the highest was 84°, at Fort Laramie on the 19th and 20th, and the lowest, zero, at Daniel on the 1st. The average precipitation was 1.19, or 0.48 below normal; the greatest monthly amount, 2.24, occurred at Red Bank, and the least, 0.10, at Centennial.

Plowing and seeding were in progress during the month over much of the State, but cold, freezing weather delayed the work. The cold rain and snow on the 21st and 22d was severe on sheared sheep, and many perished during the storm.—*W. S. Palmer.*

SPECIAL CONTRIBUTIONS.

STUDIES ON THE STATICS AND KINEMATICS OF THE ATMOSPHERE IN THE UNITED STATES.

By Prof. Frank H. Bigelow.

IV.—REVIEW OF FERREL'S AND OBERBECK'S THEORIES OF THE LOCAL AND THE GENERAL CIRCULATIONS.

GENERAL COMPARISON OF FERREL'S AND OBERBECK'S THEORIES.

In order to discuss the theories which have been proposed to account for the circulation of the atmosphere in cyclones and anticyclones, and in general over an hemisphere of the earth, it will be convenient to confine our attention to the views propounded by Ferrel and Oberbeck because their treatment is quite complete, and also because they represent a large number of writers who agree with them more or less perfectly. There is another theory of quite a different type which can be taken up profitably after some critical remarks have been made on the validity of these earlier views. In their treatment of the general circulation of the atmosphere both Ferrel and and Oberbeck adopt the "canal theory" of the circulation, and work out their solutions along that line. Oberbeck places his origin of coordinates at the center of the rotating earth, develops the equations of motion, and transforms to the surface when they are employed in the evaluation of the resulting velocities. He also deduces the terms in the pressure due to the absolute motion of the earth, and to the relative motions of the atmosphere. Ferrel places his origin of coordinates at the surface of the earth, transforms his equations to include a temperature term through the variations of the density, and discusses the meaning of the equations under special limitations, with illustrations from the observed motions of the atmosphere. It may be remarked that von Helmholtz introduces the temperature terms into the equations of motion, not through the density, but through the pressure, by using the equation of elasticity, $p\ v = R\ T$. This procedure is probably a better method of solution. There is not much difference in the results as derived from the analysis by these authors, but there is serious difficulty in making them agree with the modern observations of the motions of the atmosphere in the higher strata, as determined by the international cloud work.

In their treatment of the cyclone Ferrel and Oberbeck diverge radically from each other, though they start with the same physical principle, namely, a local overheated mass of air which in rising through its own buoyancy produces the cyclonic circulation. Ferrel assumes a fixed cylindrical boundary about his cyclone, and considers a warm-center cyclone (circulation anticlockwise), surrounded by a pericyclonic ring (circulation clockwise), in the Northern Hemisphere, the two portions being separated by a surface where the gyratory velocity vanishes. By maintaining a cold mass of air in the center instead of a warm mass the circulation is reversed, and a cold-center cyclone is developed. Oberbeck does not assume any external boundary to the circulating mass of air, but in the central region, bounded by a cylindrical surface, there is a vertical component, while outside of it there is no such vertical ascent of the air. At this boundary there is a discontinuity in the vertical velocity, and at the same distance from the center the gyratory velocity about the axis is a maximum; this falls away to zero at the center and also at some indefinite distance in the outer region. It is essential to the existence of these two theories, although they differ so radically from each other, to establish the fact that such local centers of heated air in the warm-center cyclones do occur in nature, for without them these two theories entirely fail of applicability to our meteorology. They are both possible forms of vortex motion, but it is necessary to show that the antecedent physical conditions prevail, before they can be accepted as explanations of the observed cyclonic motions.

Both of these authors have experienced much difficulty in accounting for the anticyclones. Ferrel explained that the interference of two of his pericyclonic rings would heap up the air and produce an area of high pressure with a clockwise outflow, but this theory is so far from being in conformity with the facts, that it is now, by general consent of meteorologists, considered to be of only historical value. Oberbeck sought, by simply reversing the sign of the vertical component of velocity, to invert his cyclone into an anticyclone. He met with a stumbling-block in the mathematical analysis, but was relieved of this by Pockels, who correctly evaluated the constant of integration. No attempt was made to show that the resulting stream lines conform to the motions of the air in high areas of pressure. Indeed, since the modern observations have given us more correct lines of flow, it is quite certain that the anticyclone can not be explained in this way.

THE SUPPLY OF LOCAL CENTERS OF HEAT.

It is evident, therefore, that the first practical question to decide is whether such local masses of air exist, heated in the under strata and more or less cylindrical in form, as will produce either of the above forms of cyclone. Meteorologists have generally supposed that this is the case, and they have usually attributed the source of the vertical convection to the latent heat of condensation. Dr. J. Hann, in 1890, and again in his Lehrbuch der Meteorologie, has shown in great detail the inadequacy of this source of heat to produce cyclones, and he has indicated that the source of cyclonic action consists rather in horizontal convection currents. As this agrees with the view which I have already advocated, since it seems to me to be in conformity with the observations, I will therefore make a résumé of my remarks on this topic in the International Cloud Report. It will be a great gain if meteorologists can be persuaded to reject the old condensation theory, which has an apparent but really illusory plausibility, in favor of the really efficient source of dynamic action contained in the long, horizontal currents which flow between the Tropics and the polar regions in the middle strata of the atmosphere, as illustrated in the preceding Paper III.

There is, in fact, a fundamental difficulty in accounting for the local supply of heat which is assumed to set the vertical convection in operation. Ferrel himself doubted the efficiency of the latent heat of condensation, for he says in his Meteorological Researches, Appendix No. 10, United States Coast and Geodetic Survey, Part II, page 201: "The condensation of aqueous vapor, therefore, plays an important part in cyclonic disturbances, but is by no means a primary or a principle cause of cyclones." Professor Loomis asserted in Silliman's Journal, July, 1877: "Rainfall is not essential to the formation of areas of low barometer, and is not the principle cause of their formation or of their progressive motion." Indeed, a reasonable familiarity with the United States weather maps proves conclusively that there are many deep, fully-developed storms which form near the north Pacific coast and advance to the Gulf of St. Lawrence without any precipitation worth mentioning. Also, cyclones form frequently in the southern Rocky Mountain districts and advance into the lower Mississippi Valley without any important rainfall; from that region onward in their course the precipitation and intensity of the storm often greatly increase, since the latent heat derived from the inflowing moist air of the Gulf of Mexico undoubtedly assists the vertical convection in the center of the cyclone. If the horizontal currents which converge upon a cyclonic center are bearers of moisture, the vertical motion caused by the dynamic action condenses the aqueous vapor; but if such currents are dry, the cyclone advances unattended by precipitation. Hence, it follows that rainfall is a secondary phenomenon, and is not sufficient of itself to produce true cyclonic gyrations. There are, on the other hand, many cases of copious precipitation without any attendant low pressure. Thus, on the front of an advancing cold wave there is often a long band of rain area stretching from the Great Lakes to the Gulf of Mexico, but without cyclonic formation, the precipitation being in fact caused by the upward lift of a warm southerly current which overflows the wedge-shaped cold wave in its southward movement. This is a dynamic uplift by overflow, instead of by vortical gyration, and it is sufficient to cause condensation and precipitation by the mechanical action of an underflowing stratum of very low temperature. Furthermore, on one side of a mountain range, as the Alps, rainfall is observed to occur in the midst of the high pressure, while on the other side of the mountains the atmosphere is clear and the pressure is relatively low, thus reversing the required conditions. In the summer season, local thunderstorms are quite as likely to happen in the midst of an area of high pressure as in that of low pressure, but here the vertical convection distinctly exists and arises from a superheating of the lower strata. If buoyancy of the lighter air is the principal cause of the gyration of cyclones, then we should expect to find a similar rotatory motion developed in the formation of cumulus clouds and thunderstorms in hot summer weather, when the vertical component is evidently strong. But, on the contrary, while the ascension of the heated air is clearly visible in these clouds, there is usually no evidence of gyration of the cyclonic type. It has been found by Hann's mountain observations and by the Berlin balloon ascensions that the temperature of the central portions of the cyclone is colder than the temperature in the midst of the anticyclone at the same levels. Hence, if the relative density of the air column is the source of cyclonic gyration, we perceive that this fact is in direct contradiction to the requirements of the condensation theory, which demands that the central column of the cyclone shall be warmer than its surroundings.

