move from east-northeast in the hurricane season. We shall not omit mentioning the direction that the upper clouds have at San Juan, Porto Rico. In general it can be stated that the current from the second quadrant predominates, the times at which the southwest direction is observed being relatively few.

There will hardly be found a series of observations of clouds so extended and so complete as that of the observatory at Belén College, Havana. The currents of the atmosphere have been observed very carefully at intervals of two hours during a period of 50 years or more.

In the paper that its director, Father Gangoiti, published in 1904 we find the following results for the period 1892–1902.

Mean direction of upper clouds at Havana, from the observations for 11 years (1892–1902).

	Cirrus.	Cirro-stratus.		Cirrus.	Cirro-stratus.
January	S. 81 W.	S. 79 W.	July	N. 28 E.	N. 52 E.
February	S. 83 W.	S. 80 W.	August	N. 48 E.	N. 54 E.
March	N. 88 W.	S. 89 W.	September	N. 27 E.	N. 89 E.
April	N. 89 W.	S. 89 W.	October	N. 87 W.	S. 61 W.
May	N. 83 W.	S. 84 W.	November	S. 77 W.	S. 72 W.
June	N. 49 W.	N. 84 W.	December	S. 80 W.	S. 73 W.

Conclusion.—There do not exist sound arguments in favor of the theory of the antitrade wind as advanced by many writers. In the observations in Ecuador, Bogota, Mexico, and the Antilles that upper current is not found to be constant and permanent.

THE DEVELOPMENT OF METEOROLOGY AS ILLUSTRATIVE OF THE RÔLE OF MATHEMATICS IN THE PROGRESS OF SCIENCE.[1]

By EDGAR W. WOOLARD.

[Weather Bureau, Washington, D. C., Dec. 10, 1923.]

Mathematics plays a singularly fundamental rôle in all domains of exact scientific thought. In the words of Spottiswoode,[2] "Conterminous with space and coeval with time is the kingdom of Mathematics; within this range her dominion is supreme; otherwise than according to her order nothing can exist; in contradiction to her laws nothing takes place. On her mysterious scroll is to be found written for those who can read it, that which has been, that which is, and that which is to come." An irrefutable proof of this claim is provided by an analysis of the rôle which mathematics has actually played in the development of the sciences,[3] and in the present paper we shall use the history and present status of the science of Meteorology as an illustration.

Man quickly finds by experience that his greatest good and comfort come from an understanding of the environment in which he finds himself—as Bacon said, Knowledge means power and control over Nature; besides, he naturally feels an innate longing to know the explanations of the phenomena that he sees going on about him.[4] Hence the very beginnings of natural science are found among the great nations of remote antiquity—the Egyptians, Assyrians, and Babylonians.

Atmospheric phenomena must have been among the first to attract attention, and meteorology, as a branch of knowledge, is probably as old as mankind, particularly since in early times primitive man lived largely in the open, as hunter or agriculturist, and was forced to watch the weather closely for the sake of his own welfare.[5] Weather lore existed among the Chaldeans and the Babylonians two or three thousand years before the Christian era.

The first major period in the history of meteorology may be taken to be that from antiquity to about 1600 A. D. Early knowledge was cultivated only for immediate practical needs, and in very primitive times the mere observation of facts without inquiry as to causes was as far as it was carried; such knowledge does not constitute true science: We must generalize from our observations; the ultimate goal of scientific research is the discovery of fundamental and comprehensive laws, and the demonstration of how the phenomena and their laws result from a few simple underlying principles. Simple generalizations were made at an early stage by the ancients, but it was left to the Greeks to initiate abstract thought and to invent explanatory hypotheses. Greek thought culminated in the great system of Aristotle (384–322 B. C.), who gathered together and systematized all knowledge then existing, and subdivided Science, which at that time was only a part of Philosophy, into various special subjects. Aristotle's *Meteorologica* [6] was the first treatise on meteorology. Outside of the activities of the Alexandrian School, the Arabs, and the Moors, there is little to record in the history of any science during the ensuing 2,000 years of scholasticism and introspection.

