DEPARTMENT OF COMMERCE
R. P. LAMONT, Secretary
U. S. COAST AND GEODETIC SURVEY
R. S. PATTON, Director

MANUAL OF FIRST-ORDER LEVELING

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no. 140
1935

BY

HENRY G. AVERS
Senior Mathematician

Special Publication No. 140

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Price 15 cents

UNITED STATES
GOVERNMENT PRINTING OFFICE
WASHINGTON : 1935

National Oceanic and Atmospheric Administration

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January 1, 2006

FOREWORD

As a result of the many advances in the practice of first-order leveling, the general instructions of March, 1910, as published in United States Coast and Geodetic Survey Special Publications Nos. 18 and 22, have been revised to conform to the present accepted practice for this class of work.

It is the purpose of this publication to give the general instructions in revised form and also to summarize the methods employed by the United States Coast and Geodetic Survey in executing first-order leveling and in making the office computations. An endeavor has been made to collect from all available sources material that will be of use to the field engineer or the office mathematician who may be assigned to this class of work. The publication really is the outgrowth of the reports and recommendations of those of this bureau who have been closely associated with the work done during the past decade or longer.

The author wishes to express his indebtedness to Dr. William Bowie, chief of the division of geodesy, for his many valuable suggestions and for his liberal contribution to the discussion of errors. Acknowledgment is due also to C. M. Durgin, hydrographic and geodetic engineer, and to H. S. Rappleye, associate mathematician, for their advice and criticism.

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MANUAL OF FIRST-ORDER LEVELING

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Part I.—FIELD METHODS

INTRODUCTION

First-order leveling by the United States Coast and Geodetic Survey began with the transcontinental line of levels in 1879. Previous to this time the bureau had done some leveling, but it was used mostly to control the trigonometric leveling and, while it served its purpose, it was not of a high grade of accuracy compared with the standards of to-day.

The transcontinental line of levels differed both in purpose and extent from former work of this kind, for besides furnishing vertical control for the great scheme of triangulation extending across the continent, it provided bench marks at frequent intervals for the use of engineers and others who might be in need of accurate elevations.

The instruments and methods in use in 1879 were continued until 1899, when it was found that the leveling was subject to a systematic error depending upon the azimuth of the level line. About 4,300 miles of leveling had been done by this bureau up to that time. The behavior of the systematic error was determined from an adjustment of all the first-order leveling by this and other organizations. The leveling of the Coast and Geodetic Survey was corrected for the systematic error.

In 1898–99 the Superintendent of the United States Coast and Geodetic Survey, upon the recommendation of a committee on first-order leveling appointed by him, adopted the instruments and methods which, with only a few changes, have remained in use up to this time.

Leveling is of various degrees of accuracy, depending upon the sensitiveness of the instruments used, the care with which the observations are made, and the refinement of the computations.

Leveling of the highest degree of accuracy is now designated first-order leveling to conform with the recommendation of the Federal Board of Surveys and Maps made in May, 1925. Previous to that time this class of leveling was designated "leveling of high precision" and "precise leveling," and those terms are still used by foreign countries.

The limits for errors in precise (first-order) leveling were first fixed in 1867 by the Association for the Measurement of Degrees in Central Europe (later called the International Geodetic Association). Leveling was called precise (first-order) if the probable error of the difference in elevation between two points 1 kilometer apart did not exceed 3 millimeters on an average and was not greater than 5 millimeters as a maximum.

In 1912 the International Geodetic Association at its meeting in Hamburg, Germany, introduced another class of leveling called "leveling of high precision," for which the limits of errors are plus or minus 1 millimeter per kilometer for the probable accidental error and plus or minus 0.2 millimeter per kilometer for the probable systematic error. This narrowing of the limits was due to important advances made in the art of leveling since the first limits were established.

RESOLUTION OF THE INTERNATIONAL GEODETIC ASSOCIATION

The following is a translation of the resolution adopted by the International Geodetic Association, at its Seventeenth General Conference, held at Hamburg, Germany, September, 1912:

Because of the important advances made in the art of leveling since the year 1867, when limits were first fixed for errors permissible in precise leveling, and because of the benefit accruing to the greater needs of geodesy in creating a new class of more precise leveling; that is, one with narrower limits of error and with its errors, both accidental and systematic, computed according to uniform rules:

Therefore, the Seventeenth General Conference of the International Geodetic Association, still preserving unchanged the limits of error of 1867 for precise leveling, decides to place hereafter in a new class of leveling, to be termed "leveling of high precision," every line, set of lines, or net which is run twice in opposite directions on different dates as far as possible, and whose errors, accidental and systematic, computed by the formulas hereinafter given, do not exceed—

	± 1 mm. per km. for the probable accidental error,
or	± 1.5 mm. per km. for the mean accidental error;
	± 0.2 mm. per km. for the probable systematic error,
or	± 0.3 mm. per km. for the mean systematic error.

If L denote the length of an unconnected line, or the length of the side of a polygonal circuit in the case of a net;

ΣL, the aggregate length of the set of lines, or of the net under consideration;

Δ, the discrepancy between the results of the two runnings between consecutive bench marks;

r, the distance between these two bench marks;

s, the entire systematic discrepancy between the results of the two runnings, either for a whole line or for the side of a circuit;

f, the error of closure of a circuit of the net after the orthometric correction has been applied;

Σf^2, the sum of the squares of the errors of closure of the circuits, including the closing error, Σf, of the outside circuit;

Then the probable or mean errors are to be computed by the following formulas:

1. For the probable accidental error, η_r, in the case of a set of lines, whether or not they form circuits:

$$\text{I.}\quad \eta_r^2=\frac{1}{9}\left[\frac{\Sigma\Delta^2}{\Sigma L}-\frac{\Sigma r^2}{(\Sigma L)^2}\Sigma\frac{s^2}{L}\right].$$

2. For the probable systematic error, σ_r or σ_R:

(*a*) In the case of a set of lines not forming a net:

$$\text{II.}\quad \sigma_r^2=\frac{1}{9\Sigma L}\Sigma\frac{s^2}{L}.$$

(*b*) In the case of a net allowing at least 10 circuits:

$$\text{III.}\quad \sigma_R^2=\frac{1}{\Sigma L^2}\left[\frac{2}{9}\Sigma f^2-\eta_r^2\Sigma L\right].$$

3. For the mean accidental error, or for the mean systematic error, the same formulas as above, after multiplying the second member by $\frac{9}{4}$.

All the first-order leveling done by the United States Coast and Geodetic Survey since 1899 falls within the limits prescribed for this class of leveling at the Hamburg meeting.

In the 1912 adjustment of the first-order level net the average probable accidental error per kilometer was plus or minus 0.71 millimeter and the average probable systematic error was plus or minus 0.08 millimeter.

The present instructions for first-order leveling insure that the results of the work will meet the requirements of this class of leveling. The results that are being obtained, however, do not bring out to the fullest extent the extreme accuracy of which the instruments and methods are capable. Our observers are instructed to approach the limits by lengthening the sights and by increasing the rapidity of the observing, even to the extent of doing a moderate amount of rerunning, rather than to use extreme precautions in attempting to obtain small differences in the backward and forward measures of a section. In this way it is possible to increase the progress considerably and still keep within the limits of accuracy prescribed for first-order leveling. The problem in this country, where there are vast regions as yet untraversed by first-order leveling, is to obtain the greatest amount of progress and the lowest unit costs consistent with the required accuracy.

GENERAL INSTRUCTIONS FOR FIRST-ORDER LEVELING

METHODS

1. In general, a line of levels will begin and end on previously established bench marks. At least two bench marks which, as shown by the new leveling, have not changed their relative elevations must be recovered at each end of the line.

2. Except when specific instructions are given to proceed otherwise, all lines are to be leveled independently in both the forward and backward directions.

3. The backward measurement on each section should be made under different atmospheric conditions from those which occur on the forward measurement.

It is especially desirable to make the backward measurement in the afternoon if the forward measurement was made in the forenoon, and vice versa. The observer is to secure as much difference of conditions between the forward and backward measurements as is possible without materially delaying the work for that purpose.

4. The spaces between the permanent bench marks of a line of levels should be broken by temporary bench marks into sections from 1 to 2 kilometers in length, except where special conditions make shorter sections advisable.

5. In general, when working along a railroad, the top of the spike on the inside of the rail, which holds the rail in place, should be used as a turning point. If the railroad is in poor condition and the spikes are loose, a nail should be driven into the center of a tie for the turning point. When working along a road or running a spur line away from the railroad, metal pins should be used.

6. The instrument shall be shaded from the direct rays of the sun, both during the observations and when moving from one instrument station to another.

7. At each city along the line, the leveling should be connected with at least two stable bench marks which are connected with the city datum. Connection should be made with all stable bench marks of other organizations which may be found along the route.

8. The elevation of the top of the railroad rail in front of each railroad station along the line of levels and opposite each milepost is to be determined with a check. This should be done by taking an extra foresight to it on both the backward and forward runnings, or by taking extra foresights to it from two instrument stations near it on one of the runnings of the line.

9. When working along a highway, the height of the intersection with each main road and at the principal forks of the road should be determined with a check in the same manner as outlined in paragraph 8 for the determination of the top of the railroad rail in front of each railroad station.

10. The thermometers on the rods should be read to the nearest degree centigrade at the beginning and end of each section and the temperatures recorded. Where the sections are continuous the readings at the beginning of each section are sufficient except that on the last section the temperatures at the end should also be recorded.

11. At least once each month, during the progress of the leveling, a test must be made of the adjustment of the plumbing levels of the rods. A statement should be inserted in the record showing the manner in which the test was made, the error that was found, and whether an adjustment was made. With the bubble of the level rod held at the center, the deviation of the face and the edge of the rod from the vertical must be determined. If the deviation from the vertical in either direction exceeds 10 millimeters on a 3-meter length of the rod, the rod level must be adjusted.

12. When the leveling has been completed to a point which is to hold the elevation of the line over night, or longer, a check point should be established at least one set-up of the instrument away. When the leveling is continued from or to such a pair of points, the instrument should be set up between them and rod readings taken on each point. The same arrangement of points should be used at the completed end or ends of any detached portion of the line of levels. Either one of the two points may be used for carrying along the elevation, with the other used only as a check against mistakes in reading the rod or against a discrepancy caused by a disturbance of one or both of the marks. The records should show clearly which one of the two points was used to carry the elevation, and it is believed to be a good policy always to use the same point (backward or forward) as far as may be practicable. It is believed that, by employing this method, no mistake of a meter or a decimeter made in reading the rod, held on a bench mark, will escape detection.

SUPPLEMENT TO SPECIAL PUBLICATION NO. 140

The General Instructions for First-order Leveling as given on pages 3 to 9 of Special Publication No. 140 were amended on December 6, 1933, as follows:

Paragraph 1.—Add the following:

Failure of a first-order line between two old bench marks to check the original observed difference in elevation within the limit 4.0 mm $\sqrt{k}$ (where k is the distance in kilometers, measured along the route followed by the leveling, between the bench marks) shall be construed to indicate movement of at least one of the marks. Two bench marks located on the same structure or so close together that they may both be affected by a single disturbing influence are not to be considered as a proper check for beginning or ending a line even though the new observed difference in elevation may happen to check the old observed difference in elevation.

Paragraph 7.—Add the following:

It should be kept in mind that every time a connection is made to the work of other organizations, the leveling of that organization can then be placed on a mean-sea-level datum and becomes available as control leveling of an order of accuracy determined by the accuracy of the leveling itself. A half day spent in making a connection to a bench mark on an assumed datum may bring many miles of levels into agreement with the control net. When there is any question as to whether or not the work necessary to make certain connections, not specifically provided for in the special instructions for the project, is justified, the matter should be referred to the office.

Paragraph 8.—Strike out the paragraph and substitute the following:

When working along railroads the elevation of the top of the rail opposite each station, opposite each milepost and at the crossing of each public road or highway is to be determined with a check. This should be done by taking an extra foresight to it on each running of the section in which it is located. When working through a city or village where there are several crossings close together it is not necessary to determine the elevation of the top of the rail at each crossing but only at the more important crossings. In taking the elevation of the top of the rail at points located on curves a note should be made in the record showing which rail the rod was held on, since the two rails on curves are not at the same elevation.

Paragraph 9.—Add the following:

In determining elevations at road intersections the rod should be held at the intersection of the center lines of the traveled ways, and notes in the record should state this fact or any deviation from standard practice. At complicated intersections, such as are now being constructed where main highways join, a sketch plan of the road junction showing just where the rod was held should be inserted in the record.

Paragraph 26.—Strike out the paragraph and substitute the following:

Permanent bench marks should be established at intervals not exceeding 2 miles except that, when transportation of materials is particularly difficult and there are no outcrops of rock or existing structures of suitable character in which to set the marks, this interval may be increased to 3 miles on the portion of the line where the unusual conditions are encountered. In determining the frequency of bench marks all bench marks of other organizations which are well and permanently marked in a manner closely similar to those established by this Bureau may be counted in complying with the above requirements. Other types of marks, chiseled squares, bolts, rivets, unmarked points, etc., are not to be counted in complying with the above spacing requirement.

Paragraph 35.—Following this paragraph, insert:

35A. Use Form 685 for reporting on the condition of all old marks searched for in starting or ending new lines of leveling, and for all marks searched for in making connection to old lines where the new lines cross old ones already in the control net. In general, Form 685 will be used only for marks for which original observed differences are furnished by the office.

35B. A brief description should be entered in the "Remarks" column of the leveling record opposite extra foresights at crossings so that a person unfamiliar with the original field work may be able to positively identify the crossing referred to.

13. Except in rare cases, the permanent bench marks should be established before or during the first running of the line. It is believed to be inadvisable to delay the tying in of permanent bench marks until after the line has been run, even if in only one direction. When it is impracticable to establish a permanent bench mark before or during the first measurement of the line, an acceptable manner of tying in the permanent bench mark or including it in the main line of levels is to establish a temporary bench mark on each side of the proposed location of the permanent bench mark and to leave the distance between them unleveled until the permanent bench mark has been set. The arrangement of the temporary bench marks established for this purpose should be similar to that described in paragraph 12 above. This would provide for two points, the difference in elevation between which are known, on each side of the permanent bench mark, and the distance between the two pairs of points makes a section in the main line of levels. A diagram showing the arrangement of the temporary bench marks and the permanent bench mark is shown below:

—O—X—O—X—□—X—O—X—O—

The positions of the instrument are shown by X, the positions of the temporary bench marks by O, and the position of the permanent bench mark by □.

14. As far as possible, all the permanent bench marks should be in the main line of levels and not on spur or branch lines. One of the exceptions to this rule is where the line runs several miles off the railroad to the mark of a triangulation station. In such a case the spur, or branch line, is the more economical way of doing the work and will be satisfactory. Whenever a permanent bench mark is established by means of a spur or branch line, which has only one set-up, the forward and backward lines of the spur or branch should be run at different times of the day or on different days, if practicable. If it should be necessary to have the two runnings made one immediately after the other, the height of the instrument should be materially changed in making the second measure. This will help to prevent any mistake in reading one or both rods.

DETERMINATION OF COLLIMATION ERROR

15. Once during each day of observation the error of the level should be determined in the regular course of the leveling and the observations recorded in a separate opening of the record book as follows: After the ordinary observations at an instrument station are completed, transcribe the last foresight reading as part of the error determination; call up the back rod man and have the rod placed about 10 meters back from the instrument; read the rod; move the instrument to a position about 10 meters behind the front rod; read the front rod and then the back rod. (The two instrument stations are between the two rod points.) The rod readings must be taken with the bubble in the middle of its tube. The required constant C to be determined, namely, the ratio of the required correction to any rod reading to the corresponding subtended interval, is

$$C=\frac{(\text{sum of near rod readings})-(\text{sum of distant rod readings})}{(\text{sum of distant rod intervals})-(\text{sum of near rod intervals}}.$$

The total correction for curvature and refraction must be applied to each of the distant rod readings before using it in this formula. The level should not be adjusted if C is less than 0.005. If C is between 0.005 and 0.010, the observer is advised not to adjust the level, but if C exceeds 0.010, the adjustment must be made. If a new adjustment of the level is made, C should at once be redetermined. It is desirable to have the determination of C made under the usual conditions as to length of sight, character of ground, elevation of line of sight above ground, etc. The adjustment of the instrument to reduce C, must be made by moving the level vial, not by moving the reticle. (See form of record, p. 11.)

LENGTH OF SIGHTS

16. The maximum length of sight shall be 150 meters, and this maximum is to be attained only under the most favorable conditions. (See also par. 18.)

17. The difference in length between a foresight and the corresponding backsight must not exceed 10 meters. The difference is to be made as small on each pair of sights as is feasible by the use of good judgment and care, without any expenditure of time for this particular purpose. The telescope should not be refocused between sights if possible to avoid it.

18. Chiefs of party should keep the length of sight great enough to make it necessary to do a moderate amount of rerunning. If an observer is extremely cautious and confines all his observations to sights sufficiently short to insure easy reading of the rod, it is possible to work month after month with almost no rerunning, but the progress will be slow. On the other hand, it is certain that an attempt to take sights of the limiting length, 150 meters, at all times would lead to a very large amount of rerunning and the progress would not be rapid. It is believed that the maximum speed consistent with the required degree of accuracy will be secured by continually keeping the length of sight such that the amount of rerunning will be from 5 to 15 per cent. An extremely small percentage of rerunning indicates an excess of caution on the part of the observer. The occurrence of a moderate amount of rerunning is due largely to an attempt on the part of the observer to obtain the maximum progress consistent with the required degree of accuracy and not to inability to secure such observations that little or no rerunning would be necessary. Observers have found that a convenient rule in fixing the length of sight is to shorten the sights whenever the upper and lower thread intervals subtended on the rod are found to differ frequently by more than a selected limit. Each observer should fix this limit from his own experience by noting the relation between a provisional limit and the amount of rerunning found to be necessary while using it. Such a rule is based upon the idea that the additional errors which are encountered when the length of sight is increased are, in the main, those due to the increased accidental errors in reading the rod.

ALLOWABLE DIVERGENCE OF RUNNINGS

19. On all sections upon which the forward and backward measures differ by more than 4.0 $\sqrt{K}$ in millimeters (in which K is the length of the section in kilometers), the section should be rerun until any two runnings in opposite directions check within the limits of the formula. (See table, page 75.)

20. On all sections 0.5 kilometer or less in length, a discrepancy of not more than 2.8 millimeters between the backward and forward lines will be considered a satisfactory check regardless of the limits provided by the formula in paragraph 19. This provision, however, should not influence the chief of party in determining the lengths of sections.

21. It is thought to be inadvisable to state definitely in these instructions the allowable limit on the rate of divergence between the forward and backward lines (B–F), but this should be kept small. (See pp. 23–26.)

REJECTIONS

22. If any measure over a section gives a result differing by more than 6 millimeters from the mean of all the measures over that section, this measure shall be rejected. No rejection shall be made on account of a residual smaller than 6 millimeters unless there is some good reason for suspecting an error in this particular measure, and in such cases the reason for rejection must be fully stated in the record.

MISTAKES

23. Whenever a mistake, such as a misreading of 1 decimeter or 1 meter, or an interchange of sights (the backsight being recorded for the foresight), is discovered in any measure after its completion and the necessary correction applied, such measure may be retained provided there are at least two other measures over the same section which are not subject to any such uncertainty. When it is found, however, that the mistake was made on the last instrument station of the second running of a section and the mistake is corrected on the same day and before beginning work on an adjacent section, such measure may be retained, and no further measures are required on account of the mistake, but a note describing the procedure must be made in the record.

PROGRAM OF OBSERVING

24. The program of observation at each instrument station is to be as follows: Set up and level the instrument. Read the three lines of the diaphragm as seen projected against the face of the No. 1 rod, each reading being taken to the nearest millimeter (estimated) and the bubble being held continuously in the middle of the tube; that is, both ends reading the same. (See pp. 22 and 23.) Then, after the rod has been turned, read the middle wire of the diaphragm, as seen projected against the back of the rod, to the nearest hundredth of a foot[1] (estimated). As soon as possible thereafter repeat the operation with the telescope pointed at the No. 2 rod. The foot graduations on the back of the rod need not be read on more than one running of a section.

25. At instrument stations of odd numbers the backsight is to be taken before the foresight, and at those of even numbers the foresight is to be taken before the backsight. As the same rod is held on a rod station for both the foresight and backsight, the effect of this method is that the same rod is read first at each set-up, it being the rod used for the backsight at the first instrument station of the day's running.

PERMANENT BENCH MARKS

26. Permanent bench marks should be established at 3-mile intervals or less on an average. The distance between successive permanent bench marks shall nowhere exceed 5 miles. In determining the frequency of bench marks all substantial ones with which the leveling is directly connected, regardless of whether they are new bench marks or old ones established by other organizations, should be considered.

27. At places from which it is likely that future leveling will be extended, such as a railroad junction point, or the crossing of a main highway, at least three bench marks should be established within a radius of about ½ mile, but far enough apart not to be affected by the same disturbing cause or causes.

28. Every new permanent bench mark must be marked by a standard metal tablet (see fig. 8), so fastened in the rock, brick, or concrete as to resist extraction, change of elevation, or rotation.

29. Traverse stations which are easily accessible and have been marked in a permanent manner should be used as permanent bench marks.

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[1] Since the preparation of this manuscript the backs of the level rods have been graduated in thirds and thirtieths of a meter. The new graduation allows an easy comparison between the middle-thread readings on the face and the check readings of the same thread on the back of the rod. The check readings on the new graduations, or any quantities derived from them, are three times as large as the values with which they are compared.

All references in this publication to the foot graduation on the back of the rods apply equally to the new graduation, bearing in mind, of course, the difference in the conversion factors.

30. A traverse station reference mark should not be used as a permanent bench mark. As a rule, the reference mark is shallow and does not have the desired stability.

DESIGNATIONS OF BENCH MARKS

31. Permanent bench marks should be designated by a capital letter followed by a number, the letters to follow in alphabetical order and the number to change as each alphabet is exhausted. This designation, together with the year in which the mark was established, must be stamped upon the tablet, preferably before it is set. The system of numbering is separate and distinct for each State. The chief of party will be furnished with the designation for the first bench mark in each State in which he may work. If two or more parties are working in the same State, each party will be assigned a certain number of alphabets. The letters "I" and "O" should be omitted from each alphabet, as they have been the source of considerable confusion in requests for data received from persons finding the marks. The designation of two sets of three consecutive bench marks is given below:

H 2 1927	J 2 1927	K 2 1927
M 2 1927	N 2 1927	P 2 1927

32. All recovered bench marks are to be called by their old designations, whether the bench mark was established by this or some other organization. The designation, in the records, of a bench mark of another organization is, however, to be followed by the initials of the organization that established the mark.

33. Triangulation and traverse stations connected with the leveling shall be called by their names and not given designating letters.

34. Each temporary bench mark is to be designated by a number, the numbers to be continuous throughout the line.

DESCRIPTIONS OF BENCH MARKS

35. A description must be written on Form 638 for every permanent bench mark established or connected with. Temporary bench marks which may possibly be recovered for a few years after their establishment should also be described on Form 638. No duplicates of the descriptions need be furnished the office.

RECORDS AND COMPUTATIONS

36. The record and the preliminary or field computation of first-order leveling must conform to the examples given on pages 39, 41 and 42 of this publication.

37. The recorder shall keep a record of the rod intervals subtended by the extreme lines of the diaphragm on each backsight, together with the continuous sum of such intervals between each two contiguous bench marks (temporary or permanent), and a similar record shall be kept for the foresights. The two continuous sums shall be kept as nearly equal as is feasible, without the expenditure of extra time for that purpose, by setting the instrument beyond (or short of) the middle point between the back and front rods when the sums are unequal. The two continuous sums for a section shall not be allowed to differ by more than a quantity corresponding to a distance of 20 meters.

38. Whenever the instrument is set up at some point other than on the railroad track, the number of the instrument station on the left-hand page of the record should be inclosed in parentheses. Similarly, on the right-hand page of the record, the designating number for the foresight rod (215, 216, etc.) shall be

inclosed in parantheses, if said rod is not supported on the railroad spike or on a nail driven into a tie. If the length of any portion of the level line run off the railroad is 25 meters or more greater than the railroad distance between the points of departure from and return to the railroad, then the distance along the track between these two points must be shown in the record. The purpose of these requirements is to furnish the office a means of detecting blunders in the leveling, by plotting the level line on the profile of the railroad.

39. The field computations and abstracts should be kept up as the work progresses. As soon as each book of the original record is out of use it is to be sent to the office by registered mail. The corresponding computation sheets must be retained until it is certain that there is no possibility of their being forwarded in the same mail as the records. No duplicates of the original records are to be made.

40. Notes for future use in studying leveling errors shall be inserted in the record. These should give the time of beginning and ending the work of each section, the weather conditions, especially as to cloudiness and strength of wind, and also the direction of the wind and sun with respect to the line of levels. Such other notes should be made as promise to be of value in studying errors.

MISCELLANEOUS

41. Should the experience of a chief of party indicate to him that a change or changes in these instructions would facilitate the work in the field, he is urged to communicate with this office regarding such changes.

42. When cases arise which are not provided for by these general instructions or by specific instructions, the chief of party will use his own judgment in the matter.

INSTRUMENTS

GEODETIC LEVEL

The geodetic level now in use is a modification of the one adopted by this bureau in 1899 and fully described in Appendix 3, Report for 1903. A complete description of the new model is given in United States Coast and Geodetic Survey Special Publication No. 129. Each of these models was designed by E. G. Fischer, former chief of the instrument division, United States Coast and Geodetic Survey, and the first one was constructed by him.

The earlier level was made of an alloy of one part grain nickel and two parts cast iron with a thermal coefficient of 0.000004 per degree centigrade. The telescope of this level was supported by two pivot screws near its object end. The latest type of geodetic level is made of invar with a thermal coefficient not exceeding 0.000001 per degree centigrade, thus insuring a better constancy of the relation between the level and the line of collimation. The telescope is pivoted at a point on the vertical axis of the instrument. The earlier type of level with the telescope pivoted 9.7 centimeters forward of the axis introduced systematic differences in the height of the instrument between the backsight and foresight if the observer habitually adjusted the vertical axis out of the normal in one particular direction.

Two views of the geodetic level are shown in Figures 1 and 2.

Length of bubble.—The level vial of the instrument has a small air chamber at one end, by means of which the length of the level bubble can be adjusted through the proper tilting of the instrument. To make the adjustment, hold the instrument in the hands with the leveling screws uppermost and with one end of the telescope higher than the other. If the air chamber is at the lower end, air bubbles will be seen to rise through the liquid in the vial, thus increasing the length of the bubble. If the air chamber is at the upper end, the bubble will grow smaller.

The length of the level bubble should be such that the bubble ends will be not more than one or two divisions away from the position marks on the vial. A long bubble is difficult to control, and a short bubble is too sluggish to respond immediately to the movements of the micrometer screw.

Parallax.—Before using the instrument the telescope should be carefully examined for parallax. This is done by pointing the telescope toward the sky or any light surface and focusing the eyepiece on the cross wires so that they appear sharp and black. Next focus the objective on a level rod about 100 meters distant and test for parallax by moving the eye slowly across the eyepiece. If the wires appear to move over the image of the graduations, parallax is present. The parallax may be due to poor focusing of either the eyepiece or the object glass. To eliminate the parallax, refocus the object glass until no parallax is present, then, if the image of the object is not distinct, move the eyepiece the small amount necessary to make it appear distinct. The image of the object is now in the plane of the cross wires, and both are in the focus of the eyepiece.

This adjustment should be tested frequently while observing, for, as the eyes of the observer tire, the focal distance of the lens of the eye changes, causing a blurring of the cross wires, which should be corrected by refocusing the eyepiece.

Reversing point.—The circular level, as a rule, is so nearly in adjustment that it can be used to level the instrument approximately for determining the reversing point of the micrometer screw.

The reversing point of the micrometer screw is the setting of the micrometer head at which the bubble of the level vial, when the instrument is level, will remain in the center when the instrument is rotated about its axis. The instrument should be leveled as well as possible with the circular level and the bubble of the level vial brought to the center by turning the micrometer-screw head. This will give the approximate reversing point. The exact reversing point is then found by turning the instrument 180° in azimuth, taking up one-half the error of the level bubble by turning the micrometer head and the remainder by the foot screws of the instrument. This

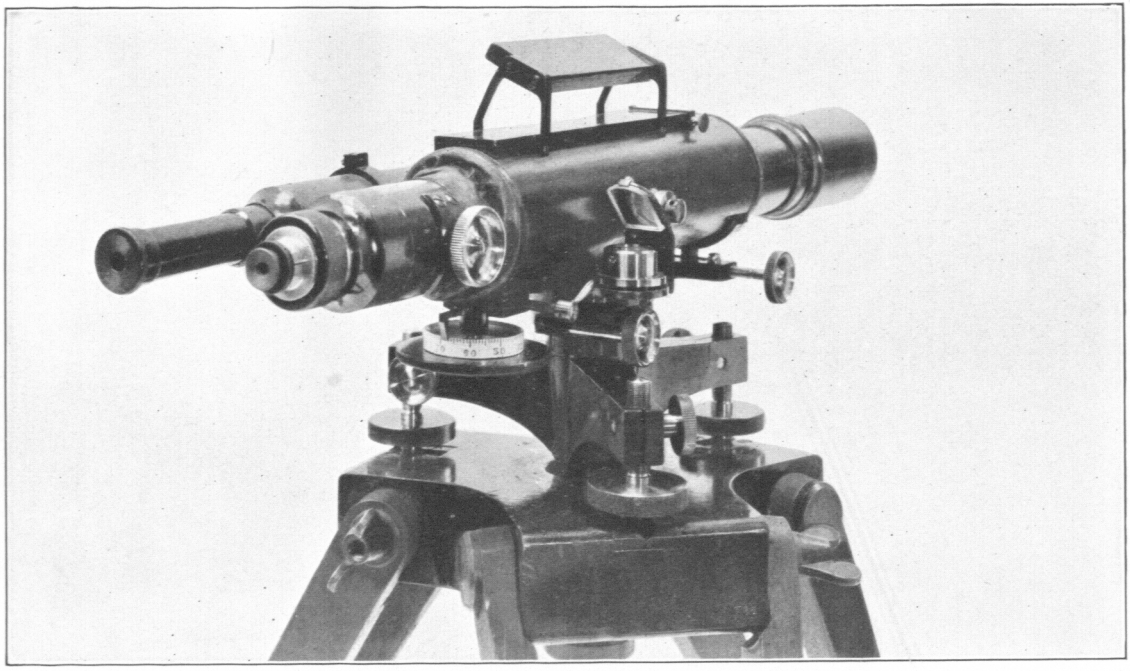

FIGURE 1.—GEODETIC LEVEL, UNITED STATES COAST AND GEODETIC SURVEY MODEL, RIGHT SIDE

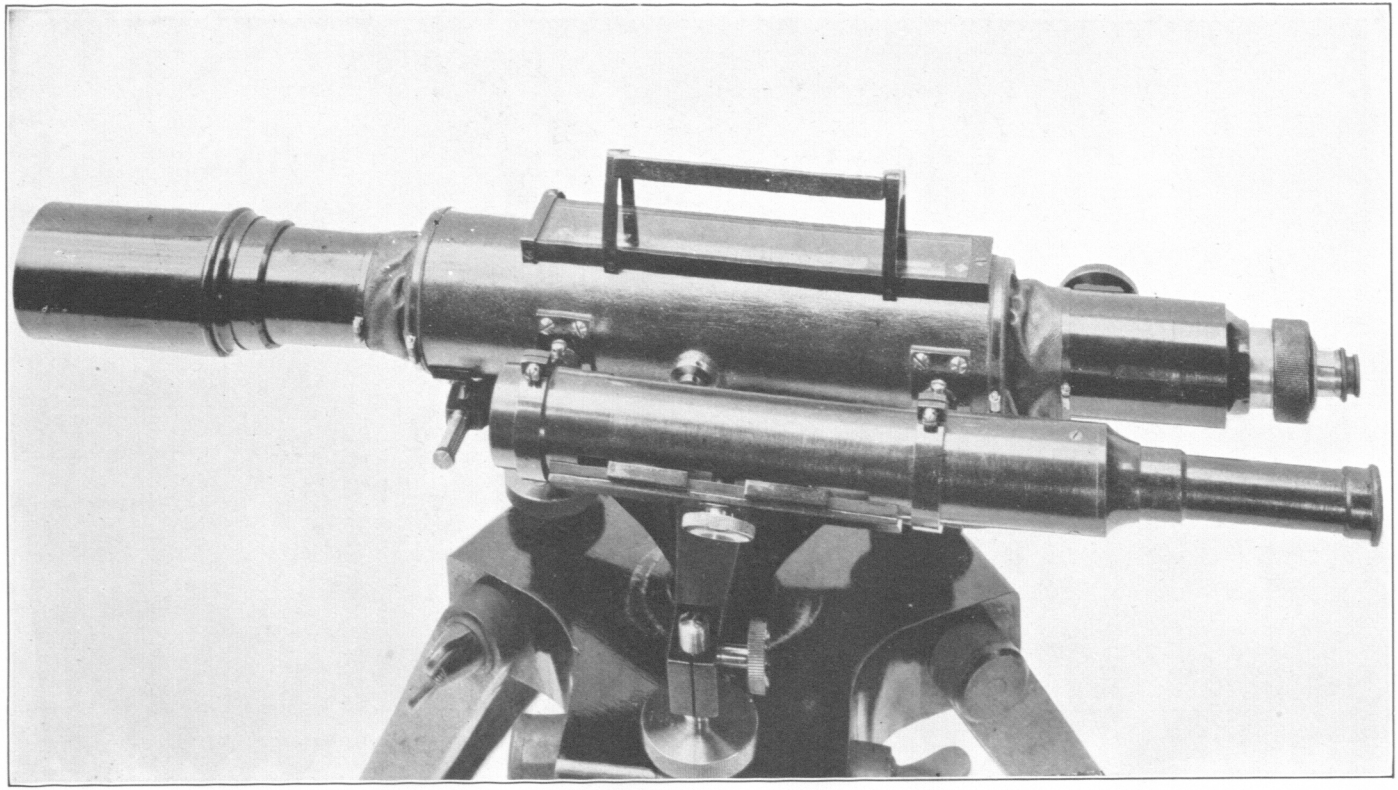

FIGURE 2.—GEODETIC LEVEL, UNITED STATES COAST AND GEODETIC SURVEY MODEL, LEFT SIDE

process should be repeated until the bubble stays exactly in the center of the vial as the instrument is shifted in azimuth.

