SECTION II.—GENERAL METEOROLOGY.

THE PROBABLE GROWING SEASON.[1]

By WILLIAM GARDNER REED.

[Dated U. S. Office of Farm Management, Washington, July 24, 1916.]

Although the term "growing season" has been used to indicate the number of days between the last killing frost in Spring and the first killing frost in Fall,[2] the average length of this interval is not the time available for the growth of planted crops. The accompanying map in figure 2 (W. G. R., fig. 2, Chart XLIV–121) is an attempt to show the "probably available growing season" or the number of days without killing frost for which the chance is about four in five, the chance being computed in the same manner as insurance risks are determined. If losses from frost occur more frequently than one year in five, in the long run, the farmer is not likely to succeed. The second map (W. G. R., fig. 3) shows the dates on which begin the periods indicated in the map of the "available growing season" (fig. 2). This map shows the date on which the chance of killing frost falls to one in ten, or when the chance that there will be no later spring frost becomes nine in ten. The third map (W. G. R., fig. 4) shows the probable end of the growing season. The dates are those on which the chance of killing frost in Fall rises to one in ten, or when the chance of safety becomes less than nine in ten.

The average number of days without killing frost is considerably longer that that available for the growth of planted annuals, as may be easily seen from an examination of the characteristics of frost occurrence. For example, the frost record of Keokuk, Iowa, Table 1 and figure 1, shows that the average date of the last killing frost in Spring is April 15 and the average date of first killing frost in Fall is October 15; therefore, the average number of days without killing frost is 183. If the record is carefully examined, it will be seen, however, that a killing frost occurred in Spring later than the average date in 20 of the 43 years. This means that on the average date there has been only a 53 per cent (23 in 43) chance of safety. The study of all the available records for the United States shows that this approximately even chance of safety is a general condition; that is, crops above ground on the average date will be killed by spring frost in about half the years. This leaves only half the crops to be carried through the summer.[3]

The conditions obtaining for fall frosts are similar. The first killing frost in Fall at Keokuk occurred before the average date (October 15) in 21 of the 43 years. Of the 20 years when there was no killing frost in the spring after the average date, the first killing frost in Fall occurred before the average date in 8 years. In other words, the "average season without killing frost" was available for planted crops in only 12 of the 43 years, or in 28 per cent of the years. Unless the time required to mature the crop is much shorter than the "average season without killing frost" planting near the average date of last killing frost in Spring is out of the question even for high value crops. Successful farming can not be carried on if the loss from frost is as great as one in two, to say nothing of three in four.

The problem for the farmer is to select a planting date late enough to afford reasonable assurance of safety from spring frost and early enough to give the crop time to mature before the danger from Fall frost becomes too great. The "probably available growing season" then is the period between the time of reasonable safety in Spring and reasonable safety in Fall.