Early science accepted its hypotheses without any adequate verificatory inquiry—its explanations did not have to explain more than they were directly invented to explain, and they did not have to cohere with previously acquired knowledge nor be able to predict undiscovered facts; much of this "science" was therefore quite fantastic. The *modern* trained man of science "uses his powers of observation to discover the facts of nature, his inventive ingenuity to propose various possible hypotheses for the explanation of the facts, his power of logical reflection to think out, or deduce, from each hypothesis, in accordance with previously acquired, pertinent knowledge, just what ought to happen if the hypothesis were true, and his impartial faculty of verification to decide which hypothesis, if any, is competent to explain the observed facts;"[7] and he must constantly be on his guard against the numerous errors that may creep in at any stage of the procedure. In the sixteenth century there was a general reconstruction and reorganization of thought; modern methods in science began with Galileo, and the complete Scientific Method came into prominence under the leadership of Francis Bacon (1561–1626). In philosophy, a reaction "back to nature" took place, away from the barren scholasticism of the Middle Ages; the special sciences left the philosophic fold, and became differentiated one from another; the great geographical discoveries of the fifteenth and sixteenth centuries, and the revolution in astronomy occasioned by the work of Copernicus (1473–1543) and Kepler (1571–1630), contributed to a broadening of man's mental horizon that ushered in a new area.

The second major period of meteorological history is that from 1600 to about 1800. The first essential in any

[1] Presented before the Fourteenth meeting of the Maryland-Virginia-District of Columbia Section of the Mathematical Association of America, Annapolis, Md., December 8, 1923.

[2] Wm. Spottiswoode: Presidential address before the British Association. *Rept. Brit. Assoc. Adv. Sci., Dublin, 1878.* pp. 1–32. London, 1879.

[3] See R. D. Carmichael: The provision made by mathematics for the needs of science. *Science*, (N. S.), 45, 465–474, 1917.

[4] *Cf.* R. D. Carmichael: Motives for the cultivation of mathematics. *Scientific Monthly*, 8, 160–178, 1919.

[5] See G. Hellmann. The dawn of meteorology. *Quar. Jour. Roy. Met. Soc.*, 34, 221–232, 1908.

[6] See E. W. Webster: The works of Aristotle translated into English: *Meteorologica*. Oxford, 1923.

[7] W. M. Davis: The reasonableness of science. *Scientific Monthly*, 15, 193–214, 1922.

natural science is to gather data by actual observation; the meagre observations of the ancients had been very crude, mostly made without the help of instruments, and much influenced by superstition and imagination. With the invention of the thermometer by Galileo in 1590, and of the barometer by Torricelli in 1643, accurate meteorological observations became possible. Many meteorological stations were established in Europe; and the major features of the general structure of the atmosphere, the heating of this atmosphere by the sun, and the distribution of winds, temperature, etc. over the globe, became well known.[8] The variable winds of the temperate zones, and in particular the storms and changes of weather, however, remained but little understood, on account of their irregularity and complexity, although in 1760 Benjamin Franklin discovered the regular travel of storms in eastern United States.[9] The physical properties of air, such as mass, weight, pressure, resistance, inertia, density, viscosity, expansion with heat, etc., were elucidated in the sixteenth and seventeenth centuries, and the chemical nature of the atmosphere discovered near the close of the eighteenth century.

Throughout this period, meteorology was in its early infancy, an almost purely observational and empirical science, greatly lacking in helpful theories. Only after much labor does law become apparent in the investigation of natural phenomena, particularly when these phenomena are as complex as those of our atmosphere. However, we can not doubt that the processes of weather are simply examples of the operation of ordinary physical laws, although special methods may be required for the treatment of the special problems involved. Meteorology proper (i. e., exclusive of the mere description of weather, climates, and atmospheric phenomena [10]) is essentially the *physics of the earth's atmosphere*,[11] and fundamentally it is the *mechanics of* the atmosphere, this term being taken to embrace both dynamics and thermodynamics; all other aspects are of minor importance. In addition to the meteorological information which had been accumulated up to the time we are now considering, there was a not inconsiderable knowledge of physics, but no one perceived the relationship between meteorological phenomena and physical laws, and dared to combine them, until Halley in 1686 and Hadley in 1735 [12] attempted to explain the cause of the trade winds; the latter gave an essentially correct explanation, although it needs revision from the dynamical point of view.[13] By a similar process, Astronomy at one stroke became an exact science, but meteorology took only one short step in this direction; the transformation in the case of the latter could not really begin in earnest until after extensive further advances in both observational meteorology and theoretical physics.

Analytical mechanics was brought to a high degree of perfection during the eighteenth century through the labors of Euler, Lagrange, Poisson, and Laplace. The third major period in the history of meteorology extended from 1800 to about 1850, and was particularly marked by numerous efforts to give logical explanations, largely *qualitative*, for the observed facts. Meteorological observations were increasing in number and in accuracy, and during this period notable progress was made by Brandes, Dove, Redfield, Reid, and Espy in the elucidation of the phenomena of storms.