Most of the instruments have a horizontal line on the micrometer head index which gives at once the approximate reversing point, within one turn of the micrometer screw, when the top of the micrometer-screw head is made even with it.

When the reversing point has been found and the leveling of the instrument made perfect, the circular level should be adjusted to agree.

Collimation.—The collimation adjustment, or what is commonly referred to as the determination of C, is made in accordance with paragraph 15 of the general instructions (p. 5), where the procedure is given in sufficient detail.

A specimen of the record of the determination of C is given below:

Determination of C

[4.43 p. m., Oct. 5, 1926, C. M. Thomas, *Observer*]

(Left-hand page)					(Right-hand page)			
Number of station	Thread reading backsight	Mean	Thread interval		Rod	Thread reading foresight	Mean	Thread interval
	Mm.	*Mm.*				*Mm.*	*Mm.*	
	1,674		12			2,076		128
A	1,662	1,662.0	12		219	1,948	1,947.7	129
	1,650		24			1,819		257
	1,882		11			1,715		130
B	1,871	1,871.0	11		220	1,585	1,584.7	131
	1,860		22			1,454		261
		3,533.0	46				3,532.4	518
		−3,531.2					−1.2*	46
		+1.8					3,531.2	472
	$C=+1.8\div472=+0.004$							

* This is twice the curvature and refraction correction for a 95-meter sight.

Each of the distant rod readings must be corrected for curvature and refraction. The two corrections may be combined as indicated in the computation above.

Note that if the figures are transferred from page to page as indicated all subtractions are made right side up.

Do not carry C to more than three decimal places.

When the instrument must be adjusted, due to too large a value for C, it should be done by raising or lowering one end of the level vial and not by moving the reticle.

The adjustment is made as follows: Point to a distant rod with the bubble in the middle of its tube, and read. Move the telescope so as to raise the middle line by an amount equal to C times the rod interval. With the telescope in this position, bring the bubble to the middle of the tube by raising or lowering one end of the level vial. If C is negative, the middle line must, of course, be lowered on the rod.

In the example given the adjustment of the instrument would require increasing the mean thread reading for each long sight about 1 millimeter.

DETERMINATION OF STADIA INTERVAL

The stadia interval of the instrument should be determined since from it are obtained the lengths of the sections of leveling, which are used in the formula for computing the allowable divergence between the backward and forward runnings.

It will be sufficient for the purpose of this work to obtain the mean stadia interval from the thread intervals when the rod is held at distances of 25, 50, 75, 100, 125, and 150 meters, respectively, from the instrument.

Lay off horizontal distances of 25, 50, 75, 100, 125, and 150 meters respectively, from the instrument. Read the rod at each distance and compute the sum of the thread intervals. The stadia interval then is the sum of the distances divided by the total sum of the thread intervals.

The record of a determination of the stadia interval is shown below. The sum of the distances is 525 meters, and the total sum of the thread intervals is 1,423 millimeters. The stadia interval therefore is $\frac{525}{1,423}$, or 0.369. For this instrument the total sum of the thread intervals in millimeters for any section should be multiplied by 0.369 to obtain the length of the section in meters.

Determination of stadia interval

[Geodetic level No. 18, Oct. 21, 1926]

Distance from level	Thread readings	Mean	Thread intervals
M.	*Mm.*	*Mm.*	*Mm.*
25	1,688 1,655 1,622	1,655.0	33 33 66
50	1,736 1,669 1,602	1,669.0	67 67 134
75	1,771 1,670 1,568	1,669.7	101 102 203
100	1,898 1,763 1,627	1,762.7	135 136 271
125	1,873 1,703 1,532	1,702.7	170 171 341
150	1,856 1,653 1,448	1,652.3	203 205 408
Sum=525			Sum=1,423

$$\text{Stadia interval} = \frac{525}{1,423} = 0.369$$

RODS

The present type of geodetic level rod, designed and first constructed in the office of the United States Coast and Geodetic Survey, was adopted in 1916.

The rod (see fig. 3) consists of a graduated metallic strip rigidly attached to a metal foot piece, the latter being fastened to a wooden staff which supports the strip.

The graduated strip, 25 millimeters wide and 1 millimeter thick, is made of invar, and has a thermal coefficient seldom exceeding 0.000001 per degree centigrade. It is fastened to the cast-steel foot piece by a brass machine screw and two bronze dowel pins.

The staff is made of a single piece of well-seasoned, straight-grained white or sugar pine, free from knots or other defects. The metal foot piece is securely fastened to the staff by four brass bolts through the face and two brass wood screws in the end of the staff. A shallow groove slightly wider than the invar strip is routed out through the entire length of the face of the staff and deep enough so that the strip is free to move underneath a number of brass washers recessed to set flush with the face. In order to eliminate any error due to sagging of the strip when the rod is held erect it is placed under tension by a stiff spring, set into a recess in the top of the staff and bearing against a small brass angle plate attached to the top of the strip.

The staff serves to hold the graduated strip straight. On the face of the staff are the numbers indicating the decimeter graduations, while the back of the staff carries the graduations to a tenth of a foot (see note, p. 7) which are used for making the check readings. A thermometer is mounted in the staff just back of the strip. A circular level enables the rodman to hold the rod vertically.

To prevent swelling, the wooden staff was formerly immersed in boiling paraffin, but this was found to add very materially to the weight and to destroy the resilience of the wood to such an extent that it frequently took a permanent set. The staff is now treated by applying a heavy coat of raw linseed oil, three coats of orange shellac, and two coats of white-lead paint. This treatment furnishes an excellent protection against moisture and in no way affects the resilience.

The invar strip carries the alternating black and white centimeter graduation of the usual form. The graduating of the strip is done by a machine and process recently perfected by D. L. Parkhurst, chief of the instrument division of the United States Coast and Geodetic Survey.

The foot piece is shaped so as to permit the use as turning points of of the spikes which hold the rail to the ties.

A complete description of the rods and the process by which they are graduated is given in United States Coast and Geodetic Survey Special Publication No. 129

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TURNING POINTS

When leveling along a railroad the tops of the spikes which hold the rail to the ties should be used as turning points.

The spike selected for a turning point should be near the center of the rail in order to be away from the rail joints where, as a rule, the spikes are loosened by the passage of trains. Only spikes with well-defined high points should be used, and the top should be made free of grease or other accumulations by the use of a steel brush or by scraping the sole of one's shoe over it. If the precaution of cleaning the top of the spike is not taken, the rod support may be lower for the back sight than for the foresight.

Also, the bottom of the rod may accumulate grease and dirt, making its surface irregular and thus affecting the length of the rod. The foot of the rod should always be wiped off before setting it on the spike.

When the leveling follows a railroad, foot pins are used as turning points only when it is necessary to extend the line to a bench mark at some distance from the track or when it is found necessary to hold an elevation for a few minutes due to a passing train or other interruption to the work. When the leveling follows a highway foot pins are used exclusively for turning points. The foot pin, Figure 4, recently added to the leveling equipment, is a commercial rivet punch 14 inches long. It replaces the bronze pin with a spherical top, formerly used, which was found to be so soft and brittle that it was soon battered or even broken when used continuously.

CARE OF LEVEL AND RODS

The level and rods should at all times be accorded the care and attention that delicate scientific instruments of this type merit.

Keep the level in the box when it is not in use and when transporting it to and from the working grounds.

The telescope should not rest upon the point of the micrometer screw when the instrument is being carried about or when it is in the box.

Keep the lenses of the telescope clean by the frequent use of a camel's-hair brush.

See that the object glass is firmly in place.

Keep the leveling screws and the micrometer screw clean by oiling and rubbing.

The rods should be carried by the handles. If the rods are always carried with the face down or up, they will become slightly bowed and the rod readings will be too large. In order to keep the rod straight, the rodman should carry it in its natural position with the face down for a certain period and then shift the rod to the shoulder, with the face up, and carry it in that position for the same period.

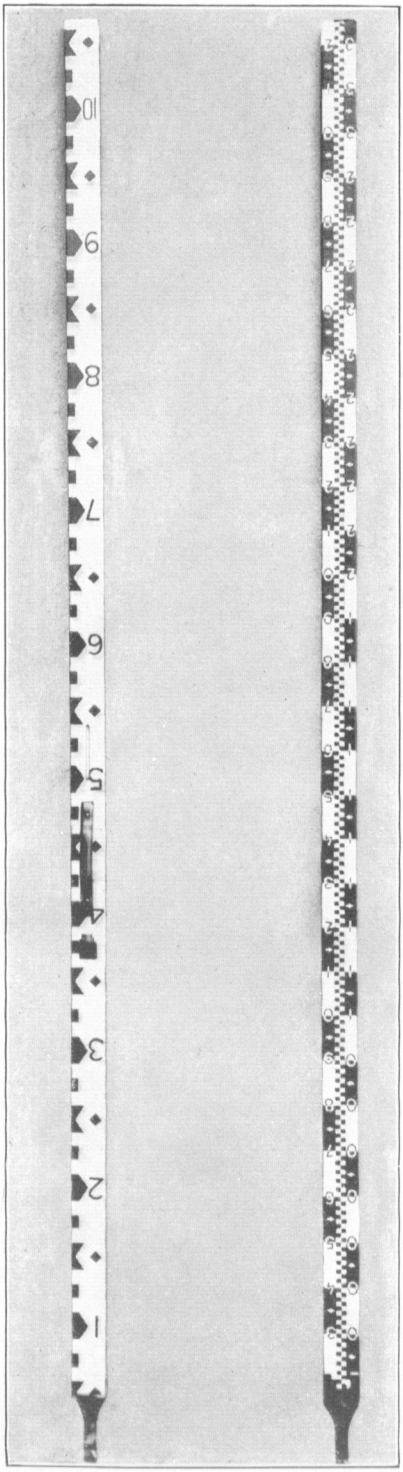

FIGURE 3.—UNITED STATES COAST AND GEODETIC SURVEY LEVEL ROD

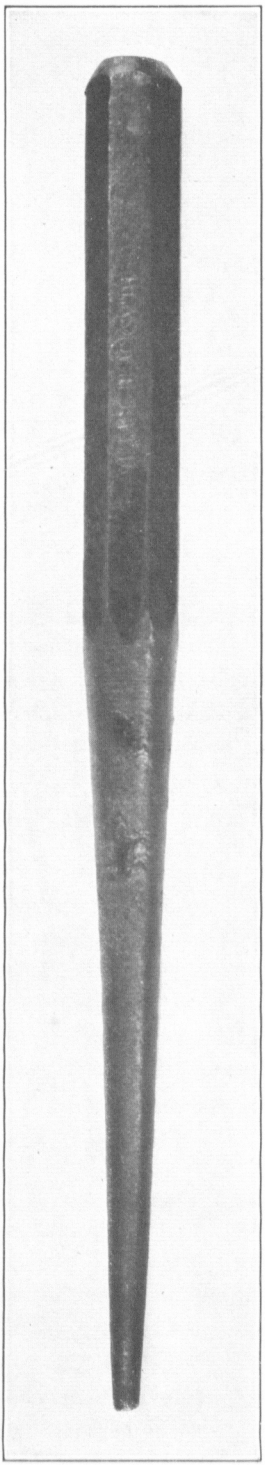

FIGURE 4.—FOOT PIN

In traveling to and from the working grounds the rods should be placed edges down on carrying brackets, with all bearing parts heavily cushioned and the faces of the rods separated by heavy cloth or burlap.

The foot of the rod, though firmly attached to the wooden staff, should be inspected frequently to see that it has not become bent so the plane of the foot is no longer normal to the face of the rod.

The rod, because of its length, is so easily struck against objects that the greatest care should be used in handling it. The numbers on the wood and the graduations on the invar strips should be protected in some way to prevent the scaling of the paint from the wood and the metal. A badly scarred rod means shorter sights, slower observing, greater unit costs, and decrease in accuracy. A rod should be returned to the office for repainting as soon as the observer finds that it is scarred so badly as to interfere with the rapid reading of the rod.

ORGANIZATION AND EQUIPMENT OF PARTY

It has been the practice in the United States Coast and Geodetic Survey, during a number of years, to have a prospective chief of party attached to the party of an experienced officer for a short time, usually a month, in order that the former may become acquainted with the management of the party, manipulation of the instrument, and the proper way of making out records and computations. By this method the work done by different observers is remarkably uniform in character and accuracy. Occasionally the training is done by an older officer who assists the new chief of party to equip and organize his party. It is believed to be the better plan to have the new chief of party serve a short time in the party of an experienced officer.

The instructions require that the records be sent to the office as soon as they are abstracted. These records are inspected under the direction of the chief of the division of geodesy, who calls the attention of the chief of party to any departure from what are considered the best methods of observing and computing. It is usually the case that the letters which go directly from the chief of the division of geodesy to the chief of party, during the first month or two, touch upon most of the causes of trouble to the new leveler. The latter should follow the general instructions very carefully, both as to observing and computing.

Besides the chief, the party consists of five men or more, depending upon what means the party has for transportation and upon other circumstances of the work. For the actual leveling only five men at the most, besides the chief of party, are required. One man does the recording, 2 act as rodmen, 1 holds the sunshade and 1 the windshield. These men should be young and active and, preferably, should live in the general locality of the work. There is not very

much advantage in having men of previous experience, except in the case of the recorder. Men of the right kind will soon learn their duties. The recorder should have at least a high-school education, and the other men should have sufficient education to enable them to assist in checking the books and abstracts. The chief of party should replace any man as soon as he shows inaptitude for the work. The party must work as a team, and its rate of progress is dependent on each man.

INSTRUMENTS AND OUTFIT

The outfit of instruments and accessories furnished by the central office consists of the following:

Adding machine	1
Bench-mark tablets (additional ones to be ordered as required)	50
Compass, pocket	1
Foot pins, for rod support	2
Fountain pens, for recording purposes	2
Level, geodetic (including tripod)	1
Levels, hand (used when leveling along highway)	2
Rods, level, geodetic	2
Stamps, steel, set	1
Stationery case	1
Sunshade	1
Tape, pocket, 50-foot steel (with metric graduations on reverse side)	1
Typewriter	1
Watch (for recording purposes)	1

The necessary equipment, usually purchased by the chief of party, includes a post-hole digger, shovel, spade, blacksmith's hammer, carpenter's hammer, rock drill, and saw.

A windshield will also have to be provided to protect the instrument during strong winds. An extra sunshade to shield the tripod legs will be sufficient for moderate winds, but for strong winds a canvas mounted on a suitable framework will be necessary. When leveling along a railroad in windy weather, observers have found it advantageous to set the instrument on the ground at the leeward side of the track and thus have the lower part of the tripod protected from the wind by the track ballast.

Practically all the first-order leveling of the Coast and Geodetic Survey has been done along the railroads, as in general it is the more economical route, although in the last few years improved highways have been used to a certain extent where railroads were not available. Along railroads the spikes are used as turning points, thus obviating the necessity of establishing such points. The route to be followed is always given in the instructions issued to the chief of party, but should he find after arriving in the field that a change in the route would expedite the work, he should communicate such information to the director with a recommendation as to what changes should be made.

Each first-order leveling party will be provided with a truck. The truck has come into general use for transporting the party to and from the working grounds and for moving the equipment and baggage when headquarters are changed. There should be an experienced driver for the truck and he, as an extra man on the party, can be used to set bench marks and to perform other duties when the party is leveling.

For a number of years motor velocipede cars were used exclusively when the party worked along the railroad, the cars serving to transport the party to the working grounds and during the actual leveling operations. This was probably one of the best and most economical methods for operating a first-order leveling party that has ever been adopted, but in recent years it has become increasingly difficult to obtain the permission of the railroads to operate motor velocipede cars over their tracks without having an employee of the railroad in the party to act as flagman and to watch for approaching trains. The flagman adds considerably to the cost of the work, not only on account of his pay but also because the type of motor velocipede cars used will not conveniently accommodate the extra man.

The instructions issued to the chief of party at the beginning of the field season always specify what means of transportation will be used.

The members of a party may live either in a hotel or in a camp. The method of living is left almost entirely to the judgment of the chief of party. Many chiefs of party have found it most convenient and practicable to live in tents and obtain meals at a boarding house or hotel. If this method of living is decided upon, tents, cots, and bed tarps are provided for the men.

If tents are obtained from storage, each one should be carefully inspected and any needed repairs made. All weak ropes should be replaced. If the tents are old, it is advisable to have on hand a piece of canvas, sail needles, thread, and sewing palm for emergency repairs.

If new tents are to be purchased, they should be of the standard type of center-pole tent, specifications for which can be obtained from the office. These standard tents are more strongly reinforced than ordinary stock tents. The standard tent for leveling work is 9 by 9 feet, and it will comfortably hold two men or, in an emergency, three men. A center-pole tent can be pitched by one man even in very windy weather and will stand in a strong wind better than a ridgepole tent. The center pole itself is usually made in two or three sections held together by ferrules of brass tubing, thus making it easy to pack.

MANAGEMENT OF PARTY AND PROCEDURE IN FIELD

The management of the party and the procedure in the field are governed entirely by the route of the work and the means of transportation. The methods adapted to working along railroads are cov-

ered below in detail, but it will only be necessary in most cases to change the wording a little to make the same methods apply to leveling along a highway.

Instead of referring to the rods by their serial number, it is customary to designate one rod as the No. 1 rod and the man carrying it as the No. 1 rodman. The other rod is designated as the No. 2 rod and the man carrying it as the No. 2 rodman. The rods retain their respective designations throughout the season, but a man's designation may be changed from rodman No. 1 to rodman No. 2, and vice versa, at will. Rod No. 1 is always read first at each instrument station. The designations "No. 1 rod" and "No. 2 rod" should not be confused with the "front rod" and "back rod." One rod is always the "No. 1 rod," but it alternates from "front rod" to "back rod" or from "back rod" to "front rod" with successive instrument stations.

In starting the leveling from a bench mark the instrument is set up between the rails at a distance from the bench mark depending upon the observing conditions. The No. 1 rodman is sent to the bench. The No. 2 rodman goes ahead a distance equal to that between the bench mark and the instrument, where he makes a mark with chalk, keel, or some other material, around the spike on the right-hand rail, on which he holds the rod. While this is being done the observer levels his instrument.

In setting up the instrument the tripod should be placed with two legs in a line parallel to the track and the other at right angles to this line. In standing at the instrument looking at No. 1 rod the two legs should be next to the right-hand rail. Setting up in this way places the telescope in the best position for the observer in lifting the instrument to his shoulder after reading the No. 2 rod.

The leveling of the instrument is first done approximately with the small universal level attached to the side of the telescope, after which it is perfected with the level proper. The point on the head of the micrometer screw, called the reversing point, which indicates the required position of the screw to make the line of sight revolve in a horizontal plane when the instrument is leveled, is supposed to have been determined and marked with a soft pencil. In leveling the instrument at a station, the head of the micrometer screw is placed in that position and the telescope is turned at right angles to the railway track. The bubble of the universal level is then brought to the center, after which the adjustment in a plane perpendicular to the track is perfected with the leveling screw, which now lies under the eye end of the telescope, by bringing the bubble of the level proper to the middle of the tube. The telescope is then turned parallel to the track, directed toward the No. 1 rod, and the adjustment is perfected in that direction, using the two leveling screws that are in line parallel to the track. The telescope is now nearly in position for the

reading on the No. 1 rod, and, as stated above, the observer will be in position to lift the instrument when the No. 2 rod has been read. If this method of setting up the instrument is followed, the number of separate motions required by the observer and the time needed at a station will be reduced to a minimum. It need only be remembered that the tripod should be set with two legs in a line parallel to the track and next to the right-hand rail when looking at the No. 1 rod; that is, the rod upon which the first reading at the station is to be made. By following this practice there is less liability of reading the No. 2 rod first, since to do so would seem awkward to the observer.

The observer is now ready to make the rod readings. He sights at the No. 1 rod and reads the three cross wires, calling out the three readings to the recorder, thus: "Two three, four one; two one, two nine; one nine, one seven." Then, for a check, the first two figures of the middle wire are called out, thus: "Twenty-one." The check reading is called out loud enough to be heard by the rodman, who then turns his rod for the reading in feet on the back of the rod. (See note, p. 7.) The No. 1 rodman then takes his rod down, starts forward to the next turning point, and as he passes the observer he is told the number of rails that will be used for the next length of sight. Arriving at the No. 2 rodman, he starts counting off on the left-hand rail double the number of rails to be used as the length of sight and selects a spike on the right-hand rail opposite the joint on the left-hand rail where he has completed his count. After the No. 2 rodman has perfected the leveling of his rod, it is read and checked in a manner similar to that described above for the No. 1 rod. The recorder makes sure that there is no error in the readings and calls "check." The observer, or whoever is to carry the instrument, picks it up and proceeds to the next station.

The procedure at each instrument station is the same except that at the instrument stations of even number the No. 2 rodman goes ahead and establishes the new turning point for the next instrument station.

To guard against the harmful effects of the rodman shifting his position or of the observer moving about, the rodman should not stand on the tie holding the spike that is being used and the legs of the tripod of the instrument should not rest against a tie.

In connecting with the bench mark at the end of a section, it usually happens that the length of sight must be changed from what had been used regularly through the section. The rodman should count the rails from the last turning point to the bench mark and call back the number to the observer, who will then be able to place the instrument midway between the turning point and the mark; or if the sum of the lengths of the backsights for the section differs too much from that of the foresights, he can place the instrument in such position as to even up the two sums. (See par. 37, p. 8.)

When motor velocipede cars are used, they serve to transport the party not only to and from the working grounds but also during the actual leveling operations. The instrument can be mounted on one of the cars in the manner shown in Figures 5 and 6 or it can be carried on the car by the observer. One car is used by the observer, windshield man, and sunshade man, and the other car by the recorder and the two rodmen. The recorder's car should be fitted with a rack for carrying the rods. The method of moving the party from one instrument station to the next can be easily worked out by the chief of party. Although there probably is an advantage in having the observer's car ahead, it may be that the relative positions of the cars will have to be reversed should it be found that they have an influence on the accumulation of the "B–F." If the recorder's car is always behind the observer's car, the line of sight will have to pass over it on every backsight. The presence of the car near the line of sight is known to cause a slight refraction, and observers have found that by reversing the position of the cars on the forward and backward runnings of a section the B–F has shown a less inclination to accumulate. The line of sight should be kept as far away as possible from intervening objects, and the sunshade man should be extremely cautious in holding the sunshade to keep it away from the line of sight.

It is a distinct advantage for the observer to know where the permanent bench marks are located. These are usually set by some other member of the party, but the observer should have a copy of their descriptions so that he can regulate his sights when approaching them. The bench-mark setter should make a mark on the tie or rail opposite the bench mark in order that it can be found easily. A paint or other mark on a telegraph pole or on the end of a tie one set-up of the instrument to either side of a permanent bench mark is of assistance to the observing party

In working along highways the procedure is the same as when working along a railroad without motor velocipede cars except that there are no rails to regulate the lengths of the sights. The sights have to be regulated by pacing, and where the grades of the highway are not uniform an extra man is needed to pace distances and establish turning points. Locating the turning points at the proper elevation is accomplished by means of a hand level.

Each chief of party will use his own judgment as to the best method of moving his party and equipment from one town to another. When using motor velocipede cars the party can make the move on the cars and do some leveling en route, but the baggage and equipment not in actual use will have to be shipped forward by freight or express.

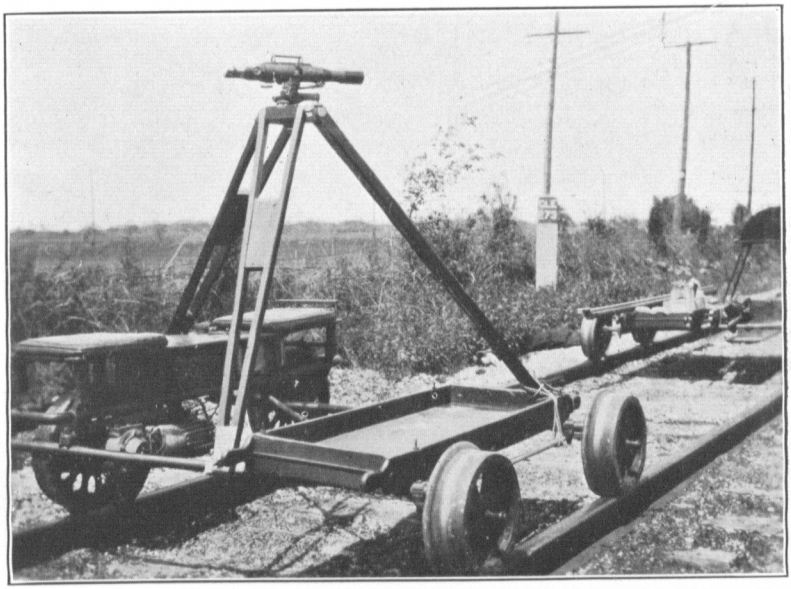

FIGURE 5.—LEVEL AND TRIPOD MOUNTED ON SIDE-DRIVE MOTOR VELOCIPEDE CAR

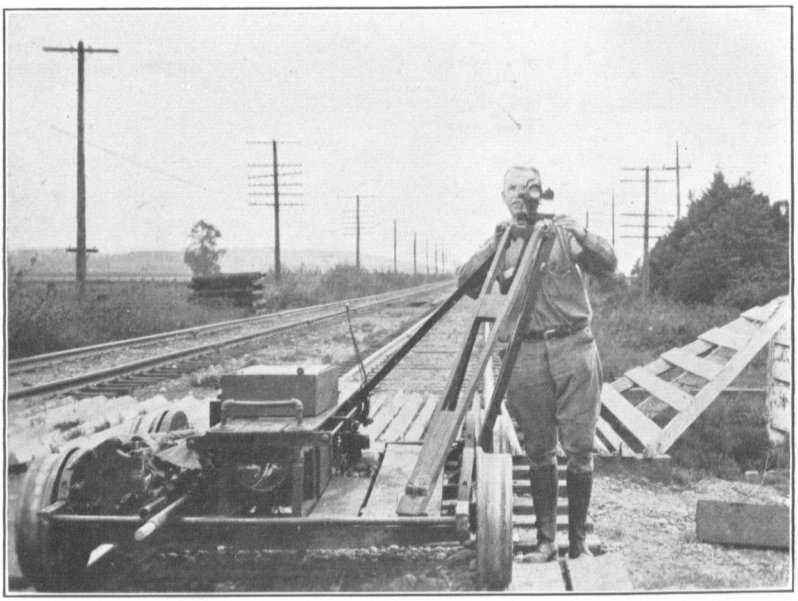

FIGURE 6.—LEVEL AND TRIPOD MOUNTED ON CENTER-DRIVE MOTOR VELOCIPEDE CAR

ERRORS OF LEVELING

The general instructions for first-order leveling by the United States Coast and Geodetic Survey, shown on pages 3 to 9, are designed to minimize or eliminate from a line of levels the effect of all known sources of accidental and systematic errors. It is the purpose of this section, however, to explain those paragraphs which are not considered to be in sufficient detail, to point out those errors which have been found most commonly to occur, and to suggest the precautions that observers should take to make their results of the required accuracy in the minimum time and at the lowest unit costs.

Many of the sources of errors in leveling of any kind are self-evident, and the observer will guard against them by properly adjusting his instrument and rods and by the use of firm rod supports.

MISTAKES OR BLUNDERS

Mistakes or blunders are especially to be guarded against, as they may cause great expense and annoyance on engineering projects before they are discovered. It may also be necessary to rerun the line to locate the error. Mistakes are usually caused by—

1. Reading a rod 1 meter or 1 decimeter wrong on both the forward and backward runnings of a section.
2. Undetected disturbance of a temporary bench mark.
3. The recorder reversing the backsights and foresights at a station on each of the two runnings of a section.
4. Not connecting a permanent bench mark with the line of levels until after the two runnings have been made.

A mistake in rod reading is most likely to occur at a decided change in grade or when changing at a bench mark from the forward to the backward running, or vice versa. The instructions provide that after the first running to a bench mark the second must not be immediately started back from that mark. While running on a long even grade both the observer and the recorder should be on the alert for a change in grade when the reading for the foresight and probably for the backsight also may fall on a different meter space on the rod from where it had been for a number of previous stations. A mistake of a whole meter is the only one likely to occur at the change of grade. The backs of the rods are graduated in feet (see note, p. 7), and by reading the feet at each station on at least one running it is believed that the probability of an undetected mistake of a meter or a decimeter is very small.

The instructions, paragraph 12 (p. 4), state that any temporary bench mark which is required to hold the elevation of the forward end of the line should have a check point at least one set-up of the instrument away. Both points are connected with when the leveling is advanced from them later, and any discrepancy between their

original and new differences in elevation is noted before proceeding. With this method any disturbance of the mark is detected.

The transposition of backsight and foresight will probably never remain undetected in Coast and Geodetic Survey leveling, for it is not customary on the second running of a section for the rods to be held at the same points as on the first running. Consequently, should such an error be made on each of the two runnings of a section, it is most improbable that the two errors would be so nearly the same that the two runnings would agree within the prescribed limit of divergence.

Paragraph 13 of the general instructions (p. 5) provides a method of tying in a bench mark to a completed line, which should prevent blunders. A mistake made in determining the elevation of a bench mark connected with a completed line will not affect the elevation as carried ahead, but it may cause serious trouble in any future leveling based on that mark.

ACCIDENTAL AND SYSTEMATIC ERRORS

With an accurately adjusted instrument of the type of the Coast and Geodetic Survey level, which is made of invar of a very low coefficient of thermal expansion, and with carefully adjusted rods, and firm rod supports there would seem to be no source of systematic error in the mean of the forward and backward runnings of a line of levels if the observing is done in accordance with the general instructions. The closing errors of the circuits show, however, that the systematic errors are not entirely eliminated, and the divergence of the backward and forward lines clearly indicates some systematic error in a single forward or backward line, though the mean of the two lines may be close to the truth.

In paragraph 24 of the general instructions (p. 7) and throughout this manual, where reference is made to the position of the level bubble during observations, the observer has been instructed to hold the bubble continuously in the middle of the tube. The difficulty of attaining this ideal position of the bubble, except with the expenditure of a large amount of time for the purpose, is apparent to any observer who has used a sensitive bubble under the conditions prevailing in our field work.

Although the bubble can easily be brought to the middle of the vial and held there, by means of the micrometer screw, until the beginning of the rod readings, it is often found to be slightly off center when the observer checks its position after making the rod readings. This creep of the bubble should not cause any concern so long as it is small. A change of 0.1 division in the position of the bubble will change the rod reading only 0.1 millimeter on a sight 100 meters in length. Since this creep may be in either direction, its effect is largely accidental.

It is not intended to suggest that leveling of the required accuracy can be obtained by deliberately keeping the bubble off center, but if there is a creep of the bubble during the observations the observer should know how much creep can be permitted before it is necessary to relevel and repeat the rod readings. If any relationship should develop between the bubble creep and the accumulation of B–F or the percentage of rerunning, the limits of the bubble creep should be chosen accordingly.

The accidental errors will be present under any method of operation, and other things being equal they will increase with the increase in the rapidity of the progress of the work beyond a certain normal amount. The observers in the United States Coast and Geodetic Survey are directed to do as much leveling as possible of a certain accuracy (as indicated by the agreement of the two separate measures of each section of a line) rather than a smaller amount of a greater accuracy. They make the lines of sight as long as the atmospheric conditions will permit, up to a maximum of 150 meters. The greatest sources of accidental errors are believed to be in estimating the tenths of the centimeter graduations on the rod and in the varying vertical refraction on the backsights and foresights. Another source of error (probably entirely accidental under the method of leveling prescribed) is the inertia of the liquid in the level vial. The observer should bring the bubble to rest at the center and then wait a few seconds to see whether or not it creeps to another position. If it moves away from the central position, it should be brought back before making the reading of the rod. An error, probably mostly accidental, may be caused by refocusing the rod image between the two sights at an instrument station. This probably changes the relation between the axis of the bubble and the line of collimation.

Paragraph 21 of the instructions refers to the accumulation of the discrepancy or the divergence between the elevations as carried by the forward and backward runnings of a line of levels. It is rarely the case that both runnings are made on the same day, but the interval of time between them is very seldom more than one week.

It has been our experience that early in a line of levels run by a new party the value of the discrepancy between the backward and forward measures (B–F) in millimeters per kilometer tends to be large, but after about the first 20 kilometers it begins to decrease. About 0.25 millimeter per kilometer, as the line progresses, is a good value, tending to decrease to the end of the line. Any noticeable tendency of the discrepancy to be of one sign is a thing to be guarded against, studied, and, if possible, counteracted.

There is no reason for the office computations to affect the value of the B–F systematically in either direction. The two principal corrections for rod length and for temperature are usually about

equal on the backward and forward lines and, therefore, do not affect the B–F.

The effects of the sun, the grade of the line, the average temperature, and the azimuth of the line on the accumulation of B–F are not known. Therefore the methods are designed to eliminate them from the observations or at least to reduce them by providing that the running of the backward and forward lines shall be under different conditions. Any rapid accumulation, in most cases, has been found to be due to the habits of the rodman or to the methods of supporting the rods.

When the accumulation is large the observer should make various experiments, such as (1) the occasional use of the foot pins for a few miles; (2) a change of rodmen; (3) requiring the rodmen to try various changes in their methods of locating and holding their rods and especially requiring them to be unusually careful in placing the rod on the highest point of the spike and in avoiding the placing of an excess of pressure on the rod by leaning on it or by bracing against wind pressure; (4) changing the program of running forward in the forenoon and backward in the afternoon to backward in the forenoon and forward in the afternoon. Then he should study the relation of the accumulated discrepency, first, to each of the above-mentioned changes; second, to the weather; third, to changes in the character of the ballast; fourth, to changes of grade of the railroad. By such studies the source of the trouble will probably be located or else the accumulation will cease without the reason for its doing so being discovered.

