NOAA Technical Memorandum NOS NGS-43

PERFORMING CENTIMETER-LEVEL SURVEYS IN SECONDS WITH GPS CARRIER PHASE: INITIAL RESULTS

Benjamin W. Remondi

Rockville, MD
October 1985

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PERFORMING CENTIMETER-LEVEL SURVEYS IN SECONDS WITH GPS CARRIER PHASE: INITIAL RESULTS

Benjamin W. Remondi
National Geodetic Survey
Charting and Geodetic Services
National Ocean Service, NOAA
Rockville, MD 20852

ABSTRACT. Two kinematic surveys were performed, using TI-4100s, which substantiate the author's earlier claim that centimeter-accuracy relative surveys can be accomplished within seconds. The first test was carried out in Arkansas using wide-bandwidth tracking. Although noisy, these data permitted the concepts and the data processing software to be verified. Based on results of this survey, a definitive kinematic survey experiment was designed and executed at the National Bureau of Standards, Gaithersburg, MD. The results of this experiment show that centimeter-level relative surveys can be performed in seconds. A priori geodetic coordinates of the initial location of the "roving" antenna are not required since they, too, can be determined within seconds.

INTRODUCTION

This paper is a continuation of a developmental activity documented in a number of earlier papers (Remondi 1985a; Remondi 1985b; Hoffman-Wellenhof and Remondi 1985). For this reason only a brief introduction will be given.

This research is rooted in preliminary discussions at the National Geodetic Survey between Bruce Douglas, James Lucas, and myself, in 1982, with regard to decimeter-level aircraft trajectory determination using the Global Positioning System (GPS) carrier (beat) phase. It was later realized that other investigators had achieved preliminary results in this area (Evans et al. 1981). By 1983, when centimeter-accuracy static surveys became a routine activity (Bock et al. 1984; Goad and Remondi 1984; Remondi 1984), the problem presented itself differently. That is, it seemed productive and natural to spend 1 to 3 hours performing static relative surveys for base lines 10 to 100 km long using L-band carrier phase measurements. However, it seemed unnatural to use that same technology to measure a 50-meter base line vector. This provided the motivation to develop technology for performing kinematic centimeter-level relative surveys in seconds.

Although the theory and programs are similar to a moving platform trajectory application, such as aircraft trajectory determination, there are differences. The following are examples of these differences: (1) Loss of lock is a more serious problem on a moving platform application than on a kinematic survey, so wider bandwidth tracking must be sustained; in kinematic surveying the bandwidth can be collapsed when the target survey mark is reached and subsequently reopened before departure, in order to maintain lock. (2) In kinematic surveying the initial coordinates of the roving antenna can be precisely determined by the simple artifice of moving what is normally the stationary antenna to the initial location of the roving antenna while, at the same time, moving the roving antenna from its initial location to the location of the stationary antenna. This procedure will, hereafter, be referred to as "exchanging" or "swapping" antennas. Such an exchange requires continuous phase tracking of four or more GPS satellites during its execution and would take a minute, or less, to perform; the antennas might be swapped back to their initial positions for a redundant measurement. This procedure could be performed at the beginning and end of an aircraft trajectory application but it would be more cumbersome. (3) Although the trajectory is determined as a byproduct of kinematic surveying, it is of secondary interest. Consequently, incomplete modeling of receiver processes which result in systematic errors while traveling between marks, but do not result in systematic errors when static mode is achieved (at the survey mark), can be tolerated. When the trajectory, itself, is the object of interest, more attention to the receiver processes is required.

Throughout this report a somewhat restricted definition of kinematic survey will be employed. It will be assumed that the carrier phase tracking of four or more GPS satellites will be maintained at every instant of the experiment and, specifically, without loss of lock. Kinematic surveying, here, will be restricted to relative kinematic surveying, where, except for antenna "exchanges," one antenna will remain stationary and one antenna will be a rover. It should be clear that many of these constraints are not required but are being imposed to facilitate the discussion.

In the following discussion, two kinematic survey experiments are described and the results of the data processing are reported. The objective, here, is to demonstrate that it is possible to perform centimeter-accuracy relative surveys within seconds based on the GPS carrier phase observable. The geodetic instruments employed in these experiments were Texas Instruments, Inc. TI-4100 GPS receivers.

THE MODEL

The modeling equations upon which the results reported here are based have been documented elsewhere (Remondi 1985a; Remondi 1985b). For this reason only a brief theoretical presentation will be included.

Suppose GPS orbital data and carrier frequencies are errorless. If a GPS receiver is scheduled to take carrier phase measurements at time t_i, but these are actually taken at time $t_i+\delta t_i$, then the transmission time of this phase event can be computed as:

$$t^j_{T_i} = t_i + \delta t_i - \tau^j(i)$$

where $\tau^j(i)$ is the signal transit time from the GPS transmitter to the GPS receiver (corrected for Earth rotation during transit). Let us symbolize a "one-way" carrier phase measurement between receiver R and GPS satellite S at true time $t_i+\delta t_i$ by $m^S_R(i)$. Then let us generate a triple difference observation based on satellites j and k, receivers 1 and 2, and receipt times i and i+1 as follows:

$$OT_{1,2}(j,k,i,i+1) = \left[\left(m^k_2(i+1)-m^k_2(i)\right) - \left(m^k_1(i+1)-m^k_1(i)\right)\right]$$

$$- \left[\left(m^j_2(i+1)-m^j_2(i)\right) - \left(m^j_1(i+1)-m^j_1(i)\right)\right]$$

It has been shown in the cited references that a simplified model of this generated observation is (f represents frequency):

$$CT_{1,2}(j,k,i,i+1) = f\cdot\left[\left(\tau^k_1(i+1)-\tau^k_2(i+1)\right) - \left(\tau^k_1(i)-\tau^k_2(i)\right)\right]$$

$$- f\cdot\left[\left(\tau^j_1(i+1)-\tau^j_2(i+1)\right) - \left(\tau^j_1(i)-\tau^j_2(i)\right)\right]$$

OT and CT represent the observed and computed triple differences, respectively. This model reflects a GPS receiver which measures satellite phase minus receiver phase; the signs in the model would be reversed for a receiver that measures receiver phase minus satellite phase.

