings, and sent messengers to Governor Stuyvesant to inform him of the demands of Lord Baltimore. Stuyvesant thereupon sent Augustine Hermen and Rosevelt Waldron to the authorities of Maryland to try to arrange the matter. These Dutch ambassadors, upon being shown the charter of 1632, immediately caught at the phrase "hactenus inculta" in the preamble thereof and at once claimed that the charter specified that the lands granted to Lord Baltimore were only such as were then uncultivated and inhabited only by certain tribes of savage Indians, and that the Dutch settlements on the Delaware antedated the charter.

Here for the first time attention was called to this weak point of Lord Baltimore's charter and the argument advanced which was later used with such persistence to tear the Delaware shore from Maryland. This matter will be further discussed in connection with the grants to William Penn.

The negotiations having come to nothing, Lord Baltimore complained to the Dutch West India Company, in Europe, of the invasion of his dominions by the servants of the company. But this wealthy and powerful society, feeling secure in the armed forces with which it occupied its settlements, took no notice of such appeals, and Baltimore, perhaps feeling that the logic of events at least was against him, seems to have made little effort, except occasional futile remonstrances, to clear his territory of the intruders. Matters thus remained for several years, neither side acknowledging the justice of the other's claims.

But in 1664, as if in commentary on the theory that England recognized a trifling Dutch settlement as a bar to an English grant, King Charles II granted to his brother, the Duke of York, all the territory between the Connecticut and Delaware rivers, although this region had been in the hands of the Dutch for more than fifty years and although the two nations were then at peace.

The Duke at once organized an expedition, which was entirely successful in its results, and New Netherlands became an English province under the name of New York.

The Duke of York's grant was only to the east bank of the Delaware, and in that same year, 1664, he granted to Lord John Berkley and Sir George Carteret the whole of New Jersey, so that his proper territory did not extend south of New York.

But by virtue of the conquest of the Dutch provinces generally the agents of the Duke appear to have exercised a sort of quasi jurisdiction over the Dutch settlements west of the Delaware.

It does not appear that Lord Baltimore protested against this state of affairs; and it has been alleged that he forfeited his rights by such omission.

It is more likely, however, that he was governed by motives of prudence and policy. He was no longer a favorite at court, and he may easily have surmised that he would fare ill, no matter how just his cause, in a controversy with a royal duke, the heir presumptive to the throne. At all events, he seems to have held amicable intercourse for several years with his new neighbors on the Delaware.

But in 1673 the Dutch reconquered the province, and during the brief period of their rule, which lasted less than a year and a half, the Maryland authorities seized the opportunity to assert their territorial rights, and for that purpose sent an armed force against the settlement at Hoornkill, which had been reestablished by the Dutch.

In spite of this more formidable expedition no good results seem to have come to Lord Baltimore.

In 1674 the New Netherlands were again surrendered to the English by the treaty of Westminster. The Duke of York, to perfect his title, obtained a new grant from the King for his former territories, and the western shore of the Delaware seems to have been considered his property by all but the Marylanders. And at this point we approach the origin of the boundary which is the subject of this sketch, a line of demarcation which was first formulated in the latter part of the seventeenth century, and which, after two hundred years of uncertainty and misconception, has at last been marked by enduring monuments in this last decade of the nineteenth century. On March 4, 1681, King Charles II granted to William Penn a tract of land to the westward of the Delaware and to the northward of Maryland. This grant was in partial payment of claims against the Crown which Penn had inherited from his father, Admiral Penn. The negotiations preliminary to the issue of this charter were very protracted, extending over several months, for the English Government had begun to realize the difficulties which might arise from the large and rather indefinite grants which had been so common. Besides, it was known that the new province for which Penn asked a charter was likely to interfere to some extent with the territorial rights of the Duke of York and of Lord Baltimore.

And yet, curiously enough, in view of all this care, the conflict over the boundaries of Penn's territory was more bitter and more protracted than any other similar trouble in the English colonies.

So it was ordered that the Duke and the Lord Proprietor should be consulted.

Lord Baltimore had no objections to a grant of land to Penn so long as his northern boundary of the fortieth parallel was respected, and the Duke of York expressed his willingness to yield his claims to the almost unsettled shore of the upper Delaware provided he should have reserved to himself a sufficient distance between his town of Newcastle and the boundary of the new province. He suggested that 20 miles would be a convenient and suitable distance.

Penn, however, was reluctant to be pushed so far up the Delaware, and upon his urgent representations to the Duke a distance of 12 miles was finally agreed upon. At that time it was thought that the fortieth parallel crossed the Delaware between Newcastle and Chester (then called Upland), and it was therefore decided that the southern boundary of Pennsylvania should follow a circular curve, at 12 miles distance from Newcastle, northward and westward from the river Delaware, until it came to the fortieth parallel, and that it should then follow that parallel westward to its limit of longitude. This description, which was soon found to be an impossible one, is thus expressed in the charter:

* * * and the said Lands to bee bounded on the * * * South by a Circle drawne at twelve miles distance from New Castle Northward and Westward unto the beginning of the fortieth degree of Northern Latitude, and thence by a straight line Westward to the Limit of Longitude above mentioned.

As a matter of fact, the most northern part of a circle of 12 miles radius from Newcastle court-house is almost exactly on the parallel of 39° 50′ north latitude, and it could, therefore, never intersect the parallel of 40°.

We here find the first mention of this singular boundary line, almost unique in its circular shape. It is probable that both Penn and the Duke of York thought that this circular boundary between their dominions would soon strike the fortieth parallel and hence would be of small extent, and it is hardly likely that either of them then thought that it would afterwards play so important a part in the location of the boundary between Maryland and Pennsylvania.

After granting the charter to Penn, the King commanded him and Lord Baltimore to arrange their boundary.

Accordingly Lord Baltimore met Penn's kinsman and deputy, Markham, at Upland, in September, 1681, when it was found by a precise observation that the fortieth parallel was several miles north of Upland, instead of being somewhat to the south of it, as formerly supposed. No doubt both parties were somewhat surprised, but Lord Baltimore at once claimed the land to his charter limit of forty degrees, wherever it might lie.

Markham, on the other hand, declined to proceed with the delimitation of the provinces and reported the disappointing state of affairs to Penn, who was still in England. This news made Penn, who had all along been dissatisfied with his province as being too difficult of access, still more anxious to secure control of the shore of the lower Delaware. He therefore importuned the Duke of York for the transfer to himself of the Duke's claims on that region. This land had never been granted to the Duke, and his possession was only a sort of "squatter sovereignty."

As a historical writer has recently expressed it, "Penn asked for that which he knew to be within the boundaries of Maryland, and beyond the power of the Duke to grant." Penn had a great influence with both the Duke of York and the King on account of the services of his father, Admiral Penn, under the Duke himself, in the naval wars with the Dutch. Lord Baltimore therefore had heavy odds against him.

Probably with the idea of strengthening his own claims by bolstering the Duke's shadowy title, Penn obtained from York a quitclaim deed, dated August 21, 1682, relinquishing to Penn any claim which the Duke might have to the province of Pennsylvania.

It is worthy of note that Penn had been contented with his title under the charter of March 4, 1681, until, on the eve of bargaining with the Duke for part of Lord Baltimore's territory, he suddenly perceived that his own title was defective through the Duke's claims. Three days later, August 24, 1682, the Duke gave Penn two deeds of feoffment for the Delaware shore. The first of these conveyed the town of Newcastle and a 12-mile circle around the same. The second conveyed all the lands south of that circle as far as Cape Henlopen.

Much doubt seems to have existed, however, as to Penn's legal rights under these deeds. The Duke had no title of record. His deeds to Penn were never confirmed by King Charles, who died soon afterwards, nor by King James himself during his short and troubled reign. Much difficulty was therefore experienced by Penn's agents in the collection of rents.

After arranging these matters Penn sailed from England to visit his province. He arrived at the Capes on October 24, 1682 (O. S.), and first landed at Newcastle, afterwards going to Upland, now Chester.

For nearly twenty years after the organization of the new government the lawmaking body was a joint assembly for the province of Pennsylvania and the "territories" of "three lower counties on the Delaware." But dissensions arose between the united provinces, and upon the revision by Penn in 1701 of the charter of government granted by him the territories insisted so strongly upon a separate assembly that Penn was obliged to yield to their wishes.

In this same year, 1701, and perhaps in consequence of this legislative division of the provinces, the circular boundary line between Chester and Newcastle counties was run out upon the ground under a warrant from Penn. The work was done by Isaac Tailer, of Chester County, and Thomas Pierson, of Newcastle County, under the direction of the county officials, in November, 1701. Their method of work is described in their field notes, which are in the possession of the Historical Society of Pennsylvania.

According to their record, they began work at "the end of the horse dike" at Newcastle and ran a traverse to the northward with compass and chain until they reached a point which, from their computations, they supposed to be exactly 12 miles north of their starting point. By some mistake, however, they came out a mile or more too far to the west and about 2,000 feet too far from Newcastle. The excess in distance may have been due to their chain being too long, though the size of the error (2 feet to each chain) seems unlikely. This supposition is further borne out by the fact that the curve actually run out by them had, as nearly as can be ascertained, a radius of about 13 miles instead of 12, as should have been the case. The excess of westing might be accounted for by supposing that they used the magnetic meridian as their standard instead of the true north, but the declination of 8° 30′ west, observed at Philadelphia in 1701, would have carried them a half mile or more still farther to the west. It is very likely that their compass needle was a poor one and that it was much affected by local attraction, which is very noticeable in the vicinity of the Brandywine. The extremity of the radius so determined fell upon land then occupied by a certain Israel Helm and now owned by one Goodley, in a peculiar bend formed by Brandywine Creek. Tailer and Pierson found there a white oak tree, which they marked with twelve notches. They next laid off a line at right angles to their supposed true radius and marked on it the distance corresponding to the chord of 1 degree of a circle of 12 miles radius. This distance they computed to be 67 perches, a value sufficiently precise for their purpose (more exactly, 67.018), but if, as seems likely, these measurements also overran, their chords were probably 68 or 69 perches in length.

One-half of this chord was laid off on the east side of their radius and the other half to the west. Then, starting from the eastern end of the first chord, they ran the curve to the Delaware River by successive chords of 67 perches, making a uniform deflection to the right of 1 degree by compass at the end of each chord. Forty-three chords brought them to the Delaware, where they found that their line struck the north side of a house close to the shore, then occupied by one Daniel Lamplugh.

The surveyors then retraced their steps to the farm of Israel Helm and in a precisely similar manner ran their curve to the westward from the first chord until they had completed 77 chords, which, together with the 43 chords east of their starting point, made up the total of 120 chords, or "two-thirds of a semicircle," called for by their instructions. They note that their line ended at a stream, a branch of Christiana Creek, and that they "well marked" a hickory tree. This point can no longer be identified, but it was most likely in the present State of Maryland, to the westward or northwestward of the "triangular stone" on the boundary between Delaware and Maryland. The stream referred to was probably one called Persimmon Creek on some recent maps. The course of the boundary line was indicated by notches cut in trees near which it ran.

It will be noticed that this boundary laid out by William Penn between two portions of his domain has no connection and little apparent relation to the boundary between the lands of Penn and those of Lord Baltimore, though subsequently complicated and confounded with the surveys of the latter line.

At ye Req of ye Inhabitants of ye County of Chester & County of New Castle that I would Grant them a Warrt for ye Running a Dividing Line Between the two sd Counties that the Inhabitants of ye Respec= tive Counties which are in Question may Know to what JurisDiction they Belong

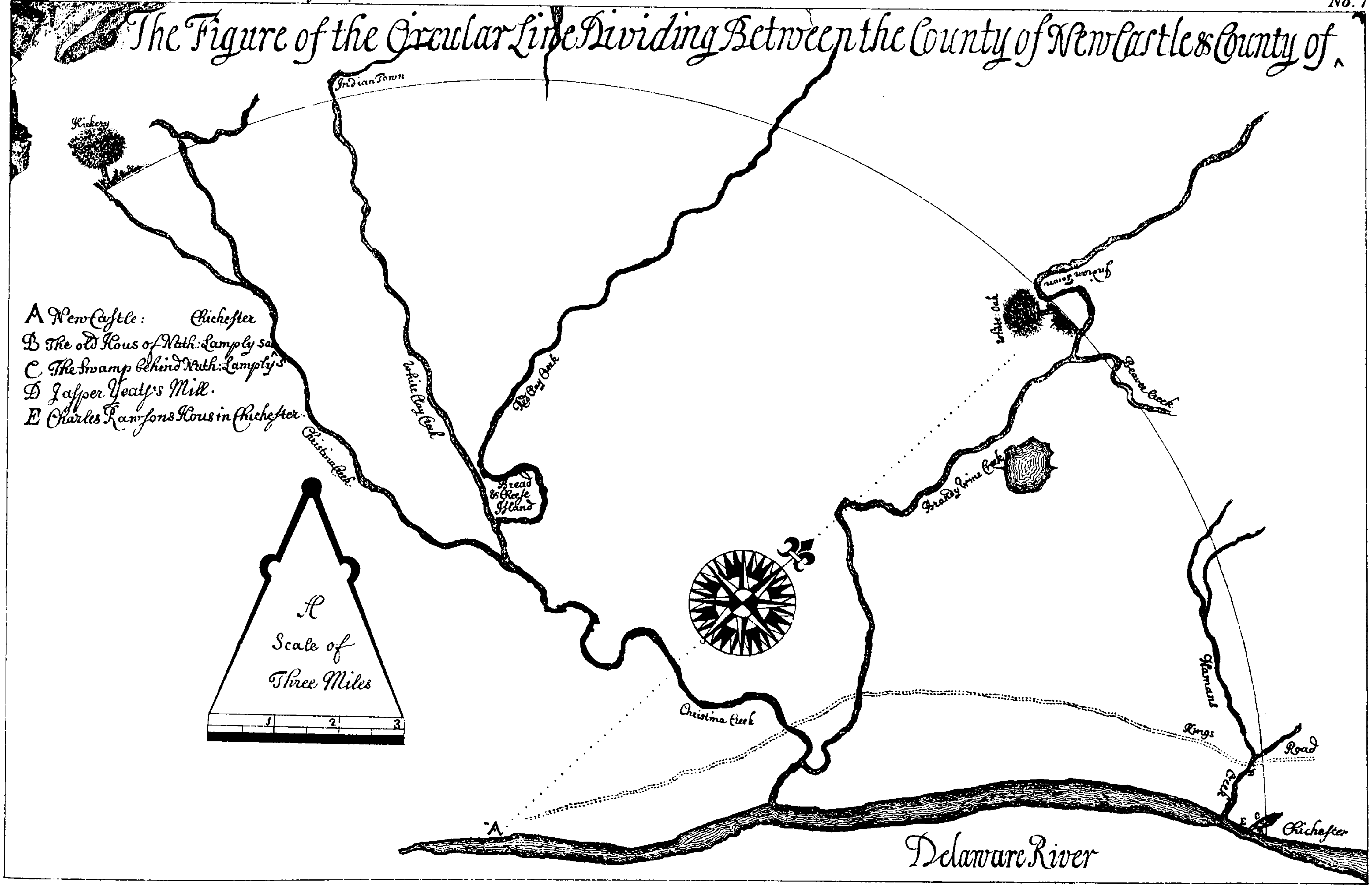
I Hereby Nominate Appoint & Authorize thee Isaack Tayler of ye County of Chester in ye Province of Pennsylvania and thou Thomas Pierson of ye County of New Castle in ye Territories to Accompany the Magistrates of Each Coun= ty or Any three of them within ye space of forty days after ye Date hereof to Admeasure & Survey from ye Town of New Castle the Distance of twelve Miles on a Right Line by ye River Dellaware Upwards & from the sd Distance to Divide Between the sd Counties By a Circular Line Extending according to ye Kings Letters Pattents & Deeds of Enfeoffment From the Duke for ye same & ye sd Circular Line to be well marked two third parts of ye Semicircle & make a true Return hereof into my Secretary's office to Remain uppon Record & for your so doing this shall be your Warrt Given under my Hand & Seale this 28 day of ye 8th month 1701

Wm Penn

Recorded in the Rolls office at Phila in ~~Patent~~ book C. 2. vol. 3 page 166. 167. ye 30th 8th mo. 1701

21 lines -- 1.9d

By me Tho. Story Mr


The Figure of the Circular Line Dividing Between the County of New Castle & County of ^
A New Castle: Chichester
B The old Hous of Nath: Lamply sa
C The Swamp behind Nath: Lamply's
D Jasper Yeat's Mill.
E Charles Ramsons Hous in Chichester
Hickery
Indian Town
White Clay Creek
Red Clay Creek
Bread & Cheese Island
Christina Creek
White Oak
Indian Town
Beaver Creek
Brandy Wine Creek
Hamans
Creek
Kings
Road
Chichester
A Scale of Three Miles
1
2
3
Delaware River

By virtue of a Warrt from William Penn Proprietary & Governour of the Province of Pensilvania & Counties Annexed Bearing Date the 28th day of ye 8th Month 1701 Authorizing us to Accompany the Magistrates of ye County of New Castle & County of Chester or any three of them within the space forty days after ye sd Date to Admeasure & Survey from the Town of New Castle the Distance of twelve Miles on a Right Line up ye sd River & from ye sd Distance to Divide Between the two sd Counties By a Circular Line Extending according to ye Kings Letters Pattents & Deeds of Enfeofment from the Duke & ye sd Circular Line to be well marked two third parts of ye Semicircle.

These are to Certifie that on ye twenty fifth day of ye Ninth Month 1701 Wee met at New Castle with Cornelius Empson Richard Halliwell & John Richardson Justices of ye County of New Castle & Caleb Pusey Philip Roman & Robert Pile Justices of the County of Chester who Did Unanimously Conclude that the Begining should be at the End of the Horse Dike next ye sd Town of New Castle and from thence to Measure Due North the sd Distance of twelve Miles & at the Extent thereof to Run the said Circular Line first Eastward Down to the River & then to Return to ye sd Extent of twelve Miles North & to Run the said Circle Westward untill it should Compleat the two third parts of the sd Semicircle And Accordingly the twenty sixth day of ye sd Month we did begin in ye Presence of ye sd Justices at the sd End of ye Horse Dike and Measured Due North twelve Miles to a White Oak Marked with twelve Notches standing on ye West side of Brandy Wine Creek in the Land of Israel Helm & from the sd White Oak Wee Ran Eastward Circularly changing our Course from the East Southward one Degree at the End of Every Sixty Seven Perches which is the Chord of one Degree to a twelve Miles Radius & at ye End of forty three Chords wee Came to Delaware River on ye upper Side of Nathaniel Lamply's Old Hous at Chichester & then wee Returned to the sd White Oak in Israel Helms Land & from thence we Ran Westward Changing our Course one Degree from the West Southward at ye End of Every Sixty Seven Perches as before untill we had Extended Seventy Seven Chords (which being Added to forty three Chords make two third parts of the Semicircle to a twelve Mills Radius) all which sd Circular Line being well marked with three Notches on Each side the Trees to a Marked Hickery Standing Neare ye Western Branch of Christina Creek Surveyed the 4th day of the 10th month 1701 By us.

Isaac Taylor
& Tho: Pierson

These may Certifie that Isaac Taylor & Thomas Pierson Did accompany us at ye Town of New Castle ye 25th day of ye 9th month 1701 together with Richard Halliwell being all Justices of ye Peace where we did unanimously agree & conclude that in order to Admeasure & Survey the twelve Miles Distance from New Castle Town for the Dividing the County of New Castle from the County of Chester according to ye Proprietarys Warrt the Begining should be at ye End of ye Horse Dike next the sd Town & then to Run Due North twelve Miles & from ye sd Extent thereof to Divide the sd Counties by a Circular Line as is above Certified & that at ye End of the sd Horse Dike ye sd Isaac Taylor & Thomas Pierson did begin to Measure the sd twelve miles in ye Presence of us all together with Richard Halliwell & from that time we were some time five but never less then four all ye Running ye sd North Line & also the two thirds of ye Semicircle till it was Compleated according to ye above Certificate & ye whole was finished ye 4th of the Instant to this Certificate we Do Subscribe our Names ye 13th of ye 10th mo 1701

Corneli Empson Caleb pusey
John Richardson Philip Roman
Robert Pyle

Although the line had been run out, little heed seems to have been given to it in the issue of patents for land. Over some small part of the boundary east of the Brandywine the patent lines were made to conform to the circular boundary, nominally at least, though it is noticeable that the old deeds pay no regard to the curvature of the line. The description of the bounds usually states that the line runs on a certain course a specified number of perches "along the circular boundary."

Except in this one district the land patents pass over the boundary without reference to it.

So for years and generations this line slumbered in obscurity, perpetuated for a time in local memory and tradition by reference to oak or hickory trees blazed or notched by the surveyors or by fences which some settler had built, as he supposed, upon the line. But year by year these witness marks decayed and passed from sight, until their very location became uncertain and until it has come to pass that at the present time the tolerably authentic relics of the old survey may be counted upon the fingers of one hand.

Meanwhile a far more troublesome question of boundary lines was arising from the conflicting claims of William Penn and Lord Baltimore to the fertile fields of the peninsula and the valley of the Susquehanna. Grants were given by both sides to lands in the disputed territory, and for many years the border was the scene of disputation and of conflict carried at times to the verge of open war. If Baltimore had the better title, Penn had the greater influence at court, and moreover held possession of a large tract claimed by Baltimore.

Several efforts were made by the proprietors to come to some agreement in this matter, but for one reason or another the negotiations repeatedly miscarried.

As early as 1685 Penn had succeeded in obtaining from the committee of trade and plantations, to which the matter had been referred by the King in council, an order that the peninsula should be divided between him and Calvert to the northward of a line running west from Cape Henlopen.

In presenting his case before the committee, we find Penn very shrewdly and skillfully availing himself of the Dutch claims through the early settlement by De Vries at the Hoornkill to support his own title obtained from the Duke of York and derived from the latter's conquest of the Dutch settlements.

In Lord Baltimore's charter of 1632 the descriptive phrase "hactenus inculta" (heretofore uncultivated) is applied to the territory so granted. This expression, found only in the preamble, was no doubt inserted merely to denote that the part of Virginia conveyed to Lord Baltimore had not been settled as part of that colony. It seems clear that it was not intended to impose a condition, for it was not repeated in the body of the charter, and it was not held to substantiate the seemingly valid claims of William Claiborne, who was actually settled on Kent Island at the time of the grant.

It is certainly hardly supposable that King Charles I intended to to recognize any prior claim of a feeble Dutch settlement located on territory claimed by England by right of discovery and conquered later by force of arms. And even if this unlikely supposition were true, the lands exempted under the words "hactenus inculta," could only be those of the actual settlement and could hardly be extended to cover the present State of Delaware.

But, justly or unjustly, Penn, who was high in favor, prevailed over Lord Baltimore, who found it prudent to yield for a time lest worse evils should befall him.

And thus we see the little village of Swaanendael, so soon swept away in fire and blood, rising from its ashes to sever the Delaware shore from Maryland.

But though Lord Baltimore submitted, he made no haste to carry out the order, and before anything had been done the revolution which drove King James from the throne also overturned the proprietary governments of Maryland and Pennsylvania.

The latter was soon restored to Penn, but Maryland remained a Crown province till 1716.

When Queen Anne succeeded to the throne, Penn managed to obtain an order in council for the enforcement of the decision of 1685. This was in 1708; but nothing was done toward carrying out this order, and in 1718 Penn died, leaving the dispute to his heirs.

The matter dragged along till 1732, when the heirs of Penn and the Lord Baltimore of that day joined in a deed to settle their boundaries. This called for a line running due west across the peninsula from Cape Henlopen, from the exact middle of which line a second line should be drawn to the northward in such a manner as to be tangent to a circle drawn 12 miles from Newcastle. From the tangent point a due north line was to be drawn, reaching to within 15 miles from Philadelphia, and from the terminus of this line the boundary to the westward should be a parallel of latitude. It was further stipulated that Newcastle County should in any event have the full area included within the 12-mile circle, even if part of it lay to the westward of the due-north line from the tangent point.

This proviso seems to have been added on account of the lack of information as to the direction which the tangent line was likely to take and for fear that the meridian line might seriously curtail the area of the circle. Had the parties to the deed known, as we know at present, that the segment of the circle west of the meridian line from the tangent point contains less than 14 acres, as the lines were marked on the ground, they might have concluded that so small an area was hardly worth considering and we might have been spared one complication in this historic interstate boundary.

But though matters thus seemed settled, this was really far from being the case. Difficulties and disputes arose as to carrying out upon the ground the provisions of the deed. The question as to the proper point in Newcastle which should be taken for the center of the 12-mile circle occasioned long debate. One rather quaint solution was the suggestion that this point should be the center of gravity of a paper plat of the town, the center of gravity having been found by experiment by balancing the plat on a pin.

Lord Baltimore's friends insisted on the absurd theory that the "12 miles" meant the periphery of the circle, while the Penns, of course, claimed that length of radius. A dispute also arose as to the true location of Cape Henlopen, as intended in the deed.

In consequence of all these difficulties nothing was done to carry out the deed.

The next move was made by Lord Baltimore, who, in spite of his own deed of 1732, applied to King George II for a grant to confirm his title according to the original charter of 1632. Naturally enough, this was refused by the King, and the matter was thrown into the court of chancery. In 1750 Lord Chancellor Hardwicke decided in favor of the Penns on every point of dispute, ruling that the center of the circle must be the middle of Newcastle as nearly as that point could be ascertained, that the "12 miles" meant the radius of the circle, and that the true Cape Henlopen was not the southern point of Delaware Bay, but the point claimed by the Penns, about 25 miles farther south and now called Fenwicks Island. This last decision seems rather a strange one, for though it appears that there was some confusion of usage among the Swedes and the English of the name "Henlopen," the present cape of that name seems to be clearly indicated in William Penn's deed from the Duke of York, which reads thus:

> * * * All that tract of land upon Delaware river and bay, beginning twelve miles south from the town of New Castle, otherwise called Delaware, and extending south to the Whore-kills, otherwise called Cape Henlopen. * * *

William Penn himself seems to recognize this limit in his "Act of Union" of December 7, 1682, where he describes the "territories" as—

> All that tract of land, from twelve miles northward of New Castle, on the river Delaware, down to the south-cape, commonly called Cape Henlopen, and by the Proprietary and Governor now called Cape James, lying on the west side of the said river and bay * * *

But even the decree of Lord Hardwicke did not end the interminable controversy. It would almost seem that Lord Baltimore and his friends, despairing of obtaining what they no doubt considered their just rights, had set about making all possible trouble for their successful opponents, even at the cost of time, money, and good repute to themselves.

In order to lessen as much as possible the amount of territory which must be yielded to the Penns, Lord Baltimore's partisans contended that the 12 miles should be measured upon the surface of the ground and not horizontally. Another appeal was made to the lord chancellor, and in March, 1751, he ordered that horizontal measurements should be employed.

After this decision by the lord chancellor it appears that the location of the boundaries was begun by commissioners and surveyors appointed for that purpose. The "base line," or east and west line, across the peninsula was laid out and measured. For this purpose a gap, or "visto," as the old records have it, was cut through the forest. The line was ranged out by poles set up in the "visto," and the distances were measured with a Gunter's chain 66 feet long, which was kept as nearly horizontal as possible. The whole length of this "base line" was found to be 69 miles and 298 perches, a value probably about a mile and a quarter greater than the real distance.

But at the distance of 66 miles and 24½ perches from the eastern end of the line the surveyors came to the shore of Slaughters Creek, a branch of the Hudson or Little Choptank River, separating Taylors Island from the peninsula.

Lord Baltimore's commissioners at once raised another question. Evidently the shorter they could make the base line the farther to the east would its middle point fall and the smaller would be the territory yielded by Maryland to Pennsylvania. So the Marylanders claimed that the line should stop at these first waters of the Chesapeake which were met in coming west from the ocean. The friends of the Penns, on the other hand, insisted that the line should continue to the open bay. It was so completed, as noted above, but the Marylanders would not accept it, and once more the dispute was referred to Lord Chancellor Hardwicke.

Before his decision had been rendered Charles Lord Baltimore died and the proceedings came to nothing.

When they were renewed, with the new proprietor, Frederick Lord Baltimore, made a party to them, he refused to be bound in any way by the acts of his predecessor. At last, however, on July 4, 1760, another deed was executed by the interested parties, and the long dispute was at an end so far as concerned the rival claims of Pennsylvania and Maryland.

The boundary called for by the deed of 1760 was substantially the same as that of 1732. The parallel of latitude forming the northern boundary of Maryland was to be 15 miles south of the most southern part of Philadelphia. The "base line" was to cross Taylors Island to the open bay, as claimed by the Penns. The court-house at Newcastle was accepted as the center of the circle.

Under the deed of 1760 commissioners were appointed by each side to supervise the work of demarcation. These commissioners held their first meeting at Newcastle on November 19, 1760. They employed a number of surveyors, who proceeded to complete the work begun in 1751. These surveyors appear to have accepted the work of that year on the "base line," and for the next three years, until the latter part of October, 1763, they were engaged in running trial lines for the tangent line, starting from the middle point of the base line, and for the radius, which should meet the tangent at right angles 12 miles from Newcastle court-house. After their three years of labor no solution had been reached, though it afterwards appeared that they had located the radius with considerable precision, considering their rude method of work. In October, 1763, the commissioners had just reached an agreement to report the state of the work to their respective principals and to ask further instructions, when they received information from the proprietors that two skillful mathematicians had been engaged by them to assist the commissioners in their labors. Further proceedings were therefore suspended until the arrival of these new surveyors, Charles Mason and Jeremiah Dixon.

On December 1, 1763, the commissioners met at Philadelphia and read the articles of agreement between the proprietors and the surveyors. They also made the necessary arrangements with the latter for the conduct of the work.

Messrs. Mason and Dixon, who thus appear upon the scene, were employed in the boundary surveys for the next four years.

Their first task was to determine the latitude of the most southern part of the city of Philadelphia. The mayor and other city officials were called upon for information in regard to the true southern limit of the city. They conducted the commissioners and Messrs. Mason and Dixon to the street called Cedar, or South street, and there pointed out a certain house occupied at the time by Thomas Plumsted and Joseph Huddle, situated on the south side of the street. The north wall of this house, marking the south side of the street, was stated by them to have been ever considered the most southern part of the city of Philadelphia. Though the position of this house is not stated, it was probably very near the river, as Cedar street runs a little south of east and the most southern part of its south side would be where it struck the shore.

Mason and Dixon built an observatory, and by observations with a zenith sector determined the latitude of this most southern point of Philadelphia to be 39° 56′ 29″·1.

From the latest survey of the water front of Philadelphia the latitude of the most southern part of the south side of Cedar or South street is about 39° 56′ 26″·6, a value differing from that of Mason and Dixon by only 2½ seconds of arc, and showing that their work was very carefully done.

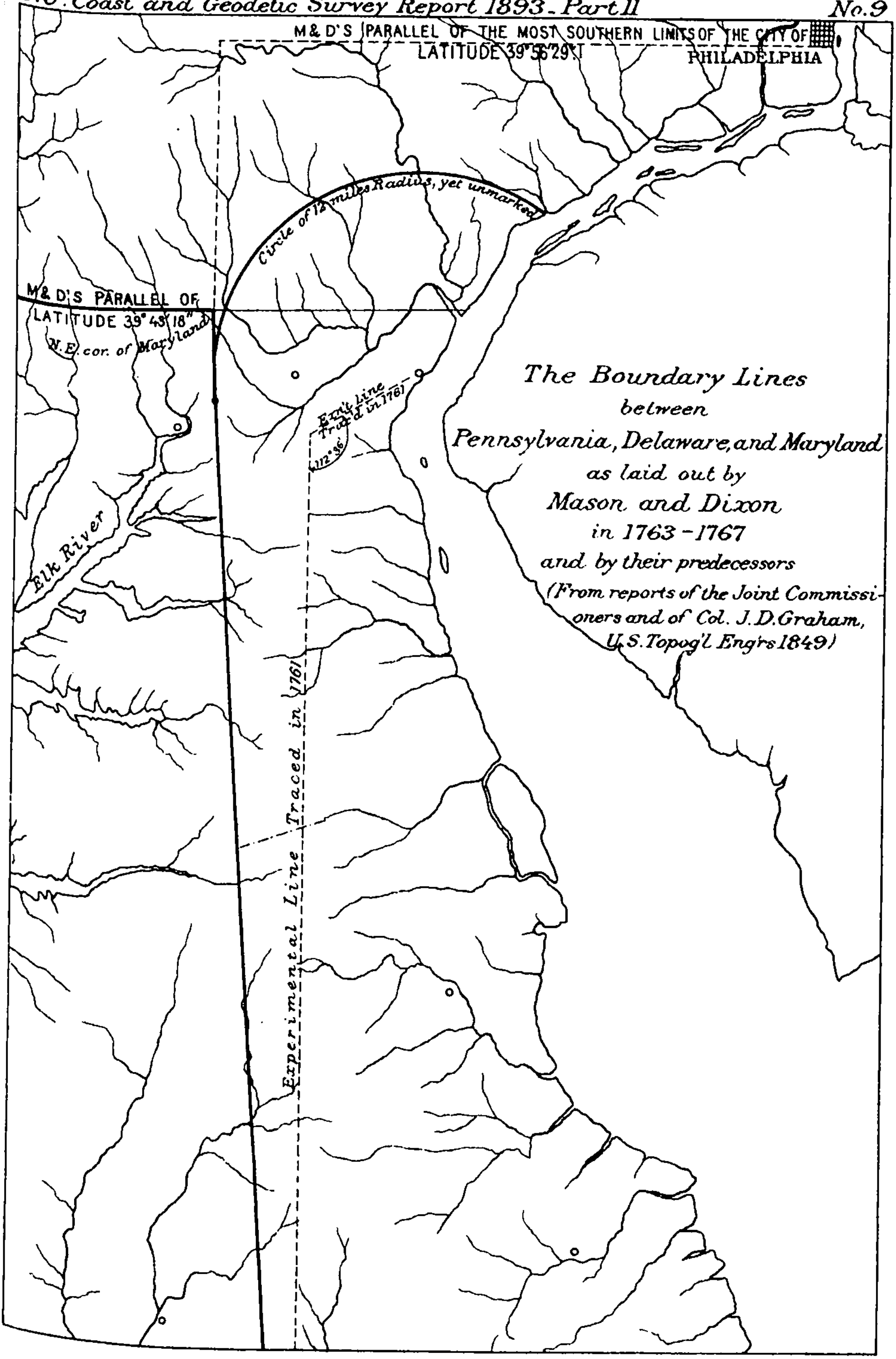
They next removed their instruments to a new station about 27 miles to the westward and near the forks of Brandywine Creek, where they again observed for latitude and located a point which they supposed to be in the same latitude as their first observatory. It is said that a white stone, locally known as "the stargazers' stone," still marks this second station. From this point they opened a line running due south through the forest and measured a distance of 15 miles in that direction. This measurement was designed to locate the parallel of latitude which by the deed of 1760 was to be laid out 15 miles south of Philadelphia to form the northern boundary of Maryland. Either through an error in the latitude of the second station as compared with that of the first, or from errors of chaining, or both, this line was carried too far south.

The difference of latitude between the end of South street and the northeast corner of Maryland is now about 13′ 6″, or about 5 chains more than 15 miles, so that the northern boundary of Maryland was put about 330 feet too far south.

From the southern end of their 15-mile meridian Mason and Dixon began laying off a parallel of latitude to the westward. After running several miles of this line, which was temporarily marked by wooden posts, the surveyors left it for a time and turned their attention to the southern portions of the boundary. Accepting as settled the "base line" which had already been measured across the peninsula by their predecessors, and the middle point marked by the same persons, Mason and Dixon endeavored to run out the tangent line from that middle point of the base to the tangent point. This extremity of the 12-mile radius laid out by the former surveyors was also accepted by Mason and Dixon, who found that it was nearly at right angles with the line which they laid out between its western end and the middle point of the base line. As will be shown later, this tangent point is really about 108 feet too far from the belfry of Newcastle court-house.

In laying out the tangent line Mason and Dixon were much assisted by the trial lines run by their predecessors. From these abortive lines they computed the direction which the line should follow, and then ran it out by transit until they reached the tangent point. They found that their line ran 16 feet 7 inches east of that stake. They then computed the proper offsets for each mile of their line to bring it to the true line, and moved their posts accordingly. This done, they reported to the commissioners that the posts so placed were, as nearly as practicable, in the true tangent line. Next, in accordance with the deed, a true north line was laid off from the tangent point to an intersection with the parallel of latitude 15 miles south of Philadelphia, which had already been partially surveyed, as related above. The point of intersection of the meridian and parallel became the northeastern corner of Maryland. The temporary mileposts already placed in this line, and referring to the south end of the 15-mile meridian as an origin, were replaced by new posts counting westward from this corner of Maryland.

The next thing to be done was the marking of that portion of the 12-mile circle which lay to the westward of the due-north line from the tangent point. And here Mason and Dixon fell into an error in computing the length of this small arc. As was pointed out by Col. J. D. Graham in 1850, they seem to have obtained their angle of deflection from the tangent to the due-north line, upon which their computation


M & D'S PARALLEL OF THE MOST SOUTHERN LIMITS OF THE CITY OF
LATITUDE 39° 56' 29".1
PHILADELPHIA
Circle of 12 miles Radius, yet unmarked
M & D'S PARALLEL OF
LATITUDE 39° 43' 18"
N.E. cor. of Maryland
Exp't Line Trac'd in 1761
112° 36'
Elk River
Experimental Line Traced in 1761
The Boundary Lines
between
Pennsylvania, Delaware, and Maryland
as laid out by
Mason and Dixon
in 1763-1767
and by their predecessors
(From reports of the Joint Commissioners and of Col. J. D. Graham, U.S. Topog'l Eng'rs 1849)

of the chord depended, by measuring the angle between their due-north line and the visible portion of the radius laid out by their predecessors. They had previously found that this radius was sensibly perpendicular to their own tangent line. Evidently, however, one of these angular measures was considerably in error. They computed their chord and offsets with a deflection angle of 3° 28′, while the actual angle between the tangent and the due-north line was found by Colonel Graham to be 3° 36′ 06″. As the arc cut off by the due-north line would be twice the deflection angle, this made an error of 16′ 12″ in the angular value of the intercepted arc and shortened the chord about 300 feet. Owing to the flatness of the curve the middle ordinate was not greatly in error, and the area of the segment was only about an acre too small.

Mason and Dixon would probably have obtained better results if they had directly measured the angle of deflection of the due-north line from their own tangent line. By an error in chaining they made a smaller mistake in the opposite direction, the actual length of their chord, according to Colonel Graham, being 84 feet greater than the value given by them. Their chain measurements, probably intrusted to careless employees, seem in general to have been irregular and inaccurate. Distances on the ground are almost always greater than their nominal length. The tangent line, supposed to be a little less than 82 miles long, is probably about 84½ miles in actual length. Curiously enough, all of the errors in measurement made by Mason and Dixon or their predecessors seem to have resulted in loss of territory by Maryland, except for the trifling error in the area of the circular segment. On the base line the distances seem to have underrun, contrary to the general rule; but if this error was cumulative it was probably distributed with some degree of uniformity and would not greatly affect the position of the middle point.

But the error in locating the northern boundary of Maryland, putting it about 5 chains too far south, meant the loss of a strip of that width along the whole length of the boundary, about 196 miles. This area would amount to nearly 8,000 acres.

The error in measuring the radius from Newcastle court-house, which placed the tangent point 108 feet too far from the center, took a strip of that width from the eastern border of Maryland to the northward of the tangent point, while south of that point, assuming that the southwest corner of Delaware was correctly placed, Maryland lost a wedge-shaped strip about 84½ miles long and 108 feet wide at the base.

After completing the arc above mentioned, Mason and Dixon took up the extension of their parallel of latitude to the westward, and their work does not further affect the matter under consideration.

As these various lines were located by Mason and Dixon they were marked by suitable stone monuments which were generally one mile apart. This work was done by other persons under the supervision of the commissioners. These stones were made in England from oölitic limestone, and were sent out from time to time as they were needed. They are stout square posts surmounted by a rather flat pyramid. Upon the side facing Maryland the letter M is cut, and on the opposite side the letter P. As Delaware was then a part of Pennsylvania the whole length of the line was marked in the same way. Every fifth milestone, however, was more elaborately marked, having the arms of the respective proprietors carved upon the opposite sides in place of the initials. It speaks well for the durability of the stone used that after being exposed to the elements for more than a century and a quarter these inscriptions are still distinctly legible and the stones are in so good condition as to have quite a modern appearance. While I was one day examining the eightieth milestone on the tangent line I was approached by a farmer who lived in a house not far way. He asked me the meaning of the stone, and on being told something of its history seemed much surprised and said that he had thought it a farm boundary mark placed there in recent years.

One of these stones carved with the arms of the proprietors has found its way by some strange chance into the town of Newark, where it supports one of the pillars of the porch in front of a very old house. It is probably one that was intended to mark the eighty-fifth mile of the tangent line; but this being only 82 miles long, as measured, it was not used, though it might well have been placed at the intersection of the arc with the due-north line instead of the rough, unmarked stone which stood there till 1849.

The monuments placed at the middle point of the base line, now the southwest corner of Delaware, and at the northeast corner of Maryland, differed from any of the others in having a coat of arms on each side thereof. Upon their north and east sides were carved the arms of the Penns and upon their south and west sides the arms of Lord Baltimore.

The stones placed at the tangent point and at some other points on the small part of the circle laid out by Mason and Dixon were of different and far less durable material—a dark granite rock of very poor quality. It appears from the minutes of the commissioners, under date of June 17, 1765, that it was intended to replace these stones with more durable monuments marked with the arms of the proprietors. For some reason this never was done, and most of the old stones still remain. The top of each is rounded to indicate that it is on the circle. The arms of the proprietors can barely be perceived upon the old stone at the tangent point, the only one which was so marked.

Not long after the completion of this celebrated work, the fruit of so many years of contention and of so great labor and expense, did the proprietors enjoy the quiet possession of their heritage. The storm of the Revolution, which even then was gathering, soon swept away the proprietary governments and severed the connection between Pennsylvania and the "three lower counties," this disaffected portion of the province becoming the State of Delaware, or, as its legal title first read, "the Delaware State."

By this separation a large portion of the boundary line over which the Penns and the Calverts had so long contended, the controversy over which had so embittered neighboring communities and which had at last been settled at so great labor and expense, became the dividing line between the States of Maryland and Delaware. Being well marked by durable monuments, no further dispute has arisen in that connection.

The boundary between Pennsylvania and the new State of Delaware, however, reverted to the old circular line between the counties of Chester and Newcastle. The survey of this line by Tailer and Pierson, 1701, has already been described. As previously noted, there was no connection whatever between Mason and Dixon's work of 1763–1767 and the old line of 1701. This line ran through a hilly and sparsely settled region, and the approximate location handed down by tradition among the country people seems to have been sufficient for their simple needs. As years passed by memories of the line of 1701 became dim, especially near its western end, and local tradition assumed that the stone at the northeast corner of Maryland marked also the western end of the circular boundary. As a corollary to this, it was held that all of the territory lying south of the parallel and east of the meridian passing through the northeast corner of Maryland, down to the tangent point, belonged to Delaware. In other words, Delaware and Maryland were thought to be coterminous up to the northeast corner of the latter State.

This popular view, which crystallized into local usage, though subsequently violently assailed and even officially renounced by the boundary commissioner of Delaware in 1850, has actually governed all interested parties even to this day. All the land titles are recorded in Delaware, the inhabitants vote and pay taxes in that State, and, as noted by Colonel Graham and others, a resident of this wedge-shaped strip between the circle and the boundary of Maryland was for some time a member of the Delaware legislature.

From a comparison of the old grants it seems a reasonable interpretation that the jurisdiction of Pennsylvania should not extend south of Mason and Dixon's parallel of latitude, and it is therefore a gratifying circumstance that one result of the last interstate commission on this subject will be to legally establish that condition.

Penn's charter of 1681 provided that the southern boundary of Pennsylvania should follow "the 12-mile circle" northward and westward until it intersected the fortieth parallel of north latitude. This was found to be impossible, but by subsequent proceedings, already described, the parallel of latitude 15 miles south of Philadelphia was substituted for the fortieth parallel. Now, the 12-mile circle had already been surveyed when Mason and Dixon marked their parallel of latitude, and it seems clear that the southern boundary of Pennsylvania resulting from these two surveys, to conform to the spirit of the charter, should have followed the circle of 1701 to its intersection with Mason and Dixon's parallel of latitude, and should thence have followed that parallel to the westward. Under Penn's deeds of 1682 from the Duke of York he claimed the 12-mile circle and the country south of it, but made no claim to any land between the circle and the southern boundary of Pennsylvania. All such land unquestionably belonged to Maryland. But in the deeds of 1732 and 1760 the Calverts so far yielded their rights in this respect as to agree to an eastern boundary running due north from the tangent point. This agreement transferred to the Penns the wedge-like area between the circle and the due-north line. This was outside of both of Penn's grants, a sort of donation from Lord Baltimore; but no good reason appears for considering this accretion a part of Pennsylvania proper rather than of the immediately contiguous territory of Newcastle County.

Even on the assumption that the territory so gained was part of Chester County, Newcastle County was entitled to claim the circle of 1701 as its boundary. That line, beyond a doubt, intersected Mason and Dixon's due-north line far to the north of the point commonly called "the junction of the three States."

The small circular arc laid out by Mason and Dixon to the northward of the tangent point was part of the boundary of Maryland under the deeds of 1732 and 1760.

The point of intersection of this arc and the due-north line, known since 1850 as "the junction of the three States," was then considered of no more importance than any other point of the Maryland line, and was perhaps the worst marked of them all. The idea of making the end of this fragmentary arc, laid out by Mason and Dixon in 1765 for one special purpose, the initial point of the circular boundary of Delaware, in utter disregard of the line actually surveyed in 1701 for that other special purpose, seems to have originated with Col. J. D. Graham, of the United States Topographical Engineers, who in 1849 and 1850 superintended a revision of a portion of Mason and Dixon's work.

This resurvey was due to the following circumstances: The monument which had been placed in 1768 at the northeastern corner of Maryland in the course of time disappeared from its place. Various stories are current as to the cause of its disappearance, but they are not important in this connection. The absence of this stone and the uncertainty as to the significance of others in the neighborhood combined with rumors of the unauthorized moving of some of the monuments, produced a general feeling of doubt in regard to the northeastern boundaries of Maryland. This condition led to the appointment in 1849 of a joint commission composed of one representative from each of the three States—Pennsylvania, Delaware, and Maryland.

It seems clear from the circumstances attending the formation of this commission that the missing stone was considered by all to have marked a point common to the three States. The resolution of the Delaware legislature authorizing the appointment of a commissioner from that State describes it as "the original boundary stone established at the point where the States of Pennsylvania, Maryland, and Delaware join each other." This resolution was adopted on the 10th of February, 1847.

The legislature of Pennsylvania did not act on the matter till 1849, when, on April 10, a bill was passed authorizing the appointment of a commissioner to act for Pennsylvania in surveying and determining the point of intersection of the three States and fixing a suitable monument at the point.

This is not quite so explicit in its designation of the particular point meant as the Delaware resolution, but it clearly indicates that a point then unmarked is to be located and suitably marked. The corner of Maryland is the only point that would answer that description, and if the view advanced by Colonel Graham had been held by the legislature of Pennsylvania it would hardly have admitted Delaware to a voice in the location of the boundary between Pennsylvania and Maryland. I can not find that the legislature of Maryland took any action toward appointing a commissioner, but one was certainly appointed by the governor of that State, and this action was subsequently ratified by the legislature, which authorized the payment of the necessary expenses.

This joint commission of 1849 obtained from the War Department the detail of Lieut. Col. J. D. Graham to conduct the necessary surveys. It is unnecessary to enter into much detail in regard to his operations, which are quite fully described in his interesting report subsequently published by each of the three States. He reestablished the corner of Maryland by producing to an intersection the north and east lines of that State as marked by monuments then in existence.

The corner thus determined was marked by a massive granite post, which is still in good condition.

He also placed new granite posts at the tangent point, at the middle point of the arc of the circle north of the tangent point, and at the point where the above arc, as laid out by Mason and Dixon, cuts the due-north line so often referred to.

This last stone was made in the shape of a triangular prism, inscribed with the initials of the three States on the appropriate sides. The names of the commissioners and the date 1849 were also cut on the north side, under the initial P.

The peculiar features of this stone were in accordance with the theory adopted by this commission of 1849, at the prompting of Colonel Graham, that this point was "the junction of the three States." It is especially surprising that the commissioner from Delaware, George Read Riddle, esq., should have assented to this encroachment upon the area and jurisdiction of his State. And the legislature had given him no authority for such surrender. The joint resolution of February

10, 1847, under which he was appointed, has already been mentioned. It is entitled a "Resolution relative to the northwest boundary stone of the State," and its text, which is given in full in the appendix to this report, clearly indicates that the legislature wished to restore the stone which had been removed from the northeast corner of Maryland, and not that other stone which was then standing at the point now occupied by the triangular prism, more than 3½ miles farther south. There was no further grant of power to the commissioner, not a hint of any authority to change a long-accepted boundary, nor to bind the State as to any details of the circular boundary between Delaware and Pennsylvania.

Yet, strangely enough, we find the Delaware commissioner accepting and signing a report and map which took from his State not only the wedge or "flatiron" south of Mason and Dixon's line, but also in the final consequences of his act, a long curved strip, or horn, about half a mile wide at its base and stretching northeastward along the circle for 11 miles, until it vanishes in a slender point at the Kennett-Pennsbury stump, near Centerville, Del.

This was due to the fact that Colonel Graham's map of 1850, signed by the three commissioners, pushed back the circular boundary from its actual intersection with Mason and Dixon's line to the theoretical 12-mile circle, regardless of the well-known rule that an actual line upon the ground is to be preferred to the written description of the same line in a deed.

It will be noticed that, although Delaware's claim to the "flatiron" seems to have been just, the common impression that the circular arc began at the corner of Maryland was erroneous.

Tailer and Pierson's line of 1701 crossed Mason and Dixon's line some 2000 feet east of that corner, and this point of intersection was the true beginning of the circular boundary.

In addition to the work above mentioned, Colonel Graham also made some trigonometric observations and calculations to obtain the distance between the tangent stone and the court house at Newcastle. He also computed the distances from the northeast corner of Maryland to the true 12-mile circle in two directions—first, on a right line, or radius, to the spire of the court-house, and, second, on Mason and Dixon's line produced. These distances, shown on the map furnished the commissioners by Colonel Graham, are, from one cause or another, considerably in error, even on the assumption that the boundary must follow the true 12-mile circle, an assumption already shown to be untenable.

As mentioned above, these apparently erroneous conclusions as to the true point of junction of the three States and as to the proper location of the circular boundary were embodied in a map which, on March 1, 1850, received the signatures of the three commissioners, H. G. S. Key, of Maryland, Joshua P. Eyre, of Pennsylvania, and George Read Riddle, of Delaware.

No subsequent acts of ratification seem to have been passed by the State legislatures, but the result was generally accepted on paper while ignored in fact. The maps showed Pennsylvania reaching a slender finger to the southward between Delaware and Maryland, but Delaware continued to exercise complete jurisdiction over that area.

In view of recent action, by which the above arrangement has been somewhat modified and the "flatiron" has been restored to Delaware, it appears that the most important effect of the map of 1850 was to commit the State of Delaware to the definite acceptance of the intersection of Mason and Dixon's line with the true 12-mile circle as the initial point of the circular boundary instead of the intersection with the circle of 1701.

This survey of 1850 called attention to the unmarked condition of this circular boundary, and while, fortunately for Delaware, the commissioners appointed at that time had no authority to undertake the work of marking it, they suggested that speedy action should be taken.

The legislature of Pennsylvania took action in the matter, and in several sections of a sort of omnibus bill, approved April 22, 1850, provided for the appointment of a commissioner, etc. The act is somewhat of a curiosity in its very matter-of-fact provisions for laying out the true 12-mile circle and for securing the titles and other vested interests of the numerous citizens of Delaware who were thus to be transferred to Pennsylvania. Aside from this prejudging of the case, its provisions seem careful and intelligent. Delaware does not seem to have cared to make so one-sided a bargain, and nothing more was done for about forty years. The increasing importance of the boundary, as the country grew in population and wealth, led to a renewed agitation of the question.

On April 25, 1889, the legislature of Delaware passed a bill appointing Hon. Thomas F. Bayard, Hon. B. L. Lewis, and Hon. John H. Hoffecker commissioners on the part of Delaware to act in conjunction with a similar commission from Pennsylvania to agree upon and mark the boundary.

On May 4, 1889, the legislature of Pennsylvania passed a similar act, authorizing the governor to appoint three commissioners to act for the Commonwealth of Pennsylvania, in conjunction with the Delaware commissioners, in "examining, surveying, and reestablishing" the boundary line between the two States. These commissioners were directed to join in marking by enduring monuments the line so reestablished.

The governor appointed Hon. Wayne MacVeagh, Hon. W. H. Miller, and Hon. R. E. Monaghan as the Pennsylvania commissioners under this act.

The proceedings of these two boards of commissioners have not yet been published, and not many details of their deliberations can be given. From the limited information at hand it appears that the commissioners from both States met in joint session at Philadelphia and selected Hon. Thomas F. Bayard as their chairman. Each commission separately employed an agent, styled, respectively, "surveyor on the part of Pennsylvania" and "surveyor on the part of Delaware." These "surveyors" acted as secretaries to their respective boards, collected information in regard to existing monuments supposed to be on the boundary line, looked up old title deeds bearing on the matter, etc. It appears also to have been expected that they would survey and mark the boundary.

Mr. Benjamin H. Smith, of Philadelphia, was the "surveyor for Pennsylvania," and Mr. Daniel Farra, of Wilmington, was the "surveyor for Delaware."

These gentlemen appear to have examined with considerable care the available documents, county records, etc., which might throw light upon the question.

Starting from the Delaware River and going westward, they seem to have been mutually satisfied that they had identified with reasonable certainty, as parts of the line of 1701, the following marks: First, the remains of the old house below Marcus Hook at which Tailer and Pierson ended the eastern section of their line; second, the boundaries of some farms west of the "Concord Turnpike" and east of Brandywine Creek, the original patents for which lands called for the circular line as their southern boundary; third, a peculiar bend of Brandywine Creek, west of the rocky promontory called Point Lookout, where the stream, flowing nearly south, touches the boundary of Delaware, but retreats again to Pennsylvania, curving back to the northeast and sweeping in a long bend around Point Lookout, when it once more trends to the southward and crosses the boundary at last near Smith's bridge; fourth, a large hickory stump, which marks the point at which the line between the townships of Kennett and Pennsbury, in Chester County, strikes the circular line of 1701. This last point is a particularly notable one. The old tree, which was standing a few years ago, was no doubt in existence in the time of William Penn. It is mentioned as a "small hickory" in a deed given in 1713 by George Harlan to his son, James Harlan, for 200 acres of land, a part of the "manor of Staneing," granted by patent by William Penn to his daughter Lætitia in 1701. The hickory was described as being "in ye eastern line of ye said manor."

To the westward of this hickory stump no marks could be found which the State surveyors were mutually willing to accept as correct. A considerable number of marks of more or less authenticity were pointed out or described at that time and subsequently by the inhabitants along the line. There was, however, no documentary evidence of their identity, and the agents of the commissioners declined to consider usage or tradition as reliable for their purposes.

One of these points, a stone at the corner of a farm in Mill Creek Hundred, was supported by title deeds dating back to about 1830, when the land of an intestate decedent was divided among his heirs. As the land lay partly in Pennsylvania and partly in Delaware, the courts of both Chester and Newcastle counties were interested in the case. A surveyor was employed, who ran out and marked the portion of the State line crossing the estate, and these marks are still in place. Each court then took jurisdiction on its own side of the line, and the estate was thus administered. There is no documentary connection between the work of this surveyor and the line of 1701, and this work of 1830 was most likely run between the nearest two traditional marks, possibly some of the old trees, which may still have been standing at that time. The marks so established had certainly been accepted as authentic by the people and the local authorities on both sides for sixty years, and might well have been considered of some value, especially as their position indicates a strong probability that they are at least very near the line of 1701. These circumstances were not, however, discovered until the joint commission had agreed to accept as the western end of the arc the intersection of Mason and Dixon's line of 1764 with the true 12-mile circle as indicated on the map of 1850. The "State surveyors," therefore, made no use of these points, which were brought to light in the course of the survey of 1892.

Although, as just mentioned, the commissioners followed the precedent furnished by the official plat of Colonel Graham's work in deciding that the circular boundary should meet Mason and Dixon's line at a point just 12 miles from the spire of Newcastle court-house, they agreed to correct so much of his work as threw into Pennsylvania the triangular area commonly called the "flatiron." The western boundary of Delaware would therefore coincide with the eastern boundary of Maryland, and the northern boundary of Delaware would run due east from the northeast corner of Maryland to a point just 12 miles from Newcastle court-house and thence would follow a curved line passing in as regular a manner as possible through the successive boundary marks accepted as authentic relics of the line of 1701.

But the commissioners and the surveyors had no reliable information as to the absolute or relative positions of these marks, and could therefore form no conclusions with regard to the curve to be passed through them. Nothing seems to have been done in the way of field work, and the matter remained in this condition until 1892.

Early in that year the joint commission, through the Hon. Thomas F. Bayard and Mr. Benjamin H. Smith, applied to the Superintendent of the United States Coast and Geodetic Survey for assistance in the matter and for the detail of an officer to execute the field work.

The consideration of this work forms the subject of the second part of this report, but to bring this historical sketch down to the present time the work of 1892 may be summarized as follows:

It was found that no single circle could be made to satisfy the conditions imposed by the commissioners. A compound curve was therefore laid out.. This is formed by two circular arcs which have a common tangent at the Kennett-Pennsbury stump, which point is very nearly midway on the whole curve. The radius of the western part of the curve is about 11·58 miles and that of the eastern part is about 12·81 miles.

Although the radius of curvature of the western part of the arc is less than 12 miles, no part of it lies within the 12-mile circle. The western end or initial point of the curve is just 12 miles from Newcastle court-house, but every other point of the line is outside of the theoretical circle. This difference increases rapidly as far as the Kennett-Pennsbury stump, where it is 1877 feet, and after that more gradually, amounting to 3137 feet at the eastern end or terminus of the line.

This line is now marked by 46 substantial monuments, as follows: An initial monument, made of dark Brandywine granite, at the western end or origin of the curve; 22 milestones marked with the initials of the names of the States and the date. 1892; 22 smaller stones marked simply ½, placed half way between the milestones, and a terminal monument near the Delaware River, at the eastern end of the line.

With the exception of the initial monument, all of these stones are of a light grayish white gneiss, from a quarry near Chester.

Thus ends for the present the history of this boundary, and although a few residents of the debatable ground near the initial monument felt aggrieved that the official location of the line placed them in Pennsylvania instead of Delaware, and although there are rumors of a contemplated appeal to the Supreme Court of the United States, it appears reasonable to suppose that the line thus marked will remain the boundary and that this will be the last chapter of the long story of border troubles outlined in the foregoing sketch.

PART II.—DETAILED ACCOUNT OF THE WORK ON THE PENNSYLVANIA-DELAWARE BOUNDARY LINE EXECUTED BY W. C. HODGKINS, ASSISTANT.

In compliance with the Superintendent's letter of instructions, dated March 8, 1892, I communicated with Messrs. Benjamin H. Smith, of Philadelphia, and Daniel Farra, of Wilmington, who had been employed by the two boards of commissioners to represent the interests of their respective States.

At the request of these gentlemen I went from Washington to Philadelphia on March 16, 1892, and met them at the office of Mr. Smith. At this conference the nature of the problem was outlined and a general plan of work was adopted.

Briefly summarized, in advance of more detailed discussion, the following were the principal features of the work to be done:

1. The accurate determination of the geographical positions of the

following points, viz: Newcastle court-house, the stone marking the northeastern corner of Maryland, and such points of the line of 1701 as could be satisfactorily identified.

2. The preparation of a drawing to show these points in their true relative positions and to indicate the various lines which might be made to satisfy more or less completely the conditions imposed by the commissioners.

3. The decision by the commissioners, after consideration of the above map, as to the character of the curve which should be adopted for the boundary.

4. The preliminary location upon the ground of the line so adopted.

5. The examination of this preliminary line by the commissioners with a view to any modifications which might become necessary.

6. The permanent marking of the line as approved by the commissioners of the two States.

It was suggested to Messrs. Smith and Farra that the most ready means of obtaining the required information would be by using the plane table, which would likewise afford a very fair degree of precision in the location of the line.

They preferred, however, to have the work done by trigonometric methods, but it was decided to make a plane table reconnaissance on a small scale (1:40000) in order to obtain an approximate idea of the positions of the guiding points with reference to each other. This would also serve for laying out the scheme of triangulation for the subsequent work.

With this understanding I returned to Washington, where I was engaged in completing my office work of the topographical survey of the District of Columbia. I at once began, however, to gather materials and to make preparations for taking the field early in April, and on the 14th and 15th of that month I sent two members of the party to Newark, Del., to begin the field work by putting up flags and searching for the station marks of the Coast Survey in that vicinity.

I expected to follow them in a few days, but before I had been able to leave Washington it appeared from my correspondence with the State surveyors that there existed a certain amount of misunderstanding or lack of definition as to the exact scope of the work which the Coast and Geodetic Survey was asked to undertake, and also as to the position which I, as the representative of the Survey, was to occupy in relation to the State surveyors.

It being deemed advisable by you that these matters should be definitely settled before I went to the field, my departure was delayed for a time. After further correspondence and a personal conference at the office of the Survey, on April 27, between the State surveyors and yourself, at which I was also present, they expressed a wish to submit the question anew to their respective commissioners for further action by them.

In pursuance of this arrangement, Messrs. Smith and Farra addressed to you a letter, dated May 5, 1892, in which they informed you that the matter had that day been submitted by them to the boundary commissions of the two States in joint session in Philadelphia, and that the joint commission had adopted a resolution, as follows:

Resolved, That Messrs. Smith and Farra be instructed to secure the survey of the line as soon as possible in accordance with the suggestions of the United States Coast and Geodetic Survey, so as to enable them to report the line approved by them to the Commission as soon as possible.

As in your opinion this resolution satisfactorily terminated the uncertainties above referred to, you verbally directed me to proceed to carry out your instructions of March 8.

I accordingly left Washington on May 11, 1892, and reached Newark, Del., on the same day. I first examined the work which had been done by the two men whom I had sent to the field in April, and who had been employed during my enforced delay in putting up flags and searching for triangulation points. They had succeeded in recovering the stations "Londonderry," "Meetinghouse Hill," and "Grandview," the last two so close together as to amount practically to one station. A large number of flags had also been put up in the vicinity of the boundary between the Maryland line and Brandywine Creek, and these were of considerable service in the reconnaissance.

I also made a personal examination of the ground in the vicinity of the northeast corner of Maryland, and, on May 18, I accompanied Messrs. Smith and Farra on a visit to the portion of the old line in the vicinity of Brandywine Creek, where some traces of the former work are still to be identified.

I was shown the old hickory stump at the southeast corner of Kennett township, the peculiar bend of the Brandywine through which Tailer and Pierson dragged their chain in 1701, and the other supposed marks referred to in the first part of this report. A tall hickory tree standing on the supposed old line at the corner common to the townships of Concord and Bethel, Delaware County, Pa., was selected by the surveyors as the reference point for that part of the line.

At a later period I also accompanied Messrs. Smith and Farra to the supposed remains of the old house on the Delaware which marked the eastern end of the line.

I now endeavored to recover some of the Coast Survey stations, which would better answer my purpose than those already found. The natural and artificial changes of the half century since that work was done had so completely changed the surroundings of the points that our search was unsuccessful for the time.

The line "Londonderry"–"Meetinghouse Hill," a side of one of the primary triangles, was adopted as the base for the triangulation, after some hesitation due to the length of the line (nearly 14 miles) and to the fact that intervening obstacles would compel me to elevate the instrument at each station. From this base line was developed a scheme of triangulation which reaches directly each of the points required to be determined except the one upon the bank of the Delaware. That also is reached indirectly, having been connected with the neighboring stations of the river triangulation, which is also connected with my work at Newcastle.

In reducing the size of the triangles from the 14-mile base line to the length required for following the circular boundary with a small scheme new points were first established at "Centerville," Del., in the village of that name, and at "White," Pa., near the village of Kemblesville. This latter station was very near the old point "Missimer," for which some ineffectual search was made in the hope of making the stations coincide.

From the line "White"-"Meetinghouse Hill" thus determined I was able to locate a station on "Grays Hill," near Elkton, Md. From this point and from "Meetinghouse Hill" observations could be made upon the spires of the Masonic Hall and Immanuel Church at Newcastle.

The court-house, being low and inconspicuous, could not be seen over the surrounding trees, but was determined by smaller triangles based upon neighboring points which had first been established from the main stations. I was then able to compute for the first time the position of this important point in terms of the standard data of the Coast and Geodetic Survey.

A great deal of trouble was experienced in making the reconnaissance for the triangulation on account of the rolling character of the country, the hills in any locality having nearly the same elevation. These hills are also generally covered with heavy timber, and as there are no commanding heights it was very difficult to secure intervisible points suitably located.

The atmospheric conditions were likewise unfavorable, the air being remarkably thick throughout the season.

All of these obstacles were overcome at last, but to accomplish it required much time and a great deal of hard work. In several cases stations had to be moved time after time to meet the conditions imposed by new ones selected further on, and this process had to be repeated until all the necessary lines of sight were arranged. In the scheme finally worked out there was scarcely any cutting and comparatively few scaffold signals.

The stone at the northeast corner of Maryland was a station peculiarly difficult to bring into the scheme of triangulation, situated as it is in the bottom of a wooded ravine. I was able, however, to obtain a satisfactory determination of this important point. A portion of the line between "Londonderry" and "Meetinghouse Hill" was heavily timbered, and it was necessary to remove some of the larger trees from the line of sight to avoid an extremely high signal at "Meetinghouse Hill." To assist in opening this line and to serve in the reconnaissance for the new stations afterwards located at "Centerville" and "White," it was found necessary to put up tall poles provided with cleats for climbing high enough to make preliminary observations. By means of these the line was opened, the new points were selected, and the necessary heights of the signals were determined. Poles from 60 to 85 feet high were raised by a tackle and horse power. They were steadied by rope or wire guys, carried flags at the top, and were cleated to a height of about 60 feet. A light whip rove through a single block at the top enabled the reconnoitering telescope to be hoisted to the eye of the observer. Such a pole was also erected near the old station "Bethel," in hopes of connecting that portion of the line with the triangulation, but the heavy timber upon the dividing ridge to the westward prevented it from being seen from "Centerville." It was therefore found necessary to omit any triangulation over the 6 miles of the line between the Concord and Philadelphia turnpikes.

By the end of June the reconnaissance was so far advanced that I felt able to proceed to build the main signals, where scaffolds of some height were necessary. To expedite matters by getting these signals up while I was completing the reconnaissance, I engaged Mr. Joseph Willis, a carpenter and builder of Newark, to build these for me as rapidly as possible; and I expected to begin the angular measurements as soon as they were done.

Mr. Willis took the work at a reasonable price and put up the signal at Grays Hill as planned. Just at this stage of the work Mr. Farra, who, with Mr. Smith, had been kept informed of the progress of events, paid me a visit and notified me that the commissioners were not agreed upon the desirability of the triangulation, and that they particularly objected to the expense of building signals. He therefore requested me to go to no further expense of the kind until the matter could be carefully considered by the joint commission.

After the particular desire which had been manifested in the beginning of the work to have everything done by triangulation, I was much surprised at Mr. Farra's communication. As the survey was being made for the commissioners, however, I felt obliged to yield to their request. As matters turned out, this action was particularly unfortunate in its results.

I therefore continued my reconnaissance until the scheme was completed and the points to be determined were all shown on the plane-table sheet. I then furnished tracings of the sheet to the commissioners from each State. These tracings showed the projected scheme of triangulation, the locations of the standard points desired, the true "12-mile circle," and the curves which most nearly fitted all the actually existing points.

Along with these I also furnished revised estimates for the expenses of the survey, the estimated total being $3100. It may be well to discuss this point a little further in this connection. Before I left Washington I had prepared estimates calling for a much smaller sum. These were based upon what I was told in the office of the probable extent of the work and were made before I had been on the ground. I had been in the field but a short time when I perceived that the ideas of the work that I had received from others were quite inadequate and that supplemental estimates would be necessary. At the request of Messrs. Smith and Farra I deferred them until the reconnaissance was finished, when they were promptly submitted. It is true that the final expense amounted to about 17 per cent more than the amount named, but I am confident that this moderate increase would have been unnecessary but for a remarkable combination of untoward circumstances. With regard to the total amount expended, Mr. Smith told me that his own estimate of the cost had been considerably above my highest estimate. I think the total very reasonable for the amount of work which was done.

By your authority I next visited Washington and explained to you the condition of the work, estimates, etc.

Upon my return to the field, as there appeared no prospect of a meeting of the commission, I took up the work at the eastern end of the line and connected the "Ruins" of the old house with the triangulation of Delaware River. This point could not be seen from the neighboring stations on account of several good-sized trees which stood close to the edge of the river. The owner of the land objected so strongly to the destruction of these trees that another station was interpolated at the windmill, a short distance to the northeast of the "Ruins," and the latter point was determined by an azimuth and measured distance from the mill.

After visiting Newcastle, where observations were prevented by the dense haze, which rendered the nearest signals invisible, I returned to Newark and began putting up signals and marking stations. Owing to the large number of stations, the hilly nature of the country, and the extreme heat of the weather, which was most exhausting both for men and horses, this work was somewhat retarded, but in the last three weeks of the month sixteen tripod signals and four 15-foot scaffolds were built. More time was lost in building the 46-foot tripod and scaffold at "Meetinghouse Hill," owing to the inexperience of my party in heavy carpenter work, and I found it necessary to again employ Mr. Willis to build the signals at "White" and "Centerville." Owing to other conflicting engagements, he was unable to finish them as promptly as desired, and some trouble was thus occasioned. I was obliged to give up building the signal at "Londonderry" on account of the unexpected opposition of the owner of the land. This person had offered no objections when consulted on the subject, but when the lumber dealers were ready to deliver the material he forbade them to do so, and when asked for an explanation demanded an extravagant bonus for the privilege of entry. In view of the delays already experienced, it was decided to do without that station and to "conclude" the triangles on the reconnoitering pole which was standing there, a correction being applied to each angle for eccentricity of the pole.

Though less satisfactory than if the station had been occupied, this method gave very good results.

The dense haze which prevailed throughout September was most unfavorable for observations of angles, and but little such work could be done.

The same trouble continued throughout October, and on the rare occasions when the air was somewhat clearer the wind was so violent as to prevent work. Though every possible opportunity was utilized to advance the work, it was often impossible to see stations only 1 or 2 miles away, and many of the lines were 6 or 8 miles long. Under these adverse conditions observations were made as rapidly as possible, work being continued till after sunset whenever any signals could be seen.

Early in November, having made a preliminary computation of the elements of the curve from the partially completed observations, I began to lay out the line upon the ground. This line, which Messrs. Smith and Farra had agreed to recommend to the commission, was composed of parts of two circles, the eastern part of such a radius that it would pass through the three guiding points—the "Ruins," the "Concord-Bethel" tree, and the "Kennett-Pennsbury" stump—and the western part of such a radius that, having a common tangent with the other at "Kennett-Pennsbury," it should pass through the point at which the eastern prolongation of the northern boundary of Maryland, considered as a parallel of latitude through the corner stone, would cut the true 12-mile circle around Newcastle court-house.

The weather now turned cold, with much rain and some snow, but the work was pushed as fast as possible. One great trouble in this work, as in the triangulation, was the obstruction caused by woods. In the surveys of Mason and Dixon and of Colonel Graham clear lines of sight were opened through the woods and no compensation was made; but in this survey the commission objected to cutting trees if it could possibly be avoided, and when it sometimes became necessary to clear a little, some of the landowners demanded exorbitant damages.

The work of running the line was thus continued, with observations of horizontal angles at intervals, as the weather permitted, until the 20th day of January, when the preliminary marking of the line was completed. On account of the almost arctic severity of the weather prevailing at that time and for several weeks before, work was then suspended until such time as the commission might select for their inspection of the line.

In the meantime the initial and terminal monuments had been erected early in December, their positions having been carefully determined. Each stone was securely planted in a pit filled in around it with a concrete base of broken stone and cement. The stones for marking the first 13 miles of the line were put on the ground ready for setting before the worst weather set in and prevented the delivery of the remainder for some time.

The winter weather during the last month of the work was of almost unprecedented severity for that latitude, the temperature for many days at no time rising above the freezing point and falling to 35° or 40° F. below freezing at night.

The deep snow, remaining a fine, dry powder in this intense cold, was continually blown through the air in blinding clouds by the high winds which prevailed, and many of the roads were completely blocked to travel by the drifts thus formed.

Field work under such conditions was attended by many hardships, but was continued until each of the monument stations had been marked.

After the suspension of field work above referred to, I returned to Washington and was occupied with office work while waiting for the joint commission to inspect the line and to listen to any objections which might be advanced against approving it.

It seemed to be the opinion of the State surveyors that the commission would perhaps make local modifications of the line in order to meet the views of the inhabitants.

Unfortunately, the weather during February continued too severe for this inspection to be made by the commission, and early in March I received new instructions directing me to prepare for duty on the survey of the boundary of southeastern Alaska.

After communicating with Messrs. Smith and Farra and finding that it would be impossible to secure the inspection by the commission before the date on which it would become necessary for me to start for the western coast, I returned to Delaware and went over the line with the State surveyors, pointing out the position of the stubs and putting in additional reference marks, etc.

Owing to the limited time at my disposal, I was not able to fully apply a system of refined checks to test the exact positions of the monuments, which I had intended to employ if the commission decided not to move the line laid out. All of these positions, however, are undoubtedly quite near their true values, especially where the work was controlled by triangulation for 16 miles from the initial point.

Between the 16-mile point and the 22-mile point there was no triangulation, and the preliminary line had been run out with a transit by half-mile chords measured by telemeter.

This method, while rapid and close enough for preliminary work, was somewhat lacking in precision, and I had intended to check it in case the commission wished to adhere to the circle by running out the long chord between the ends of this 6-mile arc and then laying off rectangular ordinates from the chord to the monument stations.

Finding now that I should not have time to do this myself, I submitted the question to Messrs. Smith and Farra for their decision as to whether this work should be left to them after the inspection by the commission or whether some person should be employed to do it at once under my direction.

They preferred the latter course, and I engaged for the purpose Mr. W. B. Carswell, of Wilmington, the only surveyor whom I could obtain at once. He had not finished his work when I had to start for Alaska, and I was therefore compelled to turn him over to the supervision of Messrs. Smith and Farra. I was later informed by Mr. Smith that after becoming involved in some serious errors Mr. Carswell had corrected his work and it had been accepted.

I left the boundary on the 1st of April, and immediately afterwards the remainder of the monuments were delivered on the ground under the supervision of a member of my party.

A few days later the joint commission went over the line, and after hearing the protests of some of the dissatisfied inhabitants of the border strip decided to accept the boundary as staked out. The monuments at the mile and half-mile points were planted immediately afterwards.

The work thus completed was one that gave me much labor and anxiety. It would have perhaps been more satisfactorily arranged in two seasons, the first for the reconnaissance and the triangulation, and then, after the complete reduction of the observations and the computation of the results, a second shorter season for laying out and marking the line.

As it was, I was compelled, while engrossed and exhausted with the field work, to reduce my work and go through the voluminous computations necessary to compute the curves and to provide for their location. This work was, for the most part, done at night, so as not to interfere with the field work.

These computations moreover had to be revised from time to time as additional observations were obtained, since in order to meet the calls of the commission for immediate results preliminary computations had to be made from reconnoitering angles.

Throughout the season neither I nor the members of my party spared our best endeavors to advance the progress of the work, and while it took much longer than was originally expected, the increased time was due in part to the fact that the large amount of real work to be done was not understood or appreciated in the beginning and in part to a series of unfortunate circumstances quite beyond my control.

Having thus reviewed the principal features of my survey, I will proceed to explain with some detail the methods of observation, of computation, and of location of monuments.

As already stated, the reconnaissance was made with the plane table, supplemented by other instruments. The angular measures were made with Repeating Theodolite No. 153, an excellent instrument, having an

8-inch horizontal circle divided to five minutes of arc and provided with three equidistant verniers which read to five seconds of arc. The telescope, which is lifted from the Y's in reversal, has an object glass of 2 inches diameter, a focal length of 16 inches, and a magnifying power of about 22 diameters. Another eyepiece of higher power was provided, but was rarely used. All of the principal angles were measured in sets of six repetitions, direct and reverse, the telescope being reversed in the middle of each set. The explement (or the difference between the angle and 360°) was always measured immediately after the angle itself, and the two results were combined to one by applying the correction necessary to make the sum of the two equal 360°. In general all the possible angles at each station were measured and the results were combined to give an approximate station adjustment. The excellence of the instrument is shown by the small corrections required in this adjustment as well as by the small errors in closing the triangles.

The signals were carefully centered, and gas-pipe poles of 2 inches diameter were used on the shorter lines, as combining economy and clearness of definition. After measuring the angles the sides of the triangles were successively computed, starting from the line "Londonderry" to "Meetinghouse Hill," used as a base. The length of this line is 22194·9^m, equal to 72817·7 feet, or about 13·8 miles. It is the longest line observed in the work, except that between "Grays Hill" and "Centerville," which is over 3 miles longer, being 27753·6^m, or 91054·6 feet.

The shortest line of the regular scheme, "Smith to Northeast Corner of Maryland," is only 168·1^m, or 551·5 feet. The whole number of triangles in the scheme is 104, of which 22 belong to the river triangulation of Assistant R. Meade Bache, executed about twelve years before.

In 36 of the principal triangles of my work the largest error of closure is 5″·4, the smallest error zero, and the mean error was 1″·66.

The triangle sides are computed by the usual formulæ:

$$b = a \sin B \operatorname{cosec} A; \quad c = a \sin C \operatorname{cosec} A;$$

after applying the corrections for spherical excess and for error of closure.

The lengths of the triangle sides having been computed, the latitude and longitude of each station were obtained by the geodetic formulæ

$$-d\,L = K \cos Z.B + K^2 \sin^2 Z.C + (\delta\,L)^2\,D - h\,K^2 \sin^2 Z.E$$

or

$$-d\,L = K \cos Z.B + K^2 \sin^2 Z.C + h^2 D, \text{ for short lines;}$$

$$d\,M = \frac{K \sin Z.A'}{\cos L'}$$

and

$$-d\,Z = d\,M \frac{\sin \lambda}{\cos \frac{1}{2}\,d\,L}$$

or

$$-d\,Z = d\,M \sin \lambda, \text{ for short lines;}$$

the derivations of which formulæ, together with the factors $A, B, C, D, E,$

are given in the Appendix No. 7 of the Report of the Superintendent of the Coast and Geodetic Survey for the year 1884.

These position computations depend on the standard data for "Meetinghouse Hill" and "Londonderry" furnished by the computing division of the Coast and Geodetic Survey. Each station was twice computed by independent determinations from different points already known in order to obtain a comparison and avoid errors.

The number of stations so computed was 54. Next, from these geographical positions I computed by the "inverse solution" of the geodetic formulæ the position of the point which, having the same latitude as the northeast corner of Maryland, should be exactly 12 miles from the spire of Newcastle court-house. In a precisely similar way I found the distance and azimuth of this initial point of the curve from the neighboring triangulation point "Whiteman," established for this special purpose, and with these values known, the initial point was marked on the ground.

By the same inverse solution were obtained the distances and directions of the lines joining the standard points of the curve, as follows: "Initial point to Kennett–Pennsbury," "Kennett–Pennsbury to Concord–Bethel," "Concord–Bethel to Ruins," "Ruins to Kennett–Pennsbury." The triangle formed by the last three lines may be considered a plane triangle, its spherical excess being less than one-tenth of a second of arc.

The radius of the circumscribing circle may then be computed by the formula

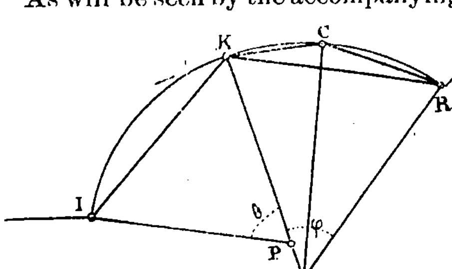
$$r_1 = \frac{S}{4 \cos \frac{1}{2} A \cos \frac{1}{2} B \cos \frac{1}{2} C}$$

in which S is the half sum of the three sides and A, B, C are the three angles of the triangle.

The radius thus found for the eastern part of the curve is $20620{\cdot}44^{m}$, equal to 67652·3 feet, or 12·81 miles. The angle at the center of the curvature between "Kennett–Pennsbury" and "Ruins" was found by the formula

$$\operatorname{Sin} \frac{1}{2} \varphi = \frac{\frac{1}{2} \text{ chord}}{r_1}$$

As will be seen by the accompanying figure, the equal angles at "Kennett–Pennsbury" and "Ruins," between the chord and the radii, are then known, each being equal to $90° - \frac{1}{2} \varphi$. These three angles must each be increased by one-third of the computed spherical excess to obtain the true spherical angles. This correction, however, is only 0″·3 for each angle. The azimuth of the center of



curvature from each extremity of the arc may next be obtained, and the latitude and longitude of that point may be computed.

We next wish to find the radius of the western part of the curve, which, starting from "Kennett–Pennsbury," with a tangent common to the eastern arc, must gradually approach the true 12-mile circle until it cuts it at the initial point. Obviously, as the two arcs have a common tangent at "Kennett–Pennsbury," their centers of curvature will lie in the same line perpendicular to that tangent. Referring once more to the figure, it will be seen that the angle at "Kennett–Pennsbury" between the chord of the arc and the center of curvature is equal to the difference of azimuth of these two directions, the azimuths of which are already known. The corresponding angle at the initial point has the same value and the angle at the center will be equal to the difference between 180° plus the spherical excess (seven-tenths of a second) and the sum of the above angles. The chord and the three angles being thus known, the radius is readily computed by the formula

$$r_2 = \frac{\frac{1}{2}\text{ chord}}{\sin \frac{1}{2}\theta}$$

It is equal to 18640·3^{m}, 61155·6 feet, or about 11·58 miles.

The latitude and longitude of this second center of curvature are computed in the same way as for the first.

The length of each part of the curve was computed separately by the formula

$$\text{arc } n^{\circ} = n \frac{2\pi r}{360}$$

The western part is 10·8977 miles long and the eastern part 11·6749 miles. Thus the total length of the curve is 22·5726 miles, and if to this we add the distance from the initial point to the northeast corner of Maryland, which is 0·7893 mile, the whole length (on dry land) of the boundary between Pennsylvania and Delaware is found to be 23·3619 miles.

After reaching the Delaware the circular boundary is supposed to continue to the New Jersey shore, as Delaware claims all of the water within the circle.

The angular value in each part of the curve of an arc one mile long may be found by dividing the number of degrees in that part by the length of the same in miles. For half-mile arcs the value will be one-half as great. As the boundary monuments are one-half mile apart, this last angle is equal to the angle of deflection from each chord to the next one. In the western arc it equals 2° 28′ 24″ and in the eastern arc 2° 14′ 09″.

The chord of the half-mile arc is equal to twice the product of the radius and the sine of one-half the angle subtended by the arc; i. e.,

$$ch = 2r \sin \tfrac{1}{2}\delta$$

The chord is less than the half mile by 0·20 foot in the western arc and by 0·17 foot in the eastern arc.

Fractional chords are computed by similar methods. The geographical positions of each of the points one-half mile apart throughout the curve, counting from the initial point, can now be computed by the aid of the data obtained as above. Then by the "inverse solution" the distances and directions of these monument stations from the triangulation points nearest to them can be computed.

All of the computations above described will be found in full in Part IV of this Report.

For applying the results of these computations to the location upon the ground of the positions for the monuments five methods were employed at different times, according to circumstances.

The first method was to put up a flag as near as possible to the probable point and so as to be visible from two stations of the triangulation. The flag being determined from these known points, the correction to the desired point of the curve was computed and applied.

The second method consisted in running out with the transit two computed azimuth lines from neighboring triangulation points to the desired point, which is fixed by their intersection.

The third method was to run out a single azimuth line as above, and to lay off on that line by direct measurement or by résection on signals the computed distance to the desired point.

The fourth method was to run out the curve by half-mile chords, making the constant deflection at the end of each chord.

In the fifth method longer chords (of 1 and 6 miles) were run out, and the intermediate points were determined by rectangular offsets.

The engineer's transit with which this work was done was unfortunately not a very good one, the transverse plate level being particularly unreliable; but it was sufficiently good for the preliminary purposes intended at the time.

The monuments, except the initial and terminal, the setting of which has been described, were put into the ground by contractors after I left the work.

In addition to the above work, necessary for the location of the boundary line, an opportunity was taken during hours unsuitable for other work to determine the position of the monument at the "tangent point."

This determination was desirable in itself, as that of a point of the greatest importance; but my principal reason for wishing it made was the desire to explain, if possible, the discrepancy between the value given by Colonel Graham for the distance from the corner of Maryland to the 12-mile circle and that which I found to exist.

The longitude of the tangent point was found to be almost identical with the longitude of the northeast corner of Maryland, showing that the line between those points is practically a meridian, as it was intended to be. The distance from the tangent point to Newcastle court-house proved to be considerably longer than it had been considered, being about 108 feet more than 12 miles and 110 feet more than the value obtained by Colonel Graham. His results were apparently deduced from insufficient data, and as he does not give his method it is difficult to see how he obtained such figures for this line and for the other distances relating to the circle which he furnished.

The above correction in the distance from the court-house to the tangent point, however, will no doubt account for a large part of the discrepancy.

From this it will be seen that the whole of the Maryland boundary is outside of the true 12-mile circle and that the Pennsylvania boundary touches that circle only at the initial point of the curve.

Circular boundary between Pennsylvania and Delaware.

Stations.	Latitude.			Longitude.			Azimuth.			Back azimuth.			To stations.	Distance.	Logarithms.
	°	′	″	°	′	″	°	′	″	°	′	″		m.	
Londonderry, Pa.	39	51	50·507	75	52	50·442	319	04	45·57	139	11	16·15	Meetinghouse Hill.	22194·9	4·3462538
Meetinghouse Hill, Del.	39	42	46·280	75	42	40·121									
Centerville, Del.	39	49	14·97	75	37	02·50	33	51	57	213	48	21	Meetinghouse Hill	14431·9	4·159325
							102	05	59	281	55	52	Londonderry	23042·7	4·362533
White, Pa.	39	45	00·43	75	48	02·35	298	18	12	118	21	38	Meetinghouse Hill	8717·7	3·940402
							151	34	27	331	31	22	Londonderry	14384·8	4·157903
Grays Hill, Md.	39	36	47·69	75	47	51·69	213	51	25	33	54	44	Meetinghouse Hill	13321·5	4·124554
							179	02	37	359	02	31	White	15198·4	4·181797
Grandview, Del.	39	42	42·65	75	42	22·71	105	07	12	285	07	01	Meetinghouse Hill	429·6	2·63311
							35	38	46	215	35	16	Grays Hill	13466·0	4·129239
Masonic Hall, Del.	39	39	36·69	75	33	54·21	115	03	30	294	57	54	Meetinghouse Hill	13829·0	4·140791
							165	54	21	345	52	20	Centerville	18389·5	4·264571
Emanuel church spire, Del.	39	39	38·77	75	33	46·83	114	30	59	294	25	18	Meetinghouse Hill	13961·9	4·144944
							70	02	05	250	02	00	Masonic Hall	187·1	2·27218
Gow, Del.	39	42	13·80	75	33	06·61	94	14	38	274	08	32	Meetinghouse Hill	13698·6	4·136677
							13	10	44	193	10	14	Masonic Hall	4976·3	3·696908
Newcastle court-house, Del.	39	39	35·33	75	33	50·13	113	28	44	293	28	41	Masonic Hall	106·0	2·02534
							191	58	30	11	58	58	Gow	4996·3	3·69865
Deepwater Point, N. J.	39	41	44·45	75	30	30·58	50	56	44	230	54	34	Masonic Hall	6250·9	3·795941
							103	42	07	283	40	27	Gow	3826·1	3·582755
Sellers, Del.	39	44	53·75	75	29	56·39	42	34	55	222	32	53	Gow	6697·5	3·825914
							7	56	40	187	56	18	Deepwater Point	5894·7	3·770462
French, N. J.	39	43	59·28	75	28	31·80	63	35	52	243	32	56	Gow	7309·4	3·863880
							129	50	09	309	49	15	Sellers	2622·6	3·418727

Ridgway, N. J.	39	46	24·78	75	26	51·11	57	32	13	237	30	14	Sellers	5228·1	3·718342
							28	07	14	208	06	10	French	5087·4	3·706492
Taggart, Del.	39	47	29·86	75	27	37·03	34	34	25	214	32	56	Sellers	5846·5	3·766899
							331	25	48	151	26	17	Ridgway	2285·5	3·358977
Naamans Creek 2, Del.	39	48	17·55	75	26	01·00	18	55	30	198	54	58	Ridgway	3676·7	3·565461
							57	14	09	237	13	08	Taggart	2717·2	3·434114
Green, N. J.	39	47	01·73	75	25	13·49	104	16	03	284	14	31	Taggart	3524·2	3·547055
							154	12	12	334	11	42	Naamans Creek 2	2597·5	3·414558
Sanderson, Pa.	39	50	12·86	75	23	50·91	41	02	06	221	00	43	Naamans Creek 2	4713·8	3·673368
							18	26	07	198	25	14	Green	6213·4	3·793332
Raccoon Island, N. J.	39	48	59·32	75	22	42·07	44	49	04	224	47	27	Green	5111·7	3·708567
							144	10	59	324	10	15	Sanderson	2797·1	3·446709
Sloan's windmill, Pa.	39	48	27·91	75	25	29·37	351	54	19	171	54	29	Green	2684·7	3·428904
							256	18	07	76	19	54	Raccoon Island	4095·7	3·612325
Ruins, Pa. and Del.	39	48	27·65	75	25	31·00	258	11	48	78	11	49	Sloan's windmill	39·65	1·59824
Terminal monument, Pa. and Del.	39	48	27·92	75	25	31·53									
Maharty, Del.	39	44	10·99	75	45	12·86	15	29	27	195	27	46	Grays Hill	14185·8	4·151853
							110	42	54	290	41	06	White	4313·9	3·634867
Rankin, Del.	39	43	12·21	75	45	00·37	19	00	59	198	59	10	Grays Hill	12542·2	4·098373
							127	37	10	307	35	14	White	5469·6	3·737955
Smith, Del.	39	43	17·86	75	47	13·49	4	19	51	184	19	27	Grays Hill	12067·4	4·081615
							240	17	32	60	18	49	Maharty	3307·1	3·519445
Northeast Corner Maryland	39	43	19·91	75	47	20·03	292	04	10	112	04	14	Smith	168·1	2·225526
							242	30	29	62	31	50	Maharty	3413·6	3·533216
Pearson, Pa.	39	43	54·78	75	46	48·60	6	31	07	186	30	27	Grays Hill	13257·2	4·122452
							257	37	41	77	38	42	Maharty	2333·9	3·368078
Whiteman, Pa.	39	43	20·85	75	46	30·77	230	11	12	50	12	02	Maharty	2415·1	3·382939
							157	54	59	337	54	48	Pearson	1129·3	3·052807
Initial Point, Pa. and Del.	39	43	19·91	75	46	26·69	290	57	02	111	05	05	Newcastle C. H.	19311·96	4·2858264
							224	43	29	44	48	49	Kennett-Pennsbury	16898·15	4·2278391

Circular boundary between Pennsylvania and Delaware—Continued.

Stations.	Latitude.	Longitude.	Azimuth.	Back azimuth.	To stations.	Distance.	Logarithms.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″		m.	
Walnut, Del.	39 45 10·50	75 45 09·89	321 15 45	141 17 21	Meetinghouse Hill	5675·1	3·755970
			85 41 17	265 39 27	White	4117·4	3·614619
Mitchell, Del.	39 46 00·69	75 43 07·45	353 48 16	173 48 34	Meetinghouse Hill	6031·0	3·780391
			62 02 24	242 01 05	Walnut	3300·2	3·518546
Hoopes, Pa.	39 47 23·05	75 45 39·14	305 07 03	125 08 40	Mitchell	4414·0	3·644836
			350 20 02	170 20 21	Walnut	4146·8	3·617710
Foulk, Pa.	39 47 44·90	75 44 17·22	332 40 22	152 41 07	Mitchell	3617·5	3·558409
			70 56 03	250 55 10	Hoopes	2062·4	3·314366
O'Neill, Del.	39 47 39·62	75 42 35·50	13 59 35	193 59 15	Mitchell	3144·2	3·497511
			93 51 47	273 50 42	Foulk	2425·5	3·384796
Stephen, Del.	39 47 31·43	75 40 43·09	50 50 43	230 49 11	Mitchell	4430·8	3·646479
			95 24 14	275 23 02	O'Neill	2686·4	3·429170
McCarty, Pa.	39 48 19·57	75 42 06·36	306 50 38	126 51 32	Stephen	2475·5	3·393663
			29 22 17	209 21 58	O'Neill	1413·8	3·150402
Cloud, Pa.	39 49 26·71	75 39 39·57	23 02 11	203 01 30	Stephen	3863·4	3·586972
			59 20 33	239 18 59	McCarty	4059·6	3·608485
Gregg, Del.	39 49 24·58	75 37 18·81	307 23 02	127 23 13	Centerville	488·0	2·68843
			91 08 11	271 06 41	Cloud	3348·2	3·524811
Kennett-Pennsbury, Pa. and Del.	39 49 48·90	75 38 06·63	303 23 52	123 24 23	Gregg	1362·4	3·134318
			72 48 08	252 47 08	Cloud	2313·5	3·364275
Hamorton, Pa.	39 52 22·05	75 37 55·56	347 39 45	167 40 19	Centerville	5905·9	3·771289
			24 34 50	204 33 43	Cloud	5946·1	3·774234
Leach, Del.	39 50 07·19	75 33 38·94	71 37 00	251 34 49	Centerville	5101·8	3·707721
			124 18 30	304 15 45	Hamorton	7383·3	3·868252
Twaddell, Pa.	39 50 38·75	75 33 44·62	61 14 47	241 12 40	Centerville	5368·3	3·729837
			352 06 06	172 06 10	Leach	982·7	2·99241

Granogue, Del.	39	49	47·52	75	34	50·81	224	52	58	44	53	40	Twaddell	2230·2	3·34834
							250	26	59	70	27	45	Leach	1813·7	3·25856
Point Lookout, Pa.	39	50	22·22	75	35	25·85	258	01	43	78	02	48	Twaddell	2460·4	3·39100
							280	19	10	100	20	19	Leach	2584·2	3·41232
Talley, Del.	39	50	16·35	75	34	35·50	240	16	00	60	16	33	Twaddell	1393·3	3·14403
							98	36	00	278	35	28	Point Lookout	1210·7	3·08302
Perkins, Pa.	39	50	38·05	75	33	01·65	91	12	58	271	12	30	Twaddell	1021·8	3·00936
							42	58	26	222	58	02	Leach	1300·7	3·11418
Concord–Bethel, Pa. and Del.	39	50	20·29	75	32	27·60	289	17	08	109	21	35	Ruins	10499·06	4·0211504
							83	10	57	263	07	20	Kennett–Pennsbury	8119·47	3·9095275
Seal, Pa.	39	45	58·99	75	44	54·84	13	28	16	193	28	06	Walnut	1537·8	3·18689
							157	52	37	337	52	09	Hoopes	2798·6	3·44694
Southwood, Pa.	39	47	08·72	75	43	34·51	342	56	08	162	56	25	Mitchell	2194·7	3·341382
							260	14	31	80	16	21	Stephen	4138·2	3·616816
Crow, Md.	39	41	47·87	75	47	40·18	255	49	45	75	52	57	Meetinghouse Hill	7371·7	3·867567
							1	49	59	181	41	52	Grays Hill	9261·8	3·966694
Williams (flag in tree), Del.	39	38	26·67	75	45	10·45	51	34	26	231	32	43	Grays Hill	4909·8	3·691060
							150	06	31	330	04	55	Crow	7158·3	3·854811
Iron Hill 2, Del.	39	38	20·42	75	45	08·16	53	46	00	233	44	16	Grays Hill	4836·2	3·684508
							150	29	19	330	27	42	Crow	7352·9	3·866457
Road, Md.	39	39	00·10	75	47	30·60	7	01	31	187	01	18	Grays Hill	4114·2	3·61429
							287	08	03	107	09	33	Williams	3497·0	3·54369
Flat, Md.	39	38	50·03	75	47	29·16	282	16	38	102	18	07	Williams	3384·9	3·52954
							173	41	43	353	41	42	Road	312·3	2·49458
Tangent Monument, Del. and Md.	39	38	56·95	75	47	20·04	180	00	07	0	00	07	N. E. Corner Md.	8109·95	3·9090182
							266	25	14	86	33	50	Newcastle C. H.	19344·8	4·2865646
Center Eastern Arc.	39	39	13·77	75	33	36·02									
Center Western Arc.	39	40	14·78	75	34	01·95									
Boundary Monument No. ½	39	43	44·58	75	46	15·69	290	14	46	110	22	35	Center Western Arc	18639·97	4·270445
							19	00	34	199	00	27	Initial Point	804·60	2·90558

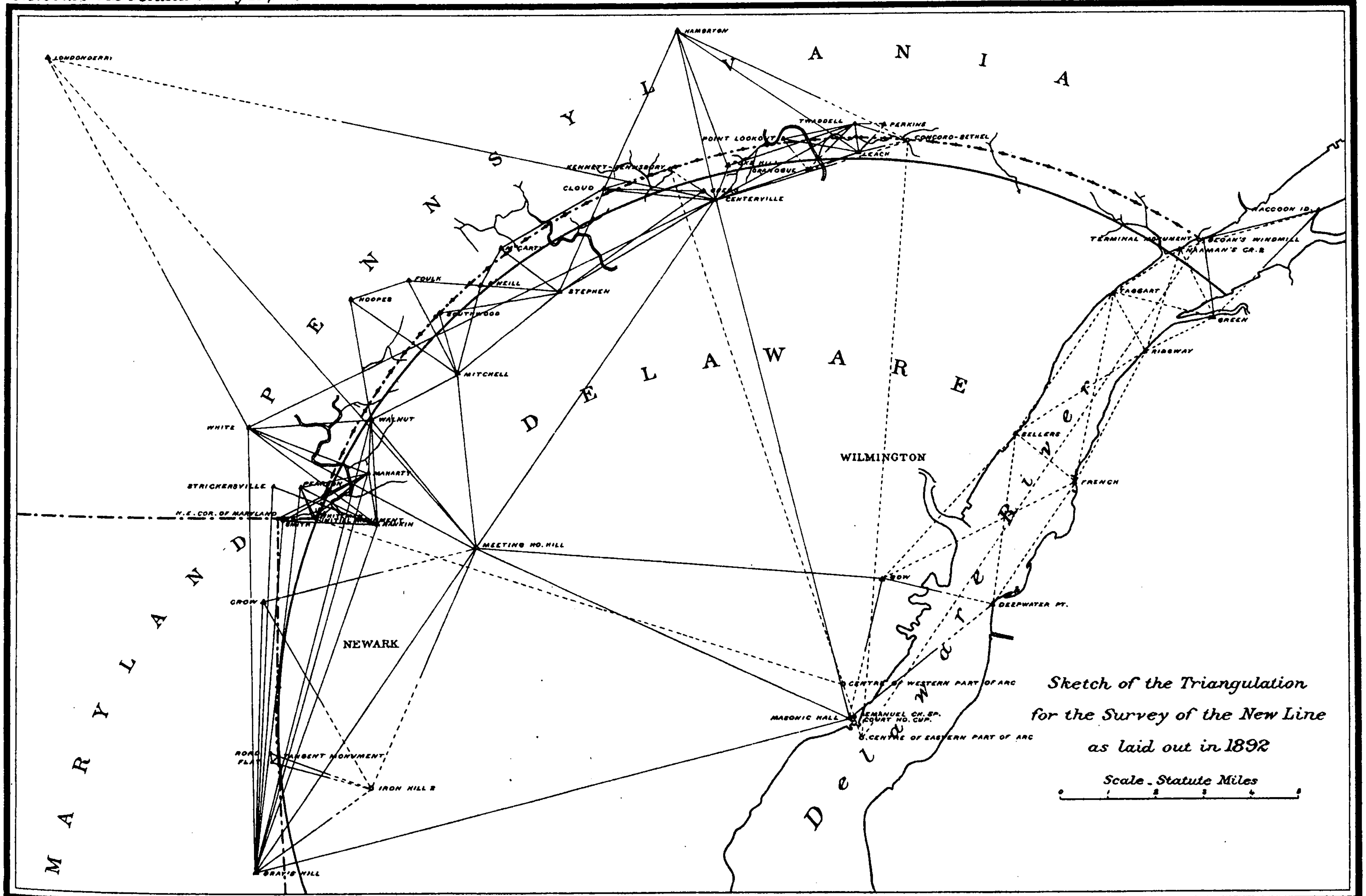
Circular boundary between Pennsylvania and Delaware—Continued.

Stations.	Latitude.			Longitude.			Azimuth.			Back azimuth.			To stations.	Distance.	Logarithms.
	°	′	″	°	′	″	°	′	″	°	′	″		m.	
Boundary Monument No. 1	39	44	08·85	75	46	03·31	21	29	06	201	28	58	Monument No. ½	804·60	2·90558
Boundary Monument No. 1½	39	44	32·70	75	45	49·59	23	57	39	203	57	30	Monument No. 1	804·60	2·90558
Boundary Monument No. 2	39	44	56·06	75	45	34·55	26	26	13	206	26	03	Monument No. 1½	804·60	2·90558
Boundary Monument No. 2½	39	45	18·89	75	45	18·21	28	54	47	208	54	37	Monument No. 2	804·60	2·90558
Boundary Monument No. 3	39	45	41·16	75	45	00·60	31	23	22	211	23	11	Monument No. 2½	804·60	2·90558
Boundary Monument No. 3½	39	46	02·83	75	44	41·77	33	51	59	213	51	47	Monument No. 3	804·60	2·90558
Boundary Monument No. 4	39	46	23·84	75	44	21·73	36	20	36	216	20	23	Monument No. 3½	804·60	2·90558
Boundary Monument No. 4½	39	46	44·17	75	44	00·54	38	49	14	218	49	00	Monument No. 4	804·60	2·90558
Boundary Monument No. 5	39	47	03·77	75	43	38·23	41	17	52	221	17	38	Monument No. 4½	804·60	2·90558
Boundary Monument No. 5½	39	47	22·61	75	43	14·83	43	46	31	223	46	16	Monument No. 5	804·60	2·90558
Boundary Monument No. 6	39	47	40·65	75	42	50·41	46	15	11	226	14	55	Monument No. 5½	804·60	2·90558
Boundary Monument No. 6½	39	47	57·86	75	42	24·99	48	43	51	228	43	35	Monument No. 6	804·60	2·90558
Boundary Monument No. 7	39	48	14·20	75	41	58·63	51	12	32	231	12	15	Monument No. 6½	804·60	2·90558
Boundary Monument No. 7½	39	48	29·65	75	41	31·37	53	41	14	233	40	57	Monument No. 7	804·60	2·90558
Boundary Monument No. 8	39	48	44·18	75	41	03·28	56	09	56	236	09	38	Monument No. 7½	804·60	2·90558
Boundary Monument No. 8½	39	48	57·75	75	40	34·39	58	38	39	238	38	20	Monument No. 8	804·60	2·90558
Boundary Monument No. 9	39	49	10·35	75	40	04·77	61	07	22	241	07	03	Monument No. 8½	804·60	2·90558

Bound.ry Monument No. 9½	39	49	21·95	75	39	34·46	63	36	05	243	35	46	Monument No. 9	804·60	2·90558
Boundary Monument No. 10	39	49	32·53	75	39	03·54	66	04	50	246	04	30	Monument No. 9½	804·60	2·90558
Boundary Monument No. 10½	39	49	42·07	75	38	32·05	68	33	34	248	33	14	Monument No. 10 Center Eastern Arc	804·60 20620·41	2·90558 4·314299
Boundary Monument No. 11	39	49	50·55	75	38	00·05	71	02	01	251	01	40	Monument No. 10½	804·62	2·90559
Boundary Monument No. 11½	39	49	58·03	75	37	27·63	73	21	01	253	20	40	Monument No. 11	804·62	2·90559
Boundary Monument No. 12	39	50	04·52	75	36	54·86	75	35	31	255	35	10	Monument No. 11½	804·62	2·90559
Boundary Monument No. 12½	39	50	10·02	75	36	21·78	77	50	01	257	49	40	Monument No. 12	804·62	2·90559
Boundary Monument No. 13	39	50	14·52	75	35	48·45	80	04	31	260	04	10	Monument No. 12½	804·62	2·90559
Boundary Monument No. 13½	39	50	18·00	75	35	14·91	82	19	02	262	18	40	Monument No. 13	804·62	2·90559
Boundary Monument No. 14	39	50	20·48	75	34	41·22	84	33	33	264	33	11	Monument No. 13½	804·62	2·90559
Boundary Monument No. 14½	39	50	21·94	75	34	07·44	86	48	03	266	47	41	Monument No. 14	804·62	2·90559
Boundary Monument No. 15	39	50	22·37	75	33	33·60	89	02	34	269	02	12	Monument No. 14½	804·62	2·90559
Boundary Monument No. 15½	39	50	21·79	75	32	59·77	91	17	04	271	16	42	Monument No. 15	804·62	2·90559
Boundary Monument No. 16	39	50	20·19	75	32	25·99	93	31	35	273	31	13	Monument No. 15½	804·62	2·90559
Boundary Monument No. 16½	39	50	17·57	75	31	52·32	95	46	06	275	45	44	Monument No. 16	804·62	2·90559
Boundary Monument No. 17	39	50	13·93	75	31	18·81	98	00	36	278	00	14	Monument No. 16½	804·62	2·90559
Boundary Monument No. 17½	39	50	09·29	75	30	45·51	100	15	06	280	14	45	Monument No. 17	804·62	2·90559
Boundary Monument No. 18	39	50	03·65	75	30	12·48	102	29	36	282	29	15	Monument No. 17½	804·62	2·90559

Circular boundary between Pennsylvania and Delaware—Continued.

Stations.	Latitude.			Longitude.			Azimuth.			Back azimuth.			To stations.	Distance.	Logarithms.
	°	′	″	°	′	″	°	′	″	°	′	″		m.	
Boundary Monument No. 18½	39	49	57·01	75	29	39·75	104	44	06	284	43	45	Monument No. 18	804·62	2·90559
Boundary Monument No. 19	39	49	49·40	75	29	07·39	106	58	36	286	58	15	Monument No. 18½	804·62	2·90559
Boundary Monument No. 19½	39	49	40·81	75	28	35·44	109	13	05	289	12	45	Monument No. 19	804·62	2·90559
Boundary Monument No. 20	39	49	31·27	75	28	03·95	111	27	34	291	27	14	Monument No. 19½	804·62	2·90559
Boundary Monument No. 20½	39	49	20·78	75	27	32·97	113	42	03	293	41	43	Monument No. 20	804·62	2·90559
Boundary Monument No. 21	39	49	09·37	75	27	02·54	115	56	31	295	56	12	Monument No. 20½	804·62	2·90559
Boundary Monument No. 21½	39	48	57·05	75	26	32·72	118	10	59	298	10	40	Monument No. 21	804·62	2·90559
Boundary Monument No. 22	39	48	43·84	75	26	03·55	120	25	27	300	25	08	Monument No. 21½	804·62	2·90559



Sketch of the Triangulation for the Survey of the New Line as laid out in 1892

UNITED STATES COAST AND GEODETIC SURVEY.

APPENDIX No. 9—REPORT FOR 1893—PART II.

PROCEEDINGS OF THE GEODETIC CONFERENCE,

HELD AT

WASHINGTON, D. C.,

JANUARY 9 TO FEBRUARY 28, 1894.

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SUPPLEMENT.

LETTERS FROM ASSISTANTS AND OTHERS, ADDRESSED TO THE CHAIRMAN OF THE CONFERENCE.

ILLUSTRATIONS.

GEODETIC CONFERENCE.

GENERAL REPORT.

UNITED STATES COAST AND GEODETIC SURVEY,
Washington, D. C., February 28, 1894.

SIR: The Geodetic Conference called by you convened at Washington on January 9, 1894. The details of its organization are appended as a preface to the reports of the committees to which the duty was assigned of collating necessary facts and formulating them for the consideration of the Conference.

An invitation was extended to members of the office and field force not members of the Conference, as well as to others interested in the subjects under consideration, to give expression to their views on matters relating to the geodetic operations of the Survey.

The Conference desires here to make grateful acknowledgment to those who responded in writing, as well as to those who, by personal attendance and verbal communications, gave their valuable time.

Professor Harkness, U. S. N., addressed the Conference on the various methods of determining the earth's figure. Professor Woodward, of Columbia College, New York, gave an account of base measurement with steel tapes, and discussed, in response to questions, various subjects relating to the work of the Survey. Professor Gore, of Columbian University, Washington, D. C., evinced his interest by being present at several meetings, in which he took part, while Professors Buchanan, of Tennessee, and Hoag, of Minnesota, Acting Assistants, United States Coast and Geodetic Survey, shared the labors of the Conference for part of the time during which it was in session.

The task assigned to the members of the Conference was the consideration of the geodetic methods as now practiced, the discussion of the application of these methods to the duties assigned by law to the Coast and Geodetic Survey, and the comparison of the means employed in foreign countries for similar purposes, with the object of suggesting improvements in regard to accuracy and economy.

The general plan of the geodetic operations followed by the Survey is that outlined by Mr. F. R. Hassler and approved by President Jefferson and Mr. Madison, Secretary of State, and subsequently prescribed by law in the "Basis for the reorganization of the Survey" (approved by President Tyler, March, 1845).

The wise prevision of those who promoted and approved these plans has been shown in this, that the latter proved themselves sufficient, without material modification, to meet the changed conditions caused by the acquisition of new coast line by the United States, the extension of the triangulation inland, and for the determination of geographical positions in the interior.

In the early days of our Republic the great migration of peoples in and to the United States, and the settling of vast areas hardly known previously to geography, rendered a sole reliance on geodetic methods inapplicable to surveys, the main purpose of which was the demarcation of Government lands for settlement. To meet the necessities of the case the excellent system of rectangular land surveys was applied, which is well adapted to the large areas of level land in this country. For certain purposes it is, however, insufficient, as shown by the important trigonometric surveys of the Mississippi and Missouri rivers, where systematic improvements ordered by Congress could only be based on extensive and accurately correlated and detailed surveys, and where, therefore, trigonometric methods were found necessary in order to supplement the system and adapt it to the ordinary use for which surveys are made. The older States neither received nor required the rectangular subdivision, and, therefore, such States, using the work done by the Survey as a basis, have adopted the trigonometric method, which, moreover, is also being applied in States subdivided on the rectangular plan.

In view of these facts, and of the experience of all European countries, not to mention India, the French possessions in northern Africa, the English in southern Africa, and the Dutch East Indies, that trigonometric surveys are necessary, it seems safe to assume that the inland geodetic operations of this Survey will, in addition to meeting an existing demand, benefit future generations in this country as much as those along the coasts have already benefited the present.

The practical bearing of these views on the subjects submitted to the consideration of the Conference has been taken into account in the discussion of all the questions as to the location of the trigonometric schemes already called for in the work of the Survey, as well as those likely to be required for the purpose of relating the local surveys to the general map of the country.

The need of association of bodies pursuing the higher branches of culture in the various countries was early felt in the case of geodesy, and it must be considered fortunate and beneficial for both parties that the United States, by the liberality of Congress in 1889, was able to join the International Geodetic Association for the determination of the earth's figure and size, and thus add its influence and contribute its share toward the realization of this object. It must not be supposed that the labors of this organization were rather theoretical than practical. On the contrary, their eminently practical character throws light on all branches of geodesy and surveying; and it was made the duty of the Conference convened by you, and one of its objects, to compare the various methods pursued by other nations with those used at home.

The Conference has applied itself to this labor with a view of pointing out the means by which, or directions in which, the system pursued by the Survey would in any way be improved in accuracy and rapidity of work as well as in economy, so as to produce results at once available as well as enduring.

The subject-matter submitted to the Conference comprises a wide range of applied science, dealing with the geodetic (inclusive of hypsometry), the astronomic, the magnetic, and the gravitation work of the Survey. The suggestions made as the outcome of the deliberations cover not only the present conditions of the work, but prepare the way for such future work as the legitimate development of the Survey would seem to demand.

To expedite business and to secure its best consideration the Conference distributed the work among a number of committees, as already stated, particularly selected to include those members who had more special knowledge or experience of the subject assigned to them. The members of each committee contributed to the preparation of the report, which, after full discussion, was submitted by its chairman to the conference at large, where it received its final consideration before adoption by a majority vote.

The committee reports thus prepared will be submitted in full. The deductions drawn from them by the Conference and the conclusions reached are herewith briefly stated for your convenience. The reasons for them can, in general, be gathered from the reports themselves, though many facts and details which were considered have not found a place in them.

ABSTRACT OF REPORT OF THE COMMITTEE ON RECONNAISSANCE.

The committee uses the term "reconnaissance" to embrace all those investigations of a region to be triangulated which precede the field work, base measurement, and the measurement of angles, and to comprise the selection of the most feasible chain or network of geometric figures, the location of base lines, the character of the necessary preparations, and the collection of information concerning the facilities available for carrying out the proposed scheme.

All reconnaissance should be thorough and exhaustive, and develop all possible schemes of triangulation. It should afford information by which economy and rapidity of execution can be obtained. When exhaustive, it will lead to the simplification of the geometric figures. In no case should geometric elegance of a scheme be sought at the expense of reasonable economy. In the interior of the country the highest peaks are frequently covered with clouds and should not be used as triangulation points if it can be avoided.

A simple chain of triangles is recommended as the most easily adapted to the most complex orography. Pentagons or quadrilaterals, with central points, also readily conform to the configuration of the country, however complex or difficult it may seem.

It is recommended that sites for the location of bases should be looked for at every eighth to tenth figure of the scheme of triangulation, according to its character and scope.

Before taking the field the officer should study all available maps of the region and familiarize himself with the lines of travel, locations of towns and villages.

Notes in the field can not be too full. Horizontal and vertical measures should be taken on all prominent objects. Approximate latitudes and azimuths should be observed, as well as the magnetic bearings of every notable object. The entire horizon should be sketched, and rough topographical sketches made to show main ridges, water courses, etc.

The atmospheric conditions should be noted and the facilities for obtaining water and forage.

The committee concludes with some suggestions in regard to instrumental outfit.

ABSTRACT OF REPORT OF THE COMMITTEE ON BASE LINES.

The committee classifies the base apparatus in use under three general heads:

Contact apparatus, comprising compensating, bimetallic, and mono metallic bars.

Optical apparatus, comprising bimetallic, monometallic, and bars in melting ice.

Steel tape and wire apparatus.

They then give a condensed statement of recent measures in Europe and India.

Brief descriptions, instances of their application and possibilities, are given for the Colby, Bessel, Brunner, Struve, Austrian, and United States Lake Survey Repsold apparatus. The various forms of apparatus used on or devised for the Coast and Geodetic Survey are mentioned, such as the Hassler bar, the Bache-Wurdemann, the Schott, the secondary contact, the Woodward steel bar in melting ice, the Eimbeck duplex, and the steel tape. As detailed descriptions of all these forms appear in the Coast Survey reports, they are not repeated here, but their principles are stated, results of their use, and, when possible, instances of their comparative uses are quoted and recommendations made concerning the particular classes of bases they are adapted for.

Foreign reports do not contain sufficient data upon which to base statement of cost, and only in two instances do we find any record of the number of officers and men required for a measurement abroad.

Of the more recent forms of apparatus tried at the Coast and Geodetic Survey, particular mention is made of the Woodward steel bar in melting ice, used in four measures of one kilometre of the Holton Base, with this expression of opinion: "The salient facts resulting from the experiments made, then, appear to be that with the 5m steel bar in melting ice, whereby the temperature error is eliminated, a base line can be measured with a degree of accuracy hitherto unapproached. In economy of operation it is little, if any, inferior to the Brunner apparatus now used abroad."

An account is given of the details and results of the measurement of the Holton Base with the secondary contact slide apparatus and the steel tape. The committee considers that with the contact slide apparatus a degree of precision is obtained beyond the needs of the triangulation, and the contact slide is commended. The cost of measuring with the secondary contact slide apparatus is about the same as that with the tape.

In the description of the steel tapes used on the Holton Base the opinion is expressed that, "carefully and judiciously handled, the steel tape apparatus will doubtless attain good standing throughout the United States, where so many extensive and independent surveys are being carried on, and especially in all newly undertaken projects."

Comparisons of the earlier and later base measurements of the Coast and Geodetic Survey are given, and notes on the site of the base lines and the procedure of field organization and measurements.

The accuracy attainable in base measurements is based upon practice in the field, comparison with the standard, and the accuracy of the new metric prototypes. After enumerating the conditions under which comparisons of standards are made, the committee says:

> We may conclude that no geodetic standard can be known with a higher degree of accuracy than 1 part in 5 000 000 of its length, in terms of the international unit.
>
> That it is possible to measure a base line repeatedly with the same apparatus with a surprising accordance between measures indicates that the elimination of accidental errors has been successfully met by the different apparatus in use, and it is also believed that the various methods of observing successive lengths of the same bar or systems of bars are sufficiently precise.
>
> On the other hand, that constant errors exist which are in the main due to a defective knowledge of the temperature of the bars is a fact commonly assumed, or proven by the lag of mercurial or bimetallic thermometers used on various apparatus. These constant errors are not easily determined, but are now the principal sources of errors.

Examples of recent measurements and discrepancies are given. No definite limit is assigned to the degree of accuracy, because the measure of a base line depends largely on the object it has to subserve and on the apparatus, time, and money available.

The performance of the contact slide in its various adaptations now in use on this Survey, as well as that of the tape, warrant the conclusion that our present methods of measuring bases are unexcelled in point of economy, rapidity, and accuracy.

"With the perfection of means now available a line may be readily measured with a probable error of 1:1 000 000, so far as mere measurement is concerned." But this accuracy is rapidly dissipated by the known and unknown errors, and especially through the three or more steps in the triangulation required to reach the first line of average length. Great accuracy in the angular measurements is needed if an error of 1:150 000 is to be maintained throughout the network of triangulation.

For secondary triangulation the fraction of the length 1:100 000 to 1:50 000 may be suitable, and the degree of accuracy in the base may be graduated accordingly.

For tertiary triangulation an average uncertainty or limit of 1:10 000, or even 1:5 000 may be allowable for the special purpose of the work.

The subject of frequency of base lines is referred to by this and another committee.

It is recommended that whenever the steel tape is used in primary work at least four measurements be made and each section be measured under rising and falling temperatures. It is particularly recommended that all base lines be measured at least twice.

In conclusion, the committee recommends—

That further experiments with the steel tape be made, especially with a view to a better determination of its temperature. This recommendation seems to be warranted by the results of the measurements of the Holton Base.

2. That the new duplex apparatus just completed by the Survey be given a thorough and careful trial as soon as practicable.

3. That the iced bar be used to lay off a 100^{m} distance as a comparator at the bases where the above recommendations are followed.

4. That in all measurement due regard be paid to rising and falling temperatures, so as to eliminate as much as possible the errors due to lack of knowledge of temperature of the measuring apparatus.

5. That the tripods for supporting bars be made of metal, and embrace the details which experience has shown to be conducive to accuracy and rapidity of measurement.

6. That ordinarily the base line be divided into sections one-half kilometre in length.

7. That a 100^{m} comparator be established in Washington for the purpose of testing steel tapes and contact slide apparatus.

8. That in future use of the iced steel bar the water in the Y trough be not drained off below the surface of the bar, in order to remove any possibility of doubt as to the actual temperature of the bar.

ABSTRACT OF THE REPORT OF THE COMMITTEE ON TRIANGULATION.

The committee has classified the various subject-matters discussed by it under eighteen heads, with a map to exhibit the area of triangulations already executed, that under progress, and projected lines.

The principal subjects are: the object of triangulation; classification; main and primary, secondary and tertiary triangulations; adaptation to the surface of the ground; general form of main triangulation; geometrical composition of a triangulation; frequency and length of base lines and their connection with the triangulation; accuracy of a triangulation, and comparisons with foreign work; international and interstate boundaries. Then follow instrumental outfit, method of observation, marking stations, signals, etc.

In the execution of a scheme of triangulation the character of the work will range from what is purely geodetic to plane surveying.

The main triangulation comprises the principal series of geometrical figures which compass along the shortest line the whole extent of territory under consideration. The subordinate divisions, with primary, secondary, and tertiary, are more technical. The primary is characterized by the greatest development of length of sides and by the greatest accuracy of measurement, wherein the geodetic positions depend on the mean of numerous astronomical latitudes, longitudes, and azimuths, and the initial, intermediate and terminal base lines are directly measured. The committee recommends that observations for latitude and azimuth be made according to the character of the project. In the primary triangulation these have been in some localities made at every station, but frequently every other station has been so occupied. Further particulars are enumerated, and then follow descriptions of the other schemes.

The frequency and length of base lines and their connection with the triangulation depend not only upon the orographical features of the country, but upon the required accuracy of the triangulation. Because base lines can be measured with much greater accuracy than the triangulation can maintain, it is recommended to increase the number of base lines rather than to increase their length.

In arriving at the desirable accuracy of a triangulation the object of the work must be considered. From an economical standpoint the degrees of accuracy in the measurement of a base line for different projects are given, ranging from 1:200 000 to 1:1 000 000 or a still smaller fraction. But to maintain an accuracy of 1:150 000, or even 1:100 000 part of the length, is a matter of difficulty in an extended triangulation.

The accuracy of the base is rapidly dissipated in the adjacent base figure, and hence it is not expedient to strain after excessive refinement in base measurement.

The committee then examines 73 triangles of the main and primary triangulation, and shows that the work of the United States Coast and Geodetic Survey stands well in the front rank. In the triangulation just referred to, the Survey inaugurated a scheme on a scale so large that there was no previous experience to guide it or suggest the attainable accuracy of the work. A critical consideration of the angle measures shows that the number of observations might have been reduced by one-half and still give results which will rank with the best foreign work. The period of observation at a station must be somewhat shortened, but a high authority is quoted to warn against hurrying observations lest the atmospheric conditions introduce larger errors than may be attributable to the instrument. This is a very important subject and should be carefully considered by the observers.

The number of series of observations recommended at a principal triangulation station is about 31, while the number of positions of the circle should be, in general, much smaller.

In the eastern part of the transcontinental triangulation, where the sides average 26 kilometres (16 miles) in length, a discussion of several hundred triangles suggests the reduction of the number of observations to about one-half or two-thirds the number recommended for the primary triangulation.

In comparison with thousands of triangles of this class of work in Europe, that of the Coast and Geodetic Survey must be rated as among the best.

In tertiary triangulation the demands of accuracy may vary from 1:20 000 even to 1:5 000 part of the length.

The committee treats of the determination of international and interstate boundaries under the principal heads: Boundary along a meridian, along a parallel, along oblique lines. The methods of tracing each are presented in some detail, with illustrations.

Under instrumental outfit are considered the merits and demerits of direction and repeating instruments for the main and primary triangulations.

Whenever practicable, the large theodolites should be mounted upon concrete or masonry piers.

In order to secure uniform size of heliotrope images at all distances a formula is given for determining the size of the heliotrope for a given distance. All heliotropes should be centered with the same accuracy as the theodolite. Their superiority on long lines is pointed out.

The method of measurement of angles with the repeating instrument is drawn up from the experience of the observers, and directions are given for making the records, etc.

The committee has also discussed the different forms of signals and elevated structures for observing, the marking of stations below and at the surface of the ground, etc.

Vertical angles should be measured at all main and primary triangulation stations, and the direction of all prominent, natural, and artificial objects in the horizon should be observed.

It is recommended that magnetic observations be made at all triangulation stations.

ABSTRACT OF REPORT OF ASTRONOMICAL COMMITTEE.

The committee has called attention to the various methods of determining longitude as required for different purposes and suited to different conditions. The most important of these is the telegraphic method, which was introduced by the Coast Survey. The mode of operation was much simplified in 1878, and a brief method of field computation adopted. The number of nights of observations required is stated to be from six to ten, the observers exchanging places. Comparisons with the results obtained in foreign countries show that the Survey maintains an equally high standard, even when the number of nights has been but one-half of that used abroad.

The expense of operating a longitude party with the present outfit has been reduced to a minimum, and without the extension of time at a station observations are also made for latitude and the magnetic elements. With the addition of another observer to the party it is proposed to observe for gravity.

The committee suggests that the longitudes of the Aleutian Islands may be determined by the use of terrestrial signals and the local time at different stations, and that the chain be subdivided and certain stations connected chronometrically with Sitka as the base station. This method is recommended.

For latitude observations the committee has shown that the number of pairs of stars and the number of nights of observations have been gradually decreased on the Survey. This has arisen in a large measure from the better determinations of the star places as given in the later catalogues. It is believed that ten or twelve pairs of stars, observed upon three or four nights with the zenith telescope or meridian instrument, will give the latitude of a station with a probable error not exceeding a tenth of a second. This is so far below the average local deflections of the vertical that greater accuracy is not deemed necessary, except in such special cases as the measurement of arcs and the determination of international boundaries.

For latitude observations the committee recommends the continued use of the Talcott method by the zenith telescope or meridian instrument as affording greater accuracy than by other methods. It recalls the competitive observations instituted by Superintendent Bache with the zenith sector, zenith telescope, prime vertical transit, and vertical circle. It is suggested that an enlarged catalogue of latitude stars is greatly needed.

The committee suggests that the four principal nations publishing Ephemerides should combine in the expense of preparing and issuing a special star list of greater extent than is now issued, so that the number of time stars may be increased to an average of one for each two minutes of time; and also that the American Ephemeris should extend the additional star places so as to cover the period now falling in daylight; also that more azimuth stars should be added, and whenever practicable grouped by differences of nearly twelve hours in right ascension. The subpolar places should be given in order of right ascension. With close circumpolars this would be useful in determining the value of a micrometer screw.

ABSTRACT OF REPORT OF THE COMMITTEE ON HYPSOMETRY.

The committee states the degree of accuracy demanded in topography, physical hydrography, base-line reduction to sea level, gravity observations, tidal planes, meteorological investigations, and engineering operations, and gives an account of the Y-level of civil engineers and the geodetic level of the Coast and Geodetic Survey.

About 9 800 kilometres (6 100 miles) of precise levels have already been run by different corps of the United States to meet the demands of special investigations and in the regular work of the Coast and Geodetic Survey. The desirability of a consistent scheme of lines of control of the whole country, to which all future work may lend itself, has made itself apparent. The committee presents considerations for such a scheme, as follows:

To provide a means, the most direct and economical, for connecting the many tidal stations of the Atlantic, Gulf, and Pacific seaboards.

To connect these tidal planes by routes which will best overcome the uncertainties arising from crossing mountain chains, etc.

To form closed figures that will best determine the degree of accuracy of the work.

To make the lines of level conform as nearly as possible to existing or proposed schemes of triangulation, and especially of arc measures.

To take advantage of all work of a like nature heretofore executed, and to distribute judiciously over the country bench marks which may serve as points of departure for hypsometric work of all kinds.

Work of this class has been executed by almost all European nations, and the committee gives an extended report of the operations and the methods. Descriptions of the instruments are given. Throughout France operations of the most exhaustive character have been or are being carried out. In fact, the whole of Europe is covered with lines of precise leveling. According to the report of the International Geodetic Association, the total length is 102 800 kilometres (64 250 miles). This does not include Great Britain. It is shown how carefully methods and means have been studied and applied so as to attain the highest degree of accuracy possible. Then follows an account of precise levels in the United States by the Corps of Engineers in the survey of the Great Lakes, by the Mississippi and Missouri River Commissions, and by the Coast and Geodetic Survey. The methods of procedure are stated and the conclusions reached to account for the cumulative errors which are found in all series of levelings.

In 1892 an elaborate series of experiments were undertaken by the Survey to investigate the system of leveling and the character of the instruments. These have not been fully discussed, but have resulted in valuable suggestions of a practical nature.

Results of long lines of leveling by the engineer's Y-level in the United States are presented, and the conclusions deduced from work done in Prussia with similar instruments are quoted.

Experiments with the geodetic level and with the Y-level are in progress by the Survey.

The subject of trigonometrical leveling is discussed and a few of the results in the United States and India are presented.

The determination of differences of elevation by observations of atmospheric pressure is at times the only means available, and the method of observing the boiling point of water is recommended as suitable for exploration surveys.

The committee concludes that the measure of trigonometrical heights should form part of the work of all triangulation stations. For this work they suggest that a new form of instrument be devised to provide for the measurement of larger arcs by micrometer differences;

That the standard bench marks throughout the country should be determined with the greatest degree of accuracy attainable;

That subsidiary lines may be leveled by less precise methods;

That if, upon investigation, it shall appear that less elaborate methods or instruments than those now employed on the Survey will, by the use of small circuits, produce satisfactory results with an increase of economy, purely theoretical considerations should not prevent their adoption;

That the present geodetic level of the Survey is as perfect as any yet devised.

Recommendations are made for comparisons of rods with the standard, and for various details of the rod and supports.

ABSTRACT OF REPORT OF THE COMMITTEE ON ALASKA.

The committee presents an account of the extraordinary growth of this but partially explored country, with its valuable resources on land and the limitless wealth in its adjacent waters. Statements are made of the extent and wealth of the forests, the fisheries, the mining, the furs, with statistics of the commerce and population. The surveys already made are considered, and due credit given for the hydrography, triangulation, and astronomical work that has been done. The astronomical work covers the chronometric determinations of several seasons; but the systematic trips of the last two years and the latitude work executed at the longitude stations occupied have added materially to our astronomical control of the work in southeastern Alaska.

The traffic through the great inland water passages of southeastern Alaska made their immediate survey a matter of the most pressing importance. This work is now nearly accomplished; but the committee considers that the present standard of worth should be made uniform throughout. It advises that the topographic features should receive more attention, and recommends that a topographic reconnaissance be made to supply what is needful in this respect.

The question of the order of sequence in which the Alaskan work should be taken up was considered, and the conclusions reached are herein briefly recounted:

First. The great chain of the Aleutian Islands is known to be very imperfectly laid down on existing maps, and the demands of the whaling fleet, the naval vessels of the United States and Great Britain, the cod-fishing fleet, and the shipping and trading vessels require that the passes through this fog-covered line of islands should be better determined.

It is proposed by the committee to establish certain base-longitude stations for chronometric determinations from Sitka, and to connect all the islands by means of terrestrial signals and local time. The heights of the islands combine to render this feasible, and the method is second in accuracy only to the telegraphic method. These longitude determinations with latitude measures will give certain positions on some of the islands. Small triangulation, and perhaps topographic reconnaissance, will then give their outline with the necessary precision for the present.

Second. The great industries that are centered about Kadiak and the western end of the Aleutian peninsula make a survey of this section only second in importance to the one just mentioned.

Third. The welfare and safety of the great fleet of American vessels engaged in the Arctic whaling industry make it necessary and a duty of the Government to establish a good longitude station at their rendezvous, Port Clarence.

Fourth. The necessity for opening a ready means of entrance into the heart of Alaska, that its great possibilities may be thoroughly explored and tested, and the returns that will reward our citizens when the great wealth of salmon that abounds in its waters can be reached, make it important that a survey should be made of the mouth of the Yukon River—the Mississippi of this vast territory—and which is now practically useless to us on account of our almost complete ignorance of the channels and waterways through its delta.

The committee submits plans for the longitude work which would control the independent surveys to be made between Kadiak and Attu. It also suggests arrangements which are possible in connection with the work at Port Clarence and about the mouth of the Yukon, and which might materially reduce the cost of both enterprises.

Its final proposition is a scheme for a triangulation through Clarence and Chatham straits, the great channels of the Archipelago Alexander, which would reenforce and control all the work in southeastern Alaska. In this scheme the character and main features of the work are considered with relation to the meteorological conditions which must be expected and the economy that should be borne in mind in suggesting a plan of operations.

ABSTRACT OF REPORT OF THE COMMITTEE ON INSTRUMENTS.

The committee has confined its attention to the consideration of instruments for astronomical work and the measurement of horizontal angles, because other committees will necessarily consider instruments special to their work.

A list of 52 astronomical instruments and 62 theodolites (namely, 21 direction and 41 repeating instruments) are given. The character and condition of the instruments are described in general terms. Comparison is made between the instruments used in the Coast and Geodetic Survey and in Europe for the determination of latitudes and of telegraphic longitudes; and the committee recommends a lighter form of chronograph.

The 30^{cm} (12-inch) direction theodolites, No. 145 and No. 146, lately constructed at the office, have been tested and are believed to be of superior character.

The committee recommends that all instruments whose graduation is defective should be sent to the office for regraduation, the purchase of 10^{cm} and 18^{cm} (4 and 7 inch) instruments for the work in Alaska, and that erratic motions of the level bubble be studied.

It makes suggestions in regard to protecting instruments in the field against sudden changes of temperature.

In conclusion, the committee states that the instruments for astronomical work and horizontal measures are in general very good.

ABSTRACT OF REPORT OF THE COMMITTEE ON OFFICE AND FIELD RELATIONS.

The committee states the necessity for the effective cooperation between the field force and the office; and, in order to still further promote this object, recommends that the regulations of 1887, with amendments, and the circulars since issued, should be carefully studied and their provisions minutely followed.

The preparation, duplication, and transmission of records should be thoroughly systematic. They, together with a summary report of the season's work, accompanied by a sketch and statistics, should be promptly turned into the office at the close of the season. The field computations should follow as soon as practicable. The condition and character of all instruments should be clearly stated when sent to or from the office. Stations in the vicinity of any field party should be visited when practicable, and their condition reported. If reported as irretrievably lost, the names should be stricken from the office list of geographic positions available for field work.

As the field parties are cut off from easy access to current publications bearing upon their work, the committee recommends that some person be appointed in the office to prepare brief extracts or headlines of matter germane to the Survey and its operations, and that copies of these be transmitted to the parties in the field and at distant stations.

The Superintendent referred to the Conference a paper prepared at his request by the disbursing agent, upon the correlation of the operating department and accounting system of the Coast and Geodetic Survey. This very interesting paper, referred by the Conference to the committee, is appended in full to its report, and should be carefully studied.

The committee recommends the preparation of a manual of observations, records, and computations, embodying the conclusions of the Conference, and a manual of accounts, in accordance with the valuable suggestions contained in the paper of the disbursing agent.

ABSTRACT OF REPORT OF THE COMMITTEE ON THE MEASUREMENT OF ARCS.

The committee shows that the measurements of arcs of the meridian, of the parallel and oblique to these, has been incidental to the extended operations of the trigonometrical survey of the United States. They are moreover of great importance in order to furnish the shape and dimensions of that geometric figure which best represents the particular area to be surveyed and to secure exact measures of geographic position on the earth's surface. Beyond this, the combination of the arcs measured by different countries is indispensable for the determination of the geometric figure which shall, in size and shape, most closely approximate to the figure of the earth as a whole.

For this latter purpose the Government of the United States became a member of the International Geodetic Association for the measurement of arcs, and thereby incurred certain scientific obligations which have in part been fulfilled by the triangulation schemes already executed. The further and necessary extension of these projects of triangulation in the prosecution of the Survey will furnish additional material for utilization in our country and for the International Association.

The arcs in process of execution are mentioned, and a map has been prepared to exhibit the prospective triangulations which may be developed, and which will be, in great measure, available for the location of boundaries, for bases of State surveys, and for geographical positions.

ABSTRACT OF REPORT OF THE COMMITTEE ON MAGNETICS.

The committee states the necessity for a knowledge of the laws governing the magnetic force, in order that the charts of the Survey shall be furnished with the magnetic variation at the dates of issue and the prospective annual change.

The distribution of magnetism is dependent mainly upon geographical position and time, but is influenced by so many local disturbing causes that in order to produce the lines defining the direction and intensity seaward for a certain epoch it is necessary to extend the observations a sufficient distance inland.

Moreover, there is such a constant demand made upon the Survey by surveyors, engineers, and courts of law in every part of the country for information looking to the recovery of old lines or landmarks that a more complete series of observations and study of the distribution of the declination is necessary over the whole area of the United States.

A demand for an accurate knowledge of the magnetic dip and intensity has been made by mariners and electricians.

By observation in the regular progress of the work, the collection of observations from the earliest to those of the present time, and by other means, the Survey has given all necessary information for the charts, and in addition very much material which has been published in print and graphically for the use of surveyors, engineers, etc.

The committee presents general descriptions of the magnetic instruments used in other countries and the recent ones for use in the Survey.

While much of the work of magnetic observation is executed incidentally by the different parties in triangulation and longitude, the committee calls attention to the necessity for a special series of observations, particularly through California, Oregon, Washington, Idaho, Montana, and the Dakotas; also to the necessity of more observations in Alaskan waters.

It recommends the use of compass declinometers by every triangulation party, and that the main and primary triangulation and telegraphic longitude parties determine all the magnetic elements. It also recommends that a second edition of the isoclinic, isodynamic, and isogonic curves be published for an epoch close at hand—say, 1895 or 1900—together with the data, method of discussion, and explanation of the results and their uses.

ABSTRACT OF REPORT OF THE COMMITTEE ON GRAVITY.

The committee states that recent developments in the instruments and methods will enable the Survey to enter successfully upon extended gravimetric research at less cost than was possible with the older processes, and this without lowering the standard of accuracy. The committee refers to the development of the earlier pendulum research on the Survey with respect to character of instruments, their shape, size, and material, as well as the methods of observing.

All nations engaged in geodetic work have considered this subject an important and necessary branch of a geodetic survey, as affording a means of determining the figure of the earth. Our relation with the International Geodetic Association renders it desirable that the Survey should conform, as far as practicable, with the general plan of work of that body.

Five hundred stations have been occupied by foreign observers, while by the Coast and Geodetic Survey 27 have been occupied in this country and 29 abroad. In many of the leading European countries pendulum investigations have been vigorously prosecuted in recent years. In India the English have carried on a very systematic scheme of gravity work in connection with their great trigonometrical survey.

Maps are presented showing the stations occupied in the United States, including Alaska, and in Europe.

The improved pendulum apparatus of the Survey is described. The method of observing permits a ready application of the telegraphic method of determining the difference in the force of gravity at two stations which are connected by wire.

A still more portable apparatus is now being constructed, in which the pendulum has a period of but one-fourth second.

The proposed outline of investigations contemplates the determination of the geographical distribution of gravity within the United States with respect to latitude, elevation, and geological structure, including, for instance, a series of gravity measures along the valley of the Mississippi, and another at right angles to it along the thirty-ninth parallel from the Atlantic to the Pacific.

It is proposed to establish, in the course of a few years, a number of base stations for gravimetric and hypsometric operations of the Survey. These will be determined with the greatest precision. The absolute force of gravity is such an important physical constant and of such great scientific interest as to justify its measurement at a few base stations.

In observations intended for the determination of the figure of the earth it is proposed to restrict the stations to the sea border of the continents and islands of the United States, so as to obviate a reduction to the sea level. The extended range of latitude in the limits of the United States is favorable to this measure.

It is not yet deemed practicable to state what degree of precision may or should be reached in either absolute or relative work, but observations at a station should be continued only long enough to reasonably eliminate the known errors of observation. Multiplication of stations is to be preferred to a high degree of accuracy.

In 1882 a conference on gravity determinations was held at the office of the Coast and Geodetic Survey, and the conclusions which were then reached are given at the close of the committee's report.

ABSTRACT OF REPORT OF THE COMMITTEE ON EQUIPMENT.

The committee has considered this subject under two principal heads, operations conducted by land and operations conducted by water. Each is then considered under several subheads.

The committee recommends the lightest practicable outfits of men, animals, and material, but in conclusion states that owing to the great variety of orographic, climatic, and economic conditions, covering so large an area as this country does, it is impracticable to recommend and specify the details. The results of the experience of officers of the Survey, who have conducted work under nearly all these conditions, are condensed in this report.

While attention to economy calls for simplicity in equipage, the operators ought not to be deprived of such conveniences as tend to prolong their working ability, by securing shelter against the elements, needful rest, and good food. Equipment which may suffice in emergencies will be insufficient for work requiring long sustained efforts, which may be protracted through successive years.

Having thus referred to the reports of the committees in brief terms, the Conference begs to state that several committees have given attention to cognate subjects which may well be incorporated in future editions of the manuals of instructions and the publications of methods, and have made recommendations therefor in their reports.

During the sessions of the Conference facilities have been cheerfully extended by the office in furnishing data and copies of its proceedings, and its investigations have been aided by ready access to the well-arranged library of the Survey.

As ex-officio member of each committee, you have been invited to the meetings of the Conference, and the members of the committees beg leave to acknowledge the advice you were pleased to extend to them.

We thank you for this special opportunity of our meeting each other and personally comparing our experiences.

We feel that the interests of the Survey have been greatly advanced and that the esprit de corps of the Survey has been intensified by this instructive contact.

Very respectfully,

GEORGE DAVIDSON,
Chairman.
O. H. TITTMANN,
Secretary.

T. C. MENDENHALL, LL. D.,
Superintendent United States Coast and Geodetic Survey, Washington, D. C.

PROCEEDINGS OF THE CONFERENCE.

The Conference met at the office of the United States Coast and Geodetic Survey, Washington, D. C., on January 9, 1894, and after completing its labors it adjourned on February 28.

The following officers of the Survey were members of the Conference:

Prof. George Davidson,
Mr. Charles A. Schott,
Mr. George A. Fairfield,
Mr. William Eimbeck,
Mr. O. H. Tittmann,
Mr. J. J. Gilbert,
Mr. F. W. Perkins,
Mr. Edwin Smith,
Mr. J. F. Pratt,
Mr. C. H. Sinclair,
Mr. E. F. Dickins,
Mr. Stehman Forney,
Mr. J. E. McGrath,
Mr. Isaac Winston,
Mr. P. A. Welker,
Mr. C. H. van Orden,
Mr. Fremont Morse,
Mr. W. B. Fairfield,
Mr. F. A. Young,
Mr. G. R. Putnam,
Mr. A. L. Baldwin,
Mr. O. B. French,
Mr. R. L. Faris,
Mr. S. B. Tinsley.

Prof. A. H. Buchanan, of Tennessee, and Prof. W. R. Hoag, of the University of Minnesota, Acting Assistants United States Coast and Geodetic Survey, joined the Conference on February 3, and participated in its deliberations until February 14.

In opening the Conference, Superintendent Mendenhall, while distinctly disclaiming any desire to restrict its deliberations, suggested the following subjects for its consideration, in the treatment of which advantage should be taken of the experience of the several corps engaged upon kindred work in the United States and foreign countries, bearing in mind the advisability of reporting its conclusions in a form which can be utilized in revising and bringing up to date the various handbooks on field methods and results issued by the Survey:

Base-line measurement.—Consider appliances, recent investigations of line measures and methods, and their adaptability to the varying conditions encountered in trigonometrical work. Compare the relative values in a scheme of triangulation of a few bases, measured with a high degree of accuracy, with frequent bases determined with less refinement.

Triangulation.—Define with greater exactness the various classes of trigonometrical work. Discuss the instruments, methods, and precision desirable for each class. Consider the relation of the number of observations to the degree of accuracy demanded by the character of the work, with the object of deciding if any material reduction can be made in the number of observations now taken, especially at primary stations. Consider reconnaissance and signal building, forms of day signals, and the use of night signals. Consider methods of observing and instruments. What besides the necessary measurement of angles should be done at a triangulation station. Submit schemes of triangulation necessary and desirable for a complete survey of the whole country, bearing in mind their utility in fixing boundary lines (State and national) and as furnishing bases for more detailed State surveys. Consider especially the character and scale of work desirable for the survey of Alaska.

Astronomical work.—Methods and instruments, giving consideration to the different classes of work and standards of accuracy. What work, other than astronomical, may be done profitably in connection with the latter.

Hypsometry.—Instruments to be used, and accuracy to be aimed at. Vertical angles; barometric work. Submit a scheme of lines of precise leveling controlling the whole area of the country.

Magnetic work.

Gravitation work.

General.

Party organization.—Camps and outfits—possibility of reduction in their size and cost. Records—their preparation, duplication, and transmission to the office. Field computations—degree of accuracy required. Accounts—relation of field officers to the disbursing agent.

At the close of his address he appointed Prof. George Davidson permanent chairman and Mr. O. H. Tittmann secretary of the Conference.

Mr. G. A. Fairfield was appointed chairman of the committee of the whole. Owing to his inability to act, on account of sickness, he was succeeded by Mr. Gilbert, who acted during the Conference.

A committee on rules, consisting of Messrs. Schott, Sinclair, and Winston, was formed, and their recommendations of an order of business and rules were adopted.

A committee to assist the chairman in the formation of committees, consisting of Messrs. Schott, Sinclair, Pratt, Smith, Eimbeck, and Morse, was appointed; and, in accordance with their recommendation, committees, to which was assigned the task of preparing reports on the various topics under consideration, were organized as follows:

(A) *Reconnaissance.*—Eimbeck (chairman), Dickins, Forney, French, Perkins, Welker.

(B) *Base lines.*—Davidson (chairman), Eimbeck, French, Pratt, Schott, Smith, Tittmann.

(C) *Triangulation and schemes of triangulation for whole country.*—Schott (chairman), Davidson, Dickins, Eimbeck, G. A. Fairfield, W. B. Fairfield, Faris, Gilbert, Perkins, Sinclair, Van Orden, Welker, Young.

(D) *Astronomy.*—Sinclair (chairman), Davidson, Gilbert, Morse, Putnam, Smith.

(E) *Hypsometry and scheme of leveling for whole country.*—Perkins (chairman), G. A. Fairfield, McGrath, Pratt, Van Orden, Winston, Young.

(F) *Alaska.*—McGrath (chairman), Baldwin, Pratt, Tittmann, Morse.

(G) *Instruments.*—Smith (chairman), Davidson, Eimbeck, Gilbert, Pratt, Tittmann, Van Orden, Winston.

(H) *Office and field relations.*—G. A. Fairfield (chairman), Forney, Sinclair, Schott, Winston.

(I) *Measurement of arcs.*—Schott (chairman), Davidson, Eimbeck, McGrath, Putnam, Sinclair.

(J) *Magnetics.*—Gilbert (chairman), Faris, McGrath, Putnam, Schott, Tinsley.

(K) *Gravity measures.*—Superintendent (chairman), Eimbeck (acting chairman), Putnam, Tinsley.

(L) *Equipment.*—Pratt (chairman), Baldwin, Dickins, W. B. Fairfield, Forney, Sinclair, Van Orden, Welker.

Special committee on arrangements.—Tittmann (chairman), Sinclair, Smith, Winston.

To these were added, near the close of the Conference, a committee composed of the Chairman, Professor Davidson, Messrs. Schott and Tittmann, to formulate, for the consideration of the Conference, the general results of its deliberations, in a report to the Superintendent, and an editing committee, composed of Messrs. Tittmann (chairman), Schott, G. A. Fairfield, McGrath, and Putnam.

During the proceedings temporary committees were appointed from time to time for some special purposes, but these need not be more particularly specified.

Under the general distribution of subjects above given, each committee prepared a list of topics to be discussed in their reports. These lists were submitted to the Conference, and after they had been considered and approved the committees proceeded to discuss the subjects assigned to them on the lines thus indicated. The reports prepared in pursuance of this plan by the several committees were discussed in committee of the whole, and were finally adopted by a majority vote of the Conference.

At the beginning of the sessions invitations were extended to the officers of the Survey who were not members of the Conference and to others interested in its labors to submit their views on matters relating to the purpose for which it was convened.

The Conference was also favored by the personal attendance, on various occasions, of Prof. J. H. Gore, of Columbian University, Washington, D. C. Prof. R. S. Woodward, of Columbia College, New York, came to Washington, at the sacrifice of his time and convenience, to reply verbally to questions proposed to him by the Conference. Prof. William Harkness, U. S. N., addressed the Conference on February 7. The following outlines of their remarks are appended:

SYNOPSIS OF PROF. R. S. WOODWARD'S REMARKS.

Addressing himself to a reply of the first question proposed to him by the Conference, namely, the number of measures of an angle necessary in triangulation of the highest order of precision with given instrumental means, Professor Woodward discussed the following heads:

Relation between the magnifying power of telescopes and the reading power of the micrometers; Errors of pointing; Graduation; Instruments; Atmosphere; Centering.

He advocated the use of a magnifying power of about 60 for telescopes, so as to preserve the proper relation between their pointing power and the reading power of the microscopes. He spoke of high power as diminishing the personal equation due to phase of signals and as serving as a test for the steadiness of the atmosphere. He advocated the use of flat signals to diminish phase. He spoke of the instability of the mounting of the theodolites, and advised the return to the measurement of angles (Gauss) rather than the measurement of directions (Bessel), and thought that in general nothing would be gained by ocular micrometer pointings in addition to the simple pointings. He stated that owing to the perfection of modern instruments a large number of positions is no longer necessary, and discussed this matter at some length. In reply to a question, he said that steadiness of atmosphere might be taken as an indication of the absence of lateral refraction. He spoke of the necessity for the careful centering of instruments, especially at base stations, and gave a detailed account of base measurement with steel tapes.

SYNOPSIS OF PROFESSOR HARKNESS'S REMARKS.

Professor Harkness addressed the Conference on the methods of determining the earth's ellipticity and the values deduced from them. He considered successively the following five methods:

Geodetic methods; Pendulum measures; Precession and nutation; Perturbations of the moon; The moon's parallax.

He called attention to the fact that an arc whose middle latitude is in 54° 45′ gives the value for the equatorial radius, while a more equatorial arc—that is, one in latitude 35° 15′—gives the polar radius, and gave two equations in which the numerical coefficients showed the relative effect of known arcs on the determination of the two radii. He stated that American arcs had not been introduced in the equations for deriving the figure of the earth, although the data for two, resulting from the operations of the Lake Survey, had been published.[1]

After discussing the pendulum methods, he presented a tabulation of values derived from pendulum experiments at various times. He explained that the values derived from precession and nutation are vitiated by want of knowledge of the internal constitution of the earth, and that the theoretically exact value from lunar perturbations is rendered questionable by the uncertainty attaching to the observed values of the perturbations themselves, and in illustration gave the observed values of certain perturbations of the moon on which the figure of the earth derived from them depends. In regard to the method by the moon's parallax, he expressed the hope that the cooperation of observatories would lead to a satisfactory determination of the ellipticity by this method.

In his opinion pendulum and astronomical methods will give the most reliable value of the ratio of the earth's semidiameters, and geodetic arc measures will give the linear values with great exactness. He stated that in order to determine the reciprocal of Σ with a probable error not exceeding one unit, the probable error of each of the semidiameters must be reduced to $\pm$ 164·4 feet ($50{\cdot}11^{m}$).

[1] The United States Coast Survey published the results of two arcs and deduced a result for the size and figure of the earth. (Appendix 6, Report for 1877.)

Flattening of the earth as found from pendulum experiments.

[A table prepared by Prof. William Harkness, U. S. N.]

Date.	Author.	1/Σ.
1799	La Place	335·78
1816	Matthieu	317·4
1818	Bessel	310·11
1821	Biot	306·75
1825	Sabine	289·1
1827	Saigey	281·62
1829	Pontecoulant	340·16
1829	Schmidt: Lehrbuch, T. 1, p. 372, 47 E.	288·20
1830	Airy	282·82
1833	Poisson	287·31
1834	Baily	285·26
1841	Peters, C. A. F.,	290·99
1842	Borenius	289·
1853	Paucker	288·38
1869	Unferdinger	289·15
1872	Nyrén	287·73
1876	Fischer, A.,	284·4
1880	Clarke	292·2
1884	Helmert	299·26
1884	Hill, G. W.,	287·73

FEBRUARY 5, 1894.

The Conference, having completed its labors, met for the last time on February 28. The general report to the Superintendent having been read and adopted, a committee, consisting of Messrs. Welker, Morse, and Faris, was appointed to notify the Superintendent of the readiness of the Conference to submit its final report. Upon the arrival of the Superintendent the chair ordered the reading of the report, upon the conclusion of which the Superintendent produced the minutes of the Board of Organization of 1843, on which he commented, saying that they contained evidence of independence of opinion which he thought had also been manifested in the present Conference, in which perfect freedom of discussion was accompanied by the utmost good fellowship. He regretted that other duties had prevented his more frequent attendance at the meetings, and expressed his gratification at the outcome of the Conference, which had already caused a feeling of regret in him that he had not summoned it at an earlier period of his superintendency.

After the departure of the Superintendent, Mr. Sinclair offered the following resolutions, which were unanimously adopted:

Resolved, That this Conference herein expresses its high appreciation of the action of the Superintendent in calling it together, realizing that great benefit has accrued to its members by the interchange of ideas, which can not but result in the increased efficiency of the entire corps;

That the thanks of the Conference be extended to its chairman and to the chairman of the committee of the whole for the courtesy with which they have presided over its deliberations;

That the thanks of the Conference be extended to the secretary for the very satisfactory manner in which he has performed the arduous labors imposed upon him.

Resolutions referring to the loss sustained by the Survey by the recent death of Assistant James S. Lawson were adopted by a rising vote. It was ordered that they be spread upon the minutes and that a copy be sent to his family. The Conference then adjourned.

REPORT OF COMMITTEE A, ON RECONNAISSANCE.

The term "reconnaissance" as here used embraces all those investigations of a region to be triangulated which precede the fieldwork of construction, base measurement, and the measurement of angles, and comprises the selection of the most feasible chain or net of geometric figures, the location of the base lines, the determination of the preparations and appliances necessary, and the collection of information as to the facilities available for carrying out the proposed scheme.

An exhaustive report upon such a subject appeared to your committee to be neither practicable nor advisable; and after a careful review of the articles already published,[1] and their comparison with the experience of those who have been most extensively engaged upon this class of work, it concluded to restrict its labors to the formulation of a few practical suggestions, which it is hoped will be useful in promoting economy and dispatch in the execution of triangulation work in the future, viz:

(1) Reconnaissance, forming as it does the basis of triangulation, should always be thorough and exhaustive, developing all possible schemes and comprising all classes of information affecting the economy and facility of the operations to follow.

(2) Simplification of the geometric configuration of schemes by the more frequent introduction of well-conditioned simple triangles in all cases where the substitution of complex figures, as quadrilaterals with open diagonals or polygons, would necessitate either the undue contraction or expansion of the scheme or the erection of high and costly scaffoldings.

(3) Avoidance, as far as compatible with the requirements of geodetic triangulation, of elevated structures for any purpose except that of overcoming obstacles or lifting the triangulation above the level of highly heated and disturbed atmosphere. In exceptional cases structures of moderate elevation may also prove necessary to preserve symmetry or to attain proper figure conditions, but in no case should mere geometrical elegance of a scheme of triangulation be sought at the expense of reasonable economy.

(4) Avoidance, as far as expedient, of the longest lines or the highest peaks in the mountains of the interior. On account of the clouds which frequently envelop these peaks and the uncertainties in the

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[1] Appendix No. 20, Coast and Geodetic Survey Report, 1876; Appendix No. 9, Coast and Geodetic Survey Report, 1882; Appendix No. 10, Coast and Geodetic Survey Report, 1885.

seeing, lines of a greater length than 200 kilometres (124 miles) invariably tend to interrupt and delay the progress of the work. In addition to this, the occupation of the highest mountains is always expensive by reason of the preparations required to make them accessible and of the difficulties of transportation. Lines that pass closely along the slopes of mountains or hills or near the vertical surface of any object, as large buildings, or that pass through narrow avenues cut through timber should be avoided if possible, as they are particularly liable to lateral refraction.

(5) Attention is also called to the degree of adaptability possessed by several typical figures commonly used in triangulation of the first order. These simple triangles, or chains of triangles, easily adapt themselves to topography of the most complex character, whereas quadrilaterals, with observable diagonals, possess this quality in the least degree, and on that account will frequently be found impracticable figures. Difficult stretches of country may, therefore, always be most easily crossed by simple triangles. Pentagons, or quadrilaterals with central points, also readily conform to the configuration, however complex or difficult it may seem. The hexagon, however, on account of the disposition of the stations, tends to retard rapid progress, and should not, therefore, be included in a chain of triangulation. This figure finds its most advantageous application when large areas are to be covered, as in the survey of a whole State.

(6) It is recommended that in all extensive chains of triangulation sites for base lines be considered and selected at intervals of from 250 to 320 kilometres (155 to 200 miles), or at every eighth, ninth, or tenth figure, according to the length of the bases, the character and scope of the scheme, and that their connection with the main chain be carefully developed in order that these connections may be simultaneously executed with the main work.

(7) Whenever the purpose of a chain of triangulation requires the traversing of flat or rolling country its trend should, when practicable, rather follow than cross the drainage. Experience has shown that when the drainage of a country has to be crossed by the triangulation it can usually be done only by contracting the scheme or by elevating the stations by means of scaffoldings. Similar conditions are met with in crossing high table-lands or flat-topped or double-crested ranges of mountains, as, for example, the Sierra Nevada, in the region about Lake Tahoe.

(8) Before taking the field the reconnoitering officer should, by a careful study of all available maps, make himself as thoroughly acquainted as possible with the character of the country to be treated, the lines of travel, the location and relative importance of towns and villages, and, above all, the drainage, as the latter will in a great measure determine the size of the scheme.

(9) The field notes of the reconnoiterer should be exhaustive. They can never be too full for himself or his successors.

Horizontal and vertical angles should be taken on all prominent peaks and objects, even though they are not directly included in the scheme, for they are often invaluable for purposes of orientation. Approximate latitude and azimuth observations should also be made, and the magnetic bearing of prominent points be noted. Specify the difficulties of the country, describe and carefully sketch the entire horizon, particularly every opening or notch through which more distant mountains can be seen. Every high point or possible station should be especially noted.

The plotting should be kept up from day to day, and a rough topographical sketch made, showing the main ridges, water courses, roads, trails, and habitations. Comprehensive notes with regard to means of transportation, subsistence for man and animals, help, materials, and accessibility are invaluable. Remarks with reference to weather and climatic conditions are important and desirable.

(10) On completion of the reconnaissance, or of the season's work, a report, together with sketches or diagrams illustrating the schemes developed, should be prepared. This report, containing full and explicit statements setting forth the economical and other advantages and disadvantages, is to be promptly transmitted to the Superintendent for his consideration and action.

OUTFIT AND INSTRUMENTS REQUIRED.

The camp outfit will necessarily vary with the character of country to be traversed, and should be as light and portable as possible for reconnaissance work in a mountainous or unsettled country.

The outfit of instruments is simple, but must be adequate for the purpose, viz:

Two aneroid barometers.
One 4-inch theodolite with vertical circle and tripod.
One reconnoitering telescope of 3-inch aperture.
Azimuth compass, hand level, good binocular.
Pocket box of drawing instruments, protractor and scale.
Steel tape.
Two heliotropes for testing the doubtful intervisibility of stations.
Best maps of the country available.
Projection with river courses and known roads drawn thereon.
Small photographic apparatus, with films.
Note and sketch books.
A gradienter will be found a valuable instrument for reconnaissance.

WM. EIMBECK,
Chairman of Committee.
OWEN B. FRENCH,
Secretary.

REPORT OF COMMITTEE B, ON BASE LINES.

The Committee on Base Lines makes the following report to the Conference:

It is not necessary to enter into any historical account of the different apparatus that have been used at various periods.

A short statement of the instruments in use, or lately in use, in foreign countries and in the United States, and the results obtained from them, will be given, but in the time at our disposal it can not be complete nor will the material in print offer means for an exhaustive comparison.

The following classification of base apparatus covers all appliances in general use:

(1) Contact apparatus, comprising compensating, bimetallic, and monometallic.

(2) Optical apparatus, comprising bimetallic, monometallic, and bars in ice.

(3) Steel tape and wire apparatus.

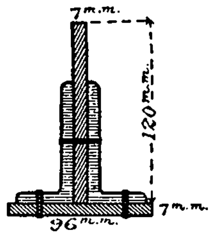
The following is a brief description of the more prominent appliances recently in use, but it must be distinctly remembered that all the relative errors given, excepting those of the Great Trigonometrical Survey of India, Lake Survey, and Coast and Geodetic Survey, are merely the errors of measurement, and do not include the error of reduction to sea level, nor the error of the unit of length of the apparatus, the latter being undoubtedly one of the largest sources of error in the determination of the length of a base line.

Base apparatus now in use—bimetallic and monometallic (*optical*).

SPAIN.

The Spanish bases have been measured with two appliances. The earlier, designed by Brunner, is 4^{m} long and consists of two bars, one of platinum, whose cross section is 5^{mm} by 21^{mm}, and the other is of brass, of the same dimensions. These bars are fastened together in the middle, and at each end Borda scales are attached to obtain the difference of length due to changes of the temperature. They are mounted on rollers in the carrying case to allow longitudinal movement. The microscopes are mounted on independent tripods. One base has been measured with this apparatus with an average relative error of measurement of $\frac{1}{5\,865\,800}$. (See "Memorias del Instituto Geographico," Vol. V, 1884, p. 99.)

Subsequent to the year 1865 an apparatus was devised by Colonel Ibañez, which consists of a 4^{m} iron bar in the form of an inverted "T," of two bars of iron 7^{mm} thick, the one in the vertical position being 120^{mm} high and the one in the horizontal 96^{mm} wide. They are fastened together throughout their entire length with angle irons bolted through the two bars. Four mercurial thermometers are placed in metallic contact with the bars. The graduation trace for the micrometer pointing is made on the top surface of the vertical member of the bar. In the field measurement the apparatus is uncovered, but the operations are conducted under a wooden shelter.



Eight bases have been measured with this apparatus with relative errors of measurement ranging from $\frac{1}{1\,244\,999}$ to $\frac{1}{4\,181\,300}$. (See Memorias del Instituto Geographico, Vol. III, 1881.)

Three bases have been measured in Switzerland with this apparatus. Their lengths are 2·4, 2·5, and 3·2 kilometres, and the relative errors of measurement are, respectively, $\frac{1}{2\,700\,000}$, $\frac{1}{1\,960\,000}$, and $\frac{1}{2\,460\,000}$.

Base apparatus—bimetallic compensating (optical)

INDIA.

The bases of the Great Trigonometrical Survey of India have all been measured with the Colby apparatus, which is compensating and optical, the unit of measurement being the yard. The bars of brass and iron are 10·1 feet (3·08^{m}) long, 0·55 by 1·5 inches (14·0^{mm} by 38·1^{mm}) in cross section, and 1·3 inches (33·0^{mm}) apart, rigidly connected at their centers by a pair of small steel cylinders. These bars are free to expand from or contract toward their centers independently of each other. Across each extremity is pivoted a short compensating lever projecting beyond the iron bar. The compensation point is marked on a silver pin near the extremity of each lever. The distance of this point from the axes of the pivots of attachment to the brass and iron bars is intended to be exactly in the same proportion as the coefficient of expansion of the two metals are to each other.

The compound bar is inclosed in a wooden case, and each component rests at one-fourth and three-fourths of its length on brass rollers which are fixed to the bottom of the box and have raised flanges to prevent lateral motion. Longitudinal motion is prevented by means of a stay fixed firmly to the bottom of the box at its center and projecting upward between the two steel cylinders. Here a spirit level is attached parallel to the direction of the bars, and is read through a glass cover in the top of the box. The compensation levers project about 2 inches beyond the sides of the box nearest the iron bar. There are six of these compound bars, designated *A*, *B*, *C*, *D*, *E*, and *H*, supported on strong brass tripods which are capable of communicating motion in a lateral, longitudinal, and vertical direction. The points of compensation of these bars are placed about 6 inches apart, and the distance between is read by a pair of microscopes attached to two parallel bars of brass and iron, both being free to expand or contract toward their centers. The adjustments are so made that the outer foci of the object glasses are compensation points exactly 6 inches apart.

As the measures are invariably made in the horizontal position of the bars, the ground has to be prepared very carefully and vertical offsets are made from time to time as the inclination of the ground demands. The apparatus requires 8 officers. The number of men is not stated.

Ten bases average between 6 and 7 miles in length, and were measured with an average speed of about 1,000 feet (305^{m}) per day. Excluding all constant errors, the probable error of a single measurement is $\pm\frac{1}{667000}$. Including all known and estimated errors, the probable error of a single measurement of a base is taken at $\pm\frac{1}{384000}$. (See Report Great Trigonometrical Survey of India, Vol. I.)

Base apparatus.

BAVARIA.

In Bavaria the Reichenbach apparatus was used in the early part of the century. It consisted of five iron bars each 4^{m} long and 22^{mm} square. The ends of these iron rods terminated in blunt knife edges, that at one end being in the vertical and the other in the horizontal plane. During the process of measurement a vertical and horizontal knife edge are brought within about 4^{mm} of each other, the distance between them being measured with a carefully constructed, graduated, long tapering wedge of hardened steel. The temperature was obtained with mercurial thermometers brought into contact with the metal of the bar.

The second longest base on record, nearly 20 kilometres long, was measured with this apparatus.

SWEDEN.

Prof. Edward Jäderin, of Stockholm, has experimented quite extensively with metallic wires and tapes, using two different metals, such as brass and iron, for the purpose of obtaining the temperature correction from them. No base lines of note have, however, thus far been measured in Europe with such apparatus. His results are published under the following titles:

"Geodätische Längenmessungen mit Stahlbändern und Metalldrähten, von Edw. Jäderin, Stockholm, 1885;" also "Exposé élémentaire de la nouvelle méthode de M. Édouard Jäderin pour la mesure des droites géodesiques au moyen de Bandes d'acier et de fils métalliques, par P. E. Bergstrand, ingénieur au bureau central d'arpentage, à Stockholm, 1885;" and "Marklig Längdförändring hos geodetiska basmätningsstrangar at Edv. Jädrin, 1893, Stockholm."

Base apparatus—bimetallic, noncompensating, contact.

THE BESSEL BASE APPARATUS.

As this appliance has been used in several countries, a general description will answer. In the usual form of the apparatus the measuring bar is of iron, of rectangular cross section, on the upper side of which rests a similar bar of zinc, but somewhat shorter. One end of this zinc bar is securely fastened to the iron bar near one of its extremities, the other end being free to expand. At each end of the iron measuring bar and on its upper surface are permanently fixed blocks of metal, the outer ends of which terminate in hardened knife edges, situated in the vertical plane. The fixed block on the end at which the free end of the zinc element rests has another vertical knife edge pointing toward the free end of the zinc bar, which also terminates with a vertical knife edge, leaving a small space between these two edges. The difference of expansion between the iron and zinc for temperature correction to the iron bar is here measured by inserting a carefully constructed graduated glass wedge. The customary method is to use four of these bars, placing their ends near each other and measuring the small intervening space with the glass wedge previously mentioned.

This apparatus has been used in the following countries:

In Belgium two bases have been measured, being, respectively, 2·3 to 2·5 kilometres in length, with relative errors of measurement of $\frac{1}{1\,700\,000}$ and $\frac{1}{2\,215\,000}$.

In Prussia eight bases have been measured, ranging in length from 900 toises (1·7 kilometres) to 7·0 kilometres, with relative errors of measurement ranging from $\frac{1}{863\,000}$ to $\frac{1}{1\,400\,000}$.

In Denmark one base 1385 toises (2·7 kilometres) long has been measured with relative error of measurement of $\frac{1}{560\,500}$.

In Italy nine bases have been measured with it, ranging in length from 340 (0·66 kilometres) to 5,150 toises (10·0 kilometres), with relative errors of measurement ranging from $\frac{1}{590\,000}$ to $\frac{1}{2\,280\,000}$.

Base apparatus—bimetallic, noncompensating (optical).

THE BRUNNER APPARATUS.

As this apparatus has also been used in several countries, only a general description is attempted.

It is a single bar 4^{m} long. As used by the French and Germans the measuring bar is of platinum-iridium, attached to which is a brass one. The two form a Borda scale for determining the difference of the lengths of the bars due to changes of temperature. The microscopes mounted on independent supports have an arrangement for making optical cut-offs and also an attachment for aligning the bars. The measurements are now made under canvas-covered frames, which are brought forward in sections.

At present this apparatus is the one that is in general favor on the Continent.

It has been used in the following countries:

In Prussia three bases of 1198 toises (2·3 kilometres), 1417 toises (2·8 kilometres), and 2·5 kilometres have been measured, with respective relative errors of measurement of $\frac{1}{4\,600\,000}$, $\frac{1}{4\,600\,000}$, and $\frac{1}{4\,600\,000}$.

In France three bases have been measured with it, for only one of which the relative error is given. This is the Paris Base, which is 7·2 kilometres long, with a relative error of measurement of $\frac{1}{2\,400\,000}$. This base was measured by two details of officers and men, each detachment consisting of 4 officers and 25 men. The operations were conducted with a rapidity of about 300^{m} per day.

Base apparatus—Mono metallic contact.

RUSSIA.

The principal apparatus used is that of Struve, which consists of an iron bar 12 French feet (nearly 4^{m}) in length and 15 by 15 lines (about 34^{mm} by 34^{mm}) in cross section. One end of the bar terminates in a steel plane and the other has a lever pivoted to it. This lever is so arranged that the bar coming in contact with it acts as a fulcrum and its longest free end moves over a divided scale.

Mercurial thermometers, whose bulbs are placed in cavities in the iron bars, are used.

Seven bases have been measured with this apparatus, ranging in length from 1·15 to 2·9 kilometres. The same relative error of measurement is given for all of them, viz, $\frac{1}{1\,240\,000}$.

For the measurement of secondary bases the Jäderin apparatus has been used, and is considered sufficiently accurate for the purpose—namely, cartographic operations.

Contact scale—Mono-Metallic.

AUSTRIA.

In Austria-Hungary an apparatus is used consisting of an iron bar of rectangular cross section, which rests on 12 brass plates fastened to the top surface of a wooden beam which has a wooden cover. The ends of these iron measuring bars, of which 4 are used in regular succession, have plane end surfaces. During the operation of measurement the bars are so placed that their ends are a short distance apart, and the distance between is measured with a short scale made in two parts, which slide on each other, the respective ends of which come in contact with the bars.

Two mercurial thermometers are used for determining the temperature of each bar. Five observers and 16 men are required to use this apparatus. Nineteen bases have been measured with this appliance; all of them twice in opposite directions. They range from 2·4 kilometres to 9·5 kilometres in length. The relative error of measurement is only given for one, that of the d'Ilidze base, which is $\frac{1}{3\,700\,000}$.

Base apparatus—bimetallic, contact, compensating.

THE LAKE SURVEY OF THE UNITED STATES.

The first refined appliance used by this organization was a copy of the Bache-Wurdemann contact compensating apparatus, as used by the Coast Survey at that time. These bars are 15 feet ($4{\cdot}57^{m}$) long, and were used, between the years 1867 and 1875, in the measurement of five bases, whose lengths ranged from 4·9 to 8·8 kilometres, with relative errors ranging from $\frac{1}{530000}$ to $\frac{1}{918000}$.

OPTICAL BIMETALLIC, NONCOMPENSATING.

Subsequently the Lake Survey Repsold optical bimetallic apparatus was devised and used. This consists of a steel measuring bar 4^m long, by the side of which is a similar one of zinc. The two are firmly fastened together in the middle. Their unequal expansion is observed upon scales at both ends for determining the temperature of the steel bar. This combination of the two metals is supported by a system of rollers adjusted inside the carrying tube so as to keep them in their proper relation to each other and to allow free expansion. There were also two mercurial thermometers at each end, with their bulbs inside the tube cylinder. The bar is provided with a telescope for alignment and a sector for measuring the inclinations. In measuring, the bar is supported at the extreme ends of the carrying tube on trestles whose heads are provided with movements in three directions, by which the tube is placed in position under microscopes which are mounted on independent stands.

Five officers and recorders and 12 laborers were required to make the measurements, an average day's work being about 300^m.

Three bases were measured with this apparatus between the years 1877 and 1879, ranging in length from 6·2 to 7·6 kilometres, with relative errors of from $\frac{1}{1029000}$ to $\frac{1}{1071000}$. (See Comstock's report "Triangulation of the Great Lakes," professional papers, Corps of Engineers, U. S. A., 1892.)

COAST AND GEODETIC SURVEY.

The Coast and Geodetic Survey has used various appliances. As early as 1817 a single bar 8^m long, made up of four iron bars 2^m each in length, clamped end to end, was devised and constructed by Mr. Hassler. The relative errors with this apparatus were about $\frac{1}{380000}$. This was the first optical monometallic apparatus used in this country. It also had the aligning sector attached to the top of the carrying case and the terminal micrometer microscopes identical in principle with those now in use. (American Philosophical Transactions, Vol. II, New Series, pp. 273–286.)

In 1845–46 the Bache-Wurdemann lever contact compensating apparatus was devised, and was in use up to 1873. The best results that have been obtained with it have a relative error of about $\frac{1}{800000}$. Since then the Schott compensating apparatus with contact slide has been used on two different bases, only one of which has been finally reduced, viz, the Yolo, 17½ kilometres in length, with a relative error of $\frac{1}{1821000}$.

In 1891 a steel bar in ice, with optical apparatus, was used in measuring a kilometre of the Holton Base, with a relative error of about $\frac{1}{2300000}$.

For secondary bases the contact slide monometallic apparatus with mercurial thermometers has been used since 1855. It has demonstrated its possibilities up to a degree of accuracy of one part in 600000.

Since 1890 two bases of 5·5 and 3·8 kilometres have been measured with the standard tape with relative errors not exceeding one part in 500000.

Wire measurements have been in use since the year 1848, varying in degree of accuracy as required by the conditions to be fulfilled.

The Eimbeck duplex apparatus, although suggested in February, 1885, has been only recently constructed by the Coast and Geodetic Survey. Its principle is to make the bars themselves determine the correction due to changes of temperature without having recourse to the uncertain use of thermometers in the field, excepting it be to furnish additional data.

The later forms of apparatus in the United States are more fully referred to and discussed individually.

WOODWARD STEEL BAR IN MELTING ICE.

For a test of the performance of different kinds of apparatus we turn to the experiences gained in this country by the measurement of the Holton base in Indiana, on a section of which, for the first time, the temperature effect was eliminated by the measurement of 1 kilometre of the base with a steel bar in ice. The section was then measured with a 100^m steel tape apparatus, and, thirdly, with the secondary contact slide apparatus, forms of apparatus so radically different that an agreement between the results obtained by them increases very greatly the probability of correctness of the several results.

The detailed report of the experiments, measurement, and reduction of the Holton Base will be found in the United States Coast and Geodetic Survey Report for 1892, Part II, Appendix No. 8.

The salient facts resulting from the experiments made there, appear to be that with the 5^m steel bar in melting ice, whereby the temperature error is eliminated, a base line can be measured with a degree of accuracy hitherto unapproached.

One kilometre of the Holton Base was measured four times with this bar in eight working days. After proper preparation 800 bars were laid in forty-two and one-half hours, at an average rate of 19 bars, or 95^m, per hour, and a maximum rate of 30 bars, or 150^m, per hour. This rate of measurement is satisfactory. In cost of operation this apparatus is little, if any, inferior to the Brunner apparatus now used abroad.

THE SECONDARY CONTACT SLIDE APPARATUS.

In the measurement of the Holton Base with this apparatus, after comparisons with the 100^m comparator there established, a degree of precision was obtained far beyond the needs of the triangulation dependent thereon. The length of the base line is 5·5 kilometres, and it was measured twice in fourteen days. Two thousand six hundred bars were laid in eighty-three and one-half hours, with an average rate of 31 bars, 155^{m}, per hour, and a maximum of 41 bars, or 205^{m}, in one hour.

No grading, and very little preparation of the ground, is necessary for this apparatus. The cost of measuring a base with it is about the same as with the steel tape.

It is believed that if the bar be made to rest upon rollers and the thermometers be placed in better metallic contact with the bar the apparatus will be improved, with very trifling addition to the weight.

This apparatus should be compared, before and after any base measurement, with the 100^{m} comparator, or directly with the 5^{m} steel bar in ice.

In the measurements of bases by bars the contact slide has been used with sufficient frequency to establish its efficiency for rapid and accurate measurement. The range of error in making a coincidence does not exceed one-twentieth millimetre, and in the large number of contacts made in one measurement of a base the probability is that the plus and minus errors will balance each other.

STEEL TAPE.

The steel tapes, graduated to 100^{m}, at present in use on the Coast and Geodetic Survey are $101·01^{m}$ long, $6·34^{mm}$ by $0·47^{mm}$ in cross section, and weigh 22·3 grammes per metre of length. They are subdivided into 20^{m} spaces by graduations ruled on the tape itself. The end graduations fall about $0·5^{m}$ from the tape ends, which terminate in loops formed by annealing and riveting the tape back on itself. The surface of the tape, where it is not polished to receive the graduation, is of a dull black color. When not in use the tapes are rolled up on reels not less than $0·3^{m}$ in diameter, and may thus be transported with ease and safety. The stretchers used with these tapes are fully described in Professor Woodward's report of the Holton Base, as also the method of preparing the line, making the measurement, protecting the thermometers, etc. The determination of the temperature is, as in all other forms of base apparatus, the most uncertain element in the operation, and needs further study and investigation.

The experiments and measures at the Holton and St. Alban's bases, which were measured six and five times, respectively, have shown relative errors of 1 : 1 300 000, which are very satisfactory, and may be diminished when the temperature is more accurately determined. Judging from these experiences, the steel tape commends itself for accuracy, and over many varieties of surface and classes of ground illy adapted to an apparatus supported on trestles, great economy may be expected from its use.

Carefully and judiciously handled, the steel-tape apparatus will doubtless attain a good standing throughout the United States, where so many extensive and independent surveys are being carried on, and especially in all newly undertaken projects.

The published accounts of foreign base measurements do not give sufficient data to judge of the cost of the measurement nor of the number of officers and men necessary, save in two instances; neither are the rates of measurement always specified, although it may be assumed to be very close to 300^{m} per diem.

The officers and men belong to the military arm of the Government service, and even if we knew their numbers the cost of measurement would not be readily comparable with civilian work. It is very likely that more soldiers are detailed for the work of measurement than are absolutely necessary.

For example, we find for the measurement of the Bonn Base the following detail: One civilian chief, 6 officers, 8 officials, 2 subalterns, 11 pioneers, and 42 infantry soldiers, a total of 70 persons. And in the measurement of the Paris Base 58 officers and soldiers served in two detachments at different times.

The following tables have been appended to give an idea of the number, length, etc., of the bases measured in foreign countries and in the United States.

Table I shows the number of geodetic bases measured in the various countries doing geodetic work.

Table II shows the approximate length of these bases.

Table III gives the principal bases of the United States, together with approximate length, relative error, and apparatus used.

Tables IV and V give more completely, statistics of a few of the most important bases of the United States.

TABLE I.—*Table showing the number of geodetic bases measured by the various countries doing geodetic work.*

Name of country.	Number.	Name of country.	Number.
Austria-Hungary	19	Norway	4
Belgium	2	Peru	1
Denmark	2	Portugal	1
France and Algiers	13	Russia	20
Germany	15	Spain	9
Great Britain	7	Sweden	6
India and Cape Colony	13	Switzerland	5
Greece	1	United States	29
Holland	1		
Italy	9	Total	157

TABLE II.—*Table showing lengths of geodetic bases compiled from 157 bases of the world.*

Lengths.	Number.	Lengths.	Number.
0– 1 kilometres	1	11–12 kilometres	12
1– 2 "	5	12–13 "	6
2– 3 "	28	13–14 "	3
3– 4 "	15	14–15 "	3
4– 5 "	13	15–16 "	--
5– 6 "	16	16–17 "	--
6– 7 "	10	17–18 "	3
7– 8 "	10	18–19 "	--
8– 9 "	14	19–20 "	2
9–10 "	5	20–21 "	--
10–11 "	10	21–22 "	1

TABLE III.—*A table of the most important bases in the United States.*

Name of base.	State.	Date.	Observer.	Apparatus.	Approximate length.	Probable error.	Relative error.
U. S. Coast and Geodetic Survey.					*km.*	*mm.*	
Fire Island	New York	1834	F. R. Hassler	Hassler 8^m bars	14·0	58·5	1–240 000
Kent Island	Maryland	1844	J. Ferguson	" "	8·7	38·1	1–228 000
Boston and Providence R. R.	Massachusetts	1844	E. Blunt	" "	17·3	35·8	1–484 000
Dauphin Island	Alabama	1847	A. D. Bache	Bache-Wurdemann 6^m	10·6	26	1–410 000
Bodies Island	North Carolina	1848	"	" "	10·8	25·5	1–425 000
Edisto Island	South Carolina	1850	"	" "	10·7	25·6	1–419 000
Key Biscayne	Florida	1855	"	" "	5·8	12·7	1–456 000
Cape Sable	"	1855	"	" "	6·4	15·7	1–410 000
Epping Plains	Maine	1857	"	" "	8·7	15·8	1–552 000
Craney Island	Virginia	1869	R. E. Halter	6^m contact slide bars	5·1	37·0	1–140 000
Portsmouth Island	North Carolina	1870	G. A. Fairfield	" "	9·0	49·1	1–181 000
Peach Tree Ridge	Georgia	1872–3	C. O. Boutelle	Bache-Wurdemann	9·3	16·6	1–562 000
Lebanon	Tennessee	1877	A. H. Buchanan	6^m contact slide bars	7·3	14·7	1–490 000
Spring Green	Wisconsin	1878	J. E. Davies	4^m " "	4·7	17·8	1–263 000
Louisville	Kentucky	1879	G. A. Fairfield	6^m " "	8·2	32·0	1–256 000
El Paso	Colorado	1879	O. H. Tittmann	6^m " "	11·3	18·6	1–607 000
Greenville	Mississippi	1880	C. H. Boyd	4^m " "	2·1	9·7	1–216 000
Yolo	California	1881	G. Davidson	5^m Schott compensating	17·5	9·57	1–1 820 000
St. Paul, Snelling Avenue	Minnesota	1888	C. O. Boutelle	6^m contact slide bars	8·7	-----	-----------
Los Angeles	California	1889	G. Davidson	5^m Schott compensating	17·5	-----	-----------
Holton	Indiana	1891	O. H. Tittmann	5^m contact slide	5·5	3·50	1–1 570 000
"	"	"	R. S. Woodward	100^m steel tape	5·5	4·00	1–1 370 000
St. Albans	West Virginia	1892	"	" "	3·87	3·0	1–1 290 000
U. S. Lake Survey.							
Minnesota Point	Minnesota	1870	C. B. Comstock	Bache-Wurdemann	6·1	11·4	1–530 000
Fond du Lac	Wisconsin	1872	E. S. Wheeler	" "	7·4	11·4	1–649 000
Keweenaw	Michigan	1873	"	" "	8·8	10·6	1–830 000
Sandy Hook	New York	1874	"	" "	4·9	5·3	1–918 000
Buffalo	"	1875	"	" "	6·8	7·6	1–889 000
Chicago	Illinois	1877	"	Repsold	7·5	7·4	1–1 052 000
Sandusky	Ohio	1878	"	"	6·2	5·4	1–1 148 600
Olney	Illinois	1878	"	"	6·6	6·4	1–1 013 000

TABLE IV.—*United States Coast and Geodetic Survey—Measurements of primary base lines.*

[Prepared by Assistant Edward Goodfellow.]

THE BACHE-WURDEMANN COMPENSATING BASE APPARATUS.

Year of measure.	Locality, etc.	Length of base.	Probable error of length.	Proportional part of length.	Days occupied.	Average length per day.	Mean temperature of measure.
		m.	*m.*			*m.*	F°.
1847	Dauphin, Island, Ala. Sand, low grass, rushes, etc.	10661·8376	±0·0260	$\frac{1}{410071}$	17	627·17	84·5
1848	Bodies Island, N. C. Sea beach, sandy and marshy.	10841·7524	±0·0255	$\frac{1}{425167}$	10	1084·17	52·0
1850	Edisto Island, S. C. Cultivated fields, clay and loam.	10721·4231	±0·0286	$\frac{1}{418806}$	13	824·72	59·5
1855	Key Biscayne, Fla. Calcareous soil, coarse grass, palmettoes, etc.	5789·2262	±0·0127	$\frac{1}{455845}$	9	664·32	82·9
1855	Cape Sable, Fla. Calcareous soil, grass, samphire weed, etc.	6431·5913	±0·0157	$\frac{1}{409656}$	8	803·95	87·9
1857	Epping Plains, Me. Rolling, sandy plain, many irregular ridges.	8715·9422	±0·0158	$\frac{1}{551642}$	8	1089·49	70·0

NOTE.—1894, January 27—The data given here and on the following page for the seven primary bases measured with the Bache-Wurdemann apparatus have been taken from the annual reports, except for Epping Base, which was furnished by Assistant Schott.

TABLE V.—*Base line statistics, February, 1894—United States Coast and Geodetic Survey Conference.*

Date.	Name, locality, etc.	Apparatus.	Number of men used.	Number of times measured.	Length.	Probable error.	Proportional part of length.	Number days measuring.	Number hours actually laying bars (*b*).	Average number bars per hour or (*a*) (*b*).	Number of bars laid (*a*).	Greatest number bars laid in 1 hour.	Remarks.
					m.	*m.*							
1872	Atlanta Base, Peach Tree Ridge, near Atlanta, Ga.—Soil, loam and clay.	Bache-Wurdemann bimetallic compensating (6^m).	22	3	9,338	0·0166	$\frac{1}{562\,539}$	41	252	19	4,747	30	
1879	El Paso Base, El Paso County, Colo.	Secondary bars and thermometers Nos. 13 and 14, monometallic (6^m).	10	2	11,289	0·0186	$\frac{1}{606\,941}$	23	125	$30\frac{1}{2}$	3,822	43	
1881	Yolo Base, Yolo County, Cal.—Soil, dark loam, stiff clay, and some sand.	Schott compensating 5^m zinc and steel bars with thermometers.	19	2	17,486	0·0096	$\frac{1}{1\,821\,512}$	46	247	$34\frac{1}{2}$	8,494	57	
1889	Los Angeles Base, California.	do.	18	3	17,495			50	300	35	10,597	64	Highest rate, 400 bars in $7^h\ 24^m$.
1891	Holton Base, Ripley County, Ind.	Secondary bars and thermometers Nos. 3 and 4 (5^m).	9	2	5,500	0·0035	$\frac{1}{1\,570\,000}$	14	83·5	31	2,600	41	
1891	do.	Iced steel bar No. 17 and microscopes.	88	4	1,000		$\frac{1}{2\,300\,000}$	8	42·5	19	800	30	
1891	do.	100^m steel tapes Nos. 85 and 88.	12	6	5,500	*0·004	$\frac{1}{1\,375\,143}$	17	29·5	17	†508	30	10 in 20 minutes.
1892	St. Albans Base, West Virginia.	do.	12	5	3,870	*0·0030	$\frac{1}{1\,290\,133}$	4	11·4	17	†195	28	Most of the tape measures were made at night.

* Obtained from Professor Woodward's computations.

† Number of tape lengths.

From the six earlier bases measured with the heavy Bache-Wurdemann apparatus given in Table IV we condense the following statistics: Each base was measured but once, and the total of the lengths measured was 53 kilometres, at the rate of 849^m per diem, and the highest daily average was 1089^m. No hours are given, and the number of officers and men is not mentioned. The relative error varied between 1:410 000 and 1:553 000; but these included the error of unit of standard and reduction to sea level.

From the later bases of the Coast and Geodetic Survey, given in Table V, the following statistics are obtained:

Five bases each employed 17 men, who measured 150^m per hour, and reached 2000^m in seven and one-half hours (in third measurement). The reduced measures give relative errors from 1:563 000 to 1:1 822 000. Two bases measured with steel tapes employed 12 men, who measured the bases six and five times, respectively, at the rate of 1280^m per hour. These gave relative errors of 1:1 375 000 and 1:1 290 000. The relative errors of all these measurements include the unit of standard, etc.

In the measurement of Yolo and Los Angeles bases with the Schott apparatus the following officers and men were employed: Six officers, 6 men at bars, tripod, and plates, 5 men pushing movable cover, etc. Besides these necessary people, there were 1 man for bridges, extra driver, etc., 1 watchman, 1 driver, 3 cooks and waiters.

THE SITE OF THE BASE LINE.

In a mountainous region the selection of a site is frequently a very difficult matter. It should be located by the reconnaissance party, and, if practicable, more than one should be selected.

One of the factors in the rapidity of measurement of the base line, whether primary or secondary, is the character of the site. In any case it should be prepared with only sufficient care to permit effective measurement by the apparatus used. If bars and trestles are used, the surface should be no more disturbed than to permit the prompt placing of the tripod supports.

FIELD PROCEDURE.

The method of procedure in the field must necessarily vary according to the apparatus used and the local conditions of the country. An example of the method used with the slide contact bars can be found in reports on the Yolo and Los Angeles bases in the United States Coast and Geodetic Survey Report for 1882, Appendix 8, and Report for 1889, Appendix 10. For the method used with the iced steel bar and 100^m steel tapes see Professor Woodward's reports on the Holton and St. Albans bases.

In the work of measurement the first measure requires time for drilling all persons in precision and rapidity of action, and a repetition of the measure can be made in much less time than the first and very probably with more accuracy. The rapidity of measurement conduces to accuracy when movable tripod supports are used, because it lessens the liability of change in the position of the bar.

For transferring the end of the bar to the ground when measurement is suspended various forms of apparatus have been used. For end contact bars some form of transit sector is generally used. It is set up at right angles to the base, opposite to the end of the bar, the terminal of which is transferred by means of it to a scale placed on the ground mark. When the optical measuring apparatus is used, the Repsold cut-off is preferred, a description of which can be found in the Report of the Lake Survey.

THE ACCURACY ATTAINABLE IN BASE MEASUREMENTS.

The great importance to geodesy of the adoption of the metre as the international unit of length need not be pointed out, but the establishment of an International Bureau of Weights and Measures for the comparison of standards is here adverted to, not only in recognition of its eminent services to geodesy, but because its work must now be taken as the standard of attainable accuracy in metrological work.

Attention may also be called to the results of a study made recently at the International Bureau of Weights and Measures of nickel and certain alloys in regard to their suitability for line standards. Nickel bronze and aluminium bronze showed a decided change in length after repeated heating to 100° and cooling to 0°. Phosphor bronze showed no such change. A short nickel bar was found to answer all the requirements considered, namely, price, hardness, susceptibility to a high polish, high modulus of elasticity, and resistance to deteriorating effects of moisture and of such chemical agencies as are commonly used in laboratories. Nickel bars of suitable lengths for geodetic standards have, however, as yet not been made.

From the report of the International Conference on the Construction and Comparison of the New Metric Prototypes, of which the United States has two, it appears that the probable error of the result of the comparison of two prototypes was only $\pm 0{\cdot}04$ micron at the temperature at which the comparisons were made. Taking into account, however, the coefficient of expansion of 𝔓𝔱, the final estimate of the accuracy reached is stated in the following words:

> It may be concluded, therefore, that the equations of the prototypes lead to a knowledge of their absolute lengths with an average probable uncertainty which, under the temperature conditions usual in metrological operations—that is, between 20° and 25° C.—lies between 0·1 micron and 0·2 micron, and at a higher temperature it may slightly exceed the last-mentioned limit.

The belief is also recorded that if the comparisons between all the prototypes were gone over again the mean differences found would be of that order of magnitude.

When it is considered that the comparisons above referred to were made under the most favorable conditions, such as uniformity of temperature, identity of material, perfect illumination, and that slight imperfections in the latter alone introduce very material discrepancies, we may conclude that no geodetic standard can be known with a higher degree of accuracy than 1 part in 5 000 000 of its length in terms of the international unit.

Since all the operations involved in base measurement tend to decrease this degree of accuracy, we may assume that 1 part in 5 000 000 of its length is a higher degree of accuracy than can ever be attained in practice by known methods. If, in contradistinction to this estimate, the probable errors of certain famous base measures, like that of the Madridjos Base, are cited ($\frac{1}{5\,865\,800}$), it must be remembered that they do not refer to the absolute length of the base.

That it is possible to measure a base line repeatedly with the same apparatus with a surprising accordance between measures, indicates that the elimination of the accidental errors has been successfully met by the different forms of apparatus in use; and it is also believed that the various methods of observing successive lengths of the same bar or systems of bars are sufficiently precise. On the other hand, that constant errors exist which are in the main due to a defective knowledge of the temperature of the bars is a fact commonly assumed or proven by the lag of mercurial or bimetallic thermometers used on various apparatus. These constant errors are not easily determined, but are now the principal sources of error.

Recently experiments to elucidate this point have been made abroad, as well as in this country, by measuring the same base with different forms of apparatus. Thus, a base at Bonn has been measured with the Bessel apparatus and again with the Brunner.

The results of the measurement with the Bessel apparatus, made twice with rising and twice with falling temperatures, indicate a relative error of $\frac{1}{4\,500\,000}$. The same base, which is 2·5 kilometres long, was then measured with the Brunner apparatus, and it is stated that the agreement with the former measures is excellent, if we regard a discrepancy of 1^{cm}, or $\frac{1}{250\,000}$, as a constant error, the cause of which is not, however, known and is being investigated.

We also have the results of the measurements of the Undine Base, as obtained by the Austrians with their apparatus and by the Italians with the Bessel apparatus. Here the difference is only 4^{mm} in a total length of 3·248 kilometres, or $\frac{1}{800\,000}$.

The Brunner apparatus, elsewhere described, is the one used most recently in France and by General Derrecagaix. It is considered a model form of a modern base apparatus, and his opinion was apparently shared by other geodesists, for the Spanish, French, and Germans have adopted it, notwithstanding its great cost and the slowness of its manipulation.

The relative error of measurement for the Brunner apparatus, as used by the French, is about $\frac{1}{2\,400\,000}$, but the experience gained at Bonn indicates that the measure of its real accuracy is still open to question, as above mentioned.

The measurement of the Holton Base with the 100^{m} steel tape and 5^{m} secondary bars in the United States also shows a marked difference. Each considered alone gives a relative error of less than $\frac{1}{1\,000\,000}$, yet differ by 1 part in 350 000.

NUMBER OF MEASUREMENTS OF A BASE LINE.

As a test of the accuracy of the work and to afford an opportunity for eliminating errors of temperature each base should be measured at least twice. It has already been stated that, owing to the experience gained in a first measurement, the second can be made much more rapidly than the first. On this account, and because a large part of the cost of such operations is the expense of preparing the line, putting the party and equipment into the field and taking them out, the second measure increases the cost of the work but slightly, while the advantage derived from it is very great.

Whenever the steel tape is used for primary work it is recommended that at least four measures be made, using two different tapes, and in such a manner as to eliminate the effect of thermometer lag as nearly as possible; i. e., by measuring each section with each tape under both rising and falling temperatures.

FREQUENCY OF BASE LINES.

In many cases the length of the triangle sides and the nature of the country constrain the economical introduction of bases. Areas like the great plains, however, afford many opportunities for introducing bases economically, and where that can be done it is advisable to obtain accuracy by a multiplication of bases rather than by refinements in the triangulation.

RELATION OF THE ACCURACY OF BASE MEASUREMENT TO TRIANGULATION.

It stands to reason that the degree of accuracy to be aimed at in the measure of a base line depends largely on the object the base has to subserve, on the apparatus and the time available, and on the money allotted. Hence, no definite limit can be assigned. With the perfection of the means available a line may readily be measured with a probable error of $\frac{1}{1\,000\,000}$ of its length so far as mere measurement is concerned. On the other hand, when all the errors which may enter have contributed their share, the actual uncertainty may be considerably greater. It may be, say, twice or three times the above; but as no additional expense is involved by carefully attending to detail, the observer will in all cases do his best with the means on hand. The high degree of accuracy ordinarily within his reach at present is, however, rapidly dissipated through the angular measure of the two, three, or more steps required to refer the length of the base to the first line of average length of the triangulation. It may be further remarked that high accuracy in the angular measures of the latter is needed if an error, say $\frac{1}{150000}$, is to be maintained or not to be exceeded throughout the chain of triangles.

In tertiary triangulation an average uncertainty or limit of $\frac{1}{10000}$, or even $\frac{1}{5000}$, may be satisfactory for the special purpose. For intermediate secondary triangulation the fraction of the length $\frac{1}{100000}$ to $\frac{1}{50000}$ may be suitable, and the degree of accuracy for the base supporting such work should be graduated accordingly, yet with the precaution above adverted to.

The observer should pay especial attention to the minute and careful centering of the stations which constitute the base figure.

RECOMMENDATIONS.

In conclusion, the committee recommend that further experiments with the steel tape be made especially with a view to a better determination of its temperature. This recommendation seems to be warranted by the results of the measurements of the Holton Base.

2. That the new duplex apparatus just completed by the Survey be given a thorough and careful trial as soon as practicable.

3. That the iced bar be used to lay off a 100^{m} comparator at the bases where the above recommendations are followed.

4. That in all measurement due regard be paid to rising and falling temperatures, so as to eliminate as much as possible the errors due to lack of knowledge of temperature of the measuring apparatus.

5. That the tripods for supporting bars be made of metal, and embrace the details which experience has shown to be conducive to accuracy and rapidity of measurement.

6. That ordinarily the base line be divided into sections one half kilometre in length.

7. That a 100^{m} comparator be established in Washington for the purpose of testing steel tapes and contact slide apparatus.

8. That in future use of the steel bar in ice the water in the Y trough be not drained off below the surface of the bar, in order to remove any possibility of doubt as to the actual temperature of the bar.

GEORGE DAVIDSON, *Chairman.*
OWEN B. FRENCH, *Secretary.*

REPORT OF COMMITTEE C, ON TRIANGULATION.

CONTENTS.

REPORT.

(1) *Object of triangulation.*—The general object of triangulation is well understood. It is primarily to furnish absolute or relative geographic positions of a series of prominent points spread over extended areas, and, secondarily, to provide the necessary data for topographic and hydrographic surveys. It furnishes the three coordinates of each place, viz, the latitude, the longitude, and the altitude.

(2) *Classification.*—In its execution great diversity exists, depending upon the special object of the work, as, for instance, whether designed as a contribution to the measure of the figure of the earth or simply to give the relative positions necessary for the traverse work of a county or for the production of a city map. The larger operation is purely geodetic in character; the smaller one refers to plane surveying. The methods of observing, the instruments, the method of computation, the accuracy, etc., are different for different classes of work; hence, for convenience certain more or less well-defined subdivisions have been made, such as main and subordinate triangulations, primary, secondary and tertiary triangulations.

By the term main triangulation we understand the principal series of triangles which compass along the shortest line the whole extent of the surface or country under consideration, and by subordinate triangulation any series of triangles which depend for their support on the main series and which may, if small enough, be contained within it or branch off from it in any direction. The other classification, into primary, secondary, and tertiary, is more technical.

(3) *Primary triangulation* is characterized by the greatest development of length of sides practicable (depending on the heights of mountains) and by the greatest accuracy of measure. The geodetic positions will depend on direct observation of astronomical latitudes, longitudes, and azimuths. Refined base lines furnish the linear measures. The length of sides may ordinarily vary between 50 kilometres (say, 31 statute miles) and 150 kilometres (say, 93 statute miles), but may reach 250 kilometres (or 155 statute miles), and objects distant 300 kilometres (or 186 statute miles) and over have been sighted. In computing triangulations developed on such large scales (as in California, Nevada, and Utah) we have to consider the spheroidal excess; the reduction to sea level of the horizontal directions on account of height of station sighted; the reduction of the astronomical latitude for height of station occupied, the vertical being a curved line; the employment of formulæ for the computation of position of greater accuracy than Puissant's modified formulæ, ordinarily used on the Survey; the use of logarithmic tables of eight places of decimals, etc.

(4) *Tertiary triangulation* is designed to furnish positions of a sufficient number of points as a basis for developing topographic or hydrographic surveys, and its extent is so limited that the area covered may be taken as that of a plane surface; hence the length of its sides is usually between 1 and 15 or 20 kilometres (say, ½ to 9 or 12 statute miles). Five places of decimals suffice for the computation of its sides.

(5) *Secondary triangulation* bears an intermediate character between the other two classes and simply effects the connection between them; i. e., the descent from the long sides of a primary triangulation to the short ones of the tertiary, constitutes its character. Its sides may therefore vary considerably; that is, between the lower limit of the former and the upper limit of the latter class. The spherical excess and Legendre's theorem are introduced and seven places of decimals are generally employed in the computation of its sides.

It may be remarked here that in our middle latitudes (about 37°) an area of 196·8 square kilometres (or 76 square statute miles) corresponds to a spherical excess of 1″.

(6) *Adaptation to the surface of the ground.*—In general, the most elevated peaks and mountain ranges, up to the limit of accessibility, are the most favorable positions for stations. Two parallel ranges inclosing a valley offer great facilities, as the latter can generally be straddled by well-proportioned figures; so also do high buildings in the case of small triangulations. On the other hand, flat regions, especially when wooded, are to be avoided if possible; also ridges when wooded and of equal height. These and related considerations, however, belong to the subject of reconnaissance (which see). Whether it will be better to build high structures to overcome obstructions or to cut lanes or

avenues through forests is largely a matter of economy of money and time, but high structures do not affect the accuracy of the survey.

(7) *General form of main triangulation.*—Two systems have been employed in the triangulation of countries, the so-called gridiron and central systems. The former starts out with a series of parallel chains of triangles at certain intervals, intersected by other chains at right angles to them. The rectangularly shaped open spaces are then to be filled up by subordinate triangles. The latter system starts from the center of the country and extends radially in all directions, thus growing by concentric rings of triangles which eventually will cover the whole area. The first system is far preferable, as it lends itself better to computation (adjustment) and admits of preference being given to the advancement or completion of the survey at certain places. Practically the two systems may alternate or merge into one another.

(8) *Geometrical composition of a triangulation.*—Triangulation chains may be made up of a series of single connected triangles, a series of hexagons (or other polygons) hinged together on one side, or a series of quadrilaterals placed together. Their relative advantages are: For the triangle series, least number of stations and rapidity of measure—hence economy; for the polygon series, great extent of surface covered, and for the quadrilaterals, great accuracy.* In practice the geometrical figures will be found distorted and generally lengthened out in the direction of the axis of the triangulation; likewise interlaced and combined in a variety of ways so as to take advantage of favorable points presented.* In the case of *single* triangles the angle opposite to the base side should not be too small for favorable intersection, and in complex cases the observer should have a well-defined and uniformly strong scheme, avoiding complications of lines.

(9) *Frequency and length of base lines and their connection with the triangulation.*—The number of base lines to be introduced into a triangulation, as well as the length of the bases, depends on the average length of the sides of the triangulation and upon the degree of accuracy desired for the latter. Owing to the fact that base lines can be measured, and are rightly measured, with much greater accuracy than can be sustained through a triangulation, it is plain that in order to increase its accuracy we must introduce more base lines, or increase the number rather than their length. The transfer of the comparatively short length of a base to the greater length of a side of the triangulation is generally effected by several steps so as to avoid too acute angles, and consequently loss of accuracy. The figure of the base net connecting the base with the triangulation is therefore one of importance, and, ideally, may be described as a series of quadrilaterals with diagonals intersecting at right angles, the length of these diagonals increasing ordinarily in a ratio of 1 to 2 or 3, thus requiring

*See Annual Report United States Coast and Geodetic Survey for 1876, Appendix No. 20.

several steps to ascend to the length of a main line. Ordinarily two or three may suffice.

No precise rule for deciding a priori on the length of a base can be given. Any of the fractions one-tenth to one-sixth of the length of an average side of the triangulation may be useful for an estimate, since the actual length of base lines varies between wide limits. For a triangulation of the first order of magnitude the length may be 15 kilometres (9·3 statute miles) or slightly more; for a third order triangulation it may be 1 kilometre (0·6 statute miles) or even less. Base lines between 5 and 8 kilometres (say 4 to 5 statute miles) in length are the most common. They should be measured at least twice, once with rising and once with falling temperature—a very important condition, only to be omitted when a "bar in ice" apparatus is used. For subdivisions of a base distances of 0·5 to 1 kilometre are suitable. These are needed for furnishing the data for that part of the probable error of a base which depends on the measurement alone. The more accurate the angular measures in a chain of triangulation the greater may be the distance apart of its base lines. This distance varies accordingly between wide limits, but ordinarily may be from 20 to 40 times (or more) the combined length of the bases.

Nevertheless there may be conditions in the orography of the country which constrain the location of base lines, as, for example, on the Pacific Coast and through the region of the Sierra Nevada and the Rocky Mountains. Thus, to avoid the unfavorable country between the Sacramento Valley and the Salt Lake region, a distance of about 550 miles, the triangulation has been expanded to a great size and is supported by a long base in the former section. This is an instance where the Survey inaugurated a scheme of triangulation on a scale larger than had ever been attempted elsewhere, and consequently there was no experience to guide or suggest the attainable accuracy in the triangulation and length and frequency of bases. Similar conditions constrained the location of a base in the Los Angeles plains, and other like examples might be given. In a region where base-line sites can be more readily obtained the number of bases will depend mainly on the degree of accuracy desired for the triangulation and to a less extent upon their length.

(10) *Accuracy of a triangulation.*—From an economical standpoint we may confine our inquiry to answering the question, What may be considered *sufficient* accuracy in the measures of the separate operations and in the results of a triangulation as a whole?

A base line can readily be measured with a probable error of 1/250 000 part of its length, and by application of superior apparatus, of several measures, and greater care—hence, at an increased cost—the

*Cf. Die Geodätischen Hauptpunkte, etc., Von G. Zachariae. Translation by Dr. E. Lamp, Berlin, 1878, Art. 37. Also Jordan's Handbuch der Vermessungskunde, Vol. III. Third edition, Stuttgardt, 1890.

probable uncertainty may be reduced to 1/500 000 or less. Although this last fraction may be taken as a practical limit worth aiming at, extreme values of accuracy are recorded as having been reached, such as probable errors of *measure* of 1/1 000 000 or of a still smaller fraction. Now, to maintain an accuracy of, say, 1/150 000 or even 1/100 000 part of the length, on the average, in an extended triangulation, is a matter of difficulty, as is manifested whenever we compare the length of a junction line derived from two independent chains of triangles. The excess of accuracy of a base is lost partly in the base figure, and is further rapidly dissipated in the adjacent triangles; hence the inexpediency of straining at an extravagant degree of accuracy in the measure of a base becomes evident.* What is really important is a knowledge of the true length of the measuring apparatus in terms of the unit of length (on this survey, the international metre).

Respecting primary triangulation employing superior theodolites (40 to 50 centimetres diameter) the probable error of a *single measure of a direction*—i. e., the mean of two pointings on the heliotrope, telescope direct and reversed, with readings of three microscopes on two graduation marks each—has been found to be $\pm 0{\cdot}64''$ (with variations between $0{\cdot}45''$ and $0{\cdot}90''$), derived from 22 stations in California and Nevada. At these same stations the average number of series or measures of a direction was 63 and the resulting *probable* error of a direction as shown by the *station adjustments* is $\pm 0{\cdot}08''$ (with variations between $0{\cdot}06''$ and $0{\cdot}10''$). Hence, *mean* error of an observed angle $\left(\frac{3}{2}\right) 0''{\cdot}08 \sqrt{2} = \pm 0''{\cdot}17$, and we may expect the triangles to close within $0{\cdot}17\sqrt{3}$ or $\pm 0''{\cdot}29$ on the average; but we find the mean closing error (73 cases), as demanded by the triangles composing the figure, to be $\pm 0''{\cdot}61$, which shows the presence of other adverse influences than those arising from the graduation and pointing errors. The most potent of these is the lateral refraction, composed of a constant and a variable part. Large local deflections of the vertical at a station also have their influence. Taking our figures as typical, we conclude that a less number of series would suffice, provided the same variety of weather is experienced, without detriment to the work.† Suppose the number reduced to 31 (a prime number), the resulting *probable* error of a direction would rise to $\pm 0''{\cdot}12$, the *mean* error of an angle to $0''{\cdot}25$, and the expectation for closing of triangles would rise to $\pm 0''{\cdot}43$. This would probably leave the large scale work of the Survey still in the front rank for accuracy.

*See Appendix No. 9, United States Coast and Geodetic Survey Report for 1885.

†Respecting the time devoted to the observations, General Walker, of the Great Trigonometrical Survey of India, remarks (Vol. II, p. 70): "Any neglect of these precautions, any hurrying over the prescribed tale of observations with the utmost possible rapidity, even at the time when the signals are apparently very steady and favorable, is liable to introduce larger errors than those which are partly attributable to any defect in the instruments of this survey." It is also remarked that it is considered a great misfortune to use instruments of inferior order.

The mean error of an angle as derived from adjusted triangulation is frequently used as a convenient measure of the relative accuracy of different triangulations. For the above case we have

$$m = \frac{\cdot 61}{\sqrt{3}} = \pm\ 0''\cdot 35$$

Taking as a second type of triangulation part of the transcontinental triangulation east of the Rocky Mountains, with sides averaging 25 kilometres (about 15½ miles) in length west of the Mississippi and 29 kilometres (about 18 miles) east of it, we can form the following table:

Central Kansas	200 km.	(about	124 miles),	from	43 triangles,	$m =$	$\pm 0''\cdot 76$
St. Louis to eastern Kansas	595 "	"	370 "	"	137 "	"	$\pm 0\cdot 78$
St. Louis to Indiana	177 "	"	110 "	"	33 "	"	$\pm 0\cdot 66$
Indiana	235 "	"	146 "	"	24 "	"	$\pm 0\cdot 70$
Western Ohio to Chesapeake Bay	864 "	"	537 "	"	451 angles,	"	$\pm 0\cdot 97$

The following table is added for comparison with foreign triangulations.

In work of this character instrument circles of 25 to 35^{cm} (diameter) are suitable. The number of positions should not exceed 17, and the number of series may be about one-half to two-thirds the number recommended for primary work, according to its importance. Special attention should be paid to the centering of the instrument and of the heliotropes and to phases of signals.

Chains of triangulation in Europe, India, and Africa.

Name of country.	Epoch.	Length of sides.	Instruments. Maker.	Diam. of circ.	Micr. or ver.	No. of Δs.	$m = \sqrt{\frac{\Sigma\Delta^2}{3n}}$
		km.		*cm.*			″
Austria-Hungary	1850–88	12 to 60	Starke, Reichenbach	27·0 32·5		674	0·910
Bavaria and Palatinate	1804–53	10 " 40	Borda, Reichenbach, Ertel	40·5 32·5 21·5		132	1·778
Belgium	1851–73	10 " 30	Gambey, Breaulieu	?		219	0·892
Denmark	1817–70	10 " 40	Ertel, Reichenbach	40·5 32·5		87	0·742
Spain	1860–84	15 " 130	Pistor, Ertel, Repsold	28 37 32	2 *micr.* " "	325	0·886
France and Algiers	1792–1880	10 " 120	Gambey & Brunner	?	*ver.*	914	1·29 (best 0·27)
Great Britain	1787–1865	10 " 85	Ramsden, Troughton & Simms	81 and 61 46	5 *micr.* 3 "	552	1·83
Greece	?	10 " 40	Starke & Kammerer	26	2 "	109	0·77
Italy	1863–90	10 " 65	Pistor, Reichenbach, Gambey, Starke	32 32 27 27	2 " 2 " 2 " 2 *ver.*	514	0·920
Norway	1853–63	10 " 80	Reichenbach, Olsen, Repsold, Ertel	36 30 19		179	0·718
Portugal	1864–88	10 " 40	Troughton, Repsold	35·6 19·6	2 *micr.*	139	1·29
Prussia A	1847–77	10 " 50	Pistor, Pistor & Martins	27 27		79	0·734
B	1832–91	10 " 50	do.	32 38	----	690	0·554
Roumania	1855–86	8 " 25	Starke, Reichenbach	32·5 32·5		36	1·736

Russia	1816–86	10 " 25	Brauer, Troughton, Ertel	21 to 37	----	147	1·495
Saxony	1867–78	10 " 25	Repsold	31	2 *micr.*	197	0·350
Sweden	1819–80	10 " 45	do.	32	2 "	304	1·09
Switzerland	1854–68	10 " 40	Starke, Reichenbach, Ertel	24 32 21		?	0·856
India	1860–80	15 " 40	Troughton & Simms, Barrow, Waugh	81 61 46	5 *micr.* 5 " 3 "	1,417	1.003

Any triangulation in which m does not exceed 1″ may generally be classed as being of a high order of accuracy, and, according to circumstances, double that amount may still be taken as of sufficient accuracy for the purpose for which the triangulation was made. In linear measure an accuracy of 1/50 000 or 1/75 000 part of the length is ordinarily considered a satisfactory one,* and 1/30 000 may be sufficient in many cases. If by reason of great distance from the base the accuracy in a triangulation should have been reduced below the standard originally set, the introduction of a new base at the weakest point, or near it, would be the proper remedy.

Tertiary triangulation demands no high degree of accuracy, and may vary from 1/20 000 to 1/5 000 part of the length, according to requirements.

When dealing with mean or probable errors the following rough-and-ready rule to judge of the admissibility of apparently large individual deviations from the mean value may often be found of service, viz: With the usual very limited number of observations, any that may be outside the total *range* of 5 times the mean error or 7 times the probable error should be looked upon with suspicion, and, conversely, the mean and probable errors may be guessed at to be about one-fifth and one-seventh of the observed range, respectively.

(11) *Interstate and international boundaries.*—Apart from the astronomical measures that may be needed, the most accurate method of locating boundary lines, and the one that best preserves the monuments, is triangulation, because all the prominent natural features and many of the artificial ones may be connected with the boundary marks as objects of reference. When, in addition to the triangulation, the topography is also carefully executed, the labor of replacing lost marks in their proper position is reduced to a minimum. This necessarily presumes that the triangulation points have been well marked.

The expense of triangulating a boundary and the length of time required to execute it has in the past generally precluded the adoption of this method in the United States.

The purpose of this paper is rather to treat of the best practical method of running boundary lines than to make a historical review of boundaries that have been already run and a minute examination of the details of their survey.

In all boundary work due regard must be paid to the legal definition of the line, whether it is to depend on astronomical measures alone or on geodetic measures when the ends or termini are to be connected by a straight line (the geodetic or shortest line), and, in the case of an arc of a parallel, whether it is to be a mean parallel or an astronomical parallel.

In lines depending upon astronomical observations the local deflection of the vertical enters as a disturbing element. This is due either to

* One sixty-three thousandths corresponds to 1 inch in a mile, nearly.

mountain masses, proximity to the ocean, or irregularity in density of the matter beneath the surface of the earth. It may amount to a few seconds, and in extreme cases to very much more.

On the survey of the Northern Boundary west of the Lake of the Woods, on which 41 latitudes were observed in a distance of 1374 kilometres (853·5 miles), the average local deflection or difference from the mean parallel was 2·15″. The observing error for geographical positions is generally far within this quantity. At 56 stations observed on the oblique arc along the Atlantic coast the average difference between the geodetic and astronomical latitudes was 2·2″, which is almost wholly due to station deflection.

The so-called regular boundaries may be divided into several kinds: (1) Boundaries along a meridian; (2) boundaries along a parallel; (3) boundaries along oblique lines. Even a circular boundary has been traced. There are others of an irregular nature, that follow natural topograpic features, such as streams, mountain summits, divides, or crest lines. These latter do not come within our scope for treatment, as they depend on ordinary trigonometric and topographic methods of surveying.

Boundaries along a meridian.—These are the simplest forms and present the least difficulties when the observer is provided with the proper instrumental outfit. It is necessary to determine the true meridian and trace it out, preferably by back and fore sights. As often as may be found necessary check azimuths should be observed. The frequency of these checks will depend on the character of the country. If the line is to be carried over a plain, where the sights are short, an astronomic check should be secured every 25 to 30 kilometres (15 or 19 miles), but if the country is rolling or mountainous, affording sights from 5 to 30 kilometres (3 to 19 miles) in length, from 60 to 100 kilometres (37 to 62 miles) may be run before checking.

The party should be provided with a 20 to 25^{cm} (8 or 10 inch) theodolite, with a diaphragm arranged for observing time as well as for measuring angles, and a sidereal chronometer. Azimuths may be determined in the usual way by observing Polaris at any hour angle in connection with a terrestrial mark. Three sets of observations on one night will generally give the desired accuracy. The meridian may be laid off on the horizontal limb of the theodolite, a signal put in line, and the angle then accurately measured by repetitions. If there should be an error worth correcting, the signal may be moved to its proper place, knowing the distance and angular deviation.

This instrument should be used for measuring angles and azimuths, ranging out the line, and interpolating points on intermediate prominences. There should also be a 10 or 15^{cm} (4 or 6 inch) theodolite, with a vertical circle, having on the diaphragm parallel lines or threads for reading telemeters at road and stream crossings.

In running a meridian boundary line, where long distances may be obtainable, say 30 to 100 kilometres (19 to 62 miles) or even more, it is suggested that the meridian instrument affords a good means for projecting the line not only to the distant stations, but to such intermediate stations as are visible. The position of the instrument, in its relation to the meridian, is to be determined by the observation of time and azimuth stars.

For the topographic work we measure distances either by a small triangulation or telemeter or tape line. The error of telemeter lines, as shown by the Northern Boundary Survey, is given as 1/300 of the length for the average of an entire season; but on selected days, with great care, this may be reduced to 1/1500. General Comstock states that on ordinary ground 1/700 may be secured. In 1893, on the Mexican Boundary Survey, in a line of 57·3 kilometres, 1/1 400 was reached; in a line of 73 kilometres the discrepancy was 1/1 900 of the true length.

For short sights, slender range poles, and for long distances—from 5 to 30 kilometres (3 to 19 miles) or more—pocket heliotropes may be used. To illustrate the accuracy with which a meridian may be run it may be stated that in 1883, while running the boundary between West Virginia and Pennsylvania, the meridian was checked by an azimuth after ranging out 18 kilometres (11 miles) and the line was found to be apparently in error $2''\cdot5$ (21^{cm}). Beyond this point a series of long sights were obtained, and the next check azimuth was observed after running 71 kilometres (44 miles) more, when the line was found to be apparently $2''\cdot4$ (8^{cm}) in error and in the other direction. Both of these errors were inappreciable and no corrections were made, as the theodolite was only graduated to $5''$. Some portion of the error may have been due to local deflection. Of course any error in the line should be distributed proportionately.

Boundaries along a parallel of latitude.—These may be run either by chords or by tangents. The latter method is preferable on account of its greater adaptability to topographic features; the former is more simple, especially when the adjacent astronomical stations are intervisible.

It is necessary to determine the meridian and lay off a tangent in the prime vertical at the point where the azimuth and latitude are observed. Before attempting to run the parallel astronomical latitude stations should be located approximately at distances 15 to 30 kilometres apart. Distances from the latitude stations are to be laid off along this tangent from which the ordinates to the parallel are measured.

If the topographic features permit, regular distances should be determined along the tangent and offsets made at the proper angle from the tangent, so that they may be either normal to the small circle of latitude or normal to the tangent. The Coast and Geodetic Survey formulæ for geographical positions will be found very useful in computing the azimuths and offsets for known distances, which distances may be irregular if necessary, and generally they are so in a broken country.

An azimuth may be observed at the end of the tangent to determine any error in ranging it out, and this and the correction due to difference of local deflection at the two stations are to be distributed along the offsets.

If the *astronomical* parallel should be laid out, it would be an irregular curve in consequence of local deflections, but it would have the

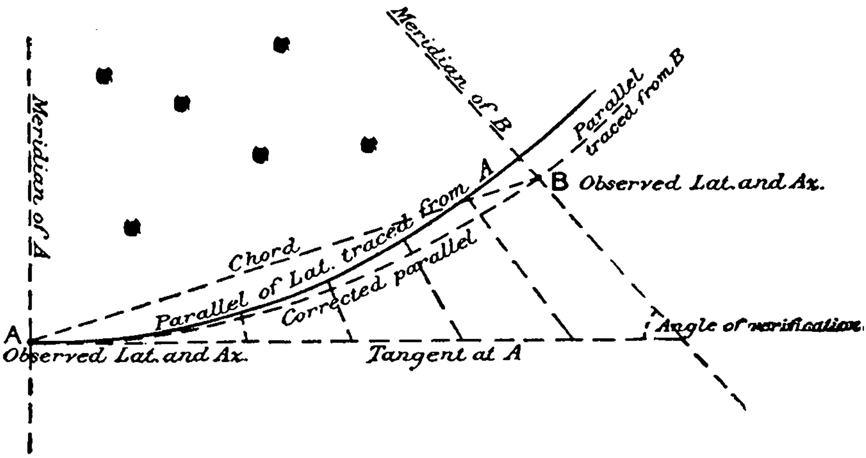


FIG. 1.

advantage of being more readily reproduced than the mean parallel if it should become necessary to restore the boundary marks. The latter may be traced out by means of triangulation. Figures 1 and 2 illustrate the method of procedure.

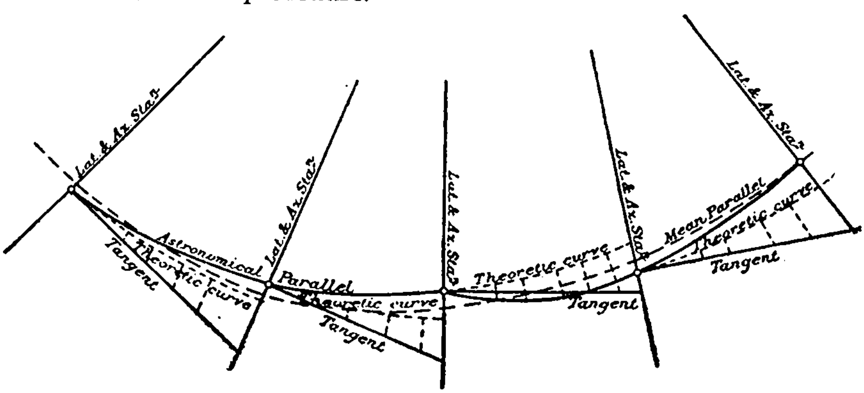


FIG. 2.—Method of tracing a parallel of latitude, showing tangents, offsets. Theoretic curve drawn through each astronomical station.

The tangents and offsets may be corrected for error of ranging out after checking them by the regular azimuth observed at each astronomical station.

Boundaries along an oblique line.—These are, like the other regular boundaries, referred to the meridian. If the latitude and longitude of the termini are known, the azimuth or direction of the line may be computed, and then by means of azimuth observations the angle can be laid off and the line traced out. Any error in running the line is to be distributed proportionately.

An example of oblique boundary is that run in 1874 by Professor Bowser between New Jersey and New York with a Coast and Geodetic Survey 20^{cm} theodolite. The Coast and Geodetic Survey located the termini—one by triangulation, the other by astronomical measures.

For tracing out the line and locating monuments at suitable intervals, so as to be readily intervisible also at intersections of roads, streams, and crest lines, two methods are available—one starting from the fixed initial station with a given azimuth to reach the opposite end and proceeding by backward and forward sighting, the other by first connecting the end points by a triangulation and the insertion of the monuments in the places computed. By the first method the error in the direction of the line when the opposite end is reached must be evenly divided along the entire line. In mountainous countries this method of ranging can not dispense with occasional small triangulation in order to get around an inaccessible height. On the other hand, the triangulation method and subsequent laying out of the points in line, may be more laborious and demand more time, but admits of the tracing out of a line not subject to any material extent to the local deflections of the vertical, which necessarily come in at all stations where a transit line is used. The preference for one or the other method thus depends on circumstances.

In countries difficult of access and where land is of little value, and the expense of tracing a boundary throughout its entire length is disproportionate to the advantages to be gained, important points may be located, and either connected or not, according to the public demands. In Alaska this method is being used.

Instrumental outfit.—For the measurement of horizontal angles two kinds of instruments are in use, the repeating instrument and the direction instrument. Theoretically the former should give the better results, but experience has shown that with the improved methods of graduation now in use the reverse is true. It may be stated in general that when the graduation is good and the optical conditions commensurate, the method of directions should be used; with poor graduation, repetitions will give the better results. Thus in the large instruments used in the primary or large scale triangulation great refinement is demanded in the graduation of the circle; and a direction instrument should therefore be preferred, though a repeating instrument may be allowable and has been found under some circumstances to give equally good results. In secondary and tertiary triangulation the repeating instrument will usually give satisfactory results, with the great advantage that it admits of rapid work both in the field and in the office. For the more refined work of the primary triangulation the instrument should have a 40 to 50^{cm} (16 to 20 inch) circle, graduated to $5'$, reading by three micrometer microscopes to $1''$ each and by estimation to $0''\cdot1$.

The telescope should have an objective of the best class, of 7·5 to 8·5^{cm} (3 to 3¼ inches) diameter, three eyepieces ranging in power from 40 to 100; also two very light vertical circles or finders for azimuth stars and an ocular micrometer for special or occasional use. The latter may be used for azimuth work as well as micrometric angular measures when the part of the day suitable for favorable operations is very short and when the atmospheric conditions are adverse. The instrument should have slides to provide for radial illumination of circle by a right-angled prism. The instrument should be mounted on a position circle cemented to the top of a pier whenever practicable. As the observations for azimuth should, if possible, have the same degree of precision as those for horizontal directions, the telescope should have sufficient optical power to observe stars of 6·5 magnitude.

In order to secure images of uniform size at all distances it is necessary to vary the size of the heliotrope according to the distance to be observed. For ordinary atmospheric conditions and for distances of 10 miles and over the formula $x = \cdot 046d$ may be used for this purpose, where x is the side of the square mirror in inches and d is the distance to be observed in miles, or $x = \frac{5}{7}k$, where x is the result in millimetres and k the distance in kilometres.

The following table contains the length of side of square mirror for various distances:

Distance.	Side.	Distance.	Side.	Distance.	Side.
Miles.	*Inches.*	*Miles.*	*Inches.*	*Miles.*	*Inches.*
10	0·46	60	2·8	120	5·5
20	0·92	70	3·2	140	6·4
30	1·37	80	3·7	160	7·3
40	1·83	90	4·1	180	8·3
50	2·3	100	4·6	200	9·2

Referring to the theodolite, the effect of an outstanding error (c) in collimation on the measure of a horizontal direction may in general be kept small, thus: Supposing $c = 10''$ and the angle of elevation of the object less than 3°, this effect is but 0''·01 at most, and it is the same for the case of $c = 1'$ and the elevation less than 1°.

The effect of an error (i) in the horizontality of the revolving axis of the telescope is in general much larger, as it depends on the tangent of the angle (α) of elevation, thus for $i = 10''$ and $\alpha = 1°$ it amounts to 0''·2, and with $\alpha = 2°$ it is 0''·4. Both defects are eliminated in the measures by the reversal of the telescope and azimuth circle.

The effect of an error (v) in the verticality of the theodolite axis can not be eliminated by any method of observing. The amount for any angle depends on v and the relation of the angle to the plane which contains the true and instrumental zenith point. With repeating theodolites we have to deal with two vertical axes which should coincide. Close attention to this last source of error must be given by careful leveling.

The effect of the eccentricity of a circle is eliminated by the reading of any number of *equidistant* microscopes. The examination of the graduation of a circle for periodic and irregular errors should be made at the office.

All heliotropes should be centered over their respective stations with the same care as the theodolite; and when poles, targets, or cones are used, the data needed for correcting for phase where required should be given (size and shape of object and local time or else the azimuth of sun). If plain poles are used, their diameter should be graduated for the distance; i. e., it should be smaller the shorter the distance from the station. It should also be stated whether the object was seen by reflected or diffused solar light.

(14) *Method of observation.*—In order to secure the best results with either form of instrument the observer should make the observations at different times of the day, say a. m. and p. m., and on different days and under varying conditions of atmosphere, but rather exceptionally during day and night, and he should refrain from observing under any manifestly unsuitable or doubtful conditions. In no case in primary and secondary triangulations should the observations be finished in one day; but several days, embracing morning and afternoon observations, should be devoted to them, in order to avoid cases of lateral refraction, which have occasionally been experienced to the extent of many seconds.* In mounting the instrument regard should be had to proper shelter for it as well as for personal comfort while observing. Pointings should be made as rapidly as possible consistent with a clear and decided bisection of the signal. In using a *direction* instrument the method usually adopted is to divide the circle into a number of equal parts, known as positions. This number should be prime,† so that no microscope may fall upon the same graduation in measuring upon the same object in different positions or after reversal of the circle. Having established an initial direction, one or more series are observed in each position, each series consisting of a pointing and reading upon each of the signals in the order of the graduation, and then, after reversing the telescope and turning the alidade 180° in azimuth, of another pointing and reading upon the signals in the

* The observations for lateral refraction made by Dr. Fr. Pfaff with a theodolite mounted in the tower of his house, and extending throughout a whole year (Publikation des König. Preuss, Geodatischen Institutes), may here be referred to as evidence of the existence of this disturbing feature in the measure of horizontal directions. Yet for future special observation two or more fixed telescopes, cemented to a low stone pier and provided with eyepiece micrometers, would prove more effective and less troublesome than Dr. Pfaffs' arrangement. The lateral variation of more than one direction at a station could be thus investigated.

† Any other number that will accomplish the same purpose may be used.

reverse* order. The number of positions to be used depends upon the accuracy of the graduation and upon the degree of refinement desired in the results. Experience tends to show that with the best instruments now in use on the primary triangulation the effect of atmospheric conditions upon the result, after a certain number of positions have been used, is much greater than the effect due to errors in graduation. It is probable that 31 series are needed to secure the desired accuracy, and about this number should be obtained. It should be left to the judgment of the observer, taking into account the character of the work and of the instrument used, to decide in how many positions of the instrument these observations shall be made.

In the past, 23 positions have been the maximum number used. There is no objection to decreasing this number, or increasing it to the full number of series desired.

It has also been suggested to take a very small number of positions and exhaust the circle, and to repeat the same number of positions, but with a different initial reading of the graduation and subdividing the former spaces. A third and fourth group of such readings can be added if greater accuracy should be demanded. The advantage claimed for this procedure is the easy comparability of the results of the series making up a group.

In high mountain regions in certain cases it has been found that midday observations of horizontal directions are obtainable with the aid of the ocular micrometer. This has only been used in primary work and should be restricted to special cases.

When a repeating instrument is used each set of repetitions should consist of a certain number of measures of the angle (α), say 3, followed by an equal number of measures with telescope reversed; then the explement of the angle ($360-\alpha$) should be immediately measured in the same manner. Two sets of 6 repetitions ($3\,D+3\,R$) are preferable to one set of 12 repetitions ($6\,D+6\,R$), as something may occur to interrupt the observations during the longer time required for the latter, thus vitiating the whole set. Enough sets should be taken to obtain the desired accuracy, from 2 to 6 probably being sufficient in most cases, according to the precision required by the character of the triangulation. In regard to the number of angles to be measured at a station, it may be stated in general that there should be a check on every angle measured besides that of closing the horizon in the manner referred to above. Although for the highest degree of accuracy all of the sum angles might be measured, this should rarely be done, especially when the number of signals is large, as the increase in the accuracy of the result is not commensurate with the increased time and labor spent. It is preferable to measure only those angles which actually occur in the figure of the triangulation, and this consideration should have some weight in selecting

* This is to correct for any azimuthal change or twist during observations in the scaffold or supporting structure.

those sum angles which are to be used as checks and which will, as nearly as may be, equalize the number of pointings.

Lost motion and stress in a repeating theodolite.—The repeating circles in use on the Survey, in common with all theodolites of their date, are lacking in rigidity, and flexure is probably a fertile source of error, the effects of which, together with the lost motion in the numerous movable parts and particularly in the clamping arrangement, can only be eliminated by careful manipulation and the adoption of a method of observing which will make them always of the same sign.

In practice it is found that by making all movements in one direction the error of closure obtained by measuring an angle and its explement will, within the probable errors of pointing and reading, remain constant for any particular condition of the instrument irrespective of the size of the angle; and since the angles measured according to this practice and obtained under conditions which give closing errors of wide range, when corrected by half the closing error, show a very close accord, it seems probable that the method largely eliminates errors from these sources.

For use upon towers, scaffolds, and like supports which are subject to an azimuthal movement, caused by the diurnal motion of the sun, the repeating theodolite is especially adapted, since the short time occupied in the measurement of a single angle reduces the effect to a minimum and the method of procedure above referred to eliminates it.

When the collimation is well adjusted it is not always necessary to reverse the telescope in the middle of a set unless there is a large difference, say one exceeding 1° of elevation, between the objects observed upon. The error of collimation should be frequently tested, and corrected if perceptible.

It is desirable to secure, *in the end*, an equal number of measures with telescope direct and with telescope reversed. Instruments with eccentric telescopes must be used in both positions of the telescope.

Respecting the manner of measuring and recording, whether with or against the graduation of the circle, the former practice is supposed preferable; but in all cases the object first sighted or pointed on should be recorded first in order. Thus in measuring the angle, A to B indicates motion direct as understood by the observer, but B to A indicates motion reversed, and the entry should be made accordingly. The observer should indicate in the preface of his record book the direction in which his instrument is graduated, and also give a diagram showing the directions to the stations seen.

Outside objects to be determined.—Besides the regular triangulation marks visible at a station, directions or angles should be carefully measured whenever practicable on all objects, as light-houses, beacons, buoys, and other aids to navigation, on all international, State, and county boundary monuments, township and section corners of the United States Land Survey, State capitols and court-house domes or cupolas, church steeples and all prominent buildings, and all outlying rocks, shoals, or breakers. A round of angles should also be taken on all prominent peaks and other landmarks, and tangents should be taken to all the headlands along the coast.

The observations for the magnetic declination to be made at each station are referred to in the report of the Committee on Terrestrial Magnetism.

(15) *Signals and scaffolds.*—The term "signal," as used in the Coast and Geodetic Survey in connection with triangulation, includes all devices, appliances, and instruments employed as objects to designate to the observer the position of a station mark to be pointed upon by him, and includes all structures intended to elevate such objects or the instruments employed in observing.

Although the term "signal," as here used, properly relates only to structures or devices especially constructed, observers have at one time or another used almost every class of object of sufficient prominence to be identified from distant points, such as mountain peaks, hilltops, headlands, rocks, trees, cairns or pyramids constructed of various materials, poles or staffs, flags, lozenges, targets of various forms, cones of tin, globes of glass, etc., heliotropes, revolving or fixed, bonfires, rockets, blue lights, lamps of various forms, lamps in combination with reflectors, magnesium and electric lights; and for supporting their instruments, mounds of earth, trunks of trees in original position, chimneys and light-houses, church towers, etc., tripods and scaffolds, towers of stone, adobe, etc.

Sometimes high structures are needed in order to maintain the general proportions of the triangulation or for the purpose of overcoming an obstruction in the line of sight, but ordinarily the supports are of moderate height, only as a means of carrying the line of sight above the highly disturbed stratum of air near the surface of the ground. Where the difference in expense is not too considerable, lines in a wooded country should be carried above the tops of the forests rather than through long, opened lanes.

The question of the desirability of portable or permanent structures, being very largely governed by the comparative cost of material and transportation, is in general not difficult to solve.

At all high structures the instrument rests on a central tripod of strong and well-braced timbers and is entirely separated from the surrounding light scaffold which supports the observer and serves, when boarded in or wrapped with canvas, as a protection to the instrument against wind or sun.

The very detailed article upon "Construction of observing tripods and scaffolds,"* as published by the Survey, makes it unnecessary to go into the subject here.

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* Appendix No. 10, United States Coast and Geodetic Report for 1882.

The visibility of objects sighted depends upon the brightness of the light, as heliotropes or artificial lights, or it depends upon contrast of the target, illuminated by ordinary daylight, with the background upon which it projects. The size of the reflecting surface in the use of heliotropes should be graduated to the distance from the station, for which see the scale proposed in another part of this paper.

In the case of targets or lozenges of various materials the relation of surface to distance will generally have to be determined by experiment. In the Coast and Geodetic Survey manual on triangulation the angular limit desirable for a signal has been stated as 1″; but as it is not practicable to maintain this limit at long distances, it will in practice be found necessary to increase the width sufficiently to preserve the area in some measure. In the case of lozenge-shaped targets the reflecting surface may be increased to any desired extent by multiplying their number. Lozenge-shaped targets of muslin have been found very satisfactory. The diagonals should be twice the linear value of 1″·5 for the distances. For lines of about 65 kilometres (40 miles) three or four lozenges, both black and white, symmetrically disposed along the pole, have proved satisfactory in all kinds of seeing and for different positions of the sun. Signals, such as rounded poles, which present to the observer two or more planes of varying relative distinctness are objectionable because they require phase correction, and should not be employed on work demanding a high degree of accuracy.

It may be well to call attention to the economy of the use of the heliotrope in consequence of its greater capacity for penetration of smoky or hazy atmosphere than could be had by a signal illuminated only by ordinary daylight. The heliotrope lights frequently enable the observations to be carried on when other signals would fail, and consequently they afford an opportunity to utilize the varying conditions of the atmosphere to a greater extent than is generally the case. This remark applies to relatively short lines; for long ones the heliotrope becomes indispensable.

The fact that all those who have made extensive experiments with night signals have reported favorably upon them should lead to their use whenever it is deemed advantageous. On lines of 50 kilometres and less they furnish beautiful steady objects for a greater number of hours, particularly before midnight, than day signals. Colonel Perrier, after his careful discussion of the results from night and day observations presented to the International Conference (Report of 1891), states as his conclusion—

> That the results of night observations satisfy better the geometrical conditions to which all triangulations are subject, or, in other words, that the errors, whether due to observation or to lateral refraction, which latter has never yet been well determined, compensate each other better in night observations than in those made in daytime.

In this connection attention is called to the extremely interesting experiments made by the Survey at Pioche, in Nevada, in 1883, and at

Mount Nebo, Utah, in 1887, to test the availability of the moon's light for night signals.

The selenotrope, as the instrument used has been called, differs from the heliotrope only in the greater size of the mirror used, and is operated in exactly the same way.

At Pioche, in 1883, a mirror 12·7^{cm} square (5 by 5 inches) was used on a line 35 kilometers (22 miles) in length, giving very satisfactory results with the moon.

In 1887, while occupying Mount Nebo, in Utah, with the view of testing the efficiency of the selenotrope upon much longer lines, mirrors 15 × 20^{cm} (6 by 8 inches), 20 × 25^{cm} (8 by 10 inches), and 30 × 46^{cm} (12 by 18 inches) were sent to Draper, Onaqui, and Ogden, respectively—77, 113, and 156 kilometers (48, 70, and 97 miles) distant—and the heliotropers were instructed to show two hours each night from June 29 to July 4, commencing forty-five minutes after sunset, or as soon as the shadow on the vanes became distinct. The weather was unfavorable except on the 2d and 3d of July, when Draper and Onaqui were plainly visible in the illuminated field of the telescope, "distinct, steady, mere dots of white light and of ideal perfection for precise pointing." Ogden, 156 kilometers (97 miles) distant, for some reason was not seen.

(16) *Marking of stations.*—The main objects in marking a station are to secure its permanency and to render it easy of recovery. If the station is located on a ledge or rock not likely to be disturbed, a copper bolt with a cross on its top to mark the center, secured in a drill hole several inches deep, or two or three short bolts placed one over the other, so as to be more difficult of extraction, form a suitable station mark. Two or three arrows pointing to the center mark may also be cut in the rock. Where excavation is possible, there should be one mark at the surface and another buried 3 feet below the surface. For primary stations a stone embedded in cement, with a copper bolt for center mark, forms the best subsurface mark. If an observing pier or terminal of a base line is used, it should be built of stone, brick, or concrete, with a cross mark in the top and also one at the surface, and another below the ground, to indicate the center of the station, and two openings should be left in the pier at the base, at right angles to each other, to give access to the surface mark.

In secondary and tertiary work a bottle, crock, or flowerpot filled with ashes may be used for a subsurface mark, with a stone and cross at the surface. Special conditions of soil require marks suited to their particular needs. In all cases there should be suitable witness or reference marks in addition to the central one, preferably grooves or drill holes (filled with sulphur) in adjacent rocks, to which the distances and bearings should be carefully noted.

Experience has shown that stations are frequently lost by reason of the thoughtless meddling of ignorant and irresponsible persons, as well as by some whose cupidity had been excited by the material used in marking them, as, for instance, lead and copper in a country inhabited by Indians.

Hence a good general rule is that stations should be marked by objects having little or no value. And it is important that the attention of the passer-by should not be attracted to the location of the station by reason of the prominence of the surface, reference, and witness marks. The aim should be to mark the spot in such a manner that there will be no difficulty for one who has its description to find it; but a casual observer should not have his attention attracted to it.

The description of a station should include a topographical sketch of the ground and its approaches, a sketch showing the relative positions of the station mark and various points of reference, the best route by which to reach it from the nearest town, and any information which might prove of value to a party subsequently occupying the station, either in finding the station or in locating a camp or obtaining supplies. It is desirable that the name of the trigonometric station be a short one.

(17) *Incidental observations.*—In reply to the question, "What other observations than those of angular measures should be made by a triangulation party?" it may be remarked that this includes the measure of vertical angles, unless there be good reason why they should be omitted. In primary work it is understood that such observations for latitude and azimuth should be made as pertain to the general needs of a triangulation for astronomic data. No special rules can be given, though frequently every other station has been an astronomic station. It may often happen that a detached piece of triangulation is to be done; here astronomic observations, at least at one station, are demanded.

Observations for the approximate determination of the magnetic declination should be made at every station. For particulars see Report of Committee on Magnetics. In general no other meteorological observations than those required for the description of the weather in the daily report or what are directly demanded by the work in hand need be made. Complete meteorological observations are essential in special experimental work respecting laws of diurnal variation of refraction or for comparative value of measures of heights by barometer and other means. These cases are always covered by instructions. Regular barometric observations at stations of great elevation are recommended, but are not obligatory unless specified in instructions. In all cases of doubt on the part of the observer he is advised to ask for special instructions.

(18) *Explanatory note to accompany map showing state of the triangulation in the United States in January, 1894.*—The object of the map (illustration No. 11) is to give at a glance the present extent of the triangulation and to make some suggestions for its future prosecution.

The map shows the work done by the Coast and Geodetic Survey, and also areas which have been either reconnoitered or which are in a state of incompleteness.

State of the Triangulation in January 1894, completed, commenced, or in progress and prospective.

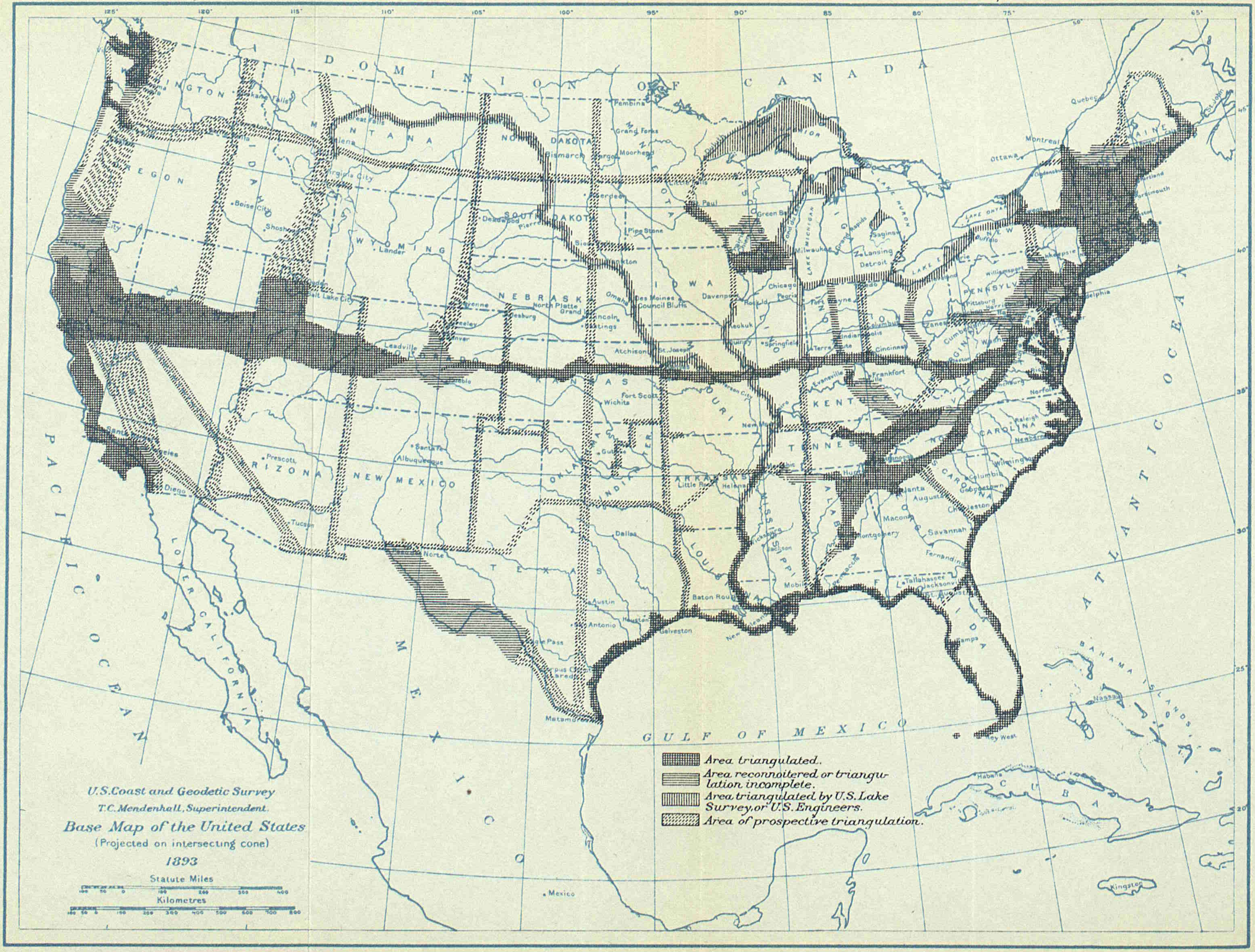


Photo-Lith. by A. HOEN & CO., Balto., Md.

The triangulations made by the United States Lake Survey and the United States Mississippi and Missouri River Commissions are also shown.

Other shaded areas are intended to show either proposed triangulations or to serve as suggestions for future consideration. They are not intended to mark out exact positions, since they can only serve to illustrate the general idea concerning the manner in which the required data for any contemplated surveys of States or their boundaries may readily be supplied. According to this view, all such surveys would depend on the standard geodetic latitude, longitude, and azimuth of the whole country, and they would use the same spheroid of development, thus securing uniformity and consistency; and in general no further astronomical observations or the measure of new base lines would be required except for special verification.

CHAS. A. SCHOTT, *Chairman.*
R. L. FARIS, *Secretary.*

REPORT OF COMMITTEE D ON ASTRONOMY.

TIME AND LONGITUDE.

The subject of astronomy, as applied in the Coast and Geodetic Survey, may suitably be treated under the four heads adopted in the Manual Appendix No. 14, United States Coast and Geodetic Survey Report for 1880, which gives very fully the methods employed up to that date and leaves but little to be added to bring it up to the present. In a cursory review of these subjects it is almost impossible to separate them entirely, as in some respects kindred operations run through all of them.

Time is used for latitude, azimuth, magnetic, gravity and longitude determinations, whether the latter falls under the head of (1) astronomic phenomena, such as eclipses, occultations, etc., (2) flashing signals, (3) chronometric, or (4) telegraphic.

The precision of the time required will indicate the character of the instrument to be used, such as the sextant, vertical circle, or transit in its various forms.

The number of time and circumpolar stars in the American Ephemeris is not sufficient for field astronomy, and it is necessary to resort to the Berlin Jahrbuch and other star catalogues to prevent otherwise unavoidable delays.

It is suggested that the four principal nations publishing nautical almanacs combine in the expense of preparing and issuing a special star list of greater extent than we now have, so that the number of time stars may be increased to an average of one for each two minutes of time, and also that the American Ephemeris should extend the

"Additional Star Places" so as to cover the period now falling in daylight; also that more azimuth stars should be added, and whenever practicable grouped by differences of nearly twelve hours. The star places should be given in order of right ascension. This would also be useful for determining value of micrometer.

Time and longitude are so intimately associated that the consideration of the latter necessarily involves the former.

Beyond the reach of the telegraph is the vast region of Alaska, which must have its longitudes determined where water transportation is available by chronometric expeditions, and far inland, beyond the navigable streams, by the observation of celestial phenomena. Works on astronomy give these methods in sufficient detail for practical use. Suffice it to say that the transportation of chronometers, when carefully conducted, gives satisfactory results within the limits expected. A thorough test of these chronometers should be made before using them. This could be done advantageously by using the new pendulum apparatus, employing an invariable pendulum, whose period is known, as a standard.

A few examples, showing the probable errors of results, are given:

Year.	Stations.	Trips.	Number of chronometers.	Days in a voyage.	Probable errors.
					s.
1855	Liverpool Cambridge	6	52	12	0·19
1856	Savannah Fernandina	8	10	1	·07
1892	Sitka Tacoma	12	8	6	·10

The cable determination between Greenwich and Cambridge differed from the chronometric result, in which 1,065 chronometers were used, by about $0·20^s$.

It is suggested that the longitude of the different Aleutian islands may be determined by flashing signals from one to the other and checking the end of a subdivision by chronometric expeditions. Most of these islands are sufficiently close to each other for the use of this method. As an example of flashing signals, the probable error of the longitude determination between Tetica, Spain, and M'Sabiha, Algiers, is given as $\pm$ $·013^s$.

The most important method of modern times for determining longitude, and the one most widely used, is the comparison of the local times of different points by means of the electric telegraph. This method of longitude determination has been so systematized and perfected in the last fifteen years that but little remains to be desired, and the slight modifications made from time to time during that period have been chiefly in the direction of equipments. A further advance may be practicable when the variable quantity, known as personal equation, can be eliminated by the photographic record of star transits, which is yet in its experimental stage.

The telegraphic method of determining longitudes was devised and introduced by the Coast Survey. In 1878 the mode of operation was much simplified; and in the same year was applied the present brief method of field computation, which enables an observer to complete the duplicate record, involving the sheet reading and time computation, in three or four hours. These field computations are of great assistance in the final office reduction.

These recent longitudes were determined by observations on from six to ten nights, the observers interchanging stations after half the number of nights were obtained, to eliminate personal equation. The local times were determined by observing the same stars whenever practicable at both stations each night, to eliminate errors of star place. Twenty stars were used each night. They were divided into two time sets of ten stars each, containing two azimuth and eight time stars, with reversal of the transit axis at the middle of each time set. Arbitrary signals were exchanged in both directions, to compare the chronometers, as near as practicable, midway between the two time sets, using the telegraph circuit for that purpose for about three minutes. The transmission and armature times were thus eliminated. The results compare favorably with the European longitudes, where observations were sometimes made on from sixteen to twenty nights, using three similar groups of stars and two exchanges of signals each night. All of the foreign determinations were not so elaborate, however. Many of recent date were determined just as ours are.

Twenty-nine European lines (see table), the best that were available, determined between 1881 and 1889, in which the observers and sometimes the instruments also were exchanged, show an average probable error of $\pm 0{\cdot}009^s$. Fifty-three lines (see tables) determined in the United States between 1880 and 1892 give a probable error of $\pm {\cdot}009^s$. When the best class of work is compared in both Europe and the United States the error of closing circuits is about the same (see table). It rarely exceeds $0{\cdot}10^s$, and in the majority of cases is lower. There are a few cases of abnormally large errors of closing in both countries that have not been fully explained. As a curious instance of a constant difference of results may be mentioned the double determination of the difference of longitude between Greenwich and Paris in 1888 by two sets of observers, making really four determinations of six nights each, which show a difference of $0{\cdot}21^s$. The work was repeated in 1892, with a difference of $0{\cdot}18^s$, or practically the same, but no explanation has been given by the observers. From the comparison above made it is clear that our work possesses the requisite accuracy.

