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U. S. COAST AND GEODETIC SURVEY

E. LESTER JONES, DIRECTOR

USE OF GEODETIC CONTROL FOR CITY SURVEYS

By

HUGH C. MITCHELL
Mathematician

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USE OF GEODETIC CONTROL FOR CITY SURVEYS.

By HUGH C. MITCHELL, *Mathematician, U. S. Coast and Geodetic Survey.*

INTRODUCTION.

While the use of geodetic control and methods by the cities of this country has grown very slowly in favor with city engineering officials, yet that growth has been sufficient to indicate a strong tendency toward the ideal of precise and comprehensive surveys for all city planning work. The object of this publication is to present to municipal engineers a comprehensive report on the methods by which geodetic control, already covering a large portion of our country, can be utilized and to suggest the use of geodetic methods to other engineers.

A recognition of the value and of the need of the precise city survey is thus clearly stated by a recent writer on municipal engineering: [1]

> At some time in the life of a city a careful, comprehensive, precise survey of the entire city should be made, and the earlier this is done the less trouble to property owners, city engineers, and all concerned * * *. Unlimited trouble is caused by inconsistencies in the surveys of neighboring tracts, by undiscovered errors, and other conditions known or unknown which can be discovered and rectified only by a precise survey of the entire city. This should be a complete survey of undeveloped as well as developed areas, which will not only check and connect up previous surveys but furnish information on which to base future city planning.

This need has been expressed in another way in the report of the planning commission of one of our eastern cities.[2] In connection with the review of subdivision plans it was stated:

> The engineer in the City Plan Commission's office becomes familiar, if he is not already, with the topographical features of the land proposed for subdivision, even if necessary making a hurried survey.

In a paper [3] read before the Institution of Surveyors in Sydney, N. S. W., nine years ago, it was stated:

> Every city or town of any dimensions requires a standard survey if its public services are to be economically and methodically administered, * * *.

Perhaps the most notable example in recent years of recognition of the relationship between the city plan and precise surveys is in Australia, where the formation, early in the present century, of the Commonwealth of Australia, and the enactment of the Seat of Government act of 1908 brought into existence a new capital city, Canberra, which started with a precise topographic survey and whose administration plans called for a department of surveys, with triangulation and topographical branches.

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[1] Municipal Engineering Practice, by A. Prescott Folwell, p. 215.
[2] Report of City Planning Commission, Syracuse, N. Y., 1919, p. 36.
[3] "Notes on a proposed standard survey for Sydney," by T. G. Wilson, jr., published in The Surveyor, Sydney, N. S. W., June 30, 1913.

Geodetic data, when available, are already largely used in this country by engineers outside the Federal service. This is particularly true of the precise level bench marks established over the country by this and other organizations, as was clearly shown in 1916 by the replies to a letter of inquiry sent out to chief engineers of cities, States, and public-service corporations, asking information as to the use and sentiment regarding the precise level elevations of the survey and the value of mean sea level as a datum for elevations. A digest of the replies received was given to the public in U. S. Coast and Geodetic Survey Special Publication No. 41, entitled "The Use of Mean Sea Level as the Datum for Elevations." It showed conclusively that mean sea level is generally regarded as the ultimate datum for all precise level operations, and that it is desirable to use elevations on this datum as initials for all engineering works whenever accurate elevations are required. In the conclusion of that publication it was stated:

> The survey, whose obligation it is to extend the precise leveling net over the country for the use of Federal, State, city, and other engineering, surveying, and mapping work, feels that the expressions of opinion of the leading engineers of the country point to only one conclusion. This is that the Federal Government should provide the fundamental elevations in sufficient number that the various Federal engineering and surveying organizations and the smaller political units such as States, counties, and cities, as well as private organizations and individuals, may have available fundamental elevations upon which to start leveling, precise or otherwise, for detailed operations.

The adoption of mean sea level as the datum for all city levels does two things: It serves first to correlate all elevations of the different city organizations and subdivisions on one permanent datum and second to coordinate these elevations with the elevations of outside organizations which operate partially within the city, such as railways and State highway commissions. Where this correlation on a single datum does not exist great confusion must result. Even where the city has all its elevations on a single but arbitrary datum the railroads entering it may be using a different datum, and the chances of error in planning improvements would be very great.

The same conditions which make desirable the use of standard geodetic control for vertical surveys obtain with regard to horizontal surveys. A careful consideration of the situation must convince the engineer that he can not properly relate the surveys of various portions of his territory, nor place them in their true positions on any map, unless those surveys are standardized in form and are coordinated by unified geodetic control. As a result of lack of coordination, any comprehensive undertaking reaching to far parts of the map will suffer, both in planning and in execution.

The necessity for this correlation and coordination is well illustrated in the work of the U. S. Coast and Geodetic Survey. In the beginning many detached pieces of triangulation were executed, independent in geographic position, orientation, and length control, but, in so far as was feasible, standard in measures of length and methods of observation. As these various pieces of work were expanded they touched and there developed discrepancies in length, latitude, longitude, and azimuth. The discrepancies in position and azimuth were to be expected as each separate survey had entering directly into it the effects of the local deflections of the plumb line. The discrepancies in length were usually well within allowable limits of error and required

only specific mathematical operations for their elimination. In order to eliminate the discrepancies in position and azimuth, however, it was necessary to reject the many independent astronomic data which controlled them and substitute therefor one standard geodetic datum—a datum where all the station errors in any one class due to deflections of the vertical would, as far as practicable, sum up to zero. Such a datum is the North American datum, which is now the official datum not only for the work of the United States surveying bureaus but also for the geodetic work of the Canadian and Mexican Governments. It was adopted in 1901 by the U. S. Coast and Geodetic Survey under the name of the United States standard datum, but this name was changed in 1913 to that of the North American datum, when its adoption by the Mexican and Canadian Governments gave it an international character. The engineer using the publications of the U. S. Coast and Geodetic Survey should therefore keep in mind that the geographic positions based on the United States standard datum are exactly coordinate with those based on the North American datum.

A geodetic datum is best defined in terms of the latitude and longitude of a given point and the azimuth of a line from that point as computed on an adopted spheroid of reference. A station may then be said to be on that datum when its position and the azimuths from it are derived by computation through a continuous triangulation from the position and azimuth given in the definition, using the adopted spheroid of reference.

The North American datum has been defined in terms of the latitude and longitude of the triangulation station *Meades Ranch* in Kansas and an azimuth from that station as follows: Latitude, 39° 13′ 26″.686; longitude, 98° 32′ 30″.506; azimuth to Waldo, 75° 28′ 14″.52. Points may then be said to be on the North American datum when they are connected with the station *Meades Ranch* by continuous triangulation through which the corresponding latitudes, longitudes, and azimuths have been computed on the Clarke spheroid of 1866 as expressed in meters, starting from the above data.

As shown on the map (p. 76) great arcs of triangulation, all based on the single datum, now form a vast network over this country. This triangulation is being expanded rapidly and it is expected that before many years it will be within 50 miles of every point in the United States. Supplementing this triangulation and attaining a corresponding accuracy, lines of precise traverse are used to carry the control into flat and wooded country where it would be excessively expensive to carry on triangulation owing to the necessity of having high signals to elevate the surveying instruments above intervening obstructions.

The value of having all precise horizontal control on one datum must be apparent at a glance. For the purposes of a national survey it was necessary that there be a connection between the charts on the Atlantic and Pacific coasts, and the first geodetic surveys made in the interior of the country were for the purpose of establishing that connection. If interstate surveys are to be properly correlated and interstate work well planned, then the State surveys must be on the same datum as the national surveys, and if county and city surveys are to properly aid in State projects, they too must be on the same datum. Binding together the work of the cities and counties are large public and semipublic undertakings, such as railroads,

highways, canals, and river improvements, which demand, for best results, that they be planned on coordinate data. Just as surely as the only datum for elevations which will serve every use is the sea-level datum, so for horizontal surveys the only datum on which the work can be properly coordinated for any extensive undertaking is the North American datum.

It might be further noted that for general military purposes and for the use of Federal engineering organizations and public-service corporations comprehensive maps of large areas are often required in a minimum of time. Such maps are necessarily made largely by compilation. This compilation is most easily, quickly, and economically made if the separate maps used are not only standard in their elements but are also on the same horizontal and vertical data.

It is hoped that this publication may grow in value through use and the evolution which a technical manual in use always undergoes. For this reason it is desired that the engineer making use of it will always feel free to write to this office of any special problems that are incompletely covered herein and to offer any suggestions which he feels would tend to make later editions of this report more useful.

Acknowledgment is here made of the help received in the preparation of this publication from various members of the division of geodesy. Many valuable suggestions and helpful criticisms have been received from William Bowie, chief of the division; C. V. Hodgson, assistant chief; W. F. Reynolds, chief geodetic mathematician; H. G. Avers, C. A. Mourhess, and C. H. Swick, mathematicians, all of the division of geodesy, U. S. Coast and Geodetic Survey; and from R. N. Ashmun, chief computer of the International Boundary Commission.

A number of publications have been consulted freely and quoted from occasionally. Effort has been made to properly credit such quotations. There are, however, in the publications of the U. S. Coast and Geodetic Survey a number of descriptions and definitions which have, through repetition, become standardized. It has not always been possible to credit such paragraphs and in some cases they may have been used without quotation marks.

Part I.—USES OF GEODETIC METHODS AND CONTROL.

QUESTIONNAIRE.

For the purpose of determining to what extent the cities of our country are making use of geodetic methods and control the following questionnaire was sent out early in December, 1921, to the city engineers of all cities of more than 10,000 population, and to a group of other engineers who were known to be engaged or interested in municipal engineering. This questionnaire was designed also to discover, as far as possible, the requirements of the different cities in precise survey work. It was mailed to 897 engineers throughout the country; 223 replies were received.

1. Are you using geodetic methods in any of your city survey work?
2. Is any of your city survey work being based on geodetic control?
3. If so, what constitutes this control? And how and when was it executed?
4. What are the standards of your geodetic control considered under the following heads?
   - A. Triangulation:
     - (a) Accuracy?
     - (b) Frequency of stations?
     - (c) Character and depth of station marks?
   - B. Traverse:
     - (a) Accuracy?
     - (b) Frequency of stations?
     - (c) Character and depth of station marks?
   - C. Levels:
     - (a) Accuracy?
     - (b) Frequency of bench marks?
     - (c) Character and depth of bench marks?
5. Does this geodetic control satisfy the demands upon it?
6. Will you please indicate what you consider the most desirable standards for geodetic control of city surveys to serve the following purposes:
   - I. Control for accurate topographic surveys and maps which are to be used in city planning (as of sewers, parks, roadways, viaducts, etc.).
     - A. Triangulation:
       - (a) Accuracy?
       - (b) Frequency and distribution of stations?
     - B. Traverse:
       - (a) Accuracy?
       - (b) Frequency and distribution of stations?
     - C. Levels:
       - (a) Accuracy?
       - (b) Frequency and distribution of bench marks?
   - II. Control for cadastral surveys.
     - A. Triangulation:
       - (a) Accuracy?
       - (b) Frequency and distribution of stations?
     - B. Traverse:
       - (a) Accuracy?
       - (b) Frequency and distribution of stations?
7. In some countries the national geodetic survey cooperates with the city governments in extending precise control over city areas. Since it is impracticable at the present time for this bureau to execute detailed triangulation for cities, what is the minimum amount of precise horizontal and vertical control which should be established in a city by this bureau while carrying the control through it or near it?
8. Will you please state any special problems or solutions of problems which have come up in your practice and which might be of interest and value to city engineers making use of geodetic control?

Twenty affirmative and 203 negative replies were received to the first question; 25 affirmative and 190 negative to the second.

The replies to question 3 showed that only 8 cities had local triangulation. For starting data 1 city was using U. S. Lake Survey triangulation and 7 were using U. S. Coast and Geodetic Survey triangulation. Only 5 cities had used geodetic methods in running level lines, but 27 were basing their levels on sea-level elevations established by the U. S. Coast and Geodetic Survey, the U. S. Geological Survey, and other organizations.

There were 12 replies to question 6 as to what constituted the standards of the geodetic control. A number of these replies were based on experience in using surveys which had been executed, in some cases, as much as a decade ago. The city engineer of Cincinnati stated that the triangulation of that city, which was executed in 1912 with an average length of line of 4 miles, does not satisfy the demands upon it, but that lines averaging 2 miles in length are required. In New York city, too, where the lines are 2 to 3 miles in length, it is now felt that for cadastral control a station every mile in each direction is desirable, though topographic survey needs would be met with stations averaging 3 miles apart. The city engineer of Detroit, planning extensive surveys, feels that for both topographic and cadastral surveys a triangulation station every 3 miles in each direction would suffice. The engineers of other cities favor triangulation stations as close together as 1,000 feet.

As to the accuracy of the triangulation, New York city, after more than 10 years' use of a triangulation having an accuracy of 1 part in 25,000 or greater, feels the need for an accuracy of 1 part in 100,000. The Cincinnati triangulation, with an accuracy of 1 part in about 50,000, must be satisfactory, since 1 in 40,000 is now set as a desirable standard by its engineer. The difference between the desirable standard of 1 part in 100,000 stated by Greater New York and 1 in 40,000 proposed by Cincinnati represents the relation existing between the requirements of the largest city and a large city. Other standards given in the replies run as low as 1 part in 2,500.

It is recognized that a control which will serve cadastral purposes is sufficiently good for any topographic surveying, and the standard of accuracy required for a cadastral survey is therefore the standard to be sought. Furthermore, the controlling factor in making a cadastral survey should be land values, and since, as a rule, the larger the city the higher the value of a unit of land, therefore the larger the city the greater will be the accuracy required. (See p. 14.)

In regard to the standards of accuracy for traverse control there was a wide divergency of replies recommending accuracies ranging from 1 part in 100,000 down to 1 part in 1,000. The replies dealing with the standards desirable for level lines were also widely divergent. It was decided not to include here a detailed analysis of these replies but they have been carefully considered in preparing the report.

Extracts from some of the replies are given below.

The engineer of one of the smaller mid-west cities very succinctly stated:

> Am in favor of any method that will insure greater accuracy.

Another engineer in an eastern city wrote:

I believe that the legislature should pass a law requiring all municipalities to make their level and meridian data conform to the U. S. Government for the district. It would wonderfully make surveying easier and more accurate and elevate surveying to the place where the layman would have more confidence in it.

The following came from an eminent city planning architect:

I have had repeated occasion to appreciate the advantages of adequate geodetic control for city surveys and the disadvantages of its absence, and have again and again urged its importance upon cities which have consulted me.

A consulting engineer in municipal practice in one of the southern States, speaking of providing geodetic control for town and county surveys, wrote:

The work is indeed worth while, and I wish that we had more frequent points.

Perhaps one of the most pertinent statements made in any of the replies received was by a topographic engineer of wide experience in the making of accurate maps for planning purposes:

All the serious problems relative to surveys that perplex the city engineer have their origin either in the diversity of data used at different times, or even during the same period, or the lack of standardizing the tapes, with the consequent different tape lengths used over different periods, and last but not least the lack of proper geodetic control to prevent the accumulation of errors and permit the coordination of the various fragmental surveys. If the different cities will get on a sea-level datum, keep their tapes standardized, and tie into and use geodetic methods and the control available, then indeed the way of the surveyor would be easy, and the burden on the taxpayer's pocketbook considerably lightened.

GEODETIC SURVEYS OF NEW YORK AND CINCINNATI.

It may be of interest and of some value to most city engineers to give here a more complete statement of the surveys of New York and of Cincinnati, already referred to, since these constitute two notable attempts on the part of city officials to secure comprehensive and accurate surveys.

The geodetic survey of Greater New York may be taken as being more nearly standard than any other city survey made in this country. The triangulation was executed under a cooperative agreement whereby the U. S. Coast and Geodetic Survey planned and supervised the survey, using personnel from the city engineer's office. The computation of the results was made by the force of the computing division (now division of geodesy) of the Washington office of the U. S. Coast and Geodetic Survey. The standards used throughout were those of the precise work of this bureau. The following statistics give a comprehensive view of the triangulation:

Work commenced in May, 1903, and final computations completed in December, 1908.

Number of points determined	183
Total cost of triangulation about $73,000, or per point	$400
Average closure of the 168 triangles in main scheme	2″.72
Average closure of the 857 triangles embraced in entire scheme	3″.33
Average correction to the 685 directions observed with direction theodolite	1″.42
Average correction to the 816 directions observed with the repeating theodolite	1″.87

Attention is called to the difference in size of the corrections to observed directions using the two different types of instrument. The repeating theodolite was used in places such as the roof of a building where the instability of the instrument support precluded the use of the direction theodolite, and as such locations were naturally most common in the built-up sections of the city, the difficulties

due to smoke, gases, hot-air vents, and lines grazing various structures were naturally greater than in the outlying sections of the city, where towers could be built and the direction theodolite used. Dr. John F. Hayford stated in the Report on the Triangulation of Greater New York (p. 438):

> In general, then, the observations with repeating theodolites were made under the difficult conditions necessarily encountered in making a triangulation in a thickly settled metropolitan district, and the observations with the direction theodolite were made under conditions intermediate between these difficult city conditions and the conditions which are ordinarily encountered in the open country in the primary triangulation of the Coast and Geodetic Survey and which are relatively very favorable to the attainment of a high degree of accuracy.

In 1912 the writer was called to Cincinnati to plan and execute a geodetic and topographic survey of that city under somewhat different conditions from those which obtained in making the New York survey. The immediate need of the city was for an accurate topographic survey to serve as a basis for extensive sewer planning, but with rare forethought, H. M. Waite, the chief engineer of the Department of Public Works, sought a map which would serve for general city planning as well. It was highly desirable that data be made available as speedily as possible for the sewer planning engineers and that the survey be completed, if possible, within less than two years. The survey was started late in April, 1912, and by the end of 1913 all work was completed, and the finished sheets of the survey were in the hands of the engraver with their reproduction fully contracted for.

Some statistics for the Cincinnati triangulation follow:

Work was commenced in May, 1912, and all field work completed in that calendar year. The office computations were completed during the next year.

Average closure of a triangle	2″75
Average correction to 84 primary observed directions	0″84
Probable error of an observed direction	0″97
Total cost $5,325.92, or per primary point	$254

PURPOSE OF THE GEODETIC SURVEY.

A geodetic survey may serve as a basis for a number of different operations in the city. It may furnish control for an accurate cadastral survey, a topographic survey, an underground survey, and for less general surveys, such as for tunnels and harbors. Outside the city it may serve as a basis for boundary surveys, canal surveys, road surveys, land subdivision, and to connect up detached portions of land surveys. It may be easily used for general land survey purposes. Some of these various uses for the geodetic control will be briefly discussed in the following pages.

CADASTRAL SURVEY.

A cadastral survey may be considered as made up of two parts or operations: (*a*) The surveying and mapping of the territory under consideration on a scale and with an accuracy, depending on local conditions, which will enable the engineer to determine within required limits of accuracy the exact location and area of the land contained in a given parcel; and (*b*) the official valuing of the land for governmental purposes. In this publication we shall be concerned with only the first operation.

Up to the present time, in this country at least, the use of geodetic control for cadastral purposes has been limited to a very few instances. In the earlier days of civilization, when land had comparatively small value and the need for precise methods of measurement did not exist, crude methods were naturally used. But as land has advanced in value the need for more and more accurate surveys has developed. In many cases litigation has arisen through the destruction of property bounds which were set by approximate survey methods and which, therefore, could not be accurately relocated. In some instances it has been found expedient to legalize erroneous data in order to avoid endless legal controversy. For example, in Pittsburgh, many years ago, a measuring bar [4] was used in city survey work which was later found to be too long. This bar was made a legal standard for its nominal length, so that now in some lot surveys a tape nominally 100 feet in length must have a standard length of 100.1 feet.

In the paper referred to on page 1, in the discussion of the functions which a proposed standard survey of Sydney would fulfill, is the statement:

> To my mind the first and most important (function) is that of being a survey of record for land titles, and by a system of permanent marking and establishment of muniments to give a settled and unchallengeable basis for the delimitation of boundaries.

That the liability to destruction of land monuments was recognized even by the ancients is indicated by the following passages from the preface of Geodaesia or the Art of Surveying, by John Love, published in London in 1688:

> But (Necessity being the Mother of Invention) we find the *Egyptians* by reason of the *Nyles* over-flowing, which either washed away all their Bound-Marks, or covered them over with *mud*, brought this Measuring of Land into an Art, and Honoured much the Professors of it.
>
> From *Egypt*, this Art was brought into *Greece* by *Thales*, and was for a long time called *Geometry;* but that being too comprehensive a name for the Mensuration of a Superficies only, it was afterwards called Geodaesia; * * *.

It is confidently believed that geodetic control will become more and more used for cadastral surveys as its value becomes recognized. There is but one point at the intersection of any given meridian and any given parallel on the earth's surface. The intersection of a geodetic parallel and meridian can be accurately determined by astronomic observations only where the deflection of the vertical is known. Since it is also wholly impracticable to determine by astronomic measures the position of each cadastral monument desired, these positions must be determined by geodetic methods and referred to a geodetic datum. From the definition of a geodetic datum (p. 3) we know that the points to be determined must be connected through triangulation with points already established on that datum. How closely we may establish the position of a station on a geodetic datum depends primarily upon two things—the number of triangulation figures between the new station and the station defining the geodetic datum and the accuracy of the triangulation through which the connection is made.

The accuracy with which a point which has been determined by geodetic methods from geodetic control can be replaced when lost

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[4] See Engineering News, Dec. 21, 1911, p. 734.

is one of the fundamental reasons for the use of geodetic control for cadastral surveys. Where a city has its cadastral monuments determined by geodetic methods, many of those monuments may be lost, but if there remain two undisturbed control points, then all the lost points may be restored therefrom with an accuracy that will generally meet the most rigid cadastral requirements. It may well be noted here that while it is stated above that two undisturbed control points are required, it is absolutely unsafe to trust to two points for this purpose. In all cases a check on the recovery should be secured by the recovery and use of at least three of the original control stations. (See p. 47.)

TOPOGRAPHIC SURVEY.

Geodetic control is of great assistance in city surveying in the making of topographic surveys and maps. Topographic surveying as ordinarily done, if extended very far from the starting point, must be checked by connection to fixed stations in order to maintain the required accuracy and to prevent blunders and the accumulation of errors. The best and most economical way to provide these fixed-control stations is by geodetic methods. A topographic map which is based on geodetic control can be made accurate, reliable, and satisfactory in every way.

A fundamental principle in the use of maps, that is often ignored, is that a map may always be reduced to one of smaller scale, but should never be enlarged and used as if the new or enlarged scale were of increased accuracy. The enlargement of the scale of a map does not of itself increase the accuracy. Attention may be called to an extreme case which was noted some years ago. The engineer of one of our most populous counties was asked what he did for a map on which to make highway plans. He replied that he simply took the mile-to-the-inch Federal survey maps, had them photographed up to a scale of 1 to 2,400, and that he then had a map on which he could easily make any plans.

In this publication we will consider the geodetic control required for the more accurate classes of surveys, assuming that the engineer whose requirements are not so rigid may easily prepare therefrom maps of the class which he needs.

UNDERGROUND SURVEY.

A third type of city survey which is aided by accurate geodetic control is the underground survey. The underground survey is essential to the proper extension of the city plan. The map resulting from that survey should show on a scale sufficiently large to avoid confusion all structures existing under streets and open city property and all public service underground structures no matter where located. This will include subways, tunnels, sewers, water pipes, gas mains, and telephone, telegraph, and electric-light conduits. In a built-up section of a city this information is necessary for the proper maintenance of the streets and of the underground structures themselves, and in partly built-up areas the information is necessary for planning extensions. If geodetic horizontal control has been

established over a city in connection with the topographic and cadastral surveys, it can be used also for the underground survey and the one control, if properly planned and executed, will thus serve all three detail surveys. The same is true in regard to the vertical control.

As a specific example of the value of geodetic control for underground surveys may be mentioned sewer work, where accuracy is very essential in making both the horizontal and the vertical surveys. If geodetic control points have been established, traverses can be run between these points, and the closure obtained will be an index of the accuracy of the traverse. Where the underground map has a scale of 1 inch equals 40 feet (1 part in 480), a scaling unit of one-eightieth inch will represent 0.5 foot on the ground. If the geodetic control has an accuracy of 1 part in 15,000 or better, there will be no difficulty in maintaining an accuracy of the detail survey within the above scaling unit of 0.5 foot. In case the geodetic control is reduced to a system of rectangular coordinates, computations of length and bearing of a sewer may be quickly made with an accuracy of result which will often exceed that to be obtained by direct measurement using ordinary survey methods and all appreciable accumulation of errors of either length or azimuth will be prevented.

So, too, with precise level bench marks, supplemented by primary levels, a minimum of leveling will be required for the sewer work and there will be the assurance that no gross errors can accumulate through the recurrence of small systematic errors.

A special form of the underground survey occurs in some of our larger cities where the necessity of constructing tunnels for various purposes sometimes arises. Lines impossible to measure directly may be determined by triangulation with an accuracy commensurate with that obtained by precise traverse. If the geodetic control is computed on a rectangular system of coordinates, line stakes may be set in any open place directly over the tunnel's axis and a direct connection and check between the triangulation and the tunnel alignment survey can be made through a vertical drill hole. The drill hole should be near enough vertical so that the point on the surface may be projected vertically downwards by means of a heavy plummet or by using the vertical collimator. (See p. 24.) For vertical control a ground elevation can be obtained on the rim of the drill hole, and this elevation referred to the floor of the tunnel by use of a long wire or tape properly standardized.

ROAD SURVEYS.

Road surveys may be for the purpose of mapping a road already in existence in connection with new construction work or relocation, or they may be for locating a new road. In either case, if the surveys, both horizontal and vertical, are based on geodetic control, the resulting data and maps will be accurate and free from distortion and the necessary plans can be easily and quickly made. If desirable, the positions determined by the triangulation and traverse may be referred to rectangular coordinates (see p. 44) and the work of laying out the plans on the ground will thus be greatly simplified.

LAND SUBDIVISION.

In subdividing land adjacent to a city, with the view of making it a part of the city itself, there are three fundamental considerations which must be kept in mind: (1) The uses to which the land being subdivided are to be put; (2) the connections between the streets being planned and the streets already in existence; (3) the relation of the street plan to the topography.

Factories and large industries naturally keep to the valleys and low level lands; their problems are to a great extent those of transportation and of spaces for large buildings. Desirable residence sites may be planned on hills too steep for factory sites, and an ideal park may be laid out in rough land broken with steep hillsides and cut with deep ravines where the engineer endeavors to harmonize his work with the surroundings. If the topographic survey is based on accurate geodetic control, the street layout for a new subdivision may be planned directly on the map with the assurance that when it is transferred to the ground by reference to the control points it will fit the topography accurately.

CONNECTION OF DETACHED LAND SURVEYS

It usually happens that a survey of a city or county, or even of a country, proceeds in a more or less haphazard fashion, depending on the exigencies in the particular case. Thus in the survey of a coast, the harbors and their approaches are naturally mapped first, then shore-line surveys connect up the harbor surveys, and finally arcs of precise triangulation connect up distant parts of the shore line. So, too, in the survey of a county contemporary surveys will often be miles apart on the ground with no direct connecting link at first. When these surveys finally meet, unless unusual care has been taken not only in the original surveys but in placing marks to hold them on the ground and in wording the description of the bounds, troublesome overlaps, gaps, and offsets will be found and expensive litigation may result. This can be avoided by the use of triangulation to tie together in true relationship every parcel of land in the county. The use of plane rectangular coordinates (see p. 44), based on the triangulation, to fix the corners of each parcel will make the work of the engineer final. It is not unusual for the surveys of a considerable area to depend upon a few marks preserved from some original survey. The survey of a parcel of land within such an area will consist in traversing from some accepted mark to get a start and then in the actual surveying of the land in question. Of these two parts the first is often the major one, and much work could be saved by the use of triangulation and rectangular coordinates.

HARBOR SURVEYS.

In many cases a precise city survey must include a survey of a river, harbor, or other body of water. The triangulation of a small body of water is ordinarily a simple matter, as the lines across the water are usually clear, but frequently the shore line is so built up with projecting structures that it is difficult to obtain clear lines along it. It is necessary to have a clear line along one side of the river or the

other for each figure of the triangulation, and this can often be accomplished only by placing the stations closer together along the shore, a procedure which may not materially affect the accuracy of the lengths carried ahead but because of the introduction of additional angles will lessen the accuracy of the azimuths. This loss of accuracy in the azimuths may be obviated by carrying the azimuth through longer lines across several figures of the triangulation.

INDEPENDENT GEODETIC DATUM.

Where no control points of the precise geodetic survey of the United States are available, it will still be good practice to make a geodetic survey of a city, although based on an independent datum. It may be possible, by precise astronomic work and the use of good judgment, to secure a datum such that when the geodetic survey of the country does finally reach the city, as is sure to happen eventually, the changes incurred on the maps based on the independent datum will be small. As already seen (p. 3), a geodetic datum may be defined in terms of the latitude and longitude of some point and an azimuth from that point. The determination of latitude and longitude with the necessary precision is not possible with the ordinary instrumental equipment of the average city, though in some cities, the seats of colleges giving technical courses, the required instruments may be borrowed. A complete statement of methods of observing and computing will be found in U. S. Coast and Geodetic Survey Special Publication No. 14, Determination of Time, Longitude, Latitude, and Azimuth. Fairly accurate azimuth determinations may be made even with the ordinary engineer's transit by using the methods given in the above publication. The U. S. Coast and Geodetic Survey may be able to cooperate with the city in establishing starting data; at least it is always glad to advise in such matters.

