with hygrometric data obtained at the regular observations of the Weather Bureau Office at Philadelphia. By means of the barograph and thermograph trace sheets from Philadelphia and the corresponding weather maps, the bog minimum temperatures on those days which clearly showed that they were not of radiation origin were eliminated. Thus the writer could feel quite sure he was using data representative, or fairly so, of radiation conditions. In all, 61 observations were used.

"Dot charts were made, using bog minimum temperature data for Whitesbog and hygrometric data for Philadelphia, following the same plan for plotting these data as was used in Figures 1 and 2. These charts are shown in Figures 3 to 8, inclusive.

"From the arrangement of the dots on each chart, a straight line appears to be the best fit and the equation ($y=a+bR$) has been calculated, the coefficients of which have been entered on the respective charts.

TABLE 7.—*Distribution of departures of forecast bog minimum temperatures made from hygrometric data at Philadelphia, Pa., from actual bog minimum temperatures at Whitesbog (New Lisbon) N. J., using Figures 3 to 8, inclusive.*

Departures.	Fig. 3.	Fig. 4.	Fig. 5.	Fig. 6.	Figs. 5 and 6 combined.	Fig. 7.	Fig. 8.	Figs. 7 and 8. combined.
0	16	9	8	4	12	5	5	10
±1	9	22	6	12	18	17	5	22
±2	16	10	8	3	11	6	7	13
±3	7	7	10	0	10	4	2	6
±4	4	4	1	0	1	3	0	3
±5	4	5	3	1	4	2	0	2
±6	2	0	1	2	3	1	2	3
±7	1	3	0	1	1	0	1	1
±8	0	1	1	0	1	0	1	1
±9	2	0	0	0	0	0	0	0
0	16	9	8	4	12	5	5	10
+	22	25	13	9	22	14	8	22
−	23	27	17	10	27	19	10	29
Total	61	61	38	23	61	38	23	61

"Figures 3 and 4 show the data charted for the total number of 12:00 noon and 8:00 p. m. observations for both the spring and fall seasons. Figures 5 to 8, inclusive, were charted to ascertain if the accuracy of the bog minimum temperatures forecast could be increased if the spring and fall seasons were separately considered for both the noon and evening data. The accuracy was slightly increased but not so much as was hoped for, although some of the largest departures were eliminated. Table 7 gives in summarized form the remarkable results obtained.

"The data in Table 7 show that the forecast bog minimum temperatures for Whitesbog, based on hygrometric data at Philadelphia, were within 2° F. of the actual bog minimum temperatures from 65 to 75 per cent of the time, while departures in excess of 4° were very few indeed."

CONCLUSION.

(1) In conclusion it may be said that for the best possible results in forecasting minimum temperatures for cranberry bogs a good hygrometric formula is necessary for use with the weather map. (2) That when the weather map indicates the radiation conditions to be good, the hygrometric formula will give a closer and more uniformly consistent estimate of the probable bog minimum temperatures than it is possible to obtain otherwise. (3) That a hygrometric formula, correlated between the bog minima and data from a near-by Weather Bureau station, will give fairly accurate and reliable results, and is a valuable aid to the forecaster. (4) That a hygrometric formula adapted to a given locality, if intelligently and accurately applied by the grower, would be a fairly reliable safeguard without other information. (5) That in the latter case, when general radiations were not so good as they appeared to be locally, the formula might give too low a temperature and cause the grower to flood the bogs unnecessarily and at some loss, which the aid of the weather map would in most cases avoid. (6) That under ideal radiation conditions the error of a well-calculated hygrometric formula seems to be no larger than the personal equation in obtaining the data. (7) That the data for at least 50 good, or fairly good, radiation nights should be used in calculating the formula.

A SIMPLE GEOMETRIC DERIVATION OF THE LAWS OF REFRACTION OF LIGHT INCLINED TO A PRINCIPAL PLANE OF A PRISM.

By W. J. HUMPHREYS.

[Weather Bureau, Washington, D. C., November 23, 1922.]

In the theory of halos, as also in any general discussion of the action of refracting prisms on light, it is necessary to consider the course of rays inclined at various angles to a principal plane—that is, a plane normal to both the face through which the light enters the prism and the face through which it leaves the prism, or, what comes to the same thing, normal to the intersection of these two faces.

This problem was first discussed fully by the French astronomer Auguste Bravais, and the laws found (often called Bravais' laws of refraction) used in his classical memoir on halos.[1] They have also been variously derived by other writers, most recently by Terpstra,[2] and, with mathematical elegance and dispatch, by Laville.[3] However, none, presumably, of these various derivations, some of which are tedious to follow, is readily available to the average reader of the REVIEW. It may be worth while, therefore, to give those naturally most interested in halos a straightforward discussion of the refraction of light rays inclined to a principal plane.

In what follows it will be necessary to remember the well-known facts:

1. The angle of incidence is the angle between the incident ray and the normal to the surface at the point of incidence.

2. The angle of refraction is the angle between the refracted ray and the normal (within the prism) to the surface at the point of refraction.

3. The plane of the incident and refracted rays is normal to the incident surface at the point of incidence. Similarly, the plane of the refracted and exit rays is normal to the exit surface at the point of exit.

4. If i is the angle of incidence and r the angle of refraction, then $\sin i=\mu \sin r$, where μ, called the index of refraction, is the ratio of the velocity of light in the surrounding medium, air, say, to its velocity within the prism.