Since the advocates of the condensation theory of cyclones usually regard the generation of the tropical hurricanes as the best example of that source of gyratory energy, it may be proper to state that the observed facts do not appear to sustain the theory. For (1) there is no evidence of a decided increase in the local temperature at the center of hurricanes. In this connection it is believed that the sudden rise in temperature in the Manila hurricane of October 20, 1882, was due to the direct radiation of the sun through the calm eye of the storm; (2) the winds are not sufficiently changed in direction at the feeble ring of high pressure to conform to the Ferrel pericyclone; they should be turned through at least 90° more in azimuth; (3) the conditions of heated, saturated air prevail in the Tropics throughout the year, but the hurricanes are produced at certain seasons only, and these are the times when the counter currents of the trades are most active at their northern and southern limits. Dr. Hann rejects the rain theory, and adopts the counter current theory for huricanes: Lehrbuch der Meteorologie, pp. 563–566. It can be proved conclusively from observations that two counter currents flow together at the places where tornadoes are formed, where the tropical hurricanes are generated, and also where the cyclones of the middle latitudes are produced. These currents are especially active in the strata one or two miles above the ground, and this is probably the reason why they have not received due attention in constructing the theory of storms. It may be concluded that the local overheated central region does not exist in cyclones as the chief cause of their motion, and that the theories fail which depend upon it. There are, however, other serious difficulties of a mathematical nature to which attention must be directed.

FERREL'S LOCAL CYCLONE.

On page 595, and following, of the International Cloud Report, the fundamental formulæ and assumptions, as employed by Professor Ferrel in his discussion of the local cyclone, are summarized, and an abstract for our purposes, in the notation already described in Paper II of this series, Monthly Weather Review, February, 1902, p. 81, is given in the following lines:

Cylindrical equations of motion applicable to the local circulation in cyclones. See International Cloud Report, p. 502.

185.
$$-\frac{1}{\rho}\frac{\partial P}{\partial \varpi} = \frac{du}{dt} - \left(2n\cos\theta + \frac{v}{\varpi}\right)v + ku.$$
$$-\frac{1}{\rho}\frac{\partial P}{\varpi\partial\varphi} = \frac{dv}{dt} + \left(2n\cos\theta + \frac{v}{\varpi}\right)u + kv.$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial z} = \frac{dw}{dt} + g.$$

Assumptions made in duscussing these equations:

1. The temperature is a function of ϖ only, varies along the radius, but not with the altitude, and is symmetrical about the center.

2. The local cyclone is symmetrical about the axis of gyration, and is bounded by a cylindrical surface whose constant radius is ϖ_0.

3. The friction is proportional to the relative velocity of two adjoining strata.

4. The assumed law of the variation of temperature along the radius is as follows, the isotherms being circles about the center:

$$t = A_0 + \frac{1}{2}(t_c - t_0)\cos\frac{\varpi}{\varpi_0}\pi.$$

5. In integrating for the law of the preservation of areas it is assumed that there is no friction between the air and the surface of the earth.

6. All forces depending upon the vertical velocity of the currents can be neglected, $w = 0$.

7. $P_0 =$ the pressure for $h_0 = 0$.

The equations of motions become the following by applying these assumptions and transforming the pressure term:

397*b*. (1.)
$$-\frac{\partial \log P_0}{a\partial\varpi} = \frac{du}{dt} - \left(2n\cos\theta + \frac{v}{\varpi}\right)v + ku$$
$$- \frac{gh\,a}{\varpi_0(1+\alpha t)}\frac{1}{2}(t_c - t_0)\pi\sin\frac{\varpi}{\varpi_0}\pi;$$

(2.)
$$0 = \frac{dv}{dt} + \left(2n\cos\theta + \frac{v}{\varpi}\right)u + kv;$$

where $a = \dfrac{1}{gl(1+\alpha t)}$, by Table 64, 23; $t_c =$ temperature at the center; and $t_0 =$ temperature at the outer boundary of the cyclone.

As the result of the discussion of these two equations Ferrel deduces his cyclone which is represented in fig. 11. The corresponding cold-center cyclone is shown in fig. 12.

The first of the above assumptions regarding the distribution of the local temperature does not sufficiently conform to the data on the weather maps to be satisfactory, because the southeast section of a cyclone in the United States is usually much warmer than the northwest section. The symmetrical distribution of pressure about the center, where $-\frac{1}{\rho}\frac{\partial P}{\varpi \partial \varphi}=0$, is found in highly developed cyclones, and may be admitted in the analysis. The friction term is of minor importance with respect to the general theory of a cyclone which we are considering, and the vertical force derived from w may be neglected, though not the vertical velocity itself.

There are two entirely different methods of treating the second equation of motion,

$$397\,b.\ (2.)\qquad \frac{dv}{dt}+\left(2n\cos\theta+\frac{v}{\varpi}\right)u+kv=0,$$

and this is the parting of the ways between (1) Ferrel's theory and (2) the German theory. The primary question to be kept in mind is, does the result of the observations conform exactly to either of these theories? This equation can be integrated by omitting the friction term kv and *assigning an outer boundary;* or it may be solved by a simple transformation, *since two roots can be found*, and the discussion of the group of general equations of motion carried forward with these. The former method is Ferrel's, and the latter is that of the German school, namely, Guldberg and Mohn, Sprung, Oberbeck, Pockels, and others.

FERREL'S SOLUTION.

Neglecting the friction term, the equation 397*b* (2) can be transformed, by substituting $u=\frac{\partial \varpi}{\partial t}$ and multiplying by ϖ, into

$$2\,n\cos\theta\,.\,\varpi\frac{\partial\varpi}{\partial t}+\varpi\frac{dv}{dt}+v\frac{\partial\varpi}{\partial t}=0.$$

It should be noted that $\frac{dv}{dt}=\frac{\partial v}{\partial t}+\frac{u\partial v}{\partial\varpi}+\frac{v\partial v}{\varpi\partial\varphi}$.

