TABLE 2.—*Vapor pressures at pyrheliometric stations on days when solar radiation intensities were measured.*

Washington, D. C.			Madison, Wis.			Lincoln, Nebr.			Santa Fe, N. Mex.		
Dates.	A. M.	P. M.	Dates.	A. M.	P. M.	Dates.	A. M.	P. M.	Dates.	A. M.	P. M.
1917.	*mm.*	*mm.*	1917.	*mm.*	*mm.*	1917.	*mm.*	*mm.*	1917.	*mm.*	*mm.*
Apr. 3	4.17	2.62	Apr. 2	2.74	3.00	Apr. 8	3.15	3.63	Apr. 6	2.26	1.96
4	4.57	4.75	6	2.62	2.87	9	3.45	4.57	7	1.52	1.52
7	4.37	3.30	9	2.74	5.36	18	7.29	10.21	20	2.36	1.60
9	3.30	2.36	12	4.37	3.63	21	5.16	5.16	21	3.00	1.19
10	2.16	2.74	13	1.68	3.30	....	......	......	23	1.78	2.62
11	2.06	3.99	21	4.75	6.02	....	......	......	24	2.74	1.96
14	2.87	3.30	....	......	......	....	......	......	25	3.81	3.99
16	3.45	4.37	....	......	......	....	......	......	26	3.15	2.06
17	4.57	5.16	....	......	......	....	......	......	30	3.81	2.16
23	9.14	9.47	....	......	......	....	......	......	....	......	......
27	4.57	7.29	....	......	......	....	......	......	....	......	......
30	9.14	9.83	....	......	......	....	......	......	....	......	......

TABLE 3.—*Daily totals and departures of solar and sky radiation during April, 1917.*

[Gram-calories per square centimeter of horizontal surface.]

Day of month.	Daily totals.			Departures from normal.			Excess or deficiency since first of month.		
	Washington.	Madison.	Lincoln.	Washington.	Madison.	Lincoln.	Washington.	Madison.	Lincoln.
	calories.	*calories.*	*calories.*	*calories.*	*calories.*	*calories.*	*calories.*	*calories.*	*calories.*
Apr. 1	487	77	308	100	−307	−73	100	−307	−73
2	299	535	383	−91	149	1	9	−158	−72
3	539	317	203	147	−71	−180	156	−229	−252
4	538	158	147	144	−232	−237	300	−461	−489
5	64	167	460	−332	−225	75	−32	−686	−414
6	202	595	358	−197	200	−27	−229	−486	−441
7	590	281	64	189	−116	−322	−40	−602	−763
8	280	631	551	−124	231	164	−164	−371	−599
9	596	529	522	189	126	134	25	−245	−465
10	618	513	425	208	108	36	233	−137	−429
11	589	524	336	177	116	−54	410	−21	−483
12	358	539	508	−57	128	117	353	107	−366
13	167	621	371	−251	207	−20	102	314	−386
14	598	418	493	178	1	101	280	315	−285
15	420	559	60	−3	139	−333	277	454	−618
16	606	186	249	180	−236	−144	457	218	−762
17	522	497	305	94	72	−89	551	290	−851
18	476	325	408	45	−103	13	596	187	−838
19	422	292	305	−12	−138	−90	584	49	−928
20	430	394	245	−7	−39	−151	577	10	−1,079
Decade departure.				....	....	....	344	147	−650
21	388	601	527	−51	166	130	526	176	−949
22	587	440	509	145	2	111	671	178	−838
23	558	480	428	113	40	30	784	218	−808
24	448	562	232	1	120	−167	785	338	−975
25	232	104	438	−218	−340	38	567	−2	−937
26	106	392	91	−347	−64	−309	220	−66	−1,246
27	502	549	76	46	102	−325	266	36	−1,571
28	134	274	46	−324	−175	−356	−58	−139	−1,927
29	266	77	234	−195	−374	−169	−253	−513	−2,096
30	574	103	160	110	−350	−244	−143	−863	−2,340
Decade departure.				....	....	....	−720	−873	−1,261
Excess or deficiency since first of year: calories				....	....	....	−1,500	+675	−2,708
Excess or deficiency since first of year: Per cent				....	....	....	−4.2	+1.9	−7.2

551.593 : 551.501

EQUATION OF HORIZONTAL RAINBOWS.[1]

[Paper read before the Tokyo Mathematico-physical Society, Jan. 20, 1917.]

By KŌKICHI OTOBE.