Remembering these facts, let ABC, Figure 1, be a principal plane of a prism; let DEF be the plane, perpen-

[1] Journal de l'Ecole Royale Polytechnique, 18, 1847.
[2] Zeit. f. den phys. und chem. Unterricht, p. 80, March, 1922.
[3] Journal de Phys. et le Radium, 2, p. 62, 1921.

dicular to the face of incidence, determined by the incident and interior portions of a ray entering the prism at O and leaving it at O'; let GH be the intersection of these two planes: and let ON be normal to the face of incidence at O. Draw OM normal to the principal plane, and connect M with L and K, the points on the intersection GH determined by the interior ray and the incident ray extended, respectively.

Since GH is the intersection of two planes both of which are normal to the incident face, it also is normal to that face and, therefore, parallel to ON. Hence the

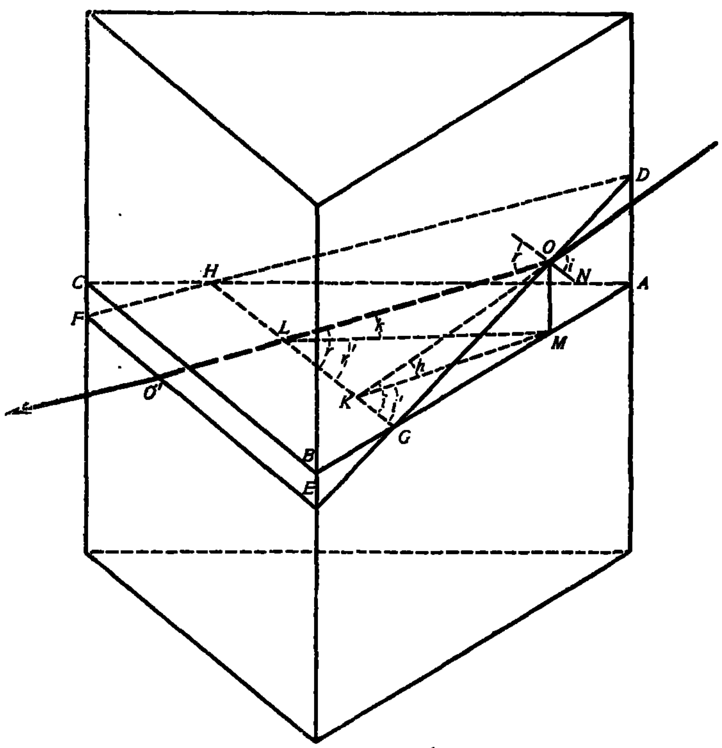

FIG. 1.—Path of a ray inclined to the principal plane.

angle OKG is equal to the angle of incidence, i, and the angle OLG equal to the angle of refraction, r.

Clearly, then, since

$$\sin i = \mu \sin r,$$

if, in length, $KO = 1$, it follows that

$$LO = \mu,$$

and $\sin h = \mu \sin k$, in which h and k are the angles between the principal plane and the incident and interior rays respectively.

The angles between the principal plane and the incident and interior rays, respectively, are connected by the law of sines.

Similarly,

$$\sin h' = \mu \sin k',$$

in which h' and k' are the angles between the principal plane and the exit and interior rays, respectively. But, obviously,

$$k' = k, \text{ hence } h' = h.$$

The incident and the exit rays are equally inclined to the principal plane.

Finally, if i' and r' are the projections of i and r, respectively, onto the principal plane, then

$$\mu \cos k \sin r' = \cos h \sin i',$$

or

$$\frac{\sin i'}{\sin r'} = \mu' = \mu \frac{\cos k}{\cos h}.$$

A ray inclined to the principal plane of a prism is so bent that its projection on this plane is the course a ray in this plane would take if the refractive index of the medium were increased by the ratio of the cosines of the angles between this plane and the internal and the incident (or exit) rays, respectively.

RARE HALO OF ABNORMAL RADIUS.

By A. F. PIPPO, Observer.

[Madison, Wis., May 12, 1922.]

On April 27, 1922, there was observed at Madison, Wis. (lat. 43° 05′ N, long. 89° 23′ W.), a very distinct form of solar halo of abnormal radius occurring simultaneously with a very brilliant halo of 22°. Halos of abnormal radius of less than 10° have been recorded probably less than a half dozen times in the United States, references to those on record in the MONTHLY WEATHER REVIEW being **43**:213, **43**:592, **47**:120, **47**:340.

The rare occurrence of such halos warrants special record being made thereof. Photographs of the halo in a black convex mirror were made. These failed due to lack of filters. Approximate theodolite readings establish the radius at about 8° 12′.

The appearance of the halo was first noted by the writer at 2:15 p. m., 90th meridian time. The sky was nearly overcast with a thin, whitish cirro-stratus veil (west) except where patches of cirro-cumuli appeared apparently at a lower level (WNW.). The smaller halo appeared as a distinct white ring of not more than 1° in width, the accompanying 22° halo being brighter than usual.

There was little change in the conditions of the phenomenon until 2:50 p. m., when for a short time it attained its greatest brilliancy. The 22° halo showed greater coloring at its upper and lower parts, appearing slightly elliptical. However, no measurements to indicate the distortion were made. The 8° halo continued as a whitish ring except that in the upper righthand quadrant could be seen the reddish blue tinge. Sun's altitude 47° 18′ at approximately 2:50 p. m. The phenomenon was easily visible until 3:10 p. m., when it disappeared behind a dark patch of cirro-cumulus. The 22° halo of average brightness was recorded at 7:30 a. m. and again at 6:15 p. m.