It is believed that the larger accumulated discrepancies can not possibly arise from any systematic error in reading the rods. Especially is this true in leveling such as that called for by the general instructions (par. 25, p. 7), where the backsight is read first at stations of odd numbers and the foresight first at the remaining stations. This makes it practically a symmetrical process in so far as the observer is concerned. (The primary purpose of this feature of the program is to eliminate from the computed elevations the effect of any systematic tendency of the instrument to rise or settle during observations and to balance out the effect of varying refraction.)

There is a possibility of an accumulated discrepancy being produced by refraction on lines having steep grades. If the conditions in regard to refraction be the same on the two runnings of a line, there would be no divergence from that cause; but it is probable that the refraction is different on the higher sight than the lower one and that this difference changes during the day as the relative temperatures of the ground and air vary. The refraction on the higher sight (up the slope) is no doubt different in the morning with a rising temperature from what it is in the afternoon with a falling temperature, while

the lower sight (down the slope) which comes well above the surface of the ground will not vary so much between the forenoon and afternoon. Consequently, if the observer systematically leveled forward early in the day and backward late in the day, refraction might cause an accumulated discrepancy. If it does, there should be a change of sign in the discrepancy when the program is reversed by leveling backward in the morning and forward in the afternoon. Also, without a change of program a change of sign should occur after passing a summit or the lowest point in a valley.

It is difficult to see how a large accumulated discrepancy could be caused by errors due to a difference in the appearance of the rods, one being more brightly illuminated than the other, but if such is the case a change in the program of forward in the morning and backward in the afternoon to forward in the afternoon and backward in the morning would tend to change the sign. The discrepancy, if due to the appearance of the rods, would not increase on portions of a line run in both directions on cloudy days, and the change of program would probably not be effective on a line running north and south.

A test similar to that mentioned above might be made to show whether the accumulation is due to the tendency of the bubble to crawl toward the source of heat—the sun.

Among the cases of large accumulations of discrepancy between the two runnings of a line are: (1) A section of the line from Ogden, Utah, to Pocatello, Idaho, where a discrepancy of 63.4 millimeters occurred on 138.4 kilometers, a rate of 0.46 millimeter per kilometer. (2) On the line Red Desert to Azusa, Wyo., there was an accumulated discrepancy of 89.1 millimeters on 170.4 kilometers, or 0.52 millimeter per kilometer.

If there is a systematic error due to the direction of running which causes the accumulated discrepancy, it is probable that this error, with different signs in the two runnings, is practically eliminated from the mean of the two runnings.

If there is a systematic error in a line, due to its having been run in the forenoon, which is of the same size but of different sign from a systematic error in the same line, caused by its having been run in the afternoon, then, if there are no other sources of error, the two lines will differ by twice the systematic error in the single line, but the mean will be free from this error. Now, if one-half of the above line is measured forward in the forenoon and the other half forward in the afternoon, and if the backward running is made half in the forenoon and half in the afternoon, then the difference in elevation of the ends of the line from each of the two runnings will be free from systematic error and they will give the same difference in elevation for the two ends of the line. It seems to be reasonably certain that, in so far as the errors due to forenoon and afternoon runnings and to the direc-

tion in which the line is run are concerned, the amount of the accumulated discrepancy is not an indication of the size of the systematic error which may be present in the line, for the systematic errors in each of the backward and the forward runnings due to these causes may be made to accumulate or eliminate themselves by a change of program without affecting the difference in elevation as given by the mean of the two lines.

In making certain astronomic observations where extreme accuracy is required it has been found that most observers have a personal equation in bisecting a star with the wire. This may be called the "bisection error." This error is eliminated from the results by means of a reversing prism, which if turned 90° reverses the image formed on the retina of the eye, and one-half the observations are made in each of the two positions. It is no doubt true that there is a "reversing error" for some observers in reading the fractions of a division of the level rod, but as the same error would presumably be made on both the front and back rods these errors would not come into the line of levels. It may be the case, however, that the "bisection error" on a rod illuminated by the direct rays of the sun is different from that on a rod the face of which is in shadow. It is possible that this phenomenon is one of the causes or the cause of the accumulation of the difference (B–F) between the forward and backward runnings of a line when one of the runnings has been made entirely toward the sun, while the other has been made in a direction away from the sun. On an east and west line the effect of this error (if it exists) will probably be eliminated from the mean of the two runnings, but this would not be the case on a line which is nearer north and south than east and west. It may be desirable where extreme accuracy is required to have a reversing prism on the level and to have readings made on the rod in both positions of the prism.

After an observer has taken every known precaution to eliminate the effect of errors on the elevations of a line it will usually be found that the closing error of the loop or loops involving the line is greater than can be attributed to the purely accidental errors. There must be systematic errors in the leveling which affect the mean of the forward and backward runnings, although the instrument may be in perfect adjustment, the constants of the rods accurately determined, the rod supports entirely satisfactory, and the two runnings made on different days and under different atmospheric conditions as far as rising and falling temperatures are concerned.

ERRORS DUE TO ATMOSPHERIC CONDITIONS

There is usually some systematic error in a single running of a line, the size and sign of which probably depend upon the direction of the running with respect to the sun, the time of day, or the movement of

the air or possibly upon all three of these causes combined. Observers have reported that the two runnings of a section on a clear day or days agree more closely when the wind has a decided velocity than when the wind is very light or it is calm. This is probably due to the fact that when the air is in motion the temperature is more likely to be uniform throughout the layers through which the line of sight passes, and the refraction should be more nearly the same on the back and forward sights.

The errors outside of the instrument and the observer which may affect a single running in a systematic manner can be made to act as accidental errors by running the second measure under the opposite conditions. For instance, if the first running has been made in the forenoon with a rising temperature and with the sun to the eastward, the second measure should be made in the afternoon with a falling temperature and with the sun to the westward. This does not apply to cloudy days, when it is believed the systematic errors due to atmospheric conditions are negligible.

It is possible that there is an error in the mean of the two measures of a section where one running is made on a clear and the other on a cloudy day, but it would be difficult to show that this is true, for any systematic error on a short section of a line (say 1 kilometer) is very small compared with the accidental errors.

SYSTEMATIC ERRORS ON SLOPES

It is believed that systematic errors are more likely to occur on steep grades if the observing is done at times when the surface of the ground and the air have different temperatures. If the air has a uniform temperature, or temperatures varying uniformly with the height above sea level, then the density of any layer of air at a certain elevation above sea level will be the same throughout the layer. The effect of barometric pressure and the hygrometric condition of the atmosphere at different points within the distance considered (300 meters between the two rods) may be considered as having no effect in causing a different density in the layer. Under this condition of uniform temperature, or temperatures varying uniformly with regard to elevation above sea level, there will be the same amount of refraction on the backsights and foresights when these are of equal length. But when the air and the surface of the ground have different temperatures, the layer of air in contact with the surface will assume a temperature nearly that of the ground, and consequently the density of the air will tend to vary with its distance from the surface of the ground instead of with the elevation above sea level. (This will, of course, be true for only a short distance from the ground, say 1 or 2 meters.) If the ground is level there will be no difference in the amount of refraction on the two sights, but if the ground is sloping the sight up the

slope will pass through layers of air of different densities, while the lower sight will go through air in a normal state so far as the arrangement of its densities is concerned. The effect of the refraction on the rod reading on the upper sight will be of one sign if the surface of the ground is colder than the air, and the opposite sign if the surface is warmer than the air. These errors are no doubt smaller when the wind is blowing and the air stirred. On cloudy days the temperature of the air and the surface of the ground are supposed to be more nearly the same, and consequently the refraction error on the slope would not be so large as on a clear day. The refraction error on a clear day should be at a minimum during the several hours in the middle of the day, when the temperatures of the air and the surface of the ground are nearly the same, and greatest in the morning and the late afternoon, but should have opposite signs during those two periods.

As stated above, this refraction error should be entirely absent if the ground is level or nearly so and the backsights and foresights at a station are equal in length. It should reach a maximum on steep grades where the line of sight up the slope comes very close to the bottom of the rod. This error may be made small by never letting the line of sight come near the ground. It is impossible to predict what the safe height is, but it is believed that if the lower one of the three wires reads more than 30 centimeters above the ground most of the trouble will be avoided.

BENCH MARKS

PERMANENT BENCH MARKS

The value of a line of levels depends upon the distribution and permanency of the bench marks. The bench marks are the only visible evidence of the work, and the necessary time and money should be spent to perpetuate the line by making the marks of a substantial character and by placing them in locations where they will be free from the usual causes of disturbance.

Railroad bridges and culverts are not, as a rule, entirely satisfactory places to set permanent bench marks. These structures should be used, however, as they afford locations that are very accessible, and the bench marks so placed are easily found. In determining the frequency of bench marks to meet the requirements of paragraph 2 of the general instructions (p. 3), these bench marks should be given a weight of only one-third that of bench marks set in more favorable locations.

Only substantial buildings should be selected upon which to place tablets. Business buildings, even when substantially built, are usually altered or removed within a period of from 15 to 25

years. Public buildings, such as schools and courthouses, usually have a longer tenure of life. The marks should not be placed on steps which are not supported on a foundation extending below the frost line. They should preferably be placed in a part of the foundation or on the water table of the foundation where the elevation will be as permanent as possible. The mark may be set either in a vertical or a horizontal surface. When set in a vertical surface it is probably less liable to be disturbed, but it does not afford so convenient a setting for the rod. In any case the tablet should be countersunk in the stone and well leaded or cemented in. If cemented in a vertical surface, care should be taken that the tablet, because of its weight, does not loosen itself before the cement has firmly set. When setting a tablet on a railroad depot it should be placed in the wall near the entrance to the ticket office, where it will be conspicuous and easily discovered by the railroad surveyors.

In setting the tablet in rock outcrop care should be taken that the rock in which it is set is hard and that it is a part of the main ledge, not a detached fragment. The tablet should be countersunk and well cemented in.

When a tablet is set in a bowlder the latter should be of durable material and of cross section, area, and depth below the surface not less than the standard concrete mark described on page 30.

Where posts must be set for the mark the tablet will usually be set in the top of the standard concrete mark described on page 30. Occasionally, however, the availability of stone posts will recommend their use, especially as they may prove more durable than the concrete. When stone posts are used they should be not less than 8 by 8 inches in cross section and extend to the depth specified for the standard concrete mark. In addition, the base of the post should be embedded in concrete at least 6 inches in depth.

Concrete or stone posts should always be set in fairly level ground. Steep slopes should be avoided, for they are subject to a sliding of the surface material after rains or as a result of frost action. In setting the posts along railroads they should be placed near the edge of the right of way far enough from the tracks to reduce to a minimum their liability of being destroyed by new track construction and the straightening of curves.

In a city or town it is best to set the posts in the ground around schools and churches. A corner of a cemetery is a particularly good spot, as cemeteries generally have a long existence. In every case the bench mark should be set near a topographic feature or some other object to which it can be referred for description.

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STANDARD CONCRETE BENCH MARKS

(1) **Shape.**—The bench mark shall be larger at the base than at the surface of the ground. It may take the form of the frustum of a pyramid or cone or be a post with an enlarged base. If it is built in the form of a frustum, there should be a batter of not less than 1 inch to 1 foot. It is believed that a post with an enlarged base will offer just as great resistance to the lifting effect of freezing and thawing as a conical post and will be more economical of concrete. The base should be not less than 4 inches larger in horizontal cross section than the post itself, and the enlarged portion should be at least 6 inches thick vertically. Care should be taken that the outside edges of the base are not rounded, so as to prevent any change in elevation due to a sifting in of sand particles or dirt around the edges during a thawing period. The portion of the mark above the frost line should be molded to lessen the lifting effect of the surface layers of the ground when freezing. These molds may be of sheet iron or wood and should have a slope of at least 1 inch to a foot. The sheet iron may be cut to proper size and shape, bent for a lock seam, and carried flat until needed.

Extreme care should be taken that the molded portion of the concrete mark has no projecting corners, which would increase the lifting effect of freezing. The concrete beneath the mold should have a sufficiently large area that no portion of the mold extends outside the concrete on which it rests.

(2) **Material.**—The main considerations in making concrete are to have clean materials, mix them well before adding water, have the mixture not too wet, and tamp well into the form. Each streak of dirt in concrete means a line of cleavage. Where rough aggregate is available, the proportions may very well vary from 1–2–4 to 1–3–5, but the top 12 inches of the mark should be of considerably richer mixture. Where only cement and sand are available, the lower part of the mark should be proportioned 1 part of cement to 3 parts of sand, and the upper part should be 1 part of cement to 2 parts of sand.

With a mark of the proper size it will not be necessary to reinforce the concrete with metal rods or wire. To avoid cracking of the concrete, due to rapid drying, it should be covered with paper or cloth and then with earth or other material for a period of at least 48 hours.

(3) **Size and depth.**—Where lack of transportation does not make the cost prohibitive, the post should be not less than 12 inches in least cross-section dimension except at the top, where it is shaped by the mold, and that should not be less than 10 inches. The top should extend from 2 to 4 inches above the surface. It should not be placed in a line of traffic.

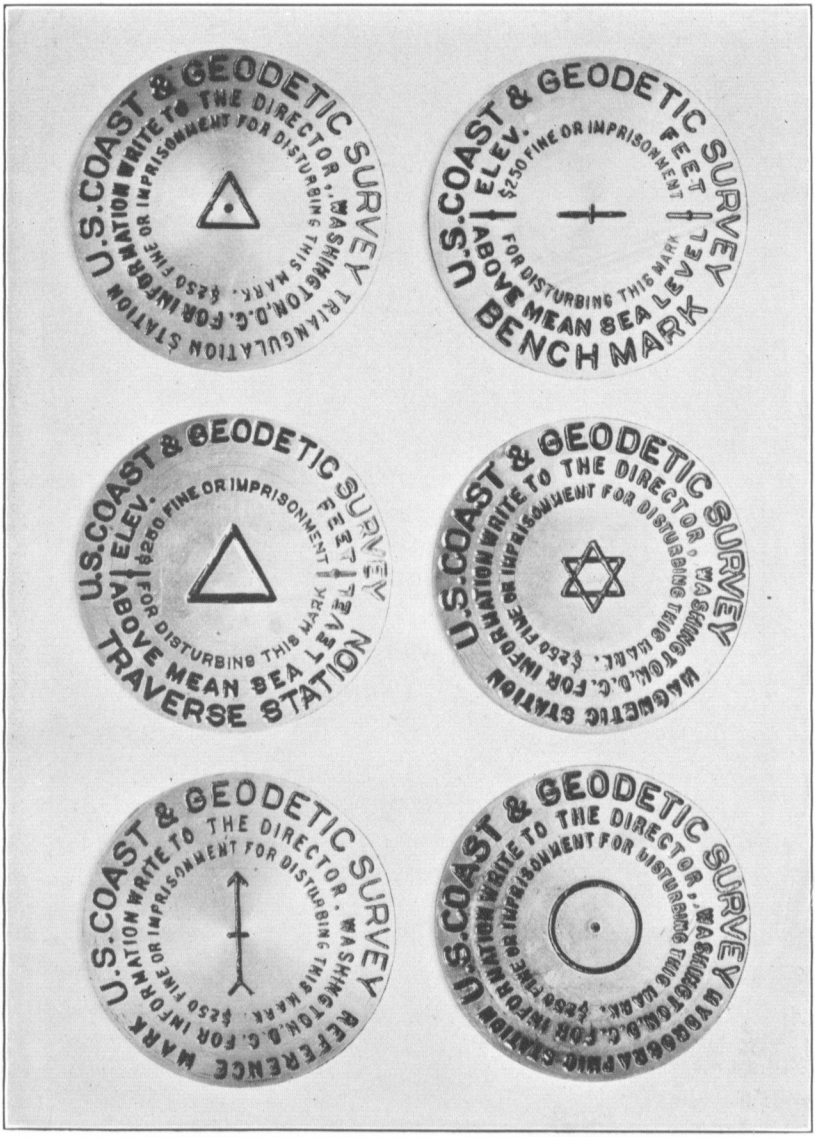


FIGURE 7.—STANDARD MARKS OF THE UNITED STATES COAST AND GEODETIC SURVEY

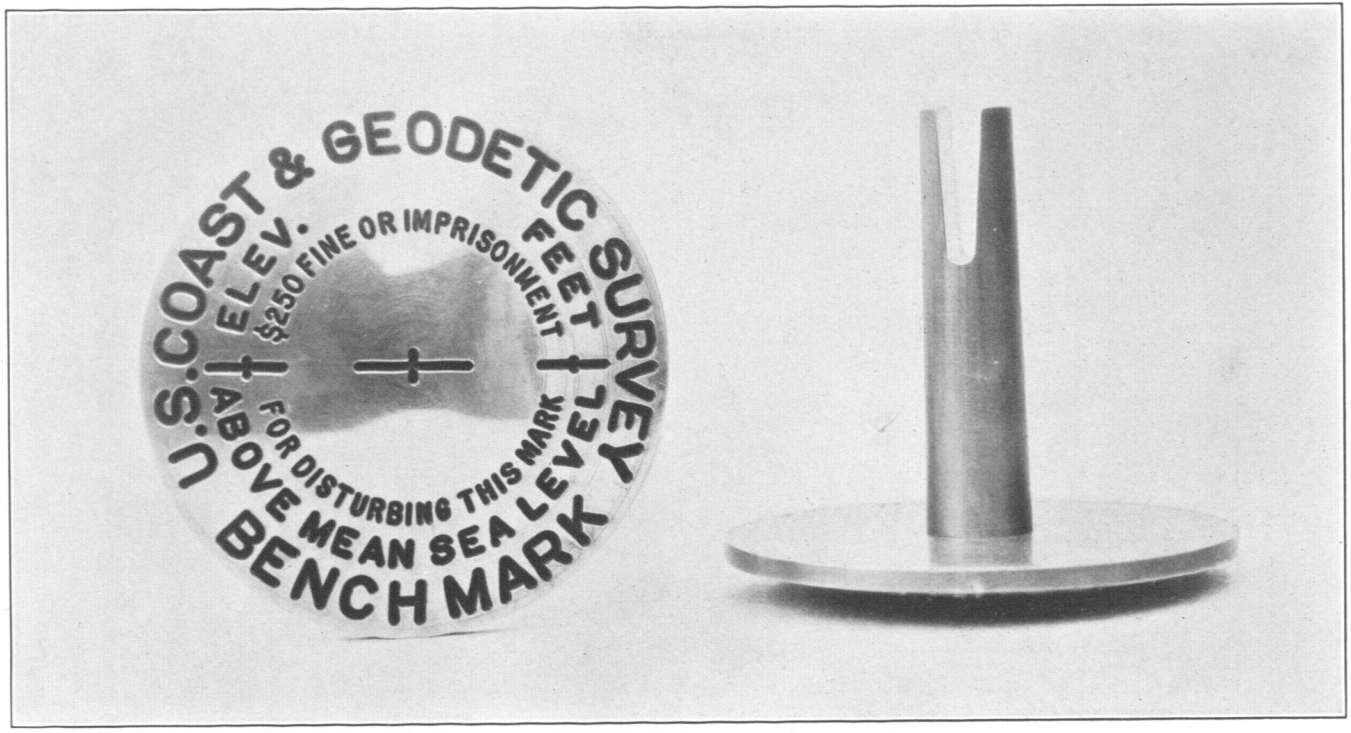


FIGURE 8.—BENCH-MARK TABLET

The marks should extend below frost depth. except where the frost penetrates to a greater depth than 4 feet. The minimum depth of any bench mark should be 36 inches below the surface, and the maximum depth should be 48 inches. It is believed that with a mushroom base a mark extending 48 inches below the surface will not be elevated by the action of frost, even though frost should extend below that depth. The instructions to the chief of the leveling party will in all cases specify the depth to which bench marks should be set.

SUPPLEMENTARY BENCH MARKS

As stated in paragraph 4 of the general instructions (p. 4), each line of levels is to be broken into sections from 1 to 2 kilometers in length. The ends of these sections, when not marked by permanent bench marks, are to be marked by supplementary bench marks consisting of spikes in poles, bolts on bridges, or chisel marks on masonry.

Railroad spikes have been in general use for supplementary bench marks, but it is believed that a lag screw makes a much better mark. It can be easily identified, especially if the letters "C. & G. S." are stamped on it, and it has an exceptional permanency when driven in a milepost, telegraph pole, or tree.

Chisel marks on masonry should be in the form of square cuts, outlined squares, or crosses, cut sufficiently deep into the stone or concrete to insure their being easily found for a number of years.

DESCRIPTIONS OF BENCH MARKS

PERMANENT BENCH MARKS

The descriptions of the permanent bench marks are of the greatest importance, for it is only by means of them that the marks can be recovered by the surveyor or engineer.

The descriptions should be clear, concise, and complete. The first part should enable one to go with certainty to the immediate vicinity of the mark, while the latter part, the detailed description, by its measured distances to reference points and its description of the character of the mark must inform the searcher of the exact location of the mark and make its identification certain.

The original description should be written in the observation record book while the party is at the bench mark. These notes are then to be transferred to Form 638, on a typewriter if practicable. A single copy only need be sent to the office. The form, after being completely filled out, should be read over carefully to see that there are no reversed directions and that no part of the description is vague, ambiguous, or erroneous. Form 638 is so arranged that, if properly filled out, all the essentials of a good description will be present.

The distance and direction from the nearest town on the railroad or highway should be given in miles and decimals of a mile. The

distances in the detailed description of location are to be measured and given in feet. The equivalent distances in meters are not necessary.

The direction of the bench mark from the track, highway, or other reference object or point must be a compass direction, a pocket compass being carried for this purpose, but not more than eight points of the compass need be used. The terms "right" or "left" are never to be used. The word "opposite" is also confusing. It is better to use the expressions "across the track (or highway) from" or "on the same side of the track (or highway) as."

In describing the character of the mark the expressions "tablet in building" and "tablet in top of concrete (or stone) post," should be used. Not only the character of the mark but also where it is set should be stated, as it is very often impossible from the detailed description of the location, especially if the references are to building corners, to determine if the tablet is on the building or in a post near the building.

RECOVERED BENCH MARKS

A complete description on Form 638 must be furnished of all bench marks of other organizations with which the work is connected. The description should state clearly the name of the organization which established the mark, give any letters or numbers found stamped on the mark or on the surface around the mark, and state exactly the character of the mark, whether it is a square cut, chisel mark, iron post, or tablet, etc. A statement should also be made in regard to the condition of the mark and whether there is any evidence of its having been disturbed or moved from its original position.

Recovered bench marks of the United States Coast and Geodetic Survey are also to be described on Form 638. If the bench mark is found as described, only a skeleton description sufficient to positively identify it need be supplied, but in every case the present condition of the mark should be stated. If the description furnished no longer fits the facts, a complete new description should be written, with such parts of the old description repeated as still state the facts. It will not be sufficient to state that the mark was found as described and then note the exceptions to the furnished description. If there are any changes required in the description, a complete new description should be written.

SUPPLEMENTARY BENCH MARKS

Supplementary bench marks need be described only when there is a possibility of their being recoverable for a number of years and when they are so placed that a good description is possible. These descriptions should be furnished on Form 638, and they are to be as

complete as those for the permanent bench marks. The numbered designation of the supplementary bench mark will insure against its being treated as a permanent bench mark.

INSTRUCTIONS FOR RIVER CROSSINGS

With the extension of first-order level lines along rivers and over routes not generally used for this class of work, there has arisen the need for an accurate and economical method for carrying the lines across rivers and other spaces where ordinary lengths of sights are not sufficient.

The method described below is one that was devised when it became necessary to extend a line of levels across the Mississippi River in the vicinity of New Orleans, La. The river is about 1,200 meters

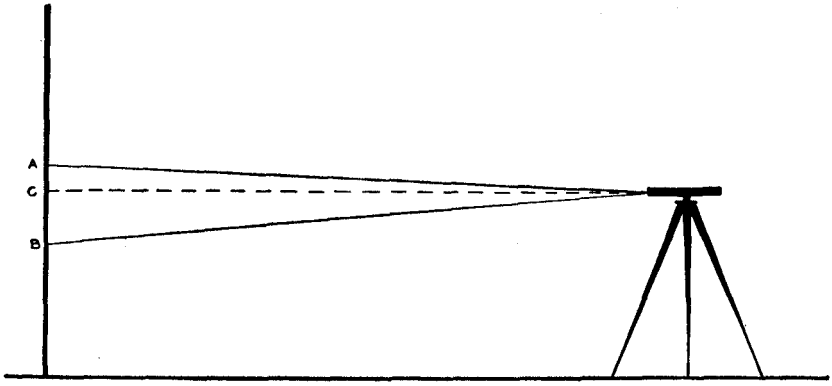


FIGURE 9.—River crossing, principle of the method

wide at that point, with no bridge across it. The method has been used a number of times since with very satisfactory results.

In this method two fixed targets are used. It is essentially a modified form of vertical angle method of leveling into which the absolute values of the angles do not enter.

The general scheme of this method of observing is that simultaneous reciprocal observations are made at each of the two points on the opposite sides of the river. Instead of moving a single target into the line of sight, two stationary targets are set on each rod, one above and the other below the line of sight when the bubble is in the middle of the tube. The micrometer head is read with the telescope in three positions: First, with the middle wire bisecting the top target; second, with the telescope level; and third, with the middle wire bisecting the bottom target. The reading on the rod given by the line of sight when the telescope is level can be determined, for the distances from this point to the two targets will be proportional to the corresponding differences in the readings on the micrometer head.

Figure 9 illustrates the principle of the method. Let A and B be the positions of the upper and lower targets on the rod and let C be the point where the line of sight intersects the rod when the telescope is level. Let the reading of the micrometer head, when the telescope is pointed at A and B, be 25.8 and 40.2, respectively, and let the reading of the micrometer head, when the telescope is level, be 30.6. Then, if the targets are at a distance of 0.4000 meter apart, the distance from target A to the point of intersection of the line of sight when the telescope is level will be $\frac{30.6-25.8}{40.2-25.8}(0.4000)=0.1333$ meter. If the setting of target A is 1.8930 meters, then the line of sight intersects the rod at 1.7597 meters.

The targets are made of galvanized iron 12 inches long and 6 inches wide, and are painted black with a horizontal white band 1 inch wide across the center. Figure 10 shows a front view and Figure 11 a back view of the targets. These targets can be requisitioned from the office.

Each rod, with its targets, is guyed into place over the bench mark. The targets are set one above and one below the line of sight, after the approximate intersection of the line of sight with the rod has been obtained, upon signal from the observer, by moving one of the targets along the rod. Figure 12 shows a rod with the targets in position. They should be spaced so as to require a turning of 15 to 25 divisions of the micrometer head to change the pointing of the instrument from the lower to the upper target.

The position of the targets on the rods can be obtained by stepping off the distance from a known graduation to the center of each target with a pair of dividers and then stepping off the same amounts on a centimeter scale or the rod graduation itself.

The observations should be made in accordance with the following instructions:

Read the near rod once for each set of 25 observations on the distant rod.

Set the middle wire of the diaphragm on the top target and read the micrometer head.

Read the micrometer head with the telescope level.

Set the middle wire of the diaphragm on the bottom target and read the micrometer head.

The three readings of the micrometer head are to be repeated in the above order until 25 of each are obtained. The observers are then to change places, taking their instruments with them but leaving the tripods in place.

Four sets of 25 observations each (two sets for each observer at each station) constitute a day's observing. These sets should be obtained in the following order:

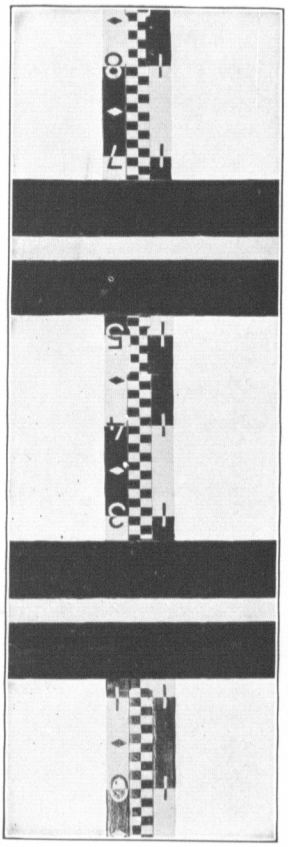


FIGURE 10.—TARGETS FOR RIVER CROSSING, FRONT VIEW

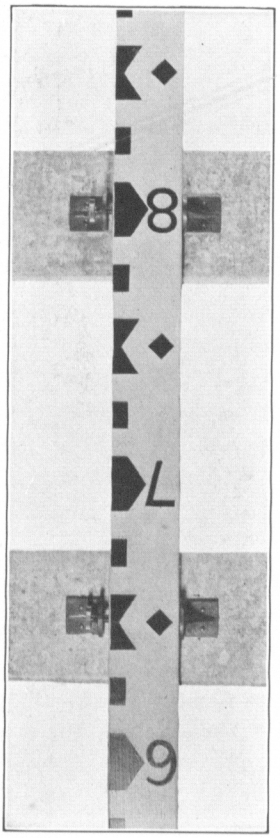


FIGURE 11.—TARGETS FOR RIVER CROSSING, BACK VIEW

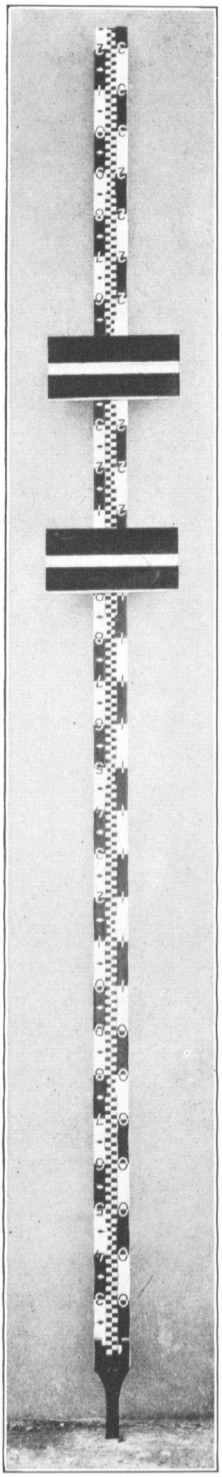


FIGURE 12.—ROD, WITH TARGETS FOR RIVER CROSSING IN POSITION

Let A and B be the two stations and numbers 1 and 2 the observers. Observer No. 1 will take a set of readings at station A, transport his instrument to station B, take two sets of readings, return to station A and take one set of readings. Observer No. 2, of course, will carry out a similar program, starting at station B.

The instrument should be adjusted as perfectly as possible and the program of observing arranged to avoid all unnecessary changing of focus; that is, at the first station read the near rod, before taking the first set of observations and, at the second station, make the near-rod reading between the two sets of observations. The day's program for each observer will then be: Read the near rod, take set of observations, go to the second station, take set of observations, read near rod, take set of observations, return to the first station, take set of observations, read near rod.

These instructions as outlined are designed to cover the case where both men observe with practically the same rapidity. Observing should begin and end at each station at the same instant, and if it is found that the observers do not work with equal rapidity the observing should start at the same time and continue until the slower observer has 25 sets of readings, both observers stopping at the same time although they do not have an equal number of sets.

It is believed that a very accurate determination of the difference in elevation can be obtained from the results of two days of observations. If, for any reason, the results of the two days differ by an amount greater than specified in special instructions, observations should be made on a third day.

The success of the method, like that of all reciprocal leveling, depends to a large extent on the equality of the refraction over the line in opposite directions. To secure this equality, the conditions on the two sides of the river should be as nearly the same as possible, with the line of sight at least 10 feet above the surface of the water.

INSTRUCTIONS FOR STAMPING ELEVATIONS ON BENCH MARKS

The figures giving the elevation of the bench mark are not stamped on the tablet until after the office computation of the field results has been made and the adjusted elevation of each bench mark has been determined.

The stamping will be carried out in accordance with the following instructions:

Visit the site of each bench mark for which you have a description.

Stamp on each Coast and Geodetic Survey bench mark, where the character of the mark permits, the elevation of the mark in feet to

the nearest hundredths of a foot, as given in the description. The selection of the nearest hundredths of a foot is to be made as follows:

Rounding off extra decimal place in stamping elevations

Elevation	Stamp
642.835	642.84
642.845	642.84
1,926.736	1,926.74
2,043.202	2,043.20
1,871.199	1,871.20

That is, if the digit in the second decimal place is odd and followed by a 5 in the third decimal place, increase it by 1. If the digit in the second decimal place is even and followed by a 5 in the third decimal place, use it as it stands. If the digit in the third decimal place is greater than 5, always increase by 1 the digit in the second decimal place.

While at the mark and after the elevation has been stamped on it, fill out the form provided for bench-mark recovery notes. This form is to be filled out for each bench mark visited, whether the elevation is stamped on it or not. Copies of this form can be obtained from the office.

Bench marks of the United States Geological Survey, with which connection has been made, should be stamped with the adjusted elevation to tenths of a foot. If the mark has been stamped with an elevation which does not agree with the one on your list, the elevation should be obliterated, or corrected, if it can be done clearly. The designation should be stamped on the mark in cases where it has been omitted. In stamping, care should be taken not to disturb the mark by the hammer blows.

If the tablet of a bench mark has become loose, it should be cemented back into place. It is believed that the mark can be reset in its original position within the limits of error of precise leveling.

A loose concrete or stone post, if still upright, should be preserved by filling in around it. If the post is leaning, it should be destroyed. In every case where it appears that the mark has been disturbed in such a way as to change its original elevation by as much as 0.05 foot the mark should be destroyed.

A copy of the mark as stamped is desired and should be made as follows: Paint the top of the disk with a thin coating of printer's ink, being careful not to fill the indentations, or paint the disk by using a stamp pad and stamp-pad ink; place a piece of blotting paper on it and rub the top of the paper with a burnisher, using considerable pressure.

