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DEPARTMENT OF COMMERCE

U. S. COAST AND GEODETIC SURVEY

E. LESTER JONES, Superintendent

DESCRIPTION

OF THE

U. S. COAST AND GEODETIC SURVEY TIDE-PREDICTING MACHINE No. 2

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UNITED STATES COAST AND GEODETIC SURVEY TIDE-PREDICTING MACHINE NO. 2.

GENERAL STATEMENT.

One of the important functions of the Coast and Geodetic Survey is the preparation and publication of a tidal calendar, issued annually, from one to two years in advance, under the name of "Tide Tables." These tables give to the mariner, the engineer, and others interested, information as to the time and magnitude of the tidal fluctuations of the sea. They show directly the predicted time to the minute and the height to the nearest tenth of a foot of every high and low water in the year at most of the principal seaports of the world, and give indirectly, by means of an appended auxiliary table, the same information for more than 3,000 intermediate ports or stations.

While the civil calendar deals only with the positions of the important heavenly bodies, the preparation of the data for a tidal calendar involves also a knowledge of the effect upon the sea due to the force of attraction of the moon and sun. The laws governing the motion of liquid bodies under the influence of the force of attraction are well known; the tides upon a sphere covered completely by water of uniform depth could be foretold from astronomical data alone. But the continents, islands, and the great difference in the depths of the oceans prevent such ideal tides, as regards both time and magnitude; so that it is necessary to resort to observation in order to ascertain the elements characteristic of each station, before the height and time of future tides can be computed.

These observations are made mainly by means of automatic-recording tide gauges, in which the rise and fall of the water is made to move a pencil back and forth across a strip of paper, the latter being kept moving at uniform speed by clockwork, thus producing a continuous tide curve.

Owing to the extremely complicated nature of the motions of the moon and earth it was practically impossible to take account of more than the most important elements in predicting tides until there was devised, about 1867, by Sir William Thomson (later Lord Kelvin) a system for the reduction of tides known as "harmonic analysis." This system assumes, in the place of the apparent sun with varying

speed in an elliptical orbit inclined to the equator about 23½°, a ficticious or theoretical sun moving with uniform speed in a circular orbit in the plane of the equator, and likewise, for the real moon, which moves with varying speed in an elliptical orbit inclined about 5° to the ecliptic, a theoretical moon with uniform speed in a circular orbit also in the equatorial plane is assumed. It further provides for additional imaginary bodies or components, each with its own uniform speed and period of revolution, so designed that the whole, when combined in proper relation to each other, will represent accurately the very complex motions of the real sun and moon.

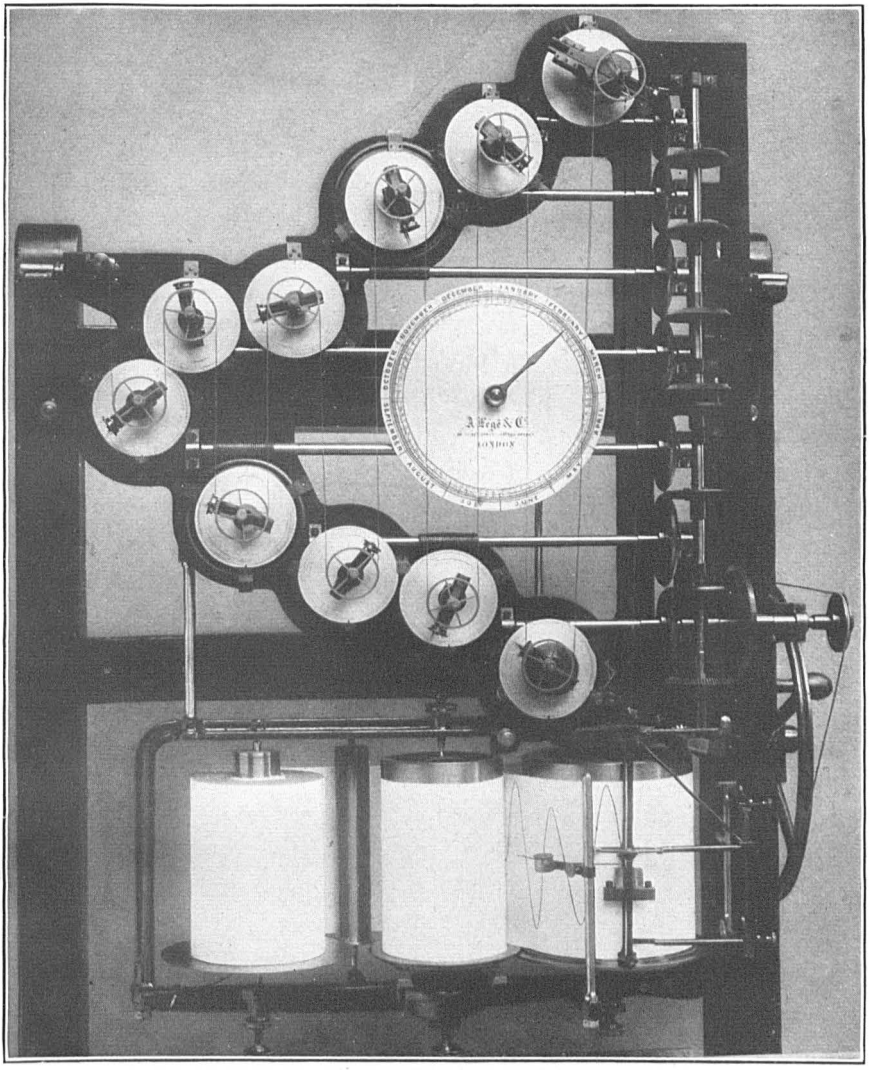
All the important theoretical bodies or components were carefully determined by Sir William Thomson and published in a table, later extended and improved by Sir George H. Darwin, in which their speeds are expressed in degrees and fractions to the seventh decimal place per mean solar hour. Each component is known by a symbol which is part of a system of notation adopted for the sake of convenience. This table has since been used the world over by all who have to deal with tidal computations.

As these components, properly applied to the theoretical mean sun and moon, practically correct their regular but incorrect motions to the irregular actual motions of the real sun and moon, they also act as correctives to the forces represented by them, and it is permissible to regard each of them as an independent tide-producing body.

Knowing the time element and consequently the period of revolution of each component, the complicated curve traced and marked into mean solar hour spaces by the tide gauge can be decomposed into its harmonic elements; provided, of course, that it extend over a sufficient length of time, preferably not less than one year, so that the effects of windstorms, freshets, etc., may be eliminated.

For convenience, the periods of revolution of the components are regarded as component days, and the twenty-fourth part of each period as a component hour. The length of the component day, expressed in mean solar hours, is equal to 360° divided by the hourly speed of the component, taken from Thomson's table above mentioned. A twenty-fourth part is, of course, the length in mean solar time of the component hour.

By starting from any convenient point at the beginning of the tide curve and measuring the heights of the tide throughout its length for a year or more at intervals of a component hour we obtain all the heights at the first, second, third, and so on to the twenty-fourth hour. These readings reduced to 24 means, expressed in feet and fractions, represent quite accurately the effect upon the sea due to the particular component at the station where the curve was made. In a general sense, these 24 hourly means may be regarded as representing a component tide curve. The semirange of this curve, or the



BRITISH ASSOCIATION TIDE-PREDICTING MACHINE, OR BRITISH TIDE PREDICTOR NO. 1.

height above mean sea level, is called the "amplitude" of the component and is a constant for that station.

Were the effects of the tidal forces upon the sea instantaneous, the time of high water would always coincide with that of the meridian passage of the tide-producing body. Inertia, friction, bodies of land, etc., however, retard the motion of the tidal wave, and unequally so, at different stations. The lagging of an observed high water behind the time when the tide-producing cause is in the meridian may also be considered as the effect of the combined laggings of all the individual components behind their respective meridian passages. To find the lag of a component, therefore, it is only necessary to determine the time from the highest point on the curve, expressed by the 24-hourly means, to the instant of the meridian passage of the component. This time, expressed in degrees of arc, is called the "epoch," and is, like the amplitude, a constant for the component at the station.

The amplitudes and epochs of the components entering into the tides have been determined by this analysis for practically all the important stations in the world, and have become, like astronomical data, through courtesy, exchange, and publication, common property. With their aid the tides can now be computed in advance by the summation of the formula—

$$h = H_0 + A \cos (a\,t + \alpha) + B \cos (b\,t + \beta) + C \cos (c\,t + \gamma) + \text{----------},$$

in which h is the height sought, H_0 is a constant expressing the height of mean sea level above the datum line, which usually represents the plane to which the soundings given on sailing charts are referred: A, B, C . . . . are the amplitudes of the successive components, expressed in feet and fractions, and a, b, c . . . . the hourly speeds of the same components, expressed in degrees of arc; t is the time in mean solar hours from the beginning of the prediction, usually January 1, midnight, to the instant for which the height is required. α is the interval from the beginning of the prediction back to the preceding high water of the component A, expressed in degrees; β, γ . . . . being like intervals for the components B, C . . . ..

Such computation of the tides, however, involves a very great amount of labor of a kind particularly wearing and subject to errors to an extent requiring additional labor in checking. The idea of carrying out these computations mechanically, therefore, soon suggested itself, both for reasons of economy as well as accuracy, and was materialized by the construction of various machines, the latest of which is here described.

The new United States Coast and Geodetic Survey tide-predicting machine which takes the place of the old Ferrel machine, formerly used for the prediction of the tides, was designed and constructed by

the Survey. It combines the essential principle of similar machines that had previously been built and includes also a number of new features.

The British tide machines merely trace a curve of the predicted tides from which the high and low waters are scaled off and tabulated for the printer.

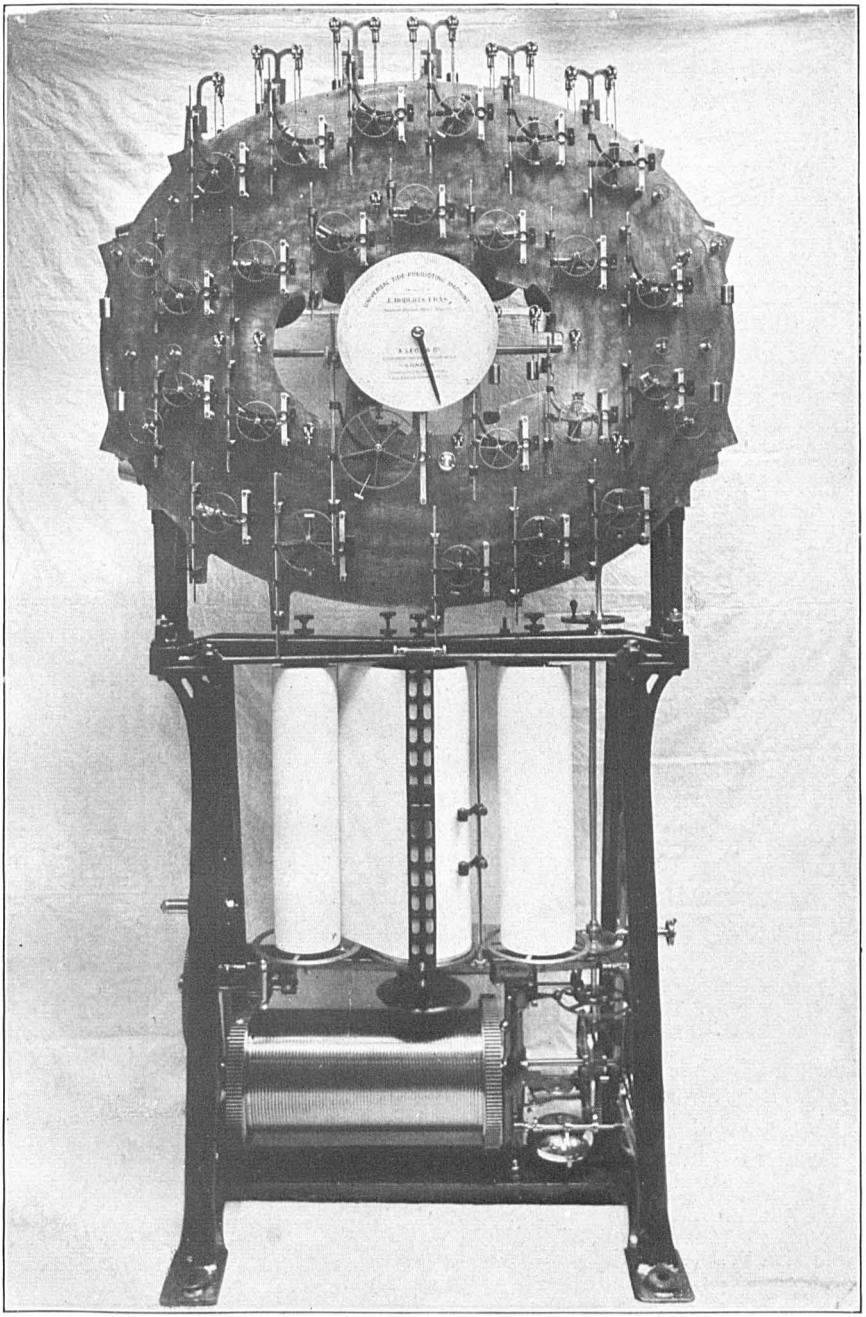
The Ferrel tide machine of the United States Coast and Geodetic Survey gives the time and height of the tide on its face, but does not trace any curve.

The United States Coast and Geodetic Survey tide-predicting machine No. 2 combines both features, for the time and height of the tide is shown upon dials, as in the Ferrel machine, and it traces a curve similar to that of the British machines.

The main part of the machine is in three sections, each consisting of two hard-rolled brass plates, upright, parallel, and about 6 or 7 inches apart. These plates support a system of sliding frames, shafts, gearing, and dials for the production and recording of the harmonic motion that represents the several elementary components of the tide. The sections are securely fastened to a heavy cast-iron base, and the entire machine occupies a space about 11 feet long, 2 feet wide, and 6 feet high, and is surrounded by a glass case sufficiently large to enable the operator to move around inside while adjusting the mahine.

The tide is assumed to be composed of many elementary components, each of which is due to the mean motion of the moon or of the sun or to some periodic variation in the motion of either. Each component may be represented by a simple harmonic motion, or graphically by a cosine curve which is defined by its amplitude and its epoch, or difference in time between the astronomical cause and the following high water of the component. The amplitudes and epochs of the components are known as harmonic constants and are obtained from the actual tidal record for any station by analysis. The harmonic constants for any one station remain practically the same for all time, but each station has a different set of constants.

The tide-predicting machine mechanically sums these elementary components for the station for which predictions are being made. The machine is designed to take account of 37 such components. For each component there is a crank attached to a shaft that is geared to rotate at a speed that is proportional to the actual speed of the astronomical cause producing that component. Each crank contains an adjustable pin, which may be set within certain limits at any distance from the axis of rotation. Before beginning the predictions for any station these pins are all set in accordance with the amplitudes of the components included in the tide at that station, and the initial positions of the cranks are set in accordance with the



ROBERTS TIDE-PREDICTING MACHINE, OR BRITISH TIDE PREDICTOR NO. 2.

epochs of the components, modified to correspond to the beginning of the period for which the predictions are to be made.

