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DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
E. LESTER JONES, DIRECTOR

# MANUAL OF TRIANGULATION COMPUTATION AND ADJUSTMENT

BY

WALTER F. REYNOLDS
Mathematician

Special Publication No. 138

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## FOREWORD

The purpose of this publication is to explain the methods used in the United States Coast and Geodetic Survey in the computation and adjustment of triangulation. It will not only serve as a guide to the younger mathematician just learning to make triangulation computations but will tend to standardize the methods of computation and adjustment of triangulation so that greater efficiency and economy will result.

Beginning on page 215 will be found a number of suggestions and general rules which have been formulated as the result of many years of experience in the adjustment of triangulation. Those just starting work on such computations will find it helpful to study these rules and suggestions before attempting to study the volume as a whole.

Acknowledgment is gratefully made to C. H. Swick, Dr. O. S. Adams, mathematicians, and O. P. Sutherland, associate mathematician, of the division of geodesy of this bureau, and to R. N. Ashmun, mathematician of the International Boundary Commission, who have carefully reviewed the entire manuscript and offered many valuable suggestions.

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# MANUAL OF TRIANGULATION COMPUTATION AND ADJUSTMENT

By WALTER F. REYNOLDS, *Mathematician, United States Coast and Geodetic Survey*

## GENERAL STATEMENT

For many years most of the computations and adjustments of triangulation in the United States Coast and Geodetic Survey have been more or less standardized, but, as there have been no printed instructions for the work, the standard methods have not been used as consistently as is desirable. This publication, giving the methods of computation resulting from years of experience by the various mathematicians, will tend to more consistency and thus to greater accuracy and speed.

The theory of least squares as applied to the adjustment of triangulation is not covered in this publication, since that is fully treated in Special Publication No. 28, Application of the Theory of Least Squares to the Adjustment of Triangulation. Instead, there is shown, step by step, how the computation of the triangulation is carried on from the time the field observations are received in the office until the final results are published. Examples of each part of the computation are given. The methods used in the computation and adjustment of traverse are not shown in this publication, since they are contained in Special Publication No. 137, Manual of First-Order Traverse.

# CHAPTER 1.—PRELIMINARY COMPUTATIONS

## ABSTRACT OF HORIZONTAL DIRECTIONS

The instructions for field work require that the lists of directions giving the observed horizontal directions or angles shall be made out and checked in the field, so that ordinarily the office computations should begin with the checked directions, and the mathematician should not have to go back to the original field record books.

But occasionally, due to a rush of work or a shortage of personnel, the field directions are not checked in the field, and this must be done in the office. The method of computing the list of directions is therefore shown here. As the methods of forming the lists of directions for triangulation of the first and second order (formerly precise and primary) and for triangulation of the third order (formerly secondary) are somewhat different, examples of both methods are given.

Below is given a sample abstract of observed directions on triangulation of the first order, as received in the office, from which the list of directions is computed. This abstract and the instructions for making out the list of directions for first-order triangulation are taken from Special Publication No. 120, Manual of First-Order Triangulation.

It is important that this form be made out carefully, because the mean directions derived from the abstract of horizontal directions constitute the basis for all the later computations. Every position observed at a station, except observations on objects where only one or two positions are taken, should appear on the abstract, the rejected readings being indicated by the letter R. Sample forms are shown in Figures 1 and 2.

Where more than one station is used as an initial, there will frequently be different ways in which the observations can be combined to give the directions from some one initial station. Figures 1 and 2 will illustrate the proper way to form the combined direction in a number of typical cases.

At station Granite both South Base and Westedge were used as initials in the observations, but South Base was chosen as the initial for the list of directions. A supplemental abstract of directions, Figure 2, was first made out for the observations in which Westedge was used as initial, and the abstract, shown in Figure 1, was then made out for such observations as had South Base for initial. It was then necessary to transfer the observations made with Westedge

as initial to equivalent values with South Base as initial, marking such transferred directions with the letter T on the abstract to show their origin. For example, in Figure 2 the direction of Floyd from Westedge, position 1, is 271° 11′ 44″.0, while the direction of Westedge from South Base, position 1, is 17° 17′ 49″.5, and the sum of the two

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 470

ABSTRACT OF DIRECTIONS

State: Arizona

Station: Granite  Computed by: C. P. S.  Date: July 25–Aug. 6
Observer: C. V. H.  Checked by: W. F. R.  Incl. No.

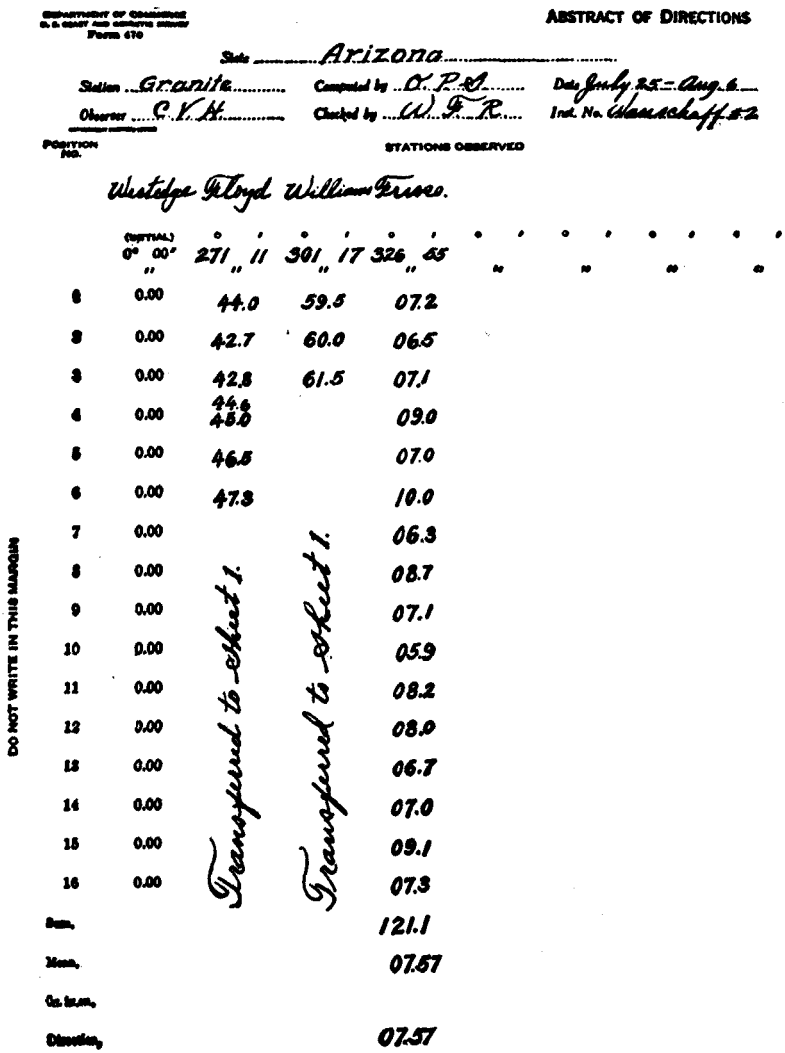
| POSITION NO. | STATIONS OBSERVED | | | | | | |
|---|---|---|---|---|---|---|---|
| | South Base (INITIAL) | Westedge | Union | Floyd | N. Base | Williams | Fraser |
| | 0° 00′ 00″ | 17° 17′ 17″ | 89° 58′ 89″ | 288° 29′ 288″ | 318° 34′ 318″ | 325° 54′ 325″ | 344° 12′ 344″ |
| 1 | 0.00 | 49.5 | 01.3 | 33.5T | 47.8 (41.6)R | 49.0 | 57.0 |
| 2 | 0.00 | 53.0 | 00.4 | 35.7T | 46.8 | 53.0T | 54.5 |
| 3 | 0.00 | 49.6 | 59.6 | 32.4T | 49.3 | 51.1T | 56.7 |
| 4 | 0.00 | 51.5 | 59.8 | 36.5T | 47.3 | 53.3 | 59.6 |
| 5 | 0.00 | 50.8 | 02.2 | 37.3T | 46.2 | 51.3 | 58.3 |
| 6 | 0.00 | 51.0 | 02.8 | 38.3T 36.6 } 37.4 | 48.0 (42.5)R | 51.8 (57.2)R | 60.7 |
| 7 | 0.00 | 48.2 | 58.5 | 32.8 | 45.8 | 52.8 | 56.5 |
| 8 | 0.00 | 50.4 | 03.0 | 36.8 | 46.8 | 52.2 | 60.0 |
| 9 | 0.00 | 52.2 | 05.0 | 35.5 | 45.8 | 53.0 | 54.5 |
| 10 | 0.00 | 52.0 | 00.5 | 36.2 | 47.7 | 52.8 | 56.7 |
| 11 | 0.00 | 50.5 | 00.6 | 35.7 | 46.4 | 52.2 | 55.7 |
| 12 | 0.00 | 51.7 | 00.7 | 35.3 | 48.3 | 53.8 | 59.7 |
| 13 | 0.00 | 50.5 | 02.5 | 32.3 | 48.0 | 51.0 | 56.5 |
| 14 | 0.00 | 50.7 | 03.8 | 35.4 | 47.2 | 54.0 | 59.2 |
| 15 | 0.00 | 48.8 | 59.2 | 34.2 | 47.0 | 52.8 | 59.6 |
| 16 | 0.00 | 51.7 | 04.6 | 33.7 | 47.0 | 52.2 | 57.8 |
| Sum, | | 812.1 | 24.5 | 560.5 | 755.1 | 836.3 | 923.0 |
| Mean, | | 50.76 | 01.53 | 35.03 | 47.19 | 52.27 | 57.69 |
| Cor. for ex., | | | −0.26 | | | | |
| Direction, | | 50.76 | 01.27 | 35.03 | 47.19 | 52.27 | 57.69 |

DO NOT WRITE IN THIS MARGIN

FIG. 1.—Sample abstract of directions

directions, 288° 29′ 33″.5, is the direction from South Base to Floyd, as shown for position 1, Figure 1. Similarly, the values for the other positions for Floyd and Williams are transferred from the supplemental abstract to the combined one, using for each position the corresponding value of the angle between South Base and Westedge.

It is not necessary to transfer Frisco from the supplemental abstract shown in Figure 2 to the combined abstract for the reason that a complete set was observed on that station from each initial. The mean of the directions on Frisco, with Westedge as initial, viz, 326° 55′ 07″.57, plus 17° 17′ 50″.76, the mean of the directions on Westedge



ABSTRACT OF DIRECTIONS

State Arizona

Station Granite — Computed by C. P. S. — Date July 25–Aug. 6

Observer C. V. H. — Checked by W. F. R. — Ind. No. Wanschaff 52

| POSITION NO. | (INITIAL) 0° 00′ | Westedge | Floyd | Williams | Frisco | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 271° 11′ | 301° 17′ | 326° 55′ | | | | | |
| 1 | 0.00 | 44.0 | 59.5 | 07.2 | | | | | |
| 2 | 0.00 | 42.7 | 60.0 | 06.5 | | | | | |
| 3 | 0.00 | 42.8 | 61.5 | 07.1 | | | | | |
| 4 | 0.00 | 44.6 45.0 | | 09.0 | | | | | |
| 5 | 0.00 | 46.5 | | 07.0 | | | | | |
| 6 | 0.00 | 47.3 | | 10.0 | | | | | |
| 7 | 0.00 | | | 06.3 | | | | | |
| 8 | 0.00 | Transferred to Sheet 1. | Transferred to Sheet 1. | 08.7 | | | | | |
| 9 | 0.00 | | | 07.1 | | | | | |
| 10 | 0.00 | | | 05.9 | | | | | |
| 11 | 0.00 | | | 08.2 | | | | | |
| 12 | 0.00 | | | 08.0 | | | | | |
| 13 | 0.00 | | | 06.7 | | | | | |
| 14 | 0.00 | | | 07.0 | | | | | |
| 15 | 0.00 | | | 09.1 | | | | | |
| 16 | 0.00 | | | 07.3 | | | | | |
| Sum. | | | | 121.1 | | | | | |
| Mean. | | | | 07.57 | | | | | |
| Co. to m. | | | | | | | | | |
| Direction. | | | | 07.57 | | | | | |

FIG. 2.—Sample abstract of directions for missing signals

with South Base as initial, gives 344° 12′ 58″.33. The mean of this value of the direction to Frisco and that obtained with South Base as initial, 344° 12′ 57″.69, is used in the list of directions shown in Figure 3. If more than 10 or 12 acceptable positions are obtained on any one night for a direction, that night should be given unit

weight with any other night in determining the mean direction. In general, where 10 or more positions of a direction have been measured on each of two or more nights no one night's observations should be rejected unless it is more than one-half second from the mean of all the values for that direction. If the divergence from the mean is greater than one-half second, that night's observations should be selected which will best close the triangles, provided that at least 12 acceptable positions are available for the retained direction.

It will be noted that since the angles were measured from South Base to Westedge, from Westedge to Frisco, and from South Base to Frisco, the proper value of the direction from South Base to the other two points could be secured most accurately by a least-squares adjustment (see pp. 8–16). In most cases, however, the results obtained by this station adjustment do not justify the time required to make the computation, but a mean value for the sum angles can usually be obtained by arbitrary methods which will meet sufficiently well the final demands for accuracy. When a number of sum angles are measured, however, and especially when the means obtained by different combinations vary considerably, a station adjustment may be made.

The direction to triangulation station Floyd, Figure 1, has two acceptable values for position 6. In such cases the mean is taken of all values for a position and that mean given unit weight in the final mean, on the theory that a symmetrical distribution of the readings around the circle is essential to accuracy. With an accurately graduated circle it is probable that the variation due to the graduation is not quite so large as that due to errors in reading, but the rule of unit weight for each position is the safest to follow as a uniform procedure.

### REJECTION OF OBSERVATIONS

The chief difficulty in making out the form lies in deciding what observations to reject. The usual formulæ for the rejection of observational quantities are too cumbersome to apply and are not satisfactorily applicable to a short series of observations. It is, therefore, customary to apply an arbitrary limit of rejection, determined empirically from previous experience with the instrument used or with one of similar qualities. For observations with the type of theodolite usually used on first-order triangulation the rejection limit for the angular value of a direction on any one position of the circle may ordinarily be taken as four seconds from the mean.

The following rules will be a sufficient guide to the rejection of observed directions:

1. No reading should be rejected if it falls within the limit of retention (in the sample this limit is $\pm 4''.0$ from the mean) unless rejected at the time of taking the observation. The observer's

reason for rejection should then appear in the original record. This rule will not apply to the case where one set of observations of a direction is rejected in favor of another set of 12 or more positions, as provided for on page 5.

2. If two or more readings have been taken for a single position, the mean should be used if all readings come within the limit of retention.

3. If one reading of a position falls without the limit and one within the limit, do not use a mean even though the mean be within the limit. Use instead the single reading within the limit.

4. If both readings of a position fall without the limit, reject the position entirely, using the remaining positions to compute the mean direction.

5. In case the 16 readings seem to fall in two groups, the mean of one group differing considerably from the mean of the other, extreme care is necessary in making the rejections.

6. Before computing a trial mean any observations so far from the approximate mean as to be very evidently the result of blunders should be rejected. After a trial mean is obtained and the rejection limit applied, the observations so rejected should not be again included even though the new mean would bring them within the limit of rejection.

7. The results obtained by applying rigorously the limit of rejection, even though the quantities rejected are just outside the limit, will probably be but little different from those derived after long consideration, and much time can be saved by a strict application of the rule.

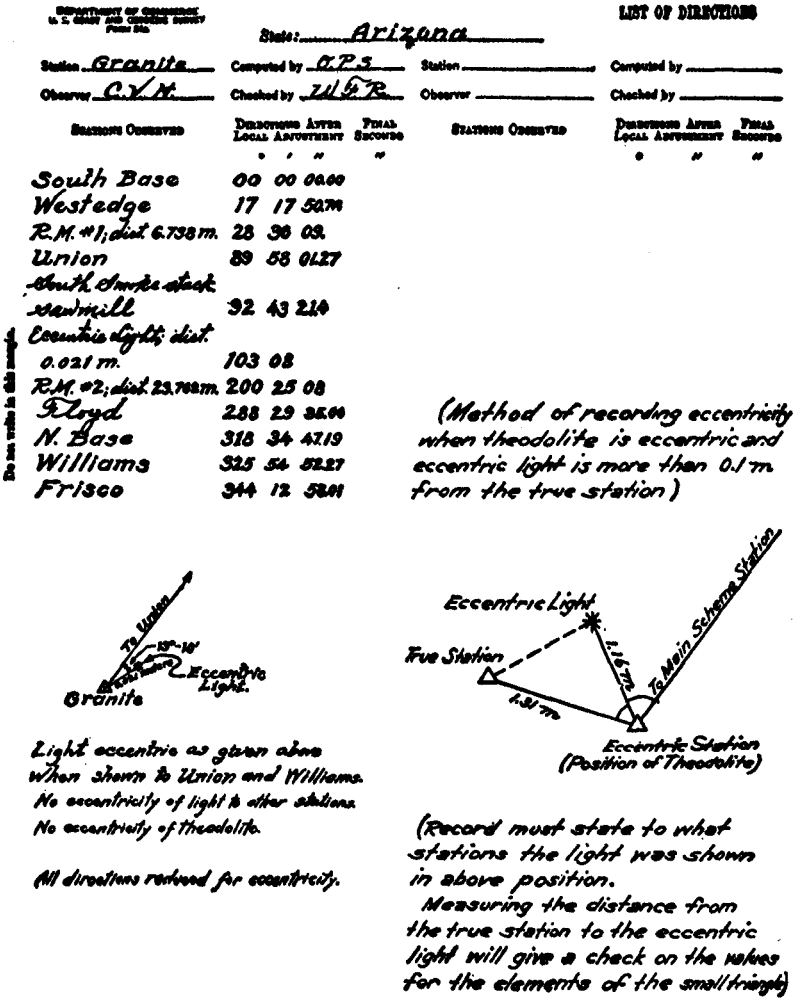
## LIST OF DIRECTIONS

On the list of directions, Figure 3, the mean directions of all unrejected observations are arranged in order of azimuth from some one selected initial. Not only the mean directions to the principal stations as listed and computed on the abstract of directions should be shown, but also the directions to intersection points and reference marks.

The data on this form constitute the material upon which the office computations are based, and these data should be so completely checked in the field that there will be no need in the office to resort to the record book or the abstract of directions. The only exception to this rule is where there is not sufficient time in the field to make all the eccentric reductions without delay to the progress of the party.

On the back of the form for the list of directions are instructions for its preparation. (See fig. 5.) Only two points covered by those instructions need be emphasized here, viz, the number of decimal places to be shown in the mean angle and the treatment of eccentric directions. As regards the first, on first-order triangulation the directions to main-scheme stations should be carried to hundredths

of a second, directions to second-order stations and to sharply defined permanently marked intersection stations to tenths of seconds, and directions to other points, such as mountain peaks, to seconds. Directions to near-by objects, such as witness or reference marks, need be taken to the nearest 10 seconds only. In general, two uncertain figures should be given; that is, the third digit from the right in the



LIST OF DIRECTIONS

State: Arizona

Station: Granite — Computed by: C.P.S.
Observer: C.V.N. — Checked by: W.G.R.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| South Base | 00 00 00.00 | |
| Westedge | 17 17 50.71 | |
| R.M. #1; dist. 6.738 m. | 28 36 03. | |
| Union | 89 58 01.27 | |
| South Smoke stack | | |
| sawmill | 92 43 21.4 | |
| Eccentric light; dist. 0.021 m. | 103 08 | |
| R.M. #2; dist. 23.768 m. | 200 25 08 | |
| Floyd | 288 29 35.00 | |
| N. Base | 318 34 47.19 | |
| Williams | 325 54 52.27 | |
| Frisco | 344 12 53.01 | |

(Method of recording eccentricity when theodolite is eccentric and eccentric light is more than 0.1 m. from the true station)



Light eccentric as given above when shown to Union and Williams.
No eccentricity of light to other stations.
No eccentricity of theodolite.

(All directions reduced for eccentricity.



(Record must state to what stations the light was shown in above position.
Measuring the distance from the true station to the eccentric light will give a check on the values for the elements of the small triangle)

FIG. 3.—Sample list of directions from horizontal directions

number denoting the direction should not be in error more than one unit.

The second point to be emphasized in the preparation of the list of directions is the computation of the eccentricity and the reduction of the observed directions to center. If a direction has not been reduced to center, the seconds pertaining to that direction should

not be written on the form in ink, but in pencil. This rule should be invariably followed, for otherwise an unreduced direction may be used for a reduced one.

### LIST OF DIRECTIONS FROM HORIZONTAL ANGLES

For making out the list of directions from observed horizontal angles in third-order triangulation, the method is slightly different from that used with observed horizontal directions. As shown below, the angles at the station are simply corrected for the closing of the horizon. Since each angle was observed in the same manner, its weight is unity, and the correction to each angle is obtained by dividing the difference between 360° and the sum of all the angles by the number of angles involved.

*Observed angles, Vance Mt.*

| Observed stations | Angle | | | Correction | Final seconds |
|---|---|---|---|---|---|
| | ° | ′ | ″ | ″ | ″ |
| Neal-Tomah Mt. | 22 | 18 | 41.5 | −0.5 | 41.0 |
| Tomah Mt.-Spruce Mt. | 59 | 51 | 12.0 | −.5 | 11.5 |
| Spruce Mt.-Mt. Henry | 116 | 54 | 37.1 | −.5 | 36.6 |
| Mt. Henry-Brandy Hill | 60 | 36 | 00.3 | −.5 | 59.8 |
| Brandy Hill-Oak | 45 | 23 | 46.2 | −.6 | 45.6 |
| Oak-Neal | 54 | 55 | 46.1 | −.6 | 45.5 |
| | 360 | 00 | 03.2 | −3.2 | 00.0 |

The corrections to close the horizon are usually applied in the Horizontal Angle Record Book, and the list of directions is made directly from that record on Form 24A, as shown in Figure 4. Complete instructions for making out the list of directions, which are given on the back of Form 24A, are given in Figure 5.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 24A

State: Maine

Station Vance Mt. Computed by O.P.S.

Observer C.V.H. Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment | Final Seconds |
|---|---|---|
| | ° ′ ″ | ″ |
| Neal | 0 00 00.0 | |
| Tomah Mt. | 22 18 41.0 | |
| Spruce Mt. | 82 09 52.5 | |
| Mt. Henry | 199 04 29.1 | |
| Brandy Hill | 259 40 28.9 | |
| Oak | 305 04 14.5 | |

FIG. 4.—Sample list of directions from horizontal angles

### STATION ADJUSTMENT, DIRECTION METHOD

Under the present system of observing, no local adjustments are necessary, and all the computations, such as the taking of means and the closing of the horizon, are made in the record book in the field. But as the mathematician has to deal occasionally with observations made a number of years ago, when it was the custom to measure as

many angles as possible, including sum angles, the method used in computing the list of directions at a station where local adjustment is necessary is given here.

As explained on page 8, the angles at each station must be corrected for the closing of the horizon. If no sum angles are observed, this is the only condition; but if sum angles are observed, new condi-

This form, properly filled out and checked, must be furnished by field parties. *To be acceptable it must contain every direction observed.*

It is to be used for observations with repeating theodolites, as well as direction theodolites.

Start each new station at the head of a new column.

If a repeating theodolite is used, do not abstract the angles in tertiary triangulation. The local adjustment corrections (to close horizon only) are to be written in the Horizontal Angle Record, and the List of Directions is to be made from that record directly.

Choose as an initial for Form 24A some station involved in the local adjustment, and preferably one which has been used as an initial for a round of directions on objects not in the main scheme. Use but one initial at a station. Call the direction of the initial 0° 00′ 00.″00, and by applying the corrected angles to this, fill in opposite each station its direction reckoned *clockwise* around the whole circumference regardless of the direction of graduation of the instrument. The clockwise reckoning is necessary for uniformity and to make the directions comparable with azimuths.

*State: Kansas.*

Station Chase — Computed by A. T. M.

Observer A. T. M. — Checked by A. R. L.

| Stations Observed | Directions After Local Adjustment | | | Final Seconds |
|---|---|---|---|---|
| | ° | ′ | ″ | ″ |
| Central | 0 | 00 | 00.00 | Do not write in this column. It is for Office computation only. |
| White church spire, 8 miles | 6 | 28 | 56.4 | |
| Chase M. E. church, white spire | 18 | 10 | 11.9 | |
| Little River | 18 | 20 | 10.78 | |
| Lyons, salt works, center hoist | 24 | 33 | 53.0 | |
| Lyons, white spire, short | 27 | 19 | 39.7 | |
| Lyons, courthouse | 27 | 55 | 34.2 | |
| Lyons, white spire, slim | 28 | 02 | 54.2 | |
| Gilmore | 53 | 32 | 33.44 | |
| Savage | 83 | 59 | 57.32 | |
| Reference mark distant 66.65 meters. | 171 | 34 | | |
| Section 3, T. 20, R. 10 W., NW. corner stone, distant 252.6 meters. | 290 | 37 | 36 | |
| Bossing | 314 | 52 | 23.61 | |

If a station has been occupied eccentrically, reduce to the center and enter in this form, in ink, the resulting directions at the center. If the reduction is not made for some directions, they should be entered in pencil, with a footnote to that effect.

Directions in the main scheme should be entered to hundredths of seconds in primary triangulation; otherwise, to tenths only. Points observed upon but once, direct and reverse, should be carried to tenths in primary and secondary triangulation, and in tertiary triangulation to even seconds only. In general, but two uncertain figures should be given.

It is recommended that the following simple plan of observing be used with a repeating instrument: Measure each single angle in the scheme at each station and the outside angle necessary to close the horizon. Measure no sum angles. Follow each measurement of every angle immediately by a measurement of its explement. Six repetitions are to constitute a measurement. The local adjustment will consist simply of the distribution of the error of closure of the horizon.

In next to the last paragraph above the designation "primary" should be changed to "first-order," "secondary" to "second-order" and "tertiary" to "third-order."

FIG. 5.—Back of Form 24A, giving instructions for making list of directions

tions are imposed on the station adjustment. If all angles are observed with the same accuracy, then the weight of each is unity; but when the angles are measured with different numbers of sets, they must be weighted accordingly. A set consists of six measures of the angle with the telescope direct and six measures of the explement of the angle with the telescope reversed.

In the example below the angles were measured with different numbers of sets, and so are weighted.

*Observed angles, Mag*

| Observed station | Angle | Weight $p$ | $v$ | Adjusted final seconds [1] |
|---|---|---|---|---|
| | ° ′ ″ | | | |
| Chamcook-Cooper | 115 58 09.8, 10.4, 10.0, 10.2 } 10.1 | 4 | $v_1$ | 10.1 |
| Cooper-Rye | 34 29 21.3, 20.5, 21.0 } 20.9 | 3 | $v_2$ | 20.8 |
| Rye-Middlemiss | 46 01 29.3, 28.1 } 28.7 | 2 | $v_3$ | 28.4 |
| Middlemiss-Collins | 80 07 16.5, 16.1 } 16.3 | 2 | $v_4$ | 16.8 |
| Rye-Mohannas | 16 16 06.7, 05.5 } 06.1 | 2 | $v_5$ | 06.3 |
| Anderson-Mohannas | 11 59 06.7, 06.1 } 06.4 | 2 | $v_6$ | 05.6 |
| Collins-Chamcook | 83 23 44.3, 43.5, 44.0 } 43.9 | 3 | $v_7$ | 43.9 |
| Rye-Collins | 126 08 44.8, 45.4, 45.0 } 45.1 | 3 | $v_8$ | 45.2 |
| Anderson-Middlemiss | 41 44 27.0, 26.6 } 26.8 | 2 | $v_9$ | 27.7 |
| Mohannas-Collins | 109 52 39.4, 39.7 } 39.6 | 2 | $v_{10}$ | 38.9 |

This column is filled out after the adjustment is completed. (See p. 16.)

*List of directions, Mag*

| Observed station | Direction | Adjusted final seconds [1] |
|---|---|---|
| | ° ′ ″ | |
| Chamcook | 0 00 00.0 | 00.0 |
| Cooper | 115 58 10.1+$v_1$ | 10.1 |
| Rye | 150 27 31.0+$v_1$+$v_2$ | 30.9 |
| Anderson | 154 44 30.7+$v_1$+$v_2$+$v_5$−$v_6$ | 31.6 |
| Mohannas | 166 43 37.1+$v_1$+$v_2$+$v_5$ | 37.2 |
| Middlemiss | 196 28 59.7+$v_1$+$v_2$+$v_3$ | 59.3 |
| Collins | 276 36 16.0+$v_1$+$v_2$+$v_3$+$v_4$ | 16.1 |

[1] This column is filled out after the adjustment is completed. (See p. 16.)

The complete list of directions has been formed using six of the angles, the remaining four not being necessary. As each of these angles not used gives rise to a condition, there will be four conditions. The equations expressing these conditions are formed as follows:

| | | ° | ′ | ″ |
|---|---|---|---|---|
| Angle Collins–Chamcook, observed, | = | 83 | 23 | $43.9+v_7$ |
| Angle Collins–Chamcook, from list, | = | 83 | 23 | $44.0-v_1-v_2-v_3-v_4$ |
| Condition 1, | | $0=-0.1+v_1+v_2+v_3+v_4+v_7$ | | |
| Angle Rye–Collins, observed, | = | 126 | 08 | $45.1+v_8$ |
| Angle Rye–Collins, from list, | = | 126 | 08 | $45.0+v_3+v_4$ |
| Condition 2, | | $0=+0.1-v_3-v_4+v_8$ | | |
| Angle Anderson–Middlemiss, observed, | = | 41 | 44 | $26.8+v_9$ |
| Angle Anderson–Middlemiss, from list, | = | 41 | 44 | $29.0+v_3-v_5+v_6$ |
| Condition 3, | | $0=-2.2-v_3+v_5-v_6+v_9$ | | |
| Angle Mohannas–Collins, observed, | = | 109 | 52 | $39.6+v_{10}$ |
| Angle Mohannas–Collins, from list, | = | 109 | 52 | $38.9+v_3+v_4-v_5$ |
| Condition 4, | | $0=+0.7-v_3-v_4+v_5+v_{10}$ | | |

After the condition equations are formed, they are tabulated in correlates as shown below.

*Correlate equations*

| | $\frac{a}{p}$ | 1 | 2 | 3 | 4 | $\Sigma_c$ | $v$ * | Adopted $v$ † |
|---|---|---|---|---|---|---|---|---|
| 1 | 3 | +1 | ------- | ------- | ------- | +1 | −0.024 | 0.0 |
| 2 | 4 | +1 | ------- | ------- | ------- | +1 | −.032 | −.1 |
| 3 | 6 | +1 | −1 | −1 | −1 | −2 | −.342 | −.3 |
| 4 | 6 | +1 | −1 | ------- | −1 | −1 | +.510 | +.5 |
| 5 | 6 | ------- | ------- | +1 | +1 | +2 | +.168 | +.2 |
| 6 | 6 | ------- | ------- | −1 | ------- | −1 | −.852 | −.8 |
| 7 | 4 | +1 | ------- | ------- | ------- | +1 | −.032 | .0 |
| 8 | 4 | ------- | +1 | ------- | ------- | +1 | +.084 | +.1 |
| 9 | 6 | ------- | ------- | +1 | ------- | +1 | +.852 | +.9 |
| 10 | 6 | ------- | ------- | ------- | +1 | +1 | −.684 | −.7 |

* This column is filled out after the adjustment is completed. (See p. 15.)
† See explanation of this column on p. 15.

In the second column above, headed $\frac{a}{p}$, $a$ is some constant, and $p$ is the weight of a given $v$. It is best to take $a$ as the least common multiple of all the weights, so as to make all the values in this column integers, provided this can be done without making the quantities too large. Thus, in the example above $a=12$. The values of the $p$'s are given in the table on page 10. The quantities in the column headed $\Sigma_c$ are obtained by adding across algebraically the quantities in columns 1, 2, 3, and 4, in the same horizontal line.

After forming the correlate equations, the normal equations are formed as shown in the table below:

*Normal equations*

| | 1 | 2 | 3 | 4 | $\eta$ | $\Sigma_n$ | $C$ * |
|---|---|---|---|---|---|---|---|
| 1 | +23 | −12 | −6 | −12 | −0.1 | −7.1 | −0.0075 |
| 2 | | +16 | +6 | +12 | +.1 | +22.1 | +.0206 |
| 3 | | | +24 | +12 | −2.2 | +33.8 | +.1417 |
| 4 | | | | +24 | +.7 | +36.7 | −.1140 |

* See p. 13 for values in this column.

The normal equations are obtained by taking the algebraic sums of $\frac{a}{p}$ times the products of the various columns in the correlates. For example, normal equation No. 1 may be expressed as: $\Sigma\left(\frac{a}{p}\cdot 1\cdot 1\right) + \Sigma\left(\frac{a}{p}\cdot 1\cdot 2\right) + \Sigma\left(\frac{a}{p}\cdot 1\cdot 3\right) + \Sigma\left(\frac{a}{p}\cdot 1\cdot 4\right) + \eta_1 + \left[\Sigma\left(\frac{a}{p}\cdot 1\cdot \Sigma_c\right) + \eta_1\right]$, normal equation No. 2 as $\Sigma\left(\frac{a}{p}\cdot 2\cdot 2\right) + \Sigma\left(\frac{a}{p}\cdot 2\cdot 3\right) + \Sigma\left(\frac{a}{p}\cdot 2\cdot 4\right) + \eta_2 + \left[\Sigma\left(\frac{a}{p}\cdot 2\cdot \Sigma_c\right) + \eta_2\right]$, normal equation No. 3 as $\Sigma\left(\frac{a}{p}\cdot 3\cdot 3\right) + \Sigma\left(\frac{a}{p}\cdot 3\cdot 4\right) + \eta_3 + \left[\Sigma\left(\frac{a}{p}\cdot 3\cdot \Sigma_c\right) + \eta_3\right]$, and normal equation No. 4 as $\Sigma\left(\frac{a}{p}\cdot 4\cdot 4\right) + \eta_4 + \left[\Sigma\left(\frac{a}{p}\cdot 4\cdot \Sigma_c\right) + \eta_4\right]$, in which the $\Sigma$ before each parenthesis indicates the sum of the products in the parentheses and should be distinguished from the $\Sigma_c$ and the $\Sigma_n$ of the preceding tables. $\eta$ is the constant term for the corresponding condition equation.

In each symbolized normal equation above, the part in the square brackets should equal the corresponding $\Sigma_n$ in the preceding table, the $\Sigma_n$ being the sum of the values in columns 1, 2, 3, 4, and $\eta$, in the same horizontal line, including the omitted terms as explained below. This gives a check on the formation of the normals. In obtaining the $\Sigma_n$ it must be remembered that due to symmetry, as explained below, certain coefficients have been omitted in the preceding table of normal equations and that these must be taken into consideration. The term in square brackets of the first normal equation is $\left[\Sigma\left(\frac{a}{p}\cdot 1\cdot \Sigma_c\right) + \eta_1\right] = -7.1$. Adding the coefficients of the terms in the first normal equation as given in the table we have $+23-12-6-12-0.1=-7.1$, which checks the formation of this equation.

In the same way the term in square brackets of the second normal equation $\left[\Sigma\left(\frac{a}{p}\cdot 2\cdot \Sigma_c\right) + \eta_2\right] = 22.1$, and this checks the sum of the coefficients of the terms in this equation, $-12+16+6+12+0.1=+22.1$. The third and fourth normal equations are checked in the same way.

In the preceding table of normal equations the coefficients occuring before what is called the "diagonal term" are omitted, as the equations are symmetrical with regard to the diagonal line shown in the table below. Thus the table above, if written in full, would be as follows:

*Normal equations in full*

| | 1 | 2 | 3 | 4 | $\eta$ | $\Sigma_n$ |
|---|---|---|---|---|---|---|
| 1 | +23 | −12 | −6 | −12 | −0.1 | −7.1 |
| 2 | −12 | +16 | +6 | +12 | +0.1 | +22.1 |
| 3 | −6 | +6 | +24 | +12 | −2.2 | +33.8 |
| 4 | −12 | +12 | +12 | +24 | +0.7 | +36.7 |

It can be readily seen from this table that all the coefficients to the left of the diagonal line may be omitted and each equation may be read from the top down to the diagonal term and then across the page.

### SOLUTION OF NORMAL EQUATIONS

In the solution of the normal equations the Doolittle method is used. As a full discussion of this method is given in Adjustment of Observations, by Wright and Hayford, second edition, page 114 et seq., no attempt is made here to discuss it or give any of the theory concerning it. A complete solution of the preceding normal equations is given below, followed by an explanation of the computation.

*Forward solution*

| 1 | 2 | 3 | 4 | $\eta$ | $\Sigma_n$ |
|---|---|---|---|---|---|
| +23 | −12 | −6 | −12 | −0.1 | −7.1 |
| $C_1$ | +.5217 | +.2609 | +.5217 | +.0043 | +.3087 |
| | +16 | +6 | +12 | +.1 | +22.1 |
| 1 | −6.260 | −3.131 | −6.260 | −.052 | −3.703 |
| | +9.740 | +2.869 | +5.740 | +.048 | +18.397 |
| | $C_2$ | −.2946 | −.5893 | −.0049 | −1.8888 |
| | | +24 | +12 | −2.2 | +33.8 |
| | 1 | −1.565 | −3.130 | −.026 | −1.852 |
| | 2 | −.845 | −1.691 | −.014 | −5.419 |
| | | +21.590 | +7.179 | −2.240 | +26.529 |
| | | $C_3$ | −.3325 | +.1038 | −1.2287 |
| | | | +24 | +.7 | +36.7 |
| | | 1 | −6.260 | −.052 | −3.703 |
| | | 2 | −3.383 | −.028 | −10.842 |
| | | 3 | −2.387 | +.745 | −8.821 |
| | | | +11.970 | +1.365 | +13.334 |
| | | | $C_4$ | −.1140 | −1.1140 |

*Back solution (computation of C's)*

| 4 | 3 | 2 | 1 |
|---|---|---|---|
| −0.1140 | +0.1038 | −0.0049 | +0.0043 |
| | +.0379 | +.0672 | −.0595 |
| | | −.0417 | +.0370 |
| | +0.1417 | | +.0107 |
| | | +0.0206 | |
| | | | −0.0075 |

EXPLANATION OF SOLUTION

The forward solution is computed as follows:

Normal equation No. 1 is written down and divided by its diagonal term, +23, all the signs being changed. $C_1$ is thus given in terms of $C_2$, $C_3$, $C_4$, and the constant term. Normal equation No. 2 is next set down, and since it has a coefficient of −12 for $C_1$, the divided coefficients of equation No. 1 are multiplied by −12, and the products are placed under equation No. 2. The quantities in each column are then added algebraically and are divided by the new diagonal term, +9.740, all the signs being changed. $C_2$ is thus given in terms of $C_3$, $C_4$, and the constant term. Normal equation No. 3 is then written down, and since it has a coefficient of −6 for $C_1$ and +2.869 for $C_2$, the divided coefficients of equation No. 1 are multiplied by −6 and those of equation No. 2 by +2.869, giving products which are set down under equation No. 3. The quantities in each column are added algebraically and these sums divided by the new diagonal term, all the signs being changed. $C_3$ is thus given in terms of $C_4$ and the constant term. Normal equation No. 4 is then set down, and since it has a coefficient of −12 for $C_1$, +5.740 for $C_2$, and +7.179 for $C_3$, the divided coefficients of equation No. 1 are multiplied by −12, those of equation No. 2 by +5.740, and those of equation No. 3 by +7.179. The quantities in each column are then added algebraically and these sums divided by the new diagonal term, all the signs being changed. The value of $C_4$ is thus obtained.

As can readily be seen from the forward solution, $C_3 = -0.3325C_4 + 0.1038$; $C_2 = -0.2946C_3 - 0.5893C_4 - 0.0049$; and $C_1 = +0.5217C_2 + 0.2609C_3 + 0.5217C_4 + 0.0043$. These $C$'s can be obtained most conveniently by arranging the back solution in the form shown on page 13. There will be as many columns as there are $C$'s to be determined, and they will be headed in reverse order from the forward solution. Thus, in this particular solution, the columns will be headed 4, 3, 2, and 1.

On the first line is written the constant term for each divided equation of the forward solution; that is, the quantities in the column headed $\eta$. Then the value of $C_4$, −0.1140, is multiplied into each of the quantities of the divided equations in the column headed 4 of the forward solution, these products being placed in the second line of the back solution beginning with the column headed 3. The quantities in the column headed 3 of the back solution are then added algebraically to give $C_3$. The value of $C_3$, +0.1417, is then multiplied into each of the quantities of the divided equations in the column headed 3 of the forward solution, these products being placed in the third line of the back solution, beginning with the column headed 2. The quantities in the column headed 2 of the back solution are then added algebraically to give $C_2$. The value of

$C_2$, $+0.0206$, is multiplied into the quantity of the divided equation in the column headed 2 of the forward solution, this product being placed in the fourth line of the back solution in the column headed 1. The quantities in the column headed 1 of the back solution are then added algebraically to give $C_1$.

### COMPUTATION OF $v$'s

After the $C$'s are determined, the next step is to compute the $v$'s by substituting the values of the $C$'s in the correlate equations tabulated on page 11, taking into account the weights as shown in the column headed $\frac{a}{p}$. It can be easily seen from this table that $v_1=3C_1$, $v_2=4C_1$, $v_3=6\ (C_1-C_2-C_3,-C_4)$, $v_4=6\ (C_1-C_2-C_4)$, $v_5=6\ (C_3+C_4)$, $v_6=-6C_3$, $v_7=4C_1$, $v_8=4C_2$, $v_9=6C_3$, and $v_{10}=6C_4$.

The $v$'s are best obtained by means of a table as shown below, which has a column for each $v$. The values of $C_1$, $C_2$, $C_3$, and $C_4$ are placed in the different columns to correspond with the set of correlate equations on page 11. These are then added algebraically in each column, and each sum is then multiplied by the corresponding weight for that $v$ to give the final $v$.

As the constant terms of the condition equations are carried to the nearest tenth of a second, it is customary to round off the $v$'s to the nearest tenth of a second. Occasionally, when the $v$'s are substituted in the condition equations, one or more equations may fail by a tenth of a second due to this dropping of hundredths of seconds in the $v$'s.

For instance, if in condition equation (1) we substitute the values of the $v$'s computed to the nearest tenth of a second, we have $0=-0.1+0.0+0.0-0.3+0.5+0.0$, or $0=+0.1$; and in equation (3) we have $0=-2.2+0.3+0.2+0.9+0.9$, or $0=+0.1$.

In order that the equations may check exactly and the results be consistent, it is customary to adopt a set of $v$'s, a few of which may not be the same to the nearest tenth of a second as the computed values. These adopted $v$'s are shown at the bottom of each column in the table of $v$'s below. In this set of adopted $v$'s, $v_2$ has been given the value $-0.1$ instead of 0.0, and $v_6$ $-0.8$ instead of $-0.9$, in order that equations (1) and (3) may be satisfied.

*Computation of v's*

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| −0.008<br>3 | −0.008<br>4 | −0.008<br>−.021<br>−.142 | −0.008<br>−.021<br>+.114 | +0.142<br>−.114 | −0.142<br>6 | −0,008<br>4 | +0.021<br>4 | +0.142<br>6 | −0.114<br>6 |
| −.024<br>.0 | −.032<br>−.1 | +.114<br>−.057<br>6 | +.085<br>6 | +.028<br>6 | −.852<br>−.8 | −.032<br>.0 | +.084<br>+.1 | +.852<br>+.9 | −.684<br>−.7 |
| | | −.342<br>−.3 | +.510<br>+.5 | +.168<br>+.2 | | | | | |

The adopted values of the $v$'s are substituted in the table of observed angles and in the list of directions on page 10 to give the values in the column headed "Adjusted final seconds" in each table.

It will be found now that if the list of directions is formed from the corrected observed angles, it will be consistent; that is, the direction at each station will be the same no matter from which angles it is computed.

The list of directions after the local adjustment should appear in the following form:

*List of directions, Mag*

| Observed station | Direction after local adjustment | | | Final seconds |
|---|---|---|---|---|
| | ° | ′ | ″ | |
| Chamcook | 0 | 00 | 00.0 | |
| Cooper | 115 | 58 | 10.1 | |
| Rye | 150 | 27 | 30.9 | |
| Anderson | 154 | 44 | 31.6 | |
| Mohannas | 166 | 43 | 37.2 | |
| Middlemiss | 196 | 28 | 59.3 | |
| Collins | 276 | 36 | 16.1 | |

## STATION ADJUSTMENT, ANGLE METHOD, OBSERVATIONS OF EQUAL WEIGHT

The method of station adjustment explained on pages 8–16 is used in cases of first-order triangulation, and particularly where the observations are of unequal weight and the adjustment is involved. When the observations are of equal weight and the number of sum angles observed is not great, the adjustment can be much simplified by using the method shown in the example below:

In the record book of horizontal angles, at the end of the observations of angles for each station, there is always given an abstract of the observed angles. Where the station adjustment is made by the angle method, the corrections are applied directly to the observed angles in this abstract, and the list of directions is made directly from the abstract.

The following example illustrates the method. The abstract below is found in the record book:

*Station Cora*

| Observed stations | Observed angle | | | Final seconds[1] |
|---|---|---|---|---|
| | ° | ′ | ″ | ″ |
| (1) Decision—Mac | 16 | 11 | 53.6+$v_1$ | 53.5 |
| (2) Decision—Nation | 99 | 42 | 50.6+$v_2$ | 50.9 |
| (3) Mac—Nation | 83 | 30 | 57.6+$v_3$ | 57.4 |
| (4) Howard—Nation | 74 | 11 | 11.0+$v_4$ | 11.1 |
| (5) Mac—Howard | 9 | 19 | 46.1+$v_5$ | 46.3 |
| (6) Nation—Decision | 260 | 17 | 08.8+$v_6$ | 09.1 |

[1] This column is filled in after the adjustment is completed. See p 18.

In the above table $v_1$, $v_2$, $v_3$, $v_4$, $v_5$, and $v_6$ represent corrections to be applied to the angles to make them consistent in themselves. Angles (2) and (3) are sum angles, and angle (6) gives a horizon closure with (2). There are, therefore, 3 condition equations, which are formed in the following manner:

| | | |
|---|---|---|
| 1st | $(1)-(2)+(3)$ | gives $0=+0.6+v_1-v_2+v_3$. |
| 2d | $-(3)+(4)+(5)$ | gives $0=-0.5-v_3+v_4+v_5$. |
| 3d | $(2)+(6)$ | gives $0=-0.6+v_2+v_6$. |

The correlate and normal equations below are formed in the same manner, as explained on pages 11–13. The normals are solved and the corrections computed in the manner explained on pages 13–16.

*Correlate equations*

| | 1 | 2 | 3 | $\Sigma_c$ | $v$* | Adopted $v$† |
|---|---|---|---|---|---|---|
| 1 | +1 | -------- | -------- | +1 | −0.062 | −0.1 |
| 2 | −1 | -------- | +1 | 0 | +.331 | +.3 |
| 3 | +1 | −1 | -------- | 0 | −.208 | −.2 |
| 4 | -------- | +1 | -------- | +1 | +.146 | +.1 |
| 5 | -------- | +1 | -------- | +1 | +.146 | +.2 |
| 6 | -------- | -------- | +1 | +1 | +.269 | +.3 |

* This column is filled in after the adjustment is completed.
† See explanation of this column on p. 15.

*Normal equations*

| | 1 | 2 | 3 | $\eta$ | $\Sigma_n$ | $C$ |
|---|---|---|---|---|---|---|
| 1 | +3 | −1 | −1 | +0.6 | +1.6 | −0.0616 |
| 2 | | +3 | -------- | −.5 | +1.5 | +.1462 |
| 3 | | | +2 | −.6 | +.4 | +.2692 |

*Solution of normal equations*

| 1 | 2 | 3 | $\eta$ | $\Sigma_n$ |
|---|---|---|---|---|
| +3 | −1 | −1 | +0.6 | +1.6 |
| $C_1$ | +.33333 | +.33333 | −.2 | −.53333 |
| | +3 | | −.5 | +1.5 |
| 1 | −.3333 | −.3333 | +.2 | +.5333 |
| | +2.6667 | −.3333 | −.3 | +2.03334 |
| | $C_2$ | +.125 | +.1125 | −.7625 |
| | | +2 | −.6 | +.4 |
| | 1 | −.3333 | +.2 | +.5333 |
| | 2 | −.0417 | −.0375 | +.2542 |
| | | +1.6250 | −.4375 | +1.1875 |
| | | $C_3$ | +.26923 | −.73077 |

*Back solution*

| 3 | 2 | 1 |
|---|---|---|
| +0.26923 | +0.1125<br>+.0337 | −0.2<br>+.0897<br>+.0487 |
| | +.1462 | −.0616 |

*Computation of corrections (v's)*

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| −0.062 | +0.062<br>+.269 | −0.062<br>−.146 | +0.146 | +0.146 | +0.269 |
| −.062<br>−.1 | +.331<br>+.3 | −.208<br>−.2 | +.146<br>+.1 | +.146<br>+.2 | +.269<br>+.3 |

After the final seconds are placed on the observed angles in the abstract in the record book, the directions are made out on form 24A. (See fig. 6.)

DEPARTMENT OF COMMERCE
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FORM 24A

*State:*.....

Station Cora  Computed by O. P. S.

Observer C.V.H.  Checked by W. F. R.

| STATIONS OBSERVED | DIRECTIONS AFTER LOCAL ADJUSTMENT ° | ′ | ″ | FINAL SECONDS ″ |
|---|---|---|---|---|
| Decision | 0 | 00 | 00.0 | |
| Mac | 16 | 11 | 53.5 | |
| Howard | 25 | 31 | 39.8 | |
| Nation | 99 | 42 | 50.9 | |

FIG. 6.—List of directions resulting from station adjustment by angle method

## REDUCTION TO CENTER

When a station is not occupied centrally, the directions or angles observed at the eccentric point must be corrected to what they would have been if the instrument had been centered over the station mark. Also, if the signal observed upon is eccentric, the directions and angles involving this station must be corrected to what they would have been if the station itself had been observed upon. The computation for the reduction to center is made on Form 382. The instructions given on that form are repeated below, and examples of the two different cases of eccentricity are shown.

First are given a list of directions for a station occupied eccentrically (fig. 7) and the computations necessary to obtain the corrections for reducing the directions to center.

### INSTRUCTIONS

The required reduction to center is, in seconds, $c=\frac{d \sin a}{s \sin 1''}$, in which $d$ is the distance from the eccentric station to the true station, and $s$ is the length in meters of the line between the true stations involved, and, therefore, log $s$ is taken directly from the computation of triangle sides. $a$ is the direction of the distant station involved, reckoned in a clockwise direction as usual but referred to the direction from the eccentric to the true station, or center, taken as zero. This definition of $a$ is true for the case in which the object pointed upon is eccentric, as well as for the case in which the instrument is eccentric.

DEPARTMENT OF COMMERCE
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FORM 24A

State:

LIST OF DIRECTIONS

Station Ken Ecc. Computed by O.P.S.
Observer C.V.H. Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° | ′ | ″ | Final Seconds Cor. "for eccentr. | Stations Observed | Directions After Local Adjustment ° | ′ | ″ | Final Seconds ″ |
|---|---|---|---|---|---|---|---|---|---|
| Chevy | 0 | 00 | 00.00 | | Chevy | 0 | 00 | 00.00 | |
| Tank west of Δ Dulce | 29 | 03 | 37.0 | -62.5 | Tank west of Δ Dulce | 29 | 02 | 34.5 | |
| Ken (center) 3.469 m. | 176 | 42 | | | Forest Glen Standpipe | 313 | 28 | 01.5 | |
| Forest Glen Standpipe | 313 | 24 | 53.0 | +188.5 | Home | 326 | 32 | 09.4 | |
| Home | 326 | 31 | 30.21 | +39.2 | Bu. of Stand. wireless pole | 352 | 17 | 33.8 | |
| Bu. of Stand. wireless pole | 352 | 17 | 20.8 | +13.0 | Reno | 357 | 28 | 54.78 | |
| Reno | 357 | 28 | 48.63 | +6.15 | | | | | |

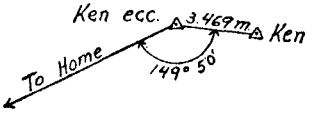


FIG. 7.—Lists of directions before and after reduction for eccentric station

Carry $a$ to minutes only and all logarithms to five decimal places only. Do not in any case carry the derived reductions to more than two decimal places. There is no advantage in carrying them to more decimal places than the directions to which they are to be applied are carried on Form 24A.

### REDUCTIONS FOR AN ECCENTRIC INSTRUMENT

If the instrument is eccentric, the first column of Form 382 should contain the names of the stations observed from that eccentric position of the instrument.

The values in the fifth column are derived by subtracting those in the fourth column from those in the third. The values in the

fourth column may need to be derived by successive approximations from the triangle side computations if the eccentric reductions are large. The values in the sixth column are obtained from those in the fifth by adding log $\frac{d}{\sin 1''}$, derived as indicated in the heading of the form, if $d$ is expressed in meters. If $d$ is expressed in feet, to the other two logarithms add also 9.48402 to convert to meters. To obtain a direction as shown on Form 24A, subtract the reduction $c$ for the station which is the initial on Form 24A from the reduction $c$ for the required direction and apply the difference to the observed direction. Similarly, the correction to any angle is the difference of the reductions on this form to the two directions involved in that angle.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 302

**REDUCTION TO CENTER**

*Eccentric Station:* Ken

$d =$ 3.469 meters

Log $d$ = 0.54020
Colog sin 1″ = 5.3 1 4 4 3
Sum = 5.8 5 4 6 3

| STATIONS | a ° | ′ | Log sin a | Log s | Log sin a / s | Logarithm of Reduction in Seconds | Reduction = c ″ |
|---|---|---|---|---|---|---|---|
| Center | 0 | 00 | | | | | |
| Chevy | 183 | 18 | 8.76015 | 3.75097 | 5.00918 | 0.86391 | -7.31 |
| Tank west of Δ Dulce | 212 | 22 | 9.72863 | 3.73984 | 5.98953 | 1.84488 | -69.86 |
| Forest Glen standpipe | 136 | 43 | 9.83608 | 3.43240 | 6.40368 | 2.25861 | +181.29 |
| Home | 149 | 50 | 9.70115 | 4.05161 | 5.64955 | 1.50696 | +31.91 |
| Bu. of Standards wireless pole | 175 | 35 | 8.88654 | 3.98487 | 4.90167 | 0.75630 | +5.71 |
| Reno | 180 | 47 | 8.13581 | 3.92635 | 4.20945 | 0.06418 | -1.16 |

FIG. 8.—Reduction to center of eccentric station

In order to compute the corrections to the directions due to eccentricity, it is necessary to know the logarithm of the distance from the station itself to each of the other stations. For this purpose preliminary triangles are computed. (See fig. 9.) The logarithms of the distances from Ken to Home, Reno, and Chevy are computed from the triangles Ken-Home-Reno and Ken-Home-Chevy. In computing these triangles it is not necessary to use the angles at Ken eccentric, as in each triangle the other two angles are known, and the concluded angles at Ken may be computed and used. This method of computing the logarithms of the lengths will give values which are more nearly the true values than if the eccentric angles were used.

However, as the other three stations observed from Ken eccentric, namely, "Tank near Dulce," "Forest Glen Standpipe," and "Bureau of Standards wireless pole," were not occupied, it is necessary to use

the eccentric angles at Ken for the preliminary triangles used in computing the logarithms of the lengths.

After the logarithms of the lengths have been computed, all the data necessary to compute the eccentric corrections are known, and the computation can be made as shown in Figure 8.

The corrections thus determined are now applied to the corresponding directions observed at Ken eccentric, as shown in Figure 7. As it is desired to keep the initial direction (in this case to station Chevy) 0° 00′ 00″.0, the correction at Chevy is subtracted algebraically from each of the other corrections and these differences applied to the other directions. The following corrections are therefore applied: At Chevy, 0″.00; at Tank, west of Dulce, $-69''.86-(-7''.31)=-62''.5$; at Forest Glen Standpipe, $+181''.26-(-7''.31)=+188''.6$; at Home, $+31''.91-(-7''.31)=+39''.22$; at Bureau of Standards wireless pole, $+5''.71-(-7''.31)=+13''.0$; at Reno, $-1''.16-(-7''.31)=+6''.15$. The corrected directions are shown in Figure 7.

When the eccentric corrections are large, the logarithms of the lengths computed by the use of the eccentric angles are usually not sufficiently accurate to give the exact corrections. In this case the triangles must be recomputed by using the corrected list of directions (see p. 19) and more accurate logarithms of the lengths obtained. These logarithms are then used to compute new eccentric corrections which are applied to the directions in the list of directions to give the final corrected directions. Ordinarily the first computation of the eccentric corrections is sufficiently accurate, but occasionally two and sometimes three computations are required.

### REDUCTIONS FOR AN ECCENTRIC OBJECT OBSERVED

If the object observed is eccentric the heading "Eccentric station ——" on Form 382, should be changed to "Eccentric observed object at station ——," the first column should contain the names of the stations from which this eccentric object was observed, and in each case *a* is the direction from the eccentric object to the distant station involved, reckoned in a clockwise direction as usual but referred to the direction from the eccentric object to the true station, or center, taken as zero. (No distinction need be made between the direction from the eccentric object to the distant station and the direction from the true station to the distant station except when the eccentric reduction is more than one minute.) The remainder of the computation on Form 382 is made in the manner indicated above with reference to an eccentric instrument. The reductions to directions are, however, to be applied to observed directions, at the stations named in the first column, to the eccentric object at the station named in the heading. The directions to which these reduc-

DEPARTMENT OF COMMERCE
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FORM 23

## COMPUTATION OF TRIANGLES

*State:* Maryland

| | NO. | STATIONS | OBSERVED ANGLE | | | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLES AND DISTANCES | LOGARITHM |
|---|---|---|---|---|---|---|---|---|---|---|
| | 2-3 | Home-Reno | | | | | | | | 3.772214 |
| c | 1 | Ken | 30 | 56 | 45.3 ~~55.6~~ | +3.5 | 48.8 | | | 31 0.288~~80~~ |
| | 2 | Home | 47 | 09 | 57.7 | +3.4 | 61.1 | | | 304 9.865~~298~~ |
| | 3 | Reno | 101 | 53 | 06.7 | +3.4 | 10.1 | | | 7 9.990~~58~~ |
| | 1-3 | Ken-Reno | | | | | | | | 49 3.926~~330~~ |
| | 1-2 | Ken-Home | | | | | | | | 32 4.051~~610~~ |
| | | | | | 49.7 | | | | | |
| | 2-3 | Home-Chevy | | | | | | | | 3.860903 |
| c | 1 | Ken | 33 | 27 | 50.6 ~~57.8~~ | +2.3 | 52.9 | | | 15 0.258~~501~~ |
| | 2 | Home | 25 | 20 | 54.6 | +2.2 | 56.8 | | | 78 9.631~~569~~ |
| | 3 | Chevy | 121 | 11 | 08.1 | +2.2 | 10.3 | | | 4 9.932~~217~~ |
| | 1-3 | Ken-Chevy | | | | | | | | 96 3.750~~975~~ |
| | 1-2 | Ken-Home | | | | | | | | 32 4.051~~621~~ |
| | | | | | 53.3 | | | | | |
| | 2-3 | Ken-Reno | | | | | | | | 49 3.926~~330~~ |
| c | 1 | Tank near Dulce (111 | 07 ~~06~~ | 53.7 ~~46.2~~ ) | | | | | | 232 0.030~~175~~ |
| | 2 | Ken | 31 | 33 ~~34~~ | 39.7 ~~47.3~~ | | | | | 8839 9.719~~970~~ |
| | 3 | Reno | 37 | 18 | 26.6 | | | | | 9.782538 |
| | 1-3 | Tank-Reno | | | | | | | | 420 3.675~~560~~ |
| | 1-2 | Tank-Ken | | | | | | | | 119 3.739~~650~~ |
| | 2-3 | Reno-Ken | | | | | | | | 49 3.926~~330~~ |
| c | 1 | Forest Glen Standpipe (119 | 49 ~~46~~ | 22.6 ~~19.0~~ ) | | | | | | 697 0.061~~476~~ |
| | 2 | Reno | 16 | 09 | 44.2 | | | | | 9.444605 |
| | 3 | Ken | 44 | 00 ~~05~~ | 53.2 ~~56.0~~ | | | | | 1837 9.842~~897~~ |
| | 1-3 | Forest Glen Standpipe - Ken | | | | | | | | 651 3.432~~401~~ |
| | 1-2 | Forest Glen Standpipe - Reno | | | | | | | | 29933 3.830~~083~~ |
| | 2-3 | Reno-Ken | | | | | | | | 49 3.926~~330~~ |
| c | 1 | Bu. of Stand. wireless pole | (31 | 22 | 14.8 ~~06.0~~ ) | | | | | 17 0.283~~545~~ |
| | 2 | Reno | 143 | 26 | 24.2 | | | | | 9.775001 |
| | 3 | Ken | 5 | 11 | 21.0 ~~29.0~~ | | | | | 381 8.956~~565~~ |
| | 1-3 | Wireless pole-Ken | | | | | | | | 7 3.984~~865~~ |
| | 1-2 | Wireless pole-Reno | | | | | | | | 247 3.166~~481~~ |

FIG. 9.—Preliminary computation of triangles for reduction of eccentric station

tions are to be applied are therefore found in various of the lists of directions on Form 24A, not all in one list, as is the case when the instrument is eccentric.

On page 24 is given an example of a computation for reduction to center, when the observed object is eccentric as seen from several other

DEPARTMENT OF COMMERCE
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FORM 24A

LIST OF DIRECTIONS

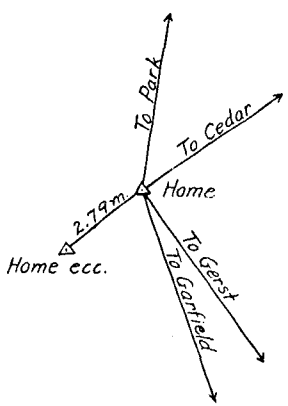
State: Maryland

Station Home — Computed by O.P.S.
Observer C.V.H. — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Park | 0 00 00.0 | |
| Cedar | 47 07 49.6 | |
| Gerst | 137 42 15.9 | |
| Garfield | 152 25 42.7 | |
| Home Ecc. 2.79 m. | 222 55 20. | |

Station Park — Computed by O.P.S.
Observer C.L.G. — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Cedar | 0 00 00.0 | |
| Wallace | 44 45 54.8 | |
| Home | 116 24 38.6 | |
| Home Ecc | 116 26 12.1 | |
| Reno | 173 33 28.9 | |




Station Cedar — Computed by O.P.S.
Observer C.L.G. — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Wallace | 0 00 00.0 | |
| Home | 43 02 10.6 | |
| Home Ecc | 43 02 07.4 | |
| Takoma | 58 59 17.6 | |
| Park | 59 29 28.5 | |

Station Gerst — Computed by O.P.S.
Observer C.L.G. — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Garfield | 0 00 00.0 | |
| Stanton | 12 13 22.0 | |
| Home | 46 21 38.0 | |
| Home Ecc | 46 20 53.2 | |

Station Garfield — Computed by O.P.S.
Observer C.L.G. — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Insane | 0 00 00.0 | |
| Home | 62 58 17.2 | |
| Home Ecc | 62 57 25.9 | |
| Stanton | 88 40 09.0 | |
| Potomac | 93 11 08.1 | |

FIG. 10.—Lists of directions for eccentric observed object

stations. Station Home is eccentric as seen from stations Park, Cedar, Gerst, and Garfield.

In Figure 10 there is given a list of directions at station Home, showing directions to stations Park, Cedar, Gerst, Garfield, and Home eccentric, and also lists of directions at stations Park, Cedar,

Gerst, and Garfield, at which station Home is seen eccentrically. The computation is made in the same way as where the station is occupied eccentrically. The directions used in the computation are obtained from the lists of directions at station Home, using Home eccentric as initial, remembering, however, that 180° must be added to these directions, since the directions are taken at the station itself and not at the eccentric station.

As in the case of an eccentric instrument, the approximate logarithms of the lengths are obtained by a preliminary computation of triangles. As the details of this computation are given on pages 20–21, it is not necessary to repeat them here.

After the eccentric corrections are determined, they are applied, not to the directions observed at station Home, but to the direction Home eccentric in each of the lists of directions for stations Park, Cedar, Gerst, and Garfield. (See fig. 10.)

DEPARTMENT OF COMMERCE
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FORM 202

**REDUCTION TO CENTER**

observed object at
*Eccentric Station:* Home

$d =$ 2.79 meters

Log $d$ = 0. 44 5 6 0
Colog sin 1″ = 5. 3 1 4 4 3
Sum = 5. 76 0 0 3

| STATIONS | $a$ ° | | Log sin $a$ | Log $s$ | Log $\frac{\sin a}{s}$ | Logarithm of Reduction in Seconds | Reduction $-\sigma$ ″ |
|---|---|---|---|---|---|---|---|
| Center | 0 | 00 | | | | | |
| Cedar | 4 | 12 | 8.86474 | 4.12189 | 4.74285 | 0.50288 | +3.18 |
| Gerst | 94 | 46 | 9.99850 | 4.10683 | 5.89167 | 1.65170 | +44.84 |
| Garfield | 109 | 30 | 9.97435 | 4.02427 | 5.95008 | 1.71011 | +51.30 |
| Park | 317 | 06 | 9.83297 | 3.62200 | 6.21097 | 1.97100 | -93.54 |

FIG. 11.—Reduction to center of eccentric observed object

## REDUCTION OF HORIZONTAL DIRECTIONS TO SEA LEVEL

In case the elevation of a triangulation station is very great, a correction must be applied to the observations upon that station to reduce them to sea level. It is only necessary to compute this correction for triangulation of the first order, as it usually amounts to only a few hundredths of a second. The correction, expressed in seconds, is

$$\frac{e^2 h \sin 2\alpha \cos^2 \phi}{2\rho \sin 1''},$$

where $e^2 = \frac{a^2 - b^2}{a^2}$, $h$ = the height of the station observed and $\alpha$ = its azimuth reckoned in a clockwise direction from south, $\rho$ = the radius of curvature of the earth in a plane normal to the meridian at the station from which the direction is measured, and $\phi$ = the latitude of the station from which the direction is measured.

Below is given an example of the computations required to obtain the corrections by means of the formula. The computations are arranged in a table for convenience.

*Computation of sea-level corrections using formula*

[Station Bull, latitude 48° 20′]

| Observed station | $h$ | $\alpha$ | | $2\alpha$ | | Log sin $2\alpha$ | Log $h$ | Log $\frac{e^2 \cos^2 \phi}{2\rho \sin 1''}$ | Log correction | Correction |
|---|---|---|---|---|---|---|---|---|---|---|
| | *m.* | ° | ′ | ° | ′ | | | | | ″ |
| Snake | 813 | 50 | 45 | 101 | 30 | 9.991 | 2.910 | 5.684 | 8.585 | +0.04 |
| Gladys | 761 | 221 | 52 | 83 | 44 | 9.997 | 2.881 | 5.684 | 8.562 | +.04 |
| Bonetrail | 740 | 239 | 21 | 118 | 43 | 9.943 | 2.869 | 5.684 | 8.496 | +.03 |
| Williston | 723 | 293 | 57 | 227 | 54 | 9.870 | 2.859 | 5.684 | 8.413 | −.03 |
| Buford | 756 | 353 | 42 | 347 | 24 | 9.339 | 2.879 | 5.684 | 7.902 | −.01 |

Since $\phi$ is a constant for a particular station, $e^2$ does not vary, and $\rho$ does not vary enough for any given latitude to affect the result, the same value of the factor $\frac{e^2 \cos^2 \phi}{2\rho \sin 1''}$ can be used for all the directions at any given station. The values of $h$ are obtained from the vertical angle computations, and the values of $\phi$ and $\alpha$ are obtained from the geographic position computations which, necessarily, must be made before the sea-level corrections can be determined. To obtain the corrections to the directions it is necessary to add the sum of log $h$ and log sin $2\alpha$ for each direction to log $\frac{e^2 \cos^2 \phi}{2\rho \sin 1''}$ which, as was stated, is the same for all directions at any given station. The sign of each correction is determined by the sign of sine $\alpha$.

Since the sea-level correction is always small and since a large part of the above formula may be considered a constant for a given station, it is possible to make use of a table which greatly facilitates the computations. Such a table is given on page 26.

*Correction to horizontal direction for elevation of mark*

[Corr. $= +\frac{e^2 h}{2\rho \sin 1''} \cos^2 \phi \sin 2\alpha$; $\frac{e^2}{2\rho \sin 1''} \cos^2 \phi \sin 2\alpha$ is tabulated below for the *sixth decimal place.* It is to be multiplied by $h$ in meters. The sign of the correction is + for azimuths in the first and third quadrants and − for azimuths in the second and fourth quadrants.]

| | Azimuth of direction | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ° | ° | ° | ° | ° | ° | ° | ° | ° | ° | ° | ° | ° | ° | ° |
| + | 0 | 5 | 10 | 12.5 | 15 | 17.5 | 20 | 22.5 | 25 | 27.5 | 30 | 32.5 | 35 | 40 | 45 |
| | 90 | 85 | 80 | 77.5 | 75 | 72.5 | 70 | 67.5 | 65 | 62.5 | 60 | 57.5 | 55 | 50 | 45 |
| − | 90 | 95 | 100 | 102.5 | 105 | 107.5 | 110 | 112.5 | 115 | 117.5 | 120 | 122.5 | 125 | 130 | 135 |
| | 180 | 175 | 170 | 167.5 | 165 | 162.5 | 160 | 157.5 | 155 | 152.5 | 150 | 147.5 | 145 | 140 | 135 |
| + | 180 | 185 | 190 | 192.5 | 195 | 197.5 | 200 | 202.5 | 205 | 207.5 | 210 | 212.5 | 215 | 220 | 225 |
| | 270 | 265 | 260 | 257.5 | 255 | 252.5 | 250 | 247.5 | 245 | 242.5 | 240 | 237.5 | 235 | 230 | 225 |
| − | 270 | 275 | 280 | 282.5 | 285 | 287.5 | 290 | 292.5 | 295 | 297.5 | 300 | 302.5 | 305 | 310 | 315 |
| | 360 | 355 | 350 | 347.5 | 345 | 342.5 | 340 | 337.5 | 335 | 332.5 | 330 | 327.5 | 325 | 320 | 315 |
| $\phi$ | | | | | | | | | | | | | | | |
| 20 | 0 | 17 | 33 | 41 | 48 | 55 | 62 | 68 | 74 | 79 | 84 | 88 | 91 | 95 | 97 |
| 21 | 0 | 17 | 33 | 40 | 48 | 55 | 61 | 67 | 73 | 78 | 83 | 86 | 90 | 94 | 95 |
| 22 | 0 | 16 | 32 | 40 | 47 | 54 | 60 | 67 | 72 | 77 | 81 | 85 | 88 | 93 | 94 |
| 23 | 0 | 16 | 32 | 39 | 46 | 53 | 60 | 66 | 71 | 76 | 80 | 84 | 87 | 91 | 93 |
| 24 | 0 | 16 | 31 | 39 | 46 | 52 | 59 | 65 | 70 | 75 | 79 | 83 | 86 | 90 | 91 |
| 25 | 0 | 16 | 31 | 38 | 45 | 52 | 58 | 64 | 69 | 74 | 78 | 81 | 84 | 89 | 90 |
| 26 | 0 | 15 | 30 | 37 | 44 | 51 | 57 | 63 | 68 | 72 | 77 | 80 | 83 | 87 | 88 |
| 27 | 0 | 15 | 30 | 37 | 43 | 50 | 56 | 61 | 67 | 71 | 75 | 79 | 82 | 86 | 87 |
| 28 | 0 | 15 | 29 | 36 | 43 | 49 | 55 | 60 | 65 | 70 | 74 | 77 | 80 | 84 | 85 |
| 29 | 0 | 15 | 29 | 35 | 42 | 48 | 54 | 59 | 64 | 69 | 73 | 76 | 79 | 82 | 84 |
| 30 | 0 | 14 | 28 | 35 | 41 | 47 | 53 | 58 | 63 | 67 | 71 | 74 | 77 | 81 | 82 |
| 31 | 0 | 14 | 28 | 34 | 40 | 46 | 52 | 57 | 62 | 66 | 70 | 73 | 76 | 79 | 80 |
| 32 | 0 | 14 | 27 | 33 | 39 | 45 | 51 | 56 | 60 | 64 | 68 | 71 | 74 | 78 | 79 |
| 33 | 0 | 13 | 26 | 33 | 38 | 44 | 49 | 54 | 59 | 63 | 67 | 70 | 72 | 76 | 77 |
| 34 | 0 | 13 | 26 | 32 | 38 | 43 | 48 | 53 | 58 | 62 | 65 | 68 | 71 | 74 | 75 |
| 35 | 0 | 13 | 25 | 31 | 37 | 42 | 47 | 52 | 56 | 60 | 64 | 67 | 69 | 72 | 73 |
| 36 | 0 | 12 | 25 | 30 | 36 | 41 | 46 | 51 | 55 | 59 | 62 | 65 | 67 | 71 | 72 |
| 37 | 0 | 12 | 24 | 30 | 35 | 40 | 45 | 49 | 53 | 57 | 60 | 63 | 66 | 69 | 70 |
| 38 | 0 | 12 | 23 | 29 | 34 | 39 | 44 | 48 | 52 | 56 | 59 | 62 | 64 | 67 | 68 |
| 39 | 0 | 12 | 23 | 28 | 33 | 38 | 42 | 47 | 51 | 54 | 57 | 60 | 62 | 65 | 66 |
| 40 | 0 | 11 | 22 | 27 | 32 | 37 | 41 | 45 | 49 | 53 | 56 | 58 | 60 | 63 | 64 |
| 41 | 0 | 11 | 21 | 26 | 31 | 36 | 40 | 44 | 48 | 51 | 54 | 56 | 59 | 61 | 62 |
| 42 | 0 | 10 | 21 | 26 | 30 | 35 | 39 | 43 | 46 | 50 | 52 | 55 | 57 | 60 | 60 |
| 43 | 0 | 10 | 20 | 24 | 29 | 34 | 38 | 41 | 45 | 48 | 51 | 53 | 55 | 58 | 59 |
| 44 | 0 | 10 | 19 | 24 | 28 | 32 | 36 | 40 | 43 | 46 | 49 | 51 | 53 | 56 | 57 |
| 45 | 0 | 10 | 19 | 23 | 27 | 31 | 35 | 39 | 42 | 45 | 47 | 50 | 51 | 54 | 55 |
| 46 | 0 | 9 | 18 | 22 | 26 | 30 | 34 | 37 | 40 | 43 | 46 | 48 | 50 | 52 | 53 |
| 47 | 0 | 9 | 17 | 22 | 25 | 29 | 33 | 36 | 39 | 42 | 44 | 46 | 48 | 50 | 51 |
| 48 | 0 | 9 | 17 | 21 | 25 | 28 | 31 | 35 | 38 | 40 | 42 | 44 | 46 | 48 | 49 |
| 49 | 0 | 8 | 16 | 20 | 24 | 27 | 30 | 33 | 36 | 39 | 41 | 43 | 44 | 46 | 47 |
| 50 | 0 | 8 | 15 | 19 | 23 | 26 | 29 | 32 | 35 | 37 | 39 | 41 | 42 | 45 | 45 |
| 51 | 0 | 8 | 15 | 18 | 22 | 25 | 28 | 31 | 33 | 36 | 38 | 39 | 41 | 43 | 43 |
| 52 | 0 | 7 | 14 | 18 | 21 | 24 | 27 | 29 | 32 | 34 | 36 | 38 | 39 | 41 | 41 |
| 53 | 0 | 7 | 14 | 17 | 20 | 23 | 25 | 28 | 30 | 32 | 34 | 36 | 37 | 39 | 40 |
| 54 | 0 | 7 | 13 | 16 | 19 | 22 | 24 | 27 | 29 | 31 | 33 | 34 | 36 | 37 | 38 |
| 55 | 0 | 6 | 12 | 15 | 18 | 21 | 23 | 25 | 28 | 29 | 31 | 33 | 34 | 35 | 36 |
| 56 | 0 | 6 | 12 | 14 | 17 | 20 | 22 | 24 | 26 | 28 | 30 | 31 | 32 | 34 | 34 |
| 57 | 0 | 6 | 11 | 14 | 16 | 19 | 21 | 23 | 25 | 27 | 28 | 29 | 31 | 32 | 32 |
| 58 | 0 | 5 | 11 | 13 | 15 | 18 | 20 | 22 | 24 | 25 | 27 | 28 | 29 | 30 | 31 |
| 59 | 0 | 5 | 10 | 12 | 15 | 17 | 19 | 21 | 22 | 24 | 25 | 26 | 27 | 29 | 29 |
| 60 | 0 | 5 | 9 | 12 | 14 | 16 | 18 | 19 | 21 | 22 | 24 | 25 | 26 | 27 | 27 |
| 61 | 0 | 4 | 9 | 11 | 13 | 15 | 17 | 18 | 20 | 21 | 22 | 23 | 24 | 25 | 26 |
| 62 | 0 | 4 | 8 | 10 | 12 | 14 | 15 | 17 | 18 | 20 | 21 | 22 | 23 | 23 | 24 |
| 63 | 0 | 4 | 8 | 10 | 11 | 13 | 14 | 16 | 17 | 18 | 20 | 20 | 21 | 22 | 23 |
| 64 | 0 | 4 | 7 | 9 | 11 | 12 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 21 |
| 65 | 0 | 3 | 7 | 8 | 10 | 11 | 13 | 14 | 15 | 16 | 17 | 18 | 18 | 19 | 20 |
| 66 | 0 | 3 | 6 | 8 | 9 | 10 | 12 | 13 | 14 | 15 | 16 | 16 | 17 | 18 | 18 |
| 67 | 0 | 3 | 6 | 7 | 8 | 10 | 11 | 12 | 13 | 14 | 14 | 15 | 16 | 16 | 17 |
| 68 | 0 | 3 | 5 | 6 | 8 | 9 | 10 | 11 | 12 | 13 | 13 | 14 | 14 | 15 | 15 |
| 69 | 0 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 12 | 13 | 13 | 14 | 14 |
| 70 | 0 | 2 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 10 | 11 | 12 | 12 | 13 | 13 |
| 71 | 0 | 2 | 4 | 5 | 6 | 7 | 7 | 8 | 9 | 10 | 10 | 11 | 11 | 11 | 12 |
| 72 | 0 | 2 | 4 | 4 | 5 | 6 | 7 | 7 | 8 | 9 | 9 | 9 | 10 | 10 | 10 |
| 73 | 0 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 7 | 8 | 8 | 8 | 9 | 9 | 9 |
| 74 | 0 | 1 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 7 | 7 | 8 | 8 | 8 | 8 |
| 75 | 0 | 1 | 3 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 6 | 7 | 7 | 7 | 7 |

The arguments used in this table are the latitude of the station at the left and the azimuth of the direction at the top. For convenience the computation should be made in tabular form as shown below.

*Computation of sea-level corrections using table*

[Station Bull, latitude 48° 20′]

| Observed station | $h$ | $\alpha$ | | $\frac{e^2 \sin 2\alpha \cos^2\phi}{2\rho \sin 1''}$ | Correction |
|---|---|---|---|---|---|
| | *m.* | ° | ′ | | ″ |
| Snake | 813 | 50 | 45 | 0.000047 | +0.04 |
| Gladys | 761 | 221 | 52 | .000048 | +.04 |
| Bonetrail | 740 | 239 | 21 | .000042 | +.03 |
| Williston | 723 | 293 | 57 | .000036 | −.03 |
| Buford | 756 | 353 | 42 | .000011 | −.01 |

In the first and second columns are given the name and height of the station observed, in the third column the azimuth of the observed station from the occupied station, in the fourth column the factor

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 24A

LIST OF DIRECTIONS

*State:* North Dakota

| Station Bull | Computed by O.P.S. | Station Bull | Computed by O.P.S. |
|---|---|---|---|
| Observer C.V.H. | Checked by W.F.R. | Observer C.V.H. | Checked by W.F.R. |

| Stations Observed | Directions After Local Adjustment | Final Seconds | Stations Observed | Directions After Local Adjustment | Final Seconds |
|---|---|---|---|---|---|
| | ° ′ ″ | ″ | | ° ′ ″ | ″ |
| Williston | 0 00 ~~00.00~~ 59.97 | | Williston | 0 00 00.00 | |
| Buford | 59 45 10.08 | 5 | Buford | 59 45 10.08 | |
| Snake | 116 48 35.32 | 6 | Snake | 116 48 35.39 | |
| Gladys | 287 55 21.13 | 6 | Gladys | 287 55 21.19 | |
| Bonetrail | 305 24 33.80 | 3 | Bonetrail | 305 24 33.86 | |

FIG. 12.—List of directions corrected for sea-level reduction

$\frac{e^2 \sin 2\alpha \cos^2 \phi}{2\rho \sin 1''}$ as taken from the table on page 26, and in the last column the sea level correction which is obtained by multiplying the factor in the fourth column by the height in the second column. The correction is plus for azimuths in the first (0° to 90°) and third (180° to 270°) quadrants and minus for azimuths in the second (90° to 180°) and fourth (270° to 360°) quadrants.

After the sea level corrections have been determined, they are applied to the corresponding directions at station Bull. The list of directions is then rewritten to make the reading of the initial station, Williston, 0° 00′ 00″.00. (See fig. 12.)

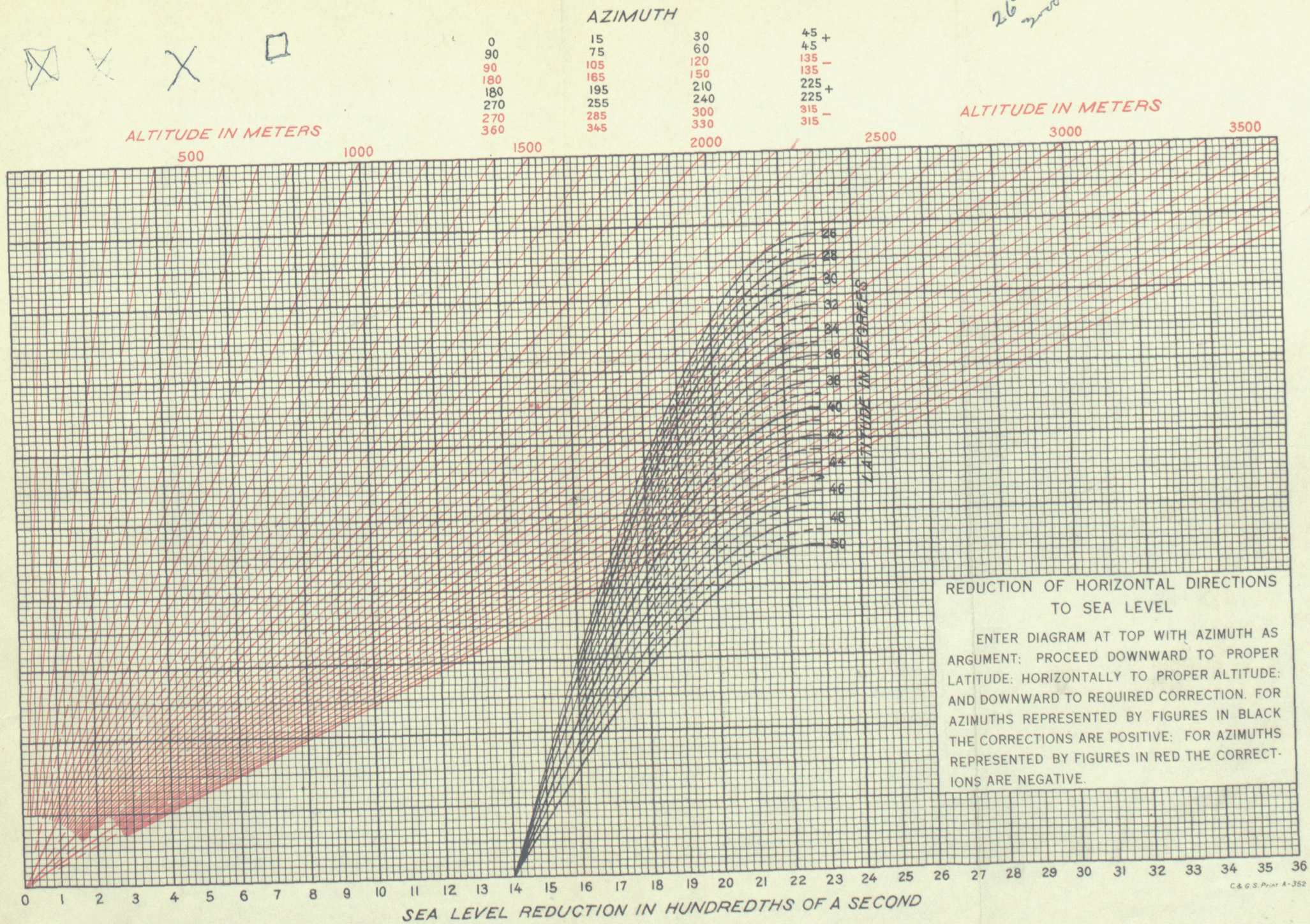
The sea-level corrections may also be determined by means of the nomogram shown in Figure 13 which was designed by H. S. Rappleye, associate mathematician of the division of geodesy of this bureau. Directions for using this nomogram are given on the figure.

## LIST OF DIRECTIONS, ALL PRELIMINARY CORRECTIONS APPLIED

After all corrections have been applied to the observed directions or angles as the case may be, the list of directions is made out on form 24A, as shown in Figure 12. The directions are arranged by giving the initial direction a value of 0° 00′ 00.″00 and continuing in a clockwise direction around the horizon.

The list of directions should then be checked and initialed by the checker before it is used in taking out the angles for the triangles.

26° max mtn




REDUCTION OF HORIZONTAL DIRECTIONS TO SEA LEVEL

ENTER DIAGRAM AT TOP WITH AZIMUTH AS ARGUMENT: PROCEED DOWNWARD TO PROPER LATITUDE: HORIZONTALLY TO PROPER ALTITUDE: AND DOWNWARD TO REQUIRED CORRECTION. FOR AZIMUTHS REPRESENTED BY FIGURES IN BLACK THE CORRECTIONS ARE POSITIVE: FOR AZIMUTHS REPRESENTED BY FIGURES IN RED THE CORRECTIONS ARE NEGATIVE.

# CHAPTER 2.—ADJUSTMENT OF A QUADRILATERAL

## SKETCH

Before starting the adjustment of a net of triangulation the mathematician should make a good clear sketch showing all the lines over which observations were made. The unobserved directions should be shown by dotted lines. A sketch of a typical quadrilateral is shown in Figure 14. In this figure C and A can not be seen from D.

## TRIANGLES

After the figure is drawn the triangles should be written out in *clockwise* order on Form 25. In the quadrilateral above with the line AB fixed the four triangles should be written as follows starting at station C: CAB, DAB, DAC, and DBC; or starting with station D, DAB, CDA, CDB, and CAB. That is, at each station not on the fixed line all the triangles formed by connecting it with stations on the fixed line or previously named stations should be written in clockwise order.

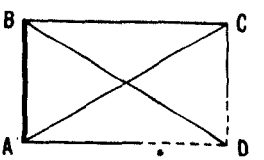


FIG. 14.—Typical quadrilateral

After the local conditions, that is, those arising from the relations of the angles at each station to one another, are satisfied (see pp. 8–18) there are general conditions arising from the geometrical relations of the various parts forming a closed figure which must be satisfied.

To illustrate the method of adjusting triangulation, it seems well to start with a simple quadrilateral, and give in detail the various steps of the adjustment. The adjustment of a larger figure or net of triangulation, involving all the various conditions which enter into such an adjustment, is shown on pages 50–109.

In the sample given below, a quadrilateral of first-order triangulation was selected for illustrating the methods. The adjustment of triangulation of the lower orders is similar except that the angles, lengths, and logarithms are not carried to as many decimal places. In the adjustment of triangulation of the first order, the angles are carried to hundredths of seconds, and the logarithms are carried to eight places in the equations and to seven places in the final lengths used in the triangles.

There are given below the lists of directions for stations Roman, Spencer, Yellow, and Fairview, the four stations making up the sample quadrilateral. These directions are assumed to have been corrected for sea-level reduction and for any local adjustment required and to have been checked. In the adjustment, the method of

directions is used; that is, an angle is considered as the difference of two directions. The geographic positions (latitudes and longitudes) of stations Roman and Spencer are considered fixed and also the length and azimuth of the line joining them.

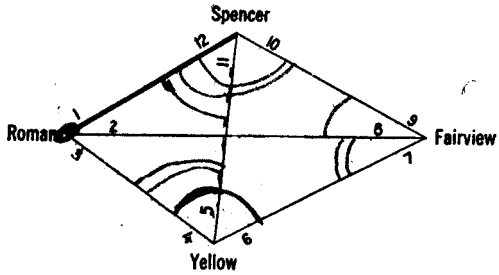



FIG. 15.—Quadrilateral used for sample adjustment

The sketch showing the relative positions of the stations is drawn and the directions numbered as shown in Figure 15. It is not necessary to spend much time in making the sketch absolutely to scale, although it should be approximately correct in order to give an idea of the relative size of the angles, the sketch being used as an aid in forming the equations.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 24A

State: Oregon

LIST OF DIRECTIONS

Station Roman — Computed by W.F.R. — Observer O.B.French — Checked by O.P.S.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Spencer | 0 00 00.00 | 59.96 |
| Fairview | 31 04 11.58 | 11.86 |
| Yellow | 65 12 45.72 | 45.48 |
| Mary | 291 34 34.04 | |
| Peterson | 321 25 23.53 | |
| Twin | 330 41 33.42 | |

Station Spencer — Computed by W.F.R. — Observer O.B.French — Checked by O.P.S.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Peterson | 0 00 00.00 | |
| Twin | 1 08 19.38 | |
| Fairview | 131 12 05.23 | 04.91 |
| Yellow | 197 25 26.30 | 26.80 |
| Roman | 251 46 38.49 | 38.31 |
| Ranch | 270 37 26.91 | |
| Willamette South Base | 311 51 09.89 | |
| Mary | 318 12 01.16 | |
| Ridge | 319 15 00.47 | |
| Willamette North Base | 328 26 41.12 | |

Station Yellow — Computed by W.F.R. — Observer O.B.French — Checked by O.P.S.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| White | 0 00 00.00 | |
| Roman | 178 40 38.63 | 38.69 |
| Spencer | 239 06 47.80 | 47.52 |
| Fairview | 297 46 09.74 | 09.96 |
| Scott | 337 15 43.07 | |

Station Fairview — Computed by W.F.R. — Observer O.B.French — Checked by O.P.S.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| White | 0 00 00.00 | |
| Scott | 23 26 03.35 | |
| Yellow | 54 53 23.69 | 23.22 |
| Roman | 81 39 24.54 | 24.90 |
| Spencer | 110 00 45.96 | 46.06 |
| Black | 310 44 55.62 | |

FIG. 16.—Lists of directions for stations of quadrilateral

The triangles are then written out in clockwise order on Form 25 as described on page 29. (See fig. 17.)

## EXPLANATION OF TRIANGLES

In the first column of Form 25 is given the designation of the angle, in the second the name of the station, in the third the observed angle at the station, in the fourth the correction to the angle as determined

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Oregon

| 1 | 2 | | 3 | | | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|---|
| NO. | STATIONS | | OBSERVED ANGLE | | | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLES AND DISTANCES | LOGARITHM |
| | 2-3 | Roman-Spencer | | | | | | | | 4.7176307 |
| -4+5 | 1 | Yellow | 60 | 26 | 09.17 | -0.34 | 08.83 | 1.95 | 06.88 | 0.0605813 |
| -1+3 | 2 | Roman | 65 | 12 | 45.72 | -0.20 | 45.52 | 1.96 | 43.56 | 9.9580218 |
| -11+12 | 3 | Spencer | 54 | 21 | 12.19 | -0.68 | 11.51 | 1.95 | 09.56 | 9.9098872 |
| | 1-3 | Yellow-Spencer | | | | -1.22 | | 5.86 | | 4.7362338 |
| | 1-2 | Yellow-Roman | | | | | | | | 4.6880992 |
| | | | | | 07.08 | | | | | |
| | 2-3 | Yellow-Roman | | | | | | | | 4.6880992 |
| -7+8 | 1 | Fairview | 26 | 46 | 00.85 | +0.83 | 01.68 | 2.19 | $\overline{59.49}$ | 0.3464441 |
| -4+6 | 2 | Yellow | 119 | 05 | 31.11 | +0.16 | 31.27 | 2.19 | 29.08 | 9.9414345 |
| -2+3 | 3 | Roman | 34 | 08 | 34.14 | -0.52 | 33.62 | 2.19 | 31.43 | 9.7491538 |
| | 1-3 | Fairview-Roman | | | | +0.47 | | 6.57 | | 4.9759778 |
| | 1-2 | Fairview-Yellow | | | | | | | | 4.7836971 |
| | | | | | 06.10 | | | | | |
| | 2-3 | Yellow-Spencer | | | | | | | | 4.7362338 |
| -7+9 | 1 | Fairview | 55 | 07 | 22.27 | +0.57 | 22.84 | 2.39 | 20.45 | 0.0859876 |
| -5+6 | 2 | Yellow | 58 | 39 | 21.94 | +0.50 | 22.44 | 2.39 | 20.05 | 9.9314862 |
| -10+11 | 3 | Spencer | 66 | 13 | 21.07 | +0.82 | 21.89 | 2.39 | 19.50 | 9.9614757 |
| | 1-3 | Fairview-Spencer | | | | +1.89 | | 7.17 | | 4.7537076 |
| | 1-2 | Fairview-Yellow | | | | | | | | 4.7836971 |
| | | | | | 05.28 | | | | | |
| | 2-3 | Roman-Spencer | | | | | | | | 4.7176307 |
| -8+9 | 1 | Fairview | 28 | 21 | 21.42 | -0.26 | 21.16 | 2.15 | 19.01 | 0.3233636 |
| -1+2 | 2 | Roman | 31 | 04 | 11.58 | +0.32 | 11.90 | 2.15 | 09.75 | 9.7127133 |
| -10+12 | 3 | Spencer | 120 | 34 | 33.26 | +0.14 | 33.40 | 2.16 | 31.24 | 9.9349835 |
| | 1-3 | Fairview-Spencer | | | | +0.20 | | 6.46 | | 4.7537076 |
| | 1-2 | Fairview-Roman | | | | | | | | 4.9759778 |
| | | | | | 06.26 | | | | | |

FIG. 17.—Triangle computation for stations of quadrilateral

by the adjustment, in the fifth the adjusted spherical angle at the station, in the sixth the spherical excess or difference between the plane and spherical angle, in the seventh the plane angle, and in the eighth column the logarithms of the distances and the logarithms of

the sines of the plane angles (cologarithm of the sine of the first angle).

It is apparent that to begin with, we have only the data in the first, second, and third columns. The other columns are filled in after the adjustment is completed. The first and second columns can be filled in directly from the sketch. The observed angles in the third column are obtained from the lists of directions, an angle being the difference of two directions. For example, the angle at Yellow, between Roman and Spencer, is obtained from the list of directions for station Yellow (fig. 16) by subtracting the direction to Roman from the direction to Spencer.

To obtain the angle at Roman between Peterson and Fairview, it is necessary to add 360° to the direction to Fairview before subtracting the direction to Peterson; $(31° \ 04' \ 11''.58+360°)-321° \ 25' \ 23''.53 = 69° \ 38' \ 48''.05$. In other words, if a direction is less than the one which is to be subtracted from it, then 360° must be added to it before the subtraction is made.

As an angle is obtained by subtracting the direction to the left hand station from the direction to the right hand station, it is designated by the two numbers representing these directions on the sketch, the number of the left hand direction having a minus sign because that direction is subtracted from the other. For example, the designation of the angle at Yellow between Roman and Spencer is $-4+5$, 4 and 5 being the designations of the directions from Yellow to Roman and to Spencer, respectively.

### SPHERICAL EXCESS

The total spherical excess for each triangle as given on Form 25 is obtained by the formula (see Special Publication No. 8, p. 7)

$$\epsilon=\frac{a_1 \ b_1 \ \sin \ C_1 \ (1-e^2 \ \sin^2 \ \phi)^2}{2 \ a^2 \ (1-e^2) \ \sin \ 1''}=a_1 \ b_1 \sin \ C_1 \times m.$$

where $\epsilon$ is the spherical excess; $a_1$, $b_1$ and $C_1$ are the two sides and the included angle, respectively, of the corresponding triangle; $e$ is the eccentricity and $a$ the semimajor axis of the spheroid of reference; and $\phi$ is the mean latitude of the three vertices of the triangle. The letter $m$ is used to designate that part of the expression above which depends only on the latitude and the dimensions of the spheroid and the values of log $m$ are given with the latitude as an argument in the table on page 234.

To compute the spherical excess of a triangle it is necessary to make a preliminary computation of the triangle to obtain the logarithms of the sines of the angles and the logarithms of the lengths. In this preliminary computation, the logarithms need be carried out to only

4 places of decimals. (See fig. 17a.) As an example, the spherical excess of the triangle, Yellow–Roman–Spencer, is computed in the following manner:

In this triangle the length Roman-Spencer may be called $a_1$, and Yellow–Spencer $b_1$. Then the angle at Spencer will be $C_1$. Log $m$ is taken out of the table for latitude 43° 49′, which is approximately the mean latitude of the three vertices. We then have

$$\begin{aligned}
\log a_1 &= 4.7176 \\
\log b_1 &= 4.7362 \\
\log \sin C_1 &= \underline{9.9099-10} \\
&\phantom{=}\ ^{1}\ 9.3637 \\
\log m &= 1.4041-10 \\
\hline
\log \epsilon &= 0.7678 \\
\epsilon &= 5''.859
\end{aligned}$$

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 115

COMPUTATION OF TRIANGLES

State: Oregon

| NO. | STATION | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|
| 2-3 | | | Roman-Spencer | | | | 4.7176 |
| 1 | Yellow | 60 26 09.17 | | | | | 0.0606 |
| 2 | Roman | 65 12 45.72 | | | | | 9.9580 |
| 3 | Spencer | 54 21 12.19 | | | | | 9.9099 |
| 1-3 | | | Yellow-Spencer | | | | 4.7362 |
| 1-2 | | | Yellow-Roman | | | | 4.6881 |

FIG. 17a.—Preliminary triangle computation to obtain lengths for spherical excess computation

One third of the total spherical excess for the triangle is applied to each of the three angles as shown in the sixth column of the triangle computation in Figure 17. In like manner the spherical excess for the other three triangles of the quadrilateral can be obtained and applied.

The total spherical excess of a quadrilateral should have the same value when computed by adding together the sperical excesses of either pair of triangles which cover its area. In the preceding example, the sum of the spherical excesses of the two triangles, Yellow–Roman–Spencer and Fairview–Yellow–Spencer, should equal the sum of the spherical excesses of the two triangles Fairview–Yellow–Roman and Fairview–Roman–Spencer, that is, $5.86+7.17=6.57+6.46$.

In computing the spherical excesses, it is sometimes found that these sums fail to check by 0″.01, due to the dropping of the third decimal place. They should always be made to check by arbitrarily changing the spherical excess of one of the triangles by 0″.01.

[1] This sum can be obtained directly from the triangle computation without recopying the individual values.

### CLOSURES OF TRIANGLES

The sum of the three angles of a triangle should equal 180° plus the spherical excess of the triangle. This rarely happens when the observed angles are used, and consequently a triangle closure arises. The triangle closure (closing correction) is obtained by subtracting the sum of the three angles of the triangle from 180° plus the spherical excess. For example, the sum of the three angles of the triangle Yellow–Roman–Spencer is 180° 00′ 07″.08, and 180° plus the spherical excess is 180° 00′ 05″.86, so the triangle closure is 180° 00′ 05″.86 − 180° 00′ 07″.08 = −1″.22.

In a similar manner the closures are obtained for the other three triangles of the quadrilateral. The closure for the whole quadrilateral should be the same when computed by adding together the closures of either pair of triangles which cover its area. For example, the algebraic sum of the closing corrections of the two triangles, Yellow–Roman–Spencer and Fairview–Yellow–Spencer should equal the algebraic sum of the closing corrections of the two triangles Fairview–Yellow–Roman and Fairview–Roman–Spencer, that is, $-1.22+1.89=+0.47+0.20$. This check should always be applied before beginning the adjustment.

### NUMBER AND SELECTION OF EQUATIONS

After the first, second, third, and sixth columns of the triangle computation (see fig. 17) are filled in as already explained, the quadrilateral is ready to be adjusted. The first thing to be done is to determine the number of equations in the adjustment. In a simple quadrilateral where the length and azimuth of only one line are fixed, we have two kinds of equations, angle and side equations.

Condition equations must be included in the adjustment to eliminate the closing errors of the triangles, that is, to make the sum of the angles of each triangle exactly 180° plus the spherical excess of the triangle. These are called angle equations. A condition equation must also be included to insure that the lines at the pole (the point around which the equation is formed) pass through the same point (see Special Publication No. 28, p. 14). This is called a side equation.

The formulas to be used in computing the number of equations in the adjustment of a triangulation net are

$$\text{number of angle equations} = n' - S' + 1,$$
$$\text{number of side equations} = n - 2S + 3,$$

in which $n$ is the total number of lines, $n'$ is the number of lines sighted over in both directions, $S$ is the total number of stations, and $S'$ is the number of occupied stations. In using these formulas allowance must be made for lines or triangles fixed by previous adjustments. (See p. 59 for another method of determining the number of equations.)