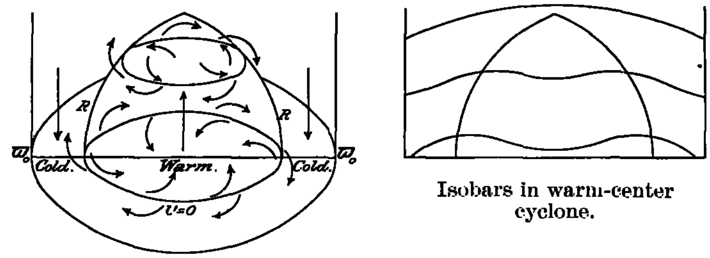
Ferrel integrates as if $\frac{dv}{dt}=\frac{\partial v}{\partial t}$, omitting the two other terms; indeed, Ferrel was neglectful about the distinction between the total and the partial differentials in several portions of his work. The integration gives $\varpi^2\left(n\cos\theta+\frac{v}{\varpi}\right)=c$, for each particle. Assigning an outer boundary ϖ_0 as the limit of integration, then, for the entire rotating mass, we deduce,

$$\varpi^2\left(n\cos\theta+\frac{v}{\varpi}\right)=\frac{1}{2}\varpi^2_0\,n\cos\theta;\ \text{and hence,}$$

$$v=\left(\frac{\varpi^2_0}{2\,\varpi^2}-1\right)\varpi\ n\cos\theta,$$

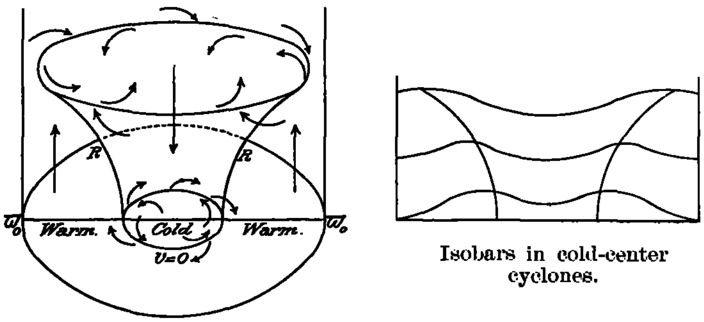
where $v=$ the tangential velocity at the distance ϖ from the axis of rotation. If $v=0$, $R=0.707\,\varpi_0$, where R is the radius of the circle at which the gyratory velocity reverses its direction. The locus of this R is indicated on fig. 11 for the warm-center cyclone and on fig. 12 for the cold-center cyclone; these two figures also show, in a general way, the circulation in this type of vortices. It will not be necessary to explain it further in this connection, but it is especially important to observe that Ferrel came to this vortex by the demands of his integration, and that he sought to uphold it by resorting to such physical sources of energy as seemed to be available. He had already applied an entirely similar process to his discussion of the circulation of the atmosphere of the earth over an entire hemisphere, but in that case it was, at least in part, justified by the fact that the air on the hemisphere continues to remain the same mass, so that integration between the pole and the plane of the equator was a proper procedure. Yet, in comparing this vortex with the circulation as displayed in figs. 6 and 7 of Paper III, we must consider the other objections besides the difficulty of accounting for local supply of heat in the central portions which is needed to keep the vortex in motion.

(1) Ferrel conceived the general cyclone on the hemisphere to be one with a cold center, since the poles are cold and the Tropics warm; and then the modification was made that the local cyclone is one with a warm center with the edges cooler than the middle portions. If a quantity of water be placed in a cylindrical vessel, and sawdust or some other material be scattered in it to show the lines of the circulation, and if this



Isobars in warm-center cyclone.

FIG. 11.—Ferrel's circulation in warm-center cyclones.



Isobars in cold-center cyclones.

FIG. 12.—Ferrel's circulation in cold-center cyclones.

be rotated on a turntable a form of motion can be produced quite similar to the one Ferrel proposed to explain the mechanism of storms. This circulation can be generated by any agency which will make a vertical current in the center of the fluid, as a lamp on the lower side, or a paddle screw at the top. A lump of ice on the center of a rotating plane will give a circulation which is like that of the general cyclone over the hemisphere. Now this experiment is open to at least three objections of a very serious nature when it is attempted to apply the lines of the model to the processes in nature. It is not enough to show that there is an inflow at the bottom and an outflow at the top, in logarithmic spiral curves, to conclude that the analogue is satisfactory. Therein lies an assumption which in fact begs the entire question. The great difficulty is that the circulation in the general cyclone consists of the same mass of air, which repeatedly passes through certain paths in consequence of the boundary conditions. The limited mass of fluid in the cylindrical vessel is also the same mass set in circulation, being bounded by the top and bottom surfaces and the curved sides, corresponding with the ground and top of the moving air and the plane of the equator. It has by no means been shown that the air concerned in the local cyclone consists of the same air moving over and over again in similar paths, and it is first necessary to do this in order to establish an analogue of that kind. The evidence from the cloud circulations proves that the cyclone is a form of circulation of the stationary type of configuration, through which fresh portions of the atmosphere continue to stream. If such is the case the analogue described above is inapplicable, and the deductions which have been so commonly drawn from it are quite incorrect. (2) There is no pericyclone discoverable in the records based upon many storms. Ferrel tried to show that the high-area pressures observed on the maps are the resultants of two or more overlapping pericyclones. But the detailed construction shown in Charts 15–35 of the International Cloud Report gives no support to this form of circulation. (3) Evidence of true cyclonic outflow at the top at some distance above the ground is probably entirely lacking. The cyclonic components of

Table 10, paper III, prove that the radial velocity is inward from the ground to the top of the cyclone. It is not our purpose at this point to explain the principles of the circulation that actually exists, but simply to indicate that the Ferrel cyclone, though perfectly possible under certain conditions, is not the type which storms follow in their construction. It is certain that the supposed analogue between the local and the general cyclone is not sustained by the evidence, and if the observed movement of the atmosphere can be accounted for on other principles, in conformity with the observations, it will only add to the force of the position here taken that the Ferrel local cyclone is merely one of many idealized cases. For these reasons we therefore are obliged to conclude that the Ferrel cyclone by no means conforms to the natural circulation, and need not be further considered. Indeed, Ferrel's teaching regarding the origin of cyclones and anticyclones should be eliminated from modern meteorology.

THE GERMAN SOLUTION.

If we make the abbreviation $\lambda = 2n \cos \theta$, retain the friction term, and make $\frac{dv}{dt} = \frac{\partial v}{\partial t}$, thus rejecting the two small terms, equation 397*b* (2) becomes:

422 $$\frac{\partial v}{\partial t} + \frac{uv}{\varpi} + \lambda u + kv = 0.$$

Taking $\frac{\partial v}{\partial t} = \frac{\partial v}{\partial \varpi}\,\frac{\partial \varpi}{\partial t} = u\,\frac{\partial v}{\partial \varpi}$, we obtain

437 $$u\,\frac{\partial v}{\partial \varpi} + \frac{uv}{\varpi} + \lambda u + kv = 0.$$

There are two solutions of this equation, as shown on pages 598 and 599 of the Cloud Report, namely:

First solution (inner).	Second solution (outer).
$u = -\frac{c}{2}\varpi.$	$u = -\frac{c}{\varpi}.$
$v = +\frac{\lambda}{k-c}\cdot\frac{c}{2}\varpi = -\frac{\lambda}{k-c}u.$	$v = +\frac{\lambda}{k}\cdot\frac{c}{\varpi} = -\frac{\lambda}{k}u.$

These can be expressed in two general laws:

(1.) Parabolic law.	(2.) Hyperbolic law.
$\frac{u}{\varpi} = -\frac{c}{2} =$ constant.	$u\varpi = -c =$ constant.
$\frac{v}{\varpi} = +\frac{\lambda}{k-c}\cdot\frac{c}{2} =$ constant.	$v\varpi = +\frac{\lambda}{k}c =$ constant.

