PROBLEMS IN METEOROLOGY.

By C. F. VON HERRMANN, Section Director. Dated Baltimore, Md., June 9, 1906.

The use of mathematics in meteorology has often been discust, either with reference to the application of methods of higher analysis to the solution of the intricate problems presented by the dynamics of the atmosphere, or to the introduction of problems in meteorology as illustrative examples in courses of higher mathematics. Even in elementary work, however, for purposes of serious instruction in meteorology, in which many officials of the Weather Bureau are now engaged, precision and dignity would be given to a course by the introduction, as laboratory work, in addition to the usual exercises in map making, etc., of examples requiring only elementary mathematics for their solution. What student could forget that the coefficient of expansion of air is 0.00367 or 1/273, if he were required to calculate the weight of a cubic meter of air at different temperatures? Or who could forget that the adiabatic rate of decrease of temperature with elevation for dry air is 1° C. for 100 meters, if he has been taught, by simple mathematical analysis, how the result is obtained? Those who are carrying on courses of instruction in meteorology (in distinction from popular lecture work) will find that the use of numerous examples will greatly stimulate the interest of the student, and help to elevate the subject to the rank of an exact science.

Unfortunately there are no text-books of elementary meteorology which give examples for solution. In Ferrel's "Recent advances in meteorology", Annual Report of the Chief Signal Officer for 1885, numerous examples are given, but they are generally too advanced for elementary work, tho many of them may readily be simplified. For the purpose suggested a number of examples have been collected, requiring only the elements of algebra and trigonometry for their solution; these are stated below. It is advantageous in all problems to use the centigrade degree, the metric system of measurements, and as the unit of heat the small calorie, which is more definite than the British thermal unit. The solutions are stated in the most elementary language, but more advanced problems will follow if these are favorably received.

Problem 1.—Calculate the mass of the atmosphere.

Solution.—If the atmosphere had the same density thruout which it has under the standard conditions ordinarily adopted (pressure 760 mm., temperature 0° C., and latitude 45°), its height would be 7991 meters (h), which is the height of a homogeneous atmosphere of air. One cubic meter of air of that density weighs 1.29305 kilograms.

From geometry, the volume of a sphere is $4/3\,\pi R^3$, in which π is 3.1416, and R the mean radius of the earth in meters or 6370191 meters (Bigelow).

The volume of the earth including the atmosphere, less the volume of the earth alone, will give the volume of the atmosphere in cubic meters, or $4/3\,\pi\,(R+h)^3 - 4/3\,\pi\,R^3$ equals volume of atmosphere in cubic meters.

Factoring: $4/3\ \pi\,(3\,h\,R^2 + 3\,h^2\,R + h^3)$, or $4/3 \times 3.1416\ (3 \times 7991 \times 6370191^2 + 3 \times 7991^2 \times 6370191 + 7991^3)$, which is equal to 4080×10^{15} cubic meters.

Since 1 cubic meter of air weighs 1.293 kilograms, then the weight of the atmosphere is $4080 \times 10^{15} \times 1.293$, or $5{,}275.46 \times 10^{15}$ kilograms.

This is $\frac{1}{1125000}$ of the mass of the solid earth. (MONTHLY WEATHER REVIEW, February, 1899, page 58–59.)[1]

The weight of the atmosphere, found in the manner above described, is somewhat greater than the result found in the MONTHLY WEATHER REVIEW, February, 1899, because the mean barometric pressure is here assumed to be 760 millimeters or 29.92 inches, instead of 29.90 inches.

According to Hann, Lehrbuch, second edition, page 9, if the heights of the continents are taken into consideration, the normal pressure would reduce to 740 millimeters (homogeneous atmosphere 7790 meters), but this should be increased about 0.48 per cent for the decrease of gravity with elevation (giving homogeneous atmosphere of 7827 meters); with this figure the mass of the atmosphere is 5200×10^{15} kilograms.

Problem 2.—The density of hydrogen is 0.0696; calculate the height of a homogeneous atmosphere of hydrogen.

Solution.—Let the standard atmospheric pressure, or height of the mercurial column in centimeters, be 76.

Let the density of mercury, or the weight of a cubic centimeter in grams, be 13.596 (Regnault).

Let the relative density of hydrogen, that of air being 1, at temperature 0°C and under standard pressure, be 0.0696.

Let the density of air under standard conditions, or the weight of a cubic centimeter in grams, be 0.001293.

Then 0.001293×0.0696 is the weight of a cubic centimeter of hydrogen, i. e., 0.00008993 grams.

Since the height of a column of gas of uniform density and the height of the mercurial column are inversely as the densities, we have the height of a homogeneous column of hydrogen,

$$\frac{76 \times 13.596}{0.00008993} = 11{,}481{,}066 \text{ centimeters or } 114{,}811 \text{ meters.}$$

For air, the weight of a cubic centimeter is 0.001293; so that the height of a homogeneous atmosphere of air is

$$\frac{76 \times 13.596}{0.001293} = 7991.04 \text{ meters.}$$

	Density.		Meters.
Nitrogen	0.96737	homogeneous atmosphere	8,261.
Oxygen	1.10535	homogeneous atmosphere	7,229.
Argon	1.37752	homogeneous atmosphere	5,801.
Carbon dioxid..	1.5291	homogeneous atmosphere	5,226.
Helium	0.1406	homogeneous atmosphere	56,834.
Aqueous vapor..	0.622	homogeneous atmosphere	12,847.