In the quadrilateral Roman-Spencer-Fairview-Yellow (see fig. 15 $n=6$, $n'=6$, $S=4$, $S'=4$. Therefore

$$\text{number of angle equations} = 6-4+1=3,$$
$$\text{number of side equations} = 6-8+3=1.$$

The number of equations in a net having been determined, it is necessary to carefully select the equations so that all discrepancies will be eliminated by the adjustment. In forming the side equations it is necessary to select the pole so that the small angles will be used in the equation, as their tabular differences are proportionately much less affected by the dropping of decimal places than those of the larger angles. On the other hand, the triangles with the large angles should be used in forming the angle equations. (This rule in regard to angle equations need not be followed for a simple quadrilateral.)

The corrections to directions are designated by $v_1, v_2, \ldots\ldots v_n$, but for convenience it is customary to drop the $v$'s, and simply write (1), (2), (3). . . . etc., in which the numbers are not quantities but subscripts of the corresponding $v$'s.

### ANGLE EQUATIONS

The angle equations then for the quadrilateral shown in Figure 15 are formed as follows:

Angle equation 1, $-(1)+(3)-(4)+(5)-(11)+(12)=-1.22$, or
as usually written $0=+1.22-(1)+(3)-(4)+(5)-(11)+(12)$,
angle equation 2, $0=-0.47-(2)+(3)-(4)+(6)-(7)+(8)$,
angle equation 3, $0=-1.89-(5)+(6)-(7)+(9)-(10)+(11)$.

Since there are four closed triangles (see fig. 17), one might suppose that there could be four angle equations. However, by studying the formation of the angle equations, it can be seen that the fourth equation would not be independent but would be a combination of the other three. For example, the fourth angle equation of the quadrilateral would read

$$0=-0.20-(1)+(2)-(8)+(9)-(10)+(12).$$

But equation 1 is, $0=+1.22-(1)+(3)-(4)+(5)-(11)+(12)$,
and equation 2 is $0=-0.47-(2)+(3)-(4)+(6)-(7)+(8)$.

Therefore equation 1 − equation 2 is,

$$0=+1.69-(1)+(2)+(5)-(6)+(7)-(8)$$
$$-(11)+(12).$$

But equation 3 is,

$$0=-1.89-(5)+(6)-(7)+(9)-(10)+(11),$$

therefore equation 1 − equation 2 + equation 3 is,

$$0=-0.20-(1)+(2)-(8)+(9)-(10)+(12)$$

which is identical with the fourth equation above. This shows that the fourth equation is simply a combination of the first three and so will necessarily be satisfied by any values of the $v$'s that satisfy the other three.

### SIDE EQUATIONS

In order to include in the side equation the small angles of the quadrilateral, designated by $-7+8$ and $-8+9$, respectively, the pole must be taken at Roman. The equation is formed by expressing the condition that the lines Roman-Spencer, Roman-Fairview, and Roman-Yellow meet in a point; that is,

$$\frac{\text{Roman-Spencer}}{\text{Roman-Fairview}} \times \frac{\text{Roman-Fairview}}{\text{Roman-Yellow}} \times \frac{\text{Roman-Yellow}}{\text{Roman-Spencer}} = 1.$$

For the sides of the triangles the sines of the opposite angles may be substituted, and the equation becomes

$$\frac{\sin[-8+9]\ \sin[-4+6]\ \sin[-11+12]}{\sin[-10+12]\ \sin[-7+8]\ \sin[-4+5]} = 1.$$

Or for computation by logarithms [1] we have

$$\log \sin[-8+9] + \log \sin[-4+6] + \log \sin[-11+12] = \log \sin[-10+12] + \log \sin[-7+8] + \log \sin[-4+5]$$

For convenience in computing, the equation is arranged in tabular form as shown on page 37. The designations of the angles are placed in the first and fifth columns, the angles themselves in the second and sixth columns, the logarithms of the sines of the angles in the third and seventh columns and the tabular differences of the logarithms of the sines for 1 second of the angles in the fourth and eighth columns. The sums of the logarithms in the third and seventh columns are then taken, and the constant term of the side equation is obtained by subtracting the sum in column 7 from the sum in column 3 and pointing off this difference in units of the sixth decimal place. The quantities in columns 4 and 8 are the coefficients of the quantities in columns 1 and 5, respectively, and the rest of the equation is formed by multiplying together the quantities in columns 4 and 1 and those in columns 8 and 5, and changing the signs of the latter products.

The designations of the angles in the first and fifth columns are taken directly from the sketch (see fig. 15), and the angles themselves in the second and sixth columns are obtained from the triangle computation (fig. 17). Each value in the fourth and eighth columns is the amount of change in the logarithm of the sine of the angle corresponding to a change of 1″ in the angle and this multiplied by

[1] It is customary in all the computations to designate logarithm by *log*, sine by *sin* and cosine by *cos*.

the $v$ applying to the angle gives the change in the logarithm of the sine of the angle produced by the $v$. In the work of the United States Coast and Geodetic Survey, the tabular differences of the logarithms are taken in units of the sixth place of decimals.

*Side equation*

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| | ° ′ ″ | | | | ° ′ ″ | | |
| −8+9 | 28 21 21.42 | 9.67664586 | +3.90 | −10+12 | 120 34 33.26 | 9.93498097 | −1.24 |
| −4+6 | 119 05 31.11 | 9.94143208 | −1.18 | −7+8 | 26 46 00.85 | 9.65356160 | +4.18 |
| −11+12 | 54 21 12.19 | 9.90989120 | +1.51 | −4+5 | 60 26 09.17 | 9.93942144 | +1.19 |
| | | 9.52796914 | | | | 9.52796401 | |

4. $0 = +5.13 + 2.37(4) - 1.19(5) - 1.18(6) + 4.18(7) - 8.08(8) + 3.90(9)$
$- 1.24(10) - 1.51(11) + 2.75(12)$

## EXPLANATION OF COMPUTATION

The equations can now be entered in the table of correlates as shown on page 38. This table is arranged like that shown on page 11, except that there is no column for weights as all the directions were considered as observed in the same manner and therefore of equal weight. The formation and solution of the normals and the computation of the $v$'s are also made in the same manner as shown on pages 11–16. After the $v$'s are determined, the extra decimal places are dropped to give the adopted values. In first-order work the adopted $v$'s are taken to hundredths of a second.

As a check to insure that the $C$'s were properly substituted in the correlates in computing the $v$'s, the $v$'s on all directions around a point are added together. The sum should equal zero, unless there is at the point a fixed direction to which no correction is applied. For example, at Roman the corrections on directions (1), (2), and (3) are (see p. 38) $-0.039+0.284-0.245=0$; at Yellow on (4), (5), and (6) they are $+0.056-0.282+0.226=0$; at Fairview on (7), (8), and (9) they are $-0.466+0.362+0.104=0$; and at Spencer on (10), (11), and (12) they are $-0.317+0.498-0.181=0$.

As was the case for the station adjustment (see p. 15) the adopted $v$'s are not simply the computed $v$'s taken to the nearest hundredth of a second. In order to satisfy all the angle equations exactly, it is necessary on some of the directions to adopt the hundredth above or below the computed value. In doing this it is well, if possible, to change those values which involve the smallest change in the thousandth decimal place. (See the adopted value of $v_6$ on p. 38.)

The full solution, both forward and backward, of the four normal equations, is given on page 38. The computation of the $v$'s is given on page 39. As a solution similar to this was fully explained on page 14, it is not necessary to explain this one.

*Correlate equations*

| | 1 | 2 | 3 | 4 | $\Sigma_c$ | $v$ | Adopted $v$ |
|---|---|---|---|---|---|---|---|
| 1 | −1 | ------- | ------- | ------- | −1.00 | −0.039 | −0.04 |
| 2 | ------- | −1 | ------- | ------- | −1.00 | +.284 | +.28 |
| 3 | +1 | +1 | ------- | ------- | +2.00 | −.245 | −.24 |
| 4 | −1 | −1 | ------- | +2.37 | +.37 | +.056 | +.06 |
| 5 | +1 | ------- | −1 | −1.19 | −1.19 | −.282 | −.28 |
| 6 | ------- | +1 | +1 | −1.18 | +.82 | +.226 | +.22 |
| 7 | ------- | −1 | −1 | +4.18 | +2.18 | −.466 | −.47 |
| 8 | ------- | +1 | ------- | −8.08 | −7.08 | +.362 | +.36 |
| 9 | ------- | ------- | +1 | +3.90 | +4.90 | +.104 | +.10 |
| 10 | ------- | ------- | −1 | −1.24 | −2.24 | −.317 | −.32 |
| 11 | −1 | ------- | +1 | −1.51 | −1.51 | +.498 | +.50 |
| 12 | +1 | ------- | ------- | +2.75 | +3.75 | −.181 | −.18 |

*Normal equations*

| | 1 | 2 | 3 | 4 | $\eta$ | $\Sigma_n$ | $C$ |
|---|---|---|---|---|---|---|---|
| 1 | +6 | +2 | −2 | +0.70 | +1.22 | +7.92 | +0.03928 |
| 2 | | +6 | +2 | −15.81 | −.47 | −6.28 | −.28422 |
| 3 | | | +6 | −.54 | −1.89 | +3.57 | +.41561 |
| 4 | | | | +117.7744 | +5.13 | +107.2544 | −.08003 |

*Solution of normal equations*

| 1 | 2 | 3 | 4 | $\eta$ | $\Sigma_n$ |
|---|---|---|---|---|---|
| +6 | +2 | −2 | +0.70 | +1.22 | +7.92 |
| $C_1$ | −.33333 | +.33333 | −.11667 | −.20333 | −1.32 |
| | +6 | +2 | −15.81 | −.47 | −6.28 |
| 1 | −.6667 | +.6667 | −.2333 | −.4067 | −2.64 |
| | +5.3333 | +2.6667 | −16.0433 | −.8767 | −8.92 |
| | $C_2$ | −.50001 | +3.00814 | +.16438 | +1.67251 |
| | | +6 | −.54 | −1.89 | +3.57 |
| | 1 | −.6667 | +.2333 | +.4067 | +2.64 |
| | 2 | −1.3333 | +8.0217 | +.4383 | +4.46 |
| | | +4.0000 | +7.7150 | −1.0450 | +10.67 |
| | | $C_3$ | −1.92875 | +.26125 | −2.6675 |
| | | | +117.7744 | +5.13 | +107.2544 |
| | | 1 | −.0817 | −.1423 | −.9240 |
| | | 2 | −48.2605 | −2.6372 | −26.8326 |
| | | 3 | −14.8803 | +2.0155 | −20.5798 |
| | | | | | 79 |
| | | | +54.5519 | +4.3660 | +58.9180 |
| | | | $C_4$ | −.08003 | −1.08003 |

*Back solution*

| 4 | 3 | 2 | 1 |
|---|---|---|---|
| −0.08003 | +0.26125 | +0.16438 | −0.20333 |
| | +.15436 | −.24079 | +.00933 |
| | | −.20781 | +.13854 |
| | +.41561 | | +.09474 |
| | | −.28422 | |
| | | | +.03928 |

In the forward and back solutions of the normal equations for a simple quadrilateral it is not necessary ordinarily to use as many decimal places as are used in the example above. Three decimal places for the multiplied terms, four for the division terms, and four for the back solution are usually sufficient. In some cases two, three, and three decimal places, respectively, may be used and the desired accuracy still be obtained.

*Computation of corrections (v's)*

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| −0.039<br>−.04 | +0.284<br>+.28 | +0.039<br>−.284 | −0.039<br>+.284<br>−.189 | +0.039<br>−.416<br>+.095 | −0.284<br>+.416<br>+.094 | +0.284<br>−.416<br>−.334 | −0.284<br>+.646 | +0.416<br>−.312 | −0.416<br>+.099 | −0.039<br>+.416<br>+.121 | +0.039<br>−.220 |
| | | −.245<br>−.24 | +.056<br>+.06 | −.282<br>−.28 | +.226<br>+.22 | −.466<br>−.47 | +.362<br>+.36 | +.104<br>+.10 | −.317<br>−.32 | +.498<br>+.50 | −.181<br>−.18 |

## COMPUTATION OF TRIANGLES

After adopting values for the $v$'s the next step is to substitute these values in column 4 of the triangle computation. (See fig. 17.) For instance, in the triangle Yellow-Roman-Spencer, the correction to angle Yellow is $-4+5=-(+0.06)+(-0.28)=-0.34$; the correction to angle Roman is $-1+3=-(-0.04)+(-0.24)=-0.20$; and the correction to angle Spencer is $-11+12=-(+0.50)+(-0.18)=-0.68$. These three corrections should sum up to the closure of the triangle, $-1''.22$, and we find this to be the case for $-0.34-0.20-0.68=-1.22$. Likewise, the corrections to the angles in each of the other three triangles should sum up to the closure of that triangle. The corrections should always be written in the triangles in *pencil* until it is certain that all the results will check.

### DISTRIBUTION OF SPHERICAL EXCESS

The spherical angles in column 5 (see fig. 17) are next computed. The spherical excess of each triangle is then distributed among the three angles, one-third of it being placed on each angle. If it is not exactly divisible by 3, the spherical excess is so distributed that the small angles will have their correct share as nearly as possible, since changes in the small angles affect the lengths to a greater degree than changes in the large angles. That is, if the spherical excess of the triangle were only $0''.02$, then $0''.01$ should be placed on each of the two smaller angles and $0''.00$ on the largest. If the total spherical excess were $0''.04$, then $0''.02$ should be placed on the largest angle, and $0''.01$ on each of the two smaller angles; or if the spherical excess were $0''.05$, then $0''.02$ should be placed on each of the smaller angles and $0''.01$ on the largest angle.

### COMPUTATION OF LOGARITHMS OF LENGTHS

The plane angles in column 7 (see fig. 17) can now be computed and finally the logarithms (or co-logarithms) of the sines of these angles are placed in column 8. (In the case of the first angle of each triangle the co-logarithm of the sine of the angle is used.) In each triangle, the logarithm of the length 2–3 is added to the co-logarithm of the sine of angle 1 and the logarithm of the sine of angle 2 to give the logarithm of the length 1–3; and the logarithm of the length 2–3 is added to the co-logarithm of the sine of angle 1 and the logarithm of the sine of angle 3 to give the logarithm of the length 1–2.

After the logarithms of the lengths for each of the four triangles are computed, the logarithm of each length will appear in two different triangles. These should be the same except possibly for a difference of 1 in the last place of decimals, which may be due to accumulation. In a flat triangle, however, having one or two very small angles the discrepancies in lengths may amount to several units of the last place of decimals used.

Where there is a difference of one or more in the last place of decimals in the adjusted logarithms of the lengths, the logarithms should be made consistent before going ahead with the work. The question naturally arises: To which logarithm should the correction be applied? Other things being equal, the correction should be applied to that logarithm which was computed through the smallest angles. However, in taking out the logarithms of the sines, one more decimal place than is necessary should be taken out and placed in small figures to the right of each logarithm. It can then readily be seen where the adding or dropping of units in this decimal place has accumulated enough to change any particular logarithm of a length one in the last decimal place used, and the correction can be applied accordingly.

### CORRECTIONS TO DIRECTIONS

After the correctness of the $v$'s has been checked by the closures of the triangles and the agreement of the lengths, the corrections should be applied to the directions and the final values placed in the list of directions. (See fig. 16.)

The first step is to put the computed and adopted values of the $v$'s in the columns intended for them in the table of correlates. (See p. 38.) Then using the sketch, Figure 15, for the designations of the directions, the corresponding corrections should be applied to the directions in the list of directions.

If the $v$ to be applied to a direction is negative it sometimes happens that the minutes of the final direction will be one less than the minutes of the observed direction. Where this occurs a bar should be placed over the value in the final seconds column. For example, in Figure 16, in the list of directions at Roman, the observed direction at

Spencer is 0° 00′ 00.″00 and the final seconds are $\overline{59.96}$, that is, the final direction is 359° 59′ 59.″96.

At station Yellow the directions to Roman, Spencer, and Fairview are numbered 4, 5, and 6, respectively. In the table of correlates, page 38, we find the values of the adopted $v$'s for 4, 5, and 6 are +0.06, −0.28, and +0.22, respectively. Then in the list of directions at Yellow, Figure 16, we apply these corrections to the directions Roman, Spencer, and Fairview as follows: To Roman, 38.″63 + 0.″06 = 38.″69; to Spencer, 47.″80 − 0.″28 = 47.″52; to Fairview, 09.″74 + 0.″22 = 09.″96.

The values of the final seconds in the list of directions should be checked by using them to verify the corrected angles in the triangle computation. The directions at Yellow should be checked, therefore, by the adjusted angles at Yellow in the various triangles. In the triangle Yellow–Roman–Spencer, Figure 17, the spherical angle at Yellow is 60° 26′ 08.″83. From the list of directions at Yellow the final angle between Roman and Spencer is 239° 06′ 47.″52 − 178° 40′ 38.″69 = 60° 26′ 08.″83 which checks the adjusted angle in the triangle. In the triangle Fairview–Yellow–Roman, the adjusted spherical angle at Yellow is 119° 05′ 31.″27. From the list of directions at Yellow the final angle between Roman and Fairview is 297° 46′ 09.″96 − 178° 40′ 38.″69 = 119° 05′ 31.″27 which also checks the angle from the triangle. In the triangle Fairview-Yellow-Spencer the adjusted angle at Yellow is 58° 39′ 22.″44. From the list of directions at Yellow the final angle between Spencer and Fairview is 297° 46′ 09.″96 − 239° 06′ 47.″52 = 58° 39′ 22.″44, which again checks the adjusted angle from the triangle.

In the same manner all the adjusted angles at Roman, Spencer, and Fairview in the triangles will be found to be checked by the angles from the lists of directions, and this shows that all the angles are consistent.

It is important not to omit the placing of the corrections in the list of directions after an adjustment is completed. The final values in the list show that the adjustment has been made, and they are the values of the directions that must be used if other adjustments are made depending on this one.

## COMPUTATION OF GEOGRAPHIC POSITIONS

After the adjustment is completed and all the triangles are computed and made consistent, the geographic positions of the two new points, Yellow and Fairview, can be computed by starting from the fixed positions of Roman and Spencer and the fixed azimuth, Roman-Spencer.

Since all the angles and lengths in the quadrilateral are now consistent due to the adjustment, any triangle may be used in the com-

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 30

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Roman | to | Spencer | 261 | 04 | 53.49 |
| ∠ | | Spencer | & | Yellow | +65 | 12 | 45.52 |
| α | 2. | Roman | to 1. | Yellow | 326 | 17 | 39.01 |
| Δα | | | | | + | 13 | 53.26 |
| | | | | | 180 | | |
| α′ | 1. | Yellow | to 2. | Roman | 146 | 31 | 32.27 |
| | | | | First Angle of Triangle | 60 | 26 | 08.83 |

| | | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|---|
| φ | | 43 | 54 | 45.041 | 2. | Roman | λ | 123 | 44 | 14.987 |
| Δφ | | − | 21 | 56.195 | s= | | Δλ | − | 20 | 05.419 |
| φ′ | | 43 | 32 | 48.846 | 1. | Yellow | λ′ | 123 | 24 | 09.568 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.688 0992 | s² | 9.37620 | | | −h | 3.1187 |
| cos α | 9.920 0699 | sin² α | 9.48848 | (Δφ)² | 6.2384 | s² sin² α | 8.8647 |
| B | 8.510 5516 | C | 1.38763 | D | 2.3922 | E | 6.1886 |
| h | 3.118 7207 | | 0.25231 | | 8.6306 | | 8.1720 |
| | ″ | | ″ | | | | |
| 1st term. | +1314.3792 | 3d term. | +0.0427 | | | (Δλ)² | 9.245 |
| 2d term. | + 1.7878 | 4th term. | −0.0149 | | | F | 7.844 |
| | +1316.1670 | | | | | | 7.092 |
| 3d and 4th terms. | +0.0278 | s | 4.688 0992 | | | | |
| −Δφ | +1316.1948 | sin α | 9.744 2376 | Arg. | | Δλ | 3.081 1382 |
| ½(φ+φ′) | 43 43 46.94 | A′ | 8.509 0278 | s | −42 | sin ½(φ+φ′) | 9.839 6397 |
| | | sec φ′ | 0.139 7754 | Δλ | +25 | sec ½(Δφ) | 22 |
| | | | 3.081,1389 | Corr. | −17 | | 2.920 7801 |
| | | Δλ | −1205.4194 | | | −Δα | −833.259 |
| | | | | | | | − 0.001 |
| | | | | | | | −833.260 |

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Spencer | to | Roman | 81 | 31 | 39.25 |
| ∠ | | Yellow | & | Roman | −54 | 21 | 11.51 |
| α | 3. | Spencer | to 1. | Yellow | 27 | 10 | 27.74 |
| Δα | | | | | − | 12 | 46.64 |
| | | | | | 180 | | |
| α′ | 1. | Yellow | to 3. | Spencer | 206 | 57 | 41.10 |

| | | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|---|
| φ | | 43 | 59 | 00.715 | 3. | Spencer | λ | 123 | 05 | 41.248 |
| Δφ | | − | 26 | 11.869 | s= | | Δλ | + | 18 | 28.320 |
| φ′ | | 43 | 32 | 48.846 | 1. | Yellow | λ′ | 123 | 24 | 09.568 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.736 2338 | s² | 9.47247 | | | −h | 3.1960 |
| cos α | 9.949 2049 | sin² α | 9.31926 | (Δφ)² | 6.3928 | s² sin² α | 8.7917 |
| B | 8.510 5462 | C | 1.38869 | D | 2.3923 | E | 6.1902 |
| h | 3.195 9849 | | 0.18042 | | 8.7851 | | 8.1779 |
| | ″ | | ″ | | | | |
| 1st term. | +1570.3083 | 3d term. | +0.0610 | | | (Δλ)² | 9.134 |
| 3d term. | + 1.5150 | 4th term. | −0.0151 | | | F | 7.848 |
| | +1571.8233 | | | | | | 6.982 |
| 3d and 4th terms. | + 0.0459 | s | 4.736 2338 | | | | |
| −Δφ | +1571.8692 | sin α | 9.659 6311 | Arg. | | Δλ | 3.044 6650 |
| ½(φ+φ′) | 43 45 54.78 | A′ | 8.509 0278 | s | −52 | sin ½(φ+φ′) | 9.839 9209 |
| | | sec φ′ | 0.139 7754 | Δλ | +21 | sec ½(Δφ) | 32 |
| | | | 3.044 6650 | Corr. | −31 | | 2.884 5891 |
| | | Δλ | + 1108.3196 | | | −Δα | +766.636 |
| | | | | | | | + 001 |
| | | | | | | | +766.637 |

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FIG. 18.—Position computation for new stations of quadrilateral

588539°—28——4

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ | |
|---|---|---|---|---|---|---|---|---|
| α | | Yellow | to | Spencer | 206 | 57 | 41.10 | |
| ∠ | | Spencer | & | Fairview | +58 | 39 | 22.44 | |
| α | 2. | Yellow | to 1. | Fairview | 265 | 37 | 03.54 | |
| Δα | | | | | + | 31 | 01.50 | |
| | | | | | 180 | | | |
| α′ | 1. | Fairview | to 2. | Yellow | 86 | 08 | 05.04 | +1 |
| | | | | First Angle of Triangle | 55 | 07 | 22.84 | |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 43 | 32 | 48.846 | 2. | Yellow | λ | 123 | 24 | 09.568 |
| Δφ | + | 2 | 21.607 | s= | | Δλ | − | 45 | 00.954 |
| φ′ | 43 | 35 | 10.453 | 1. | Fairview | λ′ | 122 | 39 | 08.614 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.783 6971 | s² | 9.56739 | | | −h | 2.1774 |
| cos α | 8.883 1608 | sin² α | 9.99746 | (½φ)² | 4.3022 | s² sin² α | 9.5648 |
| B | 8.510 5798 | C | 1.38211 | D | 2.3920 | E | 6.1805 |
| h | 2.177 4377 | | 0.94696 | | 6.6942 | | 7.9227 |
| | ″ | | ″ | | | | |
| 1st term. | −150.4658 | 3d term. | +0.0005 | | | (Δλ)² | 0.295 |
| 2d term. | +8.8503 | 4th term. | +0.0084 | | | F | 7.851 |
| | −141.6155 | | | | | | 8.146 |
| 3d and 4th term. | +0.0089 | s | 4.783 6971 | | | | |
| −Δφ | −141.6066 | sin α | 9.998 7284 | Arg. | | Δλ | 3.431 5172 |
| ½(φ+φ′) | 43 33 59.65 | A′ | 8.509 0268 | s | −65 | sin ½(φ+φ′) | 9.838 3433 |
| | | sec φ′ | 0.140 0590 | Δλ | +124 | sec ½(Δφ) | 0 |
| | | | 72 3.431 5172 | Corr. | +59 | | 3.2698 605 |
| | | Δλ | ″ −2700.9540 | | | −Δα | −1861.439 |
| | | | | | | | − 0.014 |
| | | | | | | | −1861.503 |

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DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY.
FORM 26

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Spencer | to | Yellow | 27 | 10 | 27.74 |
| ∠ | | Fairview | & | Yellow | − 66 | 13 | 21.89 |
| α | 3. | Spencer | to 1. | Fairview | 320 | 57 | 05.85 |
| Δα | | | | | + | 18 | 22.04 |
| | | | | | 180 | | |
| α′ | 1. | Fairview | to 3. | Spencer | 141 | 15 | 27.89 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 43 | 59 | 00.715 | 3. | Spencer | λ | 123 | 05 | 41.248 |
| Δφ | − | 23 | 50.262 | s= | | Δλ | − | 26 | 32.635 |
| φ′ | 43 | 35 | 10.453 | 1. | Fairview | λ′ | 122 | 39 | 08.613 +1 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.753 7076 | s² | 9.50742 | | | −h | 3.1545 |
| cos α | 9.890 2054 | sin² α | 9.59865 | (½φ)² | 6.3108 | s² sin² α | 9.1061 |
| B | 8.510 5460 | C | 1.38869 | D | 2.3923 | E | 6.1902 |
| h | 3.154 4592 | | 0.49476 | | 8.7031 | | 8.4508 |
| | ″ | | ″ | | | | |
| 1st term. | +1427.1158 | 3d term. | +0.0505 | | | (Δλ)² | 9.606 |
| 2d term. | + 3.1244 | 4th term. | −0.0282 | | | F | 7.848 |
| | +1430.2402 | | | | | | 7.454 |
| 3d and 4th term. | + 0.0223 | s | 4.753 7076 | | | | |
| −Δφ | +1430.2625 | sin α | 9.799 3242 | Arg. | | Δλ | 3.202 1162 |
| ½(φ+φ′) | 43 47 05.58 | A′ | 8.509 0268 | s | −57 | sin ½(φ+φ′) | 9.840 0764 |
| | | sec φ′ | 0.140 0590 | Δλ | +43 | sec ½(Δφ) | 26 |
| | | | 62 3.202 1162 | Corr. | −14 | | 3.0421 952 |
| | | Δλ | ″ −1592.6348 | | | −Δα | −1102.035 |
| | | | | | | | − 0.003 |
| | | | | | | | −1102.038 |

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FIG. 18.—Position computation for new stations of quadrilateral—Continued

putation and the position of the point computed should be the same. It is best, however, to use the triangle with the shorter lengths, as there will be less trouble in making the computation over the two lines to the new point agree.

The positions for first-order triangulation are computed on form 26. The formulas used in the computation of geographic positions, the development of these formulas, and the tables used in the position computation, are found in United States Coast and Geodetic Survey Special Publication No. 8, and are not repeated here.

The computation of the geographic positions of stations Yellow and Fairview are given on pages 42 and 43.[2] (See fig. 18.)

It should be noted that the angles used in the computation of the geographic positions are the spherical angles, the seconds of which appear in column 5 of the table of triangles (fig. 17). The first angle of the triangle, although not used in the actual computation of the position, should always be written down in its proper place on the form since it is used as a check on the computed azimuth.

For example, in computing the geographic position of station "Yellow," the angles used are 60° 26′ 08.″83, 65° 12′ 45.″52, and 54° 21′ 11.″51. After the computation is finished the azimuth 1 to 2 plus the first angle of the triangle should equal the azimuth 1 to 3; that is, 146° 31′ 32.″27 + 60° 26′ 08.″83 should equal 206° 57′ 41.″10, which they do.

Care should be taken to make sure that the latitude and longitude of the new station as computed from the two different stations check and that the azimuths as computed from the two stations differ exactly by the amount of the first angle of the triangle. Occasionally these values will fail to check by one unit in the last place of decimals used, due to accumulation in the next decimal place, but if the discrepancy is greater than one the computation should be checked over to see whether a mistake has been made.

If no error has been made in the computation and the latitudes (or longitudes) fail to check by one unit in the last place of decimals used, then the two values should be made to agree by adding one unit of the last place of decimals to one value or by subtracting one unit from the other value. Which value should be corrected depends upon which one should be changed to make the next position computation check. (See p. 77.)

In the same manner if the azimuth of station 1 to station 2 plus the first angle of the triangle fails to check the azimuth of station 1

[2] After the manuscript of this publication was prepared new forms with the same numbers (26 and 27) as the old forms were prepared for the computation of geographic positions of the first and third orders. Although there has been no change in the formulas used for the computation, the different terms of the formulas have been rearranged in a more compact form, which, it is believed, will expedite the computations. These new forms have not yet been finally adopted by this bureau, but are at present being tried out in the division of geodesy A copy of each of these forms with sample computations of geographic positions are shown in Figures 19 and 20.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 27

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ | | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| α | 2 | Outer | to 3 | Hard | 144 | 43 | 42.5 | α | 3 | Hard | to 2 | Outer | 324 | 42 | 28.9 |
| 2d ∠ | | Hard | & | Parson | + 23 | 51 | 16.1 | 3d ∠ | | Parson | & | Outer | − 52 | 56 | 25.1 |
| α | 2 | Outer | to 1 | Parson | 168 | 34 | 58.6 | α | 3 | Hard | to 1 | Parson | 271 | 46 | 03.8 |
| Δα | | | | | − | | 20.7 | Δα | | | | | + | | 52.9 |
| | | | | | 180 | 00 | 00.0 | | | | | | 180 | 00 | 00.0 |
| α′ | 1 | Parson | to 2 | Outer | 348 | 34 | 37.9 | α′ | 1 | Parson | to 3 | Hard | 91 | 46 | 56.7 |
| | | | | First Angle of Triangle | 103 | 12 | 18.8 | | | | | | | | |

| | ° | ′ | ″ | | | | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| φ | 40 | 35 | 18.742 | 2 | Outer | λ | 73 | 36 | 33.964 | φ | 40 | 37 | 20.514 | 3 | Hard | λ | 73 | 38 | 27.008 |
| Δφ | + | 1 | 59.853 | | | Δλ | + | | 31.763 | Δφ | − | | 01.919 | | | Δλ | − | 1 | 21.281 |
| φ′ | 40 | 37 | 18.595 | 1 | Parson | λ′ | 73 | 37 | 05.727 | φ′ | 40 | 37 | 18.595 | 1 | Parson | λ′ | 73 | 37 | 05.727 |

| | Logarithms | Values in seconds | | | ° ′ ″ | | |
|---|---|---|---|---|---|---|---|
| s | 3.576526 | | | ½(φ+φ′) | 40 36 18.7 | | |
| Cos α | 9.991320 | | | | Logarithms | Values in seconds | |
| B | 8.510807 | | | s | 3.576526 | | |
| h | 2.078653 | 1st term | +119.8541″ | Sin α | 9.296554 | | |
| s² | 7.15305 | | | A′ | 8.509103 | | |
| Sin²α | 8.59311 | | | sec φ′ | 0.119745 | | |
| C | 1.33731 | | | Δλ | 1.501928 | | +31.7635″ |
| | 7.08347 | 2d term | +0.0012 | Sin ½(φ+φ′) | 9.813476 | | |
| h² | 4.1573 | | | −Δα | 1.315404 | | +20.67 |
| D | 2.3872 | | | | | | |
| | 6.5445 | 3d term | +0.0004 | | | | |
| | | −Δφ | −119.8525 | | | | |

| | Logarithms | Values in seconds | | | ° ′ ″ | | |
|---|---|---|---|---|---|---|---|
| s | 3.281346 | | | ½(φ+φ′) | 40 37 19.6 | | |
| Cos α | 8.489222 | | | | Logarithms | Values in seconds | |
| B | 8.510804 | | | s | 3.281346 | | |
| h | 0.281372 | 1st term | +1.9115″ | Sin α | 9.999793 | | |
| s² | 6.56269 | | | A′ | 8.509103 | | |
| Sin²α | 9.99959 | | | sec φ′ | 0.119745 | | |
| C | 1.33783 | | | Δλ | 1.909987 | | −81.2806″ |
| | 7.90011 | 2d term | +0.0079 | Sin ½(φ+φ′) | 9.813626 | | |
| h² | 0.5627 | | | −Δα | 1.723613 | | −52.92 |
| D | 2.3873 | | | | | | |
| | 2.9500 | 3d term | + 0 | | | | |
| | | −Δφ | +1.9194 | | | | |

U. S. GOVERNMENT PRINTING OFFICE

FIG. 19.—Sample position computation, third-order triangulation, new form

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 26

## POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| α | 2 Mt. Nebo | to 3 Tushar | 20 | 05 | 36.08 | α | 3 Tushar | to 2 Mt. Nebo | 199 | 41 | 08.58 |
| 3d∠ | Tushar | & Wheeler Peak | +48 | 04 | 05.50 | 3d∠ | Wheeler Peak | & Mt. Nebo | −88 | 16 | 30.92 |
| α | 2 Mt. Nebo | to 1 Wheeler Peak | 68 | 09 | 41.58 | α | 3 Tushar | to 1 Wheeler Peak | 111 | 24 | 37.66 |
| Δα | | | −1 | 37 | 01.48 | Δα | | | −1 | 11 | 20.22 |
| | | | 180 | 00 | 00.00 | | | | 180 | 00 | 00.00 |
| α′ | 1 Wheeler Peak | to 2 Mt. Nebo | 246 | 32 | 40.10 | α′ | 1 Wheeler Peak | to 3 Tushar | 290 | 13 | 17.44 |
| | | | 43 | 40 | 37.34 | | | | | | |

First Angle of Triangle

| | ° | ′ | ″ | | | ° | ′ | ″ | | ° | ′ | ″ | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| φ | 39 | 48 | 38.316 | 2 Mt. Nebo | λ | 111 | 45 | 56.235 | φ | 38 | 25 | 09.551 | 3 Tushar | λ | 112 | 24 | 42.168 |
| Δφ | − | 49 | 29.300 | | Δλ | +2 | 32 | 50.783 | Δφ | + | 33 | 59.467 | | Δλ | +1 | 54 | 04.851 |
| φ′ | 38 | 59 | 09.016 | 1 Wheeler Peak | λ′ | 114 | 18 | 47.018 | φ′ | 38 | 59 | 09.018 | 1 Wheeler Peak | λ′ | 114 | 18 | 47.019 |

| | Logarithms | +/− | ″ | | Logs | | ° ′ ″ | | Logarithms | | ° ′ ″ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| s | 5.3761505 | (1) | +2867.8000 | ½ | 9.699 | ½(φ+φ′) | 39 23 53.67 | | | | |
| cos α | 9.5705323 | (2) | + 103.0505 | s² | 10.752 | | Logarithms | | | | |
| B | 8.5108661 | Sum | +2970.8505 | K | 2.013 | s | 5.3761505 | | | | |
| (1)=h | 3.4575489 | (3) | + 0.2140 | E | 6.100 | sin α | 9.9676586 | | | | |
| s² | 10.75230 | (4) | − 1.7599 | (5) | 8.564 | A′ | 8.5091440 | | | | |
| sin² α | 9.93532 | (5) | − 0.0366 | 3 | 0.477 | sec φ′ | 0.1094105 | | | | |
| C | 1.32543 | (6) | + 0.0152 | cos² α | 9.141 | Arc−sin corr. | + 428 | | | | |
| (2)=K | 2.01305 | (7) | + 0.0167 | (6) | 8.182 | Δλ | 3.9624064 | | | | |
| (bφ)² | 6.9452 | −Δφ | +2969.2999 | (cbφ) E | 13.900 | sin ½(φ+φ′) | 9.8025731 | | | | |
| D | 2.3852 | Δφ/2 | −1484.64 | A²arc1″/3 | 5.912 | sec Δφ/2 | 113 | | | | |
| (3) | 9.3304 | | | sec²φ | 0.229 | (approx.) −Δα | 3.7649908 | | | | |
| −h | 3.4575 | | | (7) | 8.223 | do | +5820.909″ | | | | |
| s² sin² α | 10.6876 | | | | | (8) | + 0.571″ | | | | |
| E | 6.1004 | Arc−sin corr. | | | | −Δα | +5821.480″ | | | | |
| (4) | 0.2455 | for s | −1003 | (Δλ)² | 11.887 | | | | | | |
| | | for Δλ | +1431 | F | 7.870 | Δλ | +9170.7830″ | | | | |
| | | Total | + 428 | (8) | 9.757 | | | | | | |

| | Logarithms | −/+ | ″ | | Logs | | ° ′ ″ | | Logarithms | | ° ′ ″ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| s | 5.2478407 | (1) | −2094.8860 | ½ | 9.699 | ½(φ+φ′) | 38 42 09.28 | | | | |
| cos α | 9.5623485 | (2) | + 54.6500 | s² | 10.496 | | Logarithms | | | | |
| B | 8.5109712 | Sum | −2040.2360 | K | 1.738 | s | 5.2478407 | | | | |
| (1)=h | 3.3211604 | (3) | + 0.1000 | E | 6.072 | sin α | 9.9689446 | | | | |
| s² | 10.49568 | (4) | + 0.6708 | (5) | 8.005 | A′ | 8.5091440 | | | | |
| sin² α | 9.93789 | (5) | − 0.0101 | 3 | 0.477 | sec φ′ | 0.1094105 | | | | |
| C | 1.30402 | (6) | + 0.0040 | cos² α | 9.125 | Arc−sin corr. | + 242 | | | | |
| (2)=K | 1.73759 | (7) | + 0.0046 | (6) | 7.607 | Δλ | 3.8353640 | | | | |
| (bφ)² | 6.6191 | −Δφ | −2039.4667 | (cbφ) E | 13.928 | sin ½(φ+φ′) | 9.7960730 | | | | |
| D | 2.3807 | Δφ/2 | +1019.73 | A²arc1″/3 | 5.912 | sec Δφ/2 | 53 | | | | |
| (3) | 8.9998 | | | sec²φ | 0.212 | (approx.) −Δα | 3.6314423 | | | | |
| −h | 3.3212 | | | (7) | 7.659 | do | +4279.985″ | | | | |
| s² sin² α | 10.4336 | | | | | (8) | 0.240″ | | | | |
| E | 6.0718 | Arc−sin corr. | | | | −Δα | +4280.225″ | | | | |
| (4) | 9.8266 | for s | −555 | (Δλ)² | 11.506 | | | | | | |
| | | for Δλ | +797 | F | 7.874 | Δλ | +6844.8508″ | | | | |
| | | Total | +242 | (8) | 9.380 | | | | | | |

FIG. 20.—Sample position computation, first-order triangulation, new form

to station 3 by one unit in the last decimal place, a correction must be applied to one of the azimuths to make the result consistent before proceeding with the next position. (See p. 43.)

## LIST OF GEOGRAPHIC POSITIONS

After the geographic positions of the stations are computed a list of these geographic positions, together with the azimuths and logarithms of the distances to the other stations, is made on Form 28B. (See fig. 21.) The azimuths from each station to the other stations should be arranged in clockwise order. The names of the stations in the first and sixth columns of the list of positions can be filled in most easily in proper order from the sketch, since this shows the arrangement of the stations in regard to azimuth. For each station the azimuth and logarithm of the distance to each of two stations can be obtained directly from the position computation and should be written in the list before the other azimuths and logarithms of distances which are obtained from the tables of triangles are entered on the form. In case the quadrilaterals are complete, that is, all the lines are included, it is possible to get a check on the computation of all extra azimuths in the list.

At station Fairview, for example, the azimuths and back azimuths to Yellow and Spencer are obtained directly from the position computation, but the azimuth and back azimuth to Roman must be computed from the triangles by using the two triangles Fairview–Yellow–Roman and Fairview–Roman–Spencer. They are computed in the following manner: The azimuth, Fairview to Yellow already in the list (fig. 21) is 86° 08′ 05″.05 and the angle at Fairview from Yellow to Roman in the triangle Fairview–Yellow–Roman is 26° 46′ 01″.68, so the azimuth of Fairview to Roman as derived from the first triangle is 86° 08′ 05″.05 + 26° 46′ 01″.68 = 112° 54′ 06″.73. In a similar manner the azimuth Fairview to Spencer already in the list is 141° 15′ 27″.89, and the angle at Fairview from Roman to Spencer in the triangle Fairview–Roman–Spencer is 28° 21′ 21″.16, so the azimuth of Fairview to Roman as derived from the second triangle is 141° 15′ 27″.89 − 28° 21′ 21″.16 = 112° 54′ 06″.73, which checks the value above.

The back azimuth Roman to Fairview is computed as follows: The azimuth Roman to Yellow already in the list (fig. 21) is 326° 17′ 39″.01, and the angle at Roman from Fairview to Yellow in the triangle Fairview–Yellow–Roman is 34° 08′ 33″.62, so the azimuth of Roman to Fairview as derived from the first triangle above is 326° 17′ 39″.01 − 34° 08′ 33″.62 = 292° 09′ 05″.39. Again, the azimuth Roman–Spencer already in the list is 261° 04′ 53″.49, and the angle at Roman from Spencer to Fairview in the triangle Fairview–Roman–Spencer is 31° 04′ 11″.90, so the azimuth Roman to Fairview as derived from the second triangle is 261° 04′ 53″.49 + 31° 04′ 11″.90 = 292° 09′ 05″.39, which checks the other value.

The extra azimuths, that is, those computed from the triangles, should *always* be checked in every possible triangle, as this will show whether or not the adjustment has made all the angles consistent. It should be borne in mind that the angles used in the computation of the azimuths are the *spherical angles*. One can readily tell whether to add or subtract the angles in the computation of the azimuths by referring to the triangulation sketch on which the relative positions of the stations are shown.

After all the azimuths and logarithms of distances have been entered in the list of positions, the other columns are filled out. The quantities in the column headed "Seconds in meters" are obtained by means of the tables in Special Publication No. 5, entitled "Tables for a polyconic projection of maps," in which the value in meters corresponding to 1 second of either latitude or longitude is given for all latitudes from 0° to 90°. Next, under the general heading "Distance," the column headed "Meters" is filled out by taking the antilogarithms of the values in the preceding column, and finally the one headed "Feet" is obtained by conversion of the meter values.

In the first column of the list of positions, in addition to the name of the station, there should be given the year of the first establishment of the station, the date when the station was last visited, and letters to indicate whether the station is described and marked or not. If the station is described but not marked the letter "d" is used; if it is marked and not described the letters "m. n. d." are used; and if it is described and marked the letters "d. m." are used. Other letters used in this column are: "n. d." which means that the station is not described, but whether or not it is marked is not known; "r" which means that the station was recovered but no description was furnished of its condition; "r. d." which means that the station was recovered and described; and "r. d. m." which means that the station was recovered, described, and marked.

The column headed "Seconds in meters" is ordinarily not filled out for first-order triangulation. The values in this column are computed for the convenience of the draftsman in constructing charts along the coast but as first-order triangulation is mostly in the interior of the country it is not used in making the charts.

The adjustment of the quadrilateral which has been given in detail on pages 29–41, fully illustrates the various steps that are necessary in an adjustment when only the geographic positions (latitudes and longitudes) of two initial stations, and the distance and azimuth between these stations are fixed. Where a number of connected triangles and quadrilaterals are to be adjusted, but the only fixed conditions are those given above, the steps in the adjustment are exactly the same as shown for the single quadrilateral, the only difference being in the number of equations, both angle and side.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 20 B

# GEOGRAPHIC POSITIONS

*Accession No. of Computation:* 35.607

*Locality* Eugene *Datum* North American *State* Oregon

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| STATION. | | LATITUDE AND LONGITUDE. ° ′ ″ | | | SECONDS IN METERS. | AZIMUTH ° ′ ″ | | | BACK AZIMUTH. ° ′ ″ | | | TO STATION. | DISTANCE. LOGARITHM (METERS). | METERS. | FEET. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Yellow | | 43 | 32 | 48.846 | 1507.5 | 146 | 31 | 32.27 | 326 | 17 | 39.01 | Roman | 4.688 0992 | 48763.99 | 159986.5 |
| 1904 | d.m. | 123 | 24 | 09.568 | 214.8 | 206 | 57 | 41.10 | 27 | 10 | 27.74 | Spencer | 4.736 2338 | 54479.59 | 178738.5 |
| Fairview | | 43 | 35 | 10.453 | 322.6 | 86 | 08 | 05.05 | 265 | 37 | 03.54 | Yellow | 4.783 6971 | 60771.10 | 199379.9 |
| 1904 | d.m. | 122 | 39 | 08.614 | 193.2 | 112 | 54 | 06.73 | 292 | 09 | 05.39 | Roman | 4.975 9778 | 94618.87 | 310428.7 |
| | | | | | | 141 | 15 | 27.89 | 320 | 57 | 05.85 | Spencer | 4.753 7076 | 56716.26 | 186076.6 |

FIG. 21.—List of geographic positions for new stations of quadrilateral

# CHAPTER 3.—ADJUSTMENT OF A NET OF TRIANGULATION BY THE DIRECTION METHOD

## CONDITIONS INVOLVED

In the adjustment of the single quadrilateral just considered, only angle and side equations were required. The adjustment of a net of triangulation is given on the following pages for which are required not only angle and side equations, but condition equations to eliminate the closing errors in length, azimuth, latitude, and longitude. The direction method will be used, as first-order triangulation is always adjusted by that method in this bureau. On pages 110–146 is shown a net of third-order triangulation adjusted by the angle method.

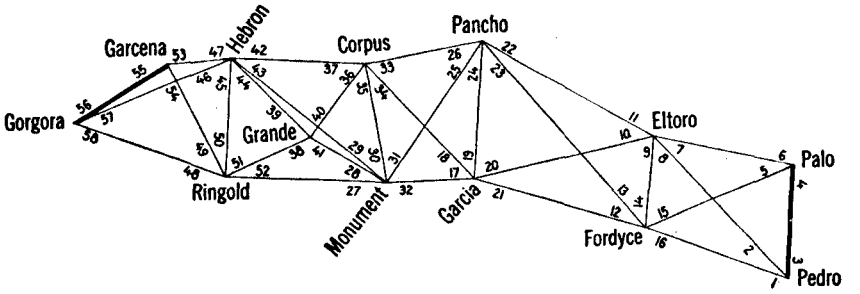


FIG. 22.—Triangulation net used in sample adjustment

As shown in the sketch of this net (fig. 22), the lines Garcena-Gorgora and Palo-Pedro are fixed in length and azimuth and the stations Garcena, Gorgora, Palo, and Pedro are fixed in position (latitude and longitude). The angle and side condition equations for this net are formed in the same manner as explained on pages 35–37 for the single quadrilateral.

The length equation is formed by starting with the fixed length Palo-Pedro and computing through the observed angles of a single chain of triangles the length of the line Garcena-Gorgora. This computed length will usually differ from the fixed length of the line and the length condition is thus obtained.

The azimuth equation is formed by starting with the fixed azimuth of the line Palo-Pedro and computing through the observed angles of the triangles an azimuth for the line Garcena-Gorgora. This computed azimuth will usually differ from the fixed azimuth of the line and the azimuth condition is thus obtained.

The latitude and longitude equations are formed by starting with the fixed position (latitude and longitude) of either Palo or Pedro

and computing through the observed angles of a single chain of triangles the geographic position (latitude and longitude) of Garcena. This computed position of Garcena will usually differ from the fixed position and the latitude and longitude conditions are thus obtained.

All the steps in the adjustment of this net of triangulation are given in detail on the following pages. Although the net is only a small one all the conditions that would appear in a large net are present, so that an understanding of this small adjustment will enable one to adjust a large net.

The formation of the equations representing all the conditions considered above are shown below in detail. If the lists of directions at the various stations have not been made out in the field they are made in the form shown on page 7, with the column "Final seconds" left blank.

A sketch (fig. 22) is then drawn showing the relative positions of the stations and all the observed directions between the stations, full lines representing directions observed from both stations at the ends of the line and lines dotted at one end representing directions observed from the station at the full end of the line but not from the other station. The directions are numbered in clockwise order on the sketch as shown in Figure 22. The triangles (fig. 24) are next written out and the angles filled in from the list of directions in Figure 23. Particular attention should be paid to the order in which the triangles are written. (See p. 29.)

As full instructions for determining the spherical excess and triangle closures, and for checking the triangle closures are given on pages 32–34 for the single quadrilateral, they will not be repeated here.

## NUMBER OF EQUATIONS

After the closures of the triangles have been determined, the next step is to determine the number of equations in the net. The formulas for computing the number of equations to be used in the adjustment of a triangulation net are given on page 34. In the net considered here, $n=29$, $n'=29$, $s=13$, and $s'=13$. The number of angle equations is therefore $n'-s'+1=29-13+1=17$, and the number of side equations is $n-2s+3=29-26+3=6$. Since there are fixed azimuths and lengths at the two ends of the net, there will also be one azimuth and one length equation. In addition to these, due to the fixed position (latitude and longitude) at each end of the net, there will be one latitude and one longitude equation. Altogether then there are 17 angle, 6 side, 1 azimuth, 1 length, 1 latitude, and 1 longitude equations, or a total of 27 equations.

Another way of determining the number of angle and side equations in a net is to build up the figure point by point, starting from the fixed line at one end, and count the number of equations at each

(Text continued on p. 59)

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 34a

LIST OF DIRECTIONS

*State:* Texas

Station Palo — Computed by A.C.D.
Observer C.V.H. 1917 — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Pedro | 0 00 00.00 | 59.95 |
| Fordyce | 61 27 31.53 | 31.71 |
| Eltoro | 99 37 25.05 | 24.93 |

Station Pedro — Computed by A.C.D.
Observer C.V.H. 1917 — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Fordyce | 0 00 00.00 | 00.29 |
| Eltoro | 26 37 19.81 | 19.61 |
| Palo | 70 32 18.94 | 18.86 |

Station Fordyce — Computed by A.C.D.
Observer C.V.H. 1917 — Checked by W.F.R.

A

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Garcia | 0 00 00.00 | 59.99 |
| Pancho | 33 42 51.34 | 51.75 |
| Eltoro | 76 47 59.67 | 60.01 |
| Palo | 140 50 53.52 | 53.15 |
| Pedro | 188 51 03.46 | 03.08 |

Station Eltoro — Computed by A.C.D.
Observer C.V.H. 1917 — Checked by W.F.R.

B

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Palo | 0 00 00.00 | 00.43 |
| Pedro | 36 27 36.43 | 36.47 |
| Fordyce | 77 47 14.58 | 14.28 |
| Garcia | 149 39 01.51 | 01.57 |
| Pancho | 194 18 20.42 | 20.19 |

Station Garcia — Computed by A.C.D.
Observer C.V.H. 1917 — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Monument | 0 00 00.00 | 00.46 |
| Corpus | 51 15 33.50 | 33.51 |
| Pancho | 89 53 49.28 | 49.12 |
| Eltoro | 160 46 14.32 | 14.39 |
| Fordyce | 192 06 27.73 | 27.35 |

Station Pancho — Computed by A.C.D.
Observer C.V.H. 1917 — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Eltoro | 0 00 00.00 | 00.43 |
| Fordyce | 20 23 46.79 | 46.52 |
| Garcia | 64 28 16.62 | 16.92 |
| Monument | 92 56 19.84 | 19.79 |
| Corpus | 142 14 59.23 | 58.83 |

Station Monument — Computed by A.C.D.
Observer C.V.H. 1917 — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Kingold | 0 00 00.00 | 59.64 |
| Grande | 25 00 27.23 | 27.85 |
| Hebron | 31 47 59.55 | 59.97 |
| Corpus | 80 32 48.42 | 48.61 |
| Pancho | 118 19 55.00 | 54.61 |
| Garcia | 179 58 03.72 | 03.25 |

Station Corpus — Computed by A.C.D.
Observer C.V.H. 1917 — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Pancho | 0 00 00.00 | 00.68 |
| Garcia | 63 35 03.71 | 03.37 |
| Monument | 92 54 15.91 | 15.83 |
| Grande | 139 12 15.04 | 15.26 |
| Hebron | 186 42 58.48 | 58.00 |

FIG. 23.—Lists of directions for stations of net

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 24A

LIST OF DIRECTIONS

State: Texas

Station Grande — Computed by A.C.D.
Observer C.V.H. 1917 — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Ringold | 0 00 00.00 | 00.09 |
| Hebron | 63 08 11.31 | 11.49 |
| Corpus | 152 41 08.92 | 08.87 |
| Monument | 230 50 49.05 | 48.82 |

Station Hebron — Computed by A.C.D.
Observer C.V.H. 1917 — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Corpus | 0 00 00.00 | 00.55 |
| Monument | 37 26 30.55 | 30.03 |
| Grande | 42 56 20.48 | 20.61 |
| Ringold | 90 19 02.33 | 02.02 |
| Gorgora | 161 52 16.28 | 16.58 |
| Garcena | 173 35 47.89 | 47.73 |

Station Ringold — Computed by A.C.D.
Observer C.V.H. 1917 — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Gorgora | 0 00 00.00 | 00.03 |
| Garcena | 41 19 43.99 | 44.21 |
| Hebron | 70 44 38.44 | 38.50 |
| Grande | 140 13 45.67 | 45.83 |
| Monument | 166 04 06.90 | 06.45 |

Station Garcena — Computed by A.C.D.
Observer C.L.G. 1917 — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Hebron | 0 00 00.00 | 00.16 |
| Ringold | 67 18 20.19 | 20.25 |
| Gorgora | 162 14 05.66 | 05.43 |

Station Gorgora — Computed by A.C.D.
Observer C.L.G. 1917 — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° ′ ″ | Final Seconds ″ |
|---|---|---|
| Garcena | 0 00 00.00 | 00.57 |
| Hebron | 6 02 24.09 | 24.18 |
| Ringold | 43 44 32.07 | 31.41 |

FIG. 23.—Lists of directions for stations of net—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Texas

| NO. | | STATION | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|---|
| | 2-3 | Palo - Pedro | | | | | | 3.9781520 |
| -15+16 | 1 | Fordyce A | 48 00 09.94 | -0.01 | 09.93 | 0.08 | 09.85 | 0.1289079 |
| -4+5 | 2 | Palo D | 61 27 31.53 | +0.23 | 31.76 | 0.09 | 31.67 | 9.9437288 |
| -1+3 | 3 | Pedro C | 70 32 18.94 | -0.37 | 18.57 | 0.09 | 18.48 | 9.9744497 |
| | 1-3 | Fordyce - Pedro | | -0.15 | | 0.26 | | 4.0507887 |
| | 1-2 | Fordyce - Palo | | | | | | 4.0815096 |
| | | | 00.41 | | | | | |
| | 2-3 | Palo - Pedro | | | | | | 3.9781520 |
| -7+8 | 1 | P Eltoro | 36 27 36.43 | -0.39 | 36.04 | 0.09 | 35.95 | 0.2260226 |
| -4+6 | 2 | Palo | 99 37 25.05 | -0.07 | 24.98 | 0.09 | 24.89 | 9.9938449 |
| -2+3 | 3 | Pedro | 43 54 59.13 | +0.12 | 59.25 | 0.09 | 59.16 | 9.8411144 |
| | 1-3 | Eltoro - Pedro | | -0.34 | | 0.27 | | 4.1980195 |
| | 1-2 | Eltoro - Palo | | | | | | 4.0452890 |
| | | | 00.61 | | | | | |
| | 2-3 | Palo - Fordyce | | | | | | 4.0815096 |
| -7+9 | 1 | Eltoro | 77 47 14.58 | -0.73 | 13.85 | 0.07 | 13.78 | 0.0099416 |
| -5+6 | 2 | Palo | 38 09 53.52 | -0.30 | 53.22 | 0.07 | 53.15 | 9.7909358 |
| -14+15 | 3 | Fordyce | 64 02 53.85 | -0.71 | 53.14 | 0.07 | 53.07 | 9.9538377 |
| | 1-3 | Eltoro - Fordyce | | -1.74 | | 0.21 | | 3.8823870 |
| | 1-2 | Eltoro - Palo | | | | | | 4.0452889 +1 |
| | | | 01.95 | | | | | |
| | 2-3 | Pedro - Fordyce | | | | | | 4.0507887 |
| Eltoro -8+9 | 1 | Elton | 41 19 38.15 | -0.34 | 37.81 | 0.07 | 37.74 | 0.1802208 |
| -1+2 | 2 | Pedro | 26 37 19.81 | -0.49 | 19.32 | 0.06 | 19.26 | 9.6513775 |
| -14+16 | 3 | Fordyce | 112 03 03.79 | -0.72 | 03.07 | 0.07 | 03.00 | 9.9670100 |
| | 1-3 | Eltoro - Fordyce | | -1.55 | | 0.20 | | 3.8823870 |
| | 1-2 | Eltoro - Pedro | | | | | | 4.1980195 |
| | | | 01.75 | | | | | |
| | 2-3 | Eltoro - Fordyce | | | | | | 3.8823870 |
| -20+21 | 1 | Garcia | 31 20 13.41 | -0.45 | 12.96 | 0.09 | 12.87 | 0.2839387 |
| -9+10 | 2 | Eltoro | 71 51 46.93 | +0.36 | 47.29 | 0.09 | 47.20 | 9.9778678 |
| -12+14 | 3 | Fordyce | 76 47 59.67 | +0.35 | 60.02 | 0.09 | 59.93 | 9.9883711 |
| | 1-3 | Garcia - Fordyce | | +0.26 | | 0.27 | | 4.1441935 |
| | 1-2 | Garcia - Eltoro | | | | | | 4.1546968 |
| | | | 00.01 | | | | | |

FIG. 24.—Triangle computation for stations of net

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 25

COMPUTATION OF TRIANGLES

*State:* Texas

| NO. | STATION | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|
| 2-3 | Eltoro - Fordyce | | | | | | 3.8823870 |
| -22+23 1 | Pancho | 20 23 46.79 | -0.70 | 46.09 | 0.08 | 46.01 | 0.4577866 |
| -9+11 2 | Eltoro | 116 31 05.84 | +0.07 | 05.91 | 0.09 | 05.82 | 9.9517221 |
| -13+14 3 | Fordyce | 43 05 08.33 | -0.07 | 08.26 | 0.09 | 08.17 | 9.8344781 |
| 1-3 | Pancho - Fordyce | | -0.70 | | 0.26 | | 4.2918957 |
| 1-2 | Pancho - Eltoro | | | | | | 4.1746517 +1 |
| | | 00.96 | | | | | |
| 2-3 | Eltoro - Garcia | | | | | | 4.1546968 |
| -22+24 1 | Pancho | 64 28 16.62 | -0.13 | 16.49 | 0.13 | 16.36 | 0.0446159 |
| -10+11 2 | Eltoro | 44 39 18.91 | -0.29 | 18.62 | 0.12 | 18.50 | 9.8468552 |
| -19+20 3 | Garcia | 70 52 25.04 | +0.23 | 25.27 | 0.13 | 25.14 | 9.9753391 |
| 1-3 | Pancho - Garcia | | -0.19 | | 0.38 | | 4.0461679 |
| 1-2 | Pancho - Eltoro | | | | | | 4.1746518 |
| | | 00.57 | | | | | |
| 2-3 | Fordyce - Garcia | | | | | | 4.1441935 |
| -23+24 1 | Pancho | 44 04 29.83 | +0.57 | 30.40 | 0.13 | 30.27 | 0.1576402 |
| -12+13 2 | Fordyce | 33 42 51.34 | +0.42 | 51.76 | 0.13 | 51.63 | 9.7443342 |
| -19+21 3 | Garcia | 102 12 38.45 | -0.22 | 38.23 | 0.13 | 38.10 | 9.9900620 |
| 1-3 | Pancho - Garcia | | +0.77 | | 0.39 | | 4.0461679 |
| 1-2 | Pancho - Fordyce | | | | | | 4.2918957 |
| | | 59.62 | | | | | |
| 2-3 | Pancho - Garcia | | | | | | 4.0461679 |
| -31+32 1 | Monument | 61 38 08.72 | -0.08 | 08.64 | 0.06 | 08.58 | 0.0555446 |
| -24+25 2 | Pancho | 28 28 03.22 | -0.35 | 02.87 | 0.05 | 02.82 | 9.6782083 |
| -17+19 3 | Garcia | 89 53 49.28 | -0.62 | 48.66 | 0.06 | 48.60 | 9.9999993 |
| 1-3 | Monument - Garcia | | -1.05 | | 0.17 | | 3.7799207 |
| 1-2 | Monument - Pancho | | | | | | 4.1017118 |
| | | 01.22 | | | | | |
| 2-3 | Pancho - Garcia | | | | | | 4.0461679 |
| -33+34 1 | Corpus | 63 35 03.71 | -1.02 | 02.69 | 0.07 | 02.62 | 0.0478917 |
| -24+26 2 | Pancho | 77 46 42.61 | -0.70 | 41.91 | 0.07 | 41.84 | 9.9900438 |
| -18+19 3 | Garcia | 38 38 15.78 | -0.17 | 15.61 | 0.07 | 15.54 | 9.7954581 |
| 1-3 | Corpus - Garcia | | -1.89 | | 0.21 | | 4.0341034 |
| 1-2 | Corpus - Pancho | | | | | | 3.8895177 |
| | | 02.10 | | | | | |

FIG. 24.—Triangle computation for stations of net—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 88

COMPUTATION OF TRIANGLES

State: Texas

| NO. | STATION | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|
| 2-3 | Pancho - Monument | | | | | | 4.1017118 |
| -33+25 1 | Corpus | 92 54 15.91 | -0.76 | 15.15 | 0.07 | 15.08 | 0.0005581 |
| -25+26 2 | Pancho | 49 18 39.39 | -0.35 | 39.04 | 0.06 | 38.98 | 9.8798168 |
| -30+31 3 | Monument | 37 47 06.58 | -0.58 | 06.00 | 0.06 | 05.94 | 9.7872478 |
| 1-3 | Corpus - Monument | | -1.69 | | 0.19 | | 3.9820867 |
| 1-2 | Corpus - Pancho | | | | | | 3.8895177 |
| | | 01.88 | | | | | |
| 2-3 | Garcia - Monument | | | | | | 3.7799207 |
| -34+35 1 | Corpus | 29 19 12.20 | +0.26 | 12.46 | 0.05 | 12.41 | 0.3100800 |
| -17+18 2 | Garcia | 51 15 33.50 | -0.45 | 33.05 | 0.05 | 33.00 | 9.8920860 |
| -30+32 3 | Monument | 99 25 15.30 | -0.66 | 14.64 | 0.05 | 14.59 | 9.9941028 |
| 1-3 | Corpus - Monument | | -0.85 | | 0.15 | | 3.9820867 |
| 1-2 | Corpus - Garcia | | | | | | 4.0841035 |
| | | 01.00 | | | | | |
| 2-3 | Corpus - Monument | | | | | | 3.9820867 |
| -40+41 1 | Grande | 78 09 40.13 | -0.1839.95 | | 0.05 | 39.90 | 0.0093379 |
| -35+36 2 | Corpus | 46 17 59.13 | +0.3059.43 | | 0.04 | 59.39 | 9.8591173 |
| -28+30 3 | Monument | 55 32 21.19 | -0.4320.76 | | 0.05 | 20.71 | 9.9161972 |
| 1-3 | Grande - Monument | | -0.31 | | 0.14 | | 3.8505419 |
| 1-2 | Grande - Corpus | | | | | | 3.9076218 |
| | | 00.45 | | | | | |
| 2-3 | Corpus - Monument | | | | | | 3.9820867 |
| -42+43 1 | Hebron | 37 26 30.55 | -1.07 | 29.48 | 0.09 | 29.39 | 0.2161314 |
| -35+37 2 | Corpus | 93 48 42.57 | -0.40 | 42.17 | 0.10 | 42.07 | 9.9990382 |
| -29+30 3 | Monument | 48 44 48.87 | -0.23 | 48.64 | 0.10 | 48.54 | 9.8761042 |
| 1-3 | Hebron - Monument | | -1.70 | | 0.29 | | 4.1972563 |
| 1-2 | Hebron - Corpus | | | | | | 4.0743223 |
| | | 01.99 | | | | | |
| 2-3 | Corpus - Grande | | | | | | 3.9076318 |
| -42+44 1 | Hebron | 42 56 20.48 | -0.42 | 20.06 | 0.06 | 20.00 | 0.1667139 |
| -36+37 2 | Corpus | 47 30 43.44 | -0.70 | 42.74 | 0.06 | 42.68 | 9.8677132 |
| -39+40 3 | Grande | 89 32 57.61 | -0.23 | 57.38 | 0.06 | 57.32 | 9.9999866 |
| 1-3 | Hebron - Grande | | -1.35 | | 0.13 | | 3.9420489 |
| 1-2 | Hebron - Corpus | | | | | | 4.0743223 |
| | | 01.53 | | | | | |

FIG. 24.—Triangle computation for stations of net—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 20

**COMPUTATION OF TRIANGLES**

*State:* Texas

| NO. | | STATION | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|---|
| | 2-3 | Monument - Grande | | | | | | 3.8505419 |
| -43+44 | 1 | Hebron | 5 29 49.93 | +0.65 | 50.58 | 0.01 | 50.57 | 1.0136334 |
| -28+29 | 2 | Monument | 6 47 32.32 | -0.20 | 32.12 | 0.01 | 32.11 | 9.0728735 |
| -39+41 | 3 | Grande | 167 42 37.74 | -0.41 | 37.33 | 0.01 | 37.32 | 9.3280810 |
| | 1-3 | Hebron - Grande | | +0.04 | | 0.03 | | 3.9420488 +1 |
| | 1-2 | Hebron - Monument | | | | | | 4.1972563 |
| | | | 59.99 | | | | | |
| | 2-3 | Hebron - Grande | | | | | | 3.9420489 |
| -50+51 | 1 | Ringold | 69 29 07.23 | +0.10 | 07.33 | 0.05 | 07.28 | 0.0284539 |
| -44+45 | 2 | Hebron | 47 22 41.85 | -0.44 | 41.41 | 0.04 | 41.37 | 9.8667328 |
| -38+39 | 3 | Grande | 63 08 11.31 | +0.09 | 11.40 | 0.05 | 11.35 | 9.9504064 |
| | 1-3 | Ringold - Grande | | -0.25 | | 0.14 | | 3.8372856 |
| | 1-2 | Ringold - Hebron | | | | | | 3.9209092 |
| | | | 00.39 | | | | | |
| | 2-3 | Hebron - Monument | | | | | | 4.1972563 |
| -50+52 | 1 | Ringold | 95 19 28.46 | -0.51 | 27.95 | 0.09 | 27.86 | 0.0018779 |
| -43+45 | 2 | Hebron | 52 52 31.78 | +0.21 | 31.99 | 0.09 | 31.90 | 9.9016361 |
| -27+29 | 3 | Monument | 31 47 59.55 | +0.78 | 60.33 | 0.09 | 60.24 | 9.7217750 |
| | 1-3 | Ringold - Monument | | +0.48 | | 0.27 | | 4.1007703 |
| | 1-2 | Ringold - Hebron | | | | | | 3.9209092 |
| | | | 59.79 | | | | | |
| | 2-3 | Grande - Monument | | | | | | 3.8505419 |
| -51+52 | 1 | Ringold | 25 50 21.23 | -0.61 | 20.62 | 0.03 | 20.59 | 0.3606682 |
| +38-41 | 2 | Grande | 129 09 10.95 | +0.32 | 11.27 | 0.04 | 11.23 | 9.8895602 |
| -27+28 | 3 | Monument | 25 00 27.23 | +0.98 | 28.21 | 0.03 | 28.18 | 9.6260755 |
| | 1-3 | Ringold - Monument | | +0.69 | | 0.10 | | 4.1007703 |
| | 1-2 | Ringold - Grande | | | | | | 3.8372856 |
| | | | 59.41 | | | | | |
| | 2-3 | Hebron - Ringold | | | | | | 3.9209092 |
| -53+54 | 1 | Garcena | 67 18 20.19 | -0.10 | 20.09 | 0.03 | 20.06 | 0.0349980 |
| -45+47 | 2 | Hebron | 83 16 45.56 | +0.15 | 45.71 | 0.03 | 45.68 | 9.9970055 |
| -49+50 | 3 | Ringold | 29 24 54.45 | -0.16 | 54.29 | 0.03 | 54.26 | 9.6911991 |
| | 1-3 | Garcia - Ringold | | -0.11 | | 0.09 | | 3.9529127 |
| | 1-2 | Garcia - Hebron | | | | | | 3.6471063 |
| | | | 00.20 | | | | | |

FIG. 24.—Triangle computation for stations of net—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 55

## COMPUTATION OF TRIANGLES

*State:* Texas

| NO. | | STATION | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|---|
| | 2-3 | Garcena - Hebron | | | | | | 3.6471063 |
| -56+57 | 1 | Gorgora | 6 02 24.09 | -0.48 | 23.61 | 0.01 | 23.60 | 0.9773983 |
| -53+55 | 2 | Garcena | 162 14 05.66 | -0.39 | 05.27 | 0.01 | 05.26 | 9.4844665 |
| -46+47 | 3 | Hebron | 11 43 31.61 | -0.46 | 31.15 | 0.01 | 31.14 | 9.3079664 |
| | 1-3 | Gorgora - Hebron | | -1.33 | | 0.03 | | 4.1094711 |
| | 1-2 | Gorgora - Garcena | | | | | | 3.9329710 |
| | | | 01.36 | | | | | |
| | 2-3 | Garcena - Ringold | | | | | | 3.9529127 |
| -56+58 | 1 | Gorgora | 43 44 32.07 | -1.23 | 30.84 | 0.07 | 30.77 | 0.1602639 |
| -54+55 | 2 | Garcena | 94 55 45.47 | -0.29 | 45.18 | 0.07 | 45.11 | 9.9983908 |
| -48+49 | 3 | Ringold | 41 19 43.99 | +0.19 | 44.18 | 0.06 | 44.12 | 9.8197944 |
| | 1-3 | Gorgora - Ringold | | -1.33 | | 0.20 | | 4.1115674 |
| | 1-2 | Gorgora - Garcena | | | | | | 3.9329710 |
| | | | 01.53 | | | | | |
| | 2-3 | Hebron - Ringold | | | | | | 3.9209092 |
| -57+58 | 1 | Gorgora | 37 42 07.98 | -0.75 | 07.23 | 0.08 | 07.15 | 0.2135649 |
| -45+46 | 2 | Hebron | 71 33 13.95 | +0.61 | 14.56 | 0.09 | 14.47 | 9.9770934 |
| -48+50 | 3 | Ringold | 70 44 38.44 | +0.03 | 38.47 | 0.09 | 38.38 | 9.9749970 |
| | 1-3 | Gorgora - Ringold | | -0.11 | | 0.26 | | 4.1115675 -1 |
| | 1-2 | Gorgora - Hebron | | | | | | 4.1094711 |
| | | | 00.37 | | | | | |

FIG. 24.—Triangle computation for stations of net—Continued

point. The rules to follow in this case are: At each point the number of angle equations is one less than the number of full lines (observed at both ends) and the number of side equations is two less than the total number of lines, counting both the full lines and those dotted at one end.

For example, in the net to be adjusted, start at the fixed line Palo-Pedro. Then at station Fordyce there are 2 full lines to stations already considered and so there are one angle and no side equations; at station Eltoro there are 3 full lines to stations already used and so there are 2 angle and 1 side equations; at Garcia there are 2 full lines and therefore 1 angle and no side equations; at Pancho there are 3 full lines and therefore 2 angle and 1 side equations. Continuing this process of building up the figure point by point until the final point, Gorgora, is reached, we have the following number of angle and side equations:

*Number of angle and side equations in net*

| Station | Equations | |
|---|---|---|
| | Angle | Side |
| Fordyce | 1 | 0 |
| Eltoro | 2 | 1 |
| Garcia | 1 | 0 |
| Pancho | 2 | 1 |
| Monument | 1 | 0 |
| Corpus | 2 | 1 |
| Grande | 1 | 0 |
| Hebron | 2 | 1 |
| Ringold | 2 | 1 |
| Garcena | 1 | 0 |
| Gorgora | 2 | 1 |
| Total | 17 | 6 |

A total of 17 angle and 6 side equations is thus obtained and this agrees with the number of equations determined by means of the formulas. In a complicated figure for which it is difficult to determine the correct number of equations the number should be checked by using both methods.

## FORMATION OF CONDITION EQUATIONS

The angle equations, which are shown on page 60, are obtained in the same manner as in the adjustment of the single quadrilateral. (See p. 35.) The side equations are also formed in the same manner as the one used in the adjustment of the single quadrilateral. One should be careful at all times to choose the pole in each side equation so that the two smallest angles will be included for, otherwise, the final adjusted logarithms of the distances in the triangle computation may not check.

The angle and side equations should be formed by building up the figure point by point in the same manner as for counting the equations and writing out the equations at each new point as the figure is built up. In this way not only will the correct number of equations be obtained, but the introduction of "identical" equations will be avoided. "Identical" equations are sometimes introduced accidentally and not discovered until the solution of the normals is considerably advanced. (An illustration of an "identical" equation is given on p. 181.)

*Angle equations*

1. $0=+0.15-(1)+(3)-(4)+(5)-(15)+(16)$
2. $0=+0.34-(2)+(3)-(4)+(6)-(7)+(8)$
3. $0=+1.74-(5)+(6)-(7)+(9)-(14)+(15)$
4. $0=-0.26-(9)+(10)-(12)+(14)-(20)+(21)$
5. $0=+0.19-(10)+(11)-(19)+(20)-(22)+(24)$
6. $0=-0.77-(12)+(13)-(19)+(21)-(23)+(24)$
7. $0=+1.05-(17)+(19)-(24)+(25)-(31)+(32)$
8. $0=+1.89-(18)+(19)-(24)+(26)-(33)+(34)$
9. $0=+1.69-(25)+(26)-(30)+(31)-(33)+(35)$
10. $0=+0.31-(28)+(30)-(35)+(36)-(40)+(41)$
11. $0=-0.69-(27)+(28)+(38)-(41)-(51)+(52)$
12. $0=+1.70-(29)+(30)-(35)+(37)-(42)+(43)$
13. $0=+1.35-(36)+(37)-(39)+(40)-(42)+(44)$
14. $0=+0.25-(38)+(39)-(44)+(45)-(50)+(51)$
15. $0=+0.11-(49)+(50)-(45)+(47)-(53)+(54)$
16. $0=+1.33-(48)+(49)-(54)+(55)-(56)+(58)$
17. $0=+0.11-(45)+(46)-(48)+(50)-(57)+(58)$

*Side equations*

| Symbol | Angle | Logarithm | Tabular difference | Symbol | Angle | Logarithm | Tabular difference |
|---|---|---|---|---|---|---|---|
| | ° ′ ″ | | | | ° ′ ″ | | |
| −15+16 | 48 00 09.94 | 9.87109231 | +1.90 | −4+5 | 61 27 31.53 | 9.94372863 | +1.15 |
| −8+9 | 41 19 38.15 | 9.81978015 | +2.40 | −1+2 | 26 37 19.81 | 9.65137982 | +4.20 |
| −5+6 | 38 09 53.52 | 9.79093675 | +2.67 | −14+15 | 64 02 53.85 | 9.95383853 | +1.02 |
| −2+3 | 43 54 59.13 | 9.84111431 | +2.19 | −7+8 | 36 27 36.43 | 9.77397878 | +2.85 |
| | | 9.32292352 | | | | 9.32292576 | |

[1] 18. $0=-2.24+4.20\ (1)-6.39\ (2)+2.19\ (3)+1.15\ (4)-3.82\ (5)+2.67\ (6)+2.85\ (7)-5.25\ (8)+2.40\ (9)+1.02\ (14)-2.92\ (15)+1.90\ (16)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| −20+21 | 31 20 13.41 | 9.71606319 | +3.46 | −12+14 | 76 47 59.67 | 9.98837099 | +0.49 |
| −22+24 | 64 28 16.62 | 9.95538436 | +1.00 | −19+20 | 70 52 25.04 | 9.97533904 | +.73 |
| −13+14 | 43 05 08.33 | 9.83447846 | +2.25 | −22+23 | 20 23 46.79 | 9.54221776 | +5.66 |
| | | 9.50592601 | | | | 9.50592779 | |

19. $0=-1.78+0.49\ (12)-2.25\ (13)+1.76\ (14)+0.73\ (19)-4.19\ (20)+3.46\ (21)+4.66\ (22)-5.66\ (23)+1.00\ (24)$

[1] This equation is formed with the pole at the intersection of the two diagonals. (See explanation on p. 185.)

*Side equations*—Continued

| Symbol | Angle | Logarithm | Tabular difference | Symbol | Angle | Logarithm | Tabular difference |
|---|---|---|---|---|---|---|---|
| | ° ′ ″ | | | | ° ′ ″ | | |
| −34+35 | 29 19 12.20 | 9.68991918 | +3.74 | −30+32 | 99 25 15.30 | 9.99410258 | −0.34 |
| −24+26 | 77 46 42.61 | 9.99004413 | +.45 | −33+34 | 63 35 03.71 | 9.95210943 | +1.05 |
| −31+32 | 61 38 08.72 | 9.94445563 | +1.13 | −24+25 | 28 28 03.22 | 9.67820974 | +3.89 |
| | | 9.62441894 | | | | 9.62442175 | |

20. $0 = -2.81 + 3.44\,(24) - 3.89\,(25) + 0.45\,(26) - 0.34\,(30) - 1.13\,(31) + 1.47\,(32) + 1.05\,(33) - 4.79\,(34) + 3.74\,(35)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| −28+29 | 6 47 32.32 | 9.07287722 | +17.68 | −43+44 | 5 29 49.93 | 8.98135262 | +21.88 |
| −51+52 | 25 50 21.23 | 9.63933451 | +4.34 | −27+28 | 25 00 27.23 | 9.62607119 | +4.52 |
| −44+45 | 47 22 41.85 | 9.86678373 | +1.93 | −50+51 | 69 29 07.23 | 9.97154607 | +.78 |
| | | 8.57899546 | | | | 8.57896988 | |

[1] 21. $0 = +25.58 + 4.52\,(27) - 22.20\,(28) + 17.68\,(29) + 21.88\,(43) - 23.81\,(44) + 1.93\,(45) + 0.78\,(50) - 5.12\,(51) + 4.34\,(52)$. (This equation should be divided by 5 before entering it in the correlates.)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| −44+45 | 47 22 41.85 | 9.86678373 | +1.93 | −50+51 | 69 29 07.23 | 9.97154607 | +0.78 |
| −36+37 | 47 30 43.44 | 9.86771468 | +1.93 | −42+44 | 42 56 20.48 | 9.83328715 | +2.27 |
| −28+30 | 55 32 21.19 | 9.91619788 | +1.45 | −35+36 | 46 17 59.13 | 9.85911681 | +2.01 |
| −51+52 | 25 50 21.23 | 9.63933451 | +4.34 | −27+28 | 25 00 27.23 | 9.62607119 | +4.52 |
| | | 9.29003080 | | | | 9.29002122 | |

[2] 22. $0 = +9.58 + 4.52(27) - 5.97(28) + 1.45(30) + 2.01(35) - 3.94(36) + 1.93(37) + 2.27(42) - 4.20(44) + 1.93(45) + 0.78(50) - 5.12(51) + 4.34(52)$.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| −48+49 | 41 19 43.99 | 9.81979414 | +2.39 | −56+58 | 43 44 32.07 | 9.83973894 | +2.20 |
| −45+47 | 83 16 45.56 | 9.99700544 | +.25 | −49+50 | 29 24 54.45 | 9.69119975 | +3.74 |
| −56+57 | 6 02 24.09 | 9.02211142 | +19.90 | −46+47 | 11 43 31.61 | 9.30797112 | +10.14 |
| | | 8.83891100 | | | | 8.83890981 | |

[1] 23. $0 = +1.19 - 0.25(45) + 10.14(46) - 9.89(47) - 2.39(48) + 6.13(49) - 3.74(50) - 17.70(56) + 19.90(57) - 2.20(58)$. (This equation should be divided by 5 before entering it in the correlates.)

[1] The coefficients of the different equations of a net should be approximately the same size. If the coefficients of an equation are too large, they should be divided by a factor to bring them to the proper size.
[2] In a central point figure with one diagonal observed, which requires 2 side equations, one of the side equations should be written with the pole at the center and should be carried entirely around the figure.

## EXPLANATION OF AZIMUTH EQUATION

In the formation of the azimuth equations differences between forward and back azimuths are used and it is necessary, therefore, to make a preliminary computation of the geographic positions of the various stations to determine these azimuths. This same computation is used in determining the latitude and longitude closures and an explanation of it will be found on pages 66 and 69. We shall consider here that these positions have already been computed.

Starting with the fixed azimuth, Palo-Pedro, and using the observed directions and the differences between the azimuths and back azimuths through the shortest route, the azimuth of the line Garcena-Gorgora is determined. It is well to go through the angles in a clockwise direction, wherever possible. The angles will then be added instead of subtracted.

*Computation of azimuth equation*

| | ° | ′ | ″ | Final seconds* |
|---|---|---|---|---|
| Palo-Pedro (fixed azimuth) | 12 | 02 | 25.00 | 25.00 |
| −4+6 | +99 | 37 | 25.05 | 24.98 |
| Palo-Eltoro | 111 | 39 | 50.05 | 49.98 |
| | − | 2 | 45.12 | 45.11 |
| Eltoro-Palo | 291 | 37 | 04.93 | 04.87 |
| −7+11 | +194 | 18 | 20.42 | 19.76 |
| Eltoro-Pancho | 125 | 55 | 25.35 | 24.63 |
| | − | 3 | 14.32 | 14.32 |
| Pancho-Eltoro | 305 | 52 | 11.03 | 10.31 |
| −22+26 | +142 | 14 | 59.23 | 58.40 |
| Pancho-Corpus | 88 | 07 | 10.26 | 08.71 |
| | − | 2 | 04.55 | 04.55 |
| Corpus-Pancho | 268 | 05 | 05.71 | 04.16 |
| −33+37 | +186 | 42 | 58.48 | 57.32 |
| Corpus-Hebron | 94 | 48 | 04.19 | 01.48 |
| | − | 3 | 10.09 | 10.09 |
| Hebron-Corpus | 274 | 44 | 54.10 | 51.39 |
| −42+47 | +173 | 35 | 47.89 | 47.18 |
| Hebron-Garcena | 88 | 20 | 41.99 | 38.57 |
| | − | 1 | 11.31 | 11.31 |
| Garcena-Hebron | 268 | 19 | 30.68 | 27.26 |
| −53+55 | +162 | 14 | 05.66 | 05.27 |
| Garcena-Gorgora (computed azimuth) | 70 | 33 | 36.34 | 32.53 |
| Garcena-Gorgora (fixed azimuth) | 70 | 33 | 32.53 | 32.53 |
| Closing error | | | +3.81 | 0.00 |

24. $0 = +3.81\dagger - (4) + (6) - (7) + (11) - (22) + (26) - (33) + (37) - (42) + (47) - (53) + (55)$

The azimuth equation above is made up as follows (see p. 50): Starting with the azimuth Palo-Pedro, 12° 02′ 25″.00, and adding the angle at Palo from Pedro to Eltoro (designated by −4+6), 99° 37′ 25″.05, we get 111° 39′ 50″.05, which is the azimuth of the line Palo-Eltoro. Now in order to carry the azimuth ahead, it is necessary to obtain the back azimuth of this line or the azimuth

* This column is filled out after the adjustment is completed.

† The azimuth constant should be corrected by the amount $\sin \phi_m (\lambda_n - \lambda_n')$, where $\phi_m$ is the mean latitude and $\lambda_n - \lambda_n'$ is the discrepancy in longitude.

$\phi_m = 26° 40' 49''$ and $\lambda_n - \lambda_n' = -0''.006$

log 0.006 = 7.7782-10 (negative).

log sin 26° 40′ 49″ = 9.6523-10

log correction = 7.4305-10 (negative).

correction = −0″.003.

This correction is not large enough to affect the azimuth constant.

Eltoro-Palo. This is obtained by adding 180° and (algebraically) the difference of azimuth due to convergence of meridians as computed on page 71, to the azimuth Palo-Eltoro. That is,

| | ° | ′ | ″ |
|---|---|---|---|
| Azimuth, Palo-Eltoro = | 111 | 39 | 50.05 |
| Difference of azimuth and back azimuth = | 180 | 00 | 00.00 |
| | − | 2 | 45.12 |
| Azimuth, Eltoro-Palo = | 291 | 37 | 04.93 |

Then using the azimuth Eltoro-Palo as given above, and adding the angle at Eltoro from Palo to Pancho, we obtain the azimuth Eltoro-Pancho.

In this manner, as shown on page 62, the azimuth is carried along until the azimuth Garcena-Gorgora is determined. It must be remembered that the observed angles are used in forming the azimuth equation. They can be obtained from either the list of directions or the triangle computation. The designation of each angle used should be placed to the left of the corresponding angle as shown on page 62. At the end of the equation the fixed azimuth of Garcena-Gorgora should be placed directly under the computed azimuth. The azimuth equation may then be written by placing the fixed azimuth equal to the computed azimuth plus all of the v's denoted by the designations of the angles in the computation of the azimuth equation. For convenience all terms are transferred to the right side of the equation, and the difference of the two azimuths (computed minus fixed) is used as the constant term of the equation.

## EXPLANATION OF LENGTH EQUATION

The specifications for horizontal control for first-order triangulation (see Special Publication No. 120, p. 2) require that the closure in length upon a measured base or a line of adjusted triangulation must not exceed that represented by an error of $\frac{1}{25000}$ after the angle and side equations have been satisfied in the adjustment. The closure in length can easily be expressed as a ratio by dividing the discrepancy in the logarithm by 0.4343, which is the modulus of the common system of logarithms. For instance, in the following length equation the discrepancy in length is 2.27 in the sixth place of logarithms and therefore the closure in length before adjustment is $\frac{0.00000227}{0.4343}$ or approximately $\frac{1}{191000}$. After the angle and side equations are satisfied in the adjustment the discrepancy in length is 7.25 in the sixth place of logarithms (see p. 91), that is, the closure in length after adjustment is $\frac{0.00000725}{0.4343}$ or approximately $\frac{1}{60000}$.

The length equation is formed as shown in the table on page 65. It differs from a side equation in that it involves the logarithms of two lengths. Starting with the line Palo-Pedro the length is computed through the *best* chain of triangles (see fig. 22) until the computed length of the line Garcena-Gorgora is obtained. The angles turned through in carrying the length through the scheme are indicated by small arcs near the vertices of the triangles in Figure 25.

In most cases it will not be necessary to compute the $\Sigma R$ (see p. 236) in order to get the best chain of triangles. An experienced mathematician will be able to tell from an inspection of the triangles which is the strongest chain. The beginner, however, when in doubt about the best triangles in any particular quadrilateral of the chain, should compute the $\Sigma R$ for that quadrilateral. A good general rule to follow is to avoid the small angles in the formation of the length equation.

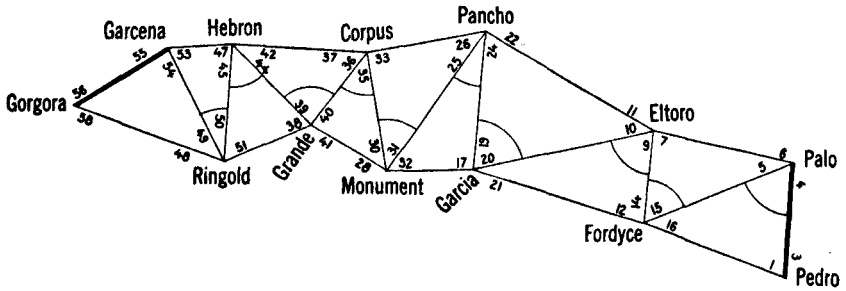


FIG. 25.—Triangulation net, showing triangles used in forming latitude and longitude equations

The logarithms of the fixed lengths are written in the first line of the length equation computation in the columns headed "logarithm." The logarithms of the fixed lengths used in the length equation must be corrected for the difference between arc and sine. A table of these corrections is given on page 231. This correction is $-16$ in the eighth place of logarithms for the length Palo-Pedro and $-13$ for the length Garcena-Gorgora. These corrections should be placed just above the logarithms of the lengths themselves. (See p. 65.)

The arc-sine correction may be explained as follows: In the formation of a length equation it is assumed that small arcs are proportional to their sines. Similarly, in the derivation of the position computation formulas it is assumed that small arcs, $s$ and $\Delta\lambda$, are proportional to their sines. As this assumption introduces a slight error in the computations a correction must be applied to take account of the error. In the length equations we are concerned only with the corrections corresponding to log $s$, which are always negative. In the case of the position computation, however, the differences are taken out for both the arguments log $s$ and log $\Delta\lambda$, the first with a negative, the second with a positive sign, and their algebraic sum is applied as a correction to log $\Delta\lambda$. A table similar to the one on page 231 is given on page 17 of Special Publication No. 8, sixth edition, but as

it is only carried to seven places of decimals it can not be used for first-order work, which should be carried to eight places of decimals.

Columns 1 and 5 of the length equation, headed "symbol," can be filled out directly from Figure 22. After determining through which triangles the length is to be carried, the following rule can be used in filling out these columns: In the first column is placed the designation of the angle adjacent to the starting length but opposite the side through which the length is next carried, while in the fifth column is placed the designation of the angle opposite the starting length. For instance, in Figure 25 the line Palo-Pedro is the starting length and the line Palo-Fordyce is the line through which the length is to be carried next, so $-1+3$ is placed in the first column, and $-15+16$ in the fifth column. Palo-Fordyce now becomes the starting line and Eltoro-Fordyce becomes the next line through which the length is carried so $-5+6$ and $-7+9$ are placed in the first and fifth columns, respectively. In a similar manner, the rest of the first and fifth columns are filled out.

The remainder of the computation is handled in exactly the same manner as for a side equation. The constant term for the length equation is obtained by subtracting the sum of the quantities in the seventh column from the sum of the quantities in the third column. The quantities in the fourth and eighth columns are the coefficients of the quantities in the first and fifth columns, respectively, as in a side equation. It must be remembered that the coefficients from the eighth column change sign, the right side of the table being subtracted from the left side.

*Computation of length equation*

| 1 Symbol | 2 Angle | | | 3 Logarithm | 4 Tabular difference | 5 Symbol | 6 Angle | | | 7 Logarithm | 8 Tabular difference |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Palo-Pedro | | | | −16* 3.9781520 | ---------- | Garcena-Gorgora | | | | −13* 3.9329710 | ---------- |
| | ° | ′ | ″ | | | | ° | ′ | ″ | | ′ |
| −1+3 | 70 | 32 | 18.94 | 9.97445004 | +0.74 | −15+16 | 48 | 00 | 09.94 | 9.87109231 | +1.90 |
| −5+6 | 38 | 09 | 53.52 | 9.79093675 | +2.67 | −7+9 | 77 | 47 | 14.58 | 9.99005871 | +.45 |
| −12+14 | 76 | 47 | 59.67 | 9.98837099 | +.49 | −20+21 | 31 | 20 | 13.41 | 9.71606319 | +3.46 |
| −10+11 | 44 | 39 | 18.91 | 9.84685608 | +2.13 | −22+24 | 64 | 28 | 16.62 | 9.95538436 | +1.00 |
| −17+19 | 89 | 53 | 49.28 | 9.99999930 | .00 | −31+32 | 61 | 38 | 08.72 | 9.94445563 | +1.13 |
| −25+26 | 49 | 18 | 39.39 | 9.87981749 | +1.81 | −33+35 | 92 | 54 | 15.91 | 9.99944177 | −.11 |
| −28+30 | 55 | 32 | 21.19 | 9.91619788 | +1.45 | −40+41 | 78 | 09 | 40.13 | 9.99066223 | +.45 |
| −36+37 | 47 | 30 | 43.44 | 9.86771468 | +1.93 | −42+44 | 42 | 56 | 20.48 | 9.83328715 | +2.27 |
| −38+39 | 63 | 08 | 11.31 | 9.95040642 | +1.06 | −50+51 | 69 | 29 | 07.23 | 9.97154607 | +.78 |
| −45+47 | 83 | 16 | 45.56 | 9.99700544 | +.25 | −53+54 | 67 | 18 | 20.19 | 9.96500209 | +.88 |
| −48+49 | 41 | 19 | 43.99 | 9.81979414 | +2.39 | −56+58 | 43 | 44 | 32.07 | 9.83973894 | +2.20 |
| | | | | 3.00970105 | | | | | | 3.00970332 | |

* See p. 64.

25. $$\begin{aligned}0 = &-2.27-0.74(1)+0.74(3)-2.67(5)+2.67(6)+0.45(7)-0.45(9)\\ &-2.13(10)+2.13(11)-0.49(12)+0.49(14)+1.90(15)-1.90(16)\\ &+3.46(20)-3.46(21)+1.00(22)-1.00(24)-1.81(25)+1.81(26)\\ &-1.45(28)+1.45(30)+1.13(31)-1.13(32)-0.11(33)+0.11(35)\\ &-1.93(36)+1.93(37)-1.06(38)+1.06(39)+0.45(40)-0.45(41)\\ &+2.27(42)-2.27(44)-0.25(45)+0.25(47)-2.39(48)+2.39(49)\\ &+0.78(50)-0.78(51)+0.88(53)-0.88(54)+2.20(56)-2.20(58)\end{aligned}$$

## LATITUDE AND LONGITUDE EQUATIONS

The angle, side, azimuth, and length equations having been formed, the latitude and longitude equations should now be formed. The development of condition equations for latitude and longitude closures is given very fully in Special Publication No. 28. Only the actual mechanical operation of forming these equations will be given here.

Until a few years ago the usual practice was to adjust a net of triangulation for only angle, side, length, and azimuth conditions at first and then, using the adjusted angles and lengths, to compute the geographic positions and determine the latitude and longitude closures. A new adjustment was then made to eliminate the discrepancies in latitude and longitude. In this second adjustment, it was necessary to hold the constant terms of all the equations, except the latitude and longitude equations, to zero, since all closing errors except in latitude and longitude were eliminated by the first adjustment.

The present practice is to adjust the latitude and longitude equations along with the other equations. When this is done it is necessary to compute preliminary positions through a selected chain of triangles in order to determine the latitude and longitude closures.

In this chain of triangles the observed angles are used for the length angles (the angles in each triangle through which the length is carried). The azimuth angle in each triangle is concluded by subtracting the sum of the two length angles from 180° plus the spherical excess. This angle should in every case be placed in parentheses to show that it is concluded. However, if one of the length angles is not observed, then the observed azimuth angle must be used, and the unobserved length angle concluded.

The chain of triangles through which the preliminary positions are computed is shown in Figure 26. It should be noted particularly that since the azimuth angle is concluded, it should be designated by the concluded correction symbols. For instance, in the triangle Fordyce-Palo-Pedro (see fig. 26), the angle at Palo and its correction symbol are obtained as follows:

| | | ° | ′ | ″ |
|---|---|---|---|---|
| −15+16 | Fordyce | 48 | 00 | 09.94 |
| −1+3 | Pedro | 70 | 32 | 18.94 |
| −1+3−15+16 | sum of Fordyce and Pedro angles | 118 | 32 | 28.88 |
| | 180°+spherical excess | 180 | 00 | 00.26 |
| +1−3+15−16 | Palo | 61 | 27 | 31.38 |

The concluded azimuth angles of all the triangles and their corresponding correction symbols are obtained in a similar manner. These triangles are given in Figure 26,

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Texas

| | NO. | STATIONS | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLES AND DISTANCES | LOGARITHM |
|---|---|---|---|---|---|---|---|---|
| | | 2-3 Palo-Pedro | | | | | | 3.978 1520 |
| B-15.16 | 1 | Fordyce | 48 00 09.94 | | | 0.08 | 09.86 | 0.128 9079 |
| C | 2 | Palo | (61 27 31.38) | | | 0.09 | 31.29 | 9.943 7283 |
| A -1+3 | 3 | Pedro | 70 32 18.94 | | | 0.09 | 18.85 | 9.974 4500 |
| | 1-3 | Fordyce-Pedro | | | | 0.26 | | 4.050 7882 |
| | 1-2 | Fordyce-Palo | | | | | | 4.081 5099 |
| | | 2-3 Palo-Fordyce | | | | | | 4.081 5099 |
| B -7+9 | 1 | Eltoro | 77 47 14.58 | | | 0.07 | 14.51 | 0.009 9414 |
| A -5+6 | 2 | Palo | 38 09 53.52 | | | 0.07 | 53.45 | 9.790 9366 |
| C | 3 | Fordyce | (64 02 52.11) | | | 0.07 | 52.04 | 9.953 8366 |
| | 1-3 | Eltoro-Fordyce | | | | 0.21 | | 3.882 3879 |
| | 1-2 | Eltoro-Palo | | | | | | 4.045 2879 |
| | | 2-3 Eltoro-Fordyce | | | | | | 3.882 3879 |
| B -20.21 | 1 | Garcia | 31 20 13.41 | | | 0.09 | 13.32 | 0.283 9371 |
| C | 2 | Eltoro | (71 51 47.19) | | | 0.09 | 47.10 | 9.977 8678 |
| A -12.14 | 3 | Fordyce | 76 47 59.67 | | | 0.09 | 59.58 | 9.988 3710 |
| | 1-3 | Garcia-Fordyce | | | | 0.27 | | 4.144 1928 |
| | 1-2 | Garcia-Eltoro | | | | | | 4.154 6960 |
| | | 2-3 Eltoro-Garcia | | | | | | 4.154 6960 |
| B -22.24 | 1 | Pancho | 64 28 16.62 | | | 0.13 | 16.49 | 0.044 6158 |
| A -10.11 | 2 | Eltoro | 44 39 18.91 | | | 0.12 | 18.79 | 9.846 8559 |
| C | 3 | Garcia | (70 52 24.85) | | | 0.13 | 24.72 | 9.975 3388 |
| | 1-3 | Pancho-Garcia | | | | 0.38 | | 4.046 1677 |
| | 1-2 | Pancho-Eltoro | | | | | | 4.174 6506 |
| | | 2-3 Pancho-Garcia | | | | | | 4.046 1677 |
| B-31.32 | 1 | Monument | 61 38 08.72 | | | 0.06 | 08.66 | 0.055 5444 |
| C | 2 | Pancho | (28 28 02.17) | | | 0.05 | 02.12 | 9.678 2055 |
| A -17.19 | 3 | Garcia | 89 53 49.28 | | | 0.06 | 49.22 | 9.999 9993 |
| | 1-3 | Monument-Garcia | | | | 0.17 | | 3.779 9176 |
| | 1-2 | Monument-Pancho | | | | | | 4.101 7114 |

FIG. 26.—Triangle computation to obtain latitude and longitude closures of net

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

## COMPUTATION OF TRIANGLES

State: Texas

| | NO. | STATION | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|---|
| | 2-3 | Pancho-Monument | | | | | | 4.101 7114 |
| B -33+35 | 1 | Corpus | 92 54 15.91 | | | 0.07 | 15.84 | 0.000 5582 |
| A -25+26 | 2 | Pancho | 49 18 39.39 | | | 0.06 | 39.33 | 9.879 8174 |
| C | 3 | Monument | (37 47 04.89) | | | 0.06 | 04.83 | 9.787 2448 |
| | 1-3 | Corpus-Monument | | | | 0.19 | | 3.982 0870 |
| | 1-2 | Corpus-Pancho | | | | | | 3.889 5144 |
| | 2-3 | Corpus-Monument | | | | | | 3.982 0870 |
| B -40+41 | 1 | Grande | 78 09 40.13 | | | 0.05 | 40.08 | 0.009 3378 |
| C | 2 | Corpus | (46 17 58.82) | | | 0.04 | 58.78 | 9.859 1161 |
| A -28+30 | 3 | Monument | 55 32 21.19 | | | 0.05 | 21.14 | 9.916 1978 |
| | 1-3 | Grande-Monument | | | | 0.14 | | 3.850 5409 |
| | 1-2 | Grande-Corpus | | | | | | 3.907 6226 |
| | 2-3 | Corpus-Grande | | | | | | 3.907 6226 |
| B -42+44 | 1 | Hebron | 42 56 20.48 | | | 0.06 | 20.42 | 0.166 7130 |
| A -36+37 | 2 | Corpus | 47 30 43.44 | | | 0.06 | 43.38 | 9.867 7146 |
| C | 3 | Grande | (89 32 56.26) | | | 0.06 | 56.20 | 9.999 9865 |
| | 1-3 | Hebron-Grande | | | | 0.18 | | 3.942 0502 |
| | 1-2 | Hebron-Corpus | | | | | | 4.074 3221 |
| | 2-3 | Hebron-Grande | | | | | | 3.942 0502 |
| B -50+51 | 1 | Ringold | 69 29 07.23 | | | 0.05 | 07.18 | 0.028 4539 |
| C | 2 | Hebron | (47 22 41.60) | | | 0.04 | 41.56 | 9.866 7831 |
| A -38+39 | 3 | Grande | 63 08 11.31 | | | 0.05 | 11.26 | 9.950 4064 |
| | 1-3 | Ringold-Grande | | | | 0.14 | | 3.837 2872 |
| | 1-2 | Ringold-Hebron | | | | | | 3.920 9105 |
| | 2-3 | Hebron-Ringold | | | | | | 3.920 9105 |
| B -53+54 | 1 | Garcena | 67 18 20.19 | | | 0.03 | 20.16 | 0.034 9979 |
| A -45+47 | 2 | Hebron | 83 16 45.56 | | | 0.03 | 45.53 | 9.997 0054 |
| C | 3 | Ringold | (29 24 54.34) | | | 0.03 | 54.31 | 9.691 1992 |
| | 1-3 | Garcena-Ringold | | | | 0.09 | | 3.952 9138 |
| | 1-2 | Garcena-Hebron | | | | | | 3.647 1076 |

FIG. 26.—Triangle computation to obtain latitude and longitude closures of net—Continued

After the preliminary triangles have been computed, the preliminary geographic positions are computed to determine the latitude and longitude closures. As the full explanation of the computation of geographic positions and the development of the formulas for computing them are given in Special Publication No. 8, it will not be necessary to repeat the explanation here. The actual computation of the positions are shown in Figure 27.