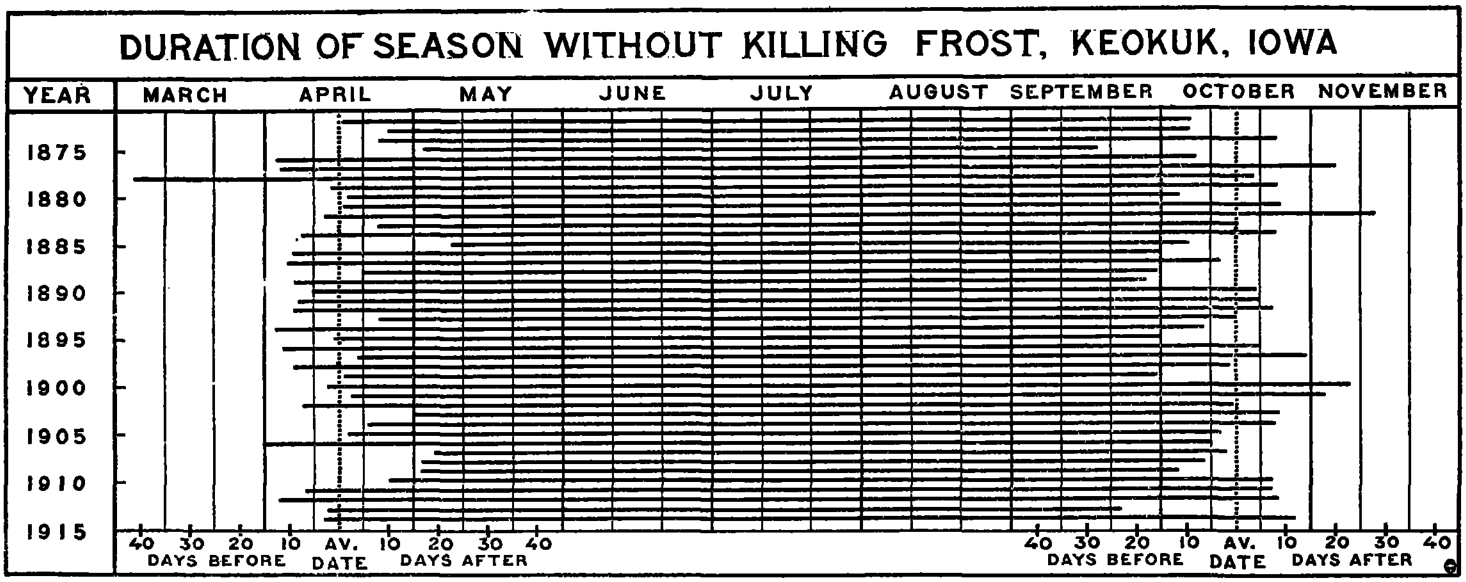

FIG. 1.—Duration of growing seasons at Keokuk, Iowa, 1872–1914, inclusive. (Average number of days without killing frost is 183.)

[1] Prof. W. J. Spillman, Chief of the Office of Farm Management, made the original suggestion leading to the study of the characteristics of the frequency distributions of *climatic phenomena important in farm management investigations. This paper is* one result of the study. The development and application of the mathematical theory is the work of Mr. H. R. Tolley of the Office of Farm Management.

Accompanied by Charts XLIV–121 to XLIV–123.

[2] *Day, P. C.* Frost data of the United States and length of the crop-growing season. (Weather Bur. Bul. V), Washington, 1911.

Fassig, O. L. The period of safe plant growth in Maryland and Delaware. MONTHLY WEATHER REVIEW 42: 152–158, Washington, 1914.

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[3] *Reed, W. G., & Tolley, H. R.* Weather as a business risk in farming. Geographical review, New York, 1916, 2: 48–53. *Abstract in* MONTHLY WEATHER REVIEW, June, 1916, 44: 354–5.

TABLE 1.—*Correlation of the last killing frost in Spring and the first killing frost in Fall, with the length of the season without killing frost at Keokuk, Iowa, 1872–1914, inclusive.*