Differences of longitude in Europe.

Stations.	Number of nights.	Difference of longitude.		Probable error.	Date.
		°	′	s.	
Paris–Milan	Not able to find data as to number of nights in most of these.	27	24·954	±·007	1881
Paris–Nice		19	51·225	·007	"
Paris–Leyden		8	35·213	·016	1884
Berlin–Swinemunde		3	28·969	·011	1883
Kiel–Swinemunde		16	28·203	·013	"
Konigsberg–Swinemunde		24	55·166	·010	1884
Konigsberg–Varsovie		2	08·300	·011	"
Berlin–Varsovie		30	32·477	·007	"
Berlin–Breslau		14	33·007	·007	1885
Konigsberg–Breslau		13	50·278	·008	"
Konigsberg–Rugard		28	11·819	·009	"
Kiel–Rugard		13	11·592	·009	1886
Kiel–Berlin		12	59·241	·010	"
Rauenberg–Berlin		0	06·393	·005	"
Konigsberg–Memel		2	24·228	·008	1887
Konigsberg–Goldaperberg		7	11·147	·006	"
Berlin–Schneekoppe	8 and 7	9	23·084	·007	1888
Breslau–Schneekoppe	7 and 8	5	10·803	·010	"
Breslau–Trockenberg	5 and 4	7	21·694	·008	1889
Breslau–Schonsee	4 and 5	7	26·812	·011	"
Trockenberg–Schonsee	4 and 5	0	05·190	·008	"
Konigsberg–Schonsee		6	23·441	·012	"
Breslau–Rosenthal		0	00·039	·007	"
Schonsee–Springberg		9	07·583	·010	1890
Berlin–Springberg		12	53·113	·016	"
Stockholm–Goteborg		24	22·73	·016	1885
Lund–Goteborg		4	53·72	·007	1886
Stockholm–Hermosand		0	24·45	·007	1888
Haparanda–Hermosand		24	45·51	·010	1889

Differences of longitude in the United States.

[Many of these are taken from the field results.]

Stations.	Number of nights.	Difference of longitude.	Probable error.	Date.
		h. m. s.	s.	
Cape May–Washington, D. C.	5 and 5	0 08 29·072	±·007	1881
Strasburg–Washington, D. C.	3 and 3	5 14·087	·007	"
Cincinnati–Washington, D. C.	4 and 5	29 29·262	·013	"
Cincinnati–Nashville	4 and 4	9 26·646	·006	"
St. Louis–Nashville	4 and 4	13 41·183	·011	"
Vincennes–Nashville	3 and 4	2 57·88	·017	"
Vincennes–St. Louis	4 and 3	10 43·232	·004	"
Little Rock–Galveston	5 and 5	10 04·260	·011	1885
Little Rock–Kansas City	5 and 4	9 15·644	·003	"
Colorado Springs–Kansas City	5 and 4	40 55·347	·010	"
Colorado Springs–Santa Fé	5 and 5	4 30·113	·011	1886
Colorado Springs–Gunnison	4 and 4	8 25·340	·004	"
Colorado Springs–Grand Junction	4 and 4	14 58·908	·012	"
Colorado Springs–Salt Lake City	5 and 5	28 18·470	·007	"
Ogden–Salt Lake City	5 and 5	0 24·546	·011	"
San Francisco–Salt Lake City	5 and 5	42 07·690	·011	1887
San Francisco–Washington, Lafayette Park	5 and 5	0 04·426	·006	"
San Francisco–Portland	5 and 5	1 00·006	·013	"
Wallawalla–Portland	5 and 5	17 19·517	·011	"
Wallawalla–Salt Lake City	5 and 5	25 48·187	·008	"
Wallawalla–Port Townsend	5 and 5	1 40·108	·012	1888
Wallawalla–Seattle	4 and 5	15 57·028	·009	"
Helena–Spokane Falls	4 and 4	21 34·437	·009	"
San Francisco–Mount Hamilton	6 and 5	3 09·041	·013	"
San Francisco–Sacramento	4 and 4	3 44·479	·007	{1888 1889
San Francisco–Point Arena	5 and 5	5 04·239	·008	1889
Sacramento–Point Arena	5 and 5	8 48·689	·006	"
Sacramento–Marysville	5 and 5	0 22·801	·008	"
Sacramento–Los Angeles	4 and 4	12 56·803	·007	"
San Francisco–Los Angeles	5 and 5	16 41·252	·009	"
Needles–Los Angeles	5 and 5	14 36·769	·014	"
Sacramento–Verdi	4 and 4	6 02·874	·007	"
Carson City–Verdi	4 and 4	0 52·558	·011	"
Carson City–Virginia City	4 and 4	0 28·180	·010	"
Carson City–Genoa	4 and 4	0 18·522	·016	"
Carson City–Austin	4 and 3	10 45·169	·015	"
Eureka–Austin	4 and 4	4 27·327	·006	"
Eureka–Salt Lake City	4 and 4	16 15·337	·007	"
Washington–Altoona	3 and 3	5 20·675	·012	1890
Salt Lake–Helena	5 and 5	0 33·583	·012	"
Bismarck–Helena	5 and 5	45 00·852	·013	"
Bismarck–Minneapolis	5 and 5	30 11·078	·007	"
Albany–Cape May	5 and 5	4 43·088	·010	1891
Albany–Detroit	5 and 5	37 11·894	·007	"
Chicago–Detroit	5 and 5	18 17·638	·006	"
Chicago–Minneapolis	5 and 5	22 27·414	·011	"
Omaha–Minneapolis	5 and 5	10 49·269	·009	"
San Diego–Los Angeles	5 and 5	4 22·802	·008	1892
San Diego–Yuma	5 and 5	10 09·127	·007	"
Los Angeles–Yuma	5 and 5	14 31·986	·006	"
Nogales–Yuma	5 and 5	14 43·698	·007	"
Nogales–El Paso	5 and 5	17 48·532	·008	"
Helena–Yellowstone Lake	5 and 5	6 33·835	·009	"

Double determinations and closing of circuits in Europe.

Stations.	Difference of longitude.		Closing error.	Stations.	Closing error.
	m.	s.	s.		s.
Paris-Algiers	2	50·374			
		·494	·126	Kiel-Berlin-Swinemunde	·007
		·368		Kiel-Berlin-Rugard	·069
Paris-Nice	27	24·964		One circuit of four lines	·042
		·957	·007		
Other closing errors are:					
Rome-Genoa	14	14·842			·085
		15·042	·200		·058
					·020
Rome-Padua	2	27·119			·004
		·131	·012		·034
					·072
Berlin-Breslau	14	33·887			·110
		·936	·049		·140
					·002
Pulkowa-Varsovie	37	11·30			·001
		·57	·27		

Double determinations and closing of circuits in the United States since 1881.

[Some of these are taken from the field computations.]

Stations.	Closing error.
	s.
Nashville-Vincennes-St. Louis	·085
Omaha-Kansas City-St. Louis	·005
Salt Lake City-San Francisco-Portland-Wallawalla	·018
Circuit of five lines	·041
" " three "	·091
" " three "	·004
" " three "	·033
" " three "	·034
" " three "	·057
" " six "	·068
" " three "	·020
Double determination of Little Rock, Ark., from San Francisco via Salt Lake City, Colorado Springs, Kansas City, and from San Francisco via Los Angeles, Yuma, Nogales, El Paso, differs	·04*

*The office computation may increase this considerably. About 20 lines are involved in this polygon; more are to be introduced.

There is no reasonable doubt that the expense of operating the longitude parties with the present outfit has been reduced to a minimum. The parties generally consist of two observers only, and determine latitude and the magnetic elements in addition to longitude without any extension of time at a station. It is also proposed to do gravity work; but this may either require a third observer or a longer detention of the parties at a station. Only such stations as are free from the jars of locomotives, street railways, and passing vehicles would be suitable for this work.

While the weight of the longitude outfit may be lessened by the use of smaller combination instruments for both longitude and latitude and by procuring lighter chronographs, the outfits on hand are too valuable to be discarded.

A revised edition of the manual should contain an example of the more recent longitude work, including a time set, computed by both the field and the least square methods.

Of the main scheme of longitude work laid out some years ago to embrace the United States there remains unfinished one long or two short seasons' work in the Southwest. It is also highly desirable to connect Montreal, Canada, with Cambridge, Mass., and Albany, N. Y., in order to utilize the last transatlantic cable determination, and to connect Cambridge, Mass., with the new Naval Observatory, Washington, D. C.

If the old and new Naval Observatories have not been satisfactorily connected, that should be done before the old station is destroyed.

It would very materially aid cartography if the geographical positions of the State capitals, important cities, county seats, and most of the larger towns along railroads were determined. Towns near national and State boundaries would be especially useful. This work might precede the triangulation many years.

Any surveys conducted by the States or by other authorities would derive great benefit from these established points.

In order to give an additional test to the accuracy of the longitude determinations it is recommended that one or more lines already determined by one set of observers be redetermined by other observers. The constant difference between Greenwich and Paris, as shown in the determination by different observers, illustrates the desirability of making this test.

ASTRONOMICAL DIFFERENCES OF LONGITUDE FROM LATITUDES AND RECIPROCAL AZIMUTHS; ALSO DIFFERENCES OF LONGITUDES FROM AZIMUTHS AND RECIPROCAL ZENITH DISTANCES.

In cases where the telegraphic method or the method by flash light is inapplicable for any reasons whatever, it is suggested that differences of longitude may be determined between intervisible points by reciprocal azimuths or by reciprocal zenith distances.

Under the most favorable conditions the method by azimuths is susceptible of a degree of accuracy equal to that of the telegraphic method. It is independent of geodetic elements of the earth.

The method of reciprocal zenith distances affords only approximate results, as it rests upon the assumed dimensions of the earth and requires a tolerably accurate knowledge of the coefficient of refraction.

INSTRUMENTS USED FOR LONGITUDE WORK ON THE CONTINENT OF EUROPE.

These instruments, with but two exceptions, so far as examined, are of the broken telescope type and resemble each other in general design. The size of the objective ranges from 63^{mm} to 77^{mm} and the focal distances from 700^{mm} to 870^{mm}, and the power of the eyepieces used averages 80. Two are as high as 90 and one as low as 60. So far as noted, a reticule of 13 threads has been used. It will be seen that at all points these figures are less than those describing the longitude instruments of the Survey.

The following table gives in compact form all the information available regarding these instruments:

Transits.

Where used.	Maker.	Objective.	Focal distance.	Power.	Telescope.	Threads.	Remarks.
		mm.	mm.				
Austria	Starke & Kammerer	66	?	90	Broken	13	Has reversing apparatus.
"	Pistor & Martins	68	870	?	"	13	Reversing and hanging level.
"	G. Starke	66	710	80	"	13	
"	Troughton & Simms	63	738	80	Straight	13	Reversing apparatus.
"	Repsold	68	835	80	"	13	Reversing gear and hanging level.
Munich	Van Ertel	77	812	60	Broken		" "
Milan	Repsold	70	800	?	?		
Padua	Van Ertel	66	700	40	Broken		
Paris	Rigaud	63	788	62	"		
Strassburg	Pistor & Martins	68	870	90	"		
Spain and Algiers	Brunner	61	775	?	Straight		Vertical circle, 415^{mm}.

The new longitude transits of the Coast and Geodetic Survey have a focal length of 95^{cm} ($37\frac{1}{2}$ inches), a power of about 100 with the present eyepiece, and a glass diaphragm with three tallies of 3, 5, and 3 lines each. The frame is so arranged that the azimuth and level adjustments are made at the base.

LATITUDE.

METHODS OF OBSERVATION AND INSTRUMENTS.

Under the head of latitude the inferior grades may be passed over with a few remarks. These are used for magnetics, reconnaissance, and explorations, and may be determined with a sextant or some form of vertical circle.

The better grades of latitude are now observed in the United States with the zenith telescope in some of its forms and by Talcott's method.

There are three classes of these instruments with telescopes, as follows:

1st.	66^{cm} (26-inch)	telescope with	57^{mm} ($2\frac{1}{4}$-inch)	objective;	power about	30–60
2d.	79^{cm} (31 ")	" "	57^{mm} ($2\frac{1}{4}$ ")	"	" "	60–90
3d.	114^{cm} (45 ")	" "	89^{mm} ($3\frac{1}{2}$ ")	"	" "	100

The main point to be considered is the accuracy desirable, and hence the number of pairs of stars to be used and the size and form of instruments. With good instruments, furnished with improved levels and micrometer screws, such as are now made, and catalogues with well-determined star places, the number of pairs of stars may be greatly reduced. On account of inferior star places, it was formerly the custom to observe thirty or forty pairs on six or seven nights. That number has been gradually reduced to fifteen or twenty pairs on from three to five nights, and may be further reduced and still give results sufficiently accurate for geodetic purposes.

An examination of the local deflections of the vertical at latitude stations shows that the determination of astronomic latitudes with an accuracy of about a quarter of a second is quite sufficient for most schemes of triangulation, being within the limits of ordinary deflections.

A greater precision is necessary in determining arcs and locating State and national boundaries, while for the purpose of investigating the variations of latitude the greatest degree of precision that may be obtained by the employment of the most refined instrument and methods is required.

The bearing of these last observations upon geodetic surveying is of the utmost importance, as it will enable us to determine corrections by which latitudes observed at various times may be reduced to the normal latitude.

With the instruments now in use the probable error of one observation for latitude is about one-third of a second. The probable error of the mean declination of a pair of stars, as furnished by the Coast and Geodetic Survey Office, is about one-quarter of a second. Ten or twelve pairs of stars observed on three or four nights should give a latitude with a probable error of about one-tenth of a second.

To further illustrate the idea that the number of observations may be reduced, a series of latitude observations was examined and the means were taken out for twenty pairs, observed on six, four, and two nights, respectively; then of fifteen pairs, ten pairs, and five pairs, observed for the same time. The sets chosen were by different observers, and both meridian instruments and zenith telescopes were used. Twenty pairs on six nights were taken as the standard, and the difference from this standard of fewer pairs on different nights is shown.

The star lists were prepared in the usual way, so that the differences of the zenith distance were balanced.

	1st set.*	2d set.*	3d set.*	4th set.†	5th set.*	Mean.
	"	"	"	"	"	"
20 pairs, 6 nights	0·00	0·00	0·00	0·00	0·00	0·00
20 " 4 "	·16	·01	·04	·05	·14	·08
20 " 2 "	·14	·11	·07	·20	·18	·14
15 " 6 "	·00	·01	·00	·05	·01	·01
15 " 4 "	·17	·03	·01	·05	·12	·08
15 " 2 "	·14	·06	·16	·09	·16	·12
10 " 6 "	·03	·14	·01	·13	·02	·07
10 " 4 "	·13	·09	·03	·22	·14	·12
10 " 2 "	·10	·16	·21	·15	·13	·15
5 " 6 "	·29	·07	·11	·12	·21	·16
5 " 4 "	·17	·03	·05	·12	·47	·17
5 " 2 "	·18	·01	·10	·13	·53	·19

*Z. T. No. 1, 117cm (46-inch) focal length, 83cm (3¼-inch) aperture.
†Meridian Inst. No. 16, 79cm (31-inch) focal length, 63cm (2½-inch) aperture.

The vertical circle has been used, and is still being used, with satisfaction for latitude observations on some of the European surveys. Under equal conditions it is not believed that it possesses any advantage over the zenith telescope. A series of tests of the various methods and instruments was made under Professor Bache in 1847, and the zenith telescope (using the Talcott method) was declared to be so far superior to the others for our work that it was adopted and has been used ever since. It is growing in favor with the Europeans. They offered a strong testimonial to its merits by using it for the precise observations necessary in determining the variations of latitude. Colonel Clarke, in his Geodesy, says of this instrument, "As made by Wurdemann, it is an instrument of extreme precision and most pleasant to observe with." And again: "The simplicity of construction of the zenith telescope exempts it from several of the recognized sources of instrumental errors, while its portability and ease of manipulation eminently fit it for geodetic purposes. It is exclusively adopted for latitudes in the United States, and it is probable that no one who has used it would return to graduated circles for latitude."

An enlarged catalogue of latitude stars is greatly needed; one that includes stars from the pole to 10° or 12° south of the equator, and down to the seventh magnitude. In the elevated regions of the United States there is but little trouble in observing these small stars.

As to the question of increasing the number of astronomical stations, especially those for latitude and azimuth determinations in the schemes of triangulation already planned, some remarks have been made in the reports of the committees on Arcs and on Triangulation. It may be stated in general terms that but few more of these stations are needed in the transcontinental scheme and in the triangulations on the Atlantic and Pacific coasts beyond those already contemplated in the uncompleted portions of the work.

For the purpose of studying the local deflections of the vertical, however, it is desirable to have as many latitudes and azimuths as can reasonably be observed without increasing the expense, and so distributed with regard to the orographic and geological features that any point of suspected disturbance may be examined. The local deflections along the Atlantic average $2''\cdot2$, but they are somewhat greater on the Pacific Slope.

When a scheme of triangulation starts on one of the principal lines of the regular network designed to cover the entire United States and terminates on another of these lines, no intermediate astronomical stations will be required.

When a scheme of triangulation is entirely independent of all other schemes both initial and terminal astronomic stations will be required.

In both of these cases the remarks on the subject of local deflections are applicable.

When no triangulation at all is contemplated the work falls under the head of geographical positions, which has been already considered.

The vertical circle used abroad in latitude work is also of the broken telescope variety. This form of instrument has been recommended for the ease and comfort to the observer, as his eye remains in the same position; also for its stability, owing to the low Y's and large base. It seems to have given no better results than instruments used in the Coast and Geodetic Survey.

The tendency toward adopting some form of zenith telescope on the Continent is presumptive evidence that it possesses advantages over all forms of vertical circles.

The latitude instrument used by Dr. Marcuse in the Hawaiian Islands was a zenith telescope of nearly the same objective, focal length, and power as those used in the Coast and Geodetic Survey for a similar purpose, but it is more massive in its parts and heavier to transport. The base is very heavy, the telescope is broken, but the prism is very near the focus.

There are two latitude levels of the best make. The horizontal axis is 20^{cm} and the vertical axis is 34^{cm} in length. The focal length of the telescope is 87^{cm} and the diameter of the objective is $6\cdot8^{cm}$

Vertical circles.

	Maker.	Objective.	Focal length.	Power.	Telescope.	Circles. H.	Circles. V.
		mm.				cm.	cm.
Austria	G. Starke	46	?	60	Broken	32	26
	"	53	?	60	"	34	34
Netherlands	Repsold	67	?	68	"	32	
Geneva and Strassburg	?	60	?	?	"	Objective at end of axis.	

AZIMUTH.

Azimuths are of different grades, according to the objects for which they may be required, such as magnetics, reconnaissance, and explorations, in which a 4 or 6 inch altazimuth may be used; for running meridian lines or other lines depending thereon; for tertiary triangulation, in which 8 or 10 inch instruments may be used; for secondary and primary triangulation, in which the best class of instruments, from 30^{cm} to 51^{cm} (12 to 20 inches), are required, and an accuracy compatible with the scheme of triangulation must be reached. The azimuth should be measured with the same precision as the horizontal angles in the scheme, which in the case of primary triangulation will in general be accomplished by observing azimuth in as many positions and series, or by as many sets of repetitions, according to the type of instruments employed, as are used in the horizontal angle work of the triangulation.

Under the assumption of careful and proper manipulation a primary azimuth may be determined with the more refined type of modern instruments in about four or five days' observations, with a probable error not exceeding about $\pm 0''\cdot 15$.

The azimuth is reduced to the normal meridian.

Accompanying this report is a map (illustration No. 12) showing the distribution of the principal astronomical stations occupied by the United States Coast and Geodetic Survey for the determination of latitude, longitude, and azimuth to January, 1894.

C. H. SINCLAIR, *Chairman.*
G. R. PUTNAM, *Secretary.*

REPORT OF COMMITTEE E, ON HYPSOMETRY.

A knowledge of relative elevations on the earth's surface is a fundamental necessity in the investigation of many physical questions and in engineering operations of various kinds. In the United States Coast and Geodetic Survey work of this nature has been done to meet the needs of the topographer, the triangulator, the physical hydrographer, and for the use of the engineers engaged on the improvement of our rivers.

So long as the operations were confined to the immediate vicinity of the seacoast, where connection could be made with tidal stations by means of short lines, the requisite degree of accuracy could be attained by methods of moderate precision; but when it became necessary to extend them considerable distances inland the demand for greater refinement became apparent, and there resulted what have been called the "standard levels of the Survey." These have been undertaken primarily to reduce the geodetic operations to sea level and to aid, by comparing tidal planes along the coast of the United States, in

Distribution of the principal astronomic stations occupied by the U.S. Coast and Geodetic Survey for Latitude, Longitude, and Azimuth to January 1894

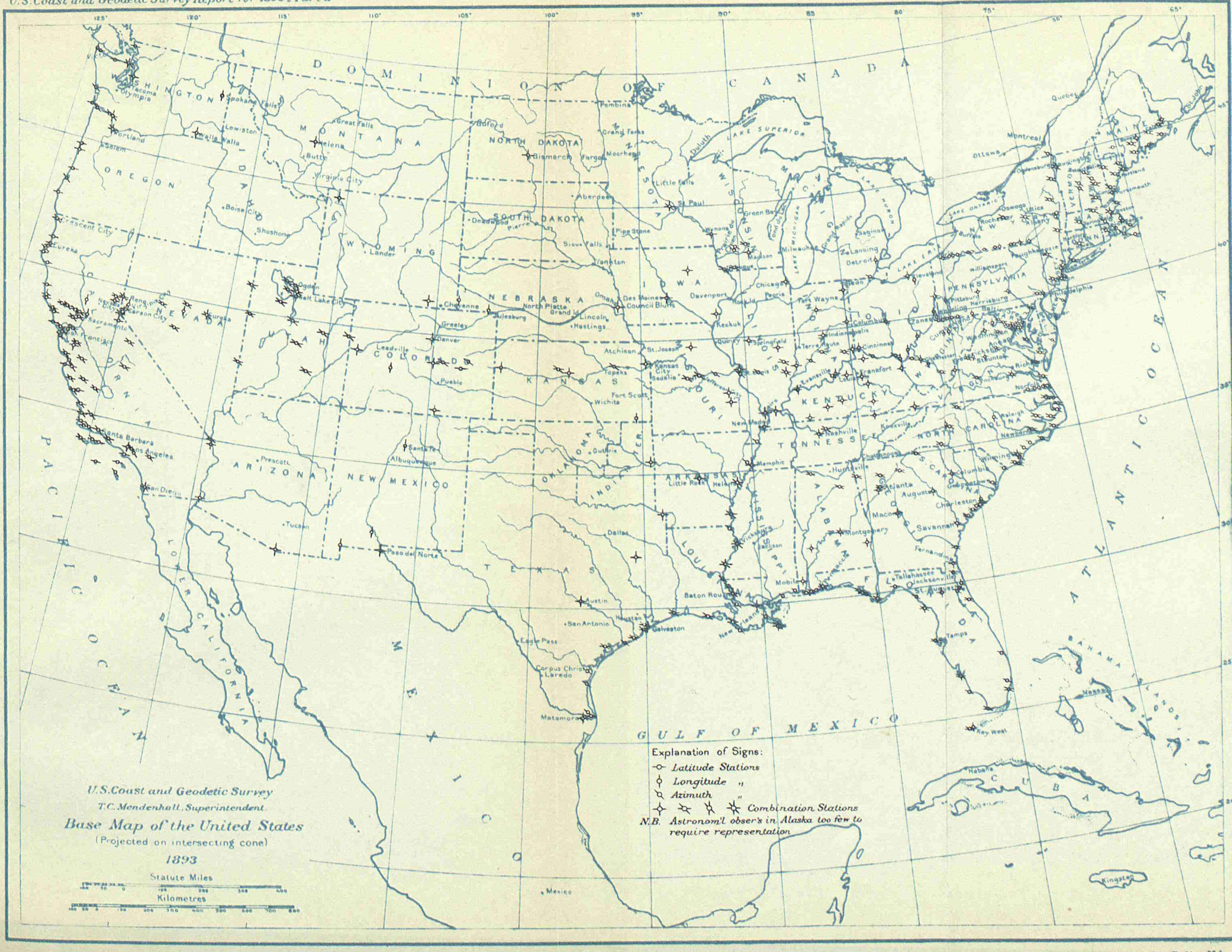


Photo-Lith by A. HOEN & CO. Balto. Md

the solution of important physical questions in regard to the ocean; to form the basis of and to bring into accord all topographical surveys of the country, whether under State or Federal patronage; to furnish data required in connection with gravity observations, and incidentally to supply planes of reference for engineering operations of all kinds.

As it is evident that these operations must ultimately be extended to all parts of the country, it becomes necessary to consider the requirements to be met and the degree of accuracy demanded. The objects most apparent and the degree of accuracy desirable in each may be stated as follows:

For topography: Within 1 metre. (See Annual Report Superintendent United States Coast and Geodetic Survey, 1891, Part II, p. 634, par. 17.)

For physical hydrography: With the utmost degree of accuracy.

For base-line reduction: Within 0·5 metre.

For gravity operations: Within six-tenths of a metre ($0{\cdot}6^{m}$).

For tidal planes: With the utmost degree of accuracy.

For meteorological investigations: To the nearest foot.*

For the study of strata and the flow of underground water in the arid regions an accurate system of levels is demanded.

For engineering operations: With great accuracy.

As the bench marks determined by the standard levels of the Survey must be used as base points for all of these objects, it is evident that a very high degree of accuracy is demanded, a degree as high as there is any hope of our attaining by the most precise instruments and methods at our command.

Two types of instruments are in use, the wye level of civil engineers and the geodetic or level of precision. They differ in construction, but more particularly in the methods employed in their use.

In general it may be stated that in wye levels the adjustments are made and supposed to remain constant during a day or a portion of a day, and the accuracy of the results depends upon the correctness of this assumption, the records containing no clue to changes, although in the best work the elimination of errors from adjustment is sought by making the F. S. and B. S. always equal.

The "geodetic level" of the Survey is a specially devised instrument. The errors in the various adjustments of the instrument are by the method used recorded at each station, and are eliminated by making the F. S. and B. S. equal by reversals of telescope and of the striding level, readings in each position by means of a micrometer screw being made and a mean taken, which is true in spite of any want of adjustment.

Other methods have been used in lines of precise levels elsewhere, and will be referred to under another head.

* Stated by Professor Harrington, Chief of United States Weather Bureau, in letter dated February 17, 1894.

About 9 750 kilometres (6 100 miles) of precise levels having already been run by different corps in the United States to meet the demands of special investigations and in the regular work of the Coast and Geodetic Survey, the desirability of a consistent scheme of lines to control the whole country to which all future work may be made to lend itself has become apparent. In presenting such a scheme the considerations governing its selection may be briefly stated as follows:

To provide a means, the most direct and economical, for connecting the many tidal stations on our Atlantic, Gulf, and Pacific seaboards.

To connect these several tidal planes by routes which will best overcome the uncertainties arising from crossing mountain chains, etc.

To form closed figures that will best determine the degree of accuracy of the work.

To make the lines of levels conform as nearly as possible to existing or proposed schemes of triangulation and especially of arc measures.

To take advantage of all work of a like nature heretofore executed, and to distribute judiciously over the country bench marks which may serve as points of departure for hypsometric work of all kinds.

The great advantages offered by graded ground for operations of this class, together with considerations of economy and facility of transportation, practically confine them to railway lines where they can be followed. Fortunately in most parts of the United States they are so numerous as to offer every facility.

On the accompanying sketch map (illustration No. 13), which is practically the same as presented to the Conference at its first meeting, the details of a general scheme are displayed which, it is believed, in a great measure fulfill the requirements mentioned.

It consists primarily of three east and west lines connecting on the Atlantic and Pacific seaboards with lines conforming to the general direction of those coasts and crossed between the ninetieth and ninety-eighth meridians by north and south lines of closed figures.

The four great figures thus formed are to be further subdivided into lesser ones as opportunity offers or special demands permit.

These lines should follow the most direct railway routes along the Atlantic seaboard from Maine to St. Augustine, Fla., across the peninsula of Florida to the Gulf of Mexico, which they skirt to Corpus Christi, thence to Laredo, Tex., and by the Southern Pacific route to the Pacific Slope, and northward, by the lines of railway conforming most nearly to the coast, to Seattle, whence they return by the Great Northern and the most direct routes to St. Paul, Chicago, Toledo, Buffalo, Oswego, and the Atlantic coast.

The central east and west line has been completed to Kansas City and the route from there to San Francisco definitely determined upon.

Lines have been run on the Atlantic coast from Sandy Hook to Fort Monroe, and the ninetieth meridian was selected for the north and south line of closed figures because the greater portion of the work has already been done or is in course of completion.

The distinction has been made between precise levels and levels as run by civil engineers for construction purposes. In the latter no attempt is generally made to secure a greater degree of accuracy than is required by the work in hand.

Precise levels, on the other hand, are intended to furnish, as nearly as physical conditions and the means at our disposal will admit, the true differences in elevations between points widely separated.

Work of this class has been done by almost all European nations, and to a lesser extent by three organizations under the United States Government. No pains have been spared that seemed necessary to give permanent value to the results either for the objects immediately in view or which might arise in the future.

While a high degree of accuracy has been attained, it nowhere equals that aimed at or hoped for, and the work may be said to be still in a measure experimental.

The following papers upon "Leveling in foreign countries," "Precise levels in the United States," and "Recent work with the engineer's precise wye level," which were prepared by different members of this committee, present the subject as fully as the time at our disposal would admit.

LEVELING IN FOREIGN COUNTRIES.

The record we find in history of vast schemes for reclamation of great areas of marsh lands, the successful projects for mighty systems of canal and river regulation, and the aqueduct systems of great water users like the Romans and Greeks testify to the early knowledge the ancients had of devices which enabled them to determine differences of levels with a fair means of exactness. The Egyptians must have had a fair idea of the relative differences of the Red Sea and the Mediterranean when they determined to connect the two seas by a canal, for it is doubtful if the Pharaoh who ordered it would have considered his project feasible, despite his mighty resources, if he had thought, as the Europeans did from the time of Napoleon's expedition to Egypt until Bourdaloué's leveling survey in 1847, that low tide in the Red Sea was 8·12^{m} higher than the same tide in the Mediterranean.

Precise leveling, however, with its present degree of exactness, was not a possibility before the invention of air-bubble levels, and this discovery only dates back to 1666. In the Journal des Savants for November 15 of that year is a paper describing the new apparatus, under the heading, "A new machine for the regulating of waterways, for building construction, for navigation, and many other arts."

The notice contains a diagram and a description of the instrument, but does not give the name of the inventor. In 1682 Melchisedech Thévenot, in an account of his travels, refers to the instrument as one he had invented fourteen or fifteen years previously, and he is now generally credited with this very important discovery.

The difficulty attending the exact construction of the level vial, particularly the calibration of the interior, prevented the utilization of the new instrument until about a century later, when the ingenuity of the French engineer Chezy overcame the difficulty and devised means for obtaining a regular curvature of the inside of the tube and enabled the mechanician to obtain any desired degree of sensibility.

The first leveling instruments which utilized the level vial in connection with the telescope were modeled after the old level with sighting vanes. The vanes were replaced by the telescope; but no attempt was made to make the line of horizontality of the level tube parallel with the line of vision, and their adjustments had to be repeated frequently and were very laborious.

Chezy, in France, and Ramsden, in England (about a hundred years after the invention of the spirit level), were the first to design instruments in which the modern principles of construction were introduced, and the main features of those constructed after their designs or by them are to be found in many used at the present time.

The first large scheme of leveling was undertaken in the latter part of the seventeenth century by order of Louis XIV in connection with the schemes for the improvement of French waterways. The perfecting of means for this purpose engaged the attention of such scientists as La Hire, Mariotte, and Huyghens, but the apparatus used was from a design submitted by the Abbé Picard, who was the first to suggest placing cross wires in the focus of the telescope, and who was given charge of the leveling operations. This level consisted of a box about $1{\cdot}25^{m}$ long inclosing a plumb line which was arranged at right angles on the tube of a telescope furnished with cross wires. In this box the plumb line was preserved from agitation by currents of air. A fiducial line engraved on an interior side of the box, and visible through an opening, enabled the observer to determine when the optical axis of the telescope was horizontal. The reticule of his instrument was adjusted by experiment so as to give a horizontal line when observing.

La Hire showed that the error of a sight with this instrument and method in the field need not exceed 1/36 000 of the length of a level sight, which would be nearly equivalent to $0{\cdot}003^{m}$ for 100^{m}. But this precision was only obtained at the expense of long and tedious trials.

The first general scheme for leveling over the area of a great country was set forth in a volume written by a French scientist, M. Ducarla, in 1782. Again, in 1805, P. S. Girard, the engineer in chief of the Ponts et Chaussées, presented a memoir in which he described a project for a system of levelings which would give all the curves necessary to show the configuration of the ground over the whole of France. This idea does not seem to have taken hold until the Restoration, when it was decided to prepare a new map of France, and in connection with this scheme a general system of trigonometrical leveling was ordered.

The disagreement (in some cases amounting to 2^m) that was so often found between the results of this leveling and the spirit levels of engineers in all parts of the country caused a great deal of unfavorable comment, and in 1847 the distinguished French engineer Bourdalouë, in an account of some levelings between Lyons, Nimes, Marseilles, and Valence, proposed a new scheme of levels of control and described methods by which the work could be done with a rapidity and degree of exactness surpassing anything yet attained in extended work.

Bourdalouë had already distinguished himself by several notable undertakings, having, among other important duties, been charged with the study of the topography of the Isthmus of Suez, where he had shown the error of the level determinations between the Red Sea and the Mediterranean made by the engineers of the First Napoleon.

For several years he occupied himself especially with the question of the general leveling of France. In the words of Breton de Champ, "A practical man above all, he wished to know at once what would be the cost of bringing so vast a work to a satisfactory conclusion. He was the possessor of a fortune acquired by the most honorable means, and a part of this was devoted to making an experimental leveling of the entire Department of le Cher, the one in which he was born. In July 15, 1857, the minister of public works, with the advice of the general council of bridges and roads, intrusted this work (the general survey of France) to M. Bourdalouë, 'who, by his previous experience, his capacity, and disinterestedness,' gave every guaranty that could be desired for the proper prosecution of an operation of this great importance."

Perhaps undue attention has been given to leveling in France; but it has seemed worthy of remark because the art we are considering seems to owe its origin and development (as now practiced) mainly to the scientists and far-sighted administrators of that country.

The telescope used in this work was $0{\cdot}50^m$ long, the diameter of the objective was $0{\cdot}04^m$, the focal length was $0{\cdot}48^m$, and the magnifying power used was 36. The levels used were attached to the frame of the instrument and had values of from $3''$ to $7''$ for 1^{mm} division of the level tube. By means of a screw the height of the rear end of the telescope could be modified when adjusting the instrument.

The rods were of the self-reading type, made of wood, 4^m in length, and divided generally into double decimetre and double centimetre divisions. Each double decimetre contains two groups of five divisions of $0{\cdot}02^m$ each, one on the left and one on the right side of the rod.

The width of the rod was $0{\cdot}07^m$ and that of the divisions $0{\cdot}025^m$, leaving $0{\cdot}045^m$ for the figures. In each group there were three figures, two in black and the third red. The third figure was 0 in the lower half of each double decimetre and V in the upper half. As two observers read the rod, and the sum of their readings was the true value of the length taken for back or fore sight, the divisions were conventional. The rods had two handles at the sides and were supplied with watch levels to insure their being held perpendicular. They were shod at the bottom and were held on portable steel spikes that were driven in the ground with a hammer or maul. The methods of equal distances of stations from instrument and keeping the level bubble in the center when observing were followed.

Each party consisted of four persons, one called the "observer," one called the "reader," and two rodmen. When the rods were in place and the instrument leveled the observer directed his telescope toward the back rod and silently noted the reading (while the reader watched the level bubble and saw it was exactly central), and then entered it in his notebook. The reader then took his place and silently marked his reading, while the observer watched the bubble. The instrument was then directed to the fore rod and the same procedure followed. Then the level and telescope were reversed, and commencing on the fore sight the same number of observations and in the same order were repeated on the two rods. When all was completed the reader announced his results, and if they agreed within the limits of tolerance with those of the observer the instrument was taken up and carried to the next station. This method, it was considered, compensated materially for the effect of sinking in the instrument or for any change which might be due to fine particles of dust getting into the bearings of the instrument, and practically eliminated all chances for recording a wrong reading. Sights were limited to 125 or 130^{m}, except for river crossings. Each line was leveled over three times in the manner described. Bourdaloué's field work was executed between 1857 and 1860, and the results issued in 1864. The lines leveled comprised a length of 14 980 kilometres, and the cost of the work was about 50 francs per kilometre. The error of closing of the polygons in Bourdaloué's work is said to be represented by the expression $1^{mm}\sqrt{k}$, where k was the length of line in kilometres. The difference allowed between any two measurements of a line of levels is said to be "very approximately" $2^{mm}\sqrt{k}$.

These values are given in Breton de Champ's treatise on leveling (3d ed., p. 331), but it is hardly credible that the Bourdaloué lines would be now classed as secondary if such precision was attained.

Bourdaloué's work was not carried out to the extent that was first planned in 1850, a neglect entailing, according to the estimate of a writer in the Genie Civil, a loss to France of at least a half milliard of francs ($90 000 000). In 1884 the necessity for greater refinement in the knowledge of the differences of levels between points of the first importance and of a more general system resulted in a new scheme for a general leveling, which, when presented to the Chamber of Deputies for action, elicited a report from M. Sadi-Carnot (the present President of the Republic), who was chairman of the committee charged with its consideration, in which he says: "Your committee is unanimous in agreeing to the projects of the Government. All its members have received from the bureaus they represent instructions to work for the success of an enterprise which concerns in the highest degree the prosperity of agriculture, the economy of public work (departmental, communal, and corporative), and the defense of our territory as well as the scientific renown of France." The appropriation for the work was set at 22 000 000 francs. Nineteen million francs were to be used for field work, and 3 000 000 francs were set aside for the production of a map (on a scale of 1:50 000) which would show the results of the leveling. The new scheme was to consist of 12 000 kilometres of lines of first-class precision and 800 000 kilometres of lines of the second class. The same instrument and general methods were to be used for both classes of work; but the allowance for discrepancies and probable error is about double for the second class what it is for the first, and there are no reversals of telescope and level required for second-class lines. The results of the levels are checked by connections with 12 first-class self-registering gauges on the channel, Atlantic, and Mediterranean.

This new leveling scheme was put in charge of a special commission consisting of delegates from the bridges and roads service, the army, and the mining engineers. The instrument used, which weighs 12 kilogrammes, has a telescope with an aperture of 36^{mm}, a focal length of 36^{cm}, and a magnifying power of 25; radius of curvature of level tube 50^{m}, giving a value of about $4''\cdot1$ per millimetre.

The instrument is carried on a spherical bearing, which permits its being leveled approximately before using the leveling screws. By means of reflecting prisms the two ends of the bubble, with their accompanying graduations, are visible in the telescope to the observer, enabling him to verify the accuracy of the level centering at the instant of getting the rod reading. The reflecting prisms are ground to different curvatures, so that both ends of the level are reflected with equal distinctness and size. The rods are the so-called compensating staves, designed by Colonel Goulier of the commission. They are of the self-reading varieties and made of wood. The divisions are centimetre, 5^{mm}, and 2^{mm}. The peculiarity of the rod is the provision made for its ready comparison. This consists of a Borda scale of iron and brass bars, used only for determining the length of the wooden rod fastened to it at the lower end and allowed to move freely in the opposite direction. The ends of the rods carry fine scales, the relative positions of which, as well as their comparison with a similar one on the wooden part of the rod, are read three times a day and noted in the record to determine the variations in the length of the rod.

When the wooden rod is graduated no care is taken to divide it with great exactness. A systematic error is allowed, and the correction for each division is determined by comparison with a standard scale; and the list so obtained is reserved for the computers and is purposely kept secret from the observers. After the standardizing of the rods a table, called an abacus, is prepared, by means of which by a rapid operation the computer finds for the apparent difference of level readings between the two rods the correction to be applied on account of the irregularities of the rod division, shown by the standardizing and the variation of length shown by the reading of the scales of compensation.

For this new work in France two field parties have been employed, each consisting of four men—an observer, an assistant, and two rodmen. The first thing done is the selection of the places for bench marks. They are then cemented into place, and after that the leveling begins. A day's work is restricted to the leveling between two fixed bench marks, and the line is to be run forward and backward. Pickets are driven in the ground to support the rods, and in their tops hemispherical-headed nails are driven for the rod to stand on. The same pickets are used for both measurements. Two rods are used in each party. After the end of a day's field work the record for the day must be mailed to the central office for computation, and if any errors are found beyond the limit of tolerance, the line is re-run. The fixed bench marks are plates of oxidized iron or bronze carrying a hemispherical projection on the flat top of the piece projecting from the wall in which it is fastened. On this the rod is set when the bench mark's height is determined. A vertical porcelain plate is also attached to the wall, which carries letters and numbers representing the section and place in it of the bench mark; also the inscription "Nivellement General" and the figures which represent the height of the bench mark above mean sea level. Another form of bench mark is a hemispherical-topped bronze bolt, which is cemented into the portals and base courses of certain buildings and bridges.

A special and original system of pay has been devised for this work by M. Lallemand, to whom more than anyone else the credit for the great success of this leveling is due. The pay of the field men increases nearly as the square of the length of lines run by them. Bourdaloüe's work, which had an accuracy only about one-third as good as the current system, cost 50 francs per kilometre. When the new work was taken up, in 1884, the price was 41 francs per kilometre, and now it has fallen to 33 francs per kilometre, while at the same time the rodman's pay has increased from 6 francs 50 centimes to 12 francs per day.

The instruments, rods, and methods of this work show a marked difference from those employed in India, Switzerland, and Germany, but apparently they are impressing others with their worth. In an article in the Genie Civil it is stated that they have been adopted for the survey of Algiers and Tunis by the Service Geographique de l'Armee. They are to be employed in the new leveling survey of Belgium. The Military Geographical Institute of Florence has announced its intention of substituting these instruments and methods for the Swiss and German ones hitherto exclusively used; and the Prussian Military Topographical Service is also going to make a trial of them (1890).

The French work is made up of closed polygons which average about 380 kilometres in perimeter. The probable error amounts on the average to $0{\cdot}9^{mm}$ per kilometre. The mean sea level at Marseilles is the datum plane adopted, and in addition to the self-registering gauge at that port 11 other of the new-form tide-registering gauges known as mediamaremeters are employed in connection with the leveling. In 1863 the results of the French work showed that the elevation assumed for the initial point of the Swiss leveling system was $2{\cdot}59^{m}$ too high. Values obtained from the lines of railroad levels which converged at Bâle gave a result which pointed to an error of $2{\cdot}11^{m}$; and a discussion of the trigonometrical results showed that the fundamental altitude of the Chasseral, which had been selected as the point of reference for that work, by a mistake was taken $0{\cdot}97^{m}$ too high, and consequently all these heights were too great by that quantity.

The consideration of these matters led to the appointment of a commission to provide for Switzerland a new and precise system of levels. This committee, through its representative, M. Hirsch, in 1864 presented a resolution to the International Geodetic Conference, which, being adopted, recommended a general system of precise levels over the greater part of Europe in which the method of leveling by equal sights and providing for the control of the work by a combination of closed polygons was to be followed. It was provided that each concurring nation should establish a permanent zero to which all of its heights would refer, the mean level of the sea to be determined at the greatest possible number of points by means of self-registering apparatus, and the zero point of the tide gauges to be comprised in the primary leveling. Upon the completion of the work a plan of comparison for all the heights in Europe was to be decided upon.

The instrument adopted by the Swiss was made by Kern and was of the type which is so generally known by his name. Two parties were selected for the work, and two instruments almost identical were supplied to them. The apertures of the objectives were $0{\cdot}033^{m}$ and $0{\cdot}035^{m}$, their focal length $0{\cdot}411^{m}$ and $0{\cdot}406^{m}$, and their magnifying powers 42 and 45, respectively. At first a Repsold level bubble with a value of $1''{\cdot}5$ for a length of division $2{\cdot}26^{mm}$ was used, but it was found impossible to utilize it in the open air, and an Ertel level of double the value was substituted for it. The level values were obtained by means of the 3-foot meridian circle in the Neufchâtel Observatory.

In the first instrument the reticule had one fixed and one movable wire, and the distance of the fixed thread, from 2^{cm} divisions between which it came, was measured by the movable wire connected with the micrometer attached to the telescope. But it was found that the errors of the micrometer readings were as great or greater than those made in dividing the rod by estimation, and three fixed horizontal wires were attached to the reticule. In a series of experiments the rods were set at a number of points varying in distance from the instrument between

10^m and 100^m, and readings taken, respectively, with micrometer and with the three wires.

For the first instrument and observer the mean error of a determination by use of fixed thread was $\pm$ 0''·838, and for the micrometer thread it was $\pm$ 1''·112.

For the second instrument and observer these values were, respectively, $\pm$ 1''·112 and $\pm$ 1''·578. The mean error of a reading in the field made with the first instrument and observer when the micrometer method was used was $\pm$ 0·69^{mm}. In the following year, when the three threads were used, this was reduced to $\pm$ 0·48^{mm}.

One rod was used in each party. These were made of pine wood, and were very carefully constructed by M. Kern, who attended personally to their graduation and painted the lines on them with his own hands. This was done with such precision that the errors of the divisions did not exceed the limits of errors of observation. The rods were 3^m long, 8^{cm} wide, 2·2^{cm} thick. To secure strength they had each a dorsal rib 4·8^{cm} thick, 2^{cm} wide. The division was into centimetres alternately black and white in the center of the rod. Outside them were white spaces on which the numbers were painted, on one side even and odd on the other. The rods had box levels and small projecting metal brackets. From the upper one the plumb bob was suspended when tests were made, and the lower one carried a small pyramid, the coincidence of whose point with the point of the plumb bob proved the perpendicularity of the rod. The end of the rod was shod like those used for precise leveling work in the Coast Survey. The foot plates were also similar. A light tripod was used to support the rod when occasion required it.

A large umbrella was used to shade the instrument. Generally the only check on the character of the work was the closing of the polygons, but in some special cases double lines were run. It was hoped that the reading of the three fixed lines would remove all danger of large errors of reading, but in the reports, nevertheless, there are instances of mistakes. In one case an error of a decimetre was made; in a second, one of 2·6^{dm}; in a third, of a whole metre.

A special point was made of separating the observations from the computations. Each evening the observer was required to make a copy of his day's record and compare it with his rodman. Then one copy was sent to the observatory at Neufchâtel from the first post-office available, and when an acknowledgment was had the other copy was sent to Geneva and two independent computations were made, one at each observatory.

The collimation and level adjustments are tested and the inequality of pivots is determined after mounting and before dismounting the instrument. Generally, in case the instrument receives any jar, they are to be determined before the instrument is used, and in any event they must be determined once a day.

The following is the manner of observing at a station: Having made the telescope horizontal and having set the vertical wire of the telescope on the center of the rod, the observer reads the level, noting the position of the ends—reading them to tenths. Then on signal from the rodman that his staff is vertical he reads the position of the three horizontal wires on the rod, beginning with the lowest, giving the result to decimillimetres. Then he examines his reading to see that there is no error of a centimetre or more, and for the second time he reads the level. On favorable ground the mean error per kilometre was about ± 0·66^{mm}, but over some of the high mountains, where rods had to be set up in springy, grassy ground, probable errors of 4·57^{mm} were found.

Two kilometres per day seems to have been the average rate of progress.

Before beginning the leveling work, in 1865, two bolts were securely cemented into a rock in front of the observatory at Neufchâtel which differed about 2·9^{m} in elevation, and each season the comparative lengths of the rods were tested by being held on them, with the instrument set up exactly between them. In addition to this test, the rods were tested every winter by comparison with the Swiss standard in Berne, and the results of the operations for fifteen years show a mean variation in the length of the rods (determined on the bench marks at Neufchâtel) of ± 0·064^{mm} per metre; and for rod number I, determined at Berne, of ± 0·058^{mm}, for rod number II of 0.066^{mm}, or a mean of ± 0·062^{mm}.

The observations at Berne were nearly all made at a mean temperature in winter, while those at Neufchâtel were at the beginning or end of a field season in temperatures varying from 2°·6 to 24° C. and with an air saturation varying from 0·57 to 0·98. The remarkable accordance of the result gave the commission, it states, a new guaranty that the rods, even when exposed in the field to still greater extremes, would not experience variations sensibly greater than those determined in the experiments.

The only information on the subject of English levels of precision that was attainable was the report on the work done in India in connection with the Great Trigonometrical Survey. Here for many years differences of height were obtained trigonometrically, but in 1858 the Survey began a line of spirit levels to connect points in central India with the mean sea level in Karachi Harbor and this work has since continued.

The instrument adopted was the Troughton & Simms level, described in Simms's Treatise on Instruments, of 1844, and is the ordinary Y-level, except that the level is partly embedded in the telescope tube. The focal length of the instrument was 53^{cm} (21 inches) and its magnifying power about 42. The levels had a value of about 1″.7 for a division (length not given), and in observing the ends of the bubble were read.

The rods were of wood, 10 feet long, and divided into feet, tenths and hundredths, one face having a white ground with black figures and the other a black ground with white figures.

On the first face the feet were numbered from 0 to 10 and on the other from 5·55 to 15·55. Both faces were read at each station, and if the horizontal wire intersected the commencement of a foot on one face it would intersect the middle of a different foot on the other face, and the observer could not be biased to repeat in the second reading a mistake made in his first, any error in either reading being shown by the deviation of the difference of the two values from the normal amount 5·55, or, in practice, by the difference in the resulting rise or fall obtained from the pairs of black and white face readings, which should give very nearly identical results.

"The rods were furnished with plumb bobs, let into their sides and visible through glass doors. Swivels were fixed on the top of the rods for guy ropes, to adjust them to the perpendicular and keep them steady. In order that the results obtained at each station by successive observers might be rigorously compared, it was necessary that the successive rods should invariably be set up on constant points, never on uneven surfaces. This was secured by driving a hemispherical brass brad in the head of each of the pins that were used for marking out the line of levels."

The distances of the rods from the instrument were measured with a chain. They were invariably made of equal length, and at the time of the report (1862), when levels had been extended nearly 2 000 miles and over every kind of country, involving the occupation of 12 000 stations, the rule had not been transgressed in a single instance. In the field two or three observers went over a line with different instruments and rods, but all using the same pegs.

The instruments were carefully shielded from the sun, but seem to have been dismounted at every station.

At an early period in this work the ever-present terror of the leveler, "cumulative error," showed itself, and as it had been supposed that the refinement of the instruments and methods employed left no room for any such difference, and the surveyors had no knowledge of the results which had perplexed earlier investigators, this factor caused much anxiety and troublesome investigation. After the first season's work an account of Professor Whewell's discussion of the line of levels run from the Bristol Channel to the English Channel in 1837–38 for the British Association relieved the Indian surveyors to some extent from the fear of unusual errors in their leveling.

One of the means employed to overcome this trouble was running alternate sections in different directions by the two observers and beginning the observations alternately with the back and fore rod. When this method is used, the black face is read first when the back rod is taken first, and the white face is read first when the fore rod is first taken.

The rods are read to the third decimal place, and if, after the leveling correction is applied, there is a difference of 0·006 of a foot in the results at a station, it must be releveled; and if the discrepancy remains, the first observer is recalled to remeasure the station, unless it appears that the fore peg has been disturbed, which would at once be shown by a corresponding change in the results obtained at the next station. All the results are used in the final computation. The bench marks are stone posts which are put in the earth at average distances of about 10 miles. The average daily rate of progress for each party is 4 miles in open, level country. The average annual output of work is 354 miles of double or treble line.

The rods are set up at distances of 8 to 10 chains (of links) from the instrument in the morning and 4 to 5 chains later in the day. A portable iron bar, whose length is known in terms of the standard of the trigonometrical survey, is taken in the field and compared at intervals with the rods.

In the German precise levels there has been no uniformity in type of instrument or rod used, although it is worthy of note that, in common with the Swiss, Italians, French, Dutch, Belgians, and English, wooden rods seem to be exclusively employed.

In the precise leveling in the Elbe Valley and in the lines from Swinemunde (on the Baltic) to Amsterdam and to Lake Constanz instruments made by Breithaupt & Son were used. The diameters of the objectives were 42^{mm}, the focal lengths were 460^{mm}, and the value of one division of level for a length of $2{\cdot}26^{mm}$ was $5''{\cdot}2$. Glass diaphragms were used, and the instruments had two eyepieces, allowing magnifying power of 42 and 32. The telescope rested on steel prisms and could be revolved about its axis, and could be changed end for end. The level could also be changed end for end. The rods used were of wood and of the type called by the Germans "reversible rods." One face was graduated from the bottom to the top and the other from the top down.

The Germans have paid great attention to the subject of the sinking of the rods and instrument, and to variations in the relations of the telescope and level while the observations are being made at a station. The manner of reading the rods and the setting up of the instrument have been considered in relation to these possible disturbances. On the Elbe work, where an extremely elaborate method was adopted, the procedure at a station was as follows: First, the cross wires were directed to the front face of back rod *a* and reading taken and the level ends read; second, front face of rod *b* read and level read; third, rear face of rod *b* read and level reading taken; fourth, read rear face of rod *b* and the level. Then, after reversal of telescope and level, these were repeated, and the fifth operation corresponded to the third, the sixth to the fourth, the seventh to the first, and the eighth to the second.

In setting up the tripod it was deemed advisable to follow a regular plan, in which it was provided that if the center leg of the tripod was set up at station 1, point forward, at station 2 it should point backward, at station 3 forward, and at station 4 backward again. This was intended to offset the disturbances due to the observer moving around the instrument.

An iron metre bar which was compared with the 290^{cm} Berne Bar, the standard adopted for rod comparisons for all the European precise leveling, was carried for regular daily comparisons. The rod was first compared for every single division error before going to the field, and after that the whole metre comparisons were considered sufficient to find the daily variation in the rod length.

The instrument used in the Prussian work by Dr. Jordan is of the Y type. The diameter of the objective is 41^{mm} and the focal length of the telescope is 420^{mm}; its magnifying power 30^{dm}. Box levels are used on the instrument and tripod for the first rough leveling of the instrument. One level division of $2{\cdot}26^{mm}$ in length corresponded to $4''{\cdot}1$.

The rod used was a hollow wooden one, 3^{m} long, 11^{cm} wide, and $3{\cdot}5^{cm}$ in thickness. The divisions were in half centimetres (subdivided into 10 parts), painted on the center of the face On one side of the marks were numbers, increasing from the bottom of the rod toward the top; on the other side the numbers increased from the top toward the bottom. Small silver stubs were inserted, on the faces of which were drawn lines marking the end of each metre. Each party was supplied with a field standard, which was used for making comparisons with the rod once or twice a day. This custom was introduced in the German surveys in 1878, and is now the universal rule. The time required is but short, as Jordan claims it can be done in five minutes daily.

The maximum length of sight in Jordan's work was 50^{m}, and benches were put in at every 2 000^{m}.

The lines were leveled in both directions and in the following manner: If the line started at bench mark *T* and was carried to stubs *a*, *b*, *c*, *d*, when returning on the same day the work was taken up at *c* and carried back to *T*. On the next day it was taken up at *d* and carried forward as on the day before, but on the return the work was carried through to *c*.

In an article in the Zeitschrift für Vermessungswesen Dr. Jordan gives as a sample of the rate of progress made by him the line from Germersheim–Bretten to Strassburg–Kniebis, 103·8 kilometres in length, and says it was leveled twice by him in twenty-seven days. The party seems to have consisted of an observer, assistant, and two rodmen. His experience showed that the leveling was more rapid with short sights than with long ones. By experiment he found that a half kilometre was leveled in 23·4 minutes when the instrument was set up six times and in 23·17 minutes when the instrument was set up seven times.

In this line a very interesting experience bearing upon one of the factors, that of sinking, which enters into the matter of cumulative error, was encountered. Dr. Jordan in one day leveled a stretch of 6 kilometres, in which he established three bench marks. A light rain was falling at times and the seeing was particularly good. The reversed leveling on this stretch was made on a day when the road was firm and dry, and the difference between the two measurements was 56^{mm}, or $9{\cdot}3^{mm}$ per kilometre. The differences between the lines on the different bench marks were 23^{mm}, 19^{mm}, and 14^{mm}, all with the same sign. Another measurement showed that the second line was correct. This was, of course, an exceptional case, and Dr. Jordan thinks that $\pm 1^{mm}$ for this cause would usually be an extreme value. The Swiss levels gave results which showed a probable value in their lines of $\pm 0{\cdot}5^{mm}$, and Seibt's Weichsel levels gave a value of $\pm 0{\cdot}3^{mm}$ for this error.

In comparing the procedure of foreign countries with our own one is struck with the stress that is there laid upon frequent comparisons of their rods with standards of length. This is of course partly due to the fact that wooden rods (the French rod must be considered such, as the metal rules in it are only used for checking the length of the wooden section) are exclusively used. Still the experience of levelers abroad seems to be that this necessity for frequent comparison is not a serious objection to the use of the wooden rod, and there seems to be no disposition on their part to abandon its use or to think its use inconsistent with obtaining the best results.

In this connection we may cite the results of Colonel Goulier's experiments on wooden rods, which are given in Kalmar's reports on the results of European levels to the International Geodetic Association in 1893. They are:

1. That the influence of temperature is not affected by the methods of preparation, and that for wood of the pinus or coniferæ varieties the rate of expansion is 9μ per metre per degree centigrade.

2. That of all woods the pine is least affected by moisture. The effect of moisture is in the direction of the vertical to the fibers. Boiling in oil has little effect in reducing the changes due to moisture, but a repeated painting with cold white lead reduces the variations considerably.

The variation in length on account of moisture is proportional to the increase in saturation until 60 per cent is reached; after that, there is very little change for any degree of saturation above this.

For painted pine wood rods the coefficient of expansion per metre for 1 per cent increase in humidity amounts to 18μ.

As the rods may vary from -5° C. to $+45^\circ$ C. in temperature, and the percentage of saturation may change from 15 to 95 per cent in the course of the season, the gross changes for the first cause may be 450μ per metre and for the second 810μ. But these causes generally affect the rod in opposite directions; when the moisture increases the temperature falls and vice versa, and experience has shown that the greatest differences do not exceed 500μ.

Lallemand furnishes a table, which is published in the report referred to above, on pages 190 and 191, giving the variations of six pairs of rods tested in the years between 1884 and 1891. There are 28 sets, and the observations were made in each case on each set in the beginning of summer and at the end of fall or beginning of winter. Only in one case is there a difference between the maximum and minimum of any rods for a season reaching 500μ per metre. The average seems to be about 250μ. Captain Kalmar calls attention to the very important fact, that has been shown by the records of the French, Prussian, and Swedish rod examinations, that the maximum or minimum values of rods used together in a season and tested together were found to be attained in almost every case on the same days for both rods. Hence, there is no chance for the individuality (as he expresses it) of any rod of a pair militating against securing good results.

In closing this short résumé of foreign methods and instruments, it may be interesting to quote from the report of the International Geodetic Association the length of precise levels in Europe. It amounted to 102 800 kilometres at the end of 1891. This includes no returns from Great Britain, and does not include Bourdaloue's levels in France, which are now considered of the second order, or the old lines in Prussia and Belgium.

PRECISE LEVELS IN THE UNITED STATES.

In the survey of the Great Lakes their elevation was determined by a combination of water levels, wye levels, and precise levels. The bench mark at Greenbush, N. Y., on which this work depends, was determined by the Coast Survey in 1856–57, but the accuracy of the determination would not at this date be regarded as satisfactory.

Starting from Greenbush, two lines of wye levels (in the same direction) were run by different parties to Oswego. Two rods were used in each party. A limit of discrepancy between the two lines of $19^{mm}\sqrt{k}$ was established, and the accumulated discrepancy amounted to $0{\cdot}293^{m}$ in the 400 kilometres.

From Oswego, N. Y., the elevation was carried by water levels across Lake Ontario to Port Dalhousie, Canada, and thence to Port Colborne by precise levels; thence across Lake Erie to Gibraltar, Mich., by water levels; thence to Lakeport by precise levels; thence across Lake Huron and as far as Escanaba, on Lake Michigan, by water levels, and thence to Marquette, Mich., by precise levels.

The precise levels referred to above may be described as follows:

Kern levels.—Wooden rods graduated to $0{\cdot}01^{m}$; position of three horizontal wires were read on the rod, estimating to $0{\cdot}001^{m}$; sights equal within 10^{m} and did not exceed 100^{m}. Correction applied to rod reading for collimation, inclination, irregularity of collars, and absolute length of rod. Two lines in the same direction by two independent parties; two rods were used in each party. The agreement is generally very good. The limit of tolerance was fixed at $5^{mm}\sqrt{k}$ and afterwards increased to $10^{mm}\sqrt{k}$. These lines are all short, but they show the gradual accumulation of error or divergence of the lines which so generally accompany levels of precision.

It was assumed that the mean level of Lakes Huron and Michigan was identical from May 19 to August 31, 1875. A theoretical discussion was made of the condition existing at the narrow junction of the two lakes and a deduced correction applied.

In the survey for the improvement of the Mississippi River an extensive system of precise levels has been executed. From New Orleans to Greenville the work was done by the Coast and Geodetic Survey for the Mississippi River Commission, and is fully described in Appendix No. 11 to Report for 1888. From Greenville they were extended by the Commission north to St. Louis, and thence to Savanna, Ill., along the river, and thence along the railroad to Chicago. From Savanna a line was carried along the river to St. Paul, and thence by railroad to Duluth, while a line along the Missouri River connects St. Louis with Kansas City and Sioux City. Various details in the methods were changed from time to time, but in general the following description applies:

Kern levels.—Wooden rods graduated to $0{\cdot}01^{m}$ and read by their wires to $0{\cdot}001^{m}$ (estimated); no target; kept vertical by circular level; foot plates and pins, but in general foot plates; spur on base of rod sometimes a plane surface supported on a spherical knob in a socket on the plate, and at other times a spur ending in a rounded point supported in a socket on plate. Conclusion reached: "If but one kind of support is to be used under all conditions, foot plates are preferable."

Lines.—In general two lines in opposite directions by different observers. Tents were used to shade and protect the instrument. Observations usually made from 6 to 8 a. m. and from 4 to 7 p. m., and never during the middle of the day except in cloudy weather. Corrections applied to rod readings for inclination, collimation, inequality of collars, and absolute length of rod. Progress about 3 kilometres per working day.

Cost.—Biloxi to New Orleans, 87 miles, $32 per mile; Keokuk to Fulton, 171 miles, $19 per mile; not given for other parts of line.

An elaborate mathematical discussion of the results has been made by Assistant Engineer L. L. Wheeler, who deduced the following conclusions:

1. The results of leveling may be affected by cumulative errors, which vary with different observers and do not always remain constant with the same observer.

2. The mean of several results obtained by the same or different observers may require a considerable correction.

3. That these cumulative errors are nearly proportional to the distances leveled and in some cases are independent of the nature of the ground, the direction in which the work is done, the season, or the manner of supporting the rods.

4. That in order, so far as possible, to eliminate the effect of such errors each observer should duplicate his own work in opposite directions under the same conditions.

5. That long lines of levels, even if leveled in duplicate, should be indisputably checked.

Mr. Wheeler also discusses personal errors of different observers in a very elaborate manner, and assigns the following values for those engaged on the work:

Personal errors of a single observation—

	mm.
J. A. Paige	± 4·27
A. D. Frost	4·14
E. H. Sankey	2·66
J. B. Johnson	3·07

(See pages 2551 and 2552, Report Chief of Engineers, 1884.)

While we may not indorse these conclusions, they seem to be very important in considering the character and accuracy with which the work was done, as they are the deliberate utterances of one who helped make the observations and afterwards discussed the results.

The instruments and methods used in the Coast Survey are described in Appendixes Nos. 15 and 16, Report for 1879. This method of observing has been followed since the beginning and the instruments at present in use are essentially the same, but many of the details have been changed, as experience suggested.

Weight of instrument, 23 pounds. The glass diaphragm has been replaced by spider lines. The vertical axis is now entirely above the tripod head. The level vial rests upon two points at either end in its inclosing tube and is held in position by a spring bearing on a third point at each end. The striding level is so constructed that it will bear with equal weight on each collar. Zylonite bands have been placed on the telescope, so that it can be revolved without touching the metal with the hands. A milled rubber head and zylonite reading head have been placed on the micrometer screw. The brass scale on the rods is secured to the brass boss at their base and left free to expand upward only. The thermometers have been attached to and brought, by means of brass filings, in metallic contact with the back of the brass scale near its center, and are read through an opening cut in the wood, which is closed by a brass slide.

In determining inequality of collars the telescope is adjusted so that it will bear with equal weight on each wye.

The field procedure has from time to time been as follows:

1. Simultaneous double line in one direction by the same observer.

2. Simultaneous double line, alternate sections in opposite directions, by the same observer.

3. Simultaneous double lines in one direction in sections by different observers.

4. Single lines with two rods in opposite directions by the same observer.

5. Single lines with two rods in opposite directions by different observers.

The present method is a simultaneous double line by the same observer in one direction, with a (proposed) similar check line in the opposite direction. With these exceptions, the description in the appendix referred to still applies.

The limit of error adopted is $5^{mm} \sqrt{k}$.

To suspend work during the middle of the day was the exception and not the rule.

In 1892 an elaborate series of experiments were undertaken under the personal direction of the Superintendent to investigate our system of leveling as well as the instruments employed. These have not been finally discussed, but they have resulted in valuable suggestions of a practical nature. Certain errors, the causes of which are still uncertain, remain uneliminated. Improvements in the appliances are proposed, and it appears particularly desirable that the qualifications of an observer be thoroughly tested before being intrusted with work of this class.

RECENT WORK WITH THE ENGINEER'S PRECISE LEVEL.

Few opportunities offer in this country for direct comparison between the "wye level" and the "geodetic level," the latter having been exclusively used upon closed circuits of the highest order; but the unexpected accordance in the double line of 200 miles of State work between Boston and Albany run by an assistant of the Coast and Geodetic Survey with the wye level during the past season, the close agreement at Springfield between the tidal planes brought by 99 miles of this work from Boston with that brought by the Engineer Corps line from Long Island Sound, together with the results from many trials of the two types over the test circuit at Washington, while inconclusive, all point to a high degree of precision with the wye level.

On the line between Boston and Albany above referred to an instrument which has been named the "engineer's precise wye level," designed by Buff and Berger in 1892–93, was used, which may be thus described:

A wye level resting in a cradle suspended upon an axis at center of instrument; micrometer screw with opposing spring under eye end of cradle. Telescope 38^{mm} ($1\frac{1}{2}$ inches) clear aperture, 38^{cm} (15 inches) focal length, Steinheil inverting eyepiece power of 35. Value of one division of the level (2^{mm}) equals 7″.

Rods, wooden, made of white pine, T-shaped, impregnated with paraffin, graduated to feet and hundredths, lower ends a flat shoe of hard steel, target with vernier reading to thousandths of a foot, cross levels on rod. The party consisted of one observer, one bubble tender, and two rodmen; instrument protected by large umbrella; double simultaneous line in one direction along a railroad; back and fore sight kept equal by counting rails; turning points, round-headed spikes; bubble kept in center; instrumental adjustments tested one or more times a day.

The terminal benches were at nearly the same elevation. The summit elevation passed over was 1 458 feet. While the two lines cross and recross each other many times, they only at one point departed by as much as 12^{mm} and are only 3^{mm} apart at the final bench mark.

The 322 kilometres of double line cost, including salaries and all other expenses, less than $5.50 per kilometre; and in a letter addressed to the chairman, published in the Massachusetts Topographical Survey Report of 1893, Dr. Mendenhall says:

> I have examined with great interest the profile of the line of levels recently run between Boston, Mass., and Albany, N. Y. The agreement between the two simultaneous lines is remarkably close, giving evidence that the whole is an excellent piece of work. I have recently tested the instrument and method used * * * and the result is such as to give me great confidence in the line. * * * I do not believe as long a line has ever before been run combining so high a degree of accuracy with so small a cost.

The wye level is used by Dr. Jordan in the Prussian work, and Professor Boersch, who has had much experience, after a careful discussion of the precise methods, says (Zeitschrift für Vermessungswesen):

> From all of the aforesaid it appears that with the expenditure of great care, labor, and cost no better results, but only the appearance of a so-called scientific treatment of the subject, can be shown. The simpler the method of observation, and the fewer the figures required without decreasing the accuracy, the better results one will obtain and the less will one be exposed to observation and computation errors. It remains, therefore, always preferable in field observations where the tripod is used to employ bubbles which come to rest, and which, during the pointing upon the rod through the telescope, can be maintained in the middle of the level scale by an ordinary assistant, whose services are required anyway.

The same author, in the same work, says in regard to a "bubble tender":

> It only remains, after what has been said, to provide an additional observer for the reading of the level. There will always be a man among the laborers employed in the work who can be trained to keep in the middle the bubble of a less sensitive level, besides performing the duties of an instrument carrier. * * *

He also objects to a mirror on the ground of parallax and strain to the observer's eyes. In a later number of this publication the following appears:

> Professor Boersch advocates * * * the use of a second assistant for tending the bubble during the observation on the rod. In this, one is free from the hypothesis that the movement of the instrument is proportional to the time of the observations. * * * This method was used in the Bavarian leveling when the wind was high.

The writer then calls attention to the effect upon the level in the change of center of gravity of the observer while leveling, even though he does not "change the position of his feet".

DETACHED POINTS.

TRIGONOMETRICAL LEVELING.

Although for the determination of differences of elevation no method presents itself which equals in precision that of the more delicate forms of leveling instruments, there are others which for the determination of the elevations of detached points offer decided advantages, and some of which produce results of a degree of accuracy sufficient to meet many of the objects for which data of this class are sought. These are included under two heads, viz:

1. By angles of elevation and depression of points between which the distance is known; and
2. By observations of the relative pressure of the atmosphere.

The first have mainly been employed in connection with schemes of triangulation which, consisting of lines of known length connecting intervisible points, offer the most favorable conditions. When the absolute difference in elevation between the instrument and any other visible point is known, and all the objects to be observed are in nearly the same horizontal plane, they may be determined with a good degree of accuracy by means of micrometric differences, the value of the results depending upon the accuracy of the pointing, stability and perfection of adjustment of the instrument and the difference of refraction in any two lines at the time of their observation. In the cases of objects beyond the range of the micrometer screw, or when the elevation of only one point is known, recourse must be had to the vertical circle for the determination of the double zenith distance or to the spirit level in connection with a vertical arc for the measurement of the angle of elevation or depression. In either of these cases errors of graduation, etc., and the uncertainty in amount of vertical refraction must be added to those mentioned as affecting the results from micrometric differences.

While the very excellent experiments already made for the determination of vertical refraction have taught geodesists much, they serve rather to point out the great changes to which it is subject than to give confidence in our ability to apply an adequate correction. The probable error in the determination of a single point, as derived from the adjustment of large figures, is still considerable, ranging from $0{\cdot}39^{m}$ to $2{\cdot}95^{m}$; yet the law of compensation seems to hold good, and the difference in elevation between widely separated terminals, as determined by vertical angles extending through a scheme of triangulation, differs but little from the results by the more precise methods. We may cite the work across California, Nevada, and Utah, some eight figures, covering about 1 000 or 1 100 kilometres, where the height of Ogden, as determined by trigonometrical leveling (double zenith distances on ten or more days at each station), agrees with that derived by Gannett from railway levels within $4{\cdot}27^{m}$, and that across the State of New

York from Albany to Oswego, six figures, covering some 240 kilometres, where the difference in elevation as determined by the trigonometrical method (observations on some five days at each station) agrees with the result of the double line of levels by the United States Engineers within 2·37^{m}.

Colonel Walker (Vol. I, p. 103, Report G. T. S. of India, 1870) gives the following results of comparisons between spirit levels and trigonometrical heights derived through long chains of triangles, viz:

From Karachi to Attok,	706 miles,	difference	—3·2 feet
Attok to Dehra Doon,	416 "	"	+5·1
Dehra Doon to Sironj,	429 "	"	+1·8
Karachi to Sironj,	669 "	"	+2·1
Sironj to Calcutta,	680 "	"	—4·6

All of these examples are from mountain work; in the first case with points of great elevation and extremely long lines and in the latter points of moderate height and lines of moderate length. In both results the effect of differences of refraction and error from deflection of the plumb line remains.

On the 39th parallel trigonometrical survey in western Missouri occurs a case which offers a particularly good opportunity for the examination of results in a rolling country. In that work the scheme coincided so nearly with the transcontinental line of precise levels that nine consecutive points, covering some 185 kilometres and forming the northern line of the scheme, were determined with the spirit level, and a comparison of the results by the two methods shows at no point a difference of more than 0·6^{m} and a final error between terminals of only 0·3^{m}. In this work the practice was to observe on five days at each station the double zenith distances of two points, and the difference in elevation between these and between all other visible points was determined by micrometric differences measured on six days.

In computing, the micrometric differences seem to have been taken as a standard and the zenith distances made to conform to them.

The methods and instruments to be adopted in this class of work will depend upon the degree of accuracy sought, and this upon the objects to which the results are to be applied.

It would appear from what has gone before that any of the methods mentioned produce results sufficiently accurate for the use of the geographer, and the better ones for the determination of bench marks for the topographer. To what extent they may in the future be required or applicable to the elucidation of physical problems does not appear; but since the difference in cost as between the rough and refined methods will rarely amount to more than 1 per cent of the expense of field operations of parties in which vertical measures constitute a very inconsiderable part of the duty, it would seem to be good policy to maintain a standard high enough to meet any requirements of the future.

The determination of differences of elevation by observations of atmospheric pressure is often convenient, and at times the only method available. The results from a limited number of observations for widely separated points can not, however, be depended upon within some hundreds of feet, and in a mountainous country are very unreliable for even moderate distances. This is aside from errors of observation of phenomena within our reach, which, with our present instruments and knowledge, are very great, and arise from atmospheric disturbances of a more or less local nature, the conditions in neighboring valleys rarely being the same, and during rapid changes often very different on different slopes and in different parts of the same valley. As pointed out by Ferrel, a cyclonic disturbance not sufficient to amount to a storm may produce a variation in the barometric gradient between points a few hundred miles apart amounting to 100 feet or more, and Williamson's computations of the difference of daily results for elevation during the year 1862 between St. Bernard and Geneva (some 50 kilometres) show errors as great as 60^{m} for a single day, the determination after forty years of most careful observations under M. Plantamour being $2{\cdot}7^{m}$ in error.

The results from careful observations in this country, ranging from one to six years, give resulting errors of from 16 to 37 feet, but with contrary signs, so that there seems no reason to doubt the constants used in the reductions.

It behooves us, however, to consider the relative degree of accuracy of the several instruments employed for the purpose and the degree and conditions of their usefulness. There are a few rules which apply when only a limited number of observations can be taken, whatever instrument is used, and which should be borne in mind by the observer, viz:

Observations should be made at such hours as give most nearly the mean temperature of the day, or, better, of the month.

Observations should not be taken when there is fog or mist.

In computing results it is desirable to use the normal temperature of the vicinity of the lower station if obtainable.

The observations should be simultaneous when practicable.

The mercurial barometer in its most improved form was probably until a late date the most suitable instrument at our command for the determination of atmosphere pressure, and from its simplicity and the ease of its manipulation must always be a favorite, but for work in a rough country the extreme care required in transporting it is a serious drawback. The marked advance in our knowledge of thermometry within the last decade, however, suggests possibilities from observations of the boiling point which equal and may surpass* in accuracy

*See Von Jordan: "Vergleich zweier Siede-Thermometer mit Quecksilber-Barometer," Zeitschrift für Instrumenten kunde, Jahrg. 1890, S. 341-347. Herner "Siede Thermometer und Quecksilber Barometer," Zeitschrift für Vermessungswesen, Bd. 21 (1892), S. 30-31.

the results obtained with the barometer, and in view of the greater portability of the apparatus required it seems desirable that its use be encouraged.

The aneroid barometer is essentially portable, but unfortunately this is true only as regards the integrity of its several parts and not of the instrument as a whole, since, while it will survive very rough usage, the relations of its parts are so easily disturbed as practically to produce a different instrument. The magnitude of these changes, and the tendency of the parts to resume given relations, differs very widely in aneroids from the same maker and intended to be of the same class.

This results from mechanical defects, but so minute and so hard to trace that to correct them would magnify the cost of construction beyond all bounds. Thus it happens that one which gives excellent results in the comparing room may prove utterly worthless in the field. It is, however, so extremely useful for purposes of reconnaissance that it well repays the labor bestowed in testing it under the conditions actually occurring in the field work on which it is intended to be used. With a well-selected instrument, carefully handled, profiles may be traced in a moderately flat or rolling country with sufficient accuracy for all purposes of reconnaissance; and how well it may be relied upon for the determination of accidents of surface where a sufficient number of controlling points are available is amply shown by very characteristic topographic maps produced by this method by the Geological Survey.

In concluding this report, and after examining accounts of work of the various kinds and in the several countries referred to, your committee beg to present the following as their conclusions, viz:

That trigonometric heights, when observed during the hours of minimum vertical refraction and under varying atmospheric conditions, can be obtained with a sufficient degree of accuracy to make them valuable for many purposes, and, when not impracticable, their determination should form part of the work at all triangulation stations.

For this purpose there is suggested a new form of instrument or such changes or additions to those now in use as will permit of the measurement of micrometric differences of larger arcs than is possible with the micrometric eye piece attached to our theodolites, and of the convenient use of a delicate level in connection with it. The gradienter as now constructed seems in a great measure to fulfill the requirements for short lines, but the optical power is too low for use over long ones, and other improvements seem possible.

That with modern thermometers and methods the atmospheric pressure may be determined by observations of the boiling point with a precision equal, if not superior, to that obtained with the less portable mercurial mountain barometer, and for purposes of exploration their use is recommended.

That the standard bench marks throughout the country should be determined with the greatest degree of accuracy attainable.

That subsidiary lines may be leveled by less precise methods.

That if upon investigation it shall appear that less elaborate methods or instruments than those now employed on the Survey will by the use of small circuits produce satisfactory results, with an increase of economy, purely theoretical considerations should not prevent their adoption.

That the leveling instruments at present in use on the Survey seem as perfect as any yet devised, but the use of prisms by which the level may be read without removing the eye from the eyepiece appears to be desirable, and we would recommend that one of the French instruments to which they have been applied be procured for experimental purposes.

That the level should have a value of about 2″ to a millimetre.

That a watch level or some other device is desirable to facilitate the preliminary rough leveling of the instrument.

That the rods be frequently compared in the field with a standard.

That if, as seems possible, wood can to a great degree be protected against hygrometric changes, it is preferable to metal for leveling rods.

We recommend a plane surface for the terminal of the rods, and the use of foot plates with hemispherical centers for supporting the rods, and retaining walls to prevent any undue lateral movement.

The scale, if of metal, should be protected from rapid changes of temperature due to varying position. It should be thin enough to respond quickly to changes in temperature, of a form that will secure the requisite rigidity, and so arranged as to be readily compared with a standard.

If wooden rods are to be used, the measuring portion should be carefully selected, homogeneous, straight-grained, thoroughly seasoned wood of the pinus family. It should be carefully impregnated with paraffin and perfectly coated to protect it from moisture as far as possible. It should be so attached to the supporting rod as to secure protection from abrasion of the surfaces and with an air space sufficient to prevent direct contact or the harboring of moisture. Certain of the graduations might be marked by lines on metal pins embedded in the wood. The face of the rod should be over the center of support.

The target, if one is used, and its advisability seems questionable, should travel as nearly as may be along a line passing through the center of support of the rod, and carry a vane by which the observer at the instrument may assure himself of the rod's verticality.

The rod level should be placed at angles of 45° with the face of the rod and as near together as possible.

Notes on observed cases of abnormal refraction in lines passing near the ground, which emphasize the possibility of errors from this source, are given below.

FRANK WALLEY PERKINS, *Chairman.*
F. A. YOUNG, *Secretary.*

MEMORANDUM FOR THE USE OF THE COMMITTEE ON HYPSOMETRY.

I offer the following evidences of local refraction close to the surface of the ground, believing that one of the principal sources of errors in leveling arises from that condition.

San Pedro Base Line—1853.

In measuring this base an aligning flag was sent forward about 200 yards, and in trying to place it in line it appeared and disappeared and changed place in such a curious and irregular manner that I personally went forward to ascertain the cause. As I approached, the flag that I had seen disappeared, and it was lying upon the ground and had been during the confused signals. The aid had misunderstood my signals, which he said were very confusing, and had stepped aside. As I returned to the base the flag again reappeared in the air to the height of about 4 or 5 feet. On these plains, in the preceding December, I had witnessed remarkable effects of mirage at midday.

On the base-line site at Port Townsend, in 1854, a target 4 feet square, standing on the ground, was apparently raised by local refraction more than its height, as seen from a distance of 250 yards.

Yolo Base Line—1881.

When the party was going out to work one morning there was a beautiful exhibition of mirage ahead of us, wherein all objects were lifted up into the air.

We drove into and through this warm stratum of air, and when in it saw the mirage effects all around us.

Los Angeles Base Line—1889.

In driving to work we frequently came into streaks of quite warm air and then into streaks of quite cold air. On different mornings, in passing over the same depressions, the temperatures were not relatively the same.

San Francisco.

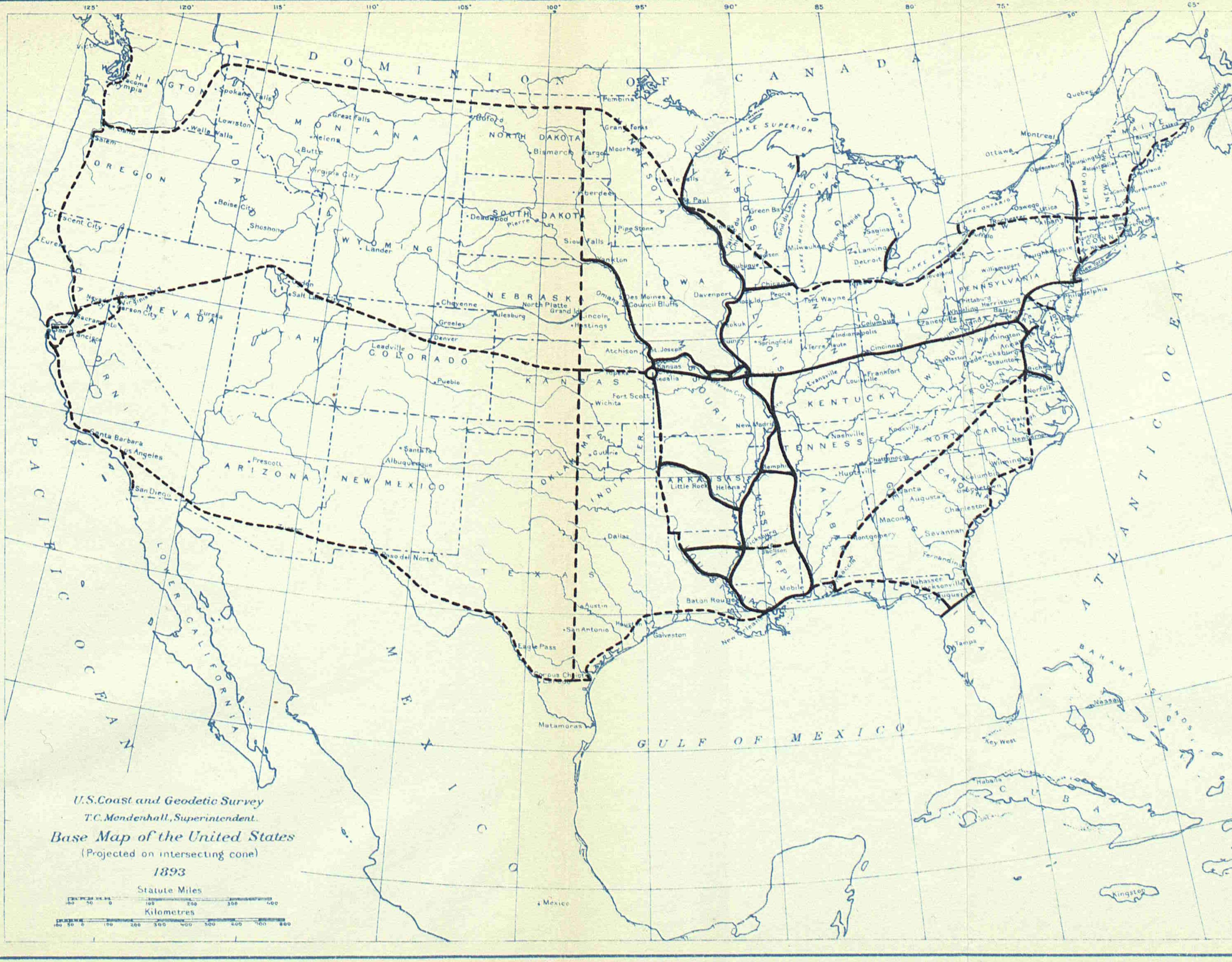
On some of the streets of San Francisco the exhibition of mirage is so marked that it has been illustrated in the newspapers. And upon Washington street, near the Lafayette Park Astronomical Station, where the block is nearly level, with a decline at each end, I frequently see the mirage along the whole block as my line of vision reaches the height of the street.

There can be no doubt that such conditions would give very wild leveling results; and it naturally suggests that abnormal conditions of the surface layer of air may not be visible, yet the effects be inimical to good results.

GEORGE DAVIDSON.

FEBRUARY 11, 1894.

General Scheme of Standard Levels for the United States.



Finished work ——————

Proposed " - - - - - - -

Photo-Lith by A. HOEN & CO. Balto. Md

REPORT OF COMMITTEE F, ON ALASKA.

The extraordinary growth of this but partially explored territory, with its valuable resources on land and the almost limitless wealth in its waters, demands greater attention than has heretofore been accorded it and makes it imperative that general and comprehensive aids to its navigation and commerce be supplied.

This vast region contains about 600 000 square miles, being about twelve and a half times the area of the State of New York. It has approximately 26 000 miles of shore line, which exceeds that of the Atlantic, Pacific, and Gulf coasts by over 11 200 miles, while the islands along its coast are estimated to be 1 100 in number. A course parallel with the trend of its shore from Cape Muzon, its most southerly point, to Point Barrow, its most northerly one, is about 2 800 miles. The Aleutian chain of islands is about 1 100 miles long, and Attu, the most westerly one of this group, is about 2 200 miles west of Sitka.

There are immense forests in Alaska, densely covering every part of the country and climbing steep mountain sides to heights of 2 000 and 2 500 feet above sea level, and which extend as far west as Kadiak Island, being a continuous stretch of a thousand miles.

They consist mainly of spruce, hemlock, and cedar, one variety of the latter, the yellow, being very valuable in the construction of small vessels on account of its durable qualities.

The commerce of Alaska is, and doubtless always will be, carried on by water, owing to the peculiar formation of the country; and being so varied and largely conducted by nonresidents and by vessels hailing from so many different ports, it is difficult to obtain an exact idea of its extent. The internal commerce is carried on through about 126 agencies, located in 104 towns and settlements situated along its coast and among its islands.

The exports consist mainly of furs, ivory, Indian curios, gold and silver bullion and ore, and the products of the whale, cod, and salmon fisheries.

During the earlier occupancy of the country its commerce depended almost exclusively on the fur trade; but since other industries dependent upon the actual necessities of man sprang up; this important factor, although of great value, has already fallen to a third place in importance. From 1868 to 1891 the total value of the furs exported is estimated at $50 124 500, and the annual yield for the last-mentioned year amounted to about $1 605 000.

In 1892 there were sixteen gold and silver mines in operation, and up to that date the total output amounted to about $6 000 000. The traffic dependent upon the necessities of the small army already engaged in this comparatively new enterprise is considerable, and will undoubtedly increase.

The salmon industry commenced in 1878, and from that date up to 1890 the pack had amounted to $9 612 000. In 1878 the entire product was valued at $59 416, while that of 1890 was $2 731 000. The salmon-canning industry of this country is confined to the waters of California, Oregon, Washington, and Alaska. In years past the Columbia River has been the principal source of supply, but the run in all the sections south of British Columbia has become smaller from year to year. In the year 1887 the total pack for the entire Pacific Coast was 969 200 cases, of which the Columbia River furnished 430 000. In 1890 the output of the western coast was about 1 223 955 cases, of which Alaska alone furnished 688 322, or more than half the entire product of the United States. The capital invested in the Alaska salmon fisheries, including permanent improvements, vessels, etc., is something more than $4 000 000. There were, in 1890, 37 canneries between Dixon Entrance and Bristol Bay (25 of which are west of Sitka), and about 6 000 persons were employed during the fishing season, using 66 vessels for the purpose.

Judging from the rate of increase during the past ten years and the enormous field yet to be developed, the commerce depending upon this single industry will be one of the most notable interests of the Pacific Coast. Three-fourths of it is now beyond the region reconnoitered, and is rapidly crowding northward into uncharted localities enormously rich in fish. It is interesting to note that the two newer industries, mining and salmon fishing, have grown so rapidly that while in 1880 both these industries were insignificant and completely overshadowed by the fur trade, by 1890 their products amounted in value to $15 000 000, or more than twice the purchase price of the territory.

The Pacific and Arctic whaling catch, though not confined strictly to Alaskan waters, is conducted by American vessels, and all but a very small percentage of it is secured in waters contiguous to the Alaskan coast. The total value of oil, bone, and ivory of the catch between 1874 and 1890 was $11 204 465. There are about 50 vessels engaged in this industry, their port of call being Port Clarence. The charts of the tracks and rendezvous of these vessels are simply compilations of early explorations, very crude and inaccurate.

Of the food fish of Alaska, the codfish stand next in commercial importance to the salmon. The eastern part of Bering Sea is a great reservoir of cod, and the area within the limits of 50 fathoms depth is no less than 18 000 square miles. In this sea fishing must be done, as it is off Newfoundland, without harbors of refuge, but in a much less depth of water. The fishing banks along the south shores of the Aleutian chain will add about 45 000 more square miles, making a total of 63 000 square miles, this being about four times the area of the banks in the region of Newfoundland. Though over twenty years have elapsed since the inception of this industry, it must still be considered in its infancy. The value of the catch during the last twenty-seven years has amounted to about $8 900 000. It is carried on without regard to the abundant supply, but solely in accordance with the demands of the local and limited market on the Pacific coast of America.

It is evident, with the numerous transcontinental railways, with the increasing population along their lines and growing tributaries, that the demand will constantly and permanently increase, so that this interest will alone crowd the waters of the Gulf of Alaska and Bering Sea with sails.

The shores contiguous to these fishing grounds and the waters covering them are imperfectly and incorrectly delineated on the compiled charts, handed down to us principally from the early Russian explorers, and should be corrected to conform with the demands of modern navigation at as early a date as possible. Although the fishing for halibut, herring, etc., is at present only for local consumption, these industries are capable of wonderful development.

The value of merchandise shipped to Alaska from Pacific Coast ports from 1868 to 1890 amounted to $15 594 086, while during the same period the exports amounted to $75 213 929. During 1880 the merchandise received from Pacific Coast ports was valued at $463 226, while during 1890 it amounted to $1 635 494, showing a gain of nearly 300 per cent in amount in ten years.

During the period from the time of its purchase, in 1867, to 1890 a conservative estimate of the value of products shipped from this detached territory was about ten and one-half times the price paid for it.

It is regrettable that our sources of information for late Alaskan statistics are confined to the brief summaries of the governor's reports, and that for a comprehensive study of all the wealth-producing industries of the territory we have to go to the publications of the census for 1890.

There was considerable interest in this new territory at the time of its purchase from Russia by the treaty of June, 1867, and Secretary Seward arranged for a geographical reconnaissance in the summer of that year under the charge of the Coast Survey, from which the first Coast Pilot of southeastern Alaska was compiled. This work included only the more important points from Dixon Entrance to Unalaska.

With the exception of the astronomical work of 1869 and a rough reconnaissance by a small party in western Alaska during the summers of 1871, 1872, 1873, and 1874, nothing of importance was accomplished until 1882, when a trigonometric and hydrographic reconnaissance of the inland waters of southeastern Alaska was commenced, and, with the exception of one year, this has been continued during the summer months to the present time.

In the summer of 1892 the longitude of Sitka was very satisfactorily determined (chronometrically) as a base station, and during that and the succeeding years four other points were equally well determined from it for the international boundary work in the southeastern portion of the territory.

With the foregoing data the conditions are favorable for carrying on the triangulation where necessary, and also extending the astronomical work into localities considered in immediate need of it.

A glance at the progress sheet of southeast Alaska shows that the survey of the inland passages is nearly completed. To finish it requires a survey from Sitka northward through Peril Strait, and thence north and south along Chatham Strait to join the work of 1879 and 1890. This is estimated to require two seasons. The survey of Icy Strait, Glacier Bay, and Cross Sound will occupy about two seasons more.

The character of the shore-line work that is now being done in the inland passages is of a sufficiently precise nature for cartographic purposes and furnishes results which supply all that is at present demanded in the line of exactness; but it is advisable that the survey of the sections completed before the methods of work were brought to their present degree of accuracy should be remade and that all the work be brought to a uniform degree of worth. The topography on our present charts of the Inland Passage has not been treated in the detail that its importance merits. A characteristic representation of the main topographical features is of vital importance for charts which are intended to satisfy the demands of a coasting trade, and nowhere is this more important than in a region where the meteorological conditions are so bad as they are in Alaska, often leaving the pilot dependent for a check on his position upon fleeting views of limited sections of the land. It is very important that the defects in this respect should be supplied by a topographical reconnaissance made at as early a date as will be found practicable. Practically the entire outside coast, from Dixon Entrance to Cross Sound, remains unsurveyed except so far as it has been done in the rapid exploratory work of the early voyagers.

In considering the necessity for active prosecution and an early completion of the work of surveying southeast Alaska, the character of the traffic to be benefited by it must be taken into account. Nearly all of the carrying trade of southeast Alaska is by steamers. The intricate passages, strong tidal currents, and deep waters render navigation by sailing vessels difficult and dangerous. For these reasons, and on account of the unfavorable meteorological conditions, the steamers running in this trade are and must be, even after a complete survey has been made, supplied with pilots having accurate local knowledge of all the waters through which they run. Numerous fisheries and canneries are already located off the main line of steamer travel on the west side of Prince of Wales and Baranof islands, and these are visited at intervals during the season by the regular steamers which carry supplies and at the end of the season transport the pack to southern markets. It needs no arguments to show that wherever the ships of our merchant marine touch along our coasts accurate charts should be provided. But inasmuch as in Alaska we have a vast extent of shore line, of which the existing charts are poor and misleading, if not actually dangerous, and as the work of remedying this state of affairs must naturally be protracted through a term of years, it is plain that the surveys of the different sections should be taken up in the order of their importance.

In contrast with the steam carrying trade of southeast Alaska is that by sailing and steam vessels to the westward from Cooks Inlet and Kadiak, along the Aleutian Islands and northward to the Arctic. The various passes through the Aleutian chain need immediate attention. These are used annually by the Arctic whaling fleet, the supply ships of the various companies trading along the shores of Bering Sea and supplying the interior by way of the Yukon and other rivers, the cod-fishing fleet, and also by the combined squadrons of United States and British vessels that patrol and guard the waters adjacent to the Seal Islands.

Of all these passes but three—Unalga, Akutan, and Unimak—are in common use, owing to the imperfect surveys, and ships are often compelled to go far out of their way to make a known pass. Even in the three passes mentioned we have little knowledge of the velocity and set of the currents.

Next in importance is probably the vicinity of Kadiak Island and Cooks Inlet, where a large salmon industry has been built up within the past ten or twelve years and where surveys are very urgently required.

The Shumagin Islands and the western end of the Alaskan Peninsula would follow in the order of prominence. These regions have already attained a commercial importance which makes a new chart of them a pressing necessity. Off the first-mentioned islands the *Albatross* has developed a great codfish bank of 4 400 square miles in area. Unga is being exploited for its mines of precious metals, and the coal indications about Port Möller were sufficiently promising to justify the Alaska Commercial Company in building a small railroad to develop them. Thin Point, Sand Point, Belkovsky, and Sannakh have long been among the most profitable stations of the Alaska Commercial Company, if we except those on the Seal Islands.

As the great avenue of communication with the interior of the territory, it is important that the Yukon, and particularly its delta, should receive early attention. Without doubt, if a deep-water channel could be traced through the flats, which at present, on account of our complete ignorance of the condition of the mouth of the river, are an insuperable bar to the navigation through them of any but light-draft, flat-bottomed steamboats, it would give a tremendous impetus to the examination of the great possibilities that we have good reason to hope exist in this vast region. And there can be no possible doubt of the value of the great fisheries that would at once be established here if a survey would show the possibility of entrance and departure for the vessels that would be required for bringing up labor, material, and supplies and taking away the product.

Our present representation of the mouth of the Yukon is the result of the examination made by a merchant marine captain in command of one of the Western Union Telegraph experiment ships in 1865. The astronomical determination of Port Clarence, the rendezvous for the

Arctic whaling fleet, would be very valuable. At present the only station available for determining the chronometer errors for these vessels and the Revenue-Marine steamers which patrol the northern part of Bering Sea is at Plover Bay, in Siberia, a station established in the fifties by the English while searching for traces of Sir John Franklin's expedition. The nature and period of this determination predicate such a low degree of accuracy for it, and our interests in these waters are now so weighty, that any delay in furnishing one or two improved positions will subject our country to a charge of serious neglect.

The continental shore line from Cross Sound to Cooks Inlet is of such a nature and interest that there seems to be no immediate demand for its survey for commercial purposes now that such work has been completed in Yakutat Bay, the most important locality along this stretch.

We conclude that the work along the Aleutian Islands and in the vicinity of Kadiak Island and Cooks Inlet should be undertaken at the very earliest opportunity possible.

For this work it is not considered advisable to prescribe or suggest rigidly defined methods of procedure. The wild character of much of the shore line, the adverse meteorological conditions, and the limitations which economical considerations put on the completion of the schemes suggested before a good reconnaissance has been made, are all arguments against offering any minutely prescribed plans for approval for actual execution.

Your committee would, however, suggest that the degree of accuracy such as the Conference deems sufficient for tertiary work should be considered satisfactory for the surveys proposed in the preceding paragraphs.

Certain general suggestions about methods of work are offered as follows, but they are to be considered as subject to the wide discretion which we consider should be allowed to the officers charged with making those surveys:

One of the first considerations in the extension of the survey westward to and along the Aleutian Islands is the astronomical determinations. These are of prime importance for the reason that it is impracticable to carry a scheme of triangulation along the chain, and hence the survey must be made of the different islands independently and their relative position determined by astronomical operations. It is suggested that along this chain of islands the method of longitude determinations by terrestrial signals would probably be feasible, accurate, and also the most economical. The islands are near enough together to allow such signals to be exchanged. For the survey of the separate islands base lines must be located and measured on each, and its outline delineated by a local survey.

The Aleutian chain extends nearly east and west for a distance of about 1 100 miles. Among the islands making up the group there are three, Unimak, Unalaska, and Umnak, which are each about 60 miles long; two, Atka and Amlia, about 40 miles long; three, Adakh, Amchitka, and Attu, about 30 miles long; three, Kanaga, Tanaga, and Kyska, about 25 miles long; four, Unga, Sannak, Akutan, and Agatu, about 15 miles long, besides a host of lesser islets.

To control the longitude of this chain it is suggested that six stations be determined chronometrically, viz, Kadiak, Sand Point, Unalaska, Seguam Island (or some other in the vicinity of Amutka Pass), one of the Rat Islands, and Attu. This would give a series of stations along the chain at an average distance of about 300 miles apart. Advantage might be taken of the regular steamer running during the summer months from Sitka westward to establish one or even three of these stations (Kadiak, Sand Point, and Unalaska) during the coming season if funds are available.

Having once located the six base stations, a single party with proper transportation facilities could rapidly locate intermediate stations by the exchange of signals. On the larger islands enumerated above two or more stations, one at either end and the other intermediate, would seem to be essential, and in cases where it would be impracticable to find intervisible points it would probably be feasible to obtain results by noting the times of signals made from a vessel located far enough off the island to be visible from its ends or from the intermediate station; to supplement the method by signals under favorable circumstances, and when practicable the differences of longitudes may also be determined from latitude and reciprocal astronomical azimuths. In the high latitudes of the Aleutian Islands this method will give the differences of longitude with very great accuracy.

No doubt the greatest obstacle to all surveying work in this vicinity would be the dense and persistent fogs that are so prevalent, and, as bearing on this subject, the following table from Dall's Alaska (p. 444), will give something of an idea of the number of favorable days that may be expected during the months of May, June, July, August, and September. The table shows the number of wholly clear, partly clear, and wholly cloudy days that occurred at Unalaska during a period of seven years.

	May.	June.	July.	August.	September.
Clear days	2	6	0	5	2
Partly clear	105	95	118	106	107
Cloudy	104	109	99	106	101

This gives an average of less than one wholly clear day and about fifteen each of partly clear and cloudy days per month. The execution of the work about the mouth of the Yukon might be intrusted to a party which could be taken to St. Michaels by the revenue cutter, and, being supplied with a steam launch, could be employed from about July

1 to about September 13, and then return on the same vessel, a course of procedure that would entail a small cost to the Government.

For the determination of Port Clarence an observer might go up to that point on the revenue cutter, and if observations were being made contemporaneously at Unalaska a set of chronometers could be taken from that place on the tender which visits Port Clarence with supplies and to receive the accumulated bone and oil of the whaling fleet about midsummer, and by comparison with the chronometers, checked by observations at Port Clarence, the longitude of this important station could be determined. It is very probable that the public-spirited managers of the Pacific Steam Whaling Company would give such assistance to this project that it could be perfected at a very small expense.

PROPOSED SCHEME OF TRIANGULATION FOR SOUTHEAST ALASKA.

In order to properly connect and coordinate the work of reconnaissance triangulation that has been carried on in southeastern Alaska for the past twelve years, it is important that a main system of triangulation should extend from Dixon Entrance to Chilcat. Such a system would naturally extend up Clarence Strait to Sumner Strait, along Sumner Strait westward, cross the southern part of Kuiu Island in the vicinity of Tebenkof Bay to Chatham Strait, thence up Chatham Strait and Lynn Canal to the head of the latter. From this main system, as a base, secondary systems could in time be carried along the outside coast.

In executing the main triangulation, and, in fact, any triangulation, in southeastern Alaska, the work must necessarily be carried along the shore line of the passages, and for the following reasons:

1. Owing to the rugged nature of the country and the dense undergrowth of the forests that cover it, it would involve too great an expenditure of time and money to attempt to carry a triangulation along the mountain tops.

2. Even if the mountain tops were accessible, there would be great loss of time in attempting to occupy them. Clouds hang for days about the summits, when lower down, near the shore line, there would be no difficulty in observing.

3. Again, a scheme looking to the occupation of mountain peaks would unquestionably involve long lines on which heliotropes would be necessary, and as clear and sunny days are exceptional this again would be a source of delay and expense. On the other hand, a triangulation extending along the shore line would involve sides of an average length of about 10 miles. For these distances signal poles only need be used, and with the instrument properly protected from rain it would be possible to observe on many days when light rain was falling.

BASE LINES.

Bases for this work will be difficult of location and will necessarily be shorter than those usually measured for a main scheme. Generally speaking, the sites for bases must be sought in river bottoms, near the shoreline, and on such stretches of beach as can be found. In the latter case advantage can be taken of flats left bare by the tide at low water.

From the nature of the country suggested above as suitable for base-line sites, it is evident that the employment of base apparatus in measurements is scarcely practicable, and advantage must be taken of tapes, by means of which accurate and quick work can be done, as has been clearly demonstrated experimentally.

In consequence of the special value that gravity experiments in the high latitudes of Alaska would have, it is recommended that these observations be made by the astronomical parties either as part of their regular work or incidentally, as circumstances may dictate.

The astronomical parties should also be required to make observations for the magnetic declination, dip, and intensity at each station, and the triangulation parties should observe for declination by noting the magnetic bearings of the sides of the triangulation.

JOHN E. MCGRATH, *Chairman.*
A. L. BALDWIN, *Secretary.*

REPORT OF COMMITTEE G, ON INSTRUMENTS.

The committee has thought proper to confine its attention to the consideration of instruments for astronomical work and the measurement of horizontal angles, as other committees, having to report on special work, will necessarily consider the instruments used in such operations.

The following list of instruments now in possession of the Survey has been prepared by the instrument division:

FOR ASTRONOMICAL WORK.

6 transits of about 114^{cm} (45-inch) focus and 70^{mm} ($2\frac{3}{4}$-inch) objective, with power of about 100.

2 transits of 95^{cm} ($37\frac{1}{2}$-inch) focus and 82^{mm} ($3\frac{1}{4}$-inch) objective, with a power of about 90.

4 meridian telescopes of 79^{cm} (31-inch) focus and 63^{mm} ($2\frac{1}{2}$-inch) objective, and powers of 60 to 90.

3 meridian telescopes of 66^{cm} (26-inch) focus and 57^{mm} ($2\frac{1}{4}$-inch) objective, with powers of 50 to 70.

1 Repsold vertical circle, with microscopes.

4 zenith telescopes of about 114^{cm} (45-inch) focus and 76^{mm} (3-inch) objectives, with powers of about 100.

1 zenith telescope of 66^{cm} (26-inch) focus and 57^{mm} ($2\frac{1}{4}$-inch) objective.
8 cylinder chronographs (Fauth & Co.).
4 sets of longitude telegraphic apparatus.
19 sidereal breaks-circuit chronometers, 10 of which are by Negus and 9 of these of very recent date.

FOR HORIZONTAL ANGLES.

Direction theodolites:

5 51^{cm} (20-inch), 3	microscopes,	Wurdemann.
1 46^{cm} (18-inch), 3	"	T. & S.
1 41^{cm} (16-inch), 3	"	F. & Co.
1 36^{cm} (14-inch), 2	"	Wurdemann.
1 30^{cm} (12-inch), 2	"	Brunner.
2 30^{cm} (12-inch), 2	"	F. & Co.
2 30^{cm} (12-inch), 3	"	F. & Co.
2 30^{cm} (12-inch), 3	"	Office.
6 20^{cm} (8-inch), 2	"	F. & Co.

Repeating theodolites:

1 36^{cm} (14-inch), Brunner.
3 30^{cm} (12-inch), Gambey, 1 with 30^{cm} (12-inch) vertical circle.
9 25^{cm} (10 inch), Gambey, 1 " 25^{cm} (10-inch) " "
4 20^{cm} (8-inch), Gambey, 3 " 20^{cm} (8-inch) " "
6 20^{cm} (8-inch), Office, 3 " 15^{cm} (6-inch) " " and micrometer eyepieces, and all with compass declinometers.
8 15^{cm} (6-inch), Gambey.
10 15^{cm} (6-inch), Brunner, 1 with vertical circle.

A number of other instruments in the possession of the Survey might have been added to this list, and some of them will undoubtedly still be used, but in general they are so antiquated or of such inferior character that it is thought scarcely worth while to consider them here.

Types of all instruments named in this list have been set up in the instrument division and examined by the committee and other members of the Conference. In general, they are good instruments of their kind.

ASTRONOMICAL INSTRUMENTS.

The six 114^{cm} (45-inch) transits were made by Troughton & Simms, of London, 1845–1856, and were used in telegraphic longitude work of the Survey prior to 1888. Three of these have recently been reconstructed and improved in the office. These instruments, though excellent, are now considered too large and heavy to be economically used in the general work of the Survey, and it is not likely that they will be used in the future except at stations where transportation is not an important item or where they may be required for a long time, as at the astronomical stations at Washington and San Francisco.

The two 85^{cm} (37-inch) transits were made in the instrument shop of the Survey in 1887–88, and have been used with success in the telegraphic longitude work since that date.

The four 79^{cm} (31-inch) and three 66^{cm} (26-inch) meridian telescopes were made in this country by Wurdemann, Kubel, and Fauth & Co. since 1868. They have been used for the determination of latitude by Talcott's method and for time observations with success, and are, in fact, the most popular instruments in the possession of the Survey for general field astronomical work. Two of the larger ones have recently been reconstructed and improved at the office, and another is now in hand. It is recommended that the others be similarly improved as far as practicable. These instruments were designed by an Assistant of the Survey.

The four 114^{cm} (45-inch) zenith telescopes were made by Troughton & Simms, of London, about 1850. As originally constructed they were not satisfactory. One of them was changed, and the other three for many years were set aside. In 1890 three of these were reconstructed in the instrument shop of the Survey and used at Rockville, Md., San Francisco, Cal., and Honolulu, Hawaii, for observations for the investigation of the variation of latitude. One of the three was provided with a new objective and eyepieces, and with a further improvement of the mounting of the levels it is believed it will equal the best zenith telescope extant. It must be said of these instruments, as of the large transits, that they are too heavy for economical use in the general work of the Survey, and they are not likely to be used in the future except for special investigations.

The 66^{cm} (26-inch) zenith telescope was made by Wurdemann in 1854, and is a good instrument.

The eight cylinder chronographs were made by Fauth & Co., of Washington, the first about 1878. They were specially designed for the work of the Survey and have been used with success; but it must be said that their great weight is a serious objection. Many chronographs of American, English, French, and German design are equally heavy, and do not seem to meet the requirements of the field work of the Coast and Geodetic Survey so well as those now in use. When new chronographs are needed, the questions of their weight and of some improvements on the present ones should be particularly considered.

The astronomical instruments employed by other surveys in this country are very similar to those in use in the Coast and Geodetic Survey. Those in Europe, however, are quite different. There the broken transit has been extensively used in the telegraphic longitude work, and the illustrations of various forms are found in the publications relating to this work. There seem to have been some difficulties with these instruments in the flexure of the axis and the displacement and distortion of the prism. It is said, however, that these difficulties have been overcome. Whether such an instrument should be adopted in the Coast and Geodetic Survey the committee deem best to leave to future consideration.

It is only of late years that the Talcott method has been used in Europe, and for this purpose a special zenith telescope has been made by Bamberg, of Berlin, for the International Geodetic Association. This instrument is illustrated in the report of Dr. Marcuse's observations at Honolulu, 1891–92. It has a broken telescope, but the prism is small and placed very near the eye end. It has about the same power as the 114^{cm} (45-inch) zenith telescope of the Coast and Geodetic Survey. It is undoubtedly a fine instrument, but its great weight renders it unfit for general field work, and it should be properly classed with instruments for a fixed observatory. A similar instrument is now in use at Columbia College, New York, where observations are being made with it to investigate the variation of latitude.

The vertical circle, especially as made by the Repsolds, has been and is being used in European surveys for the determination of latitudes. The Coast and Geodetic Survey has one such instrument, but it has not met with favor, and it is not deemed advisable to further introduce it in the Survey, as it is believed the Talcott method is superior and can be used with quite as great facility.

THEODOLITES.

The Survey has such a number and variety of theodolites that it seems inexpedient to now introduce uniformity of design without an expense which is not warranted by its present needs.

The five 51^{cm} (20-inch), the 46^{cm} (18-inch), 41^{cm} (16-inch), and 36^{cm} (14-inch) position theodolites are of good design. Five of the 30^{cm} (12-inch) instruments are faulty in that the circles move upon the collars in shifting position. It is recommended that these circles be fixed and the instruments be used with position stands. The other two 30^{cm} (12-inch) direction theodolites were recently constructed at the office. They were designed with great care and the workmanship is of the very best. They have double centers, the outer one of cast iron and the inner of hardened steel. The inner center and socket are made with great precision. The outer center and socket are well made, but with less precision, as this center serves only for shifting the position of the circle. The alidade, supported on the inner center, is of aluminum as far as practicable, and the friction upon the center is very small. In their construction no relieving spring was deemed necessary; but the committee, considering this a point of vital importance, calls attention to the necessity of studying this omission in its effect on the instrument after it has been subjected to transportation and wear. The circles were divided on the Coast and Geodetic Survey engine. These instruments have been examined and practically tested by Assistant R. S. Woodward, and a preliminary report shows No. 145 to be an instrument of a very superior order. It is recommended that as soon as the new circle of No. 146 has been added, examined, and found satisfactory both instruments should be sent to the field as soon as practicable.

The six 20^{cm} (8-inch) direction theodolites are also faulty in that the circles move upon the collars, and some of them are held in position only by friction, no clamp whatever being provided. The micrometers of the microscopes are poor. These instruments have not been used for many years, and it is recommended that, if they are to be used in the future, the circles be fixed, position stands provided, and also new micrometers.

Since 1873 the 51^{cm} (20-inch) theodolites have been mostly used in the great triangulation of the Survey, where the length of the lines have been 100 kilometres (60 miles) and upward, while the others have been used in smaller work.

Nearly all the repeating theodolites now in use by the Survey were made by Gambey and Brunner, of Paris, many years ago. They are still as good instruments as ever. Originally they all had small and low-power telescopes. Larger and better telescopes have been added to some of the 25^{cm} (10-inch) and 30^{cm} (12-inch) Gambey instruments. The construction of these instruments is such that they are so light that they will not admit of any very great weight being added to them. If larger telescopes are needed, they should be made of aluminum. These instruments have been mostly used in the smaller triangulation of the Survey. The 30^{cm} (12-inch) instruments have, however, been successfully used in triangulation with lines as great as 60 to 80 kilometres (40 to 50 miles).

Although all the theodolites named in the list above given are considered good instruments, it is well known that the graduations of some of them are defective, and it is recommended that such circles be regraduated as soon as practicable. The new 30^{cm} (12-inch) theodolites recently made at the office show that the Coast and Geodetic Survey dividing engine in its present condition will do very satisfactory work. The graduation of these circles will compare favorably with the best modern circles. It may also be said that the regraduation of a circle at the office is neither a difficult nor an expensive operation, and a faulty or injured graduation should not be allowed to stand.

In the Great Trigonometrical Survey of India the triangulation was executed with position theodolites with circles from 46^{cm} (18-inch) to 91^{cm} (36-inch), some of them having 5 microscopes. They were made by Troughton & Simms, of London. In design they are similar to some that were formerly used in the Coast and Geodetic Survey. With the exception of one 46^{cm} (18-inch) instrument, these theodolites have been discarded by the Coast and Geodetic Survey as being unnecessarily large and heavy. The theodolites that have been used in other geodetic surveys in this country are very similar to those used on the Coast and Geodetic Survey.

Many suggestions have been made to the committee as to improvements on the theodolites of the Survey and for new instruments, and it is recommended that the officers of the Survey submit these suggestions in writing to the Superintendent.

The committee wishes to recommend the purchase as soon as practicable of some 10^{cm} (4-inch) and 18^{cm} (7-inch) theodolites for use in Alaska, and the officers interested should at once submit their views on such instruments in writing.

In the European surveys a variety of instruments has been used. The greater number of them have been universal instruments, with circles of from 25 to 30^{cm} in diameter.

As regards the instruments for astronomic work and the measurement of horizontal angles, the committee considers the present equipment of the Survey as very good. Some of these instruments are among the best of their class at this date, and others, although such as would not be constructed now, are too valuable to be abandoned. With such repairs and modifications as can be made at the office shop they will render excellent service for many years. Some new instruments will, however, be needed from time to time, and their construction should receive careful attention.

In conclusion, the committee begs leave to call the attention of observers to the necessity of bestowing at all times the proper care and protection upon instruments, not only with the view to their safety against injury from accident during transportation but also during use in the field.

While using the highest grade of instruments upon the finest class of work the observer can not be too careful and circumspect in providing a perfectly stable foundation of masonry or iron when practicable, and in protecting them as much as possible against unequal or sudden changes of temperature.

An observer's tent, when a tent is used, of double walls and roofing, will be found a most serviceable and efficient protection against the radiant heat from the sun, direct or reflected. Lamps or candles should never be kept near an instrument at any time without a screen for the interception of radiant heat. It is also recommended that observers in the field should study the erratic movements of the level for the purposes of ascertaining the cause and suggesting a remedy. It is especially important to note the period and extent of the oscillations of the bubble.

EDWIN SMITH, *Chairman.*
C. H. VAN ORDEN, *Secretary.*

REPORT OF COMMITTEE H, ON OFFICE AND FIELD RELATIONS.

It is evident that it is very important and for the best interests of the Survey that the relation between the office and field forces should be as harmonious as possible. In order to effect this much-to-be-desired object your committee thinks it is especially necessary that the "Regulations of 1887," adopted and issued by the then Secretary of the Treasury (with amendments since added), and all circulars that have been issued from time to time by the Superintendent, shall be carefully studied and their provisions faithfully carried out. By this means alone can effective cooperation be secured. To attain this end the following recommendations are made:

1. RECORDS—THEIR PREPARATION, DUPLICATION, AND TRANSMISSION TO THE OFFICE.

Following are articles 31, 32, 33, on page 33 of the Regulations of 1887, relating to records and their transmission to the office; also article 35, relating to transcripts from the records:

31. The original journals of observations and original topographical and hydrographical sheets must in every case be deposited in the office of the Survey at Washington. The journals, records, all field notes, and original data of every description must be kept in the office; and all persons employed in making observations are required to furnish copies thereof to the office at the close of each season's work. Each original topographic or hydrographic sheet must be accompanied by a descriptive report in writing and in duplicate of the locality to which the sheet refers, in accordance with the Superintendent's pamphlet circular of April 11, 1887, entitled "Instructions and Memoranda for Descriptive Reports to Accompany Original Sheets."*

32. All books containing official data, all topographical and hydrographical sheets, with their accompanying descriptive reports, and all other records of field work, both original and in duplicate (when required), must be forwarded to the Superintendent, indorsed with contents of package, and must always be accompanied by a transmitting letter to the same address, stating definitely what is sent, with the necessary explanations, but with no other references. This course in regard to transmitting letters must be pursued with respect to instruments and all other articles sent to or from the office.

33. Every transmitting letter must specify in detail every article sent. Of each book or paper of records the general contents must be stated; of each topographical, hydrographic, or other sheet and accompanying report, its character and limits; of each instrument, its character, general dimensions, and condition; of each box of bottom specimens, the number it contains; and so on, for every item sent. No other matter must be referred to in the transmitting letter.

35. Except to persons employed in the work of the Survey, transcripts from the records or from notes or sketches shall not be communicated without the authority of the Superintendent.

The term "original record" means the record as originally made, and not a fair copy.

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*This circular of April 11, 1887, has been superseded by the Superintendent's circular of July 3, 1890, prescribing one descriptive report.

The duplicate records should be made in the field when practicable, and sent to the office as soon as possible.

No computation whatever should be made in the duplicate records.

Full details of instruments, of observing and recording methods, and all data useful to the computer should be inserted in the preface to all records.

Progress sketches for the use of the computer should be made of the prescribed scale and size, and conformable to facts. (See Superintendent's Circular, February 15, 1888.)

The regulation in regard to the prompt transmission at the close of the season of summary reports, with sketch and statistics, should be enforced.

While observing, any change made in the instrument, voluntarily or otherwise, should be noted in the record under head of "Remarks." The record should be full and as definite as possible, keeping in view that the computer is necessarily ignorant of many details familiar to the observer.

The record should be made complete and according to the form provided, and the names of stations should be written plainly. No recorder should be employed unless he can make a plain record.

Descriptions of triangulation stations should always be in a separate volume and not in the preface of the observations, except in primary triangulation.

All determined points, of whatever character, should, when practicable, be permanently marked and described, as the office is often unable to furnish descriptions called for of well-determined but unoccupied points.

2. FIELD COMPUTATIONS—DEGREE OF ACCURACY REQUIRED.

Computations should be made in the field while observing and before leaving the station, if possible, to make certain that the observations are satisfactory and to insure that no necessary data are omitted in the original record.

No computation should be made by the observers of a greater degree of accuracy than is sufficient for the above purpose.

The observer's abstract should be complete, showing every measured angle of any kind, and should follow the printed forms.

The triangle side computation should be complete, showing all lines determined, and a regular system should be followed; that is, all triangles upon any one point should follow each other consecutively.

The names of stations in the triangle side and position computations should be written from left to right, or in the order of the azimuth.

Angles of the nearest whole seconds and logarithms to five places are sufficient for the computation of tertiary triangulation.

As soon as practicable after the close of the field season the chiefs of parties should turn in to the office their completed computations.

3. ACCOUNTS.

It is believed that by complying strictly with the requirements contained in the Regulations and the directions printed on the different forms of vouchers all friction between the field force and the accounting division will be avoided. This matter will be referred to again in our "Conclusions," at the end of the report.

4. INSTRUMENTS—THEIR SHIPMENT TO AND FROM THE FIELD.

A history or sketch of instruments owned by the Survey should be prepared. A book should be kept for this purpose in the instrument division, in which space could be assigned to each instrument and its present condition, with full details as to the aperture of telescope, focal length, magnifying power, etc., data concerning constants, graduation, etc., and, in fact, every necessary detail should be stated, so that a reference to the instrument by kind and number would be sufficient to identify any fact concerning it in case any observer neglects to give necessary data in his records. Any reasonable change suggested by an observer should be noted in this book, and the action of the instrument board on this suggestion should be recorded. Any change whatever in the instrument made by the instrument division should be carefully entered, and the date on which the change was made should be stated. If this recommendation is carried out it will save much time and labor in the computing division.

A record of the condition of each instrument sent to the field at the time of leaving the office should be kept in the instrument division, so that the cause of any injury in transportation could be more definitely ascertained.

A report of anything objectionable to the observer discovered when the instrument is unpacked in the field should be made to the office, so that the error, if any, may not be repeated.

The regulation requiring a report on the condition of the instrument when it leaves the field should be enforced, so that the cause of injury in transportation from the field can be more definitely ascertained. This report should state in detail the defects and repairs needed.

The constants of some instruments—thermometers, barometers, magnetic instruments, all level vials, tape lines, base apparatus, and instruments of like character—should be sent to the field with the instrument. The observers should be informed of any change in the parts of an instrument, and this information should be made a part of the record.

5. MISCELLANEOUS.

All data furnished by the office for the use of field parties should be returned to the office when no longer necessary to field operations.

In order to make all records more uniform and to secure necessary data which may be omitted, a careful and critical examination should be made by a competent person as soon as the records are received, and a report made to the Superintendent, showing all defects, together with a statement as to whether the work appears good or otherwise.

In the prosecution of the field work, when a station can not be found, a full report should be made, showing in detail the steps taken to recover the station, and stating the officer's opinion as to loss of the station, with the reasons therefor.

A record of lost stations should be prepared and kept in the drawing division, and these stations should be taken off the list of geographical positions available for field work, and data concerning them should not be furnished to anyone who does not specially request it with the knowledge that the station can not be recovered.

When any station is visited, a statement of its condition should be communicated to the Superintendent. Whenever practicable without expense, the chiefs of parties should visit marks established by the Survey and report their condition to the Superintendent. A list of circulars covering the headings considered above is submitted herewith.

Inasmuch as many members of the force are, from the nature of their duties and the isolation of their stations, cut off from ready access to the large number of current publications rich in matter which is of professional interest to them, and an acquaintance with which is essential to their highest usefulness, it is recommended that some competent person be charged with the examination of certain standard publications, of which he shall prepare brief extracts or headlines, with references to the publications in which they appear, and that copies of these abstracts be forwarded from time to time to the different members of the corps.

CIRCULARS.

April 1, 1892. Concerning photographs: Negatives to be regarded as part of the original records.

October 31, 1891. Storage of property.

October 29, 1891. Articles which may be purchased when urgently needed.

March 31, 1891. Remains of aboriginal articles to be reported.

February 27, 1891. Regulations in regard to preparation of records.

August 8, 1890. Selection of geographic names.

July 1, 1890. Allowance of subsistence for Superintendent when visiting parties in the field.

April 3, 1890. Regulations in regard to preparation of records.

March 21, 1889. Treasury Department, No. 30.—Regulations in regard to sending telegrams.

March 13, 1888. Directions for the survey, condemnation, appraisement, and sale of Coast Survey property.

July 12, 1888. Preservation of triangulation points.

June 18, 1888. Statistics of field work.

February 15, 1888. Regulating progress sketches.

September 8, 1887. Treasury Department, No. 101.—Shipment of freight and payment of transportation over land-grant railroads.
September 3, 1887. Inking topographic sheets.
March 28, 1887. Requiring inventories to be rendered.
January 17, 1887. Concerning shipment of property to the office.
September 13, 1886. Proper manner of communicating with the office.
March 26, 1886. Executive order of the President.—Requiring bonds of Assistants in order to secure advances.
July 15, 1884. Treasury Department, No. 108.—Indorsement and payment of Treasury drafts.
August 16, 1886. Concerning accounts.
August 11, 1886. " "
June 21, 1886. " "
June 16, 1886. " "
June 10, 1886. " "
May 29, 1886. " "
May 1, 1886. Transportation on bonded railroads.
April 1, 1886. Treasury Department, No. 36.—Preparation and rendition of accounts.
April 3, 1886. Ink for plane-table sheets.
March 19, 1886. Transfer of property.

In conclusion, the committee desires to express its appreciation of the letter from Mr. E. H. Fowler, draftsman, dated January 11, 1894, referred to it by the Conference, and of the letter of Mr. J. B. Boutelle, computer, dated January 8, 1894, presented by Assistant Schott.

The committee recommends the preparation of a manual of observations, records, and computations which shall embody the conclusions of the Conference upon these points, and a manual of accounts, in accordance with the valuable suggestions contained in the paper (appended to this report) of the disbursing agent of the Coast and Geodetic Survey, which the committee indorses.

GEORGE A. FAIRFIELD, *Chairman.*
ISAAC WINSTON, *Secretary.*

CORRELATION OF THE OPERATING DEPARTMENT AND ACCOUNTING SYSTEM OF THE COAST AND GEODETIC SURVEY.

The reciprocal relations existing between the barren, uninviting, and verbose details of an accounting system, and the researches, deliberations, and results of a scientific commission, such as the Geodetic Conference now assembled in Washington, are not at a first glance apparent. The two elements, if they may be so termed, in fact appear to be widely at variance. No connection is immediately perceptible, yet they are closely allied, almost if not absolutely inseparable, and to a greater degree than is at once conceivable. For the purposes of this paper it is necessary at the outset to define the union between the two and to show their correlation. To do this understandingly it must be assumed that the Conference, as a whole, is representative of the *work* of the Survey, and that the accounting system is the embodiment of the *productive means* by which the work is accomplished.

It will probably be conceded that all enterprises or undertakings, no matter upon what scale they may have originally been planned, under every condition obtaining in life, require for their prosecution the providing of certain means or assistance whereby the work to be accomplished may be brought to a successful termination, or, as is sometimes the case, its impracticability completely demonstrated. The work of the Survey, therefore, as represented by the Geodetic Conference, may for the purposes of illustration be deemed an undertaking in the furtherance of which grants of money are made by Congress from time to time for its prosecution, while, on the other hand, the accounting system may be said to represent the vehicle through which the means or assistance so provided becomes available for the purposes of the work. The connection, therefore, between the operative department of the Survey and its accounting system would appear from the foregoing to be not only correlative but in fact inseparable, inasmuch as the existence of the former depends under prior legislative enactment upon the sustenance provided for its maintenance through the instrumentality of the latter.

It may be permissible to say that doubtless no point is likely to arise during the progress of the deliberations of this assemblage which will convey the impression of a closer or more essential connection as a natural result of the labors of the Conference than that of a comprehensive application and clear understanding of the laws, rules, regulations, and requirements governing the disbursement of public funds in their relation to the prosecution of the work of the Coast and Geodetic Survey, and properly and intelligently accounting for the moneys so expended. This statement is made with the more freedom, inasmuch as it appears to be a well-established fact that the *work* of the Survey in its execution, its correctness, and its methods has never been successfully assailed or impeached, while its disbursements, on the other hand, have been the subject of almost continual criticism and animadversion from the days of the first Superintendent down to the present writing.

The special and distinctive characteristics of the work of the Coast and Geodetic Survey, both in the field and office, compared with what may be called—there being few exceptions—the ordinary plain business transactions of other Bureaus of the Government, manifest themselves continually and persistently in the varied, and in many instances novel, character of its expenditures. This is naturally the case, and can not be avoided in a great governmental bureau which is notably engaged in the investigation and application of the most refined methods of science with a view to the betterment and amelioration of the conditions surrounding mankind, the diffusion of general knowledge, and the protection and development, by means of its perfect surveys of navigable waters, of the commerce of a great and growing country. But these commendable and praiseworthy features are frequently lost to view in the maelstrom of criticism and censure which has at various times in the history of the Survey followed a so-called analysis of the expenditures made for the work. To the average accounting clerk, possessed of little or no knowledge of the operations of the Survey, with acquired predilections for hypercriticism, reeinforced doubtless in some instances by superior official insistence, the accounts of the Bureau appear to present unusual opportunities for the promulgation of dogmatic opinions as to the propriety and necessity for particular items of expenditure, the usefulness and import of which are as unknown to him as would be the application of the most abstruse scientific problem. The result of such criticism is an almost continual correspondence between the officials of the Bureau and the reviewing officers of the Department with reference to the allowance of disputed items; and this condition has obtained, to a more or less extent, with the accounts of every disbursing officer from the time of Capt. W. H. Swift, Corps of Engineers, the first disbursing agent, to the present. The remedy, in part, is permanency in the tenure of office of the accounting clerks and reviewing officials. Each man becomes gradually educated up to the needs of the service and in time comprehends and appreciates the system of accounting. A change in the office, however, and the schoolmaster's work begins anew. It is this feature which is largely responsible for the main body of criticism to which our accounts have been subjected. The Survey finds no fault with the expression of a fair, honest difference of opinion as to the propriety of any particular item of expenditure. Errors of judgment are to be expected; but when discovered should be discussed and criticised in a spirit of fairness and not with a view to casting suspicion upon individual integrity.

The foregoing remarks naturally lead up to a point which seems to present the desirability of a still further advance in that degree of knowledge and familiarity with the laws relating to the disbursement of public funds which is so essential a requirement in the avoidance of adverse criticism and in securing a prompt and accurate audit of our accounts. With this object in view, and taking advantage of the assemblage of so many officers in attendance at the Conference, the Superintendent of the Survey has suggested the preparation of a paper in which reference should be made to the technicalities necessary to be observed, with a brief analysis in each case of their necessity and import. The subject is a large one and can not readily be confined within the limits of a brief paper. An effort will be made, however, to define the more important points and to show their relation to the system of accountability now in force.

Many of the requirements now obtaining in the accounting system of the Survey are a natural outgrowth of the provisions and terms of the "Plan of Reorganization of 1843." Subsequent legislation has from time to time added others, various decisions of the accounting officers of the Treasury have contributed their quota, and others again have arisen through the executive action of the different Superintendents. Regulations and directions almost innumerable in number, embracing all these various sources of authority, have been issued and promulgated as occasion demanded. It may readily be seen, therefore, that the field officer of the Coast and Geodetic Survey charged with the responsibility of making disbursements of public funds must, in addition to his other qualifications, become a walking digest of law, regulations, and decisions, if he desires to avoid criticism and individual financial responsibility in the audit and settlement of his accounts. For the purposes of this paper, a reference to the latest edition of the Regulations (1887) will be first in order. As the enacting clauses of the appropriation acts of recent years have invariably contained the provision that the appropriations therein made were to be expended in accordance with the regulations adopted from time to time by the Secretary of the Treasury, it would seem that such regulations, when so adopted, possess all the force of statutory law.

To quote from paragraph 3: "The Superintendent shall direct and superintend the work in general, and be responsible * * * for the proper and economical expenditure of the appropriations made therefor." There is food for much thought in the terms of this regulation. It is clear that the Superintendent is directly responsible for the correctness of the disbursements.

The chief of the party, therefore, in the matter of his expenditures, is to be considered as being primarily responsible to the Superintendent, although the provision contained in paragraph 46, which reads, "Chiefs of parties shall be held responsible for the expenditures of their respective parties," would seem to require modification to meet this condition. The interpolation of the words "to the Superintendent" after the word "responsible" would properly accentuate the responsibility here referred to. Responsibility should be centralized and not scattered. Divided responsibility is unsafe and should not be tolerated. Much more could be said in this connection, but the point involved is obvious.

Paragraph 30 of the Regulations provides, in brief, that all moneys received from the sales of old property, etc., shall be paid to the Assistant in charge of the office and topography. In many cases this is not done, the moneys being deposited in some subtreasury or other governmental depository or forwarded direct to the Superintendent. The regulation is mandatory and admits of no discretion. It is another case of divided responsibility; but with this we have nothing to do. Such moneys should be forwarded as directed. This may be done by means of check, money order, or by making a transfer through the instrumentality of the disbursing agent.

Paragraph 45 refers to estimates and allotments, and provides that no expenses of any description, except for the purpose of saving life or property or in other sudden emergency, shall be incurred before estimates have been submitted to the Superintendent and approved by him. The saving clause here is composed of the words "or in other sudden emergency," of which emergency the Superintendent, in his executive capacity, is the judge. Were it not for this clause all expenditures of every description would necessarily have to be first estimated for and an allotment made to cover them before they could be incurred. In this connection chiefs of parties should be careful never to deviate, in the matter of rates of compensation or per diem allowances for subsistence, from the terms of their approved estimates. There is some leeway, so to speak, in the matter of items for contingent expenses, but in those specifically referred to close adherence must be had to the terms of the estimates. If any departure is made therefrom, it should be the subject of an explanatory letter. In preparing estimates the printed directions on the face should be followed. These are plain and require no further explanation in this place.

Paragraph 47 requires chiefs of parties to transmit with their accounts a list of all articles that may be purchased for public use. This requirement is frequently overlooked, and to save time in the adjustment of the accounts the lists are often made out in the disbursing office. This provision of law admits of no discretion. Moreover, the lists of purchases, when made out by the chief of party, afford him an opportunity of stating what disposition, if any, has been made of the property purchased. It is sometimes at once expended, and this statement made on the list avoids possible trouble in the settlement of inventory accounts, which would not be the case when the lists are made out in the disbursing office.

Paragraphs 58, 59, and 60 refer to allowances for commutation and to expenses for actual subsistence. These two conditions should not be misinterpreted. They are, however, frequently misapplied. Commutation of subsistence can never be allowed while traveling, except in cases of field duty involving travel with brief stoppages. Only the actual cost of subsistence can be allowed while traveling under the usual conditions obtaining in the Survey. The error is frequently made of charging commutation or per diem while so traveling. Under the regulation this is not permissible and merely serves to invite disallowance or suspension of the amounts so charged. The amounts paid for actual subsistence when traveling should be so charged, and if for expenses at a hotel or other lodging place, should be supported by receipt. When meals are procured at restaurants or on dining cars while traveling, the prices paid should be charged in detail on the traveling voucher, and receipts may be dispensed with when the account is duly sworn to. The regulation, however, requires that all items not supported by receipts should be accompanied by a statement showing the impracticability of obtaining such receipts. The hurry and confusion incident to travel is a necessary accompaniment of such impracticability.

The conditions governing the various allowances for commutation of subsistence are so clearly and plainly stated in the regulations that further explanation in this place would seem to be unnecessary. The entire question is one that is optional with the Superintendent as to the amount allowable within the maximum sum fixed by the appropriation act.

The foregoing paragraphs referring to the Regulations cover those points in which chiefs of parties are most frequently at fault.

The vouchers used by the Survey in its system of accounting will now be taken up for description. These forms carry with them all necessary printed directions for their proper preparation. A careful consideration of the instructions contained in the directions when filling out vouchers would avoid at times much unnecessary labor in the disbursing office in the examination and audit of accounts. Many of the paragraphs of these directions are self-explanatory and need no further reference to them in this place. The purpose and intent of a few of the leading paragraphs will, however, be given in order to show the bearing which they have upon the final audit of the accounts at the Treasury Department. Paragraph 5 of the Directions, Form 2—General Voucher—Field, reads as follows: "Always state specifically the purpose of every expenditure. If for services, the capacity in which employed and the work upon which engaged; if for articles, state use for which intended." The last sentence is particularly applicable to the suspension of an item in a statement of differences recently received from the accounting officers of the Treasury, which reads as follows:

"The following items for ice are suspended for explanation as to necessity for purchase of ice in such large quantities," the total being $101.25. The ice accounted for here was purchased for the purpose of tempering the base bars of the apparatus used in the measurement of the Holton Base line. A mere statement to that effect upon the face of the voucher would doubtless have been sufficient to satisfy the accounting clerk of the Treasury that the article purchased was not intended for the gratification of individual tastes or other indulgence, but was used solely for public purposes. Again, in the measurement of the Yolo Base line, the vouchers covering expenditures for brick and cement and for the hire of bricklayers, expenditures necessary to the erection of the piers at the ends of the line, were returned to the office for the specific approval and administrative scrutiny of the Superintendent upon the ground that the disbursement was of an extraordinary character and required full explanation and the highest executive sanction before it could be passed. As before, a simple statement upon the vouchers of the purport of the expenditure would probably have been satisfactory, and unnecessary correspondence would have been avoided. These two illustrations would seem to clearly indicate the propriety of stating specifically, at least in the case of items somewhat outside the usual routine, "the purpose of the expenditure."

Paragraph 4 of the Directions, Form 2, requires that all items on the personal vouchers of chiefs of parties or their subordinates should be supported by receipts whenever practicable to obtain them; and when not practicable to obtain the receipts, to so state and give the reason therefor. This paragraph is not observed to the extent that it should be in order to satisfy the requirements of the accounting officers, as the following suspensions in our accounts will indicate:

"The following items for lumber, freight, cartage, laths, nails, wire, tools, oil can, oil, eyebolts, damages, oak plank, services, luncheons, post-office registration fee, etc., are suspended for subvouchers (receipts) needed to perfect," the total amounting to about \$17, and the items of expenditure ranging from 10 cents to about 75 cents or \$1. Individually, the sums involved are very small; in the aggregate their amount is considerable in the adjustment of an account for a full year's expenditures. In such cases, after so long a lapse of time, it is hardly possible that the necessary receipts can be obtained by the various chiefs of parties, and hence recourse must be had to an appeal for equity and to any other prevailing feature which may aid in securing the passage of the disputed item. The point might be raised in this connection that such items, unsupported by receipts, should not be allowed to pass the disbursing office. But it may be stated here in general terms that, apart from the labor involved in striking out such items, the office has always attached weight to the certificate of a chief of party "that the account was correct and just," and hence was not disposed to be critical in relation to the small sums here alluded to. It would seem to be the easiest course, however, for all concerned, to obtain receipts for every item, however small, in support of charges on personal vouchers, whenever possible to procure them, and when not so possible, to briefly state why. The only exception which may be made to this rule, if exception it may be called, is in the case of vouchers for traveling expenses, which are usually sworn to before competent legal authority. The irksomeness of this requirement as to subreceipts is patent to all; but as it is based upon the direct action of the accounting officials of the Treasury, as indicated by the foregoing citations, it is not clear that remonstrance would result in relief.

The three forms of vouchers most commonly in use by chiefs of parties are: "Form 2—General Voucher—Field," "Form 3—Transportation Voucher," and "Form 5—Abstract of Expenditures." For the purpose of bringing before the Conference, as a matter of record and for its official action, if any is deemed necessary, the particular features of the printed directions heretofore alluded to, they are here incorporated in regular order, as follows:

FORM 2—GENERAL VOUCHER—FIELD.

DIRECTIONS.

1. Signatures of firms or individuals signed "per" will not be passed.

2. Whenever practicable, have vouchers made out by the persons signing them.

3. Assign numbers to subvouchers or receipts, and refer to them in the voucher by number.

4. When used as a personal voucher by chiefs of parties or their subordinates for other than their own services and subsistence, support all items by subvouchers or receipts whenever practicable to obtain them. When not practicable, so state and give the reason therefor.

5. Always state specifically the purpose of every expenditure. If for services, the capacity in which employed and the work upon which engaged. If for articles, state use for which intended.

6. To avoid a multiplicity of vouchers it is suggested that in numerous instances receipted bills could be obtained and the amounts thereof charged upon the personal voucher of the chief of party, referring to the bill by number, as heretofore stated.

7. All subvouchers or receipts should, if possible, be signed in ink. An extra effort should be made to this end.

8. Vouchers for commutation of subsistence must be rendered in strict conformity with the Regulations.

9. Make all explanations in writing upon the face of the vouchers.

10. Signatures by "X" are only to be made by persons unable to write their own names. When made by "X" they must be witnessed. This applies to subvouchers and receipts as well as to regular forms of vouchers.

11. Vouchers for the purchase of instruments, for repairs of instruments, or for ink and mucilage must be rendered in strict conformity to the Superintendent's circular of October 29, 1891. Personal vouchers of chiefs of parties should be rendered for such expenditures, the items being supported by the receipted bills.

12. All vouchers must be itemized as completely as possible. Charges in "lump" sums will not be allowed.

13. Charges for telegrams can only be allowed at Government rates, as per Department circulars issued from time to time. The number of words and names of places from and to which sent must be given on the voucher, or copies of the telegrams must be furnished.

14. The provisions of paragraph 53 of the Regulations, in relation to accounts, must be strictly adhered to by all chiefs of parties to avoid suspensions or disallowances in their vouchers.

15. All calculations for parts of a month must be made according to the number of days of which the month consists.

16. In vouchers for hauling and moving equipments and materials give the number of loads and distance from place to place.

17. Per diem employees or hands can not be paid salaries for Sundays unless service is actually rendered on that day. When service is so rendered by per diem employees (and charged for), a certificate to that effect must be written on the face of the voucher.

18. All vouchers must bear date in the column on the left.

19. The price per unit of weight or measure must be stated in all cases whenever practicable.

20. In cases where damages for opening views, etc., are paid for, either support the charge by a written agreement (with the person claiming the damages), stating the nature and extent of damages, and his acceptance of a stated sum as a full relief

to the Government, or give in detail, on the face of the voucher, the full particulars concerning the account and the circumstances which demanded the expenditure.

21. When rendered as a subvoucher in an abstract, the briefing on the back of this form must invariably be filled out.

22. Hereafter secure invoices and bills from firms and individuals furnishing supplies, etc., or rendering services (not personal services), in addition to the regular form of voucher, and attach them to their appropriate vouchers before transmitting the accounts to the disbursing agent.

FORM 3—TRANSPORTATION VOUCHER.

DIRECTIONS.

1. Copies or extracts from letters of instructions must be written on separate sheets of paper and attached to the vouchers. Be careful to give a full copy or extract, as the case may be.

2. Give full name of railroad or steamboat company furnishing the transportation. When transported by other conveyances, so state.

3. The fare actually expended over each route must be charged. Do not "lump" the expenditures.

4. Charges for transportation over bonded and land-grant railroads will be disallowed.

5. Support all charges for meals and transportation of baggage by subvouchers or receipts whenever practicable to obtain them. When not practicable so state and give the reason therefor.

6. Assign numbers to subvouchers or receipts, and refer to them in the voucher by number.

7. Only actual expenses of board and lodging are allowed while traveling. Secretary's circular of August 24, 1886, governs the present rates allowed for field officers.

8. Subvouchers or receipts must be furnished for all hotel expenses and carriage hire. Hotel bills must clearly show the time of beginning and ending of the service charged for.

9. Expenditures for local field transportation must state specifically the purpose for which each item of expenditure was made, and must be confined strictly to the immediate locality of field work. When the distance traveled is over 50 miles from locality of work, full explanation of the necessity therefor must be made.

10. In organizing or disbanding a party, the members thereof can not be furnished transportation for a greater distance than 50 miles without previous special authority from the Superintendent.

11. When transportation expenses of employees of a party are charged in the personal voucher of the chief of party (and this course is recommended as avoiding confusion), give their names and furnish their acknowledgments, to be attached to the voucher, that they have received the transportation charged for.

12. When employees of a party are traveling alone upon special duty, away from the main party (within the limit heretofore stated), they must render to the chief of party vouchers in their own names, duly sworn to by them, *and supported by written orders* from the chief of party. In these orders the Superintendent's instructions must be quoted and the extract certified to *by the chief of party*.

13. When charges for actual expenses while traveling and for commutation of subsistence become interwoven upon any particular day an adjustment must be effected upon the basis of four parts to a day—breakfast, dinner, supper, and lodging.

14. Actual expenses of board and lodging will not be allowed while traveling locally in the routine of field work to those receiving commuted or regular rates of subsistence incidental to field operations.

15. Traveling expenses between home and the office or suboffices not allowable.

16. The approval certificate of the Superintendent on the face of the voucher is intended to be filled out by him only when the voucher is rendered separately and not included in an abstract.

17. All traveling-expense accounts must be sworn to before competent legal authority.

FORM 5—ABSTRACT OF EXPENDITURES.

DIRECTIONS.

1. In entering vouchers upon the abstract, arrange them in alphabetical order regardless of dates, writing the surname first in the column headed "To whom paid."
2. When working under two or more appropriations, render separate abstracts for each.
3. When vouchers are suspended and returned for correction, always transmit supplemental abstracts (in duplicate), dated the same as the originals, to cover the suspended vouchers.
4. Monthly abstracts must be given date of the last day of the month in the receipts. When rendered for portions of a month, the latest date in the subvouchers may be used.
5. Vouchers for expenditures made in one month must not be included in the abstract for another month. Render a separate abstract for such vouchers, and attach thereto a written explanation of the delay in transmitting the account.
6. Be careful to observe that the abstracts are signed before transmitting the accounts to the disbursing clerk. Abstracts must be signed by chiefs of parties in their official capacity.
7. The blank space in the center of the face of the abstract must not be written over by chiefs of parties, nor must the briefing on the back of the abstracts be filled out by them.
8. The dates of the vouchers must be inserted in the abstract, notwithstanding that they are arranged alphabetically.
9. All vouchers embraced within abstracts must be briefed on the back and numbered, beginning with the number 1 (one) in each abstract.
10. Separate abstracts must be rendered for "party expenses" and "repairs of vessels."

It has been the custom of the disbursing office from time to time, as points arose in the adjustment of accounts or new decisions thereon were made by the accounting officials, to change the terms of these directions so as to have them correspond to the new conditions. Such changes are made usually when a new edition of any particular form is required for issue.

A few additional suggestions or comments may not be out of place in this connection, in view of their bearing upon individual responsibility and the saving of much unnecessary loss of time and useless labor.

Whenever practicable, it is recommended that chiefs of parties deposit the moneys advanced to them for party and other expenses in some regularly designated governmental depository, if too remote to open an account with the Treasurer or a subtreasury. A list of designated depositories for public moneys among the national banks located in every State and Territory in the Union will be furnished by the disbursing office upon application. The particular advantage of this plan is that it will save chiefs of parties much trouble in their dealings with incorporated or unincorporated companies in the matter of obtaining certificates of authority for officers to sign for the companies. Such accounts, when paid by a check drawn on the Treasurer, Assistant Treasurer, or other depository of the United States, in the name of the corporation as payee, stating such fact on the face of the voucher and giving the number of the check, will not require evidence of authority for signature to be filed with them. One of the advantages of keeping moneys advanced on deposit with a governmental depository is thus made apparent. Another is that it affords a complete record of all disbursements, as the checks drawn on a governmental depository, when presented and paid, are not returned to the officer issuing them, but are permanently retained in the files of the depository. It is, of course, understood that in many cases this course of making deposits can not be adopted, owing to the remote and isolated locations of the field work. Under such conditions the chief of party accepts the full responsibility under his bond for the safe-keeping of the public funds intrusted to him.

In the case of original checks lost, stolen, or destroyed, the Revised Statutes prescribe that a duplicate may be issued after the expiration of six months from the date of issue of the original check, but only under such regulations as may be adopted by the Secretary of the Treasury. The depository upon which drawn should be at once notified of the loss of a check, and request made that payment thereof be stopped; after which, chiefs of parties should make application to the Superintendent for blank forms and the necessary instructions as to the method of procedure to be followed in securing a duplicate of the lost check.

In the rendition of accounts to the disbursing agent for settlement, chiefs of parties should state the balances, on account of moneys advanced, due that officer, or, if there is no such balance due, they should state the amount which may be due them upon settlement. In other words, each chief of party when rendering his monthly accounts should transmit with them an account current, transcribed from his books, showing his financial status with the Survey based upon the moneys advanced him and the amount of his accounts as rendered. Form 10, letter transmitting accounts, will be found by chiefs of parties a convenient means of complying with the routine here suggested.

When forwarding moneys as an advance to chiefs of parties, or when making deposits to their official credit at any governmental depository, the disbursing agent should, in his transmitting letters and notifications of such action, invariably state the amount due him on account of advances, at the date thereof, according to his books; and chiefs of parties should be careful to respond to such statements and acknowledge or correct the amounts therein stated. Such acknowledgments are of great value to the disbursing agent in the event of an examination of his office by Treasury officials. They are, moreover, an excellent check in the event of errors occurring in the statement of balances.

Correctness in entries, extensions and additions of all abstracts and other vouchers, will invariably insure a more speedy settlement of the accounts.

If the observations made herein in relation to the disbursement of public moneys and the methods of accounting for the same prove of service, the purpose of this paper will have been attained.

JOHN W. PARSONS,
Disbursing Agent.

REPORT OF COMMITTEE I, ON THE MEASUREMENT OF ARCS.

The measurements of arcs of the earth's surface are indispensable for the determination of a geometrical figure which in shape and size should approximate most closely to the figure of the earth as a whole. Such measures have been undertaken, either expressly or only indirectly, in connection with surveys made by leading nations within the boundaries of their countries; and in the latter case either the general figure or one more closely fitting the actual region was made use of for the development of the triangulation on the limited part of the surface.

It is otherwise when large areas, such as the surface of North America, or even that covered by the territory of the United States, are concerned. Here the importance of the measurement of arcs becomes apparent in order to furnish the shape and dimensions of that geometric osculatory figure which best represents the particular surface in question and upon which it is desired to develop the triangulation, which latter is the foundation for exact measures of relative geographic position on the earth's surface.

Until the representative or special figure can be determined, and as a means of furnishing the needed material for its elucidation, the work of the triangulation, constituting the basis of a survey, can (as in the case of the Coast and Geodetic Survey of the United States) be prosecuted by making use of a spheroid fairly approximating to the earth's figure. The resulting positions can and do have a satisfactory degree of accuracy. Hence it will be seen that the measurement of arcs may be regarded as in a measure incidental to the operations of a trigonometrical survey of an extended country.

In this way originated the several arcs developed by the Coast and Geodetic Survey up to this time. Other prospective arcs, more or less desirable or feasible, will be pointed out further on.

A further reason why this Conference has taken into consideration the matter of arc measures is the circumstance that by joining the International Geodetic Association for the measurement of arcs*—in other words, for the determination of the earth's figure—the United States have incurred certain scientific obligations which demand attention from this Survey.

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* See act of Congress of February 5, 1889.

For convenience and distinction arcs may be classified as arcs of the meridian, of the parallel, and oblique arcs. Of these the first and second are now, since the introduction of the telegraphic method of measuring differences of longitude, of nearly equal importance as far as obtainable accuracy is concerned. The third class may be regarded, theoretically at least, as composed of part of the first and part of the second kind.

The value of an arc depends mainly on its extent, position (with respect to latitude), accuracy of measure, and number of subdivisions. The more numerous the latter the greater the chance that the local deflections of the vertical may be neutralized and the results be freed from a source of error which up to this time has been the main cause of discord subsisting between the several arc measures as well as between their several parts when combined for a resulting geometric figure.

This paper has no space nor is there the least necessity for going into the history of arc measures, or even to refer to the great measures accomplished by England, France, and Russia, and our remarks will be confined to the subject-matter in relation to the Coast and Geodetic Survey.

A few arcs have so far been measured in this country, but none have been completed, or, at least, all are capable of extension. Appendix No. 6 of the Coast Survey Report for 1877 contains an account of two, both of the meridian: The Nantucket arc, $3^{\circ}\cdot37$ in length, with 6 subdivisions; the Pamlico-Chesapeake arc, $4^{\circ}\cdot52$ in length, with 13 subdivisions; and in order to obtain some information as to the curvature of the surface in these parts these were combined with the Peruvian arc, $3^{\circ}\cdot12$ in extent and without subdivisions. No specific difference in the curvature of the Western Hemisphere from that of the Eastern was indicated, but a change from the use of the Besselian spheroid of revolution (of 1841) to that of Clarke (of 1866) was indicated by the above combination as desirable. The substitution of the latter figure for the development of our triangulation was approved by the Superintendent on February 4, 1880. No publication has yet been made of the oblique arc extending along the North Atlantic coast from the Canadian boundary to the Gulf of Mexico. The length of the completed part between Eastport, Me., and Montgomery, Ala., is nearly $21\frac{1}{4}^{\circ}$, or about 2 252 kilometres. It will, when extended to the Gulf, reach $22\frac{1}{2}^{\circ}$, and possibly it may be extended northeasterly through the Dominion of Canada to Cape Breton Island. A preliminary computation of part of this arc also favored the change of the spheriod of reference mentioned above. In volume No. 24 of Professional Papers by the United States Engineers* the United States Lake Survey gives the results of the measure of two arcs, one a meridional one, between

*Primary Triangulation, U. S. Lake Survey, Washington, D. C., 1882.

St. Ignace, on the northern shore of Lake Superior, and Parkersburg, Ind. Its length is 10°·21, with 9 subdivisions. The other arc is considerably inclined to the parallel and extends over 11°·79 of longitude, from Willow Spring, near Chicago, Ill., to Mannsville, at the east end of Lake Ontario, New York. It is composed of 3 sub-arcs. The meridional arc will in time be extended to the Gulf of Mexico.

The old arc of 1764 in Maryland and Delaware, known as Mason and Dixon's Line, here deserves but a passing mention as the first arc measured in North America. It was 1°·48 in length, and was measured with wooden rods throughout.*

The desirability and necessity of a geodetic connection of the triangulations of the Atlantic and Pacific coasts were first pointed out in 1870 by the then Superintendent of the Survey, Prof. Benjamin Peirce. (See Coast Survey Report for 1870, p. 4.) In the following year the measurement of the parallel of 39° between Cape May, New Jersey, and Point Arena, California, was commenced, and at the time of writing (January, 1894) but a small gap of triangulation west of Pikes Peak, Colorado, remains to be filled up. The total length of this arc of the parallel is 48°·78 of longitude, or about 4 226 kilometres (very nearly 2626 statute miles). It is subdivided at present into 19 subarcs, for all of which the telegraphic longitude work is completed. Interspersed in this triangulation are a great number of astronomical latitude and azimuth stations; and a series of base lines, some yet to be measured, sustain the accuracy of the linear dimensions.

For the measure of the earth's curvature at right angles to the above parallel an arc of the meridian in about longitude 98° west of Greenwich has been proposed. Of this central arc 22°·92, or 2 544 kilometres, will be within the boundaries of the United States, between the Rio Grande and the northern boundary. It is capable of extension southward 10°, through Mexico to the Pacific Ocean at Point Sacrificios, and northward through Canada and the British Possessions to an unknown distance. An arc of the meridian in longitude 112° may be suggested; of this, 3° have already been measured in the vicinity of Salt Lake, Utah.

On the western coast an arc of the meridian can be established in longitude 120½°, from the Santa Barbara Channel to the northern boundary of the United States, a distance of about 15°. That part of the boundary of California and Nevada oblique to the meridian would lend itself well to an inclined arc about 5¼° in length, with a capacity of extension to 15° within our boundary.

Besides the central arc of the parallel already referred to, two other arcs of the parallel of great longitudinal extent have been projected, viz, the northern one, in latitude 46°, and the southern one, in latitude 33°. (See accompanying map, illustration No. 14, on which the several arcs are located.)

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* Phil. Trans. Roy. Soc. for 1768.

Arcs, measured and projected within the limits of the United States or close to its border

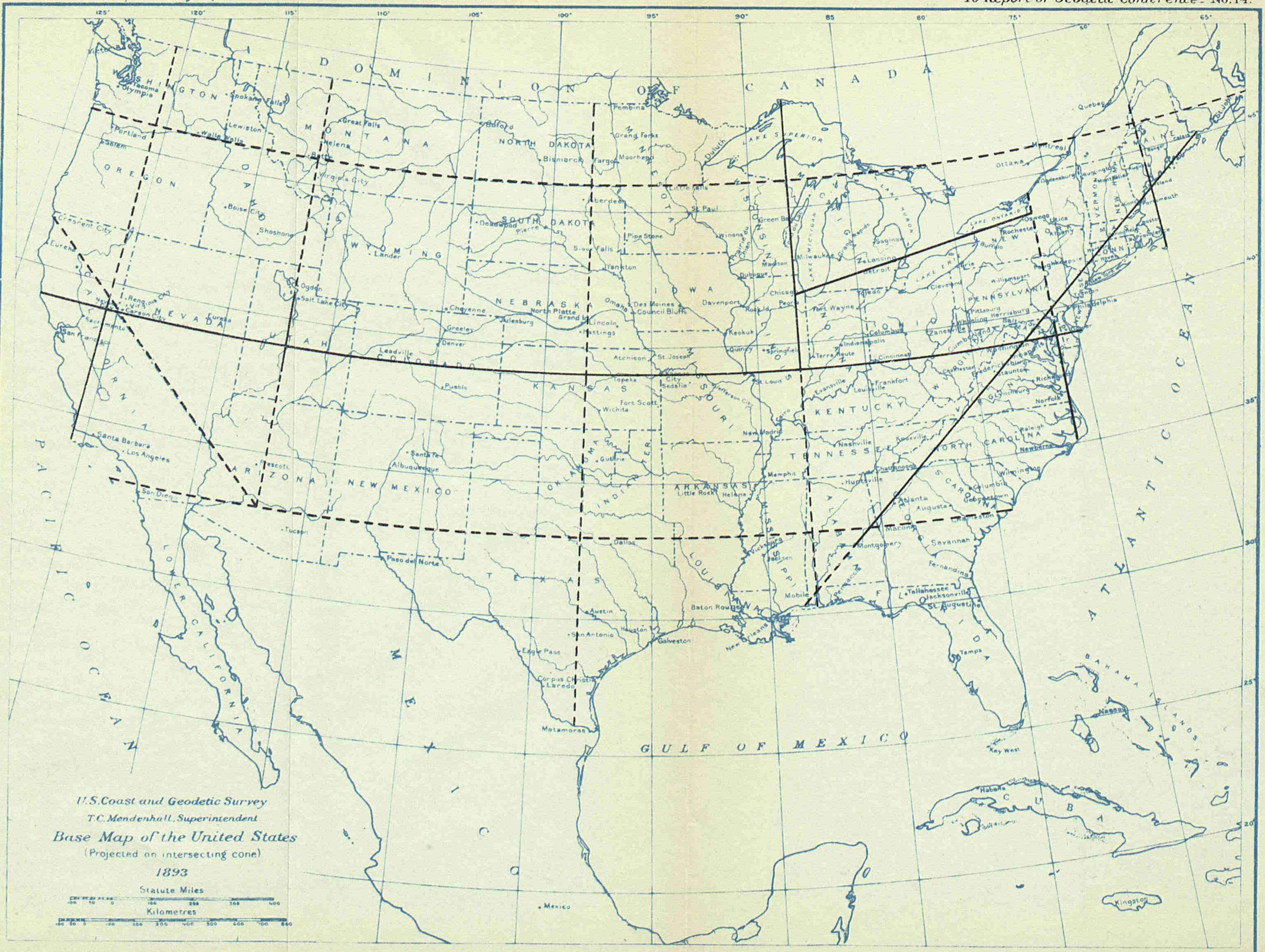

Photo-Lith by A. HOEN & CO., Balto., Md

The desirability of a remeasurement and extension of the arc of Peru (1735–1743) was pointed out in 1877,* and again in 1889.† Fortunately for the progress of geodesy, this arc appears to be nearly correct; otherwise, by reason of the possible effect of local deflections of the vertical at its two terminal latitude stations, it might have exercised a retarding influence.

Researches respecting the earth's figure made since the publication of Captain Clarke's Dimensions of the Spheroid (London, 1866)‡, which were adopted by this Survey, give smaller values than $\frac{1}{294 \cdot 97}$ for the compression $\frac{a - c}{a}$.

Dr. Helmert, in 1887, gives for his reference spheroid $\frac{1}{299 \cdot 15}$. F. Tisserand, in his Traité de Méchanique Céleste, Tome II, Paris, 1891, shows that under certain plausible hypotheses as to increase of density with depths and as to original fluidity the flattening can not exceed $\frac{1}{297 \cdot 3}$, and Prof. W. Harkness in his essay, "The solar parallax and its related constants," Washington, D. C., 1891, arrives at the result $\frac{1}{300 \cdot 205 \pm 2 \cdot 964}$, thus approximating again to the values assigned in earlier times by Airy (1830) and Bessel (1841), about $\frac{1}{299 \cdot 2}$.

The Conference recommends that the spheroid adopted by the Survey be adhered to as being sufficiently close to any other value that could now be assigned or is likely to be assigned in the near future, and that the subject of the arc measures be kept in prominent view in connection with the progress of the Survey; also that all linear measures be expressed in terms of the prototype metre and that a direct comparison be made of the lengths of the committee metre and the national prototype.

Respectfully submitted to the Geodetic Conference.

CHAS. A. SCHOTT, *Chairman.*
G. R. PUTNAM, *Secretary.*

JANUARY 23, 1894.

REPORT OF COMMITTEE J ON MAGNETICS.

A study of the laws governing the various magnetic forces is naturally connected with the work of the Coast and Geodetic Survey, and a thorough knowledge of them is indispensable in order—

1. To supply its charts with information of the magnetic variations at the dates of issue, together with the prospective annual change.

2. Incidentally to facilitate the proper adjustment of the compass

* Coast Survey Report for 1877, p. 95.

† Coast and Geodetic Survey Report for 1889, Appendix No. 7.

‡ Viz: $a = 6378206 \cdot 4^{m}$, $c = 6356583 \cdot 8$ } which we now take as expressed in terms of the International Metre.

in ascertaining local deviation on board ship, for heeling, and different positions of the vessel.

For these and other purposes it is necessary to study the laws of terrestrial magnetism as well as determine absolutely its several forms or components of declination, dip, and intensity. The distribution of this force is dependent not alone upon the time and the geographical position, but is influenced by many local disturbances; hence the imperative necessity, in order to supply our charts with compass bearings, that the study of magnetism should cover at least the entire seacoast of the United States; and in order to produce the lines defining the direction and intensity of these forces to the required limit seaward for a certain epoch there arises the further need of extending the observations a sufficient distance inland.

3. To meet the constant demand made upon the Survey by surveyors, engineers, and courts of law in every part of the country for information, generally for the recovery of old lines or landmarks. And for this purpose a further and more complete study of these forces, covering the *entire area* of the country, is demanded.

4. To meet the necessity for an accurate knowledge of the dip and intensity of the magnetic force arising from the researches of science and the practical application of them by electricians in the measurement of the closely allied forces of electricity.

To meet these demands it is obviously important to construct from time to time (say once each ten years) isomagnetic charts to represent the then existing state of distribution of this still mysterious force.

The Survey has already—

1. Made direct observations of declination, dip, and horizontal force in many widely distributed places.

2. It has carefully collected observations from all available sources whatever, from the earliest to the present time.

3. It has made a special study of the laws of terrestrial magnetism by means of photographic registration at especially selected places, to be changed after about seven years of continuous occupation to a new place (so as to cover at least more than one-half of the sun-spot period), and always placed in localities most remote from those where the best magnetic observations had previously been made. Except the occasional changing of stations for photographic registration, this is in conformity with the practice in other countries.

4. It has afforded assistance to magnetic surveys undertaken by States or private individuals by the loan of instruments and in other ways.

5. It has endeavored to elucidate the multiplicity of laws governing these forces and to disseminate them for general information in the publications of the Survey.

METHODS AND INSTRUMENTS OF THE COAST AND GEODETIC SURVEY AS COMPARED WITH THOSE OF OTHER COUNTRIES.

Since the time of the great physicist Gilbert of Colchester, who showed the earth to be a great magnet, about the year 1600, magnetic theories and graphical results have been diligently worked out; and as time has passed these efforts have been more minute and complete.

Magnetic observations have been systematically carried on in Great Britain, France, Austria, Germany, Russia, Japan, India, Australia, Mexico, Canada, the United States, and other countries. They also formed part of the programme of every scientifically organized Arctic expedition; and the United States, in the years 1881–1884, assumed the responsibility of occupying two of the stations of the cordon which, under international auspices, girdled the pole for the purpose of studying, among other points of physical interest, the laws of magnetism in that important region.

In this country, in addition to the work of the Coast and Geodetic Survey, New Jersey has been surveyed magnetically by the State, and the State of Missouri by individual enterprise. A report on the last has been published by Prof. F. E. Nipher.

The instruments employed in these surveys are essentially similar in principle, although differing in the detail of construction as well as in size and weight.

There is no means of deciding as to the relative values of the results obtained in different countries, and any comparison must deal with methods and instruments only.

A description of some of the instruments used in other countries may be interesting, and for this purpose some of the instruments used in Russia, England, Italy, and France have been selected. All are of recent date.

The Russian Universal instrument is described by Dr. Wild in the publications of the Imperial Academy of Sciences.* It is remarkable for great size among other instruments of the kind, carrying 20^{cm} (8-inch) azimuth and vertical circles, the torsion head rising a full metre above the leveling foot screws. The telescope is firmly attached to an elliptical collar at one end of its greater axis. A hollow cylindrical counterpoise is attached to the opposite end of the axis. The axes of the telescope are at the ends of the shorter axes and rest in Y's. The axes are extended beyond the Y's, and carry on one end a vertical circle and on the other two long arms immovably secured to the axis, which carry two microscopes 180° apart for pointing upon the ends of the needle when observing the dip or inclination of the needle. A broad horizontal support at the base of the Y supports carries at one end, outside the Y's, a clamp and verniers, and at the opposite end a circular metallic case in which is mounted the dip needle, while on the

*Publications of Russian Imperial Academy of Sciences, 1872, Part III, No. 2.

center of the support, and rising through the elliptical collar of the telescope, is the box containing a declination needle, surmounted by the long tube supporting the needle. The stirrup carrying the magnet is provided with a mirror, perpendicular and at right angles to the axis of the magnet. When observing, a pointing is made by observing the cross threads, illuminated by a ray of light (which enters the telescope through a slit near the eye), reflected by the mirror to the eye. The magnet is rectangular, 6^{cm} ($2\frac{1}{3}$ inches) long, 6^{mm} ($\frac{1}{4}$ inch) wide, and about $1{\cdot}5^{mm}$ ($\frac{1}{16}$ inch) thick, fitting snugly in an opening designed for it. A collar fitted to the stirrup, nearly 1 inch above the magnet, carries the inertia ring during oscillations. To reverse the magnet the stirrup and mirror are reversed. A scale in the focus of the eyepiece of the telescope serves to measure the length of vibrations, etc. In observations for inclination the telescope is turned in the Y's until the microscopes at one end of the axis are directly over the poles of the needle, when the angle is read on the vertical circle on the other end of the axis. The dip needle is easily lifted and reversed by a clever device without opening the case, which has a glass front. When observing for azimuth, or even upon the mark, the box containing magnet suspended must be removed. No weight is given, but it is evident that such an instrument, properly boxed, would perhaps not weigh less than 70 kilogrammes (154 pounds).

The objections to this instrument, if the brief description available has been rightly understood, are—

1. Its great weight.

2. The necessity of removing box to observe mark as well as to observe the azimuth.

3. The necessity of removing suspension tube in order to suspend or remove the magnet.

4. The instability of the microscopes for pointing on the dip needle

*The English instrument** is mounted on a principle essentially the same as the Coast and Geodetic Survey magnetometer, differing, however, in one important particular—the telescope is fixed in a horizontal position and pointing to the center of suspension of the declination magnet, and the azimuth of the sun is obtained by observing its image reflected in a mirror. The magnets are similar in size and construction to those heretofore used in this country. The only objections to this instrument are: The necessity in this country for securing accurate time for azimuth by observations, thus requiring another instrument; the necessity of placing the mark in the horizontal plane of the telescope, which is seldom convenient, and the difficulty and annoyance attending the adjustment of the mirror.

The Kew dip circle, used both in England and America, is too familiar to require any description. The weight of the magnetometer in its box is about 23 kilogrammes (50 pounds).

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* Encyclopædia Britannica.

*The magnetometer used by the French** is quite different in detail from any other. It is also an altazimuth instrument, and requires no changing or removing of any of its parts. The circles are 8^{cm} (3·15 inches) in diameter, graduated to half degrees, and read by verniers to minutes. A rectangular frame on the axis of the azimuth circle carries all the other parts on one side, and outside the telescope is mounted, and attached to its axis is the vertical circle. Also attached to the axis of the telescope, and rigidly adjusted parallel to it, is an index arm, at the end of which is a silvered index having three equidistant lines drawn perpendicularly on its outer face, and a single line just opposite the center line on its inner surface. The magnet is a solid cylinder 6·5^{cm} long, 4^{mm} in diameter, and weighs about 7·5 grammes. Its ends are slightly concave and polished to a reflecting surface. There is a microscope for pointing on each end of the magnet. A single fiber of silk about 11^{cm} (4·33 inches) long suspends the magnet. Observations for declination are made by bringing the line on the inside of the index, as reflected from the mirror end of the magnet, to coincide with the middle line on the outside of the index when seen through the microscopes; and pointings are made on the mark or on the sun for azimuth with the telescope. Inasmuch as the needle and the mark are observed with different lines of sight, it is evident that with the best adjustment there must remain some uncertainty of their parallelism; and this index error could only be determined by observing at a well-determined station. The principal mechanical objection to this instrument is this index error and the danger that it may not remain constant. Another objection is the use of glass to protect the magnet from air currents, dust, etc.

The French dip circle has also two circles, vertical and azimuth, each 8^{cm} (3·15 inches) in diameter. An arm, carrying concave mirrors, is swung around under the point of the needle, and the circle, to which the microscopes are rigidly attached, is turned by a slow-motioned screw until the points, as reflected from the zero mark on the mirrors, coincide with the point seen direct, thus insuring a most accurate pointing, free from parallax. This is most excellent in theory, but in practice it is probable that the unsteadiness of the needle will exceed the probable error of a pointing on it by other methods. This instrument, packed in its case, weighs only 2 kilogrammes (4 pounds).

A magnetometer used in Italy† presents at least one novel feature. It is an altazimuth instrument, constructed much like ours, except that a broken telescope is used. This permits the telescope to be revolved in its Y's, and a mark may be observed at a point opposite to the magnet without disturbing the box.

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* Moreaux, Magnetic Elements in France, 1885. See also Nature for Jan. 12, 1888.
† Publications of the Royal Observatory at Modena, 1893, No. 1.

The magnetometer of the Coast and Geodetic Survey in its usual form has been described elsewhere.* The latest instruments are provided with magnets, octagonal in form, thus facilitating their manipulation when being reversed and making it easier to place the inertia ring. Another improvement is the removal of the glass from the box front, and the employment of a shutter at the back of the box, which may be opened when observing on the mark.† The weight of the new magnetometers, boxed, the box containing also the tripod head, is about 18 kilogrammes (40 pounds). The dip circle weighs about 12 kilogrammes (25 pounds). All the instruments examined have suitable provision for deflections.

It will be seen that the French instruments are by far the lightest of those examined. They are also the simplest in construction. They could easily be carried as hand baggage when moving from station to station.

Next in order of weight and facility of manipulation come the instruments of the Coast and Geodetic Survey.

The lightness and simplicity of the French instruments highly commend them for work in a country like ours, where cost of transportation is a most important part of the expense of magnetic surveys; and in order to give them a fair test by actual use we recommend the purchase of one or more of them for the use of the Coast and Geodetic Survey.

It may be well to allude to the compass declinometer, which is the old azimuth compass in a slightly new dress, designed to measure declinations only. Properly in adjustment, and its index error known, this instrument gives very accordant and satisfactory results.

The index error is often very large (more than 1°,) but this is of no consequence. It is, however, liable to change, and as it can not be determined in the field it is an objectionable feature.

Comparison has been made at the office respecting the moment of inertia of the magnet and its appendages, as depending on computation from known dimensions and weights, with the indirect method from oscillations, with and without the inertia ring. Results thus far have proved that the two methods agree within about 2 per cent of the whole value. Further investigation in this direction will be made.

METHODS OF MAKING MAGNETIC SURVEYS IN DIFFERENT COUNTRIES.

Countries have been surveyed magnetically, either by a rapidly executed survey, covering the entire area of the country in a brief period, with the expectation of repeating the surveys after an interval of one-fourth to one-third of a century, thus bringing out by comparison the secular changes due to the interval; or observations have been continuously made, and the results were gradually collected, reduced to the epoch adopted, and discussed. The former process answers well for a country of limited extent, as, for instance, England or France, where

* See Coast Survey Report, 1881, Appendix 8.

† See magnetometer used in Japan, described in the Journal of the College of Sciences, Imperial University of Japan, Vol. II, Part 3, 1888.

this method has been employed; but for the great area of the United States it could not be carried out within a reasonable time, say in from two to five years. The second method was therefore adopted of necessity. It carries with it the continued study of the laws of secular variation.

It is very important in selecting sites for stations to avoid all probable disturbing influences from railroads, telegraph and telephone wires, or electric car or light wires; and also to consider the probabilities of their recovery for future occupation. These considerations point to the necessity for the selection of stations outside of cities or villages.

We recommend the issue as soon as may be practicable of a second edition of Directions for Measurement of Terrestrial Magnetism, containing such modifications and additions as new or improved methods or instruments may have suggested.

That more systematic observations of all the magnetic forces be made at many points throughout the country where data are now lacking; in particular throughout the States of California, Oregon, Washington, Idaho, Montana, and the Dakotas, preferably by someone who shall devote his whole time to this work; also, that no opportunity be allowed to pass for securing observations along the vast coast regions of Alaska, and for reoccupying stations for collecting necessary data for determining the secular change.

That as soon as may be desirable a second edition of the isoclinic, isodynamic, and isogonic curves be published for an epoch close at hand, say 1895 or 1900, together with the data, the method of discussion, and explanations of the results and their uses.

We recommend that each main triangulation and astronomical party be supplied with a complete instrumental outfit for determining declination, dip, and intensity; and also that all other triangulation parties be furnished with a compass declinometer, and that observations of declination be made at each station occupied.

J. J. GILBERT, *Chairman.*
R. L. FARIS, *Secretary.*

REPORT OF COMMITTEE K ON GRAVITY.

The study of the force of gravity as a part of the geodetic problem has received the attention of the Coast and Geodetic Survey for some years, and although its work in the past has in a measure been experimental it has developed instruments and methods of observation which will enable it to enter successfully upon extended gravimetric research at less cost than would have been possible with the older processes, and that without lowering the standard of accuracy.*

*See Determinations of Gravity, by T. C. Mendenhall, Appendix No. 15, C. and G. Survey Report for 1891, Part II.

Ever since the promulgation by Clairaut of his celebrated theorem, one hundred and fifty years ago, the pendulum has been regarded as a most efficient means for the investigation of the shape of the earth. It appears to be the general opinion of the enlightened nations of the world engaged in geodetic operations that any survey that would disregard gravimetric research as an important and necessary branch of inquiry would fall short of a complete geodetic survey.

The earlier gravitational work of the Survey was of necessity of an experimental character, as stated above, involving such considerations as the character of the pendulum, with respect to absolute and relative measures, its best form, size, and material, as well as the method of observing. The several instrumental reductions to normal condition had to be studied both theoretically and practically.

At this stage of the work differential comparison of gravity between American and European standard stations of absolute measures were instituted, and in connection with this work of the Survey, in 1875–76, it had become evident that the flexure of the pendulum support during its swings was a grave source of error. It accordingly received a thorough investigation so that corrections could be given to observations made on certain stands and under certain conditions.

At the instance of the Superintendent of the Survey, a conference on gravitation measures was held at Washington, D. C., in May, 1882, having for its object to devise a plan for the prosecution of the observations and for the improvement of the pendulum apparatus.

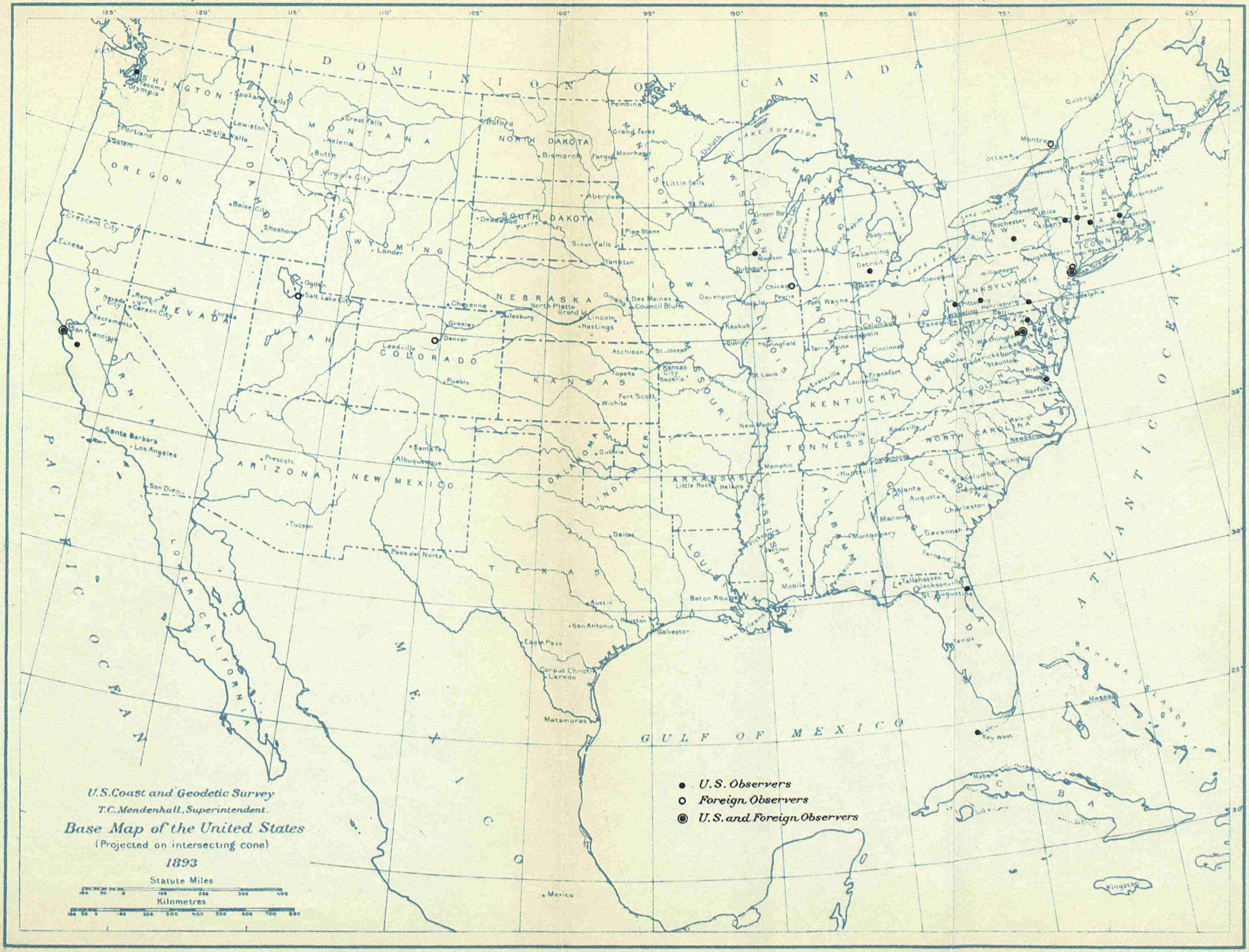
Your committee is of the opinion that the time for an active prosecution of field work has come, especially as the Survey is now well prepared, both by experience and equipment, to carry on these investigations. Its relation to the International Geodetic Association renders it desirable that this Survey should conform, as far as may be practicable, to the general plan of work followed, and that it should therefore contribute its share to the gravity research.

That determinations of gravity are essential to a complete geodetic survey, as well as of great interest in connection with geological problems, is sufficiently attested by the action shown by many of the leading nations in extending their gravimetric surveys in recent times.

Thus far over 500 stations have been determined in various parts of the world, of which a large proportion were occupied in the last few years, showing the active increase of interest in these researches. In the United States 27 stations have been determined by the Coast and Geodetic Survey and 9 by foreign observers. The Survey has also occupied 29 stations in foreign countries, taking advantage of various astronomical expeditions.

The English have made a series of determinations in their country, and have, moreover, sent expeditions for this purpose to various parts of the world. In India they have carried out a very systematic scheme of gravity work in connection with the Great Trigonometrical Survey.

United States Pendulum Stations



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REPORT

OF THE

SUPERINTENDENT

OF THE

U. S. COAST AND GEODETIC SURVEY

FOR THE

FISCAL YEAR ENDING JUNE 30, 1893,

IN TWO PARTS.

PART II.

APPENDICES RELATING TO THE METHODS, DISCUSSIONS, AND RESULTS OF THE COAST AND GEODETIC SURVEY.

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Annual Report of the Superintendent of the Coast Survey

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PREFATORY NOTE.

The text of this Report for the fiscal year 1892 has been arranged for publication in two parts, like that for the fiscal year preceding.

Part I, in quarto form, contains the historical portion, presenting reports of progress in the field and office operations of the Survey; estimates for future progress, and statements of expenditures during the fiscal year. Maps of general progress, and sketches showing localities of field work, exhibit graphically the advance of the Survey to June 30, 1893.

Part II, it will be observed, is in octavo, and includes the professional papers relating to the methods, discussions, and results of the Survey which have been approved for publication during the year. Such illustrations as are needed accompany them.

The octavo form is more convenient and suitable for the scientific and professional papers, while the quarto form appears to be demanded for the statistical matter and the progress sketches. Since the latter are of less general interest than the former, in the future distribution of the Report, Part II only will be sent, as it is believed that this will include all that is generally desired, and in a much more compact and convenient form than that of the old quarto.

In special cases, where both parts are desirable, they will be sent.

T. C. MENDENHALL,
Superintendent.

U. S. COAST AND GEODETIC SURVEY REPORT FOR 1893.—PART II.

TITLES OF APPENDICES.

U. S. COAST AND GEODETIC SURVEY REPORT FOR 1893.—PART II.

ILLUSTRATIONS.

APPENDIX No. 1—1893.

STATE LAWS AUTHORIZING OFFICERS OF THE UNITED STATES COAST AND GEODETIC SURVEY TO ENTER UPON LANDS WITHIN STATE LIMITS FOR THE PURPOSES OF THE SURVEY.

LETTER OF TRANSMISSION.

UNITED STATES COAST AND GEODETIC SURVEY,
Washington, D. C., December 29, 1893.

SIR: In accordance with your suggestion that it is desirable to have published, as an appendix to the report for 1893, the laws of several of the States enacted for the protection of Coast Survey parties working therein, I have had the accompanying typewritten copies made, for the printer's use, of all such laws that we have any record of in this office. According to this record, laws have been passed in the following nineteen States: California, Connecticut, Georgia, Illinois, Indiana, Maine, Maryland, Massachusetts, Minnesota, Missouri, New Hampshire, New Jersey, Ohio, Oregon, South Carolina, Tennessee, Vermont, Virginia, and West Virginia.

In order to insure the accuracy of the manuscript copies of the laws on file in this office, I took them over to the Law Library of Congress and compared them with the printed laws on file there.

Finding that they were incomplete and full of inaccuracies, I carefully corrected them, and from the corrected text Miss Hein then made typewritten copies. These I have carefully compared, and now submit them (55 pages) as exact copies, following strictly the punctuation and capitalization of the printed laws.

Respectfully, yours,

GEORGE A. FAIRFIELD,
Assistant in Charge of State Surveys.

DR. T. C. MENDENHALL,
Superintendent Coast and Geodetic Survey.

STATE LAWS

FOR THE

PROTECTION OF COAST SURVEY PARTIES.

CALIFORNIA.

CHAPTER LXXV.

An Act to authorize persons engaged in the United States Coast Survey, upon the Coast of California, to enter on lands within this State, for the purpose of said Survey; to protect the operations of the same from injury and molestation; to ascertain the mode of assessing damages caused to any property in the progress of the same, and to provide for the punishment of offenders against the provisions of this Act, and for other purposes.

The People of the State of California, represented in Senate and Assembly, do enact as follows:

SEC. 1. That from and after the passing of this Act, any and every person employed under and by virtue of an Act of Congress of the United States, passed the tenth day of February, one thousand eight hundred and seven, and the supplements thereto concerning the United States Coast Survey, may enter upon Lands and clear and cut the timber, within this State, upon the same, and may erect any works, buildings, or appendages requisite for the purpose of exploring, surveying, triangulation, leveling, or doing any other act requisite to effect the object of said Act of Congress, without being considered as a trespasser: *Provided*, no unnecessary injury be done thereto.

SEC. 2. That if the parties interested—namely, party or parties representing the Government of the United States Coast Survey on the Coast of California, and the owners or possessors, of the Land so entered upon, and to which damage may have been done—cannot agree together upon the amount to be paid for the damages caused by doing any of the acts aforesaid, either of them may complain, in a summary manner, to the nearest Justice of the Peace for the District of the County where the damages may have been committed, who shall associate with himself two disinterested freeholders of the said County, one to be named by each party interested, who shall, upon hearing the parties, and with or without view of the premises, as they may determine, proceed to assess and award any damages which may have accrued to the owners or possessors of the Land so entered upon: *Provided*, nevertheless, that the party complaining as, aforesaid, shall serve upon the opposite party interested, ten days notice, in writing, of the time and place where said complaint is to be heard, and the name of the freeholder by him selected.

SEC. 3. That the said magistrate and freeholders shall, without unreasonable delay, file in the Office of the Clerk of the County Court of the County where the said complaint may have been heard, a report of their proceedings, which report shall be conclusive against the parties, and be evidence of their assent to the same; unless either of them shall, within ten days after filing of the said report, file a general or special objection to the same in the office of the said Clerk, of which the other party shall have notice; whereupon an issue shall be made up and a trial had at the next term of the County Court of said County, in the same manner in which civil cases are tried; except that the judgment shall be rendered and the damages assessed at the first term.

SEC. 4. That any person so entering upon Land, as aforesaid, for the purposes aforesaid, may tender to the party injured sufficient amends for any damages done upon said Land; and if, upon examination before the Justice of the Peace and freeholders as aforesaid, or upon trial before the County Court, the damages finally assessed shall not exceed the amount so tendered, the person who has so entered and tendered the amount, shall recover his costs.

SEC. 5. That the Justices of the Peace and Freeholders aforesaid, upon complaint made to them as aforesaid, and decision given, shall receive the same costs to which, by law, Justices of the Peace are entitled in a civil case from summons to judgment; and upon the trial in the County Court the costs shall be taxed by analogy to the Bill of Costs in said Court, established by law.

SEC. 6. That if any person or persons shall wilfully or wantonly injure, deface, or remove any instrument, signal, monument, building, or any appendage thereto, used or constructed in the State of California, under and by virtue of the Act of Congress aforesaid, he and they shall be liable to indictment for the same, under this Statute, for each and every offence, and upon conviction, shall be sentenced to pay a fine of two hundred dollars, one-half of which shall go to the prosecutor, and the remainder shall be appropriated according to the Laws of this State regulating the disposal of such fines, or shall be imprisoned not more than one month, or both, at the discretion of the Court before which such conviction shall take place, and he and they shall also be liable for all damages sustained by the United States of America, by reason of any such injury, defacement, or removal; to be recovered by action on the case in any Court of competent jurisdiction.

SEC. 7. This Act shall take effect from and after its passage

Approved April 2, 1852.

CONNECTICUT.

CHAPTER XI.

An Act relating to the Survey of the Coast of Connecticut.

Be it enacted by the Senate and House of Representatives in General Assembly convened:

SEC. 1. Persons employed under an act of the Congress of the United States, passed the tenth day of February, in the year eighteen hundred and seven, and the supplements thereto, may enter upon lands within this state, for any purpose which may be necessary to effect the objects of said act, and may erect works, stations, buildings, or appendages for that purpose, doing no unnecessary injury.

SEC. 2. If the parties interested cannot agree upon the amount to be paid for damages caused thereby, either of them may petition the county commissioners of the county in which the land lies, who shall appoint a time for a hearing, as soon as may be, and order at least fourteen days' notice to all persons interested, and, with or without a view of the premises, as they may determine, hear the parties and their witnesses and assess the damages.

SEC. 3. The county commissioners shall file in the office of the clerk of the superior court of the county in which the land is situated, a report of their doings, which shall be conclusive, unless one of the parties shall, within thirty days after the filing of such report, file a petition to said court for a new hearing to be had in such superior court; in which case, after such notice of such petition to the opposite party as the said superior court, if in session, or, if in vacation, as any judge thereof or of the supreme court, or any county commissioner of the county in which such petition is pending, shall direct, a trial shall be had in said court, in the same manner as other civil actions are tried, and such hearing shall take precedence of all other civil actions.

SEC. 4. The person so entering upon land may tender to the party injured amends therefor; and, if the damages finally assessed do not exceed the amount tendered, the person entering shall recover costs; otherwise, the prevailing party shall recover costs.

SEC. 5. The costs to be taxed and allowed in all such cases, either before the county commissioners or the superior court, shall be the same as are ordinarily taxed, according to the rules and practice in the superior court.

SEC. 6. Whoever wilfully injures, defaces or removes any signal, monument, building or appendage thereto, erected, used or constructed, under said acts of Congress, shall forfeit the sum of fifty dollars for each offense; and shall be liable for damages sustained by the United States, to be recovered in an action of tort.

Approved June 5th, 1861.

GEORGIA.

[From "The Code of the State of Georgia, 1882."]

Part I—Title I—Chapter I.

ARTICLE III.

COAST SURVEY.

[Act of 1847, Code, p. 155.

§ 23. (23.) (25.) *Coast surveyors.*—Any person employed under the Act of the Congress of the United States, providing for a survey of the coasts, may enter upon lands and clear or cut timber within this State upon the same, for any purpose legitimately connected with, and requisite to effect, the said object: *Provided*, no unnecessary injury be done thereby, and all damages to the owner of the land be promptly paid.

[Act of 1847, Code, p. 155.]

§ 24. (24.) (26.) *Damage to land-owners.*—If the parties representing the Government of the United States, and the owner or possessor of the land so entered upon, cannot agree upon the amount to be paid for said damages, either party may complain in a summary manner to the nearest Justice of the Peace of the county in which the land lies, who shall associate with him two disinterested freeholders of the county—one to be named by each party interested—who shall, upon hearing the parties, and with or without view of the premises, as they may determine, proceed to assess and award the damages, if any: *Provided*, the party complaining shall give the opposite party ten days' notice, in writing, of the time and place when and where said complaint is to be heard, and the name of the freeholder by him selected.

[Act of 1847, Code, p. 156.]

§ 25. (25.) (27.) *Award and objections thereto.*—The said assessors, without unreasonable delay, shall file their award in the office of the Ordinary of the county, which shall be conclusive upon both parties, unless objections are filed to the same within ten days after the filing of the award. If objections are filed, the other party shall have written notice; whereupon an issue shall be made and tried at the first term thereafter of said Court, under the same rules as other civil cases.

[Act of 1847, Code, p. 156.]

§ 26. (26.) (28.) *Damages, tender of.*—The person so entering upon lands may tender such amount as he chooses for the damage done, and if the damages finally assessed shall not exceed the sum tendered, the party complaining shall pay all costs.

[Act of 1847, Code, p. 156.]

§ 27. (27.) (29.) *Costs.*—The costs before an Ordinary shall be the same as are allowed in civil cases in said Courts.

Part IV—Title I—Division XII.

[(a) Acts of 1865–6, p. 233.]

§ 4619. (4529.) (4483.) *Injuries to coast survey fixtures.*—Any person who shall willfully or wantonly injure, deface or remove any signal, monument, building, or any other appendage thereto, erected within this State by virtue of any Act of Congress authorizing a coast survey, shall be guilty of a misdemeanor, and, on conviction, [shall be punished as prescribed in section 4310 of this Code.] (*a*)

Part IV—Title I—Division III.

§ 4310. (4245.) (4209.) *Punishment of accessories after the fact.*—Accessories after the fact, except where it is otherwise ordered in this Code, shall be punished by a fine not to exceed one thousand dollars, imprisonment not to exceed six months, to work in the chain-gang on the public works, or on such other works as the county authorities may employ the chain-gang, not to exceed twelve months, and any one or more of these punishments may be ordered in the discretion of the Judge: *Provided,* that nothing herein contained shall authorize the giving the control of convicts to private persons, or their employment by the county authorities in such mechanical pursuits as will bring the products of their labor into competition with the products of free labor.

ILLINOIS.

SURVEY.

UNITED STATES COAST AND GEODETIC SURVEY.

An Act relating to the operations of the United States coast and geodetic survey.

SECTION 1. *Be it enacted by the People of the State of Illinois, represented in the General Assembly,* That any person employed under and by virtue of an act of congress of the United States, approved the tenth day of February, one thousand eight hundred and seven, and of the supplements thereto, for the survey of the coasts of the United States, or, under the direction of congress, to form a geodetic connection between the Atlantic and Pacific coasts, and to furnish triangulation points for state surveys, may enter upon lands within this state, for the purpose of exploring, triangulating, leveling, surveying, and of doing any other act which may be necessary to carry out the object of said laws, and may erect any works, stations, buildings and appendages requisite for that purpose, doing no unnecessary injury thereby.

§ 2. If the parties interested cannot agree upon the amount to be paid for damages caused thereby the United States of America may proceed to condemn said land, as provided by "An act to provide for the exercise of the right of eminent domain," approved April 10, 1872, in force July 1, 1872.

§ 3. If any person shall wilfully deface, injure or remove any signal, monument, building, or other property of the United States coast and geodetic survey, constructed or used under or by virtue of the act of congress aforesaid, he shall forfeit a sum not exceeding fifty dollars for each offense, and shall be liable for damages sustained by the United States, in an action on the case in any court of competent jurisdiction.

Approved April 21, 1881.

INDIANA.

Chapter XCV.

An Act relating to the operations of the United States Coast and Geodetic Survey in the State of Indiana, and declaring an emergency.

Section 1. *Be it enacted by the General Assembly of the State of Indiana*, That any person employed under and by virtue of an act of Congress of the United States, passed the tenth day of February, one thousand eight hundred and seven, and of the supplements thereto, for the survey of the coasts of the United States, or under the direction of Congress, to form a geodetic connection between the Atlantic and Pacific coasts, and to furnish triangulation points for State surveys, may enter upon lands within this State for the purpose of exploring, triangulating, leveling, surveying and doing any other act which may be necessary to carry out the objects of said laws, and may erect any works, stations, buildings and appendages requisite for that purpose, doing no unnecessary injury thereby.

Sec. 2. If the parties interested can not agree upon the amount to be paid for damages caused thereby, either of them may petition the Circuit Court in the county in which the land is situated, which Court shall appoint a time for a hearing as soon as may be, and order at least fourteen days notice to be given to all parties interested and with or without a view of the premises, as the Court may determine, hear the parties and their witnesses and assess damages.

Sec. 3. The person so entering upon land, may tender to the party injured, amends therefor, and if, in case of appeal to the Circuit Court, the damages finally assessed do not exceed the amount tendered, the person entering shall recover costs, otherwise the prevailing party shall recover costs.

Sec. 4. The costs to be allowed in all such cases shall be the same as allowed according to rules by the Court.

Sec. 5. If any person shall wilfully deface, injure, or remove, any signal, monument, building, or other property of the United States coast survey, constructed, or used under or by virtue of the acts of Congress, aforesaid, he shall forfeit a sum not exceeding fifty dollars, for each offense, and shall be liable for damages sustained by the United States in consequence of such defacing, injury, or removal, to be recovered in an action on the case in any Court of competent jurisdiction.

Sec. 6. Whereas an emergency exists for the immediate taking effect of this act, therefore, the same shall take effect and be in force from and after its passage.

Approved April 9, 1891.

STATE OF MAINE.

Chapter 181.

An Act relating to the survey of the coast of Maine.

Be it enacted by the Senate and House of Representatives in Legislature assembled, as follows:

Section 1. Any person employed under and by virtue of an act of the congress of the United States, passed the tenth day of February, one thousand eight hundred and seven, and the supplements thereto, may enter upon lands within this state for the purpose of exploring, surveying, triangulating, leveling and doing any other act which may be necessary to effect the objects of said act, and may erect any works, stations, buildings or appendages, requisite for that purpose, doing no unnecessary injury thereby.

SECT. 2. If the parties interested cannot agree upon the amount to be paid for the damages caused by doing any of the acts aforesaid, either of them may petition the commissioners of the county in which the land entered upon is situated, to hear the parties and assess any damages, which in the opinion of the commissioners has accrued to the owner or possessor of the land so entered upon.

SECT. 3. The commissioners as soon as may be, shall hear the parties either with or without a view of the premises, as the commissioners shall determine, and before any hearing shall be had, shall order notice to be given to all persons interested, at least fourteen days before the time of hearing.

SECT. 4. The commissioners shall file in the office of the clerk of the district court for said county, a report of these doings, which report shall be conclusive upon the parties unless one of them shall file within thirty days after the term of said court, which shall be held next after said report shall be filed, a petition to the said court that a trial shall be had in the case in said court, and after notice to the opposite party a trial shall be had in said court in the same manner in which other civil cases are there tried.

SECT. 5. The person so entering upon land as aforesaid, may tender to the party injured sufficient amends therefor, and if the damages finally assessed shall not exceed the amount so tendered, the person so entering shall recover his costs, and in all other cases the prevailing party shall recover his costs.

SECT. 6. In the taxation and allowance of costs in the district court upon a trial of the case, the proceedings of the said court shall hold the same relation to the report of the commissioners, as proceedings of the same court hold to judgments of justices of the peace, in cases of appeal from said judgments, and the costs shall be taxed accordingly.

SECT. 7. If any person shall wilfully injure, deface, or remove any signal, monument, building, or any appendage thereto, used and constructed under and by virtue of the act of congress aforesaid, he shall forfeit a sum not exceeding fifty dollars for each offense, to be recovered by indictment for the use of the person prosecuting, and shall also be liable for all damages sustained by the United States of America, to be recovered in an action on the case in any court of competent jurisdiction.

SECT. 8. This act shall take effect from and after its approval by the Governor.

Approved June 16, 1846.

AMENDMENT.

CHAPTER 125.

An Act to amend the second chapter of the revised statutes, relating to the coast survey.

Be it enacted by the Senate and House of Representatives, in Legislature assembled, as follows:

SECTION 1. The second chapter of the revised statutes is hereby amended by striking out the eighth section thereof, and inserting instead the following, viz:

SECT. 8. The person so entering upon land, may tender to the party injured sufficient amends therefor, and if the damages finally assessed do not exceed the tender, judgment shall be rendered against the owner for costs. The costs recovered by the prevailing party shall be taxed as in case of appeal from the judgment of a justice of the peace.

SECT. 2. This act shall take effect when approved by the governor.

Approved January 27, 1860.

MARYLAND.

An Act concerning the Survey of the Coast of Maryland.

SECTION 1. *Be it enacted by the General Assembly of Maryland*, That it shall and may be lawful for any person or persons employed under and by virtue of an act of the Congress of the United States, passed the tenth of February in the year eighteen hundred and seven, and of the supplement thereto, at any time hereafter to enter upon lands within this State for the purpose of exploring, surveying, triangulating or levelling or doing any other matter or thing which may be necessary to affect the objects of said act, and to erect any works, stations, buildings or appendages requisite for that purpose, doing no unnecessary injury to private or other property.

SEC. 2. *And be it enacted*, That in case the person or persons employed under the act of congress aforesaid, cannot agree with the owners or possessors of the land so entered upon and used as to the amount of damage done thereto by reason of the removal of fences, cutting of trees or injury to the crop or crops growing on the same, it shall and may be lawful for the said parties or either of them to apply to the chief justice for the time being or one of the associate judges of the judicial district in which such land may be situated, who shall thereupon appoint three disinterested and judicious freeholders, residents of the same judicial district, to proceed with as much despatch as possible to the examination of the matter in question, and the faithful assessment of the damages sustained by the owners or possessors aforesaid, and the said freeholders or a majority of them, having first taken and subscribed an oath or affirmation before the chief or associate justice aforesaid or other person duly authorized to administer the same, that they will well and truly examine and assess as aforesaid, and having given five days notice to both parties of the time of their meeting, shall proceed to the spot, and then and there upon their own view and if required upon the evidence of witnesses, (to be by them sworn or affirmed and examined) shall assess the said damages, and shall afterwards make report thereof and of their proceedings in writing under their hands and seals and file the same within five days thereafter in the office of the clerk of the county in which the land aforesaid is situated, subject to an appeal by either party to the county court of the said county within ten days after filing as aforesaid, and the said report so made as aforesaid, if no appeal as aforesaid be taken, shall be held to be final and conclusive as between the said parties, and the amount so assessed and reported shall be paid to the said owners or possessors of the land so damaged within twenty days after the filing of said report, and the said chief or associate justice as aforesaid, shall have authority to tax and allow upon the filing of said report, such costs, fees and expenses to the said freeholders for the performance of their duty as he shall think equitable and just, which allowance shall be paid by the person or persons employed under the act of congress aforesaid, within the time last above limited, but if an appeal as aforesaid be taken, the case shall be set down for hearing at the first term of county court aforesaid, ensuing upon and after said appeal, and it shall be lawful for either party immediately after the entry of such appeal, to take out summons for such witnesses as may be necessary to be examined upon the hearing aforesaid, and the said court shall have power in its discretion to award costs against which ever the final judgment shall be entered, and such appeal at the option of either party may and shall be heard before and the damages assessed by a jury of twelve men to be taken from the regular pannel and elected as in other cases.

SEC. 3. *And be it enacted*, That if any person or persons shall wilfully injure or deface or remove any signal, monument or building or any appendage thereto, erected, used or constructed under and by virtue of the act of congress aforesaid, such person or persons so offending shall severally forfeit and pay the sum of fifty dollars, with costs of suit to be sued for and recovered by any person who shall first prosecute the same before any justice of the peace of the county where the person so offending may reside, and shall also be liable to pay the amount of damages thereby sustained, to be recovered with costs of suit in an action on the case, in the name and for the use of the United States of America in any court of competent jurisdiction.

Passed March 9, 1842.

COMMONWEALTH OF MASSACHUSETTS.

CHAPTER 192.

An Act relating to the Survey of the Coast of Massachusetts.

Be it enacted by the Senate and House of Representatives, in General Court assembled, and by the authority of the same, as follows:

SECT. 1. Any person employed under and by virtue of an act of the Congress of the United States, passed the tenth day of February, in the year eighteen hundred and seven, and the supplement thereto, may enter upon lands within this State, for the purpose of exploring, surveying, triangulating, levelling, or doing any other act which may be necessary to effect the objects of said act, and may erect any works, stations, buildings or appendages, requisite for that purpose, doing no unnecessary injury thereby.

SECT. 2. If the parties interested cannot agree upon the amount to be paid for the damages caused by doing any of the acts aforesaid, either of them may petition the commissioners of the county in which the land entered upon is situated, to hear the parties and assess any damages which, in the opinion of the commissioners, has accrued to the owner or possessor of the land so entered upon.

SECT. 3. The commissioners, as soon as may be, shall hear the parties either with or without a view of the premises, as the commissioners shall determine, and before any hearing shall be had, shall order notice to be given to all persons interested, at least fourteen days before the time of hearing.

SECT. 4. The commissioners shall file in the office of the clerk of the court of common pleas for said county, a report of their doings, which report shall be conclusive upon the parties, unless one of them shall file, within thirty days after the term of said court, which shall be held next after said report shall be filed, a petition to the said court, that a trial be had in the case in said court; and after notice to the opposite party, a trial shall be had in said court, in the same manner in which other civil cases are there tried.

SECT. 5. The person so entering upon land as aforesaid, may tender to the party injured, sufficient amends therefor, and if the damages finally assessed shall not exceed the amount so tendered, the person so entering shall recover his costs; and, in all other cases, the prevailing party shall recover his costs.

SECT. 6. In the taxation and allowance of costs in the court of common pleas, upon a trial of the case, the proceedings of the said court shall hold the same relation to the report of the commissioners, as proceedings of the same court hold to judgments of justices of the peace, in cases of appeal from said judgments, and the costs shall be taxed accordingly.

SECT. 7. If any person shall wilfully injure, deface or remove any signal, monument, building, or any appendage thereto erected, used or constructed under and by virtue of the act of Congress aforesaid, he shall forfeit the sum of fifty dollars for each offence, to be recovered by indictment, to the use of the person prosecuting; and shall also be liable for all damages sustained by the United States of America, to be recovered in an action on the case, in any court of competent jurisdiction.

SECT. 8. This act shall take effect from and after its passage.

Approved by the Governor, March 25, 1845.

MINNESOTA.

CHAPTER 60.

[S. F. No. 219.]

An Act to provide for surveys authorized by Congress of the United States in the State of Minnesota.

Be it enacted by the Legislature of the State of Minnesota:

SECTION 1. Any person employed in the execution of any survey authorized by the congress of the United States may enter upon lands within this state for the purpose of exploring, triangulating, leveling, surveying, and of doing any work which may be necessary to carry out the objects of then existing laws relative to surveys, and may establish permanent station marks, and erect the necessary signals and temporary observatories, doing no unnecessary injury thereby.

SEC. 2. If the parties interested cannot agree upon the amount to be paid for damages caused thereby, either of them may petition the district court in the county in which the land is situated, which court shall appoint a time for a hearing as soon as may be, and order at least twenty (20) days' notice to be given to all parties interested, and, with or without a view of the premises, as the court may determine, hear the parties and their witnesses and assess damages.

SEC. 3. The person so entering upon land may tender to the injured party damages therefor, and if, in case of petition or complaint to the court, the damages finally assessed do not exceed the amount tendered, the person entering shall recover costs; otherwise, the prevailing party shall recover costs.

SEC. 4. The costs to be allowed in all such cases shall be the same as allowed according to the rules of the court, and provisions of law relating thereto.

SEC. 5. If any person shall wilfully deface, injure or remove any signal, monument, building or other property of the U. S. coast and geodetic survey, constructed or used under or by virtue of the act of congress aforesaid, he shall forfeit a sum not exceeding fifty (50) dollars for each offense, and shall be liable for damages sustained by the United States in consequence of such defacing, injury or removal, to be recovered in a civil action in any court of competent jurisdiction.

SEC. 6. This act shall take effect from and after its passage.

Approved April 2, 1889.

MISSOURI.

GEODETIC SURVEY.

An Act to provide for the protection of citizens of the State of Missouri, the interests of the United States, and persons engaged in the triangulation of the State of Missouri, under an act of Congress to form a geodetic connection between the Atlantic and Pacific Coasts.

Be it enacted by the General Assembly of the State of Missouri, as follows:

SECTION 1. Persons employed under an Act of Congress of the United States, passed the tenth day of February, 1807, and the supplement thereto, may, upon making satisfactory amends, enter upon lands within this State for any purpose which may be necessary to effect the object of said act, and may erect works, stations, buildings or appendages for that purpose, doing no unnecessary injury.

SECTION 2. If the parties interested cannot agree upon the amount to be paid for damages caused thereby, either of them may petition the County Court in the county in which the land is situated, which court shall appoint a time for a hearing as soon as may be, and order at least fourteen days' notice to be given to all persons interested, and, with or without a view of the premises, as the Court may determine, hear the parties and their witnesses and assess damages.

SECTION 3. The person so entering upon land may tender to the party injured amends therefor, and if, in case of appeal to the county court, the damages finally assessed do not exceed the amount tendered, the person entering shall recover costs; otherwise the prevailing party shall recover costs.

SECTION 4. The costs to be allowed in all such cases shall be the same as allowed according to the rules by the circuit court.

SECTION 5. Whosoever wilfully injures, defaces or removes any signal, monument, building or appendage thereto, erected, used or constructed under said acts of Congress, shall forfeit a sum not exceeding fifty dollars for each such offence, and shall be liable for damages sustained by the United States in consequence of such injuring, defacing or removing, to be recovered in an action before the circuit court of the county in which such offense is committed.

SECTION 6. Any party to the proceeding under the provisions of this act, who may feel aggrieved by the decision of any county court, may take an appeal to the circuit court, in the same term, in the same manner, and with like effect, as in other proceedings in the county courts of this State; *Provided*, that no appeal herein provided for shall prevent the continuation of the work referred to in this act.

SECTION 7. This act to take effect and be in force from and after its passage.

Approved March 9, 1872.

STATE OF NEW HAMPSHIRE.

CHAPTER 337.

An Act relating to the survey of the coast of New Hampshire.

SECTION 1. *Be it enacted by the Senate and House of Representatives in General Court convened*, That any person employed under and by virtue of an act of the congress of the United States, passed the 10th day of February, one thousand eight hundred and seven, and the supplements thereto, may enter upon lands within this state for the purpose of exploring, surveying, triangulating, levelling, or doing any other act which may be necessary to effect the objects of said acts, and may erect any works, buildings, stations or appendages requisite for that purpose, doing no unnecessary damage thereby.

SEC. 2. If the parties interested cannot agree upon the amount to be paid for the damages caused by doing any of the acts aforesaid, either of them may petition the court of common pleas for the county in which the land entered upon is situated, for an assessment of said damages, who shall refer the same to the road commissioners for such county, who shall hear the parties and make report, as in the case of assessing damages for land taken for highways, upon which the court shall render judgment as in other cases: *Provided*, that either of the parties dissatisfied with the amount of damages so assessed may appeal to the court of common pleas next to be holden in said county and not afterwards, and thereupon said court shall assess the damages of such party by a jury.

SEC. 3. The person so entering upon land as aforesaid may tender to the party injured sufficient amends therefor; and if the damages finally assessed shall not exceed the amount so tendered, the person so entering shall recover his costs, and in all cases the party prevailing shall recover his costs.

SEC. 4. If any person shall wilfully deface, injure or remove any signals, monuments, buildings, or any appendage thereto, used or constructed under and by virtue of the acts of congress aforesaid, he shall forfeit the sum of fifty dollars for each offence, to be recovered by indictment to the use of the party prosecuting, and shall also be liable for all damages sustained by the United States of America, to be recovered in an action on the case in any court of competent jurisdiction.

SEC. 5. This act shall take effect from and after its passage.

Approved, June 30, 1846.

CHAPTER XXIX.

An Act in co-operation with the United States Coast Survey, in the triangulation of the State.

Be it enacted by the Senate and House of Representatives in General Court convened:

SECTION 1. The acting assistant, in charge of the triangulation now being carried on in this state by the United States coast survey, is hereby authorized to set such signals as may be necessary to render this survey complete, and of the greatest service and benefit for future use in the construction of a map of the state, at an expense not exceeding twenty dollars in any town or city of the state, and to draw upon the state treasurer for the sums so expended.

SECT. 2. The state treasurer is hereby directed to pay out of any money in the treasury such expenses as may be incurred in carrying out the object named in the preceding section, the bills for the same having been previously approved by the governor.

SECT. 3. This act shall take effect on its passage.

Approved July 3, 1872.

NEW JERSEY.

ACTS OF THE SIXTY-FIFTH GENERAL ASSEMBLY OF THE STATE OF NEW JERSEY.

An Act concerning the survey of the coast of New Jersey.

SECTION 1. *Be it enacted by the Council and General Assembly of this State, and it is hereby enacted by the authority of the same,* That it shall and may be lawful for any person or persons, employed under and by virtue of the act of the Congress of the United States entitled, "An act to provide for surveying the coasts of the United States," passed the tenth day of February, in the year of our Lord eighteen hundred and seven, at any time hereafter to enter upon any lands within this state, for the purpose of exploring, surveying, or levelling, or doing any other matter or thing which may be necessary to effect the objects of the said act, and to erect any works, stations, buildings, and appendages necessary for that purpose, doing no unnecessary injury to private or other property.

SECTION 2. *And be it enacted,* That in case the person or persons so employed under the said act cannot agree with the owners or possessors of the said land so entered upon, for the use of the same, or upon the amount of the damage done thereto, it shall and may be lawful for the person or persons so employed, or the owners or possessors of the said lands, to apply to one of the justices of the supreme court of this state, who shall thereupon appoint three disinterested and judicious freeholders resident in the county wherein the said lands do lie, which said freeholders, having first severally taken and subscribed an oath or affirmation, before some person duly authorized to administer the same, faithfully to examine the matter in question, and assess the damages sustained by the owners or possessors of the lands so occupied, by reason of such occupation thereof, according to the best of their skill and understanding; and the said freeholders, or a majority of them, having given to the owners or possessors of the said lands, and to the person or persons so employed, five days' notice of the time and place of meeting, shall proceed upon the testimony of witnesses, to be by them sworn or affirmed and examined, or upon their own view, or both, to assess the said damages; and shall make report thereof in writing, under their hands and seals, and file the same within five days thereafter in the office of the clerk of the county in which the said lands do lie; which report, as between the said parties, shall be final and conclusive, and the amount so assessed and reported be paid to the said owners or possessors of the said lands within ten days after the filing of the said report; and upon default of such payment, any person or persons so entering upon the said lands shall forfeit all his or their right of entry given by this Act, and shall be taken and considered as guilty of trespass, in like manner as if this act had not been passed; and the said justice of the said supreme court shall, on application of either party, tax and allow such costs, fees, and expenses, to any person or persons performing any of the duties prescribed in this act, as he shall think equitable and just, which shall be paid by the person or persons employed under the said act, within the time above limited.

SECTION 3. *And be it enacted,* That, if any person or persons shall wilfully injure, deface, or remove any signal, station, monument, or building, or any appendage thereto erected, used, or constructed under the said act of the Congress of the United States, or under this act, such person or persons so offending shall severally forfeit and pay the sum of one hundred dollars, with costs of suit, to be sued for and recovered by any person who shall first sue for the same in any court having cognizance thereof; one half thereof for the use of the said prosecutor, and the other half thereof to be paid to the overseers of the poor of the township in which the offence was committed, for the use of the poor of said township, and shall be also liable to pay the amount of damages thereby sustained, to be recovered, with costs of suit, in an action on the case, in the name and for the use of the United States of America, in any court of competent jurisdiction.

SECTION 4. *And be it enacted,* That this act shall go into effect immediately after the passage thereof.

Passed March 11, 1841.

OHIO.

An Act relating to surveys authorized by the congress of the United States, in the state of Ohio.

SECTION 1. *Be it enacted by the General Assembly of the State of Ohio,* That any person employed in the execution of any survey authorized by the congress of the United States, may enter upon lands within this state for the purpose of exploring, triangulating, leveling, surveying, and of doing any work which may be necessary to carry out the objects of existing laws, and may establish permanent stations, marks, and erect the necessary signals and temporary observatories, doing no unnecessary injury thereby.

SEC. 2. If the parties interested cannot agree upon the amount to be paid for damages caused thereby, either of them may petition the probate court in the county in which the land is situated, which court shall appoint a time for a hearing as soon as may be, and order at least fourteen days' notice to be given to all parties interested, and with or without a view of the premises, as the court may determine, hear the parties and their witnesses, and assess damages.

SEC. 3. The person so entering upon land may tender to the injured parties damages therefor, and if, in case of appeal to the probate court, the damages finally assessed do not exceed the amount tendered, the persons entering shall recover costs; otherwise the prevailing party shall recover costs.

SEC. 4. The costs to be allowed in all such cases shall be the same as allowed according to the rules of the court.

SEC. 5. If any person shall wilfully deface, injure, or remove any signal, monument, building, or other property of the United States coast survey constructed or used under or by virtue of the acts of congress aforesaid, he shall forfeit a sum not exceeding fifty dollars for each offense, and shall be liable for damages sustained by the United States in consequence of such defacing, injury, or removal, to be recovered in an action in the case in any court of competent jurisdiction.

SEC. 6. This act shall take effect from and after its passage.

JAMES E. NEAL,
Speaker of the House of Representatives.
JAMES W. OWENS,
President pro tem. of the Senate.

Passed April 14, 1879.

OREGON.

An Act relating to Surveys Authorized by the Congress of the United States in the State of Oregon.

Be it enacted by the Legislative Assembly of the State of Oregon:

SECTION 1. Any person employed in the execution of any survey authorized by the congress of the United States may enter upon lands within this State for the purpose of exploring, triangulating, leveling, surveying, and of doing any work which may be necessary to carry out the objects of existing laws, and may establish permanent station marks and erect the necessary signals and temporary observatories, doing no unnecessary injury thereby, having first paid or tendered to the owner thereof the compensation or damages hereinafter prescribed.

SECTION 2. If the parties interested cannot agree upon the amount to be paid for damages caused thereby, either of them may petition the county court in the county in which the land is situated, which court shall appoint a time for a hearing as soon as may be, and order at least fourteen days' notice to be given to all parties interested and, with or without a view of the premises, as the court may determine, hear the parties and their witnesses and assess damages.

SECTION 3. The person so entering upon land may tender to the injured party damages therefor, and if in case of appeal to the county court the damages finally assessed do not exceed the amount tendered, the person entering shall recover costs; otherwise the prevailing party shall recover costs.

SECTION 4. The costs to be allowed in all such cases shall be the same as allowed according to the rules of the court.

SECTION 5. If any person shall wilfully deface, injure or remove any signal monument, building or other property of the U. S. coast survey, constructed or used under or by virtue of the Acts of congress aforesaid, he shall forfeit a sum not exceeding fifty dollars for each offense, and shall be liable for damages sustained by the United States in consequence of such defacing, injury or removal, to be recovered in an action on the case in any court of competent jurisdiction.

SECTION 6. Inasmuch as there is no law on this subject, this Act shall be in force from and after its approval by the Governor.

Approved February 25, 1889.

SOUTH CAROLINA.

An Act relating to the Survey of the Coast of South Carolina under the authority of the United States. No. 3021.

I. *Be it enacted, by the Senate and House of Representatives, now met and sitting in General Assembly, and by the authority of the same,* That any person employed under and by virtue of an Act of the Congress of the United States, passed the tenth day of February, in the year of our Lord one thousand eight hundred and seven, and the supplements thereto, may enter upon lands within this State, for the purpose of exploring, surveying, triangulation, leveling, or doing any other act which may be necessary to effect the object of the said Act of Congress, doing no unnecessary injury thereby, so that the dwelling house, yard, garden, graveyard, or ornamental trees, of any person be not invaded without his consent: *And provided,* that before such entry, the person so employed as aforesaid, shall enter into bond, with sufficient security, in such sum as may be agreed upon by and between the said persons so employed as aforesaid, and the owner of the said lands, conditioned to pay whatever damages may be done after such entry; and in case of disagreement of the parties as to the amount of the penalty of the bond, the same may be determined by any Judge of the Court of Common Pleas of this State in chambers or open court, upon application to him, after ten days' notice to the opposite party, which application may be supported or answered by affidavit.

II. If the parties interested cannot agree upon the amount to be paid for the damages caused by doing any of the acts aforesaid, either of them may petition the Court of Common Pleas for the district in which the damage has been done for the appointment of five commissioners, a majority of whom shall value and fix the amount of the said damage, either upon view or upon competent testimony, as the said commissioners may deem best. And the said commissioners, before they act, shall severally take an oath before some magistrate, faithfully and impartially to discharge the duty assigned them, and shall return their proceedings, with a full description of the damage done, under the hands and seal of a majority of them, to the Court from which the commission issued, there to remain of record.

III. In case either party shall appeal from the valuation of the damage so fixed by the said commissioners, or a majority of them, to the Court at its next sitting thereafter, and give fifteen days' notice to the opposite party, of such appeal, the Court shall order a new valuation to be made by a jury, who shall be charged therewith in the same term or as soon as practicable, and their verdict shall be final and conclusive between the parties, unless a new trial shall be granted.

IV. If any person shall wilfully and maliciously destroy, or in any manner hurt, damage, or obstruct, or shall wilfully and maliciously cause, or aid, or assist, or counsel, or advise any other person or persons to destroy or in any manner to hurt, damage, injure or obstruct any signal, monument, building, or any appendage thereto, used or constructed under and by virtue of the Act of Congress aforesaid, he shall be liable to be indicted therefor, and on conviction shall be imprisoned not more than one month, or pay a fine not exceeding fifty dollars, or both, at the discretion of the Court before which such conviction shall take place, and shall be further liable to pay all expenses of repairing the same, and it shall not be competent for any person so offending, to defend himself, by pleading or giving in evidence that he was the owner, or agent, or servant of the owner of the land where such damage was done or caused at the time the same was caused or done.

In the Senate House, the seventeenth day of December, in the year of our Lord one thousand, eight hundred and forty-eight,[1] and in the seventy-second year of the Sovereignty and Independence of the United States of America.

R. F. W. ALLSTON,
President of the Senate pro. tem.
W. F. COLCOCK,
Speaker of the House of Representatives.

TENNESSEE.

CHAPTER XXIV.

An Act relating to the United States Coast Survey in the State of Tennessee.

SECTION 1. *Be it enacted by the General Assembly of the State of Tennessee*, That any person employed under and by virtue of an Act of Congress of the United States, passed the tenth day of February, one thousand eight hundred and seven and of the supplements thereto, or under the direction of Congress to form a Geodetic connection between the Atlantic and Pacific coasts, and to furnish triangulation points for State Surveys, may enter upon such lands within this State for the purpose of exploring, triangulating, leveling, surveying and of doing any other act which may be necessary to carry out the objects of said laws, and may erect any works, stations, buildings and appendages requisite for that purpose, doing no unnecessary injury thereby.

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[1] Seven.

SEC. 2. *Be it further enacted*, That if the person or persons, over whose lands the survey has been made, or upon whose lands monuments, stations or buildings have been erected, or who has in any way sustained damage by such survey, cannot agree with the officer of the Coast Survey as to the damage sustained, the amount of such damage may be ascertained in the manner provided by Chapter II, of Title 8, Code of Tennessee, providing for taking private property for public uses.

SEC. 3. *Be it further enacted*, That if any person shall wilfully deface, injure or remove any signals, monuments, buildings or other property of the United States Coast Survey, constructed or used under or by virtue of the Acts of Congress aforesaid, he shall forfeit a sum not exceeding fifty dollars for each offense, and shall be liable for damages sustained by the United States in consequence of each defacing, injury or removal, in an action on the case in any Court of competent jurisdiction.

SEC. 4. *Be it further enacted*, That this Act shall take effect from and after its passage, the public welfare requiring it.

Passed March 17, 1877.

HUGH M. MCADOO,
Speaker of the Senate.
EDWIN T. TALIAFERRO,
Speaker of the House of Representatives.

Approved March 21, 1877.

JAMES D. PORTER, *Governor.*

VERMONT.

No. 251. An Act relating to the Survey of Lake Champlain, and to the operations connected therewith, in the State of Vermont.

It is hereby enacted by the General Assembly of the State of Vermont:

SEC. 1. Any person employed under and by virtue of an act of the Congress of the United States, passed the tenth day of February, one thousand eight hundred and seven, and the supplements thereto, may enter upon lands within this state for the purpose of exploring, surveying, triangulating, levelling, and doing any other act which may be necessary to effect the object of said act, or of the act of Congress passed the fifteenth day of July, one thousand eight hundred and seventy, and may erect any works, stations, buildings, and appendages requisite for that purpose, doing no unnecessary injury thereby.

SEC. 2. If the parties interested cannot agree upon the amount to be paid for the damages caused by doing any of the acts aforesaid, either of them may petition a judge of the county court of the county where such land is situated for the appointment of commissioners to appraise such damages; and such judge shall give reasonable notice to the parties interested of the time when and place where he will hear the parties in such petition; and such judge may appoint three judicious and disinterested persons commissioners to ascertain the damages to such land-owner. And such commissioners shall notify the parties interested, and shall proceed to ascertain and appraise the damages to the land-owner, and shall make a report thereof to the county court then next to be held in the same county; and said court may, for sufficient reasons, accept or reject said report, in whole or in part, and may render judgment in favor of the person interested in the land for such damages as it shall appear he has sustained, and may tax costs as said court shall judge just and equitable, and shall issue execution therefor.

SEC. 3. The person so entering upon the land and doing any of the acts aforesaid, may tender to the parties injured sufficient amends therefor, and if the damages finally assessed shall not exceed the amount so tendered, the person so entering shall recover his costs.

SEC. 4. If any person shall wilfully deface, injure, or remove any signals, monuments, buildings, or any appendage thereto, used and constructed under and by virtue of the acts of Congress aforesaid, he shall forfeit a sum not exceeding fifty dollars for each offense, to be recovered by indictment for the use of the party prosecuting, and shall also be liable for all damages sustained by the United States of America, to be recovered in an action on the case in any court of competent jurisdiction.

SEC. 5. This act shall take effect from and after its approval by the governor.

Approved November 8, 1870.

VIRGINIA.

LAWS OF VIRGINIA PASSED IN 1843-4—PAGE 65. CHAPTER 85.

§ 1 of Title I, Chap. 2. refers to "Places purchased by the United States for forts and other buildings."

§ 2. Any person employed under the act of congress providing for a survey of the coasts of the *United States* approved the tenth of February, eighteen hundred and seven, or under any act supplemental thereto, may, for the purpose of exploring, surveying, triangulating or leveling, to effect the objects of the first mentioned act, enter upon any lands within this state, remove the fences, cut down trees, or do any other matter or thing necessary to effect those objects.

§ 3. The damages sustained by removal of the fences, cutting of trees, injury to the crops, or otherwise, if the same be not agreed upon, shall be ascertained either on the application of the person so employed, or of the owner or possessor of the land, as follows, that is to say: notice shall be given by one of them to the other for ten days that at a certain time and place he will apply to a justice to appoint persons to assess the damages. Upon its being shewn to the justice at such time and place that such notice has been given, the justice shall appoint three intelligent, disinterested and impartial freeholders to make such assessment. They shall be duly sworn, and after giving five days' notice of the time of their meeting, both to the applicant and the other party, shall go upon the premises, and then and there, upon their own view and the evidence of such witnesses as may be adduced, to be by them sworn and examined, shall assess the damages.

§ 4. They shall make a report of their proceedings, under their hands, and file the same within five days thereafter in the office of the clerk of the court of the county wherein the land is situated.

§ 5. Within ten days after the same be filed, either party may file with the clerk a written notice stating that he appeals from the assessment to the county court.

§ 6. If no such notice be filed, the county court shall at the first term thereafter confirm the report, make a reasonable allowance to the freeholders for their services, and order payment to be made of the amount so assessed, of such allowance, of the officers' fees and of what the witnesses may be entitled to for their attendance.

§ 7. If such notice be filed, either party may thereupon take out subpoenas for witnesses; and at the first term at which the same can conveniently be done, the case shall be heard. If either party desire it, a jury may be impanelled to assess the damages; but if this be not asked, the court shall itself hear the witnesses and make such assessment as may seem to it proper. And the court shall give such directions in regard to the costs as it may deem right.

§ 8. If any person shall wilfully injure, deface or remove any signal, monument or building or any appendage thereof, erected, used or constructed under the act of congress aforesaid, such person shall forfeit fifty dollars to any person who shall sue for the same, and shall also be liable to the *United States* for the damages thereby sustained.

Code of Virginia, published in 1849, pp. 60, 61.

WEST VIRGINIA.

CHAPTER LXXXIV.

An Act concerning the United States Coast and Geodetic Survey in this State.

Be it enacted by the Legislature of West Virginia:

1. That it shall and may be lawful for any person or persons employed under and by virtue of an act of the Congress of the United States, passed February the tenth, one thousand eight hundred and seven, and all acts supplemental thereto, at any time hereafter to enter upon lands within this state for the purpose of exploring, surveying, triangulating or leveling, or doing any other matter or thing which may be necessary to effect the objects of said act; and to erect any works, stations, buildings or appendages requisite for that purpose, doing no unnecessary injury to private or other property.

2. That in case the person or persons employed under the act of Congress aforesaid, or acts supplemental thereto, cannot agree with the owners or possessors of the land so entered upon and used, as to the amount of damages done thereto by reason of the removal of fences, cutting of trees, or injury to the crop or crops growing on the same, it shall and may be lawful for the said parties, or either of them, to apply to the circuit court of the county to have the same condemned, and such application shall be proceeded in, tried and determined, in all respects, as provided in chapter forty two of the code of West Virginia.

3. That if any person or persons shall wilfully injure or deface or remove any signal, monument, or building, or any appendage thereto, erected, used or constructed under and by virtue of the act of congress aforesaid, or any act or acts supplemental thereto, such persons so offending shall severally forfeit and pay the sum of fifty dollars with the costs of suit, to be sued for and recovered by any person who shall first prosecute the same before any justice of the peace of the county where the person so offending may reside, and shall also be liable to pay the amount of damages thereby sustained, to be recovered with costs of suit in an action on the case, in the name and for the use of the United States of America, in any court of competent jurisdiction.

Passed March 14, 1881.
Approved March 16, 1881.

[Note by the Clerk of the House of Delegates.]

The foregoing act takes effect from its passage, two-thirds of the members elected to each House, by a vote taken by yeas and nays, having so directed.

APPENDIX No. 2—1893.

ON THE RESULTING HEIGHTS FROM GEODETIC LEVELING ALONG THE TRANSCONTINENTAL LINE OF LEVELS BETWEEN ST. LOUIS AND JEFFERSON CITY, MO., EXECUTED IN THE YEARS 1882 AND 1888, BY ANDREW BRAID AND GERSHOM BRADFORD, ASSISTANTS, AND ISAAC WINSTON, SUBASSISTANT.

Discussion and report by CHARLES A. SCHOTT, Assistant and Chief of the Computing Division.

Submitted for publication August 29, 1893.

The report which I have the honor to submit herewith gives the resulting heights from geodetic leveling along the transcontinental line of levels between St. Louis and Jefferson City, Mo., executed in the years 1882 and 1888 by Assistants Andrew Braid and Gershom Bradford.

In Appendix No. 11, Report for 1880, Assistant Braid explains the method of leveling then in use, viz: Two parallel lines were run simultaneously and in the same direction, one using (say) Staff E, the other Staff F, the rods being placed at slightly different distances from the intrument; *alternate parts* of the double line were run in opposite directions. On level ground or where the slope did not interfere, the average distance between the staves was 220 metres, the instrument being as near as may be midway between them. This method was afterwards found unsatisfactory and was superseded in 1885 and 1886 by the better one of running two *independent lines*, one forward, the other backward. The latter method was employed in 1888 by Assistant Bradford, who usually took the forward and Subassistant Winston the backward measures.

Route of levels.—The line starts from the Coast and Geodetic Survey bench mark J_3, as marked by a bronze plate on the western land pier of the Great Bridge across the Mississippi at St. Louis, and identical in level with bench mark K_3, known as the St. Louis Directrix, which is used by city surveyors and United States engineers. (For description see Appendix No. 11, C. and G. Survey Report for 1882, p. 556.) The line of 1882 follows the Missouri Pacific Railroad track to New Haven and a few miles beyond to Etlah, at which point it was taken up and carried, in 1888, along the same road to Moreau Creek (secondary bench mark XXV), a few miles east of Jefferson City. Total development of line of levels from St. Louis mark K_3 to temporary mark XXV, 194·5 kilometres, or 120·86 statute miles. (See illustration No. 1.)

Observers and dates of leveling.—Assistant A. Braid carried the line from St. Louis to Etlah between October 15 and December 6, 1882, and Assistant G. Bradford, aided by Subassistant I. Winston, extended it to the vicinity of Jefferson City between April 19 and June 30, 1888.

Instruments and rods.—Geodetic spirit level No. 1 was used by Assistant Braid; it is described and illustrated in Appendix No. 11, Report for 1880. The metric rods E and F are of the pattern shown on plate 23, Coast and Geodetic Survey Report for 1879, Appendix No. 15. Assistant Bradford used almost exclusively spirit level No. 2 and No. 3 on only four days; the rods A (A_1), B, C, D were used at one time or another.[1] Both instruments are described in Appendix No. 15, Report for 1879. The instrumental constants are as follows:

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[1] A_1 and B from April 19 to May 18; then C and D until May 28; then B and D until June 7; after that date A_1 and B, but from June 23 to June 30 C and D were again used, their broken thermometers having been replaced by new ones.

Geodetic Micrometer Level No. 1.	*Geodetic Micrometer Level No. 2.*	*Geodetic Micrometer Level No. 3.*
Aperture of telescope, $3{\cdot}5^{cm}$	$4{\cdot}3^{cm}$	$4{\cdot}3^{cm}$
Focal length of telescope, $40{\cdot}7^{cm}$	$41{\cdot}0^{cm}$	$41{\cdot}0^{cm}$
Magnifying power of telescope, 26	Used with low-power eyepiece, 25·6	Used with low-power eyepiece, 25·6
Value of 1 div. of striding level, $5''{\cdot}29$	$3''{\cdot}37$	Value of striding level, $4''{\cdot}48$ " " chambered level,† $2''{\cdot}73$
Determined by A. Braid, Apr. 25, 1879.	Determined by J. B. Weir, Apr. 2, 1887.	Determined by { I. Winston, Apr. 2 and 17, 1888. Office determination, 1888. }
Collar inequality, object-end large,* $2''{\cdot}74$ Determined by A. Braid, Dec. 9, 1882.	Object-end large, $0''{\cdot}25$ and $0''{\cdot}24$. Determined by I. Winston, Apr. 18, July 10, 1888.	Object-end small, $0''{\cdot}03$ and $0''{\cdot}41$. Determ'd by I. Winston, Apr. 18, 19, July 10, 1888.
Telemeter diaphragm of 3 horizontal spider lines. Upper to middle thread, $16'\ 52''{\cdot}7$ Lower " " " $16'\ 35''{\cdot}3$	Three equidistant telemeter threads. Angular distance, $16'\ 39''{\cdot}3$	Three equidistant telemeter threads. Angular distance, $14'\ 00''{\cdot}7$
Value of 1 turn = 100 divisions of microm., $443''{\cdot}1$ and $442''{\cdot}9$.	Value adopted,‡ $257''{\cdot}5$	Value adopted,‡ $257''{\cdot}5$
determined by O. H. Tittmann, Aug. and Sept., 1877, and A. Braid, May 21, 23, 1879.	determined by McGrath, Winston } in 1887.	determined by Tittmann, Winston, McGrath } in 1879-'80-'87.
Weight of instrument and stand, 10·4 kg.	20·4 kg.	20·4 kg.
Increasing t's and d's of microm. correspond to depressing object-end of telescope.	Increasing turns depress object-end of telescope.	

Rods E and F are each 3^m long.
The graduation of these rods is of standard length at $62^\circ.1$ and $66^\circ{\cdot}1$ Fah. or $16^\circ{\cdot}7$ and $18^\circ{\cdot}9$ C.

Rods A_1, B, C, D are each 3^m long (see App. No. 9, Rep. for 1887).
The graduation of A_1 is standard at $67^\circ{\cdot}0$ Fah. or $19^\circ{\cdot}4$ C.

B " "	" 71·7 "	" 22·1 "
C " "	" 68·4 "	" 20·3 "
D " "	" 58·1 "	" 14·4 "

Coefficient of expansion of brass for { Fahrenheit scale, 0·000010; Centigrade " 0·000018 }

Index corr. of E (Oct., 1883), $64{\cdot}0^{mm}$
F ("), $61{\cdot}0^{mm}$

Index corr. of A_1 (in Apr, and July, 1888), $77{\cdot}2^{mm}$
B (" " " " "), $77{\cdot}0^{mm}$
C (" May " July, "), $60{\cdot}4^{mm}$
D (" " " " "), $60{\cdot}0^{mm}$

* It was but $1''.01$ in the period April, 1881, to June, 1882, as computed by H. Farquhar from records by A. Braid.
† Used when determining collar inequality in July, 1888, the tube of striding level broken. ‡ For particulars, see Appendix No. 14, Report for 1887, p. 276.

N. B.—This difference between the terminal point of the rods and the zero of the "brass scale" does not ordinarily come into consideration. None of these rods have undergone any change since their construction except that due to an acciden. to rod A in August, 1881, and that due to wear of supporting surface. Comparisons for lengths of rods C and D were made by J. J. Clark, September 21, 1880, and August 30, 1882, and computed by H. Farquhar.

Method of observing.—As already stated, the method employed for the part of the line between St. Louis and Etlah was that of running simultaneously two parallel lines, but this was changed for the remainder of the line to the better practice of running two independent lines—one forward, the other backward. In the latter work, before taking the micrometer reading for "horizon," the bubble of the level was always brought to the center of the scale.

Computations.—The field computation was made by the observer, and the office computation of the 1882 work by Subassistant J. F. Pratt and Mr. H. Farquhar, with results drawn up by Mr. A. S. Christie.[1] The observations of 1888 were reduced by the observers, and the office computation was made by Mr. F. M. Little in November, 1888, and completed by Subassistant J. Nelson, in April, 1893. The usual corrections were made for micrometric difference when pointing to horizon and to target of staff; for effect of collar inequality; for curvature and refraction; for length of staff at various temperatures, and for index error where necessary.

Results.—They are given here in the usual tabular form, but instead of starting from the sea level the results are given differentially with respect to the St. Louis bench mark K_3. Its height above the ocean is at present not known with precision, but the value given in the Annual Report for 1882, page 554, appears too high, to judge from the two independent lines of levels now extending to the Gulf. Until the fieldwork is completed, and if temporarily approximate results of the bench marks west of St. Louis be required, we may take for the height of this mark 126 metres, or 413·4 feet, nearly.

[1] Results reported by me, August 25, 1883.

Results from geodetic spirit leveling in Missouri—First part from St. Louis to New Haven (and Etlah), 1882.

Date, 1882.		Bench mark.		Distance between successive bench marks.	Distance from initial mark K_3.	Difference of height between bench marks.			Discrepancy.		Height of mark above St. Louis bench mark K_3.
		From—	To—			E or first line.	F or second line.	Mean.	Partial E—F.	Total accumulated.	
				km.	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
			K_3	-----	0·000	-------	-------	-------	---	0·0	0·0000
Oct.	15	K_3	181	0·372	0·372	+13·5064	+13·5061	+13·5062	+0·3	+ 0·3	+13·5062
	22	181	187	2·059	2·431	— 3·1293	— 3·1224	— 3·1259	—6·9	— 6·6	10·3803
	22	187	188	0·885	3·316	+18·9172	+18·9179	+18·9175	—0·7	— 7·3	29·2978
	25	188	189	1·339	4·655	—15·7988	—15·7945	—15·7966	—4·3	—11·6	13·5012
	25	189	190	2·562	7·217	+ 9·7236	+ 9·7250	+ 9·7243	—1·4	—13·0	23·2255
Nov.	2	190	191	2·849	10·066	—10·9293	—10·9311	—10·9302	+1·8	—11·2	12·2953
	2	191	192	3·954	14·020	+ 7·1345	+ 7·1342	+ 7·1344	+0·3	—10·9	19·4297
	3	192	193	0·952	14·972	— 5·7505	— 5·7473	— 5·7489	—3·2	—14·1	13·6808
	3	193	194	2·101	17·073	+17·1256	+17·1265	+17·1260	—0·9	—15·0	30·8068
	3	194	195	2·012	19·085	+15·2657	+15·2582	+15·2620	+7·5	— 7·5	46·0688
	3	195	196	1·153	20·238	+10·8348	+10·8342	+10·8345	+0·6	— 6·9	56·9033
	4	196	197	2·336	22·574	+10·5820	+10·5790	+10·5805	+3·0	— 3·9	67·4838
	4	197	198	0·928	23·502	— 0·4905	— 0·4892	— 0·4898	—1·3	— 5·2	66·9940
	9	198	204	2·196	25·698	—17·8612	—17·8662	—17·8637	+5·0	— 0·2	49·1303
	9	204	203	3·634	29·332	—29·1377	—29·1404	—29·1390	+2·7	+ 2·5	19·9913
	7	203	202	1·102	30·434	— 9·1403	— 9·1363	— 9·1383	—4·0	— 1·5	10·8530
	7	202	201	1·680	32·114	— 6·8468	— 6·8444	— 6·8456	—2·4	— 3·9	4·0074
	7	201	200	2·134	34·248	+ 0·1142	+ 0·1181	+ 0·1162	—3·9	— 7·8	4·1236
	7	200	199	2·217	36·465	— 0·1627	— 0·1669	— 0·1648	+4·2	— 3·6	3·9588
	13	199	205	1·907	38·372	+ 1·1323	+ 1·1364	+ 1·1344	—4·1	— 7·7	5·0932
	13	205	206	2·063	40·435	+ 3·1798	+ 3·1756	+ 3·1777	+4·2	— 3·5	8·2709
	13	206	X	1·979	42·414	— 2·4040	— 2·3983	— 2·4012	—5·7	— 9·2	5·8697
	13	X	207	2·141	44·555	+ 1·2115	+ 1·2148	+ 1·2132	—3·3	—12·5	7·0829
	14	207	208	1·827	46·382	+ 0·9690	+ 0·9705	+ 0·9698	—1·5	—14·0	8·0527

Results from geodetic spirit leveling in Missouri—First part from St. Louis to New Haven (and Etlah), 1882—Continued.

Date, 1882.		Bench mark. From—	Bench mark. To—	Distance between successive bench marks.	Distance from initial mark K_3.	Difference of height between bench marks. E or first line.	Difference of height between bench marks. F or second line.	Difference of height between bench marks. Mean.	Discrepancy. Partial E—F.	Discrepancy. Total accumulated.	Height of mark above St. Louis bench mark K_3.
				km.	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
Nov.	14	208	209	1·830	48·212	+ 3·9695	+ 3·9629	+ 3·9662	+6·6	− 7·4	12·0189
	14	209	210	1·712	49·924	+ 3·4154	+ 3·4143	+ 3·4148	+1·1	− 6·3	15·4337
	18	210	215	2·177	52·101	+ 5·4122	+ 5·4089	+ 5·4105	+3·3	− 3·0	20·8442
	18	215	216	2·325	54·426	+ 6·0574	+ 6·0593	+ 6·0583	−1·9	− 4·9	26·9025
	18	216	XI	0·925	55·351	− 6·7483	− 6·7480	− 6·7482	−0·3	− 5·2	20·1543
	18	XI	217	2·655	58·006	− 4·7312	− 4·7287	− 4·7299	−2·5	− 7·7	15·4244
	18	217	218	0·640	58·646	− 0·8689	− 0·8680	− 0·8685	−0·9	− 8·6	14·5559
	21	218	224	2·428	61·074	+ 1·1732	+ 1·1719	+ 1·1725	+1·3	− 7·3	15·7284
	21	224	223	1·573	62·647	+11·7666	+11·7637	+11·7652	+2·9	− 4·4	27·4936
	21	223	222	2·045	64·692	+14·6175	+14·6112	+14.6144	+6·3	+ 1·9	42·1080
	20	222	221	0·217	64·909	+ 1·5192	+ 1·5185	+ 1·5188	+0·7	+ 2·6	43·6268
	20	221	220	3·051	67·960	+23·9667	+23·9673	+23·9670	−0·6	+ 2·0	67·5938
	20	220	219	1·890	69·850	−14·3620	−14·3641	−14·3631	+2·1	+ 4·1	53·2307
	20	219	214	1·928	71·778	−15·4343	−15·4353	−15·4348	+1·0	+ 5·1	37·7959
	16	214	213	1·569	73·347	−12·9388	−12·9412	−12·9400	−2·4	+ 7·5	24·8559
	15	213	212	2·476	75·823	− 3·2351	− 3·2341	− 3·2346	−1·0	+ 6·5	21·6213
	15	212	211	3·309	79·132	− 0·4045	− 0·3990	− 0·4017	−5·5	+ 1·0	21·2196
	22	211	225	2·329	81·461	+ 1·5481	+ 1·5503	+ 1·5492	−2·2	− 1·2	22·7688
	22	225	226	2·450	83·911	− 0·5850	− 0·5792	− 0·5821	−5·8	− 7·0	22·1867
	22	226	227	1·943	85·854	+ 0·8756	+ 0·8736	+ 0·8746	+2·0	− 5·0	23·0613
	23	227	XII	0·861	86·715	+ 0·2358	+ 0·2395	+ 0·2377	−3·7	− 8·7	23·2990
	23	XII	228	2·026	88·741	− 0·1652	− 0·1704	− 0·1678	+5·2	− 3·5	23·1312
	23	228	229	1·305	90·046	+ 1·1348	+ 1·1383	+ 1·1365	−3·5	− 7·0	24·2677
	25	229	L_3	0·451	90·497	+15·9322	+15·9332	+15·9327	−1·0	− 8·0	40·2004

	29	229	237	1·265	91·311	− 1·2030	− 1·2052	− 1·2041	+2·2	− 4·8	23·0636
	29	237	236	2·416	93·727	+ 1·4947	+ 1·4987	+ 1·4967	−4·0	− 8·8	24·5603
	29	236	235	2·057	95·784	− 0·4924	− 0·4891	− 0·4908	−3·3	−12·1	24·0695
	28	235	234	1·869	97·653	+ 1·4992	+ 1·4960	+ 1·4976	+3·2	− 8·9	25·5671
	28	234	233	2·079	99·732	+ 1·0356	+ 1·0373	+ 1·0365	−1·7	−10·6	26·6036
	28	233	232	2·025	101·757	− 0·4750	− 0·4763	− 0·4756	+1·3	− 9·3	26·1280
	27	232	231	2·485	104·242	+ 0·9905	+ 0.9834	+ 0·9869	+7·1	− 2·2	27·1149
	27	231	230	3·095	107·337	+ 0·4680	+ 0·4674	+ 0·4677	+0·6	− 1·6	27·5826
Dec.	4	230	238	2·040	109·377	− 0·3351	− 0·3330	− 0·3341	−2·1	− 3·7	27·2485
	4	238	M_3	1·103	110·480	+ 1·9052	+ 1·9015	+ 1·9034	+3·7	0·0	29·1519
	4	M_3	XIII	0·129	110·609	− 0·8890	− 0·8887	− 0·8889	−0·3	− 0·3	28·2630
	4	XIII	239	1·080	111·689	+ 0·6239	+ 0·6260	+ 0·6250	−2·1	− 2·4	28·8880
	6	239	240	2·126	113·815	+ 0·1278	+ 0·1268	+ 0·1273	+1·0	− 1·4	29·0153
	6	240	XIV	2·347	116·162	+ 0·0659	+ 0·0674	+ 0·0666	−1·5	− 2·9	29·0819

Description of primary and secondary bench marks between St. Louis, Mo., and Etlah, Mo.

K_3.—This mark is known at St. Louis as the "City Directrix." It has been in use for many years in connection with the levels of the city. It was originally the top surface of a pedestal of a monument which stood on Front street, near Market. The monument shaft was destroyed at the time of the great fire in that locality, but the pedestal remained. It is now (1882) level with the curbstone and forms a part thereof. A T mark has since been cut to indicate the point used for a bench mark. The large bronze-plate bench marks I_3, on south face of the eastern land pier of the Great Bridge at East St. Louis, Ill., and J_3, on the western land pier of the bridge, were placed, as near as possible, on the same level with the City Directrix mark K_3. (See C. and G. Survey Report for 1882, p. 554; also report of the Miss. River Commission for 1883.)

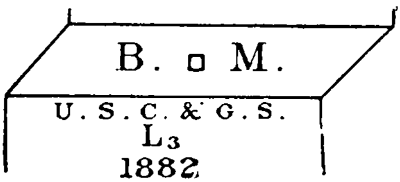


FIG. 1.

Secondary B. M. X.—Is cut on the upper surface of the middle top stone of the south side of the east abutment of railroad bridge (Missouri Pacific) at St. Paul, Mo. It is marked thus: B. □ M.

Secondary B. M. XI.—Is cut on top of the south side of the west abutment of the Missouri Pacific Railroad bridge at Allenton, Mo. It is marked thus: B □ M.

U. S. C. & G. S.
B. ✚ M.
M_3
1882

FIG. 2.

Secondary B. M. XII.—Is a cross on the head of a copper bolt inserted in the face of a perpendicular rocky bluff about three-eighths of a mile west of South Point Station (Mo. Pac. R. R.). The bolt was inserted by the United States engineers at work on improvement of Missouri River.

Primary B. M. L_3.—Is cut on the horizontal surface of the stone ledge under the windows of the east face of the German Catholic church at Washington, Mo. It is marked as shown in fig. 1.

Primary B. M. M_3.—Is cut on the northeast corner of the building occupied by the "New Haven Merchandise Company," at New Haven, Mo. The building stands a short distance south of the Missouri Pacific Railroad track and west of the railroad station. The B. M. is marked as shown in fig. 2.

Secondary B. M. XIII.—Is cut on the north side of the east abutment of railroad culvert (Mo. Pac. R. R.) about one-eighth mile west of New Haven, Mo. It is marked thus: B. ▫ M.

Secondary B. M. XIV.—Is cut on the top surface of the north end of the east abutment of a small railroad bridge or culvert (Mo. Pac. R. R.) about one-fourth mile east of Etlah Station, Mo. It is

U. S.
marked thus: B. ▫ M.
XIV.

(See route diagram.)

Results from geodetic spirit leveling in Missouri—Second part from New Haven to vicinity of Jefferson City, 1888.

