(figure 2) the snowfall in the south was interrupted during a temporary rise of temperature, but that in the Ohio valley increased in intensity as the pressure fell. By the morning of the 12th, the southern cyclone had appeared and in connection with it, snow was falling over a large area (figure 3). The snowfall as it began over the Middle Atlantic States is described as having come from a hazy sky. No. 405 of Mr. Bentley's (4) photographs of snow crystals shows the small tabular snow-crystals which fell in Jericho, Vt., on February 13, 1899 (5). As the cyclone increased in strength and moved up the coast the snowfall area became more localized and the snowfall heavier (see U. S. Daily Weather Map, February 13, 8 a. m., and figure 4). By February 14 at 8 a. m. the cyclone had advanced to Nova Scotia and snow had stopped falling over most of the eastern United States, as is evident from figure 5.

Figure 6 shows the distribution of the snowfall of the whole storm. Two maxima of 44 inches each occurred in south-central New Jersey and in southeastern Pennsylvania. Depths of more than 30 inches were reported from southeastern Massachusetts, eastern Pennsylvania, Delaware, eastern Maryland and northern Virginia. Thus, the snowfall was at a maximum where the strongest pressure gradient occurred and where local topography had the greatest cooling effect on the snow-bearing winds. The distribution of snowfall in this storm is characteristic of the northeast snowstorms of the Atlantic coast,—the heavy snowfall being generally confined to a belt about 200 miles wide along the coast.

Snowstorm of February 20–23, 1912.

The snowstorm of February 20–23, 1912, attended a well-developed elliptical cyclone which moved in a nearly straight path from the western Gulf States to the Gulf of St. Lawrence. In front of the cyclone was the characteristic sirocco with heavy rain and thunderstorms; in the rear followed the cold-wave with snowflurries, and on the north, was the heavy northeast snowstorm. The isotherm of 32° F. passed in a general northeast-southwest direction through the center of the cyclone, dividing the rain- from the snow-area. Far in front, with the usual southward bend of the 32° isotherm, the snowfall area (figure 7) extended south of the track of the center of the cyclone. Nearer the center, rain fell. (See U. S. Daily Weather Map for February 21, 1912, 8 a. m.)

The snowfall chart for February 21 presented in figure 8 shows the large southward extent of the light snowfall of west-wind snow-flurries. Figure 9 for the next day shows this area farther east. The central Appalachians are marked by heavier snowfall on the windward side and none on the leeward. Likewise the western Adirondacks had more snowfall than the eastern: this was the reverse of the conditions of the day before, when an east wind was bringing the snow. On February 23 (fig. 10) the west winds of the eastern Great Lakes made heavy snowfall on the leeward shores and mountains. Around Lakes Huron and Michigan, snowfall with southerly winds was beginning with the advance of another cyclone.

Taking the storm as a whole (fig. 11), the snowfall, although patchy, occurred in belts as was the case in February, 1899. The belt of maximum snowfall was, on the average, 150 to 200 miles north of the track of the center of the cyclone. In this belt the heaviest snowfall, 30 inches, occurred on the west shore of Lake Huron, most of it falling on the 21st with the easterly gale. The next heaviest snow, 24 inches, fell on the southeast shore of Lake Michigan with a north and northwest wind (6). Thus, both areas of maximum snowfall were located where cyclonic and local effects made the strongest combination.

SUMMARY.

As illustrated by the great snowstorms of February 10–14, 1899 and February 20–23, 1912, the distribution of snowfall in cyclones of the eastern United States is controlled by cyclonic action, temperature, topography and proximity to large sources of moisture. This distribution is roughly as follows:

1. The snowfall is spread over a wide territory on each side of the track of the cyclonic center.
2. The heaviest snowfall comes with northeast winds and occurs in a belt about 100 to 200 miles north of the track.
3. The northwest winds in the southwest quadrant sprinkle light snowfall over the country to a distance of about 300 miles south of the track of the center of the cyclone.
4. The effects of local topography and geography make the distribution of snowfall patchy.

The writer wishes to acknowledge the courtesy of Mr. R. H. Weightman of the Forecast Division, U. S. Weather Bureau, in sending tracings of some of the 8 p. m. weather maps.

REFERENCES.

(1) Contributions to Meteorology, 1882.
(2) Monthly Weather Review, 1911, pp. 1609–1616.
(3) Monthly Weather Review, May 1901; do., Ann. Sum. 1902.
(4) Monthly Weather Review, Ann. Sum. 1902, Plate II (XXX–119).
(5) Detailed accounts of this storm and its human effects are given in the Monthly Weather Review, February 1899, and in the Weather Bureau Climate & Crop Reports, February 1899, for the different States.
(6) For a study of snowstorms with reference to wind direction and cyclonic action see A. B. Crane, Snowstorms at Chicago. Am. Met. Journ. 1892, pp. 63–66.

ON THE INFLUENCE OF THE DEVIATING FORCE OF THE EARTH'S ROTATION ON THE MOVEMENT OF THE AIR.(1)

[Communicated to the International Meteorological Congress at Chicago, Ill., August, 1893.][1]

By DR. NILS EKHOLM.

(Dated Statens Meteorologiska Centralanstalt, Stockholm, 1893. Revised by the author, June, 1914.)

I. ON RELATIVE MOTION IN GENERAL.

Let there be a system of material particles or points at which observers are stationed; the earth's surface constitutes such a system. An observer at one point of this system can detect a motion of the other points only by means of the changes in their mutual distances and directions and will thus conclude that the whole system is at rest. This perception of motion among the particles of a system is called relative motion.

Now suppose the whole system of material particles to include not only the earth but also the whole solar system; then all motions are considered as relative to this system, which as a whole is supposed to be at rest.

If the system include the whole universe, then the latter must necessarily be considered as at rest. Since we have no more general system of points, therefore, motion relative to the universe as a whole is the most general of which

[1] See MONTHLY WEATHER REVIEW, February, 1914, **42**: 93.

we are able to conceive and may be called *absolute motion*. More strictly speaking, absolute motion may be defined as motion relative to a system at rest, but such a motion is only an imagination.

In meteorological researches motion is generally considered relative to the terrestrial system, and our present purpose will be to determine the general laws of such motion.

The movement of a material system consists either of a translation or a rotation, or of both simultaneously. Translation takes place if the straight line joining any two points of the system always remains of the same length and parallel to itself; or, in other words, when all points of the system have simultaneously equal and parallel velocities and accelerations, although they may vary with the time.

Now, with respect to translatory motion, we know from experience that the relative motion of the points in a system is quite independent of any motion of the system as a whole. Thus, an observer at any point in the system, unless he has a motionless point of reference outside the system, will be unable to detect the motion of the system as a whole; but, on the other hand, he will not need to know this latter motion in order to determine the laws of relative motion within the system.

This is not true of rotatory motion, which is of such a character that a straight line joining any two points of the system is in general continually changing its direction. Now, suppose all these connecting lines to be of constant length, then the system will be in relative rest, but the laws of motion of a point moving relative to the system will be essentially different from those for a nonrotating system. In fact, we will soon prove that in a rotating system the relative motions within the system will be governed not only by the *true* or *absolute accelerations*, or *forces*, but also by two apparent forces produced by the rotation of the system.

Generally, the motion of a system is *simultaneously* both rotatory and translatory; but in all theoretical researches, in accordance with a well-known mechanical principle, we may suppose these two species of motion to take place successively and independently. For instance, we may suppose, first, that in any system the point M moves to its final position M' by means of translation only; and, second, that all other points of the system are brought to their final positions by rotating the whole system about M'. Also, since any translatory motion of the system does not influence the relative motions within it, we may, for the sake of simplifying the study of relative motions, imagine the system brought to rest by giving it a translatory motion equal and opposite to the actual translatory motion.