THE ARKANSAS KINEMATIC SURVEY (JULY 4, 1985)

The National Geodetic Survey's TI-4100 survey team was instructed to establish (with compass and tape) five survey marks at an airport during their survey operation in Arkansas. They used approximately 100-foot spacings between marks and oriented the figure approximately in the cardinal directions. Figure 1 depicts the configuration of marks (A to E).

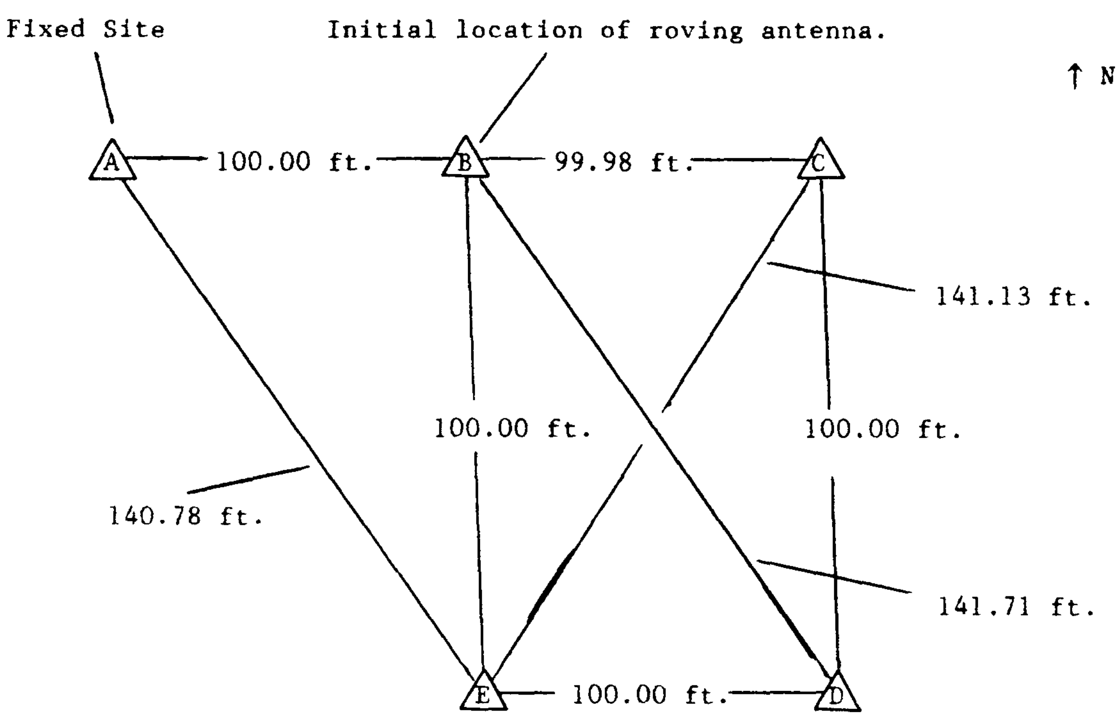

Figure 1.--Arkansas test network, West Helena, AR.

The stationary GPS receiver was to occupy Mark A for the duration of this experiment (Arkansas, User 4). The moving GPS receiver was to commence tracking at Mark B ond occupy Marks B through E according to the sequence B-C-D-E-B-C-D-E-B. L-band carrier phase tracking was maintained on four GPS satellites for the duration of the test. This experiment was later repeated (Arkansas, User 3) using a reduced tracking bandwidth.

The designators "User 3" and "User 4" indicate the dynamic level the receiver software is able to accommodate. User 3 represents 8 Hz tracking, and User 4 represents 16 Hz tracking. In comparison, the bandwidth setting for static mode is typically 0.75 Hz. To translate these into an intuitive realm, 16 Hz tracking permits approximately 4 g's of acceleration (22 Hz is the bandwidth limit before cycle ambiguities predominate). These values come from Henson (1985) and Sims (1985). Although 4 g's of acceleration are quite accommodative for the kinematic surveying application, this level can be exceeded with sudden jerks. 8 Hz tracking permits a smooth increase to approximately 1 g of acceleration. In practice it is very difficult to maintain carrier tracking while in motion at 8 Hz.

Figure 2 shows the results of processing data from the Arkansas User 4 experiment. The L_2 plots are not included because they are very similar to the L_1 plots; however they are nearly twice as noisy. Figure 2a displays the reduced horizontal trajectory. Figures 2b, 2c, and 2d show the northerly, easterly, and vertical components as a function of time. Figure 2e displays the noise penalty one must pay for wide-bandwidth tracking. Figure 3 is similar to figure 2 but comprises the results of the Arkansas User 3 experiment. One can see that halving the bandwidth has halved the noise, as expected. In the User 4 experiment the square was circumnavigated twice, whereas in the User 3 experiment the loop was made just once. This accounts for the two north to south and east to west cycles in figures 2b and 2c and the eight vertical cycles in figure 2d but only half as many in figure 3. The horizontal trajectory plots of figures 2a and 3a reflect this as well. The odd-shaped trajectory seen in figure 2a reflects the fact that the antenna cable got caught in the survey van.

This experiment was performed to provide data to the investigator to verify processing software and was not intended to be a definitive survey. In fact, geodetic truth was neither desired nor established. The correct geodetic coordinates of the marks could be ascertained, however, if desired. This was not of primary interest because the kinematic survey performed at the National Bureau of Standards (NBS) (see below) was designed to answer the question of accuracy.

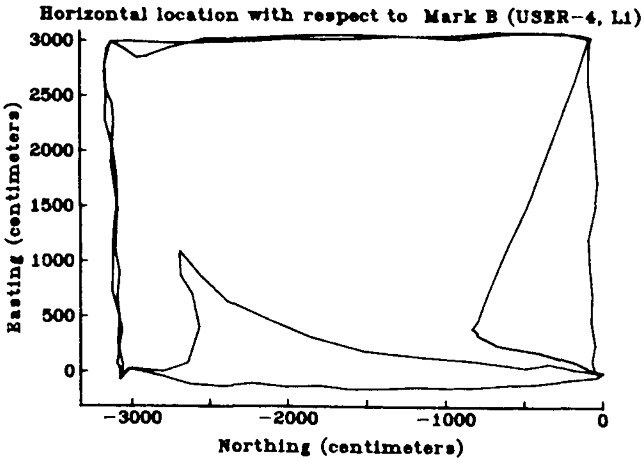

Figure 2a.