Date, 1888.		Bench mark. From—	Bench mark. To—	Distance between successive bench marks.	Distance from initial mark K_3.	Difference of height between bench marks. Forward measure.	Backward measure.	Mean.	Discrepancy. Partial F—B.	Total accumulated.	Height of mark above St. Louis mark K_3.
				km.	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
1882.			M_3	-----	110·480				---	---	+29·1519
Dec. 4	Dec. 4	M_3	XIII	0·129	110·609	—0·8890	—0·8887	—0·8889	---	---	28·2630
4	4	XIII	239	1·080	111·689	+0·6239	+0·6260	+0·6250	---	---	28·8880
6	6	239	240	2·126	113·815	+0·1278	+0·1268	+0·1273	---	---	29·0153
6	6	240	XIV	2·347	116·162	+0·0659	+0·0674	+0·0666	---	---	29·0819
1888.			M_3	-----	110·480				---	0·0	+29·1519
Apr. 19	May 2	M_3	1	0·521	111·001	—0·6548	—0·6541	—0·6544	—0·7	— 0·7	28·4975
19	1	1	2	0·956	111·957	+0·3198	+0·3169	+0·3184	+2·9	+ 2·2	28·8159
19	1	2	3	0·831	112·788	+0·2576	+0·2548	+0·2562	+2·8	+ 5·0	29·0721
20	1	3	4	0·505	113·293	—0·0502	—0·0528	—0·0515	+2·6	+ 7·6	29·0206
20	1	4	5	0·724	114·017	—0·8910	—0·8885	—0·8898	—2·5	+ 5·1	28·1308
20	1	5	6	0·830	114·847	+0·4366	+0·4375	+0·4370	—0·9	+ 4·2	28·5678
Apr. 21	Apr. 30	6	7	0·759	115·609	+0·0287	+0·0227	+0·0268	+2·6	+ 6·8	28·5946
May 2	May 2			0·766		+0·0275	+0·0283				
Apr. 21	Apr. 30	7	XIV	0·490	116·099	+0·4901	+0·4891	+0.4896	+1·0	+ 7·8	29·0842
	Mean		XIV	-----	116·131					+ 7·8	29·0830
Apr. 21	Apr. 30	XIV	8	0·744	116·875	—0·0784	—0·0746	--0·0765	—3·8	+ 4·0	29·0065
23	30	8	9	0·662	117·537	—0·1600	—0·1607	—0·1604	+0·7	+ 4·7	28·8461
23	30	9	10	0·858	118·395	+0·1742	+0·1779	+0·1760	—3·7	+ 1·0	29·0221
24	28	10	11	0·782	119·177	—0·6261	—0·6286	—0·6274	+2 5	+ 3·5	28·3947
24	28	11	12	0·911	120·088	+0·4952	+0·4950	+0·4951	+0·2	+ 3·7	28·8898
25	28	12	13	0·818	120·906	+0·4874	+0·4837	+0·4856	+3·7	+ 7·4	29·3754
25	27	13	14	0·936	121·842	—0·3931	—0·3926	—0·3928	—0·5	+ 6·9	28·9826
26	27	14	15	0·782	122·624	—0·0657	—0·0664	—0·0660	+0·7	+ 7·6	28·9166

May 26 3 10	May 27 9 10	15	16	0·834 0·831 0·835	123 457	—0·1302 —0·1293 —0·1232	—0·1247 —0·1217 —0·1229	—0·1254	—4·5	+ 3·1	28·7912
Apr. 26	Apr. 26	16	XV	0·114	123·571	+1·0344	+1·0339	+1·0342	+0·5	+ 3·6	29·8254
May 4	May 9	16	17	0·927	124·384	+0·3828	+0·3825	+0·3826	+0·3	+ 3·4	29·1738
4	9	17	18	0·985	125·369	+0·3205	+0·3186	+0·3196	+1·9	+ 5·3	29·4934
4	9	18	19	0·982	126·351	—0·2762	—0·2740	—0·2751	—2·2	+ 3·1	29·2183
5 10	9 10	19	20	0·693 0·692	127·043	+0·7521 +0·7564	+0·7561 +0·7549	+0·7548	—1·3	+ 1·8	29·9731
5	9	20	21	0·805	127·848	—0·1655	—0·1630	—0·1642	—2·5	— 0·7	29·8089
5	8	21	22	1·028	128·876	+0·4504	+0·4515	+0·4510	—1·1	— 1·8	30·2599
5	8	22	23	1·106	129·982	—0·0178	—0·0196	—0·0187	+1·8	0·0	30·2412
7	8	23	24	1·038	131·020	+0·1218	+0·1210	+0·1214	+0·8	+ 0·8	30·3626
7	8	24	25	1·062	132·082	+0·4491	+0·4536	+0·4513	—4·5	— 3·7	30·8139
7	5	25	26	0·852	132·934	—0·0794	—0·0802	—0·0798	+0·8	— 2·9	30·7341
7	7	26	N_3	0·052	132·986	+1·4628	+1·4632	+1·4630	—0·4	— 3·3	32·1971
7	12	26	27	0·677	133·611	—0·1193	—0·1209	—0·1201	+1·6	— 1·3	30·6140
11	12	27	28	0·968	134·579	+0·9055	+0·9081	+0·9068	—2·6	— 3·9	31·5208
11	12	28	29	0·896	135·475	+0·3169	+0·3180	+0·3174	—1·1	— 5·0	31·8382
11	12	29	30	0·934	136·409	+0·1725	+0·1769	+0·1747	—4·4	— 9·4	32·0129
11 14	12 14	30	31	1·066 1·066	137·475	+0·5562 +0·5644	+0·5663 +0·5644	+0·5628	—5·1	—14·5	32·5757
11, 14	15	31	XVI	1·052	138·527	+0·6489	+0·6450	+0·6470	+3·9	—10·6	33·2227
14	15	XVI	32	0·998	139·525	+0·3731	+0·3766	+0·3748	—3·5	—14·1	33·5975
14 June 8	25 June 8	32	33	0·944 0·944	140·469	—0·2549 —0·2593	—0·2633 —0·2581	—0·2589	+3·6	—10·5	33·3386
May 16 25	May 23 25	33	34	1·184 1·184	141·653	+0·8881 +0·8815	+0·8744 +0·8849	+0·8822	+5·2	— 5·3	34·2208
16	23	34	35	1·210	142·863	—0·6930	—0·6888	—0·6909	—4·2	— 9·5	33·5299
16 25	23 25	35	36	0·995 1·004	143·863	+0·0598 +0·0670	+0·0693 +0·0661	+0·0656	—4·3	—13·8	33·5955
16	23	36	XVII	0·656	144·519	—0·1385	—0·1373	—0·1379	—1·2	—15·0	33·4576

Results from geodetic spirit leveling in Missouri—Second part from New Haven to vicinity of Jefferson City, 1888—Continued.

Date, 1888.		Bench mark.		Distance between successive bench marks.	Distance from initial mark K_3.	Difference of height between bench marks.			Discrepancy.		Height of mark above St. Louis mark K_2.
		From—	To—			Forward measure.	Backward measure.	Mean.	Partial F—B.	Total accumulated.	
1888.				*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
May 16, 18	May 22, 23	XVII	XVIII	0·583	145·102	+0·9390	+0·9389	+0·9390	+0·1	−14·9	34·3966
18	22, 23	XVIII	37	1·144	146·246	−0·8794	−0·8799	−0·8796	+0·5	−14·4	33·5170
18 31	22 June 2	37	38	1·492 1·495	147·740	−0·1412 −0·1321	−0·1331 −0·1383	−0·1362	−0·9	−15·3	33·3808
May 18 31 June 4	May 22 May 31, June 2 June 4	38	39	1·566 1·564 1·564	149·305	+0·5681 +0·5775 +0·5824	+0·5816 +0·5786 +0·5763	+0·5774	−2·8	−18·1	33·9582
May 18	May 21	39	40	1·553	150·858	+0·2375	+0·2436	+0·2406	−6·1	−24·2	34·1988
18, 19	21	40	XIX	1·278	152·136	+0·3278	+0·3275	+0·3276	+0·3	−23·9	34·5264
19	21	XIX	41	0·870	153·006	−0·2010	−0·1991	−0·2000	−1·9	−25·8	34·3264
19 24 June 4	21 June 1 4	41	42	0·914 0·914 0·911	153·919	−0·2624 −0·2595 −0·2547	−0·2522 −0·2565 −0·2534	−0·2564	−4·9	−30·7	34·0700
May 19	May 21	42	43	0·974	154·893	+1·0060	+1·0090	+1·0075	−3·0	−33·7	35·0775
19, 24	June 1	43	44	1·211	156·104	+1·1416	+1·1437	+1·1426	−2·1	−35·8	36·2201
May 24	1	44	45	1·364	157·468	+0·0470	+0·0455	+0·0462	+1·5	−34·3	36·2663
24	May 29 June 1	45	46	1·082	158·550	−1·0914	−1·0948	−1·0931	+3·4	−30·9	35·1732
26 June 1	May 29 June 1	46	47	0·960 0·962	159·511	+0·4225 +0·4287	+0·4284 +0·4233	+0·4257	−0·2	−31·1	35·5989
May 26	May 29	47	48	0·848	160·359	−0·2077	−0·2035	−0·2056	−4·2	−35·3	35·3933
26	28	48	49	0·900	161·259	+1·4371	+1·4345	+1·4358	+2·6	−32·7	36·8291
26	28	49	50	1·039	162·298	−0·3258	−0·3272	−0·3265	+1·4	−31·3	36·5026
26	28	50	51	1·061	163·359	−0·0518	−0·0538	−0·0528	+2·0	−29·3	36·4498

June 5, 28	28	51	52	0·726 0·812	164·128	+0·5028 +0·5046	+0·5017	+0·5027	+2·0	—27·3	36·9525
May 28	May 28	52	O_3	0·092	164·220	+0·2664	+0·2661	+0·2662	+0·3	—27·0	37·2187
June 5	June 11	52	53	0·820	164·948	—0·0465	—0·0464	—0·0464	—0·1	—27·4	36·9061
5	11	53	54	0·832	165·781	—0·3289	—0·3239	—0·3283	—2·6	—30·0	36·5778
22	22			0·834		—0·3304	—0·3301				
5	11	54	55	0·928	166·709	—0·2724	—0·2698	—0·2711	—2·6	—32·6	36·3067
5	11	55	56	0·986	167·695	+0·8934	+0·8934	+0·8934	0·0	—32·6	37·2001
6	11	56	57	0·905	168·600	—0·3543	—0·3498	—0·3520	—4·5	—37·1	36·8481
6	11	57	58	0·820	169·420	+0·7511	+0·7553	+0·7532	—4·2	—41·3	37·6013
6	9	58	59	0·817	170·237	—0·1016	—0·1022	—0·1019	+0·6	—40·7	37·4994
6	9	59	60	0·942	171·179	+0·0600	+0·0610	+0·0605	—1·0	—41·7	37·5599
7	9	60	61	0·870	172·049	—0·0732	—0·0748	—0·0740	+1·6	—40·1	37·4859
7	9	61	XX	0·314	172·363	+0·4994	+0·4985	+0·4990	+0·9	—39·2	37·9849
7	9	XX	62	0·739	173·102	—0·0079	—0·0080	—0·0080	+0·1	—39·1	37·9769
7	9	62	XXI	1·223	174·325	—0·1606	—0·1593	—0·1600	—1·3	—40·4	37·8169
7, 12	18	XXI	63	1·002	175·327	+0·8696	+0·8746	+0·8721	—5·0	—45·4	38·6890
12	18	63	64	1·084	176·411	+0·4163	+0·4159	+0·4161	+0·4	—45·0	39·1051
12	18	64	65	0·896	177·307	—0·0178	—0·0220	—0·0199	+4·2	—40·8	39·0852
12	18	65	XXII	0·786	178·093	—0·7713	—0·7691	—0·7702	—2·2	—43·0	38·3150
12	18	XXII	66	0·844	178·937	+1·7270	+1·7287	+1·7278	—1·7	—44·7	40·0428
12, 13	16, 18	66	67	1·039	179·976	+0·4371	+0·4389	+0·4380	—1·8	—46·5	40·4808
13	16	67	68	1·078	181·054	—0·8449	—0·8461	—0·8455	+1·2	—45·3	39·6353
13	16	68	69	0·806	181·860	—0·7478	—0·7454	—0·7466	—2·4	—47·7	38·8887
13	16	69	70	0·842	182·702	+0·2703	+0·2710	+0·2706	—0·7	—48·4	39·1593
13	16			0·836		+0·3512	+0·3590				
21	21	70	71	0·838	183·539	+0·3518	+0·3556	+0·3528	—3·9	—52·3	39·5121
21	21			0·837		+0·3493	+0·3495				
14	16	71	72	1·192	184·731	+0·1626	+0·1612	+0·1619	+1·4	—50·9	39·6740
14	15	72	P_3	0·097	184·828	+2·3964	+2·3966	+2·3965	—0·2	—51·1	42·0705
14	15	P_3	73	0·656	185·484	—2·5319	—2·5302	—2·5310	—1·7	—52·8	39·5395
14	15	73	74	0·778	186·262	+0·2088	+0·2078	+0·2083	+1·0	—51·8	39·7478
14	15	74	75	0·838	187·100	+0·1475	+0·1513	+0·1494	—3·8	—55·6	39·8972

Results from geodetic spirit leveling in Missouri—Second part from New Haven to vicinity of Jefferson City, 1888—Continued.

Date, 1888.		Bench mark.		Distance between successive bench marks.	Distance from initial mark K_3.	Difference of height between bench marks.			Discrepancy.		Height of mark above St. Louis mark K_3.
		From—	To—			Forward measure.	Backward measure.	Mean.	Partial F − B.	Total accumulated.	
1888.				*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
June 14, 19	June 20	75	76	0·911	188·011	+0·1597	+0·1572	+0·1584	+2·5	−53·1	40·0556
19	20	76	77	0·906	188·917	+0·5809	+0·5782	+0·5796	+2·7	−50·4	40·6352
19	20	77	78	0·832	189·749	+0·5384	+0·5410	+0·5397	−2·6	−53·0	41·1749
19	20	78	79	0·789	190·538	−0·0681	−0·0668	−0·0674	−1·3	−54·3	41·1075
25	20, 23	79	XXIII	0·640	191·178	−0·3141	−0·3080	−0·3094	−2·3	−56·6	40·7981
26	26			0·638		−0·3101	−0·3081				
28	28			0·643		−0·3072	−0·3084				
25	23	XXIII	XXIV	0·421	191·599	−0·3978	−0·3964	−0·3971	−1·4	−58·0	40·4010
25	20, 23	79	80	0·534	191·072	−4·8475	−4·8501	−4·8488	+2·6	−51·7	36·2587
26	26	80	81	0·381	191·453	−0·2260	−0·2348	−0·2292	+8·4	−43·3	36·0295
26	26			0·381		−0·2239	−0·2320				
25	23	81	XXIV	0·301	191·754	+4·3750	+4·3750	+4·3750	0·0	−43·3	40·4045
	Mean		XXIV		191·676					−50·6	40·4028
27	28	XXIV	82	0·938	192·619	+4·0272	+4·0195	+4·0222	+2·1	−48·5	44·4250
30	30			0·948		+4·0194	+4·0230				
27	28	82	83	0·882	193·501	−0·0624	−0·0602	−0·0613	−2·2	−50·7	44·3637
27	30	83	84	1·214	194·715	−1·5940	−1·5944	−1·5942	+0·4	−50·3	42·7695
29	30	84	XXV	0·191	194·906	+0·8160	+0·8143	+0·8152	+1·7	−48·6	43·5847

Description of primary and secondary bench marks between Etlah, Mo., and vicinity of Jefferson City, Mo.

Secondary B. M. XV.—Berger, Franklin County, Mo. A limestone post 1·7 feet long, rough at the bottom and dressed to 6 by 6 inches at the top to a depth of 6 inches, was used as this B. M. It is buried 1·5 feet in the ground. It is situated on the west side of Mrs. M. M. Schaub's house, close to the wall of the foundation and 3·3 feet from the southwest corner of the house. This house is quite close to the track of the Missouri Pacific Railroad (50 feet), just north of the point where the main street of the village crosses it. Both corners of the stone on the south side are chipped off, and the stone appears to be rather soft.

Primary B. M. N_3.—Hermann, Gasconade County, Mo. A cross cut on the northeast corner (east side) of the stone foundation of the "White House" hotel, A. C. Leisner, proprietor, at Hermann, Gasconade County, Mo., and the center of this cross was used as the bench mark. The cross is 1·24 feet south of the corner and 1·26 feet above the surface of the ground. This bench was marked as follows:

U. S.
\+
B. M.
N_3.
1888.

Secondary B. M. XVI.—Gasconade County, Mo. This bench is the bottom of a square hole cut in the top of the stone abutment to the iron bridge on the Missouri Pacific Railroad across Coles Creek. The bridge rests on a portion of the abutment which is about 4 feet lower than that portion where the bench is cut. Near the corner of the stone a cross is cut with the letters B. + M.

B. M. XVI is 0·750 metre east of this, on the same stone. It is on the east side of the creek and is north of the railroad. Mr. Eaffner lives near the creek, on the west side. The bench is marked as follows:

U. S.
□
B. M.

Secondary B. M. XVII.—Gasconade County, Mo. This bench is the bottom of a square hole cut in the top of the middle stone pier of the Missouri Pacific Railroad bridge over the Gasconade River. It is south of the track and marked as follows:

U. S.
□
B. M.

Secondary B. M. XVIII.—Gasconade County, Mo. This bench is the bottom of a square hole cut in the center of the top of a limestone post which was set in the ground in the yard of Mr. J. Wolter's dwelling and storehouse at Gasconade Station. The post is about 0·7 metre from the southeast corner of the house, which is situated about 100 feet south of the Missouri Pacific Railroad and about

200 feet west of the railroad station house. The post is dressed to 6 by 6 inches at the top and is 18 inches long, buried 15 inches in the ground. It is marked on top:

U. S.
□
B. M.

Secondary B. M. XIX.—Gasconade County, Mo. This bench is the bottom of a small square hole cut in the top of the stone foundation to H. Binkholter & Co.'s grain elevator at Morrison Station, Missouri Pacific Railroad. The building is about 6 inches inside the outer face of the foundation, and the bench is on this ledge, near the northeast corner of the building, which is situated quite near the track, on the south side. The stone is soft and the letters are roughly cut; the bottom of the hole is smooth. This bench is marked as follows:

U. S.
□
B. M.

Primary B. M. O_3.—Chamois, Osage County, Mo. This bench is the bottom of a square hole cut in the top of the stone across the bottom of the side door to the saloon on the northwest corner of Main and Pacific streets. This door is on the Pacific street side and faces the railroad. The building is a two-story brick, with imitation stone foundation. The bench is near the west side of the door and is marked as follows:

U. S.
□
B. M.
O_3.
1888.

Secondary B. M. XX.—St. Aubert, Osage County, Mo. This bench is the bottom of a square hole cut in the top of the stone abutment of the Missouri Pacific Railroad bridge across ——— Creek, opposite the village of St. Aubert. The bench is on the east abutment, and is south of the track. The letters are very roughly cut. The bridge is about one-fourth mile west of the depot. The bench is marked as follows:

U. S.
□
B. M.

Secondary B. M. XXI.—Near St. Aubert, Osage County, Mo. Is on the north side of east abutment of the first trestle west of mile post 106 on the Missouri Pacific Railroad, and is about 1 mile west of St. Aubert Station and between it and Isbell Station. The B. M. is a spot surrounded by a square trench (about an inch square), with the letters

U S
□
B M

rudely and slightly cut.

Secondary B. M. XXII.—Isbell, Osage County, Mo. This bench is the bottom of a square hole cut in the top of the stone abutment of the Missouri Pacific Railroad bridge over Loose Creek. It is on the east abutment, and is north of the track. It is situated on the step in the abutment on which the bridge rests, and is about 5 feet below the track. Near the corner of the stone a cross is cut, with the letters B. M., thus: B. + M. This bench is about one-half a mile east of Isbell Station. Marked as follows:

U S
□
B M

The stone is a soft sandstone and the letters are roughly cut.

Primary B. M. P_3.—At Bonnot's Mill, Osage County, Mo. It is on the northwest corner of a brick building used as a store and owned by Mrs. L. Bonnot, and is on the limestone block forming the corner stone, which is about 8 inches square at the end and projects some 4 inches. The mark is a square cavity in center of projection, and has on upper surface U. S. □ B. M. and on western face P_3. 1888. The stone is 35 paces south of railroad. The exact B. M. is the bottom of the square cavity.

Secondary B. M. XXIII.—This B. M. is the surface of the stone inside a square (□) cut on top of the fourth pier (from east bank) of the Missouri Pacific Railroad bridge over the Osage River at Osage, Mo. The B. M. is under the center of the track and about the center of the top of the pier.

Secondary B. M. XXIV.—This B. M. is the bottom of a square cavity cut in the top of a stone post set in the southwest corner of Mrs. Rassler's boarding house yard at Osage, Mo. The stone post (limestone) is dressed to 6 by 6 inches at the top and 6 inches below; it is about 2 feet long and is set 22 inches in the ground. The top of the post is lettered as follows:

U S
□
B M

Secondary B. M. XXV.—This is the bottom of a square cavity cut in the capstone on south end of west abutment of first trestle west of mile post 119 on the Missouri Pacific Railroad, between the Osage River and the Moreau Creek. The letters

U S
□
B M

are placed thus, and roughly cut.

(See route diagram, illustration No. 1.)

Accuracy of the preceding results for heights.

The temporary marks of the line between St. Louis and New Haven are fairly regularly distributed, with an average distance apart of 1·9 kilometres; hence we may assume the weights for these partial lines to be equal and the probable error of a difference of height of 1 kilometre from a double measure (here two simultaneous measures) becomes

$$r_{\prime\prime} = 0{\cdot}675\sqrt{\frac{[dd]}{4\,[s]}}$$

and the probable error for height of a terminal point at the distance $S=[s]$ will be

$$r=0\cdot675\sqrt{\frac{[dd]}{4}}$$

These expressions suppose the two measures to be independent of one another; this, however, is not the case with simultaneous lines, the condition of the atmosphere at the time being the same for both, and this is also partially true of the condition of the instrument, so that the weight of results from two simultaneous lines is but little better than that for one line. Experience showed that in case of two simultaneous lines the above probable error should be increased by its one-fourth part in order to approximate to a more correct value.

We have $[dd]=695\cdot1$ and $[s]=116\cdot2$;
hence $r_{,,}=\pm 0\cdot83^{mm}$ and adding one-fourth, the corrected value $=\pm 1\cdot04^{mm}$, also $r=\pm 11\cdot2^{mm}$.

In the line between New Haven and vicinity of Jefferson the temporary marks are also regularly distributed, but only 0·9 kilometre apart on the average; here we have $[dd]=719\cdot9$ and $[s]=84\cdot4$; hence $r_{,,}=\pm 0\cdot98^{mm}$ and $r=\pm 9\cdot06^{mm}$.

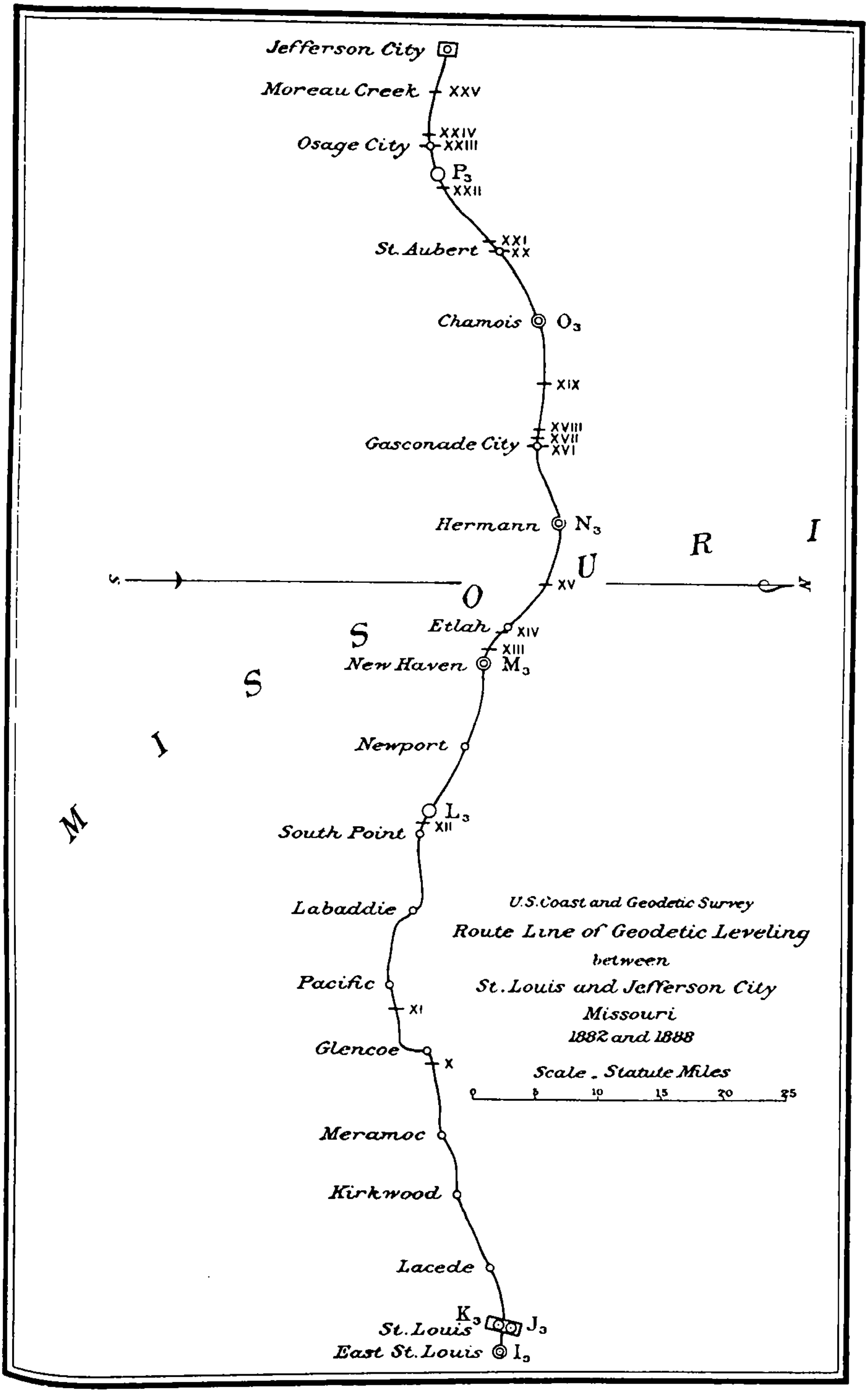
Also for first part of line $m_{,}=\sqrt{\frac{[dd]}{2\,[s]}}$, or the mean error of a single leveling of one kilometre, after increasing the r by its fourth part

$$m_{,}=\pm 2\cdot16^{mm}$$

and for second part $\qquad m_{,}=\pm 2\cdot06$

The probable error of the difference of height between St. Louis (K_3) and vicinity of Jefferson (XXV)

$$\pm 11\cdot2 \pm 9\cdot1 = \pm 14\cdot4^{mm}$$


Jefferson City
Moreau Creek
XXV
XXIV
Osage City
XXIII
P_3
XXII
XXI
St. Aubert
XX
Chamois
O_3
XIX
XVIII
XVII
Gasconade City
XVI
Hermann
N_3
S.
N.
MISSOURI
XV
Etlah
XIV
XIII
New Haven
M_3
Newport
L_3
South Point
XII
Labaddie
Pacific
XI
Glencoe
X
Meramoc
Kirkwood
Lacede
K_3
St. Louis
J_3
East St. Louis
I_3
U.S. Coast and Geodetic Survey
Route Line of Geodetic Leveling
between
St. Louis and Jefferson City
Missouri
1882 and 1888
Scale - Statute Miles
0
5
10
15
20
25

APPENDIX No. 3—1893.

PHOTOTOPOGRAPHY AS PRACTICED IN ITALY UNDER THE AUSPICES OF THE ROYAL MILITARY GEOGRAPHICAL INSTITUTE, AND AS PRACTICED IN THE DOMINION OF CANADA UNDER THE AUSPICES OF THE DEPARTMENT OF THE INTERIOR. ALSO A SHORT HISTORICAL REVIEW OF OTHER PHOTOGRAPHIC SURVEYS AND PUBLICATIONS ON THE SUBJECT.

Submitted for publication December 9, 1893, by J. A. FLEMER, Assistant.

PREFACE.

A topographic survey of a large area or of an entire country has been and still is a very laborious, time-absorbing, and expensive undertaking. Nearly all the European countries have such surveys of a more or less elaborate and detailed nature, which are the fruits of ceaseless work, begun many years ago, and in most instances the topographic work is continued to this day, in order to maintain the value of the maps, particularly for military purposes, by making frequent resurveys, covering all changes subsequent to the time at which the original surveys had been completed.

The completion of a topographic survey of the United States, executed on a scale to be useful for general purposes, if undertaken now, could not be witnessed by many of the present generation. With a practical people like the Americans such an undertaking would probably be looked upon with more favor if the generation that begins this work would also reap some of the benefits thereof.

The topography of this country is so diversified and the population is so unevenly distributed over the same that the methods to be employed for such a survey should also be diversified; the character and value of the different sections should govern the accuracy and amounts of detail of the survey, in order to reach the quickest yet practically useful and valuable results.

Minute and detailed methods, with ensuing accurate results, should be applied to cities and all closely settled regions, to the coast, larger rivers, and lakes, and the work should be platted on a large scale. Arid, barren, and mountainous regions, as well as prairies and swamp lands, should be more generalized in their cartographic representations and platted on a small scale.

The new survey of Italy demonstrates this fully, and it is there that phototopography, the subject to be considered in this paper, has reached a high state of perfection under the auspices of the Military Geographical Institute of that country.

Photogrammetry proper (or *metrophotography*) should be applied to the art of taking perspective views of buildings with a photographic camera for the purpose of constructing therefrom the elevations and ground plans of buildings. It is used chiefly for architectural purposes (remodeling, illustrating, copying, etc.).

The term *phototopography* should be generally adopted for all topographic surveys based on perspective views of the terrene obtained by means of the camera.

Photographic survey, finally, could then be applied to all surveys based on photographic data which do not include the delineation of the terrene (nonhypsometric surveys).

We have endeavored to give in the following pages a short review of the more important photographic surveys, and of some of the publications on photogrammetry and phototopography, as well as a concise description of the general methods and principles of phototopography as practiced in Europe and in the Dominion of Canada, in order that this branch of surveying may become more generally known, tested, and amplified also in this country.

SHORT REVIEW OF PHOTOGRAPHIC SURVEYS AND PUBLICATIONS.

In Europe the possibility of applying photography for constructive and surveying purposes was recognized many years ago.

Photographs obtained by aid of lenses ground specially with a view toward reducing astigmatic aberration as much as possible and giving a uniform extension of definition and depth over a strictly flat field will represent geometrically true perspectives.

Photogrammetry, or metrophotography, is the art of ascertaining graphically the true dimensions of objects from their perspectives, in which the relative dimensions of the objects are changed and distorted (chiefly foreshortened) and can not be ascertained by direct linear measurements in consequence of being represented in perspective view on a plane surface.

The study of constructing geometrical views and ground plans of objects represented in perspective can be divided into two groups or chapters.

1. To construct geometrical plans from perspectives, composed of regular figures and taken from points of view close to the objects thus represented, for instance, to construct the elevations and ground plans of buildings, machines, and the like from photographs taken from stations sufficiently close to the same to delineate all details. This art

may properly be termed photogrammetry or metrophotography; it is of interest only to constructors, architects, paleologists, artists, etc.

2. The objects represented in perspective are of irregular shape and at various distances from the stations or points of view, like distant landscapes, and it is desired to construct therefrom, graphically, a topographic map of the terrene, projected in horizontal plan. This art may be termed phototopography and it interests topographers, geographers, geologists, explorers, hydrographers, etc.

Descriptive geometry teaches the laws which are to be followed when representing objects by drawings on plane surfaces. The eye receives the natural image or the view of an object by aid of the rays of light—termed visual rays—which emanate from the illuminated parts of an object facing the spectator.

If we regard the eye as a fixed point and imagine the rays of light, emanating from different points of the object in view, intercepted by a vertical plane, we will obtain a central projection or a perspective view of the object in the vertical plane.

The greater the distance of the object from the eye, the less great will be the deviation of the extreme visual rays from the direction of the central ray; for an infinite length of the central ray all the rays will become parallel.

If the picture or image of the object is given us as a true perspective in a plane, we can, inversely, construct therefrom a geometrical projection of the object in a plane placed at right angles to the picture plane, if we know the distance and relative position of the point of view with reference to the picture plane, and if we have views taken from a sufficient number of stations in space to envelop the irregularly formed object in question.

Regarding a photograph as a geometrically true perspective, photogrammetry will be the art of reconstructing geometrical horizontal projections from given perspective views.

The theoretical fundamental ~~principles upon~~ which such reconstructions rest were known to Lambert in 1759, but the first practical application of the same ~~was made by~~ the celebrated French savant and hydrographer, Beautemps-Beaupré, while on a scientific expedition during the years 1791 to 1793. Although the camera had not yet been invented, it is ~~said that~~ Beautemps-Beaupré was an expert sketcher, and he made perspective drawings and sketches of coast regions while on that expedition, from which, at a later period, he constructed topographic maps of a part of Van Diemen's Land (now Tasmania) and of the island of Santa Cruz. Notwithstanding Beautemps-Beaupré's frequent allusions to the feasibility of this method of making reconnaissance surveys and topographic maps, nothing more was accomplished until Laussedat, major in the French army, took the study of this subject up in 1850, using, however, the camera to obtain the perspectives.

In 1839, shortly after Daguerre had presented his memorial upon photography to the Academy of Sciences in Paris through Arago, the latter called attention to the possibilities of photography in the Chamber of Deputies, where he said:

* * * Nous pourrions, par exemple, parler de quelques idées qu'on a eu sur les moyens rapides d'investigation, que le topographe pourra emprunter à la photographie.

Gay-Lussac similarly called attention to the probable adaptability of photography to topographic surveys.

In 1858 Chevallier had an instrument patented which he called a "planchette photographique." This photographic plane table is mentioned and described by Alophe (1861), d'Abbadie, Baté (1862), Jouart (1866), Tronquoy, etc.

Jouart, Wiganowski, Baté, and others also made practical tests and topographic surveys with Chevallier's photographic plane table.

Captain Cannette used the sextant and photographic camera to make topographic surveys, chiefly of fortifications.

In 1851 Laussedat constructed a "camera clara," which in 1858 was superseded by the "camera obscura" with additional improvements for surveying purposes, by Reynault. Laussedat, as "chef du génie corps," made numerous experimental surveys and studies with Reynault's improved "camera obscura," partly on his own behalf and partly under the direction of the French ministry of war. He also was the first to make topographical surveys with the aid of balloon photography. During the years 1863 to 1870 he had the assistance of Captain, now Commandant, Javary, who improved the French phototheodolite and made experimental surveys in the mountains of the Dauphiné and Savoie, in the Vosges, and in Alsatia. The first practical survey of a more extended character made with the aid of photography in France was made by Laussedat in 1861, when he mapped a portion of Paris and also the town of Grenoble under the auspices of the ministry of war. The area covered by this survey was 0·4 square mile; the field-work consumed sixty hours, and the office work was accomplished in two months. a

Pujo and Fourcede published an article in Les Mondes, 1865, on "Goniométrie photographique."

Other publications in French are:

Comptes Rendus de l'Académie des Sciences, Paris, XLIX. 1859; L, 1860; LI, 1860; LIX, 1864; 1885 and III, p. 729-732, 1890.

Magasin Pittoresque, XXIX, 1861.

Annales du Conservatoire National des Arts et Métiers, 2e série, IV, 1892.

Comptes Rendus du Congrès de Pau et Revue Scientifique de 1892.

"Éléments de Photogrammétrie," in Bulletin de la Soc. d'Éditions Scient., Paris, 1891, by V. Legros.

Application de la Photographie à la Topographie Militaire, par E. Paté. 1862.

Mémorial de l'Officier du Génie, No. 16, 1854; No. 17, 1864; No. 22, 1874.

Bulletin de la Société de Géographie de Paris, Déc. 1862.

Application de la Photographie aux Levers Militaires, par A. Jouart, 1866.

La Photographie Appliquée aux Études Géographiques, par Jules Girard, 1872.

De la Photographie et ses Applications aux Besoins de l'Armée, par Fl. Dumas, 1872.

La Photographie Appliquée au Lever des Plans, par J. Bornecque, 1886.

La Photographie dans les Armées, par Alfred Hanot, 1875.

La Photographie sans Objectif, par R. Colson, 1887.

La Nature (Paris).

La Revue d'Artillerie (Paris).

Bulletin de la Société Française de Photographie (Paris).

Les Levers Photographiques et la Photographie en Voyage, par le Dr. Gustave Le Bon, 1889.

Annales du Conservatoire des Arts et Métiers. Édouard Monet: Principes Fondamentaux de la Photogrammétrie, published by La Société d'Éditions Scientifiques, No. 4 Rue Antoine-Dubois, Paris.

In recent years the French ministry of war has had numerous experiments made with balloon surveying (using both the captive and free balloon), balloon photography being better adapted for military and secret surveys.

France had an exhibit at the World's Columbian Exposition in Chicago, 1893, showing photographic instruments and specimens in illustration of topographic and astronomical results, gained chiefly under the direction of Col. A. Laussedat and taken from the collection of the Conservatoire National des Arts et Métiers, in Paris, of which Bureau Col. A. Laussedat is the director.

The first German publication bearing on this subject ~~is probably the~~ article in Horn's Photographische Mittheilungen, April, 1863, being a German translation of A. Laussedat's explanations ~~and~~ descriptions, as given by him on January 9, 1863, in a meeting of the French Photographic Society.

Dr. A. Meydenbaur's first publication on this subject is in the June edition of the Photographische Mittheilungen of 1863, where he uses the term "photometrography," which was subsequently changed into "photogrammetry."

Vogel published, in the March number of the same magazine for 1866, an article on the use of Johnson's photographic instrument for making topographic surveys, showing the method of obtaining horizontal and vertical angles from the perspectives.

Ever since Dr. Meydenbaur first became interested in this method he endeavored to interest German private and Government surveyors in photogrammetry. He has been recently appointed director of the Photogrammetrical Institute in Berlin, founded by the Prussian Government as a branch bureau of the ministry of culture. May 4, 1893, Dr. Meydenbaur (royal counsilor), gave a lecture on metrophotography, or photogrammetry, in the ministerial building, under the auspices of Mr. Bosse, minister of culture.

Although this photogrammetrical institute was founded several years ago, no official publications or reports have been issued yet to the public.

Professor Jordan, Dr. Doergens, Dr. Stolze, Dr. Vogel, and Dr. Hauck have done much toward popularizing the photographic methods of surveying in Germany and Austria.

Professor Jordan published a treatise upon "The application of photography for geometrical representations" in the Zeitschrift für Vermessungswesen, 1876, Heft 2, Bd. V, and he points out the future importance of photogrammetry in his closing remark: "Photogrammetry can be applied with the greatest advantage in certain cases, e. g., for the survey of inaccessible mountain groups and ranges, on scientific expeditions," etc.

The first attempt at a photogrammetric survey in Germany was made under the direction of the Prussian ministry of war and commerce in 1867, when a survey of the town of Freiburg and also an architectural survey of the cathedral in Freiburg were made. The fieldwork was continued through four days and the area surveyed comprised about 0·04 square mile. The office work for the construction of the map consumed three weeks, while it took one week to draw the ground plan, one side, and one front elevation of the cathedral.

During the Franco-Prussian war phototopography was called into service by the German army, and a detachment of the engineer corps, under Dr. Doergens (later professor of geodesy in Berlin), was formed to obtain certain distances about the city of Strasburg with the aid of a camera, during the siege of that city. This detachment made a map on the scale of 1 to 25,000 of the besieged front of the city. However, the result was not utilized by the army, the city having capitulated before the map was fully platted.

Professor Jordan, as member of Rohlf's African exploring expedition in 1873–74, made a phototopographic survey of the Oasis Gassr Dachel in the Libyan desert.

In 1874 Dr. Stolze used a Meydenbaur camera-theodolite to make a survey of the ruins of Persepolis, and also an architectural survey of the mosque of Djumâht, in Shiraz, Persia.

In 1885 the students of the technical high school in Berlin, under direction of Professor Pietsch, used two instruments specially constructed to obtain views also under an inclined position of the optical axis of the camera. They obtained satisfactory results from various ascensions made in a free balloon, as well as from views taken on the ground for architectural purposes.

Photography has also not only found practical application in topographical surveys of Austria, principally in Steiermark and Kärnten, but the art of phototopography has made rapid strides in gaining public favor in that country, owing to the treatises and works published on this subject by Pollack, Hofferl, Steiner, and others, as well as to recent improvements in the instruments.

The following are the principal publications in German on photographic surveying:

Photographisches Archiv, Septr., 1865.

Zeitschr. für Bauwesen, 1867.

Archiv für die Offiziere des k. preuss. Artillerie- und Ingenieur-Corps, Bd. 63, 1868.

Deutsche Bauzeitung, 1872.

Zeitschr. f. Vermessungswesen, Heft 23 and 24, 1887.

Journal für die reine und angewandte Mathematik, Bd. 95.

Das Licht. S. G. Stein. Heft 5, 1887. (Photogrammetrie, von V. Stolze.)

Lechner's Mittheilungen aus dem Gebiete der Photographie und Kartographie. R. Lechner, Graben 31, Wien.

Dr. C. Koppe: Die Photogrammetrie, oder Bildmesskunst. Weimar, 1889.

V. Pollock: Die photographischen Terrainaufnahmen mit Berücksichtigung der Arbeit in Steiermark. R. Lechner, Wien, 1891.

V. Pollock: Photogrammetrie und Phototopographie. Mittheilungen der k. k. geogr. Gesellsch., 1891 (pages 175–195), Wien.

Fr. Steiner: Das Problem der fünf Punkte, eine Aufgabe der Photogrammetrie, 1891. Wochenschr. d. östr. Ing.- und Archt.-Vereins (pages 214–217).

Fr. Steiner: Die Photographie im Dienste des Ingenieurs. Ein Lehrbuch der Photogrammetrie. R. Lechner, Wien, 1891.

Fr. Schiffner: Die photographische Messkunst, oder Photogrammetrie, Bildmesskunst und Phototopographie. Wilhelm Knapp, Halle a. S., 1892.

Dr. A. Meydenbaur: Das photographische Aufnehmen zu wissenschaftlichen Zwecken, ins besondere das Messbildverfahren. Unte's Verlags-Anstalt, Berlin, 1892.

Gustav Fritsch, in Dr. G. Neumayer's Anleitung zu wissenschaftlichen Beobachtungen auf Reisen. Robert Oppenheim, Berlin, 1888.

Fr. Schiffner: Ueber die photogrammetrische Aufnahme einer Küste im Vorbeifahren. Mittheilungen aus dem Gebiete des Seewesens, 1890, pages 412–417.

F. Hafferl: Ueber Photogrammetrie. Vortrag. Wochenschrift des östr. Ing.- u. Archit.-Vereins, 1890, pages 199–203.

V. Pollock: Ueber Anwendung der Photogrammetrie im Hochgebirge. Vortrag. Wochenschrift d. östr. Ing.- u. Archit.-Vereins, 1890, pages 207–209.

Volkmer: Das Wesen der Photogrammetrie. Wochenschrift des östr. Ing.- u. Archit.-Vereins, Vol. XIV, page 157.

Jordan: Vermessungskunde. II. Feld-Landmessen. Metzler'sche Verlagsbuchhandlung, Stuttgart, 1893.

In Italy we find, as previously mentioned, that phototopography has been brought to a high state of perfection in recent years.

Porro spent much time, labor, and energy in perfecting photography as applied to tachymetry and topography. The results of his labors were published in Il Politecnico, Vols. X and XI, under "Applicazione della Fotografia alla Geodesia," 1853, Saldini, Milano. Porro's instruments have all been preserved by Salmairaghi, director of the Polytechnic Institute at Milan, of which Porro was a member.

In 1875 Manzi Michele, officer of the Military Geographical Institute of Italy, utilized some photographic views of the "Abruzzi" to supplement his plane-table survey of the "Gran Sasso." In 1876 the same officer continued the practical application of photography for the topographical survey of "Mont Cenis" (Bart Glacier).

The Military Geographical Institute then decided to suspend all photographic work indefinitely, as many maintained that photographical data for topographic purposes were unreliable.

In 1878 General Ferrero, chief of the geodetic department of the institute, called the attention of the directory of the institute to the desirability of resuming the studies in photogrammetry; and in the same year L. P. Paganini, engineer geographer of the institute, was commissioned to proceed to the Alps, near Apua, to resume the studies in photography applied to topographic surveys, with a view to ascertain whether phototopography was economical and expedient for practical work.

During Paganini's first season he obtained 17 cycloramic views, composed of 110 perspectives. A number of these perspectives were used to construct a map, in Florence, on a scale of 1 to 25,000, of the marble quarries at Colonnata (Carrara), with hypsometric contours in intervals of 5 metres.

In 1879 Paganini (using bromo-gelatin plates instead of wet plates, as heretofore, also having improved the camera theodolite) surveyed the Serra dell' Argentera, which was platted on a scale of 1 to 25,000, with contours in 10 metres intervals. This survey was based upon panoramic views obtained from fifteen stations, on elevated points, comprised 113 perspectives, and was the result of a field season of two months and a half. Also, this map was constructed in Florence during the following winter, and it represents an area of 28 square miles. The contours were controlled by 490 points, the elevations of which had been ascertained.

In 1880 the same officer commenced the survey of the area bounded by the valleys of the Orco, the Valsoana, the Cogne, and the Valsavaranche, representing an area of about 386 square miles. The survey of this area was finished in 1885.

However, since 1884 Paganini used an improved instrument, made by Galileo for the institute after plans submitted by Paganini. He also invented three instruments which greatly facilitate and accelerate the otherwise tedious graphic operations of the map construction from the perspectives, and which will be described later on.

Paganini's results proved the efficiency of phototopography for Alpine work, to be platted on a scale of 1 to 25,000 or 1 to 50,000, and the technical solution of the problem has been fully established. Owing to the untiring efforts of the officers of the Military Geographical Institute, and the good results which they obtained, phototopography has been adopted as an auxiliary to the plane table for the new survey of Italy.

In a more recent report on phototopographic work by Paganini to the first geographical congress in Italy, he described his latest improvements to the camera theodolite. An extract from this report, made by Fenner, can be found in the Zeitschr. f. Verm., 1893.

A German translation, by A. Schepp, of Paganini's "La fototopografia in Italia" can be found in the same periodical for 1891 and 1892.

C. W. Verner, "Notes on military topography," 1891. Also, Mechanics, Vol. II, p. 168, "Application of photography to surveying."

A short article on photogrammetry has also been published by Henry A. Reed, lieutenant, United States Army, "Topographical drawing and Photography applied to surveying." Another work in English has been issued by Allen, in London.

Civil and Military Photogrammetry (read before the American Philosophical Society, May 6, 1892), by R. Meade Bache, assistant, United States Coast and Geodetic Survey.

The following works are very explicit and full of details:

Photography Applied to Surveying, by Lieut. Henry A. Reed, United States Army. John Wiley & Sons, 15 Astor place, 1889.

Photographic Surveying, by E. Deville, surveyor-general, Canada. Ottawa, 1889. This work, unfortunately, is out of print. Only a limited number of copies had been printed, chiefly to supply the Dominion land surveyors.

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INTRODUCTION.

Topography (description of locality) serves to represent and describe, in horizontal plan, limited areas of the earth's surface, showing vertical and horizontal distances (the relief) between points of the area thus represented. On every topographic map, therefore, the characteristic lines and forms of the terrene, including natural objects which appear on the earth's surface, must be recognizable with more or less minute detail, according to the scale and purpose of the map.

Generally speaking, the scales of topographic maps vary from 1 to 10,000 to 1 to 200,000. On a larger scale than 1 to 10,000 such maps are chiefly used for constructive (engineering) purposes; they show all artificial as well as natural objects on the earth's surface, and they are the result of special surveys for special purposes.

Maps on a smaller scale than 1 to 200,000 are designated as geographic maps. Their topography is generalized to show only prominent and characteristic features (mountain ranges, valleys, plateaus, seas, lakes, rivers, etc.).

The topographer inspects the area to be mapped and intuitively separates characteristic features from minor detail. He immediately, on inspection, recognizes what is to be shown on the map, and consequently embodies only such topographic features of the terrene as will harmonize with the power of representation of the adopted scale for the work. It is useless to locate features during the instrumental survey which are too small to be represented on the scale of the map. An expert topographer will make a topographic map of a given area in the least time necessary, yet show all characteristic features that can be drawn and represented in harmony with the scale of the map.

After a system of triangulation has been extended over a country and the topographic details of the same area have been gathered by a series of photographic panorama views, taken from these trigonometric stations, and supplemented, where needed, by views taken from other points (the latter to be subsequently connected with the triangulation by resecting or otherwise), an experienced topographic draftsman can select all characteristic topographic features from such photographs, for mapping purposes, with the same good result as the topographer would select them in nature from the same area, as shown in the photographs, were he to draw the map in the field (plane-table method).

If by a simple graphic method such selected characteristic points on the photographs can be platted upon the map, a great deal would be gained in the survey of certain regions and under certain conditions toward a saving of time and money, compared with the instrumental methods of topographic surveying as generally practiced. At present phototopography can not replace instrumental topography; still, experience in Italy, France, Canada, Austria, and Germany has proven it to be a very valuable adjunct to the plane table and transit for topographic surveys of rugged mountainous regions, if they are not too wooded.

The following description of photographic instruments and methods is chiefly a condensed extract from the previously mentioned article printed in the Rivista di Topografia e Catasto, 1889, by L. P. Paganini, engineer of the Royal Italian Military Geographical Institute, followed by a description of instruments as used and methods as practiced in Canada, with a short reference to German, French, and Austrian instruments.

Chapter I.

THE PHOTOGRAMMETRIC APPARATUS—PRINCIPAL COMPONENT PARTS OF THE PERSPECTIVES.

Figure A shows Paganini's Italian photogrammetric apparatus. It has a tripod which can be dismembered into three "alpenstocks" *A*, *A*, and *A*, a theodolite, and a camera *C*. All three parts can be firmly united by means of spiral springs and screws.

The three screws S_1 of the tripod head *H* (but two shown in drawing) support the theodolite, and the camera *C* is supported by another set of three screws S_1', S_2', and S_3', which are connected with the horizontal limb of the theodolite in such a manner that the camera can be revolved about the vertical axis of the theodolite.

A spirit level *L* and telescope *T* are supported by an upright piece *U*, placed at right angles to the horizontal limb of the theodolite and at one side of, but close to, the camera.

The telescope *T* is provided with two cross hairs (one vertical and the other horizontal), in the usual adjustable manner. The camera box is made of hardened pasteboards, which are stiffened by a metal skeleton casing *B*. The camera is provided with an aplanatic objective (antiplanet), made by Steinheil, the focal length of which is $244{\cdot}5^{mm}$ and the aperture in the diaphragm has a diameter of 5^{mm}.

Regarding the general arrangement of the camera, it may be said that—

1. The optical axis of the photographic lens (objective) is vertical to the image plate *I*.

2. The intersection *O* of the optical axis and image plate *I* is marked on the latter by the point of intersection of two very fine platinum wires p_1 and p_2, placed at right angles to each other and very near the image plate *I*; when in adjustment one p_1 of these fine wires is horizontal and the other p_2 vertical.

The optical axis of the camera can be adjusted in horizontal plane by means of the three milled head screws S_1', S_2', and S_3' which support the camera. The horizontal wire p_1 is adjusted (after the instrument has been carefully leveled up) in horizontal plane by finding some easily identified point on the ground-glass plate *I*, which is bisected by this wire, and by gently revolving the camera about the vertical axis of the instrument; if the wire p_1 is in horizontal plane, the observed point will be seen to move along p_1 during the revolving motion of the camera. Should the bisected point appear above or

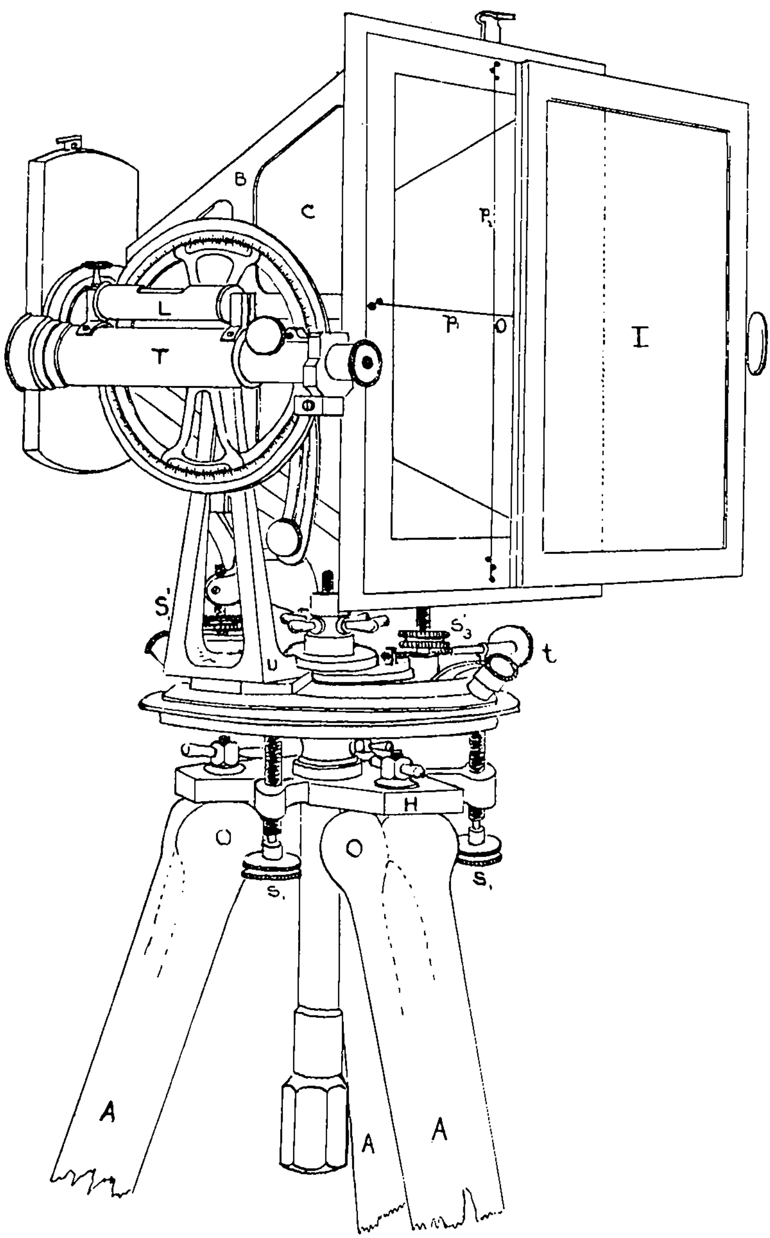

Fig A

ITALIAN PHOTOGRAMMETRIC APPARATUS.

below the wire p_1 at any time during the revolution of the camera, the same must be adjusted in horizontal plane by aid of the two forward screws S_1' and S_2' which support the camera.

The camera is provided with a short tangent screw t, by means of which the same can be slightly moved in azimuth, while the telescope and horizontal limb of the theodolite remain stationary. This will enable the observer to place the optical axis of the camera parallel to that of the telescope T, provided both are adjusted in horizontal plane. This correction is made by observing some distant point in the intersection of the cross wires of the telescope and then clamping the theodolite. The camera is now moved by means of the tangent screw t to the right or left until the same point appears in the intersection O of the two wires p_1 and p_2, it being already bisected by the camera wire p_1, as described in the preceding adjustment.

The points of the camera cross wires p_1 and p_2 appear upon every plate taken with the camera, and as these plates are vertical to the optical axis of the camera, the perspectives obtained after the camera had been adjusted, as described in the preceding paragraphs,* are in vertical plan and each shows the principal point of view O, as well as the two axes, p_1 and p_2, intersecting each other in O at right angles. The line p_1 represents the horizon of the station whence the picture was taken.

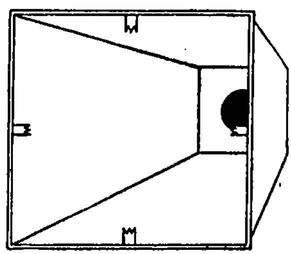

FIG. 1.

Instead of the fixed platinum wires p_1 and p_2, some recent instruments (among others those used in Canada) have a set of teeth attached to the camera, close to the plate I, as shown in figure 1. If solar prints are used for the map construction instead of the plates, this arrangement is more desirable than that of the fixed wires, as the prints will unavoidably be a little distorted. The lines p_1 and p_2 are preferably drawn in red ink on the prints after their positions in regard to the teeth have been experimentally ascertained or checked. Great care must be exercised in locating these lines properly, as they form a rectangular system of coordinates to which every point in the picture is referred during the process of the subsequent map construction. They also aid in ascertaining the value of the constant focal length of the camera.

Figure 2 shows the longitudinal section of a camera with the diaphragm AB in position, the aperture of which is 5^{mm} in diameter. Only such rays of light, emanating from a point, N, in nature, will reach the point n on plate I which form a cone around the central ray nON

*After a perspective has been taken and it appears desirable to obtain a view of the terrene immediately below (or above) the same, the construction of the instrument will permit a depression (or an elevation) of the optical axis of 30° below (or above) the horizon.

with apex in n and base in O of 5^{mm} diameter. (In our diagram, figure 2, this base will be an ellipse with 5^{mm} length for the short axis; the cone of rays emanating from a point, C, would be intercepted by the plane of diaphragm AB in a circle of 5^{mm} diameter.) The camera lenses are so focused that even for the largest diaphragm aperture used all

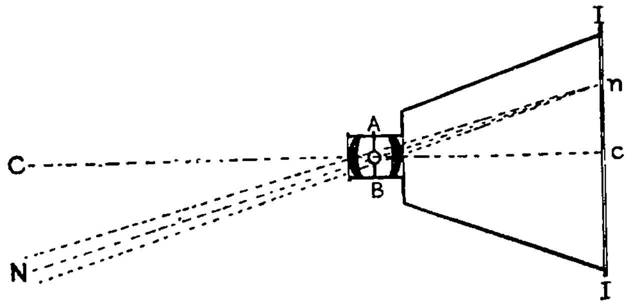

FIG. 2.

points from 10 metres to infinite distance from O, will be clearly photographed with a maximum error of 6^{mm}, as will readily be seen from the following discussion:

a=distance of object from O (10 metres to infinite distance).
f=principal focal distance (240^{mm}).
b=focal distance, variable for different lengths of a.

From the well-known relation:

$$\frac{1}{f}=\frac{1}{a}+\frac{1}{b}$$

we find

$$b=\frac{af}{a-f} \tag{1}$$

By adopting 240^{mm} as value for f, and substituting different values, from 1 metre to 300 metres, for a, in formula (1), we obtain the following values for b:

a (in m.) ...	1	10	20	30	40	50	75	100	200	300
b (in mm.)	315·8	245·9	242.9	241·9	241·4	241·1	240·7	240·5	240·2	240·02

The error, therefore, in maintaining the focal distance constant is 6^{mm}, if the object is 10^{m} distant from the image point; is 1^{mm} if the object is 50–100^{m} distant from the image point; is inappreciable if the object is 300^{m} or more distant from the image point.

The value $\frac{x}{2}$ of the error (distortion), maintaining a constant focal distance = 240^{mm} in the photograph for points at different distances, can be seen from the following:

Assuming again that the plate I is held in a fixed position and 200^{mm} distant from the "image point" (principal focus), it is evident that the plane of image I will intersect some of the cones of rays (passing through aperture of diaphragm) in a circle (or in an ellipse) instead of intercepting their apex. From the foregoing table we see that this circle of diffused rays will increase in size with the decreasing

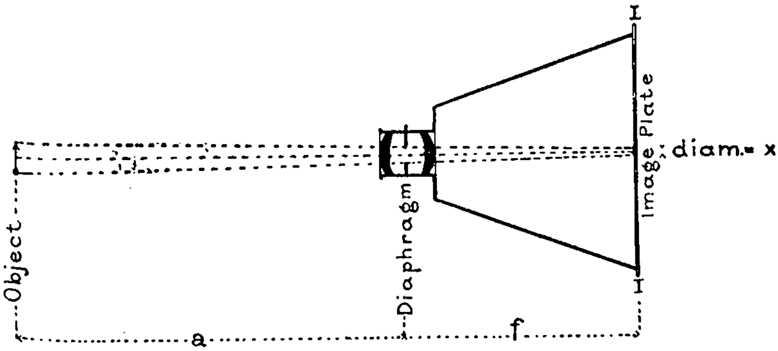

Fig. 3.

distance of the object photographed. The length of the diameter, x, of this circle (or ellipse) can be ascertained from the following relation (fig. 3):

$$x : O = f : a$$

$$x = \frac{f\,O}{a}.$$

If we assume this aperture, O, in the diaphragm, to be 5^{mm} in diameter, and assume the same values for f and a as in the preceding, we find the respective values for x as follows:

a (in $m.$) =	10	20	30	40	50	75	100	200	300	400	500	700	1000
x (in $mm.$) =	0·12	0·06	0·04	0·03	0·025	0·016	0·012	0·006	0·004	0·003	0·003	0·002	0·001

The diameter, x, of the circle (or ellipse) is evidently quite small, and the maintaining of a constant focal distance can be well carried out for all practical purposes without appreciable error.*

* The apparatus described in this paper is provided with a metal graduation plate, extending in the direction of the camera axis, and bearing a scratch to mark the focal length of the camera when focused upon objects at infinite distance. From this scratch, toward the sensitive plate of the camera, a millimetre graduation is engraved upon the plate, by means of which the observer can directly measure the focal length of the camera if the same is changed at any time for any pictures. The objective cylinder can be moved in the direction of the camera axis by means of a spiral groove cut into a second cylinder which is firmly attached to the camera, and as one turn of this screw is equal to an axial motion of 1 millimetre, the change of focus can readily be ascertained to one-tenth of a millimetre, as will be described in the following:

By revolving the external cylinder, the metal strip or plate attached to the same, glides around the outer surface of the inner cylinder (firmly attached to the camera), which bears a circular scratch lying in a plane vertical to the camera axis and passing through the constant focus (or through a point whose distance from the image plate equals the principal focal length). This circular scratch is divided into ten

The distance of the point of view from the perspective plate, the principal point of view on the perspective, and the line of horizon can always be ascertained or rectified by instrumental observations and computations, or graphically, as has been indicated, and as will be shown more fully.

We have described how the optical axes of the telescope and of the camera can be brought into two vertical parallel planes. Both can be kept in this position and yet be revolved about the vertical axis of the instrument. The horizontal limb of the theodolite is divided into 360°, with subdivisions of 20′, and by means of two verniers 30″ can be read. The vertical circle is provided with the same graduation. Thus the means are given to ascertain the azimuthal positions of the optical axis for each perspective; or, in other words, the means for orientation are thus provided for. The magnetic azimuth of the principal ray of the perspectives (i. e., direction of optical axis for each exposure), or the horizontal angle made with any other line passing through the station and some known point (e. g., trigonometrical point), can readily be ascertained.

All the perspectives which are to be used for mapping must be obtained from stations with known geographical positions. Generally trigonometrical points are selected for photographic stations, but if other points have to be occupied the elements needed (horizontal and vertical angles) to determine their positions with respect to surrounding triangulation points, can readily be observed by aid of the theodolite before leaving the camera station.

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equal parts, and the metal scale, passing over this graduated ring when the objective tube is moved toward the sensitive plate, will indicate by its position on this circular scale the number of tenths of millimetres it has moved beyond the number of millimetres which are read off on the metal scale first mentioned.

The focal length is a very important factor in all phototopographic work, and it is advisable to verify at the beginning of operations the reading of the metal scale, and if the principal focal length is changed, the difference must be entered into the notebook, so that the proper correction can be applied later on.

Chapter II.

FUNDAMENTAL PRINCIPLES OF PHOTOTOPOGRAPHY.

It was comparatively easy to obtain a close connection between the phototopographic stations and the new triangulation of Italy, as the committee who had charge of this triangulation has provided Italy with a generous and harmonious disposition of triangulation points, which have been very carefully located, their exact positions computed and permanently marked in the field, irrespective of the character of the surrounding topography or of the order of triangulation to which they belong.

This great number of triangulation points not only facilitates the application of the camera and assures the accurate determination of the panorama stations, but it also simplifies the subsequent map construction, as the greater part of the perspective contains one, two, or more triangulation points, although the instrument commands a field view of but 42° horizontally.

Thus all cardinal points of the perspectives can readily be adjusted, the pictures are easily orientated for the map construction, and the salient topographical features, deduced from the perspectives, can be frequently checked.

The camera used for this work gave pictures of $18{\cdot}5 \times 14^{cm}$ (the plates were $19 \times 24{\cdot}5^{cm}$); the lens controlled a field of 42° horizontally and 52° vertically (26° above and 26° below the horizon). Recently, however, the lens of the camera has been exchanged for one with a principal focal length of 240^{mm} in order to use plates of 18×24^{cm}, which size is readily supplied by photographic dealers.

For an exact determination of the primary points of the perspectives it is necessary to measure the "coordinates" of the points in question upon the perspectives as accurately as possible. To insure good results these measurements should be made upon the negatives with a pair of dividers made especially for this purpose, by means of which the desired distances are obtained in millimetres and tenths.

Secondary and tertiary points need not be obtained with the same degree of accuracy as the primary points (needed for the control principally), and as the plates are not well adapted for the making of direct measurements thereon, nor for the marking of identical points with numerals or other characters, it will be better to take all measurements for secondary and tertiary points from the prints, as the permanent changes which such prints undergo, compared with the negatives, are, when some care is exercised, hardly ever greater than the irregularities which can be discovered in any drawing on paper, no matter how carefully made.

The perspectives (panoramic views) which are to subserve the map making, and some of which are also to be reproduced as illustrations to accompany certain maps of the Alpine ranges, are obtained by ten successive exposures. After the camera has been adjusted at a panorama station, it will have to be revolved about the vertical axis through an angle of 36° after each exposure, and, as each plate subtends a horizontal angle of 42°, the two ends of adjoining plates will lap over by a vertical strip of the horizon of 3°, or each one of two adjoining plates will have a vertical margin of 15^{mm} width reproduced on the neighboring plate.

Figure 4 shows the horizontal projection of the positions of ten plates ($P^1 P^2$ . . . . P^{10}) at a panorama station. $V P^1 = V P^2 =$ . . . . $V P^{10} = f =$ principal focal length of camera $= 244{\cdot}5^{mm}$.

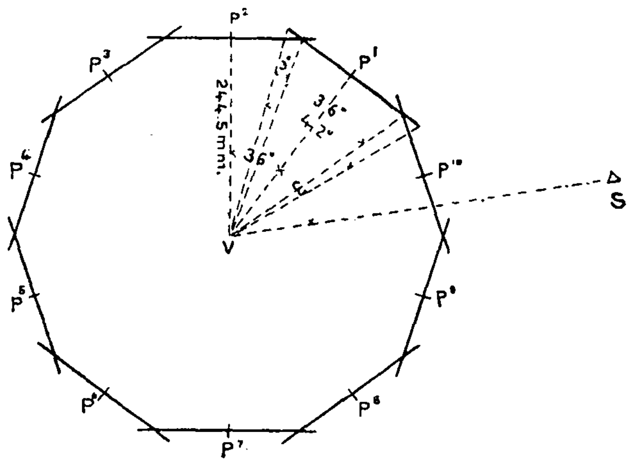


FIG. 4.

The common margins of two adjoining plates serve principally to ascertain whether the adjustments of the instrument have been disturbed during the occupancy of a station. Should, for instance, the position of the horizontal thread have been disturbed by some cause, this would be shown by different results when measuring the ordinates of identical points of two adjoining margins. These margins also serve admirably for the correct trimming of the edges of adjoining pictures if they are to be fitted together for the panorama.

The horizontal projection of the ten plates exposed from one panorama station is a regular decagon (fig. 4), with a radius of the inscribed circle equal to the principal focal length of the camera.

After the position of one panoramic view has been found on the map (i. e., after the angle ω, figure 4, has been plotted from the station V to the triangulation point, shown in the figure), the orientation of all the other plates is accomplished by adding 36°, 72°, 108° . . . . to the angle ω; thus the entire decagon can be plotted after the horizontal projection of plate P^1 has been oriented with reference to the station V and the triangulation point plotted on the map.

The orientation in horizontal plan having been made, it will be an easy matter to draw lines of direction from the platted camera station to points photographed in the panorama.

The plate $m\ m'\ n'\ n$, figure 5, may represent a vertical and oriented perspective and OO' be the line of the horizon of said plate. V is the point of view; ω' represents the angle of orientation for this plate in

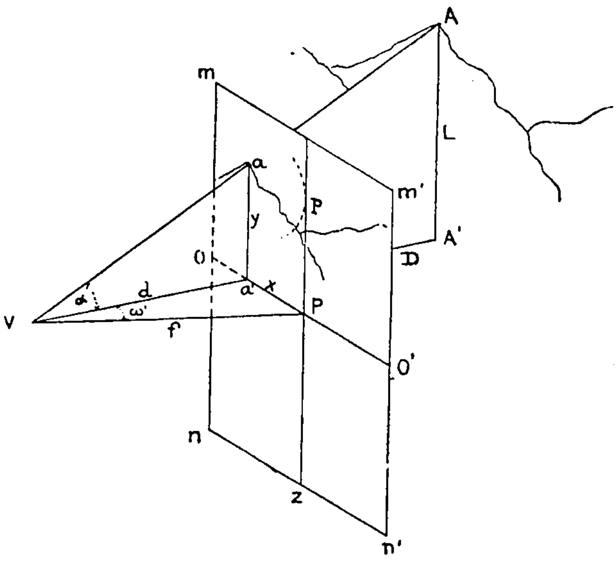


FIG. 5.

reference to the secondary point A (i. e., the horizontal angle between the direction from V to a' and the vertical ray $V\ P$ to principal point of view P of the perspective).

$VP = f$ is the principal focal length. If A represents a point of the terrene in space and a is the representation of the same on the plate, and if the vertical $a\ a'$ has been drawn from a upon the line of horizon $O\ O'$, then $P\ a' = x$ will be the abscissa of the point a.

From the rectangular triangle $V\ Pa'$ we find:

$$x = f \tan \omega'$$

Figure 6 shows this triangle as $V'\ P'\ a'$ in the plane of the drawing; P' = horizontal projection of the principal point of view (P in fig. 5) of the horizontal projection $O\ O'$ of the plate ($m\ m'\ n'\ n$ in fig. 5) which had been oriented in the plane of the map by the direction $V\ S$ (or rather by the horizontal angle ω, fig. 6) to the triangulation point S.

In order to draw the horizontal projection of the ray from V to the point A (fig. 5) the distance $P' a'$, (fig. 6) $= P a'$ (fig. 5) $= x$ is laid off upon $O O'$ (fig. 6) from P', in the sense of direction to A (whether to the right or left of the vertical thread). This abscissa x is taken from the picture by means of a pair of dividers as the radius $a p$ (fig. 5). After drawing a line from V' through a' (fig. 6), this line $V' a'$ will be the horizontal direction of the ray $V A$ (fig. 5).

The position of A' (horizontal projection of A plotted on the map) will be in the intersection of two or more lines of direction, obtained, in a similar manner, from other pictures containing a and taken from other stations. The same refers to all other points of the terrene, if their pictures can be identified upon plates taken from different panorama stations.

Every perspective containing the picture of a triangulation point will give evidence of the grade of accuracy for the relative plotted positions of the horizontal projections of the point of view and of the perspective, as well as for the orientation of the plate, by laying off upon $O O'$ (fig. 6) the abscissa x' of the triangulation point S and drawing the line $V' S'$, which should bisect the plotted point S.

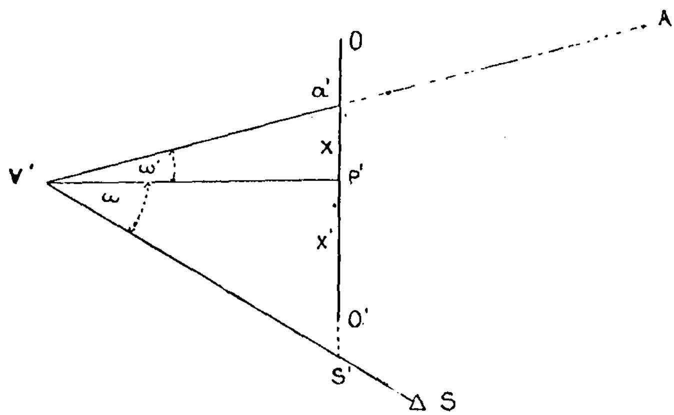


FIG. 6.

If the horizontal projection of a point, A, has been determined by the intersection of two rays from two stations, V^1 and V^2, a check is obtained if a third plate, taken from another camera station, V^3, shows the same point, as all rays to the same object, A, must intersect each other in the same point on the map.

The hypsometric determinations of the terrene can readily be accomplished after the selected points have been determined and plotted in horizontal plan in the foregoing manner. If the elevation of the station V (fig. 5) is known, the elevation of the line of horizon $O O'$ on the plate $m m' n' n$ can be easily obtained by adding the height of the instrument to the elevation of the station V. The elevations of all the points on the plate which are bisected by the line $O\ O'$ have the same elevation as the optical axis of the instrument at the station V, disregarding the effects of curvature and refraction. The apparent elevations of other secondary points, selected from the plate for the construction of the map, are obtained by determining their elevations above or their depression below the line $O\ O'$.

If D = horizontal distance of a point A from the station V;
= VA' (to be measured on the map) (fig. 5);
L = difference of elevation between point A and station V;
= $A\,A'$ (A' is the projection of A upon the horizontal plane through optical axis of instrument at station V);
d = horizontal distance of the picture of A from V;

Then we have from the similar triangles $V a' a$ and $V A' A$ the relation:

$$L : D = y : d \tag{1}$$

$$L = \frac{Dy}{d} \tag{2}$$

From the rectangular triangle $V P a'$ we find:

$$d = \frac{f}{\cos \omega'} = f \sec \omega' \tag{2^a}$$

whence:

$$L = \frac{Dy}{f \sec \omega'} \tag{3}$$

Should the point A be bisected by the vertical thread then $\omega' = O$ and $\sec \omega' = 1$, or:

$$L = \frac{Dy}{f} \tag{3^a}$$

(Formula 3^a would answer for all points of the perspective if the image plate were a cylindrical surface of radius $= f$ instead of being a plane, if the decagon were a circle.)

The differences of elevation, taken from the perspectives, are positive or negative according to the relative positions of their points in respect to the line of horizon $O\ O'$, whether above or below the same. In order to obtain the apparent elevations of these points above mean sea level their ordinates must be added to or subtracted from the elevation of the camera station. Two graphical instruments have been constructed based upon the formulas (1) and (2). The one serves to locate the points in horizontal plan by intersections after their abscissæ (x) have been measured upon the perspectives. The other serves to ascertain the differences of their elevations compared with the height of instrument after the abscissæ (x) and the ordinates (y) of the points have been measured upon the perspectives.

The use of these auxiliary instruments enables the draftsman to dispense with the construction of the decagons (representing the horizontal projections of the panoramas about the plotted camera stations), and as these polygons must be very carefully drawn, the dispensing with them altogether, especially if the area to be plotted is extensive and the stations numerous, will save much time and labor. It is particularly desirable to avoid the drawing of these polygons if the plotting is to be done on 1 to 25,000 or 1 to 50,000 scales, as the numerous decagons will intermingle in such a manner that great care and painstaking must be exercised to avoid confusion in selecting the proper lines from the intricate network of lines upon the face of the drawing. An attempt has been made to overcome this difficulty by drawing the different polygons in lines of different colors, but even this expedient failed when plotting on small scales.

Also, the hypsometric determinations of secondary points can be checked by comparing the elevations obtained in the preceding manner with the results obtained in the same way from other stations and on other plates.

Furthermore, any perspective containing one or more triangulation points (the elevations of which are determined by double zenith distances, or by any other instrumental method) will serve to check the horizontal adjustments of the instrument during the exposure of such plate, by comparing the elevation obtained from the perspective with the elevation of the trigonometrical point obtained by former instrumental measurements. Should the elevations of the triangulation points be unknown, this check can still be made by using the vertical circle of the instrument and measuring the vertical angles (α, fig. 5) from the panorama station to the surrounding triangulation points. By observing vertical angles from every camera station, a check on the measured values of the ordinates on the perspectives is obtained, inasmuch as then these ordinates can be computed and compared with those obtained by direct measurements on the perspectives.

From the triangles $V\,A'A$ and $V\,a'\,a$ we find (fig. 5):

$$\tan A\,VA' = \tan \alpha = \frac{L}{D} = \frac{y}{d}$$

According to formula (2″) we had

$$d = \frac{f}{\cos \omega'}$$

therefore:

$$\tan \alpha = \frac{y}{f} \cos \omega' \tag{4}$$

where ω' is the horizontal angle formed at V by the vertical plane containing the line of direction from the camera station V to the triangulation point and the vertical plane containing the line drawn from V to the principal point P of the perspective, which horizontal angle can also be measured directly on the horizontal limb of the camera theodolite in the field before leaving the station. If these different values are not in accord, the horizontal line on the perspective must be adjusted by determining the value of the ordinate y by aid of the following formula (5) derived from (4):

$$y = \frac{f}{\cos \omega'} \tan \alpha \tag{5}$$

From the preceding the necessity of the precise determination of the value for f is evident, and this value can readily be found if the area to be surveyed is provided with a number of triangulation points, marked by signals, and if the secondary points are of such a character (for instance, when surveying mountain ranges) that the differences of their elevations, compared with the elevation of the panorama station, are sufficiently great to give their ordinates on the perspectives, lengths sufficient to be readily measured. This will permit the determination of f by means of the line of horizon $O\ O'$, the latter being an element obtainable from the perspectives with a great degree of accuracy.

The instrument is placed over any well-determined point and adjusted, then it is turned in azimuth until the vertical thread bisects a geodetic point, which can readily be identified upon the image plate (desirable also that the ordinate y of such point be sufficiently long to assure a correct measurement).

Then will be given: the difference of elevation of bisected point and panorama station $= L$; the horizontal distance between these two $= D$, and $y =$ ordinate of bisected point; and from equation (3^a) we find for

$$f = \frac{Dy}{L}$$

a fairly accurate value, if the adjustments for securing the horizontal position of the instrument were carefully made and if the ordinate y was measured upon the negative plate correct within 0.1^{mm}.

The value for f can also be found, if the perspective contains several other points (besides the one bisected by the vertical thread) which can readily be identified, by measuring both the vertical and horizontal angles (by observing all the points, including the one bisected). The values for α and ω thus obtained, including the values of the abscissæ (x) and ordinates (y), measured upon the image or negative plate, will enable us to compute f by means of equations (1) and (4). By using the mean of these different determinations for f the computation, based upon the new values for x and y, can be repeated until an agreement is found.

Although the Italian pictures commanded an angle of but 42°, the greater part of them contain one or more triangulation points. In all similar cases, simultaneously with the determination of the value f (principal focal length), it can be ascertained whether the picture of the intersection of the crosswires coincides with the principal point of view P upon the perspective (fig. 7):

S' and S represent the pictures of two triangulation points upon the perspective $m\ n$;

$V =$ station point or point of view;

$S'\ O'$ and $S\ O =$ vertical lines from S' and S upon the line of horizon $O\ O' =$ ordinates y' and y of these two points S' and S.

It is desired to find $V P$ (vertical to plane $m\ n$) and the position of the point P in regard to O and O' or the abscissæ x and x' of the two triangulation points S and S'.

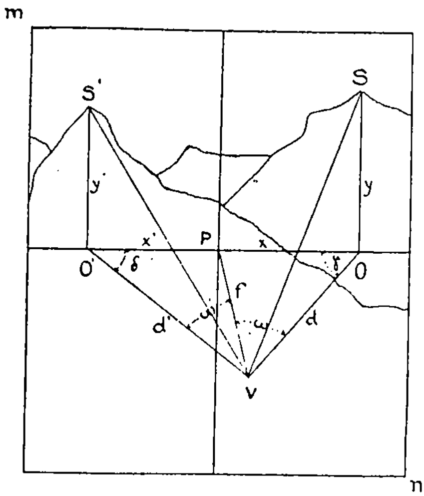


FIG. 7.

L and L' represent the differences of elevation between the camera station (V) or the horizon (OO') and the triangulation points S and S'. D and D' are the horizontal distances of S and S' from the camera station (V).

L, L', D, D', as well as y and y', are known or can be measured upon the chart projection and negative plate; therefore the horizontal distances d and d' of the pictured triangulation points S and S' from the point of view V can be computed from the following equations (see fig. 5):

$$d = \frac{D\,y}{L}$$

$$d' = \frac{D'\,y'}{L'}$$

The horizontal angle $O'\ V\ O\ (= \omega + \omega')$ being observed in the field, the other two angles, γ and δ, of the horizontal triangle $O'\ V\ O$ (fig. 7) can be computed as follows:

$$\tan\frac{\gamma - \delta}{2} = \frac{d' - d}{d + d'}\cot\frac{O'\ V\ O}{2}$$

By substituting

$$N \text{ for } \frac{\gamma - \delta}{2} \text{ and}$$

$$M \text{ for } \frac{\gamma + \delta}{2} = 90^\circ - \frac{O'\ V\ O}{2}$$

we will have:

$$\gamma = M + N$$
$$\delta = M - N$$

From the two triangles $O'\ P\ V$ and $O\ P\ V$ (both are rectangular at P) we find (fig. 7):

$$f = d \sin\gamma = d' \sin\delta$$
$$x = f\cot\gamma\ ; \quad x' = f\cot\delta$$

also the angles of orientation

$$\omega = 90^\circ - \gamma$$
$$\omega' = 90^\circ - \delta$$

The sum $x + x'$ must be equal to the value $O\ O'$ found by careful measurement upon the plate, and it must also be equal to the value of $O\ O'$ obtained from the following formula:

$$OO' = \frac{(d + d') \sin \frac{O'VO}{2}}{\cos \frac{\delta - \gamma}{2}}$$

Should the horizontal angle $O V O'$ not have been measured in the field, then the angles γ and δ can be computed by carefully measuring $O'O$ on the negative plate and using the following well-known formulas:

$$\tan \frac{\delta}{2} = \sqrt{\frac{(p - d')\ (p - OO')}{p\ (p - d)}} \text{ and}$$

$$\tan \frac{\gamma}{2} = \sqrt{\frac{(p - d)\ (p - OO')}{p\ (p - d')}}$$

where $p = \dfrac{d + d' + OO'}{2}$

The angles of elevation α and α', which are either obtained by direct measurement in the field or computed from the formulas:

$$\tan \alpha = \frac{L}{D}$$
$$\tan \alpha' = \frac{L'}{D'}$$

serve to obtain check values for the values of y and y', measured upon the negative plate, by using the formulas:

$$y = \frac{f}{\cos \omega} \tan \alpha \text{ and}$$

$$y' = \frac{f}{\cos \omega'} \tan \alpha'$$

The value for f in above formulas being the same as found (page 60) from the equation:

$$f = d \sin \gamma = d' \sin \delta.$$

By repeating the computation with these new values for y and y' the true value for f can be obtained very closely. For all practical purposes, however, it will suffice to take several pictures with a constant focal length, and to take the mean value of the different f determined from these pictures.*

* A comparison of the value read off on the graduated metal plate on the objective cylinder with the result obtained by computation will give the correction to be applied to the reading on said graduation.

Examples showing the application of the methods described in the foregoing pages.

I.

In the panorama obtained from the "Punta Percia," September 19, 1884 (this peak is on the divide between the valleys of the "Rhêmes" and the "Valsavaranche"), two stations, "Punta Rouletta" and "Gran Punta di Nomenon," of the new Italian geodetic triangulation appear upon the same plate (see fig. 8).

The following values are given for the computation:

Elevation of Punta Rouletta $= 3384 \cdot 10^{m}$
Elevation of Gran Punta di Nomenon $= 3488 \cdot 42^{m}$
} Taken from the catalogue of triangulation points.

Elevation of Punta Percia $= 3202 \cdot 3 =$ horizon of panorama station.

Distance: Percia-Rouletta $= D = 3250^{m}$
Distance: Percia-Nomenon $= D' = 9720^{m}$
} Measured graphically upon the projection on drawing board, scale: $\frac{1}{50000}$.

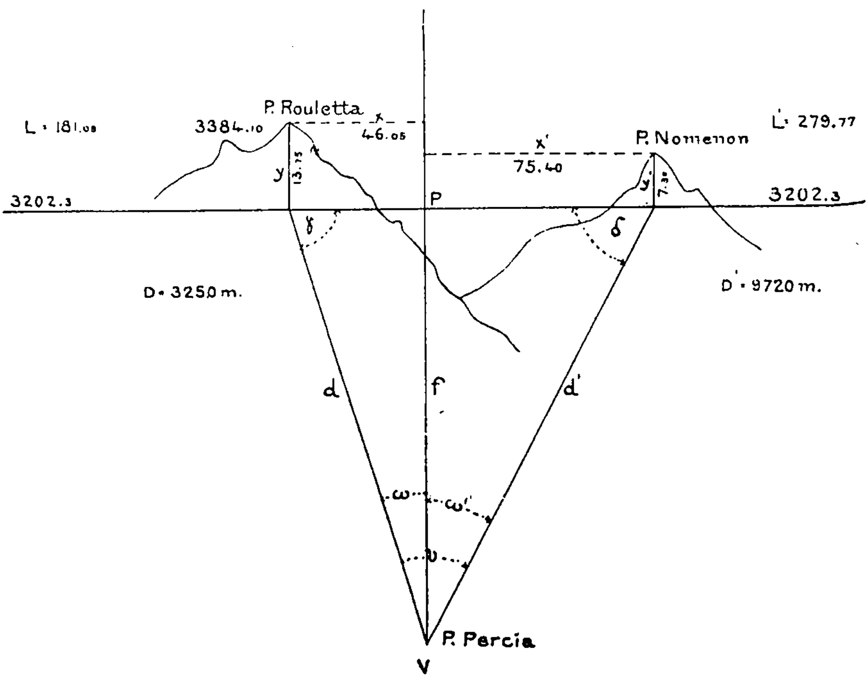


FIG. 8.

The angle V (fig. 8) at Percia included by the horizontal directions to Rouletta and Nomenon $= 28^{\circ}\ 02'\ 30''$.

It is desired to find:

(1) Focal distance $= f$, the preliminary value, read off on the graduation on plate attached to objective cylinder, is found to be about $244 \cdot 5^{mm}$.

(2) The position of the principal point of view, which is fixed by the determination of the abscissæ x and x'.

(3) The position of the line of horizon, which will be located by ascertaining the values for y and y'.

The difference in elevation of Percia and Rouletta, according to the given data, is:

$$3384{\cdot}10 - 3202{\cdot}30 = 181{\cdot}80^{m}$$

The difference in elevation between Percia and Nomenon is

$$3488{\cdot}42 - 3202{\cdot}30 = 286{\cdot}12^{m}.$$

If we consider that the distances D and D' are great, and that the above values are the apparent elevations, we will have to make a correction for curvature and refraction to obtain the true values of L and L', as follows:

$$\begin{aligned} \text{Difference of elevations: Percia–Rouletta,} &= 181{\cdot}80^{m} \\ \text{Correction for curvature and refraction,} &= -0{\cdot}70^{m} \\ L &= 181{\cdot}09^{m} \\ \text{Difference of elevations: Percia–Nomenon,} &= 286{\cdot}12^{m} \\ \text{Correction for curvature and refraction,} &= -6{\cdot}35^{m} \\ L' &= 279{\cdot}77^{m} \end{aligned}$$

By measuring the ordinates (y and y') and the abscissæ (x and x') upon the negative plate with a millimetre scale, provided with a vernier, which enables the computer to read $\frac{5}{100}^{mm}$ (the vernier being divided to read $\frac{1}{20}$ of the graduation), we find:

The coordinates of Punta–Rouletta: $x = 46{\cdot}05^{mm}$ $y = 13{\cdot}75^{mm}$

The coordinates of Punta–Nomenon: $x' = 75{\cdot}40^{mm}$ $y' = 7{\cdot}30^{mm}$

The value of d is found from the following formula:

$$d = \frac{Dy}{L} \text{ (see page 60).}$$

$$\begin{aligned} \log D = \log 3250 &= 3{\cdot}5118834 \\ \log y = \log 0{\cdot}01375 &= 8{\cdot}1383027 \\ \text{co.log } L = \text{co.log } 181{\cdot}09 &= 7{\cdot}7421055 \\ \hline \log d &= 9{\cdot}3922916 \\ d &= 0{\cdot}24677^{m} = 246{\cdot}77^{mm} \end{aligned}$$

d is similarly found:

$$\begin{aligned} \log D' = \log 9720 &= 3{\cdot}9876663 \\ \log y' = \log 0{\cdot}00730 &= 7{\cdot}8633229 \\ \text{co.log } L' = \text{co.log } 279{\cdot}77 &= 7{\cdot}5531989 \\ \hline \log d' &= 9{\cdot}4041881 \\ d' = 0{\cdot}25362^{m} &= 253{\cdot}62^{mm} \\ d + d' &= 0{\cdot}50039^{m} \\ d' - d &= 0{\cdot}00685^{m} \end{aligned}$$

The angles γ and δ (fig. 8) are computed by aid of the formula:

$$\tan \frac{\gamma-\delta}{2} = \frac{d'-d}{d+d'} \cot. \frac{O'VO}{2} \text{ (page 60)}$$

as follows:

$$V = 28^\circ\ 02'\ 30'' ; \qquad \frac{V}{2} = 14^\circ\ 01'\ 15''$$

$$\gamma+\delta = 180^\circ - V = 151^\circ\ 57'\ 30'' ; \frac{\gamma+\delta}{2} = 75^\circ\ 58'\ 45'' = M$$

$$\log (d'-d) = \log 0{\cdot}00685 \qquad = 7{\cdot}8356906$$

$$\text{log.cot } \frac{V}{2} = \log \cot\ 14^\circ\ 01'\ 15'' = 0{\cdot}6025567$$

$$\text{co.log } (d+d') = \text{co.log } 0{\cdot}50039 \qquad = 0{\cdot}3006914$$

$$\log \frac{\gamma-\delta}{2} \qquad = 8{\cdot}7389387$$

$$\frac{\gamma-\delta}{2} = 3^\circ\ 08'\ 16''{\cdot}1 = N$$

Hence $M + N = \gamma = 79^\circ\ 07'\ 01''{\cdot}1$ and
$M - N = \delta = 72^\circ\ 50'\ 28''{\cdot}9$

The computation of f based on the formulas (page 60)

$$f = d \sin \gamma \text{ and } f = d' \sin \delta,$$

gives the following two values:

$$\log d = 9{\cdot}3922916 \qquad \log d' = 9{\cdot}4041881$$
$$\log \sin \gamma = 9{\cdot}9921180 \qquad \log \sin \delta = 9{\cdot}9802269$$

$$\log f = 9{\cdot}3844096 \qquad \log f = 9{\cdot}3844150$$
$$f = 0{\cdot}242331^{\text{m}} \qquad f = 0{\cdot}242334^{\text{m}}$$

mean value for $f = 242{\cdot}332^{\text{mm}}$.

The abscissæ x and x' are computed by aid of the two formulas (page 60)

$$x = f \tan \omega \qquad x' = f \tan \omega'$$
$$\omega = 90^\circ - \gamma = 10^\circ\ 52'\ 58''.9 \qquad \omega' = 90^\circ - \delta = 17^\circ\ 09'\ 31''.1$$
$$\log f = 9{\cdot}3844123 \text{ (mean log)} \qquad \log f' = 9{\cdot}3844123 \text{ (mean log)}$$
$$\log \tan \omega = 9{\cdot}2838945 \qquad \log \tan \omega' = 9{\cdot}4896221$$

$$\log x = 8{\cdot}6683068 \qquad \log x' = 8{\cdot}8740344$$
$$x = 46{\cdot}59^{\text{mm}} \qquad x' = 74{\cdot}82^{\text{mm}}$$
$$x \text{ measured on plate} = 46{\cdot}05^{\text{mm}} \qquad \text{measured } x' = 75{\cdot}40^{\text{mm}}$$
$$\text{diff.} = {\cdot}54^{\text{mm}} \qquad \text{diff.} = {\cdot}42^{\text{mm}}$$

Computation of the ordinates y and y':

$$y = \frac{f}{\cos\omega}\tan\alpha \qquad y' = \frac{f}{\cos\omega'}\tan\alpha' \quad \text{(page 61).}$$

α = angle of elevation of Punta Rouletta $= 3^\circ\ 11'\ 30''$
α' = angle of elevation of Punta Nomenon $= 1^\circ\ 38'\ 30''$ } from station Percia. See model in Supplement.

$\log f =$	$9{\cdot}3844123$	$\log f =$	$9{\cdot}3844123$
$\log\tan\alpha =$	$8{\cdot}7463444$	$\log\tan\alpha' =$	8.4572812
$\text{co.log}\cos\omega =$	$0{\cdot}0078820$	$\text{co.log}\cos\omega' =$	$0{\cdot}0197731$
$\log y =$	$8{\cdot}1386387$	$\log y' =$	$7{\cdot}8614666$
$y =$	$13{\cdot}761^{mm}$	$y' =$	$7{\cdot}269^{mm}$
y measured on plate $=$	$13{\cdot}75^{mm}$	measured $y' =$	$7{\cdot}30^{mm}$
diff. $=$	0.01^{mm}	diff. $=$	$0{\cdot}03^{mm}$

II.

Owing to the fact that the distances D and D' in the example treated in Subject I are large, while the ordinates y and y' are quite small (due to the small difference in the elevations of the two points Rouletta and Nomenon compared with the camera station Percia), it will be preferable first to determine f by means of the abscissæ and then to compute the values for the ordinates (y and y'), based upon this value of f and the observed angles of orientation ω_1 and ω_1' (see Supplement, remarks to station Percia).

The direction to the main point of view P of the perspective containing the pictures of Rouletta and Nomenon is $= 350^\circ\ 00'\ 00''$

Direction to point Nomenon (signal) $= 332^\circ\ 42'\ 00''$
Direction to point Rouletta (signal) $= 0^\circ\ 44'\ 30''$

Direction to Rouletta $= 360^\circ\ 44'\ 30''$
Direction to point P $= 350^\circ\ 00'\ 00''$

Hence, $\omega_1 = 10^\circ\ 44'\ 30''$

Direction to point P $= 350^\circ\ 00'\ 00''$
Direction to point Nomenon $= 332^\circ\ 42'\ 00''$

Hence, $\omega_1' = 17^\circ\ 18'\ 00''$

(a) Computation of f:

$$f = \frac{x}{\tan\omega_1}$$

$\log x = \log 46{\cdot}05 = 1{\cdot}6632296$
$\text{co.log}\tan\omega_1 = \text{co.log}\tan 10^\circ\ 44'\ 30'' = 0{\cdot}7219207$

$\log f = 2{\cdot}3851503$
$f = 242{\cdot}745^{mm}$

(b)

$$f = \frac{x}{\tan \omega_1'}$$

$$\log x' = \log 75{\cdot}40 = 1{\cdot}8773713$$
$$\text{co.log} \tan \omega_1' = \text{co.log} \tan 17^\circ\, 18'\, 00'' = 0{\cdot}5065903$$

$$\log f = 2{\cdot}3839616$$
$$f = 242{\cdot}082^{\text{mm}}$$

$$\text{Mean value for } f = 242{\cdot}41^{\text{mm}}$$
$$\text{and from computation I we had } f = 242{\cdot}33^{\text{mm}}$$

$$\text{diff.} = 0{\cdot}08^{\text{mm}}$$

Computation of the ordinates y and y':

$$y = \frac{f}{\cos \omega_1} \tan \alpha$$

$$\log f = \log 242{\cdot}41 = 2{\cdot}3845505$$
$$\log \tan \alpha = \log \tan\ 3^\circ\, 11'\, 30'' = 8{\cdot}7463444$$
$$\text{co.log} \cos \omega_1 = \text{co.log} \cos 10^\circ\, 44'\, 30'' = 0.0076774$$

$$\log y = 1{\cdot}1385723$$
$$y = 13{\cdot}758^{\text{mm}}$$
$$\text{and computation I gave } y = 13{\cdot}761^{\text{mm}}$$

$$\text{diff.} = 0{\cdot}003^{\text{mm}}$$

$$y' = \frac{f}{\cos \omega^1{}'} \tan \alpha'$$

$$\log f = \log 242{\cdot}41 = 2{\cdot}3845505$$
$$\log \tan \alpha' = \log \tan 1^\circ\, 38'\, 30'' = 8{\cdot}4572812$$
$$\text{co.log} \cos \omega' = \text{co.log} \cos 17^\circ\, 18'\, 00'' = 0{\cdot}0201054$$

$$\log y' = 0{\cdot}8619371$$
$$y' = 7{\cdot}277^{\text{mm}}$$
$$\text{and computation I gave: } y' = 7{\cdot}269^{\text{mm}}$$

$$\text{diff.} = 0{\cdot}008^{\text{mm}}$$

III.

The following computation is of greater interest, the camera station having been selected over a trigonometrical point of the Italian geodetic triangulation system, thus admitting a direct comparison between the elements of the perspective and the exact values of these same elements, taken from the data of the triangulation work.

In the round of perspectives, obtained on September 21, 1884, vertically above the trigonometrical point near the Royal Hunting Lodge of Valsavaranche, there is one plate (P^5) which contains the pictures of two triangulation points, "Punta Ruja" and "Gran Punta di Nomenon," of the new geodetic triangulation.

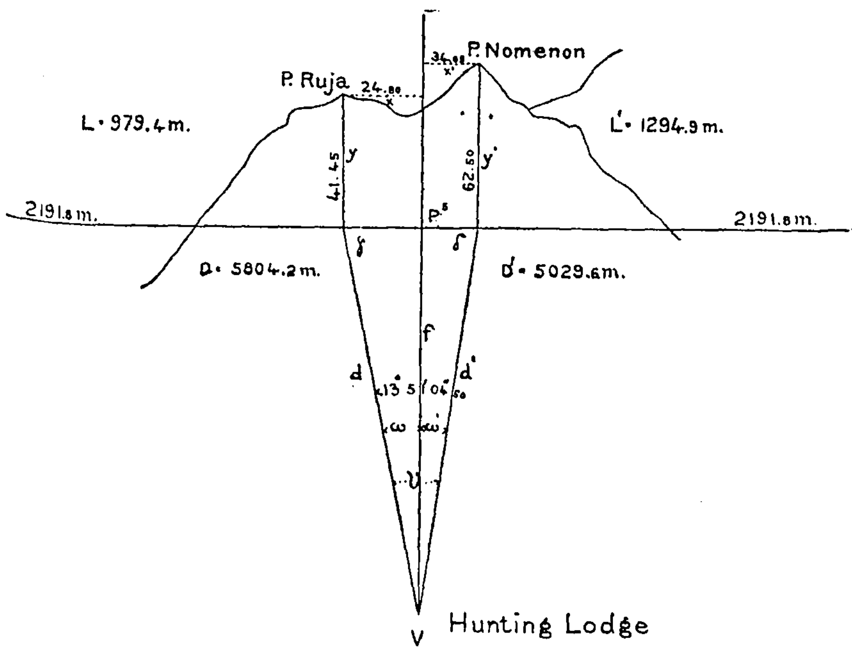


FIG. 9.

The following data are given for the computation:

Elevation of Punta Ruja (signal)	$= 3173 \cdot 5^m$
Elevation of Gran Punta di Nomenon (signal)	$= 3488 \cdot 4^m$
Elevation of the horizon of panorama station near the Royal Hunting Lodge of Valsavaranche	$= 2191 \cdot 8^m$
The triangle side Hunting Lodge — Ruja $= D$	$= 5804 \cdot 2^m$
The triangle side Hunting Lodge — Nomenon $= D'$	$= 5029 \cdot 6^m$

The angle, V, between Ruja and Nomenon is $= 13^\circ\ 51'\ 04''\cdot 50$. (See fig. 9.)

It is desired to find:

(1) The focal length, f, approximately found by reading the graduation on objective tube. $= 244 \cdot 5^{mm}$.

(2) The position of the principal point of view (located by the abscissæ x and x').

(3) The true position of the line of horizon (located by the ordinates y and y').

By careful measurements (made as in the preceding) we find:

The coordinates of Punta Ruja: $x = 24{\cdot}80^{mm}$; $y = 41{\cdot}45^{mm}$

The coordinates of Punta Nomenon: $x' = 34{\cdot}05^{mm}$; $y' = 63{\cdot}50^{mm}$

Elevation of Punta Ruja	$= 3173.5^{m}$
Elevation of point V	$= 2191{\cdot}8^{m}$
Apparent difference of elevation	$= 981{\cdot}7^{m}$
Correction for refraction and curvature	$= -\ 2{\cdot}3^{m}$
True difference of elevation	$= 979{\cdot}4^{m} = L$
Elevation of Punta di Nomenon	$= 3488{\cdot}4^{m}$
Elevation of point of view (V)	$= 2191{\cdot}8^{m}$
Apparent difference of elevation	$= 1296{\cdot}6^{m}$
Correction for curvature and refraction	$= -\ 1{\cdot}7^{m}$
True difference of elevation	$= 1294{\cdot}9^{m} = L'$

Computation of $d = \dfrac{Dy}{L}$

$$\begin{aligned}
\log D &= \log\ 5804{\cdot}2 &&= 3{\cdot}7637424\\
\log y &= \log\ 0{\cdot}04145 &&= 8{\cdot}6175245\\
\text{co.}\log L &= \text{co.}\log\ 979{\cdot}4 &&= 7{\cdot}0090399\\
&& \log d &= 9{\cdot}3903068\\
&& d &= 0{\cdot}245644^{m} = 245{\cdot}644^{mm}
\end{aligned}$$

Computation of $d' = \dfrac{D'y'}{L'}$

$$\begin{aligned}
\log D' &= \log\ 5029{\cdot}6 &&= 3{\cdot}7015334\\
\log y' &= \log\ 0{\cdot}06350 &&= 8{\cdot}8027737\\
\text{co.}\log L' &= \text{co.}\log\ 1294{\cdot}9 &&= 6{\cdot}8877638\\
&& \log d' &= 9{\cdot}3920709\\
&& d' &= 0{\cdot}246644^{m} = 246{\cdot}644^{mm}
\end{aligned}$$

$$d + d' = 492{\cdot}29^{mm}$$
$$d' - d = 1{\cdot}00^{mm}$$

Computations ' the angles γ and δ:

$$\tan\frac{\gamma-\delta}{2} = \frac{d'-d}{d+d'}\cot\frac{V}{2}$$

$$V = 13^\circ\ 51'\ 04''{\cdot}50$$

$$\frac{V}{2} = 6^\circ\ 55'\ 32''{\cdot}25$$

$$\gamma + \delta = 180^\circ - V = 166^\circ\ 08'\ 55''{\cdot}50$$

$$\frac{\gamma+\delta}{2} = 83^\circ\ 04'\ 27''{\cdot}75 = M$$

$$\log (d' - d) = \log \quad 0{\cdot}00100 \quad = 7{\cdot}0000000$$

$$\log \cot \frac{V}{2} = \log \cot 6^\circ\ 55'\ 32''{\cdot}3 = 0{\cdot}9155406$$

$$\text{co.log}\ (d + d') = \text{co.log} \quad 0{\cdot}49229 \quad = 0{\cdot}3077790$$

$$\log \tan \frac{\gamma - \delta}{2} = 8{\cdot}2233196$$

$$\frac{\gamma - \delta}{2} = 0^\circ\ 57'\ 29''{\cdot}10 = N$$

$$M + N = 84^\circ\ 01'\ 56''{\cdot}9 = \gamma$$
$$M - N = 82^\circ\ 06'\ 58''{\cdot}7 = \delta$$

Computation of $f = d \sin \gamma = d' \sin \delta$

$$\log d = \quad \log 0{\cdot}24564 \quad = 9{\cdot}3903068$$
$$\log \sin \gamma = \log \sin 84^\circ\ 01'\ 56''{\cdot}9 = 9{\cdot}9976401$$

$$\log f = 9{\cdot}3879469$$
$$f = 0{\cdot}244313$$

$$\log d' = \quad \log 0{\cdot}24664 \quad = 9{\cdot}3920709$$
$$\log \sin \delta = \log \sin 82^\circ\ 06'\ 58''{\cdot}7 = 9{\cdot}9958757$$

$$\log f = 9{\cdot}3879466$$
$$f = 0{\cdot}244313^{\text{m}}$$

Mean value for $f = 244{\cdot}31^{\text{mm}}$

Computation of the abscissæ:

$$x = f \tan \omega$$
$$\omega = 90^\circ - \gamma = 5^\circ\ 58'\ 03''{\cdot}1$$
$$\log (\text{mean value of } f) = 9{\cdot}3879468$$
$$\log \tan \omega = \log \tan 5^\circ\ 58'\ 03''{\cdot}1 = 9{\cdot}0192462$$

$$\log x = 8{\cdot}4071930$$
$$x = 0{\cdot}025538^{\text{m}} = 25{\cdot}54^{\text{mm}}$$

$$x' = f \tan \omega'$$
$$\omega' = 90^\circ - \delta = 7^\circ\ 53'\ 01''{\cdot}3$$
$$\log f = 9{\cdot}3879468$$
$$\log \tan \omega' = \log \tan 7^\circ\ 53'\ 01''{\cdot}3 = 9{\cdot}1413601$$

$$\log x' = 8{\cdot}5293069$$
$$x' = 0{\cdot}033830 = 33{\cdot}83^{\text{mm}}$$

$x = 25{\cdot}54^{\text{mm}}$	$x' = 33{\cdot}83^{\text{mm}}$
measured $x = 24{\cdot}80^{\text{mm}}$	measured $x' = 34{\cdot}05^{\text{mm}}$
diff. $= 0{\cdot}74^{\text{mm}}$	diff. $= 0{\cdot}22^{\text{mm}}$

Mean difference $= 0.48^{\text{mm}}$ = correction for principal point of view (vertical thread) on the plate.

IV.

The plate (P^5) treated of, under Subject III, is the fifth perspective of the ten plates forming the panorama. These perspectives were obtained, as previously mentioned, by moving the optical axis of the camera successively by 36° in horizontal plan, as indicated in the examples for phototopographic stations in the Supplement.

The orientation for the entire panorama (set of ten plates) was obtained by the exposure of the first plate (P') by directing the optical axis for this exposure to the trigonometrical point "Punta Chandellei" (i. e., by bisecting the signal at Punta Chandellei with the vertical thread of the camera). (See fig. 10.)

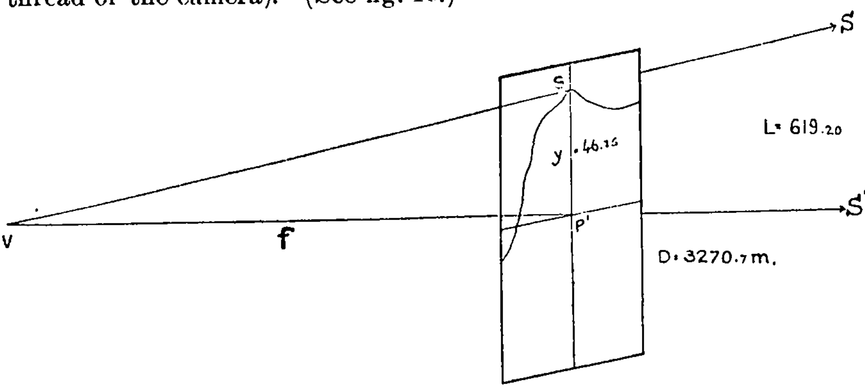


FIG. 10.

We find, therefore, on plate P' (fig. 11) the picture of the signal Punta Chandellei (S in fig. 10), bisected by the vertical thread, and this fact enables us to obtain the value for f more readily from this plate by means of the following formula (fig. 10):

$$VP' = f = \frac{VS' \times P'S}{SS'}$$

For this computation the following data are given:

D = triangle side: Royal Lodge (signal)—Punta Chandellei (signal) = VS' = 3270·7 m

The elevation of Punta Chandellei = 2811·72 m

The elevation of Royal Lodge—elevation of horizon of V = 2191·80 m

The ordinate y of image (S) of P. Chandellei, measured on plate P' = 46·25 mm

Computation of L = SS':

Elevation of Punta Chandellei	= 2811·72 m
Elevation of point V (camera horizon)	= 2191·80 m
Apparent difference of elevation	= 619·92 m
Correction for curvature and refraction	= —0·72 m
True difference of elevation	= 619·20 m = L.

Computation of $f = \frac{Dy}{L} = \frac{VS \times P'S}{SS'}$

$$\begin{aligned}
\log D &= \log 3270{\cdot}7 &&= 3{\cdot}5146407 \\
\log y &= \log 0{\cdot}04625 &&= 8{\cdot}6651117 \\
\text{co.}\log L &= \text{co.}\log 619{\cdot}20 &&= 7{\cdot}2081691 \\
\hline
\log f & &&= 9{\cdot}3879215
\end{aligned}$$

$$f = 0{\cdot}24430 = 244{\cdot}30^{\text{mm}}$$

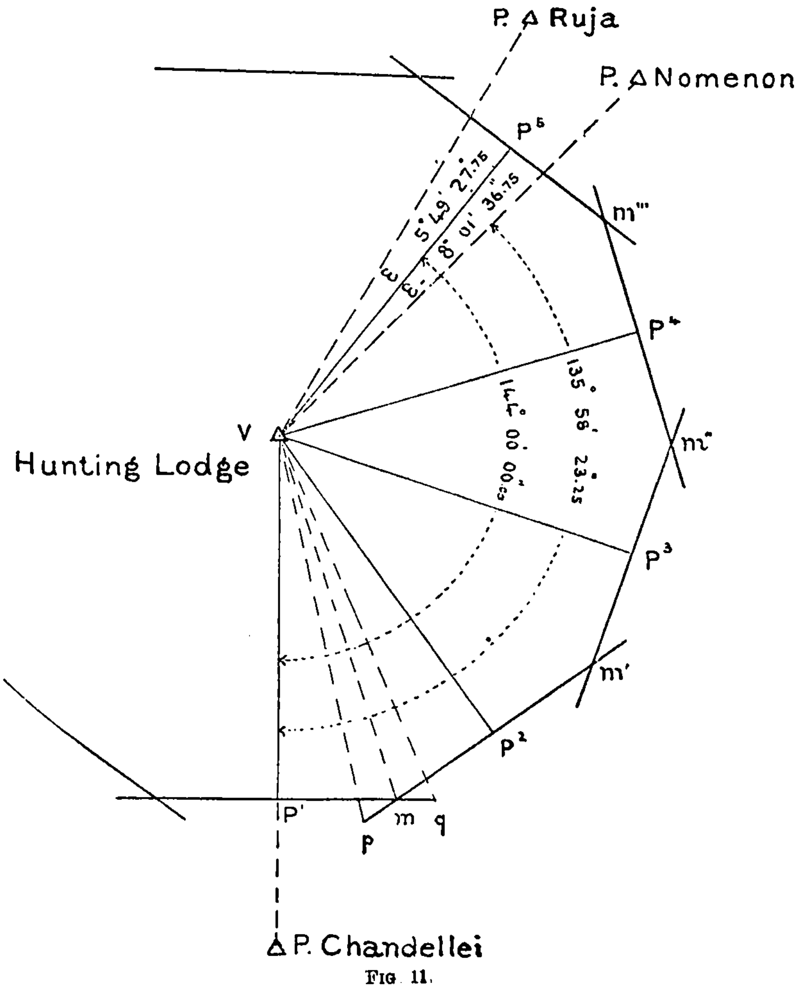


FIG. 11.

V.

Perspective P^5 (fig. 11) shows, as previously mentioned, the pictures of Punta Ruja and Punta Nomenon, while the vertical thread of plate P' bisects the picture of the signal Punta Chandellei, and as the horizontal angles $P'V\ P^2$, $P^2V\ P^3$, $P^3V\ P^4$ . . . . are uniformly $=36°$, the orientation of plate P^5, for instance, will be:

$$P'\ VP^5 = 4 \times 36° = 144°$$

From the geodetic records we take the angle: Chandellei-Royal Lodge-Nomenon $= 135°\ 58'\ 23''{\cdot}25$ and the angles: P^5VP'—(Nomenon-Royal Lodge-Chandellei)$=\omega'$

$$\text{(Ruja-Royal Lodge-Nomenon)} - \omega' = \omega$$

With aid of these values for ω, ω' and the value for f, see Subject III, we can compute the values for x and y very closely. (See fig. 11.)

The angle: (Chandellei-Lodge-P^5)$=P'\ VP^5$	$=144°\ 00'\ 00''$
The angle: (Chandellei-Lodge Nomenon)$=P'\ V$ Nomenon.	$=135°\ 58'\ 23''{\cdot}25$
Therefore ω'	$=\ \ 8°\ 01'\ 36''{\cdot}75$

$$\omega = \text{(Ruja-Lodge-Nomenon)} - \omega' = 13°\ 51'\ 04''{\cdot}50 - 8°\ 01'\ 36''{\cdot}75 = 5°\ 49'\ 27''{\cdot}75$$

Computation of the abscissæ:

$$x = f \tan \omega$$

$\log f = \log 244.31 = 9{\cdot}3879468$ (see computation Subject III)

$\log\tan\omega = \log\tan 5°\ 49'\ 27''{\cdot}75 = 9{\cdot}0086263$

$\log x = 8{\cdot}3965731$

$x = 24{\cdot}92^{\mathrm{mm}}$

by measurement: $x = 24{\cdot}80^{\mathrm{mm}}$

diff. $= 0{\cdot}12^{\mathrm{mm}}$

$$x' = f \tan \omega'$$

$\log f = 9{\cdot}3879468$

$\log \tan \omega' = \log \tan 8°\ 01'\ 36''{\cdot}75 = 9{\cdot}1492780$

$\log x' = 8{\cdot}5372248$

$x' = 34{\cdot}45^{\mathrm{mm}}$

by measurement: $x' = 34{\cdot}05^{\mathrm{mm}}$

diff. $= 0{\cdot}40^{\mathrm{mm}}$

The mean difference: $\dfrac{0{\cdot}12 + 0{\cdot}40}{2} = 0{\cdot}26^{\mathrm{mm}}$ is the correction which should be applied to the vertical axis of plate P^5.

Computation of the ordinates:

$$y = \frac{fL}{\cos \omega D}$$

$$\begin{aligned}
\log f &= \log 0{\cdot}24431 &&= 9{\cdot}3879468 \\
\log L &= \log 979{\cdot}4 &&= 2{\cdot}9909601 \\
\text{co.log} \cos \omega &= \text{co.log} \cos 5^\circ\, 49'\, 27''{\cdot}75 &&= 0{\cdot}0022478 \\
\text{co.log}\, D &= \text{co.log}\, 5804{\cdot}2 &&= 6{\cdot}2362576
\end{aligned}$$

$$\log y = 8{\cdot}6174123$$

$$y = 0{\cdot}041439$$
$$y = 41{\cdot}44^{\text{mm}}$$
$$\text{by measurement: } y = 41{\cdot}45^{\text{mm}}$$

$$y' = \frac{f\,L'}{\cos \omega'\, D'}$$

$$\begin{aligned}
& & \log f &= 9{\cdot}3879468 \\
\log L' &= & \log 1294{\cdot}9 &= 3{\cdot}1122362 \\
\text{co.log} \cos \omega' &= & \text{co.log} \cos 8^\circ\, 01'\, 36''{\cdot}75 &= 0{\cdot}0042759 \\
\text{co.log}\, D' &= & \text{co.log}\, 5029{\cdot}6 &= 6{\cdot}2984666
\end{aligned}$$

$$\log y' = 8{\cdot}8029255$$
$$y' = 0{\cdot}063522^{\text{m}} = 63{\cdot}52^{\text{mm}}$$
$$\text{by measurement: } y' = 63{\cdot}50^{\text{mm}}$$

These five examples will elucidate the various relations between parts of the perspectives and the terrene, as well as give the means to judge of the degree of accuracy of phototopography.

In practical work it would become too time consuming to make such computations for all the plates, or even for all the panoramas, with the necessary minute graphical measurements.

If the camera has been carefully constructed it is generally accepted that its optical axis is vertical to the image plate, and the value for f for any, or for all, panoramas which were obtained with the same objective and with the same constant focal length (that is to say, obtained with the same reading of the scale on the objective tube) is computed in practice in the following manner:

As the horizontal shiftings

$$P'VP^2,\ P^2VP^3,\ P^3VP^4\ \ \ldots\ldots$$

are all $= 36^\circ$, the angles

$$P'Vm,\ mVP^2,\ P^2Vm'\ \ \ldots\ldots$$

will be $= 18^\circ$ each. The value of $P'm = mP^2 = P^2m' = \ldots\ldots$ is $= f \tan 18^\circ = x^{\text{m}} =$ the value of the greatest abscissa of the plates; hence,

$$f = \frac{x^{\text{m}}}{\tan 18^\circ}$$

In the preceding (page 54) it has been stated that two adjoining plates have a common margin, representing the terrene included by an angle $p\,V q$ (fig. 11). If the negative plates are sufficiently clear, it will be an easy matter to identify a point, m (fig. 11), on the two strips $p\ q$ of two adjoining plates, which will be on or near the line of horizon, and which will be $m\,P'$ distant from the vertical axis of plate P' and $m\,P^2$ distant from the vertical axis of plate P^2. If we select such points m', m^2, m^3, ... which can be identified upon two adjoining plates, $P^2—P^3$, $P^3—P^4$, $P^4—P^5$, ... we will obtain a mean value, x^m, for the entire panorama, by aid of which a good value for f can be obtained from the formula:

$$f = \frac{x^m}{\tan 18^\circ}$$

For example: By means of ten negatives of a panorama station, occupied with the latest improved Italian apparatus, it was found:

$$\left.\begin{array}{llcl}
 & & & mm \\
x^m \text{ for} & P' - P^2 & = & 77{\cdot}10 \\
x^m & P^2 - P^3 & = & 77{\cdot}15 \\
x^m & P^3 - P^4 & = & 77{\cdot}00 \\
x^m & P^4 - P^5 & = & 77{\cdot}40 \\
x^m & P^5 - P^6 & = & 77{\cdot}40 \\
x^m & P^6 - P^7 & = & 77{\cdot}20 \\
x^m & P^7 - P^8 & = & —— \\
x^m & P^8 - P^9 & = & —— \\
x^m & P^9 - P^{10} & = & 77{\cdot}40. \\
x^m & P^{10} - P' & = & 76{\cdot}90
\end{array}\right\} x^m = 77{\cdot}194^{mm} = \text{mean value.}$$

$$\begin{aligned}
\log 77{\cdot}194 &= 1{\cdot}8875835 \\
\text{co.log} \tan 18^\circ &= 0{\cdot}4882240 \\
\hline
\log f &= 2{\cdot}3758075 \\
f &= 237{\cdot}6^{mm}
\end{aligned}$$

The above values were obtained by using the negative plates and reading the measurements, obtained by means of dividers, off the graduated rulers of the graphical instruments of the Royal Military Geographical Institute.

Using the positives (prints) of the same panorama, the following results were obtained:

$$\begin{array}{llcl}
 & & & mm \\
x^m \text{ for} & P' - P^2 & = & 76{\cdot}25 \\
x^m & P^2 - P^3 & = & 76{\cdot}20 \\
x^m & P^3 - P^4 & = & 76{\cdot}10 \\
x^m & P^9 - P^{10} & = & 76{\cdot}70 \\
x^m & P^{10} - P' & = & 76{\cdot}00
\end{array}$$

From this the mean value for x^{m} is found $= 76{\cdot}25^{\text{mm}}$

$$\log 76.25 = 1{\cdot}8822398$$
$$\text{co.log}\tan 18^\circ = 0{\cdot}4882240$$
$$\log f = 2{\cdot}3704628$$
$$f = 234{\cdot}67^{\text{mm}} = 234{\cdot}7^{\text{mm}}$$

The negatives gave $x^{\text{m}} = 77{\cdot}19$

The positives gave $x^{\text{m}} = 76{\cdot}25$

diff. — $0{\cdot}94^{\text{mm}}$

This shortening of the greatest abscissa of half a millimetre at either side of the vertical thread on the prints is due to shrinkage of the 24 x 18 cm paper. The positive prints being extensively used in the map construction, this shrinkage must be taken into account.

Chapter III.

THE EXECUTION OF THE FIELDWORK.

By a close inspection of the various panoramas upon which the construction of the map is based, it readily becomes evident that not all of the perspectives are adapted for illustrative purposes (to be used to illustrate the Alpine character).

For cartographic purposes the panoramic views should not be taken at too great a distance from the terrene which is to be delineated, in order to preserve and show as much as possible of the topographic details and also that the selected triangulation points may appear sufficiently clear and well defined in the two or three views of the panorama set which contain their pictures.

It will be best to select the distance from which views for illustrative purposes are to be taken in such a manner that the camera station may command an extensive field of the terrene. Illustrative views should therefore, be taken from isolated prominent points and from such that can readily be recognized upon the topographic map containing the section photographed, thus assuring a rapid orientation and giving the student of the map the means to form a correct opinion of the topographic character of the terrene by comparing such illustrative views with the map.

With this object in view, a selection was made from the numerous panoramas obtained during former years and the selected perspectives were copied with pen and ink by expert draftsmen, whose drawings were reproduced by photozincography and published with the addition of all data needed to identify the camera station and to enable the student to orient each view properly upon the map.

The requirements which the camera theodolite constructed under the auspices of the Royal Military Geographical Institute was to satisfy have been previously mentioned, and as a result of the improvements suggested and made by practical experience the apparatus now in use in Italy furnishes the elements of the panorama station in such completeness that little needs to be added by extra operations and computations before the map construction is begun, and, with due reference to the rough character of the terrene, the apparatus can easily be dismembered into pieces small enough to be taken to the most inaccessible points. Three small-sized knapsacks, each weighing 7 to 8 kilogrammes, contain the theodolite, the camera, and ten negative plates. They are carried by two soldiers and one guide, each bearing one tripod leg, to be used as an alpenstock.

The fieldwork consists in the fitting up of a small laboratory, conveniently located with due regard to communication, to a central position, to facilities of transportation, accessibility of good water, etc. A sufficient number of bromo-gelatin plates are kept on hand packed in air and water tight cases. From this laboratory the camp outfit is taken to the neighborhood of the stations which are to be occupied. The observer and party take daily excursions from this camp to the surrounding mountain peaks, replacing the plates exposed during the day every evening by new ones to be used the next day.

After the camera theodolite has been put together and placed in position at a station, with favorable weather and light, precluding unforeseen accidents to the corrections and adjustments of the instrument, an experienced observer will execute the panorama and determine the camera station within an hour.

To secure the position of a camera station at least three or four directions to surrounding geodetic points should be taken, as, if so many are not visible, that number of horizontal directions must be taken to some other points which have previously been determined as phototopographic stations, and which were provided with signals before leaving them. The vertical angles at these points are recorded in a notebook. After the terrene to be photographed has been focused upon, the circle reading of the focal length on the graduated metal plate on the objective tube is also recorded in this book if the principal focal distance has not been used.

The panorama is obtained by clamping the instrument, after the direction of the optical axis of the first perspective P^1 has been secured, by bisecting a geodetic point (see fig. 11), and then revolving the camera 36° for each successive exposure in order to obtain the directions of the optical axes of the following perspectives: P^2, P^3. . . . .

In the notebook (Model No. 1, supplement) all data are recorded which may be deemed useful or necessary for the selection of subsequent camera stations, also the general incidents of the fieldwork at each station (time or duration of exposure of the different plates, according to the character of illumination, in order to gather the means for regulating the subsequent developing of the plates). Finally, a pencil outline sketch of the terrene, with valuable notes for the map (names, roads, paths, buildings, etc.), is made before leaving the station.

After all the stations around this camp have been completed, and if all required data have been gathered during the several traverses through the country, from the camp to the stations, the camp is packed and the party returns to the laboratory, where the recently obtained negatives are developed and the occupied camera stations are plotted and marked upon the chart projection. All the finished panoramas left at the laboratory are catalogued and labeled. Then the party, with a new supply of plates, proceeds to another camping ground to continue the work as before.

In order to save time and trouble, it will be advisable to regulate the general progress of the work in such a manner that the elevated points are visited in the most favorable season (i. e., when the snow has least depth, when the passes are free from snowdrifts, and when the glaciers can be passed over with the least risk). The lower regions, being nearer to civilization, require less time and can be occupied at leisure at any time during the season.

A good selection of the camera station is important, and should be well considered and be made dependent upon the elevation and distance of the points of the terrène to be surveyed, upon the scale of the chart, and upon the character of the country (a diversified and broken terrene will need more stations to control the same than an undulating and a more regular section), still, with due regard to the limited length of the working season in these elevated regions, it will also be advisable to occupy no more stations than are really needed to develop the terrene properly. The stations, finally, should be selected in such a manner that the smallest area of the represented terrene is visible from three or more stations. If any part is visible from two stations only, and if it is of minor importance, its determination by two directions only may be accepted if the points of the same are determined by good intersections (if the two lines of direction intersect each other at an angle of 60°–90°).

Regarding the most favorable hours for exposing the plates, one must be guided by local conditions; generally speaking, the trend of the valleys in comparison with the course of the sun is important; slopes totally in shadow should not be photographed; neither is it advisable to execute panoramas when the sun is low or near the horizon. In the latter case an additional source of trouble would arise from the fact that one or two perspectives, taken in the direction toward the sun, will be cloudy and have a double set of cross wires, one set being printed by rays which penetrate into the camera and the other in the usual manner by the refracted rays through the lenses of the objective. Generally speaking, it will be found that the best results are obtained from exposures made during the latter part of the forenoon, the atmosphere having a greater percentage of vapor in the afternoon, particularly in mountainous countries.

From every negative plate obtained at least two positive prints are made. One serves to determine the panorama necessary for the location of the secondary points, while the other is used to measure the abscissæ and ordinates needed for the map construction. All measurements should be made upon loose prints, as the pictures become greatly distorted by being pasted on cardboards.

Chapter IV.

THE HORIZONTAL PROJECTION.

In order to obtain the map, based upon the panoramas, two drawing boards are covered with paper (gummed down on the edges). One is used as a constructing board (to make all graphical determinations of points) and the other for the drawing of the finished charts; both are provided with a chart projection upon which the trigonometric points which were used during the field season are plotted by means of their coordinates.

With the aid of a specially constructed graphical protractor and tracing paper the directions obtained with the theodolite in the field can readily be plotted upon both sheets. This protractor is shown in figure 12. It consists of two concentric circles *A A* and *B B*. The former can be moved concentric within the latter about the axis *C*, secured in the center of *A A*. This rotary motion is applied to *A A* by means of two projecting ribs *S* and *S* on the plate *a a*, the latter being secured to the movable circle.

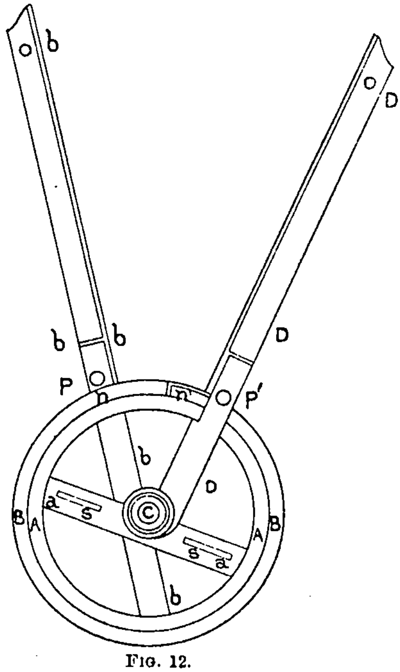


Fig. 12.

The inner circle *A A* has a graduation, divided into degrees and half degrees, while the outer circle *B B* bears a vernier *n*, the zero of which lies in the prolongation of the fiducial edge of an arm *b b*,' which is securely fastened to the outer circle and in a radial position to the same. This vernier *n* reads to half minutes. The clamp screw *P* serves to secure the two circles in any position.

An alidade ruler, *D D*, the fiducial edge of which lies also in the direction of a diameter of the circles, is revolvable about the axis *C* and slides upon the upper surfaces of the circles which are in the same plane. This alidade bears a vernier *n'*, graduated like *n*, the zero of which.

coincides with the fiducial edge of *D D*. The clamp screw *P'* serves to clamp this movable arm *D D* to the outer circle *B B*.

The axis *C* is a hollow screw bolt with conical interior, at the bottom of which a thin piece of isinglass is secured in such a manner that it can be renewed. This has a small puncture to indicate the center of the circles and revolving axis.

When an ordinary protractor is used to lay off the different directions from one camera station which were obtained by the theodolite in the field all the necessary additions and subtractions to be made in order to obtain the successive angles between these lines of directions absorb much valuable time, especially when plotting a series of panorama stations.

The protractor shown in figure 12 can be used not only as any ordinary protractor (by making the zero of the inner circle coincide with the zero of the outer circle and clamping the two circles in that position by means of the clamp screw *P*), but it can also be used to plot the directions upon the map in the same manner as they were obtained in the field, by aid of the theodolite; that is to say, they can be referred to zero or any other direction as the beginning, and then be plotted in successive order. To do this, the inner circle is revolved until the zero of the outer circle (vernier *n*) gives the same reading upon the graduation of the movable circle as the theodolite reading for the prime direction; then both circles are clamped together by the same clamp screw *P*. The line of prime direction is drawn along the fiducial edge of the fixed ruler *b b* upon the drawing (or upon the tracing paper, if the station is to be fixed or located upon the tracing of the lines), while the center of the instrument coincides with the point representing the station upon the paper.

The zero of the vernier *n'* of the alidade *D D* is then successively brought, upon the inner circle graduation, to the readings of the other directions radiating from the station point under the center of the protractor; each successive direction is plotted by drawing a pencil line along the edge of the alidade *D D*. Care must be taken not to change the primary position of the instrument as defined by the first line, during these motions of the alidade.

The tracings of the lines radiating from the stations are obtained with great accuracy by means of this instrument. If we have a sufficient number of directions to well-determined points which are evenly distributed about the station, their corresponding intersections upon both drawing boards can be located with as much rapidity and accuracy as a graphical construction will admit of.

This protractor serves also to locate points on the construction board which on account of importance or for reasons of control had been bisected from numerous stations with the theodolite, and also, as will be shown, to orient a perspective view upon the board, if such perspective contains no trigonometrical point, or if the image of such is blurred and not sufficiently clear to be identified with precision.

After all stations, including such secondary points as have been determined by theodolite directions from these camera stations, have been plotted upon the two boards, the work of determining upon the construction board such secondary points as seem needed to complete the map is taken up. For this purpose the various elements of the perspectives are corrected and adjusted in the manner previously indicated, and all secondary points are selected and marked by searching for well-defined points which are common to two or more plates, carefully selecting therefrom only such as seem to be the most useful either for drawing the contours or for tracing the general trend of mountain ranges, torrents and streams, boundary lines of glaciers, etc. The number to be selected depends chiefly upon the adopted scale and upon the accuracy to be attained. All such points are marked upon the prints (perspectives) by numerals or letters in red ink.

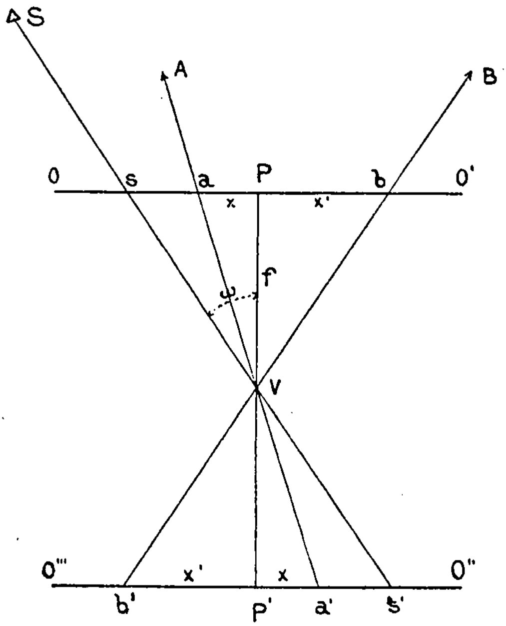


FIG. 13.

Instead of drawing the horizontal projections of all perspectives (or the polygons of the panoramas) upon the construction board, much time can be saved by using the instrument represented in figure 14. With this instrument we can draw directly upon the construction board the horizontal directions to the pairs or trios of points marked upon the prints without drawing the horizontal projections of said prints.

In figure 13, V represents the station point plotted upon the board, $O\ O'$ the horizontal projection of a perspective (which has been oriented with reference to a signal point S, a known and plotted point of the terrene).

$V\ P$ is vertical to $O\ O'$.

f = focal length for the perspective $O\ O'$.

P = principal point of view of the perspective.

$P\ s$ upon $O\ O'$ is the measure of orientation ($= \omega$) of the perspective.

We now prolong $V\ P$ through V by $V\ P = V\ P' = f$ and erect a perpendicular to $VP' = O'''\ O''$ in P'. Likewise, prolong $V\ B$, $V\ A$, $V\ S$ to their intersections with $O'''\ O''$, which intersections are marked b', a', s', respectively. $V\ P' = V\ P$ and $O\ O'$ parallel to $O'''\ O''$; hence the rectangular triangles $V P' a'$, $V P' b'$, and $V P' s'$ are congruent with $V P a$, $V P b$, and $V P s$, respectively; therefore:

$$P'a' = Pa = x$$
$$P'b' = Pb = x' \text{ and}$$
$$P's' = Ps, \text{ giving also the measure of orien-}$$

tation ($= \omega$) of the perspective.

The construction of the graphic sector (fig. 14) is based on the preceding consideration, and it serves to draw from the station point, in the plane of drawing, the various horizontal directions to secondary points of the perspectives.

The metal plate $V S S'$, shaped like a sector, can be revolved in the plane of drawing about the center of a strong needle, puncturing the station point in center (r) of sector. This needle passes through an oblong opening (of the same width as the needle) of a revolvable button, r, secured in V, and through a similar slot in the metal plate $V S S'$ at V. The metal ruler $R R'$ is revolvable about V, gliding with the end R' over the arc $S S'$ of the sector, and the fiducial edge of the ruler passing through the center of V.

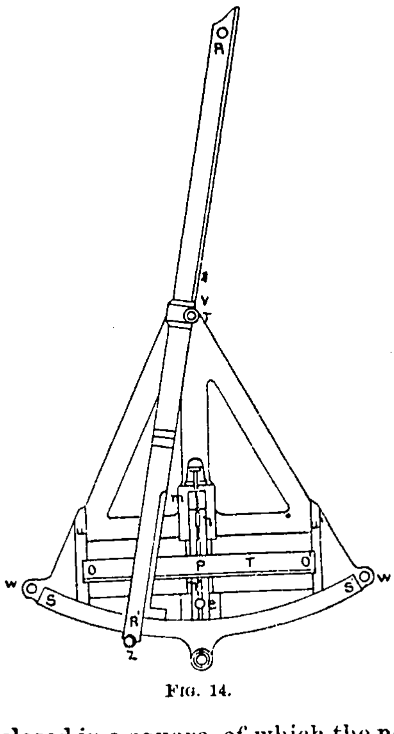


FIG. 14.

This ruler is secured to the revolvable button r by means of a cylinder, the bottom of which also bears a slot similar to those in the button and sector plate.

When the ruler and button are in a certain position the three slots in sector plate, button, and ring of ruler will coincide, and the needle can be inserted into the station point under V, the center of rotation, through the three slots. By a quarter turn of the button r the needle will become inclosed in a square, of which the needle circumference forms the inscribed circle. The entire instrument can now be revolved about the needle center in V.

The lever screw m serves to move the prolongation arms $n\ n'$ in a suitable slide in the direction of the middle line of the sector. The axis of this instrument coincides with that radius of the sector which falls together with the middle line, passing through the center of V. The steel ruler T is perpendicular to this axis, and is secured to the prolongation arms $n\ n'$; it can be moved up or down, while maintaining a position perpendicular to the axis of the instrument, by means of the screw m to positions parallel to each other. During such movements of T the ruler ends O and O' glide over two graduated metal strips $u\,u'$, which are parallel to the axis of the instrument and upon which the distance of a line coinciding with the fiducial edge $O\,O'$ of T from the center of the needle in V can be read off to $0{\cdot}1^{mm}$.

If the edge $O\,O'$ of the steel ruler T is brought to a distance $=f$ from the camera station in center of V (by means of the screw m) it will represent the line of horizon or the horizontal projection of a perspective, obtained with the focal length $=f$ (in inverse position, like the line of horizon, viewed upon the ground-glass plate of the camera).

The point P, intersection of the axis of the instrument with $O\,O'$, is the principal point of view, and it is accentuated by a small conical cavity to receive the point of one arm of the divider.

The screw c serves to give the steel ruler T a permanent position after it has been brought to the desired distance from the center of rotation V. Two thumbscrews W and W' (into which fine needles can be inserted and held in place by clamp screws) serve to secure the metal sector in any desired position upon the drawing board. The arc $S\,S'$ of the sector is graduated to ten minutes and the zero of this graduation coincides with the axis $V\,P$ of the instrument, giving readings from $0°$ to $25°$ on either side of $V\,P$ upon this arc.

The ruler or alidade $R\,R'$ bears a vernier to read fractions of the arc graduation. It is graduated to read half minutes. The thumbscrew (clamp screw) Z of the alidade has a counterplate at its lower end and it serves to secure the end R' of the alidade upon the arc of the sector and upon the steel ruler T.

In order to draw the lines of direction upon the construction board to a point of the terrene, the picture of which has been selected and marked upon the perspective, the instrument is placed with its center of rotation, V, over the needle, marking the camera station on the board and giving the button r a quarter turn (care must be taken that the side bearings of the button r of the instrument have no loose play about the needle), then $O\,O'$ is moved by turning the screw m until $O\,O'$ is distant from center of $V=f$, whereupon the orientation of the instrument is accomplished as follows:

$V\,P$ is directed to bisect a plotted triangulation point, the image of which appears on the perspective with sufficient distinctness; its abscissa is taken from the print by means of a pair of dividers and plotted in the inverse direction, upon the line $O\,O'$, from the puncture in P; the alidade $R\,R'$ is now gently brought into contact with the other point of the dividers and secured in this position by clamping the screw Z. Now the entire instrument is revolved about V until the other end R of the alidade bisects the plotted point. The instrument is held in this position by pressing the small needles W and W' into the drawing board; the end R' of the alidade is now released and the abscissæ of all the desired points of the perspective are transferred to the drawing along the line $O\ O'$ from P, by means of the dividers, in their successive order, but in inverse direction (the fiducial edge of the alidade being gently brought into contact with the divider point each time), and the lines of direction are drawn with a sharp pencil along the fiducial edge of the alidade end R.

Should the image of the triangulation point appear blurred upon the perspective, the instrument will have to be oriented upon the drawing by means of the angle of orientation $= \omega$ of the perspective, which angle is taken from the field book (Model No. 1, Supplement). The end R' of the alidade is placed and secured in such a position that the alidade $R\ R'$ forms the angle ω with the axis $V\ P$ of the instrument, which angle is read off (in the inverse direction) on the arc $S\ S'$ of the sector. The instrument is then revolved about the needle in V, the same as before, until the end R of the ruler passes through the trigonometrical point in question. The instrument is then secured upon the board in this position by means of the screws W and W' and the horizontal directions are drawn to the secondary points along R, in the manner just described.

If a plate has been exposed while the vertical wire bisected a trigonometrical point, the orientation of such perspective is accomplished by making the zero of alidade-vernier coincide with the zero of the arc graduation, $S\ S'$, clamping $R\ R'$ in this position and directing the end R to bisect the plotted trigonometrical point in question.

Should, finally, the perspective contain no images of points previously located and plotted, then the zero of alidade is again made to coincide with the zero of the arc graduation and the instrument is revolved about the center of the needle until the fiducial edge R of the alidade coincides with a line (which had been drawn by means of the previously described protractor) which forms an angle in the station-point V with the direction to a triangulation point, which is equal to the angle of orientation ($= \omega$) of the plate and which is taken from the field notebook (Model No. 1, Supplement).

After the horizontal directions have been drawn to the different points of the panorama, they are provided with numerals or symbols corresponding with the characters affixed to the points upon the panoramic views, in order to facilitate their identification when seeking for the subsequent intersections with lines to the same points from other camera stations. In this manner, shown in the preceding, the positions of the secondary points in the plan of the drawing are secured by intersections, which will serve to make up the control of the map. It is well to transfer to the fair drawing, by means of tracings, which are oriented by the plotted trigonometrical points and previously located panorama stations, all the different points obtained by intersections upon the construction board, in order to erase therefrom all lead pencil lines, which served for their determination and to obscure the subsequent constructions for the positions of other points of the terrene as little as possible.

Chapter V.

THE HYPSOMETRICAL WORK.

After the position of the most important points of the second order is well under control it remains to ascertain the elevations of the various station and secondary points of the perspectives, in order to enable the draftsman to interpolate the contours between these points.

By means of the graphical hypsometer, figure 15, the elevations of the plotted camera stations can be ascertained, by means of their graphically measured distances from triangulation points and the corresponding angles of elevation of said points (measured with the theodolite), which are recorded in the field notebook (Model No. 1, Supplement). The elevations of all secondary points are determined with the same instrument by means of their graphically measured distances from the camera station and their corresponding ordinates, y, to be measured upon the perspectives.

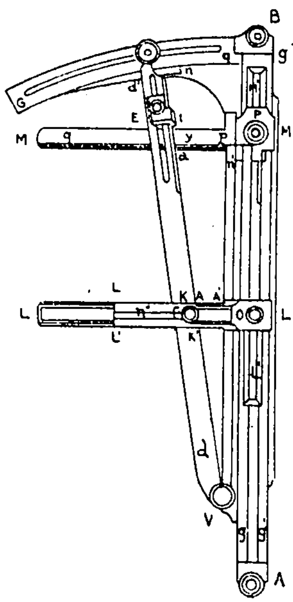


Fig. 15.

Two rulers, $L\,L'$ and $M\,M'$ (fig. 15), can be made to glide with their ends along a ruler, $A\ B$, but always maintaining a perpendicular position to the same, for which purpose their ends are secured to two sleighs, M' and L', which glide in two parallel grooves, g and g', along $A\ B$. The motion of L' is free, and is accomplished by pushing the button O up or down the ruler $A\ B$.

M' is provided with a ratchet and screw P. By turning the latter in one direction or the other the ruler $M\,M'$ is gradually moved up or down $A\ B$, the latter being provided with a row of fine teeth into which the ratchet wheel of M' bites while P is being revolved. The alidade ruler $d'd$ is secured with one end, d, in V, in such a manner that $d'd$ can be revolved about the axis of V as a center, while the other end, d', passes over a graduated arc, $G\,g\,g'$. The plug in V is similarly constructed to the one in V of the graphical sector, figure 14, previously described (it is provided with a revolvable button which contains a slot in such a manner that the ruler $A\ B$ can be revolved simultaneously with the alidade $d'd$ about a needle, marking the station point on the construction board). In this instrument the plug, the revolvable button, and the alidade have each a slot, which intersect each other in the center of rotation V, and through which the needle can be passed when they have a certain position and then be secured in place by a quarter turn of the button. The entire instrument can be revolved about the needle, the center of which lies in the directions of the fiducial edges of the ruler $A\ B$ and alidade $d'd$.

The alidade is provided with a vernier, n, graduated to read half minutes, on the graduation of the limb $G\ g\ g'$. This vernier serves to lay off angles from V between the fiducial edges of $A\ B$ and $d'd$. When $d'd$ is brought close to and in contact with $A\ B$, the zero of the vernier n and the zero of the arc graduation will coincide. The axis of this instrument is represented by that edge of $A\ B$ (lying toward $d'd$) which passes through the center of rotation V, and which passes through the zero of the graduated arc $G\ g\ g'$; it also passes through the point p of the line $p\ q$, which is marked upon the ruler $M\ M'$. This line $p\ q$ corresponds with the zero of a vernier, n', which is attached to the ruler $M\ M'$ and which glides along the groove g when $M\ M'$ is moved up or down $A\ B$. $A\ B$ has a millimetre graduation, and by means of the vernier n' the distance of the line $p\ q$ from center of V can be read to 0.1^{mm}.

When this line $p\ q$ is brought to the distance $=f$ from V, by means of the fine ratchet movement at M', the line $p\ q$ can be regarded as the axis of abscissæ drawn upon the perspective, while the point P represents the principal point of view of the perspective (see fig. 5).

In this case the line $p\ q$ can also be regarded as the axis of the ordinates of the perspective $m\ n$ (fig. 5), provided the vertical plane (containing $V\ P$ and axis of ordinates) is supposed to be rotated about $V\ P$ until it coincides with the horizontal plane $V\ P\ O$.

The point p is marked upon the line $p\ q$ (in the same way as described for the sector) by a small puncture, which serves to receive one point of the dividers, when such are used to lay off the abscissæ and ordinates, taken from the perspectives.

After $p\ q$ has been secured at a distance $=f$ from the center V and the abscissa x of a point a, taken from the perspective $m\ n$, has been transferred to the line $p\ q$ from p, the second point of the dividers upon $p\ q$ will represent the horizontal projection a' of the point a. If we now move the alidade, $d'd$, until its fiducial edge touches the second point of the dividers, the triangle formed by the edge of the alidade $d'd$, the edge of the ruler $A\ B$, and line $a'\ p$ will represent the triangle $V\ P\ a'$ of figure 5.

The end d' of the alidade is provided with a steel index mark, i which can be moved along $d'\ d$ by means of a revolvable button, E, a ratchet movement, and teeth in a groove along $d'\ d$. If this index mark is moved to a' (the intersection of fiducial edge of the alidade and line $p\ q$ on $M\ M'$), the distance $V\ a'$ cut off on $d'\ d$ will represent the horizontal distance of the point a' of the perspective from V (i. e., the value d in figure 5).

Maintaining this index mark i (fig. 15) in this position on d' d and revolving d' d about V until its fiducial edge coincides with the edge p V of A B (i. e., with the axis of the hypsometer), then moving the ruler $M M'$ away from V (by turning the ratchet button P) until the line p q coincides with the index mark i, we will have transferred the distance d (fig. 5) upon the hypsometer axis; we will have brought the line p q (engraved upon $M M'$) to a distance from the center of rotation (of the needle) in V equal to d, and by transferring the ordinate y (fig. 5), measured on the perspective m n with a pair of dividers, upon the line p q (while the latter is still in the position just described), by inserting one point of the dividers into the cavity p and bringing the fiducial edge of the alidade d' d gently into contact with the other point of the dividers, resting on the line p q (fig. 15), then the triangle $V p$ a of the hypsometer will also represent the vertical triangle V a' a of figure 5, except that it is now turned about $V a'$, as axis, into horizontal plan.

The movable ruler L L', which will always remain perpendicular to the hypsometer axis, consists of two plates joined firmly together at their ends, between which the alidade d' d (fig. 15) can glide when revolved about V. The upper plate of L L' is slotted like the handle of a penknife, and the edges L L and L L' are beveled and provided with a millimetre graduation, the numerals of which correspond with a scale of 1:50000 (50 m = 1 mm). A ratchet screw, c, serves to move a plate (K O K') with two index marks K and K', which can be made to coincide with the intersections of the fiducial edge of the alidade d' d and the two sharp graduated edges L L and L L'. The index plate K O K' also has a double vernier, n'', on the opposite side of the ratchet screw c, graduated to read $\frac{1}{50}^{\mathrm{mm}}$ (i. e., to read metres for the $\frac{1}{50000}$ scale) in connection with the millimetre scales L L and L L'.

When the zeros of this double vernier n'' coincide with the zeros of the graduated edges L L and L L', the marks of the double index K and K' will coincide with the edge $V p$ of A B (i. e., with the axis of the instrument) and also with the fiducial edge of alidade d' d, the zero of the vernier n of the alidade also coinciding with the zero of the arc graduation G g g' (i. e., the fiducial edge of d d' will fall together with the axis p V of the instrument).

In figure 5 A represents a point of the terrene, the image of which is designated by a in the perspective m n. If A' is the projection of A in the horizontal plane passing through V, then A A' will represent the difference of elevation = L between the points A and V. V A' will be the horizontal distance = D of the point A from the camera station V, which distance is represented by $\frac{D}{50000}$ for a scale of map of 1:50000.

Returning to figure 15, we imagine the hypsometer revolved about the needle center in V until the hypsometer axis p V passes through a plotted point A' in the drawing. If the ruler M M' had previously been secured in such a position that the distance of p from $V = d$ and if d d'

had been set to lay off the ordinate y upon $p\,q$ from p, and if we now bring the index mark K in a position to mark the intersection of the fiducial edge of the alidade with the edge $L\,L$, then the triangle $V\,A\,A'$ (fig. 15) will also represent (in the scale of 1:50000) the triangle $V\,A'\,A$ of figure 5. The index mark K indicating on the edge $L\,L$ the length $\frac{L}{50000}$, we will find the difference of elevation between the point A and camera station V by reading the corresponding vernier n''

The triangles $V\,p\,a$ and $V\,A'\,A$ (fig. 15) being similar ones, we will have:

$$\frac{A\,A'}{V\,A'} = \frac{P\,a}{V\,p} = \frac{y}{d}$$

we found (page 58).

$$\frac{y}{d} = \frac{L}{D}$$

hence

$$\frac{A\,A'}{V\,A'} = \frac{L}{D}$$

and as $V\,A' = \frac{D}{50000}$, we have

$$A\,A' = \frac{L}{50000}$$

$$L = 50000 \times A\,A'.$$

The numerals of the graduation of the edges $L\,L$ and $L\,L'$ and of the double vernier n'' give the value $A\,A'$ multiplied by 50,000, which is the difference of elevation.

It has been previously shown (page 58) that

$$\tan\,\alpha = \frac{L}{D} = \frac{y}{d}$$

and therefore

$$\tan\,\alpha = \frac{A\,A'}{V\,A'}$$

Hence, if we have the angle of elevation of a point A of the terrene we need only to lay off this angle upon the graduated arc $G\,g\,g'$ by means of the alidade vernier n, from g, and place the index mark K upon the intersection of the fiducial edge of alidade and edge $L\,L$ (the instrument having been placed upon the drawing in such a position that the hypsometer axis passes through the plotted point A'), and then read off on L and corresponding vernier n'' the difference of elevation between camera station and point A.

This case becomes very much simplified when the image A' of A is bisected by the vertical thread of perspective (axis of y), as then:

$$x = o \text{ and } d = f.$$

The alidade is placed so as to lay off the ordinate y of the point a upon pq from p, after the ruler $M\,M'$ had been secured in a position at a distance $=f$ from V; then the index mark K or K' is brought into the point of intersection of the fiducial edge of $d\ d'$, with edge $L\ L$ or $L\ L'$ of the ruler $L\ L'$ (the axis of hypsometer passing through the plotted point A'), and the difference of elevation between A and V is read off either on the vernier corresponding to the graduation $L\ L$ or to the graduation $L\ L'$. The correction for curvature and refraction to be applied to these differences of elevation is taken from the ordinary field tables.

A special list (Model No. 2, Supplement) is made for the secondary points, in which they are tabulated according to the numerals or symbols with which they were characterized on the perspectives, and they are catalogued according to the panorama and perspective to which they belong. This list also contains the differences of elevation between them and the two or more stations whence they were determined, as well as their absolute elevations, the latter being the mean of the values obtained from the different stations and corrected for curvature and refraction.

The elevations of the camera stations are the mean results of the values obtained by adding or subtracting the difference of elevation (obtained by means of the graphical hypsometer) to or from the known elevations of the triangulation points (using the vertical angles observed with the theodolite and the graphically measured horizontal distances between plotted camera station and triangulation points).

After the secondary points, including their subscribed elevations, have been transferred from the construction board to the final plan, it remains only to interpolate the contours between these points, in harmony with the affixed figures, to sketch in the details and everything that is needed, and to give the terrene its proper character, all based upon frequent reference to the perspectives of the terrene in question.

CONCLUSION.

As the work at a phototopographic station can be finished within an hour or an hour and a half, and as two or three well-selected stations will control the horizontal and vertical representation of an extended area, the fieldwork will take no more time for a detailed survey than for a general survey, as the perspectives will be the same for both. The difference in the time needed for obtaining a more or less detailed topographic map depends upon the office work only, every panorama giving the means to construct therefrom an unlimited number of horizontal directions from the camera station to surrounding points, as the number of secondary points selected from the perspectives can be increased indefinitely to the limit of the patience and ability of the draftsman.

The panoramic perspectives, however, are not only important aids for the construction of the map, but they can also serve as subsequent checks upon the work of the draftsman, and if they are preserved, together with the catalogues containing the numerals, symbols, etc., of their secondary points, their elevations, etc., they can serve for future illustrations of the mapped terrene, giving the relief modeler important details to enhance and complete the natural character of the model, etc.

From the foregoing it is evident that phototopography is especially well adapted for topographical surveys of mountainous regions, as the ordinary topographical methods for such regions can be carried on only during a few summer months each year with advantage. Even in favorable seasons the weather will be very vacillating; clouds will obscure during the warmer hours of nearly every bright day the more elevated peaks, winds will carry misty vapors from one valley to another, etc., so that the camera can obtain in a short bright interval more topographical data than could be obtained in weeks of time with the other instrumental methods. Even if the selected camping ground is most favorably situated for the work, the ordinary topographer will have to traverse long and difficult distances before he will reach a favorable point for a topographical station. He can not leave camp before daybreak, and he can not risk a late return on account of the danger to life and limb (not mentioning his instruments) attending a tramp through rough mountain regions by night. He will arrive at the selected station in a fatigued and nervous condition, have but a short time to spend there, and consequently will hurry through his observations. As is well known, the topographer can, under general circumstances, determine prominent points by the intersections of horizontal directions from a number of stations or by telemeter readings from one station. In order to secure the details, however, he will have to traverse the country quite extensively and make numerous sketches in order to give it the proper character and to delineate the terrene by horizontal contours. Not many horizontal and vertical angles can be obtained in a single day either with the plane table, the theodolite, the tachemeter, or other instruments, as the topographer will have to spend a good deal of his time, at the station, in making sketches in order to identify the points of the lines of direction for subsequent lines to the same objects from other stations (in order to get the correct intersections).

The work will be still less encouraging if the use of the telemeter is depended upon to determine secondary and tertiary points, on account of the slow progress, the danger to life and limb of the telemeter men, etc. If we add to this the low temperature and snow, it will be impossible to work more than a few hours at one station with the plane table, theodolite, etc., to get directions and make sketches. It appears certain, at best, that, under the conditions which always prevail in mountains of an alpine character, the positions and elevations of prominent points only will be determined (and often only such as are absolutely necessary), the telemeter will be discarded, and the characteristic forms of the terrene, which are only seen at a distance, are sketchily represented. If details are mapped at all they will be unreliable, and the hours at which they can be seen to the best advantage are few. These facts render topographic surveys of such mountains not only tedious, difficult, and expensive, but also unreliable. They explain why so few maps give the true representation of such regions, and they also show the great advantage to be derived by applying photography to the surveys of all regions which are difficult of access or inaccessible, and where snow, ice, and bad weather prevail the entire year. The photographic perspectives will not only reproduce the terrene before one's eyes at any place, and at any time, but we can also construct a topographic map, based on such views, with the utmost correctness that may be demanded by science or industry. It is evident, therefore, that phototopography is to be recommended in the following cases:

1. For all mountains of an alpine character where, if the ordinary topographic methods are followed, the lack of control will give but mediocre results.

2. For extensive scientific expeditions and explorations, for reconnaissance in times of war, for topographic surveys in unhealthy localities along the frontiers of belligerent nations, etc.

3. For surveys for geological studies, for projected railroads through mountains, for hydrographical surveys for river ameliorations; in short, for surveys for all purposes where correct representation and character of the terrene, as well as full details, are desired.

4. For naval purposes or on board of vessels fitted out for explorations, to obtain coast views, topographic and hydrographic sketches of hostile or barren coasts. (Two or more shore stations are selected from the deck of the vessel and panoramic views are taken therefrom, care being had to include in these perspectives the vessel, anchored boats, buoys, moored flags and other secured objects which served to control the soundings, simultaneously carried on with the topographic survey.)

A special apparatus, for use on shipboard, has been invented by Paganini. It furnishes a vertical photographic perspective of a known focal length and at the same time gives the magnetic azimuth of the optical axis of the camera for each perspective. The azimuths of all the points along the coast shown on the perspective can be taken directly from the perspective.

Pio Paganini, engineer geographer and director of the phototopographical work in Italy, in a report recently made to the First Geographical Congress in Italy, says the following, relating to the improvements of his camera theodolite (a German translation, by Fenner, of this report has been published in the Zeitschrift für Vermessungswesen, 1892):

The principal improvement to the camera theodolite consists in dropping the eccentric telescope of the theodolite (Fig. A) and changing the instrument so that the "*photographic camera in itself will serve as a centrally located telescope.*"

Paganini accomplished this by replacing the ground-glass plate of the camera by an opaque plate which has a Ramsden ocular lens in the center. This new apparatus has all the details of a transit, with a centrally located telescope. The same instrument serves to obtain the photographic panorama as well as to measure the horizontal angles necessary to orient the panorama or needed for the determination of the camera station by resection, and to measure the vertical angles for the determination of the elevations.*

The plates Nos. 6 and 7 of the new map of Italy, comprising the terrene to the north of Chiavenna to Splügen, were obtained in 1889 by means of the former instrument (Fig. A), and they are now completed and have been published. A comparison between a recent edition (scale $\frac{1}{50000}$, with contours of 50 metres interval, excepting the lowlands, where the interval is 10 metres) and the adjoining sheet of the Swiss "Dufour Atlas" shows that the former appears to represent the terrene more true to nature, and although the Swiss map ranks higher from an artistic point of view, it also evinces a certain uniform undisputable neglect of characteristic topographic features.

During the exposition of charts and maps at Vienna (in 1891), under the auspices of the Ninth Congress of German Geographers, this Italian map was generally praised and declared by competent judges to deserve the first rank above all other exhibits.

In 1890 Paganini, assisted by the topographer Rimbotti, began the work of phototopographing the elevated parts of the terrene of plate No. 29 of the new Italian map, which comprises the difficult group of Monte Rosa, with elevations of 4,600 metres. They used two instruments, one of the older pattern and one of the latest construction. This work, however, had to be interrupted in 1891 in order to do "more important work for military purposes." Paganini also mentions that he had been engaged in the same year upon an "important military work," to accomplish which he doubtless would not have succeeded without the aid of photogrammetry.

Concluding his report, Paganini made some very interesting remarks concerning a recently invented instrument, which, however, is not yet constructively finished. It is also a photographic instrument, but to be used on shipboard, and which he terms a "photographic azimutale."

Formerly the perspectives used to illustrate portions of the coasts in order to facilitate the identification of such portions by sailors when approaching the coast from the sea, were published with and upon the charts or in the coast-pilot books. They were obtained in the following manner:

From the deck of a vessel at anchor a free-hand perspective drawing would be made of the desired part of the coast, including all prominent

* Paganini proposes to publish a detailed description of this new apparatus shortly.

features, particularly light-houses and navigation marks. (The use of an ordinary camera was precluded on account of the rocking motion of the vessel.)

The angles formed by the lines of direction from the vessel to the various prominent points shown in the perspective were measured with a sextant, and the local magnetic azimuth of one of these lines would also be determined (giving the local magnetic azimuth of all other lines of direction to points on the perspective).

These magnetic bearings were inscribed in the drawing above the points to which they referred. The place of anchorage had to be determined as accurately as possible and plotted upon the coast chart.

Such perspectives (it is said that Porro showed a remarkable skill in making such views) would naturally be obtained more readily and far more accurately if a photographic instrument could be constructed to be used on shipboard for this purpose.

Paganini (having been an officer in the Italian navy until 1877) had for several years made studies and investigations with the above object in view, particularly since the instantaneous process in photography had been developed to the present degree of perfection.

The "photographic azimutale," the construction of which is now well under way, if not already completed, is the direct result of Paganini's studies in this direction. This instrument can be called a transit, the telescope of which is replaced by a photographic camera, which can be converted into a telescope by replacing the ground-glass plate by an opaque plate with an ocular lens in the center. This instrument differs from Paganini's latest improved camera theodolite by its mounting and by the additional attachment of a dial compass.

Regarding the mounting of this "photographic azimutale," we will say that it rests upon a plate which swings in gimbals; both are connected by a central clamp screw, which has a heavy weight attached to secure a permanent horizontal position of the horizontal limb or the vertical position of image plate.

The compass resembles the Schmalkalder or azimuth compass and is placed centrally above the horizontal plate and within the ring-shaped alidade. The magnetic bearing of the optical axis for every perspective is secured by photographing directly upon the image plate simultaneously with the picture of the coast (and immediately below the vertical wire) that part of the compass graduation which lies in the direction of the view photographed.

The zero diameter of the dial compass always being in the magnetic meridian, the compass reading designated by that graduation mark which we find bisected by the prolonged vertical thread under the picture will represent the magnetic azimuth of the optical axis of the instrument at the moment of the exposure, or it will indicate the angle of orientation for the picture.

This picture of the compass graduation, caught simultaneously with that of the coast view, is obtained by means of a small secondary camera placed immediately above the compass and below the main camera. The optical axes of these two cameras are at right angles with each other. The image of the compass graduation in the secondary camera is reflected by means of a suitably placed prism upon the image plate of the main camera.

In order to obtain the pictures of both cameras simultaneously, the shutters of both are operated automatically and at the same moment.

The "photographic azimutale" is to be permanently secured to the captain's bridge, forming a part of the instrumental outfit of every naval vessel. By replacing the gimbal support by a tripod the instrument can be used for work on land. It is also tested and adjusted on shore in order to adjust the horizontal thread by means of the sea horizon.

Paganini mentions that this instrument is well adapted to photograph the illuminated sectors of light-houses and the range of visibility of navigation marks. He also believes that the same can be employed with advantage for the topographic and hydrographic surveys of harbors, wharves, seldom-frequented coasts, for military or scientific expeditions, for the determination of the geographical latitude of a vessel's position by means of the image of the sun, which can readily be obtained with sufficient sharpness, including the illuminated sea horizon, to give good results, etc.

From every picture showing the image of the sun we can find the sun's declination and azimuth, and the time being known we can compute the geographical position. Whether such semigraphical determinations are sufficiently accurate for practical use and whether sextant observations will be supplanted by these, time and experience can only teach; at present there are no comparative results to communicate.

The preceding chapters show that photographic surveying is being pushed to a high degree of perfection in Italy, and we are particularly indebted to Paganini for the numerous improvements so recently made in photographic and graphical instruments, including methods of use for topographic and hydrographic surveys.

SUPPLEMENT.

Model No. 1.

Station on Punta Bivula (trig. pt.), on the ridge between the valleys of the Valsavaranche and Rêmes.

[September 18, 1884.]

Orientation of the panorama.	Perspectives belonging to the panorama.	Directions to the principal points of view.	Focal distance.	Remarks.
		° ′		
Punta Gran Paradiso: 78° 27′ 00″	P^1	78 27	244·5^{mm}	Time of exposure: 10^s, with smallest diaphragm, No. 7.
	P^2	114 27	Steinheil's objective: "Antiplanate."	10^s.
	P^3	150 27		9^s.
	P^4	186 27		12^s.
	P^5	222 27		9^s.
Punta Della Grivola: 123° 47′ 00″	P^6	258 27		10^s.
	P^7	294 27		9^s. Fine weather.
	P^8	330 27		10^s.
	P^9	6 27		10^s.
	P^{10}	42 27		10^s.

Directions and vertical angles of the trigonometrical points.	Computation of elevation of station and elevation of line of horizon.

Station upon the half-destroyed signal. Elev. of instr. = 2·30^m.
Geodetic point, elevation = 3413·69^m.
Elev. of instr. = 2·30^m.
Elevation of lines of horizon of panorama = 3415·99^m = 3416·0^m.

The adjoining page is utilized for topographic sketch from station, detailed remarks, names of roads, etc.

MODEL No. 1.

Station on Punta Percia, on ridge between the valleys of the Valsavaranche and the Rêmes.

[September 19, 1884.]

Orientation of the panorama.	Perspectives belonging to the panorama.	Directions to the principal points of view.	Focal distance.	Remarks.
		° ″		
Punta dell' Erbetet: 282° 04′ 00″	P1	170 00	244·5mm	Time of exposure: 6s. Shorter exposure than before on account of the great reflection of surrounding glacier.
	P2	206 00	Steinhiel's objective: "Antiplanate."	7s.
	P3	242 00		8s.
	P4	278 00		9s.
	P5	314 00		10s.
	P6	350 00		8s.
	P7	26 00		9s.
	P8	62 00		9s. Fine weather.
	P9	98 00		10s. Diaphragm No. 7.
	P10	184 00		7s.

Directions and vertical angles of the trigonometrical points.	°	′	″	Computation of elevation of station and elevation of line of horizon.		m.
Cima di Breuil,	220	54	00	Elevation Invergnan	=	3607·72
Elevation,	1	33	00	Diff. of elev. + corr.	=	406·15
						3201·57
Punta dell' Erbetet,	282	04	10			
Elevation,	3	36	30	Elevation Nomenon	=	3488·42
Cima di Nomenon,	222	42	00	Diff. of elev. + corr.	=	284·94
Elevation,	1	38	30			3202·48
Cima di Rouletta,	0	44	30			
Elevation,	3	11	30	Elevation Toss	=	3302·24
				Diff. of elev. + corr.	=	99·84
Punta dell' Invergnan,	80	07	00			
Elevation,	3	42	00			3202·40
Cima di Toss,	34	11	30	Elevation Breuil	=	3454·62
Elevation,	0	30	30	Diff. of elev. + corr.	=	252·64
						3201·98
				Elevation Rouletta	=	3384·10
				Diff. of elev. + corr.	=	182·28
						3201·82
				Elev. of line of horizon	=	3202·3

Model No. 2.

Elevations of secondary points of the panorama.

Names or numbers of points.	Stations whence they were derived.	Elevations of stations.	Diff. of elevations.	Elevation of point.	Remarks.

PHOTOGRAPHIC INSTRUMENTS AND METHODS EMPLOYED FOR TOPOGRAPHIC SURVEYS IN THE DOMINION OF CANADA.

The phototopography of the Rocky Mountain region in the Northwest Territory of the Dominion of Canada proved a success, and several of the Dominion topographic and land surveyors (J. J. McArthur, W. S. Drewry, etc.), under the direction of the surveyor general, Capt. E. Deville, have acquired skill and valuable experience in this branch of surveying, as is well proven by Deville's topographic map of the Rocky Mountains along the Canadian Pacific Railroad, based on triangulation and phototopography, plotted on 1:20000 and published on 1:40000 scale, and which was on exhibition at the Columbian World's Fair.

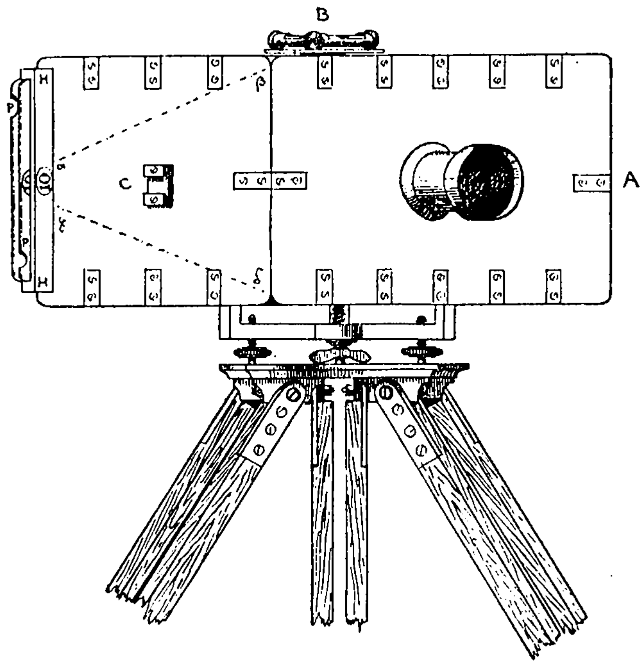


FIG. 16.

Under the direction of Dr. W. F. King, Alaskan boundary commissioner to Her Majesty, phototopography has been successfully employed for the topographic survey of southeastern Alaska, as far as this topographic reconnaissance has been executed under the Government of the Dominion of Canada.

The views taken from the camera stations of the Dominion surveys are not complete or full sets of panoramic views, and when the stations are close together, even those few plates which are exposed from one station do not always comprise adjoining pictures. According to the desired greater vertical or horizontal extension of the view, the camera can be placed with either the long or short side in an upright position upon the tripod.

The camera is a rectangular box of well-seasoned mahogany (fig. 16), strongly bound in brass and very carefully constructed, with opposite sides parallel and adjoining sides at right angles to each other. The camera has neither telescope, horizontal nor vertical circles, as it is used in conjunction with a transit, the same tripod serving for both instruments. All angles are observed with this transit, either before

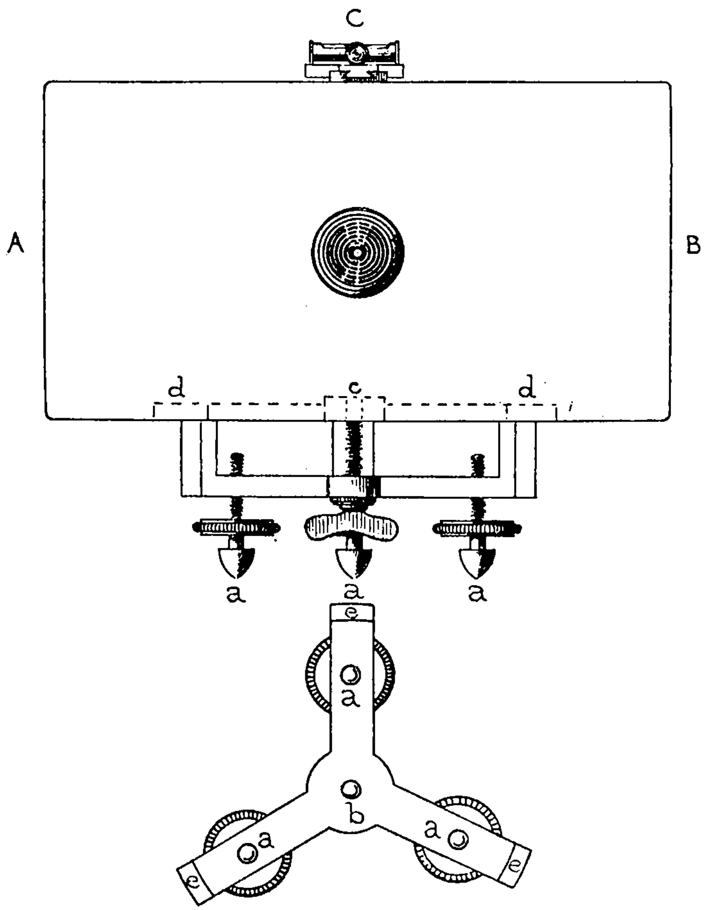


FIG. 17.

or after the exposure of the plates has been made. Care must be exercised not to disturb the tripod when changing the instruments. The camera is secured to the tripod by means of a separate triangular support (fig. 17); the three screws marked *a* serve to level up the camera before each exposure of a plate. A brass plate, with two spirit levels placed at right angles to each other, can be attached to the uppermost side of the camera, and this pair of levels is used for the leveling up. The central clamp screw, b, serves also as vertical axis when revolving the camera in azimuth.

The camera box $A\ B\ C$ (fig. 17) is provided with two nuts inserted into and made flush with the face of the camera, one in the center of a small side and the other in the center of an adjoining long side. These nuts receive the central clamp screw, b (fig. 17), of the triangular camera support, and a circular brass plate inserted into these same sides, with the nuts as centers, forms the bearings for the three camera rests, c (fig. 17), when revolving the camera horizontally. The clamp screw, b, of the camera support is drawn only tight enough still to permit the camera to be rotated in horizontal plane, and after the double levels have been inserted into the slide of the uppermost side of the camera, the latter leveled up and oriented, this central screw, b, is tightened sufficiently to secure the fixed position of the camera when the slide is drawn and the plate exposed.

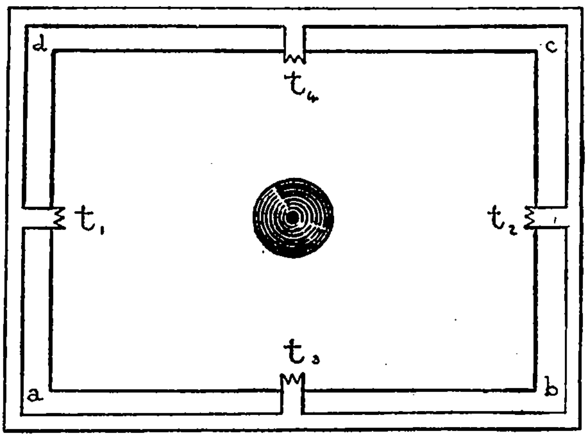


FIG. 18.

Each camera is provided with six double plate holders, $H\ H$ (fig. 16), bearing a number on each side (from 1 to 12) to enable the operator to keep trace of the plates. The latter are made by B. J. Edwards & Co., The Grove, Hackney, London, England. They are the so-called isochromatic instantaneous plates of $4\frac{3}{4}$ by $6\frac{1}{2}$ inches (old English half plate), all of one emulsion and made as uniformly in every respect as possible.

Four sets of teeth (fig. 18), each set about one-eighth of an inch wide, are securely fastened to the camera box, as close as possible to the plate-holder slides, in such a manner that the lines (horizon and principal lines) joining the middles of two opposite sets are parallel to the faces of the camera box. These metal teeth $t_1\ t_2\ t_3\ t_4$ (fig. 18) are placed close enough to the plates to give sharp and well-defined prints of the same. After the camera has been leveled up the plates are vertical, the line $t_1\ t_2$ is horizontal, $t_3\ t_4$ is vertical, and the "principal point" (the intersection of $t_1\ t_2$ and $t_3\ t_4$) is in the optical axis of the camera. Capt. E. Deville has changed these teeth, as they were too long, and inasmuch as the lens, levels, sunshade, etc., are carried within the camera box during transportation, and the jarring motion to which the pack is exposed is liable to dislodge, or at least to bend, the teeth, he advocates their being placed farther back, as shown in figure 18*a*.

Some of the Canadian cameras have a revolvable plate, with lens eccentrically located, so that the width of the picture remains the same throughout, but the horizon can be elevated or lowered by turning this revolvable plate. This plate, being of wood, swells in damp weather and then can not be moved. Then, too, every movable part of an instrument is a source of uncertain errors. Capt. E. Deville's experience teaches that the best results are obtained with a camera that is perfectly rigid in all its parts, of a constant focal length, immovable lens, situated in one-third the length of one short side from either long side and midway between two short sides of the camera as indicated in figure 18*a*. This arrangement will enable the surveyor to elevate or depress the horizon by resting the camera on the face *A B* or *C D* (fig. 18*a*).

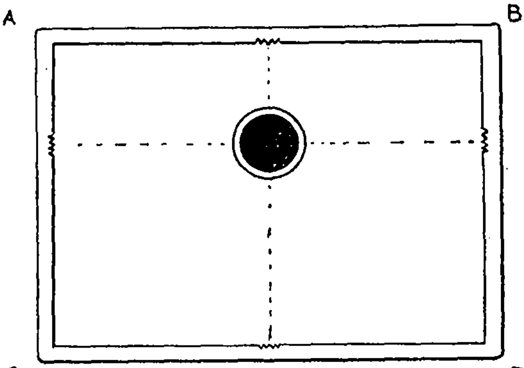


FIG. 18*a*.

A square diaphragm, *a b c d* (fig. 18), placed within the camera box admits only the light needed for the development of the negative, excluding side lights or rays which may possibly be reflected from the camera sides.

A small mahogany box, with a shutter made like a venetian blind, can readily be secured to the tube of the camera lens in case it becomes necessary to exclude the direct sunlight and shade the lens (fig. 19).

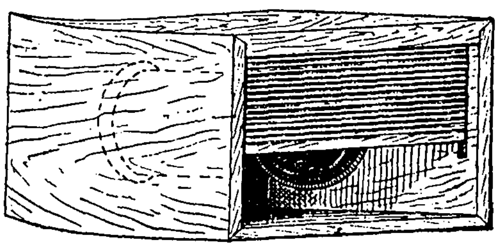


FIG. 19.

The camera faces, which are provided with the level attachment (*B* and *C*, fig. 16), also show two converging lines, $\alpha\ \beta$ and $\gamma\ \delta$ (side *C*, fig. 16), which indicate the range of the lens in horizontal and vertical plane. One set of these lines will appear on the upper face (*B*, fig. 16) of the camera when in use, while the other set ($\alpha\ \beta$ and $\gamma\ \delta$, side *C*) will appear on one vertical side. This arrangement enables the surveyor to see what part of the panorama he is taking during the exposure by sighting along the two lines of the horizontal face, and also up and down the two lines $\alpha\ \beta$ and $\gamma\ \delta$ marked on one of the vertical faces of the camera, thus dispensing with the use of the ground-glass plate and shade cloth altogether. After the camera has been oriented by means of these sight lines and leveled up, as mentioned before, the central clamp screw is tightened and the plate exposed.

The lens is a wide-angle lens, No. 1*a*, of 5⅝-inch focus, made by J. H. Dallmeyer, in London. It really is a combination of two similar lenses, between which the diaphragm is inserted. The aperture of the latter (the stop) used is always the same for all pictures. That end of the lens tube which faces the negative is closed by a planoparallel plate of a yellow or orange color to lessen the actinic action of the blue and violet rays upon the isochromatic plates, thus securing a sharp outline of distant mountain ranges and ridges.

With the plates used, this lens gives an angle of about 45° for the small side and 60° for the long side of the picture.

The camera, six double plate holders (including twelve plates), sunshade, levels, camel's-hair brush for removing dust particles from the slides, etc., are securely packed into a sole-leather case, which has straps attached to it in such a way that the whole can easily be carried on the back like a knapsack.

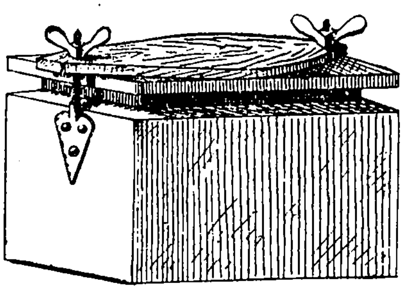


FIG. 20.

The triangular support of the camera is packed with the transit, and the case of the latter is also inclosed in a sole-leather knapsack, with straps for the extension tripod, both being carried together on the back.

The cameras are made by J. H. Dallmeyer, No. 25 Newman street, London, W.

The transits and tripods are made by Troughton & Sims, 138 Fleet street, London, E. C.

Every evening the surveyor replaces the exposed plates in his dark tent by new ones, using a ruby-colored light. He marks the exposed plates in one corner, before their removal from the holder, with his initials, the number of the dozen and of the plate, using a soft lead-pencil. e. g., III 5 means plate No. 5 of the third dozen. (The plates are packed in sets of a dozen each.)

The exposed and marked plates are placed into a double tin box (fig. 20) which can be closed hermetically and which will float when filled with two dozen plates, if by accident it should be thrown into water. These boxes are shipped to the head office, in Ottawa, where the plates are developed by a specialist (Mr. Topley).

The outline sketches of the different perspectives are designated by the same numerals as the plates to which they belong. They show the peaks, saddles, and points to which horizontal directions were taken with the transit (or altazimuth), and they also contain remarks about the weather, illumination, time of exposure, names of localities and features, and any other needed data.

The data obtained with aid of the transit for triangulation purposes are recorded in the usual manner.

The length of exposure for the plates is determined experimentally, as it may be assumed that the same length of exposure will suit a similar subject under similar conditions, with a light of equal intensity, the plates being all of one emulsion.

So-called photometers are used to measure the intensity of the light. They consist of an endless strip of sensitized paper incased in a small metal box—like a small tapeline—a short portion of the paper being exposed to the light and the time noted (in seconds) which it takes to bronze the exposed part of the paper. The nature and coloring of the subject vary but little in phototopography, and the time of exposure should be regulated with reference to the shadows or dark colors of the distant landscape; the darker these are, the longer the time of exposure should be (ten to forty seconds).

On the southeastern Alaskan boundary survey, Mr. O. J. Klotz, Dominion topographical surveyor, received the exposed plates from the different shore parties, and by way of test developed one plate out of every set of a dozen plates in a dark room fitted up for this purpose on the steamer *Thistle* to see that no bad plates had crept in.

The other plates were shipped to Ottawa, where the photographic specialist developed them and also made the enlarged prints (four times the size of the original negative) on heavy bromide paper, which enlarged prints are preferably used for the map construction, as they permit a greater precision in making direct linear measurements and the drawing of construction lines, which would become too minute and intricate if done on the small contact prints. However, if the loss of detail becomes a serious objection, larger cameras should be used or the enlargements should be made on glass. Glass transparencies (enlarged from the small negatives) show minute details in the shadows as well as in the high lights and assure more accurate results, there being no irregular expansion and contraction, as will always be more or less the case with paper prints. The only objection against enlargements on glass lies in the fact of their being less handy in manipulation during the process of the map construction than paper prints. Still the latter could be used for the location of points of detail and minor importance, while all data forming the control of the map are preferably deduced from the enlarged glass prints. Captain Deville is greatly in favor of dispensing with the use of paper prints altogether, and advocates the use of glass enlargements exclusively, the ensuing loss of time being outweighed by far by the great gain in accuracy.

The horizontal angles observed with the transit (or altazimuth) to the points of the terrene marked on the outline sketch which accompanies each negative serve not only for the orientation of the horizontal projection of the plate on the plan (the picture trace), but they also aid to counteract in a measure and to ascertain the distortion of the paper prints. The vertical angles, with the plotted distances, are used to check and verify the position of the horizon line on the different photographs.

To test whether the plate is vertical after the camera has been leveled up the following process is carried out:

Insert a piece of plate glass or a plano parallel mirror of $4\frac{3}{4}$ by $6\frac{1}{2}$ inches into the plate holder, open the rear slide of the latter, and level the camera carefully. Now set up a level (altazimuth or transit, with the vernier of the vertical circle set at 0°) near the back of the camera and revolve in azimuth until a well-defined and distant point of the landscape is covered by the intersection of the cross wires of the level telescope; the reflection of the same point must also be visible in the reflecting surface at the back of the camera from the level station. If on directing the level toward this reflected image the latter is also

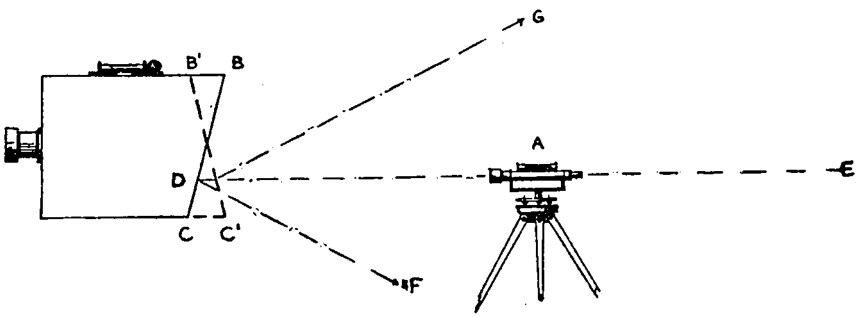


FIG. 21.

bisected by the horizontal wire of the level the plate in the camera will be vertical. (See fig. 21.)

If *A* is the position of the level and *E* the selected point of the terrene on level with the elevation of the instrument and *B C* the reflecting surface at the back of the camera, then the line *A E* will be horizontal, and if the plate *B C* is vertical *E* will be reflected in *B C* at *D* and *D A* and *E* will be in the same horizontal plane.

Should the top of *B C* be inclined toward *A* (as in fig. 21), then the ray *E D* will be reflected in the direction of *D F*, and the reflected point *D* will no longer be bisected by the horizontal wire of the level, but will fall below the same. Should the plate *B C* incline upward, as shown in *B′ C′*, the reflection *D* of *E* would appear above the horizontal thread of the level telescope, *E D* being now reflected in the direction *D G*.

Should the plate be thus found not to be vertical after the camera has been leveled up, the inclination must be changed by means of the leveling screws of the camera until *D* falls upon the horizontal thread of the level telescope, and the level, which is at right angles to the camera plate, must now be adjusted to conform with this corrected position of *B C*.

The plate holders must of course be well made, and all be exactly alike, so that the above conditions are fulfilled by every one of them, and that the distance of the sensitive plate from the lens be the same; i. e., the sensitive surface of every plate should fall into the focal plane of the camera lens.

The focal length of the camera, which has a constant value for every camera, must be determined directly if the negatives are to be used for plotting; but if prints are to subserve the construction of the map, this determination should be made from a print.

It has been previously mentioned that the prints rarely correspond in size with the negatives. They either expand or contract, sometimes both, and the distortion is greater in one direction of the paper than in the other. If this distortion is uniform in all directions the print will be similar to the negative and correspond to the perspective of the same landscape on a vertical plane (parallel to the plate), but nearer the lens when contracted and farther from the lens when expanded. The prints have either a shorter or a longer distance line (focal length) than the camera plate.

As (enlarged) prints are used for the map construction of the Canadian survey in southeastern Alaska, the constants required for this construction of the horizontal plan (i. e., the focal length, the horizontal and principal lines) are obtained from such a print.

This is done by taking a picture of some large building or any landscape with well-defined points from a station of which the distances to said points are known or can be ascertained by direct measurements in the field. From the same station vertical and horizontal angles are measured to the selected points, and the points as well as the station are plotted on a sheet of paper, and radials are drawn from the plotted station through the selected and plotted points marked on the print and plotted on the paper.

On a strip of paper, one edge of which is made perfectly straight, the points marked on the photograph are laid off, and this strip is moved over the plotted radials until the lines bisect the corresponding points marked off on the straight edge. A line is now drawn along this edge on the drawing sheet and a perpendicular dropped on this line from the plotted station. (See fig. 22.)

The line *H H'*, representing the paper edge, will be the picture trace, the perpendicular line *O P* will be the distance line, and *P* will represent the horizontal projection of the principal point.

The paper is now again laid on *H H'* in such a manner that the radials bisect the points marked on the straight edge, and *P* is marked off on the latter.

From the known distances of the reference points from the station and their vertical angles the elevations of these points, above or below the horizon of the station, are computed and laid off on the photograph. This will enable the draftsman to draw the horizon line on the photograph, and after projecting the marked reference points upon this line the strip of paper is placed on this line in such a manner that the corresponding points cover each other, and P is transferred to the picture. A vertical to $H H'$ through P will represent the principal line $V V'$ on the print. The points where the principal and horizon line bisects the comb marks are noted and will serve to draw these lines on all other similar prints without any new determinations being necessary.

In order to lay off the elevations of the selected points above or below the horizon of the station an approximate horizon line had to be drawn on the photograph. This is done by setting the vernier of the vertical circle of the transit (which replaced the camera in such a way that the

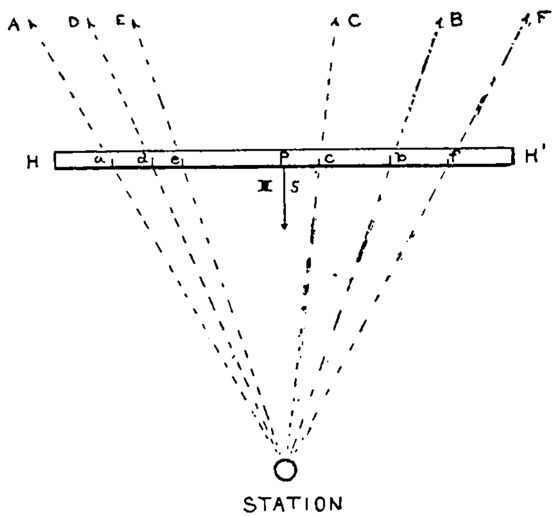


FIG. 22.

optical axes of camera and level telescope were in the same horizontal plane) at 0° and noting several points of the building or landscape which were bisected by the horizontal thread of the telescope while level and by drawing a line through the same points pictured on the photograph.

PLOTTING.

The field data of the Canadian surveys in Alaska are plotted on a scale of 1:80000, with a contour interval of 250 feet, indicating the 1,000-foot contours by heavier lines.

From the original negatives copies are made, four times enlarged on heavy bromide paper (9½ by 13 inches), which are used for the construction of the maps.

The triangulation points, obtained by means of a 4-inch transit, are plotted, and their elevations (obtained barometrically, and checked by vertical angles whenever convenient) are added in red ink.

Now the principal line and the horizon line are drawn on the prints through the proper points of the comb marks, or in proper position with reference to the position of the lens during the exposure of the original plate, if the lens was attached to a revolvable plate, an arrangement not approved by Capt. E. Deville, as a surveying camera should be rigid in all its parts in order to obtain uniform results.

The horizon line, obtained by means of the comb marks, is checked and corrected, if necessary, by means of pictured points of known elevation, each print containing two or more points which have been determined trigonometrically and which have previously been plotted on the working sheet and the elevations of which are inscribed in red ink.

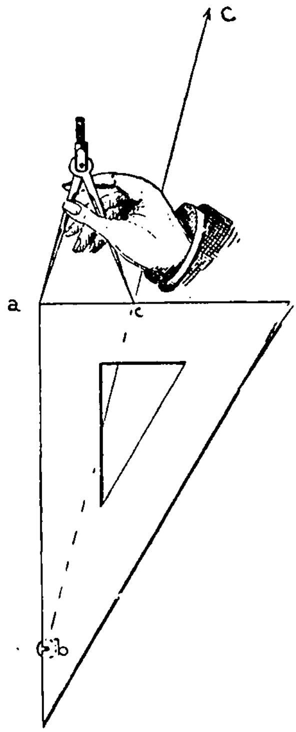


FIG. 23.

The picture traces are plotted on the working sheet as follows (fig. 23):

Take a triangle of hard rubber or wood and mark off along one side the focal distance of the print a b from the right angle a, and carefully notch b so that the center of a needle inserted into this notch will be at a distance from a equal to this focal length. From the outline sketch select a point, C (fig. 23a), to which angles have been read, and take the abscissa of $C = C_0 P_0$ between the points of a pair of dividers. Insert a fine needle into the plotted station (b, fig. 23) whence the picture was taken and place the triangle notch b close to this needle; now move the triangle with the left hand so that the line of direction to the plotted point c will be covered by the triangle, and with the right hand hold the dividers, moving one point over this line of direction until a c is equal to C_0 P_0 (the triangle must be moved likewise until this takes place). The triangle is securely held in this position and lines are drawn on the working sheet along a b and a c; prolong a c beyond a, and now check the distance a c again with the dividers (to be $= C_0$ P_0). Then b a = horizontal projection of principal ray.

a c = horiz. projection of picture plane.
$= H$ H' = picture trace.
a = principal point.

The trace of the principal plane ($= a$ b) is only marked by a short line bearing an arrow pointing toward the plotted station whence the picture was taken, and the point a is marked the same as the print to which it refers; e. g., III 5 (fig. 22). The prints are marked with the name of the camera station whence they were taken, with the numerical designation of their negative, and with the elevation of the horizon line above the datum plane; e. g., *N N*, III 2, 4,860, means name of camera station, second plate of third set of a dozen plates (or really the twenty-sixth plate), and elevation of camera horizon = 4,860 feet above mean sea level.

After the traces of the principal and corresponding picture planes have been plotted, the draftsman selects from the prints a pair of photographs which are represented on the plan by their traces, taken from two different camera stations and overlooking common ground. On these pairs he identifies as many points as possible, marking all corresponding points alike with numbers or letters and with a dot in red ink. Of course such points are chiefly selected which indicate characteristic features of and changes in the terrene like knolls and depressions in ridges, peaks, bends in streams, coves in the shore line of lakes and rivers, buildings, etc.

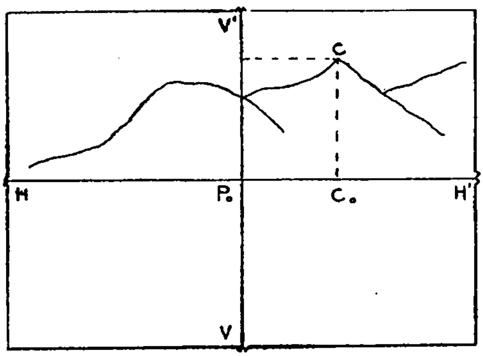


FIG. 23*a*.

After enough pictures have been selected to develop a certain area and the identification and marking of corresponding points have been completed, projections of all these points on the horizon-line are marked (their abscissas are measured) and transferred to the straight edge of a strip of paper, including the principal point of every photograph.

These strips are now placed upon the plotted traces (fig. 22) to which they belong in such a manner that the principal points of trace and paper strip coincide, and in this position they are held on the working sheet by means of small thumb tacks or paper weights.

To plot the horizontal projections of a point shown and marked on two prints we insert two fine needles into the plotted stations of these two prints and attach a fine silk thread to each needle. The other end of the thread is connected with a small paper weight by means of a thin rubber band (fig. 23*b*).

The weighted thread attached to station needle *I* is now moved over the weighted strip (indicating the picture trace) until it bisects the projection a' of the sought point *A*. The weight is now placed upon the paper, holding this thread under slight tension in this position. The second thread connected with the needle in Station *II* is placed over the projection a'' of the sought point, also under tension. The point of intersection of these two threads will be the desired point *A* plotted on the map. After this position of point A on the plan has been checked, in the same manner, by means of another photograph taken from a third station and containing the picture of this point, its plotted position is marked by a dot and its designation, as given on the prints, in red ink.

After a sufficient number of points have been plotted in this manner by intersections, and after they have been supplied with the letters or numerals (in red ink) as given on the prints, their elevations are deter-

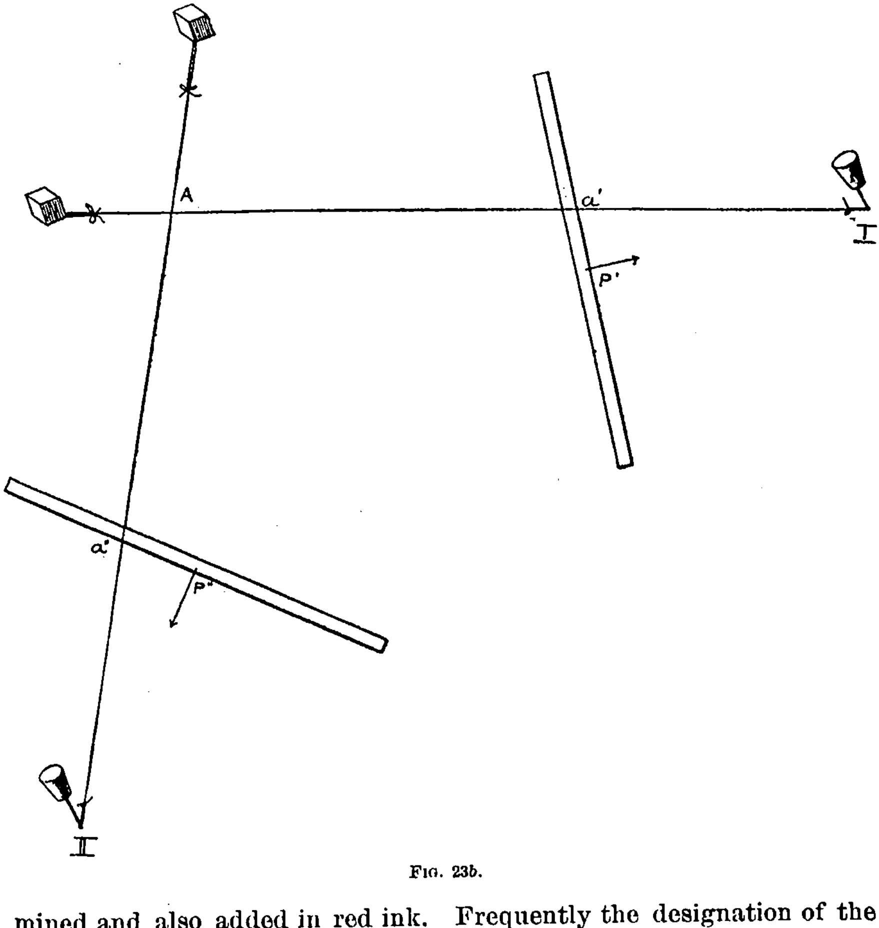


FIG. 23b.

mined and also added in red ink. Frequently the designation of the points by letters or numerals are only added in pencil, to be erased after the elevations have been added in red ink.

ELEVATIONS.

All points of the prints which are bisected by the horizon line H H' have the same elevation as the horizon of the camera station, which fact will greatly assist in drawing in the contours on the plan. The latter can be plotted or drawn in with the same precision as is attainable in other "irregular methods" of contouring if only enough points can be identified on the prints and established by intersections and their elevations to cover the area sufficiently close to leave no place for doubt. Ridges, which appear in profile on the prints, will also facilitate contouring, inasmuch as lines of directions drawn to characteristic points of these ridges can be regarded as tangents to the contours passing through such points. The heights of the points fixed by intersections are found by means of a so-called "scale of heights" (fig. 24).

$S\,x\,P$ = straight line divided into equal parts.

$S\,P$ = focal length of prints.

$P\,P'$ perpendicular to $S\,P$ and divided into equal parts.

Erect verticals to $S\,P$ in the points of division. Join the points of division along $P\,P'$ to S.

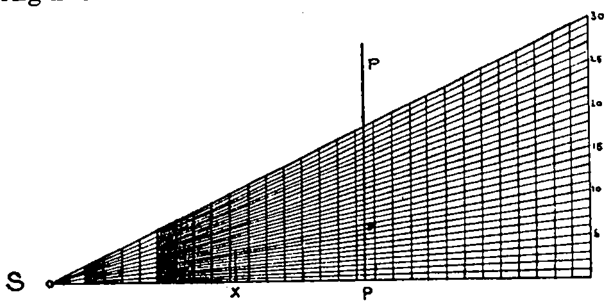


FIG. 24.

This scale is used as follows:

Take, with a pair of dividers, from the photograph the ordinate of any point bisected by the principal line of which the elevation is sought, transfer this length to $P\,P'$ from P. Suppose it corresponds to $P\pi = 11$ parts of the graduation of $P\,P'$.

Now take from the plan with the dividers the distance of the horizontal projection of the point (previously plotted by intersections) from the picture trace and lay this length off on $S\,P$ from P to the right or left of P according to the position of the point on the plan in regard to the picture trace if beyond or within the trace and station. Suppose the point was between the plotted station and picture trace and it fell on x. Then the distance from x to a point vertically above x on the ray $S\,\pi$ (= 11) and measured on the plotting scale will represent the elevation of the point above or below the camera horizon. If the point on the photograph was above $H\,H'$ this length will have to be added to the camera elevation and the sum is entered on the plan in pencil close to the plotted point. After it has been checked by a second photograph, and the discrepancy between these two heights is within the permissible limit of error, the mean is entered in red ink on the plan and the pencil marks are erased. After the elevations of all the points plotted on the working sheet have been determined and entered on the drawing in red ink, the streams, ridges, bluffs, and shore lines are drawn in, using intersections and tangents, whenever possible, to identify and locate their characteristic bends, their terminals, etc.

Now the contour lines are drawn in by estimation between the established points of known elevation ("irregular method"), having the shore lines, streams, and ridges as guides, and studying the photographs as much as possible to modify the contours so as to represent minor inequalities and accidents of the terrene.

As long as a sufficient number of points is obtained by intersections there will be little difficulty in drawing in the contour lines, but in a rapid reconnaissance it may happen that the points which can be plotted are too few and too far apart for defining the surface, when it will become necessary to resort to so-called "tricks of trade" and less accurate methods. The perspectograph and similar instruments to convert perspectives into plan drawings and vice versa are too complicated (the numerous movable parts are sources of too much lost motion) to give results sufficiently accurate for topographical maps.

THE PHOTOGRAPH BOARD.

(Fig. 25.)

So many lines are needed and drawn for the constructions on the photographs that it is advisable to prepare a special drawing board on which as many of the construction lines are drawn, once for all, as would have to be repeated for the different prints of uniform size. This so-called "photograph board" is an ordinary drawing board covered with tough drawing paper, the surface of which is to represent the picture plane, and it is used in conjunction with the photographs.

Two lines, $H H'$ and $V' V$, are drawn at right angles to each other; they represent the horizon and principal lines, while $V P_0 = H P_0 = V' P_0 = H' P_0 =$ focal length of prints. By revolving the horizontal plane about $H H'$ we obtain the upper and lower distance points V' and V in the picture plane, and by turning the principal plane about the line $V' V$ into the picture plane we obtain the left and right distance points H and H'.

The photograph is put on the middle of the board in such a manner that the principal line coincides with $V' V$ and the horizon line with $H H'$. The four scales, forming the sides of the square $T U R S$ (a little larger than the photograph which falls within $T U R S$), can be used to draw parallels to the horizon and principal lines, without obscuring the print by too many pencil lines, by placing a ruler on the corresponding graduation marks of two opposite scales. Also for marking the "ground line" for any station by joining the graduations of the vertical scales representing the height of the station.

At a suitable distance from H, outside of the photograph field, a perpendicular, $K L$ to $H H'$, is erected, on which line are marked, by means of a table of tangents, the angles formed with $H H'$ by lines drawn from the left-hand distance point H. This graduation, $K\,L$, serves for measuring the horizontal angles or the altitudes of points selected on the photographs, as will be explained in the following:

From V as a center describe a circular arc, $P_0\,C$, with $V\,P_0 =$ focal length as radius, and divide this arc into any number of equal parts. Through the points of division, between $P_0\,C$ and $P_0\,H'$, radials are drawn from V as center.

In order to obtain the elevations of points marked on the photographs the radial distance from the station to the horizontal projection of the point in the picture trace must be known. If the sensitive plate formed

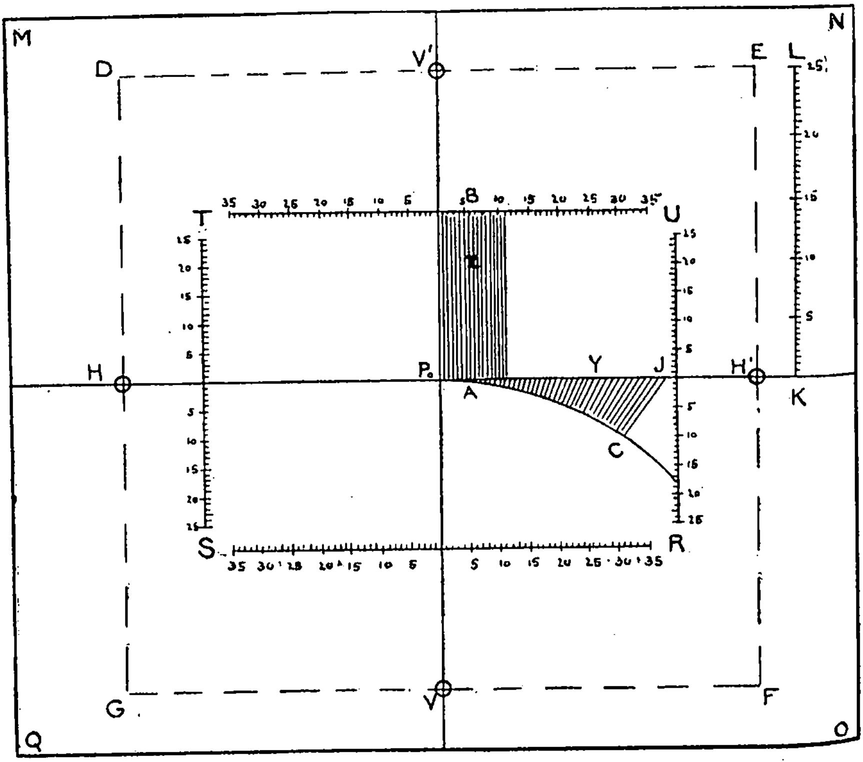


FIG. 25.

a part of a vertical cylinder with radius = focal length, then these distances would all be constant and equal to the focal length of the print. The arc $P_0\,C$ (divided into any number of equal parts) and the line $P_0\,H'$ cuts off pieces of the radials drawn from the station V to the various horizontal projections of pictured points, which must be added to the focal length $V\,P_0 = V\,C$ to give the horizontal distance of the station to the projected point on the photograph.

The equidistant lines $A\,B$, drawn parallel to the principal line, are also drawn sufficiently close together and cover a space in width equal to the largest radial difference $C\,J$. All these lines ($A\,B$ and $A\,C\,J$)

are used in connection with the scale of degrees and minutes $L K$; e. i., on a print, $T U S R$ (fig. 26), we wish to obtain the elevation $c c'$, the vertical and horizontal angles to c having been observed in the field and noted on outline sketch. From H (fig. 25) we draw a line through that division mark on $K L$ which corresponds to the vertical angle of c, say $10\frac{1}{2}°$. Now the abscissa $P_0 c' = x_c$ is laid off (on the photograph board) along $P_0 H'$ from P_0; the second point of the dividers may fall upon the twentieth radial difference (counting from P_0 toward c). The length of the difference between the radius of the arc $P_0 c$ and the distance $V C$ (from the plotted camera station V to the plotted point c) is now laid off along $P_0 H'$ from P_0, which may fall midway between the fourth and fifth line (counting from P_0 as zero) of the set $A B$. Then the distance from the line $P_0 H'$, taken midway between the fourth and fifth $A B$ line to the line $H\ 10\frac{1}{2}°$ (on degree scale $K L$) and measured on the plotting scale, will be the difference of elevation $c c'$, which, added to the elevation of the camera horizon, will give the elevation of the point c above the datum plane. Sometimes the angle between a point, a, on the print and the principal and horizon lines (altitude and azimuthal angles) may be wanted.

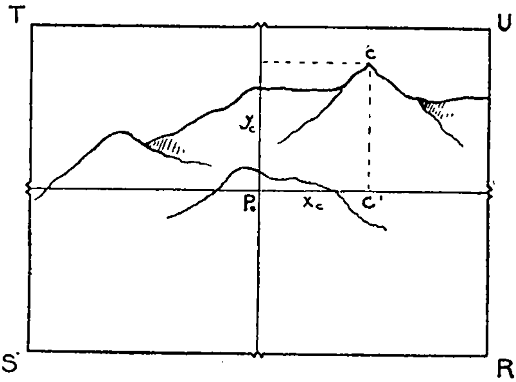


FIG. 26.

The azimuthal angle $P_0 P a_0$ (fig. 27) can be found directly by joining the plotted station P to the horizontal projection a_0 of the point a'. To find the same in degrees and minutes we transfer the abscissa $P_0 a_0$ of the point (fig. 27) a' to $P_0 V'$ (on the photograph board) from P_0 and draw a line from H through this point on $P_0 V'$. Where this line intersects the scale $K L$ will be the reading indicating the value of the azimuthal angle in degrees and minutes.

The altitude of the point a' on the photograph (fig. 27) is represented by the angle $a' P a_0$, and to find its value we transfer $P_0 a_0$ the abscissa of a' (fig. 27) to $P_0 H'$ from P_0 on the photograph board; say, equal to $P_0 Y$. With the same pair of dividers we take the radial difference at Y (distance of Y to the arc $P_0 C$) and transfer the same to $P_0 H'$ from P_0, and note which of the verticals $A B$ falls upon the second point of the dividers. On this vertical we transfer the ordinate of $a' = a' a_0$ from $P_0 H'$; say, equal to $A Z$. If we draw a line through Z from H, this line will indicate on the divisions of scale $K L$ the value of the angle $a' P a_0$ (fig. 27) in degrees and minutes.

The foregoing is a general description of the plotting methods employed by the topographers under Dr. W. F. King, boundary commissioner for the southeastern Alaskan survey. There remain, of course, some minor details which serve in a measure to facilitate the work of plotting and which every draftsman acquires by practical application, and which are not touched upon in the preceding pages.

The enlarged prints are made upon positive bromide paper, using a good copying lens to secure as much detail as possible. The prints are developed and dried in the usual manner, and classified. The essential requirement for a true copy or correct enlargement is that the sensitized paper be parallel to the negative. Both the printing camera and the easel upon which the paper rests are provided with graduations on their slides to facilitate giving the correct distances both for enlarging and reducing. An inclined position of the easel and negative is the

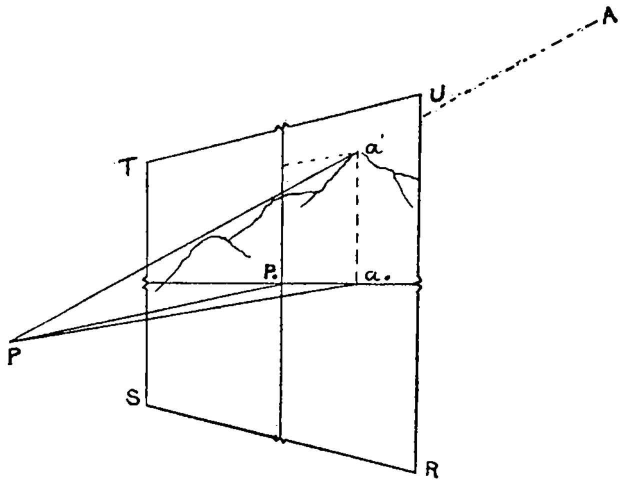


FIG. 27.

essential feature of the Canadian printing apparatus in order to give the negative the full skylight and not have a part of the plate illuminated by reflected rays from the earth, thus giving the entire plate a uniform light during the time of printing.

In France, Germany, and Italy the tendency has been toward combining the transit and camera into one instrument, while in Canada these instruments have been kept separate. The reason, probably, is that in the European countries mentioned, photographic surveys were made on a large scale and the means for safe transportation were comparatively within easy reach, while in the Canadian surveys the work was done on a small scale and the instruments had to be carried over rough country, frequently on the human back, thus making it essential to reduce the weight to a minimum (which could be best done by keeping the instruments separate) and to make them rigid and easy of adjustment.

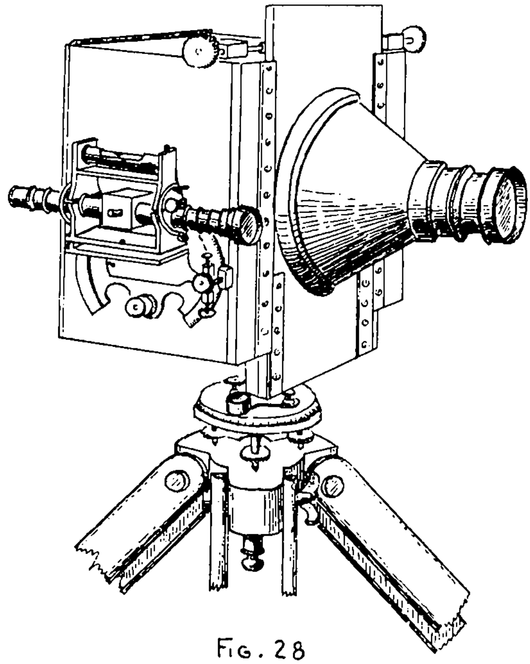


FIG. 28

FRENCH PHOTO-THEODOLITE.

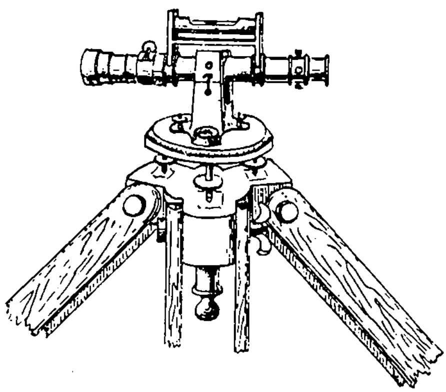


FIG. 29

THE SAME WITH CAMERA DETACHED.

Figures 28 and 29 show the latest model, in its general form, of the French "phototheodolite," which was on exhibition at the World's Columbian Fair, in Chicago. Figure 28 shows the complete phototheodolite, and figure 29 shows the theodolite without the camera, in which form it is used for trigonometrical purposes, the triangulation being made before the phototopography is begun.

This phototheodolite has a "declinatoire" (compass) and a pair of sights, which will enable the observer to direct the optical axis of the camera quickly toward any desired object or point of the panorama. The "declinatoire" is principally used for traverse work, to note the magnetic meridian from time to time. This compass and the sights are not visible in the figure; they are on the side opposite to the one with the telescope.

The near side of the instrument (fig. 28) shows the telescope, level and vertical circle for making angular measurements, in addition to the previously executed triangulation, in order to determine the position of the camera station (by the three-point problem) with reference to surrounding triangulation points or for running traverse lines between triangulation stations. The horizontal circle is under the camera proper and carries a box level. The optical axis can be elevated or depressed (maintaining a horizontal position) by means of a slide or shutter to which the camera lens is attached.

Dr. Meydenbaur's camera theodolite is a camera of constant focal length, constructed in metal throughout, with neither telescope nor vertical circle, but it is provided with a horizontal limb and mounted on a tripod.

After the instrument is leveled up, the panorama surrounding the station can be surveyed (photographed) by making six consecutive exposures, revolving the camera 60° in azimuth after each exposure by means of the horizontal circle, which is graduated to read to single minutes.

The lens is a pantoscopic one, made by E. Busch, in Rathenow, Prussia. It commands an angle of about 100°, but by excluding the external ring of this lens by means of a small stop in the diaphragm, pictures are obtained subtending a horizontal angle of about 66°, thus giving two consecutive plates a common margin of 3° width, horizontally.

The horizontal projections of the six picture planes, representing the panorama surrounding one station, form a regular hexagon, and after one picture trace has been plotted and oriented on the plan, the remaining five are readily plotted by constructing this hexagon of which one side is given.

To use this instrument it is necessary that the country to be sursurveyed photographically be well supplied with a generous number of carefully selected triangulation points of a recently made triangulation, as it is necessary that the signals shall be still intact and standing during the photographic operations. The triangulation must also include the hypsometrical determinations of all the stations, as no direct measurements of vertical angles can be made with this camera theodolite. The elevations of all the other points needed for the topographic development of terrene will have to be obtained by constructions from the negatives or photographic prints by means of the elevations of the horizon line, obtained from the given elevation of the station (triangulation point) and the height of the instrument above the ground. The survey will have to be plotted on a large scale and numerous camera stations will have to be occupied.

The recently made small traveling camera theodolite of Dr. Meydenbaur dispenses with plate holders, inasmuch as the plates are placed directly against the rear frame of the camera by an ingenious arrangement with which the plates can be exchanged under exclusion of light.

One of the more recent productions of photographic surveying instruments in Austria is Captain Hübl's plane table photogrammeter, made by R. Lechner in Vienna, and described in "Lechner's Mittheilungen aus dem Gebiete der Photographie und Kartographie." Wien, Graben No. 31.

The camera proper has also been modified like the latest Meydenbaur camera by discarding the plate holders. Captain Hübl places the sensitive plate directly against the camera casing, where it is held in position by means of springs, thus securing a constant permanent focal length. The camera box is cube shaped and has sides of 21^{cm} length. The plates are 12 by 16^{cm}, but the pictures are only 10 by 14^{cm}. The camera alone weighs $3\frac{1}{2}$ kilogrammes; with knapsack, including entire outfit for one day's work and stout tripod, the weight is $11\frac{1}{2}$ kilogrammes and the cost in Vienna is 400 florins.

This instrument is the result of Captain Hübl's efforts to reduce the weight and cost of the camera theodolite and to simplify the adjustments and manipulation of the same.

For this reason the theodolite has been replaced by a plane table and small alidade.

The upper surface (horizontal) of the camera, 21 by 21^{cm}, serves as plane table; it is provided with a pivot with which the alidade is connected. By means of special appliances the picture trace, principal line, and point, as well as horizontal directions to known (triangulation) points, for the orientation of the picture trace, are drawn directly on the paper, resting on the upper horizontal surface of the camera.

Each negative can thus be accompanied by a small plane-table sheet showing a bunch of rays radiating from the station point to a number of known points (triangulation points) in correct relation and position to the picture trace and principal line, besides numerous data which can be sketched and inscribed upon the paper.

The results obtained with this photographic plane table are easily transferred to the working sheet containing a plot of the triangulation. The adjustments of this instrument are few and simple.

APPENDIX No. 4—1893.

ON PHOTOGRAPHY AS APPLIED TO OBTAIN AN INSTANTANEOUS RECORD OF LUNAR DISTANCES FOR DETERMINATIONS OF LONGITUDE.

By C. RUNGE.

Translated and submitted for publication December 9, 1893, by J. A. FLEMER, Assistant.

Since Dr. F. Stolze's treatise on photographical determination of geographical positions without the use of chronometers was published, Mr. C. Runge, in Hanover, Prussia, has made experimental observations with an ordinary camera, such as travelers and explorers generally carry in their outfits, to develop a photographic method for obtaining the geographical longitude and latitude of a place, as well as the local time by means of photography.

Photographic determinations of the latitude and the local time of a place, however, do not offer great advantages, compared with the general methods heretofore in use for obtaining these values, as nearly every explorer will carry in his outfit instruments which can readily be used for this purpose, the ordinary methods for astronomical latitude and time observations being comparatively simple and easily applied. We will, therefore, in the following pages, consider only Mr. Runge's method for determining the geographical longitude photographically.

The desirability of developing a method for finding the longitude without the use of chronometers, which, when the geographical longitude of a place had been determined by means of chronometer readings, could also be used to check the latter, had not only been recognized by Mr. Runge, but he also felt convinced that if the method should find favor with explorers, the necessity of making astronomical observations—such as would be necessary, for instance, for longitude determinations based on lunar distances—could be avoided.

A full description of Mr. C. Runge's first application of photography, made June 17, 1893, in Hanover, for this purpose, will be found in the Zeitschr. f. Verm., Heft 15, 1893, of which the following is a free translation.

The camera, placed upon a window sill and its position secured, as well as possible, against accidents, was directed upon the new moon at 10 p. m. (June 17, 1893) and eight successive short exposures of the same plate were made; at 10^h, $10^h\ 2^m$, $10^h\ 4^m$, $10^h\ 8^m$, $10^h\ 10^m$, $10^h\ 12^m$, $10^h\ 14^m$, and at $10^h\ 23^m$ p. m., an ordinary watch being used for timing the latter.

The camera, with objective closed, was left undisturbed in the same position until the constellation Leonis appeared in the same part of the firmament where the moon had been photographed. At $10^h\ 51^m$ (by the same watch) the objective of the camera was uncovered and the plate remained exposed, short interruptions excepted, until $12^h\ 45^m$.

These interruptions, of five seconds' duration each, were effected by means of a dark cloth, with which the objective was covered, without having been brought into contact with the same.

Twenty such interruptions of the exposure were made in toto, and their times of occurrence were carefully noted by the same watch and recorded. Between $10^h\ 51^m$ and $11^h\ 00^m$ two such interruptions took place. From 11^h till 12 p. m. interruptions were made every five minutes, beginning at the full minute and lasting five seconds (12 breaks); from 12 p. m. till $12^h\ 40^m$ the breaks occurred in the same manner, but at intervals of ten minutes (4 interruptions), and two breaks were made at odd times, one at $11^h\ 37^m$ and the other at $11^h\ 54^m$.

After the plate had been developed the moon's crescent appeared eight times, as was to be expected, in the central portion of the plate. Above and below this row of moon pictures the star traces were plainly visible in the shape of smooth curves of a regular curvature.

From the relative positions of these curves and the positions of their beginnings and end points it could readily be conjectured which star of the constellation Leonis belonged to each curve. The star traces of α, β, γ, δ, ε, ζ, η, and ϑ Leonis, besides some other faint star traces, were plainly distinguishable upon the developed plate. The trace made by δ Leonis was the most distinct of all, this star having been farther north (in a darker portion of the firmament) than the other bright stars of the constellation.

The traces of β and δ Leonis, scrutinized under a microscope, distinctly showed the gaps, corresponding with the recorded interruptions, made during the exposure. These breaks were less clearly shown in the other star traces, partly because they appeared less bright, having been nearer the horizon, and partly because their light was dimmed by the illuminated western horizon.

The positions of the two breaks, corresponding with the two interruptions of the exposure, made at random (at $11^h\ 37^m$ and $11^h\ 54^m$) among the regular series, formed characteristic pointers toward identifying the breaks with their corresponding recorded time observations (by the watch).

On the lower part of the plate the upper outlines of two buildings were shown, one with a lightning rod and the other with a flag pole.

In order to ascertain the Greenwich time by means of this plate, Mr. Runge employed different methods of mensuration. The mensuration was done with an instrument used heretofore for making measurements on photographic plates in analytical investigations of spectra (with the spectroscope). Its principal parts are a frame over which a sleigh can be moved horizontally by means of a horizontal screw having a very fine thread.

The plate is placed upon the movable holder (sleigh) and illuminated from below by means of an inclined mirror. A microscope with cross wires is secured to the stationary frame in such a manner that the plate can be studied through the same while the plate is passed underneath in a horizontal plane by turning the screw passing through the plate holder (sleigh). The horizontal linear change in position of the plate when thus moved is measured by the number of turns of the screw. This screw has two threads per 1^{mm} length, and is supplied with a micrometer at one end, divided into 100 parts, and a vernier. Thus, $\frac{1}{200}^{mm}$ can be read with the index mark, and by using the vernier $\frac{1}{2000}^{mm}$ can be measured. A registering apparatus marks the full turns of the screw.

With the aid of this micrometer the following measurements on the plate were taken:

1. *To determine the right ascensions of the pictured crescents*, we find and mark on the trace of δ Leonis that point which corresponds with the meridian of one of the crescents—for instance, the first one of the eight shown on the plate—and by means of the gaps shown in this star trace the time of transit (the time as shown by the watch when δ Leonis had reached that particular spot on the trace) of δ Leonis at this marked point is ascertained.

The time which had elapsed from the moment of photographing the first position of the moon (10^h) until δ Leonis had reached the point in question on its trace gives in watch time the difference of the hour angles of the moon and δ Leonis.

This civil time interval converted into sidereal time will represent the difference of the moon's and δ Leonis's right ascensions; and after the value for the right ascension of δ Leonis has been taken from the fixed star catalogue of the Ephemeris, we will thus have found the right ascension of the moon at the time of the first exposure.

As only short time intervals enter into consideration, the quality of the watch is immaterial.

In detail the mensuration was made as follows:

By means of the difference between the right ascensions of δ and β Leonis, as taken from the Ephemeris, two points were located on the corresponding two star traces, situated as nearly as possible upon the same hour circle, which two points were joined by a fine line scratched into the film of gelatin on the plate by means of a fine needle and straight edge.

The photograph being a true perspective, all meridians are represented by straight lines on the same, and this scratch, if carefully made, should represent a meridian line. If the direction of this scratch does not appear perfectly correct, its deviation from the true position can be ascertained by measuring the distances from two corresponding breaks in the two star traces (of δ and β Leonis) to the intersections of the scratch with these two curves, and the necessary correction can be applied.

After this has been done, the distance between the crescents and the scratch are measured; and as the distance between the breaks—shown in the star traces of δ and β Leonis—between which the crescents are situated are known, we can compute the time interval corresponding with a certain length of the moon's circle of declination.

If the scratch passes close to the picture of the moon, a small error in the value with which the distance is to be multiplied in order to obtain the time interal will barely affect the result.

As only the edge of the moon could be measured in our case, a correction for the moon's semidiameter had to be applied in order to obtain the right ascension of the moon's center, which was done by measuring a chord and corresponding height of the crescent's arc. The resulting right ascension is free from atmospheric refraction, as it has been determined from the relative position of the moon with reference to the pictured stars on the plate, which were also subjected to the same atmospherical influences (and are likewise affected by refraction). In order to reduce this right ascension to the center of the earth the declination and the local time must be known or will have to be found.

2. *Mensuration of the declination of the lunar pictures.*—For this purpose the plate was placed upon the movable holder in such a manner that the direction of its course under the microscope (when moved in a horizontal plane by turning the micrometer) was vertical to the star traces.

Pointings were now made to as many of the star traces as possible, as well as to the edge of the crescent; for instance, the traces of ζ, δ, γ, β, and α Leonis were bisected, and also the edge of the crescent falling between γ and β.

If we now write opposite the micrometer values for the bisected star traces the declinations of the corresponding stars, we can regard the latter as a linear function of the micrometer readings. The two unknown values of this linear function are computed by means of the method of least squares, and after substituting the micrometer readings for the edge of the moon we find her declination.

The following example will show the degree of precision obtained in this manner:

	Micrometer reading.	Declination, taken from the Ephemeris.			Declination computed.			Diff.
		°	′	″	°	′	″	″
α Leonis.	249·0	12	29	26	12	29	15	+11
β "	2515·1	15	10	12	15	10	31	−19
Edge of moon.	6441·1				19	49	53	
δ Leonis.	7515·9	21	6	38	21	6	23	+15
ζ "	9916·7	23	57	07	23	57	13	−6

We believe that the declination of the moon's edge can be found by this method within a limit of error of 20″, and if the star curves are all close to the lunar picture even a more close value may be obtained.

The semidiameter of the moon having been determined as mentioned above, we can now compute the declination of the moon's center. The declinations having yet to be reduced to the center of the earth, we will need for this purpose, besides the right ascensions, also the local time, which is found as follows:

3. *Determination of the local time.*—We could assume the local time to be known, as the explorer will generally determine the same astronomically, especially if he intends to determine the longitude of the place of observation by means of chronometer readings. He can readily find the local time by observing sun, moon, or star altitudes with a sufficient degree of accuracy for practical purposes.

Whenever the photographic plate contains the image of a fixed terrestrial point—for instance, the lightning rod, gable, or chimney of a distant building, the peak of a distant mountain, etc.—such point can be utilized in the same manner as a star. As mentioned before, the exposed plate contained a picture of a conspicuous lightning rod. The circle of declination and hour circle of this point were determined in the same manner as shown for the lunar pictures (by means of civil time as indicated by the watch). The following day (June 18) the elevation of this point (of the lightning rod) was determined from the place occupied by the camera in the preceding night. From this elevation and the declination we can determine the hour angle, if the geographical latitude be known, and we are thus enabled to compute the difference between the time indicated by the watch and the local time.

After the local time has been found in this manner, the determined values for the right ascension and declination can be reduced to the center of the earth, the altitude of the lunar pictures being known if the local time and the declination are given.

4. *Measuring a lunar distance.*—We measured the distance between the first lunar picture and that break in the stellar curve belonging to δ Leonis which was nearest the crescent without being intersected by the same hour circle.

The angle corresponding to a measured length on the plate can be computed if we know the distance between the traces of two stars. It is true, the plate represents the area of the firmament on a variable scale (it being a perspective representation of the photographed area of the firmament), yet for a small portion in the center of the plate we can assume that there is no distortion. In the present case the distance amounted to only 2½°. A star, the image of which would have appeared at the time of exposure for the first lunar picture on that part of the plate indicated by the break referred to, would have to have had the same declination as δ Leonis, and its right ascension would be found from the time which would have elapsed until δ Leonis had reached the same place.

By means of this imaginary star point we can compute in the same manner as for a true star its lunar distance for any given Greenwich time, and also from the measured lunar distance, reduced to the center of the earth, we can interpolate the Greenwich time.

The following tabulated results were obtained by these three methods:

Mean of the R. A's of the first three lunar picture centers reduced to the center of the earth.	Mean time for Greenwich.	Local time.	Difference of time.
h. m. s.	*h. m.*	*h. m.*	*m.*
9 23 40·7	8 56·3	9 35·4	39·1

Declination of the eight lunar picture centers reduced to the center of the earth.	Mean Greenwich time.	Local time.	Difference of time.
° ′ ″	*h. m.*	*h. m.*	*m.*
20 19 33	8 54·1	9 33·4	39·3
18 58	8 57·1	35·4	38·3
18 28	8 59·6	37·4	37·8
18 1	9 1·8	41·4	39·6
17 27	9 4·7	43·4	38·7
17 22	9 5·1	45·4	40·3
16 50	9 7·8	47·4	39·6
15 9	9 17·2	56·4	39·2
		Mean	39·1(±0·2)

Lunar distances measured and reduced.	Lunar distances computed.	Mean Greenwich time.	Interpolated mean Greenwich time.	Local time.	Diff. of time.
° ′ ″	° ′ ″	*h. m.*	*h. m.*	*h. m.*	*m.*
2 35 19	2 39 34	8 45	8 54·8	9 33·4	38·6
	2 38 20	9 0			

According to these three methods we find the three corresponding differences of time:

	m.
	39·1
	39·1
	38·6
Mean	38·93

The true difference of time for the "market tower" in Hanover City is, according to Gauss:

m.
38·943

And as the place of observation was 650 metres west from this tower, the difference of time for the camera station would be:

m.
38·90

This close result seems to partake of the nature of a coincidence. Still, we believe that an error of more than $0{\cdot}2^{m}$ is precluded if the measurements on the plate are made as carefully as the one just described. In order to obtain an equally good result by the ordinary astronomical methods, the observations would have to be made within 6 seconds. We believe, however, that this photographic method can be raised to a still higher degree of precision without having to add many mechanical devices.

In this first practical attempt the stellar pictures in reference to the pictured crescents were not very favorably situated, more favorable positions being of frequent occurrence. For instance, if we photograph the moon at the moment when she has the same apparent declination as the star, and after the moon has passed this point allow the star to trace its path (making suitable breaks in the star trace by means of short interruptions of the exposure) over the plate until this moon picture is bisected by the star trace, it will be possible to obtain the right ascension with the same degree of precision with which the breaks in the star trace can be measured (bisected with the cross wires in the microscope). If we assume that the tenth part of such a gap or break can be bisected in the microscope (which is feasible), then a single reading will be correct within 0·5 seconds of time and the right ascension equally as close; consequently the Greenwich time can be ascertained in this case within 15 seconds.

If we wonder how it is possible to reach such close results with the crude means employed, we find that this crudeness is only an apparent one. The work has simply been divided in a happy manner. The entire work of mensuration has been separated from that part by means of which the observations are gained and recorded. Photography simply records instantaneously the relative positions of the stars and moon at fixed-time intervals, which positions can afterwards be studied and measured at leisure.

The described measuring apparatus takes the place of the theodolite or sextant, with its graduated limb, arc, verniers, and micrometers.

This division of work is of great value for geographical exploration parties, as the actual measuring can be done by experts at any subsequent time after the plates have been shipped to the mother country.

In regard to the camera used for the foregoing described experimental observations, it may be said that the objective was a so-called antiplanetic group lens, of a focal length of about 24^{cm}, made by Steinheil, in Munich, Bavaria. The stop used had a diameter of 17^{mm}. This objective really consists of four lenses—i. e., two cemented pairs—which are placed as closely together as the interposition of the diaphragm (with the 17^{mm} stop) will admit of. The peculiarities of this lens combination have been utilized in increasing the depth and the field of the objective without sacrificing uniformity of definition and an even distribution of light.

The constants of the camera need not be known, as they do not enter into the work. All we need to know is the geographical latitude of the station and at what time periods (given in civil time) the breaks in the star traces were made and when the lunar pictures were obtained. All remaining data can be culled from the plate.

APPENDIX No. 5—1893.

ON THE MEASUREMENT OF BASE LINES WITH STEEL TAPES AND WITH STEEL AND BRASS WIRES.

By EDV. JÄDERIN.

Translated and submitted for publication by Prof. J. HOWARD GORE, November 27, 1893.

In the report on a method for measuring geodetic base lines with steel tapes, which was published in Öfversigt af. K. Vetenskapsakaemiens förhandlingar, No. 9, 1879, only the first attempts at such measuring were described. Since these experiments were not sufficiently general to give an accurate and detailed account of the practical application of the method, and as the plans pursued were not satisfactory in every respect, especially as they could not be carried out under certain atmospheric conditions—conditions which may always be expected during any extensive measurement—with any hope of materially diminishing the error, I found it necessary within the past years to continue my experiments on a larger scale.

In the report which here follows I have, for the sake of continuity, deemed it advisable to repeat in outline the principles of this method.

The steel tapes which can be purchased in the stores are of various kinds and grades. Those which are best suited for geodetic measuring, in accordance with the method hereinafter described, are about 13^{mm} broad and 20^{m} or more long. They are divided into centimetres, with the first decimetre further divided into millimetres.

If it should be desired to make use of a steel tape in the field, where it is not possible to provide a smooth support, it is necessary to support only the two end points, allowing the tape to hang freely throughout the rest of its length. In such a case the exact distance between the two ends can be mathematically determined. For this a tension is needed at both ends, exerted either by means of a spring balance or by weights. This use of the tape requires two corrections—a negative correction, increasing the length of the tape, which arises from the stretching of the tape due to the applied tension, and a positive correction, shortening the straight-line distance between the ends, which amounts to the difference between the length of the curve formed by the freely hanging tape and its chord. Should it be desired to have as the normal length of the tape—that is, the straight-line distance from the zero mark on one end to the similar mark on the other—a length exactly equal to the distance between these marks while the tape is lying on a smooth support throughout its entire length, it is only necessary that the sum of the two corrections named be equal to zero or that a tension be found sufficient to give to the hanging tape its normal length. From this it can be seen that this tension takes the place of the support.

It is better to provide the line which is to be measured with tripods than with stakes, since the latter are less stable and not so convenient as the former. If the tripod should be placed on rocks or on ground too hard for the feet to secure a firm hold, the requisite stability can be obtained by placing a stone in a sling attached to the under part of the tripod. In measuring, the tripods are placed so that the distance between the fine needles which are fixed in their upper surfaces is some centimetres less than the full length of the tape. The zero mark on the rear end of the tape is brought into coincidence with the needle on the rear tripod. The spring balance is attached to the forward end and a tension predetermined upon is applied. The reading on the tape is then made to the tenth of a millimetre by approximation, and recorded. (The forward tripod now becomes the rear one, and the operation just described is repeated.)

The length of the line is computed by considering, besides the readings just referred to, the following corrections: The constant correction to the length of the tape, obtained by measuring with it a line of known length—the comparator; the reduction to the horizontal projection, determined from the difference in the elevation of the tripod heads for each tape length, and the correction for temperature, that of the tape being taken as the temperature of the air.

When the measurement is prosecuted in this manner the weather occasionally interferes. The steel tapes, which are usually 13^{mm} in breadth, offer in a length of 20^{m}, when turned flat side to the wind, a surface of one-fourth square metre, approximately. A strong wind, therefore, causes a waving motion and moves the millimetre scale along the needle so rapidly as to make a reading inaccurate. Again, it is easy to see that in the sunshine the temperature of the air is not the same as that of the tape or even that of the thermometer, which makes the thermometric reading very unreliable. For this reason the application of this method of obtaining the temperature of the tape should be made use of only when the sky is clouded and the air perfectly still or in a gentle wind.

To obviate these difficulties was the purpose of the following experiments:

The effect of the wind is diminished partly by selecting a tape as small in cross section as possible and partly by increasing the tension.

In the matter of cross section there is a minimum which must not be exceeded in order to avoid an appreciable stretching in applying the tension. Therefore, as the width is diminished the thickness must be increased, which leads to the fact that a circular cross section is the best, and instead of tapes wires should be used. When the tension is increased it is necessary to know the length correction for this increased tension; to find this the comparator is measured with that tension. Whenever the entire length of the tape is not employed, it is best to determine the tension which will make the correction to the length proportional to the length utilized.

In order to be able to safely determine the temperature under all conditions, there appears to be no better way than to employ two wires—for example, one of steel and one of brass—whose coefficients of expansion are different. If these wires are of the same size and have a similar surface—nickel plated, for instance—and are handled in the same manner, there is no apparent reason why, on the average, they should not have for the entire measurement the same temperature; and this temperature can be determined from the differences in the readings of the two wires stretched successively between the same two fixed points.

After making these suggestions and before passing on to a description of the instruments and a report of the results, I shall give the development of the requisite formulas.

The following notation is employed:

L_0 = the normal length of the tape or wire, or the length which it indicates when supported and without tension.

L = the straight-line distance between the two zero marks when the tape or wire is supported at both ends, hangs freely throughout its length, and is subjected to a tension I.

g = gravity, force of.

m = mass of a unit's length of the tape.

w = weight of a unit's length of the tape.

s = the extension of a unit's length due to a unit tension.

c = correction to the length due to a curvature of the tape.

c_1 = correction to the length due to tension.

One has then

$$w = m\,g$$

The curve line which the tape forms when it hangs suspended from its two end points has, according to Sturm's Cours de Mécanique, vol. 2, p. 48, the following equations:

$$\frac{y}{k} = \frac{e^{\frac{x}{k}} + e^{-\frac{x}{k}}}{2} \qquad (a)$$

$$\frac{l}{k} = \frac{e^{\frac{x}{k}} - e^{-\frac{x}{k}}}{2} \qquad (b)$$

$$\frac{\rho}{k} = \left(\frac{y}{k}\right)^2 \tag{c}$$

$$y^2 = k^2 + l^2 \tag{d}$$

$$T = w\,y \tag{e}$$

in which the y axis is vertical and the x axis horizontal, the former passing through the curve at its lowest point and the latter at a distance, k, below it; l the length of the curve reckoned from the point where the y axis cuts the same, ρ the radius of curvature, and T the tension exerted in the direction of its length.

When the two ends are at the same height the ordinates will be equal, while the abscissas will be equal but with opposite signs. If L be the length of the tape and a the straight-line distance between the end points, then

$$L = 2\,k, \frac{e^{\frac{a}{2\,k}} - e^{-\frac{a}{2\,k}}}{2}$$

or expressed in a series

$$L = 2\,k\left(\frac{a}{2\,k} + \frac{1}{1.\,2.\,3}\left(\frac{a}{2\,k}\right)^3 + \frac{1}{1.\,2.\,3.\,4.\,5}\left(\frac{a}{2\,k}\right)^5 \cdots\cdots\right)$$

and $$L - a = \frac{a^3}{24.\,k^2} + \frac{a^5}{1920.\,k^4} + \cdots\cdots \tag{f}$$

In the experiments it was found that all the terms after the first would fall between $0{\cdot}0001^{mm}$ and $0{\cdot}001^{mm}$; hence it can be assumed that

$$L - a = \frac{a^3}{24\,k^2}$$

From (e) $$1 = \frac{T}{w\,y}$$

or $$k = \frac{T\,k}{w\,y}$$

From a series of tabulated values it was found that $\frac{k}{y}$ changed value with k, but only to the extent of $\frac{1}{2000}$ of the whole, or that the factor $\frac{k}{y} = 1$. This gives:

$$k = \frac{T}{w}$$

and $$L - a = \frac{a^3 w^2}{24\,T^2}$$

or $$c = a - L = -\frac{a^3 w^2}{24\,T^2}$$

or placing $$L_0 = a^3$$

and $$w = mg$$

we have $$c = -\frac{L_0^3 m^2 g^2}{24\, T^2}$$

Likewise $$c_1 = s\, L_0\, T$$

from which we have

$$L = L_0 - \frac{L_0^3\, m^2\, g^2}{24\; T^2} + s\, L_0\, T \; \ldots\ldots \qquad (1)$$

For the force T_0 (the normal tension), which makes L equal to L_0, it would give

$$T_0 = \sqrt[3]{\frac{L_0^2\, m^2\, g^2}{24 s}} = \sqrt[3]{\frac{V^2}{24 s}} \; \ldots\ldots \qquad (2)$$

in which V (or $w\, L_0$) is the weight of the entire tape.

If it should be necessary to employ a fractional part—say, the nth part of the tape—and we wished to know the corresponding part of the normal length, we would have to introduce a tension, $T^{0\prime}$, which is obtained by substituting in (2) $n\, L_0$ for L_0, from which we obtain

$$T^{0\prime} = \sqrt[3]{\frac{n^2\, L_0^2\, m^2\, g^2}{24 s}} = T_0 \sqrt[3]{n^2} \; \ldots\ldots \qquad (3)$$

If, on the other hand, a greater tension, T, is made use of, in order to diminish the effect of the wind, the distance between the zero points will be greater than L_0. We therefore place

$$L = L_0\, (1 + f)$$

or the factor $$f = \frac{L - L_0}{L_0}$$

From (1) we obtain

$$f = -\frac{L_0^2\, m^2\, g^2}{24\; T^2} + sT = -\frac{L_0^2\; w^2}{24\; T^2} + sT$$

or

$$f = -\frac{V^2}{24\; T^2} + sT \; \ldots\ldots \qquad (4)$$

When only a part of the tape is utilized, the corresponding or nth part of f is obtained by first finding from T the value T'. From (1) we have

$$nL_0\, (1 + f) = nL_0 - \frac{n^3\, L_0^3\; m^2\, g^2}{24\;\; T'^2} + nsL_0T',$$

from which

$$f = -\frac{n^2\, V^2}{24\;\, T'^2} + sT'$$

which combined with (4) gives

$$T'^3 + \left(\frac{V^2}{24\; sT^2} - T\right) T'^2 = \frac{V^2}{24\; s}\, n^2,$$

or

$$T' = T - \frac{V^2}{24\; sT^2} + \frac{V^2}{24\; s}\, n^2, \frac{1}{T'^2} \; \ldots\ldots \qquad (5)$$

where the value of T' is given by approximation.

If p is the sag of the tape at its lowest point, or the height of the segment of the circle formed by the tape, we have

$$p = \tfrac{1}{8}\frac{L_0^2 w}{T} = \tfrac{1}{8}\frac{L_0 V}{T}$$

For three steel tapes = A and B (both 20^m long, the latter nickel plated) and C, nearly 30^m in length, but used as a 20^m tape, I found, using the metre as a unit of length and a kilogramme as the unit of weight:

A,	$w = 0{\cdot}01977^k$,	$s = 0{\cdot}0000214 \pm 6$,	$sw = 0{\cdot}000000424 \pm 11$
B,	$0{\cdot}01898$	$0{\cdot}0000226 \pm 6$,	$0{\cdot}000000429 \pm 12$
C,	$0{\cdot}01653$,	$0{\cdot}0000234 \pm 5$,	$0{\cdot}000000387 \pm 8$

The determination of s and sw were made while subjecting the tapes when horizontal to a tension by means of a dynamometer; also when vertical by suspending them from a high tower and attaching weights to the lower ends. The mean temperature was 6° (C.).

From these values, by means of (2), we obtain

for A,	$T_0 = 6{\cdot}72^k$
B,	$6{\cdot}43$
C,	$5{\cdot}80$.

For the elucidation of the formulas given above, the following table is given for A, which was used in the experimental determinations of length:

n	nL	T_0, $T_0=6{\cdot}72^k$, $f=0$	T: $T=10^k$, $f=0{\cdot}000149$	$T=15^k$, $f=0{\cdot}000293$
	$m.$	$k.$	$k.$	$k.$
0·00	0	0·00	6·96	13·65
0·05	1	0·91	6·98	13·65
0·10	2	1·45	7·02	13·67
0·15	3	1·88	7·10	13·69
0·20	4	2·30	7·20	13·71
0·25	5	2·67	7·32	13·75
0·30	6	3·01	7·45	13·79
0·35	7	3·34	7·60	13·84
0·40	8	3·65	7·77	13·90
0·45	9	3·95	7·94	13·96
0·50	10	4·24	8·11	14·03
0·55	11	4·51	8·30	14·11
0·60	12	4·78	8·48	14·19
0·65	13	5·05	8·67	14·28
0·70	14	5·30	8·86	14·37
0·75	15	5·55	9·05	14·47
0·80	16	5·79	9·24	14·57
0·85	17	6·03	9·43	14·67
0·90	18	6·27	9·62	14·78
0·95	19	6·50	9·81	14·89
1·00	20	6·72	10·00	15·00

If the length L_0 is known when the tension T_0 was used, it is possible by means of the formula already given to determine the length under a tension, T, introducing for that purpose the factor f. However, it is preferable to ascertain from the comparator this length with the application of the new tension.

The disturbing influence of the wind, as has been said, chiefly shows itself in adding to the difficulty of reading the scale on the tape. If this interference of the wind is regarded as manifesting itself in a constant thrust sidewise, or a lateral curving of the tape, it is possible by means of equation (1) to determine the length, introducing in the place of g a factor representing the strength of the wind. From this it will be seen that the distortion in question is proportional to the square of the force of the wind and inversely proportional to the square of the tension. But the force of the wind is proportional to the exposed surface of the tape, from which it is apparent that to avoid these obstacles the use of wires is well-nigh imperative.

In case the measurement takes place under a different latitude or at a greater elevation than that of the comparator, it is necessary, in order to be able to compute the effect of gravity upon the results of the measuring, to take into consideration two distinct cases:

1. When the tension is effected by means of a spring balance.
2. When the tension is effected by means of weights.

In the first instance the tension is unchanged, although the weight of the entire tape or wire is different at the two places. If the length of the wire was found from the comparator to be L under a tension of T, and at the base line the same tension was applied, then the straight-line distance between the zero marks, L^1, can be obtained from the equation

$$L^1 - L = \frac{L_0^3\, m^2\, g^2}{24\, T^2} \left(1 - \frac{g'^2}{g^2}\right)$$

in which g and g' represent the gravity at the comparator and the base line, respectively. This equation can also be written

$$L^1 - L = \frac{L_0^3\, V^2}{24\, T} \times \frac{g+g'}{g^2}\left(g - g'\right)$$

or with sufficient approximation

$$L^1 - L = \frac{L_0^3\, V^2}{12\, T^2}\left(\frac{g-g'}{g}\right) \cdots\cdots \qquad (6)$$

In the second case, where the tension is exerted by means of weights, their tensive force, as well as the weight of the wire or tape, will vary. If M represent the mass of the object used to produce the tension, then its weight (taking the place of T in formula (1)) at the two places would be $M\,g$ and $M\,g^1$; this would give

$$L^1 - L = -s L_0 M g \left(1 - \frac{g'}{g}\right).$$

or

$$L^1 - L = -s L_0 T\, \frac{g-g'}{g} \cdots\cdots \qquad (7)$$

For the numerical results, see pages 133–134.

DESCRIPTION OF THE INSTRUMENTS.

In the measurements, wires were chiefly made use of, and steel tapes employed only when the distance between two tripods happened to be less than the length of a wire. In such cases it was necessary to use a tape, since it was not possible to graduate a wire throughout its entire length.

As has already been remarked, wires were used in pairs—one of steel and one of brass. Piano wire was found to be the best size for the steel. The brass wire was repeatedly pulled through a draw plate so as to give to it its greatest possible ductility. In selecting a wire one must see to it that there are no longitudinal cracks in it, a fault which may be found by cold hammering even when invisible to the naked eye.

When it becomes necessary to roll up the wires, it is advisable to give to the rings the size which, if left to themselves, they would naturally take, so that they may not suffer a permanent change in length through bending. The winding and unwinding must be done with the greatest care, and every possible precaution taken that no kinks are formed. Should any occur, it will be necessary to redetermine its length before continuing the measurement. It may be remarked that whenever any slight unevenness occurs on the wire the error in length resulting therefrom is always the same under equal tension. The danger, therefore, arising from such misfortune is, as experience shows, considerably less than one would be inclined to imagine.

The wires are provided at both their ends with scales 1 decimetre in length, divided into millimetres. These, marked *s* in figure 2, are attached longitudinally to brass tubes. The free end of the twisted wire is inserted into the outer terminals of this tube and firmly soldered thereto. To prevent the wire from twisting when in use, a turn buckle, *r*, is inserted, which also makes it possible to bring the scale into the most favorable position with respect to the needle in the top of the tripod for reading.

Of the ten wires made by the mechanician, Fr. J. Berg, of Stockholm, eight have been repeatedly investigated on the comparator, which will be described later on. All these wires have a length of 25^{m}. This length was made necessary by the desire to employ each wire as often as possible on the 100^{m} comparator. Otherwise, wires of 30^{m} or more would have been used.

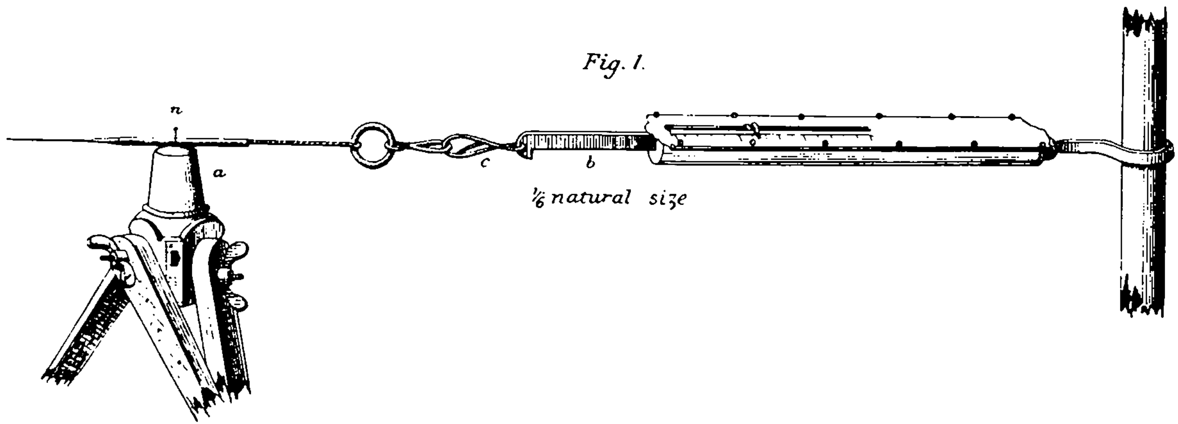
The pair *A B* was utilized since March, 1882, *C D* since April, *E F* since September, and *M N* since April, 1883. The reason why I employed so many wires was to determine from experience what diameter is the best, and also to confirm or disprove the oft-expressed fear of a continuous change in the length of tapes or wires under use.

The following wires were nickel plated:

For *A* (steel) and *B* (brass) before plating the weights were found to be

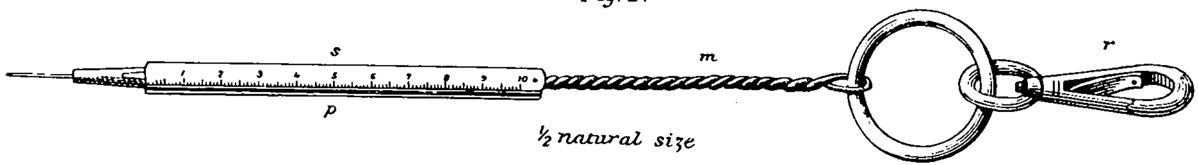
$$A \quad w = 0{\cdot}01459^{k}$$
$$B \qquad 0{\cdot}01624$$

Fig. 1.



1/6 natural size

Fig. 2.



1/2 natural size

After plating

$$A \quad w = 0{\cdot}01469^{k}$$
$$B \quad 0{\cdot}01629\,*$$

Likewise it was found that under tension, with a temperature of +1° (Celsius), the following wires gave:

$$A\ldots,\ s = 0{\cdot}00002799,\ sw = 0{\cdot}000000411$$
$$B\ldots,\ 0{\cdot}00005769,\ 0{\cdot}000000940$$

Since the wires are always employed throughout their entire length, which is not the case with tapes, it is of slight importance whether these quantities are known, even if approximate values are at times needed. A knowledge of such values can be obtained with sufficient accuracy if it is supposed that for the same metal w is proportional to the surface of a transverse section or to the square of the diameter ($a\,d^2$, in which a represents a constant and d the diameter); also, that $s\,w$ is a constant and s inversely proportional to the square of the diameter, or equal to $\frac{b}{d}$. For A, $d = 1{\cdot}53^{mm}$, and for B, $d = 1{\cdot}57^{mm}$. If a be expressed in millimetres, we would have:

for steel, $a = 0{\cdot}006275^{k}$; $w = ad^2$........[log., 7·7976]

$b = 0{\cdot}0000655^{m}$; $s = \frac{b}{d^2}$........[log., 5·8162]

for brass, $a = 0.006609^{k}$................[log., 7·8201]

$b = 0{\cdot}0001422^{m}$...............[log., 6·1529]

From these two values for a the specific gravity was found to be for steel, 7·99, and for brass, 8·41.

After having determined the diameters of the other wires, the remaining quantities were found to have the following approximate values:

Wire.		d	w	s	sL_0	V	T_0	T	p
		mm.	k.			k.	k.	k.	m.
Steel.	A	1·53	0·01469	0·00002799	0·000700	0·367	5·85	10	0·11
Brass.	B	1·57	0·01629	0·00005769	0·001442	0·407	4·92	10	0·13
Brass.	C	1·52	0·0153	0·0000616	0·00154	0·38	4·6	10	0·12
Steel.	D	1·51	0·0143	0·0000287	0·00072	0·36	5·7	10	0·11
Steel.	E	2·04	0·0270	0·0000157	0·00039	0·65	10·4	10	0·20
Brass.	F	2·02	0·0270	0·0000349	0·00087	0·67	8·2	10	0·21
Steel.	M	2·66	0·0444	0·0000091	0·00023	1·11	17·7	15	0·23
Brass.	N	2·66	0·0468	0·0000201	0·00050	1·17	14·2	15	0·24

* If for any reason w, and likewise V, suffered any change—for example, through the wear of the plating or removal of the plating—c took on a new value, which was determined from

$$d\,c = -\frac{L_0\,V}{12\,T^2}\,d\,V$$

Also s would change, giving

$$d\,c_1 = t\,L_0\,T\,ds.$$

From these equations it can be seen how small the dreaded changes from these sources are.

We readily obtain from (2)

$$T_0 = d^2 \sqrt[3]{\frac{L_0^2 a^2}{24b}} \cdots\cdots \tag{8}$$

and

$$T_0 = w \sqrt[3]{\frac{L_0^2}{24ab}} \cdots\cdots \tag{9}$$

With a wire it is easy to obtain d by measuring, and in a tape w can be found by weighing; therefore the formulas just given can serve in determining an approximate value for T_0. The values given in the eighth column of the above table were obtained in this way. The knowledge of T_0 is of no direct value, yet for the sake of comparing values of it with the arbitrarily taken values of T, as shown in column 9, they are here given. If it is desired to obtain T_0 more easily, it can be done, from formulas (8) and (9), expressing T_0 and w in kilogrammes, and d in millimetres.

Steel, $L = 20^m$, $T_0 = 2{\cdot}16\ d^2 = 344\ w$
Brass, $T_0 = 1{\cdot}72\ d^2 = 261\ w$
Steel, $L = 25^m$, $T_0 = 2{\cdot}50\ d^2 = 399\ w$
Brass, $T_0 = 2{\cdot}00\ d^2 = 303\ w$

If we take the values given in the above table and insert them in formulas (6) and (7), we will obtain for the influence of the varying force of gravity the following, in millimetres:

1. *Tension with spring balance.*

A	*B*	*C*	*D*	*E*	*F*	*M*	*N*
+2·8	+3·5	+3·0	+2·7	+8·8	+9·4	+11·4	+12·7

2. *Tension from weights.*

A	*B*	*C*	*D*	*E*	*F*	*M*	*N*
−7·0	−14·4	−15·4	−7·2	−3·9	−8·7	−3·5	−7·5

These values have not been multiplied by the factor $\frac{g - g'}{g}$; but as this factor never exceeds 0·0052, it would give a correction of only $0{\cdot}066^{mm}$ for N and 0·080 for C.