With respect to rotatory motion, however, it may be that there always exists in a rotating system a series of points located on one and the same straight line that remains in the same position for at least an instant, thus forming an *instantaneous axis of rotation*. Generally, this axis is changing its position and direction with time, relative to both absolute space and the rotating system. As to the terrestrial system, however, the change in the direction of the instantaneous axis of rotation is so extremely slow that its influence on relative motion within the terrestrial system may be neglected; also, the rotatory or angular velocity of this system is constant; and thus our investigations are very much simplified.

In investigating the laws of motion of a particle we have to deal chiefly with its *velocity* and *acceleration* of motion, both of which are determined at any moment by their respective magnitudes and directions, the latter of which may be either positive or negative.

The laws of velocity are easily determined, for the absolute velocity is always the geometrical resultant of the velocity of the particle in its relative path combined with the velocity of the point of the system with which the moving particle at that moment coincides. The proof of this is given immediately by constructing the parallelogram of velocities. Similarly, we find that the relative velocity is the resultant of the absolute velocity and the reversed velocity of the point of the system with which the moving particle is coincident.

As to the earth's motion, different parts of it, that is, different points in the terrestrial system, generally have different absolute velocities on account of the rotation; and accordingly we conclude that a particle moving relative to the earth, since on account of its inertia it tends to maintain its absolute velocity, must generally tend to change its relative velocity with reference both to its magnitude and direction. This reasoning, if correctly followed, will lead to the exact determination of the influence of the earth's rotation on the relative motions of the air. On the contrary, by means of an incomplete but very common deduction that considers only the changes of magnitude of the relative velocity, neglecting the changes of direction, we are lead to the old but inexact explanation first given by Hadley in 1735 (2), and afterwards reproduced by Dove and several other modern writers, especially in popular treatises.

The complete and exact solution of this problem was first given by G. Coriolis in 1835 as a purely analytical deduction (3); but the first extended application of Coriolis' theorem to air motions was made by Ferrel (4), and later by Guldberg and Mohn and others.

Ferrel's deduction, like that of Coriolis, is purely analytical, and, as it seems, is an independent one. Guldberg and Mohn (5) as well as several other later meteorologists, merely cite the required theorems as known from treatises on theoretical dynamics. The purely analytical deduction, although it is exact and general, is not so perspicuous as a geometrical demonstration, and I therefore think it may be well to give here a perfectly rigid and general, and very simple proof of the theorem of Coriolis, probably due to Delaunay (6), and so much the more as several elementary proofs, e. g. those of Sprung (7), and others, are neither rigid and general, nor simple.

II. CORIOLIS' THEOREM.

Lemma. The acceleration of a mobile mass may be geometrically determined as follows:

Let AB, figure 1, represent a part of the path that a mobile mass describes in the infinitely short space of time dt, and v its velocity at the point A. If there were no acceleration the mobile mass would, in the time dt, describe the path $AC=vdt$, AC being tangent to AB at A. In order that it may come to the point B, there is also required a component of motion, CB, and to produce this a constant acceleration, $\frac{2CB}{dt^2}$, must act during the time dt. Now, dt being infinitely small, the acceleration during this time is necessarily constant.

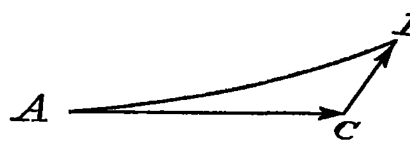

Fig. 1.

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Coriolis' theorem.—Now let AB, fig. 2, represent the absolute position in space at the time t, of the path of a mobile M, which we will suppose to be a particle of air

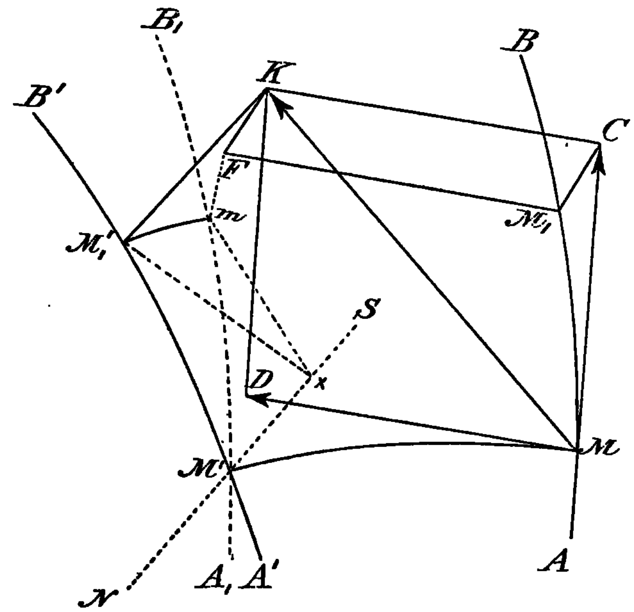

Fig. 2.

moving in the terrestrial system (8); $A'B'$ its position at the time $t+dt$; MM' the position of the path of M relative to a fixed point in the terrestrial system; hence MM' is an infinitely small part of a parallel of latitude on the earth. The point M of the terrestrial system will, by the earth's rotation at the time $t+dt$, be carried to M', and, similarly, the whole path AB will have been transferred to $A'B'$. Now, let v represent the absolute, and v_r the relative velocity of the mobile M at the time t, and v_s the velocity of that point in the terrestrial system with which M coincides at this time. By taking the points M_1 and M_1' in AB and $A'B'$, so that

$$MM_1' = M'M_1' = v_r dt,$$

then M_1' will be the actual position of the mobile at the time $t+dt$, and MM_1' the absolute path during the time dt.

But, according to the principle stated in Section 1, we may suppose the absolute motion from M to M_1' performed as follows: First, let the earth receive a translatory motion, so as to carry AB to A_1B_1, then M to M', and M_1 to m. It follows that $M'm$ is equal and parallel to $MM_1 = v_r dt$. Then let the earth rotate about the axis $NM'S$, parallel to the earth's axis (in the figure N is directed northward); therefore A_1B_1 will be carried to $A'B'$, and m to M_1', and thus they will come to their proper positions at the time $t+dt$. The path mM_1' will be an infinitely small arc of a circle whose plane is perpendicular to NS (also to the earth's axis, and parallel to the plane of the earth's equator), and whose center, x, is on NS.

Let ω represent the angular velocity of the earth, and γ ($=mM'S$ or $M_1'M'S$) be the angular inclination of the path of the mobile to the earth's axis at the time t: then the angle of rotation, $m \times M_1'$ will be ωdt, and the radius mx will equal $v_r dt \sin\gamma$. Hence, the arc $mM_1' = \omega v_r \sin \gamma dt^2$, or if for the sake of abbreviation we put

$$\varphi_\omega = 2\omega v_r \sin\gamma, \tag{1}$$

we will have

$$\varphi_\omega = \frac{2mM_1'}{dt^2}.$$

Let us draw at M tangents to the curves AB and MM', and on these take the lengths $MC = v_r dt$, and $MD = v_s dt$, and complete the parallelogram $MCKD$: then the direction of the diagonal MK coincides with that of the absolute velocity v, and the length $MK = vdt$ is equal. Now, according to the lemma, the absolute acceleration which we will represent by φ is equal to $\frac{2KM_1'}{dt^2}$.