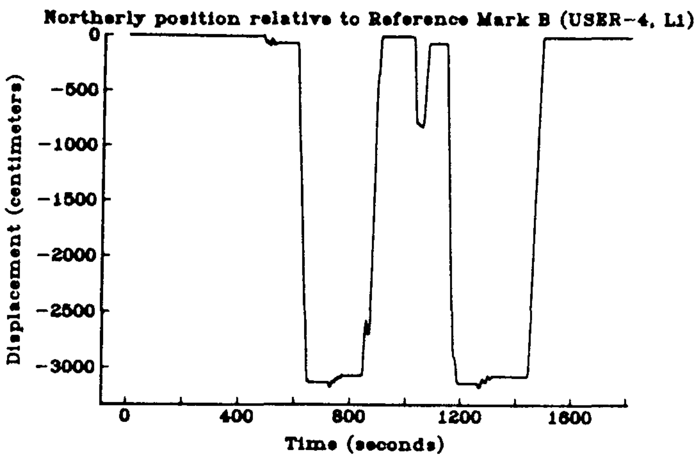

Figure 2b.

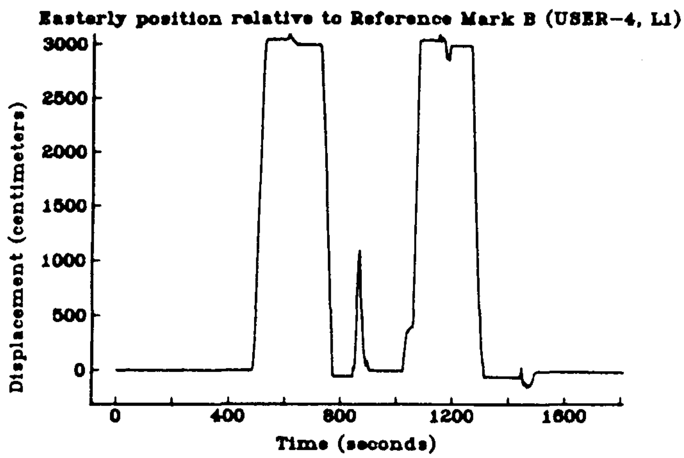

Figure 2c.

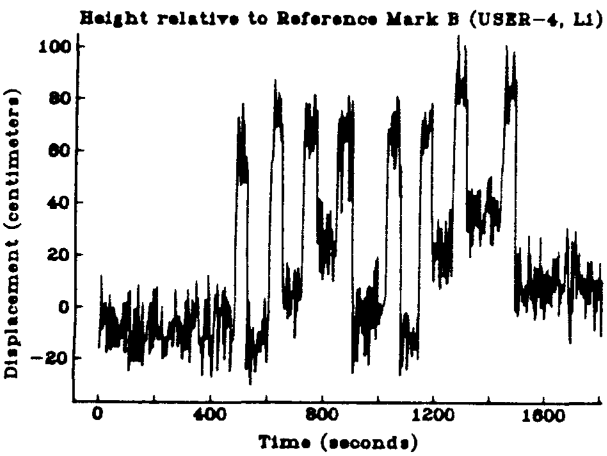

Figure 2d.

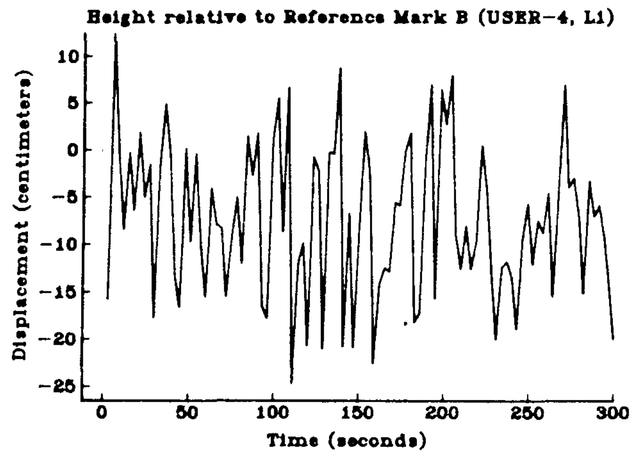

Figure 2e.

Figure 2.--Arkansas "User 4" results.

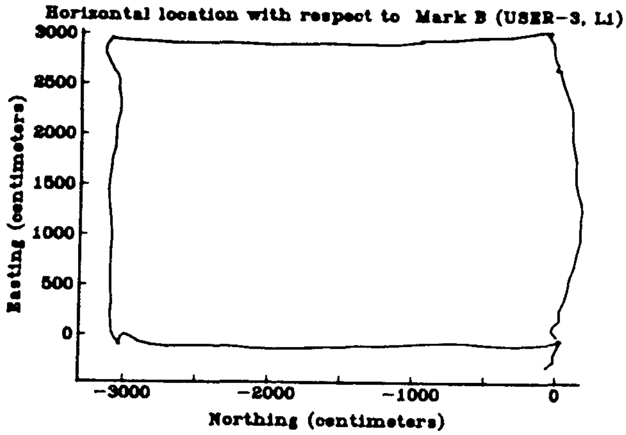

Figure 3a.

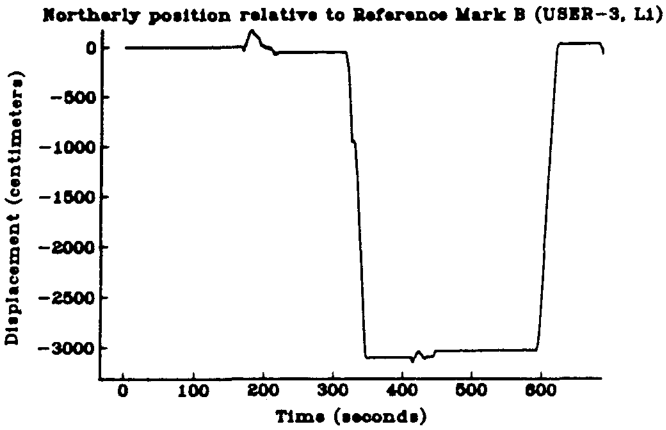

Figure 3b.

