NOAA Technical Memorandum NOS NGS 23

USE OF AUXILIARY ELLIPSOIDS IN HEIGHT-CONTROLLED SPATIAL ADJUSTMENTS

Rockville, Md.
November 1979

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UNITED STATES DEPARTMENT OF COMMERCE / NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION Richard A. Frank, Administrator / National Ocean Survey Herbert R. Lippold, Jr., Director

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ABSTRACT: In an adjustment of a geodetic network in the height-controlled three-dimensional system it is convenient to use auxiliary ellipsoids. These surfaces are used for preserving correct heights of points and for simplifying the transformation of adjusted Cartesian coordinates to the geographic system.

1. INTRODUCTION

Previous reports (Vincenty and Bowring 1978, Vincenty 1979) dealt with methods of adjusting networks in a height-controlled three-dimensional system, with heights and directions of gravity held fixed and without reducing the observations to the ellipsoid. In these adjustments the positional shifts of points can be expressed as changes in geographic coordinates B, L, or in Cartesian coordinates X, Y, Z, or in horizon coordinates in the plane perpendicular to the direction of gravity at the point in question.

The use of geographic coordinate shifts dB, dL is no longer recommended because it necessitates transformation between geographic and Cartesian coordinate systems in iterations of an adjustment. These transformations are actually unnecessary because the equations used in the adjustment can be constructed so that the adjusted spatial positions of points will remain at the prescribed heights H. This is accomplished by using auxiliary ellipsoids for the points of the network.

2. DEFINITIONS OF AUXILIARY ELLIPSOIDS

The Cartesian coordinates of a point in space are obtained from B, L, H by

$$X = (N + H) \cos B \cos L \tag{1a}$$

$$Y = (N + H) \cos B \sin L \tag{1b}$$

$$Z = [N(1 - e^2) + H] \sin B, \tag{1c}$$

where $N = a/(1 - e^2 \sin^2 B)^{1/2}$, a is equatorial radius, and e is first eccentricity. We postulate that the height of the point above the auxiliary ellipsoid be zero and that the normal to this ellipsoid at the point in question be coincident with the normal to the reference ellipsoid. The Cartesian coordinates of the point can be computed also by equations (1a), (1b), and

$$Z = (N + H)\ (1 - e_0{}^2) \sin B. \tag{2}$$

To derive the expression for e_0, the right sides of (1c) and (2) are set equal, which gives

$$e_0{}^2 = e^2/(1 + H/N). \tag{3}$$

The equation of this ellipsoid is then

$$X^2 + Y^2 + Z^2/(1 - e_0{}^2) = a_0{}^2 \tag{4}$$

and its equatorial radius a_0 is obtained by placing the correct X, Y, Z values for the point at height H on the left side of the equation. This ellipsoid can be considered as a surface of constant height over a large area of some hundreds of kilometers in extent.

If we define the auxiliary eccentricity by

$$e_0{}^2 = e^2/(1 + H/a), \tag{5}$$

the normals to the reference ellipsoid and to the auxiliary ellipsoid will no longer coincide exactly. It can be shown that the angle between the two normals is approximately

$$\gamma = e^4 H \sin^3 B \cos B/(2a), \tag{6}$$

or less than 0.003" if H = 10 km. Accordingly, the surface of this auxiliary ellipsoid departs from the constant height surface faster, the separation amounting to less than 0.001 m at 50 km from the starting point.

3. APPLICATIONS

3.1 Adjustment in Horizon Coordinates

In this version of the height-controlled adjustment each point acquires differential shifts dx and dy in the plane of the local astronomical horizon. These shifts could be converted to increments dB and dL and applied to previous B, L values, after which new X, Y, Z coordinates and corresponding geographic coordinates could be computed. A simpler method (Vincenty 1979) converts dx, dy to equivalent three-dimensional shifts by

$$\begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix} = \begin{bmatrix} -\sin\phi\cos\lambda, & -\sin\lambda \\ -\sin\phi\sin\lambda, & \cos\lambda \\ \cos\phi, & 0 \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix} \tag{7}$$

where ϕ and λ are astronomic latitude and longitude. The adjusted X, Y, Z coordinates define a height that is slightly different from the fixed value, and it might be argued that this inconsistency should be rectified so that the error in height is reduced from some millimeters or centimeters to within a millimeter.