### FORMATION OF LATITUDE AND LONGITUDE EQUATIONS

The formation of the latitude and longitude equations from the preceding preliminary positions is shown in Figure 28, form 496 being used for this purpose.

The formulas for the latitude and longitude equations are as follows (see Special Publication No. 28, p. 31):

Latitude: $$0=7238.24\ (\phi_n-\phi_{n'})''+\Sigma[(\phi_n-\phi_c)'\ \delta_A\ (v_A)-(\phi_n-\phi_c)'\ \delta_B\ (v_B)]+\Sigma[\pm a_1\ (\lambda_n-\lambda_c)'\ (v_C)]$$

Longitude: $$0=7238.24\ (\lambda_n-\lambda_{n'})''+\Sigma[(\lambda_n-\lambda_c)'\ \delta_A\ (v_A)-(\lambda_n-\lambda_c)'\ \delta_B\ (v_B)]+\Sigma[\pm a_2\ (\phi_n-\phi_c)'\ (v_C)]$$

In these equations $v_A$, $v_B$, and $v_C$ are replaced by their correction symbols, care being taken to use $v_C=-v_A-v_B$, if the azimuth angle has been concluded in carrying the position computation through the chain. For convenience in computing, it is important to arrange the computation in the form of a table as shown on page 81.

### EXPLANATION OF COMPUTATION, LATITUDE AND LONGITUDE EQUATIONS

In the column headed "station" is placed the name of the station corresponding to the $C$ angle of each triangle of the preliminary set of triangles. (See p. 67.) The values in the columns headed $\phi_c$ and $\lambda_c$ are obtained directly from the position computations, as also the values for $\phi_n$ and $\lambda_n$ in the upper right-hand corner of the form. The columns headed $\phi_n-\phi_c$ and $\lambda_n-\lambda_c$ are self-explanatory. In the column headed $\delta_A$ is placed the tabular difference of the length angle adjacent to the starting length in each triangle, and in the column $-\delta_B$ is placed the tabular difference, with minus sign, of the length angle opposite the starting length in each triangle. If the position computations are carried through the same triangles as the length equation, as they should be, then the values in the columns headed $\delta_A$ and $-\delta_B$ can be taken directly from the columns headed tabular difference in the length equation computation (see p. 65) by simply changing the signs of the quantities placed in the column headed $-\delta_B$.

In the columns headed A, B, and C are placed the designations of the angles of the various triangles passed through in forming the lati-

(Text continued on p. 80)

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Palo | to | Pedro | 12 | 02 | 25.00 |
| ∠ | | Pedro | & | Fordyce | + 61 | 27 | 31.38 |
| α | 2. | Palo | to 1. | Fordyce | 73 | 29 | 56.38 |
| Δα | | | | | − | 3 | 04.84 |
| | | | | | 180 | | |
| α′ | 1. | Fordyce | to 2. | Palo | 253 | 26 | 51.54 |
| | | | | First Angle of Triangle | 48 | 00 | 09.94 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 26 | 19 | 38.951 | 2. | Palo | λ | 98 | 27 | 48.248 |
| Δφ | − | 1 | 51.518 | s= | | Δλ | + | 6 | 56.990 |
| φ′ | 26 | 17 | 47.433 | 1. | Fordyce | λ′ | 98 | 34 | 45.238 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.081 5099 | s′ | 8.16302 | | | −h | 2.0467 |
| cos α | 9.453 3675 | sin′ α | 9.96347 | (Δφ)′ | 4.0947 | s′ sin′ α | 8.1265 |
| B | 8.511 8083 | C | 1.10025 | D | 2.2921 | E | 5.8511 |
| h | 2.046 6857 | | 9.22674 | | 6.3868 | | 6.0243 |
| 1st term. | ″ +111.3488 | M term. | ″ +0.0002 | | | (Δλ)′ | |
| 2d term. | + 0.1686 | 4th term. | −0.0001 | | | F | |
| | +111.5174 | | | | | | |
| M and 4th terms. | + 0.0001 | s | 4.081 5099 | | | | |
| −Δφ | +111.5175 | sin α | 9.981 7347 | Arg. | | Δλ | 2.620 1257 |
| ½(φ+φ′) | 26 18 43.19 | A′ | 8.509 4379 | s | −3 | tan ½(φ+φ′) | 9.646 6575 |
| | | sec φ′ | 0.047 4432 | Δλ | +3 | sec ½(Δφ) | |
| | | | 2.620 1257 | Corr. | 0 | | 2.266 7832 |
| | | Δλ | +416.9901 | | | −Δα | +184.836 |

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Pedro | to | Palo | 192 | 01 | 53.35 |
| ∠ | | Fordyce | & | Palo | − 70 | 32 | 18.94 |
| α | 3. | Pedro | to 1. | Fordyce | 121 | 29 | 34.41 |
| Δα | | | | | − | 2 | 32.93 |
| | | | | | 180 | | |
| α′ | 1. | Fordyce | to 3. | Pedro | 301 | 27 | 01.48 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 26 | 14 | 36.740 | 3. | Pedro | λ | 98 | 28 | 59.722 |
| Δφ | + | 3 | 10.693 | s= | | Δλ | + | 5 | 45.516 |
| φ′ | 26 | 17 | 47.433 | 1. | Fordyce | λ′ | 98 | 34 | 45.238 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.050 7882 | s′ | 8.10153 | | | −h | 2.2306 |
| cos α | 9.717 9972 | sin′ α | 9.86160 | (Δφ)′ | 4.5607 | s′ sin′ α | 7.9632 |
| B | 8.511 8134 | C | 1.09865 | D | 2.2911 | E | 5.8497 |
| h | 2.280 5988 | | 9.06183 | | 6.8518 | | 6.0935 |
| 1st term. | ″ − 190.8090 | M term. | ″ +0.0007 | | | (Δλ)′ | |
| 2d term. | + 0.1153 | 4th term. | +0.0001 | | | F | |
| | − 190.6937 | | | | | | |
| M and 4th terms. | + 0.0008 | s | 4.050 7882 | | | | |
| −Δφ | − 190.6929 | sin α | 9.930 7988 | Arg. | | Δλ | 2.538 4681 |
| ½(φ+φ′) | 26 16 12.08 | A′ | 8.509 4379 | s | −2 | tan ½(φ+φ′) | 9.646 0134 |
| | | sec φ′ | 0.047 4432 | Δλ | +2 | sec ½(Δφ) | |
| | | | 2.538 4681 | Corr. | 0 | | 2.184 4815 |
| | | Δλ | + 345.5160 | | | −Δα | + 152.926 |

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FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|
| α | Palo | to | Fordyce | 73 | 29 | 56.38 |
| ∠ | Fordyce | & | Eltoro | + 38 | 09 | 53.52 |
| α | 2. Palo | to 1. | Eltoro | 111 | 39 | 49.90 |
| Δα | | | | - | 2 | 45.12 |
| | | | | 180 | | |
| α′ | 1. Eltoro | to 2. | Palo | 291 | 37 | 04.78 |
| | | | First Angle of Triangle | 77 | 47 | 14.58 |

| | ° | ′ | ″ | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| φ | 26 | 19 | 38.951 | 2. Palo | λ | 98 | 27 | 48.248 |
| Δφ | + | 2 | 13.006 | s= | Δλ | + | 6 | 12.057 |
| φ′ | 26 | 21 | 51.957 | 1. Eltoro | λ′ | 98 | 34 | 00.305 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.045 2879 | s² | 8.09058 | | | −h | 2.1243 |
| cos α | 9.567 2154 | sin² α | 9.93637 | (Δφ)² | 4.2477 | s² sin² α | 8.0270 |
| B | 8.511 8083 | C | 1.10025 | D | 2.2921 | E | 5.8511 |
| h | 2.1243116 | | 9.12720 | | 6.5398 | | 6.0024 |
| | | | | | | | |
| 1st term. | −133.1409 | 3d term. | +0.0003 | | | (Δλ)² | |
| 2d term. | + 0.1340 | 4th term. | +0.0001 | | | F | |
| | −133.0069 | | | | | | |
| 3d and 4th terms. | + 0.0004 | s | 4.045 2879 | | | | |
| −Δφ | −133.0065 | sin α | 9.968 1866 | Arg. | | Δλ | 2.570 6090 |
| ½(φ+φ′) | 26 20 45.45 | A′ | 8.509 4365 | s | −2 | sin ½(φ+φ′) | 9.647 1776 |
| | | sec φ′ | 0.047 6980 | Δλ | +2 | sec ½(Δφ) | |
| | | | 2.570 6090 | Corr. | 0 | | 2.217 7866 |
| | | Δλ | + 372.0566 | | | −Δα | +165.115 |

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Department of Commerce
U. S. Coast and Geodetic Survey
Form 26

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|
| α | Fordyce | to | Palo | 253 | 26 | 51.54 |
| ∠ | Eltoro | & | Palo | − 64 | 02 | 52.11 |
| α | 3. Fordyce | to 1. | Eltoro | 189 | 23 | 59.43 |
| Δα | | | | + | | 19.93 |
| | | | | 180 | | |
| α′ | 1. Eltoro | to 3. | Fordyce | 9 | 24 | 19.36 |

| | ° | ′ | ″ | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| φ | 26 | 17 | 47.433 | 3. Fordyce | λ | 98 | 34 | 45.233 |
| Δφ | + | 4 | 04.524 | s= | Δλ | − | | 44.933 |
| φ′ | 26 | 21 | 51.957 | 1. Eltoro | λ′ | 98 | 34 | 00.305 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.882 3879 | s² | 7.76473 | | | −h | 2.3883 |
| cos α | 9.994 1291 | sin² α | 8.42610 | (Δφ)² | 4.7766 | s² sin² α | 6.1909 |
| B | 8.511 8102 | C | 1.09965 | D | 2.2918 | E | 5.8506 |
| h | 2.388 5272 | | 7.29053 | | 7.0684 | | 4.4298 |
| | | | | | | | |
| 1st term. | −244.5272 | 3d term. | +0.0012 | | | (Δλ)² | |
| 2d term. | + 0.0020 | 4th term. | 0.0000 | | | F | |
| | −244.5252 | | | | | | |
| 3d and 4th terms. | + 0.0012 | s | 3.882 3879 | | | | |
| −Δφ | −244.5240 | sin α | 9.213 0480 | Arg. | | Δλ | 1.652 5703 |
| ½(φ+φ′) | 26 19 49.69 | A′ | 8.509 4365 | s | −1 | sin ½(φ+φ′) | 9.646 9406 |
| | | sec φ′ | 0.047 6980 | Δλ | 0 | sec ½(Δφ) | |
| | | | 1.652 5703 | Corr. | −1 | | 1.299 5109 |
| | | Δλ | − 44.9335 | | | −Δα | − 19.930 |

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FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Eltoro | to | Fordyce | 9 | 24 | 19.36 |
| ∠ | | Fordyce | & | Garcia | +71 | 51 | 47.19 |
| α | 2. | Eltoro | to 1. | Garcia | 81 | 16 | 06.55 |
| Δα | | | | | - | 3 | 45.95 |
| | | | | | 180 | | |
| α′ | 1. | Garcia | to 2. | Eltoro | 261 | 12 | 20.60 |
| | | | | First Angle of Triangle | 31 | 20 | 13.41 |

| | ° | ′ | ″ | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| φ | 26 | 21 | 51.957 | 2. Eltoro | λ | 98 | 34 | 00.305 |
| Δφ | - | 1 | 10.686 | s= | Δλ | + | 8 | 28.973 |
| φ′ | 26 | 20 | 41.271 | 1. Garcia | λ′ | 98 | 42 | 29.278 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.154 6960 | s′ | 8.30939 | | | −h | 1.8478 |
| cos α | 9.181 2846 | sin² α | 9.98987 | (Δφ)² | 3.6987 | s² sin² α | 8.2993 |
| B | 8.511 8060 | C | 1.10095 | D | 2.2925 | E | 5.8517 |
| h | 1.847 7866 | | 9.40021 | | 5.9912 | | 5.9988 |
| | ″ | | ″ | | | | |
| 1st term. | +70.4347 | 3d term. | +0.0001 | | | (Δλ)² | |
| 2d term. | + 0.2513 | 4th term. | -0.0001 | | | F | |
| | +70.6860 | | | | | | |
| 3d and 4th terms. | 0.0000 | s | 4.154 6960 | | | | |
| −Δφ | +70.6860 | sin α | 9.994 9374 | Arg. | | Δλ | 2.706 6946 |
| ½(φ+φ′) | 26 21 16.61 | A′ | 8.509 4369 | s | -4 | sin ½(φ+φ′) | 9.647 3100 |
| | | sec φ′ | 0.047 6243 | Δλ | +4 | sec ½(Δφ) | |
| | | | 2.706 6946 | Corr. | 0 | | 2.354 0046 |
| | | Δλ | + 508.9728 | | | −Δα | +225.946 |

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DEPARTMENT OF COMMERCE
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FORM 26

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Fordyce | to | Eltoro | 189 | 23 | 59.43 |
| ∠ | | Garcia | & | Eltoro | - 76 | 47 | 59.67 |
| α | 3. | Fordyce | to 1. | Garcia | 112 | 35 | 59.76 |
| Δα | | | | | - | 3 | 25.75 |
| | | | | | 180 | | |
| α′ | 1. | Garcia | to 3. | Fordyce | 292 | 32 | 34.01 |

| | ° | ′ | ″ | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| φ | 26 | 17 | 47.433 | 3. Fordyce | λ | 98 | 34 | 45.238 |
| Δφ | + | 2 | 53.838 | s= | Δλ | + | 7 | 44.040 |
| φ′ | 26 | 20 | 41.271 | 1. Garcia | λ′ | 98 | 42 | 29.278 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.144 1928 | s′ | 8.28839 | | | −h | 2.2407 |
| cos α | 9.584 6639 | sin² α | 9.93060 | (Δφ)² | 4.4303 | s² sin² α | 8.2190 |
| B | 8.511 8102 | C | 1.03965 | D | 2.2918 | E | 5.3506 |
| h | 2.240 6669 | | 9.31864 | | 6.7721 | | 6.3103 |
| | ″ | | ″ | | | | |
| 1st term. | -174.0471 | 3d term. | +0.0006 | | | (Δλ)² | |
| 2d term. | + 0.2083 | 4th term. | +0.0002 | | | F | |
| | -173.8388 | | | | | | |
| 3d and 4th terms. | + 0.0008 | s | 4.144 1928 | | | | |
| −Δφ | -173.8380 | sin α | 9.965 3008 | Arg. | | Δλ | 2.666 5549 |
| ½(φ+φ′) | 26 19 14.35 | A′ | 8.509 4369 | s | -3 | sin ½(φ+φ′) | 9.646 7901 |
| | | sec φ′ | 0.047 6243 | Δλ | +4 | sec ½(Δφ) | |
| | | | 2.666 5548 | Corr. | +1 | | 2.313 3450 |
| | | Δλ | + 464.0395 | | | −Δα | +205.752 |

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FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 26

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|
| α | Eltoro | to | Garcia | 81 | 16 | 06.55 |
| ∠ | Garcia | & | Pancho | + 44 | 39 | 18.91 |
| α | 2. Eltoro | to | 1. Pancho | 125 | 55 | 25.46 |
| Δα | | | | - | 3 | 14.32 |
| | | | | 180 | | |
| α′ | 1. Pancho | to | 2. Eltoro | 305 | 52 | 11.14 |
| | | | First Angle of Triangle | 64 | 28 | 16.62 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 26 | 21 | 51.957 | 2. Eltoro | | λ | 98 | 34 | 00.305 |
| Δφ | + | 4 | 44.835 | s= | | Δλ | + | 7 | 16.977 |
| φ′ | 26 | 26 | 36.792 | 1. Pancho | | λ′ | 98 | 41 | 17.282 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.174 6506 | s′ | 8.34930 | | | −h | 2.4549 |
| cos α | 9.768 4220 | sin² α | 9.81675 | (Δφ)² | 4.9092 | s′ sin² α | 8.1660 |
| B | 8.511 8060 | C | 1.10095 | D | 2.2925 | E | 5.8517 |
| h | 2.454 8786 | | 9.26700 | | 7.2017 | | 6.4726 |
| 1st term. | ″ −285.0222 | 3d term. | ″ +0.0016 | | | (Δλ)² | |
| 2d term. | +0.1849 | 4th term. | +0.0003 | | | F | |
| | −284.8373 | | | | | | |
| 3d and 4th terms. | + 0.0019 | s | 4.174 6506 | | | | |
| −Δφ | −284.8354 | sin α | 9.908 3770 | Arg. | | Δλ | 2.640 4581 |
| ½(φ+φ′) | 26 24 14.37 | A′ | 8.509 4349 | s | −4 | sin ½(φ+φ′) | 9.648 0648 |
| | | sec φ′ | 0.047 9957 | Δλ | +3 | sec ½(Δφ) | |
| | | | 2.640 4581 | Corr. | −1 | | 2.288 5229 |
| | | Δλ | ″ + 436.9765 | | | −Δα | ″ + 194.322 |

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|
| α | Garcia | to | Eltoro | 261 | 12 | 20.60 |
| ∠ | Pancho | & | Eltoro | − 70 | 52 | 24.85 |
| α | 3. Garcia | to | 1. Pancho | 190 | 19 | 55.75 |
| Δα | | | | + | | 32.01 |
| | | | | 180 | | |
| α′ | 1. Pancho | to | 3. Garcia | 10 | 20 | 27.76 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 26 | 20 | 41.271 | 3. Garcia | | λ | 98 | 42 | 29.278 |
| Δφ | + | 5 | 55.521 | s= | | Δλ | - | 1 | 11.996 |
| φ′ | 26 | 26 | 36.792 | 1. Pancho | | λ′ | 98 | 41 | 17.282 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.046 1677 | s′ | 8.09233 | | | −h | 2.5509 |
| cos α | 9.992 9000 | sin² α | 8.50742 | (Δφ)² | 5.1017 | s′ sin² α | 6.5998 |
| B | 8.511 8072 | C | 1.10057 | D | 2.2923 | E | 5.8513 |
| h | 2.550 8749 | | 7.70032 | | 7.3940 | | 5.0020 |
| 1st term. | ″ −355.5289 | 3d term. | ″ +0.0025 | | | (Δλ)² | |
| 2d term. | + 0.0050 | 4th term. | 0.0000 | | | F | |
| | −355.5239 | | | | | | |
| 3d and 4th terms. | + 0.0025 | s | 4.046 1677 | | | | |
| −Δφ | −355.5214 | sin α | 9.253 7118 | Arg. | | Δλ | 1.857 3101 |
| ½(φ+φ′) | 26 23 39.03 | A′ | 8.509 4349 | s | −2 | sin ½(φ+φ′) | 9.647 9148 |
| | | sec φ′ | 0.047 9957 | Δλ | 0 | sec ½(Δφ) | |
| | | | 099<br>1.857 3101 | Corr. | −2 | | 1.505 2249 |
| | | Δλ | ″ − 71.9963 | | | −Δα | ″ − 32.006 |

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FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 96

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Pancho | to | Garbia | 10 | 20 | 27.76 |
| ∠ | | Garcia | & | Monument | +28 | 28 | 02.17 |
| α | 2. | Pancho | to 1. | Monument | 38 | 48 | 29.93 |
| Δα | | | | | - | 2 | 07.02 |
| | | | | | 180 | | |
| α′ | 1. | Monument | to 2. | Pancho | 218 | 46 | 22.91 |
| | | | | First Angle of Triangle | 61 | 38 | 08.72 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 26 | 26 | 36.792 | 2. | Pancho | λ | 98 | 41 | 17.282 |
| Δφ | - | 5 | 20.109 | s= | | Δλ | + | 4 | 45.680 |
| φ′ | 26 | 21 | 16.683 | 1. | Monument | λ′ | 98 | 46 | 02.962 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.101 7114 | s′ | 8.20342 | | | −h | 2.5052 |
| cos α | 9.891 6751 | sin′ α | 9.59414 | (δφ)′ | 5.0106 | s′ sin′ α | 7.7976 |
| B | 8.511 8012 | C | 1.10250 | D | 2.2934 | E | 5.8529 |
| h | 2.505 1877 | | 8.90006 | | 7.3040 | | 6.1557 |
| 1st term. | +320.0278 | 3d term. | +0.0020 | | | (Δλ)′ | |
| 2d term. | + 0.0794 | 4th term. | -0.0001 | | | F | |
| | +320.1072 | | | | | | |
| 3d and 4th terms. | + 0.0019 | s | 4.101 7114 | | | | |
| −Δφ | +320.1091 | sin α | 9.797 0714 | Arg. | | Δλ | 2.455 8805 |
| ½(φ+φ′) | 26 23 54.74 | A′ | 8.509 4367 | s | -3 | sin ½(φ+φ′) | 9.647 9901 |
| | | sec φ′ | 0.047 6612 | Δλ | +1 | sec ½(Δφ) | |
| | | | 2.455 8805 | Corr. | -2 | | 2.103 8706 |
| | | Δλ | +285.6805 | | | −Δα | +127.020 |

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Garcia | to | Pancho | 190 | 19 | 55.75 |
| ∠ | | Monument | & | Pancho | -89 | 53 | 49.28 |
| α | 3. | Garcia | to 1. | Monument | 100 | 26 | 06.47 |
| Δα | | | | | - | 1 | 34.84 |
| | | | | | 180 | | |
| α′ | 1. | Monument | to 3. | Garcia | 280 | 24 | 31.63 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 26 | 20 | 41.271 | 3. | Garcia | λ | 98 | 42 | 29.278 |
| Δφ | + | | 35.412 | s= | | Δλ | + | 3 | 33.684 |
| φ′ | 26 | 21 | 16.683 | 1. | Monument | λ′ | 98 | 46 | 02.962 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.779 9176 | s′ | 7.55983 | | | −h | 1.5497 |
| cos α | 9.257 9717 | sin′ α | 9.96551 | (δφ)′ | 3.0983 | s′ sin′ α | 7.5453 |
| B | 8.511 8072 | C | 1.10057 | D | 2.2923 | E | 5.8513 |
| h | 1.549 6965 | | 8.64591 | | 5.3906 | | 4.9463 |
| 1st term. | - 35.4566 | 3d term. | 0.0000 | | | (Δλ)′ | |
| 2d term. | + 0.0442 | 4th term. | 0.0000 | | | F | |
| | - 35.4124 | | | | | | |
| 3d and 4th terms. | 0.0000 | s | 3.779 9176 | | | | |
| −Δφ | - 35.4124 | sin α | 9.992 7570 | Arg. | | Δλ | 2.329 7725 |
| ½(φ+φ′) | 26 20 58.98 | A′ | 8.509 4367 | s | -1 | sin ½(φ+φ′) | 9.647 2352 |
| | | sec φ′ | 0.047 6612 | Δλ | +1 | sec ½(Δφ) | |
| | | | 2.329 7725 | Corr. | 0 | | 1.977 0077 |
| | | Δλ | + 213.6842 | | | −Δα | + 94.844 |

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FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net—Continued

588530°—28——6

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 26

## POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Pancho | to | Monument | 38 | 48 | 29.93 |
| ∠ | | Monument | & | Corpus | +49 | 18 | 39.39 |
| α | 2. | Pancho | to 1. | Corpus | 88 | 07 | 09.32 |
| Δα | | | | | − | 2 | 04.55 |
| | | | | | 180 | | |
| α′ | 1. | Corpus | to 2. | Pancho | 268 | 05 | 04.77 |
| | | | | First Angle of Triangle | 92 | 54 | 15.91 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| φ | 26 | 26 | 36.792 | 2. | Pancho | λ | 98 | 41 | 17.282 |
| Δφ | − | | 08.345 | s= | | Δλ | + | 4 | 39.706 |
| φ′ | 26 | 26 | 28.447 | 1. | Corpus | λ′ | 98 | 45 | 56.988 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.889 5144 | s′ | 7.77903 | | | −h | 0.9174 |
| cos α | 8.516 1292 | sin² α | 9.99963 | (Δφ)² | 1.8428 | s′ sin² α | 7.7786 |
| B | 8.511 8012 | C | 1.10250 | D | 2.2934 | E | 5.8529 |
| h | 0.917 4448 | | 8.88106 | | 4.1362 | | 4.5489 |
| 1st term. | +8.2688″ | 3d term. | 0.0000″ | | | (Δλ)′ | |
| 2d term. | +0.0760 | 4th term. | 0.0000 | | | F | |
| | +8.3448 | | | | | | |
| 3d and 4th terms. | 0.0000 | s | 3.889 5144 | | | | |
| −Δφ | +8.3448 | sin α | 9.999 7660 | Arg. | | Δλ | 2.446 7024 |
| ½(φ+φ′) | 26 26 32.62 | A′ | 8.509 4350 | s | −1 | sin ½(φ+φ′) | 9.648 6505 |
| | | sec φ′ | 0.047 9870 | Δλ | +1 | sec ½(Δφ) | |
| | | | 2.446 7024 | Corr. | 0 | | 2.095 3529 |
| | | Δλ | +279.7064 | | | −Δα | +124.553 |

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## POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Monument | to | Pancho | 218 | 46 | 22.91 |
| ∠ | | Corpus | & | Pancho | − 37 | 47 | 04.89 |
| α | 3. | Monument | to 1 | Corpus | 180 | 59 | 18.02 |
| Δα | | | | | + | | 02.66 |
| | | | | | 180 | | |
| α′ | 1. | Corpus | to 3. | Monument | 0 | 59 | 20.68 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| φ | 26 | 21 | 16.683 | 3. | Monument | λ | 98 | 46 | 02.952 |
| Δφ | + | 5 | 11.764 | s= | | Δλ | − | | 05.974 |
| φ′ | 26 | 26 | 28.447 | 1. | Corpus | λ′ | 98 | 45 | 56.988 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.982 0870 | s′ | 7.96417 | | | −h | 2.4938 |
| cos α | 9.999 9354 | sin² α | 6.47352 | (Δφ)² | 4.9877 | s′ sin² α | 4.4377 |
| B | 8.511 8066 | C | 1.10076 | D | 2.2924 | E | 5.8515 |
| h | 2.493 8290 | | 5.53845 | | 7.2801 | | 2.7830 |
| 1st term. | −311.7662″ | 3d term. | +0.0019″ | | | (Δλ)′ | |
| 2d term. | 0.0000 | 4th term. | 0.0000 | | | F | |
| | −311.7662 | | | | | | |
| 3d and 4th terms. | + 0.0019 | s | 3.982 0870 | | | | |
| −Δφ | −311.7643 | sin α | 8.236 7617 | Arg. | | Δλ | 0.776 2705 |
| ½(φ+φ′) | 26 23 52.56 | A′ | 8.509 4350 | s | −2 | sin ½(φ+φ′) | 9.647 9722 |
| | | sec φ′ | 0.047 9870 | Δλ | 0 | sec ½(Δφ) | |
| | | | 0.776 2705 | Corr. | −2 | | 0.424 2427 |
| | | Δλ | − 5.9741 | | | −Δα | −2.656 |

Do not write in this margin.

FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 26

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Corpus | to | Monument | 0 | 59 | 20.68 |
| ∠ | | Monument | & | Grande | + 46 | 17 | 58.82 |
| α | 2. | Corpus | to 1. | Grande | 47 | 17 | 19.50 |
| Δα | | | | | - | 1 | 36.34 |
| | | | | | 180 | | |
| α′ | 1. | Grande | to 2. | Corpus | 227 | 15 | 44.16 |
| | | | | First Angle of Triangle | 78 | 09 | 40.13 |

| | | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|---|
| φ | | 26 | 26 | 28.447 | 2. | Corpus | λ | 98 | 45 | 56.988 |
| Δφ | | - | 2 | 58.221 | s= | | Δλ | + | 3 | 34.298 |
| φ′ | | 26 | 23 | 30.226 | 1. | Grande | λ′ | 98 | 49 | 31.286 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.907 6226 | s′ | 7.81525 | | | −h | 2.2508 |
| cos α | 9.831 4244 | sin′ α | 9.73232 | (δφ)′ | 4.5019 | s′ sin′ α | 7.5476 |
| B | 8.511 8013 | C | 1.10240 | D | 2.2934 | E | 5.8529 |
| h | 2.250 8483 | | 8.64997 | | 6.7953 | | 5.6513 |
| 1st term. | +178.″1756 | M term. | +0.″0006 | | | (Δλ)′ | |
| 2d term. | + 0.0447 | 4th term. | 0.0000 | | | F | |
| | +178.2203 | | | | | | |
| M and 4th terms. | +0.0006 | | | | | | |
| −Δφ | +178.2209 | s | 3.907 6226 | | | | |
| ½(φ+φ′) | 26 24 59.34 | sin α | 9.866 1582 | Arg. | | Δλ | 2.331 0173 |
| | | A′ | 8.509 4359 | s | -1 | sin ½(φ+φ′) | 9.648 2555 |
| | | sec φ′ | 0.047 8006 | Δλ | +1 | sec ½(Δφ) | |
| | | | 2.331 0173 | Corr. | 0 | | 1.979 2728 |
| | | Δλ | + 214.2976 | | | −Δα | + 95.″339 |

Do not write in this margin.

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Monument | to | Corpus | 180 | 59 | 18.02 |
| ∠ | | Grande | & | Corpus | − 55 | 32 | 21.19 |
| α | 3. | Monument | to 1. | Grande | 125 | 26 | 56.83 |
| Δα | | | | | - | 1 | 32.54 |
| | | | | | 180 | | |
| α′ | 1. | Grande | to 3. | Monument | 305 | 25 | 24.29 |

| | | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|---|
| φ | | 26 | 21 | 16.683 | 3. | Monument | λ | 98 | 46 | 02.962 |
| Δφ | | + | 2 | 13.543 | s= | | Δλ | + | 3 | 28.324 |
| φ′ | | 26 | 23 | 30.226 | 1. | Grande | λ′ | 98 | 49 | 31.286 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.850 5409 | s′ | 7.70108 | | | −h | 2.1258 |
| cos α | 9.763 4128 | sin′ α | 9.82192 | (δφ)′ | 4.2512 | s′ sin′ α | 7.5230 |
| B | 8.511 8066 | C | 1.10076 | D | 2.2924 | E | 5.8515 |
| h | 2.125 7603 | | 8.62376 | | 6.5436 | | 5.5003 |
| 1st term. | -133.″5858 | M term. | +0.″0003 | | | (Δλ)′ | |
| 2d term. | + 0.0421 | 4th term. | 0.0000 | | | F | |
| | -133.5437 | | | | | | |
| M and 4th terms. | + 0.0003 | | | | | | |
| −Δφ | -133.5434 | s | 3.850 5409 | | | | |
| ½(φ+φ′) | 26 22 23.45 | sin α | 9.910 9608 | Arg. | | Δλ | 2.318 7382 |
| | | A′ | 8.509 4359 | s | -1 | sin ½(φ+φ′) | 9.647 5940 |
| | | sec φ′ | 0.047 8006 | Δλ | +1 | sec ½(Δφ) | |
| | | | 2.318 7382 | Corr. | 0 | | 1.966 3322 |
| | | Δλ | + 208.3235 | | | −Δα | + 92.″541 |

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FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| α | | Corpus | to | Grande | 47 | 17 | 19.50 |
| ∠ | | Grande | & | Hebron | + 47 | 30 | 43.44 |
| α | 2. | Corpus | to 1. | Hebron | 94 | 48 | 02.94 |
| Δα | | | | | − | 3 | 10.09 |
| | | | | | 180 | | |
| α' | 1. | Hebron | to 2. | Corpus | 274 | 44 | 52.85 |
| | | | | First Angle of Triangle | 42 | 56 | 20.48 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 26 | 26 | 28.447 | 2. Corpus | | λ | 98 | 45 | 56.988 |
| Δφ | + | | 32.094 | s= | | Δλ | + | 7 | 06.826 |
| φ' | 26 | 27 | 00.541 | 1. Hebron | | λ' | 98 | 53 | 03.814 |
| | | | −1 | | | | | | +1 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.074 3221 | s² | 8.14864 | | | −h | 1.5088 |
| cos α | 8.922 6842 | sin² α | 9.99695 | (Δφ)² | 3.0128 | s² sin² α | 8.1456 |
| B | 8.511 8013 | C | 1.10240 | D | 2.2934 | E | 5.8529 |
| h | 1.508 8076 | | 9.24799 | | 5.3062 | | 5.5073 |
| 1st term. | −32.″2706 | 3d term. | 0.″0000 | | | (Δλ)² | |
| 2d term. | + 0.1770 | 4th term. | 0.0000 | | | F | |
| | −32.0936 | | | | | | |
| 3d and 4th terms. | 0.0000 | s | 4.074 3221 | | | | |
| −Δφ | −32.0936 | sin α | 9.998 4737 | Arg. | | Δλ | 2.630 2512 |
| ½(φ+φ') | 26 26 44.49 | A' | 8.509 4348 | s | −3 | sin ½(φ+φ') | 9.648 7008 |
| | | sec φ' | 0.048 0206 | Δλ | +3 | sec ½(Δφ) | |
| | | | 2.630 2512 | Corr. | 0 | | 2.278 9520 |
| | | Δλ | + 426.8263 | | | −Δα | + 190.″087 |

Do not write in this margin.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 26

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| α | | Grande | to | Corpus | 227 | 15 | 44.16 |
| ∠ | | Hebron | & | Corpus | − 89 | 32 | 56.26 |
| α | 3. | Grande | to 1. | Hebron | 137 | 42 | 47.90 |
| Δα | | | | | − | 1 | 34.57 |
| | | | | | 180 | | |
| α' | 1. | Hebron | to 3. | Grande | 317 | 41 | 13.33 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 26 | 23 | 30.226 | 3. Grande | | λ | 98 | 49 | 31.286 |
| Δφ | + | 3 | 30.314 | s= | | Δλ | + | 3 | 32.529 |
| φ' | 26 | 27 | 00.540 | 1. Hebron | | λ' | 98 | 53 | 03.815 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.942 0502 | s² | 7.88410 | | | −h | 2.3230 |
| cos α | 9.869 1069 | sin² α | 9.65582 | (Δφ)² | 4.6458 | s² sin² α | 7.5399 |
| B | 8.511 8043 | C | 1.10146 | D | 2.2928 | E | 5.8521 |
| h | 2.322 9614 | | 8.64138 | | 6.9386 | | 5.7150 |
| 1st term. | −210.″3591 | 3d term. | +0.″0009 | | | (Δλ)² | |
| 2d term. | + 0.0438 | 4th term. | +0.0001 | | | F | |
| | −210.3153 | | | | | | |
| 3d and 4th terms. | + 0.0010 | s | 3.942 0502 | | | | |
| −Δφ | −210.3143 | sin α | 9.827 9122 | Arg. | | Δλ | 2.327 4178 |
| ½(φ+φ') | 26 25 15.38 | A' | 8.509 4348 | s | −1 | sin ½(φ+φ') | 9.648 3234 |
| | | sec φ' | 0.048 0206 | Δλ | +1 | sec ½(Δφ) | |
| | | | 2.327 4178 | Corr. | 0 | | 1.975 7412 |
| | | Δλ | + 212.5288 | | | −Δα | + 94.″567 |

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FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Hebron | to | Grande | 317 | 41 | 13.33 |
| ∠ | | Grande | & | Ringold | + 47 | 22 | 41.60 |
| α | 2. | Hebron | to 1. | Ringold | 5 | 03 | 54.93 |
| Δα | | | | | − | | 11.81 |
| | | | | | 180 | | |
| α′ | 1. | Ringold | to 2. | Hebron | 185 | 03 | 43.12 |
| | | | | First Angle of Triangle | 69 | 29 | 07.23 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 26 | 27 | 00.540 | 2. | Hebron | λ | 98 | 53 | 03.815 |
| Δφ | − | 4 | 29.783 | s= | | Δλ | + | | 26.546 |
| φ′ | 26 | 22 | 30.757 | 1. | Ringold | λ′ | 98 | 53 | 30.361 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.920 9105 | s′ | 7.84182 | | | −h | 2.4310 |
| cos α | 9.998 3007 | sin² α | 7.89183 | (Δφ)² | 4.8620 | s² sin² α | 5.7336 |
| B | 8.511 8008 | C | 1.10257 | D | 2.2935 | E | 5.8530 |
| h | 2.431 0129 | | 6.83622 | | 7.1555 | | 4.0176 |
| 1st term. | +269.7814″ | 3d term. | +0.0014″ | | | (Δλ)² | |
| 2d term. | + 0.0007 | 4th term. | 0.0000 | | | F | |
| | +269.7821 | | | | | | |
| M and 4th terms. | + 0.0014 | | | | | | |
| | | s | 3.920 9105 | | | | |
| −Δφ | +269.7835 | sin α | 8.945 9131 | Arg. | | Δλ | 1.423 9983 |
| ½(φ+φ′) | 26 24 45.65 | A′ | 8.509 4363 | s | −1 | sin ½(φ+φ′) | 9.648 1973 |
| | | sec φ′ | 0.047 7385 | Δλ | 0 | sec ½(Δφ) | |
| | | | 1.423 9983 | Corr. | −1 | | 1.072 1956 |
| | | Δλ | + 26.5459 | | | −Δα | + 11.809 |

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Department of Commerce
U. S. Coast and Geodetic Survey
Form 96

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Grande | to | Hebron | 137 | 42 | 47.90 |
| ∠ | | Ringold | & | Hebron | − 63 | 08 | 11.31 |
| α | 3. | Grande | to 1. | Ringold | 74 | 34 | 36.59 |
| Δα | | | | | − | 1 | 46.24 |
| | | | | | 180 | | |
| α′ | 1. | Ringold | to 3. | Grande | 254 | 32 | 50.35 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 26 | 23 | 30.226 | 3. | Grande | λ | 98 | 49 | 31.236 |
| Δφ | − | | 59.469 | s= | | Δλ | + | 3 | 59.075 |
| φ′ | 26 | 22 | 30.757 | 1. | Ringold | λ′ | 98 | 53 | 30.361 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.837 2872 | s′ | 7.67457 | | | −h | 1.7739 |
| cos α | 9.424 7935 | sin² α | 9.96814 | (Δφ)² | 3.5486 | s² sin² α | 7.6427 |
| B | 8.511 8043 | C | 1.10146 | D | 2.2928 | E | 5.8521 |
| h | 1.773 8850 | | 8.74417 | | 5.8414 | | 5.2687 |
| 1st term. | +59.4135″ | 3d term. | +0.0001″ | | | (Δλ)² | |
| 2d term. | + 0.0555 | 4th term. | 0.0000 | | | F | |
| | +59.4690 | | | | | | |
| M and 4th terms. | +0.0001 | | | | | | |
| | | s | 3.837 2872 | | | | |
| −Δφ | +59.4691 | sin α | 9.934 0716 | Arg. | | Δλ | 2.378 5336 |
| ½(φ+φ′) | 26 23 00.49 | A′ | 8.509 4363 | s | −1 | sin ½(φ+φ′) | 9.647 7515 |
| | | sec φ | 0.047 7385 | Δλ | +1 | sec ½(Δφ) | |
| | | | 2.378 5336 | Corr. | 0 | | 2.026 2851 |
| | | Δλ | + 239.0747 | | | −Δα | +106.239 |

Do not write in this margin.

FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 96

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|
| α | | Hebron | to Ringold | 5 | 03 | 54.93 |
| ∠ | | Ringold | & Garcena | + 83 | 16 | 45.56 |
| α | 2. | Hebron | to 1. Garcena | 88 | 20 | 40.49 |
| Δα | | | | − | 1 | 11.31 |
| | | | | 180 | | |
| α′ | 1. | Garcena | to 2. Hebron | 268 | 19 | 29.18 |
| | | | First Angle of Triangle | 67 | 18 | 20.19 |

| | ° | ′ | ″ | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| φ | 26 | 27 | 00.540 | 2. Hebron | λ | 98 | 53 | 03.815 |
| Δφ | − | | 04.190 | s= | Δλ | + | 2 | 40.096 |
| φ′ | 26 | 26 | 56.350<br>56.345* | 1. Garcena | λ′ | 98 | 55 | 43.910<br>43.916* |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.647 1076 | s² | 7.29422 | | | −h | 0.6196 |
| cos α | 8.460 7250 | sin² α | 9.99964 | (Δφ)² | 1.2444 | s² sin² α | 7.2939 |
| B | 8.511 8008 | C | 1.10257 | D | 2.2935 | E | 5.8530 |
| h | 0.619 6334 | | 8.39643 | | 3.5379 | | 3.7665 |
| 1st term. | +4.1652″ | 2d term. | 0.0000″ | | | (Δλ)′ | |
| 2d term. | + 0.0249 | 4th term. | 0.0000 | | | F | |
| 3d and 4th terms. | +4.1901<br>0.0000 | s | 3.647 1076 | | | | |
| −Δφ | +4.1901 | sin α | 9.999 8187 | Arg. | | Δλ | 2.204 3774 |
| ½(φ+φ′) | 26 26 58.44 | A′ | 8.509 4348 | s | 0 | sin ½(φ+φ′) | 9.648 7599 |
| | | sec φ′ | 0.048 0163 | Δλ | 0 | sec ½(Δφ) | |
| | | | 2.204 3774 | Corr. | 0 | | 1.853 1373 |
| | | Δλ | + 160.0949 | | | −Δα | + 71.308 |

* Fixed position.

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|
| α | | Ringold | to Hebron | 185 | 03 | 43.12 |
| ∠ | | Garcena | & Hebron | − 29 | 24 | 54.34 |
| α | 3. | Ringold | to 1. Garcena | 155 | 38 | 48.78 |
| Δα | | | | − | | 59.41 |
| | | | | 180 | | |
| α′ | 1. | Garcena | to 3. Ringold | 335 | 37 | 49.37 |

| | ° | ′ | ″ | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| φ | 26 | 22 | 30.757 | 3. Ringold | λ | 98 | 53 | 30.361 |
| Δφ | + | 4 | 25.593 | s= | Δλ | + | 2 | 13.549 |
| φ′ | 26 | 26 | 56.350 | 1. Garcena | λ′ | 98 | 55 | 43.910 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.952 9138 | s² | 7.90583 | | | −h | 2.4242 |
| cos α | 9.959 5286 | sin² α | 9.23055 | (Δφ)² | 4.8484 | s² sin² α | 7.1364 |
| B | 8.511 8053 | C | 1.10115 | D | 2.2926 | E | 5.8519 |
| h | 2.424 2477 | | 8.23753 | | 7.1410 | | 5.4125 |
| 1st term. | −265.6120″ | 2d term. | +0.0014″ | | | (Δλ)′ | |
| 2d term. | + 0.0173 | 4th term. | 0.0000 | | | F | |
| 3d and 4th terms. | −265.5947<br>+ 0.0014 | s | 3.952 9138 | | | | |
| −Δφ | −265.5933 | sin α | 9.615 2756 | Arg. | | Δλ | 2.125 6403 |
| ½(φ+φ′) | 26 24 43.55 | A′ | 8.509 4343 | s | −2 | sin ½(φ+φ′) | 9.648 1885 |
| | | sec φ′ | 0.048 0163 | Δλ | 0 | sec ½(Δφ) | |
| | | | 2.125 6405 | Corr. | −2 | | 1.773 8288 |
| | | Δλ | + 133.5489 | | | −Δα | + 59.406 |

Do not write in this margin.

FIG. 27.—Preliminary position computation to obtain latitude and longitude closures of net—Continued

tude and longitude equations. In columns A and B are placed the designations of the length angles, that of the adjacent angle in A and that of the opposite angle in B, and in column C is placed the designation of the azimuth angle. It should be noted particularly that the designations in column C are of the concluded angles. (See p. 67.) The symbols in columns A and B may be taken directly from the column headed symbol in the length equation computation. The signs of the symbols in column C depend upon whether the azimuth angle is turned to the right or to the left in computing through the triangles. If it is turned to the right, then the symbols in column C are the combined symbols of columns A and B but with opposite signs. If the azimuth angle is turned to the left the symbols in column C are the same as in columns A and B combined and with the same signs.

As a guide in obtaining the correct signs for the symbols in column C, it is well to indicate to the left of each name in the "station" column the direction in which the azimuth angle is turned, denoting the right or clockwise turn by $+$, and the left or anti-clockwise turn by $-$. The directions of the turns are easily obtained from Figure 25.

The quantities $a_1$ and $a_2$ are obtained from the table on the right-hand side of form 496, using as argument the computed latitude, $\phi_n$, of the fixed point at the terminal of the arc, which in this case is 26°.45.

The quantities in the three columns headed "Lat. equation" and in the three columns headed "Long. equation," in Figure 28 are, as indicated in the headings, simply products of quantities in other columns previously filled out. The three columns headed Lat. equation $(\phi_n-\phi_c)\ \delta_A$, Lat. equation $(\phi_n-\phi_c)\ (-\delta_B)$ and Long. equation $(\phi_n-\phi_c)\ a_2$, respectively, should be filled out at the same time, since each of the three products contains the multiplier $(\phi_n-\phi_c)$, which can be set up on a multiplying machine and used without change for the three multiplications. In a similar manner the three columns headed Long. equation $(\lambda_n-\lambda_c)\ \delta_A$, Long. equation $(\lambda_n-\lambda_c)\ (-\delta_B)$ and Lat. equation $(\lambda_n-\lambda_c)\ a_1$, respectively, should be filled out at the same time, since they contain the multiplier $(\lambda_n-\lambda_c)$.

This completes the table on form 496 and the latitude and longitude equations themselves can now be formed.

In forming the latitude equation from the table it should be noted that the quantities in the column headed Lat. equation $(\phi_n-\phi_c)\ \delta_A$ are the coefficients of the symbols in column A, the quantities in the column headed Lat. equation $(\phi_n-\phi_c)\ (-\delta_B)$ are the coefficients of the symbols in column B, and the quantities in the column headed Lat. equation $(\lambda_n-\lambda_c)\ a_1$ are the coefficients of the symbols in the column headed C.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 496

## LATITUDE AND LONGITUDE ADJUSTMENT

$\varphi_n$: 26° 26'.94 $\lambda_n$: 98° 55'.73

| | Station | $\varphi_c$ | $\lambda_c$ | $\varphi_n - \varphi_c$ | $\lambda_n - \lambda_c$ | $+\delta_A$ | $-\delta_B$ | Lat. equation $(\varphi_n - \varphi_c)\delta_A$ | $A$ | Long. equation $(\lambda_n - \lambda_c)\delta_A$ | Lat. equation $(\varphi_n - \varphi_c)(-\delta_B)$ | $B$ | Long. equation $(\lambda_n - \lambda_c)(-\delta_B)$ | Lat. equation $(\lambda_n - \lambda_c)a_1$ | $C$ $(+R \text{ or } -L)$ | Long. equation $(\varphi_n - \varphi_c)a_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| + | Palo | 26° 19.65 | 98° 27.80 | +7.29 | +27.93 | +0.74 | -1.90 | +5.39 | -1 + 3 | +20.67 | -13.85 | -15 + 16 | -53.07 | +52.79 | + 1 - 3<br>+15 - 16 | -17.06 |
| - | Fordyce | 26 17.79 | 98 34.75 | +9.15 | +20.98 | +2.67 | -0.45 | +24.43 | -5 + 6 | +56.02 | - 4.12 | - 7 + 9 | - 9.44 | +39.65 | -9 + 6<br>-7 + 9 | -21.41 |
| + | Eltoro | 26 21.87 | 98 34.01 | +5.07 | +21.72 | +0.49 | -3.46 | +2.48 | -12 + 14 | +10.84 | -17.54 | -20 + 21 | -75.15 | +41.05 | +12 - 14<br>+20 - 21 | -11.86 |
| - | Garcia | 26 20.69 | 98 42.49 | +6.25 | +13.24 | +2.13 | -1.00 | +13.31 | -10 + 11 | +28.20 | - 6.25 | -22 + 24 | -13.24 | +25.02 | -10 + 11<br>-22 + 24 | -14.62 |
| + | Pancho | 26 26.61 | 98 41.29 | +0.33 | +14.44 | 0.00 | -1.13 | 0 | -17 + 19 | 0 | - 0.37 | -31 + 32 | -16.32 | +27.29 | +17 - 19<br>+31 - 32 | - 0.77 |
| - | Monument | 26 21.28 | 98 46.05 | +5.66 | +9.68 | +1.81 | +0.11 | +10.24 | -25 + 26 | +17.52 | + 0.62 | -33 + 35 | + 1.06 | +18.30 | -26 + 26<br>-33 + 35 | -13.24 |
| + | Corpus | 26 26.47 | 98 45.95 | +0.47 | +9.78 | +1.45 | -0.45 | +0.68 | -28 + 30 | +14.18 | - 0.21 | -40 + 41 | - 4.40 | +18.48 | +38 - 39<br>+40 - 41 | -1.10 |
| - | Grande | 26 23.50 | 98 49.52 | +3.44 | +6.21 | +1.93 | -2.27 | +6.64 | -36 + 37 | +11.99 | - 7.81 | -42 + 44 | -14.10 | +11.74 | -38 + 39<br>-42 + 44 | -8.05 |
| + | Hebron | 26 27.01 | 98 53.06 | -0.07 | +2.67 | +1.06 | -0.78 | -0.07 | -38 + 39 | +2.83 | + 0.05 | -50 + 51 | - 2.08 | + 5.05 | +50 - 51 | +0.16 |
| - | Ringold | 26 22.51 | 98 53.51 | +4.43 | +2.22 | +0.25 | -0.88 | +1.11 | -45 + 47 | +0.56 | - 3.90 | -53 + 54 | - 1.95 | + 4.20 | -45 + 47<br>-53 + 54 | -10.37 |

Garcena

$\varphi_n$ = 26° 26' 56".350 $\lambda_n$ = 98° 55' 43".910

$\varphi_n'$ = 26 26 56.345 $\lambda_n'$ = 98 55 43.916

$(\varphi_n - \varphi_n')$ = +0.005 $(\lambda_n - \lambda_n')$ = -0.006

$a_1$ = +1.89

$a_2$ = -2.34

×7238.24 ×7238.24

+36.19120 -43.42944

26. $\varphi$.

0 = +36.19120 + 47.40 (1) - 47.40 (3) - 64.08 (5) + 64.08 (6) - 35.53 (7) + 35.53 (9) - 38.33 (10) + 38.33 (11) + 38.57 (12)
- 38.57 (14) + 66.64 (15) - 66.64 (16) + 27.29 (17) - 27.29 (19) + 58.59 (20) - 58.59 (21) - 18.77 (22) + 18.77 (24) - 28.54 (25)
+ 28.54 (26) + 17.80 (28) - 17.80 (30) + 27.66 (31) - 27.66 (32) - 18.92 (33) + 18.92 (35) - 18.38 (36) + 18.38 (37) + 5.12 (38)
- 5.12 (39) + 18.69 (40) - 18.69 (41) - 3.93 (42) + 3.93 (44) - 5.31 (45) + 5.31 (47) + 5.00 (50) - 5.00 (51) - 0.30 (53) + 0.30 (54).

(This equation should be divided by 10 before entering it in the correlates.)

27. $\lambda$.

0 = -43.42944 - 37.73 (1) + 37.73 (3) - 34.61 (5) + 34.61 (6) + 30.85 (7) - 30.85 (9) - 13.58 (10) + 13.58 (11) - 22.50 (12) + 22.50 (14)
+36.01 (15) - 36.01 (16) - 0.77 (17) + 0.77 (19) + 63.29 (20) - 63.29 (21) + 27.86 (22) - 27.86 (24) - 4.28 (25) + 4.28 (26) - 15.28 (28)
+15.28 (30) + 15.55 (31) - 15.55 (32) + 12.18 (33) - 12.18 (35) - 3.94 (36) + 3.94 (37) - 2.67 (38) + 2.67 (39) + 3.30 (40) - 3.30 (41)
+22.15 (42) - 22.15 (44) + 9.81 (45) - 9.81 (47) + 2.24 (50) - 2.24 (51) + 12.32 (53) - 12.32 (54).

(This equation should be divided by 10 before entering it in the correlates.)

| $\varphi$ | $a_1$ | $a_2$ |
|---|---|---|
| 0 | 2.12 | 2.09 |
| 1 | 2.12 | 2.08 |
| 2 | 2.12 | 2.08 |
| 3 | 2.12 | 2.09 |
| 4 | 2.11 | 2.09 |
| 5 | 2.11 | 2.10 |
| 6 | 2.10 | 2.10 |
| 7 | 2.10 | 2.11 |
| 8 | 2.10 | 2.11 |
| 9 | 2.09 | 2.12 |
| 10 | 2.09 | 2.12 |
| 11 | 2.08 | 2.13 |
| 12 | 2.07 | 2.13 |
| 13 | 2.06 | 2.15 |
| 14 | 2.06 | 2.16 |
| 15 | 2.05 | 2.17 |
| 16 | 2.04 | 2.18 |
| 17 | 2.03 | 2.19 |
| 18 | 2.01 | 2.20 |
| 19 | 2.00 | 2.21 |
| 20 | 1.99 | 2.23 |
| 21 | 1.98 | 2.24 |
| 22 | 1.97 | 2.26 |
| 23 | 1.96 | 2.27 |
| 24 | 1.95 | 2.28 |
| 25 | 1.93 | 2.31 |
| 26 | 1.92 | 2.32 |
| 27 | 1.91 | 2.33 |
| 28 | 1.90 | 2.35 |
| 29 | 1.88 | 2.37 |
| 30 | 1.87 | 2.39 |
| 31 | 1.85 | 2.40 |
| 32 | 1.84 | 2.42 |
| 33 | 1.82 | 2.43 |
| 34 | 1.81 | 2.45 |
| 35 | 1.79 | 2.46 |
| 36 | 1.78 | 2.48 |
| 37 | 1.76 | 2.49 |
| 38 | 1.74 | 2.51 |
| 39 | 1.73 | 2.52 |
| 40 | 1.71 | 2.54 |
| 41 | 1.69 | 2.56 |
| 42 | 1.67 | 2.57 |
| 43 | 1.66 | 2.59 |
| 44 | 1.64 | 2.60 |
| 45 | 1.62 | 2.62 |
| 46 | 1.60 | 2.64 |
| 47 | 1.59 | 2.65 |
| 48 | 1.57 | 2.67 |
| 49 | 1.55 | 2.69 |
| 50 | 1.53 | 2.71 |
| 51 | 1.51 | 2.73 |
| 52 | 1.49 | 2.74 |
| 53 | 1.47 | 2.76 |
| 54 | 1.45 | 2.78 |
| 55 | 1.43 | 2.80 |
| 56 | 1.41 | 2.82 |
| 57 | 1.39 | 2.83 |
| 58 | 1.37 | 2.85 |
| 59 | 1.34 | 2.87 |
| 60 | 1.32 | 2.89 |
| 61 | 1.30 | 2.91 |
| 62 | 1.27 | 2.93 |
| 63 | 1.25 | 2.95 |
| 64 | 1.22 | 2.97 |
| 65 | 1.20 | 2.99 |
| 66 | 1.17 | 3.01 |
| 67 | 1.15 | 3.03 |
| 68 | 1.12 | 3.05 |
| 69 | 1.09 | 3.06 |
| 70 | .97 | 3.15 |
| 71 | .94 | 3.17 |
| 72 | .91 | 3.19 |
| 73 | .88 | 3.21 |
| 74 | .85 | 3.23 |
| 75 | .82 | 3.25 |
| 76 | .79 | 3.27 |
| 77 | .76 | 3.29 |
| 78 | .73 | 3.31 |
| 79 | .69 | 3.33 |

$\frac{M}{60} \times 10^9 = 7238.24$

FIG. 28.—Formation of latitude and longitude equations of net

*Correlate equations*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | α 24 | l 25 | φ 26 | λ 27 | $\Sigma_c$* | v* | Adopted v | $v^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | −1 | | | | | | | | | | | | | | | | | +4.20 | | | | | | | −0.74 | +4.74 | −3.77 | +3.43 | +0.287 | +0.29 | 0.0824 |
| 2 | | −1 | | | | | | | | | | | | | | | | −6.39 | | | | | | | | | | −7.39 | −.201 | −.20 | .0404 |
| 3 | +1 | +1 | | | | | | | | | | | | | | | | +2.19 | | | | | | | +.74 | −4.74 | +3.77 | +3.96 | −.086 | −.08 | .0074 |
| 4 | −1 | −1 | | | | | | | | | | | | | | | | +1.15 | | | | | | −1 | | | | −1.85 | −.054 | −.05 | .0029 |
| 5 | +1 | | −1 | | | | | | | | | | | | | | | −3.82 | | | | | | | −2.67 | −6.41 | −3.46 | −16.36 | +.176 | +.18 | .0310 |
| 6 | | +1 | +1 | | | | | | | | | | | | | | | +2.67 | | | | | | +1 | +2.67 | +6.41 | +3.46 | +18.21 | −.121 | −.12 | .0146 |
| 7 | | −1 | −1 | | | | | | | | | | | | | | | +2.85 | | | | | | −1 | +.45 | −3.55 | +3.08 | −.17 | +.430 | +.43 | .1849 |
| 8 | | +1 | | | | | | | | | | | | | | | | −5.25 | | | | | | | | | | −4.25 | +.041 | +.04 | .0017 |
| 9 | | | +1 | −1 | | | | | | | | | | | | | | +2.40 | | | | | | | −.45 | +3.55 | −3.08 | +2.42 | −.302 | −.30 | .0912 |
| 10 | | | | +1 | −1 | | | | | | | | | | | | | | | | | | | | −2.13 | −3.83 | −1.36 | −7.32 | +.058 | +.06 | .0034 |
| 11 | | | | | +1 | | | | | | | | | | | | | | | | | | | +1 | +2.13 | +3.83 | +1.36 | +9.32 | −.227 | −.23 | .0515 |
| 12 | | | | −1 | | −1 | | | | | | | | | | | | | +0.49 | | | | | | −.49 | +3.86 | −2.25 | −.39 | −.008 | −.01 | .0001 |
| 13 | | | | | | +1 | | | | | | | | | | | | | −2.25 | | | | | | | | | −1.25 | +.415 | +.41 | .1722 |
| 14 | | | −1 | +1 | | | | | | | | | | | | | | +1.02 | +1.76 | | | | | | +.49 | −3.86 | +2.25 | +1.66 | +.341 | +.34 | .1163 |
| 15 | −1 | | +1 | | | | | | | | | | | | | | | −2.92 | | | | | | | +1.90 | +6.66 | +3.60 | +9.24 | −.370 | −.37 | .1369 |
| 16 | +1 | | | | | | | | | | | | | | | | | +1.90 | | | | | | | −1.90 | −6.66 | −3.60 | −9.26 | −.378 | −.38 | .1429 |
| 17 | | | | | | | −1 | | | | | | | | | | | | | | | | | | | +2.73 | −.08 | +1.65 | +.462 | +.46 | .2134 |
| 18 | | | | | | | | −1 | | | | | | | | | | | | | | | | | | | | −1.00 | +.010 | +.01 | .0001 |
| 19 | | | | | −1 | −1 | +1 | +1 | | | | | | | | | | | +0.73 | | | | | | | −2.73 | +.08 | −1.92 | −.158 | −.16 | .0250 |
| 20 | | | | −1 | +1 | | | | | | | | | | | | | | −4.19 | | | | | | +3.46 | +5.86 | +6.33 | +11.46 | +.068 | +.07 | .0046 |
| 21 | | | | +1 | | +1 | | | | | | | | | | | | | +3.46 | | | | | | −3.46 | −5.86 | −6.33 | −10.19 | −.381 | −.38 | .1452 |
| 22 | | | | | −1 | | | | | | | | | | | | | | +4.66 | | | | | −1 | +1.00 | −1.88 | +2.79 | +4.57 | +.430 | +.43 | .1849 |
| 23 | | | | | | −1 | | | | | | | | | | | | | −5.66 | | | | | | | | | −6.66 | −.271 | −.27 | .0734 |
| 24 | | | | | +1 | +1 | −1 | −1 | | | | | | | | | | | +1.00 | +3.44 | | | | | −1.00 | +1.88 | −2.79 | +2.53 | +.300 | +.30 | .0900 |
| 25 | | | | | | | +1 | | −1 | | | | | | | | | | | −3.89 | | | | | −1.81 | −2.85 | −.43 | −8.98 | −.054 | −.05 | .0029 |
| 26 | | | | | | | | +1 | +1 | | | | | | | | | | | +.45 | | | | +1 | +1.81 | +2.85 | +.43 | +8.54 | −.404 | −.40 | .1632 |
| 27 | | | | | | | | | | | −1 | | | | | | | | | | +0.90 | +4.52 | | | | | | +4.42 | −.358 | −.36 | .1282 |
| 28 | | | | | | | | | | −1 | +1 | | | | | | | | | | −4.44 | −5.97 | | | −1.45 | +1.78 | −1.53 | −11.61 | +.615 | +.62 | .3782 |
| 29 | | | | | | | | | | | | −1 | | | | | | | | | +3.54 | | | | | | | +2.54 | +.421 | +.42 | .1772 |
| 30 | | | | | | | | | −1 | +1 | | +1 | | | | | | | | −.34 | | +1.45 | | | +1.45 | +1.78 | +1.53 | +3.31 | +.190 | +.19 | .0361 |
| 31 | | | | | | | −1 | | +1 | | | | | | | | | | | −1.13 | | | | | +1.13 | +2.77 | +1.56 | +4.33 | −.396 | −.39 | .1568 |
| 32 | | | | | | | +1 | | | | | | | | | | | | | +1.47 | | | | | −1.13 | −2.77 | −1.56 | −2.99 | −.472 | −.47 | .2228 |
| 33 | | | | | | | | −1 | −1 | | | | | | | | | | | +1.05 | | | | −1 | −.11 | −1.89 | +1.22 | −2.73 | +.676 | +.68 | .4570 |
| 34 | | | | | | | | +1 | | | | | | | | | | | | −4.79 | | | | | | | | −3.79 | −.342 | −.34 | .1170 |
| 35 | | | | | | | | | +1 | −1 | | −1 | | | | | | | | +3.74 | | +2.01 | | | +.11 | +1.89 | −1.22 | +5.53 | −.077 | −.08 | .0059 |
| 36 | | | | | | | | | | +1 | | | −1 | | | | | | | | | −3.94 | | | −1.93 | −1.84 | −.39 | −8.10 | +.219 | +.22 | .0480 |
| 37 | | | | | | | | | | | | +1 | +1 | | | | | | | | | +1.93 | | +1 | +1.93 | +1.84 | +.39 | +9.09 | −.476 | −.48 | .2266 |
| 38 | | | | | | | | | | | +1 | | | −1 | | | | | | | | | | | −1.06 | +.51 | −.27 | −.82 | +.096 | +.09 | .0092 |
| 39 | | | | | | | | | | | | | −1 | +1 | | | | | | | | | | | +1.06 | −.51 | +.27 | +.82 | +.184 | +.18 | .0339 |
| 40 | | | | | | | | | | −1 | | | +1 | | | | | | | | | | | | +.45 | +1.87 | +.33 | +2.65 | −.049 | −.05 | .0024 |
| 41 | | | | | | | | | | +1 | −1 | | | | | | | | | | | | | | −.45 | −1.87 | −.33 | −2.65 | −.231 | −.23 | .0534 |
| 42 | | | | | | | | | | | | −1 | −1 | | | | | | | | | +2.27 | | −1 | +2.27 | −.39 | +2.22 | +3.37 | +.554 | +.55 | .3069 |
| 43 | | | | | | | | | | | | +1 | | | | | | | | | +4.37 | | | | | | | +5.37 | −.516 | −.52 | .2663 |
| 44 | | | | | | | | | | | | | +1 | −1 | | | | | | | −4.76 | −4.20 | | | −2.27 | +.39 | −2.22 | −13.06 | +.133 | +.13 | .0177 |
| 45 | | | | | | | | | | | | | | +1 | −1 | | −1 | | | | +.39 | +1.93 | −0.05 | | −.25 | −.53 | +.98 | +1.47 | −.310 | −.31 | .0961 |
| 46 | | | | | | | | | | | | | | | | | +1 | | | | | | +2.03 | | | | | +3.03 | +.298 | +.30 | .0888 |
| 47 | | | | | | | | | | | | | | | +1 | | | | | | | | −1.98 | +1 | +.25 | +.53 | −.98 | −.18 | −.159 | −.16 | .0253 |
| 48 | | | | | | | | | | | | | | | | −1 | −1 | | | | | | −.48 | | −2.39 | | | −4.87 | +.628 | +.03 | .0008 |
| 49 | | | | | | | | | | | | | | | −1 | +1 | | | | | | | +1.23 | | +2.39 | | | +3.62 | +.219 | +.22 | .0480 |
| 50 | | | | | | | | | | | | | | −1 | +1 | | +1 | | | | +.15 | +.78 | −.75 | | +.78 | +.50 | +.22 | +2.68 | +.056 | +.06 | .0031 |
| 51 | | | | | | | | | | | −1 | | | +1 | | | | | | | −1.02 | −5.12 | | | −.78 | −.50 | −.22 | −7.64 | +.154 | +.16 | .0237 |
| 52 | | | | | | | | | | | +1 | | | | | | | | | | +.87 | +4.34 | | | | | | +6.21 | −.456 | −.45 | .2079 |
| 53 | | | | | | | | | | | | | | | −1 | | | | | | | | | −1 | +.88 | −.03 | +1.23 | +.08 | +.161 | +.16 | .0259 |
| 54 | | | | | | | | | | | | | | | +1 | −1 | | | | | | | | | −.88 | +.03 | −1.23 | −2.08 | +.067 | +.06 | .0045 |
| 55 | | | | | | | | | | | | | | | | +1 | | | | | | | | +1 | | | | +2.00 | −.227 | −.23 | .0515 |
| 56 | | | | | | | | | | | | | | | | −1 | | | | | | | −3.54 | | +2.20 | | | −2.34 | +.571 | +.57 | .3260 |
| 57 | | | | | | | | | | | | | | | | | −1 | | | | | | +3.98 | | | | | +2.98 | +.086 | +.09 | .0074 |
| 58 | | | | | | | | | | | | | | | | +1 | +1 | | | | | | −.44 | | −2.20 | | | −.64 | −.657 | −.66 | .4316 |

* In the $\Sigma_c$ column the sum of the values for all directions around each point (except the points at which some of the directions are not numbered, due to having been fixed by a previous adjustment) is equal to zero. For example, (1)+(2)+(3)=+3.43−7.39+3.96=0. The same thing applies to the $v$ column, in which the corresponding values are +0.287−0.201−0.086=0.

In the same way in forming the longitude equation the quantities in the column headed Long. equation $(\lambda_n - \lambda_c)\ \delta_A$ are the coefficients of the symbols in column A, the quantities in the column headed Long. equation $(\lambda_n - \lambda_c)\ (-\delta_B)$ are the coefficients of the symbols in column B, and the quantities in the column headed Long. equation $(\phi_n - \phi_c)\ a_2$ are the coefficients of the symbols in column C.

The absolute terms for the latitude and longitude equations are obtained by taking the products 7238.24 $(\phi_n - \phi_n')$ and 7238.24 $(\lambda_n - \lambda_n')$, respectively, as shown on page 69, $\phi_n$ and $\lambda_n$ denoting the computed latitude and longitude, and $\phi_n'$ and $\lambda_n'$ the fixed latitude and longitude of the terminal point.

If there is room on form 496 the latitude and longitude equations should be written on it directly below the table.

## FORMATION OF CORRELATE AND NORMAL EQUATIONS

After all the equations have been formed they are tabulated as shown on pages 82–83. In order to have the coefficients in the different equations of about the same size, it is usually well to divide some of the equations by a constant factor before entering them in the correlates. For example, equation 21 on page 61 is divided by 5, and equations 26 and 27 on page 81 are divided by 10. When dividing a correlate equation by a constant factor, one should be careful to divide the constant term of that equation by the same factor before using it in the normal equation.

The table of correlate equations should be checked by another mathematician, especially if the set is large, before it is used for forming the normal equations. The normal equations are formed in the same manner as for the station adjustment on pages 11–13. As stated there the equations are checked automatically by the values in the $\Sigma_n$ column. This does not apply, however, to the constant terms in the "$\eta$" column which should be checked by another mathematician. It should be remembered that the constant term of a divided equation (see p. 61) must be divided by the same factor before entering it in the table of normal equations. The table of normals is given on the folded page facing this page.

## SOLUTION OF NORMAL EQUATIONS

The solution of the normal equations is shortened considerably if the equations are taken in the proper order. In a net of triangulation composed of simple quadrilaterals, the rule is to eliminate the three angle equations and the one side equation of each quadrilateral in order. If the net is more involved, the order of solution should be such that each succeeding equation will introduce the fewest new terms.

## *Normal equations*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | α 24 | l 25 | φ 26 | λ 27 | η | $\Sigma_n$ | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | +6 | +2 | −2 | | | | | | | | | | | | | | | −2.16 | | | | | | +1 | −4.99 | −29.21 | −3.12 | +0.15 | −32.33 | −0.33919 |
| 2 | | +6 | +2 | | | | | | | | | | | | | | | +2.00 | | | | | | +3 | +2.96 | +5.22 | +4.15 | +.34 | +27.67 | +.11311 |
| 3 | | | +6 | −2 | | | | | | | | | | | | | | +2.10 | −1.76 | | | | | +2 | +5.85 | +30.44 | +2.11 | +1.74 | +46.48 | −.73456 |
| 4 | | | | +6 | −2 | +2 | | | | | | | | | | | | −1.38 | +8.92 | | | | | | −7.62 | −26.82 | −6.44 | −.26 | −29.60 | −.46475 |
| 5 | | | | | +6 | +2 | −2 | −2 | | | | | | | | | | | −8.58 | +3.44 | | | | +2 | +5.72 | +20.01 | +3.39 | +.19 | +28.17 | −.70103 |
| 6 | | | | | | +6 | −2 | −2 | | | | | | | | | | | +6.65 | +3.44 | | | | | −3.97 | −5.11 | −6.95 | −.77 | −.71 | +.37423 |
| 7 | | | | | | | +6 | +2 | −2 | | | | | | | | | | −.27 | −4.73 | | | | | −3.07 | −15.73 | −.60 | +1.05 | −21.35 | −.50643 |
| 8 | | | | | | | | +6 | +2 | | | | | | | | | | −.27 | −8.83 | | | | +2 | +2.92 | +.13 | +2.08 | +1.89 | +5.92 | −.00951 |
| 9 | | | | | | | | | +6 | −2 | | −2 | | | | | | | | +6.24 | | +0.56 | | +2 | +3.52 | +14.03 | −1.55 | +1.69 | +28.49 | −.89215 |
| 10 | | | | | | | | | | +6 | −2 | +2 | −2 | | | | | | | −4.08 | +4.44 | +1.47 | | | −.04 | −11.03 | +3.23 | +.31 | −3.70 | −.26948 |
| 11 | | | | | | | | | | | +6 | | | −2 | | | | | | | −3.45 | −1.03 | | | −1.28 | +4.66 | −1.25 | −.69 | −1.04 | −.05744 |
| 12 | | | | | | | | | | | | +6 | +2 | | | | | | | −4.08 | +0.83 | −.90 | | +2 | +1.00 | −1.44 | +.92 | +1.70 | +8.03 | −.46382 |
| 13 | | | | | | | | | | | | | +6 | −2 | | | | | | | −4.76 | −.60 | | +2 | −1.29 | +6.84 | −3.60 | +1.35 | +3.94 | −.33785 |
| 14 | | | | | | | | | | | | | | +6 | −2 | | −2 | | | | +3.98 | +.23 | +0.70 | | +2.58 | −2.94 | +3.30 | +.25 | +6.10 | −.28950 |
| 15 | | | | | | | | | | | | | | | +6 | −2 | +2 | | | | −.24 | −1.15 | −3.91 | +2 | −2.87 | +1.62 | −4.20 | +.11 | −4.64 | −.36726 |
| 16 | | | | | | | | | | | | | | | | +6 | +2 | | | | | | +4.81 | +1 | +1.26 | −.03 | +1.23 | +1.33 | +15.60 | −.52368 |
| 17 | | | | | | | | | | | | | | | | | +6 | | | | −.24 | −1.15 | −2.61 | | +1.22 | +1.03 | −.76 | +.11 | +5.60 | +.16843 |
| 18 | | | | | | | | | | | | | | | | | | +140.9338 | +1.7952 | | | | | −1.33 | +7.3852 | +13.4924 | +1.2067 | −2.24 | +161.8033 | +.01377 |
| 19 | | | | | | | | | | | | | | | | | | | +93.2120 | +3.4400 | | | | −4.66 | −22.1867 | −58.6049 | −35.2972 | −1.78 | −19.3916 | −.01822 |
| 20 | | | | | | | | | | | | | | | | | | | | +68.7558 | | +7.0244 | | −.60 | +1.2803 | +17.3235 | −15.5894 | −2.81 | +70.2246 | +.06951 |
| 21 | | | | | | | | | | | | | | | | | | | | | +76.7816 | +60.4347 | −.1320 | | +18.0583 | −9.3813 | +18.0000 | +5.116 | +169.4373 | −.01198 |
| 22 | | | | | | | | | | | | | | | | | | | | | | +153.6336 | −.6815 | −.34 | +41.1156 | +.7959 | +28.7425 | +9.58 | +297.7402 | −.08962 |
| 23 | | | | | | | | | | | | | | | | | | | | | | | +38.9152 | −1.98 | −3.8006 | −1.3979 | +1.7264 | +.238 | +31.8776 | +.06390 |
| 24 | | | | | | | | | | | | | | | | | | | | | | | | +12 | +4.30 | +23.20 | −5.88 | +3.81 | +47.52 | +.29643 |
| 25 | | | | | | | | | | | | | | | | | | | | | | | | | +116.2130 | +117.4082 | +121.6126 | −2.27 | +401.0159 | +.12416 |
| 26 | | | | | | | | | | | | | | | | | | | | | | | | | | +451.5432 | +91.8144 | +3.6191 | +641.4826 | −.01684 |
| 27 | | | | | | | | | | | | | | | | | | | | | | | | | | | +235.2684 | −4.3429 | +429.2015 | −.01628 |

The solution of the normal equations is given in full on pages 86–92. The method used is the same as for the adjustment of the quadrilateral on page 38, and it is therefore unnecessary to describe each step of this solution.

The check furnished by the $\Sigma$ column in the forward solution make any errors comparatively easy to find. However, there is no corresponding check on the back solution, and an error is not likely to be detected until the $v$'s are substituted in the triangles. As considerable recomputation is then required to correct for the error, it is well worth while, especially in the case of a large set of equations, to be particularly careful with the back solution. The back solution, or computation of the $C$'s, is shown on page 93.

*Solution of normal equations*

| 1 | 2 | 3 | 18 | 4 | 5 | 6 | 19 | α 24 | l 25 | φ 26 | λ 27 | η | $\Sigma_n$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| +6 | +2 | −2 | −2.16 | | | | | +1 | −4.99 | −29.21 | −3.12 | +.15 | −32.33 |
| $C_1$ | −.33333 | +.33333 | +.36 | | | | | −.16667 | +.83167 | +4.86833 | +.52 | −.025 | +5.38833 |
| | +6 | +2 | +2.00 | | | | | +3 | +2.96 | +5.22 | +4.15 | +.34 | +27.67 |
| 1 | −.6667 | +.6667 | +.72 | | | | | −.3333 | +1.6633 | +9.7367 | +1.04 | −.05 | +10.7767 |
| | +5.3333 | +2.6667 | +2.72 | | | | | +2.6667 | +4.6233 | +14.9567 | +5.19 | +.29 | +38.4467 |
| | $C_2$ | −.50001 | −.51000 | | | | | −.50001 | −.86687 | −2.80440 | −.97313 | −.05438 | −7.20880 |
| | | +6 | +2.10 | −2 | | | −1.76 | +2 | +5.85 | +30.44 | +2.11 | +1.74 | +46.48 |
| | | 1 −.6667 | −.72 | | | | | +.3333 | −1.6633 | −9.7367 | −1.04 | +.05 | −10.7767 |
| | | 2 −1.3333 | −1.36 | | | | | −1.3333 | −2.3117 | −7.4783 | −2.595 | −.145 | −19.2233 |
| | | +4.0000 | +.02 | −2 | | | −1.76 | +1.0000 | +1.8750 | +13.2250 | −1.5250 | +1.645 | +16.4800 |
| | | $C_3$ | −.005 | +.5 | | | +.44 | −.25 | −.46875 | −3.30625 | +.38125 | −.41125 | −4.12000 |
| | | | +140.9338 | −1.38 | | | +1.7952 | −1.33 | +7.3852 | +13.4924 | +1.2067 | −2.24 | +161.8033 |
| | | 1 | −.7776 | | | | | +.3600 | −1.7964 | −10.5156 | −1.1232 | +.054 | −11.6388 |
| | | 2 | −1.3872 | | | | | −1.3600 | −2.3579 | −7.6280 | −2.6469 | −.1479 | −19.6079 |
| | | 3 | −.0001 | +.01 | | | +.0088 | −.0050 | −.0094 | −.0661 | +.0076 | −.0082 | −.0824 |
| | | | +138.7689 | −1.37 | | | +1.8040 | −2.3350 | +3.2215 | −4.7173 | −2.5558 | −2.3421 | +130.4742 |
| | | | $C_{18}$ | +.00987 | | | −.01300 | +.01683 | −.02321 | +.03399 | +.01842 | +.01688 | −.940232 |
| | | | | +6 | −2 | +2 | +8.92 | | −7.62 | −26.82 | −6.44 | −.26 | −29.60 |
| | | | 3 | −1 | | | −.88 | +.50 | +.9375 | +6.6125 | −.7625 | +.8225 | +8.24 |
| | | | 18 | −.0135 | | | +.0178 | −.0231 | +.0318 | −.0466 | −.0252 | −.0231 | +1.2881 |
| | | | | +4.9865 | −2 | +2 | +8.0578 | +.4769 | −6.6507 | −20.2541 | −7.2277 | +.5394 | −20.0719 |
| | | | | $C_4$ | +.40108 | −.40108 | −1.61592 | −.09564 | +1.33374 | +4.06179 | +1.44945 | −.10817 | +4.02525 |



| | 5 | 6 | 19 | 7 | 8 | 9 | 20 | α 24 | l 25 | φ 26 | λ 27 | η | $\Sigma_n$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | +6 | +2 | −8.58 | −2 | −2 | | +3.44 | +2 | +5.72 | +20.01 | +3.39 | +.19 | +28.17 |
| | −.8022 | +.8022 | +3.2318 | | | | | +.1913 | −2.6675 | −8.1236 | −2.8989 | +.2163 | −8.0505 |
| | +5.1978 | +2.8022 | −5.3482 | −2 | −2 | | +3.44 | +2.1913 | +3.0525 | +11.8864 | +.4911 | +.4063 | +20.11954 |
| | $C_5$ | −.53911 | +1.02894 | +.38478 | +.38478 | | −.66182 | −.42158 | −.58727 | −2.28681 | −.09448 | −.07817 | −3.870734 |
| | | +6 | +6.65 | −2 | −2 | | +3.44 | | −3.97 | −5.11 | −6.95 | −.77 | −.71 |
| | 4 | −.8022 | −3.2318 | | | | | −.1913 | +2.6675 | +8.1236 | +2.8989 | −.2163 | +8.0505 |
| | 5 | −1.5107 | +2.8833 | +1.0782 | +1.0782 | | −1.8546 | −1.1814 | −1.6456 | −6.4081 | −.2648 | −.2190 | −10.8466 |
| | | +3.6871 | +6.3015 | −.9218 | −.9218 | | +1.5854 | −1.3727 | −2.9481 | −3.3945 | −4.3159 | −1.2053 | −3.5061 |
| | | $C_6$ | −1.70907 | +.25001 | +.25001 | | −.42999 | +.37230 | +.79957 | +.92064 | +1.17054 | +.32690 | +.95091 |
| | | | +93.2120 | −.27 | −.27 | | +3.4400 | −4.66 | −22.1867 | −58.6049 | −35.2972 | −1.78 | −19.3916 |
| | | 3 | −.7744 | | | | | +.44 | +.8250 | +5.8190 | −.6710 | +.7238 | +7.2512 |
| | | 18 | −.0234 | | | | | +.0303 | −.0419 | +.0613 | +.0332 | +.0305 | −1.6962 |
| | | 4 | −13.0208 | | | | | −.7706 | +10.7470 | +32.7291 | +11.6794 | −.8716 | +32.4347 |
| | | 5 | −5.5030 | −2.0579 | −2.0579 | | +3.5396 | +2.2547 | +3.1408 | +12.2303 | +.5053 | +.4181 | +20.7015 |
| | | 6 | −10.7697 | +1.5754 | +1.5754 | | −2.7096 | +2.3460 | +5.0385 | +5.8014 | +7.3762 | +2.0600 | +5.9922 |
| | | | +63.1207 | −.7525 | −.7525 | | +4.2700 | −.3596 | −2.4773 | −1.9638 | −16.3741 | +.5808 | +45.29187 |
| | | | $C_{19}$ | +.01192 | +.01192 | | −.06765 | +.00570 | +.03925 | +.03111 | +.25941 | −.00919 | −.717543 |
| | | | | +6 | +2 | −2 | −4.73 | | −3.07 | −15.73 | −.60 | +1.05 | −21.35 |
| | | | 5 | −.7696 | −.7696 | | +1.3236 | +.8432 | +1.1745 | +4.5736 | +.1890 | +.1563 | +7.7415 |
| | | | 6 | −.2305 | −.2305 | | +.3964 | −.3432 | −.7370 | −.8486 | −1.0790 | −.3013 | −.8765 |
| | | | 19 | −.0090 | −.0090 | | +.0509 | −.0043 | −.0295 | −.0234 | −.1952 | +.0069 | +.5399 |
| | | | | +4.9909 | +.9909 | −2 | −2.9591 | +.4957 | −2.6620 | −12.0284 | −1.6852 | +.9119 | −13.94573 |
| | | | | $C_7$ | −.19854 | +.40073 | +.59290 | −.09932 | +.53337 | +2.41007 | +.33765 | −.18271 | +2.79415 |
| | | | | | +6 | +2 | −8.83 | +2 | +2.92 | +.13 | +2.08 | +1.89 | +5.92 |
| | | | | 5 | −.7696 | | +1.3236 | +.8432 | +1.1745 | +4.5736 | +.1890 | +.1563 | +7.7415 |
| | | | | 6 | −.2305 | | +.3964 | −.3432 | −.7370 | −.8486 | −1.0790 | −.3013 | −.8765 |
| | | | | 19 | −.0090 | | +.0509 | −.0043 | −.0295 | −.0234 | −.1952 | +.0069 | +.5399 |
| | | | | 7 | −.1967 | +.3971 | +.5875 | −.0984 | +.5285 | +2.3881 | +.3346 | −.1810 | +2.7687 |
| | | | | | +4.7942 | +2.3971 | −6.4716 | +2.3973 | +3.8565 | +6.2197 | +1.3294 | +1.5709 | +16.09365 |
| | | | | | $C_8$ | −.50000 | +1.34988 | −.50004 | −.80441 | −1.29734 | −.27729 | −.32767 | −3.35687 |

*Solution of normal equations*—Continued

| | 9 | 20 | 10 | 12 | 11 | 13 | 22 | 21 | α 24 | l 25 | φ 26 | λ 27 | η | $\Sigma_n$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | +6 | +6.24 | −2 | −2 | | | +.56 | | +2 | +3.52 | +14.03 | −1.55 | +1.69 | +28.49 |
| 7 | −.8015 | −1.1858 | | | | | | | +.1986 | −1.0667 | −4.8201 | −.6753 | +.3654 | −5.5883 |
| 8 | −1.1985 | +3.2358 | | | | | | | −1.1986 | −1.9283 | −3.1099 | −.6647 | −.7854 | −8.0467 |
| | +4 | +8.29 | −2 | −2 | | | +.56 | | +1 | +.5250 | +6.1000 | −2.89 | +1.27 | +14.855 |
| | $C_9$ | −2.0725 | +.5 | +.5 | | | −.14 | | −.25 | −.13125 | −1.525 | +.7225 | −.3175 | −3.71375 |
| | | +68.7558 | −4.08 | −4.08 | | | +7.0244 | | −.60 | +1.2803 | +17.3235 | −15.5894 | −2.81 | +70.2246 |
| | 5 | −2.2767 | | | | | | | −1.4502 | −2.0202 | −7.8666 | −.3250 | −.2689 | −13.3153 |
| | 6 | −.6817 | | | | | | | +.5902 | +1.2676 | +1.4596 | +1.8558 | +.5183 | +1.5076 |
| | 19 | −.2889 | | | | | | | +.0243 | +.1676 | +.1328 | +1.1077 | −.0392 | −3.0639 |
| | 7 | −1.7545 | | | | | | | +.2939 | −1.5783 | −7.1316 | −.9991 | +.5407 | −8.2682 |
| | 8 | −8.7359 | | | | | | | +3.2361 | +5.2058 | +8.3959 | +1.7945 | +2.1205 | +21.7243 |
| | 9 | −17.1810 | +4.1450 | +4.1450 | | | −1.1606 | | −2.0725 | −1.0881 | −12.6422 | +5.9895 | −2.6321 | −30.7870 |
| | | +37.8371 | +.0650 | +.0650 | | | +5.8638 | | +.0218 | +3.2347 | −.3286 | −6.1660 | −2.5707 | +38.0221 |
| | | $C_{20}$ | −.00172 | −.00172 | | | −.15497 | | −.00058 | −.08549 | +.00868 | +.16296 | +.06794 | −1.0048990 |
| | | | +6 | +2 | −2 | −2 | +1.47 | +4.44 | | −.04 | −11.03 | +3.23 | +.31 | −3.70 |
| | | 9 | −1 | −1 | | | +.28 | | +.5 | +.2625 | +3.05 | −1.445 | +.635 | +7.4275 |
| | | 20 | −.0001 | −.0001 | | | −.0101 | | .0000 | −.0056 | +.0006 | +.0106 | +.0044 | −.0653 |
| | | | +4.9999 | +.9999 | −2 | −2 | +1.7399 | +4.44 | +.5 | +.2169 | −7.9794 | +1.7956 | +.9494 | +3.6622 |
| | | | $C_{10}$ | −.19998 | +.40001 | +.40001 | −.34799 | −.88802 | −.10000 | −.04338 | +1.59591 | −.35913 | −.18988 | −.73245 |
| | | | | +6 | | +2 | −.90 | +.83 | +2 | +1.00 | −1.44 | +.92 | +1.70 | +8.03 |
| | | | 9 | −1 | | | +.28 | | +.5 | +.2625 | +3.05 | −1.445 | +.635 | +7.4275 |
| | | | 20 | −.0001 | | | −.0101 | | −.0000 | −.0056 | +.0006 | +.0106 | +.0044 | −.0653 |
| | | | 10 | −.2000 | +.4000 | +.4000 | −.3480 | −.8879 | −.1000 | −.0434 | +1.5958 | −.3591 | −.1899 | −.7324 |
| | | | | +4.7999 | +.4000 | +2.4000 | −.9781 | −.0579 | +2.4 | +1.2135 | +3.2064 | −.8735 | +2.1495 | +14.6598 |
| | | | | $C_{12}$ | −.08334 | −.50001 | +.20378 | +.01206 | −.50001 | −.25282 | −.66801 | +.18198 | −.44782 | −3.05419 |



| | 11 | 13 | 14 | 22 | 21 | 15 | 17 | 23 | α 24 | l 25 | φ 26 | λ 27 | η | $\Sigma_n$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | +6 | | −2 | −1.03 | −3.45 | | | | | −1.28 | +4.66 | −1.25 | −0.69 | −1.04 |
| 10 | −.8000 | −0.8000 | | +.6960 | +1.7760 | | | | +0.2000 | +.0868 | −3.1918 | +.7183 | +.3798 | +1.4649 |
| 12 | −.0333 | −.2000 | | +.0815 | +.0048 | | | | −.2000 | −.1011 | −.2672 | +.0728 | −.1791 | −1.2217 |
| | +5.1667 | −1.0 | −2 | −.2525 | −1.6692 | | | | .0000 | −1.2943 | +1.2010 | −.4589 | −.4893 | −.79685 |
| | $C_{11}$ | +.19355 | +.38709 | +.04887 | +.32307 | | | | .0000 | +.25051 | −.23245 | +.08882 | +.09470 | +.15416 |
| | | +6 | −2 | −.60 | −4.76 | | | | +2 | −1.29 | +6.84 | −3.60 | +1.35 | +3.94 |
| | 10 | −.8 | | +.6960 | +1.7760 | | | | +.2 | +.0868 | −3.1918 | +.7183 | +.3798 | +1.4649 |
| | 12 | −1.2 | | +.4891 | +.0290 | | | | −1.2 | −.6068 | −1.6032 | +.4368 | −1.0749 | −7.3299 |
| | 11 | −.1936 | −.3871 | −.0489 | −.3231 | | | | .0000 | −.2505 | +.2324 | −.0888 | −.0947 | −.1542 |
| | | +3.8064 | −2.3871 | +.5362 | −3.2781 | | | | +1.0 | −2.0605 | +2.2774 | −2.5337 | +.5602 | −2.0792 |
| | | $C_{13}$ | +.62713 | −.14087 | +.86121 | | | | −.26272 | +.54133 | −.59831 | +.66564 | −.14717 | +.54624 |
| | | | +6 | +.23 | +3.98 | −2 | −2 | +0.70 | | +2.58 | −2.94 | +3.30 | +.25 | +6.10 |
| | | 11 | −.7742 | −.0977 | −.6461 | | | | .0000 | −.5010 | +.4649 | −.1776 | −.1894 | −.3083 |
| | | 13 | −1.4970 | +.3363 | −2.0558 | | | | +.6271 | −1.2922 | +1.4282 | −1.5889 | +.3513 | −1.3039 |
| | | | +3.7288 | +.4686 | +1.2781 | −2 | −2 | +.70 | +.6271 | +.7868 | −1.0469 | +1.5335 | +.4119 | +4.48789 |
| | | | $C_{14}$ | −.12567 | −.34276 | +.53637 | +.53637 | −.18773 | −.16818 | −.21101 | +.28076 | −.41126 | −.11046 | −1.203587 |
| | | | | +153.6386 | +60.4347 | −1.15 | −1.15 | −.6815 | −.34 | +41.1156 | +.7959 | +28.7425 | +9.58 | +297.7402 |
| | | | 9 | −.0784 | | | | | −.14 | −.0735 | −.8540 | +.4046 | −.1778 | −2.0797 |
| | | | 20 | −.9087 | | | | | −.0034 | −.5013 | +.0509 | +.9556 | +.3984 | −5.8925 |
| | | | 10 | −.6055 | −1.5451 | | | | −.1740 | −.0755 | +2.7767 | −.6247 | −.3304 | −1.2744 |
| | | | 12 | −.1993 | −.0118 | | | | +.4891 | +.2473 | +.6534 | −.1780 | +.4380 | +2.9873 |
| | | | 11 | −.0123 | −.0816 | | | | .0000 | −.0633 | +.0587 | −.0224 | −.0239 | −.0389 |
| | | | 13 | −.0755 | +.4618 | | | | −.1409 | +.2903 | −.3208 | +.3569 | −.0789 | +.2929 |
| | | | 14 | −.0589 | −.1606 | +.2513 | +.2513 | −.0880 | −.0788 | −.0989 | +.1316 | −.1927 | −.0518 | −.5640 |
| | | | | +151.7000 | +59.0974 | −.8987 | −.8987 | −.7695 | −.3880 | +40.8407 | +3.2924 | +29.4418 | +9.7536 | +291.170910 |
| | | | | $C_{22}$ | −.38957 | +.00592 | +.00592 | +.00507 | +.00256 | −.26922 | −.02170 | −.19408 | −.06430 | −1.919340 |

*Solution of normal equations*—Continued

| | 21 | 15 | 17 | 16 | 23 | $\alpha$ 24 | $l$ 25 | $\phi$ 26 | $\lambda$ 27 | $\eta$ | $\Sigma_n$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | +76.7816 | −0.24 | −0.24 | ---------- | −0.1320 | ---------- | +18.0583 | −9.3813 | +18.0000 | +5.116 | +169.4373 |
| 10 | −3.9428 | ---------- | ---------- | ---------- | ---------- | −0.4440 | −.1926 | +7.0358 | −1.5945 | −.8431 | −3.2521 |
| 12 | −.0007 | ---------- | ---------- | ---------- | ---------- | +.0290 | +.0146 | +.0387 | −.0105 | +.0259 | +.1768 |
| 11 | −.5393 | ---------- | ---------- | ---------- | ---------- | .0000 | −.4182 | +.3880 | −.1483 | −.1581 | −.2573 |
| 13 | −2.8231 | ---------- | ---------- | ---------- | ---------- | +.8612 | −1.7745 | +1.9613 | −2.1820 | +.4824 | −1.7906 |
| 14 | −.4381 | +.6855 | +.6855 | ---------- | −.2399 | −.2150 | −.2397 | +.3588 | −.5256 | −.1412 | −1.5383 |
| 22 | −23.0226 | +.3499 | +.3499 | ---------- | +.2996 | +.1513 | −15.9102 | −1.2824 | −11.4696 | −3.8000 | −113.4315 |
| | +46.0150 | +.7954 | +.7954 | ---------- | −.0723 | +.3825 | −.4923 | −.8311 | +2.0695 | +.6819 | +49.34430 |
| | $C_{21}$ | −.01729 | −.01729 | ---------- | +.00157 | −.00831 | +.01070 | +.01806 | −.04497 | −.01482 | −1.07235 |
| | | +6 | +2 | −2 | −3.91 | +2 | −2.87 | +1.62 | −4.20 | +.11 | −4.64 |
| | 14 | −1.0727 | −1.0727 | ---------- | +.3755 | +.3364 | +.4220 | −.5615 | +.8225 | +.2209 | +2.4071 |
| | 22 | −.0053 | −.0053 | ---------- | −.0046 | −.0023 | +.2419 | +.0195 | +.1744 | +.0578 | +1.7250 |
| | 21 | −.0138 | −.0138 | ---------- | +.0012 | −.0036 | +.0085 | +.0144 | −.0358 | −.0118 | −.8529 |
| | | +4.9082 | +.9082 | −2 | −3.5379 | +2.3275 | −2.1976 | +1.0924 | −3.2389 | +.3769 | −1.366812 |
| | | $C_{15}$ | −.18504 | +.40748 | +.72081 | −.47421 | +.44774 | −.22257 | +.65990 | −.07679 | +.277332 |
| | | | +6 | +2 | −2.61 | ---------- | +1.22 | +1.03 | −.76 | +.11 | +5.00 |
| | | 14 | −1.0727 | ---------- | +.3755 | +.3364 | +.4220 | −.5615 | +.8225 | +.2209 | +2.4071 |
| | | 22 | −.0053 | ---------- | −.0046 | −.0023 | +.2419 | +.0195 | +.1744 | +.0578 | +1.7250 |
| | | 21 | −.0138 | ---------- | +.0012 | −.0036 | +.0085 | +.0144 | −.0358 | −.0118 | −.8529 |
| | | 15 | −.1681 | +.3701 | +.6543 | −.4207 | +.4033 | −.2021 | +.5993 | −.0597 | +.2519 |
| | | | +4.7401 | +2.3701 | −1.5833 | −.1032 | +2.2990 | +.3003 | +.8004 | +.3072 | +9.131706 |
| | | | $C_{17}$ | −.50001 | +.33402 | +.02177 | −.48501 | −.03335 | −.16883 | −.03481 | −1.92625 |
| | | | | +6 | +4.81 | +1 | +1.23 | −.03 | +1.23 | +1.33 | +15.00 |
| | | | 15 | −.8150 | −1.4416 | +.9434 | −.8355 | +.4451 | −1.3198 | +.1536 | −.5546 |
| | | | 17 | −1.1350 | +.7916 | +.0516 | −1.1495 | −.1501 | −.4002 | −.1536 | −4.5654 |
| | | | | +4 | +4.16 | +2.00 | −.7850 | +.2650 | −.49 | +1.33 | +10.48 |
| | | | | $C_{16}$ | −1.04 | −.5 | +.19625 | −.06625 | +.1225 | −.3325 | −2.62 |

*Solution of normal equations*—Continued

| | 23 | α 24 | l 25 | φ 26 | λ 27 | η | $\Sigma_n$ |
|---|---|---|---|---|---|---|---|
| | +38.9152 | −1.98 | −3.8006 | −1.3979 | +1.7264 | +0.238 | +31.8776 |
| 14 | −.1314 | −.1177 | −.1477 | +.1965 | −.2879 | −.0773 | −.8425 |
| 22 | −.0039 | −.0020 | +.2072 | +.0167 | +.1493 | +.0495 | +1.4770 |
| 21 | −.0001 | +.0006 | −.0008 | −.0013 | +.0033 | +.0011 | +.0775 |
| 15 | −2.5502 | +1.6777 | −1.5841 | +.7874 | −2.3347 | +.2717 | −.9811 |
| 17 | −.5289 | −.0345 | +.7679 | +.1003 | +.2674 | +.1026 | +3.0498 |
| 16 | −4.3264 | −2.08 | +.8164 | −.2756 | +.5096 | −1.3832 | −10.8992 |
| | +31.3743 | −2.5359 | −3.7417 | −.5739 | +.0334 | −.7976 | +23.759786 |
| | $C_{33}$ | +.08083 | +.11926 | +.01829 | −.00106 | +.02542 | −.75726 |
| | | +12 | +4.30 | +23.20 | −5.88 | +3.81 | +47.52 |
| | 1 | −.1667 | +.8317 | +4.8683 | +.52 | −.025 | +5.3883 |
| | 2 | −1.3334 | −2.3117 | −7.4785 | −2.5950 | −.1450 | −19.2237 |
| | 3 | −.2500 | −.4688 | −3.3062 | +.3812 | −.4112 | −4.1200 |
| | 18 | −.0393 | +.0542 | −.0794 | −.0430 | −.0394 | +2.1954 |
| | 4 | −.0456 | +.6361 | +1.9371 | +.6912 | −.0516 | +1.9196 |
| | 5 | −.9238 | −1.2869 | −5.0111 | −.2070 | −.1713 | −8.4820 |
| | 6 | −.5111 | −1.0976 | −1.2638 | −1.6068 | −.4487 | −1.3053 |
| | 19 | −.0020 | −.0141 | −.0112 | −.0933 | +.0033 | +.2580 |
| | 7 | −.0492 | +.2644 | +1.1947 | +.1674 | −.0906 | +1.3851 |
| | 8 | −1.1987 | −1.9284 | −3.1101 | −.6647 | −.7855 | −8.0474 |
| | 9 | −.2500 | −.1312 | −1.5250 | +.7225 | −.3175 | −3.7138 |
| | 20 | −.0000 | −.0019 | +.0002 | +.0036 | +.0015 | −.0219 |
| | 10 | −.0500 | −.0217 | +.7979 | −.1796 | −.0949 | −.3662 |
| | 12 | −1.2000 | −.6068 | −1.6032 | +.4368 | −1.0748 | −7.3299 |
| | 11 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 |
| | 13 | −.2627 | +.5413 | −.5983 | +.6656 | −.1472 | +.5462 |
| | 14 | −.1055 | −.1323 | +.1761 | −.2579 | −.0693 | −.7548 |
| | 22 | −.0010 | +.1045 | +.0084 | +.0753 | +.0249 | +.7447 |
| | 21 | −.0032 | +.0041 | +.0069 | −.0172 | −.0057 | −.4102 |
| | 15 | −1.1037 | +1.0421 | −.5180 | +1.5359 | −.1787 | +.6455 |
| | 17 | −.0022 | +.0501 | +.0065 | +.0174 | +.0067 | +.1988 |
| | 16 | −1.0000 | +.3925 | −.1325 | +.2450 | −.6650 | −5.2400 |
| | 23 | −.2050 | −.3024 | −.0464 | +.0027 | −.0645 | +1.9203 |
| | | +3.2969 | −.0828 | +7.5124 | −6.0799 | −.9395 | +3.706771 |
| | | $C_{34}$ | +.02511 | −2.27863 | +1.84413 | +.28496 | −1.124423 |
| | | | +116.2130 | +117.4082 | +121.6126 | −2.27 | +401.0159 |
| | | 1 | −4.1500 | −24.2930 | −2.5948 | +.1248 | −26.8878 |
| | | 2 | −4.0078 | −12.9656 | −4.4991 | −.2514 | −33.3284 |
| | | 3 | −.8789 | −6.1992 | +.7148 | −.7711 | −7.7250 |
| | | 18 | −.0748 | +.1095 | +.0593 | +.0544 | −3.0289 |
| | | 4 | −8.8703 | −27.0137 | −9.6399 | +.7194 | −26.7707 |
| | | 5 | −1.7926 | −6.9805 | −.2884 | −.2386 | −11.8154 |
| | | 6 | −2.3572 | −2.7141 | −3.4509 | −.9637 | −2.8034 |
| | | 19 | −.0972 | −.0771 | −.6426 | +.0228 | +1.7775 |
| | | 7 | −1.4198 | −6.4156 | −.8988 | +.4864 | −7.4380 |
| | | 8 | −3.1022 | −5.0032 | −1.0694 | −1.2637 | −12.9458 |
| | | 9 | −.0689 | −.8006 | +.3793 | −.1667 | −1.9497 |
| | | 20 | −.2765 | +.0281 | +.5271 | +.2198 | −3.2506 |
| | | 10 | −.0094 | +.3462 | −.0779 | −.0412 | −.1589 |
| | | 12 | −.3068 | −.8106 | +.2208 | −.5434 | −3.7063 |
| | | 11 | −.3242 | +.3009 | −.1150 | −.1226 | −.1995 |
| | | 13 | −1.1154 | +1.2328 | −1.3716 | +.3032 | −1.1255 |
| | | 14 | −.1660 | +.2209 | −.3236 | −.0869 | −.9470 |
| | | 22 | −10.9951 | −.8862 | −7.9264 | −2.6261 | −78.3896 |
| | | 21 | −.0053 | −.0089 | +.0221 | +.0073 | +.5279 |
| | | 15 | −.9840 | +.4891 | −1.4502 | +.1688 | −.6094 |
| | | 17 | −1.1150 | −.1456 | −.3882 | −.1490 | −4.4284 |
| | | 16 | −.1541 | +.0520 | −.0962 | +.2610 | +2.0567 |
| | | 23 | −.4462 | −.0684 | +.0040 | −.0951 | +2.8334 |
| | | 24 | −.0021 | +.1887 | −.1527 | −.0236 | +.0931 |
| | | | +73.4932 | +25.9941 | +88.5543 | −7.2452 | +180.79024 |
| | | | $C_{35}$ | −.35369 | −1.20493 | +.09858 | −2.46004 |

*Solution of normal equations*—Continued

| | $\phi$ 26 | $\lambda$ 27 | $\eta$ | $\Sigma_n$ |
|---|---|---|---|---|
| | +451.5432 | +91.8144 | +3.6191 | +641.4826 |
| 1 | −142.2039 | −15.1892 | +.7302 | −157.3931 |
| 2 | −41.9446 | −14.5548 | −.8133 | −107.8199 |
| 3 | −43.7252 | +5.0420 | −5.4388 | −54.4870 |
| 18 | −.1603 | −.0869 | −.0796 | +4.4353 |
| 4 | −82.2679 | −29.3573 | +2.1909 | −81.5278 |
| 5 | −27.1819 | −1.1230 | −.9292 | −46.0092 |
| 6 | −3.1251 | −3.9734 | −1.1097 | −3.2279 |
| 19 | −.0611 | −.5094 | +.0180 | +1.4091 |
| 7 | −28.9893 | −4.0614 | +2.1977 | −33.6092 |
| 8 | −8.0691 | −1.7247 | −2.0380 | −20.8787 |
| 9 | −9.3025 | +4.4072 | −1.9368 | −22.6539 |
| 20 | −.0029 | −.0535 | −.0223 | +.3302 |
| 10 | −12.7344 | +2.8656 | +1.5151 | +5.8445 |
| 12 | −2.1419 | +.5835 | −1.4359 | −9.7930 |
| 11 | −.2792 | +.1067 | +.1137 | +.1851 |
| 13 | −1.3626 | +1.5159 | −.3352 | +1.2440 |
| 14 | −.2939 | +.4305 | +.1156 | +1.2600 |
| 22 | −.0714 | −.6390 | −.2117 | −6.3194 |
| 21 | −.0150 | +.0374 | +.0123 | +.8912 |
| 15 | −.2431 | +.7209 | −.0839 | +.3029 |
| 17 | −.0190 | −.0507 | −.0195 | −.5785 |
| 16 | −.0176 | +.0325 | −.0881 | −.6943 |
| 23 | −.0105 | +.0006 | −.0146 | +.4346 |
| 24 | −17.1180 | +13.8538 | +2.1407 | −8.4472 |
| 25 | −9.1939 | −31.3211 | +2.5625 | −63.9465 |
| | +21.0089 | +18.7666 | +.6592 | +40.433947 |
| | $C_{26}$ | −.89327 | −.03138 | −1.92465 |
| | | +235.2684 | −4.3429 | +429.2015 |
| | 1 | −1.6224 | +.0780 | −16.8116 |
| | 2 | −5.0505 | −.2822 | −37.4137 |
| | 3 | −.5814 | +.6272 | +6.2830 |
| | 18 | −.0471 | −.0431 | +2.4030 |
| | 4 | −10.4762 | +.7818 | −29.0933 |
| | 5 | −.0464 | −.0384 | −1.9009 |
| | 6 | −5.0519 | −1.4109 | −4.1040 |
| | 19 | −4.2476 | +.1505 | +11.7489 |
| | 7 | −.5690 | +.3079 | −4.7087 |
| | 8 | −.3686 | −.4356 | −4.4626 |
| | 9 | −2.0880 | +.9176 | +10.7327 |
| | 20 | −1.0048 | −.4189 | +6.1962 |
| | 10 | −.6449 | −.3409 | −1.3152 |
| | 12 | −.1590 | +.3912 | +2.6678 |
| | 11 | −.0408 | −.0435 | −.0707 |
| | 13 | −1.6865 | +.3729 | −1.3840 |
| | 14 | −.6307 | −.1694 | −1.8457 |
| | 22 | −5.7141 | −1.8931 | −56.5106 |
| | 21 | −.0931 | −.0307 | −2.2193 |
| | 15 | −2.1374 | +.2487 | −.8982 |
| | 17 | −.1352 | −.0519 | −1.5418 |
| | 16 | −.0600 | +.1629 | +1.2838 |
| | 23 | −.0000 | +.0008 | −.0253 |
| | 24 | −11.2121 | −1.7325 | +6.8364 |
| | 25 | −106.7017 | +8.7297 | −217.8471 |
| | 26 | −16.7636 | −.5890 | −36.1191 |
| | | +58.1354 | +.9462 | +59.08156 |
| | | $C_{27}$ | −.01628 | −1.01628 |

*Back solution*

| 27 | 26 | 25 | 24 | 23 | 16 | 17 |
|---|---|---|---|---|---|---|
| −0.01628 | −0.03138 | +0.09858 | +0.28496 | +0.02542 | −0.3325 | −0.06481 |
| | +.01454 | +.01962 | −.03002 | +.00002 | −.00199 | +.00275 |
| | −.01684 | +.00596 | +.03837 | −.00031 | +.00112 | +.00107 |
| | | +.12416 | +.00312 | +.01481 | +.02437 | −.06022 |
| | | | +.29643 | +.02396 | −.14822 | +.00645 |
| | | | | +.06390 | −.06646 | +.02134 |
| | | | | | −.52368 | +.26185 |
| | | | | | | +.16843 |

| 15 | 21 | 22 | 14 | 13 | 11 | 12 |
|---|---|---|---|---|---|---|
| −0.07679 | −0.01482 | −0.06430 | −0.11046 | −0.14717 | +0.09470 | −0.44782 |
| −.01074 | +.00073 | +.00316 | +.00670 | −.01084 | −.00145 | −.00296 |
| +.00375 | −.00030 | +.00037 | −.00473 | +.01008 | +.00391 | +.01125 |
| +.05559 | +.00133 | −.03343 | −.02620 | +.06721 | +.03110 | −.03139 |
| −.14057 | −.00246 | +.00076 | −.04985 | −.07788 | 0 | −.14822 |
| +.04606 | +.00010 | +.00032 | −.01200 | −.01032 | −.00387 | −.00014 |
| −.21339 | −.00291 | +.00100 | +.08866 | +.01262 | −.00438 | −.01826 |
| −.03117 | +.00635 | −.00217 | −.19699 | −.18155 | −.11206 | +.16893 |
| | | +.00467 | +.00411 | | −.06539 | +.00479 |
| −.36726 | −.01198 | | +.01126 | −.33785 | | |
| | | −.08062 | | | −.05744 | −.46382 |
| | | | −.28950 | | | |

| 10 | 20 | 9 | 8 | 7 | 19 | 6 |
|---|---|---|---|---|---|---|
| −0.18988 | +0.06794 | −0.3175 | −0.32767 | −0.18271 | −0.00919 | +0.32690 |
| +.00585 | −.00265 | −.01176 | +.00451 | −.00550 | −.00422 | −.01906 |
| −.02688 | −.00015 | +.02568 | +.02185 | −.04059 | −.00052 | −.01550 |
| −.00539 | −.01061 | −.01630 | −.09988 | +.06622 | +.00487 | +.09927 |
| −.02964 | −.00017 | −.07411 | −.14823 | −.02944 | +.00169 | +.11036 |
| +.01064 | +.01389 | +.01255 | +.09383 | +.04121 | −.00470 | −.02989 |
| +.03119 | +.00080 | −.23191 | +.44608 | −.35751 | −.00011 | −.00238 |
| −.13514 | +.00046 | −.13474 | | +.00189 | −.00604 | −.12661 |
| −.02298 | | −.14406 | −.00951 | | | +.03114 |
| +.09275 | +.06951 | | | −.50643 | −.01822 | |
| | | −.89215 | | | | +.37423 |
| −.26948 | | | | | | |

| 5 | 4 | 18 | 3 | 2 | 1 | |
|---|---|---|---|---|---|---|
| −0.07817 | −0.10817 | +0.01688 | −0.41125 | −0.05438 | −0.025 | |
| +.00154 | −.02360 | −.00030 | −.00621 | +.01584 | −.00847 | |
| +.03851 | −.06840 | −.00057 | +.05568 | +.04723 | −.08198 | |
| −.07292 | +.16560 | −.00288 | −.05820 | −.10763 | +.10326 | |
| −.12497 | −.02835 | +.00499 | −.07411 | −.14822 | −.04941 | |
| −.04600 | +.02944 | +.00024 | −.00802 | −.00702 | +.00496 | |
| −.00366 | −.15010 | −.00459 | −.23238 | +.36729 | −.24485 | |
| −.19486 | −.28117 | | −.00007 | | −.03770 | |
| −.01875 | | +.01377 | | +.11311 | | |
| −.20175 | −.46475 | | −.73456 | | −.33919 | |
| −.70103 | | | | | | |

After the $C$'s are determined, the next step is the computation of the $v$'s, that is, the corrections to the directions. These corrections are obtained, as is the case of the single quadrilateral, by substituting the values of the $C$'s in the table of correlates. A set of $v$'s is then adopted to make all the equations consistent. (See p. 15.)