These solutions are readily verified by substitution in the second equation of motion, 397*b*, and the two forms give rise, respectively, to parabolic surfaces on the inside of a certain circle, and to hyperbolic surfaces on the outside of it. Their discussion is given on page 509; an electrical analogue is explained on page 521, and they are further illustrated on pages 619 to 622 of the International Cloud Report. A diagram of the motion is shown in fig. 13 of the present paper. The result is that there is an outer region in which there is no vertical component, $w = 0$, and an inner region in which there is a vertical component which increases with the altitude, $w = cs$; see page 621, Cloud Report. These two regions are separated by a circle where the tangential component velocity, v, is a maximum; the velocity of rotation falls away to the center by the parabolic law, and also for an unlimited distance outward by the hyperbolic law. The inner region has the isobars separated from each other by distances conforming to the law, $d_1 = R^2 - \varpi_1^2$, where R is the radius of the circle of maximum velocity, and ϖ_1 the radii of the successive isobars; on the outside the distances between isobars are determined by $d_e = 2\,R^2 \log \frac{\varpi_e}{R}$; fig. 13 shows these relative distances and velocities.

Recalling the circulation depicted in Paper III, we are induced to make the following remarks:

The theory common to the German school of meteorologists is founded upon the assumption of a vertical central current, like the electric current in a wire, which generates the cyclonic circulation in the inner and the outer parts. Now, there are a series of difficulties and objections to this view, when it is attempted to apply it to the observations of the actual motions of the atmosphere, fully as serious as those which have been urged against Professor Ferrel's theory. (1) There is no sufficient evidence that the vertical current is of definite local origin and powerful enough to influence the enormous cyclonic

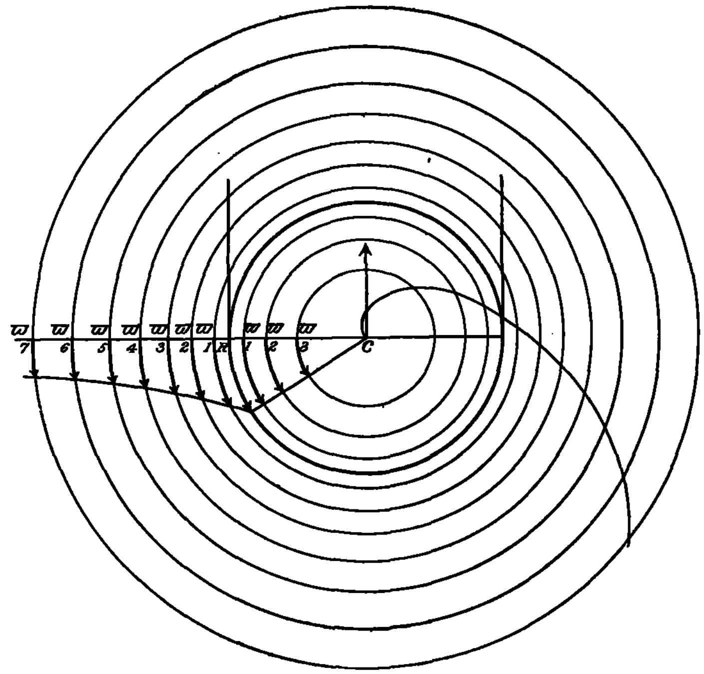


Fig. 13.—Oberbeck's circulation in warm-center cyclones.

disturbances extending horizontally to 1,000 miles in radius. These storms are very shallow compared with their width, say 3 or 4 to 1,000 at the greatest depth. An upward central current in a small inner region of 200 to 300 miles radius, even if locally produced, could hardly cause the disturbances observed on the weather maps. The chief difficulty has been to show that there is any sufficient cause for the existence of such a current, and the reasons already urged against that view hold here also, namely, that the disturbing isotherms are not circles, but their gradients lie athwart the cyclone, generally from SW to NE; that cyclones exist without precipitation; that rainfall does not necessarily produce cyclonic action; and that countercurrents from two specific directions, as NW and S, feed into the cyclone, which is not sustained from a supply equally distributed around a center. (2) The adoption of the inner and outer parts of the cyclone was due to the supposed necessity of avoiding infinitely great velocities at the center, if $u = -\frac{c}{\varpi}$ and $v = +\frac{\lambda}{k}\,\frac{c}{\varpi}$, as would occur for small values of ϖ. It will, of course, be necessary to show how this can be done by some other solution. Even if that is accomplished we find still other practical difficulties in the German solution having an inner and an outer part. This requires a maximum rotational velocity v at the boundary $\varpi = R$. But our observations give no support to this position any more than to Ferrel's theory that $v = 0$ at the boundary of the inner and the outer parts. A careful examination of our wind velocities shows that they increase steadily from the outer boundary toward the center, when a surface of discontinuity surrounding a calm center suddenly terminates the radial and tangential velocities.

The common occurrence of the central calm in hurricanes is sufficient proof of this point. Furthermore, an examination of the cyclonic components (u_2, v_2), Table 10 and fig. 10, shows that the tangential velocity v increases from the outside toward the center without any tendency to fall off. Certainly, in forecasting, no one expects to find the maximum velocities at 200 or 300 miles from the center. The two-part theory itself, although gradually reducing the velocity from a maximum at the boundary R to zero at the center, does not explain the existence of the central calm. (3) While the differential equation has two solutions which give some aspects plausible for this application, yet it is improbable that in such processes of nature as the circulation of the air there should be more than one law actually in operation. That the movement should suddenly jump from one system of forces to another is quite unlikely, unless cause can be shown for it. (4) In spite of skillful devices by which discontinuity in the rotation velocity was overcome, it is evident that there still remains a vertical discontinuity at the boundary, which becomes more and more pronounced with the increase in height above the surface, since $w=cz$. While it is hardly possible to actually observe the vertical components, yet the probabilities are that vertical motion sets in as soon as the isobars which surround the cyclone are closed up, and that all over this area there is a rising current. It may be laid down as a principle that where closed isobars exist there is an ascending or a descending current, according to the direction of the rotation. Where the isobars wander about without closing up, it may be assumed that there is no rising or descending air. In the case of cyclones this is confirmed by the general tendency of precipitation to occur over the entire region of the closed isobars. The preponderance on the eastern side over the western is due to the action of the general drift in the upper strata upon the circulation.

Hence, we conclude that while the Ferrel and the German vortices are each possible and may exist under certain conditions of boundary and distribution of heat, they do not agree with the cyclonic and anticyclonic circulation as given by the cloud observations of 1896–97. Although it is not possible to utilize the Ferrel vortex in further developments because the outer boundary is lacking, and though the German vortex, on the other hand, has apparently a closer application, yet even here it will be found difficult to avail ourselves of it without resorting to a modified method of analysis. We shall show, in part only, how this may be accomplished in the following papers, but the fact remains that the atmospheric circulation is usually too complex to be readily reduced to simple vortex motion of any kind. Hydrodynamic theories of stream lines must, on the other hand, be employed on a larger scale in the meteorology of the future than has been done in the past.