The wires A B and C D were found to be easily handled, while more trouble was experienced with M N. The latter were given a greater thickness that they might serve as standards for the others, it being accepted that they would be less subject to change than the lighter wires.

For the corrections to reduce a 25^m wire to the horizontal projection, the following table was computed from the formula

$$k = \frac{h^2}{2L} + \frac{h^4}{8L^3} + \frac{h^6}{16L^5} \cdots\cdots \tag{10}$$

in which k is the correction, always positive; L the length of the wire, and h the difference in elevation of the two tripod heads:

Correction for reduction to a horizontal projection.

h	k	h	k	h	k	h	k
m.	mm.	m.	mm.	m.	mm.	m.	mm.
0·00	0·0	0·66	8·7	1·26	31·8	1·86	69·3
0·04	0·0	0·67	9·0	1·27	32·3	1·87	70·0
0·05	0·1	0·68	9·2	1·28	32·8	1·88	70·8
0·08	0·1	0·69	9·5	1·29	33·3	1·89	71·5
0·09	0·2	0·70	9·8	1·30	33·8	1·90	72·3
0·11	0·2	0·71	10·1	1·31	34·3	1·91	73·1
0·12	0·3	0·72	10·4	1·32	34·9	1·92	73·8
0·13	0·3	0·73	10·7	1·33	35·4	1·93	74·6
0·14	0·3	0·74	11·0	1·34	36·0	1·94	75·4
0·15	0·4	0·75	11·3	1·35	36·5	1·95	76·2
0·16	0·5	0·76	11·6	1·36	37·0	1·96	76·9
0·17	0·5	0·77	11·9	1·37	37·6	1·97	77·7
0·18	0·6	0·78	12·2	1·38	38·1	1·98	78·5
0·19	0·7	0·79	12·5	1·39	38·7	1·99	79·3
0·20	0·8	0·80	12·8	1·40	39·2	2·00	80·1
0·21	0·9	0·81	13·1	1·41	39·8	2·01	80·9
0·22	1·0	0·82	13·5	1·42	40·4	2·02	81·7
0·23	1·1	0·83	13·8	1·43	40·9	2·03	82·6
0·24	1·2	0·84	14·1	1·44	41·5	2·04	83·4
0·25	1·3	0·85	14·5	1·45	42·1	2·05	84·2
0·26	1·4	0·86	14·8	1·46	42·7	2·06	85·0
0·27	1·5	0·87	15·1	1·47	43·3	2·07	86·7
0·28	1·6	0·88	15·5	1·48	43·8	2·08	87·5
0·29	1·7	0·89	15·8	1·49	44·4	2·09	88·4
0·30	1·8	0·90	16·2	1·50	45·0	2·10	89·2
0·31	1·9	0·91	16·6	1·51	45·6	2·11	90·0
0·32	2·0	0·92	16·9	1·52	46·3	2·12	90·9
0·33	2·2	0·93	17·3	1·53	46·9	2·13	91·8
0·34	2·3	0·94	17·7	1·54	47·5	2·14	92·6
0·35	2·5	0·95	18·1	1·55	48·1	2·15	93·5
0·36	2·6	0·96	18·4	1·56	48·7	2·16	94·4
0·37	2·7	0·97	18·8	1·57	49·3	2·17	95·2
0·38	2·9	0·98	19·2	1·58	50·0	2·18	96·1
0·39	3·0	0·99	19·6	1·59	50·6	2·19	97·0
0·40	3·2	1·00	20·0	1·60	51·3	2·20	97·9
0·41	3·4	1·01	20·4	1·61	51·9	2·21	98·8
0·42	3·5	1·02	20·8	1·62	52·5	2·22	99·7
0·43	3·7	1·03	21·2	1·63	53·2	2·23	100·6
0·44	3·9	1·04	21·6	1·64	53·8	2·24	101·5
0·45	4·1	1·05	22·1	1·65	54·5	2·25	102·4
0·46	4·2	1·06	22·5	1·66	55·2	2·26	103·3
0·47	4·4	1·07	22·9	1·67	55·8	2·27	103·3
0·48	4·6	1·08	23·3	1·68	56·5	2·28	104·2
0·49	4·8	1·09	23·8	1·69	57·2	2·29	105·1
0·50	5·0	1·10	24·2	1·70	57·9	2·30	106·0
0·51	5·2	1·11	24·7	1·71	58·6	2·31	106·9
0·52	5·4	1·12	25·1	1·72	59·2	2·32	107·9
0·53	5·6	1·13	25·5	1·73	59·9	2·33	108·8
0·54	5·8	1·14	26·0	1·74	60·6	2·34	109·8
0·55	6·1	1·15	26·5	1·75	61·3	2·35	110·7
0·56	6·3	1·16	26·9	1·76	62·0	2·36	111·6
0·57	6·5	1·17	27·4	1·77	62·7	2·37	112·6
0·58	6·7	1·18	27·9	1·78	63·4	2·38	113·5
0·59	7·0	1·19	28·3	1·79	64·2	2·39	114·5
0·60	7·2	1·20	28·8	1·80	64·9	2·40	115·5
0·61	7·4	1·21	29·3	1·81	65·6	2·41	116·4
0·62	7·7	1·22	29·8	1·82	66·3	2·42	117·4
0·63	7·9	1·23	30·3	1·83	67·1	2·43	118·4
0·64	8·2	1·24	30·8	1·84	67·8	2·44	119·4
0·65	8·5	1·25	31·3	1·85	68·5	2·45	120·3

Correction for reduction to a horizontal projection—Continued.

h	k	h	k	h	k	h	k
m.	*mm.*	*m.*	*mm.*	*m.*	*mm.*	*m.*	*mm.*
2·46	121·3	2·54	129·4	2·62	137·7	2·70	146·2
2·47	122·3	2·55	130·4	2·63	138·6	2·71	147·3
2·48	123·3	2·56	131·4	2·64	139·8	2·72	148·4
2·49	124·3	2·57	132·4	2·65	140·8	2·73	149·5
2·50	125·3	2·58	133·5	2·66	141·9	2·74	150·6
2·51	126·3	2·59	134·4	2·67	143·0	2·75	151·7
2·52	127·3	2·60	135·6	2·68	144·1	2·76	152·8
2·53	128·3	2·61	136·5	2·69	145·1		

The table extends only to a difference of $2{\cdot}76^{m}$ in the height of the two tripods, because a greater inclination than this—1 in 9—should be avoided. In such extreme cases an error in elevation of 1^{mm} would produce an error of $0{\cdot}1^{mm}$ in the length.

From equation (10) we obtain by differentiating

$$dk = -\frac{h^2}{2L^2}\,dL \ldots\ldots \tag{11}$$

from which it appears that when L is not exactly 25^{m}, but, as is usually the case, varies a few centimetres, owing to the scale readings, the corrections taken from the table will not be correct. If $h=3^{m}$ and $d\,L=+50^{mm}$, then the error referred to would amount to $-0{\cdot}36^{mm}$.

To compute k in those instances where L is not quite 25^{m}, as in the use of parts of a steel tape, one can proceed as follows:

Place in equation (10) a for all the terms of the second member except the first, that is

$$k=\frac{h^2}{2L}+a,$$

then

$$h=\sqrt{\sqrt{8aL^3}-2aL} \ldots\ldots \tag{12}$$

If h and L are expressed in metres and k in millimetres, then

$$k=\frac{h^2}{L}\times 500^{mm}+a \ldots\ldots \tag{12a}$$

from which it is easy to compute k by taking a from the following table:

$L(m.)$	a				
	$0·05^{mm}$	$0·15^{mm}$	$0·25^{mm}$	$0·35^{mm}$	$0·45^{mm}$
	h				
	m.	m.	m.	m.	m.
1	0·14	0·19			
2	0·24	0·31			
3	0·32	0·42			
4	0·40	0·53			
5	0·47	0·62			
6	0·54	0·71	0·81		
7	0·61	0·80	0·91		
8	0·67	0·88	1·00		
9	0·73	0·97	1·10	1·19	
10	0·79	1·05	1·19	1·29	
11	0·85	1·12	1·28	1·39	
12	0·91	1·20	1·36	1·48	
13	0·97	1·27	1·45	1·57	1·67
14	1·02	1·35	1·53	1·66	1·77
15	1·08	1·42	1·61	1·75	1·86
16	1·13	1·49	1·69	1·84	1·96
17	1·18	1·56	1·77	1·92	2·05
18	1·24	1·62	1·85	2·01	2·14
19	1·29	1·69	1·92	2·09	2·23
20	1·34	1·76	2·00	2·17	2·31
21	1.39	1·82	2·07	2·25	2·40
22	1·44	1·89	2·15	2·33	2·48
23	1·48	1·95	2·22	2·41	2·57
24	1·53	2·02	2·29	2·49	2·65
25	1·58	2·08	2·36	2·57	2·73
26	1·63	2·14	2·43	2·65	2·82
27	1·67	2·20	2·50	2·72	2·90
28	1·72	2·26	2·57	2·80	2·98
29	1·77	2·32	2·64	2·87	3·06
30	1.81	2·38	2·71	2·95	3·14

If f represent the error in the leveling rod, then for a difference of elevation h we must write $h(1+f)$; therefore from (10) we will have for the reduction to the horizon

$$k'=\frac{h^2}{2L}(1+f)^2+\frac{h^4}{8L^3}(1+f)^4+\dots\dots$$

or with sufficient approximation

$$k'=k(1+2f)$$

or

$$\Sigma k'=(1+2f)\Sigma k\dots\dots \qquad (13)$$

Both ends of the wires are provided with balances. The one at the end where the scale on the wire is read is to bring about the desired tension, while the other is to hold the counter action at the same tension and to avoid as far as possible any drag on the tripod which carries the mark indicating the terminus of the preceding wire. For the latter an ordinary spring balance such as is found in the stores will answer, but for the former purpose a more carefully constructed balance will be required. From this it will be seen that in measuring in the field two spring balances are needed—a smaller and a larger one. These balances are employed to measure the horizontal tension and not to weigh anything in a vertical position, as is supposed to be the case when the graduation is made. Therefore it is necessary to take into consideration the difference in the readings in these two cases for actual tension. This difference can be obtained by adding to half of the weight of the spring the weight of those parts—b and c—which are attached to the lower end of the spiral spring. It is easy to find that if p represents the entire weight of the dynamometer, y the difference just mentioned, x the correction to the horizontal reading for 0 k, a the reading on the balance when it hangs vertical and free, and a' the reading when the balance is inverted and suspended by its own hook, then

$$\left.\begin{aligned} x &= \frac{p-a'-a}{2} \\ \text{and} \qquad y &= \frac{p-a'+a}{2} \end{aligned}\right\} \ldots\ldots \tag{14}$$

By way of illustration, the following quantities are added:

	Old balance.	New balance.
The difference between the readings of the balance when horizontal and when vertical,	k. 0·115	k. 0·186
	$mm.$	$mm.$
The diameter of the steel wire which formed the spring,	2·25	2·62
The outer diameter of the spring,	20·5	26·0
The inner diameter of the spring,	16·0	20·8
The mean radius of the spring $=R$	9·1	11·4
Extension of spring for 1 $k=f$	8·34	10·14
The number of coils in the spring $=h$	42	39·5

We will have

$$f = k\,\frac{R^3 h}{d^4} \ldots\ldots \tag{14a}$$

where $k = 0{\cdot}0073$ for steel.

The scales on both balances were divided into tenths of a kilogramme, making it possible to read to hundredths by approximation.

If the same balances are used in measuring that were employed on the comparator, it is not necessary to determine these corrections; but if different balances are made use of, it will be absolutely necessary to investigate each balance that is employed. For this reason it is deemed well to give the correction for the two balances mentioned. In each case the vertical weighing is given with proper correction to make it the equivalent of the horizontal tension.

The old balance.

[1883, Apr. 24.]				[1884, Jan. 30.]		[1884, Feb. 3.]	
Temp.=+13°		Temp.=+6°		Temp.=+2°		Temp.=+26°	
Weight.	Corr.	Weight.	Corr.	Weight.	Corr.	Weight.	Corr.
0*k*	−0·14*k*	0*k*	−0·13*k*	0*k*	−0·115*k*	0*k*	−0·135*k*
10	−0·14	9	−0·18	2	−0·14	2	−0·195
5	−0·20	10	−0·11	4	−0·17	4	−0·245
		10	−0·10	6	−0·21	6	−0·30
		9	−0·19	9	−0·17	9	−0·27
		5	−0·19	11	−0·10	11	−0·21

The new balance.

[1883, Apr. 24.]				[1884, Jan. 30.]		[1884, Feb. 3.]	
Temp.=+13°		Temp.=+6°		Temp.=+2°		Temp.=+26°	
Weight.	Corr.	Weight.	Corr.	Weight.	Corr.	Weight.	Corr.
0*k*	−0·18*k*	0*k*	−0·18*k*	0*k*	−0·19*k*	0*k*	−0·205*k*
0	−0·18	0	−0·18	2	−0·19	2	−0·20
5	−0·18	10	−0·14	4	−0·17	4	−0·21
10	−0·18	13	−0·13	7	−0·15	6	−0·205
14	−0·14	14	−0·11	9	−0·12	9	−0·215
14	−0·14	14	−0·11	11	−0·13	11	−0·20
13	−0·17	14	−0·11	13	−0·12	13	−0·19
13	−0·16	13	−0·14	15	−0·09	15	−0·18
14	−0·13	0	−0·18				
9	−0·18	9	−0·16				
10	−0·17	10	−0·17				
0	−0·18	5	−0·17				
0	−0·19						

For both instruments the two last determinations are the mean of two series of observations.

Since the correction k can be expressed by the equation

$$k=x+my,$$

in which m is the weight suspended and x and y are constants, we have, after applying the corrections just given for reduction to horizontal readings, according to the method of least squares, the following equations:

For the old balance.

1884, Jan. 30, Temp.$=+\ 2^\circ$, $k=-0{\cdot}034-0{\cdot}0003m.$
1883, Apr. 24, " $=+\ 6^\circ$, $k=-0{\cdot}031+0{\cdot}0005m.$
" " " $=+13^\circ$, $k=-0{\cdot}036-0{\cdot}0005m.$
1884, Feb. 3, " $=+26^\circ$, $k=-0{\cdot}064-0{\cdot}0085m.$

For the new balance.

1884, Jan. 30, Temp.$=+\ 2^\circ$, $k=-0{\cdot}011+0{\cdot}0063m.$
1883, Apr. 24, " $=+\ 6^\circ$, $k=-0{\cdot}000+0{\cdot}0045m.$
" " " $=+13^\circ$, $k=-0{\cdot}001+0{\cdot}0025m.$
1884, Feb. 3, " $=+26^\circ$, $k=-0{\cdot}025+0{\cdot}0013m.$

If x changes with the temperature, it is evident that the variation is extremely slight; consequently it is regarded as practically constant.

However, the variation in y can not be made a constant function of temperature; but it is assumed that it may be expressed by the relation

$$y=u+vt,$$

in which t is the temperature and u and v are constants. In this manner it is found that for the old balance

$$y=+0{\cdot}0021-0{\cdot}000365t,$$

and $$k=-0{\cdot}041+(0{\cdot}0021-0{\cdot}000365t)m;$$

and for the new balance

$$y=+0{\cdot}0060-0{\cdot}000197t,$$

and $$k=-0{\cdot}009+(0{\cdot}0060-0{\cdot}000197t)m.$$

The temperature correction, which for the two instruments is almost the same, can not be regarded as devoid of real significance.

From what has just been given, the following table of corrections has been computed:

Temp.	The old balance tension, 10k.	The new balance.	
		Tension 10k.	Tension 15k.
—10°	+0·02k	+0·07k	+0·11k
— 5°	0·00	+0·06	+0·10
0°	—0·02	+0·05	+0·08
5°	—0·04	+0·04	+0·07
10°	—0·06	+0·03	+0·05
15°	—0·07	+0·02	+0·04
20°	—0·09	+0·01	+0·02
25°	—0·11	0·00	+0·01
30°	—0·13	—0·01	—0·01

These corrections must be applied with their proper signs to the readings of the balance in order to have the correct tension.

The correction for the length of the wire depending upon an error in tension can be found by differentiating (1), which gives

$$\frac{d\,L}{d\,T}=\frac{L_0^3\,m^2\,g^2}{12\,T^3}+sL_0=\frac{L_0\,V^2}{12\,T^3}+sL_0\ldots\ldots \qquad (15)$$

or if $$T=T_0$$

$$\frac{dL}{dT}=3sL_0\ldots\ldots \qquad (15a)$$

The equation (15) gives for a positive error of 1k in the balance the following positive corrections in the length of the wires in millimetres

for the wire	A	B	C	D	E	F	M	N
the correction	1·0	1·8	1·8	1·0	1·3	1·8	1·0	1·3

In order to secure reliable readings on the wire scale it is necessary to see to it that the friction on the interior of the balance is a minimum and that the observer is skilled in work of this character.

THE COMPARATOR.

In February, 1882, the earth not being in a frozen condition, two stones were set in the sidewalk extending along the Technology street, each of which weighed about 2,000 kilogrammes. Into the upper surface of each stone was inserted a brass rod 1 decimetre in length, and in the free ends of these rods fine holes were drilled, which marked the ends of the line. The line was intended to be exactly 100^m long as indicated by an old-standarded steel tape. The stones rested upon well-packed beds of gravel about $0{\cdot}5^m$ below ground and extended $0{\cdot}4^m$ above.

The line was measured three times in April, 1882, and twice in November with the base apparatus belonging to the Royal Academy of Sciences. Since the base bars are two toises long, 26 lengths give $101{\cdot}35^m$. The excess, $1{\cdot}35^m$, is such a large fractional part of the whole that it must be laid off with great care. For this purpose a 2^m bar of Bessemer steel was employed which had been tested both for length and graduation and compared with the standard by the Bureau of Weights and Measures.

At the time of the first measurement no comparison of the base bars with the standard was made, as I considered that no significant change had taken place since the last comparison; at least no change of sufficient importance to affect my results. However, it can be seen from what follows that this hypothesis was unjustifiable. *A*, *B*, *C*, *D* are the lengths of the four bars of the base apparatus and *N* that of the standard, all at a temperature of 13° R., or 16°·25 Celsius. The quantities are expressed in millimetres:

	1878, Oct. 10, in Stockholm.	1882, in autumn, at Jaederen.	1883, in Stockholm.	
			Nov. 9.	Nov. 15.
$A-N$	+0·0020	—0·0294	—0·0992	—0·0832
$B-N$	+0·0305	+0·1374	—0.2492	—0·2338
$C-N$	—0·1272	—0·2574	—0·2888	—0·2711
$D-N$	+0·0043	—0·0861	—0·1088	—0·0963
Mean	—0·0226	—0·0589	—0·1865	—0·1711

Before the comparisons at Jaederen were made, some flakes of rust were removed from the end surfaces of the bars, by which the lengths were somewhat shortened. Still the length of *B* had in the interim increased considerably and then diminished by not less than $0{\cdot}38^{mm}$, which revealed a change of a most significant character. I have here introduced the results of these comparisons in order to show why I attach so little weight to the measurements of April, 1882, although the results agree very well with those of November, 1883.

The two comparisons between the base bars and the standard were made immediately before and after the measurement of the comparator in November, 1883. In addition to this, a direct comparison was made on the 23d of November, 1883, between the traveling standard belonging to the base apparatus, which was used in the cases just mentioned, and the Pulkowa standard at Stockholm. It was here found that

$$N-P=-0{\cdot}0626^{mm},$$

a value almost identical with that obtained in a previous comparison.

The results of the measurements with the base apparatus are:

	m.
1882, Apr. 14,	99·9997
" "	99·9989
" "	99·9995
Mean	99·9994

	m.
1883, Nov. 12,	100·0003
" "	100·0000
Mean	100·0001

These determinations indicate that the length of the line had increased. Such an increase also became apparent in the discussion of the lengths obtained from measuring with the wires. This change of length presents nothing very surprising when it is understood that the line was laid out under the most unfavorable circumstances. The street along which the line extended was, in 1882, raised by a fill at the north end by as much as 2^m. To support this embankment a supporting wall was built, for which it was necessary to dig a foundation of considerable depth. Besides this, a number of large buildings were erected in the immediate neighborhood, during which it is quite likely that the stones received a jar or shock, and perhaps several, as they presented no evidence of violence and the change in the length of the line appeared gradual.

Inasmuch as a greater increase in length took place than is shown in the earlier determinations, and since the value which was obtained in November, 1883, can not be affected with any great error, the former determination must be regarded as having a value that is too large. As the result of seventeen trials, made between March 22, 1882, and November 17, 1883, it appears that the northern point was $0{\cdot}5853^m$ higher than the southern, while no change exceeding the probable error was apparent.

THE INVESTIGATION OF THE WIRES AND STEEL TAPES UPON THE COMPARATOR.

This investigation had for its object the determination of errors of length and the coefficient of expansion. All measurements were made when the sky was overcast or when the wires and tapes could be protected from the sun's rays, so that the temperature of the wires could be regarded as that of the air. The air temperature was ascertained from three thermometers, each held at the same distance from the ground, one being near each end of the wire and the third one at the middle. The thermometers were investigated in November, 1882, and the zero-point correction determined and errors of calibration by comparison with a standard thermometer at various temperatures. In January, 1884, the zero points were again examined and found to be unchanged.

In the epitomized results of the measurements of the comparator which follow, the reduction to the horizon has been made and likewise the errors from inaccuracies of the leveling rod (equation 13) and those of the balances (equation 15). The constant length correction of the wires for the 100^m is represented by $x_1, x_2, x_3 \ldots x_8$, while $y_1, y_2, y_3 \ldots y_8$ stand for the expansion in 100^m for 10° Celsius. The thermometer readings have been corrected and + 15° taken as the temperature at which the wires have their normal length. Therefore y will indicate the amount by which the temperature exceeds 15°; and in the majority of cases it will be negative, since the measurement of the comparator was usually made in cold weather. The length of this line is represented by x, which of course is regarded as an unknown quantity.

Wire A.

1882, Mar. 27.	$\lambda =$	$99{\cdot}9693 + x_1 - 13{\cdot}5\ y_1$,	No. of determ. = 3
Apr. 19.		$99{\cdot}9688 + x_1 - 12{\cdot}2\ y_1$,	" " = 2
" 26.		$99{\cdot}9617 + x_1 - 5{\cdot}95 y_1$,	" " = 2
June 15.		$99{\cdot}9606 + x_1 - 4{\cdot}0\ y_1$,	" " = 2
Sept. 7.		$99{\cdot}9585 + x_1 - 0{\cdot}4\ y_1$,	" " = 2
1883, Jan. 6.		$99{\cdot}9878 + x_1 - 26{\cdot}4\ y_1$,	" " = 2
" 7.		$99{\cdot}9839 + x_1 - 22{\cdot}5\ y_1$,	" " = 2
" 31.		$99{\cdot}9753 + x_1 - 13{\cdot}4\ y_1$.	" " = 2
Apr. 20.		$99{\cdot}9717 + x_1 - 11{\cdot}7\ y_1$,	" " = 2
Sept. 25.		$99{\cdot}9640 + x_1 - 4{\cdot}9\ y_1$,	" " = 2
Oct. 16.		$99{\cdot}9630 + x_1 - 3{\cdot}4\ y_1$,	" " = 2
Nov. 17.		$99{\cdot}9764 + x_1 - 15{\cdot}9\ y_1$,	" " = 2

Wire B.

1882, Mar. 22.	$\lambda =$	$99{\cdot}9215 + x_2 - 9{\cdot}7\ y_2$,	No. of determ. = 2
" 27.		$99{\cdot}9281 + x_2 - 13{\cdot}6\ y_2$,	" " = 4
Apr. 19.		$99{\cdot}9270 + x_2 - 12{\cdot}2\ y_2$,	" " = 2
June 15.		$99{\cdot}9156 + x_2 - 4{\cdot}1\ y_2$,	" " = 2
Sept. 7.		$99{\cdot}9105 + x_2 - 0{\cdot}7\ y_2$,	" " = 2
1883, Jan. 6.		$99{\cdot}9508 + x_2 - 26{\cdot}5\ y_2$,	" " = 2
" 7.		$99{\cdot}9522 + x_2 - 22{\cdot}6\ y_2$,	" " = 2
" 31.		$99{\cdot}9377 + x_2 - 13{\cdot}4\ y_2$,	" " = 2
Apr. 20.		$99{\cdot}9327 + x_2 - 11{\cdot}6\ y_2$,	" " = 2
Sept. 25.		$99{\cdot}9197 + x_2 - 4{\cdot}9\ y_2$,	" " = 2
Nov. 17.		$99{\cdot}9405 + x_2 - 15{\cdot}9\ y_2$,	" " = 3

Wire C.

1882, Apr. 19.	$\lambda = 100{\cdot}0224 + x_3 - 11{\cdot}45 y_3$,	No. of determ.	= 2
June 15.	$100{\cdot}0110 + x_3 - 4{\cdot}3\ y_3$,	" "	= 2
Sept. 7.	$100{\cdot}0070 + x_3 - 1{\cdot}2\ y_3$,	" "	= 2
1883, Jan. 6.	$100{\cdot}0530 + x_3 - 26{\cdot}0\ y_3$,	" "	= 2
" 7.	$100{\cdot}0471 + x_3 - 21{\cdot}9\ y_3$,	" "	= 2
" 31.	$100{\cdot}0330 + x_3 - 13{\cdot}4\ y_3$,	" "	= 2
Apr. 26.	$100{\cdot}0224 + x_3 - 9{\cdot}2\ y_3$,	" "	= 2
Sept. 25.	$100{\cdot}0124 + x_3 - 4{\cdot}5\ y_3$,	" "	= 2
Nov. 17.	$100{\cdot}0342 + x_3 - 15{\cdot}75 y_3$,	" "	= 2

Wire D.

1882, Apr. 19.	$\lambda = 100{\cdot}0566 + x_4 - 11{\cdot}6\ y_4$,	No. of determ.	= 3
June 15.	$100{\cdot}0509 + x_4 - 4{\cdot}15 y_4$,	" "	= 3
Sept. 7.	$100{\cdot}0498 + x_4 - 1{\cdot}75 y_4$,	" "	= 2
1883, Jan. 6.	$100{\cdot}0792 + x_4 - 26{\cdot}95 y_4$,	" "	= 2
" 7.	$100{\cdot}0752 + x_4 - 21{\cdot}8\ y_4$,	" "	= 2
" 31.	$100{\cdot}0670 + x_4 - 13{\cdot}4\ y_4$,	" "	= 2
Apr. 26.	$100{\cdot}0582 + x_4 - 8{\cdot}2\ y_4$,	" "	= 2
Sept. 25.	$100{\cdot}0556 + x_4 - 5{\cdot}2\ y_4$,	" "	= 2
Nov. 17.	$100{\cdot}0687 + x_4 - 16{\cdot}7\ y_4$,	" "	= 2

Wire E.

1882, Sept. 4.	$\lambda = 100{\cdot}0107 + x_5 + 1{\cdot}9\ y_5$,	No. of determ.	= 2
" 7.	$100{\cdot}0120 + x_5 + 0{\cdot}45 y_5$,	" "	= 3
1883, Jan. 6.	$100{\cdot}0412 + x_5 - 26{\cdot}0\ y_5$,	" "	= 2
" 7.	$100{\cdot}0384 + x_5 - 22{\cdot}25 y_5$,	" "	= 2
" 31.	$100{\cdot}0307 + x_5 - 13{\cdot}4\ y_5$,	" "	= 2
Apr. 20.	$100{\cdot}0262 + x_5 - 12{\cdot}3\ y_5$,	" "	= 2
Sept. 25.	$100{\cdot}0198 + x_5 - 5{\cdot}5\ y_5$,	" "	= 2
Nov. 17.	$100{\cdot}0278 + x_5 - 13{\cdot}1\ y_5$,	" "	= 2

Wire F.

1882, Sept. 4.	$\lambda = 100{\cdot}0139 + x_6 + 0{\cdot}9\ y_6$,	No. of determ.	= 2
" 7.	$100{\cdot}0157 + x_6 + 0{\cdot}2\ y_6$,	" "	= 2
1883, Jan. 6.	$100{\cdot}0660 + x_6 - 26{\cdot}6\ y_6$,	" "	= 3
" 7.	$100{\cdot}0584 + x_6 - 22{\cdot}5\ y_6$,	" "	= 2
" 31.	$100{\cdot}0448 + x_6 - 13{\cdot}4\ y_6$,	" "	= 2
Apr. 20.	$100{\cdot}0409 + x_6 - 12{\cdot}55 y_6$,	" "	= 2
Sept. 25.	$100{\cdot}0284 + x_6 - 5{\cdot}9\ y_6$,	" "	= 2
Nov. 17.	$100{\cdot}0431 + x_6 - 13{\cdot}4\ y_6$,	" "	= 3

Wire M.

1883, Apr. 23.	$\lambda = 100{\cdot}0567 + x_7 - 13{\cdot}8\ y_7$,	No. of determ.	= 3
Sept. 25.	$100{\cdot}0514 + x_7 - 6{\cdot}4\ y_7$,	" "	= 2
Nov. 17.	$100{\cdot}0571 + x_7 - 13{\cdot}2\ y_7$,	" "	= 2

Wire N.

1883, Apr. 23.	$\lambda = 100{\cdot}0407 + x_8 - 13{\cdot}75 y_8$,	No. of determ.	= 2
Sept. 25.	$100{\cdot}0308 + x_8 - 6{\cdot}7\ y_8$,	" "	= 2
Nov. 17.	$100{\cdot}0430 + x_8 - 13{\cdot}1\ y_8$.	" "	= 2

The total number of simple observations made in determining the length of the comparator was 136.

The following results for the length of the line were obtained after substituting the first approximate values of x and y as given below:

Date.	Temp.	*A* (steel).	*B* (brass).	*C* (brass).	*D* (steel).	*E* (steel).	*F* (brass).	*M* (brass).	*N* (brass).	Mean.
		$x = 40.6$ mm $y = 1.04$	$+87.3$ mm 1.68	-7.9 mm 1.74	-52 mm 1.02	-15.0 mm 0.99	-19.0 mm 1.75	-43.6 mm 1.02	-19.0 mm 1.70	------- -------
1882.	°									
Mar. 22	+5	--------	99·9925	--------	--------	--------	--------	--------	--------	99·9925
27	+1	99·9959	99·9926	--------	--------	--------	--------	--------	--------	99·9943
Apr. 19	+3	99·9967	99·9938	99·9946	99·9928	--------	--------	--------	--------	99·9945
26	+9	99·9961	--------	--------	--------	--------	--------	--------	--------	99·9961
June 15	+11	99·9970	99·9959	99·9956	99·9947	--------	--------	--------	--------	99·9958
Sept. 4	+16	--------	--------	--------	--------	99·9976	99·9965	--------	--------	99·9970
7	+14	99·9987	99·9966	99·9970	99·9960	99·9974	99·9970	--------	--------	99·9971
1883.										
Jan. 6	−12	100·0010	100·0015	99·9999	99·9997	99·9999	100·0004	--------	--------	100·0004
7	−7	100·0011	100·0015	100·0011	100·0010	100·0009	100·0000	--------	--------	100·0009
31	+2	100·0020	100·0025	100·0018	100·0013	100·0022	100·0024	--------	--------	100·0020
Apr. 20	+3	100·0001	100·003	--------	--------	99·9988	99·9999	--------	--------	99·9998
23	+1	--------	--------	--------	--------	--------	--------	99·9990	99·9982	99·9986
26	+7	--------	--------	99·9985	99·9978	--------	--------	--------	--------	99·9982
Sept. 25	+10	99·9995	99·9988	99·9967	99·9983	99·9992	99·9991	100·0013	100·0003	99·9991
Oct. 16	+12	100·0001	--------	--------	--------	--------	--------	--------	--------	100·0001
Nov. 17	+2	100·0005	100·0011	99·9989	99·9997	99·9986	100·0007	100·0000	100·0016	100·0003

Although only approximate values for the lengths of the wires and the expansion of the same were employed in the computation, still it unquestionably appears (1) that the variations are not altogether dependent upon temperature; (2) that the results obtained on any one day with different wires agree very well with one another; (3) that if the variations are made a function of the changes in the lengths of the wires, these changes would appear the same for all the wires; (4) that the line itself must be regarded as varying.

The especial difficulty of obtaining from the observations the errors of the wires is unquestionably to be regarded as an unfavorable exceptional case, caused by the grading and building operations already referred to. The errors also must be affected with a larger probable error than would otherwise have been the case. Hence the results in the longer lines depending upon this one must give a larger error than would ordinarily be expected rather than a smaller one.

So far as the change in the line is concerned, there was positive evidence of it during the early experiments, or until the beginning of the year 1883, but after that it seemed to remain quite constant. This cessation of change was expected about this time, since the work along the street was then completed. If the variation is regarded as a function of time, it may be obtained if we take terms up to the third involving time, as

$$\lambda = \lambda_0 + x\,t + y\cos t + z\sin t,$$

in which t is the time and x, y, and z are constants. It seems necessary to assume one expression for the period prior to the epoch 1883·0 and another for the time subsequent to it. I therefore place for the former

$$\lambda = \lambda_0 + m_1\,t + m_2\,t^2 \;\ldots\ldots \qquad (a)$$

and for the latter

$$\lambda = \lambda_0 + n_1\,t + n_2\,t^2 \;\ldots\ldots \qquad (b)$$

in which t is expressed in years dating from 1883·0 and λ_0 the length of the comparator at that time. The determination of the comparator by means of the base apparatus already referred to took place November 12, 1883, or 1883·86. Therefore the equation

$$100{\cdot}0001 = \lambda_0 + 0{\cdot}86\,n_1 + 0{\cdot}74\,n_2 \;\ldots\ldots \qquad (c)$$

must be satisfied.

Without introducing into the computation complications that are without any gain, I compute first of all, with the aid of the mean lengths of the line as obtained in the above approximations, the constants in the expressions for λ. Weights are ascribed to these means in proportion to the number of wires employed each day in finding this mean. For the period after 1883·0 we have from equations (b) and (c)

$$\lambda - 100{\cdot}0001 = (t - 0{\cdot}86)\,n_1 + (t^2 - 0{\cdot}74)\,n_2$$

and

$$
\begin{array}{llll}
+0.3 = -0.85 n_1 & -0.74 n_2, & \text{weight} & = 6 \\
+0.8 = -0.84 & -0.74 & \text{"} & = 6 \\
+1.9 = -0.78 & -0.73 & \text{"} & = 6 \\
-0.3 = -0.56 & -0.65 & \text{"} & = 4 \\
-1.5 = -0.55 & -0.64 & \text{"} & = 2 \\
-1.9 = -0.55 & -0.64 & \text{"} & = 2 \\
-1.0 = -0.13 & -0.21 & \text{"} & = 8 \\
0.0 = -0.07 & -0.12 & \text{"} & = 1 \\
+0.2 = +0.02 & +0.03 & \text{"} & = 8
\end{array}
$$

From these one finds the following most probable values:

$$n_1 = -\ 9{\cdot}67 mm$$
$$n_2 = +\ 9{\cdot}59 mm$$

and from equation (c)

$$\lambda_0 = 100{\cdot}0001 + 8{\cdot}31 mm - 7{\cdot}09 mm$$
$$= 100{\cdot}0013 m$$

The equation (a) now takes the form

$$\lambda = 100{\cdot}0013 = m_1 t + m_2 t^2,$$

from which we have

$$
\begin{array}{llll}
-8.8 mm & = -0.78 m_1 + 0.61 m_2, & \text{weight} & = 1 \\
-7.0 & = -0.77 \quad +0.59 & \text{"} & = 2 \\
-6.8 & = -0.70 \quad +0.49 & \text{"} & = 4 \\
-5.2 & = -0.69 \quad +0.48 & \text{"} & = 1 \\
-5.5 & = -0.55 \quad +0.30 & \text{"} & = 4 \\
-4.3 & = -0.33 \quad +0.11 & \text{"} & = 2 \\
-4.2 & = -0.32 \quad +0.10 & \text{"} & = 6
\end{array}
$$

which gives

$$m_1 = +\ 14{\cdot}55 mm$$
$$m_2 = +\ 6{\cdot}98 mm$$

Hence we have

before 1883·0, $\lambda = 100{\cdot}0013 m + 14{\cdot}55 mm \times t + 6{\cdot}98 mm \times t^2$
after 1883·0, $\lambda = 100{\cdot}0013 m - \ 9{\cdot}67 mm \times t + 9{\cdot}56 mm \times t^2$

in which t is the time in years reckoning from 1883·0.

With these equations the following table has been computed:

		m.
1882, Mar. 22	$\lambda =$	99·9942
" 27		·9942
Apr. 19		·9946
" 26		·9947
June 15		·9954
Sept. 4		·9973
" 7		·9974
1883, Jan. 6		100·0012
" 7		·0011
" 31		·0006
Apr. 20		99·9993
" 23		·9993
" 26		·9993
Sept. 25		·9993
Oct. 16		·9996
Nov. 17		100·0002

Upon this determination of λ rest the computation of errors of length and the expansion of the wires, by taking the approximate values given on page 145 for x and y equal to (x) and (y). One then obtains

Wire A.

mm.

$-1.7=(x_1)-14(y_1)$
$-2.1=(x_1)-12(y_1)$
$-1.4=(x_1)-\ \ 6(y_1)$
$-1.6=(x_1)-\ \ 4(y_1)$
$-1.3=(x_1)-\ \ 0(y_1)$
$+0.2=(x_1)-26(y_1)$
$0.0=(x_1)-23(y_1)$
$-1.4=(x_1)-13(y_1)$
$-0.8=(x_1)-12(y_1)$
$-0.2=(x_1)-\ \ 5(y_1)$
$-0.5=(x_1)-\ \ 3(y_1)$
$-0.3=(x_1)-16(y_1)$

Wire B.

mm.

$+1.7=(x_2)-10(y_2)$
$+1.6=(x_2)-14(y_2)$
$+0.8=(x_2)-12(y_2)$
$-0.5=(x_2)-\ \ 4(y_2)$
$+0.8=(x_2)-\ \ 1(y_2)$
$-0.3=(x_2)-27(y_2)$
$-0.4=(x_2)-23(y_2)$
$-1.9=(x_2)-13(y_2)$
$-1.0=(x_2)-12(y_2)$
$+0.5=(x_2)-\ \ 5(y_2)$
$-0.9=(x_2)-16(y_2)$

Wire C.

mm.

$0.0=(x_3)-11(y_3)$
$-0.2=(x_3)-\ \ 4(y_3)$
$+0.4=(x_3)-\ \ 1(y_3)$
$+0.3=(x_3)-26(y_3)$
$0.0=(x_3)-22(y_3)$
$-1.2=(x_3)-13(y_3)$
$+0.8=(x_3)-\ \ 9(y_3)$
$+2.6=(x_3)-\ \ 4(y_3)$
$+1.3=(x_3)-16(y_3)$

Wire D.

mm.

$+1.8=(x_4)-12(y_4)$
$+0.7=(x_4)-\ \ 4(y_4)$
$+1.4=(x_4)-\ \ 2(y_4)$
$+1.5=(x_4)-27(y_4)$
$+0.1=(x_4)-22(y_4)$
$-0.7=(x_4)-13(y_4)$
$+1.5=(x_4)-\ \ 8(y_4)$
$+1.0=(x_4)-\ \ 5(y_4)$
$+0.5=(x_4)-17(y_4)$

Wire E.

mm.

$-0.3=(x_5)+\ \ 2(y_5)$
$0.0=(x_5)+\ \ 0(y_5)$
$+1.3=(x_5)-26(y_5)$
$+0.2=(x_5)-22(y_5)$
$-1.6=(x_5)-13(y_5)$
$+0.5=(x_5)-12(y_5)$
$+0.1=(x_5)-\ \ 6(y_5)$
$+0.6=(x_5)-13(y_5)$

Wire F.

mm.

$+0.8=(x_6)+\ \ 1(y_6)$
$+0.4=(x_6)+\ \ 0(x_6)$
$+0.8=(x_6)-27(y_6)$
$+1.1=(x_6)-22(y_6)$
$-1.8=(x_6)-13(y_6)$
$-0.6=(x_6)-13(y_6)$
$+0.2=(x_6)-\ \ 6(y_6)$
$-0.5=(x_6)-13(y_6)$

Wire M.

mm.

$+0.3=(x_7)-14(y_7)$
$-2.0=(x_7)-\ \ 6(y_7)$
$+0.2=(x_7)-13(y_7)$

Wire N.

mm.

$+1.1=(x_8)-14(y_8)$
$-1.0=(x_8)-\ \ 7(y_8)$
$-1.4=(x_8)-13(y_8)$

From these we can obtain the following equations, involving the unknown quantities, the first member being in millimetres.

$$
\begin{array}{rrl}
-11{\cdot}1 = & 12(x_1) - 134(y_1), & +\ 0{\cdot}4 = 11(x_2) - 137(y_2), \\
93{\cdot}7 = & -134(x_1) + 2200(y_1), & -18{\cdot}1 = -137(x_2) + 2309(y_2), \\
+\ 5{\cdot}0 = & 9(x_3) - 106(y_3), & +\ 7{\cdot}8 = 9(x_4) - 110(y_4), \\
-56{\cdot}2 = & -106(x_3) + 1820(y_3), & -86{\cdot}3 = -110(x_4) + 1924(y_4), \\
+\ 0{\cdot}8 = & 8(x_5) - 90(y_5), & +\ 0{\cdot}4 = 8(x_6) - 93(y_6), \\
-32{\cdot}4 = & -90(x_5) + 1682(y_5), & -\ 8{\cdot}5 = -93(x_6) + 1757(y_6), \\
-\ 1{\cdot}5 = & 3(x_7) - 33(y_7), & -\ 1{\cdot}3 = 3(x_8) - 34(y_8), \\
+\ 5{\cdot}2 = & -33(x_7) + 401(y_7), & +\ 9{\cdot}8 = -34(x_8) + 414(y_8),
\end{array}
$$

From which we obtain

$$
\begin{array}{ll}
(x_1) = -1{\cdot}41, \quad x_1 = 39{\cdot}19, & (x_2) = +0{\cdot}51, \quad x_2 = +87{\cdot}81, \\
(y_1) = -0{\cdot}043, \quad y_1 = 0{\cdot}997, & (y_2) = +0{\cdot}038, \quad y_2 = 1{\cdot}718, \\
(x_3) = +0{\cdot}61, \quad x_3 = -7{\cdot}29, & (x_4) = +1{\cdot}06, \quad x_4 = -50{\cdot}94, \\
(y_3) = +0{\cdot}005, \quad y_3 = 1{\cdot}745, & \quad {\cdot}016, \quad y_4 = 1{\cdot}036, \\
(x_5) = -0{\cdot}29, \quad x_5 = -15{\cdot}29, & (x_6) = -0{\cdot}02, \quad x_6 = -19{\cdot}02, \\
(y_5) = -0{\cdot}035, \quad y_5 = 0{\cdot}955, & (y_6) = -0{\cdot}006, \quad y_6 = 1{\cdot}744, \\
x_7 = -44{\cdot}38, & x_8 = -19{\cdot}15, \\
y_7 = 0{\cdot}996, & y_8 = 1{\cdot}736.
\end{array}
$$

For the wires *M* and *N* the coefficient of expansion is taken as the mean of the preceding wires of the same metal, inasmuch as the number of determinations was too few to admit of an independent determination for each.

The wires, therefore, when hanging free and under the tension already mentioned, have the following lengths in metres from the middle of the scale at one end to the middle of the scale at the other, and also the coefficient of expansion for 1° Celsius:

Wire,	*A*	*B*	*C*	*D*
Length,	25·00980	25·02195	24·99818	24·99727
Coef. of ex.,	(0·0000)0997	1718	1745	1036
Wire,	*E*	*F*	*M*	*N*
Length,	24·99618	24·99524	24·98890	24·99521
Coef. of ex.,	(0·0000)0955	1744	0996	1736.

Since these experiments gave a noticeably smaller coefficient of expansion for steel, as well as brass, than has usually been found for these metals, I have deemed it wise to determine these coefficients independent of any geodetic operation. For this purpose I employed the proper apparatus, belonging to the school of technology, in which the wires are immersed in water of different temperatures. The changes in length were read by a telescope in a mirror, which, turned by the expansion or contraction of the wire, reflected the divisions of a large scale placed at a suitable distance from it. The result of these investigations gave for the brass wire from which *F* was taken

Coefficient of expansion = 0·00001751,

and for steel wire 1·3^{mm} in diameter

Coefficient of expansion = 0.00000979.

Some days later the experiments were repeated,

giving for the former 0·00001703

and for the latter 0·00000988;

or, taking the means we have,

Coefficient of expansion for drawn brass wire = 0·00001727
Coefficient of expansion for drawn steel wire = 0·00000984,

results which agree very closely with those found in the direct measuring.

The errors of length and coefficients of expansion for the two steel tapes *A* and *B* were also determined by measurement of the comparator; but since these tapes were only occasionally used in measuring the short overlaps at the ends of the line, a detailed account of the experiments can have no special interest. Therefore the results only will be given:

The length of the tape *A* at 15° Celsius = 20.00303^{m}.
The length of the tape *B* at 15° Celsius = 20.00493^{m}.
The coefficient of expansion for *A* = 0.00001046.
The coefficient of expansion for *B* = 0.00001030.

The length given above is for the tapes when lying on a flat surface and without tension. In addition, the errors of graduation were determined for *A*, it alone being used in the final measurements. They are given in the table below, combined with the errors of length for the normal tension and also the errors if 10^{k} or 15^{k} tension were employed.

Errors of graduation + error of length.

	$T_0=6{\cdot}72^k$		$T=10^k$		$T=15^k$	
m.	*k.*	*mm.*	*k.*	*mm.*	*k.*	*mm.*
0	0·00	0·0	6·96	0·0	13·65	0·0
1	0·91	+0·3	6·98	+0·4	13·65	+0·6
2	1·45	0·5	7·02	0·8	13·67	1·1
3	1·88	0·7	7·10	1·1	13·69	1·5
4	2·30	0·9	7·20	1·5	13·71	2·0
5	2·67	1·2	7·32	1·9	13·75	2·6
6	3·01	1·0	7·45	1·9	13·79	2·8
7	3·34	1·3	7·60	2·3	13·84	3·3
8	3·65	1·1	7·77	2·3	13·90	3·5
9	3·95	1·4	7·94	2·7	13·96	4·0
10	4·24	1·8	8·11	3·2	14·03	4·7
11	4·51	2·1	8·30	3·8	14·11	5·3
12	4·78	2·0	8·48	3·8	14·19	5·5
13	5·05	2·2	8·67	4·1	14·28	6·0
14	5·30	2·1	8·86	4·1	14·37	6·2
15	5·55	2·2	9·05	4·4	14·47	6·6
16	5·79	2·2	9·24	4·6	14·57	6·9
17	6·03	2·6	9·43	5·1	14·67	7·6
18	6·27	2·6	9·62	5·3	14·78	7·9
19	6·50	3·1	9·81	5·9	14·89	8·7
20	6·72	3·0	10·00	6·0	15·00	8·8

Scale $\frac{1}{15000}$

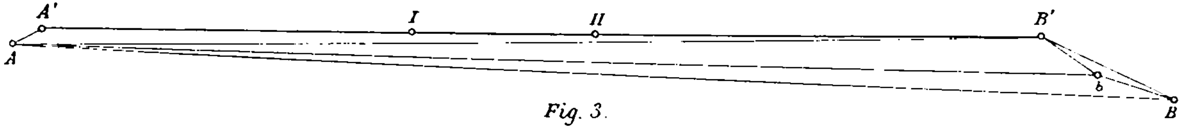


Fig. 3.

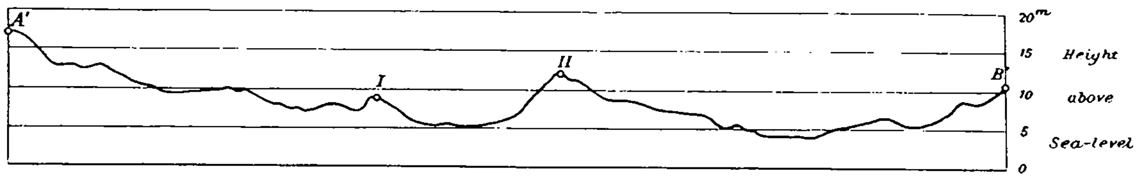


Fig. 4.

THE LONG LINE.

This line also is situated on "Ladugardsgardet," near Stockholm, but it is not the line measured in 1864 by Professor Lindhagen with the base apparatus belonging to the Academy of Sciences. One end of this line is covered by dirt, and buildings have been erected at different points along it. In figure 3 $A\ B$ is the old line and $A'\ B'$ the new one. The terminal points, as well as the two intermediate points I and II, are fixed by small holes drilled into a rod of iron firmly fastened in the upper surface of stone blocks.

The measurement in 1864 gave for $A\ B$ reduced to sea level 2320·1878^m.

In order to compare this result with the length of $A'\ B'$ obtained by wire measurement the two lines were connected by means of a triangulation.

At the beginning, in 1880, it was possible to determine $B\ B'$ by direct measurement, but shortly afterwards a house was built in the way, so that it was necessary to establish an intermediate point, b. Each of the values given below is the mean of at least two independent determinations.

LENGTHS.

				m.
1880, Apr. 26,	line $B\ b$,	measured with steel tape A,		161·6456
May 2,	" $B\ B'$,	" " " " "		294·4511
1882, May 17,	" $B'\ b$,	" " wires,		135·4757
May 19,	" $A\ A'$,	" " "		64·3883

When reduced to sea level the lines have the following values:

$$
\begin{aligned}
& \qquad\quad m. \\
A\ B &= 2320{\cdot}1878 \\
B\ b &= 161{\cdot}6453 \\
B\ B' &= 294{\cdot}4506 \\
B'\ b &= 135{\cdot}4757 \\
A\ A' &= 64{\cdot}3882
\end{aligned}
$$

ANGLE READINGS.

		°	′	″
1880, May 3,	$B\ B'\ A$, using a theodolite of Ertel reading to 10″ by means of verniers,	154	1	26·6
1882, May 20,	$A\ b\ B$, using a universal instrument of Wanschaff reading to 1″ by means of microscopes,	163	4	38·6
	$A\ b\ A'$,		51	38·9
	$A'\ b\ B'$,	31	29	16·6
	$B\ b\ B'$,	164	34	25·9
1882, June 12,	$B'\ A'\ b$, with the same instrument,	2	1	57·7
	$b\ A'\ A$,	149	35	42·2

In the triangle $B\,b\,B'$ the sides $B\,b$ and $b\,B'$ being known from direct measurement and the angle $B\,b\,B'$, one obtained from computation

$$B\,B'=294{\cdot}4538$$

By giving the measured value weight 2 and this computed result weight 1, the mean gives

$$B\,B'=294{\cdot}4517$$

And after making the corrections for triangle errors it was found that

$$B\,b=161{\cdot}6442$$
$$B'\,b=135{\cdot}4747$$

In the triangle $A\,B\,b$, $B\,b$, $A\,B$ and $A\,b\,B$ are known, from which one can find that

$$b\,A\,B=1^{\circ}\ 9'\ 43{\cdot}15''$$

and

$$A\,B\,b=15\ 45\ 38.25$$

using the known equation

$$A\,b=A\,B\cos b\,A\,B+B\,b\cos A\,b\,B=A\,B-2\,A\,B\sin^2 \tfrac{1}{2}b\,A\,B+B\,b\cos A\,b\,B$$

or

$$A\,b=2320{\cdot}1878^{m}=0{\cdot}4771-154{\cdot}6448=2165{\cdot}0659.$$

In the same manner, from $A\,B'\,b$ I found

$$A\,B'=2051{\cdot}8967^{m}$$

and from $A\,b\,A$

$$A'\,b=2109{\cdot}2877^{m}$$

And, finally, from the triangle $A\,A'\,B'$

$$A'\,B'=1995{\cdot}0148$$

and from $A'\,B'\,b$

$$A'\,B'=1995{\cdot}0168$$

the mean of which was accepted as a sufficient approximation, or

$$A'\,B=1995{\cdot}0158^{m}$$

DETERMINATION OF $A'\,B'$ WITH WIRES.

In measuring, two wires were always used, one of steel and one of brass. By means of the difference in the readings of the two a check is obtained on the reading and data for the determination of temperature.

If the two wires at the normal temperature (+15° Celsius) have lengths L and L', and if we designate by c and c' the scale readings, then after applying n wire lengths the sums of the readings would be $\Sigma\,c$ and $\Sigma\,c'$. Again, if the excess of temperature above the normal temperature be indicated by t and the coefficients of expansion by α

and β, then the length of the measured line λ is for one wire ascertained from the equation

$$\lambda = n\,L + \Sigma\,e + n\,L\,\alpha\,t$$

and for the other wire

$$\lambda = n\,L' + \Sigma\,e' + n\,L'\,\beta\,t$$

therefore

$$t = \frac{n\,L - n\,L' + \Sigma\,e - \Sigma\,e'}{n\,(L'\,\beta - L\,\alpha)}\,.$$

By substituting this expression for t in one of the equations above, we have

$$\lambda = \frac{L'\,\beta\,(n\,L + \Sigma\,e) - L\,\alpha\,(n\,L' + \Sigma\,e')}{\beta\,L' - \alpha\,L}$$

or, if the length of the line according to the two wires be represented by s and s'—that is, without reference to the temperature—then

$$\lambda = s + \frac{\alpha}{\beta - \alpha}(s - s') = s' + \frac{\beta}{\beta - \alpha}(s - s')\ldots\ldots \qquad (16)$$

or

$$\lambda = \frac{\beta}{\beta - \alpha}\,s - \frac{\alpha}{\beta - \alpha}\,e'$$

From the latter of these equations it is evident that if at each reading of the scale there was a probable error of $\pm\,r$, the probable error for the whole line from this cause would be

$$\varphi = \pm\,\frac{2}{\beta - \alpha}\,\sqrt{n\,(\beta^2 + \alpha^2)}\ldots\ldots \qquad (17)$$

a formula which is true only when the temperature, as is here regarded, is determined by the differences in the scale readings.

Over smooth ground the errors of length resulting from errors of contact and of differences of elevation are of very slight consequence. In such a case the accidental error only is found from equation (17).

Instead of using equation (16), it would be better to compute the length of the line in such a way that the temperature may be taken from a table already prepared in which the argument is the difference in the lengths of the two wires and the tabulated values degrees. In this way, by computing the length of the line for each wire, one has a check on the work. For the wires A and B, at $+\,15°$ Celsius, the difference in lengths,

$$B - A = +\,12{\cdot}15^{\mathrm{mm}}$$

and the relative expansion for 1°

$$25^{\mathrm{m}}\,(\beta - \alpha) = 0{\cdot}1802^{\mathrm{mm}}$$

from which the following table was prepared:

Differences of expansion.

t	*B−A*	*t*	*B−A*	*t*	*B−A*
°	*mm.*	°	*mm.*	°	*mm.*
−15	6·75	7	10·71	29	14·68
14	6·93	8	10·89	30	14·86
13	7·11	9	11·07	31	15·04
12	7·29	10	11·25	32	15·22
11	7·47	11	11·43	33	15·40
10	7·65	12	11·61		
9	7·83	13	11·79		
8	8·01	14	11·97	Differences.	
7	8·19	15	12·15		
6	8·37	16	12·34		
5	8·55	17	12·52	°	*mm.*
4	8·73	18	12·70	0·0	0·00
3	8·91	19	12·88	·1	·02
2	9·09	20	13·06	·2	·04
−1	9·27	21	13·24	·3	·05
0	9·45	22	13·42	·4	·07
+1	9·63	23	13·60	·5	·09
2	9·81	24	13·78	·6	·11
3	9·99	25	13·96	·7	·13
4	10·17	26	14·14	·8	·14
5	10·35	27	14·32	·9	·16
6	10·53	28	14·50	1·0	·18

The necessary personnel for measuring is as follows: One person for the spring balance, one at each end to place the scales side by side and to make the readings, one for leveling, and one for marking—or five trusty persons. For handwork there are needed one person for the small steel balance, one to hold the leveling rod, and two for carrying instruments, pegs, etc., along the line, making four workmen. Thus it will be seen that in order to have the work move along without delays nine persons are essential. In the work on "Ladugårdsgärdet" the number of assistants was not so large as desirable. Besides this, the continuous progress of the work was often hindered by passing wagons and the drilling of soldiers. This latter interruption is the reason why the line *A′ B′* was seldom measured as a whole, but in the sections *A′* I, I II, and II *B′* independently and irregularly.

During the first few days a leveling rod was used which was provided with two targets, the second one being on the reverse side and read downward, the zero being at the top. The advantage thus afforded was that the two readings should always have a constant sum, thereby furnishing a check.

But after some days one of the targets was injured, and thereafter the ordinary method was pursued with confidence, inasmuch as up to that time no erroneous readings had been made.

The following gives the measurements of A' B':

1882.	Line	Began	Finished	Remarks.
May 8.	A'I	10:50 a. m.	4:00 p. m.	Sunshine, scattering clouds.
" 9.	II I	7:45	10:10 a. m.	" wind = 3 to 4.*
	IIB	----------	3:15 p. m.	Thin clouds.
	II I	7:00 p. m.	8:10	Still.
" 10.	IIB	0:20	2:50	Sunshine, wind = 1.
	II I	6:45	7:25	"
" 11.	IA'	8:40 a. m.	10:50 a. m.	" still.
	A'I	0:30 p. m.	2:25 p. m.	" scattering clouds, wind = 1 to 2.
	IIB'	3:25	5:50	" wind = 2.
" 12.	A'I	8:40 a. m.	10:55 a. m.	Rain, wind = 2.
" 15.	IIB	3:10 p. m.	6:40 p. m.	Sunshine, wind at right angles to the wires.

In these measurements only wires A and B were employed and the tape A.

In the autumn of the same year the line was measured twice by a party of students, using the wires E and F, but unfortunately palpable errors were made, making the results useless. However, I mention them here merely to show how rapidly measurements of this kind can be done.

1882.	Line	Began	Finished	Remarks.
Sept. 8.	A'I	9:15 a. m.	0:00	Sunshine, scattering clouds.
	I II	2:30 p. m.	3:40 p. m.	
	IIB	3:40	6:05	
" 9.	B'II	8:30 a. m.	10:50 a. m.	" wind = 2.
	I II	0:40 p. m.	1:40 p. m.	
	IA'	1:40	3:50	

The three lines A' I, I II, and II B' have the lengths 740, 367, 888 metres, respectively. The rapidity of the measurement in metres is as follows:

	Line.	Time. h.	Time. m.	Lengths per hour.	Lengths per day.
1882.					
May 8	740	5	10	143	740
9	367	2	25	152	1622
	888	---	---	---	
	367	1	10	315	
10	367	0	50	440	1622
	888	2	30	355	
	367	0	40	550	
11	740	2	10	342	2368
	740	1	55	386	
	888	2	25	367	
12	740	2	15	329	740
15	888	3	30	254	888
Sept. 8	740	2	45	269	1995
	367	1	10	315	
	888	2	25	367	
9	888	2	20	381	1995
	367	1	00	367	
	740	2	10	342	

* Estimated by the meteorological scale: 0=still, 6=hurricane.

The best speed was 550^{m} in an hour, and the largest day's work was on the day when operations began at 8:40 a. m. and continued until 5:50 p. m. On this day 2368^{m} were measured.

The following is the record of one measurement of a line. The numbering is from A'. Of course the record of the measurement of the line was separate from the record of the leveling, but they are united here for illustrative purposes. With the steel tape the tension was always T_0'.

For the wires A and B, $\frac{\alpha}{\beta - \alpha} = 1{\cdot}383$ and $\frac{\beta}{\beta - \alpha} = 2{\cdot}383$,[1] which values are to be employed when the computation is made according to equation (16).

[1882, May 10.]

Point.	Wire.			Leveling.				Horizontal reduction.†	Remarks.
	A	B	Diff.	Readings.		Diff.	Height.*		
	mm.	mm.	mm.	dm.	dm.	dm.	dm.	mm.	
I				0·38			0·00		
	72·1	60·2	11·9			− 1·65		0·5	10:30 a. m.
30				2·03			− 1·65		
	90·7	78·7	12·0			− 7·40		11·0	
31				9·43			− 9·05		Sunshine.
	86·9	75·4	11·5			−10·97		24·1	
32				20·40			−20·02		
	30·5	18·3	12·2			− 3·91		3·0	
33				24·31	2·30		−23·93		
	85·2	73·4	11·8			− 0·42		0·0	
34					2·72		−24·35		
	58·6	46·8	11·8			− 1·41		0·4	
35					4·13		−25·76		
	31·7	19·3	12·4			+ 1·54		0.5	
36					2·59		−24·22		
	91·3	79·1	12·2			− 0·70		0·1	
37				16·83	3·29		−24·92		
	54·8	42·9	11·9			+ 4·24		3·6	
38				12·59			−20·68		
	38·0	26·3	11·7			+ 3·16		2·0	
39				9·43			−17·52		
	58·5	46·4	12·1			+ 8·22		13·6	
40				1·21	20·06		− 9·30		
	56·8	45·0	11·8			+11·17		25·0	
41				12·65	8·89		+ 1·87		
	17·1	5·2	11.9			+14·65		43·0	
42				1·00	18·39		+16·52		
	98·8	57·9	11·9			+18·41		67·0	
43					−0·02		+34·93		
	17·0—66·0					− 5·24		8·1‡	
II					5·22		+29·69		11:20 a. m.
	842		$=\Sigma c$				Sum =	202·8	
		674·9	$=\Sigma c'$						

The mean of the diff. = 11·94^{mm}.
The corresponding temperature § = + 13°·85.

* The check is in the differences between the first and last number in each row of readings.
† This is taken from the table on pp. 135, 136. In the record the readings on the reverse scale is omitted.
‡ Based on equation (12a) and the table following.
§ Taken from table on p. 154.
[1] The coefficients of expansion, which were determined geodetically, have been applied here.

[1882, May 10.]

Point.	Wire. A	Wire. B	Wire. Diff.	Leveling. Readings.	Leveling. Readings.	Leveling. Diff.	Leveling. Height.	Horizontal reduction.	Remarks.
	mm.	mm.	mm.	dm.	dm.	dm.	dm.	mm.	
II				5·22			0·00		
	65·3	53·1	12·2			+ 3·54		2·5	
44				1·68			+ 3·54		0:20 p. m.
	77·2	65·4	11·8			— 6·52		8·5	
45				8·20			— 2·68		
	56·4	44·1	12·3			— 9·58		18·3	
46				17·78	0·28		—12·56		
	36·0	23·7	12·3			— 5·43		5·9	
47					5·71		—17·99		
	33·6	21·5	12·1			— 2·67		1·5	
48					8·38		—20·66		
	98·7	86·4	12·3			— 3·31		2·2	
49					11·69		—23·97		
	95·6	83·7	11·9			— 3·13		1·9	
50					14·82		—27·10		
	82·6	70·8	11·8			— 3·18		2·0	
51					18·00		—30·28		
	63·2	51·6	11·6			— 2·60		1·4	
52				1·83	20·60		—32·88		
	42·0	30·0	12·0			— 1·16		0·3	
53				2·99			—34·04		
	93·3	81·2	12·1			— 6·63		8·8	
54				9·62			—40·67		
	50·0	38·0	12·0			— 2·43		1·2	
55				12·05			—43·10		
	79·0	66·8	12·2			—11·24		25·3	
56				23·29			—54·34		
	83·7	71·6	12·1			+ 3·97		3·1	
57				19·32			—50·37		
	85·4	73·5	11·9			— 6·80		9·2	
58				26·12			—57·17		
	64·8	53·0	11·8			— 3·00		1·8	
59				29·12	2·97		—60·17		
	48·4	36·5	11·9			— 3·74		2·8	
60					6·71		—63·91		
	67·7	55·6	12·1			— 1·68		0·6	
61					8·39		—65·59		Wind = 1.
	91·4	79·2	12·2			— 3·49		2·5	
62					11·88		—69·08		
	97·2	85·1	12·1			+ 4·42		3·9	
63					7·46		—64·66		Sunshine.
	42·9	30·6	12·3			+ 7·17		10·3	
64					0·29		—57·49		
	93·8	82·1	11.7			+ 0·31		0·0	
65				14·37	—0·02		—57·18		
	95·2	83·2	12·0			+ 5·70		6·5	
66				8·67			—51·48		
	45·7	33·6	12·1			+ 3·58		2·6	
67				5·09			—47·90		
	101·5	89·2	12.3			+ 3·56		2·6	
68				1·53			—44·34		
	84·4	72·7	12·1			+ 0·70		0·1	
69				0·83			—43·64		
	40·3	28·8	11·5			— 3·79		2·9	
70				4·62			—47·43		
	91·2	79·7	11·5			— 0·61		1·3	

[1882, May 10—Continued].

Point.	Wire. A	Wire. B	Wire. Diff.	Leveling. Readings.	Leveling. Readings.	Leveling. Diff.	Leveling. Height.	Horizontal reduction.	Remarks.
	mm.	*mm.*	*mm.*	*dm.*	*dm.*	*dm.*	*dm.*	*mm.*	
71				5·23			—48·04		
	71·0	59·3	11·7			— 2·48		2·0	
72				7·71			—50·52		
	61·5	49·5	12·0			+ 3·20		8·6	
73				4·51	23·99		—47·32		
	91·8	80·0	11·8			+ 6·57		23·3	
74					17·42		—40·75		
	51·2	39·1	12·1			+10·79		12·9	
75					6·63		—29·96		
	26·2	14·0	12·2			+ 8·03		0·1	
76				12·38	—1·40		—21·93		
	79·1	66·7	12·4			— 0·53		23·0	
77				12·91			—22·46		
	68·1	56·0	12·1			+10·72		1·1	
78	12·4—5·9			2·19			—11·74		
B^1				0·52		—10·07	—10·07		
	2455·8		$=\Sigma e$				Sum =	201·1	
		2035·3	$=\Sigma e'$						

The mean of the diff. =12·01^{mm}.
The temperature =+14°·2.

[May 11.]

Point.	Wire. A	Wire. B	Wire. Diff.	Leveling. Readings.	Leveling. Readings.	Leveling. Diff.	Leveling. Height.	Horizontal reduction.	Remarks.
	mm.	*mm.*	*mm.*	*dm.*	*dm.*	*dm.*	*dm.*	*mm.*	
1				1·55			0·00		
	62·6	50·5	12·1			— 4·25		3·6	
29				5·80			— 4·25		8:40 a. m.
	55·0	42·8	12·2			— 1·61		0.5	
28				7·41			— 5·86		Sunshine.
	22·6	11·0	11·6			— 0·52		0·1	
27				7·93			— 6·38		No wind.
	58·2	46·3	11·9			+ 0·16		0·0	
26				7·77			— 6·22		
	95·8	84·1	11·7			— 2·72		1·5	
25				10·49			— 8·94		
	59·2	47·3	11·9			+ 3·18		2·0	
24				7·31			— 5·76		
	35·8	24·2	11·6			+ 4·81		4·6	
23				2·50	21·48		— 0·95		
	75·2	63·1	12·1			+ 2·30		1·1	
22					19·18		+ 1·35		
	79·9	68·3	11·6			+ 4·80		4·6	
21					14·38		+ 6·15		
	58·0	45·9	12·1			+ 9·94		16·4	
20					5·24		+15·19		
	71·7	59·5	12·2			+ 1·73		0·6	
19					3·61		+16·09		
	49·1	37·0	12·1			+ 2·73		1·5	

[May 11—Continued.]

Point.	Wire.			Leveling.				Horizontal reduction.	Remarks.
	A	*B*	Diff.	Readings.		Diff.	Height.		
	mm.	*mm.*	*mm.*	*dm.*	*dm.*	*dm.*	*dm.*	*mm.*	
18					0·88		+19·65		
	44·2	32·0	12·2			+ 0·88		0·2	
17					0·00		+20·53		
	48·6	36·8	11·8			− 4·17		3·5	
16					4·17		+16·36		
	67·4	55·2	12·2			− 4·55		4·1	
15					8·72		+11·81		
	51·3	38·8	12·5			+ 3·38		2·3	
14				19·47	5·34		+15·19		
	55·3	43·2	12·1			+ 0·48		0·1	
13				18·99			+15·67		
	87·4	69·7	11·7			+ 7·10		10·2	Cloudy.
12				11·89			+22·77		
	42·3	30·6	11·7			+ 2·61		1·4	
11				9·28			+25·38		
	78·2	66·7	11·5			+ 8·21		13·5	
10				1·07	19·28		+33·59		
	73·5	61·8	11·7			+ 7·48		11·2	
9					11·80		+41·07		
	83·9	71·9	12·0			+ 6·86		9·4	
8					4·94		+47·95		
	66·2	54·7	11·5			− 0·75		0·1	
7					5·69		+47·18		
	52·7	41·1	11·6			+ 0·60		0·1	
6					5·09		+47·78		
	70·0	57·8	12·2			+ 5·09		5·2	
5					0·00		+52·87		
	99·7	87·7	12·0			− 0·51		0·1	
4				12·08	0·51		+52·36		
	54·2	42·3	11·9			+ 7·31		10·7	
3				11·77	28·59		+59·97		
	44·2	32·3	11·9					45·8	
2				13·46		+74·80	+74·80		
	64·1	52·0	12·1		+15·13			12·7	
1				5·49		+82·77	+82·77		
					+ 7·97			12·9	
0	7·6 —63·9			1·09		+87.17	+87·17		
					+ 4·40			26·0	
A	6·9 —85·2			7·04		+81.22	81·22		10:50
	1880·3 ——— =Σe				− 5·95				
	1454·6 =Σe′						Sum =	205·9	

The mean of the diff. = 11·92^{mm}.
The mean temperature = 13°·7.

COMPUTATION OF THE LINE.

In determining the length of the line it is necessary to observe the following precaution: In placing each wire put the scale so that the reading increases toward the rear end of the wire. The lengths just given refer in each case to the middle mark of the scale, the scale covering 5^{cm}, except on *M*, where it begins with 5 and stops at 15. For this reason, since these 5^{cm} enter into each reading, it is necessary to subtract 50mm from the reading, or, what is better, from the length of the wire, before multiplying it by the number of wires. Before taking from the table the temperature, by using the mean of the differences of readings as an argument, it is well to make the correction for the erroneous readings of the spring balance from the table on page 140. The rest of the computation will explain itself.

The leveling rod is too long by $\frac{1}{555}$ of its length; therefore each reading must be corrected by $+\frac{1}{555}$.

1882, MAY 10, LINE I II.

Approximate temperature $= +14^\circ$. The correction for the old balance at 10^k and $+14^\circ$ is (p. 140) $-0{\cdot}07^k$. The resulting error in the length of the wire $A = -0{\cdot}07^{mm}$, in $B = -0{\cdot}13^{mm}$. These values multiplied by 14, the number of wire lengths, give the correction to Σe and $\Sigma e'$, or

Σe	$= 842{\cdot}0^{mm}$	$\Sigma e'$	$= 674{\cdot}9^{mm}$
error	$= -1{\cdot}0$	error	$= -1{\cdot}8$
corrected Σe	$= 841{\cdot}0$	corrected $\Sigma e'$	$= 673{\cdot}1$

The difference between these values, $167{\cdot}9^{mm}$, divided by 14 gives the argument for finding the temperature.

The corrected mean of differences $= \frac{167{\cdot}9}{14} = 11{\cdot}99^{mm}$; therefore (p. 154) temp. $= 14^\circ.1$.

Wire A.	m.	*Wire B.*	m.
$14 \times 24{\cdot}95980 =$	$349{\cdot}4372$	$14 \times 24{\cdot}97195 =$	$349{\cdot}6073$
Σe (corrected) $=$	$\cdot 8410$	$\Sigma e'$ (corrected) $=$	$\cdot 6731$
	$350{\cdot}2782$		$350{\cdot}2804$
$(14^\circ{\cdot}1 - 15^\circ) \times 3{\cdot}5 \times 0{\cdot}997 =$	$-\cdot 0031$	$(14^\circ{\cdot}1 - 15^\circ) \times 3{\cdot}5 \times 1{\cdot}718 =$	$-\cdot 0054$
	$350{\cdot}2751$		$350{\cdot}2750$

	m.	
The tape A	$16{\cdot}9340$	$16{\cdot}9364$
Errors of division + errors of length	$= \cdot 0026$	
Temp. corr. $(-0^\circ{\cdot}9 \times 0{\cdot}17 \times 1{\cdot}046)$	$= -\cdot 0002$	$367{\cdot}2114$
	mm.	
Reduction to the horizon	$202{\cdot}8$	$\cdot 2035$
Corr. for error in leveling rod $+ 202{\cdot}8 \times \frac{2}{555}$	$= +07$	
	Line I II	$= 367{\cdot}0079$

The results are as follows:

Line A′ I.

	mm.	°	
1882, May 8. A' I	$739{\cdot}7708$	$T = +13{\cdot}3$	Sunshine, scattering clouds.
11. I A'	$\cdot 7723$	$14{\cdot}1$	Sunshine, still.
11. A' I	$\cdot 7695$	$13{\cdot}7$	Wind $= 1$ to 2.
12. A' I	$\cdot 7725$	$8{\cdot}0$	Heavy rain, wind $= 2$.
Mean	$= 739{\cdot}7713 \pm 0{\cdot}5$		

Probable error of a measurement = ± 0·9^{mm}.

Line I II.

		mm.		°	
1882, May 9.	I II	367·0127	$T = +$	10·7	Sunshine, wind = 3 to 4.
9.	II I	·0120		8·1	Still.
10.	I II	·0079		14·1	Sunshine.
10.	II I	·0100		8·5	Sunshine.
Mean		= 367·0106 ± 0·7			

Probable error of a measurement = ± 1·4^{mm}.

Line II B′.

		mm.		°	
1882, May 9.	II B'	888·2372	$T =$	11·5	Thin clouds.
10.	II B'	·2371		14·6	Sunshine, wind = 1.
11.	II B'	·2287		14·1	Sunshine, wind = 2.
15.	II B'	·2380		10·3	Sunshine, wind vertical to wire.
Mean		= 888·2352 ± 1·5			

Mean = 888·2352 ± 1·5^{mm}.

Probable error of a measurement = ± 2·9^{mm}.

$$\text{Line } A'\,B' = 1995{\cdot}0171^{m} \pm \sqrt{\overline{0{\cdot}5}^2 + \overline{0{\cdot}7}^2 + \overline{1{\cdot}8}^2}$$

$$= 1995{\cdot}0171^{m} \pm 1{\cdot}7^{mm}.$$

Probable error of a measurement = ± 3·4^{mm}, or 1 : 600 000.

If the measurement of May 11 of II B' were excluded, for which unfortunately there is no good reason, we would have

$$A'\,B' = 1995{\cdot}0193 \pm 0{\cdot}9^{mm}.$$

Probable error of a measurement = ± 1·7^{mm}, or 1 : 1 200 000.

The height of the three sections above sea level is

	m.
A' I	11·6
I II	7·8
II B'	7·8

which gives the following corrections for the reduction to sea level:

for A' I − 1·3^{mm}, for I II − 0·5^{mm}, for II B' − 1·1^{mm}.

This makes the lines at sea level to have the following lengths:

	m.
A' I	739 ·7700
I II	367 ·0101
II B'	888 ·2341
A' B	1995 ·0142

The result of the computation from $A\ B$ was 1995·0158, which agrees with the measurement within 1·6^{mm}, or 1 : 1 250 000.

By omitting the one result for II B already referred to, or taking for $A'\ B$ 1995·0164^{m}, the difference would be only 0·6^{mm}, or 1: 3 300 000.

The probable error committed in making a scale reading can be obtained from the record by supposing that the temperature was constant during an entire measurement. Since the temperature was changing, the variation of differences is effected by these temperature changes, and the error obtained in this manner will be somewhat too large. The record gives

		mm.
	for I II	$\pm$ 0·156
	II B'	$\pm$ 0·159
	I A'	$\pm$ 0·164
or the mean		$\pm$ 0·164

The probable error for the difference of the two readings must therefore be less than $\pm$ 0·164^{mm}, and each individual reading must be affected by an error which is smaller than

$$\pm \frac{0{\cdot}164}{\sqrt{2}} \text{ or } \pm 0{\cdot}116^{mm}$$

From equation (17) the probable error in a line 2000^{m} in length, when $n = 80$, is

$$\varphi < \pm 2{\cdot}86 \ mm.$$

or

$$\varphi < 1 : 700\,000$$

The greatest accidental error is evidently that which arises in reading the scales. For in the alignment, although made with the unaided eye, the average error can not exceed 2^{cm}, an inaccuracy which, if uniformly committed, could cause an error for each wire length of only 0·008^{mm}, or for 2000^{m}, 0·64^{mm}; and this error would always have the same sign. The error in leveling from which result errors in determining the difference in the heights of the two ends of the wire could not well make this difference erroneous by more than 3^{mm}, which, if each inclination were as much as 1 : 100, would cause an error of only 0·03^{mm}; and since the sign of this may be either plus or minus, for 80 lengths it would not exceed $0{\cdot}03 \sqrt{80} = 0{\cdot}27^{mm}$. If the inclination were always 1 : 20 the errors would be 0·15 and 1·3^{mm}, and this is less than the errors resulting from the error of measuring or from faulty scale reading.

The discussion just completed of the various sources of error which arise in the measurement of lines with wires may appear to detract from the confidence which this method appeared to beget.

The resulting probable error for the measurement finds its confirmation partly in the agreement of the measures one with another and partly in the coincidence of the final results for the whole line with the value obtained when measured with another form of apparatus.

With reference to the former of the above remarks, it is necessary to call attention to the unfavorable conditions under which the comparator was measured, for it was during this period that because of external conditions the line was very unstable. This occasioned a less accurate determination of the lengths of the wires, as well as a less reliable value for the coefficients of expansion, than under ordinary circumstances could be expected.

From the error in the determination of the lengths arises a constant error in the nature of a correction to the length as well as an error in the temperature coming from the scale readings—themselves dependent upon the lengths.

Likewise, an erroneous value for the coefficient of expansion occasions an error in the temperature reduction of the line and also a varying error in the determination of the temperature itself.

The error in the actual length of the wire has for the same line the same value, and the nearer the temperature remains constant during the measuring the nearer constant will be the error resulting from the uncertainties in the determination of the coefficient of expansion.

Since in the experiments here described the latter was the case, the total error arising from these sources may be approximately regarded as a constant correction factor, or for the line $A' B'$ it is of the same size for all the measurements.

The interagreement in the various measurements of the line $A' B'$ can not be sensibly affected by these errors.

On the other hand, in comparing the results of the measurements with wires with the results obtained with the apparatus of the Royal Academy of Sciences, the following is observed:

1. In the connection of $A' B'$ with $A B$ errors may exist which could affect our knowledge of the length of the former.

2. The point B was covered with dirt, and the stone in which it was marked might have been disarranged in the dumping.

3. The point A is in a somewhat small stone on the surface of the ground, and has for that reason an insecure position. It has been observed that this stone was raised from its position by a heavy wagon passing over it and that it afterwards fell back into its place.

4. The heads of the iron bolts in the top of the terminal stones are rusted, so that it is difficult to see distinctly the drill holes which mark A and B.

The close agreement between the measured and computed values of $A' B'$ must be regarded in a certain degree as a happy accident, but, on the other hand, it must be said that the coincidence would have been equally close if the obstacles mentioned had not been met.

Since the above communication was made to the Royal Academy of Sciences, in May of last year, the experiments have been continued and certain changes in the apparatus have been made. Therefore the description of the apparatus just given does not apply to the form now in use.

Among the experiences which have resulted from the recent experiments one at least is of importance; that is in regard to the question of a change in the lengths of wires during a long period of time.

The mean time in which the measurements of the comparator here referred to were made was 1883·0. The line was measured anew with the base apparatus of the Academy on September 3 and 4, 1884, at which time complete comparisons were made with the standard. The length of the line showed since November 12, 1882, a decrease of 0·1^{mm}. The wires *A*, *B*, *C*, *D*, *E*, and *F* were again compared on September 4 and 5 (temp. = + 16° Celsius) and once on November 12 (temp. = + 3°). The results are given below, except for *A*, which was injured in the interim. In the period from 1883·0 to 1884·8 all the wires were repeatedly unrolled and rolled.

Wire.	Change in length during 1·8 years.
	mm.
B (brass)	— 0·05
C "	+ 0·12
D (steel)	— 0·02
E "	+ 0·17
F (brass)	+ 0·01

On the average, without and with regard to the signs, the changes in the lengths were 0·07^{mm} and + 0·046^{mm}. The comparisons of September, 1884, were made by persons who had had no experience in work of this kind. The deviations in length or the apparent changes in length of the different wires fall unquestionably within or very near the probable error of the last-made comparisons.

TRANSLATOR'S NOTE.—For later experience in the use of steel tapes see an exhaustive report by Assistant R. S. Woodward, Appendix 8, Coast and Geodetic Survey Report 1892, Part II.

APPENDIX No. 6—1893.

FUNDAMENTAL STANDARDS OF LENGTH AND MASS.*

While the Constitution of the United States authorizes Congress to "fix the standard of weights and measures," this power has never been definitely exercised, and but little legislation has been enacted upon the subject. Washington regarded the matter of sufficient importance to justify a special reference to it in his first annual message to Congress (January, 1790), and Jefferson, while Secretary of State, prepared a report, at the request of the House of Representatives, in which he proposed (July, 1790) "to reduce every branch to the decimal ratio already established for coins, and thus bring the calculation of the principal affairs of life within the arithmetic of every man who can multiply and divide." The consideration of the subject being again urged by Washington, a committee of Congress reported in favor of Jefferson's plan, but no legislation followed. In the meantime the executive branch of the Government found it necessary to procure standards for use in the collection of revenue and other operations in which weights and measures were required, and the Troughton 82-inch brass scale was obtained for the Coast and Geodetic Survey in 1814, a platinum kilogramme and metre, by Gallatin, in 1821, and a troy pound from London in 1827, also by Gallatin. In 1828 the latter was, by act of Congress, made the standard of mass for the Mint of the United States, and, although totally unfit for such purpose, it has since remained the standard for coinage purposes.

In 1830 the Secretary of the Treasury was directed to cause a comparison to be made of the standards of weight and measure used at the principal custom-houses, as a result of which large discrepancies were disclosed in the weights and measures in use. The Treasury Department, being obliged to execute the constitutional provision that all duties, imposts, and excises shall be uniform throughout the United States, adopted the Troughton scale as the standard of length; the avoirdupois pound, to be derived from the troy pound of the Mint, as the unit of mass. At the same time the Department adopted the wine

* This paper was first published as Bulletin No. 26, and is republished here to give it a more permanent form. Appended to it will be found a third edition of the Tables for converting Customary and Metric Weights and Measures.

gallon of 231 cubic inches for liquid measure and the Winchester bushel of 2150·42 cubic inches for dry measure. In 1836 the Secretary of the Treasury was authorized to cause a complete set of all weights and measures adopted as standards by the Department for the use of custom-houses and for other purposes to be delivered to the governor of each State in the Union for the use of the States, respectively, the object being to encourage uniformity of weights and measures throughout the Union. At this time several States had adopted standards differing from those used in the Treasury Department, but after a time these were rejected, and finally nearly all the States formally adopted, by act of legislature, the standards which had been put in their hands by the National Government. Thus a good degree of uniformity was secured, although Congress had not adopted a standard of mass or of length, other than for coinage purposes, as already described.

The next and in many respects the most important legislation upon the subject was the act of July 28, 1866, making the use of the metric system lawful throughout the United States and defining the weights and measures in common use in terms of the units of this system. This was the first *general* legislation upon the subject, and the metric system was thus the first, and thus far the only, system made generally legal throughout the country.

In 1875 an international metric convention was agreed upon by seventeen Governments, including the United States, at which it was undertaken to establish and maintain at common expense a permanent international bureau of weights and measures, the first object of which should be the preparation of a new international standard metre and a new international standard kilogramme, copies of which should be made for distribution among the contributing Governments. Since the organization of the Bureau, the United States has regularly contributed to its support, and in 1889 the copies of the new international prototypes were ready for distribution. This was effected by lot, and the United States received metres Nos. 21 and 27 and kilogrammes Nos. 4 and 20. The metres and kilogrammes are made from the same material, which is an alloy of platinum with 10 per cent of iridium.

On January 2, 1890, the seals which had been placed on metre No. 27 and kilogramme No. 20 at the International Bureau of Weights and Measures, near Paris, were broken in the Cabinet room of the Executive Mansion by the President of the United States in the presence of the Secretary of State and the Secretary of the Treasury, together with a number of invited guests. They were thus adopted as the national prototype metre and kilogramme.

The Troughton scale, which in the early part of the century had been tentatively adopted as a standard of length, has long been recognized as quite unsuitable for such use, owing to its faulty construction and the inferiority of its graduation. For many years, in standardizing length measures, recourse to copies of the imperial yard of Great Britain had been necessary, and to the copies of the metre of the archives in the office of weights and measures. The standard of mass originally selected was likewise unfit for use for similar reasons, and had been practically ignored.

The recent receipt of the very accurate copies of the International Metric Standards, which are constructed in accord with the most advanced conceptions of modern metrology, enables comparisons to be made directly with those standards, as the equations of the national prototypes are accurately known. It has seemed, therefore, that greater stability in weights and measures, as well as much higher accuracy in their comparison, can be secured by accepting the international prototypes as the fundamental standards of length and mass. It was doubtless the intention of Congress that this should be done when the international metric convention was entered into in 1875; otherwise there would be nothing gained from the annual contributions to its support which the Government has constantly made. Such action will also have the great advantage of putting us in direct relation in our weights and measures with all civilized nations, most of which have adopted the metric system for exclusive use. The practical effect upon our customary weights and measures is, of course, nothing. The most careful study of the relation of the yard and the metre has failed thus far to show that the relation as defined by Congress in the act of 1866 is in error. The pound as there defined, in its relation to the kilogramme, differs from the imperial pound of Great Britain by not more than one part in one hundred thousand, an error, if it be so called, which utterly vanishes in comparison with the allowances in all ordinary transactions. Only the most refined scientific research will demand a closer approximation, and in scientific work the kilogramme itself is now universally used, both in this country and in England.*

In view of these facts, and the absence of any material normal standards of customary weights and measures, the Office of Weights and Measures, with the approval of the Secretary of the Treasury, will in the future regard the International Prototype Metre and Kilogramme as fundamental standards, and the customary units—the yard and the pound—will be derived therefrom in accordance with the Act of July 28,

* NOTE.—Reference to the act of 1866 results in the establishment of the following:

Equations.

$$1 \text{ yard} = \frac{3600}{3937} \text{ metre.}$$

$$1 \text{ pound avoirdupois} = \frac{1}{2{\cdot}2046} \text{ kg.}$$

A more precise value of the English pound avoirdupois is $\frac{1}{2{\cdot}20462}$ kg., differing from the above by about one part in one hundred thousand, but the equation established by law is sufficiently accurate for all ordinary conversions.

As already stated, in work of high precision the kilogramme is now all but universally used and no conversion is required.

1866. Indeed, this course has been practically forced upon this Office for several years, but it is considered desirable to make this formal announcement for the information of all interested in the science of metrology or in measurements of precision.

T. C. MENDENHALL,
Superintendent of Standard Weights and Measures.

Approved:
J. G. CARLISLE,
Secretary of the Treasury.
APRIL 5, 1893.

[United States Coast and Geodetic Survey.—Office of Standard Weights and Measures—T. C. Mendenhall, Superintendent.]

TABLES FOR CONVERTING CUSTOMARY AND METRIC WEIGHTS AND MEASURES.

OFFICE OF STANDARD WEIGHTS AND MEASURES,
Washington, D. C., March 21, 1894.

The yard in use in the United States is equal to $\frac{3600}{3937}$ of the metre.

The troy pound of the mint is the United States standard weight for coinage. It is of brass of unknown density, and therefore not suitable for a standard of mass. It was derived from the British standard troy pound of 1758 by direct comparison. The British avoirdupois pound was also derived from the latter and contains 7,000 grains troy. The grain troy is therefore the same as the grain avoirdupois, and the pound avoirdupois in use in the United States is equal to the British pound.

2·20462234 pounds avoirdupois = 1 kilogramme.

In Great Britain the legal metric equivalent of the imperial gallon is 4·54346 litres, and of the imperial bushel 36·3477 litres.

The length of a nautical mile, as given below, is that adopted by the United States Coast Survey many years ago, and defined as the length of a minute of arc of a great circle of a sphere whose surface is equal to the surface of the earth (the Clarke spheroid of 1866).

1 foot	=	0·304801 metre, 9·4840158 log.
1 fathom	=	1·829 metres.
1 Gunter's chain	=	20·1168 metres.
1 square statute mile	=	259·000 hectares.
1 nautical mile	=	1853·25 metres.
1 avoirdupois pound	=	453·5924277 grammes.
15432·35639 grains	=	1 kilogramme.

Tables for converting United States weights and measures.

[Customary to metric.]

LINEAR.

Inches.	Millimetres.	Feet.	Metres.	Yards.	Metres.	Miles.	Kilometres.
1	25·4001	1	0·304801	1	0·914402	1	1·60935
2	50·8001	2	0·609601	2	1·828804	2	3·21869
3	76·2002	3	0·914402	3	2·743205	3	4·82804
4	101·6002	4	1·219202	4	3·657607	4	6·43739
5	127·0003	5	1·524003	5	4·572009	5	8·04674
6	152·4003	6	1·828804	6	5·486411	6	9·65608
7	177·8004	7	2·133604	7	6·400813	7	11·26543
8	203·2004	8	2·438405	8	7·315215	8	12·87478
9	228·6005	9	2·743205	9	8·229616	9	14·48412

SQUARE.

Square inches.	Square centimetres.	Square feet.	Square decimetres.	Square yards.	Square metres.	Acres.	Hectares.
1	6·452	1	9·290	1	0·836	1	0·4047
2	12·903	2	18·581	2	1·672	2	0·8094
3	19·355	3	27·871	3	2·508	3	1·2141
4	25·807	4	37·161	4	3·344	4	1·6187
5	32·258	5	46·452	5	4·181	5	2·0234
6	38·710	6	55·742	6	5·017	6	2·4281
7	45·161	7	65·032	7	5·853	7	2·8328
8	51·613	8	74·323	8	6·689	8	3·2375
9	58·065	9	83·613	9	7·525	9	3·6422

CUBIC.

Cubic inches.	Cubic centimetres.	Cubic feet.	Cubic metres.	Cubic yards.	Cubic metres.	Bushels.	Hectolitres.
1	16·387	1	0·02832	1	0·765	1	0·35239
2	32·774	2	0·05663	2	1·529	2	0·70479
3	49·161	3	0·08495	3	2·294	3	1·05718
4	65·549	4	0·11327	4	3·058	4	1·40957
5	81·936	5	0·14158	5	3·823	5	1·76196
6	98·323	6	0·16990	6	4·587	6	2·11436
7	114.710	7	0·19822	7	5·352	7	2·46675
8	131·097	8	0·22654	8	6·116	8	2·81914
9	147·484	9	0·25485	9	6·881	9	3·17154

CAPACITY.

Fluid drams.	Millilitres or cubic centimetres.	Fluid ounces.	Millilitres.	Quarts.	Litres.	Gallons.	Litres.
1	3·70	1	29·57	1	0·94636	1	3·78543
2	7·39	2	59·15	2	1·89272	2	7·57087
3	11·09	3	88·72	3	2·83908	3	11·35630
4	14·79	4	118·29	4	3·78543	4	15·14174
5	18·48	5	147·87	5	4·73179	5	18·92717
6	22·18	6	177·44	6	5·67815	6	22·71261
7	25·88	7	207·02	7	6·62451	7	26·49804
8	29·57	8	236·59	8	7·57087	8	30·28348
9	33·27	9	266·16	9	8·51723	9	34·06891

Tables for converting United States weights and measures—Continued.

[Customary to metric.]

WEIGHT.

Grains.	Milli-grammes.	Avoir-dupois ounces.	Grammes.	Avoir-dupois pounds.	Kilo-grammes.	Troy ounces.	Grammes.
1	64·7989	1	28·3495	1	0·45359	1	31·10348
2	129·5978	2	56·6991	2	0·90719	2	62·20696
3	194·3968	3	85·0486	3	1·36078	3	93·31044
4	259·1957	4	113·3981	4	1·81437	4	124·41392
5	323·9946	5	141·7476	5	2·26796	5	155·51740
6	388·7935	6	170·0972	6	2·72156	6	186·62088
7	453·5924	7	198·4467	7	3·17515	7	217·72437
8	518·3914	8	226·7962	8	3·62874	8	248·82785
9	583·1903	9	255·1457	9	4·08233	9	279·93133

By the concurrent action of the principal Governments of the world, an International Bureau of Weights and Measures has been established near Paris. Under the direction of the International Committee, two ingots were cast of pure platinum-iridium in the proportion of 9 parts of the former to 1 of the latter metal. From one of these a certain number of kilogrammes were prepared; from the other a definite number of metre bars. These standards of weight and length were intercompared without preference, and certain ones were selected as international prototype standards. The others were distributed by lot, in September, 1889, to the different Governments, and are called national prototype standards. Those apportioned to the United States were received in 1890 and are in the keeping of this office.

The metric system was legalized in the United States in 1866.

The International Standard Metre is derived from the Metre des Archives, and its length is defined by the distance between two lines at 0° centigrade on a platinum-iridium bar deposited at the International Bureau of Weights and Measures.

The International Standard Kilogramme is a mass of platinum-iridium deposited at the same place, and its weight in vacuo is the same as that of the Kilogramme des Archives.

The litre is equal to a cubic decimetre, and it is measured by the quantity of distilled water which, at its maximum density, will counterpoise the standard kilogramme in a vacuum, the volume of such a quantity of water being, as nearly as has been ascertained, equal to a cubic decimetre.

Tables for converting United States weights and measures.

[Metric to customary.]

LINEAR.

Metres.	Inches.	Metres.	Feet.	Metres.	Yards.	Kilometres.	Miles.
1	39·3700	1	3·28083	1	1·093611	1	0·62137
2	78·7400	2	6·56167	2	2·187222	2	1·24274
3	118·1100	3	9·84250	3	3·280833	3	1·86411
4	157·4800	4	13·12333	4	4·374444	4	2·48548
5	196·8500	5	16·40417	5	5·468056	5	3·10685
6	236·2200	6	19·68500	6	6·561667	6	3·72822
7	275·5900	7	22·96583	7	7·655278	7	4·34959
8	314·9600	8	26·24667	8	8·748889	8	4·97096
9	354·3300	9	29·52750	9	9·842500	9	5·59233

SQUARE.

Square centimetres.	Square inches.	Square metres.	Square feet.	Square metres.	Square yards.	Hectares.	Acres.
1	0·1550	1	10·764	1	1·196	1	2·471
2	0·3100	2	21·528	2	2·392	2	4·942
3	0·4650	3	32·292	3	3·588	3	7·413
4	0·6200	4	43·055	4	4·784	4	9·884
5	0·7750	5	53·819	5	5·980	5	12·355
6	0·9300	6	64·583	6	7·176	6	14·826
7	1·0850	7	75·347	7	8·372	7	17·297
8	1·2400	8	86·111	8	9·568	8	19·768
9	1·3950	9	96·875	9	10·764	9	22·239

CUBIC.

Cubic centimetres.	Cubic inches.	Cubic decimetres.	Cubic inches.	Cubic metres.	Cubic feet.	Cubic metres.	Cubic yards.
1	0·0610	1	61·023	1	35·314	1	1·308
2	0·1220	2	122·047	2	70·629	2	2·616
3	0·1831	3	183·070	3	105·943	3	3·924
4	0·2441	4	244·094	4	141·258	4	5·232
5	0·3051	5	305·117	5	176·572	5	6·540
6	0·3661	6	366·140	6	211·887	6	7·848
7	0·4272	7	427·164	7	247·201	7	9·156
8	0·4882	8	488·187	8	282·516	8	10·464
9	0·5492	9	549·210	9	317·830	9	11·771

Tables for converting United States weights and measures—Continued.

[Metric to customary.]

CAPACITY.

Millilitres or cubic centimetres.	Fluid drams.	Centilitres.	Fluid ounces.	Litres.	Quarts.	Decalitres.	Gallons.	Hectolitres.	Bushels.
1	0·27	1	0·338	1	1·0567	1	2·6417	1	2·8377
2	0·54	2	0·676	2	2·1134	2	5·2834	2	5·6755
3	0·81	3	1·014	3	3·1700	3	7·9251	3	8·5132
4	1·08	4	1·353	4	4·2267	4	10·5668	4	11·3510
5	1·35	5	1·691	5	5·2834	5	13·2085	5	14·1887
6	1·62	6	2·029	6	6·3401	6	15·8502	6	17·0265
7	1·89	7	2·367	7	7·3968	7	18·4919	7	19·8642
8	2·16	8	2·705	8	8·4535	8	21·1336	8	22·7019
9	2·43	9	3·043	9	9·5101	9	23·7753	9	25·5397

WEIGHT.

Milligrammes.	Grains.	Kilogrammes.	Grains.	Hectogrammes.	Ounces avoirdupois.	Kilogrammes.	Pounds avoirdupois.
1	0·01543	1	15432·36	1	3·5274	1	2·20462
2	0·03086	2	30864·71	2	7·0548	2	4·40924
3	0·04630	3	46297·07	3	10·5822	3	6·61387
4	0·06173	4	61729·43	4	14·1096	4	8·81849
5	0·07716	5	77161·78	5	17·6370	5	11·02311
6	0·09259	6	92594·14	6	21·1644	6	13·22773
7	0·10803	7	108026·49	7	24·6918	7	15·43236
8	0·12346	8	123458·85	8	28·2192	8	17·63698
9	0·13889	9	138891·21	9	31·7466	9	19·84160

Quintals.	Pounds avoirdupois.	Milliers or tonnes.	Pounds avoirdupois.	Kilogrammes.	Ounces troy.
1	220·46	1	2204·6	1	32·1507
2	440·92	2	4409·2	2	64·3015
3	661·39	3	6613·9	3	96·4522
4	881·85	4	8818·5	4	128·6030
5	1102·31	5	11023·1	5	160·7537
6	1322·77	6	13227·7	6	192·9044
7	1543·24	7	15432·4	7	225·0552
8	1763·70	8	17637·0	8	257·2059
9	1984·16	9	19841·6	9	289·3567

APPENDIX No. 7—1893.

UNITS OF ELECTRICAL MEASURE.

Within but little more than a decade practical applications of electricity have developed with a rapidity unparalleled in the history of modern industries. Many millions of dollars of capital are now invested in the manufacture of machinery and various devices for the production and consumption of electricity. As it has now become a commodity of trade, its measurement is a question of the highest importance both to the producer and consumer. Both the nomenclature of electrotechnics and the methods and instruments of measure are exceptionally precise and satisfactory, but there has been lacking, up to the present time, the very important and essential element of fixed and invariable units of measure authoritatively adopted. Such units have long been in use among scientific men, but the necessity for the establishment and legalization of practical units for commercial purposes became evident in the beginning of the recent enormous development of the applications of electricity.

To meet this universally recognized want, conferences and congresses of the leading electricians of the world have been held at occasional intervals, the first being the Paris Congress of 1881. These assemblages have been international in their character, for it was wisely determined in the beginning that the new units of measure should be international and, indeed, universal in their application. It was convenient to make them so, and it was important to thus facilitate international interchange of machinery, instruments, etc. The United States was represented by official delegates in the Congress of 1881, and also in subsequent Congresses in 1884.

The difficulty of the material representation of some of the units of measure was so great at the time of holding these Congresses that no satisfactory agreement as to all of them could be arrived at. Some recommendations were made, but they at no time received the unanimous support of those interested, and were admitted by all to be tentative in their character. During the past few years the advance of knowledge and experience among electricians was such as to indicate that the time was ripe for the general adoption of the principal units of electrical measure. An International Congress of Electricians was arranged for, to meet in Chicago during the World's Columbian Exposition of 1893. In this congress the business of defining and naming units of measure was left to what was known as the "Chamber of Delegates," a body composed of those only who had been officially commissioned by their respective Governments to act as members of said chamber. The United States, Great Britain, Germany, and France were each allowed five delegates in the chamber. Other nations were represented by three, two, and in some cases one. The principal nations of the world were represented by their leading electricians, and the chamber embraced many of the most distinguished living representatives of physical science.

The delegates representing the United States have reported to the Honorable the Secretary of State, under date of November 6, 1893, giving the names and definitions of the units of electrical measure as unanimously recommended by the chamber in a resolution as follows:

Resolved, That the several Governments represented by the delegates of this International Congress of Electricians be, and they are hereby, recommended to formally adopt as legal units of electrical measure the following:

As a unit of resistance, the *international ohm*, which is based upon the ohm equal to 10^9 units of resistance of the centimetre-gramme-second system of electro-magnetic units, and is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice 14·4521 grammes in mass, of a constant cross-sectional area and of the lenght of $106{\cdot}3^{cm}$.

As a unit of current, the *international ampère*, which is one-tenth of the unit of current of the C. G. S. system of electro-magnetic units, and which is represented sufficiently well for practical use by the unvarying current which, when passed through a solution of nitrate of silver in water, and in accordance with accompanying specifications,* deposits silver at the rate of 0·001118 of a gramme per second.

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* In the following specification, the term silver voltameter means the arrangement of apparatus by means of which an electric current is passed through a solution of nitrate of silver in water. The silver voltameter measures the total electrical quantity which has passed during the time of the experiment, and by noting this time the time average of the current, or if the current has been kept constant the current itself, can be deduced.

In employing the silver voltameter to measure currents of about one ampère, the following arrangements should be adopted:

The kathode on which the silver is to be deposited should take the form of a platinum bowl not less than 10^{cm} in diameter and from 4^{cm} to 5^{cm} in depth.

The anode should be a plate of pure silver some 30 square centimetres in area and 2^{mm} or 3^{mm} in thickness.

This is supported horizontally in the liquid near the top of the solution by a platinum wire passed through holes in the plate at opposite corners. To prevent the disintegrated silver which is formed on the anode from falling onto the kathode the anode should be wrapped round with pure filter paper, secured at the back with sealing wax.

The liquid should consist of a neutral solution of pure silver nitrate, containing about 15 parts by weight of the nitrate to 85 parts of water.

The resistance of the voltameter changes somewhat as the current passes. To prevent these changes having too great an effect on the current some resistance besides that of the voltameter should be inserted in the circuit. The total metallic resistance of the circuit should not be less than 10 ohms.

As a unit of electro-motive force, the *international volt*, which is the electro-motive force that, steadily applied to a conductor whose resistance is one international ohm, will produce a current of one international ampère, and which is represented sufficiently well for practical use by $\frac{1000}{1434}$ of the electro-motive force between the poles or electrodes of the voltaic cell known as Clark's cell, at a temperature of 15° C., and prepared in the manner described in the accompanying specification.*

As a unit of quantity, the *international coulomb*, which is the quantity of electricity transferred by a current of one international ampère in one second.

As a unit of capacity, the *international farad*, which is the capacity of a condenser charged to a potential of one international volt by one international coulomb of electricity.

As a unit of work, the *joule*, which is equal to 10^7 units of work in the C. G. S. system, and which is represented sufficiently well for practical use by the energy expended in one second by an international ampère in an international ohm.

As a unit of power, the *watt*, which is equal to 10^7 units of power in the C. G. S. system, and which is represented sufficiently well for practical use by the work done at the rate of one joule per second.

As the unit of induction, the *henry*, which is the induction in a circuit when the electro-motive force induced in this circuit is one international volt, while the inducing current varies at the rate of one ampère per second.

Besides the fact that the Congress in which this important and far-reaching action was taken was held in the United States, our country has been honored by the action of the Chamber of Delegates in placing in the list of the illustrious names which are to be perpetuated in the nomenclature of electricity that of our countryman, Joseph Henry, whose splendid contributions to science, made about sixty years ago, have only in recent years met with full recognition. For these and other reasons it is extremely desirable that our Government should be among the first, if not the first, to adopt the recommendations of the Chamber. To make the use of these units obligatory in all parts of the country will require an act of Congress, but in the absence of that it is within the power of the Secretary of the Treasury to approve their adoption for use in all Departments of the Government. This, indeed, is precisely the course long ago followed in reference to the ordinary weights and measures of commerce and trade. Congress has never enacted a law fixing the value of their units, but the Secretary of the Treasury was authorized to establish and construct standards for use in the various Departments of the Government. Uniformity has followed on account of the universal adoption of these standards by the several States.

The Government is itself a large consumer of electricity and electrical machinery, and for its own protection it is important that units of measure be adopted. With the approval, therefore, of the Honorable the Secretary of the Treasury, the formal adoption by the Office of Standard Weights and Measures of the names and values of units of

* A committee, consisting of Messrs. Helmholtz, Ayrton, and Carhart, was appointed to prepare specifications for the Clark cell. Their report has not yet been received.

electrical measure as given above, the same being in accord with the recommendations of the International Congress of Electricians of 1893, is hereby announced.

T. C. MENDENHALL,
Superintendent U. S. Coast and Geodetic Survey and of Standard Weights and Measures.

Approved:

J. G. CARLISLE,
Secretary of the Treasury.

APPENDIX No. 8—1893.

[IN TWO PARTS.]

PART I.—A HISTORICAL ACCOUNT OF THE BOUNDARY LINE BETWEEN THE STATES OF PENNSYLVANIA AND DELAWARE.

By W. C. HODGKINS, Assistant.

Submitted for publication December 1, 1893.

The history of the boundary line between the States of Delaware and Pennsylvania possesses a peculiar interest to the antiquarian, the historian, and the engineer; and the consideration of its origin carried to its ultimate sources leads us far back into the colonial history of our country, to a period antedating not only this particular boundary line but even the existence and very name of the great State of Pennsylvania itself, of which province, under the proprietary government, Delaware for nearly a century formed a part.

A brief outline of the more important historical events which have left their impress in one way or another upon the long-continued controversy over the boundaries of Delaware, a controversy which has extended over more than two centuries and of which the last sounds have hardly yet been heard, may not be uninteresting nor out of place in this connection as serving to explain the somewhat intricate and frequently obscure causes which have resulted in the conditions under which this circular boundary has at last been marked by permanent monuments.

The earliest settlement by Europeans within the limits of the present State of Delaware appears to have been made by a Hollander named De Vries and a party of his countrymen, who landed in 1631 upon a tract of land near Cape Henlopen which had been purchased from the Indians two years before by another Hollander named Godyn.

De Vries called his settlement Swaanendael, and the creek on which it was situated he called the Hoornkill, after the city of Hoorn, in Holland. Swaanendael was not far from the present town of Lewes, in Sussex County, and the Hoornkill was, no doubt, the present Lewes Creek.

The name Hoornkill, it may be remarked, seems to have become more widely known than that of Swaanendael, and subsequently gave a name to the county under various corrupted forms, as Hoarkill, Horekill, or Whorekills (afterwards called New Dale, and then Sussex).

After establishing his colony De Vries returned to Holland to secure more settlers. Upon his return the following year he was horrified to find that the Indians had attacked the settlers, about thirty in number, had put them all to death, and had destroyed the village.

This tragic event, according to the best information attainable, seems to have been due to the killing by a white man of an Indian who had torn down a piece of metal ornamented with the Dutch arms from a post on which it had been placed.

There are various versions of this story, none of which are perhaps quite reliable, as they depend on the accounts of the Indians handed down by the Dutch.

It appears, however, that De Vries thought the killing not without some good reason, for he made no attempt to punish the Indians, though he parleyed with them and induced one of them to visit his ship and to give an account of the massacre.

This unfortunate affair, however, discouraged him and his party, and after a brief trip up the river they sailed to Virginia and then to New Amsterdam (New York), preferring to relinquish their dreams of wealth from a new colony on the Delaware rather than face the probability of encountering such savage foes; and for many years the Dutch made no further attempt to settle on the Delaware shore.

It might, therefore, appear that the incident thus abruptly closed could have no bearing upon the course of future settlements nor enter as a factor into the question of a boundary line between colonies subsequently established by the English.

As will be seen later, however, it proved to be an event of much moment in the conflict between the opposing claims of the proprietary governments of Pennsylvania and Maryland.

In the year 1632 King Charles I of England granted by royal letters patent to Cæcilius Calvert, second Baron Baltimore, a great tract of country on the Atlantic coast between the parallels of 38° and 40° of north latitude. This grant, which the King named Terra Mariæ, or Maryland, in honor of the Queen, Henrietta Maria, embraced not only the present area of Maryland, but the whole of Delaware and a considerable portion of the present State of Pennsylvania.

The whole of this grant, however, was included within the tract already granted by King James I for the Colony of Virginia under three charters, of 1606, 1609, and 1612, respectively. In the first charter the boundaries of Virginia are thus described:

* * * Situate, lying, or being all along the sea coasts, between four and thirty degrees of northerly latitude from the equinoctial line, and five and forty degrees of the same latitude, and in the main land between the same four and thirty and five and forty degrees and the islands thereunto adjacent, or within one hundred miles of the coast thereof. * * *

But the territory thus granted to the London and Bristol companies was materially reduced by King James himself in 1620, when he granted a charter to the Plymouth Company for all the territory between the fortieth and forty-eighth parallels, under the name of New England. In this charter we read:

We therefore * * * do grant, ordain, and establish that all that Circuit, Continent, Precincts and Limitts in America lying and being in Breadth from Fourty Degrees of Northerly Latitude from the Equinoctial Line, to Fourty eight Degrees of the said Northerly Latitude and in length by all the Breadth aforesaid throughout the Maine Land from Sea to Sea—with all the Seas, Rivers, Islands, Creekes, Inletts, Ports and Havens within the Degrees, Precincts and Limitts of the said Latitude and Longitude shall be the Limitts, and Bounds, and Precincts of the second collony—and to the end that the said Territoryes may hereafter be more particularly and certainly known and distinguished, our Will and Pleasure is, that the same shall from henceforth be nominated, termed and called by the name of New England in America.

This brought the charter limits of Virginia down to the fortieth parallel. But there was much dissatisfaction with the management of the Virginia colony by the chartered company, and in 1624 a writ of quo warranto was issued against it and the charter was forfeited.

So Virginia became a Crown Colony, and its lands were subject to the royal authority.

Sir George Calvert, the first Lord Baltimore, had received a grant of land in Newfoundland called Avalon, but finding the climate unfavorable he visited Virginia. Finding that the part of Virginia north and east of the Potomac and Chesapeake was still unsettled, he returned to England and induced the King to grant him this territory in place of the undesirable Avalon.

Before the charter was issued the first Lord Baltimore died, but the charter was confirmed and issued to his son, as already noted.

In this charter the boundaries of Maryland are, in part, described as follows:

* * * All that part of the Peninsula or Chersonese, lying in parts of America, between the ocean on the east and the Bay of Chesapeake on the west; divided from the residue thereof by a right line drawn from the promontory or headland called Watkins Point, situate upon the bay aforesaid, near the river Wighco on the west unto the main ocean on the east, and between that boundary on the south unto that part of the Bay of Delaware on the north, which lieth under the fortieth degree of north latitude from the equinoctial, where New England is terminated; and all the tract of that land within the metes underwritten (that is to say), passing from the said bay, called Delaware Bay, in a right line, by the degree aforesaid, unto the true meridian of the first fountain of the River Pattowmack; * * *

In all of these successive grants we notice a gradual reduction of the areas originally so lavishly distributed by English royalty.

By the Plymouth charter of 1620 Virginia lost a vast territory, which included the present New England States, New York, New Jersey, and the greater part of Pennsylvania, not to mention Canada and the great west. Next, by the charter of 1632, Cæcilius Calvert obtained another considerable portion of the imperial domain once included within the confines of Virginia.

So, when we find, as we shall a little later, that the barons of Baltimore themselves suffered a half century afterwards from this same trimming process, applied for the benefit of newer colonists on the Delaware, it will seem that perhaps they had little reason to complain on grounds of equity, however seriously the subsequent grants to James, Duke of York, and to William Penn may have infringed upon the letter of the Maryland charter, though legally the King might divide Virginia, as being a royal colony at the time.

The Calverts do not seem to have appreciated, until it was too late, the possible importance of the Delaware coast of the peninsula, and made no attempt to plant settlements there. Had they done so, it is not at all likely that there would be any State of Delaware to-day. They preferred to plant their new colony on the shores of Chesapeake Bay.

But the English were not alone in their attempts at colonization on this coast. As we have already seen, the Dutch had made an abortive attempt at colonizing the Lower Delaware and were firmly seated on the Hudson or North River. They had also a trading post called Fort Nassau on the eastern bank of the Delaware, near the present site of Gloucester, N. J.

The Swedish nation, then in the height of its power under the great King Gustavus Adolphus, also felt the influence of the fever for colonial aggrandizement which in the first half of the seventeenth century seems to have swept over northern Europe. Considerable preparations were made in 1627 for a Swedish colony on the Delaware, but for some reason the project was not then carried into execution.

A few years later, however, in the reign of the infant queen, Christina, the plan was again taken up by the celebrated Chancellor Oxenstiern, and in the year 1638 a Swedish expedition sailed into Delaware Bay. The commander was Peter Minuit (or Menewe), a Hollander, who had previously served the Dutch West India Company in America. The Swedes at once purchased from the Indians the whole west bank of the Delaware from Cape Henlopen to Trenton Falls. A part of their territory was included within the former purchase by Godyn, so swiftly abrogated by the massacre at Swaanendael. The whole of it was also included in the territory claimed by the English by right of discovery and disposed of by the grants to the London and Plymouth colonies and to Lord Baltimore. The English, however, had tolerated the Dutch settlements within the charter limits of New England, and the Swedes claimed that King Charles I had renounced in their favor, in 1634, any claims that England might have to that country by right of discovery. Even if this be so, it can hardly be supposed that King Charles meant to include in any such renunciation the territory of Maryland granted only two years before to his faithful servant, Lord Baltimore. This claim on the part of the Swedes affords another proof that neither they nor the English then recognized any right of the Dutch to the western bank of Delaware River and Bay.

The newcomers selected a site about where Wilmington now stands and built themselves a fort, which they named Christina, after the little Queen of Sweden. They also named the little river before them the Christina Kihl, a name still retained in the slightly altered Christiana Creek.

The Dutch governor of New Amsterdam, who claimed the country between the Connecticut and the Delaware, immediately protested against the Swedish settlement as an interference with the rights of the Dutch West India Company. The Dutch were too weak, however, to offer forcible resistance, and for nearly twenty years the Swedish settlements grew and prospered. But as their numbers and trade increased the friction between the Swedes on the western and the Dutch on the eastern bank of the Delaware became constantly greater, each colony trying to monopolize the navigation of the river. In retaliation for the Swedish pretensions, the Dutch, in 1651, repurchased from some of the Indians part of the Swedish territory below Wilmington and built a post, which they called Fort Casimir, where Newcastle now stands. The Swedes, in their turn, crossed the Jersey shore, then part of New Netherlands, and built Fort Elfsborg, about 7 miles below Fort Casimir. The Swedes were driven out of this place, however, by the swarms of mosquitoes which made life almost unendurable.

In 1654 a new Swedish governor, John Claudius Rising, arrived in the Delaware with a considerable number of colonists. One of his first acts was to compel the surrender of the Dutch at Fort Casimir, which he renamed the Fort of the Holy Trinity, on account of its having been captured on Trinity Sunday. The Dutch made no immediate reprisals, but having made thorough preparations they appeared in the Delaware at the end of August, 1655, under the redoubtable Governor Peter Stuyvesant.

Their force of seven vessels and more than 600 armed men was more than a match for the Swedes, and landing between the Fort of the Holy Trinity and Fort Christina they blocked communication between the two posts and reduced them in succession without any bloodshed. Thus the Swedish claims, such as they were, passed by conquest to the Dutch, who also claimed that the old purchase of the Hoornkill tract gave them title to the whole west bank, though that purchase was for only 30 miles of coast at Cape Henlopen.

But this conquest of the Swedes and the subsequent increase of the Dutch power on the Delaware alarmed the Lord Proprietor of Maryland, who seems to have paid little heed to the Swedish settlers, probably deeming them easy to subject to his dominion in due course of time.

In 1659 Lord Baltimore sent instructions to Maryland calling for an inquiry into the proceedings of the Dutch. A deputation was accordingly sent from Maryland to New Amstel, as Newcastle was then called, to notify the Dutch that they were unlawfully seated within the province of his lordship. The Dutch officials paid little heed to the complaints of Lord Baltimore's envoys, unbacked as they were by military force, though the spokesman of the party, Col. Nathaniel Utie, is said to have delivered his message "in a pretty harsh and bitter manner;" and the embassy came to nothing. Still the Dutch seem to have been somewhat alarmed at Utie's proceedings, and sent messengers to Governor Stuyvesant to inform him of the demands of Lord Baltimore. Stuyvesant thereupon sent Augustine Hermen and Rosevelt Waldron to the authorities of Maryland to try to arrange the matter. These Dutch ambassadors, upon being shown the charter of 1632, immediately caught at the phrase "hactenus inculta" in the preamble thereof and at once claimed that the charter specified that the lands granted to Lord Baltimore were only such as were then uncultivated and inhabited only by certain tribes of savage Indians, and that the Dutch settlements on the Delaware antedated the charter.

Here for the first time attention was called to this weak point of Lord Baltimore's charter and the argument advanced which was later used with such persistence to tear the Delaware shore from Maryland. This matter will be further discussed in connection with the grants to William Penn.

The negotiations having come to nothing, Lord Baltimore complained to the Dutch West India Company, in Europe, of the invasion of his dominions by the servants of the company. But this wealthy and powerful society, feeling secure in the armed forces with which it occupied its settlements, took no notice of such appeals, and Baltimore, perhaps feeling that the logic of events at least was against him, seems to have made little effort, except occasional futile remonstrances, to clear his territory of the intruders. Matters thus remained for several years, neither side acknowledging the justice of the other's claims.

But in 1664, as if in commentary on the theory that England recognized a trifling Dutch settlement as a bar to an English grant, King Charles II granted to his brother, the Duke of York, all the territory between the Connecticut and Delaware rivers, although this region had been in the hands of the Dutch for more than fifty years and although the two nations were then at peace.

The Duke at once organized an expedition, which was entirely successful in its results, and New Netherlands became an English province under the name of New York.

The Duke of York's grant was only to the east bank of the Delaware, and in that same year, 1664, he granted to Lord John Berkley and Sir George Carteret the whole of New Jersey, so that his proper territory did not extend south of New York.

But by virtue of the conquest of the Dutch provinces generally the agents of the Duke appear to have exercised a sort of quasi jurisdiction over the Dutch settlements west of the Delaware.

It does not appear that Lord Baltimore protested against this state of affairs; and it has been alleged that he forfeited his rights by such omission.

It is more likely, however, that he was governed by motives of prudence and policy. He was no longer a favorite at court, and he may easily have surmised that he would fare ill, no matter how just his cause, in a controversy with a royal duke, the heir presumptive to the throne. At all events, he seems to have held amicable intercourse for several years with his new neighbors on the Delaware.

But in 1673 the Dutch reconquered the province, and during the brief period of their rule, which lasted less than a year and a half, the Maryland authorities seized the opportunity to assert their territorial rights, and for that purpose sent an armed force against the settlement at Hoornkill, which had been reestablished by the Dutch.

In spite of this more formidable expedition no good results seem to have come to Lord Baltimore.

In 1674 the New Netherlands were again surrendered to the English by the treaty of Westminster. The Duke of York, to perfect his title, obtained a new grant from the King for his former territories, and the western shore of the Delaware seems to have been considered his property by all but the Marylanders. And at this point we approach the origin of the boundary which is the subject of this sketch, a line of demarcation which was first formulated in the latter part of the seventeenth century, and which, after two hundred years of uncertainty and misconception, has at last been marked by enduring monuments in this last decade of the nineteenth century. On March 4, 1681, King Charles II granted to William Penn a tract of land to the westward of the Delaware and to the northward of Maryland. This grant was in partial payment of claims against the Crown which Penn had inherited from his father, Admiral Penn. The negotiations preliminary to the issue of this charter were very protracted, extending over several months, for the English Government had begun to realize the difficulties which might arise from the large and rather indefinite grants which had been so common. Besides, it was known that the new province for which Penn asked a charter was likely to interfere to some extent with the territorial rights of the Duke of York and of Lord Baltimore.

And yet, curiously enough, in view of all this care, the conflict over the boundaries of Penn's territory was more bitter and more protracted than any other similar trouble in the English colonies.

So it was ordered that the Duke and the Lord Proprietor should be consulted.

Lord Baltimore had no objections to a grant of land to Penn so long as his northern boundary of the fortieth parallel was respected, and the Duke of York expressed his willingness to yield his claims to the almost unsettled shore of the upper Delaware provided he should have reserved to himself a sufficient distance between his town of Newcastle and the boundary of the new province. He suggested that 20 miles would be a convenient and suitable distance.

Penn, however, was reluctant to be pushed so far up the Delaware, and upon his urgent representations to the Duke a distance of 12 miles was finally agreed upon. At that time it was thought that the fortieth parallel crossed the Delaware between Newcastle and Chester (then called Upland), and it was therefore decided that the southern boundary of Pennsylvania should follow a circular curve, at 12 miles distance from Newcastle, northward and westward from the river Delaware, until it came to the fortieth parallel, and that it should then follow that parallel westward to its limit of longitude. This description, which was soon found to be an impossible one, is thus expressed in the charter:

* * * and the said Lands to bee bounded on the * * * South by a Circle drawne at twelve miles distance from New Castle Northward and Westward unto the beginning of the fortieth degree of Northern Latitude, and thence by a straight line Westward to the Limit of Longitude above mentioned.

As a matter of fact, the most northern part of a circle of 12 miles radius from Newcastle court-house is almost exactly on the parallel of 39° 50′ north latitude, and it could, therefore, never intersect the parallel of 40°.

We here find the first mention of this singular boundary line, almost unique in its circular shape. It is probable that both Penn and the Duke of York thought that this circular boundary between their dominions would soon strike the fortieth parallel and hence would be of small extent, and it is hardly likely that either of them then thought that it would afterwards play so important a part in the location of the boundary between Maryland and Pennsylvania.

After granting the charter to Penn, the King commanded him and Lord Baltimore to arrange their boundary.

Accordingly Lord Baltimore met Penn's kinsman and deputy, Markham, at Upland, in September, 1681, when it was found by a precise observation that the fortieth parallel was several miles north of Upland, instead of being somewhat to the south of it, as formerly supposed. No doubt both parties were somewhat surprised, but Lord Baltimore at once claimed the land to his charter limit of forty degrees, wherever it might lie.

Markham, on the other hand, declined to proceed with the delimitation of the provinces and reported the disappointing state of affairs to Penn, who was still in England. This news made Penn, who had all along been dissatisfied with his province as being too difficult of access, still more anxious to secure control of the shore of the lower Delaware. He therefore importuned the Duke of York for the transfer to himself of the Duke's claims on that region. This land had never been granted to the Duke, and his possession was only a sort of "squatter sovereignty."

As a historical writer has recently expressed it, "Penn asked for that which he knew to be within the boundaries of Maryland, and beyond the power of the Duke to grant." Penn had a great influence with both the Duke of York and the King on account of the services of his father, Admiral Penn, under the Duke himself, in the naval wars with the Dutch. Lord Baltimore therefore had heavy odds against him.

Probably with the idea of strengthening his own claims by bolstering the Duke's shadowy title, Penn obtained from York a quitclaim deed, dated August 21, 1682, relinquishing to Penn any claim which the Duke might have to the province of Pennsylvania.

It is worthy of note that Penn had been contented with his title under the charter of March 4, 1681, until, on the eve of bargaining with the Duke for part of Lord Baltimore's territory, he suddenly perceived that his own title was defective through the Duke's claims. Three days later, August 24, 1682, the Duke gave Penn two deeds of feoffment for the Delaware shore. The first of these conveyed the town of Newcastle and a 12-mile circle around the same. The second conveyed all the lands south of that circle as far as Cape Henlopen.

Much doubt seems to have existed, however, as to Penn's legal rights under these deeds. The Duke had no title of record. His deeds to Penn were never confirmed by King Charles, who died soon afterwards, nor by King James himself during his short and troubled reign. Much difficulty was therefore experienced by Penn's agents in the collection of rents.

After arranging these matters Penn sailed from England to visit his province. He arrived at the Capes on October 24, 1682 (O. S.), and first landed at Newcastle, afterwards going to Upland, now Chester.

For nearly twenty years after the organization of the new government the lawmaking body was a joint assembly for the province of Pennsylvania and the "territories" of "three lower counties on the Delaware." But dissensions arose between the united provinces, and upon the revision by Penn in 1701 of the charter of government granted by him the territories insisted so strongly upon a separate assembly that Penn was obliged to yield to their wishes.

In this same year, 1701, and perhaps in consequence of this legislative division of the provinces, the circular boundary line between Chester and Newcastle counties was run out upon the ground under a warrant from Penn. The work was done by Isaac Tailer, of Chester County, and Thomas Pierson, of Newcastle County, under the direction of the county officials, in November, 1701. Their method of work is described in their field notes, which are in the possession of the Historical Society of Pennsylvania.

According to their record, they began work at "the end of the horse dike" at Newcastle and ran a traverse to the northward with compass and chain until they reached a point which, from their computations, they supposed to be exactly 12 miles north of their starting point. By some mistake, however, they came out a mile or more too far to the west and about 2,000 feet too far from Newcastle. The excess in distance may have been due to their chain being too long, though the size of the error (2 feet to each chain) seems unlikely. This supposition is further borne out by the fact that the curve actually run out by them had, as nearly as can be ascertained, a radius of about 13 miles instead of 12, as should have been the case. The excess of westing might be accounted for by supposing that they used the magnetic meridian as their standard instead of the true north, but the declination of 8° 30′ west, observed at Philadelphia in 1701, would have carried them a half mile or more still farther to the west. It is very likely that their compass needle was a poor one and that it was much affected by local attraction, which is very noticeable in the vicinity of the Brandywine. The extremity of the radius so determined fell upon land then occupied by a certain Israel Helm and now owned by one Goodley, in a peculiar bend formed by Brandywine Creek. Tailer and Pierson found there a white oak tree, which they marked with twelve notches. They next laid off a line at right angles to their supposed true radius and marked on it the distance corresponding to the chord of 1 degree of a circle of 12 miles radius. This distance they computed to be 67 perches, a value sufficiently precise for their purpose (more exactly, 67.018), but if, as seems likely, these measurements also overran, their chords were probably 68 or 69 perches in length.

One-half of this chord was laid off on the east side of their radius and the other half to the west. Then, starting from the eastern end of the first chord, they ran the curve to the Delaware River by successive chords of 67 perches, making a uniform deflection to the right of 1 degree by compass at the end of each chord. Forty-three chords brought them to the Delaware, where they found that their line struck the north side of a house close to the shore, then occupied by one Daniel Lamplugh.

The surveyors then retraced their steps to the farm of Israel Helm and in a precisely similar manner ran their curve to the westward from the first chord until they had completed 77 chords, which, together with the 43 chords east of their starting point, made up the total of 120 chords, or "two-thirds of a semicircle," called for by their instructions. They note that their line ended at a stream, a branch of Christiana Creek, and that they "well marked" a hickory tree. This point can no longer be identified, but it was most likely in the present State of Maryland, to the westward or northwestward of the "triangular stone" on the boundary between Delaware and Maryland. The stream referred to was probably one called Persimmon Creek on some recent maps. The course of the boundary line was indicated by notches cut in trees near which it ran.

It will be noticed that this boundary laid out by William Penn between two portions of his domain has no connection and little apparent relation to the boundary between the lands of Penn and those of Lord Baltimore, though subsequently complicated and confounded with the surveys of the latter line.

At ye Request of ye Inhabitants of ye County of Chester & County of New Castle that I would Grant them a Warrt for ye Running a Dividing Line Between the two sd Counties that the Inhabitants of ye Respec= tive Counties which are in Question may Know to what JurisDiction they Belong

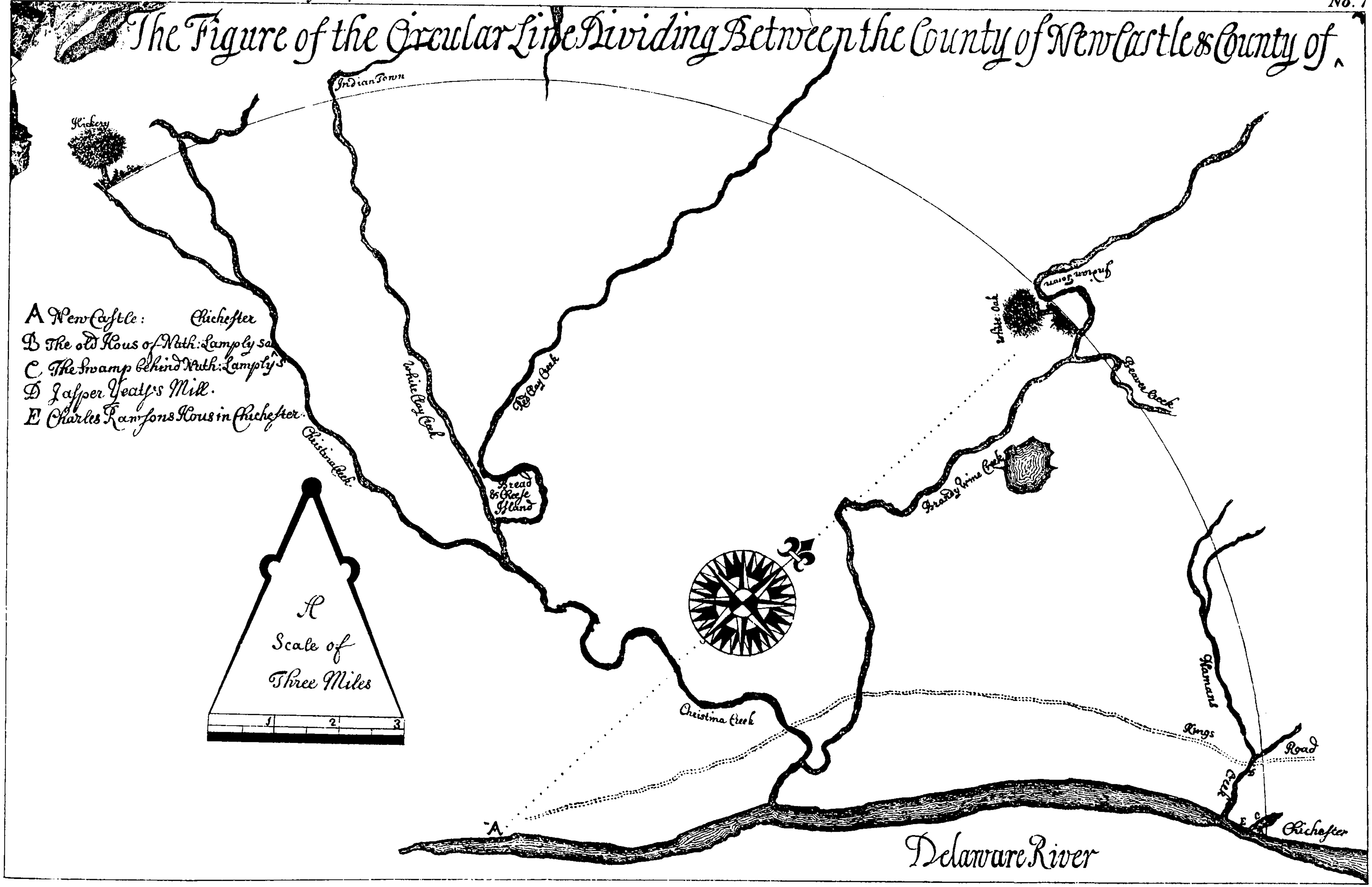
I Hereby Nominate Appoint & Authorize ~~thee~~ Isaack Tailer of ye County of Chester in ye Province of Pennsylvania and thou Thomas Pierson of ye County of New Castle in ye Territories to Accompany the Magistrates of Each Coun= ty or Any three of them within ye space of forty days after ye Date hereof to Admeasure & Survey from ye Town of New Castle the Distance of twelve Miles on a Right Line by ye River Dellaware Upwards & from the sd Distance to Divide Between the sd Counties By a Circular Line Extending according to ye Kings Letters Pattents & Deeds of Enfeoffment from the Duke for ye same & ye sd Circular Line to be well marked two third parts of ye Semicircle & make a true Return hereof into my Secretary's office to Remain uppon Record & for your so doing this shall be your Warrt Given under my Hand & Seale this 28 day of ye 8th month ~~[illegible]~~ 1701

Wm Penn

Recorded in the Rolls office at Phila in ~~[illegible]~~ book C. 2. vol. 3 page 166. 167. ye 30th 8th mo. 1701

21 lines -- 1.9d

By me Tho: Story Mr Rolls


The Figure of the Circular Line Dividing Between the County of New Castle & County of ^
Hickery
Indian Town
White Clay Creek
Red Clay Creek
Bread & Cheese Island
Christina Creek
Indian Town
White Oak
Beaver Creek
Brandy Wine Creek
Hamans
Creek
Kings
Road
Chichester
Delaware River
A
E
A New Castle: Chichester
B The old Hous of Nath: Lamply sa
C The Swamp behind Nath: Lamply's
D Jasper Yeats's Mill.
E Charles Ramsons Hous in Chichester.
A Scale of Three Miles
1
2
3

By virtue of a Warr^t from William Penn Proprietary & Governour of the Province of Pensilvania & Counties Annexed Bearing Date the 28^th day of y^e 8^th Month 1701 Authorizing us to Accompany the Magistrates of y^e County of New Castle & County of Chester or any three of them within the space forty days after y^e s^d Date to Remeasure & Survey from the Town of New Castle the Distance of twelve Miles on a Right Line up y^e s^d River & from y^e s^d Distance to Divide Between the two s^d Counties By a Circular Line Extending according to y^e Kings Letters Pattents & Deeds of Enfeofment from the Duke & y^e s^d Circular Line to be well marked two third parts of y^e Semicircle.

These are to Certifie that on y^e twenty fifth day of y^e Ninth Month 1701 Wee met at New Castle with Cornelius Empson Richard Halliwell & John Richardson Justices of y^e County of New Castle & Caleb Pusey Philip Roman & Robert Pile Justices of the County of Chester who Did Unanimously Conclude that the Begining should be at the End of the Horse Dike next y^e s^d Town of New Castle and from thence to Measure Due North the s^d Distance of twelve Miles & at the Extent thereof to Run the said Circular Line first Eastward Down to the River & then to Return to y^e s^d Extent of twelve Mills North & to Run the said Circle Westward untill it should Compleat the two third parts of the s^d Semicircle And Accordingly the twenty sixth day of y^e s^d Month we did begin in y^e Presence of y^e s^d Justices at the s^d End of y^e Horse Dike and Measured Due North twelve Miles to a White Oak Marked with twelve Notches standing on y^e West side of Brandy Wine Creek in the Land of Israel Helm & from the s^d White Oak Wee Ran Eastward Circularly changing our Course from the East Southward one Degree at the End of Every Sixty Seven Perches which is the Chord of one Degree to a twelve Miles Radius & at y^e End of forty three Chords wee Came to Delaware River on y^e upper Side of Nathaniel Lampley's Old Hous at Chichester & then wee Returned to the s^d White Oak in Israel Helms Land & from thence we Ran Westward Changing our Course one Degree from the West Southward at y^e End of Every Sixty Seven Perches as before untill we had Extended Seventy Seven Chords (which being Added to forty three Chords make two third parts of the Semicircle to a twelve Mills Radius) all which s^d Circular Line being well marked with three notches on Each side the Trees to a Marked Hickery Standing Neare y^e Western Branch of Christina Creek Surveyed the 4^th day of the 10^th month 1701 By us

Isaac Taylor
& Tho: Pierson

These may Certifie that Isaac Taylor & Thomas Pierson Did accompany us at y^e Town of New Castle y^e 25^th day of y^e 9 month 1701 together with Richard Halliwell being all Justices of y^e Peace where we did unanimously agree & conclude that in order to Remeasure & Survey the twelve Miles Distance from New Castle Town for the Dividing the County of New Castle from the County of Chester according to y^e Proprietarys Warr^t the Begining should be at y^e End of y^e Horse Dike next the s^d Town & then to Run Due North twelve Miles & from y^e s^d Extent thereof to Divide the s^d Counties by a Circular Line as is above Certified & that at y^e End of the s^d Horse Dike y^e s^d Isaac Taylor & Thomas Pierson did begin to Measure the s^d twelve miles in y^e Presence of us all together with Richard Halliwell & from that time we were some time five but never less then four all y^e Running y^e s^d North Line & also the two thirds of y^e s^d Semicircle till it was Compleated according to y^e above Certificate & y^e whole was finished y^e 4^th of the Instant to this Certificate we Do Subscribe our Names y^e 13^th of y^e 10^th mo 1701

Corneli^s Empson   Caleb pusey
John Richardson   Philip Roman
Robert Pyle

Although the line had been run out, little heed seems to have been given to it in the issue of patents for land. Over some small part of the boundary east of the Brandywine the patent lines were made to conform to the circular boundary, nominally at least, though it is noticeable that the old deeds pay no regard to the curvature of the line. The description of the bounds usually states that the line runs on a certain course a specified number of perches "along the circular boundary."

Except in this one district the land patents pass over the boundary without reference to it.

So for years and generations this line slumbered in obscurity, perpetuated for a time in local memory and tradition by reference to oak or hickory trees blazed or notched by the surveyors or by fences which some settler had built, as he supposed, upon the line. But year by year these witness marks decayed and passed from sight, until their very location became uncertain and until it has come to pass that at the present time the tolerably authentic relics of the old survey may be counted upon the fingers of one hand.

Meanwhile a far more troublesome question of boundary lines was arising from the conflicting claims of William Penn and Lord Baltimore to the fertile fields of the peninsula and the valley of the Susquehanna. Grants were given by both sides to lands in the disputed territory, and for many years the border was the scene of disputation and of conflict carried at times to the verge of open war. If Baltimore had the better title, Penn had the greater influence at court, and moreover held possession of a large tract claimed by Baltimore.

Several efforts were made by the proprietors to come to some agreement in this matter, but for one reason or another the negotiations repeatedly miscarried.

As early as 1685 Penn had succeeded in obtaining from the committee of trade and plantations, to which the matter had been referred by the King in council, an order that the peninsula should be divided between him and Calvert to the northward of a line running west from Cape Henlopen.

In presenting his case before the committee, we find Penn very shrewdly and skillfully availing himself of the Dutch claims through the early settlement by De Vries at the Hoornkill to support his own title obtained from the Duke of York and derived from the latter's conquest of the Dutch settlements.

In Lord Baltimore's charter of 1632 the descriptive phrase "hactenus inculta" (heretofore uncultivated) is applied to the territory so granted. This expression, found only in the preamble, was no doubt inserted merely to denote that the part of Virginia conveyed to Lord Baltimore had not been settled as part of that colony. It seems clear that it was not intended to impose a condition, for it was not repeated in the body of the charter, and it was not held to substantiate the seemingly valid claims of William Claiborne, who was actually settled on Kent Island at the time of the grant.

It is certainly hardly supposable that King Charles I intended to to recognize any prior claim of a feeble Dutch settlement located on territory claimed by England by right of discovery and conquered later by force of arms. And even if this unlikely supposition were true, the lands exempted under the words "hactenus inculta" could only be those of the actual settlement and could hardly be extended to cover the present State of Delaware.

But, justly or unjustly, Penn, who was high in favor, prevailed over Lord Baltimore, who found it prudent to yield for a time lest worse evils should befall him.

And thus we see the little village of Swaanendael, so soon swept away in fire and blood, rising from its ashes to sever the Delaware shore from Maryland.

But though Lord Baltimore submitted, he made no haste to carry out the order, and before anything had been done the revolution which drove King James from the throne also overturned the proprietary governments of Maryland and Pennsylvania.

The latter was soon restored to Penn, but Maryland remained a Crown province till 1716.

When Queen Anne succeeded to the throne, Penn managed to obtain an order in council for the enforcement of the decision of 1685. This was in 1708; but nothing was done toward carrying out this order, and in 1718 Penn died, leaving the dispute to his heirs.

The matter dragged along till 1732, when the heirs of Penn and the Lord Baltimore of that day joined in a deed to settle their boundaries. This called for a line running due west across the peninsula from Cape Henlopen, from the exact middle of which line a second line should be drawn to the northward in such a manner as to be tangent to a circle drawn 12 miles from Newcastle. From the tangent point a due north line was to be drawn, reaching to within 15 miles from Philadelphia, and from the terminus of this line the boundary to the westward should be a parallel of latitude. It was further stipulated that Newcastle County should in any event have the full area included within the 12-mile circle, even if part of it lay to the westward of the due-north line from the tangent point.

This proviso seems to have been added on account of the lack of information as to the direction which the tangent line was likely to take and for fear that the meridian line might seriously curtail the area of the circle. Had the parties to the deed known, as we know at present, that the segment of the circle west of the meridian line from the tangent point contains less than 14 acres, as the lines were marked on the ground, they might have concluded that so small an area was hardly worth considering and we might have been spared one complication in this historic interstate boundary.

But though matters thus seemed settled, this was really far from being the case. Difficulties and disputes arose as to carrying out upon the ground the provisions of the deed. The question as to the proper point in Newcastle which should be taken for the center of the 12-mile circle occasioned long debate. One rather quaint solution was the suggestion that this point should be the center of gravity of a paper plat of the town, the center of gravity having been found by experiment by balancing the plat on a pin.

Lord Baltimore's friends insisted on the absurd theory that the "12 miles" meant the periphery of the circle, while the Penns, of course, claimed that length of radius. A dispute also arose as to the true location of Cape Henlopen, as intended in the deed.

In consequence of all these difficulties nothing was done to carry out the deed.

The next move was made by Lord Baltimore, who, in spite of his own deed of 1732, applied to King George II for a grant to confirm his title according to the original charter of 1632. Naturally enough, this was refused by the King, and the matter was thrown into the court of chancery. In 1750 Lord Chancellor Hardwicke decided in favor of the Penns on every point of dispute, ruling that the center of the circle must be the middle of Newcastle as nearly as that point could be ascertained, that the "12 miles" meant the radius of the circle, and that the true Cape Henlopen was not the southern point of Delaware Bay, but the point claimed by the Penns, about 25 miles farther south and now called Fenwicks Island. This last decision seems rather a strange one, for though it appears that there was some confusion of usage among the Swedes and the English of the name "Henlopen," the present cape of that name seems to be clearly indicated in William Penn's deed from the Duke of York, which reads thus:

> * * * All that tract of land upon Delaware river and bay, beginning twelve miles south from the town of New Castle, otherwise called Delaware, and extending south to the Whore-kills, otherwise called Cape Henlopen. * * *

William Penn himself seems to recognize this limit in his "Act of Union" of December 7, 1682, where he describes the "territories" as—

> All that tract of land, from twelve miles northward of New Castle, on the river Delaware, down to the south-cape, commonly called Cape Henlopen, and by the Proprietary and Governor now called Cape James, lying on the west side of the said river and bay * * *

But even the decree of Lord Hardwicke did not end the interminable controversy. It would almost seem that Lord Baltimore and his friends, despairing of obtaining what they no doubt considered their just rights, had set about making all possible trouble for their successful opponents, even at the cost of time, money, and good repute to themselves.

In order to lessen as much as possible the amount of territory which must be yielded to the Penns, Lord Baltimore's partisans contended that the 12 miles should be measured upon the surface of the ground and not horizontally. Another appeal was made to the lord chancellor, and in March, 1751, he ordered that horizontal measurements should be employed.

After this decision by the lord chancellor it appears that the location of the boundaries was begun by commissioners and surveyors appointed for that purpose. The "base line," or east and west line, across the peninsula was laid out and measured. For this purpose a gap, or "visto," as the old records have it, was cut through the forest. The line was ranged out by poles set up in the "visto," and the distances were measured with a Gunter's chain 66 feet long, which was kept as nearly horizontal as possible. The whole length of this "base line" was found to be 69 miles and 298 perches, a value probably about a mile and a quarter greater than the real distance.

But at the distance of 66 miles and 24½ perches from the eastern end of the line the surveyors came to the shore of Slaughters Creek, a branch of the Hudson or Little Choptank River, separating Taylors Island from the peninsula.

Lord Baltimore's commissioners at once raised another question. Evidently the shorter they could make the base line the farther to the east would its middle point fall and the smaller would be the territory yielded by Maryland to Pennsylvania. So the Marylanders claimed that the line should stop at these first waters of the Chesapeake which were met in coming west from the ocean. The friends of the Penns, on the other hand, insisted that the line should continue to the open bay. It was so completed, as noted above, but the Marylanders would not accept it, and once more the dispute was referred to Lord Chancellor Hardwicke.

Before his decision had been rendered Charles Lord Baltimore died and the proceedings came to nothing.

When they were renewed, with the new proprietor, Frederick Lord Baltimore, made a party to them, he refused to be bound in any way by the acts of his predecessor. At last, however, on July 4, 1760, another deed was executed by the interested parties, and the long dispute was at an end so far as concerned the rival claims of Pennsylvania and Maryland.

The boundary called for by the deed of 1760 was substantially the same as that of 1732. The parallel of latitude forming the northern boundary of Maryland was to be 15 miles south of the most southern part of Philadelphia. The "base line" was to cross Taylors Island to the open bay, as claimed by the Penns. The court-house at Newcastle was accepted as the center of the circle.

Under the deed of 1760 commissioners were appointed by each side to supervise the work of demarcation. These commissioners held their first meeting at Newcastle on November 19, 1760. They employed a number of surveyors, who proceeded to complete the work begun in 1751. These surveyors appear to have accepted the work of that year on the "base line," and for the next three years, until the latter part of October, 1763, they were engaged in running trial lines for the tangent line, starting from the middle point of the base line, and for the radius, which should meet the tangent at right angles 12 miles from Newcastle court-house. After their three years of labor no solution had been reached, though it afterwards appeared that they had located the radius with considerable precision, considering their rude method of work. In October, 1763, the commissioners had just reached an agreement to report the state of the work to their respective principals and to ask further instructions, when they received information from the proprietors that two skillful mathematicians had been engaged by them to assist the commissioners in their labors. Further proceedings were therefore suspended until the arrival of these new surveyors, Charles Mason and Jeremiah Dixon.

On December 1, 1763, the commissioners met at Philadelphia and read the articles of agreement between the proprietors and the surveyors. They also made the necessary arrangements with the latter for the conduct of the work.

Messrs. Mason and Dixon, who thus appear upon the scene, were employed in the boundary surveys for the next four years.

Their first task was to determine the latitude of the most southern part of the city of Philadelphia. The mayor and other city officials were called upon for information in regard to the true southern limit of the city. They conducted the commissioners and Messrs. Mason and Dixon to the street called Cedar, or South street, and there pointed out a certain house occupied at the time by Thomas Plumsted and Joseph Huddle, situated on the south side of the street. The north wall of this house, marking the south side of the street, was stated by them to have been ever considered the most southern part of the city of Philadelphia. Though the position of this house is not stated, it was probably very near the river, as Cedar street runs a little south of east and the most southern part of its south side would be where it struck the shore.

Mason and Dixon built an observatory, and by observations with a zenith sector determined the latitude of this most southern point of Philadelphia to be 39° 56′ 29″·1.

From the latest survey of the water front of Philadelphia the latitude of the most southern part of the south side of Cedar or South street is about 39° 56′ 26″·6, a value differing from that of Mason and Dixon by only 2½ seconds of arc, and showing that their work was very carefully done.

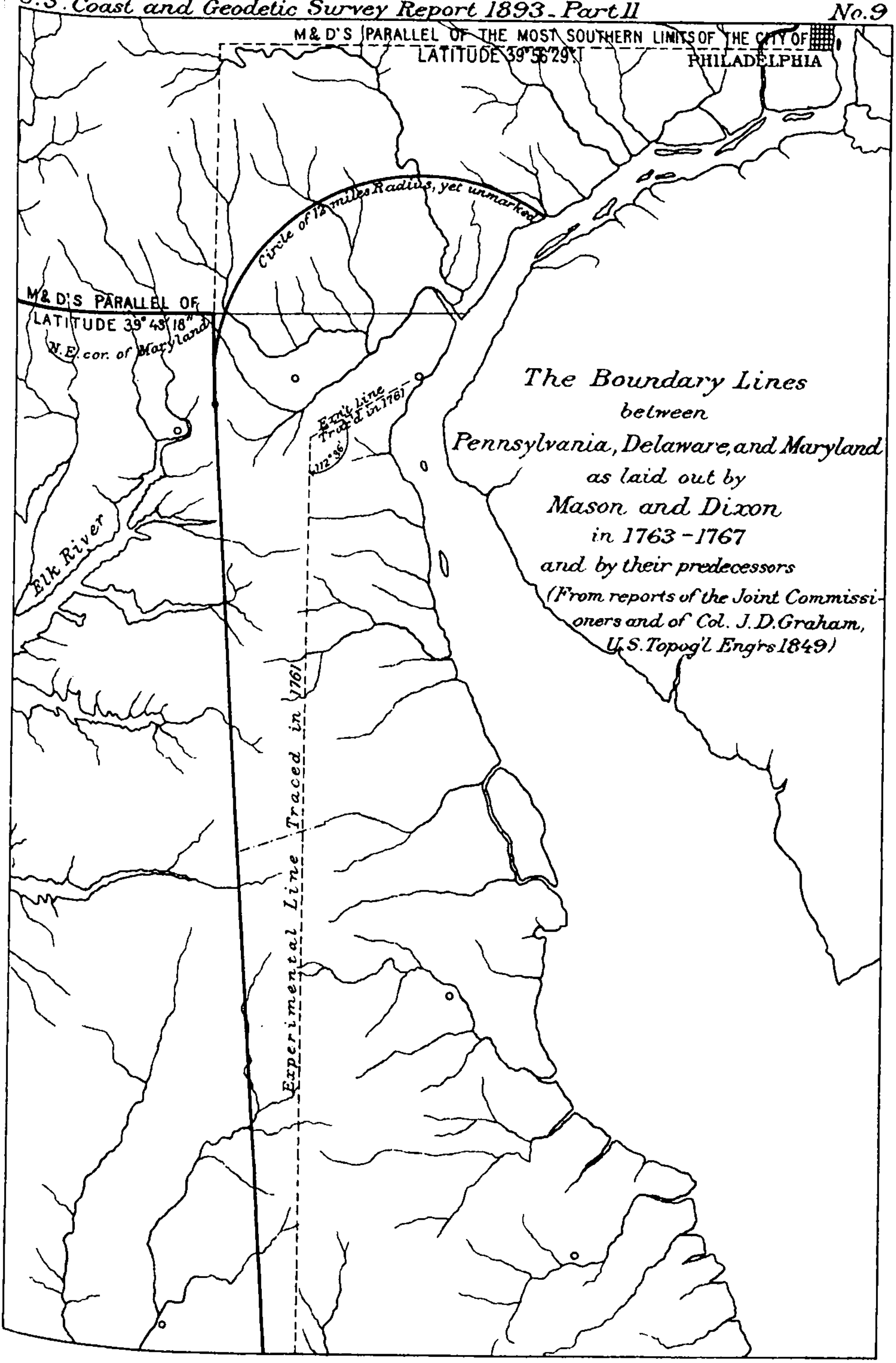
They next removed their instruments to a new station about 27 miles to the westward and near the forks of Brandywine Creek, where they again observed for latitude and located a point which they supposed to be in the same latitude as their first observatory. It is said that a white stone, locally known as "the stargazers' stone," still marks this second station. From this point they opened a line running due south through the forest and measured a distance of 15 miles in that direction. This measurement was designed to locate the parallel of latitude which by the deed of 1760 was to be laid out 15 miles south of Philadelphia to form the northern boundary of Maryland. Either through an error in the latitude of the second station as compared with that of the first, or from errors of chaining, or both, this line was carried too far south.

The difference of latitude between the end of South street and the northeast corner of Maryland is now about 13′ 6″, or about 5 chains more than 15 miles, so that the northern boundary of Maryland was put about 330 feet too far south.

From the southern end of their 15-mile meridian Mason and Dixon began laying off a parallel of latitude to the westward. After running several miles of this line, which was temporarily marked by wooden posts, the surveyors left it for a time and turned their attention to the southern portions of the boundary. Accepting as settled the "base line" which had already been measured across the peninsula by their predecessors, and the middle point marked by the same persons, Mason and Dixon endeavored to run out the tangent line from that middle point of the base to the tangent point. This extremity of the 12-mile radius laid out by the former surveyors was also accepted by Mason and Dixon, who found that it was nearly at right angles with the line which they laid out between its western end and the middle point of the base line. As will be shown later, this tangent point is really about 108 feet too far from the belfry of Newcastle court-house.

In laying out the tangent line Mason and Dixon were much assisted by the trial lines run by their predecessors. From these abortive lines they computed the direction which the line should follow, and then ran it out by transit until they reached the tangent point. They found that their line ran 16 feet 7 inches east of that stake. They then computed the proper offsets for each mile of their line to bring it to the true line, and moved their posts accordingly. This done, they reported to the commissioners that the posts so placed were, as nearly as practicable, in the true tangent line. Next, in accordance with the deed, a true north line was laid off from the tangent point to an intersection with the parallel of latitude 15 miles south of Philadelphia, which had already been partially surveyed, as related above. The point of intersection of the meridian and parallel became the northeastern corner of Maryland. The temporary mileposts already placed in this line, and referring to the south end of the 15-mile meridian as an origin, were replaced by new posts counting westward from this corner of Maryland.

The next thing to be done was the marking of that portion of the 12-mile circle which lay to the westward of the due-north line from the tangent point. And here Mason and Dixon fell into an error in computing the length of this small arc. As was pointed out by Col. J. D. Graham in 1850, they seem to have obtained their angle of deflection from the tangent to the due-north line, upon which their computation


M & D'S PARALLEL OF THE MOST SOUTHERN LIMITS OF THE CITY OF
LATITUDE 39°56'29"
PHILADELPHIA
Circle of 12 miles Radius, yet unmarked
M & D'S PARALLEL OF
LATITUDE 39°43'18"
N.E. cor. of Maryland
Exp't Line
Trac'd in 1761
112°36'
Elk River
Experimental Line Traced in 1761
The Boundary Lines
between
Pennsylvania, Delaware, and Maryland
as laid out by
Mason and Dixon
in 1763-1767
and by their predecessors
(From reports of the Joint Commissioners and of Col. J. D. Graham, U.S. Topog'l. Eng'rs 1849)

of the chord depended, by measuring the angle between their due-north line and the visible portion of the radius laid out by their predecessors. They had previously found that this radius was sensibly perpendicular to their own tangent line. Evidently, however, one of these angular measures was considerably in error. They computed their chord and offsets with a deflection angle of 3° 28′, while the actual angle between the tangent and the due-north line was found by Colonel Graham to be 3° 36′ 06″. As the arc cut off by the due-north line would be twice the deflection angle, this made an error of 16′ 12″ in the angular value of the intercepted arc and shortened the chord about 300 feet. Owing to the flatness of the curve the middle ordinate was not greatly in error, and the area of the segment was only about an acre too small.

Mason and Dixon would probably have obtained better results if they had directly measured the angle of deflection of the due-north line from their own tangent line. By an error in chaining they made a smaller mistake in the opposite direction, the actual length of their chord, according to Colonel Graham, being 84 feet greater than the value given by them. Their chain measurements, probably intrusted to careless employees, seem in general to have been irregular and inaccurate. Distances on the ground are almost always greater than their nominal length. The tangent line, supposed to be a little less than 82 miles long, is probably about 84½ miles in actual length. Curiously enough, all of the errors in measurement made by Mason and Dixon or their predecessors seem to have resulted in loss of territory by Maryland, except for the trifling error in the area of the circular segment. On the base line the distances seem to have underrun, contrary to the general rule; but if this error was cumulative it was probably distributed with some degree of uniformity and would not greatly affect the position of the middle point.

But the error in locating the northern boundary of Maryland, putting it about 5 chains too far south, meant the loss of a strip of that width along the whole length of the boundary, about 196 miles. This area would amount to nearly 8,000 acres.

The error in measuring the radius from Newcastle court-house, which placed the tangent point 108 feet too far from the center, took a strip of that width from the eastern border of Maryland to the northward of the tangent point, while south of that point, assuming that the southwest corner of Delaware was correctly placed, Maryland lost a wedge-shaped strip about 84½ miles long and 108 feet wide at the base.

After completing the arc above mentioned, Mason and Dixon took up the extension of their parallel of latitude to the westward, and their work does not further affect the matter under consideration.

As these various lines were located by Mason and Dixon they were marked by suitable stone monuments which were generally one mile apart. This work was done by other persons under the supervision of the commissioners. These stones were made in England from oölitic limestone, and were sent out from time to time as they were needed. They are stout square posts surmounted by a rather flat pyramid. Upon the side facing Maryland the letter M is cut, and on the opposite side the letter P. As Delaware was then a part of Pennsylvania the whole length of the line was marked in the same way. Every fifth milestone, however, was more elaborately marked, having the arms of the respective proprietors carved upon the opposite sides in place of the initials. It speaks well for the durability of the stone used that after being exposed to the elements for more than a century and a quarter these inscriptions are still distinctly legible and the stones are in so good condition as to have quite a modern appearance. While I was one day examining the eightieth milestone on the tangent line I was approached by a farmer who lived in a house not far way. He asked me the meaning of the stone, and on being told something of its history seemed much surprised and said that he had thought it a farm boundary mark placed there in recent years.

One of these stones carved with the arms of the proprietors has found its way by some strange chance into the town of Newark, where it supports one of the pillars of the porch in front of a very old house. It is probably one that was intended to mark the eighty-fifth mile of the tangent line; but this being only 82 miles long, as measured, it was not used, though it might well have been placed at the intersection of the arc with the due-north line instead of the rough, unmarked stone which stood there till 1849.

The monuments placed at the middle point of the base line, now the southwest corner of Delaware, and at the northeast corner of Maryland, differed from any of the others in having a coat of arms on each side thereof. Upon their north and east sides were carved the arms of the Penns and upon their south and west sides the arms of Lord Baltimore.

The stones placed at the tangent point and at some other points on the small part of the circle laid out by Mason and Dixon were of different and far less durable material—a dark granite rock of very poor quality. It appears from the minutes of the commissioners, under date of June 17, 1765, that it was intended to replace these stones with more durable monuments marked with the arms of the proprietors. For some reason this never was done, and most of the old stones still remain. The top of each is rounded to indicate that it is on the circle. The arms of the proprietors can barely be perceived upon the old stone at the tangent point, the only one which was so marked.

Not long after the completion of this celebrated work, the fruit of so many years of contention and of so great labor and expense, did the proprietors enjoy the quiet possession of their heritage. The storm of the Revolution, which even then was gathering, soon swept away the proprietary governments and severed the connection between Pennsylvania and the "three lower counties," this disaffected portion of the province becoming the State of Delaware, or, as its legal title first read, "the Delaware State."

By this separation a large portion of the boundary line over which the Penns and the Calverts had so long contended, the controversy over which had so embittered neighboring communities and which had at last been settled at so great labor and expense, became the dividing line between the States of Maryland and Delaware. Being well marked by durable monuments, no further dispute has arisen in that connection.

The boundary between Pennsylvania and the new State of Delaware, however, reverted to the old circular line between the counties of Chester and Newcastle. The survey of this line by Tailer and Pierson, 1701, has already been described. As previously noted, there was no connection whatever between Mason and Dixon's work of 1763–1767 and the old line of 1701. This line ran through a hilly and sparsely settled region, and the approximate location handed down by tradition among the country people seems to have been sufficient for their simple needs. As years passed by memories of the line of 1701 became dim, especially near its western end, and local tradition assumed that the stone at the northeast corner of Maryland marked also the western end of the circular boundary. As a corollary to this, it was held that all of the territory lying south of the parallel and east of the meridian passing through the northeast corner of Maryland, down to the tangent point, belonged to Delaware. In other words, Delaware and Maryland were thought to be coterminous up to the northeast corner of the latter State.

This popular view, which crystallized into local usage, though subsequently violently assailed and even officially renounced by the boundary commissioner of Delaware in 1850, has actually governed all interested parties even to this day. All the land titles are recorded in Delaware, the inhabitants vote and pay taxes in that State, and, as noted by Colonel Graham and others, a resident of this wedge-shaped strip between the circle and the boundary of Maryland was for some time a member of the Delaware legislature.

From a comparison of the old grants it seems a reasonable interpretation that the jurisdiction of Pennsylvania should not extend south of Mason and Dixon's parallel of latitude, and it is therefore a gratifying circumstance that one result of the last interstate commission on this subject will be to legally establish that condition.

Penn's charter of 1681 provided that the southern boundary of Pennsylvania should follow "the 12-mile circle" northward and westward until it intersected the fortieth parallel of north latitude. This was found to be impossible, but by subsequent proceedings, already described, the parallel of latitude 15 miles south of Philadelphia was substituted for the fortieth parallel. Now, the 12-mile circle had already been surveyed when Mason and Dixon marked their parallel of latitude, and it seems clear that the southern boundary of Pennsylvania resulting from these two surveys, to conform to the spirit of the charter, should have followed the circle of 1701 to its intersection with Mason and Dixon's parallel of latitude, and should thence have followed that parallel to the westward. Under Penn's deeds of 1682 from the Duke of York he claimed the 12-mile circle and the country south of it, but made no claim to any land between the circle and the southern boundary of Pennsylvania. All such land unquestionably belonged to Maryland. But in the deeds of 1732 and 1760 the Calverts so far yielded their rights in this respect as to agree to an eastern boundary running due north from the tangent point. This agreement transferred to the Penns the wedge-like area between the circle and the due-north line. This was outside of both of Penn's grants, a sort of donation from Lord Baltimore; but no good reason appears for considering this accretion a part of Pennsylvania proper rather than of the immediately contiguous territory of Newcastle County.

Even on the assumption that the territory so gained was part of Chester County, Newcastle County was entitled to claim the circle of 1701 as its boundary. That line, beyond a doubt, intersected Mason and Dixon's due-north line far to the north of the point commonly called "the junction of the three States."

The small circular arc laid out by Mason and Dixon to the northward of the tangent point was part of the boundary of Maryland under the deeds of 1732 and 1760.

The point of intersection of this arc and the due-north line, known since 1850 as "the junction of the three States," was then considered of no more importance than any other point of the Maryland line, and was perhaps the worst marked of them all. The idea of making the end of this fragmentary arc, laid out by Mason and Dixon in 1765 for one special purpose, the initial point of the circular boundary of Delaware, in utter disregard of the line actually surveyed in 1701 for that other special purpose, seems to have originated with Col. J. D. Graham, of the United States Topographical Engineers, who in 1849 and 1850 superintended a revision of a portion of Mason and Dixon's work.

This resurvey was due to the following circumstances: The monument which had been placed in 1768 at the northeastern corner of Maryland in the course of time disappeared from its place. Various stories are current as to the cause of its disappearance, but they are not important in this connection. The absence of this stone and the uncertainty as to the significance of others in the neighborhood combined with rumors of the unauthorized moving of some of the monuments, produced a general feeling of doubt in regard to the northeastern boundaries of Maryland. This condition led to the appointment in 1849 of a joint commission composed of one representative from each of the three States—Pennsylvania, Delaware, and Maryland.

It seems clear from the circumstances attending the formation of this commission that the missing stone was considered by all to have marked a point common to the three States. The resolution of the Delaware legislature authorizing the appointment of a commissioner from that State describes it as "the original boundary stone established at the point where the States of Pennsylvania, Maryland, and Delaware join each other." This resolution was adopted on the 10th of February, 1847.

The legislature of Pennsylvania did not act on the matter till 1849, when, on April 10, a bill was passed authorizing the appointment of a commissioner to act for Pennsylvania in surveying and determining the point of intersection of the three States and fixing a suitable monument at the point.

This is not quite so explicit in its designation of the particular point meant as the Delaware resolution, but it clearly indicates that a point then unmarked is to be located and suitably marked. The corner of Maryland is the only point that would answer that description, and if the view advanced by Colonel Graham had been held by the legislature of Pennsylvania it would hardly have admitted Delaware to a voice in the location of the boundary between Pennsylvania and Maryland. I can not find that the legislature of Maryland took any action toward appointing a commissioner, but one was certainly appointed by the governor of that State, and this action was subsequently ratified by the legislature, which authorized the payment of the necessary expenses.

This joint commission of 1849 obtained from the War Department the detail of Lieut. Col. J. D. Graham to conduct the necessary surveys. It is unnecessary to enter into much detail in regard to his operations, which are quite fully described in his interesting report subsequently published by each of the three States. He reestablished the corner of Maryland by producing to an intersection the north and east lines of that State as marked by monuments then in existence.

The corner thus determined was marked by a massive granite post, which is still in good condition.

He also placed new granite posts at the tangent point, at the middle point of the arc of the circle north of the tangent point, and at the point where the above arc, as laid out by Mason and Dixon, cuts the due-north line so often referred to.

This last stone was made in the shape of a triangular prism, inscribed with the initials of the three States on the appropriate sides. The names of the commissioners and the date 1849 were also cut on the north side, under the initial P.

The peculiar features of this stone were in accordance with the theory adopted by this commission of 1849, at the prompting of Colonel Graham, that this point was "the junction of the three States." It is especially surprising that the commissioner from Delaware, George Read Riddle, esq., should have assented to this encroachment upon the area and jurisdiction of his State. And the legislature had given him no authority for such surrender. The joint resolution of February

10, 1847, under which he was appointed, has already been mentioned. It is entitled a "Resolution relative to the northwest boundary stone of the State," and its text, which is given in full in the appendix to this report, clearly indicates that the legislature wished to restore the stone which had been removed from the northeast corner of Maryland, and not that other stone which was then standing at the point now occupied by the triangular prism, more than 3½ miles farther south. There was no further grant of power to the commissioner, not a hint of any authority to change a long-accepted boundary, nor to bind the State as to any details of the circular boundary between Delaware and Pennsylvania.

Yet, strangely enough, we find the Delaware commissioner accepting and signing a report and map which took from his State not only the wedge or "flatiron" south of Mason and Dixon's line, but also in the final consequences of his act, a long curved strip, or horn, about half a mile wide at its base and stretching northeastward along the circle for 11 miles, until it vanishes in a slender point at the Kennett-Pennsbury stump, near Centerville, Del.

This was due to the fact that Colonel Graham's map of 1850, signed by the three commissioners, pushed back the circular boundary from its actual intersection with Mason and Dixon's line to the theoretical 12-mile circle, regardless of the well-known rule that an actual line upon the ground is to be preferred to the written description of the same line in a deed.

It will be noticed that, although Delaware's claim to the "flatiron" seems to have been just, the common impression that the circular arc began at the corner of Maryland was erroneous.

Tailer and Pierson's line of 1701 crossed Mason and Dixon's line some 2000 feet east of that corner, and this point of intersection was the true beginning of the circular boundary.

In addition to the work above mentioned, Colonel Graham also made some trigonometric observations and calculations to obtain the distance between the tangent stone and the court house at Newcastle. He also computed the distances from the northeast corner of Maryland to the true 12-mile circle in two directions—first, on a right line, or radius, to the spire of the court-house, and, second, on Mason and Dixon's line produced. These distances, shown on the map furnished the commissioners by Colonel Graham, are, from one cause or another, considerably in error, even on the assumption that the boundary must follow the true 12-mile circle, an assumption already shown to be untenable.

As mentioned above, these apparently erroneous conclusions as to the true point of junction of the three States and as to the proper location of the circular boundary were embodied in a map which, on March 1, 1850, received the signatures of the three commissioners, H. G. S. Key, of Maryland, Joshua P. Eyre, of Pennsylvania, and George Read Riddle, of Delaware.

No subsequent acts of ratification seem to have been passed by the State legislatures, but the result was generally accepted on paper while ignored in fact. The maps showed Pennsylvania reaching a slender finger to the southward between Delaware and Maryland, but Delaware continued to exercise complete jurisdiction over that area.

In view of recent action, by which the above arrangement has been somewhat modified and the "flatiron" has been restored to Delaware, it appears that the most important effect of the map of 1850 was to commit the State of Delaware to the definite acceptance of the intersection of Mason and Dixon's line with the true 12-mile circle as the initial point of the circular boundary instead of the intersection with the circle of 1701.

This survey of 1850 called attention to the unmarked condition of this circular boundary, and while, fortunately for Delaware, the commissioners appointed at that time had no authority to undertake the work of marking it, they suggested that speedy action should be taken.

The legislature of Pennsylvania took action in the matter, and in several sections of a sort of omnibus bill, approved April 22, 1850, provided for the appointment of a commissioner, etc. The act is somewhat of a curiosity in its very matter-of-fact provisions for laying out the true 12-mile circle and for securing the titles and other vested interests of the numerous citizens of Delaware who were thus to be transferred to Pennsylvania. Aside from this prejudging of the case, its provisions seem careful and intelligent. Delaware does not seem to have cared to make so one-sided a bargain, and nothing more was done for about forty years. The increasing importance of the boundary, as the country grew in population and wealth, led to a renewed agitation of the question.

On April 25, 1889, the legislature of Delaware passed a bill appointing Hon. Thomas F. Bayard, Hon. B. L. Lewis, and Hon. John H. Hoffecker commissioners on the part of Delaware to act in conjunction with a similar commission from Pennsylvania to agree upon and mark the boundary.

On May 4, 1889, the legislature of Pennsylvania passed a similar act, authorizing the governor to appoint three commissioners to act for the Commonwealth of Pennsylvania, in conjunction with the Delaware commissioners, in "examining, surveying, and reestablishing" the boundary line between the two States. These commissioners were directed to join in marking by enduring monuments the line so reestablished.

The governor appointed Hon. Wayne MacVeagh, Hon. W. H. Miller, and Hon. R. E. Monaghan as the Pennsylvania commissioners under this act.

The proceedings of these two boards of commissioners have not yet been published, and not many details of their deliberations can be given. From the limited information at hand it appears that the commissioners from both States met in joint session at Philadelphia and selected Hon. Thomas F. Bayard as their chairman. Each commission separately employed an agent, styled, respectively, "surveyor on the part of Pennsylvania" and "surveyor on the part of Delaware." These "surveyors" acted as secretaries to their respective boards, collected information in regard to existing monuments supposed to be on the boundary line, looked up old title deeds bearing on the matter, etc. It appears also to have been expected that they would survey and mark the boundary.

Mr. Benjamin H. Smith, of Philadelphia, was the "surveyor for Pennsylvania," and Mr. Daniel Farra, of Wilmington, was the "surveyor for Delaware."

These gentlemen appear to have examined with considerable care the available documents, county records, etc., which might throw light upon the question.

Starting from the Delaware River and going westward, they seem to have been mutually satisfied that they had identified with reasonable certainty, as parts of the line of 1701, the following marks: First, the remains of the old house below Marcus Hook at which Tailer and Pierson ended the eastern section of their line; second, the boundaries of some farms west of the "Concord Turnpike" and east of Brandywine Creek, the original patents for which lands called for the circular line as their southern boundary; third, a peculiar bend of Brandywine Creek, west of the rocky promontory called Point Lookout, where the stream, flowing nearly south, touches the boundary of Delaware, but retreats again to Pennsylvania, curving back to the northeast and sweeping in a long bend around Point Lookout, when it once more trends to the southward and crosses the boundary at last near Smith's bridge; fourth, a large hickory stump, which marks the point at which the line between the townships of Kennett and Pennsbury, in Chester County, strikes the circular line of 1701. This last point is a particularly notable one. The old tree, which was standing a few years ago, was no doubt in existence in the time of William Penn. It is mentioned as a "small hickory" in a deed given in 1713 by George Harlan to his son, James Harlan, for 200 acres of land, a part of the "manor of Staneing," granted by patent by William Penn to his daughter Lætitia in 1701. The hickory was described as being "in ye eastern line of ye said manor."

To the westward of this hickory stump no marks could be found which the State surveyors were mutually willing to accept as correct. A considerable number of marks of more or less authenticity were pointed out or described at that time and subsequently by the inhabitants along the line. There was, however, no documentary evidence of their identity, and the agents of the commissioners declined to consider usage or tradition as reliable for their purposes.

One of these points, a stone at the corner of a farm in Mill Creek Hundred, was supported by title deeds dating back to about 1830, when the land of an intestate decedent was divided among his heirs. As the land lay partly in Pennsylvania and partly in Delaware, the courts of both Chester and Newcastle counties were interested in the case. A surveyor was employed, who ran out and marked the portion of the State line crossing the estate, and these marks are still in place. Each court then took jurisdiction on its own side of the line, and the estate was thus administered. There is no documentary connection between the work of this surveyor and the line of 1701, and this work of 1830 was most likely run between the nearest two traditional marks, possibly some of the old trees, which may still have been standing at that time. The marks so established had certainly been accepted as authentic by the people and the local authorities on both sides for sixty years, and might well have been considered of some value, especially as their position indicates a strong probability that they are at least very near the line of 1701. These circumstances were not, however, discovered until the joint commission had agreed to accept as the western end of the arc the intersection of Mason and Dixon's line of 1764 with the true 12-mile circle as indicated on the map of 1850. The "State surveyors," therefore, made no use of these points, which were brought to light in the course of the survey of 1892.

Although, as just mentioned, the commissioners followed the precedent furnished by the official plat of Colonel Graham's work in deciding that the circular boundary should meet Mason and Dixon's line at a point just 12 miles from the spire of Newcastle court-house, they agreed to correct so much of his work as threw into Pennsylvania the triangular area commonly called the "flatiron." The western boundary of Delaware would therefore coincide with the eastern boundary of Maryland, and the northern boundary of Delaware would run due east from the northeast corner of Maryland to a point just 12 miles from Newcastle court-house and thence would follow a curved line passing in as regular a manner as possible through the successive boundary marks accepted as authentic relics of the line of 1701.

But the commissioners and the surveyors had no reliable information as to the absolute or relative positions of these marks, and could therefore form no conclusions with regard to the curve to be passed through them. Nothing seems to have been done in the way of field work, and the matter remained in this condition until 1892.

Early in that year the joint commission, through the Hon. Thomas F. Bayard and Mr. Benjamin H. Smith, applied to the Superintendent of the United States Coast and Geodetic Survey for assistance in the matter and for the detail of an officer to execute the field work.

The consideration of this work forms the subject of the second part of this report, but to bring this historical sketch down to the present time the work of 1892 may be summarized as follows:

It was found that no single circle could be made to satisfy the conditions imposed by the commissioners. A compound curve was therefore laid out.. This is formed by two circular arcs which have a common tangent at the Kennett-Pennsbury stump, which point is very nearly midway on the whole curve. The radius of the western part of the curve is about 11·58 miles and that of the eastern part is about 12·81 miles.

Although the radius of curvature of the western part of the arc is less than 12 miles, no part of it lies within the 12-mile circle. The western end or initial point of the curve is just 12 miles from Newcastle court-house, but every other point of the line is outside of the theoretical circle. This difference increases rapidly as far as the Kennett-Pennsbury stump, where it is 1877 feet, and after that more gradually, amounting to 3137 feet at the eastern end or terminus of the line.

This line is now marked by 46 substantial monuments, as follows: An initial monument, made of dark Brandywine granite, at the western end or origin of the curve; 22 milestones marked with the initials of the names of the States and the date. 1892; 22 smaller stones marked simply ½, placed half way between the milestones, and a terminal monument near the Delaware River, at the eastern end of the line.

With the exception of the initial monument, all of these stones are of a light grayish white gneiss, from a quarry near Chester.

Thus ends for the present the history of this boundary, and although a few residents of the debatable ground near the initial monument felt aggrieved that the official location of the line placed them in Pennsylvania instead of Delaware, and although there are rumors of a contemplated appeal to the Supreme Court of the United States, it appears reasonable to suppose that the line thus marked will remain the boundary and that this will be the last chapter of the long story of border troubles outlined in the foregoing sketch.

PART II.—DETAILED ACCOUNT OF THE WORK ON THE PENNSYLVANIA-DELAWARE BOUNDARY LINE EXECUTED BY W. C. HODGKINS, ASSISTANT.

In compliance with the Superintendent's letter of instructions, dated March 8, 1892, I communicated with Messrs. Benjamin H. Smith, of Philadelphia, and Daniel Farra, of Wilmington, who had been employed by the two boards of commissioners to represent the interests of their respective States.

At the request of these gentlemen I went from Washington to Philadelphia on March 16, 1892, and met them at the office of Mr. Smith. At this conference the nature of the problem was outlined and a general plan of work was adopted.

Briefly summarized, in advance of more detailed discussion, the following were the principal features of the work to be done:

1. The accurate determination of the geographical positions of the

following points, viz: Newcastle court-house, the stone marking the northeastern corner of Maryland, and such points of the line of 1701 as could be satisfactorily identified.

2. The preparation of a drawing to show these points in their true relative positions and to indicate the various lines which might be made to satisfy more or less completely the conditions imposed by the commissioners.

3. The decision by the commissioners, after consideration of the above map, as to the character of the curve which should be adopted for the boundary.

4. The preliminary location upon the ground of the line so adopted.

5. The examination of this preliminary line by the commissioners with a view to any modifications which might become necessary.

6. The permanent marking of the line as approved by the commissioners of the two States.

It was suggested to Messrs. Smith and Farra that the most ready means of obtaining the required information would be by using the plane table, which would likewise afford a very fair degree of precision in the location of the line.

They preferred, however, to have the work done by trigonometric methods, but it was decided to make a plane table reconnaissance on a small scale (1:40000) in order to obtain an approximate idea of the positions of the guiding points with reference to each other. This would also serve for laying out the scheme of triangulation for the subsequent work.

With this understanding I returned to Washington, where I was engaged in completing my office work of the topographical survey of the District of Columbia. I at once began, however, to gather materials and to make preparations for taking the field early in April, and on the 14th and 15th of that month I sent two members of the party to Newark, Del., to begin the field work by putting up flags and searching for the station marks of the Coast Survey in that vicinity.

I expected to follow them in a few days, but before I had been able to leave Washington it appeared from my correspondence with the State surveyors that there existed a certain amount of misunderstanding or lack of definition as to the exact scope of the work which the Coast and Geodetic Survey was asked to undertake, and also as to the position which I, as the representative of the Survey, was to occupy in relation to the State surveyors.

It being deemed advisable by you that these matters should be definitely settled before I went to the field, my departure was delayed for a time. After further correspondence and a personal conference at the office of the Survey, on April 27, between the State surveyors and yourself, at which I was also present, they expressed a wish to submit the question anew to their respective commissioners for further action by them.

In pursuance of this arrangement, Messrs. Smith and Farra addressed to you a letter, dated May 5, 1892, in which they informed you that the matter had that day been submitted by them to the boundary commissions of the two States in joint session in Philadelphia, and that the joint commission had adopted a resolution, as follows:

Resolved, That Messrs. Smith and Farra be instructed to secure the survey of the line as soon as possible in accordance with the suggestions of the United States Coast and Geodetic Survey, so as to enable them to report the line approved by them to the Commission as soon as possible.

As in your opinion this resolution satisfactorily terminated the uncertainties above referred to, you verbally directed me to proceed to carry out your instructions of March 8.

I accordingly left Washington on May 11, 1892, and reached Newark, Del., on the same day. I first examined the work which had been done by the two men whom I had sent to the field in April, and who had been employed during my enforced delay in putting up flags and searching for triangulation points. They had succeeded in recovering the stations "Londonderry," "Meetinghouse Hill," and "Grandview," the last two so close together as to amount practically to one station. A large number of flags had also been put up in the vicinity of the boundary between the Maryland line and Brandywine Creek, and these were of considerable service in the reconnaissance.

I also made a personal examination of the ground in the vicinity of the northeast corner of Maryland, and, on May 18, I accompanied Messrs. Smith and Farra on a visit to the portion of the old line in the vicinity of Brandywine Creek, where some traces of the former work are still to be identified.

I was shown the old hickory stump at the southeast corner of Kennett township, the peculiar bend of the Brandywine through which Tailer and Pierson dragged their chain in 1701, and the other supposed marks referred to in the first part of this report. A tall hickory tree standing on the supposed old line at the corner common to the townships of Concord and Bethel, Delaware County, Pa., was selected by the surveyors as the reference point for that part of the line.

At a later period I also accompanied Messrs. Smith and Farra to the supposed remains of the old house on the Delaware which marked the eastern end of the line.

I now endeavored to recover some of the Coast Survey stations, which would better answer my purpose than those already found. The natural and artificial changes of the half century since that work was done had so completely changed the surroundings of the points that our search was unsuccessful for the time.

The line "Londonderry"–"Meetinghouse Hill," a side of one of the primary triangles, was adopted as the base for the triangulation, after some hesitation due to the length of the line (nearly 14 miles) and to the fact that intervening obstacles would compel me to elevate the instrument at each station. From this base line was developed a scheme of triangulation which reaches directly each of the points required to be determined except the one upon the bank of the Delaware. That also is reached indirectly, having been connected with the neighboring stations of the river triangulation, which is also connected with my work at Newcastle.

In reducing the size of the triangles from the 14-mile base line to the length required for following the circular boundary with a small scheme new points were first established at "Centerville," Del., in the village of that name, and at "White," Pa., near the village of Kemblesville. This latter station was very near the old point "Missimer," for which some ineffectual search was made in the hope of making the stations coincide.

From the line "White"-"Meetinghouse Hill" thus determined I was able to locate a station on "Grays Hill," near Elkton, Md. From this point and from "Meetinghouse Hill" observations could be made upon the spires of the Masonic Hall and Immanuel Church at Newcastle.

The court-house, being low and inconspicuous, could not be seen over the surrounding trees, but was determined by smaller triangles based upon neighboring points which had first been established from the main stations. I was then able to compute for the first time the position of this important point in terms of the standard data of the Coast and Geodetic Survey.

A great deal of trouble was experienced in making the reconnaissance for the triangulation on account of the rolling character of the country, the hills in any locality having nearly the same elevation. These hills are also generally covered with heavy timber, and as there are no commanding heights it was very difficult to secure intervisible points suitably located.

The atmospheric conditions were likewise unfavorable, the air being remarkably thick throughout the season.

All of these obstacles were overcome at last, but to accomplish it required much time and a great deal of hard work. In several cases stations had to be moved time after time to meet the conditions imposed by new ones selected further on, and this process had to be repeated until all the necessary lines of sight were arranged. In the scheme finally worked out there was scarcely any cutting and comparatively few scaffold signals.

The stone at the northeast corner of Maryland was a station peculiarly difficult to bring into the scheme of triangulation, situated as it is in the bottom of a wooded ravine. I was able, however, to obtain a satisfactory determination of this important point. A portion of the line between "Londonderry" and "Meetinghouse Hill" was heavily timbered, and it was necessary to remove some of the larger trees from the line of sight to avoid an extremely high signal at "Meetinghouse Hill." To assist in opening this line and to serve in the reconnaissance for the new stations afterwards located at "Centerville" and "White," it was found necessary to put up tall poles provided with cleats for climbing high enough to make preliminary observations. By means of these the line was opened, the new points were selected, and the necessary heights of the signals were determined. Poles from 60 to 85 feet high were raised by a tackle and horse power. They were steadied by rope or wire guys, carried flags at the top, and were cleated to a height of about 60 feet. A light whip rove through a single block at the top enabled the reconnoitering telescope to be hoisted to the eye of the observer. Such a pole was also erected near the old station "Bethel," in hopes of connecting that portion of the line with the triangulation, but the heavy timber upon the dividing ridge to the westward prevented it from being seen from "Centerville." It was therefore found necessary to omit any triangulation over the 6 miles of the line between the Concord and Philadelphia turnpikes.

By the end of June the reconnaissance was so far advanced that I felt able to proceed to build the main signals, where scaffolds of some height were necessary. To expedite matters by getting these signals up while I was completing the reconnaissance, I engaged Mr. Joseph Willis, a carpenter and builder of Newark, to build these for me as rapidly as possible; and I expected to begin the angular measurements as soon as they were done.

Mr. Willis took the work at a reasonable price and put up the signal at Grays Hill as planned. Just at this stage of the work Mr. Farra, who, with Mr. Smith, had been kept informed of the progress of events, paid me a visit and notified me that the commissioners were not agreed upon the desirability of the triangulation, and that they particularly objected to the expense of building signals. He therefore requested me to go to no further expense of the kind until the matter could be carefully considered by the joint commission.

After the particular desire which had been manifested in the beginning of the work to have everything done by triangulation, I was much surprised at Mr. Farra's communication. As the survey was being made for the commissioners, however, I felt obliged to yield to their request. As matters turned out, this action was particularly unfortunate in its results.

I therefore continued my reconnaissance until the scheme was completed and the points to be determined were all shown on the plane-table sheet. I then furnished tracings of the sheet to the commissioners from each State. These tracings showed the projected scheme of triangulation, the locations of the standard points desired, the true "12-mile circle," and the curves which most nearly fitted all the actually existing points.

Along with these I also furnished revised estimates for the expenses of the survey, the estimated total being $3100. It may be well to discuss this point a little further in this connection. Before I left Washington I had prepared estimates calling for a much smaller sum. These were based upon what I was told in the office of the probable extent of the work and were made before I had been on the ground. I had been in the field but a short time when I perceived that the ideas of the work that I had received from others were quite inadequate and that supplemental estimates would be necessary. At the request of Messrs. Smith and Farra I deferred them until the reconnaissance was finished, when they were promptly submitted. It is true that the final expense amounted to about 17 per cent more than the amount named, but I am confident that this moderate increase would have been unnecessary but for a remarkable combination of untoward circumstances. With regard to the total amount expended, Mr. Smith told me that his own estimate of the cost had been considerably above my highest estimate. I think the total very reasonable for the amount of work which was done.

By your authority I next visited Washington and explained to you the condition of the work, estimates, etc.

Upon my return to the field, as there appeared no prospect of a meeting of the commission, I took up the work at the eastern end of the line and connected the "Ruins" of the old house with the triangulation of Delaware River. This point could not be seen from the neighboring stations on account of several good-sized trees which stood close to the edge of the river. The owner of the land objected so strongly to the destruction of these trees that another station was interpolated at the windmill, a short distance to the northeast of the "Ruins," and the latter point was determined by an azimuth and measured distance from the mill.

After visiting Newcastle, where observations were prevented by the dense haze, which rendered the nearest signals invisible, I returned to Newark and began putting up signals and marking stations. Owing to the large number of stations, the hilly nature of the country, and the extreme heat of the weather, which was most exhausting both for men and horses, this work was somewhat retarded, but in the last three weeks of the month sixteen tripod signals and four 15-foot scaffolds were built. More time was lost in building the 46-foot tripod and scaffold at "Meetinghouse Hill," owing to the inexperience of my party in heavy carpenter work, and I found it necessary to again employ Mr. Willis to build the signals at "White" and "Centerville." Owing to other conflicting engagements, he was unable to finish them as promptly as desired, and some trouble was thus occasioned. I was obliged to give up building the signal at "Londonderry" on account of the unexpected opposition of the owner of the land. This person had offered no objections when consulted on the subject, but when the lumber dealers were ready to deliver the material he forbade them to do so, and when asked for an explanation demanded an extravagant bonus for the privilege of entry. In view of the delays already experienced, it was decided to do without that station and to "conclude" the triangles on the reconnoitering pole which was standing there, a correction being applied to each angle for eccentricity of the pole.

Though less satisfactory than if the station had been occupied, this method gave very good results.

The dense haze which prevailed throughout September was most unfavorable for observations of angles, and but little such work could be done.

The same trouble continued throughout October, and on the rare occasions when the air was somewhat clearer the wind was so violent as to prevent work. Though every possible opportunity was utilized to advance the work, it was often impossible to see stations only 1 or 2 miles away, and many of the lines were 6 or 8 miles long. Under these adverse conditions observations were made as rapidly as possible, work being continued till after sunset whenever any signals could be seen.

Early in November, having made a preliminary computation of the elements of the curve from the partially completed observations, I began to lay out the line upon the ground. This line, which Messrs. Smith and Farra had agreed to recommend to the commission, was composed of parts of two circles, the eastern part of such a radius that it would pass through the three guiding points—the "Ruins," the "Concord-Bethel" tree, and the "Kennett-Pennsbury" stump—and the western part of such a radius that, having a common tangent with the other at "Kennett-Pennsbury," it should pass through the point at which the eastern prolongation of the northern boundary of Maryland, considered as a parallel of latitude through the corner stone, would cut the true 12-mile circle around Newcastle court-house.

The weather now turned cold, with much rain and some snow, but the work was pushed as fast as possible. One great trouble in this work, as in the triangulation, was the obstruction caused by woods. In the surveys of Mason and Dixon and of Colonel Graham clear lines of sight were opened through the woods and no compensation was made; but in this survey the commission objected to cutting trees if it could possibly be avoided, and when it sometimes became necessary to clear a little, some of the landowners demanded exorbitant damages.

The work of running the line was thus continued, with observations of horizontal angles at intervals, as the weather permitted, until the 20th day of January, when the preliminary marking of the line was completed. On account of the almost arctic severity of the weather prevailing at that time and for several weeks before, work was then suspended until such time as the commission might select for their inspection of the line.

In the meantime the initial and terminal monuments had been erected early in December, their positions having been carefully determined. Each stone was securely planted in a pit filled in around it with a concrete base of broken stone and cement. The stones for marking the first 13 miles of the line were put on the ground ready for setting before the worst weather set in and prevented the delivery of the remainder for some time.

The winter weather during the last month of the work was of almost unprecedented severity for that latitude, the temperature for many days at no time rising above the freezing point and falling to 35° or 40° F. below freezing at night.

The deep snow, remaining a fine, dry powder in this intense cold, was continually blown through the air in blinding clouds by the high winds which prevailed, and many of the roads were completely blocked to travel by the drifts thus formed.

Field work under such conditions was attended by many hardships, but was continued until each of the monument stations had been marked.

After the suspension of field work above referred to, I returned to Washington and was occupied with office work while waiting for the joint commission to inspect the line and to listen to any objections which might be advanced against approving it.

It seemed to be the opinion of the State surveyors that the commission would perhaps make local modifications of the line in order to meet the views of the inhabitants.

Unfortunately, the weather during February continued too severe for this inspection to be made by the commission, and early in March I received new instructions directing me to prepare for duty on the survey of the boundary of southeastern Alaska.

After communicating with Messrs. Smith and Farra and finding that it would be impossible to secure the inspection by the commission before the date on which it would become necessary for me to start for the western coast, I returned to Delaware and went over the line with the State surveyors, pointing out the position of the stubs and putting in additional reference marks, etc.

Owing to the limited time at my disposal, I was not able to fully apply a system of refined checks to test the exact positions of the monuments, which I had intended to employ if the commission decided not to move the line laid out. All of these positions, however, are undoubtedly quite near their true values, especially where the work was controlled by triangulation for 16 miles from the initial point.

Between the 16-mile point and the 22-mile point there was no triangulation, and the preliminary line had been run out with a transit by half-mile chords measured by telemeter.

This method, while rapid and close enough for preliminary work, was somewhat lacking in precision, and I had intended to check it in case the commission wished to adhere to the circle by running out the long chord between the ends of this 6-mile arc and then laying off rectangular ordinates from the chord to the monument stations.

Finding now that I should not have time to do this myself, I submitted the question to Messrs. Smith and Farra for their decision as to whether this work should be left to them after the inspection by the commission or whether some person should be employed to do it at once under my direction.

They preferred the latter course, and I engaged for the purpose Mr. W. B. Carswell, of Wilmington, the only surveyor whom I could obtain at once. He had not finished his work when I had to start for Alaska, and I was therefore compelled to turn him over to the supervision of Messrs. Smith and Farra. I was later informed by Mr. Smith that after becoming involved in some serious errors Mr. Carswell had corrected his work and it had been accepted.

I left the boundary on the 1st of April, and immediately afterwards the remainder of the monuments were delivered on the ground under the supervision of a member of my party.

A few days later the joint commission went over the line, and after hearing the protests of some of the dissatisfied inhabitants of the border strip decided to accept the boundary as staked out. The monuments at the mile and half-mile points were planted immediately afterwards.

The work thus completed was one that gave me much labor and anxiety. It would have perhaps been more satisfactorily arranged in two seasons, the first for the reconnaissance and the triangulation, and then, after the complete reduction of the observations and the computation of the results, a second shorter season for laying out and marking the line.

As it was, I was compelled, while engrossed and exhausted with the field work, to reduce my work and go through the voluminous computations necessary to compute the curves and to provide for their location. This work was, for the most part, done at night, so as not to interfere with the field work.

These computations moreover had to be revised from time to time as additional observations were obtained, since in order to meet the calls of the commission for immediate results preliminary computations had to be made from reconnoitering angles.

Throughout the season neither I nor the members of my party spared our best endeavors to advance the progress of the work, and while it took much longer than was originally expected, the increased time was due in part to the fact that the large amount of real work to be done was not understood or appreciated in the beginning and in part to a series of unfortunate circumstances quite beyond my control.

Having thus reviewed the principal features of my survey, I will proceed to explain with some detail the methods of observation, of computation, and of location of monuments.

As already stated, the reconnaissance was made with the plane table, supplemented by other instruments. The angular measures were made with Repeating Theodolite No. 153, an excellent instrument, having an

8-inch horizontal circle divided to five minutes of arc and provided with three equidistant verniers which read to five seconds of arc. The telescope, which is lifted from the Y's in reversal, has an object glass of 2 inches diameter, a focal length of 16 inches, and a magnifying power of about 22 diameters. Another eyepiece of higher power was provided, but was rarely used. All of the principal angles were measured in sets of six repetitions, direct and reverse, the telescope being reversed in the middle of each set. The explement (or the difference between the angle and 360°) was always measured immediately after the angle itself, and the two results were combined to one by applying the correction necessary to make the sum of the two equal 360°. In general all the possible angles at each station were measured and the results were combined to give an approximate station adjustment. The excellence of the instrument is shown by the small corrections required in this adjustment as well as by the small errors in closing the triangles.

The signals were carefully centered, and gas-pipe poles of 2 inches diameter were used on the shorter lines, as combining economy and clearness of definition. After measuring the angles the sides of the triangles were successively computed, starting from the line "Londonderry" to "Meetinghouse Hill," used as a base. The length of this line is 22194·9^m, equal to 72817·7 feet, or about 13·8 miles. It is the longest line observed in the work, except that between "Grays Hill" and "Centerville," which is over 3 miles longer, being 27753·6^m, or 91054·6 feet.

The shortest line of the regular scheme, "Smith to Northeast Corner of Maryland," is only 168·1^m, or 551·5 feet. The whole number of triangles in the scheme is 104, of which 22 belong to the river triangulation of Assistant R. Meade Bache, executed about twelve years before.

In 36 of the principal triangles of my work the largest error of closure is 5″·4, the smallest error zero, and the mean error was 1″·66.

The triangle sides are computed by the usual formulæ:

$$b = a \sin B \operatorname{cosec} A; \quad c = a \sin C \operatorname{cosec} A;$$

after applying the corrections for spherical excess and for error of closure.

The lengths of the triangle sides having been computed, the latitude and longitude of each station were obtained by the geodetic formulæ

$$-d\,L = K \cos Z.B + K^2 \sin^2 Z.C + (\delta\,L)^2\,D - h\,K^2 \sin^2 Z.E$$

or $$-d\,L = K \cos Z.B + K^2 \sin^2 Z.C + h^2 D, \text{ for short lines;}$$

$$d\,M = \frac{K \sin Z.A'}{\cos L'}$$

and $$-d\,Z = d\,M \frac{\sin \lambda}{\cos \frac{1}{2}\,d\,L}$$

or $$-d\,Z = d\,M \sin \lambda, \text{ for short lines;}$$

the derivations of which formulæ, together with the factors $A, B, C, D, E,$

are given in the Appendix No. 7 of the Report of the Superintendent of the Coast and Geodetic Survey for the year 1884.

These position computations depend on the standard data for "Meetinghouse Hill" and "Londonderry" furnished by the computing division of the Coast and Geodetic Survey. Each station was twice computed by independent determinations from different points already known in order to obtain a comparison and avoid errors.

The number of stations so computed was 54. Next, from these geographical positions I computed by the "inverse solution" of the geodetic formulæ the position of the point which, having the same latitude as the northeast corner of Maryland, should be exactly 12 miles from the spire of Newcastle court-house. In a precisely similar way I found the distance and azimuth of this initial point of the curve from the neighboring triangulation point "Whiteman," established for this special purpose, and with these values known, the initial point was marked on the ground.

By the same inverse solution were obtained the distances and directions of the lines joining the standard points of the curve, as follows: "Initial point to Kennett–Pennsbury," "Kennett–Pennsbury to Concord–Bethel," "Concord–Bethel to Ruins," "Ruins to Kennett–Pennsbury." The triangle formed by the last three lines may be considered a plane triangle, its spherical excess being less than one-tenth of a second of arc.

The radius of the circumscribing circle may then be computed by the formula

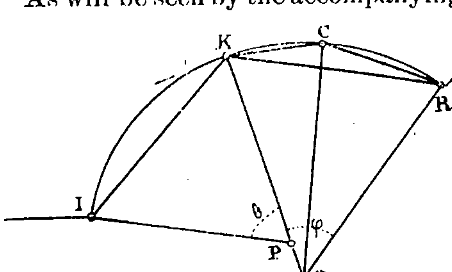
$$r_1 = \frac{S}{4 \cos\frac{1}{2} A \cos\frac{1}{2} B \cos\frac{1}{2} C}$$

in which S is the half sum of the three sides and A, B, C are the three angles of the triangle.

The radius thus found for the eastern part of the curve is $20620{\cdot}44^{\text{m}}$, equal to 67652·3 feet, or 12·81 miles. The angle at the center of the curvature between "Kennett–Pennsbury" and "Ruins" was found by the formula

$$\operatorname{Sin}\frac{1}{2}\varphi = \frac{\frac{1}{2}\text{ chord}}{r_1}$$

As will be seen by the accompanying figure, the equal angles at "Kennett–Pennsbury" and "Ruins," between the chord and the radii, are then known, each being equal to $90° - \frac{1}{2}\varphi$. These three angles must each be increased by one-third of the computed spherical excess to obtain the true spherical angles. This correction, however, is only 0″·3 for each angle. The azimuth of the center of



curvature from each extremity of the arc may next be obtained, and the latitude and longitude of that point may be computed.

We next wish to find the radius of the western part of the curve, which, starting from "Kennett-Pennsbury," with a tangent common to the eastern arc, must gradually approach the true 12-mile circle until it cuts it at the initial point. Obviously, as the two arcs have a common tangent at "Kennett-Pennsbury," their centers of curvature will lie in the same line perpendicular to that tangent. Referring once more to the figure, it will be seen that the angle at "Kennett-Pennsbury" between the chord of the arc and the center of curvature is equal to the difference of azimuth of these two directions, the azimuths of which are already known. The corresponding angle at the initial point has the same value and the angle at the center will be equal to the difference between 180° plus the spherical excess (seven-tenths of a second) and the sum of the above angles. The chord and the three angles being thus known, the radius is readily computed by the formula

$$r_2 = \frac{\frac{1}{2}\text{ chord}}{\sin \frac{1}{2}\theta}$$

It is equal to 18640·3^{m}, 61155·6 feet, or about 11·58 miles.

The latitude and longitude of this second center of curvature are computed in the same way as for the first.

The length of each part of the curve was computed separately by the formula

$$\text{arc } n^{\circ} = n \frac{2\pi r}{360}$$

The western part is 10·8977 miles long and the eastern part 11·6749 miles. Thus the total length of the curve is 22·5726 miles, and if to this we add the distance from the initial point to the northeast corner of Maryland, which is 0·7893 mile, the whole length (on dry land) of the boundary between Pennsylvania and Delaware is found to be 23·3619 miles.

After reaching the Delaware the circular boundary is supposed to continue to the New Jersey shore, as Delaware claims all of the water within the circle.

The angular value in each part of the curve of an arc one mile long may be found by dividing the number of degrees in that part by the length of the same in miles. For half-mile arcs the value will be one-half as great. As the boundary monuments are one-half mile apart, this last angle is equal to the angle of deflection from each chord to the next one. In the western arc it equals 2° 28′ 24″ and in the eastern arc 2° 14′ 09″.

The chord of the half-mile arc is equal to twice the product of the radius and the sine of one-half the angle subtended by the arc; i. e.,

$$ch = 2r \sin \tfrac{1}{2}\delta$$

The chord is less than the half mile by 0·20 foot in the western arc and by 0·17 foot in the eastern arc.

Fractional chords are computed by similar methods. The geographical positions of each of the points one-half mile apart throughout the curve, counting from the initial point, can now be computed by the aid of the data obtained as above. Then by the "inverse solution" the distances and directions of these monument stations from the triangulation points nearest to them can be computed.

All of the computations above described will be found in full in Part IV of this Report.

For applying the results of these computations to the location upon the ground of the positions for the monuments five methods were employed at different times, according to circumstances.

The first method was to put up a flag as near as possible to the probable point and so as to be visible from two stations of the triangulation. The flag being determined from these known points, the correction to the desired point of the curve was computed and applied.

The second method consisted in running out with the transit two computed azimuth lines from neighboring triangulation points to the desired point, which is fixed by their intersection.

The third method was to run out a single azimuth line as above, and to lay off on that line by direct measurement or by résection on signals the computed distance to the desired point.

The fourth method was to run out the curve by half-mile chords, making the constant deflection at the end of each chord.

In the fifth method longer chords (of 1 and 6 miles) were run out, and the intermediate points were determined by rectangular offsets.

The engineer's transit with which this work was done was unfortunately not a very good one, the transverse plate level being particularly unreliable; but it was sufficiently good for the preliminary purposes intended at the time.

The monuments, except the initial and terminal, the setting of which has been described, were put into the ground by contractors after I left the work.

In addition to the above work, necessary for the location of the boundary line, an opportunity was taken during hours unsuitable for other work to determine the position of the monument at the "tangent point."

This determination was desirable in itself, as that of a point of the greatest importance; but my principal reason for wishing it made was the desire to explain, if possible, the discrepancy between the value given by Colonel Graham for the distance from the corner of Maryland to the 12-mile circle and that which I found to exist.

The longitude of the tangent point was found to be almost identical with the longitude of the northeast corner of Maryland, showing that the line between those points is practically a meridian, as it was intended to be. The distance from the tangent point to Newcastle court-house proved to be considerably longer than it had been considered, being about 108 feet more than 12 miles and 110 feet more than the value obtained by Colonel Graham. His results were apparently deduced from insufficient data, and as he does not give his method it is difficult to see how he obtained such figures for this line and for the other distances relating to the circle which he furnished.

The above correction in the distance from the court-house to the tangent point, however, will no doubt account for a large part of the discrepancy.

From this it will be seen that the whole of the Maryland boundary is outside of the true 12-mile circle and that the Pennsylvania boundary touches that circle only at the initial point of the curve.

Circular boundary between Pennsylvania and Delaware.

Stations.	Latitude.			Longitude.			Azimuth.			Back azimuth.			To stations.	Distance.	Logarithms.
	°	′	″	°	′	″	°	′	″	°	′	″		m.	
Londonderry, Pa.	39	51	50·507	75	52	50·442	319	04	45·57	139	11	16·15	Meetinghouse Hill.	22194·9	4·3462538
Meetinghouse Hill, Del.	39	42	46·280	75	42	40·121									
Centerville, Del.	39	49	14·97	75	37	02·50	33	51	57	213	48	21	Meetinghouse Hill	14431·9	4·159325
							102	05	59	281	55	52	Londonderry	23042·7	4·362533
White, Pa.	39	45	00·43	75	48	02·35	298	18	12	118	21	38	Meetinghouse Hill	8717·7	3·940402
							151	34	27	331	31	22	Londonderry	14384·8	4·157903
Grays Hill, Md.	39	36	47·69	75	47	51·69	213	51	25	33	54	44	Meetinghouse Hill	13321·5	4·124554
							179	02	37	359	02	31	White	15198·4	4·181797
Grandview, Del.	39	42	42·65	75	42	22·71	105	07	12	285	07	01	Meetinghouse Hill	429·6	2·63311
							35	38	46	215	35	16	Grays Hill	13466·0	4·129239
Masonic Hall, Del.	39	39	36·69	75	33	54·21	115	03	30	294	57	54	Meetinghouse Hill	13829·0	4·140791
							165	54	21	345	52	20	Centerville	18389·5	4·264571
Emanuel church spire, Del.	39	39	38·77	75	33	46·83	114	30	59	294	25	18	Meetinghouse Hill	13961·9	4·144944
							70	02	05	250	02	00	Masonic Hall	187·1	2·27218
Gow, Del.	39	42	13·80	75	33	06·61	94	14	38	274	08	32	Meetinghouse Hill	13698·6	4·136677
							13	10	44	193	10	14	Masonic Hall	4976·3	3·696908
Newcastle court-house, Del.	39	39	35·33	75	33	50·13	113	28	44	293	28	41	Masonic Hall	106·0	2·02534
							191	58	30	11	58	58	Gow	4996·3	3·69865
Deepwater Point, N. J.	39	41	44·45	75	30	30·58	50	56	44	230	54	34	Masonic Hall	6250·9	3·795941
							103	42	07	283	40	27	Gow	3826·1	3·582755
Sellers, Del.	39	44	53·75	75	29	56·39	42	34	55	222	32	53	Gow	6697·5	3·825914
							7	56	40	187	56	18	Deepwater Point	5894·7	3·770462
French, N. J.	39	43	59·28	75	28	31·80	63	35	52	243	32	56	Gow	7309·4	3·863880
							129	50	09	309	49	15	Sellers	2622·6	3·418727

Ridgway, N. J.	*39*	*46*	*24·78*	*75*	*26*	*51·11*	57	32	13	237	30	14	Sellers	5228·1	3·718342
							28	07	14	208	06	10	French	5087·4	3·706492
Taggart, Del.	39	47	29·86	75	27	37·03	34	34	25	214	32	56	Sellers	5846·5	3·766899
							331	25	48	151	26	17	Ridgway	2285·5	3·358977
Naamans Creek 2, Del.	39	48	17·55	75	26	01·00	18	55	30	198	54	58	Ridgway	3676·7	3·565461
							57	14	09	237	13	08	Taggart	2717·2	3·434114
Green, N. J.	39	47	01·73	75	25	13·49	104	16	03	284	14	31	Taggart	3524·2	3·547055
							154	12	12	334	11	42	Naamans Creek 2	2597·5	3·414558
Sanderson, Pa.	39	50	12·86	75	23	50·91	41	02	06	221	00	43	Naamans Creek 2	4713·8	3·673368
							18	26	07	198	25	14	Green	6213·4	3·793332
Raccoon Island, N. J.	39	48	59·32	75	22	42·07	44	49	04	224	47	27	Green	5111·7	3·708567
							144	10	59	324	10	15	Sanderson	2797·1	3·446709
Sloan's windmill, Pa.	39	48	27·91	75	25	29·37	351	54	19	171	54	29	Green	2684·7	3·428904
							256	18	07	76	19	54	Raccoon Island	4095·7	3·612325
Ruins, Pa. and Del.	39	48	27·65	75	25	31·00	258	11	48	78	11	49	Sloan's windmill	39·65	1·59824
Terminal monument, Pa. and Del.	39	48	27·92	75	25	31·53									
Maharty, Del.	39	44	10·99	75	45	12·86	15	29	27	195	27	46	Grays Hill	14185·8	4·151853
							110	42	54	290	41	06	White	4313·9	3·634867
Rankin, Del.	39	43	12·21	75	45	00·37	19	00	59	198	59	10	Grays Hill	12542·2	4·098373
							127	37	10	307	35	14	White	5469·6	3·737955
Smith, Del.	39	43	17·86	75	47	13·49	4	19	51	184	19	27	Grays Hill	12067·4	4·081615
							240	17	32	60	18	49	Maharty	3307·1	3·519445
Northeast Corner Maryland	39	43	19·91	75	47	20·03	292	04	10	112	04	14	Smith	168·1	2·225526
							242	30	29	62	31	50	Maharty	3413·6	3·533216
Pearson, Pa.	39	43	54·78	75	46	48·60	6	31	07	186	30	27	Grays Hill	13257·2	4·122452
							257	37	41	77	38	42	Maharty	2333·9	3·368078
Whiteman, Pa.	39	43	20·85	75	46	30·77	230	11	12	50	12	02	Maharty	2415·1	3·382939
							157	54	59	337	54	48	Pearson	1129·3	3·052807
Initial Point, Pa. and Del.	39	43	19·91	75	46	26·69	290	57	02	111	05	05	Newcastle C. H.	19311·96	4·2858264
							224	43	29	44	48	49	Kennett-Pennsbury	16898·15	4·2278391

Circular boundary between Pennsylvania and Delaware—Continued.

Stations.	Latitude.			Longitude.			Azimuth.			Back azimuth.			To stations.	Distance.	Logarithms.
	°	′	″	°	′	″	°	′	″	°	′	″		m.	
Walnut, Del.	39	45	10·50	75	45	09·89	321	15	45	141	17	21	Meetinghouse Hill	5675·1	3·755970
							85	41	17	265	39	27	White	4117·4	3·614619
Mitchell, Del.	39	46	00·69	75	43	07·45	353	48	16	173	48	34	Meetinghouse Hill	6031·0	3·780391
							62	02	24	242	01	05	Walnut	3300·2	3·518546
Hoopes, Pa.	39	47	23·05	75	45	39·14	305	07	03	125	08	40	Mitchell	4414·0	3·644836
							350	20	02	170	20	21	Walnut	4146·8	3·617710
Foulk, Pa.	39	47	44·90	75	44	17·22	332	40	22	152	41	07	Mitchell	3617·5	3·558409
							70	56	03	250	55	10	Hoopes	2062·4	3·314366
O'Neill, Del.	39	47	39·62	75	42	35·50	13	59	35	193	59	15	Mitchell	3144·2	3·497511
							93	51	47	273	50	42	Foulk	2425·5	3·384796
Stephen, Del.	39	47	31·43	75	40	43·09	50	50	43	230	49	11	Mitchell	4430·8	3·646479
							95	24	14	275	23	02	O'Neill	2686·4	3·429170
McCarty, Pa.	39	48	19·57	75	42	06·36	306	50	38	126	51	32	Stephen	2475·5	3·393663
							29	22	17	209	21	58	O'Neill	1413·8	3·150402
Cloud, Pa.	39	49	26·71	75	39	39·57	23	02	11	203	01	30	Stephen	3863·4	3·586972
							59	20	33	239	18	59	McCarty	4059·6	3·608485
Gregg, Del.	39	49	24·58	75	37	18·81	307	23	02	127	23	13	Centerville	488·0	2·68843
							91	08	11	271	06	41	Cloud	3348·2	3·524811
Kennett-Pennsbury, Pa. and Del.	39	49	48·90	75	38	06·63	303	23	52	123	24	23	Gregg	1362·4	3·134318
							72	48	08	252	47	08	Cloud	2313·5	3·364275
Hamorton, Pa.	39	52	22·05	75	37	55·56	347	39	45	167	40	19	Centerville	5905·9	3·771289
							24	34	50	204	33	43	Cloud	5946·1	3·774234
Leach, Del.	39	50	07·19	75	33	38·94	71	37	00	251	34	49	Centerville	5101·8	3·707721
							124	18	30	304	15	45	Hamorton	7383·3	3·868252
Twaddell, Pa.	39	50	38·75	75	33	44·62	61	14	47	241	12	40	Centerville	5368·3	3·729837
							352	06	06	172	06	10	Leach	982·7	2·99241

Granogue, Del.	39 49 47·52	75 34 50·81	224 52 58	44 53 40	Twaddell	2230·2	3·34834
			250 26 59	70 27 45	Leach	1813·7	3·25856
Point Lookout, Pa.	39 50 22·22	75 35 25·85	258 01 43	78 02 48	Twaddell	2460·4	3·39100
			280 19 10	100 20 19	Leach	2584·2	3·41232
Talley, Del.	39 50 16·35	75 34 35·50	240 16 00	60 16 33	Twaddell	1393·3	3·14403
			98 36 00	278 35 28	Point Lookout	1210·7	3·08302
Perkins, Pa.	39 50 38·05	75 33 01·65	91 12 58	271 12 30	Twaddell	1021·8	3·00936
			42 58 26	222 58 02	Leach	1300·7	3·11418
Concord–Bethel, Pa. and Del.	39 50 20·29	75 32 27·60	289 17 08	109 21 35	Ruins	10499·06	4·0211504
			83 10 57	263 07 20	Kennett–Pennsbury	8119·47	3·9095275
Seal, Pa.	39 45 58·99	75 44 54·84	13 28 16	193 28 06	Walnut	1537·8	3·18689
			157 52 37	337 52 09	Hoopes	2798·6	3·44694
Southwood, Pa.	39 47 08·72	75 43 34·51	342 56 08	162 56 25	Mitchell	2194·7	3·341382
			260 14 31	80 16 21	Stephen	4138·2	3·616816
Crow, Md.	39 41 47·87	75 47 40·18	255 49 45	75 52 57	Meetinghouse Hill	7371·7	3·867567
			1 49 59	181 41 52	Grays Hill	9261·8	3·966694
Williams (flag in tree), Del.	39 38 26·67	75 45 10·45	51 34 26	231 32 43	Grays Hill	4909·8	3·691060
			150 06 31	330 04 55	Crow	7158·3	3·854811
Iron Hill 2, Del.	39 38 20·42	75 45 08·16	53 46 00	233 44 16	Grays Hill	4836·2	3·684508
			150 29 19	330 27 42	Crow	7352·9	3·866457
Road, Md.	39 39 00·10	75 47 30·60	7 01 31	187 01 18	Grays Hill	4114·2	3·61429
			287 08 03	107 09 33	Williams	3497·0	3·54369
Flat, Md.	39 38 50·03	75 47 29·16	282 16 38	102 18 07	Williams	3384·9	3·52954
			173 41 43	353 41 42	Road	312·3	2·49458
Tangent Monument, Del. and Md.	39 38 56·95	75 47 20·04	180 00 07	0 00 07	N. E. Corner Md.	8109·95	3·9090182
			266 25 14	86 33 50	Newcastle C. H.	19344·8	4·2865646
Center Eastern Arc.	39 39 13·77	75 33 36·02					
Center Western Arc.	39 40 14·78	75 34 01·95					
Boundary Monument No. ½	39 43 44·58	75 46 15·69	290 14 46	110 22 35	Center Western Arc	18639·97	4·270445
			19 00 34	199 00 27	Initial Point	804·60	2·90558

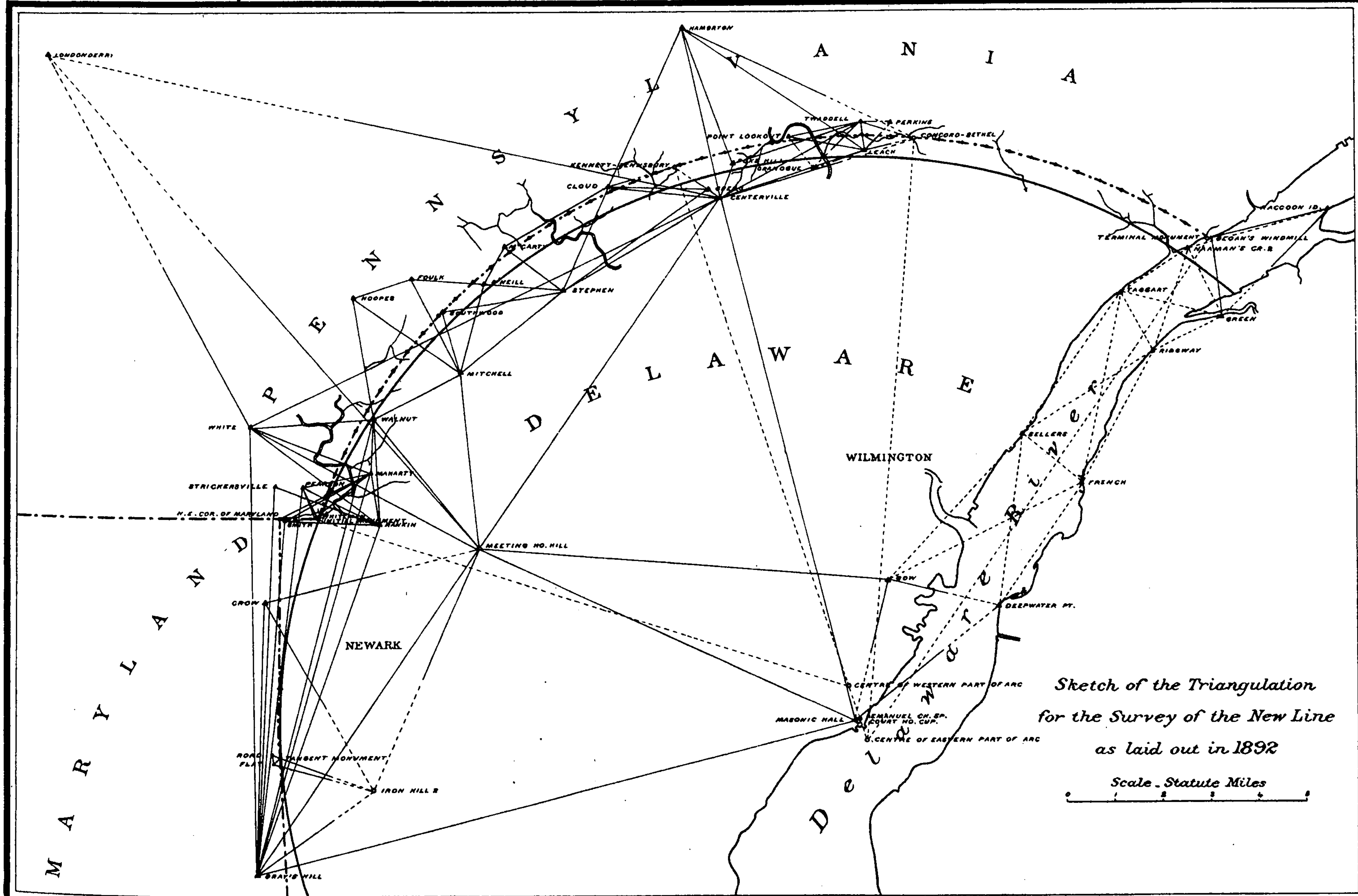
Circular boundary between Pennsylvania and Delaware—Continued.

Stations.	Latitude.			Longitude.			Azimuth.			Back azimuth.			To stations.	Distance.	Logarithms.
	°	′	″	°	′	″	°	′	″	°	′	″		*m.*	
Boundary Monument No. 1	39	44	08·85	75	46	03·31	21	29	06	201	28	58	Monument No. ½	804·60	2·90558
Boundary Monument No. 1½	39	44	32·70	75	45	49·59	23	57	39	203	57	30	Monument No. 1	804·60	2·90558
Boundary Monument No. 2	39	44	56·06	75	45	34·55	26	26	13	206	26	03	Monument No. 1½	804·60	2·90558
Boundary Monument No. 2½	39	45	18·89	75	45	18·21	28	54	47	208	54	37	Monument No. 2	804·60	2·90558
Boundary Monument No. 3	39	45	41·16	75	45	00·60	31	23	22	211	23	11	Monument No. 2½	804·60	2·90558
Boundary Monument No. 3½	39	46	02·83	75	44	41·77	33	51	59	213	51	47	Monument No. 3	804·60	2·90558
Boundary Monument No. 4	39	46	23·84	75	44	21·73	36	20	36	216	20	23	Monument No. 3½	804·60	2·90558
Boundary Monument No. 4½	39	46	44·17	75	44	00·54	38	49	14	218	49	00	Monument No. 4	804·60	2·90558
Boundary Monument No. 5	39	47	03·77	75	43	38·23	41	17	52	221	17	38	Monument No. 4½	804·60	2·90558
Boundary Monument No. 5½	39	47	22·61	75	43	14·83	43	46	31	223	46	16	Monument No. 5	804·60	2·90558
Boundary Monument No. 6	39	47	40·65	75	42	50·41	46	15	11	226	14	55	Monument No. 5½	804·60	2·90558
Boundary Monument No. 6½	39	47	57·86	75	42	24·99	48	43	51	228	43	35	Monument No. 6	804·60	2·90558
Boundary Monument No. 7	39	48	14·20	75	41	58·63	51	12	32	231	12	15	Monument No. 6½	804·60	2·90558
Boundary Monument No. 7½	39	48	29·65	75	41	31·37	53	41	14	233	40	57	Monument No. 7	804·60	2·90558
Boundary Monument No. 8	39	48	44·18	75	41	03·28	56	09	56	236	09	38	Monument No. 7½	804·60	2·90558
Boundary Monument No. 8½	39	48	57·75	75	40	34·39	58	38	39	238	38	20	Monument No. 8	804·60	2·90558
Boundary Monument No. 9	39	49	10·35	75	40	04·77	61	07	22	241	07	03	Monument No. 8½	804·60	2·90558

Boundary Monument No. 9½	39	49	21·95	75	39	34·46	63	36	05	243	35	46	Monument No. 9	804·60	2·90558
Boundary Monument No. 10	39	49	32·53	75	39	03·54	66	04	50	246	04	30	Monument No. 9½	804·60	2·90558
Boundary Monument No. 10½	39	49	42·07	75	38	32·05	68	33	34	248	33	14	Monument No. 10 Center Eastern Arc	804·60 20620·41	2·90558 4·314299
Boundary Monument No. 11	39	49	50·55	75	38	00·05	71	02	01	251	01	40	Monument No. 10½	804·62	2·90559
Boundary Monument No. 11½	39	49	58·03	75	37	27·63	73	21	01	253	20	40	Monument No. 11	804·62	2·90559
Boundary Monument No. 12	39	50	04·52	75	36	54·86	75	35	31	255	35	10	Monument No. 11½	804·62	2·90559
Boundary Monument No. 12½	39	50	10·02	75	36	21·78	77	50	01	257	49	40	Monument No. 12	804·62	2·90559
Boundary Monument No. 13	39	50	14·52	75	35	48·45	80	04	31	260	04	10	Monument No. 12½	804·62	2·90559
Boundary Monument No. 13½	39	50	18·00	75	35	14·91	82	19	02	262	18	40	Monument No. 13	804·62	2·90559
Boundary Monument No. 14	39	50	20·48	75	34	41·22	84	33	33	264	33	11	Monument No. 13½	804·62	2·90559
Boundary Monument No. 14½	39	50	21·94	75	34	07·44	86	48	03	266	47	41	Monument No. 14	804·62	2·90559
Boundary Monument No. 15	39	50	22·37	75	33	33·60	89	02	34	269	02	12	Monument No. 14½	804·62	2·90559
Boundary Monument No. 15½	39	50	21·79	75	32	59·77	91	17	04	271	16	42	Monument No. 15	804·62	2·90559
Boundary Monument No. 16	39	50	20·19	75	32	25·99	93	31	35	273	31	13	Monument No. 15½	804·62	2·90559
Boundary Monument No. 16½	39	50	17·57	75	31	52·32	95	46	06	275	45	44	Monument No. 16	804·62	2·90559
Boundary Monument No. 17	39	50	13·93	75	31	18·81	98	00	36	278	00	14	Monument No. 16½	804·62	2·90559
Boundary Monument No. 17½	39	50	09·29	75	30	45·51	100	15	06	280	14	45	Monument No. 17	804·62	2·90559
Boundary Monument No. 18	39	50	03·65	75	30	12·48	102	29	36	282	29	15	Monument No. 17½	804·62	2·90559

Circular boundary between Pennsylvania and Delaware—Continued.

Stations.	Latitude.			Longitude.			Azimuth.			Back azimuth.			To stations.	Distance.	Logarithms.
	°	′	″	°	′	″	°	′	″	°	′	″		*m.*	
Boundary Monument No. 18½	39	49	57·01	75	29	39·75	104	44	06	284	43	45	Monument No. 18	804·62	2·90559
Boundary Monument No. 19	39	49	49·40	75	29	07·39	106	58	36	286	58	15	Monument No. 18½	804·62	2·90559
Boundary Monument No. 19½	39	49	40·81	75	28	35·44	109	13	05	289	12	45	Monument No. 19	804·62	2·90559
Boundary Monument No. 20	39	49	31·27	75	28	03·95	111	27	34	291	27	14	Monument No. 19½	804·62	2·90559
Boundary Monument No. 20½	39	49	20·78	75	27	32·97	113	42	03	293	41	43	Monument No. 20	804·62	2·90559
Boundary Monument No. 21	39	49	09·37	75	27	02·54	115	56	31	295	56	12	Monument No. 20½	804·62	2·90559
Boundary Monument No. 21½	39	48	57·05	75	26	32·72	118	10	59	298	10	40	Monument No. 21	804·62	2·90559
Boundary Monument No. 22	39	48	43·84	75	26	03·55	120	25	27	300	25	08	Monument No. 21½	804·62	2·90559



Sketch of the Triangulation for the Survey of the New Line as laid out in 1892

Scale - Statute Miles

0 1 2 3 4 5

UNITED STATES COAST AND GEODETIC SURVEY.

APPENDIX No. 9—REPORT FOR 1893—PART II.

PROCEEDINGS OF THE GEODETIC CONFERENCE,

HELD AT

WASHINGTON, D. C.,

JANUARY 9 TO FEBRUARY 28, 1894.

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GEODETIC CONFERENCE.

GENERAL REPORT.

United States Coast and Geodetic Survey,
Washington, D. C., February 28, 1894.

Sir: The Geodetic Conference called by you convened at Washington on January 9, 1894. The details of its organization are appended as a preface to the reports of the committees to which the duty was assigned of collating necessary facts and formulating them for the consideration of the Conference.

An invitation was extended to members of the office and field force not members of the Conference, as well as to others interested in the subjects under consideration, to give expression to their views on matters relating to the geodetic operations of the Survey.

The Conference desires here to make grateful acknowledgment to those who responded in writing, as well as to those who, by personal attendance and verbal communications, gave their valuable time.

Professor Harkness, U. S. N., addressed the Conference on the various methods of determining the earth's figure. Professor Woodward, of Columbia College, New York, gave an account of base measurement with steel tapes, and discussed, in response to questions, various subjects relating to the work of the Survey. Professor Gore, of Columbian University, Washington, D. C., evinced his interest by being present at several meetings, in which he took part, while Professors Buchanan, of Tennessee, and Hoag, of Minnesota, Acting Assistants, United States Coast and Geodetic Survey, shared the labors of the Conference for part of the time during which it was in session.

The task assigned to the members of the Conference was the consideration of the geodetic methods as now practiced, the discussion of the application of these methods to the duties assigned by law to the Coast and Geodetic Survey, and the comparison of the means employed in foreign countries for similar purposes, with the object of suggesting improvements in regard to accuracy and economy.

The general plan of the geodetic operations followed by the Survey is that outlined by Mr. F. R. Hassler and approved by President Jefferson and Mr. Madison, Secretary of State, and subsequently prescribed by law in the "Basis for the reorganization of the Survey" (approved by President Tyler, March, 1845).

The wise prevision of those who promoted and approved these plans has been shown in this, that the latter proved themselves sufficient, without material modification, to meet the changed conditions caused by the acquisition of new coast line by the United States, the extension of the triangulation inland, and for the determination of geographical positions in the interior.

In the early days of our Republic the great migration of peoples in and to the United States, and the settling of vast areas hardly known previously to geography, rendered a sole reliance on geodetic methods inapplicable to surveys, the main purpose of which was the demarcation of Government lands for settlement. To meet the necessities of the case the excellent system of rectangular land surveys was applied, which is well adapted to the large areas of level land in this country. For certain purposes it is, however, insufficient, as shown by the important trigonometric surveys of the Mississippi and Missouri rivers, where systematic improvements ordered by Congress could only be based on extensive and accurately correlated and detailed surveys, and where, therefore, trigonometric methods were found necessary in order to supplement the system and adapt it to the ordinary use for which surveys are made. The older States neither received nor required the rectangular subdivision, and, therefore, such States, using the work done by the Survey as a basis, have adopted the trigonometric method, which, moreover, is also being applied in States subdivided on the rectangular plan.

In view of these facts, and of the experience of all European countries, not to mention India, the French possessions in northern Africa, the English in southern Africa, and the Dutch East Indies, that trigonometric surveys are necessary, it seems safe to assume that the inland geodetic operations of this Survey will, in addition to meeting an existing demand, benefit future generations in this country as much as those along the coasts have already benefited the present.

The practical bearing of these views on the subjects submitted to the consideration of the Conference has been taken into account in the discussion of all the questions as to the location of the trigonometric schemes already called for in the work of the Survey, as well as those likely to be required for the purpose of relating the local surveys to the general map of the country.

The need of association of bodies pursuing the higher branches of culture in the various countries was early felt in the case of geodesy, and it must be considered fortunate and beneficial for both parties that the United States, by the liberality of Congress in 1889, was able to join the International Geodetic Association for the determination of the earth's figure and size, and thus add its influence and contribute its share toward the realization of this object. It must not be supposed that the labors of this organization were rather theoretical than practical. On the contrary, their eminently practical character throws light on all branches of geodesy and surveying; and it was made the duty of the Conference convened by you, and one of its objects, to compare the various methods pursued by other nations with those used at home.

The Conference has applied itself to this labor with a view of pointing out the means by which, or directions in which, the system pursued by the Survey would in any way be improved in accuracy and rapidity of work as well as in economy, so as to produce results at once available as well as enduring.

The subject-matter submitted to the Conference comprises a wide range of applied science, dealing with the geodetic (inclusive of hypsometry), the astronomic, the magnetic, and the gravitation work of the Survey. The suggestions made as the outcome of the deliberations cover not only the present conditions of the work, but prepare the way for such future work as the legitimate development of the Survey would seem to demand.

To expedite business and to secure its best consideration the Conference distributed the work among a number of committees, as already stated, particularly selected to include those members who had more special knowledge or experience of the subject assigned to them. The members of each committee contributed to the preparation of the report, which, after full discussion, was submitted by its chairman to the conference at large, where it received its final consideration before adoption by a majority vote.

The committee reports thus prepared will be submitted in full. The deductions drawn from them by the Conference and the conclusions reached are herewith briefly stated for your convenience. The reasons for them can, in general, be gathered from the reports themselves, though many facts and details which were considered have not found a place in them.

ABSTRACT OF REPORT OF THE COMMITTEE ON RECONNAISSANCE.

The committee uses the term "reconnaissance" to embrace all those investigations of a region to be triangulated which precede the field work, base measurement, and the measurement of angles, and to comprise the selection of the most feasible chain or network of geometric figures, the location of base lines, the character of the necessary preparations, and the collection of information concerning the facilities available for carrying out the proposed scheme.

All reconnaissance should be thorough and exhaustive, and develop all possible schemes of triangulation. It should afford information by which economy and rapidity of execution can be obtained. When exhaustive, it will lead to the simplification of the geometric figures. In no case should geometric elegance of a scheme be sought at the expense of reasonable economy. In the interior of the country the highest peaks are frequently covered with clouds and should not be used as triangulation points if it can be avoided.

A simple chain of triangles is recommended as the most easily adapted to the most complex orography. Pentagons or quadrilaterals, with central points, also readily conform to the configuration of the country, however complex or difficult it may seem.

It is recommended that sites for the location of bases should be looked for at every eighth to tenth figure of the scheme of triangulation, according to its character and scope.

Before taking the field the officer should study all available maps of the region and familiarize himself with the lines of travel, locations of towns and villages.

Notes in the field can not be too full. Horizontal and vertical measures should be taken on all prominent objects. Approximate latitudes and azimuths should be observed, as well as the magnetic bearings of every notable object. The entire horizon should be sketched, and rough topographical sketches made to show main ridges, water courses, etc.

The atmospheric conditions should be noted and the facilities for obtaining water and forage.

The committee concludes with some suggestions in regard to instrumental outfit.

ABSTRACT OF REPORT OF THE COMMITTEE ON BASE LINES.

The committee classifies the base apparatus in use under three general heads:

Contact apparatus, comprising compensating, bimetallic, and mono metallic bars.

Optical apparatus, comprising bimetallic, monometallic, and bars in melting ice.

Steel tape and wire apparatus.

They then give a condensed statement of recent measures in Europe and India.

Brief descriptions, instances of their application and possibilities, are given for the Colby, Bessel, Brunner, Struve, Austrian, and United States Lake Survey Repsold apparatus. The various forms of apparatus used on or devised for the Coast and Geodetic Survey are mentioned, such as the Hassler bar, the Bache-Wurdemann, the Schott, the secondary contact, the Woodward steel bar in melting ice, the Eimbeck duplex, and the steel tape. As detailed descriptions of all these forms appear in the Coast Survey reports, they are not repeated here, but their principles are stated, results of their use, and, when possible, instances of their comparative uses are quoted and recommendations made concerning the particular classes of bases they are adapted for.

Foreign reports do not contain sufficient data upon which to base statement of cost, and only in two instances do we find any record of the number of officers and men required for a measurement abroad.

Of the more recent forms of apparatus tried at the Coast and Geodetic Survey, particular mention is made of the Woodward steel bar in melting ice, used in four measures of one kilometre of the Holton Base, with this expression of opinion: "The salient facts resulting from the experiments made, then, appear to be that with the 5^{m} steel bar in melting ice, whereby the temperature error is eliminated, a base line can be measured with a degree of accuracy hitherto unapproached. In economy of operation it is little, if any, inferior to the Brunner apparatus now used abroad."

An account is given of the details and results of the measurement of the Holton Base with the secondary contact slide apparatus and the steel tape. The committee considers that with the contact slide apparatus a degree of precision is obtained beyond the needs of the triangulation, and the contact slide is commended. The cost of measuring with the secondary contact slide apparatus is about the same as that with the tape.

In the description of the steel tapes used on the Holton Base the opinion is expressed that, "carefully and judiciously handled, the steel tape apparatus will doubtless attain good standing throughout the United States, where so many extensive and independent surveys are being carried on, and especially in all newly undertaken projects."

Comparisons of the earlier and later base measurements of the Coast and Geodetic Survey are given, and notes on the site of the base lines and the procedure of field organization and measurements.

The accuracy attainable in base measurements is based upon practice in the field, comparison with the standard, and the accuracy of the new metric prototypes. After enumerating the conditions under which comparisons of standards are made, the committee says:

We may conclude that no geodetic standard can be known with a higher degree of accuracy than 1 part in 5 000 000 of its length, in terms of the international unit.

That it is possible to measure a base line repeatedly with the same apparatus with a surprising accordance between measures indicates that the elimination of accidental errors has been successfully met by the different apparatus in use, and it is also believed that the various methods of observing successive lengths of the same bar or systems of bars are sufficiently precise.

On the other hand, that constant errors exist which are in the main due to a defective knowledge of the temperature of the bars is a fact commonly assumed, or proven by the lag of mercurial or bimetallic thermometers used on various apparatus. These constant errors are not easily determined, but are now the principal sources of errors.

Examples of recent measurements and discrepancies are given. No definite limit is assigned to the degree of accuracy, because the measure of a base line depends largely on the object it has to subserve and on the apparatus, time, and money available.

The performance of the contact slide in its various adaptations now in use on this Survey, as well as that of the tape, warrant the conclusion that our present methods of measuring bases are unexcelled in point of economy, rapidity, and accuracy.

"With the perfection of means now available a line may be readily measured with a probable error of 1:1 000 000, so far as mere measurement is concerned." But this accuracy is rapidly dissipated by the known and unknown errors, and especially through the three or more steps in the triangulation required to reach the first line of average length. Great accuracy in the angular measurements is needed if an error of 1:150 000 is to be maintained throughout the network of triangulation.

For secondary triangulation the fraction of the length 1:100 000 to 1:50 000 may be suitable, and the degree of accuracy in the base may be graduated accordingly.

For tertiary triangulation an average uncertainty or limit of 1:10 000, or even 1:5 000 may be allowable for the special purpose of the work.

The subject of frequency of base lines is referred to by this and another committee.

It is recommended that whenever the steel tape is used in primary work at least four measurements be made and each section be measured under rising and falling temperatures. It is particularly recommended that all base lines be measured at least twice.

In conclusion, the committee recommends—

That further experiments with the steel tape be made, especially with a view to a better determination of its temperature. This recommendation seems to be warranted by the results of the measurements of the Holton Base.

2. That the new duplex apparatus just completed by the Survey be given a thorough and careful trial as soon as practicable.

3. That the iced bar be used to lay off a 100^{m} distance as a comparator at the bases where the above recommendations are followed.

4. That in all measurement due regard be paid to rising and falling temperatures, so as to eliminate as much as possible the errors due to lack of knowledge of temperature of the measuring apparatus.

5. That the tripods for supporting bars be made of metal, and embrace the details which experience has shown to be conducive to accuracy and rapidity of measurement.

6. That ordinarily the base line be divided into sections one-half kilometre in length.

7. That a 100^{m} comparator be established in Washington for the purpose of testing steel tapes and contact slide apparatus.

8. That in future use of the iced steel bar the water in the Y trough be not drained off below the surface of the bar, in order to remove any possibility of doubt as to the actual temperature of the bar.

ABSTRACT OF THE REPORT OF THE COMMITTEE ON TRIANGULATION.

The committee has classified the various subject-matters discussed by it under eighteen heads, with a map to exhibit the area of triangulations already executed, that under progress, and projected lines.

The principal subjects are: the object of triangulation; classification; main and primary, secondary and tertiary triangulations; adaptation to the surface of the ground; general form of main triangulation; geometrical composition of a triangulation; frequency and length of base lines and their connection with the triangulation; accuracy of a triangulation, and comparisons with foreign work; international and interstate boundaries. Then follow instrumental outfit, method of observation, marking stations, signals, etc.

In the execution of a scheme of triangulation the character of the work will range from what is purely geodetic to plane surveying.

The main triangulation comprises the principal series of geometrical figures which compass along the shortest line the whole extent of territory under consideration. The subordinate divisions, with primary, secondary, and tertiary, are more technical. The primary is characterized by the greatest development of length of sides and by the greatest accuracy of measurement, wherein the geodetic positions depend on the mean of numerous astronomical latitudes, longitudes, and azimuths, and the initial, intermediate and terminal base lines are directly measured. The committee recommends that observations for latitude and azimuth be made according to the character of the project. In the primary triangulation these have been in some localities made at every station, but frequently every other station has been so occupied. Further particulars are enumerated, and then follow descriptions of the other schemes.

The frequency and length of base lines and their connection with the triangulation depend not only upon the orographical features of the country, but upon the required accuracy of the triangulation. Because base lines can be measured with much greater accuracy than the triangulation can maintain, it is recommended to increase the number of base lines rather than to increase their length.

In arriving at the desirable accuracy of a triangulation the object of the work must be considered. From an economical standpoint the degrees of accuracy in the measurement of a base line for different projects are given, ranging from 1:200 000 to 1:1 000 000 or a still smaller fraction. But to maintain an accuracy of 1:150 000, or even 1:100 000 part of the length, is a matter of difficulty in an extended triangulation. The accuracy of the base is rapidly dissipated in the adjacent base figure, and hence it is not expedient to strain after excessive refinement in base measurement.

The committee then examines 73 triangles of the main and primary triangulation, and shows that the work of the United States Coast and Geodetic Survey stands well in the front rank. In the triangulation just referred to, the Survey inaugurated a scheme on a scale so large that there was no previous experience to guide it or suggest the attainable accuracy of the work. A critical consideration of the angle measures shows that the number of observations might have been reduced by one-half and still give results which will rank with the best foreign work. The period of observation at a station must be somewhat shortened, but a high authority is quoted to warn against hurrying observations lest the atmospheric conditions introduce larger errors than may be attributable to the instrument. This is a very important subject and should be carefully considered by the observers.

The number of series of observations recommended at a principal triangulation station is about 31, while the number of positions of the circle should be, in general, much smaller.

In the eastern part of the transcontinental triangulation, where the sides average 26 kilometres (16 miles) in length, a discussion of several hundred triangles suggests the reduction of the number of observations to about one-half or two-thirds the number recommended for the primary triangulation.

In comparison with thousands of triangles of this class of work in Europe, that of the Coast and Geodetic Survey must be rated as among the best.

In tertiary triangulation the demands of accuracy may vary from 1:20 000 even to 1:5 000 part of the length.

The committee treats of the determination of international and interstate boundaries under the principal heads: Boundary along a meridian, along a parallel, along oblique lines. The methods of tracing each are presented in some detail, with illustrations.

Under instrumental outfit are considered the merits and demerits of direction and repeating instruments for the main and primary triangulations.

Whenever practicable, the large theodolites should be mounted upon concrete or masonry piers.

In order to secure uniform size of heliotrope images at all distances a formula is given for determining the size of the heliotrope for a given distance. All heliotropes should be centered with the same accuracy as the theodolite. Their superiority on long lines is pointed out.

The method of measurement of angles with the repeating instrument is drawn up from the experience of the observers, and directions are given for making the records, etc.

The committee has also discussed the different forms of signals and elevated structures for observing, the marking of stations below and at the surface of the ground, etc.

Vertical angles should be measured at all main and primary triangulation stations, and the direction of all prominent, natural, and artificial objects in the horizon should be observed.

It is recommended that magnetic observations be made at all triangulation stations.

ABSTRACT OF REPORT OF ASTRONOMICAL COMMITTEE.

The committee has called attention to the various methods of determining longitude as required for different purposes and suited to different conditions. The most important of these is the telegraphic method, which was introduced by the Coast Survey. The mode of operation was much simplified in 1878, and a brief method of field computation adopted. The number of nights of observations required is stated to be from six to ten, the observers exchanging places. Comparisons with the results obtained in foreign countries show that the Survey maintains an equally high standard, even when the number of nights has been but one-half of that used abroad.

The expense of operating a longitude party with the present outfit has been reduced to a minimum, and without the extension of time at a station observations are also made for latitude and the magnetic elements. With the addition of another observer to the party it is proposed to observe for gravity.

The committee suggests that the longitudes of the Aleutian Islands may be determined by the use of terrestrial signals and the local time at different stations, and that the chain be subdivided and certain stations connected chronometrically with Sitka as the base station. This method is recommended.

For latitude observations the committee has shown that the number of pairs of stars and the number of nights of observations have been gradually decreased on the Survey. This has arisen in a large measure from the better determinations of the star places as given in the later catalogues. It is believed that ten or twelve pairs of stars, observed upon three or four nights with the zenith telescope or meridian instrument, will give the latitude of a station with a probable error not exceeding a tenth of a second. This is so far below the average local deflections of the vertical that greater accuracy is not deemed necessary, except in such special cases as the measurement of arcs and the determination of international boundaries.

For latitude observations the committee recommends the continued use of the Talcott method by the zenith telescope or meridian instrument as affording greater accuracy than by other methods. It recalls the competitive observations instituted by Superintendent Bache with the zenith sector, zenith telescope, prime vertical transit, and vertical circle. It is suggested that an enlarged catalogue of latitude stars is greatly needed.

The committee suggests that the four principal nations publishing Ephemerides should combine in the expense of preparing and issuing a special star list of greater extent than is now issued, so that the number of time stars may be increased to an average of one for each two minutes of time; and also that the American Ephemeris should extend the additional star places so as to cover the period now falling in daylight; also that more azimuth stars should be added, and whenever practicable grouped by differences of nearly twelve hours in right ascension. The subpolar places should be given in order of right ascension. With close circumpolars this would be useful in determining the value of a micrometer screw.

ABSTRACT OF REPORT OF THE COMMITTEE ON HYPSOMETRY.

The committee states the degree of accuracy demanded in topography, physical hydrography, base-line reduction to sea level, gravity observations, tidal planes, meteorological investigations, and engineering operations, and gives an account of the Y-level of civil engineers and the geodetic level of the Coast and Geodetic Survey.

About 9 800 kilometres (6 100 miles) of precise levels have already been run by different corps of the United States to meet the demands of special investigations and in the regular work of the Coast and Geodetic Survey. The desirability of a consistent scheme of lines of control of the whole country, to which all future work may lend itself, has made itself apparent. The committee presents considerations for such a scheme, as follows:

To provide a means, the most direct and economical, for connecting the many tidal stations of the Atlantic, Gulf, and Pacific seaboards.

To connect these tidal planes by routes which will best overcome the uncertainties arising from crossing mountain chains, etc.

To form closed figures that will best determine the degree of accuracy of the work.

To make the lines of level conform as nearly as possible to existing or proposed schemes of triangulation, and especially of arc measures.

To take advantage of all work of a like nature heretofore executed, and to distribute judiciously over the country bench marks which may serve as points of departure for hypsometric work of all kinds.

Work of this class has been executed by almost all European nations, and the committee gives an extended report of the operations and the methods. Descriptions of the instruments are given. Throughout France operations of the most exhaustive character have been or are being carried out. In fact, the whole of Europe is covered with lines of precise leveling. According to the report of the International Geodetic Association, the total length is 102 800 kilometres (64 250 miles). This does not include Great Britain. It is shown how carefully methods and means have been studied and applied so as to attain the highest degree of accuracy possible. Then follows an account of precise levels in the United States by the Corps of Engineers in the survey of the Great Lakes, by the Mississippi and Missouri River Commissions, and by the Coast and Geodetic Survey. The methods of procedure are stated and the conclusions reached to account for the cumulative errors which are found in all series of levelings.

In 1892 an elaborate series of experiments were undertaken by the Survey to investigate the system of leveling and the character of the instruments. These have not been fully discussed, but have resulted in valuable suggestions of a practical nature.

Results of long lines of leveling by the engineer's Y-level in the United States are presented, and the conclusions deduced from work done in Prussia with similar instruments are quoted.

Experiments with the geodetic level and with the Y-level are in progress by the Survey.

The subject of trigonometrical leveling is discussed and a few of the results in the United States and India are presented.

The determination of differences of elevation by observations of atmospheric pressure is at times the only means available, and the method of observing the boiling point of water is recommended as suitable for exploration surveys.

The committee concludes that the measure of trigonometrical heights should form part of the work of all triangulation stations. For this work they suggest that a new form of instrument be devised to provide for the measurement of larger arcs by micrometer differences;

That the standard bench marks throughout the country should be determined with the greatest degree of accuracy attainable;

That subsidiary lines may be leveled by less precise methods;

That if, upon investigation, it shall appear that less elaborate methods or instruments than those now employed on the Survey will, by the use of small circuits, produce satisfactory results with an increase of economy, purely theoretical considerations should not prevent their adoption;

That the present geodetic level of the Survey is as perfect as any yet devised.

Recommendations are made for comparisons of rods with the standard, and for various details of the rod and supports.

ABSTRACT OF REPORT OF THE COMMITTEE ON ALASKA.

The committee presents an account of the extraordinary growth of this but partially explored country, with its valuable resources on land and the limitless wealth in its adjacent waters. Statements are made of the extent and wealth of the forests, the fisheries, the mining, the furs, with statistics of the commerce and population. The surveys already made are considered, and due credit given for the hydrography, triangulation, and astronomical work that has been done. The astronomical work covers the chronometric determinations of several seasons; but the systematic trips of the last two years and the latitude work executed at the longitude stations occupied have added materially to our astronomical control of the work in southeastern Alaska.

The traffic through the great inland water passages of southeastern Alaska made their immediate survey a matter of the most pressing importance. This work is now nearly accomplished; but the committee considers that the present standard of worth should be made uniform throughout. It advises that the topographic features should receive more attention, and recommends that a topographic reconnaissance be made to supply what is needful in this respect.

The question of the order of sequence in which the Alaskan work should be taken up was considered, and the conclusions reached are herein briefly recounted:

First. The great chain of the Aleutian Islands is known to be very imperfectly laid down on existing maps, and the demands of the whaling fleet, the naval vessels of the United States and Great Britain, the cod-fishing fleet, and the shipping and trading vessels require that the passes through this fog-covered line of islands should be better determined.

It is proposed by the committee to establish certain base-longitude stations for chronometric determinations from Sitka, and to connect all the islands by means of terrestrial signals and local time. The heights of the islands combine to render this feasible, and the method is second in accuracy only to the telegraphic method. These longitude determinations with latitude measures will give certain positions on some of the islands. Small triangulation, and perhaps topographic reconnaissance, will then give their outline with the necessary precision for the present.

Second. The great industries that are centered about Kadiak and the western end of the Aleutian peninsula make a survey of this section only second in importance to the one just mentioned.

Third. The welfare and safety of the great fleet of American vessels engaged in the Arctic whaling industry make it necessary and a duty of the Government to establish a good longitude station at their rendezvous, Port Clarence.

Fourth. The necessity for opening a ready means of entrance into the heart of Alaska, that its great possibilities may be thoroughly explored and tested, and the returns that will reward our citizens when the great wealth of salmon that abounds in its waters can be reached, make it important that a survey should be made of the mouth of the Yukon River—the Mississippi of this vast territory—and which is now practically useless to us on account of our almost complete ignorance of the channels and waterways through its delta.

The committee submits plans for the longitude work which would control the independent surveys to be made between Kadiak and Attu. It also suggests arrangements which are possible in connection with the work at Port Clarence and about the mouth of the Yukon, and which might materially reduce the cost of both enterprises.

Its final proposition is a scheme for a triangulation through Clarence and Chatham straits, the great channels of the Archipelago Alexander, which would reenforce and control all the work in southeastern Alaska. In this scheme the character and main features of the work are considered with relation to the meteorological conditions which must be expected and the economy that should be borne in mind in suggesting a plan of operations.

ABSTRACT OF REPORT OF THE COMMITTEE ON INSTRUMENTS.

The committee has confined its attention to the consideration of instruments for astronomical work and the measurement of horizontal angles, because other committees will necessarily consider instruments special to their work.

A list of 52 astronomical instruments and 62 theodolites (namely, 21 direction and 41 repeating instruments) are given. The character and condition of the instruments are described in general terms. Comparison is made between the instruments used in the Coast and Geodetic Survey and in Europe for the determination of latitudes and of telegraphic longitudes; and the committee recommends a lighter form of chronograph.

The 30^{cm} (12-inch) direction theodolites, No. 145 and No. 146, lately constructed at the office, have been tested and are believed to be of superior character.

The committee recommends that all instruments whose graduation is defective should be sent to the office for regraduation, the purchase of 10^{cm} and 18^{cm} (4 and 7 inch) instruments for the work in Alaska, and that erratic motions of the level bubble be studied.

It makes suggestions in regard to protecting instruments in the field against sudden changes of temperature.

In conclusion, the committee states that the instruments for astronomical work and horizontal measures are in general very good.

ABSTRACT OF REPORT OF THE COMMITTEE ON OFFICE AND FIELD RELATIONS.

The committee states the necessity for the effective cooperation between the field force and the office; and, in order to still further promote this object, recommends that the regulations of 1887, with amendments, and the circulars since issued, should be carefully studied and their provisions minutely followed.

The preparation, duplication, and transmission of records should be thoroughly systematic. They, together with a summary report of the season's work, accompanied by a sketch and statistics, should be promptly turned into the office at the close of the season. The field computations should follow as soon as practicable. The condition and character of all instruments should be clearly stated when sent to or from the office. Stations in the vicinity of any field party should be visited when practicable, and their condition reported. If reported as irretrievably lost, the names should be stricken from the office list of geographic positions available for field work.

As the field parties are cut off from easy access to current publications bearing upon their work, the committee recommends that some person be appointed in the office to prepare brief extracts or headlines of matter germane to the Survey and its operations, and that copies of these be transmitted to the parties in the field and at distant stations.

The Superintendent referred to the Conference a paper prepared at his request by the disbursing agent, upon the correlation of the operating department and accounting system of the Coast and Geodetic Survey. This very interesting paper, referred by the Conference to the committee, is appended in full to its report, and should be carefully studied.

The committee recommends the preparation of a manual of observations, records, and computations, embodying the conclusions of the Conference, and a manual of accounts, in accordance with the valuable suggestions contained in the paper of the disbursing agent.

ABSTRACT OF REPORT OF THE COMMITTEE ON THE MEASUREMENT OF ARCS.