*Computation of corrections (v's)*

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| +0.0614 | −0.0880 | −0.0614 | −0.2964 | +0.0563 | −0.0563 | −0.0501 | −0.0723 | +0.0561 |
| −.0798 | −.1131 | +.0798 | +.0158 | +.1079 | −.1079 | +.0598 | +.1131 | −.0598 |
| −.0919 | | +.0919 | −.1131 | −.3315 | +.3315 | +.0559 | | −.0559 |
| +.0578 | −.2011 | +.0302 | +.3392 | −.0526 | +.2964 | −.2964 | +.0408 | +.0330 |
| +.3392 | −.20 | +.1131 | | +.7346 | +.0368 | +.0393 | +.04 | +.4648 |
| | | −.3392 | −.0545 | −.3392 | −.7346 | +.7346 | | −.7346 |
| +.2867 | | | −.05 | | +.1131 | −.1131 | | |
| +.29 | | −.0856 | | +.1755 | | | | −.3024 |
| | | −.08 | | +.18 | −.1210 | +.4300 | | −.30 |
| | | | | | −.12 | +.43 | | |

| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|
| +0.0221 | −0.0221 | +0.0366 | +0.0410 | −0.0366 | −0.0586 | +0.0586 | +0.0013 | +0.0095 |
| +.0645 | −.0645 | −.0650 | +.3742 | +.0650 | −.1122 | +.1122 | −.0460 | +.01 |
| −.2645 | +.2645 | −.0608 | | +.0608 | +.2359 | −.2359 | +.5064 | |
| +.7010 | +.2964 | −.0089 | +.4152 | −.0321 | −.0402 | +.0262 | | |
| −.4648 | −.7010 | −.3742 | +.41 | +.0140 | −.7346 | −.3392 | +.4617 | |
| | | +.4648 | | −.4648 | +.3392 | | +.46 | |
| +.0583 | −.2267 | | | +.7346 | | −.3781 | | |
| +.06 | −.23 | −.0075 | | | −.3705 | −.38 | | |
| | | −.01 | | +.3409 | −.37 | | | |
| | | | | +.34 | | | | |

| 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |
|---|---|---|---|---|---|---|---|---|
| −0.0013 | −0.1031 | +0.1031 | −0.0454 | +0.1031 | +0.0454 | +0.0070 | −0.0070 | −0.4051 |
| +.0460 | −.0987 | +.0987 | +.0317 | −.3742 | −.0317 | +.0480 | −.0480 | −.0108 |
| −.0133 | +.4296 | −.4296 | +.1242 | | −.1242 | −.2247 | +.2247 | +.0574 |
| −.0095 | +.0763 | −.0630 | −.2964 | −.2711 | +.2391 | −.2704 | +.2964 | |
| −.5064 | −.7010 | +.3742 | −.0849 | −.27 | −.0182 | +.8922 | +.0313 | −.3585 |
| −.3742 | +.4648 | −.4648 | +.7010 | | +.0095 | −.5064 | −.8922 | −.36 |
| +.7010 | | | | | +.5064 | | −.0095 | |
| | +.0679 | −.3814 | +.4302 | | +.3742 | −.0543 | | |
| −.1577 | +.07 | −.38 | +.43 | | −.7010 | −.05 | −.4043 | |
| −.16 | | | | | | | −.40 | |
| | | | | | +.2995 | | | |
| | | | | | +.30 | | | |

| 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 |
|---|---|---|---|---|---|---|---|---|
| +0.0249 | −0.0424 | −0.0249 | −0.0254 | +0.0254 | −0.0199 | −0.3330 | +0.0199 | +0.0063 |
| −.0300 | +.4638 | +.0300 | −.0466 | +.0466 | +.0318 | −.0095 | −.0318 | +.0310 |
| −.1800 | | +.1800 | +.1403 | −.1403 | −.0137 | | +.0137 | −.2396 |
| +.5350 | +.4214 | −.1299 | −.0786 | +.1022 | −.2964 | −.3425 | −.1001 | +.3531 |
| +.0532 | +.42 | −.0236 | −.8922 | −.5064 | +.0730 | −.34 | +.2000 | +.3378 |
| −.0574 | | −.4638 | +.5064 | | +.8922 | | +.4638 | −.2695 |
| +.2695 | | −.2695 | | −.4725 | +.0095 | | +.2695 | |
| | | +.8922 | −.3961 | −.47 | | | −.8922 | +.2191 |
| +.6152 | | | −.39 | | +.6765 | | | +.22 |
| +.62 | | +.1905 | | | +.68 | | −.0772 | |
| | | +.19 | | | | | −.08 | |

| 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 |
|---|---|---|---|---|---|---|---|---|
| −0.0063 | +0.0044 | −0.0044 | −0.0054 | +0.0054 | −0.0361 | −0.0524 | +0.0361 | −0.0160 |
| −.0310 | −.0086 | +.0086 | −.0315 | +.0315 | +.0066 | −.4638 | −.0066 | +.0089 |
| +.2396 | −.1316 | +.1316 | +.0559 | −.0559 | +.2818 | | −.2818 | −.0310 |
| +.2964 | +.2895 | −.2895 | −.3378 | +.0574 | −.2964 | −.5162 | +.3764 | −.0032 |
| −.1730 | −.0574 | +.3378 | +.2695 | −.2695 | −.2034 | −.52 | +.0570 | −.1730 |
| −.3378 | | | | | +.3378 | | +.2895 | −.0046 |
| −.4638 | | | | | +.4638 | | −.3378 | −.1684 |
| | +.0963 | +.1841 | −.0493 | −.2311 | | | | +.3673 |
| −.4759 | +.09 | +.18 | −.05 | −.23 | +.5541 | | +.1328 | −.2895 |
| −.48 | | | | | +.55 | | +.13 | |
| | | | | | | | | −.3095 |
| | | | | | | | | −.31 |

*Computation of corrections (v's)*—Continued

| 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 |
|---|---|---|---|---|---|---|---|---|
| +0.1297 | +0.0160 | −0.2967 | +0.2967 | −0.0036 | +0.0036 | −0.3890 | −0.0200 | +0.0200 |
| +.1684 | −.0089 | −.0307 | +.0786 | −.0084 | +.0084 | −.0104 | +.0005 | −.0005 |
| | +.0310 | −.1684 | −.5237 | +.0968 | −.0968 | −.0574 | +.1093 | −.1093 |
| +.2981 | +.2964 | +.5237 | +.3673 | −.0479 | +.4589 | | −.2964 | +.5237 |
| +.30 | −.1265 | | | −.0699 | +.0122 | −.4568 | +.3673 | −.3673 |
| | −.3673 | +.0279 | +.2189 | −.0018 | −.2895 | −.45 | | |
| | | +.03 | +.22 | +.1684 | +.0574 | | +.1607 | +.0666 |
| | −.1593 | | | −.3673 | | | +.16 | +.06 |
| | −.16 | | | +.2895 | +.1542 | | | |
| | | | | | +.16 | | | |
| | | | | +.0558 | | | | |
| | | | | +.06 | | | | |

| 55 | 56 | 57 | 58 |
|---|---|---|---|
| +0.2964 | +0.2732 | +0.2543 | −0.2732 |
| −.5237 | −.2262 | −.1684 | −.0281 |
| | +.5237 | | +.1684 |
| −.2273 | | +.0859 | −.5237 |
| −.23 | +.5707 | +.09 | |
| | +.57 | | −.6566 |
| | | | −.66 |

As a check on the computation of the $v$'s they should be summed up around each point to make sure that each sum equals zero (except where one or more directions at the point are not numbered, see p. 37).

## COMPUTATION OF TRIANGLES

The adopted values of the $v$'s can now be substituted in column 4 of the triangle computation (see fig. 24) and the final triangles can be computed as explained on page 39.

In a complete quadrilateral each line is a part of two different triangles. The logarithm of each length appears therefore in two different triangles. The agreement of these two values in all cases furnishes a check on the adjustment and computation. Another check on the work is that the length as carried from the first fixed line to the second fixed line of the arc must agree with the fixed value of the latter. The elimination of the azimuth discrepancy can be checked before the positions are recomputed by substituting the adjusted angles for the corresponding observed angles on page 62.

When it is certain that all the angle, side, azimuth and length equations are satisfied, it is necessary to recompute all the geographic positions, beginning with the fixed line Palo-Pedro and ending with the fixed position Garcena. These recomputed positions are shown in Figure 29.

If the latitudes of the various stations have been only slightly changed by the adjustment, then in the recomputation of the geographic positions the second, third, and fourth terms of the $\Delta\phi$ may be taken directly from the preliminary computation of the positions and only the first term need be recomputed. In a great many cases the recomputation of positions may be made by merely correcting

(Text continued on p. 107)

Department of Commerce
U. S. Coast and Geodetic Survey
Form 26

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | Palo | to | Pedro | | 12 | 02 | 25.00 |
| ∠ | Pedro | & | Fordyce | | + 61 | 27 | 31.76 |
| α | 2. Palo | to 1. | Fordyce | | 73 | 29 | 56.76 |
| Δα | | | | | - | 3 | 04.83 |
| | | | | | 180 | | |
| α′ | 1. Fordyce | to 2. | Palo | | 253 | 26 | 51.93 |
| | | | | First Angle of Triangle | 48 | 00 | 09.93 |

| | | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|---|
| φ | | 26 | 19 | 38.951 | 2. | Palo | λ | 98 | 27 | 48.248 |
| Δφ | | - | 1 | 51.517 | s= | | Δλ | + | 6 | 56.990 |
| φ′ | | 26 | 17 | 47.434 | 1. | Fordyce | λ′ | 98 | 34 | 45.238 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| s | 4.081 5096 | s′ | 8.16302 | | | | −h | | 2.0467 |
| cos α | 9.453 3648 | sin′ α | 9.96347 | (Δφ)′ | 4.0947 | | s′ sin′ α | | 8.1265 |
| B | 8.511 8083 | C | 1.10025 | D | 2.2921 | | E | | 5.8511 |
| h | 2.046 4827 | | 9.22674 | | 6.3868 | | | | 6.0243 |
| 1st term. | + 111.3481 ″ | 2d term. | +0.0002 ″ | | | | (Δλ)′ | | |
| 2d term. | + 0.1686 | 4th term. | -0.0001 | | | | F | | |
| | + 111.5167 | | | | | | | | |
| 3d and 4th terms. | + 0.0001 | s | 4.081 5096 | | | | | | |
| −Δφ | + 111.5168 | sin α | 9.981 7349 | Arg. | | | Δλ | | 2.620 1256 |
| ½(φ+φ′) | 26 18 43.19 | A′ | 8.509 4379 | s | -3 | | sin ½(φ+φ′) | | 9.646 6575 |
| | | sec φ′ | 0.047 4432 | Δλ | +3 | | sec ½(Δφ) | | |
| | | | 2.620 1256 ″ | Corr. | 0 | | | | 2.266 7831 |
| | | Δλ | + 416.9900 | | | | −Δα | | +184.835 |

Do not write in this margin.

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Pedro | to | Palo | 192 | 01 | 53.35 |
| ∠ | | Fordyce | & | Palo | − 70 | 32 | 18.57 |
| α | 3. | Pedro | to 1. | Fordyce | 121 | 29 | 34.78 |
| Δα | | | | | - | 2 | 32.92 |
| | | | | | 180 | | |
| α′ | 1. | Fordyce | to 3. | Pedro | 301 | 27 | 01.86 |

| | | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|---|
| φ | | 26 | 14 | 36.740 | 3. | Pedro | λ | 98 | 28 | 59.722 |
| Δφ | | - | 3 | 10.694 | s= | | Δλ | + | 5 | 45.516 |
| φ′ | | 26 | 17 | 47.434 | 1. | Fordyce | λ′ | 98 | 34 | 45.238 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| s | 4.050 7887 | s′ | 8.10158 | | | | −h | | 2.2806 |
| cos α | 9.717 9984 | sin′ α | 9.86160 | (Δφ)′ | 4.5607 | | s′ sin′ α | | 7.9632 |
| B | 8.511 8134 | C | 1.09865 | D | 2.2911 | | E | | 5.8497 |
| h | 2.280 6006 | | 9.06183 | | 6.8518 | | | | 6.0935 |
| 1st term. | −190.8097 ″ | 2d term. | +0.0007 ″ | | | | (Δλ)′ | | |
| 2d term. | + 0.1153 | 4th term. | +0.0001 | | | | F | | |
| | −190.6944 | | | | | | | | |
| 3d and 4th terms. | + 0.0008 | s | 4.050 7887 | | | | | | |
| −Δφ | −190.6936 | sin α | 9.930 7983 | Arg. | | | Δλ | | 2.538 4681 |
| ½(φ+φ′) | 26 16 12.08 | A′ | 8.509 4379 | s | -2 | | sin ½(φ+φ′) | | 9.646 0134 |
| | | sec φ′ | 0.047 4432 | Δλ | +2 | | sec ½(Δφ) | | |
| | | | 2.538 4681 ″ | Corr. | 0 | | | | 2.184 4815 |
| | | Δλ | + 345.5160 | | | | −Δα | | +152.926 |

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FIG. 29.—Final position computation for stations of net

Department of Commerce
U. S. Coast and Geodetic Survey
Form 20

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Palo | to | Fordyce | 73 | 29 | 56.76 |
| ∠ | | Fordyce | & | Eltoro | + 38 | 09 | 53.22 |
| α | 2. | Palo | to 1. | Eltoro | 111 | 39 | 49.98 |
| Δα | | | | | - | 2 | 45.11 |
| | | | | | 180 | | |
| α′ | 1. | Eltoro | to 2. | Palo | 291 | 37 | 04.87 |
| | | | | First Angle of Triangle | 77 | 47 | 13.85 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 26 | 19 | 38.951 | 2. Palo | | λ | 98 | 27 | 48.248 |
| Δφ | + | 2 | 13.007 | s= | | Δλ | + | 6 | 12.057 |
| φ′ | 26 | 21 | 51.958 | 1. Eltoro | | λ′ | 98 | 34 | 00.305 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.045 2890 | s′ | 8.09058 | | | −h | 2.1243 |
| cos α | 9.567 2156 | sin² α | 9.93637 | (Δφ)² | 4.2477 | s′ sin² α | 8.0269 |
| B | 8.511 8083 | C | 1.10025 | D | 2.2921 | E | 5.8511 |
| h | 2.124 3131 | | 9.12720 | | 4.5398 | | 6.0023 |
| 1st term. | -133.″1414 | 2d term. | +0.″0003 | | | (Δλ)² | |
| 2d term. | + 0.1340 | 4th term. | +0.0001 | | | F | |
| 2d and 4th terms. | -133.0074 + 0.0004 | s | 4.045 2890 | | | | |
| −Δφ | -133.0070 | sin α | 9.968 1865 | Arg. | | Δλ | 2.570 6100 |
| ½(φ+φ′) | 26 20 45.45 | A′ | 8.509 4365 | s | -2 | sin ½(φ+φ′) | 9.647 1776 |
| | | sec φ′ | 0.047 6980 | Δλ | +2 | sec ½(Δφ) | |
| | | Δλ | 2.570 6100 | Corr. | 0 | −Δα | 2.217 7876 |
| | | | + 372.0574 | | | | +165.115 |

Do not write in this margin.

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Fordyce | to | Palo | 253 | 26 | 51.93 |
| ∠ | | Eltoro | & | Palo | − 64 | 02 | 53.14 |
| α | 3. | Fordyce | to 1. | Eltoro | 189 | 23 | 58.79 |
| Δα | | | | | + | | 19.93 |
| | | | | | 180 | | |
| α′ | 1. | Eltoro | to 3. | Fordyce | 9 | 24 | 18.72 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 26 | 17 | 47.434 | 3. Fordyce | | λ | 98 | 34 | 45.238 |
| Δφ | + | 4 | 04.524 | s= | | Δλ | - | | 44.933 |
| φ′ | 26 | 21 | 51.958 | 1. Eltoro | | λ′ | 98 | 34 | 00.305 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.882 3870 | s′ | 7.76477 | | | −h | 2.3883 |
| cos α | 9.994 1295 | sin² α | 8.42608 | (Δφ)² | 4.7766 | s′ sin² α | 6.1908 |
| B | 8.511 8102 | C | 1.09965 | D | 2.2918 | E | 5.8506 |
| h | 2.388 3267 | | 7.29050 | | 7.0684 | | 4.4297 |
| 1st term. | -244.″5269 | 2d term. | +0.″0012 | | | (Δλ)² | |
| 2d term. | + 0.0020 | 4th term. | .0000 | | | F | |
| 2d and 4th terms. | -244.5249 + 0.0012 | s | 3.882 3870 | | | | |
| −Δφ | -244.5237 | sin α | 9.213 0398 | Arg. | | Δλ | 1.652 5611 |
| ½(φ+φ′) | 26 19 49.70 | A′ | 8.509 4365 | s | -1 | sin ½(φ+φ′) | 9.646 9406 |
| | | sec φ′ | 0.047 6980 | Δλ | 0 | sec ½(Δφ) | |
| | | Δλ | 1.652 5611 | Corr. | -1 | −Δα | 1.299 5017 |
| | | | - 44.9325 | | | | - 19.930 |

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FIG. 29.—Final position computation for stations of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Eltoro | to | Fordyce | 9 | 24 | 18.72 |
| ∠ | | Fordyce | & | Garcia | +71 | 51 | 47.29 |
| α | 2. | Eltoro | to 1. | Garcia | 81 | 16 | 06.01 |
| Δα | | | | | - | 3 | 45.95 |
| | | | | | 180 | | |
| α′ | 1. | Garcia | to 2. | Eltoro | 261 | 12 | 20.06 |
| | | | | First Angle of Triangle | 31 | 20 | 12.96 |

| | ° | ′ | ″ | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| φ | 26 | 21 | 51.958 | 2. Eltoro | λ | 98 | 34 | 00.305 |
| Δφ | - | 1 | 10.687 | s= | Δλ | + | 8 | 28.974 |
| φ′ | 26 | 20 | 41.271 | 1. Garcia | λ′ | 98 | 42 | 29.279 |

-1

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.154 6968 | s² | 8.30939 | | | −h | 1.8478 |
| cos α | 9.181 2921 | sin² α | 9.98987 | (δφ)² | 3.6937 | s² sin² α | 8.2993 |
| B | 8.511 8060 | C | 1.10095 | D | 2.2925 | E | 5.8517 |
| h | 1.847 7949 | | 9.40021 | | 5.9912 | | 5.9988 |
| 1st term. | +70.4360″ | 2d term. | +0.0001″ | | | (Δλ)² | |
| 2d term. | + 0.2513 | 4th term. | −0.0001 | | | F | |
| | +70.6873 | | | | | | |
| 3d and 4th terms. | 0000 | s | 4.154 6968 | | | | |
| −Δφ | +70.6873 | sin α | 9.994 9372 | Arg. | | Δλ | 2.706 6952 |
| ½(φ+φ′) | 26 21 16.61 | A′ | 8.509 4369 | s | −4 | sin ½(φ+φ′) | 9.647 3100 |
| | | sec φ′ | 0.047 6243 | Δλ | +4 | sec ½(Δφ) | |
| | | | 2.706 6952″ | Corr. | 0 | | 2.354 0052″ |
| | | Δλ | + 508.9735 | | | −Δα | + 225.946 |

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DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 86

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Fordyce | to | Eltoro | 189 | 23 | 58.79 |
| ∠ | | Garcia | & | Eltoro | − 76 | 46 | 00.02 |
| α | 3. | Fordyce | to 1. | Garcia | 112 | 35 | 58.77 |
| Δα | | | | | - | 3 | 25.75 |
| | | | | | 180 | | |
| α′ | 1. | Garcia | to 3. | Fordyce | 292 | 32 | 33.02 |

| | ° | ′ | ″ | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| φ | 26 | 17 | 47.434 | 3. Fordyce | λ | 98 | 34 | 45.238 |
| Δφ | + | 2 | 53.836 | s= | Δλ | + | 7 | 44.041 |
| φ′ | 26 | 20 | 41.270 | 1. Garcia | λ′ | 98 | 42 | 29.279 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.144 1935 | s² | 8.28839 | | | −h | 2.2407 |
| cos α | 9.584 6588 | sin² α | 9.93060 | (δφ)² | 4.4803 | s² sin² α | 8.2190 |
| B | 8.511 8102 | C | 1.09965 | D | 2.2918 | E | 5.8506 |
| h | 2.240 5625 | | 9.31864 | | 6.7721 | | 6.3103 |
| 1st term. | −174.0454″ | 2d term. | +0.0006″ | | | (Δλ)² | |
| 2d term. | + 0.2083 | 4th term. | +0.0002 | | | F | |
| | −173.8371 | | | | | | |
| 3d and 4th terms. | + 0.0008 | s | 4.144 1935 | | | | |
| −Δφ | −173.8363 | sin α | 9.965 3017 | Arg. | | Δλ | 2.666 5565 |
| ½(φ+φ′) | 26 19 14.35 | A′ | 8.509 4369 | s | −3 | sin ½(φ+φ′) | 9.646 7901 |
| | | sec φ′ | 0.047 6243 | Δλ | +4 | sec ½(Δφ) | |
| | | | 2.666 5565″ | Corr. | +1 | | 2.313 3466″ |
| | | Δλ | + 464.0412 | | | −Δα | +205.753 |

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FIG. 29.—Final position computation for stations of net—Continued

Department of Commerce
U. S. Coast and Geodetic Survey
Form 26

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Eltoro | to | Garcia | 81 | 16 | 06.01 |
| ∠ | | Garcia | & | Pancho | +44 | 39 | 18.62 |
| α | 2. | Eltoro | to 1. | Pancho | 125 | 55 | 24.63 |
| Δα | | | | | - | 3 | 14.32 |
| | | | | | 180 | | |
| α′ | 1. | Pancho, | to 2. | Eltoro | 305 | 52 | 10.31 |
| | | | | First Angle of Triangle | 64 | 28 | 16.49 |

| | ° | ′ | ″ | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| φ | 26 | 21 | 51.958 | 2. Eltoro | λ | 98 | 34 | 00.305 |
| Δφ | + | 4 | 44.834 | s= | Δλ | + | 7 | 16.979 |
| φ′ | 26 | 26 | 36.792 | 1. Pancho | λ′ | 98 | 41 | 17.284 +1 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.174 6518 | s² | 8.34930 | | | −h | 2.4549 |
| cos α | 9.768 4195 | sin² α | 9.81676 | (δφ)² | 4.9092 | s² sin² α | 8.1661 |
| B | 8.511 8060 | C | 1.10096 | D | 2.2925 | E | 5.8517 |
| h | 2.454 8773 | | 9.26701 | | 7.2017 | | 6.4727 |
| 1st term. | −285.0213″ | 3d term. | +0.0016″ | | | (Δλ)² | |
| 2d term. | + 0.1349 | 4th term. | +0.0003 | | | F | |
| | −284.8364 | | | | | | |
| 3d and 4th terms. | + 0.0019 | s | 4.174 6518 | | | | |
| −Δφ | −284.8345 | sin α | 9.908 3784 | Arg. | | Δλ | 2.640 4607 |
| ½(φ+φ′) | 26 24 14.38 | A′ | 8.509 4349 | s | −4 | sin ½(φ+φ′) | 9.648 0648 |
| | | sec φ′ | 0.047 9957 | Δλ | +3 | sec ½(Δφ) | |
| | | | 2.640 4607 | Corr. | -1 | | 2.288 5255 |
| n—ess | | Δλ | +436.9791″ | | | −Δα | +194.324″ |

Do not write in this margin.

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Garcia | to | Eltoro | 261 | 12 | 20.06 |
| ∠ | | Pancho | & | Eltoro | − 70 | 52 | 25.27 |
| α | 3. | Garcia | to 1. | Pancho | 190 | 19 | 54.79 |
| Δα | | | | | + | 0 | 32.01 |
| | | | | | 180 | | |
| α′ | 1. | Pancho | to 3. | Garcia | 10 | 20 | 26.80 |

| | ° | ′ | ″ | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| φ | 26 | 20 | 41.270 | 3. Garcia | λ | 98 | 42 | 29.279 |
| Δφ | + | 5 | 55.522 | s= | Δλ | - | 1 | 11.994 |
| φ′ | 26 | 26 | 36.792 | 1. Pancho | λ′ | 98 | 41 | 17.285 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.046 1679 | s² | 8.09233 | | | −h | 2.5509 |
| cos α | 9.992 9004 | sin² α | 8.50740 | (δφ)² | 5.1017 | s² sin² α | 6.5997 |
| B | 8.511 8072 | C | 1.10057 | D | 2.2923 | E | 5.8513 |
| h | 2.550 8755 | | 7.70030 | | 7.3940 | | 5.0019 |
| 1st term. | −355.5293″ | 3d term. | +0.0025″ | | | (Δλ)² | |
| 2d term. | + 0.0050 | 4th term. | 0.0000 | | | F | |
| | −355.5243 | | | | | | |
| 3d and 4th terms. | +0.0025 | s | 4.046 1679 | | | | |
| −Δφ | −355.5218 | sin α | 9.253 7007 | Arg. | | Δλ | 1.857 2989 |
| ½(φ+φ′) | 26 23 39.03 | A′ | 8.509 4349 | s | −2 | sin ½(φ+φ′) | 9.647 9148 |
| | | sec φ′ | 0.047 9957 | Δλ | 0 | sec ½(Δφ) | |
| | | | 1.857 2990 | Corr. | −2 | | 1.505 2137 |
| n—ess | | Δλ | − 71.9944″ | | | −Δα | -32.005″ |

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FIG. 29.—Final position computation for stations of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Pancho | to | Garcia | 10 | 20 | 26.80 |
| ∠ | | Garcia | & | Monument | +28 | 28 | 02.87 |
| α | 2. | Pancho | to 1. | Monument | 38 | 48 | 29.67 |
| Δα | | | | | − | 2 | 07.02 |
| | | | | | 180 | | |
| α′ | 1. | Monument | to 2. | Pancho | 218 | 46 | 22.65 |
| | | | | First Angle of Triangle | 61 | 38 | 08.64 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| φ | 26° | 26′ | 36.792″ | 2. Pancho | λ | 98° | 41′ | 17.285″ |
| Δφ | − | 5 | 20.110 | s= | Δλ | + | 4 | 45.680 |
| φ′ | 26 | 21 | 16.682 | 1. Monument | λ′ | 98 | 46 | 02.965 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.101 7116 | s′ | 8.20342 | | | −h | 2.5052 |
| cos α | 9.891 6756 | sin′ α | 9.59414 | (dφ)′ | 5.0106 | s′ sin′ α | 7.7976 |
| B | 8.511 8012 | C | 1.10250 | D | 2.2934 | E | 5.8529 |
| h | 2.505 1866 | | 8.90006 | | 7.3040 | | 6.1557 |
| 1st term. | +320.0285″ | 3d term. | +0.0020″ | | | (Δλ)′ | |
| 2d term. | + 0.0794 | 4th term. | −0.0001 | | | F | |
| | + 320.1079 | | | | | | |
| 3d and 4th terms. | + 0.0019 | s | 4.101 7116 | | | | |
| −Δφ | +320.1098 | sin α | 9.797 0708 | Arg. | | Δλ | 2.455 8803 |
| ½(φ+φ′) | 26 23 56.74 | A′ | 8.509 4367 | s | −3 | sin ½(φ+φ′) | 9.647 9901 |
| | | sec φ′ | 0.047 6612 | Δλ | +1 | sec ½(Δφ) | |
| | | | 2.455 8803″ | Corr. | −2 | | 2.103 8704 |
| | | Δλ | + 285.6803″ | | | −Δα | +127.019″ |

Do not write in this margin.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 96

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Garcia | to | Pancho | 190 | 19 | 54.79 |
| ∠ | | Monument | & | Pancho | − 89 | 53 | 48.66 |
| α | 3. | Garcia | to 1. | Monument | 100° | 26 | 06.13 |
| Δα | | | | | − | 1 | 34.84 |
| | | | | | 180 | | |
| α′ | 1. | Monument | to 3. | Garcia | 280 | 24 | 31.29 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| φ | 26° | 20′ | 41.270″ | 3. Garcia | λ | 98° | 42′ | 29.279″ |
| Δφ | + | | 35.412 | s= | Δλ | + | 3 | 33.686 |
| φ′ | 26 | 21 | 16.682 | 1. Monument | λ′ | 98 | 46 | 02.965 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.779 9207 | s′ | 7.55984 | | | −h | 1.5497 |
| cos α | 9.257 9677 | sin′ α | 9.98551 | (dφ)′ | 3.0983 | s′ sin′ α | 7.5454 |
| B | 8.511 8072 | C | 1.10057 | D | 2.2923 | E | 5.8513 |
| h | 1.549 6956 | | 8.64592 | | 5.3906 | | 4.9464 |
| 1st term. | −35.4565″ | 3d term. | 0.0000″ | | | (Δλ)′ | |
| 2d term. | +0.0443 | 4th term. | 0.0000 | | | F | |
| | −35.4122 | | | | | | |
| 3d and 4th terms. | 0000 | s | 3.779 9207 | | | | |
| −Δφ | −35.4122 | sin α | 9.932 7571 | Arg. | | Δλ | 2.329 7757 |
| ½(φ+φ′) | 26 20 58.98 | A′ | 8.509 4367 | s | −1 | sin ½(φ+φ′) | 9.647 2352 |
| | | sec φ′ | 0.047 6612 | Δλ | +1 | sec ½(Δφ) | |
| | | | 2.329 7757″ | Corr. | 0 | | 1.977 0109 |
| | | Δλ | + 213.6858″ | | | −Δα | +94.844 |

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FIG. 29.—Final position computation for stations of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Pancho | to | Monument | 38 | 48 | 29.67 |
| ∠ | | Monument | & | Corpus | +49 | 18 | 39.04 |
| α | 2. | Pancho | to 1. | Corpus | 88 | 07 | 08.71 |
| Δα | | | | | − | 2 | 04.55 |
| | | | | | 180 | | |
| α′ | 1. | Corpus | to 2. | Pancho | 268 | 05 | 04.16 |
| | | | | First Angle of Triangle | 92 | 54 | 15.15 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 26 | 26 | 36.792 | 2. Pancho | λ | | 98 | 41 | 17.285 |
| Δφ | − | | 08.346 | s= | Δλ | | + | 4 | 39.709 |
| φ′ | 26 | 26 | 28.446 | 1. Corpus | λ′ | | 98 | 45 | 56.994 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.889 5177 | s′ | 7.77904 | | | −h | 0.9175 |
| cos α | 8.516 1683 | sin′ α | 9.99953 | (δφ)′ | 1.8428 | s′ sin′ α | 7.7786 |
| B | 8.511 8012 | C | 1.10250 | D | 2.2934 | E | 5.8529 |
| h | 0.917 4872 | | 8.88107 | | 4.1362 | | 4.5490 |
| 1st term. | +8.2697 | 2d term. | 0.0000″ | | | (Δλ)′ | |
| 2d term. | 40.0760 | 4th term. | 0.0000 | | | F | |
| | 48.3457 | | | | | | |
| 3d and 4th terms. | 0000 | s | 3.889 5177 | | | | |
| −Δφ | +8.3457 | sin α | 9.999 7660 | Arg. | | Δλ | 2.446 7057 |
| ½(φ+φ′) | 26 26 32.62 | A′ | 8.509 4350 | s | −1 | sin ½(φ+φ′) | 9.648 6506 |
| | | sec φ′ | 0.047 9870 | Δλ | +1 | sec ½(Δφ) | |
| | | | 2.446″7057 | Corr. | 0 | | 2.095 3562 |
| | | Δλ | + 279.7085 | | | −Δα | +124.554 |

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Monument | to | Pancho | 215 | 46 | 22.68 |
| ∠ | | Corpus | & | Pancho | − 37 | 47 | 06.00 |
| α | 3. | Monument | to 1. | Corpus | 180 | 59 | 16.65 |
| Δα | | | | | + | | 02.66 |
| | | | | | 180 | | |
| α′ | 1. | Corpus | to 3. | Monument | 0 | 59 | 19.31 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 26 | 21 | 16.682 | 3. Monument | λ | | 98 | 46 | 02.965 |
| Δφ | + | 5 | 11.764 | s= | Δλ | | − | | 05.972 |
| φ′ | 26 | 26 | 28.446 | 1. Corpus | λ′ | | 98 | 45 | 56.993 +1 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.982 0867 | s′ | 7.96417 | | | −h | 2.4938 |
| cos α | 9.999 9354 | sin′ α | 6.47319 | (δφ)′ | 4.9877 | s′ sin′ α | 4.4374 |
| B | 8.511 8066 | C | 1.10076 | D | 2.2924 | E | 5.8615 |
| h | 2.493 8287 | | 5.53813 | | 7.2801 | | 2.7827 |
| 1st term. | −311.″7660 | 2d term. | +0.0019″ | | | (Δλ)′ | |
| 2d term. | 0.0000 | 4th term. | 0.0000 | | | F | |
| | −311.7660 | | | | | | |
| 3d and 4th terms. | + 0.0019 | s | 3.982 0867 | | | | |
| −Δφ | −311.7641 | sin α | 8.236 5945 | Arg. | | Δλ | 0.776 1019 |
| ½(φ+φ′) | 26 23 52.56 | A | 8.509 4350 | s | −2 | sin ½(φ+φ′) | 9.647 9722 |
| | | sec φ′ | 0.047 9870 | Δλ | +0 | sec ½(Δφ) | |
| | | | 0.776″1032 | Corr. | −2 | | 0.424 0741 |
| | | Δλ | − 5.9718 | | | −Δα | −2.655 |

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FIG. 29.—Final position computation for stations of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Corpus | to | Monument | 0 | 59 | 19.31 |
| ∠ | | Monument | & | Grande | + 46 | 17 | 59.43 |
| α | 2. | Corpus | to 1. | Grande | 47 | 17 | 18.74 |
| Δα | | | | | - | 1 | 35.34 |
| | | | | | 180 | | |
| α′ | 1. | Grande | to 2. | Corpus | 227 | 15 | 43.40 |
| | | | | First Angle of Triangle | 78 | 09 | 39.96 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 26 | 26 | 28.446 | 2. | Corpus | λ | 98 | 45 | 56.994 |
| Δφ | - | 2 | 58.221 | s= | | Δλ | + | 3 | 34.297 |
| φ′ | 26 | 23 | 30.225 | 1. | Grande | λ′ | 98 | 49 | 31.291 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.907 6218 | s′ | 7.81524 | | | −h | 2.2508 |
| cos α | 9.831 4261 | sin′ α | 9.73231 | (δφ)² | 4.5019 | s′ sin′ α | 7.5476 |
| B | 8.511 8013 | C | 1.10240 | D | 2.2934 | E | 5.8529 |
| h | 2.250 8492 | | 8.64996 | | 6.7953 | | 5.6513 |
| 1st term. | +178.″1760 | 2d term. | +0.″0006 | | | (Δλ)² | |
| 2d term. | +0.0447 | 4th term. | 0.0000 | | | F | |
| | +178.2207 | | | | | | |
| 2d and 4th terms. | +0.0006 | | 3.907 6218 | | | | |
| −Δφ | +178.2213 | s | | | | Δλ | 2.331 0151 |
| ½(φ+φ′) | 26 24 59.34 | sin α | 9.866 1568 | Arg. | | sin ½(φ+φ′) | 9.648 2555 |
| | | A′ | 8.509 4359 | s | −1 | | |
| | | sec φ′ | 0.047 8006 | Δλ | +1 | sec ½(Δφ) | |
| | | | 2.331 0151 | Corr. | 0 | | 1.979 2706 |
| | | Δλ | +214.″2965 | | | −Δα | +95.″339 |

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DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 26

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Monument | to | Corpus | 180 | 59 | 16.65 |
| ∠ | | Grande | & | Corpus | − 55 | 32 | 20.76 |
| α | 3. | Monument | to 1. | Grande | 125 | 26 | 55.89 |
| Δα | | | | | - | 1 | 32.54 |
| | | | | | 180 | | |
| α′ | 1. | Grande | to 3. | Monument | 305 | 25 | 23.35 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 26 | 21 | 16.682 | 3. | Monument | λ | 98 | 46 | 02.966 |
| Δφ | + | 2 | 13.543 | s= | | Δλ | + | 3 | 28.325 |
| φ′ | 26 | 23 | 30.225 | 1. | Grande | λ′ | 98 | 49 | 31.290 +1 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.850 5419 | s′ | 7.70108 | | | −h | 2.1258 |
| cos α | 9.763 4101 | sin′ α | 9.82192 | (δφ)² | 4.2512 | s′ sin′ α | 7.5230 |
| B | 8.511 8066 | C | 1.10076 | D | 2.2924 | E | 5.8515 |
| h | 2.125 7586 | | 8.62376 | | 6.5436 | | 5.5003 |
| 1st term. | −133.″5853 | 2d term. | +0.″0003 | | | (Δλ)² | |
| 2d term. | +0.0421 | 4th term. | 0.0000 | | | F | |
| | −133.5432 | | | | | | |
| 2d and 4th terms. | + 0.0003 | s | 3.850 5419 | | | | |
| −Δφ | −133.5429 | sin α | 9.910 9623 | Arg. | | Δλ | 2.318 7407 |
| ½(φ+φ′) | 26 22 23.45 | A′ | 8.509 4359 | s | −1 | sin ½(φ+φ′) | 9.647 5940 |
| | | sec φ′ | 0.047 8006 | Δλ | +1 | sec ½(Δφ) | |
| | | | 2.318 7407 | Corr. | 0 | | 1.966 3347 |
| | | Δλ | + 208.″3247 | | | −Δα | +92.″541 |

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FIG. 29.—Final position computation for stations of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|
| α | | Corpus | to Grande | 47 | 17 | 18.74 |
| ∠ | | Grande | & Hebron | +47 | 30 | 42.74 |
| α | 2. | Corpus | to 1. Hebron | 94 | 48 | 01.48 |
| Δα | | | | - | 3 | 10.09 |
| | | | | 180 | | |
| α′ | 1. | Hebron | to 2. Corpus | 274 | 44 | 51.39 |
| | | | First Angle of Triangle | 42 | 56 | 20.06 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| φ | 26° | 26′ | 28.446″ | 2. | Corpus | λ | 98° | 45′ | 56.994″ |
| Δφ | + | | 32.091 | s= | | Δλ | + | 7 | 06.827 |
| φ′ | 26 | 27 | 00.537 | 1. | Hebron | λ′ | 98 | 53 | 03.821 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.074 3223 | s′ | 8.14804 | | | −h | 1.5088 |
| cos α | 8.922 6476 | sin² α | 9.99695 | (δφ)² | 3.0128 | s² sin² α | 8.1456 |
| B | 8.511 9013 | C | 1.10240 | D | 2.2934 | E | 5.8529 |
| h | 1.508 7712 | | 9.24799 | | 5.3062 | | 5.5073 |
| 1st term. | −32.2679″ | 3d term. | 0.0000″ | | | (Δλ)² | |
| 2d term. | + 0.1770 | 4th term. | 0.0000 | | | F | |
| 3d and 4th terms. | −32.0909 | | | | | | |
| | 0.0000 | s | 4.074 3223 | | | | |
| −Δφ | −32.0909 | sin α | 9.998 4739 | Arg. | | Δλ | 2.630 2516 |
| ½(φ+φ′) | 26 26 44.49 | A′ | 8.509 4348 | s | −3 | sin ½(φ+φ′) | 9.648 7008 |
| | | sec φ′ | 0.048 0206 | Δλ | +3 | sec ½(Δφ) | |
| | | | 2.630 2516 | Corr. | 0 | | 2.278 9524 |
| | | Δλ | +426.8267″ | | | −Δα | +190.087″ |

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|
| α | | Grande | to Corpus | 227 | 15 | 43.40 |
| ∠ | | Hebron | & Corpus | − 89 | 32 | 57.38 |
| α | 3. | Grande | to 1 Hebron | 137 | 42 | 46.02 |
| Δα | | | | - | 1 | 34.57 |
| | | | | 180 | | |
| α′ | 1. | Hebron | to 3. Grande | 317 | 41 | 11.45 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| φ | 26° | 23′ | 30.225″ | 3. | Grande | λ | 98° | 49′ | 31.291″ |
| Δφ | + | 3 | 30.312 | s= | | Δλ | + | 3 | 32.530 |
| φ′ | 26 | 27 | 00.537 | 1. | Hebron | λ′ | 98 | 53 | 03.821 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.942 0489 | s′ | 7.88410 | | | −h | 2.3230 |
| cos α | 9.869 1033 | sin² α | 9.65583 | (δφ)² | 4.6458 | s² sin² α | 7.5399 |
| B | 8.511 8043 | C | 1.10146 | D | 2.2928 | E | 5.8521 |
| h | 2.322 9565 | | 8.64139 | | 6.9386 | | 5.7150 |
| 1st term. | −210.3568″ | 3d term. | +0.0009″ | | | (Δλ)² | |
| 2d term. | + 0.0438 | 4th term. | +0.0001 | | | F | |
| 3d and 4th terms. | −210.3130 | | | | | | |
| | + 0.0010 | s | 3.942 0489 | | | | |
| −Δφ | −210.3120 | sin α | 9.827 9167 | Arg. | | Δλ | 2.327 4210 |
| ½(φ+φ′) | 26 25 15.38 | A′ | 8.509 4348 | s | −1 | sin ½(φ+φ′) | 9.648 3234 |
| | | sec φ′ | 0.048 0206 | Δλ | +1 | sec ½(Δφ) | |
| | | | 2.327 4210 | Corr. | 0 | | 1.975 7444 |
| | | Δλ | + 212.5304″ | | | −Δα | +94.568″ |

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FIG. 29.—Final position computation for stations of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Hebron | to | Grande | 317 | 41 | 11.45 |
| ∠ | | Grande | & | Ringold | +47 | 22 | 41.41 |
| α | 2. | Hebron | to 1. | Ringold | 5 | 03 | 52.86 |
| Δα | | | | | − | | 11.81 |
| | | | | | 180 | | |
| α′ | 1. | Ringold | to 2. | Hebron | 185 | 03 | 41.05 |
| | | | | First Angle of Triangle | 69 | 29 | 07.33 |

| | ° | ′ | ″ | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| φ | 26 | 27 | 00.537 | 2. Hebron | λ | 98 | 53 | 03.821 |
| Δφ | − | 4 | 29.783 | s= | Δλ | + | | 26.543 |
| φ′ | 26 | 22 | 30.754 | 1. Ringold | λ′ | 98 | 53 | 30.364 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.920 9092 | s′ | 7.84182 | | | −h | 2.4310 |
| cos α | 9.998 3010 | sin′ α | 7.89173 | (Δφ)′ | 4.8620 | s′ sin′ α | 5.7336 |
| B | 8.511 8008 | C | 1.10257 | D | 2.2935 | E | 5.8530 |
| h | 2.431 0110 | | 6.83612 | | 7.1555 | | 4.0176 |
| 1st term. | +269.7808″ | 2d term. | +0.0014″ | | | (Δλ)′ | |
| 3d term. | +0.0007 | 4th term. | 0000 | | | F | |
| | +269.7815 | | | | | | |
| 2d and 4th terms. | +0.0014 | | | | | | |
| | | s | 3.920 9092 | | | | |
| −Δφ | +269.7829 | sin α | 8.945 8639 | Arg. | | Δλ | 1.423 9476 |
| ½(φ+φ′) | 26 24 45.65 | A′ | 8.509 4363 | s | −1 | sin ½(φ+φ′) | 9.648 1973 |
| | | sec φ′ | 0.047 7385 | Δλ | 0 | sec ½(Δφ) | |
| | | | 1.423 9479 | Corr. | −1 | | 1.072 1449 |
| | | Δλ | +26.5429″ | | | −Δα | +11.807″ |

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Grande | to | Hebron | 137 | 42 | 46.02 |
| ∠ | | Ringold | & | Hebron | − 63 | 08 | 11.40 |
| α | 3. | Grande | to 1. | Ringold | 74 | 34 | 34.62 |
| Δα | | | | | − | 1 | 46.24 |
| | | | | | 180 | | |
| α′ | 1. | Ringold | to 3. | Grande | 254 | 32 | 48.38 |

| | ° | ′ | ″ | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| φ | 26 | 23 | 30.225 | 3. Grande | λ | 98 | 49 | 31.291 |
| Δφ | − | | 59.471 | s= | Δλ | + | 3 | 59.073 |
| φ′ | 26 | 22 | 30.754 | 1. Ringold | λ′ | 98 | 53 | 30.364 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.837 2856 | s′ | 7.67457 | | | −h | 1.7739 |
| cos α | 9.424 8085 | sin′ α | 9.96814 | (Δφ)′ | 3.5486 | s′ sin′ α | 7.6427 |
| B | 8.511 8043 | C | 1.10146 | D | 2.2928 | E | 5.8521 |
| h | 1.773 8984 | | 8.74417 | | 5.8414 | | 5.2687 |
| 1st term. | +59.4153″ | 2d term. | +0.0001″ | | | (Δλ)′ | |
| 3d term. | + 0.0555 | 4th term. | 0.0000 | | | F | — |
| | +59.4708 | | | | | | |
| 2d and 4th terms. | + 0.0001 | | | | | | |
| | | s | 3.837 2856 | | | | |
| −Δφ | +59.4709 | sin α | 9.984 0704 | Arg. | | Δλ | 2.378 5308 |
| ½(φ+φ′) | 26 23 00.49 | A′ | 8.509 4363 | s | −1 | sin ½(φ+φ′) | 9.647 7513 |
| | | sec φ′ | 0.047 7385 | Δλ | +1 | sec ½(Δφ) | |
| | | | 2.378 5308″ | Corr. | 0 | | |
| | | Δλ | +239.0732 | | | −Δα | 2.026 2821 +106.239 |

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FIG. 29.—Final position computation for stations of net—Continued

Department of Commerce
U. S. Coast and Geodetic Survey
Form 24

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Hebron | to | Ringold | 5 | 03 | 52.86 |
| ∠ | | Ringold | & | Garcena | +83 | 16 | 45.71 |
| α | 2. | Hebron | to 1. | Garcena | 88 | 20 | 38.57 |
| Δα | | | | | − | 1 | 11.31 |
| | | | | | 180 | | |
| α′ | 1. | Garcena | to 2. | Hebron | 268 | 19 | 27.26 |
| | | | | First Angle of Triangle | 67 | 18 | 20.09 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| φ | 26° | 27′ | 00.537″ | 2. | Hebron | λ | 98° | 53′ | 03.821″ |
| Δφ | − | | 04.192 | s= | | Δλ | + | 2 | 40.095 |
| φ′ | 26 | 26 | 56.345 | 1. | Garcena | λ′ | 98 | 55 | 43.916 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.647 1063 | s′ | 7.29421 | | | −h | 0.6198 |
| cos α | 8.460 8649 | sin′ α | 9.99964 | (δφ)′ | 1.2444 | s′ sin′ α | 7.2938 |
| B | 8.511 8008 | C | 1.10257 | D | 2.2935 | E | 5.8530 |
| h | 0.619 7720 | | 8.39642 | | 3.5379 | | 3.7666 |
| 1st term. | +4.1665″ | 3d term. | 0.0000″ | | | (Δλ)′ | |
| 2d term. | +0.0249 | 4th term. | 0.0000 | | | F | |
| | +4.1914 | | | | | | |
| 3d and 4th terms. | 0.0000 | | | | | | |
| | +4.1914 | s | 3.647 1063 | | | | |
| −Δφ | | sin α | 9.999 8186 | Arg. | | Δλ | 2.204 3760 |
| ½(φ+φ′) | 26 26 56.44 | A′ | 8.509 4346 | s | 0 | sin ½(φ+φ′) | 9.648 7599 |
| | | sec φ′ | 0.048 0163 | Δλ | 0 | sec ½(Δφ) | |
| | | | 2.204 3760 | Corr. | 0 | | 1.853 1359 |
| | | Δλ | +160.0944″ | | | −Δα | +71.308″ |

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POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | | Ringold | to | Hebron | 185 | 03 | 41.05 |
| ∠ | | Garcena | & | Hebron | − 29 | 24 | 54.29 |
| α | 3. | Ringold | to 1. | Garcena | 155 | 38 | 46.76 |
| Δα | | | | | − | | 59.41 |
| | | | | | 180 | | |
| α′ | 1. | Garcena | to 3. | Ringold | 336 | 37 | 47.35 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| φ | 26° | 22′ | 30.754″ | 3. | Ringold | λ | 98° | 53′ | 30.364″ |
| Δφ | + | 4 | 25.591 | s= | | Δλ | + | 2 | 13.552 |
| φ′ | 26 | 26 | 56.345 | 1. | Garcena | λ′ | 98 | 55 | 43.916 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 3.952 9127 | s′ | 7.90583 | | | −h | 2.4242 |
| cos α | 9.959 5267 | sin′ α | 9.23057 | (δφ)′ | 4.8484 | s′ sin′ α | 7.1364 |
| B | 8.511 8053 | C | 1.10115 | D | 2.2926 | E | 5.8519 |
| h | 2.424 2447 | | 8.23755 | | 7.1410 | | 5.4125 |
| 1st term. | −265.6102″ | 3d term. | +0.0014″ | | | (Δλ)′ | |
| 2d term. | +0.0173 | 4th term. | 0000 | | | F | |
| | −265.5929 | | | | | | |
| 3d and 4th terms. | + 0.0014 | | | | | | |
| | | s | 3.952 9127 | | | | |
| −Δφ | −265.6915 | sin α | 9.615 2850 | Arg. | | Δλ | 2.125 6488 |
| ½(φ+φ′) | 26 24 43.55 | A′ | 8.509 4346 | s | −2 | sin ½(φ+φ′) | 9.648 1865 |
| | | sec φ′ | 0.048 0163 | Δλ | 0 | sec ½(Δφ) | |
| | | | 2.125 6488 | Corr. | −2 | | 1.773 8573 |
| | | Δλ | + 133.5515″ | | | −Δα | + 59.407″ |

Do not write in this margin.

FIG. 29.—Final position computation for stations of net—Continued

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| a | | Hebron | to | Ringold | 5 | 03 | 52.86 |
| ∠ | | Ringold | & | Gorgora | + 71 | 33 | 14.56 |
| a | 2. | Hebron | to 1. | Gorgora | 76 | 37 | 07.42 |
| Δα | | | | | − | 3 | 21.11 |
| | | | | | 180 | | |
| a′ | 1. | Gorgora | to 2. | Hebron | 256 | 33 | 46.31 |
| | | | | First Angle of Triangle | 37 | 42 | 07.23 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| φ | 26° | 27′ | 00.537″ | 2. Hebron | λ | 98° | 53′ | 03.821″ |
| Δφ | − | 1 | 36.958 | s= | Δλ | + | 7 | 31.723 |
| φ′ | 26 | 25 | 23.579 | 1. Gorgora | λ′ | 99 | 00 | 35.544 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 4.109 4711 | s′ | 8.21894 | | | −h | 1.9857 |
| cos α | 9.364 4196 | sin′ α | 9.97609 | (δφ)′ | 3.9732 | s′ sin′ α | 8.1950 |
| B | 8.511 8008 | C | 1.10257 | D | 2.-935 | E | 5.8530 |
| h | 1.985 6914 | | 9.29760 | | 6.2667 | | 6.0337 |
| 1st term. | +96.7590 | 3d term. | +0.0002″ | | | (Δλ)″ | |
| 2d term. | + 0.1984 | 4th term. | −0.0001 | | | F | |
| | +96.9574 | | | | | | |
| 3d and 4th terms. | + 0.0001 | s | 4.109 4711 | | | | |
| −Δφ | +96.9575 | sin α | 9.988 0466 | Arg. | | Δλ | 2.654 8722 |
| ½(φ+φ′) | 26 26 12.06 | A′ | 8.509 4353 | s | −3 | sin ½(φ+φ′) | 9.648 5635 |
| | | sec φ′ | 0.047 9191 | Δλ | +4 | sec ½(Δφ) | |
| | | | 2.654 8722″ | Corr. | +1 | | 2.303 4357 |
| | | Δλ | +451.7230 | | | −Δα | +201.111 |

Do not write in this margin.

Department of Commerce
U. S. Coast and Geodetic Survey
Form 36

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| a | | Ringold | to | Hebron | 185 | 03 | 41.05 |
| ∠ | | Gorgora | & | Hebron | − 70 | 44 | 38.47 |
| a | 3. | Ringold | to 1. | Gorgora | 114 | 19 | 02.58 |
| Δα | | | | | − | 3 | 09.04 |
| | | | | | 180 | | |
| a′ | 1. | Gorgora | to 3. | Ringold | 294 | 15 | 53.54 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| φ | 26° | 22′ | 30.754″ | 3. Ringold | λ | 98° | 53′ | 30.364″ |
| Δφ | + | 2 | 52.825 | s= | Δλ | + | 7 | 05.180 |
| φ′ | 26 | 25 | 23.579 | 1. Gorgora | λ′ | 99 | 00 | 35.544 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| s | 4.111 5674 | s′ | 8.22313 | | | −h | 2.2380 |
| cos α | 9.614 6767 | sin′ α | 9.91930 | (δφ)′ | 4.4752 | s′ sin′ α | 8.1424 |
| B | 8.511 8053 | C | 1.10115 | D | 2.2926 | E | 5.8519 |
| h | 2.238 0494 | | 9.24358 | | 6.7678 | | 6.2323 |
| 1st term. | −173.0013″ | 3d term. | +0.0006″ | | | (Δλ)″ | |
| 2d term. | + 0.1752 | 4th term. | +0.0002 | | | F | |
| | −172.8261 | | | | | | |
| 3d and 4th terms. | + 0.0008 | s | 4.111 5674 | | | | |
| −Δφ | −172.8253 | sin α | 9.959 6510 | Arg. | | Δλ | 2.628 5728 |
| ½(φ+φ′) | 26 23 57.17 | A′ | 8.509 4353 | s | −3 | sin ½(φ+φ′) | 9.647 9918 |
| | | sec φ′ | 0.047 9191 | Δλ | +3 | sec ½(Δφ) | |
| | | | 2.628 5728″ | Corr. | 0 | | 2.276 5646 |
| | | Δλ | + 425.1800 | | | −Δα | +189.045 |

Do not write in this margin.

FIG. 29.—Final position computation for stations of net—Continued

the preliminary computation. Compare the positions in Figure 29 with those in Figure 27.

The computed position of Garcena should agree with the fixed position. Such being the case all the equations are now satisfied. The values of the $C$'s should now be placed in the proper column in the table of normals, facing page 84. The values of the $v$'s and the adopted $v$'s are now completely checked, and they should be entered in the proper columns in the table of correlates, pages 82–83. Next, using the sketch for the designations of the directions, the corresponding corrections should be applied to these directions in the list of directions, Figure 23. The final seconds in the list of directions should be checked by the angles in the triangles. (See p. 41.)

## PROBABLE ERROR OF A DIRECTION

In the table of correlates, pages 82–83, the last column to the right contains the squares of the adopted $v$'s. The probable error of an observed direction is computed from the formula, p. e. $= \pm 0.6745\sqrt{\frac{\Sigma v^2}{c}}$, where $\Sigma\ v^2$ is the sum of the squares of the corrections to the directions, and $c$ is the number of condition equations used in the adjustment. For the arc here given,

$$\Sigma v^2 = 5.9667$$
$$c = 27$$

Therefore the probable error $= \pm 0.6745\sqrt{\frac{5.9667}{27}} = \pm 0.''32$.

The last operation in the adjustment of an arc is making out the list of geographic positions with the azimuths and logarithms of the distances on form 28B. (See fig. 30.) The columns headed "Station" and "To station" are filled out first by using the sketch to pick out the proper order of the stations in regard to azimuth. The list of geographic positions for the present arc is given in Figure 30. For the method of computing the azimuths which can not be obtained directly from the position computation see page 47.

*Statistics showing accuracy of triangulation*

When a net of triangulation has been finally adjusted a table of statistics similar to the one below should be prepared, showing the accuracy of the observations:

| | |
|---|---|
| Total number of triangles | 23 |
| Number of triangles with plus closures | 5 |
| Number of triangles with minus closures | 18 |
| Number of concluded triangles | 0 |
| Average closure of triangles without regard to sign | 0.″82 |
| Maximum closure of a triangle | 1.″89 |
| Mean error of an angle | ±0.″59 |
| Probable error of an observed direction | ±0.″32 |

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 50 B

# GEOGRAPHIC POSITIONS

*Accession No. of Computation:* _____

*Locality* Rio Grande River *Datum* North American *State* Texas

| STATION. | | LATITUDE AND LONGITUDE. ° ′ ″ | SECONDS IN METERS. | AZIMUTH ° ′ ″ | BACK AZIMUTH. ° ′ ″ | TO STATION. | DISTANCE. LOGARITHM (METERS). | METERS. | FEET. |
|---|---|---|---|---|---|---|---|---|---|
| Fordyce | | 26 17 47.434 | 1459.8 | 253 26 51.93 | 73 29 56.76 | Palo | 4.0815096 | 12084.51 | 39581.6 |
| 1917 | d.m. | 98 34 45.238 | 1254.9 | 301 27 01.86 | 121 29 34.78 | Pedro | 4.0507537 | 11240.58 | 36878.5 |
| Eltoro | | 26 21 51.958 | 1599.0 | 291 37 04.87 | 111 39 49.98 | Palo | 4.0452890 | 11099.13 | 36414.4 |
| 1917 | d.m. | 98 34 00.305 | 8.5 | 328 04 40.91 | 148 06 54.10 | Pedro | 4.1980195 | 15776.82 | 51761.1 |
| | | | | 9 24 18.72 | 189 23 58.79 | Fordyce | 3.8823870 | 7627.58 | 25024.8 |
| Garcia | | 26 20 41.270 | 1270.1 | 261 12 20.06 | 81 16 06.01 | Eltoro | 4.1546968 | 14278.97 | 46846.9 |
| 1917 | d.m. | 98 42 29.279 | 811.8 | 292 32 33.02 | 112 35 58.77 | Fordyce | 4.1441935 | 13937.78 | 45727.5 |
| Pancho | | 26 26 36.792 | 1132.3 | 305 52 10.31 | 125 55 24.63 | Eltoro | 4.1746518 | 14950.37 | 49049.7 |
| 1917 | d.m. | 98 41 17.285 | 478.9 | 326 15 56.40 | 146 18 50.53 | Fordyce | 4.2918957 | 19583.74 | 64251.0 |
| | | | | 10 20 26.80 | 190 19 54.79 | Garcia | 4.0461679 | 11121.62 | 36488.2 |
| Monument | | 26 21 16.682 | 513.4 | 218 46 22.65 | 38 48 29.67 | Pancho | 4.1017118 | 12638.97 | 41466.4 |
| 1917 | d.m. | 98 46 02.965 | 82.2 | 280 24 31.29 | 100 26 06.13 | Garcia | 3.7799207 | 6024.50 | 19765.4 |
| Corpus | | 26 26 28.446 | 875.4 | 268 05 04.16 | 88 07 08.71 | Pancho | 3.8895177 | 7753.85 | 25439.1 |
| 1917 | d.m. | 98 45 56.994 | 1579.1 | 331 40 06.85 | 151 41 39.18 | Garcia | 4.0841034 | 12136.78 | 39818.8 |
| | | | | 0 59 19.31 | 180 59 16.65 | Monument | 3.9820867 | 9595.92 | 31482.6 |

FIG. 30.—List of geographic positions for stations of net

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 50 B

# GEOGRAPHIC POSITIONS

*Accession No. of Computation:* _____

*Locality* Rio Grande River *Datum* North American *State* Texas

| STATION. | | LATITUDE AND LONGITUDE. ° ′ ″ | SECONDS IN METERS. | AZIMUTH ° ′ ″ | BACK AZIMUTH. ° ′ ″ | TO STATION. | DISTANCE. LOGARITHM (METERS). | METERS. | FEET. |
|---|---|---|---|---|---|---|---|---|---|
| Grande | | 26 23 30.225 | 930.2 | 227 15 43.40 | 47 17 16.74 | Corpus | 3.9076218 | 8083.92 | 26522.0 |
| 1917 | d.m. | 98 49 31.291 | 867.3 | 305 25 23.35 | 125 26 55.89 | Monument | 3.8505419 | 7038.30 | 23255.5 |
| Hebron | | 26 27 00.537 | 16.5 | 274 44 51.39 | 94 48 01.48 | Corpus | 4.0743223 | 11866.49 | 38932.0 |
| 1917 | d.m. | 98 53 03.821 | 105.8 | 312 11 20.87 | 132 14 28.01 | Monument | 4.1972563 | 15749.12 | 51570.2 |
| | | | | 317 41 11.45 | 137 42 46.02 | Grande | 3.9420489 | 8750.82 | 28710.0 |
| Ringold | | 26 22 30.754 | 946.5 | 135 03 41.05 | 5 03 52.86 | Hebron | 3.9209092 | 8335.07 | 27346.0 |
| 1917 | d.m. | 98 53 30.364 | 841.7 | 254 32 48.38 | 74 34 34.62 | Grande | 3.8372856 | 6875.20 | 22556.4 |
| | | | | 280 23 09.00 | 100 26 27.68 | Monument | 4.1007703 | 12611.60 | 41376.6 |
| Garcena | | 26 26 56.345 | 1734.0 | 268 19 27.26 | 88 20 38.57 | Hebron | 3.6471063 | 4437.17 | 14557.6 |
| 1917 | d.m. | 98 55 43.916 | 1216.7 | 335 37 47.35 | 155 38 46.76 | Ringold | 3.9529127 | 8972.49 | 29437.2 |
| Gorgora | | 26 25 23.579 | 725.6 | 250 31 22.70 | 70 33 32.53 | Garcena | 3.9329710 | 8569.81 | 28116.1 |
| 1917 | d.m. | 99 00 35.544 | 984.9 | 256 33 46.31 | 76 37 07.42 | Hebron | 4.1094711 | 12866.82 | 42213.9 |
| | | | | 294 15 53.54 | 114 19 02.58 | Ringold | 4.1115674 | 12929.07 | 42418.1 |

FIG. 30.—List of geographic positions for stations of net—Continued

# CHAPTER 4.—ADJUSTMENT OF A NET OF TRIANGULATION BY THE ANGLE METHOD

## EXPLANATION OF METHOD

In the adjustment of triangulation of the first and second orders it is the practice in the United States Coast and Geodetic Survey to use the direction method, and to include all the observed lines of the main scheme. As these classes of triangulation serve as control for all other surveys, it is necessary to make the adjustment of them as rigid as possible.

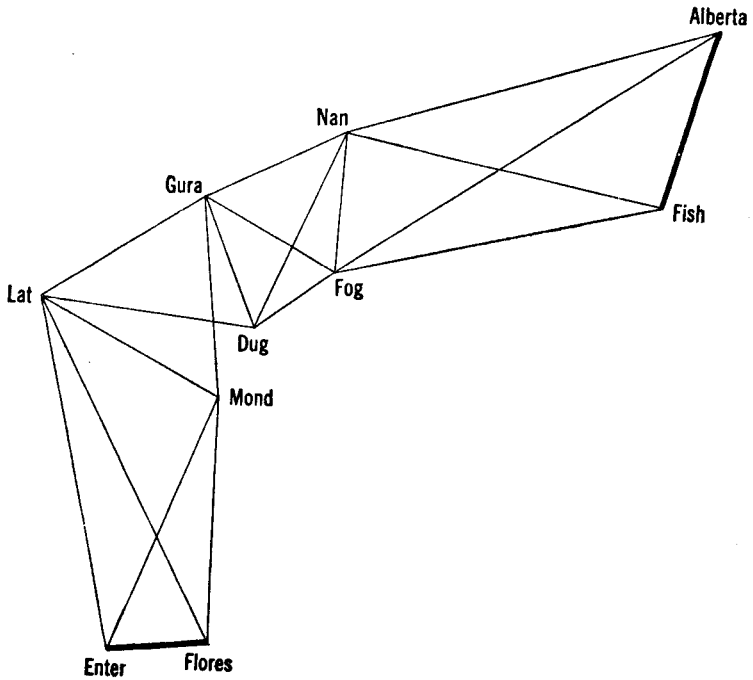


FIG. 31.—Triangulation net used in sample adjustment by angle method

However, for third-order triangulation which is fitted in between fixed points and lines of triangulation of the first and second order such a rigid adjustment is not required. Consequently in adjusting third-order triangulation the angle method should ordinarily be used.

A sample adjustment by the angle method of the arc of third-order triangulation shown in Figure 31 is given on the following pages. This small scheme requires all the different kinds of equations needed for a complete adjustment, and if these are understood, a larger scheme, which differs from this only in the number of equations, can be readily adjusted.

In Figure 31, all observed lines are shown. For the angle method of adjustment, however, it is customary to omit one diagonal in each quadrilateral and use only a chain of triangles, those of the best shape. In the example given, the diagonals Fog-Alberta, Dug-Nan, and Lat-Flores are omitted. The lines actually used in the adjustment are shown in Figure 32. After the chain of triangles has been adjusted and all the conditions are satisfied, then each omitted diagonal is computed by using the two sides and included angle of the triangle in which it occurs (see p. 139). Instructions for field work call for the observing of both diagonals of each quadrilateral in order that there may be a check on all lengths.

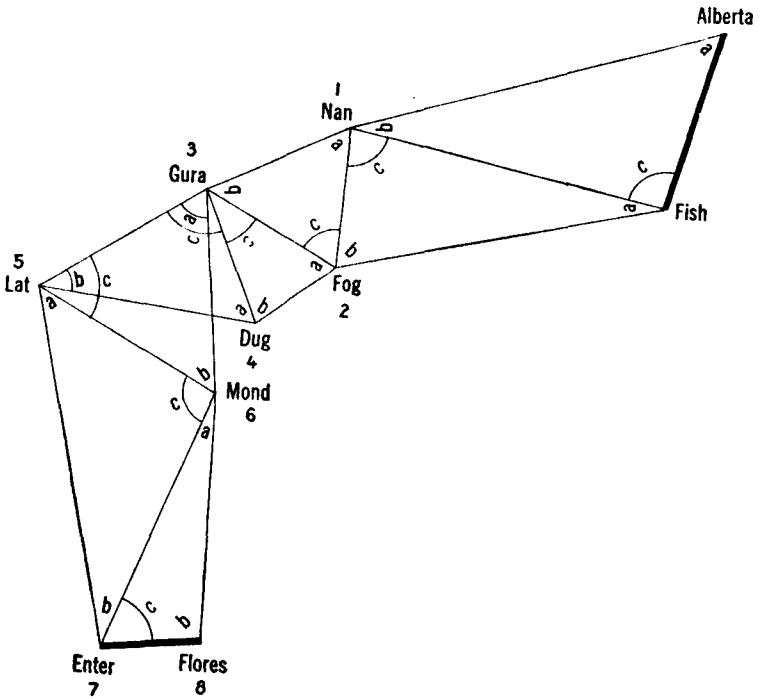


FIG. 32.—Triangulation net showing the triangles used in adjustment by angle method

The lines Alberta–Fish and Enter-Flores are fixed in length and azimuth, and the four stations at the ends of these lines are fixed in latitude and longitude.

The lists of directions for the various stations are given in Figure 33. The column headed "Final seconds" is filled out after the adjustment is completed. If the lists of directions have not been made out and checked in the field they should be computed from the horizontal angle record books in the manner shown on page 8.

The triangulation sketch (fig. 32) is numbered and lettered as follows: Starting with the fixed line Alberta-Fish and building up

the sketch point by point, each point added is given a consecutive number, Nan being 1, Fog 2, Gura 3, etc. In each triangle the angles are given the letters *a*, *b*, and *c*. The angle adjacent to the starting line but opposite the line through which the length is next carried is called *a* and the angle opposite the starting line is called *b*. These are the length angles. The azimuth angle, or the angle between the two lines through which the length is carried, is called *c*.

The following triangles (fig. 34) are laid out in exactly the same manner as for adjustment by the direction method.

As was the case for the direction method, it is necessary to compute a set of preliminary triangles (fig. 35) in order to be able to compute the geographic positions (fig. 36) and determine the latitude and longitude closures. To compute the positions properly the triangles should be closed and this is done by concluding the *c* or azimuth angle in each triangle. If one of the length angles is not observed, however, then the observed azimuth angle must be used, and the unobserved length angle concluded. As the triangles and positions are computed in exactly the same manner as when the direction method is used it is not necessary to explain the computation here.

## NUMBER AND FORMATION OF EQUATIONS

As the figure to be adjusted is simply a chain of triangles, it is obvious that there will be just as many angle equations as there are closed triangles, 8 in this case, and that there will be no side equations.

*Angle equations*

1. $0=-2.3+(1a)+(1b)+(1c)$
2. $0=-2.7+(2a)+(2b)+(2c)$
3. $0=+0.1+(3a)+(3b)+(3c)$
4. $0=+1.0+(4a)+(4b)+(4c)$
5. $0=+5.4+(5a)+(5b)+(5c)$
6. $0=+2.9+(6a)+(6b)+(6c)$
7. $0=-0.1+(7a)+(7b)+(7c)$
8. $0=+3.3+(8a)+(8b)+(8c)$

Since there are two fixed azimuths, namely, the azimuths of the lines Alberta-Fish and Enter-Flores, there will be one azimuth equation. This equation which is shown on page 125, is formed in exactly the same manner as if the direction method were used, except in the way the angles are designated.

(Text continued on p. 125)

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 34a

LIST OF DIRECTIONS

*State:* Alaska

Station Alberta — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° | ′ | ″ | Final Seconds ″ |
|---|---|---|---|---|
| Fish | 0 | 00 | 00.0 | 00.0 |
| Fog | 40 | 09 | 09.1 | 09.1 |
| Nan | 57 | 22 | 26.5 | 26.4 |

Station Fish — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° | ′ | ″ | Final Seconds ″ |
|---|---|---|---|---|
| Fog | 0 | 00 | 00.0 | 59.7 |
| Nan | 24 | 54 | 02.9 | 02.2 |
| Alberta | 118 | 48 | 46.6 | 47.5 |

Station Nan — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° | ′ | ″ | Final Seconds ″ |
|---|---|---|---|---|
| Alberta | 0 | 00 | 00.0 | 59.0 |
| Fish | 28 | 42 | 47.6 | 47.4 |
| Fog | 109 | 26 | 24.4 | 25.5 |
| Dug | 129 | 52 | 16.6 | 17.0 |
| Gura | 171 | 17 | 11.1 | 11.1 |

Station Fog — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° | ′ | ″ | Final Seconds ″ |
|---|---|---|---|---|
| Dug | 0 | 00 | 00.0 | 00.1 |
| Gura | 65 | 49 | 45.3 | 44.5 |
| Nan | 129 | 24 | 59.9 | 59.4 |
| Alberta | 182 | 45 | 14.9 | 15.7 |
| Fish | 203 | 47 | 17.6 | 18.9 |

Station Gura — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° | ′ | ″ | Final Seconds ″ |
|---|---|---|---|---|
| Nan | 0 | 00 | 00.0 | 59.9 |
| Fog | 54 | 33 | 58.8 | 59.4 |
| Dug | 93 | 03 | 00.7 | 01.1 |
| Mond | 109 | 08 | 46.2 | 46.9 |
| Lat | 172 | 30 | 45.7 | 44.0 |

Station Dug — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° | ′ | ″ | Final Seconds ″ |
|---|---|---|---|---|
| Lat | 0 | 00 | 00.0 | 02.0 |
| Gura | 61 | 35 | 02.7 | 01.7 |
| Nan | 107 | 07 | 08.4 | 06.4 |
| Fog | 137 | 16 | 16.5 | 15.6 |

Station Lat — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° | ′ | ″ | Final Seconds ″ |
|---|---|---|---|---|
| Gura | 0 | 00 | 00.0 | 01.1 |
| Dug | 38 | 57 | 17.8 | 18.6 |
| Mond | 60 | 10 | 27.4 | 27.7 |
| Flores | 94 | 59 | 06.6 | 09.3 |
| Enter | 109 | 18 | 30.8 | 28.7 |

Station Mond — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° | ′ | ″ | Final Seconds ″ |
|---|---|---|---|---|
| Flores | 0 | 00 | 00.0 | 04.7 |
| Enter | 20 | 54 | 52.5 | 50.3 |
| Lat | 115 | 25 | 02.3 | 00.6 |
| Gura | 171 | 52 | 38.3 | 36.9 |

Station Enter — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° | ′ | ″ | Final Seconds ″ |
|---|---|---|---|---|
| Lat | 0 | 00 | 00.0 | 57.9 |
| Mond | 36 | 21 | 46.8 | 47.2 |
| Flores | 100 | 12 | 19.9 | 21.6 |

Station Flores — Computed by O.P.S. — Observer C.V.H. — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° | ′ | ″ | Final Seconds ″ |
|---|---|---|---|---|
| Enter | 0 | 00 | 00.0 | 59.1 |
| Lat | 65 | 28 | 20.7 | 16.0 |
| Mond | 95 | 14 | 37.7 | 38.6 |

FIG. 33.—Lists of directions, angle method of adjustment

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Alaska

| NO. | | | STATION | OBSERVED ANGLE | | | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 2-3 | Alberta-Fish | | | | | | | | 3.669907 |
| 1 | b | 1 | Nan | 28 | 42 | 47.6 | +0.8 | 48.4 | | 48.4 | 0.313371 |
| | a | 2 | Alberta | 57 | 22 | 26.5 | -0.1 | 26.4 | | 26.4 | 9.925419 |
| | c | 3 | Fish | 93 | 54 | 43.7 | +1.6 | 45.3 | 0.1 | 45.2 | 9.998986 |
| | | 1-3 | Nan-Fish | | | | +2.3 | | 0.1 | | 3.913697 |
| | | 1-2 | Nan-Alberta | | | | | | | | 3.937264 |
| | | | | | | 57.8 | | | | | |
| | | 2-3 | Nan-Fish | | | | | | | | 3.913697 |
| 2 | b | 1 | Fog | 74 | 22 | 17.7 | +1.8 | 19.5 | | 19.5 | 0.016360 |
| | c | 2 | Nan | 80 | 43 | 36.8 | +1.3 | 38.1 | 0.1 | 38.0 | 9.994287 |
| | a | 3 | Fish | 24 | 54 | 02.9 | -0.4 | 02.5 | | 02.5 | 9.624330 |
| | | 1-3 | Fog-Fish | | | | +2.7 | | 0.1 | | 3.924344 |
| | | 1-2 | Fog-Nan | | | | | | | | 3.554387 |
| | | | | | | 57.4 | | | | | |
| | | 2-3 | Nan-Fog | | | | | | | | 3.554387 |
| 3 | b | 1 | Gura | 54 | 33 | 58.8 | +0.7 | 59.5 | | 59.5 | 0.068955 |
| | a | 2 | Nan | 61 | 50 | 46.7 | -1.1 | 45.6 | | 45.6 | 9.945312 |
| | c | 3 | Fog | 63 | 35 | 14.6 | +0.3 | 14.9 | | 14.9 | 9.952121 |
| | | 1-3 | Gura-Fog | | | | -0.1 | | 0.0 | | 3.588654 |
| | | 1-2 | Gura-Nan | | | | | | | | 3.595463 |
| | | | | | | 00.1 | | | | | |
| | | 2-3 | Gura-Fog | | | | | | | | 3.588654 |
| 4 | b | 1 | Dug | 75 | 41 | 13.8 | +0.1 | 13.9 | | 13.9 | 0.013694 |
| | c | 2 | Gura | 38 | 29 | 01.9 | -0.2 | 01.7 | | 01.7 | 9.793995 |
| | a | 3 | Fog | 65 | 49 | 45.3 | -0.9 | 44.4 | | 44.4 | 9.960151 |
| | | 1-3 | Dug-Fog | | | | -1.0 | | 0.0 | | 3.396343 |
| | | 1-2 | Dug-Gura | | | | | | | | 3.562499 |
| | | | | | | 01.0 | | | | | |

FIG. 34.—Triangle computation, angle method of adjustment

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 23

COMPUTATION OF TRIANGLES

State: Alaska

| | NO. | | STATION | OBSERVED ANGLE | | | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 2-3 | Gura-Dug | | | | | | | | 3.562499 |
| 5 | b | 1 | Lat | 38 | 57 | 17.8 | -0.3 | 17.5 | | 17.5 | 0.201551 |
| | c | 2 | Gura | 79 | 27 | 45.0 | -2.1 | 42.9 | 0.1 | 42.8 | 9.992613 |
| | a | 3 | Dug | 61 | 35 | 02.7 | -3.0 | 59.7 | | 59.7 | 9.944240 |
| | | 1-3 | Lat-Dug | | | | -5.4 | | 0.1 | | 3.756663 |
| | | 1-2 | Lat-Gura | | | | | | | | 3.708290 |
| | | | | | | 05.5 | | | | | |
| | | 2-3 | Lat-Gura | | | | | | | | 3.708290 |
| 6 | b | 1 | Mond | 56 | 27 | 36.0 | +0.3 | 36.3 | | 36.3 | 0.079094 |
| | c | 2 | Lat | 60 | 10 | 27.4 | -0.3 | 26.6 | | 26.6 | 9.938290 |
| | a | 3 | Gura | 63 | 21 | 59.5 | -2.4 | 57.1 | | 57.1 | 9.951283 |
| | | 1-3 | Mond-Gura | | | | -2.9 | | 0.0 | | 3.725674 |
| | | 1-2 | Mond-Lat | | | | | | | | 3.738667 |
| | | | | | | 02.9 | | | | | |
| | | 2-3 | Lat-Mond | | | | | | | | 3.738667 |
| 7 | b | 1 | Enter | 36 | 21 | 46.3 | +2.5 | 49.3 | | 49.3 | 0.327012 |
| | a | 2 | Lat | 49 | 08 | 03.4 | -2.4 | 01.0 | | 01.0 | 9.878658 |
| | c | 3 | Mond | 94 | 30 | 09.8 | 0.0 | 09.8 | 0.1 | 09.7 | 9.993658 |
| | | 1-3 | Enter-Mond | | | | +0.1 | | 0.1 | | 3.844337 |
| | | 1-2 | Enter-Lat | | | | | | | | 3.964337 |
| | | | | | | 00.0 | | | | | |
| | | 2-3 | Enter-Mond | | | | | | | | 3.844337 |
| 8 | b | 1 | Flores | 95 | 14 | 37.7 | +1.8 | 39.5 | | 39.5 | 0.001822 |
| | c | 2 | Enter | 63 | 50 | 33.1 | +1.3 | 34.4 | | 34.4 | 9.953077 |
| | a | 3 | Mond | 20 | 54 | 52.5 | -6.4 | 46.1 | | 46.1 | 9.552603 |
| | | 1-3 | Flores-Mond | | | | -3.3 | | 0.0 | | 3.799236 |
| | | 1-2 | Flores-Enter | | | | | | | | 3.398762 |
| | | | | | | 03.3 | | | | | |

FIG. 34.—Triangle computation, angle method of adjustment—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 35

Preliminary triangles — COMPUTATION OF TRIANGLES

*State:* ...Alaska

| NO. | | | STATION | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|---|---|
| | | 2-3 | Alberta-Fish | | | | | | 3.669907 |
| 1 | b | 1 | Nan | 28 42 47.6 | | | | | 0.318374 |
| | a | 2 | Alberta | 57 22 26.5 | | | | | 9.925419 |
| | | 3 | Fish | (93 54 46.0) | | | 0.1 | 45.9 | 9.998987 |
| | | 1-3 | Nan-Fish | | | | 0.1 | | 3.913700 |
| | | 1-2 | Nan-Alberta | | | | | | 3.987268 |
| | | 2-3 | Nan-Fish | | | | | | 3.913700 |
| 2 | b | 1 | Fog | 74 22 17.7 | | | | | 0.016361 |
| | | 2 | Nan | (80 43 39.5) | | | 0.1 | 39.4 | 9.994288 |
| | a | 3 | Fish | 24 54 02.9 | | | | | 9.624332 |
| | | 1-3 | Fog-Fish | | | | 0.1 | | 3.924349 |
| | | 1-2 | Fog-Nan | | | | | | 3.554393 |
| | | 2-3 | Nan-Fog | | | | | | 3.554393 |
| 3 | b | 1 | Gura | 54 33 58.8 | | | | | 0.088956 |
| | a | 2 | Nan | 61 50 46.7 | | | | | 9.945314 |
| | | 3 | Fog | (63 35 14.5) | | | | | 9.952121 |
| | | 1-3 | Gura-Fog | | | | | | 3.588663 |
| | | 1-2 | Gura-Nan | | | | | | 3.595470 |
| | | 2-3 | Gura-Fog | | | | | | 3.588663 |
| 4 | b | 1 | Dug | 75 41 13.8 | | | | | 0.013694 |
| | | 2 | Gura | (38 29 00.9) | | | | | 9.793993 |
| | a | 3 | Fog | 65 49 45.3 | | | | | 9.960152 |
| | | 1-3 | Dug-Fog | | | | | | 3.396350 |
| | | 1-2 | Dug-Gura | | | | | | 3.562509 |

FIG. 35.—Triangle computation to obtain latitude and longitude closures of net, angle method of adjustment

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 26

COMPUTATION OF TRIANGLES

State: Alaska

| | NO. | | STATION | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|---|---|
| | | 2-3 | Gura-Dug | | | | | | 3.562509 |
| 5 | b | 1 | Lat | 38 57 17.8 | | | | | 0.201550 |
| | | 2 | Gura | (79 27 39.6) | | | 0.1 | 39.5 | 9.992611 |
| | a | 3 | Dug | 61 35 02.7 | | | | | 9.944244 |
| | | 1-3 | Lat-Dug | | | | 0.1 | | 3.756670 |
| | | 1-2 | Lat-Gura | | | | | | 3.708303 |
| | | 2-3 | Lat-Gura | | | | | | 3.708303 |
| 6 | b | 1 | Mond | 56 27 36.0 | | | | | 0.079094 |
| | | 2 | Lat | (60 10 24.5) | | | | | 9.938287 |
| | a | 3 | Gura | 63 21 59.5 | | | | | 9.951285 |
| | | 1-3 | Mond-Gura | | | | | | 3.725684 |
| | | 1-2 | Mond-Lat | | | | | | 3.738682 |
| | | 2-3 | Lat-Mond | | | | | | 3.738682 |
| 7 | b | 1 | Enter | 36 21 46.8 | | | | | 0.227019 |
| | a | 2 | Lat | 49 08 03.4 | | | | | 9.873662 |
| | | 3 | Mond | (94 30 09.9) | | | 0.1 | 09.8 | 9.998658 |
| | | 1-3 | Enter-Mond | | | | 0.1 | | 3.944363 |
| | | 1-2 | Enter-Lat | | | | | | 3.964359 |

FIG. 35.—Triangle computation to obtain latitude and longitude closures of net, angle method of adjustment—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
3°0'TO 3°0'

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | 2 | Alberta | to 3 | Fish | 356 | 02 | 14.8 |
| 2d ∠ | | Fish | & | Nan | + 57 | 22 | 26.5 |
| α | 2 | Alberta | to 1 | Nan | 53 | 24 | 41.3 |
| Δα | | | | | − | 6 | 05.8 |
| | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Nan | to 2 | Alberta | 233 | 18 | 35.5−1 |
| | | | | | 28 | 42 | 47.6 |

FIRST ANGLE OF TRIANGLE

| | ° | ′ | ″ | | | λ | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 31 | 40.715 | 2 | Alberta | λ | 133 | 11 | 33.444 |
| Δφ | − | 3 | 07.393 | s= | | Δλ | + | 7 | 23.898 |
| φ′ | 55 | 28 | 33.322 | 1 | Nan | λ′ | 133 | 18 | 57.342 −1 |

| | ° ′ ″ | | | | | | |
|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 30 07.0 | s | 3.987268 | s² | 7.97454 | h² | 4.544 |
| | | Cos α | 9.775293 | Sin² α | 9.80936 | D | 2.363 |
| | | B | 8.509672 | C | 1.56626 | | |
| 1st term | ″ +187.1686 | h | 2.272233 | | 9.35016 | | 6.907 |
| 2d and 3d terms | + 0.2.48 | | | | 0.2240 | | 0.0008 |
| −Δφ | +187.3934 | | | | | | |

| | | | | |
|---|---|---|---|---|
| s | 3.987268 | | | |
| Sin α | 9.904681 | | | |
| A′ | 8.508727 | Δλ | 2.647283 | |
| sec φ′ | 0.246607 | Sin ½ (φ+φ′) | 9.916004 | |
| | 2.647283 | | 2.563287 | |
| Δλ | ″ + 443.8978 | −Δα | ″ + 365.84 | |

DO NOT WRITE IN THIS MARGIN

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | 3 | Fish | to 2 | Alberta | 176 | 02 | 29.9 |
| 3d ∠ | | Nan | & | Alberta | − 93 | 54 | 46.0 |
| α | 3 | Fish | to 1 | Nan | 82 | 07 | 43.9 |
| Δα | | | | | − | 6 | 20.9 |
| | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Nan | to 3 | Fish | 262 | 01 | 23.0 |

| | ° | ′ | ″ | | | λ | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 29 | 09.866 | 3 | Fish | λ | 133 | 11 | 15.042 |
| Δφ | − | | 36.544 | s= | | Δλ | + | 7 | 42.299 |
| φ′ | 55 | 28 | 33.322 | 1 | Nan | λ′ | 133 | 18 | 57.341 |

| | ° ′ ″ | | | | | | |
|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 28 51.6 | s | 3.913700 | s² | 7.82740 | h² | 3.120 |
| | | Cos α | 9.136548 | Sin² α | 9.99178 | D | 2.363 |
| | | B | 8.509676 | C | 1.56558 | | |
| 1st term | ″ + 36.3014 | h | 1.359924 | | 9.38476 | | 5.483 |
| 2d and 3d terms | + 0.2425 | | | | 0.2425 | | 0.0000 |
| −Δφ | + 36.5439 | | | | | | |

| | | | | |
|---|---|---|---|---|
| s | 3.913700 | | | |
| Sin α | 9.995889 | | | |
| A′ | 8.508727 | Δλ | 2.664923 | |
| sec φ′ | 0.246607 | Sin ½ (φ+φ′) | 9.915895 | |
| | 2.664923 | | 2.580818 | |
| Δλ | ″ + 462.2990 | −Δα | ″ + 380.91 | |

DO NOT WRITE IN THIS MARGIN

FIG. 36.—Preliminary position computation to obtain latitude and longitude closures, angle method of adjustment

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 21

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | 2 | Nan | to 3 | Fish | 262 | 01 | 23.0 |
| 2ᵃ∠ | | Fish | & | Fog | + 80 | 43 | 39.5 |
| α | 2 | Nan | to 1 | Fog | 342 | 45 | 02.5 |
| Δα | | | | | + | | 49.3 |
| | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Fog | to 2 | Nan | 162 | 45 | 52.3 |
| | | | | First Angle of Triangle | 74 | 22 | 17.7 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 28 | 33.322 | 2 | Nan | λ | 133 | 13 | 57.341 |
| Δφ | − | 1 | 50.688 | s = | | Δλ | − | 1 | 00.458 |
| φ′ | 55 | 26 | 42.634 | 1 | Fog | λ′ | 133 | 17 | 56.883 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 27 ′ 38.0 ″ | s | 3.554393 | s² | 7.10879 | | | |
| | | Cos α | 9.980014 | Sin² α | 8.94414 | h² | 4.088 |
| | | B | 8.509676 | C | 1.56344 | D | 2.363 |
| 1st term | +110.6835 | h | 2.044083 | | 7.61837 | | 6.451 |
| 2d and 3d terms | + 0.0045 | | | | 0.0042 | | 0.0003 |
| −Δφ | +110.6880 | | | | | | |

| | | | |
|---|---|---|---|
| s | 3.554393 | | |
| Sin α | 9.472069 | | |
| A′ | 8.508727 | Δλ | 1.781457 |
| sec φ′ | 0.246268 | Sin ½ (φ+φ′) | 9.915788 |
| | 1.781457 | | 1.697245 |
| Δλ | −60.4584″ | −Δα | −49.80″ |

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | 3 | Fish | to 2 | Nan | 82 | 07 | 43.9 |
| 3ᵃ∠ | | Fog | & | Nan | − 24 | 54 | 02.9 |
| α | 3 | Fish | to 1 | Fog | 57 | 13 | 41.0 |
| Δα | | | | | − | 5 | 31.0 |
| | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Fog | to 3 | Fish | 237 | 08 | 10.0 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 29 | 09.866 | 3 | Fish | λ | 133 | 11 | 15.042 |
| Δφ | − | 2 | 27.232 | s = | | Δλ | + | 6 | 41.840 |
| φ′ | 55 | 26 | 42.634 | 1 | Fog | λ′ | 133 | 17 | 56.882 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 27 ′ 56.2 ″ | s | 3.924349 | s² | 7.84870 | | | |
| | | Cos α | 9.733435 | Sin² α | 9.84942 | h² | 4.335 |
| | | B | 8.509676 | C | 1.56558 | D | 2.363 |
| 1st term | +147.0483 | h | 2.167460 | | 9.26370 | | 6.698 |
| 2d and 3d terms | + 0.1840 | | | | 0.1835 | | 0.0006 |
| −Δφ | +147.2323 | | | | | | |

| | | | |
|---|---|---|---|
| s | 3.924349 | | |
| Sin α | 9.924709 | | |
| A′ | 8.508727 | Δλ | 2.604053 |
| sec φ′ | 0.246268 | Sin ½ (φ+φ′) | 9.915814 |
| | 2.604053 | | 2.519867 |
| Δλ | +401.8398″ | −Δα | +331.03″ |

FIG. 36.—Preliminary position computation to obtain latitude and longitude closures, angle method of adjustment—Continued

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U. S. COAST AND GEODETIC SURVEY
FORM 91

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| α | 2 | Nan | to 3 | Fog | | 342 | 45 | 02.5 |
| 2ᵃ∠ | | Fog | & | Gura | | + 61 | 50 | 46.7 |
| α | 2 | Nan | to 1 | Gura | | 44 | 35 | 49.2 |
| Δα | | | | | | - | 2 | 09.6 |
| | | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Gura | to 2 | Nan | | 224 | 33 | 39.6 |
| | | | | | First Angle of Triangle | 54 | 33 | 58.8 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 28 | 33.322 | 2 | Nan | λ | 133 | 18 | 57.341 |
| Δφ | - | 1 | 30.740 | s= | | Δλ | + | 2 | 37.375 |
| φ′ | 55 | 27 | 02.582 | 1 | Gura | λ′ | 133 | 21 | 34.716 |

-1

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 27 ′ 48.0 ″ | s | 3.595470 | s² | 7.19094 | | | |
| | | Cos α | 9.652518 | Sin² α | 9.69282 | h² | 3.915 |
| | | B | 8.509676 | C | 1.56544 | D | 2.363 |
| 1st term | + 90.7119 | h | 1.957664 | | 8.44920 | | 6.278 |
| 2d and 3d terms | + 0.0283 | | | | 0.0281 | | 0.0002 |
| −Δφ | + 90.7402 | | | | | | |

| | | | |
|---|---|---|---|
| s | 3.595470 | | |
| Sin α | 9.846409 | | |
| A′ | 8.508727 | Δλ | 2.196935 |
| sec φ′ | 0.246329 | Sin ½ (φ+φ′) | 9.915803 |
| | 2.196935 | | 2.112738 |
| Δλ | + 157.3747″ | −Δα | + 129.64″ |

DO NOT WRITE IN THIS MARGIN

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| α | 3 | Fog | to 2 | Nan | | 162 | 45 | 52.3 |
| 3ᵃ∠ | | Gura | & | Nan | | - 63 | 35 | 14.5 |
| α | 3 | Fog | to 1 | Gura | | 99 | 10 | 37.8 |
| Δα | | | | | | - | 2 | 59.4 |
| | | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Gura | to 3 | Fog | | 279 | 07 | 38.4 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 26 | 42.634 | 3 | Fog | λ | 133 | 17 | 56.882 |
| Δφ | + | | 19.948 | s= | | Δλ | + | 3 | 37.833 |
| φ′ | 55 | 27 | 02.582 | 1 | Gura | λ′ | 133 | 21 | 34.715 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 26 ′ 52.6 ″ | s | 3.588663 | s² | 7.17733 | | | |
| | | Cos α | 9.202727 | Sin² α | 9.98381 | h² | 2.602 |
| | | B | 8.509678 | C | 1.56492 | D | 2.364 |
| 1st term | -20.0018 | h | 1.301068 | | 8.73106 | | 4.966 |
| 2d and 3d terms | + 0.0538 | | | | 0.0538 | | 0.0000 |
| −Δφ | -19.9480 | | | | | | |

| | | | |
|---|---|---|---|
| s | 3.588663 | | |
| Sin α | 9.994405 | | |
| A′ | 8.508727 | Δλ | 2.338124 |
| sec φ′ | 0.246329 | Sin ½ (φ+φ′) | 9.915722 |
| | 2.338124 | | 2.253846 |
| Δλ | + 217.3332″ | −Δα | + 179.41″ |