Since the results obtained from direct astronomical observations are affected by the deflection of the vertical, they require certain corrections before they can be considered as being even approximately on the North American Datum. (See p. 3.) These approximate corrections may be obtained by methods which are discussed in two publications of this bureau, entitled "Figure of the Earth and Isostasy from Measurements in the United States" and "Supplementary Investigation in 1909 of the Figure of the Earth and Isostasy." A less accurate value for these corrections may be obtained by estimation based on the deflection of the vertical at surrounding stations, though these stations be too remote for a direct triangulation connection. It must be kept in mind that until the geodetic survey of the country brings the North American datum for triangulation and the sea-level datum for elevations to the vicinity of the city, where the city may use them, that any other data for precise surveys will be only temporary and will eventually be changed. This is not said by way of discouragement, but by way of caution. Even if the national datum is not available, the city may execute a precise geodetic survey based on approximate starting data, and though the resulting map may not be in exactly its true position on the map of the country, still its various parts will be in true relation to each other and for all projects which are integral within its boundaries it will give satisfaction.

Part II.—MAKING THE SURVEY—HORIZONTAL CONTROL.

STANDARDS OF ACCURACY.

TOPOGRAPHIC MAP.

Engineers have generally agreed that for most city planning a topographic map with a scale of 1 inch equals 200 feet (1 to 2,400) will prove satisfactory. For mapping on this scale a triangulation with an accuracy of 1 part in 25,000 will more than satisfy the demands of cities of the largest areas. The distance between the triangulation stations should be from 2 to 4 miles, and the traverse control should have an accuracy of 1 part in 10,000. (See p. 16.) The traverse lines should be about 2,500 feet apart and there should be a station every 1,000 feet along the lines. These specifications are not rigid, as much depends upon the character of the country. In a steep, rough country more triangulation and less traverse is advisable, since the triangulation may be more easily executed; while in level flat country, the traverse lines may be longer and the triangulation merely serve to preserve a required degree of accuracy and prevent an accumulation of errors in the traverse.

The above standards of accuracy will meet satisfactorily the planning requirements of most cities, but not the cadastral requirements of the larger cities. Nothing is more unfortunate than to make a survey for a certain purpose and then later to condemn it because it fails to carry a heavier load which is imposed upon it. The city about to undertake an accurate topographic survey should carefully consider whether it is not better to pay a little more and secure a control which will satisfy all demands upon it, rather than pay for a survey of limited uses. Particularly in the matter of triangulation the standards adopted should be such that it will serve as a basis for all future cadastral work. The character of the equipment needed for precise triangulation, the unique professional training required for directing the work, as well as for making the computations, all dictate that only one triangulation of a city should be made, and that, of course, should be of the class which will serve the most exacting demands upon it, those of the cadastral survey. The traverse which steps down the triangulation to the more detailed operations may be of just sufficient accuracy to serve the required and immediate purpose.

CADASTRAL SURVEY.

The standards of accuracy set up for the cadastral survey of a city of any size should take into consideration the future growth of the city and future property values. It is obviously impossible, therefore, to establish a standard of accuracy which will suit all cities. In New York, for instance, a need has developed for an accuracy of 1 part in 100,000. Small cities can well be served with triangulation

of an accuracy of 1 part in 25,000. Between these two values range the various grades of accuracy required by cities of different sizes and varying land values. Each specific city will have to determine from local conditions the accuracy which it requires.

The scale of cadastral survey maps should be such that distances approximately correct may be easily scaled from them, although the final values in all cadastral work must be those which are derived from the mathematical reduction of actual observations. The cadastral map on the scale designated for general city planning, 200 feet to the inch, may have the computed data recorded on it, and where conditions are so complex that more space is required, such a map may be enlarged to two or more times its original scale and the data recorded thereon. In doing this it must be remembered that enlarging the map does not increase its accuracy. (See p. 10.)

The traverse which connects the triangulation with the cadastral monuments should be of the same grade of accuracy as the triangulation. The principal traverse stations should be spaced far enough apart to avoid an unnecessary accumulation of azimuth error and should be placed where they will best serve the purposes of accuracy.

The permanent cadastral monuments should be placed where they will be most useful regardless of traverse conditions, and it must be remembered that since the traverse stations are to be considered temporary, the cadastral monuments must be where they can be occupied with an instrument and adjacent monuments should be intervisible. They may be located with the desired degree of accuracy either directly from the traverse stations or from intermediate points on the traverse tangents. These intermediate points from which locations are to be made should be marked in a precise and semipermanent manner. In a built-up section of a city a permanent cadastral monument should be placed at each street intersection, and it may be advisable when locating this monument to determine also the position of a semipermanent mark (as a bronze bolt set in the sidewalk) at each of the three corners of the intersection other than the one on which the permanent monument is located.

UNDERGROUND SURVEY.

What has been said about the cadastral survey as to scale and accuracy is equally applicable to the underground survey, and the control which will serve the requirements of the cadastral survey will usually satisfy the needs of the underground survey.

SPECIFICATIONS FOR THE TRIANGULATION.

ACCURACY.

The triangulation of the U. S. Coast and Geodetic Survey is executed according to well-established standards, which have been developed to secure certain specified degrees of accuracy. It is therefore classified according to the accuracy which it attains. Only recently the Board of Surveys and Maps, composed of representatives of the various Federal map-making bureaus, recommended the following designations for the different classes of triangulation: Precise, primary, secondary, and tertiary. The first

three of these are, respectively, equal in accuracy to the classes primary, secondary, and tertiary, as previously designated by the U. S. Coast and Geodetic Survey. One criterion by which triangulation is judged is the limiting error of the length of a line usually expressed as a proportional part. Thus, for precise triangulation the required accuracy is 1 part in 25,000. This test of accuracy is usually obtained by comparing the length of a measured base line with the length of that line as computed through the unadjusted triangulation from the preceding base line. After the adjustment of the triangulation has been made the resulting lengths have an accuracy much greater than 1 part in 25,000. This method of stating the accuracy has one weak point; it fails through what is manifestly overstatement in the case of very short lines, and so in translating the accuracy of a very short line as stated proportionally into a linear value due allowance for this must be made.

It is believed that for cities up to 100,000 population the required accuracy of the triangulation should be 1 part in 25,000; for cities of 100,000 to 1,000,000 it should be 1 part in 50,000; and for cities of over 1,000,000 it should approach 1 part in 100,000. These are not absolute standards; only the use of triangulation by the individual cities will develop that standard for each which will be best from the viewpoint of economy and service. The traverse which locates the cadastral monuments should have the same accuracy as the triangulation. The accuracy of the traverse for locating stations to be used solely for a topographic survey will be determined by the scale of that survey, which in turn will be determined by the uses for which the proposed survey is being made. As already stated the consensus of opinion of a number of engineers who might be considered as authorities in the matter is that for the general purposes of city planning a scale of 1 inch equals 200 feet is most satisfactory. With this scale an accuracy of 1 part in 10,000, or approximately one-half foot to the mile, is satisfactory for the traverse for topographic control.

FREQUENCY OF STATIONS.

In a paper read in 1913 before the Institution of Surveyors, New South Wales,[5] on a proposed standard survey for the city of Sydney, a length of line of one-half to three-quarters of a mile was advocated for the principal triangulation of that survey. In the triangulation of Greater New York the average length of line for the Borough of Richmond was 2.5 miles, and for the Borough of the Bronx, 2.9 miles. The average length of line for the triangulation of the city of Cincinnati was 4.1 miles. The subsequent utilization of the New York and Cincinnati triangulation has developed the need for more frequent stations. In the case of Cincinnati a frequency of stations to give a maximum length of line of 2 miles is thought to be desirable, and in Greater New York a station to every square mile is now deemed advisable. It is believed, however, that even in the case of the largest city the 2-mile length will be found satisfactory if the principal traverse, joining the triangulation to the intermediate cadastral monuments, is executed according to the general instructions for precise traverse given in Special Publication No. 58 of this bureau.

[5] See footnote, p. 1.

For the control of a topographic survey, where no cadastral use of the triangulation in intended, a greater length of line will be satisfactory and more economical. A length of 2 to 4 miles is recommended.

RECONNAISSANCE.

The purpose of the survey, the required accuracy, and the frequency of stations having been determined, the next step in the precise survey of a city is making the reconnaissance or the actual selection of the stations on the ground. The reconnaissance for the triangulation must satisfy two general conditions: It must give the best possible distribution of stations to meet the needs of the detail surveys, and it must provide well-shaped figures for carrying the lengths ahead economically within the required standard of accuracy. A regular geometric arrangement of stations can be used in a flat city, but in a hilly city those points which will most easily see numerous subsidiary stations will be of greatest value. In the selection of stations the strength of figures must be considered. (See p. 20.)

BASE LINES.

It is good practice in reconnaissance for city triangulation to select the base sites first. Triangulation should never be controlled by a single base line. There should be at least two. Wherever possible a line of the precise triangulation net of the United States, or a triangle side brought by precise triangulation from that net, should be used as one of the bases, as it will furnish not only a controlling length for the triangulation but also a geodetic latitude, longitude, and azimuth. A measured base line on the opposite side of the city from this control line will afford a check on the recovery of the old stations and will also furnish a means of computing the accuracy of the lengths of the city triangulation.

The difficulties of measuring a precise base line within the limits of a city are many. It is usually necessary to measure such a line along a street or parkway where traffic interference is a constant and troublesome factor. Ordinarily it will be impossible to have triangulation stations at the extremities of the measured line, and the ends of the base must therefore be referred to stations which can be occupied with the theodolite. The difficulty of marking the tape ends with precision, the avoiding of obstacles such as trees and lamp-posts, and the carrying of the line across intersecting streets are other troublesome factors encountered.

The principal points to be considered in selecting a site for a precise base line for a city triangulation are given below.

General location.—A base can be measured with the required degree of accuracy wherever the grade of any 50-m. tape length does not exceed 10 per cent, and narrow valleys and ravines less than 50 m. wide in the direction of the base are not obstacles to measurement. The length of the base line should not be less than 2 km. or 1¼ miles. Sometimes it is necessary to introduce an angle into the measured line, but this can be done without undue loss of accuracy if the angle is measured in the same manner as an angle of precise traverse. In each base net for connecting the base to the triangulation care should be taken to have as well-shaped figures as possible,

the value of R_1 being kept well below what is allowable for a figure of the main triangulation. (See p. 20.) The stations of the base net should be so selected, if possible, that all lines may be observed.

Traffic interference.—Traffic interference can be controlled to a degree during the making of the measures by the use of traffic police or extra men in the party. Often the question of traffic interference may be solved by making the measures at some particular time of the day or night. In the precise traverse work of the U. S. Coast and Geodetic Survey it has been found when running through a city that the tape measurements are most easily made in the street 1 or 2 feet from the curb.

Marking tape stations.—In parking, stakes may be set with metal strips on top on which to mark the tape ends. In some pavements nails can be driven and the marks scratched on their heads. In the precise traverse work of the U. S. Coast and Geodetic Survey a most satisfactory tape station has been made on smoothly paved streets by placing a strip of ordinary adhesive tape on the cleaned pavement and marking thereon with a hard pencil. A finer line can be made by using a knife or other sharp point. Such a mark will remain in place for several days.

Connection of the base line with the triangulation.—The simplest condition arises where a line of the main scheme of triangulation can be measured as a base. This can seldom be done, although it is more likely to be possible in city triangulation where the standard line is quite short than on arcs in open country where economical progress demands a longer average length of line. More often, however, an expansion figure is needed to bring the short length of the measured base up to the standard length adopted for the triangulation. This expansion figure must be kept very strong. If the ends of the measured line are not coincident with the triangulation stations, the connection can sometimes be made through a very flat quadrilateral. In the triangulation of Cincinnati the base line nominally extended from a station on the tower of a high-school building to a station on the roof of another school building, and the measured line was in the street below. In the flat quadrilateral through which the connection was made between the two lines the strength of figure was $R_1=0$ (see p. 20), and there was therefore no appreciable loss of accuracy.

Frequency of base lines.—As stated on page 39, the accuracy of the lengths of a triangulation may be controlled by proper distribution of base lines. Where great accuracy is required bases must be close together, so that the high accuracy attained in their measurement may not be lost by being carried through too many triangulation figures. Where the accuracy desired is 1 part in 25,000 the value of ΣR_1 between base lines (see p. 20) should not exceed 100. Where a triangulation has been executed and the accuracy has been found to fall below that desired, the introduction of an intermediate base will materially increase the accuracy of the results.

TRIANGULATION.

Selection of stations.—The controlling factors in the selection of the stations for the main scheme are the adopted length of line, the shape of the triangulation figures to give the required strength, the inter-

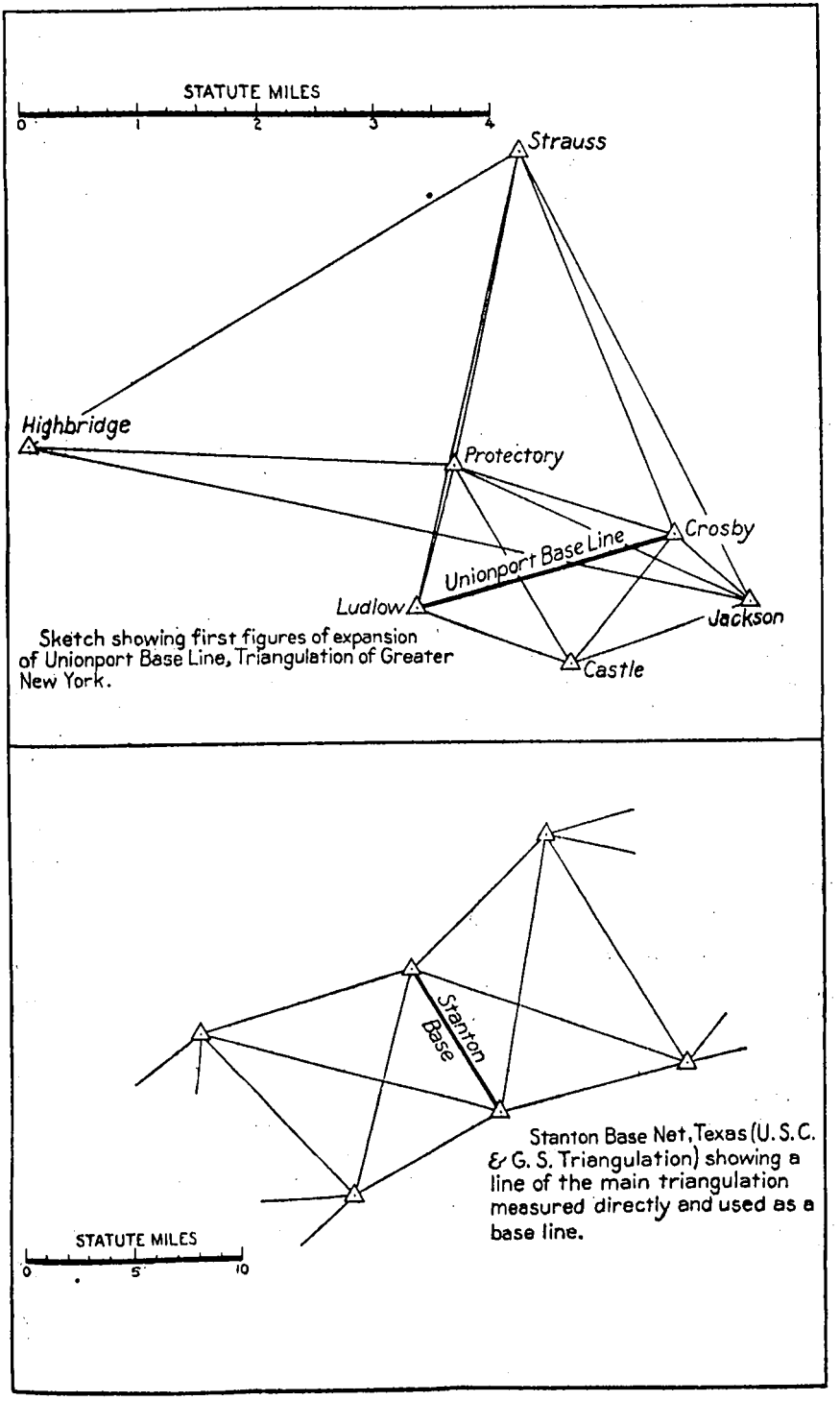


FIG. 1.—Two types of base nets.

visibility of the stations, the freedom of the proposed lines of sight from horizontal refraction, and the control which the stations are required to furnish the subsidiary surveys.

In U. S. Coast and Geodetic Survey Special Publications Nos. 19 and 93 are given general instructions for reconnaissance for precise triangulation to meet the standards established by this bureau. These instructions are designed for an arc of triangulation where progress in length is sought. In the triangulation of a city the progress must be in area, and the accuracy must be maintained in every direction. The instructions above referred to, modified to meet this different condition, are given below.

Character of figures.—The system of triangulation between base nets shall be made up of at least completed quadrilaterals or central point figures, with all stations occupied. Overlapping of figures is not only permissible, but in many cases desirable, even though the difficulties of the least-squares adjustment are thereby greatly increased. Where a central point figure is used as many diagonals should be observed as can be done without undue expense of cutting or building.

Strength of figures.—The strength of figure R is computed by the formula

$$R=\frac{D-C}{D}\Sigma[\delta_A^2+\delta_A\delta_B+\delta_B^2],$$

in which D is the number of directions observed, C is the number of conditions which must be satisfied in an adjustment of the triangulation, and δ_A and δ_B are the logarithmic differences corresponding to one second for the distance angles A and B, respectively. The distance angles in each triangle are the angles opposite the known side and the side sought. In any quadrilateral or central point figure there are two ways of computing the side sought from the known side, according to what triangles are used. The R through the best-shaped triangles is called R_1 and through the next best is called R_2.

In any one figure of precise triangulation R_1 must not exceed 10 in the sixth place of logarithms and R_2 must not exceed 20.

The values of R_1 and R_2 may be determined as follows:

In the formula stated above the two terms $\frac{D-C}{D}$ and $[\delta_A^2+\delta_A\delta_B+\delta_B^2]$ depend entirely on the figures chosen and are independent of the accuracy with which the angles are measured. The product of these two quantities is therefore a measure of the strength of the figures with respect to length, in so far as the strength depends on the selection of stations and of lines to be observed.

In the following table the values tabulated are $[\delta_A^2+\delta_A\delta_B+\delta_B^2]$.

The unit is one in the sixth place of logarithms. The two arguments given are the distance angles in degrees, the smaller distance angle being given at the top of the table.

The strength table should be used in connection with the value of $\frac{D-C}{D}$ to decide during the progress of reconnaissance which of several possible figures is the strongest, and whether a sufficiently strong scheme for the triangulation is being secured. The value of

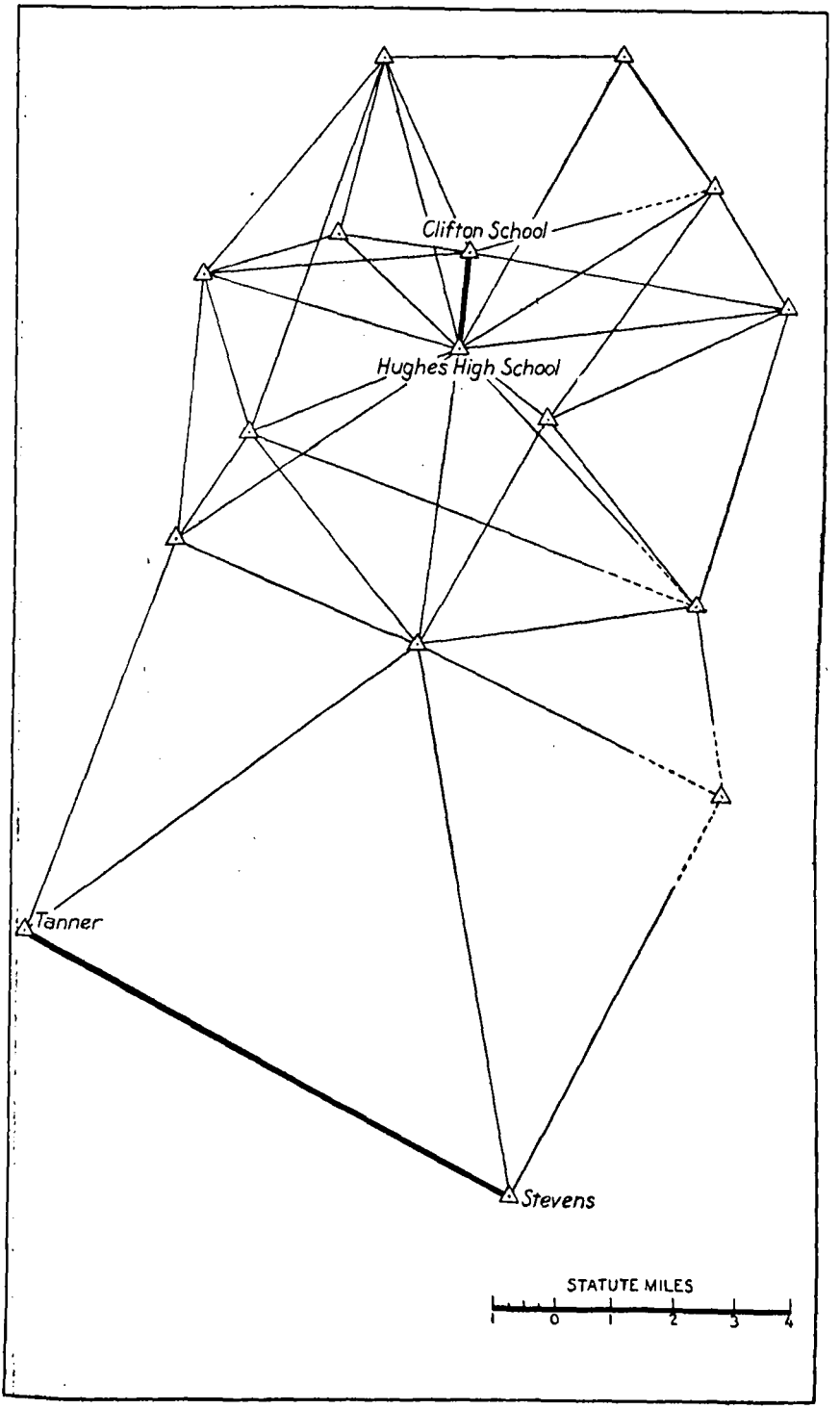


FIG. 2.—Base lines connected by triangulation.

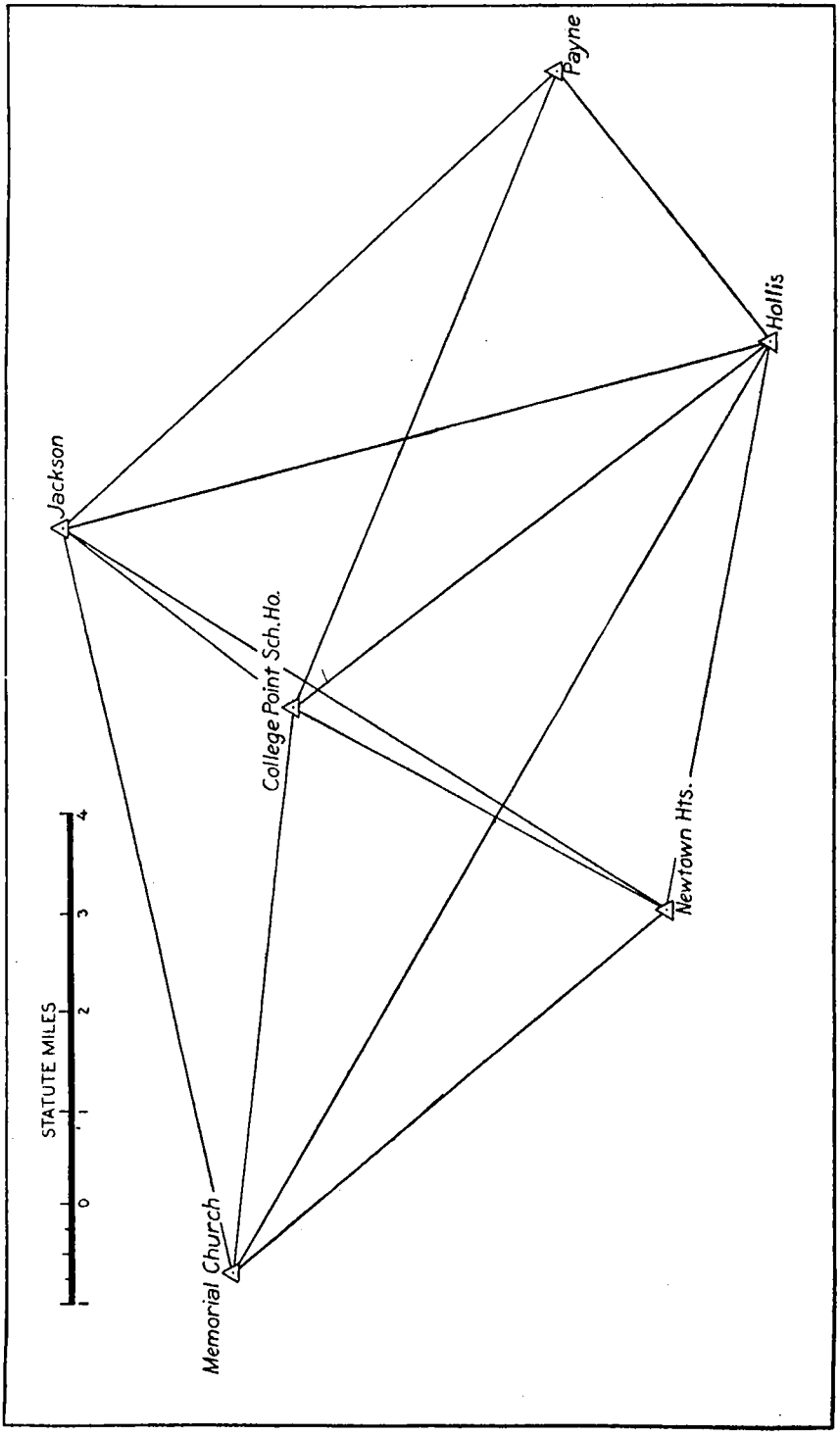


FIG. 3.—Overlapping of quadrilaterals.

the quantity $\frac{D-C}{D}$ for any figure may be readily computed if it is remembered that the starting line is considered as fixed and is therefore not included in the count for D. The following list of values of this quantity for the simpler figures may be found convenient:

For a completed quadrilateral..... $\frac{10-4}{10}=0.60$

For a three-sided central point figure..... $\frac{10-4}{10}=0.60$

For a four-sided central point figure..... $\frac{14-5}{14}=0.64$

For a five-sided central point figure..... $\frac{18-6}{18}=0.67$

For a six-sided central point figure..... $\frac{22-7}{22}=0.68$

For a four-sided central point figure, with one diagonal also observed..... $\frac{16-7}{16}=0.56$

	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°
•																							
10	428	359																					
12	359	295	253																				
14	315	253	214	187																			
16	284	225	187	162	143																		
18	262	204	168	143	126	113																	
20	245	189	153	130	113	100	91																
22	232	177	142	119	103	91	81	74															
24	221	167	134	111	95	83	74	67	61														
26	213	160	126	104	89	77	68	61	56	51													
28	206	153	120	99	83	72	63	57	51	47	43												
30	199	148	115	94	79	68	59	53	48	43	40	33											
35	188	137	106	85	71	60	52	46	41	37	33	27	23										
40	179	129	99	79	65	54	47	41	36	32	29	23	19	16									
45	172	124	93	74	60	50	43	37	32	28	25	20	16	13	11								
50	167	119	89	70	57	47	39	34	29	26	23	18	14	11	9	8							
55	162	115	86	67	54	44	37	32	27	24	21	16	12	10	8	7	5						
60	159	112	83	64	51	42	35	30	25	22	19	14	11	9	7	5	4	4					
65	155	109	80	62	49	40	33	28	24	21	18	13	10	7	6	5	4	3	2				
70	152	106	78	60	48	38	32	27	23	19	17	12	9	7	5	4	3	2	2	1			
75	150	104	76	58	46	37	30	25	21	18	16	11	8	6	4	3	2	2	1	1	1		
80	147	102	74	57	45	36	29	24	20	17	15	10	7	5	4	3	2	1	1	1	0	0	
85	145	100	73	55	43	34	28	23	19	16	14	10	7	5	3	2	2	1	1	0	0	0	0
90	143	98	71	54	42	33	27	22	19	16	13	9	6	4	3	2	1	1	1	0	0	0	0
95	140	96	70	53	41	32	26	22	18	15	13	9	6	4	3	2	1	1	0	0	0	0	
100	138	95	68	51	40	31	25	21	17	14	12	8	6	4	3	2	1	1	0	0	0		
105	136	93	67	50	39	30	25	20	17	14	12	8	5	4	2	2	1	1	0	0			
110	134	91	65	49	38	30	24	19	16	13	11	7	5	3	2	2	1	1	1				
115	132	89	64	48	37	29	23	19	15	13	11	7	5	3	2	2	1	1					
120	129	88	62	46	36	28	22	18	15	12	10	7	5	3	2	2	1						
125	127	86	61	45	35	27	22	18	14	12	10	7	5	4	3	2							
130	125	84	59	44	34	26	21	17	14	12	10	7	5	4	3								
135	122	82	58	43	33	26	21	17	14	12	10	7	5	4									
140	119	80	56	42	32	25	20	17	14	12	10	8	6										
145	116	77	55	41	32	25	21	17	15	13	11	9											
150	112	75	54	40	32	26	21	18	16	15	13												
152	111	75	53	40	32	26	22	19	17	16													
154	110	74	53	41	33	27	23	21	19														
156	108	74	54	42	34	28	25	22															
158	107	74	54	43	35	30	27																
160	107	74	56	45	38	33																	
162	107	76	59	48	42																		
164	109	79	63	54																			
166	113	86	71																				
168	122	98																					
170	143																						

Lateral refraction.—Nothing is more apt to cause large triangle closures than lateral refraction, and nowhere are conditions more productive of lateral refraction than in the vicinity of a modern city. A line of sight passing close to a heated structure, such as an active chimney or a hot-air vent, will have a lateral bend in it, and where the cause of the refraction is close to the point of observation this bending can assume very large angular proportions. Much depends upon the direction of the wind. (See p. 34.) If it is found that a line is affected by lateral refraction, acceptable observations may sometimes be obtained by establishing an eccentric station at one or at both ends of the line.