1. The subject of this paper was suggested to me when the horizontal rainbows were being demonstrated [*cf.* MONTHLY WEATHER REVIEW, Jan., 1917, **45**:5]. My object is:

(*a*) To deduce the general equation of the horizontal rainbows due to a source of light at a finite distance from the observer.

(*b*) To show that the curves are not in general the conic sections;

(*c*) But when the straight line passing through the source and the observer's eye is perpendicular to the plane sheet of fine water drops, the curves become concentric circles.

(*d*) To verify the general equation by deducing some special cases actually observable with parallel rays of the sun.

(*e*) To confirm the above results by experiment.

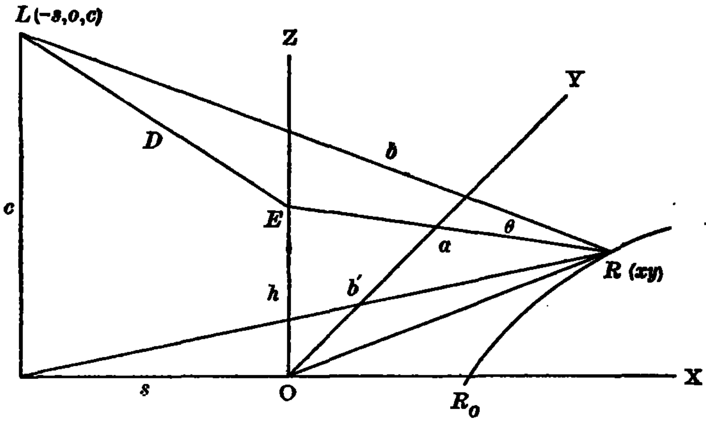

FIG. 1.

2. Let the rectangular coordinate axes be drawn as in figure 1, $(-s,0,c)$, $(0,0,h)$ and $(x,y,0)$ being the coordinates of a source of light, L, of an observer's eye E, and of a point R in the arc R_0R of the rainbow, respectively.

And let θ denote the angle between the incident ray LR and the effective ray RE.

Then we obtain from the triangle ERL,

$$D^2 = a^2 + b^2 - 2ab\cos\theta \qquad (1)$$

where

$$\left.\begin{aligned} a^2 &= h^2 + r^2 \\ b^2 &= c^2 + b'^2 \\ b'^2 &= (s+x)^2 + y^2 \\ r^2 &= x^2 + y^2 \end{aligned}\right\} \qquad (2)$$

Substituting (2) in (1), and remembering the relation

$$D^2 = (c-h)^2 + s^2,$$

or

$$D^2 - c^2 - h^2 - s^2 = -2ch,$$

we have

$$(ch + xs + x^2 + y^2)^2 = (h^2 + x^2 + y^2)(c^2 + s^2 + x^2 + y^2 + 2xs)\cos^2\theta, \qquad (3)$$

as the required equation.

The curve is symmetrical about the axis of X, but not so simple as to be traced. Since the equation contains the angle θ and the coordinates of the eye, E, the position of the rainbow will be different as we look with the right eye or with the left. This was actually the case even with the horizontal rainbow observed on the surface of the moat.

3. When both the source of light, L, and the observer's eye, E, are in the Z axis, we may put $s=0$, so that we have from (3)

$$(ch + r^2)^2 = (c^2 + r^2)(h^2 + r^2)\cos^2\theta,$$

hence

$$\begin{aligned}(ch + r^2)^2 \sin^2\theta &= [(c^2 + r^2)(h^2 + r^2) - (ch + r^2)^2]\cos^2\theta, \\ &= (c-h)^2 r^2 \cos^2\theta,\end{aligned}$$

therefore

$$ch + r^2 = \pm (c-h) r \cot\theta.$$

The plus or minus sign should be taken according as $c > h$ or $c < h$.

[1] *Reprinted from* Jour. Meteorol. Soc. Japan, Feb., 1917, **36**: 7-14.

Suppose $c>h$ as in figure 1, then we have

$$r^2-(c-h)r\cot\theta+ch=0. \qquad (4)$$

Solving for r we obtain

$$r=\tfrac{1}{2}[(c-h)\cot\theta\pm\sqrt{(c-h)\cot^2\theta-4\,ch}]. \qquad (5)$$

There are two values of r. For example,

if $c=200$ cm,
$h=10$ cm,
$\theta=42°$,

we have

$$r_1=19.7 \text{ cm.}; \qquad r_2=191.4 \text{ cm.}$$

In my former demonstration the circular rainbow of radius r_1 was actually observed, while that of radius r_2 was beyond the extent of the blackened plate.