DO NOT WRITE IN THIS MARGIN

FIG. 36.—Preliminary position computation to obtain latitude and longitude closures, angle method of adjustment—Continued

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U. S. COAST AND GEODETIC SURVEY
Form 97

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | |
|---|---|---|---|---|---|---|---|---|
| a | 2 | Gura | to 3 | Fog | | 279 | 07 | 38.4 |
| 3d ∠ | | Fog | & | Dug | | + 38 | 29 | 00.9 |
| a | 2 | Gura | to 1 | Dug | | 317 | 36 | 39.3 |
| Δa | | | | | | + | 1 | 55.3 |
| | | | | | | 180 | 00 | 00.00 |
| a′ | 1 | Dug | to 2 | Gura | | 137 | 38 | 34.6 |
| | | | | | | 75 | 41 | 13.8 |

First Angle of Triangle

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 27 | 02.582 | 2 | Gura | λ | 133 | 21 | 34.715 |
| Δφ | - | 1 | 27.237 | a= | | Δλ | - | 2 | 19.979 |
| φ′ | 55 | 25 | 35.345 | 1 | Dug | λ′ | 133 | 19 | 14.736 |

| | ° ′ ″ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 26 19.0 | a | 3.562509 | a² | 7.12502 | | | |
| | | Cos a | 9.368400 | Sin² a | 9.65753 | h² | 3.381 | |
| | | B | 8.509678 | C | 1.56501 | D | 2.364 | |
| | ″ | h | 1.940587 | | 8.34756 | | 6.245 | |
| 1st term | + 87.2142 | | | | 0.0223 | | 0.0002 | |
| 2d and 3d terms | + 0.0225 | | | | | | | |
| −Δφ | + 87.2367 | | | | | | | |

| | | | |
|---|---|---|---|
| a | 3.562509 | | |
| Sin a | 9.828764 | | |
| A′ | 8.508728 | Δλ | 2.146063 |
| sec φ′ | 0.246062 | Sin ½ (φ+φ′) | 9.915674 |
| | 2.146063 | | 2.061737 |
| Δλ | -139.9790″ | −Δa | -115.27″ |

DO NOT WRITE IN THIS MARGIN

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | |
|---|---|---|---|---|---|---|---|---|
| a | 3 | Fog | to 2 | Gura | | 99 | 10 | 37.8 |
| 3d ∠ | | Dug | & | Gura | | − 65 | 49 | 45.3 |
| a | 3 | Fog | to 1 | Dug | | 33 | 20 | 52.5 |
| Δa | | | | | | - | 1 | 04.1 |
| | | | | | | 180 | 00 | 00.00 |
| a′ | 1 | Dug | to 3 | Fog | | 213 | 19 | 48.4 |

| | ° | ′ | ″ | | | | ° | ′ | |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 26 | 42.634 | 3 | Fog | λ | 133 | 17 | 56.882 |
| Δφ | - | 1 | 07.288 | a= | | Δλ | + | 1 | 17.854 |
| φ′ | 55 | 25 | 35.346 | 1 | Dug | λ′ | 133 | 19 | 14.736 |

| | ° ′ ″ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 26 09.0 | a | 3.396350 | a² | 6.79270 | | | |
| | | Cos a | 9.921867 | Sin² a | 9.48029 | h² | 3.656 | |
| | | B | 8.509678 | C | 1.56492 | D | 2.364 | |
| | ″ | h | 1.827895 | | 7.83791 | | 6.020 | |
| 1st term | + 67.2814 | | | | 0.0069 | | 0.0001 | |
| 2d and 3d terms | + 0.0070 | | | | | | | |
| −Δφ | + 67.2884 | | | | | | | |

| | | | |
|---|---|---|---|
| a | 3.396350 | | |
| Sin a | 9.740143 | | |
| A′ | 8.508728 | Δλ | 1.891283 |
| sec φ′ | 0.246062 | Sin ½ (φ+φ′) | 9.915659 |
| | 1.891283 | | 1.806942 |
| Δλ | + 77.8544″ | −Δa | + 64.11″ |

DO NOT WRITE IN THIS MARGIN

FIG. 36.—Preliminary position computation to obtain latitude and longitude closures, angle method of adjustment—Continued

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | 2 | Gura | to 3 | Dug | 317 | 36 | 39.3 |
| 3d ∠ | | Dug | & | Lat | + 79 | 27 | 39.6 |
| α | 2 | Gura | to 1 | Lat | 37 | 04 | 18.9 |
| Δα | | | | | - | 2 | 24.1 |
| | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Lat | to 2 | Gura | 217 | 01 | 54.8 |
| | | | | First Angle of Triangle | 38 | 57 | 17.8 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 27 | 02.582 | 2 | Gura | λ | 133 | 21 | 34.715 |
| Δφ | - | 2 | 11.836 | ε- | | Δλ | + | 2 | 55.042 |
| φ′ | 55 | 24 | 50.746 | 1 | Lat | λ′ | 133 | 24 | 29.757 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 25 ′ 56.7 ″ | s | 3.708303 | s² | 7.41661 | | |
| | | Cos α | 9.901937 | Sin² α | 9.56037 | h² | 4.240 |
| | | B | 8.509678 | C | 1.56501 | D | 2.364 |
| 1st term | + 131.8008 | h | 2.119918 | | 8.54199 | | 6.604 |
| 2d and 3d terms | + 0.0352 | | | | 0.0348 | | 0.0004 |
| −Δφ | + 131.8360 | | | | | | |

| | | | |
|---|---|---|---|
| s | 3.708303 | | |
| Sin α | 9.780186 | | |
| A′ | 8.508728 | Δλ | 2.243143 |
| sec φ′ | 0.245926 | Sin ½ (φ+φ′) | 9.915641 |
| | 2.243143 | | 2.158784 |
| Δλ | + 175.0423 ″ | −Δα | + 144.14 ″ |

DO NOT WRITE IN THIS MARGIN

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 57

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | 3 | Dug | to 2 | Gura | 137 | 38 | 34.6 |
| 3d ∠ | | Lat | & | Gura | - 61 | 35 | 02.7 |
| α | 3 | Dug | to 1 | Lat | 76 | 03 | 31.9 |
| Δα | | | | | - | 4 | 19.4 |
| | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Lat | to 3 | Dug | 256 | 59 | 12.5 +1 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 25 | 35.346 | 3 | Dug | λ | 133 | 19 | 14.736 |
| Δφ | - | | 44.600 | ε- | | Δλ | + | 5 | 15.021 |
| φ′ | 55 | 24 | 50.746 | 1 | Lat | λ′ | 133 | 24 | 29.757 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 25 ′ 13.0 ″ | s | 3.756670 | s² | 7.51334 | | |
| | | Cos α | 9.381882 | Sin² α | 9.97403 | h² | 3.296 |
| | | B | 8.509680 | C | 1.55463 | D | 2.364 |
| 1st term | +44.4869 | h | 1.648232 | | 9.05200 | | 5.660 |
| 2d and 3d terms | +0.1127 | | | | 0.1127 | | 0.0000 |
| −Δφ | +44.5996 | | | | | | |

| | | | |
|---|---|---|---|
| s | 3.756670 | | |
| Sin α | 9.987015 | | |
| A′ | 8.508728 | Δλ | 2.498339 |
| sec φ′ | 0.245926 | Sin ½ (φ+φ′) | 9.915578 |
| | 2.498339 | | 2.413917 |
| Δλ | + 315.0207 ″ | −Δα | + 259.37 ″ |

DO NOT WRITE IN THIS MARGIN

FIG. 36.—Preliminary position computation to obtain latitude and longitude closures, angle method of adjustment—Continued

58853°—28——9

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U. S. Coast and Geodetic Survey
Form 27

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| c | 2 | Lat | to 3 | Gura | | 217 | 01 | 54.8 |
| 2d ∠ | | Gura | & | Mond | | + 60 | 10 | 24.5 |
| a | 2 | Lat | to 1 | Mond | | 277 | 12 | 19.3 |
| Δa | | | | | | + | 4 | 14.3 |
| | | | | | | 180 | 00 | 00.00 |
| a′ | 1 | Mond | to 2 | Lat | | 97 | 16 | 33.6 |
| | | | | | | 56 | 27 | 36.0 |

First Angle of Triangle

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 24 | 50.746 | 2 | Lat | λ | 133 | 24 | 29.757 |
| Δφ | - | | 22.329 | a= | | Δλ | - | 5 | 08.905 |
| φ′ | 55 | 24 | 28.417 | 1 | Mond | λ′ | 133 | 19 | 20.852 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 24 ′ 39.6 ″ | | 3.738682 | | 7.47736 | | | |
| | | Cos a | 9.098388 | Sin² a | 9.99311 | h² | 2.694 |
| | | B | 8.509681 | C | 1.56443 | D | 2.364 |
| 1st term | +22.2204 ″ | h | 1.348751 | | 9.03490 | | 5.058 |
| 2d and 3d terms | +0.1084 | | | | 0.1084 | | 0.0000 |
| −Δφ | +22.3288 | | | | | | |

| | | | |
|---|---|---|---|
| a | 3.738682 | | |
| Sin a | 9.996557 | | |
| A′ | 8.508728 | Δλ | 2.489825 |
| sec φ′ | 0.245858 | Sin ½ (φ+φ′) | 9.915529 |
| | 2.489825 | | 2.405354 |
| Δλ | -308.9050 | −Δa | -254.30″ |

DO NOT WRITE IN THIS MARGIN

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| a | 3 | Gura | to 2 | Lat | | 37 | 04 | 18.9 |
| 3d ∠ | | Mond | & | Lat | | - 53 | 21 | 59.5 |
| a | 3 | Gura | to 1 | Mond | | 333 | 42 | 19.4 |
| Δa | | | | | | + | 1 | 50.2 |
| | | | | | | 180 | 00 | 00.00 |
| a′ | 1 | Mond | to 3 | Gura | | 153 | 44 | 09.6 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 27 | 02.582 | 3 | Gura | λ | 133 | 21 | 34.715 |
| Δφ | - | 2 | 34.165 | a= | | Δλ | - | 2 | 13.863 |
| φ′ | 55 | 24 | 28.417 | 1 | Mond | λ′ | 133 | 19 | 20.852 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 25 ′ 45.5 ″ | | 3.725684 | | 7.45137 | | | |
| | | Cos a | 9.952564 | Sin² a | 9.29278 | h² | 4.376 |
| | | B | 8.509673 | C | 1.56501 | D | 2.364 |
| 1st term | +154.1438 ″ | h | 2.187926 | | 8.30916 | | 6.740 |
| 2d and 3d terms | + 0.0209 | | | | 0.0204 | | 0.0005 |
| −Δφ | +154.1647 | | | | | | |

| | | | |
|---|---|---|---|
| a | 3.725684 | | |
| Sin a | 9.646391 | | |
| A′ | 8.508728 | Δλ | 2.126661 |
| sec φ′ | 0.245858 | Sin ½ (φ+φ′) | 9.915625 |
| | 2.126661 | | 2.042286 |
| Δλ | -133.8631 | −Δa | -110.23″ |

DO NOT WRITE IN THIS MARGIN

FIG. 36.—Preliminary position computation to obtain latitude and longitude closures, angle method of adjustment—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 27

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | 2 | Lat | to 3 | Mond | 277 | 12 | 19.3 |
| 3d ∠ | | Mond | & | Enter | + 49 | 08 | 03.4 |
| α | 2 | Lat | to 1 | Enter | 326 | 20 | 22.7 |
| Δα | | | | | + | 3 | 58.4 |
| | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Enter | to 2 | Lat | 146 | 24 | 21.1 |
| | | | | | 36 | 21 | 46.8 |

FIRST ANGLE OF TRIANGLE

| | ° | ′ | ″ | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| φ | 55 | 24 | 50.746 | 2 Lat | λ | 133 | 24 | 29.757 |
| Δφ | − | 4 | 08.033 | s = | Δλ | − | 4 | 49.720 |
| φ′ | 55 | 20 | 42.713 | 1 Enter | λ′ | 133 | 19 | 40.037 |
| Fixed | 55 | 20 | 42.730 | | | 133 | 19 | 40.049 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 22 ′ 46.7 ″ | | s | 3.964359 | s² | 7.92872 | | |
| | | | Cos α | 9.920300 | Sin² α | 9.48744 | h² | 4.789 |
| | | | B | 8.509681 | C | 1.56443 | D | 2.364 |
| 1st term | | +247.9362 | h | 2.394340 | | 8.98059 | | 7.153 |
| 2d and 3d terms | | + 0.0970 | | | | 0.0956 | | 0.0014 |
| −Δφ | | +248.0332 | | | | | | |

| | | | |
|---|---|---|---|
| s | 3.964359 | | |
| Sin α | 9.743720 | | |
| A′ | 8.508730 | Δλ | 2.461979 |
| sec φ′ | 0.245170 | Sin ½ (φ+φ′) | 9.915365 |
| | 2.461979 | | 2.377344 |
| Δλ | −289.7203 ″ | −Δα | −238.42 ″ |

DO NOT WRITE IN THIS MARGIN

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|
| α | 3 | Mond | to 2 | Lat | 97 | 16 | 33.6 |
| 3d ∠ | | Enter | & | Lat | − 94 | 30 | 09.9 |
| α | 3 | Mond | to 1 | Enter | 2 | 46 | 23.7 |
| Δα | | | | | − | | 15.8 |
| | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Enter | to 3 | Mond | 182 | 46 | 07.9 |

| | ° | ′ | ″ | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| φ | 55 | 24 | 28.417 | 3 Mond | λ | 133 | 19 | 20.852 |
| Δφ | − | 3 | 45.703 | s = | Δλ | + | | 19.185 |
| φ′ | 55 | 20 | 42.714 −1 | 1 Enter | λ′ | 133 | 19 | 40.037 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 22 ′ 35.6 ″ | | s | 3.844363 | s² | 7.68873 | | |
| | | | Cos α | 9.999491 | Sin² α | 7.36939 | h² | 4.707 |
| | | | B | 8.509681 | C | 1.56433 | D | 2.364 |
| 1st term | | +225.7018 | h | 2.353535 | | 6.62245 | | 7.071 |
| 2d and 3d terms | | + 0.0016 | | | | 0.0004 | | 0.0012 |
| −Δφ | | +225.7034 | | | | | | |

| | | | |
|---|---|---|---|
| s | 3.844363 | | |
| Sin α | 8.684697 | | |
| A′ | 8.508730 | Δλ | 1.282960 |
| sec φ′ | 0.245170 | Sin ½ (φ+φ′) | 9.915349 |
| | 1.282960 | | 1.198309 |
| Δλ | + 19.1849 ″ | −Δα | + 15.79 ″ |

DO NOT WRITE IN THIS MARGIN

FIG. 36.—Preliminary position computation to obtain latitude and longitude closures, angle method of adjustment—Continued

*Azimuth equation*

| | ° | ′ | ″ | Final seconds[1] |
|---|---|---|---|---|
| Fish-Alberta | 176 | 02 | 29.9 | 29.9 |
| −1c | −93 | 54 | 43.7 | 45.3 |
| Fish-Nan | 82 | 07 | 46.2 | 44.6 |
| | − | 6 | 20.9 | 20.9 |
| Nan-Fish | 262 | 01 | 25.3 | 23.7 |
| +2c | +80 | 43 | 36.8 | 38.1 |
| Nan-Fog | 342 | 45 | 02.1 | 01.8 |
| | + | | 49.8 | 49.8 |
| Fog-Nan | 162 | 45 | 51.9 | 51.6 |
| −3c | −63 | 35 | 14.6 | 14.9 |
| Fog-Gura | 99 | 10 | 37.3 | 36.7 |
| | − | 2 | 59.4 | 59.4 |
| Gura-Fog | 279 | 07 | 37.9 | 37.3 |
| +4c+5c | +117 | 56 | 46.9 | 44.6 |
| Gura-Lat | 37 | 04 | 24.8 | 21.9 |
| | − | 2 | 24.1 | 24.1 |
| Lat-Gura | 217 | 02 | 00.7 | 57.8 |
| +6c | +60 | 10 | 27.4 | 26.6 |
| Lat-Mond | 277 | 12 | 28.1 | 24.4 |
| | + | 4 | 14.3 | 14.3 |
| Mond-Lat | 97 | 16 | 42.4 | 38.7 |
| −7c | −94 | 30 | 09.8 | 09.8 |
| Mond-Enter | 2 | 46 | 32.6 | 28.9 |
| | − | | 15.8 | 15.8 |
| Enter-Mond | 182 | 46 | 16.8 | 13.1 |
| +8c | +63 | 50 | 33.1 | 34.4 |
| Enter-Flores (computed) | 246 | 36 | 49.9 | 47.5 |
| Enter-Flores (fixed) | 246 | 36 | 47.5 | 47.5 |
| | | | +2.4 | 0.0 |

9. $0 = +2.4 - (1c) + (2c) - (3c) + (4c) + (5c) + (6c) - (7c) + (8c)$

Since there are two fixed lengths namely, Alberta to Fish and Enter to Flores, there will be one length equation. This equation, which is shown below, is formed in exactly the same manner as if the direction method were used, except in the way the angles are designated.

*Length equation*

+2.02 2.02

| Fish-Alberta | | | | 3.669907 | −2.02 | Enter-Flores | | | | 3.398762 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | ° | ′ | ″ | | | | ° | ′ | ″ | | |
| 1a | 57 | 22 | 26.5 | 9.9254194 | +1.3 | 1b | 28 | 42 | 47.6 | 9.6816265 | +3.8 |
| 2a | 24 | 54 | 02.9 | 9.6243321 | +4.5 | 2b | 74 | 22 | 17.7 | 9.9836394 | +.6 |
| 3a | 61 | 50 | 46.7 | 9.9453135 | +1.1 | 3b | 54 | 33 | 58.8 | 9.9110442 | +1.5 |
| 4a | 65 | 49 | 45.3 | 9.9601516 | +.9 | 4b | 75 | 41 | 13.8 | 9.9863060 | +.5 |
| 5a | 61 | 35 | 02.7 | 9.9442440 | +1.1 | 5b | 38 | 57 | 17.8 | 9.7984497 | +2.6 |
| 6a | 63 | 21 | 59.5 | 9.9512853 | +1.1 | 6b | 56 | 27 | 36.0 | 9.9209058 | +1.4 |
| 7a | 49 | 08 | 03.4 | 9.8786625 | +1.8 | 7b | 36 | 21 | 46.8 | 9.7729807 | +2.9 |
| 8a | 20 | 54 | 52.5 | 9.5526388 | +5.5 | 8b | 95 | 14 | 37.7 | 9.9981786 | −.2 |
| | | | | 2.4519542 | 17.3 | | | | | 2.4518929 | 13.1 |

1.98 [illegible]

30.4.6 26/61.3

30.8 to get 61.3

10. $0 = +61.3 + 1.3(1a) - 3.8\ (1b) + 4.5(2a) - 0.6(2b) + 1.1(3a) - 1.5(3b) + 0.9(4a) - 0.5(4b) + 1.1(5a) - 2.6(5b) + 1.1(6a) - 1.4(6b) + 1.8(7a) - 2.9(7b) + 5.5(8a) + 0.2(8b)$

[1] The values in this column are filled in after the adjustment is completed.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 496

## LATITUDE AND LONGITUDE ADJUSTMENT

$\varphi_n$: 55° 20′.71 ... $\lambda_n$: 133° 19′.67 ...

| | Station | $\varphi_s$ | $\lambda_s$ | $\varphi_n-\varphi_s$ | $\lambda_n-\lambda_s$ | $+\delta_\lambda$ | $-\delta_\varphi$ | Lat. equation $(\varphi_n-\varphi_s)\delta_\lambda$ | $A$ | Long. equation $(\lambda_n-\lambda_s)\delta_\varphi$ | Lat. equation $(\varphi_n-\varphi_s)(-\delta_\varphi)$ | $B$ | Long. equation $(\lambda_n-\lambda_s)(-\delta_\varphi)$ | Lat. equation $(\lambda_n-\lambda_s)\alpha_1$ | $C$ $(+R \text{ or } -L)$ | Long. equation $(\varphi_n-\varphi_s)\alpha_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| - | Fish | 55° 29′.16 | 133° 11′.25 | -8.45 | +8.42 | +1.3 | -3.8 | -11.0 | 1 a | +10.9 | +32.1 | 1 b | -32.0 | +10.1 | +1a + 1b | +31.3 |
| + | Man | 55 28.56 | 133 18.96 | -7.85 | +0.71 | +4.5 | -0.6 | -35.3 | 2 a | + 3.2 | + 4.7 | 2 b | - 0.4 | + 0.9 | +2a - 2b | +29.0 |
| - | Fog | 55 26.71 | 133 17.95 | -6.00 | +1.72 | +1.1 | -1.5 | - 6.6 | 3 a | + 1.9 | + 9.0 | 3 b | - 2.6 | + 2.1 | +3a +3b | +22.2 |
| + | Gura | 55 27.04 | 133 21.58 | -6.33 | -1.91 | +0.9 | -0.5 | - 5.7 | 4 a | - 1.7 | + 3.2 | 4 b | + 1.0 | - 2.3 | -4a - 4b | +23.4 |
| + | Gura | 55 27.04 | 133 21.58 | -6.33 | -1.91 | +1.1 | -2.6 | - 7.0 | 5 a | - 2.1 | +16.5 | 5 b | + 5.0 | - 2.3 | -5a - 5b | +23.4 |
| + | Lat | 55 24.85 | 133 24.50 | -4.14 | -4.83 | +1.1 | -1.4 | - 4.6 | 6 a | - 5.3 | + 5.8 | 6 b | + 6.8 | - 5.8 | -6a - 6b | +15.3 |
| - | Mond | 55 24.47 | 133 19.35 | -3.76 | +0.32 | +1.8 | -2.9 | - 6.8 | 7 a | + 0.6 | +10.9 | 7 b | - 0.9 | + 0.4 | +7a + 7b | +13.9 |
| | Enter | 55 20.71 | 133 19.67 | | | | | | | | | | | | | |

$\phi_n$ = 55° 20′ 42″.713 $\lambda_n$ = 133° 19′ 40″.037

$\phi_n'$ = 55 20 42.730 $\lambda_n'$ = 133 19 40.049

$(\phi_n-\phi_n')$ = -0.017 $(\lambda_n-\lambda_n')$ = -0.012

x7238.24 x7238.24

-123.0508 -86.8588

$a_1$ = +1.20

$a_2$ = -3.70

11. $\phi$.

$$0 = -123.0501 + 0.9\,(1\,a) + 42.2\,(1\,b) - 36.2\,(2\,a) + 3.8\,(2\,b) - 4.5\,(3\,a) + 11.1\,(3\,b) - 3.4\,(4\,a) + 5.5\,(4\,b) + 4.7\,(5\,a) + 18.8\,(5\,b) + 1.2\,(6\,a) + 11.6\,(6\,b) - 6.4\,(7\,a) + 11.3\,(7\,b).$$

(This equation should be divided by 10 before entering it in the correlates.)

12. $\lambda$.

$$0 = -86.8589 + 42.2\,(1\,a) - 0.7\,(1\,b) - 25.8\,(2\,a) - 29.4\,(2\,b) + 24.1\,(3\,a) + 19.6\,(3\,b) - 25.1\,(4\,a) - 22.4\,(4\,b) - 25.5\,(5\,a) - 18.4\,(5\,b) - 20.6\,(6\,a) - 8.5\,(6\,b) + 14.5\,(7\,a) + 13.0\,(7\,b).$$

(This equation should be divided by 10 before entering it in the correlates.)

| $\varphi$ | $a_1$ | $a_2$ |
|---|---|---|
| ° | + | - |
| 0 | 2.13 | 2.09 |
| 1 | 2.12 | 2.09 |
| 2 | 2.12 | 2.09 |
| 3 | 2.12 | 2.09 |
| 4 | 2.11 | 2.10 |
| 5 | 2.11 | 2.10 |
| 6 | 2.11 | 2.10 |
| 7 | 2.10 | 2.11 |
| 8 | 2.10 | 2.11 |
| 9 | 2.09 | 2.12 |
| 10 | 2.09 | 2.12 |
| 11 | 2.08 | 2.13 |
| 12 | 2.07 | 2.14 |
| 13 | 2.06 | 2.15 |
| 14 | 2.05 | 2.16 |
| 15 | 2.05 | 2.17 |
| 16 | 2.04 | 2.18 |
| 17 | 2.03 | 2.19 |
| 18 | 2.01 | 2.20 |
| 19 | 2.00 | 2.21 |
| 20 | 1.99 | 2.22 |
| 21 | 1.98 | 2.24 |
| 22 | 1.96 | 2.26 |
| 23 | 1.95 | 2.27 |
| 24 | 1.93 | 2.29 |
| 25 | 1.92 | 2.31 |
| 26 | 1.90 | 2.33 |
| 27 | 1.89 | 2.35 |
| 28 | 1.87 | 2.37 |
| 29 | 1.85 | 2.39 |
| 30 | 1.83 | 2.42 |
| 31 | 1.81 | 2.44 |
| 32 | 1.79 | 2.47 |
| 33 | 1.77 | 2.50 |
| 34 | 1.74 | 2.53 |
| 35 | 1.72 | 2.56 |
| 36 | 1.71 | 2.59 |
| 37 | 1.69 | 2.63 |
| 38 | 1.67 | 2.66 |
| 39 | 1.64 | 2.70 |
| 40 | 1.62 | 2.74 |
| 41 | 1.60 | 2.78 |
| 42 | 1.57 | 2.82 |
| 43 | 1.55 | 2.87 |
| 44 | 1.53 | 2.92 |
| 45 | 1.50 | 2.97 |
| 46 | 1.47 | 3.02 |
| 47 | 1.44 | 3.08 |
| 48 | 1.42 | 3.14 |
| 49 | 1.39 | 3.20 |
| 50 | 1.36 | 3.27 |
| 51 | 1.33 | 3.34 |
| 52 | 1.30 | 3.41 |
| 53 | 1.27 | 3.49 |
| 54 | 1.24 | 3.57 |
| 55 | 1.21 | 3.65 |
| 56 | 1.17 | 3.74 |
| 57 | 1.13 | 3.83 |
| 58 | 1.12 | 3.92 |
| 59 | 1.08 | 4.02 |
| 60 | 1.05 | 4.20 |
| 61 | 1.02 | 4.34 |
| 62 | .98 | 4.48 |
| 63 | .96 | 4.63 |
| 64 | .92 | 4.80 |
| 65 | .88 | 4.98 |
| 66 | .85 | 5.17 |
| 67 | .82 | 5.34 |
| 68 | .78 | 5.67 |
| 69 | .74 | 5.87 |
| 70 | .72 | 6.15 |
| 71 | .69 | 6.46 |
| 72 | .65 | 6.81 |

$\frac{M}{60} \times 10^6 = 7238.24$

FIG. 37.—Formation of latitude and longitude equations, angle method of adjustment

Since the lines Alberta-Fish and Enter-Flores are fixed in position (latitude and longitude) there will be one latitude and one longitude equation. These equations, which are shown in Figure 37, are formed in exactly the same manner as when the direction method is used, except in the way the angles are designated.

In the adjustment of this net of triangulation there will be 8 angle, 1 azimuth, 1 length, 1 latitude, and 1 longitude equations, or a total of 12.

As the correlate and normal equations are formed exactly as in the case of the direction method (see p. 84), they are given below without explanation.

*Correlate equations*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | α 9 | $l$ 10 | $\phi$ 11 | λ 12 | $\Sigma_0$ | $v$ | Adopted $v$ | $v^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1a | +1 | | | | | | | | | +1.3 | −0.09 | +4.22 | +6.43 | −0.102 | −0.1 | 0.01 |
| b | +1 | | | | | | | | | −3.8 | +4.22 | −.07 | +1.35 | +.778 | +.8 | .64 |
| c | +1 | | | | | | | | −1 | | | | 0 | +1.623 | +1.6 | 2.56 |
| 2a | | +1 | | | | | | | | +4.5 | −3.62 | −2.58 | −0.70 | −.425 | −.4 | .16 |
| b | | +1 | | | | | | | | −.6 | +.38 | −2.94 | −2.16 | +1.821 | +1.8 | 3.24 |
| c | | +1 | | | | | | | +1 | | | | +2.00 | +1.302 | +1.3 | 1.69 |
| 3a | | | +1 | | | | | | | +1.1 | −.45 | +2.41 | +4.06 | −1.063 | −1.1 | 1.21 |
| b | | | +1 | | | | | | | −1.5 | +1.11 | +1.96 | +2.57 | +.697 | +.7 | .49 |
| c | | | +1 | | | | | | −1 | | | | 0 | +.266 | +.3 | .09 |
| 4a | | | | +1 | | | | | | +.9 | −.34 | −2.51 | −.95 | −.902 | −.9 | .81 |
| b | | | | +1 | | | | | | −.5 | +.55 | −2.24 | −1.19 | +.096 | +.1 | .01 |
| c | | | | +1 | | | | | +1 | | | | +2.00 | −.192 | −.2 | .04 |
| 5a | | | | | +1 | | | | | +1.1 | −.47 | −2.55 | −.92 | −2.964 | −3.0 | 9.00 |
| b | | | | | +1 | | | | | −2.6 | +1.88 | −1.84 | −1.56 | −.324 | −.3 | .09 |
| c | | | | | +1 | | | | +1 | | | | +2.00 | −2.114 | −2.1 | 4.41 |
| 6a | | | | | | +1 | | | | +1.1 | +.12 | −2.06 | +.16 | −2.400 | −2.4 | 5.76 |
| b | | | | | | +1 | | | | −1.4 | +1.16 | −.85 | −.09 | +.331 | +.3 | .09 |
| c | | | | | | +1 | | | +1 | | | | +2.00 | −.831 | −.8 | .64 |
| 7a | | | | | | | +1 | | | +1.8 | −.64 | +1.45 | +3.61 | −2.370 | −2.4 | 5.76 |
| b | | | | | | | +1 | | | −2.9 | +1.13 | +1.30 | +.53 | +2.455 | +2.5 | 6.25 |
| c | | | | | | | +1 | | −1 | | | | 0 | +.014 | 0 | 0 |
| 8a | | | | | | | | +1 | | +5.5 | | | +6.50 | −6.470 | −6.4 | 40.96 |
| b | | | | | | | | +1 | | +.2 | | | +1.20 | +1.835 | +1.8 | 3.24 |
| c | | | | | | | | +1 | +1 | | | | +2.00 | +1.337 | +1.3 | 1.69 |
| | | | | | | | | | | | | | | | Total | 88.84 |

$$\text{p.e.} = \pm 0.674\sqrt{\frac{\Sigma v^2}{c}} = \pm 0.674\sqrt{\frac{88.84}{12}} = \pm 1''.83$$

*Normal equations*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | α 9 | $l$ 10 | $\phi$ 11 | λ 12 | $\eta$ | $\Sigma_n$ | $C$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | +3 | | | | | | | | −1 | −2.5 | +4.13 | +4.15 | −2.3 | +5.48 | +0.8125 |
| 2 | | +3 | | | | | | | +1 | +3.9 | −3.24 | −5.52 | −2.7 | −3.56 | +2.1131 |
| 3 | | | +3 | | | | | | −1 | −.4 | +.66 | +4.37 | +.1 | +6.73 | −.5449 |
| 4 | | | | +3 | | | | | +1 | +.4 | +.21 | −4.75 | +1.0 | +.86 | +.6186 |
| 5 | | | | | +3 | | | | +1 | −1.5 | +1.41 | −4.39 | +5.4 | +4.92 | −1.3028 |
| 6 | | | | | | +3 | | | +1 | −.3 | +1.28 | −2.91 | +2.9 | +4.97 | −.0205 |
| 7 | | | | | | | +3 | | −1 | −1.1 | +.49 | +2.75 | −.1 | +4.04 | −.7967 |
| 8 | | | | | | | | +3 | +1 | +5.7 | | | +3.3 | +13.00 | +2.1475 |
| 9 | | | | | | | | | +8 | | | | +2.4 | +12.40 | −.8106 |
| 10 | | | | | | | | | | +94.34 | −46.7380 | −5.7790 | +61.3 | +107.3230 | −1.5670 |
| 11 | | | | | | | | | | | +39.7198 | +5.3068 | −12.3050 | −9.0764 | −1.4152 |
| 12 | | | | | | | | | | | | +72.7274 | −8.6859 | +57.2693 | +.2360 |

## SOLUTION OF NORMAL EQUATIONS

The solution of the normals is much simpler than when the direction method of adjustment is used. This is due to the fact that no two angle equations involve the same $v$'s and the first 8 equations in the example below are eliminated by simply dividing by 3 in each case. There are only 4 equations which are much involved.

*Solution of normal equations*

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | $\eta$ | $\Sigma_n$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| +3 | | | | | | | | −1 | −2.5 | +4.13 | +4.15 | −2.3 | +5.48 |
| $C_1$ | | | | | | | | +.33333 | +.83333 | −1.37667 | −1.38333 | +.76667 | −1.82667 |
| | +3 | | | | | | | +1 | +3.9 | −3.24 | −5.52 | −2.7 | −3.56 |
| | $C_2$ | | | | | | | −.33333 | −1.3 | +1.08 | +1.84 | +.9 | +1.18667 |
| | | +3 | | | | | | −1 | −.4 | +.66 | +4.37 | +.1 | +6.73 |
| | | $C_3$ | | | | | | +.33333 | +.13333 | −.22 | −1.45667 | −.93333 | −2.243334 |
| | | | +3 | | | | | +1 | +.4 | +.21 | −4.75 | +1.0 | +.86 |
| | | | $C_4$ | | | | | −.33333 | −.13333 | −.07 | +1.58333 | −.33333 | −.286676 |
| | | | | +3 | | | | +1 | −1.5 | +1.41 | −4.39 | +5.4 | +4.92 |
| | | | | $C_5$ | | | | −.33333 | +.5 | −.47 | +1.46333 | −1.8 | −1.64 |
| | | | | | +3 | | | +1 | −.3 | +1.28 | −2.91 | +2.9 | +4.97 |
| | | | | | $C_6$ | | | −.33333 | +.1 | −.42667 | +.97 | −.96667 | −1.65667 |
| | | | | | | +3 | | −1 | −1.1 | +.49 | +2.75 | −.1 | +4.04 |
| | | | | | | $C_7$ | | +.33333 | +.36667 | −.16333 | −.91667 | +.03333 | −1.34667 |
| | | | | | | | +3 | +1 | +5.7 | | | +3.3 | +13.00 |
| | | | | | | | $C_8$ | −.33333 | −1.9 | | | −1.1 | −4.33333 |
| | | | | | | | | +8 | | | | +2.4 | +12.40 |
| | | | | | | | 1 | −.3333 | −.8333 | +1.3767 | +1.3833 | −.7667 | +1.8267 |
| | | | | | | | 2 | −.3333 | −1.3 | +1.08 | +1.84 | +.9 | +1.1867 |
| | | | | | | | 3 | −.3333 | −.1333 | +.22 | +1.4567 | +.0333 | +2.2433 |
| | | | | | | | 4 | −.3333 | −.1333 | −.07 | +1.5833 | −.3333 | −.2867 |
| | | | | | | | 5 | −.3333 | +.5 | −.47 | +1.4633 | −1.8 | −1.64 |
| | | | | | | | 6 | −.3333 | +.1 | −.4267 | +.97 | −.9667 | −1.6567 |
| | | | | | | | 7 | −.3333 | −.3667 | +.1633 | +.9167 | −.0333 | +1.3467 |
| | | | | | | | 8 | −.3333 | −1.9 | | | −1.1 | −4.3333 |
| | | | | | | | | +5.3336 | −4.0666 | +1.8733 | +9.6133 | −1.6667 | +11.08679 |
| | | | | | | | | $C_9$ | +.76245 | −.35122 | −1.80240 | +.31249 | −2.078698 |
| | | | | | | | | | +94.34 | −46.7380 | −5.7790 | +61.3 | +107.3230 |
| | | | | | | | | 1 | −2.0833 | +3.4417 | +3.4583 | −1.9167 | +4.5667 |
| | | | | | | | | 2 | −5.07 | +4.212 | +7.176 | +3.51 | +4.6280 |
| | | | | | | | | 3 | −.0533 | +.088 | +.5827 | +.0133 | +.8973 |
| | | | | | | | | 4 | −.0533 | −.028 | +.6333 | −.1333 | −.1147 |
| | | | | | | | | 5 | −.75 | +.705 | −2.195 | +2.7 | +2.460 |
| | | | | | | | | 6 | −.03 | +.1280 | −.291 | +.2900 | +.4970 |
| | | | | | | | | 7 | −.4033 | +.1797 | +1.0083 | −.0367 | +1.4813 |
| | | | | | | | | 8 | −10.83 | | | −6.27 | −24.70 |
| | | | | | | | | 9 | −3.1006 | +1.4283 | +7.3296 | −1.2708 | +8.4532 |
| | | | | | | | | | +71.9662 | −36.5833 | +11.9232 | +58.1858 | +105.49169 |
| | | | | | | | | | $C_{10}$ | +.50834 | −.16568 | −.80852 | −1.465856 |
| | | | | | | | | | | +39.7198 | +5.3068 | −12.3050 | −9.0764 |
| | | | | | | | | | 1 | −5.6856 | −5.7132 | +3.1663 | −7.5441 |
| | | | | | | | | | 2 | −3.4992 | −5.9616 | −2.916 | −3.8448 |
| | | | | | | | | | 3 | −.1452 | −.9614 | −.022 | −1.4806 |
| | | | | | | | | | 4 | −.0147 | +.3325 | −.070 | −.0602 |
| | | | | | | | | | 5 | −.6627 | +2.0633 | −2.538 | −2.3124 |
| | | | | | | | | | 6 | −.5461 | +1.2416 | −1.2373 | −2.1205 |
| | | | | | | | | | 7 | −.0800 | −.4492 | +.0163 | −.6599 |
| | | | | | | | | | 9 | −.6579 | −3.3764 | +.5854 | −3.8940 |
| | | | | | | | | | 10 | −18.5968 | +6.0611 | +29.5783 | +53.6260 |
| | | | | | | | | | | +9.8316 | −1.4565 | +14.2580 | +22.6331 |
| | | | | | | | | | | $C_{11}$ | +.14814 | −1.45022 | −2.30208 |
| | | | | | | | | | | | +72.7274 | −8.6859 | +57.2693 |
| | | | | | | | | | | 1 | −5.7408 | +3.1817 | −7.5807 |
| | | | | | | | | | | 2 | −10.1568 | −4.968 | −6.5504 |
| | | | | | | | | | | 3 | −6.3656 | −.1456 | −9.8034 |
| | | | | | | | | | | 4 | −7.5208 | +1.5833 | +1.3616 |
| | | | | | | | | | | 5 | −6.4240 | +7.902 | +7.1996 |
| | | | | | | | | | | 6 | −2.8227 | +2.813 | +4.8209 |
| | | | | | | | | | | 7 | −2.5208 | +.0917 | −3.7033 |
| | | | | | | | | | | 9 | −17.3270 | +3.0041 | −19.9830 |
| | | | | | | | | | | 10 | −1.9754 | −9.6402 | −17.4777 |
| | | | | | | | | | | 11 | −.2158 | +2.1122 | +3.3530 |
| | | | | | | | | | | | +11.6577 | −2.7517 | +8.905960 |
| | | | | | | | | | | | $C_{12}$ | +.23604 | −.76396 |

The back solution, as well as the forward solution, is much shorter than when the direction method is used, as only 4 of the $C$'s are carried back through all the equations.

*Back solution*

| 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| +0.2360 | −1.4502 | −0.8085 | +0.3125 | −1.1 | +0.0333 | −0.9667 | −1.8 | −0.3333 | −0.0333 | +0.9 | +0.7667 |
| | +.0350 | −.0391 | −.4254 | -------- | −.2163 | +.2289 | +.3454 | +.3737 | −.3438 | +.4342 | −.3265 |
| | ——— | −.7194 | +.4971 | -------- | +.2311 | +.6038 | +.6651 | +.0991 | +.3113 | −1.5284 | +1.9483 |
| | −1.4152 | ——— | −1.1948 | +2.9773 | −.5746 | −.1567 | −.7835 | +.2089 | −.2089 | +2.0371 | −1.3058 |
| | | −1.5670 | ——— | +.2702 | −.2702 | +.2702 | +.2702 | +.2702 | −.2702 | +.2702 | −.2702 |
| | | | −.8106 | ——— | ——— | ——— | ——— | ——— | ——— | ——— | ——— |
| | | | | +2.1475 | −.7967 | −.0205 | −1.3028 | +.6186 | −.5449 | +2.1131 | +.8125 |

The $C$'s determined by the back solution are substituted in the correlate equations to determine the $v$'s, which in this case are corrections to the angles and not to directions. The adopted $v$'s are obtained in much the same manner as when the direction method is used and consequently may not be the same as the computed values to the nearest tenth of a second. It is necessary to adopt $v$'s which may differ slightly from the computed values in order to make the triangles consistent. Compare the computed and adopted values for 8a in the following table:

*Computation of corrections ($v$'s)*

| 1a | b | c | 2a | b | c | 3a | b | c | 4a | b | c |
|---|---|---|---|---|---|---|---|---|---|---|---|
| +0.812 | +0.812 | +0.812 | +2.113 | +2.113 | +2.113 | −0.545 | −0.545 | −0.545 | +0.619 | +0.619 | +0.619 |
| −2.037 | +5.955 | +.811 | −7.052 | +.940 | +.811 | −1.724 | +2.350 | +.811 | −1.410 | +.784 | −.811 |
| +.127 | −5.972 | ——— | +5.123 | −.538 | ——— | +.637 | −1.571 | ——— | +.481 | −.778 | ——— |
| +.996 | −.017 | +1.623 | −.609 | −.694 | +1.302 | +.569 | +.463 | +.266 | −.592 | −.529 | −.192 |
| ——— | ——— | +1.6 | ——— | ——— | +1.3 | ——— | ——— | +.3 | ——— | ——— | −.2 |
| −.102 | +.778 | | −.425 | +1.821 | | −1.063 | +.697 | | −.902 | +.096 | |
| −.1 | +.8 | | −.4 | +1.8 | | −1.1 | +.7 | | −.9 | +.1 | |

| 5a | b | c | 6a | b | c | 7a | b | c | 8a | b | c |
|---|---|---|---|---|---|---|---|---|---|---|---|
| −1.303 | −1.303 | −1.303 | −0.020 | −0.020 | −0.020 | −0.797 | −0.797 | −0.797 | +2.148 | +2.148 | +2.148 |
| −1.724 | +4.074 | −.811 | −1.724 | +2.194 | −.811 | −2.821 | +4.544 | +.811 | −8.618 | −.313 | −.811 |
| +.665 | −2.661 | ——— | −.170 | −1.642 | ——— | +.906 | −1.599 | ——— | ——— | ——— | ——— |
| −.602 | −.434 | −2.114 | −.486 | −.201 | −.831 | +.342 | +.307 | +.014 | −6.470 | +1.835 | +1.337 |
| ——— | ——— | −2.1 | ——— | ——— | −.8 | ——— | ——— | .0 | −6.4 | +1.8 | +1.3 |
| −2.964 | −.324 | | −2.400 | +.331 | | −2.370 | +2.455 | | | | |
| −3.0 | −.3 | | −2.4 | +.3 | | −2.4 | +2.5 | | | | |

## COMPUTATION OF TRIANGLES

The adopted $v$'s or corrections are now substituted in the column headed "Corrections" in the triangles in Figure 34, and the triangles are computed in the manner already explained. If the computed length of the fixed line at the end of the scheme agrees with the fixed length, the length equation is satisfied.

Next the corrected angles are substituted for the observed angles in the formation of the azimuth equation (p. 125) to see if that equation is satisfied.

After making sure that the angle, length, and azimuth equations are all satisfied, it is necessary to recompute the geographic positions of the stations using the data of the corrected triangles. (See fig. 38.) This recomputation can usually be made quite easily and quickly by simply correcting the preliminary positions as explained on page 95. It is only after obtaining the corrected position of Enter and comparing it with the fixed position that one can be certain that the adjustment is correct, that is, that the latitude and longitude as well as the other equations have been satisfied.

## CORRECTIONS TO DIRECTIONS

The angle, length, azimuth, latitude, and longitude equations are now satisfied and the corrections can be applied to the directions in Figure 33. Since the corrections determined in the angle method of adjustment are corrections to angles and not to directions, the manner of applying them is somewhat more complicated.

The sample below with the explanation following it shows in detail how the corrections are applied at station Nan. The list of directions before adjustment is given in the first two columns.

*Station Nan*

| Station | Observed direction | | | Preliminary seconds | Final seconds |
|---|---|---|---|---|---|
| | ° | ′ | ″ | ″ | ″ |
| Alberta | 0 | 00 | 00.0 | 00.0 | 59.0 |
| Fish | 28 | 42 | 47.6 | 48.4 | 47.4 |
| Fog | 109 | 26 | 24.4 | 26.5 | 25.5 |
| Dug | 129 | 52 | 16.6 | | |
| Gura | 171 | 17 | 11.1 | 12.1 | 11.1 |

The corrected directions which go in the column headed "Final seconds" are determined as follows: From the adjusted triangles, Figure 34, using the angles at Nan, we have

| | ° | ′ | ″ |
|---|---|---|---|
| Triangle 1, angle Alberta to Fish | 28 | 42 | 48.4(*A*) |
| Triangle 2, angle Fish to Fog | 80 | 43 | 38.1 |
| Angle Alberta to Fog | 109 | 26 | 26.5(*B*) |
| Triangle 3, angle Fog to Gura | 61 | 50 | 45.6 |
| Angle Alberta to Gura | 171 | 17 | 12.1(*C*) |

The values of the seconds for the angles (*A*), (*B*), and (*C*) are placed in the column headed "Preliminary seconds" in the table above. Opposite the initial station, Alberta, is placed 00.0. As the direction "Dug" was not included in the adjustment, there is no correction to be applied to it.

In this method of applying the corrections, the direction at Alberta remains unchanged and so does not receive its share of the correction.

(Text continued on p. 138)

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| α | 2 | Alberta | to 3 | Fish | | 356 | 02 | 14.8 |
| 2d ∠ | | Fish | & | Nan | | + 57 | 22 | 26.4 |
| α | 2 | Alberta | to 1 | Nan | | 53 | 24 | 41.2 |
| Δα | | | | | | − | 6 | 05.8 |
| | | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Nan | to 2 | Alberta | | 233 | 18 | 35.4 −1 |
| | | | | | | 28 | 42 | 48.4 |

FIRST ANGLE OF TRIANGLE

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 31 | 40.715 | 2 | Alberta | λ | 133 | 11 | 33.444 |
| Δφ | − | 3 | 07.392 | ε− | | Δλ | + | 7 | 23.394 |
| φ′ | 55 | 28 | 33.323 | 1 | Nan | λ′ | 133 | 18 | 57.338 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 30 ′ 07.0 ″ | ε | 3.987264 | ε² | 7.97453 | | | |
| | | Cos α | 9.775293 | Sin² α | 9.80936 | h² | 4.544 | |
| | | B | 8.509672 | C | 1.56626 | D | 2.363 | |
| 1st term | + 187.1669 | h | 2.272229 | | 9.35015 | | 6.907 | |
| 2d and 3d terms | + 0.2247 | | | | 0.2239 | | 0.0008 | |
| −Δφ | + 187.3916 | | | | | | | |

| | | | |
|---|---|---|---|
| ε | 3.987264 | | |
| Sin α | 9.904681 | | |
| A′ | 8.508727 | Δλ | 2.647279 |
| sec φ′ | 0.246607 | Sin ½ (φ+φ′) | 9.916004 |
| | 2.647279 | | 2.563283 |
| Δλ | + 443.8937 | −Δα | + 365.83 |

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U. S. COAST AND GEODETIC SURVEY
FORM 27

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| α | 3 | Fish | to 2 | Alberta | | 176 | 02 | 29.9 |
| 3d ∠ | | Nan | & | Alberta | | − 93 | 54 | 45.3 |
| α | 3 | Fish | to 1 | Nan | | 82 | 07 | 44.6 |
| Δα | | | | | | − | 6 | 20.9 |
| | | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Nan | to 3 | Fish | | 262 | 01 | 23.7 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 29 | 09.866 | 3 | Fish | λ | 133 | 11 | 15.042 |
| Δφ | − | | 36.543 | ε− | | Δλ | + | 7 | 42.296 |
| φ′ | 55 | 28 | 33.323 | 1 | Nan | λ′ | 133 | 18 | 57.338 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 28 ′ 51.6 ″ | ε | 3.913697 | ε² | 7.82739 | | | |
| | | Cos α | 9.136537 | Sin² α | 9.99178 | h² | 3.120 | |
| | | B | 8.509676 | C | 1.56558 | D | 2.363 | |
| 1st term | + 36.3003 | h | 1.559910 | | 9.38475 | | 5.483 | |
| 2d and 3d terms | + 0.2425 | | | | 0.2425 | | 0.0000 | |
| −Δφ | + 36.5428 | | | | | | | |

| | | | |
|---|---|---|---|
| ε | 3.913697 | | |
| Sin α | 9.995889 | | |
| A′ | 8.508727 | Δλ | 2.664919 |
| sec φ′ | 0.246607 | Sin ½ (φ+φ′) | 9.915895 |
| | 2.664920 | | 2.580814 |
| Δλ | + 462.2959 | −Δα | + 380.90 |

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FIG. 38.—Final position computation, angle method of adjustment

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| α | 2 | Nan | to 3 | Fish | | 262 | 01 | 23.7 |
| 2d ∠ | | Fish | & | Fog | | + 80 | 43 | 38.1 |
| α | 2 | Nan | to 1 | Fog | | 342 | 45 | 01.8 |
| Δα | | | | | | + | | 49.8 |
| | | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Fog | to 2 | Nan | | 162 | 45 | 51.6 |
| | | | | | | 74 | 22 | 19.5 |

First Angle of Triangle

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 28 | 33.323 | 2 | Nan | λ | 133 | 18 | 57.338 |
| Δφ | − | 1 | 50.687 | s = | | Δλ | − | 1 | 00.458 |
| φ′ | 55 | 26 | 42.636 | 1 | Fog | λ′ | 133 | 17 | 56.880 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 27 ′ 38.0 ″ | s | 3.554387 | s² | 7.10877 | | | |
| | | Cos α | 9.930014 | Sin² α | 8.94415 | h² | 4.088 | |
| | | B | 8.509676 | C | 1.56544 | D | 2.363 | |
| 1st term | +110.6820 | h | 2.044077 | | 7.61936 | | 6.451 | |
| 2d and 3d terms | + 0.0045 | | | | 0.0042 | | 0.0003 | |
| −Δφ | +110.6865 | | | | | | | |

| | | | |
|---|---|---|---|
| s | 3.554387 | | |
| Sin α | 9.472073 | | |
| A′ | 8.508727 | Δλ | 1.731454 |
| sec φ′ | 0.246268 | Sin ½ (φ+φ′) | 9.915788 |
| | 1.781455 | | 1.697242 |
| Δλ | -60.4582 | −Δα | -49.80 |

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Form 27

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| α | 3 | Fish | to 2 | Nan | | 82 | 07 | 44.6 |
| 3d ∠ | | Fog | & | Nan | | − 24 | 54 | 02.5 |
| α | 3 | Fish | to 1 | Fog | | 57 | 13 | 42.1 |
| Δα | | | | | | − | 5 | 31.0 |
| | | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Fog | to 3 | Fish | | 237 | 08 | 11.1 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 29 | 09.866 | 3 | Fish | λ | 133 | 11 | 15.042 |
| Δφ | − | 2 | 27.230 | s = | | Δλ | + | 6 | 41.837 |
| φ′ | 55 | 26 | 42.636 | 1 | Fog | λ′ | 133 | 17 | 56.879 +1 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 27 ′ 56.3 ″ | s | 3.924344 | s² | 7.84869 | | | |
| | | Cos α | 9.733432 | Sin² α | 9.84942 | h² | 4.335 | |
| | | B | 8.509676 | C | 1.56558 | D | 2.363 | |
| 1st term | +147.0456 | h | 2.167452 | | 9.26369 | | 6.698 | |
| 2d and 3d terms | + 0.1840 | | | | 0.1835 | | 0.0005 | |
| −Δφ | +147.2296 | | | | | | | |

| | | | |
|---|---|---|---|
| s | 3.924344 | | |
| Sin α | 9.924711 | | |
| A′ | 8.508727 | Δλ | 2.604049 |
| sec φ′ | 0.246268 | Sin ½ (φ+φ′) | 9.915815 |
| | 2.604050 | | 2.519864 |
| Δλ | +401.8370 | −Δα | + 331.03 |

FIG. 38.—Final position computation, angle method of adjustment—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 21

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| α | 2 | Nan | to 3 | Fog | | 342 | 45 | 01.3 |
| 2ª∠ | | Fog | & | Gura | | + 61 | 50 | 45.6 |
| α | 2 | Nan | to 1 | Gura | | 44 | 35 | 47.4 |
| Δα | | | | | | - | 2 | 09.6 |
| | | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Gura | to 2 | Nan | | 224 | 33 | 37.3 |
| | | | | | | 54 | 33 | 59.5 |

First Angle of Triangle

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 28 | 33.323 | 2 | Nan | λ | 133 | 18 | 57.338 |
| Δφ | - | 1 | 30.740 | s= | | Δλ | + | 2 | 37.371 |
| φ′ | 55 | 27 | 02.583 | 1 | Gura | λ′ | 133 | 21 | 34.709 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 27 ′ 48.0 ″ | s | 3.595463 | s² | 7.19092 | | | |
| | | Cos α | 9.852522 | Sin² α | 9.69281 | h² | 3.915 | |
| | | B | 8.509676 | C | 1.56544 | D | 2.363 | |
| 1st term | + 90.7112 | h | 1.957661 | | 8.44917 | | 6.278 | |
| 2d and 3d terms | + 0.0283 | | | | 0.0281 | | 0.0002 | |
| −Δφ | + 90.7395 | | | | | | | |

| | | | | |
|---|---|---|---|---|
| s | 3.595463 | | | |
| Sin α | 9.846405 | | | |
| A′ | 8.508727 | Δλ | 2.196923 | |
| sec φ′ | 0.246329 | Sin ½ (φ+φ′) | 9.915803 | |
| | 2.196924 | | 2.112726 | |
| Δλ | + 157.3708″ | −Δα | + 129.64″ | |

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POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| α | 3 | Fog | to 2 | Nan | | 162 | 45 | 51.6 |
| 3ª∠ | | Gura | & | Nan | | - 63 | 35 | 14.9 |
| α | 3 | Fog | to 1 | Gura | | 99 | 10 | 36.7 |
| Δα | | | | | | - | 2 | 59.4 |
| | | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Gura | to 3 | Fog | | 279 | 07 | 37.3 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 26 | 42.636 | 3 | Fog | λ | 133 | 17 | 56.880 |
| Δφ | + | | 19.947 | s= | | Δλ | + | 3 | 37.829 |
| φ′ | 55 | 27 | 02.583 | 1 | Gura | λ′ | 133 | 21 | 34.709 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 26 ′ 52.6 ″ | s | 3.538654 | s² | 7.17731 | | | |
| | | Cos α | 9.202713 | Sin² α | 9.98881 | h² | 2.602 | |
| | | B | 8.509678 | C | 1.56492 | D | 2.364 | |
| 1st term | -20.0007 | h | 1.301045 | | 8.73104 | | 4.966 | |
| 2d and 3d terms | + 0.0538 | | | | 0.0538 | | 0.0000 | |
| −Δφ | -19.9469 | | | | | | | |

| | | | | |
|---|---|---|---|---|
| s | 3.538654 | | | |
| Sin α | 9.994405 | | | |
| A′ | 8.508727 | Δλ | 2.338114 | |
| sec φ′ | 0.246329 | Sin ½ (φ+φ′) | 9.915722 | |
| | 2.338115 | | 2.253836 | |
| Δλ | + 217.8286″ | −Δα | + 179.41″ | |

FIG. 38.—Final position computation, angle method of adjustment—Continued

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| α | 2 | Gura | to 3 | Fog | | 279 | 07 | 37.3 |
| 3rd ∠ | | Fog | & | Dug | | + 38 | 29 | 01.7 |
| α | 2 | Gura | to 1 | Dug | | 317 | 36 | 39.0 |
| Δα | | | | | | + | 1 | 55.3 |
| | | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Dug | to 2 | Gura | | 137 | 38 | 34.3 |
| | | | | | | 75 | 41 | 13.9 |

FIRST ANGLE OF TRIANGLE

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 27 | 02.583 | 2 | Gura | λ | 133 | 21 | 34.709 |
| Δφ | − | 1 | 27.234 | s = | | Δλ | − | 2 | 19.976 |
| φ′ | 55 | 25 | 35.349 | 1 | Dug | λ′ | 133 | 19 | 14.733 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 26 ′ 19.0 ″ | | s Cos α | 3.562499 | s² Sin² α | 7.12500 | h² | 3.881 |
| | | | B | 9.863399 | C | 9.65753 | D | 2.364 |
| 1st term | +37.2120 | | h | 1.940576 | | 8.34754 | | 6.245 |
| 2d and 3d terms | +0.0225 | | | | | 0.0223 | | 0.0002 |
| −Δφ | +37.2345 | | | | | | | |

| | | | |
|---|---|---|---|
| s | 3.562499 | | |
| Sin α | 9.828765 | | |
| A′ | 8.508728 | Δλ | 2.146053 |
| sec φ′ | 0.246062 | Sin ½ (φ+φ′) | 9.915674 |
| | 2.146054 | | 2.061727 |
| Δλ | -139.9761 ″ | −Δα | -115.27 ″ |

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DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 57

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| α | 3 | Fog | to 2 | Gura | | 99 | 10 | 36.7 |
| 3rd ∠ | | Dug | & | Gura | | − 65 | 49 | 44.4 |
| α | 3 | Fog | to 1 | Dug | | 33 | 20 | 52.3 |
| Δα | | | | | | − | 1 | 04.1 |
| | | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Dug | to 3 | Fog | | 213 | 19 | 48.2 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 26 | 42.636 | 3 | Fog | λ | 133 | 17 | 56.880 |
| Δφ | − | 1 | 07.287 | s = | | Δλ | + | 1 | 17.853 |
| φ′ | 55 | 25 | 35.349 | 1 | Dug | λ′ | 133 | 19 | 14.733 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 26 ′ 09.0 ″ | | s Cos α | 3.396343 | s² Sin² α | 6.79268 | h² | 3.656 |
| | | | B | 9.921868 | C | 9.43028 | D | 2.364 |
| 1st term | +67.2805 | | h | 1.827389 | | 7.83788 | | 6.020 |
| 2d and 3d terms | +0.0070 | | | | | 0.0069 | | 0.0001 |
| −Δφ | +67.2875 | | | | | | | |

| | | | |
|---|---|---|---|
| s | 3.396343 | | |
| Sin α | 9.740142 | | |
| A′ | 8.508723 | Δλ | 1.891274 |
| sec φ′ | 0.246062 | Sin ½ (φ+φ′) | 9.915659 |
| | 1.891275 | | 1.806933 |
| Δλ | +77.8529 ″ | −Δα | +64.11 ″ |

DO NOT WRITE IN THIS MARGIN

FIG. 38.—Final position computation, angle method of adjustment—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 21

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| α | 2 | Gura | to 3 | Dug | | 317 | 36 | 39.0 |
| 2d ∠ | | Dug | & | Lat | | + 79 | 27 | 42.9 |
| α | 2 | Gura | to 1 | Lat | | 37 | 04 | 21.9 |
| Δα | | | | | | − | 2 | 24.1 |
| | | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Lat | to 2 | Gura | | 217 | 01 | 57.8 |
| | | | | | First Angle of Triangle | 38 | 57 | 17.5 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 27 | 02.583 | 2 | Gura | λ | 133 | 21 | 34.709 |
| Δφ | − | 2 | 11.831 | s = | | Δλ | + | 2 | 55.040 |
| φ′ | 55 | 24 | 50.752 | 1 | Lat | λ′ | 133 | 24 | 29.749 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 25 ′ 56.7 ″ | s | 3.708290 | s² | 7.41658 | | | |
| | | Cos α | 9.901933 | Sin² α | 9.56039 | h′ | 4.240 | |
| | | B | 8.509678 | C | 1.56501 | D | 2.364 | |
| 1st term | +131.7956 | h | 2.119901 | | 8.54198 | | 6.604 | |
| 2d and 3d terms | + 0.0352 | | | | 0.0348 | | 0.0004 | |
| −Δφ | +131.8308 | | | | | | | |

| | | | |
|---|---|---|---|
| s | 3.708290 | | |
| Sin α | 9.780194 | | |
| A′ | 8.508728 | Δλ | 2.243138 |
| sec φ′ | 0.245926 | Sin ½ (φ+φ′) | 9.915641 |
| | 2.243138 | | 2.158779 |
| Δλ | +175.0403 ″ | −Δα | +144.14 ″ |

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POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| α | 3 | Dug | to 2 | Gura | | 137 | 38 | 34.3 |
| 3d ∠ | | Lat | & | Gura | | − 61 | 34 | 59.7 |
| α | 3 | Dug | to 1 | Lat | | 76 | 03 | 34.6 |
| Δα | | | | | | − | 4 | 19.4 |
| | | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Lat | to 3 | Dug | | 255 | 59 | 15.2 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 25 | 35.349 | 3 | Dug | λ | 133 | 19 | 14.733 |
| Δφ | − | | 44.597 | s = | | Δλ | + | 5 | 15.016 |
| φ′ | 55 | 24 | 50.752 | 1 | Lat | λ′ | 133 | 24 | 29.749 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 25 ′ 13.1 ″ | s | 3.756663 | s² | 7.51332 | | | |
| | | Cos α | 9.281859 | Sin² α | 9.97403 | h′ | 3.296 | |
| | | B | 8.509680 | C | 1.56463 | D | 2.364 | |
| 1st term | +44.4838 | h | 1.643202 | | 9.05198 | | 5.660 | |
| 2d and 3d terms | +0.1127 | | | | 0.1127 | | 0.0000 | |
| −Δφ | +44.5965 | | | | | | | |

| | | | |
|---|---|---|---|
| s | 3.756663 | | |
| Sin α | 9.987016 | | |
| A′ | 8.508728 | Δλ | 2.498332 |
| sec φ′ | 0.245926 | Sin ½ (φ+φ′) | 9.915578 |
| | 2.498333 | | 2.413910 |
| Δλ | + 315.0163 ″ | −Δα | + 259.36 ″ |

FIG. 38.—Final position computation, angle method of adjustment—Continued

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| a | 2 | Lat | to 3 | Gura | | 217 | 01 | 57.8 |
| 2d ∠ | | Gura | & | Mond | | + 60 | 10 | 36.6 |
| a | 2 | Lat | to 1 | Mond | | 277 | 12 | 24.4 |
| Δa | | | | | | + | 4 | 14.3 |
| | | | | | | 180 | 00 | 00.00 |
| a′ | 1 | Mond | to 2 | Lat | | 97 | 16 | 38.7 |
| | | | | | | 56 | 27 | 36.3 |

FIRST ANGLE OF TRIANGLE

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 24 | 50.752 | 2 | Lat | λ | 133 | 24 | 29.749 |
| Δφ | - | | 22.332 | s = | | Δλ | - | 5 | 08.893 |
| φ′ | 55 | 24 | 28.420 | 1 | Mond | λ′ | 133 | 19 | 20.856 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 24 ′ 39.6 ″ | | s | 3.738667 | s² | 7.47733 | | | |
| | | | Cos a | 9.098473 | Sin² a | 9.99311 | h² | 2.694 | |
| | | | B | 8.509681 | C | 1.56443 | D | 2.364 | |
| 1st term | + 22.2239 | | h | 1.346821 | | 9.03487 | | 5.058 | |
| 2d and 3d terms | + 0.1084 | | | | | 0.1084 | | 0.0000 | |
| −Δφ | + 22.3323 | | | | | | | | |

| | | | |
|---|---|---|---|
| s | 3.738667 | | |
| Sin a | 9.996555 | | |
| A′ | 8.508728 | Δλ | 2.439808 |
| sec φ′ | 0.245858 | Sin ½ (φ+φ′) | 9.915529 |
| | 2.439808 | | 2.405337 |
| Δλ | -308.3930″ | −Δa | -254.29″ |

DO NOT WRITE IN THIS MARGIN

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 27

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| a | 3 | Gura | to 2 | Lat | | 37 | 04 | 21.9 |
| 3d ∠ | | Mond | & | Lat | | − 63 | 21 | 52.1 |
| a | 3 | Gura | to 1 | Mond | | 333 | 42 | 24.8 |
| Δa | | | | | | + | 1 | 50.2 |
| | | | | | | 180 | 00 | 00.00 |
| a′ | 1 | Mond | to 3 | Gura | | 153 | 44 | 15.0 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 27 | 02.583 | 3 | Gura | λ | 133 | 21 | 34.709 |
| Δφ | - | 2 | 34.163 | s = | | Δλ | - | 2 | 13.853 |
| φ′ | 55 | 24 | 28.420 | 1 | Mond | λ′ | 133 | 19 | 20.856 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 25 ′ 45.6 ″ | | s | 3.725674 | s² | 7.45135 | | | |
| | | | Cos a | 9.952569 | Sin² a | 9.29274 | h² | 4.376 | |
| | | | B | 8.509678 | C | 1.56501 | D | 2.364 | |
| 1st term | +154.1420 | | h | 2.137921 | | 8.30910 | | 6.740 | |
| 2d and 3d terms | + 0.0209 | | | | | 0.0204 | | 0.0005 | |
| −Δφ | +154.1629 | | | | | | | | |

| | | | |
|---|---|---|---|
| s | 3.725674 | | |
| Sin a | 9.646368 | | |
| A′ | 8.508728 | Δλ | 2.126628 |
| sec φ′ | 0.245858 | Sin ½ (φ+φ′) | 9.915625 |
| | 2.126628 | | 2.042253 |
| Δλ | -133.8530″ | −Δa | -110.22″ |

DO NOT WRITE IN THIS MARGIN

FIG. 38.—Final position computation, angle method of adjustment—Continued

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| α | 2 | Lat | to 3 | Mond | | 277 | 12 | 24.4 |
| $2^{\alpha}\angle$ | | Mond | & | Enter | | + 49 | 08 | 01.0 |
| α | 2 | Lat | to 1 | Enter | | 326 | 20 | 25.4 |
| Δα | | | | | | + | 3 | 58.4 |
| | | | | | | 180 | 00 | 00.00 |
| α′ | 1 | Enter | to 2 | Lat | | 146 | 24 | 23.8 |
| | | | | | | 36 | 21 | 49.3 |

First Angle of Triangle

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 24 | 50.752 | 2 | Lat | λ | 133 | 24 | 29.749 |
| Δφ | − | 4 | 08.022 | s− | | Δλ | − | 4 | 49.700 |
| φ′ | 55 | 20 | 42.730 | 1 | Enter | λ′ | 133 | 19 | 40.049 |
| Fixed | 55 | 20 | 42.730 | | | | 133 | 19 | 40.049 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 22 ′ 46.7 ″ | s | 3.964337 | s² | 7.92867 | | | |
| | | Cos α | 9.920303 | Sin² α | 9.48742 | h′ | 4.789 | |
| | | B | 8.509681 | C | 1.56443 | D | 2.364 | |
| 1st term | +247.9254 | h | 2.394321 | | 8.98052 | | 7.153 | |
| 2d and 3d terms | + 0.0970 | | | | 0.0956 | | 0.0014 | |
| −Δφ | +248.0224 | | | | | | | |

| | | | |
|---|---|---|---|
| s | 3.964337 | | |
| Sin α | 9.743712 | | |
| A′ | 8.508730 | Δλ | 2.461949 |
| sec φ′ | 0.245170 | Sin ½ (φ+φ′) | 9.915365 |
| | 2.461949 | | 2.377314 |
| Δλ | −289.7003 | −Δα | −238.40 |

DO NOT WRITE IN THIS MARGIN

Department of Commerce
U. S. Coast and Geodetic Survey
Form 57

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

| | | | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| α | 3 | Mond | to 2 | Lat | | 97 | 16 | 38.7 |
| $3^{\alpha}\angle$ | | Enter | & | Lat | | − 94 | 30 | 09.8 |
| α | 3 | Mond | to 1 | Enter | | 2 | 46 | 28.9 |
| Δα | | | | | | − | | 15.8 |
| | | Enter | | Mond | | 180 | 00 | 00.00 |
| α′ | 1 | | to 3 | | | 182 | 46 | 13.1 |

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| φ | 55 | 24 | 28.420 | 3 | Mond | λ | 133 | 19 | 20.856 |
| Δφ | − | 3 | 45.690 | s− | | Δλ | + | | 19.194 |
| φ′ | 55 | 20 | 42.730 | 1 | Enter | λ′ | 133 | 19 | 40.050 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ½(φ+φ′) | 55 ° 22 ′ 35.6 ″ | s | 3.844337 | s² | 7.68867 | | | |
| | | Cos α | 9.999491 | Sin² α | 7.36985 | h′ | 4.707 | |
| | | B | 8.509681 | C | 1.56423 | D | 2.364 | |
| 1st term | + 225.6883 | h | 2.353509 | | 6.62275 | | 7.071 | |
| 2d and 3d terms | + 0.0016 | | | | 0.0004 | | 0.0012 | |
| −Δφ | + 225.6899 | | | | | | | |

| | | | |
|---|---|---|---|
| s | 3.844337 | | |
| Sin α | 8.684923 | | |
| A′ | 8.508730 | Δλ | 1.283160 |
| sec φ′ | 0.245170 | Sin ½ (φ+φ′) | 9.915349 |
| | 1.283160 | | 1.198509 |
| Δλ | + 19.1938 | −Δα | + 15.79 |

DO NOT WRITE IN THIS MARGIN

FIG. 38.—Final position computation, angle method of adjustment—Continued

A mean correction is therefore computed by taking the differences between the directions in the column headed "Observed directions" and those in the column headed "Preliminary seconds."

| | |
|---|---|
| At Alberta, the correction is | 0. 0 |
| At Fish, the correction is | +0. 8 |
| At Fog, the correction is | +2. 1 |
| At Gura, the correction is | +1. 0 |
| Total | +3. 9 |
| Average correction | +1. 0 |

The average correction +1.″0, with sign changed, is then applied to each of the directions in the "Preliminary seconds" column to obtain the values in the "Final seconds" column.

The rule to be followed, then, is: Keeping the seconds of the initial station 00.0, compute the seconds of the other directions by adding the adjusted spherical angles of the triangles in the proper order. Place these values in a column headed "Preliminary seconds." Take the algebraic sum of the differences between these values and the values in the preceding column and divide this sum by the number of directions involved. Add algebraically with opposite sign the mean thus obtained to each of the values in the column "Preliminary seconds," and place the resulting values in the "Final seconds" column.

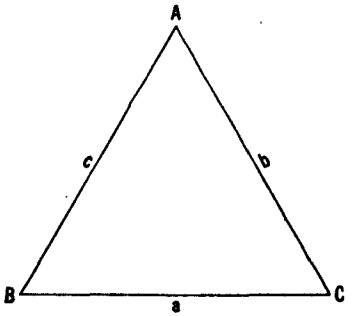


FIG. 39.—Triangle as lettered for computation using two sides and included angle

In order to see whether the corrections have been applied properly, check these final seconds from the triangles. For example, the angle at Nan between Alberta and Fish as taken from the "Final seconds" column is 28° 42′ 48.″4, which checks the angle at Nan in triangle 1. Similarly, the angle at Nan between Fish and Fog is 80° 43′ 38.″1, which checks the angle at Nan in triangle 2, and the angle at Nan between Fog and Gura is 61° 50′ 45.″6, which checks the angle at Nan in triangle 3.

## CORRECTIONS TO DIRECTIONS OMITTED IN ADJUSTMENT

After the final seconds have been determined for all directions included in the adjustment, the corrections to the directions for the diagonals omitted from the adjustment are computed. The three

diagonals omitted were Alberta-Fog, Nan-Dug, and Lat-Flores. These are computed by the two-sides-and-included-angle method. (See pp. 141–143.)

### TRIANGLE COMPUTATION BY TWO-SIDES-AND-INCLUDED-ANGLE METHOD

The diagonal Alberta-Fog is computed as follows. The formulas used are

$$\tan(45^\circ+\phi)=\frac{a}{b}, \text{ where } a>b$$

$$\tan 1/2\ (A-B)=\tan\phi\ \tan\left(90^\circ-\frac{C}{2}\right)$$

$$1/2\ (A+B)=90^\circ-\frac{C}{2}$$

in which $a$ and $b$, the two sides, and $C$, the included angle, of the triangle are known, and $\phi$ is an auxiliary angle.

The logarithms of the sides Fish-Alberta and Fish-Fog (see fig. 31) are given in the triangles, Figure 34, as 3.669907 and 3.924344, respectively. The angle at Fish between Fog and Alberta is the sum of the two adjusted angles between Fog and Nan, and between Nan and Alberta, namely, 24° 54′ 02″5 plus 93° 54′ 45″3, or 118° 48′ 47″8.

The triangle, Alberta-Fish-Fog, is written as shown in Figure 40. The asterisk is placed opposite the fixed angle and the logarithms of the fixed lengths are placed in the last column. As there is 0″1 spherical excess, this is applied to the large angle making the plane angle at Fish 118° 48′ 47″8 − 0″1, or 118° 48′ 47″7.

The problem then is to determine $A$, the angle at Alberta; $B$, the angle at Fog; and $c$ the line Fog-Alberta. The computation is shown in Figure 40.

These angles $A$ and $B$ as thus determined are placed in the triangle Alberta-Fish-Fog on the lower part of the form (fig. 40) and the triangle is computed in the usual way. The logarithm of the length, Alberta-Fog, is thus obtained. The computed logarithm of the fixed length, Alberta-Fish, should check the given value probably within one in the last place.

The value of the final seconds for the direction on Fog at station Alberta is now obtained from the triangle by simply adding the angle at Alberta between Fish and Fog, 40° 09′ 09″1, to the fixed direction Fish, 0° 00′ 00″0. At station Fog, the final seconds for the direction on Alberta are obtained by subtracting the angle between Alberta and Fish from the fixed direction on Fish; that is, 203° 47′ 18″9 −21° 02′ 03″2 = 182° 45′ 15″7.

In order that the length, Fog-Alberta, as determined from the two sides and included angle computation, may have a check the triangle Fog-Nan-Alberta (see fig. 40) is also computed. The three angles of this triangle are fixed by the computation of the first triangle and can be taken out directly from the lists of directions at the three stations. The logarithm of the length, Alberta-Fog, should agree in the two triangles.

In the same manner the diagonals Nan-Dug and Lat-Flores are computed, using first a triangle with two sides and the included angle known and then a second triangle, whose angles are fixed by the computation of the first triangle, to check the length of the line in question.

### COMPUTATION OF THE LENGTH AND AZIMUTH OF THE OMITTED DIAGONAL BY THE METHOD OF APPROXIMATION

If the diagonals omitted in the adjustment have been observed it may be advisable or expedient at times to resort to the method of approximation in computing them after the adjustment has been completed.

An example of this method is given on page 144. (See fig. 41.) In the quadrilateral Lat-Mond-Flores-Enter (see fig. 31) the diagonal Lat-Flores was omitted in the adjustment. It may be computed from the triangle Lat-Mond-Flores in the following manner: The logarithms of the lengths Mond-Flores and Lat-Mond and the angle at Mond between Flores and Lat have been fixed by the adjustment. The observed angles at Lat and Flores are obtained from the lists of directions. (See fig. 33.) The sum of the two observed angles at Lat and Flores and the adjusted angle at Mond equals 179° 59′ 52″.0, so in order to close the triangle 4″.0 must be added to each of the angles at Lat and Flores.

Using the fixed logarithm of the length Mond-Flores, 3.799236, the logarithm of the length Lat-Mond as computed to seven decimal places through the approximate angles at Lat and Flores is 3.7386565. This value differs from the fixed logarithm of Lat-Mond, 3.738667, by 10.5 in the sixth place of logarithms. The tabular differences of the logarithmic sines of the angles at Lat and Flores for one second change in angle are +3.03 and +3.68, respectively. The discrepancy in the logarithms of the length divided by the algebraic sum of these two tabular differences, that is, $\frac{10.5}{6.71}=1''.6$, is the correction which must be applied to the angles at Lat and Flores which were used in the preliminary computation.

This correction is applied with the negative sign to the angle at Lat and with the positive sign to the angle at Flores. The triangle is then recomputed, using the corrected angles, and the final logarithm

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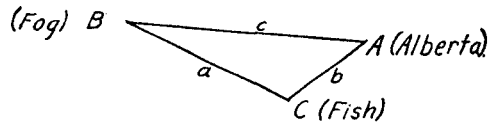
## TRIANGLE COMPUTATION USING TWO SIDES AND INCLUDED ANGLE

$\left[\frac{a}{b} = \tan(45° + \phi)\right.$ (Call longer side $a$) : $\tan \frac{1}{2}(A_p - B_p) = \tan \phi \tan \frac{1}{2}(A_p + B_p)$ : $\left. c = \frac{a \sin C_p}{\sin A_p}\right]$ *

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C_s$ | 118° | 48′ | 47.8″ | Log $a$ | 3.924344 | | | | Log $m$ | 1.403 |
| $\frac{\text{Sph. excess}}{3}$ | | | 0.1 | Log $b$ | 3.669907 | | | | Log sin $C_s$ | 9.943 |
| $C_p$ | 118 | 48 | 47.7 | Log tan $(45+\phi)$ | 0.254437 | | | | Log $a$ | 3.924 |
| $\frac{1}{2} C_p$ | 59 | 24 | 23.85 | $(45° + \phi)$ | 60° | 53′ | 54.88″ | | Log $b$ | 3.670 |
| $90° - \frac{1}{2}C_p = \frac{1}{2}(A_p + B_p)$ | 30 | 35 | 36.15 | $\phi$ | 15 | 53 | 54.88 | | Log sph. ex. | 8.940 |
| $\frac{1}{2}(A_p - B_p)$ | 9 | 33 | 32.92 | Log tan $\phi$ | 9.4545867 | | | | Sph. excess | 0.09 |
| Sum $= A_p$ | 40 | 09 | 09.07 | Log tan $\frac{1}{2}(A_p + B_p)$ | 9.7717656 | | | | | |
| Diff. $= B_p$ | 21 | 02 | 03.23 | Log tan $\frac{1}{2}(A_p - B_p)$ | 9.2263523 | | | | | |
| $C_p$ | | | | | (Sketch) | | | | | |

| | |
|---|---|
| Log $a$ | 3.924344 |
| Log sin $C_p$ | 9.942601 |
| Colog sin $A_p$ | 0.190558 |
| Log $c$ | 4.057503 |




### CHECK COMPUTATION

| NO. | STATION | SPHERICAL ANGLE | SPHERICAL EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|
| 2-3 | Fish-Fog | | | | 3.924344 |
| 1 | Alberta | 40 09 09.1 | | 09.1 | 0.190558 |
| 2 | Fish | 118 48 47.8 | 0.1 | 47.7 | 9.942601 |
| 3 | Fog | 21 02 03.2 | | 03.2 | 9.555004 |
| 1-3 | Alberta-Fog | | | | 4.057503 |
| 1-2 | Alberta-Fish | | | | 3.669907 |
| | | | | | |
| 2-3 | Nan-Alberta | | | | 3.987264 |
| 1 | Fog | 53 20 16.3 | | 16.3 | 0.095733 |
| 2 | Nan | 109 26 26.5 | 0.1 | 26.4 | 9.974506 |
| 3 | Alberta | 17 13 17.3 | | 17.3 | 9.471389 |
| 1-3 | Fog-Alberta | | | | 4.057503 |
| 1-2 | Fog-Nan | | | | 3.554387 |

* The subscripts $s$ and $p$ on this form refer to spherical and plane angles respectively.

FIG. 40.—Triangle computation using two sides and included angle

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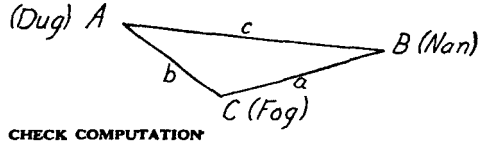
FORM 665

## TRIANGLE COMPUTATION USING TWO SIDES AND INCLUDED ANGLE

$\left[\frac{a}{b} = \tan(45° + \phi)\right.$ (Call longer side $a$): $\tan \frac{1}{2}(A_p - B_p) = \tan \phi \tan \frac{1}{2}(A_p + B_p)$: $\left.c = \frac{a \sin C_p}{\sin A_p}\right]$*

| | ° | ′ | ″ | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $C_s$ | 129 | 24 | 59.3 | Log $a$ | 3.554387 | | | Log $m$ | 1.403 |
| Sph. excess / 3 | | | 0.0 | Log $b$ | 3.396343 | | | Log sin $C_s$ | 9.888 |
| $C_p$ | 129 | 24 | 59.3 | Log tan $(45+\phi)$ | 0.158044 | | | Log $a$ | 3.554 |
| $\frac{1}{2} C_p$ | 64 | 42 | 29.65 | $(45° + \phi)$ | 55 | 12 | 08.93 | Log $b$ | 3.396 |
| $90° - \frac{1}{2}C_p = \frac{1}{2}(A_p + B_p)$ | 25 | 17 | 30.35 | $\phi$ | 10 | 12 | 08.93 | Log sph. ex. | 8.241 |
| $\frac{1}{2}(A_p - B_p)$ | 4 | 51 | 38.90 | Log tan $\phi$ | 9.2552076 | | | Sph. excess | 0.02 |
| Sum $= A_p$ | 30 | 09 | 09.25 | Log tan $\frac{1}{2}(A_p + B_p)$ | 9.6744220 | | | | |
| Diff. $= B_p$ | 20 | 25 | 51.45 | Log tan $\frac{1}{2}(A_p - B_p)$ | 8.9296296 | | | | |
| $C_p$ | | | | | (Sketch) | | | | |

| | |
|---|---|
| Log $a$ | 3.554387 |
| Log sin $C_p$ | 9.887927 |
| Colog sin $A_p$ | 0.299033 |
| Log $c$ | 3.741347 |




### CHECK COMPUTATION

| NO. | STATION | | SPHERICAL ANGLE | | | SPHERICAL EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|---|
| 2-3 | | Fog-Dug | | | | | | 3.396343 |
| 1 | Nan | | 20 | 25 | 51.5 | 0.0 | 51.5 | 0.457077 |
| 2 | Fog | | 129 | 24 | 59.3 | 0.0 | 59.3 | 9.887927 |
| 3 | Dug | | 30 | 09 | 09.2 | 0.0 | 09.2 | 9.700967 |
| 1-3 | | Nan-Dug | | | | | | 3.741347 |
| 1-2 | | Nan-Fog | | | | | | 3.554387 |
| 2-3 | | Gura-Nan | | | | | | 3.595463 |
| 1 | Dug | | 45 | 32 | 04.7 | 0.0 | 04.7 | 0.146500 |
| 2 | Gura | | 93 | 03 | 01.2 | 0.0 | 01.2 | 9.999384 |
| 3 | Nan | | 41 | 24 | 54.1 | 0.0 | 54.1 | 9.820535 |
| 1-3 | | Dug-Nan | | | | | | 3.741347 |
| 1-2 | | Dug-Gura | | | | | | 3.562499 |

* The subscripts $s$ and $p$ on this form refer to spherical and plane angles respectively.

FIG. 40.—Triangle computation using two sides and included angle—Continued

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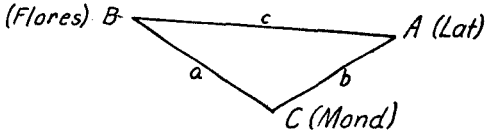
Form 665

## TRIANGLE COMPUTATION USING TWO SIDES AND INCLUDED ANGLE

$\left[\frac{a}{b} = \tan(45° + \phi)\right.$ (Call longer side $a$): $\tan \frac{1}{2}(A_p - B_p) = \tan \phi \tan \frac{1}{2}(A_p + B_p)$: $\left.c = \frac{a \sin C_p}{\sin A_p}\right]$*

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $C_s$ | 115° | 24′ | 55.9″ | Log $a$ | 3.799236 | | | Log $m$ | 1.403 | |
| Sph. excess / 3 | | | 0.1 | Log $b$ | 3.738667 | | | Log sin $C_s$ | 9.956 | |
| $C_p$ | 115 | 24 | 55.8 | Log tan $(45+\phi)$ | 0.060569 | | | Log $a$ | 3.799 | |
| $\frac{1}{2} C_p$ | 57 | 42 | 27.9 | $(45°+\phi)$ | 48° | 58′ | 56.98″ | Log $b$ | 3.739 | |
| $90° - \frac{1}{2}C_p = \frac{1}{2}(A_p + B_p)$ | 32 | 17 | 32.1 | $\phi$ | 3 | 58 | 56.98 | Log sph. ex. | 8.897 | |
| $\frac{1}{2}(A_p - B_p)$ | 2 | 31 | 09.5 | Log tan $\phi$ | 8.8427327 | | | Sph. excess | 0.08″ | |
| Sum $= A_p$ | 34 | 48 | 41.6 | Log tan $\frac{1}{2}(A_p + B_p)$ | 9.8007064 | | | | | |
| Diff. $= B_p$ | 29 | 46 | 22.6 | Log tan $\frac{1}{2}(A_p - B_p)$ | 8.6434391 | | | | | |
| $C_p$ | | | | | (Sketch) | | | | | |

| | |
|---|---|
| Log $a$ | 3.799236 |
| Log sin $C_p$ | 9.955793 |
| Colog sin $A_p$ | 0.243456 |
| Log $c$ | 3.998485 |




### CHECK COMPUTATION

| NO. | STATION | SPHERICAL ANGLE | SPHERICAL EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|
| 2-3 | Mond-Flores | | | | 3.799236 |
| 1 | Lat | 34 48 41.6 | | 41.6 | 0.243456 |
| 2 | Mond | 115 24 55.9 | 0.1 | 55.8 | 9.955793 |
| 3 | Flores | 29 46 22.6 | | 22.6 | 9.695975 |
| 1-3 | Lat-Flores | | 0.1 | | 3.998485 |
| 1-2 | Lat-Mond | | | | 3.738667 |
| | | | | | |
| 2-3 | Enter-Lat | | | | 3.964337 |
| 1 | Flores | 65 28 16.9 | | 16.9 | 0.041076 |
| 2 | Enter | 100 12 23.7 | | 23.7 | 9.993072 |
| 3 | Lat | 14 19 19.4 | | 19.4 | 9.393350 |
| 1-3 | Flores-Lat | | 0.0 | | 3.998485 |
| 1-2 | Flores-Enter | | | | 3.398762 |

* The subscripts $s$ and $p$ on this form refer to spherical and plane angles respectively.

FIG. 40.—Triangle computation using two sides and included angle—Continued

of the length Lat-Flores is thus obtained. The computed logarithm of the length Lat-Mond should now agree with its fixed value.

The rule to be followed in this approximation method may be stated briefly as follows: Write the triangle which includes the omitted diagonal and fill in the data fixed by the previous adjustment; that is, the logarithms of the two sides and the value of the included angle. Put in the observed values of the other two angles as obtained from the list of directions. If the sum of the three angles, after applying the spherical excess, does not equal 180°, apply one half of the triangle closure to each of the observed angles to close the triangle. Then, starting with the logarithm of one fixed length compute the logarithm of the other fixed length through the triangle, taking out the tabular differences with their proper signs for the logarithms of the sines corresponding to a change of 1 second in the two angles yet to be fixed. Next divide the discrepancy between the computed and the fixed

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Form 25

COMPUTATION OF TRIANGLES

*State:* .....

| NO. | STATION | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|
| Tabular dif. | | | | | | | |
| 2-3 | Mond-Flores | | | | | | 3.739236 |
| +3.03 1 | Lat | 34 48 39.2 | | 41.6 | | 41.6 ~~42.0~~ | 6 0.2434510 |
| 2 | Mond | 115 25 02.3 | -6.4 | 55.9 | 0.1 | 55.8 | 9.955793 |
| +3.68 3 | Flores | 29 46 17.0 | | 22.6 | | 22.6 ~~21.0~~ | 75 9.6959~~696~~ |
| +6.71 1-3 | Lat-Flores | | | | | | 3.998485 |
| 1-2 | Lat-Mond | | | | | | ~~565~~ 3.738667 |

FIG. 41.—Triangle computation using two sides and included angle, method of approximation

logarithm of the second fixed length by the algebraic sum of the tabular differences, and apply this quotient with the same sign as a correction to the approximate angle which appears as the third angle in the triangle and with the opposite sign as a correction to the approximate angle which appears as the first angle in the triangle. Finally, recompute the triangle using the corrected angles.

The probable error of an observed angle is determined by the same formula as that used to obtain the probable error of a direction. (See p. 107.) In the table of correlates on page 127, $\Sigma v^2 = 88.84$, and since there are 12 equations,

$$\text{p. e.} = \pm 0.674 \sqrt{\frac{88.84}{12}} = \pm 1''.83$$

After the adjustment has been completed and the omitted diagonals have been computed, the list of geographic positions is made out on form 28B in exactly the same manner as the list on page 108. (See fig. 42.)

DEPARTMENT OF COMMERCE
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Form 30 B

# GEOGRAPHIC POSITIONS

*Accession No. of Computation:* _____

*Locality* Prince of Wales Island *Datum* North American *State* Alaska

| STATION. | | LATITUDE AND LONGITUDE. ° | ′ | ″ | SECONDS IN METERS. | AZIMUTH ° | ′ | ″ | BACK AZIMUTH. ° | ′ | ″ | TO STATION. | DISTANCE. LOGARITHM (METERS). | METERS. | FEET. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nan | | 55 | 28 | 33.323 | 1030.5 | 233 | 18 | 35.3 | 53 | 24 | 41.2 | Alberta | 3.987264 | 9711.0 | 31860 |
| 1907 | d.m. | 133 | 18 | 57.338 | 1007.2 | 262 | 01 | 23.7 | 82 | 07 | 44.6 | Fish | 3.913097 | 8197.8 | 26896 |
| Fog | | 55 | 26 | 42.636 | 1318.6 | 162 | 45 | 51.6 | 342 | 45 | 01.8 | Nan | 3.554387 | 3584.2 | 11759 |
| 1907 | d.m. | 133 | 17 | 56.880 | 1000.0 | 216 | 06 | 07.9 | 36 | 11 | 23.9 | Alberta | 4.057503 | 11415.7 | 37453 |
| | | | | | | 237 | 08 | 11.1 | 57 | 12 | 42.1 | Fish | 3.924344 | 8401.3 | 27563 |
| Oura | | 55 | 27 | 02.583 | 79.9 | 224 | 33 | 37.8 | 44 | 35 | 47.4 | Nan | 3.595463 | 3939.7 | 12925 |
| 1907 | d.m. | 133 | 21 | 34.709 | 610.1 | 279 | 07 | 37.3 | 99 | 10 | 36.7 | Fog | 3.588654 | 3878.4 | 12724 |
| Dug | | 55 | 25 | 35.349 | 1093.2 | 137 | 38 | 34.3 | 317 | 36 | 39.0 | Oura | 3.562499 | 3651.7 | 11981 |
| 1907 | d.m. | 133 | 19 | 14.733 | 259.1 | 183 | 10 | 39.0 | 3 | 10 | 53.3 | Nan | 3.741347 | 5512.5 | 18086 |
| | | | | | | 213 | 19 | 48.2 | 33 | 20 | 52.3 | Fog | 3.396343 | 2490.8 | 8172 |
| Lat | | 55 | 24 | 50.752 | 1569.6 | 217 | 01 | 57.8 | 37 | 04 | 21.9 | Oura | 3.708290 | 5108.5 | 16760 |
| 1907 | d.m. | 133 | 24 | 29.749 | 523.3 | 255 | 59 | 15.2 | 76 | 03 | 34.6 | Dug | 3.756663 | 5710.4 | 18735 |
| | | | | | | 312 | 01 | 06.0 | 132 | 06 | 51.8 | Flores | 3.998485 | 9965.2 | 32694 |
| | | | | | | 326 | 20 | 25.4 | 146 | 24 | 23.8 | Enter | 3.964337 | 9211.6 | 30222 |
| Mond | | 55 | 24 | 28.420 | 878.9 | 341 | 51 | 42.8 | 161 | 53 | 14.4 | Flores | 3.799236 | 6298.5 | 20664 |
| 1907 | d.m. | 133 | 19 | 20.856 | 367.0 | 2 | 46 | 28.9 | 182 | 46 | 13.1 | Enter | 3.844337 | 6987.7 | 22925 |
| | | | | | | 97 | 16 | 38.7 | 277 | 12 | 24.4 | Lat | 3.738667 | 5478.6 | 17974 |
| | | | | | | 153 | 44 | 15.0 | 333 | 42 | 24.8 | Oura | 3.725674 | 5317.1 | 17445 |

FIG. 42.—List of geographic positions, angle method of adjustment

*Statistics showing accuracy of triangulation*

| | |
|---|---|
| Total number of triangles | 8 |
| Number of triangles with plus closures | 3 |
| Number of triangles with minus closures | 5 |
| Number of concluded triangles | 0 |
| Average closure of triangles without regard to sign | 2″.2 |
| Maximum closure of a triangle | 5″.4 |
| Mean error of an angle | ±1″.6 |
| Probable error of an observed angle | ±1″.8 |

## COMPARISON OF DIRECTION AND ANGLE METHODS OF TRIANGULATION ADJUSTMENT

Comparing the direction and angle methods of adjustments it is seen that each has its advantages and disadvantages. The direction method gives a more rigid adjustment by making use of all the directions observed, and when the adjustment is completed the azimuths and lengths of all the lines are immediately available. There is no necessity for two-sides-and-included-angle computations. The disadvantage of the method is that the solution of the normals is more laborious, especially where the scheme of triangulation is much involved. However, for first-order and second-order triangulations, the adjustment should be as rigid as possible, and the direction method should, therefore, always be used.

The angle method of adjustment has the advantage that the number of equations is reduced considerably, there being one less angle equation in each full quadrilateral and no side equations at all. The solution of the normal equations is very simple as all the angle equations are independent of each other, and the azimuth equation does not, ordinarily, involve the same $v$'s as the length equation. The disadvantage of this method is that a number of two-sides-and-included-angle computations must be made after the adjustment is completed to obtain the azimuths and lengths of the lines omitted in the adjustment. However, the advantages of this method far outweigh the disadvantages in the adjustment of third-order triangulation and it should ordinarily be used for this class of triangulation.

# CHAPTER 5.—COMPUTATION AND ADJUSTMENT OF ELEVATIONS FROM ZENITH-DISTANCE OBSERVATIONS

## GENERAL STATEMENT

In connection with the observation of horizontal directions for triangulation it is customary to observe zenith distances, in order that the elevations of the stations may be determined. These elevations are needed for reducing the horizontal directions to sea level and, in some cases, if precise elevations are not available, to reduce the base lines to sea level. As there are usually two or more lines observed to each station, a rigid adjustment of the observed differences of elevation should be made in order to remove the inconsistencies and obtain the best possible elevations from the observations.

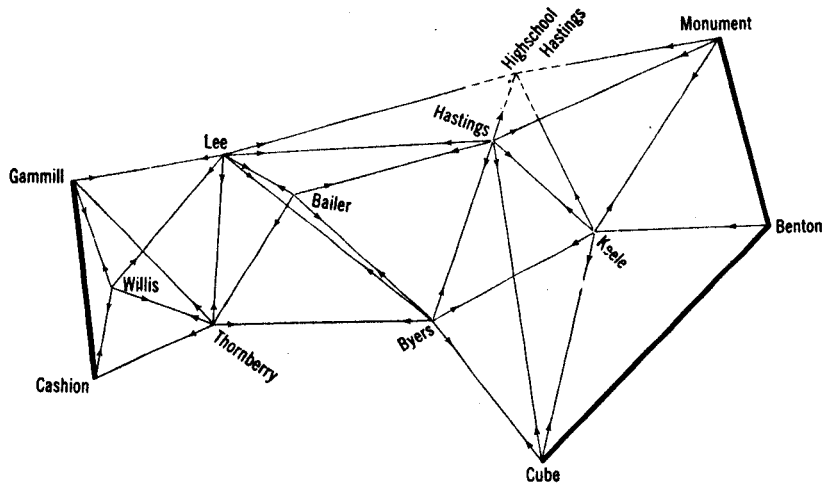


FIG. 43.—Triangulation net used in sample computation of elevations

A sample computation and least-squares adjustment of elevations based on zenith distances is given on the following pages. The triangulation stations at which the zenith distances were measured are shown in Figure 43. Arrowheads pointing away from each station denote the lines over which zenith distances were observed from that station. Arrowheads at both ends of a line indicate reciprocal observations or that zenith distances were observed in both directions over that line. Where only one arrowhead appears on a line, zenith distances were observed only in the direction in which the arrowhead points. All the stations shown in Figure 43, except "Hastings high school," are main scheme stations. The elevation of Hastings high school is computed after the elevations of the other stations are fixed.

All zenith distances are abstracted on Form 29 and checked in the field. The abstracts, Figure 44, therefore, contain the starting

data for the office computation. It is sometimes necessary, however, to compute in the office the values in the column headed "Reduction to line joining stations." This column is used only when the observations are reciprocal. Each value in the column is an angle which in seconds equals $\frac{t-o}{s \sin 1''}$, $t$ being the height of the telescope above the station mark, $o$ the height of the object above its station mark, and $s$ the horizontal distance between the stations concerned. This really represents a vertical eccentric reduction which is applied as a correction to the observed zenith distance to obtain the zenith distance referred to the station marks. Only four places of logarithms should be used in computing the values.

If the observations are nonreciprocal, that is, are made in one direction only over a line, then the vertical eccentric reduction is not needed. In this case, the difference $t—o$, is applied as a correction to the computed difference of elevation, as indicated on Form 29 B (see fig. 47).

As the development of the formulas for the computation of elevations from reciprocal and nonreciprocal observations is fully shown in special publication No. 28, it is not given here.

The formula for the computation of elevations from reciprocal observations is

$$h_2 - h_1 = s \tan\left(\frac{\zeta_2 - \zeta_1}{2}\right) A\ B\ C \qquad (1)$$

in which $h_1$ is the elevation above mean sea level of station 1 and $h_2$ that of station 2; $s$ is the horizontal sea level distance between the two stations; $\zeta_1$ is the zenith distance of station 2 as observed from station 1; $\zeta_2$ is the zenith distance of station 1 from station 2; and $A$, $B$, and $C$ are correction factors whose values are close to unity and whose logarithms are given in the tables on pages 232. The station designated 1 is the station whose elevation is already computed.

The formula for the computation of elevations from non-reciprocal observations is

$$h_2 - h_1 = s \cot\left[\zeta_1 - (0.5 - m)\ \frac{s}{\rho \sin 1''}\right] A\ B\ C \qquad (2)$$

in which $h_2$ and $h_1$ are elevations of the two stations, $s$ the horizontal distance between the stations, $\zeta_1$ the mean corrected zenith distance at the station occupied, $m$ the coefficient of refraction, $\rho$ the radius of curvature of the earth's surface in the mean latitude of the stations and in the azimuth of the line observed (see table on pp. 220–222), and $A$, $B$, and $C$ the correction factors.