A line crossing a section of freight yards where switching is constantly in progress may be completely obscured for long periods of the day by the smoke from engines shifting back and forth. The smoke from a factory stack will often produce serious interference. There was one such line in the Cincinnati triangulation where it seemed as if observations would always be prevented by smoke from a large industrial plant, but a two-day holiday over Sunday and Labor Day with a complete banking of the fires permitted the required observations to be made.

Connection of triangulation and traverse.—Another important consideration in selecting triangulation stations is the feasibility of making connections with the traverse system. The simplest connection is where the triangulation station is in open ground and is marked by a ground monument to which the traverse measures may be carried directly. But if the triangulation station is on the roof of a tall building and the traverse line is in the street below, it may even be impossible to see the triangulation signal from near-by traverse stations, or if they are intervisible, the vertical angle between them may be so great as to require extraordinary precautions in making a connection. In some cases it may be possible to construct a slight projection over the eaves or cornice of the roof on which the triangulation station is located and establish thereon a point which can be connected directly with the triangulation station, and then by means of the vertical collimator refer this point to the ground below and so make a connection with the traverse station.

A satisfactory connection in position will not necessarily give a strong azimuth connection, but both may sometimes be attained by the execution of a small scheme of triangulation especially designed for the purpose. Ordinarily the points nearest the triangulation station should be used to give the strongest connection in position and the more distant points to give the best connection in azimuth.

SIGNALS.

Targets and heliotropes.—If observations of horizontal angles are made in the daytime, the pointings may be made either on a target or on a heliotrope. If heliotropes are used, the brightness of the image observed on may be controlled by modifying the effective size of the mirror with a mask having a circular hole or by reducing the intensity of the illumination by stretching a piece of fine gauze over the mirror. Where a target is used, special precautions must be observed to avoid phase. The target may be made of thin metal of suitable size and shape, painted a color which will contrast with

the background and so fastened to a pole that it can be adjusted to make the center line of the target coincide with the vertical through the station. Another form of target is a board of suitable width painted and nailed to a pole with the vertical center line of the face in the vertical through the station. The back edges of this board are cut away or beveled, leaving the front edges very thin. On the precise traverse work of the U. S. Coast and Geodetic Survey this type of target has been successfully used. Both types of targets should be adjusted to face the observer squarely, but their thin edges are a provision against any asymmetry of image where the target is not exactly at right angles to the line of sight. In the triangulation of Yonkers, N. Y., a 4 by 4 inch pole was mounted on a turntable and could readily be adjusted into a position facing any station.

Electric signal lamps.—For the most precise work of the U. S. Coast and Geodetic Survey night observations are made on lights. For the long lines a very satisfactory electric signal lamp has been devised, which consists of an ordinary automobile headlight, with a specially constructed bulb having a condensed filament in the focus of its parabolic reflector. This produces a light, in the case of the larger lamps, having a beam candlepower of 250,000 at a distance of 100 feet, and has been seen at a distance of 152 miles, appearing like a star of the magnitude of Polaris.

For shorter lines a small electric signal lamp has been designed.[6] The following description of it is taken from U. S. Coast and Geodetic Survey Special Publication No. 81, Geodetic Operations in the United States, January 1, 1912, to December 31, 1921:

> It is made up of an automobile headlight with a parabolic reflector 17 cm. in diameter pivoted for adjustment in elevation in a wooden box frame 28 by 26 by 17 cm., which is mounted on the triangulation point by means of a center screw, permitting of adjustment in azimuth. A small rheostat, switch, and sighting tube are provided as for the larger lamp. The lamp bulb is likewise specially made from specifications furnished by the survey. The filament, being designed for a current of only 3 to 4 volts and 0.6 ampere, is concentrated into a space of only about a cubic millimeter and when adjusted into the focus of the reflector produces a beam very nearly parallel. The test showed an apparent beam candlepower at a distance of 100 feet (30 m.) of 74,000, with a current of 0.64 ampere furnished by three dry cells connected in series. The lamp has been seen with the unaided eye a distance of 80 miles (128 km.).
>
> Both the large and small signal lamps are arranged so as to permit the use of either the 2.4 ampere or the 0.6 ampere lamp bulb.

Even this small electric signal lamp may give too bright a light for the short lines of a city triangulation. The brightness may be reduced by using an ordinary flash-light bulb in place of the specially constructed bulb, and also by masking the face of the lamp. Neither of these methods will greatly reduce the apparent size of the light, but will merely decrease its intensity. One effect of too bright a light is to "burn out" the lines of the diaphragm, but this effect can sometimes be overcome by merely increasing the illumination on the diaphragm.

Lamps should be used on very short lines with extreme caution. Tests should be made to determine if there be either eccentricity or asymmetry of image of a magnitude which will show in the angle measures. Remember that 0.3 inch subtends an angle of 1 second at a distance of 1 mile. For the very short lines of a city triangula-

[6] The small lamp in use by the U. S. Coast and Geodetic Survey is constructed from a standard automobile headlight.

tion it might be well to consider some form of signal such as is sometimes used as an azimuth mark by the U. S. Coast and Geodetic Survey, and is shown in Figure 27 of Special Publication No. 14.

The considerations which should guide the engineer as regards the type of signal to be used are the lengths of the lines to be observed, the condition of the atmosphere through which these lines pass, the type of diaphragm and power of the telescope of the instrument used, and possibly certain observing characteristics of the observer.

Towers and flagpoles on buildings.—It is sometimes advisable to use as a triangulation station some mark or object already in existence, as a tower on a building or a flagpole, even though it may be necessary to place the theodolite for observing the horizontal angles eccentric to it. Where the relation of the occupied eccentric station to the station itself is accurately and definitely determined, the process of reducing the observations to center is simple and practically no loss of accuracy results therefrom. The one great objection to eccentric stations is that they afford an additional opportunity for blunders, and they should therefore be avoided as far as is practicable. One is not justified, however, in devising elaborate signals in order to avoid eccentric stations. A very satisfactory method of establishing a station on a large building is to construct thereon a small observing tripod and scaffold signal and determine it as the station. From such a signal any other points on the roof which it is advisable to locate, such as flagstaffs, chimneys, or architectural ornaments, may be determined by angles and direct measures of distances. Care must be used in constructing an instrument support on a roof to make it stable, as the roof itself may not have the required degree of stability. In Figure 4 is shown a signal on the tower of Hughes High School, Cincinnati. The roof of the tower is of tar and gravel on boards, but some feet below this is a concrete floor. The legs of the observing tripod were inserted through the roof and placed securely on the concrete floor below. This gave a stability which permitted the use of a direction theodolite at this important station. The places where the tripod legs pierced the roof were, of course, protected against the entry of rain. The legs of the observing scaffold rested directly on the roof.

Signal towers.—Not only will it be difficult to plan the triangulation of a city so that no high observing towers are required, but, although high towers are not in themselves desirable, a certain number of stations in open ground are desirable for the reason that such stations are the only ones that can be marked so permanently that there will be little danger of destruction.

The service which is required of the standard double-tower triangulation signal is that the outer tower, called the scaffold, shall support the observer and the tent which protects him and the instrument from sun and wind, and that the inner tower, called the tripod, shall support the instrument with such stability that its motion in azimuth shall never be so rapid or irregular as to seriously affect the accuracy of the measurements of angles, its disturbance in level never so rapid as to inconvenience the observer by making frequent adjustments necessary, and its vibration due to wind seldom so great as to interfere seriously with accurate pointing. The signals must, moreover, be strong enough to stand without injury in all ordinary winds and in most storms. It is not economical, however, to build

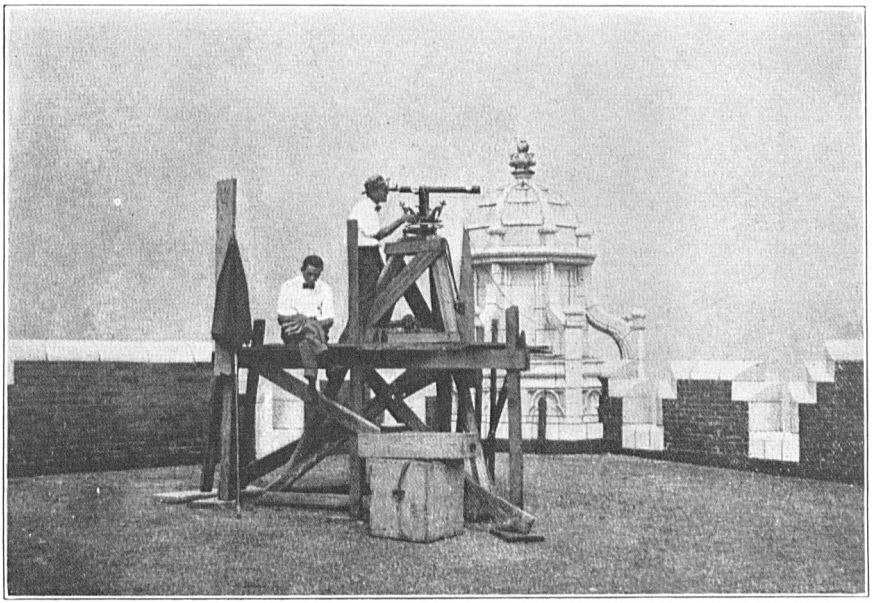


FIG. 4.—TRIANGULATION STATION ON HUGHES HIGH SCHOOL, CINCINNATI, OHIO.

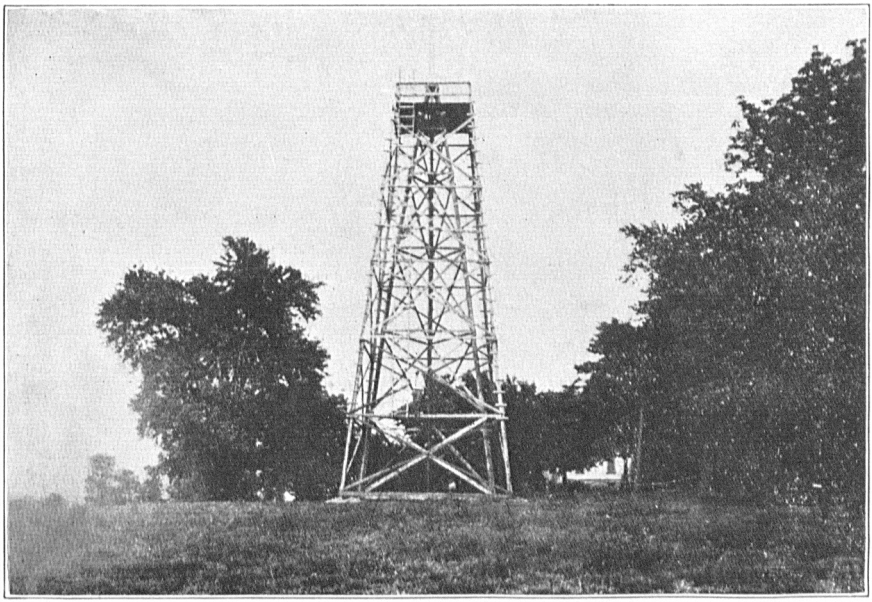


FIG. 5.—DOUBLE-TOWER TRIANGULATION SIGNAL AT STATION "OBSERVATORY" OF THE CINCINNATI SURVEY.

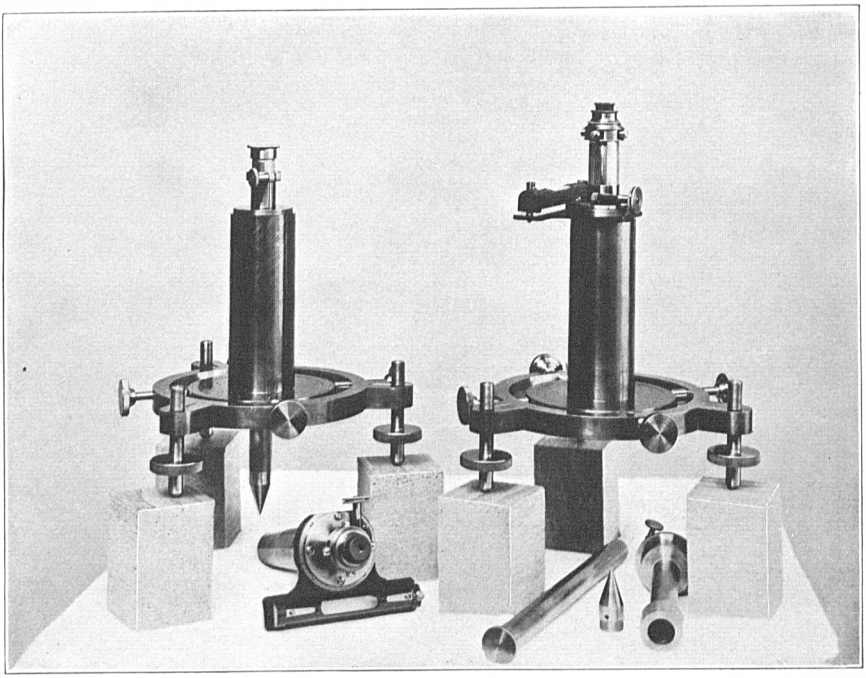


FIG. 6.—VEPTICAL COLLIMATOR. TWO VIEWS.

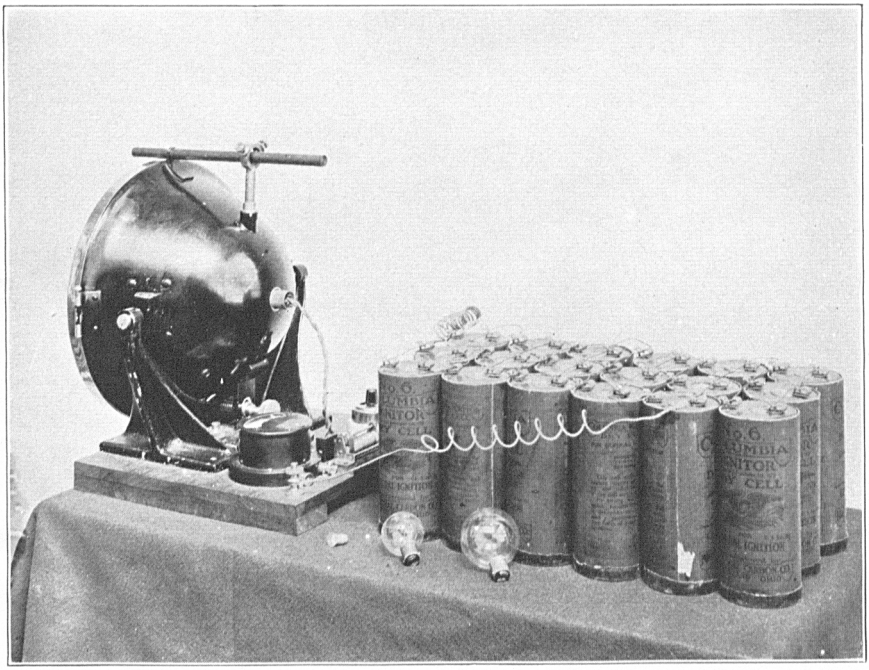


FIG. 7.—LARGE-SIZE ELECTRIC SIGNAL LAMP. A MORE RECENT TYPE OF ELECTRIC SIGNAL LAMP REQUIRES FROM ONE TO SIX CELLS ONLY.

them strong enough to withstand the most violent storms. The planning and erection of such signals is carefully considered in Appendix 4, U. S. Coast and Geodetic Survey Report for 1903, and in Special Publication No. 93, now in preparation.

MARKING STATIONS.

The marking of a triangulation station has just one object—the preservation of the exact location of the point. Where the station is the center of a cupola, a spire, or a flagpole, there is often no way of placing a mark at the center. When placing a mark directly below such an object used as a station it should be so located, if possible, as not to be subject to the same destroying agencies as the station itself. For instance, if the station is the finial on a cupola, the mark may be placed directly under it and some floors below. An important thing to guard against is the possibility of a slight change in the station, such as might be caused by the rebuilding of a spire or the replacing of a flagpole. Such a change might be detected only in the final adjusted results of later work and give the appearance of poor observing. It may be guarded against by the use of reference marks so placed as to be as free as possible from influences which might affect the station itself.

Reference marks serve two purposes—they aid in and verify the recovery of the station, and where a station has been destroyed they make possible its restoration. They should therefore have the same permanent character as the station mark, and since there is greater freedom in the selection of locations for them, they can be made more secure from destruction than can the station mark itself.

For stations on the ground it is well to adopt some standard method of marking. An underground mark over which is a monumental ground mark, situated in a small plot of ground owned by the city, is probably the nearest approach to a really permanent survey station. This use of a small plot of ground does not interfere with its use as a playground or minor park. The city triangulation should have a sufficient number of these stations to enable it to reproduce its other stations, should they be destroyed, without the necessity for a complete new survey. Every precaution should be taken to preserve these points on the ground. They have a distinct function to perform in future surveys as well as a present value. They represent a definite investment of the peoples' money, and their preservation is a matter of the utmost importance. Some of the types of marks already in use are referred to below.

Specific types of marks used.—The usual type of station mark used on the trigonometrical survey of Greater New York is described in the report on that work as follows:

> The station was marked by the regular U. S. C. & G. S. brass marking bolt set in a granite post 6 by 6 by 24 inches set in concrete 3 by 3 by 4 feet. The exact point is a small hole $\frac{1}{100}$ foot in diameter drilled in top of bolt.

The marking of a main station of the Cincinnati survey is thus described:

> Center is marked by a cross cut in bronze disk set in top of a concrete block. The block which was molded in place is triangular in form, extends 3 feet below surface of ground, about 1½ feet above, is 2 feet on edge, and has in each of its three vertical faces a triangle cast sunken.

The standard ground mark for the triangulation of Yonkers, N. Y., is a solid block of concrete about 4 feet deep and from 3 to 4 feet square. The station point is marked by a one-eighth inch brass plug driven in a hole 1 inch deep drilled in a rod of Swedish iron 1 inch in diameter embedded in the concrete.

In the U. S. Coast and Geodetic Survey the wide diversity of conditions which must be met in marking triangulation stations has developed a rather large list of types of marks. Many of these are of no special interest to the city engineer, but he may find some valuable suggestions in meeting what may be a condition peculiar to his city by consulting the list of standard notes on the marking of stations. Such a list will be found in any of the recent reports on results of triangulation by the survey, and can be obtained upon request from the Director, U. S. Coast and Geodetic Survey.

BASE MEASUREMENT.

The process of measuring a precise base line by the U. S. Coast and Geodetic Survey has been well standardized in the past two decades. In 1906 tests were made of invar tapes for precise base measures,[7] and these tests led to the adoption by this bureau of invar tapes for the measurement of all precise base lines and precise traverse lines. Invar is a nickel-steel alloy with a low coefficient of expansion which for the invar tapes is only about one twenty-fifth that of steel. The tapes are, therefore, practically unaffected by temperature conditions. The instructions issued to field parties of the U. S. Coast and Geodetic Survey for precise base measurements, as printed in Special Publication No. 19, are as pertinent in general to the measuring of a base line along a street in one of our populous cities as in a sparsely settled locality. They are given below in slightly modified form. General instructions for precise traverse work are available in complete and compact form in U. S. Coast and Geodetic Survey Special Publication No. 58, General Instructions for Precise and Secondary Traverse.

GENERAL INSTRUCTIONS FOR PRECISE BASE MEASUREMENT.

Very little increase in accuracy of the lengths of the triangle sides of the triangulation will result from increasing the accuracy of the base measurements beyond that represented by a probable error of 1 part in 500,000 in the length of the base. The methods devised are therefore such as will keep within this limit of accuracy and at the same time avoid any additional expenditure of time or money in an effort to attain a greater degree of accuracy.

At least two tapes should be used in the measures, and these tapes should be standardized at the Bureau of Standards, Washington, D. C., both before and after the measurements and as near to them in point of time as possible. The measurements may be made either in broad daylight or at night if invar tapes are used, but if it should be necessary to use steel tapes, the measurements should be made only at night.

[7] See Appendix 4, Report of 1907, "Six primary bases measured with steel and invar tapes."

A base line should be measured in sections of 1 km. in length, except that there may be one section of shorter length. Each section should be measured at least twice with different tapes and in opposite directions. No more than two measures of a section should be made unless the discrepancy between the forward and backward measures of the section exceeds 10 mm. per kilometer, in which case additional measures should be made until two are obtained which agree within this limit.

Such precautions should be taken to secure accurate horizontal and vertical alignment of the tapes, to apply the correct tension, and to ascertain their exact temperatures as will insure that errors arising from these sources shall each be less than 1 part in 1,000,000. In the case of steel tapes the temperature correction is very important, the coefficient of expansion being approximately 0.000011 per degree centigrade, against 0.0000004 for the invar tapes.

Wind blowing against the tapes is apt to cause trouble, but its effect may be lessened by using five points of support instead of three for a 50-m. tape. Where the tape is supported throughout its length the wind effect will, of course, be negligible. In any event precautions should be taken to keep the error from such a source down to 1 part in 1,000,000, or 0.05 mm. per 50-m. tape length.

Accurate results can not be obtained when the tape is supported throughout on some smooth surface, such as a paved street or the top of a rail, if the surface is wet. The suction between the tape and its support is then so great that the tension is not uniform from one end of the tape to the other, and so the conditions are not identical with those under which the tape was standardized.

The two cardinal rules from which there should be no departure are: The base tapes must be standardized both before and after the measurement of the base lines as near in time as possible and without any intervening use of the tapes; and the tapes must always be used on the base lines in exactly the same manner in regard to tension and support as that in which they were standardized. As it is sometimes necessary to use several different methods of support on a base, the reconnaissance should determine how the standardizations are to be made.

EQUATION OF A BASE TAPE.

The complete equation of an invar base tape T_{516} is given in the following form:

Under a tension of 15 kg., $T_{516} = 50$ m. $+ (12.382$ mm. ± 0.016 mm.$)$ $+ (0.0178$ mm. ± 0.0007 mm.$) \times (t - 25.°8C.)$. In this equation t is the temperature at which the tape is being used. The temperature of the tape should be determined for each tape length of the measurement. This is done by reading two thermometers which are fastened to the tape, one near each end, and protected from breakage by specially constructed guards. Thermometers so used should be calibrated in order that the corrections to reduce observed temperatures to standard temperatures may become known.

Base tapes are usually standardized under a tension of 15 kg., and the same tension must be applied while making the measures. At the beginning and end of each day's work the spring balances

used should be tested with a standard balance, or preferably with a standard weight kept solely for that purpose. It is desirable to have the index arm of the balance fixed and to determine and allow for the index error rather than to endeavor, by means of an adjustable arm, to keep the index error down to a negligible quantity. Where a standard weight is used for testing the balances, it must be remembered that the balances are used in approximately a horizontal position.

GRADE CORRECTION.

In making precise measures of length the tape is not held horizontal but is stretched directly between the marking strips or tables, whether they be on top of stakes or attached directly to the pavement. It is therefore necessary to determine the corrections required to reduce the inclined length to the horizontal. For this purpose spirit levels are run over the line, and the grades between tape ends thus determined. A check should be provided by a second line of levels in a backward direction. The horizontal distance between tape ends is $\sqrt{l^2-h^2}$, where l is the inclined distance and h the difference in elevation of the two ends of the tape. The correction to be applied to the inclined distance then becomes $-(l-\sqrt{l^2-h^2})$. A tabulation of this quantity for 25 m. lengths and various differences of elevation is given on pages 29 to 34 of U. S. Coast and Geodetic Survey Special Publication No. 26, General Instructions for the Field Work of the U. S. Coast and Geodetic Survey. A similar table for 50 m. lengths is given in Special Publication No. 58, General Instructions for Precise and Secondary Traverse. It should be noted that the correction for a 50 m. length is one-half that for a 25 m. length for the same difference in elevation of the ends. Where the tape is supported on stakes, the middle of the tape is allowed to rest freely on a nail driven horizontally in the middle stake on line between the end supports. If the middle support is above or below the line between the end supports, a level reading must be taken on it and the two halves of the tape reduced to the horizontal separately.

SETUPS AND SETBACKS.

Setups and setbacks should be avoided as far as possible by a careful setting of the stakes or marking tables. For measuring a small setup or setback a finely divided scale and a pair of dividers may be used, while a properly divided steel tape may be used for measuring longer setups or setbacks. A half-tape length may be closely determined, where there is no standardization of the 25 m. point, by stretching each half of the tape over the distance and using the mean of the markings.

SEA-LEVEL REDUCTION.

In order that the lengths in the triangulation depending on the base line may be reduced to sea level, a correction must be applied

to the measured length of the base. This correction is given by the formula

$$C = -S\frac{h}{\rho} + S\frac{h^2}{\rho^2} - S\frac{h^3}{\rho^3} + \cdots\cdots$$

where C is the required reduction to sea level, S is the length of the section, h its elevation above sea level, and ρ the radius of curvature of the sea-level section. The logarithm of the radius of curvature is tabulated, for latitudes 0° to 72° and for different azimuths, on pages 66 to 71 of U. S. Coast and Geodetic Survey Special Publication No. 26. It will be readily seen that since ρ is so very much larger than h, the second term of the above formula will be very small and the third term practically always negligible. Using an approximate value for ρ of 6,370,000 m. we see that although an elevation of 1 m. above sea level affects a line 1 m. long by only 0.00016 mm., a line 1,000 m. long and 100 m. (328 feet) above sea level will require a correction of −16 mm.

It is sometimes necessary to consider the reverse problem. Given the length of a line at sea level, what is its actual length on the ground? The precise triangulation of the U. S. Coast and Geodetic Survey is all computed on mean sea level as a standard surface. In a cadastral survey the length with which one is concerned is the actual length at the average elevation of the ground. A 100-foot lot means 100 feet measured at the actual elevation of the lot in question. Therefore, when a cadastral survey is based on triangulation which has been computed on mean sea level, the lengths must be corrected for elevation of ground above that level. The length on the ground can be computed with sufficient accuracy from the formula

$$S' = S + S\frac{h}{\rho}$$

where S' is the length on the ground, S the length at mean sea level, h the elevation of the line above mean sea level, and ρ the radius of curvature of the mean sea-level line. As a rule a mean radius of curvature equal to 6,370,000 m. may be used with no appreciable error.

The question has been asked, "Why not compute the triangulation directly on an approximate average elevation for the city, and thus obtain lengths which can be considered as surface lengths?" While there are very few instances where the relief of the city terrain is so great that this procedure would introduce measurable discrepancies, there may be noted the following objections: All published tables for computing geographic positions and for radius of curvature are based on mean sea level and any data obtained from the triangulation net of the United States are also based on mean sea level. The proper coordination of the city survey with surveys of adjacent areas can only be done if they are all reduced to the same standard surface.

Below is a specimen page of a precise base line measure, and following it is shown the computation of the final length of a section of the same base line:

Sample record of base measurement, Deming, N. Mex.

Stake. From—	Stake. To—	Thermometers. 14130	Thermometers. 14135	Setback.	Setup.	Remarks.
		°C.	°C.	m.	m.	
140	141	9.5	9.9			Chief of party: J. S. H. Date: January 15, 1910. Start: 10.20 a. m. Finish: 10.49 a. m. Tape used: No. 516. Balance: No. 166. Weather: No wind, partly cloudy Direction of measure: South.
141	142	9.5	9.9	0.0518		
142	143	9.0	9.8			
143	144	9.5	9.8			
144	145	9.4	9.7			
145	146	9.5	9.7	.0484		
146	147	9.7	9.8			
147	148	9.7	9.6			
148	149	9.9	10.0	.0405		
149	150	10.4	10.7			
150	151	10.6	10.7			
151	152	10.6	10.6	.0431		
152	153	14.0	14.0			
153	154	13.2	12.1			
154	155	12.5	12.1			
155	156	12.4	13.0	.0486		
156	157	12.5	12.5			
157	158	12.5	12.9			
158	159	11.8	12.1			
159	160	12.3	12.3			
Mean of thermometers			10.99	Sum.. 0.2324	0.0000	
Correction			.07			
Mean temperature			10.92			

Sample computation of a section of base line, Deming, N. Mex.[8]

Section	VIII: 140-160	
Date and hour, 1910	Jan. 15, 10.34 a. m.	Jan. 15, 1.39 p. m.
Direction of measure	S.	N.
Tape number	516	517
Weather	Cloudy, calm.	Cloudy, calm.
Temperature	Rising.	Falling.
Temperature, mean corrected	10°92 C.	14°00 C.
Temperature correction to reduce to t_0	−0.0053 m.	−0.0038 m.
Setup or setback	−0.2324	−0.2395
Grade correction	−0.0054	−0.0054
Tape correction	+0.2476	+0.2554
Reduction to sea level	−0.2001	−0.2001
Reduced length of section	999.8044	999.8066
Adopted length of section	999.8055 m.	
Residuals (v)	+1.1 mm.	−1.1 mm.
(vv)	1.21	1.21

PROBABLE ERROR OF BASE LINE.