The machine is operated by turning a large hand crank, which, by a system of gearing, sets in motion all the component cranks to which reference has been made. The pins attached to these smaller cranks raise and lower a system of sliding frames, the motion of which is strictly harmonic. To each frame is attached a small pulley. A slender chain, one end of which is fixed, passes alternately downward under one pulley and upward over the next, connecting all, and transmitting the resultant motion of all the components to a dial in the face of the machine, where the height of the tide for any given time may be read directly. Other dials on the face of the machine indicate the time to which the heights correspond. As the machine is being operated, the time pointers pass rapidly around the dials indicating successive intervals of time, and simultaneously the height pointer moves back and forth over a circular scale, indicating the corresponding height of the tide. The height may be read on the dial at any time desired. The predictions as directly given in the tide tables are limited to the high and low waters, and the times and heights of these are the only ones that are usually recorded by the operator.

The machine also constructs automatically a curve, which graphically represents the tide and which may be used to verify the high and low waters as tabulated by the operator, and also to obtain the height of the tide at any intermediate hour that may be desired. This curve is drawn on a roll of paper about 6 inches wide and about 370 feet long, which is sufficient for the record for one year at the station. This paper moves through the machine at the rate of 1 foot for every day of record. A fixed pen draws a line representing mean sea level near the middle of the paper, and at the beginning of each hour makes a little jog in the line. Another pen moves up and down with the rising and falling of the tide, and this, together with the forward motion of the paper, produces a tide curve from which the height of the tide referred to mean sea level may be obtained for any desired time.

Another important feature of the machine is that which utilizes the fact well known in mathematics that the first derivative of a function becomes zero when the function itself is at a maximum or minimum. The high and low waters are the maxima and minima of the tides. The first derivative of the harmonic terms representing the tide gives a similar set of harmonic terms with initial points 90° distant from the original terms. Attached to the same shafts containing the component cranks already referred to, but on the opposite side of the machine is a similar set of cranks arranged at 90° or at right angles to the other. On this side there is a similar set of pins,

sliding frames, pulleys, and a summation chain. This system is operated simultaneously and with the same hand crank as for the first side, and indicates by means of an electric circuit and electromagnets the times of the high and low waters. At these times the machine is automatically stopped by a brake, which prevents the operator from thoughtlessly running by these points.

To set up the machine with the harmonic constants, predict, and tabulate the high and low waters for a year at any station requires from 10 to 15 hours, depending upon the complexity of the tides at the station.

PREVIOUS TIDE-PREDICTING MACHINES.

The machine to be described here, like almost every contrivance, apparatus, or machine in practical use, is based very largely upon what has been accomplished by others who previously labored in the same field.

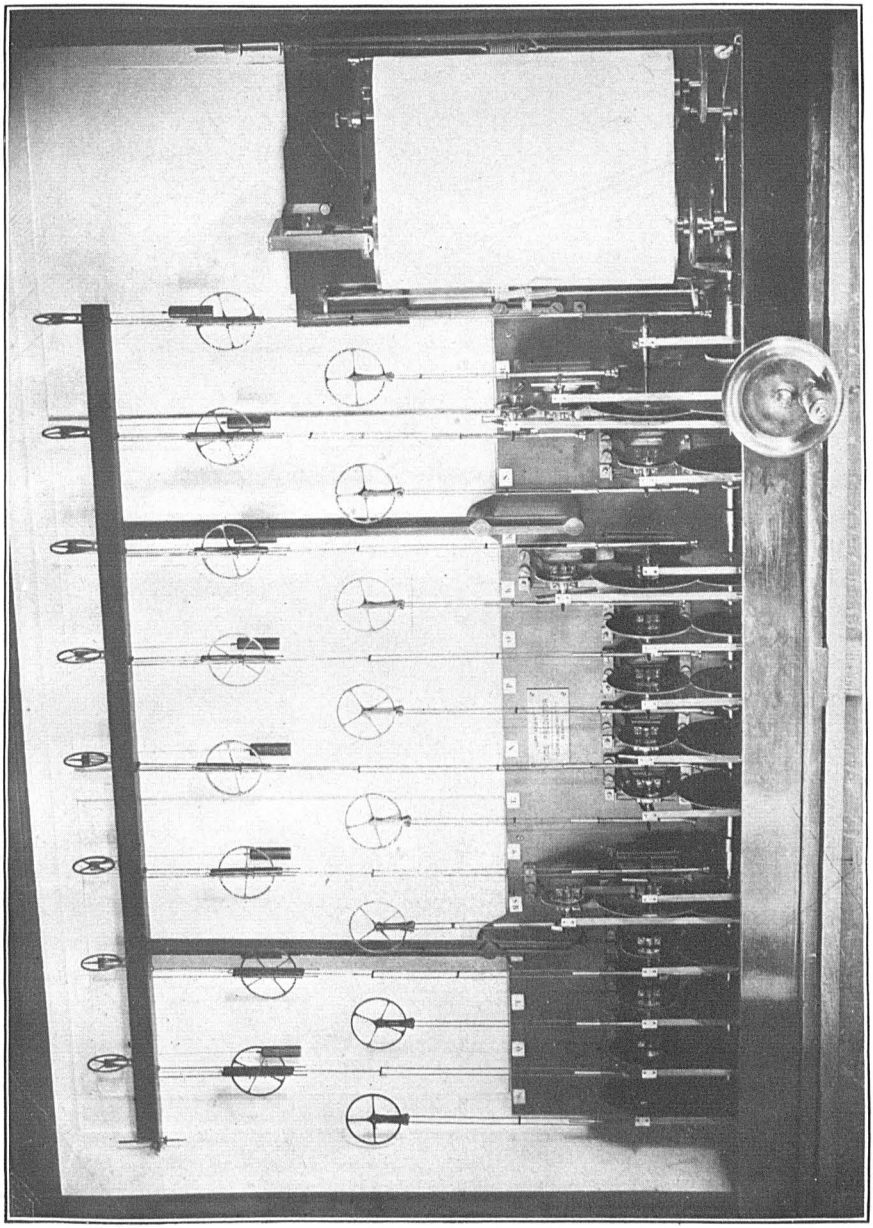
When harmonic analysis had been applied to the reduction of tidal observations, and it became practicable to take into account more than the most important elements entering into the composition of the tides, the need of mechanical means for relieving the brain of the methodical and wearying labor involved in their predictions became at once apparent.

BRITISH TIDE PREDICTOR NO. 1.

In 1872 Sir William Thomson (later Lord Kelvin) had made a rough model, with 8 components, for showing the feasibility of predicting tides mechanically. Under the auspices of the British Association he designed and had made in 1873 by A. Légé & Co., of London, the first tide-predicting machine, containing the 10 components M_2, S_2, N_2, K_2, Q_1, K_1, L_2, P_1, M_4, and $(MS)_4$. He gives credit to "A description of a machine for finding the numerical roots of equations, etc.," by the Rev. F. Bashford, read before the British association in 1845, for suggesting to him the means of summing mechanically the series expressing the tidal fluctuations of the sea employed in his machine. It is known as the British Association tide-predicting machine or tide predictor No. 1. (See plate 1.) It is described in Thomson and Tait's Natural Philosophy, second edition, Part I, pages 479, 480. Having been used but little, it was deposited in the South Kensington Museum.

BRITISH TIDE PREDICTOR NO. 2.

In 1879 a second tide-predicting machine was designed by E. Roberts, of the British Almanac office, and constructed under his direction for the Government of India by A. Légé & Co. It had originally 20 components, but in 1891 the component λ_2 was replaced by T_2, and $2N$, MN, $2MK$, and MK were added, making in all



BRITISH TIDE PREDICTOR NO. 3—NOW FRENCH TIDE PREDICTOR.

the 24 components given in Table B. The summing mechanism of the first machine was improved upon in this, at the suggestion of Sir William Thomson, by giving true linear harmonic motion to its elements. It is generally known under the name of the Roberts tide-predicting machine or British tide predictor No. 2. (See plate 2.) A description and drawings were published in The Engineer (London) of December 19, 1879, later in Volume XVI, pages 331–335, of the Great Trigonometrical Survey of India, and also in the supplement of the Scientific American, January 23, 1904.

It was used exclusively at first for the prediction of the tides for India, but later was used to predict tides for all the British Empire, with the exception of Great Britain and Ireland, the tidal predictions for which are still made from old tables. This tide predictor No. 2 was placed under the immediate charge of E. Roberts until the year 1903, when it was removed to the National Physical Laboratory.

BRITISH TIDE PREDICTOR NO. 3 (NOW IN FRANCE).

Some time later a third machine, designed by Sir William Thomson, was constructed by Kelvin & James White (Ltd.), Glasgow. It was fitted with 15 components (see Table B) and was known as British tide predictor No. 3. Greater rigidity than was attained with the conical gearing employed in the former machines seems to have been considered desirable by the designer, because spur wheels on short and strong shafts in place of conical gears were used for driving the components. It was described briefly in Thomson's Popular Lectures and Addresses, volume 3, pages 184–187, and also in the supplement of the Scientific American of January 23, 1904.

This machine, after being used a number of years by Lord Kelvin for experimental work, was acquired by Kelvin & James White (Ltd.), who generally overhauled it and added the component T_2 (see Table B). From this firm it went into the possession, about 1900, of the hydrographic service of the French Government for use in the preparation of the French tide tables. (See plate 3.)

BRITISH TIDE PREDICTOR NO. 4 (NOW IN BRAZIL).

About 1909 or 1910 Kelvin & James White (Ltd.), constructed another machine upon the same model for the Brazilian Government. It differs from the French machine only in the number of components, of which 12 only are provided. (See Table B.)

BRITISH TIDE PREDICTOR NO. 5 OR ROBERTS' UNIVERSAL TIDE PREDICTOR.

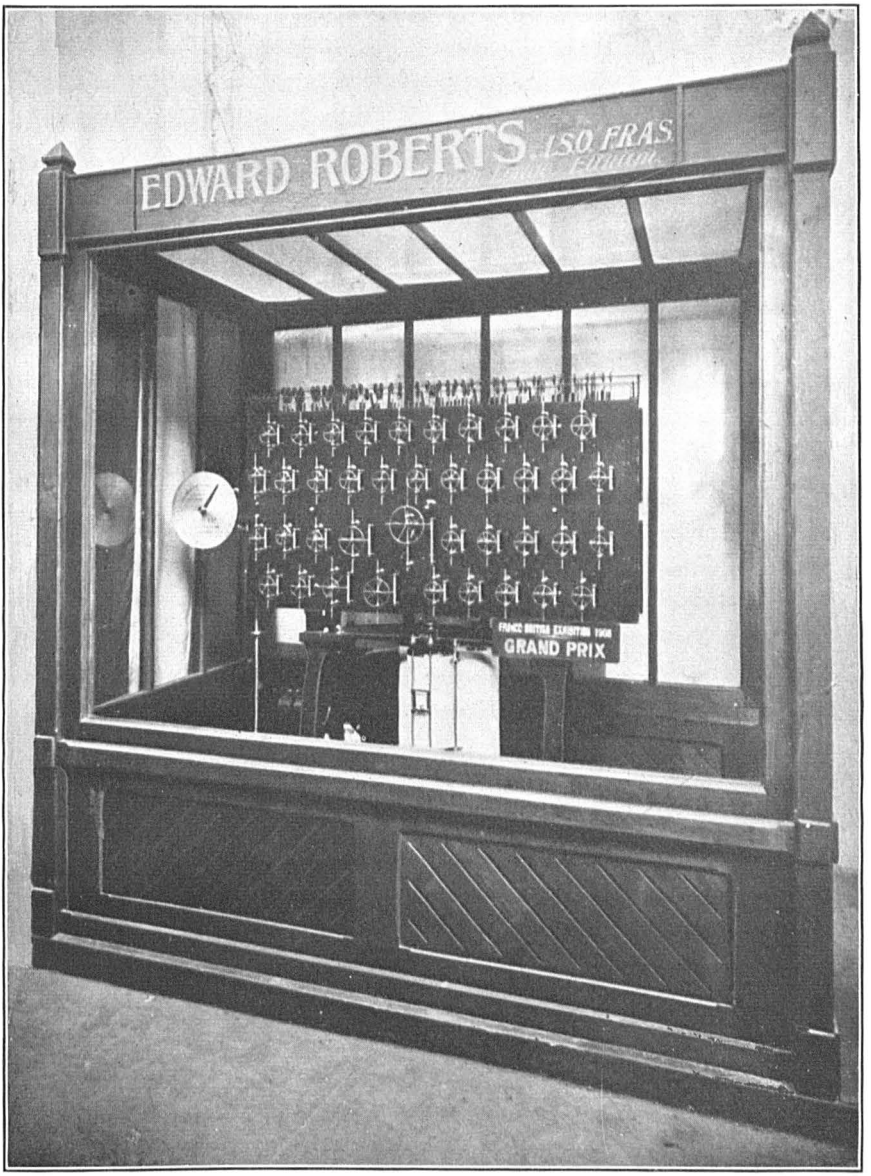
During the preparation of this paper, information was received from Edward Roberts of the existence of another tide-predicting machine, designed by him in 1906 for his own private use and exhib-

ited in 1908 at the Franco-British Exhibition in London, where it was awarded the *grand prix*. It is called by him the Universal tide predictor. The principles of its construction, as will be seen from the illustration (plate 4), "are the same as in the India office tide predictor." It provides for 40 components, "33 of which are actually geared up and vacant places for the gearing of the remaining seven components have been left for the insertion of compound tides which may be found sufficiently large to warrant their inclusion in the predictions." The components now represented are shown in Table B.

All these machines sum mechanically by means of a wire or chain fixed at one end and laid under and over the pulleys representing the speeds, amplitudes, and epochs of the components, the well known series $A \cos (at + \alpha) + B \cos (bt + \beta) + C \cos (ct + \gamma) + \ldots$, the free end of the chain, provided with a suitable pen, moving vertically and thus tracing the tide curve upon a strip of paper moving laterally at uniform speed. This tide curve must be read or scaled off to obtain the heights of the tides. The times of the high and low waters may also be obtained approximately from the sheet by applying the rate of motion of the paper under the curve pen; for the exact times the machines require a second setting and tracing of a curve representing the first derivative of the above series.

THE FERREL OR UNITED STATES COAST AND GEODETIC SURVEY TIDE-PREDICTING MACHINE NO. 1.

To measure these curves and prepare the results for the printer requires far more time than was consumed in their production. This led Prof. William Ferrel of the United States Coast and Geodetic Survey, when the needs of the Survey required a tide-predicting machine, to plan an apparatus which, instead of tracing a curve at all, would indicate to the operator for copying directly upon the printer's form the numerical data required for the tide tables. The contract for its construction was given to Fauth & Co., of Washington, D. C. It was completed in 1882 and used for preparing the tide tables published by the Survey from 1883 to 1910. A description with illustrations is given in Coast and Geodetic Survey Report, 1883, pages 253–272. This machine, called by Prof. Ferrel the maxima and minima tide-predicting machine (see plate 5), differs materially from those made in England. Its components, instead of moving at their respective speed ratios, move at speeds representing the differences between them and that of the principal lunar component M_2, the motion of the latter being represented by a pointer on the face of the machine. The components represented are given in Table B. The most noteworthy innovation introduced in this machine was the addition to the set of cranks summing the cosine series, as used in



ROBERTS UNIVERSAL TIDE PREDICTOR.

the British machines, of another set of cranks, pulleys, and chain for summing simultaneously the first derivative of the former, namely, $Aa \sin (at + \alpha) + Bb \sin (bt + \beta) + Cc \sin (ct + \gamma) + \ldots$, which, by an ingenious device, points out upon the dial the time when the value of the derived series is equal to zero, and, consequently, that of the cosine series a maximum or a minimum. Theoretically considered the machine requires separate settings for the times and heights, though practically, for the simpler tides, such as those of the eastern coast of the United States and the western coast of Europe, a single setting is made to suffice.