All but three of the main-scheme lines in Figure 43 are observed from both ends and the differences of elevations for these lines are, therefore, computed by formula (1). The three lines Benton-Keele, Bailer-Thornberry and Thornberry-Gammill are observed from one end only and these differences of elevations must be computed by formula (2). (Text continued on p. 159)

## Abstract of Zenith Distances

Station Monument State Oklahoma

Observer E.O.H. Instrument V.C. 109

| Date | Hour | Object Observed | Object above Station =o | Telescope above Station =t | Diff. of Heights t—o | Reduction to Line joining Stations | Observed Zenith Distance | Corrected Zenith Distance |
|---|---|---|---|---|---|---|---|---|
| 1923 | | | Meters | Meters | Meters | " | ° ′ ″ | ° ′ ″ |
| 5/19 | 1:16 | Benton | 1.17 | 2.115 | +0.945 | -11.8 | 90 10 20.5 | |
| | 2:19 | | | | | | 90 10 19.0 | |
| | 2:29 | | | | | | 90 10 15.4 | |
| | | | | | | | 18.3 | 90 10 06.5 |
| | 1:24 | Hastings | 1.24 | 2.115 | +0.875 | -8.6 | 90 08 01.7 | |
| | 1:25 | | | | | | 90 08 05.0 | |
| | 1:35 | | | | | | 90 08 12.6 | |
| | 1:39 | | | | | | 90 08 05.8 | |
| | 2:09 | | | | | | 90 08 11.1 | |
| | | | | | | | 07.2 | 90 07 58.6 |
| | 1:42 | Keele | 1.92 | 2.115 | +0.195 | -2.0 | 90 08 23.4 | |
| | 1:46 | | | | | | 90 08 23.9 | |
| | 1:49 | | | | | | 90 08 25.2 | |
| | 1:51 | | | | | | 90 08 32.7 | |
| | 1:54 | | | | | | 90 08 22.3 | |
| | | | | | | | 25.5 | 90 08 23.8 |
| | 2:52 | Cupola of | | 2.115 | | | 90 04 49.4 | |
| | 2:55 | Schoolhouse | | | | | 90 04 58.3 | |
| | | Hastings. | | | | | 53.8 | |

Station Benton State Oklahoma

Observer E.O.H. Instrument V.C. 109

| Date | Hour | Object Observed | Object above Station =o | Telescope above Station =t | Diff. of Heights t—o | Reduction to Line joining Stations | Observed Zenith Distance | Corrected Zenith Distance |
|---|---|---|---|---|---|---|---|---|
| 1923 | | | Meters | Meters | Meters | " | ° ′ ″ | ° ′ ″ |
| 5/21 | 1:14 | Keele | 1.92 | 1.375 | -0.545 | +7.4 | 90 01 56.7 | |
| | 1:19 | (Helio) | | | | | 90 02 02.5 | |
| | 1:24 | | | | | | 90 01 56.3 | |
| | 1:29 | | | | | | 90 01 58.0 | |
| | | | | | | | 90 01 58.4 | 90 02 05.8 |
| | 1:57 | Cube | 9.37 | 1.375 | -7.995 | +58.2 | 90 06 41.0 | |
| | 2:09 | (Helio) | | | | | 90 06 35.6 | |
| | | | | | | | 38.3 | 90 07 36.5 |

Fig. 44.—Abstract of zenith distances for sample computation of elevations

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29

## ABSTRACT OF ZENITH DISTANCES

Station Cube State Texas

Observer E.O.H. - A.F.B. Instrument V.C. 109

| Date | Hour | Object Observed | Object Above Station =o | Telescope Above Station =t | Diff. of Heights t−o | Reduction to Line Joining Stations | Observed Zenith Distance | Corrected Zenith Distance |
|---|---|---|---|---|---|---|---|---|
| 1923 | | | Meters | Meters | Meters | ″ | ° ′ ″ | ° ′ ″ |
| 5/28 | 2:06 | Keele | 1.92 | 9.575 | +7.655 | -79.2 | 90 04 30.6 | |
| | 2:08 | (Helio) | | | | | 90 04 29.9 | |
| | 2:10 | | | | | | 90 04 29.1 | |
| 5/29 | 3:33 | | | | | | 90 04 16.2 | |
| | 3:38 | | | | | | 90 04 08.9 | |
| | 3:44 | | | | | | 90 04 15.2 | |
| | | | | | | | 21.6 | 90 03 03.4 |
| 5/28 | 2:18 | Byers | 3.61 | 9.575 | +5.965 | -79.5 | 90 01 09.0 | |
| | 2:26 | (Helio) | | | | | 90 01 07.3 | |
| | 3:32 | | | | | | 90 01 04.1 | |
| | 3:37 | | | | | | 90 01 09.1 | |
| 5/29 | 3:07 | | | | | | 90 01 07.0 | |
| | 3:12 | | | | | | 90 01 02.2 | |
| | 3:20 | | | | | | 90 00 55.2 | |
| | 3:25 | | | | | | 90 00 51.7 | |
| | | | | | | | 90 01 03.2 | 89 59 43.7 |
| 5/28 | 2:56 | Hastings | 1:24 | 9.575 | +8.335 | -61.3 | 90 06 37.9 | |
| | 3:21 | (Helio) | | | | | 90 06 34.5 | |
| | 3:33 | | | | | | 90 06 30.1 | |
| | 3:45 | | | | | | 90 06 37.4 | |
| 5/29 | 4:02 | | | | | | 90 06 07.4 | |
| | 4:12 | | | | | | 90 06 07.0 | |
| | 4:22 | | | | | | 90 05 55.4 | |
| | 4:34 | | | | | | 90 06 01.1 | |
| | | | | | | | 90 06 18.8 | 90 05 17.5 |

FIG. 44.—Abstract of zenith distances for sample computation of elevations—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29

## ABSTRACT OF ZENITH DISTANCES

Station Keele State Oklahoma

Observer E.O.H. - A.F.B. Instrument V.C. 109

| Date | Hour | Object Observed | Object above Station =o | Telescope above Station =t | Diff. of Heights t—o | Reduction to Line joining Stations | Observed Zenith Distance | Corrected Zenith Distance |
|---|---|---|---|---|---|---|---|---|
| 1923 | | | Meters | Meters | Meters | | ° ′ ″ | ° ′ ″ |
| 5/22 | 2:06 | Byers | 3.61 | 2.125 | -1.485 | +20.2 | 90 01 33.8 | |
| | 2:09 | (Helio) | | | | | 90 01 36.4 | |
| | 2:12 | | | | | | 90 01 33.0 | |
| | | | | | | | 34.4 | 90 01 54.6 |
| | 2:16 | Hastings | 1.24 | 2.125 | +0.885 | -15.6 | 90 02 37.8 | |
| | 2:20 | (Helio) | | | | | 90 02 30.0 | |
| | 2:28 | | | | | | 90 02 37.0 | |
| | 2:40 | | | | | | 90 02 32.5 | |
| | | | | | | | 34.3 | 90 02 18.7 |
| | 2:46 | Cube | 9.37 | 2.125 | -7.245 | +74.9 | 90 06 04.9 | |
| | | (Helio) | | | | | 90 05 52.8 | |
| | | | | | | | 90 05 54.9 | |
| | | | | | | | 90 05 57.5 | 90 07 12.4 |
| | 3:02 | Monument | 1.91 | 2.125 | +0.215 | -2.2 | 90 02 03.4 | |
| | 3:10 | (Helio) | | | | | 90 02 04.6 | |
| | | | | | | | 90 01 58.6 | |
| | | | | | | | 02.2 | 90 02 00.0 |
| | 4:07 | Cupola of | | 2.125 | | | 90 00 32.8 | |
| | | Schoolhouse | | | | | 90 00 26.4 | |
| | | Hastings | | | | | 29.6 | |

FIG. 44.—Abstract of zenith distances for sample computation of elevations—Continued

## ABSTRACT OF ZENITH DISTANCES

Station Hastings State Oklahoma

Observer E.O.H. - A.F.B. Instrument V.C. 109

| Date | Hour | Object Observed | Object above Station =o | Telescope above Station =t | Diff. of Heights t−o | Reduction to Line joining Stations | Observed Zenith Distance | Corrected Zenith Distance |
|---|---|---|---|---|---|---|---|---|
| 1923 | | | Meters | Meters | Meters | " | ° ′ ″ | ° ′ ″ |
| 5/24 | 1:27 | Monument | 1.21 | 1.445 | -0.465 | +4.5 | 90 02 32.3 | |
| | 1:34 | (Helio) | | | | | 90 02 41.5 | |
| | 1:40 | | | | | | 90 02 32.1 | |
| | | | | | | | 35.3 | 90 02 39.8 |
| | 1:47 | Keele | 1.92 | 1.445 | -0.475 | +8.4 | 90 03 26.1 | |
| | 1:50 | (Helio) | | | | | 90 03 26.8 | |
| | 1:52 | | | | | | 90 03 27.8 | |
| | | | | | | | 26.9 | 90 03 35.3 |
| | 2:06 | Bailer | 1.21 | 1.445 | +0.235 | -2.7 | 90 04 37.9 | |
| | 2:12 | (Helio) | | | | | 90 04 43.9 | |
| | 2:15 | | | | | | 90 04 44.2 | |
| | | | | | | | 42.0 | 90 04 39.3 |
| | 2:47 | Byers | 3.61 | 1.445 | -2.165 | +27.7 | 90 02 16.4 | |
| | 2:50 | (Helio) | | | | | 90 02 24.8 | |
| | 2:59 | | | | | | 90 02 16.7 | |
| | 3:24 | | | | | | 90 02 13.4 | |
| | 3:40 | | | | | | 90 02 03.4 | |
| | 3:37 | | | | | | 90 02 11.9 | |
| 5/26 | 2:40 | | | | | | 90 02 23.2 | |
| | 2:45 | | | | | | 90 02 15.7 | |
| | 3:52 | | | | | | 90 02 12.8 | |
| | | | | | | | 15.4 | 90 02 43.1 |
| 5/24 | 2:22 | Cube | 9.37 | 1.445 | -7.925 | +58.3 | 90 07 41.3 | |
| | 2:25 | | | | | | 90 07 39.0 | |
| | 2:27 | | | | | | 90 07 47.6 | |
| | | | | | | | 42.6 | 90 08 40.9 |
| 5/26 | 2:57 | Lee | 1.09 | 1.445 | +0.355 | -3.1 | 90 04 60.1 | |
| | 3:04 | (Helio) | | | | | 90 04 53.6 | |
| | 3:19 | | | | | | 90 04 55.2 | |
| | 3:30 | | | | | | 90 04 50.3 | |
| | | | | | | | 54.8 | 90 04 51.7 |
| 5/24 | 4:48 | Cupola | | 1.445 | | | 89 53 05.9 | |
| | 4:54 | Schoolhouse | | | | | 89 52 59.0 | |
| 5/25 | 3:26 | Hastings | | | | | 89 52 51.3 | |
| | 3:30 | | | | | | 89 52 53.5 | |
| | | | | | | | 89 52 57.4 | |

FIG. 44.—Abstract of zenith distances for sample computation of elevations—Continued

## ABSTRACT OF ZENITH DISTANCES

Station Byers State Texas

Observer A.F.B. - E.O.H. Instrument V.C. 109

| Date | Hour | Object Observed | Object above Station =o | Telescope above Station =t | Diff. of Heights t−o | Reduction to Line joining Stations | Observed Zenith Distance | Corrected Zenith Distance |
|---|---|---|---|---|---|---|---|---|
| 1923 | | | Meters | Meters | Meters | " | ° ′ ″ | ° ′ ″ |
| 5/30 | 1:23 | Cube | 9.37 | 3.815 | -5.555 | +74.1 | 90 06 58.6 | |
| | 1:31 | (Helio) | | | | | 90 07 05.7 | |
| | 1:37 | | | | | | 90 06 54.5 | |
| | 1:43 | | | | | | 90 06 55.7 | |
| | | | | | | | 90 06 58.6 | 90 06 12.7 |
| | 1:53 | Bailer | 1.21 | 3.815 | +2.605 | -33.2 | 90 05 42.5 | |
| | 2:00 | (Helio) | | | | | 90 05 33.4 | |
| | 2:06 | | | | | | 90 05 25.3 | |
| | 2:11 | | | | | | 90 05 35.1 | |
| | 2:13 | | | | | | 90 05 30.4 | |
| | 3:15 | | | | | | 90 05 39.4 | |
| 5/31 | 1:35 | | | | | | 90 05 44.4 | |
| | 1:40 | | | | | | 90 05 47.8 | |
| | 1:45 | | | | | | 90 05 45.6 | |
| | | | | | | | 38.2 | 90 05 05.0 |
| | 2:27 | Thornberry | 3.01 | 3.815 | +0.805 | -9.0 | 90 04 33.0 | |
| | | (Helio) | | | | | 90 04 36.7 | |
| | | | | | | | 90 04 36.5 | |
| 6/4 | 1:55 | | | | | | 90 04 20.5 | |
| | 2:00 | | | | | | 90 04 29.8 | |
| | 2:04 | | | | | | 90 04 35.3 | |
| | | | | | | | 32.0 | 90 04 23.0 |
| 5/31 | 2:43 | Keele | 1.92 | 3.815 | +1.895 | -25.8 | 90 06 02.1 | |
| | | (Helio) | | | | | 90 06 06.3 | |
| | | | | | | | 90 06 01.5 | |
| | | | | | | | 03.3 | 90 05 37.5 |
| | 2:56 | Hastings | 1.24 | 3.815 | +2.575 | -32.9 | 90 05 47.7 | |
| | 3:03 | (Helio) | | | | | 90 05 48.2 | |
| | | | | | | | 90 05 47.9 | |
| | | | | | | | 47.9 | 90 05 15.0 |
| 6/4 | 2:33 | Lee | 6.12 | 3.815 | -2.305 | +21.0 | 90 04 40.7 | 90 05 01.7 |
| | 2:38 | (Top of stand) | 6.21 | | -2.395 | +21.8 | 90 04 46.8 | 90 05 08.6 |
| | 2:44 | (Helio) | 6.21 | | | | 90 04 51.8 | 90 05 13.6 |
| | | | | | | | | 90 05 08.0 |

FIG. 44.—Abstract of zenith distances for sample computation of elevations—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29

## ABSTRACT OF ZENITH DISTANCES

Station Bailey State Oklahoma

Observer A.F.B. Instrument V.C. 109

| Date | Hour | Object Observed | Object above Station =o | Telescope above Station =t | Diff. of Heights t−o | Reduction to Line joining Stations | Observed Zenith Distance | | | Corrected Zenith Distance | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1923 | | | Meters | Meters | Meters | " | ° | ′ | ″ | ° | ′ | ″ |
| 6/2 | 2:20 | Thornberry | 3.01 | 1.415 | -1.595 | +25.7 | 90 | 01 | 16.8 | | | |
| | 2:23 | (Helio) | | | | | 90 | 01 | 18.1 | | | |
| | 2:28 | | | | | | 90 | 01 | 19.3 | | | |
| | | | | | | | | | 18.1 | 90 | 01 | 43.8 |
| | 2:42 | Byers | 3.61 | 1.415 | -2.195 | +28.0 | 90 | 02 | 24.6 | | | |
| | 2:46 | (Helio) | | | | | 90 | 02 | 29.2 | | | |
| | 2:50 | | | | | | 90 | 02 | 25.2 | | | |
| | | | | | | | | | 26.3 | 90 | 02 | 54.3 |
| | 2:55 | Hastings | 1.24 | 1.415 | +0.175 | -2.0 | 90 | 05 | 01.5 | | | |
| | 3:00 | (Helio) | | | | | 90 | 04 | 59.7 | | | |
| | 3:05 | | | | | | 90 | 04 | 55.1 | | | |
| | | | | | | | 90 | 04 | 58.8 | 90 | 04 | 56.8 |
| | 3:12 | Lee | 1.09 | 1.415 | +0.325 | -10.2 | 89 | 58 | 56.1 | | | |
| | 3:17 | (Helio) | | | | | 89 | 58 | 51.8 | | | |
| | 3:21 | | | | | | 89 | 58 | 48.9 | | | |
| | | | | | | | | | 52.3 | 89 | 58 | 42.1 |

FIG. 44.—Abstract of zenith distances for sample computation of elevations—Continued

## ABSTRACT OF ZENITH DISTANCES

Station Lee State Oklahoma

Observer A.F.B. Instrument V.C. 109

| Date | Hour | Object Observed | Object above Station =o | Telescope above Station =t | Diff. of Heights t−o | Reduction to Line joining Stations | Observed Zenith Distance | Corrected Zenith Distance |
|---|---|---|---|---|---|---|---|---|
| 1923 | | | Meters | Meters | Meters | " | ° ′ ″ | ° ′ ″ |
| 6/5 | 2:10 | Byers | 3.61 | 6.415 | +2.805 | -25.6 | 90 06 15.5 | |
| | 2:15 | (Helio) | | | | | 90 06 21.3 | |
| | 2:18 | | | | | | 90 06 20.7 | |
| | | | | | | | 19.2 | 90 05 53.6 |
| | 2:25 | Bailer | 1.21 | 6.415 | +5.205 | -164.0 | 90 07 53.2 | |
| | 2:29 | (Helio) | | | | | 90 07 55.2 | |
| | 2:34 | | | | | | 90 07 59.4 | |
| | | | | | | | 55.9 | 90 05 11.9 |
| | 2:43 | Gammill (Helio) | 9.23 | 6.415 | -2.815 | +45.7 | 89 58 47.0 | 89 59 32.7 |
| | 2:48 | (Top of stand) | 9.14 | | -2.725 | +44.2 | 89 58 43.4 | 89 59 27.6 |
| | 2:52 | | 9.14 | | | | 89 58 35.5 | 89 59 19.7 |
| | 3:00 | | 9.14 | | | | 89 58 23.5 | 89 59 07.7 |
| | 4:00 | | 9.14 | | | | 89 58 33.5 | 89 59 17.7 |
| | | | | | | | 33.5 | 89 59 21.1 |
| | 3:13 | Willis | 1.77 | 6.415 | +4.645 | -65.8 | 90 00 23.1 | |
| | 3:25 | (Helio) | | | | | 90 00 28.2 | |
| | 3:31 | | | | | | 90 00 29.4 | |
| | | | | | | | 26.9 | 89 59 21.1 |
| | 3:50 | Thornberry | 3:01 | 6.415 | +3.405 | -49.3 | 90 04 29.5 | |
| | 3:52 | (Helio) | | | | | 90 04 31.1 | |
| | 3:56 | | | | | | 90 04 27.6 | |
| | | | | | | | 29.4 | 90 03 40.1 |
| | 4:10 | Hastings | 1.24 | 6.415 | +5.175 | -45.7 | 90 07 25.4 | |
| | 4:13 | (Helio) | | | | | 90 07 33.3 | |
| | 4:28 | | | | | | 90 07 23.2 | |
| | 4:34 | | | | | | 90 07 35.3 | |
| | | | | | | | 29.3 | 90 06 43.6 |
| | 5:33 | School House | | 6.415 | | | 90 05 41.4 | |
| | 5:40 | top, Hastings | | | | | 90 05 43.1 | |
| | | | | | | | 42.2 | |

FIG. 44.—Abstract of zenith distances for sample computation of elevations—Continued

## ABSTRACT OF ZENITH DISTANCES

Station Thornberry State Texas

Observer A.F.B. - E.O.H. Instrument V.C. 109

| Date | Hour | Object Observed | Object above Station =0 | Telescope above Station =1 | Diff. of Heights 1-0 | Reduction to Line joining Stations | Observed Zenith Distance | Corrected Zenith Distance |
|---|---|---|---|---|---|---|---|---|
| 1923 | | | Meters | Meters | Meters | " | ° ′ ″ | ° ′ ″ |
| 6/1 | 1:53 | Byers | 3.61 | 3.215 | -0.395 | +4.4 | 90 04 34.4 | |
| | 1:56 | (Helio) | | | | | 90 04 35.5 | |
| | 2:02 | | | | | | 90 04 31.0 | |
| | | | | | | | 33.6 | 90 04 38.0 |
| | 2:07 | Lee | 1.09 | 3.215 | +2.125 | -30.8 | 90 04 47.4 | |
| | 2:11 | (Helio) | | | | | 90 04 44.7 | |
| | 2:19 | | | | | | 90 04 48.3 | |
| | | | | | | | 46.8 | 90 04 16.0 |
| | 2:29 | Cashion | 1.16 | 3.215 | +2.055 | -37.5 | 90 02 28.3 | |
| | 2:34 | (Helio) | | | | | 90 02 34.8 | |
| | 2:37 | | | | | | 90 02 26.6 | |
| | 2:45 | | | | | | 90 02 35.3 | |
| | 3:23 | | | | | | 90 02 18.2 | |
| | | | | | | | 28.6 | 90 01 51.1 |
| | 3:00 | Gammill | 9.23 | 3.215 | -6.015 | +71.8 | 90 00 01.3 | |
| | 3:08 | (Helio) | | | | | 90 00 05.9 | |
| | 3:12 | | | | | | 90 00 14.1 | |
| | 3:20 | | | | | | 90 00 04.2 | |
| | | | | | | | 06.4 | 90 01 13.2 |
| | 3:30 | Willis | 1.68 | 3.215 | +1.535 | -35.0 | 89 55 39.1 | |
| | 3:45 | (Top of stand) | | | | | 89 55 37.3 | |
| | 3:50 | | | | | | 89 55 40.3 | |
| | | | | | | | 39.2 | 89 55 04.2 |

FIG. 44.—Abstract of zenith distances for sample computation of elevations—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29

## ABSTRACT OF ZENITH DISTANCES

Station Willis State Oklahoma

Observer E.O.H. - A.F.B. Instrument V.C. 109

| Date | Hour | Object Observed | Object above Station =o | Telescope above Station =t | Diff. of Heights t—o | Reduction to Line joining Stations | Observed Zenith Distance | Corrected Zenith Distance |
|---|---|---|---|---|---|---|---|---|
| 1923 | | | Meters | Meters | Meters | ″ | ° ′ ″ | ° ′ ″ |
| 6/6 | 1:55 | Gammill | 9.23 | 1.975 | -7.255 | +147.6 | 90 01 21.7 | |
| | 2:01 | (Helio) | | | | | 90 01 24.0 | |
| | 2:05 | | | | | | 90 01 22.3 | |
| | | | | | | | 22.7 | 90 03 50.3 |
| | 2:12 | Lee | 6.21 | 1.975 | -4.235 | +60.0 | 90 06 48.1 | |
| | 2:19 | (Helio) | | | | | 90 06 40.4 | |
| | 3:09 | | | | | | 90 06 58.1 | |
| | | | | | | | 48.9 | 90 07 48.9 |
| | 2:55 | Cashion | 1.16 | 1.975 | +0.815 | -20.8 | 90 08 22.0 | |
| | 2:58 | (Helio) | | | | | 90 08 21.8 | |
| | 3:02 | | | | | | 90 08 22.8 | |
| | | | | | | | 22.2 | 90 08 01.4 |
| 6/7 | 8:10 | (Light) | 1.26 | 1.975 | +0.715 | -18.3 | 90 07 56.7 | |
| | 8:20 | | | | | | 90 07 50.9 | |
| | 8:25 | | | | | | 90 07 46.9 | |
| | 9:15 | | | | | | 90 07 45.9 | |
| | | | | | | | 50.1 | 90 07 31.8 |
| | | | | | | | | 29.6 |
| | 8:40 | Thornberry | 3.11 | 1.975 | -1.135 | +25.9 | 90 07 54.1 | |
| | 8:45 | (Light) | | | | | 90 08 01.0 | |
| | 8:52 | | | | | | 90 08 16.7 | |
| | 9:00 | | | | | | 90 08 13.5 | |
| | 9:05 | | | | | | 90 08 13.6 | |
| | 9:08 | | | | | | 90 08 08.8 | |
| | | | | | | | 90 08 08.0 | 90 08 33.9 |
| | | | | | | | | 29.6 |
| | | | | | | | | 90 09 03.5 |

FIG. 44.—Abstract of zenith distances for sample computation of elevations—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29

## ABSTRACT OF ZENITH DISTANCES

Station Gammill State Oklahoma

Observer A.F.B. - E.O.H. Instrument V.C. 109

| Date | Hour | Object Observed | Object Above Station = o | Telescope Above Station = t | Diff. of Heights t − o | Reduction to Line Joining Stations | Observed Zenith Distance | | | Corrected Zenith Distance | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1923 | | | Meters | Meters | Meters | " | ° | ′ | ″ | ° | ′ | ″ |
| 6/18 | 4:49 | Lee | 6.12 | 9.435 | +3.315 | -53.8 | 90 | 07 | 26.9 | | | |
| | 5:12 | (Top of stand) | | | | | 90 | 07 | 28.3 | | | |
| | 5:14 | | | | | | 90 | 07 | 31.7 | | | |
| | | | | | | | | | 29.0 | 90 | 06 | 35.2 |
| | 5:26 | Willis | 1.68 | 9.435 | +7.755 | -157.8 | 90 | 03 | 36.3 | | | |
| | 5:30 | (Top of stand) | | | | | 90 | 03 | 36.2 | | | |
| | 5:31 | | | | | | 90 | 03 | 33.8 | | | |
| | | | | | | | | | 35.4 | 90 | 00 | 57.6 |

Station: Cashion State: Texas

Observer: A.F.B. - E.O.H. Instrument: V.C. 109

11—616

| Date | Hour | Object Observed | Object Above Station = o Meters | Telescope Above Station = t Meters | Diff. of Heights t − o Meters | Reduction to Line Joining Stations " | Observed Zenith Distance | | | Corrected Zenith Distance | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1923 | | | | | | | | | | | | |
| 6/11 | 1:07 | Willis | 1.77 | 1.365 | -0.405 | +10.3 | 89 | 56 | 21.3 | | | |
| | 1:10 | (Helio) | | | | | 89 | 56 | 14.0 | | | |
| | 1:14 | | | | | | 89 | 56 | 16.0 | | | |
| | | | | | | | | | 17.1 | 89 | 56 | 27.4 |
| | 1:25 | Gammill | 9.23 | 1.365 | -7.865 | +92.3 | 90 | 01 | 43.1 | | | |
| | 1:35 | (Helio) | | | | | 90 | 01 | 53.0 | | | |
| | 1:40 | | | | | | 90 | 01 | 45.0 | | | |
| | 1:49 | | | | | | 90 | 01 | 36.0 | | | |
| | | | | | | | 90 | 01 | 42.3 | | | |
| | | | | | | | | | 43.9 | 90 | 03 | 16.2 |
| | 4:26 | Thornberry | 3.01 | 1.365 | -1.645 | +30.0 | 90 | 04 | 01.5 | | | |
| | 4:36 | (Helio) | | | | | 90 | 03 | 59.6 | | | |
| | 4:41 | | | | | | 90 | 04 | 00.7 | | | |
| | | | | | | | 90 | 04 | 00.6 | 90 | 04 | 30.6 |

FIG. 44.—Abstract of zenith distances for sample computation of elevations—Continued

## COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS

Form 29A is used in computing differences of elevation from reciprocal observations. (See fig. 45.) The coefficient of refraction, $m$, which is needed in computing the nonreciprocal observations is also obtained on this form. The computations for all the reciprocal lines are given in Figure 45.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 29 A

COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS.

| | | | | | | |
|---|---|---|---|---|---|---|
| Station 1, occ. | Monument | Cube | | Monument | Cube | Keele |
| Station 2, obs. | Keele | Keele | | Hastings | Hastings | Hastings |
| | ° ′ ″ | ° ′ ″ | | ° ′ ″ | ° ′ ″ | ° ′ ″ |
| $\zeta_1$ | 90 08 23.5 | 90 03 02.4 | | 90 07 58.6 | 90 05 17.5 | 90 02 18.7 |
| $\zeta_2$ | 90 02 00.0 | 90 07 12.4 | | 90 02 39.8 | 90 08 40.9 | 90 03 35.3 |
| $\zeta_2-\zeta_1$ | − 6 23.5 | + 4 10.0 | | − 5 18.8 | + 3 23.4 | + 1 16.6 |
| $\frac{1}{2}(\zeta_2-\zeta_1)$ | − 3 11.8 | + 2 05.0 | | − 2 39.4 | + 1 41.7 | + 0 38.3 |
| $\frac{1}{2}(\zeta_2-\zeta_1)$ in secs. | − 191.8 | + 125.0 | | − 159.4 | + 101.7 | + 38.3 |
| log ditto | 2.28285 | 2.09691 | | 2.20249 | 2.00732 | 1.58320 |
| T | 4.68557 | 4.68557 | | 4.68557 | 4.68557 | 4.68557 |
| log $s$ | 4.30550 | 4.29977 | | 4.32395 | 4.44750 | 4.06840 |
| log $[s \tan \frac{1}{2}(\zeta_2-\zeta_1)]$ | 1.27392 | 1.08225 | | 1.21201 | 1.14039 | 0.33717 |
| log $A$ | + 2 | + 2 | | + 2 | + 2 | + 2 |
| log $B$ | 0 | 0 | | 0 | 0 | 0 |
| log $C$ | 0 | 0 | | 0 | 0 | 0 |
| log $(h_2-h_1)$ | 1.27394 | 1.08227 | | 1.21203 | 1.14041 | 0.33719 |
| $h_2-h_1$ | −18.79 | +12.08 | | −16.29 | +13.82 | +2.17 |
| $h_1$ | 329.90 | 298.80 | | 329.90 | 298.80 | 310.99 |
| $h_2$ | 311.11 | 310.88 | | 313.61 | 312.62 | 313.16 |
| 2 log $s$ | 8.6110 | 8.5995 | | 8.6479 | 8.8950 | 8.1368 |
| log $p=9-2$ log $s$ | 0.3890 | 0.4005 | | 0.3521 | 0.1050 | 0.8632 |
| $p$ of $(h_2-h_1)$ | 2.45 | 2.51 | | 2.25 | 1.27 | 7.30 |
| $\alpha$ and mean $\Phi$ | 38.7° 34.2″ | 18.5° 34.0″ | | 71.6° 34.2″ | 2.2° 34.1″ | 39.1° 34.1″ |
| $\zeta_1+\zeta_2-180°$ | 10′ 23.5″ | 10′ 14.8″ | | 10′ 38.4″ | 13′ 58.4″ | 05′ 54.0″ |
| $\zeta_1+\zeta_2-180°$ in sec. | 623.5″ | 614.8″ | | 638.4″ | 838.4″ | 354.0″ |
| log ditto | 2.79484 | 2.78873 | | 2.80509 | 2.92345 | 2.54900 |
| log $p$ | 6.80394 | 6.80334 | | 6.80496 | 6.80314 | 6.80394 |
| colog $s$ | 5.69450 | 5.70023 | | 5.67605 | 5.55250 | 5.93160 |
| log $\frac{\sin 1''}{2}=\bar{4}.38454$ | 4.38454 | 4.38454 | | 4.38454 | 4.38454 | 4.38454 |
| log $(0.5-m)$ | 9.67782 | 9.67684 | | 9.67064 | 9.66363 | 9.66908 |
| $(0.5-m)$ | 0.4762 | 0.4752 | | 0.4684 | 0.4609 | 0.4667 |
| $p$ of $(0.5-m)$ | 4.08 | 3.98 | | 4.45 | 7.85 | 1.37 |

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310.99 313.19

FIG. 45.—Computation of elevations and refractions from reciprocal observations

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 A

COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS.

| Station 1, occ. | Keele | Cube | Hastings | | Byers | Hastings |
|---|---|---|---|---|---|---|
| Station 2, obs. | Byers | Byers | Byers | | Bailer | Bailer |
| $\zeta_1$ | 90° 01′ 54.6″ | 89° 59′ 43.7″ | 90° 02′ 43.1″ | | 90° 05′ 05.0″ | 90° 04′ 39.3″ |
| $\zeta_2$ | 90 05 37.5 | 90 08 12.7 | 90 05 15.0 | | 90 02 54.3 | 90 04 56.8 |
| $\zeta_2-\zeta_1$ | + 3 42.9 | + 8 29.0 | + 2 31.9 | | − 2 10.7 | + 0 17.5 |
| $\frac{1}{2}(\zeta_2-\zeta_1)$ | + 1 51.4 | + 4 14.5 | + 1 16.0 | | − 1 05.4 | + 0 08.8 |
| $\frac{1}{2}(\zeta_2-\zeta_1)$ in secs. | + 111.4 | + 254.5 | + 76.0 | | − 65.4 | + 8.8 |
| log ditto | 2.04689 | 2.40569 | 1.88081 | | 1.81558 | 0.94448 |
| T | 4.68557 | 4.68558 | 4.68557 | | 4.68557 | 4.68557 |
| log $s$ | 4.18110 | 4.18949 | 4.20738 | | 4.20903 | 4.25792 |
| log [$s$ tan $\frac{1}{2}(\zeta_2-\zeta_1)$] | 0.91356 | 1.28076 | 0.77376 | | 0.71018 | 9.88797 |
| log $A$ | + 2 | + 2 | + 2 | | + 2 | + 2 |
| log $B$ | 0 | 0 | 0 | | 0 | 0 |
| log $C$ | 0 | 0 | 0 | | 0 | 0 |
| log ($h_2-h_1$) | 0.91358 | 1.28078 | 0.77378 | | 0.71020 | 9.88799 |
| $h_2-h_1$ | + 8.20 | + 19.09 | + 5.94 | | − 5.13 | + 0.77 |
| $h_1$ | 310.99 | 298.80 | 313.19 | | 318.73 | 313.19 |
| $h_2$ | 319.19 | 317.89 | 319.13 | | 313.60 | 313.96 |
| 2 log $s$ | 8.3622 | 8.3790 | 8.4148 | | 8.4181 | 8.5158 |
| log $p$=9−2 log $s$ | 0.6378 | 0.6210 | 0.5852 | | 0.5819 | 0.4842 |
| $p$ of ($h_2-h_1$) | 4.34 | 4.18 | 3.85 | | 3.82 | 3.05 |
| $\alpha$ and mean $\phi$ | 68°.5 34′.1 | 30°.3 34′.0 | 24°.7 34′.1 | | 43°.5 34′.1 | 80°.8 34′.2 |
| $\zeta_1+\zeta_2-180°$ | 7′ 32″.1 | 07′ 56″.4 | 07′ 58″.1 | | 07′ 59″.3 | 09′ 36″.1 |
| $\zeta_1+\zeta_2-180°$ in sec. | 452″.1 | 476″.4 | 478″.1 | | 479″.3 | 576″.1 |
| log ditto | 2.65523 | 2.67797 | 2.67952 | | 2.68061 | 2.76050 |
| log $\rho$ | 6.80488 | 6.80365 | 6.80349 | | 6.80409 | 6.80511 |
| colog $s$ | 5.81890 | 5.81051 | 5.79262 | | 5.79097 | 5.74208 |
| log $\frac{\sin 1''}{2}$ = $\bar{4}$.38454 | 4.38454 | 4.38454 | 4.38454 | | 4.38454 | 4.38454 |
| log (0.5−$m$) | 9.66355 | 9.67667 | 9.66017 | | 9.66021 | 9.69223 |
| (0.5−$m$) | 0.4608 | 0.4750 | 0.4573 | | 0.4573 | 0.4923 R |
| $p$ of (0.5−$m$) | 2.30 | 2.39 | 2.60 | | 2.62 | 3.28 |
| | 318.73 | | | | 313.76 | |

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FIG. 45.—Computation of elevations and refractions from reciprocal observations—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 A

COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS.

| Station 1, occ. | Byers | Hastings | Bailer | | Byers | Lee |
|---|---|---|---|---|---|---|
| Station 2, obs. | Lee | Lee | Lee | | Thornberry | Thornberry |
| $\zeta_1$ | 90° 05′ 08″.0 | 90° 04′ 51″.7 | 89° 58′ 42″.1 | | 90° 04′ 23″.0 | 90° 03′ 40″.1 |
| $\zeta_2$ | 90 05 53.6 | 90 06 43.6 | 90 05 11.9 | | 90 04 38.0 | 90 04 16.0 |
| $\zeta_2-\zeta_1$ | + 0 45.6 | + 1 51.9 | + 6 29.8 | | + 0 15.0 | + 0 35.9 |
| $\frac{1}{2}(\zeta_2-\zeta_1)$ | + 0 22.8 | + 0 56.0 | + 3 14.9 | | + 0 07.5 | + 0 18.0 |
| $\frac{1}{2}(\zeta_2-\zeta_1)$ in secs. | + 22.8 | + 56.0 | + 194.9 | | + 7.5 | + 18.0 |
| log ditto | 1.35793 | 1.74819 | 2.28981 | | 0.87506 | 1.25527 |
| T | 4.68557 | 4.68557 | 4.68557 | | 4.68557 | 4.68557 |
| log $s$ | 4.35434 | 4.36805 | 3.81606 | | 4.26766 | 4.15328 |
| log [$s$ tan $\frac{1}{2}(\zeta_2-\zeta_1)$] | 0.39784 | 0.80181 | 0.79144 | | 9.82829 | 0.09412 |
| log $A$ | + 2 | + 2 | + 2 | | + 2 | + 2 |
| log $B$ | 0 | 0 | 0 | | 0 | 0 |
| log $C$ | 0 | 0 | 0 | | 0 | 0 |
| log $(h_2-h_1)$ | 0.39786 | 0.80183 | 0.79146 | | 9.82831 | 0.09414 |
| $h_2-h_1$ | + 2.50 | + 6.34 | + 6.19 | | + 0.67 | + 1.24 |
| $h_1$ | 318.73 | 313.19 | 313.76 | | 318.73 | 320.01 |
| $h_2$ | 321.23 | 319.53 | 319.95 | | 319.40 | 321.25 |
| 2 log $s$ | 8.7087 | 8.7361 | 7.6321 | | 8.5353 | 8.3066 |
| log $p$=9−2 log $s$ | 0.2913 | 0.2639 | 1.3679 | | 0.4647 | 0.6934 |
| $p$ of $(h_2-h_1)$ | 1.96 | 1.84 | 23.33 | | 2.92 | 4.94 |
| $\alpha$ and mean $\phi$ | 47°.2 34′.1 | 88°.2 34′.2 | 56°.5 34′.2 | | 86°.2 34′.1 | 7°.5 34′.1 |
| $\zeta_1+\zeta_2-180°$ | 11′ 01″.6 | 11′ 35″.3 | 03′ 54″.0 | | 09′ 01″.0 | 07′ 56″.1 |
| $\zeta_1+\zeta_2-180°$ in sec. | 661″.6 | 695″.3 | 234″.0 | | 541″.0 | 476″.1 |
| log ditto | 2.82060 | 2.84217 | 2.36922 | | 2.73320 | 2.67770 |
| log $\rho$ | 6.80422 | 6.80516 | 6.80455 | | 6.80515 | 6.80317 |
| colog $s$ | 5.64566 | 5.63195 | 6.18394 | | 5.73234 | 5.84672 |
| log $\frac{\sin 1''}{2}=\bar{4}.38454$ | 4.38454 | 4.38454 | 4.38454 | | 4.38454 | 4.38454 |
| log $(0.5-m)$ | 9.65502 | 9.66382 | 9.74225 | | 9.65523 | 9.71213 |
| $(0.5-m)$ | 0.4519 | 0.4611 | 0.5524 R | | 0.4521 | 0.5154 R |
| $p$ of $(0.5-m)$ | 5.11 | 5.45 | 0.43 | | 3.43 | 2.03 |

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320.01 320.56

FIG. 45.—Computation of elevations and refractions from reciprocal observations—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 A

COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS.

| | | | | | |
|---|---|---|---|---|---|
| Station 1, occ. | Lee | Thornberry | | Lee | Willis |
| Station 2, obs. | Willis | Willis | | Gammill | Gammill |
| $\zeta_1$ | 89° 59′ 21″.1 | 89° 55′ 04″.2 | | 89° 59′ 21″.1 | 90° 03′ 50″.3 |
| $\zeta_2$ | 90 07 48.9 | 90 09 03.5 | | 90 06 35.2 | 90 00 57.6 |
| $\zeta_2-\zeta_1$ | + 8 27.8 | +13 59.3 | | + 7 14.1 | - 2 52.7 |
| $\frac{1}{2}(\zeta_2-\zeta_1)$ | + 4 13.9 | + 6 59.6 | | + 3 37.0 | - 1 26.4 |
| $\frac{1}{2}(\zeta_2-\zeta_1)$ in secs. | + 253.9 | + 419.6 | | + 217.0 | - 86.4 |
| log ditto | 2.40466 | 2.62284 | | 2.33646 | 1.93651 |
| T | 4.68558 | 4.68558 | | 4.68558 | 4.68557 |
| log $s$ | 4.16315 | 3.95628 | | 4.10401 | 4.00605 |
| log [$s$ tan $\frac{1}{2}(\zeta_2-\zeta_1)$] | 1.25339 | 1.26470 | | 1.12605 | 0.62813 |
| log $A$ | + 2 | + 2 | | + 2 | + 2 |
| log $B$ | 0 | 0 | | 0 | 0 |
| log $C$ | 0 | 0 | | 0 | 0 |
| log ($h_2-h_1$) | 1.25341 | 1.26472 | | 1.12607 | 0.62815 |
| $h_2-h_1$ | +17.92 | +18.40 | | +13.37 | - 4.25 |
| $h_1$ | 320.01 | 320.56 | | 320.01 | 338.67 |
| $h_2$ | 337.93 | 338.96 | | 333.38 | 334.42 |
| 2 log $s$ | 8.3263 | 7.9126 | | 8.2080 | 8.0121 |
| log $p$=9−2 log $s$ | 0.6737 | 1.0874 | | 0.7920 | 0.9879 |
| $p$ of ($h_2-h_1$) | 4.72 | 12.23 | | 6.19 | 9.73 |
| $\alpha$ and mean $\phi$ | 44°.1 34′.1 | 56°.2 34′.1 | | 87°.1 34′.2 | 14°.7 34′.1 |
| $\zeta_1+\zeta_2-180°$ | 07′ 10″.0 | 04′ 07″.7 | | 05′ 56″.3 | 04′ 47″.9 |
| $\zeta_1+\zeta_2-180°$ in sec | 430″.0 | 247″.7 | | 356″.3 | 287″.9 |
| log ditto | 2.63347 | 2.39393 | | 2.55182 | 2.45924 |
| log $\rho$ | 6.80411 | 6.80483 | | 6.80515 | 6.80326 |
| colog $s$ | 5.83685 | 6.04372 | | 5.89599 | 5.99395 |
| log $\frac{\sin 1''}{2}$=$\bar{4}$.38454 | 4.38454 | 4.38454 | | 4.38454 | 4.38454 |
| log (0.5−$m$) | 9.65897 | 9.62702 | | 9.63750 | 9.64099 |
| (0.5−$m$) | 0.4560 | 0.4237 | | 0.4340 | 0.4375 |
| $p$ of (0.5−$m$) | 2.12 | 0.82 | | 1.61 | 1.03 |

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338.67 334.02

FIG. 45.—Computation of elevations and refractions from reciprocal observations—Continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 A

COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS.

| | | | | | | |
|---|---|---|---|---|---|---|
| Station 1, occ. | Thornberry | Willis | | | | |
| Station 2, obs. | Cashion | Cashion | | | | |
| $\zeta_1$ | 90° 01′ 51.″1 | 90° 08′ 01.″4 | | | | |
| $\zeta_2$ | 90 04 30.6 | 89 56 27.4 | | | | |
| $\zeta_2-\zeta_1$ | + 2 39.5 | − 11 34.0 | | | | |
| $\frac{1}{2}(\zeta_2-\zeta_1)$ | + 1 19.3 | − 5 47.0 | | | | |
| $\frac{1}{2}(\zeta_2-\zeta_1)$ in secs. | + 79.3 | 347.0 | | | | |
| log ditto | 1.90200 | 2.54033 | | | | |
| T | 4.68557 | 4.68558 | | | | |
| log $s$ | 4.05335 | 3.90728 | | | | |
| log [$s$ tan $\frac{1}{2}(\zeta_2-\zeta_1)$] | 0.64092 | 1.13319 | | | | |
| log $A$ | + 2 | + 2 | | | | |
| log $B$ | 0 | 0 | | | | |
| log $C$ | 0 | 0 | | | | |
| log $(h_2-h_1)$ | 0.64094 | 1.13321 | | | | |
| $h_2-h_1$ | + 4.37 | − 13.59 | | | | |
| $h_1$ | 320.56 | 338.67 | | | | |
| $h_2$ | 324.93 | 325.08 | | | | |
| 2 log $s$ | 8.1067 | 7.8146 | | | | |
| log $p$=9−2 log $s$ | 0.8933 | 1.1854 | | | | |
| $p$ of $(h_2-h_1)$ | 7.82 | 15.32 | | | | |
| $\alpha$ and mean $\phi$ | 68°.7 34°.0 | 16°.2 34°.1 | | | | |
| $\zeta_1+\zeta_2-180°$ | 06′ 21.″7 | 04′ 28.″8 | | | | |
| $\zeta_1+\zeta_2-180°$ in sec. | 381.″7 | 268.″8 | | | | |
| log ditto | 2.58172 | 2.42943 | | | | |
| log $\rho$ | 6.80489 | 6.80329 | | | | |
| colog $s$ | 5.94665 | 6.09272 | | | | |
| log $\frac{\sin 1''}{2}$=$\bar{4}$.38454 | 4.38454 | 4.38454 | | | | |
| log (0.5−$m$) | 9.71780 | 9.70998 | | | | |
| (0.5−$m$) | 0.5222 R | 0.5128 R | | | | |
| $p$ of (0.5−$m$) | | | | | | |

11—5040

325.03

FIG. 45.—Computation of elevations and refractions from reciprocal observations—Continued

## EXPLANATION OF COMPUTATION

The mean corrected zenith distances, $\zeta_1$ and $\zeta_2$, are taken from Form 29. (See fig. 44.) Log $s$, the logarithm of the length of the given line, $\alpha$, the azimuth of the line, and "mean $\phi$" the mean latitude of the two stations concerned, are obtained from the list of geographic positions. If the list of positions has not been made out, log $s$ can be obtained directly from the triangle computations and $\alpha$ and "mean $\phi$" from the position computations.

The following rules should be observed: Carry all angles to tenths of seconds only, and all logarithms to five decimal places only except "2 log $s$" and "9 − 2 log $s$" which should be carried to four decimal places only. If the zenith distance is to some indefinite object, such as a mountain peak for which there was no well-defined point on which to sight, the angles should be carried only to even seconds. The quantity $h_2-h_1$ should be carried to centimeters only. Log $(0.5-m)$ should be computed to five decimal places and $(0.5-m)$ to four decimal places. The weights should be carried to two decimal places.

To convert $\frac{1}{2}(\zeta_2-\zeta_1)$ from minutes and seconds, to seconds, use the tables at the bottom of pages 2–185 of the Vega Logarithmic Tables. The logarithm of the value in seconds and the corresponding value of $T$ are found on the same page as the conversion, in each case, and should be taken out at the same time. Log $[s \tan \frac{1}{2}(\zeta_2-\zeta_1)]$ is the sum of the three logarithms next above it. Log $A$, log $B$, and log $C$ are obtained from the tables on page 232, which are self-explanatory. The sum of these three logarithms and log $[s \tan \frac{1}{2}(\zeta_2-\zeta_1)]$ gives log $(h_2-h_1)$. The elevation $h_1$ in each case is obtained from preceding computations.

The relative weights to be assigned to the various values of $h_2-h_1$ in the least squares adjustment are inversely proportional to $s^2$, and for convenience are computed by the formula log $p=9-2$ log $s$, as shown on the computation directly below $h_2$. By this formula a line 31.6 kilometers long is given unit weight.

After the value of $h_2$ and its weight, $p$, have been determined, the coefficient of refraction is computed by the formula,

$$0.5-m=\frac{(\zeta_1+\zeta_2-180^\circ)\rho \sin 1''}{2s}$$

in which $m$, the coefficient of refraction, is the ratio of the mean angle at the two stations, between the tangent to the line of sight and the chord joining the two stations, to the angle between the lines of gravity at the two stations. The azimuth $\alpha$ of the line and the mean latitude, $\phi$, of the two stations are taken out to the nearest tenth of a degree only. The radius of curvature, $\rho$, is taken from the tables on pages 220–222 with $\alpha$ and "mean $\phi$" as arguments. The relative weight,

$p$, to be assigned to each determination of $(0.5-m)$ is proportional to $s^2$. The logarithm of $s^2$ (or 2 log $s$) has already been taken out for use in computing the relative weights of $(h_2-h_1)$. As the numbers corresponding to (2 log $s$) would usually be large quantities, it is customary to divide them by a power of 10. For example, in the computations shown above they are all divided by $10^8$. The value of $(0.5-m)$ is only for use in computing elevations from nonreciprocal observations and need not be computed unless such observations are made. All values of $(0.5-m)$ which are greater than 0.5 or which are nearly 0.5 should be rejected.

After the elevation of a given station has been determined from two or more stations, the weighted mean should be taken before it is used in determining the elevation of some other station. For example, the elevation of Keele (fig. 45) as determined from Monument is 311.11 meters with a weight of 2.45, and as determined from Cube is 310.88 with a weight of 2.51. The elevation to be used for Keele is found as follows:

$$\begin{array}{ll} 2.45\times 311.11 & =762.2195 \\ 2.51\times 310.88 & =780.3088 \\ \hline 4.96 & \quad\quad 1542.5283 \end{array}$$

$$\frac{1542.5283}{4.96}=310.99 \text{ meters.}$$

This elevation is placed at the bottom of the computation for Keele. The elevations of the other stations are obtained in a similar manner.

After the elevations of the various stations connected by reciprocal observations have been determined and the value of $(0.5-m)$ obtained at each, the differences of elevation for the nonreciprocal observations are computed.

## COMPUTATION OF ELEVATIONS FROM NONRECIPROCAL OBSERVATIONS

The formula for computing elevations from nonreciprocal observations is given on page 148. This formula may be rewritten in the form

$$h_2-h_1=s \cot (\zeta_1-k)\, ABC=s \tan (90^\circ-\zeta_1+k)\, ABC$$

where $k=\frac{(0.5-m)\, s}{\rho \sin 1''}$. The quantity, $\zeta_1$, is the mean observed zenith distance of the object sighted, no vertical eccentric reduction being made in the case of nonreciprocal observations. The other quantities in the formula have already been defined.

Form 29B is used for computing differences of elevation from nonreciprocal observations. (See fig. 46.) The quantities $\zeta_1$ and $(t-o)$ are obtained from Form 29. (See fig. 44.) The quantities log $s$, $\alpha$ and mean $\phi$ are all obtained from the list of geographic positions, or from the triangle and position computations. Colog $\rho$ is determined by subtracting the value of log $\rho$ as taken from the tables on pages 220–222 from 10. Log $A$, log $B$, and log $C$ are obtained from the tables on page 232. The value of $(0.5-m)$ is obtained from the computation of the reciprocal observations involving the same station. For example, for the observations on station Keele from station Benton the $(0.5-m)$ used should be a weighted mean of the $(0.5-m)$'s as determined in the computations of the reciprocal observations involving Benton. It happens in this case that the elevation of Benton was fixed to start with and as there were no reciprocal observations from this station there are no computations of $(0.5-m)$. The log value, $9.63246-10$, given on page 63 of Special Publication No. 26, was therefore issued.

In the computation of the nonreciprocal observations on Thornberry from Bailer, the log $(0.5-m)$ is obtained as follows: In Figure 45 it is seen that $(0.5-m)$ was determined three times in the computation of the reciprocal observations involving station Bailer. In the Hastings-Bailer computation the value determined is 0.4923, and in the Bailer-Lee computation it is 0.5524. As the value is very close to 0.5 in one of these computations, and in the other it is greater than 0.5, these values are rejected. In the Byers-Bailer computation the value is 0.4573 and this is the one used, the logarithm being 9.66021.

In the computation of station Gammill from Thornberry, the value of $(0.5-m)$ is obtained as follows: The value of $(0.5-m)$ for station Thornberry is determined four times in the computation of the reciprocal observations in Figure 45. For the Lee-Thornberry line the value of $(0.5-m)$ is 0.5154 and for the Thornberry-Cashion line it is 0.5222. Both of these values are rejected since they are greater than 0.5. For the Byers-Thornberry line $(0.5-m)$ is 0.4521 with a weight of 3.43, and for the Thornberry-Willis line it is 0.4237 with a weight of 0.82. The weighted mean value is then

$$\frac{(0.4521\times 3.43)+(0.4237\times 0.82)}{3.43+0.82}=0.4466.$$

The log of this value, $9.64992-10$, is used in the computation of the elevation of Gammill from Thornberry.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 B

COMPUTATION OF ELEVATIONS FROM NONRECIPROCAL OBSERVATIONS.

| | | | | | | |
|---|---|---|---|---|---|---|
| Station 1, occ. | Benton | Bailer | Thornberry | | | |
| Station 2, obs. | Keele | Thornberry | Gammill | | | |
| Object sighted | Helio | Helio | Helio | | | |
| $\zeta_1$ | 90° 01′ 58.4″ | 90° 01′ 18.1″ | 90° 00′ 06.4″ | | | |
| $\alpha$ and mean $\phi$ | 88.1 34.1 | 34.9 34.1 | 38.9 34.1 | | | |
| log $(0.5-m)$ | 9.63246 | 9.66021 | 9.64992 | | | |
| log $s$ | 4.18146 | 4.10702 | 4.23756 | | | |
| colog $\rho$ | 3.19485 | 3.19620 | 3.19606 | | | |
| colog sin 1″ | 5.31443 | 5.31443 | 5.31443 | 5.31443 | 5.31443 | 5.31443 |
| log ($k$ in secs.) | 2.32320 | 2.27786 | 2.39797 | | | |
| $k$ in secs. | 210.″5 | 189.″6 | 250.″0 | | | |
| $(90°-\zeta_1+k)$ in secs. | +92.″1 | +111.″5 | +243.″6 | | | |
| log ditto | 1.96426 | 2.04727 | 2.38668 | | | |
| T | 4.68557 | 4.68557 | 4.68558 | | | |
| log $s$ | 4.18146 | 4.10702 | 4.23756 | | | |
| log [$s$ tan $(90°-\zeta_1+k)$] | 0.83129 | 0.83986 | 1.30982 | | | |
| log $A$ | + 2 | + 2 | + 2 | | | |
| log $B$ | 0 | 0 | 0 | | | |
| log $C$ | 0 | 0 | 0 | | | |
| log $(h_2-h_1)$ | 0.83131 | 0.83988 | 1.30984 | | | |
| $h_2-h_1$ | + 6.78 | + 6.92 | + 20.41 | | | |
| $h_1$ | | | | | | |
| $t-o$ | -0.54 | -1.60 | -6.02 | | | |
| Corrected $(h_2 - h_1)$ | + 6.24 | + 5.32 | +14.39 | | | |
| log $p=9-2$ log $s$ | 0.6371 | 0.7860 | 0.5249 | | | |
| $p$ | 4.34 | 6.11 | 3.35 | | | |
| Weighted mean elevation of sta. obs. | | | | | | |

11—3043

FIG. 46.—Computation of elevations from nonreciprocal observations

## LEAST-SQUARES ADJUSTMENT OF DIFFERENCES OF ELEVATIONS

After all differences of elevations have been computed, both from the reciprocal and the nonreciprocal observations, and their weights determined, the next step is the adjustment of these differences of elevation by the method of least squares. An example of such an adjustment, from the formation of the equations to the determination of the final elevations, is given in detail on the following pages for the differences of elevation represented in Figure 43.

The adjustment of vertical observations is made by means of observation equations. Elevations in even meters approximating the final values are first assumed for the different stations. To these assumed values are added $x$'s to be determined by the adjustment. Then observation equations are formed by comparing the differences of the assumed elevations with the differences determined by computation.

In the first one of the following tables are given the stations already fixed in elevation and their elevations. In the second table are given the names of the stations whose elevations are to be fixed by the adjustment and in the second column of this table their assumed elevations and correction symbols. The last two columns of this table are filled in after the adjustment is completed.

If any of the stations of the scheme have been determined in elevation directly from first-order leveling their elevations should be held fixed in the adjustment. The mathematician making the adjustment should be very careful at the start to ascertain whether there are any direct connections with first-order leveling.

*Fixed elevations*

| Station | Elevation | |
|---|---|---|
| | Meters | Feet |
| Monument | 329.90 | 1,082.3 |
| Benton | 301.40 | 988.8 |
| Cube | 298.80 | 980.3 |
| Gammill | 333.35 | 1,093.7 |
| Cashion | 324.65 | 1,065.1 |

*Assumed and adjusted elevations*

| Station | Elevation | | |
|---|---|---|---|
| | Assumed, plus correction | Adjusted [1] | |
| | | Meters | Feet |
| | *Meters* | | |
| Keele | $311+x_1$ | 310.41 | 1,018.4 |
| Hastings | $313+x_2$ | 312.81 | 1,026.3 |
| Byers | $319+x_3$ | 318.47 | 1,044.8 |
| Bailer | $314+x_4$ | 313.60 | 1,028.9 |
| Lee | $320+x_5$ | 319.75 | 1,049.0 |
| Thornberry | $321+x_6$ | 319.90 | 1,049.5 |
| Willis | $339+x_7$ | 338.05 | 1,109.1 |

[1] This column is filled in after the adjustment is completed.

## FORMATION OF OBSERVATION EQUATIONS

There are just as many observation equations as there are computed differences of elevation. They are tabulated in the form shown below.

*Formation of observation equations*

| | Station 1 | Station 2 | Weight $p$ | Assumed difference of elevation $h_2-h_1$ | Observed difference of elevation $h_2-h_1$ | Assumed minus observed | Symbol | Adjusted difference of elevation $h_2-h_1$ | Adjusted minus observed $v$ | $pv$ | $v^2$ | $pv^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Monument | Keele | 2.45 | −18.90 | −18.79 | −0.11 | $+x_1$ | −19.49 | −0.70 | −1.72 | 0.4900 | 1.2005 |
| 2 | Benton [1] | Keele | 1.45 | +9.60 | +6.24 | +3.36 | $+x_1$ | +9.01 | +2.77 | +4.02 | 7.6729 | 11.1257 |
| 3 | Cube | Keele | 2.51 | +12.20 | +12.08 | +.12 | $+x_1$ | +11.61 | −.47 | −1.18 | .2209 | .5545 |
| 4 | Monument | Hastings | 2.25 | −16.90 | −16.29 | −.61 | $+x_2$ | −17.09 | −.80 | −1.80 | .6400 | 1.4400 |
| 5 | Keele | Hastings | 7.30 | +2.00 | +2.17 | −.17 | $-x_1+x_2$ | +2.40 | +.23 | +1.68 | .0529 | .3862 |
| 6 | Cube | Hastings | 1.27 | +14.20 | +13.82 | +.38 | $+x_2$ | +14.01 | +.19 | +.24 | .0361 | .0458 |
| 7 | Hastings | Byers | 3.85 | +6.00 | +5.94 | +.06 | $-x_2+x_3$ | +5.66 | −.28 | −1.08 | .0784 | .3018 |
| 8 | Keele | Byers | 4.34 | +8.00 | +8.20 | −.20 | $-x_1+x_3$ | +8.06 | −.14 | −.61 | .0196 | .0851 |
| 9 | Cube | Byers | 4.18 | +20.20 | +19.09 | +1.11 | $+x_3$ | +19.67 | +.58 | +2.42 | .3364 | 1.4062 |
| 10 | Hastings | Bailer | 3.05 | +1.00 | +.77 | +.23 | $-x_2+x_4$ | +.79 | +.02 | +.06 | .0004 | .0012 |
| 11 | Byers | Bailer | 3.82 | −5.00 | −5.13 | +.13 | $-x_3+x_4$ | −4.87 | +.26 | +.99 | .0676 | .2582 |
| 12 | Hastings | Lee | 1.84 | +7.00 | +6.34 | +.66 | $-x_2+x_5$ | +6.94 | +.60 | +1.10 | .3600 | .6624 |
| 13 | Bailer | Lee | 23.33 | +6.00 | +6.19 | −.19 | $-x_4+x_5$ | +6.15 | −.04 | −.93 | .0016 | .0373 |
| 14 | Byers | Lee | 1.96 | +1.00 | +2.50 | −1.50 | $-x_3+x_5$ | +1.28 | −1.22 | −2.39 | 1.4884 | 2.9173 |
| 15 | Lee | Thornberry | 4.94 | +1.00 | +1.24 | −.24 | $-x_5+x_6$ | +.15 | −1.09 | −5.38 | 1.1881 | 5.8692 |
| 16 | Bailer [1] | Thornberry | 2.04 | +7.00 | +5.32 | +1.68 | $-x_4+x_6$ | +6.30 | +.98 | +2.00 | .9604 | 1.9592 |
| 17 | Byers | Thornberry | 2.92 | +2.00 | +.67 | +1.33 | $-x_3+x_6$ | +1.43 | +.76 | +2.22 | .5776 | 1.6866 |
| 18 | Lee | Willis | 4.72 | +19.00 | +17.92 | +1.08 | $-x_5+x_7$ | +18.30 | +.38 | +1.79 | .1444 | .6816 |
| 19 | Thornberry | Willis | 12.23 | +18.00 | +18.40 | −.40 | $-x_6+x_7$ | +18.15 | −.25 | −3.06 | .0625 | .7644 |
| 20 | Lee | Gammill | 6.19 | +13.35 | +13.37 | −.02 | $-x_5$ | +13.60 | +.23 | +1.42 | .0529 | .3275 |
| 21 | Thornberry [1] | Gammill | 1.12 | +12.35 | +14.39 | −2.04 | $-x_6$ | +13.45 | −.94 | −1.05 | .8836 | .9896 |
| 22 | Willis | Gammill | 9.73 | −5.65 | −4.25 | −1.40 | $-x_7$ | −4.70 | −.45 | −4.38 | .2025 | 1.9703 |
| 23 | Willis | Cashion | 15.32 | −14.35 | −13.59 | −.76 | $-x_7$ | −13.40 | +.19 | +2.91 | .0361 | .5531 |
| 24 | Thornberry | Cashion | 7.82 | +3.65 | +4.37 | −.72 | $-x_6$ | +4.75 | +.38 | +2.97 | .1444 | 1.1292 |
| | | | | | | | | | | | $\Sigma\, pv^2$ | 36.3529 |

[1] Computed from nonreciprocal observations. Weight used here is one-third of that determined by the computation on p. 167.

The data in the second, third, fourth, and sixth columns are obtained directly from the computations of the differences of elevations in Figures 45 and 46, except that where the difference of elevation, $h_2-h_1$, is determined from nonreciprocal observations the weight, $p$, used in the adjustment is one-third of the weight determined in the computations. The quantities in the fifth column are obtained from the data given in the tables of fixed and assumed elevations on page 168.

The observation equations are formed in the following manner: First, take the equation for the difference of elevation of Monument and Keele.

| | | |
|---|---|---|
| (1) Monument, fixed elevation | $=329.90$ | |
| (2) Keele, assumed elevation + correction | $=311 \quad +x_1$ | |
| $h_2-h_1$ (assumed) | $=-18.90+x_1$ | |
| $h_2-h_1$ (observed) | $=-18.79+v_1$ | |
| assumed − observed | $=-0.11+x_1-v_1=0$ | |
| | $v_1=-0.11+x_1$ | (1) |

For the difference of elevation of Benton and Keele, we have

| | | |
|---|---|---|
| (1) Benton, fixed elevation | $=301.40$ | |
| (2) Keele, assumed elevation + correction | $=311 \quad +x_1$ | |
| $h_2-h_1$ (assumed) | $=+9.60+x_1$ | |
| $h_2-h_1$ (observed) | $=+6.24+v_2$ | |
| assumed − observed | $=+3.36+x_1-v_2=0$ | |
| | $v_2=+3.36+x_1$ | (2) |

The other 22 observation equations are formed in a similar manner. The constant terms of the equations ($-0.11$ in equation 1, $+3.36$ in equation 2, etc.) are placed in the column "Assumed minus observed" and the symbols for the unknown corrections ($+x_1$ in equation 1, $+x_1$ in equation 2, etc.) are placed in the column "Symbol."

After the first eight columns of the preceding table of observation equations are filled in, the table below in which the observation equations are written in horizontal lines with the $x$'s in their respective columns is made out for convenience in forming the normals. In the second column of this table are given the weights, $p$, and in the third column the constant terms, $N$, of the observation equations. In the last column the products, $pN$, are given.

*Data for formation of normal equations*

| | $p$ | $N$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | $pN$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.45 | −0.11 | +1 | | | | | | | −0.2695 |
| 2 | 1.45 | +3.36 | +1 | | | | | | | +4.8720 |
| 3 | 2.51 | +.12 | +1 | | | | | | | +.3012 |
| 4 | 2.25 | −.61 | | +1 | | | | | | −1.3725 |
| 5 | 7.30 | −.17 | −1 | +1 | | | | | | −1.2410 |
| 6 | 1.27 | +.38 | | +1 | | | | | | +.4826 |
| 7 | 3.85 | +.06 | | −1 | +1 | | | | | +.2310 |
| 8 | 4.34 | −.20 | −1 | | +1 | | | | | −.8680 |
| 9 | 4.18 | +1.11 | | | +1 | | | | | +4.6398 |
| 10 | 3.05 | +.23 | | −1 | | +1 | | | | +.7015 |
| 11 | 3.82 | +.13 | | | −1 | +1 | | | | +.4966 |
| 12 | 1.84 | +.66 | | −1 | | | +1 | | | +1.2144 |
| 13 | 3.33 | −.19 | | | | −1 | +1 | | | −4.4327 |
| 14 | 1.96 | −1.50 | | | −1 | | +1 | | | −2.9400 |
| 15 | 4.94 | −.24 | | | | | −1 | +1 | | −1.1856 |
| 16 | 2.04 | +1.68 | | | | −1 | | +1 | | +3.4272 |
| 17 | 2.92 | +1.33 | | | −1 | | | +1 | | +3.8836 |
| 18 | 4.72 | +1.08 | | | | | −1 | | +1 | +5.0976 |
| 19 | 12.23 | −.40 | | | | | | −1 | +1 | −4.8920 |
| 20 | 6.19 | −.02 | | | | | −1 | | | −.1238 |
| 21 | 1.12 | −2.04 | | | | | | −1 | | −2.2848 |
| 22 | 9.73 | −1.40 | | | | | | | −1 | −13.6220 |
| 23 | 15.32 | −.76 | | | | | | | −1 | −11.6432 |
| 24 | 7.82 | −.72 | | | | | | −1 | | −5.6304 |

**FORMATION OF NORMAL EQUATIONS**

The normals are formed in the same manner as for condition equations (see p. 11), except that the $pN$ values are multiplied by the coefficients in each numbered column in turn and the sums taken for the constant terms in the normals. For example, the constant term of the first normal equation is $(+1\times-0.2695)+(+1\times+4.8720)+(+1\times+0.3012)+(-1\times-1.2410)+(-1\times-0.8680)=-0.2695+4.8720+0.3012+1.2410+0.8680=+7.0127$.

The constant terms of the other normal equations are obtained in a similar manner. The complete set of normal equations is given below.

*Normal equations*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | $\eta$ | $\Sigma$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | +18.05 | −7.30 | −4.34 | | | | | +7.0127 | +13.4227 |
| 2 | | +19.56 | −3.85 | −3.05 | −1.84 | | | −4.2778 | −.7578 |
| 3 | | | +21.07 | −3.82 | −1.96 | −2.92 | | +2.5626 | +6.7426 |
| 4 | | | | +32.24 | −23.33 | −2.04 | | +2.2036 | +2.2036 |
| 5 | | | | | +42.98 | −4.94 | −4.72 | −9.9465 | −3.7565 |
| 6 | | | | | | +31.07 | −12.23 | +18.9324 | +27.8724 |
| 7 | | | | | | | +42.00 | +25.4708 | +50.5208 |

## SOLUTION OF NORMAL EQUATIONS

These normal equations are solved in the same manner as the normal equations for the adjustment of horizontal directions. (See p. 38.) The complete forward and back solutions are given below.

*Solution of normal equations*

| 1 | 2 | 3 | 5 | 6 | 7 | 4 | η | Σ |
|---|---|---|---|---|---|---|---|---|
| +18.05 | −7.30 | −4.34 | | | | | +7.0127 | +13.4227 |
| $x_1$ | +.40443 | +.24044 | | | | | −.38851 | −.74364 |
| | +19.56 | −3.85 | −1.84 | | | −3.05 | −4.2778 | −.7578 |
| 1 | −2.9523 | −1.7552 | | | | | +2.8361 | +5.4286 |
| | +16.6077 | −5.6052 | −1.84 | | | −3.05 | −1.4417 | +4.6708 |
| | $x_2$ | +.33751 | +.11079 | | | +.18365 | +.08681 | −.28124 |
| | | +21.07 | −1.96 | −2.92 | | −3.82 | +2.5626 | +6.7426 |
| | 1 | −1.0435 | | | | | +1.6861 | +3.2274 |
| | 2 | −1.8918 | −.6210 | | | −1.0294 | −.4866 | +1.5764 |
| | | +18.1347 | −2.5810 | −2.92 | | −4.8494 | +3.7621 | +11.5464 |
| | | $x_3$ | +.14232 | +.16102 | | +.26741 | −.20745 | −.63670 |
| | | | +42.98 | −4.94 | −4.72 | −23.33 | −9.9465 | −3.7565 |
| | | 2 | −.2039 | | | −.3379 | −.1597 | +.5175 |
| | | 3 | −.3673 | −.4156 | | −.6902 | +.5354 | +1.6433 |
| | | | +42.4088 | −5.3556 | −4.72 | −24.3581 | −9.5708 | −1.5957 |
| | | | $x_5$ | +.12629 | +.11130 | +.57436 | +.22568 | +.03763 |
| | | | | +31.07 | −12.23 | −2.04 | +18.9324 | +27.8724 |
| | | | 3 | −.4702 | | −.7808 | +.6057 | +1.8592 |
| | | | 5 | −.6764 | −.5961 | −3.0760 | −1.2086 | −.2015 |
| | | | | +29.9234 | −12.8261 | −5.8968 | +18.3295 | +29.53010 |
| | | | | $x_6$ | +.42863 | +.19706 | −.61255 | −.986856 |
| | | | | | +42.00 | | +25.4708 | +50.5208 |
| | | | | 5 | −.5253 | −2.7110 | −1.0652 | −.1776 |
| | | | | 6 | −5.4976 | −2.5275 | +7.8566 | +12.6576 |
| | | | | | +35.9771 | −5.2385 | +32.2622 | +63.0008 |
| | | | | | $x_7$ | +.14561 | −.89674 | −1.75113 |
| | | | | | | +32.24 | +2.2036 | +2.2036 |
| | | | | | 2 | −.5601 | −.2648 | +.8578 |
| | | | | | 3 | −1.2968 | +1.0060 | +3.0876 |
| | | | | | 5 | −13.9903 | −5.4971 | −.9165 |
| | | | | | 6 | −1.1620 | +3.6120 | +5.8193 |
| | | | | | 7 | −.7628 | +4.6976 | +9.1733 |
| | | | | | | +14.4680 | +5.7573 | +20.22513 |
| | | | | | | $x_4$ | −.39793 | −1.39793 |

*Back solution*

| 4 | 7 | 6 | 5 | 3 | 2 | 1 |
|---|---|---|---|---|---|---|
| −0.39793 | −0.89674 | −0.61255 | +0.22568 | −0.20745 | +0.08681 | −0.38851 |
| | −.05794 | −.07842 | −.22856 | −.10641 | −.07308 | −.12655 |
| −.39793 | | −.40920 | −.10626 | −.17715 | −.02748 | −.07740 |
| −.40 | −.95468 | | −.13894 | −.03531 | −.17764 | |
| $x_4$ | −.95 | −1.10017 | | | | −.59246 |
| | $x_7$ | −1.10 | −.24808 | −.52632 | −.19139 | −.59 |
| | | $x_6$ | −.25 | −.53 | −.19 | $x_1$ |
| | | | $x_5$ | $x_3$ | $x_2$ | |

The values of the $x$'s obtained from the back solution are the quantities to be added to the assumed elevations to give the adjusted elevations which are placed in the third column of the table on page 168. These elevations which are in meters are then converted to feet for the last column to the right of the same table.

After the final elevations of the various stations have been determined, the remainder of the columns in the table on page 169 are

filled out; that is, all the columns to the right of the one headed "Symbol." The column headed "Adjusted difference of elevation $h_2-h_1$" is filled out from the data in the tables on page 168. Each $v$ in the next column is simply the adjusted $(h_2-h_1)$ minus the observed $(h_2-h_1)$ and is obtained by subtracting the quantity in column 6 from the corresponding quantity in column 9.

The column headed $pv$ is obtained by multiplying column 10 by column 4, and the column headed $pv^2$ is obtained by multiplying column 12 by column 4.

As a check on the computation the sum of all the $pv$'s for any station in columns 2 and 3 should equal zero, except for a possible discrepancy of a few hundredths due to dropping decimal places in adopting values for the $x$'s. In computing this check the sign of the $x$ must be taken into account. For example, for station Keele, correction symbol $x_1$, we have

$$\Sigma pv=(+1\times-1.72)+(+1\times+4.02)+(+1\times-1.18)+(-1\times+1.68)$$
$$+(-1\times-0.61)=-1.72+4.02-1.18-1.68+0.61=+0.05.$$

For station Hastings, correction symbol $x_2$, we have,

$$\Sigma pv=(+1\times-1.80)+(+1\times+1.68)+(+1\times+0.24)+(-1\times-1.08)$$
$$+(-1\times+0.06)+(-1\times+1.10)=-1.80+1.68+0.24+1.08-0.06$$
$$-1.10=+0.04.$$

All the remaining $x$'s can be checked in a similar manner.

## COMPUTATION OF PROBABLE ERROR

The probable error of an observation of unit weight derived from the adjustment is determined from the formula:

$$\text{Probable error}=\pm 0.6745\sqrt{\frac{\Sigma pv^2}{N_o-N_u}}$$

in which $N_o$ is the number of observed zenith distances, $N_u$ is the number of unknowns, and $\Sigma$ indicates as usual "the sum of."

$$\Sigma\, pv^2=36.3529 \quad \text{(See p. .169)}$$
$$N_o-N_u=24-7=17 \quad \text{(See Fig. 43.)}$$

$$\log 36.3529=1.56054$$
$$\text{colog } 17=\underline{8.76955-10}$$
$$\text{og } \frac{\Sigma pv^2}{N_o-N_u}=0.33009$$
$$\log\sqrt{\frac{\Sigma pv^2}{N_o-N_u}}=0.16504$$
$$\log 0.6745=\underline{9.82898-10}$$
$$\log \text{ probable error}=9.99402-10$$

Probable error (observation of unit weight) $=\pm 0.99$ m.

This means that the reciprocal observations over a line 31.6 kilometers (19⅔ miles) long, this being the length of line corresponding to unit weight, determined the difference of elevation of two points with such a degree of accuracy that it is an even chance whether the error is greater or less than 0.99 meter.

It is the general practice in the United States Coast and Geodetic Survey to compute the probable error of the elevation for that station of the net which is the farthest away from a fixed elevation and whose elevation, therefore, is least accurately determined. The probable error of this elevation can be very readily determined if the equation involving it is eliminated last in the solution of the normals. For example, in the net here considered the elevation of station Bailer is assumed to be least accurately determined. The correction to the elevation of Bailer is designated by $x_4$ in the adjustment, so the equation containing $x_4$ is eliminated last. (See p. 172.)

The formula used in computing the probable error of the elevation of any particular station is

$$(p.\ e.)_s = \pm\sqrt{\frac{(p.\ e.)_1^2}{C_w}}$$

in which $(p.\ e.)_s$ is the desired probable error of the elevation, $(p.\ e.)_1$ is the probable error for an observation of unit weight and $C_w$ is the weight coefficient of the elevation for the station in question.

The weight coefficient for any $x$ is the corresponding diagonal term before division in the solution of the normal equations. For $x_4$, the correction symbol for station Bailer, it is $+14.468$. (See p. 172.) The probable error of the elevation of Bailer is therefore

$$\pm\sqrt{\frac{(0.99)^2}{14.468}}$$

$$\log 0.99 = 9.99402 - 10 \quad \text{(See p. 173.)}$$
$$\log (0.99)^2 = 9.98804 - 10$$
$$\log 14.468 = \underline{1.16041}$$
$$\log \frac{(0.99)^2}{14.468} = 8.82763 - 10$$
$$\log\sqrt{\frac{(0.99)^2}{14.468}} = 9.41382 - 10$$

Probable error of elevation of Bailer $= \pm 0.26$ meter.

## COMPUTATION OF ELEVATIONS OF INTERSECTION STATIONS

Figure 43 shows that Hastings High School was not occupied, but was sighted upon from four stations, Monument, Keele, Hastings, and Lee. After the final elevations of these main scheme stations have been determined, the elevation of Hastings High School may be computed from the non-reciprocal observations on it from the four stations.

(See fig. 47.) The values of $(0.5-m)$ used in this computation are obtained in the manner explained on page 166. For the final elevation of Hastings High School a weighted mean of the elevations determined from the four stations is taken.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 B

COMPUTATION OF ELEVATIONS FROM NONRECIPROCAL OBSERVATIONS.

| | | | | | | |
|---|---|---|---|---|---|---|
| Station 1, occ. | Monument | Keele | Hastings | Lee | | |
| Station 2, obs. | Cupola H.S. | Cupola H.S. | Cupola H.S. | Cupola H.S. | | |
| | Hastings | Hastings | Hastings | Hastings | | |
| Object sighted | Top | Top | Top | Top | | |
| $r_1$ | 90° 04′ 53″.3 | 90° 00′ 29″.6 | 89° 52′ 57″.4 | 90° 05′ 42″.2 | | |
| $\alpha$ and mean $\phi$ | 86.8 34.2 | 19.5 34.2 | 20.8 34.2 | 78.9 34.2 | | |
| log $(0.5-m)$ | 9.67403 | 9.67367 | 9.66502 | 9.65706 | | |
| log $s$ | 4.25195 | 4.19443 | 3.78252 | 4.41427 | | |
| colog $\rho$ | 3.19485 | 3.19663 | 3.19660 | 3.19491 | | |
| colog sin 1″ | 5.31443 | 5.31443 | 5.31443 | 5.31443 | 5.31443 | 5.31443 |
| log ($k$ in secs.) | 2.43526 | 2.37916 | 1.95857 | 2.58067 | | |
| $k$ in secs. | 272″.4 | 239″.4 | 90″.9 | 380″.8 | | |
| $(90°-r_1+k)$ in secs. | -21″.4 | +209″.8 | + 513″.5 | + 38″.6 | | |
| log ditto | 1.33041 | 2.32181 | 2.71054 | 1.58659 | | |
| T | 4.68557 | 4.68558 | 4.68558 | 4.68557 | | |
| log $s$ | 4.25195 | 4.19443 | 3.78252 | 4.41427 | | |
| log $[s \tan (90°-r_1+k)]$ | 0.26793 | 1.20182 | 1.17864 | 0.68643 | | |
| log $A$ | 2 | 2 | 2 | 2 | | |
| log $B$ | 0 | 0 | 0 | 0 | | |
| log $C$ | 0 | 0 | 0 | 0 | | |
| log $(h_2-h_1)$ | 0.26795 | 1.20184 | 1.17866 | 0.68645 | | |
| $h_2-h_1$ | -1.85 | +15.92 | +15.09 | + 4.86 | | |
| $h_1$ | 329.90 | 310.41 | 312.81 | 319.75 | | |
| $t-o$ | +2.12 | + 2.12 | + 1.44 | + 6.42 | | |
| Corrected elevation | 330.17 | 328.45 | 329.34 | 331.03 | | |
| log $p=9-2 \log s$ | 0.4961 | 0.6111 | 1.4350 | 0.1715 | | |
| $p$ | 3.13 | 4.08 | 27.23 | 1.48 | | |
| Weighted mean elevation of sta. obs. | 329.38 | | | | | |

FIG. 47.—Computation of elevation of intersection station

As the final step in the computation of elevations from zenith distances, a table is prepared giving the elevations of the stations, both in meters and feet. The elevations in feet should be given to one less place of decimals than the elevations in meters. The table should be placed at the end of the computation where it may be readily found.

*Table of elevations*

| Station | Elevation | |
|---|---|---|
| | Meters | Feet |
| Monument | 329.90 | 1,082.3 |
| Benton | 301.40 | 988.8 |
| Cube | 298.80 | 980.3 |
| Gammill | 333.35 | 1,093.7 |
| Cashion | 324.65 | 1,065.1 |
| Keele | 310.41 | 1,018.4 |
| Hastings | 312.81 | 1,026.3 |
| Byers | 318.47 | 1,044.8 |
| Bailer | 313.60 | 1,028.9 |
| Lee | 319.75 | 1,049.0 |
| Thornberry | 319.90 | 1,049.5 |
| Willis | 338.05 | 1,109.1 |
| Hastings High School cupola | 329.4 | 1,081 |

# CHAPTER 6.—SPECIAL PROBLEMS CONNECTED WITH THE COMPUTATION OF TRIANGULATION

## LIST OF DIRECTIONS WITH SOME DIRECTIONS PREVIOUSLY FIXED

It frequently happens that in making out a list of directions for a particular station, some of the directions from the station have already been fixed from previous adjustments. Care must be taken to correct these directions before they are used with the other directions in another adjustment, since the new adjustment will not be consistent with the old one if the observed directions are used. Below is an example of the method used in applying corrections to the observed values of these adjusted directions to make them consistent. At station "Monument" the angle from Grande to Corpus as fixed from a previous adjustment is 55° 32′ 21″.88 and the angle from Corpus to Garcia is 99° 25′ 16″.20. The observed values of the three directions involved are given in column 2 of Figure 48.

*Computation of mean correction, station Monument*

| 1<br>Station | 2<br>Observed direction | | | 3<br>Preliminary corrected direction | 4<br>Difference (3–2) | 5<br>Final corrected direction |
|---|---|---|---|---|---|---|
| | ° | ′ | ″ | ″ | ″ | ″ |
| Grande | 25 | 00 | 27.23 | 27.23 | 0.00 | 26.47 |
| Corpus | 80 | 32 | 48.42 | 49.11 | +.69 | 48.35 |
| Garcia | 179 | 58 | 03.72 | 05.31 | +1.59 | 04.55 |

Using Grande as an initial and adding successively the two fixed angles given above we obtain the values given in column 3. The differences between these values and the observed seconds are placed in column 4. The mean of the three differences is $\frac{+2''.28}{3} = +0''.76$. This mean is then applied with opposite sign to each of the values in column 3 to obtain the final values in column 5, which are the values to be placed in the final seconds column of the list of directions at Monument. (See fig. 48.) As a check on the computation the direction to Grande should be subtracted from the direction to Corpus and the direction to Corpus from the direction to Garcia,

using the values in the final seconds column. The angles thus obtained should be identical with the fixed values given above.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 24A

State:

Station Monument — Computed by A.C.D.

Observer C.V.H. 1917 — Checked by W.F.R.

| Stations Observed | Directions After Local Adjustment ° | ′ | ″ | Final Seconds ″ |
|---|---|---|---|---|
| Ringold | 0 | 00 | 00.00 | |
| Grande | 25 | 00 | 27.23 | 26.47 |
| Hebron | 31 | 47 | 59.55 | |
| Corpus | 80 | 32 | 48.42 | 48.35 |
| Pancho | 118 | 19 | 55.00 | |
| Garcia | 179 | 58 | 03.72 | 04.55 |

FIG. 48.—List of directions, with some directions previously fixed

The rule to be followed then in making the directions already fixed consistent with the new directions is as follows: Take the observed direction at one of the fixed stations as an initial and add the fixed angles in order to obtain a set of preliminary corrected directions for the fixed stations. Take an algebraic mean of the differences obtained by subtracting the observed directions from these corrected directions and apply it with opposite sign to the latter to obtain the values for the final seconds column.

## NUMBER OF EQUATIONS IN ADJUSTMENT OF UNUSUAL FIGURES

*It frequently happens that the mathematician* in forming equations for an adjustment makes a mistake in using either too many or too few. Figure 49 is an example where the number of equations necessary to adjust it is not at once apparent.

There are only two triangles in this figure and both have one concluded angle. It would appear at first as if there were only one side equation and no angle equations. There is an angle equation, however, due to the fact that the sum of the interior angles of a quadrilateral must equal 360° plus the spherical excess of the quadrilateral. This equation is formed as follows:

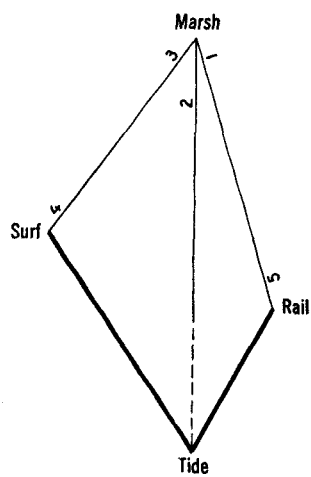



FIG. 49.—First example of unusual triangulation figure

| | ° | ′ | ″ |
|---|---|---|---|
| Angle at Tide, Surf to Rail (fixed) | 61 | 09 | 52.0 |
| Angle at Rail, Tide to Marsh (+5) | 135 | 09 | 30.6 |
| Angle at Marsh, Rail to Surf (−1+3) | 53 | 44 | 33.5 |
| Angle at Surf, Marsh to Tide (−4) | 109 | 56 | 13.2 |
| Sum −(1)+(3)−(4)+(5) | 360 | 00 | 09.3 |

As there is no spherical excess in this quadrilateral, we have

$$360° \; 00' \; 00''.0 = 360° \; 00' \; 09''.3 - (1) + (3) - (4) + (5)$$

or

$$0 = +9.3 - (1) + (3) - (4) + (5)$$

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 26

COMPUTATION OF TRIANGLES

State: .....

| NO. | STATION | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|
| 2-3 | Rail - Tide | | | | | | 3.437982 |
| -1+2 1 | Marsh | 16 25 20.7 | | | | | |
| +5 2 | Rail | 135 09 30.6 | | | | | |
| +1 -2-5 3 | Tide | (28 25 08.7) | | | | | |
| 1-3 | Marsh - Tide | | | | 0.0 | | |
| 1-2 | Marsh - Rail | | | | | | |
| 2-3 | Tide - Surf | | | | | | 3.544409 |
| -2+3 1 | Marsh | 37 19 12.3 | | | | | |
| +2 - 3+4 2 | Tide | (32 44 34.0) | | | | | |
| -4 3 | Surf | 109 56 13.2 | | | | | |
| 1-3 | Marsh - Surf | | | | 0.0 | | |
| 1-2 | Marsh - Tide | | | | | | |

FIG. 50.—Triangle computation for Figure 49

Or the equation may be formed in another way. The sum of the two concluded angles should equal the fixed angle at Tide. That is

| | ° | ′ | ″ |
|---|---|---|---|
| Angle at Tide, Marsh to Rail (+1−2−5) | 28 | 25 | 08.7 |
| Angle at Tide, Surf to Marsh (+2−3+4) | 32 | 44 | 34.0 |
| Angle at Tide, Surf to Rail (+1−3+4−5) | 61 | 09 | 42.7 |

Since the fixed angle at Tide, Surf to Rail, is 61° 09′ 52″.0, we have

$$61° \; 09' \; 42''.7 + (1) - (3) + (4) - (5) = 61° \; 09' \; 52''.0$$

or

$$0 = +9.3 - (1) + (3) - (4) + (5)$$

In Figure 51 is given another example, occasionally encountered, in which the proper number of equations for the adjustment is not very apparent. Ordinarily one would think that there are two angle and one side equations in the quadrilateral *C D F E* and two angle and one side equations in the quadrilateral *A B D C*, or a total of six equations. However, if the figure is built up point by point, it is seen that there are seven equations, as shown below:

| Station | Number of equations | |
|---|---|---|
| | Angle | Side |
| C | 1 | 0 |
| D | 1 | 1 |
| B | 1 | 0 |
| A | 2 | 1 |
| Total | 5 | 2 |

The additional angle equation is obtained by the closure of either of the two quadrilaterals *A D F C* or *A D E C*. If the latter one is used then the equation is, angle *C A D* + angle *A D E* + angle *D E C* + angle *E C A* − the closure = 360° + spherical excess.

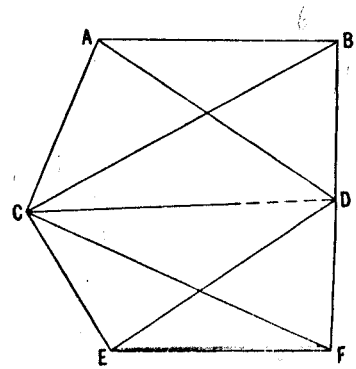



FIG. 51.—Second example of unusual triangulation figure

## SIDE EQUATION IN FORM OF LENGTH EQUATION

It frequently happens in the adjustment of an intersection station from three fixed points, that the length and azimuth between two of the fixed points are not known. In this case the side equation will take the form of a length equation. An example of such an adjustment is given below. (See fig. 52.)

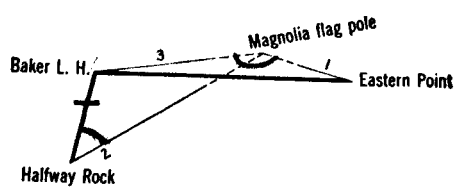



FIG. 52.—Triangulation figure requiring length equation instead of ordinary side equation

*Side equation*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Baker L.H.—Halfway Rock | | 3. 588491 | ---------- | Baker L.H.—Eastern Point | | −2 4. 045264 | ---------- |
| | ° ′ ″ | | | | ° ′ ″ | | |
| +2 | 47 46 55. 1 | 9. 8695797 | +1. 91 | −2+3 | 22 43 22. 5 | 9. 5868966 | +5. 03 |
| −1+3 | 159 14 06. 0 | 9. 5496601 | −5. 55 | +1 | 13 44 18. 9 | 9. 3756496 | +8. 61 |
| | | 3. 0077308 | | | | 3. 0078100 | |

$$0 = -79.2 - 3.06(1) + 6.94(2) - 10.58(3)$$

*Correlate equation*

| | | $v$ | Adopted $v$ |
|---|---|---|---|
| 1 | −3.06 | −1.430 | −1.4 |
| 2 | +6.94 | +3.244 | +3.2 |
| 3 | −10.58 | −4.945 | −4.9 |

*Solution of normal equation*

$$0 = -79.2 + 169.4636\ C$$
$$C = +0.4674$$

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

*State:* .....

| NO. | STATION | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|
| | 2-3 Eastern Point-Baker L.H. | | | | | | 4.045264 |
| | 1 Magnolia flag pole | 159 14 06.0 | -3.5 | | | 02.5 | 0.450320 |
| +1 | 2 Eastern Point | 13 44 18.9 | -1.4 | | | 17.5 | 9.375637 |
| -3 | 3 Baker L.H. | 7 01 35.1 | +4.9 | | | 40.0 | 9.087606 |
| | 1-3 Magnolia flag pole-Baker L.H. | | | | | | 3.871221 |
| | 1-2 Magnolia flag pole-Eastern Point | | | | | | 3.583190 |
| | 2-3 Halfway Rock-Baker L.H. | | | | | | 3.588491 |
| | 1 Magnolia flag pole | 22 43 22.5 | -8.1 | 14.4 | | 14.4 | 0.413144 |
| +2 | 2 Halfway Rock | 47 46 55.1 | +3.2 | 58.3 | | 58.3 | 9.869586 |
| -3 | 3 Baker L.H. | 109 29 42.5 | +4.9 | 47.4 | 0.1 | 47.3 | 9.974356 |
| | 1-3 Magnolia flag pole-Baker L.H. | | | | 0.1 | | 3.871221 |
| | 1-2 Magnolia flag pole-Halfway Rock | | | | | | 3.975991 |

FIG. 53.—Triangle computation for Figure 52

## IDENTICAL EQUATIONS

One must be very careful in the selection of equations for an adjustment, especially if the scheme is much involved, to avoid what is known as an "identical equation," which often is not discovered until the solution of equations is made. In the solution, the diagonal term of the identical equation will become zero, or nearly so, and the error in selecting the equation thus becomes known.

Below is given an illustration of a set of five equations, three angle and two side, in which one of the side equations is an "identical equation." The solution of the equations is carried as far as the diagonal term of the identical equation. It is seen that the diagonal term of the fifth equation is practically zero, and this means that the fifth equation is not an independent equation.

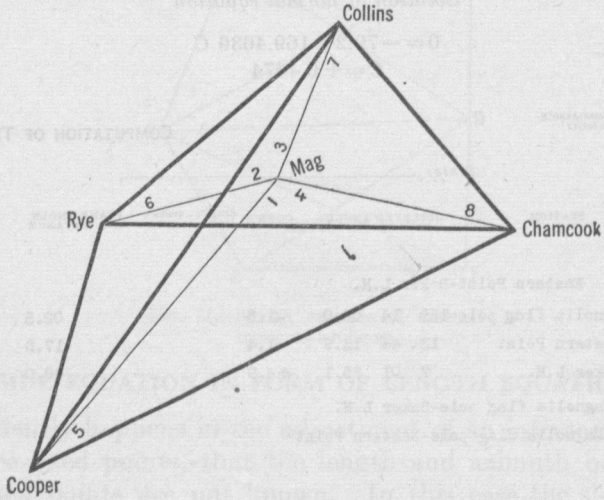



FIG. 54.—Triangulation figure for which "identical equation" was selected

*Angle equations*

1. $0=+3.7-(1)+(2)+(5)-(6)$
2. $0=-2.4-(2)+(3)+(6)-(7)$
3. $0=+1.1-(3)+(4)+(7)-(8)$

*Side equations*

| | ° ′ ″ | | | | ° ′ ″ | | |
|---|---|---|---|---|---|---|---|
| −7 | 27 53 43.7 | 9.6701159 | +3.98 | +6 | 25 57 29.1 | 9.6411899 | +4.32 |
| −6 | 16 52 35.5 | 9.4628623 | +6.94 | +8 | 12 39 55.0 | 9.3409495 | +9.37 |
| −8 | 37 22 42.4 | 9.7832437 | +2.76 | +7 | 59 13 35.2 | 9.9340924 | +1.25 |
| | | 8.9162219 | | | | 8.9162318 | |

4. $0=-9.9-11.26(6)-5.23(7)-12.13(8)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | −13 | | | | −8 | |
| Rye-Chamcook | | 4.4295047 | ---------- | Rye-Collins | | 4.3145841 | ---------- |
| +8 | 12 39 55.0 | 9.3409495 | +9.37 | +2−4 | 150 27 31.0 | 9.6928929 | −3.72 |
| −2+3 | 126 08 45.1 | 9.9071522 | −1.54 | −7 | 27 53 43.7 | 9.6701159 | +3.98 |
| | | 3.6776051 | 7.83 | | | 3.6775921 | +0.26 |

5. $0=+13.0+5.26(2)-1.54(3)-3.72(4)+3.98(7)+9.37(8)$

13.0 / 8.09 = 1.61

7.83
0.26
8.09

*Correlate equations*

| | 1 | 2 | 3 | 4 | 5 | $\Sigma_0$ |
|---|---|---|---|---|---|---|
| 1 | −1 | | | | | −1.00 |
| 2 | +1 | −1 | | | +5.26 | +5.26 |
| 3 | | +1 | −1 | | −1.54 | −1.54 |
| 4 | | | +1 | | −3.72 | −2.72 |
| 5 | +1 | | | | | +1.00 |
| 6 | −1 | +1 | | −11.26 | | −11.26 |
| 7 | | −1 | +1 | −5.23 | +3.98 | −1.25 |
| 8 | | | −1 | −12.13 | +9.37 | −3.76 |

*Normal equations*

| | 1 | 2 | 3 | 4 | 5 | $\eta$ | $\Sigma_n$ |
|---|---|---|---|---|---|---|---|
| 1 | +4 | −2 | | +11.26 | +5.26 | +3.7 | +22.22 |
| 2 | | +4 | −2 | −6.03 | −10.78 | −2.4 | −19.21 |
| 3 | | | +4 | +6.90 | −7.57 | +1.1 | +2.43 |
| 4 | | | | +301.2774 | −134.4735 | −9.9 | +169.0339 |
| 5 | | | | | +147.5149 | +13.0 | +12.9514 |

*Solution of normal equations*

| 1 | 2 | 3 | 4 | 5 | $\eta$ | $\Sigma_n$ |
|---|---|---|---|---|---|---|
| +4 | −2 | | +11.26 | +5.26 | +3.7 | +22.22 |
| $C_1$ | +.5 | | −2.815 | −1.315 | −.925 | −5.555 |
| | +4 | −2 | −6.03 | −10.78 | −2.4 | −19.21 |
| 1 | −1 | | +5.63 | +2.63 | +1.85 | +11.11 |
| | +3 | −2 | −.40 | −8.15 | −.55 | −8.10 |
| | $C_2$ | +.66667 | +.13333 | +2.71667 | +.18333 | +2.70 |
| | | +4 | +6.90 | −7.57 | +1.1 | +2.43 |
| | 2 | −1.3333 | −.2667 | −5.4333 | −.3667 | −5.40 |
| | | +2.6667 | +6.6333 | −13.0033 | +.7333 | −2.97 |
| | | $C_3$ | −2.4875 | +4.87625 | −.2750 | +1.11375 |
| | | | +301.2774 | −134.4735 | −9.9 | +169.0339 |
| | | 1 | −31.6969 | −14.8069 | −10.4155 | −62.5493 |
| | | 2 | −.0533 | −1.0867 | −.0733 | −1.0800 |
| | | 3 | −16.5003 | +32.3456 | −1.8242 | +7.3878 |
| | | | +253.0269 | −118.0215 | −22.2130 | +112.7924 |
| | | | $C_4$ | +.46644 | +.08779 | −.44577 |
| | | | | +147.5149 | +13.0 | +12.9514 |
| | | | 1 | −6.9169 | −4.8655 | −29.2193 |
| | | | 2 | −22.1409 | −1.4941 | −22.0050 |
| | | | 3 | −63.4073 | +3.5759 | −14.4825 |
| | | | 4 | −55.0500 | −10.3611 | +52.6104 |
| | | | | −.0002 | −.1448 | −.1450 |

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Maine

| NO. | STATION | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|
| | 2-3 | Cooper-Rye | | | | | 4.1907270 |
| -1+2 | 1 Mag | 34 29 20.9 | | | 0.1 | | |
| +5 | 2 Cooper | 25 52 15.7 | | | 0.1 | | |
| -6 | 3 Rye | 119 38 27.5 | | | 0.2 | | |
| | 1-3 | Mag-Rye | +3.7 | | 0.4 | | |
| | 1-2 | Mag-Cooper | | | | | |
| | 2-3 | Cooper-Collins | | | | | 4.5386319 |
| -1+3 | 1 Mag | 160 38 06.0 | | | 0.1 | | |
| +5 | 2 Cooper | 6 09 35.8 | | | | | |
| -7 | 3 Collins | 13 12 19.7 | | | 0.1 | | |
| | 1-3 | Mag-Collins | -1.3 | | 0.2 | | |
| | 1-2 | Mag-Cooper | | | | | |
| | 2-3 | Rye-Collins | | | | | 4.3145841 |
| -2+3 | 1 Mag | 126 08 45.1 | | | 0.1 | | |
| +6 | 2 Rye | 25 57 29.1 | | | 0.1 | | |
| -7 | 3 Collins | 27 53 43.7 | | | 0.1 | | |
| | 1-3 | Mag-Collins | +2.4 | | 0.3 | | |
| | 1-2 | Mag-Rye | | | | | |
| | 2-3 | Collins-Chamcook | | | | | 4.2624872 |
| -3+4 | 1 Mag | 83 23 43.9 | | | 0.2 | | |
| +7 | 2 Collins | 59 13 35.2 | | | 0.1 | | |
| -8 | 3 Chamcook | 37 22 42.4 | | | 0.1 | | |
| | 1-3 | Mag-Chamcook | -1.1 | | 0.4 | | |
| | 1-2 | Mag-Collins | | | | | |
| | 2-3 | Chamcook-Cooper | | | | | 4.5299291 |
| +1-4 | 1 Mag | 115 58 10.1 | | | 0.3 | | |
| +8 | 2 Chamcook | 39 11 30.8 | | | 0.3 | | |
| -5 | 3 Cooper | 24 50 17.6 | | | 0.3 | | |
| | 1-3 | Mag-Cooper | +2.4 | | 0.9 | | |
| | 1-2 | Mag-Chamcook | | | | | |
| | 2-3 | Chamcook-Rye | | | | | 4.4295047 |
| +2-4 | 1 Mag | 150 27 31.0 | | | 0.1 | | |
| +8 | 2 Chamcook | 12 39 55.0 | | | | | |
| -6 | 3 Rye | 16 52 35.5 | | | 0.1 | | |
| | 1-3 | Mag-Rye | -1.3 | | 0.2 | | |
| | 1-2 | Mag-Chamcook | | | | | |

FIG. 55.—Triangle computation for Figure 54

A careful inspection of the side equations on page 182 will show that both equations relate to the quadrilateral Rye-Mag-Chamcook-Collins, and they differ only in that Mag is the pole in the fourth equation and Rye in the fifth. The fifth equation should have been formed for the quadrilateral Mag-Chamcook-Cooper-Rye, with the pole at Mag. Although it is easy to see in a simple figure such as the one above when an "identical equation" is selected, it is much more difficult to see in the case of more complicated figures and all equations should therefore be selected very carefully.

## ADJUSTMENT OF QUADRILATERAL WITH POLE AT INTERSECTION OF DIAGONALS

In the adjustment of quadrilaterals it frequently happens that it is impossible to include the two smallest angles in the formation of the side equations by taking the pole at any one of the four vertices. In such a case the pole should be taken at the intersection of the diagonals. An example of such an adjustment is given below. (See fig. 56.)

The angle equations are formed in exactly the same manner as in the ordinary quadrilateral. (See p. 35.) The side equation is formed as follows: Call the intersection of the diagonals O. (See fig. 56.)

Then

$$\frac{\text{O}-\text{Tall}}{\text{O}-\text{Muketeo L. H.}}\times\frac{\text{O}-\text{Ridge}}{\text{O}-\text{Tall}}\times\frac{\text{O}-\text{Stump}}{\text{O}-\text{Ridge}}\times\frac{\text{O}-\text{Muketeo L. H.}}{\text{O}-\text{Stump}}=1.$$

Substituting the sines of the angles opposite the sides for the sides we have

$$\frac{\sin(-3+4)}{\sin(-5+6)}\times\frac{\sin(-6+7)}{\sin(-8+9)}\times\frac{\sin(-9+10)}{\sin(-1+2)}\times\frac{\sin(-2)}{\sin(+3)}=1.$$

The equation is then tabulated as explained on page 36 except in one particular. In the example given there the designation of the angle in the denominator appears on the same horizontal line in the tabulated equation as the designation of the angle in the numerator, and the angles corresponding to these designations are taken from the same triangle. In the example given here, however, this is not true, that is, the angle designated by $(-5+6)$ is not in the same triangle as the angle designated by $(-3+4)$, etc. It is best, however, to arrange the tabulated side equation so that the two angles from the same triangle are on the same horizontal line. This can be done as follows: With the pole at the intersection of the diagonals there will be 4 lines for the tabulated equation. Put the designation of the angles in the numerators of the first, second, third, and fourth fractions on lines 1, 2, 3, and 4, respectively, of the left side of the equation, and the designations of the angles in the denominators of the same fractions in lines 4, 1, 2, and 3, respectively, of the right side

of the equation. That is, in the example given here $(-3+4)$, $(-6+7)$, $(-9+10)$, and $(-2)$ are placed in lines 1, 2, 3, and 4, respectively, of the left side of the equation, and $(-5+6)$, $(-8+9)$, $(-1+2)$, and $(+3)$ are placed in lines 4, 1, 2, and 3, respectively, of the right side of the equation. The two angles in the same horizontal line in the side equation can then be taken out of the same triangle.

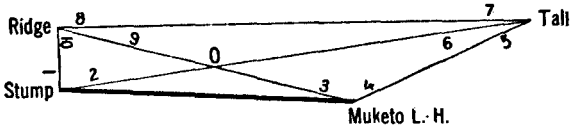



FIG. 56.—Quadrilateral with pole at intersection of diagonals

The remainder of the adjustment of the quadrilateral is similar to the one previously adjusted on pages 29–41.

