to permit observation for centering ray of light. With lunar halos it is rarely possible to center by means of the ray of light. In this case the pointer is set at zero and centered on moon by adjustment at F. Set screw at F is clamped, and pointer moved to limb of halo as in the other case. Brass strap over tube holds it in place and may be tightened by set screw G to hold scale board in place. Axle at M is a pin used for a maximum thermometer with old-style thermometer support, flattened at end and driven into scale board, furnishing a handle for turning or holding the board. Pointer D is about one-third inch thick material and three-fourths inch wide—width to provide place for level. Nephoscope level has been used and works well. I have not yet arranged a satisfactory mounting for level to insure that scale board is in vertical plane when that is desired.

For halos.—By shifting of base H and angle of inclination at F, center ray of light from sun A passing through small aperture at B on point C at center of back of tube visible through cut out. If halo is complete no shifting of board X is necessary; if but a portion is visible, board may be turned on axis BM, so that the pointer D will bear on the segment visible. Sight along pointer D turning on pivot O until it bears on the limb of the halo. Angle of radius of halo will be shown by pointer on scale.

Angular elevation of any heavenly body.—Set pointer D at zero; set small level on pointer D; turn at F as necessary to level pointer. Clamp F.—This establishes horizon. Turn pointer on object whose elevation is desired and position of pointer on scale will show angle.

Angles in horizontal plane may be measured by leveling board X and clamping at F and G to hold stationary. If desired, C on the scale may be set true north or south by means of compass. Sighting along pointer D at objects whose angular relation is desired and noting reading on scale in each case gives data desired.

RECENT CONTRIBUTIONS TO DYNAMICAL METEOROLOGY.

By EDGAR W. WOOLARD.

[Weather Bureau, Washington, D. C., April 13, 1922.]

In the fifth century B. C., Anaximander of Ionia gave a scientific definition of wind, which is still valid, viz., "The wind is a flowing of air," although there should perhaps be added the words "relative to the surface of the earth." Sometimes a wind is defined as air in motion near the earth's surface and nearly parallel to the latter, all other motions of masses of air being spoken of as air currents;[1] but this distinction is not always recognized, and it is unimportant for purposes of dynamical meteorology.

Dynamical meteorology considers the mass motions taking place in the earth's atmosphere; such motions must of necessity obey the ordinary laws of dynamics. Now, it may be demonstrated[2] directly from the kinetic theory that although a gas is composed of discrete particles, its mass motions will obey the ordinary hydrodynamical equations derived[3] explicitly on the very different assumption of continuity.

Therefore the starting point of dynamical meteorology will be the hydrodynamical equations of motion of a fluid covering a rotating globe.[4] Any quantitative theory of the various winds involves first of all a general account of the fundamental dynamical principles common to all winds, and a classification of winds based on dynamical principles, with a subsequent elaboration of the theory of each type. A comprehensive treatise on dynamical meteorology, written for the student with extensive mathematical training, has not yet appeared, although such a work is much to be desired; of great importance in this connection, however, are the recent papers of Jeffreys, in the latest of which[5] he has given a classification of winds, based on a discussion of the relative importance of the various relevant physical factors which determine the characteristics of each type as expressed in the differential equations of motion.

The only forces acting on any mass of air are gravity, hydrostatic pressure, and friction; the acceleration of the mass is composed of two parts—acceleration relative to the surface of the earth, which we observe, and the acceleration common to this surface itself; by the laws of motion, the sum of the two parts of this actual acceleration is equal to the sum of the accelerations produced by the three forces. Each term or set of terms in the general equations represents one of the five rates of change of momentum corresponding to the five accelerations.