Problem 3.—The twilight arch disappears when the sun is 18° below the western horizon; calculate the height of the atmosphere.

Solution.—See fig. 1. At the moment when twilight ceases, the last visible particle of air will be just halfway between the observer and the point nearest the sun where it is just setting.

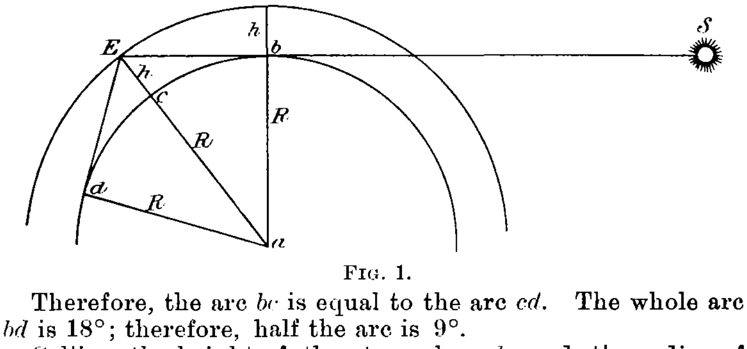

FIG. 1.

Therefore, the arc bc is equal to the arc cd. The whole arc bd is 18°; therefore, half the arc is 9°.

Calling the height of the atmosphere h, and the radius of the earth R, we have from the right-angled triangle abe, by simple definition in trigonometry, ae/ab is the secant of bae.

$$ae = ab \times \text{secant } bae.$$

Since $ae = R + h$, and $ab = R$ we have

$$R + h = R \times \text{secant } 9°$$

$$h = R \text{ secant } 9° - R = R\ (\text{secant } 9° - 1).$$

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[1] The figures in Monthly Weather Review, Vol. XXVII, p. 59, require the following corrections: For 198,940,000 read 196,940,000 square miles; for 10,392 read 11,602; for 1/1,125,000 read 1/1,132,400. The mass of the atmosphere would, therefore, be $11{,}602 \times 10^{15}$ pounds, or $5{,}263 \times 10^{15}$ kilograms. The difference between this older computation and that in the above text is traceable to the differences in the assumed data, some of which are slightly uncertain.—EDITOR.

Secant 9° is 1.0125; therefore, the last expression reduces to $h = 0.0125\ R$.

In which $R = 6{,}370{,}191$ meters or 20,899,600 feet.[2] $6370191 \times 0.0125 = 79627.4$ meters or 80 kilometers—about 50 miles.

This must be reduced by about 1/5 on account of refraction, making the height of the atmosphere about 40 miles. See Young's General Astronomy, 1889, pages 68–69.

Problem 4.—From the known rate of increase of temperature with increasing depth in the earth's crust, calculate the heat annually received at the surface and the thickness of ice which it will melt.

Solution.—The calculation of the heat received from the interior is made by multiplying the temperature gradient by the average thermal conductivity of the soil. This latter is about 0.006 gram-calories per square centimeter per second. The gradient is 1° C. for 35 meters, or 0.000286° C. for each centimeter. This multiplied by 0.006 gives the amount of heat received per second on each square centimeter of the earth's surface from the internal heat. It is equal to 0.000001716 gram-calories.

As the year has 31,556,926 seconds,[3] the amount of heat received per year on each square centimeter is $0.000001716 \times 31{,}556{,}926$, or 54.2 gram-calories.

The thickness of ice melted or water evaporated by 54.2 calories is based on the number of heat units required to melt a cubic centimeter of ice or evaporate a cubic centimeter (gram) of water.

The latent heat of fusion of ice is 80.02 calories, which is the amount of heat required to melt 1 gram. A cubic centimeter of ice, however, only weighs 0.917 gram, and to melt it requires only 80×0.917, or 73.4 calories.

Then the heat received per annum per square centimeter from the interior, or 54.2 calories, will melt only 54.2/73.4 or 0.74 cubic centimeters of ice, i. e., a piece one centimeter square and only 7 millimeters thick.

The latent heat of vaporization of water is in round numbers about 600 calories, so 54.2 calories would evaporate only 54.2/600 or about 0.09 grams of water per annum. See Hann, Lehrbuch der Meteorologie, first edition, page 23.

Problem 5.—Given, in certain cases, the temperature gradient in the soil and its conductivity, calculate the amount of heat transmitted to the air, and how much the air may be warmed thereby.

Solution.—At Tiflis in January the mean temperature of the soil at a depth of 0.1 meter is 1.1° C.; at 0.2 meters it is 1.6° C., and at 0.4 meters it is 2.9° C. Therefore the temperature increases with depth at the rate of 2.5° C. per 40 centimeters, or 0.06 °C. per centimeter.

The calorimetric conductivity of the soil, i. e., the quantity of heat in calories which will pass in one second thru a centimeter cube when the difference in temperature of the two faces is 1° C., is 0.006; this gives 0.36 calories per minute.

The amount of heat conducted to the surface by the soil is equal to the temperature gradient, multiplied by the conductivity of the soil, multiplied by the time.

For the case given: $0.36 \times 0.06 \times 1440$, which is equal to 31.1 calories per day.

The specific heat of air is 0.238 calories, i. e., one gram of air requires 0.238 calories to increase its temperature 1° C. One cubic centimeter of air weighs only 0.001293 grams, and requires, therefore, only 0.001293×0.238, or 0.000307 calories to raise its temperature 1° C.