The copies of the top of the tablets so obtained are to be sent to the chief, division of geodesy, at the end of each day's work. They

are to be placed in an envelope and forwarded without transmitting letter.

In filling out the recovery cards the following information should be supplied in the spaces provided:

1. The designation of the mark as given in the furnished description; your initials; date; State in which the mark is located.
2. Letters and numbers, other than the usual legend, found on the mark.
3. Nearest town to the bench mark and the county in which the town (not the bench mark) is located.
4. Distance and direction from nearest town along the route of the levels.
5. If the furnished description is sufficient, so state; otherwise, write an entirely new description.
6. State whether the mark is a stone or concrete post, iron pipe, disk, square cut, or outlined square. The exact point to which the elevation refers should be described; for example, disk in top of stone post or square cut in top of stone post.
7. Condition of mark as actually found. Does the mark appear stable? May it have been disturbed since its establishment? Is the disk loose? State exactly what means, if any, were taken to preserve the mark.
8. On bench marks of the United States Geological Survey, where it is necessary to change the stamped elevation, state exactly what you did.

Any additional remarks may be written on the back of the form.

Should conditions arise which are not anticipated by these instructions, the office should be communicated with by telegram or letter.

SEASON'S REPORT

After the completion of a line of levels the chief of party should submit a season's report, covering in more or less detail the following points:

1. Purpose and scope of work, as shown by excerpts from instructions or by a condensed statement.
2. Locality, with description of such topographic and climatic and transportation conditions as affected organization or progress of party.
3. Organization of party—personnel, equipment, transportation, etc.
4. Chronology of progress of work.
5. Progress sketch and statistics of field work (Form 21).
6. Statement of costs.
7. Recommendations, or special features.

The report should be clearly written, logically arranged, and concise. It may vary in length from a 2-page report covering a routine piece of work to a 15 or 20 page report, with photographs illustrating phases of the season's activities when those activities are diversified. Attention should be called to all matters connected with the work which, in the judgment of the chief of party, will help others to execute leveling at smaller unit costs in time and money. Even a short report, however, will necessarily embrace the features listed in the outline above, although the text relating to some one or more of them might be condensed to a single paragraph or sentence.

The progress sketch should show the route of the work and the towns passed through, but it is not necessary to indicate the location of the bench marks. Parallels of latitude and meridians of longitude should be shown on the sketch. Sections of one or more State maps of convenient scale may be used for the basis of the sketch.

A statement of costs of a project is an essential part of any report upon it. In order to secure uniformity and to insure that the statement shall contain all of the necessary data, it is suggested that the forms given below be followed.

Truck statistics

Reo truck No. 47:	
Season's mileage	6, 531
Cost of operating truck, including depreciation	$762. 90
Cost per mile	0. 117

Cost data

Total miles of levels	359
Total number of permanent bench marks	155
Party expenses	$4, 094. 69
Officers' salaries (including amount for prorated leave)	1, 000. 00
Truck depreciation	490. 82
Total	5, 585. 51

	Party expenses	Officers' salaries	Truck depreciation	Total
Cost per mile	$11. 41	$2. 78	$1. 37	$15. 56
Cost per bench mark	26. 42	6. 45	3. 17	36. 04

FIELD RECORDS AND COMPUTATIONS

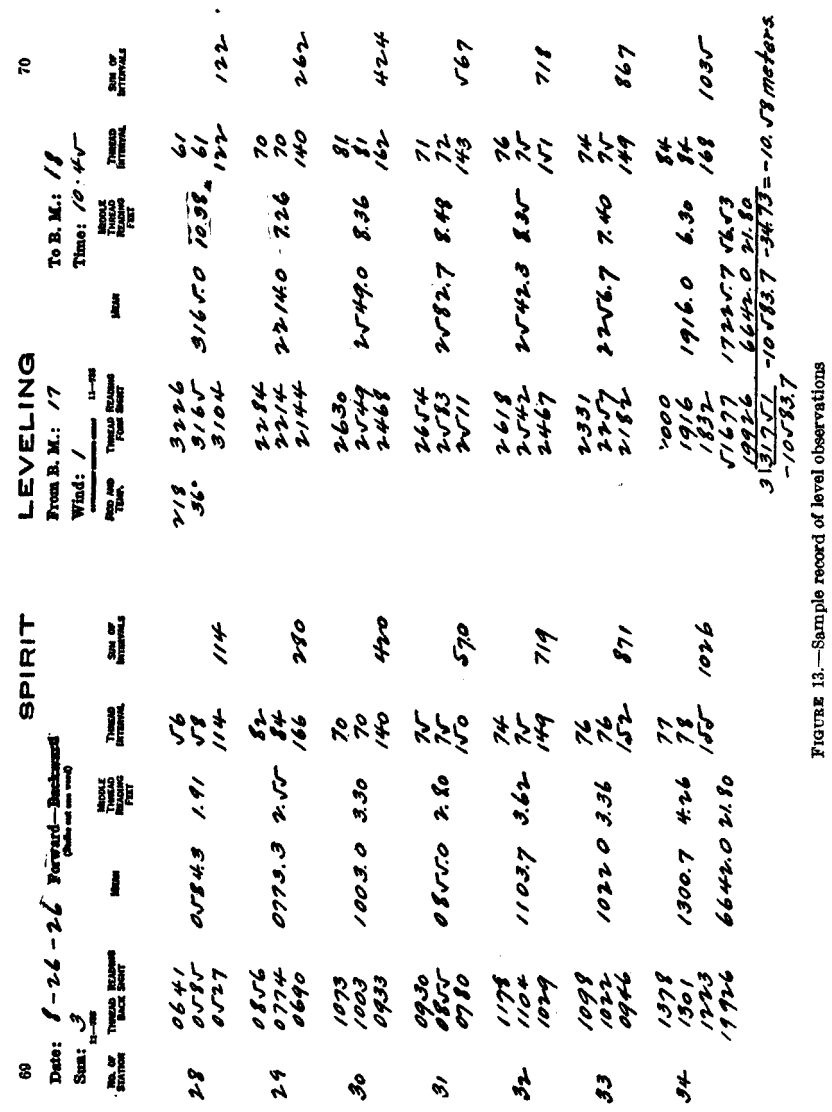
All observations for first-order leveling are recorded in Form 257, Spirit-Level Observations.

The following examples of the record will serve to illustrate the method of recording and also to explain some details of the method of observation. The record should be made in ink and should be as neat and clear as possible. In cold and rainy weather when a fountain pen does not work satisfactorily it is permissible to use a hard-lead pencil. No erasures are to be made. If a figure is found to be in error, it must be crossed out and the correct figure written above.

Figure 13 shows the record of leveling between bench marks B. M. 17 and B. M. 18. The top of the right-hand page should be filled out to indicate the direction of progress of the work, regardless of whether the running is a backward or forward one—that is, if the line runs from B. M. 17 to B. M. 18, as in the example, it should be so recorded, and if the running is in the direction of progress of the entire line of levels, it is recorded as a forward running, the word "backward" at the top of the left-hand page being struck out. The backward running of this section would be designated "B. M. 18 to B. M. 17,"

with the word "forward" struck out. On spur lines the forward running is always the one toward the free end of the spur.

The explanation of the numbers used after the words "sun" and "wind" is given at the bottom of Form 45. (See fig. 15.) The unit in the columns headed "Thread reading," "Mean," "Thread interval," and "Sum of intervals" is the millimeter. The unit in the col-



69 SPIRIT LEVELING 70

Date: 8-26-26 Forward—Backward (Strike out one word)

Sun: 3 (1-10)

From B. M.: 17 To B. M.: 18

Wind: 1 (1-10)

Time: 10·45

No. of Station	Thread Reading Back Sight	Thread Reading Forward Sight	Mean	Middle Thread Reading Feet	Thread Interval	Sum of Intervals	Pro and Tem.	Thread Reading Foresight	Mean	Middle Thread Reading Feet	Thread Interval	Sum of Intervals
							718	3226			61	
							36°	3165			61	
28	0641		0584.3	1.91	56			3104	3165.0	10.58	122	122
	0585				58							
	0527				114	114						
	0856				52			2284			70	
29	0774		0775.3	2.55	84			2214	2214.0	7.26	70	262
	0690				166	280		2144			140	
	1073				70			2630			81	
30	1003		1003.0	3.30	70			2549	2549.0	8.36	81	424
	0933				140	420		2468			162	
	0930				75			2654			71	
31	0855		0855.0	2.80	75			2583	2582.7	8.48	72	567
	0780				150	570		2511			143	
	1178				74			2618			76	
32	1104		1103.7	3.62	75			2542	2542.3	8.35	75	718
	1029				149	719		2467			151	
	1098				76			2331			74	
33	1022		1022.0	3.36	76			2257	2256.7	7.40	75	867
	0946				152	871		2182			149	
	1378				77			2000			84	
34	1301		1300.7	4.26	78			1916	1916.0	6.30	84	
	1223				155	1026		1832			168	1035
	19926		6642.0	21.80				51677	17225.7	56.53		
								19926	6642.0	21.80		
							3	31751	-10583.7	-34.73	= -10.58 meters	
								-10583.7				

FIGURE 13.—Sample record of level observations

umn headed "Middle-thread reading" is the foot. (See note, p. 7.) The reading of the middle thread to hundredths of feet is for checking purposes, and it is not used otherwise in the computations.

The instrument stations (not turning points) are numbered consecutively throughout the day, as shown in column 1.

The columns headed "Thread interval" show the intervals between the upper and middle threads as seen projected on the rod, and between the middle and lower threads, and finally the total intervals. The columns headed "Sum of intervals" show the continuous sum of the total intervals, and as these values are proportional to the sums of the backsight distances and foresight distances, respectively, they enable the observer to keep these sums nearly equal at all times as required by the instructions, for the purpose of eliminating instrumental errors. (See par. 37, p. 8.)

The columns headed "Mean" show the means of the three thread readings for each rod reading. The recorder uses, as a short method of computing the mean of the three thread readings, the fact that the difference of the upper and lower intervals divided by three is the correction to be applied with the proper sign to the middle-thread reading to give the mean of the three.

The rod number and its temperature are recorded for the first backsight in each section. If it is found that the readings of the two rod thermometers differ, the number of each rod and its thermometer reading should be recorded, but the rod that is in backsight should be recorded first. A rod once placed at a turning point is kept there, for both the foresight of one instrument station and the backsight of the next, each rodman thus being front and back rodman alternately. To carry out the requirement of the general instructions, that at stations of odd numbers the backsight is to be taken before the foresight and that at even stations the foresight is to be taken first, it is only necessary to remember that this is equivalent to the statement that one particular rodman must always show his rod first after each placing of the instrument.

Such portions of the computation as form a part of the record (see fig. 13) are kept up by the recorder as the work progresses. The instrument is not moved forward from any station until the recorder announces that the readings at that station, as shown by the rod intervals, check properly.

The difference in elevation for the ends of each section is computed in the record. This difference is equal to the sum of the backsights minus the sum of the foresights, the sign of the difference being plus if the sum of the backsights is the larger. The difference between the sums of the columns headed "Mean" equals the difference in elevation, but, as a check, the difference in elevation should also be computed from the thread readings by subtracting the sum of the backsight thread readings from the sum of the foresight readings and dividing this difference by three. The difference as computed from the sum of the thread readings may differ from that of the means by two or three tenths of a millimeter, depending upon the number of hundredths of a millimeter dropped or added in computing the means of the three thread readings.

As an additional check upon the difference in elevation for the section and to guard against an erroneous reading of the rod in meters and decimeters, a reading of the middle thread is made, as already

89 SPIRIT LEVELING 90

Date: 8-12-26 Forward—~~Backward~~ (Strike out one word)

Sun: 11—23

From B. M.: DW-4179 To B. M.: V-12

Wind: 1 11—23

Time: 10:21

No. of Station	Thread Reading Back Sight	Mean	Middle Thread Reading Feet	Thread Interval	Sum of Intervals	Rod and Temp.	Thread Reading Fore Sight	Mean	Middle Thread Reading Feet	Thread Interval	Sum of Intervals
	2344			85		218	1676			95	
28	2259	2259.0	7.41	85		29.	1581	1580.7	5.18	96	
	2174			170	170		1485			191	191
	1920						1943			77	
29	1834	1833.7	6.02	86			1866	1866.0	6.12	77	
	1747	4692.7		87			1789			154	345
				173	343						
							2015			110	
							1905	1905.0	6.25	110	
							1795	3485.7		220	
29	Top of rail, depot Minnekanta										
							1890			84	
30	1619	1530.3	5.02	88			1806	1806.3	5.92	83	
	1531			90			1723			167	512
	1441			178	521						
							2008			61	
31	1714	1654.7	5.42	59			1947	1947.0	6.39	61	
	1655			60			1886			122	624
	1595			119	640						
							2759			50	
32	1739	1692.0	5.55	47			2709	2709.0	8.89	50	
	1692			47			2659			100	734
	1645			94	734		2559	9904.0	32.50		
	26909	8969.7	29.42				29721				
							26909	8969.7	29.42		
							32818	-939.3	-3.08	= -0.939 meter	
							-939.3				

FIGURE 14.—Sample record showing recording of extra foresight

explained, on the back of the rod which is graduated in feet. (See note, p. 7.) The difference in elevation is computed from these readings and the conversion made into meters as indicated in Figure 13.

Quite frequently the reading for the first backsight or the last foresight of a section is made upon a tape. This is the case where the bench mark is in the wall of a building where it is impossible to hold a rod upon it. The record should indicate clearly what readings are made on a tape and the number of the tape.

Figure 14 illustrates the method of recording an extra foresight, such as is made to obtain the elevation of the top of the rail in front of a depot or in the middle of a bridge. The space in the backsight column should be left blank unless it is desirable to place a note there in explanation of the foresight. The number of the instrument station must always be given, as it is the only way one can be sure from the record to connect the extra foresight with the corresponding backsight, the fact that the extra foresights should always follow the regular foresight at an instrument station not being sufficient.

If a regular foresight is upon a point whose elevation is to be determined, a notation designating the foresight used and an abbreviated description of the point should be inserted in the record.

Where the level line follows the same railroad for a considerable distance, it is not necessary to insert the name of the railroad when giving the name of the depot in recording the extra foresight for the top of the rail, but where there can be any doubt as to which depot is referred to, the name of the railroad should also be given.

Form 45, Figure 15, should show the separate results of each section of leveling together with the date and time of the running, the position of the sun, the strength of the wind, the mean temperature of the rods, the length of the section, and the distance of the second bench mark of the section from the initial bench mark of the line.

On this form the sections are arranged in their order along the line. The spur lines and the elevations determined from extra foresights follow the bench marks of the main line with which they are connected. Do not assemble spur lines and the results of leveling for supplementary elevations on separate sheets. The computation of a line of levels is progressive and to have all the sections in their proper order will not only facilitate the computation, but will make errors much less likely to occur in the computation of the elevations.

A heavy horizontal line should be drawn across the page at the beginning and end of a spur line in order that the sections on a spur will not be added into the main line of levels.

The lengths of the sections in column 8 are obtained by multiplying the total sum of the rod intervals for each section by the stadia constant of the instrument. The length of each section, whether in the main line or on a spur line should be given, but it is not necesssary to give the lengths involved in the determination of the elevations of points by extra foresights. It is well, though, to record the time for the section in which an extra foresight occurs, as this provides an easy

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 45

COMPUTATION OF FIRST-ORDER LEVELS

Line Sioux Falls – Edgemont, S.D. Rods 221, & 222

Level No. 18 Observer C.M. Thomas Year 1925

Date (1)	Designation of Section (2)	Forward or Backward (3)	Time (4)	Sun (5)	Wind (6)	Temp. of Rods (7)	Length of Section (8) km.	Distance from B.M. (9) km.	ROD READINGS ΣB (10) meters	ROD READINGS ΣF (11) meters	Difference of Elevation Each Section (12) meters	DIVERGENCE (B–F) Partial (13) mm.	DIVERGENCE (B–F) Total Accumulated (14) mm.	Mean Difference of Elevation (15) meters	Designation of Bench Mark (16)	Elevation of Bench Mark (17) meters	CORRECTIONS Index Error (18) mm.	CORRECTIONS Length of Rod (19) mm.	CORRECTIONS Temp. of Rod (20) mm.	REMARKS (21)
Oct 29	272-273	F	11:05	6	2	-2°	1.13	431.51	9.2054	11.8120	-2.6066 (R)	+0.4	-14.3	-2.6152	273	745.9839				running rejected
21		B	9:20	2	2	14			7.7334	5.1211	+2.6123									
29		F	12:20	6	2	0			10.4200	13.0354	-2.6154									
29		B	11:40	0	2	0			11.6623	9.0446	+2.6177									
29	273-X9	F	12:45	2	2	2	1.35	432.86	15.5624	13.1507	+2.4117	+1.1	-13.2	+2.4122	X9	748.3961				
21		B	8:40	2	1	12			6.9946	9.4074	-2.4128									
		F																		
		B																		
24	X9-Y9	F	9:15	7	1	3	0.56	433.42	5.2240	3.0970	+2.1270	-1.6	-14.8	+2.1262	Y9	750.5223				
24		B	8:50	3	1	4			3.2703	5.3957	-2.1254									
		F																		
		B																		
24	Y9-Rail	F	9:35						3.2577	1.8497	+1.4080	+1.0		+1.4085	Kadoka	751.9303				Spur Line
24		B	4:30						2.1643	3.5733	-1.4090									
		F																		
		B																		
24	Y9-Z9	F	9:35	6	2	4	1.67	435.09	12.1491	14.1500	-2.0009	-2.8	-17.6	-2.0023	Z9	748.5200				
24		B	4:30	4	2	8			15.4287	13.4250	+2.0037									
		F																		
		B																		
24	Z9-280	F	10:50	6	2	3	1.28	436.37	4.9731	11.9276	-6.9545	+0.1	-17.5	-6.9544	280	741.5655				
24		B	3:40	4	2	9			14.8967	7.9423	+6.9544									
		F																		
		B																		

Abbreviations for sun: O=cloudy. With sunshine, show direction of progress in relation to sun by numbers in diagram.
Abbreviations, strength of wind: O=calm; 1=moderate; 2=strong.

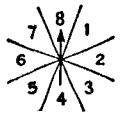


FIGURE 15.—Sample field computation of results

means of finding the observations in the record book. In the computation of the top of the rail at Kadoka (fig. 15) the times indicate at once that one extra foresight occurs in the forward running of the section Y 9—Z 9, while the other extra foresight occurs in the backward running of that section.

Columns 10, 11, and 12 (fig. 15) are transcribed directly from the record book, the results being those obtained from using the means of the thread readings.

In computing the mean differences in elevation shown in column 15, all values in column 12 that differ more than 6 millimeters from the mean are to be rejected. In the results shown for section 272–273, the mean of the four values is 2.6130. As the first forward running differs by more than 6 millimeters from this mean, it is rejected. In computing the mean of the three remaining values, the backward and forward measures are treated separately, as they should be in all cases, for the partial divergence must show the difference between the backward and forward measures. The forward measure of section 272–273 is −2.6154, the mean of the two backward measures is +2.6150, the mean difference in elevation is −2.6152, and the partial divergence +0.4. The sign of the partial discrepancy is always the same as that of the smaller difference in elevation.

Where there are only two runnings of a section a straight mean is taken of the values given in column 12, and the partial discrepancy, of course, is the difference between the two values with the sign of the smaller.

Column 16 contains the designation of the bench mark whose elevation is given in column 17. This designation should be the same as the second one in column 2 of the same line.

The elevation of the initial bench mark of the line is always furnished the chief of party, and the elevations in column 17 are obtained by applying the successive differences in elevation between bench marks, given in column 15, to the elevation of the initial bench mark.

Columns 18, 19, and 20 are for office use only, as none of these corrections form a part of the field computation.

Chiefs of parties should endeavor to keep the computations as nearly up to date as possible. For the immediate field work only the results in columns 8, 12, 13, and 14 are required, but the computation should be carried through to completeness. By computing the elevation of each point determined, it is often possible to detect blunders in the computations, especially if connections are made with previously established bench marks.

Part II.—OFFICE COMPUTATIONS

CHECKING FIELD DATA

The office computation of a line of first-order levels consists of a revision of the field computation and the addition of the index correction, the rod and temperature corrections, and finally of the orthometric correction.

It will not be necessary in the office to check the additions in the level record books, but the difference in elevation of each section as obtained from one-third the sum of the thread readings, from the sum of the mean-thread readings and from the sum of the middle-thread readings in feet (see note, p. 7) converted into meters should be compared for agreement. All subtractions involved in the computation of each difference in elevation in the record book should be checked.

The index, rod, and temperature corrections should be inserted in red ink on the field computation submitted on Form 45 (fig. 16). The transcription of the values in the record book to columns 10, 11, and 12 of Form 45, as made in the field, should be checked. It is not necessary to check the transcription of the values in the other columns, as the field check is considered sufficient.

CORRECTIONS

The index correction, or the difference in the lengths of the space between the foot of the rods and the 2-decimeter graduation is never applied unless this difference amounts to more than 0.1 millimeter. The index correction of all the rods used by this bureau is practically zero, and in any event this correction is only applied when there is an odd number of instrument stations in a section or when a tape is used on one of the bench marks. It has not been necessary to apply the index correction to the work of this bureau for a number of years, as in graduating the rods all measurements are made from the foot of the rod and not from any particular graduation.

The rod correction is caused by the meter spaces not being exactly 1 meter in length. The rods are tested frequently by the United States Bureau of Standards and the exact length determined. In the computation facing this page, the mean length of the rods was 3.00015 meters, or an excess in length of 0.05 millimeter per meter. This excess is multiplied by the difference in elevation shown in

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 43

COMPUTATION OF FIRST-ORDER LEVELS

Line Saugus to Bakersfield, Calif. Rods 228 & 229

Date 1926	Designation of Section (2)	Forward or Backward (3)	Time (4)	Sun (5)	Wind (6)	Temp. of Rods (7)	Length of Section (8) km.	Distance from B.M. T43 (9) km.	ROD READINGS ΣB (10) meters	ROD READINGS ΣF (11) meters	Difference of Elevation Each Section (12) meters	DIVERGENCE (B–F) Partial (13) mm.	DIVERGENCE (B–F) Total Accumulated (14) mm.
8-26	10-11	F	3:50	7	1	38.7	1.31	39.96	76.9092	11.1532	+65.7560	-2.1	-2.6
8-27		B	8:58	6	0	35.0			11.7713	77.5252	-65.7539		
		F											
		B											
8-30	11-R52	F	8:50	3	2	34.0	0.95	40.91	60.0209	7.5821	+52.4388	+1.7	-0.9
8-30		B	3:52	3	2	28.0			10.2569	62.6974	-52.4405		
		F											
		B											
8-30	R52-12	F	10:13	3	2	26.5	0.83	41.74	47.5994	5.4642	+42.1352	+3.4	+2.5
8-30		B	2:40	3	2	29.9			6.7728	48.9132	-42.1404		
		F											
9-4		B	8:38	6	0	20.0			6.9525	49.0892	-42.1367		
8-30	12-13	F	11:21	4	2	27.3	0.89	42.63	61.2843	6.5895	+54.6948	-1.7	+0.8
8-30		B	4:26	1	2	26.3			6.7275	61.4325	-54.7050	R	
		F											
9-1		B	4:13	3	1	33.0			7.4011	62.0942	-54.6931		
8-31	13-S52	F	8:50	3	1	28.0	1.13	43.76	55.1937	8.6198	+46.5739	+2.9	+3.7
8-31		B	3:28	4	0	28.9			10.9530	57.5298	-46.5768		
		F											
		B											
8-31	S52-14	F	10:06	3	2	32.0	0.95	44.71	25.4220	14.0771	+11.3449	+2.6	+6.3
8-31		B	2:40	3	0	37.0			13.6930	25.0405	-11.3475		
		F											
		B											

Abbreviations for sun: O=cloudy. With sunshine, show direction of progress in relation to sun by numbers in diagram.
Abbreviations, strength of wind: O=calm; 1=moderate; 2=strong.

COMPUTATION OF FIRST-ORDER LEVELS

Level No. 17 Observer J. M. Smook Year 1926

Mean Difference of Elevation (15) meters	Designation of Bench Mark (16)	Elevation of Bench Mark (17) meters	CORRECTIONS Index Error (18) mm.	CORRECTIONS Length of Rod (19) mm.	CORRECTIONS Temp. of Rod (20) mm.	REMARKS (21)
+65.7550	11	674.8441		+3.3	+1.6	
				-3.3	-1.3	
+52.4396	R52	727.2837		+2.6	+0.2	
				-2.6	-0.5	6 mi. N. of Castaic
+42.1369	12	769.4206		+2.1	+0.3	
				-2.1	-0.5	
				-2.1	+0.1	
+54.6940	13	824.1146		+2.7	+0.4	
				-2.7	-0.4	
				-2.7	-0.9	
+46.5754	S52	870.6900		+2.3	+0.5	5 mi. S. of Nat. Forest Inn.
				-2.3	-0.5	
+11.3462	14	882.0362		+0.6	+0.2	
				-0.6	-0.3	

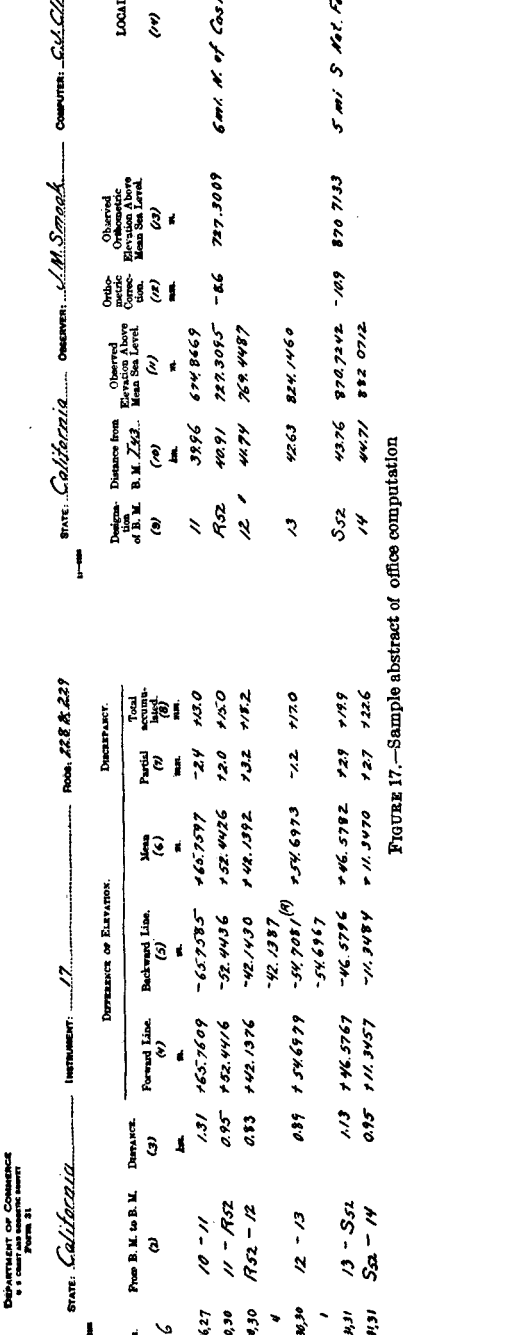
FIGURE 16.—Sample office computation of results

column 15 of Form 45 (fig. 16) and the result in millimeters is placed in column 19. If the rod is too long, the sign of the correction is the same as the sign of the difference in elevation and, conversely, if the rod is too short, the sign of the correction is opposite to that of the difference in elevation.

The rods are usually standardized at a temperature of 21° to 23° C. Any variation from the standardization temperature while the rods are in use will make necessary the application of a correction. The coefficient of expansion of the rods (invar strips) used in the work shown in Figure 16 is 0.0000014 per degree centigrade, and the rods were standardized at 21.1° C. The correction shown in column 20 is obtained by multiplying the difference in elevation by the difference between the standard temperature and the temperature of the rods when used and then multiplying this product by the coefficient of expansion of the invar. The correction increases the difference in elevation when the temperature of the rods is higher than that at which they were standardized.

The computation on Form 45 should next be abstracted on Form 31 as shown in Figure 17. Columns 1, 2, 3, 9, and 10 on this form correspond with columns 1, 2, 8, 16, and 9, respectively, of Form 45.

The values in columns 4 and 5 of Form 31 are obtained by applying, with the proper sign, the corrections in columns 18, 19, and 20 of Form 45 to the values in column 12 of the same form.



DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 31

STATE: California INSTRUMENT: 17 Rods: 228 & 229

Date (1)	From B. M. to B. M. (2)	Distance (3)	Forward Line (4)	Backward Line (5)	Mean (6)	Partial (7)	Total accumulated (8)
1926			Difference of Elevation			Discrepancy	
		Km.	m.	m.	m.	mm.	mm.
Aug. 26, 27	10 - 11	1.31	+65.7609	-65.7585	+65.7597	-2.4	+13.0
" 30, 30	11 - R52	0.95	+52.4416	-52.4436	+52.4426	+2.0	+15.0
" 30, 30	R52 - 12	0.83	+42.1376	-42.1430	+42.1392	+3.2	+18.2
Sept. 4				-42.1387			
Aug. 30, 30	12 - 13	0.89	+54.6979	-54.7081 (8)	+54.6973	-1.2	+17.0
Sept. 1				-54.6967			
Aug. 31, 31	13 - S52	1.13	+46.5767	-46.5796	+46.5782	+2.9	+19.9
" 31, 31	S52 - 14	0.95	+11.3957	-11.3984	+11.3970	+2.7	+22.6

STATE: California OBSERVER: J. M. Smith COMPUTER: Carl Clifford

Designation of B. M. (9)	Distance from B. M. Zero (10)	Observed Elevation Above Mean Sea Level (11)	Orthometric Correction (12)	Observed Orthometric Elevation Above Mean Sea Level (13)	LOCALITY (14)
	Km.	m.	mm.	m.	
11	3.896	674.8469			
R52	4.891	727.3095	-8.6	727.3009	6 mi. N. of Cos Bate
12	4.874	769.4987			
13	5.863	824.1460			
S52	6.876	870.7242	-1.09	870.7133	5 mi. S. Nat. Forest line
14	6.971	882.0712			

FIGURE 17.—Sample abstract of office computation

Columns 6, 7, 8, and 11 of Form 31 are obtained in the manner described for columns 15, 13, 14, and 17, respectively, in connection with the field computation of the work. (See p. 43.)

After the completion of the abstract through column 11 the observed elevations of each bench mark and of other points along the line are available. The orthometric correction (see p. 47) is next computed for each permanent bench mark. This correction eliminates from the observed results the effect of the convergence of level surfaces as the poles of the earth are approached, and the resulting elevations represent the vertical distances of the points above mean sea level.

There is shown below a sample form of the computation of the orthometric correction. Column 1 contains the designation of the bench mark, column 2 gives the latitude of the bench mark obtained by scaling from a reliable map, and column 3 shows the mean elevation of the section of leveling between the bench marks.

Computation of orthometric correction

[San Fernando, Saugus, Bakersfield, Mohave, Saugus]

Bench mark	Latitude	Elevation of section	Correction	Total correction	Bench mark	Latitude	Elevation of section	Correction	Total correction
	° ′	*Meters*	*Mm.*	*Mm.*		° ′	*Meters*	*Mm.*	*Mm.*
T 43	34 17			0.0	O 52	34 28	340	−0.5	−5.9
P 53	18	350	−0.5	−0.5	P 52	30	395	−1.1	−7.0
R. S. 10	18			−0.5	Q 52	31	500	−0.7	−7.7
O 53	19	375	−0.5	−1.0	R 52	32	650	−0.9	−8.6
N 53	19			−1.0	S 52	34	800	−2.3	−10.9
USGS 50	19			−1.0	USGS	34			−10.9
M 53	20	415	−0.6	−1.6	T 52	35	875	−1.3	−12.2
L 53	21	445	−0.6	−2.2	U 52	35			−12.2
USGS 48	22	420	−0.6	−2.8	V 52	36	960	−1.4	−13.6
K 53	22			−2.8	W 52	37	1,000	−1.4	−15.0
J 53	24	375	−1.1	−3.9	Q 53	38	1,100	−1.6	−16.6
USGS Saugus	24			−3.9	X 52	38			−16.6
J 52	25	355	−0.5	−4.4	R 53	38			−16.6
K 52	26	335	−0.5	−4.9	S 53	39	1,140	−1.6	−18.2
L 52	26			−4.9	T 53	40	1,130	−1.6	−19.8
M 52	27	320	−0.5	−5.4	U 53	41	1,160	−1.7	−21.5
N 52	27			−5.4	V 53	43	1,240	−3.6	−25.1

The values in column 4 are the computed orthometric corrections for the sections and are obtained by multiplying the mean elevation of a section by the difference in minutes between the latitudes of the bench marks at the ends of the section and then multiplying this product by the factor *C*, corresponding to the mean latitude of the section, which is obtained from the table on pages 78–81.

The sign of the correction is plus where the direction of progress of a section is from north to south and minus where it is from south to north.

Each value in column 5 is the accumulated correction from the beginning of the line of levels for the corresponding bench mark in column 1. These values are to be transferred to column 12 of Form

31 (fig. 17) and applied as corrections to the elevations of the bench marks in column 11 of that form. The resulting elevation of each mark is its unadjusted orthometric elevation above mean sea level.

ORTHOMETRIC AND DYNAMIC CORRECTIONS[1]

As stated on page 46, it has been found necessary to apply a correction to the observed elevations on account of the nonparallelism of level surfaces.

A level surface is one which is everywhere perpendicular to the direction of gravity as defined by the plumb line. The surface of still water at a uniform temperature is level. A level surface is also called an equipotential surface, meaning thereby that every point on it has the same gravity potential. The difference in potential between two equipotential surfaces is equal to the work done by or against gravity [2] in moving a unit mass from a point on one surface to a point on the other surface. The work or difference of potential is independent of the particular points that may be chosen or of the path between them. The work in passing from one point to another on the same level surface is zero. Potential and equipotential surfaces and their properties are discussed in many books, such as Helmert's Die Mathematischen und Physikalischen Theorieen der Höheren Geodäsie: II Theil. In ordinary spirit leveling it is assumed that, if the field work could be made sufficiently exact, the same difference of elevation between two points would be obtained, irrespective of the route followed. It is implied that level surfaces are parallel to each other.