In the case of atmospheric motions, the general equations reduce to—

$$\left.\begin{aligned} \frac{du}{dt}-2\omega v\cos\theta &= -\frac{1}{\rho}\frac{\partial p}{\partial x}+k\frac{\partial^2 u}{\partial z^2} \\ \frac{dv}{dt}+2\omega u\cos\theta &= -\frac{1}{\rho}\frac{\partial p}{\partial y}+k\frac{\partial^2 v}{\partial z^2} \\ 0 &= -\frac{1}{\rho}\frac{\partial p}{\partial z}-g \end{aligned}\right\}$$

We then have three possible cases: (1) Eulerian winds, in which the rotational and frictional terms are so small in comparison with the accelerational term that they may be neglected—the observable acceleration corresponds to to the horizontal pressure gradient, as in ordinary elementary hydrodynamics; (2) geostrophic winds, in which the accelerational and frictional terms are negligible in comparison with the rotational term—the velocity is at right angles to the pressure gradient; (3) antitriptic winds, in which the rotational and accelerational terms are negligible in comparison with the frictional term—these winds are driven by, and blow in the direction of the pressure gradient, but the velocity does not increase throughout the whole journey.

A consideration of the factors shows that winds on a scale comparable with the size of the British Isles, or larger, are geostrophic; tropical cyclones, and all cyclostrophic winds, as tornadoes, are Eulerian; land and sea breezes, and mountain and valley winds, are mainly antitriptic. However, in order to explain seasonal pressure changes at the earth's surface, the accelerational term must be retained in the equations for the geostrophic wind; temperature differences are capable of accounting for the annual pressure variation in Asia and probably for the permanent winds of Antarctica. Jeffreys has worked out a mathematical theory of some of the antitriptic winds which agrees well with the facts. A fundamental part is played by the deviation of the actual average temperature lapse rate from the adiabatic value.

[1] *Cf.* W. I. Milham, *Meteorology*, New York, 1912, p. 136; W. M. Davis, *Elementary Meteorology*, 1894, p. 93.
[2] J. H. Jeans, *The Dynamical Theory of Gases*, 3 ed., Cambridge, 1921, pp. 165–175.
[3] See P. Appell, *Traité de Mécanique Rationnelle*, Tome 3, 3 ed., Paris, 1921; H. Lamb, *Hydrodynamics*, 4 ed., Cambridge Press, 1916.
[4] Lamb, *op. cit.*, p. 318.
[5] Harold Jeffreys, On the Dynamics of Wind. *Quar. Jour. Roy. Met'l Soc.*, 48 : 29–47, 1922.

Classical hydrodynamics, however, does not afford a satisfactory means of dealing with the general equations of motion derived above, for they involve the density, and wherever, as a consequence, it is necessary to take into account the physical properties of the fluid, the classical theory practically always assumes an equation of state of the form $f(p, \rho)=0$, the density being a function of the pressure only.[6] In the actual cases of nature, particularly in meteorology and hydrography, many other independent variables enter, such as temperature, humidity, salinity, etc. The hydrodynamical theory of *baroclinic* fluids—i. e., fluids in which other independent variables than the pressure also affect the density—has been worked out by Bjerknes and has recently been made easily accessible in elementary form by Appell.[7] In such fluids surfaces of equal density are not always surfaces of equal pressure, and the formation and annihilation of vortices are possible.

The application of this theory to the dynamics of the earth's atmosphere has also been largely the work of Bjerknes, having been worked out in parallel with the well-known empirical investigations of the Bergen meteorologists. There has recently appeared a comprehensive and up-to-date summary of the whole subject,[8] which constitutes a most valuable memoir on theoretical meteorology.

[6] See Lamb, *op. cit.*, art. 8; *cf.* Appell, *op. cit.*, art. 627.

[7] Appell, *op. cit.*, chap. xxxii, pp. 562–605.

[8] V. Bjerknes, On the Dynamics of the Circular Vortex, with applications to the Atmosphere and Atmospheric Vortex and Wave Motions. *Geofysiske Publikationer*, Vol. II, No. 4, Kristiania, 1921. 4to, 88 pp.

SHORT METHOD OF OBTAINING A PEARSON COEFFICIENT OF CORRELATION, AND OTHER SHORT STATISTICAL PROCESSES.

By FRANK M. PHILLIPS, Ph. D.

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The usual method of obtaining a Pearson coefficient of correlation is somewhat long and tedious, especially if there be a large number of paired measures and if the measures or the averages of these happen to be such as to involve either large numbers or numbers running out to two or three decimal places. It is the purpose of this paper to derive and illustrate a shorter method, which at the same time will tend to eliminate errors likely to creep into a solution by the ordinary method. The formulas given in this article have all been derived by purely mathematical processes and do not involve any approximations; neither the average nor the deviations are used in computations by them; they shorten the work materially when solving for average deviation, standard deviation, coefficient of variability, and coefficient of correlation.