FERREL'S THEORY OF THE GENERAL CIRCULATION OVER A HEMISPHERE.

We can only briefly mention the principles which prevail in Ferrel's and in Oberbeck's solution for the circulation of the atmosphere over a hemisphere of the earth. In this case the boundaries are fixed, namely, the earth's surface, the plane of the equator, the topmost stratum of the atmosphere, and the pole of rotation. The heat distribution is such that the polar regions are cold and the Tropics warm. The primary idea adopted in the mathematical analysis is that of the so-called canal circulation, as, for example, when fluid in a long vessel with rectangular sides is artificially heated at one of its ends, so that the fluid rises at that end, falls at the other, moves in a horizontal direction from the warm end toward the cold end in the upper layers, but from the cold end to the warm end in the lower layers. In the same way the atmosphere is assumed to rise at the Tropics, move northward in the upper strata, fall in the polar zones, and flow southward along the surface of the earth. The effect of the contraction of the meridians, together with the rotation of the earth, is to introduce a complex torque effect, which causes the air to flow rapidly eastward in the temperate zones, especially in the upper strata, and westward in the tropical zones, especially in the lower strata. The general result is shown on fig. 14, for Ferrel's solution; and on fig. 15 the relative component velocities are given for Oberbeck's solution. These two methods of solution have some features in common, and also some of the results agree, and yet there is wide divergence in other respects, as will be indicated. The most conspicuous feature for us to note is that a neutral plane of velocity for the components u along the meridian is obtained, where there is no northward or southward velocity, but above it an increasing northward, and below it an increasing southward velocity proportional to the distance from this plane. We shall have to compare this view with the results of the observations as given in the data of the year 1896–97. The main features of Ferrel's solution of the general cyclone are contained in the following extracts from pages 588–590, International Cloud Report:

Polar equations of motion applicable to the general circulation over a hemisphere.

$$200. \quad -\frac{1}{\rho}\frac{\partial P}{r\partial\theta} = \frac{du}{dt} - \cos\theta\,(2n+\nu)\,v + \frac{uw}{r}; \text{ where } \nu = \frac{v}{r\sin\theta}.$$

$$-\frac{1}{\rho}\frac{\partial P}{r\sin\theta\,\partial\lambda} = \frac{dv}{dt} + \cos\theta\,(2n+\nu)\,u + \sin\theta\,(2n+\nu)\,w.$$

$$-\frac{1}{\rho}\frac{\partial P}{\partial r} = \frac{dw}{dt} - \sin\theta\,(2n+\nu)\,v - \frac{u^2}{r} + g.$$

Assumptions are made precisely analogous to those for the local cyclone, except that the temperature is expressed by the equation,

$$t = \Sigma A_s \cos s\theta, \text{ where,}$$

$$A_0 = 8.50°. \quad A_1 = -1.75°. \quad A_2 = -20.95°. \quad A_3 = -1.00°. \quad A_4 = -2.66°.$$

The equations of general motion take the form,

$$397a. \quad -\frac{\partial \log P_0}{a\partial x} = \frac{du}{dt} - \cos\theta\,(2n+\nu)\,v + ku - \frac{gh\,a}{r\,(1+at)}\,4\,A_2\sin\theta\cos\theta.$$

$$0 = \frac{dv}{dt} + \cos\theta\,(2n+\nu)\,u + kv.$$

The second equation admits of integration between the pole and the plane of the equator for the entire rotating mass of air, with the resulting equation for the velocity v,

$$v = \left(\frac{2}{3}\cdot\frac{1}{\sin\theta} - \sin\theta\right) r\,n.$$

If $v=0$, $\theta = 54°\ 44'$, and the latitude $\varphi = 35°\ 16'$ where the velocity reverses direction at the surface.

The locus of $v=0$, above the surface forms an arch over the equator, as shown in fig 14. This is analogous to the curve R of fig. 12 in the cold-center cyclone.

TABLE 13.—*Theoretical west-east velocities.*

Latitude ϕ.	v in miles per hour.
90°	+ ∞ eastward.
80°	+3807 "
70°	+1669 "
60°	+ 865 "
50°	+ 410 "
40°	+ 108 "
35° 16'	0 "
30°	— 100 westward.
20°	— 239 "
10°	— 320 "
Equator 0°	— 346 "

Professor Ferrel was governed in his method of integration by the theorem of the preservation of areas, $\varpi\, v$=constant, depending chiefly upon the velocity v along the parallels of latitude, in order that the sum of the momenta might be equal to zero, $\Sigma mv=0$, for the entire earth, which is a necessary result. However, it led to impossible velocities, v, as shown in Table 13, where excessive westward velocities prevail in the Tropics, and enormous eastward velocities in the polar regions. If we may assume that the location of the neutral plane is determined by the fact that half the air moves northward over it, and half the air southward under it, then the height of this plane should be about 6 kilometers=3.7 miles above the ground, as given in Table 14.

TABLE 14.—*Vertical diminution of pressure.*

Height. H.	Pressure. B.	Per cent.	Height. H.	Pressure. B.	Per cent.
km.	*mm.*		*km.*	*mm.*	
0	760	100.0	8	280	36.9
1	671	88.3	9	247	32.6
2	591	77.8	10 (Cirrus)	218	28.7
3	523	68.9	11	193	25.3
4	461	60.7	12	170	22.4
5	407	53.6	13	150	19.8
6 (A. Cu.)	359	47.3	14	133	17.5
7	317	41.8	15	117	15.4

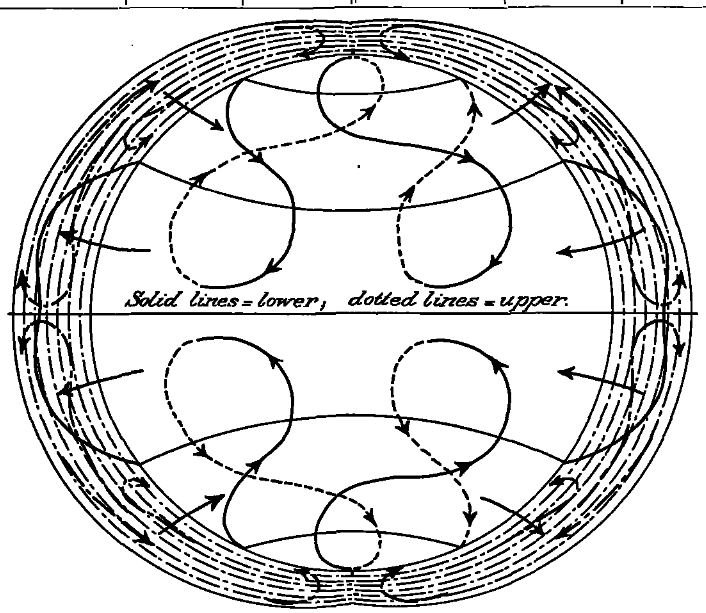


FIG. 14.—Ferrel's general cyclone.

