21. **Marvin, C. F.** The Flight of Aircraft and the Deflective Influence of the Earth's Rotation. Monthly Weather Review, this issue, p. 75.
22. **Humphreys, W. J.** Physics of the Air. Chapter VIII and Appendix. In *Journal of the Franklin Institute*. 1917–18.
23. *Scientific American*. Editorial: The Doubtful Factors in the Problem of Trans-Atlantic Flight. March 1, 1919.

SUPPLEMENTARY BIBLIOGRAPHY.

24. **Riach, M. A. S.** Atlantic Flight. A Matter of Mathematical Analysis. *Scientific American Supplement*. January 18, 1919.
25. **Tucker, Frank T.** Winds and the Trans-Atlantic Flight. *Aviation*. February 15, 1919.
26. Trans-Atlantic Flight Number of *Flying*. July, 1918.
27. Possibilities of Airship Transport Services. *Aeronautics*. February 19, 1919.
28. **D'Orcy, Baron Ladislas.** Possibilities and Conditions of Crossing the Atlantic by Airship. *Scientific American*. August 26, 1916.
29. **Boykow, Lieut. H.** Trans-Atlantic Flight. A Discussion of Possible Routes. *Scientific American Supplement*. July 25, 1914.
30. **Hobbs, William H.** A proposed aeroplane route across the Atlantic. *Flying*, Apr. 1919, p. 243.

THE FLIGHT OF AIRCRAFT AND THE DEFLECTIVE INFLUENCE OF THE EARTH'S ROTATION.

By CHARLES F. MARVIN, Chief of Weather Bureau.

[Paper presented before the Philosophical Society of Washington, March 29, 1919.]

It is well known that objects moving freely over the earth's surface are deflected constantly to the right by a force given by the equation

$$f = 2mv\omega \sin \phi \qquad (1)$$

in which m = mass of the body, v its velocity, ω = the angular velocity of the earth's rotation, and ϕ the latitude. For each gram of matter moving at airplane speeds, say, 90 miles per hour (4,000 cm. p. s.) and at latitude 50° the force is,

$$f = 2 \times 4{,}000 \frac{2\pi}{86164} \sin \phi = .447 \text{ dynes.}$$

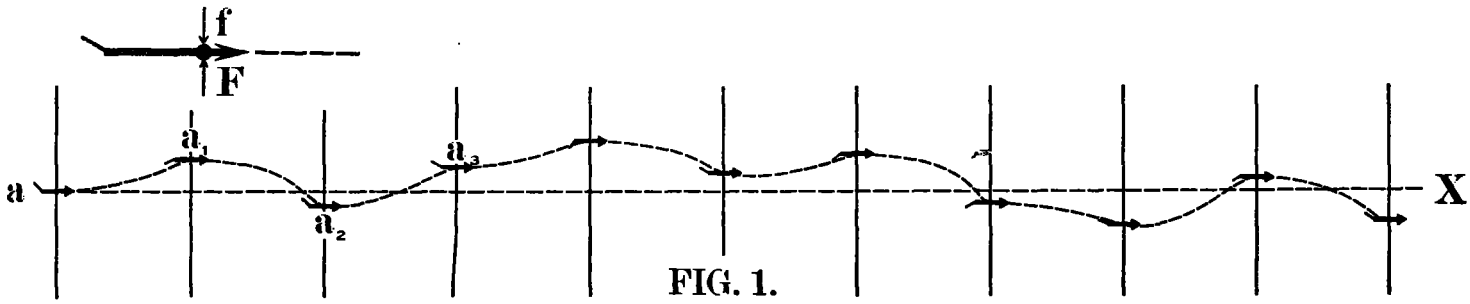

FIG. 1.

An aircraft is a body subject to such an influence even though its course be continuously controlled by a pilot and it is interesting to ascertain the quantitative effects of this small force under practical conditions of flight, as, for example, over trackless wastes where positions must be determined from the compass aided by processes of dead reckoning. For this study we can entirely disregard the influence of any winds or similar atmospheric motions, because if the conditions are known these effects can all be accounted for separately and on their own merits. Such effects on our aircraft are quite independent of the deflective influences with which alone we are now concerned. The actions we wish now to consider are such as might be observed in still air.

To ascertain how the deflective influence modifies the course of an aircraft it is necessary to make some assumptions as to how it is directed and controlled. For this purpose we assume a pilot desires to fly from a to X (fig. 1). The compass direction of X from a is supposed fully known, that is, all corrections, variations, errors, etc., are duly accounted for.