Let—

n = number of independent, or of paired, measures.
n_- = number of measures below the average.
n_+ = number of measures above the average.
Σm = sum of independent measures.
Σm_- = sum of measures below the average.
Σm_+ = sum of measures above the average.
S = measures of "subject."
R = measures of "relative."
a = average of the "subject."
c = average of the "relative."

Then the usual process of getting the coefficient of correlation may be represented as follows:

S	R	x	y	x^2	y^2	xy.
S_1...	R_1..	S_1-a.	R_1-c..	$S_1^2-2S_1a+a^2$....	$R_1^2-2R_1c+c^2$....	$S_1R_1-S_1c-R_1a+ac$.
S_2...	R_2..	S_2-a	R_2-c..	$S_2^2-2S_2a+a^2$....	$R_2^2-2R_2c+c_2$....	$S_2R_2-S_2c-R_2a+ac$.
S_3...	R_3..	S_3-a	R_3-c..	$S_3^2-2S_3a+a^2$....	$R_3^2-2R_3c+c^2$....	$S_3R_3-S_3c-R_3a+ac$.
......	......	......	......	......	......	......
......	......	......	......	......	......	......
S_n..	R_n.	S_n-a	R_n-c..	$S_n^2-2S_na+a^2$...	$R_n^2-2R_nc+c^2$...	$S_nR_n-S_nc-R_na+ac$.
ΣS..	ΣR	....	....	$\Sigma S^2-2\Sigma Sa+na^2$..	$\Sigma R^2-2\Sigma Rc+nc^2$.	$\Sigma SR-\Sigma Sc-\Sigma Ra+nac$.

$$r=\frac{\Sigma xy}{n\sigma_S\sigma_R}=\frac{\Sigma xy}{\sqrt{\Sigma x^2\cdot\Sigma y^2}}$$

Now, since $\Sigma S=na$, and $\Sigma R=nc$, then $\Sigma Sc=nac$, $\Sigma Ra=nac$, and $\Sigma SR-\Sigma Sc-\Sigma Ra+nac=\Sigma SR-nac$; furthermore, since $\Sigma Sa=na^2$, and $\Sigma Rc=nc^2$, then $\Sigma S^2-2\Sigma Sa+na^2=\Sigma S^2-na^2$, and $\Sigma R^2-2\Sigma Rc+nc^2=\Sigma R^2-nc^2$.

Therefore, we have—

$$r=\frac{\Sigma xy}{\sqrt{\Sigma x^2\cdot\Sigma y^2}}=\frac{\Sigma SR-\Sigma Sc-\Sigma Ra+nac}{\sqrt{(\Sigma S^2-2\Sigma Sa+na^2)\,(\Sigma R^2-2\Sigma Rc+nc^2)}}$$
$$=\frac{\Sigma SR-nac}{\sqrt{(\Sigma S^2-na^2)\,(\Sigma R^2-nc^2)}},$$

a formula much better adapted to numerical computation.

To illustrate the advantage of this method, let us take the following seven pairs of related measures and solve for a coefficient of correlation by the usual method:

S	R	x	x^2	y	y^2	xy
4	5	−3	9	−7	49	21
5	8	−2	4	−4	16	8
6	11	−1	1	−1	1	1
7	11	0	0	−1	1	0
8	13	1	1	1	1	1
9	18	2	4	6	36	12
10	18	3	9	6	36	18
49 $a=7$	84 $c=12$	....	28	....	140	61

$$r=\frac{61}{28\times 140}=0.974.$$

Now, let us apply the short method to the same series of paired measures:

S	R	S^2	R^2	SR
4	5	16	25	20
5	8	25	64	40
6	11	36	121	66
7	11	49	121	77
8	13	64	169	104
9	18	81	324	162
10	18	100	324	180
49	84	371	1,148	649
....	....	343	1,008	588
$a=7$	$c=12$	28	140	61

$nac=588.$
$na^2=343.$
$nc^1=1,008.$

$$r=\frac{61}{\sqrt{28\times 140}}=0.974.$$