Also, since KF is equal and parallel to CM_1, the relative acceleration may be represented by $\varphi_r = \frac{2KF}{dt^2}$, and finally, Fm being equal and parallel to DM', since M_1F and M_1m are equal and parallel to MD and MM', respectively, the acceleration of the point in the terrestrial system may be represented by

$$\varphi_s = \frac{2DM'}{dt^2} = \frac{2Fm}{dt^2} \tag{2}$$

Now, according to the well-known theorem of the composition of forces or velocities, we see by inspection of the skew=skew quadrilateral $KFmM_1'$ [that is, a quadrilateral whose sides are not in one plane], that KM_1' is the geometrical resultant of KF, Fm, and mM_1'. Since the same reasoning applies to any quadrilateral similar to $KFmM_1'$ and similarly situated, and calling to mind the mechanical significance deduced above for the sides φ, φ_r, φ_s, and φ_ω, which are $\frac{2}{dt^2}$ times the sides of the above quadrilateral, we have

$$\varphi = \text{geometrical resultant of } \varphi_r,\ \varphi_s, \text{ and } \varphi_\omega; \tag{3}$$

that is, the absolute acceleration φ is the geometrica resultant of the relative acceleration φ_r, the acceleration φ_s, of the point on the terrestrial system, and the acceleration, φ_ω, defined by equation (1). This is Coriolis' theorem, the meaning of which will now be more closely examined.

The component of acceleration, φ_ω, is called by Despérioux *complementary acceleration* (accélération complementaire); Coriolis himself less conveniently calls it the *composed centripetal acceleration* (accélération centripète composée) and calls the acceleration in the opposite direction, of which we will soon speak, the *composed centrifugal acceleration* (accélération centrifuge composée). According to equation (1) the magnitude of the complementary acceleration, φ_ω, is equal to twice the product of the angular velocity of the earth, ω, into the projection of the relative velocity upon the equator plane ($=v_r \sin\gamma$); and by inspection of figure 2 we find that since the direction of φ_ω, including its sign, plus or minus, coincides with mM', it is perpendicular to the relative path as well as to the axis of rotation, and acts in the same direction and with the same sign as the rotation. If there is no rotation ($\omega=0$), φ_ω will vanish, as in fact we know it must, since the motion is then a translatory one.

On account of the simple rotatory motion of the earth we may easily deduce a general and simple expression for the acceleration, φ_s, of the point on the terrestrial system. Its direction and sign coincide with DM', figure 2, and its magnitude is given by equation (2). We see from figure 2 that DM' is parallel to the equator plane and, as dt is infinitely small, it is also perpendicular to the path MM' and thus is directed inward toward the earth's axis.

Let us redraw this part of the figure in figure 3. Let $M'L$ be tangent to MM' at M'. Since the arc MM' $= v_s dt$, is a part of a parallel circle of radius r_1, we shall have

$$MM' = v_s dt = r_1 \omega\, dt;$$

from which we obtain, $v_s = r_1 \omega$.

Also the angle DLM' formed by the two tangents at M and M' is equal to ωdt; and since in the infinitely nar-

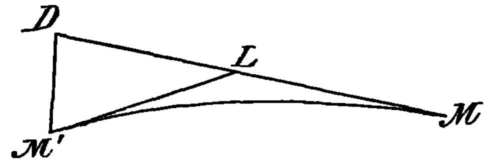

Fig. 3

row triangle DLM' the two angles at D and M' may be considered right angles and therefore equal

$$LD = LM' = LM = \tfrac{1}{2}v_s dt = \tfrac{1}{2}\omega r_1 dt;$$

also, the side DM' may be considered equal to the arc DM', the center of which is at L; consequently

$$DM' = DL\omega dt = \tfrac{1}{2}\omega^2 r_1 dt^2 = \tfrac{1}{2}\frac{v_s^2}{r_1}dt^2$$

and thus by equation (2)

$$\varphi_s = \omega^2 r_1 = \frac{v_s^2}{r_1}. \tag{4}$$

Let λ be the geocentric latitude and r the corresponding radius (9) of the earth; then

$$r_1 = r \cos \lambda,$$

and thus finally

$$\varphi_s = \omega^2 r \cos \lambda = \frac{v_s^2}{r \cos \lambda}. \tag{5}$$

φ_s is called the centripetal acceleration of the earth; it forces the parts of the earth to move in parallel circles around its axis instead of in straight lines in accordance with the law of inertia. The force required is taken from gravity, and therefore the remaining component of gravity, called *apparent gravity*, the only component that we are able to observe directly on the earth, is a little less than the *absolute gravity*, as that calculated by astronomers is called, and acts in a slightly different direction.

In order to conveniently apply the theorem of Coriolis to meteorological phenomena, we must reverse it, as a closer inspection of the above-named accelerations will show.

Among all these, the acceleration of the point in the terrestrial system, φ_s, is the one that may be *most easily and exactly* determined, since it depends only upon the dimensions and motion of the earth and upon the geographical position of the mobile, as is seen from equations (4) and (5). Next come the absolute accelerations φ, such as the absolute gravity, the gradients of atmospheric pressure, and friction. Generally these may either be calculated, or determined by experiment or observation. More difficult is the determination of the complementary acceleration φ_ω; it may be calculated, but only when the relative velocity v_r is known, as will be seen from equation (1); but v_r may be determined by observation. Finally, for the relative acceleration, φ_r, there is at present no method of experiment or observation to determine it, and no other method of calculating it than by means of Coriolis' theorem. Thus in all meteorological researches φ_r is to be regarded as unknown in equation (3), which must be solved with respect to it.

For this purpose let us draw in figure 4 the skew quadrilateral $ABCD$, whose sides, AB, AD, DC, and CB, are equal to and have the same direction as φ, φ_r, φ_s, and φ_ω, respectively;[2] this quadrilateral is therefore similar and similarly situated to $KFmM_1'$ in figure 2.

[2] In figure 4 the side BC has been lettered φ_w instead of φ_ω.—EDITOR.

By inspection AD is equal to the resultant of AB, BC, and CD, as regards magnitude, direction, and sign. Now, $AD = \varphi_r$, $AB = \varphi$, and BC and CD are φ_ω and φ_s *reversed*. Let the former be denoted by ψ, and the latter by ψ_s in respect to magnitude, direction, and sign; then

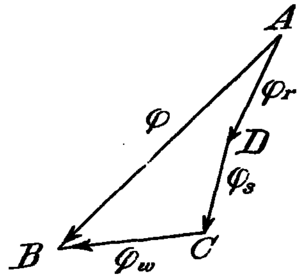

Fig. 4

See footnote.

$$\varphi_r = \text{resultant of } \varphi, \psi, \text{ and } \psi_s \tag{6}$$

This theorem, which is only another form of Coriolis' theorem, when referred to relative air motion, by putting force instead of acceleration proportional to the relative motion, and hereafter representing the relative velocity simply by v, may be expressed in words in the following manner:

The external impressed forces, φ_r, which accelerate an air particle have the following components:

1. *The absolute impressed forces, φ (absolute gravity, pressure gradients, and friction).*
2. *The centrifugal force of the earth, ψ_s, at the geographical point of the particle, ψ_s, being equal to $\omega^2 r \cos \lambda$, meeting the earth's axis perpendicularly, and acting outward.*
3. *The deviating force, ψ, which is equal to $2\omega v \sin \gamma$, is parallel to the equator plane and perpendicular to the path of the particle, and acts in a direction opposite to that of the earth's rotation.*

We call to mind that ω is the angular velocity of the earth and r the radius drawn from its center to the particle, λ the geocentric latitude of the particle, and γ the angular inclination of the path of the particle to the earth's axis.