The probable error r_0 of the length of each section of a base due to accidental errors of measure is computed by the formula

$$r_0 = \pm 0.6745\sqrt{\frac{[vv]}{n(n-1)}},$$

in which n is the number of measures, v the residual from the mean, and $[vv]$ is the sum of the squares of the residuals.

The probable error of the length of a section due to the uncertainty of the lengths of the tapes is assumed to be the number of tape

[8] The computation has here been arranged in vertical form for convenience in printing; the usual arrangement is horizontally across a double page.

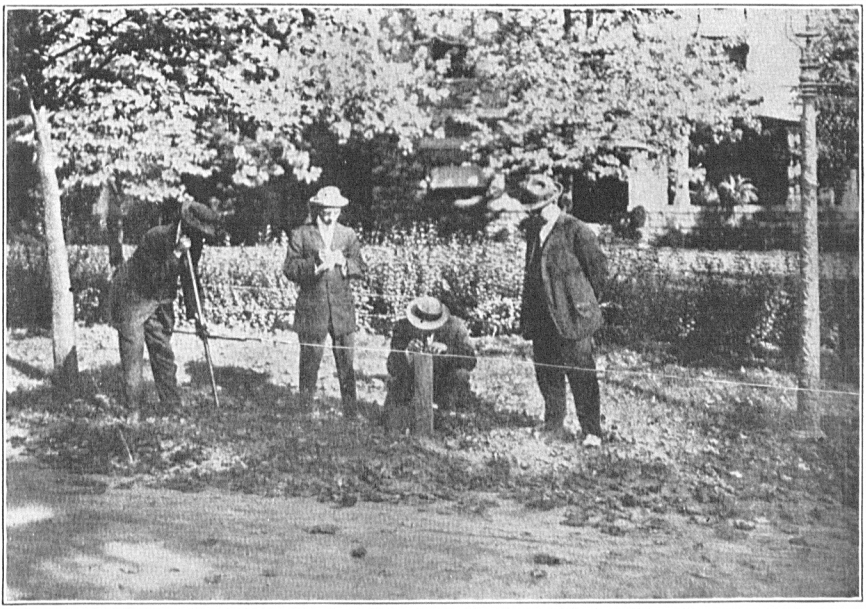


FIG. 8.—BASE TAPE STRETCHER, FORWARD END OF TAPE.

FIG. 9.—BASE TAPE STRETCHER, REAR END OF TAPE.

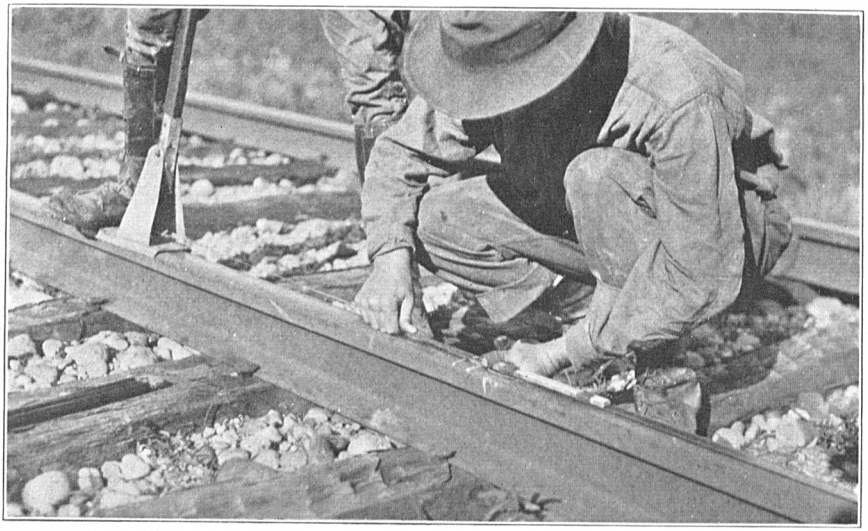


FIG. 10.—TYPE OF TAPE STRETCHER USED ON TRAVERSE WORK.

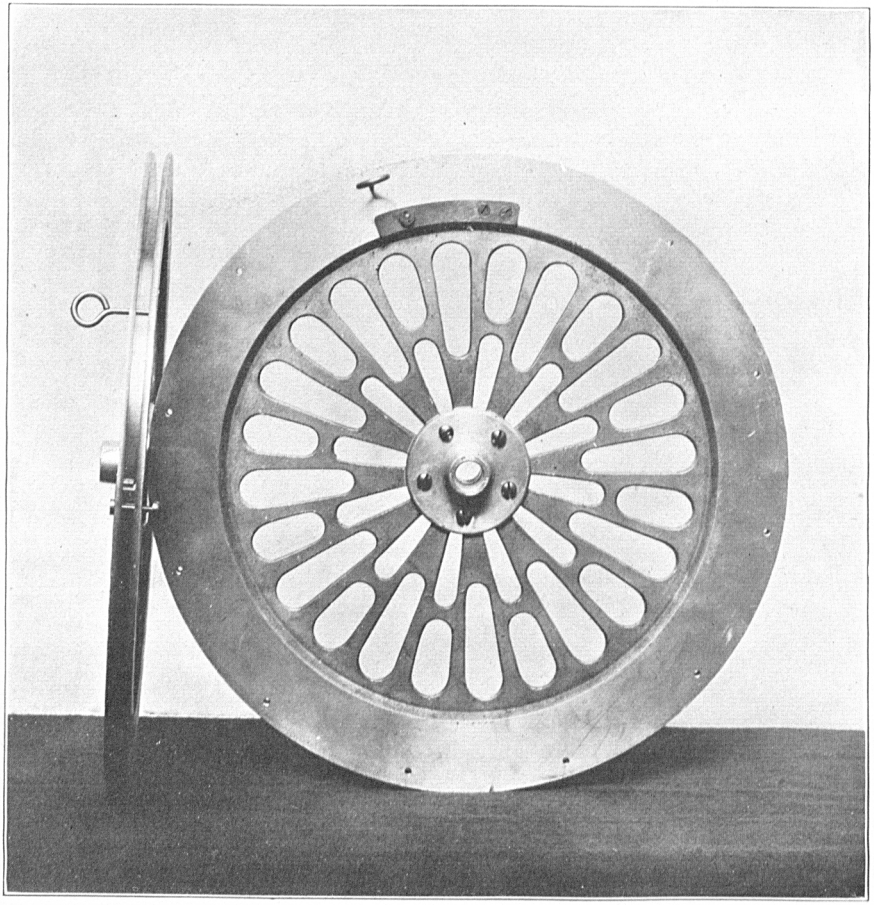


FIG. 11.—INVAR BASE TAPE ON ALUMINUM REEL, TWO VIEWS.

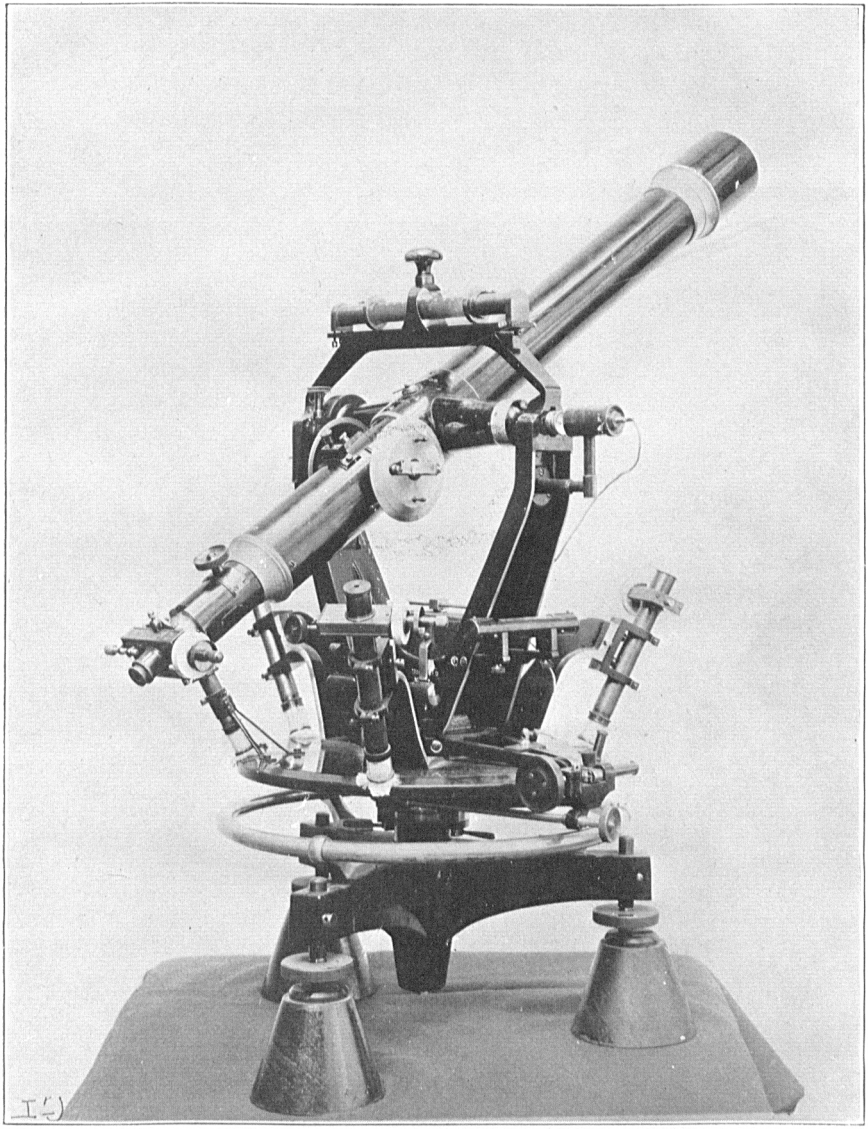


FIG. 12.—TWELVE-INCH DIRECTION THEODOLITE.

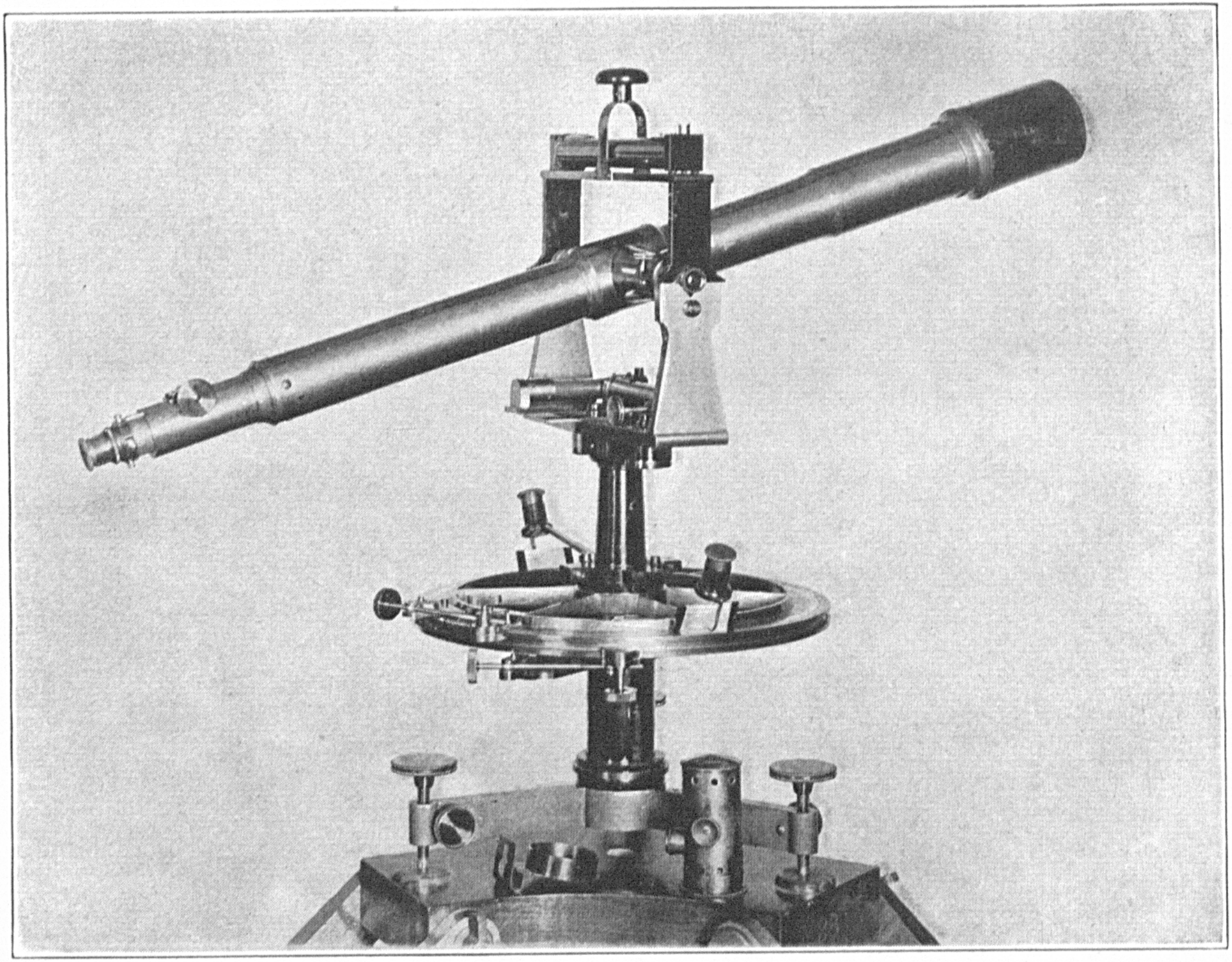


FIG. 13.—TEN-INCH REPEATING THEODOLITE, OLD TYPE, USED ON THE NEW YORK AND CINCINNATI SURVEYS.

lengths in the section multiplied by one-half the square root of the sum of the squares of the probable errors of the lengths of the two tapes used in measuring the section. The probable error of the length of a section due to the uncertainty of the coefficients of expansion of the tapes is assumed to be the number of tape lengths in the section multiplied by the difference between the temperature of the standardization and the average temperature of the measurement multiplied by one-half the square root of the sum of the squares of the probable errors of the temperature coefficients of the two tapes used in measuring the section.

For the section, the computation of which is shown above, the probable errors from the various sources are as follows:

Probable error due to uncertainties in tape lengths	±0.22 mm.
Probable error due to uncertainties in coefficients of expansion	±0.13 mm.
Probable error due to accidental errors of measure	±0.74 mm.
Probable error of length of section (square root of the sum of the squares of the various probable errors)	±0.78 mm.

The probable error of the total length of the base line may be obtained by a similar combination, as it is equal to the square root of the sums of the squares of the probable errors of the separate sections.

ANGLE MEASURES.

ACCURACY DESIRED.

The instructions for precise triangulation by the U. S. Coast and Geodetic Survey require that such instruments and methods of observation be used as will give an average closing error for the triangles of about one second, and a maximum closure for a single triangle of three seconds. As a rule the results show that with this standard for closure an accuracy is obtained of the lengths far in excess of the required 1 part in 25,000. (See p. 16.) This is along arcs through open country. In a city triangulation the same instruments and methods can not be expected to give as accurate results. In the triangulations of Greater New York and of Cincinnati the average closing errors were between two and three seconds, but because of well-shaped figures, many adjustment conditions, and the proximity of bases, the desired accuracy of line was obtained. This evidence indicates that the criterion for an accuracy of 1 part in 25,000 in city work should be an average closure of a triangle of two seconds. It shows also that more time should be given to the reconnaissance in a city than in open country. It is not enough, in the reconnaissance for city triangulation, to know that the lines are unobstructed, that the stations are well placed for future uses, that the triangulation figures are well shaped, and that no very apparent source of unusual refraction exists near the lines, but in addition a minute examination should be made for obscure and hidden sources of refraction. It is recommended that the reconnaissance be plotted on the best obtainable map of the city and that a careful study of structures and conditions along each line be made.

Observing conditions.—There are several interesting facts which may puzzle the inexperienced observer who follows closely the results of his observing with relation to atmospheric conditions. When the atmosphere is clear and the seeing apparently the finest, less accordant

results may be obtained than when a brisk wind is blowing or when there is a tendency to fogginess. When the observing seems most difficult the number of observations which must be rejected may be smaller than usual. When the atmosphere is quiet and very clear there are apt to be rising air currents from the paved streets and certain buildings, and these currents will distort the air strata and cause the line of sight to be refracted. Also when a very light breeze is blowing across the line of sight in a direction from a heated structure or object toward the line of sight the observing may be very poor, but if the wind is in the opposite direction the observing may be good. When a brisk wind is blowing the air is well mixed up and of fairly uniform temperature and there is no decided refraction at any point. This is also the case when the air is slightly foggy.

In city triangulation it is advisable to make the observations at night and rather late, after the disturbed atmospheric conditions which naturally accompany the change from day to night have mostly disappeared.

INSTRUMENTS.

There are two general types of instruments for the measuring of angles in precise triangulation, the direction instrument having a horizontal circle read by micrometer microscopes to single seconds in the case of the highest grade instruments, and the repeating theodolite where the circle is read to two, three, or five seconds by means of verniers.

In any city triangulation economic considerations will make the use of both types of instruments desirable. The direction theodolite is the better type for precise work, but can be used successfully only where the weight of the observer can be so carried apart from the support of the theodolite that his ordinary movements around the instrument will not affect it in azimuth. The repeating theodolite, on the contrary, can be used on buildings where the support is more or less unstable, where the observer's movements may possibly disturb the instrument, provided the observer maintains one position of footing while making the two pointings of an angle.

Both instruments are made in various sizes and are designated by the diameter of the horizontal circle. The accuracy of either instrument depends upon the excellence of its design, the skilled workmanship used in its construction, the precision of the graduating engine used in graduating the circles, and finally, within certain limits, upon the diameter of the horizontal circle. It has been found by tests that, owing to temperature distortion and other strains in the metal, a 20-inch circle for a theodolite gives no better results than a 12-inch circle, and it is therefore considered inadvisable to exceed 12 inches for the diameter, especially as the weight of the instrument becomes excessive beyond this limit.

The diaphragm best suited for precise work, according to experience in the U. S. Coast and Geodetic Survey, consists of two vertical threads 20 to 30 seconds apart. It has been found that an observer can very closely bisect the space between such lines, and that where he has a tendency to observe a little to one side or the other of the mid-point this tendency will become characteristic with him and he will make all pointings alike, provided the image of each object, as seen in the telescope, is about the same size as the others in relation

to the distance between the pointing lines. There will naturally be less personal equation on a pointing where the image is nearly the same width as the distance between the pointing lines than where the image is very small. If the personal equation is the same on all pointings, it will, of course, disappear from the observed angles.

Direction theodolite.—The standard instrument of this type used by the U. S. Coast and Geodetic Survey on precise work is one having a 12-inch circle and three micrometer microscopes reading to single seconds, though for several years past an 8-inch instrument with two micrometer microscopes reading to two seconds has been successfully used. With either of these instruments the program of observing should be as follows: The instrument shall be used in 16 positions of the circle in order that each angle shall be measured on 16 different parts of the horizontal circle. The micrometer *A* is set to read as shown in the table given below for each of the various positions of the circle designated as I, II, III, etc., and the initial signal is bisected by means of the lower motion. All micrometers are read and recorded for this initial setting, and then with the circle clamped in position the telescope is pointed successively on the various signals, moving always from left to right and ending on the initial station. All micrometers are read and recorded at each pointing. The telescope is then plunged or reversed by lifting it from its wyes, but keeping the same pivot in the same wye, and again pointed on the initial signal and successively on the other signals, moving from right to left and finishing on the initial signal. The clockwise pointings with the circle in positions II, IV, VI, etc., are made with the telescope reversed to avoid the necessity of changing the telescope back to the direct position after the completion of the anticlockwise pointings for positions I, III, V, etc. If the setting of the circle is always made with the *A* micrometer this program automatically shifts the circle about 195° between settings instead of about 15°, as the table below would indicate, and tends to eliminate any constant errors due to unequal heating of the circle. The mean of the readings, direct and reverse, on any signal gives the result for that signal independent of collimation, and the difference between the results for any two signals gives the angle between them. These results, when final means for all positions of the circle have been taken, should be tabulated as directions in clockwise order, left to right, from the signal used as an initial, whose direction will be called 0° 00′ 00″.00.

The various settings of the circle (micrometer *A*) for the required 16 positions are given in the following table:

Position.	Setting.			Position.	Setting.		
	°	′	″		°	′	″
I	0	00	40	IX	128	00	40
II	15	01	50	X	143	01	50
III	30	03	10	XI	158	03	10
IV	45	04	20	XII	173	04	20
V	64	00	40	XIII	192	00	40
VI	79	01	50	XIV	207	01	50
VII	94	03	10	XV	222	03	10
VIII	109	04	20	XVI	237	04	20

If any signals are not showing at the time a series of observations is made, they should be passed by and observed later in connection

with the chosen initial or with some other one, and only one, of the stations already observed in that series. With this system of observing no local adjustment is necessary.

Any observed direction differing more than five seconds from the mean shall be rejected and a new observation made to replace it.

The following is a specimen of observations with a direction theodolite. It is taken from the report on the triangulation of Greater New York.

Horizontal directions.

[Station: *Fort Tompkins 2.* Observer: *F. W. Koop.* Instrument: *Theodolite No. 168.*[9] Date: *June 20, 1906.*]

Position.	Objects observed.	Time.	Telescope direct or reversed.	Micrometer.	Reading.			Mean.	Mean, direct and reversed.	Direction.
						Backward.	Forward.			
		h. m.			° ′	″	″	″	″	″
I	Bogart (distinct, unsteady).	3 30	D	A	00 00	33	33			
				B		35	35			
				C		39	39	35.67		
	Bogart	4 00	R	A	180 00	43	43			
				B		33	33			
				C		52	52	42.67	39.17	00.00
	Centennial T.	3 40	D	A	208 37	38	38			
				B		35	35			
				C		54	54	42.33		
	Centennial T.	3 51	R	A	28 37	50	50			
				B		54	54			
				C		55	55	53.00	47.66	08.49
	Sandy Hook lighthouse.	3 42	D	A	256 42	53	53			
				B		52	52			
				C		63	63	56.00		
	Sandy Hook lighthouse.	3 49	R	A	76 43	04	04			
				B		01	01			
				C		03	03	02.67	59.34	20.17

[9] Theodolite No. 168 is a 12-inch direction instrument with three micrometers reading to single seconds. These micrometers have double reading wires, 4 minutes apart, to permit a backward and a forward reading with only one turn of the wheel. The run of each micrometer should be tested once a month and the error corrected if it exceeds 4 seconds. In the above table the agreement between the forward and backward readings of the micrometer indicates that "run" had been completely eliminated through adjustment.

Repeating theodolite.—In observing a horizontal angle with a repeating theodolite care must be taken that the instrument is not disturbed by the observer's shifting his position or by any other cause while the upper motion or plate clamp is loose. A set of observations should consist of six repetitions of the angle with the telescope in the direct position and six repetitions of the explement of the angle with the telescope in the reversed position. The method used in making a set of observations is as follows: Set the circle approximately at zero and record the initial reading. Point on the left-hand object by means of the lower motion and then unclamp the upper motion and point on the right-hand object. Record the approximate reading of the circle. This completes the first repetition of the angle. Next loosen the lower motion and point again on the left-hand object, then unclamp the upper motion and point again on the right-hand object, and so on. A careful reading of the circle must be made and recorded after three repetitions of the angle and again after six repetitions. After six repetitions reverse the telescope about the horizontal axis and by means of the lower motion repoint

on the right-hand object. Then loosen the upper motion and point on the left-hand object. This constitutes one repetition of the explement of the angle. After six repetitions of the explement the instrument should be back very nearly to the initial setting. The circle should then be carefully read and recorded as before. (See sample record below.) Slightly change the setting of the circle at the completion of each set of observations and make an entirely independent reading for the initial of the next set. When two or more sets of observations are made on the same angle the initial setting for each set should differ by an amount approximately equal to 180° divided by the number of sets. For example, if an angle is to be measured with two sets of observations, the initial settings should be about 90° apart; if with three sets, 60° apart, etc. If some of the stations observed upon are much higher or lower than the station occupied, it is necessary to keep the horizontal axis of the instrument level, in order to avoid large and troublesome errors. (See table on p. 40.) With any repeating theodolite measure only the single angles between adjacent lines of the main scheme and the angle necessary to close the horizon. In the comparatively rare case in which the failure of adjacent signals to show at the same time prevents carrying out this program, make as near an approach to it as possible and then take the remaining signals in another series together with some one, and only one, of the signals observed in the first series, and measure in the new series only the single angles between adjacent signals and the angle necessary to close the horizon. With this scheme of observing, no local adjustment is necessary, except to distribute each horizon closure uniformly among the angles measured in that series.

As a general rule the 10-inch repeating theodolite reads to 3 or 5 seconds and does not transit, while the 7-inch instrument reads to 10 seconds and does transit.

With a repeating theodolite having a 10-inch horizontal circle there should be five sets of observations on each angle of precise triangulation. The initial settings of the circle should be 0°, 36°, 72°, 108°, and 144°.

In many cases a 10-inch repeating theodolite is not available, and the magnitude of the work or other conditions preclude the expenditure of the time or money required to secure one. In the triangulation of Greater New York two 7-inch theodolites of a standard make were tried out on the measure of angles at supplementary stations and proved unsatisfactory, while a 6-inch theodolite (repeater) of another make was successfully used on secondary stations and on the shorter main scheme lines. Much depends on the instrument, but with a well-made 7-inch theodolite work of precision can probably be done by increasing the number of sets to six or perhaps more. After observations have proceeded to a point where it is possible to form an opinion based on triangle closures, the program may be altered if necessary.

The following is a specimen of the record of observation of a horizontal angle with a repeating theodolite. It is taken from the published report of the triangulation of Greater New York. It shows the earlier practice of reversing the telescope in the middle of each set, instead of between the measures of the angle and of its explement as is now done.

Sample record of horizontal angles.

[Station: *Idlewild Eccentric No. 1.* State: *New York.* County: *Queens.* Date: *April 14, 1908.* Observer: *W. V. B.* Instrument: *Theodolite No. 63.*]

Objects observed.	Time.	Telescope, direct or reversed.	Repetitions.	Circle.	Verniers. A	Verniers. B	Mean of verniers.	Arc passed over.	Angle, mean direct and reversed.	Mean of 12 repetitions.
	h. m.			° ′	″	″	″	° ′ ″	° ′ ″ ″	° ′ ″
Hollis (circle set to)				0 00	00	00	00			
Metropolitan			1	3 35	50					
Metropolitan	2 25	3D and 3R	6	21 34	40	40	40	21 34 40	3 35 46.67 +0.41	3 35 47.08
Metropolitan										
Hollis	2 42	3R and 3D	6	359 59	55	55	55	2,138 25 15	356 24 12.50 −0.42	
									Sum 359 59 59.17 Correction +0.83	

INCREASED ACCURACY.

The foregoing instructions are designed to secure an accuracy of closure between bases of at least 1 part in 25,000. (See p. 16.) It is sometimes desirable to obtain an accuracy as great as 1 part in 100,000, as in the case of control for cadastral surveys of the largest cities. How is this higher accuracy to be obtained? Increasing the number of observations will only slightly increase the accuracy of the result. The possibilities of the direction instrument are more or less exhausted by a 16-position program, though some slight increase of accuracy might result in the case of the repeating instrument through the addition of one or two sets of angle measures.

An increased accuracy can best be obtained by increasing the refinements of operation at every point of the work and by increasing the frequency of base lines. For an accuracy of 1 part in 50,000 the value of R_1 for the best chain of triangles between bases should not exceed 50, and for an accuracy of 1 part in 100,000 R_1 should not exceed 25. Extreme care must be taken in centering stations; that is, in centering the instrument and the mark observed on exactly over the station mark. The instrument should be carefully selected. Great care must be taken in the reconnaissance to avoid lines on which there is even a suggestion of lateral refraction. For marks on which to observe, signals specially constructed for the purpose must be used. (See p. 24.) The observations should be made at night and sufficiently late to avoid the atmospheric adjustments which ordinarily accompany the change from day to night. Even then a further increase in accuracy may be obtained by making the observations over any line on several nights rather than on a single night. The effect on an observed direction of the inclination of the horizontal axis of the telescope must be considered unless it is actually known to be negligible. (See p. 40.) The strength, R_1, of the selected best chain of triangles in any one figure should not exceed 5.

COMPUTATION OF TRIANGULATION.

CORRECTION TO A DIRECTION DUE TO INCLINATION OF HORIZONTAL AXIS.

