CERTAIN UNUSUAL HALOS.

By W. J. HUMPHREYS.

[Weather Bureau, Washington, D. C., November 23, 1922.]

Satisfactory theories exist for all, or nearly all, the more common halos. Several of the rarer halos, however, are not so well understood. It is the purpose of this article to supply, in part, this lacking information.

Halo of 90°.—Occasionally a faint halo, sometimes called the "Halo of Hevelius," is seen at about 90° from the sun. Several explanations of this halo have been suggested, but none is satisfactory.

In seeking to explain this or any other halo, it is essential that our attention be given first to simple forms of discrete ice crystals, because there is little or no evidence that any special grouping of crystals ever occurs in sufficient abundance and predominance to produce a halo of any kind.

Now, only two kinds of small ice crystals are known to occur naturally, namely, (*a*) hexagonal columns, long

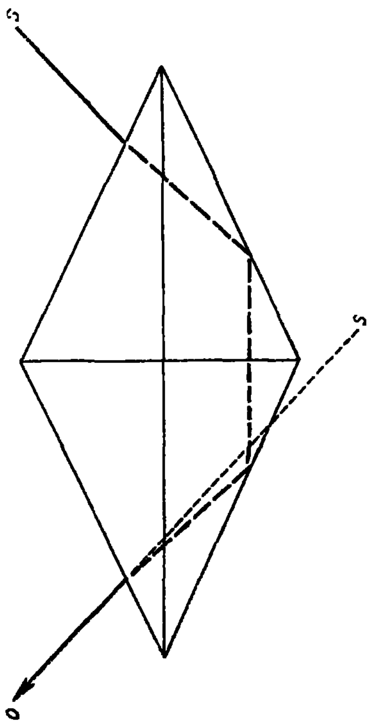

FIG. 1.—Path of a ray through a pyramidal prism giving a 90°-halo.

(needles) or short (tablets), with flat bases perpendicular to the sides, and (*b*) hexagonal abutting pyramids, truncated or not, and with or without an intervening hexagonal column.

The 90°-halo seems to be owing to the presence of randomly directed bipyramidal crystals whose faces are inclined 24° 51′, or thereabouts, to the longitudinal axis.

Apparently no exact measurements of pyramidal ice crystals have ever been made.[1] However, by X-ray analysis it has been shown [2] that the oxygen atoms of an ice crystal are arranged in hexagonal patterns, and so spaced that the ratio of the longitudinal to the lateral axes is very close to 1.62.

Hence, from the laws of crystallography, the ratio of the height of the pyramidal end of an ice crystal to the inner radius of its base, a lateral axis, must also be 1.62, or some multiple or submultiple thereof expressible in either a small whole number or a fraction whose numerator and denominator both are small whole numbers.

If, then, we multiply 1.62 by 4/3, a factor entirely allowable, we obtain a pyramid whose sides are inclined 24° 51′ to the longitudinal axis; and since this value satisfies both the 90°-halo, and also several other halos of unusual radii, as explained below, it will be provisionally accepted as a value that actually occurs in nature.

Light from any source S, Figure 1, entering a face of such a crystal (truncated or pointed, and with or without an intervening hexagonal column), in or near a plane determined by the longitudinal axis and a normal to that face from this axis, will, over a wide range (42° 06′) of the angle of incidence, undergo two internal total reflections and pass out the corresponding face of the abutting pyramid in such direction that an observer at O will see the image S_1 very nearly 90° from the source; the total range being, for light of refractive index 1.31, from 89° 28′ where the concentation is greatest to 88° 02′ where it is least. Light outside this range is relatively too faint to be considered, being enfeebled by at least one reflection that is not total. Minimum refraction, hence, in this case, maximum deviation (turning of the ray by reflection minus its turning by refraction) and maximum concentration, occurs when that portion of the internal ray that lies between the points of reflection is parallel to the longitudinal axis. This, as above stated, puts the brighter edge of the halo at nearly 90° from the sun or moon. Clearly, too, the red of this halo, contrary to rule, is on the side away from, and not the side nearest to, the parent luminary, and still nearer 90° therefrom than the above angular distance, 89° 28′, corresponding to greenish yellow light, though always, perhaps, too faint to arouse a distinct color sensation.