Ferrel attempted to show that the excessive east-west velocities could be reduced to proper proportions by introducing a coefficient of friction, and considering that the sum of the moments Σmvk above the neutral plane must be much greater than the Σmv_0k below that plane. The excess of energy $\Sigma mvk - \Sigma mv_0k = E$, must be used up in overcoming the motion of the atmosphere, employing the term friction to include the forces that retard circulation by internal turbulent motion, or by the action of the adjacent discontinuous surfaces of the larger streams. It is evident, however, even supposing this theory correct, that this source of retardation is by no means sufficient to overcome the great amount of energy which must be consumed in motion to equalize the heat energy derived from the solar radiation in the Tropics. Professor Ferrel never executed a complete integration involving all the component equations of motion, but merely discussed his several equations under different relative conditions, and thus drew out of them certain conceptions of the general circulation of the atmosphere which it was easy to show harmonized fairly well with many of the facts which were known to him at the time of his study, now

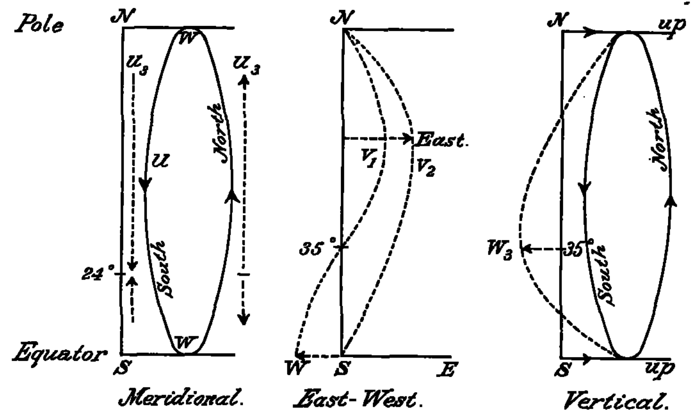


FIG. 15.—Oberbeck's component motions in the general cyclone.

nearly twenty years ago. It has become increasingly difficult, however, to believe that this solution is really as satisfactory as was then supposed.

OBERBECK'S SOLUTION OF THE GENERAL CIRCULATION.

Oberbeck attacked the same problem by a more complete analysis, and reached conclusions which in general accord with those of Ferrel, but differ from his in important particulars. He subdivided the total pressure of the atmosphere into partial pressures, and deduced a series of component velocities which could be computed by means of coefficients distributed at equidistant intervals throughout the atmosphere. An upper boundary was assumed for the atmosphere, but the solution was conducted in such a manner that this limiting stratum, whose height is H, could be changed in relation to the radius of the earth, R. The equations of motion were constructed for an origin at the center of the earth, while Ferrel's origin was on the surface, but the two systems of equations can be shown to be equivalent, so that the mathematical starting point is practically the same in both. Oberbeck held all the components together in one system, and hence, by not discussing them separately, could arrive at some conclusions which are really more instructive than Ferrel's. Yet it will be easily seen that these do not conform sufficiently well to the data of observation to be accepted as the complete solution of the problem.

Taking the component equations and notation given on pages 591–593 of the International Cloud Report, Oberbeck's solution for the component velocities is as follows:

South:

$$u = C\, 6 \cos\theta \sin\theta \frac{\sigma}{48}(6h^2 - 15h\sigma + 8\sigma^2).$$

$$u_3 = \frac{2n}{\kappa} R^3 \cos\theta \sin\theta\, (\nu_1 + 2\nu_2 - 7\nu_1 \cos^2\theta) \frac{\sigma}{48}(6h^2 - 15h\sigma + 8\sigma^2).$$

East:

$$v_1 = D \sin\theta\, (1 - 3\cos^2\theta) \frac{\sigma}{480}(-9h^5 + 15h^2\sigma^3 - 15h\sigma^4 + 4\sigma^5).$$

$$v_2 = D\ 6 \cos^2\theta \frac{\sigma}{960}(20h^2\sigma^2 - 25h\sigma^3 + 8\sigma^4).$$

Zenith:

$$w = C\,(1 - 3\cos^2\theta) \frac{\sigma}{8}(h - \sigma)(3h\sigma - 2\sigma^2).$$

$$w_3 = \frac{2n}{\kappa} R^3 [\nu_1 + 2\nu_2 - 6\,(4\nu_1 + \nu_2)\cos^2\theta + 35\nu_1 \cos^4\theta]$$
$$\times \frac{\sigma^2}{48}(h - \sigma)(3h - 2\sigma.)$$

$$h = \frac{H}{R} = \frac{1}{100};\ \sigma = \left(\frac{0}{10}, \frac{1}{10}, \frac{2}{10} \cdots \frac{10}{10}\right)h; \quad C = 0.5429\ R^2.$$

$D = 0.00008532\ R^4$.

$r = R + Rh = R + H =$ radius of earth + height of atmosphere.

$r = R + R\sigma = R + \frac{n}{10} Rh =$ radius of earth + height of the stratum σ; h is the maximum value of σ.

Assumed law of relative angular velocity, $\nu = \nu_1 \cos^2 \theta - \nu_2$.

Tables 15, 16, 17, give the relative coefficients of the component velocities in the direction of the meridians, the parallels of latitude, and in the vertical, respectively. By assigning values to H the different velocities may be computed.

I have not been successful in obtaining such velocities as will harmonize at all well with the known movements of the atmosphere, and I have, therefore, been led to distrust the

TABLE 15.

I. *Components on the meridians due to the rotation of the earth.*

u = horizontal currents on meridians. + = south, — = north.

Coefficients of $C\frac{H^3}{R^3} = .5429 \frac{H^3}{R} = \begin{cases} 2.203 \times 10^7 \text{ for } H = 63700^m. \\ 2.203 \times 10^4 \text{ for } H = 6370. \end{cases}$

θ	0°	10°	20°	30°	45°	50°	60°	70°	80°	90°
σ = 1.0	.0000	—.0213	—.0401	—.0540	—.0614	—.0614	—.0540	—.0401	—.0213	.0000
.9	.0000	— 196	— 368	— 496	— 564	— 564	— 496	— 368	— 196	.0000
.8	.0000	— 151	— 283	— 382	— 434	— 434	— 382	— 283	— 151	.0000
.7	.0000	— 87	— 164	— 221	— 251	— 251	— 221	— 164	— 87	.0000
.6	.0000	— 15	— 29	— 39	— 44	— 44	— 39	— 29	— 15	.0000
.5	.0000	+ 53	+ 100	+ 135	+ 154	+ 154	+ 135	+ 100	+ 53	.0000
.4	.0000	+ 110	+ 206	+ 278	+ 316	+ 316	+ 278	+ 206	+ 110	.0000
.3	.0000	+ 143	+ 268	+ 361	+ 411	+ 411	+ 361	+ 268	+ 143	.0000
.2	.0000	+ 142	+ 266	+ 359	+ 408	+ 408	+ 359	+ 266	+ 142	.0000
.1	.0000	+ 97	+ 183	+ 247	+ 281	+ 281	+ 247	+ 183	+ 97	.0000
.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

II. *Components on the meridians due to the relative motion of the atmosphere.*

u_3 = horizontal currents on meridians. + = south, — = north.