As this machine was designed to be set upon a table or desk, its dimensions as a whole (18 by 14 by 24 inches) necessitated those of its parts, as pitch and diameter of gear wheels, diameters of shafts, etc., to be such as failed to provide sufficient rigidity when set up with amplitudes for tides of the larger ranges. Also, in predicting the more complicated tides, the device for pointing out upon the dial the times of the maxima and minima failed mechanically in certain positions and required the operator's attention and assistance. Besides, almost constant use during 12 years had developed considerable wear, and it was necessary to make frequent repairs. For these reasons it was deemed advisable to replace it by a new machine.

Sir William Thomson, engaged for a number of years with the problem of mechanically analyzing observed tide curves, saw in the reversal of this process and the production of a tide curve from its component elements the complete solution of the problem of predicting tides. It would seem that for this reason the British machines, including those made for the French and Brazilian Governments and the Roberts 33-component machine, are merely "tide curve predictors." Prof. Ferrel conceived the idea of making the machine also bridge over the gap which lies between the predicted tide curve and the numerical data required for the tide table, by introducing an additional mechanism for summing the sine terms of the derived series, a system of time dials and a height scale, thus producing what may justly be called the first real tide predictor.

This machine fell short of filling all requirements, because it gave the times of the maxima and minima only and, as stated before, required resetting and a separate operation for heights of the complicated tides.

UNITED STATES COAST AND GEODETIC SURVEY TIDE-PREDICTING MACHINE NO. 2.

A study of these two types could not fail to reveal that slight changes and simplification of the principles underlying them could be made the basis of a new machine, capable of satisfying all demands.

The conversion of the motion of the curve-tracing pen in the Thomson type into that of a pointer indicating numerically the

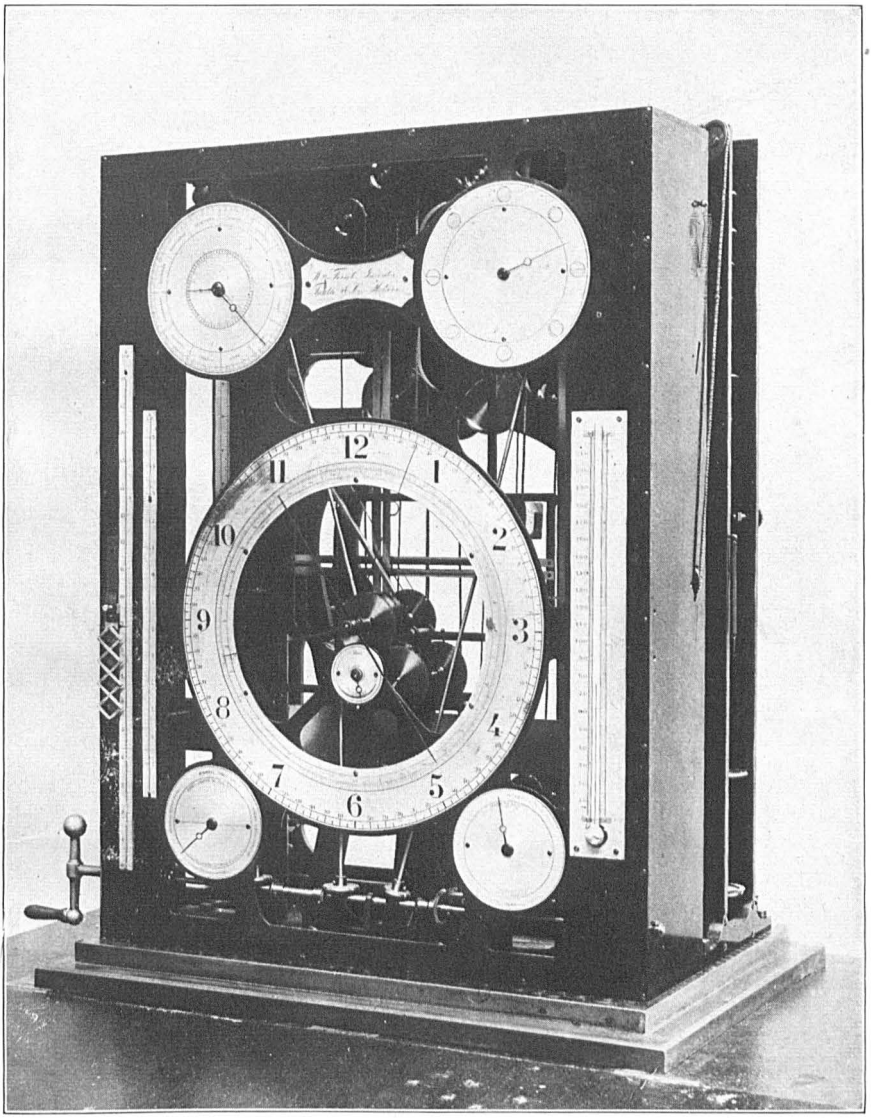
heights at any time, thus avoiding the long process of measuring a curve, suggests itself at the first study of the subject.

Time dials with their pointers in fixed relations to the driving shafts of the components were already used in the Ferrel machine.

A simpler and more direct use of the means already employed in the Ferrel machine for pointing out the maxima and minima of the height function was found in the coincidence of a marked link in the moving chain which sums the first derivative of the height series, with a fixed index in view of the operator. At the instant when this marked link is at the index the sum of all the derivative heights is zero and, consequently, the height either a maximum or a minimum. When the marked link approaches the index from the right it is a high water and when it approaches it from the left it is a low water.

In order to have at the office a record of the predictions copied from the face of the Ferrel machine directly upon the forms to be sent to the printer, careful copies were made of them by hand for some years, after which blue prints or photostat copies of the original tabulations were used. Tide curves, as produced by the British machines, serving as such records, suggested the idea of saving the time and expense of making copies by having the free end of the height summing chain, in addition to pointing out the heights on the dial, also move a pen upon a moving strip of paper and produce a tide-curve record automatically. The marking of the latter into time spaces is a process employed for many years in various ways in the chronograph, meteorological recording instruments, etc., and was accomplished in the British tide predictor No. 3 by means of the momentary lateral displacement of the curve tracing pen.

Before making the first plans the subject of mechanical summation was exhaustively studied with the hope of improving upon the chain and pulley, but all the methods suggesting themselves, when developed in detail, led back to that, the simplest and most efficient form of summing harmonic series. A number of practical tests were also made for the purpose of studying the relative merits of bevel and spur gears. The former style of gear wheels having been replaced in the third British machine by the latter to obtain greater rigidity, and the Ferrel machine showing some flexure when set up with large amplitudes, made caution as to this point necessary; especially as the machine was to provide for so large a number of components. Besides, the shape and dimensions of the machine as a whole depended largely upon the decision as to this point. Having decided in favor of the use of bevel gears and upon the dimensions of the detail parts necessary to insure rigidity, and a half-inch unit for the height amplitudes having been generally accepted as most suitable, a drawing was prepared, upon the scale of 1:10, which was submitted to and accepted by the instrument board sometime in 1895, and the work of construction was begun in 1896.



FERREL TIDE-PREDICTING MACHINE, OR UNITED STATES COAST AND GEODETIC SURVEY TIDE-PREDICTING MACHINE NO. 1.

Owing to various causes it was not until February, 1910, that the machine was completed, excepting as to polishing, plating, and lacquering. It was immediately put into use for predicting the more complicated tides for the 1912 and 1913 tide tables, thus affording opportunity for dismounting, finishing, and reassembling in time for predicting the 1914 tide tables.

DESCRIPTION OF THE MACHINE.

The names or symbols of the components represented in this machine are given in Tables B and C.

As regards the general form of the machine, the reader is referred to plates 6 and 7. The first presents a full view of the right side, showing a desk supporting the dial case; behind it, regarding the desk end as the front of the machine, a tall frame mounted on a depression in the base, called the front component frame; and behind this another long and lower one, called the rear component frame. Plate 7 shows the front of the dial case with the dials, pointers, curve sheet, pens, etc., and, looking toward the rear, the left side of the component frames. This side carries the mechanism for summing the cosine or height series; its parts will hereafter be referred to as the height cranks, height pulleys, height chain, etc. Upon the other or right side are disposed the cranks, pulleys, chain, etc., for summing the sine terms of the derived or time series, hereafter designated as time cranks, pulleys, chain, etc.

A fair idea of the size of the machine as a whole is obtained from the following dimensions: Horizontal distance from front edge of desk to rear edge of rear base plate, 129¼ inches; vertical distance from floor to top of front component frame, 74¼ inches; extreme width—i. e., width of base plates—24 inches.

The dial case is mounted upon an iron base 2½ feet high to permit of the use of a wooden covering (the latter was not in place when the photographs were taken) for the desk top. This case is made up of two hard-rolled brass plates $\frac{5}{16}$ inch thick, 19 inches wide, and 24½ inches high, secured to each other by six brass posts $\frac{11}{16}$ inch diameter. The shafts of the hour and minute and the day and month dials have their bearings in these plates (plate 9). The vertical shaft, ⅜ inch diameter, runs in brackets secured to the rear plate, the intermediate shafts being mounted in the same manner.

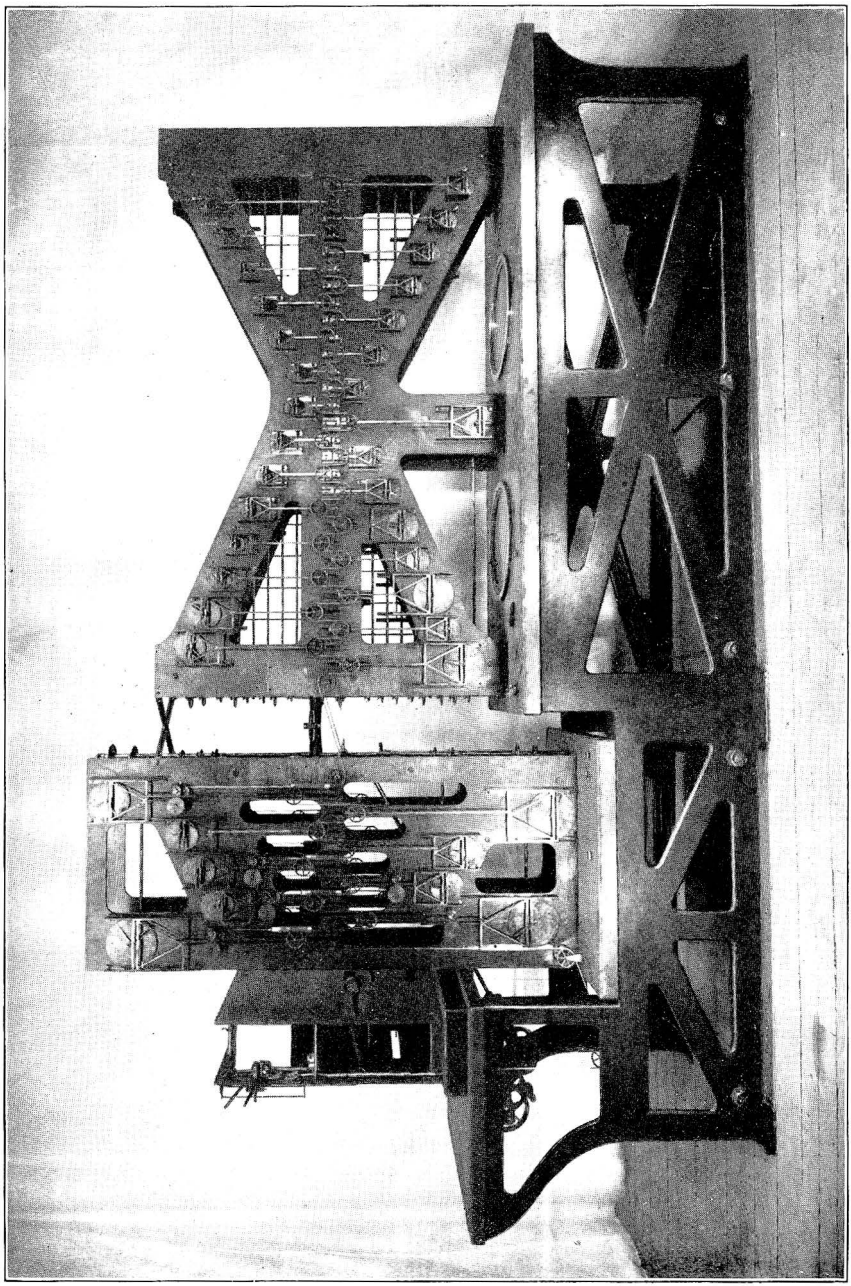
The front component frame is made up of two hard-rolled brass plates $\frac{11}{32}$ inch thick, 24 inches wide, and 56¼ inches high, secured to each other, 7 inches apart, by ten ¾-inch brass posts.

Each of the sides of the rear component frame, of the same thickness and material as those of the front frame, is composed of two pieces joined horizontally at the medial line. The sides thus formed held together 7 inches apart by fifteen ¾-inch posts, are 40¼ inches

high at their ends and 56¼ inches long. The top of the base plate supporting them is 25¾ inches above the floor. The two component frames are secured to their base plates with a space of 7 inches between them, thus permitting access to the gear wheels on the vertical shafts, and are secured to each other by two cross braces, one at the top and one slightly above the middle of the rear frame, the lower one serving to support a stud gear for transmitting motion from the vertical shaft of the front to that of the rear frame.

Distribution of motion.—The general plan of distribution of motion to the various parts of the machine is as follows: By means of the operating crank at the left side of the desk, through a system of spur and bevel gears (plate 8), motion is imparted to the inclined driving shaft (plates 8 and 6), at the rate of one turn of the latter to three turns of the crank. From the inclined shaft, under the desk, through a system of bevel gears with effective ratio of 1:1, motion is conveyed to the vertical driving shaft within the dial case operating the time dials and the paper-winding mechanism of the curve-tracing apparatus (plates 3 and 4). The upper end of the inclined shaft drives by means of bevel gears, also of ratio 1:1, the vertical driving shaft of the front component frame (plate 10). The latter, through spur and stud gears of ratio 1:1, moves the forward vertical driving shaft of the rear component frame (plate 11), which in turn, through a strong horizontal shaft and 1:1 bevel gears, moves the rear vertical driving shaft of the same component frame. Thus, all the vertical driving shafts rotate at the same rate of speed. The one within the dial case conveys motion by means of two pairs of bevel wheels 1:2 and 1:1 to the pointer of the 24-hour dial at the left of the center of the dial face (plates 7 and 9), so that two revolutions of this vertical shaft and the three mounted within the component frames correspond to one revolution of the hour pointer or one mean solar day. The pointer of the minute dial, to the right of the former, is driven by two pairs of bevel gears, 3:1 and 4:1.