*Angle equations*

1. $0=+4.9-(1)+(3)-(9)+(10)$
2. $0=-2.0-(2)+(4)-(5)+(6)$
3. $0=+3.4-(3)+(4)-(5)+(7)-(8)+(9)$

*Side equation*

| | ° ′ ″ | | | | ° ′ ″ | | |
|---|---|---|---|---|---|---|---|
| −3+4 | 139 26 09.8 | 9.8131113 | −2.5 | −8+9 | 16 03 51.3 | 9.4420329 | +7.3 |
| −6+7 | 7 26 41.4 | 9.1125095 | +16.1 | −1+2 | 80 24 45.5 | 9.9938914 | +.4 |
| −9+10 | 76 04 52.1 | 9.9870570 | +.5 | +3 | 11 53 01.8 | 9.3137156 | +10.0 |
| −2 | 11 37 25.5 | 9.3042404 | +10.2 | −5+6 | 17 03 20.9 | 9.4673165 | +6.9 |
| | | 8.2169182 | | | | 8.2169564 | |

4. $0=-38.2+0.4(1)-10.6(2)-7.5(3)-2.5(4)+6.9(5)-23.0(6)+16.1(7)+7.3(8)-7.8(9)+0.5(10)$ (This equation should be divided by 5 before entering it in the table of correlates.)

*Correlate equations*

| | 1 | 2 | 3 | 4 | $\Sigma_o$ | $v$ | Adopted $v$ |
|---|---|---|---|---|---|---|---|
| 1 | −1 | ------- | ------- | +0.08 | −0.92 | +2.202 | +2.2 |
| 2 | ------- | −1 | ------- | −2.12 | −3.12 | −2.541 | −2.6 |
| 3 | +1 | ------- | −1 | −1.50 | −1.50 | −.801 | −.8 |
| 4 | ------- | +1 | +1 | −.50 | +1.50 | −.242 | −.3 |
| 5 | ------- | −1 | −1 | +1.38 | −.62 | +.543 | +.5 |
| 6 | ------- | +1 | ------- | −4.60 | −3.60 | +.243 | +.2 |
| 7 | ------- | ------- | +1 | +3.22 | +4.22 | −.786 | −.8 |
| 8 | ------- | ------- | −1 | +1.46 | +.46 | +2.386 | +2.4 |
| 9 | −1 | ------- | +1 | −1.56 | −1.56 | −.245 | −.2 |
| 10 | +1 | ------- | ------- | +.10 | +1.10 | −2.141 | −2.1 |

*Normal equations*

| | 1 | 2 | 3 | 4 | $\eta$ | $\Sigma_n$ | $C$ |
|---|---|---|---|---|---|---|---|
| 1 | +4 | ------- | −2 | +0.08 | +4.9 | +6.98 | −2.1752 |
| 2 | | +4 | +2 | −4.36 | −2.0 | −.36 | +1.8162 |
| 3 | | | +6 | −.18 | +3.4 | +9.22 | −1.8869 |
| 4 | | | | +45.0088 | −7.64 | +32.9088 | +.3420 |

*Solution of normal equations*

| 1 | 2 | 3 | 4 | $\eta$ | $\Sigma_n$ |
|---|---|---|---|---|---|
| +4 | ------- | −2 | +0.08 | +4.9 | +6.98 |
| $C_1$ | | +.5 | −.02 | −1.225 | −1.745 |
| | +4 | +2 | −4.36 | −2.0 | −.36 |
| | $C_2$ | −.5 | +1.09 | +.5 | +.09 |
| | | +6 | −.18 | +3.4 | +9.22 |
| | 1 | −1 | +.04 | +2.45 | +3.49 |
| | 2 | −1 | +2.18 | +1.00 | +.18 |
| | | +4 | +2.04 | +6.85 | +12.89 |
| | | $C_3$ | −.51 | −1.7125 | −3.2225 |
| | | | +45.0088 | −7.64 | +32.9088 |
| | | 1 | −.0016 | −.098 | −.1396 |
| | | 2 | −4.7524 | −2.180 | −.3924 |
| | | 3 | −1.0404 | −3.4935 | −6.5739 |
| | | | +39.2144 | −13.4115 | +25.8029 |
| | | | $C_4$ | +0.34200 | −.65800 |

*Back solution*

| 4 | 3 | 2 | 1 |
|---|---|---|---|
| +0.3420 | −1.7125 | +0.5 | −1.225 |
| | −.1744 | +.3728 | −.0068 |
| | | +.9434 | −.9434 |
| | −1.8869 | | |
| | | +1.8162 | −2.1752 |

*Computation of v's*

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| +2.175 | −1.816 | −2.175 | +1.816 | −1.816 | +1.816 | −1.887 | +1.887 | +2.175 | −2.175 |
| +.027 | −.725 | +1.887 | −1.887 | +1.887 | −1.573 | +1.101 | +.499 | −1.887 | +.034 |
| | | −.513 | −.171 | +.472 | | | | −.533 | |
| +2.202 | −2.541 | | | | +.243 | −.786 | +2.386 | | −2.141 |
| +2.2 | −2.6 | −.801 | −.242 | +.543 | +.2 | −.8 | +2.4 | −.245 | −2.1 |
| | | −.8 | −.3 | +.5 | | | | −.2 | |

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

*State:* Massachusetts

| | NO. | | STATION | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|---|---|
| | | 2-3 | Muketeo L. H. - Stump | | | | | | 3.526870 |
| | -9+10 | 1 | Ridge | 76 04 52.1 | -1.9 | | | 50.2 | 0.012944 |
| 1 | +3 | 2 | Muketeo L.H. | 11 53 01.8 | -0.8 | | | 01.0 | 9.313708 |
| | -1 | 3 | Stump | 92 02 11.0 | -2.2 | | | 08.8 | 9.999726 |
| | | 1-3 | Ridge - Stump | | -4.9 | | | | 2.853512 |
| | | 1-2 | Ridge - Muketeo L.H. | | | | | | 3.539540 |
| | | | | 04.9 | | | | | |
| | | 2-3 | Muketeo L.H. - Stump | | | | | | 3.526870 |
| | -5+6 | 1 | Tall | 17 03 20.9 | -0.3 | | | 20.6 | 0.532685 |
| 2 | +4 | 2 | Muketeo L.H. | 151 19 11.6 | -0.3 | | | 11.3 | 9.681169 |
| | -2 | 3 | Stump | 11 37 25.5 | +2.6 | | | 28.1 | 9.304267 |
| | | 1-3 | Tall - Stump | | +2.0 | | | | 3.740724 |
| | | 1-2 | Tall - Muketeo L.H. | | | | | | 3.363822 |
| | | | | 58.0 | | | | | |
| | | 2-3 | Muketeo L.H. - Ridge | | | | | | 3.539540 |
| | -5+7 | 1 | Tall | 24 30 02.3 | -1.3 | | | 01.0 | 0.382268 |
| 3 | -3+4 | 2 | Muketeo L.H. | 139 26 09.8 | +0.5 | | | 10.3 | 9.813110 |
| | -8+9 | 3 | Ridge | 16 03 51.3 | -2.6 | | | 48.7 | 9.442014 |
| | | 1-3 | Tall - Ridge | | -3.4 | | | | 3.734918 |
| | | 1-2 | Tall - Muketeo L.H. | | | | | | 3.363822 |
| | | | | 03.4 | | | | | |
| | | 2-3 | Stump - Ridge | | | | | | 2.853522 |
| | -6+7 | 1 | Tall | 7 26 41.4 | -1.0 | | | 40.4 | 0.887506 |
| 4 | -1+2 | 2 | Stump | 80 24 45.5 | -4.8 | | | 40.7 | 9.993890 |
| | -8+10 | 3 | Ridge | 92 08 43.4 | -4.5 | | | 38.9 | 9.999696 |
| | | 1-3 | Tall - Ridge | | -10.3 | | | | 3.734918 |
| | | 1-2 | Tall - Stump | | | | | | 3.740724 |
| | | | | 10.3 | | | | | |

FIG. 57.—Triangle computation for Figure 56

## ADJUSTMENT OF QUADRILATERAL HAVING ONE TRIANGLE WITH TWO CONCLUDED ANGLES

It sometimes becomes necessary to make an adjustment of a quadrilateral in which one triangle has two concluded angles. In such an adjustment the thing to be particularly careful about is the proper designation of the angles. In the following example (see fig. 58) the triangle, Ramparts flag-Bond Hill-Ten Pound Island L. H. has two concluded angles, one at Ramparts flag and the other at Ten Pound Island L. H. In triangle No. 4, Figure 59, the angle at Ten Pound Island L. H. is the sum of the two angles at Ten Pound Island L. H. between Eastern Pt. L. H. and Bond Hill, and between Ramparts flag and Eastern Pt. L. H. In triangles Nos. 1 and 3,

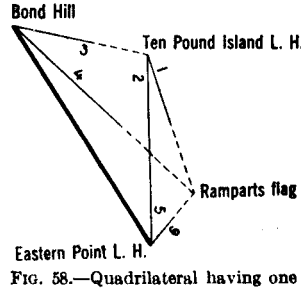


FIG. 58.—Quadrilateral having one triangle with two concluded angles

| | ° | ′ | ″ |
|---|---|---|---|
| +3−5 (Ten Pound Id. L. H. between Eastern Pt. L. H. and Bond Hill) = | 104 | 53 | 11. 1 |
| −1+2 (Ten Pound Id. L. H. between Ramparts flag and Eastern Pt. L. H.) = | 16 | 09 | 19. 0 |
| −1+2+3−5 (Ten Pound Id. L. H. between Ramparts flag and Bond Hill) = | 121 | 02 | 30. 1 |

The angle at Ramparts flag in triangle No. 4 is obtained by subtracting from 180° 00′ 00″.0 plus the spherical excess (which in this case is 0″.0) the sum of the angles at Bond Hill and Ten Pound Island L. H., using the proper designations as shown below.

| | | ° | ′ | ″ |
|---|---|---|---|---|
| −3+4 | (Bond Hill) = | 30 | 52 | 02. 6 |
| −1+2+3−5 | (Ten Pound Id. L. H.) = | 121 | 02 | 30. 1 |
| −1+2+4−5 | (sum of 2 angles) | 151 | 54 | 32. 7 |
| | | 180 | 00 | 00. 0 |
| +1−2−4+5 | (Ramparts flag) | 28 | 05 | 27. 3 |

After these angles have been computed and properly designated, the adjustment of the quadrilateral is exactly the same as that of any other quadrilateral, and so no further explanation is necessary here.

*Side equation*

| | ° | ′ | ″ | | | | ° | ′ | ″ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| −1+2+3−5 | 121 | 02 | 30. 1 | 9. 9328756 | −1. 3 | −3+4 | 30 | 52 | 02. 6 | 9. 7101620 | +3. 5 |
| −4 | 13 | 58 | 30. 0 | 9. 3829144 | +8. 5 | +6 | 70 | 59 | 35. 3 | 9. 9756521 | +0. 7 |
| −5+6 | 40 | 43 | 19. 0 | 9. 8145064 | +2. 4 | −1+2 | 16 | 09 | 19. 0 | 9. 4444218 | +7. 3 |
| | | | | 9. 1302964 | | | | | | 9. 1302359 | |

0 = +60. 5+8. 6(1) −8. 6(2) +2. 2(3) −12. 0(4) −1. 1(5) +1. 7(6)

*Correlate equation*

| | $'$ | $v$ | Adopted $v$ |
|---|---|---|---|
| 1 | +8.6 | −1.73 | −1.7 |
| 2 | −8.6 | +1.73 | +1.7 |
| 3 | +2.2 | −.44 | −.4 |
| 4 | −12.0 | +2.41 | +2.4 |
| 5 | −1.1 | +.22 | +.2 |
| 6 | +1.7 | −.34 | −.3 |

*Solution of normal equation*

$$0=+60.5+300.86C$$
$$C=-0.2011$$

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 26

COMPUTATION OF TRIANGLES

*State:* Massachusetts

| NO. | STATION | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|
| | 2-3 Eastern Pt. L.H.- Bond Hill | | | | | | 3.517770 |
| +3-5 | 1 Ten Pound Id.L.H. | 104 53 11.1 | -0.6 | | | 10.5 | 0.014826 |
| 1. +5 | 2 Eastern Pt.L.H. | 30 16 16.3 | +0.3 | | | 16.5 | 9.702512 |
| -3 | 3 Bond Hill | 44 50 32.6 | +0.4 | | | 33.0 | 9.848288 |
| | 1-3 Ten Pound Id.L.H.-Bond Hill | | | | | | 3.235108 |
| | 1-2 Ten Pound Id.L.H.-Eastern Pt.L.H. | | | | | | 3.380884 |
| | 2-3 Eastern Pt.L.H. - Bond Hill | | | | | | 3.517770 |
| +4-6 | 1 Ramparts flag | 95 01 54.7 | +2.7 | | | 57.4 | 0.001677 |
| 2. +6 | 2 Eastern Pt.L.H. | 70 59 35.3 | -0.3 | | | 35.0 | 9.975652 |
| -4 | 3 Bond Hill | 13 58 30.0 | -2.4 | | | 27.6 | 9.338894 |
| | 1-3 Ramparts flag - Bond Hill | | | | | | 3.495099 |
| | 1-2 Ramparts flag - Eastern Pt.L.H. | | | | | | 2.902341 |
| | 2-3 Eastern Pt.L.H.-Ten Pound Id.L.H. | | | | | | 3.380884 |
| +1-2 +5-6 | 1 Ramparts flag | (123 07 22.0) | -2.9 | | | 19.1 | 0.077010 |
| 3. -5+6 | 2 Eastern Pt.L.H. | 40 43 19.0 | -0.5 | | | 18.5 | 9.814505 |
| -1+2 | 3 Ten Pound Id.L.H. | 16 09 19.0 | +3.4 | | | 22.4 | 9.444447 |
| | 1-3 Ramparts flag-Ten Pound Id.L.H. | | | | | | 3.272399 |
| | 1-2 Ramparts flag-Eastern Pt.L.H. | | | | | | 2.902341 |
| | 2-3 Bond Hill - Ten Pound Id.L.H. | | | | | | 3.235108 |
| +1-2 -4+5 | 1 Ramparts flag | (28 05 27.3) | -5.6 | | | 21.7 | 0.327119 |
| 4. -3+4 | 2 Bond Hill | 30 52 02.6 | +2.8 | | | 05.4 | 9.710172 |
| -1+2 +3-5 | 3 Ten Pound Id.L.H. | (121 02 30.1) | +2.8 | | | 32.9 | 9.932872 |
| | 1-3 Ramparts flag-Ten Pound Id.L.H. | | | | | | 3.272399 |
| | 1-2 Ramparts flag-Bond Hill | | | | | | 3.495099 |

FIG. 59.—Triangle computation for Figure 58

## THREE-POINT PROBLEM

A triangulation station is sometimes determined by means of directions observed at that station to the three fixed points of a triangle, the sides and angles of which are either known or can be computed. This is called the "three-point problem."

The computations are made on Form 655 as shown in Figure 61. Three cases are illustrated on this form, depending upon the location of the point, designated $P$, with reference to the sides of the triangle. If $P$ is on the circumference of the circle which passes through the vertices of the fixed triangle, the problem is indeterminate, since any point on this circumference would have the same values of the angles $P'$ and $P''$ (or the supplement of one of the angles).

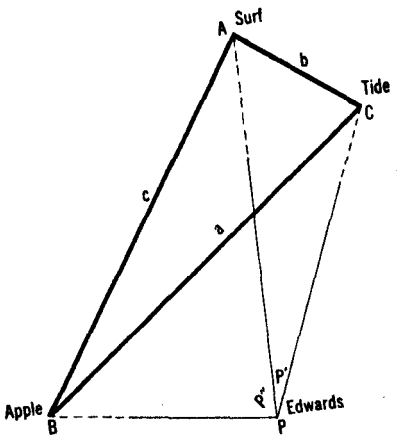



FIG. 60.—Quadrilateral, three-point problem

The formulas used in the computation are as follows: From the known sides $a$, $b$, $c$, and the known angle $A$ of the triangle $A\ B\ C$, and the observed angles $A\ P\ C = P'$ and $A\ P\ B = P''$, the problem is to find the angles $A\ B\ P$ and $A\ C\ P$.

$$\text{For cases 1 and 2, let } S = 180° - {}^1/_2\,(A + P' + P''),$$
$$\text{and for case 3, let } S = {}^1/_2\,(A - P' - P'');$$
$$\text{Let } \tan Z = \frac{C \sin P'}{b \sin P''}$$
$$\text{and } \tan \epsilon = \cot\,(Z + 45°)\,\tan S$$

Then angle $A\ B\ P = S + \epsilon$, and angle $A\ C\ P = S - \epsilon$ if $\tan \epsilon$ is positive, and angle $A\ B\ P = S - \epsilon$, and angle $A\ C\ P = S + \epsilon$ if $\tan \epsilon$ is negative.

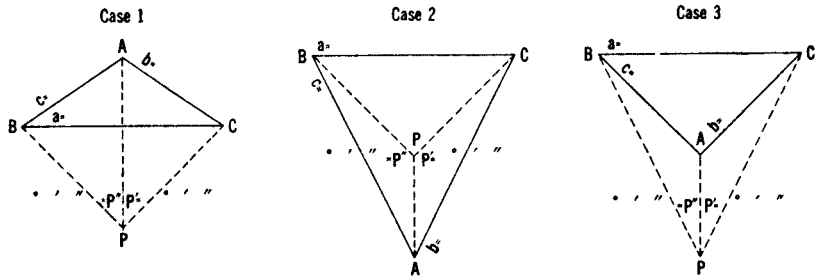
After the angles $A\ B\ P$ and $A\ C\ P$ are computed all the angles of the triangles can be obtained, and since the length of one side in each triangle is known all the remaining lengths can be computed. As each computed length is obtained in two different triangles, the agreement of the two values for the logarithm of each length within one, or possibly two, in the last place of decimals gives a check on the computation.

The example given below is for case 1 on Form 655. In the triangle Tide-Apple-Surf all the angles and sides are known and at Edwards the angles Apple to Surf and Surf to Tide are observed. The starting data are: $P' = 21°\ 38'\ 06''.8$, $P'' = 84°\ 12'\ 57''.9$, $A = 86°\ 29'\ 42''.3$, $\log c = 4.109221$, and $\log b = 3.644409$. The

problem is to find the angle at Apple between Surf and Edwards and the angle at Tide between Edwards and Surf. After these angles have been determined, the logarithms of the lengths are computed on Form 25.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 655

COMPUTATION OF THREE-POINT PROBLEM




| Cases 1 and 2 | | | | Case 3 | |
|---|---|---|---|---|---|
| P′ | 21 | 38 | 06.8 | P′ | |
| P″ | 84 | 12 | 57.9 | P″ | |
| A | 86 | 29 | 42.3 | | |
| Sum | 192 | 20 | 47.0 | Sum | |
| ½ Sum | 96 | 10 | 23.5 | A | |
| | | | | A − sum | |
| S = 180° − ½ sum = | 83 | 49 | 36.5 | S = ½ (A − sum) = | |

Log c = 4.109221
Log sin P′ = 9.566668 -10
Colog b = 6.355591 -10
Colog sin P″ = 0.002217

Sum = log tan Z = 0.033697

| | ° | ′ | ″ |
|---|---|---|---|
| Z = | 47 | 13 | 14.0 |
| Z + 45° = | 92 | 13 | 14.0 |

Log cot (Z + 45°) = $8.588556_m$ 10
Log tan S = 0.965928

Sum = log tan ε = $9.554484_n$-10 (sign − )

| | ° | ′ | ″ |
|---|---|---|---|
| ε | 19 | 43 | 21.1 |
| S | 83 | 49 | 36.5 |
| (Tan ε +) S + ε = angle ABP | 64 | 06 | 15.4 |
| S − ε = angle ACP | 103 | 32 | 57.6 |

(Tan ε −)
S − ε = angle ABP
S + ε = angle ACP

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| BPA | 84 | 12 | 57.9 | APC | 21 | 38 | 06.8 | PCB | 29 | 18 | 51.3 |
| ABP | 64 | 06 | 15.4 | PCA | 103 | 32 | 57.6 | CBP | 44 | 50 | 04.0 |
| PAB | 31 | 40 | 46.7 | CAP | 54 | 48 | 55.6 | BPC | 105 | 51 | 04.7 |

(For explanation of this form see Special Publication No. 26, 2d edition, paragraph 108, page 57)

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FIG. 61.—Computation of three-point problem

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Maine

| NO. | STATION | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|
| 2-3 | Apple-Surf | | | | | | 4.109221 |
| 1 | Tide | 74 14 10.9 | -4.6 | | | 06.3 | 0.016651 |
| 2 | Apple | 19 16 12.3 | -0.9 | | | 11.4 | 9.518537 |
| 3 | Surf | 86 29 45.9 | -3.5 | 42.4 | 0.1 | 42.3 | 9.999187 |
| 1-3 | Tide-Surf | | -9.0 | | 0.1 | | 3.644409 |
| 1-2 | Tide-Apple | | | | | | 4.125059 |
| 2-3 | Apple-Surf | | | | | | 4.109221 |
| 1 | Edwards | 84 12 57.9 | | | | 57.9 | 0.002217 |
| 2 | Apple | 64 06 | | 15.5 | 0.1 | 15.4 | 9.954045 |
| 3 | Surf | 31 40 | | 46.8 | 0.1 | 46.7 | 9.720299 |
| 1-3 | Edwards-Surf | | | | 0.2 | | 4.065483 |
| 1-2 | Edwards-Apple | | | | | | 3.831737 |
| 2-3 | Apple-Tide | | | | | | 4.125059 |
| 1 | Edwards | 105 51 04.7 | | | | 04.7 | 0.016837 |
| 2 | Apple | 44 50 | | 04.1 | 0.1 | 04.0 | 9.848236 |
| 3 | Tide | 29 18 | | 51.4 | 0.1 | 51.3 | 9.689841 |
| 1-3 | Edwards-Tide | | | | 0.2 | | 3.990122 |
| 1-2 | Edwards-Apple | | | | | | 3.831737 |
| 2-3 | Surf-Tide | | | | | | 3.644409 |
| 1 | Edwards | 21 38 06.8 | | | | 06.8 | 0.433331 |
| 2 | Surf | 54 48 | | | | 55.6 | 9.912382 |
| 3 | Tide | 103 32 | | 57.7 | 0.1 | 57.6 | 9.987742 |
| 1-3 | Edwards-Tide | | | | 0.1 | | 3.990122 |
| 1-2 | Edwards-Surf | | | | | | 4.065482 +1 |

FIG. 62.—Triangle computation for three-point problem

## ADJUSTMENT OF A FIGURE WITH AN $x$-CORRECTION ON ONE DIRECTION

In the adjustment of triangulation, cases sometimes arise where it is possible to obtain an approximate value for an unknown direction which is needed in the adjustment. By designating the correction to this direction as "$x$," it is possible to make an adjustment in which the $x$ is eliminated and new values of the observed directions are obtained which will make the lengths consistent, as computed in the different triangles. An example of such an adjustment is given below.

In Figure 63 the triangles Tide-Apple-Surf and Rail-Tide-Surf (see triangles 1 and 2, fig. 64) are fixed and the angles and lengths are known. At station Edwards directions have been observed on stations Apple, Surf, Rail, and Tide, but no directions on Edwards have been observed at any of the four fixed stations. It is desired to make an adjustment of the figure so that all the lengths will be consistent.

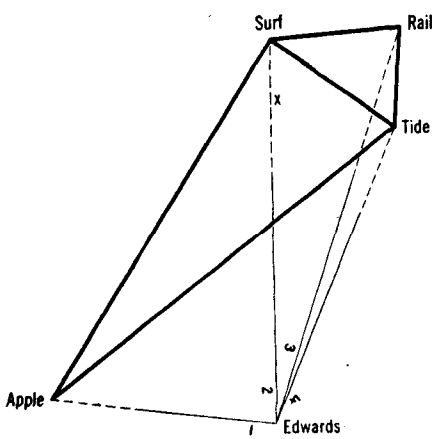



FIG. 63.—Triangulation figure for which an "$x$" direction is used

The triangles Edwards-Apple-Surf, Edwards-Apple-Tide, and Edwards-Surf-Tide (triangles 3, 4, and 6) are first solved by means of the three-point problem (see p. 191). Next the directions at Edwards are numbered as in an ordinary adjustment and one of the unobserved directions at one of the fixed stations is designated "$x$." In this example, the direction to Edwards from Surf is designated "$x$." By using the fixed angles of the triangles Tide-Apple-Surf and Rail-Tide-Surf and the angles in triangles 3, 4, and 6 computed by the three-point problem, it is possible to obtain the angles of the triangles Edwards-Surf-Rail and Edwards-Rail-Tide. Care must be taken in computing these angles to obtain their proper designations.

The angles of the triangle Edwards-Surf-Rail are computed as follows:

| | ° | ′ | ″ |
|---|---|---|---|
| Angle at Surf, Tide to Apple, | 86 | 29 | 42. 4 |
| Angle at Surf, Rail to Tide, | 37 | 52 | 36. 0 |
| Angle at Surf, Rail to Apple, | 124 | 22 | 18. 4 |
| Angle at Surf, Edwards to Apple ($-x$) | 31 | 40 | 46. 8 |
| Angle at Surf, Rail to Edwards ($+x$) | 92 | 41 | 31. 6 |
| Angle at Edwards, Surf to Rail ($-2+3$) | 18 | 18 | 16. 0 |
| Sum ($-2+3+x$) | 110 | 59 | 47. 6 |
| 180° + spherical excess, | 180 | 00 | 00. 1 |
| Angle at Rail, Edwards to Surf ($+2-3-x$) | 69 | 00 | 12. 5 |

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 20

COMPUTATION OF TRIANGLES

State: Maine

| | NO. | STATION | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|---|
| | 2-3 | Apple-Surf | | | | | | 4.109221 |
| | 1 | Tide | 74 14 10.9 | -4.6 | | | 06.3 | 0.016651 |
| 1. | 2 | Apple | 19 16 12.3 | -0.9 | | | 11.4 | 9.518537 |
| | 3 | Surf | 86 29 45.9 | -3.5 | 42.4 | 0.1 | 42.3 | 9.999187 |
| | 1-3 | Tide-Surf | | -9.0 | | 0.1 | | 3.644409 |
| | 1-2 | Tide-Apple | | | | | | 4.125059 |
| | 2-3 | Tide-Surf | | | | | | 3.644409 |
| | 1 | Rail | 80 57 35.0 | -3.0 | | | 32.0 | 0.005430 |
| 2. | 2 | Tide | 61 09 53.5 | -1.5 | | | 52.0 | 9.942508 |
| | 3 | Surf | 37 52 36.7 | -0.7 | | | 36.0 | 9.788143 |
| | 1-3 | Rail-Surf | | -5.2 | | 0.0 | | 3.592347 |
| | 1-2 | Rail-Tide | | | | | | 3.437982 |
| | 2-3 | Apple-Surf | | | | | | 4.109221 |
| -1+2 | 1 | Edwards | 84 12 57.9 | +0.1 | 58.0 | | 58.0 | 0.002217 |
| 3. +1-2+x | 2 | Apple | 64 06 15.5 | +2.9 | 18.4 | 0.1 | 18.3 | 9.954048 |
| -x | 3 | Surf | 31 40 46.8 | -3.0 | 43.8 | 0.1 | 43.7 | 9.720289 |
| | 1-3 | Edwards-Surf | | | | | | 4.065486-1 |
| | 1-2 | Edwards-Apple | | | | 0.2 | | 3.831727 |
| | 2-3 | Apple-Tide | | | | | | 4.125059 |
| -1+4 | 1 | Edwards | 105 51 04.7 | -0.2 | 04.5 | | 04.5 | 0.016837 |
| 4. +1-2+x | 2 | Apple | 44 50 04.1 | +2.9 | 07.0 | 0.1 | 06.9 | 9.848233 |
| +2-4-x | 3 | Tide | 29 18 51.4 | -2.7 | 48.7 | 0.1 | 48.6 | 9.689831 |
| | 1-3 | Edwards-Tide | | | | 0.2 | | 3.990129-1 |
| | 1-2 | Edwards-Apple | | | | | | 3.831727 |
| | 2-3 | Surf-Rail | | | | | | 3.592347 |
| -2+3 | 1 | Edwards | 18 18 16.0 | 0.0 | 16.0 | | 16.0 | 0.502979 |
| 5. +x | 2 | Surf | 92 41 31.6 | +3.0 | 34.6 | 0.1 | 34.5 | 9.999520 |
| +2-3-x | 3 | Rail | 69 00 12.5 | -3.0 | 09.5 | | 09.5 | 9.970159 |
| | 1-3 | Edwards-Rail | | | | 0.1 | | 4.094846 |
| | 1-2 | Edwards-Surf | | | | | | 4.065485 |

FIG. 64.—Triangle computation for Figure 63

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25

COMPUTATION OF TRIANGLES

State: Maine

| | NO. | | STATION | OBSERVED ANGLE | | | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE AND DISTANCE | LOGARITHM |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 2-3 | Surf-Tide | | | | | | | | 3.644409 |
| | -3+4 | 1 | Edwards | 21 | 38 | 06.8 | -0.3 | 06.5 | | 06.5 | 0.433333 |
| 6. | +x | 2 | Surf | 54 | 48 | 55.6 | +3.0 | 58.6 | | 58.6 | 9.912386 |
| | +2-4-x | 3 | Tide | 103 | 32 | 57.7 | -2.7 | 55.0 | 0.1 | 54.9 | 9.987743 |
| | | 1-3 | Edwards-Tide | | | | | | 0.1 | | 3.990128 |
| | | 1-2 | Edwards-Surf | | | | | | | | 4.065485 |
| | | 2-3 | Rail-Tide | | | | | | | | 3.437982 |
| | -3+4 | 1 | Edwards | 3 | 19 | 50.8 | -0.3 | | | 50.5 | 1.235832 |
| 7. | -2+3+x | 2 | Rail | 11 | 57 | 19.5 | +3.0 | | | 22.5 | 9.316316 |
| | +2-4-x | 3 | Tide | 164 | 42 | 49.7 | -2.7 | | | 47.0 | 9.421033 |
| | | 1-3 | Edwards-Tide | | | | | | 0.0 | | $3.990130_{-2}$ |
| | | 1-2 | Edwards-Rail | | | | | | | | $4.094847_{-1}$ |

FIG. 64.—Triangle computation for Figure 63—Continued

The angles of the triangle Edwards-Rail-Tide with their proper designations may be obtained in the same manner. In this triangle the angle at Edwards is an observed angle, the angle at Rail is obtained by subtracting the angle at Rail in triangle No. 5 from the fixed angle at Rail in triangle No. 2, and the angle at Tide is obtained by adding the angles at Edwards and Rail, and subtracting this sum from 180° plus the spherical excess of the triangle.

To make sure that the angles of the triangles have been correctly taken out they are checked as follows before the adjustment is made. The angles in triangles 3, 4, and 6, determined by the three-point problem, are checked by computing the lengths. (See fig. 64.) The other two triangles are checked by adding the angle at Tide in triangle No. 6 to the fixed angle at Tide in triangle No. 2 to obtain the angle at Tide in triangle No. 7.

| | ° | ′ | ″ |
|---|---|---|---|
| Angle at Tide, Surf to Rail, | 61 | 09 | 52. 0 |
| Angle at Tide, Edwards to Surf $(+2-4-x)$ | 103 | 32 | 57. 7 |
| Angle at Tide, Edwards to Rail $(+2-4-x)$ | 164 | 42 | 49. 7 |

This checks the angle obtained by the computation of triangle 7, and as triangle 5 was used in computing triangle 7, both triangles are verified by this check.

After the angles in the triangles have thus been checked the adjustment can be made, the "$x$" correction being treated just the same as the numbered corrections in forming the equations. Since there are four lines from station Edwards, there will be two side

equations, one involving directions 1, 2, 4, and $x$, the other 2, 3, 4, and $x$. As both these equations contain $x$, it is possible to eliminate the $x$ and combine the two equations into one equation involving directions 1, 2, 3, and 4. (See below.) From this point the adjustment is similar to that of any other quadrilateral and so no further explanation is needed.

*Side equations*

| Surf-Tide | ° ′ ″ | 3.644409 | ---------- | Surf-Apple | ° ′ ″ | $4.109221^{-3}$ | ---------- |
|---|---|---|---|---|---|---|---|
| $+x$ | 54 48 55.6 | 9.9123816 | +1.48 | $-2+4$ | 21 38 06.8 | 9.5666685 | +5.31 |
| $+2-4-x$ | 29 18 51.4 | 9.6898412 | +3.75 | $+1-2+x$ | 44 50 04.1 | 9.8482267 | +2.12 |
| $-1+2$ | 84 12 57.9 | 9.9977834 | +.21 | $-x$ | 31 40 46.8 | 9.7202996 | +3.41 |
| | | 3.2444152 | | | | 3.2444155 | |

1. $0=-0.3-2.33\ (1)+11.39\ (2)-9.06\ (4)-0.98\ (x)$

Or $(x)=-0.3061-2.3775\ (1)+11.6224\ (2)-9.2449\ (4)$

| Tide-Rail | | 3.437982 | ---------- | Tide-Surf | | 3.644409 | ---------- |
|---|---|---|---|---|---|---|---|
| $-2+3+x$ | 11 57 19.5 | 9.3162860 | +9.94 | $-3+4$ | 3 19 50.8 | 8.7641784 | +36.18 |
| $-2+4$ | 21 38 06.8 | 9.5666685 | +5.31 | $+x$ | 54 48 55.6 | 9.9123816 | +1.48 |
| | | 2.3209365 | | | | 2.3269690 | |

2. $0=-32.5-15.25\ (2)+46.12\ (3)-30.87\ (4)+8.46\ (x)$

Multiplying equation 1 by $\frac{8.46}{0.98}$ $(=8.633)$ and adding to equation 2 we obtain the combined equation with the $x$ eliminated.

$0=-\ 2.59-20.11\ (1)+98.32\ (2)\qquad -78.21\ (4)-8.46(x)$

2. $0=-32.5\qquad -15.25\ (2)+46.12\ (3)-30.87\ (4)+8.46\ (x)$

$0=-35.09-20.11\ (1)+83.07\ (2)+46.12\ (3)-109.08\ (4)$. (This equation shoud be divided by 10 before entering it in the table of correlates.)

*Correlate equation*

| | 1 | $v$ | Adopted $v$ |
|---|---|---|---|
| 1 | −2.01 | −0.033 | 0.0 |
| 2 | +8.31 | +.136 | +.1 |
| 3 | +4.61 | +.076 | +.1 |
| 4 | −10.91 | −.179 | −.2 |
| $x$ | | +3.008 | +3.0 |

*Solution of normal equation*

$$0=213.3764\ C-3.509$$
$$C=+0.0164$$

If an approximate value for the direction designated by "$x$" in Figure 63 can be obtained by inverse computation from the fixed data and the field computations it is not necessary to solve the three-point problem. For example, if a field position were available

for Edwards, the approximate direction for Surf to Edwards could be obtained by an inverse position computation between Surf and Edwards instead of by the three-point problem. It is only necessary to know this direction closely enough so that the tabular difference of the logarithm of the sine will be practically the same for the approximate angle as for the final angle. If the $x$-correction obtained by the adjustment is large, it is no indication that the observations are poor but only that the computed direction is considerably in error. In some cases involving very small angles it may be necessary to make a second adjustment unless the changes in the small angles necessary to make the lengths check are estimated and the tabular differences are corrected accordingly before the solution of the equations.

## ADJUSTMENT OF INTERSECTION STATIONS

Observations are sometimes made upon an intersection station from four or more main scheme stations. Only three of the observed lines should be used in the adjustment but these three should be selected to give the strongest intersection at the new station unless the field computation indicates that a more accurate result can be obtained by using some other combination of three lines. In forming the side equation the pole should be so selected as to include the two smallest angles in the adjustment. In Figure 65 are shown four examples of intersection stations. The way in which the pole is selected in each is explained below.

In case No. 1, the smallest two angles being $P\ A\ C$ and $P\ B\ C$, the pole should be taken at $P$. In case No. 2, $A\ P\ B$ and $A\ B\ P$ are the smallest angles, and so the pole should be taken at $A$, the fixed lengths $A\ C$ and $A\ B$ being used in the formation of the equation. In case No. 3, $C\ P\ A$ and $C\ A\ P$ are the smallest angles and so the pole should be taken at $C$, the fixed lengths $C\ B$ and $C\ A$ being used in the formation of the equation, as in case No. 2. In case No. 4, the smallest angles are $P\ B\ C$ and $P\ A\ B$, and so the pole should be taken at $P$.

It should be noted that in all examples like cases 3 and 4, where one of the four points is inside the triangle formed by the other three points, the pole should always be taken at the inside point, since the smallest angles will then be included in the equation.

The size of the corrections to the angles to be expected in the adjustment of intersection stations depends largely upon the instruments used, the manner in which the field work was done, and upon the size and definiteness of the point observed. For example, in the computation of mountain peaks from angles observed by sextants, large corrections must be expected. The number of decimal places used for the angles and logarithms in the adjustment will depend upon the nature of the work.

## SOLUTION OF NORMAL EQUATIONS INCLUDING TERMS USUALLY OMITTED

In the forward solution of normal equations a check on the work is obtained each time an equation is eliminated. The column headed $\Sigma$ is for this purpose alone. For example, in the second line of the solution on page 201, the quantity $-1.32$ in the $\Sigma$ column should equal the algebraic sum of all the other terms in that line, that is, $-1-0.33333+0.33333-0.11667-0.20333$ should equal $-1.32$, which

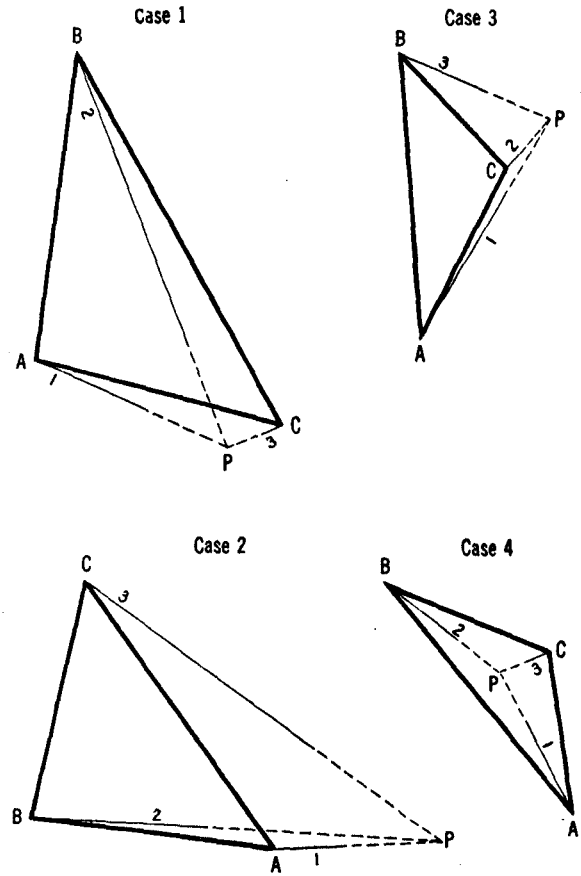



FIG. 65.—Typical figures in adjustment of intersection stations

it does, exactly. When making this check for the "divided" line, or the last line in the solution of each equation, one must always remember to include the $-1$ obtained by dividing the diagonal term by itself and changing the sign, as this quantity is never set down in the solution.

In the solution of the second equation of the following example the line containing the sums of the terms in the various columns should be checked before it is divided by the diagonal term. The

sum of $+5.3333+2.6667-16.0433-0.8767$ should equal the $-8.92$ in the $\Sigma$ column, which it does, exactly. In the next line, after division by the diagonal term, we have $-1-0.50001+3.00814+0.16438$, which equals $+1.67251$ and checks exactly the value in the $\Sigma$ column.

If the line containing the sum terms does not check exactly but the discrepancy is so small as to indicate that no blunder has been made in the computation then the quantity in the $\Sigma$ column should be made to agree exactly with the sum of the other terms before the line is divided by the diagonal term. For example, in the elimination of the fourth equation in the line containing the sum terms we have $+54.5519+4.3660$ and their sum equals $+58.9179$. But as the quantity in the $\Sigma$ column is $+50.9180$ this should be changed to $+58.9179$ before the division is made. In the same way the quantity in the $\Sigma$ column of the divided line should always be made to check the sum of the other terms in the line.

In a great many cases when a blunder has been made and the solution fails to check it is possible to find the trouble by inspection, especially if the mistake is a wrong sign on some term or the transposition of a decimal point. Sometimes, however, an error is difficult to find. In such cases, each line in the elimination of an equation can be checked. To illustrate this let us take the solution of the fourth equation on page 201 where all the usually omitted terms are shown in bold-faced type. In the second line of the solution we find that the sum of $-0.70-0.2333+0.2333-0.0817-0.1423=-0.9240$ which checks the value in the $\Sigma$ column. Likewise, in the third line, $+16.0433+8.0217-48.2605-2.6372=-26.8327$ which checks the value in the $\Sigma$ column within 1 in the last decimal place. In the fourth line, $-7.7150-14.8803+2.0155=-20.5798$ which checks exactly the quanty in the $\Sigma$ column.

It should be carefully noted that all the quantities shown in bold-faced type on page 201 appear in the solution as ordinarily carried out and that it is not necessary to carry along this part of the solution to get the check on each line. For example, in the second line of the elimination of the fourth equation, the term $+0.2333$ appears also in column 4 in the second line of the elimination of the third equation; $-0.2333$ appears also in column 4 in the second line of the elimination of the second equation; and $-0.70$ is $-1$ times the term $+0.70$ in column 4 and equation 1. It is seen, therefore, that all the terms used in the checking of the individual lines in the elimination of equation 4 are given either in the lines themselves or in column 4. For the first line of the solution of equation 4 all the omitted terms except one are given in column 4 in the first lines of the solution of the other equations. The one exception is the quantity in column 4 in the first equation which must be multiplied by $-1$.

In the same way, for the second line of the solution of equation 4, all the omitted terms except one will be found in column 4 in the second line of the solution of the other equations. The exception is the quantity in the sum line of equation 2 which must be multiplied by $-1$. The omitted terms for the third line of the solution of equation 4 are obtained in a similar manner.

Particular care should be taken in the solution of normal equations to guard against compensating errors since they usually cause a great deal of trouble and may not be found until the triangles are computed. For example, if a mistake is made in one of the constant terms in the $\eta$ column and a counterbalancing mistake in the $\Sigma$ column, the solution will check and the errors will not be discovered until the triangles are solved.

*Solution of normal equations, including terms usually omitted*

| 1 | 2 | 3 | 4 | $\eta$ | $\Sigma_n$ |
|---|---|---|---|---|---|
| +6 | +2 | −2 | +0.70 | +1.22 | +7.92 |
| −1 $C_1$ | −.33333 | +.33333 | −.11667 | −.20333 | −1.32 |
| +2 | +6 | +2 | −15.81 | −.47 | −6.28 |
| −2 (1) | −.6667 | +.6667 | −.2333 | −.4067 | −2.64 |
| 0 | +5.3333 | +2.6667 | −16.0433 | −.8767 | −8.92 |
| 0 | −1 $C_2$ | −.50001 | +3.00814 | +.16438 | +1.67251 |
| −2 | +2 | +6 | −.54 | −1.89 | +3.57 |
| +2 | +.6667 (1) | −.6667 | +.2333 | +.4067 | +2.64 |
| 0 | −2.6667 (2) | −1.3333 | +8.0217 | +.4383 | +4.46 |
| 0 | 0 | +4 | +7.7150 | −1.0450 | +10.67 |
| 0 | 0 | −1 $C_3$ | −1.92875 | +.26125 | −2.6675 |
| +.70 | −15.81 | −.54 | +117.7744 | +5.13 | +107.2544 |
| −.70 | −.2333 | +.2333 (1) | −.0817 | −.1423 | −.9240 |
| 0 | +16.0433 | +8.0217 (2) | −48.2605 | −2.6372 | −26.8326 |
| 0 | 0 | −7.7150 (3) | −14.8803 | +2.0155 | −20.5798 |
| 0 | 0 | 0 | +54.5519 | +4.3660 | +58.91979 |
| 0 | 0 | 0 | −1 $C_4$ | −.08003 | −1.08003 |

## INVERSE POSITION COMPUTATION

It sometimes becomes necessary in the adjustment of triangulation to compute the azimuths and length of a line joining two stations which are fixed in position, but which have not been directly connected by the observations. In order to compute this line an inverse or back computation must be made. This computation can be made on Form 26 or Form 27 (see Special Publication No. 8, p. 141), but it can be made more easily and simply on Form 662.

An example of an inverse position computation on Form 662 is given in Figure 66. The formulas for the computation are given at the top of the form. The table for the correction of arc to sine is given on the back of the form. (Do not confuse this table with the table given on page 17 of Special Publication No. 8, sixth edition, which is an entirely different table.) Triangulation stations Spencer and

Peterson are fixed in position (latitude and longitude) and it is desired to determine the azimuth, back-azimuth, and length of the line Spencer to Peterson. The form is self-explanatory and needs no

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 662

**INVERSE POSITION COMPUTATION**

$$s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{\Delta\lambda_1 \cos\phi_m}{A_m}$$

$$s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{-\Delta\phi_1 \cos\frac{\Delta\lambda}{2}}{B_m}$$

$$-\Delta\alpha=\Delta\lambda \sin\phi_m \sec\frac{\Delta\phi}{2}+F(\Delta\lambda)^2$$

in which $\log \Delta\lambda_1=\log(\lambda'-\lambda)-$correction for arc to sin*; $\log \Delta\phi_1=\log(\phi'-\phi)-$correction for arc to sin*; and $\log s=\log s_1+$ correction for arc to sin*.

| | | | | NAME OF STATION | | | | |
|---|---|---|---|---|---|---|---|---|
| 1. $\phi$ | 43° | 59′ | 00.715″ | Spencer | $\lambda$ | 123° | 05′ | 41.248″ |
| 2. $\phi'$ | 44 | 30 | 38.293 | Peterson | $\lambda'$ | 122 | 58 | 05.537 |
| $\Delta\phi$ $(=\phi'-\phi)$ | + | 31 | 37.578 | | $\Delta\lambda$ $(=\lambda'-\lambda)$ | | - 7 | 35.711 |
| $\frac{\Delta\phi}{2}$ | + | 15 | 48.789 | | $\frac{\Delta\lambda}{2}$ | | - 3 | 47.856 |
| $\phi_m\left(=\phi+\frac{\Delta\phi}{2}\right)$ | 44 | 14 | 49.504 | | | | | |
| $\Delta\phi$ (secs.) | + | 1897.578 | | | $\Delta\lambda$ (secs.) | | -455.711 | |

| | | | | | | |
|---|---|---|---|---|---|---|
| $\log \Delta\phi$ | 3.2781997 | | | $\log \Delta\lambda$ | 2.6586896 m | |
| cor. arc−sin | − 15 | | | cor. arc−sin | − 1 | |
| $\log \Delta\phi_1$ | 3.2781982 | | | $\log \Delta\lambda_1$ | 2.6586895 m | |
| $\log \cos\frac{\Delta\lambda}{2}$ | 9.9999997 | | | $\log \cos\phi_m$ | 9.8551177 | |
| colog $B_m$ | 1.4894742 | | | colog $A_m$ | 1.4909902 | |
| $\log\left\{s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$ | 4.7676721 | (opposite in sign to $\Delta\phi$) | | $\log\left\{s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$ | 4.0047974 m | |
| | | | | $\log\left\{s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$ | 4.7676721 m | |
| $\log \Delta\lambda$ | 2.6586896 m | 3 log $\Delta\lambda$ | 7.976 | $\log \tan\left(\alpha+\frac{\Delta\alpha}{2}\right)$ | 9.2371253 | |
| $\log \sin\phi_m$ | 9.8437024 | log F | 7.848 | $\alpha+\frac{\Delta\alpha}{2}$ | 189° 47′ | 40.69″ |
| $\log \sec\frac{\Delta\phi}{2}$ | 0.0000046 | log b | 5.824 | $\log \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)$ | 9.2307483 | |
| log a | 2.5023966 m | | | $\log \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)$ | 9.9936230 | |
| a | -317.98 | | | $\log s_1$ | 4.7740491 | |
| b | 0.00 | | | cor. arc−sin | + 16 | |
| $-\Delta\alpha$ (secs.) | -317.98″ | | | log s | 4.7740507 | |
| $-\frac{\Delta\alpha}{2}$ | -158.99″ | | | s | 59436.15 | |
| | - 2′ 38.99″ | | | | | |
| $\alpha+\frac{\Delta\alpha}{2}$ | 189° 47′ 40.69″ | | | | | |
| $\alpha$ (1 to 2) | 189° 45′ 01.70″ | | | | | |
| $\Delta\alpha$ | + 5′ 17.98″ | | | | | |
| | 180 | | | | | |
| $\alpha'$ (2 to 1) | 9° 50′ 19.68″ | | | | | |

* Use the table on the back of this form for correction of arc to sin.

NOTE.—For log s up to 4.52 and for $\Delta\phi$ or $\Delta\lambda$ (or both) up to 10′, omit all terms below the heavy line except those printed in heavy type or those underscored, if using logarithms to 6 decimal places.

FIG. 66.—Inverse position computation, Form 662

detailed explanation. It should be noted, however, that the quadrant in which the angle $\left(\alpha+\frac{\Delta\alpha}{2}\right)$ occurs depends upon the algebraic signs of the quantities $\sin\left(\alpha+\frac{\Delta\alpha}{2}\right)$ and $\cos\left(\alpha+\frac{\Delta\alpha}{2}\right)$. In the following ex-

ample the sine and cosine are both minus and therefore the angle is in the third quadrant. Log $s$ is obtained in two ways, first by subtracting $\log \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)$ from $\log\left\{s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$ and second by subtracting $\log \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)$ from $\log\left\{s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$.

*Table of arc-sin corrections for inverse position computations*

| log $s_1$ | Arc-sin correction in units of seventh decimal of logarithms | log $\Delta\phi$ or log $\Delta\lambda$ | log $s_1$ | Arc-sin correction in units of seventh decimal of logarithms | log $\Delta\phi$ or log $\Delta\lambda$ | log $s_1$ | Arc-sin correction in units of seventh decimal of logarithms | log $\Delta\phi$ or log $\Delta\lambda$ |
|---|---|---|---|---|---|---|---|---|
| 4. 177 | 1 | 2. 686 | 5. 223 | 124 | 3. 732 | 5. 525 | 497 | 4. 034 |
| 4. 327 | 2 | 2. 836 | 5. 234 | 130 | 3. 743 | 5. 530 | 508 | 4. 039 |
| 4. 415 | 3 | 2. 924 | 5. 243 | 136 | 3. 752 | 5. 534 | 519 | 4. 043 |
| 4. 478 | 4 | 2. 987 | 5. 253 | 142 | 3. 762 | 5. 539 | 530 | 4. 048 |
| 4. 526 | 5 | 3. 035 | 5. 260 | 147 | 3. 769 | 5. 543 | 541 | 4. 052 |
| 4. 566 | 6 | 3. 075 | 5. 269 | 153 | 3. 778 | 5. 548 | 553 | 4. 057 |
| 4. 599 | 7 | 3. 108 | 5. 279 | 160 | 3. 788 | 5. 553 | 565 | 4. 062 |
| 4. 628 | 8 | 3. 137 | 5. 287 | 166 | 3. 796 | 5. 557 | 577 | 4. 066 |
| 4. 654 | 9 | 3. 163 | 5. 294 | 172 | 3. 803 | 5. 561 | 588 | 4. 070 |
| 4. 677 | 10 | 3. 186 | 5. 303 | 179 | 3. 812 | 5. 566 | 600 | 4. 075 |
| 4. 697 | 11 | 3. 206 | 5. 311 | 186 | 3. 820 | 5. 570 | 613 | 4. 079 |
| 4. 716 | 12 | 3. 225 | 5. 318 | 192 | 3. 827 | 5. 575 | 625 | 4. 084 |
| 4. 734 | 13 | 3. 243 | 5. 326 | 199 | 3. 835 | 5. 579 | 637 | 4. 088 |
| 4. 750 | 14 | 3. 259 | 5. 334 | 206 | 3. 843 | 5. 583 | 650 | 4. 092 |
| 4. 765 | 15 | 3. 274 | 5. 341 | 213 | 3. 850 | 5. 587 | 663 | 4. 096 |
| 4. 779 | 16 | 3. 288 | 5. 349 | 221 | 3. 858 | 5. 591 | 674 | 4. 100 |
| 4. 792 | 17 | 3. 301 | 5. 356 | 228 | 3. 865 | 5. 595 | 687 | 4. 104 |
| 4. 804 | 18 | 3. 313 | 5. 363 | 236 | 3. 872 | 5. 600 | 702 | 4. 109 |
| 4. 827 | 20 | 3. 336 | 5. 369 | 243 | 3. 878 | 5. 604 | 716 | 4. 113 |
| 4. 857 | 23 | 3. 366 | 5. 376 | 251 | 3. 885 | 5. 608 | 729 | 4. 117 |
| 4. 876 | 25 | 3. 385 | 5. 383 | 259 | 3. 892 | 5. 612 | 743 | 4. 121 |
| 4. 892 | 27 | 3. 401 | 5. 390 | 267 | 3. 899 | 5. 616 | 757 | 4. 125 |
| 4. 915 | 30 | 3. 424 | 5. 396 | 275 | 3. 905 | 5. 620 | 771 | 4. 129 |
| 4. 936 | 33 | 3. 445 | 5. 403 | 284 | 3. 912 | 5. 624 | 785 | 4. 133 |
| 4. 955 | 36 | 3. 464 | 5. 409 | 292 | 3. 918 | 5. 628 | 800 | 4. 137 |
| 4. 972 | 39 | 3. 481 | 5. 415 | 300 | 3. 924 | 5. 632 | 814 | 4. 141 |
| 4. 988 | 42 | 3. 497 | 5. 422 | 309 | 3. 931 | 5. 636 | 829 | 4. 145 |
| 5. 003 | 45 | 3. 512 | 5. 428 | 318 | 3. 937 | 5. 640 | 845 | 4. 149 |
| 5. 017 | 48 | 3. 526 | 5. 434 | 327 | 3. 943 | 5. 644 | 861 | 4. 153 |
| 5. 035 | 52 | 3. 544 | 5. 440 | 336 | 3. 949 | 5. 648 | 877 | 4. 157 |
| 5. 051 | 56 | 3. 560 | 5. 446 | 345 | 3. 955 | 5. 652 | 893 | 4. 161 |
| 5. 062 | 59 | 3. 571 | 5. 451 | 354 | 3. 960 | 5. 656 | 909 | 4. 165 |
| 5. 076 | 63 | 3. 585 | 5. 457 | 364 | 3. 966 | 5. 660 | 925 | 4. 169 |
| 5. 090 | 67 | 3. 599 | 5. 462 | 373 | 3. 971 | 5. 663 | 941 | 4. 172 |
| 5. 102 | 71 | 3. 611 | 5. 468 | 383 | 3. 977 | 5. 667 | 957 | 4. 176 |
| 5. 114 | 75 | 3. 623 | 5. 473 | 392 | 3. 982 | 5. 671 | 973 | 4. 180 |
| 5. 128 | 80 | 3. 637 | 5. 479 | 402 | 3. 988 | 5. 674 | 989 | 4. 183 |
| 5. 139 | 84 | 3. 648 | 5. 484 | 412 | 3. 993 | 5. 678 | 1005 | 4. 187 |
| 5. 151 | 89 | 3. 660 | 5. 489 | 422 | 3. 998 | | | |
| 5. 163 | 94 | 3. 672 | 5. 495 | 433 | 4. 004 | | | |
| 5. 172 | 98 | 3. 681 | 5. 500 | 443 | 4. 009 | | | |
| 5. 183 | 103 | 3. 692 | 5. 505 | 453 | 4. 014 | | | |
| 5. 193 | 108 | 3. 702 | 5. 510 | 464 | 4. 019 | | | |
| 5. 205 | 114 | 3. 714 | 5. 515 | 474 | 4. 024 | | | |
| 5. 214 | 119 | 3. 723 | 5. 520 | 486 | 4. 029 | | | |

FIG. 67.—Arc-sine corrections for inverse position computation, back of Form 662

The values of $\alpha$ and log $s$ determined by this inverse computation may be checked in the following manner. Starting with the azimuth, Peterson to Spencer, 9° 50′ 19.″68, the logarithm of the length, Peterson to Spencer, 4.7740507, and the fixed latitude and longitude of Peterson, 44° 30′ 38.″293 and 122° 58′ 05.″537, the latitude and longitude of Spencer are computed on Form 26. (See fig. 68.) The values thus obtained should check the fixed values of the latitude and longitude of Spencer within one in the last place of decimals.

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

| | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|
| $\alpha$ | | to | | | | |
| $\angle$ | | & | | + | | |
| $\alpha$ | 2. Peterson | to 1. | Spencer | 9 | 50 | 19.68 |
| $\Delta\alpha$ | | | | - | 5 | 17.98 |
| | | | | 180 | | |
| $\alpha'$ | 1. Spencer | to 2. | Peterson | 189 | 45 | 01.70 |

First Angle of Triangle

| | ° | ′ | ″ | | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| $\varphi$ | 44 | 30 | 38.293 | 2. | Peterson | $\lambda$ | 122 | 58 | 05.537 |
| $\Delta\varphi$ | - | 31 | 37.579 | $s=$ | | $\Delta\lambda$ | + | 7 | 35.711 |
| $\varphi'$ | 43 | 59 | 00.714 +1 | 1. | Spencer | $\lambda'$ | 123 | 05 | 41.248 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $s$ | 4.7740507 | | $s'$ | 9.54810 | | | | $-h$ | 3.2781 |
| $\cos\alpha$ | 9.9935651 | | $\sin^2\alpha$ | 8.46537 | $(\delta\varphi)^2$ | 6.5564 | | $s'\sin^2\alpha$ | 8.0135 |
| B | 8.5105055 | | C | 1.39664 | D | 2.3925 | | E | 6.2020 |
| h | 3.2781213 | | | 9.41011 | | 8.9489 | | | 7.4936 |
| | | ″ | | ″ | | | | | |
| 1st term. | | +1897.2358 | 3d term. | +0.0889 | | | | $(\Delta\lambda)^2$ | 7.976 |
| 2d term. | | +0.2571 | 4th term. | -0.0031 | | | | F | 7.343 |
| | | +1897.4929 | | | | | | | 5.319 |
| 3d and 4th terms. | | +0.0858 | $s$ | 4.7740507 | | | | | |
| $-\Delta\varphi$ | | +1897.5787 | $\sin\alpha$ | 9.2326831 | Arg. | | | $\Delta\lambda$ | 2.6586899 |
| $\frac{1}{2}(\varphi+\varphi')$ | 44 | 14 49.53 | A′ | 8.5090166 | $s$ | -63 | | $\sin\frac{1}{2}(\varphi+\varphi')$ | 9.8437025 |
| | | | $\sec\varphi'$ | 0.1429454 | $\Delta\lambda$ | +4 | | $\sec\frac{1}{2}(\Delta\varphi)$ | 46 |
| | | | | 2.6586899 | Corr. | -59 | | | 2.5023970 |
| | | | $\Delta\lambda$ | + 455.7114 | | | | $-\Delta\alpha$ | +317.979 |

FIG. 68.—Position computation to check inverse computation

## LAPLACE AZIMUTHS

A triangulation station at which both astronomic longitude and astronomic azimuth observations have been made is called a Laplace point, and the azimuth is called a Laplace azimuth.

The geodetic determinations of latitude, longitude, and azimuth at a station are referred to the point on the celestial sphere defined by the normal to the Clarke spheroid at the station; while the astronomic determinations of latitude, longitude, and azimuth at the same station are referred to the point on the celestial sphere defined by the plumb line at the station. These points of reference on the celestial sphere are called the geodetic and astronomic zeniths, re-

spectively. By a comparison of the astronomic and geodetic determinations of latitude, longitude, and azimuth at a station, it is possible to determine the deflection of the plumb line from the normal to the Clarke spheroid. This deflection of the vertical may be expressed by the angular distance between the geodetic and the astronomic zeniths and the azimuth of the line joining them, or by the components of the deflection along the meridian and the prime vertical.

At each Laplace point the prime vertical component of the deflection of the vertical should be the same, except for errors of observation, whether derived from the observed longitude or from the observed azimuth. This relation may be expressed as follows: If $\phi_G$ is the geodetic latitude; $\lambda_A$ and $\lambda_G$ the astronomic and geodetic longitudes respectively; $\alpha_A$ and $\alpha_G$ the astronomic and geodetic azimuths, respectively, then

$$\cos \phi_G(\lambda_A - \lambda_G) = -\cot \phi_G(\alpha_A - \alpha_G),$$
$$\text{or } (\alpha_A - \alpha_G) + \sin \phi_G (\lambda_A - \lambda_G) = 0,$$
$$\text{or } \alpha_G = \sin \phi_G (\lambda_A - \lambda_G) + \alpha_A,$$

which is known as the Laplace equation, since it was first used by Laplace. (For full development of this equation see Spec. Pub. No. 110, pp. 90–91.)

The accuracy of all the data used in determining the true geodetic azimuth at a Laplace point has been thoroughly considered in the various investigations of the figure of the earth and isostasy. It is shown from these investigations that the astronomic longitudes and the astronomic azimuths are each subject to probable errors which are, upon an average, not greater than $\pm 0''.50$. The geodetic longitude anywhere in the United States is subject to a probable error of less than $\pm 0''.50$, due to all causes other than errors in geodetic azimuth. However, the geodetic azimuths as computed through the triangulation are subject to probable errors as great as $\pm 5.''$

It is clearly seen then that at each Laplace point all the quantities used in the formula for computing the true geodetic azimuth are known with a much higher degree of accuracy than the geodetic azimuth at that point is known. Therefore the true geodetic azimuth computed by the formula above is more reliable than the geodetic azimuth as computed through the triangulation, and consequently is the one held fixed in the adjustment of the triangulation.

If there were no deflections of the vertical or station errors, as they are sometimes called, the determination of the correct relative positions of different points on the surface of the earth would be a simple matter and could be done by astronomic observations alone.

However, the relation between the difference of astronomic and geodetic longitude and the difference of astronomic and geodetic azimuth, as expressed by the Laplace equation, makes it possible to

correct for the deflection of the vertical and to determine the true geodetic azimuth at a Laplace station, and hence to obtain the accumulated error in the geodetic azimuth as carried through the triangulation.

The method followed by the United States Coast and Geodetic Survey in determining the true geodetic azimuth is to establish Laplace stations; that is, make astronomic observations for both longitude and azimuth at various stations along the continuous arcs of triangulation. Then at each Laplace station the observed azimuth is corrected for the deflection of the vertical by means of the Laplace equation, and the true geodetic azimuth obtained. The true geodetic azimuth is then held fixed in the adjustment of the triangulation, and in each case the discrepancy between the true geodetic azimuth and the geodetic azimuth as carried through the triangulation is distributed by means of an azimuth equation. (See p. 62.)

An example of the computation necessary to obtain the true geodetic azimuth from the observed azimuth and longitude is shown below. The astronomic longitude of triangulation station Parkersburg and the astronomic azimuth of the line Parkersburg to Denver have been determined by star observations, and the geodetic longitude of Parkersburg and the geodetic azimuth of the line Parkersburg to Denver have been computed through the triangulation. It is desired to obtain the true geodetic azimuth of the line Parkersburg to Denver, and hence determine the accumulated error developed in the geodetic azimuth as computed through the triangulation from the next preceding Laplace azimuth.

The observed astronomic azimuth of the line Parkersburg to Denver is 143° 16′ 15″.55; the observed astronomic longitude of Parkersburg is 88° 01′ 48″.30; the geodetic longitude (that computed through the triangulation) of Parkersburg is 88° 01′ 49″.00; the geodetic latitude of Parkersburg is 38° 34′ 51″.52; and the geodetic azimuth of the line Parkersburg to Denver is 143° 16′ 15″.64.

The computation necessary to obtain the true geodetic azimuth of the line Parkersburg to Denver can best be arranged as follows:

| | | ° | ′ | ″ |
|---|---|---|---|---|
| Astronomic longitude of Parkersburg, $\lambda_A$ | = | 88 | 01 | 48.30 |
| Geodetic longitude of Parkersburg, $\lambda_G$ | = | 88 | 01 | 49.00 |
| $\lambda_A - \lambda_G$ | = | | | −0.70 |
| Sine of geodetic latitude of Parkersburg, $\sin \phi_G$ | = | | | +0.624 |
| $\sin \phi_G (\lambda_A - \lambda_G)$ | = | | | −0.44 |
| Astronomic azimuth, Parkersburg to Denver, $\alpha_A$ | = | 143 | 16 | 15.55 |
| Laplace azimuth, Parkersburg to Denver, $\alpha_G$ | = | 143 | 16 | 15.11 |
| Geodetic azimuth, Parkersburg to Denver | = | 143 | 16 | 15.64 |
| Correction to geodetic azimuth | = | | | −0.53 |

The true geodetic, or Laplace, azimuth of the line Parkersburg to Denver, 143° 16′ 15″.11, is the azimuth to be held in the adjustment of the triangulation, and −0″.53 is the accumulated error developed in computing the azimuth through the triangulation, or the discrepancy which must be distributed through the triangulation by the azimuth equation in the least-squares adjustment. (See p. 63.)

## COMPUTATION OF LONG DISTANCES

The formulas on Form 662 (see fig. 66) may be used in computing distances up to approximately 200 miles between points whose latitudes and longitudes are known. When it is necessary to compute distances much greater than this, such as distances between widely separated cities, the following method should be used. (See pp. 88 and 89 of the Figure of the Earth and Isostasy from Measurements in the United States.) Let $\phi$, $\lambda$, and $\phi'$, $\lambda'$ be the latitudes and longitudes of the given stations. It is required to find the distance, $s$, in meters between the stations ($\theta$ being the arc distance) and the azimuth $\alpha_F$ and the back azimuth $\alpha_B$ of the line joining the stations. Let, $a=90°-\phi$, $c=90°-\phi'$, $x=\frac{1}{2}(a-c)\,e^2\sin^2\frac{1}{2}(a+c)$,

$$a'=a-x,\ c'=c+x,\ B=\lambda'-\lambda,$$

$$\tan\tfrac{1}{2}(A'-C')=\sin\tfrac{1}{2}(a'-c')\csc\tfrac{1}{2}(a'+c')\cot\tfrac{1}{2}B,$$

$$\tan\tfrac{1}{2}(A'+C')=\cos\tfrac{1}{2}(a'-c')\sec\tfrac{1}{2}(a'+c')\cot\tfrac{1}{2}B,$$

$$\sin b=\sin B\sin a'\csc A'=\sin B\sin c'\csc C',$$

$$\text{and } b=\theta$$

$$\text{Then } \alpha_F=180°-C',$$

$$\alpha_B=180°+A',$$

$$\text{and } s\ (\text{in meters})=(\theta\ \text{in seconds})\left(\frac{1}{A}\right)$$

$$\text{or } s\ (\text{in miles})=(\theta\ \text{in seconds})\left(\frac{1}{A}\right)\times 0.00062137$$

In the formulas above $e$ is the eccentricity of the spheroid ($\log e^2=7.83050$), and $A$ is a factor whose logarithm is tabulated for each minute of latitude in Special Publication No. 8. Log $A$ should be taken out for the mean latitude of the two stations.

In making the computation the work should be arranged in a convenient form as shown in the example below. Taking the latitude and longitude of Mexico City as 19° 27′ 20″.0 and 99° 08′ 37″.0, respectively, and the latitude and longitude of Washington, D. C., as 38° 53′ 23″.0 and 77° 00′ 34″.0, respectively, the distance and azimuths between the two cities are computed as follows.

*Sample computation of long distance*

| | ° | ′ | ″ |
|---|---|---|---|
| $\phi$ | 19 | 27 | 20. 0 |
| $\lambda$ | 99 | 08 | 37. 0 |
| $\phi'$ | 38 | 53 | 23. 0 |
| $\lambda'$ | 77 | 00 | 34. 0 |
| $a=90°-\phi$ | 70 | 32 | 40. 0 |
| $c=90°-\phi'$ | 51 | 06 | 37. 0 |
| $\frac{1}{2}(a+c)=\frac{1}{2}(a'+c')$ | 60 | 49 | 38. 5 |
| $\frac{1}{2}(a-c)$ | 9 | 43 | 01. 5 |
| $B=\lambda'-\lambda$ | −22 | 08 | 03. 0 |
| $\frac{1}{2}B$ | −11 | 04 | 01. 5 |
| $\frac{1}{2}(a-c)$ in seconds | | | 34981. 5 |
| log $\frac{1}{2}(a-c)$ in seconds | | | 4. 54384 |
| log $\sin^2 \frac{1}{2}(a+c)$ | | | 9. 88218–10 |
| log $e^2$ | | | 7. 83050–10 |
| log $x$ | | | 2. 25652 |
| $x$ | | | 180″.5 |
| | ° | ′ | ″ |
| $a'=a-x$ | 70 | 29 | 39. 5 |
| $c'=c+x$ | 51 | 09 | 37. 5 |
| $\frac{1}{2}(a'-c')$ | 9 | 40 | 01. 0 |
| log $\sin \frac{1}{2}(a'-c')$ | | | 9. 22510–10 |
| log $\csc \frac{1}{2}(a'+c')$ | | | 0. 05891 |
| log $\cot \frac{1}{2}B$ | | | 0. 70864$_n$ |
| log $\tan \frac{1}{2}(A'-C')$ | | | 9. 99265$_n$–10 |
| log $\cos \frac{1}{2}(a'-c')$ | | | 9. 99379–10 |
| log $\sec \frac{1}{2}(a'+c')$ | | | 0. 31208 |
| log $\cot \frac{1}{2}B$ | | | 0. 70864$_n$ |
| log $\tan \frac{1}{2}(A'+C')$ | | | 1. 01451$_n$ |

| | ° | ′ | ″ |
|---|---|---|---|
| $\frac{1}{2}(A'-C')$ | −44 | 30 | 55 |
| $\frac{1}{2}(A'+C')$ | −84 | 28 | 33 |
| $A'$ | −128 | 59 | 28 |
| $C'$ | −39 | 57 | 38 |
| $\alpha_F=180°-C'$ | 219 | 57 | 38 |
| $\alpha_B=180°+A'$ | 51 | 00 | 32 |
| log sin $B$ | | | 9. 57608$_n$–10 |
| log sin $a'$ | | | 9. 97433–10 |
| log csc $A'$ | | | 0. 10944$_n$ |
| log sin $\theta$ | | | 9. 65985–10 |
| log sin $B$ | | | 9. 57608$_n$–10 |
| log sin $c'$ | | | 9. 89148–10 |
| log csc $C'$ | | | 0. 19229$_n$ |
| log sin $\theta$ | | | 9. 65985–10 |
| | ° | ′ | ″ |
| $\theta$ | 27 | 11 | 21 |
| $\theta$ (in seconds) | | | 97881″ |
| log ($\theta$ in seconds) | | | 4. 99070 |
| colog $A$ | | | 1. 49062 |
| log $s$ (in meters) | | | 6. 48132 |
| log 0.00062137 | | | 6. 79335–10 |
| log $s$ (in miles) | | | 3. 27467 |
| $s$ (in meters) | | | 3029145 |
| $s$ (in miles) | | | 1882. 2 |

## SIDE EQUATION TEST

Frequently in the adjustment of triangulation it is found that although the triangle closures in some particular quadrilateral are very small, the side equation for that quadrilateral has a large discrepancy. This indicates that one or more of the directions must be in error and that the small triangle closures may be due to errors that counterbalance each other. By testing the quadrilateral by means of the side equation one is usually able to find the direction or directions which should be rejected in the adjustment.

The side equation test should be made in the following manner: Add together the coefficients of the terms of the side equation, disregarding their signs. Divide the constant term of the equation by this sum. The result is the approximate average correction that must be applied to a direction to eliminate the discrepancy in the figure and should not be greater than 0″.4 for first-order work.

For the following example the quadrilateral used (fig. 69) is one taken from an arc of first-order triangulation executed in 1925, and it illustrates very clearly the value of the side equation test.

The four triangles in this quadrilateral (see fig. 70) have closures of 0".05, 0".15, 0".49, and 0".39, or an average of 0".27, disregarding the signs. These small closures seemed to indicate accurate values for the angles. In forming the side equation for the office computation, however, it was noticed that the constant term was very large in relation to the coefficients of the various terms. In order to see just what corrections would be required to eliminate the discrepancy in the side equation, the quadrilateral was adjusted. (See p. 212.) It was found that the angles at Anarchist between Gillespie and Spur and between Spur and Oroville required corrections of −3".28 and +3".37, respectively, and that the angles at Spur between Anarchist and Gillespie and between Oroville and Anarchist required corrections of +3".78 and −3".51. This was sufficient proof that some of the angles in the quadrilateral were in error, since the small triangle closures did not justify such large corrections to the angles to eliminate the side discrepancy. Had the side equation test been made in the field the error could have been found and corrected.

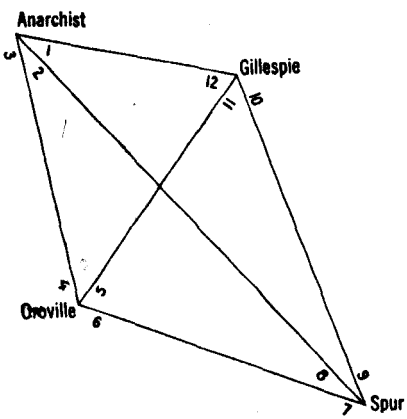


FIG. 69.—Quadrilateral used in side equation test

The smallest two angles in the quadrilateral (see fig. 69) are at Spur between Oroville and Anarchist and between Anarchist and Gillespie, so in writing the side equation the pole should be taken at Anarchist. The constant term of the side equation is then 38.3, and the sum of the coefficients without regard to sign is 30.9. Dividing 38.3 by 30.9 we obtain 1".24, which is the approximate average correction that must be applied to a direction in order to eliminate the side equation discrepancy. This is entirely too large considering that the average closure of the four triangles is only 0".27, and that the maximum is only 0".49.

Although the side equation test as made in this manner shows that some angle or angles must be in error, it does not show which ones are wrong. It is possible, however, to apply the test further by taking the pole of the side equation at some other vertex and bringing in some angles not used in the first test. If the side discrepancy is within the limit in this second test, then the angles used in it are probably correct and the error must be in some of the angles used in the first test, but not in the second test.

If necessary, the test may be further extended by using the third and fourth vertices as poles in forming other side equations. In most cases, however, especially if only one direction is in error, it can probably be located by the second test.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
FORM 35

COMPUTATION OF TRIANGLES

State: Washington

| NO. | STATIONS | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLES AND DISTANCES | LOGARITHM |
|---|---|---|---|---|---|---|---|
| | 2-3 Gillespie - Spur | | | | | | |
| -5+6 | 1 Oroville | 77 58 50.51 | | | | | |
| -10+11 | 2 Gillespie | 54 50 54.68 | | | | | |
| -7+9 | 3 Spur | 47 10 15.78 | | | | | |
| | 1-3 Oroville - Spur | | -0.05 | | 0.92 | | |
| | 1-2 Oroville - Gillespie | | | | | | |
| | | 00.97 | | | | | |
| | 2-3 Gillespie - Spur | | | | | | |
| -1+2 | 1 Anarchist | 35 39 53.10 | | | | | |
| -10+12 | 2 Gillespie | 123 44 27.67 | | | | | |
| -8+9 | 3 Spur | 20 35 40.14 | | | | | |
| | 1-3 Anarchist - Spur | | -0.15 | | 0.76 | | |
| | 1-2 Anarchist - Gillespie | | | | | | |
| | | 00.91 | | | | | |
| | 2-3 Gillespie - Oroville | | | | | | |
| -1+3 | 1 Anarchist | 64 31 19.43 | | | | | |
| -11+12 | 2 Gillespie | 68 53 32.99 | | | | | |
| -4+5 | 3 Oroville | 46 35 08.71 | | | | | |
| | 1-3 Anarchist - Oroville | | -0.49 | | 0.64 | | |
| | 1-2 Anarchist - Gillespie | | | | | | |
| | | 01.13 | | | | | |
| | 2-3 Spur - Oroville | | | | | | |
| -2+3 | 1 Anarchist | 28 51 26.33 | | | | | |
| -7+8 | 2 Spur | 26 34 35.64 | | | | | |
| -4+6 | 3 Oroville | 124 33 59.22 | | | | | |
| | 1-3 Anarchist - Oroville | | -0.39 | | 0.80 | | |
| | 1-2 Anarchist - Spur | | | | | | |
| | | 01.19 | | | | | |

FIG. 70.—Observed angles and closures for triangles used in side equation test

In exceptional cases the side equation tests will show that some angles are in error, but will not indicate which ones. The example given here is such a case. As shown above the first test with the pole at Anarchist gave an approximate average correction to a direc-

tion of $\frac{38.3}{30.9}=1''.24$. The second test with the pole at Gillespie gave an approximate average correction of $\frac{33.4}{22.0}=1''.52$, the third with the pole at Spur, a correction of $\frac{25.5}{23.1}=1''.10$, and the fourth with the pole at Oroville, a correction of $\frac{30.4}{20.6}=1''.48$. In this example, therefore, all four tests show large average corrections to the directions, indicating that some of the angles are in error, but they do not disclose the erroneous angles.

It happens that this quadrilateral appears in an arc of triangulation through which a length equation was carried. It was found that if the length was carried through the triangles, Oroville-Gillespie-Spur and Anarchist-Gillespie-Oroville, the discrepancy in length was 1 part in 400,000, but if the length equation was carried through the triangles Anarchist-Gillespie-Spur and Oroville-Anarchist-Spur, the discrepancy was 1 part in 13,000. This test indicated that the line Anarchist-Spur was probably in error, and by about the same amount at both ends. Either direction 2 (at Anarchist) or direction 8. (at Spur) appears in all four side equations and consequently all four equations have large discrepancies.

This example, however, is an exceptional case as both ends of a line are seldom in error by approximately the same amount. Ordinarily the side equation test will indicate which angles are in error.

*Side equation with pole at Anarchist*

| | ° ′ ″ | | | | ° ′ ″ | | |
|---|---|---|---|---|---|---|---|
| −10+12 | 123 44 27.67 | 9.9198919 | −1.41 | −8+9 | 20 35 40.14 | 9.5462360 | +5.60 |
| −7+8 | 26 34 35.64 | 9.6506895 | +4.21 | −4+6 | 124 33 59.22 | 9.9156471 | −1.45 |
| −4+5 | 46 35 08.71 | 9.8611781 | +1.99 | −11+12 | 68 53 32.99 | 9.9698381 | +.81 |
| | | 9.4317595 | | | | 9.4317212 | |

$$0=+38.3-3.44(4)+1.99(5)+1.45(6)-4.21(7)+9.81(8)-5.60(9)+1.41(10)+0.81\,(11)-2.22\,(12)$$

$$\frac{38.3}{30.9}=1''.24$$

*Side equation with pole at Gillespie*

| | ° ′ ″ | | | | ° ′ ″ | | |
|---|---|---|---|---|---|---|---|
| −7+9 | 47 10 15.78 | 9.8653329 | +1.95 | −5+6 | 77 58 50.51 | 9.9903732 | +0.45 |
| −4+5 | 46 35 08.71 | 9.8611781 | +1.99 | −1+3 | 64 31 19.43 | 9.9555679 | +1.00 |
| −1+2 | 35 39 53.10 | 9.7656995 | +2.94 | −8+9 | 20 35 40.14 | 9.5462360 | +5.60 |
| | | 9.4922105 | | | | 9.4921771 | |

$$0=+33.4-1.94(1)+2.94(2)-1.00(3)-1.99(4)+2.44(5)-0.45(6)-1.95(7)+5.60(8)-3.65(9)$$

$$\frac{33.4}{22.0}=1''.52$$

*Side equation with pole at Spur*

| | ° ′ ″ | | | | ° ′ ″ | | |
|---|---|---|---|---|---|---|---|
| −4+6 | 124 33 59.22 | 9.9156471 | −1.45 | −2+3 | 28 51 26.33 | 9.6836143 | +3.82 |
| −1+2 | 35 39 53.10 | 9.7656995 | +2.94 | −10+12 | 123 44 27.67 | 9.9198919 | −1.41 |
| −10+11 | 54 50 54.68 | 9.9125583 | +1.48 | −5+6 | 77 58 50.51 | 9.9903732 | +.45 |
| | | 9.5939049 | | | | 9.5938794 | |

$$0=+25.5-2.94(1)+6.76(2)-3.82(3)+1.45(4)+0.45(5)-1.90(6)-2.89(10)+1.48(11)+1.41(12)$$

$$\frac{25.5}{23.1}=1''.10$$

*Side equation with pole at Oroville*

| | ° ′ ″ | | | | ° ′ ″ | | |
|---|---|---|---|---|---|---|---|
| −1+3 | 64 31 19.43 | 9.9555679 | +1.00 | −11+12 | 68 53 32.99 | 9.9698381 | +0.81 |
| −10+11 | 54 50 54.68 | 9.9125583 | +1.48 | −7+9 | 47 10 15.78 | 9.8653329 | +1.95 |
| −7+8 | 26 34 35.64 | 9.6506895 | +4.21 | −2+3 | 28 51 26.33 | 9.6836143 | +3.82 |
| | | 9.5188157 | | | | 9.5187853 | |

$$0=+30.4-1.00(1)+3.82(2)-2.82(3)-2.26(7)+4.21(8)-1.95(9)-1.48(10)+2.29(11)-0.81(12)$$

$$\frac{30.4}{20.6}=1''.48$$

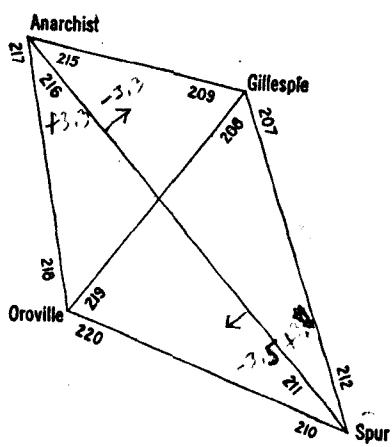



FIG. 71.—Quadrilateral used in side equation test with directions renumbered for office computation

The following adjustment of the quadrilateral shows what large corrections are required to make it consistent if all observed directions are used. The numbers on the directions are those used in the office adjustment of the arc.

*Angle equations*

1. $0=+0.05-(207)+(208)-(210)+(212)-(219)+(220)$
2. $0=+0.39-(210)+(211)-(216)+(217)-(218)+(220)$
3. $0=+0.49-(208)+(209)-(215)+(217)-(218)+(219)$

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 23

COMPUTATION OF TRIANGLES

State: Washington

| NO. | STATIONS | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLES AND DISTANCES | LOGARITHM |
|---|---|---|---|---|---|---|---|
| 2-3 | Gillespie - Spur | | | | | | 4.3872464 |
| 219+220[1] | Oroville | 77 58 50.51 | +0.94 | 51.45 | 0.31 | 51.14 | 0.0096264 |
| -207+208[2] | Gillespie | 54 50 54.68 | -1.26 | 53.42 | 0.31 | 53.11 | 9.9125560 |
| -210+213[3] | Spur | 47 10 15.78 | +0.27 | 16.05 | 0.30 | 15.75 | 9.3653328 |
| 1-3 | Oroville - Spur | | -0.05 | | 0.92 | | 4.3094288 |
| 1-2 | Oroville - Gillespie | | | | | | 4.2622056 |
| 2-3 | Gillespie - Spur | | | | | | 4.3872464 |
| -215+216[1] | Anarchist | 35 39 53.10 | -3.2849.82 | | 0.25 | 49.57 | 0.2343109 |
| -207+209[2] | Gillespie | 123 44 27.67 | -0.6527.02 | | 0.25 | 26.77 | 9.9198931 |
| -211+212[3] | Spur | 20 35 40.14 | +3.7843.92 | | 0.26 | 43.66 | 9.5462557 |
| 1-3 | Anarchist - Spur | | -0.15 | | 0.76 | | 4.5414504 |
| 1-2 | Anarchist - Gillespie | | | | | | 4.1678130 |
| 2-3 | Gillespie - Oroville | | | | | | 4.2622056 |
| -215+217[1] | Anarchist | 64 31 19.43 | +0.0919.52 | | 0.21 | 19.31 | 0.0444321 |
| -208+209[2] | Gillespie | 68 53 32.99 | +0.6133.60 | | 0.21 | 33.39 | 9.9698384 |
| -218+219[3] | Oroville | 46 35 08.71 | -1.1907.52 | | 0.22 | 07.30 | 9.8611753 |
| 1-3 | Anarchist - Oroville | | -0.49 | | 0.64 | | 4.2764761 |
| 1-2 | Anarchist - Gillespie | | | | | | 4.1678130 |
| 2-3 | Spur - Oroville | | | | | | 4.3094288 |
| -216+217[1] | Anarchist | 28 51 26.33 | +3.3729.70 | | 0.27 | 29.43 | 0.3163738 |
| -210+211[2] | Spur | 26 34 35.64 | -3.5132.13 | | 0.27 | 31.86 | 9.6506736 |
| -218+220[3] | Oroville | 124 33 59.22 | -0.2558.97 | | 0.26 | 58.71 | 9.9156479 |
| 1-3 | Anarchist - Oroville | | -0.39 | | 0.80 | | 4.2764762 |
| 1-2 | Anarchist - Spur | | | | | | 4.5414504 |

FIG. 72.—Triangle computation for quadrilateral used in side equation test

*Side equation*

| | ° ′ ″ | | | | ° ′ ″ | | |
|---|---|---|---|---|---|---|---|
| −207+208 | 54 50 54.68 | 9.91255829 | +1.48 | −219+220 | 77 58 50.51 | 9.99037327 | +0.45 |
| −210+211 | 26 34 35.64 | 9.65068952 | +4.21 | −216+217 | 28 51 26.33 | 9.68361433 | +3.82 |
| −218+219 | 46 35 08.71 | 9.86117810 | +1.99 | −208+209 | 68 53 32.99 | 9.96983806 | +0.81 |
| −215+216 | 35 39 53.10 | 9.76569945 | +2.94 | −211+212 | 20 35 40.14 | 9.54623598 | +5.60 |
| | | 9.19012536 | | | | 9.19006164 | |

4. $0 = +63.72 - 1.48(207) + 2.29(208) - 0.81(209) - 4.21(210) + 9.81(211) - 5.60(212) - 2.94(215) + 6.76(216) - 3.82(217) - 1.99(218) + 2.44(219) - 0.45\ (220)$

*Correlate equations*

| | 1 | 2 | 3 | 4 | $\Sigma_c$ | $v$ | Adopted $v$ |
|---|---|---|---|---|---|---|---|
| 207 | −1 | ------- | ------- | −1.48 | −2.48 | +0.636 | +0.64 |
| 208 | +1 | ------- | −1 | +2.29 | +2.29 | −.622 | −.62 |
| 209 | ------- | ------- | +1 | −.81 | +.19 | −.014 | −.01 |
| 210 | −1 | −1 | ------- | −4.21 | −6.21 | +1.081 | +1.08 |
| 211 | ------- | +1 | ------- | +9.81 | +10.81 | −2.435 | −2.43 |
| 212 | +1 | ------- | ------- | −5.60 | −4.60 | +1.354 | +1.35 |
| 215 | ------- | ------- | −1 | −2.94 | −3.94 | +1.067 | +1.06 |
| 216 | ------- | −1 | ------- | +6.76 | +5.76 | −2.221 | −2.22 |
| 217 | ------- | +1 | +1 | −3.82 | −1.82 | +1.154 | +1.15 |
| 218 | ------- | −1 | −1 | −1.99 | −3.99 | +.478 | +.48 |
| 219 | −1 | ------- | +1 | +2.44 | +2.44 | −.707 | −.71 |
| 220 | +1 | +1 | ------- | −.45 | +1.55 | +.229 | +.23 |

*Normal equations*

| 1 | 2 | 3 | 4 | $\eta$ | $\Sigma_n$ | $C$ |
|---|---|---|---|---|---|---|
| +6 | +2 | −2 | −0.51 | +0.05 | +5.54 | −0.21996 |
| | +6 | +2 | +4.98 | +.39 | +15.37 | +.32198 |
| | | +6 | +.45 | +.49 | +6.94 | −.24124 |
| | | | +232.4606 | +63.72 | +301.1006 | −.28102 |

## RECTANGULAR COORDINATES

The results of the triangulation computed by the United States Coast and Geodetic Survey are always given in geographic coordinates, since the triangulation usually covers a large area, and must necessarily be computed by means of geographic coordinates (latitudes and longitudes) on account of the curvature of the earth.

For the survey of a relatively small area, such as a city or small county, it is much more convenient to use plane coordinates, since the computations are simpler. If in the small area being surveyed, however, there are triangulation stations, these should be used to control the local survey and to connect it to the triangulation of the country.

In order to make its results of value to city and county surveyors using plane coordinates the United States Coast and Geodetic Survey has issued Special Publication No. 71, entitled "Relation between plane rectangular coordinates and geographic positions," in which tables are given that make the computations for transforming geographic to plane rectangular coordinates or vice versa very simple. This publication can be purchased from the Superintendent of Documents, Washington, D. C., for 10 cents.

## CHAPTER 7.—GENERAL RULES AND SUGGESTIONS

All computations can be made much more rapidly and can be checked more easily if the work is arranged in a systematic manner. This bureau has printed forms for nearly all triangulation computations and these should be used whenever possible, as they expedite the work and lessen considerably the chances for errors.

The mathematician should bear in mind that accuracy is desired above everything else. Of course speed is desired also, but accuracy should not be sacrificed to it. Nothing is gained if a piece of work is done in an unusually short time, if as a consequence it afterwards needs considerable revision.

For a great many computations no fixed rules can be laid down, but the proper procedure depends upon the judgment of the mathematician doing the work. For instance, it is impossible to specify the number of decimal places to be used for the numbers and logarithms in all the various computations. This depends upon the particular piece of work being computed.

All work should be done neatly and all figures should be written carefully and legibly. Do not write one figure over another. Either erase entirely the one which is superseded, or draw a line through it and write the correct figure above it.

Every computation, unless self-checking, should be checked by a mathematician other than the one who made the original computation. The mathematician who does the checking should first make his changes in pencil, and then he and the mathematician making the original computation should agree on the proper values before the final corrections are made in ink.

An inexperienced mathematician should feel free at all times to consult the man under whom he is working or, in an emergency, any of the more experienced mathematicians in regard to the work. It is well to make sure you are right before carrying a computation too far. This consultation suggestion should not be abused, however. Beginners should proceed upon their own initiative in the work assigned them, unless there is doubt as to the proper method. Knowledge obtained by study and hard work is much more easily retained.

In all triangulation computations meters are used, but the final results are converted to feet, and both meters and feet are published.

In an adjustment of a large net of triangulation the selection of the equations, as well as the formation, should be checked before the work is carried ahead, as often an "identical" equation (see p. 181) will

not become apparent until the forward solution of the normals is made, and considerable recomputation will then be required to substitute the proper equation.

Intersection stations, that is, unoccupied points, unless they are main scheme stations, should be adjusted by using only the three lines which give the strongest angles at the vertex.

The mathematician making the first check of a list of geographic positions should also check the angles in the triangles for all "no-check" points.

In those adjustments in which the logarithms of certain lengths each appear in only one triangle these logarithms should be carefully checked before the list is made out.

The mathematician making out a list of geographic positions and the one doing the checking should scan it carefully for errors that may easily be discovered by inspection. For example, it is easy to see whether the azimuth and back azimuth differ by approximately 180° as they should, or whether the number of figures in a length correspond to the characteristic of the logarithm from which it is taken.

All final lists of geographic positions should be checked by two mathematicians, who should put their initials at the bottom of each sheet. As soon as a list has been properly checked, a photostat copy of it should be obtained for the files of the division of geodesy. If the triangulation is along the coast another photostat copy of the list should be obtained for the files of the division of charts.

Pages of geographic positions and cards of descriptions of the stations should not be taken out of the files of the division of geodesy without leaving a memorandum of such withdrawals with the mathematician in charge of the files.

In the adjustment of a central point figure where there is more than one side equation, one of these equations should be written with the pole at the center and carried completely around the figure.

In an adjustment having several side equations, if the closure for one is large the directions used in that equation should be investigated. If the closure can be improved considerably by omitting a certain direction that should be done.

Whenever a field computation of triangles is available it is not necessary to make a preliminary office computation of triangles, but the logarithms of lengths from the field computation can be used to compute the spherical excess.

In the adjustment of triangulation by the angle method the azimuth equation should always be formed by using the $c$ or azimuth angles, as then the azimuth equation will not ordinarily involve the same $v$'s as the length equation, and the solution of the normal equations will be simplified.

In using the Shortrede logarithmic tables, the tables of proportional parts at the right of each page and the tabular difference for one second at the bottom of each minute column should be used except for very small angles. For small angles, where the tabular difference for one second is changing rapidly, the difference should be taken out for the particular second used.

In forming the correlate equations, the coefficients of some of the equations should, in some cases, be divided by 5, 10, or 100 to make them of approximately the same size as those in the other equations. This makes it possible to obtain a given accuracy with a smaller number of decimal places in the solution of the normal equations.

In the computation of a geodetic position for which the signs of the $\Delta\lambda$'s as computed over the two lines are the same, and the two values are approximately the same size, the resulting values of $\lambda$ may check and yet the computation be wrong. This is due to the fact that the values of $A'$ and sec $\phi'$ are the same for both lines used in the computation, and if there is an error in either of these terms, it will affect both $\Delta\lambda$'s by about the same amount. The error will not be apparent until this position is used in computing some other position for which the longitude will necessarily fail to check.

In the computation of a geodetic position all signs, whether plus or minus, should be indicated for all the terms. This saves much time and avoids confusion.

Eternal (true black carbon) ink should be used for making out all lists of geographic positions since this permits much better photostat copies to be made.

The last mathematician to leave a room, in which there are computing machines, at the end of a day should see that all the machines are covered. This prevents dust from entering and injuring the delicate parts of the machine.

In making out lists of geographic positions, the mathematician should always insert at the top of each sheet, in the blanks provided, the locality and State in which the triangulation is located, and also the datum on which the work is computed. This will avoid confusion if the sheet should become misplaced from the files or computation cahier.

Before starting the adjustment of a net of triangulation, the mathematician should find out just how the net is connected in position, length or azimuth to previously adjusted triangulation. If any of the points or lines of the new net are identical with points or lines of previously adjusted triangulation, they should be held fixed in the new adjustment, in order that the new triangulation may be made consistent with the existing fixed triangulation. Occasionally, however, a station already adjusted may be allowed to take a new posi-

tion, if the old data on which its position was based are not considered very accurate or reliable. The final decision in a matter of this kind should rest with the more experienced judgment of the older mathematicians.

In many of the operations in the computation of triangulation more decimal places are used in the course of the computation than are necessary when the final values are reached. In dropping the extra decimal places no question arises if the figures dropped represent either more or less than one-half a unit of the last decimal place retained, for if they are less the last figure retained remains unchanged and if more it is increased one. For example, if the two numbers 0.2273 and 0.3366 are rounded off to three decimal places, then the adopted numbers should be 0.227 and 0.337, respectively.

When the figures in the dropped decimal places, however, represent exactly one-half a unit of the last decimal place retained, then the number adopted may have two values both of which are equally correct. In order to avoid confusion the United States Coast and Geodetic Survey has arbitrarily adopted the plan of using the nearest *even* figure for the last decimal place retained. For example, in rounding off to three decimal places, the numbers 0.4215 and 0.6245, the adopted numbers should be 0.422 and 0.624, respectively.

# CHAPTER 8.—CONSTANTS, FORMULAS, AND TABLES

## CONSTANTS AND FORMULAS

Dimensions of the earth according to Clarke's spheroid of reference (1866):

Equatorial radius, $a$, $=6378206.4$ meters

$\log a = 6.80469857$

Polar semi-axis, $b$, $=6356583.8$ meters

$\log b = 6.80322378$

Eccentricity, $e$, $=\sqrt{\frac{a^2-b^2}{a^2}}$,

$e^2 = 0.006768658$,

$\log e^2 = 7.83050257 - 10$

Base of Naperian logarithms, $\epsilon$, $=2.71828183$

$\log \epsilon = 0.43429448$

Modulus of common logarithms, M, $=0.43429448$

$\log M = 9.63778431 - 10$

$\pi = 3.14159265$

$\log \pi = 0.49714987$

$\log \sin 1'' = 4.68557487 - 10$

$\log \tan 1'' = 4.68557487 - 10$

1 kilometer = 0.621370 statute mile = 0.539593 nautical mile.

1 meter = 0.000621370 statute mile = 0.000539593 nautical mile.

1 statute mile = 1609.35 meters = 1.60935 kilometers.

1 nautical mile = 1853.25 meters = 1.85325 kilometers.

1 nautical mile = 1.151553 statute miles.

1 statute mile = 0. 868393 nautical mile.

1 meter = 39.37 inches (law of July 28, 1886).

1 meter = 3.28083333 feet.

log. 3.28083333 = 0.51598417.

1 foot = 0.30480061 meter.

log. 0.30480061 = 9.48401583 − 10.

Probable error of an observation, $r = 0.6745\sqrt{\frac{\Sigma v^2}{n-1}}$

Probable error of result, $r_0 = \frac{r}{\sqrt{n}} = 0.6745\sqrt{\frac{\Sigma v^2}{n(n-1)}}$

Probable error of an observation of unit weight, $\mu_1 = 0.6745\sqrt{\frac{\Sigma pv^2}{n-1}}$

Probable error of an observation of weight $p_1$, $r_1 = \frac{\mu_1}{\sqrt{p_1}} = 0.6745\sqrt{\frac{\Sigma pv^2}{p_1(n-1)}}$

Probable error of an observed direction, $d = 0.6745\sqrt{\frac{\Sigma v^2}{c}}$ where $\Sigma v^2$ = sum of squares of corrections to directions, and $c$ is the number of conditions.

Mean error of an angle, $\alpha = \sqrt{\frac{\Sigma\Delta^2}{3n}}$,

where $\Sigma\Delta^2$ is the sum of the squares of the closing errors of the triangles, and $n$ is the number of triangles.