When the elevations of the stations differ greatly, it is necessary to keep the horizontal axis of the instrument level in order to avoid large and troublesome errors. The magnitude of these errors for various conditions is shown in the following table. Of course, releveling should only be done between position or sets. Where the level graduations increase continuously from one end to the other, the inclination of the horizontal axis is $\frac{1}{4}[(w+e)-(w'+e')]\times$ the angular value of one division of the level. Where the graduations are from the center outwards in both directions, the inclination is $\frac{1}{4}[(w+w')-(e+e')]\times$ the angular value of one division. In both cases w and e are the readings of the striding level before reversal and w' and e' are the readings after reversal. The correction to a horizontal direction due to this inclination is $i \tan h$, in which i is the inclination of the horizontal axis and h the vertical angle to the object observed. For a circle graduated in clockwise direction this correction is negative if the right-hand end of the horizontal axis is the higher and the object observed is higher than the instrument.

Corrections to horizontal directions due to inclination of horizontal axis of telescope.

Inclination of the horizontal axis (i).	Vertical angle of the line of collimation (h).	Correction to horizontal direction ($i \tan h$).	Inclination of the horizontal axis (i).	Vertical angle of the line of collimation (h).	Correction to horizontal direction ($i \tan h$).	Inclination of the horizontal axis (i).	Vertical angle of the line of collimation (h).	Correction to horizontal direction ($i \tan h$).
"	'	"	"	'	"	"	'	"
10	20	0.06	10	40	0.12	10	60	0.17
20	20	0.12	20	40	0.23	20	60	0.35
30	20	0.17	30	40	0.35	30	60	0.52

ECCENTRIC REDUCTION.

Where it is impracticable to set the theodolite directly over or under the triangulation station, as in the case of a flagpole, the angle measures must be made at a point near-by called the eccentric station, and computations made to reduce the observed directions to what they would have been had the instrument been centered on the station. The same thing is true if the object pointed on is eccentric.

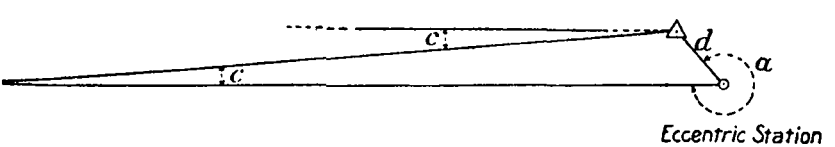


FIG. 14.—Eccentric reduction.

The computations involve the solution of a triangle with a very acute angle, and since for very small angles the sine and the arc are practically in direct proportion, this solution may be reduced to the following compact form:

$$c = \frac{d \sin \alpha}{s \sin 1''}$$

where c is the correction to an observed direction, d is the distance from the true station to the point occupied, α is the observed direction of the distant station involved, reckoned in a clockwise direction as usual, but referred to the direction from the eccentric station to the true station taken as zero, and s is the distance between the true triangulation stations involved.

The following is a sample eccentric computation for an eccentric instrument at the station for which a partial record of observations is shown on page 38.

Sample eccentric reduction.

[Eccentric station: *No. 1, Idlewild.* c (in seconds) $= \frac{d \sin \alpha}{s \sin 1''}$. $d = 1.270$ *m.*]

$$\text{Log } d = 0.10380$$
$$\text{Colog } \sin 1'' = 5.31443$$
$$\text{Log } \frac{d}{\sin 1''} = 5.41823$$

Station.	α		Log sin α.	Log s.	Log $\frac{\sin \alpha}{s}$	Log reduction in seconds.	Reduction=c.
	°	′					″
Center	0	00					
Hollis	54	43	9.91105	3.98490	5.92615	1.34438	+22.10
Metropolitan	58	10	9.92921	3.67449	6.25472	1.67295	+47.09
Far Rockaway schoolhouse	175	16	8.91655	3.72701	5.18954	0.60777	+ 4.05
Seaside Hotel	243	08	9.95039n	3.81138	6.13901n	1.55724n	−36.08

If the object observed is eccentric, the heading "Eccentric station ——" should be changed to "Eccentric observed object at station ——," the first column should contain the names of the stations from which this eccentric object was observed, and in each case α is the direction from the eccentric object to the distant station involved, reckoned in a clockwise direction as usual, but referred to the direction from the eccentric object to the true station, or center, taken as zero. (No distinction need be made between the direction from the eccentric object to the distant station and the direction from the true station to the distant station except when the eccentric reduction is more than one minute.) The remainder of the computation on this form is made in the manner indicated above with reference to an eccentric instrument. The reductions to directions are, however, to be applied to observed directions at the stations named in the first column to the eccentric object at the station named in the heading. The directions to which these reductions are to be applied are therefore found in the various lists of directions and not all in one list as is the case when the instrument is eccentric.

The preceding computation gives the correction to an observed direction. The correction to reduce an eccentrically observed angle to center is the difference (algebraic) between the corrections to the two directions which form the angle. Thus for the angle at Idlewild between Hollis and Metropolitan the correction is $+47.''09-22.''10=+24.''99$ and between Far Rockaway schoolhouse and Seaside Hotel is $-36.''08-4.''05=-40.''13$.

In computing the eccentric corrections it is sometimes convenient to scale an approximate value of the distance s from the best chart or map available and, by using this in the eccentric computation, obtain an approximate correction to the direction. A triangle can then be solved to obtain a better value for the length s. The eccentric computation can now be corrected for this new value of the length and a better value for the correction to the direction will be obtained. A still more accurate length can now be obtained by recomputing the triangle. It will seldom be necessary to make more than two such approximations to obtain a value of c which is correct to the second decimal place. Where the angle at the station occupied eccentrically can be concluded and a triangle solved for the desired length, the first approximation will usually be all that is required.

LISTS OF DIRECTIONS.

After taking out the final means in the record book and applying the corrections for eccentrically occupied or observed stations, all

observations at a given station are then assembled in the form shown below. By thus relating all observations at a given station to one initial and putting them in the form of directions any desired angle may be readily obtained by a simple operation as it is the difference between two directions. The assembling of all the observations of a given angle or direction before entering it in the list of directions is usually done on a form called an *Abstract of horizontal angles* or *Abstract of horizontal directions*, depending on whether a repeating or direction theodolite was used. The following partial list of observed directions is taken from the report on the triangulation of Greater New York.

Sample list of observed directions.

[Station: *Idlewild*. Observer: *W. V. B.* Computed by: *A. T. M.* Checked by: *J. H. S.*]

Station observed.	Direction.		
	°	′	″
Far Rockaway schoolhouse	0	00	00.00
Edgemere Hotel	20	59	55.74
Arverne Hotel	48	04	39.31
Seaside Hotel	67	52	09.69
Hollis	239	19	07.01
Metropolitan	242	55	19.11

REDUCTION TO MEAN SEA LEVEL.

The observed directions must be reduced to what they would be if observed at mean sea level if the computation of the triangulation is to be referred to the sea-level surface. This correction is usually very small and often negligible, but in precise work it must be considered. At stations of elevation of 2,000 m. it will often exceed 0″.10. It is given by the formula[10]

$$\frac{e^2 h \sin 2\alpha \cos^2 \phi}{2\rho \sin 1''}$$

where $e^2 = \frac{(a^2 - b^2)}{a^2}$; a and b are, respectively, the major and minor semidiameters of the spheroid of reference; h is the height of the station sighted; ρ is the radius of curvature of the sea-level surface in a plane normal to the meridian; ϕ is the latitude; and α is the azimuth counted from the south to the westward.

TRIANGLES.

Writing down the names of the three vertices of a triangle in a given order does not seem a very important operation, yet it is one which must be done by system else grave errors may result. In the work of the U. S. Coast and Geodetic Survey all azimuths and all directions increase in a clockwise direction. Azimuths start with south as the initial point or zero and increase through the west (90°) to the north (180°) and east (270°) back to south (360° or 0°). Tri-

[10] See U. S. Coast and Geodetic Survey Special Publication No. 19, p. 42. Log. $e^2 = 7.8305026 - 10$. See U. S. Coast and Geodetic Survey Special Publication No. 8 (6th edition), p. 19.

angles should *always* be written in the same clockwise order, starting at the new station in every case. It is advisable, but not so essential, that the triangles in a figure be written down according to a definite system to avoid overlooking any of them. In Figure 15, considering *AB* as the base or known line, such a systematic procedure would be as follows: Starting at point *C*, the triangle on the known side would be *CAB*. Next at point *D*, considering only the starting line and the point *C*, already used, the triangles would be written in the following order: *DAB*, *DAC*, and *DBC*. Finally, at point *E*, all the remaining triangles would be written as follows: *EAB*, *EAC*, *EAD*, *EBC*, *EBD*, and *ECD*. It will be easily seen that with such a system there will be small chance of overlooking any of the triangles in the most complex figure.

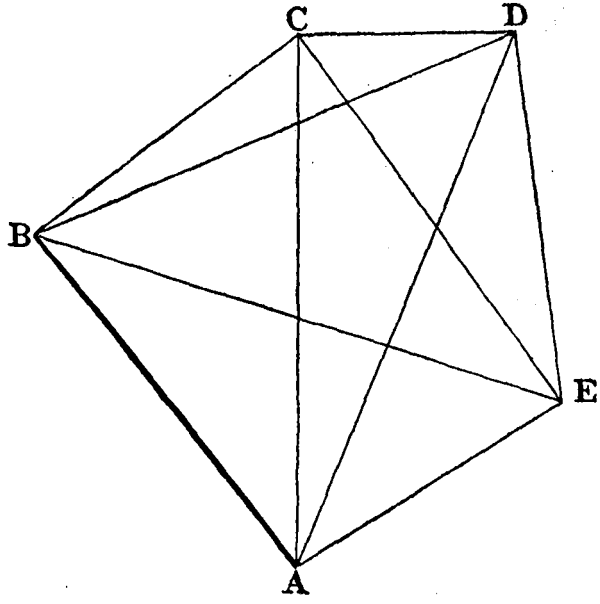


FIG. 15.—*Five-sided figure with all lines observed.*

The standard form for the solution of a triangle is given below:

Sample computation of triangles.

Number of directions.	Station.	Observed angle.	Correction.	Spherical angle.	Spherical excess.	Plane angle.	Logarithms of sines and distances.
		° ′ ″	″	″	″	″	
	Jackson-Ludlow						3.6460219
3-4	Straus	40 22 24.18	−1.85	22.33	0.02	23.31	0.1885865
14-15	Jackson	62 18 30.92	−0.20	30.72	.02	30.70	9.9471703−10
6-7	Ludlow	77 19 07.91	−0.89	07.02	.03	06.99	9.9892744−10
					0.07		
	Straus-Ludlow						3.7817787
	Straus-Jackson						3.8238828

In the above form the first column headed "Number of directions" refers to the numbers given the directions by way of designation in making the least-squares adjustment, and in the column headed "Correction" is entered the correction to the observed angle obtained from the least-squares computation. The spherical excess is computed from the formula

$$\text{Spherical excess}^{11} = \frac{a_1 b_1 \sin C_1 (1 - e^2 \sin^2 \phi)^2}{2a^2(1 - e^2) \sin 1''} = a_1 b_1 \sin C_1 \times m,$$

where a_1, b_1, and C_1 are two sides and the included angle, respectively, of the triangle, e is the eccentricity of the spheroid of reference and a the corresponding major semiaxis, and ϕ is the mean latitude of the three vertices of the triangle. That part of the expression which depends only on the latitude and the dimensions of the spheroid may be designated by a single letter, m, as shown, and will be found tabulated on page 16 of Special Publication No. 8.

It will be seen that the above tabular computation is simply a compact and convenient arrangement for the solution of the triangle by the law of sines. The following points should be noted in regard to the logs in the last column: That it is the colog sin which is given in the second line; that the log in the fifth line is the sum of those in the first three lines; and that the log in the last line is the sum of those in the first two and fourth lines.

GEOGRAPHIC POSITIONS AND RECTANGULAR COORDINATES.

After the solution of the triangles the next step is the computation of the geographic positions and rectangular coordinates. For the computation of geographic positions the engineer is referred to U. S. Coast and Geodetic Survey Special Publication No. 8, Formulæ and Tables for the Computation of Geodetic Positions. The necessary formulæ and tables for the computation of rectangular linear coordinates will be found in Special Publication No. 71, Relation Between Plane Rectangular Coordinates and Geographic Positions. These tables are prepared for a sea-level surface, and the triangulation must therefore be reduced to that surface if the tables are used. (See pp. 30 and 42.)

In making use of rectangular coordinates the engineer is advised to consult the table on page 6 of Special Publication No. 71, in which is shown the accuracy of plane coordinates at various distances from the origin. By dividing the territory being surveyed into areas of limited size and establishing a separate origin of coordinates for each area, a desired standard of accuracy may be maintained. These areas will not be coordinate, however, and so will not "match up" along their common boundaries.

LEAST-SQUARES ADJUSTMENT OF TRIANGULATION.

The least-squares adjustment of triangulation[12] is beyond the scope of this publication, but it is well to state here what such an

[11] See U. S. Coast and Geodetic Survey Special Publication No. 8 (6th edition), p. 7.
[12] The matter is fully treated in U. S. Coast and Geodetic Survey Special Publication No. 28, Application of the Theory of Least Squares to the Adjustment of Triangulation.

adjustment accomplishes and to direct the engineer desirous of going more fully into the matter to those publications [13] which will best serve his purpose.

In computing a chain of triangles extending from one base line to another, the following discrepancies will develop:

First. The sum of the three observed angles of a triangle will seldom equal exactly 180° plus the spherical excess of that triangle.

Second. In a quadrilateral the length of an unknown side will have two different values when computed through the two different sets of triangles.

Third. The measured length of the second base line will not agree with its length as computed through the triangulation from the first base.

Fourth. When the three kinds of discrepancies just mentioned are cared for by arbitrary changes of the angles, and the latitudes, longitudes, and azimuths of the triangulation are computed starting with points fixed in position, discrepancies in latitude, longitude, and azimuth will be developed at the other fixed points to which the triangulation is connected.

It is the function of the least-squares adjustment of a triangulation to eliminate all these discrepancies. It can be mathematically demonstrated that the corrections to the unadjusted quantities thus obtained are the most probable ones. In other words, the least-squares adjustment of a triangulation net will determine the most probable and mutually consistent values of the lengths, azimuths, and positions. The values obtained through an arbitrary distribution of discrepancies are only tentative.

PROBABLE ERROR OF AN OBSERVED DIRECTION.

A probable error of an observation may be defined as an error such that the chances are even as to whether the actual error is greater or less than that amount. The most critical test which can be applied to the accuracy of triangulation is to determine the probable error of an observed direction. This is given by the formula

$$d = 0.674\sqrt{\frac{\Sigma v^2}{c}},$$

where d is the probable error to an observed direction, Σv^2 is the sum of the squares of the corrections to the directions, and c is the number of conditions.

PROBABLE ERROR OF THE LENGTH OF A TRIANGLE SIDE.

The square of the probable error p of the logarithm of a triangle side as computed through a chain of triangles is given by the equation

$$p^2 = \frac{4}{3}\left(d^2\right)\frac{D-C}{D}\Sigma[\delta_A{}^2 + \delta_A\delta_B + \delta_B{}^2],$$

in which d is the probable error of an observed direction in seconds, D is the number of directions observed, C is the number of geometric

[13] See bibliography, p. 76.

conditions that must be satisfied in the figure, and δ_A and δ_B are the differences in the sixth place of the log sines corresponding to a difference of 1″ in the angles A and B, A being opposite the known side of the triangles and B opposite the computed side.

As an example, take the simplest case, a point determined by a single triangle from a known base, all angles being measured. In this triangle there will be six directions and only one condition, that required to make the sum of the three angles equal to 180° plus the spherical excess. Assume that the limiting value of R for this simple figure is 15 in the sixth place of logarithms. (See p. 20.) Then the following table gives the probable error of a triangle side for various lengths of side and various probable errors of an observed direction.

Probable error of the length of a triangle side.

[R=15.]

Probable error of an observed direction.	Length of triangle side.				
	1 mile (1,609 m.).	2 miles (3,218 m.).	3 miles (4,828 m.).	4 miles (6,437 m.).	5 miles (8,046 m.).
″	*mm.*	*mm.*	*mm.*	*mm.*	*mm.*
0.2	3	7	10	13	17
0.3	5	10	15	20	25
0.4	7	13	20	26	33
0.5	8	17	25	33	41
0.6	10	20	30	40	50
0.7	12	23	35	46	58
0.8	13	26	40	53	66
0.9	15	30	45	60	74
1.0	17	33	50	66	83

For a line 3 miles in length and a probable error of 0.″5 for an observed direction the resulting probable error for the length of the line will be 25 mm. or approximately 1 part in 190,000. Assuming that this is also the probable error of the resulting position, there is an even chance that if the point thus located is lost it can be relocated within 25 mm. (about 1 inch).

In using the above table it must be remembered that the values are computed on the assumption that the point is determined by a single triangle having a value for R equal to 15. Reducing the value of R will reduce the probable errors above tabulated, and vice versa

ACCURACY WITH WHICH A TRIANGULATION STATION MAY BE REESTABLISHED.

A triangulation station which has been destroyed may be replaced from near-by stations as follows: Occupy two of the recovered stations with a theodolite and set a point on the ground close to the position of the lost point by turning off from each station the angle between the other recovered station and the lost station as taken from the adjusted computation. In some cases marks may remain on the ground which will enable one to identify an approximate location for the lost point without this additional work. Establish a signal in the approximate location thus found and make a standard determination of its position by observations from at least three

recovered stations. The coordinates (spherical or plane) of the new point are then computed by a least-squares adjustment and compared with the coordinates of the lost point. The differences in the coordinates are reduced to an azimuth and a distance, which can be readily laid off on the ground with practically no error, and the original point is thus reestablished. If the lost point and its immediate vicinity are not visible from the recoverable stations, the determination of the position of the approximate station must be made through a system of triangulation based on recovered points.

Actual tests of the closeness with which a station may be replaced are available from data of the U. S. Coast and Geodetic Survey obtained when new triangulation is started from previously established stations. While new triangulation may actually be based on only two old stations, the third is required to prove the recovery of the other two. For example, in 1920 an arc of precise triangulation was extended from the vicinity of Mount Shasta, Calif., to the eastern border of Oregon. At its California end this arc depended for length, azimuth, and position on stations of the 1904 California-Washington arc. In the triangle of the 1904 work which was recovered, one angle was remeasured in 1920 with 11 positions of the direction instrument, a second angle was remeasured with a single set of observations with a repeating theodolite, and the third angle was concluded. A comparison of the computed lengths showed the new lengths to be shorter than the old by 1 part in 310,000 (4.0 inches) and 1 part in 217,000 (10.9 inches) on lines, respectively, 19.6 and 37.5 statute miles in length.

At the eastern end of the same arc, where in 1920 two angles of the recovered triangle (1916) were remeasured with the full 16 positions of the direction instrument and the third angle concluded, the 1920 lengths were found to be shorter than the 1916 adjusted lengths by 1 part in 434,000 (4.4 inches) and 1 part in 869,000 (2.9 inches) on lines, respectively, 30.4 and 39.6 statute miles in length.

For the connections just described only sufficient observations were made in 1920 to verify the recovery of the old stations. Had it been desired to reestablish one of the old stations from the other two, all three angles of the triangle would have been observed with the full 16 positions of the direction instrument.

PRINCIPAL TRAVERSE.

The primary purpose of the principal traverse in city surveying is to make the geodetic control readily available for the detail surveys by determining the positions of numerous stations in accessible locations.

In addition to this temporary value primary traverse has a permanent value in connection with all future plans and surveys within the area of the city, for it provides permanent stations with accurately determined positions within a short distance of any point in that area.

Principal traverse may be measured in much the same manner as a base line but with such modifications as to increase the speed without allowing the accuracy to fall below that of the triangulation which controls it. The use of invar tapes stretched flat on the pavement is recommended, and also the use of markers for the tape ends which can be quickly and cheaply put in place. (See p. 18.) In

U. S. Coast and Geodetic Survey Special Publication No. 58 will be found general instructions for precise and secondary traverse.

In making connection between a triangulation station on a high building, which can not be plumbed down to the ground, and the traverse, angles must be observed at three or more of the traverse stations in order that the distances and directions to the triangulation station may be computed with checks. The vertical angle of the triangulation station above the traverse stations will probably be large and it will therefore be necessary to give careful consideration to the correction for inclination of the horizontal axis of the telescope. (See p. 40.) After the instrument has been put in as close adjustment as is possible the stride level should be used to determine the actual inclination of the axis in order to apply a correction therefor.

COMPUTATION OF TRAVERSE.

Although traverse is one of the simplest of surveying operations, the computation of traverse involves problems which are rather difficult of exact solution. In triangulation, where only an occasional base line is measured and the other sides are obtained through the mathematical relationship existing between triangle sides and sines of opposite angles, condition equations can be formed based on this relationship. In traverse work, however, there is no mathematical relationship existing between the angles and the distance measures. A traverse which extends indefinitely from a starting point without closing either on itself or on some known base is not even susceptible of being corrected for accidental errors.

In a traverse which closes either on itself or on adjusted stations, quantitative errors are developed. Often it is impossible to tell what proportion of these closing errors are due to angle measures and what proportion to length measures, except that corrections to the angles are generally required to close on the fixed line. It is sometimes possible to say that an error of closure is due more to length errors than to angle errors, as, for example, when all the sections of a traverse extend in one general direction and the closure is in the same direction. In such a case the major portion of the correction is obviously a length correction. With a knowledge of the accuracy attained in the angle and length measures, the computer can often, by the exercise of good judgment, determine within satisfactory limits how much of the closing discrepancy to attribute to angle measures and how much to length measures.

A number of different ways have been proposed for eliminating the discrepancies which will occur in any closed traverse. One of the simplest is to place equal corrections on each angle, and then using the angles thus obtained to compute the latitudes and departures, and to distribute the discrepancies of closure in proportion to the computed quantities. With the coordinates thus obtained for the various points new azimuths and distances between stations must be computed.

This method is satisfactory provided that all angles have been measured in such a way as to be of equal weight, that all measured lines are of an accuracy proportional to their lengths, and that there exists a consistency between the standards for measuring angles and those for measuring distances. This last condition may be illus-

trated as follows: For an angle of 45°, both the sine and cosine change 1 in the sixth place of logarithms for a change of one-half second of arc, while 1 in the sixth place of the logarithm of a distance represents 1 part in 434,294. Therefore, to keep the errors of measurement down to 1 part in 50,000, the errors of the angles must not exceed 4.5 seconds of arc.

A method which has been developed in the U. S. Coast and Geodetic Survey for adjusting traverse computed on geographic coordinates is given in Special Publication No. 79, entitled "Precise Traverse and Triangulation in Indiana."

Always remember in precise work that traverse lengths as measured on the ground can not be adjusted to triangulation which has been computed on the sea-level surface until sea-level corrections have been applied to the traverse. (See p. 30.)

It is usually desirable to reduce traverse positions in a city to a rectangular system of coordinates. A standard set of traverse tables will be found convenient for this purpose.[14]

THE CITY MAP.

One of the final results of a survey is a map of the area covered. The framework of this map is constructed by plotting the known positions, determined by the geodetic survey, on some form of map projection. The best kind of projection to be used is a vital question and must be viewed from several angles. The representation on a plane of an area which approximates a portion of a sphere must involve some kind of distortion.[15] If it is desired to make the areas on the map proportional to the areas on the earth, form must be sacrificed and an "equal-area" projection used, but if a conformal representation is more to be desired, then a distortion of scale must be accepted. In general, some one element of the map, as area, form, or azimuth, may be held close to the truth, but always at the expense of the other elements. Fortunately the area covered by the average city is so small that its departure from a plane, when reduced to the scale of a map, is generally negligible, and for this reason what is generally termed a rectangular linear projection will nearly always serve. For this projection a center of coordinates is adopted, usually a triangulation station near the center of the area involved, and the plane coordinates of the remaining triangulation stations are computed with reference to this center and the meridian through it. The transformation from geographic to plane coordinates may be easily made by using U. S. Coast and Geodetic Survey Special Publication No. 71, Relation Between Plane Rectangular Coordinates and Geographic Positions. On page 6 of that publication is given a table of maximum error which occurs through the use of this projection for small areas in latitudes not exceeding 50°. This table shows that for a point whose distance from the origin of coordinates is 18.6 miles, the accuracy is 1 part in 100,000 or greater.

For general use, as for the construction of a wall map on a small scale, a geographic projection is desirable. Such a projection is the polyconic projection, which has had many years of use in Federal

[14] A very satisfactory set of tables is Traverse Tables by R. L. Gurden. These tables give latitudes and departures to four places of decimals for every minute of azimuth and for lengths of 1 to 100 units.
[15] See U. S. Coast and Geodetic Survey Special Publication No. 68, Elements of Map Projection.

surveys and is easily constructed. A description of the methods of construction and the required tables are given in U. S. Coast and Geodetic Survey Special Publication No. 5, Tables for a Polyconic Projection of Maps. For a small area this projection is virtually a rectangular projection.

GENERAL CONCLUSIONS.

The main advantages of a geodetic control for city survey work are *accuracy*, *convenience*, and *economy*. With geodetic control, mapping operations may be started at a number of different places at the same time with the assurance that when a junction is effected between any two pieces of work there will be complete coordination within the specified limits of accuracy. It may be desirable to map a city in somewhat irregular fashion, surveying first those sections where the need is most pressing, or it may be advisable to survey the entire city within a limited time. In either case geodetic control is very necessary and useful.

Part III.—MAKING THE SURVEY—VERTICAL CONTROL.

GENERAL REMARKS.

There are very few engineering undertakings which do not require the determination of elevations or of differences of elevation. The required accuracy varies from that needed in building a mountain trail, where a hand level or perhaps the unaided eye may be used, to the precise requirements of sewer designing in a city of flat contour. In the modern city the need for a system of accurately determined elevations is very great. The sewer system is but one of a number of projects which emphasizes this need. A sewer system is essentially a system of gravity drains, and there is a well-established relationship between the size of sewer and the gradient for a given quantitative service. Faulty sewer design is often due to the lack of a complete and accurately coordinated system of elevations. In fact, there is no more prolific source of error in city work than bench marks based on several different datum planes. The precise level survey should connect with bench marks of all preexisting systems and determine equations for all the various datum planes which may be in use in various parts of the city or by different organizations. Quite often in the past an engineer needing a datum for elevations simply assumed one, but practically all engineers at the present time agree that the ultimate datum for all elevations should be mean sea level.[16]

PRECISE LEVEL INSTRUMENT.

The instructions on page 53 are for use with a U. S. Coast and Geodetic Survey type of level instrument. This instrument fully satisfies the requirements of the most precise class of work and is easily and rapidly handled. Its use is no longer limited to this bureau, as the demand for it by other organizations has made it possible for the private engineer to secure one for his own use, if he so desires, without unreasonable expense or bother.

An ideal level instrument is one having a sensitive and accurately graduated level so connected to a telescope that the line of collimation and the tangent to the center of the level tube remain accurately parallel. The instrument must also be so constructed that the bubble may be held at the center of its tube while the rod reading is made.

In 1896 a committee was appointed by the Superintendent of the U. S. Coast and Geodetic Survey to study the various methods and instruments in use for precise leveling. As a result of the work of this committee a new precise leveling instrument was designed and constructed by E. G. Fischer, chief of the instrument section of this bureau. This type of instrument has been in constant use by the U. S. Coast and Geodetic Survey for more than two decades, and was

[16] See U. S. Coast and Geodetic Survey Special Publication No. 41, Use of Mean Sea Level as the Datum for Elevations.

used on the precise level surveys of New York City and Cincinnati. The excellence of the original design has been well proven. Only two improvements have been made in the instrument, one a slight change in design and the other a change in the material of which it is constructed. (See below.)

In designing this instrument the fundamental problem was to insure a constant relationship between the level vial and the line of collimation. This was accomplished by sinking the level vial into the main telescope of the instrument as near to its center as could be done without interfering with the pointing line, and by using invar for those parts of the instrument upon which depended the above relationship. Within the last year or two it has become possible to so manipulate invar that all parts of the instrument can be made of it, and during the past season an all-invar precise level has been in use by the U. S. Coast and Geodetic Survey. This new instrument has one small change from the design of the original instrument, namely, the telescope is pivoted at its middle point instead of at a point a short distance ahead of the center. This eliminates any possible systematic error which raising or lowering the telescope on the micrometer screw might have tended to introduce.

In general design the precise level is a binocular instrument (see Figs. 16 and 17), in which the secondary or left-hand telescope provides an easy means of viewing the level bubble while making the pointing. The tripod is made of such a height that the observer stands erect. In operation the instrument is first approximately leveled with the attached circular level, and then the micrometer screw is used to bring the bubble easily and quickly to the middle of its tube. The observer then watches the bubble with one eye and holds it at the center by means of the micrometer screw while with the other eye he reads the three lines of the diaphragm as seen projected on the level rod. A complete description of this instrument is given in Appendix 3, U. S. Coast and Geodetic Survey Report for 1903, and also in the report, Precise Leveling in New York City.

OTHER LEVEL INSTRUMENTS.

