REFERENCES.

(1) Air masses have been computed from the equation

$$m=\frac{\text{atmospheric refraction (in seconds)}}{58.36'' \times \sin Z}$$

(2) Ball, Frederick. Altitude tables. London, 1907.
(3) Abbot, C. G., and Aldrich, L. B. Smithsonian pyrheliometry revised. Smithsonian Misc. Collection. v. 60, No. 18. Washington, 1913.
(4) See Table 2. Bulletin Mount Weather Observatory. Washington, 1912. v. 5, p. 303–311.
(5) Kimball, Herbert H. The dense haze of June 10–11, 1912. Bull. Mount Weather Obsy., Washington, 1912. v. 5, p. 161–165.

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STANDARD UNITS IN AEROLOGY.

The views and practice of some American physicists are probably well presented in the following extracts and articles from Profs. T. W. Richards and A. E. Kennelly, both of Harvard University, and which we now publish with their permission.—EDITOR.

1. [*Extract from* "New method for determining compressibility," by Theodore William Richards and Wilfred Newsome Stull. Carnegie Institution of Washington. Publication No. 7. Washington. December, 1903. p. 42–43.]

"It is a matter of great regret that the scientific world has not agreed upon a less arbitrary unit of pressure than the 'atmosphere.' The difficulty is now increased by the frequent technical use of this word to designate the pressure of a kilogram per square centimeter. The growing tendency toward the adoption of the C. G. S. system suggests the use of a consistent unit for this dimension also. Might not the pressure of a dyne per square centimeter be suitably called a *bar* (Greek *βαρος*, pressure, weight)? This suggestion is made because the practical use of a unit is always much facilitated by a definite verbal designation. In this case the pressure of a megadyne per square centimeter would be called a *megabar*, a name no more cumbrous than 'atmosphere,' and far more definite. This unit, though unnamed, has long been advocated by Ostwald (Grundriss Allgem. Chem., p. 54, 1899) as a more scientific one than the present standard. The megabar is 1,000 ÷ 980.6 = 101.98 per cent of a kilogram per square centimeter, or 101.98 ÷ 1033.2 = 98.703 per cent of an atmosphere, or the pressure measured by 75.015 centimeters of mercury at 0° C. at sea level, and latitude 45°. This pressure is more nearly the average atmospheric pressure at the laboratories of the world than the arbitrary 'atmosphere' usually taken. A megabar, acting through the volume of a cubic centimeter or milliliter, performs a megerg of work, or one-tenth of a joule."

2. [*Extract from* "The convection of heat from small copper wires." By A. E. Kennelly, C. A. Wright, and J. S. Van Bylevelt, in Proc. Am. instit. electr. eng., June, 1909, v. 28, p. 706.]

"*Air pressure in absolute measure.*—In column II of the foregoing table the air pressure in the tank is recorded in megabars. The C. G. S. unit of pressure, 1 dyne per square centimeter has been called the 'bar'; so that a megabar is 10^6 dynes per square centimeter. According to the recently published data of the Bureau International des Poids et Mesures (Les Recents Progrés de 1907, pp. 30–31), a column of mercury 760 mm. (29.92 inches) high, at sea level, in latitude 45°, exerts a pressure of 1.0132 megabars. Consequently 1 megabar represents the pressure of a column of mercury of 750.09 mm. (29.53 inches) under the same conditions. For most practical purposes, therefore, a megabar may be taken as 1 atmosphere. *It is actually* 0.987 *of an atmosphere of* 760 mm. [under apparent gravity] at sea level and 45° latitude."

3. STANDARD UNITS IN AEROLOGY.

By Prof. A. E. KENNELLY.

[Dated Cambridge, Mass., Mar. 25, 1914.]

In "Science" for March 13, 1914 (p. 391), Prof. Alexander McAdie calls attention to the confusion which is likely to be produced in scientific literature by the use of the term "bar" as a unit of pressure, with two distinct significations. I beg the privilege of indorsing in your columns the views there expressed, and of adding a few remarks.

It is generally agreed that the "bar" should be the name of a unit of pressure, in some simple numerical relation of dynes per square centimeter. The question is as to whether it should be applied to the C. G. S. unit (1 dyne per square centimeter) or to a pressure one million times greater. If it is given to the C. G. S. unit, then the standard atmospheric pressure, as hitherto adopted, would be the megabar of 750.09 mm. of mercury. On the other hand, if it is given to this latter standard atmosphere, then the C. G. S. unit of pressure would become equal to a microbar.