The pilot is assumed to set his craft true to course. Presently, however, close attention to the compass shows him his craft is headed a greater or less angular amount away from course. The craft is again set true to course, only presently again to be found off course. These operations of rectifying the flight of the craft are repeated many times on a long course, with the supposed result that the craft practically follows the line aX.

The short arrows in figure 1 indicate the successive positions and alignment of the *axis* of the machine just after the course has been rectified. The axis of the machine is defined to be a line through the center of mass which coincides with the instantaneous path of that center when the machine is flying normally through a stationary atmosphere over a stationary earth, or over the rotating earth near the Equator.

With the enlarged arrow in figure 1 is shown the horizontal force F which is the resultant of all the horizontal motor or air forces, resistances and reactions to which the machine is subject when in horizontal flight at a uniform velocity. The other force f represents the deflecting force due to the earth's rotation. If F and f are opposite and equal the path of the plane will be rigorously a straight line,—a great circle.

Except when he wishes to execute a relatively short turn the pilot has no control over, or knowledge of the residual force F or the direction of its action, which falls to the right or the left of the course in an entirely accidental manner. When the pilot sets his craft on a straight-away course he imagines the force F is zero, and in occasional cases this may be true. In general, however, F will have a finite value which, although relatively small, will generally be several times the value of f. The larger force will therefore dominate the flight in all cases and swerve the machine to the right or the left of the course somewhat as shown in figure 1.

In so far as the force F itself is concerned, it is believed we may properly assume that its nature is such that perturbations from course due to it are quite accidental, and in the many rectifications which must be made in any case the influences of F will tend to be automatically eliminated or averaged out. That is, ΣF for even a comparatively small number of cases continually tends to be zero. Accordingly, in a long flight

such as indicated in figure 1, if executed at or near the Equator, where f is zero or inappreciable, the machine would actually fly true to course. In higher latitudes, however, the force f is larger, and in the Northern Hemisphere acts in a manner to swerve the machine always to the right.

The assumption that $\Sigma F=0$ is nearly equivalent (so far as the effects of the force f are concerned) to the assumption that on the average, when the pilot sets his machine true to course, $F=0$; that is, there is no accidental error inherent in the setting, and accordingly the machine each time must fly true to the course set except for the deflective influence.

Possible side slipping has not been considered as yet. Except for possible faulty arrangement of wings or controls, side slipping in normal flight should be an entirely accidental feature, and as in the case of F, the sum of effects of side slipping must tend to average out.

There now remains only one final point of assumption, which, however, is probably the most important of all, namely, what is the average interval of time between necessary rectifications of the course? The value of the average angle of deviation at the time of rectification is also of some consequence, but the accidental influences tend to make this angle large and thus cause very frequent rectifications of the course which might otherwise be infrequent. For further analysis let t = the average interval in seconds between rectifications of the course.

Under the foregoing assumption and conditions everything which can modify or change the horizontal course of an aircraft, except the earth's deflective influence, has been considered and disposed of. The theory of the operation of the deflective influence may, therefore, be developed as follows:

Since the deflection of a body moving over the earth's surface is due to the turning of the earth under the line of motion of the body, which line tends to maintain a constant direction in *space*, an airplane will seem to turn as its course seems to curve, and will not be deflected by side-slipping. The so-called "deflective force" is equal to the force which would be required to make a body move in the same compass direction over the surface of the moving earth, just as "centrifugal force" is equal to the force required to keep a body moving on a curved path from flying off on a tangent. The path followed will then be that of a freely moving body acted upon by no other deflecting force than that of the influence of the earth's rotation.[1] Its radius of curvature is given by the equation:

$$r=\frac{v}{2\omega \sin \varphi} \qquad (2)$$

For distances r measured in meters and for velocities v in meters per second, the equation becomes

$$r=\frac{(\log^{-1}3.83612)v}{\sin \varphi} \qquad (3)$$

In figure 2, aX is the desired course of the aircraft and it sets out on this course at a. After the brief interval of time t, just at rectification, the craft, on the average, will occupy a position a_2 at the right of the course and it will be moving in the direction $a_2\ X'$ which makes an angle 2θ with the desired course.

[1] Relative to the earth's surface, the deflection has a vertical as well as a horizontal component. An eastward moving body will weigh less than an identical westward moving body, since their absolute speeds are increased or decreased respectively. In this paper only the horizontal component of deflection will be considered.