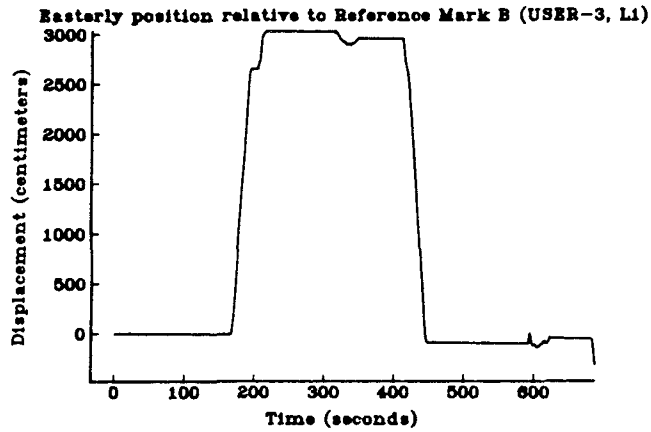

Figure 3c.

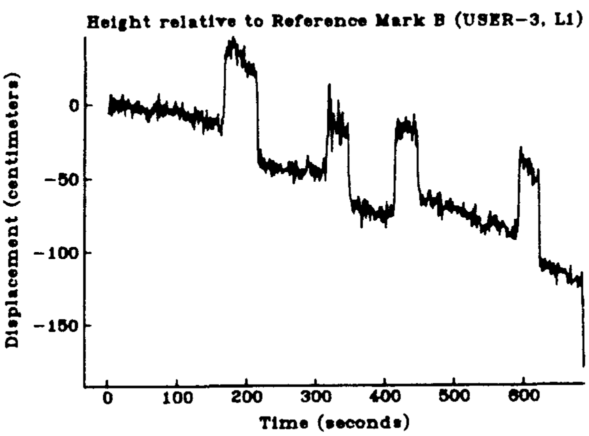

Figure 3d.

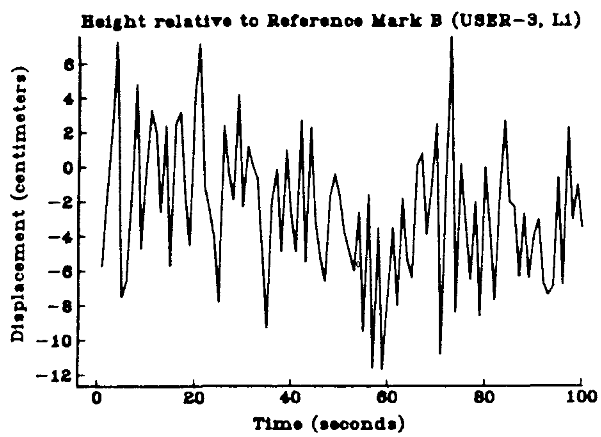

Figure 3e.

Figure 3.--Arkansas "User 3" results.

The Arkansas experiment was considered successful in that:

(1) It established 16 Hz as the appropriate bandwidth for this particular geodetic instrument based on its current design.
(2) It pointed to the need to be able to collapse the bandwidth once the antenna reached static mode over a geodetic mark, to reduce the noise.
(3) It pointed to the need to include, in the survey design, an additional step for the determination of the initial geodetic coordinates of the moving receiver.
(4) It pointed to the need to revisit the moving receiver's initial mark (or <u>any</u> precisely known mark) between visits to unknown marks. This is considered a temporary measure; it is expected to be unnecessary when the modeling is complete and the technique is mature.
(5) It allowed the data processing software to be verified, and suggested enhancements.

Two final comments with regard to the Arkansas survey. The lack of closure in the vertical component is a result of two factors: (1) coordinates for Mark B were imprecisely known -- only the length was precisely known; (2) the simplified model used in the data reduction, reported here, does not account for a small linear drift in the vertical component and a much smaller linear drift in the horizontal components. This point will be discussed later.

THE NBS KINEMATIC SURVEY (AUGUST 12/13, 1985)

On August 12/13, 1985 a kinematic survey similar to the Arkansas survey was performed at the National Bureau of Standards (NBS), Gaithersburg, MD. The relationship of the survey marks is depicted in figure 4. They are within 1 km of each other. Marks NBS-1, NBS-2, NBS-3, and NBS-4 were precisely known a priori. Marks NBS-3a and NBS-3b were established on August 12 using terrestrial methods, specifically as part of this experiment. NBS-3 is in an open field and, for this reason, was chosen as the stationary site. Marks NBS-3a and NBS-3b were established to allow the demonstration of an important aspect of this kinematic surveying method: the ability to determine the starting coordinates of the moving receiver within seconds. We would be free from this minor inconvenience if it were possible for the stationary antenna and the roving antenna to share a common starting point. This aspect of the experiment proved to be completely successful in that the reduced geodetic coordinates of NBS-3b were correctly determined at the millimeter level. The precise coordinates for the NBS survey

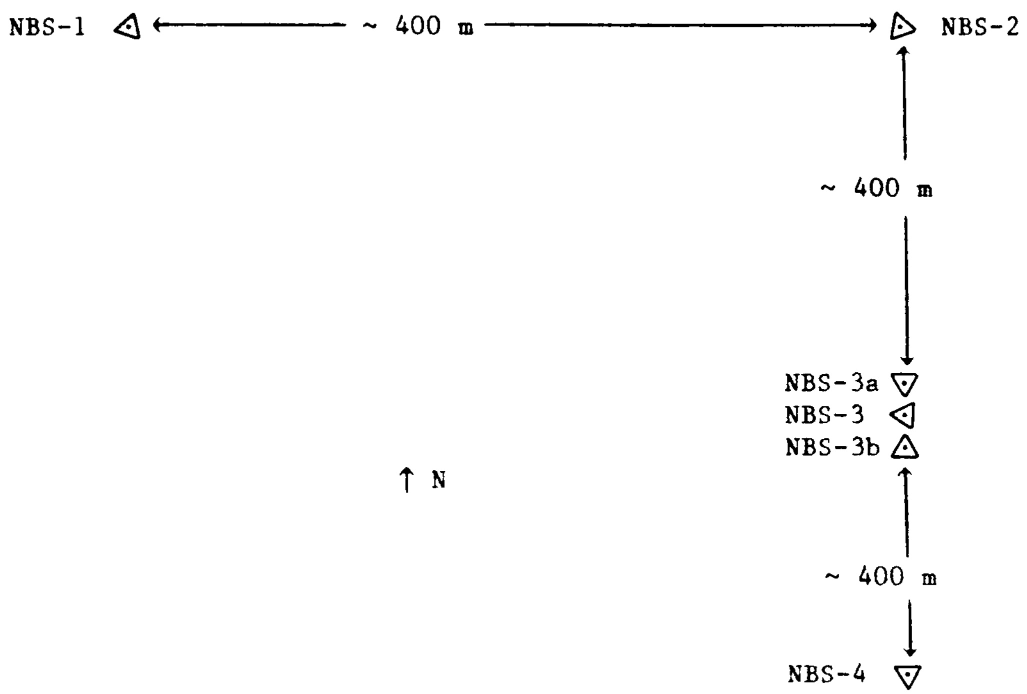

Figure 4.--Kinematic survey test network at the National Bureau of Standards, Gaithersburg, MD.

marks, relevant to this test, are included in table 1. In all cases the coordinates are known to within a few millimeters.