It is submitted that in view of (1) the history of the term, (2) of scientific consistency, (3) of existing usage, the "bar" should be adopted as the name of the C. G. S. unit, making the standard atmosphere a megabar.

History.—Prof. McAdie has pointed out that the term "barad" was proposed for the C. G. S. unit by a committee of the British Association in 1888. The International Physical Congress of Paris, in 1900, reported in favor of the "barie" as the name of the C. G. S. unit, (see vol. I of Proceedings, p. 100). The following is quoted from page 31 of Guillaume's "Recents Progres du Systeme Métrique" (Paris, Gauthier-Villars, 1907), a report presented to the Fourth Convention of Weights and Measures in Paris October, 1907:

Cette relation permet de calculer immédiatement la valeur en baries (unite C. G. S. de pression, egale à une dyne par centimetre carré) de la pression exercée par une colonne de mercure de la hauteur normale de 76 cm. dans les conditions de la pesanteur qui résultent de l'ensemble des stations considerées par M. Helmert. On trouve ainsi

$$P \text{ normal} = 1\,013\,211 \text{ baries.}$$

On peut calculer aussi, en posant P égal a l'unité, la hauteur de mercure qui exerce l'unité de pression. On trouve ainsi 0.75009μ. La megabarie normale serait donc exerçée par une colonne de mercure de 750.09 mm., à la témperature de la glace fondante, sous la latitude de 45°, et au niveau de la mer; l'intensité de pesanteur pour laquelle la colonne de mercure, exerçant une pression égale à une megabarie serait de 750.09 mm. devrait avoir la valeur:

$$g = 980.738 \text{ cm. sec}^{-2}.$$

In 1903 Prof. T. W. Richards independently originated and adopted the name "bar" for the C. G. S. unit of pressure in his chemical work.

Scientific consistency.—It is generally admitted that the C. G. S. system is the most generally and internationally recognized physical system of units in use at the present time, and the system most frequently employed in theoretical discussions of physical quantities. The system is strengthened when its unit magnitudes receive internationally recognized names. It necessarily becomes weakened when such names are assigned to unit magnitudes outside the system, even if decimally connected therewith. For example, the C. G. S. system became weakened when the name "ohm" was assigned to an electrical resistance unit magnitude of 10^9 C. G. S. units, and when the "volt" was assigned to an electromotive force unit magnitude of 10^8 C. G. S. units; because in order to maintain a simple relation between these units, an entire system of corresponding unit magnitudes—the "practical" electrical system of the volt, ohm, ampere, coulomb, joule, watt, and henry—all distinct from the C. G. S. system, and so related, as Maxwell showed in his treatise, that the "practical" unit of length became equal to a quadrant of the earth, and the "practical" unit of mass 10^{-11} gram. If the C. G. S. unit of e. m. f. had been named the volt and the C. G. S. unit of resistance the ohm, engineers would be using megavolts for the present hundredth volt and megohms for the present thousandth ohm in their practical work, just as they actually use microfarads to-day, and the entire engineering system would have remained identical with the C. G. S. system. It is too late to make such a change to-day in electrical unit magnitudes; but we can hope to avoid such sectionalizing of units, in other and new directions, by keeping unit names in the C. G. S. system.

Usage.—The "bar" as the C. G. S. unit or 1 dyne per square centimeter has been used in various papers on physico-chemistry by Richards in this country [see 1 above], and also in papers of my own [see 2 above]. It also appears in recent textbook literature as the name of the C. G. S. unit.[1]

4. [*Extract from* letter by Prof. A. E. Kennelly, Cambridge, Mass., Apr. 6, 1914.]

"Many thanks for your kind letter of April 3, inclosing a most interesting Northern Hemisphere chart. The numerical values of the isobars [viz, millibars of Bjerknes] are so convenient on this chart that the question as to whether they should be called kilobars or millibars occupies a lesser place in the mind. I note that the statement is correctly made that these are given in standard pressure. If they were expressed as kilobars, they would not only be in standard pressure but also in absolute pressure. [See 5 below.] I was not aware that the use of the bar had been adopted officially by the United States Weather Bureau as a standard atmosphere. * * *" [See 6 below.]

5. The bar and millibar introduced into dynamic atmospherics by Bjerknes are simply names for pressures expressed in units of dynes per square centimeter, and are therefore *strictly* absolute units or *units of absolute pressure* in the C. G. S. system.—[C. A.]