Let the latitude be 50° and let $v=40$ meters per second, which must be reckoned as velocity over the earth and not relative to the air.

For these conditions $r=358{,}000$ meters.

The arc traversed in time $t=vt$.

The angle subtended $=360°\frac{vt}{2\pi r}=2\theta$

$$\therefore\ \theta=\frac{90vt}{\pi r} \text{ or } \frac{360°}{86164}\,t \sin \varphi, = (\log^{-1}3.62097)\ t \sin \varphi. \qquad (4)$$

The angle θ becomes greater in proportion to the lapse of time t between the rectification of the course of the machine, and diminishes as the latitude diminishes.

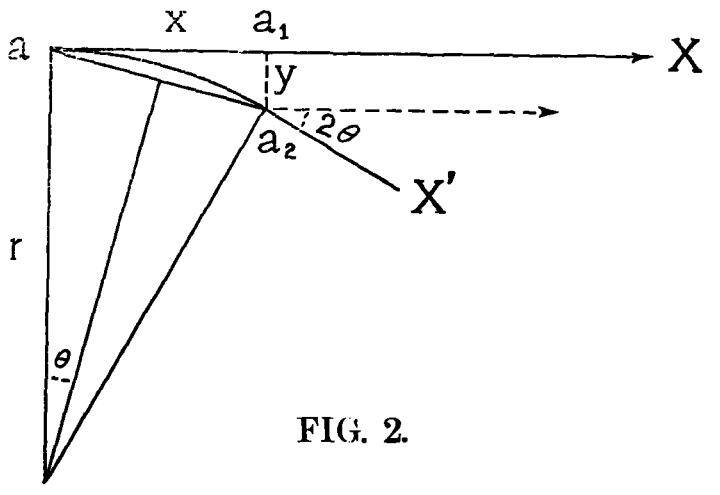

FIG. 2.

For purposes of illustration we may assume $t=60$ seconds whence $\theta=11.52'$ and $2\theta=23'$. This small angle is obviously entirely beyond detection by the pilot, nevertheless in every interval of 60 seconds between rectifications the machine swerves to the right by this angular amount.

The values of x and y, figure 2, are

$x=r \sin 2\theta = vt$ very nearly.

$y=x \tan \theta$ or $2r \sin^2 \theta = vt \sin \theta$ very nearly.

The lateral displacement from the course when the flight is rectified once each minute is, therefore,

$y=8.04$ meters per minute.

From this result it follows that a machine flying in latitude about 50° on a 10-hours' run with rectification of course no more nor less frequently than once a minute will tend systematically to miss its destination by nearly 5,000 meters to the right thereof. This is doubtless a small error in a run of this magnitude, but the displacements y from course increase rapidly should the length of time between rectifications increase.

Occasionally, as has been mentioned, the force F may be actually zero after some of the rectifications and the craft will then seem to hold its course for several minutes, that is, longer than usual, because the force f alone is too small to swerve the machine out of course quickly. From (4) we have

$$t=\frac{(\log^{-1}2.37903)\theta}{\sin \varphi} \qquad (5)$$

which is the time required for the deflective force to produce an angular deviation from course of 2θ

If we assume that the pilot either does not perceive or that he disregards deflections from course smaller than, say

one-quarter point on the compass, that is, about 3°, then from (5)

$$t = 468.7 \text{ seconds} = 7.81 \text{ minutes}$$

which is the time required under conditions assumed for the deflective influence acting alone to produce an angular deflection from course of 3°.[1] The value of y in this case is

$$y = 490.7 \text{ meters.}$$

If controlled according to this condition the pilot would miss port at the end of a 10-hour flight by over 29 kilometers which would of course be a very serious matter.

It is easy to see from the foregoing that if a pilot sets out to go to a given point X from a, figure 3, and governs his flight according to the conditions assumed he will not traverse the line $a\ X$, but will tend to follow such a line as $a, a_1, a_2 \ldots\ldots\ldots a_n \ldots X'$

From the theory already stated the angle θ subtends one-half the arc traversed by the machine under the deflective influence during the time between rectifications of the course. To reach the point X, therefore, the pilot must lay his course to the left of the known direction by an angular amount $=\theta$ which in a sense is a small angle having the nature of a compass correction.

As may be seen from equation (4) this angular correction depends only on the latitude and the time interval t between rectifications of the course. It is, therefore, independent of the velocity of the craft. Moreover, the whole theory applies with equal force to marine as well as aerial navigation.