Table 1.--Geodetic truth (in WGS-72) at the antenna phase center.

Survey Mark	North latitude	East longitude	Ellipsoid height
NBS-1	39.133689736	282.786495208	106.822
NBS-2	39.13385466	282.790696732	105.381
NBS-3	39.130829886	282.790787626	105.774
NBS-3a	39.130985798	282.790782946	105.774
NBS-3b	39.130674044	282.790760425	105.635
NBS-4	39.12760489	282.790224959	95.564

Receiver UT09 was initially situated at NBS-3b; receiver UT04 was initially situated approximately 17 m north at NBS-3. The applications software in the TI-4100 was a modified version of the Arkansas User 4 software with an enhancement provided by Mrs. Laura Hawkins of NSWC which allowed one to toggle between 16 Hz (high dynamic (HD) mode) tracking and 0.75 Hz (low dynamic (LD) mode) tracking. Data were recorded every 5 seconds so as to limit the number of observations. It was felt that minimizing the random component was of little concern at this stage; this improvement will be there when it is required. Table 2 shows the log of times and events.

Table 2.--Log of events for kinematic survey at National Bureau of Standards.

Event	Date/Time
(1) UT04 and UT09 begin tracking in LD mode.	1985/08/12/23/09/55
(2) UT04 and UT09 go into HD mode.	23/17/00
(3) UT04 and UT09 go into LD mode.	23/18/50
(4) UT04 and UT09 go into HD mode.	23/20/05
(5) UT04 departs from NBS-3 for NBS-3b.	23/21/00
(6) UT09 departs from NBS-3b for NBS-3.	23/21/00
(7) UT04 arrives at NBS-3b.	23/21/40
(8) UT09 arrives at NBS-3.	23/21/40
(9) UT04 and UT09 go into LD mode.	23/21/45
(10) UT04 and UT09 go into HD mode.	23/22/35
(11) UT04 departs from NBS-3b for NBS-3.	23/22/55
(12) UT09 departs from NBS-3 for NBS-3b.	23/22/55
(13) UT04 arrives at NBS-3.	23/23/30
(14) UT09 arrives at NBS-3b.	23/23/30
(15) UT04 and UT09 go into LD mode.	23/23/45
(16) UT09 goes into HD mode.	23/24/35
(17) UT09 departs from NBS-3b for NBS-3a	23/25/00
(18) UT09 arrives at NBS-3a	23/25/45
(19) UT09 goes into LD mode.	23/25/55
(20) UT09 goes into HD mode.	23/27/30
(21) UT09 departs from NBS-3a for NBS-4	23/28/00
(22) UT09 arrives at NBS-4.	23/31/55
(23) UT09 goes into LD mode.	23/32/10
(24) UT09 goes into HD mode.	23/33/45
(25) UT09 departs from NBS-4 for NBS-3b.	23/34/00
(26) UT09 arrives at NBS-3b.	23/36/45
(27) UT09 goes into LD mode.	23/36/55
(28) UT09 goes into HD mode.	23/38/40
(29) UT09 departs from NBS-3b for NBS-3a.	23/39/00
(30) UT09 arrives at NBS-3a.	23/40/00
(31) UT09 goes into LD mode.	23/40/05
(32) UT09 goes into HD mode.	23/41/00
(33) UT09 departs from NBS-3a for NBS-3b.	23/41/10
(34) UT09 arrives at NBS-3b.	23/42/15

Table 2.--Log of events for kinematic survey at National Bureau of Standards. (continued)

Event	Date/Time
(35) UT09 goes into LD mode.	23/42/20
(36) UT04 and UT09 go into HD mode.	23/43/40
(37) UT04 departs from NBS-3 for NBS-3b.	23/44/00
(38) UT09 departs from NBS-3b for NBS-3.	23/44/00
(39) UT04 arrives at NBS-3b.	23/44/40
(40) UT09 arrives at NBS-3.	23/44/40
(41) UT04 and UT09 go into LD mode.	23/44/45
(42) UT04 and UT09 go into HD mode.	23/45/30
(43) UT04 departs from NBS-3b for NBS-3.	23/46/00
(44) UT09 departs from NBS-3 for NBS-3b.	23/46/00
(45) UT04 arrives at NBS-3.	23/46/30
(46) UT09 arrives at NBS-3b.	23/46/30
(47) UT04 and UT09 go into LD mode.	23/46/45
(48) UT09 goes into HD mode.	23/48/20
(49) UT09 departs from NBS-3b for NBS-2	23/49/00
(50) UT09 arrives at NBS-2	23/51/55
(51) UT09 goes into LD mode.	23/52/05
(52) UT09 goes into HD mode.	23/53/35
(53) UT09 departs from NBS-2 for NBS-1.	23/54/00
(54) UT09 arrives at NBS-1.	23/56/40
(55) UT09 goes into LD mode.	23/56/50
(56) UT09 goes into HD mode.	23/57/45
(57) UT09 departs from NBS-1 for NBS-3a.	23/58/00
(58) UT09 arrives at NBS-3a.	1985/08/13/00/01/30
(59) UT09 goes into LD mode.	00/01/45
(60) UT09 goes into HD mode.	00/06/45

A few comments are in order. Tracking began in LD mode. For the remainder of the experiment, every departure was preceded by a toggle to HD and every arrival was followed by a toggle back to LD. The only events that are unusual, and deserve special mention, were the four occasions where the antennas of UT09 and UT04 were exchanged. Only one of these was required, but repeat measurements seemed appropriate. These antenna swaps provide a quick and accurate way of determining the initial coordinates of the roving antenna. Such a swap yields exactly twice the vector from one mark to the other. This swapping event would normally be the only motion that the otherwise stationary antenna would undergo for the remainder of the survey. The speed of travel between the survey marks did not exceed 10 to 15 miles per hour since the terrain was rough. The procedure was to remove the antenna from the tribrach and hand it to an individual on top of the survey vehicle. Care was exercised so as not to block the GPS signals. It should be mentioned that the receiver display was monitored at all times for the possibility of loss of lock. This was tedious and a loss-of-lock buzzer would be required in an operational mode. No effort was made to orient the antennas in the same direction. This fact, alone, could lead to several millimeters of error.