Year.	Last killing frost in Spring.			First killing frost in Fall.			$d_s d_a$	Season without killing frost.			Year.
	s	d_s	d_s^2	a	d_a	d_a^2		l	d_l	d_l^2	
								Days.			
1872	Apr. 16	+1	1	Oct. 6	−9	81	− 9	173	−10	100	1872
1873	Apr. 25	+10	100	Oct. 6	−9	81	− 90	164	−19	360	1873
1874	Apr. 23	+8	64	Oct. 23	+8	64	+ 64	183	0	......	1874
1875	May 2	+17	289	Sept. 18	−27	729	− 459	139	−44	1,936	1875
1876	Apr. 2	−13	169	Oct. 7	−8	64	+ 104	188	+5	25	1876
1877	Apr. 3	−12	144	Nov. 5	+21	441	− 252	216	+33	1,089	1877
1878	Mar. 4	−42	1,764	Oct. 19	+4	16	− 168	229	+46	2,116	1878
1879	Apr. 13	−2	4	Oct. 24	+9	81	− 18	194	+11	121	1879
1880	Apr. 17	+2	4	Oct. 4	−11	121	− 22	170	−13	169	1880
1881	Apr. 16	+1	1	Oct. 24	+9	81	+ 9	191	+8	64	1881
1882	Apr. 12	−3	9	Nov. 13	+29	841	− 87	215	+32	1,024	1882
1883	Apr. 24	+9	81	Oct. 15	0	0	0	174	−9	81	1883
1884	Apr. 8	−7	49	Oct. 23	+8	64	− 56	198	+15	225	1884
1885	May 8	+23	529	Oct. 6	−9	81	− 207	151	−32	1,024	1885
1886	Apr. 6	−9	81	Oct. 1	−14	196	+ 126	178	−5	25	1886
1887	Apr. 5	−10	100	Oct. 12	−3	9	+ 30	190	+7	49	1887
1888	Apr. 20	+5	25	Sept. 29	−16	256	− 80	162	−21	441	1888
1889	Apr. 6	−9	81	Sept. 27	−18	324	+ 162	174	−9	81	1889
1890	Apr. 10	−5	25	Oct. 19	+4	16	− 20	192	+9	81	1890
1891	Apr. 7	−8	64	Oct. 20	+5	25	− 40	196	+13	169	1891
1892	Apr. 6	−9	81	Oct. 23	+8	64	− 72	200	+17	289	1892
1893	Apr. 23	+8	64	Oct. 15	0	0	0	175	−8	64	1893
1894	Apr. 2	−13	169	Oct. 9	−6	36	+ 78	190	+7	49	1894
1895	Apr. 14	−1	1	Sept. 30	−15	225	+ 15	169	−14	196	1895
1896	Apr. 4	−11	121	Oct. 20	+5	25	− 55	199	+16	256	1896
1897	Apr. 19	+4	16	Oct. 29	+14	196	+ 56	193	+10	100	1897
1898	Apr. 6	−9	81	Oct. 14	−1	1	+ 9	191	+8	64	1898
1899	Apr. 16	+1	1	Sept. 29	−16	256	− 16	166	−17	289	1899
1900	Apr. 13	−2	4	Nov. 8	+24	576	− 48	209	+26	676	1900
1901	Apr. 18	+3	9	Nov. 3	+19	361	+ 57	199	+16	256	1901
1902	Apr. 8	−7	49	Oct. 14	−1	1	+ 7	189	+6	36	1902
1903	May 1	+16	256	Oct. 24	+9	81	+ 144	176	−7	49	1903
1904	Apr. 21	+6	36	Oct. 23	+8	64	+ 48	185	+2	4	1904
1905	Apr. 17	+2	4	Oct. 12	−3	9	− 6	178	−5	25	1905
1906	Apr. 1	−14	196	Oct. 10	−5	25	+ 70	192	+9	81	1906
1907	May 4	+19	361	Oct. 13	−2	4	− 38	162	−21	441	1907
1908	May 2	+17	289	Oct. 9	−6	36	− 102	160	−23	529	1908
1909	May 2	+17	289	Oct. 4	−11	121	− 187	155	−28	784	1909
1910	Apr. 25	+10	100	Oct. 22	+7	49	+ 70	180	−3	9	1910
1911	Apr. 9	−6	36	Oct. 22	+7	49	− 42	196	+13	169	1911
1912	Apr. 3	−12	144	Oct. 23	+8	64	− 96	203	+20	400	1912
1913	Apr. 13	−2	4	Sept. 22	−23	529	+ 46	162	−21	441	1913
1914	Apr. 12	−3	9	Oct. 27	+12	144	− 36	198	+15	225	1914
Sums	....	−30	5,904	....	+5	6,487	−1,111	......	+35	14,612	Sums
Means	Apr. 15	.....	......	Oct. 15	......	......	....	183	......	....	Means

Symbols.

In order to employ the data of Table 1 in mathematical investigations it is necessary to employ the shorthand of mathematics, viz, symbols. Those used in this paper include the conventional ones generally adopted for investigations in "probability," and they are listed below for convenience of reference. In a previous similar study [4] the author made use of other symbols which did not employ Greek letters; these previous symbols are given in [] after the appropriate explanation.

- s is the date of last killing frost in Spring in any year.
- a is the date of first killing frost in Fall in any year.
- l is the number of days between s and a in any year.
- A_s is the average date of last killing frost in Spring. [M_s]
- A_a is the average date of first killing frost in Fall. [M_a]
- A_l is the average number of days without killing frost. [M_l]
- A_s' is some arbitrary number near A_s. [M_s']
- A_a' is some arbitrary number near A_a. [M_a']
- A_l' is some arbitrary number near A_l. [M_l']
- d_s is the departure of s from A_s'. [X_s']
- d_a is the departure of a from A_a'. [X_a']
- d_l is the departure of l from A_l'. [X_l']
- n is the number of observations. [n]
- σ_s is the standard deviation of s. [D]
- σ_a is the standard deviation of a.
- σ_l is the standard deviation of l.
- r is the coefficient of correlation.
- E_r is the probable error of r.
- Σd_s, etc., algebraic sums of d_s, etc. [r]
- Σd_s^2, etc., algebraic sums of d_s^2, etc. [$S(X')^2$]