As stated above, this is not true, but the error introduced by this assumption in the results of leveling which is not of a high grade is small compared with the errors of leveling and may be neglected.

In first-order leveling a correction must be applied to the observed differences in elevation on account of the nonparallelism of level surfaces, and this is most important on north-and-south lines, especially when the average elevation of the line is great. This correction would be small and probably negligible on a line run along a coastal plain.

The surface of the sea and other level surfaces above or below it are approximately spheroidal in shape, but each of the surfaces above sea level has a greater proportional flattening than the sea surface, and consequently such a surface will be at a shorter distance from sea level at the poles than at the Equator. A level surface 1,000 meters above the sea at the Equator would be only 995 meters above the sea at the poles. The polar convergence of other level surfaces

[1] Lallemand discusses at length the orthometric and dynamic corrections on pp. 358-387 of his "Nivellement de Haute Précision" in the Encyclopédie des Travaux Publics, Paris et Liège, 1912.

[2] Gravity is considered to be the resultant of gravitation (the attraction of the earth's mass) and the centrifugal force due to the earth's rotation.

toward sea level is approximately in the same proportion to their elevation; for instance, a level surface 500 meters above the sea at the Equator would be 497½ meters above it at the poles, etc.

If a line were drawn whose tangent everywhere coincided with the direction of gravity—that is, with the perpendicular at every point to the level surface through that point—this line would not be straight but slightly curved and concave toward the earth's axis. Strictly speaking, there is therefore some ambiguity in the expression "elevation" of a point above sea level. The distance between a point and the sea level might be measured (1) along a curved line of the kind just mentioned passing through the point; (2) along a straight line perpendicular to the level surface at the point in question and extending down to sea level; (3) along a straight line perpendicular to the sea-level surface and passing through the point in question. This third kind of line is the one commonly used in geodesy for reducing to sea level to obtain a point on the sea-level surface corresponding to a point above it. But as long as only the lengths of these lines are considered it makes no difference which length is measured, since if one of these lengths be 10,000 meters—greater than the elevation of the highest mountain on the earth—the others will differ from it by less than a thousandth of a millimeter. This source of ambiguity may, therefore, be neglected, but the ambiguity arising from the route followed remains to be considered.

The manner in which discordant differences of level between two points are obtained by following different routes between them may be illustrated by a simple ideal case. Suppose a still lake, with axis lying north and south, is situated on the edge of a level plateau near the sea in the Northern Hemisphere. Let B be a point at the lake's surface near the middle and A a point at sea level in the same latitude as B. Let one line of levels be carried by water leveling north from A to C, a point at sea level and in the same latitude as the north end of the lake, then directly to D, a point on the surface at the north end of the lake, then by water leveling along the lake southward to B. The difference of level between A and B, or between A and any point of the lake's surface will come out equal to the elevation of D above C. Let another line of levels be run southward from A by water levels to E, a point at sea level and in the same latitude as the south end of the lake, then directly from E to F, a point on the surface at the south end of the lake, and northward by water levels on the lake to B. The difference of level between A and B, or between A and any point on the lake's surface, will now come out equal to the elevation of F above E and will be greater than the apparent elevation by the first route, since the two level surfaces approach each other as they near the north pole; and neither result will agree with the result of measuring directly from A to B.

There are two methods of correcting this ambiguity, so that except for errors of observation one may always arrive at the same result for the same point. One method is to correct the difference of level in such a manner that one may obtain the actual vertical distance between B and the sea level or, in the general case, between a point and the geoid. This elevation is called the orthometric elevation of B, and the correction to the measured difference of elevation to obtain the orthometric elevation from the observed results of leveling is called the orthometric correction. It is to be noted (1) that one may speak, not of the orthometric correction to an elevation, but of the orthometric correction to a difference of elevation for a given route; (2) that points on the same level surface have different orthometric elevations if they lie in different latitudes, and that therefore, conversely, points in different latitudes, having the same orthometric elevation, lie on different level surfaces. This inconvenience has led to the second method, which discards the simple conception of measured vertical distances altogether and gives to each surface a number of its own. Instead of the elevation of a point above sea level, a serial number is given to the level surface on which it lies. The points F, B, and D on the lake-level surface would bear the same number. For convenience, the system of numbering these surfaces is such that the number of a level surface is not very different from the height (in the unit chosen) of any point in the surface. The serial number of a level surface is called its dynamic number [3] and is defined as follows: In the metric system the dynamic number of a point is the work required to raise a mass of 1 kilogram against the force of gravity from sea level to the level surface passing through the point, the work being measured in standard kilogram-meters at sea level in latitude 45°. If the English system be used, the kilogram in the preceding statement is replaced by the pound and the kilogram-meter by the corresponding standard foot-pound. More generally, to get the dynamic number in any system of units, the work which is necessary to raise a unit mass from sea level to the level surface in question is expressed in absolute units and the result divided by g_{45}, where g_{45} is the normal acceleration of gravity at sea level in latitude 45°.

The quantity which must be added to the orthometric elevation of a point to obtain the dynamic number is called the dynamic correction.

The exact determination of orthometric and dynamic corrections would require a knowledge of the force of gravity at every point of the line of levels. To get a sufficient approximation to this knowledge by direct observation at numerous points on the line is, at present,

[3] The phrase "dynamic number" is an attempt to render into English the French expression "cote dynamique," or its equivalent, "dynamische Kote," as used by German writers.

impracticable. If it is assumed that the force of gravity is given by one of the usual mathematical expressions, a fair approximation to the truth will, in general, be obtained. The following expression [4] is taken for the force of gravity g at any point in latitude ϕ and at an orthometric elevation h:

$$g=g_{45}(1-\alpha\cos 2\phi+\beta\cos^2 2\phi-kh); \tag{1}$$

g_{45} has the meaning previously given; α, β, and k are constants, α and β being independent of the unit used and equal, respectively, to 0.002644 and 0.000007; $g_{45}=980.624$ dynes and $k=0.0000003147$, if h is in meters; or, more strictly,

$$k=\frac{3147}{10^{10}}\left(1+\frac{71}{10^5}\cos 2\phi-\frac{23}{10^8}h\right),$$

or for brevity, $k=k'(1+\gamma\cos 2\phi-ch)$*

Let W be the work of raising a unit mass from sea level to a point at orthometric elevation h and H the dynamic number of the level surface on which the point lies. The curved vertical of length h is chosen as the most convenient path for passing from sea level to the given point.

Then

$$W=\int_0^h g\,dh=g_{45}\int_0^h(1-\alpha\cos 2\phi+\beta\cos^2 2\phi-kh)dh$$

$$W=g_{45}(1-\alpha\cos 2\phi+\beta\cos^2 2\phi)h-k'\frac{h^2}{2}\left(1+\gamma\cos 2\phi-\frac{2}{3}ch\right)$$

By definition $H=\frac{W}{g_{45}}$, or

$$H=h-(\alpha\cos 2\phi-\beta\cos^2 2\phi)h-k'\frac{h^2}{2}\left(1+\gamma\cos 2\phi-\frac{2}{3}ch\right) \tag{2}$$

This gives the connection between the orthometric elevation and dynamic number. Since H is by definition constant for a level surface, equation (2) may be regarded as the equation of the level surface in coordinates ϕ and h. Let there be a neighboring point on the surface with coordinates $\phi+d\phi$ and $h+dh$. The relation that $d\phi$ and dh must satisfy will be found by differentiating (2), noting that H is constant. It is

$$dh=\frac{-2h(\alpha\sin 2\phi-\beta\sin 4\phi+\frac{1}{2}k'h\gamma\sin 2\phi)d\phi}{1-\alpha\cos 2\phi+\beta\cos^2 2\phi-k'h(1+\gamma\cos 2\phi-ch)} \tag{3}$$

or

$$dh=-2h\alpha\sin 2\phi\left[1+\left(\alpha-\frac{2\beta}{\alpha}\right)\cos 2\phi+k'h\left(1+\frac{\gamma}{2\alpha}\right)+\text{-----}\right]d\phi \tag{4}$$

[4] Equivalent to the expression given by Helmert.

* Sitzungsberichte der Königliche Preussischen Akademie der Wissenschaften, 1903, first Halbband, p. 651.

The term in $k'h$ may be neglected as insignificant in comparison with uncertainties due to errors of observation. On the line of levels from San Diego to Seattle, with an average elevation of about 1,000 meters, and stretching through a difference of latitude of about 15°, the orthometric correction amounted to about one and one-quarter meters, while the term in question had an effect of less than half a millimeter. The neglected terms in (4), involving higher powers in α, β, and $k'h$, have even less effect on the result, so for all practical purposes the equation may be written—

$$dh = -2h\alpha \sin 2\phi\left[1+\left(\alpha-\frac{2\beta}{\alpha}\right)\cos 2\phi\right]d\phi \tag{5}$$

By this equation one may compute the orthometric correction to an observed difference in elevation. After corrections have been applied for curvature and refraction, the two points sighted on the level rods are on the same level surface. The rods measure the vertical distances from this level surface to the points on which they rest, but since the point sighted on the more northerly rod (speaking for the Northern Hemisphere) is nearer sea level than the corresponding point of the level surface on the other rod by an amount which may be calculated from equation (5), the observed difference in rod readings must be corrected by equation (5) to obtain the required difference in elevation above sea level. Equation (5) may be extended to far greater distances than those of a single sight by using for h the average height of the instrument between the points whose difference in elevation is to be found.

Tables on pages 78 to 82 contain quantities useful in computing from equations (2) and (5) the orthometric and dynamic corrections.

In Table IV the argument is the latitude; the quantities tabulated in the second and third columns are for computing by equation (5) and consist, respectively, of the factor $2\alpha \sin 2\phi\left[1+\left(\alpha-\frac{2\beta}{\alpha}\right)\cos 2\phi\right]\sin 1'$, called C, and its logarithm; $\sin 1'$ is included so that $d\phi$ may be taken in minutes of arc. The orthometric correction to the difference of elevation between two points (elevation of second point minus elevation of starting point) is then $-Chd\phi$, where h is the average elevation of the instrument between the points, and $d\phi$ is the difference of latitude in minutes, positive when the second point is north of the first. Any unit of length may be used for h; the orthometric correction computed as above is then expressed in the same unit.

The fourth and fifth columns of Table IV contain, respectively, the quantities $\alpha \cos 2\phi - \beta \cos^2 2\phi$ (called D_1) and its logarithm.

The expression $-(\alpha \cos 2\phi - \beta \cos^2 2\phi)h = -D_1h$ constitutes the dynamic correction for latitude and is expressed in the same unit as h. The remaining term of equation (2), $-k'\frac{h^2}{2}\left(1+\gamma \cos 2\phi-\frac{2}{3}ch\right)$,

constitutes the dynamic correction for elevation. In the first table on page 78, $\frac{k'}{2}\left(1+\gamma \cos 2\phi-\frac{2}{3}ch\right)$, abbreviated as D_2, is tabulated under the two arguments ϕ and h (expressed in meters), and in the second table log D_2 is given for the same arguments. The total dynamic correction is then $-D_1h-D_2h^2$.

Example 1: If the elevation above sea level of Lake Michigan be 177 meters at Chicago, latitude 41° 53′, what is it at Milwaukee, in latitude 43° 03′? In making the computation the mean elevation of the lake, h, may be assumed to be 177 meters. $d\phi = +70'$. The tables give C for the middle latitude, 42° 28′, as .000001532; whence the elevation at Milwaukee $= 177 - Chd\phi = 177-0.0190 =$ 176.9810 meters.

Example 2: Suppose levels are to be carried from A, in south latitude 30° 35′, to a point B, in south latitude 30° 17′, and that the average elevation of the instrument between these points is 600 meters. For the middle latitude 30° 26′ the table gives $C=$ $-$ .000001342, the negative sign following from the rule at the head of the table, and $d\phi = -30° 17' - (-30° 35') = +18'$. The correction to the quantity (observed elevation at B $-$ observed elevation at A) is then $-600 \times (-.000001342) \times 18 = +.0145$ meters.

An idea of how closely the actual gravity at a point conforms to the value derived from formula (1), which gives the value in "free-air," may be obtained by consulting pages 59 and 60 of Special Publication No. 99 of the United States Coast and Geodetic Survey.

It may be noted that, if numerous observations of the relative intensity of gravity have been made along the line of leveling, the dynamic numbers may be obtained, free from any hypothesis as to the change in the intensity of gravity, much more readily than the orthometric elevations. Suppose it is assumed that the dynamic number of a certain point is known and that it is desired to determine by leveling the dynamic number of a second point, not far distant. For this purpose, besides the observed difference of level, there is needed the average intensity of gravity along the line, which may be taken as the value of gravity at a point whose projection on the geoid is the middle point of the line joining the projections of the two points and whose elevation is the mean of the elevations, which need not be known with the greatest exactness. This value could be obtained by direct observation or by interpolation from adjacent gravity stations. Let this value be called g and the observed difference of elevation $\Delta h'$. Then the difference in dynamic numbers is $\frac{\Delta h' g}{g_{45}}$.

A similar process, using theoretical instead of observed gravity, could be used to obtain the dynamic number of one point from another without first computing the orthometric elevations. For this purpose, if g now represents theoretical gravity, the formula can be written $\frac{g}{g_{45}}\Delta h' = \Delta h' - (D_1 + kh)\Delta h'$. The last term serves as a correction to the observed $\Delta h'$ to obtain the difference in the dynamic numbers. The value (approximate) of k is $\frac{3147}{10^{10}}$. (See p. 50.)

But should the law of variation of gravity with elevation and latitude not be known, and the orthometric elevation of a point be desired with only observed values of gravity available, then its dynamic number must first be found, and next both the distance and gravity must be measured along the vertical into the earth to the surface of the geoid, the dynamic number being derived at each step. The dynamic number zero marks the surface of the geoid, and the distance between the given point and the point on the surface whose dynamic number is zero is the orthometric elevation of the former.

CORRECTIONS FOR COLLIMATION AND FOR CURVATURE AND REFRACTION

The corrections for collimation and for curvature and refraction are not applied in general. Observers are cautioned to keep the difference in the lengths of the backsights and foresights as nearly equal as possible and within the limits prescribed in paragraphs 17 and 37 of the general instructions. It is only in exceptional cases that the total effect of these corrections on a line of levels is appreciable. The corrections are applied so seldom that no columns are provided for them on Form 45.

A rapid inspection of the record books will show to what extent the backsights and foresights have been kept equal. The collimation correction is obtained by multiplying the difference between the total backsight intervals and the total foresight intervals for a section by the collimation constant C, due regard being given to the sign of each factor. The sign of the first factor is positive if the backsight intervals are the larger.

The correction for curvature and refraction depends upon the difference in length between corresponding backsights and foresights. The correction can be easily computed by means of Table III, page 76.

ADJUSTMENT OF ELEVATIONS

There have been four general adjustments of the first-order level net of the United States, each succeeding adjustment having become necessary by the addition of new lines to the net. The fourth and last adjustment was made in 1912, the net at that time being composed of about 29,000 miles of leveling. The successive adjustments

necessarily gave different values for the elevations of the bench marks, and the changes in certain sections of the country were of such size that the older values could not be held. The last adjustment showed that the net was sufficiently extended and the leveling of such strength and accuracy that the elevations could be considered as standard. So

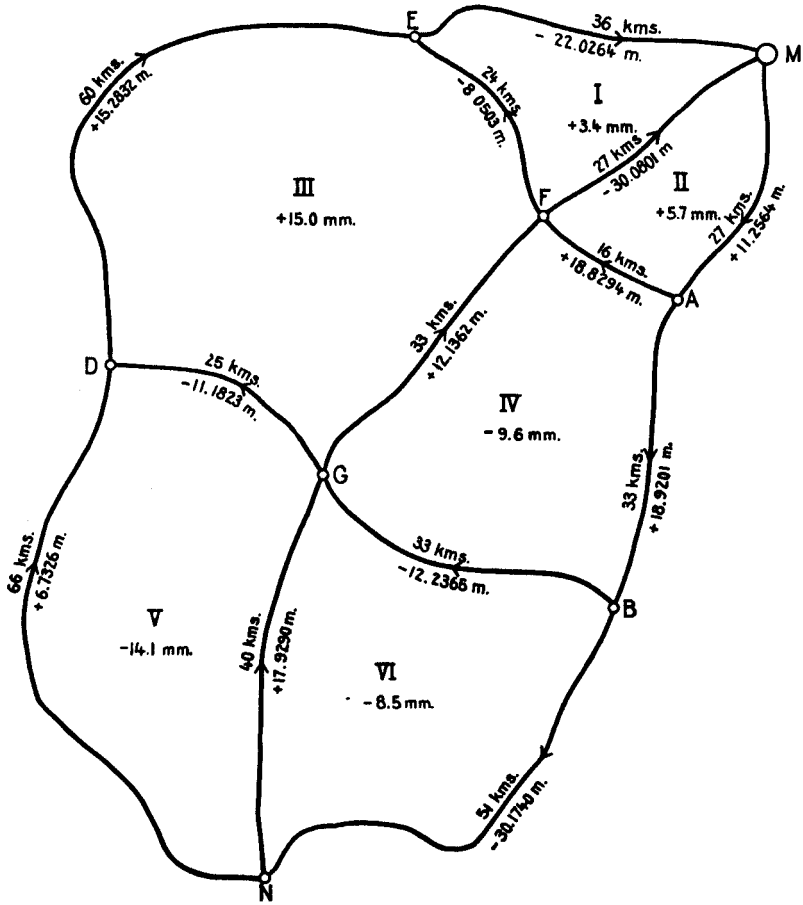


FIGURE 18.—Sample level net

far as surveying and engineering purposes are concerned these elevations may be held for an indefinite period.

The lines run since the last adjustment have been fitted to the net without in any way disturbing the previously adopted elevations unless the results showed that certain adopted elevations were appreciably in error on account of blunders in the leveling or disturbance of the bench marks.

Where only one new line of leveling is to be fitted to the existing net the adjustment consists of distributing the closing error uniformly

along the new line in proportion to the distance from the terminal bench marks.

Where two or more new lines of leveling which are in the same locality and joined together are to be fitted to the net it will become necessary to make a small adjustment to determine the best elevations for the new junction points. This adjustment can be made by the methods outlined below by varying the conditions to fit the particular net which it is desired to adjust.

In order to explain fully the methods employed in this office in the adjustment of a level net, the adjustment of the sample level net shown in Figure 18 will be given in detail.

The complete data for the level net are shown on the sketch. For each line there are given its length in kilometers and the difference in elevation of its terminal points. The sign of the difference in elevation corresponds with the direction in which the line was run as indicated by the arrows. For example, the line M A is 27 kilometers long and A is 11.2564 meters above M.

For convenience of identification each circuit is given a Roman numeral. The closure of the circuit obtained by summing up the differences in elevation in a clockwise direction is given in millimeters in the space within the circuit. For example, the closure of circuit VI is $(-30.1740+17.9290+12.2365)=-8.5$ millimeters.

Two cases of the adjustment of the net are here considered, and each case is adjusted by two methods.

CASE I

In this case the net is supposed to swing free on M; that is, M is the only fixed elevation. It is desired to find the most probable elevations of each of the other points from the given observations.

FIRST METHOD

In this method equations of condition are used. Each circuit closure is a condition which must be satisfied by the adjustment, and the sum of the corrections to the lines forming the circuit must equal the circuit closure with the opposite sign, so that after the adjustment each circuit closure will be zero.

The 13 differences in elevation shown on the sketch are tabulated below with the symbol of the correction that is used in the adjustment.

Data used in adjustment

No.	Line	Difference in elevation	Correction
		Meters	
1	A-M	+11.2564	$+v_1$
2	F-M	+30.0801	$+v_2$
3	F-A	+18.8294	$+v_3$
4	E-M	+22.0264	$+v_4$
5	E-F	−8.0503	$+v_5$
6	B-A	+18.9201	$+v_6$
7	G-F	−12.1362	$+v_7$
8	G-B	−12.2365	$+v_8$
9	D-E	−15.2832	$+v_9$
10	D-G	−11.1823	$+v_{10}$
11	N-D	−6.7326	$+v_{11}$
12	N-G	−17.9290	$+v_{12}$
13	N-B	−30.1740	$+v_{13}$

The circuit closures are then computed from the above data, and the equations are formed as follows:

Formation of equations

CIRCUIT I

M-E	−22.0264	$-v_4$
E-F	−8.0503	$+v_5$
F-M	+30.0801	$+v_2$
	+3.4	

Equation (1) $+v_2-v_4+v_5+3.4=0$

CIRCUIT II

M-F	−30.0801	$-v_2$
F-A	+18.8294	$+v_3$
A-M	+11.2564	$+v_1$
	+5.7	

Equation (2) $+v_1-v_2+v_3+5.7=0$

CIRCUIT III

F-E	+8.0503	$-v_5$
E-D	+15.2832	$-v_9$
D-G	−11.1823	$+v_{10}$
G-F	−12.1362	$+v_7$
	+15.0	

Equation (3) $-v_5+v_7-v_9+v_{10}+15.0=0$

CIRCUIT IV

A-F	−18.8294	$-v_3$
F-G	+12.1362	$-v_7$
G-B	−12.2365	$+v_8$
B-A	+18.9201	$+v_6$
	−9.6	

Equation (4) $-v_3+v_6-v_7+v_8-9.6=0$

CIRCUIT V

G–D D–N N–G	+11.1823 +6.7326 −17.9290	$-v_{10}$ $-v_{11}$ $+v_{12}$
	−14.1	

Equation (5) $\quad -v_{10}-v_{11}+v_{12}-14.1=0$

CIRCUIT VI

B–G G–N N–B	+12.2365 +17.9290 −30.1740	$-v_8$ $-v_{12}$ $+v_{13}$
	−8.5	

Equation (6) $\quad -v_8-v_{12}+v_{13}-8.5=0$

The equations are then written in the correlate form. The coefficient of each v is 1, and the weight (p) of each line is taken as $\frac{50}{L}$, in which L is the length of the line in kilometers.

All of the leveling in the net is of the same order of accuracy, and it is assumed that all the errors are of the accidental class. The accidental errors tend to accumulate as the square root of the length of the lines, so it is customary to take the weights of the lines as inversely proportional to their length. The weight of the line M–A, then, is 50/27, the 50 in the numerator being an arbitrary factor selected to facilitate the computations.

The entries in the Σ column of the table of correlates are the algebraic sums of the entries under the equation headings on the corresponding line of the table.

Correlate equations

No. of v	$1/p$	1	2	3	4	5	6	Σ
1	0.54		+1					+1
2	0.54	+1	−1					0
3	0.32		+1		−1			0
4	0.72	−1						−1
5	0.48	+1		−1				0
6	0.66				+1			+1
7	0.66			+1	−1			0
8	0.66				+1		−1	0
9	1.20			−1				−1
10	0.50			+1		−1		0
11	1.32					−1		−1
12	0.80					+1	−1	0
13	1.02						+1	+1

In the table above, the first column contains the number of each v, or unknown, the second column contains the reciprocal of the weight of each unknown, and the succeeding columns, headed by the

numbers of the equations, contain the coefficients of the v's in the given equations. Each coëfficient is placed on the horizontal line corresponding to its v.

The normal equations for the above set are formed and arranged in the following manner:

Normal equations

No.	1	2	3	4	5	6	N	Σ
1	+1.74	−0.54	−0.48				+3.4	+4.12
2		+1.40		−0.32			+5.7	+6.24
3			+2.84	−0.66	−0.50		+15.0	+16.20
4				+2.30		−0.66	−9.6	−8.94
5					+2.62	−0.80	−14.1	−12.78
6						+2.48	−8.5	−7.48

The value, +1.74, in the first column is obtained by squaring the coefficients in the first correlate equation, multiplying each by its weight, and taking the sum $(0.54+0.72+0.48)=1.74$. The values +1.40, +2.84, +2.30, +2.62, and +2.48 are obtained in the same manner and are called the diagonal terms.

The remainder of the values in columns 2 to 6 of the normal equations are product terms resulting from multiplying each correlate equation by each of the other correlate equations and by the corresponding weights. For example, equation (1) times equation (2) times the weight of the coefficients equals $(+1)\times(-1)\times(0.54)=-0.54$. Equation (4) times equation (6) times the weight equals $(+1)\times(-1)\times(0.66)=-0.66$.

Column N contains the constant term of each equation, which in this case is the closing error of the corresponding circuit.

Each entry in the Σ column of the table of normal equations is found by multiplying the corresponding equation in the table of correlates by the "Σ equation" in that table and by the corresponding weights and adding in the constant term of the normal equation. If any normal equation is added down the column to the diagonal term and then across, including the N term, the result should equal the corresponding entry in the Σ column. This serves as a check on the formation of the normal equations. For example, in equation (4), $-0.32-0.66+2.30-0.66-9.6=-8.94$.

The Σ column permits carrying a check on the forward solution of the normal equations and in a large set of equations is a very valuable aid in their solution.

The solution of the normal equations is most easily accomplished by the Doolittle method, as shown on page 59.

Forward solution of normal equations

1	2	3	4	5	6	N	Σ
+1.740	−0.540	−0.480				+3.400	+4.120
C_1	+0.3103	+0.2759				−1.9540	−2.3678
	+1.400		−0.320			+5.700	+6.240
1	−0.168	−0.149				+1.055	+1.278
	+1.232	−0.149	−0.320			+6.755	+7.518
	C_2	+0.1209	+0.2597			−5.4830	−6.1023
		+2.840	−0.660	−0.500		+15.000	+16.200
	1	−0.132				+0.938	+1.137
	2	−0.018	−0.039			+0.817	+0.909
		+2.690	−0.699	−0.500		+16.755	+18.246
		C_3	+0.2599	+0.1859		−6.2286	−6.7829
			+2.300		−0.660	−9.600	−8.940
		2	−0.083			+1.754	+1.952
		3	−0.182	−0.130		+4.355	+4.742
			+2.035	−0.130	−0.660	−3.491	−2.246
			C_4	+0.0639	+0.3243	+1.7155	+1.1037
				+2.620	−0.800	−14.100	−12.780
			3	−0.093		+3.115	+3.392
			4	−0.008	−0.042	−0.223	−0.144
				+2.519	−0.842	−11.208	−9.532
				C_5	+0.3343	+4.4494	+3.7840
					+2.480	−8.500	−7.480
				4	−0.214	−1.132	−0.728
				5	−0.281	−3.747	−3.187
					+1.985	−13.379	−11.395
					C_6	+6.7401	+5.7406

The first line of the solution is the first equation of the set of normals and is copied directly from the table of normals. The second line is derived by dividing the first line by its first term with the sign changed. This operates to make the coefficient of C_1 -1.0000. The solution can now be checked to this point by summing the second line across to and including the N term. The summation should equal the entry in the Σ column. When these checks are made it will be found that the two values do not always agree exactly, owing to a slight amount of uncertainty in the last decimal place introduced by dropping decimals through the various steps of the solution. Except for this small uncertainty the two values should agree in each case.

The third line is obtained by copying equation (2) from the table of normal equations. The fourth line is derived by multiplying the values in line 1, columns 2, 3, N, and Σ by $+0.3103$, entering each new product on line 4 in the corresponding column.

The fifth line is obtained by adding together the two lines above. When the fifth line is added across to and including the N term this total should check the Σ column entry in that line. The sixth line is derived by dividing the fifth line through by -1.232. Taking the coefficient of C_2 equal to -1.0000, the sixth line is then added across to and including the N term to obtain a check with the Σ value.

The seventh line is equation (3) copied from the table of normals. The eighth line is derived by multiplying the first line, columns 3,

N, and Σ by $+0.2759$. The ninth line is derived by multiplying the fifth line, columns 3, 4, N, and Σ by $+0.1209$. The tenth line is obtained by taking the sum of the three lines above and is checked as usual by adding it across to and including the N term and comparing it with the value in the Σ column. The eleventh line is derived from the tenth line by dividing through by minus the first term. The check is obtained by adding across as before.

Each succeeding equation is brought into the solution and handled in the same manner until all have been introduced and computed. The computation is purely mechanical, and if the checks at the end of each step are carefully made the solution can be carried through with every assurance of its being accurately done.

The back solution shown below is computed as follows:

Back solution

1	2	3	4	5	6
−1.9540	−5.4830	−6.2286	+1.7155	+4.4494	+6.7401
----------	----------	----------	+2.1858	+2.2532	+6.7401
----------	----------	+1.2460	+0.4283	+6.7026	----------
----------	+1.1244	+1.1253	+4.3296	----------	----------
−1.0642	−0.4663	−3.8573	----------	----------	----------
−1.4972	−4.8249	----------	----------	----------	----------
−4.5154	----------	----------	----------	----------	----------

The first line of the back solution is derived by entering in the proper columns the values in the "C" lines of the N column of the forward solution. Then the value in column 6 is multiplied by each of the entries in the "C" lines of column 6 of the forward solution to obtain the second line of the back solution; that is, $+6.7401$ is multiplied by $+0.3343$ and by $+0.3243$, obtaining $+2.2532$ and $+2.1858$, which are placed in columns 5 and 4, respectively. Column 5 of the back solution is added, and the sum is multiplied by each of the "C" line entries in column 5 of the forward solution to obtain the remainder of the third line of the back solution. The rest of the back solution is computed in this same manner.

The next step in the solution is the computation of the v's or corrections which are to be applied to the various observed differences to render the circuits consistent among themselves. The computation of the v's is carried out as follows:

Computation of v's

1	2	3	4	5	6	7
−4.825	−4.515	−4.825	+4.515	−4.515	+4.330	−3.857
	+4.825	−4.330		+3.857		−4.330
−4.825	+0.310	−9.155	+4.515	−0.658	+4.330	−8.187
×0.54	×0.54	×0.32	×0.72	×0.48	×0.66	×0.66
−2.606	+0.167	−2.930	+3.251	−0.316	+2.858	−5.403

8	9	10	11	12	13
+4.330	+3.857	−3.857	−6.703	+6.703	+6.740
−6.740		−6.703		−6.740	+6.740
−2.410	+3.857	−10.560	−6.703	−0.037	
×0.66	×1.20	×0.50	×1.32	×0.80	×1.02
−1.591	+4.628	−5.280	−8.848	−0.030	+6.875

The results of the addition of the various columns in the back solution are known as the "C's." C_6 is the total for column 6 of the back solution, and so on. These C's are now applied to the entries in the table of correlates to obtain the desired v's. For example, $v_2 = (+1 \times C_1) + (-1 \times C_2)$; or $v_{10} = (+1 \times C_3) + (-1 \times C_5)$, etc. After the v's have been computed in this manner, each must be multiplied by its corresponding $1/p$ as shown in the second column of the table of correlates. In this step of the computation the C's may be rounded off to one decimal place less than the number of places to which they were computed.

The v's are now collected in tabular form and rounded off to one decimal place as shown below.

Table of v's

No.	v
1	−2.6
2	+0.2
3	−2.9
4	+3.3
5	−0.3
6	+2.9
7	−5.4
8	−1.6
9	+4.6
10	−5.3
11	−8.8
12	0.0
13	+6.9

These v's, when substituted in the original equations of condition shown on pages 56 and 57, should satisfy the equations.

Substitution of v's in condition equations

1	2	3	4	5	6
+0.2	−2.6	+0.3	+2.9	+5.3	+1.6
−3.3	−0.2	−5.4	+2.9	+8.8	0.0
−0.3	−2.9	−4.6	+5.4	0.0	+6.9
		−5.3	−1.6		
−3.4	−5.7			+14.1	+8.5
		−15.0	+9.6		
+3.4	+5.7			−14.1	−8.5
		+15.0	−9.6		
0.0	0.0			0.0	0.0
		0.0	0.0		

Adjusted differences

No.	Line	Difference in elevation	Correction	Adjusted difference
		Meters	*Mm.*	*Meters*
1	A-M	+11.2564	−2.6	+11.2538
2	F-M	+30.0801	+0.2	+30.0803
3	F-A	+18.8294	−2.9	+18.8265
4	E-M	+22.0264	+3.3	+22.0297
5	E-F	−8.0503	−0.3	−8.0506
6	B-A	+18.9201	+2.9	+18.9230
7	G-F	−12.1362	−5.4	−12.1416
8	G-B	−12.2365	−1.6	−12.2381
9	D-E	−15.2832	+4.6	−15.2786
10	D-G	−11.1823	−5.3	−11.1876
11	N-D	−6.7326	−8.8	−6.7414
12	N-G	−17.9290	0.0	−17.9290
13	N-B	−30.1740	+6.9	−30.1671

From these adjusted values of the differences the elevations of the junction points may be computed and checked as follows:

	Meters
Fixed elev. of M =	103.7620
Dif. of elev., M to A =	+11.2538
Elev. of A =	115.0158
Dif. of elev., A to F =	+18.8265
Elev. of F =	133.8423
Dif. of elev., F to M =	−30.0803
Elev. of M =	103.7620 (check)
Elev. of F =	133.8423
Dif. of elev., F to E =	− 8.0506
Elev. of E =	125.7917
Dif. of elev., E to M =	−22.0297
Elev. of M =	103.7620 (check)
Elev. of B =	133.9388
Dif. of elev., B to N =	−30.1671
Elev. of N =	103.7717
Dif. of elev., N to G =	+17.9290
Elev. of G =	121.7007 (check)

	Meters
Elev. of A =	115.0158
Dif. of elev., A to B =	+18.9230
Elev. of B =	133.9388
Dif. of elev., B to G =	−12.2381
Elev. of G =	121.7007
Dif. of elev., G to F =	+12.1416
Elev. of F =	133.8423 (check)
Elev. of G =	121.7007
Dif. of elev., G to D =	−11.1876
Elev. of D =	110.5131
Dif. of elev., D to E =	+15.2786
Elev. of E =	125.7917 (check)
Elev. of D =	110.5131
Dif. of elev., D to N =	− 6.7414
Elev. of N =	103.7717 (check)

Adjusted elevations of junction points

Junction point	Elevation	Junction point	Elevation
	Meters		*Meters*
M	103.7620	B	133.9388
A	115.0158	G	121.7007
F	133.8423	D	110.5131
E	125.7917	N	103.7717

SECOND METHOD

In this method, observation equations are used. The first step in the adjustment by the method of observation equations is to list the junction points to be adjusted, assume elevations for them, and apply to these elevations symbolic corrections which are called x's in this case.