The two forces ψ_s and ψ are called *apparent external impressed forces;* these having been introduced into the equations of dynamical meteorology, we may treat atmospheric motions according to the usual mechanical methods, quite as if the earth were at rest. The great advantage of this method of treatment is evident from the fact that the earth seems to our immediate perception to be at rest and consequently the air motions to be absolute motions. Therefore, since Coriolis' theorem affords a quite exact method of treating the meteorological phenomena according to our natural perception of them, it is of the highest importance to meteorological science. In fact, this method has already been generally accepted; but certainly in an incomplete manner, as several components of forces have been neglected at random without determining the errors thereby introduced.

III. APPLICATION OF CORIOLIS' THEOREM TO DYNAMIC METEOROLOGY.

First, the centrifugal force ψ_s may be very briefly treated, for as has already been said this force and the absolute gravity form the two components of *the apparent gravity*. Under the action of this force the earth's once fluid sphere has taken the form of an oblate spheroid whose surface, being at all points normal to the direction of apparent gravity, constitutes a level surface (surface de niveau). At the present time this applies directly only to the surface of the sea, although it also applies to the *ideal* sea surface drawn through the continents, which has been determined by a system of levels, and is the surface to which barometric observations are reduced. Thus the horizontal component of the centrifugal force is accounted for. The vertical component is considered by taking into account the variations in apparent gravity with latitude and altitude, this variation being caused partly by the centrifugal force and partly by the spheroidal form of the earth and the consequent variation in the distance from the center with latitude. Thus we obtain the well-known formulas for reducing air pressure to normal gravity (gravity at lat. 45° and at sea level). With regard to the distance between two level surfaces, we must remember that two such surfaces are nearer each other at the poles than at the equator, their mutual distances always being inversely proportional to the corresponding intensities of apparent gravity (10). We need not here enter into the details of this question.

Second, as to the deflecting force

$$\psi = 2\omega v \sin \gamma; \qquad (7)$$

its intensity and direction depend upon the position of the relative path and on the relative velocity, and it must always be treated as an external impressed force.

Let us first refer the motion to the earth's axis and the equator plane. Let figure 5 present a north polar pro-

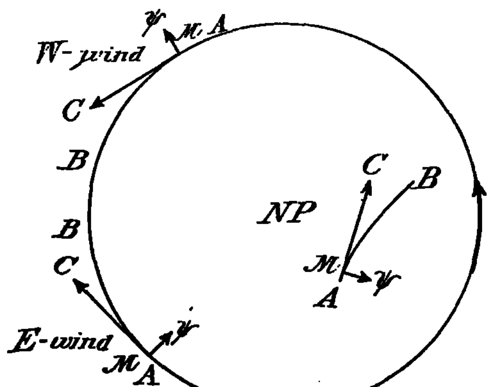

Fig. 5.

jection of our globe, AB the projection of the relative path, and $MC = v \sin \gamma$ that of the velocity, v, of the particle M. Then with respect to magnitude, $\psi = 2\omega\, MC$. If now we imagine an observer placed at M, parallel to the earth's axis and with his head towards the north pole (that is, perpendicularly upward in the figure), and looking toward C, he will see the earth rotate before him from his right to his left hand; and therefore ψ, which is directed in the opposite direction, will urge the particle toward his right hand and perpendicularly to MC in the equator plane, as drawn in the figure. In the Southern Hemisphere ψ acts similarly toward the left. If the path is parallel to the equator ($\gamma = 90°$), ψ will have its maximum value $2\omega v$, which is a constant for the whole earth. If the path is perpendicular to the equator ($\gamma = 0°$), ψ vanishes at all points on the earth.

Second, let us refer the motion to the zenith and the horizon.

As has just been shown, in the *neighborhood of the pole* in a *horizontal current*, ψ is always equal to $2\omega v$, is independent of the azimuth of the motion and is directed *horizontally* to the right at the north pole, and to the left at the south pole; there is no vertical deviation, and consequently *a vertical current is not deviated at all*.

In the *neighborhood of the equator in a north-south horizontal current*, the path being perpendicular to the equator plane, *the deviation is zero;* but in an *east-west current* the path is parallel to this plane. Therefore $\psi = 2\omega v$, is directed vertically upward in a wind from the west and vertically downward in a wind from the east, as will be immediately seen from figure 5. If a horizontal current blows from some other azimuth, the projection MC represents the east-west component of v; and thus the deviation is always vertical and proportional to the east-west component, being directed upward in a west wind and downward in an east wind. Also, we find immediately that in a vertical upward current ψ is directed horizontally westward, and in a vertical downward current it is directed horizontally eastward, and that $\psi = 2\omega v$.

Now let us consider *the deviation at any latitude* λ.

1. *Air current horizontal.*

(a) If the current is directed poleward, as in figure 6*a*, ψ will be directed horizontally eastward, and the projection of v on the equator plane being $v \sin \lambda$, we obtain

$$\psi = 2\omega v \sin \lambda.$$

The same value is obtained if v is directed toward the equator, but ψ will be directed westward.

(b) If v is directed eastward (west wind, fig. 6*b*), ψ is in the meridian plane parallel to the equator and directed

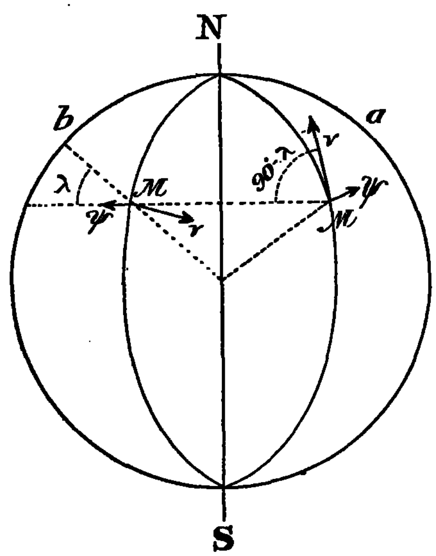

Fig. 6.

upward, thus forming, with the vertical, the angle λ; and v being parallel to the equator, we get

$$\psi = 2\omega v.$$

Thus the horizontal component of ψ, which we will call ψ_h, will be directed toward the equator, and we shall have for its magnitude

$$\psi_h = 2\omega v \sin \lambda;$$

the vertical component, which we will represent by ψ_z, will be directed upward toward the zenith, and we shall have for its magnitude

$$\psi_z = 2\omega v \cos \lambda.$$

If v is directed westward (east wind), we get the same values for ψ, ψ_h, and ψ_z, but the directions are reversed; ψ toward the earth's axis, ψ_h toward the pole, and ψ_z downward (toward the nadir).

(c) If v is directed in any azimuth, it may be resolved into two components, as presented in figure 7, the one, $v_x = MX$, with a north-south direction; the other,

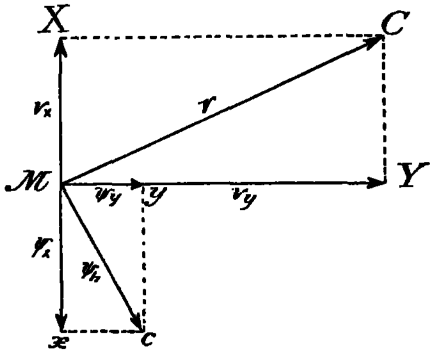

Fig. 7

$v_y = MY$, with an east-west direction. The effect of the component v_x will be, as shown in (a), to produce a component of deviation directed along MY, which may be expressed by the following equation:

$$M_y = \psi_y = 2\omega v_x \sin \lambda;$$

and the effect of the component v_y will be [as shown in (b)] to produce a horizontal component of deviation, which may be expressed by

$$M_x = \psi_x = 2\omega v_y \sin \lambda.$$

If the resultant of ψ_x and ψ_y is represented by ψ_h, this latter will be the horizontal component of the force of deviation. Now, by the above formulas,

$$\psi_y : \psi_x = v_x : v_y;$$

and since the right-angled triangles Myc and MXC are similar, therefore also $\psi_h : \psi_y = v : v_x$, and consequently

$$\psi_h = 2\omega v \sin \lambda. \qquad (8)$$

Also, the angle CMc is right-angled, and it is immediately seen that ψ_h is directed to the right in the Northern Hemisphere (for which the figure is drawn), and to the left in the Southern Hemisphere. This is the theorem, already generally known, on the horizontal force of deviation in a horizontal air current.