Dial for months and days.—The dial for indicating the days and months, visible through a curved opening above the hour and minute dials and read by an index just below the opening, is fixed upon a shaft which carries a worm wheel with 366 teeth; the worm screw engaging with this wheel is driven by the vertical shaft through a pair of bevel gears 1:2. The worm wheel with 366 teeth provides, of course, for the 29th of February, which also has its place on the day dial. The index or pointer just below the curved opening (plate 7), which curves into the latter and close to the face of the dial, is secured to a short shaft, which carries at its inner end a lever arm with a pin reaching under the lower edge of the day dial, toward which it is pressed by a light spring. A portion of the edge of the dial equal to the angular distance from January 1 to February 28 is of a slightly



UNITED STATES COAST AND GEODETIC SURVEY TIDE-PREDICTING MACHINE NO. 2.

larger radius, so that the pin pressing against it rises and throws the day pointer to the right one day when this portion has passed by or, in other words, the pointer "jumps" one day. This condition is indicated by a smaller pointer, upon the same center as the former, pointing to the right of a plate carrying the legend "Common year." When this small pointer is thrown over to the left or toward the words "Leap year," the day pointer is locked and can not "jump" the 29th of February. (The plates with the legends referred to were not in place when the photograph was taken.) The day dial can be released by loosening the milled head nut immediately above the large dial ring, and quickly moved and clamped to any desired position.

Connecting shafts.—From the vertical driving shafts within the component frames motion is imparted to the component shafts which run in bearings in the plates forming the frames by means of horizontal intermediate shafts (best seen in plate 12), and two pairs of gear wheels each, excepting in the case of the components *Ssa*, *Mm*, *Sa*, *Msf*, and *Mf*, which move at speeds so slow as to require resort to the use of one pair of gears and a worm screw and wheel. The rear vertical shaft in the rear frame drives 15 and the forward vertical shaft 16 of the 31 components placed in the rear frame. The vertical shaft in the front component frame gives motion to 9 component shafts, 3 of which are duplicates introduced for reasons to be stated further on, which represent the remaining 6 components of the 37 provided for.

The gears in these trains of motion are set in such relation to each other that all the vertical shafts move clockwise when viewing the machine from above and all the component shafts clockwise when viewing the machine from the time side.

Dimensions of parts.—The exact dimensions of the parts transmitting motion from the crank shaft to the various parts of the machine, as traced out in the foregoing are not of general interest, but are here given for the sake of completeness. Diametral pitch is used in designating the sizes of gear teeth, the number of the pitch being the number of teeth per inch diameter.

Driving crank with throw of 6 inches (plate 8) on crank shaft $\frac{9}{16}$-inch diameter. On inner end of crank shaft, spur gear 40 teeth, 24-pitch, $\frac{9}{16}$-inch face, to spur-stud gear, 120 teeth, $\frac{1}{2}$-inch face, to spur gear 120 teeth, on cross shaft $\frac{1}{2}$-inch diameter. On same shaft, bevel gear, 72 teeth, 24-pitch, $\frac{13}{32}$-inch face, to bevel gear, 72 teeth, on lower end of inclined shaft, $\frac{1}{2}$-inch diameter. On same shaft, under desk top, bevel gear, 75 teeth, 30-pitch, $\frac{3}{8}$-inch face, to bevel gear, 75 teeth, on lower end of short vertical shaft through desk top, $\frac{3}{8}$-inch diameter. On same shaft, upper end, inside of dial case (plate 9), bevel gear, 75 teeth, 30-pitch, $\frac{3}{8}$-inch face, to bevel gear,

75 teeth, on inner end of short horizontal shaft, $\frac{3}{8}$-inch diameter. On same shaft, outer end, bevel gear, 75 teeth, 30-pitch, 0.27-inch face, to bevel gear, 75 teeth on lower end of vertical shaft within dial case, $\frac{3}{8}$-inch diameter.

On vertical shaft, next above preceding, releasable bevel gear, 60 teeth, 48 pitch, 0.17 inch face, to bevel gear, 120 teeth, on outer end of intermediate shaft, 0.15 inch diameter. On same shaft, inner end, bevel gear, 84 teeth, 48 pitch, 0.17 inch face, to bevel gear, 84 teeth on hour pointer shaft, 0.15 inch diameter.

On vertical shaft, next above preceding, releasable bevel gear, 180 teeth, 48 pitch, 0.17 inch face, to bevel gear, 60 teeth, on outer end of inclined intermediate shaft, 0.15 inch diameter. On same shaft, inner end, bevel gear 240 teeth, 48 pitch, 0.17 inch face, to bevel gear, 60 teeth, on minute pointer shaft, 0.15 inch diameter.

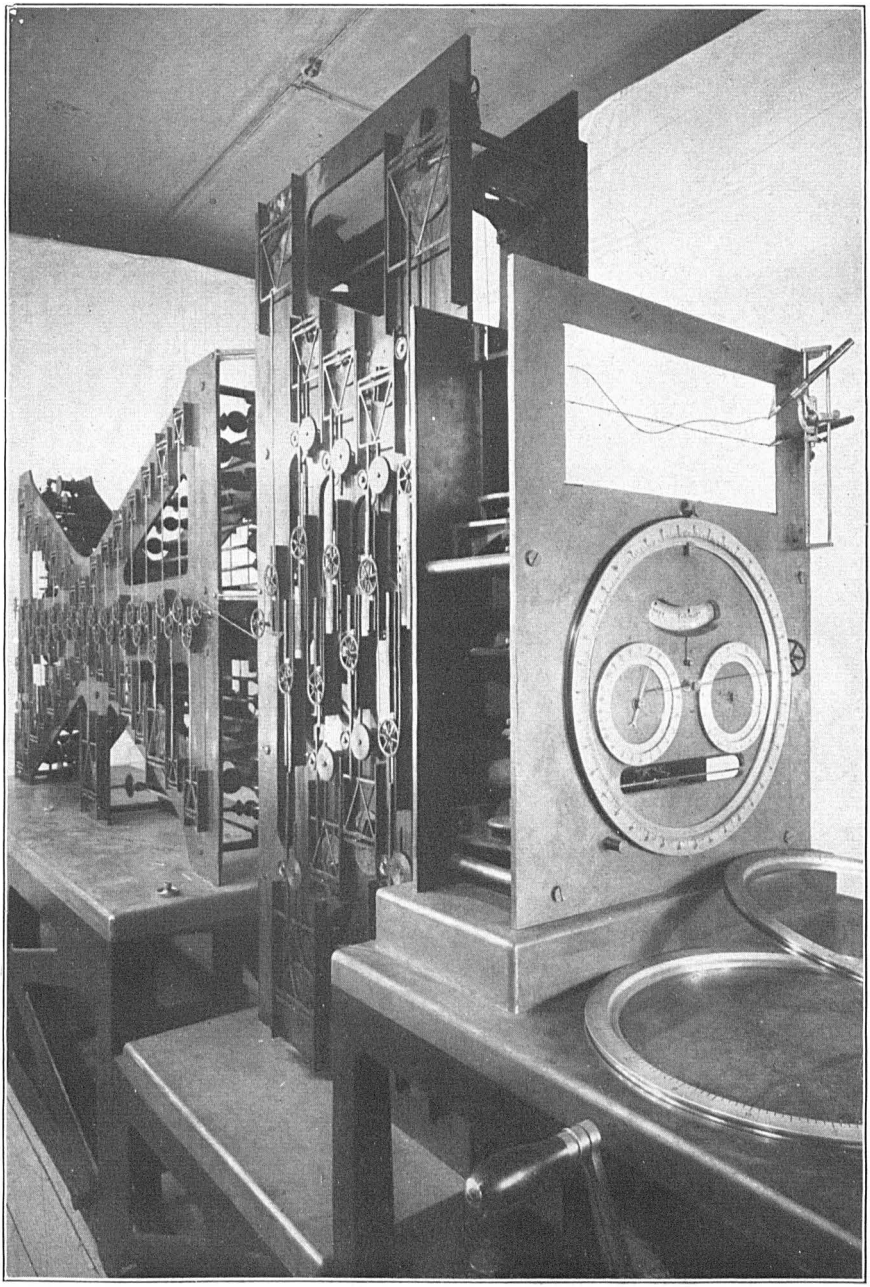
On vertical shaft, next above preceding, releasable bevel gear, 60 teeth, 48 pitch, 0.17 inch face, to bevel gear, 120 teeth, on outer end of intermediate shaft, 0.15 inch diameter. On same shaft, inner end, worm screw $\frac{9}{16}$ inch diameter, 18 threads per inch, to worm wheel, 366 teeth, 6.47 inches diameter, on day dial shaft, $\frac{5}{16}$ inch diameter.

Inclined shaft, upper end (plate 10), bevel gear, 72 teeth, 24 pitch, $\frac{13}{32}$ inch face, to bevel gear, 72 teeth, on vertical shaft of front component frame, $\frac{7}{16}$ inch diameter. On same shaft, next above preceding, spur gear, 110 teeth, 30 pitch, $\frac{3}{8}$ inch face, to stud gear on cross brace (plates 10 and 11), 110 teeth, to spur gear, 110 teeth, on front vertical shaft of rear component frame, $\frac{3}{8}$ inch diameter. On same shaft, next below preceding, bevel gear, 75 teeth, 30 pitch, $\frac{9}{32}$ inch face, to bevel gear, 75 teeth, on front end of long horizontal shaft within rear component frame, $\frac{1}{2}$ inch diameter. On same shaft, rear end (plate 12), bevel gear, 75 teeth, 30 pitch, $\frac{9}{32}$ inch face, to bevel gear, 75 teeth, on rear vertical shaft of rear component frame, $\frac{3}{8}$ inch diameter.

From the vertical shaft in the front component frame (plate 10) motion is conveyed by releasable bevel gears to bevel gears on horizontal intermediate shafts $\frac{3}{8}$ inch diameter, by bevel gears at their inner ends to bevel gears on component shafts $\frac{5}{16}$ inch diameter, all the bevel gears in the front component frame are of 40 pitch and 0.24 inch face.

From the front vertical shaft in the rear component frame (plate 11) motion is conveyed by releasable bevel gears to bevel gears on horizontal intermediate shafts, $\frac{5}{16}$ inch diameter, by bevel gears at their inner ends to bevel gears on 16 component shafts.

From the rear vertical shaft of the rear component frame (plate 12) motion is conveyed by releasable bevel gears to bevel gears on horizontal intermediate shafts, $\frac{5}{16}$ inch diameter, by bevel gears at their inner ends to bevel gears on 10 component shafts. The 5 remaining



UNITED STATES COAST AND GEODETIC SURVEY TIDE-PREDICTING MACHINE NO. 2, DIAL END.

components driven from this vertical shaft, *Ssa* and *Mm* at the top, and *Sa*, *Msf*, and *Mf* at the bottom, receive their motion from releasable bevel gears to bevel gears on horizontal intermediate shafts, $\frac{5}{16}$ inch diameter, by worm screws at their inner ends, $\frac{1}{2}$ inch diameter, 13 threads per inch, to worm wheels, 48 pitch, on shafts carrying spur gears 48 pitch, to intermediate sliding spur gears on stud shaft to spur gears on component shafts. The intermediate sliding gears are provided for disconnecting the component shafts from the slow-moving worm-screw gears, thus permitting of more rapid setting of the epochs.

All the component shafts in the rear component frame are $\frac{5}{16}$ inch diameter, and all the bevel gears are of 48 pitch and 0.20 inch face.

Durability of bearings.—The intermediate shafts in both the component frames run in bracket bearings made of hard cast brass, secured to one of the plates of the component frame. They can readily be replaced, one at a time, should looseness due to wear require their renewal, without putting the machine out of use. Should wear become too great in the bearings of the component shafts, however, which are in the place of the frames themselves, considerable time and expense would be required to repair the machine. For this reason the bearings in the plates were bushed or lined with hard cast-iron rings, forced into place and riveted. It is thought that 50 years' constant use will not cause sufficient wear to require the renewal of the bushings.

The vertical shafts also run in bracket bearings made of hard cast brass, the one in the front component frame being supported by seven (plate 10), that in the front of the rear frame by six (plate 11), and the rear one (plate 12) by five such bearings. One of the bearings near the middle of each vertical shaft is fitted with a clamping device, consisting of a double-armed steel block fitting over the bracket bearing, with a powerful hexagon-headed steel screw. By means of a large milled-head wrench the screw can be tightened and the shaft clamped so firmly as to prevent its moving under the stresses incident to the loosening and tightening of the releasing gears. The number of teeth in each of the gears conveying motion from the vertical shafts to the component shafts depend upon the speeds of the components.

Speeds of components.—The speeds of the components are their angular motions expressed in degrees per mean solar hour. They are derived from astronomical data, and were first determined by a committee of the British association, appointed in 1872, "for the examination of the question of Harmonic Analysis of Tidal Observations," Sir William Thomson being chairman of the committee. Sir George H. Darwin, having taken charge of this work some time later, extended and perfected the list of components, which was finally published in

the Report of the British Association for the Advancement of Science for 1883, and has since, together with the system of notation adopted by that committee, come into universal use.

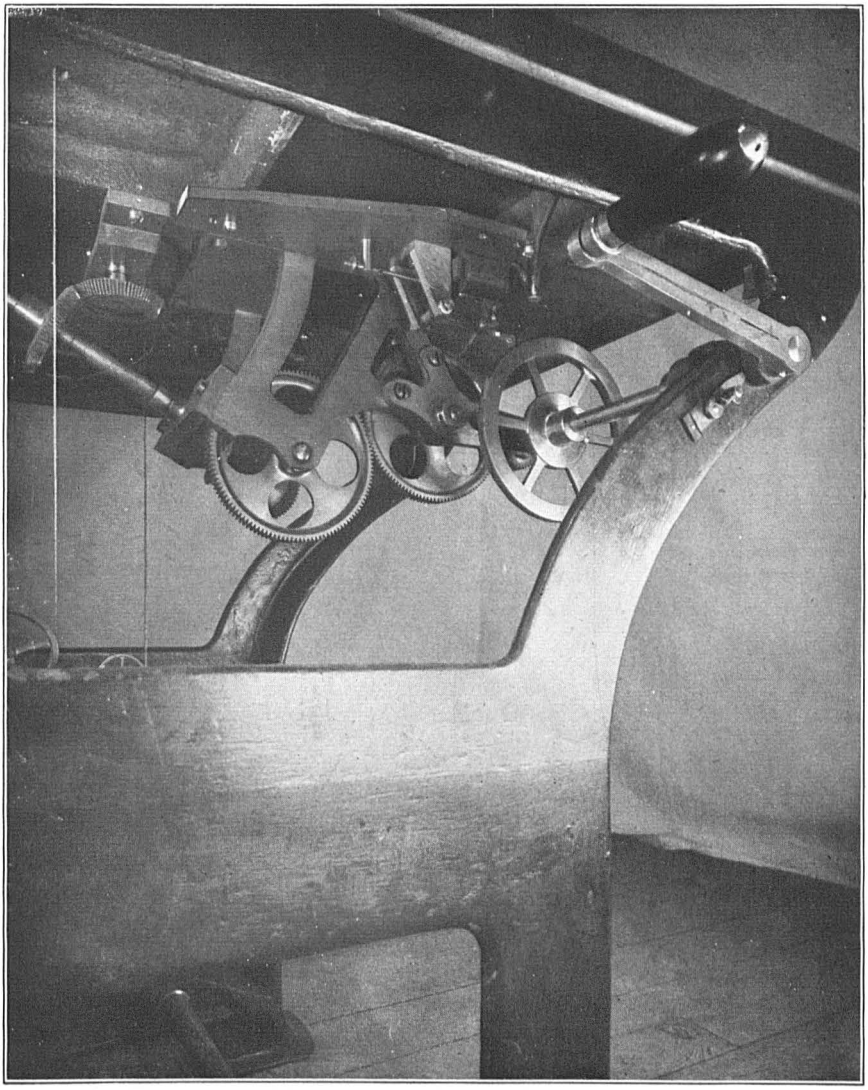
The gear ratios or the number of teeth of each of the gear wheels conveying motion from the vertical shafts to the respective component shafts were obtained by converting the decimal expression of the speed ratio given in Darwin's table to the nearest unit in the seventh place into two vulgar fractions, which, when multiplied by 30, the angular speed of the vertical shafts in degrees per mean solar hour, will most nearly approach the former decimal expression. By refactoring these ratios it was possible to obtain such diameters of the respective gear wheels as permitted of considerable saving of space when locating the components.