Therefore the heat given to the air per square centimeter in this case would raise the temperature of 31.1/0.000307, or approximately 100,000 cubic centimeters of air, by 1° C. in one day—provided it were all absorbed by the air and not lost by radiation. This is equivalent to a horizontal layer one kilometer deep. See Hann, Lehrbuch, page 85.

Problem 6.—Calculate the heat received annually by the entire earth, assuming the solar constant to be 3 calories per square centimeter per minute.

Solution.—The solar constant 3 means that each square centimeter would receive per minute 3 small calories of heat, if there were no atmosphere, assuming the receiving surface to be perpendicular to the sunbeam.

The amount received per square centimeter per annum would evidently be 3×60 (minutes) $\times 24$ (hours) $\times 365\frac{1}{4}$ (days) $= 1{,}577{,}880$ calories.

Since the sun shines at one time on only one-half of the earth, its rays are perpendicular over an area represented by the area of a great circle or πR^2. Hence the above figure must be multiplied by $6{,}370{,}191 \times 6{,}370{,}191 \times 3.1416$, which gives $20{,}116 \times 10^{16}$ gram calories. See Hann, Lehrbuch, first edition, page 26. The amount there given is $20{,}116 \times 10^{20}$, possibly a typographical mistake for 2.0116×10^{20}.

The amount of ice which this will melt may be ascertained easily, as follows: Three calories per square centimeter per minute are 180 calories per hour. This would melt 180/73.4 or 2.45 cubic centimeters of ice in an hour. In a year, therefore, $2.45 \times 24 \times 365\frac{1}{4}$ or 21,476.7 cubic centimeters of ice would be melted for each square centimeter of surface. If the heat were uniformly distributed over the earth's surface it would cover 4 great circles, hence the above figure must be divided by 4, which gives a depth of about 5370 cubic centimeters of ice, or 54 meters or 177 feet per year.

Problem 7.—Prove that the intensity of insolation varies as the sine of the angle of incidence of the sun's rays.

Solution.—See fig. 2. The surface $A'\ B$ receives less insolation in proportion as this surface is larger than the surface $C'\ B$ at right-angles to the pencil of rays S. The intensity (I') of the insolation on $A'\ B$ is to the intensity (I) on $C'\ B$ inversely as the lengths of those lines, or

[2] A sphere whose surface has the same area as Clarke's spheroid of 1866 (whose $a = 20{,}926{,}062$ and $b = 20{,}855{,}121$ feet) would have $R = 20{,}902{,}490$ feet. Its surface would be 196,940,000 square miles. (See Woodward, Smithsonian Geographical Tables, 1894). Not only the dimensions of the globe but the relation between the meter and the foot have been subject to numerous investigations, and the results as given by different geodesists are gradually becoming more reliable. Besides the above-given values by Clarke, the following values may be mentioned:

Bessel, 1842, $a = 6{,}377{,}397$ and $b = 6{,}356{,}079$ meters.
Fischer, 1868, $a = 6{,}378{,}238$ and $b = 6{,}356{,}230$ meters.
Faye, 1889, $a = 6{,}378{,}393$ and $b = 6{,}356{,}549$ meters.

The mean radius of the earth may be described as the radius of a perfect sphere whose surface is equal to that of the spheroidal earth, or again, that of a sphere whose volume is equal to that of the earth, or again, that of a sphere whose radius is the average of all terrestrial radii. These three values differ slightly among themselves. The first value is that above given in connection with Clarke's spheroid. The International Meteorological Tables of 1900 adopt the a and b of Bessel's spheroid, and the mean radius R equals 6,371,104 meters, equals 20,902,950 English feet. The values of a and b adopted in Bigelow's Cloud Report are those of Bessel's spheroid, and the average R equals 6,370,191 meters, equals 20,899,600 feet.

The relation between the meter and the English foot adopted by the International Meteorological Tables, namely, 1 meter equals 3.28089917 feet, or 1 foot equals 0.30479449 meter, was Kater's value of 1818; it has lately been more accurately determined (see Monthly Weather Review for December, 1896); namely, 1 meter equals 3.2808429 feet, and 1 foot equals 0.30479973 meter. All these refinements in decimals imply equal refinements in definitions and other matters that are still under discussion, and need not trouble the elementary student, who should for consistency's sake use either the system adopted by the International Meteorological Tables or that adopted by Professor Bigelow, or that adopted by the International Bureau of Weights and Measures.—EDITOR.

[3] According to S. Newcomb, Compendium of Spherical Astronomy, 1906, p. 393, the Julian year has 31,557,600, but the correct mean solar year has 31,556,926.0 seconds.—EDITOR.

$$I' : I :: C'B : A'B.$$
$$I' = I(C'B / A'B)$$

$C'B / A'B$ is the cosine of $90° - h$, or the sine of h, which is the angle of incidence of the sun's rays to the horizontal surface, or the angular elevation of the sun above the horizon.

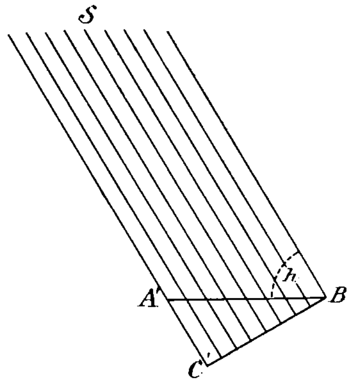

FIG. 2.