In conclusion there is given below a small table of compass corrections resulting from these studies.

Compass corrections to overcome the deflective influence of the earth's rotation on the navigation of marine and aerial craft.

(Based on a rectification of the course at intervals of 60 seconds. Corrections for other time intervals may be found by multiplying the tabular correction by the ratio of the desired interval, expressed in seconds, to 60 seconds.)

Latitude.								
° 10	° 20	° 30	° 40	° 50	° 60	° 70	° 80	° 90
Compass corrections in minutes of arc.								
′ 2.61	′ 5.14	′ 7.52	′ 9.67	′ 11.52	′ 13.03	′ 14.13	′ 14.81	′ 15.04

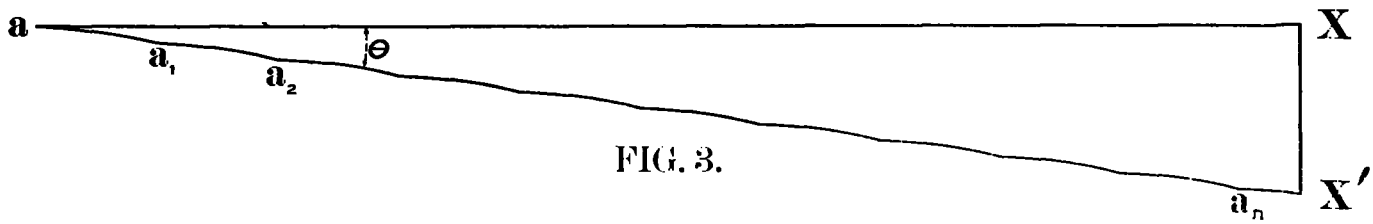

FIG. 3.

In the Northern Hemisphere the course must be laid to the left of the uncorrected course by the angular amounts indicated, and vice versa in the Southern Hemisphere.

[1] If the machine were left entirely uncontrolled, that is, not rectified as to course, still assuming $F=0$, we have the case of the craft in continuous flight at uniform velocity, but acted on solely by the deflective influence of the earth's rotation. Making $\theta=180°$ in (5) and reducing time to hours, T, we get,

$$T=\frac{\log^{-1} 1.07800}{\sin\varphi}=15.62 \text{ hrs. for } \varphi=50°$$

which is the time required for any freely-moving body to execute a complete gyration, which will be nearly but not quite a closed circle.

The general equation for the motion of unit mass (1 gram) of a body moving in any of the curved paths in figure 1 is

$$\frac{v^2}{r}=F\pm f=F\pm 2r\omega\sin\varphi \qquad \text{(a)}$$

Plus for right-hand turns in the Northern Hemisphere, minus for left-hand turns.

Solving (a) for v we get

$$v=\sqrt{Fr+(r\omega\sin\varphi)^2}\pm r\omega\sin\varphi$$

From (a) we may also get, with (4)

$$\left(\frac{F}{f}\pm 1\right)=\frac{v^2}{rf}=\frac{v}{2r\omega\sin\varphi}=\frac{86164\,\theta}{360\,t\sin\varphi}$$

in which 2θ is the arc traversed in time t seconds. For $\theta=1.°5$ and $t=60$ seconds,

$$\left(\frac{F}{f}\pm 1\right)=\log^{-1} 0.98272, \text{ or } 7.811$$

$$\therefore F=(6.811f) \text{ for right-hand motion and } (8.811f)$$

for left-hand turns of same radius and same velocity.

These differences of force F at same velocity and radii of turn are possible only when associated with different expenditures of power. See also pp. 88-89.

UNITED STATES STILL HOLDS AIRPLANE ALTITUDE RECORD.

[Reprinted from Aviation, New York, Mar. 15, 1919, Vol. 6, No. 4, p. 234.]

America still holds the world's airplane altitude record with 28,900 feet, set September 18 last by Maj. E. W. Schroeder, A. S., at Wilbur Wright Field, Dayton, Ohio.

Since January 2 it had been believed that Capt. Lang of the Royal Air Force had topped this climb by 1,600 feet in reaching a height of 30,500 feet in England. On March 11, however, a cable message from the London Graphic to the authorities of the Aeronautical Exposition in Madison Square Garden said that a rereading of Capt. Lang's instruments had fixed his altitude at 27,000 feet, leaving a clear margin in Maj. Schroeder's favor. The American made his climb in an American-built Bristol airplane, equipped with a 300-horsepower Hispano-Suiza engine.