If crops are to be planted intelligently, the farmer must know the chance of frost occurrence on any date in both Spring and Fall. There would be no difficulty in determining the chance in the long run if the records were of sufficient length—that is, at least 100 years, or still better 500. For the United States, however, the longest frost record is but 59 years, and there are but 671 complete records of 20 years or more, although there are 116 additional 20-year records with one or more years missing. [The theoretical probability can be determined satisfactorily from numerous short records, but it must be recognized that in nature the actual weather conditions over a short period may and usually do differ widely from the theoretical ones for a long period.] It has been shown in another place [5] that the chance of occurrence of frost can be determined with usable accuracy from a 20-year record on the basis of the theory of probability. The method by which this may be done is indicated by Table 2. It is applicable to frost data because the dispersion of these data follows closely that of the "normal frequency curve."

Computation of the "standard deviation," etc.

When the standard deviations of the dates of last killing frost in Spring (σ_s) or of the first killing frost in Fall (σ_a) are known, the date when the chance of occurrence falls to a given per cent and the average interval between unfavorable occurrences may easily be determined from the Spillmann curve [6] or from Table 2 below. The computation of the standard deviation and other quantities referred to later is as follows.

It appears from the last two lines of Table 1 that—

$\Sigma d_s = -30$	$\Sigma d_a = +5$	$\Sigma d_s d_a = -1111$	$\Sigma d_l = +35$
$A_s' =$ April 15	$A_a' =$ Oct. 15		$A_l' = 183$
$A_s =$ April 15	$A_a =$ Oct. 15		$A_l = 183$

The value of the standard deviation, σ, is computed by the formula

$$\sigma = \sqrt{\frac{\Sigma d^2}{\text{n}} - \left(\frac{\Sigma d}{\text{n}}\right)^2} \qquad (1)$$

and successive substitutions in (1) give the following values:

$$\sigma_s = \sqrt{\frac{5904}{43} - 0.49} = 11.7 \text{ days} \qquad \sigma_a = \sqrt{\frac{6487}{43} - 0.01} = 12.3 \text{ days} \qquad \sigma_l = \sqrt{\frac{14612}{43} - 0.66} = 18.4 \text{ days.}$$

[4] *Spillmann, W. J. & others.* The average interval curve and its application to meteorological phenomena. MONTHLY WEATHER REVIEW, April, 1916, 44: 197-200, with plate.

[5] *Reed, W. G. & Tolley, H. R.*, op. cit.
Spillman, W. J., & others, op. cit.

[6] *See* MONTHLY WEATHER REVIEW, April, 1916, 44, chart facing p. 198.

A possible relation between these values may be revea by computing the correlation coefficient, r, according to the usual formula which is stated in (2).

$$r=\left\{\frac{\Sigma d_s d_a}{n}-\left(\frac{\Sigma d_s}{n}\right)\left(\frac{\Sigma d_a}{n}\right)\right\}\div\sigma_s\sigma_a, \qquad (2)$$
$$=\left\{\frac{-1111}{43}-\left(\frac{-30}{43}\right)\left(\frac{5}{43}\right)\right\}\div(11.7\times12.3),$$
$$=-0.179.$$

One may also determine the standard deviation of the season without killing frost, σ_l, from the following equation given by Yule[7]:

$$\sigma_l^2=\sigma_s^2+\sigma_a^2-2r\sigma_s\sigma_a, \qquad (3)$$
$$=136.89+151.29-(-51.52),$$
$$=339.70$$

therefore

$$\sigma_l=\sqrt{339.70}=18.4.$$

The existence of the residual $2r\sigma_s\sigma_a$ shows that mathematically the dates of last spring and first fall frost can not be regarded as independent. However, if E_r, the probable error of r, is calculated from the usual formula[8] it will be seen to be large when compared with the value of r.