By this method the intensity is decomposed after the manner of a force in mechanics, as first proposed by Halley in 1693; the same law may be obtained in an entirely different way from the principle of the inverse square of the distance. See Meech, L. W., On the Relative Intensity of the Heat and Light of the Sun, upon Different Latitudes of the Earth, 1856, pp. 13, 14.

Problem 8.—Given the coefficients of expansion of brass and mercury, deduce the corrections to be applied for the temperature of the scale and of the mercury in a mercurial barometer.

Solution: (Metric system).—1. As the brass scale divisions and their numbers rise with increase of temperature, at any temperature above freezing (where the scale has its standard length), opposite a fixed point, the scale reading would be too low or the length of the scale would be too great.

Let n be the coefficient of linear expansion of brass; then unit length of the scale at 0° is 1; at 1° C. it becomes $1 + n$; at 2° it becomes $1 + 2n$, or in general at $t°$ it bec omes $1 + tn$

2. If B is the mercurial column (barometric height) as measured with the scale at a temperature $t°$, then the height as measured with the scale at the temperature 0° would be greater, since the length of each division would then be less, in the ratio of 1 to $1 + tn$, so that the number of divisions corresponding to a given length will be increased in the ratio $(1 + tn)$ to 1. Hence, if B_n is the barometer reading corrected for the expansion of the scale, then

$$B_n = B\ (1 + tn) \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

See Watson's Physics, page 159.

3. Here B_n is the height of the mercurial column at the temperature $t°$, and we have to find what would be the height if the temperature of the mercury were 0°.

If D_t is the density of mercury at $t°$, and D_0 its density at 0°, and m the coefficient of expansion of mercury, then 1 cubic meter of mercury at 0° becomes $(1 + m)$ cubic meters at 1°, $(1 + 2m)$ cubic meters at 2°, or in general $(1 + tm)$ cubic meters at $t°$. Or if V_0 is the volume at 0° and V_t the volume at $t°$, $V_t = V_0\ (1 + tm)$.

Since the mass of the mercury remains the same, the volume at 0°, V_0, multiplied by the density of mercury at 0°, D_0, i. e., the mass, M, must equal the volume at $t°$, V_t, multiplied by the density at $t°$, D_t.

Substituting for V_t its value $(1 + tm)\ V_0$ gives

$$M = V_0\ D_0 = V_t\ D_t = (1 + tm)\ V_0\ D_t.$$
$$V_0\ D_0 = (1 + tm)\ V_0\ D_t.$$
$$D_0 = (1 + tm)\ D_t.$$
$$\frac{D_t}{D_0} = \frac{1}{(1 + tm)}.$$

The height of a column of mercury supported by a given pressure being inversely proportional to the density of the liquid, therefore

B_n (height of mercurial column at $t°$) : B_0 (height at 0°) :: D_0 (density of mercury at 0°) : D_t (density of mercury at $t°$), from which

$$\frac{B_0}{B_n} = \frac{D_t}{D_0} = \frac{1}{(1 + tm)} \qquad B_0 = \frac{B_n}{(1 + tm)} \ldots\ldots\ldots\ldots (2)$$

Substituting in equation (2) the value for B_n found by equation (1), gives

$$B_0 = \frac{B\ (1 + nt)}{(1 + mt)}.$$

Dividing by $(1 + mt)$ gives

$$B_0 = B\left(1 - \frac{(m - n)\ t}{(1 + mt)}\right); \text{ or } B_0 - B = -\frac{(m - n)\ t}{(1 + mt)}\ B.$$

See equation (17), Bigelow's Report on Barometry, page 62.

The coefficient of expansion of brass for 1° C. is 0.0000184, or approximately 0.00002. For mercury, $m = 0.0001818$.

By assuming that $1/(1 + mt)$ is equal to $(1 - mt)$, which can be done, as the higher powers of m are very small, the above equation will approximate

$$B_0 = B\left(1 - (m - n)\ t\right),$$

or substituting the constants, $B_0 = B\ (1 - 0.000163\ t)$. The correction is very closely $-0.000163\ tB$. See Hann, Lehrbuch, page 164.

Example.—Observed reading of the barometer 745.6 millimeters at a temperature of 25° C. Corrected reading will be found by subtracting $0.000163 \times 25 \times 745.6$, or 3.05 millimeters, which corresponds closely with the correction found from the usual tables.

Problem 9.—Obtain the formula in the English system for the correction of the mercurial barometer for temperature.

In obtaining the formula for the English system it must be remembered that the brass scale is normal at 62° F. and the mercury has its normal density at 32° F. The equations in solution of problem 8 may readily be modified accordingly. See Abbe, Treatise on Meteorological Apparatus and Methods, 1887.[4]

Problem 10.—From well known physical relations deduce the law that ascending dry air cools 1° C for each 100 meters of ascent.

Solution.—It is necessary to know the following data:

1. The unit of heat, the small calorie, is the amount required to raise the temperature of 1 gram of pure water 1° C. Engineers use a large calorie, which is the amount of heat required to raise 1 kilogram of water 1° C.; this is 1000 times the small calorie.[5]

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[4] Where the formulas are:

$$B' = B''\ [\ 1 + \beta\ (t - 62)\ ]$$
$$H_0 = \frac{B'}{1 + \gamma\ (t - 32)}$$
$$H_0 - h = H_0 - B'' = B'\left(\frac{1 + \beta\ (t - 62)}{1 + \gamma\ (t - 32)}\right) - B''$$
$$= B''\left(\frac{\beta\ (t - 62) - \gamma\ (t - 32)}{1 + \gamma\ (t - 32)}\right)$$

From Report of the Chief Signal Officer, 1887, part 2, pp. 124–126.