Fixed elevation

Junction point	Elevation
	Meters
M	103.7620

Assumed elevations

Junction point	Elevation
	Meters
A	$115.02+x_1$
F	$133.84+x_2$
E	$125.79+x_3$
B	$133.94+x_4$
G	$121.70+x_5$
D	$110.51+x_6$
N	$103.77+x_7$

The following table is for the formation of the observation equations and is explained on page 64:

Table for formation of observation equations

Line	Junction points (1)	Junction points (2)	Difference of elevation: Assumed h_2-h_1	Difference of elevation: Observed h_2-h_1	Difference of elevation: Assumed minus observed	Weight p	Symbol	Adjusted difference in elevation h_2-h_1	v	pv
			Meters	*Meters*	*Meter*			*Meters*	*Mm.*	
1	A	M	−11.2580	−11.2564	−0.0016	1.85	$-x_1$	−11.2538	+2.6	+4.8
2	F	M	−30.0780	−30.0801	+.0021	1.85	$-x_2$	−30.0803	−0.2	−0.4
3	F	A	−18.8200	−18.8294	+.0094	3.12	$-x_2+x_1$	−18.8265	+2.9	+9.0
4	E	M	−22.0280	−22.0264	−.0016	1.39	$-x_3$	−22.0296	−3.2	−4.4
5	E	F	+8.0500	+8.0503	−.0003	2.08	$-x_3+x_2$	+8.0507	+0.4	+0.8
6	B	A	−18.9200	−18.9201	+.0001	1.52	$-x_4+x_1$	−18.9230	−2.9	−4.4
7	G	F	+12.1400	+12.1362	+.0038	1.52	$-x_5+x_2$	+12.1416	+5.4	+8.2
8	G	B	+12.2400	+12.2365	+.0035	1.52	$-x_5+x_4$	+12.2381	+1.6	+2.4
9	D	E	+15.2800	+15.2832	−.0032	.83	$-x_6+x_3$	+15.2785	−4.7	−3.9
10	D	G	+11.1900	+11.1823	+.0077	2.00	$-x_6+x_5$	+11.1876	+5.3	+10.6
11	N	D	+6.7400	+6.7326	+.0074	.76	$-x_7+x_6$	+6.7415	+8.9	+6.8
12	N	G	+17.9300	+17.9290	+.0010	1.25	$-x_7+x_5$	+17.9291	+0.1	+0.1
13	N	B	+30.1700	+30.1740	−.0040	.98	$-x_7+x_4$	+30.1672	−6.8	−6.7

In the preceding table the 13 observed differences or lines are numbered in the left-hand column. In the second and third columns are given the letters designating the junction points connected by these lines. The fourth column contains the assumed elevation of junction point (2) minus the assumed elevation of junction point (1) and the fifth column contains the corresponding values for the observed differences. The sixth column is obtained by subtracting the values in the fifth column from those in the fourth. The weights in the seventh column are derived as explained on page 57. Each is equal to $\frac{50}{L}$, where L is the length of the line in kilometers. In the eighth column are the symbols of the corrections to be applied to the assumed elevations of the junction points involved in each line. Manifestly, the difference in elevation of two junction points will be changed by the algebraic sum of the corrections to be applied to the elevations of those points. M is fixed in elevation and has no correction symbol. Each of the other junction points has an x as shown in the table of assumed elevations. The eighth column therefore contains the x's corresponding to the junction points designated in the second and third columns. A minus sign is prefixed to the x's corresponding to the junction points in the second column and a plus sign to the others. The last three columns to the right are filled in after the forward and back solutions have been completed.

The normal equations are formed by means of the following table:

Table for formation of normal equations

Line	Weight p	x_1	x_2	x_3	x_4	x_5	x_6	x_7	N	pN	Σ	$p\Sigma$
									Mm.			
1	1.85	−1							−1.6	−2.960	−2.6	−4.810
2	1.85		−1						+2.1	+3.885	+1.1	+2.035
3	3.12	+1	−1						+9.4	+29.328	+9.4	+29.328
4	1.39			−1					−1.6	−2.224	−2.6	−3.614
5	2.08		+1	−1					−0.3	−0.624	−0.3	−0.624
6	1.52	+1			−1				+0.1	+0.152	+0.1	+0.152
7	1.52		+1			−1			+3.8	+5.776	+3.8	+5.776
8	1.52				+1	−1			+3.5	+5.320	+3.5	+5.320
9	0.83			+1			−1		−3.2	−2.656	−3.2	−2.656
10	2.00					+1	−1		+7.7	+15.400	+7.7	+15.400
11	0.76						+1	−1	+7.4	+5.624	+7.4	+5.624
12	1.25					+1		−1	+1.0	+1.250	+1.0	+1.250
13	0.98				+1			−1	−4.0	−3.920	−4.0	−3.920

The columns headed with the x's in the preceding table contain the coefficients of the corresponding x's as obtained from the table for the formation of the observation equations on page 63. The values in the N column are taken from the "assumed minus observed" column of the table on page 63 but are pointed off to read in millimeters instead of meters.

The Σ column of the table is derived by adding across each line, beginning with the x_1 column and including the N column. The columns headed pN and $p\Sigma$ are obtained by multiplying each entry in the N and Σ columns by its corresponding weight in the second column. These two columns are for use in the formation of the normals.

The following normal equations are formed in the same manner as those for the condition equations on page 57 except that the N column is treated the same as the columns headed by the x's.

Normal equations

x_1	x_2	x_3	x_4	x_5	x_6	x_7	N	Σ
+6.49	−3.12		−1.52				+32.440	+34.290
	+8.57	−2.08		−1.52			−28.061	−26.211
		+4.30			−0.83		+0.192	+1.582
			+4.02	−1.52		−0.98	+1.248	+1.248
				+6.29	−2.00	−1.25	+5.554	+5.554
					+3.59	−0.76	−7.120	−7.120
						+2.99	−2.954	−2.954

The first entry in the preceding table is found by squaring each entry in the x_1 column of the table on page 64, multiplying each square by the corresponding weight, and adding the results. Practically, this amounts to summing up the weights opposite the entries in the x_1 column. For example, $(-1\times-1\times+1.85)+(+1\times+1\times+3.12)+(+1\times+1\times+1.52)=+6.49$. The other "diagonal terms" are obtained in a similar manner from the entries in the corresponding columns of the table on page 64.

The second entry on line one of the preceding table is found by multiplying together any entries opposite each other in columns x_1 and x_2 of the table on page 64 and then multiplying each product by the corresponding weight, thus: $(+1\times-1\times+3.12)=-3.12$. If there are more than one of these products, they should be summed up.

Columns x_1 and x_3 of the table on page 64 do not have entries opposite each other, so the first line of column x_3 in the preceding table is blank. The value in column x_4 of the preceding table is found by multiplying together opposite entries in columns x_1, x_4, and the weight column of the table on page 64, thus: $(+1\times-1\times+1.52)=-1.52$.

Column x_1 of the table on page 64 has no entries opposite those in columns x_5, x_6, or x_7, so the corresponding entries in the preceding table of normals are lacking. The entry under N in line one of the preceding table of normals is computed in the same manner as the other entries, thus: $(-1\times-1.6\times+1.85)+(+1\times+9.4\times+3.12)+(+1\times+0.1\times+1.52)=+32.440$.

By using the pN and $p\Sigma$ columns of the table on page 64 the complete table of normal equations can be computed by simple addition and subtraction. For example, by the use of the pN column the

above entry can be computed, thus: $(-1\times-2.960)+(+1\times+29.328)+(+1\times+0.152)=+32.440$. Similarly, the entry in line one of the Σ column may be computed, thus: $(-1\times-4.810)+(+1\times+29.328)+(+1\times+0.152)=+34.290$.

The first normal equation can now be checked by adding across, thus: $+6.49-3.12-1.52+32.440=34.290$ (check).

The remainder of the table of normal equations is computed in the same manner and each equation can be checked by adding down and across as in the case of the normals formed from condition equations. (See p. 58.)

After the normals have been formed, the forward and back solutions are carried through in exactly the same manner as for condition equations. (See pp. 59 and 60.) This computation is shown below without added explanation.

Solution of normal equations

x_1	x_2	x_3	x_4	x_5	x_6	x_7	N	Σ
+6.490	−3.120	----------	−1.520	----------	----------	----------	+32.440	+34.290
x_1	+0.4807	----------	+0.2342	----------	----------	----------	−4.9985	−5.2835
	+8.570	−2.080	----------	−1.520	----------	----------	−28.061	−26.211
1	−1.500	----------	−0.731	----------	----------	----------	+15.594	+16.483
	+7.070	−2.080	−0.731	−1.520	----------	----------	−12.467	−9.728
	x_2	+0.2942	+0.1034	+0.2150	----------	----------	+1.7634	+1.3760
		+4.300	----------	----------	−0.830	----------	+0.192	+1.582
	2	−0.612	−0.215	−0.447	----------	----------	−3.668	−2.862
		+3.688	−0.215	−0.447	−0.830	----------	−3.476	−1.280
		x_3	+0.0583	+0.1212	+0.2251	----------	+0.9425	+0.3471
			+4.020	−1.520	----------	−0.980	+1.248	+1.248
		1	−0.356	----------	----------	----------	+7.597	+8.031
		2	−0.076	−0.022	----------	----------	−1.289	−1.006
		3	−0.013	−0.157	−0.048	----------	−0.203	−0.075
			+3.575	−1.699	−0.048	−0.980	+7.353	+8.198
			x_4	+0.4752	+0.0134	+0.2741	−2.0568	−2.2931
				+6.290	−2.000	−1.250	+5.554	+5.554
			2	−0.327	----------	----------	−2.680	−2.092
			3	−0.054	−0.101	----------	−0.421	−0.155
			4	−0.807	−0.023	−0.466	+3.494	+3.896
				+5.102	−2.124	−1.716	+5.947	+7.203
				x_5	+0.4163	+0.3363	−1.1656	−1.4118
					+3.590	−0.760	−7.120	−7.120
				3	−0.187	----------	−0.782	−0.288
				4	−0.001	−0.013	+0.099	+0.110
				5	−0.884	−0.714	+2.476	+2.999
					+2.518	−1.487	−5.327	−4.299
					x_6	+0.5905	+2.1156	+1.7073
						+2.990	−2.954	−2.954
					4	−0.269	+2.015	+2.247
					5	−0.577	+2.000	+2.422
					6	−0.878	−3.146	−2.539
						+1.266	−2.085	−0.824
						x_7	+1.6469	+0.6508

Back solution

x_1	x_2	x_3	x_4	x_5	x_6	x_7
−4.9985	+1.7634	+0.9425	−2.0568	−1.1656	+2.1156	+1.6469
			+0.4514	+0.5539	+0.9725	+1.6469
		+0.6951	+0.0414	+1.2856	+3.0881	
	+0.1449	+0.0817	+0.3202	+0.6739		
−0.2913	−0.1286	−0.0725	−1.2438			
	+0.4845	+1.6468				
+1.0884	+2.2642					
−4.2014						

The back solution gives the values of the seven x's or corrections to the assumed elevations of the junction points. The adjusted elevations of the junction points are computed in the following table:

Adjusted elevations of junction points

Junction point	Assumed elevation	Correction	Adjusted elevation
	Meters	*Mm*	*Meters*
A	115.02	−4.2	115.0158
F	133.84	+2.3	133.8423
E	125.79	+1.6	125.7916
B	133.94	−1.2	133.9388
G	121.70	+0.7	121.7007
D	110.51	+3.1	110.5131
N	103.77	+1.6	103.7716

It is seen that these results agree exactly with the results of the adjustment by condition equations on page 63 except in the cases of E and N, which differ only 1 in the last decimal place. It will be noticed that in the back solution above only a slight change would be required to make each x for these two points +1.7 mm. instead of +1.6 mm. and bring both solutions into perfect agreement. If these solutions had been carried out one or two more decimal places, it is probable that the agreement would have been perfect.

Ordinarily, an adjustment is not made in two ways as in this sample, and if the second method is used it must be checked by computing the v's and summing them around each point. To do this the column headed "Adjusted difference in elevation h_2-h_1" in the table on page 63 is first filled in. The v's are the values in this column minus those in the column headed "Observed h_2-h_1." It will be noted that these v's, except for variations of 1 in the last decimal place, are the same as the v's determined by the adjustment using condition equations. The reversal of signs throughout is due to the reversal of the lines. The lines in the condition equations ran A to M, F to M, etc., but in the observation equations they run M to A, M to F, etc.

The *pv* column in the table on page 63 is obtained by multiplying each value in the "*v*" column by its corresponding weight. The *pv*'s around each point adjusted are checked by adding together all values opposite the same correction symbol in the eighth column, taking account of the sign of the symbol, thus:

Approximate check on solution

x_1	x_2	x_3	x_4	x_5	x_6	x_7
−4.8 +9.0 −4.4	+0.4 −9.0 +0.8 +8.2	+4.4 −0.8 −3.9	+4.4 +2.4 −6.7	−8.2 −2.4 +10.6 +0.1	+3.9 −10.6 +6.8	−6.8 −0.1 +6.7
−0.2	+0.4	−0.3	+0.1	+0.1	+0.1	−0.2

Although this check is not exact, owing to the fact that the *v*'s are only carried to tenths of millimeters and then are multiplied by weights as high in this case as 3.12, nevertheless it serves as a rough check on the whole adjustment and prevents any large blunder from remaining undetected.

CASE II

The other case of adjustment explained in this publication is where more than one of the elevations in the net are held fixed.

FIRST METHOD

The same net as used for Case I is adjusted but, in addition to holding *M* fixed, *N* is also held fixed at an elevation of 103.7713 meters.

The 13 differences in elevation shown on the sketch and tabulated for the adjustment by condition equations remain the same. The six equations of condition necessary to close the six circuits also remain the same. To hold the extra fixed elevation one additional equation must now be formed as follows:

Formation of additional equation

[See pp. 56 and 57 for other equations]

M-F F-G G-N N-M	−30.0801 +12.1362 +17.9290 +0.0093	$-v_2$ $-v_7$ $-v_{12}$
	−5.6	

Equation (7) $\quad -v_2-v_7-v_{12}-5.6=0.$

The table of correlates is made up as before except that there are seven equations instead of six.

Correlate equations

No. of v	$1/p$	1	2	3	4	5	6	7	Σ
1	0.54		+1						+1
2	0.54	+1	−1					−1	−1
3	0.32		+1		−1				0
4	0.72	−1							−1
5	0.48	+1		−1					0
6	0.66				+1				+1
7	0.66			+1	−1			−1	−1
8	0.66				+1		−1		0
9	1.20			−1					−1
10	0.50			+1		−1			0
11	1.32					−1			−1
12	0.80					+1	−1	−1	−1
13	1.02						+1		+1

The normal equations are made up from the table of correlates in exactly the same manner as in the first adjustment.

Normal equations

No.	1	2	3	4	5	6	7	N	Σ
1	+1.74	−0.54	−0.48				−0.54	+3.4	+3.58
2		+1.40		−0.32			+0.54	+5.7	+6.78
3			+2.84	−0.66	−0.50		−0.66	+15.0	+15.54
4				+2.30		−0.66	+0.66	−9.6	−8.28
5					+2.62	−0.80	−0.80	−14.1	−13.58
6						+2.48	+0.80	−8.5	−6.68
7							+2.00	−5.6	−3.60

Forward solution of normal equations

1	2	3	4	5	6	7	N	Σ
+1.740	−0.540	−0.480				−0.540	+3.400	+3.580
C_1	+0.3103	+0.2759				+0.3103	−1.9540	−2.0575
	+1.400		−0.320			+0.540	+5.700	+6.780
1	−0.168	−0.149				−0.168	+1.055	+1.111
	+1.232	−0.149	−0.320			+0.372	+6.755	+7.891
	C_2	+0.1201	+0.2597			−0.3019	−5.4830	−6.4050
		+2.840	−0.660	−0.500		−0.660	+15.000	+15.540
	1	−0.132				−0.149	+0.938	+0.989
	2	−0.018	−0.038			+0.045	+0.811	+0.948
		+2.690	−0.698	−0.500		−0.764	+16.749	+17.477
		C_3	+0.2595	+0.1859		+0.2840	−6.2264	−6.4970
			+2.300		−0.660	+0.660	−9.600	−8.280
		2	−0.083			+0.097	+1.754	+2.049
		3	−0.181	−0.130		−0.198	+4.346	+4.535
			+2.036	−0.130	−0.660	+0.559	−3.500	−1.696
			C_4	+0.0639	+0.3242	−0.2746	+1.7191	+0.8330
				+2.620	−0.800	−0.800	−14.100	−13.580
			3	−0.093		−0.142	+3.114	+3.249
			4	−0.008	−0.042	+0.036	−0.224	−0.108
				+2.519	−0.842	−0.906	−11.210	−10.439
				C_5	+0.3343	+0.3597	+4.4502	+4.1441
					+2.480	+0.800	−8.500	−6.680
				4	−0.214	+0.181	−1.135	−0.550
				5	−0.261	−0.303	−3.748	−3.490
					+1.985	+0.678	−13.383	−10.720
					C_6	−0.3416	+6.7421	+5.4005
						+2.000	−5.600	−3.600
					1	−0.168	+1.055	+1.111
					2	−0.112	−2.038	−2.382
					3	−0.217	+4.757	+4.963
					4	−0.154	+0.961	+0.466
					5	−0.326	−4.032	−3.755
					6	−0.232	+4.572	+3.662
						+0.791	−0.325	+0.466
						C_7	+0.4109	−0.5891

Back solution

1	2	3	4	5	6	7
−1.9540 +0.1275	−5.4830 −0.1241	−6.2264 +0.1167	+1.7191 −0.1128 +2.1403 +0.4348	+4.4502 +0.1478 +2.2069	+6.7421 −0.1404	+0.4109
		+1.2650 +1.0851			+6.6017	+0.4109
−1.0373 −1.5430	+1.0859 −0.4515	−3.7596	+4.1814	+6.8049		
−4.4068	−4.9727					

Computation of v's

1	2	3	4	5	6	7
−4.973	−4.407 +4.973 −0.411	−4.973 −4.181	+4.407	−4.407 +3.760	+4.181	−3.760 −4.181 −0.411
−4.973 ×0.54	+0.155 ×0.54	−9.154 ×0.32	+4.407 ×0.72	−0.647 ×0.48	+4.181 ×0.66	−8.352 ×0.66
−2.685	+0.083	−2.929	+3.173	−0.311	+2.759	−5.512

8	9	10	11	12	13
+4.181 −6.602	+3.760	−3.760 −6.805	−6.805	+6.805 −6.602 −0.411	+6.602
−2.421 ×0.66	+3.760 ×1.20	−10.565 ×0.50	−6.805 ×1.32	−0.208 ×0.80	+6.602 ×1.02
−1.598	+4.512	−5.282	−8.983	−0.166	+6.734

Table of v's

No.	v	No.	v	No.	v
1	−2.7	6	+2.8	11	−9.0
2	+0.1	7	−5.5	12	−0.2
3	−2.9	8	−1.6	13	+6.7
4	+3.2	9	+4.5		
5	−0.3	10	−5.3		

Substitution of v's in condition equations

(1)	(2)	(3)	(4)	(5)	(6)	(7)
+0.1 −3.2 −0.3	−2.7 −0.1 −2.9	+0.3 −5.5 −4.5 −5.3	+2.9 +2.8 +5.5 −1.6	+5.3 +9.0 −0.2	+1.6 +0.2 +6.7	−0.1 +5.5 +0.2
−3.4	−5.7	−15.0	+9.6	+14.1	+8.5	+5.6
+3.4	+5.7	+15.0	−9.6	−14.1	−8.5	−5.6
0.0	0.0	0.0	0.0	0.0	0.0	0.0

Adjusted differences

No.	Line	Difference in elevation	Correction	Adjusted difference
		Meters	*Mm.*	*Meters*
1	A-M	+11.2564	−2.7	+11.2537
2	F-M	+30.0801	+0.1	+30.0802
3	F-A	+18.8294	−2.9	+18.8265
4	E-M	+22.0264	+3.2	+22.0296
5	E-F	−8.0503	−0.3	−8.0506
6	B-A	+18.9201	+2.8	+18.9229
7	G-F	−12.1362	−5.5	−12.1417
8	G-B	−12.2365	−1.6	−12.2381
9	D-E	−15.2832	+4.5	−15.2787
10	D-G	−11.1823	−5.3	−11.1876
11	N-D	−6.7326	−9.0	−6.7416
12	N-G	−17.9290	−0.2	−17.9292
13	N-B	−30.1740	+6.7	−30.1673

From these adjusted values of the differences the elevation of the junction points may be computed and checked as follows:

	Meters
Fixed elev. of M =	103.7620
Dif. of elev., M to A =	+11.2537
Elev. of A =	115.0157
Dif. of elev., A to F =	+18.8265
Elev. of F =	133.8422
Dif. of elev., F to M =	−30.0802
Elev. of M =	103.7620 (check)
Dif. of elev., M to E =	+22.0296
Elev. of E =	125.7916
Dif. of elev., E to F =	+8.0506
Elev. of F =	133.8422 (check)
Dif. of elev., F to G =	−12.1417
Elev. of G =	121.7005
Dif. of elev., G to B =	+12.2381
Elev. of B =	133.9386
Dif. of elev., B to A =	−18.9229
Elev. of A =	115.0157 (check)

	Meters
Elev. of B =	133.9386
Dif. of elev., B to N =	−30.1673
Elev. of N =	103.7713 (check)
Dif. of elev., N to D =	+6.7416
Elev. of D =	110.5129
Dif. of elev., D to G =	+11.1876
Elev. of G =	121.7005 (check)
Dif. of elev., G to N =	−17.9292
Elev. of N =	103.7713 (check)
Elev. of D =	110.5129
Dif. of elev., D to E =	+15.2787
Elev. of E =	125.7916 (check)

Adjusted elevations of junction points

Junction point	Elevation	Junction point	Elevation
	Meters		*Meters*
A	115.0157	B	133.9386
F	133.8422	G	121.7005
E	125.7916	D	110.5129

SECOND METHOD

An adjustment by the method of observation equations is given below for this same case of two fixed elevations. The fixed and assumed elevations are as follows:

Fixed elevations

Junction point	Elevation
	Meters
M	103.7620
N	103.7713

Assumed elevations

Junction point	Elevation	Junction point	Elevation
	Meters		*Meters*
A	$115.02+x_1$	B	$133.94+x_4$
F	$133.84+x_2$	G	$121.70+x_5$
E	$125.79+x_3$	D	$110.51+x_6$

Table for formation of observation equations

Line	Junction points (1)	Junction points (2)	Difference of elevation: Assumed h_2-h_1	Difference of elevation: Observed h_2-h_1	Difference of elevation: Assumed minus observed	Weight p	Symbol	Adjusted difference in elevation h_2-h_1	v	pv
			Meters	*Meters*	*Meter*			*Meters*	*Mm.*	
1	A	M	−11.2580	−11.2564	−0.0016	1.85	$-x_1$	−11.2537	+2.7	+5.0
2	F	M	−30.0780	−30.0801	+0.0021	1.85	$-x_2$	−30.0802	−0.1	−0.2
3	F	A	−18.8200	−18.8294	+0.0094	3.12	$-x_2+x_1$	−18.8265	+2.9	+9.0
4	E	M	−22.0280	−22.0264	−0.0016	1.39	$-x_3$	−22.0296	−3.2	−4.4
5	E	F	+8.0500	+8.0503	−0.0003	2.08	$-x_3+x_2$	+8.0506	+0.3	+0.6
6	B	A	−18.9200	−18.9201	+0.0001	1.52	$-x_4+x_1$	−18.9229	−2.8	−4.3
7	G	F	+12.1400	+12.1362	+0.0038	1.52	$-x_5+x_2$	+12.1417	+5.5	+8.4
8	G	B	+12.2400	+12.2365	+0.0035	1.52	$-x_5+x_4$	+12.2381	+1.6	+2.4
9	D	E	+15.2800	+15.2832	−0.0032	0.83	$-x_6+x_3$	+15.2787	−4.5	−3.7
10	D	G	+11.1900	+11.1823	+0.0077	2.00	$-x_6+x_5$	+11.1876	+5.3	+10.6
11	N	D	+6.7387	+6.7326	+0.0061	0.76	$+x_6$	+6.7416	+9.0	+6.8
12	N	G	+17.9287	+17.9290	−0.0003	1.25	$+x_5$	+17.9292	+0.2	+0.2
13	N	B	+30.1687	+30.1740	−0.0053	0.98	$+x_4$	+30.1673	−6.7	−6.6

Table for formation of normal equations

Line	p	x_1	x_2	x_3	x_4	x_5	x_6	N	pN	Σ	$p\Sigma$
1	1.85	−1						−1.6	−2.960	−2.6	−4.810
2	1.85		−1					+2.1	+3.885	+1.1	+2.035
3	3.12	+1	−1					+9.4	+29.328	+9.4	+29.328
4	1.39			−1				−1.6	−2.224	−2.6	−3.614
5	2.08		+1	−1				−0.3	−0.624	−0.3	−0.624
6	1.52	+1			−1			+0.1	+0.152	+0.1	+0.152
7	1.52		+1			−1		+3.8	+5.776	+3.8	+5.776
8	1.52				+1	−1		+3.5	+5.320	+3.5	+5.320
9	0.83			+1			−1	−3.2	−2.656	−3.2	−2.656
10	2.00					+1	−1	+7.7	+15.400	+7.7	+15.400
11	0.76						+1	+6.1	+4.636	+7.1	+5.396
12	1.25					+1		−0.3	−0.375	+0.7	+0.875
13	0.98				+1			−5.3	−5.194	−4.3	−4.214

Normal equations

x_1	x_2	x_3	x_4	x_5	x_6	N	Σ
+6.49	−3.12		−1.52			+32.440	+34.290
	+8.57	−2.08		−1.52		−28.061	−26.211
		+4.30			−0.83	+0.192	+1.582
			+4.02	−1.52		−0.026	+0.954
				+6.29	−2.00	+3.929	+5.179
					+3.59	−8.108	−7.348

Solution of normal equations

x_1	x_2	x_3	x_4	x_5	x_6	N	Σ
+6.490	−3.120		−1.520			+32.440	+34.290
x_1	+0.4807		+0.2342			−4.9985	−5.2835
	+8.570	−2.080		−1.520		−28.061	−26.211
1	−1.500		−0.731			+15.594	+16.483
	+7.070	−2.080	−0.731	−1.520		−12.467	−9.728
	x_2	+0.2942	+0.1034	+0.2150		+1.7634	+1.3760
		+4.300			−0.830	+0.192	+1.582
	2	−0.612	−0.215	−0.447		−3.668	−2.862
		+3.688	−0.215	−0.447	−0.830	−3.476	−1.280
		x_3	+0.0583	+0.1212	+0.2251	+0.9425	+0.3471
			+4.020	−1.520		−0.026	+0.954
		1	−0.356			+7.597	+8.031
		2	−0.076	−0.157		−1.289	−1.006
		3	−0.013	−0.026	−0.048	−0.203	−0.075
			+3.575	−1.703	−0.048	+6.079	+7.904
			x_4	+0.4764	+0.0134	−1.7004	−2.2109
				+6.290	−2.000	+3.929	+5.179
			2	−0.327		−2.680	−2.092
			3	−0.054	−0.101	−0.421	−0.155
			4	−0.811	−0.023	+2.896	+3.765
				+5.098	−2.124	+3.724	+6.697
				x_5	+0.4166	−0.7305	−1.3137
					+3.590	−8.108	−7.348
				3	−0.187	−0.782	−0.288
				4	−0.001	+0.081	+0.106
				5	−0.885	+1.551	+2.790
					+2.517	−7.258	−4.740
					x_6	+2.8836	+1.8832

Back solution

x_1	x_2	x_3	x_4	x_5	x_6
−4.9985	+1.7634	+0.9425	−1.7004	−0.7305	+2.8836
		+0.6491	+0.0386	+1.2013	
	+0.1012	+0.0571	+0.2243		+2.8836
−0.3367	−0.1486	−0.0838		+0.4708	
	+0.4604		−1.4375		
+1.0462		+1.5649			
	+2.1764				
−4.2890					

Adjusted elevations of junction points

Junction point	Assumed elevation	Correction	Adjusted elevation	Junction point	Assumed elevation	Correction	Adjusted elevation
	Meters	*Mm.*	*Meters*		*Meters*	*Mm.*	*Meters*
A	115.02	−4.3	115.0157	B	133.94	−1.4	133.9386
F	133.84	+2.2	133.8422	G	121.70	+0.5	121.7005
E	125.79	+1.6	125.7916	D	110.51	+2.9	110.5129

It is seen that in this case the adjusted elevations of the six junction points agree exactly by the two different methods of adjustment.

To compute the check of the Σpv's around each point we must fill in the additional column in the table of observation equations on page 72 and then add together all values opposite the same correction symbol.

Approximate check on solution

x_1	x_2	x_3	x_4	x_5	x_6
−5.0	+0.2	+4.4	+4.3	−8.4	+3.7
+9.0	−9.0	−0.6	+2.4	−2.4	−10.6
−4.3	+0.6	−3.7	−6.6	+10.6	+6.8
	+8.4			+0.2	
−0.3		+0.1	+0.1		−0.1
	+0.2			0.0	

After the elevations of the junction points are fixed as set forth in the preceding pages, the intermediate bench marks take corrections in proportion to their distances from the junction points. In other words, the v of any line is distributed in proportion to distance to bring the elevations of intermediate bench marks into agreement with those of the junction points.

TABLES

TABLE I.—*Allowable divergence between backward and forward runnings of a section*

Length of section	Allowable divergence	Length of section	Allowable divergence
Kilometers	*Mm.*	*Kilometers*	*Mm.*
0 to 0.51	2.8	1.14 to 1.18	4.3
0.52 to 0.54	2.9	1.19 to 1.24	4.4
0.55 to 0.58	3.0	1.25 to 1.29	4.5
0.59 to 0.62	3.1	1.30 to 1.35	4.6
0.63 to 0.66	3.2	1.36 to 1.41	4.7
0.67 to 0.70	3.3	1.42 to 1.47	4.8
0.71 to 0.74	3.4	1.48 to 1.53	4.9
0.75 to 0.79	3.5	1.54 to 1.59	5.0
0.80 to 0.83	3.6	1.60 to 1.66	5.1
0.84 to 0.88	3.7	1.67 to 1.72	5.2
0.89 to 0.93	3.8	1.73 to 1.79	5.3
0.94 to 0.97	3.9	1.80 to 1.86	5.4
0.98 to 1.03	4.0	1.87 to 1.93	5.5
1.04 to 1.08	4.1	1.94 to 2.00	5.6
1.09 to 1.13	4.2		

TABLE II.—*Total correction for curvature and refraction*

[This table of total correction for curvature and refraction is for use in computing C and in general wherever the total correction is required. In computing this table the refraction was assumed to be equal to one-eighth the curvature]

Distance	Correction to rod reading	Distance	Correction to rod reading
Meters	*Mm.*	*Meters*	*Mm.*
0 to 27	0.0	160	−1.8
28 to 47	−0.1	170	−2.1
48 to 60	−0.2	180	−2.3
61 to 72	−0.3	190	−2.6
73 to 81	−0.4	200	−2.8
82 to 90	−0.5	210	−3.0
91 to 98	−0.6	220	−3.3
99 to 105	−0.7	230	−3.7
106 to 112	−0.8	240	−4.0
113 to 118	−0.9	250	−4.3
119 to 124	−1.0	260	−4.7
125 to 130	−1.1	270	−5.0
131 to 136	−1.2	280	−5.4
137 to 141	−1.3	290	−5.8
142 to 146	−1.4	300	−6.2
147 to 150	−1.5		

TABLE III.—*Differential correction for curvature and refraction*

[The table for the differential correction for curvature and refraction is for use in deriving the corrections that are necessary when the sights are of unequal length. The table was computed upon the assumption that the refraction is one-eighth of the curvature and that the stadia interval for the instrument is such that the distance from the instrument to the rod in meters is one-third of the interval subtended upon the rod in millimeters. An inspection of the table will show that it is sufficiently accurate for use even though the stadia interval differs from that stated by 10 per cent. The sign of this correction is positive when the foresight is the longer; that is, when the stadia interval subtends more divisions on the rod for the foresight than for the backsight]

Mean length of sight in rod interval in millimeters	Difference of sights in rod interval in millimeters																												
	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58
10	.0	.0	.0	.0	.0																								
20	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0																			
30	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0														
40	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0									
50	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0				
60	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1
70	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1
80	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1
90	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
100	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
110	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
120	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
130	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
140	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
150	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
160	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
170	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2
180	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2
190	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2
200	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2
210	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2
220	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2
230	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2
240	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2
250	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2
260	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2
270	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2
280	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2
290	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.3
300	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.3	.3
310	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.3	.3	.3
320	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.3	.3	.3	.3
330	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.3	.3	.3	.3	.3
340	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.3	.3	.3	.3	.3
350	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.3	.3	.3	.3	.3	.3
360	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.3	.3	.3	.3	.3	.3	.3
400	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.3	.3	.3	.3	.3	.3	.3	.3	.4
440	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.3	.3	.3	.3	.3	.3	.3	.3	.4	.4	.4
480	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.3	.3	.3	.3	.3	.3	.4	.4	.4	.4	.4	.4
520	.0	.0	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.3	.3	.3	.3	.3	.3	.3	.4	.4	.4	.4	.4	.4	.4

The differential correction for curvature and refraction may also be taken from the following table, which is sometimes more convenient. It shows, for a given difference of rod intervals, the lower limiting values of the mean rod interval for which the correction is (to the nearest tenth of a millimeter) 0.1 millimeter, 0.2 millimeter, etc.