Figure 5a shows the computed trajectory including the antenna swaps referred to above.

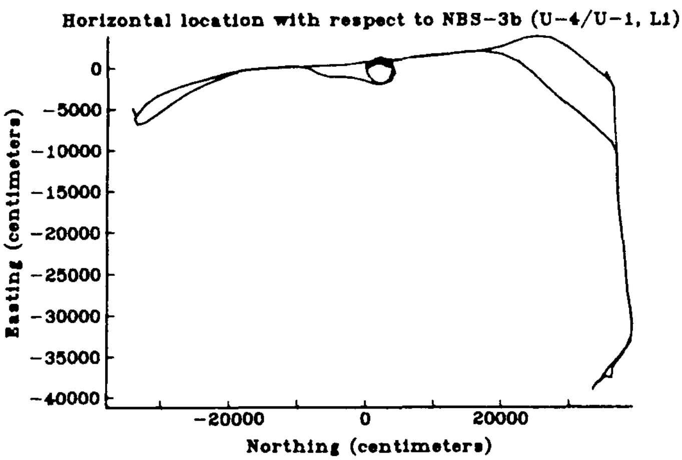

Figure 5a.

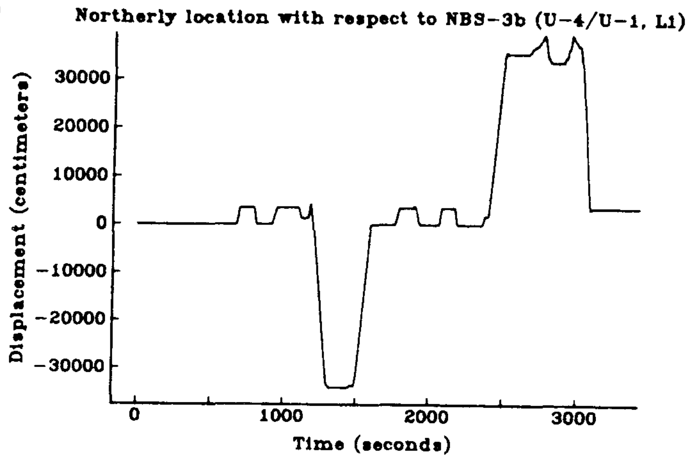

Figure 5b.

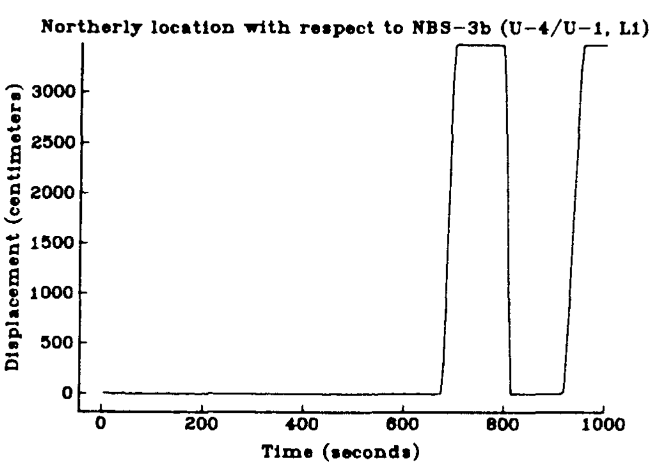

Figure 5c.

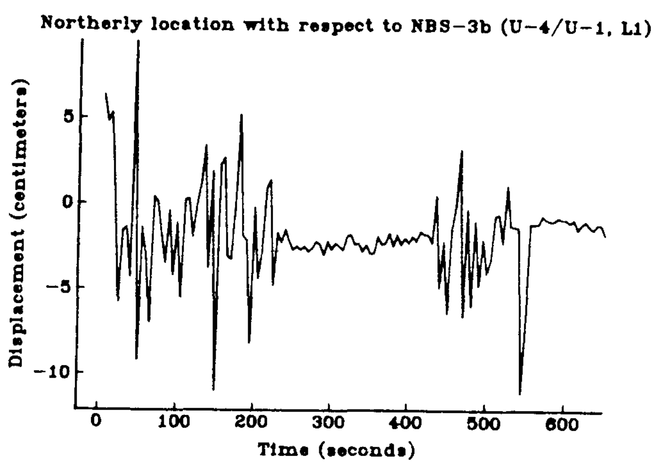

Figure 5d.

Figure 5.--High dynamic/low dynamic results from the National Bureau of Standards test.

As before, only the L_1 plots are displayed since the L_2 plots are similar. These swapping events yield an exaggerated trajectory of UT09. The distortion ends as soon as the stationary receiver returns to its site (NBS-3). Figure 5b shows the northerly component of the trajectory including the distortions during the swapping events. Figure 5c provides an expanded scale of the northerly component. Notice, for example, that the 17 m swapping event appears as a 34 m event (first peak). The subsequent event was not a swap; it reflects the transition of UT09 to NBS-3a while UT04 was fixed. Just by coincidence NBS-3a is approximately 34 m north of NBS-3b whereas NBS-3 is approximately 17 m north of NBS-3b. The result is that the 17 m swap appears nearly identical to the 34 m transition from NBS-3b to NBS-3a. Figure 5d provides yet another scale change. The transitions between 16 Hz and 0.75 Hz tracking are apparent.

Figure 6 displays information similar to figure 5 for the easterly and vertical components. The apparent spikes in figure 6b reflect two facts: (1) NBS-3 is almost directly north of NBS-3b; (2) during the swap UT09 traveled wide to the east (from NBS-3b to NBS-3) whereas UT04 traveled wide to the west (from NBS-3 to NBS-3b). The motions, again, appear to be distorted because in the computations UT04 was assumed to be stationary -- even though it was not.