The committee shows that the measurements of arcs of the meridian, of the parallel and oblique to these, has been incidental to the extended operations of the trigonometrical survey of the United States. They are moreover of great importance in order to furnish the shape and dimensions of that geometric figure which best represents the particular area to be surveyed and to secure exact measures of geographic position on the earth's surface. Beyond this, the combination of the arcs measured by different countries is indispensable for the determination of the geometric figure which shall, in size and shape, most closely approximate to the figure of the earth as a whole.

For this latter purpose the Government of the United States became a member of the International Geodetic Association for the measurement of arcs, and thereby incurred certain scientific obligations which have in part been fulfilled by the triangulation schemes already executed. The further and necessary extension of these projects of triangulation in the prosecution of the Survey will furnish additional material for utilization in our country and for the International Association.

The arcs in process of execution are mentioned, and a map has been prepared to exhibit the prospective triangulations which may be developed, and which will be, in great measure, available for the location of boundaries, for bases of State surveys, and for geographical positions.

ABSTRACT OF REPORT OF THE COMMITTEE ON MAGNETICS.

The committee states the necessity for a knowledge of the laws governing the magnetic force, in order that the charts of the Survey shall be furnished with the magnetic variation at the dates of issue and the prospective annual change.

The distribution of magnetism is dependent mainly upon geographical position and time, but is influenced by so many local disturbing causes that in order to produce the lines defining the direction and intensity seaward for a certain epoch it is necessary to extend the observations a sufficient distance inland.

Moreover, there is such a constant demand made upon the Survey by surveyors, engineers, and courts of law in every part of the country for information looking to the recovery of old lines or landmarks that a more complete series of observations and study of the distribution of the declination is necessary over the whole area of the United States.

A demand for an accurate knowledge of the magnetic dip and intensity has been made by mariners and electricians.

By observation in the regular progress of the work, the collection of observations from the earliest to those of the present time, and by other means, the Survey has given all necessary information for the charts, and in addition very much material which has been published in print and graphically for the use of surveyors, engineers, etc.

The committee presents general descriptions of the magnetic instruments used in other countries and the recent ones for use in the Survey.

While much of the work of magnetic observation is executed incidentally by the different parties in triangulation and longitude, the committee calls attention to the necessity for a special series of observations, particularly through California, Oregon, Washington, Idaho, Montana, and the Dakotas; also to the necessity of more observations in Alaskan waters.

It recommends the use of compass declinometers by every triangulation party, and that the main and primary triangulation and telegraphic longitude parties determine all the magnetic elements. It also recommends that a second edition of the isoclinic, isodynamic, and isogonic curves be published for an epoch close at hand—say, 1895 or 1900—together with the data, method of discussion, and explanation of the results and their uses.

ABSTRACT OF REPORT OF THE COMMITTEE ON GRAVITY.

The committee states that recent developments in the instruments and methods will enable the Survey to enter successfully upon extended gravimetric research at less cost than was possible with the older processes, and this without lowering the standard of accuracy. The committee refers to the development of the earlier pendulum research on the Survey with respect to character of instruments, their shape, size, and material, as well as the methods of observing.

All nations engaged in geodetic work have considered this subject an important and necessary branch of a geodetic survey, as affording a means of determining the figure of the earth. Our relation with the International Geodetic Association renders it desirable that the Survey should conform, as far as practicable, with the general plan of work of that body.

Five hundred stations have been occupied by foreign observers, while by the Coast and Geodetic Survey 27 have been occupied in this country and 29 abroad. In many of the leading European countries pendulum investigations have been vigorously prosecuted in recent years. In India the English have carried on a very systematic scheme of gravity work in connection with their great trigonometrical survey.

Maps are presented showing the stations occupied in the United States, including Alaska, and in Europe.

The improved pendulum apparatus of the Survey is described. The method of observing permits a ready application of the telegraphic method of determining the difference in the force of gravity at two stations which are connected by wire.

A still more portable apparatus is now being constructed, in which the pendulum has a period of but one-fourth second.

The proposed outline of investigations contemplates the determination of the geographical distribution of gravity within the United States with respect to latitude, elevation, and geological structure, including, for instance, a series of gravity measures along the valley of the Mississippi, and another at right angles to it along the thirty-ninth parallel from the Atlantic to the Pacific.

It is proposed to establish, in the course of a few years, a number of base stations for gravimetric and hypsometric operations of the Survey. These will be determined with the greatest precision. The absolute force of gravity is such an important physical constant and of such great scientific interest as to justify its measurement at a few base stations.

In observations intended for the determination of the figure of the earth it is proposed to restrict the stations to the sea border of the continents and islands of the United States, so as to obviate a reduction to the sea level. The extended range of latitude in the limits of the United States is favorable to this measure.

It is not yet deemed practicable to state what degree of precision may or should be reached in either absolute or relative work, but observations at a station should be continued only long enough to reasonably eliminate the known errors of observation. Multiplication of stations is to be preferred to a high degree of accuracy.

In 1882 a conference on gravity determinations was held at the office of the Coast and Geodetic Survey, and the conclusions which were then reached are given at the close of the committee's report.

ABSTRACT OF REPORT OF THE COMMITTEE ON EQUIPMENT.

The committee has considered this subject under two principal heads, operations conducted by land and operations conducted by water. Each is then considered under several subheads.

The committee recommends the lightest practicable outfits of men, animals, and material, but in conclusion states that owing to the great variety of orographic, climatic, and economic conditions, covering so large an area as this country does, it is impracticable to recommend and specify the details. The results of the experience of officers of the Survey, who have conducted work under nearly all these conditions, are condensed in this report.

While attention to economy calls for simplicity in equipage, the operators ought not to be deprived of such conveniences as tend to prolong their working ability, by securing shelter against the elements, needful rest, and good food. Equipment which may suffice in emergencies will be insufficient for work requiring long sustained efforts, which may be protracted through successive years.

Having thus referred to the reports of the committees in brief terms, the Conference begs to state that several committees have given attention to cognate subjects which may well be incorporated in future editions of the manuals of instructions and the publications of methods, and have made recommendations therefor in their reports.

During the sessions of the Conference facilities have been cheerfully extended by the office in furnishing data and copies of its proceedings, and its investigations have been aided by ready access to the well-arranged library of the Survey.

As ex-officio member of each committee, you have been invited to the meetings of the Conference, and the members of the committees beg leave to acknowledge the advice you were pleased to extend to them.

We thank you for this special opportunity of our meeting each other and personally comparing our experiences.

We feel that the interests of the Survey have been greatly advanced and that the esprit de corps of the Survey has been intensified by this instructive contact.

Very respectfully,

GEORGE DAVIDSON,
Chairman.
O. H. TITTMANN,
Secretary.

T. C. MENDENHALL, LL. D.,
Superintendent United States Coast and Geodetic Survey,
Washington, D. C.

PROCEEDINGS OF THE CONFERENCE.

The Conference met at the office of the United States Coast and Geodetic Survey, Washington, D. C., on January 9, 1894, and after completing its labors it adjourned on February 28.

The following officers of the Survey were members of the Conference:

Prof. George Davidson,
Mr. Charles A. Schott,
Mr. George A. Fairfield,
Mr. William Eimbeck,
Mr. O. H. Tittmann,
Mr. J. J. Gilbert,
Mr. F. W. Perkins,
Mr. Edwin Smith,
Mr. J. F. Pratt,
Mr. C. H. Sinclair,
Mr. E. F. Dickins,
Mr. Stehman Forney,
Mr. J. E. McGrath,
Mr. Isaac Winston,
Mr. P. A. Welker,
Mr. C. H. van Orden,
Mr. Fremont Morse,
Mr. W. B. Fairfield,
Mr. F. A. Young,
Mr. G. R. Putnam,
Mr. A. L. Baldwin,
Mr. O. B. French,
Mr. R. L. Faris,
Mr. S. B. Tinsley.

Prof. A. H. Buchanan, of Tennessee, and Prof. W. R. Hoag, of the University of Minnesota, Acting Assistants United States Coast and Geodetic Survey, joined the Conference on February 3, and participated in its deliberations until February 14.

In opening the Conference, Superintendent Mendenhall, while distinctly disclaiming any desire to restrict its deliberations, suggested the following subjects for its consideration, in the treatment of which advantage should be taken of the experience of the several corps engaged upon kindred work in the United States and foreign countries, bearing in mind the advisability of reporting its conclusions in a form which can be utilized in revising and bringing up to date the various handbooks on field methods and results issued by the Survey:

Base-line measurement.—Consider appliances, recent investigations of line measures and methods, and their adaptability to the varying conditions encountered in trigonometrical work. Compare the relative values in a scheme of triangulation of a few bases, measured with a high degree of accuracy, with frequent bases determined with less refinement.

Triangulation.—Define with greater exactness the various classes of trigonometrical work. Discuss the instruments, methods, and precision desirable for each class. Consider the relation of the number of observations to the degree of accuracy demanded by the character of the work, with the object of deciding if any material reduction can be made in the number of observations now taken, especially at primary stations. Consider reconnaissance and signal building, forms of day signals, and the use of night signals. Consider methods of observing and instruments. What besides the necessary measurement of angles should be done at a triangulation station. Submit schemes of triangulation necessary and desirable for a complete survey of the whole country, bearing in mind their utility in fixing boundary lines (State and national) and as furnishing bases for more detailed State surveys. Consider especially the character and scale of work desirable for the survey of Alaska.

Astronomical work.—Methods and instruments, giving consideration to the different classes of work and standards of accuracy. What work, other than astronomical, may be done profitably in connection with the latter.

Hypsometry.—Instruments to be used, and accuracy to be aimed at. Vertical angles; barometric work. Submit a scheme of lines of precise leveling controlling the whole area of the country.

Magnetic work.

Gravitation work.

General.

Party organization.—Camps and outfits—possibility of reduction in their size and cost. Records—their preparation, duplication, and transmission to the office. Field computations—degree of accuracy required. Accounts—relation of field officers to the disbursing agent.

At the close of his address he appointed Prof. George Davidson permanent chairman and Mr. O. H. Tittmann secretary of the Conference.

Mr. G. A. Fairfield was appointed chairman of the committee of the whole. Owing to his inability to act, on account of sickness, he was succeeded by Mr. Gilbert, who acted during the Conference.

A committee on rules, consisting of Messrs. Schott, Sinclair, and Winston, was formed, and their recommendations of an order of business and rules were adopted.

A committee to assist the chairman in the formation of committees, consisting of Messrs. Schott, Sinclair, Pratt, Smith, Eimbeck, and Morse, was appointed; and, in accordance with their recommendation, committees, to which was assigned the task of preparing reports on the various topics under consideration, were organized as follows:

(A) *Reconnaissance.*—Eimbeck (chairman), Dickins, Forney, French, Perkins, Welker.

(B) *Base lines.*—Davidson (chairman), Eimbeck, French, Pratt, Schott, Smith, Tittmann.

(C) *Triangulation and schemes of triangulation for whole country.*—Schott (chairman), Davidson, Dickins, Eimbeck, G. A. Fairfield, W. B. Fairfield, Faris, Gilbert, Perkins, Sinclair, Van Orden, Welker, Young.

(D) *Astronomy.*—Sinclair (chairman), Davidson, Gilbert, Morse, Putnam, Smith.

(E) *Hypsometry and scheme of leveling for whole country.*—Perkins (chairman), G. A. Fairfield, McGrath, Pratt, Van Orden, Winston, Young.

(F) *Alaska.*—McGrath (chairman), Baldwin, Pratt, Tittmann, Morse.

(G) *Instruments.*—Smith (chairman), Davidson, Eimbeck, Gilbert, Pratt, Tittmann, Van Orden, Winston.

(H) *Office and field relations.*—G. A. Fairfield (chairman), Forney, Sinclair, Schott, Winston.

(I) *Measurement of arcs.*—Schott (chairman), Davidson, Eimbeck, McGrath, Putnam, Sinclair.

(J) *Magnetics.*—Gilbert (chairman), Faris, McGrath, Putnam, Schott, Tinsley.

(K) *Gravity measures.*—Superintendent (chairman), Eimbeck (acting chairman), Putnam, Tinsley.

(L) *Equipment.*—Pratt (chairman), Baldwin, Dickins, W. B. Fairfield, Forney, Sinclair, Van Orden, Welker.

Special committee on arrangements.—Tittmann (chairman), Sinclair, Smith, Winston.

To these were added, near the close of the Conference, a committee composed of the Chairman, Professor Davidson, Messrs. Schott and Tittmann, to formulate, for the consideration of the Conference, the general results of its deliberations, in a report to the Superintendent, and an editing committee, composed of Messrs. Tittmann (chairman), Schott, G. A. Fairfield, McGrath, and Putnam.

During the proceedings temporary committees were appointed from time to time for some special purposes, but these need not be more particularly specified.

Under the general distribution of subjects above given, each committee prepared a list of topics to be discussed in their reports. These lists were submitted to the Conference, and after they had been considered and approved the committees proceeded to discuss the subjects assigned to them on the lines thus indicated. The reports prepared in pursuance of this plan by the several committees were discussed in committee of the whole, and were finally adopted by a majority vote of the Conference.

At the beginning of the sessions invitations were extended to the officers of the Survey who were not members of the Conference and to others interested in its labors to submit their views on matters relating to the purpose for which it was convened.

The Conference was also favored by the personal attendance, on various occasions, of Prof. J. H. Gore, of Columbian University, Washington, D. C. Prof. R. S. Woodward, of Columbia College, New York, came to Washington, at the sacrifice of his time and convenience, to reply verbally to questions proposed to him by the Conference. Prof. William Harkness, U. S. N., addressed the Conference on February 7. The following outlines of their remarks are appended:

SYNOPSIS OF PROF. R. S. WOODWARD'S REMARKS.

Addressing himself to a reply of the first question proposed to him by the Conference, namely, the number of measures of an angle necessary in triangulation of the highest order of precision with given instrumental means, Professor Woodward discussed the following heads:

Relation between the magnifying power of telescopes and the reading power of the micrometers; Errors of pointing; Graduation; Instruments; Atmosphere; Centering.

He advocated the use of a magnifying power of about 60 for telescopes, so as to preserve the proper relation between their pointing power and the reading power of the microscopes. He spoke of high power as diminishing the personal equation due to phase of signals and as serving as a test for the steadiness of the atmosphere. He advocated the use of flat signals to diminish phase. He spoke of the instability of the mounting of the theodolites, and advised the return to the measurement of angles (Gauss) rather than the measurement of directions (Bessel), and thought that in general nothing would be gained by ocular micrometer pointings in addition to the simple pointings. He stated that owing to the perfection of modern instruments a large number of positions is no longer necessary, and discussed this matter at some length. In reply to a question, he said that steadiness of atmosphere might be taken as an indication of the absence of lateral refraction. He spoke of the necessity for the careful centering of instruments, especially at base stations, and gave a detailed account of base measurement with steel tapes.

SYNOPSIS OF PROFESSOR HARKNESS'S REMARKS.

Professor Harkness addressed the Conference on the methods of determining the earth's ellipticity and the values deduced from them. He considered successively the following five methods:

Geodetic methods; Pendulum measures; Precession and nutation; Perturbations of the moon; The moon's parallax.

He called attention to the fact that an arc whose middle latitude is in 54° 45′ gives the value for the equatorial radius, while a more equatorial arc—that is, one in latitude 35° 15′—gives the polar radius, and gave two equations in which the numerical coefficients showed the relative effect of known arcs on the determination of the two radii. He stated that American arcs had not been introduced in the equations for deriving the figure of the earth, although the data for two, resulting from the operations of the Lake Survey, had been published.[1]

After discussing the pendulum methods, he presented a tabulation of values derived from pendulum experiments at various times. He explained that the values derived from precession and nutation are vitiated by want of knowledge of the internal constitution of the earth, and that the theoretically exact value from lunar perturbations is rendered questionable by the uncertainty attaching to the observed values of the perturbations themselves, and in illustration gave the observed values of certain perturbations of the moon on which the figure of the earth derived from them depends. In regard to the method by the moon's parallax, he expressed the hope that the cooperation of observatories would lead to a satisfactory determination of the ellipticity by this method.

In his opinion pendulum and astronomical methods will give the most reliable value of the ratio of the earth's semidiameters, and geodetic arc measures will give the linear values with great exactness. He stated that in order to determine the reciprocal of Σ with a probable error not exceeding one unit, the probable error of each of the semidiameters must be reduced to $\pm$ 164·4 feet (50·11^{m}).

[1] The United States Coast Survey published the results of two arcs and deduced a result for the size and figure of the earth. (Appendix 6, Report for 1877.)

Flattening of the earth as found from pendulum experiments.

[A table prepared by Prof. William Harkness, U. S. N.]

Date.	Author.	$^1/\Sigma$.
1799	La Place	335·78
1816	Matthieu	317·4
1818	Bessel	310·11
1821	Biot	306·75
1825	Sabine	289·1
1827	Saigey	281·62
1829	Pontecoulant	340·16
1829	Schmidt: Lehrbuch, T. 1, p. 372, 47 E.	288·20
1830	Airy	282·82
1833	Poisson	287·31
1834	Baily	285·26
1841	Peters, C. A. F.,	290·99
1842	Borenius	289·
1853	Paucker	288·38
1869	Unferdinger	289·15
1872	Nyrén	287·73
1876	Fischer, A.,	284·4
1880	Clarke	292·2
1884	Helmert	299·26
1884	Hill, G. W.,	287·73

FEBRUARY 5, 1894.

The Conference, having completed its labors, met for the last time on February 28. The general report to the Superintendent having been read and adopted, a committee, consisting of Messrs. Welker, Morse, and Faris, was appointed to notify the Superintendent of the readiness of the Conference to submit its final report. Upon the arrival of the Superintendent the chair ordered the reading of the report, upon the conclusion of which the Superintendent produced the minutes of the Board of Organization of 1843, on which he commented, saying that they contained evidence of independence of opinion which he thought had also been manifested in the present Conference, in which perfect freedom of discussion was accompanied by the utmost good fellowship. He regretted that other duties had prevented his more frequent attendance at the meetings, and expressed his gratification at the outcome of the Conference, which had already caused a feeling of regret in him that he had not summoned it at an earlier period of his superintendency.

After the departure of the Superintendent, Mr. Sinclair offered the following resolutions, which were unanimously adopted:

Resolved, That this Conference herein expresses its high appreciation of the action of the Superintendent in calling it together, realizing that great benefit has accrued to its members by the interchange of ideas, which can not but result in the increased efficiency of the entire corps;

That the thanks of the Conference be extended to its chairman and to the chairman of the committee of the whole for the courtesy with which they have presided over its deliberations;

That the thanks of the Conference be extended to the secretary for the very satisfactory manner in which he has performed the arduous labors imposed upon him.

Resolutions referring to the loss sustained by the Survey by the recent death of Assistant James S. Lawson were adopted by a rising vote. It was ordered that they be spread upon the minutes and that a copy be sent to his family. The Conference then adjourned.

REPORT OF COMMITTEE A, ON RECONNAISSANCE.

The term "reconnaissance" as here used embraces all those investigations of a region to be triangulated which precede the fieldwork of construction, base measurement, and the measurement of angles, and comprises the selection of the most feasible chain or net of geometric figures, the location of the base lines, the determination of the preparations and appliances necessary, and the collection of information as to the facilities available for carrying out the proposed scheme.

An exhaustive report upon such a subject appeared to your committee to be neither practicable nor advisable; and after a careful review of the articles already published,[1] and their comparison with the experience of those who have been most extensively engaged upon this class of work, it concluded to restrict its labors to the formulation of a few practical suggestions, which it is hoped will be useful in promoting economy and dispatch in the execution of triangulation work in the future, viz:

(1) Reconnaissance, forming as it does the basis of triangulation, should always be thorough and exhaustive, developing all possible schemes and comprising all classes of information affecting the economy and facility of the operations to follow.

(2) Simplification of the geometric configuration of schemes by the more frequent introduction of well-conditioned simple triangles in all cases where the substitution of complex figures, as quadrilaterals with open diagonals or polygons, would necessitate either the undue contraction or expansion of the scheme or the erection of high and costly scaffoldings.

(3) Avoidance, as far as compatible with the requirements of geodetic triangulation, of elevated structures for any purpose except that of overcoming obstacles or lifting the triangulation above the level of highly heated and disturbed atmosphere. In exceptional cases structures of moderate elevation may also prove necessary to preserve symmetry or to attain proper figure conditions, but in no case should mere geometrical elegance of a scheme of triangulation be sought at the expense of reasonable economy.

(4) Avoidance, as far as expedient, of the longest lines or the highest peaks in the mountains of the interior. On account of the clouds which frequently envelop these peaks and the uncertainties in the

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[1] Appendix No. 20, Coast and Geodetic Survey Report, 1876; Appendix No. 9, Coast and Geodetic Survey Report, 1882; Appendix No. 10, Coast and Geodetic Survey Report, 1885.

seeing, lines of a greater length than 200 kilometres (124 miles) invariably tend to interrupt and delay the progress of the work. In addition to this, the occupation of the highest mountains is always expensive by reason of the preparations required to make them accessible and of the difficulties of transportation. Lines that pass closely along the slopes of mountains or hills or near the vertical surface of any object, as large buildings, or that pass through narrow avenues cut through timber should be avoided if possible, as they are particularly liable to lateral refraction.

(5) Attention is also called to the degree of adaptability possessed by several typical figures commonly used in triangulation of the first order. These simple triangles, or chains of triangles, easily adapt themselves to topography of the most complex character, whereas quadrilaterals, with observable diagonals, possess this quality in the least degree, and on that account will frequently be found impracticable figures. Difficult stretches of country may, therefore, always be most easily crossed by simple triangles. Pentagons, or quadrilaterals with central points, also readily conform to the configuration, however complex or difficult it may seem. The hexagon, however, on account of the disposition of the stations, tends to retard rapid progress, and should not, therefore, be included in a chain of triangulation. This figure finds its most advantageous application when large areas are to be covered, as in the survey of a whole State.

(6) It is recommended that in all extensive chains of triangulation sites for base lines be considered and selected at intervals of from 250 to 320 kilometres (155 to 200 miles), or at every eighth, ninth, or tenth figure, according to the length of the bases, the character and scope of the scheme, and that their connection with the main chain be carefully developed in order that these connections may be simultaneously executed with the main work.

(7) Whenever the purpose of a chain of triangulation requires the traversing of flat or rolling country its trend should, when practicable, rather follow than cross the drainage. Experience has shown that when the drainage of a country has to be crossed by the triangulation it can usually be done only by contracting the scheme or by elevating the stations by means of scaffoldings. Similar conditions are met with in crossing high table-lands or flat-topped or double-crested ranges of mountains, as, for example, the Sierra Nevada, in the region about Lake Tahoe.

(8) Before taking the field the reconnoitering officer should, by a careful study of all available maps, make himself as thoroughly acquainted as possible with the character of the country to be treated, the lines of travel, the location and relative importance of towns and villages, and, above all, the drainage, as the latter will in a great measure determine the size of the scheme.

(9) The field notes of the reconnoiterer should be exhaustive. They can never be too full for himself or his successors.

Horizontal and vertical angles should be taken on all prominent peaks and objects, even though they are not directly included in the scheme, for they are often invaluable for purposes of orientation. Approximate latitude and azimuth observations should also be made, and the magnetic bearing of prominent points be noted. Specify the difficulties of the country, describe and carefully sketch the entire horizon, particularly every opening or notch through which more distant mountains can be seen. Every high point or possible station should be especially noted.

The plotting should be kept up from day to day, and a rough topographical sketch made, showing the main ridges, water courses, roads, trails, and habitations. Comprehensive notes with regard to means of transportation, subsistence for man and animals, help, materials, and accessibility are invaluable. Remarks with reference to weather and climatic conditions are important and desirable.

(10) On completion of the reconnaissance, or of the season's work, a report, together with sketches or diagrams illustrating the schemes developed, should be prepared. This report, containing full and explicit statements setting forth the economical and other advantages and disadvantages, is to be promptly transmitted to the Superintendent for his consideration and action.

OUTFIT AND INSTRUMENTS REQUIRED.

The camp outfit will necessarily vary with the character of country to be traversed, and should be as light and portable as possible for reconnaissance work in a mountainous or unsettled country.

The outfit of instruments is simple, but must be adequate for the purpose, viz:

Two aneroid barometers.
One 4-inch theodolite with vertical circle and tripod.
One reconnoitering telescope of 3-inch aperture.
Azimuth compass, hand level, good binocular.
Pocket box of drawing instruments, protractor and scale.
Steel tape.
Two heliotropes for testing the doubtful intervisibility of stations.
Best maps of the country available.
Projection with river courses and known roads drawn thereon.
Small photographic apparatus, with films.
Note and sketch books.
A gradienter will be found a valuable instrument for reconnaissance.

WM. EIMBECK,
Chairman of Committee.
OWEN B. FRENCH,
Secretary.

REPORT OF COMMITTEE B, ON BASE LINES.

The Committee on Base Lines makes the following report to the Conference:

It is not necessary to enter into any historical account of the different apparatus that have been used at various periods.

A short statement of the instruments in use, or lately in use, in foreign countries and in the United States, and the results obtained from them, will be given, but in the time at our disposal it can not be complete nor will the material in print offer means for an exhaustive comparison.

The following classification of base apparatus covers all appliances in general use:

(1) Contact apparatus, comprising compensating, bimetallic, and monometallic.

(2) Optical apparatus, comprising bimetallic, monometallic, and bars in ice.

(3) Steel tape and wire apparatus.

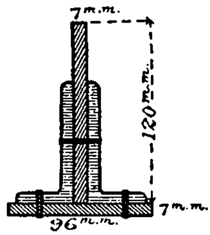
The following is a brief description of the more prominent appliances recently in use, but it must be distinctly remembered that all the relative errors given, excepting those of the Great Trigonometrical Survey of India, Lake Survey, and Coast and Geodetic Survey, are merely the errors of measurement, and do not include the error of reduction to sea level, nor the error of the unit of length of the apparatus, the latter being undoubtedly one of the largest sources of error in the determination of the length of a base line.

Base apparatus now in use—bimetallic and monometallic (optical).

SPAIN.

The Spanish bases have been measured with two appliances. The earlier, designed by Brunner, is 4^{m} long and consists of two bars, one of platinum, whose cross section is 5^{mm} by 21^{mm}, and the other is of brass, of the same dimensions. These bars are fastened together in the middle, and at each end Borda scales are attached to obtain the difference of length due to changes of the temperature. They are mounted on rollers in the carrying case to allow longitudinal movement. The microscopes are mounted on independent tripods. One base has been measured with this apparatus with an average relative error of measurement of $\frac{1}{5865800}$. (See "Memorias del Instituto Geographico," Vol. V, 1884, p. 99.)

Subsequent to the year 1865 an apparatus was devised by Colonel Ibañez, which consists of a 4^{m} iron bar in the form of an inverted "T," of two bars of iron 7^{mm} thick, the one in the vertical position being 120^{mm} high and the one in the horizontal 96^{mm} wide. They are fastened together throughout their entire length with angle irons bolted through the two bars. Four mercurial thermometers are placed in metallic contact with the bars. The graduation trace for the micrometer pointing is made on the top surface of the vertical member of the bar. In the field measurement the apparatus is uncovered, but the operations are conducted under a wooden shelter.



Eight bases have been measured with this apparatus with relative errors of measurement ranging from $\frac{1}{1\,244\,999}$ to $\frac{1}{4\,161\,300}$. (See Memorias del Instituto Geographico, Vol. III, 1881.)

Three bases have been measured in Switzerland with this apparatus. Their lengths are 2·4, 2·5, and 3·2 kilometres, and the relative errors of measurement are, respectively, $\frac{1}{2\,700\,000}$, $\frac{1}{1\,960\,000}$, and $\frac{1}{2\,460\,000}$.

Base apparatus—bimetallic compensating (optical)

INDIA.

The bases of the Great Trigonometrical Survey of India have all been measured with the Colby apparatus, which is compensating and optical, the unit of measurement being the yard. The bars of brass and iron are 10·1 feet ($3·08^{m}$) long, 0·55 by 1·5 inches ($14·0^{mm}$ by $38·1^{mm}$) in cross section, and 1·3 inches ($33·0^{mm}$) apart, rigidly connected at their centers by a pair of small steel cylinders. These bars are free to expand from or contract toward their centers independently of each other. Across each extremity is pivoted a short compensating lever projecting beyond the iron bar. The compensation point is marked on a silver pin near the extremity of each lever. The distance of this point from the axes of the pivots of attachment to the brass and iron bars is intended to be exactly in the same proportion as the coefficient of expansion of the two metals are to each other.

The compound bar is inclosed in a wooden case, and each component rests at one-fourth and three-fourths of its length on brass rollers which are fixed to the bottom of the box and have raised flanges to prevent lateral motion. Longitudinal motion is prevented by means of a stay fixed firmly to the bottom of the box at its center and projecting upward between the two steel cylinders. Here a spirit level is attached parallel to the direction of the bars, and is read through a glass cover in the top of the box. The compensation levers project about 2 inches beyond the sides of the box nearest the iron bar. There are six of these compound bars, designated *A*, *B*, *C*, *D*, *E*, and *H*, supported on strong brass tripods which are capable of communicating motion in a lateral, longitudinal, and vertical direction. The points of compensation of these bars are placed about 6 inches apart, and the distance between is read by a pair of microscopes attached to two parallel bars of brass and iron, both being free to expand or contract toward their centers. The adjustments are so made that the outer foci of the object glasses are compensation points exactly 6 inches apart.

As the measures are invariably made in the horizontal position of the bars, the ground has to be prepared very carefully and vertical offsets are made from time to time as the inclination of the ground demands. The apparatus requires 8 officers. The number of men is not stated.

Ten bases average between 6 and 7 miles in length, and were measured with an average speed of about 1,000 feet (305^{m}) per day. Excluding all constant errors, the probable error of a single measurement is $\pm \frac{1}{567000}$. Including all known and estimated errors, the probable error of a single measurement of a base is taken at $\pm \frac{1}{384000}$. (See Report Great Trigonometrical Survey of India, Vol. I.)

Base apparatus.

BAVARIA.

In Bavaria the Reichenbach apparatus was used in the early part of the century. It consisted of five iron bars each 4^{m} long and 22^{mm} square. The ends of these iron rods terminated in blunt knife edges, that at one end being in the vertical and the other in the horizontal plane. During the process of measurement a vertical and horizontal knife edge are brought within about 4^{mm} of each other, the distance between them being measured with a carefully constructed, graduated, long tapering wedge of hardened steel. The temperature was obtained with mercurial thermometers brought into contact with the metal of the bar.

The second longest base on record, nearly 20 kilometres long, was measured with this apparatus.

SWEDEN.

Prof. Edward Jäderin, of Stockholm, has experimented quite extensively with metallic wires and tapes, using two different metals, such as brass and iron, for the purpose of obtaining the temperature correction from them. No base lines of note have, however, thus far been measured in Europe with such apparatus. His results are published under the following titles:

"Geodätische Längenmessungen mit Stahlbändern und Metalldrähten, von Edw. Jäderin, Stockholm, 1885;" also "Exposé élémentaire de la nouvelle méthode de M. Édouard Jäderin pour la mesure des droites géodesiques au moyen de Bandes d'acier et de fils métalliques, par P. E. Bergstrand, ingénieur au bureau central d'arpentage, à Stockholm, 1885;" and "Marklig Längdförändring hos geodetiska basmätningsstrangar at Edv. Jädrin, 1893, Stockholm."

Base apparatus—bimetallic, noncompensating, contact.

THE BESSEL BASE APPARATUS.

As this appliance has been used in several countries, a general description will answer. In the usual form of the apparatus the measuring bar is of iron, of rectangular cross section, on the upper side of which rests a similar bar of zinc, but somewhat shorter. One end of this zinc bar is securely fastened to the iron bar near one of its extremities, the other end being free to expand. At each end of the iron measuring bar and on its upper surface are permanently fixed blocks of metal, the outer ends of which terminate in hardened knife edges, situated in the vertical plane. The fixed block on the end at which the free end of the zinc element rests has another vertical knife edge pointing toward the free end of the zinc bar, which also terminates with a vertical knife edge, leaving a small space between these two edges. The difference of expansion between the iron and zinc for temperature correction to the iron bar is here measured by inserting a carefully constructed graduated glass wedge. The customary method is to use four of these bars, placing their ends near each other and measuring the small intervening space with the glass wedge previously mentioned.

This apparatus has been used in the following countries:

In Belgium two bases have been measured, being, respectively, 2·3 to 2·5 kilometres in length, with relative errors of measurement of $\frac{1}{1\ 700\ 000}$ and $\frac{1}{2\ 215\ 000}$.

In Prussia eight bases have been measured, ranging in length from 900 toises (1·7 kilometres) to 7·0 kilometres, with relative errors of measurement ranging from $\frac{1}{863\ 000}$ to $\frac{1}{1\ 400\ 000}$.

In Denmark one base 1385 toises (2·7 kilometres) long has been measured with relative error of measurement of $\frac{1}{560\ 500}$.

In Italy nine bases have been measured with it, ranging in length from 340 (0·66 kilometres) to 5,150 toises (10·0 kilometres), with relative errors of measurement ranging from $\frac{1}{590\ 000}$ to $\frac{1}{2\ 280\ 000}$.

Base apparatus—bimetallic, noncompensating (optical).

THE BRUNNER APPARATUS.

As this apparatus has also been used in several countries, only a general description is attempted.

It is a single bar 4^{m} long. As used by the French and Germans the measuring bar is of platinum-iridium, attached to which is a brass one. The two form a Borda scale for determining the difference of the lengths of the bars due to changes of temperature. The microscopes mounted on independent supports have an arrangement for making optical cut-offs and also an attachment for aligning the bars. The measurements are now made under canvas-covered frames, which are brought forward in sections.

At present this apparatus is the one that is in general favor on the Continent.

It has been used in the following countries:

In Prussia three bases of 1198 toises (2·3 kilometres), 1417 toises (2·8 kilometres), and 2·5 kilometres have been measured, with respective relative errors of measurement of $\frac{1}{4\ 600\ 000}$, $\frac{1}{4\ 600\ 000}$, and $\frac{1}{4\ 600\ 000}$, In France three bases have been measured with it, for only one of which the relative error is given. This is the Paris Base, which is 7·2 kilometres long, with a relative error of measurement of $\frac{1}{2\,400\,000}$. This base was measured by two details of officers and men, each detachment consisting of 4 officers and 25 men. The operations were conducted with a rapidity of about 300^{m} per day.

Base apparatus—Mono metallic contact.

RUSSIA.

The principal apparatus used is that of Struve, which consists of an iron bar 12 French feet (nearly 4^{m}) in length and 15 by 15 lines (about 34^{mm} by 34^{mm}) in cross section. One end of the bar terminates in a steel plane and the other has a lever pivoted to it. This lever is so arranged that the bar coming in contact with it acts as a fulcrum and its longest free end moves over a divided scale.

Mercurial thermometers, whose bulbs are placed in cavities in the iron bars, are used.

Seven bases have been measured with this apparatus, ranging in length from 1·15 to 2·9 kilometres. The same relative error of measurement is given for all of them, viz, $\frac{1}{1\,240\,000}$.

For the measurement of secondary bases the Jäderin apparatus has been used, and is considered sufficiently accurate for the purpose—namely, cartographic operations.

Contact scale—Mono-Metallic.

AUSTRIA.

In Austria-Hungary an apparatus is used consisting of an iron bar of rectangular cross section, which rests on 12 brass plates fastened to the top surface of a wooden beam which has a wooden cover. The ends of these iron measuring bars, of which 4 are used in regular succession, have plane end surfaces. During the operation of measurement the bars are so placed that their ends are a short distance apart, and the distance between is measured with a short scale made in two parts, which slide on each other, the respective ends of which come in contact with the bars.

Two mercurial thermometers are used for determining the temperature of each bar. Five observers and 16 men are required to use this apparatus. Nineteen bases have been measured with this appliance; all of them twice in opposite directions. They range from 2·4 kilometres to 9·5 kilometres in length. The relative error of measurement is only given for one, that of the d'Ilidze base, which is $\frac{1}{3\,700\,000}$.

Base apparatus—bimetallic, contact, compensating.

THE LAKE SURVEY OF THE UNITED STATES.

The first refined appliance used by this organization was a copy of the Bache-Wurdemann contact compensating apparatus, as used by the Coast Survey at that time. These bars are 15 feet ($4{\cdot}57^{m}$) long, and were used, between the years 1867 and 1875, in the measurement of five bases, whose lengths ranged from 4·9 to 8·8 kilometres, with relative errors ranging from $\frac{1}{530000}$ to $\frac{1}{918000}$.

OPTICAL BIMETALLIC, NONCOMPENSATING.

Subsequently the Lake Survey Repsold optical bimetallic apparatus was devised and used. This consists of a steel measuring bar 4^m long, by the side of which is a similar one of zinc. The two are firmly fastened together in the middle. Their unequal expansion is observed upon scales at both ends for determining the temperature of the steel bar. This combination of the two metals is supported by a system of rollers adjusted inside the carrying tube so as to keep them in their proper relation to each other and to allow free expansion. There were also two mercurial thermometers at each end, with their bulbs inside the tube cylinder. The bar is provided with a telescope for alignment and a sector for measuring the inclinations. In measuring, the bar is supported at the extreme ends of the carrying tube on trestles whose heads are provided with movements in three directions, by which the tube is placed in position under microscopes which are mounted on independent stands.

Five officers and recorders and 12 laborers were required to make the measurements, an average day's work being about 300^m.

Three bases were measured with this apparatus between the years 1877 and 1879, ranging in length from 6·2 to 7·6 kilometres, with relative errors of from $\frac{1}{1020000}$ to $\frac{1}{1071000}$. (See Comstock's report "Triangulation of the Great Lakes," professional papers, Corps of Engineers, U. S. A., 1892.)

COAST AND GEODETIC SURVEY.

The Coast and Geodetic Survey has used various appliances. As early as 1817 a single bar 8^m long, made up of four iron bars 2^m each in length, clamped end to end, was devised and constructed by Mr. Hassler. The relative errors with this apparatus were about $\frac{1}{380000}$. This was the first optical monometallic apparatus used in this country. It also had the aligning sector attached to the top of the carrying case and the terminal micrometer microscopes identical in principle with those now in use. (American Philosophical Transactions, Vol. II, New Series, pp. 273–286.)

In 1845–46 the Bache-Wurdemann lever contact compensating apparatus was devised, and was in use up to 1873. The best results that have been obtained with it have a relative error of about $\frac{1}{800000}$. Since then the Schott compensating apparatus with contact slide has been used on two different bases, only one of which has been finally reduced, viz, the Yolo, 17½ kilometres in length, with a relative error of $\frac{1}{1821000}$.

In 1891 a steel bar in ice, with optical apparatus, was used in measuring a kilometre of the Holton Base, with a relative error of about $\frac{1}{2300000}$.

For secondary bases the contact slide monometallic apparatus with mercurial thermometers has been used since 1855. It has demonstrated its possibilities up to a degree of accuracy of one part in 600000.

Since 1890 two bases of 5·5 and 3·8 kilometres have been measured with the standard tape with relative errors not exceeding one part in 500000.

Wire measurements have been in use since the year 1848, varying in degree of accuracy as required by the conditions to be fulfilled.

The Eimbeck duplex apparatus, although suggested in February, 1885, has been only recently constructed by the Coast and Geodetic Survey. Its principle is to make the bars themselves determine the correction due to changes of temperature without having recourse to the uncertain use of thermometers in the field, excepting it be to furnish additional data.

The later forms of apparatus in the United States are more fully referred to and discussed individually.

WOODWARD STEEL BAR IN MELTING ICE.

For a test of the performance of different kinds of apparatus we turn to the experiences gained in this country by the measurement of the Holton base in Indiana, on a section of which, for the first time, the temperature effect was eliminated by the measurement of 1 kilometre of the base with a steel bar in ice. The section was then measured with a 100^m steel tape apparatus, and, thirdly, with the secondary contact slide apparatus, forms of apparatus so radically different that an agreement between the results obtained by them increases very greatly the probability of correctness of the several results.

The detailed report of the experiments, measurement, and reduction of the Holton Base will be found in the United States Coast and Geodetic Survey Report for 1892, Part II, Appendix No. 8.

The salient facts resulting from the experiments made there, appear to be that with the 5^m steel bar in melting ice, whereby the temperature error is eliminated, a base line can be measured with a degree of accuracy hitherto unapproached.

One kilometre of the Holton Base was measured four times with this bar in eight working days. After proper preparation 800 bars were laid in forty-two and one-half hours, at an average rate of 19 bars, or 95^m, per hour, and a maximum rate of 30 bars, or 150^m, per hour. This rate of measurement is satisfactory. In cost of operation this apparatus is little, if any, inferior to the Brunner apparatus now used abroad.

THE SECONDARY CONTACT SLIDE APPARATUS.

In the measurement of the Holton Base with this apparatus, after comparisons with the 100^m comparator there established, a degree of precision was obtained far beyond the needs of the triangulation dependent thereon. The length of the base line is 5·5 kilometres, and it was measured twice in fourteen days. Two thousand six hundred bars were laid in eighty-three and one-half hours, with an average rate of 31 bars, 155^m, per hour, and a maximum of 41 bars, or 205^m, in one hour.

No grading, and very little preparation of the ground, is necessary for this apparatus. The cost of measuring a base with it is about the same as with the steel tape.

It is believed that if the bar be made to rest upon rollers and the thermometers be placed in better metallic contact with the bar the apparatus will be improved, with very trifling addition to the weight.

This apparatus should be compared, before and after any base measurement, with the 100^m comparator, or directly with the 5^m steel bar in ice.

In the measurements of bases by bars the contact slide has been used with sufficient frequency to establish its efficiency for rapid and accurate measurement. The range of error in making a coincidence does not exceed one-twentieth millimetre, and in the large number of contacts made in one measurement of a base the probability is that the plus and minus errors will balance each other.

STEEL TAPE.

The steel tapes, graduated to 100^m, at present in use on the Coast and Geodetic Survey are $101{\cdot}01^m$ long, $6{\cdot}34^{mm}$ by $0{\cdot}47^{mm}$ in cross section, and weigh 22·3 grammes per metre of length. They are subdivided into 20^m spaces by graduations ruled on the tape itself. The end graduations fall about $0{\cdot}5^m$ from the tape ends, which terminate in loops formed by annealing and riveting the tape back on itself. The surface of the tape, where it is not polished to receive the graduation, is of a dull black color. When not in use the tapes are rolled up on reels not less than $0{\cdot}3^m$ in diameter, and may thus be transported with ease and safety. The stretchers used with these tapes are fully described in Professor Woodward's report of the Holton Base, as also the method of preparing the line, making the measurement, protecting the thermometers, etc. The determination of the temperature is, as in all other forms of base apparatus, the most uncertain element in the operation, and needs further study and investigation.

The experiments and measures at the Holton and St. Alban's bases, which were measured six and five times, respectively, have shown relative errors of 1 : 1 300 000, which are very satisfactory, and may be diminished when the temperature is more accurately determined. Judging from these experiences, the steel tape commends itself for accuracy, and over many varieties of surface and classes of ground illy adapted to an apparatus supported on trestles, great economy may be expected from its use.

Carefully and judiciously handled, the steel-tape apparatus will doubtless attain a good standing throughout the United States, where so many extensive and independent surveys are being carried on, and especially in all newly undertaken projects.

The published accounts of foreign base measurements do not give sufficient data to judge of the cost of the measurement nor of the number of officers and men necessary, save in two instances; neither are the rates of measurement always specified, although it may be assumed to be very close to 300^{m} per diem.

The officers and men belong to the military arm of the Government service, and even if we knew their numbers the cost of measurement would not be readily comparable with civilian work. It is very likely that more soldiers are detailed for the work of measurement than are absolutely necessary.

For example, we find for the measurement of the Bonn Base the following detail: One civilian chief, 6 officers, 8 officials, 2 subalterns, 11 pioneers, and 42 infantry soldiers, a total of 70 persons. And in the measurement of the Paris Base 58 officers and soldiers served in two detachments at different times.

The following tables have been appended to give an idea of the number, length, etc., of the bases measured in foreign countries and in the United States.

Table I shows the number of geodetic bases measured in the various countries doing geodetic work.

Table II shows the approximate length of these bases.

Table III gives the principal bases of the United States, together with approximate length, relative error, and apparatus used.

Tables IV and V give more completely, statistics of a few of the most important bases of the United States.

TABLE I.—*Table showing the number of geodetic bases measured by the various countries doing geodetic work.*

Name of country.	Number.	Name of country.	Number.
Austria-Hungary	19	Norway	4
Belgium	2	Peru	1
Denmark	2	Portugal	1
France and Algiers	13	Russia	20
Germany	15	Spain	9
Great Britain	7	Sweden	6
India and Cape Colony	13	Switzerland	5
Greece	1	United States	29
Holland	1		
Italy	9	Total	157

TABLE II.—*Table showing lengths of geodetic bases compiled from 157 bases of the world.*

Lengths.	Number.	Lengths.	Number.
0– 1 kilometres	1	11–12 kilometres	12
1– 2 "	5	12–13 "	6
2– 3 "	28	13–14 "	3
3– 4 "	15	14–15 "	3
4– 5 "	13	15–16 "	--
5– 6 "	16	16–17 "	--
6– 7 "	10	17–18 "	3
7– 8 "	10	18–19 "	--
8– 9 "	14	19–20 "	2
9–10 "	5	20–21 "	--
10–11 "	10	21–22 "	1

TABLE III.—*A table of the most important bases in the United States.*

Name of base.	State.	Date.	Observer.	Apparatus.	Approximate length.	Probable error.	Relative error.
U. S. Coast and Geodetic Survey.					*km.*	*mm.*	
Fire Island	New York	1834	F. R. Hassler	Hassler 8^m bars	14·0	58·5	1–240 000
Kent Island	Maryland	1844	J. Ferguson	" "	8·7	38·1	1–228 000
Boston and Providence R. R.	Massachusetts	1844	E. Blunt	" "	17·3	35·8	1–484 000
Dauphin Island	Alabama	1847	A. D. Bache	Bache-Wurdemann 6^m	10·6	26	1–410 000
Bodies Island	North Carolina	1848	"	" "	10·8	25·5	1–425 000
Edisto Island	South Carolina	1850	"	" "	10·7	25·6	1–419 000
Key Biscayne	Florida	1855	"	" "	5·8	12·7	1–456 000
Cape Sable	"	1855	"	" "	6·4	15·7	1–410 000
Epping Plains	Maine	1857	"	" "	8·7	15·8	1–552 000
Craney Island	Virginia	1869	R. E. Halter	6^m contact slide bars	5·1	37·0	1–140 000
Portsmouth Island	North Carolina	1870	G. A. Fairfield	" "	9·0	49·1	1–181 000
Peach Tree Ridge	Georgia	1872–3	C. O. Boutelle	Bache-Wurdemann	9·3	16·6	1–562 000
Lebanon	Tennessee	1877	A. H. Buchanan	6^m contact slide bars	7·3	14·7	1–490 000
Spring Green	Wisconsin	1878	J. E. Davies	4^m " "	4·7	17·8	1–263 000
Louisville	Kentucky	1879	G. A. Fairfield	6^m " "	8·2	32·0	1–256 000
El Paso	Colorado	1879	O. H. Tittmann	6^m " "	11·3	18·6	1–607 000
Greenville	Mississippi	1880	C. H. Boyd	4^m " "	2·1	9·7	1–216 000
Yolo	California	1881	G. Davidson	5^m Schott compensating	17·5	9·57	1–1 820 000
St. Paul, Snelling Avenue	Minnesota	1888	C. O. Boutelle	6^m contact slide bars	8·7	-----	-----------
Los Angeles	California	1889	G. Davidson	5^m Schott compensating	17·5	-----	-----------
Holton	Indiana	1891	O. H. Tittmann	5^m contact slide	5·5	3·50	1–1 570 000
"	"	"	R. S. Woodward	100^m steel tape	5·5	4·00	1–1 370 000
St. Albans	West Virginia	1892	"	" "	3·87	3·0	1–1 290 000
U. S. Lake Survey.							
Minnesota Point	Minnesota	1870	C. B. Comstock	Bache-Wurdemann	6·1	11·4	1–530 000
Fond du Lac	Wisconsin	1872	E. S. Wheeler	" "	7·4	11·4	1–649 000
Keweenaw	Michigan	1873	"	" "	8·8	10·6	1–830 000
Sandy Hook	New York	1874	"	" "	4·9	5·3	1–918 000
Buffalo	"	1875	"	" "	6·8	7·6	1–889 000
Chicago	Illinois	1877	"	Repsold	7·5	7·4	1–1 052 000
Sandusky	Ohio	1878	"	"	6·2	5·4	1–1 148 600
Olney	Illinois	1878	"	"	6·6	6·4	1–1 013 000

TABLE IV.—*United States Coast and Geodetic Survey—Measurements of primary base lines.*

[Prepared by Assistant Edward Goodfellow.]

THE BACHE-WURDEMANN COMPENSATING BASE APPARATUS.

Year of measure.	Locality, etc.	Length of base.	Probable error of. length.	Proportional part of length.	Days occupied.	Average length per day.	Mean temperature of measure.
		m.	*m.*			*m.*	F°.
1847	Dauphin, Island, Ala. Sand, low grass, rushes, etc.	10661·8376	±0·0260	$\frac{1}{410\,071}$	17	627·17	84·5
1848	Bodies Island, N. C. Sea beach, sandy and marshy.	10841·7524	±0·0255	$\frac{1}{425\,167}$	10	1084·17	52·0
1850	Edisto Island, S. C. Cultivated fields, clay and loam.	10721·4231	±0·0286	$\frac{1}{374\,875}$	13	824·72	59·5
1855	Key Biscayne, Fla. Calcareous soil, coarse grass, palmettoes, etc.	5789·2262	±0·0127	$\frac{1}{455\,845}$	9	664·32	82·9
1855	Cape Sable, Fla. Calcareous soil, grass, samphire weed, etc.	6431·5913	±0·0157	$\frac{1}{409\,655}$	8	803·95	87·9
1857	Epping Plains, Me. Rolling, sandy plain, many irregular ridges.	8715·9422	±0·0158	$\frac{1}{551\,642}$	8	1089·49	70·0

NOTE.—1894, January 27—The data given here and on the following page for the seven primary bases measured with the Bache-Wurdemann apparatus have been taken from the annual reports, except for Epping Base, which was furnished by Assistant Schott.

TABLE V.—*Base line statistics, February, 1894—United States Coast and Geodetic Survey Conference.*

Date.	Name, locality, etc.	Apparatus.	Number of men used.	Number of times measured.	Length.	Probable error.	Proportional part of length.	Number days measuring.	Number hours actually laying bars (b).	Average number bars per hour or (a) (b).	Number of bars laid (a).	Greatest number bars laid in 1 hour.	Remarks.
					m.	m.							
1872	Atlanta Base, Peach Tree Ridge, near Atlanta, Ga.—Soil, loam and clay.	Bache-Wurdemann bimetallic compensating (6^m).	22	3	9,338	0·0166	$\frac{1}{562\,539}$	41	252	19	4,747	30	
1879	El Paso Base, El Paso County, Colo.	Secondary bars and thermometers Nos. 13 and 14, monometallic (6^m).	10	2	11,289	0·0186	$\frac{1}{606\,941}$	23	125	$30\frac{1}{2}$	3,822	43	
1881	Yolo Base, Yolo County, Cal.—Soil, dark loam, stiff clay, and some sand.	Schott compensating 5^m zinc and steel bars with thermometers.	19	2	17,486	0·0096	$\frac{1}{1\,821\,512}$	46	247	$34\frac{1}{2}$	8,494	57	
1889	Los Angeles Base, California.	do.	18	3	17,495			50	300	35	10,597	64	Highest rate, 400 bars in $7^h\ 24^m$.
1891	Holton Base, Ripley County, Ind.	Secondary bars and thermometers Nos. 3 and 4 (5^m).	9	2	5,500	0·0035	$\frac{1}{1\,570\,000}$	14	83·5	31	2,600	41	
1891	do.	Iced steel bar No. 17 and microscopes.	88	4	1,000		$\frac{1}{2\,300\,000}$	8	42·5	19	800	30	
1891	do.	100^m steel tapes Nos. 85 and 88.	12	6	5,500	*0·004	$\frac{1}{1\,375\,143}$	17	29·5	17	†508	30	10 in 20 minutes.
1892	St. Albans Base, West Virginia.	do.	12	5	3,870	*0·0030	$\frac{1}{1\,290\,133}$	4	11·4	17	†195	28	Most of the tape measures were made at night.

* Obtained from Professor Woodward's computations.

† Number of tape lengths.

From the six earlier bases measured with the heavy Bache-Wurdemann apparatus given in Table IV we condense the following statistics: Each base was measured but once, and the total of the lengths measured was 53 kilometres, at the rate of 849^{m} per diem, and the highest daily average was 1089^{m}. No hours are given, and the number of officers and men is not mentioned. The relative error varied between 1:410 000 and 1:553 000; but these included the error of unit of standard and reduction to sea level.

From the later bases of the Coast and Geodetic Survey, given in Table V, the following statistics are obtained:

Five bases each employed 17 men, who measured 150^{m} per hour, and reached 2000^{m} in seven and one-half hours (in third measurement). The reduced measures give relative errors from 1:563 000 to 1:1 822 000. Two bases measured with steel tapes employed 12 men, who measured the bases six and five times, respectively, at the rate of 1280^{m} per hour. These gave relative errors of 1:1 375 000 and 1:1 290 000. The relative errors of all these measurements include the unit of standard, etc.

In the measurement of Yolo and Los Angeles bases with the Schott apparatus the following officers and men were employed: Six officers, 6 men at bars, tripod, and plates, 5 men pushing movable cover, etc. Besides these necessary people, there were 1 man for bridges, extra driver, etc., 1 watchman, 1 driver, 3 cooks and waiters.

THE SITE OF THE BASE LINE.

In a mountainous region the selection of a site is frequently a very difficult matter. It should be located by the reconnaissance party, and, if practicable, more than one should be selected.

One of the factors in the rapidity of measurement of the base line, whether primary or secondary, is the character of the site. In any case it should be prepared with only sufficient care to permit effective measurement by the apparatus used. If bars and trestles are used, the surface should be no more disturbed than to permit the prompt placing of the tripod supports.

FIELD PROCEDURE.

The method of procedure in the field must necessarily vary according to the apparatus used and the local conditions of the country. An example of the method used with the slide contact bars can be found in reports on the Yolo and Los Angeles bases in the United States Coast and Geodetic Survey Report for 1882, Appendix 8, and Report for 1889, Appendix 10. For the method used with the iced steel bar and 100^{m} steel tapes see Professor Woodward's reports on the Holton and St. Albans bases.

In the work of measurement the first measure requires time for drilling all persons in precision and rapidity of action, and a repetition of the measure can be made in much less time than the first and very probably with more accuracy. The rapidity of measurement conduces to accuracy when movable tripod supports are used, because it lessens the liability of change in the position of the bar.

For transferring the end of the bar to the ground when measurement is suspended various forms of apparatus have been used. For end contact bars some form of transit sector is generally used. It is set up at right angles to the base, opposite to the end of the bar, the terminal of which is transferred by means of it to a scale placed on the ground mark. When the optical measuring apparatus is used, the Repsold cut-off is preferred, a description of which can be found in the Report of the Lake Survey.

THE ACCURACY ATTAINABLE IN BASE MEASUREMENTS.

The great importance to geodesy of the adoption of the metre as the international unit of length need not be pointed out, but the establishment of an International Bureau of Weights and Measures for the comparison of standards is here adverted to, not only in recognition of its eminent services to geodesy, but because its work must now be taken as the standard of attainable accuracy in metrological work.

Attention may also be called to the results of a study made recently at the International Bureau of Weights and Measures of nickel and certain alloys in regard to their suitability for line standards. Nickel bronze and aluminium bronze showed a decided change in length after repeated heating to 100° and cooling to 0°. Phosphor bronze showed no such change. A short nickel bar was found to answer all the requirements considered, namely, price, hardness, susceptibility to a high polish, high modulus of elasticity, and resistance to deteriorating effects of moisture and of such chemical agencies as are commonly used in laboratories. Nickel bars of suitable lengths for geodetic standards have, however, as yet not been made.

From the report of the International Conference on the Construction and Comparison of the New Metric Prototypes, of which the United States has two, it appears that the probable error of the result of the comparison of two prototypes was only $\pm 0{\cdot}04$ micron at the temperature at which the comparisons were made. Taking into account, however, the coefficient of expansion of 𝔐, the final estimate of the accuracy reached is stated in the following words:

> It may be concluded, therefore, that the equations of the prototypes lead to a knowledge of their absolute lengths with an average probable uncertainty which, under the temperature conditions usual in metrological operations—that is, between 20° and 25° C.—lies between 0·1 micron and 0·2 micron, and at a higher temperature it may slightly exceed the last-mentioned limit.

The belief is also recorded that if the comparisons between all the prototypes were gone over again the mean differences found would be of that order of magnitude.

When it is considered that the comparisons above referred to were made under the most favorable conditions, such as uniformity of temperature, identity of material, perfect illumination, and that slight imperfections in the latter alone introduce very material discrepancies, we may conclude that no geodetic standard can be known with a higher degree of accuracy than 1 part in 5 000 000 of its length in terms of the international unit.

Since all the operations involved in base measurement tend to decrease this degree of accuracy, we may assume that 1 part in 5 000 000 of its length is a higher degree of accuracy than can ever be attained in practice by known methods. If, in contradistinction to this estimate, the probable errors of certain famous base measures, like that of the Madridjos Base, are cited ($\frac{1}{5\,865\,800}$), it must be remembered that they do not refer to the absolute length of the base.

That it is possible to measure a base line repeatedly with the same apparatus with a surprising accordance between measures, indicates that the elimination of the accidental errors has been successfully met by the different forms of apparatus in use; and it is also believed that the various methods of observing successive lengths of the same bar or systems of bars are sufficiently precise. On the other hand, that constant errors exist which are in the main due to a defective knowledge of the temperature of the bars is a fact commonly assumed or proven by the lag of mercurial or bimetallic thermometers used on various apparatus. These constant errors are not easily determined, but are now the principal sources of error.

Recently experiments to elucidate this point have been made abroad, as well as in this country, by measuring the same base with different forms of apparatus. Thus, a base at Bonn has been measured with the Bessel apparatus and again with the Brunner.

The results of the measurement with the Bessel apparatus, made twice with rising and twice with falling temperatures, indicate a relative error of $\frac{1}{4\,500\,000}$. The same base, which is 2·5 kilometres long, was then measured with the Brunner apparatus, and it is stated that the agreement with the former measures is excellent, if we regard a discrepancy of 1^{cm}, or $\frac{1}{250\,000}$, as a constant error, the cause of which is not, however, known and is being investigated.

We also have the results of the measurements of the Undine Base, as obtained by the Austrians with their apparatus and by the Italians with the Bessel apparatus. Here the difference is only 4^{mm} in a total length of 3·248 kilometres, or $\frac{1}{800\,000}$.

The Brunner apparatus, elsewhere described, is the one used most recently in France and by General Derrecagaix. It is considered a model form of a modern base apparatus, and his opinion was apparently shared by other geodesists, for the Spanish, French, and Germans have adopted it, notwithstanding its great cost and the slowness of its manipulation.

The relative error of measurement for the Brunner apparatus, as used by the French, is about $\frac{1}{2\,400\,000}$, but the experience gained at Bonn indicates that the measure of its real accuracy is still open to question, as above mentioned.

The measurement of the Holton Base with the 100^{m} steel tape and 5^{m} secondary bars in the United States also shows a marked difference. Each considered alone gives a relative error of less than $\frac{1}{1\,000\,000}$, yet differ by 1 part in 350 000.

NUMBER OF MEASUREMENTS OF A BASE LINE.

As a test of the accuracy of the work and to afford an opportunity for eliminating errors of temperature each base should be measured at least twice. It has already been stated that, owing to the experience gained in a first measurement, the second can be made much more rapidly than the first. On this account, and because a large part of the cost of such operations is the expense of preparing the line, putting the party and equipment into the field and taking them out, the second measure increases the cost of the work but slightly, while the advantage derived from it is very great.

Whenever the steel tape is used for primary work it is recommended that at least four measures be made, using two different tapes, and in such a manner as to eliminate the effect of thermometer lag as nearly as possible; i. e., by measuring each section with each tape under both rising and falling temperatures.

FREQUENCY OF BASE LINES.

In many cases the length of the triangle sides and the nature of the country constrain the economical introduction of bases. Areas like the great plains, however, afford many opportunities for introducing bases economically, and where that can be done it is advisable to obtain accuracy by a multiplication of bases rather than by refinements in the triangulation.

RELATION OF THE ACCURACY OF BASE MEASUREMENT TO TRIANGULATION.

It stands to reason that the degree of accuracy to be aimed at in the measure of a base line depends largely on the object the base has to subserve, on the apparatus and the time available, and on the money allotted. Hence, no definite limit can be assigned. With the perfection of the means available a line may readily be measured with a probable error of $\frac{1}{1\,000\,000}$ of its length so far as mere measurement is concerned. On the other hand, when all the errors which may enter have contributed their share, the actual uncertainty may be considerably greater. It may be, say, twice or three times the above; but as no additional expense is involved by carefully attending to detail, the observer will in all cases do his best with the means on hand. The high degree of accuracy ordinarily within his reach at present is, however, rapidly dissipated through the angular measure of the two, three, or more steps required to refer the length of the base to the first line of average length of the triangulation. It may be further remarked that high accuracy in the angular measures of the latter is needed if an error, say $\frac{1}{150000}$, is to be maintained or not to be exceeded throughout the chain of triangles.

In tertiary triangulation an average uncertainty or limit of $\frac{1}{10000}$, or even $\frac{1}{5000}$, may be satisfactory for the special purpose. For intermediate secondary triangulation the fraction of the length $\frac{1}{100000}$ to $\frac{1}{50000}$ may be suitable, and the degree of accuracy for the base supporting such work should be graduated accordingly, yet with the precaution above adverted to.

The observer should pay especial attention to the minute and careful centering of the stations which constitute the base figure.

RECOMMENDATIONS.

In conclusion, the committee recommend that further experiments with the steel tape be made especially with a view to a better determination of its temperature. This recommendation seems to be warranted by the results of the measurements of the Holton Base.

2. That the new duplex apparatus just completed by the Survey be given a thorough and careful trial as soon as practicable.

3. That the iced bar be used to lay off a 100^m comparator at the bases where the above recommendations are followed.

4. That in all measurement due regard be paid to rising and falling temperatures, so as to eliminate as much as possible the errors due to lack of knowledge of temperature of the measuring apparatus.

5. That the tripods for supporting bars be made of metal, and embrace the details which experience has shown to be conducive to accuracy and rapidity of measurement.

6. That ordinarily the base line be divided into sections one half kilometre in length.

7. That a 100^m comparator be established in Washington for the purpose of testing steel tapes and contact slide apparatus.

8. That in future use of the steel bar in ice the water in the Y trough be not drained off below the surface of the bar, in order to remove any possibility of doubt as to the actual temperature of the bar.

GEORGE DAVIDSON, *Chairman.*
OWEN B. FRENCH, *Secretary.*

REPORT OF COMMITTEE C, ON TRIANGULATION.

CONTENTS.

REPORT.

(1) *Object of triangulation.*—The general object of triangulation is well understood. It is primarily to furnish absolute or relative geographic positions of a series of prominent points spread over extended areas, and, secondarily, to provide the necessary data for topographic and hydrographic surveys. It furnishes the three coordinates of each place, viz, the latitude, the longitude, and the altitude.

(2) *Classification.*—In its execution great diversity exists, depending upon the special object of the work, as, for instance, whether designed as a contribution to the measure of the figure of the earth or simply to give the relative positions necessary for the traverse work of a county or for the production of a city map. The larger operation is purely geodetic in character; the smaller one refers to plane surveying. The methods of observing, the instruments, the method of computation, the accuracy, etc., are different for different classes of work; hence, for convenience certain more or less well-defined subdivisions have been made, such as main and subordinate triangulations, primary, secondary and tertiary triangulations.

By the term main triangulation we understand the principal series of triangles which compass along the shortest line the whole extent of the surface or country under consideration, and by subordinate triangulation any series of triangles which depend for their support on the main series and which may, if small enough, be contained within it or branch off from it in any direction. The other classification, into primary, secondary, and tertiary, is more technical.

(3) *Primary triangulation* is characterized by the greatest development of length of sides practicable (depending on the heights of mountains) and by the greatest accuracy of measure. The geodetic positions will depend on direct observation of astronomical latitudes, longitudes, and azimuths. Refined base lines furnish the linear measures. The length of sides may ordinarily vary between 50 kilometres (say, 31 statute miles) and 150 kilometres (say, 93 statute miles), but may reach 250 kilometres (or 155 statute miles), and objects distant 300 kilometres (or 186 statute miles) and over have been sighted. In computing triangulations developed on such large scales (as in California, Nevada, and Utah) we have to consider the spheroidal excess; the reduction to sea level of the horizontal directions on account of height of station sighted; the reduction of the astronomical latitude for height of station occupied, the vertical being a curved line; the employment of formulæ for the computation of position of greater accuracy than Puissant's modified formulæ, ordinarily used on the Survey; the use of logarithmic tables of eight places of decimals, etc.

(4) *Tertiary triangulation* is designed to furnish positions of a sufficient number of points as a basis for developing topographic or hydrographic surveys, and its extent is so limited that the area covered may be taken as that of a plane surface; hence the length of its sides is usually between 1 and 15 or 20 kilometres (say, ½ to 9 or 12 statute miles). Five places of decimals suffice for the computation of its sides.

(5) *Secondary triangulation* bears an intermediate character between the other two classes and simply effects the connection between them; i. e., the descent from the long sides of a primary triangulation to the short ones of the tertiary, constitutes its character. Its sides may therefore vary considerably; that is, between the lower limit of the former and the upper limit of the latter class. The spherical excess and Legendre's theorem are introduced and seven places of decimals are generally employed in the computation of its sides.

It may be remarked here that in our middle latitudes (about 37°) an area of 196·8 square kilometres (or 76 square statute miles) corresponds to a spherical excess of 1″.

(6) *Adaptation to the surface of the ground.*—In general, the most elevated peaks and mountain ranges, up to the limit of accessibility, are the most favorable positions for stations. Two parallel ranges inclosing a valley offer great facilities, as the latter can generally be straddled by well-proportioned figures; so also do high buildings in the case of small triangulations. On the other hand, flat regions, especially when wooded, are to be avoided if possible; also ridges when wooded and of equal height. These and related considerations, however, belong to the subject of reconnaissance (which see). Whether it will be better to build high structures to overcome obstructions or to cut lanes or

avenues through forests is largely a matter of economy of money and time, but high structures do not affect the accuracy of the survey.

(7) *General form of main triangulation.*—Two systems have been employed in the triangulation of countries, the so-called gridiron and central systems. The former starts out with a series of parallel chains of triangles at certain intervals, intersected by other chains at right angles to them. The rectangularly shaped open spaces are then to be filled up by subordinate triangles. The latter system starts from the center of the country and extends radially in all directions, thus growing by concentric rings of triangles which eventually will cover the whole area. The first system is far preferable, as it lends itself better to computation (adjustment) and admits of preference being given to the advancement or completion of the survey at certain places. Practically the two systems may alternate or merge into one another.

(8) *Geometrical composition of a triangulation.*—Triangulation chains may be made up of a series of single connected triangles, a series of hexagons (or other polygons) hinged together on one side, or a series of quadrilaterals placed together. Their relative advantages are: For the triangle series, least number of stations and rapidity of measure—hence economy; for the polygon series, great extent of surface covered, and for the quadrilaterals, great accuracy.* In practice the geometrical figures will be found distorted and generally lengthened out in the direction of the axis of the triangulation; likewise interlaced and combined in a variety of ways so as to take advantage of favorable points presented.* In the case of *single* triangles the angle opposite to the base side should not be too small for favorable intersection, and in complex cases the observer should have a well-defined and uniformly strong scheme, avoiding complications of lines.

(9) *Frequency and length of base lines and their connection with the triangulation.*—The number of base lines to be introduced into a triangulation, as well as the length of the bases, depends on the average length of the sides of the triangulation and upon the degree of accuracy desired for the latter. Owing to the fact that base lines can be measured, and are rightly measured, with much greater accuracy than can be sustained through a triangulation, it is plain that in order to increase its accuracy we must introduce more base lines, or increase the number rather than their length. The transfer of the comparatively short length of a base to the greater length of a side of the triangulation is generally effected by several steps so as to avoid too acute angles, and consequently loss of accuracy. The figure of the base net connecting the base with the triangulation is therefore one of importance, and, ideally, may be described as a series of quadrilaterals with diagonals intersecting at right angles, the length of these diagonals increasing ordinarily in a ratio of 1 to 2 or 3, thus requiring

*See Annual Report United States Coast and Geodetic Survey for 1876, Appendix No. 20.

several steps to ascend to the length of a main line. Ordinarily two or three may suffice.

No precise rule for deciding a priori on the length of a base can be given. Any of the fractions one-tenth to one-sixth of the length of an average side of the triangulation may be useful for an estimate, since the actual length of base lines varies between wide limits. For a triangulation of the first order of magnitude the length may be 15 kilometres (9·3 statute miles) or slightly more; for a third order triangulation it may be 1 kilometre (0·6 statute miles) or even less. Base lines between 5 and 8 kilometres (say 4 to 5 statute miles) in length are the most common. They should be measured at least twice, once with rising and once with falling temperature—a very important condition, only to be omitted when a "bar in ice" apparatus is used. For subdivisions of a base distances of 0·5 to 1 kilometre are suitable. These are needed for furnishing the data for that part of the probable error of a base which depends on the measurement alone. The more accurate the angular measures in a chain of triangulation the greater may be the distance apart of its base lines. This distance varies accordingly between wide limits, but ordinarily may be from 20 to 40 times (or more) the combined length of the bases.

Nevertheless there may be conditions in the orography of the country which constrain the location of base lines, as, for example, on the Pacific Coast and through the region of the Sierra Nevada and the Rocky Mountains. Thus, to avoid the unfavorable country between the Sacramento Valley and the Salt Lake region, a distance of about 550 miles, the triangulation has been expanded to a great size and is supported by a long base in the former section. This is an instance where the Survey inaugurated a scheme of triangulation on a scale larger than had ever been attempted elsewhere, and consequently there was no experience to guide or suggest the attainable accuracy in the triangulation and length and frequency of bases. Similar conditions constrained the location of a base in the Los Angeles plains, and other like examples might be given. In a region where base-line sites can be more readily obtained the number of bases will depend mainly on the degree of accuracy desired for the triangulation and to a less extent upon their length.

(10) *Accuracy of a triangulation.*—From an economical standpoint we may confine our inquiry to answering the question, What may be considered *sufficient* accuracy in the measures of the separate operations and in the results of a triangulation as a whole?

A base line can readily be measured with a probable error of 1/250 000 part of its length, and by application of superior apparatus, of several measures, and greater care—hence, at an increased cost—the

*Cf. Die Geodätischen Hauptpunkte, etc., Von G. Zachariae. Translation by Dr. E. Lamp, Berlin, 1878, Art. 37. Also Jordan's Handbuch der Vermessungskunde, Vol. III. Third edition, Stuttgardt, 1890.

probable uncertainty may be reduced to 1/500 000 or less. Although this last fraction may be taken as a practical limit worth aiming at, extreme values of accuracy are recorded as having been reached, such as probable errors of *measure* of 1/1 000 000 or of a still smaller fraction. Now, to maintain an accuracy of, say, 1/150 000 or even 1/100 000 part of the length, on the average, in an extended triangulation, is a matter of difficulty, as is manifested whenever we compare the length of a junction line derived from two independent chains of triangles. The excess of accuracy of a base is lost partly in the base figure, and is further rapidly dissipated in the adjacent triangles; hence the inexpediency of straining at an extravagant degree of accuracy in the measure of a base becomes evident.* What is really important is a knowledge of the true length of the measuring apparatus in terms of the unit of length (on this survey, the international metre).

Respecting primary triangulation employing superior theodolites (40 to 50 centimetres diameter) the probable error of a *single measure of a direction*—i. e., the mean of two pointings on the heliotrope, telescope direct and reversed, with readings of three microscopes on two graduation marks each—has been found to be $\pm 0{\cdot}64''$ (with variations between $0{\cdot}45''$ and $0{\cdot}90''$), derived from 22 stations in California and Nevada. At these same stations the average number of series or measures of a direction was 63 and the resulting *probable* error of a direction as shown by the *station adjustments* is $\pm 0{\cdot}08''$ (with variations between $0{\cdot}06''$ and $0{\cdot}10''$). Hence, *mean* error of an observed angle $\left(\frac{3}{2}\right) 0''{\cdot}08 \sqrt{2} = \pm 0''{\cdot}17$, and we may expect the triangles to close within $0{\cdot}17 \sqrt{3}$ or $\pm 0''{\cdot}29$ on the average; but we find the mean closing error (73 cases), as demanded by the triangles composing the figure, to be $\pm 0''{\cdot}61$, which shows the presence of other adverse influences than those arising from the graduation and pointing errors. The most potent of these is the lateral refraction, composed of a constant and a variable part. Large local deflections of the vertical at a station also have their influence. Taking our figures as typical, we conclude that a less number of series would suffice, provided the same variety of weather is experienced, without detriment to the work.† Suppose the number reduced to 31 (a prime number), the resulting *probable* error of a direction would rise to $\pm 0''{\cdot}12$, the *mean* error of an angle to $0''{\cdot}25$, and the expectation for closing of triangles would rise to $\pm 0''{\cdot}43$. This would probably leave the large scale work of the Survey still in the front rank for accuracy.

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*See Appendix No. 9, United States Coast and Geodetic Survey Report for 1885.

†Respecting the time devoted to the observations, General Walker, of the Great Trigonometrical Survey of India, remarks (Vol. II, p. 70): "Any neglect of these precautions, any hurrying over the prescribed tale of observations with the utmost possible rapidity, even at the time when the signals are apparently very steady and favorable, is liable to introduce larger errors than those which are partly attributable to any defect in the instruments of this survey." It is also remarked that it is considered a great misfortune to use instruments of inferior order.

The mean error of an angle as derived from adjusted triangulation is frequently used as a convenient measure of the relative accuracy of different triangulations. For the above case we have

$$m = \frac{\cdot 61}{\sqrt{3}} = \pm\ 0''\cdot 35$$

Taking as a second type of triangulation part of the transcontinental triangulation east of the Rocky Mountains, with sides averaging 25 kilometres (about 15½ miles) in length west of the Mississippi and 29 kilometres (about 18 miles) east of it, we can form the following table:

									$''$
Central Kansas	200 km.	(about	124 miles),	from	43	triangles,	$m=$		$\pm$0·76
St. Louis to eastern Kansas	595 "	"	370 "	"	137	"	"		$\pm$0·78
St. Louis to Indiana	177 "	"	110 "	"	33	"	"		$\pm$0·66
Indiana	235 "	"	146 "	"	24	"	"		$\pm$0·70
Western Ohio to Chesapeake Bay	864 "	"	537 "	"	451	angles,	"		$\pm$0·97

The following table is added for comparison with foreign triangulations.

In work of this character instrument circles of 25 to 35^{cm} (diameter) are suitable. The number of positions should not exceed 17, and the number of series may be about one-half to two-thirds the number recommended for primary work, according to its importance. Special attention should be paid to the centering of the instrument and of the heliotropes and to phases of signals.

Chains of triangulation in Europe, India, and Africa.

Name of country.	Epoch.	Length of sides.	Instruments. Maker.	Diam. of circ.	Micr. or ver.	No. of Δs.	$m = \sqrt{\frac{\Sigma\Delta^2}{3n}}$
		km.		*cm.*			″
Austria-Hungary	1850–88	12 to 60	Starke, Reichenbach	27·0 32·5		674	0·910
Bavaria and Palatinate	1804–53	10 " 40	Borda, Reichenbach, Ertel	40·5 32·5 21·5		132	1·778
Belgium	1851–73	10 " 30	Gambey, Breaulieu	?		219	0·892
Denmark	1817–70	10 " 40	Ertel, Reichenbach	40·5 32·5		87	0·742
Spain	1860–84	15 " 130	Pistor, Ertel, Repsold	28 37 32	2 *micr.* " "	325	0·886
France and Algiers	1792–1880	10 " 120	Gambey & Brunner	?	*ver.*	914	1·29 (best 0·27)
Great Britain	1787–1865	10 " 85	Ramsden, Troughton & Simms	81 and 61 46	5 *micr.* 3 "	552	1·83
Greece	?	10 " 40	Starke & Kammerer	26	2 "	109	0·77
Italy	1863–90	10 " 65	Pistor, Reichenbach, Gambey, Starke	32 32 27 27	2 " 2 " 2 " 2 *ver.*	514	0·920
Norway	1853–63	10 " 80	Reichenbach, Olsen, Repsold, Ertel	36 30 19		179	0·718
Portugal	1864–88	10 " 40	Troughton, Repsold	35·6 19·6	2 *micr.*	139	1·29
Prussia A	1847–77	10 " 50	Pistor, Pistor & Martins	27 27		79	0·734
B	1832–91	10 " 50	do.	32 38	----	690	0·554
Roumania	1855–86	8 " 25	Starke, Reichenbach	32·5 32·5		36	1·736

Russia	1816–86	10 " 25	Brauer, Troughton, Ertel	21 to 37	----	147	1·495
Saxony	1867–78	10 " 25	Repsold	31	2 *micr.*	197	0·350
Sweden	1819–80	10 " 45	do	32	2 "	304	1·09
Switzerland	1854–68	10 " 40	Starke, Reichenbach, Ertel	24 32 21		?	0·856
India	1860–80	15 " 40	Troughton & Simms, Barrow, Waugh	81 61 46	5 *micr.* 5 " 3 "	1,417	1.003

Any triangulation in which m does not exceed 1″ may generally be classed as being of a high order of accuracy, and, according to circumstances, double that amount may still be taken as of sufficient accuracy for the purpose for which the triangulation was made. In linear measure an accuracy of 1/50 000 or 1/75 000 part of the length is ordinarily considered a satisfactory one,* and 1/30 000 may be sufficient in many cases. If by reason of great distance from the base the accuracy in a triangulation should have been reduced below the standard originally set, the introduction of a new base at the weakest point, or near it, would be the proper remedy.

Tertiary triangulation demands no high degree of accuracy, and may vary from 1/20 000 to 1/5 000 part of the length, according to requirements.

When dealing with mean or probable errors the following rough-and-ready rule to judge of the admissibility of apparently large individual deviations from the mean value may often be found of service, viz: With the usual very limited number of observations, any that may be outside the total *range* of 5 times the mean error or 7 times the probable error should be looked upon with suspicion, and, conversely, the mean and probable errors may be guessed at to be about one-fifth and one-seventh of the observed range, respectively.

(11) *Interstate and international boundaries.*—Apart from the astronomical measures that may be needed, the most accurate method of locating boundary lines, and the one that best preserves the monuments, is triangulation, because all the prominent natural features and many of the artificial ones may be connected with the boundary marks as objects of reference. When, in addition to the triangulation, the topography is also carefully executed, the labor of replacing lost marks in their proper position is reduced to a minimum. This necessarily presumes that the triangulation points have been well marked.

The expense of triangulating a boundary and the length of time required to execute it has in the past generally precluded the adoption of this method in the United States.

The purpose of this paper is rather to treat of the best practical method of running boundary lines than to make a historical review of boundaries that have been already run and a minute examination of the details of their survey.

In all boundary work due regard must be paid to the legal definition of the line, whether it is to depend on astronomical measures alone or on geodetic measures when the ends or termini are to be connected by a straight line (the geodetic or shortest line), and, in the case of an arc of a parallel, whether it is to be a mean parallel or an astronomical parallel.

In lines depending upon astronomical observations the local deflection of the vertical enters as a disturbing element. This is due either to

* One sixty-three thousandths corresponds to 1 inch in a mile, nearly.

mountain masses, proximity to the ocean, or irregularity in density of the matter beneath the surface of the earth. It may amount to a few seconds, and in extreme cases to very much more.

On the survey of the Northern Boundary west of the Lake of the Woods, on which 41 latitudes were observed in a distance of 1374 kilometres (853·5 miles), the average local deflection or difference from the mean parallel was 2·15″. The observing error for geographical positions is generally far within this quantity. At 56 stations observed on the oblique arc along the Atlantic coast the average difference between the geodetic and astronomical latitudes was 2·2″, which is almost wholly due to station deflection.

The so-called regular boundaries may be divided into several kinds: (1) Boundaries along a meridian; (2) boundaries along a parallel; (3) boundaries along oblique lines. Even a circular boundary has been traced. There are others of an irregular nature, that follow natural topograpic features, such as streams, mountain summits, divides, or crest lines. These latter do not come within our scope for treatment, as they depend on ordinary trigonometric and topographic methods of surveying.

Boundaries along a meridian.—These are the simplest forms and present the least difficulties when the observer is provided with the proper instrumental outfit. It is necessary to determine the true meridian and trace it out, preferably by back and fore sights. As often as may be found necessary check azimuths should be observed. The frequency of these checks will depend on the character of the country. If the line is to be carried over a plain, where the sights are short, an astronomic check should be secured every 25 to 30 kilometres (15 or 19 miles), but if the country is rolling or mountainous, affording sights from 5 to 30 kilometres (3 to 19 miles) in length, from 60 to 100 kilometres (37 to 62 miles) may be run before checking.

The party should be provided with a 20 to 25^{cm} (8 or 10 inch) theodolite, with a diaphragm arranged for observing time as well as for measuring angles, and a sidereal chronometer. Azimuths may be determined in the usual way by observing Polaris at any hour angle in connection with a terrestrial mark. Three sets of observations on one night will generally give the desired accuracy. The meridian may be laid off on the horizontal limb of the theodolite, a signal put in line, and the angle then accurately measured by repetitions. If there should be an error worth correcting, the signal may be moved to its proper place, knowing the distance and angular deviation.

This instrument should be used for measuring angles and azimuths, ranging out the line, and interpolating points on intermediate prominences. There should also be a 10 or 15^{cm} (4 or 6 inch) theodolite, with a vertical circle, having on the diaphragm parallel lines or threads for reading telemeters at road and stream crossings.

In running a meridian boundary line, where long distances may be obtainable, say 30 to 100 kilometres (19 to 62 miles) or even more, it is suggested that the meridian instrument affords a good means for projecting the line not only to the distant stations, but to such intermediate stations as are visible. The position of the instrument, in its relation to the meridian, is to be determined by the observation of time and azimuth stars.

For the topographic work we measure distances either by a small triangulation or telemeter or tape line. The error of telemeter lines, as shown by the Northern Boundary Survey, is given as 1/300 of the length for the average of an entire season; but on selected days, with great care, this may be reduced to 1/1500. General Comstock states that on ordinary ground 1/700 may be secured. In 1893, on the Mexican Boundary Survey, in a line of 57·3 kilometres, 1/1 400 was reached; in a line of 73 kilometres the discrepancy was 1/1 900 of the true length.

For short sights, slender range poles, and for long distances—from 5 to 30 kilometres (3 to 19 miles) or more—pocket heliotropes may be used. To illustrate the accuracy with which a meridian may be run it may be stated that in 1883, while running the boundary between West Virginia and Pennsylvania, the meridian was checked by an azimuth after ranging out 18 kilometres (11 miles) and the line was found to be apparently in error $2''{\cdot}5$ (21^{cm}). Beyond this point a series of long sights were obtained, and the next check azimuth was observed after running 71 kilometres (44 miles) more, when the line was found to be apparently $2''{\cdot}4$ (8^{cm}) in error and in the other direction. Both of these errors were inappreciable and no corrections were made, as the theodolite was only graduated to $5''$. Some portion of the error may have been due to local deflection. Of course any error in the line should be distributed proportionately.

Boundaries along a parallel of latitude.—These may be run either by chords or by tangents. The latter method is preferable on account of its greater adaptability to topographic features; the former is more simple, especially when the adjacent astronomical stations are intervisible.

It is necessary to determine the meridian and lay off a tangent in the prime vertical at the point where the azimuth and latitude are observed. Before attempting to run the parallel astronomical latitude stations should be located approximately at distances 15 to 30 kilometres apart. Distances from the latitude stations are to be laid off along this tangent from which the ordinates to the parallel are measured.

If the topographic features permit, regular distances should be determined along the tangent and offsets made at the proper angle from the tangent, so that they may be either normal to the small circle of latitude or normal to the tangent. The Coast and Geodetic Survey formulæ for geographical positions will be found very useful in computing the azimuths and offsets for known distances, which distances may be irregular if necessary, and generally they are so in a broken country.

An azimuth may be observed at the end of the tangent to determine any error in ranging it out, and this and the correction due to difference of local deflection at the two stations are to be distributed along the offsets.

If the *astronomical* parallel should be laid out, it would be an irregular curve in consequence of local deflections, but it would have the

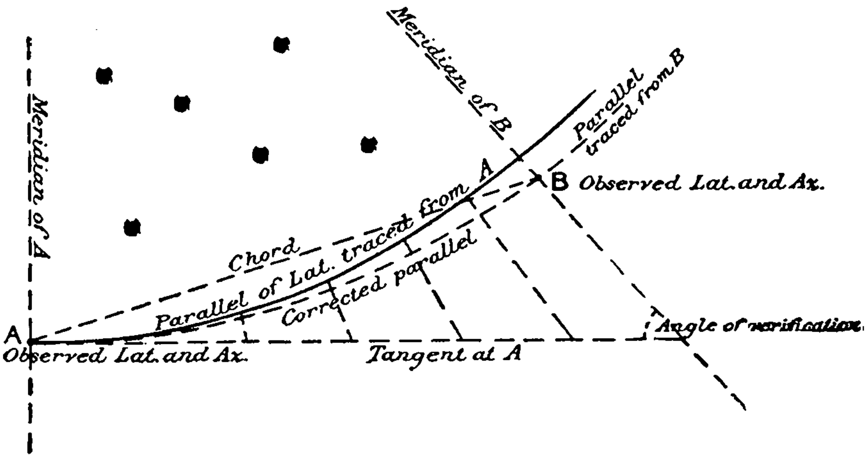

FIG. 1.

advantage of being more readily reproduced than the mean parallel if it should become necessary to restore the boundary marks. The latter may be traced out by means of triangulation. Figures 1 and 2 illustrate the method of procedure.

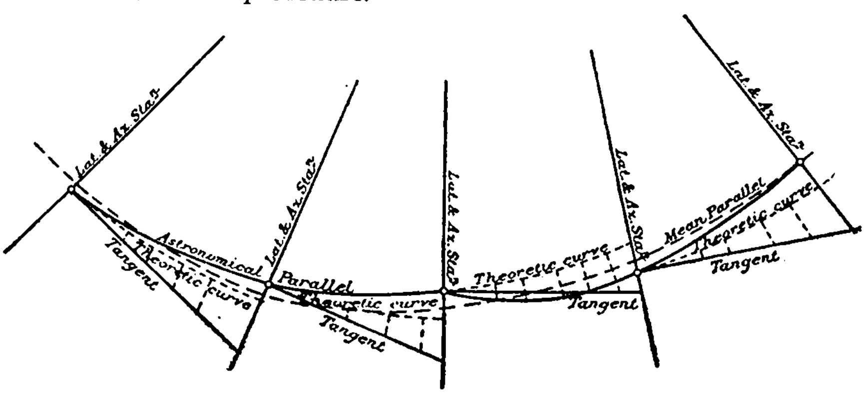

FIG. 2.—Method of tracing a parallel of latitude, showing tangents, offsets. Theoretic curve drawn through each astronomical station.

The tangents and offsets may be corrected for error of ranging out after checking them by the regular azimuth observed at each astronomical station.

Boundaries along an oblique line.—These are, like the other regular boundaries, referred to the meridian. If the latitude and longitude of the termini are known, the azimuth or direction of the line may be computed, and then by means of azimuth observations the angle can be laid off and the line traced out. Any error in running the line is to be distributed proportionately.

An example of oblique boundary is that run in 1874 by Professor Bowser between New Jersey and New York with a Coast and Geodetic Survey 20^{cm} theodolite. The Coast and Geodetic Survey located the termini—one by triangulation, the other by astronomical measures.

For tracing out the line and locating monuments at suitable intervals, so as to be readily intervisible also at intersections of roads, streams, and crest lines, two methods are available—one starting from the fixed initial station with a given azimuth to reach the opposite end and proceeding by backward and forward sighting, the other by first connecting the end points by a triangulation and the insertion of the monuments in the places computed. By the first method the error in the direction of the line when the opposite end is reached must be evenly divided along the entire line. In mountainous countries this method of ranging can not dispense with occasional small triangulation in order to get around an inaccessible height. On the other hand, the triangulation method and subsequent laying out of the points in line, may be more laborious and demand more time, but admits of the tracing out of a line not subject to any material extent to the local deflections of the vertical, which necessarily come in at all stations where a transit line is used. The preference for one or the other method thus depends on circumstances.

In countries difficult of access and where land is of little value, and the expense of tracing a boundary throughout its entire length is disproportionate to the advantages to be gained, important points may be located, and either connected or not, according to the public demands. In Alaska this method is being used.

Instrumental outfit.—For the measurement of horizontal angles two kinds of instruments are in use, the repeating instrument and the direction instrument. Theoretically the former should give the better results, but experience has shown that with the improved methods of graduation now in use the reverse is true. It may be stated in general that when the graduation is good and the optical conditions commensurate, the method of directions should be used; with poor graduation, repetitions will give the better results. Thus in the large instruments used in the primary or large scale triangulation great refinement is demanded in the graduation of the circle; and a direction instrument should therefore be preferred, though a repeating instrument may be allowable and has been found under some circumstances to give equally good results. In secondary and tertiary triangulation the repeating instrument will usually give satisfactory results, with the great advantage that it admits of rapid work both in the field and in the office. For the more refined work of the primary triangulation the instrument should have a 40 to 50^{cm} (16 to 20 inch) circle, graduated to $5'$, reading by three micrometer microscopes to $1''$ each and by estimation to $0''\cdot1$.

The telescope should have an objective of the best class, of 7·5 to 8·5^{cm} (3 to $3\frac{1}{4}$ inches) diameter, three eyepieces ranging in power from 40 to 100; also two very light vertical circles or finders for azimuth stars and an ocular micrometer for special or occasional use. The latter may be used for azimuth work as well as micrometric angular measures when the part of the day suitable for favorable operations is very short and when the atmospheric conditions are adverse. The instrument should have slides to provide for radial illumination of circle by a right-angled prism. The instrument should be mounted on a position circle cemented to the top of a pier whenever practicable. As the observations for azimuth should, if possible, have the same degree of precision as those for horizontal directions, the telescope should have sufficient optical power to observe stars of 6·5 magnitude.

In order to secure images of uniform size at all distances it is necessary to vary the size of the heliotrope according to the distance to be observed. For ordinary atmospheric conditions and for distances of 10 miles and over the formula $x = \cdot046d$ may be used for this purpose, where x is the side of the square mirror in inches and d is the distance to be observed in miles, or $x = \frac{3}{4}k$, where x is the result in millimetres and k the distance in kilometres.

The following table contains the length of side of square mirror for various distances:

Distance.	Side.	Distance.	Side.	Distance.	Side.
Miles.	*Inches.*	*Miles.*	*Inches.*	*Miles.*	*Inches.*
10	0·46	60	2·8	120	5·5
20	0·92	70	3·2	140	6·4
30	1·37	80	3·7	160	7·3
40	1·83	90	4·1	180	8·3
50	2·3	100	4·6	200	9·2

Referring to the theodolite, the effect of an outstanding error (c) in collimation on the measure of a horizontal direction may in general be kept small, thus: Supposing $c = 10''$ and the angle of elevation of the object less than 3°, this effect is but $0''\cdot01$ at most, and it is the same for the case of $c = 1'$ and the elevation less than 1°.

The effect of an error (i) in the horizontality of the revolving axis of the telescope is in general much larger, as it depends on the tangent of the angle (α) of elevation, thus for $i = 10''$ and $\alpha = 1°$ it amounts to $0''\cdot2$, and with $\alpha = 2°$ it is $0''\cdot4$. Both defects are eliminated in the measures by the reversal of the telescope and azimuth circle.

The effect of an error (v) in the verticality of the theodolite axis can not be eliminated by any method of observing. The amount for any angle depends on v and the relation of the angle to the plane which contains the true and instrumental zenith point. With repeating theodolites we have to deal with two vertical axes which should coincide. Close attention to this last source of error must be given by careful leveling.

The effect of the eccentricity of a circle is eliminated by the reading of any number of *equidistant* microscopes. The examination of the graduation of a circle for periodic and irregular errors should be made at the office.

All heliotropes should be centered over their respective stations with the same care as the theodolite; and when poles, targets, or cones are used, the data needed for correcting for phase where required should be given (size and shape of object and local time or else the azimuth of sun). If plain poles are used, their diameter should be graduated for the distance; i. e., it should be smaller the shorter the distance from the station. It should also be stated whether the object was seen by reflected or diffused solar light.

(14) *Method of observation.*—In order to secure the best results with either form of instrument the observer should make the observations at different times of the day, say a. m. and p. m., and on different days and under varying conditions of atmosphere, but rather exceptionally during day and night, and he should refrain from observing under any manifestly unsuitable or doubtful conditions. In no case in primary and secondary triangulations should the observations be finished in one day; but several days, embracing morning and afternoon observations, should be devoted to them, in order to avoid cases of lateral refraction, which have occasionally been experienced to the extent of many seconds.* In mounting the instrument regard should be had to proper shelter for it as well as for personal comfort while observing. Pointings should be made as rapidly as possible consistent with a clear and decided bisection of the signal. In using a *direction* instrument the method usually adopted is to divide the circle into a number of equal parts, known as positions. This number should be prime,† so that no microscope may fall upon the same graduation in measuring upon the same object in different positions or after reversal of the circle. Having established an initial direction, one or more series are observed in each position, each series consisting of a pointing and reading upon each of the signals in the order of the graduation, and then, after reversing the telescope and turning the alidade 180° in azimuth, of another pointing and reading upon the signals in the

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* The observations for lateral refraction made by Dr. Fr. Pfaff with a theodolite mounted in the tower of his house, and extending throughout a whole year (Publikation des König. Preuss, Geodatischen Institutes), may here be referred to as evidence of the existence of this disturbing feature in the measure of horizontal directions. Yet for future special observation two or more fixed telescopes, cemented to a low stone pier and provided with eyepiece micrometers, would prove more effective and less troublesome than Dr. Pfaffs' arrangement. The lateral variation of more than one direction at a station could be thus investigated.

† Any other number that will accomplish the same purpose may be used.

reverse* order. The number of positions to be used depends upon the accuracy of the graduation and upon the degree of refinement desired in the results. Experience tends to show that with the best instruments now in use on the primary triangulation the effect of atmospheric conditions upon the result, after a certain number of positions have been used, is much greater than the effect due to errors in graduation. It is probable that 31 series are needed to secure the desired accuracy, and about this number should be obtained. It should be left to the judgment of the observer, taking into account the character of the work and of the instrument used, to decide in how many positions of the instrument these observations shall be made.

In the past, 23 positions have been the maximum number used. There is no objection to decreasing this number, or increasing it to the full number of series desired.

It has also been suggested to take a very small number of positions and exhaust the circle, and to repeat the same number of positions, but with a different initial reading of the graduation and subdividing the former spaces. A third and fourth group of such readings can be added if greater accuracy should be demanded. The advantage claimed for this procedure is the easy comparability of the results of the series making up a group.

In high mountain regions in certain cases it has been found that midday observations of horizontal directions are obtainable with the aid of the ocular micrometer. This has only been used in primary work and should be restricted to special cases.

When a repeating instrument is used each set of repetitions should consist of a certain number of measures of the angle (α), say 3, followed by an equal number of measures with telescope reversed; then the explement of the angle ($360-\alpha$) should be immediately measured in the same manner. Two sets of 6 repetitions ($3\,D+3\,R$) are preferable to one set of 12 repetitions ($6\,D+6\,R$), as something may occur to interrupt the observations during the longer time required for the latter, thus vitiating the whole set. Enough sets should be taken to obtain the desired accuracy, from 2 to 6 probably being sufficient in most cases, according to the precision required by the character of the triangulation. In regard to the number of angles to be measured at a station, it may be stated in general that there should be a check on every angle measured besides that of closing the horizon in the manner referred to above. Although for the highest degree of accuracy all of the sum angles might be measured, this should rarely be done, especially when the number of signals is large, as the increase in the accuracy of the result is not commensurate with the increased time and labor spent. It is preferable to measure only those angles which actually occur in the figure of the triangulation, and this consideration should have some weight in selecting

* This is to correct for any azimuthal change or twist during observations in the scaffold or supporting structure.

those sum angles which are to be used as checks and which will, as nearly as may be, equalize the number of pointings.

Lost motion and stress in a repeating theodolite.—The repeating circles in use on the Survey, in common with all theodolites of their date, are lacking in rigidity, and flexure is probably a fertile source of error, the effects of which, together with the lost motion in the numerous movable parts and particularly in the clamping arrangement, can only be eliminated by careful manipulation and the adoption of a method of observing which will make them always of the same sign.

In practice it is found that by making all movements in one direction the error of closure obtained by measuring an angle and its explement will, within the probable errors of pointing and reading, remain constant for any particular condition of the instrument irrespective of the size of the angle; and since the angles measured according to this practice and obtained under conditions which give closing errors of wide range, when corrected by half the closing error, show a very close accord, it seems probable that the method largely eliminates errors from these sources.

For use upon towers, scaffolds, and like supports which are subject to an azimuthal movement, caused by the diurnal motion of the sun, the repeating theodolite is especially adapted, since the short time occupied in the measurement of a single angle reduces the effect to a minimum and the method of procedure above referred to eliminates it.

When the collimation is well adjusted it is not always necessary to reverse the telescope in the middle of a set unless there is a large difference, say one exceeding 1° of elevation, between the objects observed upon. The error of collimation should be frequently tested, and corrected if perceptible.

It is desirable to secure, *in the end*, an equal number of measures with telescope direct and with telescope reversed. Instruments with eccentric telescopes must be used in both positions of the telescope.

Respecting the manner of measuring and recording, whether with or against the graduation of the circle, the former practice is supposed preferable; but in all cases the object first sighted or pointed on should be recorded first in order. Thus in measuring the angle, *A* to *B* indicates motion direct as understood by the observer, but *B* to *A* indicates motion reversed, and the entry should be made accordingly. The observer should indicate in the preface of his record book the direction in which his instrument is graduated, and also give a diagram showing the directions to the stations seen.

Outside objects to be determined.—Besides the regular triangulation marks visible at a station, directions or angles should be carefully measured whenever practicable on all objects, as light-houses, beacons, buoys, and other aids to navigation, on all international, State, and county boundary monuments, township and section corners of the United States Land Survey, State capitols and court-house domes or cupolas, church steeples and all prominent buildings, and all outlying rocks, shoals, or breakers. A round of angles should also be taken on all prominent peaks and other landmarks, and tangents should be taken to all the headlands along the coast.

The observations for the magnetic declination to be made at each station are referred to in the report of the Committee on Terrestrial Magnetism.

(15) *Signals and scaffolds.*—The term "signal," as used in the Coast and Geodetic Survey in connection with triangulation, includes all devices, appliances, and instruments employed as objects to designate to the observer the position of a station mark to be pointed upon by him, and includes all structures intended to elevate such objects or the instruments employed in observing.

Although the term "signal," as here used, properly relates only to structures or devices especially constructed, observers have at one time or another used almost every class of object of sufficient prominence to be identified from distant points, such as mountain peaks, hilltops, headlands, rocks, trees, cairns or pyramids constructed of various materials, poles or staffs, flags, lozenges, targets of various forms, cones of tin, globes of glass, etc., heliotropes, revolving or fixed, bonfires, rockets, blue lights, lamps of various forms, lamps in combination with reflectors, magnesium and electric lights; and for supporting their instruments, mounds of earth, trunks of trees in original position, chimneys and light-houses, church towers, etc., tripods and scaffolds, towers of stone, adobe, etc.

Sometimes high structures are needed in order to maintain the general proportions of the triangulation or for the purpose of overcoming an obstruction in the line of sight, but ordinarily the supports are of moderate height, only as a means of carrying the line of sight above the highly disturbed stratum of air near the surface of the ground. Where the difference in expense is not too considerable, lines in a wooded country should be carried above the tops of the forests rather than through long, opened lanes.

The question of the desirability of portable or permanent structures, being very largely governed by the comparative cost of material and transportation, is in general not difficult to solve.

At all high structures the instrument rests on a central tripod of strong and well-braced timbers and is entirely separated from the surrounding light scaffold which supports the observer and serves, when boarded in or wrapped with canvas, as a protection to the instrument against wind or sun.

The very detailed article upon "Construction of observing tripods and scaffolds,"* as published by the Survey, makes it unnecessary to go into the subject here.

* Appendix No. 10, United States Coast and Geodetic Report for 1882.

The visibility of objects sighted depends upon the brightness of the light, as heliotropes or artificial lights, or it depends upon contrast of the target, illuminated by ordinary daylight, with the background upon which it projects. The size of the reflecting surface in the use of heliotropes should be graduated to the distance from the station, for which see the scale proposed in another part of this paper.

In the case of targets or lozenges of various materials the relation of surface to distance will generally have to be determined by experiment. In the Coast and Geodetic Survey manual on triangulation the angular limit desirable for a signal has been stated as 1″; but as it is not practicable to maintain this limit at long distances, it will in practice be found necessary to increase the width sufficiently to preserve the area in some measure. In the case of lozenge-shaped targets the reflecting surface may be increased to any desired extent by multiplying their number. Lozenge-shaped targets of muslin have been found very satisfactory. The diagonals should be twice the linear value of 1″·5 for the distances. For lines of about 65 kilometres (40 miles) three or four lozenges, both black and white, symmetrically disposed along the pole, have proved satisfactory in all kinds of seeing and for different positions of the sun. Signals, such as rounded poles, which present to the observer two or more planes of varying relative distinctness are objectionable because they require phase correction, and should not be employed on work demanding a high degree of accuracy.

It may be well to call attention to the economy of the use of the heliotrope in consequence of its greater capacity for penetration of smoky or hazy atmosphere than could be had by a signal illuminated only by ordinary daylight. The heliotrope lights frequently enable the observations to be carried on when other signals would fail, and consequently they afford an opportunity to utilize the varying conditions of the atmosphere to a greater extent than is generally the case. This remark applies to relatively short lines; for long ones the heliotrope becomes indispensable.

The fact that all those who have made extensive experiments with night signals have reported favorably upon them should lead to their use whenever it is deemed advantageous. On lines of 50 kilometres and less they furnish beautiful steady objects for a greater number of hours, particularly before midnight, than day signals. Colonel Perrier, after his careful discussion of the results from night and day observations presented to the International Conference (Report of 1891), states as his conclusion—

That the results of night observations satisfy better the geometrical conditions to which all triangulations are subject, or, in other words, that the errors, whether due to observation or to lateral refraction, which latter has never yet been well determined, compensate each other better in night observations than in those made in daytime.

In this connection attention is called to the extremely interesting experiments made by the Survey at Pioche, in Nevada, in 1883, and at

Mount Nebo, Utah, in 1887, to test the availability of the moon's light for night signals.

The selenotrope, as the instrument used has been called, differs from the heliotrope only in the greater size of the mirror used, and is operated in exactly the same way.

At Pioche, in 1883, a mirror 12·7^{cm} square (5 by 5 inches) was used on a line 35 kilometers (22 miles) in length, giving very satisfactory results with the moon.

In 1887, while occupying Mount Nebo, in Utah, with the view of testing the efficiency of the selenotrope upon much longer lines, mirrors 15 × 20^{cm} (6 by 8 inches), 20 × 25^{cm} (8 by 10 inches), and 30 × 46^{cm} (12 by 18 inches) were sent to Draper, Onaqui, and Ogden, respectively—77, 113, and 156 kilometers (48, 70, and 97 miles) distant—and the heliotropers were instructed to show two hours each night from June 29 to July 4, commencing forty-five minutes after sunset, or as soon as the shadow on the vanes became distinct. The weather was unfavorable except on the 2d and 3d of July, when Draper and Onaqui were plainly visible in the illuminated field of the telescope, "distinct, steady, mere dots of white light and of ideal perfection for precise pointing." Ogden, 156 kilometers (97 miles) distant, for some reason was not seen.

(16) *Marking of stations.*—The main objects in marking a station are to secure its permanency and to render it easy of recovery. If the station is located on a ledge or rock not likely to be disturbed, a copper bolt with a cross on its top to mark the center, secured in a drill hole several inches deep, or two or three short bolts placed one over the other, so as to be more difficult of extraction, form a suitable station mark. Two or three arrows pointing to the center mark may also be cut in the rock. Where excavation is possible, there should be one mark at the surface and another buried 3 feet below the surface. For primary stations a stone embedded in cement, with a copper bolt for center mark, forms the best subsurface mark. If an observing pier or terminal of a base line is used, it should be built of stone, brick, or concrete, with a cross mark in the top and also one at the surface, and another below the ground, to indicate the center of the station, and two openings should be left in the pier at the base, at right angles to each other, to give access to the surface mark.

In secondary and tertiary work a bottle, crock, or flowerpot filled with ashes may be used for a subsurface mark, with a stone and cross at the surface. Special conditions of soil require marks suited to their particular needs. In all cases there should be suitable witness or reference marks in addition to the central one, preferably grooves or drill holes (filled with sulphur) in adjacent rocks, to which the distances and bearings should be carefully noted.

Experience has shown that stations are frequently lost by reason of the thoughtless meddling of ignorant and irresponsible persons, as well as by some whose cupidity had been excited by the material used in marking them, as, for instance, lead and copper in a country inhabited by Indians.

Hence a good general rule is that stations should be marked by objects having little or no value. And it is important that the attention of the passer-by should not be attracted to the location of the station by reason of the prominence of the surface, reference, and witness marks. The aim should be to mark the spot in such a manner that there will be no difficulty for one who has its description to find it; but a casual observer should not have his attention attracted to it.

The description of a station should include a topographical sketch of the ground and its approaches, a sketch showing the relative positions of the station mark and various points of reference, the best route by which to reach it from the nearest town, and any information which might prove of value to a party subsequently occupying the station, either in finding the station or in locating a camp or obtaining supplies. It is desirable that the name of the trigonometric station be a short one.

(17) *Incidental observations.*—In reply to the question, "What other observations than those of angular measures should be made by a triangulation party?" it may be remarked that this includes the measure of vertical angles, unless there be good reason why they should be omitted. In primary work it is understood that such observations for latitude and azimuth should be made as pertain to the general needs of a triangulation for astronomic data. No special rules can be given, though frequently every other station has been an astronomic station. It may often happen that a detached piece of triangulation is to be done; here astronomic observations, at least at one station, are demanded.

Observations for the approximate determination of the magnetic declination should be made at every station. For particulars see Report of Committee on Magnetics. In general no other meteorological observations than those required for the description of the weather in the daily report or what are directly demanded by the work in hand need be made. Complete meteorological observations are essential in special experimental work respecting laws of diurnal variation of refraction or for comparative value of measures of heights by barometer and other means. These cases are always covered by instructions. Regular barometric observations at stations of great elevation are recommended, but are not obligatory unless specified in instructions. In all cases of doubt on the part of the observer he is advised to ask for special instructions.

(18) *Explanatory note to accompany map showing state of the triangulation in the United States in January, 1894.*—The object of the map (illustration No. 11) is to give at a glance the present extent of the triangulation and to make some suggestions for its future prosecution.

The map shows the work done by the Coast and Geodetic Survey, and also areas which have been either reconnoitered or which are in a state of incompleteness.

State of the Triangulation in January 1894, completed, commenced, or in progress and prospective.

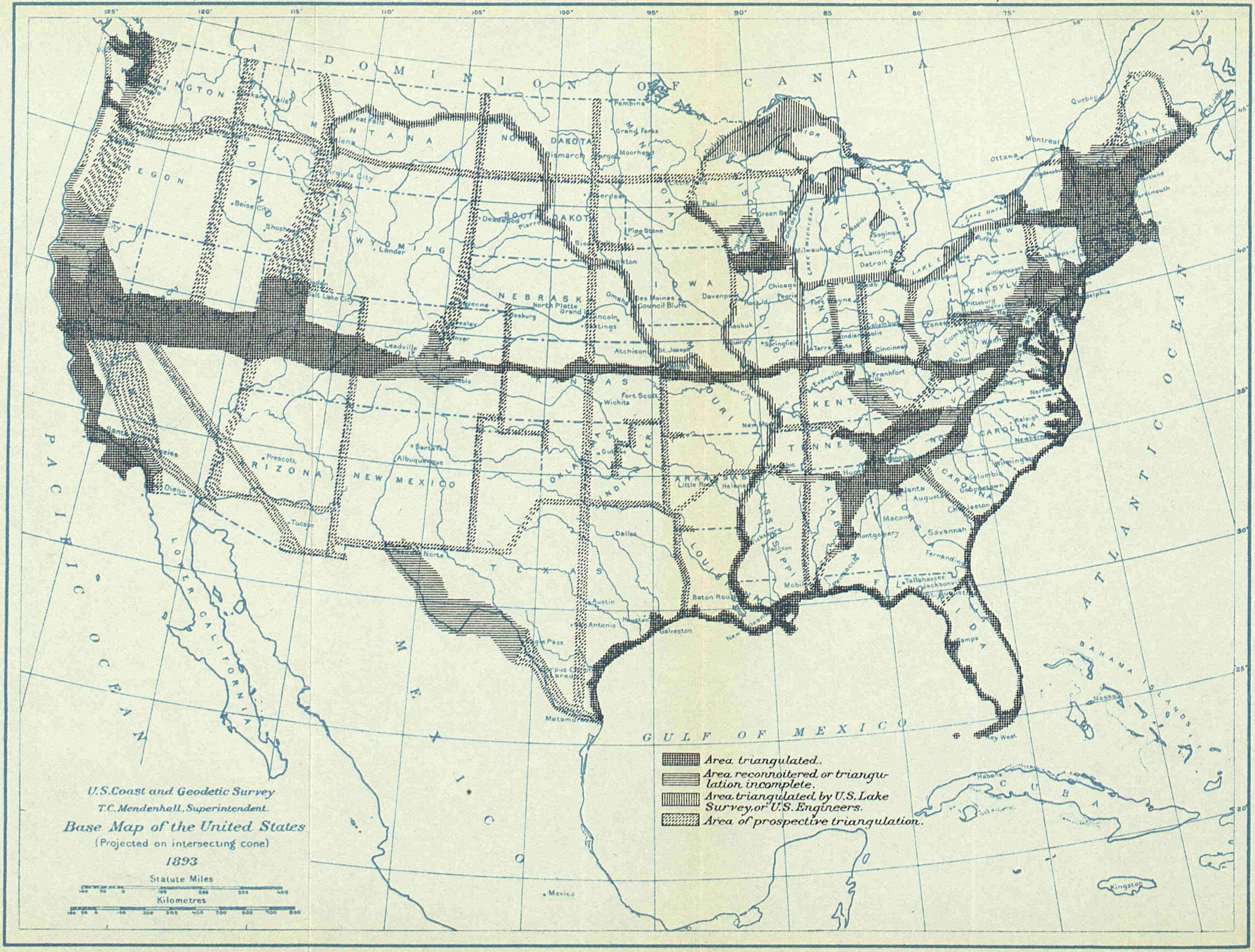

Photo-Lith. by A. HOEN & CO., Balto., Md.

The triangulations made by the United States Lake Survey and the United States Mississippi and Missouri River Commissions are also shown.

Other shaded areas are intended to show either proposed triangulations or to serve as suggestions for future consideration. They are not intended to mark out exact positions, since they can only serve to illustrate the general idea concerning the manner in which the required data for any contemplated surveys of States or their boundaries may readily be supplied. According to this view, all such surveys would depend on the standard geodetic latitude, longitude, and azimuth of the whole country, and they would use the same spheroid of development, thus securing uniformity and consistency; and in general no further astronomical observations or the measure of new base lines would be required except for special verification.

CHAS. A. SCHOTT, *Chairman.*
R. L. FARIS, *Secretary.*

REPORT OF COMMITTEE D ON ASTRONOMY.

TIME AND LONGITUDE.

The subject of astronomy, as applied in the Coast and Geodetic Survey, may suitably be treated under the four heads adopted in the Manual Appendix No. 14, United States Coast and Geodetic Survey Report for 1880, which gives very fully the methods employed up to that date and leaves but little to be added to bring it up to the present. In a cursory review of these subjects it is almost impossible to separate them entirely, as in some respects kindred operations run through all of them.

Time is used for latitude, azimuth, magnetic, gravity and longitude determinations, whether the latter falls under the head of (1) astronomic phenomena, such as eclipses, occultations, etc., (2) flashing signals, (3) chronometric, or (4) telegraphic.

The precision of the time required will indicate the character of the instrument to be used, such as the sextant, vertical circle, or transit in its various forms.

The number of time and circumpolar stars in the American Ephemeris is not sufficient for field astronomy, and it is necessary to resort to the Berlin Jahrbuch and other star catalogues to prevent otherwise unavoidable delays.

It is suggested that the four principal nations publishing nautical almanacs combine in the expense of preparing and issuing a special star list of greater extent than we now have, so that the number of time stars may be increased to an average of one for each two minutes of time, and also that the American Ephemeris should extend the

"Additional Star Places" so as to cover the period now falling in daylight; also that more azimuth stars should be added, and whenever practicable grouped by differences of nearly twelve hours. The star places should be given in order of right ascension. This would also be useful for determining value of micrometer.

Time and longitude are so intimately associated that the consideration of the latter necessarily involves the former.

Beyond the reach of the telegraph is the vast region of Alaska, which must have its longitudes determined where water transportation is available by chronometric expeditions, and far inland, beyond the navigable streams, by the observation of celestial phenomena. Works on astronomy give these methods in sufficient detail for practical use. Suffice it to say that the transportation of chronometers, when carefully conducted, gives satisfactory results within the limits expected. A thorough test of these chronometers should be made before using them. This could be done advantageously by using the new pendulum apparatus, employing an invariable pendulum, whose period is known, as a standard.

A few examples, showing the probable errors of results, are given:

Year.	Stations.	Trips.	Number of chronometers.	Days in a voyage.	Probable errors.
					s.
1855	Liverpool Cambridge	6	52	12	0·19
1856	Savannah Fernandina	8	10	1	·07
1892	Sitka Tacoma	12	8	6	·10

The cable determination between Greenwich and Cambridge differed from the chronometric result, in which 1,065 chronometers were used, by about $0{\cdot}20^s$.

It is suggested that the longitude of the different Aleutian islands may be determined by flashing signals from one to the other and checking the end of a subdivision by chronometric expeditions. Most of these islands are sufficiently close to each other for the use of this method. As an example of flashing signals, the probable error of the longitude determination between Tetica, Spain, and M'Sabiha, Algiers, is given as $\pm\ {\cdot}013^s$.

The most important method of modern times for determining longitude, and the one most widely used, is the comparison of the local times of different points by means of the electric telegraph. This method of longitude determination has been so systematized and perfected in the last fifteen years that but little remains to be desired, and the slight modifications made from time to time during that period have been chiefly in the direction of equipments. A further advance may be practicable when the variable quantity, known as personal equation, can be eliminated by the photographic record of star transits, which is yet in its experimental stage.

The telegraphic method of determining longitudes was devised and introduced by the Coast Survey. In 1878 the mode of operation was much simplified; and in the same year was applied the present brief method of field computation, which enables an observer to complete the duplicate record, involving the sheet reading and time computation, in three or four hours. These field computations are of great assistance in the final office reduction.

These recent longitudes were determined by observations on from six to ten nights, the observers interchanging stations after half the number of nights were obtained, to eliminate personal equation. The local times were determined by observing the same stars whenever practicable at both stations each night, to eliminate errors of star place. Twenty stars were used each night. They were divided into two time sets of ten stars each, containing two azimuth and eight time stars, with reversal of the transit axis at the middle of each time set. Arbitrary signals were exchanged in both directions, to compare the chronometers, as near as practicable, midway between the two time sets, using the telegraph circuit for that purpose for about three minutes. The transmission and armature times were thus eliminated. The results compare favorably with the European longitudes, where observations were sometimes made on from sixteen to twenty nights, using three similar groups of stars and two exchanges of signals each night. All of the foreign determinations were not so elaborate, however. Many of recent date were determined just as ours are.

Twenty-nine European lines (see table), the best that were available, determined between 1881 and 1889, in which the observers and sometimes the instruments also were exchanged, show an average probable error of $\pm 0{\cdot}009^s$. Fifty-three lines (see tables) determined in the United States between 1880 and 1892 give a probable error of $\pm {\cdot}009^s$. When the best class of work is compared in both Europe and the United States the error of closing circuits is about the same (see table). It rarely exceeds $0{\cdot}10^s$, and in the majority of cases is lower. There are a few cases of abnormally large errors of closing in both countries that have not been fully explained. As a curious instance of a constant difference of results may be mentioned the double determination of the difference of longitude between Greenwich and Paris in 1888 by two sets of observers, making really four determinations of six nights each, which show a difference of $0{\cdot}21^s$. The work was repeated in 1892, with a difference of $0{\cdot}18^s$, or practically the same, but no explanation has been given by the observers. From the comparison above made it is clear that our work possesses the requisite accuracy.

Differences of longitude in Europe.

Stations.	Number of nights.	Difference of longitude.		Probable error.	Date.
		°	′	*s.*	
Paris–Milan	Not able to find data as to number of nights in most of these.	27	24·954	±·007	1881
Paris–Nice		19	51·225	·007	"
Paris–Leyden		8	35·213	·016	1884
Berlin–Swinemunde		3	28·969	·011	1883
Kiel–Swinemunde		16	28·203	·013	"
Konigsberg–Swinemunde		24	55·166	·010	1884
Konigsberg–Varsovie		2	08·300	·011	"
Berlin–Varsovie		30	32·477	·007	"
Berlin–Breslau		14	33·007	·007	1885
Konigsberg–Breslau		13	50·278	·008	"
Konigsberg–Rugard		28	11·819	·009	"
Kiel–Rugard		13	11·592	·009	1886
Kiel–Berlin		12	59·241	·010	"
Rauenberg–Berlin		0	06·393	·005	"
Konigsberg–Memel		2	24·228	·008	1887
Konigsberg–Goldaperberg		7	11·147	·006	"
Berlin–Schneekoppe	8 and 7	9	23·084	·007	1888
Breslau–Schneekoppe	7 and 8	5	10·803	·010	"
Breslau–Trockenberg	5 and 4	7	21·694	·008	1889
Breslau–Schonsee	4 and 5	7	26·812	·011	"
Trockenberg–Schonsee	4 and 5	0	05·190	·008	"
Konigsberg–Schonsee		6	23·441	·012	"
Breslau–Rosenthal		0	00·039	·007	"
Schonsee–Springberg		9	07·583	·010	1890
Berlin–Springberg		12	53·113	·016	"
Stockholm–Goteborg		24	22·73	·016	1885
Lund–Goteborg		4	53·72	·007	1886
Stockholm–Hermosand		0	24·45	·007	1888
Haparanda–Hermosand		24	45·51	·010	1889

Differences of longitude in the United States.

[Many of these are taken from the field results.]

Stations.	Number of nights.	Difference of longitude.			Probable error.	Date.
		h.	*m.*	*s.*	*s.*	
Cape May–Washington, D. C.	5 and 5	0	08	29·072	±·007	1881
Strasburg–Washington, D. C.	3 and 3		5	14·087	·007	“
Cincinnati–Washington, D. C.	4 and 5		29	29·262	·013	“
Cincinnati–Nashville	4 and 4		9	26·646	·006	“
St. Louis–Nashville	4 and 4		13	41·183	·011	“
Vincennes–Nashville	3 and 4		2	57·88	·017	“
Vincennes–St. Louis	4 and 3		10	43·232	·004	“
Little Rock–Galveston	5 and 5		10	04·260	·011	1885
Little Rock–Kansas City	5 and 4		9	15·644	·003	“
Colorado Springs–Kansas City	5 and 4		40	55·347	·010	“
Colorado Springs–Santa Fé	5 and 5		4	30·113	·011	1886
Colorado Springs–Gunnison	4 and 4		8	25·340	·004	“
Colorado Springs–Grand Junction	4 and 4		14	58·908	·012	“
Colorado Springs–Salt Lake City	5 and 5		28	18·470	·007	“
Ogden–Salt Lake City	5 and 5		0	24·546	·011	“
San Francisco–Salt Lake City	5 and 5		42	07·690	·011	1887
San Francisco–Washington, Lafayette Park	5 and 5		0	04·426	·006	“
San Francisco–Portland	5 and 5		1	00·006	·013	“
Wallawalla–Portland	5 and 5		17	19·517	·011	“
Wallawalla–Salt Lake City	5 and 5		25	48·187	·008	“
Wallawalla–Port Townsend	5 and 5		1	40·108	·012	1888
Wallawalla–Seattle	4 and 5		15	57·028	·009	“
Helena–Spokane Falls	4 and 4		21	34·437	·009	“
San Francisco–Mount Hamilton	6 and 5		3	09·041	·013	“
San Francisco–Sacramento	4 and 4		3	44·479	·007	1888 1889
San Francisco–Point Arena	5 and 5		5	04·239	·008	1889
Sacramento–Point Arena	5 and 5		8	48·689	·006	“
Sacramento–Marysville	5 and 5		0	22·801	·008	“
Sacramento–Los Angeles	4 and 4		12	56·803	·007	“
San Francisco–Los Angeles	5 and 5		16	41·252	·009	“
Needles–Los Angeles	5 and 5		14	36·769	·014	“
Sacramento–Verdi	4 and 4		6	02·874	·007	“
Carson City–Verdi	4 and 4		0	52·558	·011	“
Carson City–Virginia City	4 and 4		0	28·180	·010	“
Carson City–Genoa	4 and 4		0	18·522	·016	“
Carson City–Austin	4 and 3		10	45·169	·015	“
Eureka–Austin	4 and 4		4	27·327	·006	“
Eureka–Salt Lake City	4 and 4		16	15·337	·007	“
Washington–Altoona	3 and 3		5	20·675	·012	1890
Salt Lake–Helena	5 and 5		0	33·583	·012	“
Bismarck–Helena	5 and 5		45	00·852	·013	“
Bismarck–Minneapolis	5 and 5		30	11·078	·007	“
Albany–Cape May	5 and 5		4	43·088	·010	1891
Albany–Detroit	5 and 5		37	11·894	·007	“
Chicago–Detroit	5 and 5		18	17·638	·006	“
Chicago–Minneapolis	5 and 5		22	27·414	·011	“
Omaha–Minneapolis	5 and 5		10	49·269	·009	“
San Diego–Los Angeles	5 and 5		4	22·802	·008	1892
San Diego–Yuma	5 and 5		10	09·127	·007	“
Los Angeles–Yuma	5 and 5		14	31·986	·006	“
Nogales–Yuma	5 and 5		14	43·698	·007	“
Nogales–El Paso	5 and 5		17	48·532	·008	“
Helena–Yellowstone Lake	5 and 5		6	33·835	·009	“

Double determinations and closing of circuits in Europe.

Stations.	Difference of longitude.		Closing error.	Stations.	Closing error.
	m.	s.	s.		s.
Paris-Algiers	2	50·374			
		·494	·126	Kiel-Berlin-Swinemunde	·007
		·368		Kiel-Berlin-Rugard	·069
Paris-Nice	27	24·964		One circuit of four lines	·042
		·957	·007		
Other closing errors are:					
Rome-Genoa	14	14·842			·085
		15·042	·200		·058
					·020
Rome-Padua	2	27·119			·004
		·131	·012		·034
					·072
Berlin-Breslau	14	33·887			·110
		·936	·049		·140
					·002
Pulkowa-Varsovie	37	11·30			·001
		·57	·27		

Double determinations and closing of circuits in the United States since 1881.

[Some of these are taken from the field computations.]

Stations.	Closing error.
	s.
Nashville-Vincennes-St. Louis	·085
Omaha-Kansas City-St. Louis	·005
Salt Lake City-San Francisco-Portland-Wallawalla	·018
Circuit of five lines	·041
" " three "	·091
" " three "	·004
" " three "	·033
" " three "	·034
" " three "	·057
" " six "	·068
" " three "	·020
Double determination of Little Rock, Ark., from San Francisco via Salt Lake City, Colorado Springs, Kansas City, and from San Francisco via Los Angeles, Yuma, Nogales, El Paso, differs	·04*

* The office computation may increase this considerably. About 20 lines are involved in this polygon; more are to be introduced.

There is no reasonable doubt that the expense of operating the longitude parties with the present outfit has been reduced to a minimum. The parties generally consist of two observers only, and determine latitude and the magnetic elements in addition to longitude without any extension of time at a station. It is also proposed to do gravity work; but this may either require a third observer or a longer detention of the parties at a station. Only such stations as are free from the jars of locomotives, street railways, and passing vehicles would be suitable for this work.

While the weight of the longitude outfit may be lessened by the use of smaller combination instruments for both longitude and latitude and by procuring lighter chronographs, the outfits on hand are too valuable to be discarded.

A revised edition of the manual should contain an example of the more recent longitude work, including a time set, computed by both the field and the least square methods.

Of the main scheme of longitude work laid out some years ago to embrace the United States there remains unfinished one long or two short seasons' work in the Southwest. It is also highly desirable to connect Montreal, Canada, with Cambridge, Mass., and Albany, N. Y., in order to utilize the last transatlantic cable determination, and to connect Cambridge, Mass., with the new Naval Observatory, Washington, D. C.

If the old and new Naval Observatories have not been satisfactorily connected, that should be done before the old station is destroyed.

It would very materially aid cartography if the geographical positions of the State capitals, important cities, county seats, and most of the larger towns along railroads were determined. Towns near national and State boundaries would be especially useful. This work might precede the triangulation many years.

Any surveys conducted by the States or by other authorities would derive great benefit from these established points.

In order to give an additional test to the accuracy of the longitude determinations it is recommended that one or more lines already determined by one set of observers be redetermined by other observers. The constant difference between Greenwich and Paris, as shown in the determination by different observers, illustrates the desirability of making this test.

ASTRONOMICAL DIFFERENCES OF LONGITUDE FROM LATITUDES AND RECIPROCAL AZIMUTHS; ALSO DIFFERENCES OF LONGITUDES FROM AZIMUTHS AND RECIPROCAL ZENITH DISTANCES.

In cases where the telegraphic method or the method by flash light is inapplicable for any reasons whatever, it is suggested that differences of longitude may be determined between intervisible points by reciprocal azimuths or by reciprocal zenith distances.

Under the most favorable conditions the method by azimuths is susceptible of a degree of accuracy equal to that of the telegraphic method. It is independent of geodetic elements of the earth.

The method of reciprocal zenith distances affords only approximate results, as it rests upon the assumed dimensions of the earth and requires a tolerably accurate knowledge of the coefficient of refraction.

INSTRUMENTS USED FOR LONGITUDE WORK ON THE CONTINENT OF EUROPE.

These instruments, with but two exceptions, so far as examined, are of the broken telescope type and resemble each other in general design. The size of the objective ranges from 63^{mm} to 77^{mm} and the focal distances from 700^{mm} to 870^{mm}, and the power of the eyepieces used averages 80. Two are as high as 90 and one as low as 60. So far as noted, a reticule of 13 threads has been used. It will be seen that at all points these figures are less than those describing the longitude instruments of the Survey.

The following table gives in compact form all the information available regarding these instruments:

Transits.

Where used.	Maker.	Objective.	Focal distance.	Power.	Telescope.	Threads.	Remarks.
		mm.	mm.				
Austria	Starke & Kammerer	66	?	90	Broken	13	Has reversing apparatus.
"	Pistor & Martins	68	870	?	"	13	Reversing and hanging level.
"	G. Starke	66	710	80	"	13	
"	Troughton & Simms	63	738	80	Straight	13	Reversing apparatus.
"	Repsold	68	835	80	"	13	Reversing gear and hanging level.
Munich	Van Ertel	77	812	60	Broken		" "
Milan	Repsold	70	800	?	?		
Padua	Van Ertel	66	700	40	Broken		
Paris	Rigaud	63	788	62	"		
Strassburg	Pistor & Martins	68	870	90	"		
Spain and Algiers	Brunner	61	775	?	Straight		Vertical circle, 415^{mm}.

The new longitude transits of the Coast and Geodetic Survey have a focal length of 95^{cm} (37½ inches), a power of about 100 with the present eyepiece, and a glass diaphragm with three tallies of 3, 5, and 3 lines each. The frame is so arranged that the azimuth and level adjustments are made at the base.

LATITUDE.

METHODS OF OBSERVATION AND INSTRUMENTS.

Under the head of latitude the inferior grades may be passed over with a few remarks. These are used for magnetics, reconnaissance, and explorations, and may be determined with a sextant or some form of vertical circle.

The better grades of latitude are now observed in the United States with the zenith telescope in some of its forms and by Talcott's method.

There are three classes of these instruments with telescopes, as follows:

1st.	66^{cm} (26-inch)	telescope with	57^{mm} ($2\frac{1}{4}$-inch)	objective; power about	30–60
2d.	79^{cm} (31 ")	" "	57^{mm} ($2\frac{1}{4}$ ")	" " "	60–90
3d.	114^{cm} (45 ")	" "	89^{mm} ($3\frac{1}{2}$ ")	" " "	100

The main point to be considered is the accuracy desirable, and hence the number of pairs of stars to be used and the size and form of instruments. With good instruments, furnished with improved levels and micrometer screws, such as are now made, and catalogues with well-determined star places, the number of pairs of stars may be greatly reduced. On account of inferior star places, it was formerly the custom to observe thirty or forty pairs on six or seven nights. That number has been gradually reduced to fifteen or twenty pairs on from three to five nights, and may be further reduced and still give results sufficiently accurate for geodetic purposes.

An examination of the local deflections of the vertical at latitude stations shows that the determination of astronomic latitudes with an accuracy of about a quarter of a second is quite sufficient for most schemes of triangulation, being within the limits of ordinary deflections.

A greater precision is necessary in determining arcs and locating State and national boundaries, while for the purpose of investigating the variations of latitude the greatest degree of precision that may be obtained by the employment of the most refined instrument and methods is required.

The bearing of these last observations upon geodetic surveying is of the utmost importance, as it will enable us to determine corrections by which latitudes observed at various times may be reduced to the normal latitude.

With the instruments now in use the probable error of one observation for latitude is about one-third of a second. The probable error of the mean declination of a pair of stars, as furnished by the Coast and Geodetic Survey Office, is about one-quarter of a second. Ten or twelve pairs of stars observed on three or four nights should give a latitude with a probable error of about one-tenth of a second.

To further illustrate the idea that the number of observations may be reduced, a series of latitude observations was examined and the means were taken out for twenty pairs, observed on six, four, and two nights, respectively; then of fifteen pairs, ten pairs, and five pairs, observed for the same time. The sets chosen were by different observers, and both meridian instruments and zenith telescopes were used. Twenty pairs on six nights were taken as the standard, and the difference from this standard of fewer pairs on different nights is shown.

The star lists were prepared in the usual way, so that the differences of the zenith distance were balanced.

	1st set.*	2d set.*	3d set.*	4th set.†	5th set.*	Mean.
	"	"	"	"	"	"
20 pairs, 6 nights	0·00	0·00	0·00	0·00	0·00	0·00
20 " 4 "	·16	·01	·04	·05	·14	·08
20 " 2 "	·14	·11	·07	·20	·18	·14
15 " 6 "	·00	·01	·00	·05	·01	·01
15 " 4 "	·17	·03	·01	·05	·12	·08
15 " 2 "	·14	·06	·16	·09	·16	·12
10 " 6 "	·03	·14	·01	·13	·02	·07
10 " 4 "	·13	·09	·03	·22	·14	·12
10 " 2 "	·10	·16	·21	·15	·13	·15
5 " 6 "	·29	·07	·11	·12	·21	·16
5 " 4 "	·17	·03	·05	·12	·47	·17
5 " 2 "	·18	·01	·10	·13	·53	·19

* Z. T. No. 1, 117cm (46-inch) focal length, 83cm (3¼-inch) aperture.
† Meridian Inst. No. 16, 79cm (31-inch) focal length, 63cm (2½-inch) aperture.

The vertical circle has been used, and is still being used, with satisfaction for latitude observations on some of the European surveys. Under equal conditions it is not believed that it possesses any advantage over the zenith telescope. A series of tests of the various methods and instruments was made under Professor Bache in 1847, and the zenith telescope (using the Talcott method) was declared to be so far superior to the others for our work that it was adopted and has been used ever since. It is growing in favor with the Europeans. They offered a strong testimonial to its merits by using it for the precise observations necessary in determining the variations of latitude. Colonel Clarke, in his Geodesy, says of this instrument, "As made by Wurdemann, it is an instrument of extreme precision and most pleasant to observe with." And again: "The simplicity of construction of the zenith telescope exempts it from several of the recognized sources of instrumental errors, while its portability and ease of manipulation eminently fit it for geodetic purposes. It is exclusively adopted for latitudes in the United States, and it is probable that no one who has used it would return to graduated circles for latitude."

An enlarged catalogue of latitude stars is greatly needed; one that includes stars from the pole to 10° or 12° south of the equator, and down to the seventh magnitude. In the elevated regions of the United States there is but little trouble in observing these small stars.

As to the question of increasing the number of astronomical stations, especially those for latitude and azimuth determinations in the schemes of triangulation already planned, some remarks have been made in the reports of the committees on Arcs and on Triangulation. It may be stated in general terms that but few more of these stations are needed in the transcontinental scheme and in the triangulations on the Atlantic and Pacific coasts beyond those already contemplated in the uncompleted portions of the work.

For the purpose of studying the local deflections of the vertical, however, it is desirable to have as many latitudes and azimuths as can reasonably be observed without increasing the expense, and so distributed with regard to the orographic and geological features that any point of suspected disturbance may be examined. The local deflections along the Atlantic average 2″·2, but they are somewhat greater on the Pacific Slope.

When a scheme of triangulation starts on one of the principal lines of the regular network designed to cover the entire United States and terminates on another of these lines, no intermediate astronomical stations will be required.

When a scheme of triangulation is entirely independent of all other schemes both initial and terminal astronomic stations will be required.

In both of these cases the remarks on the subject of local deflections are applicable.

When no triangulation at all is contemplated the work falls under the head of geographical positions, which has been already considered.

The vertical circle used abroad in latitude work is also of the broken telescope variety. This form of instrument has been recommended for the ease and comfort to the observer, as his eye remains in the same position; also for its stability, owing to the low Y^s and large base. It seems to have given no better results than instruments used in the Coast and Geodetic Survey.

The tendency toward adopting some form of zenith telescope on the Continent is presumptive evidence that it possesses advantages over all forms of vertical circles.

The latitude instrument used by Dr. Marcuse in the Hawaiian Islands was a zenith telescope of nearly the same objective, focal length, and power as those used in the Coast and Geodetic Survey for a similar purpose, but it is more massive in its parts and heavier to transport. The base is very heavy, the telescope is broken, but the prism is very near the focus.

There are two latitude levels of the best make. The horizontal axis is 20^{cm} and the vertical axis is 34^{cm} in length. The focal length of the telescope is 87^{cm} and the diameter of the objective is $6{\cdot}8^{cm}$

Vertical circles.

	Maker.	Objective.	Focal length.	Power.	Telescope.	Circles. H.	Circles. V.
		mm.				cm.	cm.
Austria	G. Starke	46	?	60	Broken	32	26
	"	53	?	60	"	34	34
Netherlands	Repsold	67	?	68	"	32	
Geneva and Strassburg	?	60	?	?	"	Objective at end of axis.	

AZIMUTH.

Azimuths are of different grades, according to the objects for which they may be required, such as magnetics, reconnaissance, and explorations, in which a 4 or 6 inch altazimuth may be used; for running meridian lines or other lines depending thereon; for tertiary triangulation, in which 8 or 10 inch instruments may be used; for secondary and primary triangulation, in which the best class of instruments, from 30^{cm} to 51^{cm} (12 to 20 inches), are required, and an accuracy compatible with the scheme of triangulation must be reached. The azimuth should be measured with the same precision as the horizontal angles in the scheme, which in the case of primary triangulation will in general be accomplished by observing azimuth in as many positions and series, or by as many sets of repetitions, according to the type of instruments employed, as are used in the horizontal angle work of the triangulation.

Under the assumption of careful and proper manipulation a primary azimuth may be determined with the more refined type of modern instruments in about four or five days' observations, with a probable error not exceeding about $\pm 0''\cdot 15$.

The azimuth is reduced to the normal meridian.

Accompanying this report is a map (illustration No. 12) showing the distribution of the principal astronomical stations occupied by the United States Coast and Geodetic Survey for the determination of latitude, longitude, and azimuth to January, 1894.

C. H. SINCLAIR, *Chairman.*
G. R. PUTNAM, *Secretary.*

REPORT OF COMMITTEE E, ON HYPSOMETRY.

A knowledge of relative elevations on the earth's surface is a fundamental necessity in the investigation of many physical questions and in engineering operations of various kinds. In the United States Coast and Geodetic Survey work of this nature has been done to meet the needs of the topographer, the triangulator, the physical hydrographer, and for the use of the engineers engaged on the improvement of our rivers.

So long as the operations were confined to the immediate vicinity of the seacoast, where connection could be made with tidal stations by means of short lines, the requisite degree of accuracy could be attained by methods of moderate precision; but when it became necessary to extend them considerable distances inland the demand for greater refinement became apparent, and there resulted what have been called the "standard levels of the Survey." These have been undertaken primarily to reduce the geodetic operations to sea level and to aid, by comparing tidal planes along the coast of the United States, in

Distribution of the principal astronomic stations occupied by the U.S. Coast and Geodetic Survey for Latitude, Longitude, and Azimuth to January 1894

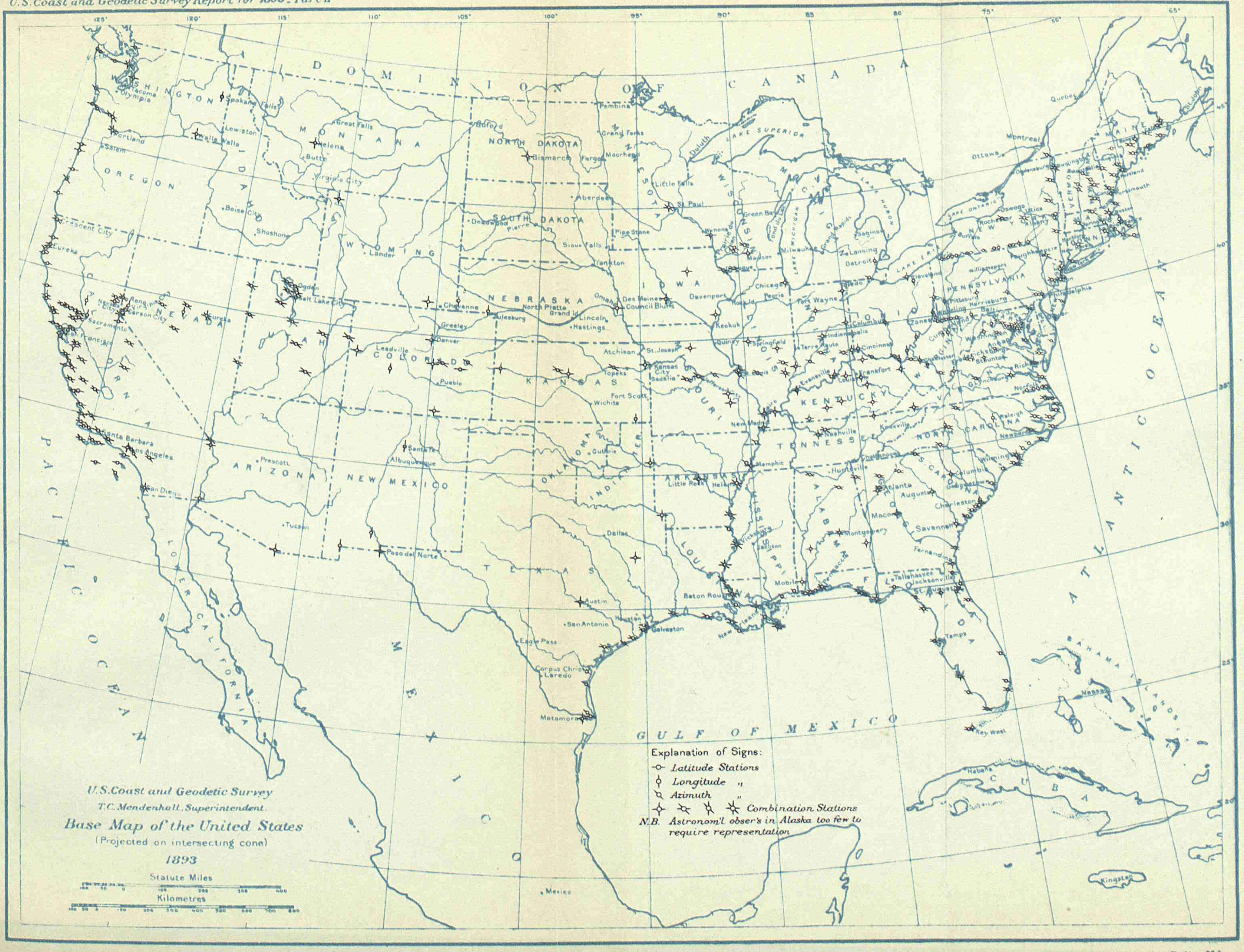

Photo-Lith by A. HOEN & CO., Balto., Md

the solution of important physical questions in regard to the ocean; to form the basis of and to bring into accord all topographical surveys of the country, whether under State or Federal patronage; to furnish data required in connection with gravity observations, and incidentally to supply planes of reference for engineering operations of all kinds.

As it is evident that these operations must ultimately be extended to all parts of the country, it becomes necessary to consider the requirements to be met and the degree of accuracy demanded. The objects most apparent and the degree of accuracy desirable in each may be stated as follows:

For topography: Within 1 metre. (See Annual Report Superintendent United States Coast and Geodetic Survey, 1891, Part II, p. 634, par. 17.)

For physical hydrography: With the utmost degree of accuracy.

For base-line reduction: Within 0·5 metre.

For gravity operations: Within six-tenths of a metre ($0{\cdot}6^{m}$).

For tidal planes: With the utmost degree of accuracy.

For meteorological investigations: To the nearest foot.*

For the study of strata and the flow of underground water in the arid regions an accurate system of levels is demanded.

For engineering operations: With great accuracy.

As the bench marks determined by the standard levels of the Survey must be used as base points for all of these objects, it is evident that a very high degree of accuracy is demanded, a degree as high as there is any hope of our attaining by the most precise instruments and methods at our command.

Two types of instruments are in use, the wye level of civil engineers and the geodetic or level of precision. They differ in construction, but more particularly in the methods employed in their use.

In general it may be stated that in wye levels the adjustments are made and supposed to remain constant during a day or a portion of a day, and the accuracy of the results depends upon the correctness of this assumption, the records containing no clue to changes, although in the best work the elimination of errors from adjustment is sought by making the F. S. and B. S. always equal.

The "geodetic level" of the Survey is a specially devised instrument. The errors in the various adjustments of the instrument are by the method used recorded at each station, and are eliminated by making the F. S. and B. S. equal by reversals of telescope and of the striding level, readings in each position by means of a micrometer screw being made and a mean taken, which is true in spite of any want of adjustment.

Other methods have been used in lines of precise levels elsewhere, and will be referred to under another head.

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*Stated by Professor Harrington, Chief of United States Weather Bureau, in letter dated February 17, 1894.

About 9 750 kilometres (6 100 miles) of precise levels having already been run by different corps in the United States to meet the demands of special investigations and in the regular work of the Coast and Geodetic Survey, the desirability of a consistent scheme of lines to control the whole country to which all future work may be made to lend itself has become apparent. In presenting such a scheme the considerations governing its selection may be briefly stated as follows:

To provide a means, the most direct and economical, for connecting the many tidal stations on our Atlantic, Gulf, and Pacific seaboards.

To connect these several tidal planes by routes which will best overcome the uncertainties arising from crossing mountain chains, etc.

To form closed figures that will best determine the degree of accuracy of the work.

To make the lines of levels conform as nearly as possible to existing or proposed schemes of triangulation and especially of arc measures.

To take advantage of all work of a like nature heretofore executed, and to distribute judiciously over the country bench marks which may serve as points of departure for hypsometric work of all kinds.

The great advantages offered by graded ground for operations of this class, together with considerations of economy and facility of transportation, practically confine them to railway lines where they can be followed. Fortunately in most parts of the United States they are so numerous as to offer every facility.

On the accompanying sketch map (illustration No. 13), which is practically the same as presented to the Conference at its first meeting, the details of a general scheme are displayed which, it is believed, in a great measure fulfill the requirements mentioned.

It consists primarily of three east and west lines connecting on the Atlantic and Pacific seaboards with lines conforming to the general direction of those coasts and crossed between the ninetieth and ninety-eighth meridians by north and south lines of closed figures.

The four great figures thus formed are to be further subdivided into lesser ones as opportunity offers or special demands permit.

These lines should follow the most direct railway routes along the Atlantic seaboard from Maine to St. Augustine, Fla., across the peninsula of Florida to the Gulf of Mexico, which they skirt to Corpus Christi, thence to Laredo, Tex., and by the Southern Pacific route to the Pacific Slope, and northward, by the lines of railway conforming most nearly to the coast, to Seattle, whence they return by the Great Northern and the most direct routes to St. Paul, Chicago, Toledo, Buffalo, Oswego, and the Atlantic coast.

The central east and west line has been completed to Kansas City and the route from there to San Francisco definitely determined upon.

Lines have been run on the Atlantic coast from Sandy Hook to Fort Monroe, and the ninetieth meridian was selected for the north and south line of closed figures because the greater portion of the work has already been done or is in course of completion.

The distinction has been made between precise levels and levels as run by civil engineers for construction purposes. In the latter no attempt is generally made to secure a greater degree of accuracy than is required by the work in hand.

Precise levels, on the other hand, are intended to furnish, as nearly as physical conditions and the means at our disposal will admit, the true differences in elevations between points widely separated.

Work of this class has been done by almost all European nations, and to a lesser extent by three organizations under the United States Government. No pains have been spared that seemed necessary to give permanent value to the results either for the objects immediately in view or which might arise in the future.

While a high degree of accuracy has been attained, it nowhere equals that aimed at or hoped for, and the work may be said to be still in a measure experimental.

The following papers upon "Leveling in foreign countries," "Precise levels in the United States," and "Recent work with the engineer's precise wye level," which were prepared by different members of this committee, present the subject as fully as the time at our disposal would admit.

LEVELING IN FOREIGN COUNTRIES.

The record we find in history of vast schemes for reclamation of great areas of marsh lands, the successful projects for mighty systems of canal and river regulation, and the aqueduct systems of great water users like the Romans and Greeks testify to the early knowledge the ancients had of devices which enabled them to determine differences of levels with a fair means of exactness. The Egyptians must have had a fair idea of the relative differences of the Red Sea and the Mediterranean when they determined to connect the two seas by a canal, for it is doubtful if the Pharaoh who ordered it would have considered his project feasible, despite his mighty resources, if he had thought, as the Europeans did from the time of Napoleon's expedition to Egypt until Bourdaloué's leveling survey in 1847, that low tide in the Red Sea was $8{\cdot}12^{m}$ higher than the same tide in the Mediterranean.

Precise leveling, however, with its present degree of exactness, was not a possibility before the invention of air-bubble levels, and this discovery only dates back to 1666. In the Journal des Savants for November 15 of that year is a paper describing the new apparatus, under the heading, "A new machine for the regulating of waterways, for building construction, for navigation, and many other arts."

The notice contains a diagram and a description of the instrument, but does not give the name of the inventor. In 1682 Melchisedech Thévenot, in an account of his travels, refers to the instrument as one he had invented fourteen or fifteen years previously, and he is now generally credited with this very important discovery.

The difficulty attending the exact construction of the level vial, particularly the calibration of the interior, prevented the utilization of the new instrument until about a century later, when the ingenuity of the French engineer Chezy overcame the difficulty and devised means for obtaining a regular curvature of the inside of the tube and enabled the mechanician to obtain any desired degree of sensibility.

The first leveling instruments which utilized the level vial in connection with the telescope were modeled after the old level with sighting vanes. The vanes were replaced by the telescope; but no attempt was made to make the line of horizontality of the level tube parallel with the line of vision, and their adjustments had to be repeated frequently and were very laborious.

Chezy, in France, and Ramsden, in England (about a hundred years after the invention of the spirit level), were the first to design instruments in which the modern principles of construction were introduced, and the main features of those constructed after their designs or by them are to be found in many used at the present time.

The first large scheme of leveling was undertaken in the latter part of the seventeenth century by order of Louis XIV in connection with the schemes for the improvement of French waterways. The perfecting of means for this purpose engaged the attention of such scientists as La Hire, Mariotte, and Huyghens, but the apparatus used was from a design submitted by the Abbé Picard, who was the first to suggest placing cross wires in the focus of the telescope, and who was given charge of the leveling operations. This level consisted of a box about $1{\cdot}25^{m}$ long inclosing a plumb line which was arranged at right angles on the tube of a telescope furnished with cross wires. In this box the plumb line was preserved from agitation by currents of air. A fiducial line engraved on an interior side of the box, and visible through an opening, enabled the observer to determine when the optical axis of the telescope was horizontal. The reticule of his instrument was adjusted by experiment so as to give a horizontal line when observing.

La Hire showed that the error of a sight with this instrument and method in the field need not exceed 1/36 000 of the length of a level sight, which would be nearly equivalent to $0{\cdot}003^{m}$ for 100^{m}. But this precision was only obtained at the expense of long and tedious trials.

The first general scheme for leveling over the area of a great country was set forth in a volume written by a French scientist, M. Ducarla, in 1782. Again, in 1805, P. S. Girard, the engineer in chief of the Ponts et Chaussées, presented a memoir in which he described a project for a system of levelings which would give all the curves necessary to show the configuration of the ground over the whole of France. This idea does not seem to have taken hold until the Restoration, when it was decided to prepare a new map of France, and in connection with this scheme a general system of trigonometrical leveling was ordered.

The disagreement (in some cases amounting to 2^{m}) that was so often found between the results of this leveling and the spirit levels of engineers in all parts of the country caused a great deal of unfavorable comment, and in 1847 the distinguished French engineer Bourdaloué, in an account of some levelings between Lyons, Nimes, Marseilles, and Valence, proposed a new scheme of levels of control and described methods by which the work could be done with a rapidity and degree of exactness surpassing anything yet attained in extended work.

Bourdaloué had already distinguished himself by several notable undertakings, having, among other important duties, been charged with the study of the topography of the Isthmus of Suez, where he had shown the error of the level determinations between the Red Sea and the Mediterranean made by the engineers of the First Napoleon.

For several years he occupied himself especially with the question of the general leveling of France. In the words of Breton de Champ, "A practical man above all, he wished to know at once what would be the cost of bringing so vast a work to a satisfactory conclusion. He was the possessor of a fortune acquired by the most honorable means, and a part of this was devoted to making an experimental leveling of the entire Department of le Cher, the one in which he was born. In July 15, 1857, the minister of public works, with the advice of the general council of bridges and roads, intrusted this work (the general survey of France) to M. Bourdaloué, 'who, by his previous experience, his capacity, and disinterestedness,' gave every guaranty that could be desired for the proper prosecution of an operation of this great importance."

Perhaps undue attention has been given to leveling in France; but it has seemed worthy of remark because the art we are considering seems to owe its origin and development (as now practiced) mainly to the scientists and far-sighted administrators of that country.

The telescope used in this work was $0{\cdot}50^{m}$ long, the diameter of the objective was $0{\cdot}04^{m}$, the focal length was $0{\cdot}48^{m}$, and the magnifying power used was 36. The levels used were attached to the frame of the instrument and had values of from $3''$ to $7''$ for 1^{mm} division of the level tube. By means of a screw the height of the rear end of the telescope could be modified when adjusting the instrument.

The rods were of the self-reading type, made of wood, 4^{m} in length, and divided generally into double decimetre and double centimetre divisions. Each double decimetre contains two groups of five divisions of $0{\cdot}02^{m}$ each, one on the left and one on the right side of the rod.

The width of the rod was $0{\cdot}07^{m}$ and that of the divisions $0{\cdot}025^{m}$, leaving $0{\cdot}045^{m}$ for the figures. In each group there were three figures, two in black and the third red. The third figure was 0 in the lower half of each double decimetre and V in the upper half. As two observers read the rod, and the sum of their readings was the true value of the length taken for back or fore sight, the divisions were conventional. The rods had two handles at the sides and were supplied with watch levels to insure their being held perpendicular. They were shod at the bottom and were held on portable steel spikes that were driven in the ground with a hammer or maul. The methods of equal distances of stations from instrument and keeping the level bubble in the center when observing were followed.

Each party consisted of four persons, one called the "observer," one called the "reader," and two rodmen. When the rods were in place and the instrument leveled the observer directed his telescope toward the back rod and silently noted the reading (while the reader watched the level bubble and saw it was exactly central), and then entered it in his notebook. The reader then took his place and silently marked his reading, while the observer watched the bubble. The instrument was then directed to the fore rod and the same procedure followed. Then the level and telescope were reversed, and commencing on the fore sight the same number of observations and in the same order were repeated on the two rods. When all was completed the reader announced his results, and if they agreed within the limits of tolerance with those of the observer the instrument was taken up and carried to the next station. This method, it was considered, compensated materially for the effect of sinking in the instrument or for any change which might be due to fine particles of dust getting into the bearings of the instrument, and practically eliminated all chances for recording a wrong reading. Sights were limited to 125 or 130^{m}, except for river crossings. Each line was leveled over three times in the manner described. Bourdaloüe's field work was executed between 1857 and 1860, and the results issued in 1864. The lines leveled comprised a length of 14 980 kilometres, and the cost of the work was about 50 francs per kilometre. The error of closing of the polygons in Bourdaloüe's work is said to be represented by the expression $1^{mm}\sqrt{k}$, where k was the length of line in kilometres. The difference allowed between any two measurements of a line of levels is said to be "very approximately" $2^{mm}\sqrt{k}$.

These values are given in Breton de Champ's treatise on leveling (3d ed., p. 331), but it is hardly credible that the Bourdaloüe lines would be now classed as secondary if such precision was attained.

Bourdaloüe's work was not carried out to the extent that was first planned in 1850, a neglect entailing, according to the estimate of a writer in the Genie Civil, a loss to France of at least a half milliard of francs ($90 000 000). In 1884 the necessity for greater refinement in the knowledge of the differences of levels between points of the first importance and of a more general system resulted in a new scheme for a general leveling, which, when presented to the Chamber of Deputies for action, elicited a report from M. Sadi-Carnot (the present President of the Republic), who was chairman of the committee charged with its consideration, in which he says: "Your committee is unanimous in agreeing to the projects of the Government. All its members have received from the bureaus they represent instructions to work for the success of an enterprise which concerns in the highest degree the prosperity of agriculture, the economy of public work (departmental, communal, and corporative), and the defense of our territory as well as the scientific renown of France." The appropriation for the work was set at 22 000 000 francs. Nineteen million francs were to be used for field work, and 3 000 000 francs were set aside for the production of a map (on a scale of 1:50 000) which would show the results of the leveling. The new scheme was to consist of 12 000 kilometres of lines of first-class precision and 800 000 kilometres of lines of the second class. The same instrument and general methods were to be used for both classes of work; but the allowance for discrepancies and probable error is about double for the second class what it is for the first, and there are no reversals of telescope and level required for second-class lines. The results of the levels are checked by connections with 12 first-class self-registering gauges on the channel, Atlantic, and Mediterranean.

This new leveling scheme was put in charge of a special commission consisting of delegates from the bridges and roads service, the army, and the mining engineers. The instrument used, which weighs 12 kilogrammes, has a telescope with an aperture of 36^{mm}, a focal length of 36^{cm}, and a magnifying power of 25; radius of curvature of level tube 50^{m}, giving a value of about $4''\cdot 1$ per millimetre.

The instrument is carried on a spherical bearing, which permits its being leveled approximately before using the leveling screws. By means of reflecting prisms the two ends of the bubble, with their accompanying graduations, are visible in the telescope to the observer, enabling him to verify the accuracy of the level centering at the instant of getting the rod reading. The reflecting prisms are ground to different curvatures, so that both ends of the level are reflected with equal distinctness and size. The rods are the so-called compensating staves, designed by Colonel Goulier of the commission. They are of the self-reading varieties and made of wood. The divisions are centimetre, 5^{mm}, and 2^{mm}. The peculiarity of the rod is the provision made for its ready comparison. This consists of a Borda scale of iron and brass bars, used only for determining the length of the wooden rod fastened to it at the lower end and allowed to move freely in the opposite direction. The ends of the rods carry fine scales, the relative positions of which, as well as their comparison with a similar one on the wooden part of the rod, are read three times a day and noted in the record to determine the variations in the length of the rod.

When the wooden rod is graduated no care is taken to divide it with great exactness. A systematic error is allowed, and the correction for each division is determined by comparison with a standard scale; and the list so obtained is reserved for the computers and is purposely kept secret from the observers. After the standardizing of the rods a table, called an abacus, is prepared, by means of which by a rapid operation the computer finds for the apparent difference of level readings between the two rods the correction to be applied on account of the irregularities of the rod division, shown by the standardizing and the variation of length shown by the reading of the scales of compensation.

For this new work in France two field parties have been employed, each consisting of four men—an observer, an assistant, and two rodmen. The first thing done is the selection of the places for bench marks. They are then cemented into place, and after that the leveling begins. A day's work is restricted to the leveling between two fixed bench marks, and the line is to be run forward and backward. Pickets are driven in the ground to support the rods, and in their tops hemispherical-headed nails are driven for the rod to stand on. The same pickets are used for both measurements. Two rods are used in each party. After the end of a day's field work the record for the day must be mailed to the central office for computation, and if any errors are found beyond the limit of tolerance, the line is re-run. The fixed bench marks are plates of oxidized iron or bronze carrying a hemispherical projection on the flat top of the piece projecting from the wall in which it is fastened. On this the rod is set when the bench mark's height is determined. A vertical porcelain plate is also attached to the wall, which carries letters and numbers representing the section and place in it of the bench mark; also the inscription "Nivellement General" and the figures which represent the height of the bench mark above mean sea level. Another form of bench mark is a hemispherical-topped bronze bolt, which is cemented into the portals and base courses of certain buildings and bridges.

A special and original system of pay has been devised for this work by M. Lallemand, to whom more than anyone else the credit for the great success of this leveling is due. The pay of the field men increases nearly as the square of the length of lines run by them. Bourdaloüe's work, which had an accuracy only about one-third as good as the current system, cost 50 francs per kilometre. When the new work was taken up, in 1884, the price was 41 francs per kilometre, and now it has fallen to 33 francs per kilometre, while at the same time the rodman's pay has increased from 6 francs 50 centimes to 12 francs per day.

The instruments, rods, and methods of this work show a marked difference from those employed in India, Switzerland, and Germany, but apparently they are impressing others with their worth. In an article in the Genie Civil it is stated that they have been adopted for the survey of Algiers and Tunis by the Service Geographique de l'Armee. They are to be employed in the new leveling survey of Belgium. The Military Geographical Institute of Florence has announced its intention of substituting these instruments and methods for the Swiss and German ones hitherto exclusively used; and the Prussian Military Topographical Service is also going to make a trial of them (1890).

The French work is made up of closed polygons which average about 380 kilometres in perimeter. The probable error amounts on the average to 0·9^{mm} per kilometre. The mean sea level at Marseilles is the datum plane adopted, and in addition to the self-registering gauge at that port 11 other of the new-form tide-registering gauges known as medianaremeters are employed in connection with the leveling. In 1863 the results of the French work showed that the elevation assumed for the initial point of the Swiss leveling system was 2·59^{m} too high. Values obtained from the lines of railroad levels which converged at Bâle gave a result which pointed to an error of 2·11^{m}; and a discussion of the trigonometrical results showed that the fundamental altitude of the Chasseral, which had been selected as the point of reference for that work, by a mistake was taken 0·97^{m} too high, and consequently all these heights were too great by that quantity.

The consideration of these matters led to the appointment of a commission to provide for Switzerland a new and precise system of levels. This committee, through its representative, M. Hirsch, in 1864 presented a resolution to the International Geodetic Conference, which, being adopted, recommended a general system of precise levels over the greater part of Europe in which the method of leveling by equal sights and providing for the control of the work by a combination of closed polygons was to be followed. It was provided that each concurring nation should establish a permanent zero to which all of its heights would refer, the mean level of the sea to be determined at the greatest possible number of points by means of self-registering apparatus, and the zero point of the tide gauges to be comprised in the primary leveling. Upon the completion of the work a plan of comparison for all the heights in Europe was to be decided upon.

The instrument adopted by the Swiss was made by Kern and was of the type which is so generally known by his name. Two parties were selected for the work, and two instruments almost identical were supplied to them. The apertures of the objectives were 0·033^{m} and 0·035^{m}, their focal length 0·411^{m} and 0·406^{m}, and their magnifying powers 42 and 45, respectively. At first a Repsold level bubble with a value of 1″·5 for a length of division 2·26^{mm} was used, but it was found impossible to utilize it in the open air, and an Ertel level of double the value was substituted for it. The level values were obtained by means of the 3-foot meridian circle in the Neufchâtel Observatory.

In the first instrument the reticule had one fixed and one movable wire, and the distance of the fixed thread, from 2^{cm} divisions between which it came, was measured by the movable wire connected with the micrometer attached to the telescope. But it was found that the errors of the micrometer readings were as great or greater than those made in dividing the rod by estimation, and three fixed horizontal wires were attached to the reticule. In a series of experiments the rods were set at a number of points varying in distance from the instrument between

10^m and 100^m, and readings taken, respectively, with micrometer and with the three wires.

For the first instrument and observer the mean error of a determination by use of fixed thread was $\pm$ 0''·838, and for the micrometer thread it was $\pm$ 1''·112.

For the second instrument and observer these values were, respectively, $\pm$ 1''·112 and $\pm$ 1''·578. The mean error of a reading in the field made with the first instrument and observer when the micrometer method was used was $\pm$ $0·69^{mm}$. In the following year, when the three threads were used, this was reduced to $\pm$ $0·48^{mm}$.

One rod was used in each party. These were made of pine wood, and were very carefully constructed by M. Kern, who attended personally to their graduation and painted the lines on them with his own hands. This was done with such precision that the errors of the divisions did not exceed the limits of errors of observation. The rods were 3^m long, 8^{cm} wide, $2·2^{cm}$ thick. To secure strength they had each a dorsal rib $4·8^{cm}$ thick, 2^{cm} wide. The division was into centimetres alternately black and white in the center of the rod. Outside them were white spaces on which the numbers were painted, on one side even and odd on the other. The rods had box levels and small projecting metal brackets. From the upper one the plumb bob was suspended when tests were made, and the lower one carried a small pyramid, the coincidence of whose point with the point of the plumb bob proved the perpendicularity of the rod. The end of the rod was shod like those used for precise leveling work in the Coast Survey. The foot plates were also similar. A light tripod was used to support the rod when occasion required it.

A large umbrella was used to shade the instrument. Generally the only check on the character of the work was the closing of the polygons, but in some special cases double lines were run. It was hoped that the reading of the three fixed lines would remove all danger of large errors of reading, but in the reports, nevertheless, there are instances of mistakes. In one case an error of a decimetre was made; in a second, one of $2·6^{dm}$; in a third, of a whole metre.

A special point was made of separating the observations from the computations. Each evening the observer was required to make a copy of his day's record and compare it with his rodman. Then one copy was sent to the observatory at Neufchâtel from the first post-office available, and when an acknowledgment was had the other copy was sent to Geneva and two independent computations were made, one at each observatory.

The collimation and level adjustments are tested and the inequality of pivots is determined after mounting and before dismounting the instrument. Generally, in case the instrument receives any jar, they are to be determined before the instrument is used, and in any event they must be determined once a day.

The following is the manner of observing at a station: Having made the telescope horizontal and having set the vertical wire of the telescope on the center of the rod, the observer reads the level, noting the position of the ends—reading them to tenths. Then on signal from the rodman that his staff is vertical he reads the position of the three horizontal wires on the rod, beginning with the lowest, giving the result to decimillimetres. Then he examines his reading to see that there is no error of a centimetre or more, and for the second time he reads the level. On favorable ground the mean error per kilometre was about $\pm$ 0·66^{mm}, but over some of the high mountains, where rods had to be set up in springy, grassy ground, probable errors of 4·57^{mm} were found.

Two kilometres per day seems to have been the average rate of progress.

Before beginning the leveling work, in 1865, two bolts were securely cemented into a rock in front of the observatory at Neufchâtel which differed about 2·9^{m} in elevation, and each season the comparative lengths of the rods were tested by being held on them, with the instrument set up exactly between them. In addition to this test, the rods were tested every winter by comparison with the Swiss standard in Berne, and the results of the operations for fifteen years show a mean variation in the length of the rods (determined on the bench marks at Neufchâtel) of $\pm$ 0·064^{mm} per metre; and for rod number I, determined at Berne, of $\pm$ 0·058^{mm}, for rod number II of 0.066^{mm}, or a mean of $\pm$ 0·062^{mm}.

The observations at Berne were nearly all made at a mean temperature in winter, while those at Neufchâtel were at the beginning or end of a field season in temperatures varying from 2°·6 to 24° C. and with an air saturation varying from 0·57 to 0·98. The remarkable accordance of the result gave the commission, it states, a new guaranty that the rods, even when exposed in the field to still greater extremes, would not experience variations sensibly greater than those determined in the experiments.

The only information on the subject of English levels of precision that was attainable was the report on the work done in India in connection with the Great Trigonometrical Survey. Here for many years differences of height were obtained trigonometrically, but in 1858 the Survey began a line of spirit levels to connect points in central India with the mean sea level in Karachi Harbor and this work has since continued.

The instrument adopted was the Troughton & Simms level, described in Simms's Treatise on Instruments, of 1844, and is the ordinary Y-level, except that the level is partly embedded in the telescope tube. The focal length of the instrument was 53^{cm} (21 inches) and its magnifying power about 42. The levels had a value of about 1″.7 for a division (length not given), and in observing the ends of the bubble were read.

The rods were of wood, 10 feet long, and divided into feet, tenths and hundredths, one face having a white ground with black figures and the other a black ground with white figures.

On the first face the feet were numbered from 0 to 10 and on the other from 5·55 to 15·55. Both faces were read at each station, and if the horizontal wire intersected the commencement of a foot on one face it would intersect the middle of a different foot on the other face, and the observer could not be biased to repeat in the second reading a mistake made in his first, any error in either reading being shown by the deviation of the difference of the two values from the normal amount 5·55, or, in practice, by the difference in the resulting rise or fall obtained from the pairs of black and white face readings, which should give very nearly identical results.

"The rods were furnished with plumb bobs, let into their sides and visible through glass doors. Swivels were fixed on the top of the rods for guy ropes, to adjust them to the perpendicular and keep them steady. In order that the results obtained at each station by successive observers might be rigorously compared, it was necessary that the successive rods should invariably be set up on constant points, never on uneven surfaces. This was secured by driving a hemispherical brass brad in the head of each of the pins that were used for marking out the line of levels."

The distances of the rods from the instrument were measured with a chain. They were invariably made of equal length, and at the time of the report (1862), when levels had been extended nearly 2 000 miles and over every kind of country, involving the occupation of 12 000 stations, the rule had not been transgressed in a single instance. In the field two or three observers went over a line with different instruments and rods, but all using the same pegs.

The instruments were carefully shielded from the sun, but seem to have been dismounted at every station.

At an early period in this work the ever-present terror of the leveler, "cumulative error," showed itself, and as it had been supposed that the refinement of the instruments and methods employed left no room for any such difference, and the surveyors had no knowledge of the results which had perplexed earlier investigators, this factor caused much anxiety and troublesome investigation. After the first season's work an account of Professor Whewell's discussion of the line of levels run from the Bristol Channel to the English Channel in 1837–38 for the British Association relieved the Indian surveyors to some extent from the fear of unusual errors in their leveling.

One of the means employed to overcome this trouble was running alternate sections in different directions by the two observers and beginning the observations alternately with the back and fore rod. When this method is used, the black face is read first when the back rod is taken first, and the white face is read first when the fore rod is first taken.

The rods are read to the third decimal place, and if, after the leveling correction is applied, there is a difference of 0·006 of a foot in the results at a station, it must be releveled; and if the discrepancy remains, the first observer is recalled to remeasure the station, unless it appears that the fore peg has been disturbed, which would at once be shown by a corresponding change in the results obtained at the next station. All the results are used in the final computation. The bench marks are stone posts which are put in the earth at average distances of about 10 miles. The average daily rate of progress for each party is 4 miles in open, level country. The average annual output of work is 354 miles of double or treble line.

The rods are set up at distances of 8 to 10 chains (of links) from the instrument in the morning and 4 to 5 chains later in the day. A portable iron bar, whose length is known in terms of the standard of the trigonometrical survey, is taken in the field and compared at intervals with the rods.

In the German precise levels there has been no uniformity in type of instrument or rod used, although it is worthy of note that, in common with the Swiss, Italians, French, Dutch, Belgians, and English, wooden rods seem to be exclusively employed.

In the precise leveling in the Elbe Valley and in the lines from Swinemunde (on the Baltic) to Amsterdam and to Lake Constanz instruments made by Breithaupt & Son were used. The diameters of the objectives were 42^{mm}, the focal lengths were 460^{mm}, and the value of one division of level for a length of $2{\cdot}26^{mm}$ was $5''{\cdot}2$. Glass diaphragms were used, and the instruments had two eyepieces, allowing magnifying power of 42 and 32. The telescope rested on steel prisms and could be revolved about its axis, and could be changed end for end. The level could also be changed end for end. The rods used were of wood and of the type called by the Germans "reversible rods." One face was graduated from the bottom to the top and the other from the top down.

The Germans have paid great attention to the subject of the sinking of the rods and instrument, and to variations in the relations of the telescope and level while the observations are being made at a station. The manner of reading the rods and the setting up of the instrument have been considered in relation to these possible disturbances. On the Elbe work, where an extremely elaborate method was adopted, the procedure at a station was as follows: First, the cross wires were directed to the front face of back rod *a* and reading taken and the level ends read; second, front face of rod *b* read and level read; third, rear face of rod *b* read and level reading taken; fourth, read rear face of rod *b* and the level. Then, after reversal of telescope and level, these were repeated, and the fifth operation corresponded to the third, the sixth to the fourth, the seventh to the first, and the eighth to the second.

In setting up the tripod it was deemed advisable to follow a regular plan, in which it was provided that if the center leg of the tripod was set up at station 1, point forward, at station 2 it should point backward, at station 3 forward, and at station 4 backward again. This was intended to offset the disturbances due to the observer moving around the instrument.

An iron metre bar which was compared with the 290^{cm} Berne Bar, the standard adopted for rod comparisons for all the European precise leveling, was carried for regular daily comparisons. The rod was first compared for every single division error before going to the field, and after that the whole metre comparisons were considered sufficient to find the daily variation in the rod length.

The instrument used in the Prussian work by Dr. Jordan is of the Y type. The diameter of the objective is 41^{mm} and the focal length of the telescope is 420^{mm}; its magnifying power 30^{dm}. Box levels are used on the instrument and tripod for the first rough leveling of the instrument. One level division of $2{\cdot}26^{mm}$ in length corresponded to $4''{\cdot}1$.

The rod used was a hollow wooden one, 3^{m} long, 11^{cm} wide, and $3{\cdot}5^{cm}$ in thickness. The divisions were in half centimetres (subdivided into 10 parts), painted on the center of the face On one side of the marks were numbers, increasing from the bottom of the rod toward the top; on the other side the numbers increased from the top toward the bottom. Small silver stubs were inserted, on the faces of which were drawn lines marking the end of each metre. Each party was supplied with a field standard, which was used for making comparisons with the rod once or twice a day. This custom was introduced in the German surveys in 1878, and is now the universal rule. The time required is but short, as Jordan claims it can be done in five minutes daily.

The maximum length of sight in Jordan's work was 50^{m}, and benches were put in at every 2 000^{m}.

The lines were leveled in both directions and in the following manner: If the line started at bench mark *T* and was carried to stubs *a*, *b*, *c*, *d*, when returning on the same day the work was taken up at *c* and carried back to *T*. On the next day it was taken up at *d* and carried forward as on the day before, but on the return the work was carried through to *c*.

In an article in the Zeitschrift für Vermessungswesen Dr. Jordan gives as a sample of the rate of progress made by him the line from Germersheim–Bretten to Strassburg–Kniebis, 103·8 kilometres in length, and says it was leveled twice by him in twenty-seven days. The party seems to have consisted of an observer, assistant, and two rodmen. His experience showed that the leveling was more rapid with short sights than with long ones. By experiment he found that a half kilometre was leveled in 23·4 minutes when the instrument was set up six times and in 23·17 minutes when the instrument was set up seven times.

In this line a very interesting experience bearing upon one of the factors, that of sinking, which enters into the matter of cumulative error, was encountered. Dr. Jordan in one day leveled a stretch of 6 kilometres, in which he established three bench marks. A light rain was falling at times and the seeing was particularly good. The reversed leveling on this stretch was made on a day when the road was firm and dry, and the difference between the two measurements was 56^{mm}, or $9{\cdot}3^{mm}$ per kilometre. The differences between the lines on the different bench marks were 23^{mm}, 19^{mm}, and 14^{mm}, all with the same sign. Another measurement showed that the second line was correct. This was, of course, an exceptional case, and Dr. Jordan thinks that $\pm 1^{mm}$ for this cause would usually be an extreme value. The Swiss levels gave results which showed a probable value in their lines of $\pm 0{\cdot}5^{mm}$, and Seibt's Weichsel levels gave a value of $\pm 0{\cdot}3^{mm}$ for this error.

In comparing the procedure of foreign countries with our own one is struck with the stress that is there laid upon frequent comparisons of their rods with standards of length. This is of course partly due to the fact that wooden rods (the French rod must be considered such, as the metal rules in it are only used for checking the length of the wooden section) are exclusively used. Still the experience of levelers abroad seems to be that this necessity for frequent comparison is not a serious objection to the use of the wooden rod, and there seems to be no disposition on their part to abandon its use or to think its use inconsistent with obtaining the best results.

In this connection we may cite the results of Colonel Goulier's experiments on wooden rods, which are given in Kalmar's reports on the results of European levels to the International Geodetic Association in 1893. They are:

1. That the influence of temperature is not affected by the methods of preparation, and that for wood of the pinus or coniferæ varieties the rate of expansion is 9μ per metre per degree centigrade.

2. That of all woods the pine is least affected by moisture. The effect of moisture is in the direction of the vertical to the fibers. Boiling in oil has little effect in reducing the changes due to moisture, but a repeated painting with cold white lead reduces the variations considerably.

The variation in length on account of moisture is proportional to the increase in saturation until 60 per cent is reached; after that, there is very little change for any degree of saturation above this.

For painted pine wood rods the coefficient of expansion per metre for 1 per cent increase in humidity amounts to 18μ.

As the rods may vary from -5° C. to $+45^{\circ}$ C. in temperature, and the percentage of saturation may change from 15 to 95 per cent in the course of the season, the gross changes for the first cause may be 450μ per metre and for the second 810μ. But these causes generally affect the rod in opposite directions; when the moisture increases the temperature falls and vice versa, and experience has shown that the greatest differences do not exceed 500μ.

Lallemand furnishes a table, which is published in the report referred to above, on pages 190 and 191, giving the variations of six pairs of rods tested in the years between 1884 and 1891. There are 28 sets, and the observations were made in each case on each set in the beginning of summer and at the end of fall or beginning of winter. Only in one case is there a difference between the maximum and minimum of any rods for a season reaching 500μ per metre. The average seems to be about 250μ. Captain Kalmar calls attention to the very important fact, that has been shown by the records of the French, Prussian, and Swedish rod examinations, that the maximum or minimum values of rods used together in a season and tested together were found to be attained in almost every case on the same days for both rods. Hence, there is no chance for the individuality (as he expresses it) of any rod of a pair militating against securing good results.

In closing this short résumé of foreign methods and instruments, it may be interesting to quote from the report of the International Geodetic Association the length of precise levels in Europe. It amounted to 102 800 kilometres at the end of 1891. This includes no returns from Great Britain, and does not include Bourdaloué's levels in France, which are now considered of the second order, or the old lines in Prussia and Belgium.

PRECISE LEVELS IN THE UNITED STATES.

In the survey of the Great Lakes their elevation was determined by a combination of water levels, wye levels, and precise levels. The bench mark at Greenbush, N. Y., on which this work depends, was determined by the Coast Survey in 1856–57, but the accuracy of the determination would not at this date be regarded as satisfactory.

Starting from Greenbush, two lines of wye levels (in the same direction) were run by different parties to Oswego. Two rods were used in each party. A limit of discrepancy between the two lines of $19^{mm}\sqrt{k}$ was established, and the accumulated discrepancy amounted to $0{\cdot}293^{m}$ in the 400 kilometres.

From Oswego, N. Y., the elevation was carried by water levels across Lake Ontario to Port Dalhousie, Canada, and thence to Port Colborne by precise levels; thence across Lake Erie to Gibraltar, Mich., by water levels; thence to Lakeport by precise levels; thence across Lake Huron and as far as Escanaba, on Lake Michigan, by water levels, and thence to Marquette, Mich., by precise levels.

The precise levels referred to above may be described as follows:

Kern levels.—Wooden rods graduated to $0{\cdot}01^{m}$; position of three horizontal wires were read on the rod, estimating to $0{\cdot}001^{m}$; sights equal within 10^{m} and did not exceed 100^{m}. Correction applied to rod reading for collimation, inclination, irregularity of collars, and absolute length of rod. Two lines in the same direction by two independent parties; two rods were used in each party. The agreement is generally very good. The limit of tolerance was fixed at $5^{mm}\sqrt{k}$ and afterwards increased to $10^{mm}\sqrt{k}$. These lines are all short, but they show the gradual accumulation of error or divergence of the lines which so generally accompany levels of precision.

It was assumed that the mean level of Lakes Huron and Michigan was identical from May 19 to August 31, 1875. A theoretical discussion was made of the condition existing at the narrow junction of the two lakes and a deduced correction applied.

In the survey for the improvement of the Mississippi River an extensive system of precise levels has been executed. From New Orleans to Greenville the work was done by the Coast and Geodetic Survey for the Mississippi River Commission, and is fully described in Appendix No. 11 to Report for 1888. From Greenville they were extended by the Commission north to St. Louis, and thence to Savanna, Ill., along the river, and thence along the railroad to Chicago. From Savanna a line was carried along the river to St. Paul, and thence by railroad to Duluth, while a line along the Missouri River connects St. Louis with Kansas City and Sioux City. Various details in the methods were changed from time to time, but in general the following description applies:

Kern levels.—Wooden rods graduated to $0{\cdot}01^{m}$ and read by their wires to $0{\cdot}001^{m}$ (estimated); no target; kept vertical by circular level; foot plates and pins, but in general foot plates; spur on base of rod sometimes a plane surface supported on a spherical knob in a socket on the plate, and at other times a spur ending in a rounded point supported in a socket on plate. Conclusion reached: "If but one kind of support is to be used under all conditions, foot plates are preferable."

Lines.—In general two lines in opposite directions by different observers. Tents were used to shade and protect the instrument. Observations usually made from 6 to 8 a. m. and from 4 to 7 p. m., and never during the middle of the day except in cloudy weather. Corrections applied to rod readings for inclination, collimation, inequality of collars, and absolute length of rod. Progress about 3 kilometres per working day.

Cost.—Biloxi to New Orleans, 87 miles, $32 per mile; Keokuk to Fulton, 171 miles, $19 per mile; not given for other parts of line.

An elaborate mathematical discussion of the results has been made by Assistant Engineer L. L. Wheeler, who deduced the following conclusions:

1. The results of leveling may be affected by cumulative errors, which vary with different observers and do not always remain constant with the same observer.

2. The mean of several results obtained by the same or different observers may require a considerable correction.

3. That these cumulative errors are nearly proportional to the distances leveled and in some cases are independent of the nature of the ground, the direction in which the work is done, the season, or the manner of supporting the rods.

4. That in order, so far as possible, to eliminate the effect of such errors each observer should duplicate his own work in opposite directions under the same conditions.

5. That long lines of levels, even if leveled in duplicate, should be indisputably checked.

Mr. Wheeler also discusses personal errors of different observers in a very elaborate manner, and assigns the following values for those engaged on the work:

Personal errors of a single observation—

	mm.
J. A. Paige	± 4·27
A. D. Frost	4·14
E. H. Sankey	2·66
J. B. Johnson	3·07

(See pages 2551 and 2552, Report Chief of Engineers, 1884.)

While we may not indorse these conclusions, they seem to be very important in considering the character and accuracy with which the work was done, as they are the deliberate utterances of one who helped make the observations and afterwards discussed the results.

The instruments and methods used in the Coast Survey are described in Appendixes Nos. 15 and 16, Report for 1879. This method of observing has been followed since the beginning and the instruments at present in use are essentially the same, but many of the details have been changed, as experience suggested.

Weight of instrument, 23 pounds. The glass diaphragm has been replaced by spider lines. The vertical axis is now entirely above the tripod head. The level vial rests upon two points at either end in its inclosing tube and is held in position by a spring bearing on a third point at each end. The striding level is so constructed that it will bear with equal weight on each collar. Zylonite bands have been placed on the telescope, so that it can be revolved without touching the metal with the hands. A milled rubber head and zylonite reading head have been placed on the micrometer screw. The brass scale on the rods is secured to the brass boss at their base and left free to expand upward only. The thermometers have been attached to and brought, by means of brass filings, in metallic contact with the back of the brass scale near its center, and are read through an opening cut in the wood, which is closed by a brass slide.

In determining inequality of collars the telescope is adjusted so that it will bear with equal weight on each wye.

The field procedure has from time to time been as follows:

1. Simultaneous double line in one direction by the same observer.

2. Simultaneous double line, alternate sections in opposite directions, by the same observer.

3. Simultaneous double lines in one direction in sections by different observers.

4. Single lines with two rods in opposite directions by the same observer.

5. Single lines with two rods in opposite directions by different observers.

The present method is a simultaneous double line by the same observer in one direction, with a (proposed) similar check line in the opposite direction. With these exceptions, the description in the appendix referred to still applies.

The limit of error adopted is $5^{mm} \sqrt{k}$.

To suspend work during the middle of the day was the exception and not the rule.

In 1892 an elaborate series of experiments were undertaken under the personal direction of the Superintendent to investigate our system of leveling as well as the instruments employed. These have not been finally discussed, but they have resulted in valuable suggestions of a practical nature. Certain errors, the causes of which are still uncertain, remain uneliminated. Improvements in the appliances are proposed, and it appears particularly desirable that the qualifications of an observer be thoroughly tested before being intrusted with work of this class.

RECENT WORK WITH THE ENGINEER'S PRECISE LEVEL.

Few opportunities offer in this country for direct comparison between the "wye level" and the "geodetic level," the latter having been exclusively used upon closed circuits of the highest order; but the unexpected accordance in the double line of 200 miles of State work between Boston and Albany run by an assistant of the Coast and Geodetic Survey with the wye level during the past season, the close agreement at Springfield between the tidal planes brought by 99 miles of this work from Boston with that brought by the Engineer Corps line from Long Island Sound, together with the results from many trials of the two types over the test circuit at Washington, while inconclusive, all point to a high degree of precision with the wye level.

On the line between Boston and Albany above referred to an instrument which has been named the "engineer's precise wye level," designed by Buff and Berger in 1892–93, was used, which may be thus described:

A wye level resting in a cradle suspended upon an axis at center of instrument; micrometer screw with opposing spring under eye end of cradle. Telescope 38^{mm} ($1\frac{1}{2}$ inches) clear aperture, 38^{cm} (15 inches) focal length, Steinheil inverting eyepiece power of 35. Value of one division of the level (2^{mm}) equals $7''$.

Rods, wooden, made of white pine, T-shaped, impregnated with paraffin, graduated to feet and hundredths, lower ends a flat shoe of hard steel, target with vernier reading to thousandths of a foot, cross levels on rod. The party consisted of one observer, one bubble tender, and two rodmen; instrument protected by large umbrella; double simultaneous line in one direction along a railroad; back and fore sight kept equal by counting rails; turning points, round-headed spikes; bubble kept in center; instrumental adjustments tested one or more times a day.

The terminal benches were at nearly the same elevation. The summit elevation passed over was 1 458 feet. While the two lines cross and recross each other many times, they only at one point departed by as much as 12^{mm} and are only 3^{mm} apart at the final bench mark.

The 322 kilometres of double line cost, including salaries and all other expenses, less than $5.50 per kilometre; and in a letter addressed to the chairman, published in the Massachusetts Topographical Survey Report of 1893, Dr. Mendenhall says:

I have examined with great interest the profile of the line of levels recently run between Boston, Mass., and Albany, N. Y. The agreement between the two simultaneous lines is remarkably close, giving evidence that the whole is an excellent piece of work. I have recently tested the instrument and method used * * * and the result is such as to give me great confidence in the line. * * * I do not believe as long a line has ever before been run combining so high a degree of accuracy with so small a cost.

The wye level is used by Dr. Jordan in the Prussian work, and Professor Boersch, who has had much experience, after a careful discussion of the precise methods, says (Zeitschrift für Vermessungswesen):

From all of the aforesaid it appears that with the expenditure of great care, labor, and cost no better results, but only the appearance of a so-called scientific treatment of the subject, can be shown. The simpler the method of observation, and the fewer the figures required without decreasing the accuracy, the better results one will obtain and the less will one be exposed to observation and computation errors. It remains, therefore, always preferable in field observations where the tripod is used to employ bubbles which come to rest, and which, during the pointing upon the rod through the telescope, can be maintained in the middle of the level scale by an ordinary assistant, whose services are required anyway.

The same author, in the same work, says in regard to a "bubble tender":

It only remains, after what has been said, to provide an additional observer for the reading of the level. There will always be a man among the laborers employed in the work who can be trained to keep in the middle the bubble of a less sensitive level, besides performing the duties of an instrument carrier. * * *

He also objects to a mirror on the ground of parallax and strain to the observer's eyes. In a later number of this publication the following appears:

Professor Boersch advocates * * * the use of a second assistant for tending the bubble during the observation on the rod. In this, one is free from the hypothesis that the movement of the instrument is proportional to the time of the observations. * * * This method was used in the Bavarian leveling when the wind was high.

The writer then calls attention to the effect upon the level in the change of center of gravity of the observer while leveling, even though he does not "change the position of his feet".

DETACHED POINTS.

TRIGONOMETRICAL LEVELING.

Although for the determination of differences of elevation no method presents itself which equals in precision that of the more delicate forms of leveling instruments, there are others which for the determination of the elevations of detached points offer decided advantages, and some of which produce results of a degree of accuracy sufficient to meet many of the objects for which data of this class are sought. These are included under two heads, viz:

1. By angles of elevation and depression of points between which the distance is known; and
2. By observations of the relative pressure of the atmosphere.

The first have mainly been employed in connection with schemes of triangulation which, consisting of lines of known length connecting intervisible points, offer the most favorable conditions. When the absolute difference in elevation between the instrument and any other visible point is known, and all the objects to be observed are in nearly the same horizontal plane, they may be determined with a good degree of accuracy by means of micrometric differences, the value of the results depending upon the accuracy of the pointing, stability and perfection of adjustment of the instrument and the difference of refraction in any two lines at the time of their observation. In the cases of objects beyond the range of the micrometer screw, or when the elevation of only one point is known, recourse must be had to the vertical circle for the determination of the double zenith distance or to the spirit level in connection with a vertical arc for the measurement of the angle of elevation or depression. In either of these cases errors of graduation, etc., and the uncertainty in amount of vertical refraction must be added to those mentioned as affecting the results from micrometric differences.

While the very excellent experiments already made for the determination of vertical refraction have taught geodesists much, they serve rather to point out the great changes to which it is subject than to give confidence in our ability to apply an adequate correction. The probable error in the determination of a single point, as derived from the adjustment of large figures, is still considerable, ranging from $0{\cdot}39^m$ to $2{\cdot}95^m$; yet the law of compensation seems to hold good, and the difference in elevation between widely separated terminals, as determined by vertical angles extending through a scheme of triangulation, differs but little from the results by the more precise methods. We may cite the work across California, Nevada, and Utah, some eight figures, covering about 1 000 or 1 100 kilometres, where the height of Ogden, as determined by trigonometrical leveling (double zenith distances on ten or more days at each station), agrees with that derived by Gannett from railway levels within $4{\cdot}27^m$, and that across the State of New

York from Albany to Oswego, six figures, covering some 240 kilometres, where the difference in elevation as determined by the trigonometrical method (observations on some five days at each station) agrees with the result of the double line of levels by the United States Engineers within 2·37^{m}.

Colonel Walker (Vol. I, p. 103, Report G. T. S. of India, 1870) gives the following results of comparisons between spirit levels and trigonometrical heights derived through long chains of triangles, viz:

From Karachi to Attok,	706 miles,	difference	—3·2 feet
Attok to Dehra Doon,	416 "	"	+5·1
Dehra Doon to Sironj,	429 "	"	+1·8
Karachi to Sironj,	669 "	"	+2·1
Sironj to Calcutta,	680 "	"	—4·6

All of these examples are from mountain work; in the first case with points of great elevation and extremely long lines and in the latter points of moderate height and lines of moderate length. In both results the effect of differences of refraction and error from deflection of the plumb line remains.

On the 39th parallel trigonometrical survey in western Missouri occurs a case which offers a particularly good opportunity for the examination of results in a rolling country. In that work the scheme coincided so nearly with the transcontinental line of precise levels that nine consecutive points, covering some 185 kilometres and forming the northern line of the scheme, were determined with the spirit level, and a comparison of the results by the two methods shows at no point a difference of more than 0·6^{m} and a final error between terminals of only 0·3^{m}. In this work the practice was to observe on five days at each station the double zenith distances of two points, and the difference in elevation between these and between all other visible points was determined by micrometric differences measured on six days.

In computing, the micrometric differences seem to have been taken as a standard and the zenith distances made to conform to them.

The methods and instruments to be adopted in this class of work will depend upon the degree of accuracy sought, and this upon the objects to which the results are to be applied.

It would appear from what has gone before that any of the methods mentioned produce results sufficiently accurate for the use of the geographer, and the better ones for the determination of bench marks for the topographer. To what extent they may in the future be required or applicable to the elucidation of physical problems does not appear; but since the difference in cost as between the rough and refined methods will rarely amount to more than 1 per cent of the expense of field operations of parties in which vertical measures constitute a very inconsiderable part of the duty, it would seem to be good policy to maintain a standard high enough to meet any requirements of the future.

The determination of differences of elevation by observations of atmospheric pressure is often convenient, and at times the only method available. The results from a limited number of observations for widely separated points can not, however, be depended upon within some hundreds of feet, and in a mountainous country are very unreliable for even moderate distances. This is aside from errors of observation of phenomena within our reach, which, with our present instruments and knowledge, are very great, and arise from atmospheric disturbances of a more or less local nature, the conditions in neighboring valleys rarely being the same, and during rapid changes often very different on different slopes and in different parts of the same valley. As pointed out by Ferrel, a cyclonic disturbance not sufficient to amount to a storm may produce a variation in the barometric gradient between points a few hundred miles apart amounting to 100 feet or more, and Williamson's computations of the difference of daily results for elevation during the year 1862 between St. Bernard and Geneva (some 50 kilometres) show errors as great as 60^{m} for a single day, the determination after forty years of most careful observations under M. Plantamour being $2 \cdot 7^{m}$ in error.

The results from careful observations in this country, ranging from one to six years, give resulting errors of from 16 to 37 feet, but with contrary signs, so that there seems no reason to doubt the constants used in the reductions.

It behooves us, however, to consider the relative degree of accuracy of the several instruments employed for the purpose and the degree and conditions of their usefulness. There are a few rules which apply when only a limited number of observations can be taken, whatever instrument is used, and which should be borne in mind by the observer, viz:

Observations should be made at such hours as give most nearly the mean temperature of the day, or, better, of the month.

Observations should not be taken when there is fog or mist.

In computing results it is desirable to use the normal temperature of the vicinity of the lower station if obtainable.

The observations should be simultaneous when practicable.

The mercurial barometer in its most improved form was probably until a late date the most suitable instrument at our command for the determination of atmosphere pressure, and from its simplicity and the ease of its manipulation must always be a favorite, but for work in a rough country the extreme care required in transporting it is a serious drawback. The marked advance in our knowledge of thermometry within the last decade, however, suggests possibilities from observations of the boiling point which equal and may surpass* in accuracy

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*See Von Jordan: "Vergleich zweier Siede-Thermometer mit Quecksilber-Barometer," Zeitschrift für Instrumenten kunde, Jahrg. 1890, S. 341–347. Horner "Siede Thermometer und Quecksilber Barometer," Zeitschrift für Vermessungswesen, Bd. 21 (1892), S. 30–31.

the results obtained with the barometer, and in view of the greater portability of the apparatus required it seems desirable that its use be encouraged.

The aneroid barometer is essentially portable, but unfortunately this is true only as regards the integrity of its several parts and not of the instrument as a whole, since, while it will survive very rough usage, the relations of its parts are so easily disturbed as practically to produce a different instrument. The magnitude of these changes, and the tendency of the parts to resume given relations, differs very widely in aneroids from the same maker and intended to be of the same class.

This results from mechanical defects, but so minute and so hard to trace that to correct them would magnify the cost of construction beyond all bounds. Thus it happens that one which gives excellent results in the comparing room may prove utterly worthless in the field. It is, however, so extremely useful for purposes of reconnaissance that it well repays the labor bestowed in testing it under the conditions actually occurring in the field work on which it is intended to be used. With a well-selected instrument, carefully handled, profiles may be traced in a moderately flat or rolling country with sufficient accuracy for all purposes of reconnaissance; and how well it may be relied upon for the determination of accidents of surface where a sufficient number of controlling points are available is amply shown by very characteristic topographic maps produced by this method by the Geological Survey.

In concluding this report, and after examining accounts of work of the various kinds and in the several countries referred to, your committee beg to present the following as their conclusions, viz:

That trigonometric heights, when observed during the hours of minimum vertical refraction and under varying atmospheric conditions, can be obtained with a sufficient degree of accuracy to make them valuable for many purposes, and, when not impracticable, their determination should form part of the work at all triangulation stations.

For this purpose there is suggested a new form of instrument or such changes or additions to those now in use as will permit of the measurement of micrometric differences of larger arcs than is possible with the micrometric eye piece attached to our theodolites, and of the convenient use of a delicate level in connection with it. The gradienter as now constructed seems in a great measure to fulfill the requirements for short lines, but the optical power is too low for use over long ones, and other improvements seem possible.

That with modern thermometers and methods the atmospheric pressure may be determined by observations of the boiling point with a precision equal, if not superior, to that obtained with the less portable mercurial mountain barometer, and for purposes of exploration their use is recommended.

That the standard bench marks throughout the country should be determined with the greatest degree of accuracy attainable.

That subsidiary lines may be leveled by less precise methods.

That if upon investigation it shall appear that less elaborate methods or instruments than those now employed on the Survey will by the use of small circuits produce satisfactory results, with an increase of economy, purely theoretical considerations should not prevent their adoption.

That the leveling instruments at present in use on the Survey seem as perfect as any yet devised, but the use of prisms by which the level may be read without removing the eye from the eyepiece appears to be desirable, and we would recommend that one of the French instruments to which they have been applied be procured for experimental purposes.

That the level should have a value of about 2″ to a millimetre.

That a watch level or some other device is desirable to facilitate the preliminary rough leveling of the instrument.

That the rods be frequently compared in the field with a standard.

That if, as seems possible, wood can to a great degree be protected against hygrometric changes, it is preferable to metal for leveling rods.

We recommend a plane surface for the terminal of the rods, and the use of foot plates with hemispherical centers for supporting the rods, and retaining walls to prevent any undue lateral movement.

The scale, if of metal, should be protected from rapid changes of temperature due to varying position. It should be thin enough to respond quickly to changes in temperature, of a form that will secure the requisite rigidity, and so arranged as to be readily compared with a standard.

If wooden rods are to be used, the measuring portion should be carefully selected, homogeneous, straight-grained, thoroughly seasoned wood of the pinus family. It should be carefully impregnated with paraffin and perfectly coated to protect it from moisture as far as possible. It should be so attached to the supporting rod as to secure protection from abrasion of the surfaces and with an air space sufficient to prevent direct contact or the harboring of moisture. Certain of the graduations might be marked by lines on metal pins embedded in the wood. The face of the rod should be over the center of support.

The target, if one is used, and its advisability seems questionable, should travel as nearly as may be along a line passing through the center of support of the rod, and carry a vane by which the observer at the instrument may assure himself of the rod's verticality.

The rod level should be placed at angles of 45° with the face of the rod and as near together as possible.

Notes on observed cases of abnormal refraction in lines passing near the ground, which emphasize the possibility of errors from this source, are given below.

FRANK WALLEY PERKINS, *Chairman.*
F. A. YOUNG, *Secretary.*

MEMORANDUM FOR THE USE OF THE COMMITTEE ON HYPSOMETRY.

I offer the following evidences of local refraction close to the surface of the ground, believing that one of the principal sources of errors in leveling arises from that condition.

San Pedro Base Line—1858.

In measuring this base an aligning flag was sent forward about 200 yards, and in trying to place it in line it appeared and disappeared and changed place in such a curious and irregular manner that I personally went forward to ascertain the cause. As I approached, the flag that I had seen disappeared, and it was lying upon the ground and had been during the confused signals. The aid had misunderstood my signals, which he said were very confusing, and had stepped aside. As I returned to the base the flag again reappeared in the air to the height of about 4 or 5 feet. On these plains, in the preceding December, I had witnessed remarkable effects of mirage at midday.

On the base-line site at Port Townsend, in 1854, a target 4 feet square, standing on the ground, was apparently raised by local refraction more than its height, as seen from a distance of 250 yards.

Yolo Base Line—1881.

When the party was going out to work one morning there was a beautiful exhibition of mirage ahead of us, wherein all objects were lifted up into the air.

We drove into and through this warm stratum of air, and when in it saw the mirage effects all around us.

Los Angeles Base Line—1889.

In driving to work we frequently came into streaks of quite warm air and then into streaks of quite cold air. On different mornings, in passing over the same depressions, the temperatures were not relatively the same.

San Francisco.

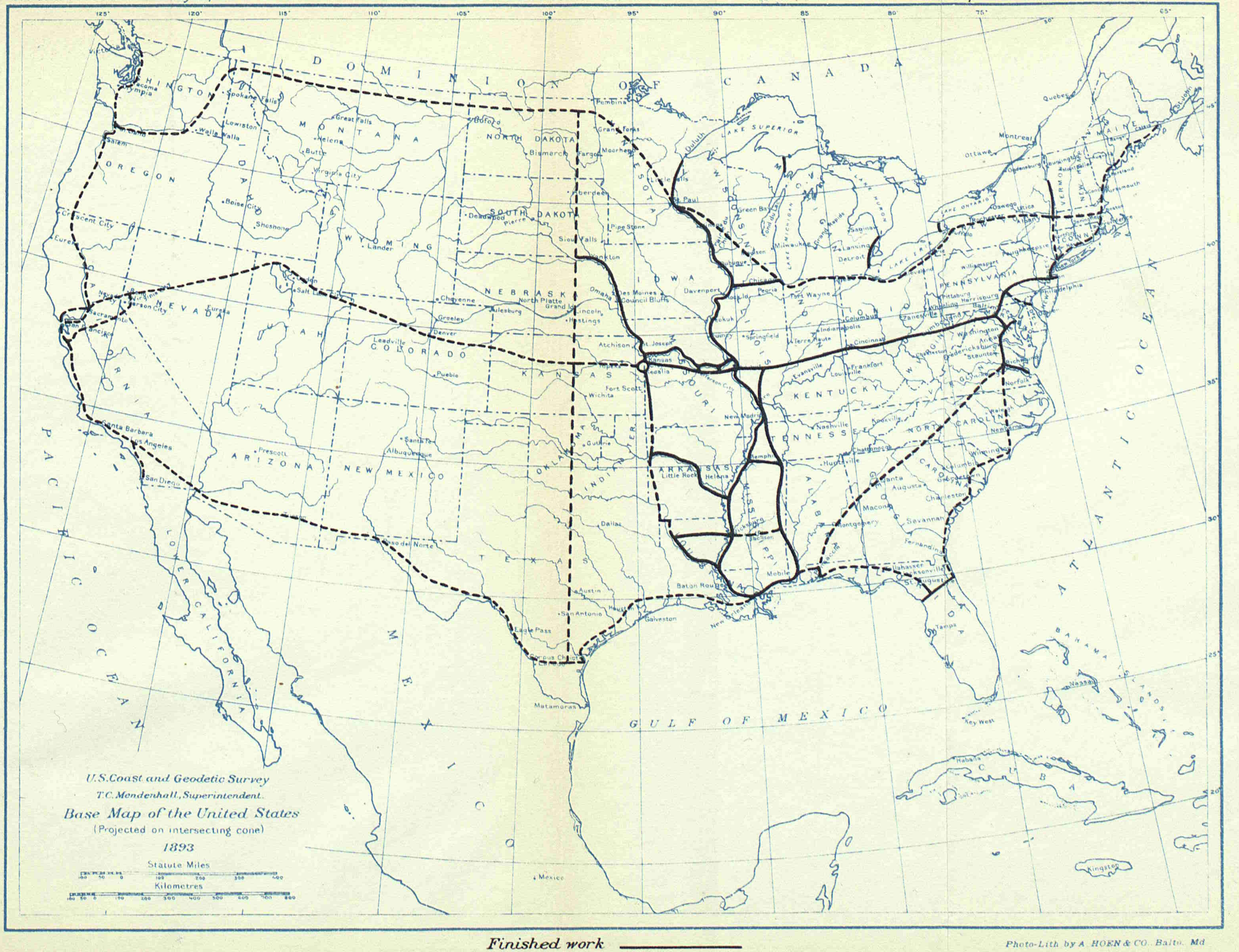
On some of the streets of San Francisco the exhibition of mirage is so marked that it has been illustrated in the newspapers. And upon Washington street, near the Lafayette Park Astronomical Station, where the block is nearly level, with a decline at each end, I frequently see the mirage along the whole block as my line of vision reaches the height of the street.

There can be no doubt that such conditions would give very wild leveling results; and it naturally suggests that abnormal conditions of the surface layer of air may not be visible, yet the effects be inimical to good results.

GEORGE DAVIDSON.

FEBRUARY 11, 1894.

General Scheme of Standard Levels for the United States.



Finished work ———
Proposed " - - - - -

Photo-Lith by A. HOEN & CO. Balto. Md.

REPORT OF COMMITTEE F, ON ALASKA.

The extraordinary growth of this but partially explored territory, with its valuable resources on land and the almost limitless wealth in its waters, demands greater attention than has heretofore been accorded it and makes it imperative that general and comprehensive aids to its navigation and commerce be supplied.

This vast region contains about 600 000 square miles, being about twelve and a half times the area of the State of New York. It has approximately 26 000 miles of shore line, which exceeds that of the Atlantic, Pacific, and Gulf coasts by over 11 200 miles, while the islands along its coast are estimated to be 1 100 in number. A course parallel with the trend of its shore from Cape Muzon, its most southerly point, to Point Barrow, its most northerly one, is about 2 800 miles. The Aleutian chain of islands is about 1 100 miles long, and Attu, the most westerly one of this group, is about 2 200 miles west of Sitka.

There are immense forests in Alaska, densely covering every part of the country and climbing steep mountain sides to heights of 2 000 and 2 500 feet above sea level, and which extend as far west as Kadiak Island, being a continuous stretch of a thousand miles.

They consist mainly of spruce, hemlock, and cedar, one variety of the latter, the yellow, being very valuable in the construction of small vessels on account of its durable qualities.

The commerce of Alaska is, and doubtless always will be, carried on by water, owing to the peculiar formation of the country; and being so varied and largely conducted by nonresidents and by vessels hailing from so many different ports, it is difficult to obtain an exact idea of its extent. The internal commerce is carried on through about 126 agencies, located in 104 towns and settlements situated along its coast and among its islands.

The exports consist mainly of furs, ivory, Indian curios, gold and silver bullion and ore, and the products of the whale, cod, and salmon fisheries.

During the earlier occupancy of the country its commerce depended almost exclusively on the fur trade; but since other industries dependent upon the actual necessities of man sprang up; this important factor, although of great value, has already fallen to a third place in importance. From 1868 to 1891 the total value of the furs exported is estimated at $50 124 500, and the annual yield for the last-mentioned year amounted to about $1 605 000.

In 1892 there were sixteen gold and silver mines in operation, and up to that date the total output amounted to about $6 000 000. The traffic dependent upon the necessities of the small army already engaged in this comparatively new enterprise is considerable, and will undoubtedly increase.

The salmon industry commenced in 1878, and from that date up to 1890 the pack had amounted to $9 612 000. In 1878 the entire product was valued at $59 416, while that of 1890 was $2 731 000. The salmon-canning industry of this country is confined to the waters of California, Oregon, Washington, and Alaska. In years past the Columbia River has been the principal source of supply, but the run in all the sections south of British Columbia has become smaller from year to year. In the year 1887 the total pack for the entire Pacific Coast was 969 200 cases, of which the Columbia River furnished 430 000. In 1890 the output of the western coast was about 1 223 955 cases, of which Alaska alone furnished 688 322, or more than half the entire product of the United States. The capital invested in the Alaska salmon fisheries, including permanent improvements, vessels, etc., is something more than $4 000 000. There were, in 1890, 37 canneries between Dixon Entrance and Bristol Bay (25 of which are west of Sitka), and about 6 000 persons were employed during the fishing season, using 66 vessels for the purpose.

Judging from the rate of increase during the past ten years and the enormous field yet to be developed, the commerce depending upon this single industry will be one of the most notable interests of the Pacific Coast. Three-fourths of it is now beyond the region reconnoitered, and is rapidly crowding northward into uncharted localities enormously rich in fish. It is interesting to note that the two newer industries, mining and salmon fishing, have grown so rapidly that while in 1880 both these industries were insignificant and completely overshadowed by the fur trade, by 1890 their products amounted in value to $15 000 000, or more than twice the purchase price of the territory.

The Pacific and Arctic whaling catch, though not confined strictly to Alaskan waters, is conducted by American vessels, and all but a very small percentage of it is secured in waters contiguous to the Alaskan coast. The total value of oil, bone, and ivory of the catch between 1874 and 1890 was $11 204 465. There are about 50 vessels engaged in this industry, their port of call being Port Clarence. The charts of the tracks and rendezvous of these vessels are simply compilations of early explorations, very crude and inaccurate.

Of the food fish of Alaska, the codfish stand next in commercial importance to the salmon. The eastern part of Bering Sea is a great reservoir of cod, and the area within the limits of 50 fathoms depth is no less than 18 000 square miles. In this sea fishing must be done, as it is off Newfoundland, without harbors of refuge, but in a much less depth of water. The fishing banks along the south shores of the Aleutian chain will add about 45 000 more square miles, making a total of 63 000 square miles, this being about four times the area of the banks in the region of Newfoundland. Though over twenty years have elapsed since the inception of this industry, it must still be considered in its infancy. The value of the catch during the last twenty-seven years has amounted to about $8 900 000. It is carried on without regard to the abundant supply, but solely in accordance with the demands of the local and limited market on the Pacific coast of America.

It is evident, with the numerous transcontinental railways, with the increasing population along their lines and growing tributaries, that the demand will constantly and permanently increase, so that this interest will alone crowd the waters of the Gulf of Alaska and Bering Sea with sails.

The shores contiguous to these fishing grounds and the waters covering them are imperfectly and incorrectly delineated on the compiled charts, handed down to us principally from the early Russian explorers, and should be corrected to conform with the demands of modern navigation at as early a date as possible. Although the fishing for halibut, herring, etc., is at present only for local consumption, these industries are capable of wonderful development.

The value of merchandise shipped to Alaska from Pacific Coast ports from 1868 to 1890 amounted to $15 594 086, while during the same period the exports amounted to $75 213 929. During 1880 the merchandise received from Pacific Coast ports was valued at $463 226, while during 1890 it amounted to $1 635 494, showing a gain of nearly 300 per cent in amount in ten years.

During the period from the time of its purchase, in 1867, to 1890 a conservative estimate of the value of products shipped from this detached territory was about ten and one-half times the price paid for it.

It is regrettable that our sources of information for late Alaskan statistics are confined to the brief summaries of the governor's reports, and that for a comprehensive study of all the wealth-producing industries of the territory we have to go to the publications of the census for 1890.

There was considerable interest in this new territory at the time of its purchase from Russia by the treaty of June, 1867, and Secretary Seward arranged for a geographical reconnaissance in the summer of that year under the charge of the Coast Survey, from which the first Coast Pilot of southeastern Alaska was compiled. This work included only the more important points from Dixon Entrance to Unalaska.

With the exception of the astronomical work of 1869 and a rough reconnaissance by a small party in western Alaska during the summers of 1871, 1872, 1873, and 1874, nothing of importance was accomplished until 1882, when a trigonometric and hydrographic reconnaissance of the inland waters of southeastern Alaska was commenced, and, with the exception of one year, this has been continued during the summer months to the present time.

In the summer of 1892 the longitude of Sitka was very satisfactorily determined (chronometrically) as a base station, and during that and the succeeding years four other points were equally well determined from it for the international boundary work in the southeastern portion of the territory.

With the foregoing data the conditions are favorable for carrying on the triangulation where necessary, and also extending the astronomical work into localities considered in immediate need of it.

A glance at the progress sheet of southeast Alaska shows that the survey of the inland passages is nearly completed. To finish it requires a survey from Sitka northward through Peril Strait, and thence north and south along Chatham Strait to join the work of 1879 and 1890. This is estimated to require two seasons. The survey of Icy Strait, Glacier Bay, and Cross Sound will occupy about two seasons more.

The character of the shore-line work that is now being done in the inland passages is of a sufficiently precise nature for cartographic purposes and furnishes results which supply all that is at present demanded in the line of exactness; but it is advisable that the survey of the sections completed before the methods of work were brought to their present degree of accuracy should be remade and that all the work be brought to a uniform degree of worth. The topography on our present charts of the Inland Passage has not been treated in the detail that its importance merits. A characteristic representation of the main topographical features is of vital importance for charts which are intended to satisfy the demands of a coasting trade, and nowhere is this more important than in a region where the meteorological conditions are so bad as they are in Alaska, often leaving the pilot dependent for a check on his position upon fleeting views of limited sections of the land. It is very important that the defects in this respect should be supplied by a topographical reconnaissance made at as early a date as will be found practicable. Practically the entire outside coast, from Dixon Entrance to Cross Sound, remains unsurveyed except so far as it has been done in the rapid exploratory work of the early voyagers.

In considering the necessity for active prosecution and an early completion of the work of surveying southeast Alaska, the character of the traffic to be benefited by it must be taken into account. Nearly all of the carrying trade of southeast Alaska is by steamers. The intricate passages, strong tidal currents, and deep waters render navigation by sailing vessels difficult and dangerous. For these reasons, and on account of the unfavorable meteorological conditions, the steamers running in this trade are and must be, even after a complete survey has been made, supplied with pilots having accurate local knowledge of all the waters through which they run. Numerous fisheries and canneries are already located off the main line of steamer travel on the west side of Prince of Wales and Baranof islands, and these are visited at intervals during the season by the regular steamers which carry supplies and at the end of the season transport the pack to southern markets. It needs no arguments to show that wherever the ships of our merchant marine touch along our coasts accurate charts should be provided. But inasmuch as in Alaska we have a vast extent of shore line, of which the existing charts are poor and misleading, if not actually dangerous, and as the work of remedying this state of affairs must naturally be protracted through a term of years, it is plain that the surveys of the different sections should be taken up in the order of their importance.

In contrast with the steam carrying trade of southeast Alaska is that by sailing and steam vessels to the westward from Cooks Inlet and Kadiak, along the Aleutian Islands and northward to the Arctic. The various passes through the Aleutian chain need immediate attention. These are used annually by the Arctic whaling fleet, the supply ships of the various companies trading along the shores of Bering Sea and supplying the interior by way of the Yukon and other rivers, the cod-fishing fleet, and also by the combined squadrons of United States and British vessels that patrol and guard the waters adjacent to the Seal Islands.

Of all these passes but three—Unalga, Akutan, and Unimak—are in common use, owing to the imperfect surveys, and ships are often compelled to go far out of their way to make a known pass. Even in the three passes mentioned we have little knowledge of the velocity and set of the currents.

Next in importance is probably the vicinity of Kadiak Island and Cooks Inlet, where a large salmon industry has been built up within the past ten or twelve years and where surveys are very urgently required.

The Shumagin Islands and the western end of the Alaskan Peninsula would follow in the order of prominence. These regions have already attained a commercial importance which makes a new chart of them a pressing necessity. Off the first-mentioned islands the *Albatross* has developed a great codfish bank of 4 400 square miles in area. Unga is being exploited for its mines of precious metals, and the coal indications about Port Möller were sufficiently promising to justify the Alaska Commercial Company in building a small railroad to develop them. Thin Point, Sand Point, Belkovsky, and Sannakh have long been among the most profitable stations of the Alaska Commercial Company, if we except those on the Seal Islands.

As the great avenue of communication with the interior of the territory, it is important that the Yukon, and particularly its delta, should receive early attention. Without doubt, if a deep-water channel could be traced through the flats, which at present, on account of our complete ignorance of the condition of the mouth of the river, are an insuperable bar to the navigation through them of any but light-draft, flat-bottomed steamboats, it would give a tremendous impetus to the examination of the great possibilities that we have good reason to hope exist in this vast region. And there can be no possible doubt of the value of the great fisheries that would at once be established here if a survey would show the possibility of entrance and departure for the vessels that would be required for bringing up labor, material, and supplies and taking away the product.

Our present representation of the mouth of the Yukon is the result of the examination made by a merchant marine captain in command of one of the Western Union Telegraph experiment ships in 1865. The astronomical determination of Port Clarence, the rendezvous for the

Arctic whaling fleet, would be very valuable. At present the only station available for determining the chronometer errors for these vessels and the Revenue-Marine steamers which patrol the northern part of Bering Sea is at Plover Bay, in Siberia, a station established in the fifties by the English while searching for traces of Sir John Franklin's expedition. The nature and period of this determination predicate such a low degree of accuracy for it, and our interests in these waters are now so weighty, that any delay in furnishing one or two improved positions will subject our country to a charge of serious neglect.

The continental shore line from Cross Sound to Cooks Inlet is of such a nature and interest that there seems to be no immediate demand for its survey for commercial purposes now that such work has been completed in Yakutat Bay, the most important locality along this stretch.

We conclude that the work along the Aleutian Islands and in the vicinity of Kadiak Island and Cooks Inlet should be undertaken at the very earliest opportunity possible.

For this work it is not considered advisable to prescribe or suggest rigidly defined methods of procedure. The wild character of much of the shore line, the adverse meteorological conditions, and the limitations which economical considerations put on the completion of the schemes suggested before a good reconnaissance has been made, are all arguments against offering any minutely prescribed plans for approval for actual execution.

Your committee would, however, suggest that the degree of accuracy such as the Conference deems sufficient for tertiary work should be considered satisfactory for the surveys proposed in the preceding paragraphs.

Certain general suggestions about methods of work are offered as follows, but they are to be considered as subject to the wide discretion which we consider should be allowed to the officers charged with making those surveys:

One of the first considerations in the extension of the survey westward to and along the Aleutian Islands is the astronomical determinations. These are of prime importance for the reason that it is impracticable to carry a scheme of triangulation along the chain, and hence the survey must be made of the different islands independently and their relative position determined by astronomical operations. It is suggested that along this chain of islands the method of longitude determinations by terrestrial signals would probably be feasible, accurate, and also the most economical. The islands are near enough together to allow such signals to be exchanged. For the survey of the separate islands base lines must be located and measured on each, and its outline delineated by a local survey.

The Aleutian chain extends nearly east and west for a distance of about 1 100 miles. Among the islands making up the group there are three, Unimak, Unalaska, and Umnak, which are each about 60 miles long; two, Atka and Amlia, about 40 miles long; three, Adakh, Amchitka, and Attu, about 30 miles long; three, Kanaga, Tanaga, and Kyska, about 25 miles long; four, Unga, Sannak, Akutan, and Agatu, about 15 miles long, besides a host of lesser islets.

To control the longitude of this chain it is suggested that six stations be determined chronometrically, viz, Kadiak, Sand Point, Unalaska, Seguam Island (or some other in the vicinity of Amutka Pass), one of the Rat Islands, and Attu. This would give a series of stations along the chain at an average distance of about 300 miles apart. Advantage might be taken of the regular steamer running during the summer months from Sitka westward to establish one or even three of these stations (Kadiak, Sand Point, and Unalaska) during the coming season if funds are available.

Having once located the six base stations, a single party with proper transportation facilities could rapidly locate intermediate stations by the exchange of signals. On the larger islands enumerated above two or more stations, one at either end and the other intermediate, would seem to be essential, and in cases where it would be impracticable to find intervisible points it would probably be feasible to obtain results by noting the times of signals made from a vessel located far enough off the island to be visible from its ends or from the intermediate station; to supplement the method by signals under favorable circumstances, and when practicable the differences of longitudes may also be determined from latitude and reciprocal astronomical azimuths. In the high latitudes of the Aleutian Islands this method will give the differences of longitude with very great accuracy.

No doubt the greatest obstacle to all surveying work in this vicinity would be the dense and persistent fogs that are so prevalent, and, as bearing on this subject, the following table from Dall's Alaska (p. 444), will give something of an idea of the number of favorable days that may be expected during the months of May, June, July, August, and September. The table shows the number of wholly clear, partly clear, and wholly cloudy days that occurred at Unalaska during a period of seven years.

	May.	June.	July.	August.	September.
Clear days	2	6	0	5	2
Partly clear	105	95	118	106	107
Cloudy	104	109	99	106	101

This gives an average of less than one wholly clear day and about fifteen each of partly clear and cloudy days per month. The execution of the work about the mouth of the Yukon might be intrusted to a party which could be taken to St. Michaels by the revenue cutter, and, being supplied with a steam launch, could be employed from about July

1 to about September 13, and then return on the same vessel, a course of procedure that would entail a small cost to the Government.

For the determination of Port Clarence an observer might go up to that point on the revenue cutter, and if observations were being made contemporaneously at Unalaska a set of chronometers could be taken from that place on the tender which visits Port Clarence with supplies and to receive the accumulated bone and oil of the whaling fleet about midsummer, and by comparison with the chronometers, checked by observations at Port Clarence, the longitude of this important station could be determined. It is very probable that the public-spirited managers of the Pacific Steam Whaling Company would give such assistance to this project that it could be perfected at a very small expense.

PROPOSED SCHEME OF TRIANGULATION FOR SOUTHEAST ALASKA.

In order to properly connect and coordinate the work of reconnaissance triangulation that has been carried on in southeastern Alaska for the past twelve years, it is important that a main system of triangulation should extend from Dixon Entrance to Chilcat. Such a system would naturally extend up Clarence Strait to Sumner Strait, along Sumner Strait westward, cross the southern part of Kuiu Island in the vicinity of Tebenkof Bay to Chatham Strait, thence up Chatham Strait and Lynn Canal to the head of the latter. From this main system, as a base, secondary systems could in time be carried along the outside coast.

In executing the main triangulation, and, in fact, any triangulation, in southeastern Alaska, the work must necessarily be carried along the shore line of the passages, and for the following reasons:

1. Owing to the rugged nature of the country and the dense undergrowth of the forests that cover it, it would involve too great an expenditure of time and money to attempt to carry a triangulation along the mountain tops.

2. Even if the mountain tops were accessible, there would be great loss of time in attempting to occupy them. Clouds hang for days about the summits, when lower down, near the shore line, there would be no difficulty in observing.

3. Again, a scheme looking to the occupation of mountain peaks would unquestionably involve long lines on which heliotropes would be necessary, and as clear and sunny days are exceptional this again would be a source of delay and expense. On the other hand, a triangulation extending along the shore line would involve sides of an average length of about 10 miles. For these distances signal poles only need be used, and with the instrument properly protected from rain it would be possible to observe on many days when light rain was falling.

BASE LINES.

Bases for this work will be difficult of location and will necessarily be shorter than those usually measured for a main scheme. Generally speaking, the sites for bases must be sought in river bottoms, near the shoreline, and on such stretches of beach as can be found. In the latter case advantage can be taken of flats left bare by the tide at low water.

From the nature of the country suggested above as suitable for base-line sites, it is evident that the employment of base apparatus in measurements is scarcely practicable, and advantage must be taken of tapes, by means of which accurate and quick work can be done, as has been clearly demonstrated experimentally.

In consequence of the special value that gravity experiments in the high latitudes of Alaska would have, it is recommended that these observations be made by the astronomical parties either as part of their regular work or incidentally, as circumstances may dictate.

The astronomical parties should also be required to make observations for the magnetic declination, dip, and intensity at each station, and the triangulation parties should observe for declination by noting the magnetic bearings of the sides of the triangulation.

JOHN E. McGRATH, *Chairman.*
A. L. BALDWIN, *Secretary.*

REPORT OF COMMITTEE G, ON INSTRUMENTS.

The committee has thought proper to confine its attention to the consideration of instruments for astronomical work and the measurement of horizontal angles, as other committees, having to report on special work, will necessarily consider the instruments used in such operations.

The following list of instruments now in possession of the Survey has been prepared by the instrument division:

FOR ASTRONOMICAL WORK.

6 transits of about 114^{cm} (45-inch) focus and 70^{mm} (2¾-inch) objective, with power of about 100.

2 transits of 95^{cm} (37½-inch) focus and 82^{mm} (3¼-inch) objective, with a power of about 90.

4 meridian telescopes of 79^{cm} (31-inch) focus and 63^{mm} (2½-inch) objective, and powers of 60 to 90.

3 meridian telescopes of 66^{cm} (26-inch) focus and 57^{mm} (2¼-inch) objective, with powers of 50 to 70.

1 Repsold vertical circle, with microscopes.

4 zenith telescopes of about 114^{cm} (45-inch) focus and 76^{mm} (3-inch) objectives, with powers of about 100.

1 zenith telescope of 66cm (26-inch) focus and 57mm ($2\frac{1}{4}$-inch) objective.
8 cylinder chronographs (Fauth & Co.).
4 sets of longitude telegraphic apparatus.
19 sidereal breaks-circuit chronometers, 10 of which are by Negus and 9 of these of very recent date.

FOR HORIZONTAL ANGLES.

Direction theodolites:

5	51cm (20-inch), 3	microscopes,	Wurdemann.
1	46cm (18-inch), 3	"	T. & S.
1	41cm (16-inch), 3	"	F. & Co.
1	36cm (14-inch), 2	"	Wurdemann.
1	30cm (12-inch), 2	"	Brunner.
2	30cm (12-inch), 2	"	F. & Co.
2	30cm (12-inch), 3	"	F. & Co.
2	30cm (12-inch), 3	"	Office.
6	20cm (8-inch), 2	"	F. & Co.

Repeating theodolites:

1 36cm (14-inch), Brunner.
3 30cm (12-inch), Gambey, 1 with 30cm (12-inch) vertical circle.
9 25cm (10 inch), Gambey, 1 " 25cm (10-inch) " "
4 20cm (8-inch), Gambey, 3 " 20cm (8-inch) " "
6 20cm (8-inch), Office, 3 " 15cm (6-inch) " " and micrometer eyepieces, and all with compass declinometers.
8 15cm (6-inch), Gambey.
10 15cm (6-inch), Brunner, 1 with vertical circle.

A number of other instruments in the possession of the Survey might have been added to this list, and some of them will undoubtedly still be used, but in general they are so antiquated or of such inferior character that it is thought scarcely worth while to consider them here.

Types of all instruments named in this list have been set up in the instrument division and examined by the committee and other members of the Conference. In general, they are good instruments of their kind.

ASTRONOMICAL INSTRUMENTS.

The six 114cm (45-inch) transits were made by Troughton & Simms, of London, 1845–1856, and were used in telegraphic longitude work of the Survey prior to 1888. Three of these have recently been reconstructed and improved in the office. These instruments, though excellent, are now considered too large and heavy to be economically used in the general work of the Survey, and it is not likely that they will be used in the future except at stations where transportation is not an important item or where they may be required for a long time, as at the astronomical stations at Washington and San Francisco.

The two 85cm (37-inch) transits were made in the instrument shop of the Survey in 1887–88, and have been used with success in the telegraphic longitude work since that date.

The four 79^{cm} (31-inch) and three 66^{cm} (26-inch) meridian telescopes were made in this country by Wurdemann, Kubel, and Fauth & Co. since 1868. They have been used for the determination of latitude by Talcott's method and for time observations with success, and are, in fact, the most popular instruments in the possession of the Survey for general field astronomical work. Two of the larger ones have recently been reconstructed and improved at the office, and another is now in hand. It is recommended that the others be similarly improved as far as practicable. These instruments were designed by an Assistant of the Survey.

The four 114^{cm} (45-inch) zenith telescopes were made by Troughton & Simms, of London, about 1850. As originally constructed they were not satisfactory. One of them was changed, and the other three for many years were set aside. In 1890 three of these were reconstructed in the instrument shop of the Survey and used at Rockville, Md., San Francisco, Cal., and Honolulu, Hawaii, for observations for the investigation of the variation of latitude. One of the three was provided with a new objective and eyepieces, and with a further improvement of the mounting of the levels it is believed it will equal the best zenith telescope extant. It must be said of these instruments, as of the large transits, that they are too heavy for economical use in the general work of the Survey, and they are not likely to be used in the future except for special investigations.

The 66^{cm} (26-inch) zenith telescope was made by Wurdemann in 1854, and is a good instrument.

The eight cylinder chronographs were made by Fauth & Co., of Washington, the first about 1878. They were specially designed for the work of the Survey and have been used with success; but it must be said that their great weight is a serious objection. Many chronographs of American, English, French, and German design are equally heavy, and do not seem to meet the requirements of the field work of the Coast and Geodetic Survey so well as those now in use. When new chronographs are needed, the questions of their weight and of some improvements on the present ones should be particularly considered.

The astronomical instruments employed by other surveys in this country are very similar to those in use in the Coast and Geodetic Survey. Those in Europe, however, are quite different. There the broken transit has been extensively used in the telegraphic longitude work, and the illustrations of various forms are found in the publications relating to this work. There seem to have been some difficulties with these instruments in the flexure of the axis and the displacement and distortion of the prism. It is said, however, that these difficulties have been overcome. Whether such an instrument should be adopted in the Coast and Geodetic Survey the committee deem best to leave to future consideration.

It is only of late years that the Talcott method has been used in Europe, and for this purpose a special zenith telescope has been made by Bamberg, of Berlin, for the International Geodetic Association. This instrument is illustrated in the report of Dr. Marcuse's observations at Honolulu, 1891–92. It has a broken telescope, but the prism is small and placed very near the eye end. It has about the same power as the 114^{cm} (45-inch) zenith telescope of the Coast and Geodetic Survey. It is undoubtedly a fine instrument, but its great weight renders it unfit for general field work, and it should be properly classed with instruments for a fixed observatory. A similar instrument is now in use at Columbia College, New York, where observations are being made with it to investigate the variation of latitude.

The vertical circle, especially as made by the Repsolds, has been and is being used in European surveys for the determination of latitudes. The Coast and Geodetic Survey has one such instrument, but it has not met with favor, and it is not deemed advisable to further introduce it in the Survey, as it is believed the Talcott method is superior and can be used with quite as great facility.

THEODOLITES.

The Survey has such a number and variety of theodolites that it seems inexpedient to now introduce uniformity of design without an expense which is not warranted by its present needs.

The five 51^{cm} (20-inch), the 46^{cm} (18-inch), 41^{cm} (16-inch), and 36^{cm} (14-inch) position theodolites are of good design. Five of the 30^{cm} (12-inch) instruments are faulty in that the circles move upon the collars in shifting position. It is recommended that these circles be fixed and the instruments be used with position stands. The other two 30^{cm} (12-inch) direction theodolites were recently constructed at the office. They were designed with great care and the workmanship is of the very best. They have double centers, the outer one of cast iron and the inner of hardened steel. The inner center and socket are made with great precision. The outer center and socket are well made, but with less precision, as this center serves only for shifting the position of the circle. The alidade, supported on the inner center, is of aluminum as far as practicable, and the friction upon the center is very small. In their construction no relieving spring was deemed necessary; but the committee, considering this a point of vital importance, calls attention to the necessity of studying this omission in its effect on the instrument after it has been subjected to transportation and wear. The circles were divided on the Coast and Geodetic Survey engine. These instruments have been examined and practically tested by Assistant R. S. Woodward, and a preliminary report shows No. 145 to be an instrument of a very superior order. It is recommended that as soon as the new circle of No. 146 has been added, examined, and found satisfactory both instruments should be sent to the field as soon as practicable.

The six 20^{cm} (8-inch) direction theodolites are also faulty in that the circles move upon the collars, and some of them are held in position only by friction, no clamp whatever being provided. The micrometers of the microscopes are poor. These instruments have not been used for many years, and it is recommended that, if they are to be used in the future, the circles be fixed, position stands provided, and also new micrometers.

Since 1873 the 51^{cm} (20-inch) theodolites have been mostly used in the great triangulation of the Survey, where the length of the lines have been 100 kilometres (60 miles) and upward, while the others have been used in smaller work.

Nearly all the repeating theodolites now in use by the Survey were made by Gambey and Brunner, of Paris, many years ago. They are still as good instruments as ever. Originally they all had small and low-power telescopes. Larger and better telescopes have been added to some of the 25^{cm} (10-inch) and 30^{cm} (12-inch) Gambey instruments. The construction of these instruments is such that they are so light that they will not admit of any very great weight being added to them. If larger telescopes are needed, they should be made of aluminum. These instruments have been mostly used in the smaller triangulation of the Survey. The 30^{cm} (12-inch) instruments have, however, been successfully used in triangulation with lines as great as 60 to 80 kilometres (40 to 50 miles).

Although all the theodolites named in the list above given are considered good instruments, it is well known that the graduations of some of them are defective, and it is recommended that such circles be regraduated as soon as practicable. The new 30^{cm} (12-inch) theodolites recently made at the office show that the Coast and Geodetic Survey dividing engine in its present condition will do very satisfactory work. The graduation of these circles will compare favorably with the best modern circles. It may also be said that the regraduation of a circle at the office is neither a difficult nor an expensive operation, and a faulty or injured graduation should not be allowed to stand.

In the Great Trigonometrical Survey of India the triangulation was executed with position theodolites with circles from 46^{cm} (18-inch) to 91^{cm} (36-inch), some of them having 5 microscopes. They were made by Troughton & Simms, of London. In design they are similar to some that were formerly used in the Coast and Geodetic Survey. With the exception of one 46^{cm} (18-inch) instrument, these theodolites have been discarded by the Coast and Geodetic Survey as being unnecessarily large and heavy. The theodolites that have been used in other geodetic surveys in this country are very similar to those used on the Coast and Geodetic Survey.

Many suggestions have been made to the committee as to improvements on the theodolites of the Survey and for new instruments, and it is recommended that the officers of the Survey submit these suggestions in writing to the Superintendent.

The committee wishes to recommend the purchase as soon as practicable of some 10^{cm} (4-inch) and 18^{cm} (7-inch) theodolites for use in Alaska, and the officers interested should at once submit their views on such instruments in writing.

In the European surveys a variety of instruments has been used. The greater number of them have been universal instruments, with circles of from 25 to 30^{cm} in diameter.

As regards the instruments for astronomic work and the measurement of horizontal angles, the committee considers the present equipment of the Survey as very good. Some of these instruments are among the best of their class at this date, and others, although such as would not be constructed now, are too valuable to be abandoned. With such repairs and modifications as can be made at the office shop they will render excellent service for many years. Some new instruments will, however, be needed from time to time, and their construction should receive careful attention.

In conclusion, the committee begs leave to call the attention of observers to the necessity of bestowing at all times the proper care and protection upon instruments, not only with the view to their safety against injury from accident during transportation but also during use in the field.

While using the highest grade of instruments upon the finest class of work the observer can not be too careful and circumspect in providing a perfectly stable foundation of masonry or iron when practicable, and in protecting them as much as possible against unequal or sudden changes of temperature.

An observer's tent, when a tent is used, of double walls and roofing, will be found a most serviceable and efficient protection against the radiant heat from the sun, direct or reflected. Lamps or candles should never be kept near an instrument at any time without a screen for the interception of radiant heat. It is also recommended that observers in the field should study the erratic movements of the level for the purposes of ascertaining the cause and suggesting a remedy. It is especially important to note the period and extent of the oscillations of the bubble.

EDWIN SMITH, *Chairman.*
C. H. VAN ORDEN, *Secretary.*

REPORT OF COMMITTEE H, ON OFFICE AND FIELD RELATIONS.

It is evident that it is very important and for the best interests of the Survey that the relation between the office and field forces should be as harmonious as possible. In order to effect this much-to-be-desired object your committee thinks it is especially necessary that the "Regulations of 1887," adopted and issued by the then Secretary of the Treasury (with amendments since added), and all circulars that have been issued from time to time by the Superintendent, shall be carefully studied and their provisions faithfully carried out. By this means alone can effective cooperation be secured. To attain this end the following recommendations are made:

1. RECORDS—THEIR PREPARATION, DUPLICATION, AND TRANSMISSION TO THE OFFICE.

Following are articles 31, 32, 33, on page 33 of the Regulations of 1887, relating to records and their transmission to the office; also article 35, relating to transcripts from the records:

31. The original journals of observations and original topographical and hydrographical sheets must in every case be deposited in the office of the Survey at Washington. The journals, records, all field notes, and original data of every description must be kept in the office; and all persons employed in making observations are required to furnish copies thereof to the office at the close of each season's work. Each original topographic or hydrographic sheet must be accompanied by a descriptive report in writing and in duplicate of the locality to which the sheet refers, in accordance with the Superintendent's pamphlet circular of April 11, 1887, entitled "Instructions and Memoranda for Descriptive Reports to Accompany Original Sheets."*

32. All books containing official data, all topographical and hydrographical sheets, with their accompanying descriptive reports, and all other records of field work, both original and in duplicate (when required), must be forwarded to the Superintendent, indorsed with contents of package, and must always be accompanied by a transmitting letter to the same address, stating definitely what is sent, with the necessary explanations, but with no other references. This course in regard to transmitting letters must be pursued with respect to instruments and all other articles sent to or from the office.

33. Every transmitting letter must specify in detail every article sent. Of each book or paper of records the general contents must be stated; of each topographical, hydrographic, or other sheet and accompanying report, its character and limits; of each instrument, its character, general dimensions, and condition; of each box of bottom specimens, the number it contains; and so on, for every item sent. No other matter must be referred to in the transmitting letter.

35. Except to persons employed in the work of the Survey, transcripts from the records or from notes or sketches shall not be communicated without the authority of the Superintendent.

The term "original record" means the record as originally made, and not a fair copy.

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*This circular of April 11, 1887, has been superseded by the Superintendent's circular of July 3, 1890, prescribing one descriptive report.

The duplicate records should be made in the field when practicable, and sent to the office as soon as possible.

No computation whatever should be made in the duplicate records.

Full details of instruments, of observing and recording methods, and all data useful to the computer should be inserted in the preface to all records.

Progress sketches for the use of the computer should be made of the prescribed scale and size, and conformable to facts. (See Superintendent's Circular, February 15, 1888.)

The regulation in regard to the prompt transmission at the close of the season of summary reports, with sketch and statistics, should be enforced.

While observing, any change made in the instrument, voluntarily or otherwise, should be noted in the record under head of "Remarks." The record should be full and as definite as possible, keeping in view that the computer is necessarily ignorant of many details familiar to the observer.

The record should be made complete and according to the form provided, and the names of stations should be written plainly. No recorder should be employed unless he can make a plain record.

Descriptions of triangulation stations should always be in a separate volume and not in the preface of the observations, except in primary triangulation.

All determined points, of whatever character, should, when practicable, be permanently marked and described, as the office is often unable to furnish descriptions called for of well-determined but unoccupied points.

2. FIELD COMPUTATIONS—DEGREE OF ACCURACY REQUIRED.

Computations should be made in the field while observing and before leaving the station, if possible, to make certain that the observations are satisfactory and to insure that no necessary data are omitted in the original record.

No computation should be made by the observers of a greater degree of accuracy than is sufficient for the above purpose.

The observer's abstract should be complete, showing every measured angle of any kind, and should follow the printed forms.

The triangle side computation should be complete, showing all lines determined, and a regular system should be followed; that is, all triangles upon any one point should follow each other consecutively.

The names of stations in the triangle side and position computations should be written from left to right, or in the order of the azimuth.

Angles of the nearest whole seconds and logarithms to five places are sufficient for the computation of tertiary triangulation.

As soon as practicable after the close of the field season the chiefs of parties should turn in to the office their completed computations.

3. ACCOUNTS.

It is believed that by complying strictly with the requirements contained in the Regulations and the directions printed on the different forms of vouchers all friction between the field force and the accounting division will be avoided. This matter will be referred to again in our "Conclusions," at the end of the report.

4. INSTRUMENTS—THEIR SHIPMENT TO AND FROM THE FIELD.

A history or sketch of instruments owned by the Survey should be prepared. A book should be kept for this purpose in the instrument division, in which space could be assigned to each instrument and its present condition, with full details as to the aperture of telescope, focal length, magnifying power, etc., data concerning constants, graduation, etc., and, in fact, every necessary detail should be stated, so that a reference to the instrument by kind and number would be sufficient to identify any fact concerning it in case any observer neglects to give necessary data in his records. Any reasonable change suggested by an observer should be noted in this book, and the action of the instrument board on this suggestion should be recorded. Any change whatever in the instrument made by the instrument division should be carefully entered, and the date on which the change was made should be stated. If this recommendation is carried out it will save much time and labor in the computing division.

A record of the condition of each instrument sent to the field at the time of leaving the office should be kept in the instrument division, so that the cause of any injury in transportation could be more definitely ascertained.

A report of anything objectionable to the observer discovered when the instrument is unpacked in the field should be made to the office, so that the error, if any, may not be repeated.

The regulation requiring a report on the condition of the instrument when it leaves the field should be enforced, so that the cause of injury in transportation from the field can be more definitely ascertained. This report should state in detail the defects and repairs needed.

The constants of some instruments—thermometers, barometers, magnetic instruments, all level vials, tape lines, base apparatus, and instruments of like character—should be sent to the field with the instrument. The observers should be informed of any change in the parts of an instrument, and this information should be made a part of the record.

5. MISCELLANEOUS.

All data furnished by the office for the use of field parties should be returned to the office when no longer necessary to field operations.

In order to make all records more uniform and to secure necessary data which may be omitted, a careful and critical examination should be made by a competent person as soon as the records are received, and a report made to the Superintendent, showing all defects, together with a statement as to whether the work appears good or otherwise.

In the prosecution of the field work, when a station can not be found, a full report should be made, showing in detail the steps taken to recover the station, and stating the officer's opinion as to loss of the station, with the reasons therefor.

A record of lost stations should be prepared and kept in the drawing division, and these stations should be taken off the list of geographical positions available for field work, and data concerning them should not be furnished to anyone who does not specially request it with the knowledge that the station can not be recovered.

When any station is visited, a statement of its condition should be communicated to the Superintendent. Whenever practicable without expense, the chiefs of parties should visit marks established by the Survey and report their condition to the Superintendent. A list of circulars covering the headings considered above is submitted herewith.

Inasmuch as many members of the force are, from the nature of their duties and the isolation of their stations, cut off from ready access to the large number of current publications rich in matter which is of professional interest to them, and an acquaintance with which is essential to their highest usefulness, it is recommended that some competent person be charged with the examination of certain standard publications, of which he shall prepare brief extracts or headlines, with references to the publications in which they appear, and that copies of these abstracts be forwarded from time to time to the different members of the corps.

CIRCULARS.

April 1, 1892. Concerning photographs: Negatives to be regarded as part of the original records.

October 31, 1891. Storage of property.

October 29, 1891. Articles which may be purchased when urgently needed.

March 31, 1891. Remains of aboriginal articles to be reported.

February 27, 1891. Regulations in regard to preparation of records.

August 8, 1890. Selection of geographic names.

July 1, 1890. Allowance of subsistence for Superintendent when visiting parties in the field.

April 3, 1890. Regulations in regard to preparation of records.

March 21, 1889. Treasury Department, No. 30.—Regulations in regard to sending telegrams.

March 13, 1888. Directions for the survey, condemnation, appraisement, and sale of Coast Survey property.

July 12, 1888. Preservation of triangulation points.

June 18, 1888. Statistics of field work.

February 15, 1888. Regulating progress sketches.

September 8, 1887. Treasury Department, No. 101.—Shipment of freight and payment of transportation over land-grant railroads.
September 3, 1887. Inking topographic sheets.
March 28, 1887. Requiring inventories to be rendered.
January 17, 1887. Concerning shipment of property to the office.
September 13, 1886. Proper manner of communicating with the office.
March 26, 1886. Executive order of the President.—Requiring bonds of Assistants in order to secure advances.
July 15, 1884. Treasury Department, No. 108.—Indorsement and payment of Treasury drafts.
August 16, 1886. Concerning accounts.
August 11, 1886. " "
June 21, 1886. " "
June 16, 1886. " "
June 10, 1886. " "
May 29, 1886. " "
May 1, 1886. Transportation on bonded railroads.
April 1, 1886. Treasury Department, No. 36.—Preparation and rendition of accounts.
April 3, 1886. Ink for plane-table sheets.
March 19, 1886. Transfer of property.

In conclusion, the committee desires to express its appreciation of the letter from Mr. E. H. Fowler, draftsman, dated January 11, 1894, referred to it by the Conference, and of the letter of Mr. J. B. Boutelle, computer, dated January 8, 1894, presented by Assistant Schott.

The committee recommends the preparation of a manual of observations, records, and computations which shall embody the conclusions of the Conference upon these points, and a manual of accounts, in accordance with the valuable suggestions contained in the paper (appended to this report) of the disbursing agent of the Coast and Geodetic Survey, which the committee indorses.

GEORGE A. FAIRFIELD, *Chairman.*
ISAAC WINSTON, *Secretary.*

CORRELATION OF THE OPERATING DEPARTMENT AND ACCOUNTING SYSTEM OF THE COAST AND GEODETIC SURVEY.

The reciprocal relations existing between the barren, uninviting, and verbose details of an accounting system, and the researches, deliberations, and results of a scientific commission, such as the Geodetic Conference now assembled in Washington, are not at a first glance apparent. The two elements, if they may be so termed, in fact appear to be widely at variance. No connection is immediately perceptible, yet they are closely allied, almost if not absolutely inseparable, and to a greater degree than is at once conceivable. For the purposes of this paper it is necessary at the outset to define the union between the two and to show their correlation. To do this understandingly it must be assumed that the Conference, as a whole, is representative of the *work* of the Survey, and that the accounting system is the embodiment of the *productive means* by which the work is accomplished.

It will probably be conceded that all enterprises or undertakings, no matter upon what scale they may have originally been planned, under every condition obtaining in life, require for their prosecution the providing of certain means or assistance whereby the work to be accomplished may be brought to a successful termination, or, as is sometimes the case, its impracticability completely demonstrated. The work of the Survey, therefore, as represented by the Geodetic Conference, may for the purposes of illustration be deemed an undertaking in the furtherance of which grants of money are made by Congress from time to time for its prosecution, while, on the other hand, the accounting system may be said to represent the vehicle through which the means or assistance so provided becomes available for the purposes of the work. The connection, therefore, between the operative department of the Survey and its accounting system would appear from the foregoing to be not only correlative but in fact inseparable, inasmuch as the existence of the former depends under prior legislative enactment upon the sustenance provided for its maintenance through the instrumentality of the latter.

It may be permissible to say that doubtless no point is likely to arise during the progress of the deliberations of this assemblage which will convey the impression of a closer or more essential connection as a natural result of the labors of the Conference than that of a comprehensive application and clear understanding of the laws, rules, regulations, and requirements governing the disbursement of public funds in their relation to the prosecution of the work of the Coast and Geodetic Survey, and properly and intelligently accounting for the moneys so expended. This statement is made with the more freedom, inasmuch as it appears to be a well-established fact that the *work* of the Survey in its execution, its correctness, and its methods has never been successfully assailed or impeached, while its disbursements, on the other hand, have been the subject of almost continual criticism and animadversion from the days of the first Superintendent down to the present writing.

The special and distinctive characteristics of the work of the Coast and Geodetic Survey, both in the field and office, compared with what may be called—there being few exceptions—the ordinary plain business transactions of other Bureaus of the Government, manifest themselves continually and persistently in the varied, and in many instances novel, character of its expenditures. This is naturally the case, and can not be avoided in a great governmental bureau which is notably engaged in the investigation and application of the most refined methods of science with a view to the betterment and amelioration of the conditions surrounding mankind, the diffusion of general knowledge, and the protection and development, by means of its perfect surveys of navigable waters, of the commerce of a great and growing country. But these commendable and praiseworthy features are frequently lost to view in the maelstrom of criticism and censure which has at various times in the history of the Survey followed a so-called analysis of the expenditures made for the work. To the average accounting clerk, possessed of little or no knowledge of the operations of the Survey, with acquired predilections for hypercriticism, reeinforced doubtless in some instances by superior official insistence, the accounts of the Bureau appear to present unusual opportunities for the promulgation of dogmatic opinions as to the propriety and necessity for particular items of expenditure, the usefulness and import of which are as unknown to him as would be the application of the most abstruse scientific problem. The result of such criticism is an almost continual correspondence between the officials of the Bureau and the reviewing officers of the Department with reference to the allowance of disputed items; and this condition has obtained, to a more or less extent, with the accounts of every disbursing officer from the time of Capt. W. H. Swift, Corps of Engineers, the first disbursing agent, to the present. The remedy, in part, is permanency in the tenure of office of the accounting clerks and reviewing officials. Each man becomes gradually educated up to the needs of the service and in time comprehends and appreciates the system of accounting. A change in the office, however, and the schoolmaster's work begins anew. It is this feature which is largely responsible for the main body of criticism to which our accounts have been subjected. The Survey finds no fault with the expression of a fair, honest difference of opinion as to the propriety of any particular item of expenditure. Errors of judgment are to be expected; but when discovered should be discussed and criticised in a spirit of fairness and not with a view to casting suspicion upon individual integrity.

The foregoing remarks naturally lead up to a point which seems to present the desirability of a still further advance in that degree of knowledge and familiarity with the laws relating to the disbursement of public funds which is so essential a requirement in the avoidance of adverse criticism and in securing a prompt and accurate audit of our accounts. With this object in view, and taking advantage of the assemblage of so many officers in attendance at the Conference, the Superintendent of the Survey has suggested the preparation of a paper in which reference should be made to the technicalities necessary to be observed, with a brief analysis in each case of their necessity and import. The subject is a large one and can not readily be confined within the limits of a brief paper. An effort will be made, however, to define the more important points and to show their relation to the system of accountability now in force.

Many of the requirements now obtaining in the accounting system of the Survey are a natural outgrowth of the provisions and terms of the "Plan of Reorganization of 1843." Subsequent legislation has from time to time added others, various decisions of the accounting officers of the Treasury have contributed their quota, and others again have arisen through the executive action of the different Superintendents. Regulations and directions almost innumerable in number, embracing all these various sources of authority, have been issued and promulgated as occasion demanded. It may readily be seen, therefore, that the field officer of the Coast and Geodetic Survey charged with the responsibility of making disbursements of public funds must, in addition to his other qualifications, become a walking digest of law, regulations, and decisions, if he desires to avoid criticism and individual financial responsibility in the audit and settlement of his accounts. For the purposes of this paper, a reference to the latest edition of the Regulations (1887) will be first in order. As the enacting clauses of the appropriation acts of recent years have invariably contained the provision that the appropriations therein made were to be expended in accordance with the regulations adopted from time to time by the Secretary of the Treasury, it would seem that such regulations, when so adopted, possess all the force of statutory law.

To quote from paragraph 3: "The Superintendent shall direct and superintend the work in general, and be responsible * * * for the proper and economical expenditure of the appropriations made therefor." There is food for much thought in the terms of this regulation. It is clear that the Superintendent is directly responsible for the correctness of the disbursements.

The chief of the party, therefore, in the matter of his expenditures, is to be considered as being primarily responsible to the Superintendent, although the provision contained in paragraph 46, which reads, "Chiefs of parties shall be held responsible for the expenditures of their respective parties," would seem to require modification to meet this condition. The interpolation of the words "to the Superintendent" after the word "responsible" would properly accentuate the responsibility here referred to. Responsibility should be centralized and not scattered. Divided responsibility is unsafe and should not be tolerated. Much more could be said in this connection, but the point involved is obvious.

Paragraph 30 of the Regulations provides, in brief, that all moneys received from the sales of old property, etc., shall be paid to the Assistant in charge of the office and topography. In many cases this is not done, the moneys being deposited in some subtreasury or other governmental depository or forwarded direct to the Superintendent. The regulation is mandatory and admits of no discretion. It is another case of divided responsibility; but with this we have nothing to do. Such moneys should be forwarded as directed. This may be done by means of check, money order, or by making a transfer through the instrumentality of the disbursing agent.

Paragraph 45 refers to estimates and allotments, and provides that no expenses of any description, except for the purpose of saving life or property or in other sudden emergency, shall be incurred before estimates have been submitted to the Superintendent and approved by him. The saving clause here is composed of the words "or in other sudden emergency," of which emergency the Superintendent, in his executive capacity, is the judge. Were it not for this clause all expenditures of every description would necessarily have to be first estimated for and an allotment made to cover them before they could be incurred. In this connection chiefs of parties should be careful never to deviate, in the matter of rates of compensation or per diem allowances for subsistence, from the terms of their approved estimates. There is some leeway, so to speak, in the matter of items for contingent expenses, but in those specifically referred to close adherence must be had to the terms of the estimates. If any departure is made therefrom, it should be the subject of an explanatory letter. In preparing estimates the printed directions on the face should be followed. These are plain and require no further explanation in this place.

Paragraph 47 requires chiefs of parties to transmit with their accounts a list of all articles that may be purchased for public use. This requirement is frequently overlooked, and to save time in the adjustment of the accounts the lists are often made out in the disbursing office. This provision of law admits of no discretion. Moreover, the lists of purchases, when made out by the chief of party, afford him an opportunity of stating what disposition, if any, has been made of the property purchased. It is sometimes at once expended, and this statement made on the list avoids possible trouble in the settlement of inventory accounts, which would not be the case when the lists are made out in the disbursing office.

Paragraphs 58, 59, and 60 refer to allowances for commutation and to expenses for actual subsistence. These two conditions should not be misinterpreted. They are, however, frequently misapplied. Commutation of subsistence can never be allowed while traveling, except in cases of field duty involving travel with brief stoppages. Only the actual cost of subsistence can be allowed while traveling under the usual conditions obtaining in the Survey. The error is frequently made of charging commutation or per diem while so traveling. Under the regulation this is not permissible and merely serves to invite disallowance or suspension of the amounts so charged. The amounts paid for actual subsistence when traveling should be so charged, and if for expenses at a hotel or other lodging place, should be supported by receipt. When meals are procured at restaurants or on dining cars while traveling, the prices paid should be charged in detail on the traveling voucher, and receipts may be dispensed with when the account is duly sworn to. The regulation, however, requires that all items not supported by receipts should be accompanied by a statement showing the impracticability of obtaining such receipts. The hurry and confusion incident to travel is a necessary accompaniment of such impracticability.

The conditions governing the various allowances for commutation of subsistence are so clearly and plainly stated in the regulations that further explanation in this place would seem to be unnecessary. The entire question is one that is optional with the Superintendent as to the amount allowable within the maximum sum fixed by the appropriation act.