Coefficients of $\frac{2n}{\kappa} R^3 \frac{H^3}{R^3} = .1571 \times 10^{-9} H^3 = \begin{cases} .4062 \times 10^5 \text{ for } H = 63700^m. \\ .4062 \times 10^2 \text{ for } H = 6370. \end{cases}$

θ	0°	10°	20°	30°	45°	50°	60°	70°	80°	90°
σ = 1.0	.0000	—.0426	—.0713	—.0782	—.0641	—.0376	—.0112	+.0050	+.0073	.0000
.9	.0000	— 392	— 655	— 718	— 588	— 346	— 103	+ 46	+ 67	.0000
.8	.0000	— 301	— 504	— 553	— 452	— 266	— 79	+ 35	+ 51	.0000
.7	.0000	— 174	— 292	— 320	— 262	— 154	— 46	+ 20	+ 30	.0000
.6	.0000	— 31	— 51	— 56	— 46	— 27	— 8	+ 4	+ 5	.0000
.5	.0000	+ 107	+ 178	+ 196	+ 160	+ 94	+ 28	— 12	— 18	.0000
.4	.0000	+ 219	+ 367	+ 402	+ 330	+ 194	+ 58	— 26	— 37	.0000
.3	.0000	+ 285	+ 477	+ 523	+ 428	+ 252	+ 75	— 33	— 49	.0000
.2	.0000	+ 283	+ 473	+ 519	+ 425	+ 250	+ 75	— 33	— 48	.0000
.1	.0000	+ 195	+ 327	+ 357	+ 293	+ 172	+ 51	— 23	— 33	.0000
.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

TABLE 16.

I. *First components on the parallels due to the rotation of the earth.*

v_1 = horizontal currents on parallels. + = east, — = west.

Coefficients of $D\frac{H^6}{R^6} = .8532 \times 10^{-4} \frac{H^6}{R^2} = \begin{cases} 1.405 \times 10^{11} \text{ for } H = 63700^m. \\ 1.405 \times 10^5 \text{ for } H = 6370. \end{cases}$

θ	0°	10°	20°	30°	45°	50°	60°	70°	80°	90°
σ = 1.0	.00000	+.00346	+.00589	+.00651	+.00509	+.00191	—.00226	—.00636	—.00934	—.01042
.9	.00000	+ 345	+ 588	+ 650	+ 510	+ 190	— 226	— 634	— 932	— 1040
.8	.00000	+ 341	+ 580	+ 641	+ 502	+ 188	— 223	— 626	— 919	— 1026
.7	.00000	+ 328	+ 559	+ 618	+ 484	+ 181	— 215	— 603	— 884	— 989
.6	.00000	+ 307	+ 522	+ 578	+ 452	+ 169	— 201	— 564	— 828	— 924
.5	.00000	+ 275	+ 467	+ 517	+ 404	+ 151	— 179	— 504	— 741	— 827
.4	.00000	+ 231	+ 395	+ 437	+ 342	+ 128	— 152	— 426	— 626	— 699
.3	.00000	+ 181	+ 307	+ 340	+ 266	+ 100	— 118	— 332	— 487	— 544
.2	.00000	+ 123	+ 210	+ 232	+ 181	+ 68	— 81	— 226	— 332	— 371
.1	.00000	+ 62	+ 106	+ 117	+ 91	+ 34	— 41	— 114	— 168	— 187
.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000

TABLE 16—Continued.

II. *Second components on the parallels due to the rotation of the earth.*

v_2 = horizontal currents on parallels. + = east, — = west.

Coefficients of $D\frac{H^5}{R^5} = .8532 \times 10^{-4}\frac{H^5}{R} = \begin{cases} 1.405 \times 10^{13} \text{ for } H = 63700^m. \\ 1.405 \times 10^{8} \text{ for } H = 6370. \end{cases}$

θ	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
σ=1.0	.00000	+.00315	+.00565	+.00702	+.00706	+.00592	+.00404	+.00206	+.00056	.00000
.9	.00000	+ 305	+ 547	+ 680	+ 683	+ 573	+ 391	+ 199	+ 54	.00000
.8	.00000	+ 276	+ 495	+ 614	+ 618	+ 518	+ 354	+ 180	+ 49	.00000
.7	.00000	+ 231	+ 415	+ 515	+ 518	+ 435	+ 297	+ 157	+ 41	.00000
.6	.00000	+ 179	+ 321	+ 398	+ 401	+ 336	+ 229	+ 117	+ 32	.00000
.5	.00000	+ 125	+ 225	+ 279	+ 281	+ 235	+ 161	+ 82	+ 22	.00000
.4	.00000	+ 76	+ 136	+ 169	+ 170	+ 142	+ 97	+ 50	+ 13	.00000
.3	.00000	+ 37	+ 67	+ 83	+ 84	+ 70	+ 48	+ 24	+ 7	.00000
.2	.00000	+ 13	+ 24	+ 29	+ 29	+ 25	+ 17	+ 9	+ 2	.00000
.1	.00000	+ 2	+ 4	+ 4	+ 5	+ 4	+ 3	+ 1	0	.00000
.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000

TABLE 17.

I. *Vertical components due to the rotation of the earth.*

w = vertical currents. + = ascending. — = descending.

Coefficients of $C\frac{H^4}{R^4} = .5429\frac{H^4}{R^2} = \begin{cases} 2.203 \times 10^{5} \text{ for } H = 63700^m. \\ 2.203 \times 10 \text{ for } H = 6370. \end{cases}$

θ	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
σ=1.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.9	—.0244	—.0233	—.0202	—.0152	—.0093	—.0029	+.0030	+.0079	+.0111	+.0122
.8	— 448	— 428	— 370	— 280	— 170	— 54	+ 56	+ 145	+ 204	+ 224
.7	— 588	— 562	— 486	— 368	— 224	— 70	+ 74	+ 191	+ 268	+ 294
.6	— 648	— 619	— 535	— 405	— 247	— 78	+ 81	+ 210	+ 295	+ 324
.5	— 624	— 596	— 515	— 390	— 237	— 75	+ 78	+ 202	+ 284	+ 312
.4	— 528	— 504	— 436	— 330	— 201	— 63	+ 66	+ 171	+ 240	+ 264
.3	— 378	— 361	— 312	— 236	— 144	— 45	+ 47	+ 123	+ 172	+ 189
.2	— 208	— 199	— 172	— 130	— 79	— 24	+ 26	+ 68	+ 95	+ 104
.1	— 64	— 61	— 53	— 40	— 24	— 8	+ 8	+ 21	+ 29	+ 32
.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

II. *Vertical components due to the relative motion of the atmosphere.*

w_3 = vertical currents. + = ascending. — = descending.