Thus

$$E_r=\pm0.67\ \frac{1-r^2}{4\sqrt{n}} \qquad (4)$$
$$=\pm0.674\frac{1-0.0321}{6.6}$$
$$=\pm0.099$$

TABLE 2.—*Chance of frost occurrence, by the theory of probability, based on a 20-year record whence the standard deviations are known.*

[See p. 510 for explanation of symbols.]

Chance of frost.	Chance of safety.	Spring.	Fall.
Per cent.	*Per cent.*		
50	50	A_s	A_a
40	60	$A_s+0.25\sigma_s$	$A_a-0.25\sigma_a$
30	70	$A_s+0.52\sigma_s$	$A_a-0.52\sigma_a$
25	75	$A_s+0.67\sigma_s$	$A_a-0.67\sigma_a$
20	80	$A_s+0.84\sigma_s$	$A_a-0.84\sigma_a$
10	90	$A_s+1.28\sigma_s$	$A_c-1.28\sigma_a$

The chance that a definite period extending through the Summer will be free from killing frosts may be computed from the chance that it will be free from killing frost at the beginning and at the end. In mathematical terms this is as follows: The probability that two independent events should both happen is the product of the separate probabilities of their happening. It can be shown that the longest period with any given chance of safety from killing frost will be that when the chance of killing frost in the early part (Spring) is equal to the chance of killing frost in later part (Fall), the middle part (Summer) being practically always frost free in the strictly agricultural regions.

The mathematical statement of the problem is as follows:

Let P' = the chance that the selected period will be frost free.
P_s = the chance of safety in Spring.
P_a = the chance of safety in Fall.
and $P_s=P_a=P$

$$P\times P=P'$$
$$P^2=P'$$
$$P=\sqrt{P'}$$

This may be applied to any case under consideration; for example, the frost record at Keokuk, Iowa, as given by Table 1, may be studied. On the average date in Spring (April 15) $P_s=0.50$ and on the average date in Fall (October 15) $P_a=0.50$.

Then
$$P=0.50$$
$$P^2=0.25$$
but $P^2=P'$
$$P'=0.25 \text{ or } 25 \text{ per cent}$$

which is not far from the 28 per cent shown by counting the cases; in fact it is within the variation to be expected as a result of the small number of observations.

It is equally easy to find the length of the season in which the chance of killing frost is even, that is, 50 per cent.

Here
$$P'=0.50$$
$$P^2=0.50$$
$$P=\sqrt{0.50}$$
$$=0.71 \text{ or } 71 \text{ per cent.}$$

Table 2 shows that the chance of safety in spring rises to 70 per cent at $0.52\sigma_s$ days after the average and falls to 70 per cent in fall at $0.52\sigma_a$ days before the average date, so that at Keokuk (see Table 1), for example—

$$A_s = \text{April 15 (14.3)*}$$
$$\sigma_s = 11.7$$
$$A_s+0.52\,\sigma_s = \text{April 21 (20.4)*}$$

$$A_a = \text{October 15 (15.1)*}$$
$$\sigma_a = 12.3$$
$$A_a-0.52\sigma_a = \text{October 8 (8.7)*}$$
$$\text{April 21 to October 8} = 170 \text{ days.}$$

Thus, at Keokuk there is an even chance that the 170 days from April 21 to October 8 will be free from killing frost. An examination of the Keokuk record shows that there actually were 26 years between 1872 and 1914 in which there were more than 170 days beginning with April 21 on which killing frost did not occur. This is a chance of 60 per cent and, if the years represent a fair sample of the conditions at Keokuk, it shows that the calculated season is a little too short. The explanation of this is to be found in the fact that during the years studied the first killing frost of Fall sometimes came late in years when the last killing frost in Spring was early, and vice versa. That is, there is a doubtful tendency toward "negative correlation," as is shown by the fact that there is a residual ($2r\sigma_s\sigma_a$) in equation (3) in which r (the coefficient of correlation[9]) has a value of -0.179. But the probable error of this coefficient is so large (0.099) when compared with its value, that correlation remains doubtful. The fact that this condition is persistent in most

[7] *Yule, G. Udney.* Introduction to the theory of statistics. 2d ed. London, 1912. p. 210–211.
[8] *Yule, G. U.* Op. cit., p. 352–353.