*Logarithms of radii of curvature of the earth's surface (in meters)*

[Based upon Clarke's spheroid of 1866 as expressed in meters].

| Azimuth (degrees) | Latitude | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0° | 1° | 2° | 3° | 4° | 5° | 6° | 7° | 8° |
| 0 | 6.80175 | 6.80175 | 6.80175 | 6.80176 | 6.80177 | 6.80178 | 6.80180 | 6.80181 | 6.80183 |
| 5 | 177 | 177 | 178 | 178 | 179 | 180 | 182 | 184 | 186 |
| 10 | 184 | 184 | 184 | 185 | 186 | 187 | 188 | 190 | 192 |
| 15 | 195 | 195 | 195 | 196 | 197 | 198 | 199 | 201 | 203 |
| 20 | 209 | 209 | 210 | 210 | 211 | 212 | 214 | 215 | 217 |
| 25 | 227 | 228 | 228 | 228 | 229 | 230 | 232 | 233 | 235 |
| 30 | 248 | 249 | 249 | 250 | 250 | 251 | 252 | 254 | 256 |
| 35 | 272 | 272 | 272 | 273 | 273 | 274 | 276 | 277 | 278 |
| 40 | 296 | 297 | 297 | 297 | 298 | 299 | 300 | 301 | 303 |
| 45 | 322 | 322 | 322 | 323 | 324 | 324 | 325 | 326 | 328 |
| 50 | 348 | 348 | 348 | 348 | 349 | 350 | 351 | 352 | 353 |
| 55 | 373 | 373 | 373 | 373 | 374 | 374 | 375 | 376 | 377 |
| 60 | 396 | 396 | 396 | 396 | 397 | 398 | 398 | 399 | 400 |
| 65 | 417 | 417 | 417 | 418 | 418 | 418 | 419 | 420 | 421 |
| 70 | 435 | 435 | 436 | 436 | 436 | 437 | 437 | 438 | 439 |
| 75 | 450 | 450 | 450 | 450 | 451 | 451 | 452 | 452 | 453 |
| 80 | 461 | 461 | 461 | 461 | 462 | 462 | 463 | 463 | 464 |
| 85 | 468 | 468 | 468 | 468 | 468 | 469 | 469 | 470 | 470 |
| 90 | 470 | 470 | 470 | 470 | 471 | 471 | 472 | 472 | 473 |

| Azimuth (degrees) | Latitude | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 8° | 9° | 10° | 11° | 12° | 13° | 14° | 15° | 16° |
| 0 | 6.80183 | 6.80186 | 6.80188 | 6.80191 | 6.80194 | 6.80197 | 6.80201 | 6.80204 | 6.80208 |
| 5 | 186 | 188 | 190 | 193 | 196 | 199 | 203 | 206 | 210 |
| 10 | 192 | 194 | 197 | 200 | 202 | 206 | 209 | 213 | 217 |
| 15 | 203 | 205 | 207 | 210 | 213 | 216 | 219 | 223 | 227 |
| 20 | 217 | 219 | 222 | 224 | 227 | 230 | 233 | 236 | 240 |
| 25 | 235 | 237 | 239 | 242 | 244 | 247 | 250 | 254 | 257 |
| 30 | 256 | 257 | 260 | 262 | 264 | 267 | 270 | 273 | 276 |
| 35 | 278 | 280 | 282 | 284 | 287 | 289 | 292 | 295 | 298 |
| 40 | 303 | 304 | 306 | 308 | 310 | 313 | 315 | 318 | 321 |
| 45 | 328 | 329 | 331 | 333 | 335 | 337 | 339 | 342 | 344 |
| 50 | 353 | 354 | 356 | 358 | 359 | 361 | 364 | 366 | 368 |
| 55 | 377 | 379 | 380 | 382 | 383 | 385 | 387 | 389 | 391 |
| 60 | 400 | 401 | 403 | 404 | 406 | 407 | 409 | 411 | 413 |
| 65 | 421 | 422 | 423 | 424 | 426 | 427 | 429 | 430 | 432 |
| 70 | 439 | 440 | 441 | 442 | 443 | 444 | 446 | 447 | 449 |
| 75 | 453 | 454 | 455 | 456 | 457 | 458 | 460 | 461 | 463 |
| 80 | 464 | 465 | 466 | 467 | 468 | 469 | 470 | 471 | 473 |
| 85 | 470 | 471 | 472 | 473 | 474 | 475 | 476 | 478 | 479 |
| 90 | 473 | 474 | 474 | 475 | 476 | 477 | 478 | 480 | 481 |

| Azimuth (degrees) | Latitude | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 16° | 17° | 18° | 19° | 20° | 21° | 22° | 23° | 24° |
| 0 | 6.80208 | 6.80213 | 6.80217 | 6.80222 | 6.80226 | 6.80232 | 6.80237 | 6.80242 | 6.80248 |
| 5 | 210 | 215 | 219 | 224 | 228 | 234 | 239 | 244 | 250 |
| 10 | 217 | 221 | 225 | 230 | 234 | 239 | 244 | 250 | 255 |
| 15 | 227 | 231 | 235 | 239 | 244 | 249 | 254 | 259 | 264 |
| 20 | 240 | 244 | 248 | 252 | 257 | 262 | 266 | 271 | 277 |
| 25 | 257 | 261 | 265 | 269 | 273 | 277 | 282 | 287 | 292 |
| 30 | 276 | 280 | 284 | 287 | 292 | 296 | 300 | 305 | 309 |
| 35 | 298 | 301 | 305 | 308 | 312 | 316 | 320 | 324 | 329 |
| 40 | 321 | 324 | 327 | 330 | 334 | 338 | 341 | 345 | 350 |
| 45 | 344 | 347 | 350 | 353 | 357 | 360 | 364 | 367 | 371 |
| 50 | 368 | 371 | 373 | 376 | 379 | 382 | 386 | 389 | 392 |
| 55 | 391 | 394 | 396 | 398 | 401 | 404 | 407 | 410 | 413 |
| 60 | 413 | 415 | 417 | 419 | 422 | 424 | 427 | 430 | 432 |
| 65 | 432 | 434 | 436 | 438 | 440 | 443 | 445 | 448 | 450 |
| 70 | 449 | 451 | 453 | 454 | 456 | 459 | 461 | 463 | 465 |
| 75 | 463 | 464 | 466 | 468 | 470 | 472 | 473 | 476 | 478 |
| 80 | 473 | 474 | 476 | 478 | 479 | 481 | 483 | 485 | 487 |
| 85 | 479 | 480 | 482 | 483 | 485 | 487 | 489 | 490 | 492 |
| 90 | 481 | 482 | 484 | 485 | 487 | 489 | 490 | 492 | 494 |

*Logarithms of radii of curvature of the earth's surface (in meters)*—Continued

| Azimuth (degrees) | Latitude | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 24° | 25° | 26° | 27° | 28° | 29° | 30° | 31° | 32° |
| 0 | 6.80248 | 6.80254 | 6.80260 | 6.80266 | 6.80272 | 6.80279 | 6.80285 | 6.80292 | 6.80299 |
| 5 | 250 | 256 | 262 | 268 | 274 | 280 | 287 | 294 | 300 |
| 10 | 255 | 261 | 267 | 273 | 279 | 285 | 292 | 298 | 305 |
| 15 | 264 | 270 | 276 | 282 | 288 | 294 | 300 | 306 | 313 |
| 20 | 277 | 282 | 288 | 293 | 299 | 305 | 311 | 317 | 324 |
| 25 | 292 | 297 | 302 | 308 | 313 | 319 | 325 | 331 | 337 |
| 30 | 309 | 314 | 319 | 324 | 330 | 335 | 340 | 346 | 352 |
| 35 | 329 | 333 | 338 | 343 | 348 | 353 | 358 | 363 | 369 |
| 40 | 350 | 354 | 358 | 362 | 367 | 372 | 377 | 382 | 386 |
| 45 | 371 | 375 | 379 | 383 | 387 | 391 | 396 | 400 | 405 |
| 50 | 392 | 396 | 399 | 403 | 407 | 411 | 415 | 419 | 423 |
| 55 | 413 | 416 | 420 | 423 | 426 | 430 | 434 | 437 | 441 |
| 60 | 432 | 435 | 438 | 442 | 445 | 448 | 451 | 455 | 458 |
| 65 | 450 | 453 | 455 | 458 | 461 | 464 | 467 | 470 | 473 |
| 70 | 465 | 468 | 470 | 473 | 475 | 478 | 481 | 484 | 486 |
| 75 | 478 | 480 | 482 | 484 | 487 | 489 | 492 | 494 | 497 |
| 80 | 487 | 489 | 491 | 493 | 495 | 498 | 500 | 502 | 505 |
| 85 | 492 | 494 | 496 | 498 | 501 | 503 | 505 | 507 | 510 |
| 90 | 494 | 496 | 498 | 500 | 502 | 504 | 507 | 509 | 511 |

| Azimuth (degrees) | Latitude | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 32° | 33° | 34° | 35° | 36° | 37° | 38° | 39° | 40° |
| 0 | 6.80299 | 6.80306 | 6.80313 | 6.80320 | 6.80327 | 6.80335 | 6.80342 | 6.80350 | 6.80357 |
| 5 | 300 | 307 | 314 | 322 | 329 | 336 | 344 | 351 | 359 |
| 10 | 305 | 312 | 319 | 326 | 333 | 340 | 348 | 355 | 363 |
| 15 | 313 | 320 | 326 | 333 | 340 | 348 | 355 | 362 | 369 |
| 20 | 324 | 330 | 337 | 343 | 350 | 357 | 364 | 371 | 378 |
| 25 | 337 | 343 | 349 | 355 | 362 | 368 | 375 | 382 | 388 |
| 30 | 352 | 358 | 364 | 370 | 376 | 382 | 388 | 394 | 401 |
| 35 | 369 | 374 | 380 | 385 | 391 | 397 | 402 | 408 | 414 |
| 40 | 386 | 392 | 397 | 402 | 407 | 412 | 418 | 423 | 429 |
| 45 | 405 | 410 | 414 | 419 | 424 | 429 | 434 | 439 | 444 |
| 50 | 423 | 428 | 432 | 436 | 441 | 445 | 450 | 454 | 459 |
| 55 | 441 | 445 | 449 | 453 | 457 | 461 | 465 | 469 | 474 |
| 60 | 458 | 462 | 465 | 469 | 472 | 476 | 480 | 484 | 487 |
| 65 | 473 | 476 | 480 | 483 | 486 | 489 | 493 | 496 | 500 |
| 70 | 486 | 489 | 492 | 495 | 498 | 501 | 504 | 507 | 510 |
| 75 | 497 | 500 | 502 | 505 | 508 | 510 | 513 | 516 | 519 |
| 80 | 505 | 507 | 510 | 512 | 515 | 517 | 520 | 523 | 525 |
| 85 | 510 | 512 | 514 | 517 | 519 | 522 | 524 | 527 | 529 |
| 90 | 511 | 514 | 516 | 518 | 521 | 523 | 526 | 528 | 531 |

| Azimuth (degrees) | Latitude | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 40° | 41° | 42° | 43° | 44° | 45° | 46° | 47° | 48° |
| 0 | 6.80357 | 6.80365 | 6.80373 | 6.80380 | 6.80388 | 6.80396 | 6.80404 | 6.80411 | 6.80419 |
| 5 | 359 | 366 | 374 | 382 | 389 | 397 | 404 | 412 | 420 |
| 10 | 363 | 370 | 378 | 385 | 393 | 400 | 408 | 415 | 423 |
| 15 | 369 | 376 | 384 | 391 | 398 | 406 | 413 | 420 | 428 |
| 20 | 378 | 385 | 392 | 399 | 406 | 413 | 420 | 427 | 434 |
| 25 | 388 | 395 | 402 | 408 | 415 | 422 | 429 | 436 | 442 |
| 30 | 401 | 407 | 413 | 420 | 426 | 433 | 439 | 446 | 452 |
| 35 | 414 | 420 | 426 | 432 | 438 | 444 | 450 | 456 | 462 |
| 40 | 429 | 434 | 440 | 446 | 451 | 457 | 462 | 468 | 474 |
| 45 | 444 | 449 | 454 | 459 | 464 | 470 | 475 | 480 | 485 |
| 50 | 459 | 464 | 468 | 473 | 478 | 482 | 487 | 492 | 496 |
| 55 | 474 | 478 | 482 | 486 | 490 | 495 | 499 | 503 | 508 |
| 60 | 487 | 491 | 495 | 499 | 502 | 506 | 510 | 514 | 518 |
| 65 | 500 | 503 | 507 | 513 | 514 | 517 | 520 | 524 | 528 |
| 70 | 510 | 514 | 517 | 520 | 523 | 526 | 529 | 532 | 536 |
| 75 | 519 | 522 | 525 | 528 | 531 | 534 | 536 | 539 | 542 |
| 80 | 525 | 528 | 531 | 534 | 536 | 539 | 542 | 544 | 547 |
| 85 | 529 | 532 | 534 | 537 | 540 | 542 | 545 | 548 | 550 |
| 90 | 531 | 533 | 536 | 538 | 541 | 544 | 546 | 549 | 551 |

*Logarithms of radii of curvature of the earth's surface (in meters)*—Continued

| Azimuth (degrees) | Latitude | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 48° | 49° | 50° | 51° | 52° | 53° | 54° | 55° | 56° |
| 0 | 6.80419 | 6.80426 | 6.80434 | 6.80442 | 6.80449 | 6.80457 | 6.80464 | 6.80471 | 6.80479 |
| 5 | 420 | 428 | 435 | 443 | 450 | 458 | 465 | 472 | 479 |
| 10 | 423 | 430 | 438 | 445 | 453 | 460 | 467 | 474 | 481 |
| 15 | 428 | 435 | 442 | 450 | 457 | 464 | 471 | 478 | 485 |
| 20 | 434 | 441 | 448 | 455 | 462 | 469 | 476 | 483 | 489 |
| 25 | 442 | 449 | 456 | 463 | 469 | 476 | 482 | 489 | 495 |
| 30 | 452 | 458 | 465 | 471 | 477 | 484 | 490 | 496 | 502 |
| 35 | 462 | 468 | 474 | 480 | 486 | 492 | 498 | 503 | 509 |
| 40 | 474 | 479 | 485 | 490 | 496 | 501 | 506 | 512 | 517 |
| 45 | 485 | 490 | 495 | 500 | 505 | 510 | 515 | 520 | 525 |
| 50 | 496 | 501 | 506 | 510 | 515 | 520 | 524 | 528 | 533 |
| 55 | 508 | 512 | 516 | 520 | 524 | 528 | 533 | 537 | 541 |
| 60 | 518 | 522 | 526 | 530 | 533 | 537 | 541 | 544 | 548 |
| 65 | 528 | 531 | 534 | 538 | 541 | 545 | 548 | 551 | 555 |
| 70 | 536 | 539 | 542 | 545 | 548 | 551 | 554 | 557 | 560 |
| 75 | 542 | 545 | 548 | 551 | 554 | 557 | 559 | 562 | 565 |
| 80 | 547 | 550 | 553 | 555 | 558 | 561 | 563 | 566 | 568 |
| 85 | 550 | 553 | 555 | 558 | 560 | 563 | 566 | 568 | 570 |
| 90 | 551 | 554 | 556 | 559 | 561 | 564 | 566 | 569 | 571 |

| Azimuth (degrees) | Latitude | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 56° | 57° | 58° | 59° | 60° | 61° | 62° | 63° | 64° |
| 0 | 6.80479 | 6.80486 | 6.80493 | 6.80500 | 6.80506 | 6.80513 | 6.80520 | 6.80526 | 6.80532 |
| 5 | 479 | 486 | 493 | 500 | 507 | 514 | 520 | 526 | 532 |
| 10 | 481 | 488 | 495 | 502 | 509 | 515 | 522 | 528 | 534 |
| 15 | 485 | 492 | 498 | 505 | 511 | 518 | 524 | 530 | 536 |
| 20 | 489 | 496 | 502 | 509 | 515 | 521 | 527 | 533 | 539 |
| 25 | 495 | 501 | 508 | 514 | 520 | 526 | 531 | 537 | 542 |
| 30 | 502 | 508 | 514 | 519 | 525 | 530 | 536 | 541 | 546 |
| 35 | 509 | 515 | 520 | 525 | 531 | 536 | 541 | 546 | 551 |
| 40 | 517 | 522 | 527 | 532 | 537 | 542 | 546 | 551 | 556 |
| 45 | 525 | 530 | 534 | 539 | 543 | 548 | 552 | 556 | 560 |
| 50 | 533 | 537 | 542 | 546 | 550 | 554 | 558 | 562 | 565 |
| 55 | 541 | 545 | 548 | 552 | 556 | 560 | 563 | 567 | 570 |
| 60 | 548 | 552 | 555 | 558 | 562 | 565 | 568 | 572 | 575 |
| 65 | 555 | 558 | 561 | 564 | 567 | 570 | 573 | 576 | 579 |
| 70 | 560 | 563 | 566 | 569 | 572 | 574 | 577 | 580 | 582 |
| 75 | 565 | 568 | 570 | 573 | 575 | 578 | 580 | 583 | 585 |
| 80 | 568 | 571 | 573 | 576 | 578 | 580 | 583 | 585 | 587 |
| 85 | 570 | 573 | 575 | 578 | 580 | 582 | 584 | 586 | 588 |
| 90 | 571 | 574 | 576 | 578 | 580 | 583 | 585 | 587 | 589 |

| Azimuth (degrees) | Latitude | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 64° | 65° | 66° | 67° | 68° | 69° | 70° | 71° | 72° |
| 0 | 6.80532 | 6.80538 | 6.80544 | 6.80550 | 6.80555 | 6.80560 | 6.80565 | 6.80570 | 6.80575 |
| 5 | 532 | 538 | 544 | 550 | 555 | 561 | 566 | 570 | 575 |
| 10 | 534 | 540 | 545 | 551 | 556 | 562 | 566 | 571 | 576 |
| 15 | 536 | 542 | 547 | 553 | 558 | 563 | 568 | 572 | 577 |
| 20 | 539 | 544 | 550 | 555 | 560 | 565 | 570 | 574 | 578 |
| 25 | 542 | 548 | 553 | 558 | 562 | 567 | 572 | 576 | 580 |
| 30 | 546 | 551 | 556 | 561 | 565 | 570 | 574 | 578 | 582 |
| 35 | 551 | 556 | 560 | 564 | 569 | 573 | 577 | 581 | 584 |
| 40 | 556 | 560 | 564 | 568 | 572 | 576 | 580 | 583 | 587 |
| 45 | 560 | 564 | 568 | 572 | 576 | 579 | 583 | 586 | 589 |
| 50 | 565 | 569 | 573 | 576 | 579 | 583 | 586 | 589 | 592 |
| 55 | 570 | 574 | 577 | 580 | 583 | 586 | 589 | 591 | 594 |
| 60 | 575 | 578 | 581 | 584 | 586 | 589 | 591 | 594 | 596 |
| 65 | 579 | 582 | 584 | 587 | 589 | 592 | 594 | 596 | 6.80598 |
| 70 | 582 | 585 | 587 | 590 | 592 | 594 | 596 | 6.80598 | 6.80600 |
| 75 | 585 | 587 | 590 | 592 | 594 | 596 | 598 | 6.80600 | 601 |
| 80 | 587 | 589 | 591 | 593 | 595 | 597 | 6.80599 | 601 | 602 |
| 85 | 588 | 590 | 592 | 594 | 596 | 598 | 6.80600 | 601 | 603 |
| 90 | 589 | 591 | 593 | 595 | 597 | 598 | 600 | 602 | 603 |

## CONVERSION TABLES

*Lengths—Feet to meters (from 1 to 1000 units)*

[Reduction factor: 1 foot=0.3048006096 meter]

| Feet | Meters | Feet | Meters | Feet | Meters | Feet | Meters | Feet | Meters |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.0 | 50 | 15.24003 | 100 | 30.48006 | 150 | 45.72009 | 200 | 60.96012 |
| 1 | 0.30480 | 1 | 15.54483 | 1 | 30.78486 | 1 | 46.02489 | 1 | 61.26492 |
| 2 | 0.60960 | 2 | 15.84963 | 2 | 31.08966 | 2 | 46.32969 | 2 | 61.56972 |
| 3 | 0.91440 | 3 | 16.15443 | 3 | 31.39446 | 3 | 46.63449 | 3 | 61.87452 |
| 4 | 1.21920 | 4 | 16.45923 | 4 | 31.69926 | 4 | 46.93929 | 4 | 62.17932 |
| 5 | 1.52400 | 5 | 16.76403 | 5 | 32.00406 | 5 | 47.24409 | 5 | 62.48412 |
| 6 | 1.82880 | 6 | 17.06883 | 6 | 32.30886 | 6 | 47.54890 | 6 | 62.78893 |
| 7 | 2.13360 | 7 | 17.37363 | 7 | 32.61367 | 7 | 47.85370 | 7 | 63.09373 |
| 8 | 2.43840 | 8 | 17.67844 | 8 | 32.91847 | 8 | 48.15850 | 8 | 63.39853 |
| 9 | 2.74321 | 9 | 17.98324 | 9 | 33.22327 | 9 | 48.46330 | 9 | 63.70333 |
| 10 | 3.04801 | 60 | 18.28804 | 110 | 33.52807 | 160 | 48.76810 | 210 | 64.00813 |
| 1 | 3.35281 | 1 | 18.59284 | 1 | 33.83287 | 1 | 49.07290 | 1 | 64.31293 |
| 2 | 3.65761 | 2 | 18.89764 | 2 | 34.13767 | 2 | 49.37770 | 2 | 64.61773 |
| 3 | 3.96241 | 3 | 19.20244 | 3 | 34.44247 | 3 | 49.68250 | 3 | 64.92253 |
| 4 | 4.26721 | 4 | 19.50724 | 4 | 34.74727 | 4 | 49.98730 | 4 | 65.22733 |
| 5 | 4.57201 | 5 | 19.81204 | 5 | 35.05207 | 5 | 50.29210 | 5 | 65.53213 |
| 6 | 4.87681 | 6 | 20.11684 | 6 | 35.35687 | 6 | 50.59690 | 6 | 65.83693 |
| 7 | 5.18161 | 7 | 20.42164 | 7 | 35.66167 | 7 | 50.90170 | 7 | 66.14173 |
| 8 | 5.48641 | 8 | 20.72644 | 8 | 35.96647 | 8 | 51.20650 | 8 | 66.44653 |
| 9 | 5.79121 | 9 | 21.03124 | 9 | 36.27127 | 9 | 51.51130 | 9 | 66.75133 |
| 20 | 6.09601 | 70 | 21.33604 | 120 | 36.57607 | 170 | 51.81610 | 220 | 67.05613 |
| 1 | 6.40081 | 1 | 21.64084 | 1 | 36.88087 | 1 | 52.12090 | 1 | 67.36093 |
| 2 | 6.70561 | 2 | 21.94564 | 2 | 37.18567 | 2 | 52.42570 | 2 | 67.66574 |
| 3 | 7.01041 | 3 | 22.25044 | 3 | 37.49047 | 3 | 52.73051 | 3 | 67.97054 |
| 4 | 7.31521 | 4 | 22.55525 | 4 | 37.79528 | 4 | 53.03531 | 4 | 68.27534 |
| 5 | 7.62002 | 5 | 22.86005 | 5 | 38.10008 | 5 | 53.34011 | 5 | 68.58014 |
| 6 | 7.92482 | 6 | 23.16485 | 6 | 38.40488 | 6 | 53.64491 | 6 | 68.88494 |
| 7 | 8.22962 | 7 | 23.46965 | 7 | 38.70968 | 7 | 53.94971 | 7 | 69.18974 |
| 8 | 8.53442 | 8 | 23.77445 | 8 | 39.01448 | 8 | 54.25451 | 8 | 69.49454 |
| 9 | 8.83922 | 9 | 24.07925 | 9 | 39.31928 | 9 | 54.55931 | 9 | 69.79934 |
| 30 | 9.14402 | 80 | 24.38405 | 130 | 39.62408 | 180 | 54.86411 | 230 | 70.10414 |
| 1 | 9.44882 | 1 | 24.68885 | 1 | 39.92888 | 1 | 55.16891 | 1 | 70.40894 |
| 2 | 9.75362 | 2 | 24.99365 | 2 | 40.23368 | 2 | 55.47371 | 2 | 70.71374 |
| 3 | 10.05842 | 3 | 25.29845 | 3 | 40.53848 | 3 | 55.77851 | 3 | 71.01854 |
| 4 | 10.36322 | 4 | 25.60325 | 4 | 40.84328 | 4 | 56.08331 | 4 | 71.32334 |
| 5 | 10.66802 | 5 | 25.90805 | 5 | 41.14808 | 5 | 56.38811 | 5 | 71.62814 |
| 6 | 10.97282 | 6 | 26.21285 | 6 | 41.45288 | 6 | 56.69291 | 6 | 71.93294 |
| 7 | 11.27762 | 7 | 26.51765 | 7 | 41.75768 | 7 | 56.99771 | 7 | 72.23774 |
| 8 | 11.58242 | 8 | 26.82245 | 8 | 42.06248 | 8 | 57.30251 | 8 | 72.54255 |
| 9 | 11.88722 | 9 | 27.12725 | 9 | 42.36728 | 9 | 57.60732 | 9 | 72.84735 |
| 40 | 12.19202 | 90 | 27.43205 | 140 | 42.67209 | 190 | 57.91212 | 240 | 73.15215 |
| 1 | 12.49682 | 1 | 27.73686 | 1 | 42.97689 | 1 | 58.21692 | 1 | 73.45695 |
| 2 | 12.80163 | 2 | 28.04166 | 2 | 43.28169 | 2 | 58.52172 | 2 | 73.76175 |
| 3 | 13.10643 | 3 | 28.34646 | 3 | 43.58649 | 3 | 58.82652 | 3 | 74.06655 |
| 4 | 13.41123 | 4 | 28.65126 | 4 | 43.89129 | 4 | 59.13132 | 4 | 74.37135 |
| 5 | 13.71603 | 5 | 28.95606 | 5 | 44.19609 | 5 | 59.43612 | 5 | 74.67615 |
| 6 | 14.02083 | 6 | 29.26086 | 6 | 44.50089 | 6 | 59.74092 | 6 | 74.98095 |
| 7 | 14.32563 | 7 | 29.56566 | 7 | 44.80569 | 7 | 60.04572 | 7 | 75.28575 |
| 8 | 14.63043 | 8 | 29.87046 | 8 | 45.11049 | 8 | 60.35052 | 8 | 75.59055 |
| 9 | 14.93523 | 9 | 30.17526 | 9 | 45.41529 | 9 | 60.65532 | 9 | 75.89535 |

*Lengths—Feet to meters (from 1 to 1000 units)*—Continued

| Feet | Meters | Feet | Meters | Feet | Meters | Feet | Meters | Feet | Meters |
|---|---|---|---|---|---|---|---|---|---|
| 250 | 76.20015 | 300 | 91.44018 | 350 | 106.68021 | 400 | 121.92024 | 450 | 137.16027 |
| 1 | 76.50495 | 1 | 91.74498 | 1 | 106.98501 | 1 | 122.22504 | 1 | 137.46507 |
| 2 | 76.80975 | 2 | 92.04978 | 2 | 107.28981 | 2 | 122.52985 | 2 | 137.76988 |
| 3 | 77.11455 | 3 | 92.35458 | 3 | 107.59462 | 3 | 122.83465 | 3 | 138.07468 |
| 4 | 77.41935 | 4 | 92.65939 | 4 | 107.89942 | 4 | 123.13945 | 4 | 138.37948 |
| 5 | 77.72416 | 5 | 92.96419 | 5 | 108.20422 | 5 | 123.44425 | 5 | 138.68428 |
| 6 | 78.02896 | 6 | 93.26899 | 6 | 108.50902 | 6 | 123.74905 | 6 | 138.98908 |
| 7 | 78.33376 | 7 | 93.57379 | 7 | 108.81382 | 7 | 124.05385 | 7 | 139.29388 |
| 8 | 78.63856 | 8 | 93.87859 | 8 | 109.11862 | 8 | 124.35865 | 8 | 139.59868 |
| 9 | 78.94336 | 9 | 94.18339 | 9 | 109.42342 | 9 | 124.66345 | 9 | 139.90348 |
| 260 | 79.24816 | 310 | 94.48819 | 360 | 109.72822 | 410 | 124.96825 | 460 | 140.20828 |
| 1 | 79.55296 | 1 | 94.79299 | 1 | 110.03302 | 1 | 125.27305 | 1 | 140.51308 |
| 2 | 79.85776 | 2 | 95.09779 | 2 | 110.33782 | 2 | 125.57785 | 2 | 140.81788 |
| 3 | 80.16256 | 3 | 95.40259 | 3 | 110.64262 | 3 | 125.88265 | 3 | 141.12268 |
| 4 | 80.46736 | 4 | 95.70739 | 4 | 110.94742 | 4 | 126.18745 | 4 | 141.42748 |
| 5 | 80.77216 | 5 | 96.01219 | 5 | 111.25222 | 5 | 126.49225 | 5 | 141.73228 |
| 6 | 81.07696 | 6 | 96.31699 | 6 | 111.55702 | 6 | 126.79705 | 6 | 142.03708 |
| 7 | 81.38176 | 7 | 96.62179 | 7 | 111.86182 | 7 | 127.10185 | 7 | 142.34188 |
| 8 | 81.68656 | 8 | 96.92659 | 8 | 112.16662 | 8 | 127.40665 | 8 | 142.64669 |
| 9 | 81.99136 | 9 | 97.23139 | 9 | 112.47142 | 9 | 127.71146 | 9 | 142.95149 |
| 270 | 82.29616 | 320 | 97.53620 | 370 | 112.77623 | 420 | 128.01626 | 470 | 143.25629 |
| 1 | 82.60097 | 1 | 97.84100 | 1 | 113.08103 | 1 | 128.32106 | 1 | 143.56109 |
| 2 | 82.90577 | 2 | 98.14580 | 2 | 113.38583 | 2 | 128.62586 | 2 | 143.86589 |
| 3 | 83.21057 | 3 | 98.45060 | 3 | 113.69063 | 3 | 128.93066 | 3 | 144.17069 |
| 4 | 83.51537 | 4 | 98.75540 | 4 | 113.99543 | 4 | 129.23546 | 4 | 144.47549 |
| 5 | 83.82017 | 5 | 99.06020 | 5 | 114.30023 | 5 | 129.54026 | 5 | 144.78029 |
| 6 | 84.12497 | 6 | 99.36500 | 6 | 114.60503 | 6 | 129.84506 | 6 | 145.08509 |
| 7 | 84.42977 | 7 | 99.66980 | 7 | 114.90983 | 7 | 130.14986 | 7 | 145.38989 |
| 8 | 84.73457 | 8 | 99.97460 | 8 | 115.21463 | 8 | 130.45466 | 8 | 145.69469 |
| 9 | 85.03937 | 9 | 100.27940 | 9 | 115.51943 | 9 | 130.75946 | 9 | 145.99949 |
| 280 | 85.34417 | 330 | 100.58420 | 380 | 115.82423 | 430 | 131.06426 | 480 | 146.30429 |
| 1 | 85.64897 | 1 | 100.88900 | 1 | 116.12903 | 1 | 131.36906 | 1 | 146.60909 |
| 2 | 85.95377 | 2 | 101.19380 | 2 | 116.43383 | 2 | 131.67386 | 2 | 146.91389 |
| 3 | 86.25857 | 3 | 101.49860 | 3 | 116.73863 | 3 | 131.97866 | 3 | 147.21869 |
| 4 | 86.56337 | 4 | 101.80340 | 4 | 117.04343 | 4 | 132.28346 | 4 | 147.52350 |
| 5 | 86.86817 | 5 | 102.10820 | 5 | 117.34823 | 5 | 132.58827 | 5 | 147.82830 |
| 6 | 87.17297 | 6 | 102.41300 | 6 | 117.65304 | 6 | 132.89307 | 6 | 148.13310 |
| 7 | 87.47777 | 7 | 102.71781 | 7 | 117.95784 | 7 | 133.19787 | 7 | 148.43790 |
| 8 | 87.78258 | 8 | 103.02261 | 8 | 118.26264 | 8 | 133.50267 | 8 | 148.74270 |
| 9 | 88.08738 | 9 | 103.32741 | 9 | 118.56744 | 9 | 133.80747 | 9 | 149.04750 |
| 290 | 88.39218 | 340 | 103.63221 | 390 | 118.87224 | 440 | 134.11227 | 490 | 149.35230 |
| 1 | 88.69698 | 1 | 103.93701 | 1 | 119.17704 | 1 | 134.41707 | 1 | 149.65710 |
| 2 | 89.00178 | 2 | 104.24181 | 2 | 119.48184 | 2 | 134.72187 | 2 | 149.96190 |
| 3 | 89.30658 | 3 | 104.54661 | 3 | 119.78664 | 3 | 135.02667 | 3 | 150.26670 |
| 4 | 89.61138 | 4 | 104.85141 | 4 | 120.09144 | 4 | 135.33147 | 4 | 150.57150 |
| 5 | 89.91618 | 5 | 105.15621 | 5 | 120.39624 | 5 | 135.63627 | 5 | 150.87630 |
| 6 | 90.22098 | 6 | 105.46101 | 6 | 120.70104 | 6 | 135.94107 | 6 | 151.18110 |
| 7 | 90.52578 | 7 | 105.76581 | 7 | 121.00584 | 7 | 136.24587 | 7 | 151.48590 |
| 8 | 90.83058 | 8 | 106.07061 | 8 | 121.31064 | 8 | 136.55067 | 8 | 151.79070 |
| 9 | 91.13538 | 9 | 106.37541 | 9 | 121.61544 | 9 | 136.85547 | 9 | 152.09550 |

*Lengths—Feet to meters (from 1 to 1000 units)*—**Continued**

| Feet | Meters | Feet | Meters | Feet | Meters | Feet | Meters | Feet | Meters |
|---|---|---|---|---|---|---|---|---|---|
| 500 | 152.40030 | 550 | 167.64034 | 600 | 182.88037 | 650 | 198.12040 | 700 | 213.36043 |
| 1 | 152.70511 | 1 | 167.94514 | 1 | 183.18517 | 1 | 198.42520 | 1 | 213.66523 |
| 2 | 153.00991 | 2 | 168.24994 | 2 | 183.48997 | 2 | 198.73000 | 2 | 213.97003 |
| 3 | 153.31471 | 3 | 168.55474 | 3 | 183.79477 | 3 | 199.03480 | 3 | 214.27483 |
| 4 | 153.61951 | 4 | 168.85954 | 4 | 184.09957 | 4 | 199.33960 | 4 | 214.57963 |
| 5 | 153.92431 | 5 | 169.16434 | 5 | 184.40437 | 5 | 199.64440 | 5 | 214.88443 |
| 6 | 154.22911 | 6 | 169.46914 | 6 | 184.70917 | 6 | 199.94920 | 6 | 215.18923 |
| 7 | 154.53391 | 7 | 169.77394 | 7 | 185.01397 | 7 | 200.25400 | 7 | 215.49403 |
| 8 | 154.83871 | 8 | 170.07874 | 8 | 185.31877 | 8 | 200.55880 | 8 | 215.79883 |
| 9 | 155.14351 | 9 | 170.38354 | 9 | 185.62357 | 9 | 200.86360 | 9 | 216.10363 |
| 510 | 155.44831 | 560 | 170.68834 | 610 | 185.92837 | 660 | 201.16840 | 710 | 216.40843 |
| 1 | 155.75311 | 1 | 170.99314 | 1 | 186.23317 | 1 | 201.47320 | 1 | 216.71323 |
| 2 | 156.05791 | 2 | 171.29794 | 2 | 186.53797 | 2 | 201.77800 | 2 | 217.01803 |
| 3 | 156.36271 | 3 | 171.60274 | 3 | 186.84277 | 3 | 202.08280 | 3 | 217.32283 |
| 4 | 156.66751 | 4 | 171.90754 | 4 | 187.14757 | 4 | 202.38760 | 4 | 217.62764 |
| 5 | 156.97231 | 5 | 172.21234 | 5 | 187.45237 | 5 | 202.69241 | 5 | 217.93244 |
| 6 | 157.27711 | 6 | 172.51715 | 6 | 187.75718 | 6 | 202.99721 | 6 | 218.23724 |
| 7 | 157.58192 | 7 | 172.82195 | 7 | 188.06198 | 7 | 203.30201 | 7 | 218.54204 |
| 8 | 157.88672 | 8 | 173.12675 | 8 | 188.36678 | 8 | 203.60681 | 8 | 218.84684 |
| 9 | 158.19152 | 9 | 173.43155 | 9 | 188.67158 | 9 | 203.91161 | 9 | 219.15164 |
| 520 | 158.49632 | 570 | 173.73635 | 620 | 188.97638 | 670 | 204.21641 | 720 | 219.45644 |
| 1 | 158.80112 | 1 | 174.04115 | 1 | 189.28118 | 1 | 204.52121 | 1 | 219.76124 |
| 2 | 159.10592 | 2 | 174.34595 | 2 | 189.58598 | 2 | 204.82601 | 2 | 220.06604 |
| 3 | 159.41072 | 3 | 174.65075 | 3 | 189.89078 | 3 | 205.13081 | 3 | 220.37084 |
| 4 | 159.71552 | 4 | 174.95555 | 4 | 190.19558 | 4 | 205.43561 | 4 | 220.67564 |
| 5 | 160.02032 | 5 | 175.26035 | 5 | 190.50038 | 5 | 205.74041 | 5 | 220.98044 |
| 6 | 160.32512 | 6 | 175.56515 | 6 | 190.80518 | 6 | 206.04521 | 6 | 221.28524 |
| 7 | 160.62992 | 7 | 175.86995 | 7 | 191.10998 | 7 | 206.35001 | 7 | 221.59004 |
| 8 | 160.93472 | 8 | 176.17475 | 8 | 191.41478 | 8 | 206.65481 | 8 | 221.89484 |
| 9 | 161.23952 | 9 | 176.47955 | 9 | 191.71958 | 9 | 206.95961 | 9 | 222.19964 |
| 530 | 161.54432 | 580 | 176.78435 | 630 | 192.02438 | 680 | 207.26441 | 730 | 222.50445 |
| 1 | 161.84912 | 1 | 177.08915 | 1 | 192.32918 | 1 | 207.56922 | 1 | 222.80925 |
| 2 | 162.15392 | 2 | 177.39395 | 2 | 192.63399 | 2 | 207.87402 | 2 | 223.11405 |
| 3 | 162.45872 | 3 | 177.69876 | 3 | 192.93879 | 3 | 208.17882 | 3 | 223.41885 |
| 4 | 162.76353 | 4 | 178.00356 | 4 | 193.24359 | 4 | 208.48362 | 4 | 223.72365 |
| 5 | 163.06833 | 5 | 178.30836 | 5 | 193.54839 | 5 | 208.78842 | 5 | 224.02845 |
| 6 | 163.37313 | 6 | 178.61316 | 6 | 193.85319 | 6 | 209.09322 | 6 | 224.33325 |
| 7 | 163.67793 | 7 | 178.91796 | 7 | 194.15799 | 7 | 209.39802 | 7 | 224.63805 |
| 8 | 163.98273 | 8 | 179.22276 | 8 | 194.46279 | 8 | 209.70282 | 8 | 224.94285 |
| 9 | 164.28753 | 9 | 179.52756 | 9 | 194.76759 | 9 | 210.00762 | 9 | 225.24765 |
| 540 | 164.59233 | 590 | 179.83236 | 640 | 195.07239 | 690 | 210.31242 | 740 | 225.55245 |
| 1 | 164.89713 | 1 | 180.13716 | 1 | 195.37719 | 1 | 210.61722 | 1 | 225.85725 |
| 2 | 165.20193 | 2 | 180.44196 | 2 | 195.68199 | 2 | 210.92202 | 2 | 226.16205 |
| 3 | 165.50673 | 3 | 180.74676 | 3 | 195.98679 | 3 | 211.22682 | 3 | 226.46685 |
| 4 | 165.81153 | 4 | 181.05156 | 4 | 196.29159 | 4 | 211.53162 | 4 | 226.77165 |
| 5 | 166.11633 | 5 | 181.35636 | 5 | 196.59639 | 5 | 211.83642 | 5 | 227.07645 |
| 6 | 166.42113 | 6 | 181.66116 | 6 | 196.90119 | 6 | 212.14122 | 6 | 227.38125 |
| 7 | 166.72593 | 7 | 181.96596 | 7 | 197.20599 | 7 | 212.44602 | 7 | 227.68606 |
| 8 | 167.03073 | 8 | 182.27076 | 8 | 197.51080 | 8 | 212.75083 | 8 | 227.99086 |
| 9 | 167.33553 | 9 | 182.57557 | 9 | 197.81560 | 9 | 213.05563 | 9 | 228.29566 |

*Lengths—Feet to meters (from 1 to 1000 units)*—Continued

| Feet | Meters | Feet | Meters | Feet | Meters | Feet | Meters | Feet | Meters |
|---|---|---|---|---|---|---|---|---|---|
| 750 | 228.60046 | 800 | 243.84049 | 850 | 259.08052 | 900 | 274.32055 | 950 | 289.56058 |
| 1 | 228.90526 | 1 | 244.14529 | 1 | 259.38532 | 1 | 274.62535 | 1 | 289.86538 |
| 2 | 229.21006 | 2 | 244.45009 | 2 | 259.69012 | 2 | 274.93015 | 2 | 290.17018 |
| 3 | 229.51486 | 3 | 244.75489 | 3 | 259.99492 | 3 | 275.23495 | 3 | 290.47498 |
| 4 | 229.81966 | 4 | 245.05969 | 4 | 260.29972 | 4 | 275.53975 | 4 | 290.77978 |
| 5 | 230.12446 | 5 | 245.36449 | 5 | 260.60452 | 5 | 275.84455 | 5 | 291.08458 |
| 6 | 230.42926 | 6 | 245.66929 | 6 | 260.90932 | 6 | 276.14935 | 6 | 291.38938 |
| 7 | 230.73406 | 7 | 245.97409 | 7 | 261.21412 | 7 | 276.45415 | 7 | 291.69418 |
| 8 | 231.03886 | 8 | 246.27889 | 8 | 261.51892 | 8 | 276.75895 | 8 | 291.99898 |
| 9 | 231.34366 | 9 | 246.58369 | 9 | 261.82372 | 9 | 277.06375 | 9 | 292.30378 |
| 760 | 231.64846 | 810 | 246.88849 | 860 | 262.12852 | 910 | 277.36855 | 960 | 292.60859 |
| 1 | 231.95326 | 1 | 247.19329 | 1 | 262.43332 | 1 | 277.67336 | 1 | 292.91339 |
| 2 | 232.25806 | 2 | 247.49809 | 2 | 262.73813 | 2 | 277.97816 | 2 | 293.21819 |
| 3 | 232.56287 | 3 | 247.80290 | 3 | 263.04293 | 3 | 278.28296 | 3 | 293.52299 |
| 4 | 232.86767 | 4 | 248.10770 | 4 | 263.34773 | 4 | 278.58776 | 4 | 293.82779 |
| 5 | 233.17247 | 5 | 248.41250 | 5 | 263.65253 | 5 | 278.89256 | 5 | 294.13259 |
| 6 | 233.47727 | 6 | 248.71730 | 6 | 263.95733 | 6 | 279.19736 | 6 | 294.43739 |
| 7 | 233.78207 | 7 | 249.02210 | 7 | 264.26213 | 7 | 279.50216 | 7 | 294.74219 |
| 8 | 234.08687 | 8 | 249.32690 | 8 | 264.56693 | 8 | 279.80696 | 8 | 295.04699 |
| 9 | 234.39167 | 9 | 249.63170 | 9 | 264.87173 | 9 | 280.11176 | 9 | 295.35179 |
| 770 | 234.69647 | 820 | 249.93650 | 870 | 265.17653 | 920 | 280.41656 | 970 | 295.65659 |
| 1 | 235.00127 | 1 | 250.24130 | 1 | 265.48133 | 1 | 280.72136 | 1 | 295.96139 |
| 2 | 235.30607 | 2 | 250.54610 | 2 | 265.78613 | 2 | 281.02616 | 2 | 296.26619 |
| 3 | 235.61087 | 3 | 250.85090 | 3 | 266.09093 | 3 | 281.33096 | 3 | 296.57099 |
| 4 | 235.91567 | 4 | 251.15570 | 4 | 266.39573 | 4 | 281.63576 | 4 | 296.87579 |
| 5 | 236.22047 | 5 | 251.46050 | 5 | 266.70053 | 5 | 281.94056 | 5 | 297.18059 |
| 6 | 236.52527 | 6 | 251.76530 | 6 | 267.00533 | 6 | 282.24536 | 6 | 297.48539 |
| 7 | 236.83007 | 7 | 252.07010 | 7 | 267.31013 | 7 | 282.55017 | 7 | 297.79020 |
| 8 | 237.13487 | 8 | 252.37490 | 8 | 267.61494 | 8 | 282.85497 | 8 | 298.09500 |
| 9 | 237.43967 | 9 | 252.67971 | 9 | 267.91974 | 9 | 283.15977 | 9 | 298.39980 |
| 780 | 237.74448 | 830 | 252.98451 | 880 | 268.22454 | 930 | 283.46457 | 980 | 298.70460 |
| 1 | 238.04928 | 1 | 253.28931 | 1 | 268.52934 | 1 | 283.76937 | 1 | 299.00940 |
| 2 | 238.35408 | 2 | 253.59411 | 2 | 268.83414 | 2 | 284.07417 | 2 | 299.31420 |
| 3 | 238.65888 | 3 | 253.89891 | 3 | 269.13894 | 3 | 284.37897 | 3 | 299.61900 |
| 4 | 238.96368 | 4 | 254.20371 | 4 | 269.44374 | 4 | 284.68377 | 4 | 299.92380 |
| 5 | 239.26848 | 5 | 254.50851 | 5 | 269.74854 | 5 | 284.98857 | 5 | 300.22860 |
| 6 | 239.57328 | 6 | 254.81331 | 6 | 270.05334 | 6 | 285.29337 | 6 | 300.53340 |
| 7 | 239.87808 | 7 | 255.11811 | 7 | 270.35814 | 7 | 285.59817 | 7 | 300.83820 |
| 8 | 240.18288 | 8 | 255.42291 | 8 | 270.66294 | 8 | 285.90297 | 8 | 301.14300 |
| 9 | 240.48768 | 9 | 255.72771 | 9 | 270.96774 | 9 | 286.20777 | 9 | 301.44780 |
| 790 | 240.79248 | 840 | 256.03251 | 890 | 271.27254 | 940 | 286.51257 | 990 | 301.75260 |
| 1 | 241.09728 | 1 | 256.33731 | 1 | 271.57734 | 1 | 286.81737 | 1 | 302.05740 |
| 2 | 241.40208 | 2 | 256.64211 | 2 | 271.88214 | 2 | 287.12217 | 2 | 302.36220 |
| 3 | 241.70688 | 3 | 256.94691 | 3 | 272.18694 | 3 | 287.42697 | 3 | 302.66701 |
| 4 | 242.01168 | 4 | 257.25171 | 4 | 272.49174 | 4 | 287.73178 | 4 | 302.97181 |
| 5 | 242.31648 | 5 | 257.55652 | 5 | 272.79655 | 5 | 288.03658 | 5 | 303.27661 |
| 6 | 242.62129 | 6 | 257.86132 | 6 | 273.10135 | 6 | 288.34138 | 6 | 303.58141 |
| 7 | 242.92609 | 7 | 258.16612 | 7 | 273.40615 | 7 | 288.64618 | 7 | 303.88621 |
| 8 | 243.23089 | 8 | 258.47092 | 8 | 273.71095 | 8 | 288.95098 | 8 | 304.19101 |
| 9 | 243.53569 | 9 | 258.77572 | 9 | 274.01575 | 9 | 289.25578 | 9 | 304.49581 |

*Lengths—Meters to feet (from 1 to 1000 units)*

[Reduction factor: 1 meter=3.280833333 feet]

| Meters | Feet | Meters | Feet | Meters | Feet | Meters | Feet | Meters | Feet |
|---|---|---|---|---|---|---|---|---|---|
| 0 | | 50 | 164.04167 | 100 | 328.08333 | 150 | 492.12500 | 200 | 656.16667 |
| 1 | 3.28083 | 1 | 167.32250 | 1 | 331.36417 | 1 | 495.40583 | 1 | 659.44750 |
| 2 | 6.56167 | 2 | 170.60333 | 2 | 334.64500 | 2 | 498.68667 | 2 | 662.72833 |
| 3 | 9.84250 | 3 | 173.88417 | 3 | 337.92583 | 3 | 501.96750 | 3 | 666.00917 |
| 4 | 13.12333 | 4 | 177.16500 | 4 | 341.20667 | 4 | 505.24833 | 4 | 669.29000 |
| 5 | 16.40417 | 5 | 180.44583 | 5 | 344.48750 | 5 | 508.52917 | 5 | 672.57083 |
| 6 | 19.68500 | 6 | 183.72667 | 6 | 347.76833 | 6 | 511.81000 | 6 | 675.85167 |
| 7 | 22.96583 | 7 | 187.00750 | 7 | 351.04917 | 7 | 515.09083 | 7 | 679.13250 |
| 8 | 26.24667 | 8 | 190.28833 | 8 | 354.33000 | 8 | 518.37167 | 8 | 682.41333 |
| 9 | 29.52750 | 9 | 193.56917 | 9 | 357.61083 | 9 | 521.65250 | 9 | 685.69417 |
| 10 | 32.80833 | 60 | 196.85000 | 110 | 360.89167 | 160 | 524.93333 | 210 | 688.97500 |
| 1 | 36.08917 | 1 | 200.13083 | 1 | 364.17250 | 1 | 528.21417 | 1 | 692.25583 |
| 2 | 39.37000 | 2 | 203.41167 | 2 | 367.45333 | 2 | 531.49500 | 2 | 695.53667 |
| 3 | 42.65083 | 3 | 206.69250 | 3 | 370.73417 | 3 | 534.77583 | 3 | 698.81750 |
| 4 | 45.93167 | 4 | 209.97333 | 4 | 374.01500 | 4 | 538.05667 | 4 | 702.09833 |
| 5 | 49.21250 | 5 | 213.25417 | 5 | 377.29583 | 5 | 541.33750 | 5 | 705.37917 |
| 6 | 52.49333 | 6 | 216.53500 | 6 | 380.57667 | 6 | 544.61833 | 6 | 708.66000 |
| 7 | 55.77417 | 7 | 219.81583 | 7 | 383.85750 | 7 | 547.89917 | 7 | 711.94083 |
| 8 | 59.05500 | 8 | 223.09667 | 8 | 387.13833 | 8 | 551.18000 | 8 | 715.22167 |
| 9 | 62.33583 | 9 | 226.37750 | 9 | 390.41917 | 9 | 554.46083 | 9 | 718.50250 |
| 20 | 65.61667 | 70 | 229.65833 | 120 | 393.70000 | 170 | 557.74167 | 220 | 721.78333 |
| 1 | 68.89750 | 1 | 232.93917 | 1 | 396.98083 | 1 | 561.02250 | 1 | 725.06417 |
| 2 | 72.17833 | 2 | 236.22000 | 2 | 400.26167 | 2 | 564.30333 | 2 | 728.34500 |
| 3 | 75.45917 | 3 | 239.50083 | 3 | 403.54250 | 3 | 567.58417 | 3 | 731.62583 |
| 4 | 78.74000 | 4 | 242.78167 | 4 | 406.82333 | 4 | 570.86500 | 4 | 734.90667 |
| 5 | 82.02083 | 5 | 246.06250 | 5 | 410.10417 | 5 | 574.14583 | 5 | 738.18750 |
| 6 | 85.30167 | 6 | 249.34333 | 6 | 413.38500 | 6 | 577.42667 | 6 | 741.46833 |
| 7 | 88.58250 | 7 | 252.62417 | 7 | 416.66583 | 7 | 580.70750 | 7 | 744.74917 |
| 8 | 91.86333 | 8 | 255.90500 | 8 | 419.94667 | 8 | 583.98833 | 8 | 748.03000 |
| 9 | 95.14417 | 9 | 259.18583 | 9 | 423.22750 | 9 | 587.26917 | 9 | 751.31083 |
| 30 | 98.42500 | 80 | 262.46667 | 130 | 426.50833 | 180 | 590.55000 | 230 | 754.59167 |
| 1 | 101.70583 | 1 | 265.74750 | 1 | 429.78917 | 1 | 593.83083 | 1 | 757.87250 |
| 2 | 104.98667 | 2 | 269.02833 | 2 | 433.07000 | 2 | 597.11167 | 2 | 761.15333 |
| 3 | 108.26750 | 3 | 272.30917 | 3 | 436.35083 | 3 | 600.39250 | 3 | 764.43417 |
| 4 | 111.54833 | 4 | 275.59000 | 4 | 439.63167 | 4 | 603.67333 | 4 | 767.71500 |
| 5 | 114.82917 | 5 | 278.87083 | 5 | 442.91250 | 5 | 606.95417 | 5 | 770.99583 |
| 6 | 118.11000 | 6 | 282.15167 | 6 | 446.19333 | 6 | 610.23500 | 6 | 774.27667 |
| 7 | 121.39083 | 7 | 285.43250 | 7 | 449.47417 | 7 | 613.51583 | 7 | 777.55750 |
| 8 | 124.67167 | 8 | 288.71333 | 8 | 452.75500 | 8 | 616.79667 | 8 | 780.83833 |
| 9 | 127.95250 | 9 | 291.99417 | 9 | 456.03583 | 9 | 620.07750 | 9 | 784.11917 |
| 40 | 131.23333 | 90 | 295.27500 | 140 | 459.31667 | 190 | 623.35833 | 240 | 787.40000 |
| 1 | 134.51417 | 1 | 298.55583 | 1 | 462.59750 | 1 | 626.63917 | 1 | 790.68083 |
| 2 | 137.79500 | 2 | 301.83667 | 2 | 465.87833 | 2 | 629.92000 | 2 | 793.96167 |
| 3 | 141.07583 | 3 | 305.11750 | 3 | 469.15917 | 3 | 633.20083 | 3 | 797.24250 |
| 4 | 144.35667 | 4 | 308.39833 | 4 | 472.44000 | 4 | 636.48167 | 4 | 800.52333 |
| 5 | 147.63750 | 5 | 311.67917 | 5 | 475.72083 | 5 | 639.76250 | 5 | 803.80417 |
| 6 | 150.91833 | 6 | 314.96000 | 6 | 479.00167 | 6 | 643.04333 | 6 | 807.08500 |
| 7 | 154.19917 | 7 | 318.24083 | 7 | 482.28250 | 7 | 646.32417 | 7 | 810.36583 |
| 8 | 157.48000 | 8 | 321.52167 | 8 | 485.56333 | 8 | 649.60500 | 8 | 813.64667 |
| 9 | 160.76083 | 9 | 324.80250 | 9 | 488.84417 | 9 | 652.88583 | 9 | 816.92750 |

*Lengths—Meters to feet (from 1 to 1000 units)*—Continued

| Meters | Feet | Meters | Feet | Meters | Feet | Meters | Feet | Meters | Feet |
|---|---|---|---|---|---|---|---|---|---|
| 250 | 820.20833 | 300 | 984.25000 | 350 | 1,148.29167 | 400 | 1,312.33333 | 450 | 1,476.37500 |
| 1 | 823.48917 | 1 | 987.53083 | 1 | 1,151.57250 | 1 | 1,315.61417 | 1 | 1,479.65583 |
| 2 | 826.77000 | 2 | 990.81167 | 2 | 1,154.85333 | 2 | 1,318.89500 | 2 | 1,482.93667 |
| 3 | 830.05083 | 3 | 994.09250 | 3 | 1,158.13417 | 3 | 1,322.17583 | 3 | 1,486.21750 |
| 4 | 833.33167 | 4 | 997.37333 | 4 | 1,161.41500 | 4 | 1,325.45667 | 4 | 1,489.49833 |
| 5 | 836.61250 | 5 | 1,000.65417 | 5 | 1,164.69583 | 5 | 1,328.73750 | 5 | 1,492.77917 |
| 6 | 839.89333 | 6 | 1,003.93500 | 6 | 1,167.97667 | 6 | 1,332.01833 | 6 | 1,496.06000 |
| 7 | 843.17417 | 7 | 1,007.21583 | 7 | 1,171.25750 | 7 | 1,335.29917 | 7 | 1,499.34083 |
| 8 | 846.45500 | 8 | 1,010.49667 | 8 | 1,174.53833 | 8 | 1,338.58000 | 8 | 1,502.62167 |
| 9 | 849.73583 | 9 | 1,013.77750 | 9 | 1,177.81917 | 9 | 1,341.86083 | 9 | 1,505.90250 |
| 260 | 853.01667 | 310 | 1,017.05833 | 360 | 1,181.10000 | 410 | 1,345.14167 | 460 | 1,509.18333 |
| 1 | 856.29750 | 1 | 1,020.33917 | 1 | 1,184.38083 | 1 | 1,348.42250 | 1 | 1,512.46417 |
| 2 | 859.57833 | 2 | 1,023.62000 | 2 | 1,187.66167 | 2 | 1,351.70333 | 2 | 1,515.74500 |
| 3 | 862.85917 | 3 | 1,026.90083 | 3 | 1,190.94250 | 3 | 1,354.98417 | 3 | 1,519.02583 |
| 4 | 866.14000 | 4 | 1,030.18167 | 4 | 1,194.22333 | 4 | 1,358.26500 | 4 | 1,522.30667 |
| 5 | 869.42083 | 5 | 1,033.46250 | 5 | 1,197.50417 | 5 | 1,361.54583 | 5 | 1,525.58750 |
| 6 | 872.70167 | 6 | 1,036.74333 | 6 | 1,200.78500 | 6 | 1,364.82667 | 6 | 1,528.86833 |
| 7 | 875.98250 | 7 | 1,040.02417 | 7 | 1,204.06583 | 7 | 1,368.10750 | 7 | 1,532.14917 |
| 8 | 879.26333 | 8 | 1,043.30500 | 8 | 1,207.34667 | 8 | 1,371.38833 | 8 | 1,535.43000 |
| 9 | 882.54417 | 9 | 1,046.58583 | 9 | 1,210.62750 | 9 | 1,374.66917 | 9 | 1,538.71083 |
| 270 | 885.82500 | 320 | 1,049.86667 | 370 | 1,213.90833 | 420 | 1,377.95000 | 470 | 1,541.99167 |
| 1 | 889.10583 | 1 | 1,053.14750 | 1 | 1,217.18917 | 1 | 1,381.23083 | 1 | 1,545.27250 |
| 2 | 892.38667 | 2 | 1,056.42833 | 2 | 1,220.47000 | 2 | 1,384.51167 | 2 | 1,548.55333 |
| 3 | 895.66750 | 3 | 1,059.70917 | 3 | 1,223.75083 | 3 | 1,387.79250 | 3 | 1,551.83417 |
| 4 | 898.94833 | 4 | 1,062.99000 | 4 | 1,227.03167 | 4 | 1,391.07333 | 4 | 1,555.11500 |
| 5 | 902.22917 | 5 | 1,066.27083 | 5 | 1,230.31250 | 5 | 1,394.35417 | 5 | 1,558.39583 |
| 6 | 905.51000 | 6 | 1,069.55167 | 6 | 1,233.59333 | 6 | 1,397.63500 | 6 | 1,561.67667 |
| 7 | 908.79083 | 7 | 1,072.83250 | 7 | 1,236.87417 | 7 | 1,400.91583 | 7 | 1,564.95750 |
| 8 | 912.07167 | 8 | 1,076.11333 | 8 | 1,240.15500 | 8 | 1,404.19667 | 8 | 1,568.23833 |
| 9 | 915.35250 | 9 | 1,079.39417 | 9 | 1,243.43583 | 9 | 1,407.47750 | 9 | 1,571.51917 |
| 280 | 918.63333 | 330 | 1,082.67500 | 380 | 1,246.71667 | 430 | 1,410.75833 | 480 | 1,574.80000 |
| 1 | 921.91417 | 1 | 1,085.95583 | 1 | 1,249.99750 | 1 | 1,414.03917 | 1 | 1,578.08083 |
| 2 | 925.19500 | 2 | 1,089.23667 | 2 | 1,253.27833 | 2 | 1,417.32000 | 2 | 1,581.36167 |
| 3 | 928.47583 | 3 | 1,092.51750 | 3 | 1,256.55917 | 3 | 1,420.60083 | 3 | 1,584.64250 |
| 4 | 931.75667 | 4 | 1,095.79833 | 4 | 1,259.84000 | 4 | 1,423.88167 | 4 | 1,587.92333 |
| 5 | 935.03750 | 5 | 1,099.07917 | 5 | 1,263.12083 | 5 | 1,427.16250 | 5 | 1,591.20417 |
| 6 | 938.31833 | 6 | 1,102.36000 | 6 | 1,266.40167 | 6 | 1,430.44333 | 6 | 1,594.48500 |
| 7 | 941.59917 | 7 | 1,105.64083 | 7 | 1,269.68250 | 7 | 1,433.72417 | 7 | 1,597.76583 |
| 8 | 944.88000 | 8 | 1,108.92167 | 8 | 1,272.96333 | 8 | 1,437.00500 | 8 | 1,601.04667 |
| 9 | 948.16083 | 9 | 1,112.20250 | 9 | 1,276.24417 | 9 | 1,440.28583 | 9 | 1,604.32750 |
| 290 | 951.44167 | 340 | 1,115.48333 | 390 | 1,279.52500 | 440 | 1,443.56667 | 490 | 1,607.60833 |
| 1 | 954.72250 | 1 | 1,118.76417 | 1 | 1,282.80583 | 1 | 1,446.84750 | 1 | 1,610.88917 |
| 2 | 958.00333 | 2 | 1,122.04500 | 2 | 1,286.08667 | 2 | 1,450.12833 | 2 | 1,614.17000 |
| 3 | 961.28417 | 3 | 1,125.32583 | 3 | 1,289.36750 | 3 | 1,453.40917 | 3 | 1,617.45083 |
| 4 | 964.56500 | 4 | 1,128.60667 | 4 | 1,292.64833 | 4 | 1,456.69000 | 4 | 1,620.73167 |
| 5 | 967.84583 | 5 | 1,131.88750 | 5 | 1,295.92917 | 5 | 1,459.97083 | 5 | 1,624.01250 |
| 6 | 971.12667 | 6 | 1,135.16833 | 6 | 1,299.21000 | 6 | 1,463.25167 | 6 | 1,627.29333 |
| 7 | 974.40750 | 7 | 1,138.44917 | 7 | 1,302.49083 | 7 | 1,466.53250 | 7 | 1,630.57417 |
| 8 | 977.68833 | 8 | 1,141.73000 | 8 | 1,305.77167 | 8 | 1,469.81333 | 8 | 1,633.85500 |
| 9 | 980.96917 | 9 | 1,145.01083 | 9 | 1,309.05250 | 9 | 1,473.09417 | 9 | 1,637.13583 |

*Lengths—Meters to feet (from 1 to 1000 units)*—Continued

| Meters | Feet | Meters | Feet | Meters | Feet | Meters | Feet | Meters | Feet |
|---|---|---|---|---|---|---|---|---|---|
| 500 | 1,640.41667 | 550 | 1,804.45833 | 600 | 1,968.50000 | 650 | 2,132.54167 | 700 | 2,296.58333 |
| 1 | 1,643.69750 | 1 | 1,807.73917 | 1 | 1,971.78083 | 1 | 2,135.82250 | 1 | 2,299.86417 |
| 2 | 1,646.97833 | 2 | 1,811.02000 | 2 | 1,975.06167 | 2 | 2,139.10333 | 2 | 2,303.14500 |
| 3 | 1,650.25917 | 3 | 1,814.30083 | 3 | 1,978.34250 | 3 | 2,142.38417 | 3 | 2,306.42583 |
| 4 | 1,653.54000 | 4 | 1,817.58167 | 4 | 1,981.62333 | 4 | 2,145.66500 | 4 | 2,309.70667 |
| 5 | 1,656.82083 | 5 | 1,820.86250 | 5 | 1,984.90417 | 5 | 2,148.94583 | 5 | 2,312.98750 |
| 6 | 1,660.10167 | 6 | 1,824.14333 | 6 | 1,988.18500 | 6 | 2,152.22667 | 6 | 2,316.26833 |
| 7 | 1,663.38250 | 7 | 1,827.42417 | 7 | 1,991.46583 | 7 | 2,155.50750 | 7 | 2,319.54917 |
| 8 | 1,666.66333 | 8 | 1,830.70500 | 8 | 1,994.74667 | 8 | 2,158.78833 | 8 | 2,322.83000 |
| 9 | 1,669.94417 | 9 | 1,833.98583 | 9 | 1,998.02750 | 9 | 2,162.06917 | 9 | 2,326.11083 |
| 510 | 1,673.22500 | 560 | 1,837.26667 | 610 | 2,001.30833 | 660 | 2,165.35000 | 710 | 2,329.39167 |
| 1 | 1,676.50583 | 1 | 1,840.54750 | 1 | 2,004.58917 | 1 | 2,168.63083 | 1 | 2,332.67250 |
| 2 | 1,679.78667 | 2 | 1,843.82833 | 2 | 2,007.87000 | 2 | 2,171.91167 | 2 | 2,335.95333 |
| 3 | 1,683.06750 | 3 | 1,847.10917 | 3 | 2,011.15083 | 3 | 2,175.19250 | 3 | 2,339.23417 |
| 4 | 1,686.34833 | 4 | 1,850.39000 | 4 | 2,014.43167 | 4 | 2,178.47333 | 4 | 2,342.51500 |
| 5 | 1,689.62917 | 5 | 1,853.67083 | 5 | 2,017.71250 | 5 | 2,181.75417 | 5 | 2,345.79583 |
| 6 | 1,692.91000 | 6 | 1,856.95167 | 6 | 2,020.99333 | 6 | 2,185.03500 | 6 | 2,349.07667 |
| 7 | 1,696.19083 | 7 | 1,860.23250 | 7 | 2,024.27417 | 7 | 2,188.31583 | 7 | 2,352.35750 |
| 8 | 1,699.47167 | 8 | 1,863.51333 | 8 | 2,027.55500 | 8 | 2,191.59667 | 8 | 2,355.63833 |
| 9 | 1,702.75250 | 9 | 1,866.79417 | 9 | 2,030.83583 | 9 | 2,194.87750 | 9 | 2,358.91917 |
| 520 | 1,706.03333 | 570 | 1,870.07500 | 620 | 2,034.11667 | 670 | 2,198.15833 | 720 | 2,362.20000 |
| 1 | 1,709.31417 | 1 | 1,873.35583 | 1 | 2,037.39750 | 1 | 2,201.43917 | 1 | 2,365.48083 |
| 2 | 1,712.59500 | 2 | 1,876.63667 | 2 | 2,040.67833 | 2 | 2,204.72000 | 2 | 2,368.76167 |
| 3 | 1,715.87583 | 3 | 1,879.91750 | 3 | 2,043.95917 | 3 | 2,208.00083 | 3 | 2,372.04250 |
| 4 | 1,719.15667 | 4 | 1,883.19833 | 4 | 2,047.24000 | 4 | 2,211.28167 | 4 | 2,375.32333 |
| 5 | 1,722.43750 | 5 | 1,886.47917 | 5 | 2,050.52083 | 5 | 2,214.56250 | 5 | 2,378.60417 |
| 6 | 1,725.71833 | 6 | 1,889.76000 | 6 | 2,053.80167 | 6 | 2,217.84333 | 6 | 2,381.88500 |
| 7 | 1,728.99917 | 7 | 1,893.04083 | 7 | 2,057.08250 | 7 | 2,221.12417 | 7 | 2,385.16583 |
| 8 | 1,732.28000 | 8 | 1,896.32167 | 8 | 2,060.36333 | 8 | 2,224.40500 | 8 | 2,388.44667 |
| 9 | 1,735.56083 | 9 | 1,899.60250 | 9 | 2,063.64417 | 9 | 2,227.68583 | 9 | 2,391.72750 |
| 530 | 1,738.84167 | 580 | 1,902.88333 | 630 | 2,066.92500 | 680 | 2,230.96667 | 730 | 2,395.00833 |
| 1 | 1,742.12250 | 1 | 1,906.16417 | 1 | 2,070.20583 | 1 | 2,234.24750 | 1 | 2,398.28917 |
| 2 | 1,745.40333 | 2 | 1,909.44500 | 2 | 2,073.48667 | 2 | 2,237.52833 | 2 | 2,401.57000 |
| 3 | 1,748.68417 | 3 | 1,912.72583 | 3 | 2,076.76750 | 3 | 2,240.80917 | 3 | 2,404.85083 |
| 4 | 1,751.96500 | 4 | 1,916.00667 | 4 | 2,080.04833 | 4 | 2,244.09000 | 4 | 2,408.13167 |
| 5 | 1,755.24583 | 5 | 1,919.28750 | 5 | 2,083.32917 | 5 | 2,247.37083 | 5 | 2,411.41250 |
| 6 | 1,758.52667 | 6 | 1,922.56833 | 6 | 2,086.61000 | 6 | 2,250.65167 | 6 | 2,414.69333 |
| 7 | 1,761.80750 | 7 | 1,925.84917 | 7 | 2,089.89083 | 7 | 2,253.93250 | 7 | 2,417.97417 |
| 8 | 1,765.08833 | 8 | 1,929.13000 | 8 | 2,093.17167 | 8 | 2,257.21333 | 8 | 2,421.25500 |
| 9 | 1,768.36917 | 9 | 1,932.41083 | 9 | 2,096.45250 | 9 | 2,260.49417 | 9 | 2,424.53583 |
| 540 | 1,771.65000 | 590 | 1,935.69167 | 640 | 2,099.73333 | 690 | 2,263.77500 | 740 | 2,427.81667 |
| 1 | 1,774.93083 | 1 | 1,938.97250 | 1 | 2,103.01417 | 1 | 2,267.05583 | 1 | 2,431.09750 |
| 2 | 1,778.21167 | 2 | 1,942.25333 | 2 | 2,106.29500 | 2 | 2,270.33667 | 2 | 2,434.37833 |
| 3 | 1,781.49250 | 3 | 1,945.53417 | 3 | 2,109.57583 | 3 | 2,273.61750 | 3 | 2,437.65917 |
| 4 | 1,784.77333 | 4 | 1,948.81500 | 4 | 2,112.85667 | 4 | 2,276.89833 | 4 | 2,440.94000 |
| 5 | 1,788.05417 | 5 | 1,952.09583 | 5 | 2,116.13750 | 5 | 2,280.17917 | 5 | 2,444.22083 |
| 6 | 1,791.33500 | 6 | 1,955.37667 | 6 | 2,119.41833 | 6 | 2,283.46000 | 6 | 2,447.50167 |
| 7 | 1,794.61583 | 7 | 1,958.65750 | 7 | 2,122.69917 | 7 | 2,286.74083 | 7 | 2,450.78250 |
| 8 | 1,797.89667 | 8 | 1,961.93833 | 8 | 2,125.98000 | 8 | 2,290.02167 | 8 | 2,454.06333 |
| 9 | 1,801.17750 | 9 | 1,965.21917 | 9 | 2,129.26083 | 9 | 2,293.30250 | 9 | 2,457.34417 |

*Lengths—Meters to feet (from 1 to 1000 units)*—Continued

| Meters | Feet | Meters | Feet | Meters | Feet | Meters | Feet | Meters | Feet |
|---|---|---|---|---|---|---|---|---|---|
| 750 | 2,460.62500 | 800 | 2,624.66667 | 850 | 2,788.70833 | 900 | 2,952.75000 | 950 | 3,116.79167 |
| 1 | 2,463.90583 | 1 | 2,627.94750 | 1 | 2,791.98917 | 1 | 2,956.03083 | 1 | 3,120.07250 |
| 2 | 2,467.18667 | 2 | 2,631.22833 | 2 | 2,795.27000 | 2 | 2,959.31167 | 2 | 3,123.35333 |
| 3 | 2,470.46750 | 3 | 2,634.50917 | 3 | 2,798.55083 | 3 | 2,962.59250 | 3 | 3,126.63417 |
| 4 | 2,473.74833 | 4 | 2,637.79000 | 4 | 2,801.83167 | 4 | 2,965.87333 | 4 | 3,129.91500 |
| 5 | 2,477.02917 | 5 | 2,641.07083 | 5 | 2,805.11250 | 5 | 2,969.15417 | 5 | 3,133.19583 |
| 6 | 2,480.31000 | 6 | 2,644.35167 | 6 | 2,808.39333 | 6 | 2,972.43500 | 6 | 3,136.47667 |
| 7 | 2,483.59083 | 7 | 2,647.63250 | 7 | 2,811.67417 | 7 | 2,975.71583 | 7 | 3,139.75750 |
| 8 | 2,486.87167 | 8 | 2,650.91333 | 8 | 2,814.95500 | 8 | 2,978.99667 | 8 | 3,143.03833 |
| 9 | 2,490.15250 | 9 | 2,654.19417 | 9 | 2,818.23583 | 9 | 2,982.27750 | 9 | 3,146.31917 |
| 760 | 2,493.43333 | 810 | 2,657.47500 | 860 | 2,821.51667 | 910 | 2,985.55833 | 960 | 3,149.60000 |
| 1 | 2,496.71417 | 1 | 2,660.75583 | 1 | 2,824.79750 | 1 | 2,988.83917 | 1 | 3,152.88083 |
| 2 | 2,499.99500 | 2 | 2,664.03667 | 2 | 2,828.07833 | 2 | 2,992.12000 | 2 | 3,156.16167 |
| 3 | 2,503.27583 | 3 | 2,667.31750 | 3 | 2,831.35917 | 3 | 2,995.40083 | 3 | 3,159.44250 |
| 4 | 2,506.55667 | 4 | 2,670.59833 | 4 | 2,834.64000 | 4 | 2,998.68167 | 4 | 3,162.72333 |
| 5 | 2,509.83750 | 5 | 2,673.87917 | 5 | 2,837.92083 | 5 | 3,001.96250 | 5 | 3,166.00417 |
| 6 | 2,513.11833 | 6 | 2,677.16000 | 6 | 2,841.20167 | 6 | 3,005.24333 | 6 | 3,169.28500 |
| 7 | 2,516.39917 | 7 | 2,680.44083 | 7 | 2,844.48250 | 7 | 3,008.52417 | 7 | 3,172.56583 |
| 8 | 2,519.68000 | 8 | 2,683.72167 | 8 | 2,847.76333 | 8 | 3,011.80500 | 8 | 3,175.84667 |
| 9 | 2,522.96083 | 9 | 2,687.00250 | 9 | 2,851.04417 | 9 | 3,015.08583 | 9 | 3,179.12750 |
| 770 | 2,526.24167 | 820 | 2,690.28333 | 870 | 2,854.32500 | 920 | 3,018.36667 | 970 | 3,182.40833 |
| 1 | 2,529.52250 | 1 | 2,693.56417 | 1 | 2,857.60583 | 1 | 3,021.64750 | 1 | 3,185.68917 |
| 2 | 2,532.80333 | 2 | 2,696.84500 | 2 | 2,860.88667 | 2 | 3,024.92833 | 2 | 3,188.97000 |
| 3 | 2,536.08417 | 3 | 2,700.12583 | 3 | 2,864.16750 | 3 | 3,028.20917 | 3 | 3,192.25083 |
| 4 | 2,539.36500 | 4 | 2,703.40667 | 4 | 2,867.44833 | 4 | 3,031.49000 | 4 | 3,195.53167 |
| 5 | 2,542.64583 | 5 | 2,706.68750 | 5 | 2,870.72917 | 5 | 3,034.77083 | 5 | 3,198.81250 |
| 6 | 2,545.92667 | 6 | 2,709.96833 | 6 | 2,874.01000 | 6 | 3,038.05167 | 6 | 3,202.09333 |
| 7 | 2,549.20750 | 7 | 2,713.24917 | 7 | 2,877.29083 | 7 | 3,041.33250 | 7 | 3,205.37417 |
| 8 | 2,552.48833 | 8 | 2,716.53000 | 8 | 2,880.57167 | 8 | 3,044.61333 | 8 | 3,208.65500 |
| 9 | 2,555.76917 | 9 | 2,719.81083 | 9 | 2,883.85250 | 9 | 3,047.89417 | 9 | 3,211.93583 |
| 780 | 2,559.05000 | 830 | 2,723.09167 | 880 | 2,887.13333 | 930 | 3,051.17500 | 980 | 3,215.21667 |
| 1 | 2,562.33083 | 1 | 2,726.37250 | 1 | 2,890.41417 | 1 | 3,054.45583 | 1 | 3,218.49750 |
| 2 | 2,565.61167 | 2 | 2,729.65333 | 2 | 2,893.69500 | 2 | 3,057.73667 | 2 | 3,221.77833 |
| 3 | 2,568.89250 | 3 | 2,732.93417 | 3 | 2,896.97583 | 3 | 3,061.01750 | 3 | 3,225.05917 |
| 4 | 2,572.17333 | 4 | 2,736.21500 | 4 | 2,900.25667 | 4 | 3,064.29833 | 4 | 3,228.34000 |
| 5 | 2,575.45417 | 5 | 2,739.49583 | 5 | 2,903.53750 | 5 | 3,067.57917 | 5 | 3,231.62083 |
| 6 | 2,578.73500 | 6 | 2,742.77667 | 6 | 2,906.81833 | 6 | 3,070.86000 | 6 | 3,234.90167 |
| 7 | 2,582.01583 | 7 | 2,746.05750 | 7 | 2,910.09917 | 7 | 3,074.14083 | 7 | 3,238.18250 |
| 8 | 2,585.29667 | 8 | 2,749.33833 | 8 | 2,913.38000 | 8 | 3,077.42167 | 8 | 3,241.46333 |
| 9 | 2,588.57750 | 9 | 2,752.61917 | 9 | 2,916.66083 | 9 | 3,080.70250 | 9 | 3,244.74417 |
| 790 | 2,591.85833 | 840 | 2,755.90000 | 890 | 2,919.94167 | 940 | 3,083.98333 | 990 | 3,248.02500 |
| 1 | 2,595.13917 | 1 | 2,759.18083 | 1 | 2,923.22250 | 1 | 3,087.26417 | 1 | 3,251.30583 |
| 2 | 2,598.42000 | 2 | 2,762.46167 | 2 | 2,926.50333 | 2 | 3,090.54500 | 2 | 3,254.58667 |
| 3 | 2,601.70083 | 3 | 2,765.74250 | 3 | 2,929.78417 | 3 | 3,093.82583 | 3 | 3,257.86750 |
| 4 | 2,604.98167 | 4 | 2,769.02333 | 4 | 2,933.06500 | 4 | 3,097.10667 | 4 | 3,261.14833 |
| 5 | 2,608.26250 | 5 | 2,772.30417 | 5 | 2,936.34583 | 5 | 3,100.38750 | 5 | 3,264.42917 |
| 6 | 2,611.54333 | 6 | 2,775.58500 | 6 | 2,939.62667 | 6 | 3,103.66833 | 6 | 3,267.71000 |
| 7 | 2,614.82417 | 7 | 2,778.86583 | 7 | 2,942.90750 | 7 | 3,106.94917 | 7 | 3,270.99083 |
| 8 | 2,618.10500 | 8 | 2,782.14667 | 8 | 2,946.18833 | 8 | 3,110.23000 | 8 | 3,274.27167 |
| 9 | 2,621.38583 | 9 | 2,785.42750 | 9 | 2,949.46917 | 9 | 3,113.51083 | 9 | 3,277.55250 |

*Corrections to log s and log Δλ for difference in arc and sine*

| Log $s$ (−) | Log difference (units of eighth decimal place) | Log Δλ (+) | Log $s$ (−) | Log difference (units of eighth decimal place) | Log Δλ (+) | Log $s$ (−) | Log difference (units of eighth decimal place) | Log Δλ (+) |
|---|---|---|---|---|---|---|---|---|
| 3.3760 | 1 | 1.8850 | 4.8270 | 799 | 3.3360 | 5.1780 | 4025 | 3.6870 |
| 3.5260 | 2 | 2.0350 | 4.8380 | 841 | 3.3470 | 5.1830 | 4119 | 3.6920 |
| 3.6140 | 3 | 2.1230 | 4.8500 | 889 | 3.3590 | 5.1880 | 4215 | 3.6970 |
| 3.6770 | 4 | 2.1860 | 4.8620 | 939 | 3.3710 | 5.1940 | 4333 | 3.7030 |
| 3.7250 | 5 | 2.2340 | 4.8710 | 979 | 3.3800 | 5.1990 | 4434 | 3.7080 |
| 3.7650 | 6 | 2.2740 | 4.8820 | 1030 | 3.3910 | 5.2040 | 4537 | 3.7130 |
| 3.7980 | 7 | 2.3070 | 4.8920 | 1078 | 3.4010 | 5.2090 | 4643 | 3.7180 |
| 3.8270 | 8 | 2.3360 | 4.9040 | 1140 | 3.4130 | 5.2140 | 4751 | 3.7230 |
| 3.8530 | 9 | 2.3620 | 4.9130 | 1188 | 3.4220 | 5.2190 | 4862 | 3.7280 |
| 3.8760 | 10 | 2.3850 | 4.9220 | 1238 | 3.4310 | 5.2240 | 4975 | 3.7330 |
| 4.0260 | 20 | 2.5350 | 4.9330 | 1303 | 3.4420 | 5.2290 | 5091 | 3.7380 |
| 4.1140 | 30 | 2.6230 | 4.9420 | 1358 | 3.4510 | 5.2330 | 5186 | 3.7420 |
| 4.1770 | 40 | 2.6860 | 4.9520 | 1422 | 3.4610 | 5.2380 | 5306 | 3.7470 |
| 4.2250 | 50 | 2.7340 | 4.9590 | 1468 | 3.4680 | 5.2420 | 5405 | 3.7510 |
| 4.2650 | 60 | 2.7740 | 4.9680 | 1530 | 3.4770 | 5.2470 | 5531 | 3.7560 |
| 4.2980 | 70 | 2.8070 | 4.9780 | 1603 | 3.4870 | 5.2520 | 5660 | 3.7610 |
| 4.3270 | 80 | 2.8360 | 4.9860 | 1663 | 3.4950 | 5.2560 | 5765 | 3.7650 |
| 4.3530 | 90 | 2.8620 | 4.9930 | 1717 | 3.5020 | 5.2600 | 5872 | 3.7690 |
| 4.3760 | 100 | 2.8850 | 5.0020 | 1790 | 3.5110 | 5.2650 | 6009 | 3.7740 |
| 4.3960 | 110 | 2.9050 | 5.0100 | 1857 | 3.5190 | 5.2690 | 6121 | 3.7780 |
| 4.4150 | 120 | 2.9240 | 5.0170 | 1918 | 3.5260 | 5.2740 | 6263 | 3.7830 |
| 4.4330 | 130 | 2.9420 | 5.0250 | 1990 | 3.5340 | 5.2780 | 6380 | 3.7870 |
| 4.4490 | 140 | 2.9580 | 5.0330 | 2064 | 3.5420 | 5.2820 | 6498 | 3.7910 |
| 4.4640 | 150 | 2.9730 | 5.0400 | 2132 | 3.5490 | 5.2860 | 6619 | 3.7950 |
| 4.4780 | 160 | 2.9870 | 5.0480 | 2212 | 3.5570 | 5.2900 | 6742 | 3.7990 |
| 4.4910 | 170 | 3.0000 | 5.0550 | 2285 | 3.5640 | 5.2940 | 6868 | 3.8030 |
| 4.5030 | 180 | 3.0120 | 5.0620 | 2359 | 3.5710 | 5.2990 | 7027 | 3.8080 |
| 4.5260 | 200 | 3.0350 | 5.0680 | 2425 | 3.5770 | 5.3030 | 7158 | 3.8120 |
| 4.5560 | 230 | 3.0650 | 5.0750 | 2505 | 3.5840 | 5.3070 | 7291 | 3.8160 |
| 4.5750 | 250 | 3.0840 | 5.0820 | 2587 | 3.5910 | 5.3110 | 7427 | 3.8200 |
| 4.5910 | 270 | 3.1000 | 5.0890 | 2672 | 3.5980 | 5.3150 | 7565 | 3.8240 |
| 4.6140 | 300 | 3.1230 | 5.0950 | 2747 | 3.6040 | 5.3190 | 7705 | 3.8280 |
| 4.6350 | 330 | 3.1440 | 5.1020 | 2837 | 3.6110 | 5.3230 | 7849 | 3.8320 |
| 4.6540 | 360 | 3.1630 | 5.1080 | 2916 | 3.6170 | 5.3270 | 7995 | 3.8360 |
| 4.6710 | 390 | 3.1800 | 5.1140 | 2998 | 3.6230 | 5.3310 | 8143 | 3.8400 |
| 4.6870 | 420 | 3.1960 | 5.1210 | 3096 | 3.6300 | 5.3350 | 8295 | 3.8440 |
| 4.7020 | 450 | 3.2110 | 5.1270 | 3183 | 3.6360 | 5.3390 | 8449 | 3.8480 |
| 4.7160 | 480 | 3.2250 | 5.1330 | 3272 | 3.6420 | 5.3430 | 8606 | 3.8520 |
| 4.7340 | 521 | 3.2430 | 5.1390 | 3364 | 3.6480 | 5.3470 | 8766 | 3.8560 |
| 4.7500 | 561 | 3.2590 | 5.1450 | 3458 | 3.6540 | | | |
| 4.7610 | 590 | 3.2700 | 5.1500 | 3538 | 3.6590 | | | |
| 4.7750 | 629 | 3.2840 | 5.1560 | 3637 | 3.6650 | | | |
| 4.7890 | 671 | 3.2980 | 5.1610 | 3722 | 3.6700 | | | |
| 4.8010 | 709 | 3.3100 | 5.1670 | 3826 | 3.6760 | | | |
| 4.8130 | 750 | 3.3220 | 5.1720 | 3916 | 3.6810 | | | |

## FACTORS USED IN THE COMPUTATION OF ELEVATIONS FROM RECIPROCAL AND NONRECIPROCAL OBSERVATIONS

The unit of length throughout these tables is the meter.

*Log A*

| Elevation of occupied station $h_1$ | Log $A$ units of fifth place | Elevation of occupied station $h_1$ | Log $A$ units of fifth place |
|---|---|---|---|
| *Meters* | | *Meters* | |
| 0 | 0.0 | 3009 | 20.5 |
| 73 | .5 | 3156 | 21.5 |
| 220 | 1.5 | 3303 | 22.5 |
| 367 | 2.5 | 3449 | 23.5 |
| 514 | 3.5 | 3596 | 24.5 |
| 661 | 4.5 | 3743 | 25.5 |
| 807 | 5.5 | 3890 | 26.5 |
| 954 | 6.5 | 4036 | 27.5 |
| 1101 | 7.5 | 4183 | 28.5 |
| 1248 | 8.5 | 4330 | 29.5 |
| 1394 | 9.5 | 4477 | 30.5 |
| 1541 | 10.5 | 4624 | 31.5 |
| 1688 | 11.5 | 4770 | 32.5 |
| 1835 | 12.5 | 4917 | 33.5 |
| 1982 | 13.5 | 5064 | 34.5 |
| 2128 | 14.5 | 5211 | 35.5 |
| 2275 | 15.5 | 5357 | 36.5 |
| 2422 | 16.5 | 5504 | 37.5 |
| 2569 | 17.5 | 5651 | 38.5 |
| 2715 | 18.5 | 5798 | 39.5 |
| 2862 | 19.5 | 5945 | 40.5 |

*Log B and log C*

| Log approximate difference of elevation $=\log s \tan\left(\frac{\zeta_2-\zeta_1}{2}\right)$* | Log $B$ units of 5th place | Log $s$ | Log $C$ |
|---|---|---|---|
| | 0.0 | | 0.0 |
| 2.167 | .5 | 4.875 | .5 |
| 2.644 | 1.5 | 5.113 | 1.5 |
| 2.866 | 2.5 | 5.224 | 2.5 |
| 3.011 | 3.5 | 5.297 | 3.5 |
| 3.121 | 4.5 | 5.352 | 4.5 |
| 3.208 | 5.5 | 5.395 | 5.5 |
| 3.281 | 6.5 | 5.432 | 6.5 |
| 3.343 | 7.5 | 5.463 | 7.5 |
| 3.397 | 8.5 | | |
| 3.445 | 9.5 | | |
| 3.489 | 10.5 | | |
| 3.528 | 11.5 | | |
| 3.565 | 12.5 | | |
| 3.598 | 13.5 | | |
| 3.629 | 14.5 | | |
| 3.658 | 15.5 | | |
| 3.685 | 16.5 | | |
| 3.711 | 17.5 | | |
| 3.735 | 18.5 | | |
| 3.758 | 19.5 | | |
| 3.779 | 20.5 | | |
| 3.800 | 21.5 | | |
| 3.820 | 22.5 | | |
| 3.839 | 23.5 | | |
| 3.857 | 24.5 | | |
| 3.874 | 25.5 | | |

*Or $\log s \cot\left[\zeta_1-(0.5-m)\frac{s}{\rho \sin 1''}\right]$ for nonreciprocal observations.

Log $B$ has the same sign as the approximate difference of elevation.
Log $C$ is always positive.

## COMPUTATION OF SPHERICAL EXCESS

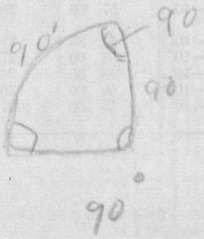
The spherical excess of a triangle is computed by the formula,

$$\epsilon = \frac{a_1 b_1 \sin C_1 (1 - e^2 \sin^2 \phi)^2}{2a^2(1 - e^2) \sin 1''} = a_1 b_1 \sin C_1 \times m.$$

In this formula $\epsilon$ is the spherical excess; $a_1$ $b_1$ and $C_1$ are two sides and the included angle, respectively, of the corresponding triangle; $e^2$ is the square of the eccentricity, and $a$ the major semiaxis of the spheriod of reference; and $\phi$ is the mean latitude of the three vertices of the triangle. That part of the above expression which depends only on the latitude and the dimensions of the spheroid may be designated by a single letter, $m$, as shown. In the following table the logarithms of $m$ are given with the latitude as an argument.

The above formula gives the spherical excess too small by one one-hundredth of a second for an equilateral triangle with 200-kilometer sides, or for a nonequilateral triangle of the same area. For an equilateral triangle of 100-kilometer sides, or an equivalent nonequilateral triangle, the excess as given by this formula is too small by less than one one-thousandth of a second.

In cases where a more accurate value of the spherical excess is required the formulas given on page 51 of Special Publication No. 4, The Transcontinental Triangulation, may be used. These formulas give a slightly unequal distribution of the spherical excess among the three angles of the triangle.




$\frac{1}{8}$

$\frac{1}{2}\pi = 90°$

$\frac{4\pi}{8}$

*Table of log m*

[Computed for the Clarke spheroid of 1866 as expressed in meters]

| Latitude | Log *m* | Latitude | Log *m* | Latitude | Log *m* | Latitude | Log *m* |
|---|---|---|---|---|---|---|---|
| ° ′ | | ° ′ | | ° ′ | | ° ′ | |
| 0 00 | 1.40695 −10 | 20 00 | 1.40626 −10 | 40 00 | 1.40452 −10 | 60 00 | 1.40253 −10 |
| 0 30 | 695 −10 | 20 30 | 623 −10 | 40 30 | 446 −10 | 60 30 | 249 −10 |
| 1 00 | 695 −10 | 21 00 | 619 −10 | 41 00 | 441 −10 | 61 00 | 244 −10 |
| 1 30 | 694 −10 | 21 30 | 616 −10 | 41 30 | 436 −10 | 61 30 | 240 −10 |
| 2 00 | 694 −10 | 22 00 | 612 −10 | 42 00 | 431 −10 | 62 00 | 235 −10 |
| 2 30 | 694 −10 | 22 30 | 608 −10 | 42 30 | 426 −10 | 62 30 | 231 −10 |
| 3 00 | 693 −10 | 23 00 | 605 −10 | 43 00 | 421 −10 | 63 00 | 227 −10 |
| 3 30 | 693 −10 | 23 30 | 601 −10 | 43 30 | 416 −10 | 63 30 | 223 −10 |
| 4 00 | 692 −10 | 24 00 | 597 −10 | 44 00 | 411 −10 | 64 00 | 219 −10 |
| 4 30 | 691 −10 | 24 30 | 594 −10 | 44 30 | 406 −10 | 64 30 | 215 −10 |
| 5 00 | 690 −10 | 25 00 | 590 −10 | 45 00 | 400 −10 | 65 00 | 210 −10 |
| 5 30 | 689 −10 | 25 30 | 586 −10 | 45 30 | 395 −10 | 65 30 | 207 −10 |
| 6 00 | 688 −10 | 26 00 | 582 −10 | 46 00 | 390 −10 | 66 00 | 203 −10 |
| 6 30 | 687 −10 | 26 30 | 578 −10 | 46 30 | 385 −10 | 66 30 | 199 −10 |
| 7 00 | 686 −10 | 27 00 | 573 −10 | 47 00 | 380 −10 | 67 00 | 195 −10 |
| 7 30 | 685 −10 | 27 30 | 569 −10 | 47 30 | 375 −10 | 67 30 | 192 −10 |
| 8 00 | 683 −10 | 28 00 | 565 −10 | 48 00 | 369 −10 | 68 00 | 188 −10 |
| 8 30 | 682 −10 | 28 30 | 560 −10 | 48 30 | 364 −10 | 68 30 | 185 −10 |
| 9 00 | 680 −10 | 29 00 | 556 −10 | 49 00 | 359 −10 | 69 00 | 181 −10 |
| 9 30 | 679 −10 | 29 30 | 552 −10 | 49 30 | 354 −10 | 69 30 | 178 −10 |
| 10 00 | 677 −10 | 30 00 | 548 −10 | 50 00 | 349 −10 | 70 00 | 174 −10 |
| 10 30 | 675 −10 | 30 30 | 544 −10 | 50 30 | 344 −10 | 70 30 | 171 −10 |
| 11 00 | 673 −10 | 31 00 | 539 −10 | 51 00 | 339 −10 | 71 00 | 168 −10 |
| 11 30 | 671 −10 | 31 30 | 534 −10 | 51 30 | 334 −10 | 71 30 | 164 −10 |
| 12 00 | 669 −10 | 32 00 | 530 −10 | 52 00 | 329 −10 | 72 00 | 1.40161 −10 |
| 12 30 | 667 −10 | 32 30 | 525 −10 | 52 30 | 324 −10 | | |
| 13 00 | 665 −10 | 33 00 | 520 −10 | 53 00 | 319 −10 | | |
| 13 30 | 663 −10 | 33 30 | 516 −10 | 53 30 | 314 −10 | | |
| 14 00 | 660 −10 | 34 00 | 511 −10 | 54 00 | 309 −10 | | |
| 14 30 | 658 −10 | 34 30 | 506 −10 | 54 30 | 304 −10 | | |
| 15 00 | 655 −10 | 35 00 | 501 −10 | 55 00 | 299 −10 | | |
| 15 30 | 653 −10 | 35 30 | 496 −10 | 55 30 | 295 −10 | | |
| 16 00 | 650 −10 | 36 00 | 491 −10 | 56 00 | 290 −10 | | |
| 16 30 | 647 −10 | 36 30 | 486 −10 | 56 30 | 285 −10 | | |
| 17 00 | 644 −10 | 37 00 | 482 −10 | 57 00 | 280 −10 | | |
| 17 30 | 642 −10 | 37 30 | 477 −10 | 57 30 | 276 −10 | | |
| 18 00 | 639 −10 | 38 00 | 472 −10 | 58 00 | 271 −10 | | |
| 18 30 | 636 −10 | 38 30 | 467 −10 | 58 30 | 266 −10 | | |
| 19 00 | 632 −10 | 39 00 | 462 −10 | 59 00 | 262 −10 | | |
| 19 30 | 1.40629 −10 | 39 30 | 1.40457 −10 | 59 30 | 1.40257 −10 | | |

When area of Δ is in sq. mi.

add 6.41330 + 0.30103 = 6.71433 to above logs.

Δ of 76.15 sq. mi. has 1″ excess at 45°

## COMPUTATION OF STRENGTH OF FIGURE

In the following table the values tabulated are $[\delta_A^2 + \delta_A\delta_B + \delta_B^2]$. The unit is one in the sixth place of logarithms. The two arguments of the table are the length angles in degrees, the smaller length angle being given at the top of the table. The length angles are the angles in each triangle opposite the known side and the side required. $\delta_A$ and $\delta_B$ are the logarithmic differences corresponding to 1 second for the length angles $A$ and $B$ of a triangle.

*Table for determining relative strength of figures in triangulation*

| | 10° | 12° | 14° | 16° | 18° | 20° | 22° | 24° | 26° | 28° | 30° | 35° | 40° | 45° | 50° | 55° | 60° | 65° | 70° | 75° | 80° | 85° | 90° |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ° | | | | | | | | | | | | | | | | | | | | | | | |
| 10 | 428 | 359 | | | | | | | | | | | | | | | | | | | | | |
| 12 | 359 | 295 | 253 | | | | | | | | | | | | | | | | | | | | |
| 14 | 315 | 253 | 214 | 187 | | | | | | | | | | | | | | | | | | | |
| 16 | 284 | 225 | 187 | 162 | 143 | | | | | | | | | | | | | | | | | | |
| 18 | 262 | 204 | 168 | 143 | 126 | 113 | | | | | | | | | | | | | | | | | |
| 20 | 245 | 189 | 153 | 130 | 113 | 100 | 91 | | | | | | | | | | | | | | | | |
| 22 | 232 | 177 | 142 | 119 | 103 | 91 | 81 | 74 | | | | | | | | | | | | | | | |
| 24 | 221 | 167 | 134 | 111 | 95 | 83 | 74 | 67 | 61 | | | | | | | | | | | | | | |
| 26 | 213 | 160 | 126 | 104 | 89 | 77 | 68 | 61 | 56 | 51 | | | | | | | | | | | | | |
| 28 | 206 | 153 | 120 | 99 | 83 | 72 | 63 | 57 | 51 | 47 | 43 | | | | | | | | | | | | |
| 30 | 199 | 148 | 115 | 94 | 79 | 68 | 59 | 53 | 48 | 43 | 40 | 33 | | | | | | | | | | | |
| 35 | 188 | 137 | 106 | 85 | 71 | 60 | 52 | 46 | 41 | 37 | 33 | 27 | 23 | | | | | | | | | | |
| 40 | 179 | 129 | 99 | 79 | 65 | 54 | 47 | 41 | 36 | 32 | 29 | 23 | 19 | 16 | | | | | | | | | |
| 45 | 172 | 124 | 93 | 74 | 60 | 50 | 43 | 37 | 32 | 28 | 25 | 20 | 16 | 13 | 11 | | | | | | | | |
| 50 | 167 | 119 | 89 | 70 | 57 | 47 | 39 | 34 | 29 | 26 | 23 | 18 | 14 | 11 | 9 | 8 | | | | | | | |
| 55 | 162 | 115 | 86 | 67 | 54 | 44 | 37 | 32 | 27 | 24 | 21 | 16 | 12 | 10 | 8 | 7 | 5 | | | | | | |
| 60 | 159 | 112 | 83 | 64 | 51 | 42 | 35 | 30 | 25 | 22 | 19 | 14 | 11 | 9 | 7 | 5 | 4 | 4 | | | | | |
| 65 | 155 | 109 | 80 | 62 | 49 | 40 | 33 | 28 | 24 | 21 | 18 | 13 | 10 | 7 | 6 | 5 | 4 | 3 | 2 | | | | |
| 70 | 152 | 106 | 78 | 60 | 48 | 38 | 32 | 27 | 23 | 19 | 17 | 12 | 9 | 7 | 5 | 4 | 3 | 2 | 2 | 1 | | | |
| 75 | 150 | 104 | 76 | 58 | 46 | 37 | 30 | 25 | 21 | 18 | 16 | 11 | 8 | 6 | 4 | 3 | 2 | 2 | 1 | 1 | 1 | | |
| 80 | 147 | 102 | 74 | 57 | 45 | 36 | 29 | 24 | 20 | 17 | 15 | 10 | 7 | 5 | 4 | 3 | 2 | 1 | 1 | 1 | 0 | 0 | |
| 85 | 145 | 100 | 73 | 55 | 43 | 34 | 28 | 23 | 19 | 16 | 14 | 10 | 7 | 5 | 3 | 2 | 2 | 1 | 1 | 0 | 0 | 0 | 0 |
| 90 | 143 | 98 | 71 | 54 | 42 | 33 | 27 | 22 | 19 | 16 | 13 | 9 | 6 | 4 | 3 | 2 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 95 | 140 | 96 | 70 | 53 | 41 | 32 | 26 | 22 | 18 | 15 | 13 | 9 | 6 | 4 | 3 | 2 | 1 | 1 | 0 | 0 | 0 | 0 | |
| 100 | 138 | 95 | 68 | 51 | 40 | 31 | 25 | 21 | 17 | 14 | 12 | 8 | 6 | 4 | 3 | 2 | 1 | 1 | 0 | 0 | 0 | | |
| 105 | 136 | 93 | 67 | 50 | 39 | 30 | 25 | 20 | 17 | 14 | 12 | 8 | 5 | 4 | 2 | 2 | 1 | 1 | 0 | 0 | | | |
| 110 | 134 | 91 | 65 | 49 | 38 | 30 | 24 | 19 | 16 | 13 | 11 | 7 | 5 | 3 | 2 | 2 | 1 | 1 | 1 | | | | |
| 115 | 132 | 89 | 64 | 48 | 37 | 29 | 23 | 19 | 15 | 13 | 11 | 7 | 5 | 3 | 2 | 2 | 1 | 1 | | | | | |
| 120 | 129 | 88 | 62 | 46 | 36 | 28 | 22 | 18 | 15 | 12 | 10 | 7 | 5 | 3 | 2 | 2 | 1 | | | | | | |
| 125 | 127 | 86 | 61 | 45 | 35 | 27 | 22 | 18 | 14 | 12 | 10 | 7 | 5 | 4 | 3 | 2 | | | | | | | |
| 130 | 125 | 84 | 59 | 44 | 34 | 26 | 21 | 17 | 14 | 12 | 10 | 7 | 5 | 4 | 3 | | | | | | | | |
| 135 | 122 | 82 | 58 | 43 | 33 | 26 | 21 | 17 | 14 | 12 | 10 | 7 | 5 | 4 | | | | | | | | | |
| 140 | 119 | 80 | 56 | 42 | 32 | 25 | 20 | 17 | 14 | 12 | 10 | 8 | 6 | | | | | | | | | | |
| 145 | 116 | 77 | 55 | 41 | 32 | 25 | 21 | 17 | 15 | 13 | 11 | 9 | | | | | | | | | | | |
| 150 | 112 | 75 | 54 | 40 | 32 | 26 | 21 | 18 | 16 | 15 | 13 | | | | | | | | | | | | |
| 152 | 111 | 75 | 53 | 40 | 32 | 26 | 22 | 19 | 17 | 16 | | | | | | | | | | | | | |
| 154 | 110 | 74 | 53 | 41 | 33 | 27 | 23 | 21 | 19 | | | | | | | | | | | | | | |
| 156 | 108 | 74 | 54 | 42 | 34 | 28 | 25 | 22 | | | | | | | | | | | | | | | |
| 158 | 107 | 74 | 54 | 43 | 35 | 30 | 27 | | | | | | | | | | | | | | | | |
| 160 | 107 | 74 | 56 | 45 | 38 | 33 | | | | | | | | | | | | | | | | | |
| 162 | 107 | 76 | 59 | 48 | 42 | | | | | | | | | | | | | | | | | | |
| 164 | 109 | 79 | 63 | 54 | | | | | | | | | | | | | | | | | | | |
| 166 | 113 | 86 | 71 | | | | | | | | | | | | | | | | | | | | |
| 168 | 122 | 98 | | | | | | | | | | | | | | | | | | | | | |
| 170 | 143 | | | | | | | | | | | | | | | | | | | | | | |

### HOW TO USE THE TABLE

To compare with each other two alternative figures, either quadrilaterals or central-point figures, in so far as the strength with which the length is carried is concerned, proceed as follows:

(*a*) For each figure take out the length angles, to the nearest degree, for the best and second-best chains of triangles through the figure. These chains are to be selected at first by estimation, and the estimate is to be checked later by the results of comparison.

(*b*) For each triangle in each chain enter the table with the length angles as the two arguments and take out the tabular value.

(*c*) For each chain, the best and second best, through each figure, take the sum of the tabular values.

(*d*) Multiply each sum by the factor $\frac{D-C}{D}$ for that figure, where $D$ is the number of directions observed and $C$ is the number of conditions to be satisfied in the figure. The quantities so obtained, namely, $\frac{D-C}{D} \Sigma\ [\delta_A{}^2+\delta_A\delta_B+\delta_B{}^2]$, will for convenience be called $R_1$ and $R_2$ for the best and second-best chains, respectively. (Examples of various triangulation figures with the corresponding values of $R_1$ and $R_2$ may be found in Special Publication No. 93, pp. 8–12.)

(*e*) The strength of the figure is dependent mainly upon the strength of the best chain through it, hence the smaller the $R_1$ the greater the strength of the figure. The second-best chain contributes somewhat to the total strength, and the other weaker and progressively less independent chains contribute still smaller amounts. In deciding between figures they should be classed according to their best chains, unless said best chains are very nearly of equal strength and their second-best chains differ greatly.

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