Coefficients of $\frac{2n}{\kappa}R^3\frac{H^4}{R^4} = .1571 \times 10^{-9}\frac{H^4}{R} = \begin{cases} .4062 \times 10^{8} \text{ for } H = 63700^m. \\ .4062 \times 10^{-1} \text{ for } H = 6370 \end{cases}$

θ	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
σ=1.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.9	+.0502	+.0444	+.0294	+.0106	—.0050	—.0128	—.0119	—.0053	+.0020	+.0050
.8	+ 927	+ 820	+ 542	+ 197	— 92	— 237	— 220	— 97	+ 37	+ 93
.7	+ 1217	+ 1077	+ 712	+ 258	— 121	— 311	— 289	— 127	+ 49	+ 122
.6	+ 1341	+ 1187	+ 785	+ 285	— 133	— 343	— 319	— 140	+ 53	+ 134
.5	+ 1294	+ 1145	+ 757	+ 275	— 129	— 331	— 307	— 135	+ 52	+ 130
.4	+ 1093	+ 967	+ 639	+ 232	— 109	— 279	— 260	— 114	+ 44	+ 110
.3	+ 782	+ 692	+ 458	+ 166	— 78	— 200	— 186	— 82	+ 31	+ 78
.2	+ 430	+ 380	+ 251	+ 91	— 43	— 110	— 102	— 45	+ 17	+ 43
.1	+ 132	+ 117	+ 77	+ 28	— 13	— 34	— 31	— 14	+ 5	+ 13
.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

validity of this "canal theory" of the circulation, as explained in my report. An inspection of the coefficients, Table 15, shows that in the case of the meridian components, u, the upper northward and the lower southward circulation as deduced for the rotating earth, is somewhat modified by the component u_3 depending on the motion of the atmosphere relative to the earth itself. The component v along the parallels of latitude, Table 16, differs radically from Ferrel's result, and it should be carefully noted. Oberbeck divides the eastward drift in higher latitudes from the westward drift in lower latitudes at the parallel 35°. The eastward v is a maximum in the neighborhood of latitude 60°, $\theta = 30°$, but vanishes at the poles. This is exactly contrary to Ferrel's result, which made the velocity v a maximum at the pole, before the assumed modification by friction was applied. Oberbeck makes the westward drift a maximum at the plane of the equator, which is certainly not in conformity with the observations. He also makes the westward velocity increase at the equator from the surface to the upper boundary, and show no sign of a reversal from westward to eastward at a moderate elevation, as is generally believed to be the fact, judging from certain well known motions of the air observed in the Tropics.

The United States Weather Bureau has been conducting a series of nephoscope observations in the West Indies for the past three years, and it is hoped that the discussion of these observations, soon to be undertaken, will give us some definite information on this important point.

The second term v_2 modifies v_1, but the two combined, $v = v_1 + v_2$, sustain the conclusions just mentioned. This feature of Oberbeck's solution is so far from conforming to the observed motions of the atmosphere that it seems to me to be inferior in value to Ferrel's for the Tropics. Ferrel's arch over the Tropics, shown in fig. 14, is probably a fact, and if this is so, then the only serious modification required in Ferrel's treatment is to show how the excessive eastward drift in the mid-latitude and polar zones can be effectively checked. It is evident that there must be a large amount of energy available for use in the construction of local cyclones and anticyclones, and that there is, therefore, no pressing need to refer the energy of these motions to any local supply of heat, as is done by those who extend to cyclones the theory of the latent heat of condensation from precipitation originally devised by Espy to explain cumulus clouds and thunderstorms. The components w and w_3, Table 17, show that there is an ascending current in the Tropics, and a descending current in the higher latitudes. Thus, as the result of the theoretical discussion in general, the canal theory has several of its features verified, and yet there are serious discrepancies inherent in both Ferrel's and Oberbeck's solutions.

My statement has suggested by implication that there exists an important principle which has been neglected by these meteorologists. They have each discussed the general and the local cyclones as if they were in a sense *independent of one another*, since separate sources of heat energy are assigned to each, and two characteristic laws of circulation are deduced therefrom. It is much more natural to suppose that these two systems are mutually interdependent, and that the excess of energy of the general cyclone is transformed into the driving forces of the local circulation; also, that the acquired motion of the local cyclone reacts upon and retards the excess of motion of the general cyclone in the temperate zones. The subject becomes, however, excessively complex, and I can only attempt to sketch in general terms in my next paper an outline of this view, hoping some other time to be able to supplement it with a more suitable mathematical analysis, when the study of the observations now in hand has been advanced more nearly to completion.

REVISION OF WOLF'S SUN-SPOT RELATIVE-NUMBERS.

By Prof. A. WOLFER, Zurich, dated March 29, 1902.

The next number (XCIII) of the Astronomische Mittheilungen will contain a new edition of Wolf's table of relative numbers, in which not only will some inaccuracies of the earlier tables—partly errors of computation, partly typographical errors—be expunged, but those older observations that have come to light since the tables were compiled, but have not yet been worked up, will be used in the revision. For the most part these new observations were made at Kremsmünster during the years 1802–1830 and have furnished a very valuable addition to the record of sun spots. I now send you a copy of this corrected and completed series (Table 1), entitled "Observed sun-spot relative-numbers." This table, extending from 1749–1901, replaces that published by Wolf in 1880 in No. L of the Astronomische Mittheilungen, as well as the various copies which afterwards appeared in the Meteorologische Zeitschrift and in the Memorie della Societá degli Spettroscopisti Italiani. [It also replaces the table on pages 505–506, MONTHLY WEATHER REVIEW, November 1901.] It contains no error, and is now to be regarded as definitive so long as the complete new reduction of the whole amount of observational material is not executed, for which the preliminary work is now going on; this will, however, apparently require five or six years more.[1]

Those numbers in the above-mentioned table that are entered in heavy-faced type depend wholly on actual observations; those in light-faced type depend in great part upon actual observations, yet also have in part been obtained by means of graphic interpolations between the days of any month that contained observations with considerable gaps between them; the interpolated numbers were combined with the observed numbers in the computation of the monthly means. Only a very few monthly means, in the eighteenth century, depend entirely upon interpolations; by far the larger number are based upon actual observations. But every monthly mean in which even a single interpolated number has been used is shown by light-faced type; in this respect the distinction may have been too rigorous rather than indulgent, and the light-faced type are, therefore, in no sense to be regarded as an indication of the unreliability of the corresponding numbers.

I have thought that it would perhaps be agreeable to you also to possess new editions of the two other tables that are based upon the preceding, namely, the table of "Smoothed relative numbers" and that of the "Epochs of maxima and minima," which are directly deduced from the preceding. In No. XLII of his Astronomische Mittheilungen Wolf has published the smoothed numbers up to 1876, inclusive, and reproductions of these are found in various periodicals and other publications. But there are some errors of computation in this table, and numerous typographical errors occur in the reproductions.

The smoothed relative numbers of Table 2 present the mean course of the spot phenomena; that is to say, without the numerous secondary short-periodical variations that really occur in addition to the 11-year variation. Investigations into the general course of the phenomena and the periods of a higher order should, therefore, be based upon these smoothed numbers and not on those observed. The method of formation of these numbers has been explained by Wolf, in No. XLII of his Astronomische Mittheilungen. The mean of every twelve consecutive observed monthly relative numbers is taken, and every pair of two consecutive means is again united into one mean value according to the following scheme:

1/12 (I + II + III . . . . + XII) = n_1, for epoch July 1.
1/12 (II + III + IV . . . . . . + XII + I) = n_2, for epoch August 1.
1/2 ($n_1 + n_2$) = r, which is the smoothed number for mid-July.

This method of smoothing is conformable to that which Wolf has used for eliminating the annual period of the variations of magnetic declination when comparing the latter with the solar spots. This consideration was not necessary for the relative numbers but the combination of twelve months into one mean has been adopted in order to secure a uniform method of treating both phenomena. Table 2, which contains these

[1] As Professor Wolfer's revision furnishes us with sun-spot numbers that replace all previous publications the Editor has reproduced them in graphic form on figs. 2 and 3. In these figures the light line represents the observed numbers of Table 1, the heavy line the smoothed numbers of Table 2.—ED.