6. The bar of Bjerknes, or the pressure corresponding to about 750 mm. of the mercurial barometer, is not proposed by the United States Weather Bureau nor by Bjerknes as a "standard atmosphere." The bar of Bjerknes, and its subdivisions, is used as a method of expressing the *absolute pressure in the atmosphere*, or the absolute pressure of the atmosphere at any place. The ordinary barometric reading, whether in millimeters or inches, expresses only the apparent pressure, and requires several corrections in order to express the absolute pressure.—[C. A.]

[1] Zeleny & Eriksen. A Manual of physical measurements. McGraw-Hill Book Co., New York, 1912, p. 220.

7. The C. G. S. system originally proposed to do away with all unnecessary, awkward, and arbitrary relations and names; it adopted a perfectly systematic series of elementary units, multiples, and combinations; the centimeter for length; the gram for mass; the mean solar second for time; each of these to be increased or diminished individually by powers of ten when any problem seemed to require the use of larger or smaller units. Eventually these new multiples and combinations, or derived units, began to be called by special names, as matters of individual pride or international courtesy, but many adhered strictly to the simple international and neutral C. G. S. system, and looked with disfavor on further innovations.

For instance, the C. G. S. unit of force is *one dyne*, or the force that can, by acting for one second on one gram, produce a change of velocity of one centimeter per second when the gram is free to move; this is a change of one unit in its momentum (which Newton called its quantity of motion) and is measured by the product of the number expressing its mass M, by the number expressing its velocity L/T, per second $1/T$. This algebraic expression is MLT^{-2} or $\frac{ML}{T^2}$

The C. G. S. unit of pressure is the pressure exerted by one dyne or 1 C. G. S. unit of force, pressing against or acting on every portion of a C. G. S. unit of area, or on one square centimeter. This C. G. S. unit of pressure may be expressed algebraically, $\frac{ML}{T}\cdot\frac{1}{T}L^2$; it is so used by many physicists in laboratory experimentation and has been called a *bar* by Ostwald, Richards, and others.

On the other hand, Bjerknes proposes a unit of pressure for use in atmospheric dynamics and recommends the pressure exerted by 1,000,000 dynes acting upon unit area of 1 sq. cm.; this he also calls a *bar*. Hence, the bar of Bjerknes is 1,000,000 times the C. G. S. unit or bar of the physicist. The 1,000th part of the Bjerknes *bar* is his *millibar*, or 1,000 times the unit *bar* of the physicist. The *bar* of Bjerknes is sufficiently large to be convenient in atmospheric studies, while the *bar* of the physicist seems more appropriate for special laboratory studies; hence the unit suggested by Bjerknes has been adopted by the recent international meteorological and aerodynamic congresses although it represents a slight departure from the established C. G. S. system of nomenclature, whereas the units adopted by the physicists agree precisely therewith. Fortunately, the *bar* of Bjerknes is the megabar or simply $10^6 = 1,000,000$ times the small unit of the physicist so there need be no confusion in the thoughts, or in the equations of the respective departments of physics, and until the various international authorities agree on appropriate names we have only to remember that the *bar of Bjerknes* represented by B, or *Bjer* or *Bjr*, and that of the physicists represented by b in the C. G. S. system, have such a ratio that the relation between the two is $B = b \times 10^6$ or $\mathrm{Bjr} = 1,000,000\ b$ or $1\ b = \mathrm{Bjr} \times 10^{-6}$.

Meteorologists need to discuss the motions of the atmosphere by using the absolute units quite as much as is done by other physicists in respect to problems in engineering, electricity, etc. We still adhere to dynes and fundamental absolute units. It is convenient for some to speak of the C. G. S. megabar as a million dynes per square centimeter, while others find it convenient to call it one bar. The bar of Bjerknes is not a "standard atmosphere," nor do we for a moment presume to alter the definition of a standard or absolute or normal atmosphere whenever physicists have occasion to use that term for a unit which departs so much from the C. G. S. system. We agree entirely with the spirit of the recommendation adopted by the British Association for the Advancement of Science, 1898; Ostwald, 1899; and the International Congress of Physicists, Paris, 1900; all of whom appear to agree that the so-called "standard atmospheric pressure" (760 mm. of pure mercury under standard gravity at sea-level and latitude 45°) is not always the most appropriate datum for use.

In this connection we note that P. W. Bridgman (Phys. rev, Lancaster, Pa., (2), Feb., 1914, v. 3, p. 126, ffg) finds it convenient to use as his C. G. S. unit of pressure not dynes per square centimeter, but kilograms per square centimeter and the corresponding kilogram-meter per gram instead of gram-calory per gram.—[C. A.]