Finally, we get also [from (b)] a vertical force

$$\psi_z = 2\omega v_y \cos \lambda, \qquad (9)$$

directed upward in a westerly wind (v_y directed eastward), and downward in an easterly wind (v_y directed westward).

2. Air current vertical.

The projection of v on the equator being $v \cos \lambda$, the deviating force will be

$$\psi = 2\ \omega v \cos \lambda, \qquad (10)$$

and it is immediately seen that it is horizontal and is directed westward in an upward, and eastward in a downward current.

3. Air current directed obliquely upward or downward.

This case may be easily reduced to the two former ones by resolving v into three components—the first, v_x, north-south; the second, v_y, east-west; and the third, v_z, vertical; taken positively to the north, east, and zenithward, respectively.

From v_x we get [by (*1.* a.)] only a horizontal east-west component of deviating force, which may be represented by the equation

$$\psi_y' = 2\ \omega v_x \sin \lambda;$$

from v_y [by case (*1.* b.)] we get a horizontal north-south force which may be expressed by

$$\psi_x = -2\ \omega v_y \sin \lambda,$$

and a vertical force that may be expressed by

$$\psi_z = 2\ \omega v_y \cos \lambda;$$

from v_z [by case *2*] we get only a horizontal east-west force which may be expressed by

$$\psi_y'' = -2\ \omega v_z \cos \lambda.$$

Thus we get the following components:

$$\psi_y' + \psi_y'' = \begin{cases} \psi_x = -2\ \omega v_y \sin \lambda; \\ \psi_y = \ \ 2\ \omega (v_x \sin \lambda - v_z \cos \lambda); \\ \psi_z = \ \ 2\ \omega v_y \cos \lambda. \end{cases} \qquad (11)$$

Formula (11) represents, as may be easily seen, the components of the deviating force for either hemisphere as to magnitude, direction, and sign, if we make λ positive at a northern latitude and negative at a southern latitude.

Now let α be the azimuth from which the wind blows, counted from south through west; α will also represent the azimuth *towards which* the wind blows; that is, the azimuth of v counted from north through east. Let β be the inclination of the path of the wind to the horizon, a positive value of β corresponding to an upward wind, and a negative value to a downward wind. Then by projecting v on the axes we obtain

$$v_x = v \cos \alpha \cos \beta; \quad v_y = v \sin \alpha \cos \beta; \quad v_z = v \sin \beta;$$

and by putting these values of v_x, v_y, and v_z, in (11), we get

$$\begin{cases} \psi_x = -2\ \omega v \sin \alpha \cos \beta \sin \lambda; \\ \psi_y = \ \ 2\ \omega v (\cos \alpha \cos \beta \sin \lambda - \sin \beta \cos \lambda); \\ \psi_z = \ \ 2\ \omega v \sin \alpha \cos \beta \cos \lambda; \end{cases} \qquad (12)$$

The complete discussion of formulas (11) and (12) being somewhat long, I will for this refer to my paper (1) and give here only the chief results.

In the Northern Hemisphere in an obliquely upward current *the horizontal component of the deviating force of a south wind is less, and that of a north wind is generally more, than in a horizontal current.* In an obliquely downward current the reverse holds good. As to the Southern Hemisphere, we need only to interchange the north and south in the above proposition. This symmetry of deviation is greatest near the equator. For instance, in the Northern Hemisphere, if the upward inclination β of a south wind is equal to the latitude λ, $\psi = 0$; (11) if $\beta > \lambda$, ψ will be directed to the left. The north and south winds are never vertically deviated.

If in an east-west current the air is obliquely rising or falling ψ_h will not be directed exactly perpendicular to v, but if the air is rising it throws the east winds a little forward and the west winds a little backward, and the reverse of this if the air is falling. The west winds are always deviated upward and the east winds downward.

IV. RELATIVE PATH OF INERTIA OF AN AIR PARTICLE.

If no external forces accelerate a mobile, it will, on account of its inertia, describe a straight line with a constant velocity; this is therefore the absolute path of inertia.

The path that the mobile under the same conditions will describe on a moving system, such as the earth, is however generally a curved line, on account of the two apparent forces ψ_s and ψ then brought into play, and it is described with varying velocity. But the simultaneous absolute path being a straight line and the absolute velocity constant, this relative path is also evidently a real path of inertia. Such a path would be described by a particle of air moving relative to the earth's surface, if the influence upon it of the absolute gravity, pressure-gradients, and friction, always balanced each other. Since the absolute path is a straight line, such a particle could never be at relative rest.

In meteorology there has, however, been introduced another definition of the path of inertia, which, while it does not well correspond to this name, may yet be of use in some investigations. Let us suppose that the influence of the *apparent* gravity, pressure-gradients, and friction, on the air particle, always balance each other. Under these conditions, if the air particle was at relative rest at the beginning it would remain at rest; and if at the beginning it had a certain relative velocity, due to some external impressed force, it would on account of its inertia maintain this velocity unaltered, and only change the direction of its relative path; for the only force acting upon it would be ψ, which has been shown to be always acting perpendicularly to the relative path. Such a path of inertia may consequently be called the *apparent path of inertia*.

1. First, let the original direction of the path be parallel to the earth's axis. Then $\psi=0$, and the relative path is a straight line parallel to the axis of the earth.

2. Second, let the original direction of the path be parallel to the equator plane, and the original velocity equal to v. Then the deviating force will be $\psi=2\omega v$, and also acting parallel to the equator plane. Since ψ is constant and perpendicular to v, the path will evidently be a circle whose radius, ρ, will be determined by putting the resistance of inertia of the particle, $\frac{v^2}{\rho}$, equal to ψ, thus:

$$\frac{v^2}{\rho}=2\omega v,$$

by which

$$\rho=\frac{v}{2\omega}.$$

The periphery being $2\pi\rho$, and v being the part of the periphery described in a second of mean solar time, the time of the whole revolution will be $\frac{2\pi\rho}{v}=\frac{\pi}{\omega}$ seconds of mean solar time; that is, one half of a siderial day (12). The radius is proportional to v, and if $v=1$ m./sec., $\rho=6\,857$ meters; if $v=10$ m./sec., $\rho=68\,570$ meters, etc. The direction of motion is opposite to that of the earth's rotation; at the pole the plane of the path is horizontal and at the equator it is vertical, the direction being eastward in the lower half of the path and westward in the upper half. At any latitude the plane of the path, which is always parallel to the plane of the equator, will be inclined to the horizon, the direction of motion being the same as at the equator.

3. Third, let the original direction of motion be inclined by an angle γ to the earth's axis. Then let v be resolved into two components, $v\cos\gamma$ and $v\sin\gamma$, parallel respectively to the earth's axis and to the equator plane. The effect of the first component alone will be to cause the particle to describe (by 1 above) a straight line parallel to the earth's axis, with the constant velocity $v\cos\gamma$; the effect of the second component taken alone will be (by 2 above) to cause the particle to describe a circle parallel to the equator plane and having the radius $\frac{v\sin\gamma}{2\omega}$. The actual path will therefore be a helix, having its axis parallel to the earth's axis.