Thus, for a difference of rod intervals of 37 millimeters, there are given in the table the numbers 88.6, 265.8, and 442.9, under the respective headings 0.1 millimeter, 0.2 millimeter, 0.3 millimeter. This means that up to 88.6 millimeters the correction is 0.0 millimeter to the nearest tenth; from that point up to 265.8 the correction is 0.1 millimeter to the nearest tenth; and between 265.8 and 442.9 it is 0.3 millimeter.

The table is computed on the same assumptions as the preceding table, and the rule for the sign of the correction is the same, namely, positive when the foresight is the longer of the two sights, negative when it is the shorter. Numbers over 460 are omitted from the table.

Differential correction for curvature and refraction

[Limiting values of the mean rod interval]

Difference o. rod intervals (millimeters)	Correction		Difference of rod intervals (millimeters)	Correction			
	0.1 mm.	0.2 mm.		0.1 mm.	0.2 mm.	0.3 mm.	0.4 mm.
8	409.7		36	91.0	273.2	455.2	
9	364.2		37	88.6	265.8	442.9	
10	327.8		38	86.3	258.8	431.3	
			39	84.0	252.1	420.2	
11	298.0		40	81.9	245.8	409.7	
12	273.2						
13	252.1		41	79.9	239.8	399.7	
14	234.1		42	78.0	234.1	390.2	
15	218.5		43	76.2	228.7	381.1	
			44	74.5	223.5	372.5	
16	204.9		45	72.8	218.5	364.2	
17	192.8						
18	182.1		46	71.3	213.8	356.3	
19	172.5		47	69.7	209.2	348.7	
20	163.9		48	68.3	204.9	341.4	
			49	66.9	200.7	334.5	
21	156.1		50	65.6	196.7	327.8	458.9
22	149.0	447.0					
23	142.5	427.5	51	64.3	192.8	321.4	449.9
24	136.6	407.9	52	63.0	189.1	315.2	441.2
25	131.1	393.3	53	61.8	185.5	309.2	432.9
			54	60.7	182.1	303.5	424.9
26	126.1	378.2	55	59.6	178.8	298.0	417.2
27	121.4	364.2					
28	117.1	351.2	56	58.5	175.6	292.7	409.7
29	113.0	339.1	57	57.5	172.5	287.5	402.5
30	109.3	327.8	58	56.5	169.5	282.6	395.6
			59	55.6	166.7	277.8	388.9
31	105.7	317.2	60	54.6	163.9	273.2	382.4
32	102.4	307.3					
33	99.3	298.0					
34	96.4	289.2					
35	93.7	281.0					

TABLE IV.—*Orthometric and dynamic corrections*

[For explanation of the use of the tables, see pp. 51 and 52]

[C is expressed in units of the ninth decimal place. D_1 is expressed in units of the sixth decimal place. -10 is to be understood with log C and log D_1. In south latitudes, if ϕ is taken negative, C is negative. The sign of D_1 is independent of the sign of the latitude]

Latitude ϕ	C	Log C	D_1	Log D_1	Latitude ϕ	C	Log C	D_1	Log D_1
° ′					° ′				
0 00	0	$-\infty$	+2637	7.4211	10 00	+525	3.7200	+2478	7.3942
10	+9	1.9506	+2637	7.4211	10	+533	3.7269	+2473	7.3932
20	+18	2.2516	+2637	7.4211	20	+542	3.7336	+2468	7.3923
30	+27	2.4277	+2637	7.4210	30	+550	3.7403	+2462	7.3913
40	+36	2.5526	+2636	7.4210	40	+558	3.7468	+2457	7.3904
50	+45	2.6495	+2636	7.4209	50	+567	3.7532	+2451	7.3894
1 00	+54	2.7287	+2635	7.4208	11 00	+575	3.7595	+2445	7.3884
10	+62	2.7956	+2635	7.4208	10	+583	3.7657	+2440	7.3873
20	+71	2.8536	+2634	7.4206	20	+591	3.7718	+2434	7.3863
30	+80	2.9047	+2633	7.4205	30	+600	3.7778	+2428	7.3852
40	+89	2.9504	+2633	7.4204	40	+608	3.7837	+2422	7.3841
50	+98	2.9917	+2632	7.4202	50	+616	3.7896	+2416	7.3831
2 00	+107	3.0295	+2631	7.4201	12 00	+624	3.7953	+2410	7.3819
10	+116	3.0642	+2629	7.4199	10	+632	3.8009	+2403	7.3808
20	+125	3.0963	+2628	7.4197	20	+640	3.8065	+2397	7.3797
30	+134	3.1262	+2627	7.4195	30	+649	3.8119	+2391	7.3785
40	+143	3.1541	+2626	7.4192	40	+657	3.8173	+2384	7.3773
50	+151	3.1804	+2624	7.4190	50	+665	3.8226	+2377	7.3761
3 00	+160	3.2051	+2623	7.4187	13 00	+673	3.8278	+2371	7.3749
10	+169	3.2285	+2621	7.4185	10	+681	3.8330	+2364	7.3736
20	+178	3.2507	+2619	7.4182	20	+689	3.8380	+2357	7.3724
30	+187	3.2718	+2617	7.4179	30	+697	3.8430	+2350	7.3711
40	+196	3.2919	+2615	7.4176	40	+705	3.8480	+2343	7.3698
50	+205	3.3111	+2614	7.4172	50	+713	3.8528	+2336	7.3685
4 00	+214	3.3294	+2611	7.4169	14 00	+720	3.8576	+2329	7.3672
10	+222	3.3470	+2609	7.4165	10	+728	3.8623	+2322	7.3658
20	+231	3.3640	+2607	7.4161	20	+736	3.8670	+2315	7.3645
30	+240	3.3802	+2605	7.4157	30	+744	3.8716	+2307	7.3631
40	+249	3.3959	+2602	7.4153	40	+752	3.8761	+2300	7.3617
50	+258	3.4110	+2600	7.4149	50	+760	3.8806	+2292	7.3602
5 00	+266	3.4256	+2597	7.4145	15 00	+767	3.8850	+2285	7.3588
10	+275	3.4396	+2594	7.4140	10	+775	3.8893	+2277	7.3573
20	+284	3.4533	+2592	7.4136	20	+783	3.8936	+2269	7.3558
30	+293	3.4665	+2589	7.4131	30	+790	3.8979	+2261	7.3543
40	+302	3.4793	+2586	7.4126	40	+798	3.9020	+2253	7.3528
50	+310	3.4917	+2583	7.4121	50	+806	3.9062	+2245	7.3513
6 00	+319	3.5038	+2580	7.4115	16 00	+813	3.9102	+2237	7.3497
10	+328	3.5155	+2576	7.4110	10	+821	3.9143	+2229	7.3481
20	+336	3.5269	+2573	7.4104	20	+828	3.9182	+2221	7.3465
30	+345	3.5380	+2570	7.4099	30	+836	3.9222	+2213	7.3449
40	+354	3.5488	+2566	7.4093	40	+843	3.9260	+2204	7.3432
50	+362	3.5593	+2563	7.4087	50	+851	3.9299	+2196	7.3416
7 00	+371	3.5696	+2559	7.4080	17 00	+858	3.9336	+2187	7.3399
10	+380	3.5796	+2555	7.4074	10	+866	3.9374	+2179	7.3382
20	+388	3.5894	+2551	7.4068	20	+873	3.9410	+2170	7.3364
30	+397	3.5989	+2547	7.4061	30	+880	3.9447	+2161	7.3347
40	+406	3.6082	+2543	7.4054	40	+888	3.9483	+2152	7.3329
50	+414	3.6173	+2539	7.4047	50	+895	3.9518	+2143	7.3311
8 00	+423	3.6262	+2535	7.4040	18 00	+902	3.9553	+2134	7.3293
10	+431	3.6350	+2531	7.4033	10	+909	3.9588	+2125	7.3274
20	+440	3.6435	+2526	7.4025	20	+917	3.9622	+2116	7.3256
30	+449	3.6519	+2522	7.4018	30	+924	3.9656	+2107	7.3237
40	+457	3.6600	+2518	7.4010	40	+931	3.9689	+2098	7.3218
50	+466	3.6680	+2513	7.4002	50	+938	3.9722	+2089	7.3198
9 00	+474	3.6759	+2508	7.3994	19 00	+945	3.9755	+2079	7.3179
10	+483	3.6836	+2503	7.3985	10	+952	3.9787	+2070	7.3159
20	+491	3.6912	+2499	7.3977	20	+959	3.9819	+2060	7.3139
30	+500	3.6986	+2494	7.3968	30	+966	3.9850	+2051	7.3119
40	+508	3.7058	+2489	7.3960	40	+973	3.9881	+2041	7.3098
50	+516	3.7130	+2484	7.3951	50	+980	3.9912	+2031	7.3077

TABLE IV.—*Orthometric and dynamic corrections*—Continued

[For explanation of the use of the tables, see pp. 51 and 52]

[C is expressed in units of the ninth decimal place. D_1 is expressed in units of the sixth decimal place. −10 is to be understood with log C and log D_1. In south latitudes, if ϕ is taken negative, C is negative. The sign of D_1 is independent of the sign of the latitude]

Latitude ϕ	C	Log C	D_1	Log D_1	Latitude ϕ	C	Log C	D_1	Log D_1
° ′					° ′				
20 00	+ 987	3.9942	+2021	7.3056	30 00	+1330	4.1240	+1320	7.1207
10	+ 994	3.9972	+2011	7.3035	10	+1335	4.1254	+1307	7.1163
20	+1000	4.0002	+2001	7.3014	20	+1339	4.1269	+1294	7.1118
30	+1007	4.0031	+1991	7.2992	30	+1344	4.1283	+1280	7.1073
40	+1014	4.0060	+1981	7.2970	40	+1348	4.1297	+1267	7.1027
50	+1021	4.0088	+1971	7.2947	50	+1352	4.1311	+1253	7.0980
21 00	+1027	4.0117	+1961	7.2925	31 00	+1356	4.1324	+1240	7.0933
10	+1034	4.0145	+1951	7.2902	10	+1361	4.1338	+1226	7.0886
20	+1040	4.0172	+1940	7.2879	20	+1365	4.1351	+1213	7.0837
30	+1047	4.0200	+1930	7.2855	30	+1369	4.1364	+1199	7.0788
40	+1054	4.0227	+1919	7.2832	40	+1373	4.1377	+1185	7.0738
50	+1060	4.0253	+1909	7.2808	50	+1377	4.1389	+1171	7.0687
22 00	+1066	4.0280	+1898	7.2784	32 00	+1381	4.1402	+1158	7.0636
10	+1073	4.0306	+1888	7.2759	10	+1385	4.1414	+1144	7.0584
20	+1079	4.0331	+1877	7.2734	20	+1389	4.1426	+1130	7.0531
30	+1086	4.0357	+1866	7.2709	30	+1393	4.1438	+1116	7.0477
40	+1092	4.0382	+1855	7.2684	40	+1396	4.1450	+1102	7.0423
50	+1098	4.0407	+1844	7.2658	50	+1400	4.1461	+1088	7.0367
23 00	+1104	4.0432	+1833	7.2632	33 00	+1404	4.1473	+1074	7.0311
10	+1111	4.0456	+1822	7.2606	10	+1407	4.1484	+1060	7.0254
20	+1117	4.0480	+1811	7.2579	20	+1411	4.1495	+1046	7.0196
30	+1123	4.0504	+1800	7.2553	30	+1414	4.1506	+1032	7.0137
40	+1129	4.0527	+1789	7.2525	40	+1418	4.1517	+1018	7.0077
50	+1135	4.0550	+1777	7.2498	50	+1421	4.1527	+1004	7.0016
24 00	+1141	4.0573	+1766	7.2470	34 00	+1425	4.1538	+ 989	6.9954
10	+1147	4.0596	+1755	7.2442	10	+1428	4.1548	+ 975	6.9891
20	+1153	4.0618	+1743	7.2413	20	+1431	4.1558	+ 961	6.9827
30	+1159	4.0640	+1732	7.2384	30	+1435	4.1568	+ 947	6.9762
40	+1165	4.0662	+1720	7.2355	40	+1438	4.1577	+ 932	6.9695
50	+1171	4.0684	+1708	7.2326	50	+1441	4.1587	+ 918	6.9628
25 00	+1176	4.0705	+1697	7.2296	35 00	+1444	4.1596	+ 903	6.9559
10	+1182	4.0726	+1685	7.2266	10	+1447	4.1605	+ 889	6.9489
20	+1188	4.0747	+1673	7.2235	20	+1450	4.1614	+ 875	6.9418
30	+1193	4.0768	+1661	7.2204	30	+1453	4.1623	+ 860	6.9345
40	+1199	4.0788	+1649	7.2173	40	+1456	4.1632	+ 846	6.9271
50	+1205	4.0808	+1637	7.2141	50	+1459	4.1640	+ 831	6.9196
26 00	+1210	4.0828	+1625	7.2109	36 00	+1462	4.1649	+ 816	6.9119
10	+1216	4.0848	+1613	7.2076	10	+1464	4.1657	+ 802	6.9040
20	+1221	4.0868	+1601	7.2044	20	+1467	4.1665	+ 787	6.8960
30	+1227	4.0887	+1589	7.2010	30	+1470	4.1673	+ 772	6.8879
40	+1232	4.0906	+1576	7.1977	40	+1472	4.1680	+ 758	6.8795
50	+1237	4.0924	+1564	7.1943	50	+1475	4.1688	+ 743	6.8710
27 00	+1243	4.0943	+1552	7.1908	37 00	+1478	4.1695	+ 728	6.8623
10	+1248	4.0961	+1539	7.1873	10	+1480	4.1703	+ 713	6.8534
20	+1253	4.0979	+1527	7.1838	20	+1482	4.1710	+ 699	6.8443
30	+1258	4.0997	+1514	7.1802	30	+1485	4.1717	+ 684	6.8350
40	+1263	4.1015	+1502	7.1766	40	+1487	4.1723	+ 669	6.8254
50	+1268	4.1032	+1489	7.1729	50	+1489	4.1730	+ 654	6.8157
28 00	+1273	4.1049	+1476	7.1692	38 00	+1492	4.1736	+ 639	6.8057
10	+1278	4.1066	+1464	7.1654	10	+1494	4.1743	+ 624	6.7954
20	+1283	4.1083	+1451	7.1616	20	+1496	4.1749	+ 609	6.7849
30	+1288	4.1100	+1438	7.1577	30	+1498	4.1755	+ 594	6.7741
40	+1293	4.1116	+1425	7.1538	40	+1500	4.1761	+ 579	6.7630
50	+1298	4.1132	+1412	7.1499	50	+1502	4.1766	+ 564	6.7516
29 00	+1303	4.1148	+1399	7.1459	39 00	+1504	4.1772	+ 549	6.7399
10	+1307	4.1164	+1386	7.1418	10	+1506	4.1777	+ 534	6.7278
20	+1312	4.1180	+1373	7.1377	20	+1507	4.1782	+ 519	6.7154
30	+1317	4.1195	+1360	7.1335	30	+1509	4.1787	+ 504	6.7026
40	+1321	4.1210	+1347	7.1293	40	+1511	4.1792	+ 489	6.6894
50	+1326	4.1225	+1334	7.1250	50	+1513	4.1797	+ 474	6.6758

TABLE IV.—*Orthometric and dynamic corrections*—Continued

[C is expressed in units of the ninth decimal place. D_1 is expressed in units of the sixth decimal place. -10 is to be understood with log C and log D_1. In south latitudes, if ϕ is taken negative, C is negative. The sign of D_1 is independent of the sign of the latitude]

Latitude ϕ	C	Log C	D_1	Log D_1	Latitude ϕ	C	Log C	D_1	Log D_1
° ′					° ′				
40 00	+1514	4.1802	+ 459	6.6617	50 00	+1516	4.1806	− 459	6.6621$_n$
10	+1516	4.1806	+ 444	6.6472	10	+1514	4.1801	− 474	6.6762$_n$
20	+1517	4.1810	+ 429	6.6321	20	+1512	4.1797	− 490	6.6899$_n$
30	+1519	4.1815	+ 413	6.6164	30	+1511	4.1792	− 505	6.7031$_n$
40	+1520	4.1819	+ 398	6.6002	40	+1509	4.1787	− 520	6.7159$_n$
50	+1521	4.1822	+ 383	6.5833	50	+1507	4.1782	− 535	6.7283$_n$
41 00	+1523	4.1826	+ 368	6.5657	51 00	+1505	4.1777	− 550	6.7404$_n$
10	+1524	4.1830	+ 353	6.5473	10	+1504	4.1771	− 565	6.7521$_n$
20	+1525	4.1833	+ 337	6.5281	20	+1502	4.1766	− 580	6.7635$_n$
30	+1526	4.1836	+ 322	6.5080	30	+1500	4.1760	− 595	6.7746$_n$
40	+1527	4.1839	+ 307	6.4869	40	+1498	4.1754	− 610	6.7854$_n$
50	+1528	4.1842	+ 292	6.4648	50	+1496	4.1748	− 625	6.7959$_n$
42 00	+1529	4.1845	+ 276	6.4414	52 00	+1493	4.1742	− 640	6.8062$_n$
10	+1530	4.1848	+ 261	6.4166	10	+1491	4.1736	− 655	6.8162$_n$
20	+1531	4.1850	+ 246	6.3904	20	+1489	4.1729	− 670	6.8260$_n$
30	+1532	4.1853	+ 230	6.3625	30	+1487	4.1723	− 685	6.8356$_n$
40	+1533	4.1855	+ 215	6.3326	40	+1485	4.1716	− 700	6.8449$_n$
50	+1534	4.1857	+ 200	6.3005	50	+1482	4.1709	− 715	6.8540$_n$
43 00	+1534	4.1859	+ 184	6.2658	53 00	+1480	4.1702	− 729	6.8629$_n$
10	+1535	4.1861	+ 169	6.2280	10	+1477	4.1695	− 744	6.8716$_n$
20	+1535	4.1862	+ 154	6.1867	20	+1475	4.1687	− 759	6.8802$_n$
30	+1536	4.1864	+ 138	6.1410	30	+1472	4.1680	− 774	6.8885$_n$
40	+1536	4.1865	+ 123	6.0899	40	+1470	4.1672	− 788	6.8967$_n$
50	+1537	4.1866	+ 108	6.0319	50	+1467	4.1664	− 803	6.9047$_n$
44 00	+1537	4.1867	+ 92	5.9650	54 00	+1464	4.1656	− 818	6.9126$_n$
10	+1537	4.1868	+ 77	5.8859	10	+1461	4.1648	− 832	6.9203$_n$
20	+1538	4.1869	+ 62	5.7890	20	+1459	4.1639	− 847	6.9279$_n$
30	+1538	4.1869	+ 46	5.6641	30	+1456	4.1631	− 862	6.9353$_n$
40	+1538	4.1870	+ 31	5.4880	40	+1453	4.1622	− 876	6.9426$_n$
50	+1538	4.1870	+ 15	5.1870	50	+1450	4.1613	− 891	6.9497$_n$
45 00	+1538	4.1870	00	$-\infty$	55 00	+1447	4.1604	− 905	6.9567$_n$
10	+1538	4.1870	− 15	5.1870$_n$	10	+1444	4.1595	− 920	6.9636$_n$
20	+1538	4.1870	− 31	5.4880$_n$	20	+1441	4.1585	− 934	6.9704$_n$
30	+1538	4.1870	− 46	5.6641$_n$	30	+1437	4.1576	− 948	6.9770$_n$
40	+1538	4.1869	− 62	5.7891$_n$	40	+1434	4.1566	− 963	6.9835$_n$
50	+1538	4.1869	− 77	5.8860$_n$	50	+1431	4.1556	− 977	6.9900$_n$
46 00	+1537	4.1868	− 92	5.9651$_n$	56 00	+1428	4.1546	− 991	6.9963$_n$
10	+1537	4.1867	− 108	6.0320$_n$	10	+1424	4.1536	−1006	7.0025$_n$
20	+1537	4.1866	− 123	6.0900$_n$	20	+1421	4.1526	−1020	7.0086$_n$
30	+1536	4.1865	− 138	6.1411$_n$	30	+1417	4.1515	−1034	7.0146$_n$
40	+1536	4.1864	− 154	6.1868$_n$	40	+1414	4.1504	−1048	7.0205$_n$
50	+1535	4.1862	− 169	6.2282$_n$	50	+1410	4.1493	−1062	7.0263$_n$
47 00	+1535	4.1860	− 184	6.2659$_n$	57 00	+1407	4.1482	−1077	7.0320$_n$
10	+1534	4.1859	− 200	6.3006$_n$	10	+1403	4.1471	−1091	7.0377$_n$
20	+1533	4.1857	− 215	6.3328$_n$	20	+1399	4.1459	−1105	7.0432$_n$
30	+1533	4.1855	− 230	6.3627$_n$	30	+1396	4.1448	−1119	7.0487$_n$
40	+1532	4.1852	− 246	6.3906$_n$	40	+1392	4.1436	−1133	7.0541$_n$
50	+1531	4.1850	− 261	6.4169$_n$	50	+1388	4.1424	−1147	7.0594$_n$
48 00	+1530	4.1848	− 276	6.4416$_n$	58 00	+1384	4.1412	−1160	7.0646$_n$
10	+1529	4.1845	− 292	6.4650$_n$	10	+1380	4.1399	−1174	7.0698$_n$
20	+1528	4.1842	− 307	6.4872$_n$	20	+1376	4.1387	−1188	7.0748$_n$
30	+1527	4.1839	− 322	6.5083$_n$	30	+1372	4.1374	−1202	7.0798$_n$
40	+1526	4.1836	− 338	6.5284$_n$	40	+1368	4.1361	−1216	7.0848$_n$
50	+1525	4.1833	− 353	6.5476$_n$	50	+1364	4.1348	−1229	7.0896$_n$
49 00	+1524	4.1829	− 368	6.5660$_n$	59 00	+1360	4.1335	−1243	7.0944$_n$
10	+1523	4.1826	− 383	6.5836$_n$	10	+1356	4.1321	−1256	7.0991$_n$
20	+1521	4.1822	− 399	6.6005$_n$	20	+1351	4.1308	−1270	7.1038$_n$
30	+1520	4.1818	− 414	6.6168$_n$	30	+1347	4.1294	−1283	7.1084$_n$
40	+1519	4.1814	− 429	6.6324$_n$	40	+1343	4.1280	−1297	7.1129$_n$
50	+1517	4.1810	− 444	6.6475$_n$	50	+1338	4.1266	−1310	7.1174$_n$

TABLE IV.—*Orthometric and dynamic corrections*—Continued

[C is expressed in units of the ninth decimal place. D_1 is expressed in units of the sixth decimal place. -10 is to be understood with log C and log D_1. In south latitudes, if ϕ is taken negative, C is negative. The sign of D_1 is independent of the sign of the latitude]

Latitude ϕ	C	Log C	D_1	Log D_1	Latitude ϕ	C	Log C	D_1	Log D_1
° ′					° ′				
60 00	+1334	4.1251	−1324	7.1218$_n$	66 00	+1145	4.0589	−1772	7.2485$_n$
10	+1329	4.1237	−1337	7.1262$_n$	10	+1139	4.0566	−1784	7.2513$_n$
20	+1325	4.1222	−1350	7.1305$_n$	20	+1133	4.0543	−1795	7.2541$_n$
30	+1320	4.1207	−1364	7.1347$_n$	30	+1127	4.0519	−1806	7.2568$_n$
40	+1316	4.1192	−1377	7.1389$_n$	40	+1121	4.0496	−1818	7.2595$_n$
50	+1311	4.1176	−1390	7.1430$_n$	50	+1115	4.0472	−1829	7.2622$_n$
61 00	+1306	4.1160	−1403	7.1471$_n$	67 00	+1109	4.0448	−1840	7.2648$_n$
10	+1302	4.1145	−1416	7.1511$_n$	10	+1102	4.0423	−1851	7.2674$_n$
20	+1297	4.1129	−1429	7.1551$_n$	20	+1096	4.0398	−1862	7.2700$_n$
30	+1292	4.1112	−1442	7.1590$_n$	30	+1090	4.0373	−1873	7.2726$_n$
40	+1287	4.1096	−1455	7.1629$_n$	40	+1083	4.0348	−1884	7.2751$_n$
50	+1282	4.1079	−1468	7.1667$_n$	50	+1077	4.0322	−1895	7.2776$_n$
62 00	+1277	4.1062	−1481	7.1705$_n$	68 00	+1071	4.0296	−1906	7.2800$_n$
10	+1272	4.1045	−1493	7.1742$_n$	10	+1064	4.0270	−1916	7.2825$_n$
20	+1267	4.1028	−1506	7.1779$_n$	20	+1058	4.0243	−1927	7.2849$_n$
30	+1262	4.1010	−1519	7.1815$_n$	30	+1051	4.0216	−1937	7.2872$_n$
40	+1257	4.0993	−1531	7.1851$_n$	40	+1045	4.0189	−1948	7.2896$_n$
50	+1252	4.0975	−1544	7.1887$_n$	50	+1038	4.0162	−1958	7.2919$_n$
63 00	+1246	4.0957	−1557	7.1922$_n$	69 00	+1031	4.0134	−1969	7.2942$_n$
10	+1241	4.0938	−1569	7.1956$_n$	10	+1025	4.0106	−1979	7.2965$_n$
20	+1236	4.0919	−1581	7.1990$_n$	20	+1018	4.0077	−1989	7.2987$_n$
30	+1230	4.0901	−1594	7.2024$_n$	30	+1011	4.0048	−1999	7.3009$_n$
40	+1225	4.0881	−1606	7.2058$_n$	40	+1004	4.0019	−2010	7.3031$_n$
50	+1220	4.0862	−1618	7.2091$_n$	50	+0998	3.9990	−2020	7.3053$_n$
64 00	+1214	4.0843	−1630	7.2123$_n$	70 00	+0991	3.9960	−2030	7.3074$_n$
10	+1209	4.0823	−1643	7.2155$_n$	10	+0984	3.9929	−2039	7.3095$_n$
20	+1203	4.0803	−1655	7.2187$_n$	20	+0977	3.9899	−2049	7.3116$_n$
30	+1197	4.0782	−1667	7.2219$_n$	30	+0970	3.9868	−2059	7.3137$_n$
40	+1192	4.0762	−1679	7.2250$_n$	40	+0963	3.9836	−2069	7.3157$_n$
50	+1186	4.0741	−1691	7.2280$_n$	50	+0956	3.9805	−2078	7.3177$_n$
65 00	+1180	4.0720	−1702	7.2311$_n$	71 00	+0949	3.9773	−2088	7.3197$_n$
10	+1175	4.0699	−1714	7.2341$_n$	10	+0942	3.9740	−2097	7.3217$_n$
20	+1169	4.0677	−1726	7.2370$_n$	20	+0935	3.9707	−2107	7.3236$_n$
30	+1163	4.0656	−1738	7.2400$_n$	30	+0928	3.9674	−2116	7.3255$_n$
40	+1157	4.0633	−1749	7.2429$_n$	40	+0921	3.9640	−2125	7.3274$_n$
50	+1151	4.0611	−1761	7.2457$_n$	50	+0913	3.9606	−2134	7.3293$_n$
					72 00	+0906	3.9572	−2144	7.3311$_n$

D_2 *in units of the tenth decimal place*

[See p. 52. D_2 is always positive]

ϕ \ h	0 meter	1,000 meters	2,000 meters	3,000 meters	4,000 meters	5,000 meters
°						
0	1575	1574	1574	1574	1574	1573
10	1575	1574	1574	1574	1574	1573
20	1574	1574	1574	1574	1573	1573
30	1574	1574	1574	1573	1573	1573
40	1574	1573	1573	1573	1573	1572
50	1573	1573	1573	1573	1572	1572
60	1573	1573	1572	1572	1572	1572
70	1573	1572	1572	1572	1572	1572
80	1572	1572	1572	1572	1572	1571
90	1572	1572	1572	1572	1571	1571

Log D_2

[See p. 52. −10 is to be supplied as part of the characteristic. D_2 is always positive]

ϕ \ h	0 meter	1,000 meters	2,000 meters	3,000 meters	4,000 meters	5,000 meters
°						
0	3.1972	3.1971	3.1970	3.1970	3.1969	3.1968
10	.1971	.1971	.1970	.1970	.1969	.1968
20	.1971	.1970	.1970	.1969	.1968	.1968
30	.1970	.1970	.1969	.1968	.1968	.1967
40	.1969	.1969	.1968	.1967	.1966	.1966
50	.1968	.1968	.1967	.1966	.1965	.1965
60	.1967	.1966	.1966	.1965	.1965	.1964
70	.1966	.1966	.1965	.1964	.1964	.1963
80	.1966	.1965	.1965	.1964	.1963	.1962
90	3.1966	3.1965	3.1964	3.1964	3.1963	3.1962

V.—CONVERSION TABLES

Lengths—Feet to meters (from 1 to 1,000 units)

[Reduction factor: 1 foot = 0.3048006096 meter]

Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters
0	0.0	50	15.24003	100	30.48006	150	45.72009	200	60.96012
1	0.30480	1	15.54483	1	30.78486	1	46.02489	1	61.26492
2	0.60960	2	15.84963	2	31.08966	2	46.32969	2	61.56972
3	0.91440	3	16.15443	3	31.39446	3	46.63449	3	61.87452
4	1.21920	4	16.45923	4	31.69926	4	46.93929	4	62.17932
5	1.52400	5	16.76403	5	32.00406	5	47.24409	5	62.48412
6	1.82880	6	17.06883	6	32.30886	6	47.54890	6	62.78893
7	2.13360	7	17.37363	7	32.61367	7	47.85370	7	63.09379
8	2.43840	8	17.67844	8	32.91847	8	48.15850	8	63.39853
9	2.74321	9	17.98324	9	33.22327	9	48.46330	9	63.70333
10	3.04801	60	18.28804	110	33.52807	160	48.76810	210	64.00813
1	3.35281	1	18.59284	1	33.83287	1	49.07290	1	64.31293
2	3.65761	2	18.89764	2	34.13767	2	49.37770	2	64.61773
3	3.96241	3	19.20244	3	34.44247	3	49.68250	3	64.92253
4	4.26721	4	19.50724	4	34.74727	4	49.98730	4	65.22733
5	4.57201	5	19.81204	5	35.05207	5	50.29210	5	65.53213
6	4.87681	6	20.11684	6	35.35687	6	50.59690	6	65.83693
7	5.18161	7	20.42164	7	35.66167	7	50.90170	7	66.14173
8	5.48641	8	20.72644	8	35.96647	8	51.20650	8	66.44653
9	5.79121	9	21.03124	9	36.27127	9	51.51130	9	66.75133
20	6.09601	70	21.33604	120	36.57607	170	51.81610	220	67.05613
1	6.40081	1	21.64084	1	36.88087	1	52.12090	1	67.36093
2	6.70561	2	21.94564	2	37.18567	2	52.42570	2	67.66574
3	7.01041	3	22.25044	3	37.49047	3	52.73051	3	67.97054
4	7.31521	4	22.55525	4	37.79528	4	53.03531	4	68.27534
5	7.62002	5	22.86005	5	38.10008	5	53.34011	5	68.58014
6	7.92482	6	23.16485	6	38.40488	6	53.64491	6	68.88494
7	8.22962	7	23.46965	7	38.70968	7	53.94971	7	69.18974
8	8.53442	8	23.77445	8	39.01448	8	54.25451	8	69.49454
9	8.83922	9	24.07925	9	39.31928	9	54.55931	9	69.79934
30	9.14402	80	24.38405	130	39.62408	180	54.86411	230	70.10414
1	9.44882	1	24.68885	1	39.92888	1	55.16891	1	70.40894
2	9.75362	2	24.99365	2	40.23368	2	55.47371	2	70.71374
3	10.05842	3	25.29845	3	40.53848	3	55.77851	3	71.01854
4	10.36322	4	25.60325	4	40.84328	4	56.08331	4	71.32334
5	10.66802	5	25.90805	5	41.14808	5	56.38811	5	71.62814
6	10.97282	6	26.21285	6	41.45288	6	56.69291	6	71.93294
7	11.27762	7	26.51765	7	41.75768	7	56.99771	7	72.23774
8	11.58242	8	26.82245	8	42.06248	8	57.30251	8	72.54255
9	11.88722	9	27.12725	9	42.36728	9	57.60732	9	72.84735
40	12.19202	90	27.43205	140	42.67209	190	57.91212	240	73.15215
1	12.49682	1	27.73686	1	42.97689	1	58.21692	1	73.45695
2	12.80163	2	28.04166	2	43.28169	2	58.52172	2	73.76175
3	13.10643	3	28.34646	3	43.58649	3	58.82652	3	74.06655
4	13.41123	4	28.65126	4	43.89129	4	59.13132	4	74.37135
5	13.71603	5	28.95606	5	44.19609	5	59.43612	5	74.67615
6	14.02083	6	29.26086	6	44.50089	6	59.74092	6	74.98095
7	14.32563	7	29.56566	7	44.80569	7	60.04572	7	75.28575
8	14.63043	8	29.87046	8	45.11049	8	60.35052	8	75.59055
9	14.93523	9	30.17526	9	45.41529	9	60.65532	9	75.89535