The notation can be easily understood by comparing these formulas with those of Problem 8.

If the temperatures of the mercury and the brass scale are not identical then the corrections for each must be calculated separately, or may be taken from the tables given on pages 1133–1137 of Appendix 59, Report of the Chief Signal Officer, for 1881.—EDITOR.

[5] As the specific heat of water varies with its temperature it is necessary to define a calorie more exactly. The practise among European physicists is to define the small calorie as the quantity of heat necessary to raise the temperature of a gram of water from 0° C. to 1° C.—EDITOR.

2. Work is the product of the force acting multiplied by the space thru which it acts.

3. By actual experiment it is found that the energy which would raise the temperature of 1 kilogram of water 1° C. would be able to raise against gravity 1 kilogram to the height 426.8 meters. (See Bigelow, Cloud Report, p. 488.) This is the mechanical equivalent of a unit of heat, or the work done by it. Standard gravity at sea level and 45° latitude is the value here used.

4. To raise the temperature of 1 kilogram of air 1° C. under constant pressure requires 0.2374 of a large calorie. This is the specific heat of air under constant pressure, and is found also by experiment.

5. Since 1 cubic meter of air weighs 1.293 kilograms, therefore the amount of heat required to raise the temperature of 1 cubic meter of air 1° C. is a little more than 0.2374 of a unit; it is evidently 0.2374 × 1.293 or 0.307 of a large calorie.

Apply heat to a cubic meter of air and allow it to expand in one direction while the pressure is kept constant. The amount of heat required to raise the temperature of the cubic meter of air 1° C. is 0.307 unit of heat. The air will at the same time be expanded 1/273 of its volume.

The resistance to be overcome by the expanding air is the pressure of a standard atmosphere on a square meter, which is 0.76 × 13,596, or 10,333 kilograms per square meter. The space thru which the resistance is overcome is 1/273 of a meter; thus the work done by the expanding air against the pressure of the atmosphere is 10,333 × 1/273 or 37.85 kilogram-meters.

If the amount of work performed by the 0.307 unit of heat which is used to expand the air be 37.85 kilogram-meters, then 1 entire unit of heat so employed to the expansion of air would do an amount of work, x, as given by the proportion

$$0.307 : 37.85 :: 1.000 : x$$
$$x = 123.28 \text{ kilogram-meters.}$$

But by paragraph 3 the whole work equivalent of 1 unit of heat is 426.8 kilogram-meters. Therefore the fraction of a heat unit doing the expansive work required when 1 cubic meter of air is heated 1° C. is to the whole unit as 123.28 to 426.8, or as 0.289 to 1. In general when a given amount of heat acts on dry air the fractional part 0.711 goes toward heating the air, and the remaining 0.289 is used in doing the work of expansion against the outside pressure of 760 millimeters.

On the other hand, if air is caused to expand by coming under diminished pressure without the addition of any heat from without, i. e., adiabatically, then in expanding 1/273 of its volume, it will require 0.289 part of a heat unit for the work. The expansion will be done at the expense of its own heat, and the air will be cooled 0.289° C. by an expansion of 1/273 part.

If the air cools 0.289° in expanding 1/273 part, then to cool 1 whole degree the air must expand x parts, as given by the proportion

$$0.289 : 1/273 :: 1 : x$$
$$x = 1/79$$

A homogeneous atmosphere would have a height of 7991 meters. If in such a homogeneous atmosphere the air ascends 1 meter the pressure would be diminished 1/7991 part, and the volume would expand 1/7991 part. Then in order to increase the volume 1/79 part (and cool the air 1°C.) the air must ascend x meters, as given by the proportion

$$1 : 1/7991 :: x : 1/79$$
$$x = 7991/79 \text{ or } 101.2 \text{ meters.}$$

Thus we see that air must ascend 101.2 meters to cool 1° C. This is 0.99° for 100 meters, or as frequently stated in round numbers 1° C for 100 meters.

This is hardly a problem, as the matter is simply reasoned out. By the use of the elements of calculus the problem is much more elegantly solved. See Ferrel's Treatise on Winds, pages 23 to 28.

Problem 11.—Deduce the simplest formula for expressing the change of pressure with elevation in the atmosphere.

Solution.—The solution of this problem requires the use of the very simplest elements of calculus, which any student can readily grasp, even if not previously familiar with the subject.