We may also attempt to determine the path of inertia of a particle moving in a horizontal plane at the latitude λ. Here, also, if λ is constant the path will be a circle having the radius equal to $\frac{v}{2\omega\sin\lambda}$; the time of revolution will be $\frac{\pi}{\omega\sin\lambda}$; that is, ½ cosec λ times a siderial day (13). Such a path of inertia can exist only when certain vertical forces, such as pressure and temperature gradients, constrain the particle to always remain in the same horizontal plane; for instance, when the vertical equilibrium of the atmosphere is very stable.

V. COMPARISON BETWEEN THE THEORY AND EXPERIENCE.

Thus far our deductions have been purely theoretical, and the results given above possess mathematical certainty; but whether the apparent forces generated by the earth's rotation are essentially important to dynamic meteorology depends upon the order of magnitude of these forces. They will be of importance only when they are of the same order of magnitude as the absolute forces, pressure gradients, and friction, which we know from observation produce and stop air motion. This question will now be examined.

In comparing these forces we shall express all in the centimeter-gram-second system.

The *unit of force*, the dyne or $\frac{\text{g. cm}}{\text{sec}^2}$, is the force which acting during one second (sec.), mean time, on a mass of one gram (g.) produces an acceleration of one centimeter (cm.). Thus pressure is measured in dynes per square centimeter $\left(\frac{\text{dyne}}{\text{cm}^2}=\frac{\text{g}}{\text{cm.sec}^2}\right)$

A normal atmosphere (measured by a column of mercury 76 cm. in height *reduced to* normal density and gravity) is equal to $1.01325\times10^6\ \frac{\text{dynes}}{\text{cm}^2}$ and thus very nearly a million dynes or one megadyne per cm².

In a horizontal air current the only moving force is the horizontal gradient $-dp/dx$, that is, the diminution of pressure per unit of length along a level surface, or a surface at all points normal to apparent gravity. Let G be this gradient in centimeters of mercury for 1° along the meridian, dp the diminution of pressure in dynes/cm² and dx the corresponding variation of length expressed in centimeters; we will then obtain

$$-\frac{dp}{dx}=\frac{1.01325\times10^6}{76}\cdot\frac{90}{10^9}\cdot G, \text{ in } \frac{\text{dynes}}{\text{cm}^3};$$

that is,

$$-\frac{dp}{dx}=0.001200G, \text{ in } \frac{\text{g}}{\text{cm}^2.\text{sec}^2}. \qquad (13)$$

This equation represents the variations in the pressure acting on 1 cubic centimeter of air moving in a horizontal path. In order to obtain the acceleration in cm./sec.², it must be divided by the mass of a cubic centimeter of air; that is, by the density of the air, δ, in $\frac{\text{g}}{\text{cm}^3}$; we thus obtain for the acceleration produced by the gradient G,

$$-\frac{1}{\delta}\frac{dp}{dx}=\frac{0.001200}{\delta}\,G, \text{ in } \frac{\text{cm}}{\text{sec}^2}. \qquad (14)$$

For example, let the air have a temperature of 17.0°C., a pressure of 75.3 cm., and a relative humidity of 70 per cent, a condition that nearly corresponds to the summer mean of northern Europe; then $\delta = 0.001200$, and the acceleration sought will be simply equal to G.

We thus see that at the mean air density, $\delta = 0.001200 \frac{g}{cm^3}$, the acceleration produced by G will be expressed by the same measure of length as the gradient itself, if this is given for a meridian degree.

Now we know by observation the relation between gradient and wind velocity in a steady motion. In a cyclone, for example, a gradient of 5 mm. (=0.5 cm.) will generally produce stormy winds with a velocity of about 30 meters per second ($v = 3\,000$ cm./sec.²). Then, since $\omega = 0.00\,007\,292$, we get by equation (7) for a current parallel to the equator $\psi = 0.44$ cm./sec.², which is nearly the same value as that for the acceleration produced by the gradient, this latter being 0.5 cm./sec.². Further, since, as we know, the horizontal deviation of the wind from the direction of the gradient amounts to at least 60°–70°, only G cos 60° to G cos 70° or about $\frac{1}{2}$ to $\frac{1}{3}$ of the gradient force will accelerate the air movement, while the remaining component, G sin 60° to G sin 70°, or about $\frac{7}{8}$ to $\frac{16}{17}$ of G, being perpendicular to the path, will unite with the horizontal component of ψ [or $\psi_h = 2\omega\, v \sin\lambda$ by equation (8)] in a resultant perpendicular to the path and tending to incurve it, thereby modifying the distribution of the mass and the pressure of the air.

The vertical component of ψ [or $\psi_z = 2\omega v_y \cos\lambda$ by equation (9) or (11)] will also tend to incurve the path, the direction being upward in a west wind (v_y positive), and downward in an east wind (v_y negative), and thereby likewise modify the distribution of the mass and the pressure of the air. This component has hitherto been very generally neglected, but as will be seen below, this will not do.

Further, as the researches of Guldberg and Mohn (**14**) have shown, the resistance of friction caused by the earth's surface on the lowest strata of the atmosphere, like ψ, is a quantity proportional to v; friction $= kv$.

The coefficient of friction, k, ranges between 0.00002 and 0.00004 for the open sea, and does not exceed 0.00012 for a very rough continental surface. Friction even at its maximum will therefore be less than ψ, or $2\omega v$, in a current parallel to the equator ($2\omega = 0.00\,014\,584$).

As to the upper strata of the atmosphere H. von Helmholtz (**15**) has shown the friction in a horizontal parallel current is so extremely small that it vanishes altogether when compared with the gradient. Hence it follows that in the upper strata even a very slight gradient—for example, 0.1 mm. (0.01 cm.)—if it act sufficiently long in the direction of motion, is able to produce a very great velocity, particularly as the acceleration is inversely proportional to the density of the air. For instance, at an altitude where the density is only ⅓ of that at sea level (about 8 km., which corresponds to the average height of the cirrus), the acceleration produced by the above-named slight gradient will be 0.03 cm./sec.². This acceleration acting in the direction of motion during 100 000 seconds, or about 28 hours, will produce a velocity of 3 000 cm./sec. = 30 m./sec.; and with this velocity ψ is, as previously shown, equal to 0.44 cm./sec.², which is about 15 times greater than the component of acceleration acting in the direction of motion.

In order more fully to examine the action of the deviating force ψ, we must decompose it by means of the formulas given in Part III. If the air current is nearly horizontal, formulas (8) and (9) will suffice. Since the horizontal component is $\psi_h = 2\omega v \sin\lambda$ and the vertical component is $\psi_z = 2\omega v_y \cos\lambda$, the former will attain its greatest importance in the higher latitudes, and the latter in the lower latitudes.

The importance of ψ_h has already been shown by several eminent investigators, especially Ferrel, Guldberg and Mohn, Köppen, Sprung, De Marchi, and von Bezold; and having thus been generally acknowledged, we need scarcely discuss this part of the problem. The following remarks may suffice. Even in the vicinity of the equator, where this component nearly vanishes, it is essential to the formation of the trade winds and monsoons. This fact at once indicates that the vertical component ψ_z, even in the higher latitudes where it is comparable with the ψ_h of the lower latitudes, must have an essential influence on air movements. To be sure, there exists a great difference between the horizontal and vertical movements of the air, inasmuch as the former may amount to hundreds of miles, while the latter can only amount to a few miles; it therefore follows that, the accelerations and the other conditions of motion being equal, the horizontal velocities must be much greater than the vertical, since the time during which the accelerating forces act will be much greater in the former case. Nevertheless, no one will deny that a vertical displacement of the air masses for a few miles may not have as much influence on the weather as a horizontal displacement of many hundreds of miles. And even where such vertical forces do not directly produce motion, they will, since the conditions of continuity of moving air have always to be satisfied, call forth opposite forces, such as vertical pressure gradients or frictional resistances, which will essentially modify the conditions of equilibrium or motion of the atmosphere.