Lengths—Feet to meters (from 1 to 1,000 units)—Continued

Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters
250	76.20015	**300**	91.44018	**350**	106.68021	**400**	121.92024	**450**	137.16027
1	76.50495	1	91.74498	1	106.98501	1	122.22504	1	137.46507
2	76.80975	2	92.04978	2	107.28981	2	122.52985	2	137.76988
3	77.11455	3	92.35458	3	107.59462	3	122.83465	3	138.07468
4	77.41935	4	92.65939	4	107.89942	4	123.13945	4	138.37948
5	77.72416	5	92.96419	5	108.20422	5	123.44425	5	138.68428
6	78.02896	6	93.26899	6	108.50902	6	123.74905	6	138.98908
7	78.33376	7	93.57379	7	108.81382	7	124.05385	7	139.29388
8	78.63856	8	93.87859	8	109.11862	8	124.35865	8	139.59868
9	78.94336	9	94.18339	9	109.42342	9	124.66345	9	139.90348
260	79.24816	**310**	94.48819	**360**	109.72822	**410**	124.96825	**460**	140.20828
1	79.55296	1	94.79299	1	110.03302	1	125.27305	1	140.51308
2	79.85776	2	95.09779	2	110.33782	2	125.57785	2	140.81788
3	80.16256	3	95.40259	3	110.64262	3	125.88265	3	141.12268
4	80.46736	4	95.70739	4	110.94742	4	126.18745	4	141.42748
5	80.77216	5	96.01219	5	111.25222	5	126.49225	5	141.73228
6	81.07696	6	96.31699	6	111.55702	6	126.79705	6	142.03708
7	81.38176	7	96.62179	7	111.86182	7	127.10185	7	142.34188
8	81.68656	8	96.92659	8	112.16662	8	127.40665	8	142.64669
9	81.99136	9	97.23139	9	112.47142	9	127.71146	9	142.95149
270	82.29616	**320**	97.53620	**370**	112.77623	**420**	128.01626	**470**	143.25629
1	82.60097	1	97.84100	1	113.08103	1	128.32106	1	143.56109
2	82.90577	2	98.14580	2	113.38583	2	128.62586	2	143.86589
3	83.21057	3	98.45060	3	113.69063	3	128.93066	3	144.17069
4	83.51537	4	98.75540	4	113.99543	4	129.23546	4	144.47549
5	83.82017	5	99.06020	5	114.30023	5	129.54026	5	144.78029
6	84.12497	6	99.36500	6	114.60503	6	129.84506	6	145.08509
7	84.42977	7	99.66980	7	114.90983	7	130.14986	7	145.38989
8	84.73457	8	99.97460	8	115.21463	8	130.45466	8	145.69469
9	85.03937	9	100.27940	9	115.51943	9	130.75946	9	145.99949
280	85.34417	**330**	100.58420	**380**	115.82423	**430**	131.06426	**480**	146.30429
1	85.64897	1	100.88900	1	116.12903	1	131.36906	1	146.60909
2	85.95377	2	101.19380	2	116.43383	2	131.67386	2	146.91389
3	86.25857	3	101.49860	3	116.73863	3	131.97866	3	147.21869
4	86.56337	4	101.80340	4	117.04343	4	132.28346	4	147.52350
5	86.86817	5	102.10820	5	117.34823	5	132.58827	5	147.82830
6	87.17297	6	102.41300	6	117.65304	6	182.89307	6	148.13310
7	87.47777	7	102.71781	7	117.95784	7	133.19787	7	148.43790
8	87.78258	8	103.02261	8	118.26264	8	133.50267	8	148.74270
9	88.08738	9	103.32741	9	118.56744	9	133.80747	9	149.04750
290	88.39218	**340**	103.63221	**390**	118.87224	**440**	134.11227	**490**	149.35230
1	88.69698	1	103.93701	1	119.17704	1	134.41707	1	149.65710
2	89.00178	2	104.24181	2	119.48184	2	134.72187	2	149.96190
3	89.30658	3	104.54661	3	119.78664	3	135.02667	3	150.26670
4	89.61138	4	104.85141	4	120.09144	4	135.33147	4	150.57150
5	89.91618	5	105.15621	5	120.39624	5	135.63627	5	150.87630
6	90.22098	6	105.46101	6	120.70104	6	135.94107	6	151.18110
7	90.52578	7	105.76581	7	121.00584	7	136.24587	7	151.48590
8	90.83058	8	106.07061	8	121.31064	8	136.55067	8	151.79070
9	91.13538	9	106.37541	9	121.61544	9	136.85547	9	152.09550

Lengths—Feet to meters (from 1 to 1000 units)—Continued

Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters
500	152.40030	550	167.64034	600	182.88037	650	198.12040	700	213.36043
1	152.70511	1	167.94514	1	183.18517	1	198.42520	1	213.66523
2	153.00991	2	168.24994	2	183.48997	2	198.73000	2	213.97003
3	153.31471	3	168.55474	3	183.79477	3	199.03480	3	214.27483
4	153.61951	4	168.85954	4	184.09957	4	199.33960	4	214.57963
5	153.92431	5	169.16434	5	184.40437	5	199.64440	5	214.88443
6	154.22911	6	169.46914	6	184.70917	6	199.94920	6	215.18923
7	154.53391	7	169.77394	7	185.01397	7	200.25400	7	215.49403
8	154.83871	8	170.07874	8	185.31877	8	200.55880	8	215.79883
9	155.14351	9	170.38354	9	185.62357	9	200.86360	9	216.10363
510	155.44831	560	170.68834	610	185.92837	660	201.16840	710	216.40843
1	155.75311	1	170.99314	1	186.23317	1	201.47320	1	216.71323
2	156.05791	2	171.29794	2	186.53797	2	201.77800	2	217.01803
3	156.36271	3	171.60274	3	186.84277	3	202.08280	3	217.32283
4	156.66751	4	171.90754	4	187.14757	4	202.38760	4	217.62764
5	156.97231	5	172.21234	5	187.45237	5	202.69241	5	217.93244
6	157.27711	6	172.51715	6	187.75718	6	202.99721	6	218.23724
7	157.58192	7	172.82195	7	188.06198	7	203.30201	7	218.54204
8	157.88672	8	173.12675	8	188.36678	8	203.60681	8	218.84684
9	158.19152	9	173.43155	9	188.67158	9	203.91161	9	219.15164
520	158.49632	570	173.73635	620	188.97638	670	204.21641	720	219.45644
1	158.80112	1	174.04115	1	189.28118	1	204.52121	1	219.76124
2	159.10592	2	174.34595	2	189.58598	2	204.82601	2	220.06604
3	159.41072	3	174.65075	3	189.89078	3	205.13081	3	220.37084
4	159.71552	4	174.95555	4	190.19558	4	205.43561	4	220.67564
5	160.02032	5	175.26035	5	190.50038	5	205.74041	5	220.98044
6	160.32512	6	175.56515	6	190.80518	6	206.04521	6	221.28524
7	160.62992	7	175.86995	7	191.10998	7	206.35001	7	221.59004
8	160.93472	8	176.17475	8	191.41478	8	206.65481	8	221.89484
9	161.23952	9	176.47955	9	191.71958	9	206.95961	9	222.19964
530	161.54432	580	176.78435	630	192.02438	680	207.26441	730	222.50445
1	161.84912	1	177.08915	1	192.32918	1	207.56922	1	222.80925
2	162.15392	2	177.39395	2	192.63399	2	207.87402	2	223.11405
3	162.45872	3	177.69876	3	192.93879	3	208.17882	3	223.41885
4	162.76353	4	178.00356	4	193.24359	4	208.48362	4	223.72365
5	163.06833	5	178.30836	5	193.54839	5	208.78842	5	224.02845
6	163.37313	6	178.61316	6	193.85319	6	209.09322	6	224.33325
7	163.67793	7	178.91796	7	194.15799	7	209.39802	7	224.63805
8	163.98273	8	179.22276	8	194.46279	8	209.70282	8	224.94285
9	164.28753	9	179.52756	9	194.76759	9	210.00762	9	225.24765
540	164.59233	590	179.83236	640	195.07239	690	210.31242	740	225.55245
1	164.89713	1	180.13716	1	195.37719	1	210.61722	1	225.85725
2	165.20193	2	180.44196	2	195.68199	2	210.92202	2	226.16205
3	165.50673	3	180.74676	3	195.98679	3	211.22682	3	226.46685
4	165.81153	4	181.05156	4	196.29159	4	211.53162	4	226.77165
5	166.11633	5	181.35636	5	196.59639	5	211.83642	5	227.07645
6	166.42113	6	181.66116	6	196.90119	6	212.14122	6	227.38125
7	166.72593	7	181.96596	7	197.20599	7	212.44602	7	227.68606
8	167.03073	8	182.27076	8	197.51080	8	212.75083	8	227.99086
9	167.33553	9	182.57557	9	197.81560	9	213.05563	9	228.29566

Lengths—Feet to meters (from 1 to 1,000 units)—Continued

Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters
750	228.60046	800	243.84049	850	259.08052	900	274.32055	950	289.56058
1	228.90526	1	244.14529	1	259.38532	1	274.62535	1	289.86538
2	229.21006	2	244.45009	2	259.69012	2	274.93015	2	290.17018
3	229.51486	3	244.75489	3	259.99492	3	275.23495	3	290.47498
4	229.81966	4	245.05969	4	260.29972	4	275.53975	4	290.77978
5	230.12446	5	245.36449	5	260.60452	5	275.84455	5	291.08458
6	230.42926	6	245.66929	6	260.90932	6	276.14935	6	291.38938
7	230.73406	7	245.97409	7	261.21412	7	276.45415	7	291.69418
8	231.03886	8	246.27889	8	261.51892	8	276.75895	8	291.99898
9	231.34366	9	246.58369	9	261.82372	9	277.06375	9	292.30378
760	231.64846	810	246.88849	860	262.12852	910	277.36855	960	292.60859
1	231.95326	1	247.19329	1	262.43332	1	277.67336	1	292.91339
2	232.25806	2	247.49809	2	262.73813	2	277.97816	2	293.21819
3	232.56287	3	247.80290	3	263.04293	3	278.28296	3	293.52299
4	232.86767	4	248.10770	4	263.34773	4	278.58776	4	293.82779
5	233.17247	5	248.41250	5	263.65253	5	278.89256	5	294.13259
6	233.47727	6	248.71730	6	263.95733	6	279.19736	6	294.43739
7	233.78207	7	249.02210	7	264.26213	7	279.50216	7	294.74219
8	234.08687	8	249.32690	8	264.56693	8	279.80696	8	295.04699
9	234.39167	9	249.63170	9	264.87173	9	280.11176	9	295.35179
770	234.69647	820	249.93650	870	265.17653	920	280.41656	970	295.65659
1	235.00127	1	250.24130	1	265.48133	1	280.72136	1	295.96139
2	235.30607	2	250.54610	2	265.78613	2	281.02616	2	296.26619
3	235.61087	3	250.85090	3	266.09093	3	281.33096	3	296.57099
4	235.91567	4	251.15570	4	266.39573	4	281.63576	4	296.87579
5	236.22047	5	251.46050	5	266.70053	5	281.94056	5	297.18059
6	236.52527	6	251.76530	6	267.00533	6	282.24536	6	297.48539
7	236.83007	7	252.07010	7	267.31013	7	282.55017	7	297.79020
8	237.13487	8	252.37490	8	267.61494	8	282.85497	8	298.09500
9	237.43967	9	252.67971	9	267.91974	9	283.15977	9	298.39980
780	237.74448	830	252.98451	880	268.22454	930	283.46457	980	298.70460
1	238.04928	1	253.28931	1	268.52934	1	283.76937	1	299.00940
2	238.35408	2	253.59411	2	268.83414	2	284.07417	2	299.31420
3	238.65888	3	253.89891	3	269.13894	3	284.37897	3	299.61900
4	238.96368	4	254.20371	4	269.44374	4	284.68377	4	299.92380
5	239.26848	5	254.50851	5	269.74854	5	284.98857	5	300.22860
6	239.57328	6	254.81331	6	270.05334	6	285.29337	6	300.53340
7	239.87808	7	255.11811	7	270.35814	7	285.59817	7	300.83820
8	240.18288	8	255.42291	8	270.66294	8	285.90297	8	301.14300
9	240.48768	9	255.72771	9	270.96774	9	286.20777	9	301.44780
790	240.79248	840	256.03251	890	271.27254	940	286.51257	990	301.75260
1	241.09728	1	256.33731	1	271.57734	1	286.81737	1	302.05740
2	241.40208	2	256.64211	2	271.88214	2	287.12217	2	302.36220
3	241.70688	3	256.94691	3	272.18694	3	287.42697	3	302.66701
4	242.01168	4	257.25171	4	272.49174	4	287.73178	4	302.97181
5	242.31648	5	257.55652	5	272.79655	5	288.03658	5	303.27661
6	242.62129	6	257.86132	6	273.10135	6	288.34138	6	303.58141
7	242.92609	7	258.16612	7	273.40615	7	288.64618	7	303.88621
8	243.23089	8	258.47092	8	273.71095	8	288.95098	8	304.19101
9	243.53569	9	258.77572	9	274.01575	9	289.25578	9	304.49581

Lengths—Meters to feet (from 1 to 1,000 units)

[Reduction factor: 1 meter=3.280833333 feet]

Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet
0		50	164.04167	100	328.08333	150	492.12500	200	656.16667
1	3.28083	1	167.32250	1	331.36417	1	495.40583	1	659.44750
2	6.56167	2	170.60333	2	334.64500	2	498.68667	2	662.72833
3	9.84250	3	173.88417	3	337.92583	3	501.96750	3	666.00917
4	13.12333	4	177.16500	4	341.20667	4	505.24833	4	669.29000
5	16.40417	5	180.44583	5	344.48750	5	508.52917	5	672.57083
6	19.68500	6	183.72667	6	347.76833	6	511.81000	6	675.85167
7	22.96583	7	187.00750	7	351.04917	7	515.09083	7	679.13250
8	26.24667	8	190.28833	8	354.33000	8	518.37167	8	682.41333
9	29.52750	9	193.56917	9	357.61083	9	521.65250	9	685.69417
10	32.80833	60	196.85000	110	360.89167	160	524.93333	210	688.97500
1	36.08917	1	200.13083	1	364.17250	1	528.21417	1	692.25583
2	39.37000	2	203.41167	2	367.45333	2	531.49500	2	695.53667
3	42.65083	3	206.69250	3	370.73417	3	534.77583	3	698.81750
4	45.93167	4	209.97333	4	374.01500	4	538.05667	4	702.09833
5	49.21250	5	213.25417	5	377.29583	5	541.33750	5	705.37917
6	52.49333	6	216.53500	6	380.57667	6	544.61833	6	708.66000
7	55.77417	7	219.81583	7	383.85750	7	547.89917	7	711.94083
8	59.05500	8	223.09667	8	387.13833	8	551.18000	8	715.22167
9	62.33583	9	226.37750	9	390.41917	9	554.46083	9	718.50250
20	65.61667	70	229.65833	120	393.70000	170	557.74167	220	721.78333
1	68.89750	1	232.93917	1	396.98083	1	561.02250	1	725.06417
2	72.17833	2	236.22000	2	400.26167	2	564.30333	2	728.34500
3	75.45917	3	239.50083	3	403.54250	3	567.58417	3	731.62583
4	78.74000	4	242.78167	4	406.82333	4	570.86500	4	734.90667
5	82.02083	5	246.06250	5	410.10417	5	574.14583	5	738.18750
6	85.30167	6	249.34333	6	413.38500	6	577.42667	6	741.46833
7	88.58250	7	252.62417	7	416.66583	7	580.70750	7	744.74917
8	91.86333	8	255.90500	8	419.94667	8	583.98833	8	748.03000
9	95.14417	9	259.18583	9	423.22750	9	587.26917	9	751.31083
30	98.42500	80	262.46667	130	426.50833	180	590.55000	230	754.59167
1	101.70583	1	265.74750	1	429.78917	1	593.83083	1	757.87250
2	104.98667	2	269.02833	2	433.07000	2	597.11167	2	761.15333
3	108.26750	3	272.30917	3	436.35083	3	600.39250	3	764.43417
4	111.54833	4	275.59000	4	439.63167	4	603.67333	4	767.71500
5	114.82917	5	278.87083	5	442.91250	5	606.95417	5	770.99583
6	118.11000	6	282.15167	6	446.19333	6	610.23500	6	774.27667
7	121.39083	7	285.43250	7	449.47417	7	613.51583	7	777.55750
8	124.67167	8	288.71333	8	452.75500	8	616.79667	8	780.83833
9	127.95250	9	291.99417	9	456.03583	9	620.07750	9	784.11917
40	131.23333	90	295.27500	140	459.31667	190	623.35833	240	787.40000
1	134.51417	1	298.55583	1	462.59750	1	626.63917	1	790.68083
2	137.79500	2	301.83667	2	465.87833	2	629.92000	2	793.96167
3	141.07583	3	305.11750	3	469.15917	3	633.20083	3	797.24250
4	144.35667	4	308.39833	4	472.44000	4	636.48167	4	800.52333
5	147.63750	5	311.67917	5	475.72083	5	639.76250	5	803.80417
6	150.91833	6	314.96000	6	479.00167	6	643.04333	6	807.08500
7	154.19917	7	318.24083	7	482.28250	7	646.32417	7	810.36583
8	157.48000	8	321.52167	8	485.56333	8	649.60500	8	813.64667
9	160.76083	9	324.80250	9	488.84417	9	652.88583	9	816.92750

Lengths—Meters to feet (from 1 to 1,000 units)—Continued

Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet
250	820.20833	**300**	984.25000	**350**	1,148.29167	**400**	1,312.33333	**450**	1,476.37500
1	823.48917	1	987.53083	1	1,151.57250	1	1,315.61417	1	1,479.65583
2	826.77000	2	990.81167	2	1,154.85333	2	1,318.89500	2	1,482.93667
3	830.05083	3	994.09250	3	1,158.13417	3	1,322.17583	3	1,486.21750
4	833.33167	4	997.37333	4	1,161.41500	4	1,325.45667	4	1,489.49833
5	836.61250	5	1,000.65417	5	1,164.69583	5	1,328.73750	5	1,492.77917
6	839.89333	6	1,003.93500	6	1,167.97667	6	1,332.01833	6	1,496.06000
7	843.17417	7	1,007.21583	7	1,171.25750	7	1,335.29917	7	1,499.34083
8	846.45500	8	1,010.49667	8	1,174.53833	8	1,338.58000	8	1,502.62167
9	849.73583	9	1,013.77750	9	1,177.81917	9	1,341.86083	9	1,505.90250
260	853.01667	**310**	1,017.05833	**360**	1,181.10000	**410**	1,345.14167	**460**	1,509.18333
1	856.29750	1	1,020.33917	1	1,184.38083	1	1,348.42250	1	1,512.46417
2	859.57833	2	1,023.62000	2	1,187.66167	2	1,351.70333	2	1,515.74500
3	862.85917	3	1,026.90083	3	1,190.94250	3	1,354.98417	3	1,519.02583
4	866.14000	4	1,030.18167	4	1,194.22333	4	1,358.26500	4	1,522.30667
5	869.42083	5	1,033.46250	5	1,197.50417	5	1,361.54583	5	1,525.58750
6	872.70167	6	1,036.74333	6	1,200.78500	6	1,364.82667	6	1,528.86833
7	875.98250	7	1,040.02417	7	1,204.06583	7	1,368.10750	7	1,532.14917
8	879.26333	8	1,043.30500	8	1,207.34667	8	1,371.38833	8	1,535.43000
9	882.54417	9	1,046.58583	9	1,210.62750	9	1,374.66917	9	1,538.71083
270	885.82500	**320**	1,049.86667	**370**	1,213.90833	**420**	1,377.95000	**470**	1,541.99167
1	889.10583	1	1,053.14750	1	1,217.18917	1	1,381.23083	1	1,545.27250
2	892.38667	2	1,056.42833	2	1,220.47000	2	1,384.51167	2	1,548.55333
3	895.66750	3	1,059.70917	3	1,223.75083	3	1,387.79250	3	1,551.83417
4	898.94833	4	1,062.99000	4	1,227.03167	4	1,391.07333	4	1,555.11500
5	902.22917	5	1,066.27083	5	1,230.31250	5	1,394.35417	5	1,558.39583
6	905.51000	6	1,069.55167	6	1,233.59333	6	1,397.63500	6	1,561.67667
7	908.79083	7	1,072.83250	7	1,236.87417	7	1,400.91583	7	1,564.95750
8	912.07167	8	1,076.11333	8	1,240.15500	8	1,404.19667	8	1,568.23833
9	915.35250	9	1,079.39417	9	1,243.43583	9	1,407.47750	9	1,571.51917
280	918.63333	**330**	1,082.67500	**380**	1,246.71667	**430**	1,410.75833	**480**	1,574.80000
1	921.91417	1	1,085.95583	1	1,249.99750	1	1,414.03917	1	1,578.08083
2	925.19500	2	1,089.23667	2	1,253.27833	2	1,417.32000	2	1,581.36167
3	928.47583	3	1,092.51750	3	1,256.55917	3	1,420.60083	3	1,584.64250
4	931.75667	4	1,095.79833	4	1,259.84000	4	1,423.88167	4	1,587.92333
5	935.03750	5	1,099.07917	5	1,263.12083	5	1,427.16250	5	1,591.20417
6	938.31833	6	1,102.36000	6	1,266.40167	6	1,430.44333	6	1,594.48500
7	941.59917	7	1,105.64083	7	1,269.68250	7	1,433.72417	7	1,597.76583
8	944.88000	8	1,108.92167	8	1,272.96333	8	1,437.00500	8	1,601.04667
9	948.16083	9	1,112.20250	9	1,276.24417	9	1,440.28583	9	1,604.32750
290	951.44167	**340**	1,115.48333	**390**	1,279.52500	**440**	1,443.56667	**490**	1,607.60833
1	954.72250	1	1,118.76417	1	1,282.80583	1	1,446.84750	1	1,610.88917
2	958.00333	2	1,122.04500	2	1,286.08667	2	1,450.12833	2	1,614.17000
3	961.28417	3	1,125.32583	3	1,289.36750	3	1,453.40917	3	1,617.45083
4	964.56500	4	1,128.60667	4	1,292.64833	4	1,456.69000	4	1,620.73167
5	967.84583	5	1,131.88750	5	1,295.92917	5	1,459.97083	5	1,624.01250
6	971.12667	6	1,135.16833	6	1,299.21000	6	1,463.25167	6	1,627.29333
7	974.40750	7	1,138.44917	7	1,302.49083	7	1,466.53250	7	1,630.57417
8	977.68833	8	1,141.73000	8	1,305.77167	8	1,469.81333	8	1,633.85500
9	980.96917	9	1,145.01083	9	1,309.05250	9	1,473.09417	9	1,637.13583

Lengths—Meters to feet (from 1 to 1,000 units)—Continued

Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet
500	1,640.41667	**550**	1,804.45833	**600**	1,968.50000	**650**	2,132.54167	**700**	2,296.58333
1	1,643.69750	**1**	1,807.73917	**1**	1,971.78083	**1**	2,135.82250	**1**	2,299.86417
2	1,646.97833	**2**	1,811.02000	**2**	1,975.06167	**2**	2,139.10333	**2**	2,303.14500
3	1,650.25917	**3**	1,814.30083	**3**	1,978.34250	**3**	2,142.38417	**3**	2,306.42583
4	1,653.54000	**4**	1,817.58167	**4**	1,981.62333	**4**	2,145.66500	**4**	2,309.70667
5	1,656.82083	**5**	1,820.86250	**5**	1,984.90417	**5**	2,148.94583	**5**	2,312.98750
6	1,660.10167	**6**	1,824.14333	**6**	1,988.18500	**6**	2,152.22667	**6**	2,316.26833
7	1,663.38250	**7**	1,827.42417	**7**	1,991.46583	**7**	2,155.50750	**7**	2,319.54917
8	1,666.66333	**8**	1,830.70500	**8**	1,994.74667	**8**	2,158.78833	**8**	2,322.83000
9	1,669.94417	**9**	1,833.98583	**9**	1,998.02750	**9**	2,162.06917	**9**	2,326.11083
510	1,673.22500	**560**	1,837.26667	**610**	2,001.30833	**660**	2,165.35000	**710**	2,329.39167
1	1,676.50583	**1**	1,840.54750	**1**	2,004.58917	**1**	2,168.63083	**1**	2,332.67250
2	1,679.78667	**2**	1,843.82833	**2**	2,007.87000	**2**	2,171.91167	**2**	2,335.95333
3	1,683.06750	**3**	1,847.10917	**3**	2,011.15083	**3**	2,175.19250	**3**	2,339.23417
4	1,686.34833	**4**	1,850.39000	**4**	2,014.43167	**4**	2,178.47333	**4**	2,342.51500
5	1,689.62917	**5**	1,853.67083	**5**	2,017.71250	**5**	2,181.75417	**5**	2,345.79583
6	1,692.91000	**6**	1,856.95167	**6**	2,020.99333	**6**	2,185.03500	**6**	2,349.07667
7	1,696.19083	**7**	1,860.23250	**7**	2,024.27417	**7**	2,188.31583	**7**	2,352.35750
8	1,699.47167	**8**	1,863.51333	**8**	2,027.55500	**8**	2,191.59667	**8**	2,355.63833
9	1,702.75250	**9**	1,866.79417	**9**	2,030.83583	**9**	2,194.87750	**9**	2,358.91917
520	1,706.03333	**570**	1,870.07500	**620**	2,034.11667	**670**	2,198.15833	**720**	2,362.20000
1	1,709.31417	**1**	1,873.35583	**1**	2,037.39750	**1**	2,201.43917	**1**	2,365.48083
2	1,712.59500	**2**	1,876.63667	**2**	2,040.67833	**2**	2,204.72000	**2**	2,368.76167
3	1,715.87583	**3**	1,879.91750	**3**	2,043.95917	**3**	2,208.00083	**3**	2,372.04250
4	1,719.15667	**4**	1,883.19833	**4**	2,047.24000	**4**	2,211.28167	**4**	2,375.32333
5	1,722.43750	**5**	1,886.47917	**5**	2,050.52083	**5**	2,214.56250	**5**	2,378.60417
6	1,725.71833	**6**	1,889.76000	**6**	2,053.80167	**6**	2,217.84333	**6**	2,381.88500
7	1,728.99917	**7**	1,893.04083	**7**	2,057.08250	**7**	2,221.12417	**7**	2,385.16583
8	1,732.28000	**8**	1,896.32167	**8**	2,060.36333	**8**	2,224.40500	**8**	2,388.44667
9	1,735.56083	**9**	1,899.60250	**9**	2,063.64417	**9**	2,227.68583	**9**	2,391.72750
530	1,738.84167	**580**	1,902.88333	**630**	2,066.92500	**680**	2,230.96667	**730**	2,395.00833
1	1,742.12250	**1**	1,906.16417	**1**	2,070.20583	**1**	2,234.24750	**1**	2,398.28917
2	1,745.40333	**2**	1,909.44500	**2**	2,073.48667	**2**	2,237.52833	**2**	2,401.57000
3	1,748.68417	**3**	1,912.72583	**3**	2,076.76750	**3**	2,240.80917	**3**	2,404.85083
4	1,751.96500	**4**	1,916.00667	**4**	2,080.04833	**4**	2,244.09000	**4**	2,408.13167
5	1,755.24583	**5**	1,919.28750	**5**	2,083.32917	**5**	2,247.37083	**5**	2,411.41250
6	1,758.52667	**6**	1,922.56833	**6**	2,086.61000	**6**	2,250.65167	**6**	2,414.69333
7	1,761.80750	**7**	1,925.84917	**7**	2,089.89083	**7**	2,253.93250	**7**	2,417.97417
8	1,765.08833	**8**	1,929.13000	**8**	2,093.17167	**8**	2,257.21333	**8**	2,421.25500
9	1,768.36917	**9**	1,932.41083	**9**	2,096.45250	**9**	2,260.49417	**9**	2,424.53583
540	1,771.65000	**590**	1,935.69167	**640**	2,099.73333	**690**	2,263.77500	**740**	2,427.81667
1	1,174.93083	**1**	1,938.97250	**1**	2,103.01417	**1**	2,267.05583	**1**	2,431.09750
2	1,778.21167	**2**	1,942.25333	**2**	2,106.29500	**2**	2,270.33667	**2**	2,434.37833
3	1,781.49250	**3**	1,945.53417	**3**	2,109.57583	**3**	2,273.61750	**3**	2,437.65917
4	1,784.77333	**4**	1,948.81500	**4**	2,112.85667	**4**	2,276.89833	**4**	2,440.94000
5	1,788.05417	**5**	1,952.09583	**5**	2,116.13750	**5**	2,280.17917	**5**	2,444.22083
6	1,791.33500	**6**	1,955.37667	**6**	2,119.41833	**6**	2,283.46000	**6**	2,447.50167
7	1,794.61583	**7**	1,958.65750	**7**	2,122.69917	**7**	2,286.74083	**7**	2,450.78250
8	1,797.89667	**8**	1,961.93833	**8**	2,125.98000	**8**	2,290.02167	**8**	2,454.06333
9	1,801.17750	**9**	1,965.21917	**9**	2,129.26083	**9**	2,293.30250	**9**	2,457.34417

Lengths—Meters to feet (from 1 to 1,000 units)—Continued

Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet
750	2,460.62500	800	2,624.66667	850	2,788.70833	900	2,952.75000	950	3,116.79167
1	2,463.90583	1	2,627.94750	1	2,791.98917	1	2,956.03083	1	3,120.07250
2	2,467.18667	2	2,631.22833	2	2,795.27000	2	2,959.31167	2	3,123.35333
3	2,470.46750	3	2,634.50917	3	2,798.55083	3	2,962.59250	3	3,126.63417
4	2,473.74833	4	2,637.79000	4	2,801.83167	4	2,965.87333	4	3,129.91500
5	2,477.02917	5	2,641.07083	5	2,805.11250	5	2,969.15417	5	3,133.19583
6	2,480.31000	6	2,644.35167	6	2,808.39333	6	2,972.43500	6	3,136.47667
7	2,483.59083	7	2,647.63250	7	2,811.67417	7	2,975.71583	7	3,139.75750
8	2,486.87167	8	2,650.91333	8	2,814.95500	8	2,978.99667	8	3,143.03833
9	2,490.15250	9	2,654.19417	9	2,818.23583	9	2,982.27750	9	3,146.31917
760	2,493.43333	810	2,657.47500	860	2,821.51667	910	2,985.55833	960	3,149.60000
1	2,496.71417	1	2,660.75583	1	2,824.79750	1	2,988.83917	1	3,152.88083
2	2,499.99500	2	2,664.03667	2	2,828.07833	2	2,992.12000	2	3,156.16167
3	2,503.27583	3	2,667.31750	3	2,831.35917	3	2,995.40083	3	3,159.44250
4	2,506.55667	4	2,670.59833	4	2,834.64000	4	2,998.68167	4	3,162.72333
5	2,509.83750	5	2,673.87917	5	2,837.92083	5	3,001.96250	5	3,166.00417
6	2,513.11833	6	2,677.16000	6	2,841.20167	6	3,005.24333	6	3,169.28500
7	2,516.39917	7	2,680.44083	7	2,844.48250	7	3,008.52417	7	3,172.56583
8	2,519.68000	8	3,683.72167	8	2,847.76333.	8	3,011.80500	8	3,175.84667
9	2,522.96083	9	2,687.00250	9	2,851.04417	9	3,015.08583	9	3,179.12750
770	2,526.24167	820	2,690.28333	870	2,854.32500	920	3,018.36667	970	3,182.40833
1	2,529.52250	1	2,693.56417	1	2,857.60583	1	3,021.64750	1	3,185.68917
2	2,532.80333	2	2,696.84500	2	2,860.88667	2	3,024.92833	2	3,188.97000
3	2,536.08417	3	2,700.12583	3	2,864.16750	3	3,028.20917	3	3,192.25083
4	2,539.36500	4	2,703.40667	4	2,867.44833	4	3,031.49000	4	3,195.53167
5	2,542.64583	5	2,706.68750	5	2,870.72917	5	3,034.77083	5	3,198.81250
6	2,545.92667	6	2,709.96833	6	2,874.01000	6	3,038.05167	6	3,202.09333
7	2,549.20750	7	2,713.24917	7	2,877.29083	7	3,041.33250	7	3,205.37417
8	2,552.48833	8	2,716.53000	8	2,880.57167	8	3,044.61333	8	3,208.65500
9	2,555.76917	9	2,719.81083	9	2,883.85250	9	3,047.89417	9	3,211.93583
780	2,559.05000	830	2,723.09167	880	2,887.13333	930	3,051.17500	980	3,215.21667
1	2,562.33083	1	2,726,37250	1	2,890.41417	1	3,054.45583	1	3,218.49750
2	2,565.61167	2	2,729.65333	2	2,893.69500	2	3,057.73667	2	3,221.77833
3	2,568.89250	3	2,732.93417	3	2,896.97583	3	3,061.01750	3	3,225.05917
4	2,572.17333	4	2,736.21500	4	2,900.25667	4	3,064.29833	4	3,228.34000
5	2,575.45417	5	2,739.49583	5	2,903.53750	5	3,067.57917	5	3,231.62083
6	2,578.73500	6	2,742.77667	6	2,906.81833	6	3,070.86000	6	3,234.90167
7	2,582.01583	7	2,746.05750	7	2,910.09917	7	3,074.14083	7	3,238.18250
8	2,585.29667	8	2,749.33833	8	2,913.38000	8	3,077.42167	8	3,241.46333
9	2,588.57750	9	2,752.61917	9	2,916.66083	9	3,080.70250	9	3,244.74417
790	2,591.85833	840	2,755.90000	890	2,919.94167	940	3,083.98333	990	3,248.02500
1	2,595.13917	1	2,759.18083	1	2,923.22250	1	3,087.26417	1	3,251.30583
2	2,598.42000	2	2,762.46167	2	2,926.50333	2	3,090.54500	2	3,254.58667
3	2,601.70083	3	2,765.74250	3	2,929.78417	3	3,093.82583	3	3,257.86750
4	2,604.98167	4	2,769.02333	4	2,933.06500	4	3,097.10667	4	3,261.14833
5	2,608.26250	5	2,772.30417	5	2,936.34583	5	3,100.38750	5	3,264.42917
6	2,611.54333	6	2,775.58500	6	2,939.62667	6	3,103.66833	6	3,267.71000
7	2,614.82417	7	2,778.86583	7	2,942.90750	7	3,106.94917	7	3,270.99083
8	2,618.10500	8	2,782.14667	8	2,946.18833	8	3,110.23000	8	3,274.27167
9	2,621.38583	9	2,785.42750	9	2,949.46917	9	3,113.51083	9	3,277.55250

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