The auxiliary ellipsoid can be used to advantage in obtaining precise X, Y, Z values regardless of the magnitudes of coordinate shifts as they occur in practice. Let

$$X' = X + dX \qquad Y' = Y + dY \qquad Z' = Z + dZ \tag{8}$$

$$J^2 = \frac{X^2 + Y^2 + Z^2/(1 - e_0^2)}{X'^2 + Y'^2 + Z'^2/(1 - e_0^2)} . \tag{9}$$

Then J is a factor by which X', Y', and Z' should be multiplied to reduce the point to the surface of constant height along the line joining the point with the center of the auxiliary ellipsoid. Here the approximation given by eq. (5) is more than adequate for all practical situations.

This refinement has been included in a recent revision of the HOACOS program (Vincenty 1979).

The reduction to the auxiliary ellipsoid can be accomplished also in other ways, for example by calculating the height of the shifted point above the auxiliary ellipsoid and then obtaining corrections to X, Y, Z. As is known, the height above the geodetic horizon is given by

$$dH = \cos B \cos L \; dX + \cos B \sin L \; dy + \sin B \; dz, \qquad (10)$$

or with the substitutions $\cos B \cos L = X/(N + H)$,
$\cos B \sin L = Y/(N + H)$, and $\sin B = Z/[(N + H)\ (1 - e_o^2)]$,

$$dH = (X\,dX + Y\,dY + \frac{Z}{1 - e_o^2}\,dZ)/(N + H). \qquad (11)$$

Substituting dX, dY, dZ from (7) in (11) and adding a correction for departure of the surface from the horizon, we get

$$(N + H)\ dH = [Z \cos\phi/(1 - e_o^2) - \sin\phi(X \cos\lambda + Y \sin\lambda)]\ dx$$

$$- (X \sin\lambda - Y \cos\lambda)\ dy + (dx^2 + dy^2)/2 = Q. \qquad (12)$$

The required corrections are then

$$\delta X = -\ (X'/a)\ dH = -\ Q \cos\phi \cos\lambda/a$$

$$\delta Y = -\ (Y'/a)\ dH = -\ Q \cos\phi \sin\lambda/a$$

$$\delta Z = -\ (Z'/a)\ dH = -\ Q\ (1 - e_o^2) \sin\phi/a. \qquad (13)$$

3.2 Adjustment in Cartesian Coordinates

The observation equations for distances and azimuths are expressed in the form

$$a_1(dX_2 - dX_1) + a_2(dY_2 - dY_1) + a_3(dZ_2 - dZ_1) + L = v. \qquad (14)$$

The three unknowns at each point must satisfy the condition

$$X\,dX + Y\,dY + \frac{Z}{(1 - e_o^2)}\,dZ = 0. \qquad (15)$$

This condition is enforced in the simplest way by eliminating one unknown and expressing it in terms of the other two (Vincenty and Bowring 1978). Equation (15) ensures that the movement of the point is restricted to the plane of the geodetic horizon. The reduction of the point to the ellipsoid can be accomplished by use of the J factor (eq. 9) or by eq. (13) with

$$Q = (dX^2 + dY^2 + dZ^2)/2. \qquad (16)$$

3.3 Transformation to Geographic System

The longitude comes from

$$\tan L = Y/X. \qquad (17)$$

For the calculation of latitude we have

$$p^2 = X^2 + Y^2 \qquad (18)$$

$$\tan B = (Z/p)/(1 - e_o^2), \qquad (19)$$

if e_o^2 is given by (3) with N for the point in question. The latitude for computing N is not known; however, eq. (19) is quite insensitive to an error in latitude used for obtaining N and e_o^2. In fact, if B' is the approximate latitude, the error in B given by eq. (19) is

$$\delta B = (H/a)\ (e^2 \sin B \cos B)^2 (B' - B). \qquad (20)$$

In other words, even if the approximation of latitude should be wrong by 10 minutes, the maximum error in B will be 0.00001" for H = 10 km. Therefore, N can be approximated by

$$N = a(1 + \tfrac{1}{2}\, e^2 \sin^2\phi) \simeq a(1 + \tfrac{1}{2}\, e^2 Z^2/a^2). \qquad (21)$$

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