It may sometimes be necessary for a city to make use of level instruments already in its possession, and where such is the case a study of the instructions given herein for precise leveling (see p. 53), and a careful study of the particular instruments available should result in work of a character which will approach the required accuracy of precise levels, but it will not be accomplished as easily nor as rapidly and economically as with the type of instrument described above. In using any type of level for precise leveling it is very important to keep the collimation error as small as possible, to equalize the lengths of sights and shorten them to a point where the target may be dispensed with (see p. 53), and to read three lines of the diaphragm as projected on the level rod instead of only one.

LEVEL ROD.

The level rod used in the work of the U. S. Coast and Geodetic Survey is a direct-reading rod, graduated metrically. For years the rod used had a symmetrical cross section of the form of a cross (✚),

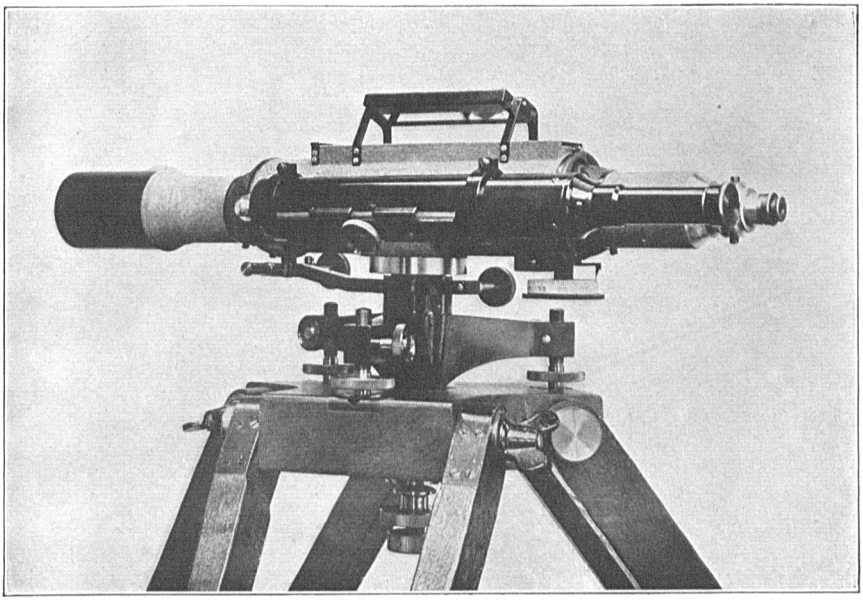


FIG. 16.—STANDARD TYPE OF U. S. COAST AND GEODETIC SURVEY PRECISE LEVEL (BEFORE 1921).

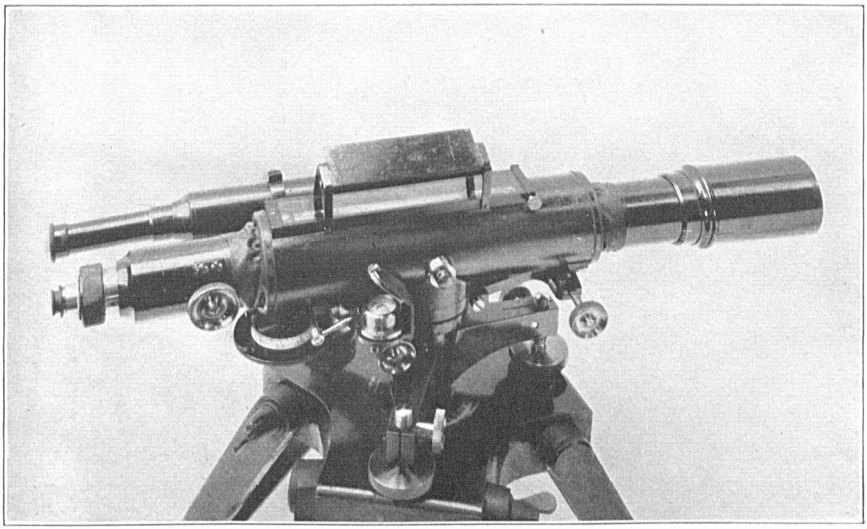


FIG. 17.—STANDARD ALL-INVAR PRECISE LEVEL NOW USED BY THE U. S. COAST AND GEODETIC SURVEY.

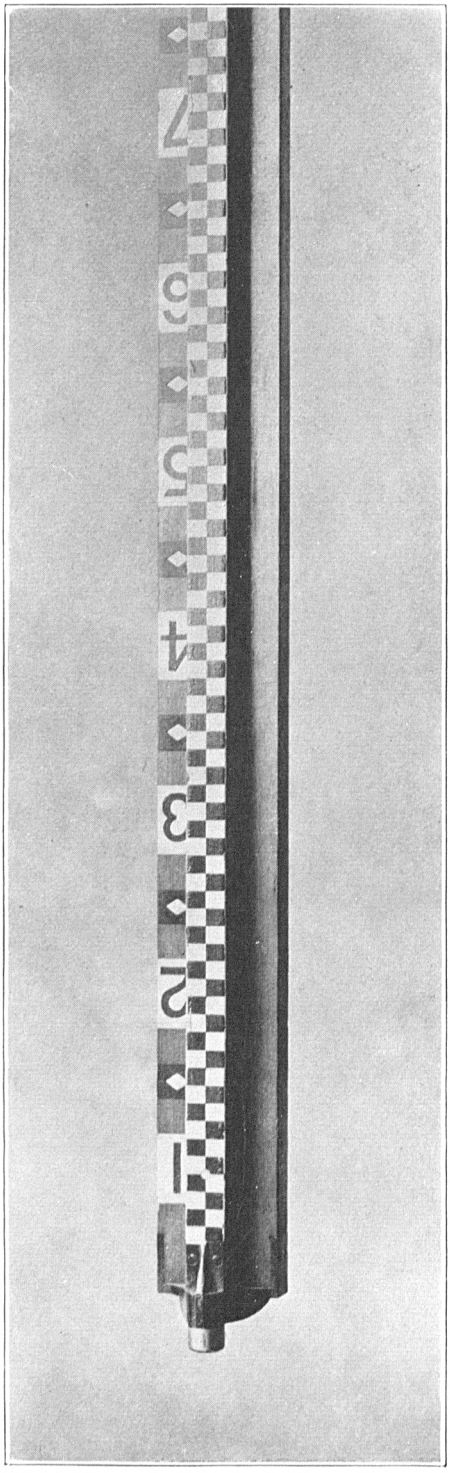


FIG. 18.—PRECISE LEVEL ROD, OLD TYPE, WITH WOODEN FACE AND CONVEX FOOT.

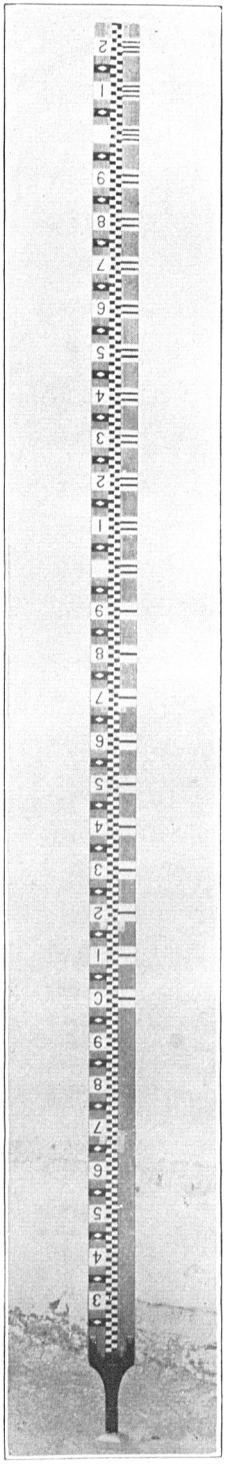


FIG. 19.—PRECISE LEVEL ROD, WITH INVAR FACE AND FLAT FOOT.

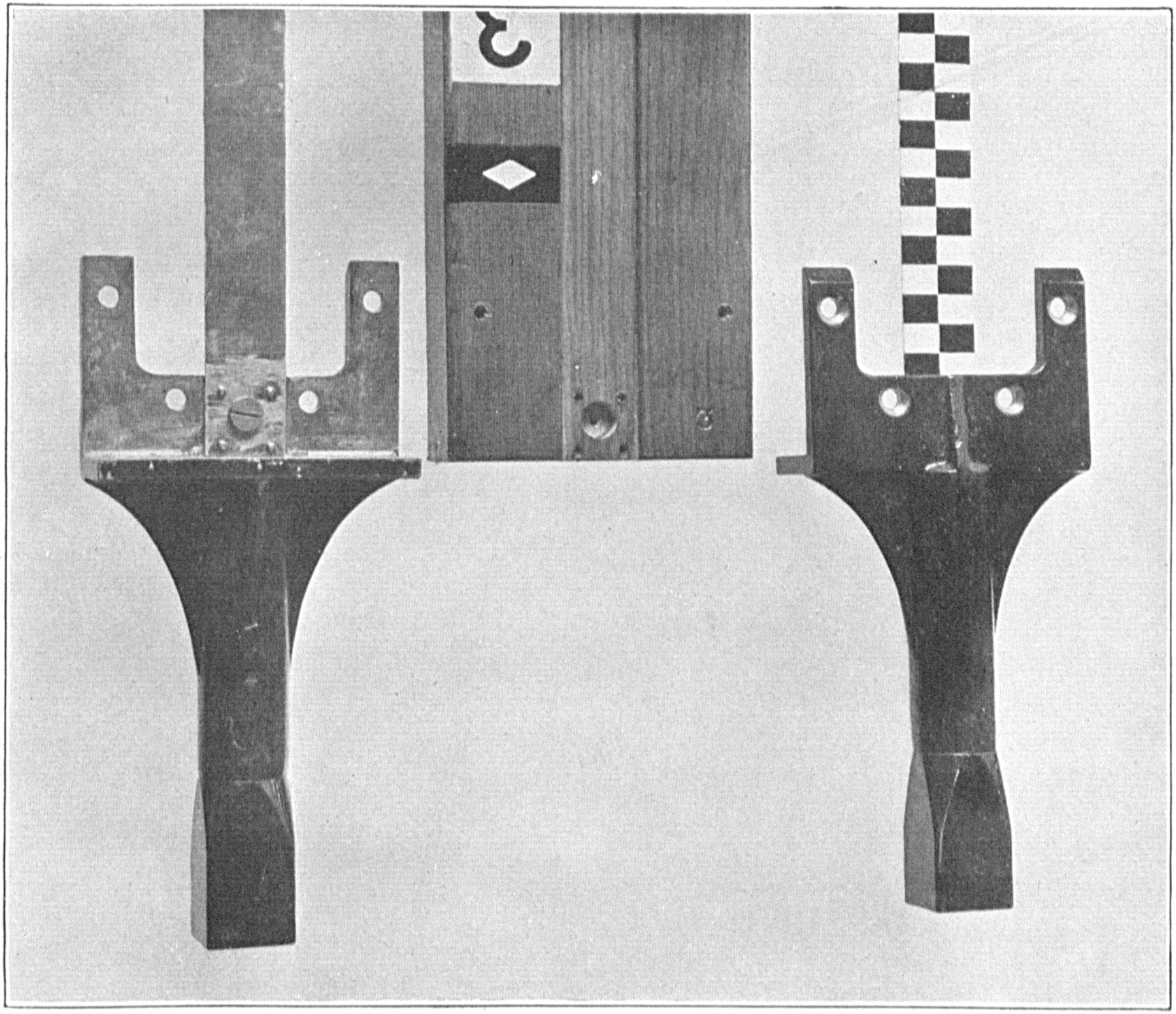


FIG. 20.—DETAIL OF ROD FOOT, SHOWING ATTACHMENT OF INVAR ROD FACE.

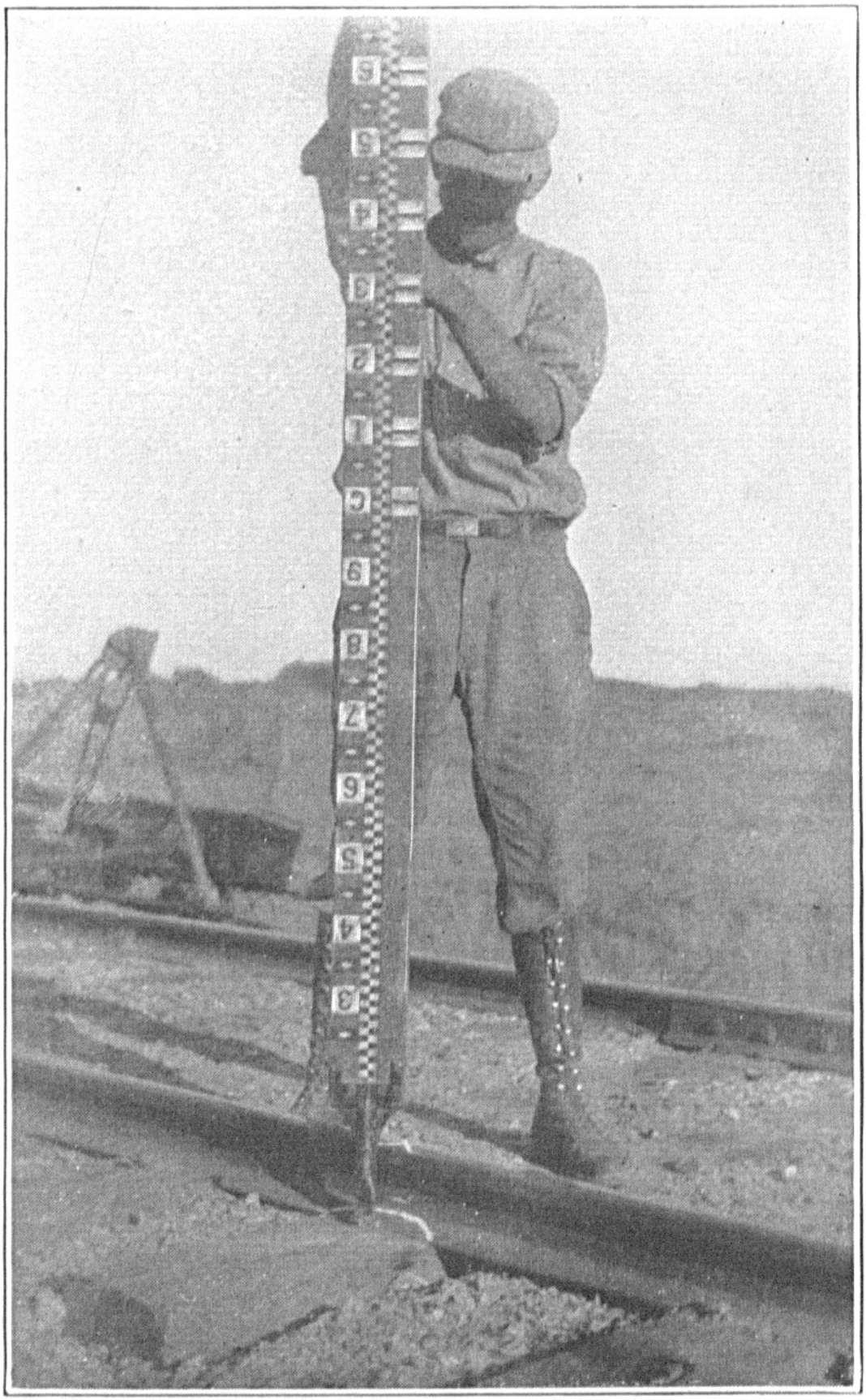


FIG. 21.—USING A RAIL SPIKE AS A TURNING POINT.

and was made of white pine impregnated with paraffine to render it impervious to moisture as far as possible. These wooden rods, however, were found to vary in length according to weather conditions. The rod in present use in this survey is made of a single piece of untreated white pine a little over 3 m. long, 83 mm. (3.27 inches) wide, and 28 mm. (1.10 inches) thick, with a strip of invar 26 mm. (1.02 inches) wide attached to it on which are the centimeter graduations. (See Figs. 19 and 20.) This invar strip is made fast to the shoe of the rod, but is so held in a groove against the rod itself as to be free to expand or contract in a longitudinal direction. The temperature coefficient of the invar rod face is only about 0.000001 per degree centigrade. The temperature of the rod is determined by a thermometer set into the wood and protected by a slide, and an attached circular level indicates when the rod is vertical. The older type of rod had a slightly convex foot, but the foot of the new rod is a plane. The system of graduations used on the rod has been developed through years of use and is very satisfactory. (See Fig. 19.)

The essentials of a good rod are accuracy, straightness, temperature control, and an easily read system of graduations. Lightness and balance might also be included. For an important survey the rod lengths should be standardized at the U. S. Bureau of Standards. A short invar tape should be kept solely for the purpose of testing the lengths of the rods during field use and discovering any unexpected changes.

There is no question as to the superiority for precise work of the direct-reading rod over the target rod, and even in ordinary wye-level work the writer believes that a direct-reading rod will prove more efficient and economical. The target rod may be read at greater distances than the direct-reading rod, but the multiplied uncertainties of refraction over the longer sights and the added difficulties of equalizing the lengths of sights overbalance this advantage. Although the direct-reading rod will undoubtedly require shorter sights and therefore more set-ups, the observer will have complete control over his progress and the time ordinarily required to make a close setting of the target will be eliminated. As Frederick W. Koop stated in the report, Precise Leveling in New York City:

> Precise leveling carried on by the national surveys and by the European Governments has conclusively demonstrated the superiority of the direct-reading rod over the target rod. With a good instrument the thread readings can be accurately taken in less time than is required to set a target, and the mean of the three readings is much more accurate than a target reading. * * *. Direct-reading rods are, for many reasons, superior to target rods, and might to advantage be more generally used than is at present the case.

INSTRUCTIONS FOR PRECISE LEVELING.

The instructions for precise leveling followed by the U. S. Coast and Geodetic Survey, and used as a basis for the precise level surveys of Cincinnati and New York City, are given below with modifications only in certain minor details which require special consideration for city work:

1. Except where special reasons exist for proceeding otherwise, all lines are to be leveled over independently in both the forward and backward directions.
2. The level net of the city is to be connected with *all* adjusted precise level marks

established within the area of the survey by the U. S. Coast and Geodetic Survey, or by any other organization.

3. The level net of the city is to be connected with at least three bench marks on each datum within the area of the survey in use by the city or by public-utility corporations.

4. All old bench marks are to be called by their original names or numbers and are to be fully described by quoting and amending the original descriptions.

5. It is desirable that the backward measure of a line be made under different atmospheric conditions from those which occur on the forward measure. This is accomplished by making the two measures on different days or in the forenoon and afternoon of the same day.

6. On all sections where the forward and backward measures differ by more than 4.0 mm.$\sqrt{\text{length of section in kilometers}}$ (or 0.017 foot $\sqrt{\text{length of section in miles}}$), both the forward and backward measures are to be repeated until the difference between two such measures falls within this limit. No one of the questioned measures is to be used with a new measure in order to secure this agreement.

7. If any measure over a section gives a result differing by more than 6 mm. (0.020 foot) from the mean of all the measures over the section, this measure shall be rejected. No rejection shall be made on account of a residual smaller than 6 mm. unless there is some good reason for suspecting an error in this particular measure, in which case the reason for the rejection should be fully stated in the record.

8. Whenever a mistake, such as the misreading of 1 dm. or 1 m., or an interchange of sights (the backsight being recorded as the foresight), is discovered in any measure after its completion and the necessary correction applied, such measure may be retained provided there are at least two other measures over the same section which are not subject to any such uncertainty.

9. The program of observation at each station should be as follows: Set up and level the instrument. Read the three lines of the diaphragm as seen projected against the front (or rear) rod, each reading being taken to the nearest millimeter (estimated), and the bubble being held continuously in the middle of the tube (i. e., both ends reading the same). As soon thereafter as possible read the three lines of the diaphragm as seen projected against the rear (or front) rod, estimating to millimeters and holding the bubble in the center of the tube as before.

10. All instrument stations are to be numbered in the record, being given consecutive numbers beginning with 1 for the first station of each day.

11. At stations of odd numbers the backsight is to be taken before the foresight, and at even numbered stations the foresight is to be read before the backsight. As the same rod is held on a rod station for both foresight and backsight, the effect of this program is that the same rod[17] is read first at each set-up, it being the rod used for the backsight at the first instrument station.

12. At each rod station the thermometer is to be read to the nearest degree centigrade and the temperature recorded.

13. The difference in length between a foresight and the corresponding backsight must not exceed 10 m., and this difference is to be made as small on each pair of sights as is feasible by the use of good judgment without any expenditure of time for this particular purpose.

14. The recorder shall keep a record of the rod intervals subtended by the extreme lines of the diaphragm on each backsight, together with their continuous sum between contiguous bench marks. A similar record shall be kept for the foresights. The two continuous sums shall be kept as nearly equal as is feasible, without the expenditure of extra time for that purpose, by changing the relative lengths of the backsights and foresights, subject always to the requirements of 13 (above). The two continuous sums for a section shall not be allowed to differ by more than a quantity corresponding to a distance of 20 m.

15. Once during each day of observation the error of the level should be determined in the regular course of the leveling and recorded in a separate opening of the record book as follows: The ordinary observations at an instrument station being completed, transcribe the last foresight reading as part of the error determination, call up the back rod and have it placed about 10 m. back from the instrument, read the rod, move the instrument to a position about 10 m. behind the front rod, read the front rod and then the back rod. (The two instrument stations are between the two rod points.) The rod readings must be taken with the bubble in the middle of the tube. The required

[17] This may be called *head rod* in contradistinction to *front rod*.

constant to be determined, namely, the ratio of the required correction for any rod reading to the corresponding subtended interval, is

$$C=\frac{(\text{sum of near rod readings}) - (\text{sum of distant rod readings})}{(\text{sum of distant rod intervals}) - (\text{sum of near rod intervals})}$$

The total correction for curvature and refraction must be applied to the sum of the distant rod readings before using it in the formula. The level should not be adjusted if C is less than 0.005. If C is between 0.005 and 0.010, the observer is advised not to adjust the level, but if C is greater than 0.010 the adjustment must be made. If a new adjustment of the level is made, C should at once be redetermined. It is desirable to have this determination of the level error (C) made under the usual conditions as to length of sight, character of ground, elevation of line above ground, etc. The adjustment of the instrument to reduce C must be made by moving the level vial, not by moving the reticle. (See p. 63.)

16. Notes for future use in studying leveling errors should be inserted in the record, indicating the time of beginning and ending the work on each section, the weather conditions, especially as to cloudiness and wind, the direction of the sun with reference to the direction of the running of the line, and traffic conditions, such as the near passage of heavy trucks and cars. Such other notes should be made as promise to be of value in studying errors.

17. The instrument shall be shaded from the direct rays of the sun, both during the observations and when moving from station to station.

18. The maximum length of sight shall be 150 m., and this maximum shall be attained only under the most favorable conditions.

19. At least once a month, during the progress of the leveling, a test must be made of the adjustment of the rod levels, and a statement should be inserted in the record showing the manner in which the test was made, whether the error was found to be outside the limit stated below, and whether an adjustment was made. With the bubble of the level rod held at the center, the deviation from the vertical of the plane intersecting the center of the face of the rod throughout its length and normal to the face of the rod must be determined. The deviation from the vertical of the plane coinciding with the face of the rod must also be determined. If the deviation from the vertical exceeds 10 mm. on a 3 m. length of rod, the rod level must be adjusted.

20. The forward and backward runnings along any line must begin and end on permanent bench marks. No temporary bench marks may be used to hold the line, even overnight.

21. All bench marks should be established a sufficiently long time before the first leveling over them to give them time to assume a permanent set.

22. The length of sight should be kept just long enough to require a moderate amount of rerunning. If an observer is extremely cautious and confines all his observations to sights sufficiently short to insure easy reading of the rod, the amount of rerunning required under paragraph 6 will be very small, but the progress will be slow. On the other hand, if the observer attempts to take sights of the limiting length, the rerunning will be large and by reason of this large amount of rerunning the progress will likewise be slow. It is believed that the maximum speed consistent with the required degree of accuracy will be secured by continually keeping the length of sight such that the amount of rerunning will be from 5 to 15 per cent. Observers will find it a convenient rule in fixing the length of sight to shorten the sights whenever the upper and lower thread intervals subtended on the rod are found to differ frequently by more than a selected limit. This limit should be selected by each observer from his own experience by noting the relation between some provisional limit and the amount of rerunning found necessary while using it. Such a rule is based on the idea that the errors which are encountered when the length of sight is increased are, in the main, those due to the accidental errors in reading the rods.

23. While it is not advisable from our present knowledge of the matter to attempt to state allowable limits for the rate of divergence between the forward and backward runnings of a line, we know that this must be kept small.

24. When cases arise which are not provided for in these instructions, the U. S. Coast and Geodetic Survey will be glad to aid with its advice.[18] It will be glad also to receive any suggestions which might tend to improve the procedure herein laid down.

RIVER CROSSINGS.

It will occasionally be necessary in making a city survey to carry a line of precise levels across a river or arm of water too wide for ordinary sighting and not spanned by a structure of sufficient stability to carry a line of levels. Ten such crossings, varying in length from 790 to 4,400 feet, were met with in the survey of New York City. To secure an accuracy comparable with what is attained in the regular leveling over land, two general methods have been developed, known as the fixed-target and the movable-target methods.

MOVABLE-TARGET METHOD.

On the New York City survey special targets were constructed for each river crossing. These targets had the same general design but differed in absolute dimensions and in some details of design, depending on the width of the crossing. They were clamped onto the regular level rods. The observations were made simultaneously from both sides of the river and required, therefore, two levels which were as nearly alike as possible. The instruments were very carefully adjusted and were set up on opposite sides of the river with the rods held on bench marks or turning points about 20 feet distant from each. After each observer had read the near rod he then sighted across the river and by prearranged signal waved the target into position at the same time that the observer on the other side of the river was performing the corresponding operation. In order to permit a change in atmospheric conditions to take place, five minutes were allowed to elapse before the targets were again waved into position for a new set. After 10 such sets had been obtained the observers changed stations, each observer taking his instrument with him, and 10 more sets were observed, followed by readings on the near rods. Such a series of observations will ordinarily give a satisfactory determination of the difference of elevation of the two rod stations. If the result should not have the accuracy desired, the atmospheric conditions should be carefully studied and more sets should be taken at such a time that the observations will be least likely to be subject to any abnormal refraction. Particular attention should be paid to seeing that the line is not too close to the water, and that observations are not made when the sun is shining brightly in patches on the water through breaks in the clouds. There should be no change in the focus nor any disturbance of the line of collimation of the instruments from the time the transriver observations are started on one side of the river until after the observers have exchanged positions and completed the transriver observations from the other side. The instruments should be handled carefully and protected against jars while being transported across the river.

It is readily seen that the mean of simultaneous observations will be free from the effects of curvature and ordinarily from those of refraction. In practice it is best to combine the 10 readings on each rod in each set by taking a straight mean before subtracting to obtain an approximate difference of elevation. This also enables one to study the actual observations with the view of determining if any of them are beyond the limit of rejection.

A safe rule to follow in rejecting observations is given in The Adjustment of Observations, by Wright and Hayford, and is as follows:

Reject each observation for which the residual exceeds five times the probable error of a single observation. Examine each observation for which the residual exceeds three and one-half times the probable error of a single observation, and reject it if any of the conditions under which the observation was made were such as to produce any lack of confidence.

The probable error of a single observation is computed from the formula

$$r = 0.6745\sqrt{\frac{\Sigma v^2}{n-1}}$$

in which r is the probable error of a single observation, Σv^2 is the sum of the squares of the residuals from the arithmetic mean, and n is the number of observations. Where an observation is rejected, it should, if possible, be replaced by another observation, in order that the computation may be kept as simple as possible by avoiding the introduction of unequal weights.

After correcting the approximate difference of elevation from the separate sets for index error of rod, rod length, and temperature, the results may be combined into one mean, giving the final difference of elevation of the rod stations. The computations may be made by combining either the means corresponding to simultaneous observations or the means representing a given instrument in its two positions. If the sets of observations are perfectly symmetrical, the final result will be independent of how the combination is made.

FIXED-TARGET METHOD.

In this method the micrometer screw of the instrument is used to make readings on the rod above and below the level line of the instrument. It is essentially a method of vertical angles into which the absolute values of the vertical angles do not enter. The stations are selected with due regard to water line and height above water surface, and the rods are held in place by guys. Two targets are clamped on each rod in such position that one is above and the other below the point on the rod intersected by the middle wire of the diaphragm when the instrument is level. A single observation consists of three readings of the micrometer, one when the middle wire bisects the upper target, another when the instrument is made level, and the third when the middle wire bisects the lower target. The positions of the targets on the rod may be closely measured, using dividers and scale. The point at which the middle wire cuts the rod when the instrument is level may then be easily computed, the intercepts on the rod being in direct proportion to the differences of the micrometer readings. The program of observing adopted by the U. S. Coast and Geodetic Survey in making a crossing over the Mississippi River at a point where the width was 1,200 m. (0.75 mile) was as follows: Two instruments were used and the observations on the two sides of the river were made simultaneously. The near rod for each instrument was first read and then, on a signal from the observer in charge, observations were begun across the river and continued until the observer in charge had made 25 sets of observations when, on signal, observing across the river ceased. The observers then changed stations, carrying their instruments with them, using

care not to disturb the focus or line of collimation, and the program was repeated except that the near rods were read last. The observers then changed stations again and a third repetition of the program was made. This constituted a day's program and was repeated on at least one other day.

The reduction of observations made by this method do not differ materially from the computation where the *movable-target* method is used.

TURNING POINTS.