1. Let v represent the volume of a given mass of air or gas at the pressure p and temperature t; and v^1 its volume, p^1 its pressure, and t^1 its temperature under standard conditions; then, since the coefficient of expansion of air is a, 1 cubic meter at zero becomes $(1 + a)$ cubic meters at 1° C., $(1 + 2a)$ cubic meters at 2°, and in general $(1 + at)$ cubic meters at t. By the law of Boyle-Gay Lussac, the volume of a gas multiplied by its pressure is constant, so that

$$p\,v = p^1 v^1 (1 + a t) \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Substituting for a its value 1/273, we have

$$p\,v = \frac{p^1 v^1 (273 + t)}{273} = \frac{p^1 v^1}{273} (273 + t).$$

Now, $(273 + t)$ is called the absolute temperature, or T, and $p^1 v^1/273$ is called the gas constant, R.

$$\text{Therefore, } p\,v = R\,T \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

2. Next find the numerical value of $R\,T$ for dry air.

The volume of gas is the reciprocal of its density; or if one cubic meter of air weighs 1.293 kilograms, then 1 kilogram will occupy 1/1.293 cubic meters of space. Calling D^1 the density of air, weight of unit volume, at 760 mm., at 0° C, then

$$v^1 = 1/D^1, \text{ or } D^1 = 1/v^1 \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Therefore, $p^1 v^1 = 1/D^1 \times p^1$, and p^1 equals the normal pressure, that is the density of mercury multiplied by the normal height of the barometer, or

$$p^1 v^1 = \frac{13.596 \times 0.760}{0.001293} = 7991.$$

This is evidently equal to the height in meters of a homogeneous atmosphere of air, or 7991.

Therefore, $p^1 v^1/273$, the gas constant for dry air, or R, is equal to

$$\frac{13.596 \times 0.760}{0.001293 \times 273} = 29.2713.$$

3. In ascending a very small distance (infinitesimal distance) in the atmosphere, in which the density is D_0, the absolute pressure changes in the inverse proportion by an infinitesimally small amount; this is exprest in the notation of calculus as follows:

$$-dp = D_0\,dh.$$

From (2) and (3), $p\,v = R\,T$, and $D_0 = 1/v$; $v = R\,T/p$; $D_0 = p/R\,T$.

Substituting, $-dp = p/R\,T\,(dh)$, or

$$-\frac{dp}{p} = \frac{dh}{R\,T}$$

From which follows by integration

$$\log_n p = \log_n p^1 - h/R\,T \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in natural logarithms.

4. Instead of the absolute pressure p and p^1, we may introduce the barometric heights, b and B_n (normal pressure), which gives:

$$\log_n b = \log_n B_n - h/7991 \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

5. To reduce to ordinary logarithms, divide the denominator, 7991, by the modulus, 0.43429, giving 18,400, the so-called barometric constant for air, giving final answer to the problem:

$$\log b = \log B_n - h/18{,}400 \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Numerical example.—What is the pressure at an elevation of 10 kilometers when sea-level pressure is 760 millimeters and temperature is 0° C.?

$\log b = \log 760 - 10{,}000/18{,}400$
$\log b = 2.88081 - 0.5435$
$\log b = 2.33731$, which corresponds to 217 millimeters.

The student should be required to work out a table of barometric pressures for a series of elevations.

6. From the above the additional problem is suggested of finding the simplest formula for calculating the altitude of a place, if the mean temperature of the air column and the pressures at the two stations are known.

By transposing (6) and introducing a temperature factor we have $h = 18{,}400\,(1 + \alpha t) \log (B_n/b)$ the simplest hypsometrical formula. See Hann, Lehrbuch, page 168.

Problem 12.—Give a formula expressing the weight of a cubic meter of dry air under varying temperature and pressure.

Solution.—Call the standard density D_0. A cubic meter of air under standard conditions (temperature 0° C., pressure 760 millimeters, and latitude 45°) weighs 1.29305 kilograms, or 1293.05 grams. The density of air diminishes as the temperature rises in the proportion of 1 to $1+\alpha t$; it also diminishes as the pressure decreases, for the air expands in proportion, or as b to 760. Therefore the density of air under other conditions is equal to its density under standard conditions, D_0, multiplied by

$$\frac{1}{1+\alpha t} \times \frac{b}{760};$$

or the weight in grams of a cubic meter of air at $t°$ C. and pressure b is equal to

$$\frac{D_0\, b}{(1+\alpha t) 760} = \frac{1293.05}{1+\alpha t} \times \frac{b}{760} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

Example.—What is the weight of a cubic meter of air under 760 millimeters pressure at the temperature of 30° C?

$\alpha = 0.00367$. Then,

$$\frac{1293.05}{1+0.00367 \times 30} \times \frac{760}{760} = \frac{1293.05}{1.1101} = 1164.9 \text{ grams.}$$

If we call the weight of a cubic meter of air at 0° unity, then at 30° C. the weight of a cubic meter will be 0.9008 of unity.

If 1 cubic meter of air at 30° weighs 0.9008 of what it does at zero, then it will require 1/0.9008 cubic meters at 30° to weigh as much as 1 cubic meter at zero, or 1.1101.

The student should be required to calculate for every 5° of temperature between −30° and 30° C. the weight of a cubic meter in grams, the density when 1 cubic meter at 0° weighs unity, and the volume whose weight equals that of 1 cubic meter at 0°—arranging the data in the form of a table, thus:

Temperature.	Weight of a cubic meter.	Density when 1 cubic meter at 0° weighs 1.	Volume which weighs the same as 1 cubic meter at 0°.
° C.	*Grams.*		*Cubic meters.*
30	1164.9	0.9008	1.1101

See Hann, Lehrbuch, first edition, pages 219, 220.

Problem 13.—Give a formula expressing the weight of a saturated cubic meter of aqueous vapor at different temperatures.

Solution.—1. The specific gravity of aqueous vapor is 0.622[6] (air = 1). Aqueous vapor obeys the same laws as to expansion with rise of temperature and decrease of pressure as does air, therefore by analogy with equation (1), problem 12, remembering, however, that the vapor is under its own saturation tension, e, the weight of a cubic meter of aqueous vapor is

$$\frac{0.622\,(1293.05)}{1+\alpha t} \times \frac{e}{760} \ldots\ldots\ldots\ldots\ldots (2)$$

Example.—What is the weight of a cubic meter of saturated vapor at 30° C?