The number of the teeth of the gear wheels are given in Table C, in the columns headed I, II, III, and IV. The product of the gear ratios I: II and III: IV of a component multiplied by 30, the angular speed of the vertical shaft per dial hour, is the hourly speed of that component as produced by the gears. The difference between this and the theoretical speed, both given in the table, multiplied by the number of hours in a year, is shown for each component in the column headed "Errors of gears per dial year."

It will be seen that the errors in the speeds of all the large components are less than a quarter degree; only a few of the unimportant ones amount to more than half a degree in a year, quantities which are altogether negligible.

Gear wheels.—In making the designs the durability or the life of the machine was kept in view. This is altogether a question of the wearing out of those parts which can not be readily renewed. The most important of these, besides the bearings of the component shafts before mentioned, are the gear wheels. These were cut with cutters specially made to order for producing teeth of the well-known involute type, but with an additional bottom clearance of about 15 per cent of the height of the tooth. This permits of adjustment for taking out play due to wear of a greater amount than can occur in many year's use.

Releasable gears.—The releasable gears heretofore mentioned are used when setting the machine for a new station, for disengaging or releasing each component from the remainder of the train and setting it to its proper amplitude and epoch, after which it is again secured to the vertical shaft. As the moving or slipping of any one of them would vitiate the result, special care was taken in designing the clamping device, which is shown in the adjoined illustrations. (Plate 13, fig. *a*.) A collar *A*, with a thread at its upper end and a flange at the bottom, is fastened to the shaft by means of three counterbored steel screws. The gear wheel, *B*, fitting closely upon this



MAIN DRIVING GEAR AND AUTOMATIC STOPPING DEVICE.

collar and resting upon the flange, has sunk into its upper surface a recess, b, which is filled by the flange of the collar C. The latter is free to move vertically, but prevented from turning by a small steel screw reaching into a vertical groove, a, in the collar A. The lower surface of the collar C is slightly "dished"; after being fitted accurately into the recess b, it is split twice, at right angles, nearly to the top. When the milled nut D is screwed down with a small pin wrench the edge of the collar C is pressed against the edge of the recess b with such force as to make slipping practically impossible.

Arrangement of components.—The arrangement of the components in relation to each other as regards location was considered carefully with the object of reducing flexure and friction to a minimum. As far as economy in space would permit the components with the largest amplitudes and greatest speeds were placed nearest to the free ends of the summing chains and connected most nearly direct with the main driving shaft, while the others were placed farther toward the fixed ends of the chains and away from the source of power. The product of the maximum amplitude and the speed of each component was taken as a measure for assigning its place.

Viewing the machine from the time side (plate 6), the components are arranged in the following order, beginning at the left or front and going toward the right or rear:

UPPER RANGE.

FRONT COMPONENT FRAME.

$$\frac{M_2}{2},\ \frac{S_2}{2},\ N_2,\ \frac{K_1}{2},\ M_4$$

REAR COMPONENT FRAME.

Driven by front vertical shaft.

$$M_6,\ MK_1,\ S_4,\ MN,\ \nu_2,\ S_6,\ \mu_2,\ 2N.$$

Driven by rear vertical shaft.

$$OO,\ \lambda_2,\ S_1,\ M_1,\ J_1,\ Mm,\ Sea.$$

LOWER RANGE.

FRONT COMPONENT FRAME.

$$\frac{M_2}{2},\ \frac{S_2}{2},\ \frac{K_1}{2},\ O_1.$$

REAR COMPONENT FRAME.

Driven by front vertical shaft.

$$(MS)_4,\ M_8,\ K_2,\ 2MK,\ L_2,\ M_3,\ 2SM,\ P_1.$$

Driven by rear vertical shaft.

$$2Q,\ R_2,\ T_2,\ Q_1,\ \rho_1,\ Mf,\ MSf,\ Sa.$$

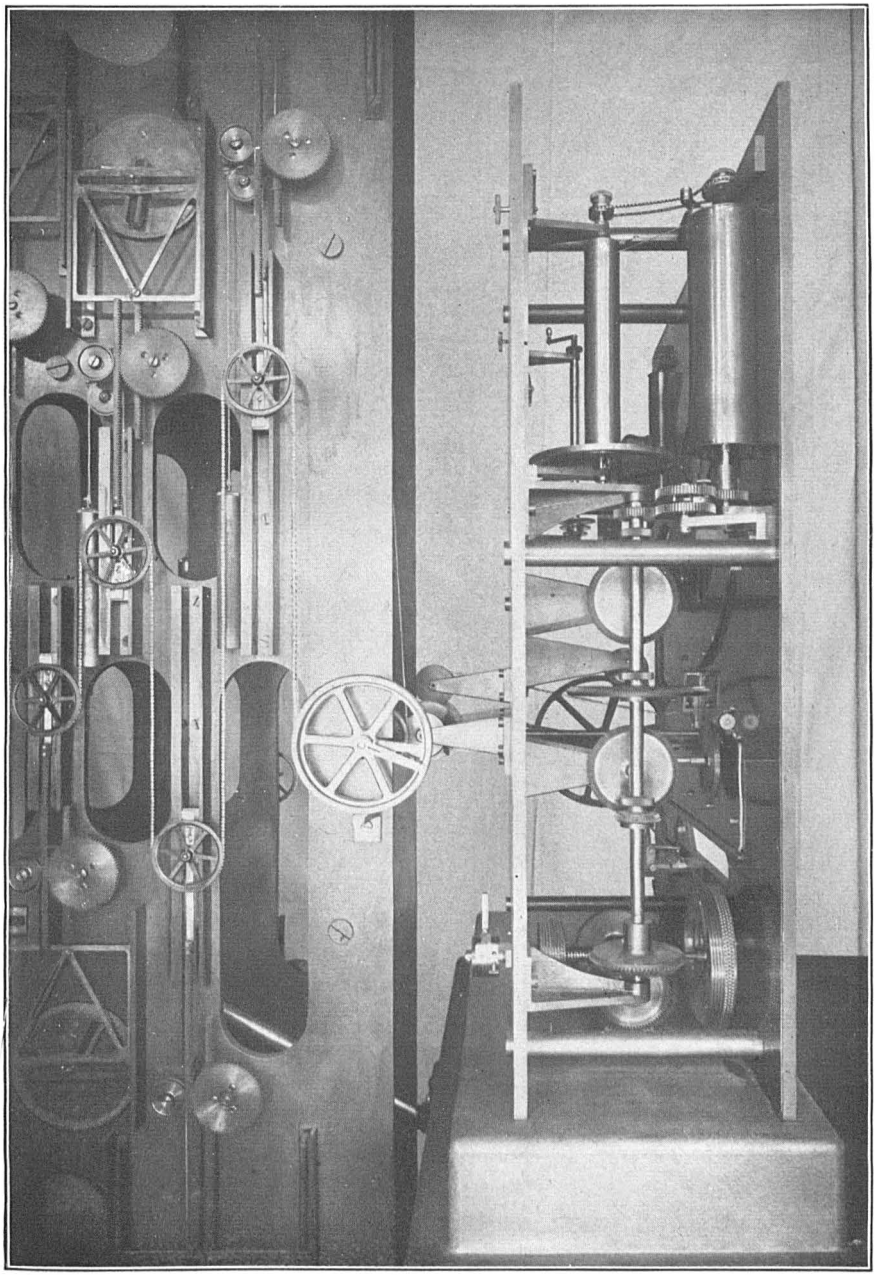
The horizontal distance between each two component shafts was, of course, fixed by the diameter of the chain pulleys, which was taken as 2 inches, measured upon the center of the chain.

Harmonic motion of pulleys.—The several views of the sides of the machine show plainly that provision was made for simple harmonic motion of all the chain pulleys and in what manner this was done. The frames with rectilinear horizonital slots 0.24 inch wide were cut from sheet steel $\frac{1}{16}$ inch thick, and drilled and filed out to shapes rendering them light, without impairing their rigidity. They move in grooves planed into the inner sides of hard brass angle pieces, which are screwed to the side plates.

Component cranks.—The lengths of the component cranks, depending upon the height unit adopted for the machine and the maximum amplitude of each component, were made ample, in order to provide for any extreme case. They are given in the last two columns of Table C.

The component cranks are situated immediately behind these frames, their hubs serving as shoulders of the component shafts to which they are securely pinned. The cranks (see adjoining illustration, plate 13, fig. *b*) are of hard brass and have ⊥ slots, *a*, milled through them, into which are fitted accurately the heads of the steel crank pins, *b*. The latter are bored out and threaded to receive the square-headed clamp screws, *c*, by means of which the crank pins can be fixed firmly in any position upon the cranks. A thin slightly curved plate of hardened steel, *d*, between the head of the crank pin and the bottom of the ⊥ slot, protects the latter from being bruised by the clamp screw and acts as a spring which prevents the crank pin from falling when released. Upon the crank pin, which is 0.19 inch diameter, and finely polished, turns freely a closely fitting rectangular block of hardened steel, *e*, which fits accurately into and slides along the slot of the steel frame. The milled head wrench for loosening and tightening the clamp screw *c* is at *B*.

Upon the side of each crank at its center is mounted a steel pointer, for setting the epoch of the component upon a silvered degree circle screwed to the side plate. The time crank is set upon the shaft 90° ahead of the height crank, and the circles are numbered and the pointers placed so that the effects upon the height chain vary as the cosines of the angles indicated by the pointer, and those upon the time chain as the sines of the same angles. For greater convenience and to avoid errors in setting, the circles of the upper and lower ranges of components were numbered alike and the pointers set upon the crank so that, facing them, all the height circles have their zeros at the right side, the numbering increasing counter clockwise. When facing the time circles, the numbering increases clockwise from the zeros at the tops of all the circles.



DIAL CASE, SIDE VIEW.

The components *Sa* and *Ssa* have height cranks only, the sine terms of their first derivatives being too small to be taken into account.

Chain pulleys.—The chain pulleys turn on steel pins 0.18 inch diameter, screwed into steel plates $\frac{1}{16}$ inch thick, $\frac{19}{32}$ inch wide, and $1\frac{19}{32}$ inch high, which slide in grooves planed into the inner sides of ⊔-shaped hard brass guides, screwed to the sides of the plates. Incidentally these guides serve to unite securely the two halves of the plates forming the rear component frame. The steel plates carrying the pulleys, or pulley slides, excepting those of some of the larger components which will be referred to further on, are connected directly with the cross-slot frames by means of steel strips $\frac{1}{16}$ inch thick and 0.2 inch wide.

One side of each pulley guide is widened to accomodate the silvered amplitude scale and its numbering, which is read by means of an index line cut upon a small strip of German silver screwed upon the pulley slide in a manner to permit of its being adjusted accurately to the zero of the scale when the crank is in a horizontal position or the amplitude is set to zero.

Amplitude scales.—Only the negative halves of the amplitude scales are ruled upon the pulley guides. To set the amplitude, the component shaft is released, the crank set vertical pointing toward its chain pulley; the crank pin is released with the aid of a milled-head wrench and moved up or down until the index on the pulley slide indicates the desired amplitude upon the scale, when the crank pin is again tightened. The unit of the height scales, 0.5 inch, is subdivided to tenths, which permits of setting amplitudes to hundredths by estimation. The time-scale unit of each component is the product of the height-scale unit and the ratio of the speed of that component to that of the mean moon, or M_2, in accordance with the coefficient of the derived series.

The maximum amplitudes of the larger components, upon the scale of 0.5 inch per height unit, require cranks so long that the machine as a whole could not have been kept within dimensions suitable for an ordinary-sized room without special provision. The maximum amplitude for M_2, for instance, requires cranks of 10 inches on the height and time sides. To accommodate these components in a reasonable space, two component shafts were provided for M_2, S_2, and K_1, and the motions of their respective chain pulleys and those of several other components were doubled by means of racks fastened to the cross-slot frames which gear into the smaller ones of pairs of 1:2 spur gears, the larger ones engaging into racks fastened to the pulley slides. This reduced the largest of the cranks to 2.5 inches. The racks are 0.2 inches wide, of 40 pitch, and are held in close gearing with the spur gears by flanged rollers which are adjustable for taking up any play due to wear. Counterpoise weights were provided for

the components in the upper range to relieve them of the undue strain of the weights of the chain pulleys, slides, and racks, the effects of which are doubled by the doubling gears. All these components are mounted in the front component frame. They are arranged as follows:

Arrangement of components in front frame.

UPPER RANGE.

Components.	Cranks.	Length of crank.	Gearing.
		Inches.	
$\frac{M_2}{2}$	Height crank	2.50	Doubling gear.
	Time crank	2.50	Do.
$\frac{S_2}{2}$	Height crank	1.23	Do.
	Time crank	1.25	Do.
N_2	Height crank	1.50	Do.
	Time crank	1.50	Do.
$\frac{K_1}{2}$	Height crank	1.38	Do.
	Time crank	1.45	Not doubled.
M_4	Height crank	2.00	Do.
	Time crank	2.00	Doubling gear.