4. When the sun's rays are perpendicular to a plane sheet of water drops, we must suppose c tends to become infinity, while h and θ remain finite. Thus writing (5) as follows

$$r=\tfrac{1}{2}[(c-h)\cot\theta]\left[1-\sqrt{1-\frac{4ch}{(c-h)^2}\tan^2\theta}\right],$$

and expanding the expression under the radical sign, we have

$$r=\frac{ch\tan\theta}{c-h}.$$

By putting $c=\infty$, we obtain

$$r = h\tan\theta. \qquad (6)$$

This might have been obtained directly from (3) by supposing r and s finite and c infinite, for then equation (3) is reduced to

$$h^2=(h^2+x^2+y^2)\cos^2\theta,$$

whence

$$x^2+y^2=h^2\tan^2\theta. \qquad (7)$$

This represents a circle of radius $h\tan\theta$, being the base of a right cone having the observer's eye as the vertex and semivertical angle $=\theta$, as shown in figure 2.

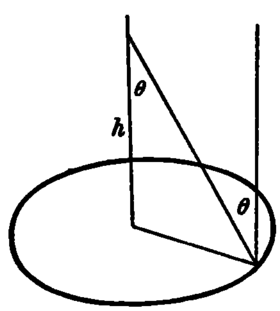

FIG. 2.

5. When the altitude of the sun is less than $\frac{\pi}{2}$, suppose c and s tend to infinity, while their ratio remains finite and equals $\tan\phi$, so that

$$c=s\tan\phi;$$

then writing (3) in the form

$$\left(x+h\tan\theta+\frac{r^2}{s}\right)^2=(h^2+r^2)\left(\sec^2\phi+\frac{2x}{s}+\frac{r^2}{s^2}\right)\cos^2\theta$$

and putting $s=-\infty$, we have

$$(x+h\tan\phi)^2=(h^2+r^2)\sec^2\phi\cos^2\theta.$$

After a few reductions we obtain

$$\begin{aligned} x^2(\cos^2\phi-\cos^2\theta)+2\,xh\sin\phi\cos\phi- \\ y^2\cos^2\theta+h^2(\sin^2\phi-\cos^2\theta)=0 \end{aligned} \qquad (8)$$

This is the equation of conic sections.

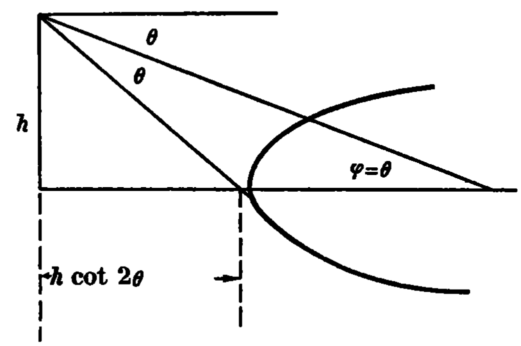

FIG. 3.

If the sun's altitude ϕ is equal to θ (see fig. 3), equation (8) becomes

$$y^2\cos^2\theta=2xh\sin\theta\cos\theta+h^2(\sin^2\theta-\cos^2\theta)$$

or

$$y^2=2h\tan\theta\,(x-h\cot 2\theta), \qquad (9)$$

which represents a parabola having its latus rectum equal to $2h\tan\theta$ and intersecting the axis of x at a point whose distance from the origin is $h\cot 2\theta$.

6. If $\phi\neq\theta$, transforming the origin to the center (see fig. 4)

$$\begin{cases} y=0 \\ x=-\dfrac{h\sin\phi\cos\phi}{\cos^2\phi-\cos^2\theta}=0, \end{cases}$$

equation (8) is reducible to

$$(\cos^2\phi-\cos^2\theta)x^2-\cos^2\theta y^2-\frac{h^2\sin^2\theta\cos^2\theta}{\cos^2\phi-\cos^2\theta}=0,$$

or

$$\frac{x^2}{\left(\dfrac{h\sin\theta\cos\theta}{\cos^2\phi-\cos^2\theta}\right)^2}-\frac{y^2}{\dfrac{(h\sin\theta)^2}{\cos^2\phi-\cos^2\theta}}=1. \qquad (10)$$

If $\cos^2\phi-\cos^2\theta>0$ or $\phi<\theta$, the above equation represents a hyperbola, which is the form of horizontal rainbow most frequently observable with the morning sun, for the altitude is probably less than $\phi\doteq 42°$. (cf. fig. 4.)