The vapor pressure, or e, at 30° C. is 31.51 millimeters. Therefore the answer is:

$$\frac{0.622 \times 1293.05 \times 31.51}{(1+0.00367 \times 30) \times 760}, \text{ or } 30.09 \text{ grams.}$$

The student should be required to construct a table, giving for every 5° C., using the accepted values of vapor pressure as determined experimentally by physicists, (1) the weight of vapor in a cubic meter of saturated space; (2) the relative weights of the vapor at $t°$ and 0° C; (3) the volume in cubic meters of an amount of vapor weighing 1 gram, viz.:

Temperature.	Vapor pressure.	Weight of vapor in a saturated cubic meter of space.	Change per 5°.	Relative weight to that of 1 cubic meter at 0°.	Volume of 1 gram of vapor.
° C.	*mm.*	*Grams.*	*mm.*		*Cubic meter.*
30	31.51	30.09	1.59	6.1408	0.0332

Problem 14.—At what temperature is the weight in grams of vapor in a cubic meter of saturated space the same as the vapor pressure exprest in millimeters of the mercurial barometer?

Solution.—Equation (2), problem 13, reduces to

$$\frac{0.622\,(1293.05)}{760} \times \frac{e}{(1+\alpha t)} = 1.058 \frac{e}{(1+\alpha t)}$$

If we put $(1+\alpha t)$ equal to 1.058, then the weight in grams of a cubic meter of saturated vapor becomes equal to e, the vapor pressure in millimeters of mercury.[7] Solving

$$1+\alpha t = 1.058 \text{ or } 1 + 0.00367\,t = 1.058$$
$$0.00367\,t = 0.058 \qquad t = 15.8° \text{ C.}$$

At 15.8° the vapor pressure is the same as the weight in grams of a saturated cubic meter of vapor; below that temperature the weight of a cubic meter is greater than the vapor pressure; above that it is less.

Example.—At what temperature is the volume of 1 gram of saturated vapor equal to 1 cubic meter? *Answer.*—At some point between −15° and −20° C.

Problem 15.—Give a formula expressing the weight of a cubic meter of saturated air.

Solution.—The weight of a cubic meter of saturated air is less than the weight of a cubic meter of dry air at the same t and b, or it is equal to the weight of the vapor at the pressure e plus that of the dry air, at the pressure $b-e$, for the addition of vapor increases the total pressure and causes an expansion of the volume when both are unconfined as in the ordinary free atmosphere. From equations (1) and (2), problems 12 and 13, we find weight in grams of a cubic meter of saturated air:

$$\frac{1293.05\,(b-e)}{(1+\alpha t)\,760} + \frac{0.622\,(1293.05)\,e}{(1+\alpha t)\,760} \text{ which reduces to}$$

$$\frac{1293.05\,(b-0.378\,e)}{1+\alpha t \quad 760} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

[6] The specific gravity of aqueous vapor relative to that of dry air at the same pressure and temperature is computed by the formula of physical chemistry more accurately than it has as yet been determined by any direct measurement. The calculation is very simple. Two volumes of hydrogen, whose weight relative to that of air is 2 × 0.06960 (Rayleigh, 1893), combine with one volume of oxygen, whose relative weight is 1.10535 (Rayleigh, 1897), to form two volumes of saturated aqueous vapor, whose relative weight is therefore 1.24455. Hence, the relative weight of one volume, or the specific gravity of aqueous vapor relative to that of air, is one-half of this, or 0.62228. This computation relates to saturated vapor, but on the assumption that vapor acts like a gas, it becomes true for any temperature and pressure; hence, its use in the above text.—EDITOR.

[7] In all dynamic problems the vapor pressure, like the air pressure, must be exprest in grams per square centimeter, or kilograms per square meter, or pounds per square foot, depending on the system of units that is employed.—EDITOR.

Example.—What is the weight of a cubic meter of saturated air at 10° C.? *Answer.*—At 10° the vapor pressure is 9.14 millimeters. By the formula

$$\frac{1293.05}{1+0.00367\times 10}\,\frac{760-0.378\times 9.14}{760}=1241.6 \text{ grams.}$$

A cubic meter of dry air at 10° weighs 1247.3 grams; the saturated air weighs 5.7 grams less than an equal volume of dry air.

The student should be required to construct a table giving the weight of a cubic meter of dry air for every 5° C. between − 30° and 35° C., and the weight of a cubic meter of saturated air, and the difference between them. The table may be arranged as follows:

Temperature.	Weight of a cubic meter of dry air.	Weight of a cubic meter of saturated air.	Difference.
°C.	*Grams.*	*Grams.*	*Grams.*
10	1247 3	1241.6	5.70

Example.—What is the difference between the weight of a cubic meter of dry air and of saturated air at − 20° and 30° C.? Will be answered by the above table, when completed.

Problem 16.—Give formulas expressing the weight of dry air and the weight of aqueous vapor in a kilogram of saturated air.