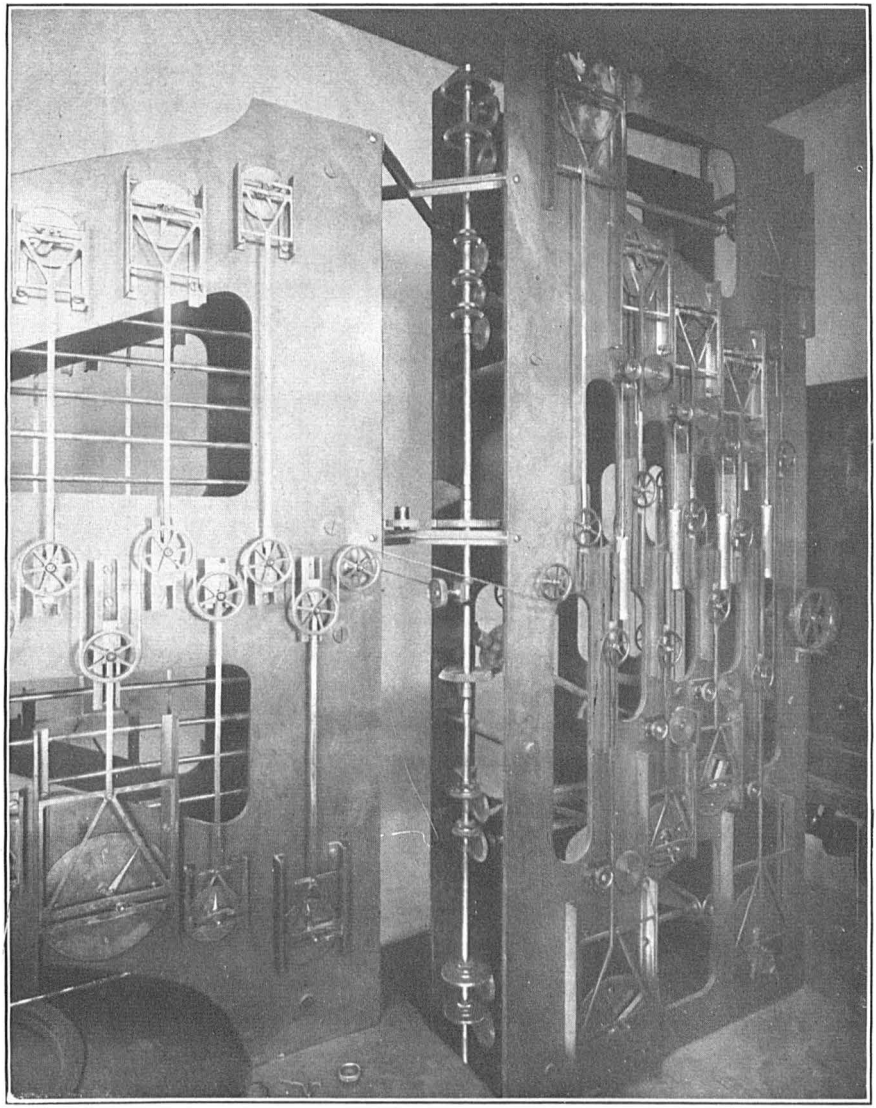
LOWER RANGE.

$\frac{M_2}{2}$	Height crank	2.50	Doubling gear.
	Time crank	2.50	Do.
$\frac{S_2}{2}$	Height crank	1.23	Do.
	Time crank	1.25	Do.
$\frac{K_1}{2}$	Height crank	1.38	Do.
	Time crank	1.45	Not doubled.
O_1	Height crank	2.25	Doubling gear.
	Time crank	2.20	Not doubled.

The solar annual component, *Sa*, mounted in the rear component frame, was also provided with a doubling gear, which permitted the use of a 2-inch instead of the 4-inch crank required by its maximum height amplitude.

The outcome of careful tests as regards the necessity for counterpoising each one of the components, made prior to deciding upon the first general plan of the machine, led to rigidity of construction rather than adding the many moving parts required for balancing the weights of all the individual summing mechanisms.

Summation chain.—The summation chain used in this machine is of the kind made for box chronometers. It is of hardened and tempered steel, single and double link plates alternating, the sides being ground slightly convex and polished. It is 0.1 inch wide, 0.05 inch thick, and has 125 links per foot. While amply flexible under strains like those of a chronometer spring, its joints were too stiff to work freely over and under 2-inch pulleys under a tension small enough to avoid undue flexure and torsion in the driving mechanism. The chain was therefore purchased long in advance of the time when needed, so that it could be made flexible and constant as to length



VERTICAL DRIVING SHAFT, FRONT COMPONENT FRAME.

by actual wear. For this purpose it was joined into one endless piece and laid over and under pulleys, some of them only one-half inch in diameter, fastened against the wall of one of the shop rooms, one free pulley being loaded with a 5-pound tension weight, while one of the fixed pulleys was kept immersed in watch oil. By means of a belt from the line shaft the chain was kept in motion during working hours for about a year.

Height chain.—The height chain is fastened at the rear end of the rear component frame (plate 12). It is riveted to a threaded rod which can be raised or lowered in a bracket, fastened to the plate, by means of two milled nuts, for purposes of accurate adjustment. From this rod the chain passes under and over all the height pulleys of the rear component frame, by means of two idler pulleys across the space between the rear and front component frames (plate 10), and over and under the height pulleys on the front component frame. After leaving the last one of these, the upper $\frac{M_2}{2}$ (plate 9), it passes down and around the thread groove of the height sum pulley, at the edge of which it ends and is fastened. The circumference of this pulley, as measured on the center of the chain, is exactly 12 inches. It can hold over seven turns of chain, the length of which is cut so that when all the components are set to zero amplitudes, the thread grooves are half filled and the pulley can either take up or pay out 45 inches of chain. The total length of the chain is 27 feet 7 inches.

The height sum pulley is mounted upon a shaft which runs in bearings secured to the plates of the front component frame. By means of a fixed tooth reaching into the threads of a screw fastened to the shaft the latter is forced, when rotating, into a screw motion with a pitch equal to that of the thread groove of the sum pulley, thus always keeping the chain between the latter and the last component pulley vertical.

From a threaded pulley secured to this shaft, of a diameter one-half that of the sum pulley, is suspended by means of a silken cord and an idler pulley mounted within and at the top of the component frame, a counterpoise weighing 12 ounces, half of which, or 6 ounces, is effective in keeping taut the chain.

A pointer at the side of the height sum pulley serves to indicate mean sea level. All the height amplitudes being set to zero, a fine line at its end is made to coincide with one upon a small bracket immediately below the pulley.

One end of the chain being fixed, the vertical motion of any one of the pulleys causes double this motion in the free end of the chain, so that the motion of a height pulley through one unit produces a motion of 1 inch at the free end, or one-twelfth revolution of the height sum pulley. The rotary motion of the latter is conveyed by

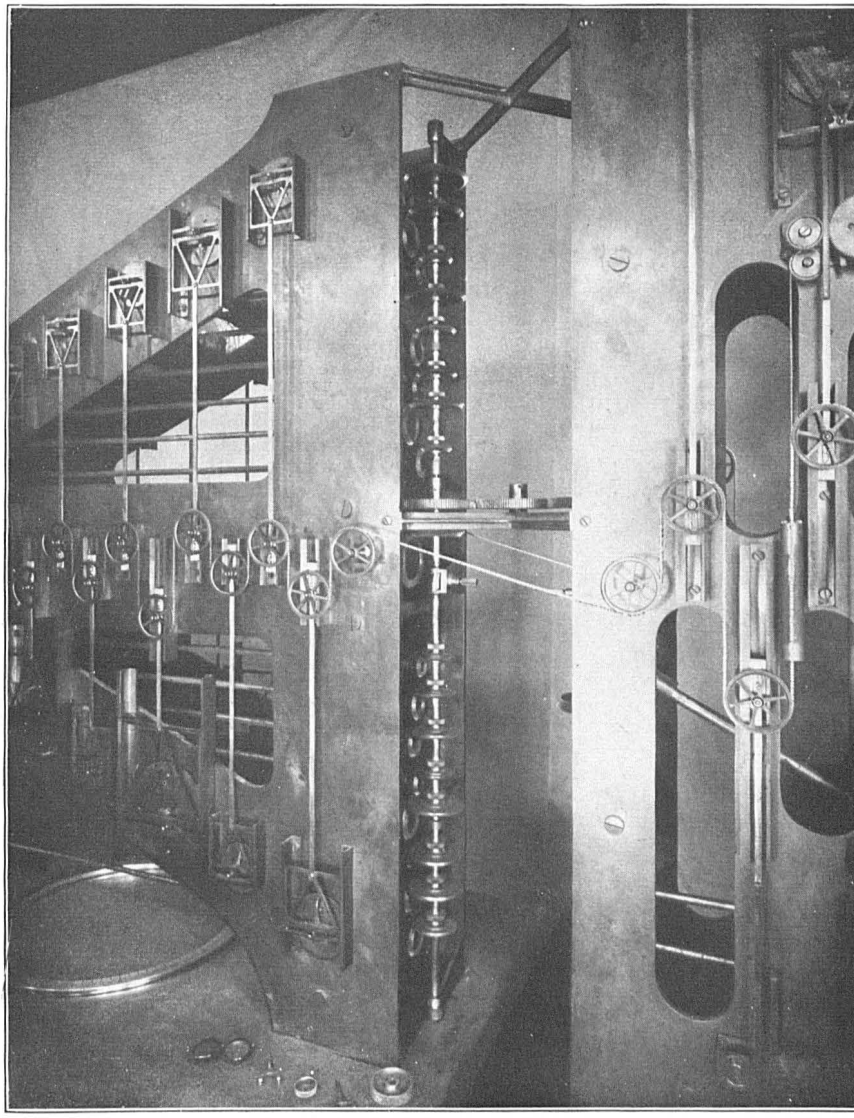
means of a spur gear, 30 teeth, 48 pitch, mounted on it, through one of 100 teeth to a horizontal shaft running in brackets fastened to the back of the dial case. A pair of bevel gears, 75 teeth each, 48 pitch, connect the latter to that carrying the height pointer on the dial face (plate 7). The height scale is 12.74 inches in diameter or very nearly 40 inches in circumference; so that, with the ratio of gears employed, 1 inch of chain motion produces 1 inch of motion of the pointer at the scale.

Height scales.—Three height scales are provided, graduated to 10, 20, and 40 feet and tenths, respectively. For predicting the smaller tides, the amplitudes are set up in the machine to four times their value and the 10-foot scale is used. For tides of average ranges the amplitudes are doubled and the 20-foot scale is put in place, while the larger tides are set up at their true amplitudes and read off the 40-foot scale, the one in place when the photograph was taken. The turning of a small button at the top of the scale permits of quick removal and accurate replacement of a scale. The pointer can be released and clamped in the desired position by means of a small milled nut at its center.

The height amplitudes and the epochs having been set for a station and the machine being set in motion, the pointer indicates the height of the sea at any time, shown on the day, hour, and minute dials.

Time chain.—The time chain is fastened at the rear end of the right side of the machine, as seen from the desk, in the same manner as the height chain (plate 6). It is passed under and over all the time pulleys of the rear and front component frames. Leaving the last one of these, it passes down around an idler pulley at the base, forward under the desk, around another idler pulley fastened to the base frame, upward, its flat side making a quarter turn, through the desk top into the dial case, over a third idler pulley across the dial case immediately behind a horizontal opening in its front plate (plate 7), exposing it to plain view of the operator. It passes around the thread grooves and ends at the side of a pulley which, with its shaft threaded pulley for a counterpoise and lateral screw motion, is an exact counterpart of the height sum pulley and its mounting (plate 9). The weight of the counterpoise, which rises and falls under the desk, and the motion of which is reduced to one-half by a movable pulley, is 18 ounces, one-fourth of which, or 4½ ounces, is effective in keeping the chain taut. The length of the time chain from the fixed end to the platinum zero link is 27 feet 1 inch and the total length 30 feet 7½ inches.

When all the time amplitudes are set to zero, a link in the chain, made of platinum with a point projecting upward, is adjusted by means of the nuts at the fixed end of the chain, to be in exact coincidence and contact with a fixed index, also of platinum, in the middle



FORWARD VERTICAL DRIVING SHAFT, REAR COMPONENT FRAME.

of the horizontal opening in the dial case. Whenever the chain, the machine being set for a station, is in this position, the sum of the harmonic motions of the time pulleys is zero, and at that instant, as pointed out on the day, hour, and minute dials, the height indicated by the height pointer is a maximum or the height of high water, or a minimum or the height of low water.

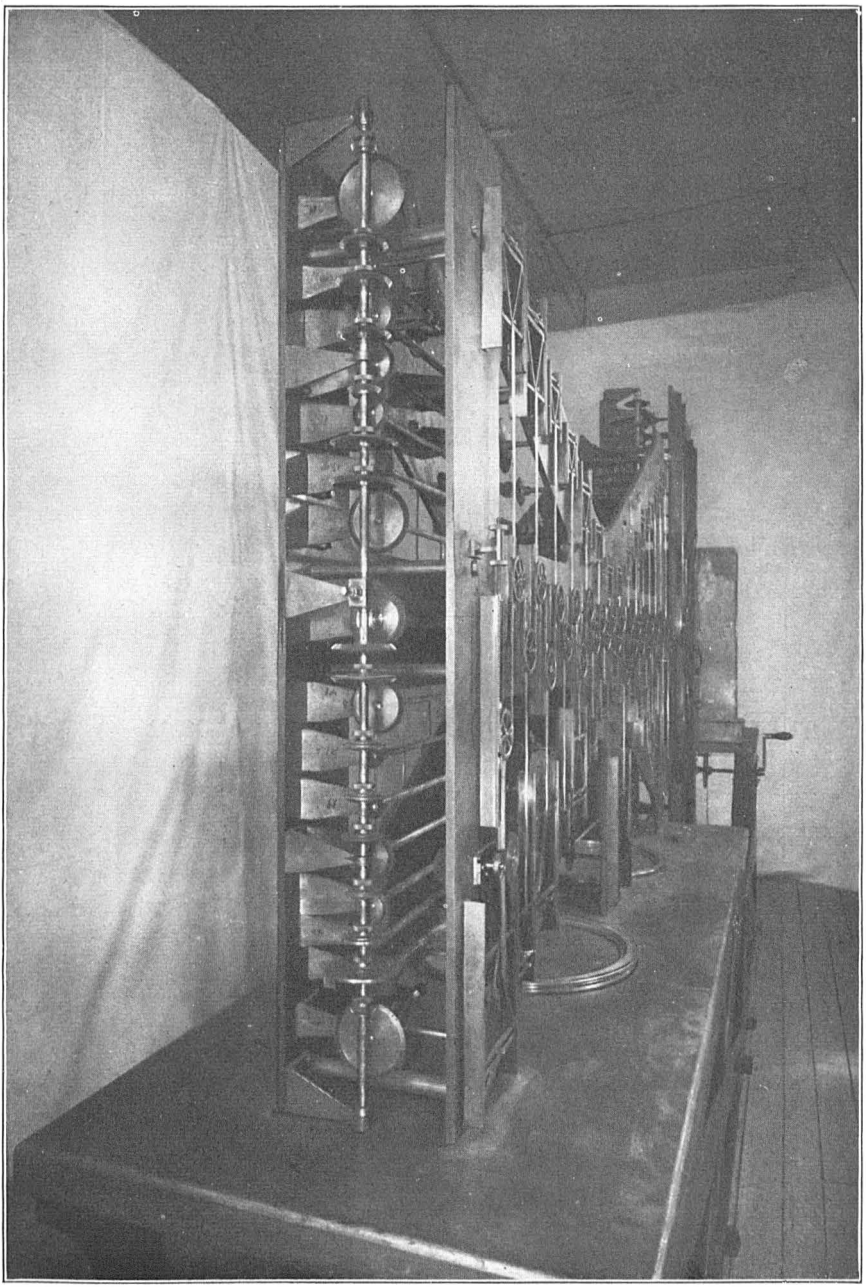
Paper-feeding mechanism.—The paper-feeding mechanism of the curve-tracing apparatus is arranged in the following manner: Upon a mandrel mounted within the dial case in the upper right-hand corner (plates 9 and 12), which can be quickly removed and replaced, is slipped the blank roll of paper holding on a 1-inch wooden core 380 feet of bond paper, 6 inches wide and 0.0024 inch thick. From the blank roll the paper passes around a 1-inch idler roller mounted in the front plate of the dial case, across the face of the latter for a distance of 13 inches and around the feed roller, which is provided near each end with 12 fine needle points for engaging the paper, into the interior of the dial case (plate 9), where it is taken up by the receiving roller, to which it has been fastened by entering its end into a narrow slit. The feeding roller is set in motion by a spur gear at the top of the vertical shaft which operates the time pointers. This spur gear, 46 teeth, 40 pitch, engages the lower one of two stud gears connected to each other by a ratchet wheel and pawl, the upper one engaging into the spur gear, 46 teeth, at the lower end of the feed roller. The circumference of the feed roller being exactly 6 inches, one revolution of the vertical shaft, which corresponds to 12 dial hours, therefore, feeds 6 inches of paper, or ½ inch per dial hour. The ratchet and pawl in the stud gear are placed so as to leave the paper at rest when for any reason the machine is turned backward. If desired, the paper feed can be thrown out of action altogether by turning a small milled head on the ratchet stud gear. A sprocket wheel with 16 teeth, held by adjustable friction to the upper end of the feed roller, drives the receiving roller by means of a chain and an 8-tooth sprocket. The ratio of the sprockets is such as to force the receiving roller to wind up the paper delivered by the feed roller with the tension furnished by the retarding friction of the driving sprocket on the feed roller. The paper exposed on the dial face is kept taut by an adjustable retarding friction under the mandrel holding the blank roll. The device works smoothly and winds up without hitch the whole of a year's tide curve. To remove the completed roll, the sprocket is lifted off the receiving roller and set upon a pin provided for the purpose; pulling out a pin at the back of the dial case permits of the removal of the upper bearing bracket, when the whole can be lifted out and the metal core of the receiving roller removed from the paper roll.