The foregoing paragraphs referring to the Regulations cover those points in which chiefs of parties are most frequently at fault.

The vouchers used by the Survey in its system of accounting will now be taken up for description. These forms carry with them all necessary printed directions for their proper preparation. A careful consideration of the instructions contained in the directions when filling out vouchers would avoid at times much unnecessary labor in the disbursing office in the examination and audit of accounts. Many of the paragraphs of these directions are self-explanatory and need no further reference to them in this place. The purpose and intent of a few of the leading paragraphs will, however, be given in order to show the bearing which they have upon the final audit of the accounts at the Treasury Department. Paragraph 5 of the Directions, Form 2—General Voucher—Field, reads as follows: "Always state specifically the purpose of every expenditure. If for services, the capacity in which employed and the work upon which engaged; if for articles, state use for which intended." The last sentence is particularly applicable to the suspension of an item in a statement of differences recently received from the accounting officers of the Treasury, which reads as follows:

"The following items for ice are suspended for explanation as to necessity for purchase of ice in such large quantities," the total being $101.25. The ice accounted for here was purchased for the purpose of tempering the base bars of the apparatus used in the measurement of the Holton Base line. A mere statement to that effect upon the face of the voucher would doubtless have been sufficient to satisfy the accounting clerk of the Treasury that the article purchased was not intended for the gratification of individual tastes or other indulgence, but was used solely for public purposes. Again, in the measurement of the Yolo Base line, the vouchers covering expenditures for brick and cement and for the hire of bricklayers, expenditures necessary to the erection of the piers at the ends of the line, were returned to the office for the specific approval and administrative scrutiny of the Superintendent upon the ground that the disbursement was of an extraordinary character and required full explanation and the highest executive sanction before it could be passed. As before, a simple statement upon the vouchers of the purport of the expenditure would probably have been satisfactory, and unnecessary correspondence would have been avoided. These two illustrations would seem to clearly indicate the propriety of stating specifically, at least in the case of items somewhat outside the usual routine, "the purpose of the expenditure."

Paragraph 4 of the Directions, Form 2, requires that all items on the personal vouchers of chiefs of parties or their subordinates should be supported by receipts whenever practicable to obtain them; and when not practicable to obtain the receipts, to so state and give the reason therefor. This paragraph is not observed to the extent that it should be in order to satisfy the requirements of the accounting officers, as the following suspensions in our accounts will indicate:

"The following items for lumber, freight, cartage, laths, nails, wire, tools, oil can, oil, eyebolts, damages, oak plank, services, luncheons, post-office registration fee, etc., are suspended for subvouchers (receipts) needed to perfect," the total amounting to about $17, and the items of expenditure ranging from 10 cents to about 75 cents or $1. Individually, the sums involved are very small; in the aggregate their amount is considerable in the adjustment of an account for a full year's expenditures. In such cases, after so long a lapse of time, it is hardly possible that the necessary receipts can be obtained by the various chiefs of parties, and hence recourse must be had to an appeal for equity and to any other prevailing feature which may aid in securing the passage of the disputed item. The point might be raised in this connection that such items, unsupported by receipts, should not be allowed to pass the disbursing office. But it may be stated here in general terms that, apart from the labor involved in striking out such items, the office has always attached weight to the certificate of a chief of party "that the account was correct and just," and hence was not disposed to be critical in relation to the small sums here alluded to. It would seem to be the easiest course, however, for all concerned, to obtain receipts for every item, however small, in support of charges on personal vouchers, whenever possible to procure them, and when not so possible, to briefly state why. The only exception which may be made to this rule, if exception it may be called, is in the case of vouchers for traveling expenses, which are usually sworn to before competent legal authority. The irksomeness of this requirement as to subreceipts is patent to all; but as it is based upon the direct action of the accounting officials of the Treasury, as indicated by the foregoing citations, it is not clear that remonstrance would result in relief.

The three forms of vouchers most commonly in use by chiefs of parties are: "Form 2—General Voucher—Field," "Form 3—Transportation Voucher," and "Form 5—Abstract of Expenditures." For the purpose of bringing before the Conference, as a matter of record and for its official action, if any is deemed necessary, the particular features of the printed directions heretofore alluded to, they are here incorporated in regular order, as follows:

FORM 2—GENERAL VOUCHER—FIELD.

DIRECTIONS.

1. Signatures of firms or individuals signed "per" will not be passed.

2. Whenever practicable, have vouchers made out by the persons signing them.

3. Assign numbers to subvouchers or receipts, and refer to them in the voucher by number.

4. When used as a personal voucher by chiefs of parties or their subordinates for other than their own services and subsistence, support all items by subvouchers or receipts whenever practicable to obtain them. When not practicable, so state and give the reason therefor.

5. Always state specifically the purpose of every expenditure. If for services, the capacity in which employed and the work upon which engaged. If for articles, state use for which intended.

6. To avoid a multiplicity of vouchers it is suggested that in numerous instances receipted bills could be obtained and the amounts thereof charged upon the personal voucher of the chief of party, referring to the bill by number, as heretofore stated.

7. All subvouchers or receipts should, if possible, be signed in ink. An extra effort should be made to this end.

8. Vouchers for commutation of subsistence must be rendered in strict conformity with the Regulations.

9. Make all explanations in writing upon the face of the vouchers.

10. Signatures by "X" are only to be made by persons unable to write their own names. When made by "X" they must be witnessed. This applies to subvouchers and receipts as well as to regular forms of vouchers.

11. Vouchers for the purchase of instruments, for repairs of instruments, or for ink and mucilage must be rendered in strict conformity to the Superintendent's circular of October 29, 1891. Personal vouchers of chiefs of parties should be rendered for such expenditures, the items being supported by the receipted bills.

12. All vouchers must be itemized as completely as possible. Charges in "lump" sums will not be allowed.

13. Charges for telegrams can only be allowed at Government rates, as per Department circulars issued from time to time. The number of words and names of places from and to which sent must be given on the voucher, or copies of the telegrams must be furnished.

14. The provisions of paragraph 53 of the Regulations, in relation to accounts, must be strictly adhered to by all chiefs of parties to avoid suspensions or disallowances in their vouchers.

15. All calculations for parts of a month must be made according to the number of days of which the month consists.

16. In vouchers for hauling and moving equipments and materials give the number of loads and distance from place to place.

17. Per diem employees or hands can not be paid salaries for Sundays unless service is actually rendered on that day. When service is so rendered by per diem employees (and charged for), a certificate to that effect must be written on the face of the voucher.

18. All vouchers must bear date in the column on the left.

19. The price per unit of weight or measure must be stated in all cases whenever practicable.

20. In cases where damages for opening views, etc., are paid for, either support the charge by a written agreement (with the person claiming the damages), stating the nature and extent of damages, and his acceptance of a stated sum as a full relief

to the Government, or give in detail, on the face of the voucher, the full particulars concerning the account and the circumstances which demanded the expenditure.

21. When rendered as a subvoucher in an abstract, the briefing on the back of this form must invariably be filled out.

22. Hereafter secure invoices and bills from firms and individuals furnishing supplies, etc., or rendering services (not personal services), in addition to the regular form of voucher, and attach them to their appropriate vouchers before transmitting the accounts to the disbursing agent.

Form 3—Transportation Voucher.

DIRECTIONS.

1. Copies or extracts from letters of instructions must be written on separate sheets of paper and attached to the vouchers. Be careful to give a full copy or extract, as the case may be.

2. Give full name of railroad or steamboat company furnishing the transportation. When transported by other conveyances, so state.

3. The fare actually expended over each route must be charged. Do not "lump" the expenditures.

4. Charges for transportation over bonded and land-grant railroads will be disallowed.

5. Support all charges for meals and transportation of baggage by subvouchers or receipts whenever practicable to obtain them. When not practicable so state and give the reason therefor.

6. Assign numbers to subvouchers or receipts, and refer to them in the voucher by number.

7. Only actual expenses of board and lodging are allowed while traveling. Secretary's circular of August 24, 1886, governs the present rates allowed for field officers.

8. Subvouchers or receipts must be furnished for all hotel expenses and carriage hire. Hotel bills must clearly show the time of beginning and ending of the service charged for.

9. Expenditures for local field transportation must state specifically the purpose for which each item of expenditure was made, and must be confined strictly to the immediate locality of field work. When the distance traveled is over 50 miles from locality of work, full explanation of the necessity therefor must be made.

10. In organizing or disbanding a party, the members thereof can not be furnished transportation for a greater distance than 50 miles without previous special authority from the Superintendent.

11. When transportation expenses of employees of a party are charged in the personal voucher of the chief of party (and this course is recommended as avoiding confusion), give their names and furnish their acknowledgments, to be attached to the voucher, that they have received the transportation charged for.

12. When employees of a party are traveling alone upon special duty, away from the main party (within the limit heretofore stated), they must render to the chief of party vouchers in their own names, duly sworn to by them, *and supported by written orders* from the chief of party. In these orders the Superintendent's instructions must be quoted and the extract certified to *by the chief of party*.

13. When charges for actual expenses while traveling and for commutation of subsistence become interwoven upon any particular day an adjustment must be effected upon the basis of four parts to a day—breakfast, dinner, supper, and lodging.

14. Actual expenses of board and lodging will not be allowed while traveling locally in the routine of field work to those receiving commuted or regular rates of subsistence incidental to field operations.

15. Traveling expenses between home and the office or suboffices not allowable.

16. The approval certificate of the Superintendent on the face of the voucher is intended to be filled out by him only when the voucher is rendered separately and not included in an abstract.

17. All traveling-expense accounts must be sworn to before competent legal authority.

FORM 5—ABSTRACT OF EXPENDITURES.

DIRECTIONS.

1. In entering vouchers upon the abstract, arrange them in alphabetical order regardless of dates, writing the surname first in the column headed "To whom paid."

2. When working under two or more appropriations, render separate abstracts for each.

3. When vouchers are suspended and returned for correction, always transmit supplemental abstracts (in duplicate), dated the same as the originals, to cover the suspended vouchers.

4. Monthly abstracts must be given date of the last day of the month in the receipts. When rendered for portions of a month, the latest date in the subvouchers may be used.

5. Vouchers for expenditures made in one month must not be included in the abstract for another month. Render a separate abstract for such vouchers, and attach thereto a written explanation of the delay in transmitting the account.

6. Be careful to observe that the abstracts are signed before transmitting the accounts to the disbursing clerk. Abstracts must be signed by chiefs of parties in their official capacity.

7. The blank space in the center of the face of the abstract must not be written over by chiefs of parties, nor must the briefing on the back of the abstracts be filled out by them.

8. The dates of the vouchers must be inserted in the abstract, notwithstanding that they are arranged alphabetically.

9. All vouchers embraced within abstracts must be briefed on the back and numbered, beginning with the number 1 (one) in each abstract.

10. Separate abstracts must be rendered for "party expenses" and "repairs of vessels."

It has been the custom of the disbursing office from time to time, as points arose in the adjustment of accounts or new decisions thereon were made by the accounting officials, to change the terms of these directions so as to have them correspond to the new conditions. Such changes are made usually when a new edition of any particular form is required for issue.

A few additional suggestions or comments may not be out of place in this connection, in view of their bearing upon individual responsibility and the saving of much unnecessary loss of time and useless labor.

Whenever practicable, it is recommended that chiefs of parties deposit the moneys advanced to them for party and other expenses in some regularly designated governmental depository, if too remote to open an account with the Treasurer or a subtreasury. A list of designated depositories for public moneys among the national banks located in every State and Territory in the Union will be furnished by the disbursing office upon application. The particular advantage of this plan is that it will save chiefs of parties much trouble in their dealings with incorporated or unincorporated companies in the matter of obtaining certificates of authority for officers to sign for the companies. Such accounts, when paid by a check drawn on the Treasurer, Assistant Treasurer, or other depository of the United States, in the name of the corporation as payee, stating such fact on the face of the voucher and giving the number of the check, will not require evidence of authority for signature to be filed with them. One of the advantages of keeping moneys advanced on deposit with a governmental depository is thus made apparent. Another is that it affords a complete record of all disbursements, as the checks drawn on a governmental depository, when presented and paid, are not returned to the officer issuing them, but are permanently retained in the files of the depository. It is, of course, understood that in many cases this course of making deposits can not be adopted, owing to the remote and isolated locations of the field work. Under such conditions the chief of party accepts the full responsibility under his bond for the safe-keeping of the public funds intrusted to him.

In the case of original checks lost, stolen, or destroyed, the Revised Statutes prescribe that a duplicate may be issued after the expiration of six months from the date of issue of the original check, but only under such regulations as may be adopted by the Secretary of the Treasury. The depository upon which drawn should be at once notified of the loss of a check, and request made that payment thereof be stopped; after which, chiefs of parties should make application to the Superintendent for blank forms and the necessary instructions as to the method of procedure to be followed in securing a duplicate of the lost check.

In the rendition of accounts to the disbursing agent for settlement, chiefs of parties should state the balances, on account of moneys advanced, due that officer, or, if there is no such balance due, they should state the amount which may be due them upon settlement. In other words, each chief of party when rendering his monthly accounts should transmit with them an account current, transcribed from his books, showing his financial status with the Survey based upon the moneys advanced him and the amount of his accounts as rendered. Form 10, letter transmitting accounts, will be found by chiefs of parties a convenient means of complying with the routine here suggested.

When forwarding moneys as an advance to chiefs of parties, or when making deposits to their official credit at any governmental depository, the disbursing agent should, in his transmitting letters and notifications of such action, invariably state the amount due him on account of advances, at the date thereof, according to his books; and chiefs of parties should be careful to respond to such statements and acknowledge or correct the amounts therein stated. Such acknowledgments are of great value to the disbursing agent in the event of an examination of his office by Treasury officials. They are, moreover, an excellent check in the event of errors occurring in the statement of balances.

Correctness in entries, extensions and additions of all abstracts and other vouchers, will invariably insure a more speedy settlement of the accounts.

If the observations made herein in relation to the disbursement of public moneys and the methods of accounting for the same prove of service, the purpose of this paper will have been attained.

JOHN W. PARSONS,
Disbursing Agent.

REPORT OF COMMITTEE I, ON THE MEASUREMENT OF ARCS.

The measurements of arcs of the earth's surface are indispensable for the determination of a geometrical figure which in shape and size should approximate most closely to the figure of the earth as a whole. Such measures have been undertaken, either expressly or only indirectly, in connection with surveys made by leading nations within the boundaries of their countries; and in the latter case either the general figure or one more closely fitting the actual region was made use of for the development of the triangulation on the limited part of the surface.

It is otherwise when large areas, such as the surface of North America, or even that covered by the territory of the United States, are concerned. Here the importance of the measurement of arcs becomes apparent in order to furnish the shape and dimensions of that geometric osculatory figure which best represents the particular surface in question and upon which it is desired to develop the triangulation, which latter is the foundation for exact measures of relative geographic position on the earth's surface.

Until the representative or special figure can be determined, and as a means of furnishing the needed material for its elucidation, the work of the triangulation, constituting the basis of a survey, can (as in the case of the Coast and Geodetic Survey of the United States) be prosecuted by making use of a spheroid fairly approximating to the earth's figure. The resulting positions can and do have a satisfactory degree of accuracy. Hence it will be seen that the measurement of arcs may be regarded as in a measure incidental to the operations of a trigonometrical survey of an extended country.

In this way originated the several arcs developed by the Coast and Geodetic Survey up to this time. Other prospective arcs, more or less desirable or feasible, will be pointed out further on.

A further reason why this Conference has taken into consideration the matter of arc measures is the circumstance that by joining the International Geodetic Association for the measurement of arcs*—in other words, for the determination of the earth's figure—the United States have incurred certain scientific obligations which demand attention from this Survey.

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* See act of Congress of February 5, 1889.

For convenience and distinction arcs may be classified as arcs of the meridian, of the parallel, and oblique arcs. Of these the first and second are now, since the introduction of the telegraphic method of measuring differences of longitude, of nearly equal importance as far as obtainable accuracy is concerned. The third class may be regarded, theoretically at least, as composed of part of the first and part of the second kind.

The value of an arc depends mainly on its extent, position (with respect to latitude), accuracy of measure, and number of subdivisions. The more numerous the latter the greater the chance that the local deflections of the vertical may be neutralized and the results be freed from a source of error which up to this time has been the main cause of discord subsisting between the several arc measures as well as between their several parts when combined for a resulting geometric figure.

This paper has no space nor is there the least necessity for going into the history of arc measures, or even to refer to the great measures accomplished by England, France, and Russia, and our remarks will be confined to the subject-matter in relation to the Coast and Geodetic Survey.

A few arcs have so far been measured in this country, but none have been completed, or, at least, all are capable of extension. Appendix No. 6 of the Coast Survey Report for 1877 contains an account of two, both of the meridian: The Nantucket arc, 3°·37 in length, with 6 subdivisions; the Pamlico-Chesapeake arc, 4°·52 in length, with 13 subdivisions; and in order to obtain some information as to the curvature of the surface in these parts these were combined with the Peruvian arc, 3°·12 in extent and without subdivisions. No specific difference in the curvature of the Western Hemisphere from that of the Eastern was indicated, but a change from the use of the Besselian spheroid of revolution (of 1841) to that of Clarke (of 1866) was indicated by the above combination as desirable. The substitution of the latter figure for the development of our triangulation was approved by the Superintendent on February 4, 1880. No publication has yet been made of the oblique arc extending along the North Atlantic coast from the Canadian boundary to the Gulf of Mexico. The length of the completed part between Eastport, Me., and Montgomery, Ala., is nearly $21\frac{1}{4}$°, or about 2 252 kilometres. It will, when extended to the Gulf, reach $22\frac{1}{2}$°, and possibly it may be extended northeasterly through the Dominion of Canada to Cape Breton Island. A preliminary computation of part of this arc also favored the change of the spheriod of reference mentioned above. In volume No. 24 of Professional Papers by the United States Engineers* the United States Lake Survey gives the results of the measure of two arcs, one a meridional one, between

*Primary Triangulation, U. S. Lake Survey, Washington, D. C., 1882.

St. Ignace, on the northern shore of Lake Superior, and Parkersburg, Ind. Its length is 10°·21, with 9 subdivisions. The other arc is considerably inclined to the parallel and extends over 11°·79 of longitude, from Willow Spring, near Chicago, Ill., to Mannsville, at the east end of Lake Ontario, New York. It is composed of 3 sub-arcs. The meridional arc will in time be extended to the Gulf of Mexico.

The old arc of 1764 in Maryland and Delaware, known as Mason and Dixon's Line, here deserves but a passing mention as the first arc measured in North America. It was 1°·48 in length, and was measured with wooden rods throughout.*

The desirability and necessity of a geodetic connection of the triangulations of the Atlantic and Pacific coasts were first pointed out in 1870 by the then Superintendent of the Survey, Prof. Benjamin Peirce. (See Coast Survey Report for 1870, p. 4.) In the following year the measurement of the parallel of 39° between Cape May, New Jersey, and Point Arena, California, was commenced, and at the time of writing (January, 1894) but a small gap of triangulation west of Pikes Peak, Colorado, remains to be filled up. The total length of this arc of the parallel is 48°·78 of longitude, or about 4 226 kilometres (very nearly 2626 statute miles). It is subdivided at present into 19 subarcs, for all of which the telegraphic longitude work is completed. Interspersed in this triangulation are a great number of astronomical latitude and azimuth stations; and a series of base lines, some yet to be measured, sustain the accuracy of the linear dimensions.

For the measure of the earth's curvature at right angles to the above parallel an arc of the meridian in about longitude 98° west of Greenwich has been proposed. Of this central arc 22°·92, or 2 544 kilometres, will be within the boundaries of the United States, between the Rio Grande and the northern boundary. It is capable of extension southward 10°, through Mexico to the Pacific Ocean at Point Sacrificios, and northward through Canada and the British Possessions to an unknown distance. An arc of the meridian in longitude 112° may be suggested; of this, 3° have already been measured in the vicinity of Salt Lake, Utah.

On the western coast an arc of the meridian can be established in longitude 120½°, from the Santa Barbara Channel to the northern boundary of the United States, a distance of about 15°. That part of the boundary of California and Nevada oblique to the meridian would lend itself well to an inclined arc about 5¼° in length, with a capacity of extension to 15° within our boundary.

Besides the central arc of the parallel already referred to, two other arcs of the parallel of great longitudinal extent have been projected, viz, the northern one, in latitude 46°, and the southern one, in latitude 33°. (See accompanying map, illustration No. 14, on which the several arcs are located.)

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* Phil. Trans. Roy. Soc. for 1768.

Arcs, measured and projected within the limits of the United States
or close to its border

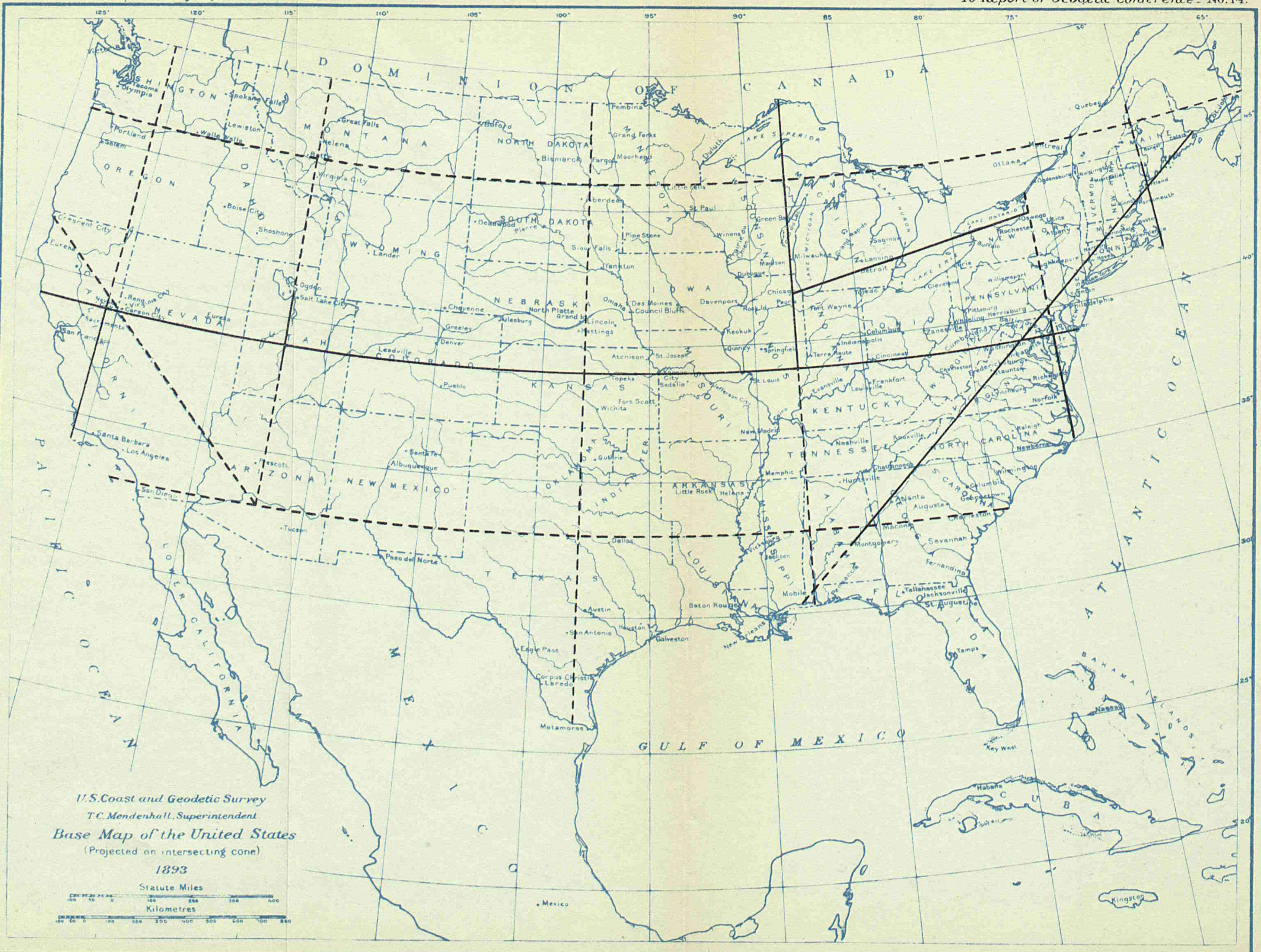

Photo-Lith by A. HOEN & CO., Balto., Md.

The desirability of a remeasurement and extension of the arc of Peru (1735–1743) was pointed out in 1877,* and again in 1889.† Fortunately for the progress of geodesy, this arc appears to be nearly correct; otherwise, by reason of the possible effect of local deflections of the vertical at its two terminal latitude stations, it might have exercised a retarding influence.

Researches respecting the earth's figure made since the publication of Captain Clarke's Dimensions of the Spheroid (London, 1866)‡, which were adopted by this Survey, give smaller values than $\frac{1}{294{\cdot}97}$ for the compression $\frac{a-c}{a}$.

Dr. Helmert, in 1887, gives for his reference spheroid $\frac{1}{299{\cdot}15}$. F. Tisserand, in his Traité de Méchanique Céleste, Tome II, Paris, 1891, shows that under certain plausible hypotheses as to increase of density with depths and as to original fluidity the flattening can not exceed $\frac{1}{297{\cdot}3}$, and Prof. W. Harkness in his essay, "The solar parallax and its related constants," Washington, D. C., 1891, arrives at the result $\frac{1}{300{\cdot}205 \pm 2{\cdot}964}$, thus approximating again to the values assigned in earlier times by Airy (1830) and Bessel (1841), about $\frac{1}{299{\cdot}2}$.

The Conference recommends that the spheroid adopted by the Survey be adhered to as being sufficiently close to any other value that could now be assigned or is likely to be assigned in the near future, and that the subject of the arc measures be kept in prominent view in connection with the progress of the Survey; also that all linear measures be expressed in terms of the prototype metre and that a direct comparison be made of the lengths of the committee metre and the national prototype.

Respectfully submitted to the Geodetic Conference.

CHAS. A. SCHOTT, *Chairman.*
G. R. PUTNAM, *Secretary.*

JANUARY 23, 1894.

REPORT OF COMMITTEE J ON MAGNETICS.

A study of the laws governing the various magnetic forces is naturally connected with the work of the Coast and Geodetic Survey, and a thorough knowledge of them is indispensable in order—

1. To supply its charts with information of the magnetic variations at the dates of issue, together with the prospective annual change.
2. Incidentally to facilitate the proper adjustment of the compass

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* Coast Survey Report for 1877, p. 95.
† Coast and Geodetic Survey Report for 1889, Appendix No. 7.
‡ Viz: $a = 6378206{\cdot}4^m$, $c = 6356583{\cdot}8$ } which we now take as expressed in terms of the International Metre.

in ascertaining local deviation on board ship, for heeling, and different positions of the vessel.

For these and other purposes it is necessary to study the laws of terrestrial magnetism as well as determine absolutely its several forms or components of declination, dip, and intensity. The distribution of this force is dependent not alone upon the time and the geographical position, but is influenced by many local disturbances; hence the imperative necessity, in order to supply our charts with compass bearings, that the study of magnetism should cover at least the entire seacoast of the United States; and in order to produce the lines defining the direction and intensity of these forces to the required limit seaward for a certain epoch there arises the further need of extending the observations a sufficient distance inland.

3. To meet the constant demand made upon the Survey by surveyors, engineers, and courts of law in every part of the country for information, generally for the recovery of old lines or landmarks. And for this purpose a further and more complete study of these forces, covering the *entire area* of the country, is demanded.

4. To meet the necessity for an accurate knowledge of the dip and intensity of the magnetic force arising from the researches of science and the practical application of them by electricians in the measurement of the closely allied forces of electricity.

To meet these demands it is obviously important to construct from time to time (say once each ten years) isomagnetic charts to represent the then existing state of distribution of this still mysterious force.

The Survey has already—

1. Made direct observations of declination, dip, and horizontal force in many widely distributed places.

2. It has carefully collected observations from all available sources whatever, from the earliest to the present time.

3. It has made a special study of the laws of terrestrial magnetism by means of photographic registration at especially selected places, to be changed after about seven years of continuous occupation to a new place (so as to cover at least more than one-half of the sun-spot period), and always placed in localities most remote from those where the best magnetic observations had previously been made. Except the occasional changing of stations for photographic registration, this is in conformity with the practice in other countries.

4. It has afforded assistance to magnetic surveys undertaken by States or private individuals by the loan of instruments and in other ways.

5. It has endeavored to elucidate the multiplicity of laws governing these forces and to disseminate them for general information in the publications of the Survey.

METHODS AND INSTRUMENTS OF THE COAST AND GEODETIC SURVEY AS COMPARED WITH THOSE OF OTHER COUNTRIES.

Since the time of the great physicist Gilbert of Colchester, who showed the earth to be a great magnet, about the year 1600, magnetic theories and graphical results have been diligently worked out; and as time has passed these efforts have been more minute and complete.

Magnetic observations have been systematically carried on in Great Britain, France, Austria, Germany, Russia, Japan, India, Australia, Mexico, Canada, the United States, and other countries. They also formed part of the programme of every scientifically organized Arctic expedition; and the United States, in the years 1881-1884, assumed the responsibility of occupying two of the stations of the cordon which, under international auspices, girdled the pole for the purpose of studying, among other points of physical interest, the laws of magnetism in that important region.

In this country, in addition to the work of the Coast and Geodetic Survey, New Jersey has been surveyed magnetically by the State, and the State of Missouri by individual enterprise. A report on the last has been published by Prof. F. E. Nipher.

The instruments employed in these surveys are essentially similar in principle, although differing in the detail of construction as well as in size and weight.

There is no means of deciding as to the relative values of the results obtained in different countries, and any comparison must deal with methods and instruments only.

A description of some of the instruments used in other countries may be interesting, and for this purpose some of the instruments used in Russia, England, Italy, and France have been selected. All are of recent date.

The Russian Universal instrument is described by Dr. Wild in the publications of the Imperial Academy of Sciences.* It is remarkable for great size among other instruments of the kind, carrying 20^{cm} (8-inch) azimuth and vertical circles, the torsion head rising a full metre above the leveling foot screws. The telescope is firmly attached to an elliptical collar at one end of its greater axis. A hollow cylindrical counterpoise is attached to the opposite end of the axis. The axes of the telescope are at the ends of the shorter axes and rest in Y's. The axes are extended beyond the Y's, and carry on one end a vertical circle and on the other two long arms immovably secured to the axis, which carry two microscopes 180° apart for pointing upon the ends of the needle when observing the dip or inclination of the needle. A broad horizontal support at the base of the Y supports carries at one end, outside the Y's, a clamp and verniers, and at the opposite end a circular metallic case in which is mounted the dip needle, while on the

*Publications of Russian Imperial Academy of Sciences, 1872, Part III, No. 2.

center of the support, and rising through the elliptical collar of the telescope, is the box containing a declination needle, surmounted by the long tube supporting the needle. The stirrup carrying the magnet is provided with a mirror, perpendicular and at right angles to the axis of the magnet. When observing, a pointing is made by observing the cross threads, illuminated by a ray of light (which enters the telescope through a slit near the eye), reflected by the mirror to the eye. The magnet is rectangular, 6^{cm} ($2\frac{1}{3}$ inches) long, 6^{mm} ($\frac{1}{4}$ inch) wide, and about $1{\cdot}5^{mm}$ ($\frac{1}{16}$ inch) thick, fitting snugly in an opening designed for it. A collar fitted to the stirrup, nearly 1 inch above the magnet, carries the inertia ring during oscillations. To reverse the magnet the stirrup and mirror are reversed. A scale in the focus of the eyepiece of the telescope serves to measure the length of vibrations, etc. In observations for inclination the telescope is turned in the Y's until the microscopes at one end of the axis are directly over the poles of the needle, when the angle is read on the vertical circle on the other end of the axis. The dip needle is easily lifted and reversed by a clever device without opening the case, which has a glass front. When observing for azimuth, or even upon the mark, the box containing magnet suspended must be removed. No weight is given, but it is evident that such an instrument, properly boxed, would perhaps not weigh less than 70 kilogrammes (154 pounds).

The objections to this instrument, if the brief description available has been rightly understood, are—

1. Its great weight.
2. The necessity of removing box to observe mark as well as to observe the azimuth.
3. The necessity of removing suspension tube in order to suspend or remove the magnet.
4. The instability of the microscopes for pointing on the dip needle

*The English instrument** is mounted on a principle essentially the same as the Coast and Geodetic Survey magnetometer, differing, however, in one important particular—the telescope is fixed in a horizontal position and pointing to the center of suspension of the declination magnet, and the azimuth of the sun is obtained by observing its image reflected in a mirror. The magnets are similar in size and construction to those heretofore used in this country. The only objections to this instrument are: The necessity in this country for securing accurate time for azimuth by observations, thus requiring another instrument; the necessity of placing the mark in the horizontal plane of the telescope, which is seldom convenient, and the difficulty and annoyance attending the adjustment of the mirror.

The Kew dip circle, used both in England and America, is too familiar to require any description. The weight of the magnetometer in its box is about 23 kilogrammes (50 pounds).

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* Encyclopædia Britannica.

*The magnetometer used by the French** is quite different in detail from any other. It is also an altazimuth instrument, and requires no changing or removing of any of its parts. The circles are 8^{cm} (3·15 inches) in diameter, graduated to half degrees, and read by verniers to minutes. A rectangular frame on the axis of the azimuth circle carries all the other parts on one side, and outside the telescope is mounted, and attached to its axis is the vertical circle. Also attached to the axis of the telescope, and rigidly adjusted parallel to it, is an index arm, at the end of which is a silvered index having three equidistant lines drawn perpendicularly on its outer face, and a single line just opposite the center line on its inner surface. The magnet is a solid cylinder 6·5^{cm} long, 4^{mm} in diameter, and weighs about 7·5 grammes. Its ends are slightly concave and polished to a reflecting surface. There is a microscope for pointing on each end of the magnet. A single fiber of silk about 11^{cm} (4·33 inches) long suspends the magnet. Observations for declination are made by bringing the line on the inside of the index, as reflected from the mirror end of the magnet, to coincide with the middle line on the outside of the index when seen through the microscopes; and pointings are made on the mark or on the sun for azimuth with the telescope. Inasmuch as the needle and the mark are observed with different lines of sight, it is evident that with the best adjustment there must remain some uncertainty of their parallelism; and this index error could only be determined by observing at a well-determined station. The principal mechanical objection to this instrument is this index error and the danger that it may not remain constant. Another objection is the use of glass to protect the magnet from air currents, dust, etc.

The French dip circle has also two circles, vertical and azimuth, each 8^{cm} (3·15 inches) in diameter. An arm, carrying concave mirrors, is swung around under the point of the needle, and the circle, to which the microscopes are rigidly attached, is turned by a slow-motioned screw until the points, as reflected from the zero mark on the mirrors, coincide with the point seen direct, thus insuring a most accurate pointing, free from parallax. This is most excellent in theory, but in practice it is probable that the unsteadiness of the needle will exceed the probable error of a pointing on it by other methods. This instrument, packed in its case, weighs only 2 kilogrammes (4 pounds).

A magnetometer used in Italy† presents at least one novel feature. It is an altazimuth instrument, constructed much like ours, except that a broken telescope is used. This permits the telescope to be revolved in its Y's, and a mark may be observed at a point opposite to the magnet without disturbing the box.

* Moreaux, Magnetic Elements in France, 1885. See also Nature for Jan. 12, 1888.
† Publications of the Royal Observatory at Modena, 1893, No. 1.

The magnetometer of the Coast and Geodetic Survey in its usual form has been described elsewhere.* The latest instruments are provided with magnets, octagonal in form, thus facilitating their manipulation when being reversed and making it easier to place the inertia ring. Another improvement is the removal of the glass from the box front, and the employment of a shutter at the back of the box, which may be opened when observing on the mark.† The weight of the new magnetometers, boxed, the box containing also the tripod head, is about 18 kilogrammes (40 pounds). The dip circle weighs about 12 kilogrammes (25 pounds). All the instruments examined have suitable provision for deflections.

It will be seen that the French instruments are by far the lightest of those examined. They are also the simplest in construction. They could easily be carried as hand baggage when moving from station to station.

Next in order of weight and facility of manipulation come the instruments of the Coast and Geodetic Survey.

The lightness and simplicity of the French instruments highly commend them for work in a country like ours, where cost of transportation is a most important part of the expense of magnetic surveys; and in order to give them a fair test by actual use we recommend the purchase of one or more of them for the use of the Coast and Geodetic Survey.

It may be well to allude to the compass declinometer, which is the old azimuth compass in a slightly new dress, designed to measure declinations only. Properly in adjustment, and its index error known, this instrument gives very accordant and satisfactory results.

The index error is often very large (more than 1°,) but this is of no consequence. It is, however, liable to change, and as it can not be determined in the field it is an objectionable feature.

Comparison has been made at the office respecting the moment of inertia of the magnet and its appendages, as depending on computation from known dimensions and weights, with the indirect method from oscillations, with and without the inertia ring. Results thus far have proved that the two methods agree within about 2 per cent of the whole value. Further investigation in this direction will be made.

METHODS OF MAKING MAGNETIC SURVEYS IN DIFFERENT COUNTRIES.

Countries have been surveyed magnetically, either by a rapidly executed survey, covering the entire area of the country in a brief period, with the expectation of repeating the surveys after an interval of one-fourth to one-third of a century, thus bringing out by comparison the secular changes due to the interval; or observations have been continuously made, and the results were gradually collected, reduced to the epoch adopted, and discussed. The former process answers well for a country of limited extent, as, for instance, England or France, where

* See Coast Survey Report, 1881, Appendix 8.

† See magnetometer used in Japan, described in the Journal of the College of Sciences, Imperial University of Japan, Vol. II, Part 3, 1888.

this method has been employed; but for the great area of the United States it could not be carried out within a reasonable time, say in from two to five years. The second method was therefore adopted of necessity. It carries with it the continued study of the laws of secular variation.

It is very important in selecting sites for stations to avoid all probable disturbing influences from railroads, telegraph and telephone wires, or electric car or light wires; and also to consider the probabilities of their recovery for future occupation. These considerations point to the necessity for the selection of stations outside of cities or villages.

We recommend the issue as soon as may be practicable of a second edition of Directions for Measurement of Terrestrial Magnetism, containing such modifications and additions as new or improved methods or instruments may have suggested.

That more systematic observations of all the magnetic forces be made at many points throughout the country where data are now lacking; in particular throughout the States of California, Oregon, Washington, Idaho, Montana, and the Dakotas, preferably by someone who shall devote his whole time to this work; also, that no opportunity be allowed to pass for securing observations along the vast coast regions of Alaska, and for reoccupying stations for collecting necessary data for determining the secular change.

That as soon as may be desirable a second edition of the isoclinic, isodynamic, and isogonic curves be published for an epoch close at hand, say 1895 or 1900, together with the data, the method of discussion, and explanations of the results and their uses.

We recommend that each main triangulation and astronomical party be supplied with a complete instrumental outfit for determining declination, dip, and intensity; and also that all other triangulation parties be furnished with a compass declinometer, and that observations of declination be made at each station occupied.

J. J. GILBERT, *Chairman.*
R. L. FARIS, *Secretary.*

REPORT OF COMMITTEE K ON GRAVITY.

The study of the force of gravity as a part of the geodetic problem has received the attention of the Coast and Geodetic Survey for some years, and although its work in the past has in a measure been experimental it has developed instruments and methods of observation which will enable it to enter successfully upon extended gravimetric research at less cost than would have been possible with the older processes, and that without lowering the standard of accuracy.*

*See Determinations of Gravity, by T. C. Mendenhall, Appendix No. 15, C. and G. Survey Report for 1891, Part II.

Ever since the promulgation by Clairaut of his celebrated theorem, one hundred and fifty years ago, the pendulum has been regarded as a most efficient means for the investigation of the shape of the earth. It appears to be the general opinion of the enlightened nations of the world engaged in geodetic operations that any survey that would disregard gravimetric research as an important and necessary branch of inquiry would fall short of a complete geodetic survey.

The earlier gravitational work of the Survey was of necessity of an experimental character, as stated above, involving such considerations as the character of the pendulum, with respect to absolute and relative measures, its best form, size, and material, as well as the method of observing. The several instrumental reductions to normal condition had to be studied both theoretically and practically.

At this stage of the work differential comparison of gravity between American and European standard stations of absolute measures were instituted, and in connection with this work of the Survey, in 1875–76, it had become evident that the flexure of the pendulum support during its swings was a grave source of error. It accordingly received a thorough investigation so that corrections could be given to observations made on certain stands and under certain conditions.

At the instance of the Superintendent of the Survey, a conference on gravitation measures was held at Washington, D. C., in May, 1882, having for its object to devise a plan for the prosecution of the observations and for the improvement of the pendulum apparatus.

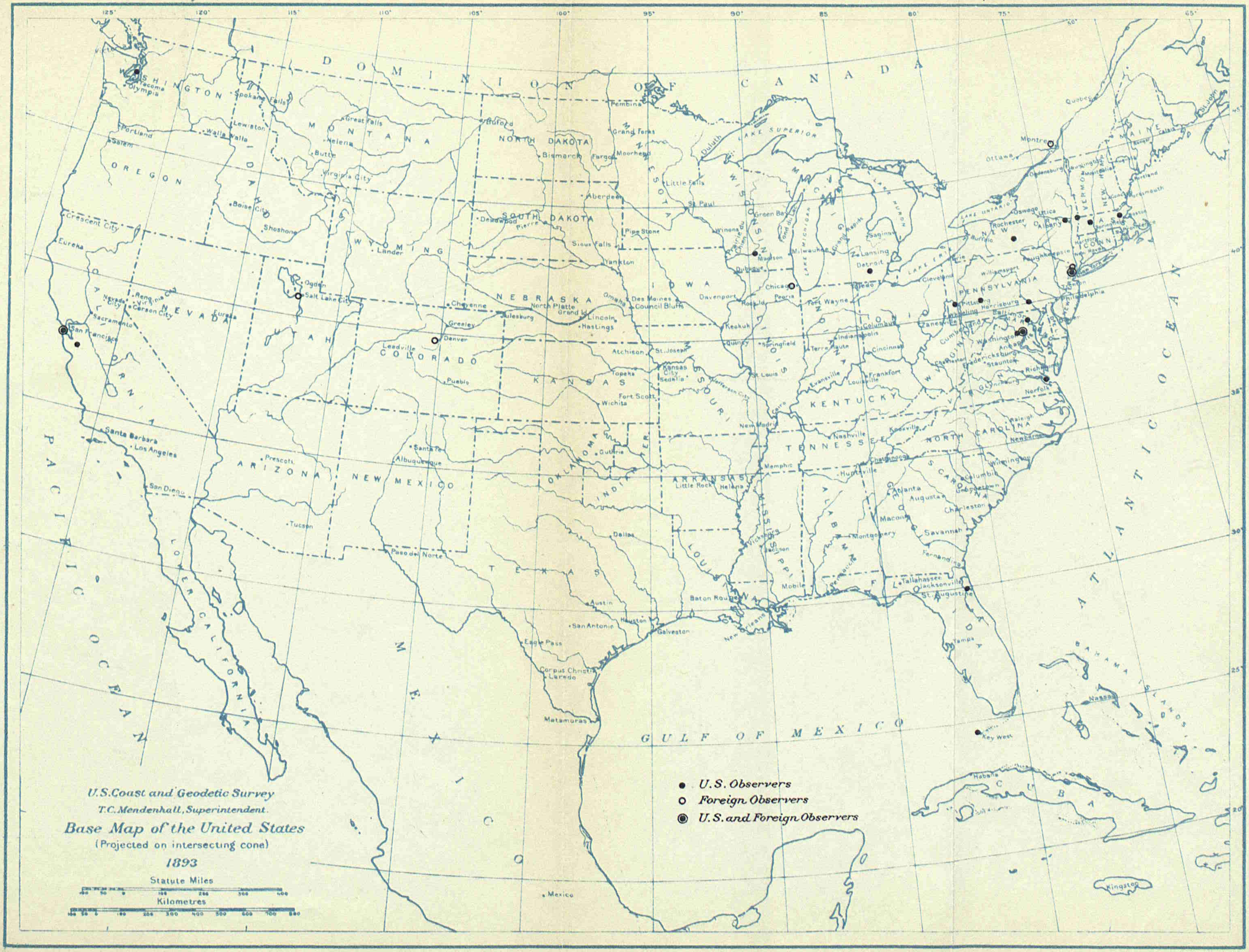
Your committee is of the opinion that the time for an active prosecution of field work has come, especially as the Survey is now well prepared, both by experience and equipment, to carry on these investigations. Its relation to the International Geodetic Association renders it desirable that this Survey should conform, as far as may be practicable, to the general plan of work followed, and that it should therefore contribute its share to the gravity research.

That determinations of gravity are essential to a complete geodetic survey, as well as of great interest in connection with geological problems, is sufficiently attested by the action shown by many of the leading nations in extending their gravimetric surveys in recent times.

Thus far over 500 stations have been determined in various parts of the world, of which a large proportion were occupied in the last few years, showing the active increase of interest in these researches. In the United States 27 stations have been determined by the Coast and Geodetic Survey and 9 by foreign observers. The Survey has also occupied 29 stations in foreign countries, taking advantage of various astronomical expeditions.

The English have made a series of determinations in their country, and have, moreover, sent expeditions for this purpose to various parts of the world. In India they have carried out a very systematic scheme of gravity work in connection with the Great Trigonometrical Survey.

United States Pendulum Stations

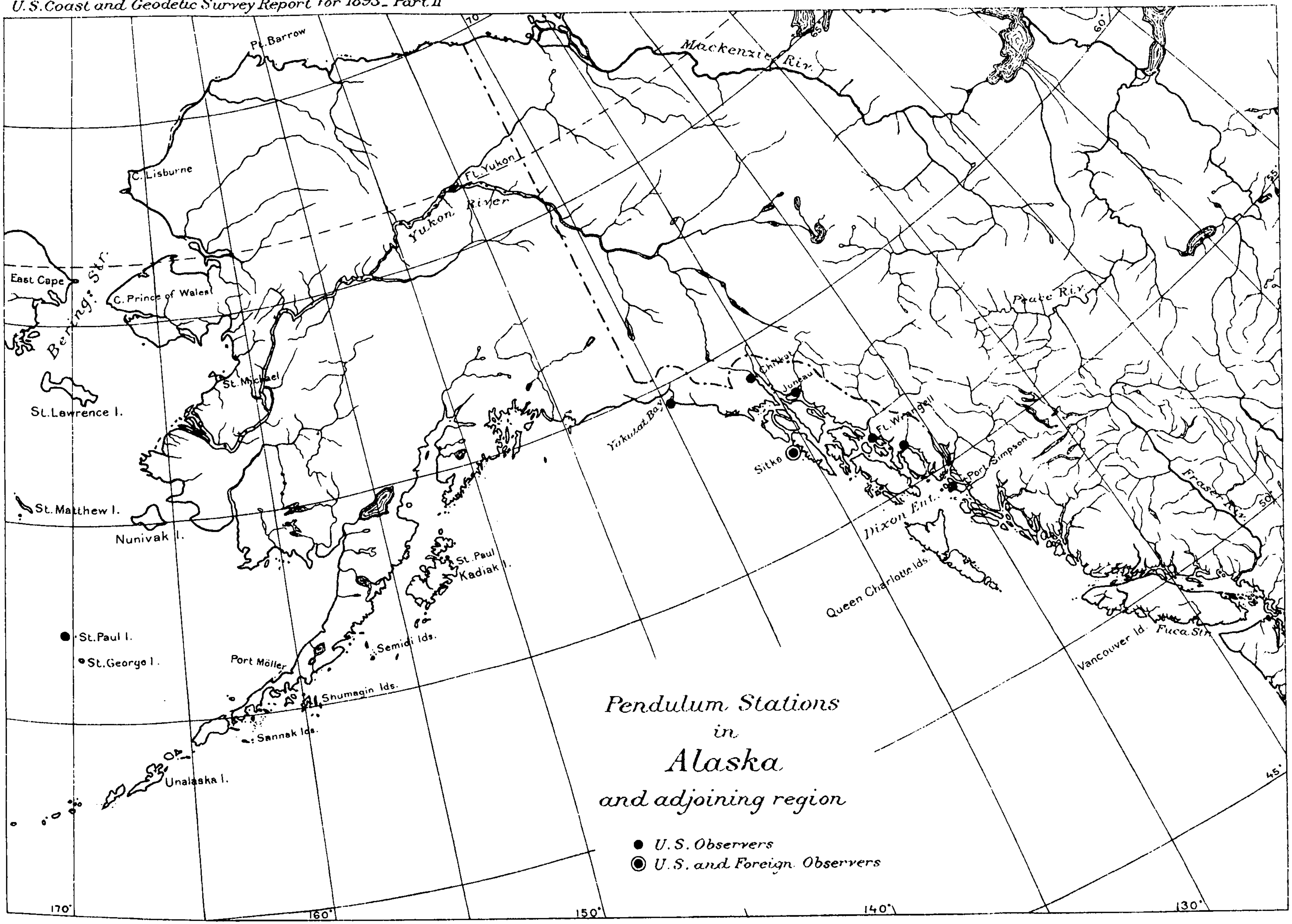


U.S. Coast and Geodetic Survey
T.C. Mendenhall, Superintendent.

Base Map of the United States
(Projected on intersecting cone)
1893

Statute Miles
Kilometres

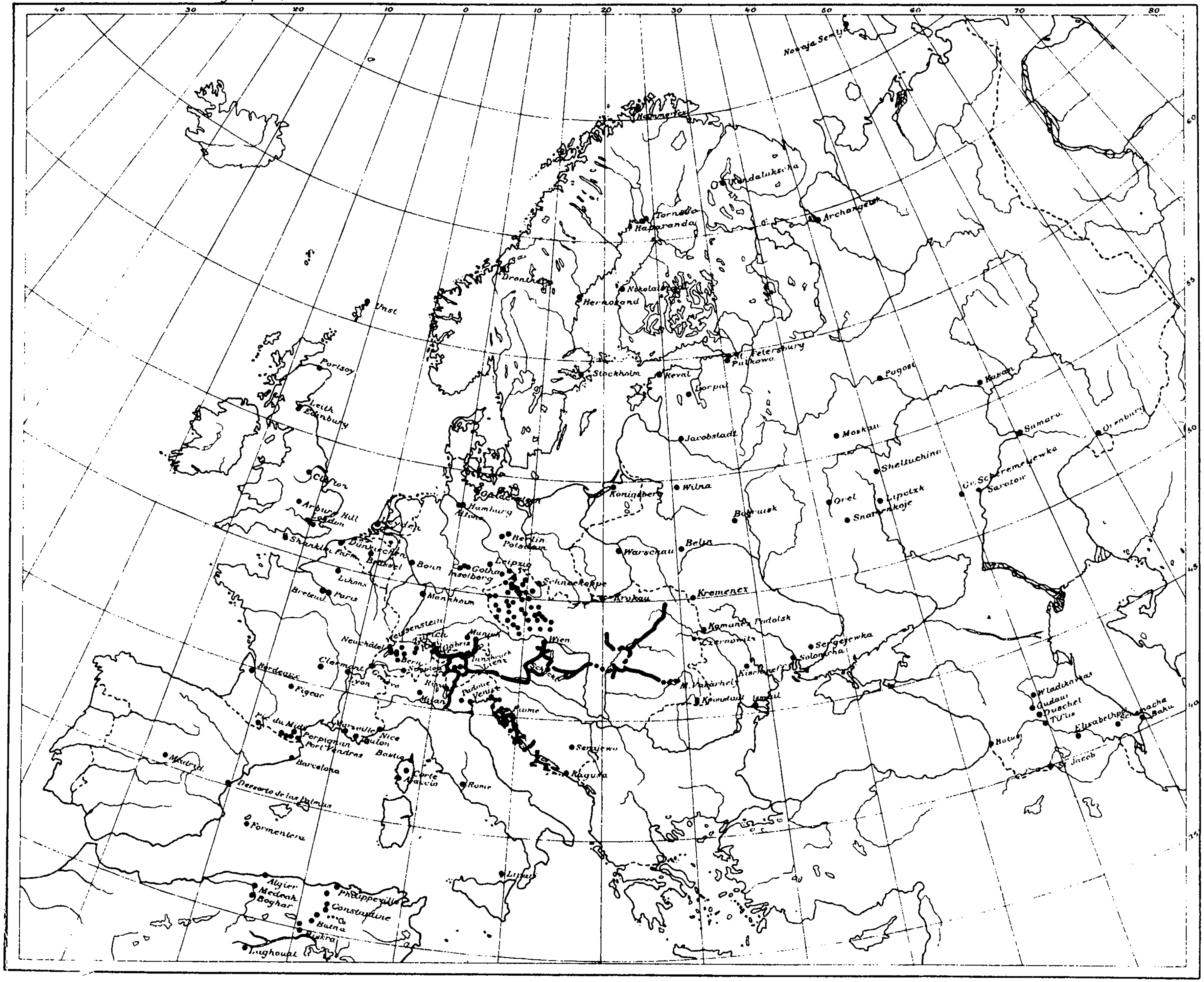
Photo-Lith by A. HOEN & CO. Balto. Md



Pendulum Stations in Alaska and adjoining region

● U.S. Observers
◉ U.S. and Foreign Observers

Pendulum Stations in Europe



In France the Service Géographique has perfected new apparatus, and has made a series of pendulum observations in various parts of France, Algeria, the United States, and other countries.

In Austria the Geographische Institut has made numerous determinations with extremely portable apparatus, and has investigated particularly the effect of gravity disturbances on lines of precise leveling, and also the question of the density of the earth. Some of its apparatus has also been sent to the far north on naval expeditions, as well as to other countries.

In Russia systematic gravity observations have been and are still being made in connection with the great triangulation scheme of that country. Italy, Sweden, Switzerland and other countries are carrying on similar investigations.

Illustrations Nos. 15, 16, 17, indicate the location of stations already determined in Europe, the United States, and Alaska.

DESCRIPTION OF APPARATUS.

The improved pendulum apparatus of the Survey was designed with a special view to its portability, and in general principles is a modification of that used in Austria by Herr von Sterneck, though differing in many details. (For full description see Appendix No. 15, Report for 1891.) The pendulums have a period of about half a second and are of the invariable type, being designed only for relative determinations. The periods are obtained with precision by observations of coincidences with a break-circuit chronometer by means of electrical and optical devices. The pendulums, of which three constitute a set, are made of alloy of copper and aluminum, with agate plane (swinging on an agate knife edge), and are swung in an air-tight receiver at or near a standard atmospheric pressure. Coefficients for temperature, pressure, and elasticity of support have been determined experimentally, so that observations may be reduced to uniform conditions. Arcs of oscillation not exceeding 40′ (total) are now used, and swings of four hours or more may be observed without difficulty. A recent improvement permits the arc to be read directly in the observing telescope. The apparatus may be mounted on a pier or operated without difficulty directly on the floor or ground.

The method of observing permits of a ready application of the telegraphic plan of determining the difference in the force of gravity between two places (already used in Europe); that is, the comparison of pendulums at distant stations connected by wire using the same timepiece, or, to eliminate change in the line, a timepiece at each station alternately. Two methods of using the apparatus are therefore available. In the first the periods of a set of pendulums will be determined at a base station and then at various other stations, the chronometer being rated by time observations. In the second two sets of pendulums at distant stations will be compared telegraphically, starting from a base station as before. As the second plan avoids the necessity for time observations and the errors due to change in rate of chronometers, it is of course to be preferred whenever practicable. In either case the determination of a station need not take more than a few days.

A still more portable apparatus is proposed and is now being constructed, the pendulums having a period of but one-fourth of a second, and the various parts being correspondingly reduced in size.

PROPOSED OUTLINE OF INVESTIGATIONS.

The following general outline of a plan for a gravimetric survey in connection with the geodetic operations of the Coast and Geodetic Survey in the United States is submitted for consideration:

First. To ascertain the geographical distribution of gravity within the United States with respect to latitude, elevation, and geological structure, it is proposed to make pendulum observations at a number of stations extending from the mouth of the Mississippi up its valley to the shores of Lake Michigan and the northern boundary, and also at a similar series of stations at distances of about 1° from each other along the thirty-ninth parallel, from Cape May to Point Arena.* It is further suggested that a number of localities of especial interest should be investigated, as the basin of the prehistoric Lake Bonneville, the two great depressions in southern California, where stations may be located several hundred feet below the sea level, some of the great eroded valleys of the western plateau, the volcanic region of the Yellowstone Park, and the vicinity of the mouths of the large silt-bearing rivers—the Mississippi and the Colorado. Under the assumption of a rigid crust, the enormous quantities of alluvium transported and deposited by these rivers must, on account of a specific gravity at least twice as great as water, show quite a sensible excess of gravity. The results from these stations, if studied in conjunction with corresponding series of observations scattered through the upper drainage basins of the Colorado and Missouri rivers and along the crest line of the Continental Divide, would probably not fail to throw much light upon the truth or fallacy of the doctrine of isostasy, as well as the questions in regard to the deflection of the plumb line. The doctrine of isostasy has claimed the attention of geologists to such an extent, in connection with the continental problem, that the proof of the existence or nonexistence of regions in which there is an excess or deficiency of gravity within the domain of the United States would not fail to be of much interest to them. The behavior of gravity over the continental plateau, rising locally to an elevation of about 3 300^{m} (11 000 feet) and more above the level of the sea, the origin of these plateaus, their permanency or rise and fall in the course of ages, are questions which, although as yet shrouded in mystery, will never cease to claim the deepest interest not

*For outline of this plan see Appendix No. 22, Report for 1882, p. 509.

only of the geologist but of man generally. In connection with this and other questions of interest, it is proposed to establish in the course of time a number of base stations for both the gravimetric and hypsometric operations of the Coast and Geodetic Survey for future reference, which will be determined with the utmost precision attainable. A number of such base stations judiciously distributed over the country and permanently marked and referred to the normal level of the oceans would, in the judgment of the committee, form a valuable heritage to hand down to posterity.

In the selection of pendulum stations the location of lines of precise levels should be kept in view, so that the effect of gravity irregularities on hypsometric measurements may be studied and corrected for.

Second. Figure of the earth. As in the present state of our knowledge the reduction to the sea level of elevated continental pendulum stations tends to introduce into the results an element of uncertainty, it is proposed to restrict the location of stations intended for the determination of the earth's compression to the sea border of the continents and islands of the United States and Alaska, so as to obviate the necessity of applying a reduction. Results thus far appear to indicate that fairly normal conditions exist along seashores. The large range of latitude embraced in the United States would be favorable to such an investigation. As the western and northern shores of Alaska are annually visited by commercial and Government vessels, it is suggested that their aid would be valuable in establishing a few stations there. Work of this nature might also be carried on in connection with astronomical stations that may be needed in the survey of the Aleutian Islands and the western coast of Alaska.

Although a precise knowledge of the absolute force of gravity is not indispensable to the application of the results of relative pendulum observations to the various purposes mentioned, yet it is so important a physical constant and of such great scientific interest as to justify the undertaking of such determinations at a few base stations, which may thereafter be used as reference points for relative observations.

When more extended pendulum research has been carried out in the United States, it would probably be desirable to strengthen the connection between the base stations in this country and those in Europe, using the same apparatus that has been employed here.

It is not yet deemed practicable to state what degree of precision may be or should be reached in either relative or absolute work. Observations at a station should be continued only long enough to reasonably eliminate the known errors of observation. In the present state of our knowledge multiplication of stations is to be preferred to great accuracy. One of the principal obstacles to rapid and correct work will be the rating and irregularity of the chronometer, and it is suggested that this is a subject that should receive careful consideration.

As of especial interest in connection with this report, the following conclusions, which were adopted by a conference held at the Coast and Geodetic Survey in 1882, are appended: *

1. The main object of pendulum research is its determination of the figure of the earth. From a sufficient number of observations suitably distributed over the surface of the earth the actual figure may be determined.

2. A complete geodetic survey should include determinations of the intensity of gravity. These determinations should be made at as many critical points of local deflection and physical structure within the area of the survey as possible; and these should be combined with others distributed over the whole globe.

3. A minute gravimetric survey of some limited region is at present of such interest as to justify its execution.

4. Extended linear gravimetric exploration is desirable, to be ultimately followed by similar work distributed over large areas.

5. Each series of such determinations should be made with the same apparatus, so that the differential results should not be affected by constant errors peculiar to the apparatus.

6. While it is inadvisable at present to strictly fix a numerical limit of the permissible probable error of pendulum work, yet such determinations ought commonly to be accurate to the $\frac{1}{200000}$ part.

7. Since different pendulums may be used in different regions, all should be compared at some central station.

8. Determinations of absolute gravity will probably prove useful in comparing the yard and the metre, and they should at any rate be made in order to test the constancy of the length of a metallic bar.

9. In the present state of our experiences unchanged pendulums are decidedly to be preferred for ordinary explorations.

WILLIAM EIMBECK, *Acting Chairman.*
S. B. TINSLEY, *Secretary.*

REPORT OF COMMITTEE L ON EQUIPMENT.

For convenience the subject of "equipment" has been considered under the following heads:

I. Operations conducted by land.
   1. In a thickly settled country;
   2. In a sparsely settled country;
   3. In an undeveloped country;
   4. In an arid country.

II. Operations conducted by water.
   5. On inland waters in a settled country;
   6. On inland waters in an unsettled country;
   7. On large or partly landlocked waters.

I. OPERATIONS CONDUCTED BY LAND.

1. *In a thickly settled country* the necessary outfit, beyond the instruments and observatories, is confined to a few tools, etc., required for signal building, mounting of instruments, and the erection of observa-

* See Appendix No. 22, Report U. S. C. and G. S. for 1882, p. 516.

tories. Transportation in the field can be hired advantageously, and board obtained near the scene of operations.

2. *For a sparsely settled country* it is not possible to make very definite suggestions, owing to the great diversity of circumstances. For most of the mountainous regions east of the Mississippi River a camping outfit will generally be needed in order to be sufficiently near the stations. The camp outfit should be only sufficient for the actual needs of the party, but should not be so restricted as to endanger health.

The equipage should be so selected that it can be stowed very compactly for ready transportation from station to station. Teams can usually be hired at reasonable rates for the purpose of moving camps.

In most of the States east of the Rocky Mountains it is possible to do away with camp outfits entirely, and to live in towns or villages, or at farmhouses near the stations. Such places, even in the Appalachian mountain region, or in the sparsely settled sections west of the Mississippi, can generally be found within 5 miles of the stations, which distance, unless observations are made at night, causes no serious inconvenience. Under such conditions the party outfit can be reduced to a minimum.

On the triangulation of the thirty-ninth parallel, in western Kansas and eastern Colorado, the work was done under the foregoing conditions. A double party was employed. One party occupied the northern stations of the scheme, while the other party occupied the southern ones, each party consisting of three persons. For means of transportation each party had a two-horse spring wagon, which carried instruments and outfit from station to station. At some places, where sleeping quarters could not be furnished, owing to the smallness of the sod houses or to the large families inhabiting them, it was found to be advisable to take along a tent or two for the sleeping accommodations of the party. At such times an additional wagon was required to transport the extra outfit.

3. *For undeveloped and unsettled country,* like the mountainous regions of the West, where roads are few and far between, and when they do exist are of the roughest character, and a great portion of the country can only be reached by trails, many of which have to be opened as the work progresses, it is of the utmost importance to have the outfit and equipment as light as possible.

For reconnaissance the party should be small, consisting of a chief, an aid, two hands, and a man who is accustomed to outdoor cooking.

All hands should be able to ride, drive, pack, and care for animals. There should be a saddle animal for each member of the party, and one pack mule for every two men, to transport the camp outfit, supplies, instruments, etc. Where roads are available, a light wagon will be of great assistance.

The camp outfit should consist of two light single-pole pyramid tents, the poles jointed for convenience in packing. In a timbered country it will be unnecessary to transport poles, as they can readily be cut when required.

Each member of the party should have two pair of blankets and a canvas cover. Clothes should be rolled with the blanket or carried in a canvas bag.

For a regular triangulation party the outfit can be increased according to the size of the party and the requirements of the work. Folding camp cots, chairs, tables, and other conveniences can be added according to the judgment of the officer in charge of the party; but it should always be remembered that transportation is one of the principal items of expense, and therefore the outfit should be kept as light as possible.

The heliotroper's equipment should consist of a small pyramid or A tent, an ax, hatchet, and the necessary cooking utensils for one or two men, as required.

In an arid and undeveloped country a party must carry its own equipage and supplies, and, on account of the difficulty and expense of transportation, should be restricted to only such things as are necessary for the prosecution of the work, subsistence and health of the members of the party, and forage for the animals.

Great care should be used in selecting the articles for such an outfit, so as to get them as light as possible and of a form convenient for packing into a small space.

It would be useless to attempt to give any specific direction for equipment for work in sections like the region under consideration, as it will necessarily be varied in different localities and must depend upon the character of the work, etc. The following general considerations, which are the fruit of the experience of the parties employed in the main triangulation across California, Nevada, Utah, and Colorado, however, will be found advantageous in nearly every section.

In order to avoid the necessity of carrying along too many tent poles, a tent requiring only a jointed single pole should be used.

Whenever practicable, all camp furniture should be made to fold and of light material. Mess outfit and cooking utensils should be selected without handles, and they should be of such a form as to be convenient for packing into nests.

All articles that are likely to break in transportation should be put into boxes of proper size to be conveniently carried on the backs of animals.

In case heating stoves are necessary, they should be made of strong sheet iron and of varying sizes, so as to allow several of them to nest in one package. Stovepipes should be made to telescope. All bedding should be carried in rolls of a convenient size for packing upon mules, and a canvas covering should be furnished for each roll. For temporary camp purposes a cooking bar or plate may be carried.

For the purpose of carrying water upon the backs of mules the ordinary 10-gallon casks or canvas water pouches will be found useful. They can easily be placed upon a common pack saddle.

If the camp equipage and outfit of instruments is large, as in case of a party engaged in primary triangulation, it will be found advisable to transport everything, as far as possible, by means of teams and wagons, which can usually be hired temporarily. The party should be supplied with one light wagon and a number of mules for driving, riding, and packing.

It will often be found necessary to carry along enough water to last for several days. In case wagons are used for transportation, the most convenient method for carrying it will be in 45-gallon barrels. One should be lashed between each set of wheels, on the outside of the wagon box, where the water will always be available without unloading the wagon.

In starting upon a long journey, which may last for a number of days, the party should be prepared to camp whenever night may overtake it, and such articles as are needed for bivouac along the road should be selected and placed in one wagon, so as to avoid the necessity of unloading all the wagons at every camping place. Among such articles might be mentioned a supply of provisions, cooking utensils, bedding, supplies for animals, cooking bar or grate, wood, etc.

In a reconnaissance or running survey large wagons may be used as bases of supplies, from which saddle and pack animals may be employed to branch off in order to reach points difficult of access.

In a country where there are no roads and it is impossible to travel with wagons and teams, a pack train, consisting of mules, equipped with the ordinary pack saddles, becomes necessary. Sometimes it even becomes necessary to carry the outfit upon the backs of men, and for that purpose pack straps should be furnished.

II. OPERATIONS CONDUCTED BY WATER.

5. *In a settled country*, where operations can be carried on by water, it will be found most convenient and economical to establish the party at some boarding place as near as possible to the field of work, and use boats for local transportation. It will cause a great saving of time and expense to furnish a steam launch for a party working in such a locality.

If boarding places can not be obtained without necessitating local transportation for a distance of 5 miles or more, the party should be furnished with floating quarters of a character suitable to the locality.

6. *Land transportation in a region that is unsettled and adjoining partly landlocked waters* is, in a great measure, impracticable. For carrying on work in such localities two methods of procedure are possible—one by using small vessels, and the other by living in camps.

A very important and troublesome feature—one that can not be well avoided in work in this kind of country—is the fact that the source of supplies is usually at a long distance from the working ground. This necessitates carrying a much greater stock of provisions, etc., than would be advantageous if near a well-settled country, so that sufficient transportation and storage capacity becomes necessary in order that the regular progress of the work be not interrupted and delayed by occasional journeys for procuring supplies.

If the field to be covered is an extensive one, it is undoubtedly more economical and expeditious to work from a vessel, equipped in the customary manner, but of sufficient size only to afford quarters for living and storage of supplies and material for the working party. Thus vessels can always be kept on or contiguous to the working ground, so that but little time will be lost in moving camps. On account of the liability of the provisions being spoiled if moved in rainy weather, camps necessarily have to be moved during weather suitable for field work, which consumes time that could otherwise be devoted to continuous operations.

After it is decided that it is best to place the party on a particular piece of work afloat, the adoption of the class and size of vessel ought to depend upon the nature of the waters—that is, whether deep or shallow and whether wholly or partially protected—and upon the size of the party. In some instances, where the waters are shoal and perfectly protected, especially on the New Jersey and Florida coasts and on the Columbia River, scows or flatboats of light draft with quarters built on them have been found to answer the purpose of party accommodation admirably. Moving from place to place can be effected by poling when the water is shoal enough, by a square sail when the wind is fair and strong, or, better still, by either a steam or naphtha launch, according to the circumstances.

In deep and exposed waters sailing or steam vessels of seaworthy qualities become necessary. In localities liable to much calm weather or strong tidal currents steam vessels would be advisable.

If the area to be surveyed is not of large extent, and a suitable vessel is not on hand or readily available, the work will necessarily have to be conducted from a camp, the outfit of which for economy and facility in moving should be so selected as to be stowed very compactly, and to be as light as the conditions of climate and exposure to vicissitudes of weather will permit without endangering the health of the various members of the party. In either case, whether working from a camp or vessel, the field transportation will be by water, and the "working boats" should be selected with regard to safety, utility, cost, and maintenance, all of which depend upon the nature and character of the waters. Scarcely anything can be added or suggested in the way of improving the pulling boats generally used on our vessels. For very shoal waters specially constructed, small, flat-bottomed boats are necessary, the design and size of which must depend upon the conditions in each particular case, and their selection must be guided largely by the experience gained in similar cases.

Experience has shown that there is great economy in the use of suitable launches, varying in size and character according to circumstances, on account of the rapidity with which they allow the field party to move about, and the facility with which material and equipage can be carried or towed by them; and their general use is recommended. In this manner the number of the crew may generally be decreased, and much valuable time be saved.

For explorations up and along rapid streams, and work similar to the Alaskan Boundary Survey, light canoes, like the *Peterboro*, but with good beam for the sake of safety, are desirable, on account of their extreme lightness, for portage from one channel or stream to another. Pulling boats of very light draft, but of strong and elastic qualities, constructed on the general plan of what are termed "St. Lawrence skiffs," where a greater carrying capacity than that of the canoes mentioned above is required, are often desirable on account of their great portability. For rough field usage and the moving of camps along rapid streams a double-shovel nosed skiff, being in model a cross between a punt and a batteau, is very good on account of its light draft and stable qualities. In many instances Indian canoes are excellent for working purposes.

In all portions of the country where there are protected or partially protected waters the triangulation, which is the framework or foundation for chart construction, is closely followed by the topography and hydrography. Experience has shown that where these waters are not more than 6 or 8 miles in width there is great economy in conducting all three classes of work by one party of proper size and equipment, operating from either a vessel or camp.

7. *On large or partly protected waters*, like the Straits of Fuca, and especially the prospective work in Alaska, stanch seagoing vessels are required.

For the Straits of Fuca and the triangulation of southeastern Alaska types of existing steamers, of the class of 100 tons or a little less, would be suitable. For their safety, on account of the strong tidal currents, they should have a speed of not less than 10 knots per hour. The present outfit of boats, etc., of this class of vessels will, speaking in general terms, meet the requirements of this kind of work.

For the work in Alaska beyond Sitka a steam vessel similar in size, speed, and equipment to the one just mentioned seems desirable. For economy in fuel, and, as a precaution of safety, to guard as much as may be against disaster due to accident to the machinery, it seems advisable that she be full schooner rigged and also a good sailer, so that during heavy weather when in exposed places she could be laid to under sail, thus saving a good deal of fuel, which would have to be brought from a long distance.

In conclusion, your committee would state that, owing to the great variety of orographic, economic, and climatic conditions in a country as large as this, it is impracticable to recommend specific details for the numerous varying conditions under which the different classes of work are executed.

J. F. PRATT, *Chairman.*
A. L. BALDWIN, *Secretary.*

SUPPLEMENT.

LETTERS FROM ASSISTANTS AND OTHERS ADDRESSED TO THE CHAIRMAN OF THE CONFERENCE.

LETTER FROM AUG. F. RODGERS, ASSISTANT.

UNITED STATES COAST AND GEODETIC SURVEY,
Suboffice, San Francisco, Cal., January 29, 1894.

Appreciating the courtesy of the chairman and gentlemen of the Geodetic Conference assembled in Washington in inviting me by the circular of January to give my views in writing on the subjects suggested to the Conference by the Superintendent for discussion, I beg to reply as follows:

BASE-LINE MEASUREMENT.

It seems to me that the most effective way to determine with certainty the relative values of bases measured by different methods, those of acknowledged high refinement of method with a number of measures determined with less refinement, is to compare the results of the various methods on some occasion when it may be required to measure a long base with the best modern appliances of refinement. I am inclined to believe in the practicability of close and even refined wire measurements, with appliances to insure perfect alignment, prevent sagging, and correct for changes of temperature. Every bar contact, while it is an element of retardation in time, is also an element of error. These two are reduced to their lowest factors by the great lengths possible in wire measurement.

It would certainly be interesting and valuable information to determine by careful field comparisons the relative values of refinement to be expected between modern bar measures and a repetition of wire measurements; the latter a method comparatively so inexpensive that if the degree of refinement could be assured, frequent bases would be practicable, while they are not now.

To define with greater exactness the various classes of trigonometrical work I suggest—

Triangulation reconnaissance.
Geodetic meridian triangulation.
Geodetic latitude triangulation.
State and interocean primary.
State and interocean secondary.
State and interocean tertiary.
Coast primary triangulation.
Coast secondary triangulation.
Coast-line triangulation.

Triangulation reconnaissance explains itself.

Geodetic meridian and *latitude* refer to schemes that, while incidentally accomplishing general connections, are specially devised to measure arcs of meridian and latitude.

State and interocean, to define relatively the geography of the several States, their boundaries, and finally to make connection between the Atlantic and Pacific coasts.

Primary, *secondary*, and *tertiary*, in the case of State and inter-ocean triangulation, would indicate the character of the work; secondary, more or less local in character, as applied to any State, and the tertiary entirely so, as being the basis for topographic detail.

Coast primary refers to triangulation which, though incidentally connecting with interior work, is specially designed to control the coast geographic detail through the coast secondary and coast-line schemes.

INSTRUMENTS, ETC.

No theodolite reading to less than 5″ should be used in the coast-line work, and the degree of minuteness of measure of course should increase with the higher grades of work, though I am inclined to think the refinement of graduation, when read with micrometer microscope, is in advance of the character of signals observed over long lines. The heliotrope, when a good object, is very excellent; but in disturbed conditions of the atmosphere is liable to distortion and enlargement, and the latter much beyond the limits of dimension requisite for precise pointing.

It is too well known to require discussion that the best observing weather, within distances in which pole signals can be defined, occurs when the sun is obscured by clouds, and on such days, if the obscuration covers large areas, they must be lost days to the observer dependent on heliotropes.

The inference to be drawn from the effects of cloud obscuration of the sun is suggestive of the superior conditions of atmosphere at night for observation of horizontal angles or directions, and, cæteris paribus, diminished loss of time as compared with daylight heliotrope observations.

Over the longer lines of the present interocean triangulation I suppose it would be necessary to devise a more powerful artificial light than any that has been heretofore used in Coast Survey work.

The reconnaissance for the main and primary triangulation of the Pacific Coast appears to have been always a weak element, and this has resulted in the necessity of reoccupation of stations and unnecessary expense to the work.

It seems remarkable that the geography of the coast and of its mountain ranges is sufficiently well known to permit authentic useful State maps to be made, always based partly on determinations by the Coast and Geodetic Survey, and yet the question of intervisibility of prominent summits available for carrying the network of precise triangulation of our Survey over the regions embraced by the maps referred to is still in many cases doubtful. I think more attention should be given to reconnaissance and the study of schemes for development of the triangulation before the latter is undertaken.

HYPSOMETRY.

The close determination of elevations by the Coast Survey in triangulation and by precise leveling would appear to afford special facilities for testing the value of hypsometric methods, and especially results attainable from mercurial and aneroid barometers.

MAGNETIC WORK.

The *magnetic work* of the Coast Survey has been, and the results attained ever will be, of value to the mariner, the land-owner, and the surveyor, and should be fostered by appropriations to secure continuity of observation, which will still be of important practical interest and value long after the present generation shall have passed away.

PARTY ORGANIZATION.

The organization of parties in the Coast Survey would appear to be as simple as it is possible to make them.

CAMP OUTFITS, ETC.

When it is remembered that our camp outfits involve protection from severe weather, and often at high altitudes, at points more or less remote from ordinary lines of travel, the question of "impedimenta" is one to be seriously considered, but from so many standpoints and under such varying conditions that it would be impracticable to limit outfits as to cost and size by any rigid rule.

RECORDS, PREPARATIONS, ETC.

To secure uniformity in the best method decided upon in any case is a desideratum.

FIELD COMPUTATION, ETC.

This is a question to be governed by the character of work in each particular case and whether the results are for field use or for transmission only to the office for revision.

ACCOUNTS—RELATIONS OF FIELD OFFICERS TO THE DISBURSING AGENT.

The present system of accounts seems to me to be generally acceptable, and my own official relations with the disbursing agent leave me nothing to suggest.

LETTER FROM A. T. MOSMAN, ASSISTANT.

SAN DIEGO, CAL., *February 1, 1894.*

In presenting to the Conference my views herewith I must confine myself almost entirely to the results of my own experience in the Survey, as I have no books or reports at hand bearing upon the geodetic work of foreign countries and very few of the reports of kindred work in the United States.

BASE-LINE MEASUREMENT.

For base lines of the first class, where the greatest attainable accuracy is sought, the apparatus used may be divided into two classes—"noncompensating," when the bars are made of one metal, and "compensating," when, by a combination of two metals of unequal coefficients of expansion, it is sought to render the length of the combined bar invariable at all temperatures within the limits usually encountered in actual work.

With *all* kinds of apparatus used to measure bases the accuracy of the final result for length depends more on the accurate determination of the temperature of the measure used than on manipulation of the apparatus. If means could be found to obtain the true temperature of the bars used, either a compensating or a noncompensating apparatus might be made to give accurate results. In practice it has been found, I think, that it is impossible to keep the two metals composing a compensating apparatus at the same temperature; therefore the invariable length of the compound bar sought is never attained. And to obtain an accurate result it is necessary to *correct* for the *difference* of temperatures of the two metals composing the bar, which can only be done after we know the actual temperature of each, and it is easier, I think, to obtain the true temperature of one bar of steel or iron than of two. The complication necessary in the mechanical arrangement of the compensating apparatus is also an argument against its use. The experience of the Lake Survey with the Bache-Wurdemann compensating apparatus, composed of bars of iron and brass, was unsatisfactory, and "its liability to change length owing to its having thirteen joints or points of contact, at any one of which change in contact by wear of or by change of adjustment may change the length of the tube," is mentioned (on page 88, Professional Papers No. 24, Corps of Engineers, U. S. A.) as an argument against its use; and the fact is stated "that in any one of the tubes of the Lake Survey apparatus the length of the tube can be changed 0·003 inch by simply tightening or loosening the screw which forms the axis of rotation of the compensating lever, the tightening twisting the original plane of rotation."

Accuracy of result depending in any form of apparatus, therefore, almost entirely upon obtaining the true temperature of the bar, either by a mercurial or by some form of metallic thermometer, it is important to know how we can best attain this end, whether by covering the tubes by several thicknesses of nonconducting material, thus making changes of temperature slow and gradual, or by exposing both bar and thermometers to the air. In either case it is supposed that the whole apparatus is sheltered from the direct rays of the sun.

By experiment it is known that there is a certain "lag" of thermometers going on during changes of temperature, and it seems to me therefore preferable to cover the tubes and thermometers as carefully as possible to prevent sudden changes of temperature and to correct the "lag" by measuring, if possible, each day as many tubes during a rising temperature as during a falling, thus getting a mean result as nearly as possible independent of "lag" of thermometers.

For the same reason I should use the same method in working with the secondary base apparatus for subsidiary bases. How we can best apply our thermometer to the bar in order that its reading shall show the temperature of the bar I do not know as none of the means used heretofore, to my knowledge, have been very successful.

By measuring in the night, when either bars or long tapes are used, we secure usually a more equable temperature and presumably obtain more nearly the actual temperature of the measure used. This is perfectly practicable when the tape is used, the stakes having been previously set over the whole base; but there are many difficulties in the way of using a primary base apparatus in the night.

The average probable error of result with primary base apparatus on the Lake Survey was about 1/1 070 000; for the Coast and Geodetic Survey I have no data at hand. The experiments of Prof. R. S. Woodward on the Holton and St. Albans bases show that with long steel tapes, used under the most favorable conditions at night, the probable error of result can be reduced to 1/1 280 000. If even an accuracy of 1/1 000 000 can be usually obtained with tape measurement, with the greatest care, it will doubtless prove the most economical method when the conditions are such as to allow the stakes to be driven along the whole line and the state of the ground such as to allow night measurements.

When extreme accuracy is not required in each individual base, as in a scheme of triangulation covering a large section of country, especially when the surface is nearly level and covered with timber, necessitating the building of high signals, frequent bases measured with a degree of accuracy equal or but little superior to that obtained from the triangulation, say 1/150 000, would seem to be a more economical and equally accurate method of checking the work. This would be particularly applicable to a section of the country like Indiana or Illinois, nearly level and heavily wooded, where lines of 15 to 20 miles can be got only by building up and cutting, and where bases of from 2 to 5 miles can be laid out without much expense.

In a mountainous country, like that of Virginia and West Virginia between the Blue Ridge and the Ohio River, it would often be impossible to find a spot suitable for a base line of even 2 miles in length, and when found it would be almost impossible to connect it with the scheme of triangulation.

In such a country bases would necessarily be few and far apart, and for that reason a greater degree of accuracy would be required in each.

TRIANGULATION.

Triangulation is usually divided into three classes, viz, primary, secondary, and tertiary. It is difficult to define exactly the last two, as under differing circumstances of country and length of lines the two latter merge into each other and are rarely kept entirely separate. Primary work may be defined as that scheme which can cover the area to be surveyed with well-conditioned figures having the smallest number of stations possible. Tertiary work has for its object the determination of a suitable number of points along or near the shore line of the coast, bay, or river to be surveyed for the use of the topographical and hydrographic parties following the triangulation. The scale to be used fixes the accuracy needed in the determination of these points. In many cases the points can be fixed by intersections from secondary or even from primary stations, as in the case of a wide river or arm of the sea, in which case there would not be a distinct chain of tertiary figures. There should always be, of course, two or more independent determinations of each of these points for a check, and in many cases, where lines are not more than 10 miles in length, a single reading of the angle direct and reverse is sufficient to fix the point with the required degree of accuracy.

Secondary triangles are those intermediate between the primary and tertiary, sometimes forming an independent chain and being connected as often as possible with the primary, and sometimes being merely a step in the determination of tertiary points from the large primary sides.

Primary work requires the greatest attainable degree of accuracy, and instruments of the highest class, with accurately divided circles and powerful telescopes, should be used.

Direction instruments having circles of from 12 to 20 inches in diameter and reading by three microscopes to single seconds, with telescopes of high power, are the best instruments to use for primary work.

For secondary work either direction or repeating theodolites of from 8 to 12 inch limbs may be used, depending upon the country. When it is necessary to occupy the tops of buildings and spires of churches, as in a triangulation covering a city or town, or when light-houses must be occupied eccentrically on the outside platform or rail, a repeating instrument must be used, as the support of the instrument is not firm enough to use a position instrument and it is frequently impossible to read the microscopes.

When a repeating theodolite is used, a set of measures should be made of the angle with telescope direct and reversed, and then a set of measures of the explement follow directly, turning the telescope in the same direction in each of the measures. The sum of the angle and its explement should equal 360°, and half the excess or deficiency applied to the direct measure gives the mean result for two sets.

For secondary work I should generally prefer to use a repeating theodolite varying in size from 6 to 10 inches in diameter of circle, according to the lengths of the lines to be observed. On the triangulation of the Delaware River two 6-inch Gambey theodolites were used, and lines of over 20 miles in length were observed from one light-house to another. Over 70 miles of river were triangulated in about two months, starting from a base joining two old light-houses about 6 miles apart and closing satisfactorily on the two light-houses at the mouth of the river, Cape May and Cape Henlopen, about 16 miles apart, and the work was also extended up the river from Bombay Hook nearly to Newcastle and joined to the upper river triangulation.

As the principal object of the work was to furnish points for the topographic and hydrographic survey of both banks of the river below Newcastle, and also to determine the position of the new light-houses built on the shoals of the lower river, great accuracy was not sought, but the rapid and economical determination of the points was the first consideration. As the only two old stations extant were the two old light-houses, the line connecting them was a little uncertain in length, as it was possible that the lanterns were not identical with those observed on by Assistant Blunt several years previously; so it was desirable to check its length at the mouth of the river by closing on the two light-houses there, whose distance apart was known. Several reasons not necessary to enumerate prevented starting the triangulation from the mouth of the river.

The plan followed was to run a scheme of triangulation consisting of quadrilaterals from one base to the other as quickly as possible, and to determine points for the topography and hydrography on each shore as often as were needed. Most of the stations were light-houses, as they were higher than any available points on shore and needed no signals. All stations of the main chain were occupied, most of them eccentrically, some light-houses in the middle of the river needing three eccentric stations, and the chimney of the lantern at each was observed on. The horizon was closed by each two successive sets, and, as I now remember, but four double sets of six repetitions each were taken at each principal station = 48 repetitions or measures. Very few tertiary stations were occupied, most of them being simply fixed by intersections from three to five of the main stations, the object being to determine each tertiary station within a metre and to have always not less than two and usually three or four separate determinations. The work was computed and points given to the topographic and hydrographic parties at once, two of each being at work on the river at the time.

Only one set of six repetitions was taken on the tertiary points, and on short lines two readings, one direct and one reverse, sufficed.

The number of observations taken on this work was less than half usually obtained, and the time correspondingly shortened; still I believe the object of the survey was fully and satisfactorily accomplished.

I think that more observations are taken on all kinds of triangulation than are needed. This is more especially the case on secondary and tertiary work, but applies in a less degree to primary work.

In the latter, when a 20-inch theodolite is used, it is usual to observe in 11 positions and take 3 series on a position. With a 12-inch theodolite 17 positions and 2 series on a position are usually taken. Each series consists of a pointing on each object, telescope direct, the reading of the 3 microscopes, and another round of pointings, telescope reversed, and the reading of the 3 microscopes. The mean of these 2 pointings on the 6 readings constitutes a series.

With a 20-inch we have, therefore, 33 series in 11 positions, and with the 12-inch 34 series in 17 positions.

As it is desirable to read on as many parts of the limb as practicable, so that the final direction shall be free from the errors of graduation, and as the only object of observing more than one series in any position is to check one series by another, I am convinced that but one series should be observed in each position and that as many positions should be observed as may be needed to gain the required degree of accuracy. Two series on the same position rarely differ more than a second, while the *mean* of the series on the different positions usually varies from 5″ on the 20-inch theodolite to 13″ on some of the 12-inch theodolites. Some experiments made with a 12-inch theodolite in connecting the Holton Base with the triangulation convinced me that the mean of 17 positions, *one* series on a position, differed but very little from 17 positions with 2 series in each position, although the probable error was much larger owing to the smaller number of results. Of course there will be a difference of opinion as to the number of observations needed on each angle, but my own opinion is that 23 positions of 1 series each will give as accurate a result as 11 positions of 3 series each or 17 positions of 2 series each, and about 33 per cent of the time spent in observing would be saved.

For secondary work of the usual character, 4 to 6 double sets of 6 repetitions each, or from 48 to 72 measures, will be ample for a repeating theodolite. For a direction instrument on the same class of work an 8-inch circle is large enough, and 7 positions of 1 series each sufficient.

SIGNALS.

For signals we have the choice of poles and heliotropes by day and lights by night. All have given good results under differing conditions. For very long lines poles can not be seen, and heliotropes must be used for day observations.

In many parts of the country, however, where it is necessary to use heliotropes during hazy weather, it is desirable to have poles also, as they can often be seen in cloudy weather when no heliotrope can be used. Often a line whose direction is nearly north and south can be observed in one direction on a pole, especially if a lozenge of white cloth is attached to it, while in the opposite direction the observer must rely upon the heliotrope entirely. My own practice has been of late years to erect a pole at each station. In case of high tripods and scaffolds being used, a central pole 4 by 4 inches and 16 feet long was erected in the center of the tripod. A lozenge of muslin 3 feet by 3 feet was nailed to the pole by a crosspiece facing toward the station being occupied. When the pole could not be seen from all the other stations, heliotrope stands were erected on the rail of the scaffold accurately in line from the center to the stations to which the heliotropes were to show. When the sun is nearly behind the observer looking toward the signal the lozenge shows white and like a faint heliotrope light, when the sun is shining. In cloudy weather neither the lozenge nor the heliotrope can be seen, but the pole itself then shows and can be pointed on directly. On a line running nearly north and south not over 25 miles long, situated in the Ohio River Valley, I never employed a heliotroper at the northern station, but used a lozenge on the pole. On occupying the northern station on the same line I posted a heliotrope at the southern. In the valley of the Ohio I have seen a lozenge a distance of 30 miles when the outlines of all hills over 10 miles distant were obscured by thick haze, and have obtained good observations on it for two or three hours. In the mountains I have frequently observed on poles 12 inches in diameter and 50 feet high (crotch of tripod 18 feet above ground) a distance of 50 to 60 miles during cloudy weather. I have never had much experience with lamps on night work, other than azimuth observations, when they have invariably given me a great deal of trouble. Where the country is flat and wooded, and high signals must be used, we can often reduce the height of the signal to be erected by observing at night on account of the increased vertical refraction. The atmosphere is probably more steady also at night, and a greater number of hours' work at a time can be obtained, and screens to cover the scaffold are not needed: but the use of lamps entails many disadvantages and some dangers. Several high signals have been burned by fires started from exploded lamps which were lighted at sunset by an attendant and left unwatched to burn themselves out. Safety seems to require that the attendant should remain at the station and watch the lamp during the time of observation, and afterwards extinguish it. In a sparsely settled country it is practicable to hire a man who will attend a heliotrope at stated hours, even if he has to walk several miles from his house to do so; but the same man would utterly refuse to remain and watch a lamp at night.

When a primary triangulation party has two observers available and many directions to measure from a station, it would be economical to observe on heliotrope and poles by day and on lamps at night, and thus be possible to finish the station in less time than if only one method were used. Several stations thus occupied would give data to decide the relative accuracy of day and night work. By furnishing the heliotroper a tent and bed, he could easily work the heliotrope by day and watch the light at night for a slightly increased compensation.

It is well to state that day observations on a high signal require the use of screens on the scaffold to protect the tripod from the sun. My experience convinces me that screens are never needed for protection from the wind. If the wind is too strong to observe without screens, setting them will make matters worse, as a much greater area is presented to the wind and there is danger of the scaffold being blown against the tripod. Under such circumstances observations should be discontinued.

Besides the necessary measurement of angles at a station, the △ should be permanently marked and a detailed description and sketch made showing the location of the station and the roads and paths necessary to travel to reach it.

RECONNAISSANCE AND SIGNAL BUILDING.

The manner of making a reconnaissance for a scheme of triangulation varies so much in different sections of the country that it is impossible to submit rules that will apply to all. The reconnaissance of a mountainous country where the peaks are detached and well marked, and the stations can be placed on the ground, requires an entirely different outfit and manner of work from that necessary to be pursued in a nearly flat, heavily wooded country, where the building of high signals is necessary. In the latter country the signal building should be kept well in advance of the observing party, and it will be more economical to have a well-organized building party, under a competent foreman, at work building the signals, under the direction of the chief of party, at the same time that the observing party is occupying stations behind.

When signals of over 100 feet are required and there are railroads that can be used for transportation, it is better to buy timber sawed to dimensions at a mill or town and ship it to the place where it is required than to attempt the building of signals from trees cut on the spot. Wire-rope guys are required on the scaffold while the tripod is being occupied, but if the signals are well constructed none are needed at other times. The △ should be marked securely on the ground as soon as the signal is completed, preferably by using a "vertical collimator" placed in the center of the square hole in the tripod through which the pole is erected. This should be done before the pole is run up into position. On occupying the station this "plumbing down" should be repeated before the theodolite is centered on the tripod to see if the head of the tripod has moved relatively to the mark on the ground, owing to unequal settlement or any other cause, and in case of movement between the time that observations have been made on the pole and the time of occupation a correction for eccentricity can be applied unless the tripod head can be shifted sufficiently to bring center of theodolite over the △ on the ground and the pole recentered on leaving the station. The final permanent marking of the station by means of stone posts or drain tile pipe filled with concrete at the center and establishment of reference marks can best be done while the station is being occupied.

ASTRONOMICAL WORK.

The determinations of latitude and azimuth at the primary stations of a scheme of triangulation can best be made at the time the station is occupied for horizontal and vertical angles. The longitude can best be done by a separate party, properly equipped for the purpose, observing at the nearest telegraph station, which can afterwards be connected with the scheme of triangulation.

LATITUDE.

The latitude can undoubtedly be best determined by the zenith telescope, and a portable instrument of 30 inches focal length, using about 100 observations, will give a result whose probable error need not exceed $0''.05$ if well-selected stars are used. These 100 observations on well-determined stars can be distributed over as many nights as we choose. The usual plan is to select a list of about 20 pairs and observe each pair on five to six nights. This method requires that the sky shall be clear at a certain time each night, or stars will be lost, and they must be observed on another night. If a list of 50 or 60 pairs are selected, running from sunset to nearly sunrise, the whole list may be observed on two exceptionally clear nights, or, in case of clouds at one part of the night and several pairs are lost, the remainder of the list can be observed.

By this method we may have only one observation on many of the pairs and not more than two or three on any, but the 100 (or more) necessary observations may be obtained in two or three nights, while the resulting latitude has been proved to be more accurate. In other words, the more *pairs* observed for the same number of results the more accurate is the result. This method necessitates the observation on the last night of such pairs to finish up as shall make the sum of the north and south differences of zenith distances balance each other for the whole list, in order that the final result shall be independent of errors in the value of micrometer used. Of course the use of so many pairs adds to the labor of computation, as the mean places of more stars must be computed (at present this is done in the office and the results sent to the field party) and more reductions from mean to apparent places computed. It has been found, however, that by computing the stars only for the dates actually observed, instead of computing the apparent places of 40 stars each on four or five dates five days apart, the labor is not increased.

If a list of well-determined stars were compiled and published for the use of the Survey, the field parties could compute their latitudes at once, and the places be still further corrected, if necessary, when the office computation was made. A list of stars was prepared by Prof. T. H. Safford for the use of the International Boundary Commission covering the limits for our work on the boundary line, from which the observer selected his pairs for each night's work and observed them at the most convenient hours of the night. In some cases observations were carried on all night and over 70 pairs observed, and the necessary 100 observations were made up on the next night. In many cases the final result for latitude of the station was obtained in from four to five days after the observations were begun. This extreme haste was necessary in order to give results for latitude and azimuth to the parties running the parallel to start the tangent at one astronomical station and to check its direction at the other.

At Stations No. 1, near El Paso, and No. 4, at the intersection of parallel 31° 47′ with the meridian section, which was also a longitude station, such haste was not necessary. So the old plan of using only 20 pairs and observing them on five or six nights each was tried for comparison. The result showed that 100 observations on 60 pairs gave a smaller probable error of result than 100 observations on 20 pairs.

The value of micrometer should be observed at each station occupied, if possible, but this is not absolutely necessary.

AZIMUTH.

The azimuth of some one line of the triangulation at each astronomical station is required. Observations on slow-moving circumpolar stars afford the best means of measuring the azimuth. Usually we need an azimuth light fixed at a distance of from 3 to 30 miles to use for a night mark, and it is connected with the lines of the main triangulation by the day observations. Collimators have been used instead of a distant light, but, so far as I am informed, they were a failure.

The most favorable time to observe on a star for azimuth is within an hour before or after elongation, when the star is rising or falling and having very little horizontal motion; although, if the time is accurately known, observations may be made at any time. When the azimuth light can be placed nearly in the vertical plane through the star at elongation, the azimuth may be observed very quickly and accurately by using the micrometer of a theodolite or meridian telescope to measure the horizontal angle between the light and the vertical plane through the star. (An example of the method is given in Bulletin No. 21, United States Coast and Geodetic Survey, December, 1890.) A set can easily be taken in twenty minutes, and three to five sets are sufficient for any one night. The number of nights' observations required depends upon the class of work, for it is usual to find more range between the mean results of the different nights than between any two of the sets taken on a single night. For primary work six nights of four sets each night will usually give a result with a probable error of result less than $\pm$ 0''.15, while for secondary work three nights of three sets will give a probable error of from $\pm$ 0''.20 to $\pm$ 0''.30, depending in both cases almost entirely upon the difference in the mean results for the different nights and not on the whole number of sets taken.

A sufficient number of sets can be taken in less than two hours for any one night, and if these are observed within less than one hour of elongation the time becomes of little consequence, so that sextant observations will give a result near enough for the chronometer error.

When the light can not be placed in the proper position for using the micrometer, the angle between the star and mark is measured the same as any horizontal angle, except that the times of the pointings on the star are marked by chronometer; and it is preferable by this method also to observe on the star near elongation. When a light can be fixed near the horizon and in the meridian plane passing through the station, the azimuth can be measured with an astronomical transit, fitted with an eyepiece micrometer, by observing the transits of close circumpolar stars both at upper and lower culmination for the azimuth error of the instrument and obtaining the error of chronometer by transits of high and low stars. If the transit is mounted over the △, it must be removed and a theodolite mounted in its place to transfer the observed azimuth of the mark to a line of the triangulation, which is often inconvenient. At Moons Mountain △, North Carolina, this difficulty was obviated by mounting the astronomical transit inside the observatory, accurately in line between the theodolite, mounted over the station, and the azimuth light placed on a hill 10 miles distant. The transit was mounted enough below the axis of the theodolite to allow the azimuth light to be seen over the axis of the transit, and the post holding the lamp at night became a signal for the theodolite during the day observations and one of the regular directions observed, so that it was combined with every direction measured from the station.

This arrangement allowed an independent set of azimuth to be observed with the theodolite in the usual manner for a check of the result by the transit.

Besides these two independent results for the azimuth of the mark, we were enabled to use the telescope of the transit for a collimator, and by setting the micrometer wire to the reading of line of collimation when the transit was playing exactly in the meridian plane (as determined by the azimuth correction obtained for the transit at night) we could point on this wire with every set taken for horizontal directions made during the day.

The result, like all other attempts to use a collimator instead of an azimuth light by me, was a failure because of the movement of the telescope in azimuth. This transit was a large 48-inch Troughton & Simms, with a heavy iron frame mounted on a brick pier.

A specially constructed collimating telescope with low wyes *cast* on a heavy iron plate, which plate was set in cement on the top of a solid brick pier, with a brick arch covering the whole collimator, and the whole instrument covered with wooden house the sides and roof of which were double, was also tried at Elliotts Knob, Virginia. The opening in the front of the house toward the theodolite was only a hole 1½ inches in diameter, and the wires of collimator were illuminated by reflecting the light of a lamp *outside* the house by means of a mirror through the collimator.

This collimator was used as an azimuth mark for night observations, and was pointed on in each set of directions measured in the day. It was found that the collimator was twisted during the day, following the sun, but returned to nearly the same position during the night.

SECONDARY ASTRONOMICAL OBSERVATIONS.

For an astronomical reconnaissance where extreme accuracy is not required, observations for time, latitude, and azimuth can be made with an altazimuth instrument, or vertical circle, as it is called on the Survey. A Gambey repeater of 10-inch circles, reading to five seconds by four verniers, and fitted with a delicate level (value 1 div.$=2''$), was used by the writer during the war to determine points for a military map of West Virginia, in 1863. The instrument was mounted on a post of wood set about 2 feet in the ground, and the operations of a night were as follows:

Observations of two east stars, measuring two sets on each star, near the prime vertical.

Observation of two west stars, of two sets each, in the same manner, each set consisting of six repetitions. The mean of these eight sets gave a result for time.

The chronometer was then taken to the telegraph office, and signals were exchanged by telegraph with the Naval Observatory, after which another determination of time was made in the same manner for rate of chronometer.

Latitude was then observed on north and south stars by circummeridian zenith distances. Twelve sets of six repetitions each, as evenly divided as possible between north and south sets, were observed. The time necessary for observing a set of six repetitions was four to five minutes, either for time or latitude. This allowed one set to be observed on a quick-moving south star, while on a slow-moving north star it was possible to measure three sets—one before culmination, one on meridian (nearly), and one after culmination.

The same method was pursued in the survey of the Tennessee, Ohio, and Mississippi rivers in 1864–65, except that the longitude was then obtained by transportation of chronometers.

The accuracy of results obtained was as follows:

Time could be determined within a probable error of $\pm 0''\cdot2$.

Latitude by one night's observations gave an average probable error of result of $\pm 0''\cdot7$.

RECORDS.

Where it is impracticable to make the original record in ink, as often happens when an observer has to read the microscopes and also keep the record, the original should be duplicated as soon as possible in ink. The means having all been taken by one person, should be carefully checked by a second, and the abstract then made out by one person and checked by another.

The original, in pencil, should be carefully compared with the duplicate in ink, and then the original should be inked and the two again compared. After this has been done, we no longer need either the original or duplicate record in the field, as the abstract gives us all the data we need for computing. When the station is finished, the original should be sent to the office, the duplicate remaining in the possession of the chief of party. On receiving notice from the office that the original has been received, the duplicate can then be sent.

FIELD COMPUTATIONS.

A rough computation of the "triangle sides," showing how the triangles close, the difference in length of sides by two determinations, etc., must be kept up to date to see how well the work is being done. For this purpose it is not necessary to apply the correction for run nor to make any least square adjustment either for station or sides and angles. The position computation should also be kept up, so as to be able to know the station errors in latitude and azimuth at each astronomical station.

At the end of the season these computations should be revised before being turned in to the office; but as the office computation is made entirely independent, and is the only official one used for results, the field computation is of use only to give approximate results more quickly and to check approximately the final office computation.

ACCOUNTS.

As the accounts of field officers must be made out promptly at the end of each month and in *addition* to their duties as surveyors, the amount of clerical work necessary to be done by the chief of party, usually with no clerical assistance, should be reduced to a minimum, and as much of this work as is possible should be done in the disbursing office. Accounting officers too often forget that the keeping of the accounts of a field party is only incidental to the scientific work done by the parties, and that the only time the chief has to devote to accounts is in the intervals between more important work; and for this reason the amount of writing and duplicating should be made as small as possible.

When originals of any accounts for a month are received by the disbursing agent, he should acknowledge them *at once*, so that the field officer may know they are received and will be placed to his credit after examination.

All decisions of the Comptroller in regard to accounts, especially new requirements, should be sent to each chief of party *as soon as known*, and he should not, as is too often the case, be left to discover changes in methods by the return of one or more sets of accounts "disallowed" or "suspended."

LETTER FROM HERBERT G. OGDEN, ASSISTANT.

UNITED STATES COAST AND GEODETIC SURVEY OFFICE,
Washington, D. C., February 6, 1894.

I have the pleasure to acknowledge the receipt of your favor transmitting an invitation from the Conference to submit my views on the subjects presented by the Superintendent for the consideration of the Conference. It is evident that a number of these question can only be satisfactorily considered after an examination of the results now on file.

BASE-LINE MEASUREMENTS.

Recent investigations of line measures confirm our former experiences in the reliability of measures with the tape in certain cases, and show that with improved appliances the tape may also be used where the greatest accuracy is desired. How far it would be practicable to substitute base lines of lesser accuracy with the tape or some other appliance, but with greater frequency than has heretofore been practiced, I think can be best determined by an examination of the schemes that have been completed under a system of base lines and angular work of great precision.

In primary work no diminution should be permitted in the accuracy of the results; but in relation to smaller work—that is, tertiary or subsidiary between the stronger schemes—it is quite possible that such work, being checked by its surroundings, could be executed with a less degree of refinement and be perfectly satisfactory.

TRIANGULATION.

I believe that the classes of trigonometrical work have heretofore been defined more by the length of the sides than by the character of the work. It has always seemed to me that this led to greater or less confusion, as many schemes embracing sides that ordinarily would be considered secondary in reality form a piece of primary work. I therefore suggest that "primary triangulation" should designate only the scheme that is recognized to be the best that could be carried through the region, limited by a minimum length for the sides of the triangles; the "secondary triangulation" to be a scheme admitted not to be the largest, nor of primary value, also limited by a minimum length of triangle sides. Anything smaller than secondary work would be tertiary, and those schemes to be used exclusively for the detail to be designated "subsidiary." In describing a piece of work under such a plan, we might have a "primary triangulation" with secondary sides or a "secondary triangulation" with primary sides. These descriptions would convey quite clearly the strength of the work and the general character of the scheme.

It will doubtless be admitted by all that primary work should have the greatest strength that can be obtained with a reasonable degree of effort and outlay. But just what is reasonable it would be hard to define by rules; and it seems to me, therefore, that the primary work of the Coast Survey must be left, as heretofore, almost entirely to the judgment of the officer executing the work. It may be that the greatly improved instruments we now have will permit of the establishment of a limit of probable error for primary angles, and that when this has been obtained in the observations no further measure of the angles should be made, of course the observations being qualified by a stated number, as the minimum of repetitions for an angle. A similar rule, or simply to limit the repetitions, might be sufficient in secondary triangulation and all smaller work.

I believe it is quite possible at times to reduce the checks required in a triangulation. That would in some instances lessen the number of stations to be occupied and the number of angles to be measured. With imperfect instruments we naturally feel that the check should be strong; but with those now available I see no reason why plain triangles should not be occasionally introduced without fear of materially affecting the reliability of the resulting work. The quadrilateral, we may admit, affords all the check that is necessary for the strongest work. Upon the reduction of the quadrilaterals we can judge of the value of a triangle and the reliability of the instrument used, and therefore where difficult ground is encountered that could be readily overcome by the use of the single triangle but would require great labor to spread a stronger figure, it seems to me that the triangle might be accepted, provided, of course, that the instrument has been demonstrated to be a reliable one.

The schemes of triangulation proposed in late years for the complete survey of the whole country are doubtless quite sufficient. The gridiron system has entirely supplanted the original suggestion of Professor Peirce, to cover the whole territory of the United States with a figure that would form one huge quadrilateral. The later projects are probably more easy of attainment, and will, I believe, afford a better distribution of the work for the subsequent delineation of details, and probably will be quite sufficient for combination with the work abroad in ascertaining the figure of the earth. The projects of other nations will doubtless be studied in this connection, to fully determine the relations that may be practicable.

My brief experience in Alaska was quite sufficient to satisfy me that any attempt to carry a primary triangulation from the mountain tops would be one of great labor and expense, arising from the great prevalence of cloud and mist, obscuring the view from any considerable elevation. For days at a time the mountain summits are in clouds, while along the water it is clear and good observing weather. A triangulation near the water, or not many hundred feet above, would probably be the most practicable if we consider time and expense; and there are many hills of 200 or 300 feet in elevation that could be utilized in such work, permitting generally a much better scheme than could be conducted from the shore line directly.

The work we now have in Alaska is doubtless quite sufficient for local purposes of cartography, and I believe it is very questionable if we require a stronger scheme laid over it until such time as the country indicates a far greater development to be possible than we now have any reason to anticipate.

From the northward and westward of Cross Sound, the northern boundary of the Alexander Archipelago, we have a long stretch of coast to the Aleutian Islands, over which it would be difficult to carry a triangulation of precision, principally in consequence of the fogs which usually prevail in the summer season; but I am firmly of the opinion that no attempt should be made to conduct work of that order along this stretch of coast. A first-class reconnaissance, I believe, will be sufficient to develop the region to the full measure of its value.

In extending the work over the Aleutian Islands the reconnaissance can be strengthened by basing it upon a triangulation, but I doubt if it should be a work any stronger than is necessary for cartographic purposes.

To the northward, along the shores of Bering Sea, I believe a first-class reconnaissance would be all sufficient, for the same reasons I have assigned for that section of the coast north and west of Cross Sound. Probably one of the most important places to be examined—the mouth of the Yukon River—is within this section. There have been in the past several reports that there is a deep-water entrance in one of the mouths of this river, and I should consider it desirable to have an examination made of the whole delta at the earliest practicable date to determine the most available entrance to the river.

IN GENERAL.

It does not seem to me possible to make any fixed rules to govern party organization, camps, outfit, etc., as each case must depend very largely upon the nature of the country and the importance of the work undertaken, especially in regard to the necessity for immediate execution. Where there is ample time, party outfit can be considerably reduced and the expenditure distributed over a longer period than would be deemed desirable if the work were in a locality that required the results to be obtained in the shortest possible time.

Rules to govern the degree of accuracy required in a field computation should be suggested by the chief of the computing division; but I think we should bear in mind the desirability of reducing the labor of the field computation to the minimum, for to produce a complete reduction frequently requires an expenditure of time on the part of the field officer in the preparations of computations, afterwards duplicated in the office, that might be delegated to the experts of the office force in the first place, thus relieving the field officer for new operations in the field.

As I have had but one season's experience in the field for a number of years, I do not feel justified in criticising the question of accounts and relations of field officers to the disbursing agent. It gives me pleasure to put on record, however, that the single experience I have had of late was entirely satisfactory to me, although I found very many changes had been made in the system of rendering accounts since my previous field service. And it seems to me that the present regulations permitting supplemental accounts for arrears that could not be included when the party accounts for the month were rendered affords every facility that is essential for the proper classification of expenditures under dates and periods.

LETTER FROM HENRY L. WHITING, ASSISTANT.

UNITED STATES COAST AND GEODETIC SURVEY,
West Tisbury, Mass., February 6, 1894.

My presence in Boston and attention to matters connected with the publication of the report of the State commission, which could not well be set aside, is my excuse for not sooner replying to your letter relating to the Geodetic Conference and the invitation to present in writing my views or comments on any of the subjects mentioned.

In response I feel induced to submit a few remarks on the topic of "Schemes of triangulation, necessary and desirable, bearing in mind their utility in fixing boundary lines (State and national)."

I am led to this not as offering suggestions to yourself and the other Assistants of the Survey who have had more experience than I in this branch of our work, but to give such information concerning the practical execution of the Massachusetts town boundary survey as my connection with it, and, in fact, direction of it, as the executive member of the State commission, has enabled me to possess, and that may bear on the subjects you have under consideration.

This survey of Massachusetts is, so far as I am aware, the first work of the kind systematically undertaken by any State, and is therefore, from lack of precedent, largely experimental in its operations.

At the outset of the Massachusetts surveys the commissioners, foreseeing the necessity of very considerable additional triangulation beyond that of the Borden survey and the far-reaching stations of the Coast and Geodetic Survey in the interior of the State, and as Congress had already provided that the Coast and Geodetic Survey should furnish necessary triangulation to any State making its topographical or geological survey, the commission applied to its superintendent to supply such additional triangulation to Massachusetts as might be needed in the proper execution of the contemplated work, to which a most cordial affirmative response was given.

The vague and inaccurate manner in which the boundary lines of many towns are described and the imperfect way in which they have been located are facts within the knowledge of nearly every selectman or those who have had to do with municipal affairs. A good topographical map, upon which the locations of town lines are accurately shown, will be a great auxiliary in identifying the positions of the various points and angles in them; but the determination of the lines should not be an incident in the survey or depend upon the plottings of the map, but should, on the contrary, be effected with such precision as to be a basis for the survey itself and form a framework, as it were, to which all other surveys within its limits could be referred.

The laws of Massachusetts relating to this matter are not sufficiently definite or comprehensive.

By the provisions of the Public Statutes (1882), chapter 27, sections 1 to 6, a duty is imposed on the selectmen of towns, under a forfeiture of $20, namely, "There shall be a perambulation of town lines, and they shall be run and the marks renewed once in every five years." There is no system of survey nor manner of running these lines prescribed, and it may and does occur that adjoining towns use different ways and means in performing this work, with different degrees of accuracy, which must of necessity give different results. In many cases the only bases for the location of a town boundary are in the monuments marking the angles in their respective lines. These are often in hidden and inaccessible places, on the summits or in the gorges of wooded mountains and hills, where their positions in relation to other points can not be accurately determined by ordinary processes of surveying.

The system of determining town lines by triangulation was a part of Mr. Simeon Borden's masterly scheme of the State survey, devised fifty years ago, and was partially carried out and several points in the State boundary line were included in his triangulation, but the determination of town boundaries was not practically carried out. Such a plan would give to the State government, and to each municipality in it, a perfect record of the exact boundary of each city and town in the Commonwealth, and in so methodical and precise a manner that any city or town engineer or ordinary land surveyors could reproduce the exact position of any point in any line. The system of record should be uniform with that of the other geographical positions determined by the triangulation of the State survey, giving the latitude and longitude of each point, probably within 1 or 2 feet of its true position; whereas, as now described, many angles and even whole lines are probably out of position hundreds of yards, and in some places hundreds of rods.

In their various reports to the commission the field officers refer to continued embarrassment in conducting the survey because of the uncertain and imperfect records of township boundaries and the difficulty of obtaining reliable information in regard to them. Many citizens officially interested in this class of public matters are unaware of the nature and detail of the present survey, and of how much the State is doing through its commissioners that should be done by its municipal officers. If the perambulation records were even geographically described, and distinguishable marks put in place to indicate where the boundary angles and corners were, the labor and cost of their accurate determination by triangulation would be much lessened. It may not be out of place in this connection briefly to describe some of the detailed operations of the present survey.

On going into a new section the officer in charge first gets from the selectmen of the towns concerned copies of the latest perambulations. In cases where the selectmen are in doubt as to a certain corner or angle, they look up older perambulations and confer with former selectmen in regard to them. After getting all the information obtainable, officers of the survey go on to the ground, taking up one town at a time and finding successively every corner in the line. They then ascertain, by climbing trees and going on the top of hills nearest each boundary point, what known stations, or available sites for new ones, will command the respective corners or boundary points, and decide as to the height of the signal necessary in each case. The height of the signals to be erected varies from 8 to 100 feet. The shorter signals are usually built directly on the monument. Where a signal has to be from 50 to 100 feet high, it is generally in consequence of wooded ground, where tall, straight young trees can be cut, and, if necessary, several of them spliced together. Three wire guys are then made fast to the signal pole at each place where it is spliced. A fall and block is made fast to the signal and to some suitable tree near it, and the signal is raised into place. Where it is not possible to make a boundary corner a point of triangulation, a station is selected as near the corner as practicable, and a traverse run to the boundary point. These traverse measurements are made with a steel tape. Where a bend in the traverse line is necessary, the angle of deflection is carefully measured with the theodolite. Each evening the angles taken during the day are checked and an abstract made of them. A rough "triangle sides" computation is also kept up.

The work of locating the town boundaries by triangulation is a much greater undertaking than was supposed at the outset. In fact, it involves a second triangulation of the State and a determination of some thousands of points in addition to the few hundred which the first survey by Mr. Borden comprehended. But its value when completed is so evident that no one who understands it can fail to see that it is a work of eminently practical economy. The present condition of the records of town boundaries is simply scandalous in a State like Massachusetts. The law of the Commonwealth, which provides for the perambulation or "running out" of all the town boundaries once in five years, was well devised to secure the intended object, namely, the preservation of town boundaries; but the implied requisition of a quinquennial resurvey of the lines is practically a dead letter, all the towns justly regarding it as involving an unreasonable and unnecessary expense.

The general custom throughout the State has been for the selectmen of every two adjacent towns to visit together the monuments of the boundary as described in previous perambulation records, and if the monuments were found standing a new perambulation record was filed as required by law, usually copied from the former one, not infrequently perpetuating gross errors and sometimes introducing new ones to be copied in future records. Since each new set of perambulation records is signed by the representatives of the two towns interested, subsequent dispute or litigation over the locations of town boundaries would seem to be debarred. But as years pass on there is great danger, even with this precaution, that confusion will ultimately arise from the uncertainty attending the positions of monuments remote and inconveniently accessible from human habitations. In fact, a comparison of the later perambulation records of many towns with ones, more ancient shows that changes have taken place in boundaries as at present recognized, quite important in the amount of territory involved, but unauthorized, by legislative action. Other perambulation records are so vague and indefinite in their descriptions of locations, directions, and distances that it would be difficult, if not impossible, to say where the real boundary was to be found. Sometimes the actual boundaries as recognized by the selectmen are essentially different from those given in somewhat recent acts of incorporation, and seem to require confirmatory legislative action.

It often happens that the knowledge of the location of a sequestered monument is confined to the memory of a single person, whose age and decrepitude make it difficult for him to go to the place and point it out. There are many boundary monuments of whose locations no available record exists sufficient to restore them if personal memories of them should be lost by the death of the person or persons possessing these memories; nor could any change of locations arising from carelessness or malicious mischief be detected. After the work now in progress is completed, whose cost, at the rates of the work already done, will not be likely to exceed the cost of a single "running out" of the lines by the older methods of chain and compass, there can be no danger of losing the position of a monument, since it can be restored with absolute certainty from the chain of triangulation points upon which the whole work is based.

QUALITY OF THE MASSACHUSETTS WORK.

In order that the town boundary survey of Massachusetts might have the benefit of the criticism of the Coast and Geodetic Survey, and its indorsement if deserved, the State commissioners requested the Superintendent to inform them of the result of the examination of the data submitted for that purpose. The following are quotations from the report of Mr. Schott upon the subject:

COMPUTING DIVISION COAST AND GEODETIC SURVEY,
Washington, D. C., January 20, 1891.

DEAR SIR: At the request of the commissioners for Massachusetts, and for their information, I have the honor to report to you the present state of the computations covering the triangulation made in the years 1885 to 1889, inclusive. The work comprises mainly triangulation needed for the determination of the boundaries of counties and towns.

The computations and adjustments were intrusted to Mr. E. H. Courtenay, of the computing division, one of the most experienced computers in this branch of the Survey. In general, the method employed is that stated by me in the introduction to the "Geographical positions in the States of Massachusetts and Rhode Island," Appendix No. 8, Report of the United States Coast and Geodetic Survey for 1885. The principal figures for adjustment by the method of least squares were laid out and attended to by Mr. Courtenay, and assistance was given him for the solution of the numerous equations involved and for all mere clerical work. The services of Mr. Daniel L. Hazard were furnished by the commission, and these were given in the most effective manner since he reported here for duty, December 13, 1889.

Two thousand five hundred and ninety-five triangles, of which about three-fourths are completed, will ultimately be placed in the hands of the commission, as well as 1 112 positions of trigonometrical stations, of which about one-half are at present completed. The boundary work of 1885-1889 thus exceeds in results all that had previously been done in the same region, and nearly equals the work previously done in the entire State by the Coast and Geodetic Survey, inclusive of Borden's; and it should be borne in mind that this new work covers but two-fifths of the area of the State.

The quality of the work is somewhat inferior to the best of the secondary work previously executed. This is owing in part to the great number of eccentric stations employed and the numerous spires observed and occupied, and yet I consider it sufficiently accurate for the purpose intended. The work is so interlaced that no serious error could exist.

Yours, respectfully,

CHAS. A. SCHOTT,
Assistant, in charge Computing Division.

Dr. T. C. MENDENHALL,
Superintendent United States Coast and Geodetic Survey.

Considering the peculiarities of a municipal boundary survey like that of Massachusetts in relation to the difficulties, labor, and expense of its execution, the determination of points—"corners" and "angles"—in the various lines, is, perhaps, the best criterion of the value and amount of work accomplished in a given time and at a given cost, rather than the number of townships surveyed or the area of country included in them.

The method of determining special points previously located, in obscure as well as in accessible places, differs essentially from that of ordinary triangulation, where the points are selected, as far as practicable, to form well-conditioned triangles, and where, if a location does not answer this purpose, another is sought for that will do so. But in the case of boundary points, many of which are in thick woodland, in hollows and swamps, and otherwise inaccessible positions, the scheme of work must be so arranged as to reach and determine each individual point. In other words, the triangulation must conform to the situation of existing points since the situation of points can not be arranged to favor the triangulation.

PARTY ORGANIZATION—CAMPS AND OUTFIT; POSSIBILITY OF REDUCTION IN THEIR SIZE AND COST.

This question is one of the most important and critical we have to deal with.

Health, strength, enthusiasm, and the consciousness of being well supported and well supplied are large elements of success in sustaining the mental and physical incentives to labor in performing the work required in the field operations of the Coast and Geodetic Survey. I need not quote the military philosopher who has said that to defeat the enemy he only asked the control of the commissariat. Good tents or quarters or cabins, good beds, cots, or berths, promoting rest and sleep, and good food should be provided. I believe it true of all the field corps of the Survey, that they are ready and willing to bear exposure and privation in emergency; but to be on "short commons" throughout a long season is not in the logic of best economy or best results.

For the transportation of the personnel, instruments, and equipments of our field parties the most approved appliances should be provided. We see this principle exemplified in organizations where efficiency and economy are the keys to the situation. In the transportation of our mails and the matter of our large express companies, the best horses, best harness, and most suitable wagons. In work that requires the use of vessels and boats, they should be adapted to the nature of the survey, the climate, and the waters in which they are to be used. Crews should be sufficient for the most effective service.

With regard to instruments, the late Topographical Conference considered and reported at length, and I think with wisdom, on that subject. I have lately received a letter in acknowledgment of a copy of the report of the Topographical Conference in which the writer, who is among our best instrument makers, says:

> From a perusal of it I find it to contain much of interest to us and to our profession, and as soon as time permits we shall be glad to act on some of the suggestions made by the experts of the Coast and Geodetic Survey so far as they relate to improvements in the instruments we manufacture.

This is, at least, one of the good points of our conference.

For the examination, repair, and adjustment of instruments intended for the field, the office of the Survey would seem to be the proper place.

I hope my colleague will bear with me in these perhaps unnecessary remarks, when they bring to mind the fact that in some of the branches of the Survey I have had an experience of a good many years, both with my own party and in visiting many others of the Survey. I can at least give this testimony, that so far as my observation has gone the parties that have been the best equipped are those that have done *the most* and the *best work.*

LETTER FROM EDWARD GOODFELLOW, ASSISTANT.

UNITED STATES COAST AND GEODETIC SURVEY,
Washington, D. C., January 27, 1894.

In response to your recent letter conveying an invitation to present in writing my views on any or all of the subjects outlined for discussion by the Geodetic Conference, I would submit a brief statement with regard to base measurement.

It may be assumed, perhaps, that as in the past there have been needed in the progress of the Survey some bases measured with a high degree of accuracy and a larger number measured with less refinement, so it will be in the future, and that the study of the highest degree of precision of measurement will continue, due regard being paid to economy in cost and time.

The table* which I present to accompany this communication, giving data in regard to the chief primary bases measured since 1845, may be of service to the Conference.

LETTER FROM F. W. PERKINS, ASSISTANT, INCLOSING A COMMUNICATION FROM FOREMAN E. E. TORREY RELATING TO THE BUILDING OF HIGH TRIPODS AND SCAFFOLDS FOR OBSERVING PURPOSES.

WASHINGTON, D. C., *February 24, 1894.*

I hand you herewith an article upon signal building which was prepared at my request by Foreman E. E. Torrey, whose extensive experience in building high tripods and scaffolds gives it special value:

NOTES ON SIGNAL BUILDING.

[By E. E. TORREY.]

The signal may be framed at any convenient place, the pieces numbered, and then hauled to the △. If the framing is to be done on the ground it is best to separate the lumber into piles of the different lengths, so that each desired piece may be readily selected when required by the carpenters. The timbers not needed in the earlier operations, such as the inside braces, floor scantlings, and stairways, should be used as sleepers to make a level surface for accurate framing. The flooring and other boards may be made into a temporary shed to shelter the tools and ropes from the weather.

Up to 75 feet in height one side of tripod, completely framed, may be raised at once. The derrick should be at least two-thirds of the height of the structure to be raised. If a suitable tree should happen to be in a convenient place, it may be utilized as a derrick and afterwards cut down and removed; but one can be quickly made by using the lower two sections of the scaffold legs, the top ends lashed securely together and the bottoms spreading about 3 feet apart, like a pair of shears. Temporary cleats should be nailed along its length to keep the two parts from spreading. This makes a good derrick and from material already at hand. Use three guys to the derrick. The back guy, which should lead directly opposite from the weight to be raised, should be very strong and preferably a tackle. The other guys, which should lead at an angle of 120° from the back guy (and from each other), may be of

*Table embodied in the report of the Committee on Base Lines.

lighter rope, as but little strain should come on them. Should the scaffold legs be of light material, it would be well to put a middle set of guys on the derrick to prevent its "buckling" under the strain. For the side guys strong posts set 4 feet in the ground at an angle inclining from the △ will be sufficient to secure to, but I would not risk such a fastening for the back guy. If there is no natural object, such as a tree trunk, in the desired direction, a pit should be dug 4 feet deep and 6 feet long at right angles to the guy rope. At the bottom of this a round log 6 inches or so in diameter is placed, with a stout strap around its middle to hook the tackle of guy to. A narrow trench should be dug sloping from the middle of the log (or "dead man") to the surface of the ground, so that the direction of the strain shall be in a straight line from the log underground to the top of the derrick.

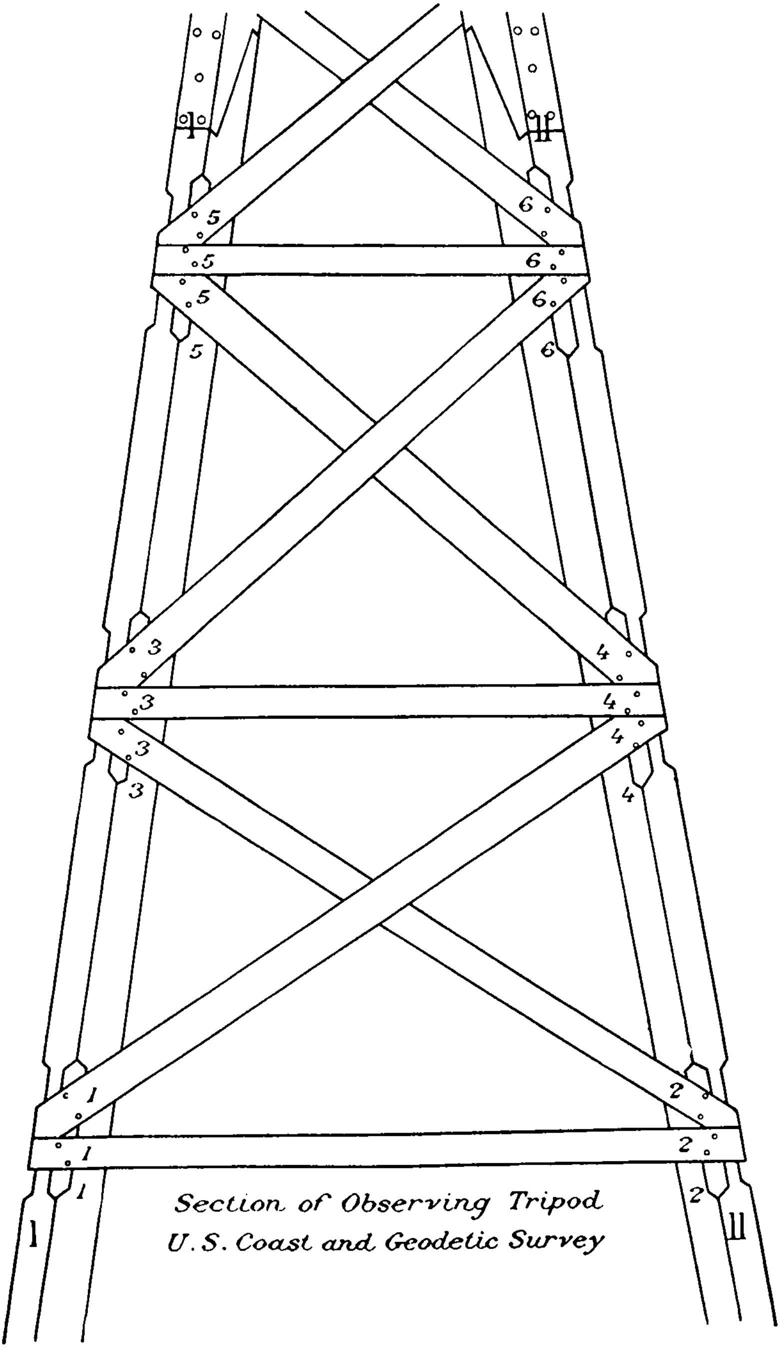
For hoisting, a portable winch with pawls, secured to the foot of the derrick, is desirable; but if the hoisting is done through a leading block with horses or other power, it should be seen to that the foot of the derrick is well secured, and it would be well to have a man with a snubbing rope on the fall to control the hoisting.

In digging the holes for the foundation do not spend time in getting the bottoms all exactly on the same level, especially if the ground is sloping or uneven, but dig one of them to the depth as required by the plan, and use it as a bench mark for the others. After the platforms are tamped in the holes and the shoes set, get a round of levels of the different platforms and saw off the lower ends of the tripod and scaffold legs either above or below the mark of the plan length, as the differences in depths of the holes require. Care must be taken to get solid foundation for the platforms to prevent settling of the signal when up. In marshy ground dig the holes deep, lay a tier of small logs across the bottom, ram them, and cover with a layer of dry earth before setting up the plank platforms.

FRAMING TRIPOD.

Make scarfing for splices as described by Assistant C. O. Boutelle; that is, 3 feet in length, with hooked ends, and use 5 bolts to each splice. When the lower lengths of the tripod legs are of larger dimensions than the upper, which they should be in a high signal, an offset should be made in the larger sticks, so that the outside faces of the completed legs may be in a continuous line. The splices should be fitted snugly together, the bolts inserted, and nail holes on ends of the scarfs bored. The nails are not driven at this time if the tripod is to be raised in sections. The legs, now bolted to their full length, are laid on the sleepers as level as possible and measured and marked for the chamfering. Measure the length of each section of leg to make certain that no part of the bracing shall come on a splice. Beginning at the top ends, make a mark across the back with a T square, and with the bevel square along the sides. The angle at which the bevel should be set may be computed or taken from the working plan. The full length of the leg, as taken from the plan, is measured carefully along the leg, and similar marks made parallel to those at the top. Beginning at the top again, the distances to each of the cross braces are measured and marked. The measures should be verified to prevent mistakes. The top ends may now be sawed off on the marks, and the bottom ends either so much above or below the mark of the plan length as the difference in levels of the foundation platforms may require. The inside of the top ends are cut to angle 120°, so that the three sticks may come snugly together at the top when up. Make chamfers 1·5 inches and 2·6 inches as bearing surfaces to nail the braces to. They do not materially weaken a 6 by 6 inch timber. It is convenient to make a pattern for this chamfer by nailing two strips of wood together edgewise, 18 inches long and the required inside measures. All the chamfers on the three legs being made, two of the legs are stretched along the sleepers with their top ends near together and the bottoms spreading apart the required distance. The legs are blocked to rest on their edges, bringing the broader chamfered spaces on top and horizontal. The cross braces, carefully measured from the working plan, are nailed temporarily, by spikes driven partly home, to the places marked for them on the legs. Cut these cross braces a little larger than the measured length, so as to give a small overhang, which lessens the chances of the ends splitting from the nail holes. It is not necessary to measure the diagonal braces, but when two of the cross braces have been fastened to the legs "square" that section. This is readily done by moving one or other of the legs until the distances between opposite diagonal corners of the section coincide. Then lay the diagonal braces in their places and mark them to be sawed so that 3 inches or so of their ends shall rest snugly against the adjacent cross brace. Nail them temporarily with one spike at each corner, and bore the holes for the second nails and at the intersections. Square and frame each section separately until a side of the tripod is completed. After numbering each piece, the side is all taken apart and the braces laid adjacent to the side where they will be required in building.

The following is a convenient way of numbering the small timbers, and if, when the braces are removed, they are laid with odd numbers to the left and even numbers to the right and facing the △, they will be in the right position to be sent aloft to the carpenters in the final building. (Illustration No. 18.) No. I leg is now put on one side, No. II is brought to the place of No. I, and No. III to the place of No. II, and the second side framed as before and the pieces laid to their proper side. No. II leg is now finished as far as the framing is concerned, and is laid aside. No. III leg is brought to No. II's position and No. I brought to where No. III was. When all three sides have been thus framed and the braces removed, nail cleats along the full length of each leg for the carpenters to climb by. These may be made from 1 by 4 inch strips sawed 14 inches long and nailed 15 inches apart.



Section of Observing Tripod
U. S. Coast and Geodetic Survey

The scaffold may be framed now or later. In showery or unsettled weather keep on framing, as it is work that can be suspended at any time, while when raising, the workmen should not leave until everything is secure. In a high signal the multiplicity of the braces on the ground may make confusion, so, if possible, commence raising the tripod as soon as it is framed. Raise and complete the tripod to its full height before building up any of the scaffold. Up to 75 feet a whole side of tripod may be raised at once, but in higher signals it should be raised in parts. It being determined to take the two lower lengths of a side at the first lift, the splices of that section should be tightly bolted and the nails at the thin ends driven, and the legs placed so that their lower ends overhang the holes that are to receive them. The cross-ties and the diagonals of the part to be raised are spiked to the legs, care being taken that each nail is driven home and in the same holes that were made at the first framing. The nails at the intersections and the second nails at the ends of the braces are tightly driven. Double guys are attached to near the upper ends of the legs and hold-backs to the feet. These may be of a single stout piece of rope or a small tackle, and are secured to posts set in the ground for the purpose, and are to be eased away during the process of hoisting so that the feet may come naturally to the shoes provided for them. A stout rope is fastened as a bridle to the two legs a short distance from the top, and the block of the hoisting tackle hooked into its bight, the free end of the fall taken to the winch, and the framework raised to its position. The guys are now tightened and secured and the structure made as rigid as possible. The third leg of the tripod is now bolted together (or as much of it as is purposed to raise, say one length longer than that already raised). No braces are fastened to this leg before raising, and with a winch a rope through a single block at the top of the part already raised is sufficient to lift it. Two sets of guys, one near the top and the other about the middle, are needed here, with a man at each set of guys to guide the timber in the raising. When set to the required inclination, it is held in that position by the guy ropes, and small blocks with hauling lines rove through them are attached to each leg a short distance above the lower section. The carpenters with their heavy hammers and extra spikes take their positions on the legs, and the lower and second cross-ties are sent up and nailed to the marks. The diagonals of these sections are also sent up by the hauling lines, and the slope of the legs adjusted by means of the guys so that the nails of the braces will come to the holes made in the first framing, and all nails in the section are driven solidly. Before carrying the bracing any higher, fill up the foundation holes, nailing solid logs as anchors to the feet of the legs and ramming the earth as it is shoveled into the holes. The small hauling lines are shifted higher on the legs and the bracing is carried up until the tripod has its three sides braced equally high and one length of the third leg projecting above the rest.

The heavy derrick is now taken down and apart and the winch secured to the third tripod leg, where it remains for all the other operations. A single block is secured to the top of the projecting III leg, and the end of fall leading down inside of tripod is bent on to a length of leg I (or II), about one-third from its top, and hoisted by the winch. When free of the framed part, guys are bent on and the hoisting continued until a man at the top of the leg I (or II) can pull the foot of the length hoisted, over to its splice, and, holding the two parts of the splice together with a few turns of rope, the guys are pulled out and hoisting rope slackened until the two parts of the splice come into place, when the splice is secured by its bolts and nails. The corresponding length is raised in the same way, and the bracing on the three sides carried up to their tops. This leaves the tripod braced to an even height, with no projecting leg to use for a derrick for continuing the building. The next length of leg is sent up as a topmast is sent up a mast, and is temporarily used as a derrick. The hoisting rope is bent on at the foot and "stopped" at intervals along its length. A man, at the top of the completed part of the tripod, bends on the guys and a single block and fall as the top of the length being raised reaches him, and takes a few loose turns of rope around both the completed leg and the part being raised to act as a "cap" or guides, while another man follows up the foot to keep the stick from getting a cant or slue while being raised. This derrick should be hoisted so that at least three-fourths of its full length is above the completed part of tripod, when the lower part must be securely lashed to the standing tripod leg, the guys tightened, and the fall of the hoisting rope sent down inside the tripod. The length to be raised by this derrick should have the hoisting rope bent on just above its center of weight, the guys handled, and the splice brought to its place on the completed part in the same way as described before. The corresponding length being raised in the same way, this side is braced to its top, the hoisting rope shifted to the higher point, and the length used as a derrick hoisted the few feet necessary to bring it to its splice, and the bracing completed evenly on the three sides. And so on until the full height is reached. Then put on the inside bracing and struts, and complete the strengthening of the tripod, which is now used as a derrick in building up the scaffold.

In the case of a very large signal, it may be found that the top of the projecting third leg in the earlier stages may be too far away as a derrick to allow the corresponding lengths to be readily brought to place. I should then send up some other convenient stick of timber (any of the upper lengths of the tripod leg are available), lash the heel of it to the foot of the projecting third leg, and with a back guy, rove through a block near top of projecting leg, and side guys to the ground, it can be given the necessary slope and held in any desired position for use as a derrick. As the building progresses, the size of the inclosed triangle becomes smaller and an inclined derrick will not be needed.

FRAMING SCAFFOLD.

Make the scarfs on scaffold legs with square ends, instead of hooked like the tripod, as in that form they are more conveniently brought to their places in the method used in building. The four legs, bolted together for their full length, are laid on the sleepers, measured for length and for distances between cross-ties, and marked from the outside corner for the required bevel across the two outside faces; tops sawed off at the marks and at the bottoms, as the foundation levels may require. Cross-ties are measured and temporarily fastened at the proper marks and each section is "squared" before putting on the diagonals. Do not mortise the legs for receiving the cross-ties, as they are kept firmly in their places by the diagonals being cut with a shoulder, as described in tripod framing. The bracing is sawed off evenly with the legs, no overhang being given. Put the bracing evenly on adjacent sides, excepting on the uppermost tier on the two sides that are to receive the floor-scantlings. As one side is framed, the bracing is numbered and removed to a convenient place, the legs transposed on the sleepers, and so on until the four sides have been framed.

RAISING SCAFFOLD.

The two large tackles are sent to the tripod head and overhauled down on opposite sides. The topmost sections of two opposite sides are solidly framed with their tops pointing to the tripod and lifted by the tackles, so that they may rest against the tripod with their lower ends on the ground. In a signal of moderate size the intermediate sections may be raised in the same way, and left resting against the tripod in the order of their position. In a large signal, where a middle leg is carried in a section, the frames are too wide and cumbersome to be handled with facility at an elevation, so merely up-end the different lengths of each scaffold leg and let them rest against the tripod till required. The lowest sections of corresponding sides are now tightly framed, with intersections bolted, and are placed with their tops close to the tripod. A strong bridle rope is attached to near the top of each leg, and the tackle hooked into its bight. A long bight should be made to moderate the tendency of the frame to buckle together, and a supplementary strap should be used from the middle leg to prevent the sagging of the frame or undue straining in the hoisting. Double guys are secured to each leg near its top and the section lifted, the lower ends dragging along the ground and guided to their platforms by a man at each foot. These two lower sections on opposite sides being lifted and their feet adjusted to their shoes, the frames are brought to the required inclination by means of the guys; the carpenters take their places on the legs (the middle legs of the other two sides having been set in their holes), and the connecting bracing between the two frames is bolted and nailed, the nails being brought to their indicated nail holes by the guy ropes, as described in tripod building. The feet may now be anchored and the holes filled and tamped. A rope through a single block at tripod head is fastened about one-third of the way down on the next length of leg to be raised, guys are fastened to it as it is hoisted clear of the frame already up, and, with a man following up its foot and bringing it to its splice, the guys give it the required inclination and the splice is bolted and nailed. The corresponding lengths of leg for this section are sent up in the same way and the braces sent up to the carpenters until the four sides are completely braced to an even height. The remaining intermediate sections are sent up in a similar manner, until there are left only the two uppermost sections, which are already framed and resting against the tripod on the ground inside the scaffold square. The bridles on these frames are fastened to the legs just above the center of weight and the large tackles hooked to them. The bight of this bridle must be short or the tackle will come to "two blocks" before the necessary height is gained. It does not matter in this piece of frame, as it is so narrow and well braced that there is no danger of its buckling. In hoisting this piece, a man should follow it up along the tripod to keep it from catching against cleats or other projections on the tripod. Guys are put on at the very top of this frame, and when it is hoisted high enough, a man at the top of each scaffold leg hauls the foot over to its splice and holds it there, with a turn of rope, while men at the guy ropes pull the top over to the required inclination, the hoisting tackle being gradually slackened, and the splices are bolted and nailed. The connecting bracing between the two sides and the diagonals between this and the next lower sections are sent up and secured, and the framework of the signal is completed.

The inside bracing of the scaffold should next be fitted and sprung into place, the floor laid, trapdoor fitted, tripod head finished off, and the stairs put up. A ladder form of stairway, with 2 by 6 inch sides and 1 by 4 inch strips 2 feet long and nailed 12 inches apart for steps, is commonly used, though a form giving a broader tread would undoubtedly be more comfortable in use. The stairways should have landings at each section and may be made to wind around the tripod or to zigzag up the widest space. An inside hand rail should be provided. All parts of the stairway are connected to the scaffold alone, and no connection of any kind is to be allowed between the tripod and the scaffold structures. The permanent wire guys should be set up when the temporary rope guys used in building are removed. The ground ends of these wire guys may be secured to tree trunks or any object sufficiently strong and permanent. A stout turn-buckle to set up and keep these guys tense, is desirable.

OUTFIT FOR BUILDING HIGH SIGNALS—ROPES, BLOCKS, ETC.

1 small portable winch, *with pawl.*

4 large double blocks, swivel hooks, with falls three-fourths inch in diameter and 600 feet long.

1 leading (snatch) block, swivel hook.

4 large single blocks, swivel hooks (for hoisting single sticks), with falls three-fourths inch diameter and 160 feet long.

4 medium-size double blocks (for watch tackles); falls can be improvised from any spare guys.

4 small single blocks (for sending up braces, etc.), falls of one-half inch rope 160 feet long.

18 guy ropes, three-fourths inch diameter, 125 feet long.

1 guy rope, 1 inch diameter, 125 feet long.

4 bridle ropes, $1\frac{1}{4}$ inches diameter, 50 feet long.

8 rope straps, assorted lengths and sizes.

Sundry odds and ends of rope, different sizes, for lashings, etc.

In building the upper part of a 150-foot signal, some of these ropes will not be long enough if used singly, but it is better to bend on extra pieces to make up the length rather than to carry in the outfit special ropes, whose weight makes it inconvenient to handle them readily.

TOOLS, ETC., REQUIRED.

In addition to the ordinary carpenter's tools that can be carried in a tool chest small enough to be easily handled, there should be 2 good axes for framing work and 1 rough ax for chopping roots under ground, 2 spades or shovels, 2 mattocks, 1 post-hole digger (for setting guy posts), and 1 medium-size crowbar. A crosscut saw, with adjustable handles, is convenient where much timber is to be felled. There should be a plentiful supply of bits or augers of the different sizes, and at least 2 braces and monkey wrenches and 3 heavy hammers. There should always be carried to the ground a few more bolts and other supply of hardware than the plan absolutely calls for, and the saws and edge tools should always be kept in good condition for work.

There are few localities where timber is abundant where there is not a sawmill of some kind within a convenient distance, though it may not always be possible to get out the longer and heavier sticks; or it may be that a signal is to be built at a point where the hauling from the mill would be too much of an expense. In such case the legs and heavier pieces may with advantage be hewn near the ground. The trees felled for this purpose should be large enough to allow the finished pieces to be all heart, with no large knots, and free from wind shakes or twist, and the timber should be inspected before being worked up. If practicable, the pieces should remain long enough to be thoroughly seasoned, but if necessary to build of partially seasoned timber, the building should succeed the framing as soon as possible, as a short time in the hot sun would twist the pieces so that it would be impossible to closely follow the framing in the building up.

The time required for building such signals and their cost will vary under different circumstances. The cases cited below are from a season's work in Indiana in 1888. The lumber used was white pine and was brought from Chicago. The cost on the ground of the material used was something more than 50 per cent of the whole cost, and the distance it had to be hauled from the nearest railroad station averaged 12 or 14 miles. Four skilled workmen were employed on monthly wages, and day laborers were hired as required.

Height of signal.	Working days.	Cost of signal.
"Green," 152 feet	18	$705·21
"Stout," 135 feet	14	583·45
*"Tripp," 100 feet	6	293·74
"Culbertson," 116 feet	11	487·58
"Reizin," 116 feet	11	488·42
"Weed Patch," 75 feet	6	274·63

* "Tripp" signal was only 1 mile from railroad station.

LETTER FROM GERSHOM BRADFORD, ASSISTANT.

UNITED STATES COAST AND GEODETIC SURVEY,
Washington, D. C., January 27, 1894.

In reply to your letter of January, 1894, inviting me to present my views in writing on any of the subjects under discussion by your Conference, I would offer a few suggestions relating to triangulation, in which, however, my experience has not been extensive, nor have I been connected with its higher branches.

1. To promote uniformity in the methods of making and recording observations and in field computations, a book of instructions of convenient size should be printed and supplied to each member of the normal force and to officers of the Navy detailed to Coast Survey work, and these instructions should cover all important points and be expressed in plain and precise language, not too technical, and should include all the methods of office computations which are likely to be of use in the field.

2. It is difficult to estimate too highly the value of a single pointing upon every signal in sight from a station under very favorable conditions, such as are likely to occur at least once during the occupation of a station, and I think the most favorable to be when the atmosphere is clear but the sky wholly overcast. When a repeating instrument is used such a sweep of the horizon should never be neglected, and should, if practicable, precede the measurement of each separate angle.

3. In much of the minor work, where the sides are very short, two pointings, each direct and reverse, the second for verification, would be amply sufficient for precision and much time would be saved, the value of such work depending much more upon the arrangement of the triangles so as to insure the best practicable conditions than upon the

number of repetitions, for one measurement under good circumstances may be worth many under less favorable ones, the latter only tending to discredit the former.

4. Emphasis should be laid upon the necessity of precision in centering signals and instrument and in referring an eccentric station to center. It is not unusual that an observer whose work has been of a less precise character fails to appreciate the importance of such precision.

5. I doubt whether a telescope should ever be lifted from its wyes for reversal, for the delicacy of manipulation required is so great that there is always a possibility of a slight unnoticed displacement of the instrument introducing an error difficult to trace to its true cause. If the telescope be of suitable length, the frame supporting it may be made high enough to allow it to make a complete revolution, but if not the point of bearing may be so altered that the forward or after end will swing clear below, the poise being equalized by weighting the short end.

LETTER FROM SPENCER C. McCORKLE, ASSISTANT.

UNITED STATES COAST AND GEODETIC SURVEY,
Washington, D. C., January 22, 1894.

I have received this morning your very courteous circular, and owing to my interest in the labor of the Geodetic Conference I take pleasure in replying at once.

I have very decided opinions upon most of the questions propounded in the "scope of deliberations," and give my views for what they may be worth and very briefly.

1. BASE-LINE MEASUREMENT.

I prefer "a few bases measured with a high degree of accuracy" as best adapted, in my opinion, to the requirements of an extended survey.

2. TRIANGULATION.

The old definitions, "primary," "secondary," and "tertiary," are, to my mind, all that is necessary, and easily understood.

3. I prefer a 6 or 8 inch theodolite for secondary and tertiary triangulation for obvious reasons. Twelve observations are sufficient, if well taken, for secondary, and six for tertiary.

4. Reconnaissance and signal building depends upon the character of the country for the first and the facility of obtaining material in the second.

5. CONSIDER METHODS OF OBSERVING.

This depends very much on the character of the instruments. After the necessary measurements of angles, the most important thing is the marking of the station, which should be done with the greatest care.

Would it not be well for the Conference to select from the various methods one that could be generally followed by every Assistant and bring out a degree of uniformity very much to be desired? Of course there are cases where the observer must decide.

6. PARTY ORGANIZATION, ETC.

I have successfully carried on a secondary triangulation when living on shore with one foreman and one hand. When living on a vessel have carried on triangulation and topography with one assistant, one mate, and six hands.

RECORDS.

As soon as the field computations are made, they, with the records, should be transmitted to the office. If necessary to retain a record of angles, a second copy should be made and kept.

I think every Assistant desires that his field computations should have the highest degree of accuracy compatible with the circumstances. The final reduction should only be made at the office.

LETTER FROM F. D. GRANGER, ASSISTANT.

39 West Twenty-sixth Street,
New York, February 2, 1894.

In response to your circular letter of January, 1894, the following remarks, embodying my views on a few of the topics suggested for discussion by the Superintendent in his opening address to the Conference, are herewith respectfully submitted.

For primary triangulation, where a high degree of accuracy is demanded, I regard the direction theodolite, read by means of three micrometer microscopes, as the best that can be used, and the well-known method of observing generally employed in the Survey by positions and series, telescope direct and reversed, as good as has been devised. Certainly the results obtained would seem to warrant this assumption.

Although I have but little to say upon the question of the relation of the number of observations to the degree of accuracy demanded by the character of the work, with the object of deciding if any material reduction can be made in the number of observations now taken, especially at a primary station, I will venture the following remarks as bearing on the subject:

In a scheme of triangles whose sides average, say, 20 miles in length, to give the same number of measures as would be necessary in a scheme whose triangle sides exceed 50 miles, supposing the same class of instrument to be employed, would seem to be an unnecessary and, to a certain extent, a useless labor. Nevertheless it is a very difficult question to draw the line definitely as to the best number of observations that should be taken. There are so many considerations to be noted in this connection, among which the general character of the country enters so largely, that I think the observer should be governed in a great measure by his own judgment. On the other hand, it might be well to have the instruments classified, their accuracy of graduation, etc., being known and recorded as suitable for work of this or that character, and further showing, if a direction instrument, in how many positions of the azimuth circle it would be well to employ it; if a repeating theodolite, how many repetitions of individual angles would be desirable. Information of this nature would be very useful, but should not influence the observer to the extent of entire disregard of his own judgment.

For secondary and tertiary triangulation the repeating theodolite has been, I believe, almost exclusively used in the Survey, and the good results obtained would seem to warrant its continued employment for that purpose. As to the best number of observations to be taken at a secondary or tertiary station, the same remarks I have made with reference to primary work are applicable.

SIGNAL BUILDING.

This subject has received considerable attention in the Survey, and I have but a few comments to make.

I think, as a rule, the class of signals termed observing tripods and scaffolds when employed for elevating the theodolite are not given sufficient stability—that is to say, are built of too light material and are insufficiently anchored and stayed; all of which give rise to variable eccentricity under the action of wind and weather. And in this connection I might add, I believe, other things being equal, that eccentricity of signal is the greatest source of error in a triangulation where wooden structures are employed.

DAY AND NIGHT SIGNALS.

Experiments for determining the relative precision of day and night observations, as is well known, were made at Sugar Loaf Mountain, Maryland, in the summer and autumn of 1879, by the late Assistant C. O. Boutelle, whose interesting report forms an appendix to the Annual Report of the Survey for the year 1880. I refer to that report, as it was my good fortune to be attached to Mr. Boutelle's party during the greater part of his occupancy of Sugar Loaf Mountain. While assisting Mr. Boutelle in his experiments I was struck with the much greater distinctness and steadiness of the night signals as compared with those of the day and with the greater accuracy of the observations. And the following winter, while carrying a small scheme of triangulation up the Mississippi River in the vicinity of Plaquemine and Baton Rouge, I tried, at the suggestion of Superintendent Patterson, the experiment of observing on lights at night. Much to my disappointment, however, the results were so very unsatisfactory I soon abandoned the attempt to continue the night work. The unsteadiness of the lights was doubtless due in a great measure to the nearness of the line of sight to the surface, and the imperfect results to unsteadiness and refraction. The measures at night differed several seconds of arc from the day measures. While these experiments were limited to a few nights only, and hence can not have much weight, yet they would seem to show that under certain conditions night observations are less accurate than those by day. The average closing error in this small work was under 2 seconds, and I attribute these satisfactory results principally to the form of day signals employed. They were exclusively of the observing tripod and scaffold type, from 12 to 16 feet in height, built after plans devised, I believe, by Assistant F. Walley Perkins. The signal poles were of gas pipe 1½ to 2½ inches in diameter, of 17 feet lengths, fitted to be easily removed from the tripod for mounting the theodolite, which was readily and quickly centered over the pole aperture in the cap block, leaving no chance for eccentricity of position. A dozen poles of this kind did service for many miles of river work by shifting them as occasion demanded. This form of signal I found to be superior to anything I had hitherto employed on small work.

With regard to what should be done at a triangulation station besides the necessary measurements of (horizontal) angles, I think that at a primary station vertical measures (double zenith distances and micrometric differences) on all the surrounding signals and principal prominent objects should always be taken, no matter how unsatisfactory the results may appear to be; and in connection with such observations readings of the barometer and hygrometer and notes on the direction and force of the wind, etc., should be made. I also think that special observations for magnetic declination should be taken when possible.

In conclusion, I would say that as the cost of a triangulation, especially of a primary triangulation, warrants unusual care in marking the stations, not only to insure their preservation for a great length of time, but also to make their identification easy, I think for the latter purpose surface marks of reference of an enduring character should always be employed; and accompanying the description of each station a carefully made sketch, or, better, a plane-table survey of the immediate surroundings.

In Kansas, where rocky soil is seldom encountered, most of the stations have been marked as follows:

A stone bottle or jug filled with ashes is set 3 or more feet under the surface of the ground, then a layer of several inches of earth; over this a stone bowl-shaped crock with small drill hole in bottom is placed bottom up; again a layer of several inches of earth, and above this, and projecting an inch or two above the surface, a marble post 2½ feet long by 6 to 8 inches square, having its top surface marked with two V-shaped grooves cut at right angles and the letters U. S. C. S. In the meridian of the station, at equal distances north and south, is set a hard limestone or sandstone post 2½ feet long by 5 inches square, marked on top with a single diagonal groove terminating in an arrowhead pointing toward the center of station. Surrounding the marble post a circular trench of from 6 to 10 feet in diameter, 8 inches wide by from 1 to 2 feet deep, is dug and partly filled with coal and covered with earth. The surface marks are then referred to all prominent natural or artificial objects in the immediate vicinity, and, when possible, to the section corners by angular and linear measurements.

ERRORS OF GRADUATION IN A THREE-MICROMETER MICROSCOPE POSITION THEODOLITE, AND A METHOD FOR ELIMINATING THEM IN THE MEASUREMENT OF HORIZONTAL ANGLES.

[A report by EDWIN SMITH, Assistant.]

In a recent examination of the two new 30^{cm} position theodolites, Nos. 145 and 146, constructed at the Coast and Geodetic Survey Office, it has been shown that the spacing errors of the graduation are very small, and it has been stated that in this respect the instruments must be considered nearly perfect. In this examination repeated measures of 72 spaces, symmetrically disposed about the circle, were made. In respect to No. 145, the greatest deviation of any one space from the mean of all the spaces measured is $0''\cdot6$ and the average deviation only $0''\cdot2$. The measures of spaces of No. 146 have not been given, as they are considered unreliable, owing to a defect of the circle, it being thin and having a warped surface. A new circle for this instrument is in course of construction and will be as nearly as possible like the one on No. 145.

Possibly the greatest accidental errors of graduation are to be found in the spacing, but it must be admitted that with the showing made with theodolite No. 145 such errors will be practically eliminated by readings on a much less number of spaces than the number examined.

In what have long been considered the best graduated circles, spacing errors of $2''$ and even greater have been found. If it is admitted that circles now in use have spacing errors so great as $3''$ from the mean of all the spaces, such errors would still be practically eliminated by readings on a comparatively small number of spaces. It certainly seems that in the mean reading on 60 symmetrical spaces on any such circle the residual error can not be greater than $0''\cdot05$, and in the better circles the error in the mean of 12 or 15 spaces will probably be no greater.

By far the most important errors of graduation which affect the measurement of angles or directions are of a periodic nature. By the use of a number of equidistant microscopes certain of such errors are eliminated in the mean of the microscope readings. In Volume II, page 53, of Chauvenet's Astronomy, it is shown that "the terms of a periodic series not eliminated by taking the mean of q equidistant microscopes are those only which involve the multiples of $q\,z$," z being the reading of the circle. It is there also shown that the mean of 3 microscopes requires a correction of the form $u \sin (3\,Z - U) - u' \sin (6\,Z - U') -$ etc., in which u u' and U U' are constants. These constants may be determined and the mean reading of the microscopes on any part of the circle thereby corrected.

The special determination of these constants for all the theodolites of the Coast and Geodetic Survey would be a useless labor, as the measurement of angles in the field may be so made on symmetrical parts of the circles as to both eliminate such error and at the same time determine these constants. The important question is, What is the number of symmetrical parts of the circle upon which angles should be measured to be certain that all classes of errors of graduation may be eliminated or reduced to so small a quantity as to be neglected?

In the examination of the new theodolites above referred to, angles were measured in groups of 4 symmetrical positions, and the agreement of the means of these groups shows that the graduation errors are practically eliminated in each group. The 4 measures of each group involve 12 symmetrical arcs of the circle 30° apart and readings on 24 spaces of the graduation.

If 4 positions are determined by dividing half the distance between the microscopes and observing the angles with microscope A set at reading of each position with telescope direct, and at 180° from each position with telescope reversed, each angle will involve 24 symmetrical arcs of the circle 15° apart and readings on 48 spaces of the graduation. Inasmuch as 8 measures (4 direct and 4 reversed) will not be sufficient for the higher class of triangulation, 8 positions may be taken upon the same principle, and each angle will then be measured 16 times (8 direct and 8 reversed), involving 48 symmetrical arcs of the circle 7°·5 apart and readings upon 96 spaces of the graduation. With such a number of readings and measures, if errors of graduation are not practically eliminated the circle must be very poor indeed.

It appears desirable to test this method in the field, as it would at once give the means of investigating the errors of graduation. The 8 positions of this method could be divided into 2 groups of 4 symmetrical positions, each group involving 24 symmetrical arcs of the circle in the measurement of each angle. A comparison of the means of these groups would give the desired information as to the value of the circles, and the constants could be computed if deemed necessary. In the method of observation generally followed on the Coast and Geodetic Survey the mean of 2 measures, 1 in the direct and 1 in the reversed positions of the telescopes and microscopes, is called a series. The report of the Triangulation Committee states 31 series for primary and one-half or two-thirds that number for secondary triangulation. By the method of positions above proposed 2 or 3 series in each position will give the necessary number for secondary triangulation and 4 series for primary; or for the latter, 2 series in each of 16 positions might be taken, involving 96 symmetrical arcs 3°·75 apart in the measurement of each angle. The accompanying diagram illustrates the positions.

Respectfully submitted to the Geodetic Conference, United States Coast and Geodetic Survey, February 27, 1894.

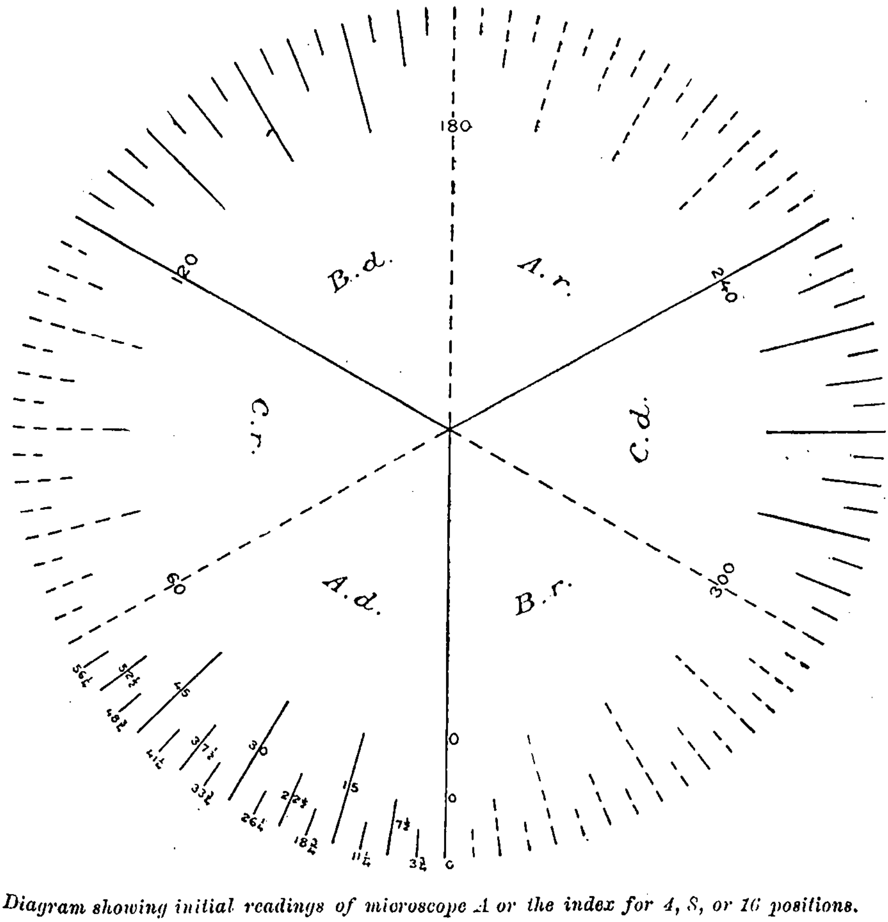


Diagram showing initial readings of microscope A or the index for 4, 8, or 16 positions.

[Full lines show initial readings of the 3 microscopes in the direct positions, and dotted lines the initial readings in reversed positions.]

4 positions.—Set microscope or index direct at 0°, 15°, 30°, and 45°.

8 positions.—Set microscope or index direct at 0°, 7½°, 15°, 22½°, 30°, 37½°, 45°, and 52½°.

16 positions.—Set microscope or index direct at 0°, 3¾°, 7½°, 11¼°, 15°, 18¾°, 22½°, 26¼°, 30°, 33¾°, 37½°, 41¼°, 45°, 48¾°, 52½°, and 56¼°.

MEMORANDUM BY W. C. HODGKINS, ASSISTANT.

CERTAIN APPLIANCES USED IN THE PRECISE LEVELING WORK OF THE COAST AND GEODETIC SURVEY, WITH SOME SUGGESTIONS OF POSSIBLE IMPROVEMENTS; SUBMITTED TO THE GEODETIC CONFERENCE IN SESSION AT WASHINGTON, D. C., JANUARY, 1894.

WASHINGTON, *January 31, 1894.*

One of the most striking features of many lines of so-called "precise levels" is the singular tendency to an accumulation of errors, as a result of which two practically simultaneous lines frequently show a gradually increasing divergence from each other. I would suggest that, in my opinion, the greater part of the difficulty may be sought in the rods used in the work.

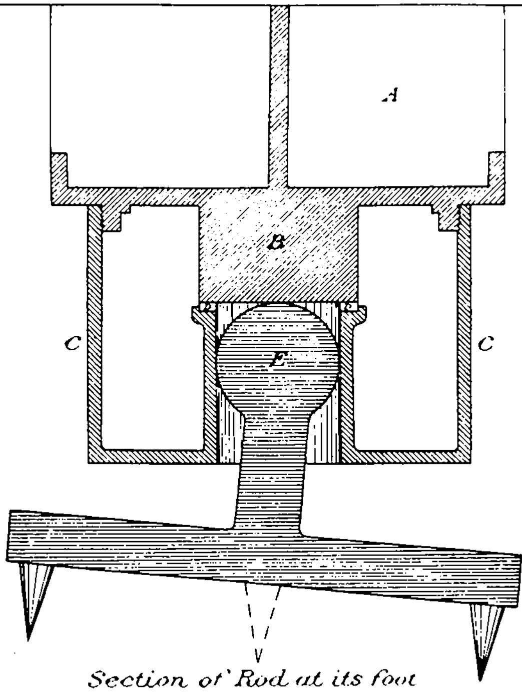
The leveling instrument now in use in the Coast and Geodetic Survey seems reasonably satisfactory both in theory and in workmanship. I am disposed to think that in the natural reaction from the excessive weight of some instruments formerly in use we have gone rather too far in the direction of lightness and small size. It is possible also that the instrument might be improved by somewhat increasing the optical power of the telescope. It might also be desirable to investigate the question whether our means of determining the angle between the target and the horizon are sufficiently refined, bearing in mind the fact that an error of one second of arc, represented by about one-third of a division of the head of the micrometer screw, would produce an error in height of nearly half a millimetre in a distance of 100^{m}.

Without further reference to the above, I turn to the rods now in use, which seem to be objectionable in several respects.

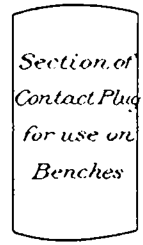
1. THE FOOT OF THE ROD AND THE FOOTPLATES OR TURNING POINTS.

Without commenting on the defects of the forms now in use, many of which are obvious, I submit a sketch of an improved form which seems to me to be more desirable. In this sectional view *A* is the rod, *B* the projecting metal foot, the lower surface of which is a plane perpendicular to the axis of the rod and is the zero or initial plane of the rod; *C* is a protective cap, consisting of a cylindrical brass box made to screw upon the foot of the rod and provided with a cylindrical opening at the center to permit the foot of the rod to be reached; *D* is a gasket or ring of any suitable packing to close the space within the cap without metallic contact with the foot of the rod; *E* is the spherical metallic head of the footplate or foot stub, the latter being preferable when it can be used.

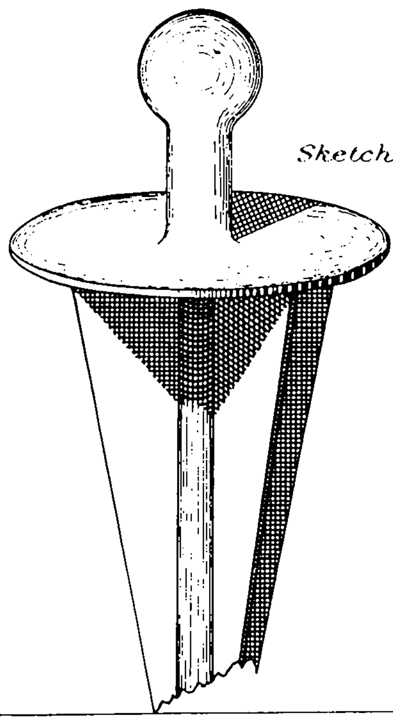
The walls of the tube in the cap prevent lateral motion of the rod, and the metal sphere upon which the rod rests insures a support in the axis of the rod and prevents any possibility of a binding between the rod and its support, even when the latter is much more out of the vertical than would ever occur in practice, as shown in the sketch. The



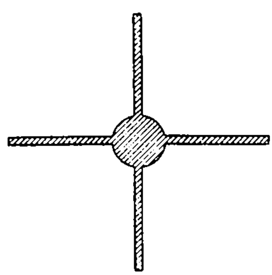
Section of Rod at its foot



Section of Contact Plug for use on Benches



Sketch of Foot stub



Section of Foot stub

foot stub, pushed firmly into the ground, should furnish a more stable support than the plate in many kinds of ground.

When the rod is to be held upon a bench mark, the protecting cap may be removed or a small cylindrical plug, made to fit the central opening and of known length, may be used for the purpose of making the contact.

2. THE MATERIAL OF THE ROD AND ITS GRADUATION.

While there are some valid objections to the use of wood for the standard of length in a leveling rod, I think that most of them may be eliminated by proper arrangements, and that rods so arranged should produce results far more reliable than any at present obtained. I think the use of a target objectionable, but if it be thought desirable to retain it provision should be made to obviate the mechanical defects by which the target may be thrown out of position when clamped. The graduation of the rod should also be directly under the target, and the vernier should be on the middle line of its face. The divisions of the rod from which the target reading is taken should be lines cut on the faces of pieces of brass firmly set in the wood, avoiding the uncertainty of a line cut in wood. This of course does not apply to the extension of these lines for the purposes of telescopic reading. The wood of which the rod is made should be carefully protected against moisture by the most approved methods and should be nearly invariable. A metallic standard, firmly united with the foot cylinder, should, however, be inclosed in the rod for purposes of comparison from day to day. This standard should be free to expand and contract, being fastened only at its lower end. Suitable arrangements for obtaining its temperature and for comparing its upper end with the surface graduation of the rod being provided, comparisons made under favorable conditions, before and after the work of the day, should afford a very precise check on the condition of the rod. I had intended to write more fully on this point, but in recent conversation with a member of the Conference I learned that a rod constructed on this principle has already been used in France.

3. METHODS OF WORK.

I think that in every party engaged in precise levels there should be four rodmen, so that a double simultaneous line may be run without waiting for the rodmen to move between the back sights and the fore sights. I think, also, that the length of sights should be limited so as not to often exceed 100^m, and that great care should be used in equalizing fore sights with back sights.

While I am fully aware that this important subject is very inadequately treated in the foregoing paper, necessarily prepared in some haste, I submit it to the consideration of the Conference in the hope that it may not be without some value in eliciting the opinions of those best qualified to decide these questions.

LETTER FROM W. IRVING VINAL, ASSISTANT.

UNITED STATES COAST AND GEODETIC SURVEY,
Washington, D. C., February 2, 1894.

In acknowledging the receipt of your letter of January, 1894, inviting me to express my views upon any of the subjects under discussion by the Geodetic Conference, I desire to call attention to the markings of stations in the secondary and minor triangulation.

Along the Atlantic and Gulf coasts the topographer or hydrographer is frequently unable to recover these positions, as the sites have been insecurely marked or imperfectly described.

It is not always possible to guard these positions by underground marks, and the surface monuments, either of iron or stone, are often disturbed and sometimes appropriated by boatmen and others.

I would suggest, in general, that in addition to the three or four reference marks usually placed near the stone or post marking the center of station other marks be set within a few hundred feet of the signal, where they will be least liable to disturbance.

A good sketch of the topography within a radius of 300 or 400^{m} of the station, with the distances and azimuths of the reference marks, together with a clear description of the locality, should ordinarily secure the position from the absolute loss which has now too often to be incurred.

LETTER FROM E. H. FOWLER, DRAFTSMAN.

UNITED STATES COAST AND GEODETIC SURVEY,
Washington, D. C., July 11, 1894.

Permit me to call your attention to a subject which causes much annoyance and loss of time both in the office and with the field parties of the United States Coast and Geodetic Survey:

The geographical positions given in the office registers include nearly everything from the beginning of the Survey up to the present time, yet many of these positions have long since been lost or are of no further use; but the draftsman has no way to distinguish between the good and bad, and is therefore obliged to plot hundreds of points, with much loss of time, which only cause great annoyance to the field officer in a fruitless search for them, as well as mutilating the sheet.

It is not my object to suggest what methods should be employed to designate the useless geographical positions in the office registers, but would respectfully refer the subject to the honorable members of the Geodetic Conference.

APPENDIX No. 10—1893.

ON THE PREPARATION AND ARRANGEMENT OF THE EXHIBIT OF THE UNITED STATES COAST AND GEODETIC SURVEY AT THE WORLD'S COLUMBIAN EXPOSITION.

Rreport by D. B. WAINWRIGHT, Assistant.
Submitted for publication May 14, 1894.

I have the honor to report that, in accordance with the instructions of the Superintendent, dated March 11, 1892, I took charge of the preparation of the exhibit of the Coast and Geodetic Survey for the World's Columbian Exposition.

Shortly after the above date I commenced to familiarize myself with what had been done at previous expositions in which the Survey participated, so as to be informed what arrangements were necessary in regard to the allotment and expenditures of money, floor space, transportation, installation, and the numerous other details which require the attention of an exhibitor long before the opening.

There was, however, very scanty material for this purpose to be obtained in the shape of reports, and I had to rely principally for information on the memory of those who had been connected in some capacity with our exhibits at other expositions.

The character and scope of the exhibit had been considered some time previous by a committee consisting of the Assistant in charge of the office, the hydrographic inspector, and the chiefs of the several divisions in the office.

Their report, which was submitted October 15, 1891, was referred to me for my guidance. It gave in detail a list of the instruments it was desirable to furnish, indicated what charts, reports, appendixes, and other publications of the Survey should be displayed, and proposed that a series of pamphlets illustrating the methods and operations of the Survey be written and printed for distribution. If practicable, it was suggested to send to Chicago one of our steamers, on which all the varieties of hydrographic work could be most fitly represented, especially that part pertaining to deep sea sounding.

The committee advised a full display of weights and measures, both customary and metric, and, as a special feature for this group, a 100-foot bench standard.

Nearly all of the foregoing material could be furnished by the office. In addition it was recommended to construct a model of a segment of the earth on a scale of 1-1 000 000, showing the United States and Alaska, and indicating on its surface the geodetic work of the Survey.

Also to furnish three globes of a large size, on two of which were to be represented in a graphic manner the direction and extent of the deflection of the magnetic needle, and on the other the historical and modern geodetic arcs completed and proposed at the present time.

Also two relief plaster models—one to form part of a series to illustrate topographic methods of surveying; the other, hydrographic methods of surveying.

This programme, as outlined by the committee, was in the main closely adhered to. All the instruments mentioned in their list could not be taken, either because of their being wanted in the field or for lack of space to properly mount them.

Instead of the pamphlets recommended, leaflets were substituted as more likely to meet the demands of the greatest number of visitors.

It was considered desirable to direct the attention more fully than had been done hitherto to the methods of the Survey in obtaining data for the prediction of tides and from which our tide tables are compiled. For this purpose the tidal model was constructed.

In order to show the current metres in operation, a large trough was also provided. From the commencement of the preparations to their close I reported to the Superintendent almost daily in order to keep him informed as to the difficulties encountered in carrying out the scheme and the progress made, to receive his instructions and advice and also to propose such additions and improvements as were suggested to me by a more intimate knowledge of the subject.

At an early stage of the preparations it became apparent that there was not sufficient room available in the office buildings for the construction and setting up of the large model of the United States and Alaska, the tidal model, and the globes. It was then decided to erect a frame building of the simplest character on the lot adjoining the carpenter shop. It was 40 feet long and 20 feet wide, sides 12 feet high, with gable roof 8 feet high.

The principal requirements were a substantial floor and plenty of light.

After Mr. French, the head carpenter, had drawn a satisfactory design, specifications were typewritten and sent to several builders. The bids received were all very much higher than had been looked for and beyond the amount it seemed wise to expend for this purpose. Some time was spent in trying to have the bids reduced to a reasonable figure, but without avail.

At this juncture Assistant Edwin Smith came to the rescue, and suggested sheathing the sides and roof with corrugated iron, and found a contractor who would undertake the floor and framework for a price within the limits of the estimate. The Pennsylvania Iron Company furnished the corrugated iron sidings and pressed-seam steel roof, and also sent a skilled laborer to put it in place.

The building was completed early in August, and was occupied by the various employees from that time up to the completion of the work, except during some of the severest weather of the winter, when, in spite of two stoves, situated one at either end, standing water would freeze.

In the meantime, to obtain a clear idea of the requirements of the large model of the United States and Alaska, I made one in miniature on a scale of one-half inch to the foot, and then further to develop the method of construction one on a scale of 1 inch to the foot of the large model.

From a study of these it was concluded to make the model in eight sections for the convenience of transportation, and to have it include very little territory outside the limits of our own possessions. The selection of a scale for the relief provoked a good deal of discussion. It was finally decided, however, to adopt the natural scale, or, in other words, there would be no exaggeration of the heights as compared to horizontal distances. Plenty of models were already in existence which showed very plainly the configuration of the mountain ranges by an increased vertical scale, and it was thought unwise on a model of this size to dwarf all the horizontal distances and to sacrifice for pictorial effect a clear perception of the true contour of the earth's curvature.

To simplify the construction of the framework, each of the principal ribs was placed so as to correspond with some meridian. In this way they all had the same radius of curvature, consequently were of the same pattern, differing merely in length. They were made of white pine seconds, 1⅜ inches thick and 10 inches wide, the length of each depending on the number of degrees of latitude included in that part of the model.

Each section had four of these longitudinal ribs, one at either side and two intermediate, and each section commenced and ended on some multiple of each fifth degree of longitude. Thus the right-hand edge of the second section corresponded to the meridian of 80° and its left-hand edge to the meridian of 90°.

The short cross ribs were arranged along some parallel of latitude and cut to conform to the curvature of the sphere at that parallel. Thus all on the same latitude were of the same pattern. Sixty pounds of angle irons were used for rigidly fastening together the different members of the sections.

Experiments were made to discover the most suitable covering for this framework. A shell of prepared paper similar to that forming the hull of the racing boats made by E. Waters & Son, of Troy, N. Y., was favorably considered; but there were reasons in the way of its adoption, notably the difficulty in obtaining it of the desired shape and at a sufficiently low cost.

Finally one-fourth-inch pine casing was used, and as it assumed very nearly the requisite curvature there was little dressing down necessary to perfect it. On this were glued in succession three thicknesses of heavy cotton cloth, which was then painted with white lead until the pores were filled and a smooth surface obtained on which a draftsman could work.

While this was in progress, maps were being collected which would furnish the natural and artificial features to be represented on the model. I did not find this an easy task, since the scale of $\frac{1}{1\,000\,000}$ is an unusual one, and for many States quite a gap exists between maps of a smaller scale and the county maps. The method I was forced to adopt was to select those of a larger scale and have them reduced by photography. Some conception of the labor this involved may be realized when it is known that 275 negatives were required. These were made by Mr. Chapman of the photographing and electrotyping branch of the office, and demanded a skill in their preparation only to be acquired after long familiarity with such work. Before this reduction was made, draftsmen first plotted and inked the thousand-foot contour lines wherever the nature of the country called for them.

Blue prints were made from the negatives, and where the height above sea level was less than 1 000 feet the topography was traced from them directly on the prepared surface of the model. The elevated areas of the continent were built up by gluing to the cloth a layer of painted cardboard twelve one-thousandths of an inch thick for each thousand feet. These layers were cut out according to the pattern indicated by the contours on the reduced maps. To further bring out the modeling of the mountain ranges and individual peaks, their eastern slopes were shaded with a neutral tint. When a section had reached this stage, it was turned over to Miss Antisell, who painted the water areas and State boundaries with appropriate colors in oil. The position of the model when exhibited was such that the axis of the imaginary sphere of which it might be conceived to form part was vertical, with the North Pole upward, the southern end of Florida nearly touching the floor and the upper part of Alaska 13 feet above.

This arrangement made a stairway necessary to obtain a clear view of the upper portions. This was built on the floor space by a Chicago firm, before the arrival of the exhibits, from a design made by Mr. Von Erichsen. It consisted of a number of steps and platforms ascending from a point opposite Florida, following the irregular outline of the model to the highest and longest platform, opposite the Aleutian Islands; thence it descended in a curve to the floor at the rear.

There were no globes to be obtained in this country 1m in diameter. Even had this not been the case, the customary globes would have proved unsatisfactory owing to the great amount of detail drawn upon them, which would have a tendency to obscure the additional lines and colors necessary to represent Assistant Schott's designs. For these reasons it was decided to construct them of plaster at the office.

Each globe was composed of a framework of light but strong semicircular wooden ribs, surrounding and fastened to a hollow brass axis. These longitudinal ribs were stayed by several series of shorter ones fastened at right angles. This skeleton was covered with wire gauze; after which the plaster surface was applied. To effect this the ends of two wooden templets one-fourth inch wide and thick were fastened at one of the poles and then bent over the framework and tacked down at the other pole of this rude sphere, so as to inclose a lune of about 30° between them. Dampened fiber was now placed in a thin layer within this space, so as to partially close the interstices of the wire gauze.

A sufficient amount of plaster of paris was then mixed, and when it had arrived at a mush-like consistency was rapidly applied over the fiber. Some portions would force their way through the gauze and serve to lock the casting to the frame. After the plaster was set, and while it was still in a soft condition, the opportunity was taken to pare down the excess, the templets serving as guides to show where to cut away.

After this was completed a templet from one side of the cast was carefully removed and secured again at the proper distance, so as to form a new space for the next casting.

In this way, section by section, the frame received a complete coating of plaster. Then by using a carefully made brass templet for reference the rough surface was turned down to the proper dimensions.

When the plaster was thoroughly dry, it was sized with a thin solution of glue, and afterwards given a coat of white lead and japan varnish. The globe was now given to a draftsman, who drew the meridians and parallels, outlined the continents, and indicated the boundaries of the different colors. The artist, Miss Antisell, then tinted the land areas a neutral color on all three globes, and on the isogonic globe the areas of western declination a buff color and eastern declination a blue. On the isoclinic globe the areas of northern dip were tinted buff and the southern dip blue. Lastly, a draftsman drew the lines indicating the amount and direction of the magnetic deflection, the historical and modern geodetic arcs, and the gravity stations on the appropriate globes.

The stands were made from designs furnished by Mr. E. G. Fischer, chief instrument maker, and attracted attention on account of the ingenuity displayed, all the framework which ordinarily surrounds a globe of this size being avoided and the view of the surface presented free of obstruction.

A stand consists of a steel shaft fitting the axis of the globe, having four spokes above and below for the purpose of revolving it. The lower end was mounted on ball bearings fitting a casting with gas-pipe feet. The axis was inclined to represent the position of the earth in relation to the plane of the ecliptic.

The form in which the tidal model was exhibited is the outgrowth of various attempts to display in a comprehensive manner the methods pursued by the Survey in procuring the data on which are based our published tide tables.

It consisted of a stout wooden box with a sloping bottom, the greater part of which was lined with lead so as to form a water-tight tank. At one end was modeled in miniature a strip of sandy beach and a number of sand dunes. At one extremity of this little piece of coast line was placed a diminutive light-house and keeper's dwelling, and at the other a fish house and wharf. That part of the tank intended to represent the sea bottom was modeled to imitate the appearance of shoals and sandbars. Along the side of the tank opposite the strand, and where the water was deepest, were placed the miniature, self-recording tide gauge and the miniature tide indicator, both designed by Assistant E. E. Haskell. This indicator was the counterpart of one located at The Narrows, New York, which is of sufficient size for those aboard passing vessels to read on its dial the height of the tide above or below mean low water, and also, by a device connected with the float, learn whether the tide is rising or falling. It was the intention to have the characteristic tides of New York and San Francisco reproduced on a reduced scale, recurring at six-minute intervals. With this in view, Assistant Haskell designed two balanced valves, one for supply and one for discharge, controlled by two cam disks, the latter being run by clockwork. This control was effected by causing a small roller attached to the end of each piston rod to travel in the grooves of the disks, so that the valves were opened and closed in proper sequence to raise and lower the water in the tank the necessary amount to produce the tidal curve.

In order to have this mechanism perform satisfactorily, it was indispensable to have a constant water pressure. Unfortunately, after the exhibit was installed, the pressure was found to vary to such an extent that it was impossible to keep the apparatus in adjustment for any length of time, and a discharging siphon had to be substituted: This reduced the representation to a much simpler form, the tank being filled and emptied to the same extent each time, without any attempt to simulate the special features of the tidal curve.

Assistant Ogden selected a series of exhibits to show the various steps taken in the production of a finished chart from the field sheets.

This series in part contained the original topographic and hydrographic sheets, the drawings from the same for engraving and lithographing, reductions by photography, lithographic stones on which these had been transferred, the engraved copper plate, a basso and alto partly separated, a basso and alto separated, and, finally, the finished chart.

To present more distinctly the relation between a published chart and the region represented, and to illustrate more clearly the meaning of the various symbols and arbitrary signs used on a chart, two plaster composition relief models were ordered of Mr. E. E. Howell. One is modeled directly from Chart No. 306 and embraces a portion of Mount Desert Island and Frenchmans Bay, Maine. The diversified character of this region, containing wooded mountains, bare cliffs, lakes, islands, and seacoast, makes it an especially fit subject for the purpose.

The other model was taken from a portion of Chart No. 5487—Carmel Bay, California, vertical and horizontal scale 1–12 000. In this vicinity is found one of those submarine valleys, peculiar to the Pacific coast, whose deep trough and precipitous slopes penetrate the continental plateau to within a short distance of the coast line. This feature affords an excellent opportunity for the explanation of the technical details of a hydrographic chart.

As a further illustration of our topographical work, a model was made of Rock Creek Park and vicinity. This area is included in the large scale survey (1–4 800) made by the officers of the Coast and Geodetic Survey for the Commissioners of the District of Columbia.

For precision of methods and elaboration of details the treatment of this region is unequaled by any other of like extent in this country. Lithographic reproductions in colors of the original sheets were made on cardboard; and as the adopted scales for horizontal and vertical distances were the same, its thickness (thirteen one-hundredths of an inch) represented 5 feet.

The usual method was pursued in building up the model. One of the prints, with all that area cut away below the lowest contour, was glued upon another print left entire, the cutting being done with a sharp-pointed knife and the cut being made so as to divide the contour line in half. Another print was then taken, and all the area below the next higher contour cut away before being glued down. In this way successive layers were superimposed, one upon the other, until the piece containing the last or highest contour was put in place, when the resulting cardboard block appeared as a faithful copy in miniature of the country covered by the print. A number of blocks were constructed in this manner, and then carefully fitted together to form the complete model. The edge of each layer was colored brown in order to show graphically the relation between the contours of a topographical map and the vertical heights.

Fourteen leaflets were prepared and printed to give an account in a popular form of the organization, operations, and methods of the Survey. They were made of a convenient size for carrying in the coat pocket. Each consisted of four pages 6½ by 4¼ inches, and contained about 1 300 words.

The following are the titles and authors:

"The Coast and Geodetic Survey," condensed from an article by Dr. T. C. Mendenhall.

"Time, latitude, and longitude," C. H. Sinclair.

"Base apparatus," R. S. Woodward.

"Triangulation and reconnaissance," W. C. Hodgkins.

"Gravity," E. D. Preston.

"Topography," H. L. Whiting.

"Hypsometry," Andrew Braid.

"Hydrography," S. M. Ackley.

"Tides and currents," A. S. Christie and E. E. Haskell.

"Description of the Coast and Geodetic Survey steamer *Blake*," C. E. Vreeland.

"Magnetics," C. A. Schott.

"Chart publications," H. G. Ogden.

"Weights and measures," O. H. Tittmann.

"Model of the United States and Alaska," D. B. Wainwright.

Three editions of 5,000 each were printed, all of which were distributed excepting a few retained for special purposes.

Among the details which occupied my attention were the designs for a show case for the display of a large portion of the exhibit of Standard Weights and Measures. It was built of cherry wood, by a Chicago firm, and completed the latter part of March.

Also rails and posts of special design for inclosing sections of the floor space were ordered in Washington and taken along. Under the direction of Assistant Edwin Smith the instruments intended for the Exposition were collected and put in order.

By the 1st of April nearly all the exhibits were completed, boxed, marked, and invoiced for shipment and loaded on the cars at the Baltimore and Ohio freight station. I arrived in Chicago April 6, with Mr. Dice, the carpenter who had been connected with the preparations since the active work commenced, in June, and Mr. William T. Oliver, who had also been employed since July.

The floor space was located in the southwest corner of the Government building, opposite the Word's Fair post-office. It consisted of two detached portions, separated by an 8-foot aisle, and together contained 2 920 square feet. The larger space was originally in the shape of a rectangle 52½ by 44 feet, but for the convenience of the Light-House Board a triangular piece in the northeast corner 15 by 12 feet was cut off and a small rectangle 9 by 14 feet added on in its place, The other portion of our space was north of the latter, across the aisle, and was in rectangular form 21 by 28 feet.

The first work undertaken was the construction of an oak partition between our space and that of the light-house exhibit, on which were displayed in oak frames original topographic and hydrographic sheets and a selection of charts and diagrams. Running along the partition was built a counter on which were disposed the plaster models of Carmel Bay, Mount Desert Island, and some of our publications. The large model of the United States and Alaska was afterwards put together and occupied a central position. To conceal the supports and braces, a series of white pine strips fastened at the upper ends to the edge of the model and at their lower ends to the floor formed a framework on which was tacked wine-colored felt. The area thus inclosed proved very useful as a receptacle for storing instrument boxes.

After vexatious delays and many fruitless trips to the railroad offices and yards, the 50-foot iron bar for the bench standard was found and brought to our floor space. On Mr. Louis Fischer's arrival, it was taken to the blacksmith's shop at the southern extremity of the grounds to be straightened, it having been bowed in transportation.

On being brought back, Mr. Fischer bored the holes for the german-silver plugs, making use of the lathe in the United States ordnance exhibit through the courtesy of the officer in charge; after which it was graduated. It was then mounted on the heavy railing which extended along the western side of our space, from its upper limits to within 6 feet of the lower edge.

The advantage of this over customary bench standards, in that its coefficient of expansion is nearly the same as that of steel tapes or chains, so that thermometer readings can be dispensed with when comparing, was explained to visiting engineers, and a number of steel tapes tested.

Mr. George W. Dorr, chief engineer of the West Side parks, became interested in having the standard retained in Chicago, where only crude devices for a similar purpose existed. In response to a resolution of the park commissioners, the Superintendent obtained the authority to leave it in their charge for the benefit of the engineers of Chicago, and accordingly after the close of the Exposition it was set up in one of the conservatories of Douglas Park.

Parallel to this exhibit, and forming a group, were arranged the compensating base-bar apparatus and the duplex apparatus.

The latter, a decided novelty, since mercurial thermometers are discarded, consists of two bars, each one containing two tubes of nearly equal length—one of steel and one of brass—and so arranged that the measurement may be conducted with and expressed in terms of either component, and the difference between the measured lengths as expressed by the two components affords a measure of average temperature of either component during the measure.

Another innovation shown in the same group is the iced-bar apparatus, where the temperature of the bar is controlled by surrounding it with melting ice. Visiting engineers were invariably attracted to these exhibits and manifested their interest by the close attention given to the explanations by the Assistant in charge, and also by the number of questions they asked.

At the northern end of the main floor space an aisle 6 feet wide was left between the bench standard and the group of astronomical instruments.

The latter occupied an inclosure 8 feet square and was composed of a meridian telescope, astronomical transit, prismatic transit, zenith telescope, telegraph keyboard, break circuit chronometer, and cylinder chronograph.

A portion of the instruments were arranged so as to show the relative positions they would occupy in a longitude field station.

To the south of these came the inclosure, surrounded by a 5-foot aisle, in which were grouped the instruments used in triangulation and hypsometry. They consisted of direction theodolites, repeating theodolites, a vertical circle, a geodetic level, and an engineer's wye level.

A short distance below these, and close to the stairway, stood the geodetic globe; and next in order came a 6-foot counter show case in which were displayed the small instruments used in reconnaissance, and also the new half-seconds pendulums.

On the southern edge of the floor space, and far enough from the base apparatus to admit of an aisle, was located an 11 by 14 inch Dee press, on which the plate printer and his helper struck off from a small engraved copper plate prepared expressly for this purpose copies of a little chart of the St. Croix River, Maine. This exhibit always attracted a large and interested throng, and the prints were eagerly sought after. As the plate printer was plied with numerous questions and had to make frequent explanations of the process, it was impossible for him to work fast enough to supply more than a fraction of the demand. However, the office provided for this deficiency by sending on some thousands of photolithograph copies.

On the eastern side of the inclosure of the plate-printing exhibit was an opening in the railing to serve as a passageway to the Marine Hospital space. Next to this was situated a screen, on both sides of which Assistant Colonna, on his arrival, arranged the publication exhibit of the engraving division of the Survey. This illustrated in a very interesting manner, by specimens, the various processes employed in producing a finished chart. It furnished the material for explaining why the Survey retained the method of copper-plate engraving for most of its charts instead of the more rapid method of reproduction by photolithography. By its aid could be shown that each photolithograph print had to be corrected by hand for every change, either natural or artificial, occuring within its limits, since such corrections can not be made on the stone itself; how they are quickly and easily made on the copper plate; and, finally, how photolithography, while at first the cheapest and most rapid, becomes the most expensive and slowest method of reproduction.

Next to this screen, and in the southeast corner of the floor space, came the relief model of Rock Creek Park already described. Adjoining this, and close up to the partition on which were suspended original topographic sheets, were placed the two plane tables and alidades—one of standard size; the other a small one for mountain work.

In the curved area at the back of the large model of the United States and Alaska were situated an office desk, a revolving bookcase containing a complete set of the Annual Reports of the Survey, and a case with fourteen drawers containing a nearly complete collection of our published charts for exhibit on special call.

Immediately north of the staircase of the large model, on an oak table, was placed the full-rigged model of the United States Coast and Geodetic Survey steamer *Blake*. It is 42 inches in length, and shows in miniature the steamer's apparatus for anchoring and sounding in great depths. Over it was a card informing visitors that the steamer herself was located at the pier near the Casino and had on board a complete outfit for hydrographic surveying.

Along the upper end of the partition was placed a large oak trough through which flowed a stream of water. In this was shown the Haskell-Ritchie direction and velocity meter in operation and several types of velocity meters.

In the small rectangular space on the extreme north was located the tidal model. It was always surrounded by a number of interested spectators. By its side were a Stierle self-registering gauge and the tide predictor.

On the east side of the detached floor space was situated a large cherry-wood show case, which contained all the smaller articles of the weights and measures exhibit; among others a copy of the international prototype metre.

On the opposite side of the space were the large and medium balances. In the northwest corner the Saxton pyrometer was arranged on two imitation piers—the steel bar and mirror close to the partition, and the telescope and scale 8 feet farther out. In the middle of the space was a railed inclosure in which were mounted New Magnetometer No. 20, one of the Survey's latest design, and, as a contrast, Magnetometer No. 3, one of the earliest designed.

The two plaster globes, on which was represented by lines and colors the present state of the magnetic elements on the earth's surface, stood near by to the eastward.

On the south was placed the comparator for testing thermometers in liquid.

The Repsold reversible pendulum and the Peirce yard and metre pendulums were arranged in brackets along the western partition.

Messrs. James S. Hunter, J. D. Cleary, and G. Ritter, draftsmen, were engaged in plotting the contours on the State maps previous to their reduction by photography, and then in drawing the details on the surface of the large model and building up the elevations.

Mr. Cleary prepared most of the sections of the Rock Creek model, and its excellence is due to his skill and patient care. He was assisted in the latter part of this work by Mr. Hunter.

Mr. Hunter also prepared the plaster globes for the artist, outlining the continents and the color boundaries, besides drawing the parallels and meridians and tracing the colored lines showing the dip, intensity, and deflection on the magnetic globes. He worked with great rapidity and intelligence.

Miss M. Antisell painted in appropriate oil colors the water areas and State boundaries on the large model and the continents and magnetic areas on the plaster globes. Her work was pronounced quite satisfactory and artistic.

Mr. R. M. C. Dice, assisted by Mr. William T. Oliver, executed the carpenter work on the large model, as well as all work of like character required in preparing the exhibits. He showed himself to be a man of energy and resource.

Mr. H. O. French, the chief carpenter of the Survey, materially assisted by solving a number of knotty problems which presented themselves during the construction of some of the models.

Mr. William T. Oliver had proved so useful in such a variety of ways that his appointment as attendant to the exhibit at Chicago was recommended and approved. Messrs. Louis L. Williams, H. G. Gassaway, S. Blake, jr., J. P. Moritz, and A. L. Wasserbach were engaged principally in building up the surface of the large model.

I was relieved from the charge of the exhibit in Chicago on June 17 by Mr. B. A. Colonna, Assistant in charge of the office, and was granted a ten days' leave of absence before proceeding to Eastport, Me., in connection with the international water boundary.

Mr. Colonna was relieved by the following officers of the Survey in turn:

W. H. Dennis, from July 6 to July 24; C. A. Schott, from July 25 to August 15; E. D. Preston, from August 16 to September 7; Edwin Smith, from September 8 to September 30; Lieut. Commander Jeff. C. Moser, from October 1 to October 10.

The following mechanicians were on duty at Chicago in connection with the exhibit:

Louis A. Fischer, from May 1 to May 28; Otto Storm, from June 1 to July 31; E. G. Fischer, from August 1 to August 31; W. R. Whitman, from September 2 to October 16. Also the following plate printers: C. J. Harlow, Eberhard Fordan, and Richard Bright.

Mr. Harlow arrived before the installation was complete and before it was practicable to commence printing the miniature charts. In the meantime he was quite zealous in forwarding the work in every way.

On October 11 I returned to Chicago, relieving Lieutenant-Commander Moser.

Previous to the close of the Exposition, on October 30, such arrangements as could be made for expediting the packing were perfected, and promptly on the morning of the 31st the work of dismounting the instruments and putting them in their boxes was begun.

The wisdom of the office was clearly shown in the detail of Mr. George W. Clarvoe for this work. His long familiarity with the handling and packing of delicate instruments enabled him to accomplish far more than an ordinary carpenter without this experience.

Although the progress made was rapid, every precaution was taken to insure the safety of the articles shipped, with the result that everything arrived in Washington in excellent condition. By November 30 the last details for shipping the exhibits were completed, and I transferred them, together with the invoices and bills of lading, to the charge of Mr. Fred. A. Stocks, representative of the United States Treasury, and returned to Washington.

There was allotted to the Coast and Geodetic Survey out of the appropriation for the United States Government exhibits the sum of $12,000.

Part of the preparation, where expert work was required on instruments and models, was necessarily performed in our instrument and carpenter shops, and no estimate is given of its cost. Other than this, the sum of $10,000 was expended, and the items and approximate amounts can be grouped under the following heads:

Labor, $4,000.—This includes the pay of draftsmen, carpenters, an artist, and the labor required in constructing the iron building.

Material, $1,800.—This includes lumber, paints, oils, hardware, plaster, globe stands, plaster relief models, mechanism for tidal model, miniature tide gauge, miniature tide indicator, etc.

Traveling expenses, $2,000.—This includes the railroad fares to and from Chicago of the officers who had charge of the exhibit at the Exposition, of the mechanicians, and of the plate printers; also their subsistence allowance while en route and in Chicago, at $2.50 per day.

Installation, $1,200.—This includes show cases, desk and chairs, railing, stairway for large model, chart frames and glass, partitions, and decoration.

Maintenance, $1,000.—This includes the pay of an attendant, of a plate printer's helper, water bills, and sundries.

About 300 accounts were presented for settlement, each account requiring duplicate and sometimes triplicate vouchers.

During the preparations in Washington requisitions signed by the Superintendent or his representative were made for all material used.

A book containing printed requisitions, with stubs and serial numbers, was furnished for this purpose.

Exclusive of the short letters transmitting vouchers and accounts, the correspondence consisted of about 150 communications and the same number of replies on various topics connected with the exhibit.

On the resignation of Assistant Secretary A. B. Nettleton, in the spring of 1893, Mr. Fred. A. Stocks, then chief clerk of the Treasury Department, was appointed the representative of the Treasury on the board of management of the Government exhibits.

It is a pleasant duty to record here his unfailing care of our interests and his invariable courtesy in a trying position.

In closing this report, some general remarks seem appropriate in order that those in charge of future exhibits may profit by our past experience.

It is a great saving of time, worry, and expense to carry along all material for installation, except possibly lumber for partitions. Even after the most careful arrangements have been made, once on the ground delays from unforeseen causes will arise. The more independent one is of supplies from the outside the smaller the opportunity for these delays to occur. Posts and railings should be designed, ordered, and made in time to ship with or before the exhibits.

The most satisfactory partition was one adopted by the Treasury Department, following the example of the National Museum. It consisted of planed pine boards with a molding running along the top. Below the molding the boards are covered with wide, heavy cotton cloth, tacked on. The lines of tacks are concealed by a narrow molding. The moldings are painted a dead or drop black, and the cloth stained a dark maroon color. The result is quite rich and effective. It would not answer to dispense with the cotton cloth and paint the boards themselves, since ugly white cracks would appear in time, due to the shrinkage of the wood.

I believe our instruments were the only ones at the Exposition displayed without the protection of glass cases. On this account they gradually lost their bright and neat appearance. On the other hand, the cost of show cases sufficiently large would have been a serious tax upon our allotment. Besides, visitors never take the same interest or show the same desire to study articles which are shut off from them in this way, compared to the interest they manifest for those more accessible.

When practicable, imitation brick or stone piers should be used for mounting all instruments, as they add very much to the effectiveness of the display. Our example in this respect was followed in several instances. All machinery or mechanism in operation has an intense fascination for the average visitor, and next to the souvenir fever may be said to be his most conspicuous mental characteristic.

The rage for souvenirs manifests itself among those of a low moral standard in an exceedingly inconvenient way for the exhibitor. It is unsafe to leave exposed small articles, even those of no intrinsic value, where they can be readily carried off by relic hunters.

The following awards were made to the Survey:

1. For the collective exhibit of charts, maps, models, instruments, and publications.

2. For the charts of the Survey, which are no doubt the best in the world on account of the perfection of survey, the short time in which they were made, the greatness of plan for doing the general survey, and the enormous resources that the Government put into the work.

3. For a number of improvements in the construction of theodolites, levels, and other implements of precision. For a large collection of geodetic instruments of the highest degree of excellence, many of which were improved by members of the corps and made in the shops of the Survey.

4. For important improvements in pendulums for gravity work, namely: The transfer of the knife edges to the supports, whereby they can be polished or sharpened without affecting the pendulum; for the means of determining the period by the principle of coincidences; for the consequent reduction of size and weight, facilitating transportation and manipulation.

5. For the instructive object lesson presented in the model of the United States, including Alaska, by which the true curvature of the earth is clearly shown and the relations of heights and distances by the employment of but one scale. It exhibits in an ingenious manner the direction of the magnetic meridians, and also the principal triangulation of the United States and the position of the base lines.

6. For various ingenious devices for securing the greatest possible accuracy in the measurement of bases, and for determining the errors incidental thereto. For carefully engraved charts, a collection of the annual reports of great interest and importance, and complete sets of tide tables and coast pilots.

7. For evidence of the fine construction and precision of the standards of length, weight, and volume constructed in the shops of the Bureau. For the representatives of the International Metre and the British Imperial yard.

GEODESY.

ON THE VARIATION OF LATITUDE AT SAN FRANCISCO, CAL.,

FROM

OBSERVATIONS MADE IN CONCERT WITH THE INTERNATIONAL GEODETIC ASSOCIATION IN 1891 AND 1892.

Observations by G. DAVIDSON, Assistant.
Discussion and report by CHAS. A. SCHOTT, Assistant.

APPENDIX No. 11—REPORT FOR 1893.

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ILLUSTRATIONS.

APPENDIX No. 11—1893.

THE VARIATION OF LATITUDE AT SAN FRANCISCO, CAL., AS DETERMINED FROM OBSERVATIONS MADE BY GEORGE DAVIDSON, ASSISTANT COAST AND GEODETIC SURVEY, BETWEEN MAY, 1891, AND AUGUST, 1892.

Discussion of results and report by CHAS. A. SCHOTT, Assistant.
Report submitted for publication June 27, 1894.

INTRODUCTORY REMARKS.

The observations for variation of latitude at San Francisco, Cal., were undertaken in cooperation with the International Geodetic Association. It is one of three stations occupied by the Survey, simultaneously, and was selected with respect to position in order to secure the best results for elucidating the phenomenon of the shifting of the earth's axis of rotation. The results at the other two stations have already been published, those on the variation of latitude at Rockville, Md., in Appendix No. 1, Coast and Geodetic Survey Report for 1892, and those reached at Waikiki, near Honolulu, Hawaiian Islands, in Appendix No. 2, Report for 1892. The number of observations at San Francisco considerably exceeds those made at the other stations combined, and their reduction was somewhat delayed for want of available computing force.

POSITION, DESCRIPTION, AND CLIMATIC CHARACTER OF STATION.

The astronomic and geodetic station known as "Lafayette Park" is situated in the northern part of the city of San Francisco, just north of the intersection of Clay and Octavia streets, where a small inclosure protects the temporary observatories, used also for telegraphic longitude work and as a pendulum station for gravitation work. The observatory is a painted wooden structure, 15 feet long east and west, and 10 feet broad north and south. The floor is 3 feet above the ground, and no part of the structure touches the piers. The one upon which the zenith telescope was mounted has stood many years, and is laid up in brick and cement from the base on the clay to the sandstone cap. The top of the pier is about 5 feet from the ground and about 30 inches above the observatory floor. The pier is subject to heat effects. On a very hot day, with the sun shining, the south side always rises. The center of the latitude instrument was 5·1 feet east of the center of the transit instrument (as used for the determination of time and longitude). The north and south openings in the roof above the zenith telescope are 1 foot wide, and the slides were usually kept 10° below the lowest star, except in winter or during strong winds. A short distance east and west of the slides there are wind breaks to protect the levels. The telescope is eccentric to the vertical axis and just clears each side of the roof opening, with the objective about 2 feet inside the building.

The geographical position* is referred to the pier supporting the transit instrument, which is approximately in latitude 37° 47′ 28″·3 and in longitude $8^h\ 09^m\ 42{\cdot}8^s$, or 122° 25′ 42″ W. of Greenwich. The top of the pier supporting the latitude instrument is 382·2 feet above the datum plane of the survey, equivalent to 379·3 feet or $115{\cdot}6^m$ above average sea level. The climatic conditions at this station are very adverse to astronomical observations. The observer states in the preface to his record, "The situation of this station, almost on the southern shore of the Golden Gate and surrounded by water on the east and the west—with the strong, cold ocean winds commingling with the warm or hot air of the day and the comparatively warm air of the night, with high and low fogs forming and dissipating through the night, and variable winds almost always from the sea or with the warm air of the city blowing over the station with a light southerly air—is perhaps the worst that can be imagined." And the observer remarks further, "There was scarcely a night when the stars were not moving slowly or quickly 2″ above or 2″ below the micrometer thread, and frequently 5″ or 6″; sometimes the atmosphere caused them to appear very diffused and nebulous."

INSTRUMENTAL OUTFIT AND METHOD OF OBSERVATION.

Observations for time were made with Transit Instrument No. 3, made by Simms in 1848. It has an aperture of 7^{cm}, a focal length of 116^{cm}, and is used with a magnifying power of 110. Transits were recorded over the 5 lines of the middle tally of the glass reticule, and time was noted by sidereal chronometer Hutton, No. 211, in part by means of a

* For convenience of reference the approximate positions (φ, λ) of the other latitude stations, as well as of Berlin, Germany, are here appended:

	ϕ	λ	
	° ′ ″	h. m. s.	° ′ ″
Waikiki, near Honolulu, Hawaiian Islands	21 16 24·4	10 31 20·1 or	157 50 01 W.
Rockville, Md.	39 05 10·5	5 08 38·5	77 09 37 W.
Berlin, Germany	52 30 16·7	0 53 34·9	13 23 44 E.

chronograph, but later by the eye and ear method. Value of one division of striding level, 1″·01, and pivot inequality clamp end 0·08ˢ larger.

Latitude observations were made by Talcott's method. Zenith Telescope No. 1 was used from the beginning of the series to December 3, 1891, when it was replaced by Zenith Telescope No. 3, which instrument was used to the close of the work.

The first-named instrument is the oldest on the Survey. It was made by Troughton & Simms in 1847. Its principal defect is a lack of stability; hence demanding great caution in handling it. A new eyepiece micrometer was supplied in 1879. The telescope has an aperture of 8·25ᶜᵐ, a focal length of 117ᶜᵐ, and magnifying power used about 53. From May to July 28, 1891, a single level was used. It is a chambered level, graduated from the middle and having a value of one division equal 0″·92 for that part of the scale most frequently used in latitude work, the general value being 0″·90 (at temperature 22°·8 C.), as derived from micrometric measures; length of one division = 1·2ᵐᵐ on ivory scale. After the above date two new levels were attached to the quadrant of the instrument. They are chambered and read from one end of the scale—the lower level from 60 to 100 divisions, the upper one from 0 to 50 divisions. One division of the lower level, marked 11, 1″·55 K & E, was found to be 1″·56 (at temperature 25°·5 C.), and one division of the upper level, marked 9, 1″·7 K & E, 1″.73 (at the same temperature) for those parts of the scale usually observed upon; length of one division = 2ᵐᵐ for both levels. The value of the micrometer screw has been determined from observations on close circumpolar stars near the eastern and western elongations. They embrace the following series:

No.	Date.	Star.	Phase.	Range of screw turns.	Subdivisions of a turn.	Resulting value of one turn.	Weight (relative).
	1891.					″	
1	May 28	λ Urs. Min.	E. Elong.	7 to 27	$\frac{1}{8}$	47·544	49
2	31	"	"	10 30	$\frac{1}{8}$	·568	42
3	June 1	"	"	12 30	$\frac{1}{8}$	·505	30
4	3	"	"	8 30	$\frac{1}{8}$	·546	49
--	5	"	"	9 13	$\frac{1}{8}$	[1·069]	0
5	12	"	"	16 25·2	$\frac{1}{8}$	·619	23
6	Aug. 5	α Urs. Min.	E. Elong.	5 36	$\frac{1}{4}$	·625	63
7	6	"	"	4 36	$\frac{1}{4}$	·590	65
8	7	"	"	4·5 37	$\frac{1}{4}$	·590	65
9	Nov. 24	λ Urs. Min.	W. Elong.	33 7	$\frac{1}{4}$	·632	53
10	25	"	"	33 7	$\frac{1}{4}$	·679	50
11	27	"	"	33 6	$\frac{1}{4}$	·608	52
12	Dec. 1	"	"	33 7	$\frac{1}{4}$	·605	51

The weighted mean value from the 12 series is 1 revolution or turn of 100 divisions=47″·596±0″·008. The relative weights depend on the number of individual observations in each series, and the parts of the screw haye been compared with the adopted value 47″·60, furnishing the following corrections to the screw readings on account of progressive irregularity in the value of one turn:

Corrections to micrometer readings for inequality in values of turns.

Reading.	Corr.	Reading.	Reading.	Corr.	Reading.
t	d	t	t	d	t
7·91	+2·6	35·29	10·93	+0·9	29·32
8·06	2·5	34·93	11·16	0·8	28·96
·22	2·4	·57	·38	0·7	·59
·37	2·3	34·21	·62	0·6	28·20
·52	2·2	33·86	·87	0·5	27·81
·68	2·1	·53	12·14	0·4	·41
·85	2·0	33·18	·42	0·3	27·00
9·02	1·9	32·82	·70	0·2	26·59
·17	1·8	·50	13·00	0·1	26·16
·35	1·7	32·14	·33	0	25·70
·53	1·6	31·79	·68	—0·1	25·24
·72	1·5	·46	14·06	0·2	24·74
·91	1·4	31·11	·48	0·3	24·19
10·10	1·3	30·75	·95	0·4	23·61
·30	1·2	·39	15·46	0·5	22·94
·50	1·1	30·04	16·09	0·6	22·19
·71	1·0	29·68	·94	0·7	21·22
	+0·9			—0·8	

A further correction for periodic inequality in a turn was applied, viz:

Reading of screw head.	Corr'n.	Reading of screw head.
d	d	d
52·8	+0·3	71·8
46·4	0·2	76·9
40·1	0·1	80·9
32·1	0·0	84·9
21·8	—0·1	89·4
11·6	—0·2	95·9
0·0	—0·3	100·0

The sidereal focus of the telescope was tested before each series. Changes of temperature do not appear to have any sensible effect upon the screw value. Trials were also made for testing the regularity of the screw without resort to star observations. The method is to remove the micrometer and fit it to the stage of a microscope; turn the microscope tube horizontal; fix a ground-glass scale, say of millimetres, at such distance from the ocular that one revolution of the screw shall cover just 20mm. The illumination is by lamplight, and the space between the ocular and the scale is covered with black velvet. After adjusting the lines of the scale and the micrometer thread, the observer moves the scale horizontally until a certain space is filled with the thread when the micrometer is reading zero. The micrometer head is then turned until the thread fills the next space, when the micrometer is again read. The process is continued to get a number of such sets. Satisfactory results were obtained in this manner. For extra meridional observations the diaphragm of Telescope No. 1 was provided with 5 vertical lines, with intervals from the middle line of $32{\cdot}0^s$, $16{\cdot}1^s$, $0{\cdot}0^s$, $16{\cdot}6^s$, and $33{\cdot}1^s$, respectively.

From and after December 13, 1891, Zenith Telescope No. 3 took the place of No. 1 for the latitude work. No. 3 is by the same makers and dates from 1848. It was, however, remodeled in 1891. It is of the same pattern as the instrument used at Rockville, Md., and shown on plate No. 2, in Appendix No. 1, Report for 1892. The improvements made consist of: New objective, by Brashear; new diagonal eyepiece and straight eyepieces, by Kahler; two new levels; a new stride level; entirely new micrometer with horizontal and vertical parallactic motions; new and larger axis of telescope, with adjustment; new and heavier base and vertical axis; new wyes for support of telescope; improved clamp for telescope, and some minor repairs. The instrument is thus practically new. Aperture of telescope, $7{\cdot}6^{cm}$; focal length, $116{\cdot}6^{cm}$; magnifying power used, about 100. The observer remarks: "Stars which the objective of No. 1 would not reach are readily observed with the new objective, and seventh magnitude stars can now be observed with greater satisfaction than sixth magnitude stars with No. 1. The double star B. A. C. 2300 is separated clean and distinctly and is easy to observe." The values of the divisions of the chambered levels were as follows: Level mounted in upper case marked 4, $1''{\cdot}8$, K & E, and numbered from 0 to 60, with 2^{mm} divisions—one division equals $1''{\cdot}805$; level in lower case marked 5, $1''{\cdot}8$ K & E, and numbered from 70 to 130, with 2^{mm} divisions—one division equals $1''{\cdot}807$ (temperature about 17° C.), as tested October 15 and 16, 1891. A long series of observations were made on close circumpolar stars to establish the value of the micrometer screw, as well as its irregularities. For determining the horizontality of the micrometer thread the instrument was collimated with a Fraunhofer telescope. A bright, star-like object was selected on the eyepiece of the collimator on which to test the motion of the screw, the same as had been done with the first or older instrument. No less than 4 443 observations were recorded for value of micrometer,* and it became necessary to make a selection of the series, omitting from computation the more or less broken and imperfect series, thus retaining 15 of the total number 31.

* Increasing numbers of micrometer correspond to decreasing zenith distances. Micrometer and levels were read by electric light.

The following table exhibits the results:

No.	Date.		Star.	Phase.	Range of screw turns.		Subdivisions of a turn.	Resulting value of one turn.	Weight (relative).
	1892.							″	
1	Feb.	8	α Urs. Min.	W. Elong.	5 to	35	¼	47·668	61
2		9	Bradley 1672	E. "	35	9½	¼	·557	50
3		9	α Urs. Min.	W. "	5	35	⅕	·633	76
4		10	Bradley 1672	E. "	35	9½	¼	·659	49
5		10	α Urs. Min.	W. "	5	35	⅕	·671	73
6	May	4	λ Urs. Min.	E. "	35	4	⅕	·624	75
7		6	δ Urs. Min.	E. "	35	5	½	·656	31
8		6	λ Urs. Min.	E. "	35	5	⅕	·620	76
9		7	δ Urs. Min.	E. "	35	5	½	·648	31
10		7	λ Urs. Min.	E. "	35	5	⅕	·621	76
11	Sept.	5	α Urs. Min.	E. "	36	4	⅕	·638	81
12		5	λ Urs. Min.	W. "	4	36	⅕	·647	81
13		6	λ Urs. Min.	W. "	4	36	⅕	·642	81
14		17	α Urs. Min.	E. "	36	4	⅕	·617	81
15		21	λ Urs. Min.	W. "	4	36	⅕	·644	81

The weighted mean value of one turn is 47″·636 ± 0″·005. The discussion of the results for inequality of screw showed that it was nearly perfect. The small corrections for inequality in values of one turn are as follows, and correspond to an adopted value, 47″·60:

Reading.	Corr.	Reading.	Reading.	Corr.	Reading.
t	d	t	t	d	t
	+2·0			+0·7	
	1·9	35·86	3·89	0·6	27·21
	1·8	35·10	4·25	0·5	26·57
	1·7	34·37	4·63	0·4	25·91
	1·6	33·67	5·04	0·3	25·24
	1·5	32·99	5·46	0·2	24·56
	1·4	32·32	5·91	0·1	23·86
	1·3	31·66	6·40	0·0	23·13
	1·2	31·02	6·93	—0·1	22·36
	1·1	30·38	7·51	0·2	21·55
	1·0	29·74	8·16	0·3	20·67
2·89	0·9	29·11	8·90	0·4	19·70
3·21	0·8	28·48	9·78	0·5	18·58
3·54	+0·7	27·84	10·99	—0·6	17·09

The correction for periodic inequality in one turn, or for eccentricity of screw head, is as below:

Reading of head.		Corr.	Reading of head.	
d	d	d	d	d
		+0·2		
22·7	33·2	+0·1		
15·3	38·9	0·0	75·5	92·8
0·0	43·6	—0·1	70·3	100·0
	48·1	—0·2	65·1	

The changes of temperature do not appear to have any sensible effect upon the value of the screw. For extra-meridional observations there were five vertical lines on the reticule at the respective distances of $18{\cdot}9^s$, $9{\cdot}4^s$, $0{\cdot}0^s$, and $9{\cdot}5^s$, the last line being missing. During the observations for latitude the condition of the atmosphere as to pressure, temperature, moisture, wind, and general aspect was noted. For recording the time of observation either sidereal chronometer Hutton, No. 211 or break-circuit (sid.) chronometer Frodsham, No. 3479 was employed.

Respecting the method of observing, the record contains the following statements: "The meridian of the instrument was determined each night before the latitude observations by an observation of a circumpolar star to fix the east stop for the north meridian, and by an observation of a low south star to fix the west stop for the south meridian. During the series of observations care was taken in reversing the instrument to come up to the stops without jar." Owing to want of horizontality of the micrometer thread when making extra-meridian pointings,* the observer remarks: "I therefore propose that all the *earlier* extra-meridional observations be rejected." The levels were read about one and one-fourth minute before the passage of the star, the pillar and telescope having been slightly tapped with a wand to remove any possible strain, and they were read again immediately after the transit. The observer's judgment is that the level readings *before* the star observations should be disregarded. Earthquake shocks were experienced on June 20, 1891, on October 11, 1891, and on April 18, 1892, without any apparent local effect on the direction of the vertical. The following is an extract from the record: "I determined to observe on every clear night two consecutive groups of three hours each; to observe in each group as many pairs as practicable, having reasonable regard to the authorities and the amount and sign of the difference of zenith distances in each pair. In selecting pairs where the proper motion was determined I was restricted to the magnitude of stars not smaller than six and one-half. I paid no regard to the (absolute) zenith distances of the pairs, and have gone as far as zenith distance 46° 38′ when there was an unfilled interval. I have taken some pairs of doubtful value, both as to authorities and large range of micrometer or shortness of interval. These may be eliminated if their weakness is shown to be detrimental to satisfactory results. * * * The difference of zenith distance of the two stars in a pair rarely exceeded 15′, and was then only adopted when no other combination could be utilized. * * * I was hampered in the selection of pairs because I had but few catalogues to collate from."

The observations for time were made by F. W. Edmonds and Subassistant F. Morse. The earlier observations were recorded on the chronograph; after that by the eye and ear. The latitude observations

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*Such pointings were introduced by the observer prior to the year 1880, and had otherwise been considered prior to 1866. (Report for 1866, p. 75.)

proper were all made by Assistant G. Davidson, aided, when observing for value of micrometer, by T. D. Davidson or by G. J. Kammerer, who read the levels, and by F. W. Edmonds, who noted the times and made the record. The series extends over fifteen months with but one noticeable break of forty days, in June and July, 1892, when the observer was absent on other official duty.

The total number of individual latitude determinations is 6 768, the total number of nights of observation 237, and the average number of pairs of stars (doublets, triplets, and other combinations included) 28½. The whole star work is arranged in 8 groups, of which 3 were reobserved after the expiration of the year. The date of the first observation is May 27, 1891, and that of the last August 19, 1892. There is, by the same observer (same place, same instrument, No. 1, same pairs of stars), a prior series of observations, in January and February, 1888, as well as a later one, commencing with November, 1893, and extending to the present year. These series will be taken up and the results communicated at some future time.

THE REDUCTION OF THE OBSERVATIONS FOR LATITUDE.

The labor connected with the computation from a record of 52 octavo books, and involving no less than 6 768 individual results for latitude, was distributed among several computers, viz: The determination of the mean places of stars, 303 in number, and the computation and discussion (with table of corrections for inequality) of the 28 series for value of micrometers were assigned to H. Farquhar; the computation of the apparent places of stars and the application of the several corrections for micrometer measures and for level and refraction were intrusted to L. Pike; and the general revision of these results, together with the reduction to the meridian and final checking, was placed in the hands of H. F. Flynn, all members of the computing division. Further occasional assistance was had by the temporary assignment of 4 computers for short times. The combination and discussion of the results are due to the writer. The whole work was accomplished substantially between May 1, 1893, and May 31, 1894.

THE MEAN PLACES OF STARS.

In making out the mean places of stars all available star catalogues were consulted, as will be seen from the appended but condensed form of presentation given to it by Mr. H. Farquhar, the high quality, as to accuracy, of whose results may be seen in the probable errors assigned to them, and better in the small reductions to the group means of the results of the individual pairs forming the groups.

Designations of authorities used.

Designation.	Observatory.	Epoch.	Conductor of observations.	Editor, etc.
a^{i}	= Armagh	1834–54	Robinson	
a^{ii}	"	59–82	"	Dreyer.
b	= Berlin.	76–78	Becker	
c^{iv}	= Cape of Good Hope	34–40	Maclear	Stone.
c^{v}	" " "	49–52	"	Gill.
c^{vii}	" " "	71–79	Stone	
d	= Bonn	45–67	Argelander	
e^{i}	= Edinburgh	34–44	Henderson	Smyth.
e^{ii}	"	54–69	Smyth	
f^{v}	= Paris	37–53	Arago	Mouchez.
f^{vi}	"	54–69	Le Verrier	"
f^{vii}	"	70–81	De Launay	"
g^{v}	= Greenwich	36–47	Airy	12y
g^{vi}	"	48–53	"	6y
g^{vii}	"	54–60	"	7y^{1}
g^{viii}	"	61–67	"	7y^{2}
g^{ix}	"	68–76	"	9y
g^{x}	"	77–86	Christie	10y
h^{i}	= Harvard	70–79	Rogers	Pickering.
h^{ii}	"	83–85	"	"
i	= Madras	50±	Jacob	Smyth.
k	= Glasgow	60–81	Grant	
l	= Leiden	64–70	Kaiser	
m	= Melbourne	63–70	Ellery	
n	= Redhill	54–56	Carrington	
o^{i}	= Radcliffe	40–54	Johnson	Main.
o^{ii}	"	54–61	"	"
o^{iii}	"	62+	Main	
p^{i}	= Pulkowa	42–49	Struve	Ver. Circ.
p^{ii}	"	40–69	"	Merid. Circ.
p^{iv}	"	63–75	Gyldén, Nyrén	Auwers (Hauptst.).
p^{v}	"	69–74		" (Zusatzst.).
p^{vi}	"	74–80	Romberg	
q	= Brussels	56–78	Quetelet	Folie.
r	= Rome (Capit.)	75–77	Respighi	
s	= Ann Arbor	79±	Schaeberle	MS.
t	= Madison	85	Holden	MS.
u	= Cordoba	72–83	Gould	
v	= Leipzig	66–70	Engelmann	Auwers, Safford.
w	= Washington	45–77		Yarnall, Frisby.
	Hamburg	36+	*Rümker*	
	Königsberg	25±	Bessel	*Weisse.*

(Last two are only quoted for a very few stars for which other authorities are lacking.)

Designations of stars.

[No special designation B. A. C.]

	Observatory.	Epoch of Cat.	Observer.	Editor.
F	Paris	1790	Lalande	Fedorenko.
G	Blackheath	1810	Groombridge	Airy.
L	Paris	1800	Lalande	Baily.
$Œ_{n}$	Argelander's northern zones			Œltzen.

*** In the table, authorities are not included when the observations are earlier than 1834. No weight is given to any such authorities when the proper motion is derived from Auwers's Bradley. When the proper motion has to be calculated (or Auwers's value corrected), earlier authorities are used, though not here quoted.

A few later authorities used by Auwers and Safford are omitted, because omitted on the sheets. Auwers's stars are those under p^{iv} and p^{v}. Most of the p^{iv} stars are p^{i} also; a considerable number of them also l and v.

Authorities.	No. *B.A.C.*, etc.	Mean *N.P.D.* 1891·0.			ϵ^2	μ'
		°	′	″		″
$a^{i}c^{i,ii}g^{v,vi,vii,viii,ix,x}h^{i,ii}o^{i}p^{ii,iv,vi}qw$	4696	25	06	11·00	0·02	−0·019
$a^{i}e^{i}g^{viii,ix,x}ko^{ii}p^{ii}q$	4724	79	23	09·31	·05	+0·167
$g^{vii,viii}io^{i}qw$	4783	51	06	50·76	·22	+0·003
$a^{ii}e^{ii}g^{ix}h^{i}ikqrw$	4797	53	18	55·28	·10	+0·01
$a^{i}c^{vii}e^{i}g^{v,vi,vii,viii,ix,x}h^{i,ii}kmo^{ii}p^{ii,iv,vi}qrw$	4808	59	08	59·84	·02	−0·111
$a^{i}be^{i}g^{v,x}h^{i}o^{i,ii}p^{ii,v,vi}qr$	4843	45	07	29·55	·02	+0·034
$a^{ii}e^{ii}io^{iii}qrw$	4863	52	46	45·27	·22	+0·06
$de^{i}g^{x}iko^{i}p^{ii,vi}qw$	4897	51	44	21·88	·04	−0·112
$a^{i}de^{i}g^{v,x}o^{i,ii}p^{ii,vi}qrw$	4937	39	55	32·15	·03	+0·227
$a^{i}e^{i}g^{v,ix,x}p^{ii,vi}qrw$	4953	64	33	38·94	·03	+0·060
$a^{i}e^{i}g^{ix,x}h^{i}o^{i,ii}$	5022	5	37	39·90	·03	−0·029
$c^{iv,vii}e^{i,ii}g^{v,vi,vii,viii,ix,x}h^{i}kmo^{i,ii}p^{ii,iv,vi}quw$	5034	98	58	49·43	·02	+0·030
$a^{i}e^{i}g^{v,vi,vii,viii,ix,x}h^{i,ii}o^{i,ii}p^{ii,iv,vi}qrw$	5097	30	39	07·17	·01	−0·013
$a^{i}g^{viii}kp^{ii}$	5120	73	34	26·43	·12	+0·007
$g^{x}h^{i}p^{ii}s$	*F* 2677	20	21	53·29	·10	−0·04
$a^{ii}g^{x}$	*L* 28716	84	12	37·02	·08	0·00
$a^{i}c^{v}e^{i}g^{viii}kp^{ii,vi}quw$	5246	92	45	36·66	·04	+0·035
$a^{i}c^{i}g^{v,vi,vii,viii,ix,x}h^{i,ii}o^{i}p^{ii,iv,vi}qw$	5285	11	52	13·60	·02	0·000
$a^{i}e^{i}g^{v,vi,ix,x}h^{i,ii}o^{i,ii}p^{ii,iv,vi}qr$	5348	31	08	36·71	·02	−0·345
$be^{i}g^{v,vi,x}p^{ii}qw$	5392	73	03	07·27	·04	+0·011
$e^{i}g^{v,vii,ix,x}io^{i}p^{ii}r$	5459	29	58	50·36	·04	−0·019
$a^{ii}e^{ii}ikq$	5504	74	24	22·71	·28	0·00
$a^{ii}e^{ii}g^{viii}iqr$	5530	67	34	12·68	·20	+0·045
$a^{i}e^{i}g^{v,ix,x}lo^{i}p^{ii,vi}r$	5574	36	52	50·85	·03	−0·024
$a^{i}\left\{\begin{matrix} e^{i}g^{v,ix,x}o^{i}p^{ii,vi} \\ g^{v,ix,x}o^{i}p^{vi} \end{matrix}\right\}r$	5575	36	51	$\left\{\begin{matrix}22·88\\24·44\end{matrix}\right\}$ 23·30	·04	−0·024
$a^{i}be^{i}g^{v,vi,x}h^{i}p^{ii}qw$	5692	79	39	17·50	·04	+0·048
$e^{i}g^{v,vii,ix,x}o^{i,ii}p^{ii,vi}q$	5740	24	41	55·28	·03	−0·047
$a^{ii}e^{i}g^{x}p^{ii}$	*L* 31117	55	03	27·87	·07	+0·014
$e^{i}g^{x}h^{i}io^{i}p^{ii,v,vi}rw$	5790	49	20	28·87	·03	+0·035
$c^{iv,vii}e^{i,ii}g^{v,vi,vii,viii,ix,x}h^{i,ii}kmo^{i,ii}p^{ii,iv,vi}qw$	5821	75	29	06·38	·01	−0·022
$a^{i}g^{ix,x}o^{i}p^{ii,v}$	*G* 2432	29	10	11·30	·03	−0·045
$a^{i}e^{i}g^{ix,x}o^{i}p^{ii}$	*G* 2433	29	12	49·24	·04	−0·014
$a^{i}be^{i}g^{v,x}ko^{ii}p^{ii}qrw$	5883	66	56	17·01	·03	+0·048
$a^{i}e^{i}g^{v,vi,vii,viii,ix,x}h^{i,ii}ko^{i,ii}p^{ii,iv,vi}qrw$	5937	37	37	04·18	·02	−0·001
$a^{i}g^{ix,x}ko^{ii}p^{ii}qr$	5988	65	25	57·15	·04	−0·042

Authorities.	No. *B.A.C.*, etc.	Mean *N.P.D.* 1891·0.			e^2	μ'
		°	′	″		″
$a^{\mathrm{i}}g^{\mathrm{viii,ix,x}}kp^{\mathrm{ii,vi}}qr$	5999	65	22	50·53	0·03	+0·118
$a^{\mathrm{i}}be^{\mathrm{i}}g^{\mathrm{v,x}}o^{\mathrm{i,ii}}p^{\mathrm{ii,vi}}qrw$	6052	39	11	35·14	·02	—0·198
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{v,vi,vii,ix,x}}p^{\mathrm{ii,vi}}rw$	6087	59	48	05·32	·05	+0·007
$e^{\mathrm{i}}g^{\mathrm{x}}o^{\mathrm{i}}p^{\mathrm{ii}}q$	*G* 2494	44	31	03·05	·12	—0·023
$e^{\mathrm{i}}g^{\mathrm{x}}o^{\mathrm{i}}p^{\mathrm{ii}}qr$	6109	44	29	37·14	·05	+0·058
$e^{\mathrm{i}}g^{\mathrm{x}}h^{\mathrm{i}}p^{\mathrm{ii}}s$	*L* 33291	57	46	43·84	·05	+0·028
$a^{\mathrm{i}}g^{\mathrm{x}}io^{\mathrm{i}}p^{\mathrm{ii}}r$	6162	46	33	06·69	·04	+0·062
$e^{\mathrm{i}}g^{\mathrm{x}}p^{\mathrm{ii}}$	*L* 33521	56	34	45·45	·08	0·00
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{viii,ix,x}}h^{\mathrm{i}}io^{\mathrm{i}}p^{\mathrm{ii,v,vi}}r$	6203	47	52	39·70	·02	+0·008
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{v,vii,ix,x}}p^{\mathrm{ii}}rw$	6235	53	59	04·06	·05	—0·020
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{viii,ix,x}}o^{\mathrm{i}}p^{\mathrm{ii}}qr$	6268	50	33	07·26	·03	+0·005
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{v,viii,ix,x}}kp^{\mathrm{ii}}qw$	6322	66	27	50·71	·03	—0·007
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{v,ix,x}}kp^{\mathrm{ii,vi}}qw$	6341	66	28	56·29	·03	+0·001
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{ix,x}}lo^{\mathrm{i}}p^{\mathrm{ii}}qrw$	6372	37	54	23·59	·03	—0·012
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{viii,ix,x}}ko^{\mathrm{ii}}p^{\mathrm{ii}}q$	6397	71	56	21·92	·04	—0·106
$g^{\mathrm{x}}h^{\mathrm{i}}p^{\mathrm{ii}}s$	*L* 35333	72	01	50·77	·05	+0·18
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{v,vii,x}}h^{\mathrm{i}}o^{\mathrm{i}}p^{\mathrm{ii}}qrt$	6496	32	19	46·48	·03	+0·064
$a^{\mathrm{i}}c^{\mathrm{vii}}e^{\mathrm{i,ii}}g^{\mathrm{v,vi,vii,viii,ix,x}}h^{\mathrm{i,ii}}kmo^{\mathrm{i,ii}}p^{\mathrm{ii,iv,vi}}qw$	6528	76	17	53·72	·01	+0·108
$g^{\mathrm{ix}}io^{\mathrm{i}}r$	6555	28	04	10·16	·09	+0·005
$de^{\mathrm{i}}g^{\mathrm{x}}io^{\mathrm{i}}p^{\mathrm{vi}}q$ } *r*	6579	40	20	58·38	·02	—0·626
$g^{\mathrm{x}}io^{\mathrm{i}}p^{\mathrm{ii,vi}}q$ }	*L* 36249	40	20	50·86	·03	—0·626
$io^{\mathrm{i}}r$	6603	40	07	16·21	·19	—0·015
(Rümker)	*L* 36485	40	20	39·	8·0	-------
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{viii,ix,x}}kp^{\mathrm{ii}}r$	6637	63	56	47·57	0·03	+0·025
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{v,vii,viii,x}}h^{\mathrm{i,ii}}ko^{\mathrm{i,ii}}p^{\mathrm{ii,v,vi}}qrw$	6734	40	01	52·26	·01	—0·245
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{viii,ix,x}}kp^{\mathrm{ii}}r$	6758	64	29	19·25	·03	—0·014
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{v,vii,x}}h^{\mathrm{i}}o^{\mathrm{ii}}p^{\mathrm{ii}}rt$	6824	37	17	18·56	·03	+0·078
$e^{\mathrm{i}}g^{\mathrm{vii,ix,x}}kp^{\mathrm{ii}}qr$	6866	67	11	42·27	·02	—0·007
a^{ii} (Rümker)	*L* 38237	70	17	44·53	·90	+0·02
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{viii,x}}ko^{\mathrm{ii}}p^{\mathrm{ii}}qw$	6901	70	19	15·81	·05	—0·081
$g^{\mathrm{v}}h^{\mathrm{i}}o^{\mathrm{i}}p^{\mathrm{vi}}qr$	6924	33	58	25·64	·04	—0·080
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{viii,ix,x}}kp^{\mathrm{ii}}qr$	6957	61	38	07·21	·04	+0·045
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{v,vii,ix}}lo^{\mathrm{i}}p^{\mathrm{ii}}rt$	6983	42	37	14·12	·03	+0·013
$g^{\mathrm{x}}h^{\mathrm{i}}o^{\mathrm{i}}p^{\mathrm{ii}}s$	*G* 3151	44	33	18·53	·05	—0·030
$a^{\mathrm{i,ii}}e^{\mathrm{i}}g^{\mathrm{vii,ix,x}}h^{\mathrm{i}}p^{\mathrm{ii}}rtw$	7067	59	59	42·05	·03	+0·015
$a^{\mathrm{i}}g^{\mathrm{v,vi,vii,ix,x}}ko^{\mathrm{iii}}p^{\mathrm{ii,vi}}qw$	7146	74	32	40·11	·06	+0·019
$c^{\mathrm{vii}}e^{\mathrm{i}}g^{\mathrm{v,vi,vii,viii,ix,x}}h^{\mathrm{i}}ko^{\mathrm{ii}}p^{\mathrm{ii,iv,vi}}qw$	7149	74	28	20·15	·02	+0·015
$a^{\mathrm{i}}g^{\mathrm{x}}iko^{\mathrm{i}}p^{\mathrm{ii}}qr$	7176	29	53	19·60	·04	—0·182
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{viii}}ko^{\mathrm{ii}}p^{\mathrm{ii,vi}}rw$	7194	59	40	41·07	·06	—0·020
$iko^{\mathrm{i}}rw$	7219	44	49	13·54	·14	+0·012
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{viii}}o^{\mathrm{i}}p^{\mathrm{ii,vi}}r$	7306	44	16	19·90	·06	+0·001
$a^{\mathrm{i}}g^{\mathrm{ix,x}}o^{\mathrm{ii}}p^{\mathrm{ii}}q$	*L* 41026	60	14	04·98	·02	+0·026
$a^{\mathrm{i}}c^{\mathrm{vii}}e^{\mathrm{i,ii}}g^{\mathrm{v,vi,vii,viii,ix,x}}h^{\mathrm{i,ii}}kmo^{\mathrm{i,ii}}p^{\mathrm{ii,iv,vi}}qrw$	7368	60	13	12·20	·01	+0·068
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{viii}}iko^{\mathrm{i}}p^{\mathrm{ii,vi}}rw$	7402	46	30	45·75	·05	+0·020
$a^{\mathrm{ii}}g^{\mathrm{x}}h^{\mathrm{i}}p^{\mathrm{ii}}s$	*L* 41554	57	51	00·80	·05	0·00
$a^{\mathrm{i}}e^{\mathrm{i}}g^{\mathrm{ix,x}}kp^{\mathrm{ii}}qrw$	7437	66	11	38·67	·04	—0·003

Authorities.	No. *B.A.C.*, etc.	Mean *N.P.D.* 1891·0.			ϵ^2	μ'
		°	′	″		″
$g^{viii}io^{i}qr$	7488	38	17	11·51	0·12	−0·072
$g^{ix}o^{i}w$	7508	19	39	31·52	·14	+0·008
$a^{i}c^{vii}e^{i}g^{vii,x}kp^{ii}qw$	7522	84	43	11·82	·05	−0·018
$a^{i,ii}e^{i}g^{viii}kp^{ii}q$	7547	84	48	58·95	·08	+0·018
$a^{i}e^{i}g^{x}p^{ii}$	*L* 42627	70	41	04·48	·07	+0·010
$a^{i}c^{viii}eg^{vi,vii,viii,ix,x}h^{i,ii}kmo^{ii}p^{ii,v,vi}qrw$	7627	64	35	15·46	·02	+0·014
$a^{ii}e^{i}g^{x}h^{i}p^{ii}qs$	*L* 42690	70	50	44·26	·04	+0·005
$a^{i}e^{i}g^{v,vii,x}h^{i}o^{i}p^{ii}rw$	7643	33	54	17·70	·02	+0·009
$e^{i}g^{ix}o^{i}qw$	7678	10	12	37·21	·09	−0·011
$a^{i}c^{v}e^{i}g^{viii}ko^{ii}p^{ii}qu$	7720	94	25	41·39	·11	+0·063
$a^{i}c^{vii}e^{i}g^{vii,x}kp^{ii,vi}qw$	7788	84	45	29·60	·04	+0·018
$e^{i}g^{x}h^{i}p^{ii}s$	*F* 4151	19	47	04·12	·06	+0·004
$a^{i}e^{i}g^{viii,ix,x}ko^{ii}p^{ii}qr$	7843	57	59	06·87	·02	+0·023
$a^{i}e^{i}g^{vii,viii,ix,x}o^{i,ii}p^{ii}qr$	7906	46	17	34·04	·02	+0·006
$a^{i}g^{vii,x}iko^{i}p^{ii}qr$	7961	34	40	31·13	·05	−0·034
$a^{i}e^{i}g^{viii,ix,x}o^{ii}p^{ii,vi}q$	7997	69	48	56·08	·03	−0·047
$a^{i}e^{i}g^{vii,viii,ix,x}o^{i}p^{ii}qw$	8039	23	22	42·09	·02	−0·010
$a^{i}c^{vii}e^{i}g^{vii,x}h^{i}kp^{ii}$	8051	81	10	45·55	·04	+0·027
$g^{viii,ix}io^{i}q$	8077	23	20	58·08	·12	−0·034
$a^{i}e^{i}g^{viii,ix,x}kp^{ii}qrw$	8097	62	20	47·01	·04	+0·016
$a^{i}e^{i}g^{viii}o^{i}p^{ii}r$	8125	41	58	21·70	·07	−0·038
$a^{i}e^{i}g^{viii}p^{ii}r$	8211	57	06	20·22	·08	−0·031
$e^{i}g^{v,vi,vii,viii,ix,x}h^{ii}o^{i}p^{ii,iv,vi}rw$	8229	47	20	07·64	·01	+0·016
$a^{ii}g^{x}h^{i}s$	*L* 46607	33	09	14·73	·14	+0·05
$a^{i}e^{i}g^{viii,ix,x}h^{i,ii}p^{ii,v,vi}q$	8299	71	29	06·47	·02	+0·054
$a^{i}e^{i}g^{v,viii,x}h^{i}o^{i}p^{ii,v,vi}rw$	8310	33	06	25·81	·02	+0·013
$g^{x}h^{i}qs$	*L* 47167	73	03	10·99	·08	+0·03
$a^{i}be^{ii}g^{v,vi,ix,x}o^{i}p^{ii,vi}qw$	8364	32	04	29·34	·03	+0·031
$a^{i}de^{ii}g^{ix}o^{i}p^{ii,vi}qw$	8372	32	10	15·87	·06	−0·033
$a^{i}e^{i}g^{viii,ix,x}o^{ii}p^{ii}q$	8	72	23	38·40	·04	+0·035
$a^{i}e^{i}f^{vi,vii}g^{vi,ix,x}h^{i}o^{i,ii}p^{ii,vi}qrw$	52	51	55	24·92	·02	+0·020
$a^{i}e^{i}f^{vi}g^{viii,ix,x}lp^{ii,vi}qrw$	67	52	38	06·84	·02	+0·038
$a^{i}e^{ii}f^{v}g^{v,ix,x}p^{ii,vi}qw$	105	13	34	55·99	·02	+0·039
$a^{i}c^{v,vii}e^{i}f^{v,vi,vii}g^{viii,ix}ko^{ii}p^{ii}quw$	147	91	06	16·42	·05	+0·062
$a^{i}e^{i}f^{vii}g^{viii}ko^{ii}p^{ii}rw$	170	69	09	34·83	·05	+0·037
$a^{i}f^{vi}g^{x}ko^{i,ii}p^{ii}qr$	201	35	22	31·04	·04	+0·012
$a^{i}e^{i,ii}f^{v,vi}g^{v,vi,vii,viii,ix,x}h^{i}o^{i,ii}p^{ii,iv,vi}qrw$	218	32	45	44·30	·03	+0·482
$a^{i}e^{i}f^{v,vi}g^{viii,ix,x}ko^{iii}p^{ii,vi}q$	247	71	24	10·54	·03	+0·019
$f^{vii}g^{x}o^{i}p^{ii}q$	*G* 205	45	52	27·27	·04	+0·036
$a^{i}e^{i}f^{v,vi,vii}g^{ix,x}h^{i}p^{ii}r$	285	58	46	52·39	·02	+0·034
$a^{i}e^{i}f^{v,vii}g^{viii,x}o^{ii}p^{ii}qr$	321	58	34	10·88	·04	+0·047
$a^{i}e^{i}g^{viii}p^{ii}r$	345	59	09	18·46	·12	+0·008
$a^{i}e^{i}f^{v,vi}g^{vii,viii,ix,x}o^{i}p^{ii,vi}rw$	404	45	02	33·86	·03	+0·012
$a^{i}be^{i}f^{v}g^{v,vi,x}o^{i}p^{ii}r$	456	31	19	39·54	·03	+0·026
$e^{i}f^{v,vii}g^{vii,viii,x}o^{ii}p^{ii}qw$	477	73	07	27·79	·05	−0·022

Authorities.	No. *B.A.C.*, etc.	Mean *N.P.D.* 1891·0.			e^2	μ'
		°	′	″		″
$e^{i}f^{vii}g^{vii,viii,ix,x}p^{ii,vi}w$	500	74	08	50·83	0·03	+0·021
$a^{i}e^{i}f^{v}g^{v,ix,x}o^{i,ii}p^{ii}qr$	515	29	59	55·50	·02	+0·027
$a^{i,ii}e^{ii}g^{v,x}io^{i}p^{ii}$	588	25	54	33·45	·05	+0·028
$c^{vii}e^{i}f^{v,vii}g^{ix,x}kp^{ii}qw$	609	78	14	03·01	·04	+0·036
$a^{i}e^{i}f^{v,vii}g^{viii,ix,x}ko^{i,ii}p^{ii}qr$	644	67	52	18·15	·03	+0·056
$a^{i}bg^{x}o^{i}p^{vi}$	653	36	40	20·64	·04	+0·055

Authorities.	No. *B.A.C.*, etc.	Mean *N.P.D.* 1892·0.			e^2	μ'
		°	′	″		″
$a^{i}e^{i}f^{vi}g^{vii,viii,ix}h^{i}o^{i,ii}p^{ii,v,vi}qr$	673	39	26	10·81	0·02	+0·177
$a^{i}f^{v,vi,vii}g^{viii}o^{ii}p^{ii}qw$	692	64	43	06·94	·07	+0·070
$a^{i,ii}e^{i}f^{v,vi}g^{v,vii,viii,ix,x}o^{i,ii}p^{ii,v,vi}qw$	707	70	35	55·44	·03	+0·002
$a^{i}g^{viii}o^{i}p^{ii}q$	733	33	52	49·61	·07	−0·005
$f^{v,vi,vii}g^{v}o^{i}r$	761	51	20	41·83	·15	+0·020
$a^{ii}g^{x}qw$	*L* 4752	53	09	39·23	·07	+0·033
$a^{i}be^{i}f^{v,vi,vii}g^{v,x}kp^{ii,vi}qr$	813	63	24	11·67	·03	+0·043
$a^{i,ii}e^{i}g^{v,vi,ix,x}h^{i,ii}o^{i,ii}p^{ii,iv,vi}qr$	827	41	13	43·65	·02	+0·097
$a^{i}e^{i}f^{vii}g^{vii,viii,ix}o^{i,ii}p^{ii,vi}qrw$	871	52	07	34·95	·03	+0·107
$a^{i}f^{vi,vii}g^{viii,ix,x}kp^{ii,vi}qw$	888	52	06	10·70	·03	+0·086
$a^{i}e^{i}f^{v,vi}g^{vi,viii,ix,x}h^{i,ii}o^{i}p^{ii,v,vi}qw$	896	11	00	31·86	·03	−0·019
$a^{ii}f^{vi}g^{x}kp^{ii}u$	*L* 5514	93	12	49·12	·07	+0·072
$a^{i,ii}c^{vii}f^{vii}g^{viii}p^{ii}qu$	943	93	18	28·08	·04	+0·002
$a^{i}e^{i,ii}f^{vi,vii}g^{v,vii,x}ko^{i,ii}p^{ii,iv,vi}qrw$	962	40	47	59·45	·02	+0·082
$a^{i}e^{i,ii}g^{viii}o^{iii}p^{ii}qrw$	980	63	31	02·96	·13	−0·057
$a^{i}f^{v,vii}g^{x}kp^{ii}$	991	83	44	46·34	·05	+0·008
$f^{v}g^{v,vii\ ix,x}o^{i}$	998	20	39	54·44	·04	+0·014
$a^{i}c^{vii}e^{i}f^{vii}g^{viii,ix,x}kp^{ii}qw$	1045	69	38	40·37	·04	+0·019
$a^{i}e^{i}f^{vi}g^{vii,ix,x}ko^{iii}p^{ii}qw$	1053	69	34	48·62	·04	+0·011
$a^{i}e^{i}g^{viii,ix}o^{i}p^{ii}qr$	1065	34	55	19·90	·08	−0·009
$a^{i}c^{iv,vii}e^{i}f^{v,vi,vii}g^{v,vi,vii,viii,ix,x}h^{i,ii}ko^{i,ii}p^{ii,iv,vi}qw$	1087	77	26	01·79	·02	+0·003
$e^{i}g^{v,vii,viii,ix,x}h^{i}o^{i}p^{ii,v,vi}rw$	1111	27	08	02·44	·02	−0·026
$a^{i}e^{i}f^{v}g^{v,vi,vii,viii,ix,x}p^{ii,iv,vi}qrw$	1138	58	03	16·12	·02	+0·029
$f^{vii}g^{x}o^{i}p^{ii}$	*G* 740	46	22	15·70	·06	−0·004
$f^{vi,vii}g^{x}p^{ii}$	*L* 7110	77	16	49·52	·07	+0·046
$a^{i}e^{i}f^{vi}g^{v,vii,viii,ix,x}o^{i}p^{ii}$	1203	27	14	42·63	·03	−0·011
$a^{i}e^{i}f^{v,vii}g^{vii,x}o^{i,ii}p^{ii}qr$	1262	62	41	30·83	·05	+0·074
$a^{i}e^{i}f^{v,vi,vii}g^{v,vii,x}o^{i}p^{ii,vi}qrw$	1287	41	51	56·71	·03	+0·032
$a^{i}e^{i}f^{v,vi}g^{v,viii,ix,x}ko^{ii}p^{ii}q$	1302	74	52	12·10	·04	+0·023
$a^{i}de^{i}f^{vii}g^{ix,x}o^{i,ii}p^{ii,vi}qrw$	1313	29	31	19·61	·02	+0·114
$a^{i}e^{i}f^{v,vii}g^{vii,ix,x}p^{ii}q$	1362	67	57	14·35	·05	+0·066
$a^{i}e^{i}f^{v,vii}g^{viii,x}o^{ii}p^{ii}q$	1363	68	02	51·28	·04	+0·063
$a^{i}bg^{v,x}ho^{i}p^{ii,v,vi}qr$	1382	36	19	28·95	·02	+0·011
$a^{i,ii}be^{i}f^{v}g^{viii,ix}o^{i}p^{ii}r$	1425	37	08	11·08	·05	+0·022
$a^{i}c^{iv,vii}e^{i}f^{v,vii}g^{v,vi,vii,viii,ix,x}h^{i,ii}ko^{i,ii}p^{ii,v,vi}qrw$	1449	67	15	02·77	·02	+0·021

Authorities.	No. *B.A.C.*, etc.	Mean *N.P.D.* 1892·0.			c^2	μ'
		°	′	″		″
$a^{\mathrm{i}}f^{\mathrm{vi}}g^{\mathrm{ix,x}}o^{\mathrm{i,ii}}qw$	1496	15	53	56·42	0·04	—0·030
$a^{\mathrm{i}}f^{\mathrm{vi,vii}}w$	*L* 9261	88	32	55·01	·60	0·00
$a^{\mathrm{i}}e^{\mathrm{i}}f^{\mathrm{v,vi,vii}}g^{\mathrm{vii,x}}p^{\mathrm{ii}}qw$	1538	88	27	08·21	·05	+0·010
$a^{\mathrm{i}}e^{\mathrm{i}}f^{\mathrm{v,vi}}g^{\mathrm{viii}}o^{\mathrm{i,ii}}p^{\mathrm{ii,vi}}qr$	1554	38	32	47·73	·05	+0·183
$a^{\mathrm{i}}e^{\mathrm{i}}f^{\mathrm{vi}}g^{\mathrm{v,vi,vii,viii,ix,x}}p^{\mathrm{ii}}qrw$	1572	65	52	41·53	·05	+0·025
$a^{\mathrm{i}}e^{\mathrm{i}}f^{\mathrm{vi,vii}}g^{\mathrm{viii,ix,x}}h^{\mathrm{i,ii}}o^{\mathrm{ii}}p^{\mathrm{ii,v,vi}}rw$	1602	51	38	38·98	·03	+0·083
(Weisse)	*L* 9902	52	40	22·	9·0	-------
$a^{\mathrm{i}}e^{\mathrm{i}}f^{\mathrm{vii}}g^{\mathrm{viii,ix,x}}p^{\mathrm{ii}}r$	1663	52	42	58·36	0·04	+0·012
$a^{\mathrm{i}}e^{\mathrm{i}}f^{\mathrm{v,vii}}g^{\mathrm{viii,ix,x}}o^{\mathrm{i,ii}}p^{\mathrm{ii,vi}}qr$	1705	32	51	21·95	·02	+0·219
$a^{\mathrm{i}}c^{\mathrm{iv}}e^{\mathrm{i}}f^{\mathrm{vi,vii}}g^{\mathrm{v,vii,viii,ix,x}}p^{\mathrm{ii}}w$	1726	71	29	12·12	·04	+0·014
$a^{\mathrm{i}}e^{\mathrm{i}}f^{\mathrm{vi}}g^{\mathrm{v,ix}}p^{\mathrm{ii}}w$	1734	71	32	14·41	·07	+0·004
$a^{\mathrm{i}}c^{\mathrm{vii}}e^{\mathrm{i}}f^{\mathrm{vi,vii}}g^{\mathrm{vi,vii,viii,x}}p^{\mathrm{i,ii,iv,vi}}q$	1749	80	08	19·37	·04	+0·013
$e^{\mathrm{ii}}f^{\mathrm{v,vi}}g^{\mathrm{vii,viii,ix,x}}io^{\mathrm{i}}p^{\mathrm{ii}}w$	1751	24	21	43·20	·05	+0·039
$a^{\mathrm{ii}}e^{\mathrm{i}}f^{\mathrm{v,vi,vii}}g^{\mathrm{v,vi,vii}}kp^{\mathrm{ii}}w$	1821	74	13	13·82	·11	+0·020
$e^{\mathrm{i}}f^{\mathrm{v}}g^{\mathrm{v,vi,vii,ix,x}}o^{\mathrm{i}}p^{\mathrm{ii}}r$	1849	30	08	13·59	·03	+0·022
$a^{\mathrm{i}}f^{\mathrm{vii}}g^{\mathrm{v,viii,x}}kp^{\mathrm{ii}},qw$	1867	69	43	35·94	·08	+0·024
$a^{\mathrm{i}}e^{\mathrm{i}}f^{\mathrm{vi,vii}}g^{\mathrm{v,vi,vii,viii,ix,x}}ko^{\mathrm{ii}}p^{\mathrm{ii,vi}}qw$	1876	69	44	40·32	·03	+0·108
$a^{\mathrm{i}}g^{\mathrm{viii,ix,x}}o^{\mathrm{i}}p^{\mathrm{ii}}r$	1887	34	41	20·02	·05	+0·090
$a^{\mathrm{i}}c^{\mathrm{vii}}e^{\mathrm{i}},f^{\mathrm{vi,vii}}g^{\mathrm{viii}}h^{\mathrm{i}}kp^{\mathrm{ii}}qw$	1928	80	21	12·32	·06	+0·044
$a^{\mathrm{i}}e^{\mathrm{i}}f^{\mathrm{v,vi}}g^{\mathrm{v,ix,x}}h^{\mathrm{i}}o^{\mathrm{i,ii}}p^{\mathrm{ii,v,vi}}q$	1952	24	15	40·46	·02	+0·045
$a^{\mathrm{ii}}e^{\mathrm{i}}f^{\mathrm{v,vi,vii}}g^{\mathrm{viii}}o^{\mathrm{iii}}p^{\mathrm{ii}}$	1989	73	50	43·58	·10	+0·013
$a^{\mathrm{i}}e^{\mathrm{i}}f^{\mathrm{v,vi}}g^{\mathrm{viii}}ko^{\mathrm{i,ii}}p^{\mathrm{ii}}q$	2009	73	49	26·61	·09	+0·008
$a^{\mathrm{i}}be^{\mathrm{i}}f^{\mathrm{v}}g^{\mathrm{x}}o^{\mathrm{i}}p^{\mathrm{ii,vi}}r$	2020	30	34	57·44	·03	—0·010
$a^{\mathrm{i}}e^{\mathrm{i}}o^{\mathrm{iii}}$	2057	86	10	55·7	1·4	+0·05
$a^{\mathrm{i}}c^{\mathrm{vii}}e^{\mathrm{i}}f^{\mathrm{v,vi,vii}}g^{\mathrm{v,vi,vii,viii,ix,x}}h^{\mathrm{i}}o^{\mathrm{ii}}p^{\mathrm{ii,vi}}qw$	2090	69	43	12·15	0·02	+0·018
$a^{\mathrm{i}}e^{\mathrm{i}}f^{\mathrm{vi}}o^{\mathrm{i,ii}}r$	2107	34	34	07·59	·15	+0·018
$g^{\mathrm{x}}h^{\mathrm{i}}p^{\mathrm{ii}}$	Œn6978	18	09	44·41	·28	-------
$a^{\mathrm{i}}g^{\mathrm{ix,x}}o^{\mathrm{i,ii}}p^{\mathrm{ii}}qr$	2143	27	59	06·42	·02	—0·023
$a^{\mathrm{i}}e^{\mathrm{i}}f^{\mathrm{v,vi}}g^{\mathrm{viii}}kp^{\mathrm{ii}}$	2230	76	27	47·88	·11	+0·013
$a^{\mathrm{i}}o^{\mathrm{i}}$	2249	32	18	00·0	·14	—0·035
$a^{\mathrm{i}}e^{\mathrm{i}}kq$	2265	72	07	24·76	0·30	+0·06
$a^{\mathrm{i}}e^{\mathrm{i}}\left\{\begin{matrix}g^{\mathrm{v}}o^{\mathrm{i}}p^{\mathrm{ii}}\\ g^{\mathrm{v}}o^{\mathrm{i}}\end{matrix}\right\}qr$	2300	37	04	$\left\{\begin{matrix}50{\cdot}60\\ 53{\cdot}50\end{matrix}\right\}$52·05	·11	+0·072
$a^{\mathrm{i}}e^{\mathrm{i}}f^{\mathrm{v,vi}}g^{\mathrm{viii,x}}kp^{\mathrm{ii}}w$	2313	67	12	04·75	·07	+0·017
$a^{\mathrm{i}}e^{\mathrm{i}}f^{\mathrm{v,vi,vii}}g^{\mathrm{vii,ix,x}}p^{\mathrm{ii,vi}}q$	2330	73	53	50·66	·04	+0·117
$a^{\mathrm{i}}c^{\mathrm{vii}}e^{\mathrm{i}}f^{\mathrm{vi,vii}}g^{\mathrm{v,vi,vii,viii,ix,x}}ko^{\mathrm{i,ii}}p^{\mathrm{ii}}w$	2362	73	39	29·80	·05	+0·046
$a^{\mathrm{i}}bf^{\mathrm{v}}o^{\mathrm{i,ii}}p^{\mathrm{ii}}$	2369	30	40	53·32	·12	+0·014
$a^{\mathrm{i}}bf^{\mathrm{v}}g^{\mathrm{ix,x}}o^{\mathrm{i}}p^{\mathrm{ii,vi}}$	2376	30	33	09·27	·02	+0·033
$b\left\{\begin{matrix}g^{\mathrm{x}}ko^{\mathrm{i}}p^{\mathrm{vi}}q\\ a^{\mathrm{i}}g^{\mathrm{x}}ko^{\mathrm{i}}p^{\mathrm{ii,vi}}q\end{matrix}\right\}$	2409	39	38	$\left\{\begin{matrix}61{\cdot}16\\ 50{\cdot}46\end{matrix}\right\}$58·81	·02	+0·053
$a^{\mathrm{i}}e^{\mathrm{i}}f^{\mathrm{v,vi,vii}}g^{\mathrm{viii,ix,x}}p^{\mathrm{ii}}r$	2431	64	44	33·07	·03	+0·029
$a^{\mathrm{i}}g^{\mathrm{v,x}}o^{\mathrm{i}}p^{\mathrm{ii}}$	*G* 1318	41	35	49·96	·06	+0·055
$a^{\mathrm{i}}c^{\mathrm{iv}}c^{\mathrm{i}}f^{\mathrm{vi,vii}}g^{\mathrm{v,vii,viii,ix,x}}o^{\mathrm{i,ii}}p^{\mathrm{ii}}qrw$	2493	62	51	53·27	·04	+0·114
$f^{\mathrm{v,vii}}g^{\mathrm{v}}o^{\mathrm{i}}p^{\mathrm{ii}}q$	*G* 1351	41	37	01·58	·22	+0·141
$a^{\mathrm{i}}e^{\mathrm{i}}f^{\mathrm{v}}g^{\mathrm{viii,ix,x}}o^{\mathrm{i}}p^{\mathrm{ii,vi}}r$	2532	39	18	41·26	·04	+0·040

Authorities.	No. *B.A.C.*, etc.	Mean *N.P.D.* 1892·0.			e^2	μ'
		°	′	″		″
$a^{i}e^{i}f^{v,vii}g^{v,vi,vii,viii,ix,x}h^{ii}o^{i}p^{ii,iv,vi}qrw$	2551	65	20	36·94	0·04	+0·067
$g^{v,vi,vii,viii,ix,x}h^{i,ii}o^{i}p^{ii,v,vi}qw$	2596	15	47	40·47	·02	+0·027
$g^{x}kp^{vi}q$	*L* 15522	88	35	05·63	·09	—0·02
$a^{i}f^{vi}g^{v,x}o^{i}p^{ii}rw$	2650	32	25	40·72	·05	+0·059
$a^{i}e^{i}f^{v,vi,vii}g^{v,vi,vii,viii,ix,x}h^{i}ko^{i,ii}p^{ii,vi}qw$	2744	72	01	36·92	·02	+0·126
$a^{i}f^{vi}g^{v,vi,vii,viii,ix,x}o^{i,ii}p^{vi}qw$	2745	72	01	40·39	·03	+0·110
$a^{i}e^{i}f^{vi,vii}kq$	2759	71	59	56·36	·22	—0·001
$a^{i}e^{i}f^{v,vi}g^{v,vi,viii}o^{i}p^{ii,vi}qrw$	2776	31	55	14·70	·04	—0·018
$a^{i}f^{v}g^{viii}ko^{ii}p^{ii}q$	2810	72	27	57·34	·26	+0·122
$a^{i}e^{i}f^{vi,vii}g^{v,ix,x}o^{ii}p^{ii,vi}qw$	2816	72	35	54·00	·04	+0·155
$a^{i}e^{i}f^{v}g^{vii,viii,ix,x}o^{i}p^{ii}$	2842	24	29	14·39	·02	+0·060
$a^{i}e^{i}f^{v}g^{viii,ix}o^{i}p^{ii}$	2876	24	36	23·38	·05	—0·097
$a^{i}e^{i}f^{vii}g^{vii,viii,x}kp^{ii}qw$	2897	79	58	11·24	·04	+0·025
$a^{i}f^{v}g^{viii}kp^{ii}w$	2902	80	02	54·15	·10	+0·006
$a^{i}e^{i}g^{vii,viii,ix,x}kp^{ii}q$	2942	76	55	56·04	·04	+0·004
$a^{i}e^{i}f^{v}g^{viii,ix,x}h^{i}o^{i,ii}p^{ii,vi}q$	2982	27	38	03·45	·01	—0·017
$a^{i}e^{i}g^{viii,ix,x}kp^{ii}qrw$	3033	56	40	28·20	·04	+0·079
$a^{i}e^{i}f^{vii}g^{v,vi,vii,viii}h^{ii}o^{i,ii}p^{ii,iv,vi}qr$	3059	47	47	24·11	·03	+0·267
$a^{i}f^{v,vi,vii}g^{viii,x}ko^{iii}p^{ii}q$	3069	61	40	20·92	·05	+0·089
$a^{i}f^{v,vii}g^{viii}o^{iii}p^{ii}$	3088	61	40	28·74	·07	+0·003
$a^{i}e^{i}f^{vii}g^{x}ko^{i}p^{ii}qr$	3150	42	43	59·33	·05	+0·006
$a^{i}bg^{ix,x}o^{i}p^{ii}qrw$	3182	39	59	46·69	·02	—0·004
$a^{i}f^{vii}g^{vii,x}ko^{iii}p^{ii}qw$	3194	64	21	21·29	·07	+0·002
$a^{i}e^{i}f^{vi,vii}g^{viii}o^{ii}p^{ii}qr$	3241	54	25	09·77	·08	+0·096
$a^{i}e^{i}f^{vi,vii}g^{x}o^{i}p^{ii,vi}r$	3265	49	53	57·87	·04	—0·010
$f^{vi,vii}g^{viii,ix,x}o^{iii}p^{vi}$	3292	69	12	57·04	·05	+0·046
$ko^{i}r$	3308	35	08	36·93	·15	+0·026
$a^{i}e^{i}f^{vi,vii}g^{ix,x}kqw$	3318	69	18	48·28	·07	+0·008
$a^{i}f^{vi,vii}g^{v,viii,ix,x}ko^{i}p^{ii}qr$	3352	49	51	56·75	·04	—0·014
$a^{ii}e^{ii}f^{vi,vii}iko^{iii}q$	3375	54	30	28·12	·27	—0·015
$a^{i}f^{vii}qw$	3456	57	51	56·97	·18	+0·072
$a^{i}e^{ii}f^{vi,vii}g^{viii,ix}ip^{ii}qrw$	3484	58	02	21·90	·12	+0·048
$a^{i}f^{vii}g^{viii,ix,x}kp^{ii}qrw$	3490	57	59	45·87	·03	+0·003
$a^{i}e^{i}f^{v,vi,vii}g^{v,vi,viii,ix,x}h^{i,ii}o^{i}p^{ii,iv,vi}qrw$	3505	46	32	47·68	·01	+0·047
$a^{i}g^{vii}o^{i}w$	*G* 1635	46	24	34·77	·60	+0·085
$a^{i}e^{i}g^{viii,ix,x}h^{i,ii}o^{i}p^{ii,v,vi}qw$	3531	23	53	15·84	·01	+0·018
$a^{i,ii}e^{i}f^{v,vi,vii}g^{v,vii,viii,x}ko^{iii}p^{ii}qw$	3561	80	39	58·84	·04	+0·035
$a^{i}e^{i}f^{v}g^{v,vi,ix}o^{i}qw$	3571	23	49	17·59	·10	+0·043
$a^{i}e^{i}f^{vi}g^{vii,x}iko^{i}p^{ii}qrw$	3607	49	01	08·42	·05	+0·016
$a^{i}e^{i}f^{vi,vii}g^{viii}kp^{ii}r$	3633	55	21	40·12	·07	—0·025
$a^{i}e^{i}f^{v,vi,vii}\left\{\begin{matrix}g^{viii,x}\\g^{viii,x}kp^{ii}q\end{matrix}\right\}$	3672	84	41	$\left\{\begin{matrix}11·04\\08·74\end{matrix}\right\}$09·59	·04	+0·022
$a^{i}dg^{x}o^{i}p^{ii,vi}qw$	*G* 1697	19	34	14·80	·04	+0·084
$a^{i}f^{v,vii}g^{viii}o^{ii}p^{ii}qw$	3736	55	23	19·94	·09	+0·058
$a^{i}e^{i}f^{v,vi,vii}g^{viii,ix,x}ko^{i,ii}p^{ii,vi}qr$	3757	48	59	35·25	·04	—0·050

Authorities	No. *B.A.C.*, etc.	Mean *N.P.D.* 1892·0.			ϵ^2	μ'
		°	′	″		″
$a^{i}c^{iv,vii}e^{i}f^{v,vi,vii}g^{v,vi,vii,viii,ix,x}h^{i}kmo^{i,ii}p^{ii,v,vi}$ qw	3788	82	04	48·71	0·03	+0·036
$e^{i}f^{vi}g^{x}p^{ii}$	Œ$_{II}$ 11453	22	12	14·91	·05	+0·030
$a^{i}e^{i}f^{vi}g^{v,vii,x}o^{i}p^{ii}qr$	3885	33	33	27·00	·03	—0·052
$a^{i}e^{i}f^{v,vi}g^{viii,ix,x}kp^{ii,vi}qw$	3915	70	59	43·86	·03	—0·013
$a^{i}e^{i}f^{v,vii}g^{vii,ix,x}p^{ii,vi}$	3937	61	37	18·97	·03	+0·010
$a^{i}e^{i}f^{vi}g^{viii,ix}o^{i}p^{ii}r$	3953	42	34	01·68	·07	+0·046
$a^{i}c^{v,vii}e^{i}f^{v,vi,vii}g^{x}kp^{ii,vi}quw$	3975	96	04	35·76	·04	+0·042
$a^{i}e^{i}f^{v,vi}g^{vii,viii,ix,x}h^{i}o^{i}qw$	4050	8	32	39·71	·05	+0·033
$a^{i}c^{v}e^{i}f^{v,vi}g^{x}kqu$	4077	92	31	47·44	·06	+0·021
$a^{i}o^{i}p^{ii,vi}q$	*F* 2015	12	00	35·82	·17	—0·09
$a^{i}g^{v,ix}kqw$	4104	85	20	36·79	·13	+0·007
$a^{i}g^{vi,vii,ix,x}lo^{i}qw$	4122	19	11	55·77	.04	+0·022
$a^{i}e^{i}g^{v,vi,vii,viii,ix,x}kmo^{i,ii}p^{ii}quw$	4137	90	11	12·57	·02	+0·025
$a^{i}e^{i}g^{vii,viii,ix,x}o^{i,ii}p^{ii}q$	4143	14	14	23·42	·03	—0·011
$a^{i}e^{i}g^{viii}ko^{ii}p^{ii,vi}qw$	4168	84	05	38·35	·05	+0·067
$a^{i}e^{i}g^{vii,viii,ix,x}o^{i,ii}p^{ii}qw$	4222	20	12	01·57	·03	+0·060
$a^{i}e^{i}g^{ix,x}o^{i}q$	4249	9	09	14·62	·03	—0·001
$a^{i}c^{v}e^{i}g^{vii,viii,ix,x}hko^{ii}p^{ii}quw$	4247	95	14	12·05	·04	+0·030
$a^{i}e^{i}g^{vii,viii,ix,x}h^{i,ii}o^{i}p^{ii,v,vi}$	4276	26	41	38·37	·02	+0·019
$a^{i}e^{i}g^{viii,ix,x}kp^{ii}q$	4292	77	27	05·25	·04	+0·033
$c^{v}qu$	4326	98	28	34·86	·15	+0·047
$a^{i}e^{i}g^{v,vi,viii,ix,x}o^{i,ii}p^{ii,vi}qw$	4339	5	59	41·83	·03	—0·025
$a^{i}e^{i}g^{v,vi,viii,ix,x}h^{i}o^{i,ii}p^{ii,vi}qw$	4342	5	59	59·91	·03	—0·022
$a^{i}e^{i}g^{v,vi,vii,viii,ix}hlp^{ii,vi}qrw$	4384	53	37	23·83	·05	—0·005
$a^{i}e^{i}g^{viii,x}o^{i}p^{ii}r$	4408	50	53	26·47	·06	+0·005
$a^{i}e^{i}g^{viii,ix,x}h^{i}o^{i}p^{ii,v,vi}r$	4415	50	55	37·55	·03	—0·034
$e^{i,ii}g^{x}h^{i}ikp^{ii}q$	4470	87	20	42·06	·06	+0·045
$a^{i}e^{i}g^{vii,viii,ix,x}h^{i}o^{i}p^{ii,v,vi}qw$	4506	17	02	51·78	·02	+0·020
q (Rümker)	4509	70	22	59·30	·38	—0·11
$a^{i}be^{i}g^{vi,x}o^{i}p^{ii}qr$	4540	34	05	52·99	·03	+0·011
$a^{i}qw$	4553	66	55	11·64	·50	+0·07
$a^{i}e^{i}g^{viii,ix,x}kp^{ii}rw$	4566	66	57	24·90	·03	+0·039
$a^{i}e^{i}g^{x}o^{i}p^{ii}q$	*G* 2036	37	23	33·73	·05	+0·021
$a^{i}g^{x}kp^{ii}qw$	4640	60	49	13·20	·06	—0 019
$a^{i}e^{i}g^{x}h^{i}p^{ii}q$	*L* 25839	43	43	25·55	·04	+0·098
$c^{v}u$	5043	98	45	04·16	·25	+0·03
$p^{iii,vi}$	*F* 2626	30	48	51·00	·21	0·00
$a^{i}g^{viii}kp^{ii}$	5126	73	37	22·18	·15	+0·038
$a^{i}e^{i}g^{x}p^{ii}$	*L* 28533	65	07	27·41	·07	+0·052
$e^{i}io^{i}p^{ii}r$	5181	39	13	27·00	·19	+0·019
g^{x}	*F* 2763	29	58	23·31	·17	-------
$a^{ii}e^{ii}ikq$	5507	74	19	42·76	·32	+0·02
$g^{vi,ix,x}o^{i}p^{ii,vi}q$	5745	24	47	47·88	·03	—0·033
g^{x}	*L* 37106	64	10	21·59	·10	+0·02
$a^{i}g^{v,vii,viii,x}ko^{i}p^{ii,vi}qrw$	6730	40	00	12·75	·03	—0·027

The apparent places depend on the preceding mean places and proper motions and on the independent star numbers of the American Ephemeris and Nautical Almanac for the respective year.