Solution.—If a cubic meter of dry air weighs 1.29305 kilograms, then 1 kilogram has a volume of 1/1.29305 cubic meters. Or in general, as one cubic meter of saturated air weighs by equation (1), problem 15,

$$\frac{1293.05\,(b-0.378\,e)}{(1+at)\,760}\text{ grams or }\frac{1.29305\,(b-0.378\,e)}{(1+at)\,760}\text{ kilograms,}$$

then 1 kilogram will occupy in cubic meters, the reciprocal of that, or 1 kilogram of saturated air occupies

$$\frac{(1+at)\,760}{1.29305\,(b-0.378\,e)}\text{ cubic meters} \ldots (1)$$

In order to know how much dry air is present in this number of cubic meters of saturated air, we must multiply the expression by the quantity of dry air in a cubic meter, given by the first part of equation (1), problem 15, or

$$\frac{(1+at)\,760}{1.293\,(b-.378\,e)}\times\frac{1.293\,(b-e)}{(1+at)\,760}=\frac{(b-e)}{(b-.378\,e)}.$$

The number of kilograms of dry air in 1 kilogram of saturated air is

$$\frac{(b-e)}{(b-.378\,e)}\ldots\ldots\ldots\ldots (2)$$

In a similar manner by multiplying the expression (1) by the second part of equation (1), problem 15, giving the quantity of aqueous vapor in a cubic meter, we get an expression giving the number of kilograms of vapor in 1 kilogram of saturated air, or

$$\frac{(1+at)\,760}{(b-.378\,e)\,1.293}\times\frac{0.622\times 1.293\times e}{(1+at)\,760}=\frac{0.622\,e}{(b-.378\,e)}.$$

The number of kilograms of vapor in a kilogram of saturated air is

$$\frac{0.622\,e}{(b-.378\,e)}\ldots\ldots\ldots\ldots (3)$$

Problem 17.—How much dry air and how much aqueous vapor are contained in a kilogram of saturated air at 10° C?

Solution.—By applying the formulas of problem 16, we get, since *e* at 10° is 9.14 mm:—

$$\text{from (2) dry air } \frac{760-9.14}{760-.378\times 9.14}=0.99247\text{ kilogram.}$$

$$\text{from (3) vapor } \frac{0.622\times 9.14}{760-.378\times 9.14}=0.00753\text{ kilogram.}$$

$$\text{Sum}=1.00000\text{ kilogram.}$$

The student should be required to construct a table giving (1) The volume which 1 kilogram of dry air occupies at different temperatures; (2) The volume which 1 kilogram of saturated air occupies; (3) The quantity of dry air in a kilogram of saturated air; (4) The quantity of vapor in a kilogram of saturated air. Example:

Temperature.	Volume of 1 kilogram of dry air.	Volume of 1 kilogram of saturated air.	Weight of dry air in 1 kilogram of saturated air.	Weight of vapor in 1 kilogram of saturated air.
°C.	*Cubic meter.*	*Cubic meter.*	*Kilogram.*	*Kilogram.*
10	0.8017	0.8054	0.99247	0.00753

An extended table of the weights of aqueous vapor in a kilogram of saturated air under various pressures, in the metric system, will be found in Bigelow's Cloud Report, pages 560 and 561. See also Marvin's tables for the Psychrometer and Smithsonian Meteorological Tables.

All these problems may also be solved for other pressures than 760 mm.

[*To be continued.*]

NOTES ON THE CLIMATE OF KANSAS.

By T. B. JENNINGS, Section Director. Dated Topeka, Kans.

[Read before the Kansas Academy of Science November 30, 1906.]

In reviewing the history of a country it is customary to divide it into prehistoric and historic periods. In writing of the climatology of this State we shall divide it into two periods, the first period extending from the earliest reliable written accounts of its weather down to the time (1887) that systematic observations and records were practically begun over the entire State. Tho the State is young, it has a few records that began in the dim past. The Fort Leavenworth record began in 1836, the Fort Riley record in 1853, the State Agricultural College record in 1858, the Kansas University record in 1868, the Independence record in 1872, and the Dodge record in 1875.

FLOODS.

The old river boatmen give an account of a flood in the eastern part of the territory and in the Missouri River in 1785 which past down that river and into the Mississippi, flooding the American bottoms across from St. Louis, and which for many years was referred to as "The Great Flood." Twenty-six years later the Missouri River bottoms were again flooded.

About the last of February or first of March, 1826, heavy rains began in what is now the southeast quarter of the State, raising the Neosho and its tributaries "out of their banks" and flooding their bottoms; heavy rains continued in the territory during the season. In June the lowlands near the mouth of the Kaw were flooded, owing to high water in the Kaw and Missouri rivers meeting; in the fall a destructive flood swept down the Neosho, carrying away wigwams, houses, and gathered and ungathered crops.

In 1844 occurred probably the worst floods eastern Kansas has ever experienced. Rev. Mr. Meeker, who was missionary to the Ottawa Indians and was living on what is now the site of the city of Ottawa, in his letters gave a graphic account of the condition of the Marais des Cygnes and the destruction wrought by it at that point. From the 7th to the 20th of May there were nine days of rain, and daily from the 23d to the 29th, inclusive, rain fell; it began again on June 7, and on the 12th the Marais des Cygnes overflowed its banks, carrying away outhouses, fences, cattle, pigs, and chickens; the river began falling on the 14th and began rising again on the 20th.

At Fort Leavenworth the rainfall for June, 1844, was 8.53 inches; for July, 12 inches; for August, 8.08 inches, aggregating 28.61 inches for the three months. (The normal annual precipitation for that place is 30.89 inches.) Mr. Richard W.