For marking upon the sheet the record of the predictions indicated to the operator by the pointers and dials there are provided two pens (plates 7 and 14), one for tracing a base line and marking the hour and day spaces and the times of the high and low waters, and the other for tracing the predicted tide curve. The curve pen uses about half the ink it can hold for a year's curve, the base-line pen somewhat less.

Base-line pen.—The base-line pen (plates 7 and 14) is mounted by means of a metal lock joint, like that of the curve pen, on a swivel arm with a spring for pressing against the paper. The swivel arm is secured to the outer end of a shaft (plate 14), which carries two armatures, one for an upper and one for a lower electromagnet. A spring keeps the armatures at equal distances from their respective electromagnets, the pen tracing a straight line upon the paper moved along under it. The swivel arm holding the pen can be raised or lowered a small amount for exact adjustment of the pen point to mean sea level of the curve. The electromagnets, together with the pen shaft and its armatures, are mounted upon an angle plate held against the inner side of the front plate of the dial case by two screws which, when loosened, permit of adjusting the pen point laterally, so that the curve pen will just clear it in its vertical movement. The unavoidable error in time, due to the fact that both pens can not pass through the same point, amounting to two or three minutes, can at any time be accurately recorded on the sheet by pulling the curve pen, when above the base line, down and across the base line, thus making it trace a vertical line, and then making a vertical line with the base-line pen by pressing against either of the armatures. The distance between these two lines is the measure of the error.

The upper electromagnet is in circuit with a battery and a current-making device, which consists of a platinum-tipped contact spring resting upon the edge of a rubber disk, in which are imbedded, accurately spaced, 24 narrow strips of platinum. This rubber disk is secured to the shaft of the hour pointer within the dial case (plate 9). An extra strip of platinum is placed close to that representing the twenty-fourth hour, or midnight. The contact spring can be brought to exact agreement with the hour and minute pointers by means of a fine-threaded screw. As each platinum strip passes under the contact spring the upper electromagnet attracts for a moment the armature, which throws the base-line pen downward for an instant and makes a short mark below the base line for each dial hour, a double mark indicating the twenty-fourth hour, or midnight.

Another circuit, including the same battery but the lower electromagnet, is closed when the projecting point of the platinum link in the time chain is in coincidence and contact with the fixed platinum index seen by the operator in the middle of the horizontal opening in



REAR VERTICAL DRIVING SHAFT, REAR COMPONENT FRAME.

the dial case. At such times the base-line pen is thrown upward, making a mark above the base line, which is a record of the time of the high or low water traced by the curve pen.

Both this circuit and the one making the hour and day breaks have one battery wire in common, which is led to a small switch at the left side of the desk, just above the crank shaft (plate 8), by means of which the battery can be cut out when the machine is not in operation.

Curve pen.—The curve pen is mounted in a swiveling arm on a light carriage which slides along two vertical steel rods. The pen, of the ordinary fountain type, is pressed against the paper by a light coil spring under the outer end of the swivel arm, but can be kept away from the paper by the turning of a small nut. Being fitted with a metal lock joint it can be quickly removed and replaced in exactly the same position. Motion is imparted to the curve pen by the height chain through the height sum pulley in the following manner.

Scale of the tide curve.—The horizontal shaft running in brackets fastened to the back of the dial case, the lower one seen on plate 4, the rotary motion of which, as before stated, is thirty one-hundredths that of the height sum pulley, carries upon it three sliding change gears of 48 pitch, with 75, 100, and 90 teeth, respectively. Immediately above this shaft is mounted another one with three gears of 75, 50, and 60 teeth, respectively, and, at its outer end, a thread grooved pulley with a circumference of 4 inches, measured on the centers of the chain fastened at its side and wound around it (plate 14). From this pulley the chain passes through the dial case around an idler pulley in its front, up past the curve pen carriage over another idler pulley to a counterpoise rising and falling within the dial case. A clamp and clamp screw at the pen carriage permits of the latter being secured to the chain at any desired point.

By means of the three different ratios of the change gears a motion of 1 inch of the free end of the chain, or one-twelfth revolution of the height sum pulley, can be converted into pen motions with either of the following values:

With gears 75:75, or 1:1, $\frac{1}{12} \times \frac{30}{100} \times \frac{75}{75} \times 4 = 0.1$ inch.

With gears 90:60, or 3:2, $\frac{1}{12} \times \frac{30}{100} \times \frac{90}{60} \times 4 = 0.15$ inch.

With gears 100:50, or 2:1, $\frac{1}{12} \times \frac{30}{100} \times \frac{100}{50} \times 4 = 0.2$ inch.

When the amplitudes set up in the machine are multiplied by four and the 10-foot scale is used, the tide curve may be traced, within the limits of the height of the sheet, upon scales of 1 to 15, 1 to 20, or 1 to 30 of nature; with amplitudes multiplied by two and the use of the

20-foot scale, 1 to 30, 1 to 40, or 1 to 60, and with amplitudes at their true value and the use of the 40-foot scale, 1 to 60, 1 to 80, or 1 to 120 of nature.

Tide curve.—The tide curve, though traced incidentally to the regular predictions for use as a record, also serves another purpose. The tides at some stations are of the form in which at times the diurnal overcomes the semidiurnal wave, thus producing vanishing or evanescent tides (plate 15, at *a*), in which frequently a theoretical high or low water is lower or higher than the preceding low or high water, respectively. In such cases it becomes necessary, in order to avoid cumbering the tide tables with theoretical data, to make such compromises as are most suitable for the practical needs of the mariner. This can be done by an experienced operator by inspection of the curve, of which more than a day is exposed at all times, as the predictions proceed, so that, when the high and low waters of the last day of the year have been written down, the printer's copy for the station is completed.

On plate 15 are shown two facsimile curves on actual scale produced by the machine. One of them shows, at *b* the manner in which the small difference in time between the curve and base line pens is ascertained.

Automatic stopping device.—For relieving the operator of the greater part of the strain due to watching the appearance of the platinum zero link in the time chain, and stopping the machine at the instant of its coincidence with the index, an automatic stopping device is provided. It consists of an electric circuit which, when closed, causes an electromagnet mounted under the desk top (plate 8) to throw down upon the edge of a ratchet wheel, 5⅞ inches diameter, 400 teeth, ⅜ inch face, secured to the crank shaft, a steel pawl, thereby arresting the motion of the crank and stopping the machine. The circuit is closed by a contact spring which rests upon a hard-rubber cylinder (plate 9) on the rear end of the shaft holding the pulley upon which the time chain ends. A small platinum plug in this cylinder comes in contact with the spring, which latter is fitted with a fine motion adjustment, when the zero link of the time chain is in coincidence with the index. The lateral screw motion of this shaft prevents the platinum plug from again making contact with the spring when the shaft has made one or more revolutions on either side of the zero position of the chain. The circuit is led through an insulated ring on the hub of the crank and a contact kept closed by a spring. A slight inward pressure against the crank handle overcomes this spring, breaks the circuit, and releases the armature and pawl, thereby permitting the machine to be moved forward to the next stop. By means of a small switch just below the crank the battery can be cut out.

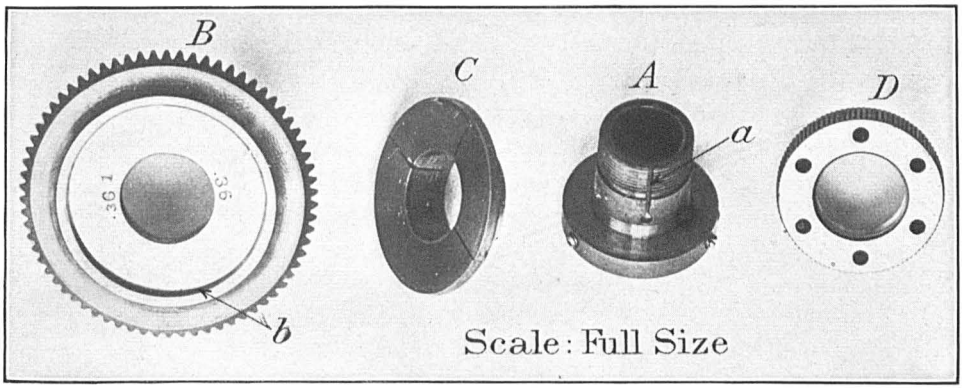


Fig. *a*.—RELEASING GEARS ON VERTICAL DRIVING SHAFTS.

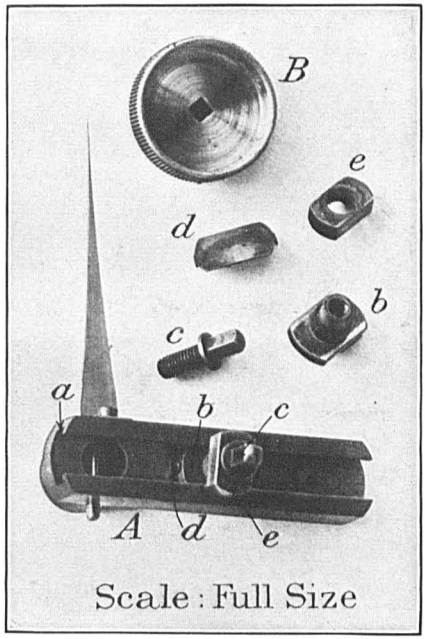


Fig. *b*.—COMPONENT CRANKS.

Upon the crank shaft, close to the bearing in the desk frame (plate 3), is secured a small ratchet wheel. A pawl, kept lifted away from this wheel by friction springs when the machine is moved forward in the right direction, is instantly thrown into engagement when the crank is accidentally turned backward. By pushing in one of the small buttons just above the crank the pawl is locked, which permits of moving the machine backward when desired. Pushing in the other button again secures the machine against accidental backward motion.

Speed of machine.—The speed with which the machine may be turned, were its object only to produce a tide curve, as is the case with the British machines, varies somewhat with the amplitudes set, a large tide requiring a slower motion of the crank than a small one. The average speed is 20 turns of the crank per minute. A dial day requiring six turns, a year's curve could be traced in 1 hour and 50 minutes; but the curve is produced incidentally only to the main purpose of the machine, which is to indicate the times and heights of the high and low waters for filling out the blank forms to be sent to the printer. As ascertained from the predictions of all the complicated tides for the years 1912 and 1913, made in the year 1910, the time of setting the machine for a station varies, according to the number of components involved, between 2 and 3 hours for one man; that of operating the machine and copying upon the forms the indicated times and heights for one year, from 7 to 12 hours. The difference in the time of the latter operation is due to the greater or less frequency of periods of diurnal or evanescent tides, which require more or less judgment on the part of the operator and therefore cause more or less delay.

Temperature error.—As regards the question of error due to the effects of changes in temperature, which would appear of moment in view of the great length of chain, it may be said that the results are practically unaffected by thermal expansion or contraction. A rough determination of the effect upon the free end of the chain, which involves the thermal coefficients of brass and steel and the doubling effect of the motions of the chain pulleys, shows a shortening by 0.055 inch or 1.4 millimeters for a rise and a lengthening of the same amount for a fall of 25° F. or 14° C. Such changes of temperature can take place only from season to season, during which time the machine has been readjusted a number of times.

A possible source of error in the results obtained with the machine, which had, however, been anticipated by the treatment of the chain before described, is that due to wear in its many joints (27.6 feet by 125 links equals 3,447) and consequent lengthening of the whole. For the purpose of testing its constancy in this respect, all the components were carefully set to zero and the position of the mean sea level

pointer of the height sum pulley and that of the platinum zero link of the time chain in relation to their respective indices noted several times during the prediction of the tides for the 1912 and 1913 tables. No change in the lengths of the chains could be noticed.

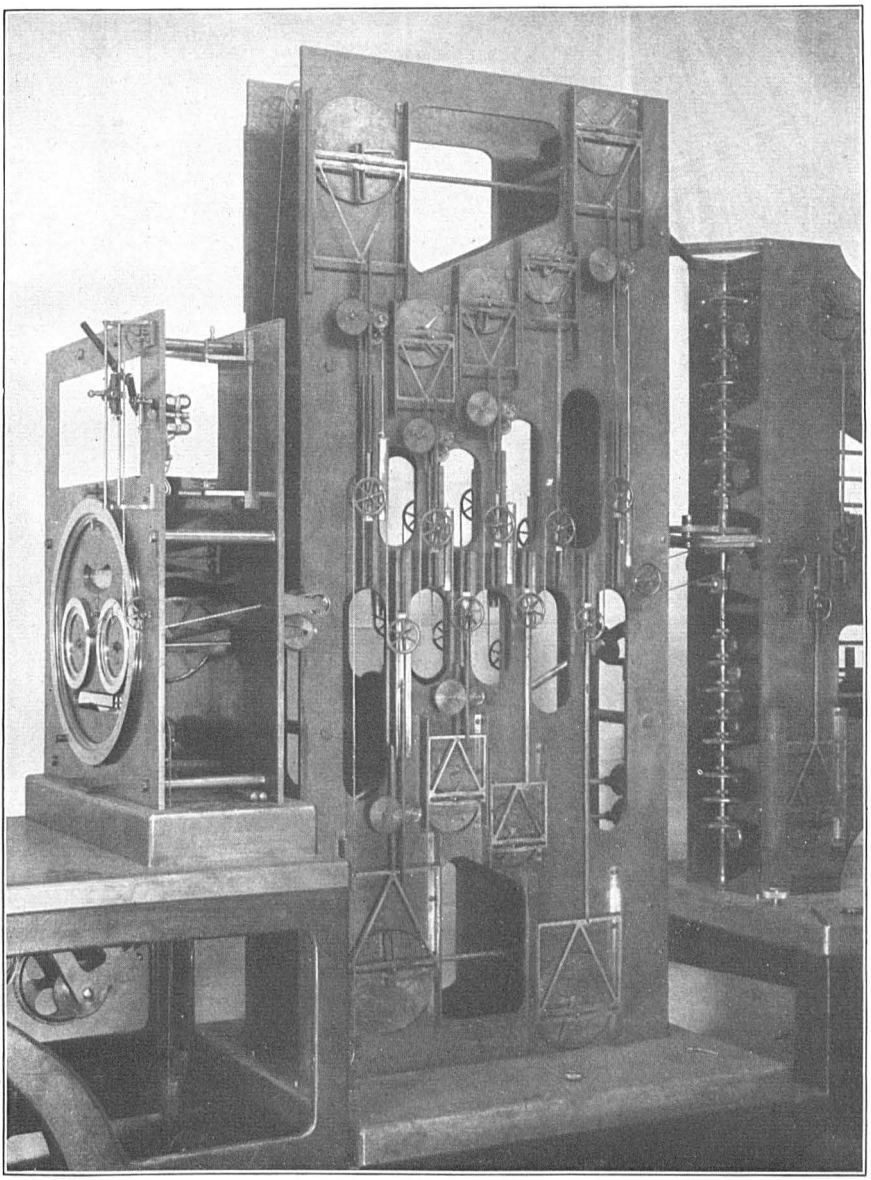
Accuracy of the machine.—At first thought one would probably suppose that the best test of the accuracy of a tide-predicting machine would be a comparison of its results with actual observation at the port for which it is set up. Such a comparison is always possible by setting the machine for a preceding period during which automatic tide-gauge readings were made. But the tides of nature are so much affected by meteorological conditions, which can not be predicted, that such a comparison must necessarily be more or less rough.