THE C. G. S. SYSTEM AND METEOROLOGY.

By Prof. VILHELM BJERKNES, Leipzig.

[Translated from Meterologische Zeitschrift, Februar, 1913, p. 67-71.]

The International Commission for Scientific Aeronautics at its meeting in Vienna (1912) adopted the following resolution:

> In the publications of the International Commission the pressure will be expressed in *bars* or in decimals thereof, such as *decibar*, *centibar*, *millibar*, instead of in millimeters of mercury; this decision will however first become effective when the International Meteorological Committee shall have communicated its agreement therewith.

* * * * *

The principal advantages of the C. G. S. system were not considered during the discussion in Vienna, but were considered by all present as well known and recognized. But the subsequent discussion has shown that even on this point there prevails a surprising confusion. It will therefore not be improper to consider the question when we can apply entirely arbitrary units without injury, and when we can not relinquish the advantages of the C. G. S. system.

So long as scientific work consists only in the registration of individual elements and the statistical discussion of the resulting numerical series, we can without harm choose the units for the individual quantities quite arbitrarily—we merely need to apply the same units at various times and places; it is in this case quite unimportant whether the units thus applied to different quantities belong to a systematic system of units.

But so soon as we pass from climatological to dynamic researches we have to meet very different demands in order to understand the quantitative relations between the different quantities. For instance, we then no longer observe the pressure in order to consider the pressure itself, but in order to compute from it accelerations and velocities; we determine forces and motions not because of interest in these quantities themselves, but in order to compute from their combinations the work that is done and the heat that is evolved.

The conditions hitherto prevailing in meteorology have been very unfavorable for the development of this dynamic side of atmospherics. The equations of dynamics and of thermodynamics relate to the three dimensions of space and remain indefinite so long as we introduce into these equations only the results of observations obtained in two dimensions. With the establishment of aerology, these conditions have entirely changed. Simultaneous aerological observations give all the data needed for the direct application of the equations of dynamics and thermodynamics to meteorological problems, and thus open a prospect for an unsuspected development of meteorological science. But this development is restricted in its most sensitive portion as long as we retain an irrational unit of pressure for our simultaneous aerological observations. A single example will suffice to show the confusion that enters into dynamic equations as soon as we fail to apply a coherent system of units.

The condition of equilibrium in the atmosphere is as follows: The pressure against the boundary surfaces of any arbitrary volume of air must have a resultant that is directed vertically upward and is equal to the weight of the volume of air. If we consider a unit volume of this air, then its weight is equal to the product of its density ρ into the acceleration of gravity g. The resultant of the pressures against the boundary surfaces of a unit of volume we call the pressure gradient G (the dynamic definition of the gradient) and the equation of equilibrium takes the form

$$G = -\rho g \qquad \text{(a).}$$

The pressure gradient G may also be defined simply as the change of pressure per unit of length which is the geometrical definition of the gradient. Therefore, if z is a distance or length measured along the vertical, we have

$$G = -\frac{\partial p}{\partial z} \qquad \text{(b).}$$

Now (a) and (b) are the classic equations to which we are led under the condition that we are using a coherent system of units like the C. G. S. system.

But if we express the pressure in millimeters of mercury and retain the C. G. S. units for the density and the acceleration of gravity, then the equations (a) and (b) no longer harmonize but are incompatible with each other and clash together, and one or the other must be modified. If we decide to retain the geometric definition (b) for the gradient, then the equation of equilibrium (a) must be written in the form

$$1.333193\, G = -\rho g \qquad \text{(a}'\text{).}$$

The property of the gradients as simply equal numerically to the product of density and acceleration of gravity, is thus ignored.

If, on the other hand, we decide to hold fast to the dynamic definition of the gradient as in equation (a), then the geometric definition of this quantity must be expressed under the form

$$G = -1.333193\frac{\partial p}{\partial z} \qquad \text{(b}'\text{).}$$

In this equation the gradient loses its property of being equal to the negative of the change of pressure. Whichever way we may decide it is evident that we lose the simplicity and harmony of the systems of equations (a) and (b).

Confusions of this or a similar kind will be introduced into every dynamic or thermodynamic equation that contains the pressure, and uses millimeters of mercury as the unit of pressure while at the same time retaining the C. G. S. units for all other quantities. In order to realize the extent of this class of difficulties with which dynamic meteorology will be burdened so long as we continue to use the millimeter of mercury as the unit, it suffices to write out in full the equations that come into use in dynamic meteorology.