Some concrete instances of the magnitude and direction of the deviating force ψ in the upper currents of the atmosphere are here presented from the results of the cloud measurements executed by Dr. K. L. Hagstrom and myself in Upsala (**16**) (lat. 59° 51.5′).

Let z_m be the mean height of the cloud, v_h its horizontal velocity, and α_ψ the azimuth of ψ, the other symbols being those used in the formulas given in Part III.

1885, May 26, 8 p. m. Barometric depression to the north of Upsala. Five successive measurements of a cirrus cloud.

$$z_m = 8\,068 \text{ m.};\ \alpha = 101.1;\ v_h = 15.4 \text{ m./sec.};\ v_z = 2.2 \text{ m./sec.};$$
$$\beta = 8.1°;\ \psi_h = 0.20 \text{ cm./sec.}^2;\ \alpha_\psi = 191.9°;$$
$$\psi_z = 0.11 \text{ cm./sec.}^2 \text{ upward.}$$

1885, May 30, 8 a. m., depression to the north of Upsala. Six successive measurements of a cirrus cloud.

$$z_m = 7\,405 \text{ m.};\ \alpha = 55.7°;\ v_h = 42.0 \text{ m.};\ v_z = 2.6 \text{ m.};$$
$$\beta = 3.6°;\ \psi_h = 0.52 \text{ cm./sec.}^2,\ \alpha_\psi = 147.4°;$$
$$\psi_z = 0.25 \text{ cm./sec.}^2 \text{ upward.}$$

1885, June 6, 1 p. m., depression to the north of Upsala. Three successive measurements of a high cirrus cloud.

$$z_m = 9\,143 \text{ m.};\ \alpha = 65.7°;\ v_h = 43.8 \text{ m.};\ v_z = 6.3 \text{ m.};$$
$$\beta = 8.1°;\ \psi_h = 0.54 \text{ cm./sec.}^2;\ \alpha_\psi = 160.2°;$$
$$\psi_z = 0.29 \text{ cm./sec.}^2 \text{ upward.}$$

With regard to the density of the air at this height, we find that for the last two measurements ψ_h is equivalent to a horizontal gradient of about 2 mm., ψ_z to an upward vertical gradient of about 1 mm., using the ordinary units.

In order to judge exactly of the influence of the vertical deviating component, ψ_z, we ought to compare it with the vertical pressure gradients. The calculation and measurements of these latter are, however, much more difficult than the calculation and measurement of the horizontal pressure gradients.

If the air is at rest we know by the principle of Archimedes that the lift by air pressure is equal to the apparent weight of a unit volume of air; thus according to the adopted designation, g being the acceleration of apparent gravity,

$$-\frac{dp}{dz}=g\delta, \text{ or } -\frac{dp}{dz}-g\delta=0.$$

If, however, these two forces are unequal, the difference will represent the vertical moving force of pressure, or the vertical pressure gradient, $-\frac{dp}{dz}-g\delta$, the acceleration of which is $-\frac{1}{\delta}\frac{dp}{dz}-g$, and ought to be compared with ψ_z.

If the vertical height z is taken positively upwards, p will decrease when z increases, and thus $-\frac{dp}{dz}$ will be a positive quantity, representing an upward pressure. Putting

$$-\frac{dp}{dz}-g\delta=\varepsilon\delta \tag{15}$$

the gradient $\varepsilon\delta$ is directed upward when positive and downward when negative. Thus ε represents the acceleration imparted by the gradient to the unit of volume of air at the height z.

The only means of determining this gradient by observation is given by the barometric measurement of heights. In fact, if in equation (15) we put $\varepsilon=0$ and integrate the equation, we obtain the barometric formula for measuring heights. The uncertainty of such a measurement, however, is not due alone or principally to placing $\varepsilon=0$, for, as the discussion of the barometric formula shows, we are unable to determine with sufficient accuracy the air pressure, and especially the mean air density, which is a function of the temperature and moisture, and even of the dust and water particles floating in the air, for every element of the column lying between the two barometers to be read at its top and bottom (17).

In the general case, ε will vary from one element to another, and in order to determine the dynamical state of the atmosphere, we must write down and solve the general hydrodynamic equations, with due regard to all pressure gradients, friction, deviating force ψ, condition of continuity, and limits. This seeming at present impossible, we must confine ourselves to the study of some simple and typical phenomena, comparing them with the observed facts.

In doing this, let us first consider the vertical component of the inflowing air current of a steady cyclone. First, as shown by Hann and others, the rising mass of air in the inflowing current of such a cyclone is very nearly in the *adiabatic or indifferent* equilibrium.

Let us first suppose such a mass of air to be at rest. Then we have

$$-\frac{dp}{dz}=g\delta.$$

If now a particle of air be thrown upward, by means of an impact for instance, both the lifting power $-\frac{dp}{dz}$ and the weight $g\delta$ of the unit of volume of air will decrease. But since the surrounding mass of air is in adiabatic equilibrium, $-\frac{dp}{dz}$ and $g\delta$ will always decrease at the same rate. For since the moving air mass has not time to give out or receive a sensible quantity of heat, its change of state will be adiabatic; consequently, during its motion it will always take the same density as the surrounding air in the same level. Thus, the vertical gradient will remain zero, and the only forces acting upon the moving air particle will be the deviating force ψ and the friction. But the latter may be neglected on account of its smallness; hence the air particle, under the influence of ψ, will perform the apparent path of inertia described in Part IV. It must be remembered, however, that if the air is saturated with moisture but not charged with fog, the adiabatic equilibrium, and consequently the path of inertia, will exist in an upward motion only, while the stable equilibrium, which will be spoken of below, will manifest itself in a downward movement.

In reality the air column of a cyclone is not in equilibrium, but is rising under the influence of a vertical upward force, which is the resultant chiefly of the vertical pressure gradient and the vertical component ψ_z; also, its different strata are rotating around a vertical axis. That the acceleration of these vertical pressure gradients will probably in most cases be smaller than ψ, and even than ψ_z, especially in the upper strata, is shown by the following reasoning. The vertical pressure gradients must generally be smaller than the horizontal ones, because the frictional resistance is less, and the flow less checked than in a horizontal current. Now since the horizontal pressure gradients are generally not greater than ψ, or even ψ_h, the vertical gradients will be less than ψ, and not greater than ψ_z.

The above results of cloud measurements confirm this conclusion. In fact, it may be shown that in those cases the mean value of ε is probably negative, and therefore directed downward, and tending to move the air in that direction. To prove this, we must compute the upward velocity, v_z, that would be produced by the action of ψ_z alone, and compare it with the observed value of v_z. Now, ψ_z is proportional to the westerly component of velocity, v_y, and this, as shown both by our own cloud measurements and those of H. H. Clayton, is very nearly proportional to the altitude z. Thus for ψ_z put c^2z, where c is a constant, we get

$$\frac{d^2z}{dt^2}=c^2z,$$

if we suppose that simultaneously $z=0$ and $\frac{dz}{dt}=v_z=0$, the first integral of this will be (18)

$$v_z^2=c^2z^2. \tag{16}$$

By means of the observations cited above, we get

1885.	May 26.	May 30.	June 6.
$c^2=$	$\frac{0.11}{806\,800}$,	$\frac{0.25}{740\,500}$,	$\frac{0.29}{914\,300}$;

thus, by equation (16), at the measured heights,

v_z (calculated) $=298$,	430,	515cm./sec.;
while v_z (observed) $=220$,	260,	630cm./sec.