After mature consideration it is found that the best test of the accuracy of a tide-predicting machine as a piece of mechanism is to compare its mechanical solution of the equations put into it with the corresponding results obtained by computation from the same elements.

Such a comparison was made, using the regular predictions for 1912 and 1913 tide tables, at the station Aden, Arabia, with 34 components, and at Hongkong, China, with 30 components, hourly heights being read off the dial, by estimation to hundredths of feet, upon a day at the end of the year for which the predictions were being made. The greatest difference between the predicted and computed 24-hourly heights was 0.02 foot in the case of the Aden tide, and 0.06 foot in that of Hongkong. The results of the latter test are shown in Table A.

TABLE A.—*Test for accuracy by comparison of predicted with computed heights—Hourly heights for Hongkong, China, Dec. 31, 1912.*

Hour.	Predicted.		Computed.	Predicted—Computed.		Hour.	Predicted.		Computed.	Predicted—Computed.	
	Dial.	Curve.		Dial.	Curve.		Dial.	Curve.		Dial.	Curve.
	Feet.	*Feet.*	*Feet.*	*Feet.*	*Feet.*		*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*
0.....	4.28	4.32	4.32	−0.04	0.00	12.....	1.51	1.54	1.56	−0.05	−0.02
1.....	4.65	4.68	4.68	−0.03	0.00	13.....	2.31	2.34	2.37	−0.06	−0.03
2.....	4.78	4.80	4.80	−0.02	0.00	14.....	3.00	3.06	3.06	−0.06	0.00
3.....	4.60	4.61	4.59	+0.01	+0.02	15.....	3.48	3.52	2.53	−0.05	−0.01
4.....	4.07	4.07	4.05	+0.02	+0.02	16.....	3.70	3.72	3.73	−0.03	−0.01
5.....	3.27	3.25	3.22	+0.05	+0.03	17.....	3.64	3.66	3.66	−0.02	0.00
6.....	2.32	2.30	2.28	+0.04	+0.02	18.....	3.38	3.38	3.40	−0.02	−0.02
7.....	1.42	1.40	1.37	+0.05	+0.03	19.....	3.04	3.03	3.07	−0.03	−0.04
8.....	0.70	0.70	0.66	+0.04	+0.04	20.....	2.73	2.73	2.75	−0.02	−0.02
9.....	0.31	0.32	0.27	+0.04	+0.05	21.....	2.50	2.50	2.51	−0.01	−0.01
10.....	0.34	0.36	0.33	+0.01	+0.03	22.....	2.46	2.47	2.47	−0.01	0.00
11.....	0.76	0.79	0.81	−0.05	−0.02	23.....	2.64	2.65	2.67	−0.03	−0.02



DIAL CASE AND FRONT COMPONENT FRAME, TIME SIDE.

The curves traced during the predictions of these hourly heights also served as a means of comparison between results as computed and given by the machine. The facsimile curve on plate 15, marked Hongkong, 1912, contains the day, December 31, for which the hourly heights are given in Table A.

In order to ascertain the certainty of action of the machine, predictions were made for a number of days a number of times in succession, the machine being turned back between each set of predictions. The agreement between the heights and times of the different sets were within 0.02 foot and 1 minute, respectively. The curves traced in these tests, being on a small scale, revealed no differences when laid upon each other. A practical agreement was also found between the predictions made for the same period by three different observers.

Amplitude unit.—A tidal calendar giving the tides for the ports and stations of all countries is, like astronomical publications, of international import. It would seem at first glance that the amplitude unit of this machine might preferably have been based upon the meter rather than the foot, but when it is considered that by far the largest number of charts used by the navigators of all nations give the soundings in fathoms or multiples of the foot, the advantage of using the foot is apparent. Should predictions in meters be required, the conversion of the metric into foot amplitudes, which need only be done once for each station, and the addition of three height-dial rings with scales of 3, 6, and 12 meters, would prepare the machine for the prediction of heights in meters.

TABLE B.—*Table of components represented in tide-predicting machines.*

Symbols of components.	British Association, or British tide predictor No. 1 (10 components).	Roberts's India Office, or British tide predictor No. 2 (24 components).	Thomson's, or British tide predictor No. 3 (15 components).	French tide predictor (16 components).	Brazilian tide predictor (12 components).	Roberts's universal tide predictor (33 components).	Ferrel's maxima and minima, or United States Coast and Geodetic Survey tide predictor No. 1 (19 components).	United States Coast and Geodetic Survey tide predictor No. 2 (37 components).
J_1	.....	J_1	.....	.....	.....	J_1	.....	J_1
K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1
K_2	K_2	K_2	K_2	K_2	K_2	K_2	K_2	K_2
L_2	L_2	L_2	L_2	L_2	L_2	L_2	L_2	L_2
λ_2	.....	.....	.....	.....	.....	λ_2	λ_2	λ_2
M_1	.....	.....	.....	.....	.....	.....	M_1	M_1
M_2	M_2	M_2	M_2	M_2	M_2	M_2	M_2	M_2
M_3	.....	.....	.....	.....	.....	M_3	.....	M_3
M_4	M_4	M_4	M_4	M_4	.....	M_4	M_4	M_4
M_6	.....	M_6	M_6	M_6	.....	M_6	M_6	M_6
M_8	.....	.....	.....	.....	.....	M_8	.....	M_8
Mf	.....	.....	.....	.....	.....	Mf	.....	Mf
MK	.....	MK	.....	.....	.....	MK	.....	MK
2MK	.....	2MK	.....	.....	.....	2MK	.....	2MK
Mm	.....	.....	.....	.....	.....	Mm	.....	Mm
MN	.....	MN	.....	.....	.....	MN	.....	MN
$(MS)_4$	$(MS)_4$	$(MS)_4$	$(MS)_4$	$(MS)_4$	.....	$(MS)_4$	$(MS)_4$	$(MS)_4$
3MS	.....	.....	.....	.....	.....	3MS	.....	.....
MSf	.....	.....	.....	.....	.....	MSf	.....	MSf
μ_2	.....	μ_2	μ_2	μ_2	μ_2	μ_2	μ_2	μ_2
N_2	N_2	N_2	N_2	N_2	N_2	N_2	N_2	N_2
2N	.....	2N	.....	.....	.....	2N	.....	2N
ν_2	.....	ν_2	ν_2	ν_2	ν_2	ν_2	ν_2	ν_2
O_1	.....	O_1	O_1	O_1	O_1	O_1	O_1	O_1
OO	.....	.....	.....	.....	.....	.....	.....	OO
P_1	P_1	P_1	P_1	P_1	P_1	P_1	P_1	P_1
Q_1	Q_1	Q_1	Q_1	Q_1	Q_1	Q_1	Q_1	Q_1
2Q	.....	.....	.....	.....	.....	.....	.....	2Q
R_2	.....	.....	.....	.....	.....	.....	.....	R_2
ρ_1	.....	.....	.....	.....	.....	.....	.....	ρ_1
S_1	.....	S_1	S_1	S_1	S_1	S_1	.....	S_1
S_2	S_2	S_2	S_2	S_2	S_2	S_2	S_2	S_2
S_4	.....	.....	.....	.....	.....	S_4	.....	S_4
S_6	.....	.....	.....	.....	.....	.....	.....	S_6
Sa	.....	Sa	.....	.....	.....	Sa	Sa	Sa
2SM	.....	2SM	.....	.....	.....	2SM	.....	2SM
3SM	.....	.....	.....	.....	.....	3SM	.....	.....
Ssa	.....	Ssa	.....	.....	.....	Ssa	.....	Ssa
T_2	.....	T_2	.....	T_2	.....	T_2	T_2	T_2
U_2	.....	.....	.....	.....	.....	.....	U_2	.....

TABLE C.—*Coast and Geodetic Survey tide-predicting machine No. 2.*

Components.	Theoretical speed per mean solar hour.	Number of teeth in gear wheels.				Gear speed per dial hour.	Error of gears per dial year.	Length of cranks.	
		Vertical shafts.	Intermediate shafts.		Component shafts.			Height.	Time.
			Outer end.	Inner end.					
		I	II	III	IV				
	°					°	°	*Inches.*	*Inches.*
J_1	15.5854433	107	90	52	119	15.5854342	+0.08	0.7	0.4
$K_1/2$	15.0410686	61	73	51	85	15.0410959	− .24	1.38×2	1.45
$K_1/2$	15.0410686	61	73	51	85	15.0410959	− .24	1.38×2	1.45
K_2	30.0821372	122	80	96	146	30.0821918	− .48	2.0	2.0
L_2	29.5284788	104	61	56	97	29.5284773	+ .01	1.2	1.2
λ_2	29.4556254	131	65	57	117	29.4556213	+ .04	0.2	0.2
M_1	14.4920521	103	85	59	148	14.4920509	+ .01	0.5	0.3
$M_2/2$	28.9841042	103	74	59	85	28.9841017	+ .02	2.50×2	2.50×2
$M_2/2$	28.9841042	103	74	59	85	28.9841017	+ .02	2.50×2	2.50×2
M_3	43.4761563	86	62	70	67	43.4761675	− .10	0.7	1.0
M_4	57.9682084	118	74	103	85	57.9682035	+ .04	2.00	2.00×2
M_6	86.9523126	140	62	86	67	86.9523351	− .20	0.5	1.5
M_8	115.9364168	118	37	103	85	115.9364070	+ .00	0.2	0.8
Mf	1.0980330	84	45	1	51	1.0980392	− .05	2.0	0.08
MK	44.0251728	120	81	105	106	44.0251572	+ .14	1.0	1.4
2MK	42.0271398	81	52	79	86	42.0271020	+ .33	0.7	1.0
Mm	0.5443747	93	41	1	125	0.5443902	− .14	1.5	0.03
MN	57.4238338	135	42	53	89	57.4237560	+ .68	0.4	0.7
$(MS)_4$	58.9841042	118	61	62	61	58.9841440	− .35	1.0	2.0
MSf	1.0158958	149	80	1	55	1.0159091	− .12	1.0	0.04
μ_2	27.9682084	125	82	74	121	27.9681516	+ .50	0.6	0.6
N_2	28.4397296	65	46	53	79	28.4397358	− .05	1.50×2	1.50×2
2N	27.8953548	68	58	46	58	27.8953627	− .07	0.5	0.5
ν_2	28.5125830	89	69	70	95	28.5125858	− .02	1.0	1.0
O_1	13.9430356	92	89	58	129	13.9430363	− .01	2.25×2	2.20
OO	16.1391016	134	131	71	135	16.1391009	+ .01	0.4	0.3
P_1	14.9589314	91	73	50	125	14.9589041	+ .24	2.4	1.3
Q_1	13.3986609	84	88	51	109	13.3986656	− .04	1.5	0.7
2Q	12.8542862	127	114	50	130	12.8542510	+ .31	0.3	0.2
R_2	30.0410686	85	50	43	73	30.0410959	− .24	0.2	0.2
ρ_1	13.4715144	69	70	41	90	13.4714286	+ .75	0.4	0.27
S_1	15.0000000	63	75	50	84	15.0000000	.00	1.0	0.6
$S_2/2$	30.0000000	70	70	70	70	30.0000000	.00	1.23×2	1.25×2
$S_2/2$	30.0000000	70	70	70	70	30.0000000	.00	1.23×2	1.25×2
S_4	60.0000000	75	45	60	50	60.0000000	.00	0.5	1.0
S_6	90.0000000	90	48	80	50	90.0000000	.00	0.5	1.5
Sa	0.0410686	51	149 1	125 60	120	0.0410738	− .05	2.00×2	.....
2SM	31.0158958	69	47	50	71	31.0158825	+ .12	0.7	0.7
Ssa	0.0821372	51	149	1	125	0.0821477	− .09	1.5	.....
T_2	29.9589314	81	50	45	73	29.9589041	+ .24	0.5	0.5

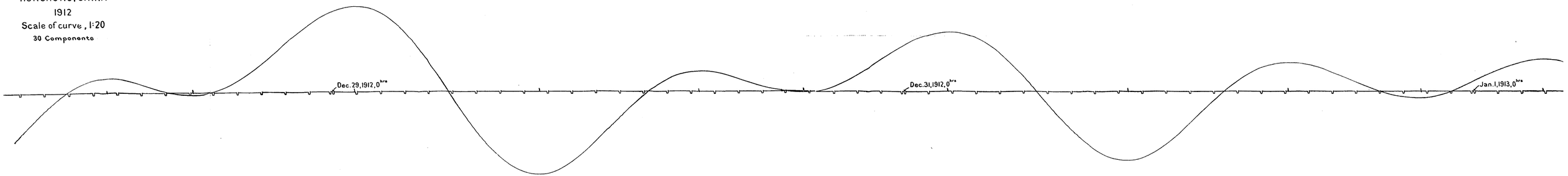
O

HONGKONG, CHINA

1912

Scale of curve, 1:20

30 Components

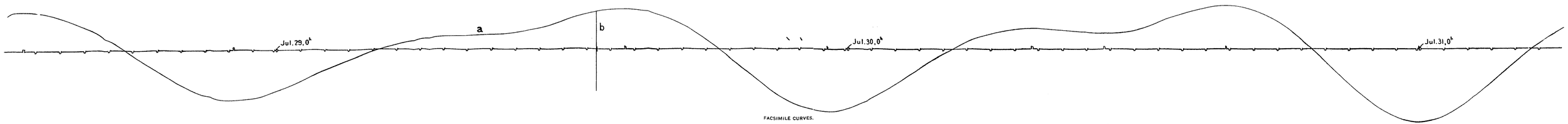


ADEN, ARABIA

1913

Scale of curve 1:30

35 Components



FACSIMILE CURVES.