* All fractions are added in Spring and dropped in Fall.
[9] *Smith, J. Warren.* Correlation. MONTHLY WEATHER REVIEW, May, 1911, 39: 792–795.

records is, perhaps, significant. At any rate, the existence of the residual due to the apparent value of r must be considered in any such comparison as this. Only when the value of r is zero does the residual disappear. In the computation of risk, however, it is better to assume that there is no negative correlation between the date of last killing frost in Spring and first killing frost in Fall, as this gives a small margin of safety.

SIGNIFICANCE OF THE CHARTS.

The limiting dates of the season with any other chance of safety may be determined in the same manner. A consideration of the agricultural conditions and of the planting dates of grain crops leads to the belief that the risk of frost damage may reasonably be carried when the chance of killing frost falls to 1 in 10 and that crops should generally be harvested before the chance of killing frost in Fall has risen much above that ratio. If these dates are observed, the available growing season is that which may be expected to occur in about four-fifths of the years. That is—

$$P_s = 0.90$$
$$P_a = 0.90$$
$$0.90 \times 0.90 = P'$$
$$P' = 0.81 \text{ or about } 4/5.$$

The results are, of course, the same if it be assumed that a season free from killing frost in about four years in five is required for successful agriculture.

Here
$$P' = 0.80$$
but
$$P' = P^2$$
then
$$P^2 = 0.80$$
$$P = \sqrt{0.80}$$
$$P = 0.894$$
$$= 9/10 \text{ (approx.)}$$
therefore
$$P_s = 9/10$$
$$P_a = 9/10$$

The maps presented as figures 3, 4, and 5 (charts XLIV–121 to 123) are intended to supplement rather than to supersede the maps showing average conditions. The usual maps of average conditions will continue to be more accurate for the information they are able to give, viz, the dates after or before which the chance of frost is 1/2 and the length of the season available in 1/4 of the years, because many more data are available for their construction. The new charts presented with this paper attempt to furnish information about more closely calculated periods, by means of which the degree of certainty of freedom from frost may be better calculated and farm practice accordingly better adapted to the natural conditions of the region. In general it appears that the chance of killing frost falls to 10 per cent between 10 and 30 days after the average date of the last killing frost in Spring; in the Fall the corresponding difference is about the same. In general any station has a dispersion in Spring similar to that in Fall (i. e., σ_s and σ_a are nearly equal).

In the attempt to use any generalized maps of frost conditions allowance must be made for local variations. Any maps of the United States as a whole on the scales practicable for this REVIEW, can show only the general conditions over wide areas. Within these areas the more favored places will be much less subject to frosts and will have much longer available growing seasons than those indicated by the map, while the less favored spots will have later spring and earlier fall frosts with resulting shorter growing seasons. The chance of killing frost or of a frost-free season of any given length for a station may be determined, from such maps as those accompanying this paper, by applying a correction for local conditions, and this correction must be determined for each place. The necessity of this local correction is not limited to these data but applies with equal force to all maps of average dates or conditions.

CERTAIN CHARACTERISTICS OF THE WINDS AT MOUNT TAMALPAIS, CAL.

By HERBERT H. WRIGHT, Assistant Observer.

[Dated: Weather Bureau, Mount Tamalpais, Cal., July 12, 1916.]

Mount Tamalpais, Cal., while only about 2,600 feet above sealevel, rises so abruptly from the low, surrounding country that it is specially adapted for securing wind data. Topography has little or no effect on the directions or velocities recorded.

In Table 1 will be found the prevailing wind directions for the months and for the year at Mount Tamalpais, computed for the 13 years 1899 to 1911, inclusive. The persistency of the northwest wind along this coast is quite marked, as this table shows. As lower levels are reached there is a tendency for the wind to blow more from the west, specially during the summer. In winter, at sealevel, the prevailing direction for a few months is southerly.

TABLE 1.—*Prevailing wind directions at Mount Tamalpais, 1899–1911 inclusive.*

Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
SE.	NW.	NW.	NW.	NW.	NW.	NW.	NW.	NW.	NW.	NW.	NW.	NW.

The southeast winds at Mount Tamalpais during January are due to the fact that it is midwinter, the period of greatest storm frequency, and the LOWS follow each other in such rapid succession that the winds prevalent during fair weather have little influence in determining the prevailing direction at this season.