The ideal turning point for the flat-footed rod is the convex head of a bolt or spike. For leveling along a railroad track the rail spikes are most satisfactory, but along a city street other types of turning points must be used. In the precise level surveys of New York City and Cincinnati the rods used were of the older type, having a cylindrical foot with slightly convex bottom, and the usual turning point was a mark made with keel on the pavement. Two specially constructed types of portable turning points, formerly used by the U. S. Coast and Geodetic Survey when the line left the rail, were used on those surveys. One type consisted of a circular disk with prongs on its lower side and a cup-shaped cavity on its upper surface to receive the foot of the rod, and the other type consisted of a pin having a similarly shaped depression in its head. Their use is explained in the report, Precise Leveling in New York City, as follows:

> The footplates were found to make the best turning points on dirt or macadam roads, or when crossing long stretches of sand, gravel, or farm lands. The method of using the footplate is simply to set it down and press it firmly into the soil; when the rod is set on the plate, it is kept there until the rodman leaves his station; that is, he is not allowed to raise his rod and then replace it.
>
> The foot pin is made of hard steel, having a broad head with spherical cavity for receiving the foot of the rod, the same as the footplate. The pins are driven with a mallet and are very useful when running over grassy soils, through wooded country, and on very steep grades.

If a rod having a flat foot is used, the cavity in the footplate or in the top of the pin must be replaced with a rounded knob on which the foot of the rod is to rest.

BENCH MARKS.

Before spending time and money on the establishment of a bench mark serious consideration should be given to such questions as the following: How deep must a bench mark be set to be safe against the action of frost and what is the best-shaped mark to resist this action? What has the character of the soil to do with the depth for setting the mark? What type of bench mark will best resist the ravages of time? What type will resist the vandalism of man? In selecting the type of bench mark to be used the question of cost must be subordinate to that of permanency. There is a standard, depending somewhat on local conditions, below which it is uneconomical to go in the marking of a triangulation station or bench mark.

The different types of bench marks in present-day use may be classified into four main groups, as follows:

1. Marks cut into some solid masonry forming part of a large structure, or marks cut in bedrock.
2. Metal disks, plates, bolts or pins set into large masonry structures or into bedrock.

3. Metal pipes or rods carrying metal caps, set into the ground and sometimes surrounded by concrete.

4. Stone or concrete posts with designating marks such as a cut, bolt, or disk.

The last two groups are sometimes set with their tops below the surface of the ground and protected by a well with a cover.

Of the type classified under 1, Frederick W. Koop in his report, Precise Leveling in New York City, has this to say:

> * * * The mark was usually made in the form of a T or + of uniform width with V-shaped indentations at least one-fourth inch deep.
>
> An advantage of this type of mark is the ease and rapidity with which it can be prepared and the absence of any probability that it will be defaced or tampered with. In general, this type more nearly meets all requirements of an ideal bench mark than does any other type.

In selecting the types of bench marks and planning their distribution the engineer should consider the following points:

The number and distribution of bench marks should be such as to meet the present and future needs of the city in the most economical manner. Precise bench marks should be available within reasonable distance of any important engineering construction which may be undertaken.

Bench marks should be *accessible*. They are more or less inaccessible if placed high up on a building or in an angle or recess where observations on them are difficult to obtain; or if they are at the bottom of a hole which will not receive the end of an ordinary level rod; or if they are in a well with a locked cover with the key not easily obtainable.

Bench marks should be *permanent*. This means that they must resist destruction by disintegration, and that they must be as secure as possible from any disturbance of position, however slight. Large, expensive, monumental marks are useless unless they are set so they can not settle or lean.

Bench marks must be of such a character that they will be recognized easily and identified with certainty. A cross cut in masonry may be difficult to identify unless the description is very carefully worded. The best assurance that the bench mark on which new work is based has been correctly identified, and that it is in its exact original position, is to connect it with another old bench mark nearby and make sure the difference in elevation is the same as given by the old work. It was the consideration of this point which prompted the setting of bench marks in pairs in the Cincinnati survey. The two marks of a pair were usually set within a block of each other, and when possible they were placed where they would not be subject to the same destroying agencies.

DISTRIBUTION OF BENCH MARKS.

It is recommended that permanent bench marks be placed in pairs as just described and that the pairs be from one-half to 1 mile apart. If possible, the two marks of a pair should be placed where they will not be subject to the same destroying agencies, and yet they should be so near together that one or two set-ups of the level instrument will enable the engineer to check between them.

CONSTRUCTION OF BENCH MARKS.

In the accompanying illustration (Fig. 23) are shown several bench marks as actually constructed. There is apparently a wide diversity of opinion as to what really constitutes a permanent bench mark. The climate, soil, drainage, and environment all contribute to render the problem a local one, and there is opportunity for pioneer experimentation along this line. The bench mark must be set sufficiently deep and so shaped as to be free from the lifting effects of frost; in most parts of the United States 4 feet is considered a sufficient depth. If a pipe is used, it should be treated with a preservative and set in concrete. A pipe set in earth without any treatment to protect it from rust will not prove to be a permanent mark. Where concrete or stone posts are used they should have metal disks cemented into their tops. Bench marks in buildings should be placed where they are reasonably certain to last as long as the building itself, and not be destroyed by ordinary remodeling. In some localities it is necessary to drive piles and construct the bench marks on their tops. It is doubtful, however, if such marks should be considered as permanent over any considerable period of time. In such a locality it is probably advisable to place the marks farther apart and spend more time and money in making them as permanent as possible. In the built-up portion of a city it may sometimes be well to set the mark in a manhole having a cover in order that it may not be in the way of traffic and yet be easily accessible.

The wisdom of setting many semipermanent bench marks at the time of the survey is doubtful. It will be possible to observe on a number of such points while leveling between the permanent marks, and several accurate elevations may be thus secured with little additional cost, but there is no assurance that the marks will remain in position, and it will ordinarily be found most economical to bend all efforts toward establishing the best class of bench marks, distributed as indicated above, and to place intermediate marks only on those structures which happen to be close to the lines run and are likely to be permanent.

The following specifications for a standard concrete bench mark, recently adopted by the U. S. Coast and Geodetic Survey, will be found a satisfactory guide in setting bench marks of this type in a city survey. These specifications cover only the concrete part of the mark, and it is understood that a metal disk is to be set in the top of each post.

STANDARD CONCRETE BENCH MARKS.

Shape.—The bench mark shall be larger at the base than at the surface of the ground. It may take the form of the frustum of a pyramid or cone or be a post with an enlarged base. If it is built in the form of a frustum, there should be a batter of not less than 1 inch to 1 foot. It is believed that a post with an enlarged base will offer just as great resistance to the lifting effect of freezing and thawing as a conical post and will be more economical of concrete. The base should be not less than 4 inches larger in horizontal cross section than the post itself, and the enlarged portion should be at least 6 inches thick vertically. Care should be taken that the outside edges of the base are not rounded, so as to prevent any change in elevation due to a sifting in of sand particles or dirt around the edges during a thawing period. The mark should be molded for the upper foot of its length to lessen the lifting effect of the surface layers of the ground when freezing. The molds may be of sheet iron or wood and should have a slope of at least 1 inch to 1 foot. The sheet iron may be cut to proper size and shape, bent for a lock seam, and carried flat until needed.

FIG. 22.—CONCRETE POST WITH DISK IN TOP, A STANDARD TYPE OF BENCH MARK USED BY THE U. S. COAST AND GEODETIC SURVEY.

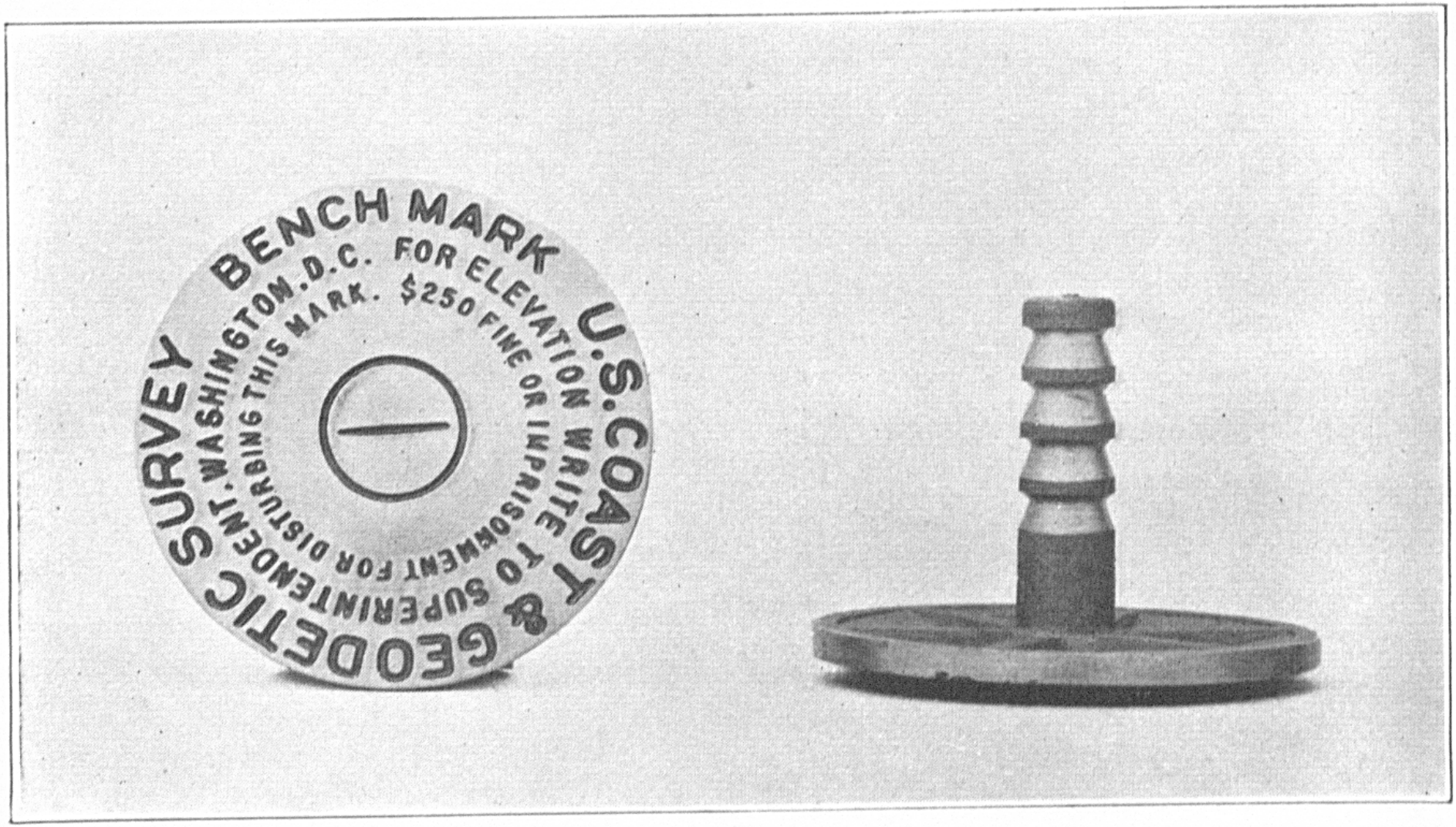


FIG. 24.—DISK BENCH MARK OF THE U. S. COAST AND GEODETIC SURVEY. (A MORE RECENT TYPE HAS A DIFFERENT LEGEND.)

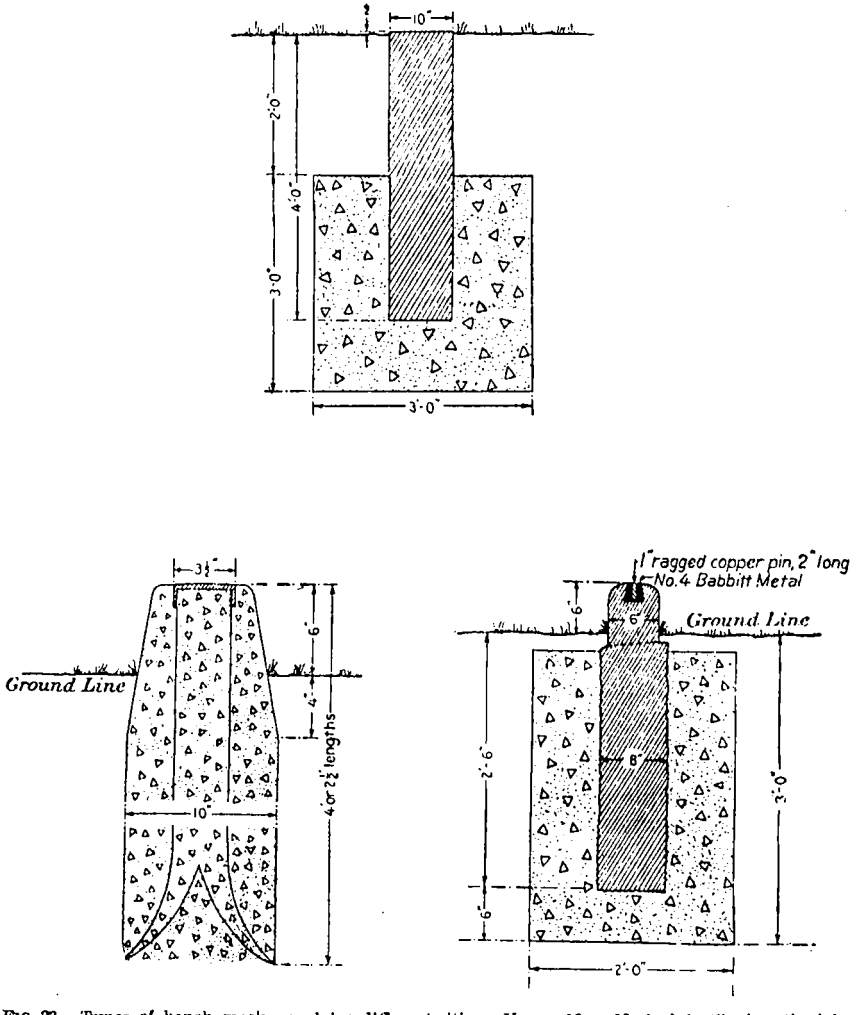


FIG. 23.—Types of bench marks used by different cities. Upper, New York; left, Cincinnati; right, New Orleans.

Extreme care should be taken that the molded portion of the concrete mark has no projecting corners, which would increase the lifting effect of freezing. The concrete beneath the mold should have a sufficiently large area that no portion of the mold extends outside the concrete on which it rests.

Material.—The main considerations in making concrete are to have clean materials, mix them well before adding water, have the mixture not too wet, and tamp well into the form. Each streak of dirt in concrete means a line of cleavage. Where rough aggregate is available, the proportions may very well vary from 1-2-4 to 1-3-5, but the top 12 inches of the mark should be of considerably richer mixture. Where only cement and sand are available, the lower part of the mark should be proportioned 1 part of cement to 3 parts of sand and the upper part should be 1 part of cement to 2 parts of sand.

With a mark of the proper size it will not be necessary to reinforce the concrete with metal rods or wire. To avoid cracking of the concrete, due to rapid drying, it should be covered with paper or cloth and then with earth or other material for a period of at least 48 hours.

Size and depth.—Where lack of transportation does not make the cost prohibitive, the post should be not less than 14 inches in least cross-section dimension, except at the top, where it is shaped by the mold, and that should not be less than 12 inches. The top should extend from 2 to 4 inches above the surface, except where there is traffic, under which conditions the mark should be placed level with the surface of the ground.

The marks should extend below frost depth, except where the frost penetrates to a greater depth than 4 feet. The minimum depth of any bench mark should be 30 inches below the surface and the maximum depth should be 48 inches. It is believed that with a mushroom base a mark extending 48 inches below the surface will not be elevated by the action of frost, even though frost should extend below that depth. The instructions to the chief of the leveling party will, in all cases, specify the depth to which bench marks should be set.

COMPUTATION OF PRECISE LEVELS.

In U. S. Coast and Geodetic Survey Special Publication No. 18, entitled "Fourth General Adjustment of the Precise Level Net of the United States and Resulting Standard Elevations," will be found a complete statement of instruments and methods used for observing and computing the precise level work of the Survey. The following text and tables are taken from that publication, with occasional modification or condensation. The engineer contemplating a precise level survey of his city should have a copy of that publication for the sake of the greater detail given therein. He will also find the report, Precise Leveling in New York City, of value.

Below is a sample determination of the value of C, the collimation error of the instrument, in accordance with section 15 of the instructions. (See p. 54.)

Sample determination of C, 8.20 a. m., August 28, 1900.

Left-hand page.

Number of station.	Thread reading, backsight.	Mean.	Thread interval.
	mm.	*mm.*	*mm.*
	1515		13
A	1528	1528.3	14
	1542		27
	2252		105
B	2357	2357.0	105
	2462		210
		0461.7	419
		2818.7	52
Cor. for curv. and ref.		−0.8	367
		2817.9	

Right-hand page.

Rod.	Thread reading, foresight.	Mean.	Thread interval.
	mm.	*mm.*	*mm.*
	0357		105
W	0162	0461.7	104
	0566		209
	1276		12
W	1288	1288.3	13
	1301		25
		1528.3	
		2816.6	
		2817.9	
		367) −1.3	(−0.004=C

In making the computations involved in the preceding determination of C the distant rod readings are corrected for curvature and refraction (see table on p. 67) and the corrections for the distant rods are combined as indicated. The distance argument for entering the table is obtained from the stadia constant for this particular instrument, for which 287 mm. subtended on the rod corresponds to a length of sight of 100 m. Note that if the transfers of figures across from page to page are made as indicated all subtractions are made right side up. C should not be carried to more than three decimal places.

When the instrument must be adjusted, due to too large a value for C (see par. 15, p. 54), it should be done by raising or lowering one end of the level vial and not by moving the reticle. The adjustment is made as follows: Point to the distant rod with the bubble in the middle of its tube and read. Move the telescope so as to raise the middle line (projected on the rod) by an amount equal to C times the rod interval. While holding the telescope in this position bring the bubble to the middle of its tube by raising or lowering one end of the level vial. If C is negative, the middle line must be lowered on the rod.

The following sample record shows the results of leveling between two temporary bench marks on the line of levels from Klamath Falls to Ontario, Oreg.

Left-hand page. Right-hand page.

Sample record of precise levels.

Date: July 30,1920. Forward—~~Backward~~, (Strike out one word.) From B. M.: 178. To B. M.: 179
Time: 2.20 p. m. Wind: 2. Sun: 6.

No. of station.	Thread reading, backsight.	Mean.	Thread interval.	Sum of intervals.	Rod and temp.	Thread reading, foresight.	Mean.	Thread interval.	Sum of intervals.
	mm.	*mm.*	*mm.*	*mm.*		*mm.*	*mm.*	*mm.*	*mm.*
	1274		126		214	1743		133	
45	1148	1147.7	127		38°	1610	1609.7	134	
	1021		253	253		1476		267	267
	1412		159			1409		160	
46	1253	1252.7	160			1249	1248.3	162	
	1093		319	572		1087		322	589
	1704		144			1718		142	
47	1560	1560.3	143			1576	1575.3	144	
	1417		287	859		1432		286	875
	1640		157			1573		150	
48	1483	1483.3	156			1423	1423.3	149	
	1327		313	1172		1274		299	1174
	2280		137			1211		138	
49	2143	2143.7	135			1073	1073.0	138	
	2008		272	1444		935		276	1450
	3)22763					3)20789	6929.6		
	7587.7	7587.7				6929.7			
		6929.6							
		+658.1							

The time of starting work on the section is shown at the top of the page. The explanation of the numbers used after the words "Sun" and "Wind" is given at the bottom of the form "Computation of precise levels" (p. 64). As a check on the taking out and addition of the means the sum of the thread readings divided by three is used, as shown above. The difference in elevation obtained from the means is used in the following computation:

Sample computation of precise levels.

[Line: Klamath Falls to Ontario, Oreg. Rods: 214, 215. Level No.: 7. Observer: F. W. Hough. Year: 1920.]

Date.	Designation of section.	Forward or backward.	Time.	Sun.*	Wind.†	Temp. of rods.	Length of section.	Distance from B. M.	Rod readings.		3‡ times difference of elevation each section.	Divergence 3‡ (B—F).		3‡ times mean difference of elevation.	Designation of bench mark.	Elevation of bench mark.	Corrections.		
									3‡ Σ B.	3‡ Σ F.		Partial.	Total accumulated.				Index error.	Length of rod.	Temp. of rod.
July.			h. m.				km.	km.	m.	m.	m.	mm.	mm.	m.		m.	mm.	mm.	mm.
30	175–176	F	1 00	6	1	42	0.7	247.4	6.8387	6.3757	+0.4630	+1.3	−34.1	+0.4636	176	1297.9966	.....	−0.1	+0.2
30		B	11 30	1	1	41			8.0387	8.5030	−0.4643						.....	+0.1	−0.2
30	176–177	F	1 20	6	1	42	1.0	248.4	12.0483	10.3940	+1.6543	Rejected.		+1.6429	177	1299.6395	.....	−0.3	+0.8
30		B	10 50	1	1	37			9.0457	10.6867	−1.6410						.....	+0.3	−0.7
30		F	4 40	6	1	37			8.9787	7.3357	+1.6430	−0.2	−34.3				.....	−0.3	+0.7
30		B	4 10	3	1	38			9.4253	11.0700	−1.6447						.....	+0.3	−0.7
30	177–178	F	1 50	6	1	40	1.1	249.5	7.3520	7.8177	−0.4657	+4.0	−30.3	−0.4637	178	1299.1758	.....	+0.1	−0.2
30		B	10 30	8	1	39			7.3447	6.8830	+0.4617						.....	−0.1	+0.2
30	178–179	F	2 20	6	2	38	1.0	250.5	7.5877	6.9297	+0.6581	−1.8	−32.1	+0.6572	179	1299.8330	.....	−0.1	+0.3
30		B	10 10	8	0	36			6.4687	7.1250	−0.6563						.....	+0.1	−0.3

* Abbreviations for sun: 0=cloudy. With sunshine, show direction of progress in relation to sun by numbers: 1 is northeast, 2 is east, 3 is southeast, 4 is south, 5 is southwest, 6 is west, 7 is northwest, and 8 is north.

† Abbreviations, strength of wind: 0=calm; 1=moderate; 2=strong.

‡ Strike out 3 when mean rod readings are used.

The above computation, except for the three columns of corrections is called the *field* computation and gives only approximate results which determine the acceptability of the observations. It should be kept up very closely behind the observing. In making the office or final computation the various necessary corrections are applied to the field computation and the resulting differences of elevation assembled on the "Abstract of precise levels" (p. 66).

In the foregoing example the corrections for curvature and refraction were considered negligible. They are practically zero when the instructions (secs. 13 and 14, p. 54) are complied with, which require that the lengths of backsights and foresights be kept as nearly equal as possible, and that the difference of the sums of the rod intervals be kept within prescribed limits. An examination of the recent leveling of the U. S. Coast and Geodetic Survey shows that in no line would this correction have accumulated to more than a small fraction of a millimeter. Where it is found that a correction for curvature and refraction is required, the correction may be applied very quickly by the use of the tables on page 70 and by making a rapid inspection of the record book. It is important to note that this is, in the main, a correction for curvature, the uncertain refraction being upon an average about one-eighth as great as the curvature. No space is provided in the computation form for the level correction, since its effect is always kept practically zero through the equalizing of the lengths of the sights. It is equal to the constant C (as defined in section 15 of the instructions) times the difference between the sum of the rod intervals for the backsights and the sum for the foresights.

The index error takes account of the fact that the zero of the graduation and the foot of the rod may not be exactly coincident. When the index errors of the two rods of a pair are the same, they need not be considered except when a tape is used on a bench mark which is not accessible with a rod. When there is an appreciable difference in the values of the index errors of the two rods forming a pair, this difference is to be applied to the resulting difference in elevation of the two ends of a section only when the rear rod of the first station and the front rod of the last station are different. When the same rod happens to be used on the two bench marks forming the terminals of a section, the effect of the index errors is zero.

The correction for length of rod is to take account of the error of graduation of the rod at the standard temperature which for the rods of the U. S. Coast and Geodetic Survey is zero degrees centigrade. The temperature correction then takes account of the expansion of the rod for a change of temperature from zero degrees centigrade (or the standard temperature) to the temperature of observation and is computed from the known coefficient of expansion of the rods. (See table on p. 71.)

Sample abstract of precise levels.

[State: Oregon. Instrument: 7. Rods: 214, 215. Observer: F. W. Hough. Computer: H. G. Avers.]

Date.	From B. M. to B. M.	Distance.	Difference of elevation.			Discrepancy.		Designation of B. M.	Distance from B. M. T4.	Observed elevation above mean sea level.	Orthometric correction.	Observed orthometric elevation above mean sea level.
			Forward line.	Backward line	Mean.	Partial.	Total accumulated.					
1920.		*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*		*km.*	*m.*	*mm.*	*m.*
July 30–30	175–176	0.7	+0.4631	−0.4644	+0.4638	+1.3	−37.6	176	247.4	1298.0147	.....	.....
30–30	176–177	1.0	* +1.6548	−1.6414	+1.6433	−0.2	−37.8	177	248.4	1299.6580	.....	.....
30–30	.....	.....	+1.6434	−1.6451	.....	.....	.....	.....	.....	.....	.....	.....
30–30	177–178	1.1	−0.4658	+0.4618	−0.4638	+4.0	−33.8	178	249.5	1299.1942	.....	.....
30–30	178–179	1.0	+0.6583	−0.6565	+0.6574	−1.8	−35.6	179	250.5	1299.8516	.....	.....

* Rejected.

It will be noted that in the preceding table there is a column for orthometric correction. This is a correction to take account of the nonparallelism of level surfaces at different elevations due to the fact that the earth is a revolving spheroid and that the intensity of gravity increases from the Equator toward either pole. This correction is most important on a north-and-south line of high average elevation. It need not be considered for city work except in rare cases. It is fully explained on pages 49 to 56 of U. S. Coast and Geodetic Survey Special Publication No. 18.

CORRECTION TABLES.

The following tables are for use in making the computations of precise levels:

The first table gives the total correction for curvature and refraction and is used in computing the value of C (see p. 62) in making river crossings, and, in general, wherever the total correction is required. In computing this table the refraction was assumed to be equal to one-eighth the curvature.

Total correction for curvature and refraction.

Distance.	Correction to rod reading.	Distance.	Correction to rod reading.
m. m.	*mm.*	*m. m.*	*mm.*
0 to 27	0.0	151 to 160	−1.8
28 to 47	−0.1	161 to 170	−2.1
48 to 60	−0.2	171 to 180	−2.3
61 to 72	−0.3	181 to 190	−2.6
73 to 81	−0.4	191 to 200	−2.8
82 to 90	−0.5	201 to 210	−3.0
91 to 98	−0.6	211 to 220	−3.3
99 to 105	−0.7	221 to 230	−3.7
106 to 112	−0.8	231 to 240	−4.0
113 to 118	−0.9	241 to 250	−4.3
119 to 124	−1.0	251 to 260	−4.7
125 to 130	−1.1	261 to 270	−5.0
131 to 136	−1.2	271 to 280	−5.4
137 to 141	−1.3	281 to 290	−5.8
142 to 146	−1.4	291 to 300	−6.2
147 to 150	−1.5		

The next table gives the differential correction for curvature and refraction. It is computed on the assumption that the refraction is one-eighth the curvature, and that a stadia interval of 300 mm. subtended on the rod corresponds to a length of sight of 100 m. The sign of this correction is positive when the foresight is the longer.

Differential correction for curvature and refraction.

Mean rod interval (in millimeters).	Difference of rod intervals in millimeters.																											
	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58
	mm	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*
10	.0	.0	.0	.0	.0																								
20	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0																			
30	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0														
40	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0									
50	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0				
60	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1
70	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1
80	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1
90	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
100	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
110	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
120	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
130	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
140	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
150	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
160	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
170	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2
180	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2
190	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2
200	.0	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2
210	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2
220	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2
230	.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2
240	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2
250	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2

260	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2
270	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2
280	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2
290	.0	.0	.0	.0	0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.3
300	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.3	.3
310	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.3	.3	.3
320	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.3	.3	.3	.3
330	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.3	.3	.3	.3	.3
340	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.3	.3	.3	.3	.3
350	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.3	.3	.3	.3	.3	.3
360	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2	.3	.3	.3	.3	.3	.3	.3
400	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.3	.3	.3	.3	.3	.3	.3	.3	.4
440	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.3	.3	.3	.3	.3	.3	.3	.3	.4	.4	.4
480	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.3	.3	.3	.3	.3	.3	.4	.4	.4	.4	.4	.4
520	.0	.0	.1	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.2	.3	.3	.3	.3	.3	.3	.3	.4	.4	.4	.4	.4	.4	.4

The differential correction for curvature and refraction may also be obtained from the following table, which may sometimes be found more convenient to use than the table above. It gives for each difference of rod intervals the lower limiting values of the mean rod interval for which the correction is 0.1, 0.2, 0.3, or 0.4 mm. to the nearest tenth of a millimeter. This table is computed on the same assumptions as the preceding table, and the rule for the sign of the correction is the same, namely, positive when the foresight is the longer.

Differential correction for curvature and refraction—Limiting values of the mean rod interval.