RESULTS OF THE NBS KINEMATIC SURVEY

Only the geodetic coordinates of NBS-3 were assumed in the data reduction. Even this was not required since the point position solution, based on pseudoranges, would have been satisfactory. Precise NBS-3 coordinates were used for ease in verifying the reduced geodetic positions of the other geodetic marks.

Starting with NBS-3 coordinates, the first data reduction task was to determine the precise coordinates of NBS-3b based on the swapping events where, recall, NBS-3b was the starting location of the kinematic survey. Geodetic coordinates of NBS-3b are subsequently required for the determination of the geodetic locations of the rest of the survey marks visited.

Table 3 includes the four independent determinations of NBS-3b based on these antenna exchanges. Included is the average of these determinations and a comparison with geodetic truth. The data reduction agreed with the geodetic truth at the 2-millimeter level. This is considered fortuitous since the variation was at the 1-centimeter level. It should be pointed out that the ability to determine the initial vector of the roving antenna within seconds is a new and significant result. The theoretical implication is that the integer biases (Bock et al. 1984; Goad and Remondi 1984; Remondi 1984) can be determined within seconds and applied throughout the survey. A more mundane implication is that it is possible to add an azimuth mark within seconds (depending on distance).

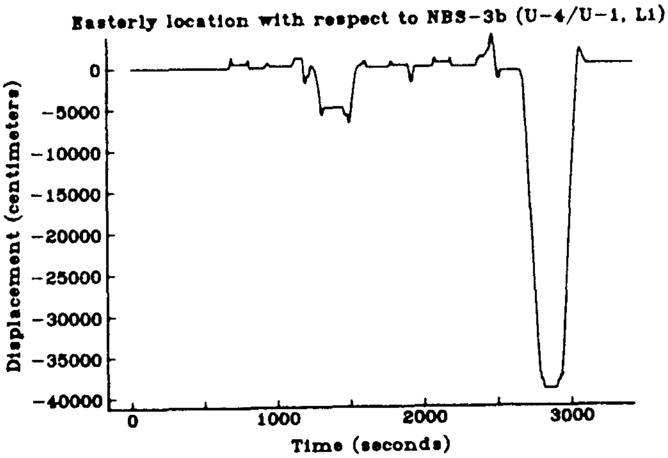

Figure 6a.

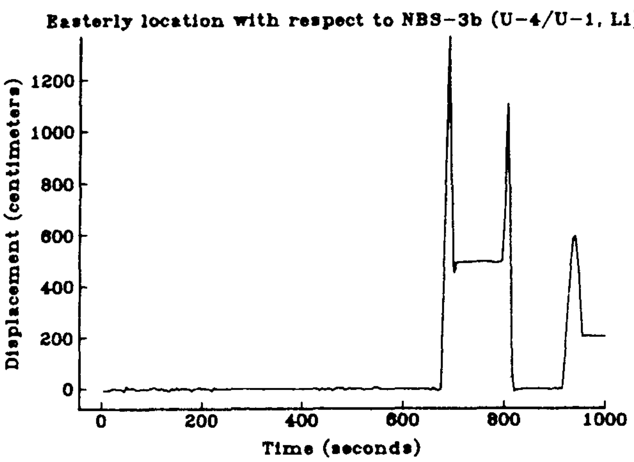

Figure 6b.

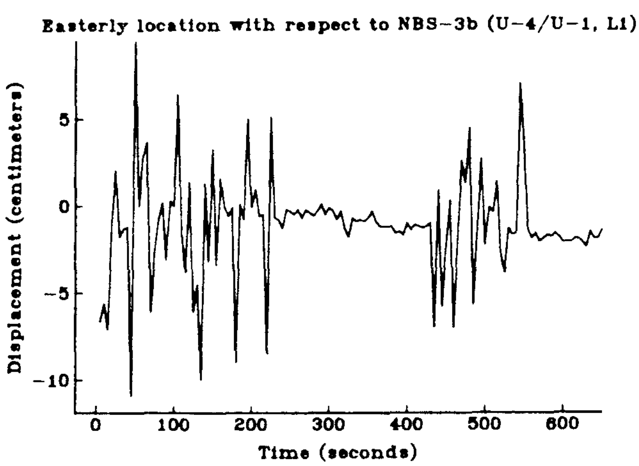

Figure 6c.

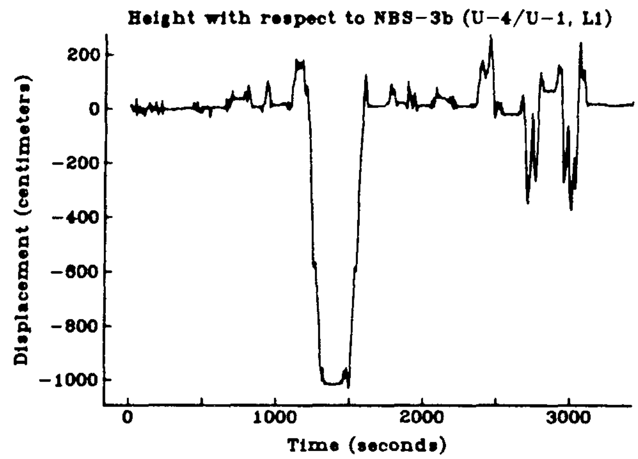

Figure 6d.

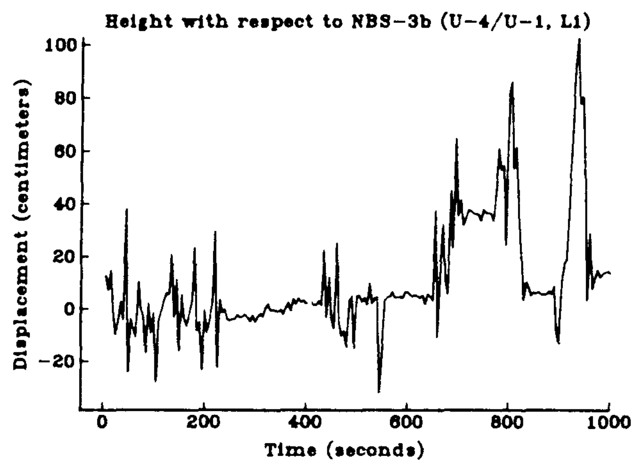

Figure 6e.