It was thought sufficient to tabulate the individual results only, there being no novelty or special advantage in publishing the detail of the record and reduction. The results are given in the order of time and separately for each of the eight groups, which extend over the calendar year. The connection of each group with that following is readily seen from the dates in the first column of the tabulation.

Results for latitude of astronomic station, Lafayette Park, San Francisco.

$\varphi = 37° \; 47' +$ tabular seconds.

PAIRS OF GROUP I.

Date.	1 4696 4724	2 4783 4797	3 4808 4843	4 4863 4897	5 4937 4953	6 5022 5034	7 5097 5120	8 F 2677 L 28716	9 5246 5285	10 5348 5392	11 5459 5504	12 5530 5574	13 5692 5740	14 L 31117 5790
1891.														
May 27								28·06		28·69				
28	29·27	27·51	28·11		28·44	28·53	26·74	29·03	28·67	27·91	29·24	28·36	28·26	27·95
29	28·53	28·58	29·44	29·29		28·50							27·80	28·34
30														27·63
31		27·90	28·02		28·22		27·89	29·05	29·65	27·56	28·44	28·52	29·54	27·82
June 1	28·75		28·92		28·86						29·32	27·98	27·31	26·80
3	29·49	28·99	28·25		29·09	29·81	27·96	28·81	28·68	27·12		26·91	28·61	28·57
5	28·63		28·00	27·39	26·58	28·61	28·29	28·52	28·45	28·52	28·47	27·73	28·71	28·01
6	28·31	29·34	28·49	28·78	28·32	28·98	28·08	28·96	29·04	27·19	27·81	27·86	28·44	28·84
7	28·81	28·47	28·05		28·34	30·01	28·18	29·49	28·13	28·02	28·53	27·85	28·41	27·51
8	28·75	28·99	29·60		27·84	29·63	28·38	28·27	28·85	27·72	28·32	28·18	27·10	29·03
12	28·88	29·30	28·54		28·33					27·84	27·56		27·96	
13	28·60	28·20	27·54		27·75		27·87	29·38	29·09	27·97	28·20	27·70	27·90	27·60
14	28·52	28·73	27·87		27·57	29·47	28·47	29·40	29·16	27·95		27·86	28·06	28·01
17	28·03	27·74	27·19		28·31	29·38	28·16	29·25	28·39	28·10	28·23	29·00	28·23	28·21
18	29·13	28·22	27·92		28·42	28·58								
19	28·59	28·84	28·00		27·92	29·75	27·80	28·61	28·49	27·75	27·66	27·60	27·97	28·02
20	28·06	29·09			28·49	29·57	27·80	29·01	29·46	28·43	28·07	27·25	28·30	27·06
21	28·44	27·68	26·99		27·59	28·31	27·82	29·11	28·76	28·14	28·85	28·20	27·94	27·93
22	28·59	28·93			28·56	28·32						27·80	27·87	28·01

24		28·80	28·04		27·61									
25	29·12	28·54	28·63		27·70	29·26	27·85	29·31	29·19	27·92	28·49	28·40	29·46	26·88
26	28·85	28·52			27·73	28·19	28·38	29·42	29·72	28·61		28·08	28·95	28·35
27	29·13	27·65	29·26		28·88	29·64	29·11	28·41	29·16	28·40	28·17	27·83	28·16	28·02
28	28·58	29·45	27·33		27·82	29·78	28·67	28·64	29·29	29·78		28·58	27·93	28·52
29	29·01	29·17	28·41		28·64		29·20	29·38	28·12	28·45	28·99	28·44	28·92	28·20
30		28·68	29·03		28·57	30·25	29·06	28·27		28·14		27·64	28·73	27·99
July 1							28·57	28·79	28·95				27·90	

N. B.—The same dates refer also to Group II.

Results for latitude of astronomic station, Lafayette Park, San Francisco—Continued.

$\varphi = 37^\circ\ 47' +$ tabular seconds.

PAIRS OF GROUP II.

Date.	1		2	3		4		5	6	7	8		9		10	11	12	13	14	
	5821 G 2432	5821 G 2433	5883 5937	5988 6052	5999 6052	6087 6109	6087 G 2494	L 33291 6162	L 33521 6203	6235 6268	6322 6372	6341 6372	6397 6496	L 35333 6496	6528 6555	6579 6637	6734 6758	6824 6866	L 38237 6924	6901 6924
1891. May 27				29·26	28·71	27·42		28·52	28·33	28·97	26·97			28·35	28·75					28·52
28	28·41		28·45	28·35	28·32	28·10		28·19	28·64	28·34	28·24	28·12		28·44	29·19	27·60	28·68		27·88	28·45
29	28·91		29·13	28·70	28·62	28·17		28·82	29·39	27·54	28·65	28·85				27·86	28·93	28·80		27·98
30		29·54	29·88	29·01	28·80	27·64		27·67	29·40	27·64	28·18	27·97	27·83	28·19	28·14	27·73	27·77	28·69		27·93
31		28·59	28·79	28·13	28·26	27·74		28·17	29·40	27·19	28·84	28·56	28·27	27·86	29·37	28·36	29·24	28·49		
June 1		29·73	29·08	29·02	29·43	27·68	28·39	27·99	29·04	27·47			28·59	28·15						
3	30·46		27·63	28·13	28·06	27·68		27·82	27·85	27·08										
5	28·62		29·01	28·57	28·36	27·62		28·59	28·51		27·95	28·25	29·31	29·03	28·78	27·62	28·55	28·21		28·78
6		29·44	28·84	27·78	27·85	27·37		28·44	29·68	27·84	28·95	27·98	28·30	28·01	28·82	28·76	29·35	28·01		27·19
7			27·85	28·81	28·83	27·55		28·46	29·08	27·64	28·68	28·95	29·04	28·77	27·35	28·06	28·50	28·97		27·56
8		29·10	28·57	27·92	29·05	26·96		27·61	29·39	28·09	28·72	28·54	27·77	28·23	28·49	27·43	28·81	29·05		28·25
12	28·90		28·68	28·91	28·77	28·34	29·21	27·93	29·43	28·33	27·63	27·21	28·77	28·27		27·29	28·43	29·02		28·04
13		29·38	29·34	28·81	28·78	27·51		28·47	28·90	28·59	28·19	27·72	28·63	28·47	28·44	28·56	28·71	29·46		28·27
14		28·63	28·34	28·65	28·97	27·38		28·60	28·76	28·22	27·70	27·66	28·62	28·65		27·33	28·15	29·19		
17		29·63	28·59		28·68	27·88		28·36	28·89	28·10	28·20	28·16	26·93	27·73	27·40	27·63	28·35	29·14		27·70
18																				
19		29·11	28·75	27·98	27·71	28·52		28·13	28·95	28·03	28·26	27·42	28·32	28·54	28·86	27·62	28·90	28·40		28·13
20		28·23	28·59	29·10	28·83	27·29		27·68	28·95	27·85	29·04	28·43	28·02	29·56	28·06	27·94	28·82	28·14		27·97
21		28·43	28·13	28·48	28·14	27·16		28·15	28·70	27·43	28·03	27·88	28·23	28·13	27·63	28·29	28·15	29·55		28·23
22		28·26	29·15	28·06	28·34	27·14		27·94		28·04	27·75	27·46	28·00	27·99	28·20	28·49	28·37	28·73		28·14
24																				
25		28·49	28·89	27·40	27·55	26·91		28·01		28·08	28·45	28·48	28·90	28·93	28·69	28·25	28·83	29·36		28·21
26		28·84	28·39	28·54	28·38	26·41		29·02	29·52	28·24	28·17	28·34	28·70	28·47	28·22	28·78	28·22	28·85		29·08
27		29·14	28·66		28·66	28·21		28·25	29·69	28·41	28·67	28·73	28·11	28·87	29·32	28·45	29·14	29·29		29·01
28		28·88	29·90	28·88	28·63	28·07		28·86	28·91	28·10	28·22	28·01		29·43	29·30	28·35	28·93	28·80		28·65

29		28·62	29·05	29·18	27·75	28·64	29·43	28·04	28·24	28·30	28·56	29·01	28·50	28·49	28·52	28·77		28·90
30	28·52	29·51	29·25	28·31	27·84	28·70	28·41	27·96	28·33	27·82	28·77	28·08	28·27	28·04	28·58	28·65		28·98
July 1		27·69		27·46	27·43	27·78												
4	30·62	28·48	28·87	28·56	27·55	28·39	28·23	28·24	27·71	27·16	27·52	27·11	27·32	28·19	28·58	28·73		28·57
5	28·26	28·86	28·54	28·50	28·33	28·65	28·56	28·00	28·33	28·10	28·71	29·07	28·68	28·52	28·49	28·64		28·78
6	29·83	28·74	28·67	28·16	27·35	27·64	28·40	27·72	27·98	27·93	27·97	28·90	27·99		29·20	29·29		28·40
7	29·64	29·96	28·94	29·50	27·66	27·68	28·85	27·69	27·32	27·18	28·55	28·83	28·10	27·46	28·99	28·48		28·71
9	29·26	29·44	29·03	28·31	27·50													
10	29·21	29·24	29·27	28·42	27·16		28·06	28·08	28·76	28·86	28·13	28·13	28·35	28·05	29·19	29·66		28·26
11	28·72	28·73	28·58	28·38	27·58	28·05	28·59	27·63	28·28	28·13	28·24	28·85	28·54	28·23	28·85	28·43		28·26
12	29·01	28·74	28·52	28·35	27·50	28·07	29·39	28·25	27·70	28·07	29·19	28·86	28·71	28·51	27·98	29·51		29·14
14			27·81	27·69	26·82	28·41	28·40	28·00	27·94	28·32	27·02	28·39	27·38	27·63	29·02	29·20		
17	28·57	28·29	28·37	28·29	27·32	28·46	28·05	28·39	28·38	28·43	28·32	28·31	28·51	27·71	28·83	28·72	27·82	28·29
18	28·20	29·06	28·83	28·75	27·54	28·62	28·71	28·88	28·56	28·51	28·03	28·17	28·43	28·28	29·95			
19	28·68				27·76	27·82	28·67	28·70	28·07	28·21	28·43	28·11	28·82	27·84	30·73	28·65	28·00	28·32
20	29·15	28·27		28·15	28·05	28·61		28·20	27·91	27·51	28·44	28·39	28·58	28·31	29·08	28·92	27·97	28·87
21	28·45	29·02			27·79	28·68	30·35	27·12	27·76	27·89	28·78	28·65	27·58	28·44	28·41	28·65		28·50
23	27·73		28·40	28·14	27·14	28·36	29·09	27·57										
24	28·21	28·11	28·68	28·10	27·37	29·00	28·16	27·67	27·92	27·17	28·26	28·00	28·80	27·87	28·98	28·64	26·93	27·48
25	28·11	28·96	29·05	28·46	27·19	28·81	28·89	27·48	27·75	28·02	29·04	28·78	27·82	27·98	29·41	29·20	28·01	28·23
30						29·15	28·37						28·82	27·95	28·21	28·19		
31		28·58	27·60	28·18	28·08	28·21	28·48	28·34	28·33	28·55	28·17	27·40			28·72	28·20		28·88
Aug. 3	28·83	28·44		27·76		28·32		26·59	28·44	27·70	28·71	28·16			28·38	28·32		28·23
5	27·91	28·60	27·48	27·53	27·50	28·22	28·57	28·65	28·12	27·95								
6	27·96	28·51	26·85	26·54	27·46	28·74	28·34	27·99	28·85	28·06								
7	27·71		27·97	28·27	26·76	27·95	27·83	27·69	28·01	27·72								
10	28·14	28·09	28·38	28·18	27·29	27·90	28·25	27·71	28·42	28·51	28·72	27·78						
11	27·99		28·31	28·27	27·54	28·24	28·71	28·29	28·19	28·03	28·35	28·22	28·93	27·61	28·27	28·32		28·54
12	27·84	27·87	27·26	26·60	28·05	27·89	29·03	27·44	28·50	28·36	29·12	28·12		28·48	27·78	28·65		28·47
13	27·45	28·21	27·98	27·91	27·71	28·90	28·53	27·92	29·19	27·95	28·33	28·51	28·26	27·81		28·54		27·13

N. B.—Connection with Group I is from May 27 to July 1, inclusive, and with Group III from July 4 to August 13, inclusive.

Results for latitude of astronomic station, Lafayette Park, San Francisco—Continued.

$\varphi = 37^\circ\ 47' +$ tabular seconds.

PAIRS OF GROUP III.

Date.	1	2	3	4	5		6	7	8		9	10	11	12	13	14
	6957 6983	G 3151 7067	7149 7176	7194 7219	7506: L 41026	7506: 7368	7402 L 41554	7437 7488	7508: 7522	7508: 7547	L 42627 7643	7678 7720	7788 F 4151	7843 7906	7961 7997	8039 8051
1891.																
July 4	27·43	27·77	28·28	28·67	27·91	28·63	28·47	28·96	28·09	28·28	27·73	29·02	28·51	28·05	27·90	
5	27·88	28·97	28·15	28·94	28·56	28·54	28·02	29·22	27·85	28·63	28·53	29·42	28·75	29·13	28·51	28·99
6	27·99	28·53	28·24	28·54	28·65	28·36	28·34	28·64			27·53	28·15	29·17	28·13	28·23	28·87
7	27·52	28·70	28·78	28·79	27·46	27·31	27·98	28·40	27·89	28·53	28·35	28·96	28·42	27·50		28·85
9																
10	28·91	29·10	28·36		28·04	27·80	28·82	29·04	28·54	28·02	28·33	29·35	28·75	27·82	28·16	29·42
11	27·17	28·68	28·07	29·06	27·92	27·99	27·70	28·99	28·83	28·87	27·59	28·75	28·20	27·80	28·33	28·19
12	28·17	28·63	27·53													
14																
17	27·76	28·08	28·92	29·15	27·90	27·60	28·35	28·51	29·02	29·23	28·20	29·68		28·01	28·43	28·25
18																
19	27·54	28·51	29·02	29·87	28·67	28·34	28·04	29·17	28·18	28·06	28·18	29·02	28·51	27·21	28·48	28·33
20	27·26	28·41	28·24	29·15	28·15	27·86	28·22	29·04	29·97	29·74	27·70	28·08	28·46	28·18	28·39	28·86
21	28·58	28·00	27·75	28·72	28·39	28·27	28·09	29·09	28·74	28·29	28·03	28·65	28·17	27·98	28·67	28·44
23																
24																
25	27·61	28·74	28·64	28·18	28·55	28·78	27·57	28·91	28·46	28·70	27·50	28·74	28·49	27·76	28·77	28·46
30	28·67	28·31	27·64	28·10	29·06	27·25			28·81	28·52						
31	27·74		27·38	29·28	27·55	27·33	28·05	29·51	28·63	28·04	28·19	29·36	28·03	28·13	28·44	28·66
Aug. 3	27·74		27·68	28·46	27·75	27·66	27·94	28·10			27·70	27·91	27·77	27·33	28·27	28·31

	5	27·65	28·29	27·83	29·11			28·48	28·94	28·79	28·43	28·34	27·77	28·38	28·49	27·31	28·44
	6	27·91	28·88	28·39	29·22	28·36	27·61	28·19	28·50	28·40	28·25	28·78	28·52	27·63	27·78	28·49	28·30
	7	27·82	28·48	28·03	30·59	28·61	27·83	28·07	28·41	28·38	28·12	27·64	28·84	28·54	27·69	28·43	28·23
	10																
	11	27·83	28·18	27·90	28·83	27·86	27·88	28·18	28·14			27·64	29·28	28·44	28·27	27·68	28·07
	12	28·46	27·43	28·29	28·41	28·03	27·36	28·23	28·45	29·12	29·02	28·20	28·57	27·94	28·06	28·37	29·00
	13	27·64	27·35	27·63	27·82	27·59	27·47	27·99	28·46	27·80	27·90	27·16	28·50	27·28		27·52	28·16
	15	28·29	27·25	28·24	28·83	27·53	27·60	28·27	29·70			27·79	27·61	27·19	28·20	28·49	28·54
	16	27·54	27·63	28·26	27·97	27·88	27·49	27·81	28·18		27·63	27·85	27·96	28·13	27·81	28·16	28·83
	17	27·37	27·35	28·04	28·39	27·80	28·28										
	20		26·54	27·48	28·78	27·93	27·21	28·85		28·30	27·89	28·07	28·12	28·74	28·08	28·65	29·03
	21			27·53	28·60	27·72	27·83	28·34				27·43	29·79		27·32	28·29	28·15
	22	27·91	28·76	29·02	27·66	28·23	28·55	27·86	28·95	28·55	28·02	28·04	28·40	28·26	28·45	28·07	28·68
	31	27·24	28·32	27·52	27·03	28·02	27·89	27·62				27·31		28·01	28·80	28·38	28·43
Sept.	5			28·57	28·42	28·63	27·88	28·82				27·37		27·66	28·20	28·10	
	6		28·02	29·03	28·18	28·49	28·27	28·31	28·85			27·36	27·76	28·47		27·82	28·82
	7		28·40	28·35	28·56	27·74	27·83	28·34	28·16	27·89	27·80	27·43	27·24	27·77	27·85	28·29	28·09
	8	28·04	28·55					27·97						27·91	27·93		28·25
	9	28·13	27·91	28·07	28·35	27·58	27·83	26·15	28·63			27·18	27·43	28·37	28·27	28·15	
	10	28·01	26·94	27·99		27·27	27·23	28·73	27·88	28·92	28·60	27·16	28·01	27·52	27·82	28·42	29·00
	12			28·97	28·09	28·27	27·93	28·01						27·51	27·88		
	15	27·55	28·02			27·72	26·69										
	16	28·01	27·70	28·24	28·35	27·59	27·02	27·78	28·79	27·81	27·96	27·72	28·39	27·82	27·77	28·15	28·68
	18	28·34	28·40	27·95	28·62	27·61	27·58	27·80	28·46			27·42	28·38	28·07		28·32	27·77
	19	28·21	27·86	27·48	27·96	28·04	27·94	27·81	27·70			27·55			27·99	28·26	28·22
	20	27·69	28·12					28·54									
	22	28·05	28·66	28·66	28·74	28·30	28·71	27·05	28·48	28·65	27·70	27·76	28·11	28·16	28·14	27·61	28·05

N. B.—On and after August 15 the group connects with Group IV.

Results for latitude of astronomic station, Lafayette Park, San Francisco—Continued.

$\varphi = 37^\circ\ 47' +$ tabular seconds.

PAIRS OF GROUP IV.

Date.	1	2	3		4		5	6	7	8	9		10	11	12	13	14
	8097	8211	8299		8364	8372	52	105	170	218	G 205		345	456	500	588	644
	8125	8229	L 46607	8310	8		67	147	201	247	285	321	404	477	515	609	653
1891.																	
Aug. 15	27·85	28·36	27·97	27·84	28·20	28·23	29·42	28·40	28·20	28·81	28·66	28·19	28·58	28·63	28·57		28·29
16	28·46	28·87	27·60				29·13	28·00	27·76	29·14			28·02				
17																	
20		29·22	28·26	28·16	28·16	28·84	29·09	27·93	28·60	28·47	29·02	28·85	28·26	28·97	27·88	28·48	27·98
21	28·25	29·00	28·15	27·95	28·19	28·96	28·28	28·37	28·56	27·84	28·76	28·56	28·30	28·21	27·68	28·11	28·37
22	28·40	27·96															
31	28·47		27·88	27·99			29·47	28·29	28·47	29·11	28·43	28·71	28·57	28·81	28·19		
Sept. 5	27·84	28·86	27·66	27·79													
6	28·32		37·50	27·87	28·36	28·56	29·17	27·94	28·41	28·53	28·11	28·24	28·04	28·31	28·03	28·50	27·63
7	28·49	28·83	28·02	28·35		28·72	29·00	26·22	29·81	29·24	27·92						
8	27·97	29·66	29·44	29·26													
9	27·43	29·12	27·51	27·60	27·89	28·72	28·86	27·94	28·76	28·53	28·73	28·18	29·08	28·47			
10	27·64	28·34	27·38	28·09	26·98	28·07	28·83	26·67	28·67	28·52	28·11	27·97	28·13	28·57			
12			27·76	28·36	28·13	28·46		27·91		28·27							
15																	
16	28·87	28·37	27·89	28·56	28·21	28·76	28·57	27·97	28·20	29·02	27·84	27·71	28·71	29·30	28·54	28·75	27·49
18	28·11	29·08	27·56	27·66	27·51	28·37	28·51	27·60		28·45	28·55	28·71	28·89	28·69	28·74	28·65	27·96
19	28·23	28·06	27·46	27·67	27·81	28·17	28·42	28·03	27·94	28·54	28·57	28·35	28·00	28·58	28·42	28·05	27·81
20																	
22	28·19	28·26	27·60	27·92	28·36	28·54		27·48	28·05	29·09	28·12	28·25	28·28	28·45	27·82	28·82	27·58

23	27·32	29·15	27·40	28·21	28·19	28·26	28·54	28·03	27·47	28·59	27·81	27·96	28·61	29·19	28·34	28·85	28·05
24		28·85	28·09	28·25		29·13	28·45	28·08	27·46	28·66	26·50	27·07	28·86	28·68	27·84	28·33	27·77
25	27·83	28·21		27·46													
27	28·25	27·97	27·98	27·87	27·79	28·30	27·92	28·15	28·81	29·08	27·84	27·67	27·95	28·69	27·79	28·13	27·82
28	27·62	28·70	28·13	27·45	27·61		28·68	28·17	27·62	28·22	27·09	27·75	28·13	28·22	28·65	28·28	27·71
29	27·74	28·40	27·43	27·86	28·74		28·35	27·63	28·11	28·11	27·72	27·32	27·82	28·29	27·90	28·20	27·41
30	27·96	29·11	27·80	28·15	28·16	28·66	29·26	27·64	28·54	28·26	27·73	27·32	27·89	28·49	27·99	28·18	
Oct. 1	28·01	28·96	27·40	27·86	28·40		28·88	28·03	28·13	28·34	28·18	27·45	27·81	29·24	28·35	28·64	27·77
2	28·12	28·24	28·35	27·99	27·79	28·87	29·07	28·04	28·96	28·23	27·80	28·18	28·46	28·63	28·30	27·99	28·06
3	27·91	27·55		27·74	28·16	28·85	28·70	28·20	28·58	28·30	28·07	28·03	28·41	29·00	28·18	28·71	27·75
4	27·96	28·04	27·39	27·52	27·97	28·72	27·90	28·22	28·16	28·45	28·31	28·44	28·16	28·46		28·44	27·61
5	28·43	28·51	28·23	27·56	27·65	27·81	29·05	28·09	27·77	28·07	27·18	28·16	28·35	28·66	28·48	28·02	29·06
10	28·09	28·23			27·81	28·86	29·41	28·82	28·55	28·61	27·98	27·96	28·11	28·99	27·80	28·18	28·80
11	27·98	28·51	27·79	27·58	27·71	28·48	28·30	28·05	28·40	28·55	28·15	27·40		29·23	28·32	28·95	28·45
16	27·95	28·90	28·03	28·38	28·29	28·75	29·06	28·38	28·63		27·55	27·19	27·47	29·08	28·03	28·27	27·10
17	27·69	28·19	26·98	26·92	27·37	27·36	28·17	28·38	28·23	28·67	27·25	27·21	28·83				
21	28·24	28·93	28·27	28·24	28·73	29·49	28·99	27·85	27·78	28·62	28·78	28·47	28·50	28·95	28·28	28·51	
24	27·45	28·59	27·77	27·75	28·66	28·90	28·47	28·20	28·81								
26	27·82		28·54	28·27													
28																	
29		28·23	28·52	28·50	28·22	28·72	29·18	28·53		27·84	28·08	27·91	26·94	28·74	27·39	27·31	28·14
30	28·15			27·87	27·97	28·55	28·96	28·41	27·61								

N.B.—From and after September 23 the group connects with Group V.

Results for latitude of astronomic station, Lafayette Park, San Francisco—Continued.

$\varphi = 37° 47' +$ tabular seconds.

PAIRS OF GROUP V.

Date.	1 673 692	2 707 733	3 761 *L.* 4752	4 813 827	5 871 888	6 896 *L.* 5514	 943	7 962 980	8 991 998
1891									
Sept. 23	28·08	28·24	27·58	28·77	29·11	28·62		27·70	28·49
24		28·81	27·79	28·33	28·47	28·81		27·71	
25									
27	27·66	28·52	26·45	27·94	29·33	28·31		28·45	27·98
28	27·80	28·39	27·39	27·94	30·06	29·24		28·06	28·03
29	27·27	28·01	28·41	28·47	28·55	28·12		27·51	28·07
30	27·03	28·31	27·99	28·11	28·73			27·55	28·07
Oct. 1	27·45	28·28	28·12	28·34	28·18	28·55		27·78	28·18
2	27·87	27·84	28·74	28·56	28·64	28·68		27·79	28·14
3	27·64	28·62	27·19	28·65	28·26	28·49		27·57	28·73
4	28·08	27·75	27·57	27·53	27·66	28·99		28·12	29·04
5	28·12	28·15		28·19	28·97				29·14
10	27·82	28·35	27·63	28·23	28·98	28·88	29·09	28·52	28·54
11	27·86	29·21	28·40	28·16	28·06	28·87	28·47	27·61	28·49
16		28·18	28·33	27·47	28·58	28·21	27·94	27·84	29·23
17									
21									
24									
26									
28							27·61		
29	27·20	28·26	27·55	28·21		28·06	27·69	27·84	28·05
30									
Nov. 5			27·75	28·19	28·10	28·12	28·22		
6			28·20	28·47	27·55		27·68	28·20	28·30
7	27·34	28·37	27·37	27·95	28·26	27·68	27·43	28·25	28·76
8	27·82	28·55	27·60	27·76	28·13	28·01	27·81	28·02	28·55
9	27·45	28·54	27·69	27·51	28·60	28·72	27·92	28·33	28·81
11									
12			28·58	28·34	28·75	28·44		27·47	28·05
13		28·75	28·52	28·18	29·05	28·52	28·16	27·57	28·25
14	27·53	28·53	28·01	28·08	27·92	28·36	27·77	28·16	28·65
15	27·59	27·89	27·09	28·17	28·43	28·66	28·33	28·39	28·17
16	27·75	28·22	27·48	28·08	28·18	28·21	27·84	27·52	28·39
17	27·75	27·96	27·83	27·68	28·21	29·01	28·06	27·98	28·00
19		28·16	29·15	29·03	29·67		27·97	28·25	28·50
20	27·21			28·18	28·72	28·58		27·71	28·83
21	27·38	28·01	27·42	28·56	28·98	28·64	27·79	28·48	28·13
22	27·48	28·56	27·99	29·09	28·96	27·67	27·07	28·05	28·01
23	27·67	28·77	27·85	28·07	28·14	28·05	27·38	27·68	27·90
24			28·12	27·75	28·38	27·43	27·40	27·92	28·29
25		29·09	27.47	28·29	27·54	28·01	27·83	28·36	28·32
26	27·63	29·97	27·45	27·96	28·10	28·05	27·20	27·86	28·15
27			28·23	27·96	28·02	28·20	28·15	28·14	28·28
Dec. 1		29·01	28·34	28·10	28·68	28·67	28·20	27·94	28·63
2*		28·37		28·42	27·98		28·05	28·22	28·57

* Last day of observations with Zen. Tel. No. 1.

Results for latitude of astronomic station, Lafayette Park, San Francisco—Continued.

$\varphi = 37^\circ\ 47' +$ tabular seconds.

PAIRS OF GROUP V—Completed.

9		10	11	12	13	14	15	16	17		18
1053	1045	1087	1138	L 7110	1262	1302	1363	1425	1496		1554
1065		1111	G 740	1203	1287	1313	1382	1449	L 9261	1538	1572
28.77		28.04	27.49	28.06	28.02	28.76	29.20	27.83		28.15	28.37
28.49		28.19	27.38		27.56	28.04	28.11	28.42			28.05
28.98		28.38	27.49	26.92	27.66	27.96	28.05	28.33		27.76	27.82
28.86	28.59	27.79	27.24	27.65	28.56	27.94	27.87	28.09	28.07	27.46	27.88
28.49		28.42	26.51	27.77	27.97	28.14	28.20	27.74	28.42	27.44	27.32
28.09	29.17	28.11	27.47	27.85	27.44	28.13	27.94	27.81		27.27	
28.53	29.51	28.44	27.33	27.63	28.15	27.84	28.13	28.27	28.38	27.90	27.89
28.86	29.46	28.22	27.52		28.24		28.02	28.26			27.72
28.50	28.97	28.36	28.29	27.94	27.89	26.92	28.08	27.60		27.98	
28.69	29.51	29.19	27.31	28.01	28.25	28.26	28.42	28.31		28.58	26.81
28.34	30.09	28.62	28.12		28.42	28.72	28.35	27.82		27.79	28.06
29.30	29.67	28.27	27.27	27.96	28.12	27.99	28.17	28.28		27.67	27.93
28.77	29.46	28.50	28.01		28.06	28.49	28.48	28.13		27.78	27.02
28.56	29.55	28.52	27.66	28.43		28.52	28.18	27.70		27.55	28.22
29.30	29.80	28.25	27.52	28.10	28.16	27.76	27.54	27.80			28.13
28.92	29.31						27.59	27.97		27.51	27.89
28.98	29.53	28.31	27.37	28.41			28.45	27.84		27.53	27.77
28.52	28.95	28.83	27.56	27.72	27.79	27.58	27.41	27.79		28.05	27.82
28.54	29.38	28.71	27.54	28.31	27.98	28.72	28.39	27.83		27.72	27.68
29.43	29.66	28.21	27.43		27.95	28.17	28.15	28.00		27.89	27.54
						27.60	27.54			27.88	
28.14		28.55	26.34	27.76	27.89	27.75	27.86	28.04		27.35	28.18
29.01	29.26	28.60	27.15	28.42	28.08	27.99	27.77	27.72		27.62	27.83
	29.72	28.71	27.95	28.04	27.26	28.02	28.45	27.97	28.43	27.74	27.86
28.61	29.42	28.46	27.89	27.33	28.72	27.87	28.13	27.72	29.10	27.70	28.35
28.83	29.34	28.49	27.20	28.19		28.57	28.56	28.30	29.43	28.00	27.83
28.25	28.81	28.22	28.19	28.36	27.88	28.01	27.66	27.49	28.51	27.50	27.60
28.67	29.58	29.37		27.97	27.99	29.02	27.47	27.52		27.31	27.85
28.88	29.33	28.46	27.36	28.15	27.47	27.79	27.45	28.18		28.26	
29.13	29.76	28.84	27.37	28.09	27.80	27.81	27.86	27.60	28.46	27.32	27.98
28.93	29.69	28.72		27.90	28.25	27.96	28.43	27.85	28.78	27.65	27.78
27.87	29.10	28.43	27.86	28.07	27.71	27.72	27.84	27.92	27.62	27.33	28.21
28.66	29.24	28.48	27.00	27.59	27.96	28.13	27.71	26.55	28.49	28.09	28.25
28.90	29.38	28.27	27.32	27.73	27.40	27.95	27.48	27.70	28.65	27.57	27.81
28.73	29.21	28.19	27.31	28.02	27.11	27.69	28.21	27.99	28.36	27.77	27.72
28.71	29.16	28.47	26.70	27.91	27.68	27.90	28.08	27.83		26.55	27.99
27.24	27.69	28.44	27.92	27.95	27.22	28.18	28.18	27.89	28.35	27.56	28.26
28.70	29.86	28.62	26.87	28.42				27.64		28.40	28.42

N. B.—From and after November 5 the group connects with Group VI.

Results for latitude of astronomic station, Lafayette Park, San Francisco—Continued.

$\varphi = 37° 47' +$ tabular seconds.

PAIRS OF GROUP VI.

Date.	1		2		3	4	5		6	7		8 (*a*)	8 (*b*)
	1602		1705		1749	1821	1867 1887	1876	1928	1989 2020	2009	2057	2090
	1663	*L* 9902	1734	1726	1751	1849			1952			Æ*6978	2107
1891.													
Nov. 5	28·63		28·18	27·71				27·68					
6	28·71				27·67	27·78		27·97	28·62	28·77		27·14	
7	28·19		27·57	27·49	27·17	27·65		27·97	28·07	28·75		27·28	
8	28·38			27·94	27·94	27·39		27·56	27·76	28·83		27·48	
9	28·67			27·90	27·71	27·96		27·61	28·29	28·63			27·27
11	28·85			28·02	28·23	28·22		27·45	28·51	28·86			27·21
12	28·09			28·12	28·21	27·56		28·05	28·37	28·30			27·47
13	28·61			27·68	28·15	27·10		26·76	28·41	28·71			27·10
14	28·31			28·16	27·93	27·91		27·44	28·18	28·57			27·40
15	27·97			27·50	28·04	27·82		27·96	28·31	28·81			27·19
16	28·29			27·78	27·49	27·93		27·87	28·03	28·95			27·36
17	28·00			27·62	28·06	28·02		27·30	27·93	28·20			27·45
19	29·22			28·59	28·13	26·81		27·10	28·18	28·52			27·02
20	28·04			28·71	28·61	27·72		28·03	28·16	28·86			27·04
21	27·93			27·52	28·52	28·27		28·35	28·13	28·78			27·64
22	28·36			27·69	27·93	27·68	28·01	27·45	28·12	28·52			27·39
23	28·95			27·82	27·75	28·13		27·98	28·13	28·60			27·70
24	27·75			27·67	27·88	27·35	28·50	27·39	28·19	28·41			27·32
25	27·76			28·01	27·88	27·06			27·61	28·88			27·43
26	27·89			27·93	28·12	27·58		27·75	27·68	28·41			27·39

27	27·67			27·96	27·82			27·94	28·00	28·24			27·26
Dec. 1	28·13			28·08	28·46	27·88	28·19	28·22	27·81	28·81			27·66
2*	27·95			28·07		27·63		27·71	29·14	28·53			
13†										28·65	28·85		
16							28·01	27·93	27·19				28·47
17	28·18			27·62	28·18	28·05	28·35	27·97	27·34				
19					28·37		27·88	27·50	27·83	28·92	28·94		27·85
20							27·97	27·77	27·35	28·71			
21					27·54	27·59		28·22	27·76	28·74	29·05		27·62
23	28·22				27·07	27·57		27·91	27·47	27·42	27·29		27·79
24	28·17			27·91	27·96	27·33	28·09	27·87	27·76	28·62	28·49		27·78
25	28·49			28·29	28·39	27·62	27·89	27·87					
27	27·94					27·92		28·34	27·78	28·69	28·76		28·02
30	28·15			27·94	28·16	27·62			27·99	29·02	28·63		28·11
92. Jan. 2	28·69			28·56	28·33	27·33		28·43	27·67	28·66	28·41		27·45
8					28·28	27·70		28·02	27·87	28·78	28·86		28·36
10	28·21			28·25	28·57	28·33		28·33		27·73	28·71		28·02
11	28·43			28·23	28·26	27·39		27·76	27·22	28·65	28·46		28·14
12													
14	27·99	28·80		28·04	27·94	27·84	28·93	28·78	28·09	28·69	29·10		27·62
15				28·02	28·22	27·43				28·34	28·24		
16	28·57	29·18		28·54	29·56	27·67	28·17	28·01	27·83	29·04	28·88		
17	28·32			28·58	28·11	27·52	27·75	27·77	27·78	28·88	28·70		27·74
18	28·71	29·78	28·31	28·09	28·36	27·97	27·99	28·01	28·03	28·73	28·75		27·63
19	28·62	29·11	28·15	27·88	28·42	27·69	28·42	28·32	27·85	28·72	28·84		27·58
20				27·84	28·04	27·75		28·20	27·73	28·98	30·28		27·92
21	28·66	29·38	28·71	28·49	28·52	27·46		28·35	28·15	29·12	28·90		28·14
22	28·06		28·48		28·25			28·34	27·92				
23	27·55			27·79	28·89	27·74		27·81	27·51	29·10	28·87		27·45

* Last day of observations with Zen. Tel. No. 1. † First day of observations with Zen. Tel. No. 3.

Results for latitude of astronomic station, Lafayette Park, San Francisco—Continued.

$\varphi = 37^\circ\ 47' +$ tabular seconds.

PAIRS OF GROUP VI—Completed.

Date.	9	10	11	12		13		14	15		16	17	18	
	2143	2249	2300	2330	2362	2330	2362	2409	*G* 1318	*G* 1351	2532	2596	2650	
	2230	2265	2313	2376		2369		2431	2493		2551	*L* 15522	2744	2759
1891. Nov. 5		28·77	28·55	27·87	27·69				28·06	28·33	28·33	28·55	27·51	
6		29·34							28·77	28·49	28·55	28·08	28·38	
7		28·59	27·99	27·98	27·96				28·19	28·15	28·81	29·11	28·47	
8		29·07	28·01	28·00	28·15	28·56	28·73		27·81	28·08		28·68	28·15	
9	28·13	29·19	28·84		28·35				28·24	28·45	28·98	28·55	28·48	
11	27·88	28·19		28·71	28·28				28·68	28·22	28·90	28·62	27·55	
12	27·51	28·97		27·36	28·56				27·93	27·63	28·56	29·02	27·98	
13	28·40	29·39	27·56	28·11	27·59				27·80	28·08	28·62	28·19	28·03	
14	27·86	29·22		27·91	27·62				27·61	27·83	28·51	28·30	28·08	
15	28·21	28·83	28·61	28·28	28·42				27·82	27·61	28·23	28·35	27·56	
16	28·23	29·16	27·81	28·37	28·14				28·36	28·58	29·12	28·69	28·22	
17	28·40	28·72		28·27	27·82				27·96	27·51	28·94	28·31	28·29	
19				28·23	27·25				27·89	27·98		29·03	28·79	
20	28·23	29·15	27·85	27·99	27·37				27·78	27·93	28·72	28·52	28·32	
21	28·63	29·02	27·98		27·57				28·50	27·95	28·88	28·25	28·54	
22	28·63	28·94		28·00	27·33				27·76	28·03	28·04	28·46	27·69	
23	27·46	28·89	27·25	27·82	27·53				28·02	28·08	28·20	28·43	28·39	
24	27·75	29·26	28·70	27·81	27·75				27·67	27·53	28·29	28·37	28·14	
25				27·32	27·46				27·65		28·11	28·61	28·45	
26	28·01	29·12	28·47	28·15	27·72				27·79	27·80	28·35	28·39	27·84	

	27														
Dec.	1	27·54	29·42	28·14	27·64	27·70				27·67	27·29	28·62	28·74	28·09	
	2*														
	13†	28·04		27·89		28·27				27·78	27·66	28·08	29·35	29·25	
	16		28·50	28·81	28·10	28·18					27·75				
	17														
	19	27·27	29·50	29·38	28·40	28·77	28·43	28·78	28·15	27·92	27·86	27·71	28·43	28·76	
	20		29·23	28·70	28·09	28·51	28·59	29·01	28·53	27·94	28·31	27·98	28·36	29·13	
	21	27·59	29·47	28·55	27·78	27·97	28·36	28·55	27·97	29·24	29·45	27·67	29·06	29·02	
	23	27·93	28·83	28·88	27·89	28·18	28·35	28·65	28·18	28·02	28·25	27·76	28·58	28·97	
	24	27·91	28·80	28·75	28·12	28·42	28·45	28·75	27·88	28·01	28·05	27·97	28·37		
	25														
	27	28·36	29·47		28·48	28·05				27·93	28·00	28·20			
	30	28·34	29·14	28·70	27·69	28·01	28·57	28·89	28·14	27·95	28·21	27·75	28·46	29·31	
1892.															
Jan.	2		27·73	28·32						28·44	28·41	28·66			
	8	28·35													
	10	28·26	27·65	28·39	27·57	28·00	28·19	28·63	27·03	28·00	27·88	28·05	28·84	29·07	
	11	27·94	29·43	28·60	27·98	28·61	28·23	28·87	27·23	27·75	27·93	28·06	28·84	29·46	
	12								27·77	27·92	28·32	28·05	28·57	28·81	
	14	28·02	29·15	28·72	28·29	28·47	28·84	29·01	27·91	28·02	28·01	27·92	29·09	29·29	
	15									28·22	28·17				
	16	28·15	29·26	28·86	27·12		27·46		27·65	28·06	28·11	28·03	28·72	29·00	28·86
	17	28·08	28·54	28·36	28·11	28·12	28·82	28·83	27·75	27·94	28·01	27·75	28·53	28·92	28·85
	18	28·29	29·21	28·57	28·36	29·08	28·64	29·36	27·69	28·21	28·67	27·80	28·57	29·24	28·90
	19	26·97	29·00	28·54	28·11	28·21	28·50	28·58	28·12	28·22	28·33	27·96	27·83	28·49	28·13
	20	28·10	29·12	28·21	28·33	28·73				28·31	29·15	28·38	28·27		
	21	28·35	29·28	28·30	28·17	28·44	28·67	28·93	28·20	28·36	28·03	28·37	28·64		
	22					28·21		28·73	28·17	28·11	27·96	28·18	28·85	29·18	28·72
	23	27·92	29·74	28·75	27·78	28·11	28·12	28·47	28·04	28·08	28·14	27·90	28·34	29·38	29·05

* Last day of observations with Zen. Tel. No. 1.

† First day of observations with Zen. Tel. No. 3.

N. B.—From and after December 17 the group connects with Group VII.

Results for latitude of astronomic station, Lafayette Park, San Francisco—Continued.

$\varphi = 37° \, 47' +$ tabular seconds.

PAIRS OF GROUP VII.

Date.	1		2				3	4	5		6	7	8	
	2776, 2810	2776, 2816	2842, 2897	2876, 2897	2842, 2902	2876, 2902	2942, 2982	3033, 3059	3069, 3150	3088, 3150	3182, 3194	3241, 3265	3292, 3308	3318, 3308
1891.														
Dec. 17			28·72	28·60			28·06	28·87				28·64		
19	28·22	28·13	28·11	28·04			28·20	28·05	27·89	28·17	28·40	28·79	28·28	28·90
20	28·47	28·34	28·31	28·31			28·49	28·80		28·50	28·20	28·75	28·00	29·11
21	28·45	28·55	28·61	28·89			28·37	28·26	27·88	28·07	27·81	28·72	27·90	28·69
23	28·23	28·22	28·05	28·10			28·14	28·08	27·47	28·01	27·99	28·60	27·80	28·59
24	28·33	28·12	28·03	28·12			27·99	27·95	27·80	27·93	28·34	28·93	27·96	28·75
25														
27														
30	28·37	28·17	28·18	28·12			28·36	28·15	28·34	28·31	27·75	28·43	28·14	28·78
1892.														
Jan. 2								28·60						
8							28·14	28·46				28·59		
10	28·54	28·64	28·78	28·36			28·70	27·95	28·33	28·52	28·30	28·76		
11	29·05	28·98	28·37	28·41			28·75	27·78		28·15	28·72	28·65	28·36	29·21
12	28·44	28·25	27·96	28·30			28·11	28·61	27·81	28·50	28·77		28·31	29·25
14	28·53	28·43	28·25	28·05			27·98	28·33	28·40	28·50	28·57	28·86	28·12	28·43
15														
16	29·30	29·03	28·37	28·36			28·51	28·26	28·36	28·54		28·98	28·35	29·10
17	28·60	28·44	28·27	28·41			28·44	28·20	28·45	28·49	28·60	28·76	28·14	28·74
18	28·60	28·41	28·08	28·38			28·51	28·54	28·30	28·55	28·20	28·80	28·51	28·82

19	28·62	28·49	28·64	28·56			26·74	28·13	27·95	27·76	28·16	28·70	28·09	28·62
20		28·46	28·63	28·58			28·41	28·63						
21	28·78	28·48	28·30	27·86	28·22	27·77	28·19	28·16	27·78	28·52	28·52	28·71	28·23	28·97
22	28·52		28·04	27·90	28·52	28·38	28·34	28·48	28·43	28·52	28·64	28·84	28·25	28·73
23	28·23	28·51	28·22	28·12	28·61	28·51	28·18	28·16	28·15	28·20	28·38	29·25	28·55	28·76
27	29·00	28·85	28·32		28·72		28·03	28·15	27·86	28·76	28·52	29·23	28·09	28·86
28	28·61	28·26	27·89	28·15	28·20	28·46	28·43	28·27	28·33	28·55	28·29	28·47	28·31	28·89
30	28·83	28·62	28·31	28·38	28·13	28·20	27·89	28·63	28·19	28·87	28·41	29·13	28·45	29·22
F b. 1	28·12	28·18	28·03	28·07	28·38	28·43	28·41	28·38	28·06	28·15	28·43	27·72	28·26	28·25
2			27·92	27·90	28·18	28·15	28·15	27·97	27·94	27·99	28·35	28·57	28·14	28·52
6	28·30	28·07	27·99	27·80	28·68	28·48	28·18	28·28	28·29	28·29	28·43	28·87	28·46	28·77
7														
8	28·33	28·20	28·25	27·93	28·33	27·99	28·37	27·97	28·34	29·03	28·34	29·05	28·26	28·38
9	28·68	28·75	28·21	28·45	28·36	28·60	28·64	28·21	28·10	28·02	28·60	28·61	28·76	28·67
10	28·57	28·57	28·41	28·29	28·65	28·53	28·27	27·78	28·36	28·44	28·53	28·68	28·15	28·85
28							28·04	28·61						29.10
Mar. 2		28·67	27·99	28·24	28·71	28·94	28·58	28·59	28·60	28·40	28·51	29·24	28·16	28·67
3	28·45		28·60	28·76	28·02	28·18	28·31	27·77	28·44	28·51		28·96	28·55	
4	28·87	28·91	28·92	28·49	29·32	28·89	27·30	28·11	27·88	28·23	28·61	28·30	28·28	28·95
5	28·28		27·78	26·73	28·44	27·39	28·21	28·46	27·87	27·91	28·11	28·97	28·39	28·73
6														
7	28·09	27·87	28·11	27·85	28·76	28·50	28·14	27·97	28·33	28·00	28·01	28·69	28·46	29·17
8	28·14	27·93	28·17	28·37	28·50	28·70	28·46	28·05	28·40	28·46	28·21	29·40	27·93	28·77

Results for latitude of astronomic station, Lafayette Park, San Francisco—Continued.

$\varphi = 37^\circ\ 47' +$ tabular seconds.

PAIRS OF GROUP VII—Completed.

Date.	9 3352 3375	10 3456 3490 3484 3505		11 3456 3490 3484 G 1635		12 3531 3571 3561		13 3607 3633	14 3672 G 1697	15 3736 3757	16 3788 Œ₀ 11453
1891.											
Dec. 17		28·90	28·03			29·27	29·08	28·19			
19	28·38	28·67	28·52	28·63	28·46	28·39	27·90	28·95	28·36	28·35	28·65
20	29·05	28·74	28·72	28·82	28·79	28·25	27·83	28·63	28·26	28·60	28·96
21	29·17	28·29	27·96	28·26	27·96	29·03	28·40	28·19	28·66	28·75	
23	28·62	28·37	28·07	28·13	27·81	28·23	27·95	28·46		28·20	28·09
24	28·76	28·35	28·15	28·31	28·08	29·06	28·42	28·85	28·07	28·28	28·22
25											
27											
30	28·64	28·59	28·60	28·37	28·36	28·66	28·42	28·28	28·31	28·23	28·44
1892.											
Jan. 2											
8					28·51	28·34	27·82				
10											
11	29·00	28·76	28·50	[illegible]	28·22	28·03	27·68	28·52		28·82	28·31
12	28·60	28·69	28·41	28·17	27·86	28·46	27·97	28·21	28·14	28·55	
14	29·00	28·55	28·36	28·24	28·05	28·50	28·23	28·49	28·59	28·62	28·34
15											
16	28·94	28·61	28·14	28·46	28·00	28·78	28·34	28·61	28·14	28·25	28·25
17	29·00	28·46	28·11	28·32	27·98	28·65	28·47	28·45	28·60	28·50	28·64
18	29·35	28·92	28·45	28·95	28·48	29·24	28·57	28·24	28·82	28·60	28·93

19	29·01	29·07	28·47		28·85	28·24		28·69	28·38	28·92	28·43	28·22	28·49
20													
21	27·97	28·54	28·17		28·90	28·55		28·27	28·21	29·11	28·59	28·32	28·55
22	29·21	29·30	28·90		28·70	28·28		29·06	28·31	28·62	28·23	28·79	28·34
23	29·47	28·75	28·43		28·56	28·27		28·65	28·22	28·74	28·24	28·10	28·21
27	28·67	28·69	28·04		28·51	27·86		28·85	28·32	29·85	28·77	28·52	28·36
28	28·75	28·85	28·66	28·77	28·53	28·33	28·43	28·78	28·42	28·32	28·94	28·26	28·53
30	28·81	28·84	28·32	28·25	29·06	28·55	28·51	28·63	27·91	28·43	28·48	28·32	28·39
Feb. 1		29·21	28·46	28·57	28·89	28·12	28·22	28·70	28·43	28·36	27·88	28·16	28·63
2	29·24	28·87	28·50	28·42	28·57	28·18	28·15	28·30	27·99	28·42	28·31	27·85	28·62
6	29·07	28·87		28·39	28·93		28·44	28·38	28·59	28·50	28·76	28·28	28·39
7											27·95	28·37	
8	28·73	28·95	28·20	28·40	28·78	28·04	28·24	28·64	28·32	28·37	28·55	28·80	28·40
9	28·89	28·31	27·96	27·77	28·51	28·13	27·94	28·38	28·28	28·83	28·65	28·53	28·79
10	28·66	28·87	28·22	28·24	28·69	28·00	28·06	28·17	27·85	28·61	28·64	28·72	28·19
28	29·60	29·06	28·73	28·60	29·06	28·74	28·75	29·02	28·06	29·00	28·86	28·70	28·82
Mar. 2	29·17	28·94	28·36	28·03	29·02	28·36	28·15	28·66	28·27	28·41		28·56	28·69
3	28·99	29·38	29·08	28·62	29·15	28·83	28·42	28·77	28·18	28·71	28·08	28·53	
4	28·62	28·51	27·99	27·76	28·54	28·01	27·78	28·51	27·96	28·30	27·59	28·29	28·38
5	28·69	28·81	28·27	28·49	28·59	28·05	28·25	28·90	28·24	28·76	28·34	28·64	
6													
7													
8		28·53	28·43	28·23	28·22	28·12	27·95	28·62	28·12	28·35	28·01	28·28	28·33

N. B.—From and after January 27 the group connects with Group VIII.

Results for latitude of astronomic station, Lafayette Park, San Francisco—Continued.

$\varphi = 37° 47' +$ tabular seconds.

PAIRS OF GROUP VIII.

Date.	1	2	3	4	5	6	7	8	9	10		11		12	13	14		15
	3885 3915	3937 3953	3975 4050	4077 F 2015	4104 4122	4137 4143	4168 4222	4247 4249	4276 4292	4326 4339	4342	4384 4408	4415	4470 4506	4509 4540	4553 G 2036	4566	4640 L 25839
1892.																		
Jan. 27		27·96	28·35	28·47	28·53	28·49	28·28	28·22	28·29					28·18	28·69		27·45	27·89
28	29·14	27·90	28·78	28·77	28·39	28·35	28·02	28·00	28·45			28·33	28·50	28·20	28·36		28·53	28·59
30	29·04	27·76	28·69	28·87	28·28	28·65	28·25	27·65	28·41		28·53	28·56	28·41	28·14	28·27		28·31	28·70
Feb. 1	28·48	27·79		28·70	28·24	28·22	28·12	28·58	28·55		28·89	27·96	27·98	28·65	28·31		28·61	28·00
2	28·52	27·78	28·34	28·28	28·28	28·67	27·87	28·19	28·31	29·33	29·27	28·33	28·24	28·55	28·72		28·14	28·13
6	28·55	27·70	28·56	28·46		28·63	27·98	28·21	28·19	29·12	29·15	28·13	28·03	28·22	28·06		28·10	28·22
7	28·73	27·78	28·81	28·51	28·60	28·64	28·28	27·77	27·76	29·13	28·98	28·11	28·52					
8	28·30	27·90	28·73	28·90	28·07	28·26	27·96	28·06	28·18	28·43	28·34	28·30	28·39	28·67	28·45		27·97	27·58
9	28·38	27·85	28·63	28·90	28·23	28·44	28·09	28·79	28·43	29·39	28·96	28·52	28·94	28·42	28·91		28·09	28·32
10	28·51	27·78	28·27	28·26	28·31	28·51	27·72	28·25	28·08		29·05	28·59	28·86	28·45	28·52		28·32	27·77
28	28·67	27·92	28·38	28·55	28·43	29·02	28·48	28·16	28·40	28·83	28·89	28·43	28·51	28·37	28·75		28·09	28·32
Mar. 2	28·75	27·81	28·23	28·35	28·71	28·92		28·53		29·02	28·78	28·20	28·37	28·70	28·38		28·20	28·14
3	28·97	27·82	28·47	28·77	28·18	28·33	28·04		29·64									
4	28·22	28·24	28·76	28·56	28·70	28·86	28·39	28·14	28·51	29·26	29·88	28·57	28·67	28·57	28·48		28·32	28·19
5	28·66	28·12	28·33	28·94	28·11	28·86	28·41	28·16	28·32	28·98	29·30	28·11	27·94	28·49	28·62	29·06	28·65	28·44
6	28·72	27·74	28·14	28·61	28·26	28·46	28·78	28·34	28·20	28·68	28·34	28·54	28·28	28·35	28·63	28·86	28·29	28·05
7																		
8	28·35	27·48	28·56	28·87	28·62	28·47	28·64	28·11	28·69	29·28	29·04	28·43	28·37	28·92	28·41	29·12	28·52	28·17
12													28·29	28·33		28·37	28·37	28·05
15		27·83	28·69			28·87	29·02	28·25	28·28	28·52	28·55	28·56	28·49	28·75		28·64	28·37	28·32
20			28·38	28·42	28·31	28·90	28·48	27·99	28·56	29·12	29·62	28·94	28·92	28·55	28·52	28·80	28·42	28·45
21	28·53	27·62	28·39	28·68	28·54	28·65	28·63	27·78	28·20			28·56	28·35	28·70	28·67	28·70	28·27	28·57
22																		
23								27·91	28·20		29·35	28·56	28·70	28·50		28·62	28·34	28·74
24		27·92		28·50	28·58	28·72	28·00	28·31	28·00			28·92	28·63	28·21	28·52	28·69	27·92	

25	29·01	28·51	29·75	28·55	28·77				28·30			28·59	28·52	28·63	28·40			28·38
27	29·21	28·60	28·11	28·82		28·23		27·93	28·70	28·60	28·54	28·39	27·24	28·53	28·86	28·75	28·24	28·04
29	28·64	27·94	28·46	29·33	28·37	28·84				29·12	29·22				28·72	28·23	28·30	28·20
30			28·84	29·07	29·53	28·57	27·88	27·84	28·24	29·19	29·13	28·25	28·27	28·17	28·49	28·67	28·30	28·56
31	28·65	27·32	29·03	29·38														
Apr. 2				28·99	28·69	28·77	28·57	27·99	28·52	29·30	29·19	28·70	28·47	28·60	28·81	28·94	28·44	28·81
3	28·94	27·92	28·69	28·59	28·07	28·72	28·20	28·50	27·71	28·90	28·51	28·20	28·19	28·52	28·62		28·03	28·40
4	28·83		27·81	28·98	28·63	28·55	28·12	28·29	26·76	29·36	28·92	28·01	28·11	28·12	28·59	28·55	28·23	28·15
5	29·38	28·76			28·75	28·34	28·37	28·36	28·64	29·00	29·29	28·74	29·51	29·03	29·32	28·78	28·68	28·33
6												28·51	28·79	29·12				
7		28·12																
9	28·63	27·29	28·66	29·14	28·73		28·29	27·57	28·62	29·57	28·85	28·70	28·55	28·90	28·90	28·98	28·57	28·51
11		28·44	28·59	28·18	28·44		28·63	27·98	28·36	28·89		28·85	28·61	28·51	29·46	28·94	28·33	28·46
13	28·86	28·29	28·33	28·94		29·18	28·87											
15	28·73	28·04	28·52	28·53	29·20	28·38						29·11		28·16	28·65	28·64	28·59	28·58
16	28·53	28·48	28·52	28·78	28·25	28·54	28·50	28·17	28·35	28·48	28·85	28·96	28·92	28·62	28·76	28·82	28·64	27·97
17		28·19	28·41	28·72	28·63	28·40		28·13	28·46	28·50	28·24	28·63	28·45	28·46	28·79	28·97	28·59	28·30
18	28·57			28·60		28·47	28·67	28·73	28·55	27·59	27·46	28·17	28·22	28·91	28·98			28·06
19	28·88	28·73	28·80	29·04	28·26	28·48		28·49	28·29			28·60	28·78	28·84	28·58	28·72	28·16	28·28
21	28·26	28·27	28·50	28·41		28·47	28·41	28·03	28·54	28·72	28·62	28·46	28·43	28·27	28·65	28·49	28·33	28·68
22	28·88		28·97	28·88		28·62	28·37	28·60	28·25	28·87	28·77	28·51	28·60	28·70	28·68	28·56	27·87	27·87
24	28·53	28·16	28·61	28·46	28·74	29·00	28·50	28·69	28·27	28·69	29·07			28·68	29·05	29·17	28·52	28·13
25	28·84	28·70	28·57	28·28	28·50		28·00	28·05	28·39	28·77	28·68	28·42	28·36	28·37	28·97	29·35	28·56	28·29
26	29·07	28·34	28·85	28·37			28·23	28·33	28·05	29·71	29·33	27·89	27·98	28·11	28·52	28·62	28·09	27·59
27	28·60	28·30																
29				29·36														
May 2																		
4	28·05	27·94																
6	29·03	28·63																
7	29·32	28·86																

N. B.—From and after March 6 the group connects with Group I, reobserved.

Results for latitude of astronomic station, Lafayette Park, San Francisco—Continued.

$\varphi = 37^\circ\ 47' +$ tabular seconds.

PAIRS OF GROUP I, REOBSERVED.

Date.	1	2	3	4	5	6		7				7 (bis)
	4696	4783	4808	4863	4937	5022		F 2626		5097		L 28533
	4724	4797	4843	4897	4953	5034	5043	5120	5126	5120	5126	5181
1892.												
Mar. 6	29·16	28·85	28·42	27·97	28·55	29·08	29·63			28·02	28·28	28·68
7												
8		28·79	28·32	27·73	28·13	28·75						
12	28·51											
15		29·08	28·31	28·31	28·87	28·89	29·92					
20	28·12	29·16	27·13	27·80		29·07	30·03			28·57	28·50	29·32
21	28·72	29·03										
22	28·50	29·13	28·24	28·50	28·83	28·39	29·74			28·40	28·51	29·04
23	28·58	28·72	28·38	27·75	28·41	29·06	29·83			28·17	28·49	29·43
24	27·84	28·65	28·00	27·90	28·99	28·06				28·10	28·14	28·79
25	28·39				28·42						28·60	29·19
27	28·41	28·81	28·29		28·89					28·44	28·43	29·21
29	28·79											
30	28·13	28·86	28·46		28·64	28·78	29·46			28·08	28·17	29·49
31												
Apr. 2		29·12	28·98	28·08	29·13	28·38	29·47			28·62	28·62	29·40
3	28·41	28·85	28·04	28·07	28·40	28·91	29·43	28·47	28·53	28·59	28·65	29·42
4	28·72	28·64	28·35	27·58	28·16	28·88	29·32	28·38	28·60	28·39	28·62	29·14
5	29·03	29·02	28·55	27·75	28·75			28·40	28·27	28·28	28·18	29·35
6												
7												
9	28·63	29·06	28·59	28·47	28·46	28·57				28·72	29·00	28·86
11	29·02	28·94	28·58	28·05	28·76	28·73	29·37			28·65	28·93	29·01
13												
15	28·68	28·80	28·31	28·39	28·82	28·21				28·70	28·74	28·72

	16	28·39	28·95	28·48	28·46	28·42	28·86	29·95	28·47	28·43	28·68	28·64	28·98
	17	28·43	28·47	27·50	28·33	28.71	28·91	29·74			28·73	28·86	29·03
	18	28·51	28·48	27·94	27·78	28·47	28·88	29·75	27·99	28·03	28·36	28·40	29·12
	19	28·91	28·82	28·44	27·97	28·68	28·95	29·57			27·95	28·23	28·89
	21	28·33	28·78	28·17		28·80	28·62	29·51	28·05	27·98	28·77	28·70	29·24
	22	28·33	28·39	28·36	27·85		28·89	29·98					28·90
	24	28·83	28·20		27·52	28·58	29·06		28·56	28·77	28·23	28·44	28·79
	25	28·49	29·04	28·24	27·86	28·41	28·66	29·58	28·74	28·60	28·86	28·75	29·56
	26	28·51	28·26	27·95	27·32	28·05	29·04	29·81	28·20	28·42	28·34	28·54	28·43
	27				28·25		28·96	29·55	28·60	28·40	28·65	28·45	29·12
	29												
May	2			29·10	28·87	29·29			28·36	28·24	28·77	28·61	29·08
	4			28·56	28·28	28·51	29·43	30·11			28·36		
	6				28·61	28·64	29·47	30·29			28·94	28·78	28·95
	7			28·35	28·11	28·70	29·32	29·59	28·46	28·42	28·78	28·73	29·08
	8	28·28	29·16	28·04		28·38	28·77	29·40			28·24	28·35	29·01
	10	28·46	28·99			29·03	29·50	29·90	28·21	28·45	28·54	28·78	29·32
	11	28·66	28·62	27·81	28·48	28·71	29·01	30·13	28·08	28·28	28·36	28·56	28·82
	12	27·80	28·43	28·26	28·11	29·01	29·09	29·66	27·99	27·93	28·08	28·01	28·86
	13	28·39	28·97	28·71	27·84	28·47	28·97	29·86			28·85	28·97	28·69
	15		28·82	28·19	27·96	28·37	28·67		28·76	28·76	29·42	29·42	29·00
	16	28·64	28·87	28·28	27·42	28·13	29·10	29·56	28·10	28·04	28·20	28·12	28·67
	17			28·25	27·86	28·96	28·75	29·33	28·06	28·09	28·33	28·39	28·74
	18	28·71	28·74	27·99	28·18	28·54	29·13	30·17	28·27	28·59	28·44	28·74	29·18
	19												
	20	28·87	28·50	28·30	28·24	28·09	29·01	29·46	28·62	28·38	28·56	28·30	29·06
	21	29·12											
	28			28·70	27·83	28·38	28·88	29·13			28·64	28·51	28·79
	29	28·65	28·08	27·94	27·59	28·39	29·39	29·96			28·75	28·68	29·03
	30	28·78	28·33	28·26	27·76	28·32	29·19	29·78	27·70	27·92	28·57	28·79	29·09
	31	28·86	28·60	28·07	28·08	28·77	28·61				29·08	29·19	29·11
June	1	28·66	28·45	28·28	28·01	28·54	29·16	29·80	28·52	27·96	28·84	28·28	29·30
	2	28·08	28·87	28·48	27·30	28·58	28·51	29·61	28·05	28·11	28·67	28·73	29·33
	4												
	5		28·77	28·16	28·28	28·95	29·14		28·65	28·63	28·86	28·85	

Results for latitude of astronomic station, Lafayette Park, San Francisco—Continued.

$\varphi = 37^\circ\ 47' +$ tabular seconds.

PAIRS OF GROUPS I REOBSERVED—Completed.

Date.	8	9	10	11				12		13		14
	F 2677	5246	5348	5459		*F* 2763		5530		5692		*L* 31117
	L 28716	5285	5392	5504	5507	5504	5507	5574	5575	5740	5745	5790
1892 Mar. 6	28·64		28·24	28·41	28·28	28·72	28·60	27·65		28·60	28·52	28·14
7												
8												
12												
15												
20	28·75	28·98	28·42	28·85		29·26		28·43		28·30	28·12	28·05
21												
22	28·63	28·83	28·41	28·28	28·41	28·74	28·87	27·81		28·35	28·11	27·90
23	28·88	28·70		28·73		29·05		27·49		28·51	28·31	27·90
24	28·45	28·51	28·84	29·20	29·27	29·08	29·14	28·12	27·99			28·14
25		29·05	28·40		29·35		29·60	28·54	28·34	28·59		
27	28·62	29·13	28·35							28·41		28·12
29												
30	28·35	28·60	28·18	28·89	29·02	29·01	29·14	27·46	27·52	28·57	28·47	27·83
31												
Apr. 2	28·60	28·83	28·60	28·42	28·61	28·80	28·99	27·96	27·92	28·41	28·52	28·19
3	28·49	28·66	28·94	28·94	28·56	29·08	28·74	27·86		28·30	28·36	27·87
4	28·77	27·81	28·62	28·23	28·58	28·18	28·54	27·56		28·91	28·61	27·89
5	28·86	28·78	28·64	28·65	28·48	29·15	29·00	28·25	28·38	28·77	28·29	28·09
6												
7												
9	28·76	28·71	28·50	28·80	29·08	29·23	29·52	28·01		28·37	28·46	28·17
11	28·82	28·88	28·63	28·67	28·61	28·76	28·68		27·67	28·67	28·56	27·91
13												
15		28·92	28·38						27·82			

	16		28·49	28·42	28·73		29·12				28·13	27·91	27·89
	17	28·88	28·60	28·87	28·65	29·01	29·08	29·43		27·60	28·39	28·31	28·08
	18	28·73	28·34	28·46	28·60	28·89	28·77	29·06		28·05	28·55	28·35	27·93
	19	28·55	27·95	28·23	28·55	28·61	28·97	29·04		27·97	28·46		27·89
	21	28·39	28·77	28·94	28·39	28·97	29·00	29·60		28·14	29·00	28·33	
	22	29·01	28·42	28·82						27·65	28·66	29·68	28·29
	24	28·82	28·89	28·92	28·80	28·85	29·07	29·15		27·91	28·78	28·45	27·89
	25		28·38	28·45	28·88	29·02	29·04	29·18		27·81	28·43	28·33	27·61
	26	28·61	28·65	28·65	28·80	28·80	29·18	29·14		27·81	28·80	28·47	27·92
	27		28·82	29·13	28·78	28·99	28·95	29·17		27·82	28·30	28·61	27·49
	29												
May	2	28·73	28·07	28·50	28·90	28·63	29·56	29·31		27·99	28·52		
	4	28·61	28·76	28·51						27·91	28·73	28·69	28·08
	6	29·05	28·82	28·93	28·68	28·73	28·87	28·92		27·76	28·56	28·62	27·45
	7		28·47	28·76	28·74	28·93	29·11	29·30		27·69	28·09	28·09	28·30
	8	28·83	28·80	28·83									28·52
	10	28·68	29·00	28·61	28·86	28·90	29.21	29·25	27·95	28·34	28·43	28·65	28·57
	11	28·48	28·52	28·26	28·50	28·48	28·90	28·90			28·33	27·93	28·36
	12	28·69	28·42	28·15		28·57		28·93	27·68	27·38	28·17	27·93	27·54
	13	28·50	28·68	28·79	28·73	28·64	28·89	28·81			28·52	28·25	28·01
	15	28·43	28·59	28·98		28·78		29·33	28·36	28·31	28·74	28·50	28·53
	16	28·73	28·37	28·53	28·59	28·54	28·61	28·57	27·92	27·71	28·55	28·00	28·17
	17	28·66	28·34	28·68	28·62	28·70	28·93	29·02	28·21	28·33			27·92
	18	29·53	28·64	28·84	28·22	28·50	28·75	29·04	27·51	27·49	28·51	28·14	27·90
	19												
	20	28·69	28·50	28·83	28·81	28·83	29·09	29·13	27·68	27·52	28·85	28·80	28·42
	21												
	28	28·95	28·38	28·95					27·87	28·01	28·35		28·36
	29	28·93	28·56	28·79	29·37	28·91	29·57	29·11	28·05	28·06	28·70	28·15	28·05
	30	29·02	28·42	28·64	28·61	29·05	28·67	29·12	27·94	27·88	28·86	28·60	28·08
	1	28·84	28·39	28·42	28·48	28·90	28·65	29·10	27·39	27·78	28·27	28·22	28·06
June	1	27·97	28·63	28·57	28·81	28·78	29·77	29·75		27·88	28·45	28·27	28·37
	2	28·92	28·92	29·11		28·86		29·21	27·94	27·77	28·36	28·00	27·98
	4												
	5	28·58	28·54	28·13	28·60	28·60	28·94	28·96			28·51	28·66	

N. B.—From and after May 10 the group connects with Group II, reobserved.

Results for latitude of astronomic station, Lafayette Park, San Francisco—Continued.

$\varphi = 37^\circ\ 47' +$ tabular seconds.

PAIRS OF GROUP II, REOBSERVED.

Date.	1		2	3		4		5	6	7	8	
	5821		5883	5988	5999	6087		*L* 33291	*L* 33521	6235	6322	6341
	G 2432	*G* 2433	5937	6052		6109	*G* 2494	6162	6203	6268	6372	
1892.												
May 10		28·49	28·51			28·21		28·64	28·71	28·40	28·28	28·18
11	28·80		28·58	28·63	28·55	27·98			28·59	28·13	28·58	28·48
12	28·36	28·34	28·47	28·00	28·41	27·60		28·52	28·33	28·49	28·63	28·36
13	29·02	28·49	28·42	28·73	28·58	27·99		28·92	28·31	28·24	28·40	28·56
15	28·96	28·54	28·88	28·62	28·48	28·08		28·74	28·88	28·53	29·01	28·73
16	29·02	28·42	28·07	28·08	28·16	27·83		28·70	28·43	28·46	28·56	
17	28·90	28·66	29·13	28·69	28·42	28·11		28·76	28·63	28·38	28·56	28·50
18	28·94	28·90	28·51	28·64	28·51	28·16		28·66	28·70	28·39	28·27	27·83
19			28·43		27·89	27·32		28·84	28·51			
20	29·16	28·75		28·10	28·22	27·47		28·38	28·58	27·89	28·81	28·46
21												
28	28·75		28·55	27·82	27·92	27·95		28·53	28·16	28·33	28·30	28·55
29	29·02	28·52	28·62	28·29	28·21	27·68		28·86	28·69	28·21	28·44	27·97
30	28·82	28·46	28·59	28·23	28·13	27·74		28·90	28·73	28·34	28·08	28·10
31	28·83	28·22	28·37	28·62	28·46		28·63	28·58	28·72	28·29	28·65	28·87
June 1	28·59	28·57	28·44	28·28	27·90	27·94		28·45	29·03	28·23	29·17	29·46
2	28·33	27·91	28·09	28·02	27·80	27·89		28·80	28·26		28·51	28·48
4	28·77	28·37	28·54	28·00	27·76	27·76		28·70	28·09	28·40	28·56	28·40
5	28·67	28·52	28·47	28·28	28·32	28·16		28·69	28·19	27·93	28·29	28·07
July 29	29·09	28·82	28·70		28·46	27·76	28·10	28·55	28·26	28·56	28·35	28·29

30	28·69	28·69	28·29	28·04	28·26	27·54		28·35	28·58	28·23	28·64	28·37
Aug. 2			28·69		27·75	28·09		28·88	28·14	27·87		28·38
3			28·34	28·30	28·43	28·05		29·26	28·68	28·49	28·11	27·90
4	27·32	28·10	28·26	28·58	28·45	28·23		28·77	28·85	27·99	28·19	28·09
8		28·73	28·56			27·83		28·50	28·50	28·29	28·46	28·84
9				28·02	28·03	27·85			28·37	27·56		28·26
10												
11	29·05	28·92	28·81	28·50	28·48	27·75	28·36	28·68	28·72	28·59	28·77	28·76
12	28·79	28·21	27·86	28·17	28·15	27·60		28·21	28·41	28·04	28·10	28·02
14	28·31		28·60	28·08	28·15	27·88		28·40	28·44	28·35		28·44
15		27·63	28·19	28·37	28·04	27·82		28·03	27·86	28·27	27·94	28·40
16	28·46		28·56	27·91	28·22			28·13	28·[illegible]7	28·75	28·33	28·72
17	29·70		27·81	27·70	27·86	27·70		28·49	28·10	28·16	28·39	28·20
18	28·93		28·06	28·22	28·18	27·17		28·38	28·72	27·80	28·51	28·65
19	28·80	•28·96	28·28	27·89	28·28	28·05		28·36	28·71	28·04	28·13	27·90

Results for latitude of astronomic station, Lafayette Park, San Francisco—Continued.

$\varphi = 37^{\circ}\ 47' +$ tabular seconds.

PAIRS OF GROUP II, REOBSERVED—Completed.

Date.	9		10	11				12		13	14	
	6397 L 35333 6496		6528 6555	6579		L 36483		6730 6734 6758		6824 6866	L 38237 6901 6924	
				6637	L 36249	6637	L 36249					
1892.												
May 10	28·88	28·81	28·93	28·47				28·20	28·44	28·86	28·38	29·11
11	28·69	28·70	28·56	28·42	27·92			28·38	28·76	28·95	28·35	28·98
12	28·98	28·78	28·26	28·18	27·99			28·95	28·67	28·65	28·01	28·94
13	28·92	28·95										
15	29·02	28·85	28·61	28·49	28·13			28·46	29·17	28·85		29·18
16	28·67	28·68	28·69	27·65	27·65			28·04	28·02	28·45	28·18	28·97
17	28·66	28·62	28·61	28·30	28·17			28·62	28·72	28·98	28·61	29·06
18	28·85	29·16	28·10	27·99	27·74				28·59	28·36		28·73
19	28·37	28·65	28·67	28·41	28·12			28·99	29·30	28·79		29·19
20	28·93	28·96	28·52	28·31	28·04			28·33	28·45	28·92	28·27	29·20
21												
28	28·65	28·37	28·35	28·14	27·33			29·22	28·65	28·68	27·89	28·64
29	28·77	28·70	28·20	28·42	28·09			28·37	28·42	29·22		
30	28·68	28·82	28·80	28·26	28·20			28·28	28·37	28·80	28·13	29·11
31	28·90	28·61	28·73	28·69	28·60				28·55	29·04	28·39	29·12
June 1	28·86	28·52	28·91	28·86	28·01				28·05	28·50	28·18	29·08
2	28·68	28·58	28·68	28·42	28·04			27·99	28·06	28·42	27·82	28·73
4	29·18	28·90	28·27	28·48	27·96					28·58	28·34	29·01
5	28·67	28·38	28·77	28·67	27·97			28·56	28·56	28·52		28·93
July 29	29·01	28·86										

30	28·76	28·89	27·92	28·08	28·07	30·38	30·35		28·55	28·47	28·44	29·16
Aug. 2												
3	28·84	28·54	28·74	28·87	27·82	30·53	29·48		29·25	28·26	28·08	28·93
4	28·84	28·61	27·79	28·50	28·74				28·17	28·16	28·16	28·81
8	28·94	28·85	28·42	28·42	28·34	30·69	30·60			28·65	28·17	28·88
9	29·09	29·27	28·78	27·75		30·36			28·09	28·73	27·89	28·76
10												
11	29·06	28·78	28·41	28·33	28·54	30·61	30·81		28·33	28·61	27·78	28·99
12	28·42	28·20	28·44	27·60	27·85	29·82	30·04	28·56	28·47	28·85	28·25	29·01
14	29·03	28·76	29·22	27·91	27·77	30·08	29·93	28·00	28·91	29·10	28·31	28·79
15	28·77	28·61	28·67	28·92	28·36	30·75	30·19	28·44	28·65	28·89		
16	29·37	29·22	28·91	28·49	28·10	29·83	30·45	28·50	28·38	28·63	28·13	28·80
17	28·45	28·43	28·78	27·90	27·77	30·36	30·25	27·99	28·13	28·59	27·66	28·63
18	28·56	28·38	28·31	28·21	28·12	30·20	30·12		28·53	28·47	28·00	29·02
19	29·00	29·05	28·30	27·73		29·95			28·58	28·58		

N. B.—From and after July 30 Group II connects with Group III, reobserved.

Results for latitude of astronomic station, Lafayette Park, San Francisco—Continued.

$\varphi = 37^\circ\ 47' +$ tabular seconds.

PAIRS OF GROUP III, REOBSERVED.

Date.	1	2	3		4	5		6	7	8		9	10	11	12	13	14	
	6957 6983	G 3151 7067	7146 7176	7149 7176	7194 7219	L 41026 7306	7368 7306	7402 L 41554	7437 7488	7522 7508	7547 7508	L 42627 7643	7678 7720	7788 F 4151	7843 7906	7961 7997	8039 8051	8077 8051
1892. July 30			28·92	28·39	28·71	28·23	28·25	28·05	28·50	27·91	27·90	28·44	29·01	28·49	28·00	28·32	29·00	
Aug. 2																		
3	27·80	28·26		28·37	28·60	28·53				28·13	28·00	27·85	28·92	28·69	27·85	28·16	28·75	
4	27·95	27·97		28·88	28·48	28·15	28·16	28·38	28·86	28·02	27·94	28·09	28·42	28·19	28·30	28·54	28·18	
8	28·13	28·45		28·51	28·63													
9	28·30	28·42		28·58	28·80	28·25	28·14	28·11	28·68	28·27	28·02	28·18	28·96	28·57				
10					28·61	28·48	28·34	28·59	29·04	28·33	28·36	28·06	28·71	28·38		28·15	28·70	
11	28·34	28·40	28·65	28·37		28·14	28·07	28·68	28·91					28·27	28·18	28·52		
12	28·07	28·44		28·84				27·65	29·26	28·31	27·88	28·12	28·22	28·38	28·32	28·15	28·43	
14	27·90	28·46		28·05	28·60	28·55	28·23	28·67	28·70	28·51	28·38	28·03	28·61	28·11	28·53	28·38		
15	28·26	28·36		28·49	29·06	28·16	27·80	28·46	28·83	28·30	28·15	27·96	29·05	28·64	28·17	28·63	28·80	28·99
16	28·26	28·47	28·24		29·05	28·17	28·27	28·41	28·63	28·85	28·61	28·13	28·59	28·48	28·23	28·85	28·93	28·76
17	27·50	27·53		28·27	28·87	27·70	27·60	27·77	28·87	28·02	28·06	27·50	28·62	28·19	28·13	28·27	28·68	28·57
18	28·15	28·15		28·16	28·91	28·74	28·68	28·77	28·95	27·94	27·85	27·57	28·19	27·85	27·70	28·44	28·04	28·21
19		28·47		27·85	27·99	27·90	27·71	28·51		28·55	28·64	27·97	28·17	28·40		28·71		

N. B.—Groups II and III, reobserved, are connected on the above dates.

COMBINATION OF THE PRECEDING RESULTS FOR VARIATION OF LATITUDE.

The method of combination must be such as not to interfere with any change in latitude that may be indicated by the observations, no matter what the character of the change may be. To secure this the same method was followed as had been used in the case of Berlin, Rockville, Waikiki, and other places. The groups had been selected by the observer; but they differ from the groups at other stations in these respects: They contain a great many more pairs; they include doublets and triplets and other combinations, and when the same group is reobserved at the expiration of a year additional pairs or combinations of pairs are introduced. This greatly added to the labor of reduction, and demanded a deviation from the method of selecting for each group all the *full* series only from which to form the mean latitude of each pair in order to reduce the star places to a uniform system of declination. Were we to depend only on the *complete* series in a group to effect this reduction, their number would be so *small* as to defeat the object in view. In fact, there is a group (VI) in which there is not a single night in which all the pairs belonging to the group were observed. The observations on each night are much more liable to occasional omissions of pairs than with a few pairs only to a group, and thus the number of complete series falls below a desired number. In order to increase the number of nights and values from which to reduce the group mean I have admitted all series in which but a *single* omission (of a pair) occurred, and have supplied the corresponding single value by temporary interpolation of a value, namely, the mean of the preceding and the following observed latitude of the pair involved. In this way very full and fair group means were obtained. In the exceptional case of there being an omission at the beginning or at the end of the observed values of a pair, the two nearest (in time) results were used for the interpolation. In cases of doublets and triplets the values received the weights two-thirds and one-half, respectively. There were, however, a great many pairs which, on account of the broken or disadvantageously distributed character of the observations, could not be admitted directly into the group mean. Thus in Group I there are 10 such pairs. The reduction to the group declination system for these pairs was effected as follows: Each value of one of the pairs was directly compared with the mean reduced value deduced from all the group pairs observed on the respective day, and the differences so obtained from all the days of observations for that pair furnished the reduction to the group system.

The following table contains the number of pairs of stars and of series from which resulted the group mean:

Group.	Pairs.	Series.	Seconds of group mean.	Total number of observations in group.
I	14	37	28·42	278 and 924
II	17	53	28·42	771 " 627
III	16	27	28·28	528 " 192
IV	17	21	28·24	497
V	21	15	28·11	682
VI	19	16	28·17	811
VII	23	23	28·44	789
VIII	18	14	28·47	669

The resulting reductions to the values for latitude by each pair of stars, in order to refer the group results to a uniform system, are as follows:

Group I.		Group II.		Group III.		Group IV.	
Pair.	Reduction.	Pair.	Reduction.	Pair.	Reduction.	Pair.	Reduction.
	″		″		″		″
1	− ·17	1_2	− ·25	1	+ ·41	1	+ ·16
2	− ·21	2	− ·18	2	+ ·04	2	− ·34
3	+ ·20	3_1	+ ·03	3_2	− ·04	3_1	+ ·41
4	+ ·44	3_2	+ ·11	4	− ·51	3_2	+ ·33
5	+ ·01	4_1	+ ·75	5_1	+ ·20	4_1	+ ·15
6_1	− ·54	5	·00	5_2	+ ·34	4_2	− ·35
7_3	+ ·08	6	− ·32	6	+ ·14	5	− ·46
8	− ·34	7	+ ·35	7	− ·46	6	+ ·23
9	− ·20	8_1	+ ·05	8_1	− ·14	7	.00
10	·00	8_2	+ ·22	8_2	− ·03	8	− ·24
11_1	− ·17	9_1	− ·12	9	+ ·38	9_1	+ ·15
12_1	+ ·50	9_2	− ·13	10	− ·35	9_2	+ ·23
13_1	− ·03	10	− ·01	11	+ ·09	10	+ ·03
14	+ ·39	11_1	+ ·22	12	+ ·28	11	− ·49
		12_2	− ·17	13	− ·05	12	+ ·02
		13	− ·33	14_1	− ·30	13	− ·18
		14	− ·16			14	+ ·38
Extra pairs.	″	*Extra pairs.*	″	*Extra pairs.*	″		
6_2	−1·23	1_1	−0·37	3_1	−0·15		
7_1	+0·16	4_2	+0·02	14_2	−0·30		
7_2	+0·16	11_2	+0·42				
7_4	−0·10	11_3	−1·87				
7_5	−0·57	11_4	−1·78				
11_2	−0·29	12_1	+0·02				
11_3	−0·50	14_1	+0·35				
11_4	−0·61						
12_2	+0·61						
13_2	+0·09						

Group V.		Group VI.		Group VII.		Group VIII.	
Pair.	Reduction.	Pair.	Reduction.	Pair.	Reduction.	Pair.	Reduction.
	″		″		″		″
1	+ ·49	1_1	— ·07	1_1	— ·10	1	— ·09
2	— ·42	2_2	+ ·21	1_2	·00	2	+ ·47
3	+ ·44	3	+ ·02	2_1	+ ·18	3	·00
4	— ·04	4	+ ·50	2_2	+ ·20	4	— ·23
5	— ·34	5_1	— ·09	3	+ ·26	5	+ ·09
6_1	— ·27	5_2	+ ·31	4	+ ·22	6	— ·16
6_2	+ ·15	6	+ ·23	5_1	+ ·33	7	+ ·17
7	+ ·04	7_1	— ·55	5_2	+ ·07	8	+ ·29
8	— ·18	8_2	+ ·66	6	+ ·13	9	+ ·22
9_1	— ·49	9	+ ·20	7	— ·33	10_1	— ·53
9_2	—1·10	10	— ·96	8_1	+ ·23	10_2	— ·42
10	— ·34	11	— ·10	8_2	— ·34	11_1	+ ·04
11	+ ·54	12_1	+ ·11	9	— ·40	11_2	+ ·05
12	+ ·18	12_2	+ ·13	10_1	— ·25	12	— ·04
13	+ ·23	15_1	+ ·24	10_2	+ ·12	13	— ·21
14	+ ·12	15_2	+ ·17	11_1	— ·14	14_1	— ·44
15	+ ·06	16	— ·05	11_2	+ ·25	14_2	+ ·15
16	+ ·20	17	— ·31	12_1	— ·20	15	+ ·24
17_1	— ·38	18_1	— ·34	12_2	+ ·17		
17_2	+ ·45			13	— ·17		
18	+ ·20			14	+ ·04		
				15	+ ·02		
				16	— ·02		
		Extra pairs.	″	*Extra pairs.*	″		
		1_2	—0·89	2_3	—0·03		
		2_1	+0·03	2_4	+0·09		
		7_2	—0·48	10_3	+0·15		
		8_1	+0·89	11_3	+0·24		
		13_1	—0·18				
		13_2	—0·55				
		14	+0·36				
		18_2	—0·47				

These corrections to the results by each pair and for each group were next applied, forming a new general table of values for latitude, and referring to a uniform declination system in each group.*

The probable error of observation of a single result for latitude may now be deduced by comparing on any one day each value with the daily mean of all values within the group. Let these differences be Δ_1 Δ_2 Δ_3...... then the probable error of observation

$$e_0 = 0{\cdot}675\sqrt{\frac{\Sigma\Delta^2}{n-n_1}}$$

where n = whole number of observations or Δ's, and n_1 the number of means or nights involved. Taking at random about 100 values in each

* This table remains in MS.

groups, we get the following results for e_0 separately for the two instruments:

Date	June, 1891	July, 1891	July, Aug., Sept., 1891	Sept., 1891	Oct., Nov., 1891	Nov., 1891
Group	I	II	III	IV	V	VI
No. of diff's.	105	105	112	102	100	110
Prob. error e_0	$\pm 0''{\cdot}38$	$\pm 0''{\cdot}36$	$\pm 0''{\cdot}26$	$\pm 0''{\cdot}25$	$\pm 0''{\cdot}24$	$\pm 0''{\cdot}19$

Average for Zenith Telescope No. 1 from 634 values $\pm 0''{\cdot}28$.

Date	Jan., 1892	Jan., Feb., Mar., 1892	Mar., Apr., 1892	May, 1892	July, Aug., 1892	Aug., 1892
Group	VI	VII	VIII	I_1	II_1	III_1
No. of diff's.	100	108	106	119	111	99
Prob. error e_0	$\pm 0''{\cdot}21$	$\pm 0''{\cdot}19$	$\pm 0''{\cdot}17$	$\pm 0''{\cdot}18$	$\pm 0''{\cdot}19$	$\pm 0''{\cdot}19$

Average for Zenith Telescope No. 1 from 643 values $e_0 = \pm 0''{\cdot}19$.

These numbers, besides giving a general idea of the accuracy of the observations, plainly show the superiority of the second instrument over the first, which is due to the improvements mentioned in detail on a preceding page.

Tabulation of the daily mean values after their reduction to the mean declination system of their respective group, together with the number of pairs of stars observed each night.

[In this table the connection of the groups is plainly shown.]

LATITUDE OF SAN FRANCISCO, CAL.

[Daily means of values reduced to group system. $\varphi = 37° 47' 20'' +$ tabular quantity.]

Date.	Group I.		Group II.		Group III.	
1891.	″		″			
May 27	8·20	2	8·45	10		
28	8·27	13	8·38	16		
29	8·64	7	8·61	14		
30	8·02	1	8·36	17		
31	8·44	11	8·47	16		
June 1	8·38	7	8·62	11		
3	8·50	12	8·14	8		
5	8·16	13	8·46	16		
6	8·46	14	8·39	17		
7	8·41	13	8·40	16		
8	8·48	13	8·36	17		
12	8·29	7	8·42	17		
13	8·16	12	8·60	17		
14	8·40	12	8·34	15		
17	8·29	13	8·21	16		
18	8·31	5				
19	8·19	13	8·34	17		
20	8·33	12	8·39	17		
21	8·10	13	8·16	17		
22	8·29	7	8·15	16		
24	8·15	3				
25	8·48	13	8·36	16		
26	8·57	11	8·48	17		
27	8·56	13	8·79	16		
28	8·67	12	8·76	16		
29	8·75	12	8·64	16		
30	8·64	10	8·47	17	″	
July 1	8·43	4	7·76	4		
4			8·23	17	8·27	15
5			8·53	17	8·63	16
6			8·38	16	8·40	14
7			8·45	17	8·24	15
9			8·80	5		
10			8·56	16	8·60	15
11			8·36	17	8·26	16
12			8·56	17	8·25	3
14			8·05	14		
17			8·30	18	8·47	15
18			8·60	15		
19			8·48	15	8·45	16
20			8·44	16	8·48	16
21			8·40	15	8·37	16
23			8·16	7		
24			8·10	18		
25			8·42	18	8·37	16
30			8·35	6	8·33	8
31			8·27	14	8·29	15
Aug. 3			8·11	12	7·90	13
5			8·13	10	8·26	14
6			8·01	10	8·33	16
7			7·87	9	8·36	16
10			8·16	12		
11			8·24	16	8·17	14
12			8·10	16	8·31	16
13			8·16	16	7·73	15

Daily means of values reduced to group system—Continued.

Date.	Group III.		Group IV.		Group V.		Group VI.	
1891.	″		″		″		″	
Aug. 15	8·12	14	8·40	16				
16	7·94	15	8·35	8				
17	7·94	6						
20	8·12	14	8·50	16				
21	8·11	10	8·33	17				
22	8·34	16	8·09	2				
31	7·96	12	8·56	12				
Sept. 5	8·28	9	8·18	4				
6	8·24	12	8·24	16				
7	7·96	15	8·45	10				
8	8·22	6	9·22	4				
9	7·88	13	8·33	14				
10	8·00	15	7·98	14				
12	8·17	7	8·24	6				
15	7·74	4						
16	7·99	16	8·40	17				
18	8·05	13	8·32	16				
19	7·95	12	8·13	17				
20	8·31	3						
22	8·18	16	8·21	16				
23			8·24	17	8·34	18		
24			8·12	15	8·15	14		
25			7·88	3				
27			8·12	17	8·05	18		
28			8·03	16	8·11	20		
29			7·96	16	7·96	19		
30			8·17	16	7·94	17		
Oct. 1			8·24	16	8·11	20		
2			8·30	17	8·22	16		
3			8·23	16	8·06	18		
4			8·11	16	8·20	19		
5			8·18	17	8·41	15		
10			8·37	15	8·33	20		
11			8·24	16	8·29	19		
16			8·21	16	8·22	18		
17			7·81	13				
21			8·52	16				
24			8·30	9				
26			8·51	3				
28					7·76	1		
29			8·14	15	8·06	18		
30			8·22	7				
Nov. 5					8·07	11	8·09	13
6					8·16	15	8·33	13
7					7·97	20	8·10	18
8					8·15	20	8·13	18
9					8·20	19	8·33	17
11					7·88	3	8·29	17
12					8·02	16	8·13	17
13					8·20	19	8·04	18
14					8·16	20	8·08	17
15					8·17	21	8·11	18
16					8·19	20	8·27	18
17					8·02	21	8·08	17
19					8·39	17	8·14	14
20					8·13	16	8·19	18
21					8·14	21	8·28	17
22					8·19	20	8·02	18
23					7·94	21	8·09	18
24					7·94	19	8·00	19
25					8·01	20	7·93	13

Daily means of values reduced to group system—Continued.

Date.	Group V.		Group VI.		Group VII.		Group VIII.		Group IX.	
1891.	″		″		″		″		″	
Nov. 26	8·01	21	8·04	18						
27	7·99	18	7·96	7						
Dec. 1	8·08	20	8·13	19						
2*	8·24	14	8·28	6						
13			8·27	10						
16			8·16	9						
17			8·11	7	8·66	10				
19			8·27	20	8·38	23				
20			8·28	16	8·54	22				
21			8·33	20	8·42	22				
23			8·03	21	8·16	22				
24			8·15	22	8·31	23				
25			8·24	6						
27			8·31	14						
30			8·29	21	8·36	23				
1892.										
Jan. 2			8·23	14	8·82	1				
8			8·39	8	8·37	6				
10			8·14	21	8·58	10				
11			8·23	22	8·51	21				
12			8·25	6	8·38	21				
14			8·38	24	8·42	23				
15			8·11	7						
16			8·30	22	8·53	22				
17			8·20	24	8·48	23				
18			8·43	26	8·63	23				
19			8·18	26	8·41	23				
20			8·45	17	8·71	5				
21			8·46	23	8·40	25				
22			8·34	14	8·57	24				
23			8·24	23	8·47	25				
27					8·56	23	8·32	12		
28					8·50	27	8·48	15		
30					8·54	27	8·44	16		
Feb. 1					8·37	26	8·38	15		
2					8·32	25	8·41	17		
6					8·47	25	8·33	16		
7					8·19	2	8·43	13		
8					8·44	27	8·27	17		
9					8·46	27	8·55	17		
10					8·43	27	8·36	16		
28					8·79	16	8·48	17		
Mar. 2					8·59	25	8·45	15		
3					8·62	23	8·59	8		
4					8·37	27	8·61	17		
5					8·32	25	8·51	18		
6							8·38	18	8·37	20
7					8·35	14				
8					8·38	26	8·54	18	8·32	5
12							8·27	5	8·34	1
15							8·51	14	8·68	6
20							8·60	16	8·47	18
21							8·46	16	8·68	2
22									8·40	21
23							8·56	9	8·44	18
24							8·41	13	8·41	19
25							8·72	11	8·69	11
27							8·38	16	8·51	12
29							8·52	12	8·62	1
30							8·51	16	8·38	21
31							8·63	4		
Apr. 2							8·67	15	8·53	21

* Last day of observation with Zenith Telescope No. 1.

Daily means of values reduced to group system—Continued.

Date.		Group VIII.		Group I_1.		Group II_1.		Group III_1.	
1892.		″		″		″		″	
Apr.	3	8·40	17	8·46	23				
	4	8·30	17	8·33	23				
	5	8·82	16	8·56	22				
	6	8·82	3						
	7	8·59	1						
	9	8·60	17	8·60	20				
	11	8·60	15	8·51	21				
	13	8·77	6						
	15	8·58	12	8·50	12				
	16	8·54	18	8·48	19				
	17	8·46	16	8·50	21				
	18	8·33	13	8·38	23				
	19	8·62	15	8·37	20				
	21	8·41	17	8·52	21				
	22	8·50	16	8·55	14				
	24	8·61	16	8·51	21				
	25	8·52	17	8·52	22				
	26	8·42	16	8·39	23				
	27	8·64	2	8·54	18				
	29	9·13	1						
May	2			8·67	17				
	4	8·19	2	8·62	13				
	6	9·02	2	8·62	18				
	7	9·28	2	8·55	20				
	8			8·46	13				
	10			8·67	22	8·58	17		
	11			8·40	22	8·57	19		
	12			8·23	22	8·45	21		
	13			8·49	20	8·59	13		
	15			8·68	20	8·72	20		
	16			8·30	24	8·35	20		
	17			8·41	20	8·65	21		
	18			8·48	24	8·48	19		
	19					8·54	14		
	20			8·51	24	8·52	20		
	21			8·95	1				
	28			8·46	15	8·38	20		
	29			8·55	22	8·47	19		
	30			8·46	24	8·48	21		
	31			8·45	21	8·63	20		
June	1			8·53	23	8·58	20		
	2			8·44	22	8·28	20		
	4					8·46	19		
	5			8·55	18	8·44	20		
July	29					8·53	13		
	30					8·43	22	8·37	15
Aug.	2					8·39	7		
	3					8·52	20	8·30	13
	4					8·35	20	8·28	16
	8					8·60	18	8·40	4
	9					8·38	15	8·41	13
	10							8·42	12
	11					8·63	23	8·48	11
	12					8·26	23	8·31	13
	14					8·45	21	8·40	15
	15					8·39	20	8·46	17
	16					8·50	21	8·50	17
	17					8·28	22	8·11	17
	18					8·36	21	8·23	17
	19*					8·36	18	8·24	12

* Close of series.

COMPARISON OF GROUP MEANS OR REDUCTION OF VALUES OF ONE GROUP TO VALUES OF THE ADJACENT GROUP DURING THE SAME PERIOD OF TIME.

By means of the preceding table, which contains the daily mean values, together with their weights—i. e., the number of pairs observed on the respective dates and within the group—we get the weighted means of simultaneous observations (on the same night) for any two adjacent groups during the period of their connection, as shown by the horizontal bars. These means, together with their relative weights (number of observations), are as follows:

Comparison of group systems.

I	II	III	IV	V	VI
″	″	″	″	″	″
28·40(278)	28·42(392)				
	28·31(379)	28·32(300)			
		28·07(228)	28·33(205)		
			28·18(292)	28·16(270)	
				28·10(412)	28·13(368)
					28·27(424)*

VII	VIII	I_1	II_1	III_1
″	″	″	″	″
*28·45(397)				
28·46(392)	28·44(265)			
28·37(40)		28·36(25)		
	28·52(440)	28·49(567)		
		28·47(357)	28·51(343)	
			28·42(271)	28·34(192)

* Continued on same line.

ESTABLISHMENT OF THE OBSERVATION EQUATIONS.

The sum of the successive differences between the groups, when, after the lapse of a year, the same system or Group I has been reobserved, should equal zero. Likewise, the differences for any one group when reobserved should be the same, and when overlapping the difference for the three groups must satisfy the condition that the sum of two differences must equal the third. Let $v_i - v_{i-1}$ be the correction to the observed difference between the means of a group (i) and the preceding group ($i-1$). The relative weights to the differences are given by a combination of the above numbers (in parenthesis) of pairs involved; whence we adopt for the reciprocal the value

$$u = \frac{1}{P} = \frac{10\,(p_1 + p_2)}{p_1\,p_2}$$

The group differences and their respective weights are as follows:

Observed differences.	$u=P^{-1}$	Observed differences.	$u=P^{-1}$
″		″	
II — I = + 0·025	·061	and I_1 — VIII = — 0·035	·040
III — II = + 0·004	·060	II_1 — I_1 = + 0·033	·057
IV — III = + 0·253	·093	III_1 — II_1 = — 0·078	·089
V — IV = — 0·025	·071	I_1 — VII = — 0·010	·650
VI — V = + 0·029	·051		
VII — VI = + 0·179	·049		
VIII — VII = — 0·020	·063		

The observation equations are:

$$\begin{cases} v_{2-1}+v_{3-2}+v_{4-3}+v_{5-4}+v_{6-5}+v_{7-6}+v_{8-7}+v_{1-8} = +\cdot 41 \\ v_{2-1} - v'_{2-1} = -\cdot 01 \\ v_{3-2} - v'_{3-2} = +\cdot 08 \\ v_{8-7}+v_{1-8} - v_{1-7} = -\cdot 04 \end{cases}$$

Forming the correlate equations and the normal equations, having regard to the above weights, we get

$$\begin{aligned} &+\cdot 488\,k_1 + \cdot 061\,k_2 + \cdot 060\,k_3 + \cdot 103\,k_4 - \cdot 41 = 0 \\ &+\cdot 061\,k_1 + \cdot 118\,k_2 + \cdot 01 = 0 \\ &+\cdot 060\,k_1 + \cdot 149\,k_3 - \cdot 08 = 0 \\ &+\cdot 103\,k_1 + \cdot 753\,k_4 + \cdot 04 = 0 \end{aligned}$$

Solving, we find

$$\begin{cases} k_1 = +\ \cdot 9287 \\ k_2 = -\ \cdot 5648 \\ k_3 = +\ \cdot 1629 \\ k_4 = -\ \cdot 1802 \end{cases}$$

and the corrections

$$\begin{aligned} v_{2-1} &= +\ \cdot 0222'' \\ v_{3-2} &= +\ \cdot 0655 \\ v_{4-3} &= +\ \cdot 0864 \\ v_{5-4} &= +\ \cdot 0659 \\ v_{6-5} &= +\ \cdot 0474 \\ v_{7-6} &= +\ \cdot 0455 \\ v_{8-7} &= +\ \cdot 0472 \\ v_{1-8} &= +\ \cdot 0299 \\ v'_{2-1} &= +\ \cdot 0322 \\ v'_{3-2} &= -\ \cdot 0145 \\ v'_{1-7} &= +\ \cdot 1171 \end{aligned}$$

and the adjusted reductions become:

$$\begin{aligned} R_{2-1} &= -\ \cdot 003'' \\ R_{3-2} &= +\ \cdot 062 \\ R_{4-3} &= -\ \cdot 167 \\ R_{5-4} &= +\ \cdot 091 \\ A_{6-5} &= +\ \cdot 018 \\ R_{7-6} &= -\ \cdot 133 \\ R_{8-7} &= +\ \cdot 067 \\ R_{1-8} &= +\ \cdot 065 \end{aligned}$$

So far as the variation of latitude is concerned, it is immaterial which group may be taken as the initial one. Taking the first, we get the final reductions of the groups as follows:

Group	″	Group	″
I	·00	V	— ·02
II	·00	VI	·00
III	+ ·06	VII	— ·13
IV	— ·11	VIII	— ·06

Applying the corrections to the respective group means of the last table of latitude results, and combining the two values of each day with regard to their weight, we find the final daily mean values as tabulated below:

Resulting latitude of San Francisco.

[Daily means between May, 1891, and August, 1892, 37° 47′ 20″ + tabular quantities.]

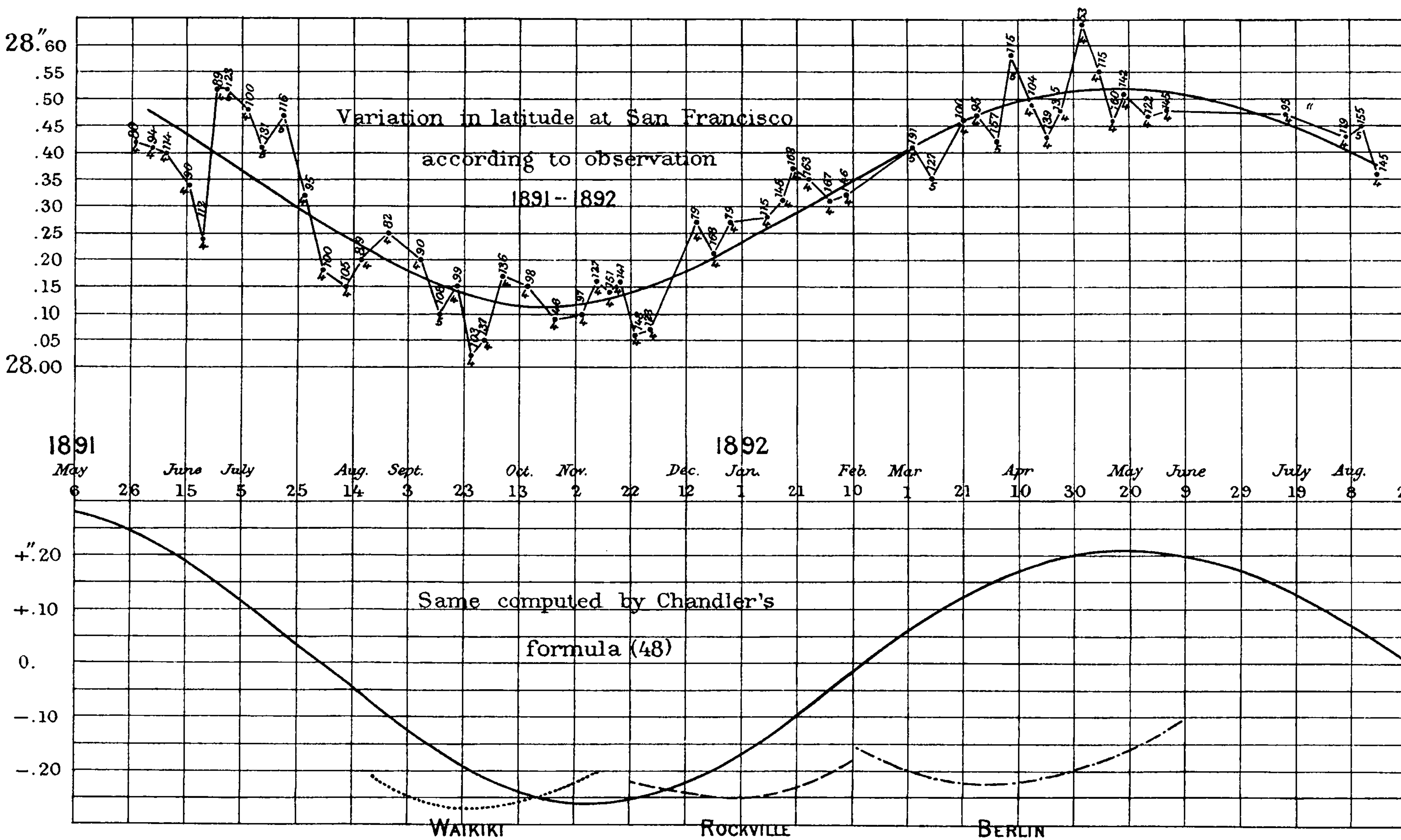
Date.	Latitude.	No. of obs.	ϕ	n	Date.	Latitude.	No. of obs.	ϕ	n
1891.	″				1891.	″			
May 27	8·41	12			Aug. 10	8·16	12		
28	·33	29	8·42	80	11	·24	30	8·15	105
29	·62	21			12	8·24	32		
30	·34	18			13	7·98	31		
31	·46	27			15	8·24	30		
June 1	·53	18	8·41	94	16	·08	23	8·20	89
3	·36	20			17	·00	6		
5	·33	29			20	·29	30		
6	·42	31			21	·20	27		
7	·40	29	8·40	114	22	·35	18	8·25	82
8	·41	30			31	·24	24		
12	·38	24			Sept. 5	·26	13		
13	·42	29			6	·20	28		
14	·37	27	8·34	90	7	·15	25	8·20	90
17	·25	29			8	·61	10		
18	·31	5			9	8·09	27		
19	·28	30			10	7·97	29		
20	·37	29	8·24	112	12	8·18	13		
21	·13	30			15	7·80	4	8·10	108
22	·19	23			16	8·17	33		
24	·15	3			18	·16	29		
25	·41	29	8·52	89	19	·02	29		
26	·51	28			20	·37	3	8·15	99
27	·69	29			22	·17	32		
28	·72	28			23	·23	35		
29	·69	28			24	8·06	29		
30	·53	27	8·52	123	25	7·77	3	8·02	103
July 1	·10	8			27	8·02	35		
4	·28	32			28	8·01	36		
5	·59	33			29	7·90	35		
6	·42	30	8·48	100	30	7·99	33	8·05	137
7	·38	32			Oct. 1	8·11	36		
9	·80	5			2	·19	33		
10	·61	31			3	·08	34		
11	·34	33			4	·10	35	8·17	136
12	·52	20	8·41	131	5	·22	32		
14	·05	14			10	·29	35		
17	·40	33			11	·21	35		
18	·60	15			16	8·15	34	8·15	98
19	·50	31			17	7·70	13		
20	·49	32	8·47	116	21	8·41	16		
21	·42	31			24	·19	9		
23	·16	7			26	8·40	3	8·09	46
24	·10	18			28	7·74	1		
25	·42	34	8·32	95	29	8·04	33		
30	·37	14			30	·11	7		
31	·31	29			Nov. 5	·07	24	8·10	97
Aug. 3	·03	25			6	·23	28		
5	·24	24	8·18	100	7	·02	38		
6	·24	26							
7	·22	25							

Resulting latitude of San Francisco—Continued.

[Daily means between May, 1891, and August, 1892, 37° 47′ 20″ + tabular quantities.]

Date.	Latitude.	No. of obs.	ϕ	n
1891.	″			
Nov. 8	8·13	38		
9	·25	36	8·16	127
11	·23	20		
12	·07	33		
13	·11	37		
14	·11	37	8·14	151
15	·13	39		
16	·22	38		
17	·04	38		
19	·27	31	8·16	141
20	·15	34		
21	·19	38		
22	·10	38		
23	8·00	39	8·01	148
24	7·96	38		
25	7·97	33		
26	8·01	39		
27	7·97	25	8·06	123
Dec. 1	8·09	39		
2	·24	20		
13	·27	10		
16	·16	9	8·27	79
17	·36	17		
19	·26	43		
20	·36	38		
21	·31	42	8·21	168
23	·03	43		
24	·17	45		
25	·24	6		
27	·31	14		
30	·26	44	8·27	79
1892.				
Jan. 2	·26	15		
8	·32	14		
10	·24	31	8·28	115
11	·30	43		
12	·25	27		
14	·34	47		
15	·11	7	8·31	145
16	·35	44		
17	·28	47		
18	·46	49		
19	·23	49	8·37	168
20	·48	22		
21	·36	48		
22	·40	38		
23	·29	48	8·36	163
27	·37	35		
28	·39	42		
30	·40	43		
Feb. 1	·27	41	8·31	167
2	·25	42		
6	·31	41		
7	·32	15		
8	·27	44	8·32	146
9	·39	44		
10	·30	43		

Date.	Latitude.	No. of obs.	ϕ	n
1892.	″			
Feb. 28	8·54	33		
Mar. 2	·43	40		
3	·50	31	8·41	191
4	·36	44		
5	·30	43		
6	·35	38		
7	·22	14		
8	·34	49	8·35	127
12	·23	6		
15	·52	20		
20	·50	34		
21	·43	18	8·46	100
22	·40	21		
23	·46	27		
24	·39	32		
25	·67	22	8·47	95
27	·40	28		
29	·47	13		
30	·41	37		
31	·57	4		
Apr. 2	·56	36	8·42	157
3	·41	40		
4	·29	40		
5	·64	38		
6	·76	3		
7	·53	1	8·58	115
9	·57	37		
11	·52	36		
13	·71	6		
15	·51	24	8·49	104
16	·48	37		
17	·46	37		
18	·34	36		
19	·45	35	8·43	139
21	·44	38		
22	·49	30		
24	·53	37		
25	·49	39	8·48	135
26	·38	39		
27	8·54	20		
29	9·07	1		
May 2	8·67	17	8·64	53
4	·55	15		
6	·65	20		
7	·61	22		
8	·46	13	8·55	115
10	·63	39		
11	·48	41		
12	·34	43		
13	·53	33	8·46	160
15	·70	40		
16	·32	44		
17	·53	41		
18	·48	43	8·51	142
19	·54	14		
20	·51	44		


Variation in latitude at San Francisco
according to observation
1891 -- 1892
28.″60
.55
.50
.45
.40
.35
.30
.25
.20
.15
.10
.05
28.00
1891
1892
May 6
June 15
July 5
Aug. 14
Sept. 3
Oct. 13
Nov. 2
Dec. 12
Jan. 1
Feb. 10
Mar 1
Apr 10
May 20
June 9
July 19
Aug. 8
Same computed by Chandler's
formula (48)
+.″20
+.10
0.
−.10
−.20
Waikiki
Rockville
Berlin

Resulting latitude of San Francisco—Continued.

[Daily means between May, 1891, and August, 1892, 37° 47′ 20″ + tabular quantities.]

Date.	Latitude.	No. of obs.	φ	n	Date.	Latitude.	No. of obs.	φ	n
1892.	″				1892.	″			
May 21	8·95	1			Aug. 3	8·44	33		
28	·41	35	8·47	122	4	·35	36	8·43	119
29	·51	41			8	·57	22		
30	·47	45			9	·42	28		
31	·54	41			10	·48	12		
June 1	·55	43	8·48	145	11	·60	34		
2	·36	42			12	·30	36	8·45	155
4	·46	19			14	·45	36		
5	·50	38			15	·45	37		
July 29	·53	13	8·47	95	16	·53	38		
30	·43	37			17	·23	39	8·36	145
Aug. 2	·39	7			18	·33	38		
					19	8·34	30		

Total number of nights of observation, 237, and of individual values for latitude, 6 768, contracted into 57 mean values, which are shown on the accompanying diagram by the zigzag line. The scales for this diagram are: For the time scale, 20 days to the centimetre; for the latitude scale, 0″·10 to the centimetre. The numbers attached to the points represent the number of observations condensed into a mean for the given date and are placed above the points; the numbers below them show the number of nights of observation. With but a few exceptions the results of 4 nights were combined, and never more than 5.

The smooth curve passing between the zigzags is the result of computation. It was derived, however, not from the preceding 57 mean values, but from a further condensation of these values to 29 normal values by uniting them two by two. The normals so obtained may be regarded as the outcome of the observations at San Francisco. They are:

No.	Date.	Latitude normals 37° 47′ +	No. of obs.	No.	Date.	Latitude normals 37° 47′ +	No. of obs.
	1891.	″			1891.	″	
1	May 30·8	28·42	174	16	Dec. 25·4	28·24	247
					1892.		
2	June 11·8	·37	204	17	Jan. 12·8	·29	260
3	June 23·0	·38	201	18	Jan. 22·2	·36	331
4	July 3·6	·50	223	19	Feb. 5·2	·32	313
5	July 16·5	·44	247	20	Mar. 6·2	·38	318
6	July 31·8	·25	195	21	Mar. 23·8	·46	195
7	Aug. 14·2	·18	194	22	Apr. 4·6	·50	272
8	Sept. 2·0	·22	172	23	Apr. 17·6	·46	243
9	Sept. 17·6	·12	207	24	Apr. 29·1	·56	188
10	Sept. 28·2	·04	240	25	May 11·5	·51	275
11	Oct. 10·8	·16	234	26	May 22·7	·49	264
12	Oct. 31·0	·10	143	27	June 24·2	·48	240
13	Nov. 12·2	·15	278	28	Aug. 9·2	·44	274
14	Nov. 21·3	·08	289	29	Aug. 17·5	·36	145
15	Dec. 7·6	·16	202				

Average number of observations forming a normal value, 233.

According to Dr. S. C. Chandler's researches* the variation in latitude of a place is dependent on two cycles: One having a dynamical basis, and supposed to be the Eulerian period lengthened out in consequence of the earth not being a rigid body, as explained by Prof. S. Newcomb; the other of an annual period, supposed to depend on climatic conditions existing in the two hemispheres. The first period is subject to slow changes, and its length at present is estimated as 431 days. The other period, of 365¼ days, is supposed less stable in amount as well as in epoch. The resultant of these two periodic fluctuations can be represented by the expression

$$\varphi = \varphi_0 + r_1 \sin\left(\frac{2\pi}{P_1}t + \alpha_1\right) + r_2 \sin\left(\frac{2\pi}{P_2}\ t + \alpha_2\right)$$

where t = number of days from the assumed epoch January 0, 1891, and $\frac{2\pi}{P_1} = \frac{360}{431}$, $\frac{2\pi}{P_2} = \frac{360}{365{\cdot}25}$. For φ_0 we put $37°\ 47'\ 28''{\cdot}33 + x$ and form the 29 observation equations of the form

$$o = \varphi_0 - \varphi + x + \sin m_1 t . y_1 + \cos m_1 t . z_1 + \sin m_2 t . y_2 + \cos m_2 t . z_2$$

where $y_1 = r_1 \cos \alpha_1$ and $y_2 = r_2 \cos \alpha_2$ also $\tan \alpha_1 = \frac{z_1}{y_1}$ and $\tan \alpha_2 = \frac{z_2}{y_2}$

$z_1 = r_1 \sin \alpha_1$ $z_2 = r_2 \sin \alpha_2$ $r_1 = \sqrt{y_1^2 + z_1^2}$ $r_2 = \sqrt{y_2^2 + z_2^2}$

The normal equations become

$$\left\{\begin{array}{l}
29x - 0{\cdot}3260y_1 - 2{\cdot}1330z_1 - 1{\cdot}0590y_2 - 4{\cdot}5440z_2 + 0{\cdot}1500 = 0 \\
\qquad + 13{\cdot}7572y_1 - 0{\cdot}4248z_1 + 7{\cdot}4752y_2 - 11{\cdot}3361z_2 - 2{\cdot}5741 = 0 \\
\qquad\qquad + 15{\cdot}2402z_1 + 10{\cdot}5193y_2 + 8{\cdot}9253z_2 - 1{\cdot}0466 = 0 \\
\qquad\qquad\qquad + 13{\cdot}6656y_2 - 0{\cdot}3927z_2 - 2{\cdot}3283 = 0 \\
\qquad\qquad\qquad\qquad + 15{\cdot}3393z_2 + 1{\cdot}4915 = 0
\end{array}\right.$$

Solving, we get

$x = + 0{\cdot}0042$ Hence:

$y_1 = + 0{\cdot}1713$

$z_1 = + 0{\cdot}0111$

$y_2 = + 0{\cdot}0692$

$z_2 = + 0{\cdot}0259$

$$\varphi = 37°\ 47'\ 28''{\cdot}334 + 0''{\cdot}171 \sin(\cdot 83526\ t + 3{\cdot}7) + 0''{\cdot}074 \sin(\cdot 98563\ t + 20°{\cdot}5) \quad \text{---}(a)$$

The observed values of φ are represented as follows:

No.	Obs'd.	Comp'd.	$C-O$.	No.	Obs'd.	Comp'd.	$C-O$.	No.	Obs'd.	Comp'd.	$C-O$.
	"	"	"		"	"	"		"	"	"
1	28·42	28·48	+ ·06	11	28·16	28·12	— ·04	21	28·46	28·46	+ ·00
2	·37	·45	+ ·08	12	·10	·12	+ ·02	22	·50	·49	— ·01
3	·38	·41	+ ·03	13	·15	·13	— ·02	23	·46	·50	+ ·04
4	·50	·36	— ·14	14	·08	·14	+ ·06	24	·56	·51	— ·05
5	·44	·32	— ·12	15	·16	·17	+ ·01	25	·51	·52	+ ·01
6	·25	·28	+ ·03	16	·24	·21	— ·03	26	·49	·52	+ ·03
7	·18	·23	+ ·05	17	·29	·27	— ·02	27	·48	·49	+ ·01
8	·22	·18	— ·04	18	·36	·29	— ·07	28	·44	·40	— ·04
9	·12	·15	+ ·03	19	·32	·33	+ ·01	29	·36	·37	+ ·01
10	·04	·13	+ ·09	20	·38	·42	+ ·04				

* Gould's Astronomical Journal—a series of articles on the variation of latitude—Vol. XI and XII, 1891–93.

While this representation may be considered quite satisfactory, even here the difference in the performance of the two instruments is perceptible, the earlier values $C-O$ being slightly larger than the later ones.

The maximum and minimum values are found from the condition

$$o = m_1\, r_1 \cos\,(m_1\, t + \alpha_1) + m_2\, r_2 \cos\,(m_2\, t + \alpha_2)$$

hence the minimum occurred for $t = 295$, or for October 22, 1891,

with the latitude $37^\circ\ 47'\ 28''\!\cdot\!118$

and the maximum occurred for $t = 501$, or for May 15, 1892,
with the latitude $37\ 47\ 28\cdot517$

giving a range in latitude of $0''\!\cdot\!40$.

The curve shown in the lower part of the accompanying illustration has been computed for the longitude of San Francisco from Chandler's formula (48),* viz:

$$\varphi-\varphi_0=-0''\!\cdot\!15\cos\,[\lambda+(t-1875,\mathrm{Nov.}1)\,0^\circ\!\cdot\!83527]-0''\!\cdot\!15\cos(\lambda+\odot-320^\circ)$$

where t is the Julian date, November 1, 1875 (being the 2 406 194th day), $\odot$ is the sun's longitude. Computation was made for intervals of 20 days. The time of minimum is about November 9, 1891, and that of maximum about May 17, 1892, with a range of $0''\!\cdot\!26+0''\!\cdot\!21=0''\!\cdot\!47$.

Comparing the two curves, that given by observation and that given by computation, the latter as the result of an extended research of the phenomenon, we find a very close accord in their general run, sufficient to inspire much confidence in the results deduced so far. We find the observed minimum 18 days earlier† and the observed maximum variation 2 days earlier than the times given by formula (48), with an observed range of $0''\!\cdot\!40$ and a computed range of $0''\!\cdot\!47$.

On the same diagram there are also shown parts near the minima of the computed curves by formula (48) for Waikiki, near Honolulu, for Rockville, near Washington, D. C., and for Berlin, Germany. These curves exhibit the gradually decreasing minima as the instantaneous pole of rotation is traveling eastward. The minima for these places occur about the time indicated by a short vertical bar.

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* See Gould's Ast. Jour., Nos. 277, 284, and 307, of Nov. 14, 1893.

† The uncertainty may amount to a week or more.

COMPARISON OF THE RESULTS FOR VARIATION OF LATITUDE AS DEDUCED FROM THE OBSERVATIONS MADE BY THE COAST AND GEODETIC SURVEY AT THE THREE STATIONS, WAIKIKI, SAN FRANCISCO, AND ROCKVILLE, 1891–92.

To bring these results under one view the observed values for latitude at *Waikiki* have been represented by two periodic functions, one of 431, the other of 365¼ days, the same as was done for San Francisco.

The data for Waikiki are taken from page 155, Appendix No. 2, Report for 1892. These, together with the results, are as follows:

No.	Date.	No. of pairs.	No. of nights.	t	Observed ϕ''	Comp'd ϕ''	$C-O$.
	1891.				"	"	"
1	June 11	94	11	162	24·48	24·48	·00
2	24	96	8	175	·50	·43	—·07
3	July 14	99	8	195	·32	·34	+·02
4	29	91	9	210	·21	·27	+·06
5	Aug. 9	100	9	221	·14	·23	+·09
6	21	93	7	233	·21	·18	—·03
7	Sept. 7	112	8	250	·18	·13	—·05
8	27	104	7	270	·09	·10	+·01
9	Oct. 18	90	9	291	·15	·09	—·06
10	Nov. 6	92	8	310	·12	·11	—·01
11	24	100	10	328	·18	·16	—·02
12	Dec. 17	119	11	351	·29	·25	—·04
13	30	91	8	364	·29	·31	+·02
	1892.						
14	Jan. 16	101	10	381	·26	·39	+·13
15	28	91	6	393	·45	·45	·00
16	Feb. 9	94	6	405	·42	·51	+·09
17	22	101	9	418	·55	·57	+·02
18	Mar. 3	101	8	428	·73	·61	—·12
19	16	81	8	441	·70	·65	—·05
20	Apr. 2	89	10	458	·76	·68	—·08
21	22	86	10	478	·67	·69	+·02
22	May 6	96	11	492	·61	·68	+·07
23	22	97	9	508	·65	·63	—·02
24	June 6	96	12	523	·55	·59	+·04
25	21	54	7	538	·56	·53	—·03
	Σ	2368	219				

The equation

$$\varphi = 21^\circ\ 16'\ 24''\cdot372 + 0''\cdot109 \sin\ (\cdot83526\ t + 87^\circ\cdot4)$$
$$+ 0''\cdot236 \sin\ (\cdot98563\ t + 330^\circ\cdot4) \ldots\ldots (b)$$

represents the observations and leaves the differences, computed minus observed, as shown in the last column of the table.

It is plain that the two periodic terms are not fully separated, owing to the shortness of the series, so that we have to take the expression for nothing more than a representation of the observations.

The probable observing error of a single determination for latitude at this station, Assistant Preston, observer, was found as follows:

	Nos.	Prob. error.	
Group I	123	$\pm 0''.24$	Mean from 1206 obser's $e_0 = \pm 0''.21$
III	274	$0''.21$	
V	257	$0''.20$	
VI	311	$0''.21$	
I	241	$0''.19$	

In deducing this value the individual results by each pair were corrected for change in latitude.

The discussion of the *Rockville, Md.*, series apparently brought out some large deviations from a regular variation, and in particular toward the end of the observations there appeared a high value of the latitude about the close of April, 1892, followed by a rapid falling off. That this was not due to any real change in latitude could be taken for granted, but its origin had to be sought in the comparative shortness of the groups and especially in their loose connections.

Returning to the table of the 14 normal values given on page 51 of Appendix No. 1, Report for 1892, which are now represented by a single periodic function of a length of 387 days, which has been taken for the rough interval at this time between two successive minima or maxima.

The formula

$$\varphi = 39^\circ\ 05'\ 10''.45 + 0''.19 \sin\ (0.935\ t - 42^\circ.1) \ldots\ldots\ldots (c)$$

represents the observations as shown by the residuals $C - O$ in the last column of the table of data below:

No.	Date.	No. of pairs.	No. of nights.	t	Observed ϕ''	Comp'd ϕ''	$C-O$
	1891.				$''$	$''$	$''$
1	June 21	90	6	172	10.81	10.62	−.19
2	July 17	126	9	198	.53	.57	+.04
3	Aug. 12	77	8	224	.47	.49	+.02
4	Sept. 17	288	19	260	.39	.38	−.01
5	Oct. 17	171	12	290	.30	.31	+.01
6	Nov. 14	190	15	318	.25	.26	+.01
7	Dec. 15	117	9	349	.43	.27	−.16
	1892.						
8	Jan. 20	91	9	385	.21	.32	+.11
9	Feb. 14	39	4	410	.34	.39	+.05
10	Mar. 20	95	11	445	.39	.50	+.11
11	Apr. 18	108	12	474	.73	.58	−.15
12	May 15	149	13	501	.71	.63	−.08
13	June 14	205	16	531	.56	.64	+.08
14	July 8	43	3	555	.45	.62	+.17
	$\Sigma =$	1789	146				

The probable observing error e_0 of a single determination for latitude was found to be $\pm 0''.17$, using 1789 values, which is equivalent to the mean error as found by the observer, Assistant E. Smith (footnote, p. 50 of Appendix No. 1, Report for 1892).

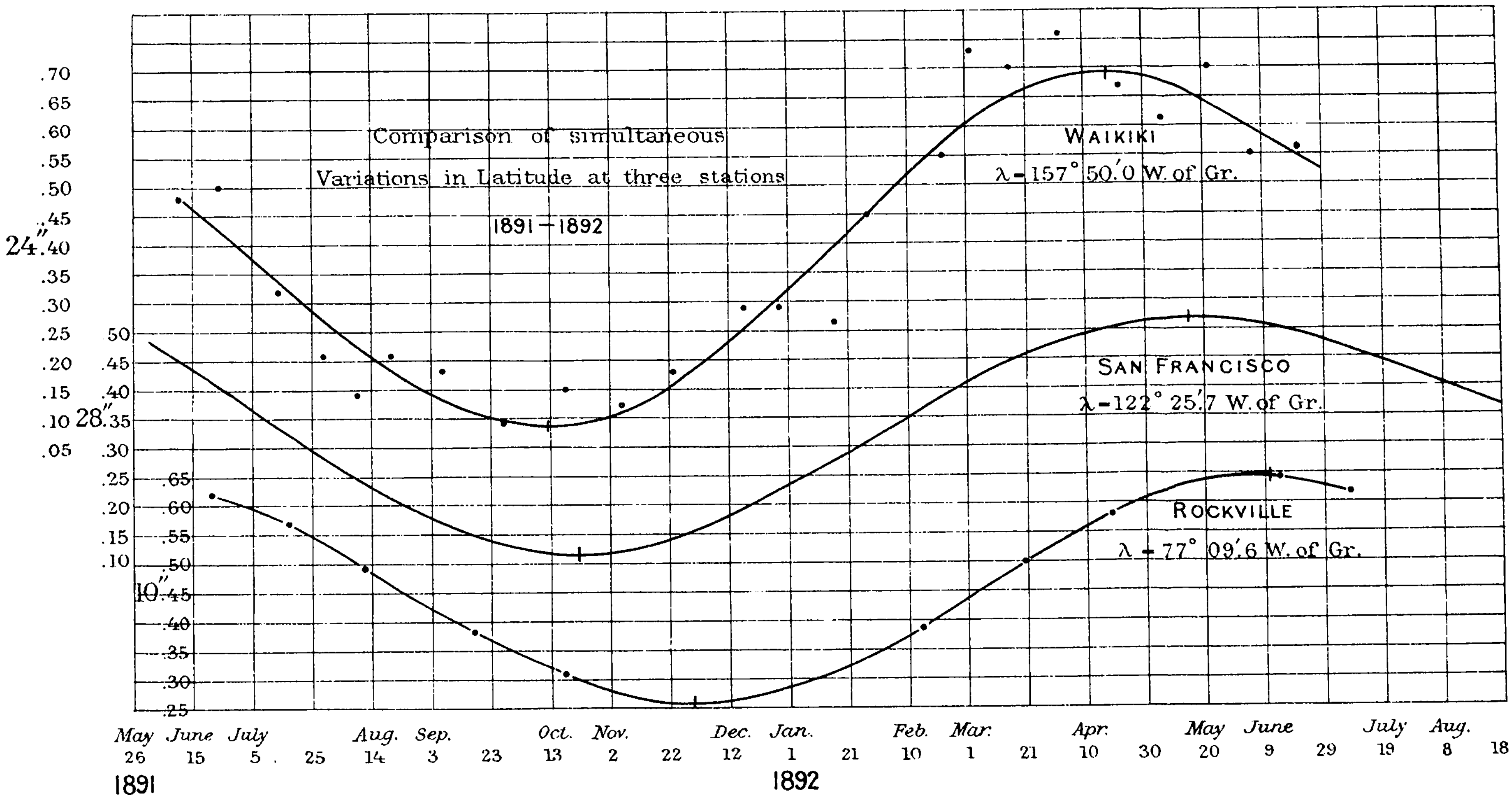
The three curves exhibiting the variation of the latitudes at the Coast Survey stations are given on Diagram No. 21, the ordinates or time scale being the same for all The dots about the Waikiki curve represent observed (mean) values. The local minima and maxima are indicated by short vertical strokes. These phases compare as follows:

	Minimum.	Maximum.	Interval.	Range in lat.
			Days.	$''$
At Waikiki*	Oct. 12, 1891	Apr. 18, 1892	189	0·61
San Francisco	Oct. 22, 1891	May 15, 1892	206	0·40
Rockville	Nov. 30, 1891	June 10, 1892	193·5	0·38
		Average	196	

*According to Assistant Preston's computation (App. 2, Rep. for 1892), using three terms of a Fourier series, the dates of the extreme values are September 14, 1891, and April 2, 1892.

We have also the time required for the minimum to shift from the longitude of Waikiki to that of Rockville, 49 days, and the corresponding time for the maximum phase is 53 days, or a mean value of 51 days for a change of longitude of $80^\circ 40'$. This motion of the phase is at a rate much too fast to be depended upon, since the uncertainty in the time of any one of the minima may be estimated at half a month or more. The observations made at Berlin* show that the minimum was reached there about May 5, 1892; i. e., 196 days after the same was noted at San Francisco, or 206 days after it occurred at Waikiki, which for the last place is at the rate of $0^\circ\!\cdot\!830$ per day. It is, however, preferable, in cases where the length of the series of observations admits of it, to deal with the two periodic terms separately, especially since the annual period is suspected to be very unstable as regards amplitude and epoch.

* Report of the International Geodetic Association, Brussels meeting, 1892.


Comparison of simultaneous
Variations in Latitude at three stations
1891 – 1892
WAIKIKI
λ = 157° 50′.0 W. of Gr.
SAN FRANCISCO
λ = 122° 25′.7 W. of Gr.
ROCKVILLE
λ = 77° 09′.6 W. of Gr.
24″.40
28″.35
10″.45
May 26
June 15
July 5
25
Aug. 14
Sep. 3
23
Oct. 13
Nov. 2
22
Dec. 12
Jan. 1
21
Feb. 10
Mar. 1
21
Apr. 10
30
May 20
June 9
29
July 19
Aug. 8
18
1891
1892

GEODESY.

DETERMINATIONS OF LATITUDE, GRAVITY, AND THE MAGNETIC ELEMENTS AT STATIONS IN THE HAWAIIAN ISLANDS, INCLUDING A RESULT FOR THE MEAN DENSITY OF THE EARTH.

1891, 1892.

A report by E. D. PRESTON, Assistant.

Submitted for publication June 30, 1894.

APPENDIX No. 12—REPORT FOR 1893.

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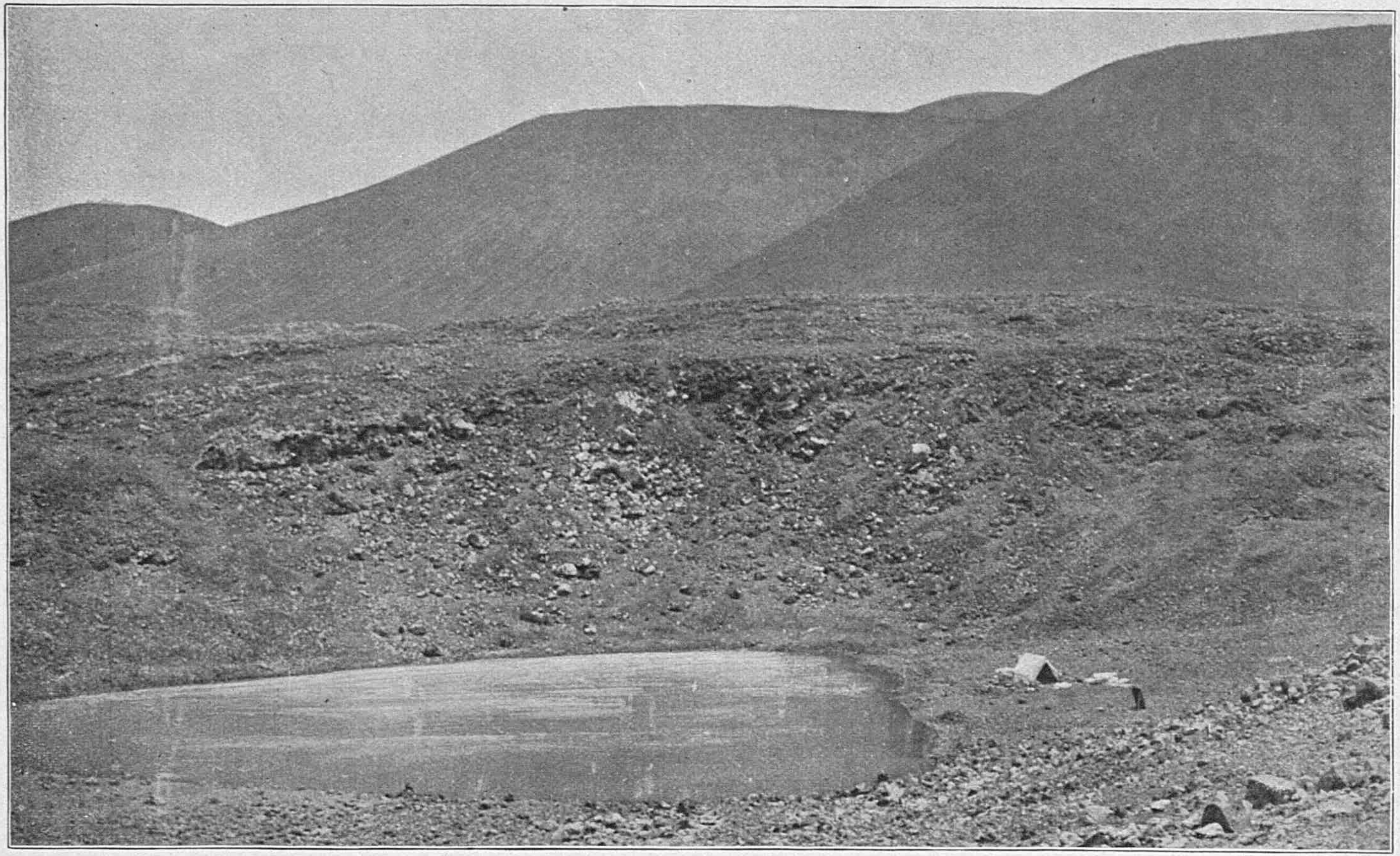
ILLUSTRATIONS.

* From negatives in archives.



GRAVITY, LATITUDE, AND MAGNETIC STATION AT WAIAU, LOOKING NORTHEAST.

Elevation 13,060 feet (3,981 meters).

APPENDIX No. 12—1893.

DETERMINATIONS OF LATITUDE, GRAVITY, AND THE MAGNETIC ELEMENTS AT STATIONS IN THE HAWAIIAN ISLANDS, INCLUDING A RESULT FOR THE MEAN DENSITY OF THE EARTH. 1891, 1892.

A report by E. D. PRESTON, Assistant.
Submitted for publication June 30, 1894.

While engaged in astronomical observations in the Hawaiian Islands in 1891–92, in cooperation with the work of the International Geodetic Association, occasion was taken to make a continuous study for one year of the force of gravity at Waikiki.

After the work had been completed at this place an expedition was undertaken to the summit of Mauna Kea, an extinct crater, having an elevation of 13 825 feet. The object of this trip was the determination of the force of gravity at the base and summit, from which the density of the mountain and the mean density of the earth might be deduced. Availing ourselves of the occupation of this unique station, magnetic, latitude, and hypsometrical observations were carried on, besides making a trigonometric and topographic survey of the great plateau at an elevation of about 12 500 feet. When this was done, some magnetic observations were made at other points of the group, notably at Napoopoo, Kealakeakua Bay, on the lee side of Hawaii, where Captain Cook made similar observations in 1779, and at Lahaina, Maui, where De Freycinet had an observatory in 1819. For an account of other work done in the Hawaiian Islands in 1891–92 the reader is referred to Appendix No. 12, Coast and Geodetic Survey Report, 1891 (Transit of Mercury); Appendix No. 13, Coast and Geodetic Survey Report, 1891 (Preliminary note on the occupation of stations in the Hawaiian Islands); Appendix No. 2, Report for 1892 (On the variation of latitude at Waikiki, near Honolulu, from observations made in connection with the International Geodetic Association), and Bulletin No. 28, on the Constant of Aberration.

The following report has to deal with—

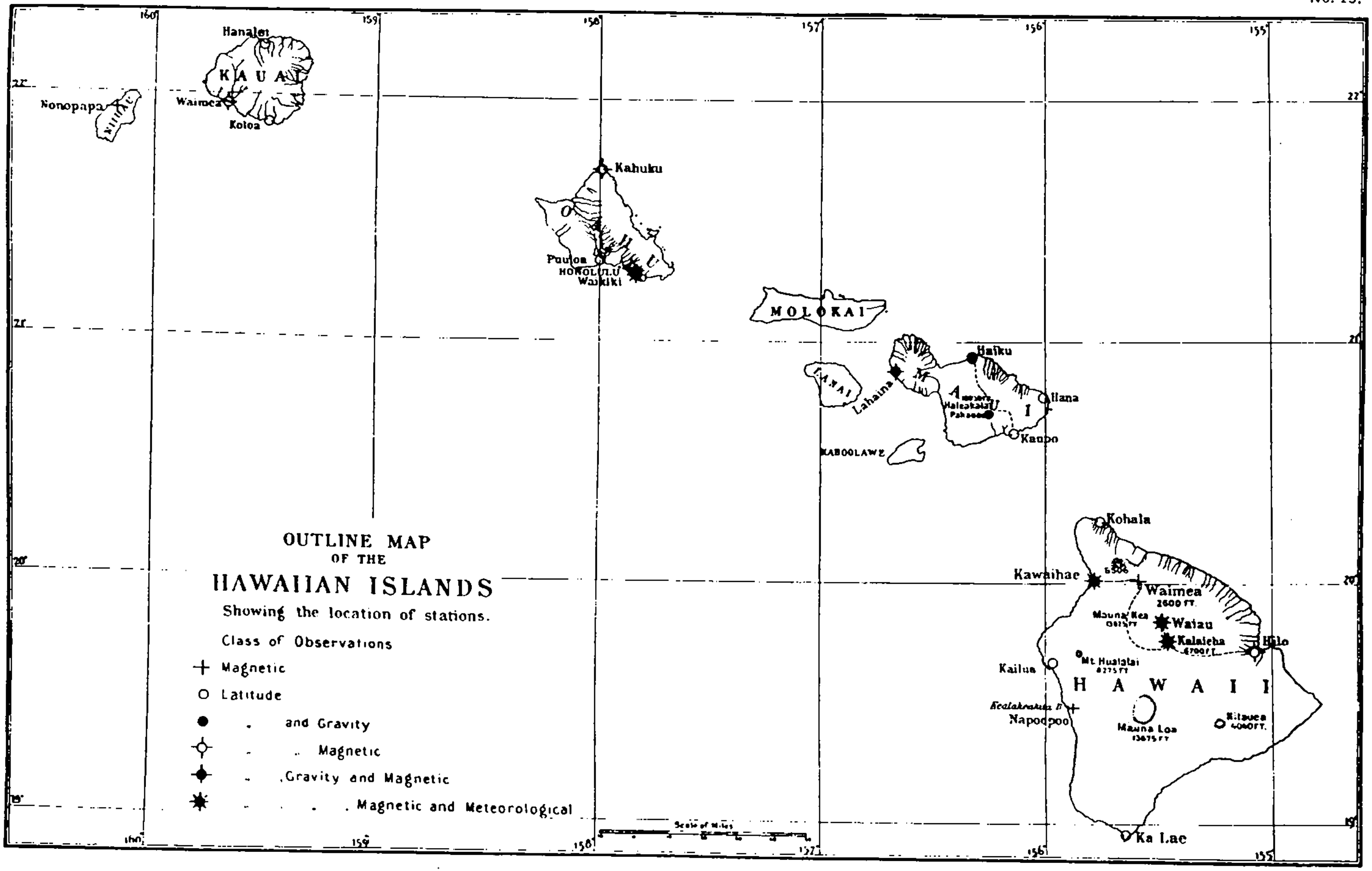
I. Gravity observations at Waikiki.
II. Gravity observations at Honolulu, Kawaihae, Kalaieha, and Waiau (summit of Mauna Kea).
III. Latitude observations at Kawaihae, Kalaieha, Waiau, and Lahaina.
IV. Magnetic observations at Kahuku, Waikiki, and Honolulu, on Oahu; at Kawaihae, Waimea, Kalaieha, Waiau, Hilo, and Napoopoo, on Hawaii; at Lahaina, on Maui; at Waimea, on Kauai, and at Nonopapa, on Niihau.
V. Hypsometrical observations at Honolulu, Hilo, Kawaihae, Waimea, Kalaieha, and Waiau.

The location of these stations is shown in Illustration No. 23.

The gravity observations at Waikiki were made in connection with the International Geodetic Association work. The subsequent determinations were carried on with the cooperation of the Hawaiian Government Survey. The greater part of the expense was borne by this Bureau, and the personnel of the party was largely composed of members of the staff. Prof. W. D. Alexander, the accomplished surveyor-general of the islands, accompanied the expedition to the island of Hawaii and remained with us at all stations except Hilo. During the occupation of the summit of Mauna Kea he assumed the difficult task of making a trigonometrical survey of the plateau. The peaks have an altitude of nearly 14 000 feet and are composed largely of scoria and red volcanic sand, which makes the ascent one requiring extraordinary endurance. In this work he was assisted by Mr. J. M. Muir, who voluntarily accompanied the expedition without compensation and whose services were of great value. The other members of the party were Mr. W. E. Wall, Mr. E. D. Baldwin, and Mr. W. W. Chamberlain, of the Government Survey staff. Mr. Louis Koch performed the duties of steward, a service of some difficulty and of great importance to a party encamped above the clouds, and Kauwe, an intelligent Kanaka, acted as guide both during the ascent and on the return. In the computations I had the help of Mr. C. C. Yates during the latter part of the work.

PRELIMINARY AND CONCLUDING OBSERVATIONS AT WASHINGTON.

The gravity work of 1891–92 was entirely of a differential character. The continuous determinations at Waikiki simply required that the pendulums should receive no accident during the year of occupation, while the observations for the density of Mauna Kea only made it necessary to guard against accident between the times of swinging at the base and summit of the mountain. It is evident, however, that if the periods of oscillation of the three pendulums are determined in Washington before leaving on the expedition and again on the return an agreement of these two determinations will give increased confidence in all the work executed during the trip. In April 1891, the following values were found for the periods of oscillation of pendulums B_1, B_2, and B_3 at the Smithsonian Institution. They are reduced to


OUTLINE MAP
OF THE
HAWAIIAN ISLANDS
Showing the location of stations.
Class of Observations
Magnetic
Latitude
and Gravity
Magnetic
Gravity and Magnetic
Magnetic and Meteorological
Hanalei
KAUAI
Waimea
Koloa
Nonopapa
Kahuku
OAHU
Puuloa
HONOLULU
Waikiki
MOLOKAI
LANAI
Lahaina
Haiku
MAUI
Haleakala
Hana
Kaupo
KAHOOLAWE
Kohala
Kawaihae
Waimea
2600 FT.
Mauna Kea
Waiau
Kalaieha
Hilo
Kailua
Mt. Hualalai
HAWAII
Napoopoo
Mauna Loa
Kilauea
Ka Lae
Scale of Miles
160
159
158
157
156
155
22
21
20
19

a temperature of 15° C., a pressure of 500^{mm} at 0° C., to an infinitely small arc and sidereal time.

$$\left.\begin{array}{l} B_1 = 0{\cdot}500\ 782\ 8^s \\ B_2 = 0{\cdot}500\ 696\ 2 \\ B_3 = 0{\cdot}500\ 632\ 5 \end{array}\right\}\text{Mean} = 0{\cdot}500\ 703\ 8^s$$

Before the return of the expedition, in October, 1892, the base station at Washington had been transferred from the Smithsonian Institution to the Coast and Geodetic Survey Office. The periods were therefore determined at the latter place.

The following is the result:

$$\left.\begin{array}{l} B_1 = 0{\cdot}500\ 779\ 4^s \\ B_2 = 0{\cdot}500\ 695\ 0 \\ B_3 = 0{\cdot}500\ 631\ 3 \end{array}\right\}\text{Mean} = 0{\cdot}500\ 701\ 9^s$$

A comparison of the above values gives a diminished period for all three pendulums. We have the excess of the time of oscillation in April, 1891, at the Smithsonian Institution, over that in December, 1892, at the Coast and Geodetic Survey Office, as follows:

$$\left.\begin{array}{l} \text{For } B_1 = +\ 0{\cdot}000\ 003\ 4^s \\ \phantom{\text{For }} B_2 = +\ 0{\cdot}000\ 001\ 2 \\ \phantom{\text{For }} B_3 = +\ 0{\cdot}000\ 001\ 2 \end{array}\right\} +\ 0{\cdot}000\ 001\ 9^s$$

These two stations were connected by simultaneous determinations of gravity by Mr. Putnam and Mr. Von der Trenck. The result gave an excess of the period at the Coast and Geodetic Survey Office over that at the Smithsonian Institution of 0·000 000 5^s. So that we have a mean decrease in the period of oscillation of the B pendulums consequent upon the work outside of the United States of 0·000 002 4^s. This is about 1 part in 400 000. When we consider that these pendulums were in continual service for more than a year at Waikiki, that they were transported on mule back to an elevation of over 13 000 feet, where the observations were made under difficult and adverse circumstances, and that numerous other stations were occupied in the Hawaiian Islands and in this country, this close agreement between the periods of oscillation before and after the expedition must be regarded as highly satisfactory.

In regard to the accuracy attainable in the general method of optical coincidences as practiced in this work, it appears to be far beyond what is necessary. If $c =$ the interval in seconds between two coincidences, and if in that time the pendulum has gained or lost one beat on the timepiece, the time (t) of one oscillation is

$$\frac{c}{2c \pm 1}$$

The accuracy with which t is determined depends both on the length of the interval and the rate of the pendulum on the timepiece, but principally on the former. Differentiating the above expression gives approximately

$$dt = \frac{\pm\, dc}{4c^2}$$

Where the pendulum period is nearly an aliquot part of that of the timepiece and the interval between two coincidences is 15 minutes, or say 1 000 seconds, an observation of c to the nearest second will only produce an error in the length of the period of one part in eight million.

The following are the observations at Washington before and after the expedition. The computations, as well as the observations, were made by Mr. G. R. Putnam, Assistant Coast and Geodetic Survey. The arc corrections were computed by Borda's formula. The temperature coefficient was determined empirically by observations at high and low temperatures in April, 1891. The increase in period for 1° C. increase in temperature was found to be:

For pendulum	B_1	0·000 004 16^s
	B_2	4 22
	B_3	4 08

The mean of these, which is 0·000 004 15^s, was used in the reductions. From the coefficient of expansion of the pendulum alloy, as determined by the Office of Standard Weights and Measures, we get 0·000 004 14^s.

The pressure coefficient was determined in April, 1891. The mean for the three pendulums was found to be 0·000 000 078 9^s, representing the increase in period for an increase of 1^mm in pressure at 0° C.

The correction to the period is therefore.

$$-K\left[\frac{P}{1+0.00367t}-500\right]$$

where P is the difference between the readings of the barometer and manometer and t is the temperature. This reduces the period to that at a pressure of 500^mm at 0° C.

The chronometer rates were determined in April, 1891, from United States Naval Observatory signals, and in December, 1892, by time observations, by Mr. Putnam.

The gravity work for all the stations is published according to the following scheme:

Column I indicates the pendulum.

Column II, the position (direct or reverse.

Column III, the number of the swing.

Column IV, the date.

Column V, the number of seconds between the first and eleventh coincidence.

Columns VI and VII indicate the arc through which the pendulum was swinging at the first and eleventh coincidence.

Column VIII indicates the corrected temperature.

Column IX, the manometer reading.

Column X, indicates the barometer reading.

Column XI, the pressure in the receiver at a temperature of 0° C.

The uncorrected periods, the different corrections, and finally the corrected period in sidereal time are given in the following table:

Pendulum observations, Washington, D. C. (Smithsonian Institution).

[Observer: G. R. Putnam.]

Pendulum.	Position.	Swing.	Date.	Time of ten coincidence intervals.	Semi-arc. Initial.	Semi-arc. Final.	Temperature.	Manometer.	Barometer.	Pressure at 0° C.
			1891.	*Seconds.*	*mm.*	*mm.*	°*C.*	*mm.*	*mm.*	*mm.*
B_1	D	1	Apr. 15	3267·0	5·35	3·85	20·93	224·2	765·1	500·6
		2	15	3266·0	5·25	3·85	20·98	223·0	765·3	501·9
		13	16	3271·0	5·85	4·10	20·93	228·6	768·7	499·9
		19	16	3260·0	5·30	3·80	21·08	228·4	766·9	498·3
B_1	R	3	15	3252·5	5·20	3·80	21·08	226·5	765·3	498·4
		4	15	3257·0	5·35	3·80	21·18	225·5	765·3	499·1
		14	16	3260·0	5·70	3·90	20·93	229·0	768·3	499·2
		22	16	3261·8	5·25	3·20	21·10	226·2	767·8	501·0
B_2	D	5	15	3645·0	5·30	3·55	21·28	203·7	765·0	519·0
		6	15	3653·0	5·40	3·85	21·36	227·2	764·6	496·7
		15	16	3600·0	5·30	3·65	20·98	229·4	767·7	498·2
		20	16	3662·0	5·30	3·75	21·08	231·0	765·5	496·3
B_2	R	7	15	3666·5	5·20	3·70	21·40	226·7	764·4	496·9
		8	15	3670·0	5·30	3·75	21·40	224·8	764·3	498·6
		16	16	3681·0	5·25	3·75	21·06	229·4	767·2	497·6
		23	17	3708·0	5·25	3·50	20·57	215·4	770·2	514·2
B_3	D	9	15	4042·5	5·20	3·25	21·40	224·6	764·4	498·8
		10	16	4044·5	5·35	3·75	20·74	227·7	769·2	501·5
		17	16	4054·5	5·00	3·20	21·08	228·2	766·9	498·4
		21	16	4047·0	5·00	3·20	21·10	230·0	767·8	497·5
B_3	R	11	16	4054·5	5·30	3·60	20·81	227·2	769·1	501·8
		12	16	4053·5	5·30	3·70	20·88	230·0	769·0	498·9
		18	16	4050·5	4·85	3·25	21·10	228·1	766·8	498·4
		24	17	4099·5	4·80	3·20	20·59	234·8	770·1	495·9

Reduction of pendulum observations, Washington, D. C. (Smithsonian Institution).

[Periods reduced to temperature, 15° C; pressure, 500mm at 0° C.; arc infinitely small; sidereal time.]

Pendulum.	Swing.	Period uncorrected.	Corrections (in seventh decimal place).				Period corrected.
			Arc.	Temperature.	Pressure.	Rate.	
		Seconds.					*Seconds.*
B_1	1	0·500 7664	—74	—246	0	+486	0·500 7830
	2	7666	—73	—248	— 1	+486	7830
	13	7655	—86	—246	0	+457	7780
	19	7680	—73	—252	+ 1	+468	7824
						Direct	0·500 7816
B_1	3	0·500 7698	—71	—252	+ 1	+486	0·500 7862
	4	7688	—74	—256	+ 1	+486	7845
	14	7680	—81	—246	+ 1	+468	7822
	22	7676	—62	—253	— 1	+468	7828
						Reverse	0·500 7839
						Mean	0·500 7828
B_2	5	0·500 6868	—68	—261	—15	+486	0·500 7010
	6	6853	—75	—264	+ 3	+457	6974
	15	6829	—70	—248	+ 1	+468	6980
	20	6836	—72	—252	+ 3	+468	6983
						Direct	0·500 6987
B_2	7	0·500 6828	—70	—266	+ 2	+457	0·500 6951
	8	6821	—72	—266	+ 1	+457	6941
	16	6801	—71	—251	+ 2	+468	6949
	23	6751	—67	—231	—11	+468	6910
						Reverse	0·500 6938
						Mean	0·500 6962
B_3	9	0·500 6192	—62	—266	+ 1	+457	0·500 6322
	10	6189	—73	—238	— 1	+457	6334
	17	6174	—59	—252	+ 1	+468	6332
	21	6185	—59	—253	+ 2	+468	6343
						Direct	0·500 6333
B_3	11	0·500 6174	—69	—241	— 1	+457	0·500 6320
	12	6175	—71	—244	+ 1	+457	6318
	18	6180	—57	—253	+ 1	+468	6339
	24	6106	—56	—232	+ 3	+468	6289
						Reverse	0·500 6316
						Mean	0·500 6324

Pendulum observations, Washington, D. C. (Coast and Geodetic Survey Office).

[Observer: G. R. Putnam.]

Pendulum.	Position.	Swing.	Date.	No. of coincidence intervals.	Time of ten coincidence intervals.	Semiarc. Initial.	Semiarc. Final.	Temperature.	Manometer.	Barometer.	Pressure at 0°C.
			1892.		*Seconds.*	*mm.*	*mm.*	°C.	*mm.*	*mm.*	*mm.*
B_1	D	13	Dec. 14	14	3295·4	5·35	3·50	12·00	232·9	764·6	508·3
		14	15	10	3297·0	5·35	4·00	11·88	239·6	767·4	504·8
		15	15	10	3296·5	5·30	3·95	11·93	239·3	767·4	505·0
		16	15	10	3295·5	5·25	3·80	11·97	237·6	766·8	505·9
		31	17	16	3292·2	5·40	3·25	12·40	237·7	762·4	500·9
		32	18	10	3294·5	5·30	3·95	11·95	240·0	763·8	500·9
B_1	R	17	15	10	3281·5	5·15	3·85	12·17	216·4	765·8	525·0
		18	15	10	3287·5	5·25	3·80	12·35	246·8	765·5	495·3
		19	15	10	3283·5	5·30	3·80	12·45	244·2	765·4	497·5
		33	18	6	3296·7	5·20	4·35	11·83	239·4	761·5	499·4
		34	18	12	3297·9	5·30	3·70	11·83	237·5	761·5	501·2
B_2	D	7	14	10	3729·0	5·40	3·80	11·20	241·5	762·6	499·6
		8	14	10	3725·0	5·40	4·00	11·30	243·1	762·8	498·2
		9	14	10	3723·0	5·40	3·90	11·40	240·4	762·5	500·2
		26	17	10	3706·0	5·35	3·80	11·93	242·8	761·9	496·4
		27	17	10	3705·0	5·30	3·80	11·95	240·0	761·6	498·8
		28	17	10	3704·5	5·30	3·80	12·00	239·2	761·0	498·9

Reduction of pendulum observations, Washington, D. C. (Coast and Geodetic Survey Office).

[Periods reduced to temperature 15° C.; pressure, 500mm at 0° C.; arc infinitely small; sidereal time.]

Pendulum.	Swing.	Period uncorrected.	Corrections (in seventh decimal place).				Period corrected.
			Arc.	Temperature.	Pressure.	Rate.	
		Seconds.					*Seconds.*
B_1	13	0·500 7598	—68	+124	— 6	+152	0·500 7800
	14	7594	—77	+129	— 4	+142	784
	15	7595	—75	+127	— 4	+142	785
	16	7598	—72	+126	— 5	+142	789
	31	7606	—65	+108	— 1	+144	792
	32	7600	—75	+126	— 1	+145	795
						Direct	0·500 7791
B_1	17	0·500 7630	—71	+117	—20	+142	0·500 7798
	18	7616	—72	+110	+ 4	+142	800
	19	7625	—73	+106	+ 2	+142	802
	33	7595	—80	+132	0	+145	792
	34	7592	—71	+132	— 1	+145	797
						Reverse	0·500 7798
						Mean	0·500 7794
B_2	7	0·500 6713	—74	+158	0	+162	0·500 6959
	8	6720	—78	+154	+ 1	+162	959
	9	6724	—76	+149	0	+162	959
	26	6755	—74	+127	+ 3	+143	954
	27	6757	—73	+126	+ 1	+143	954
	28	6758	—73	+124	+ 1	+143	953
						Direct	0·500 6956

Pendulum observations, Washington, D. C. (Coast and Geodetic Survey Office).

[Observer: G. R. Putnam.]

Pendulum.	Position.	Swing.	Date.	No. of coincidence intervals.	Time of ten coincidence intervals.	Semi-arc.		Temperature.	Manometer.	Barometer.	Pressure at 0° C.
						Initial.	Final.				
			1892.		*Seconds.*	*mm.*	*mm.*	°*C.*	*mm.*	*mm.*	*mm.*
B_2	R	10	Dec. 14	10	3730·5	5·35	3·65	11·50	243·6	762·2	496·7
		11	14	10	3726·0	5·25	3·75	11·67	241·7	762·4	498·4
		12	14	10	3723·5	5·20	3·80	11·77	244·0	762·7	496·3
		29	17	10	3713·0	5·30	3·90	12·13	244·0	760·0	493·0
		30	17	10	3709·5	5·30	3·80	12·27	241·1	760·0	495·6
B_3	D	1	12	12	4122·1	5·90	3·60	11·45	235·2	774·9	517·0
		2	13	10	4143·5	5·25	3·40	10·90	241·5	772·6	509·7
		3	13	10	4145·5	5·20	3·45	10·97	252·4	771·9	498·4
		20	15	14	4092·9	5·10	3·10	12·55	240·0	766·6	502·4
		21	16	10	4096·5	4·85	3·30	12·23	244·8	767·9	499·7
		22	16	12	4096·2	4·90	3·15	12·27	241·6	767·8	502·5
B_3	R	4	13	10	4125·0	4·85	3·30	11·13	239·6	770·7	509·2
		5	13	10	4129·0	4·85	3·30	11·25	254·9	769·8	493·6
		6	13	10	4125·0	4·85	3·30	11·33	246·7	769·2	500·7
		23	16	10	4083·5	4·85	3·30	12·40	243·5	766·8	499·6
		24	16	10	4081·0	4·85	3·30	12·45	241·4	766·5	501·2
		25	16	10	4080·0	4·85	3·30	12·45	239·6	766·4	502·8

Reduction of pendulum observations, Washington, D. C. (Coast and Geodetic Survey Office).

[Periods reduced to temperature 15° C.; pressure, 500^{mm} at 0° C.; arc infinitely small; sidereal time.]

Pendulum.	Swing.	Period uncorrected.	Corrections (in seventh decimal place).				Period corrected.
			Arc.	Temperature.	Pressure.	Rate.	
		Seconds.					*Seconds.*
B_2	10	0·500 6710	—71	+145	+ 3	+162	0·500 6949
	11	6719	—71	+138	+ 1	+162	949
	12	6723	—71	+134	+ 3	+162	951
	29	6742	—74	+119	+ 6	+143	936
	30	6748	—73	+113	+ 3	+143	934
						Reverse	0·500 6944
						Mean	0·500 6950
B_3	1	0·500 6072	—78	+147	—13	+162	0·500 6290
	2	6041	—65	+170	— 8	+162	300
	3	6038	—65	+167	+ 1	+162	303
	20	6116	—58	+102	— 2	+142	300
	21	6110	—58	+115	0	+143	310
	22	6111	—56	+113	— 2	+143	309
						Direct	0·500 6302
B_3	4	0·500 6068	—58	+161	— 7	+162	0·500 6326
	5	6062	—58	+156	+ 5	+162	327
	6	6068	—58	+152	— 1	+162	323
	23	6130	—58	+108	0	+143	323
	24	6134	—58	+106	— 1	+143	324
	25	6135	—58	+106	— 2	+143	324
						Reverse	0·500 6324
						Mean	0·500 6313

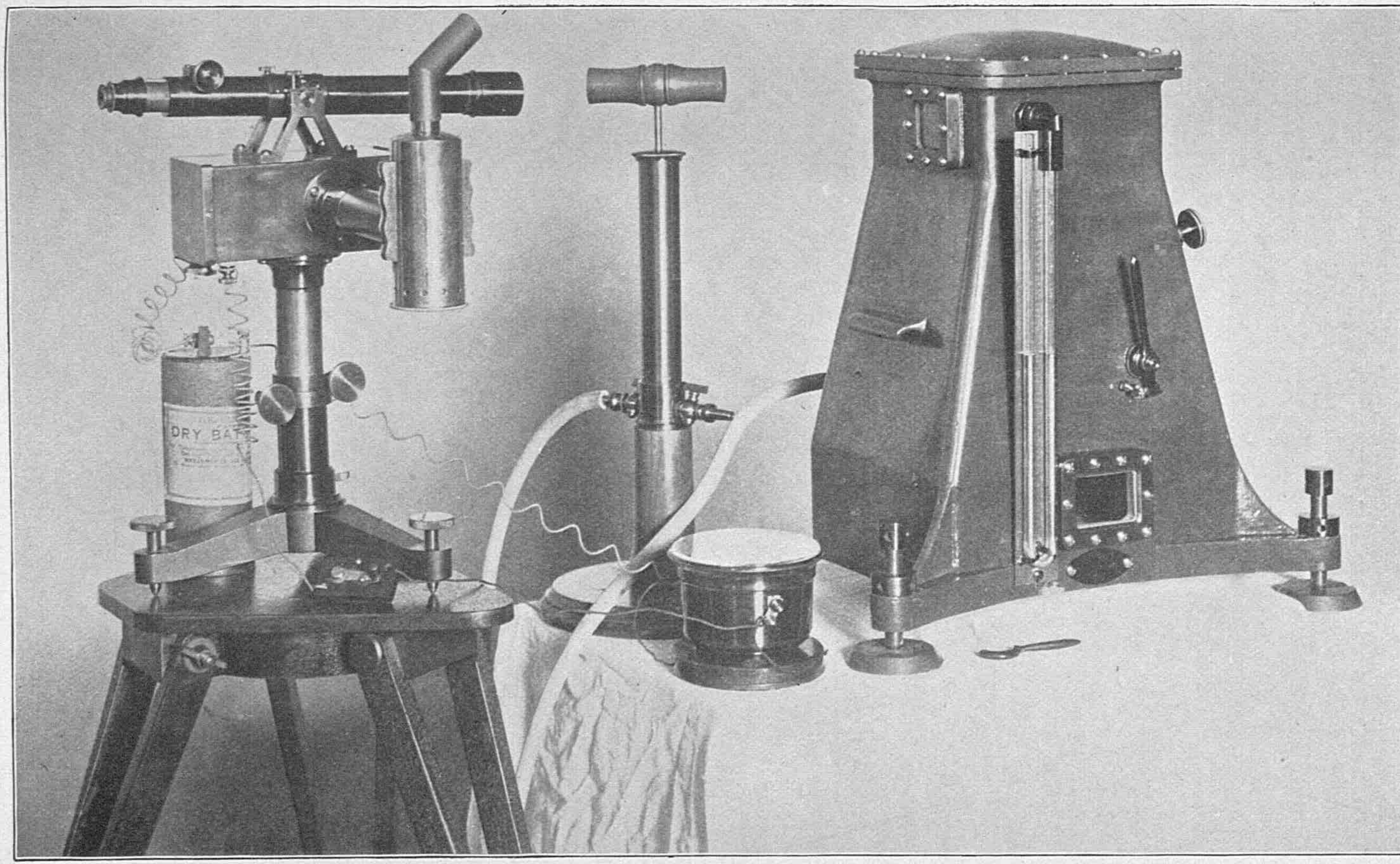
GRAVITY OBSERVATIONS AT WAIKIKI.

For a description of this station it will be sufficient to refer to Appendix No. 2, Report of Coast and Geodetic Survey for 1892. The pendulum apparatus has been described by Mr. G. R. Putman in a special report by the Superintendent on the "Determination of gravity," Appendix No. 15, Report of Coast and Geodetic Survey for 1891. This paper may be consulted for all details of construction and manipulation. A general view of the apparatus is shown in illustration No. 24.

The observations for the force of gravity at Waikiki were begun on June 9, 1891, and continued until June 11, 1892. During this time 199 nights' work were obtained. The determinations were limited to those nights on which satisfactory star observations could be made, and the pendulum was swung during the time that the regular international latitude investigations were carried on. This made the duration of the swings somewhat variable, but the pendulum in nearly every case was allowed to oscillate for several consecutive hours. The plan generally followed was this: In the early evening, after one or two pairs of stars had been obtained for latitude, one circumpolar and several time stars were observed with the instrument in the position "clamp west." The pendulum was then started, after which the meridian telescope was reversed to the position "clamp east" and a few more stars taken. A circumpolar was obtained in the second position when possible, but this was not considered essential. The advantage of the foregoing scheme is that the pendulum apparatus has time to take the evening temperature before beginning the swing, and that the beginning of the swing is referred to the epoch of the reversal of the instrument. The telescope was allowed to remain with "clamp east" until the close of the night's work, when another half set was observed.

Inasmuch as we desire only differential rates for the chronometer, we have here the corrections determined before and after the swing almost entirely independent of the instrumental constants. The effect of azimuth was entirely eliminated by observing stars north and south of the zenith. The level was directly observed, but since the instrument was not reversed during the swing, the change was inappreciable, and whatever the collimation, as there was no reversal, its effect on the difference of the clock corrections is insignificant.

The instrumental constants were quite small during the entire year. Moreover, since the same stars were observed from night to night the chronometer rates are independent of any errors in the right ascensions. It is believed that this work reaches all the accuracy attainable with this set of instruments. The arrangement of the time observations, the method employed of interchanging the pendulums, and the precautions taken in the temperature and pressure conditions seem to exhaust the precision of manipulation, so that any discrepancies must be attributed either to the construction of the instruments or to real



HALF-SECOND PENDULUM APPARATUS.
Side view (right) of receiver, flash apparatus, etc.

differences in the force of gravity. The principal defect in the apparatus was in the means of reading the amplitude of oscillation. At the beginning of the swing the half amplitude was usually about 1°, or a distance of 5^{mm} on the scale at the pendulum point, which is 297^{mm} below the knife edge. After an hour and a half this half amplitude had decreased to about $0^\circ.5$ or to 3^{mm}, and the correction to reduce the time of one oscillation to what it would have been in an infinitely small arc is $0{\cdot}000\ 005\ 6^s$. Now, an error of one-tenth of a millimetre in estimating the amplitude would produce an error of rather more than one ten-millionth of a second in the correction for arc, so that we may assume that the uncertainties arising from the scale readings do not materially affect the deduced period as much as the millionth part of a second.

The change of gravity for a small change in the latitude of the place may be obtained by differentiating the equation

$$g=C+D\sin^2\varphi$$

where C is the force of gravity at the equator, and D is the increase in this force in passing from the equator to the pole, the unit being taken in metres. For a value of $\varphi=21^\circ\ 16'$ we get

$$dg=0{\cdot}0338\ d\varphi$$

A change, therefore, of 1″ in the latitude would give a change of $\frac{0{\cdot}0338}{206\ 000}$ or $\frac{1}{6\ 100\ 000}$ of a metre in the value of $D\sin^2\varphi$ or about $\frac{1}{60\ 000\ 000}$ of the total force of gravity.

The change of latitude at Waikiki during the entire year being six-tenths of a second, the force of gravity would not be disturbed from this cause more than 1 part in 100 000 000.

The following table contains the chronometer corrections and instrumental constants from June 4, 1891, to June 11, 1892. The value of the collimation is given with its appropriate sign for the position of the telescope with the clamp on the west side. The azimuths are given for those positions in which a determination was made. When no stars of sufficiently high declination could be obtained to bring out this constant, that of a preceding or following day was used and the corresponding tabular space is left vacant.

Clock rates and instrumental constants, Waikiki, Hawaiian Islands.

[Negus sidereal chronometer No. 1825. Observer: E. D. Preston.]

Date.	Epoch (sidereal time).		Correction.		Daily rate.	Collimation W.	Azimuth.	
							E.	W.
1891.	*h.*	*m.*	*m.*	*s.*	*Seconds.*	*Seconds.*	*Seconds.*	*Seconds.*
June 4	13	30	+2	9·92	6·60	+0·06	+0·19	
6	16	00		23·82	7·06	— ·09		—0·22
9	15	40		44·90	7·34	+ ·07	— ·62	
10	15	45		52·24	7·40	— ·09	— ·62	
11	15	30		59·57	7·37	— ·18	+ ·15	— ·42
12	15	30		6·94	6·88			
13	14	30	+3	13·54	6·24	— ·25	— ·07	+ ·05
15	14	30		26·02	6·67	— ·55	— ·59	— ·80
16	13	15		32·37	6·43		+ ·71	
17	14	30		39·14	6·67	·00	+ ·58	+ ·13
18	14	30		45·81	6·97	— ·23		+ ·42
19	14	30		52·78	6·60	+ ·09	+ ·12	— ·01
22	14	30	+4	12·58	6·68	— ·16		— ·16
24	15	30		26·21	6·16	+ ·12	— ·05	+ ·33
25	16	30		32·62	6·58	+ ·18		
26	15	25		38·93	6·89	+ ·16		+ ·28
27	14	30		45·53	6·61	+ ·17	+ ·06	
28	15	30		52·42	6·36	+ ·05		·00
30	14	30	+5	4·89		+ ·10	+ ·37	
July 4	16	10	—2	29·29	6·64		+ ·30	
7	18	00		8·87	6·36	+ ·10		— ·22
8	17	50		2·56	7·04	+ ·21	+ ·04	
17	18	30	—0	59·20	7·15	+ ·22	— ·14	
18	17	50		52·26	7·20	+ ·27	— ·26	— ·60
22	18	30		23·26	6·59	+ ·27		
23	17	45		16·86	6·76	— ·15	+ ·41	
24	17	45		10·10	6·90	— ·11		— ·05
25	18	30		2·99	7·76			
26	18	00	+0	4·62	7·25	— ·03	+ ·03	+ ·04
27	17	45		11·80	7·19	— ·08	+ ·21	
29	17	45		26·18	6·63	— ·03	— ·12	
30	19	00		33·16	6·79	— ·10	+ ·13	
31	17	45		39·60	6·50	·00	— ·12	·00
Aug. 2	18	15		52·71	6·25	— ·07	+ ·01	+ ·13
3	18	15		58·96	6·57	— ·21	+ ·22	+ ·19
4	17	45	+1	5·40	6·93	— ·11	+ ·23	+ ·33
5	17	30		12·26	6·97	— ·14	+ ·25	+ ·33
6	18	00		19·38	7·09	+ ·04	+ ·21	+ ·06
7	17	45		26·40	7·24	+ ·05	+ ·34	+ ·32
10	17	30		48·07	7·14	+ ·04	*— ·57	— ·36
11	17	45		55·28	7·34	— ·05	— ·49	— ·40
13	17	30	+2	9·91	7·11	— ·04	— ·42	— ·32
14	17	30		17·02	7·02	+ ·05	— ·31	·00
15	17	30		24·04	6·96	+ ·05	— ·23	— ·31
16	17	30		31·00	6·76	— ·03	— ·14	— ·38
20	19	30		58·04	6·19	+ ·10	— ·37	— ·22
22	19	30	+3	10·42	6·13	+ ·05	·00	— ·19
23	19	30		16·55	6·16	+ ·07	— ·15	— ·38
25	22	00		29·52	6·34	— ·07	— ·37	
26	19	00		35·07	6·04	— ·06	— ·18	+ ·11
29	19	00		53·20		— ·08		+ ·12
	22	30		54·06	6·53			
31	19	00	+4	6·26		— ·07	— ·45	— ·07

*On August 8 telescope moved eastward.

Clock rates and instrumental constants, Waikiki, Hawaiian Islands—Continued.

[Negus sidereal chronometer No. 1825. Observer: E. D. Preston.]

Date.	Epoch (sidereal time).	Correction.	Daily rate.	Collimation W.	Azimuth. E.	Azimuth. W.
1891.	*h.* *m.*	*m.* *s.*	*Seconds.*	*Seconds.*	*Seconds.*	*Seconds.*
Sept. 2	19 00	−2 40.30	6.47	− .12	− .07	− .17
6	19 00	14.43		.00	− .42	− .22
	20 00	14.03	6.15			
9	19 00	−1 55.59	6.19	− .05	− .33	− .28
12	19 00	37.02	6.28	− .07	− .30	− .22
15	19 00	18.18	6.48	− .06	− .36	− .33
17	20 30	4.83	6.61	− .03	− .47	− .22
18	19 00	−0 58.63	6.35	− .02	− .41	− .22
19	19 00	52.28	6.40	− .12	− .37	− .26
23	21 00	26.17	6.61	− .05	− .32	
26	19 00	6.84	6.51	− .09	− .20	− .29
27	20 00	−0 0.06	6.61	− .09		− .42
28	19 00	+0 6.28				
Oct. 4	22 30	46.65		− .20		− .60
	22 34	46.58	6.43			
	2 13	47.46				
5	22 30	53.08		− .20		− .60
	22 34	53.00	6.56			
	2 13	53.88			− .27	
8	22 30	+1 12.75		− .10		− .60
	22 34	12.79	6.41		− .43	
	2 15	13.80				
10	22 30	25.57		− .08		+ .70
	22 34	25.56	6.60			
	0 00	26.07			+ .09	
14	22 30	51.97	6.51	+ .11	+ .13	− .11
16	22 30	+2 4.99		+ .13	+ .20	+ .13
	22 33	4.98	6.60			
	2 15	5.99				
22	22 30	44.60		+ .09		+ .10
	22 55	44.61	6.53			
	1 00	45.20				
28	22 30	+3 23.80		− .04		− .31
	22 54	23.80	6.73			
	2 04	24.53				
30	22 30	37.26		+ .12	+ .15	+ .10
	22 31	37.26	6.54			
	2 15	38.24				
31	22 30	43.80		− .12		+ .10
	22 32	43.81	6.38			
	2 15	44.78				
Nov. 1	22 30	−3 9.82		− .04	+ .07	+ .08
	22 38	9.82	6.33			
	2 15	8.84				
4	22 30	−2 50.82		− .04	− .06	
	22 38	50.82	6.33			
	2 15	49.84				
6	22 30	38.16		− .04	− .22	− .32
	22 37	38.16	6.49			
	2 15	37.06				
7	22 30	31.67	6.05			
8	23 00	25.50		− .10	− .25	− .28
	23 02	25.49	6.29			
	1 30	24.90				

Clock rates and instrumental constants, Waikiki, Hawaiian Islands—Continued.

[Negus sidereal chronometer No. 1825. Observer: E. D. Preston.]

Date.	Epoch (sidereal time).		Correction.		Daily rate.	Collimation W.	Azimuth.	
							E.	W.
1891.	*h.*	*m.*	*m.*	*s.*	*Seconds.*	*Seconds.*	*Seconds.*	*Seconds.*
Nov. 12	1	00	—1	59·86	6·38	·00	·00	+ ·35
13	22	30		54·14		— ·12	+ ·30	·00
	22	37		54·13	6·40			
	2	15		53·21				
14	22	30		47·74		— ·10	+ ·22	+ ·05
	22	37		47·78	6·62			
	2	15		46·74				
17	22	30		27·86		— ·03	— ·01	+ ·12
	22	36		27·86	6·90			
	2	15		26·75				
19	0	30	—1	13·45	6·52	+ ·13	+1·97	+1·75
	5	30		12·10				
	5	44		12·08				
21	0	30		00·40				+ ·42
	0	35		00·45	6·61			
	5	30	—0	59·00		+ ·20		
22	1	30		53·51	6·48	+ ·14	+ ·18	+ ·47
23	1	00		47·16	6·59	+ ·12	·00	+ ·27
28	1	00		14·22	6·70	[— ·04	From Nov. 30	+ ·21]
30	0	30		00·96		— ·04	+ ·67	+ ·21
	0	36		00·85	6·42			
	5	30	+0	00·11				
Dec. 1	0	30		05·46				+ ·22
	0	36		05·45	6·43			
	5	30		06·36			+ ·55	
5	1	00		31·32				+ ·41
	5	30		32·55	6·43	+ ·15		
	5	32		32·55				
6	0	30		37·63	6·60	+ ·15	From Dec. 5	+ ·44
7	1	00		44·36	6·51	— ·05	+ ·57	+ ·33
9	0	30		57·26				
	0	35		57·28	6·45			
	5	30		58·52		— ·08	— ·06	+ ·25
10	0	30	+1	03·71	6·34			·00
11	1	00		10·17	6·61	+ ·15	+ ·26	— ·11
12	1	00		16·78				
	0	33		16·78	6·35			
	5	30		17·99		— ·06	— ·36	— ·20
15	1	00		35·84				
	1	34		35·83	6·34			
	5	30		37·10		— ·24	— ·40	— ·53
17	0	30		48·38	6·15			+ ·11
19	0	30	+2	00·68		+ ·11		— ·09
	0	34		00·77	5·93			
	5	30		02·00			·00	
20	0	30		06·61	6·35			
21	0	30		12·96	6·00			
22	0	30		18·96	6·26			
23	0	30		25·22		+ ·20		— ·30
	0	33		25·21	6·67			
	5	30		26·74			— ·31	
24	0	30		31·89	6·58			— ·15
25	0	30		38·47		+ ·08	+ ·15	+ ·06
	0	38		38·48	6·43			
	5	30		39·82				

Clock rates and instrumental constants, Waikiki, Hawaiian Islands—Continued.

[Negus sidereal chronometer No. 1825. Observer: E. D. Preston.]

Date.	Epoch (sidereal time).		Correction.		Daily rate.	Collimation W.	Azimuth.	
							E.	W.
1891.	*h.*	*m.*	*m.*	*s.*	*Seconds.*	*Seconds.*	*Seconds.*	*Seconds.*
Dec. 26	1	30	+2	45·17				·00
	5	30		46·46	6·62	+0·17	—0·31	
	5	40		46·48				
27	0	30		51·52				— ·10
28	0	30		58·00	6·48			— ·16
	0	31		58·00	6·42			
	5	30		59·24		+ ·09	— ·32	
29	0	30	+3	04·42				— ·22
	0	31		04·42	5·88			
	5	30		05·62		+ ·04	— ·19	
30	0	30		10·30				+ ·16
	0	33		10·29	5·82			
	5	30		11·42		+ ·05	+ ·17	
1892.								
Jan. 1	0	30		21·94				+ ·13
2	0	30	—3	32·31	5·75			+ ·14
3	0	30		26·30	6·01			
5	1	00		14·64	5·77	+ ·02		+ ·17
	1	38		14·64				
	5	30		13·66	5·59		— ·10	
6	5	00		08·12		+ ·26		·00
10	5	00	—2	46·04	5·52	+ ·10		
11	5	30		40·34	5·59			
	5	46		40·36				
	7	30		39·85	5·71	+ ·10	+ ·20	
12	5	30		34·63				
	5	46		34·62	5·49			
	8	00		34·00		[+ ·12]	+ ·15	
16	5	30		12·66		+ ·07		+ ·40
18	5	00		01·11	5·83			— ·26
	8	00		00·58	5·79	[— ·10]		
19	5	00	—1	55·32	6·06	[— ·10]		[— ·20]
20	5	00		49·26				
	8	00		48·70		— ·10	— ·14	
	8	14		48·69	5·62			
21	5	00		43·64				
	5	02		43·60	5·26			
	8	00		42·97		— ·10		+ ·08
23	5	00		33·12				— ·25
	5	02		33·14				
	8	00		32·50	5·40	— ·10		
24	5	00		27·72				
	4	57		27·71				
	8	00		27·00	5·82	·08	+ ·41	·14
25	5	00		21·90				
	8	00		20·84				
	8	05		20·84	6·43			
27	5	00		09·04				·21
	8	00		08·33				
	8	40		08·30	5·87	+ ·02		— ·21
Feb. 2	8	00	—0	33·12				
	8	00		33·13		— ·13		
3	5	00		28·26				— ·10
	4	56		28·26	5·50			

Clockrates and instrumental constants, Waikiki, Hawaiian Islands—Continued.

[Negus sidereal chronometer No. 1825. Observer: E. D. Preston.]

Date.	Epoch (sidereal time).	Correction.	Daily rate.	Collimation W.	Azimuth. E.	Azimuth. W.
1892.	*h.* *m.*	*m.* *s.*	*Seconds.*	*Seconds.*	*Seconds.*	*Seconds.*
	8 00	—0 27·63		—0·04		
Feb. 4	5 00	22·51	5·75			—0·11
	8 00	21·72		— ·04	·00	
	8 41	21·72	5·93			
6	5 00	10·64				
	8 00	09·92	5·86			
	8 41	09·83		+ ·02		·00
8	5 00	+0 01·08				
	8 00	01·85		·00		+ ·16
	8 41	01·86	5·47			
12	5 00	22·94				
	5 30	22·82				
	8 00	23·62	5·24	·00		+ ·13
13	5 00	28·18				— ·23
	5 22	28·24	5·13			
	8 00	28·84		— ·03		
14	5 00	33·31				
	4 55	33·32	4·94			
	8 00	34·11				— ·10
19	7 30	58·51	5·02	·00		
20	5 00	+1 03·01	5·01	·00		+ ·06
21	5 00	08·02				— ·05
	4 55	c8·02				
	8 00	08·51	5·07	·00	— ·46	
22	5 00	13·09	4·98	— ·05		·00
23	5 00	18·07				
	4 55	18·07	5·47			
	8 00	18·72		— ·05		+ ·05
26	7 30	35·06	5·82	— ·16	·00	
27	7 30	40·88				
	7 37	40·88	6·01			
	12 00	41·89		— ·07		
28	7 30	46·89				
	8 10	46·89	6·02			
	11 30	47·80		·05	+ ·30	
29	7 30	52·91				
	7 40	52·90	5·91			
	11 30	53·81		— ·07	+ ·32	
Mar. 1	7 30	—3 01·18				
	7 41	01·19	5·74			
	9 00	00·84		— ·05	+ ·23	
2	7 30	—2 55·44				
	7 41	55·44	5·99			
	11 30	54·55		— ·05	·00	
4	7 30	43·45	6·07	— ·07		
5	7 30	37·38				
	7 41	37·38	5·88			
	11 30	36·41		— ·03	·00	
8	10 30	19·00		— ·03		+ ·48
9	7 30	13·82	5·91			
	7 41	13·81	6·17			
	11 30	12·84		— ·03	+ ·21	
10	7 30	07·65		— ·05	+ ·38	
	7 40	07·64				
	11 30	06·68	5·77			

Clock rates and instrumental constants, Waikiki, Hawaiian Islands—Continued.

[Negus sidereal chronometer No. 1825. Observer: E. D. Preston.]

Date.	Epoch (sidereal time).		Correction.		Daily rate.	Collimation W.	Azimuth.	
							E.	W.
1892.	*h.*	*m.*	*m.*	*s.*	*Seconds.*	*Seconds.*	*Seconds.*	*Seconds.*
Mar. 13	7	30	−1	50·33	5·88	−0·02	[+0·40]	
14	7	30		44·45	5·82	− ·05	[+ ·40]	
16	7	30		32·81	5·71	·00	·00	
19	7	30		15·67	5·70	− ·06	+ ·25	
20	7	30		09·97	5·91	− ·04	[+ ·25]	
21	7	30		04·06				
	7	39		04·07	5·82			
	12	00		02·93		− ·04	[+ ·25]	
23	9	00	−0	52·05				+0·70
	9	07		52·05	5·76			
	12	00		51·52		− ·12	+ ·50	
24	7	30		46·66		− ·04	+ ·30	
	7	39		46·65				
	11	30		45·76	5·79			
29	7	30		17·73				
	7	39		17·72	5·77			
	11	30		16·88		+ ·26	+ ·70	
30	7	30		11·96		+ ·25		
	7	38		11·98	5·86			
	11	30		11·06			+ ·54	
31	7	30		06·10		·00		
	7	38		06·11				
	11	30		05·28	5·79		+ ·54	
Apr. 6	10	15	+1	29·39	5·78			− ·20
9	10	15		46·73				
	14	45		47·74	5·85	+ ·12	− ·40	− ·18
	14	58		47·73		[+ ·10]		
10	10	15		52·58	5·86	[+ ·10]		
13	10	30		10·25	5·87	[+ ·10]		− ·18
14	10	30		16·12		[+ ·10]		− ·10
15	10	30		22·07	5·95	[+ ·10]		− ·16
16	10	30		28·12	6·05			
	14	45		29·14	5·91			
	14	57		29·17		+ ·08	− ·65	− ·68
17	10	15		33·98				
	10	13		33·98				
	12	45		34·58	5·95	[+ ·12]		− ·30
18	11	30		40·23		[+ ·16]		− ·43
19	10	30		45·80	5·81			
	14	45		46·90				
	14	57		47·90	5·93	+ ·20	− ·67	− ·60
20	10	15		51·66				− ·18
	11	45		52·08	6·08	[+ ·20]		− ·33
21	10	15		57·74		[+ ·18]		− ·30
23	10	15	+2	09·71	5·99			
	14	45		11·04		+ ·15		− ·60
	14	56		11·09	6·08			
24	10	15		15·79				
	14	45		16·90		+ ·12	− ·84	− ·38
	14	56		16·89	5·77			
25	10	15		21·56				
	14	45		22·73		+ ·20		− ·35
	14	56		22·80	5·75			
29	10	15		44·57		+ ·20		− ·10
					5·86			
May 1	10	45		56·42				

Clock rates and instrumental constants, Waikiki, Hawaiian Islands—Continued.

[Negus sidereal chronometer No. 1825. Observer: E. D. Preston.]

Date.	Epoch (sidereal time).		Correction.		Daily rate.	Collimation W.	Azimuth. E.	Azimuth. W.
1892.	*h.*	*m.*	*m.*	*s.*	*Seconds.*	*Seconds.*	*Seconds.*	*Seconds.*
May 1	14	45	+2	57·17		+0·16	−0·43	−0·04
	14	52		57·19	5·48			
2	10	15	−2	58·21	5·66	[+ ·15]		− ·13
4	10	15		46·80				
	14	45		45·70		+ ·15	+ ·06	+ ·22
	14	50		45·69	5·90			
5	10	15	−2	40·90				
	14	45		39·90		+ ·07	·00	+ ·34
	14	56		39·88	5·87			
6	10	15		35·03		+ ·07		+ ·20
7	11	15		28·69	6·09			
9	10	15		17·37	5·78			
	14	45		16·22		+ ·08	+ ·07	+ ·30
	14	50		16·23	5·98			
10	10	15		11·39				
	14	45		10·44		+ ·08		+ ·32
	10	17		11·42	5·64			
11	10	15		05·75		+ ·08		+ ·07
13	12	15	−1	53·74	5·76			+ ·25
	14	45		53·15		+ ·06	·00	
	15	00		53·12	5·92			
14	10	15		48·31				− ·22
	14	45		47·17		+ ·07	− ·20	
	15	00		47·10	5·67			
15	10	15		42·64				− ·16
	14	45		41·45		+ ·16	− ·40	
	15	00		41·43	5·84			
18	10	15		25·13				− ·07
	14	45		24·10		+ ·07	− ·27	
	15	00		24·10	5·80			
21	11	45		07·37				− ·20
	14	45		06·53		+ ·15	− ·30	
	15	00		06·45	5·97			
22	11	15		01·51				− ·50
	14	45		00·58		+ ·11	− ·16	
	15	00		00·55	5·80			
23	14	45	−0	54·87		+ ·23	− ·51	− ·13
24	14	45		49·00	5·87	+ ·12	− ·09	+ ·09
25	14	00		43·46	5·76			
	17	45		42·50		+ ·11	− ·12	− ·13
	17	57		42·51	5·74			
27	14	45		31·80		+ ·08	+ ·13	+ ·20
28	14	00		26·23	5·76			
	17	45		25·41		+ ·10	− ·16	+ ·10
	17	58		25·38	5·80			
29	14	45		20·26		+ ·04	− ·22	
					5·69			
June 3	14	00	+0	07·97		[+ ·10]		− ·06
4	14	00		13·64	5·67			·00
5	14	00		19·39	5·75	+ ·10		·00
6	14	00		25·24	5·85			
7	14	00		31·00	5·76			+ ·07
	17	45		31·99		+ ·18	− ·33	
	18	02		31·98	5·89			
11	14	00		54·57				+ ·11
	17	45		55·45		+ ·11	− ·31	
	18	02		55·44				

The following table contains the Pendulum Results for 199 nights at Waikiki between June 9, 1891, and June 11, 1892:

Pendulum observations, Waikiki, Honolulu.

[Observer: E. D. Preston.]

Pendulum.	Position.	Date.	Epoch.	No. of coincidence intervals.	Time in seconds.	Time of ten coincidence intervals.	Semi-arc. Initial.	Semi-arc. Final.	Temperature.	Manometer.	Barometer.	Pressure at 0° C.
		1891.	*h.*		*s.*	*s.*	*mm.*	*mm.*	° *C.*	*mm.*	*mm.*	*mm.*
B_3	D	June 9	14·1	24	6296·0	2623·3	5·0	2·5	26·33	206·0	767·2	510
		10	14·0	20	5256·0	2628·0	5·2	2·5	25·98	206·0	767·4	511
		10	15·7	20	5280·0	2640·0	5·2	2·5	24·92	200·0	767·4	519
		11	13·9	20	5282·0	2641·0	5·2	3·0	25·43	212·5	768·0	507
		11	15·7	20	5303·5	2651·8	5·2	2·7	24·62	205·0	767·6	515
		12	14·2	32	8440·0	2637·5	5·2	[2·1]	25·47	211·0	767·4	508
		13	14·6	44	11588·0	2633·6	5·2	1·2	25·42	208·0	766·4	510
		15	14·3	32	8416·5	2630·2	5·1	2·0	25·73	217·0	766·8	501
		16	14·7	20	5244·5	2622·2	5·2	2·5	26.33	214·0	767·6	503
		17	14·9	30	7895·5	2631·8	[5·1]	2·0	26·13	219·0	767·5	499
		18	14·9	30	7901·0	2633 7	5·2	2·2	26·18	218·0	766·4	499
		19	15·6	50	13248·0	2649·6	5·2	1·2	25·68	215·0	765·5	502
		22	15·4	40	10564·0	2641·0	5·2	1·2	25·83	215·0	767·1	503
		24	15·2	40	10543·0	2635·8	5·2	1·2	25·68	213·0	767·1	505
		25	14.2	20	5257·5	2628·8	5·2	2·5	25·73	217·0	766·5	501
	D	25	16·3	20	5274·5	2637·2	5·2	2·6	24·82	218·0	766·9	502
B_2	D	26	14·8	30	7399·0	2466·3	5·2	2.5	26·28	214·0	765·4	501
	R	26	17·0	30	7455·0	2485·0	5·2	2·3	25·12	217·0	765·6	501
B_1	D	27	14·8	30	6832·5	2277·5	5·2	2·7	26·43	218·0	765·2	497
	R	27	17·1	40	9140·5	2285·1	5·0	2·0	25·32	214·0	765·6	503
B_3	D	28	14·7	30	7883·0	2627·7	5·1	2·5	26·48	212.0	766·6	504
		28	17·0	30	7923·0	2641·0	5·1	2·4	25·42	219·0	766·6	499
		30	14·8	30	7896·0	2632·0	[5·1]	2·4	26·03	214·0	765·0	502

Reduction of pendulum observations, Waikiki, Honolulu.

[Periods reduced to temperature, 15° C.; pressure, 500mm at 0° C.; arc infinitely small; sidereal time.]

Position.	Period uncorrected.	Corrections (in seventh decimal place). Arc.	Temperature.	Pressure.	Rate.	Period corrected.
	Seconds.					*Seconds.*
D	0·500 9548	−48	−470	−8	+414	0·500 9436
	531	−50	−456	−9	+426	442
	488	−50	−412	−15	+429	440
	484	−58	−433	−6	+429	416
	445	−53	−399	−12	+427	408
	497	−44	−435	−6	+427	439
	511	−31	−432	−8	+380	420
	523	−42	−445	−1	+375	410
	552	−50	−470	−2	+373	403
	517	−42	−462	+1	+380	394
	511	−46	−464	+1	+395	397
	453	−31	−443	−2	+393	370
	484	−31	−449	−2	+385	387
	503	−31	−443	−4	+372	397
	528	−50	−445	−1	+357	389
D	498	−52	−408	−2	+370	406
					June 17	410
D	0·501 0157	−50	−468	−1	+382	0·501 0020
R	081	−47	−420	−1	+397	010
					June 26	015
D	0·501 1001	−53	−474	+2	+390	0·501 0866
R	0964	−41	−428	−2	+383	876
					June 27	871
D	0·500 9532	−49	−476	−3	+376	0·500 9380
	484	−48	−432	+1	+369	374
	517	−48	−458	−2	+377	386

Pendulum observations, Waikiki, Honolulu.

[Observer: E. D. Preston.]

Pendulum.	Position.	Date.	Epoch.	No. of coincidence intervals.	Time in seconds.	Time of ten coincidence intervals.	Semi-arc. Initial.	Semi-arc. Final.	Temperature.	Manometer.	Barometer.	Pressure at 0° C.
		1891.	*h.*		*s.*	*s.*	*mm.*	*mm.*	° *C.*	*mm.*	*mm.*	*mm.*
B_3	D	July 4	18·2	34	9014·0	2651·2	5·1	1·9	24·72	217·0	767·0	503
		7	17·8	30	7931·0	2643·7	5·1	2·2	25·12	214·0	767·0	505
		8	17·6	34	8945·0	2630·9	5·1	2·1	25·43	183·0	768·8	534
		17	17·2	18	4768·0	2648·9	5·1	3·0	25·33	219·0	767·2	500
		18	17·7	32	8442·0	2638·1	4·5	[1·7]	25·33	220·0	766·7	499
		22	17·6	32	8463·0	2644·7	5·0	2·0	26·03	219·0	766·6	499
		23	17·4	30	7925·0	2641·7	5·0	2·2	26·03	222·0	766·6	496
		24	17·6	30	7950·5	2650·2	5·0	2·0	25·73	222·0	766·4	496
		25	17·6	32	8475·0	2648·4	5·0	2·0	26·44	220·0	766·7	497
		26	17·6	34	9023·0	2653·8	5·0	1·8	26·13	222·0	767·0	496
		27	17·8	38	10105·5	2659·3	5·0	1·8	25·73	222·0	767·2	497
		29	17·6	30	7932·0	2644·0	5·0	2·1	26·28	222·0	766·2	494
		30	19·3	30	7913·0	2637·7	5·0	2·1	26·44	216·0	767·9	501
	D	31	18·4	34	8986·5	2643·1	4·3	1·7	26·08	218·0	766·4	500
B_2	D	Aug. 2	16·9	34	8383·0	2465·6	5·2	2·2	25·98	218·0	764·7	498
	R	2	19·2	30	7453·0	2484·3	5·1	2·2	24·72	221·0	765·1	497
B_1	D	3	16·4	44	9996·0	2271·8	5·1	2·0	26·94	218·0	765·8	497
	R	3	19·2	36	8197·0	2277·0	5·1	2·3	25·93	216·0	766·4	502
B_3	D	4	17·4	30	7887·5	2629·2	5·0	2·0	27·80	226·0	766·6	490
		5	17·5	30	7888·5	2629·5	5·0	2·1	27·45	222·0	768·0	495
		6	17·5	30	7919·5	2639·8	5·0	2·2	26·59	224·0	767·5	494
		7	17·5	30	7937·0	2645·7	5·0	2·2	26·33	221·0	766·8	496
		10	17·7	32	8469·5	2646·7	5·0	2·0	26·33	218·0	766·9	499

Reduction of pendulum observations, Waikiki, Honolulu.

[Periods reduced to temperature, 15° C.; pressure, 500mm at 0° C.; arc infinitely small; sidereal time.]

Position.	Period uncorrected.	Corrections (in seventh decimal place).				Period corrected.
		Arc.	Temperature.	Pressure.	Rate.	
	Seconds.					*Seconds.*
D	0·500 9448	−40	−403	−2	+385	0·500 9388
	474	−45	−420	−4	+377	382
	520	−43	−433	−27	+389	406
	456	−57	−429	0	+412	382
	495	−32	−429	+1	+416	451
	471	−41	−458	+1	+400	373
	482	−44	−458	+3	+387	370
	451	−41	−445	+3	+396	364
	457	−41	−475	+2	+425	368
	438	−38	−462	+3	+435	376
	418	−38	−445	+2	+419	356
	473	−42	−468	+5	+401	369
	496	−42	−475	−1	+390	368
D	477	−30	−460	0	+386	373
					July 17	380
D	0·501 0160	−46	−456	+2	+377	0·501 0037
R	0083	−45	−403	+2	+362	0 9999
					Aug. 2	1 0018
D	0·501 1029	−42	−496	+2	+362	0·501 0855
R	1004	−46	−454	−2	+381	0883
					Aug. 3	1 0869
D	0·500 9526	−41	−531	+8	+391	0·500 9353
	525	−42	−517	+4	+403	373
	488	−44	−481	+5	+408	376
	467	−44	−470	+3	+415	371
	464	−41	−470	+1	+418	372

Pendulum observations, Waikiki, Honolulu.

[Observer: E. D. Preston.]

Pendulum.	Position.	Date.	Epoch.	No. of coincidence intervals.	Time in seconds.	Time of ten coincidence intervals.	Semi-arc. Initial.	Semi-arc. Final.	Temperature.	Manometer.	Barometer.	Pressure at 0° C.
		1891.	*h.*		*s.*	*s.*	*mm.*	*mm.*	° *C.*	*mm.*	*mm.*	*mm.*
B_3	D	Aug. 11	17·5	30	7936·0	2645·3	5·0	2·0	26·59	219·0	766·6	497
		12	17·5	30	7916·0	2638·7	5·0	2·2	27·14	216·0	767·0	500
		13	17·3	30	7914·0	2638·0	5·0	2·1	27·04	215·0	767·8	501
		14	17·8	36	9523·5	2645·4	5·0	[1·9]	26·44	217·0	767·6	501
		15	17·5	30	7925·0	2641·7	5·0	2·1	26·59	219·0	766·5	497
		16	17·5	30	7933·5	2644·5	5·0	2·1	26·33	219·0	766·4	497
		20	20·3	40	10562·5	2640·6	5·0	1·7	25·53	217·0	765·1	500
		22	19·6	20	5270·0	2635·0	5·0	3·0	25·88	226·0	767·0	493
		23	20·3	40	10548·5	2637·1	5·0	[1·7]	26·03	218·0	766·0	499
		25	19·6	22	5786·5	2630·2	5·0	3·0	26·44	222·0	766·1	494
		26	19·9	30	7907·0	2635·7	5·0	2·3	26·33	220·0	766·6	496
		29	20·4	38	10031·0	2639·7	[4·8]	[1·7]	26·03	218·0	765·9	499
	D	31	19·6	20	5287·0	2643·5	5·0	3·0	25·83	224·0	765·8	494
B_2	D	Sept. 2	19·5	32	7894·5	2467·0	5·2	2·2	26·89	215·0	765·8	500
	R	2	22·3	32	7962·0	2488·1	5·2	2·2	25·47	223·0	766·0	495
B_1	D	6	19·6	32	7273·0	2272·8	5·1	[2·8]	26·64	219·0	766·5	497
	R	6	22·2	32	7289·5	2278·0	5·1	2·8	25·68	214·0	766·5	503
B_3	D	9	19·6	20	5240·5	2620·2	5·0	[3·0]	27·19	220·0	766·7	496
		12	20·4	40	10536·0	2634·0	5·0	[1·7]	26·74	215·0	765·6	501
		15	20·4	40	10540·0	2635·0	5·0	[1·7]	26·89	218·0	766·4	498
		18	19·6	20	5253·0	2626·5	5·0	3·0	27·09	219·0	766·5	496
		19	20·4	40	10562·0	2640·5	5·0	[1·7]	26·44	216·0	765·0	499
		23	21·9	42	11120·5	2647·7	5·0	1·5	25·93	216·0	765·6	501

Reduction of pendulum observations, Waikiki, Honolulu.

[Periods reduced to temperature, 15° C.; pressure, 500mm at 0° C.; arc infinitely small; sidereal time.]

Position.	Period uncorrected.	Corrections (in seventh decimal place). Arc.	Temperature.	Pressure.	Rate.	Period corrected.
	Seconds.					*Seconds.*
D	0·500 9468	−41	−481	+2	+420	0·500 9368
	492	−44	−504	0	+426	370
	495	−42	−500	−1	+419	371
	468	−39	−475	−1	+409	362
	481	−42	−481	+2	+405	365
	471	−42	−470	+2	+399	360
	485	−36	−437	0	+375	387
	506	−56	−452	+6	+358	362
	498	−36	−458	+1	+356	361
	523	−56	−475	+5	+357	354
	503	−45	−470	+3	+359	350
	489	−34	−458	+1	+350	348
D	475	−56	−449	+5	+379	354
					Aug. 18	364
D	0·501 0154	−46	−493	0	+377	0·500 9992
R	0068	−46	−435	+4	+375	966
					Sept. 2	979
D	0·501 1024	−54	−483	+2	+366	0·501 0855
R	0999	−54	−443	−2	+357	857
					Sept. 6	856
D	0·500 9560	−56	−506	+3	+358	0·500 9359
	509	−36	−487	−1	+362	347
	506	−36	−493	+2	+370	349
	536	−56	−502	+3	+375	356
	486	−36	−475	+1	+370	346
	460	−33	−454	−1	+377	349

Pendulum observations, Waikiki, Honolulu.

[Observer: E. D. Preston.]

Pendulum.	Position.	Date.	Epoch.	No. of coincidence intervals.	Time in seconds.	Time of ten coincidence intervals.	Semi-arc.		Temperature.	Manometer.	Barometer.	Pressure at 0° C.
							Initial.	Final.				
		1891.	*h.*		*s.*	*s.*	*mm.*	*mm.*	° *C.*	*mm.*	*mm.*	*mm.*
B_3	D	Sept. 26	20·4	40	10598·0	2649·5	5·0	[1·7]	25·98	222·0	766·0	495
		27	21·1	40	10612·0	2653·0	5·0	[1·7]	25·43	219·0	764·5	497
		28	20·4	40	10598·0	2649·5	5·0	[1·7]	25·98	222·0	764·9	494
B_2	D	Oct. 4	23·0	30	7444·0	2481·3	5·2	2·5	24·97	224·0	765·3	495
	R	4	1·5	34	8487·0	2496·2	5·2	2·4	24·14	226·0	764·6	493
B_1	D	5	22·8	30	6871·0	2290·3	5·2	2·8	24·97	221·0	764·5	496
	R	5	1·1	36	8265·0	2295·8	5·2	2·3	24·09	223·0	764·3	496
B_3	D	8	22·6	20	5301·0	2650·5	5·0	[3·0]	24·92	222·0	765·6	496
		10	22·6	20	5265·0	2632·5	5·0	3·0	26·84	226·0	766·6	491
		14	22·6	20	5303·0	2651·5	5·0	3·0	25·12	229·0	767·2	491
		16	22·6	20	5320·0	2660·0	5·0	[3·0]	25·17	227·0	765·9	492
		22	23·2	38	10149·0	2670·8	5·0	[1·8]	24·53	222·0	765·4	497
		28	23·3	40	10671·0	2667·8	5·0	[1·7]	24·63	224·0	766·3	496
		30	23·3	40	10690·0	2672·5	5·0	[1·7]	24·04	225·0	766·5	496
B_2	D	31	22·9	32	7925·0	2476·6	5·2	2·3	25·63	225·0	767·4	494
	R	31	1·2	28	6979·0	2492·5	5·2	[2·4]	24·48	224·0	767·3	497
B_1	D	Nov. 1	22·6	34	7785·5	2289·8	5·2	2·8	25·37	238·0	766·4	482
	R	1	0·8	32	7343·5	2294·8	5·2	2·5	24·24	225·0	766·3	495

Reduction of pendulum observations, Waikiki, Honolulu.

[Periods reduced to temperature, 15° C.; pressure, 500mm at 0° C.; arc infinitely small; sidereal time.]

Position.	Period uncorrected.	Corrections (in seventh decimal place). Arc.	Temperature.	Pressure.	Rate.	Period corrected.
	Seconds.					*Seconds.*
D	0·500 9454	—36	—456	+ 4	+380	0·500 9346
	441	—36	—433	+ 2	+380	354
	454	—36	—456	+ 5	+378	345
					Sept. 19	350
D	0·501 0095	—50	—414	+ 4	+373	0·501 0008
R	035	—49	—379	+ 6	+373	0 9986
					Oct. 1	0 9997
D	0·501 0940	—55	—414	+ 3	+376	0·501 0850
R	913	—47	—377	+ 3	+380	872
					Oct. 5	861
D	0·500 9450	—56	—412	+ 3	+376	0·500 9361
	515	—56	—491	+ 7	+377	352
	446	—56	—420	+ 7	+379	356
	416	—56	—422	+ 6	+380	324
	378	—38	—395	+ 2	+381	328
	389	—36	—400	+ 3	+384	340
	372	—36	—375	+ 3	+384	348
					Oct. 19	344
D	0·501 0115	—47	—441	+ 5	+375	0·501 0007
R	050	—49	—393	+ 2	+370	0 9980
					Oct. 31	0 9994
D	0·501 0941	—55	—430	+14	+367	0·501 0837
R	918	—50	—383	+ 4	+367	856
					Nov. 1	846

Pendulum observations, Waikiki, Honolulu.

[Observer: E. D. Preston.]

Pendulum.	Position.	Date.	Epoch.	No. of coincidence intervals.	Time in seconds.	Time of ten coincidence intervals.	Semi-arc. Initial.	Semi-arc. Final.	Temperature.	Manometer.	Barometer.	Pressure at 0° C.
		1891.	h.		s.	s.	mm.	mm.	° C.	mm.	mm.	mm.
B_3	D	Nov. 4	22·7	20	5309·5	2654·8	5·0	2·7	24·87	229·0	766·0	490
		6	23·6	38	10068·0	2649·5	[4·8]	[1·8]	25·73	223·0	766·1	495
		7	22·6	20	5274·0	2637·0	5·0	3·0	26·23	224·0	766·0	492
		8	0·3	30	7988·5	2662·8	5·0	2·1	24·33	222·0	763·2	496
		10	22·7	20	5323·0	2661·5	5·0	3·0	23·99	229·0	761·2	487
		12	22·7	20	5308·5	2654·2	5·0	3·0	24·43	226·0	764·4	492
		13	22·7	20	5292·0	2646·0	5·0	3·0	25·68	226·0	765·2	491
		14	22·7	20	5269·0	2634·5	5·0	3·0	26·68	229·0	766·4	488
		15	22·3	10	2648·5	2648·5	5·0	3·9	25·17	227·0	765·6	491
		17	0·1	40	10724·0	2681·0	5·0	[1·9]	24·43	220·0	765·0	499
		19	3·1	32	8605·0	2689·0	3·9	[1·1]	23·05	230·0	766·2	493
		21	2·0	20	5326·5	2663·2	5·0	3·0	24·43	231·0	767·0	490
		23	2·0	20	5320·0	2660·0	5·0	3·0	24·72	230·0	767·0	491
		30	1·9	20	5330·0	2665·0	5·0	3·0	24·18	226·0	767·6	496
B_2	D	Dec. 1	2·1	32	7956·0	2486·2	5·2	2·4	24·53	224·0	765·6	496
	R	1	4·9	30	7498·0	2499·3	[5·1]	2·5	23·44	226·0	765·4	495
B_1	D	5	1·9	22	5043·5	2292·5	5·2	[3·2]	25·02	225·0	767·4	495
	R	5	3·9	32	7343·0	2294·7	5·2	2·7	24·48	224·0	767·2	497
B_3	D	6	2·0	20	5312·0	2656·0	5·0	3·0	24·77	228·0	768·0	494
		7	1·9	20	5275·5	2637·8	5·0	3·0	24·67	120·0	768·8	592
		9	1·9	20	5340·0	2670·0	5·0	3·0	23·49	227·0	766·9	496
		10	1·9	20	5337·0	2668·5	5·0	3·0	23·84	228·0	764·2	491
		11	1·8	10	2703·0	2703·0	5·0	[3·9]	20·63	236·0	764·4	489

Reduction of pendulum observations, Waikiki, Honolulu.

[Periods reduced to temperature, 15° C.; pressure, 500mm at 0° C.; arc infinitely small; sidereal time.]

Position.	Period uncorrected.	Corrections (in seventh decimal place). Arc.	Temperature.	Pressure.	Rate.	Period corrected.
	Seconds.					*Seconds.*
D	0·500 9435	−51	−410	+ 8	+367	0·500 9349
	454	−36	−445	+ 4	+371	348
	498	−56	−446	+ 6	+363	365
	406	−42	−387	+ 3	+358	338
	411	−56	−373	+10	+358	350
	437	−56	−391	+ 6	+368	364
	466	−56	−443	+ 7	+370	344
	508	−56	−485	+ 9	+377	353
	457	−70	−422	+ 7	+384	356
	342	−39	−391	+ 1	+389	302
	314	−20	−334	+ 6	+378	344
	405	−56	−391	+ 8	+383	349
	416	−56	−403	+ 7	+379	343
	398	−56	−381	+ 3	+372	336
					Nov. 17	346
D	0·501 0075	−49	−395	+ 3	+371	0·501 0005
R	023	−49	−350	+ 4	+370	0 9998
					Dec. 1	1 0002
D	0·501 0929	−61	−416	+ 4	+373	0·501 0829
R	918	−53	−393	+ 2	+373	847
					Dec. 5	838
D	0·500 9430	−56	−405	+ 5	+378	0·500 9352
	495	−56	−401	−73	+380	345
	381	−56	−352	+ 3	+376	352
	386	−56	−367	+ 7	+368	338
	266	−70	−234	+ 9	+375	346

Pendulum observations, Waikiki, Honolulu.

[Observer: E. D. Preston.]

Pendulum.	Position.	Date.	Epoch.	No. of coincidence intervals.	Time in seconds.	Time of ten coincidence intervals.	Semi-arc. Initial.	Semi-arc. Final.	Temperature.	Manometer.	Barometer.	Pressure at 0° C.
		1891.	*h.*		*s.*	*s.*	*mm.*	*mm.*	° *C.*	*mm.*	*mm.*	*mm.*
B_3	D	Dec. 12	1·9	20	5428·0	2714·0	5·0	[3·0]	20·00	233·0	763·6	494
		15	2·7	40	10753·0	2688·2	5·0	[1·8]	22·55	228·0	769·0	498
		17	1·9	20	5330·5	2665·2	5·0	3·0	23·84	229·0	766·8	494
		19	2·0	22	5858·0	2662·7	5·0	[2·8]	24·09	227·0	768·5	496
		21	2·7	42	11193·0	2665·0	5·0	1·8	24·53	222·0	765·8	497
		22	1·4	10	2644·0	2644·0	5·0	3·9	25·12	226·5	766·8	493
		23	1·9	20	5316·0	2658·0	5·0	3·0	24·72	231·0	769·8	492
		24	1·9	20	5327·0	2663·5	5·0	3·0	24·33	231·0	770·8	494
		25	1·9	20	5339·0	2669·5	5·0	[3·0]	23·99	231·0	771·1	495
		26	1·9	20	5347·0	2673·5	5·0	[3·0]	23·50	231·0	770·4	495
		27	1·9	20	5330·0	2665·0	5·0	3·0	24·24	231·0	768·8	492
		28	1·9	20	5326·0	2663·0	5·0	3·0	24·43	228·0	768·7	494
		29	1·9	20	5315·0	2657·5	5·0	3·0	23·99	230·5	768·3	493
		30	1·9	20	5316·0	2658·0	5·0	[3·0]	23·94	231·0	768·3	492
		1892.										
B_1	D	Jan. 1	1·6	12	2749·0	2290·8	5·2	4·0	23·70	232·0	767·3	490
B_2	D	2	1·9	22	5528·0	2512·7	5·1	3·1	21·57	234·5	767·8	493
B_3	D	5	1·9	18	4822·0	2678·9	5·0	3·1	21·77	232·0	764·7	493
	R	5	5·2	24	6461·0	2692·1	5·0	2·4	20·05	172·0	764·8	550
B_1	D	6	5·4	22	5121·0	2327·7	5·2	3·2	19·70	226·0	762·4	499
	R	6	7·3	34	7946·0	2337·1	5·2	2·2	18·42	204·0	761·8	520

Reduction of pendulum observations, Waikiki, Honolulu.

[Periods reduced to temperature, 15° C.; pressure, 500mm at 0° C.; arc infinitely small; sidereal time.]

Position.	Period uncorrected.	Corrections (in seventh decimal place). Arc.	Temperature.	Pressure.	Rate.	Period corrected.
	Seconds.					*Seconds.*
D	0·500 9228	−56	−208	+ 5	+368	0·500 9337
	317	−38	−313	+ 2	+368	336
	398	−56	−367	+ 5	+363	343
	407	−52	−377	+ 3	+344	325
	398	−38	−395	+ 2	+348	315
	473	−70	−420	+ 6	+363	352
	423	−56	−403	+ 6	+387	357
	404	−56	−387	+ 5	+382	348
	383	−56	−373	+ 4	+373	331
	369	−56	−353	+ 4	+384	348
	398	−56	−383	+ 6	+376	341
	406	−56	−391	+ 5	+372	336
	425	−56	−373	+ 6	+341	343
	423	−56	−371	+ 6	+338	340
					Dec. 18	341
D	0·501 0937	−75	−361	+ 8	+333	0·501 0842
D	0·500 9969	−58	−273	+ 6	+349	0·500 9993
D	0·500 9350	−57	−281	+ 6	+324	0·500 9342
R	304	−46	−210	−39	+324	333
					Jan. 5	338
D	0·501 0763	−61	−195	+ 1	+320	0·501 0828
R	720	−46	−142	−16	+320	836
					Jan. 6	832

Pendulum observations, Waikiki, Honolulu.

[Observer: E. D. Preston.]

Pendulum.	Position.	Date.	Epoch.	No. of coincidence intervals.	Time in seconds.	Time of ten coincidence intervals.	Semi-arc. Initial.	Semi-arc. Final.	Temperature.	Manometer.	Barometer.	Pressure at 0° C.
		1892.	*h.*		*s.*	*s.*	*mm.*	*mm.*	° *C.*	*mm.*	*mm.*	*mm.*
B_2	D	Jan. 10	5·4	8	1981·0	2476·2	[5·0]	[4·2]	20·54	55·0	763·4	658
B_3	D	11	6·5	36	9722·0	2700·6	5·0	1·9	20·45	222·0	764·4	503
		12	6·3	30	8145·0	2715·0	4·5	2·0	19·90	246·0	762·8	481
B_1	D	16	5·2	18	4156·0	2308·9	5·0	3·2	22·51	229·0	762·6	491
B_2	D	18	6·4	20	5034·0	2517·0	4·8	2·9	21·23	230·0	762·6	493
	R	18	8·3	26	6574·0	2528·5	4·5	2·2	21·03	228·5	762·4	495
B_3	D	19	5·4	8	2678·8	2678·8	4·2	3·3	22·85	253·0	763·0	470
		20	6·2	30	8056·0	2685·3	4·2	2·0	22·65	227·0	765·0	496
		21	6·4	36	9612·0	2670·0	4·8	1·8	23·15	227·0	766·4	496
		23	6·4	36	9621·0	2672·5	[4·8]	[1·8]	22·95	226·0	766·2	497
		24	6·1	28	7531·0	2689·6	4·2	2·0	22·02	227·0	768·8	500
		25	6·2	30	8085·0	2695·0	4·2	[2·0]	21·92	227·0	769·8	501
		27	6·2	30	8080·0	2693·3	4·3	2·0	22·46	227·0	766·7	498
		Feb. 2	6·1	40	10720·0	2680·0	4·3	1·2	22·70	222·0	766·8	501
B_1	D	3	5·3	24	5527·0	2302·9	4·8	2·8	23·25	230·0	768·0	495
	R	3	7·0	24	5525·0	2302·3	4·8	2·8	22·85	229·0	768·0	496
B_2	D	4	5·3	22	5480·0	2490·9	4·4	2·8	23·85	226·0	767·0	497
	R	4	7·1	28	7020·5	2507·3	4·4	2·1	23·10	222·0	767·2	501

Reduction of pendulum observations, Waikiki, Honolulu.

[Periods reduced to temperature, 15° C.; pressure, 500mm at 0° C.; arc infinitely small; sidereal time.]

Position.	Period uncorrected.	Corrections (in seventh decimal place). Arc.	Temperature.	Pressure.	Rate.	Period corrected.
	Seconds.					*Seconds.*
D	0·501 0117	−75	−230	−125	+322	0·501 0009
D	0·500 9274	−39	−226	− 2	+331	0·500 9338
	225	−35	−203	+ 15	+318	320
					Jan. 12	329
D	0·501 0851	−59	−312	+ 7	+328	0·501 0815
D	0·500 9952	−52	−258	+ 6	+336	0·500 9984
R	907	−38	−250	+ 4	+336	959
					Jan. 18	972
D	0·500 9350	−50	−326	+ 24	+351	0·500 9349
	327	−33	−317	+ 3	+326	306
	381	−36	−338	+ 3	+305	315
	372	−36	−330	+ 2	+313	321
	312	−33	−291	0	+338	326
	294	−33	−287	− 1	+373	346
	299	−34	−310	+ 2	+340	297
	346	−34	−320	− 1	+340	331
					Jan. 25	324
D	0·501 0879	−50	−342	+ 4	+319	0·501 0810
R	882	−50	−326	+ 3	+319	828
					Feb. 3	819
D	0·501 0057	−45	−367	+ 2	+344	0·500 9991
R	9991	−36	−336	− 1	+344	962
					Feb. 4	976

Pendulum observations, Waikiki, Honolulu.

[Observer: E. D. Preston.]

Pendulum.	Position.	Date.	Epoch.	No. of coincidence intervals.	Time in seconds.	Time of ten coincidence intervals.	Semi-arc. Initial.	Semi-arc. Final.	Temperature.	Manometer.	Barometer.	Pressure at 0° C.
		1892.	*h.*		*s.*	*s.*	*mm.*	*mm.*	° *C.*	*mm.*	*mm.*	*mm.*
B_3	D	Feb. 6	6·6	40	10714·0	2678·5	4·4	1·4	23·40	227·0	766·6	495
		8	6·8	32	8587·0	2683·4	[4·0]	1·5	23·15	229·0	767·8	496
		11	5·5	12	3198·0	2665·0	4·4	3·1	23·00	233·0	766·2	490
		12	6·1	28	7504·5	2680·2	4·4	2·0	22·42	229·0	765·4	494
		13	6·1	28	7521·0	2686·1	4·4	2·0	21·59	231·0	763·6	493
		14	6·5	38	10110·0	2660·5	4·4	1·6	23·54	220·0	761·3	497
B_1	D	19	8·7	16	3704·0	2315·0	4·8	3·3	20·98	230·0	763·2	494
		20	7·0	32	7375·0	2304·7	4·8	2·2	22·31	226·0	764·5	496
		20	9·2	32	7388·5	2308·9	4·8	2·2	21·47	229·0	764·8	496
B_2	D	21	6·6	46	11431·0	2485·0	4·7	1·6	23·69	221·0	765·9	500
	R	21	9·3	26	6505·0	2501·9	4·4	2·2	22·31	227·0	766·1	497
B_3	D	22	6·4	36	9519·0	2644·2	4·4	1·7	24·92	223·0	767·6	497
		23	6·7	44	11655·5	2649·0	4·4	1·4	24·38	223·0	767·4	498
		26	8·6	10	2667·0	2667·0	4·2	3·1	23·44	229·0	767·7	494
		27	9·7	40	10717·0	2679·2	4·2	1·5	23·05	222·0	767·4	501
		28	9·4	30	8034·0	2678·0	4·2	[2·1]	23·05	227·0	768·6	498
		29	9·4	30	8044·5	2681·5	4·2	[2·1]	23·05	228·0	769·0	498
B_1	D	Mar. 1	7·6	18	4146·0	2303·3	4·7	3·0	23·40	230·0	768·0	494
	R	1	8·8	16	3687·5	2304·7	4·5	[3·1]	22·95	232·0	768·0	493

Reduction of pendulum observations, Waikiki, Honolulu.

[Periods reduced to temperature, 15° C.; pressure, 500^{mm} at 0° C.; arc infinitely small; sidereal time.]

Position.	Period uncorrected.	Corrections (in seventh decimal place).				Period corrected.
		Arc.	Temperature.	Pressure.	Rate.	
	Seconds.					*Seconds.*
D	0·500 9351	−27	−349	+4	+340	0·500 9319
	334	−25	−338	+3	+317	291
	398	−49	−332	+8	+317	342
	345	−35	−308	+5	+304	311
	325	−35	−373	+6	+298	221
	414	−29	−354	+2	+292	325
					Feb. 10	302
D	0·501 0822	−58	−248	+5	+291	0·501 0812
	871	−41	−303	+3	+291	821
	851	−41	−268	+3	+291	836
					Feb. 20	823
D	0·501 0081	−32	−361	0	+294	0·500 9982
R	012	−37	−303	+2	+294	968
					Feb. 21	975
D	0·500 9472	−31	−412	+2	+289	0·500 9320
	455	−27	−389	+2	+317	358
	392	−47	−350	+5	+338	338
	348	−27	−334	−1	+349	335
	353	−34	−334	+2	+349	336
	341	−34	−334	+2	+343	318
					Feb. 26	334
D	0·501 0878	−52	−349	+5	+333	0·501 0815
R	871	−51	−330	+6	333	829
					Mar. 1	822

Pendulum observations, Waikiki, Honolulu.

[Observer: E. D. Preston.]

Pendulum.	Position.	Date.	Epoch.	No. of coincidence intervals.	Time in seconds.	Time of ten coincidence intervals.	Semi-arc. Initial.	Semi-arc. Final.	Temperature.	Manometer.	Barometer.	Pressure at 0° C.
		1892.	h.		s.	s.	mm.	mm.	° C.	mm.	mm.	mm.
B_2	D	Mar. 2	9·2	30	7512·0	2504·0	4·7	2·1	22·80	225·0	767·6	499
	R	2	11·0	16	4022·0	2513·8	4·3	3·0	22·22	230·0	767·4	495
B_3	D	4	8·8	14	3750·0	2678·6	4·2	3·0	23·30	231·0	766·5	492
		5	9·8	40	10741·0	2685·2	4·2	1·5	22·56	221·0	765·0	501
		8	11·4	16	4346·0	2716·2	4·2	2·9	19·65	236·0	764·9	492
		9	9·5	32	8748·0	2733·8	4·2	1·8	18·71	231·0	765·3	498
		10	9·4	30	8179·0	2726·3	4·2	[2·0]	19·30	230·0	765·8	500
		13	9·4	30	8087·0	2695·7	4·2	[2·0]	21·38	225·0	767·7	502
		14	9·7	38	10241·0	2695·0	4·2	1·5	21·92	222·0	767·5	503
B_1	D	16	8·3	18	4149·0	2305·0	4·2	2·9	23·30	236·0	768·0	488
	R	16	9·6	14	3229·5	2306·8	4·2	3·1	22·80	238·0	768·4	488
B_2	D	19	8·4	16	3996·0	2497·5	4·0	2·9	23·40	235·0	768·1	489
	R	19	9·5	12	3012·0	2510·0	3·9	[2·9]	22·95	237·0	768·3	489
B_3	D	20	8·9	18	4830·0	2683·3	3·2	2·0	23·15	233·0	768·8	493
		21	9·5	34	9141·0	2688·5	3·4	1·3	23·10	228·0	769·5	498
		23	10·5	30	8058·0	2686·0	3·6	[1·5]	22·85	229·0	769·3	497
		24	9·4	30	8039·0	2679·7	3·5	1·4	23·89	249·0	769·2	477
		29	9·4	30	8058·0	2686·0	3·45	1·5	23·15	225·0	769·3	500
		30	9·5	34	9106·0	2678·2	3·4	1·3	23·69	224·0	769·3	500
		31	9·4	32	8578·0	2680·6	3·4	1·5	23·59	225·0	770·0	500

Reduction of pendulum observations, Waikiki, Honolulu.

[Periods reduced to temperature, 15° C.; pressure, 500mm at 0° C.; arc infinitely small; sidereal time.]

Position.	Period uncorrected.	Corrections (in seventh decimal place).				Period corrected.
		Arc.	Temperature.	Pressure.	Rate.	
	Seconds.					*Seconds.*
D	0·501 0004	—39	—324	+ 1	+347	0·500 9989
R	0 9965	—47	—300	+ 4	+347	969
					Mar. 2	979
D	0·500 9351	—46	—344	+ 6	+352	0·500 9319
	327	—27	—314	— 1	+341	326
	221	—44	—193	+ 6	+343	333
	162	—30	—154	+ 2	+358	338
	186	—33	—178	0	+335	310
	291	—33	—265	— 2	+341	332
	294	—27	—287	— 2	+338	316
					Mar. 9	325
D	0·501 0870	—44	—344	+ 9	+331	0·501 0822
R	861	—47	—324	+ 9	+331	830
					Mar. 16	826
D	0·501 0030	—42	—349	+ 9	+331	0·500 9979
R	0 9980	—41	—330	+ 9	+331	949
					Mar. 19	964
D	0·500 9334	—24	—338	+ 6	+343	0·500 9321
	316	—18	—336	+ 2	+338	302
	325	—22	—326	+ 2	+334	313
	346	—20	—369	+18	+336	311
	325	—20	—338	0	+335	302
	352	—18	—361	0	+340	313
	344	—20	—356	0	+336	304
					Mar. 26	309

Pendulum observations, Waikiki, Honolulu.

[Observer: E. D. Preston.]

Pendulum.	Position.	Date.	Epoch.	No. of coincidence intervals.	Time in seconds.	Time of ten coincidence intervals.	Semi-arc. Initial.	Semi-arc. Final.	Temperature.	Manometer.	Barometer.	Pressure at 0° C.
		1892.	*h.*		*s.*	*s.*	*mm.*	*mm.*	° *C.*	*mm.*	*mm.*	*mm.*
B_1	D	Apr. 6	11·3	18	4148·5	2304·7	5·0	3·2	22·90	231·0	768·0	494
	R	6	12·3	8	1841·0	2301·2	5·0	4·1	22·75	235·0	768·0	491
B_2	D	9	11·4	18	4465·0	2480·6	5·0	3·5	24·49	228·0	766·2	492
	R	9	13·6	42	10532·0	2507·6	5·0	2·0	23·05	224·0	765·8	498
B_3	D	13	11·6	20	5319·0	2659·5	4·5	2·9	24·28	229·0	766·5	491
		14	11·4	16	4247·0	2654·4	4·6	3·0	24·82	226·0	766·1	493
		15	11·1	12	3181·0	2650·8	4·6	3·4	24·22	228.0	766.4	492
B_1	D	16	11·3	20	4584·5	2292·2	5·0	3·1	24·77	227·0	766·8	493
	R	16	12·9	28	6434·5	2298·0	4·9	2·7	23·94	227·0	766·8	495
B_2	D	17	11·3	16	3961·0	2475·6	5·0	3·2	25·07	227·0	767·0	493
	R	17	12·3	8	1989·0	2486·2	4·9	3·1	24·53	229·0	767·0	492
B_3	D	18	12·5	26	6932·0	2666·1	4·6	2·0	24·04	225·0	767·6	496
		19	12·2	38	10129·0	2665·5	4·6	1·8	24·33	222·0	767·4	499
		20	11·5	18	4788·5	2660·3	4·6	2·8	24·77	229·0	766·8	491
		21	11·4	16	4262·0	2663·8	4·6	3·0	24·33	226·0	766·7	494
		23	11·4	16	4243·0	2651·9	4·6	3·0	25·17	226·0	765·4	492
		24	11·3	14	3724·5	2660·4	4·5	[2·9]	24·33	227·0	765·7	494
		25	11·3	14	3722·5	2658·9	4·6	3·0	24·53	227·0	766·4	493
		29	11·0	6	1590·0	2650·0	4·6	4·1	24·58	228·0	765·4	491
		May 1	12·4	32	8478·0	2649·4	4·5	[1·8]	25·63	224·0	767·0	495

Reduction of pendulum observations, Waikiki, Honolulu.

[Periods reduced to temperature, 15° C.; pressure, 500^{mm} at 0° C.; arc infinitely small; sidereal time.]

Position.	Period uncorrected.	Corrections (in seventh decimal place). Arc.	Temperature.	Pressure.	Rate.	Period corrected.
	Seconds.					*Seconds.*
D	0·501 0871	−59	−328	+5	+335	0·501 0824
R	888	−73	−322	+7	+335	835
					Apr. 6	830
D	0·501 0099	−63	−394	+6	+339	0·500 9987
R	0 9990	−41	−334	+2	+339	956
					Apr. 9	972
D	0·500 9418	−48	−385	+7	+340	0·500 9332
	436	−50	−408	+6	+345	329
	449	−56	−383	+6	+351	367
					Apr. 14	343
D	0·501 0930	−57	−405	+6	+343	0·501 0817
R	903	−50	−371	+4	+343	829
					Apr. 16	823
D	0·501 0119	−59	−418	+6	+345	0·500 9993
R	075	−56	−395	+6	+345	975
					Apr. 17	984
D	0·500 9395	−37	−375	+3	+337	0·500 9323
	397	−34	−387	+1	+344	321
	415	−48	−405	+7	+353	322
	403	−50	−387	+5	+347	318
	445	−50	−422	+6	+353	332
	415	−48	−387	+5	+335	320
	420	−50	−395	+6	+334	315
	452	−67	−398	+7	+340	334
	454	−33	−441	+4	+318	302
					Apr. 25	321

Pendulum observations, Waikiki, Honolulu.

[Observer: E. D. Preston.]