Difference of rod intervals, in millimeters.	Correction.				Difference of rod intervals, in millimeters.	Correction.			
	0.1mm.	0.2mm.	0.3mm.	0.4mm.		0.1mm.	0.2mm.	0.3mm.	0.4mm.
	mm.	*mm.*	*mm.*	*mm.*		*mm.*	*mm.*	*mm.*	*mm.*
8	409.7	.....	.....	.....	36	91.0	273.2	455.2	.....
9	364.2	.....	.....	.....	37	88.6	265.8	442.9	.....
10	327.8	.....	.....	.....	38	86.3	258.8	431.3	.....
					39	84.0	252.1	420.2	.....
11	298.0	.....	.....	.....	40	81.9	245.8	409.7	.....
12	273.2	.....	.....	.....					
13	252.1	.....	.....	.....	41	79.9	239.8	399.7	.....
14	234.1	.....	.....	.....	42	78.0	234.1	390.2	.....
15	218.5	.....	.....	.....	43	76.2	228.7	381.1	.....
					44	74.5	223.5	372.5	.....
16	204.9	.....	.....	.....	45	72.8	218.5	364.2	.....
17	192.8	.....	.....	.....					
18	182.1	.....	.....	.....	46	71.3	213.8	356.3	.....
19	172.5	.....	.....	.....	47	69.7	209.2	348.7	.....
20	163.9	.....	.....	.....	48	68.3	204.9	341.4	.....
					49	66.9	200.7	334.5	.....
21	156.1	.....	.....	.....	50	65.6	196.7	327.8	458.9
22	149.0	447.0	.....	.....					
23	142.5	427.5	.....	.....	51	64.3	192.8	321.4	449.9
24	136.6	407.9	.....	.....	52	63.0	189.1	315.2	441.2
25	131.1	393.3	.....	.....	53	61.8	185.5	309.2	432.9
					54	60.7	182.1	303.5	424.9
26	126.1	378.2	.....	.....	55	59.6	178.8	298.0	417.2
27	121.4	364.2	.....	.....					
28	117.1	351.2	.....	.....	56	58.5	175.6	292.7	409.7
29	113.0	339.1	.....	.....	57	57.5	172.5	287.5	402.5
30	109.3	327.8	.....	.....	58	56.5	169.5	282.6	395.6
					59	55.6	166.7	277.8	388.9
31	105.7	317.2	.....	.....	60	54.6	163.9	273.2	382.4
32	102.4	307.3	.....	.....					
33	99.3	298.0	.....	.....					
34	96.4	289.2	.....	.....					
35	93.7	281.0	.....	.....					

The table of temperature corrections below is computed on the assumption that the coefficient of expansion of the rod is 4 parts in 1,000,000 per degree centigrade, which is the coefficient of the old wooden rods formerly used by this bureau. If a rod with a different coefficient of expansion is used it will be found to expedite the computations to prepare a similar table for the actual temperature coefficient of the rod used.[10] The sign of this correction is the same as the measured difference of elevation when the rod temperature is above centigrade zero and of the opposite sign when below zero.

[10] The coefficient of expansion of the invar-faced rods now in use by this bureau varies from 0.0000010 to 0.0000004.

Correction for temperature.

Temperature of rod, degrees centigrade.	Difference of elevation in meters.													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	mm.	*mm.*	*mm.*	*mm.*	*mm.*	*mm.*	*mm.*	*mm.*	*mm.*	*mm.*	*mm.*	*mm.*	*mm.*	*mm.*
1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1
2	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1
3	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.1	.2	.2
4	.0	.0	.0	.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2
5	.0	.0	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.3	.3
6	.0	.0	.1	.1	.1	.1	.2	.2	.2	.2	.3	.3	.3	.3
7	.0	.1	.1	.1	.1	.2	.2	.2	.2	.3	.3	.3	.4	.4
8	.0	.1	.1	.1	.2	.2	.2	.3	.3	.3	.4	.4	.4	.4
9	.0	.1	.1	.1	.2	.2	.2	.3	.3	.4	.4	.4	.5	.5
10	.0	.1	.1	.2	.2	.2	.3	.3	.4	.4	.4	.5	.5	.6
11	.0	.1	.1	.2	.2	.3	.3	.4	.4	.4	.5	.5	.6	.6
12	.0	.1	.1	.2	.2	.3	.3	.4	.4	.5	.5	.6	.6	.7
13	.0	.1	.2	.2	.3	.3	.4	.4	.5	.5	.6	.6	.7	.7
14	.1	.1	.2	.2	.3	.3	.4	.4	.5	.6	.6	.7	.7	.8
15	.1	.1	.2	.2	.3	.4	.4	.5	.5	.6	.7	.7	.8	.8
16	.1	.1	.2	.3	.3	.4	.4	.5	.6	.6	.7	.8	.8	.9
17	.1	.1	.2	.3	.3	.4	.5	.5	.6	.7	.8	.8	.9	.9
18	.1	.1	.2	.3	.4	.4	.5	.6	.6	.7	.8	.9	.9	1.0
19	.1	.2	.2	.3	.4	.5	.5	.6	.7	.8	.8	.9	1.0	1.1
20	.1	.2	.2	.3	.4	.5	.6	.6	.7	.8	.9	1.0	1.0	1.1
21	.1	.2	.2	.3	.4	.5	.6	.7	.8	.8	.9	1.0	1.1	1.2
22	.1	.2	.3	.4	.4	.5	.6	.7	.8	.9	1.0	1.1	1.1	1.2
23	.1	.2	.3	.4	.5	.6	.6	.7	.8	.9	1.0	1.1	1.2	1.3
24	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.2	1.3
25	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3	1.4
26	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3	1.5
27	.1	.2	.3	.4	.5	.6	.8	.9	1.0	1.1	1.2	1.3	1.4	1.5
28	.1	.2	.3	.4	.6	.7	.8	.9	1.0	1.1	1.2	1.3	1.4	1.6
29	.1	.2	.4	.5	.6	.7	.8	.9	1.0	1.2	1.3	1.4	1.5	1.6
30	.1	.2	.4	.5	.6	.7	.8	1.0	1.1	1.2	1.3	1.4	1.6	1.7
31	.1	.2	.4	.5	.6	.7	.9	1.0	1.1	1.2	1.4	1.5	1.6	1.7
32	.1	.3	.4	.5	.6	.8	.9	1.0	1.2	1.3	1.4	1.5	1.7	1.8
33	.1	.3	.4	.5	.7	.8	.9	1.1	1.2	1.3	1.4	1.6	1.7	1.8
34	.1	.3	.4	.5	.7	.8	1.0	1.1	1.2	1.4	1.5	1.6	1.8	1.9
35	.1	.3	.4	.6	.7	.8	1.0	1.1	1.3	1.4	1.5	1.7	1.8	2.0

ADJUSTMENT OF THE LEVEL NET.

Because of the limited lengths of the loops of a city precise level survey the corrections known as the orthometric and the dynamic corrections, which must be applied to the large loops of a national survey in order to obtain fair closures, may be ignored. (See p. 67.) However, when the precise level survey of a city consists of a net of interrelated closed circuits, something more than an arbitrary distribution of the closing error around each loop will be required, since sections of each loop will enter into other loops where the errors of closure may be different not only in size but sometimes also in sign. As in triangulation, and in fact in the reduction of most quantitative observations, the most probable values for the quantities sought will be obtained from a least-squares reduction of the observations. The various circuit closures will thereby be eliminated and all results harmonized. The method of adjustment for trigonometrical leveling developed in Special Publication No. 28 of this bureau may readily be adapted for making this computation.

Publication of results.—It is very desirable to make the final elevations available to the engineers who are to use them in the form of a printed book of pocket size. The descriptions of the bench marks with elevations appended can be so arranged therein, according to names or numbers, as to be self-indexing, while their general distribution can be shown on an attached skeleton map of the city. To fully realize the benefit of the level survey the use of its results should be required in the planning and construction of all public works within the city. This may possibly be done by an ordinance adopting as the legal datum of the city that defined by the elevations of the various bench marks determined by the survey.

APPENDIXES.

Appendix I.—TOPOGRAPHIC SURVEY.

While somewhat beyond the contemplated scope of this publication, it seems well to include a few paragraphs on the city topographic survey, since it is one of the important detail surveys most benefited by geodetic control. That an accurate map is essential for satisfactory city planning is evidenced by the continual insistence of those interested in city planning problems. In a report on a park and street plan for the city of Pittsburgh, F. L. Olmsted [20] says:

> But a thing of greater consequence than any one of these specific improvements, a thing of vital import to every taxpaying citizen of the present and future city, is the making of comprehensive and accurate topographical maps. It is only on the basis of such maps that all municipal engineering, and indeed much other work directly managed by the city, can be planned and carried out with proper economy and efficiency. It is only on the basis of such maps that improvements in the city—details of city replanning—can be most economically determined. And in the outlying districts, where the future city is being built, such maps are absolutely essential to an intelligent planning or control which will avoid the heavy penalties that follow haphazard city growth, especially in such a hilly region. * * *. As a means of accomplishing these ends, an accurate geodetic triangulation of the district, supplemented by the necessary precise traverse work and precise leveling, all fully checked and compensated for errors (is necessary).

Where a city is covered by a net of triangulation having an accuracy of 1 part in 25,000, an accuracy easily attained by geodetic methods, there will be no appreciable accumulation of error in the results. It may seem possible to secure this accuracy by using ordinary tape and transit methods, and indeed the accidental errors may be kept down to something of this order of accuracy, and yet there may be undiscovered systematic errors far in excess of the allowable limit.

Where the scale of the topographic map is 200 feet on the ground equals 1 inch on the map, a scale which inquiry has developed to be most desirable in general city planning work, the smallest scale division which the topographer can easily handle is one-eightieth inch, equivalent to 2½ feet on the ground. On a 3-mile line this represents approximately 1 part in 6,000. The required accuracy for the control will, therefore, insure that on practically any length of line of the city geodetic control the error will be smaller than can be plotted on this scale.

INSTRUMENTS AND METHODS.

There are two general methods followed in topographic mapping: One where the map is practically made in its entirety in the field; the other where the actual construction of the map takes place in the office from notes and data gathered in notebook form in the field. The first may be called the plane-table method, while the second is the ordinary transit and stadia (or tape) method.

Plane-table method.[21]—This is the method in general use in the big mapping programs of the Federal survey bureaus and was the method used in the survey of Cincinnati. (See p. 74.) The plane-table method of mapping has two distinct advantages: The first is that the topographer maps each feature with it in plain view and thereby reduces the chances of error; and the second is the economical production resulting through the ever-increasing skill of the topographer, which makes it possible to reduce the frequency of rod stations to a certain extent without any corresponding decrease in accuracy.

Another advantage of the plane-table method is that there will be no unexpected gaps in a map made by this method. The topographer has always in front of him the area already mapped and the area yet to be surveyed. He may traverse around a thicket and go into it from each side in turn and almost automatically secure a complete map with no overlaps or gaps.

Transit and stadia method.—This method is used to advantage where a field season is short or the weather unfavorable for drafting in the field, as a mass of notes may be taken and later reduced and plotted in the shelter of the office. The objection to this method is that an error in reading the rod or circle in the field may not be discovered until the map has been completed and actually used in the field, and some-

[20] Pittsburgh—Main Thoroughfares and the Down-Town District. A report by Frederick Law Olmsted, 1910.

[21] This method is fully described in Special Publication No. 85, U. S. Coast and Geodetic Survey, entitled "A Plane Table Manual."

times not even then. This is a most important consideration where accuracy is a prime requisite, as in city mapping. As the map is drawn by one who perhaps never even saw the terrain being mapped, a much larger number of points will be required to secure the proper delineation of the surface of the ground with the transit and stadia than with the plane-table method. Even with the latter method one observer may use only half as many rod points as another and secure the same accuracy because of keener perception or greater capacity for visualization, combined perhaps with greater experience. There can be no cutting down in the number of rod points if the map is compiled in the office from notes taken in the field.

With either the plane-table or transit and stadia method the contours may be located by actually running them out on the ground, using some form of level instrument. The level rod is placed in successive positions on a contour by having the target set at proper height and moving the rod over the ground until the target is bisected by the thread of the diaphragm in the level. The topographer then sights the rod and locates its position. This is an unnecessarily laborious and costly method of contouring. A better and more generally accepted practice is to take rod readings at each break in grade of the ground, plot the elevations of the points thus determined, and interpolate the contours between them.

The advantages of the two primary methods of topographic mapping are sometimes combined by setting up a portable drawing table in the field beside the transit. The observations are then made with the transit and given directly to the draftsman, who constructs the map in the usual office manner. Sometimes a regular plane table is used in combination with the transit. The distances and elevations are then read with the transit and the directions are obtained with the alidade on the plane table. Both of these methods eliminate the principal objection to the transit method and both have been used with considerable success.

A modification of the transit method which is sometimes used may be designated as the checkerboard method. The area to be mapped is divided into squares which are staked out on the ground. Levels are run over these stakes to determine the positions of the contours, and from the stakes as control points the main cultural features are located and plotted. Against this method it may be said that topographic features are never in such regular arrangement and points so determined will seldom be placed to best advantage for developing the relief of the terrain.

When a method for topographic surveying is used which involves field mapping, the topographer is given a sheet on which are plotted to the required scale all control points determined in the area covered by the sheet. These points are plotted by coordinates and checked by scaled distances between them. In order that the required scale may be adhered to as closely as possible, paper distortion is reduced to a minimum by using well-seasoned paper of a quality that is not sensitive to hygrometric changes. Since paper will change in different ratios along its two dimensions, a plane-table sheet which has been found very satisfactory is made by mounting two pieces of drawing paper with the grain at right angles and with muslin between the sheets. This gives a reduced ratio of shrinkage with the ratio the same in both directions and therefore more easily allowed for.

The topographer is provided with the necessary descriptions for the recovery of the control points on the ground and with elevations of a sufficient number of bench marks to enable him to carry his height of instrument without exceeding the specified limits of error. A convenient form for these data is a tracing of the plane-table sheet with the description and elevation of each control point written on it. Blue prints may be made of this tracing for field use.

TOPOGRAPHIC SURVEY OF CINCINNATI.

The topographic survey of Cincinnati was executed under contract at so much per square mile. The specifications under which that work was done are given below and may prove interesting and valuable to others contemplating similar work. Although the scale of the Cincinnati map is 400 feet to 1 inch the consensus of opinion among a number of engineers interested in the subject is that 200 feet to 1 inch is a better scale. (See p. 14.)

Specifications for topographic survey of Cincinnati, Ohio. [22]

1. Survey to be on scale of 1 inch=400 feet. The contour interval to be as follows:
   2½ feet on slopes of 6° or less.
   5 feet on slopes of more than 6° and less than 12°.
   10 feet on slopes of more than 12° provided that change from one contour interval to another is not to be made in cases where revision would be made back to first interval after going a short distance.

[22] From Progress Report on a Plan for Sewerage for the City of Cincinnati, p. 84.

2. There are to be shown on the map, in addition to all contours, all streets, roads, alleys, railroads, street railways and traction lines, bench marks and topographic stations, public buildings and monuments, parks, and prominent buildings.

3. Said work to be done in pencil on sheets 24 by 31 inches, each sheet to have mapped on it an area 8,000 by 11,000 feet (20 by 27½ inches), thus leaving a margin of about 2 inches.

4. The criterions for acceptable work shall be as follows:

(a) Each contour must not be in error by more than one-half a contour interval.

(b) All horizontal distances between well-defined points must scale correct to the smallest distance which it is possible to plot on the sheet. This distance is one-eightieth inch, representing 5 feet on the ground.

(c) The allowable error in azimuth in closing on a fixed point shall be the same as given above in (b); that is, one-eightieth inch or 5 feet.

(d) Errors beyond the limits above prescribed will be allowed to the number of 10 per cent (10%) of the total number of points tested. In accepted sheets all errors found beyond these prescribed limits shall be corrected.

5. The accuracy of the work may be tested at any time by the chief engineer of the city of Cincinnati or by his duly authorized agents. The chief engineer or his duly authorized agents shall be free at any time to relieve the topographer at the plane table and personally test work already done; he may also make the observations when closing on a fixed point. Another check which may be applied is the running of a random traverse and level line across the section covered by the sheet, plotting same, and comparing with map. The matching of sheets where they join is also an acceptable test. The right is reserved by the party of the first part to apply any and all tests, subject to limits of accuracy herein presented, whether mentioned or not, whenever and wherever desired.

6. (Relates to delivery of sheets.)

7. Said city of Cincinnati is to supply said plane-table sheets with all control, both horizontal and vertical, plotted thereon. The horizontal control to be provided shall consist of topographic points about 800 feet apart located by closed traverses. The average width of such traverse shall be about 1 mile. The vertical control shall consist of elevations on these points.

As each sheet was finished it was delivered to the city. A profile was then run across some selected portion of it and its acceptability thus determined. It was believed that the inking in of contours and drainage on each sheet should be done directly under the supervision of the topographer who had made the sheet, so this was included in the contract, but the inking in of the culture and the placing of names upon the sheets was done in the office of the principal assistant city engineer having charge of the survey. While doing this a careful study of each sheet was made, and all doubtful points as to the interpretation of pencil lines and all questioned details were indicated on a piece of tracing cloth placed over the sheet. The tracing was then carried into the field and these doubtful points were settled on the ground. This proof reading of the sheet in the field was splendid assurance of its accuracy.

PHOTOGRAPHIC MAPPING.

Because of the large amount of interference on the part of buildings it is doubtful if the methods for making maps from photographs at ground stations can ever find a place in city surveying. On the contrary, the method of map making from photographs taken from aeroplanes is so promising and is as yet in so formative a state as to be worthy of much investigation. The possibilities of making precise maps of a city from aeroplane photographs are very great and great progress is being made in this line at the present time. Aeroplane photographs show the locations of those natural and artificial objects which appear most prominently from the sky and which are, therefore, very desirable points of control. Such points can be most accurately and easily determined by geodetic methods. It is believed that many maps of the future, of the city as well as of the open country, will be made from aeroplane photographs adjusted to geodetic control.

Appendix II.—COMPLETED GEODETIC CONTROL IN THE UNITED STATES.

The map facing page 76 shows the location and extent of the completed geodetic control in the United States. The hachured areas in black represent the portion of the United States that has been covered by triangulation and traverse which have been computed rigidly on the North American Datum and issued in published form. Following the numbers below, corresponding to the numbers shown on the map, are the titles of the publications that contain the published results. For example, the results of the published triangulation in the State of Oregon are contained in four different publications, as is shown by the four different hachured areas in black. These areas are numbered 19, 13, 27, and 10. Referring to these numbers below, it will be seen that the results are contained in Special Publications Nos. 31, 13, 74, and 84.

1. Appendix 8, Report for 1888.
2. Appendix 8, Report for 1893.
3. Appendix 6, Report for 1901.
4. Appendix EEE, Annual Report of the Chief of Engineers, 1902. (This publication is obtainable only from the Chief of Engineers, U. S. Army, Washington, D. C.)
5. Special Publication No. 88.
6. Appendix 9, Report for 1904.
7. Special Publication No. 86.
8. Appendix 5, Report for 1910.
9. Appendix 4, Report for 1911.
10. Special Publication No. 84.
11. Appendix 6, Report for 1911
12. Special Publication No. 11.
13. Special Publication No. 13.
14. Special Publication No. 16.
15. Special Publication No. 17.
16. Special Publication No. 19.
17. Special Publication No. 24.
18. Special Publication No. 30.
19. Special Publication No. 31.
20. Report on the Triangulation of Greater New York. (This publication is obtainable only from the city engineer, New York City.)
21. Report on Plan of Sewerage for the City of Cincinnati. (This publication is obtainable only from the city engineer, Cincinnati, Ohio.)
22. Special Publications Nos. 43 and 45.
23. Special Publication No. 46.
24. Special Publication No. 54.
25. Special Publication No. 62.
26. Special Publication No. 70.
27. Special Publication No. 74.
28. Special Publication No. 76.
29. Special Publication No. 78.
30. Special Publication No. 79.

The hachured areas in red on the map represent the portion of the United States that has been covered by triangulation and traverse which have been rigidly computed on the North American Datum, but which have not yet been issued in published form.

The heavy black lines on the map indicate the published precise level net of the United States. The red lines indicate the portion of the precise level net of the United States not yet issued in published form.

It is impracticable to designate on the map the publications containing the results of precise leveling by this bureau. However, the map does indicate where lines of levels have been run, and if the published results are desired for any particular locality, and this bureau is so advised, the proper publications will be selected and supplied.

Appendix III.—BIBLIOGRAPHY.

In the preceding pages a number of special publications of the U. S. Coast and Geodetic Survey have been referred to quite frequently. The engineer about to undertake a geodetic survey of a city will do well to obtain these various manuals and books of reference. As regards the other books of reference named below, the author has included only those which he has found suited to his needs. There are, of course, other standard works along the same lines, some of which may be fully suited to his needs and more familiar to the engineer than the ones named.

In the following list a short digest of each book is given to emphasize some of its special features.

U. S. Coast and Geodetic Survey Special Publication No. 20,[22] General Instructions for the Field Work of the U. S. Coast and Geodetic Survey. The general instructions given in that volume are for less precise work than is considered in the present volume. There are given, however, certain tables which will be of considerable value to the engineer, and also notes on the marking of stations which he may find suggestive of marks to serve his particular purpose.

U. S. Coast and Geodetic Survey Special Publication No. 58, General Instructions for Precise and Secondary Traverse. In addition to the instructions the volume contains certain tables which are needed in the computation of traverse.

U. S. Coast and Geodetic Survey Special Publication No. 93, Instructions for Reconnaissance and Signal Building. It contains the instructions for reconnaissance for triangulation across wide areas and for the construction of signals and the marking of stations. The instructions given in the present volume supplement those given in that publication.

Logarithms of Sines and Tangents for Every Second, by Robert Shortrede, F. R. A. S., revised edition, published by Charles and Edwin Layton, London. This table of logarithms to seven places, or a similar one, is essential in computing precise triangulation.

Logarithmic Tables of Numbers and Trigonometrical Functions, by Baron Von Vega, published by Weidmanns, Berlin. These tables will be found desirable for obtaining logarithms to seven places of numbers up to 100,000.

U. S. Coast and Geodetic Survey Special Publication No. 8, Formulæ and Tables for the Computation of Geodetic Positions. It contains the required tables for computing the spherical excess of triangles, and for the computation of latitudes, longitudes, and azimuths.

U. S. Coast and Geodetic Survey Special Publication No. 71, Relation Between Plane Rectangular Coordinates and Geographic Positions. It contains tables and methods of computation for reducing geographic positions to plane rectangular coordinates and vice versa.

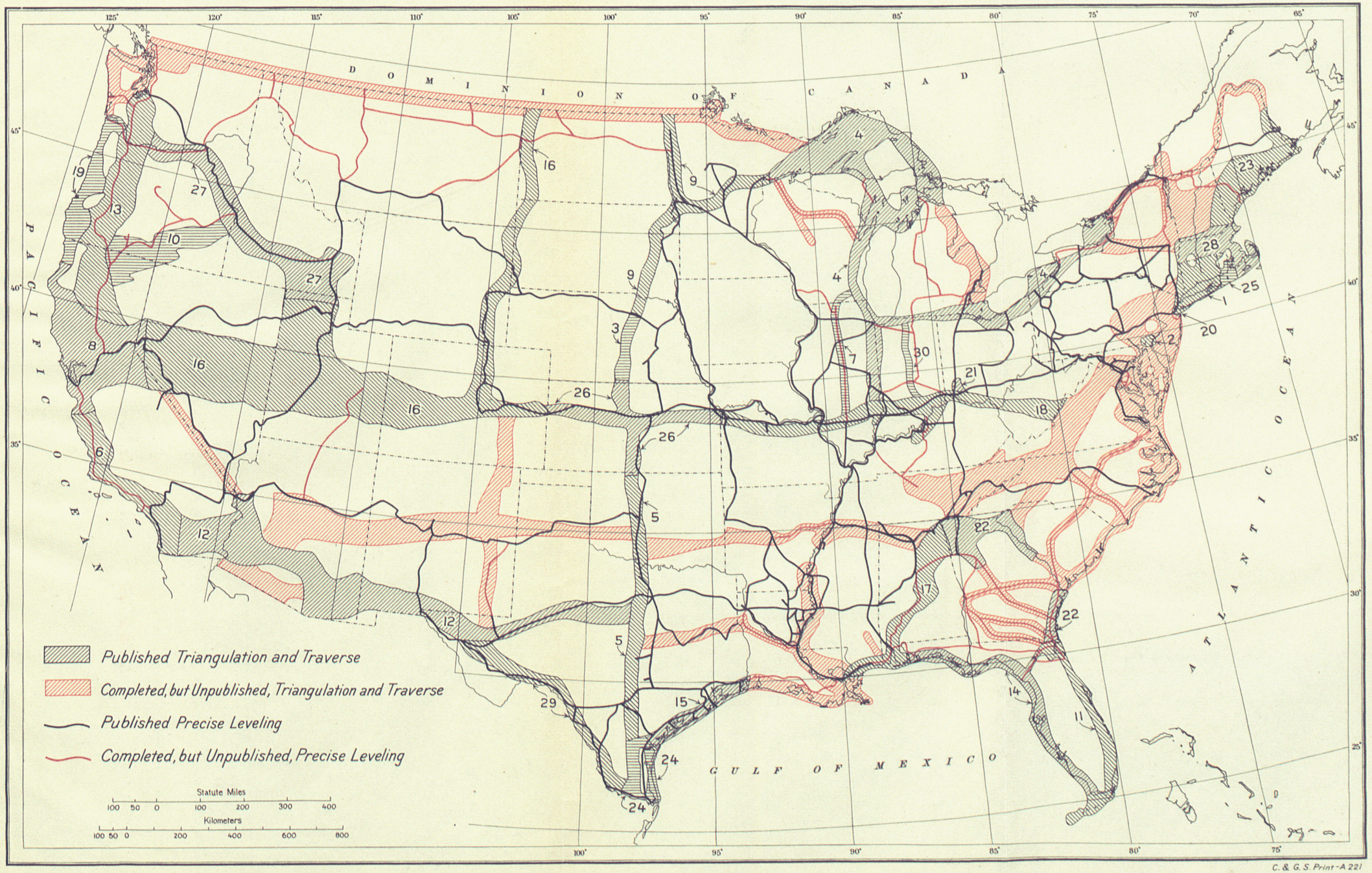
U. S. Coast and Geodetic Survey Special Publication No. 28, Application of the Theory of Least Squares to the Adjustment of Triangulation. It is a working manual for the complete adjustment of triangulation. Even with this manual it is not advisable to attempt the computation of even a simple triangulation without some special instruction and some form of machine for handling the heavy arithmetical operations which enter into every adjustment.

U. S. Coast and Geodetic Survey Special Publication No. 79, Precise Traverse and Triangulation in Indiana. It shows the high degree of accuracy attained in a traverse extending nearly the length of the State of Indiana, and gives detailed descriptions of the methods used in the field and an analysis of the computation of results.

[22] This, as well as the other publications of the U. S. Coast and Geodetic Survey referred to in this volume, may be secured from the Superintendent of Documents, Washington, D. C., upon the payment of a nominal sum.

UNITED STATES COAST AND GEODETIC SURVEY

Graphic index sheet showing: (1) Areas in the United States covered by published and unpublished triangulation and traverse which have been computed rigidly on the North American datum. (2) Net of published and unpublished precise leveling in the United States. October, 1922.



C. & G. S. Print-A 221

Fig. 25.

U. S. Coast and Geodetic Survey Special Publication No. 18, Fourth General Adjustment of the Precise Level Net of the United States and the Resulting Standard Elevations. It contains more detailed description of methods of observing and computing levels than are given in the present volume.

Dr. A. L. Crelle's Calculating Tables. B. Westerman & Co., New York. It gives the products of every two numbers from one to one thousand.

Traverse Tables, by R. L. Gurden. D. Van Nostrand Co., New York. It contains latitudes and departures to four places of decimals for every minute of angle and up to 100 units of distance.

Report on the Triangulation of Greater New York, published by the Board of Estimate and Apportionment, New York City. It contains a description of the methods used and results attained in what may well be considered the premier effort in city geodetic surveys in this country.

Report on a Plan of Sewerage for the City of Cincinnati, published by the Department of Public Service, Cincinnati, Ohio. It contains a brief description of methods and results of a topographic survey based on geodetic control.

Precise Leveling in New York City, by Frederick W. Koop, published by the Board of Estimate and Apportionment, New York City. It is a complete discussion of methods and results of a precise level survey of New York City.

U. S. Coast and Geodetic Survey Special Publication No. 68, Elements of Map Projection, with applications to map and chart construction.

U. S. Coast and Geodetic Survey Special Publication No. 5, Tables for a Polyconic Projection of Maps. It contains the absolute values at various latitudes of the lengths of unit arcs of the meridians and parallels, the coordinates of curvature of those lines, and a description of how to construct a polyconic projection.

Text Book on Geodesy and Least Squares, by Charles L. Crandall. John Wiley & Sons, New York. It is a valuable reference work on the practice of geodetic surveying and the application of least squares to the computation of triangulation.

The Adjustment of Observations by the Method of Least Squares with Applications to Geodetic Work, by T. W. Wright and J. F. Hayford. D. Van Nostrand Co., New York. It is a general reference work in which a number of particular problems are considered.

Geodesy, by George L. Hosmer. John Wiley & Sons, New York. It is a textbook on geodesy which follows rather closely the methods in use by the U. S. Coast and Geodetic Survey.

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