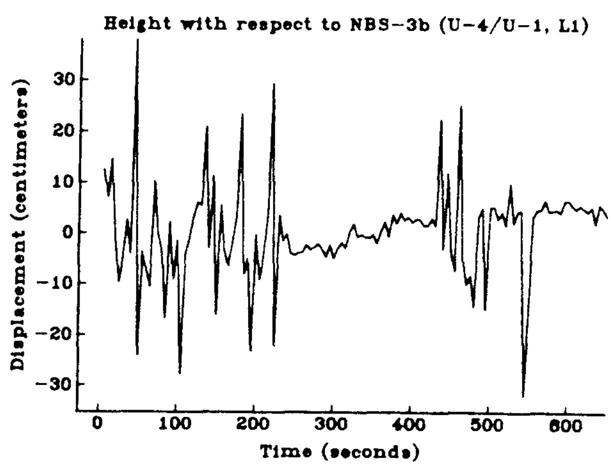

Figure 6f.

Figure 6.--Additional high dynamic/low dynamic results from the National Bureau of Standards test.

Table 3.--Solutions of the initial location of the roving antenna.

Event	N. latitude	E. longitude	Ellipsoid height
swap 1.	39.130674105	282.790760403	105.636
swap 2.	39.130674411	282.790760348	105.644
swap 3.	39.130673975	282.790760475	105.630
swap 4.	39.130674044	282.790760445	105.626
average	39.130674058	282.790760415	105.634
truth	39.130674044	282.790760425	105.635
error	1.4 mm	1.0 mm	2.0 mm

The next phase of the data processing requires approximate point position coordinates of the stationary site (NBS-3) and precise relative coordinates of the initial location of the roving antenna. Using the model and the least-squares algorithm given in the cited references, the geodetic locations of the roving antenna were determined at each measurement epoch. These solutions are tabulated, along with their times, only for those times when the receiver achieved narrow-bandwidth tracking over the target geodetic marks. About 10 to 15 seconds are required for the tracking bandwidths to collapse from 16 Hz to 0.75 Hz, as the TI-4100 is currently designed. For this reason, one may depart from a survey mark from 15 to 30 seconds after arrival. Table 4 lists the "true minus computed" values.

An examination of these results reveals a linear drift in the coordinates of NBS-3b. This drift is likely the product of an over-simplified model, and unmodeled clock drift is the suspected error source. Since the coordinates of NBS-3b are now precisely known, one is justified in removing the nearly linear drift in the coordinates of NBS-3b and subsequently applying this correction to the remainder of the survey as well. Although such a correction is expected to yield improved results for similar surveys, the primary purpose, here, is to demonstrate the accuracy one can expect with an improved model and the corresponding processing software. Naturally this will be the next refinement. Table 5 presents the results of table 4 after correcting for this linear drift. The resulting root-mean-square error is 0.6 cm in latitude, 1.0 cm in longitude, and 2.6 cm in height.

Table 4.--True minus computed geodetic coordinates for NBS test.

Date and time	Survey mark	ΔN lat (cm)	ΔE long (cm)	Δht (cm)
8/12/23/09/55	NBS-3b * †	0.0	0.0	0.0
23/19/35	NBS-3b *	0.3	1.9	-5.3
23/24/20	NBS-3b *	1.8	1.2	-6.2
23/26/55	NBS-3a	1.9	1.9	-7.9
23/33/10	NBS-4	0.9	3.3	-5.9
23/38/00	NBS-3b *	2.7	2.5	-10.0
23/40/45	NBS-3a	3.6	2.7	-14.8
23/43/15	NBS-3b *	3.4	1.9	-12.4
23/47/45	NBS-3b *	3.7	1.1	-10.0
23/53/00	NBS-2	6.0	0.2	-16.1
23/57/25	NBS-1	4.4	0.1	-15.4
8/13/00/04/00	NBS-3a	5.0	1.0	-9.7

* Initial location of roving antenna.

† Imposed initial coordinates based on results from the antenna exchanges (This point was not included in the linear fit reported in table 5.)

Table 5.--True minus computed geodetic coordinates for NBS test corrected for the linear drift observed at NBS-3b.

Date and time	Survey mark	ΔN lat (cm)	ΔE long (cm)	Δht (cm)
8/12/23/19/35	NBS-3b	-1.1	0.1	0.1
23/24/20	NBS-3b	0.0	0.6	0.3
23/26/55	NBS-3a	0.2	0.2	0.8
23/33/10	NBS-4	-1.9	1.6	2.5
23/38/00	NBS-3b	-0.7	0.7	-0.7
23/40/45	NBS-3a	-0.1	1.0	-4.7
23/43/15	NBS-3b	-0.5	0.2	-1.7
23/47/45	NBS-3b	-0.8	-0.6	1.6
23/53/00	NBS-2	1.0	-1.5	-3.2
23/57/25	NBS-1	-1.1	-1.6	-1.5
8/13/00/04/00	NBS-3a	-0.9	-0.8	4.6

The presented results are based on L_1 carrier phase measurements. The results from processing L_2 are similar but the contribution due to noise is about twice as large. The L_2 linear drift in the NBS-3b coordinates is approximately 1.6 times as large as that of L_1 and in the opposite direction. Notice that $f_1/f_2=1.283$ and the ratio of the bias frequency plan (-6000 Hz for L_1; +7600 for L_2) is 1.27. The product of these quantities is 1.625. For this reason a timing error is suspected. This modeling deficiency will doubtless be corrected but does not obscure the central fact that centimeter-level accuracy is achievable in seconds.

CONCLUSIONS

Centimeter-level relative positioning in seconds has been achieved. Furthermore, centimeter-accuracy determination of the initial relative vector of the roving antenna, within seconds, has been demonstrated. These results are new and significant. Considering the improvements which would obviously result from tracking 6 to 9 SVs, while using more than one fixed geodetic receiver, and when data are recorded at a high rate, one must conclude that the future is bright for this technique. When the model is upgraded, the artifice of revisiting the initial location of the roving antenna and removing the small linear drift will be unnecessary. Since the trajectory of the roving antenna, in this application, is of no particular interest, the main results reported here do not require the recording of data during the transition between marks. In fact, it makes sense to record measurements at a high rate at the survey marks once narrow-bandwidth tracking begins. Since the number of measurements can be kept small, they can be transferred on a real time basis. Not only does this permit real-time centimeter-level surveying in seconds, but also it would allow the user to navigate to within centimeters of a desired location in real time!

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