If $\cos^2\phi-\cos^2\theta<0$ or $\phi>\theta$, equation (10) represents an elliptical rainbow as shown in figure 5.

7. To confirm the above results recourse must be had to the following experiment. One end, E, of the glass rod supported horizontally on an adjustable vertical stand, is used to specify the position of the observer's eye. And E', the shadow of E, is marked on the blackened plate. As EE', ER, RE' and EP (the height of E above the plate) are directly measurable, the angles ϕ and θ can be easily calculated. The calculated value of θ is found to be nearly equal to that given by the exact theory of Airy.

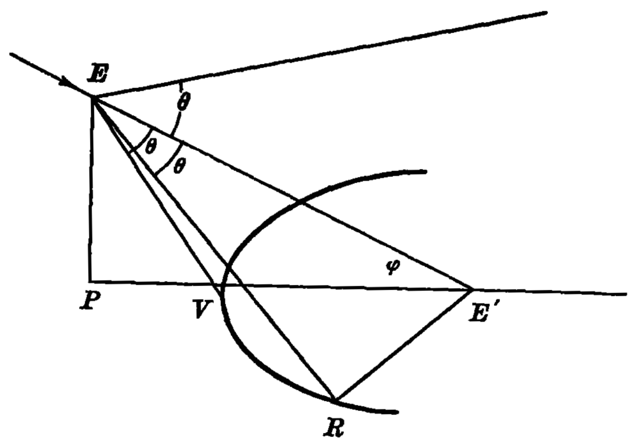

FIG. 4.

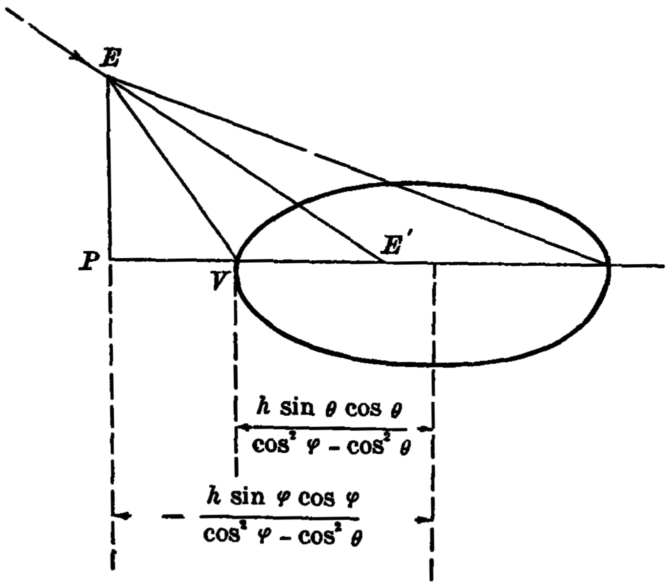

FIG. 5.

8. When the rainbow appears in the sky the air must be still and the space between the observer and the rainbow must be clear from any thick cloud or falling rain, so that the raindrops causing the rainbow fall in a vertical plane and the effective rays from these drops generate the surface of a cone having its vertex at the observer's eye and its axis parallel to the incident rays. Hence it is probable that the sky rainbow is, in general, elliptic in form though it may appear as circular.

To confirm this statement experimentally proceed as follows: When the sun shines from behind you in a laboratory, let your assistant stand at your arm's length and let him spray water drops in a vertical plane perpendicular to your line of sight. You will see a nearly complete elliptic rainbow whose major axis is vertical.

Hence the constancy of the angle between the incident and effective rays gives no more conclusive reason why the rainbow must be a circle.

551.508.7

IMPROVED KITE HYGROMETER AND ITS RECORDS.

By WILLIS RAY GREGG, Meteorologist.

[Division of Aerological Investigations, U. S. Weather Bureau, Apr. 24, 1917.]

Free-air records of relative humidity are obtained by means of the hair hygrometer. In the Marvin kite meteorograph[1] the bundle of hairs is mounted longitudinally in the horizontal screening tube, which also contains the temperature element and recently the anemometer wheel. The meteorograph is attached to the middle back rib of the kite, just behind the front cell, in such a manner that the wind always blows directly through the screening tube, thus insuring good ventilation. In spite of its good exposure, considerable difficulty has always been experienced in obtaining accurate values of humidity, because of sluggishness or "lag" in the hygrometer. Part of this trouble has been eliminated by connecting the element as directly as possible

[1] See photographs published in this REVIEW, October, 1899, Pl. I; in Bulletin, Mount, Weather Observatory, v. 1, pt. 1, Pl. I; also Weather Bureau Bulletin F (Washington 1899), fig. 1 and fig. 5.—C. A., jr.