In the first two cases the calculated value is greater than the observed, and only in the third is it a little less; but in this case the observed value may be too great on account of errors of observation, the value being calculated from only three successive observations. Generally we have not found vertical upward velocities so great as those given above, although horizontal eastward velocities amounting to more than 50 m./sec. are not uncommon, and the values of ψ_z are proportional to this.

Hence it seems that generally, at least in the southern part of the cyclone, where westerly winds blow in all the strata, the vertical upward velocity that would be produced by ψ_z and thus indirectly by these strong westerly winds, is greater than the one really observed. Now the observed velocity is produced jointly by ψ_z, the vertical pressure gradient ε, and the friction (which latter may be neglected); and thus it follows that ε must be negative. This remarkable result may be expressed in the following manner: A part of the *vis viva* of the westerly winds prevailing in the cyclone is used in pumping the air up, by means of ψ_z, against the mean vertical temperature gradient ε, which tends to make it descend.

Now Prof. Hann has shown (19) that the mean temperature of the air column in the inflowing winds of a cyclone is generally so much lower than that of an anticyclone, that the vertical temperature gradient of the former must probably be directed downward. This result, although deduced from incontestable facts, has called forth much criticism, as the incontestable fact of an upward motion in cyclones seemed then inexplicable. Hann himself has pointed out that the mechanical energy (*vis viva*) of the upper current may be able to pump up the air of the cyclone against the pressure gradient. I believe that I have now shown how this transformation takes place. Of course, there may be modes of transformation other than the above, but this evidently will accomplish much. As to the *vis viva* of the upper currents of the cyclone, it may originate partly from the general atmospheric circulation, and partly from the mechanical energy produced by the cyclone itself from the latent heat of aqueous vapor. The proportion between these two sources of energy is probably quite variable.

There is another observed fact, which is explained by the action of ψ_z. Clement Ley and Hildebrandsson have observed that the cirrus clouds are much more numerous in westerly upper currents than in easterly. Now, since ψ_z is directed upward in the former and downward in the latter, the air will generally rise in a westerly current and thereby be cooled, so that the aqueous vapor contained in it will be condensed and form the ice needles of which the cirri consist. The reverse will take place in an easterly current.

The vertically deviating component ψ_z will also have a marked influence on the propagation of the cyclone center. Considering the well-known diagrams of Clement Ley and Hildebrandsson, we find that all strata, upper and lower, have a westward component of horizontal velocity in the northwest quadrant of the cyclone, but in no other. Thus the currents of all strata in this quadrant will have a downward acceleration (ψ_z negative) which after a time will give a downward velocity. This will reach its maximum somewhere to the west of the center, in the rear of the cyclone; then the currents entering the southwest quadrant will acquire an eastward component of horizontal velocity, which will give rise to an upward acceleration (ψ_z positive), by which the downward velocity acquired in the northwest quadrant will be gradually diminished and will vanish somewhere in the southern part of the cyclone. It therefore follows that the air will tend to sink down in the western half of the cyclone and that the reverse will happen in the eastern half. Obviously this will contribute to fill up the western part and empty the eastern part of the cyclone, so as to displace the center eastward. Certainly the propagation of a cyclone is a very complicated phenomenon which may have many coöperating causes, but I think the cause above named is in most cases a very efficacious one that may not be neglected.

REFERENCES AND NOTES.

(1) Partly extracted from the memoir: "Über die Einwirkung der ablenkende Kraft der Erdrotation auf die Luftbewegung." Bihang till K. Svenska Vetensk.-Akad. Handl., Bd. 15, Afd. 1, No. 14. Stockholm, 1890.

(2) The cause of the general trade-winds. Phil. trans., London, 1735.

(3) Mémoire sur les équations du mouvement relatif des systémes de corps. Jour., École polytechnique, t. 15, cahier 24, p. 142.

(4) Motions of fluids relative to the earth's surface. Mathematical monthly (Runkle), 1859; and then in Ferrel's well-known "Researches."

(5) Études sur les mouvements de l'atmosphère. Christiania, 1876 and 1880.

(6) Traité de méchanique rationelle, par M. Ch. Delaunay. 2me. éd., Paris, 1857. p. 94. (I have not seen the first edition of this treatise.) The proof is reproduced in "Cours de mécanique" par M. Despérioux, etc. Avec des Notes par M. G. Darboux. t. 1. Paris 1884. p. 176 ffg. Also in Schell's "Theorie der Bewegung und der Kräfte" there is a geometrical proof of Coriolis' theorem, which is clear and rigid but rather long.

(7) Sprung. Lehrbuch der Meteorologie. Hamburg. 1885. p. 16.

(8) The proof is quite general; only for the sake of perspicacity we refer the position of the mobile immediately to the earth.

(9) Instead of these we may use the geographical latitude and the mean radius of the earth, without introducing any sensible error.

(10) Sprung. Lehrbuch der Meteorologie. p. 80–81.

(11) This is evident since then v is perpendicular to the equator.

(12) Because ω is equal to the quotient of 2π by a siderial day expressed in seconds of mean time $\omega = 2\pi/86\,164.09^{\text{sec.}} = 0.00\,007\,292\,1$.

(13) *See* Sprung's Lehrbuch. p. 16.

(14) Études sur les mouvements de 'atmosphère. Christiania, 1876, and Ztschr. d. Gesellsch. f. Meteorol., Wien, 1877, 12:53.

(15) Meteorologische Zeitschrift, Wien, 1888, 5. Jhrg., p. 329.

(16) These measurements number more than 2,000. Although calculated some years previous to 1893 they have not yet been published in extenso, partly for want of time and partly for other reasons.

(17) This is according to Jordan, "Handbuch der Vermessungskunde," Stuttgart, 1877. 1. Bd., p. 532. Bauernfeind's inquiries furnish for a difference of height of 1 km. a mean error of ± 5.7 meters in the difference when determined barometrically. This, if due to a vertical gradient, would require a gradient of about 50 mm., which is obviously impossible.

For the rest, it would evidently require very exact barometric observations to determine even a horizontal gradient in so short a distance as 1 km.

(18) The second integral, $t - t_0 = \frac{1}{c}$ nat. log $\frac{z}{z_0}$, gives the time necessary for the vertical movement from z_0 to z. If $z_0 = 0$, it becomes infinite, which is rational, as we have supposed both acceleration and velocity equal to 0 at the ground ($z = 0$).

(19) *See* Meteorologische Zeitschrift, Wien, 1890, 7. Jhrg. p. 226, 328, 457.

METEOROLOGY AT THE LICK OBSERVATORY.[1]

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[Dated, University of California, Department of Geography, June 10, 1914.]

INTRODUCTION.

The Lick Observatory was founded under two deeds of trust by James Lick of San Francisco, the first dated July 16, 1874, and the second, September 21, 1875. These provided for "a powerful telescope, * * * and also a suitable observatory connected therewith." After a careful consideration of various possible sites (restricted to the State of California by the terms of the trust), the choice was Mount Hamilton, lat. 37° 20′ north, long. 121° 38′ west from Greenwich, altitude,[2] 4,209 feet above sea level, located among the Coast Ranges in the eastern

[1] The writer wishes to thank the members of the staff of the observatory, without whose cooperation and assistance this study could not have been made, and in particular to thank Director W. W. Campbell for the meteorological data, the photographs, and numerous other courtesies at the observatory.