Halos of unusual radii.—In addition to the halo of 22° radius, due to randomly directed 60° refracting angles, the halo of 46° radius, due to randomly directed 90° refracting angles, and the 90°-halo, explained above, several other halos concentric about the sun or moon are occasionally seen. The radii of these are, roughly, 8°, 17°, 19°, and, perhaps, 32°. The last of these values is based on various crude estimates ranging from about 28° to 33°, or more; the other three have been measured, and probably are correct to within half a degree. Piippo,[3] using a theodoolite, obtained 8° 12′ as the radius of a certain small but well-defined halo. He reports, in addition, only the 22°-halo. Andrus[4] estimated the radius of the inner halo to be 8°–9° and that of the outer one to be 28°–29°. A crude sighting device that gave him 22° 50′ for the radius of the 22° halo, gave for the other two 17°–18° and 18°–19°, respectively.

All these halos of unusual radii, 8°, 17°, 19°, and 32° (?), doubtless are due to randomly directed pyramidal crystals; whether truncated or pointed, and with or without columns between the pyramidal ends. Such a pointed, bipyramidal crystal, with an intervening hexagonal column (a well-known type) is represented by

[1] Dobrowolski, *Arkiv för Kemi, mineralogi och geologi*, v. 6, No. 7, p. 44, 1916.

[2] A. St. John, *Proc. Nat. Acad. Sciences*, 4, p. 193, 1918; D. M. Dennison, *Phys. Rev.*, 17, p. 20, 1921; W. H. Bragg, *Proc. Phys. Soc., Lond.*, 34, p. 98, 1922.

[3] See this REVIEW, p. 534.

[4] MO. WEATHER REV., May, 1915, 43:213.

Figure 2. Light obviously can pass through such a crystal in various directions. Those courses that give refraction, and hence produce halos, are listed in the following table, in which the numerical values correspond to an inclination of 24° 51′ of a pyramidal face to the

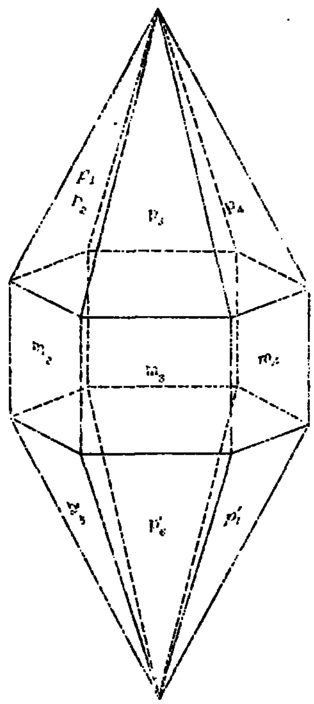

FIG. 2.—Pyramidal ice crystal giving halos of unusual radii.

longitudinal axis, the value adopted above in the discussion of the 90°-halo.

In this table the meanings are: *incident face*, that face of the crystal through which the ray in question passes in; *exit face*, that face of the crystal through which the given ray passes out; *refraction angle*, the dihedral angle between the incidence and exit faces, extended; *minimum deviation*, the least difference in direction between the incident and exit branches of any single ray—the deviation corresponding to maximum light, hence to the angular radius of a particular halo.

Circular halos about sun or moon, by pyramidal crystals, whose faces are inclined 24° 51′ to longitudinal axis.

Incidence face.	Exit face.	Refraction angle.	Minimum deviation.
p_1	p_4	49° 42′	17° 06′
p_1	p_3	76° 24′	31° 49′
p_1	m_4	24° 51′	7° 54′
p_1	m_3	63° 01′	23° 24′
p_1	p'_6	53° 58′	18° 58′
c	p′	65° 09′	24° 34′
c	m	90°	45° 44′
m_1	m_3	60°	21° 50′
p_1	p'_4	[1] 130° 18	[2] 89° 28

m, face of hexagonal column; p, face of one pyramid; p′, face of companion pyramid; c, truncate face, normal to longitudinal axis.

[1] Not really a refraction angle, but the crystal angle between the incident and exit rays.

[2] Maximum total deviation, corresponding to minimum refraction.

The pairs of faces listed in this table are merely typical since, obviously, a change in either face of any pair merely requires a corresponding change in the other.

The last member of this list is the 90°-halo explained above; the next to the last is the very common 22°-halo; and the next above that the well-known 46°-halo. The 8°-, 17°-, and 19°-halos, the unusual ones that have been tolerably well measured, are the third, first, and fifth, repectively, of this table. The second may well be the halo whose radius has been variously estimated at from 28° to 33° or more. The sixth, due to truncated bipyramids, has not certainly been reported. The 46°-halo, though listed here as producible by truncated pyramids, does not require the pyramid form—only faces at right angles to each other. The appearance, therefore, of the 46°-halo does not prove the presence of truncated pyramids; hence this halo and several others of the table may be simultaneously seen when there is no trace of the sixth. Finally, the fourth is apt to blend more or less with the 22°-halo into a band broader than usual and thereby cause the radius of the 32°-halo to be underestimated.