Pendulum.	Position.	Date.	Epoch.	No. of coincidence intervals.	Time in seconds.	Time of ten coincidence intervals.	Semi-arc. Initial.	Semi-arc. Final.	Temperature.	Manometer.	Barometer.	Pressure at 0° C.
		1892.	*h.*		*s.*	*s.*	*mm.*	*mm.*	° *C.*	*mm.*	*mm.*	*mm.*
B_1	D	May 2	11·2	14	3186·0	2275·7	5·0	3·6	26·28	225·0	767·0	492
	R	2	12·9	26	5940·0	2284·6	4·8	2·8	24·97	221·0	766·7	498
B_2	D	4	11·4	16	3992·0	2495·0	5·0	3·3	23·20	225·0	763·8	496
	R	4	12·8	22	5518·0	2508·2	4·9	2·9	22·46	223·0	763·8	498
B_3	D	5	11·4	14	3744·0	2674·3	4·6	3·1	23·10	225·0	764·7	497
		6	11·4	14	3762·0	2687·1	4·6	3·1	22·16	227·0	764·8	496
		9	12·4	38	10164·0	2674·7	4·5	1·5	23·94	226·0	768·1	496
		10	11·5	16	4249·0	2655·6	4·6	3·1	24·68	231·0	768·0	490
		11	11·7	14	3722·0	2658·6	4·5	2·9	24·72	231·0	767·2	490
		13	13·3	26	6900·0	2653·8	4·6	2·2	25·17	231·0	768·4	490
		14	11·5	16	4222·0	2638·8	4·6	2·9	26·33	227·0	768·4	493
B_1	D	15	11·2	18	4099·0	2277·2	5·0	3·1	26·13	226·0	768·5	493
	R	15	12·8	26	5937·0	2283·5	4·9	2·8	25·17	226·0	769·0	496
B_2	D	18	11·4	16	3950·0	2468·8	4·9	3·1	25·42	225·0	766·1	494
	R	18	13·1	32	7956·0	2486·2	4·9	2·1	24·67	205·0	766·4	514
B_3	D	21	13·0	32	8478·0	2649·4	4·6	1·9	25·63	221·0	766·9	498
		22	13·0	36	9554·0	2653·9	4·6	1·9	25·32	219·0	767·9	501
		23	13·6	42	11154·0	2655·7	4·6	1·5	25·27	219·0	768·3	502
		24	14·4	20	5305·0	2652·5	4·6	3·0	24·67	224·0	770·2	499

Reduction of pendulum observations, Waikiki, Honolulu.

[Periods reduced to temperature, 15° C.; pressure, 500mm at 0° C.; arc infinitely small; sidereal time.]

Position.	Period uncorrected.	Corrections (in seventh decimal place).				Period corrected.
		Arc.	Temperature.	Pressure.	Rate.	
	Seconds.					*Seconds.*
D	0·501 1010	−65	−468	+ 6	+328	0·501 0811
R	1 0967	−50	−414	+ 2	+328	833
					May 2	822
D	0·501 0040	−60	−340	+ 3	+342	0·500 9985
R	0 9987	−53	−310	+ 2	+342	968
					May 4	976
D	0·500 9365	−52	−336	+ 2	+340	0·500 9319
	321	−52	−297	+ 3	+353	328
	364	−29	−371	+ 3	+347	314
	432	−52	−402	+ 8	+327	313
	421	−48	−403	+ 8	+334	312
	439	−39	−422	+ 8	+343	329
	492	−49	−470	+ 6	+329	308
					May 10	318
D	0·501 1002	−57	−462	+ 6	+339	0·501 0828
R	1 0972	−51	−422	+ 3	+339	841
					May 15	834
D	0·501 0147	−56	−432	+ 5	+336	0·501 0000
R	075	−41	−401	−11	+336	0 9958
					May 18	0 9979
D	0·500 9454	−35	−441	+ 2	+346	0·500 9326
	438	−35	−428	− 1	+336	310
	432	−30	−426	− 2	+336	310
	443	−50	−401	+ 1	+340	333

Pendulum observations, Waikiki, Honolulu.

[Observer: E. D. Preston.]

Pendulum.	Position.	Date.	Epoch.	No. of coincidence intervals.	Time in seconds.	Time of ten coincidence intervals.	Semi-arc. Initial.	Semi-arc. Final.	Temperature.	Manometer.	Barometer.	Pressure at 0° C.
		1892.	*h.*		*s.*	*s.*	*mm.*	*mm.*	° *C.*	*mm.*	*mm.*	*mm.*
B_3	D	May 25	15·7	42	11175·0	2660·7	4·7	1·4	24·62	217·0	768·5	505
		26	14·4	20	5300·0	2650·0	4·7	2·9	24·92	224·0	768·9	498
		28	15·7	42	11167·0	2658·8	4·7	1·5	24·68	214·0	766·2	505
B_1	D	June 3	14·7	16	3655·0	2284·4	4·9	3·5	25·02	225·0	766·4	495
B_2	D	4	14·7	16	3953·0	2470·6	5·0	3·3	25·12	226·0	767·6	495
B_3	D	5	14·7	16	4243·0	2651·9	4·7	3·0	25·12	244·0	767·6	477
		7	14·7	16	4235·0	2646·9	4·6	3·0	25·48	227·0	767·9	493
		11	15·7	28	7421·0	2650·4	[4·4]	2·1	25·48	225·0	767·8	495

Reduction of pendulum observations, Waikiki, Honolulu.

[Periods reduced to temperature, 15° C.; pressure, 500mm at 0° C.; arc infinitely small; sidereal time.]

Position.	Period uncorrected.	Corrections (in seventh decimal place).				Period corrected.
		Arc.	Temperature.	Pressure.	Rate.	
	Seconds.					*Seconds.*
D	0·500 9414	−29	−399	− 4	+333	0·500 9315
	452	−50	−412	+ 2	+333	325
	421	−31	−402	− 4	+336	320
					May 25	320
D	0·501 0968	−62	−416	+ 4	+329	0·501 0823
					June 3	0·501 0823
D	0·501 0140	−60	−420	+ 4	+334	0·500 9998
					June 4	0·500 9998
D	0·500 9445	−52	−420	+18	+339	0·500 9330
	463	−50	−435	+ 6	+342	326
	450	−36	−435	+ 4	+342	325
					June 8	327

Pendulum No. 3 was invariably swung in the direct position. No. 1 and No. 2 were used both in the direct and reverse, when possible. In order to reduce the results to the same conditions, a correction is applied when only one position was obtained. This correction is deduced from all the reversals during the year, as shown in the following table:

Differences between the periods of pendulums B_1 and B_2 for the direct and reverse positions.

Date.	B_2 D—R	B_1 D—R
1891–1892.	*s.*	*s.*
June 26	+0·000 0010	—0·000 0010
Aug. 2	+ 38	— 28
Sept. 4	+ 26	— 02
Oct. 4	+ 22	— 22
31	+ 27	— 19
Dec. 3	+ 07	— 18
Jan. 6		— 08
18	+ 25	
Feb. 3	+ 29	— 18
21	+ 14	
Mar. 1	+ 20	— 14
17	+ 30	— 08
Apr. 8	+ 31	— 11
16	+ 18	— 12
May 3	+ 17	— 22
16	+ 42	— 13
Means	+0·000 0024	—0·000 0015

The signs remain the same throughout the year, and we have a mean value of

$$\text{for } B_1\ (D-R) = -0{\cdot}000\ 0015\ s.$$
$$B_2\ (D-R) = +0{\cdot}000\ 0024$$

The correction, therefore, to reduce those observations made in the direct position to the mean of those made in both is

s.
+0·000 0008 for B_1 and
—0·000 0012 " B_2

These corrections being applied, we have the following table. Mean values are taken for those periods during which continuous observations were made with each pendulum:

B_1			B_2			B_3		
Date.	Period.	No. of obs.	Date.	Period.	No. of obs.	Date.	Period.	No. of obs.
1891–'92.	*Seconds.*		1891–'92.	*Seconds.*		1891–'92.	*Seconds.*	
June 27	0·501 0871	2	June 26	0·501 0015	2	June 17	0·500 9410	16
Aug. 3	869	2	Aug. 2	1 0018	2	July 14	380	17
Sept. 6	856	2	Sept. 2	0 9979	2	Aug. 17	364	18
Oct. 5	861	2	Oct. 4	0 9997	2	Sept. 19	350	9
Nov. 1	846	2	31	0 9994	2	Oct. 19	344	7
Dec. 5	838	2	Dec. 1	1 0002	2	Nov. 17	346	14
Jan. 1	850	1	Jan. 2	0 9981	1	Dec. 18	341	19
6	832	2	10	0 9997	1	Jan. 5	338	2
16	823	1	18	72	2	12	329	2
Feb. 3	819	2	Feb. 4	76	2	26	324	8
20	831	2	21	75	2	Feb. 10	302	6
Mar. 1	822	2	Mar. 2	79	2	25	334	6
16	826	2	19	64	2	Mar. 9	325	7
Apr. 6	830	2	Apr. 9	72	2	25	309	7
16	823	2	17	84	2	Apr. 14	343	3
May 2	822	2	May 4	76	2	24	321	9
15	834	2	18	79	2	May 9	318	7
June 3	831	1	June 4	86	1	24	320	7
						June 8	327	3

Comparing the preceding values with the respective mean values for each pendulum, we get the following differences. The column headed (t) gives the excess (minus) or defect (plus) of the observed period of the pendulum over the mean value for the year in units of the seventh decimal place of sidereal seconds. The following column, headed (g), gives the excess or defect of the force of gravity, the unit being one ten-millionth part of gravity. The sign plus indicates in this column that the observed force of gravity at the given date is less than the mean value for the year. The fourth column gives mean values and dates for successive groups of three:

B_1				B_2				B_3			
Mean value, 0·501 0838s.				Mean value, 0·500 9986s.				Mean value, 0·500 9338s.			
Date.	t.	g.	Means.	Date.	t.	g.	Means.	Date.	t.	g.	Means.
1891–'92.				1891–'92.				1891–'92.			
June 27	−33	+132	Aug. 2	June 26	−29	+116	July 31	June 17	−72	+287	July 12
Aug. 3	−31	+124	+109	Aug. 2	−32	+128	+72	July 14	−42	+168	+186
Sept. 6	−18	+ 72		Sept. 2	+ 7	− 28		Aug. 17	−26	+104	
Oct. 5	−23	+ 92	Nov. 3	Oct. 4	−11	+ 44	Nov. 1	Sept. 19	−12	+ 48	Oct. 19
Nov. 1	−08	+ 32	+41	31	−08	+ 32	+47	Oct. 19	−06	+ 24	+35
Dec. 5	00	00		Dec. 1	−16	+ 64		Nov. 17	−08	+ 32	
Jan. 1	−12	+ 48	Jan. 8	Jan. 2	+ 5	− 20	Jan. 10	Dec. 18	−03	+ 12	Jan. 1
6	+06	− 24	−12	10	−11	+ 44	−10	Jan. 5	00	00	−08
16	+15	− 60		18	+14	− 56		12	+09	− 36	
Feb. 3	+19	− 76	Feb. 16	Feb. 4	+10	− 40	Feb. 19	26	+14	− 56	Feb. 10
20	+07	− 28	−56	21	+11	− 44	−37	Feb. 10	+36	−144	−72
Mar. 1	+16	− 64		Mar. 2	+07	− 28		25	+04	− 16	
16	+12	− 48	Apr. 2	19	+22	− 88	Apr. 5	Mar. 9	+13	− 52	Mar. 27
Apr. 6	+08	− 32	−47	Apr. 9	+14	− 56	−51	25	+29	−116	−49
16	+15	− 60		17	+02	− 08		Apr. 14	−05	+ 20	
May 2	+16	− 64	May 17	May 4	+10	− 40	May 19	24	+17	− 68	May 16
15	+04	− 16	−36	18	+07	− 28	−23	May 9	+20	− 80	−66
June 3	+07	− 28		June 4	00	00		24	+18	− 72	
								June 8	+11	− 44	

The above variations of gravity are shown graphically in illustration No. 25. The smallest square in the diagram represents one millionth part of gravity, and for this argument the plotting is done to the nearest line in the diagram. For the time argument the nearest day is plotted, four days representing the space between vertical lines.

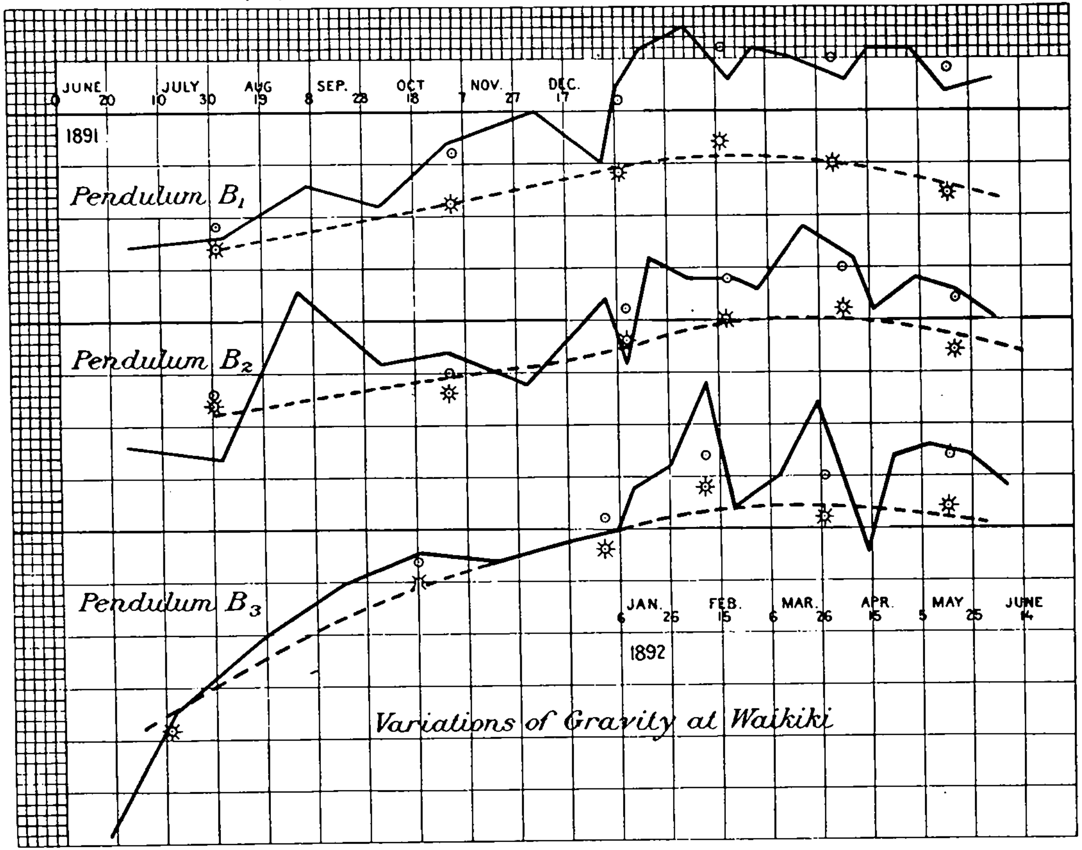
From the observations at Washington cited in the first part of this paper, it appears that the time of oscillation of the pendulums was decreased on account of use between April, 1891, and December, 1892, by the following amounts:

	s.
for B_1	0·000 0039
B_2	17
B_3	17

It is assumed that this change took place during the time the pendulums were in actual use, that is from June 9, 1891, to October 1, 1892. The above decrease in the periods correspond to an increase in the force of gravity of

15·6 millionths of g for pendulum B_1
6·8 " " " " " B_2
6·8 " " " " " B_3

Supposing that this effect was proportional to the time, we have the following table, which gives the apparent increase in gravity after cor-


JUNE 0 20
JULY 10 30
AUG 19
SEP. 8 28
OCT 18
NOV. 7 27
DEC. 17
1891
Pendulum B_1
Pendulum B_2
Pendulum B_3
JAN. 6 26
FEB. 15
MAR. 6 26
APR. 15
MAY 5 25
JUNE 14
1892
Variations of Gravity at Waikiki

recting for wear of the knives. These values are plotted in illustration No. 25 and are indicated by the sign ☼:

Pendulum.	Date.	dg.	Days from June 9, 1891.	Effect of wear.	Apparent increase of g.
	1891–'92.				
B_1	Aug. 2	+109	54	+ 17	+126
	Nov. 3	+ 41	147	+ 48	+ 89
	Jan. 8	− 12	213	+ 69	+ 57
	Feb. 18	− 56	254	+ 83	+ 27
	Apr. 2	− 47	298	+ 97	+ 50
	May 17	− 36	343	+111	+ 75
B_2	July 31	+ 72	52	+ 7	+ 79
	Nov. 1	+ 47	145	+ 21	+ 68
	Jan. 10	− 10	215	+ 31	+ 21
	Feb. 19	− 37	255	+ 36	− 1
	Apr. 5	− 51	301	+ 43	− 8
	May 19	− 23	345	+ 49	+ 26
B_3	July 12	+186	33	+ 5	+191
	Oct. 19	+ 35	132	+ 19	+ 54
	Jan. 1	− 8	206	+ 29	+ 21
	Feb. 10	− 72	246	+ 35	− 37
	Mar. 27	− 49	292	+ 41	− 8
	May 16	− 66	342	+ 49	− 17

After thus correcting for wear we then have an increase of gravity, between July, 1891, and May, 1892, of

5 millionths for B_1
5 " " B_2
21 " " B_3

giving an average of 10 millionths for the three pendulums. This is equivalent to a decrease of $\frac{1}{400000}$ of a second in the time of oscillation of a half-second pendulum. When we consider that all three pendulums indicate a slight increase of the force of gravity, it is worth while to examine the conditions under which the work was done. The influences bearing on the result were in several respects such that their effect on differences of the force of gravity would be entirely eliminated when comparisons are made between observations made one year apart. Any effect on the time of oscillation of the pendulum depending on the influence of climate should be the same at the same season of the year. The influence of temperature on the rate of the chronometer and the effect of erroneous indications of the thermometer consequent upon the failure of the pendulums to take the temperature as soon as the mercury in the thermometer, although varying with the season, should be nearly the same for June, 1891, and June, 1892. These conditions, moreover, change but little throughout the year at Waikiki. The star places can not affect the character of the result, since the same stars were observed for time during several months, and the identical ones were used for the work of June, 1891, and in the determinations twelve months later. It is true that the wear of the knives was perceptible during the year, and that this effect on the time of oscillation of the pendulums is the same as would be produced by an increase of the force of gravity. But the effect of wear will not quite account for the observed decrease in the period of the pendulums. In fact, the observed decrease in B_3 is more than three times that attributed to wear. Moreover, we must not lose sight of the fact that this effect was determined by a comparison of the times of oscillation at Washington before and after the work at Waikiki, and that this naturally assumes that the force of gravity at Washington was constant during the year, or at least that it was the same at the time that the two determinations were made. This may or may not be the case. Indeed, there is, a priori, no more reason to suppose a variation at one place than at the other, except the volcanic nature of the islands, and the effect of wear would probably best be estimated by considering alone the Waikiki observations in which one of the pendulums was swung so much more than the other two that the effect of wear on these may be neglected in the comparison. Pendulum No. 1 and pendulum No. 2 were each used on 18 nights, while pendulum No. 3 was used on 163 nights, or in the proportion of about 9 to 1. The weight of all the pendulums being the same, the agate knives being of the same material, and the support being identical, we should expect equal effects for equal amounts of work, and a tolerable accurate estimate of the effect of use may be had independent of the Washington comparisons. It is well known that the effect on the time of oscillation of the wearing off of the knife and of its blunting are in opposite directions. In the first case the pendulum is virtually made longer and hence oscillates slower. In the latter the effect is to make it oscillate more rapidly, so that there is some compensation in the total effect. Whatever may have been the effect of use, as regards the amount to be attributed to each of these causes, the pendulums do not seem to have diverged from each other more than a few millionths of a second during the year, and there appear no progressive changes depending on the time, which should be the case if these changes were a function of the wear. The following table gives a comparison of the pendulums at those times during the year when a change was made from one instrument to the other. Since B_1 and B_2 were swung between the times of swinging B_3, the mean value of B_3 before and after is compared with each of the others. The unit is one ten-millionth of a sidereal second.

Comparison of pendulums.

[The unit is in the seventh decimal place of a sidereal second.]

Date.	B_3 Before.	B_3 After.	B_3 Mean.	B_2 Mean D and R.	B_1 Mean D and R.	$B_1—B_2$	$B_1—B_3$	$B_2—B_3$
1891–'92.								
June 26	9398	9377	9388	10015	10871	856	1483	627
Aug. 2	70	63	66	18	69	851	1503	652
Sept. 4	51	53	52	9979	56	877	1504	627
Oct. 4	50	56	53	97	61	864	1508	644
31	44	48	46	94	46	852	1500	648
Dec. 3	40	48	44	10002	38	836	1494	658
Jan. 2	42	38	40	9981	50	869	1510	641
10	38	29	34	97	32	835	1498	663
18	29	28	28	72	23	851	1495	644
Feb. 3	14	05	10	76	19	843	1509	666
20	25	39	32	75	31	856	1499	643
Mar. 1	27	22	24	79	22	843	1498	655
16	24	12	18	64	26	862	1508	646
Apr. 9	08	30	19	72	30	858	1511	653
16	48	22	35	84	23	839	1488	649
May 2	18	24	21	76	22	846	1501	655
18	18	18	18	79	34	855	1516	661
June 4	22	28	25	86	31	845	1506	661
						Means	1502	650

Differences from mean value.

Date.	$B_1—B_3$	$B_2—B_3$	Date.	$B_1—B_3$	$B_2—B_3$	Date.	$B_1—B_2$	$B_2—B_3$
1891.			1892.			1892.		
June 26	+19	+23	Jan. 2	−8	+9	Mar. 16	−6	+4
Aug. 2	−1	−2	10	+4	−13	Apr. 9	−9	−3
Sept. 4	−2	+23	18	+7	+6	16	+14	+1
Oct. 4	−6	+6	Feb. 3	−7	−16	May 2	+1	−5
31	+2	+2	20	+3	+7	18	−14	−11
Dec. 3	+8	−8	Mar. 1	+4	−5	June 4	−4	−11

The fact that the pendulums were not swung at the same station in Washington before and after the work in the Hawaiian Islands can hardly throw any uncertainty on the comparisons before and after the expedition. The connection was made by the telegraphic method, the pendulums being swung simultaneously and the greatest discrepancy of the period of individual swings from the mean value was only one part in two and one-half millions. It does not seem that the corrections for amplitude, temperature, atmospheric pressure, or rate of chronometer could be in error sufficiently to account for the change of gravity noticed at Waikiki. Even supposing that the coefficients were not strictly accurate, or that the amplitude scale was not true this would not influence relative gravity, since the corrections were calculated with the same constants throughout. The rate of the chronometer was determined each night from star observations and also from observations before and after the gravity determinations, in order to detect any irregularity in the rate during the interval from night to night. The result of this investigation was that greater irregularities were found in the corrections for rate, but it is doubtful whether these variations were really in the movement of the chronometer or whether the period of two or three hours was not so short that they came from uncertainties in the corrections themselves. Under either supposition, however, the result for the variation of gravity for the year is unchanged, because when we apply these corrections, deduced from determinations before and after the pendulum work, we get practically the same result for the periods of continuous work with each pendulum. For example, for those periods when B_3 was swung continuously we have the following differences in the rate corrections by the two methods:

	s.
Oct. 5 to Oct. 30	+0·000 0007
Nov. 4 Nov. 30	+ 05
Dec. 9 Dec. 30	− 01
Jan. 20 Jan. 27	+ 11
Feb. 6 Feb. 14	− 08
23 29	+ 06
Mar. 5 Mar. 10	− 08
21 31	+ 07
Apr. 19 May 1	− 10
May 5 14	+ 12
21 28	− 08
Mean ...	+0·000 0001

It appears, then, that we can not explain the progressive change in period during the year by using clock corrections computed from short intervals extending over the pendulum work for each night, and that on the average the corrections for short intervals do not differ materially from those adopted from the daily rates.

It is now worth while to see what a change of $\frac{1}{400000}$ of a second in the period of the pendulums represents in the determination of the period and in the corrections that are applied to it. In order to have a convenient standard of reference, this effect was calculated for one-millionth of a second, and we have approximately—

A change of 0·000001^s in the deduced period is produced by an error of 5 seconds in noting the coincidence between the chronometer and pendulum (swing one hour long);

of $\frac{1}{2}$ mm. in estimating both initial and final arcs (swing from half amplitude, 5·0 mm. to 3·0 mm.);

of 0°·25 C. in noting the temperature of the pendulum;

of 12 mm. in noting height of manometer;

of 0·2^s per day in the rate of the chronometer.

An error of $\frac{1}{400000}$ of a second in the pendulum period would imply magnitudes at least twice those given above.

HIGH AND LOW WATER AT HONOLULU, HAWAIIAN ISLANDS, FROM JUNE 17, 1891, TO JUNE 30, 1892.

VARIATION OF THE SEA LEVEL.

In connection with the variations of gravity, if such variations really exist, it may not be out of place to show the variations of the sea level.

No ordinary movement of the tides, however, can sensibly affect our results, as has been shown both theoretically and practically by several eminent authorities. (See Helmert Theil, II, pp. 144, 155; Thomson and Tait, Vol. I, Part II; Woodward Bulletin No. 48, U. S. Geological Survey, etc.)

Professor Woodward's equation is

$$\frac{dg}{g}=\frac{3}{2}\frac{h}{r}\frac{\delta}{\triangle}\int_0^{\theta_1}\varphi(\theta)\cos\tfrac{1}{2}\theta\,d\theta$$

where g = acceleration of gravity.

δ = density of superposed mass.

$\triangle$ = mean density of earth.

r = radius of earth.

$h\varphi(\theta)$ = thickness of superposed lenticular mass at angular distance θ from center of mass [$h\varphi(\theta) = h_1$ for $\theta = 0$].

θ_1 = limiting angle of θ.

If we take $\varphi(\theta) = \cos 2$ and $\theta_1 = 60°$ we have

$$\frac{dg}{g}=\frac{3}{2}\frac{h}{r}\frac{\delta}{\triangle}\int_0^{60°}\cos 2\theta\cos\tfrac{1}{2}\theta\,d\theta$$

If $\frac{\delta}{\triangle}$ is taken equal to $\frac{1}{6}$ the above equation gives

$$\frac{dg}{g}=\frac{1}{10}\frac{h}{r}\text{ approximately}$$

which shows that a wave of the above form must be 21 feet high in order to change gravity by its ten-millionth part.

The following table gives the times of high and low water at Honolulu, together with the reduced height. The local mean civil time is given, the hours counting from midnight. Interpolated values are given in brackets. The heights refer to the mean sea level for the period between June 17, 1891, and June 30, 1892. This mean sea level is derived from all the staff readings for the period under consideration. The table has been furnished by the Tidal Division in the Coast and Geodetic Survey Office from records sent by the Hawaiian Government Survey. Mr. C. J. Lyons, in charge of the Government Survey Office at Honolulu, has called attention to the existence of a relation between the variations of latitude and the change in the sea level.

High and low water, Honolulu, Hawaiian Islands.

Date.	High water. Corrected time.	High water. Reduced height.	Low water. Corrected time.	Low water. Reduced height.	Date.	High water. Corrected time.	High water. Reduced height.	Low water. Corrected time.	Low water. Reduced height.
1891.	h. m.	Feet.	h. m.	Feet.	1891.	h. m.	Feet.	h. m.	Feet.
June 17			5 45	—1·00	July 12	9 45	—0·05	1 45	—0·85
	13 30	+0·60	19 30	—0·50		19 50	+0·30	12 30	—0·25
18	0 10	—0·15	5 55	—1·10	13	10 00	+0·15	3 55	—0·75
	13 30	+0·85	20 40	—0·55		19 30	+0·20	15 20	—0·10
19	0 30	—0·20	6 30	—1·00	14	11 20	+0·55	3 00	—0·75
	14 10	+1·15	21 15	—0·60		21 40	+0·20	16 50	—0·10
20	0 50	—0·40	7 30	—1·20	15	11 30	+0·80	[3 40]	[—0·75]
	15 00	+1·20	22 30	—0·80		[22 45]	[+0·05]	[18 50]	[—0·20]
21	2 30	—0·50	8 15	—1·25	16	13 00	+1·00	[4 20]	[—0·70]
	15 30	+1·30	23 00	0·85		23 50	0·00	20 50	—0·30
22	2 45	—0·55	8 30	—1·20	17	-----	------	5 00	—0·70
	16 30	+1·25	-----	------		[13 30]	[+1·25]	[21 30]	[—0·35]
23	4 20	—0·65	0 30	—1·10	18	[0 45]	[0·00]	[6 35]	[—0·65]
	17 15	+1·20	9 45	—1·35		[14 00]	[+1·45]	[22 00]	[—0·40]
24	5 20	—0·55	0 30	—1·10	19	[1 35]	[0·00]	[7 20]	[—0·75]
	17 55	+1·15	9 35	—1·30		14 30	+1·65	22 30	—0·45
25	6 25	—0·55	1 30	—1·15	20	2 25	0·00	8 00	—0·85
	[18 35]	[+1·05]	[10 35]	[—1·00]		15 00	+1·70	23 00	—0·50
26	[7 30]	[—0·35]	[1 55]	[—1·20]	21	3 30	0·00	8 30	—0·85
	19 20	+0·95	11 30	—0·85		15 45	+1·70	23 35	—0·50
27	[8 30]	[—0·05]	2 20	—1·20	22	4 20	0·00	9 20	—0·90
	20 30	+0·70	12 30	—0·70		16 30	+1·50	-----	------
28	9 30	+0·20	3 20	—1·25	23	5 10	—0·05	0·15	—0·55
	21 40	+0·45	15 10	—0·45		17 45	+1·40	10·10	—0·80
29	10 50	+0·45	4 15	—1·20	24	6 15	+0·10	0·15	—0·70
	22 00	+0·30	17 10	—0·30		18 15	+1·25	10·45	—0·60
30	12 30	+0·75	4 45	—1·10	25	6 40	+0·20	1·10	—0·70
	22 55	+0·05	17 50	—0·30		18 45	+1·00	10 45	—0·40
July 1	12 45	+1·00	5 10	—1·10	26	8 00	+0·30	2 10	—0·75
	23 30	—0·15	20 30	—0·50		19 30	+0·70	12 30	—0·10
2	13 45	+1·20	6 30	—1·15	27	9 30	+0·50	2 30	—0·75
	-----	------	20 45	—0·65		20 00	+0·40	15 35	0·00
3	1 15	—0·30	6 40	—1·20	28	11 30	+0·75	3 00	—0·75
	14 15	+1·10	21 30	—0·95		21 00	+0·10	16 45	—0·05
4	2 00	—0·50	[7 20]	[—1·20]	29	12 20	+1·10	4 10	—0·70
	14 50	+1·35	22 45	—0·75		22 30	—0·10	19 30	—0·10
5	2 50	—0·40	8 00	—1·15	30	12 45	+1·30	4 10	—0·65
	15 35	+1·30	23 00	—0·70		23 40	—0·15	21 00	—0·30
6	3 30	—0·40	8 30	—1·15	31	-----	------	5 30	—0·70
	16 00	+1·25	-----	------		13 25	+1·30	21 20	—0·40
7	4 20	—0·35	0 20	—0·65	Aug. 1	2 15	—0·20	6 00	—0·65
	16 30	+1·40	9 15	—1·00		14 10	+1·45	[21 45]	[—0·45]
8	5 00	—0·15	0 00	—0·75	2	[2 25]	[—0·25]	[6 40]	[—0·65]
	17 20	+1·10	9 30	—0·95		[14 30]	[+1·40]	[22 05]	[—0·50]
9	6 15	—0·30	0 45	—0·65	3	[3 00]	[—0·20]	[7 20]	[—0·60]
	18 20	+1·00	10 50	—0·75		[14 55]	[+1·35]	[22 30]	[—0·55]
10	6 30	—0·20	1 50	—0·65	4	[3 35]	[—0·15]	[8 00]	[—0·55]
	18 30	+0·75	11 30	—0·75		[15 20]	[+1·25]	[22 25]	[—0·55]
11	8 00	—0·15	2 00	—0·85	5	[4 00]	[—0·15]	8 45	—0·50
	19 50	+1·10	11 00	—0·50		15 45	+1·15	23 20	—0·60

High and low water, Honolulu, Hawaiian Islands—Continued.

Date.	High water. Corrected time.	High water. Reduced height.	Low water. Corrected time.	Low water. Reduced height.
1891.	*h. m.*	*Feet.*	*h. m.*	*Feet.*
Aug 6	4 15	−0·10	9 30	−0·65
	16 40	+1·00	-----	-----
7	5 30	−0·05	0 30	−0·60
	17 15	+0·75	9 45	−0·55
8	[6 00]	[+0·05]	0 35	−0·55
	17 30	+0·80	10 30	−0·25
9	6 45	+0·20	1 00	−0·45
	18 00	+1·00	11 50	−0·20
10	9 50	+0·30	1 50	−0·50
	18 00	+0·40	13 10	0·00
11	9 30	+0·45	1 40	−0·50
	19 45	+0·20	14 20	+0·15
12	11 00	+0·60	1 45	−0·50
	19 00	+0·10	17 50	0·00
13	11 30	+0·85	3 00	−0·60
	22 30	−0·10	18 15	−0·05
14	12 30	+1·15	4 20	−0·60
	23 20	−0·10	21 15	−0·30
15	-----	-----	4 40	−0·65
	13 10	+1·30	21 00	−0·40
16	1 00	−0·20	5 50	−0·75
	14 10	+1·45	21 30	−0·50
17	2 20	−0·10	7 00	−0·75
	14 30	+1·65	21 20	−0·50
18	2 30	0·00	7 40	−0·70
	15 00	+1·80	22 00	−0·45
19	3 15	+0·30	8 30	−0·80
	15 40	+1·75	23 05	−0·50
20	4 20	+0·45	9 30	−0·65
	16 30	+1·75	23 30	−0·55
21	4 35	+0·60	10 20	−0·50
	17 10	+1·40	23 45	−0·60
22	6 00	+0·75	11 10	−0·25
	17 30	+1·25	-----	-----
23	6 40	+1·00	0 20	−0·55
	[18 15]	[+1·05]	[12 20]	[0·00]
24	[7 35]	[+1·10]	[0 55]	[−0·50]
	19 00	+0·85	13 35	+0·30
25	8 30	+1·25	1 30	−0·40
	19 30	+0·50	16 00	+0·25
26	10 15	+1·40	2 30	−0·45
	20 45	+0·40	17 30	+0·25
27	11 40	+1·45	3 10	−0·35
	21 30	+0·20	18 50	+0·05
28	12 00	+1·50	4 15	−0·30
	-----	-----	20 20	−0·10
29	0 10	+0·20	4 30	−0·40
	13 20	+1·65	20 30	−0·10
30	1 30	+0·35	5 45	−0·35
	14 05	+1·65	21 30	−0·10

Date.	High water. Corrected time.	High water. Reduced height.	Low water. Corrected time.	Low water. Reduced height.
1891.	*h. m.*	*Feet.*	*h. m.*	*Feet.*
Aug. 31	2 15	+0·40	6 45	−0·30
	15 00	+1·70	21 40	−0·15
1	2 40	+0·55	7 50	−0·45
	15 10	+1·70	22 10	−0·15
2	3 10	+0·60	8 30	−0·40
	15 50	+1·50	22 30	−0·20
3	3 45	+0·70	9 00	−0·40
	16 10	+1·60	22 50	−0·25
4	4 25	+1·00	9 30	−0·30
	16 20	+1·30	22 50	−0·40
5	4 30	+0·90	10 30	−0·35
	17 10	+1·05	23 30	−0·55
6	5 30	+0·80	11 15	−0·15
	17 30	+0·95	-----	-----
7	6 30	+1·00	0 10	−0·40
	17 50	+0·80	12 30	+0·10
8	6 40	+1·10	0 30	−0·30
	17 45	+0·55	12 30	+0·10
9	7 00	+1·05	0 00	−0·40
	18 00	+0·40	15 00	+0·15
10	8 20	+1·15	1 30	−0·35
	19 50	+0·40	16 30	+0·20
11	10 00	+1·30	2 30	−0·30
	20 30	+0·10	17 50	+0·05
12	11 30	+1·35	3 00	−0·30
	23 00	+0·10	19 00	−0·05
13	-----	-----	4 30	−0·30
	12 30	+1·60	20 30	−0·10
14	0 40	+1·20	5 10	−0·30
	13 20	+1·60	20 40	−0·35
15	2 20	+0·40	6 35	−0·45
	14 10	+1·80	21 30	−0·45
16	2 30	+0·45	8 00	−0·50
	[15 00]	[+1·70]	[22 00]	[−0·50]
17	3 15	+0·70	9 10	−0·50
	15 50	+1·60	22 30	−0·55
18	3 55	+0·95	9 20	−0·35
	16 30	+1·45	22 30	−0·70
19	4 40	+1·30	10 40	−0·30
	16 50	+1·35	23 10	−0·60
20	5 30	+1·40	11 40	−0·15
	17 30	+1·05	23 20	−0·50
21	6 30	+1·55	-----	-----
	17 50	+0·80	12 45	+0·20
22	7 30	+1·55	0 00	−0·30
	17 30	+0·45	15 00	+0·10
23	9 00	+1·50	0 50	−0·35
	20 20	+0·20	16 30	+0·15
24	10 00	+1·50	1 35	−0·20
	22 00	+0·15	18 00	−0·10

High and low water, Honolulu, Hawaiian Islands—Continued.

Date.	High water.		Low water.		Date.	High water.		Low water.	
	Corrected time.	Reduced height.	Corrected time.	Reduced height.		Corrected time.	Reduced height.	Corrected time.	Reduced height.
1891.	*h. m.*	*Feet.*	*h. m.*	*Feet.*	1891.	*h. m.*	*Feet.*	*h. m.*	*Feet.*
Sept. 25	10 30	+1·30	2 45	−0·10	Oct. 20	5 30	+1·45	12 05	−0·20
	-----	------	18 40	−0·20		16 50	+0·10	22 55	−0·70
26	0 10	+0·20	4 10	−0·20	21	6 45	+1·45	14 00	−0·25
	11 50	+1·40	20 30	−0·10		18 10	−0·05	23 30	−0·30
27	0 45	+0·45	5 45	−0·20	22	7 30	+1·30	16 30	−0·20
	12 30	+1·40	20 30	−0·25		22 30	−0·10	-----	-----
28	1 50	+0·45	6 45	−0·30	23	8 25	+1·30	0 15	−0·25
	13 20	+1·45	20 30	−0·35		23 50	−0·05	17 30	−0·25
29	1 50	+0·55	7 10	0·30	24	9 45	+1·10	1 30	−0·15
	14 00	+1·40	21 00	−0·50		-----	-----	18 30	−0·35
30	2 15	+0·65	8 15	−0·50	25	0 30	+0·15	3 00	0·00
	14 30	+1·30	21 00	−0·50		12 00	+1·05	18 45	−0·40
Oct. 1	2 45	+0·80	8 50	−0·45	26	1 10	+0·30	5 00	0·00
	15 05	+1·20	21 20	−0·65		12 10	+1·00	18 50	−0·50
2	3 30	+0·95	9 10	−0·50	27	1 50	+0·50	6 20	−0·10
	15 10	+1·05	21 45	−0·60		12 45	+0·90	19 15	−0·70
3	3 45	+1·15	10 00	−0·40	28	2 00	+0·65	7 00	−0·30
	15 45	+0·90	22 00	−0·55		13 20	+0·80	20 00	−0·70
4	4 30	+1·30	10 50	−0·30	29	2 40	+0·70	7 30	−0·30
	16 15	+0·90	22 30	−0·55		13 20	+0·60	20 00	−0·75
5	5 00	+1·35	11 40	−0·25	30	2 35	+0·90	[8 15]	[−0·30]
	16 40	+0·80	23 00	−0·50		[14 05]	[+0·60]	[20 25]	[−0·70]
6	5 40	+1·40	12 30	−0·10	31	[3 05]	[+1·15]	9 00	−0·30
	17 20	+0·65	23 20	−0·50		14 50	+0·60	20 50	−0·70
7	6 30	+1·30	-----	------	Nov. 1	3 30	+1·40	9 40	−0·50
	18 20	+0·30	14 15	0·00		15 00	+0·45	21 00	−0·70
8	7 10	+1·35	0 00	−0·45	2	4 00	+1·40	11 00	−0·25
	18 45	+0·05	15 00	−0·20		16 30	+0·30	21 00	−0·60
9	8 30	+1·20	0 10	−0·40	3	4 20	+1·40	12 30	−0·10
	20 30	−0·05	16 30	−0·15		16 15	+0·30	21 40	−0·70
10	10 00	+1·20	1 00	−0·30	4	5 30	+1·50	12 50	−0·10
	23 00	−0·05	17 50	−0·45		16 40	+0·10	21 50	−0·55
11	10 30	+1·35	3 20	−0·30	5	6 10	+1·40	14 30	−0·10
	-----	------	18 30	−0·50		17 30	+0·10	21 30	−0·50
12	0 45	+0·10	4 45	−0·30	6	7 00	+1·45	16 40	−0·20
	11 50	+1·35	19 30	−0·70		19 15	−0·10	22 20	−0·30
13	0 20	+0·25	6 20	−0·45	7	8 00	+1·30	17 00	−0·30
	12 35	+1·45	19 45	−0·75		19 30	−0·15	22 40	−0·25
14	1 30	+0·60	7 10	−0·50	8	9 30	+1·20	-----	------
	13 20	+1·30	20 30	−0·75		22 45	−0·10	17 30	−0·45
15	2 00	+0·75	8 30	−0·45	9	11 00	+1·40	2 45	−0·15
	14 05	+1·25	21 00	−0·80		-----	------	18 00	−0·25
16	3 10	+1·00	8 45	−0·55	10	0 00	+0·30	3 10	+0·10
	14 50	+1·20	21 20	−0·85		10 30	+1·30	18 30	−0·30
17	3 45	+1·30	10 10	−0·45	11	0 30	+0·50	5 30	+0·05
	15 20	+0·85	21 50	−0·95		11 50	+1·10	18 50	−0·55
18	[4 10]	[+1·40]	11 00	−0·50	12	2 00	+0·90	7 10	−0·15
	16 15	+0·50	22 00	−0·90		12 00	+0·90	19 15	−0·60
19	4 35	+1·55	11 55	−0·35	13	1 30	+1·15	8 00	−0·20
	16 40	+0·20	22 30	−0·75		13 00	+0·70	20 00	−0·65

High and low water, Honolulu, Hawaiian Islands—Continued.

Date.	High water.		Low water.		Date.	High water.		Low water.	
	Corrected time.	Reduced height.	Corrected time.	Reduced height.		Corrected time.	Reduced height.	Corrected time.	Reduced height.
1891.	*h. m.*	*Feet.*	*h. m.*	*Feet.*	1891.	*h. m.*	*Feet.*	*h. m.*	*Feet.*
Nov. 14	2 45	+1·30	9 10	—0·20	Dec. 9	10 00	+1·00	4 10	—0·05
	13 30	+0·40	20 00	—0·90		-----	------	17 20	—0·60
15	3 50	+1·60	10 45	—0·30	10	0 30	+0·75	6 00	+0·15
	15 30	+0·30	21 15	—0·90		11 20	+0·70	18 00	—0·70
16	3 50	+1·60	11 30	—0·35	11	1 45	+1·20	7 00	0·00
	16 20	+0·05	21 00	—1·00		11 30	+0·55	18 50	—0·75
17	5 50	+1·65	12 40	—0·40	12	1 50	+1·45	8 00	—0·15
	17 00	0·00	21 15	—0·65		[12 35]	[+0·35]	[19 25]	[—0·85]
18	6 10	+1·70	13 45	—0·50	13	[2 30]	[+1·50]	[9 20]	[—0·35]
	17 10	—0·15	22 00	—0·65		[13 40]	[+0·10]	[19 55]	[—1·00]
19	6 45	+1·40	14 25	—0·40	14	[3 10]	[+1·60]	10 40	—0·55
	18 30	—0·20	23 20	—0·55		14 50	—0·10	20 30	—1·10
20	7 20	+1·25	15 55	—0·45	15	3 50	+1·65	11 20	—0·55
	20 10	—0·10	23 50	—0·30		15 50	—0·05	20 50	—1·00
21	8 00	+1·25	15 45	—0·45	16	4 30	+1·75	12 15	—0·60
	20 45	—0·05	23 10	—0·20		15 45	—0·05	21 25	—1·00
22	8 45	+1·20	-----	------	17	5 15	+1·60	12 30	—0·60
	23 45	+0·05	16 40	—0·50		17 20	—0·05	22 20	—0·90
23	9 50	+1·00	3 00	0·00	18	5 30	+1·60	13 10	—0·60
	-----	------	17 10	—0·40		17 30	0·00	22 50	—0·85
24	0 00	+0·25	5 00	+0·05	19	6 30	+1·40	13 45	—0·55
	11 00	+0·80	17 45	—0·45		18 30	—0·10	23 00	—0·50
25	1 00	+0·50	5 30	+0·05	20	7 30	+1·10	14 20	—0·70
	11 30	+0·70	18 00	—0·40		[19 45]	[+0·05]	[23 45]	[—0·35]
26	1 20	+0·80	7 00	+0·05	21	[8 00]	[+0·95]	-----	------
	11 20	+0·60	18 30	—0·50		20 55	+0·20	14 55	—0·55
27	1 50	+1·00	7 45	—0·10	22	8 30	+0·80	0 30	—0·20
	12 15	+0·60	19 00	—0·40		22 40	+0·30	16 00	—0·65
28	2 35	+1·25	8 30	—0·05	23	9 00	+0·45	3 50	0·00
	13 30	+0·30	19 45	—0·55		-----	------	17 00	—0·70
29	3 00	+1·30	9 10	—0·30	24	1 00	+0·25	5 30	—0·10
	14 00	+0·05	20 15	—0·80		9 30	+0·30	17 45	—0·80
30	4 05	+1·30	10 30	—0·30	25	1 15	+0·80	6 30	—0·10
	15 20	+0·15	20 30	—0·65		11 10	+0·15	18 00	—0·95
Dec. 1	3 50	+1·50	11 20	—0·35	26	1 40	+0·90	7 40	—0·35
	15 20	+0·10	21 00	—0·60		12 15	+0·10	18 30	—1·05
2	5 10	+1.70	12 15	—0·35	27	1 50	+1·15	[8 45]	[—0·40]
	15 50	+0·05	21 30	—0·75		[13 15]	[—0·05]	[18 55]	[—1·10]
3	5 15	+1·60	12 45	—0·45	28	2 45	+0·85	9 45	—0·50
	17 00	0·00	22 15	—0·80		14 15	—0·15	19 20	—1·10
4	5 50	+1·55	14 20	—0·45	29	3 10	+1·40	10 40	—0·65
	17 40	—0·20	22 30	—0·65		14 50	—0·25	20 30	—1·00
5	7 00	+1·50	14 40	—0·55	30	[3 45]	[+1·50]	[11 15]	[—0·45]
	18 30	—0·05	23 00	—0·50		[15 30]	[+0·05]	21 00	—0·15
6	8 00	+1·40	-----	------	31	4 25	+1·55	11 50	—0·25
	19 50	+0·05	14 30	—0·60		16 15	+0·35	21 30	—0·50
7	8 30	+1·30	0 50	—0·35	1892.				
	20 40	+0·20	15 30	—0·50	Jan. 1	5 15	+2·10	12 30	—0·90
8	8 30	+1·20	2 20	—0·10		16 30	—0·30	22 10	—1·15
	23 30	+0·35	16 40	—0·60	2	5 50	+1·40	13 00	—0·85
						17 50	—0·25	22 40	—1·00

High and low water, Honolulu, Hawaiian Islands—Continued.

Date.	High water.		Low water.		Date.	High water.		Low water.	
	Corrected time.	Reduced height.	Corrected time.	Reduced height.		Corrected time.	Reduced height.	Corrected time.	Reduced height.
1892.	*h. m.*	*Feet.*	*h. m.*	*Feet.*	1892.	*h. m.*	*Feet.*	*h. m.*	*Feet.*
Jan. 3	6 30	+1·20	13 50	1·00	Jan. 28	3 30	+1·35	11 00	−0·75
	18 45	−0·20	23 40	−0·80		15 40	−0·25	20 45	−1·15
4	7 00	+1·00	14 00	−0·90	29	4 30	+1·50	11 30	−0·80
	20 00	−0·10	-----	------		16 15	−0·20	21 30	−1·30
5	7 50	+0·80	0 10	−0·50	30	4 30	+1·40	11 35	−0·80
	21 30	+0·10	15 10	0·80		16 50	+0·10	22 00	−0·95
6	8 30	+0·65	2 15	−0·15	31	5 10	+1·40	12 30	−0·85
	23 20	+0·60	15 50	−0·75		17 50	+0·10	23 00	−0·85
7	8 30	+0·55	4 30	+0·10	Feb. 1	5 50	+1·30	12 55	−0·80
	-----	------	16 00	−0·65		18 30	+0·25	23 30	−0·75
8	0 20	+1·00	6 50	0·00	2	6 40	+1·00	13 20	−0·85
	10 00	+0·30	16 30	−0·70		19 30	+0·50	-----	------
9	0 30	+1·35	8 15	−0·05	3	7 15	+0·60	0 30	−0·30
	11 20	+0·05	17 20	−0·90		20 30	+0·60	13 30	−0·90
10	1 30	+1·35	9 45	−0·40	4	7 50	+0·35	2 00	0·00
	12 15	−0·15	18 40	−0·95		22 00	+0·65	14 10	−0·85
11	2 10	+1·40	10 00	−0·65	5	9 30	0·00	6 10	−0·05
	14 15	−0·30	18 40	−1·05		23 30	+0·85	15 00	−0·70
12	2 55	+1·50	11 30	0·70	6	9 00	−0·15	7 00	−0·10
	15 00	−0·25	20 00	1·05		-----	------	15 30	−0·75
13	3 45	+1·70	11 00	−0·50	7	0 20	+1·00	8 30	−0·45
	15 45	+0·10	21 10	0·80		11 30	−0·30	16 20	−0·75
14	4 10	+1·60	12 00	−0·50	8	1 20	+1·25	8 45	−0·50
	16 15	+0·15	21 40	−0·80		13 00	−0·35	18 30	−0·85
15	4 40	+1·50	12 45	−0·70	9	1 50	+1·20	9 15	−0·70
	17 30	0·00	22 10	−0·80		14 00	−0·25	19 20	−1·00
16	5 30	+1·30	13 00	−0·70	10	2 15	+1·15	10 10	−0·70
	18 30	+0·05	23 30	−0·75		14 40	−0·25	20 00	−1·10
17	6 15	+1·10	13 15	−0·75	11	3 30	+1·25	10 40	−0·90
	18 25	+0·25	23 55	−0·40		15 45	−0·20	20 50	−1·15
18	6 30	+1·10	13 40	−0·70	12	3 50	+1·10	11 00	−0·90
	19 30	+0·20	-----	------		15 50	−0·20	21 30	−1·30
19	7 20	+0·85	0 00	−0·30	13	4 20	+1·00	11 30	−0·95
	20 30	+0·20	14 20	−0·60		16 40	−0·10	21 30	−1·00
20	7 50	+0·70	1 30	0·00	14	4 45	+1·00	11 50	−0·85
	23 15	+0·25	15 00	−0·65		17 30	+0·15	22 10	−0·70
21	8 10	+0·20	4 00	−0·15	15	5 20	+0·90	12 15	−0·70
	23 15	+0·55	14 50	−0·75		18 00	+0·30	23 30	−0·50
22	9 20	+0·10	6 00	0·00	16	6 20	+0·65	13 20	−0·75
	-----	------	15 50	−0·70		19 00	+0·40	-----	------
23	0 20	+0·65	6 40	−0·25	17	6 00	+0·55	0 10	−0·40
	9 45	−0·15	16 30	−0·80		19 45	+0·40	13 15	−0·80
24	1 10	+0·80	8 00	−0·35	18	6 40	+0·20	1 30	−0·30
	11 45	−0·30	17 40	−0·95		20 50	+0·35	14 00	−0·70
25	1 30	+0·90	9 15	−0·55	19	6 30	0·00	2 30	−0·20
	13 00	−0·45	18 10	−1·00		21 20	+0·50	14 00	−0·75
26	2 15	+1·10	10 00	−0·65	20	8 50	−0·20	5 40	−0·20
	13 40	−0·40	19 00	−1·05		23 00	+0·55	14 40	−0·75
27	2 40	+1·20	9 40	−0·75	21	[9 40]	[−0·30]	6 40	[0·30]
	15 00	−0·35	19 30	−1·05		[23 35]	[+0·65]	[15 35]	[−0·80]

High and low water, Honolulu, Hawaiian Islands—Continued.

Date.	High water.		Low water.		Date.	High water.		Low water.	
	Corrected time.	Reduced height.	Corrected time.	Reduced height.		Corrected time.	Reduced height.	Corrected time.	Reduced height.
1892.	*h. m.*	*Feet.*	*h. m.*	*Feet.*	1892.	*h. m.*	*Feet.*	*h. m.*	*Feet.*
Feb. 22	10 30	—0·45	[8 05]	—0·50	Mar. 18	7 00	—0·20	3 00	—0·40
	-----	------	16 30	—0·90		19 50	+0·60	12 00	—0·70
23	0 00	+0·75	9 30	—0·75	19	7 25	—0·40	3 55	—0·35
	13 30	—0·60	18 10	—0·85		21 10	+0·65	12 30	—0·80
24	1 20	+0·95	9 45	—0·85	20	9 50	—0·55	6 30	—0·60
	13 45	—0·50	18 30	—1·00		22 15	+0·65	13 10	—0·70
25	2 30	+1·10	10 30	—0·85	21	10 10	—0·45	7 00	—0·60
	14 40	—0·30	19 45	—1·00		23 30	+0·75	14 00	—0·65
26	2 45	+1·20	10 30	—0·90	22	11 30	—0·40	8 00	—0·70
	15 30	—0·15	20 45	—1·00		-----	------	17 00	—0·65
27	3 30	+1·30	11 00	—0·85	23	0 10	+0·90	8 40	—0·60
	15 50	0·00	21 30	—1·00		12 00	—0·20	16 50	—0·70
28	4 30	+1·25	11 20	—0·95	24	0 40	+1·15	8 00	—0·70
	16 30	+0·15	22 00	—0·95		13 40	—0·05	18 25	—0·65
29	4 45	+0·90	11 20	—1·00	25	1 30	+1·20	8 30	—0·70
	17 30	+0·40	22 45	—0·80		14 15	+0·20	19 50	—0·65
Mar. 1	5 10	+0·65	12 00	—1·05	26	2 25	+1.25	9 30	—0·70
	18 20	+0·55	23 45	—0·55		15 00	+0·45	20 20	—0·60
2	5 10	+0·40	12 00	—1·10	27	2 40	+1·10	9 45	—0·75
	18 45	+0·60	-----	------		15 30	+0·65	21 15	—0·60
3	5 40	+0·05	1 00	—0·45	28	3 15	+0·90	10 15	—0·75
	20 45	+0·65	12 30	—1·05		16 15	+0·80	22 45	—0·50
4	6 00	—0·25	3 30	—0·35	29	3 50	+0·50	10 30	—0·85
	21 00	+0·60	13 00	—1·00		17 00	+0·90	23 15	—0·50
5	9 30	—0·40	7 00	—0·50	30	5 10	+0·40	11 00	—0·85
	22 50	+0·80	14 00	—0·85		18 00	+1·10	-----	------
6	10 30	—0·50	7 30	—0·55	31	5 15	0·00	1 30	—0·40
	23 50	+0·85	15 40	—0·80		19 00	+1.10	11 30	—0·90
7	-----	------	7 30	—0·75	Apr. 1	6 40	—0·20	3 15	—0·40
	12 20	—0·50	17 00	—0·65		20 30	+1·05	11 50	—0·90
8	0 50	+0·95	8 30	—0·75	2	8 30	—0·40	4 30	—0·55
	13 50	—0·40	18 30	—0·90		21 15	+1·05	13 10	—0·70
9	1 45	+0·90	9 35	—0·80	3	9 50	—0·45	6 15	—0·60
	14 00	—0·30	19 15	—0·90		21 50	+0·95	13 50	—0·55
10	2 20	+0·85	9 40	—0·80	4	11 45	—0·40	7 00	—0·75
	14 50	—0·15	20 20	—0·90		23 15	+0·90	15 45	—0·60
11	3 10	+0·90	10 20	—0·75	5	12 20	—0·30	7 30	—0·70
	15 30	—0·10	21 00	—0·95		-----	------	17 15	—0·55
12	3 45	+0·70	10 15	—0·90	6	0 15	+0·80	8 00	—0·70
	15 45	—0·05	21 15	—0·85		13 10	—0·20	18 10	—0·50
13	4 05	+0·60	10 30	—0·90	7	0 30	+0·65	8 00	—0·75
	16 30	+0·20	22 00	—0·70		14 30	+0·05	19 00	—0·55
14	5 00	+0·45	10 50	—0·75	8	1 20	+0·65	8 30	—0·70
	17 20	+0·25	22 30	—0·65		14 30	+0·20	20 00	—0·60
15	5 30	+0·20	11 15	—0·80	9	2 10	+0·65	9 00	—0·70
	18 10	+0·50	23 50	—0·55		15 00	+0·40	20 30	—0·45
16	5 50	+0·10	11 20	—0·80	10	3 00	+0·55	9 30	—0·60
	18 00	+0·50	-----	------		15 15	+0·70	21 30	—0·50
17	5 20	0.00	0 30	—0·40	11	3 30	+0·50	22 15	—0·75
	18 40	+0·55	11 45	—0·80		16 00	+0·75	22 15	—0·50

High and low water, Honolulu, Hawaiian Islands—Continued.

Date.	High water.		Low water.		Date.	High water.		Low water.	
	Corrected time.	Reduced height.	Corrected time.	Reduced height.		Corrected time.	Reduced height.	Corrected time.	Reduced height.
1892.	*h. m.*	*Feet.*	*h. m.*	*Feet.*	1892.	*h. m.*	*Feet.*	*h. m.*	*Feet.*
Apr. 12	3 30	+0·25	9 35	−0·80	May 7	0 50	+0·65	8 00	−0·70
	16 25	+0·80	23 10	−0·45		14 20	+0·70	20 20	−0·25
13	4 15	+0·20	10 10	−0·75	8	1 15	+0·50	8 15	−0·60
	16 50	+0·90	-----	------		14 30	+0·80	20 40	−0·30
14	4 30	−0·05	0 40	−0·50	9	2 30	+0·40	8 35	−0·70
	17 25	+0·90	9 45	−0·85		15 00	+1·00	21 45	−0·40
15	5 20	−0·15	1 15	−0·45	10	2 30	+0·20	8 40	−0·85
	18 40	+0·90	10 20	−0·85		15 40	+1·15	23 15	−0·45
16	6 00	−0·40	2 50	−0·50	11	3 30	+0·15	8 50	−0·80
	19 00	+0·85	10 00	−0·85		16 10	+1·25	-----	------
17	6 30	−0·45	3 30	−0·55	12	3 50	0·00	0 00	−0·50
	20 00	+0·75	11 00	−0·70		16 30	+1·25	9 20	−0·85
18	7 00	−0·60	4 30	−0·65	13	4 45	−0·15	0 20	−0·60
	21 40	+0·80	11 00	−0·65		17 30	+1·25	9 45	−0·90
19	12 00	−0·40	6 30	−0·55	14	5 40	−0·30	1 10	−0·60
	22 00	+0·80	14 30	−0·45		18 30	+1·20	10 20	−0·85
20	13 00	−0·25	7 00	−0·60	15	6 00	−0·30	2 30	−0·60
	23 10	+0·95	16 00	−0·40		18 50	+1·15	10 35	−0·75
21	-----	------	7 00	−0·60	16	7 00	−0·30	3 30	−0·60
	12 50	−0·10	17 15	−0·50		19 45	+1·00	11 15	−0·70
22	0 10	+0·85	7 30	−0·70	17	8 00	−0·40	3 30	−0·70
	13 45	+0·25	18 00	−0·50		20 45	+1·00	11 50	−0·55
23	1 00	+0·90	8 00	−0·70	18	10 45	−0·40	5 00	−0·75
	13 50	+0·45	19 40	−0·50		21 40	+0·85	13 00	−0·45
24	1 45	+0·85	8 40	−0·90	19	12 05	−0·15	5 45	−0·80
	14 30	+0·70	20 45	−0·55		22 20	+0·75	15 00	−0·30
25	2 40	+0·65	8 50	−0·95	20	12 45	+0·15	6 00	−0·80
	15 30	+1·15	21 50	−0·55		23 10	+0·65	17 30	−0·20
26	3 30	+0·35	9 00	−0·95	21	13 00	+0·45	6 40	−0·90
	16 00	+1·35	23 30	−0·50		23 45	+0·60	18 50	−0·30
27	3 55	+0·20	9 45	−1·10	22	-----	------	7 00	−0·95
	17 15	+1·40	-----	------		13 55	+0·75	20 15	−0·35
28	5 00	−0·10	1 00	−0·50	23	0 40	+0·25	7 00	−1·00
	17 50	+1·45	10 10	−1·00		14 15	+1·15	21 00	−0·50
29	5 45	−0·20	1 15	−0·60	24	2 00	+0·10	8 00	−1·10
	18 30	+1·40	10 50	−0·90		14 50	+1·40	22 25	−0·60
30	6 30	−0·30	2 30	−0·65	25	2 45	−0·05	8 30	−1·10
	19 30	+1·30	11 40	−0·75		15 30	+1·40	[23 35]	[−0·70]
May 1	7 00	−0·40	3 30	−0·60	26	[3 35]	[−0·20]	9 10	−1·20
	20 00	+1·10	11 25	−0·60		16 15	+1·50	-----	------
2	10 00	−0·45	6 30	−0·65	27	4 30	0·40	0 45	−0·80
	21 40	+1·00	14 30	−0·35		17 25	+1·50	9 45	−1·10
3	11 15	−0·25	6 15	−0·65	28	5 30	−0·50	1 45	−0·85
	22 50	+1·05	14 30	−0·20		18 30	+1·45	10 30	−0·90
4	12 30	+0·10	7 00	−0·70	29	7 00	−0·40	2 30	−0·70
	23 20	+0·85	17 20	−0·25		19 30	+1·20	11 10	−0·80
5	-----	------	7 15	−0·70	30	7 40	−0·40	3 30	−0·85
	13 30	+0·40	20 00	−0·30		20 00	+1·00	11 45	−0·60
6	0 15	+0·75	7 30	−0·80	31	9 20	−0·30	4 15	−0·80
	13 45	+0·50	19 30	−0·25		21 00	+0·85	12 30	−0·50

High and low water, Honolulu, Hawaiian Islands—Continued.

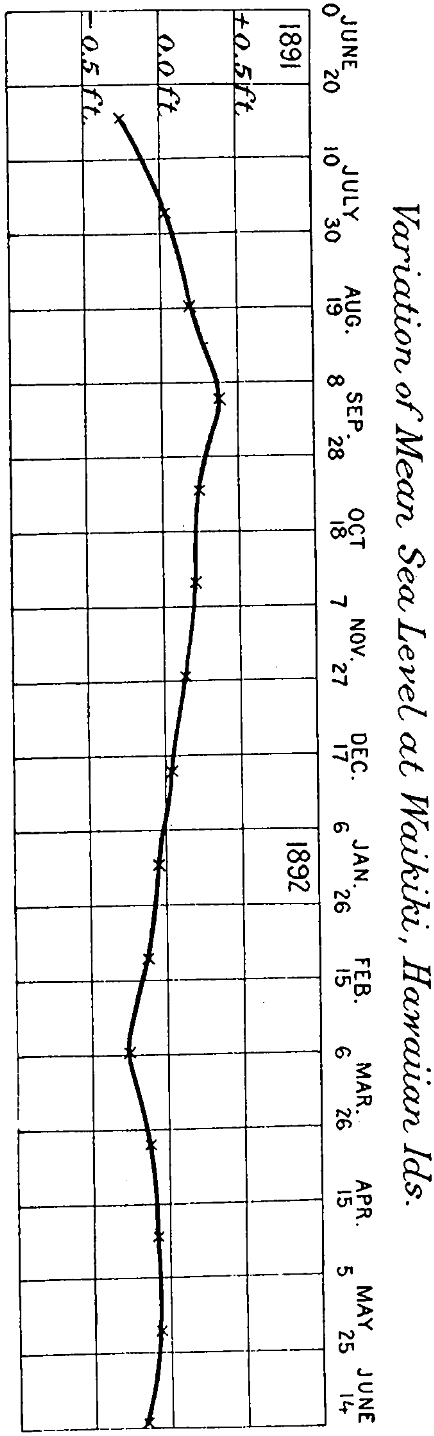
Date.	High water. Corrected time.	High water. Reduced height.	Low water. Corrected time.	Low water. Reduced height.	Date.	High water. Corrected time.	High water. Reduced height.	Low water. Corrected time.	Low water. Reduced height.
1892.	*h. m.*	*Feet.*	*h. m.*	*Feet.*	1892.	*h. m.*	*Feet.*	*h. m.*	*Feet.*
June 1	11 30	—0·15	5 00	—0·75	June 16	10 45	—0·15	3 50	—0·90
	22 00	+0·70	13 50	—0·30		22 00	+0·80	13 30	—0·40
2	[12 05]	[+0·10]	5 25	—0·80	17	11 15	+0·05	4 30	—1·00
	[23 10]	[+0·55]	[15 40]	[—0·25]		22 00	+0·60	15 40	—0·25
3	12 40	+0·40	[6 10]	[—0·80]	18	11 30	+0·40	5 00	—1·00
	-----	------	17 35	—0·20		22 30	+0·30	17 40	—0·25
4	0 20	+0·40	7 00	—0·80	19	12 40	+0·80	5 15	—1·10
	13 00	+0·55	18 40	—0·35		23 50	+0·05	18 50	—0·30
5	0 30	+0·15	6 30	—0·90	20	-----	------	6 00	—1·10
	13 30	+0·70	20 00	—0·40		13 20	+1·00	20 20	—0·50
6	1 15	0·00	7 00	—1·05	21	0 30	—0·15	6 40	—1·10
	14 00	+0·80	21 15	—0·45		14 00	+1·30	21 45	—0·50
7	1 10	—0·05	7 30	—0·95	22	1 15	—0·30	7 30	—1·20
	14 30	+0·95	22 00	—0·60		14 40	+1·45	22 30	—0·70
8	1 45	—0·30	7 30	—1·05	23	2 30	—0·50	8 00	—1·20
	15 10	+1·10	22 50	—0·55		15 20	+1·50	23 40	—0·70
9	2 30	—0·35	8 00	—0·95	24	4 00	—0·45	8 40	—1·25
	15 30	+1·20	23 45	—0·60		[16 10]	[+1·50]	-----	------
10	3 50	—0·40	8 30	—1·00	25	[4 40]	[—0·40]	[0 15]	[—0·70]
	16 30	+1·30	-----	------		17 00	+1·55	9 20	—1·10
11	4 40	—0·40	0 20	—0·60	26	5 20	—0·35	0 45	—0·65
	16 55	+1·20	9 15	—1·00		17 45	+1·35	10 00	—1·00
12	5 30	—0·50	0 40	—0·80	27	6 00	—0·35	1 10	—0·65
	17 30	+1·05	9 30	—1·00		18 30	+1·20	11 15	—0·85
13	6 45	—0·60	2 30	—0·80	28	6 45	—0·25	2 45	—0·70
	18 30	+1·10	10 30	—0·95		19 30	+0·95	11 35	—0·70
14	7 00	—0·45	2 30	—0·75	29	7 30	—0·25	3 00	—0·80
	19 00	+1·05	10 50	—0·80		[19 45]	[+0·80]	12 30	—0·55
15	8 00	—0·35	3 00	—0·75	30	[8 15]	[—0·20]	[3 45]	[—0·90]
	20 30	+0·80	12 00	—0·60		20 00	+0·60	12 40	—0·30

The means being taken for intervals of five days, and these means being again consolidated for each successive group of five, we have the following table:

Relative heights of mean sea level at Honolulu, Hawaiian Islands, from June, 1891, to June, 1892.

Mean date.	Mean sea level (five days).	Mean date.	Mean sea level (twenty-five days).	Mean date.	Mean sea level (five days).	Mean date.	Mean sea level (twenty-five days).
1891.	*Feet.*	1891.	*Feet.*	1891–'92.	*Feet.*	1891–'92.	*Feet.*
June 19	−0·22			Dec. 26	−0·10		
24	− ·42			31	+ ·03		
29	− ·22	June 29	−0·25	Jan. 5	− ·04		
July 4	− ·24			10	− ·01		
9	− ·16			15	+ ·05	Jan. 15	−0·04
14	− ·08			20	+ ·02		
19	+ ·13			25	− ·22		
24	+ ·02	July 24	+ ·03	30	− ·10		
29	+ ·07			Feb. 4	·00		
Aug. 3	·00			9	− ·18	Feb. 9	− ·11
8	·00			14	− ·16		
13	+ ·07			19	− ·12		
18	+ ·14	Aug. 18	+ ·19	24	− ·32		
23	+ ·36			29	− ·15		
28	+ ·40			Mar. 5	− ·28	Mar. 5	− ·24
Sept. 2	+ ·40			10	− ·26		
7	+ ·33			15	·19		
12	+ ·40	Sept. 12	+ ·38	20	− ·27		
17	+ ·34			25	·00		
22	+ ·44			30	− ·01	Mar. 30	− ·11
27	+ ·36			Apr. 4	− ·20		
Oct. 2	+ ·26			9	− ·05		
7	+ ·28	Oct. 7	+ ·25	14	− ·12		
12	+ ·17			19	− ·20		
17	+ ·18			24	− ·04	Apr. 24	− ·09
22	+ ·22			29	− ·08		
27	+ ·14			May 4	− ·02		
Nov. 1	+ ·18	Nov. 1	+ ·21	9	+ ·06		
6	+ ·18			14	− ·10		
11	+ ·32			19	− ·11	May 19	− ·08
16	+ ·13			24	− ·06		
21	+ ·14			29	− ·20		
26	+ ·22	Nov. 26	+ ·15	June 3	− ·09		
Dec. 1	+ ·12			8	− ·19		
6	+ ·12			13	− ·26	June 13	− ·17
11	+ ·20			18	− ·12		
16	+ ·01			23	− ·20		
21	+ ·05	Dec. 21	+ ·04				

These values are plotted in the accompanying illustration.



MAGNETIC, GRAVITY, AND LATITUDE OBSERVATIONS IN CONNECTION WITH THE EXPEDITION TO THE SUMMIT OF MAUNA KEA, AND AT SOME SUBSEQUENT STATIONS.

In June, 1892, the latitude determinations were discontinued at Waikiki, and Honolulu was occupied for magnetic and gravity observations. The magnetic work already done at Waikiki and Kahuku is, for convenience of reference, also given in what follows.

DESCRIPTION OF MAGNETIC INSTRUMENTS.

The theodolite magnetometer is the same as the one taken to Africa in 1889–'90. It is designated as No. 11, United States Coast and Geodetic Survey, and was remodeled and improved at the office in July, 1887. The telescope has an aperture of 2^{cm}. The horizontal and vertical circles are each 10^{cm} in diameter. The graduation is from left to right, and angles may be read to single minutes by means of two opposite verniers. The dimensions of the magnets are:

$$^*N\,L_{11}\ \text{length} = 7{\cdot}1^{cm},\ \text{diameter} = 0{\cdot}8^{cm}$$
$$N\,S_{11}\ \quad ``\quad = 5{\cdot}8 \qquad ``\quad = 0{\cdot}8$$

The ring used in determining the moment of mass of the intensity magnet ($N\,L_{11}$) had the following dimensions (C. S. Schott, October 31, 1889):

$$r\ = \text{inner radius} = 1{\cdot}4749^{cm}$$
$$r_1 = \text{outer}\quad `` \quad = 1{\cdot}8906$$
$$w = \text{weight} \quad = 19{\cdot}48940\ \text{grams}$$

from which

$$M_o = \tfrac{1}{2}\,(r^2 + r_1^2)\,w = 56{\cdot}0291$$

at 16°·7 C. or 62°·0 F.

Moment of mass at temperature $t = M_1$

$$M_1 = M_o\,[1 + 2\,e\,(t - t_o)]$$

where e = coefficient of expansion for 1° C. = 0·000019 and

$$\log M_1 = 1{\cdot}74841 + 0{\cdot}0000165\,(t - 16^\circ{\cdot}7)$$

The value of 1 division of scale of long magnet ($N\,L_{11}$) was determined five times during the season, giving a mean result of 3′·68. This, combined with the African value, 3′·72, gives a value of 3′·70, which is adopted in the reduction of the observations. Increasing scale readings correspond to decreasing circle readings.

The moment of mass of the long magnet ($N\,L_{11}$) determined at Washington by Mr. Braid is 95·748 ± 0·094 (C. G. S. units). This value, however, includes the moment of the small balancing ring (K). (See Appendix No. 12, Report 1890.) Deducting this we have for the

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* $N\,L_{11}$ designates new long magnet of No. 11.

moment of mass to be used in the work of 1891–'92 a value of 94·303 at a temperature of 16°·7 C.

$$\log M \text{ at temperature } t = 1{\cdot}97453 + 0{\cdot}0000106\ (t - 16^\circ{\cdot}7)$$

Temperature coefficient (q) of $N\ L_{11}$ = 0·00108 for 1° F. and 0·00194 for 1° C.

Induction factor (h) in C. G. S. units = 0·0457 ± 0·0006 (C. S. Schott, December 6, 1890).

Induction coefficient μ (= mh) (C. G. S. units) = 6·54 ± 0·08 at 62° F.

To correct oscillations for induction

$$\text{to } \log T^2 \text{ add } \log\ (1 + h\ H).$$

To correct deflections for induction

$$\text{to } \log \frac{m}{H} \text{ add } \log \left(1 + \frac{2\mu}{r^3}\right) = 0{\cdot}00020 \text{ for } r = 30{\cdot}54^{\text{cm}}$$

$$= 0{\cdot}00006 \text{ for } r = 45{\cdot}78^{\text{cm}}$$

First distribution coefficient of $N\ L_{11} = P = -4 \pm 4$ (C. G. S. units).

$$\text{For short deflecting distance } (r = 30{\cdot}54^{\text{cm}}) \log\left(1 - \frac{P}{r^2}\right) = 0{\cdot}00182$$

$$\text{" long " " } (r = 45{\cdot}78^{\text{cm}}) \text{ " " } = 0{\cdot}00082$$

Reduction of magnetic moment (m) of intensity magnet ($N\ L_{11}$) to 16°·7 C. = $m_{16^\circ{\cdot}7} = m\ [1 + (t - 16^\circ{\cdot}7) \times 0{\cdot}00194]$

$\log m_{16^\circ{\cdot}7} = \log m$ + modulus $\times$ 0·00194 (t — 16°·7) = $\log m$ + 0·00084 (t — 16°.7)

Values of magnetic moment (m) at 16°·7 C. of long magnet ($N\ L_{11}$), showing loss of magnetism.

[Observer: E. D. Preston.]

Station.	Date.	Magnetic moment at 16°·7 C. (C.G.S. units).	No. of sets.
Washington	1889·73	143·3	3
Azores Islands	·83	140·4	2
Cape Verde Islands	·86	140·7	1
Africa:			
Sierra Leone	·88	134·3	1
Gold Coast	·91	128·9	2
Loanda	·96	129·4	3
Cabiri	·97	135·0	2
Cape Town	1890·07	134·2	5
St. Helena, Jamestown	·15	133·9	2
" " Longwood	·17	133·7	3
Ascension Island, Georgetown	·22	133·8	3
" " Green Mountain	·33	133·7	3
Barbados	·33	133·3	3
Bermuda	·39	131·9	3
Hawaiian Islands, Polynesia:			
Waikiki	1891·62	129·6	3
Kahuku	·90	128·8	3
Honolulu	1892·42	127·9	3
Kawaihae	·50	128·5	3
Waimea	·52	128·0	1
"	·53	128·8	2
Kalaieha	·54	128·3	2
Waiau	·56	123·6	2
Hilo	·58	127·1	3
Napoopoo	·64	124·7	3
Lahaina	·65	125·0	3
Waimea A	·67	125·0	2
" B	·68	124·0	3
Nonopapa	·70	125·4	1

Summary of time and azimuth results in connection with magnetic observations.

Station.	Date.	Epoch.	Correction to chronometer.*	Adopted daily rate.†	Position of mark.	Adopted position of mark.
	1891–'92.	h. m.	m. s.		° ′	° ′
Waikiki	Aug. 10	8 23 p. m.	− 8 10·9			
	11	8 19	− 8 27·6	−17		
	13	8 12	− 9 2·2			
	15	8 00 a. m.			N. 1 53·0 E.	N. 1 52·2 E.
	15	5 30 p. m.			N. 1 51·4 E.	
Kahuku	Nov. 25	9 00 a. m.			S. 65 8·8 W.	S. 65 7·6 W.
	25	3 00 p. m.	− 1 54		S. 65 6·4 W.	
	27	9 30 a. m.	− 3 37	−58		
Honolulu ‡	June 10	9 00	+ 0 37	+ 2	N. 55 20·0 E.	N. 55 20·0 E.
Kawaihae	30	5 00 p. m.	+ 4 03	−10	S. 25 2·7 E.	S. 25 2·2 E.
	July 3	5 00	+ 3 33		S. 25 1·8 E.	
Waimea	9	9 30 a. m.	− 0 26		S. 81 40·2 E.	
	9	4 30 p. m.	− 0 42	−17	S. 81 51·4 E.	S. 81 50·3 E.
	11	9 30 a. m.	− 0 65		S. 81 58·3 E.	
Kalaieha	14	8 40	+ 3 50		N. 63 47·7 W.	
	15	8 40	+ 3 33	−18	N. 63 54·6 W.	N. 63 48·3 W.
	15	8 40 p. m.	+ 3 25		N. 63 45·3 W.	
Waiau	21	10 20 a. m.	+ 3 54		N. 22 8·0 W.	
	21	4 30 p. m.	+ 3 47	−27	N. 22 3·2 W.	N. 22 5·6 W.
Hilo §	30	7 50 a. m.	+18 28		N. 72 41·0 E.	
	31	9 40	+18 50	+20	N. 72 43·1 E.	
	Aug. 1	9 20	+17 46	−56	N. 72 49·7 E.	N. 72 37·6 E.
	1	3 30 p. m.	+17 45	− 1	N. 72 30·8 E.	
Napoopoo	17	4 30	− 5 48		N. 78 18·7 W.	
	19	9 10 a. m.	− 6 18	−18	N. 78 14·6 W.	N. 78 22·7 W.
	20	2 45 p. m.	− 6 45	−22	N. 78 28·8 W.	
Lahaina	23	9 30 a. m.	+ 0 10		S. 0 51·4 E.	
	24	9 00	[− 0 41]	−24	S. 1 5·3 E.	S. 1 00·4 E.
	25	9 20	− 0 38		S. 1 4·5 E.	
Waimea A	Sept. 2	8 30	+ 0 09	−18	S. 39 37·2 W.	S. 39 40·7 W.
	3	8 20	− 0 09		S. 39 44·2 W.	
Waimea B	5	8 25	− 1 02	−14	S. 74 35·3 E.	
	6	8 25	− 1 16	−22	S. 74 26·5 E.	S. 74 33·8 E.
	7	8 10	− 1 38		S. 74 39·6 E.	
Nonopapa	9	8 50	− 4 42	−18	N. 4 13·6 W.	N. 4 13·6 W.

* Bond chronometer No. 177.
† + = losing; − = gaining.
‡ At Honolulu chronometer Gowland, 3280, belonging to the Government Survey, was used. The azimuth was also furnished by Mr. C. J. Lyons, of that service.
§ At Hilo watch No. 7583 was used.

MAGNETIC OBSERVATIONS AT WAIKIKI.

These observations were made on August 11, 12, and 13, 1891. As it was necessary to continue the time, latitude, and gravity determinations throughout the year, an epoch was chosen for the magnetic work which would least interfere with the regular observations. Accordingly, about the middle of August, just before changing the groups of stars then being observed for latitude, the magnetic work was undertaken. This implied almost continual observation and computation from 6.30 a. m. to 11 p. m., and involved the determination of the magnetic declination, dip, and horizontal intensity, with three sets of observations for time and azimuth with the theodolite magnetometer, besides the regular work with the pendulum apparatus, the zenith telescope, and the meridian instrument in the observatory. This was accomplished without any assistance either by a recorder or aid. The limit of satisfactory work was probably reached in this scheme, as a full day's magnetic work, followed by five hours' night observations, about exhausts an ordinary observer's capacity for good work. The station was situated near the astronomical observatory and on the property of Mr. J. F. Brown. The sketch on page 581 gives the location.

Abstract of results, Waikiki, Hawaiian Islands.

DIP.

Date.	Needle No. 1.			Needle No. 2.		
	N.	S.	Mean.	N.	S.	Mean.
1891.	° ′	° ′	° ′	° ′	° ′	° ′
Aug. 11	40 08	39 29	39 48	39 51	40 02	39 56
12	14	28	51	40 10	01	40 06
13	20	28	54	39 50	06	39 58

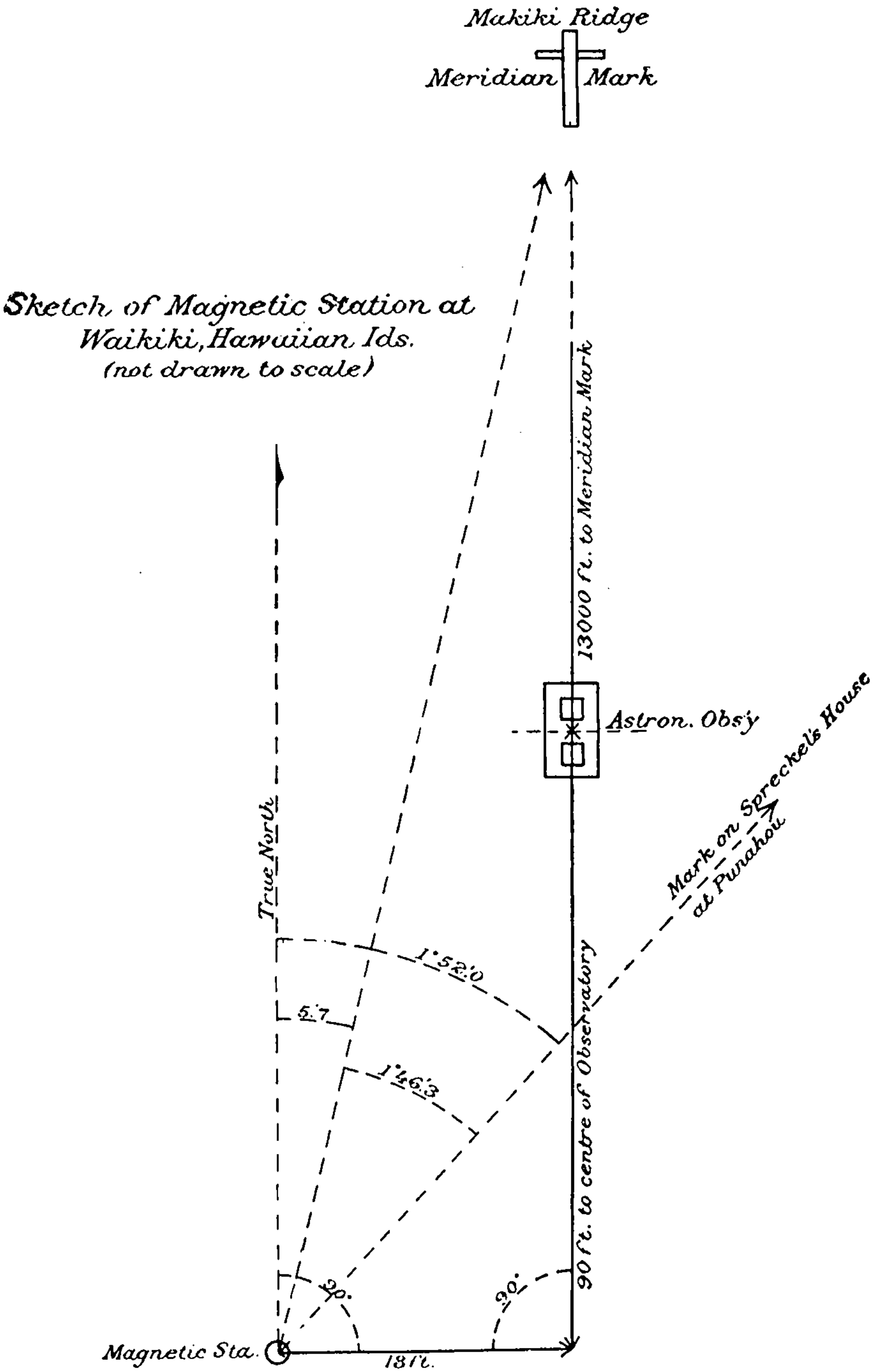
DECLINATION, DIP, AND INTENSITY.

Date.	Scale reading of axis.	Declination (east).	Mean dip.	Horizontal intensity = H.	Magnetic moment of magnet.
1891.	*d.*	° ′	° ′	*Dynes.*	
Aug. 11	28·72	10 05·3	39 52·0	0·2979	129·8
12	28·80	04·9	58·5	0·2971	129·6
13	-----	05·3	56·0	0·2992	129·4

MAGNETIC OBSERVATIONS AT KAHUKU, OAHU.

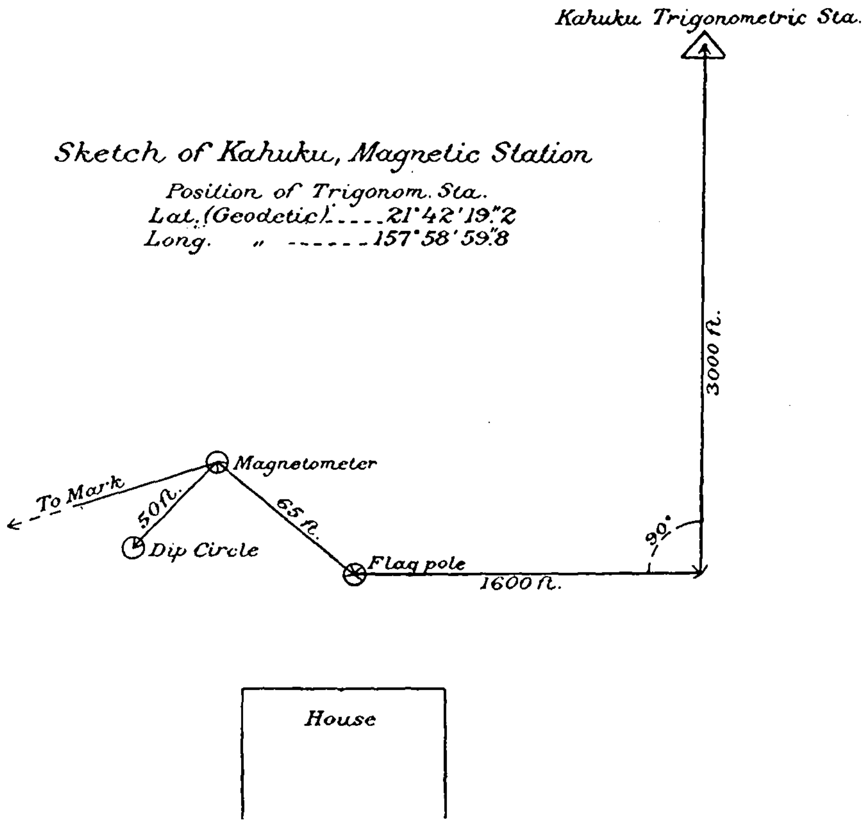
Taking advantage of an interval of one week in November, when my colleague on the international latitude work was necessarily absent from Waikiki, a set of magnetic determinations was made at the extreme north end of Oahu. Accompanied by Prof. W. D. Alexander, I left Waikiki on the morning of November 24, taking the steam cars from Honolulu to Ewa and riding from Ewa to Kahuku, a distance of 30 miles. We arrived at Kahuku late in the evening. Observations were begun early next morning and continued until about 1.30 p. m. on the 27th, when we started for Honolulu by way of the Pali. Stopping over night at Kualoa, Waikiki was reached the following afternoon and the observatory work was immediately resumed. Less than five days' absence were necessary for three full days' work at a station 40 miles from home, the distance being made both ways on horseback over a difficult road. The station occupied was situated in the front yard of the Kahuku ranch cottage. The cottage is about

[The diagrams hereafter given are not generally drawn to scale, being inserted simply to aid in reestablishing the position of the station in case it is desirable to repeat the observations at some future time.]



three-fifths of a mile south and about one-third of a mile west of the trigonometrical station on the extreme north point of the island. The astronomical latitude of this station was determined in 1887.

The following sketch gives the relative positions:



Abstract of results, Kahuku, Oahu.

DIP.

Date.	Needle No. 1.						Needle No. 2.					
	N.		S.		Mean.		N.		S.		Mean.	
1891.	°	′	°	′	°	′	°	′	°	′	°	′
Nov. 25	42	01	41	01	41	31	41	34	41	20	41	27
26	41	34		05		20		26		28		27
27		41		07		24		12		38		25

Abstract of results, Kahuku, Oahu—Continued.

DECLINATION, DIP, AND INTENSITY.

Date.	Scale reading of axis.	Declination (east).	Mean dip.	Horizontal intensity.	Magnetic moment of magnet.
1891.	*d.*	° ′	° ′	*Dynes.*	
Nov. 25	-----	10 16·4	41 29·0	0·2932	129·3
26	29·34	13·0	23·5	31	128·6
27	-----	14·7	24·5	44	128·4

OBSERVATIONS AT HONOLULU.

Gravity—Description of station.—The receiver was mounted on the window sill of the second story south room of the Kapuaiwa building, occupied by the Government Survey. Time observations were made in the observatory near by, and the chronometer used in noting the pendulum coincidences was connected with the chronograph so as to be used for the star observations in the evening. The oscillations were made generally in the daytime, as it was necessary for me to devote the evenings to either time observations at Honolulu or latitude observations at Waikiki, and, indeed, sometimes to both on the same night. The gravity determinations heretofore made at Honolulu were in 1883 and 1887. At the first-named time the station occupied was the cellar of the Young Men's Christian Association building, on the corner of Hotel and Alakea streets, and the instrument used was a pendulum of the reversible type (No. 3), measuring 1 yard between the knives In 1887 the Kapuaiwa building was occupied, and the pendulums were hung from an iron bracket embedded in the wall in the first story of the building. The determinations in this case were made with two pendulums, of which one was (No. 3) cited above, and the other (No. 4) of the same pattern, but having a distance of 1 metre between the knives. The small pendulums employed in 1891–'92 were therefore oscillated at Honolulu not only to connect the long series of observations made at Waikiki with a permanent station, but also to have an additional check on the work that was soon to follow on the Island of Hawaii. Moreover, the permanent station in Honolulu would thus be connected with our continental stations by means of a new form of instrument, which would materially strengthen the result.

The following are the pendulum observations and reductions. The mean of the direct and reverse results are combined with equal weights, irrespective of the number of individual observations in each position:

Pendulum observations. Honolulu, Hawaiian Islands.

[Observers: E. D. Preston, W. E. Wall.]

Pendulum.	Position.	Swing.	Date.	No. of coincidence intervals.	Time of ten coincidence intervals.	Semi-arc. Initial.	Semi-arc. Final.	Temperature.	Manometer.	Barometer.	Pressure at 0° C.
			1892.		*Seconds.*	*mm.*	*mm.*	° *C.*	*mm.*	*mm.*	*mm.*
B_1	D	1	June 23	10	2263·0	4·8	3·4	25·93	73·4	767·0	630
		2	24	14	2254·3	4·9	1·4	28·52	119·4	766·2	583
	R	3	24	12	2260·8	4·8	3·5	28·40	214·6	765·8	497
		4	24	16	2263·1	4·7	3·2	28·25	229·0	765·5	485
		5	24	12	2262·5	4·7	3·5	27·90	221·0	765·3	492
	R	6	24	10	2266·5	4·8	3·9	26·64	200·0	766·0	514
		7	24	10	2267·5	4·8	3·9	26·34	189·0	766·4	525
B_2	D	8	25	6	2437·5	5·0	4·2	28·60	166·8	766·0	540
	R	9	25	4	2441·2	5·0	4·4	28·80	160·6	765·9	546
		10	25	4	2438·8	5·0	4·2	28·85	161·0	765·9	545
B_3	D	11	25	6	2614·2	4·6	3·8	26·28	97·4	765·6	608
		12	25	8	2613.1	4·8	3·7	25·93	88·2	766·0	617
		13	25	6	2617·5	4·7	4·0	25·48	82·0	766·4	624
		14	25	6	2615·8	4·8	4·1	25·22	65·2	766·4	640

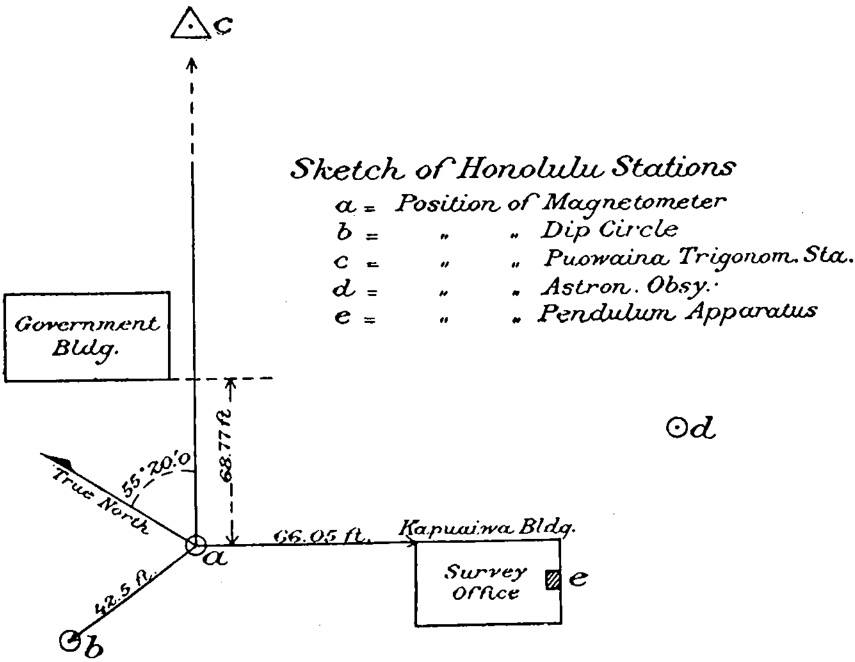
Reduction of pendulum observations, Honolulu, Hawaiian Islands.

[Periods reduced to temperature, 15° C.; pressure, 500mm at 0° C.; arc infinitely small; sidereal time.]

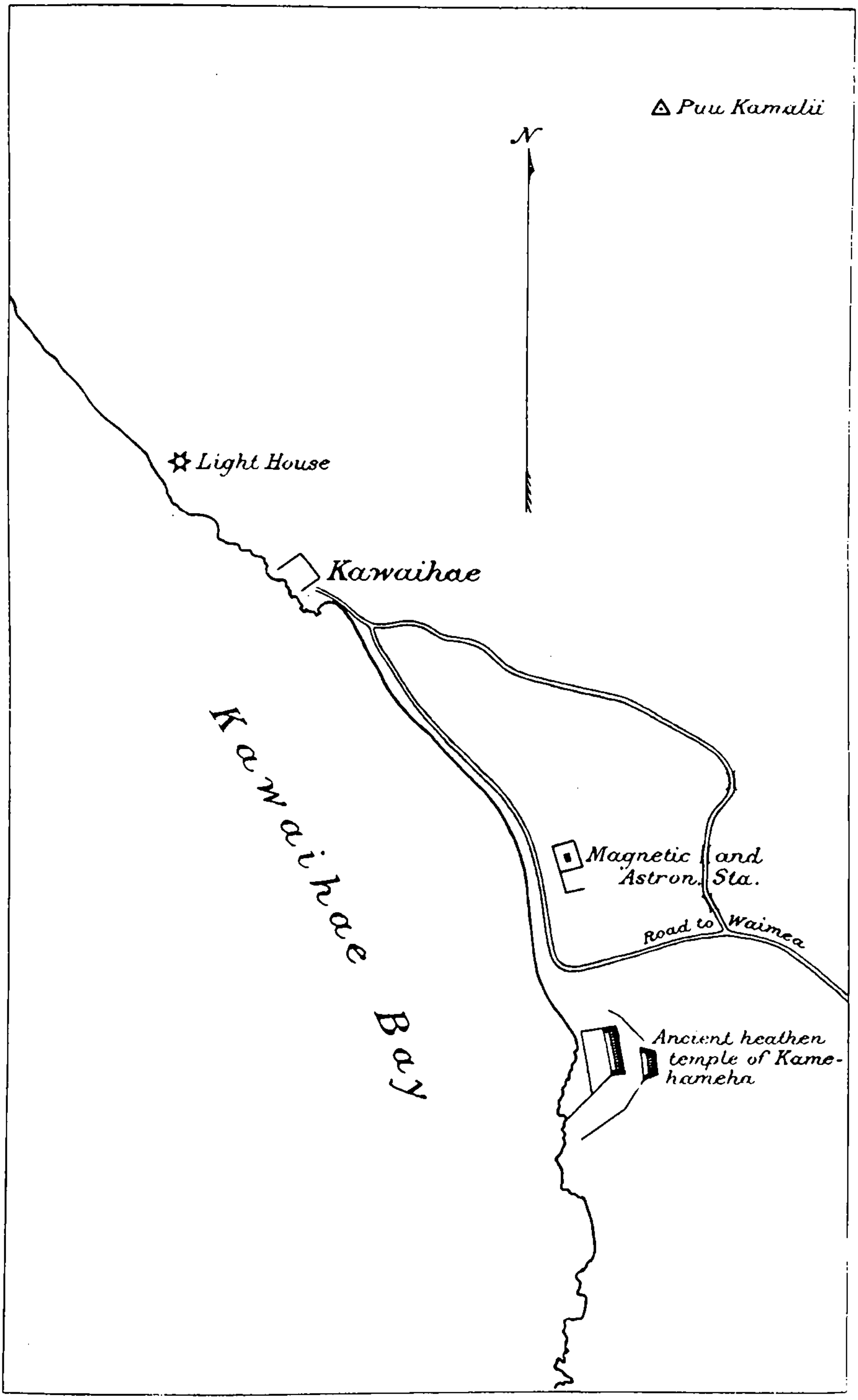
Period uncorrected.	Corrections (in seventh decimal place).				Period corrected.
	Arc.	Temperature.	Pressure.	Rate.	
Seconds.					*Seconds.*
0·501 1072	−59	−453	−103	+326	0·501 0783
1115	−31	−561	− 65	+326	784
1083	−61	−556	+ 2	+326	794
1071	−55	−549	+ 12	+326	805
1074	−59	−535	+ 06	+326	812
1055	−67	−483	− 11	+326	820
1050	−67	−471	− 20	+326	818
					0·501 0797
0·501 0277	−75	−564	− 31	+326	0·500 9933
0262	−78	−573	− 36	+326	901
0272	−75	−575	− 36	+326	912
					0·500 9920
0·500 9582	−62	−467	− 85	+349	0·500 9317
9585	−64	−453	− 92	+349	325
9569	−67	−434	− 98	+349	319
9575	−70	−424	−110	+349	320
					0·500 9320

MAGNETIC OBSERVATIONS AT HONOLULU.

The station was located in the public yard near the Government building. The magnetometer was set up in range with the northeast side of the Kapuaiwa building, and at a point which would give a clear line of sight past the Government building to the trigonometric station (Puowaina) on Punch Bowl hill. The dip circle was placed under the shade of the banyan tree, a few paces distant. Time and azimuth observations were not necessary at this station, as the standard clock



of the Survey office was kept rated for the regular time service, and the direction from the magnetic station to the trigonometric point on Punch Bowl was accurately known from the Government triangulation. The azimuth of the line magnetic station – Punch Bowl, as furnished by Mr. C. J. Lyons, in charge of the Survey office, is north 55° 20′·0 east.


Puu Kamalii
N
Light House
Kawaihae
Kawaihae Bay
Magnetic and Astron. Sta.
Road to Waimea
Ancient heathen temple of Kamehameha

MAGNETIC OBSERVATIONS.

Abstract of results, Honolulu, Hawaiian Islands.

DIP.

Date.	Needle No. 1. N.	Needle No. 1. S.	Needle No. 1. Mean.	Needle No. 2. N.	Needle No. 2. S.	Needle No. 2. Mean.
1892.	° ′	° ′	° ′	° ′	° ′	° ′
June 2	41 02	40 10	40 36	40 42	40 54	40 48
3	40 55	13	34	34	46	40
4	41 10	22	46	46	52	49

DECLINATION, DIP, AND INTENSITY.

Date.	Scale reading of axis.	Declination (east).	Mean dip.	Horizontal intensity.	Magnetic moment.
1892.	*d.*	° ′	° ′	*Dynes.*	
June 2		10 14·7	40 42·0	0·2954	127·9
3	29·12	17·4	37·0	47	·9
4	29·00	16·7	47·5	51	·9

GRAVITY OBSERVATIONS.

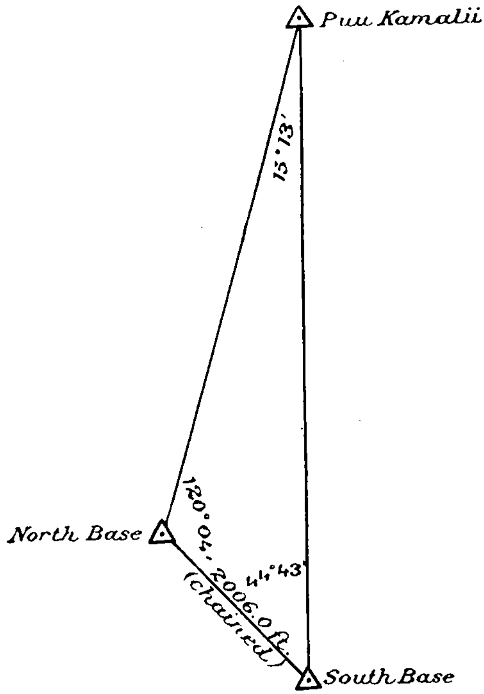
	s.
Pendulum periods	$B_1 = 0{\cdot}5010797$
(Sidereal seconds)	$B_2 = 0{\cdot}5009920$
	$B_3 = 0{\cdot}5009320$

KAWAIHAE.

Island of Hawaii.

Leaving Honolulu in the afternoon of June 28, on board the ***Kinau***, we arrived at Kawaihae on the evening of the 29th. The party consisted of Prof. W. D. Alexander, surveyor-general; Messrs. W. E. Wall, W. W. Chamberlain, Louis Koch, and myself. The first observations were made on the 30th. The work at this station consisted of gravity, latitude, time, and magnetic determinations. The station was situated on the property of the Hon. Samuel Parker, to whom, as well as to the general superintendent, Mr. Paul Jarrett, our thanks are due for many acts of kindness. The connection between the astronomical station and the triangulation of the Government survey was carefully made. The general location of the property is between the boat landing and the Heiau of Kamehameha I, and about one-third the distance from the Heiau. Illustration No. 27 gives the relative positions. Illustration No. 26 shows the triangulation station and shore line.

Position of latitude station at Kawaihae, Hawaii.



Distance from Puu Kamalii to north base = 5377·47 ft.
" " " " " south " = 6614·39 "
∠ " " " " " = 7° 34′

		°	′	″
Geodetic L Puu Kamalii	=	20	03	28·80
" dL	= −	0	01	5·00
" L' south base		20	02	23·80

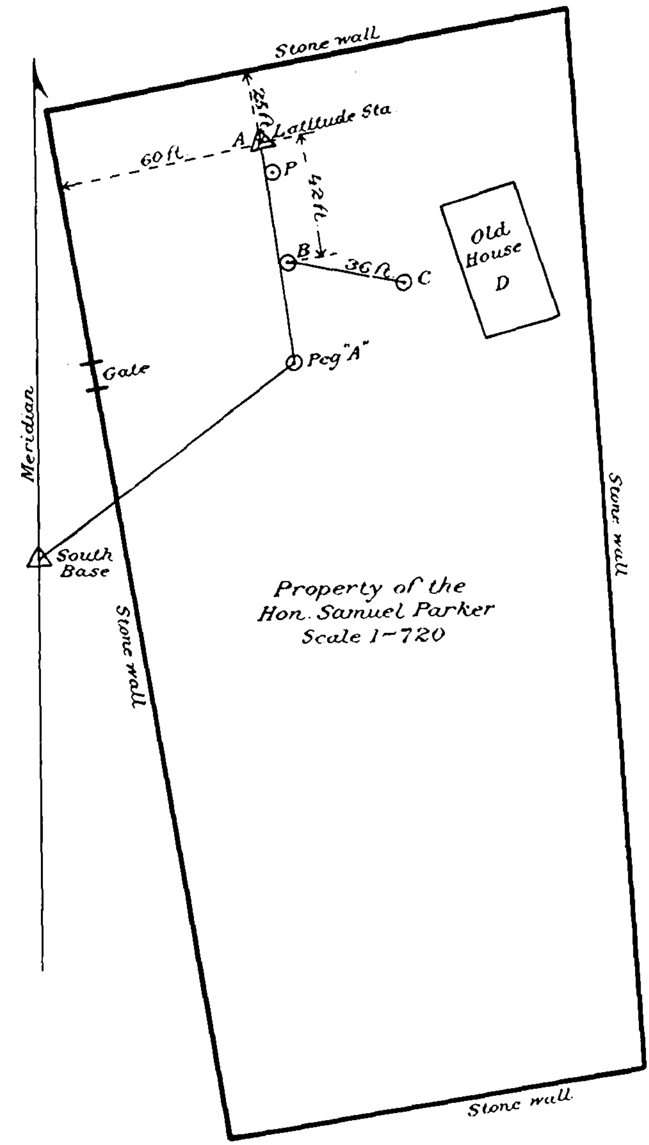
		°	′	″
Geodetic M Puu Kamalii	=	155	47	27·19
" dM	= +	0	00	09·14
" M' south base	=	155	47	36·33

Traverse, south base to Kawaihae latitude station:

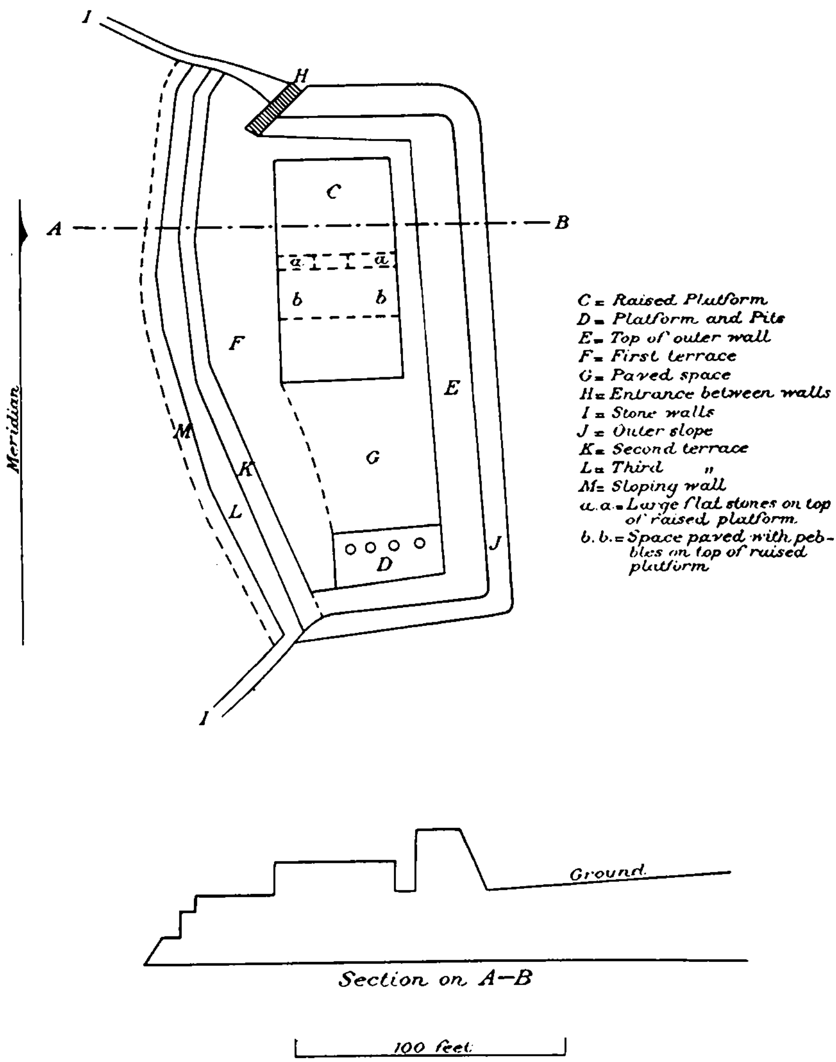
1. N 52° 43′ E., 100 ft. to peg "A"
2. N 6 55 W., 68.7 ft. to latitude station

Latitude station 128·77 ft. north of south base
" " 71·29 " east " " "

Location of Kawaihae Stations with reference to So. Base



A = Latitude pier (Meridian telescope)
B = Dip Circle
C = Magnetometer
D = Ruins of stone house (John Young)
P = Pendulum Station



Plan of Heiau of Puu Kohola at Kawaihae, Hawaii

		°	′	″
Latitude L south base	=	20	02	23·80
dL	=	+	00	1·28
" L' latitude sta.	=	20	02	25·08

		°	′	″
Longitude M south base	=	155	47	36·33
dM	=	−	00	0·70
" M' latitude sta.	=	155	47	35·63

At this station are the remains of an ancient temple, famous in early Hawaiian history as the scene of the first steps by which all the islands were consolidated under one government. It was here that Kamehameha betrayed and murdered his rival, Keoua, baked his body in an oven as a last indignity, and finally deposited it in the temple on the altar of the war god. He was henceforth recognized as the master of Hawaii. A sketch of this interesting Heiau from actual measurements has been furnished by Professor Alexander and is given here as a matter of curiosity. [Illustration No. 28.]

A remarkable feature of it is that although the early Hawaiians had no metal tools, and are to-day poor mathematical reasoners, their temples furnish examples of quite accurate right angles. One tested with a theodolite at Napoopoo was surprisingly near the truth.

Magnetic observations were made on July 1, 2, and 3. The pendulums were swung on the 3d, 4th, 5th, and 6th, and time and latitude were observed during the entire stay. We left on the morning of the 7th. The weather was generally favorable for work with the exception of one or two occasions when we had sudden gusts of wind from the mountains.* At this point preparations were made for the ascent of Mauna Kea. Packers and horses were engaged and the services of a guide secured. The connection of the astronomical station with the trigonometrical survey of the island was made at this station as well as at Kalaieha and Waiau by Professor Alexander. Meteorological observations were undertaken here in connection with those made at Honolulu and Hilo, to be used subsequently in determining the height of the mountain. The pendulum receiver was mounted on a large and massive cast-iron wheel, firmly embedded in the ground. This was part of the machinery of an old mill and was fortunately found near the spot where it was desirable to use it. The tent was erected close by and the flash apparatus was set up inside the tent. The meridian telescope was also erected so near that the tent fly served as a protection during the day, and at night was released and thrown back while the star observations were in progress. The following are the results of the pendulum observations:

* These storms are called mumúku in the native language, in contradistinction to the storms from the southeast, which receive the name of kona.

Pendulum observations, Kawaihae, Hawaiian Islands.

[Observers: E. D. Preston, W. E. Wall.]

Pendulum.	Position.	Swing.	Date.	No. of coincidence intervals.	Time of ten coincidence intervals.	Semi-arc. Initial.	Semi-arc. Final.	Temperature.	Barometer.	Pressure at 0° C.
			1892.		*Seconds.*	*mm.*	*mm.*	° *C.*	*mm.*	*mm.*
B_1	D	1	July 3	20	2184·0	4·9	3·0	26·78	765·6	695
	D	2	3	18	2187·8	4·9	2·8	25·22	765·5	697
	R	3	4	16	2173·4	4·6	2·8	26·53	766·0	696
	D	4	4	18	2165·6	4·9	2·9	27·85	766·2	693
	D	5	4	24	2156·2	4·9	2·5	29·66	766·1	688
	R	6	4	30	2143·0	4·9	2·0	31·53	765·4	683
B_2	D	7	4	14	2343·6	5·0	3·3	27·34	765·2	692
	R	8	4	16	2356·0	5·0	2·8	26·33	765·3	695
	R	9	5	18	2369·5	5·0	2·7	24·77	766·2	700
	D	10	5	20	2348·0	5·0	2·6	26·18	766·3	697
	D	11	5	18	2334·7	5·0	2·7	27·79	766·3	693
	R	12	5	28	2330·5	5·0	2·1	29·66	766·1	688
	R	13	5	36	2353·6	3·9	1·3	29·06	765·5	622
B_3	D	14	5	14	2500·4	4·6	2·8	27·49	766·0	694
	R	15	5	10	2498·5	4·6	3·0	26·84	766·2	695
	D	16	6	28	2532·2	4·2	1·7	25·88	766·9	626
	D	17	6	16	2494·4	4·8	2·8	27·64	767·0	695
	R	18	6	14	2482·9	4·7	2·8	29·06	767·2	691
	R	19	6	14	2485·7	4·7	2·3	29·56	767·2	690

Reduction of pendulum observations, Kawaihae, Hawaiian Islands.

[Periods reduced to temperature, 15° C.; pressure, 500mm at 0° C.; arc infinitely small; sidereal time.]

Period uncorrected.	Corrections (in seventh decimal place).				Period corrected.
	Arc.	Temperature.	Pressure.	Rate.	
Seconds.					*Seconds.*
0·501 1473	−54	−489	−154	+333	0·501 1109
1454	−51	−424	−155	+333	157
1529	−48	−479	−155	+333	180
1571	−53	−533	−152	+333	166
1622	−47	−608	−148	+333	152
1693	−40	−686	−144	+333	156
					157
0·501 0690	−60	−512	−151	+316	0·501 0283
0634	−52	−470	−154	+316	274
0573	−51	−405	−158	+316	275
0670	−49	−464	−155	+316	318
0731	−51	−531	−152	+316	313
0750	−42	−608	−148	+316	268
0645	−22	−583	−096	+316	260
					287
0·501 0018	−48	−518	−153	+314	0·500 9613
0026	−50	−491	−154	+314	645
0 9893	−29	−451	−099	+314	628
1 0042	−50	−525	−154	+314	627
0089	−49	−583	−151	+314	620
0078	−42	−603	−150	+314	597
					622

MAGNETIC OBSERVATIONS.

Abstract of results, Kawaihae, Hawaii.

DIP.

Date.	Needle No. 1.			Needle No. 2.		
	N.	S.	Mean.	N.	S.	Mean.
1892.	° ′	° ′	° ′	° ′	° ′	° ′
July 1	38 32	37 46	38 09	38 04	38 15	38 10
2	42	48	15	05	14	10
3	37	41	09	13	16	14

DECLINATION, DIP, AND INTENSITY.

Date.	Scale reading of axis.	Declination (east).	Mean dip.	Horizontal intensity.	Magnetic moment.
1892.	d.	° ′	° ′	Dyne.	
July 1	29·18	9 18·7	38 09·5	0·3001	129·4
2	28·88	20·5	12·5	0·3025	128·2
3	-----	22·5	11·5	0.3010	127·8

GRAVITY OBSERVATIONS.

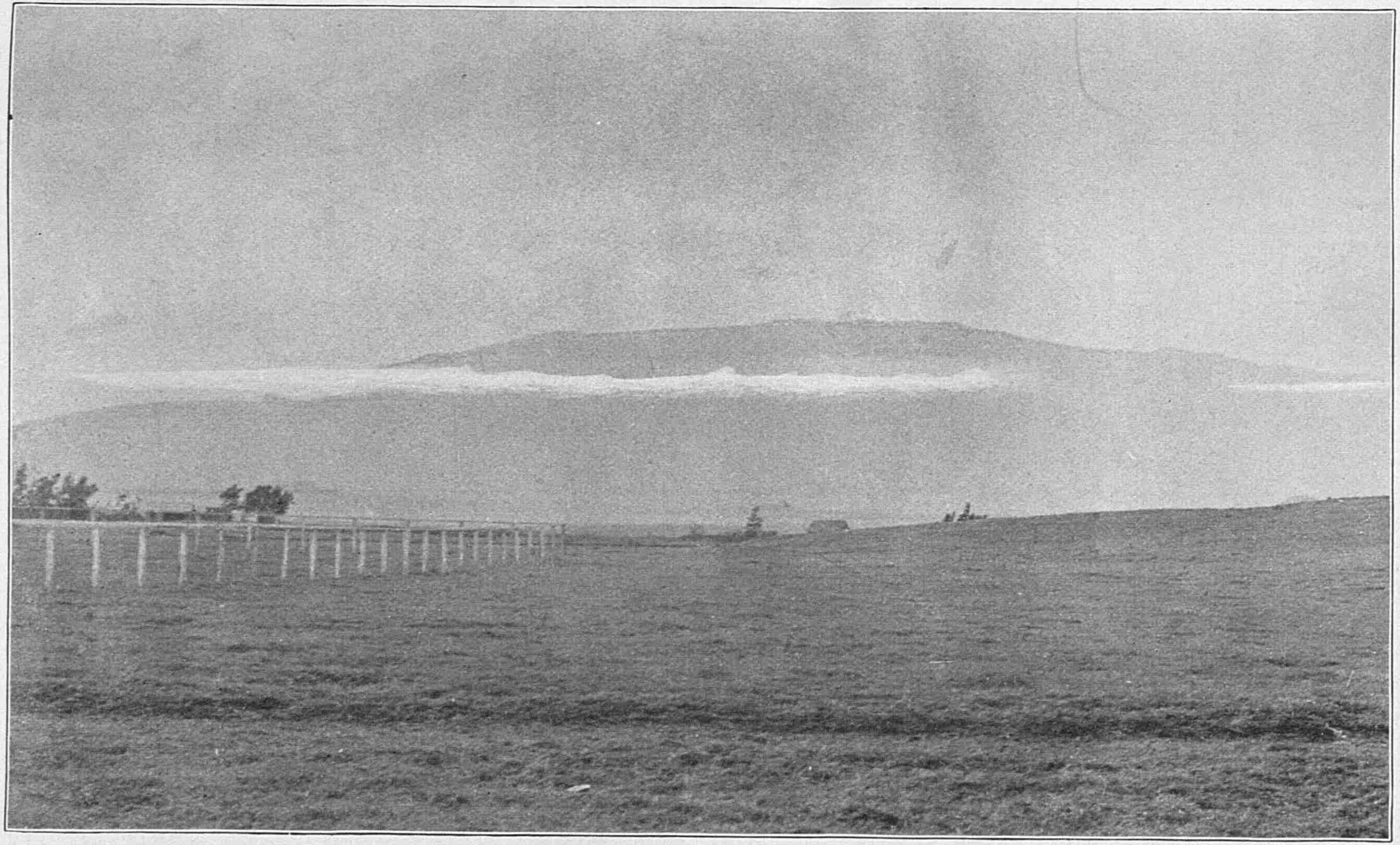
[Sidereal seconds.]

Pendulum periods

$$B_1 = 0{\cdot}5011157 \text{ s.}$$
$$B_2 = 0{\cdot}5010287$$
$$B_3 = 0{\cdot}5009622$$

DETERMINATIONS OF LATITUDE.

The latitude work at the three stations occupied for gravity was only of secondary importance. Only a limited number of pairs were selected for observation, and at Kalaieha the weather was so unfavorable that but three latitudes were obtained. The instrument used was a meridian telescope of 31 inches focal length, $2\frac{1}{2}$ inches aperture, and magnifying power of 77. One revolution of the micrometer gave an angular value of 65″·85. One division of the latitude level is equal to 1″·66, and that of the striding level is 2″·21. The instrument is known as Meridian Telescope No. 2. In making the observations only one bisection was made, and the level was not read more than once, generally after the measurement with the micrometer. The results are not comparable in point of accuracy with those made at Waikiki with the zenith telescope, partly on account of the inferior accuracy of the instrument, but principally because the pier was generally constructed under poor conditions of stability. At Kawaihae and Kalaieha only a wooden pier was available, and at Waiau the great difficulty of trans-



MAUNA KEA, AS SEEN FROM WAIMEA, DISTANT 15 MILES, LOOKING SOUTHEAST.
Cloud belt at 10,000 feet elevation.

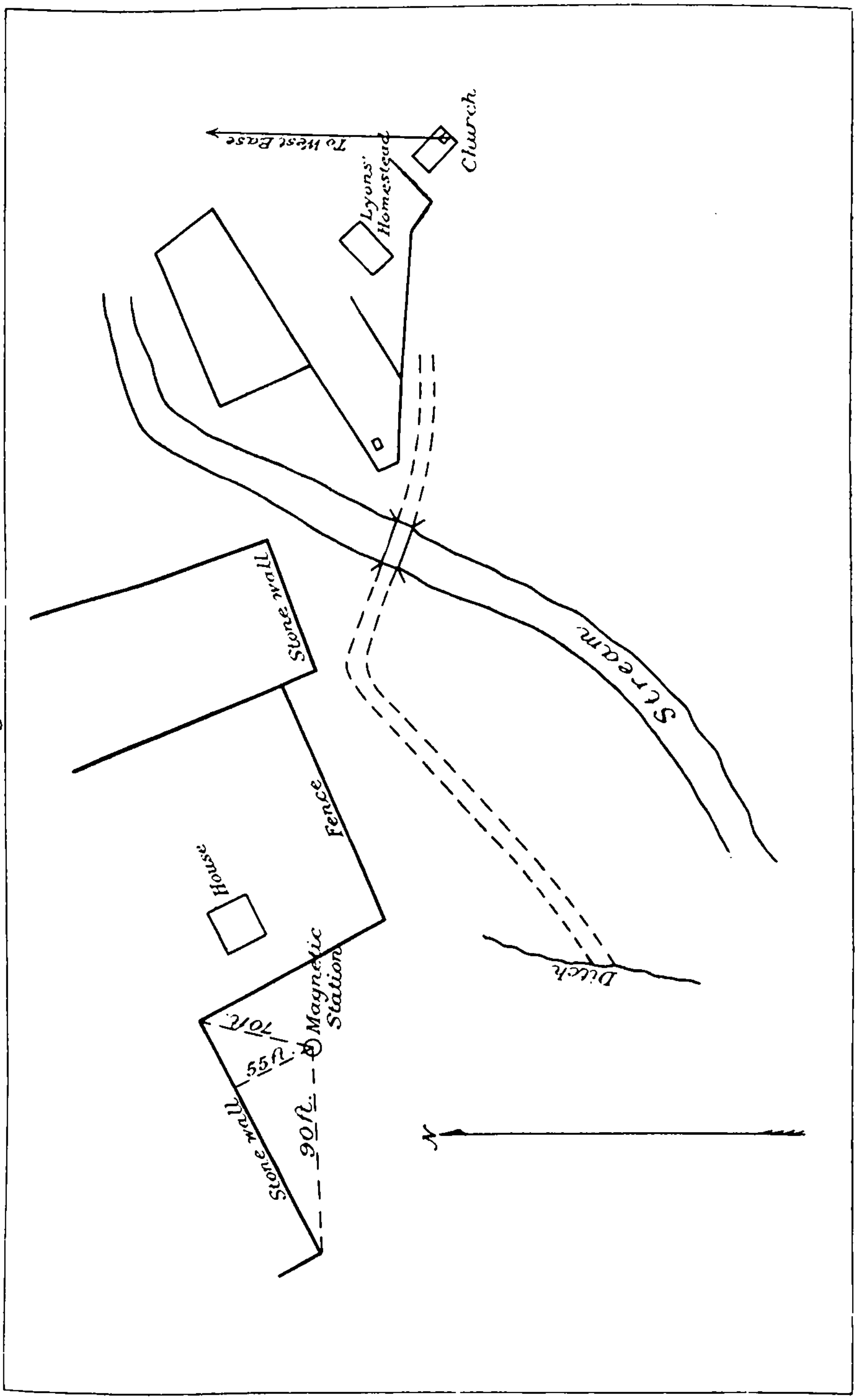
portation and labor made it necessary to construct a very small substructure. The following tables give the mean places and the individual results for latitude; e is the probable error and μ is the proper motion:

Mean places of latitude stars.

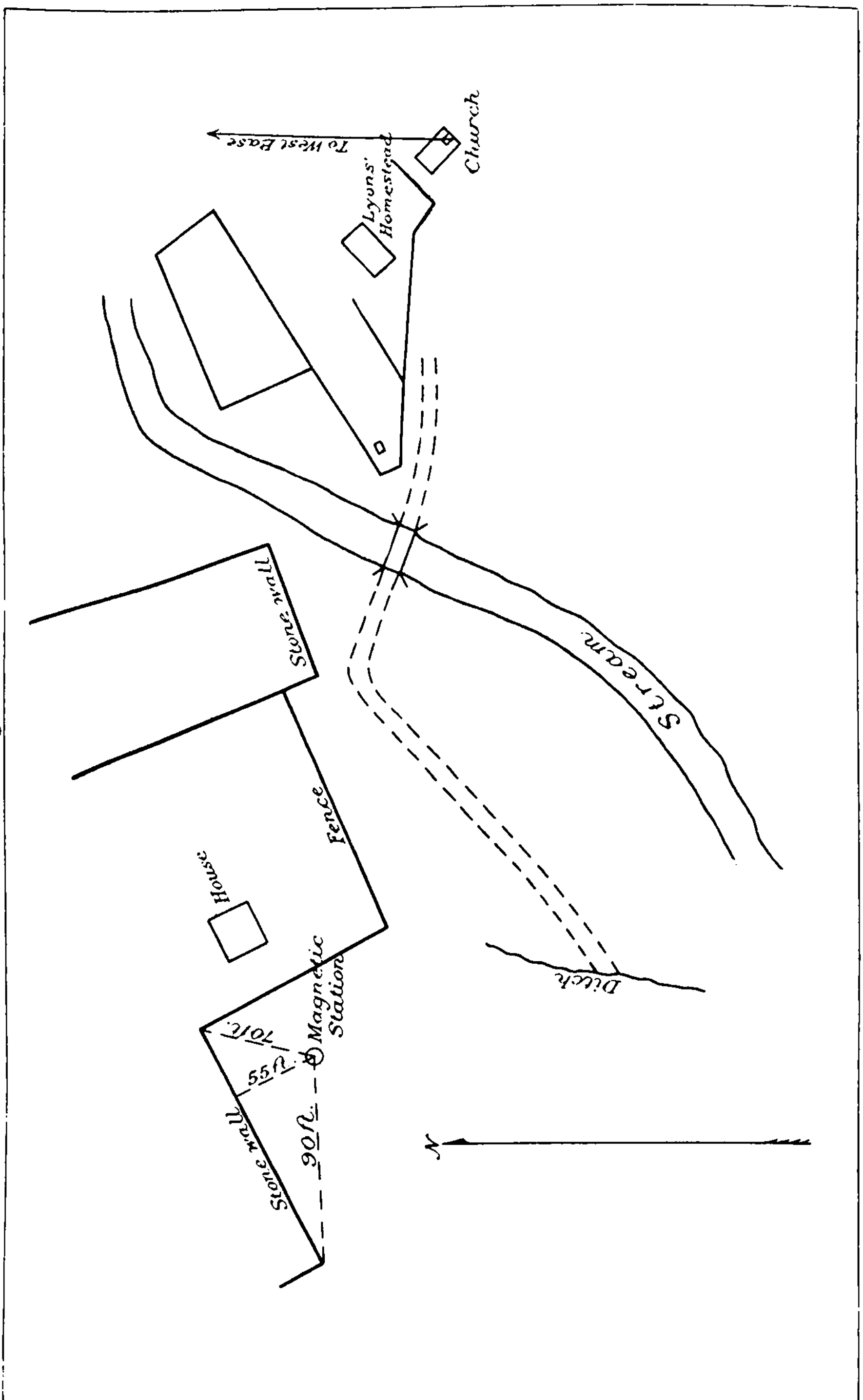
No. of star (U. S. C. and G. S. catalogue).	Mean right ascension (R. A.). 1892.0. h.	m.	s.	Mean declination (δ). 1892·0. °	′	″	e^2 ″	Proper motion (μ). ″
65	0	43	17	16	21	26·70	0·08	—0·210
73		49	01	23	02	35·70	·03	—0·042
78		51	26	22	50	03·63	·05	—0·054
1247	15	01	50	48	34	06·01	·03	+0·023
8313*		11	12	- 8	59	02·94	·02	—0·030
1257		13	48	2	10	27·21	·02	—0·524
1267		20	24	37	45	21·60	·02	+0·071
1284		30	06	27	04	41·89	·02	—0·107
1298		36	43	13	11	38·95	·08	—0·029
1302		38	12	26	38	16·52	·02	+0·029
1309		43	52	18	28	31·07	·02	—0·104
1317		46	32	21	18	09·86	·07	+0·005
1322		51	28	16	00	51·59	·02	—1·299
1333		57	39	23	06	16·19	·05	+0·025
1337	16	03	12	17	20	05·41	·03	—0·023
1342		07	55	5	17	51·29	·10	—0·005
1346		10	38	34	07	52·76	·03	—0·128
1361		21	34	37	38	24·51	·05	—0·022
1367		25	28	2	13	13·89	·01	—0·093
1383		35	49	12	36	17·63	·07	—0·022
1387		37	14	27	07	30·74	·08	—0·062
1390		39	53	34	14	17·07	·04	+0·058
1394		42	27	5	26	26·27	·10	—0·057
1416		59	58	— 0	44	36·71	·04	—0·006
1424	17	06	03	40	54	44·69	·05	+0·007
1428		09	44	14	30	49·27	·01	+0·022
1429		10	36	24	58	00·44	·04	—0·165
1436		13	33	10	58	54·60	·04	—0·094
1438		14	35	28	56	10·13	·07	—0·012
1449		19	57	37	14	44·45	·03	+0·010
1451		22	09	20	10	24·11	·04	+0·024
1457		25	56	2	48	21·10	·04	+0·014
1462		28	49	19	20	06·58	·06	—0·096
1471		33	05	24	22	27·88	·04	—0·004
1474		36	18	15	14	01·65	·50	—0·070
1477		38	03	24	37	07·43	·03	—0·118
1489		42	26	39	21	47·92	·04	0·000
1499		50	49	0	41	13·16	·09	—0·021
1522	18	02	08	8	13	13·35	·10	+0·019
1524		02	55	30	32	47·59	·04	+0·057

* Stone's Catalogue.

Waimea Magnetic Station



Waimea Magnetic Station



Abstract of magnetic results, Waimea, Hawaii.

DIP.

Date.	Needle No. 1.			Needle No. 2.		
	N.	S.	Mean.	N.	S.	Mean.
1892.	° ′	° ′	° ′	° ′	° ′	° ′
July 8	39 32	38 15	38 54	38 30	38 25	38 28
9	38 38	04	21	20	29	24
11	55	34	44	18	28	23

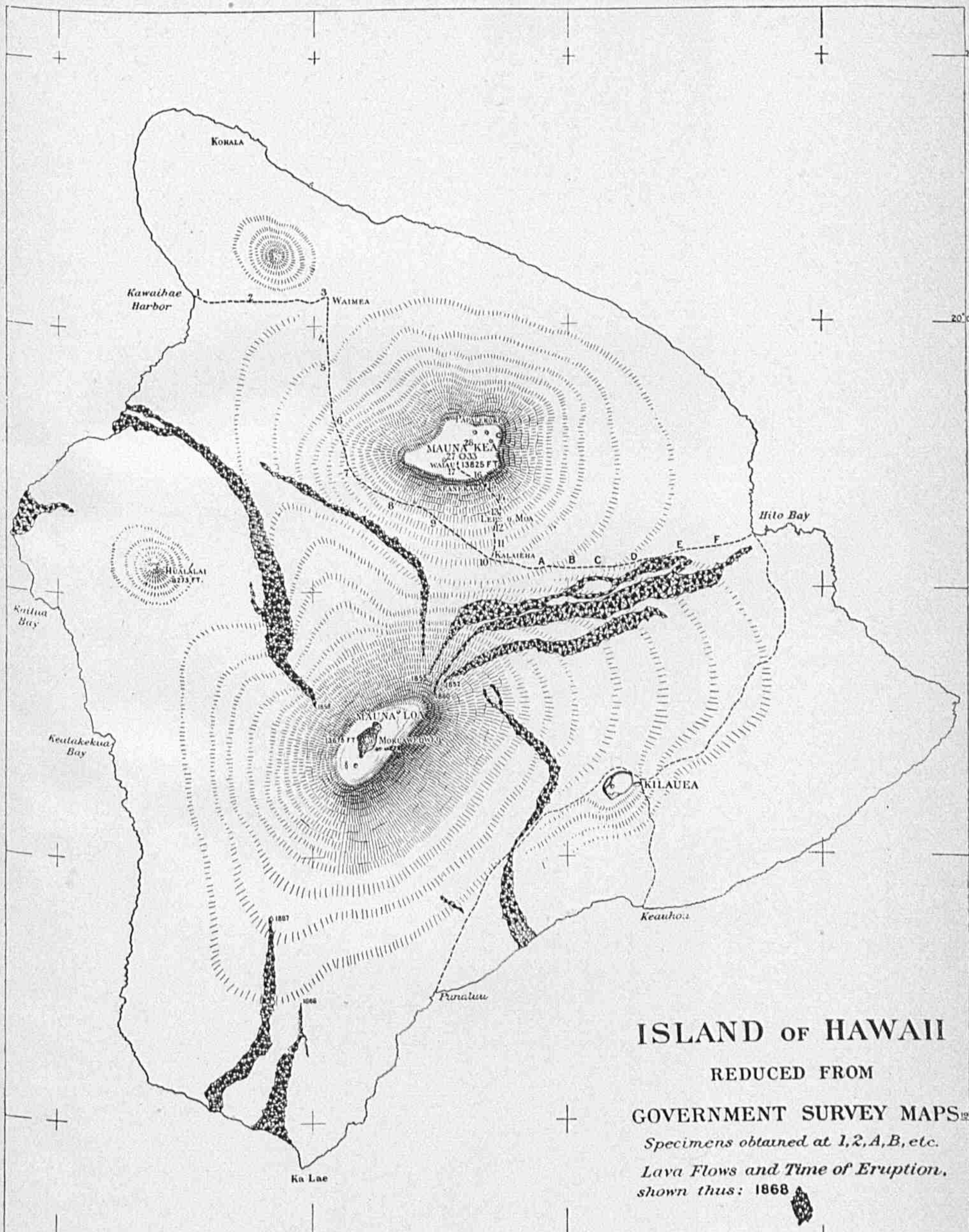
DECLINATION, DIP, AND INTENSITY.

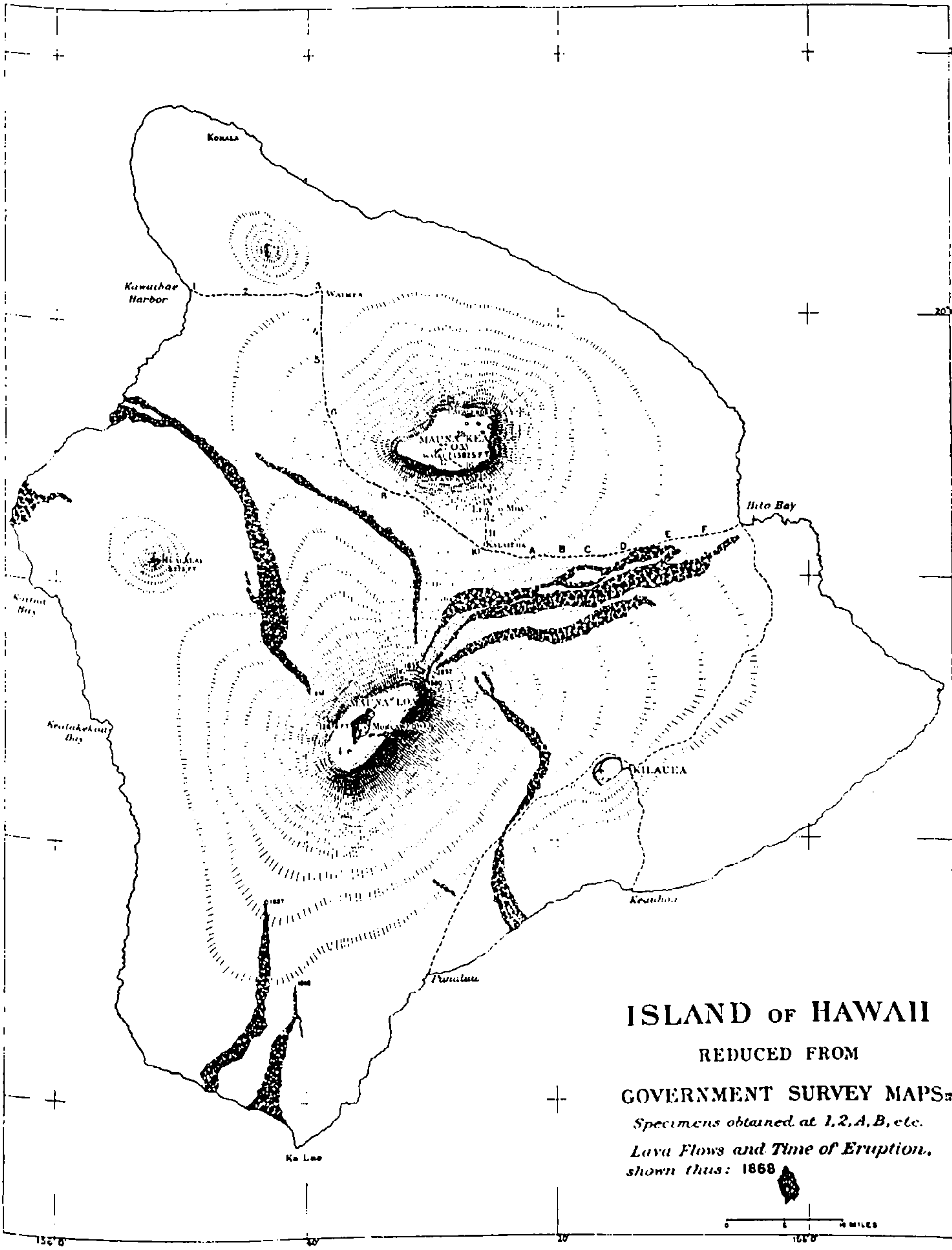
Date.	Scale reading of axis.	Declination (east).	Mean dip.	Horizontal intensity.	Magnetic moment.
1892.		° ′	° ′	*Dyne.*	
July 8	......	8 50·0	38 41·0	0·2966	128·0
9	......	9 03·0	22·5	78	129·3
11	......	04·2	33·5	86	128·4

NOTE.—On July 8 the magnetic station was at the west end of the base line of the Government survey. On July 9 and 11 the station was half a mile distant from the previous one, at a point occupied in 1872 by Mr. C. J. Lyons, of the Survey staff.

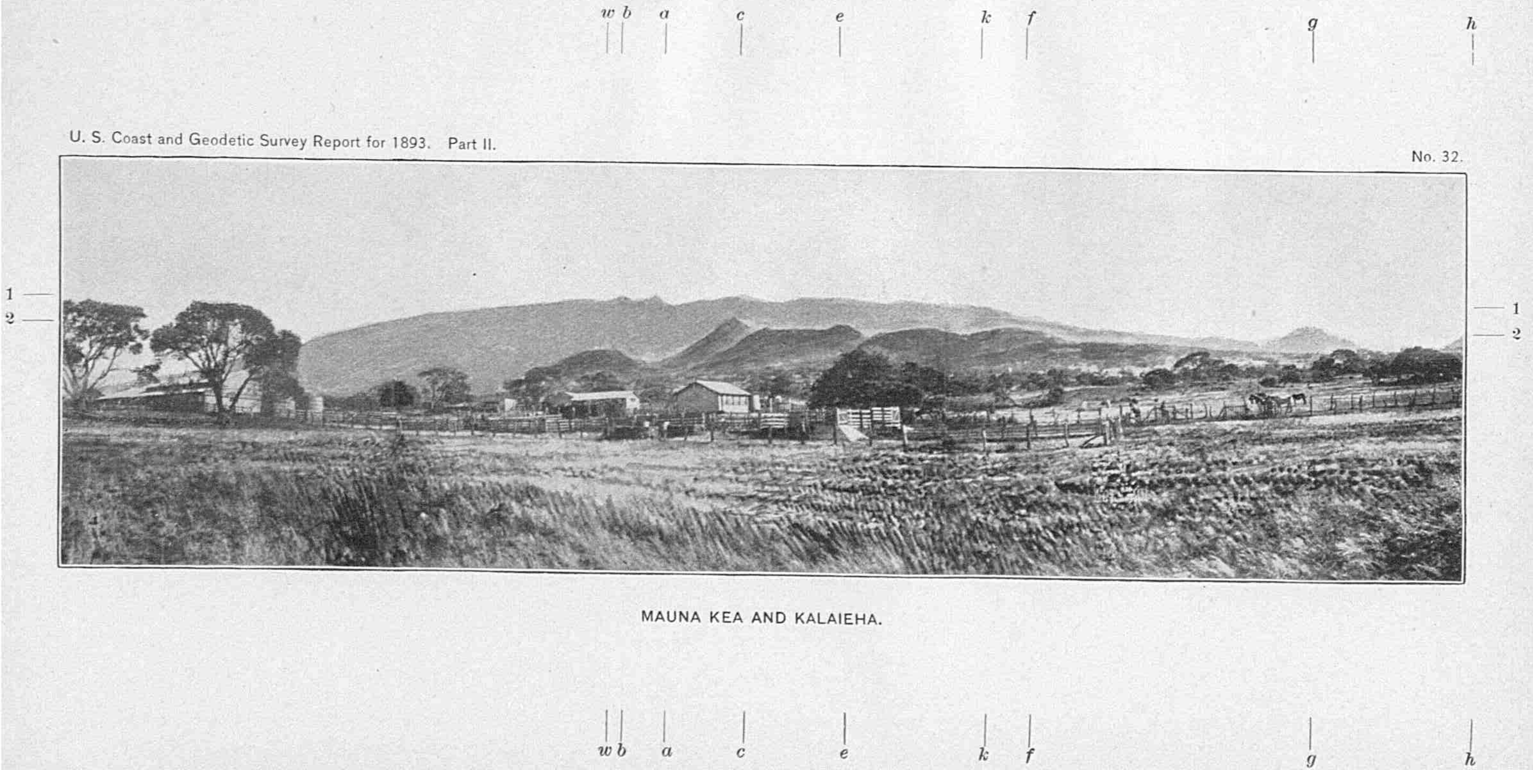
KALAIEHA.

Leaving Waimea at 7.25 a. m. on July 12, we arrived at Kalaieha at 5 o'clock in the evening, having passed the entire day in the saddle. The distance is about 30 miles. The road is not steep, as the elevation to be overcome between the two places is only 4 000 feet. This gives an average rise of 1 in 40, or an inclination of about 1½°. On the road specimens of lava were gathered at designated points in order to form a basis for estimating the average density of the rocks of the island. The route taken, as well as the points from which specimens were obtained, is shown in illustration No. 31. Kalaieha is situated on the Humuula ranch, which contains 237 000 acres, including a part of Hamakua. The tract runs down to the sea on the windward side and extends from the summit of Mauna Loa on the south to Mauna Kea on the north. At its widest region it is 20 miles broad. Its longest dimension is about 45 miles. On July 13 the stations were located and the tents and instruments put in position, and on the following day work was begun. The pendulum receiver was mounted on a large rock about 100 feet west of the house farthest to the west, and the latitude pier was within 2 or 3 feet of the pendulum in a southeast direction. The magnetic station was 200 feet due north of the pendulum. A general view of Kalaieha is shown in illustration No. 32. The prominent peaks along the path to the summit are identified by rectangular coordinates.


ISLAND OF HAWAII
REDUCED FROM
GOVERNMENT SURVEY MAPS
Specimens obtained at 1, 2, A, B, etc.
Lava Flows and Time of Eruption.
shown thus: 1868
KOHALA
Kawaihae Harbor
WAIMEA
MAUNA KEA
13825 FT.
Hilo Bay
KALAIEHA
A
B
C
D
E
F
HUALALAI
Kailua Bay
MAUNA LOA
MOKUAWEOWEO
KILAUEA
Kealakekua Bay
Keauhou
Punaluu
Ka Lae
1887
1868
1859
1855
1852
1880
156° 0'
40'
20'
155° 0'
20°00'
40'
20'
19°00'
0
5
10 MILES


Kohala
Kawaihae Harbor
Waimea
Mauna Kea
Hilo Bay
Kaiwiki
A B C D E F
Mauna Loa
Kilauea
Keauhou
Punaluu
Ka Lae
1868
ISLAND OF HAWAII
REDUCED FROM
GOVERNMENT SURVEY MAPS
Specimens obtained at 1, 2, A, B, etc.
Lava Flows and Time of Eruption, shown thus: 1868

No. 32.



MAUNA KEA AND KALAIEHA.

Geodetic Z Kalaieha, north base to latitude station = 28° 56′ 00″
Distance " " " " " = 202·5 feet

	°	′	″
Geodetic L north base =	19	42	35·23
$d\ L$ =	−	00	1·76
Geodetic latitude of latitude station =	19	42	33·47

	°	′	″
Geodetic M north base =	155	25	52·26
$d\ M$ =	+	00	1·02
Geodetic longitude of latitude station =	155	25	53·28

The above position of Omaokoili depends on the correctness of a short base measured near Aahuwela and a chain of six triangles carefully measured. The error probably does not exceed 0″·20.

Pendulum observations, Kalaieha, Hawaiian Islands.

[Observers: E. D. Preston, W. E. Wall.]

Pendulum.	Position.	Swing.	Date.	No. of coincidence intervals.	Time of ten coincidence intervals.	Semi-arc. Initial.	Semi-arc. Final.	Temperature.	Barometer.	Pressure at 0° C.
			1892.		*Seconds.*	*mm.*	*mm.*	° *C.*	*mm.*	*mm.*
B_1	D	1	July 14	12	2129·6	5·1	4·0	14·77	604·4	572
	R	2	14	8	2128·1	5·1	4·0	14·36	604·4	573
	R	3	15	16	2112·8	4·0	2·7	16·61	605·2	570
	D	4	15	18	2106·7	5·0	3·5	17·04	605·3	568
	D	5	15	34	2110·6	5·0	2.2	16·99	605·0	568
	R	6	15	30	2121·2	5·0	2·4	15·76	604·7	571
B_2	D	7	15	22	2307·5	4·9	2·8	13·61	605·2	576
	R	8	15	14	2320·0	4·8	3·2	12·51	605·7	578
	R	9	16	12	2342·9	4·3	3·1	10·44	605·7	582
	D	10	16	20	2326·8	5·0	3·1	11·40	605·8	581
	D	11	16	32	2313·0	5·0	[2·3]	12·81	605·9	579
	R	12	16	28	2301·4	5·0	2·4	14·07	605·8	575
B_3	D	13	16	28	2439·6	4·9	2·1	14·77	606·2	574
	R	14	16	14	2427·9	4·9	2·9	14·97	605·9	573
	R	15	16	20	2431·5	4·8	2·7	14·77	605·4	573
	D	16	16	24	2446·7	4·8	2·3	14·16	605·0	574
	D	17	16	18	2456·4	4·7	2·9	13·36	605·6	576
	R	18	16	14	2459·0	4·7	3·0	12·76	606·0	578

Reduction of pendulum observations, Kalaieha, Hawaiian Islands.

[Periods reduced to temperature, 15° C.; pressure, 500mm at 0° C.; arc infinitely small; sidereal time.]

Period uncorrected.	Corrections (in seventh decimal place).				Period corrected.
	Arc.	Temperature.	Pressure.	Rate.	
Seconds.					*Seconds.*
0·501 1767	−73	+ 10	−57	+258	0·501 1905
1774	−73	+ 27	−58	+258	928
1861	−39	− 67	−55	+269	969
1896	−63	− 85	−54	+269	963
1873	−44	− 83	−54	+269	961
1813	−46	− 32	−56	+269	948
					946
0·501 0858	−51	+ 58	−60	+269	0·501 1074
0799	−56	+103	−62	+269	053
0693	−48	+189	−65	+269	038
0768	−57	+149	−64	+269	065
0832	−45	+ 91	−62	+269	085
0886	−46	+ 39	−59	+269	089
					067
0·501 0269	−41	+ 10	−58	+269	0·501 0449
0318	−53	+ 1	−58	+269	477
0303	−49	+ 10	−58	+269	475
0239	−43	+ 35	−58	+269	442
0198	−51	+ 68	−60	+269	425
0188	−52	+ 93	−62	+269	436
					451

The latitude observations at this station were made with great difficulty. During the entire stay not more than four pairs could be obtained. The evenings were always either foggy or rainy, and as the telescope was mounted in the open air, it was often necessary to lift it from the Ys and take it inside the tent to be dried. The latitude was always made to give way for the time observations, as these were necessary for the success of the gravity work, which was the real objective point of the trip.

Results of latitude observations at Kalaieha, Hawaii.

No. of pair.	No. of star (U. S. C. and G. S. catalogue).	Number of observations.	Latitude.
			° ′ ″
1	1298—1302	1	19 42 03·2
2	1309—1317	1	02·5
3	65— 73	1	03·1
4	65— 78	1	01·7
		4 (Total)	19 42 02·6 (Mean)

Abstract of magnetic results at Kalaieha, Hawaii.

DIP.

Date.	Needle No. 1.						Needle No. 2.					
	N.		S.		Mean.		N.		S.		Mean.	
1892.	°	′	°	′	°	′	°	′	°	′	°	′
July 14	39	12	38	23	38	48	38	50	38	50	38	50
15		09		34		52		39		43		41
16		14		19		46		53		46		50

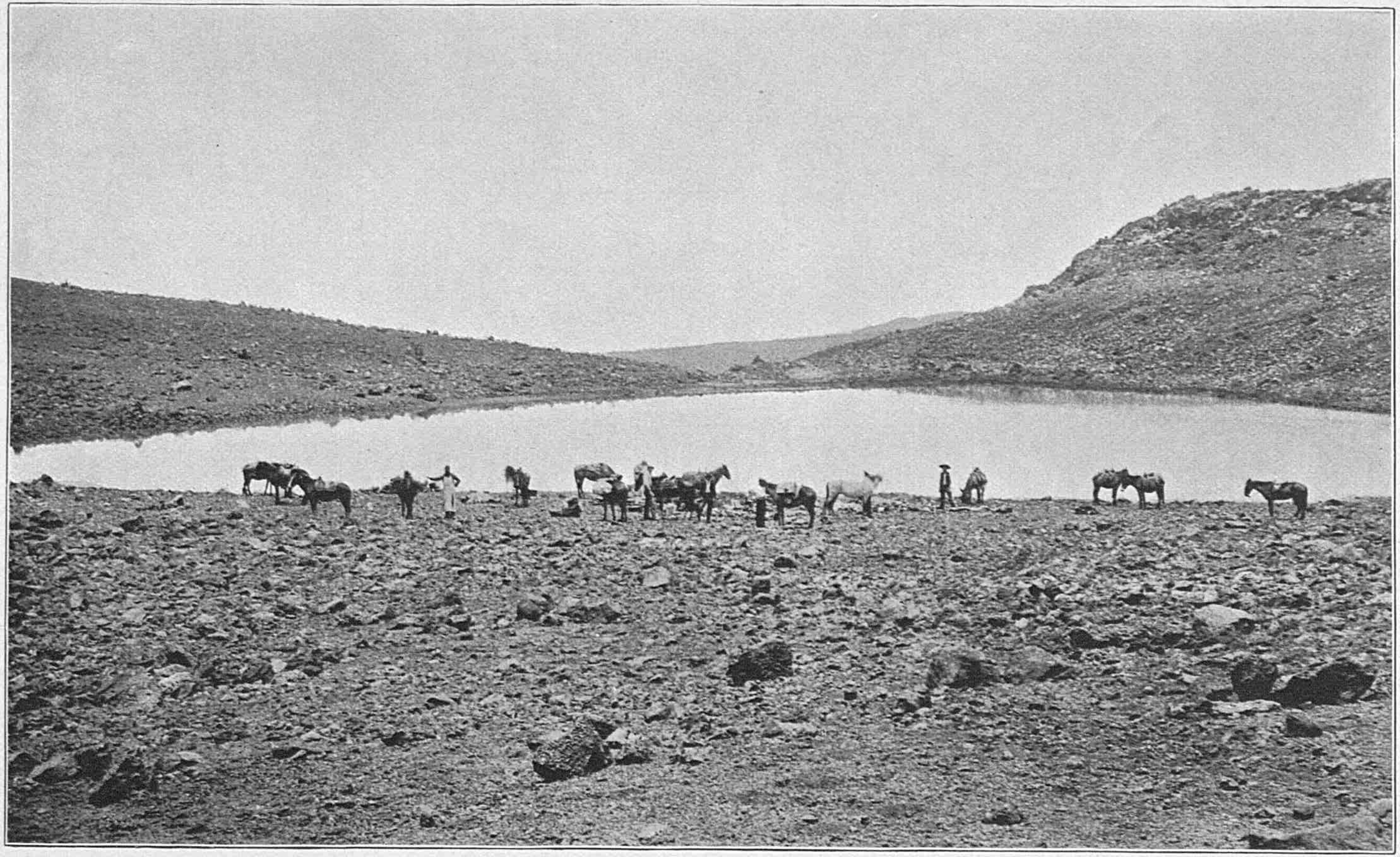
DECLINATION, DIP, AND INTENSITY.

Date.	Scale reading of axis.	Declination (east).	Mean dip.	Horizontal intensity.	Magnetic moment.
1892.	*d.*	° ′	° ′	*Dyne.*	
July 14	29.06	9 53·1	38 49·0	0·2949	128·5
15	-----	52·1	46·5	0·2958	128·1
16	-----	-------	48·0	------	-----

GRAVITY OBSERVATIONS.

[Sidereal seconds.]

$$\begin{aligned} & \quad\quad\quad\quad s. \\ \text{Pendulum } B_1 &= 0{\cdot}5011946 \\ B_2 &= 0{\cdot}5011067 \\ B_3 &= 0{\cdot}5010451 \end{aligned}$$



WAIAU LAKE, NEAR SUMMIT OF MAUNA KEA.
Elevation over 13,000 feet.

WAIAU.

[See illustration No. 33.]

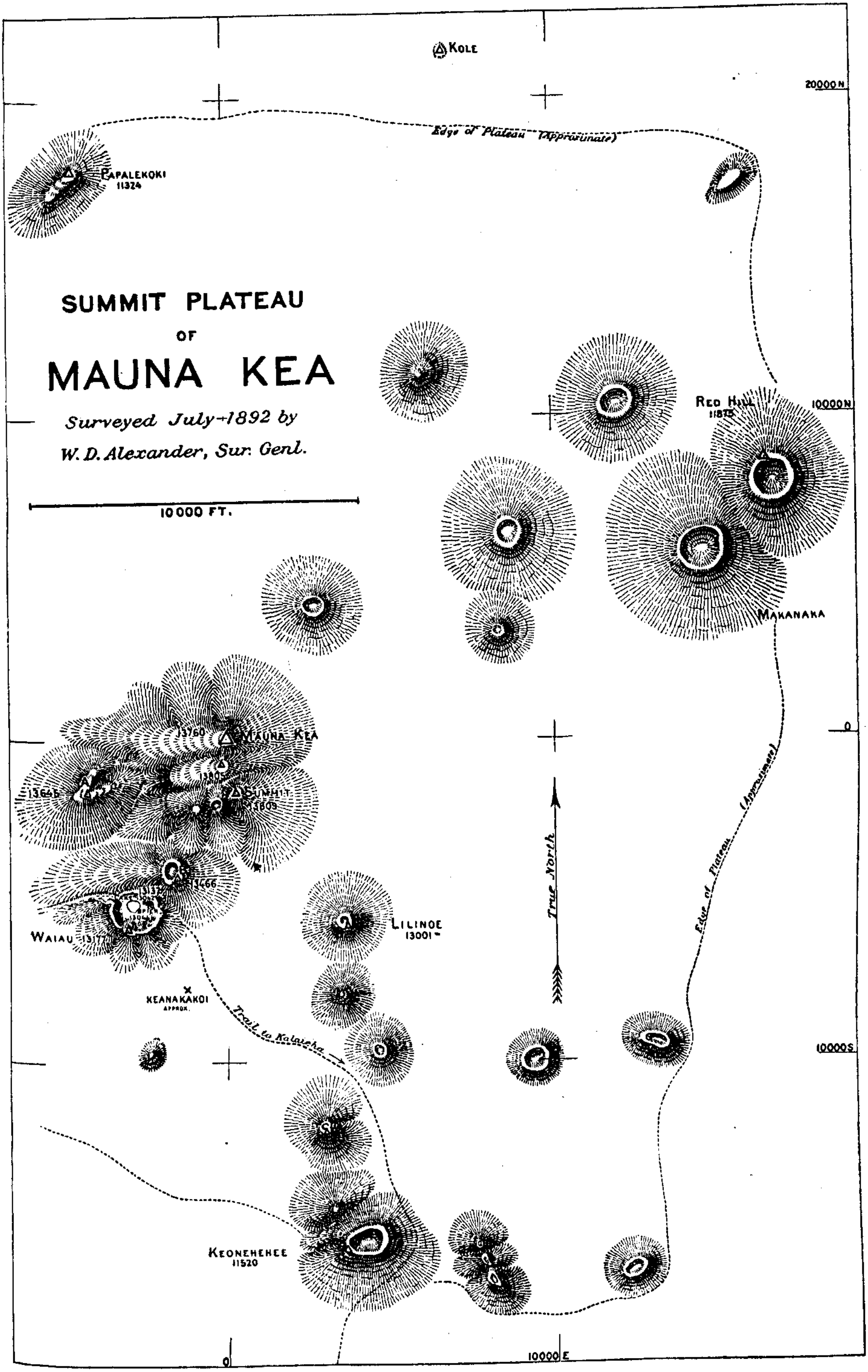
The instruments were dismounted at Kalaieha on July 18 and packed for the final ascent. The distance to the summit, in an air line, is about 7 miles, and the difference of elevation about 7 000 feet. The path, however, was about 12 miles in length, and Waiau is 700 feet below the summit, so that the average rate of rise was 1 in 11, or an angle of about $5\frac{1}{2}^{\circ}$. The amount of material to be transported to the top of the mountain was very great. Besides the astronomical, gravity, and magnetic instruments, and the provisions required to maintain the party on the summit long enough to complete the work, it was necessary to carry fuel, tents, and blankets, and enough cement to build a pier for the meridian telescope. The whole outfit was packed on 11 mules, and the party consisted of 11 persons, including 3 packers. Everything being in readiness, a start was made on the morning of July 19. Before getting well under way, however, a fog set in. Some of the pack animals became difficult to manage, and soon it was noticed that the mule carrying the magnetic instruments, probably the most delicate ones of the outfit, was missing. A halt was made and eight of the party started in search, but as the fog was now dense, our efforts were of no avail. After a couple of hours of delay it was decided to abandon the journey for the day. We all returned to Kalaieha, the animals were unpacked, and the day given up to hunting the lost instruments. The mule was found about 3 p. m. at the foot of the Omaokoili hills, some 3 miles distant.

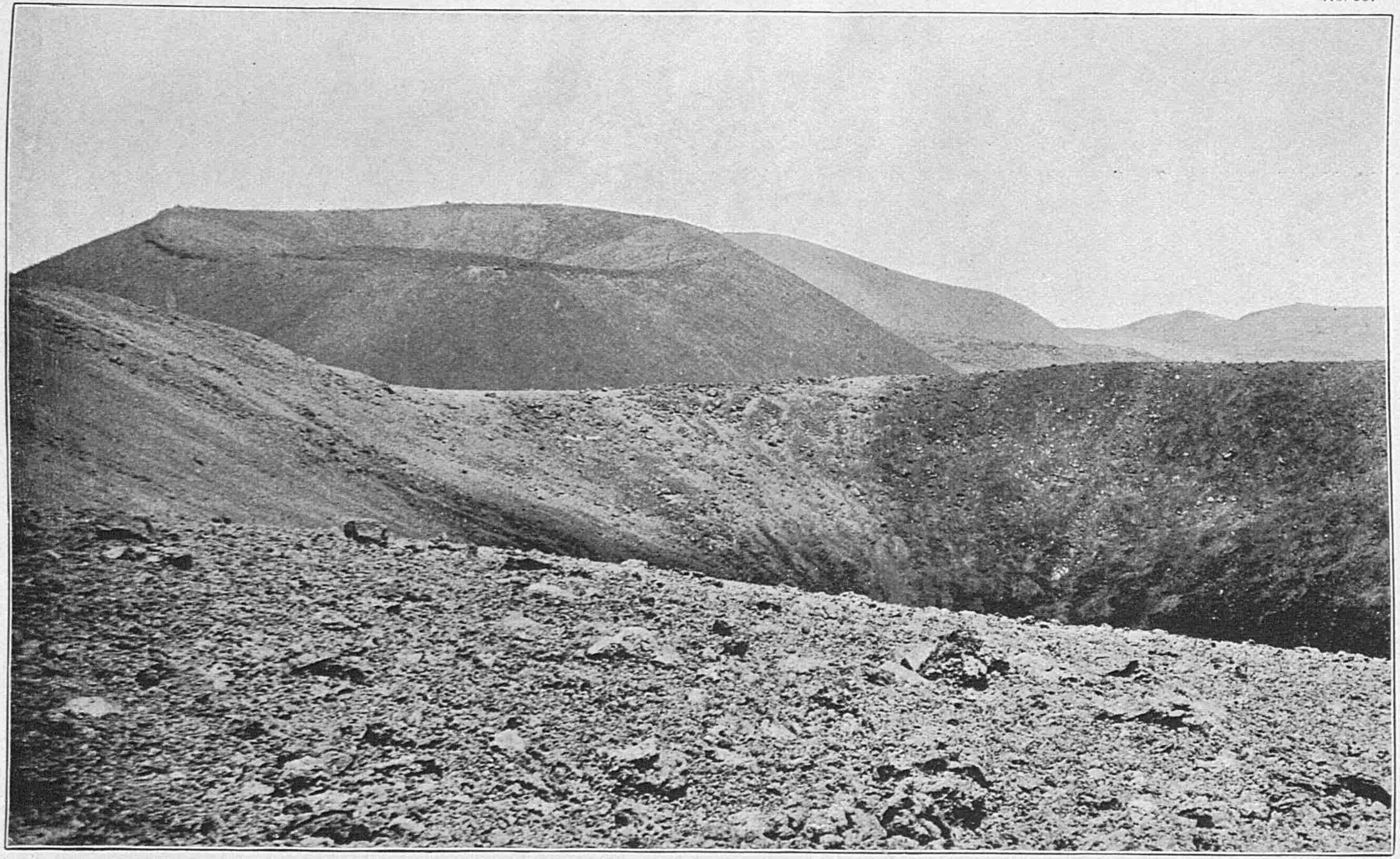
On the following day we again set out at 7.40 a. m. At 11.40 a stop was made for lunch. The route passed between Lepe a Moa on the left and Kole on the right, and we now found ourselves at an altitude of about 10 000 feet. Continuing in the direction of Keonehehee, and climbing this cinder cone in a northeast direction (see illustration No. 34), we arrived at the plateau level at 2 p. m. The elevation of this point is 11 600 feet. The mamane trees were not seen above 10 000 feet, and the raillardia, the only remaining sign of vegetation, disappeared at 11 500 feet. From this point on, the path was over an unbroken landscape of lava. Some interesting pyramids of stone, built to commemorate Queen Emma's visit, were seen on the edge of the plateau, and at an elevation of 12 000 feet was found Keanakakoi, a famous quarry opened by the natives many centuries ago for the manufacture of battle axes. At an elevation of nearly 13 000 feet, near Lilinoe, a burying ground was found, where the ancient chiefs were laid to rest in the red volcanic sand. Before reaching the plateau the animals suffered considerably from the rarity of the atmosphere. On the flank of Keonehehee it was with great difficulty that they were driven—with tongues out and sinking ankle deep in the soft scoria at every step, they presented a pitiable picture indeed and seemed utterly regardless of the drivers' urging, whether with noise or whip. Although heavily laden, they repeatedly lay down, profiting by even a few minutes, when unobserved, to snatch a few moments' rest. Some were unable to reach the destination and had to be unloaded three-quarters of a mile from Waiau and turned loose to descend to the plains below. Their loads were repacked on stronger mules, which were sent back from the summit. The endurance of these mountain animals is remarkable. On the leeward side of the island, where it seldom rains, it is a common occurrence for them to pass eight days without water, and cases are on record where two weeks have elapsed between drinking times. Our camp was established on the banks of the lake known as Waiau. [See Frontispiece.] This is a body of water formed by the melting snow and gathered from the sides of an extinct crater. It is one of the highest bodies of water in the world, being at an elevation of over 13 000 feet. At 4 p. m. the baggage was all at the station and the animals were sent back to Kalaieha, as there is no provender within many miles of the place.

The boiling point of water on the summit (illustration No. 35) is about 186° F. The ranges of temperature during our stay were from 13° F. at night to 108° F. in the daytime, the thermometer having the same position at both times. The barometer stood at 18·30 inches at 54° F. We found the trade winds blowing at the summit, although the anti-trades are supposed by some to appear much below 14 000 feet elevation. The atmosphere was very clear. Many stars were observed before sundown with a small telescope. We had, of course, ice every night on the lake. With such extreme ranges of temperature there was much discomfort. Sleeping cots were not taken, as it was entirely too cold at night to lie off the ground. It was found necessary to make sleeping bags by sewing blankets together. Although for miles in every direction around Waiau there is an unbroken landscape of lava, and apparently nothing to support life, we saw spiders, butterflies, and flies during the stay. Around the shores of the lake the following plants were found growing, although the lake itself is several thousand feet above the last limit of vegetation. They were submitted to President D. C. Gilman, of the Johns Hopkins University, who kindly forwarded the list, as follows:

Cystopteris fragilis Bernh.
Trisetum glomeratum Trin.
Poa annua L., forma vel vaz.
Deschampsia australis Nees.

The first specimen was determined by Mr. John Donnell Smith, and the last three by Dr. George Vasey. All the above plants were found growing near the same locality, at an elevation of about 13 100 feet above sea level. See illustration No. 35 for summit view.


KOLE
20000 N
Edge of Plateau (Approximate)
PAPALEKOKI
11324
SUMMIT PLATEAU
OF
MAUNA KEA
Surveyed July 1892 by
W. D. Alexander, Sur. Genl.
10000 FT.
RED HILL
11875
10000 N
MAKANAKA
0
13760
MAUNA KEA
13805
SUMMIT
13809
13646
13466
WAIAU 13117
LILINOE
13001
True North
Edge of Plateau (Approximate)
KEANAKAKOI
APPROX.
Trail to Kalaieha
10000 S
KEONEHEHEE
11520
0
10000 E



VIEW FROM KU-KA-HAU-ULA, THE SUMMIT OF MAUNA KEA, LOOKING SOUTHWEST.
Elevation 4,214 meters (13,825 feet).

The Geodetic Position of Waiau, Latitude Station.

	°	′	″
Latitude of Mauna Kea (primary triangulation)	19	50	01·63 N.
Longitude " " " " "	155	26	18·75 W.

Geodetic Z Mauna Kea to "Peak A" = 69° 26′ 00″
Distance from " " " " " log (feet) = 3·6563410
" " " " " " " log (metres) = 3·1403733

Geodetic $d\,L$ from Mauna Kea to "Peak A" = −15″·785
" L' of "Peak A" = 19° 49′ 45″·845

Geodetic $d\,M$ from Mauna Kea to "Peak A" = +44″·456
" M' of "Peak A" = 155° 27′ 03″·206

Geodetic Z "Peak A" to Waiau latitude station = 337° 18′ 45″·00
Distance from " " " " " " log (feet) = 3·583900
" " " " " " " " log (metres) = 3·067932

Geodetic $d\,L'$ from "Peak A" to Waiau latitude station = −35″·087
" latitude of Waiau latitude station = 19° 49′ 10″·758

Geodetic $d\,M$ from "Peak A" to Waiau latitude station = −15″·500
" longitude of Waiau latitude station = 155° 26′ 47″·706

Position of latitude station probably correct within 3 feet.

All the above longitudes are conventional and should be corrected by adding 2′ to them. The latitudes are derived from the main triangulation. The above computation was supplied by the Government Survey Office in Honolulu.

Pendulum observations, Waiau, Hawaiian Islands.

[Observors: E. D. Preston, W. E. Wall.]

Pendulum.	Position.	Swing.	Date.	No. of coincidence intervals.	Time of ten coincidence intervals.	Semi-arc. Initial.	Semi-arc. Final.	Temperature.	Barometer.	Pressure at 0° C.
			1892.		*Seconds.*	*mm.*	*mm.*	° *C.*	*mm.*	*mm.*
B_1	D	1	July 22	22	1992·7	5·0	3·5	10·09	477·0	459
	R	2	22	22	2020·9	5·0	3·4	7·57	477·2	464
	R	3	23	26	2049·4	5·0	3·1	3·95	477·3	470
	D	4	23	20	2011·5	5·0	3·7	7·57	477·8	464
	D	5	23	22	1977·7	5·0	3·5	11·20	478·1	458
	R	6	23	20	1955·2	5·0	3·5	13·91	478·1	454
B_2	D	7	23	24	2087·1	5·0	3·1	15·77	478·0	451
	R	8	23	24	2078·1	5·0	3·2	16·85	477·8	449
	R	9	23	16	2071·6	5·0	3·7	17·09	477·6	449
	D	10	23	20	2065·0	5·0	[3·5]	16·85	477·5	449
	D	11	23	18	2092·2	5·0	3·6	14·07	477·2	453
	R	12	23	24	2126·9	5·0	3·2	11·45	477·2	457
B_3	D	13	24	18	2221·1	4·9	3·5	13·71	478·7	456
	R	14	24	16	2238·8	4·0	3·1	12·10	478·9	458
	R	15	25	20	2359·5	4·6	2·9	3·30	478·5	472
	D	16	25	16	2338·1	4·5	3·0	4·70	478·6	470
	D	17	25	18	2309·4	4·6	3·1	7·02	478·9	466
	R	18	25	42	2272·1	4·5	1·9	10·15	479·0	461

Reduction of pendulum observations, Waiau, Hawaiian Islands.

[Periods reduced to temperature, 15° C.; pressure, 500mm at 0° C.; arc infinitely small; sidereal time.]

Period uncorrected.	Corrections (in seventh decimal place).				Period corrected.
	Arc.	Temperature.	Pressure.	Rate.	
Seconds.					*Seconds.*
0·501 2578	—63	+204	+32	+309	0·501 3060
2401	—62	+308	+28	+309	2984
2228	—57	+459	+23	+309	2962
2460	—67	+308	+28	+309	3038
2674	—63	+158	+33	+309	3111
2820	—63	+045	+36	+309	3147
					3050
0·501 2008	—57	— 32	+39	+309	0·501 2267
2058	—59	— 77	+40	+309	271
2097	—67	— 87	+40	+309	292
2136	—63	— 77	+40	+309	345
1976	—65	+ 39	+37	+260	247
1781	—59	+147	+34	+260	163
					264
0·501 1281	—62	+ 54	+35	+260	0·501 1568
1191	—44	+120	+33	+260	560
0618	—49	+486	+22	+260	337
0715	—49	+427	+23	+260	376
0849	—52	+331	+27	+260	415
1027	—34	+201	+31	+260	485
					457

Abstract of magnetic results at Waiau, summit of Mauna Kea, Hawaii.

DIP.

Date.	Needle No. 1.			Needle No. 2.		
	N.	S.	Mean.	N.	S.	Mean.
1892.	° ′	° ′	° ′	° ′	° ′	° ′
July 21	38 60	38 09	38 34	38 42	38 14	38 28
22	57	13	35	24	38	31
23	54	17	36	41	42	42

DECLINATION, DIP, AND INTENSITY.

Date.	Scale reading of axis.	Declination (east).	Mean dip.	Horizontal intensity.	Magnetic moment.
1892.	d.	° ′	° ′	Dyne.	
July 21	30·02	10 21·0	38 31·0	------	-----
22	29·24	--------	33·0	0·2950	123·5
23	-----	--------	39·0	0·2950	123·6
24	-----	10 24·4	--------	------	-----

ABSTRACT OF GRAVITY RESULTS.

[Sidereal seconds.]

$$
\begin{aligned}
& \qquad\qquad\qquad s. \\
\text{Pendulum } B_1 &= 0{\cdot}5013050 \\
B_2 &= 0{\cdot}5012264 \\
B_3 &= 0{\cdot}5011457
\end{aligned}
$$

The latitude of the station was determined by fifty-two measures on sixteen pairs of stars, as follows:

Latitude of Waiau.

No. of pair.	No. of star (U. S. C. and G. S. catalogue).	Number of observations.	Latitude.
			° ′ ″
1	1257—1267	2	19 48 51·7
2	1298—1302	3	51·0
3	1309—1317	3	54·6
4	1322—1333	3	50·8
5	1342—1346	4	51·2
6	1361—1367	3	52·3
7	1383—1387	4	53·3
8	1390—1394	4	52·4
9	1416—1424	4	54·0
10	1428—1429	4	52·2
11	1436—1438	4	52·5
12	1451—1462	4	52·1
13	1471—1474	3	50·4
14	1471—1477	2	50·6
15	1489—1499	3	53·0
16	1522—1524	2	49·4
		52 (Total)	19 48 52·0 (Mean)

The geodetic positions of the three stations on Hawaii, as communicated by Professor Alexander, are given in the third column following:

Latitudes.

Stations.	Astronomical latitude.	Geodetic latitude.	Astronomical minus geodetic.
	° ′ ″	° ′ ″	″
Kawaihae	20 02 05·9	20 02 25·1	—19·2
Kalaieha	19 42 02·6	19 42 32·4	—29·8
Waiau	19 48 52·0	19 49 10·8	—18·8

The above table shows that there is a deflection of the plumb line toward the north at all three stations, and the deflection at Kalaieha appears to be much more marked than at either of the other stations. By reference to illustration No. 31 it will be seen that we should expect a greater deflection at Kalaieha than at either of the other two. The deflection at Ka Lae, the extreme southern point of the island, appears to be about 1′ 29″. This result is from work done by myself at this point in 1887 with zenith telescope No. 1, and a subsequent triangulation by Mr. J. S. Emerson, of the Hawaiian Government Survey, previous to 1891.

FROM WAIAU TO HILO.

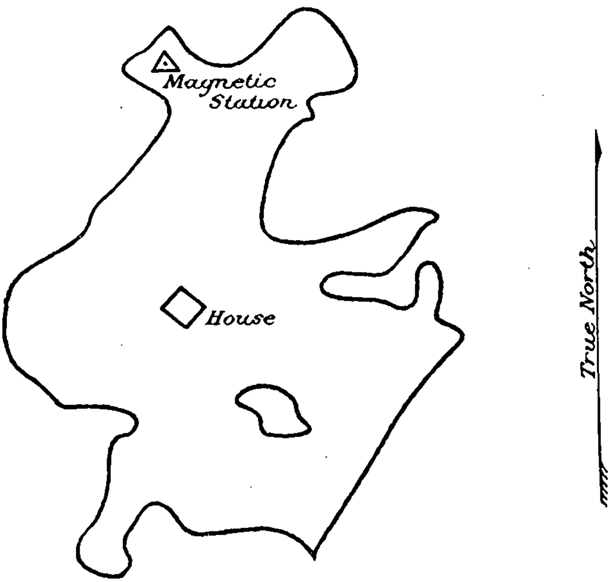
The last observations were made at Waiau on the evening of July 25. The next morning the animals arrived from Kalaieha. They were packed during the forenoon of the 26th, and at 1.30 p. m. we started down. We reached the Humuula ranch (Kalaieha) at 5.45 p. m., having stopped an hour at Keanakakoi. On the 27th the instruments and baggage were all repacked. The party separated at this place, some going down the windward side of the mountain to Hilo, and the others returning to the sea over the same route taken in the ascent. This course was necessary because magnetic observations were to be carried on at Hilo, and as it was impossible to transport the baggage to the steamer on this side of the island, it was sent to Waimea and then to Kawaihae. We left Kalaieha at 6 a. m. on July 28th with a small pack train and a guide. The path is about 30 miles long, very rough, and much of the way over sharp lava. We were supplied with horseshoeing implements. This is a requisite to everyone making the trip. The lava is so hard and sharp that if a shoe is lost the horse's foot is badly cut in a few minutes, and neither persuasion nor force will induce him to continue the route unshod. Many carcasses were seen along the road, of animals that had been killed or left to die, as there is nothing by the wayside to support life. Just before arriving at Hilo we passed through 1¾ miles of swampy woods, which consumed two hours in crossing. Hilo was reached at 7.30 in the evening, after having spent thirteen hours in the saddle.

The photographic plates exposed on the mountain were developed the next day, and on Saturday, the 30th, magnetic observations were begun.

HILO.

The station occupied was on Cocoanut Island, about a mile and a half from the court-house. This station was chosen because magnetic work had been done here by other observers. The following is a sketch of the island, showing the location of the magnetic station and its position with reference to the house near by.

Sketch of Mokuola (Cocoanut Island.)



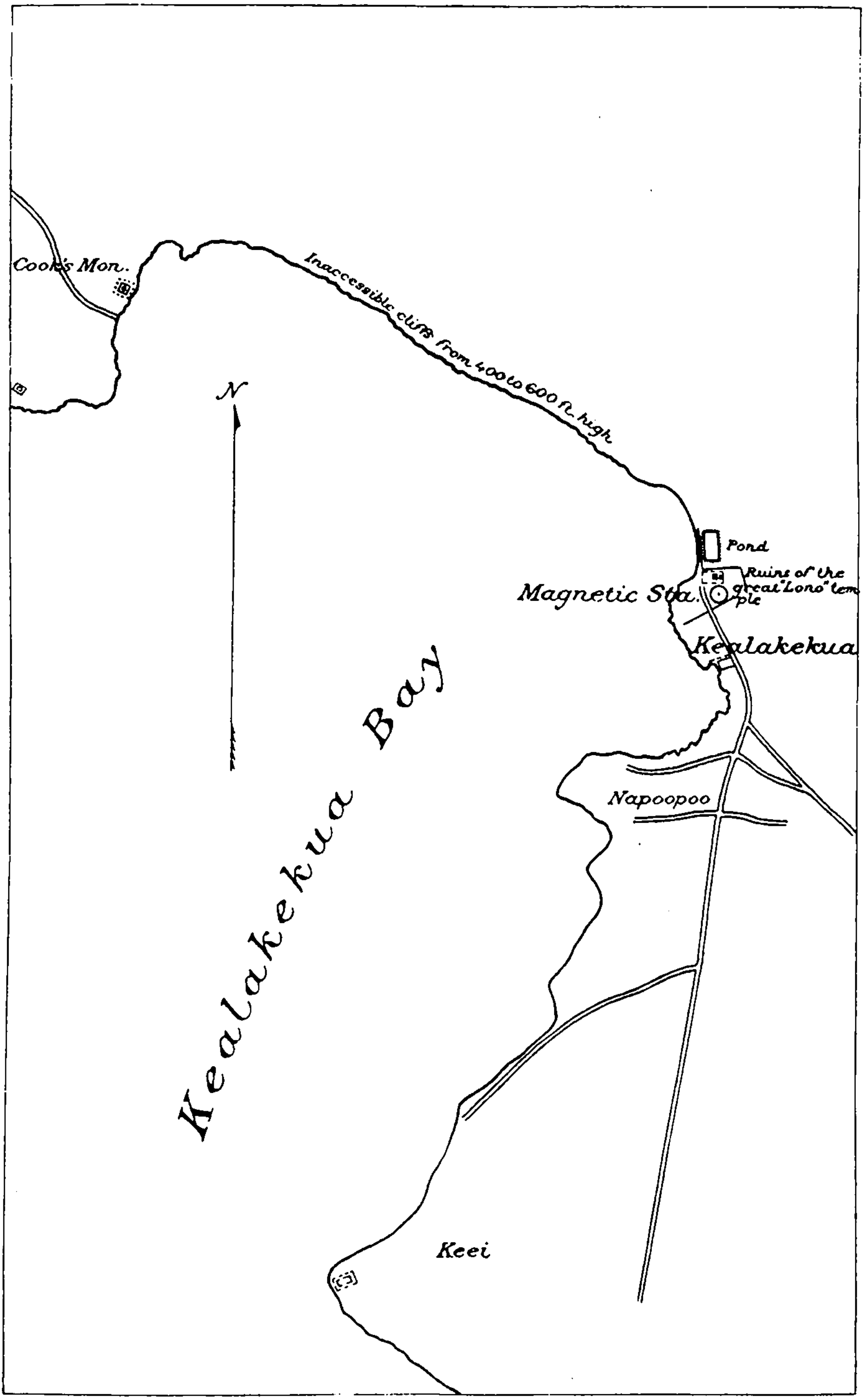
Abstract of magnetic observations at Hilo, Hawaii.

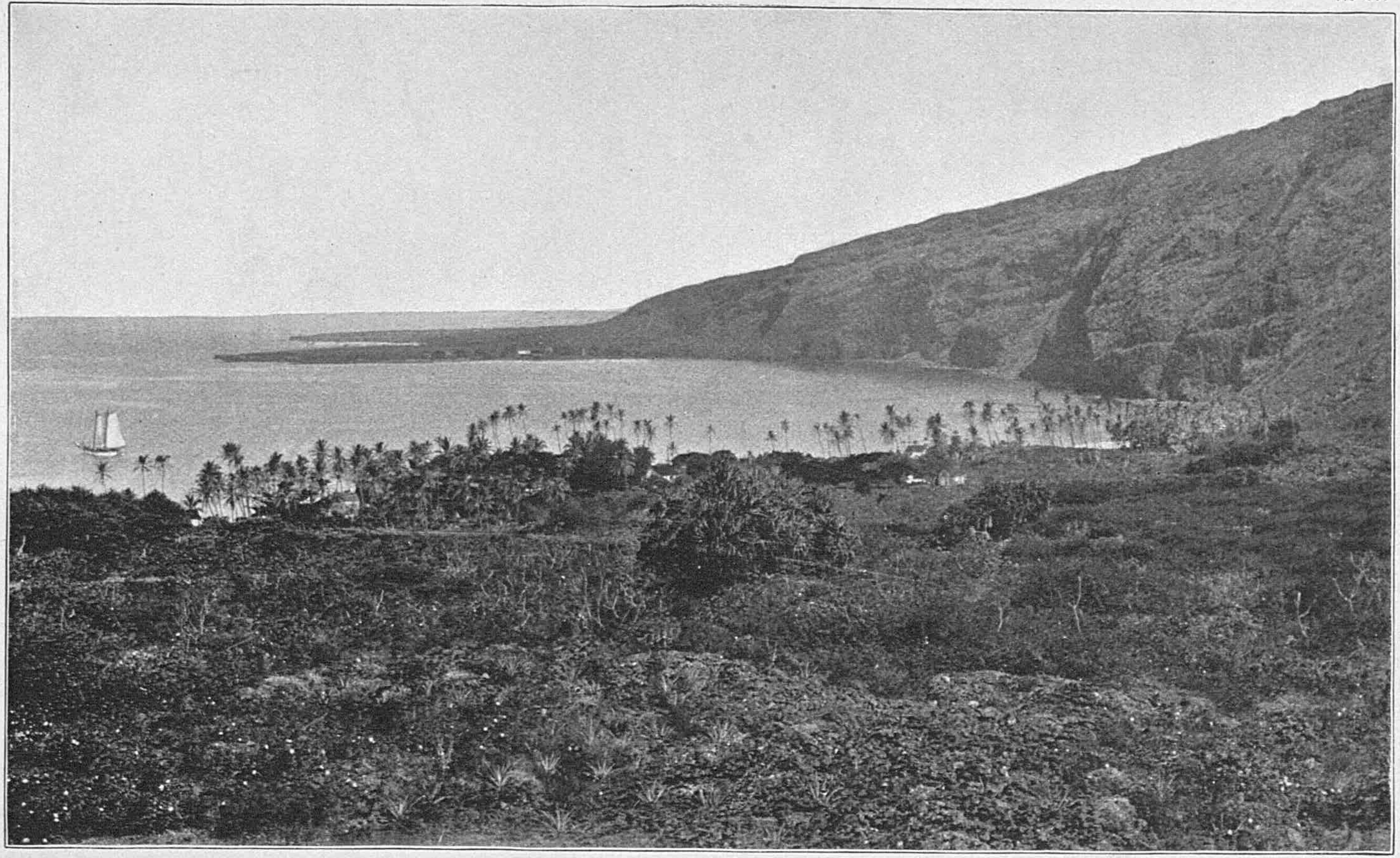
DIP.

Date.	Needle No. 1.						Needle No. 2.					
	N.		S.		Mean.		N.		S.		Mean.	
1892.	°	′	°	′	°	′	°	′	°	′	°	′
July 30	39	43	39	03	39	23	39	14	39	23	39	18
31		39		04		22		20		32		26
Aug. 1		50		12		31		25		15		20

DECLINATION, DIP, AND INTENSITY.

Date.	Scale reading of axis.	Declination (east).		Mean dip.		Horizontal intensity.	Magnetic moment.
1892.	*d.*	°	′	°	′	*Dyne.*	
July 30	-----	8	26·1	39	20·5	0·3064	127·0
31	29·00		25·3		24·0	·3064	127·5
Aug. 1	28·76		18·9		25·5	·3057	126·8
2	-----		19·4	--------		------	-----


Cook's Mon.
Inaccessible cliffs from 400 to 600 ft. high
N
Pond
Ruins of the great "Lono" temple
Magnetic Sta.
Kealakekua
Kealakekua Bay
Napoopoo
Keei



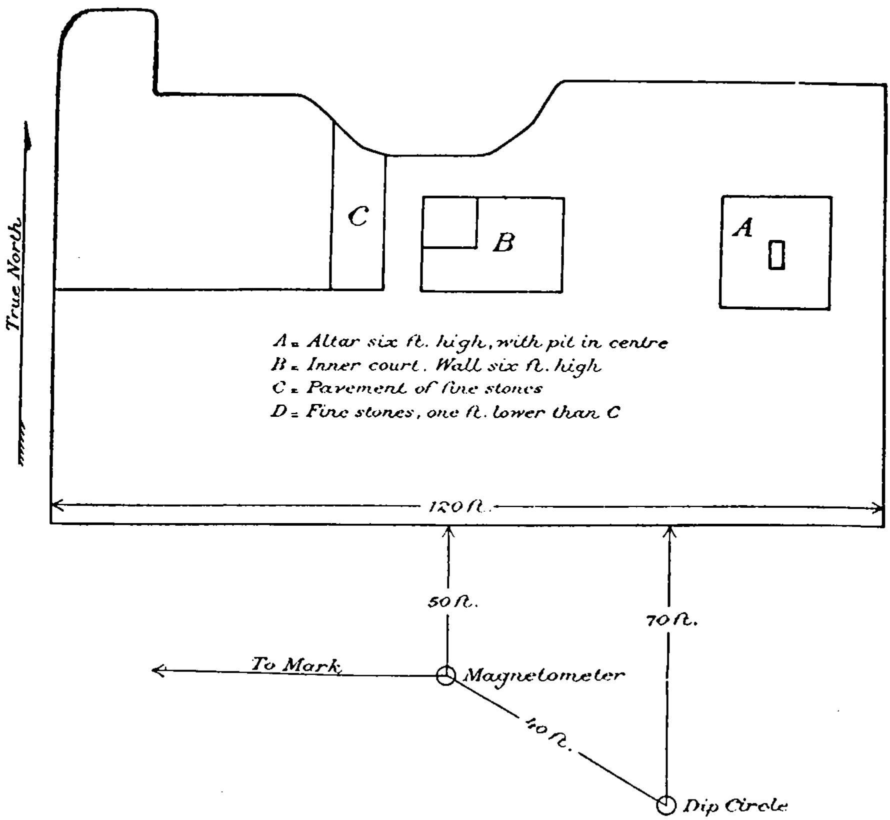
KEALAKEAKUA BAY, LOOKING NORTHWEST.

In December, 1878, Mr. C. J. Lyons found an easterly declination of 7° 40′ at this same station. A comparison with the above would give an annual increase of east declination of three minutes.

NAPOOPOO, HAWAII.

On August 16 I left Honolulu for Kealakeakua Bay, where Captain Cook had his observatory in 1779. Arriving on the 17th, the observations were begun the following day. With the aid of a map and verbal instructions at the Government Survey Office, in Honolulu, I was able to find the spot occupied by the great navigator, and the magnetic instruments were placed practically in the same locality. (See illustrations 36 and 37.)

The following sketch (which is only approximately drawn to scale) shows the situation with reference to the Heiau:



The following readings were made at the magnetometer station:

	°	′
Cook's monument	4	12
Heiau on Kalaemamo Point	353	06
Pole on Kamehameha Heiau	15	32

Abstract of magnetic results at Napoopoo, Hawaii.

DIP.

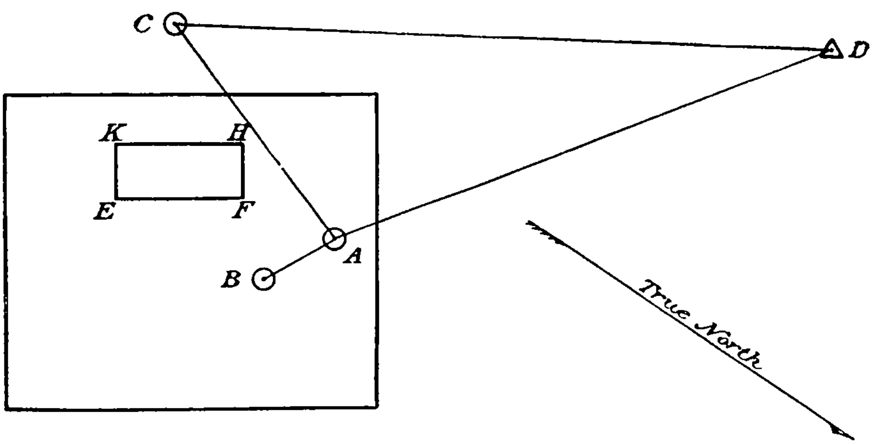
Date.	Needle No. 1.			Needle No. 2.		
	N.	S.	Mean.	N.	S.	Mean.
1892.	° ′	° ′	° ′	° ′	° ′	° ′
Aug. 18	37 56	37 22	37 39	37 44	37 34	37 39
19	37 46	36 57	22	22	46	34
20	38 08	37 10	39	54	30	42

DECLINATION, DIP, AND INTENSITY.

Date.	Scale reading of axis.	Declination (east).	Mean dip.	Horizontal intensity.	Magnetic moment.
1892.	*d.*	° ′	° ′	*Dyne.*	
Aug. 18	28·30	9 09·2	37 39·0	0·3035	124·0
19	-----	10·9	28·0	0·3027	124·7
20	28·38	05·7	40·5	0·3011	125·3
21	-----	06·6	--------	-----	-----

LAHAINA, MAUI.

Leaving Napoopoo at 3 p. m., on August 22, we arrived at Lahaina at 7 a. m. the following morning. Observations were made on the 23d, 24th, and 25th. The station chosen was in the court-house yard, as shown in the following sketch (approximately):



A = Position of magnetometer.
B = Dip circle.
C = Flag pole.
K H F E = Court-house.
D = Latitude pier and gravity station, 1883.

Readings at A.		*Distance from A.*
E =	0° 01′ (mark)	
F =	19 00	92 feet
H =	31 20	
C =	36 53	108 "
D =	128 42	
B =	313 20	35 "

Angle at *D* between *A* and *C* = 28° 45′
Azimuth of line *C D* = 336 18
Distance *C D* = 448.5 feet.

This latitude was determined in 1883 by observations on sixteen pairs of stars, with an average of five observations on each pair. The result is inserted here in order that the list of Hawaiian latitudes may be complete. As no special report was made on the single latitude determined on Maui in 1883, it has never yet been published. The result is

$$\varphi = 20^\circ\ 52'\ 22''\cdot 8$$

The following are the results from the separate pairs:

No. of pair.	Latitude.	Number of observations.
	° ′ ″	
1	20 52 22·69	4
2	24·13	4
3	22·82	5
4	23·31	5
5	24·72	5
6	23·34	5
7	23·57	5
8	22·39	6
9	21·11	5
10	21·09	7
11	22·84	4
12	22·25	4
13	22·40	1
14	22·43	6
15	22·33	7
16	22·73	7

Abstract of magnetic results at Lahaina, Maui.

DIP.

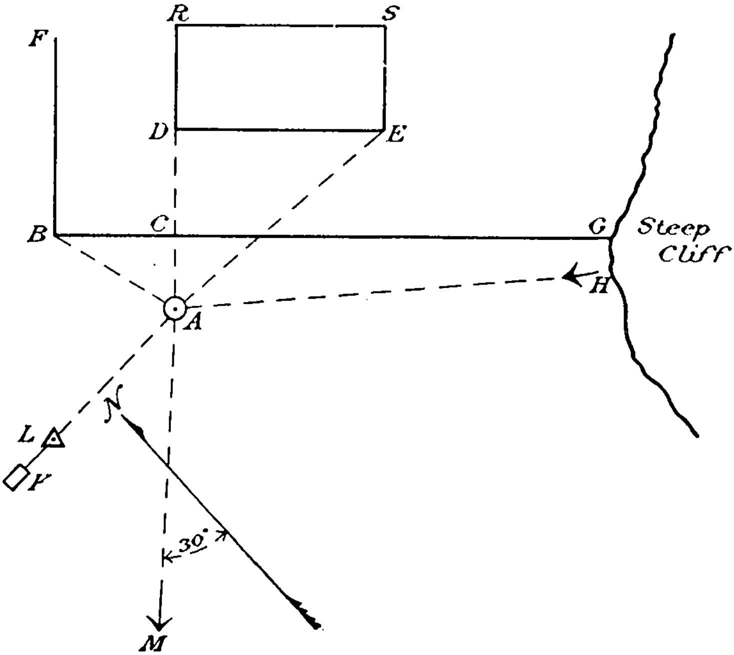
Date.	Needle No. 1.			Needle No. 2.		
	N.	S.	Mean.	N.	S.	Mean.
1892.	° ′	° ′	° ′	° ′	° ′	° ′
Aug. 23	39 43	38 24	39 04	39 24	39 22	39 23
24	48	49	18	32	36	34
25	50	51	20	28	30	29

DECLINATION, DIP, AND INTENSITY.

Date.	Scale reading of axis.	Declination (east).	Mean dip.	Horizontal intensity.	Magnetic moment.
1892.	*d.*	° ′	° ′	*Dyne.*	
Aug. 23	-----	9 08·1	39 13·5	0·2990	124·9
24	-----	09·9	26·0	0·2980	125·4
25	28·42	08·8	24·5	0·2996	124·8

WAIMEA "A," AT KAUAI.

Return...g to Honolulu on the 27th of August, preparations were made for the occupation of two stations on the Island of Kauai. The steamer left on the 30th, and I arrived at Waimea on the following day. The instruments were not landed until September 1. On the 2d observations were begun. The first station was made near the old Transit of Venus station, occupied by the English party in 1874, and reoccupied by myself in 1887 while determining astronomical latitudes for the Hawaiian Government. The following sketch explains distances and bearings:



A = Magnetic station.
K = Transit of Venus pier.
L = Latitude station, 1887.
M = Mark.
F B G = Stone wall of terrace.
H = Arrow cut in rock (said to be by Captain Cook).
R S D E = Dr. Campbell's house.

BEARINGS AND DISTANCES.

Circle readings at A.	°	′	*Distances from A.* *Feet.*
M =	0	00	....
K =	42	23	78·2
B =	140	40	35·5
D =	178	40	103·5
E =	211	25	131·0
H =	250	14	105·3
C =	...	..	30·5
K L =	...	..	11·8
B C =	...	..	16·4

The mark is the southern spire of church, about one-fourth mile southwest of magnetic station. Magnetic bearing of mark is south 30° west.

Abstract of magnetic results at Waimea, "A," Kauai.

DIP.

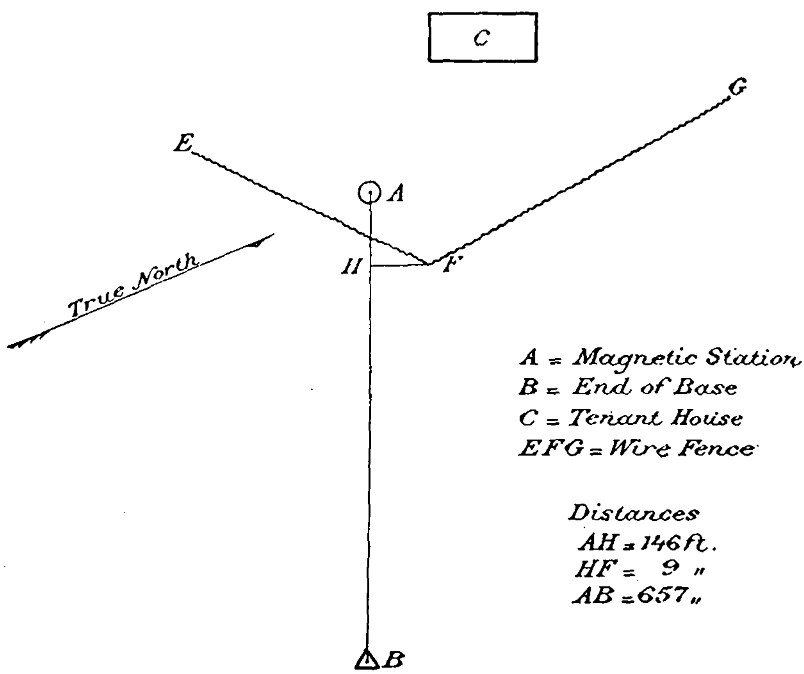
Date.	Needle No. 1.			Needle No. 2.		
	N.	S.	Mean.	N.	S.	Mean.
1892.	° ′	° ′	° ′	° ′	° ′	° ′
Sept. 2	40 48	39 35	40 12	40 13	40 25	40 19
3	56	48	22	28	29	28

DECLINATION, DIP, AND INTENSITY.

Date.	Scale reading of axis.	Declination (east).	Mean dip.	Horizontal intensity.	Magnetic moment.
1892.	d.	° ′	° ′	Dyne.	
Sept. 2	28·58	10 03·8	40 15·5	0·2886	125·2
3	-----	03·2	25·0	79	124·9

WAIMEA "B."

As it was feared that local attraction might have influenced the work at the preceding station, a second station was made at Thornycroft. This station was designated as Waimea "B," and is situated nearer the sea, on a level piece of land, with no rocks in the immediate neighborhood. It is on what is known as the Rowell property and is about one-eighth of a mile west of the house. The station is 1 015 feet north and 2 828 feet west of the Transit of Venus pier. This is taken from a large scale map made by Mr. William Rowell, and is correct within a few feet. The following sketch shows the relative positions:



Abstract of magnetic results at Waimea "B," Kauai.

DIP.

Date.	Needle No. 1.			Needle No. 2.		
	N.	S.	Mean.	N.	S.	Mean.
1892.	° ′	° ′	° ′	° ′	° ′	° ′
Sept. 5	40 45	39 54	40 20	40 24	40 16	40 20
6	36	55	16	22	34	28
7	44	64	24	36	20	28

DECLINATION, DIP, AND INTENSITY.

Date.	Scale reading of axis.	Declination (east).	Mean dip.	Horizontal intensity.	Magnetic moment.
1892.		° ′	° ′	*Dyne.*	
Sept. 5	-----	9 42·6	40 20·0	0·2934	124·0
6	-----	49·2	22·0	0·2933	124·0
7	-----	47·6	26·0	0·2926	124·0

NONOPAPA, NIIHAU.

It was originally intended to close the season's work with the stations on Kauai, but as it was necessary to wait several days for the return steamer, the time was utilized by going to Niihau and getting a few observations on this island. It is the most western one of the Hawaiian group, and is rarely visited. Our magnetic observations are undoubtedly the only ones ever made on the island, and it was fortunate that the occasion presented itself for even one day's work. The conditions, however, were not very favorable. During the entire stay the wind blew violently, which precluded the possibility of determining the axis of the magnet and made all the observations extremely difficult. The station occupied was 350 feet distant from the large crane at the landing. The direction from the crane to the magnetic station was south 18° east.

Abstract of magnetic results at Nonopapa, Niihau.

DIP.

Date.	Needle No. 1.			Needle No. 2.		
	N.	S.	Mean.	N.	S.	Mean.
1892.	° ′	° ′	° ′	° ′	° ′	° ′
Sept. 9	41 01	40 08	40 34	40 35	40 51	40 43

DECLINATION, DIP, AND INTENSITY.

Date.	Scale reading of axis.	Declination (east).	Mean dip.	Horizontal intensity	Magnetic moment.
1892.		° ′	° ′	*Dyne.*	
Sept. 9	-----	10 01·4	40 38·5	0·2928	125·4

MOUNT HAMILTON, LICK OBSERVATORY, CALIFORNIA.

Leaving Honolulu at noon on September 14, we arrived at San Francisco at 1 p. m on the 21st. Passing the instruments through the custom-house and repairing the air chamber of the pendulum apparatus consumed several days, and on the 26th I left for the Lick Observatory in order to connect this station with the work done outside of the United States. Moreover, this station had been occupied several times for the determination of the force of gravity, both with the long and short pendulums, and it was desirable to check this work as well as to swing the pendulums used in the Hawaiian Islands under exceptionally favorable conditions as regards temperature and clock corrections. Observations were made on September 28, 29, and 30. The work was much facilitated by the kindness of the director, Professor Holden, a separate time correction being made by Mr. Campbell for our use.

Pendulum observations, Mount Hamilton, California.

[Observer: E. D. Preston.]

Pendulum.	Position.	Swing.	Date.	No. of coincidence intervals.	Time of ten coincidence intervals.	Semi-arc. Initial.	Semi-arc. Final.	Temperature.	Manometer.	Barometer.	Pressure at 0° C.
			1892.		*Seconds.*	*mm.*	*mm.*	° *C.*	*mm.*	*mm.*	*mm.*
B_1	D	*1	Sept. 28	12	2843·3	[5·2]	3·8	16·15	119	653·1	502·9
	D	2	28	26	2836·3	5·4	2·7	16·15	109	653·9	513·1
	D	3	28	24	2838·5	5·4	2·7	16·20	123	654·3	500·2
	R	4	28	26	2831·2	5·2	2·6	16·40	124	654·0	498·6
	R	5	28	28	2836·1	4·5	2·1	16·60	127	653·5	494·9
	R	6	28	6	2823·3	4·9	4·4	16·75	139	653·0	482·8
	D	7	28	22	2823·6	5·0	2·7	16·65	108	652·0	511·7
B_2	D	8	29	26	3185·2	4·1	1·8	14·62	134	653·6	491·9
	D	9	29	16	3169·4	5·1	3·0	14·67	129	654·0	497·0
	R	10	29	22	3181·6	5·0	2·7	14·72	132	653·8	493·8
	R	11	29	24	3179·6	5·0	2·6	14·77	137	653·0	488·2
	D	12	29	16	3167·2	5·0	3·0	14·77	130	652·5	494·4
	D	13	29	18	3175·0	5·0	3·0	14·47	133	652·5	492·0
B_3	D	14	30	18	3509·4	4·6	2·4	12·56	130	656·0	501·9
	D	15	30	20	3500·2	5·1	2·5	12·66	136	656·6	496·5
	R	16	30	20	3479·0	5·0	2·5	12·81	129	657·0	503·3
	R	17	30	18	3476·4	5·0	2·6	12·86	130	657·4	502·6
	D	18	30	28	3493·4	5·0	1·8	13·06	128	657·6	504·4
	D	19	30	20	3492·5	5·0	2·7	12·86	132	657·8	501·1

* Swing No. 1 rejected by observer.

Reduction of pendulum observations, Mount Hamilton, California.

[Periods reduced to temperature, 15° C.; pressure, 500^{mm} at 0° C.; arc infinitely small; sidereal time.]

Period uncorrected.	Corrections (in seventh decimal place).				Period corrected.
	Arc.	Temperature.	Pressure.	Rate.	
Seconds.					*Seconds.*
0·500 8808	−71	− 48	− 2	+247	
8829	−56	− 48	−10	+247	0·500 8962
8823	−56	− 50	0	+247	964
8846	−52	− 58	+ 1	+247	984
8830	−37	− 66	+ 4	+247	978
8870	−76	− 73	+14	+247	982
8869	−51	− 68	− 9	+247	988
					976
0·500 7861	−29	+ 16	+ 6	+246	0·500 8100
7900	−57	+ 14	+ 2	+246	105
7870	−51	+ 12	+ 5	+246	082
7875	−49	+ 10	+ 9	+246	091
7906	−56	+ 10	+ 4	+246	110
7886	−56	+ 22	+ 6	+246	104
					096
0·500 7134	−42	+101	− 1	+266	0·500 7458
7153	−49	+ 97	+ 3	+266	470
7196	−48	+ 91	− 3	+266	502
7202	−49	+ 89	− 2	+266	506
7167	−38	+ 81	− 3	+266	473
7168	−51	+ 89	− 1	+266	471
					486

SUMMARY.

s.

Pendulum $B_1 = 0{\cdot}5008976$

$B_2 = 0{\cdot}5008096$

$B_3 = 0{\cdot}5007486$

BAROMETRIC DETERMINATION OF THE HEIGHTS OF WAIMEA, KALAIEHA, AND WAIAU.

Meteorological observations were made during the month of July as a check on the heights determined trigonometrically. A barometer was read at Hilo, near the sea level, on the windward side of the island, simultaneously with similar observations at the three stations mentioned above. In addition to this, Prof. A. B. Lyons, of Oahu College, made readings at Waimea, from July 18 to July 27, which furnish independent values for the elevations sought. The observations were made at 9 a. m., 3 p. m., and 9 p. m. At Waimea the station occupied by myself was not identical with that of Professor Lyons, so there is no check on the result for this place; but by combining the Waimea observations with the continuous ones at Hilo, and with those made on the mountain during its occupancy, two very concordant independent values are obtained for Kalaieha and Waiau. Barometer Green No. 3380 was carried to the summit, and No. 3353, also by Green, was used by Professor Lyons. An intercomparison of all the instruments used showed an agreement within one hundredth of an inch in the readings when under the same conditions. The station occupied by myself at Waimea was Mr. W. L. Vredenburg's house. Professor Lyons observed at the "Lyons Mansion," which is presumably about 50 feet higher. At Kalaieha the readings were made at the northwestern cottage, about 100 feet east of the pendulum station, and approximately on the same level.

The barometer was hung in the tent at Waiau and was 3 feet higher than the surface of the lake. The Hilo observations were made some distance from the sea, and the elevation of the station is given as 100 feet, although I believe this is only an estimated value. The reductions were made by means of the Smithsonian Meteorological Tables (edition of 1893) and gave the following results:

Differences of height.

Differences of height between—	Dates of observation.	Number of readings.	Probable error of mean.	Difference in feet.
			Feet.	
Waimea (V) and Hilo	July 7 to 11	11	3	2632
Waimea (L) and Hilo	18 to 25	13	5	2672
Kalaieha and Hilo	13 to 18	18	18	6700
Waiau and Hilo	21 to 25	14	23	13233
Kalaieha and Waimea (L)	18 and 27	4	13	4033
Waiau and Waimea (L)	21 to 25	12	27	10557

From the above table we derive the elevations above Hilo as follows:

Station.	Observer, A. B. Lyons.	Observer, E. D. Preston.	Mean.
	Feet.	*Feet.*	*Feet.*
Kalaieha	6705	6700	6702
Waiau	13229	13233	13231

The triangulation made by Professor Alexander (based on the elevation of Mauna Kea primary trigonometrical station) gave the following results:

	Feet.
Mauna Kea 1ry △ (above mean tide from previous triangulation)	13760·0
" " 2ry △ (by leveling from above)	13804·6
Summit △	13810·0
Peak A = Poliahu	13646·5
" B	13468·6
Waiau astronomical station	13041·4
" Lake	13040
" crater △ (on outer bank of crater)	13179·4
Lilinoe	12996·5
Kalaieha latitude station	6713·6
Keonehehee	11513

The following tables give a summary of the results for the entire season's work:

Abstract of magnetic declination observations.

Station.	Date.	Scale reading of magnetic axis.	Magnetic declination.
	1891–'92.	d.	° ′
Waikiki, Oahu	Aug. 11	28·72	10 05·3
	12	28·80	04·9
	13	-----	05·3
	Mean	28·76	10 05·2
Kahuku, Oahu	Nov. 25	-----	10 16·4*
	26	29·34	13·0
	27	-----	14·7
	Mean	29·34	10 14·7
Honolulu	June 2	-----	10 14·7
	3	29·12	17·4
	4	29·00	16·7
	Mean	29·06	10 16·3
Kawaihae, Hawaii	July 1	29·18	9 18.7
	2	28·88	20·5
	3	-----	22·5
	Mean	29·03	9 20·6
Waimea, Hawaii (west base)	July 8	-----	8 50·0
Waimea, Hawaii (Lyons 1872)	9	-----	9 03·0
	11	-----	04·2
	Mean	-----	9 03·6
Kalaieha, Hawaii	July 14	29·06	9 53·1
	15	-----	52·1
	Mean	29·06	9 52·6
	1892.	d.	° ′
Waiau (summit of Mauna Kea)	July 21	30·02	10 21·0
	22	29·24	-------
	24	-----	24·4
	Mean	29·63	10 22·7
Hilo, Hawaii	July 30	-----	8 26·1*
	31	29·00	25·3
	Aug. 1	28·76	18·9
	2	-----	19·4
	Mean	28·88	8 22·4
Napoopoo, Hawaii (Captain Cook's station, 1779)	Aug. 18	28·30	9 09·2
	19	-----	10·9
	20	28·38	05·7
	21	-----	06·6
	Mean	28·34	9 08·1
Lahaina, Hawaii (De Freycinet's station 1819)	Aug. 23	-----	9 08·1*
	24	-----	09·9
	25	28·42	08·8*
	Mean	28·42	9 08·9
Waimea A, Kauai	Sept. 2	28·58	10 03·8
	3	-----	03·2
	Mean	28·58	10 03·5
Waimea B, Kauai	Sept. 5	-----	9 42·6
	6	-----	49·2
	7	-----	47·0
	Mean	-----	9 46·3
Nonopapa, Niihau	Sept. 9	-----	10 01·4

* Only a. m. or p. m. observations.

Abstract of results of magnetic dip observations.

Station.	Date.	Dip by needle No. 1. N.	S.	N.-S.	Mean.	Dip by needle No. 2. N.	S.	N.-S.	Mean.
	1891–'92.	° ′	° ′	′	° ′	° ′	° ′	′	° ′
Waikiki, Oahu	Aug. 11	40 08	39 29	+39	39 48	39 51	40 02	−11	39 56
	12	14	28	+46	51	40 10	01	+ 9	40 06
	13	20	28	+52	54	39 50	06	−16	39 58
Kahuku, Oahu	Nov. 25	42 01	41 01	+60	41 31	41 34	41 20	+14	41 27
	26	41 34	05	+29	20	26	28	− 2	27
	27	41	07	+34	24	12	38	−26	25
Honolulu	June 2	41 02	40 10	+52	40 36	40 42	40 54	−12	40 48
	3	40 55	13	+42	34	34	46	−12	40
	4	41 10	22	+48	46	46	52	− 6	49
Kawaihae, Hawaii	July 1	38 32	37 46	+46	38 09	38 04	38 15	−11	38 10
	2	42	48	+54	15	05	14	− 9	10
	3	37	41	+56	09	13	16	− 3	14
Waimea (west base), Hawaii	8	39 32	38 15	+77	38 54	38 30	38 25	+ 5	38 28
Waimea (Lyons' 72), Hawaii	9	38 38	04	+34	21	20	29	− 9	24
Waimea (Lyons '72), Hawaii	11	55	34	+21	44	18	28	−10	23
Kalaieha, Hawaii	14	39 12	38 23	+49	38 48	38 50	38 50	0	38 50
	15	09	34	+35	52	39	43	− 4	41
	16	14	19	+55	46	53	46	+ 7	50
Waiau (summit of	21	39 00	38 09	+51	38 34	38 42	38 14	+28	38 28
Mauna Kea)	22	38 57	13	+44	35	24	38	−14	31
	23	54	17	+37	36	41	42	− 1	42
Hilo, Hawaii	30	39 43	39 03	+40	39 23	39 14	39 23	− 9	39 18
	31	39	04	+35	22	20	32	−12	26
	Aug. 1	50	12	+38	31	25	15	+10	20
Napoopoo, Hawaii	18	37 56	37 22	+34	37 39	37 44	37 34	+10	37 39
(Captain Cook's	19	46	36 57	+49	22	22	46	−24	34
station, 1779)	20	38 08	37 10	+58	39	54	30	+24	42
Lahaina, Maui (De	23	39 43	38 24	+79	39 04	39 24	39 22	+ 2	39 23
Freycinet's station,	24	48	49	+59	18	32	36	− 4	34
1819)	25	50	51	+59	20	28	30	− 2	29
Waimea A, Kauai	Sept. 2	40 48	39 35	+73	40 12	40 13	40 25	−12	40 19
	3	56	48	+68	22	28	29	− 1	28
Waimea B, Kauai	5	40 45	39 54	+51	40 20	40 24	40 16	+ 8	40 20
	6	36	55	+41	16	22	34	−12	28
	7	44	40 04	+40	24	36	20	+16	28
Nonopapa, Niihau	9	41 01	40 08	+53	40 34	40 35	40 51	−16	40 43

Summary of magnetic dip.

Station.	Date.	Needles 1–2.	Dip.	Station.	Date.	Needles 1–2.	Dip.
	1891–'92.		° ′		1892.		° ′
Waikiki, Oahu	Aug. 11	− 8	39 52·0	Waiau (summit of Mauna Kea)	July 21	+ 6	38 31·0
	12	−15	58·5		22	+ 4	33·0
	13	− 4	56·0		23	− 6	39·0
		Mean	39 55·5			Mean	38 34·3
Kahuku, Oahu	Nov. 25	+ 4	41 29·0	Hilo, Hawaii	30	+ 5	39 20·5
	26	− 7	23·5		31	− 4	24·0
	27	− 1	24·5		Aug. 1	+11	25·5
		Mean	41 25·7			Mean	39 23·3
Honolulu	June 2	−12	40 42·0	Napoopoo, Hawaii (Captain Cook's station, 1779)	18	0	37 39·0
	3	− 6	37·0		19	−12	28·0
	4	− 3	47·5		20	−18	40·5
		Mean	40 42·2			Mean	37 35·8
Kawaihae, Hawaii	July 1	− 1	38 09·5	Lahaina, Maui (De Freycinet's station, 1819)	23	−19	39 13·5
	2	+ 5	12·5		24	−16	26·0
	3	− 5	11·5		25	− 9	24·5
		Mean	38 11·2			Mean	39 21·3
Waimea (west base)	8	+26	38 41·0	Waimea A, Kauai	Sept. 2	− 7	40 15·5
Waimea (Lyons, '72)	9	− 3	38 22·5		3	− 6	25·0
Waimea (Lyons, '72)	11	+21	33·5			Mean	40 20·2
		Mean	38 28·0	Waimea B, Kauai	5	0	40 20·0
Kalaieha, Hawaii	14	− 2	38 49·0		6	−12	22·0
	15	+11	46·5		7	− 4	26·0
	16	− 4	48·0			Mean	40 22·7
		Mean	38 47·8	Nonopapa, Niihau	9	− 9	40 38·5

Abstract of magnetic horizontal intensity observations.

Station.	Date.	Horizontal intensity = H (C. G. S. units).	Magnetic moment = m of intensity magnet (C. G. S. units).
	1891–'92.		
Waikiki, Oahu	Aug. 11	·2979	129·8
	12	71	9·6
	13	92	9·4
	Mean	·2981	129·6
Kahuku, Oahu	Nov. 25	·2932	129·3
	26	31	8·6
	27	48	8·6
	27	40	8·3
	Mean	·2938	128·7
Honolulu	June 2	·2954	127·9
	3	47	7·9
	4	51	7·9
	Mean	·2951	127·9
Kawaihae, Hawaii	July 1	·3001	129·4
	2	25	8·2
	3	10	7·8
	Mean	·3012	128·5
Waimea (west base)	July 8	·2966	128·0
(Lyons, '72) Hawaii	9	·2978	129·3
(Lyons, '72)	11	86	8·4
	Mean	·2982	128·8
Kalaicha, Hawaii	July 14	·2949	128·5
	15	58	8·1
	Mean	·2954	128·3
	1892.		
Waiau (summit of Mauna Kea)	July 22	·2950	123·5
	23	50	3·1
	23	50	4·1
	Mean	·2950	123·6
Hilo, Hawaii	July 30	·3064	127·0
	31	64	7·5
	Aug. 1	57	6·8
	Mean	·3062	127·1
Napoopoo, Hawaii (Captain Cook's station, 1779)	Aug. 18	·3035	124·0
	19	27	4·7
	20	11	5·3
	Mean	·3024	124·7
Lahaina, Maui (De Freycinet's station, 1819)	Aug. 23	·2990	124·9
	24	80	5·4
	25	96	4·8
	Mean	·2989	125·0
Waimea A, Kauai	Sept. 2	·2886	125·2
	3	79	4·9
	Mean	·2882	125·0
Waimea B, Kauai	Sept. 5	·2934	124·0
	6	33	4·0
	7	26	4·0
	Mean	·2931	124·0
Nonopapa, Niihau	Sept. 9	·2928	125·4

Recapitulation of results of magnetic observations.

Station.	Latitude (north).	Longitude (west of Greenwich).	Date, 1891–'92.	Declination (east).	Dip N. end below horizon.	Horizontal intensity.	Total intensity.
	° ′	° ′		° ′	° ′	*Dyne.*	*Dyne.*
Waikiki, Oahu	21 16·4	157 49·7	Aug. 11–13	10 05·2	39 55·5	0·2981	0·3887
Kahuku, Oahu	21 42·6	157 51·7	Nov. 25–27	10 14·7	41 25·7	0·2938	0·3910
Honolulu	21 18·0	157 51·5	June 2–4	10 16·3	40 42·2	0·2951	0·3892
Kawaihae, Hawaii	20 02·4	155 47·6	July 1–3	9 20·6	38 11·2	0·3012	0·3832
Waimea (west base)	20 02·1	155 37·9	8	8 50·0	38 41·0	0·2966	0·3799
Waimea (Lyons, '72), Hawaii	20 02·0	155 37·0	9–11	9 03·6	38 28·0	0·2982	0·3809
Kalaieha, Hawaii	19 42·6	155 25·9	14–16	9 52·6	38 47·8	0·2954	0·3790
Waiau (summit of Mauna Kea)	19 49·2	155 26·8	21–25	10 22·7	38 34·3	0·2950	0·3761
Hilo, Hawaii	19 44·0	155 04·0	30–Aug. 2	8 22·4	39 23·3	0·3062	0·3962
Napoopoo, Hawaii	19 29·0	155 59·0	Aug. 18–21	9 08·1	37 35·8	0·3024	0·3819
Lahaina, Maui	20 52·0	156 40·9	23–25	9 08·9	39 20·8	0·2989	0·3865
Waimea A, Kauai	21 57·0	159 42·0	Sept. 2–3	10 03·5	40 20·2	0·2882	0·3753
Waimea B, Kauai	21 57·2	159 42·4	5–7	9 46·3	40 22·7	0·2931	0·3848
Nonopapa, Niihau	21 55·0	160 13·0	9	10 01·4	40 38·5	0·2928	0·3859

The geographical positions are given only approximately. That of Nonopapa is uncertain.

The localities, geographical positions, and elevations of the gravity stations in order of their latitude are as follows:

Station.	Locality.	Latitude.	Longitude.	Elevation.	Elevation.
		φ (+) ° ′ ″	λ (+) ° ′ ″	*Metres.*	*Feet.*
Washington	Smithsonian Institution	38 53 20	77 01 35	10	34
Washington	C. and G. Survey Office	38 53 13	77 00 31	14	45
Mount Hamilton	Lick Observatory	37 20 25	121 38 35	1282	4205
Honolulu	Kapuaiwa Building	21 18 03	157 51 46	6	20
Waikiki	J. F. Brown's	21 16 25	157 50 01	3	10
Kawaihae	S. Parker's	20 02 25	155 49 36	2	8
Mauna Kea	Waiau	19 49 11	155 28 48	3981	13060
Kalaieha	Humuula	19 42 32	155 27 53	2030	6660

The following tables give a summary of results for each pendulum and the comparative force of gravity for each station:

Summary of results.

[Periods reduced to temperature, 15° C.; pressure, 500mm at 0° C.; infinitely small arc; sidereal time.]

Station.	Pendulums.			Differences (in seventh decimal place).		
	B_1	B_2	B_3	B_1-B_2	B_2-B_3	B_1-B_3
	Seconds.	*Seconds.*	*Seconds.*			
Washington*	0·500 7828	0·500 6962	0·500 6325	+866	+637	+1503
Waikiki ('91)†	1 0871	1 0015	0 9377	+856	+638	+1494
Waikiki ('92)‡	1 0823	0 9998	0 9330	+825	+668	+1493
Honolulu	1 0797	0 9920	0 9320	+877	+600	+1477
Kawaihae	1 1157	1 0287	0 9622	+870	+665	+1535
Kalaieha	1 1946	1 1067	1 0451	+879	+616	+1495
Waiau	1 3050	1 2264	1 1457	+786	+807	+1593
Mount Hamilton	0 8976	0 8096	0 7486	+880	+610	+1490
Washington ‖	0 7794	0 6950	0 6313	+844	+637	+1481

*Smithsonian Institution, 1891.
†June 26, 27, and 28, 1891.
‡June 3, 4, and 5, 1892.
‖Coast and Geodetic Survey Office, 1892.

Station.	Mean period.	Relative periods.	Relative force of gravity.	Absolute force of gravity.
	Seconds.			*Dynes.*
Washington*	0·500 7026	1·000 000	1·000 000	980·100
Waikiki ('91)	0·501 0088	1·000 612	0·998 776	978·902
Waikiki ('92)	0·501 0050	1·000 604	0·998 792	978·917
Honolulu	0·501 0012	1·000 596	0·998 808	978·933
Kawaihae	0·501 0355	1·000 665	0·998 670	978·798
Kalaieha	0·501 1155	1·000 825	0·998 350	978·485
Waiau	0·501 2257	1·001 045	0·997 909	978·055
Mount Hamilton	0·500 8186	1·000 232	0·999 536	979·646

*Corrected for wear.

The pendulum observations made on the Island of Hawaii enable us to calculate the mean density of Mauna Kea from an assumed value for the earth's mean density; or, accepting a density of the mountain derived from a study of the rocks, we may reverse the problem and obtain a value of the mean density of the earth. An attempt will now be made to utilize the preceding results in both of these ways.

THE DENSITY OF MAUNA KEA AND THE MEAN DENSITY OF THE EARTH.

The formula usually employed in the treatment of the change of the force of gravity with elevation is:

$$\frac{dg}{g} = -2\frac{h}{r}\left(1 - \frac{3}{4}\frac{\delta}{\triangle}\right) \qquad (1)$$

where g = the force of gravity at the sea level,
h = the elevation,
r = the radius of the earth,
δ = the density of the mountain or table-land, and
$\triangle$ = the mean density of the earth.

This formula is derived by combining the earth attraction, varying inversely as the square of the distance, with the attraction of the matter lying between the sea level and the upper station. Certain suppositions are made in regard to the form of this exterior matter and it has been generally considered a sufficient approximation to regard all the matter as equivalent to that contained in a plain of infinite extent and of a thickness equal to the elevation (h). Whether this supposition is admissible depends, of course, on the relation between vertical and horizontal dimensions of the intervening mass. If we suppose this to be of a conical form, the above term depending on the attraction of the plain must receive the correction

$$+\frac{3}{4}\frac{\delta}{\triangle}\frac{h}{\sqrt{h^2+a^2}}$$

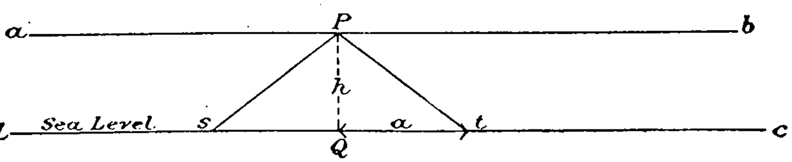
where (a) is the radius of the base.

This expression reduces to zero for an infinite value of (a) in which we have the previous case of an infinite plain. For $a = o$ the entire effect of the intervening matter disappears, as it should do in the formula. The expression for the differential of gravity at the summit of a conical mountain would therefore be:

$$dg = -\frac{2h}{r}g\left(1-\frac{3}{4}\frac{\delta}{\triangle}+\frac{3}{4}\frac{\delta}{\triangle}\frac{h}{\sqrt{h^2+a^2}}\right) \qquad (2)$$

$$= -\frac{2h}{r}g\left(1-\frac{3}{4}\frac{\delta}{\triangle}+\frac{3}{4}\frac{\delta}{\triangle}\cos\beta\right)$$

where (β) is the semivertical angle of the mountain. When (a) is very large, compared with (h), the correction for a paraboloid is $\frac{2}{3}$ and that for a sphere is $\frac{1}{2}$ that given above.* Since from the nature of the case (h) is always smaller than (a), the quantity $\frac{h}{\sqrt{h^2+a^2}}$ is never equal to unity, and the total effect of the last two terms must be essentially positive, and we have a greater value for gravity in the case of the cylinder than for the paraboloid, and a greater one for the paraboloid



than for the cone. In the figure, gravity at (Q) would be diminished by passing to (P) (if no intervening matter existed) by

$$\frac{2h}{r}g = \text{correction for distance.}$$

* Helmert, p. 172, II Theil.

If we interpose the infinite plain $(a\ b\ c\ d)$, gravity at (P) will be increased by

$$\frac{2h}{r}\,g\,\frac{3}{4}\,\frac{\delta}{\triangle} = \text{correction for plain.}$$

Since (δ) is always less than $(\triangle)$, the combined effect of these two terms will be essentially negative. The difference between the effects of the plain and the cone is seen in the last term, and the total influence exerted by matter equal in volume and position to that generated by the revolution of the trapezoid $(a\ P\ s\ d)$ around $(P\ Q)$ would be:

$$-\frac{2h}{r}\,g\,\frac{3}{4}\,\frac{\delta}{\triangle}\,\frac{h}{\sqrt{h^2+a^2}}$$

Since (a) is always very much greater than (h), the expression $\frac{h}{\sqrt{h^2+a^2}}$ is a small fraction, and the combined effect of the last two terms will be of the same sign as the first one of them; that is, their effect will be essentially positive. This is as it should be, and shows that the effect of this matter is to increase gravity at (P), whereas the effect of the first two terms would necessarily be to diminish gravity at the same point, except in the extraordinary case where the infinite plain should be one-third as heavy again as the average earth matter. A uniform density of 7·56 in an infinite plain immediately under the station would exactly counterbalance the diminution of gravity on account of distance. The negative sign attributed to the last term must not be interpreted as meaning that the actual effect of this matter is to diminish the value of gravity at (P). It rather means that, the general effect of the plain being to increase gravity at (P), the influence of $(a\ P\ s\ d)$ comes in here as expressing the difference between the effects of the plain and cone, and that the plain effect must be diminished by $\frac{3}{4}\,\frac{\delta}{\triangle}\,\frac{h}{\sqrt{h^2+a^2}}$ in order to get the cone effect. In fact, the parenthetical part of the actual plain effect being $+\frac{3}{4}\,\frac{\delta}{\triangle}$, and that of the volume $(a\ P\ s\ d)$ being $+\frac{3}{4}\,\frac{\delta}{\triangle}\,\frac{h}{\sqrt{h^2+a^2}}$, their difference, or $+\frac{3}{4}\,\frac{\delta}{\triangle} - \frac{3}{4}\,\frac{\delta}{\triangle}\,\frac{h}{\sqrt{h^2+a^2}}$, gives the actual effect for the cone as expressed in (2). For a mountain 2·5 miles high and having a radius of base equal to 30 miles $\frac{h}{\sqrt{h^2+a^2}} = \frac{1}{12}$ (nearly); so that it makes an essential difference whether we treat the mountain matter as a cone or as a plain. In this case the effect is changed by its one-twelfth part.

The formula $\frac{dg}{g} = -\frac{2h}{r}\left(1 - \frac{3}{4}\frac{\delta}{\triangle}\right)$ has long been known as Young's Rule, although it first appeared in 1749, in Bouguer's work, "La Figure de la Terre." He makes the assumption that $\triangle = 2\,\delta$, from which the total diminution of gravity on account of distance and mass would be $\frac{5}{4}\frac{h}{r}$. This correction has continually been applied in the treatment of mountains and table-lands by most modern observers. There has been some question whether this formula is the proper one to use, or whether, indeed, any correction at all should be made for continental attraction. It has been assumed that the whole cone of matter ($P\,s\,t$) might be brought down by compression to the line ($d\,c$) without materially altering the shape of the sea level. That is to say, that the vertical attraction at the point (P) before compression is approximately equal to that after compression. In other words, the intervening matter has no effect. This assumption has evidently been made in view of the earlier measurements of the force of gravity, which seemed to show a very small density for mountains. Two notable cases of this are, first, the Andes,* which appear to be not much heavier than ice, and the Island of Ascension, where the observed force of gravity at the sea level was actually less than that at the summit of Green Mountain,† indicating that the downward attraction of all the matter above the sea level was insignificant. These remarkable results have not always been confirmed by later observers, and recent work calls for a different interpretation of the effect of this matter. The condensation theory proposed by Professor Helmert, in his "Die Mathematischen und physikalischen Theorieen der höheren Geodäsie," makes it possible to treat pendulum observations in a way more consistent with the results of modern observations, and by so doing to derive a much more trustworthy value for the earth's ellipticity. Just how particular forms modify the density deduced for the mass lying between the sea level and the upper station is shown by the following table, which has been calculated for Kalaieha, at an elevation of 6 660 feet. The mean density of the earth is assumed as 5·67. The diminution of gravity in passing from Kawaihae (elevation 8 feet) to Kalaieha is 0·000 320 g (g being the value at the sea level).

* La Figure de la Terre, par M. Bouguer, Paris, 1749, p. 362.

† Memoirs Royal Astronomical Society, Vol. VII, p. 60.

Figure.	Radius of base in miles.				
	10	15	20	25	30
	Density.				
Cone	4·30	4·10	4·00	3·96	3·92
Spherical segment	4·10	3·98	3·92	3·88	3·86
Cylinder	4·01	3·92	3·87	3·85	3·84
Infinite plain	3·75	3·75	3·75	3·75	3·75
	Differences for varying radius.				
Cone	0·20	0·10	0·04	0·04	
Spherical segment	0·12	0·06	0·04	0·02	
Cylinder	0·09	0·05	0·02	0·01	
Infinite plain	0·00	0·00	0·00	0·00	
	Differences from cone, for varying figure.				
Cone					
Spherical segment	0·20	0·12	0·08	0·08	0·06
Cylinder	0·09	0·06	0·05	0·03	0·02
Infinite plain	0·26	0·17	0·12	0·10	0·09

It is seen that beyond 30 miles the effect of an increase in the radius is comparatively insignificant, and that any possible change in the form of the matter after this limit is reached can not essentially modify the resulting density. The cone and the infinite plain being the extreme cases between which all other forms fall, it is worth while to compare these two in their effect on the force of gravity for the particular case under consideration. The elevation of the pendulum station at Waiau, near the summit of Mauna Kea, is 13 060 feet. The mountain is generally gradual in its ascent, and if we assume 30 miles as the radius of the base, we have $\frac{h}{a} = \frac{2{\cdot}473}{30} = 0{\cdot}0824 = \frac{1}{12}$.

Without stopping to inquire for the present whether Young's Rule gives a density for the mountain which is absolutely correct, for the sake of comparing Mauna Kea with other mountains a rigorous formula is perhaps not necessary. If we consider the matter lying between Waiau and the sea level as an infinite plain, as has been done in other cases, we arrive at the equation:

$$\delta = 0{\cdot}536\ \triangle$$

Taking the matter as a cone of height 2·47 miles and radius of base 30 miles, we get:

$$\delta = 0{\cdot}584\ \triangle$$

Assuming $\triangle = 5{\cdot}58$ (Harkness), we have

$$\text{for infinite plain } \delta = 2{\cdot}99$$
$$\text{for cone } \delta = 3{\cdot}26$$

Either of these values is much greater than has usually been found for other mountains, and the latter one is greater than any of the rocks found on the surface.. An extensive collection of these rocks was made at different elevations, beginning at Kawaihae, on the leeward side, passing to the summit, and finally ending at Hilo, on the windward side. In all, 26 specimens were obtained. They have been carefully studied by Prof. George P. Merrill, curator of lithology at the Smithsonian Institution, who has kindly furnished the following description. The points from which they were obtained are indicated in illustration No. 31. Only 18 determinations of specific gravity were made, as this number of specimens seemed to include all the distinct types in the collection.

DESCRIPTION OF SPECIMENS.

As was the case with samples submitted by you in 1888,* and as was to be expected from papers by Dana and others† on the rocks of these islands, the samples are all basaltic lavas, differing only in degrees of crystallization, compactness, and the amount of decomposition they have undergone since their extrusion. The essential constituents of all these rocks are basic plagioclase feldspars, augites, olivines, and grains of magnetic iron, together with sporadic microscopic apatites, titanic iron, iron pyrites, and the various incipient forms of crystallization to which the convenient name of trichites may be applied. A residual glass, due to imperfect crystallization, is to be observed in greater or less abundance in nearly all cases. The microscopic characteristics of these rocks and their constituent minerals have been so well described by the above-mentioned writer that little remains to be done here but to call attention to such of their peculiarities as have a direct bearing upon the subject in hand. All stages of crystallization are to be found in the samples submitted, from those consisting almost wholly of glass (specimens D and 13), through forms carrying sharply angular or beautifully dendritic crystallizations of feldspar, augite, and olivine in a glassy base (specimen 14), to those almost wholly crystalline, exhibiting only here and there small interstitial areas of more or less devitrified glass (specimens 1, 2, 18, 23, etc.). A like variation is found in other physical properties, samples from various sources showing all gradations from highly vesicular (pumiceous) forms (specimens No. 7, 11, 13, 28, 31, 33), so light in some cases as to almost float on water, to compact glassy or crystalline varieties showing in the mass a specific gravity as high as 3·08 (specimen 2). The following notes on a few characteristic specimens will serve to show the general character of the rocks. The specific gravities given were made upon powdered material in picnometer flasks:

No. 1. A dark blue-gray rock, weathering brownish, only slightly vesicular, and showing no crystalline secretions sufficiently developed to be visible to the unaided eye. In the thin section it shows an abundance of lath-shaped plagioclases, thickly crowded together; numerous imperfectly outlined black and opaque areas, assumed to be altered augites, and an occasional yellowish-red, badly altered olivine. The ground mass is full of black opaque matter and brownish decomposition products. Specific gravity, 2·82.

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* See App. 14th Rept. U. S. Coast and Geodetic Survey for 1888, p. 529.

† See Characteristics of Volcanoes, by J. D. Dana, pp. 318–354.

No. 2. A compact dark-gray rock, with occasional small vesicles and showing phenocrysts of feldspar and occasionally augite of such size as to be visible to the naked eye. In the thin section abundant basaltic augites in irregular granular forms, interspersed with lath-shaped plagioclases, form the chief characteristics. There is the usual sprinkling of granules of iron ore, and an occasional rather small, rounded bleb of olivine. The augites and plagioclases occur in crystals of two generations. The amount of interstitial glass is proportionally small. Specific gravity, 3·08.

No. 5. A dark-gray vesicular rock, weathering brownish, and containing a comparatively large amount of iron pyrite. In the section a dense base of iron ores and small augites and plagioclases, with an abundant sprinkling of augite, olivine, and plagioclase phenocrysts. Specific gravity, 2·9.

No. 7. A finely vesicular, almost pumiceous, nearly black rock, without macroscopic constituents. In the section a dense and very opaque, partially devitrified, glassy base, carrying scattering phenocrysts of plagioclase, olivine, and augite.

No. 9. A finely vesicular, nearly black rock, with only an occasional grain of pyrite recognizable by the unaided eye. Under the microscope like 7, but with only feldspar phenocrysts. Specific gravity, 2·79.

No. 10. A compact, finely vesicular, nearly black rock, without macroscopic constituents. Under the microscope a dense, almost opaque ground mass, crowded full of granules of iron ore and lath-shaped plagioclases. Neither augites nor olivines seen in the single section examined. Specific gravity, 2·79.

No. 12. A dark-gray, nearly black, finely vesicular rock, without macroscopic constituents. Under the microscope shows only small feldspar microlites in a ground mass consisting of dark, smoky, brown glass and innumerable black opaque granules of iron ore. Between crossed nicols are seen occasional brilliantly polarizing particles, evidently olivine and augite, but too small and imperfect for absolute determination.

No. 14. In macroscopic characters like the last, but a trifle more vesicular, and with more vitreous luster; weathers brown. Under the microscope a yellow-brown glass, thickly studded with clear, colorless, sharply outlined plagioclases, olivines, and iron oxides. Specific gravity, 2·73.

No. 16. A compact, dark blue-gray rock, without macroscopic constituents, and showing in the sections under a power of 180 diameters a very dense ground mass in the form of a microgranular aggregate of minute grains of magnetic iron and nearly colorless silicates, bearing rarely greatly elongated plagioclases. The iron ore is so prominent a constituent that pieces of 0·03 gram weight were taken up by an ordinary horseshoe magnet. A rough analysis yielded—

SiO_2	40·02
Al_2O_3 and Fe_2O_3	32·80
CaO	10·20
MgO	5·19
K_2O	0·96
Na_2O (by difference)	5·19
	100·36

Specific gravity in bulk, 2·99; in powder, 3·02.

No. 17. Dark gray, nearly black, coarsely vesicular, with a few macroscopic olivines. In the section a black, smoky glass, full of iron ores and bearing abundant plagioclases in all stages of development and more rarely olivines. Specific gravity, 2·73.

No. 18. Compact, dark gray, with only an occasional small cleavage face of a feldspar sufficiently developed for determination by the unaided eye. Under the microscope a normal basalt, almost holocrystalline. Specific gravity, 2·83.

No. 19. A brownish-red, vesicular, somewhat vitreous rock, showing under the microscope a red and very opaque glassy base with numerous plagioclases.

No. 20. A brownish vesicular rock, showing under the microscope characters very similar to the last.

No. 23. A compact black rock, without macroscopic constituents. In the section a brown-black glass, injected with iron oxides and bearing numerous plagioclases and small colorless olivines. Augites quite inconspicuous. Specific gravity, 2·8.

No. 24. A close-grained dark-gray rock, showing to the naked eye only minute elongated white specks suggestive of feldspars. Under the microscope like 23.

D. A nearly black, highly vesicular rock, with frequent inclosures of pyrite. Under the microscope a yellow-brown glass with occasional large and small olivines and more numerous feldspars. Specific gravity, 2·88.

In connection with the above the following remarks may be of interest. It is a commonly accepted principle among petrologists that, the composition of the magma being the same, the physical properties of any eruptive rock are largely controlled by the conditions of cooling and crystallization, rapid cooling being conducive to the production of glassy or microcrystalline forms, while a more complete and, as a rule, coarser crystallization is developed when the cooling is more gradual. As, moreover, the conditions which govern the rate of cooling of a molten magma are mainly those of pressure from superincumbent matter—material being the same—it becomes at once apparent that those portions of any magma which are deeply buried will be the more highly crystalline. The vesicularity and consequent specific gravity (in bulk) of a lava is to a certain extent controlled by similar conditions, since this vesicularity is due to the expansion of moisture in the magma; hence the amount of moisture being the same, the superficial portions will be the more highly vesicular.

Further, it has been shown, the magma being the same, the glassy varieties of any rock are less dense than those which are crystalline. The variations are usually slight, but, as given in the published descriptions, are found to be from 0·1 to 0·3 greater in the holocrystalline than in the glassy forms.* At depths greater than have yet been rendered accessible it is possible to conceive of a still further condensation, due to a possible existence of the iron in a metallic state. Our knowledge of the physical conditions of rock masses under conditions of heat and pressure such as must exist at great depths is as yet too incomplete to afford data for anything more than speculation.† From what has been said above regarding the effects of pressure upon the physical conditions of a rock it will appear that the deeper-lying portions may be relied upon to show a greater specific gravity—the magma remaining the same—than do the superficial portions. It is obvious, therefore, that determinations on specific gravity on the samples submitted can be regarded merely as suggestive. Such determinations as I have made were, with one or two exceptions, as will be noticed, done by means of a picnometer flask, the material being first broken into small bits and weighed after the exhaustion of the air by means of a pump. The results may be regarded as in all cases a trifle under the true specific gravity of the material, since there is little probability that all the air included in the pores

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* Thus, an average of 23 determinations of specific gravity of crystalline and semi-crystalline basalt, as given in Teall's British Petrography, shows a mean of 2·86, with extremes of 2·75–3·10, while 9 determinations on basalt glass gave a mean of 2·71 with extremes of 2·69–2·76. (British Petrography, by J. J. Harris Teall, London, 1888.)

† The presence of metallic iron in the basalt of Ovifak serves to illustrate the possibility of occurrence of this sort, although unfortunately we are not able to say definitely that this may not be a superficial phenomenon due to a reduction of the preexisting ores by carbon rather than original metallic iron from the earth's interior.

and vesicles was wholly removed. Below are the results obtained in the method described:

Specimen	No. 1.	Specific	gravity	2·82
"	No. 2.	"	"	3·08
"	No. 4.	"	"	2·90
"	No. 5.	"	"	2·90
"	No. 9.	"	"	2·79
"	No. 10.	"	"	2·76
"	No. 11.	"	"	2·84
"	No. 13.	"	"	2·27 (in powder)
"	No. 13.	"	"	2·12 (in bulk)
"	No. 14.	"	"	2·73
"	No. 16.	"	"	3·02 (in powder)
"	No. 16.	"	"	2·99 (in bulk)
"	No. 17.	"	"	2·73
"	No. 18.	"	"	2·83
"	No. 23.	"	"	2·80
"	D.	"	"	2·88
"	No. 13.	"	"	2·00 (other part of piece)
"	No. 33.	"	"	1·70

The mean of the preceding values is 2·7. This, according to Professor Merrill, may be slightly under the true value. A specimen from the Island of Hawaii not included in the above collection gave a value of 3·2. In Prof. E. S. Dana's "Contributions to the Petrography of the Sandwich Islands," in the American Journal of Science, June, 1889, the results of a number of determinations of specific gravity are given from the same island. Seven specimens of basalt gave values varying from 2·82 to 3·00 (page 442). Another group of seven determinations furnished examples of even heavier lava, varying from 3·00 to 3·20 (page 447). If we take the mean of the above values we get a specific gravity of 2·90 for the specimens on the Island of Hawaii. Mauna Kea has the form of truncated cone, so that its effect on the force of gravity at its summit would be intermediate between that of a cone and an infinite plain. As the difference between the ratios of $\frac{\delta}{\triangle}$ for the two forms is only about one-twelfth of the value of either, a direct mean between them will in all probability be a close approach to the actual effect of the mountain. The attraction of an infinite plain on a point above it is entirely independent of the distance of the point from the plain, and we have for the plain effect the expression

$$2\,\pi\,\delta\,h \text{ or } 15{\cdot}54\,\delta$$

The attractions of the earth at the upper station (Waiau) and the lower station (Kawaihae) are, respectively,

$$16566{\cdot}8\,\triangle \text{ and } 16587{\cdot}6\,\triangle$$

the linear unit being 1 mile throughout.

The forces of gravity at the two places are, when corrected for difference of latitude,

$$G \text{ at Waiau} = 978{\cdot}067 \text{ dynes}$$
$$G \text{ at Kawaihae} = 978{\cdot}798 \quad \text{"}$$

The foregoing values lead to the equation

$$-0{\cdot}0007470 = -\frac{20{\cdot}8}{16587{\cdot}6} + \frac{15{\cdot}54\ \delta}{16587{\cdot}6\ \triangle}$$

$$\text{from which } \frac{\delta}{\triangle} = 0{\cdot}541$$

If we consider the mountain as a cone with an altitude of 2·473 miles and radius of base (a) of 30 miles, the attraction becomes

$$2\,\pi\,\delta\,h \left[1 - \frac{h}{\sqrt{h^2 + a^2}}\right] = 14{\cdot}26\ \delta$$

and the resulting value of $\frac{\delta}{\triangle}$ is 0·589.

We may therefore assume that the density of the earth is 1·77 times the density of the mountain, or that the

$$\text{mean density of earth} = 1{\cdot}77 \times 2{\cdot}90 = 5{\cdot}13$$

SUMMARY.

List of stations occupied in the Hawaiian Islands, with results obtained.

1883– 1887–1891-2.

No.	Station.	Island.	Date of occupation.	Height (feet).	Latitude.	Longitude.
					° ′ ″	h. m. s.
1	Nonopapa	Niihau	Sept., 1892			
2	Waimea "B"	Kauai	" "			
3	Waimea "A"	"	Feb. and Mar., 1887		21 57 00·8	
4	" "A"	"	Sept., 1892			
5	Hanalei	"	Mar., 1887		22 12 56·5	
6	Koloa	"	" "		21 52 13·2	
7	Puuloa	Oahu	Jan., "		21 19 15·6	
8	Kahuku	"	Feb., "		21 43 06·1	
9	"	"	Nov., 1892			
10	Honolulu	"	June, 1883	12		
11	"	"	Apr., 1887		21 18 02·5	
12	"	"	June, "	10		
13	"	"	" 1892	20		
14	Waikiki	"	May, 1891*			
15	"	"	June and Dec., 1891	10	21 16 24·3	
16	"	"	Jan. and June, 1892	10	21 16 24·7	10 31 18·6
17	Lahaina	Maui	June, 1883	10	20 52 22·8	
18	"	"	Aug., 1892			
19	Haiku	"	June, 1887	385	20 56 02·6	
20	Pakaoao	"	July, "	9846	20 42 51·0	
21	Kaupo	"	July and Aug., 1887		20 36 40·8	
22	Hana	"	July, "		20 45 38·9	
23	Kailua	Hawaii	June, "		19 38 20·9	
24	Napoopoo	"	Aug., 1892			
25	Kawaihae	"	July, "	8	20 02 05·9	
26	Kohala	"	Apr., 1887		20 15 29·3	
27	Ka Lae	"	May, "		18 53 51·7	
28	Waimea	"	July, 1892	2772		
29	" (west base)	"	" "			
30	Waiau	"	" "	13060	19 48 52·0	
31	Kalaieha	"	" "	6660	19 42 02·6	
32	Hilo	"	Apr. and May, 1887		19 43 11·2	
33	"	"	July and Aug., 1892			

* Transit of Mercury.

SUMMARY.

List of stations occupied in the Hawaiian Islands, with results obtained.

No.	Magnetic elements.					Force of gravity.	
	Declination (east).		Dip.		Hor. inten.	Relative.	Absolute.
	°	′	°	′	*Dyne.*		*Dynes.*
1	10	01·4	40	38·5	0·2928		
2	9	46·3	40	22·7	0·2931		
3	10	03·5	40	20·2	0·2882		
4							
5							
6							
7							
8							
9	10	14·7	41	25·7	0·2938		
10						0·998 836	978·959
11							
12						0·998 852	978·975
13	10	16·3	40	42·2	0·2951	0·998 808	978·932
14*							
15	10	05·2	39	55·5	0·2981	0·998 776	978·900
16						0·998 792	978·916
17						0·998 724	978·850
18	9	08·9	39	20·8	0·2989		
19						0·998 790	978·914
20						0·998 142	978·279
21							
22							
23							
24	9	08·1	37	35·8	0·3024		
25	9	20·6	38	11·2	0·3012	0·998 672	978·799
26							
27							
28	9	03·6	38	28·0	0·2982		
29	8	50·0	38	41·0	0·2996		
30	10	22·7	38	34·3	0·2950	0·997 913	978·055
31	9	52·6	38	47·8	0·2954	0·998 352	978·485
32							
33	8	22·4	39	23·3	0·3062		

* Time of transit = $4^h\ 43^m\ 57^s$, second and third contacts.

The relative forces of gravity refer to the value at the Smithsonian Institution in Washington. In the absolute column this is assumed to be 980·100 dynes, and the given values are the best ones attainable from the three expeditions.

SUPPLEMENTARY NOTE TO THE FOREGOING REPORT.

Since the foregoing report was written, Professor Alexander has made a comparison of the astronomical and geodetic latitudes on the three principal islands of the group. In addition to this, since the observed latitudes at Kawaihae, Hilo, and Waiau (Mauna Kea) are consistent with one another, the latter has been adopted as a standard, and a comparison has been made between the observed latitudes and the Mauna Kea standard. This brings out some interesting deflections of the plumb line.

There appears to be a disturbance of more than a minute in the direction of gravity at the south point of Hawaii (Ka Lae). At Kohala the plumb line is deflected half a minute toward the south, and at Kalaieha nearly as much toward the north, the disturbance being in both cases toward the mountain. The enormous deflection at Ka Lae (67″) is also to the northward. This is evidently caused by the great mass of Mauna Loa,* which adds its effect to that of Mauna Kea, and, moreover, is comparatively near to the astronomical station.

On Maui the same phenomenon appears. At Haiku there is a deflection toward the south, and at Kaupo there is one to the north, and, as before, the astronomical latitude determined on the top of Haleakala (10 000 feet elevation), at Pakaoao, appears to be a normal one for that island. Judging by analogy, there seems to be no reasonable doubt that Oahu would have shown the same thing had a station been made on the summit between Kahuka and Puuloa.

The mean deflection for each of the islands (leaving out Ka Lae, on the Island of Hawaii) is:

	″
Hawaii	27
Maui	29
Oahu	26

When we come to compare the mean latitudes for each island with one another, we find that Maui is too small, whether judged by the Hawaii or the Oahu standard. The amount is nearly the same in either case, so that the most probable assumption is that there is a preponderance of matter deflecting the plumb line to the northward at all the Maui stations. This supposition has been made by Professor Alexander, and seems to be the most rational interpretation of the results. The following table has been furnished by him, and is inserted here with his permission:

* Both Mauna Kea and Mauna Loa are nearly 14 000 feet high.

Hawaiian latitudes compared.

[See illustration No. 23.]

Station.	Astronomical.			Geodetic.			Diff.	Mauna Kea standard.			Diff.
Oahu.	°	′	″	°	′	″	″	°	′	″	″
Kahuku	21	43	06·1	21	42	16·1	+50·0	21	42	43·2	+22·9
Puuloa	21	19	15·6	21	19	11·8	+ 3·8	21	19	38·9	—23·3
Honolulu	21	18	02·5	21	18	02·3	+ 0·2	21	18	29·6	—27·1
Waikiki	21	16	24·5	21	16	26·8	— 2·3	21	16	53·9	—29·4
Maui.											
Lahaina	20	52	22·8	20	52	53·2	—30·4	20	52	34·5	—11·7
Haiku	20	56	02·6	20	56	04·0	— 1·4	20	55	45·3	+17·3
Pakaoao	20	42	51·0	20	43	21·6	—30·6	20	43	02·9	—11·9
Kaupo	20	36	40·8	20	37	41·0	—60·2	20	37	22·3	—41·5
Hana	20	45	38·9	20	45	47·5	— 8·6	20	45	28·8	+10·1
Hawaii.											
Kohala	20	15	29·3	20	15	17·7	+11·6	20	14	59·0	+30·3
Kawaihae	20	02	05·9	20	02	25·1	—19·2	20	02	06·4	— 0·5
Mauna Kea	19	48	52·0	19	49	10·7	—18·7	19	48	52·0	± 0·0
Kalaieha	19	42	02·6	19	42	33·5	—30·9	19	42	14·8	—12·2
Hilo	19	43	11·2	19	43	30·4	—19·2	19	43	11·7	— 0·5
Kailua	19	38	20·9	19	39	03·8	—42·9	19	38	45·1	—24·2
Ka Lae	18	53	51·7	18	55	17·7	—86·0	18	54	59·0	—67·3

Positions for Lahaina:	° ′ ″
By observation	20 52 22·8
" Oahu mean latitude	32·3
" Mauna Kea standard	34·5