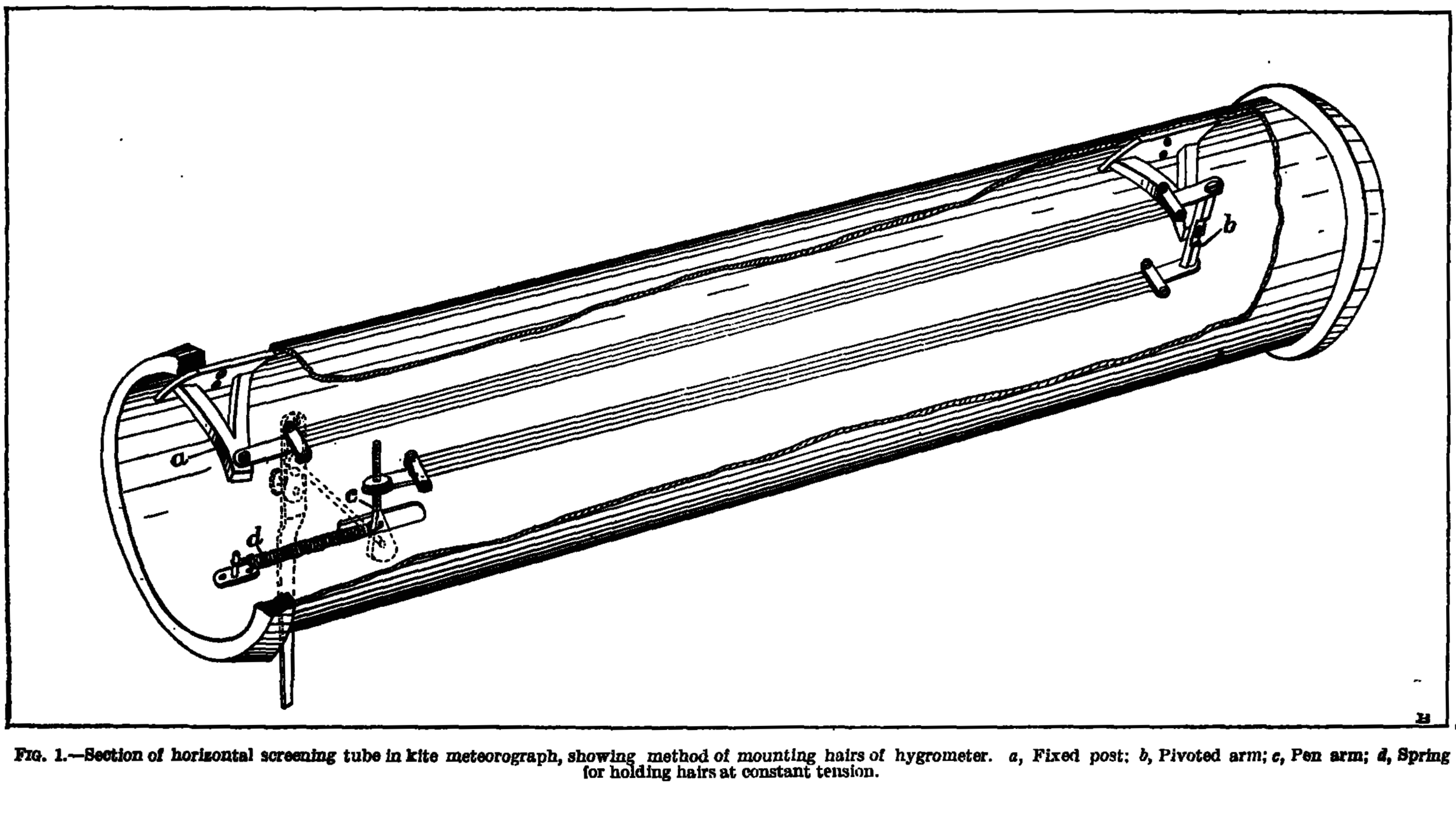

FIG. 1.—Section of horizontal screening tube in kite meteorograph, showing method of mounting hairs of hygrometer. a, Fixed post; b, Pivoted arm; c, Pen arm; d, Spring for holding hairs at constant tension.