Deep insight into the phenomena of the atmosphere began to come, however, only when meteorology, having attained vigorous youth, began to be transformed into an exact science; this transformation is still very far indeed from having been completed, but great advances have been made since the middle of the last century. There are two sides to the study of the atmosphere, which can be pursued by different people who may never meet each other: One is the observation and collection of facts, the other is the interpretation of the facts by physical and mathematical reasoning. The science of meteorology "includes the process of observing, of the first part; the compilation and summarizing, in maps or otherwise, of the facts of weather, of the second part; the application of meteorological principles, which includes the forecasting of future weather, of the third part; and the development of the science of meteorology, of the fourth part. Any one of the first three may be pursued according to recognized canons of procedure with satisfactory results: every one of them is indispensable, and all three of them may be pursued simultaneously without any effective recognition of the fourth part, which forms our only avenue to the comprehension of the secrets of the sequence of weather." [14] The mathematicians and mathematical physicists of the eighteenth century had made available for application many powerful tools, and since then further striking advances have been, and are being, rapidly made in these subjects; but in the present position of meteorology there still exist to some extent two undesirable extremes of opinion—one, to think the penetration into the secrets of the subject to be so difficult that we must be content to forego the attempt and deal with what we have; the other, to think it so easy that only observations are required, and the training of our brains of no account. However, as Sir Napier Shaw has emphasized, nothing useful can be done without real facts—brains without observations are of no avail in any problem dealing with Nature—but real facts do not, as a rule, explain themselves—observations, however numerous and excellent, will not exonerate us from the use of highly trained intelligence.

Dynamical meteorology deals essentially with the study of the behavior of a permanent gas, surrounding a rotating globe, and exposed to solar radiation, dilatable with heat and compressible with pressure, but mixed with small and variable percentages of vapors that condense to liquids or solids at ordinary low temperatures. The state of the atmosphere at any point at any instant is completely determined, so far as all weather phenomena are concerned, by the values at the time and place of the six meteorological elements—temperature, pressure, wind velocity, humidity, cloudiness, and precipitation occurring. The day-to-day fluctuations of these elements constitute weather, whereas the "normal" values at any locality, obtained by averaging a long series of observations, determine, in part, the climate. The problem before meteorology is: Given complete observations describing the state of an extensive three-dimensional portion of the atmosphere, to determine, from the laws of physics, the conditions that will result at some given time in the future. We can not rest content, nor can meteorology become most effective in a practical way, until an *exact* and *com-*

[8] See Edgar W. Woolard: Historical note on charts of the distribution of temperature, pressure, and winds over the surface of the earth. *Monthly Weather Review*, 48, 408–411, 1920. Also, Edmund Halley: On the height of the mercury in the barometer at different elevations above the surface of the earth, *Phil. Trans.*, 16, 104, 1686; and On the proportional heat of the sun in all latitudes, *Phil. Trans.*, 17, 878, 1693.

[9] Benjamin Franklin: On the northeast storms in North America. *Works*, II, pp 63–66.

[10] The standard works on descriptive meteorology are: Julius von Hann. *Lehrbuch der Meteorologie*. 3te aufl., Leipzig, 1915. (A new edition is now appearing.) And Julius von Hann, *Handbuch der Klimatologie*. 3te aufl., Stuttgart, 1908–1911. 3 vols.

[11] The only comprehensive work treating meteorology from the physical standpoint is W. J. Humphreys' *Physics of the Air*, Philadelphia, 1920.

[12] Edmund Halley: An historical account of the trade-winds and monsoons. *Phil. Trans.*, 16, 153–168, 1686. George Hadley: Concerning the cause of the general trade-winds. *Phil. Trans.*, 39, 58–62, 1735.

[13] See Sir Napier Shaw: *The Air and its Ways*, p. 53, Cambridge Press, 1923.

[14] Sir Napier Shaw: The outlook of meteorological science. *Monthly Weather Review*, 48, 34–37, 1920.

plete solution of this problem has been attained; such a solution would involve the reduction of meteorology to a mathematical science. *The progress of exact science must always wait on the progress of pure mathematics; and no science can come to even a relatively advanced state of development or perfection until it has been made to rest on a mathematical foundation and has taken on a mathematical form.* Unfortunately, at the present time the subjects of pure mathematics and mathematical physics have not been developed to an extent sufficient to handle completely the actual problems of nature involved in the study of the atmosphere, and weather forecasting as conducted by the chief meteorological services of the world has of necessity been completely *empirical.*

However, there is no justification for insisting that this shall always be the case. Meteorology has in the past gone through the various stages of development common to all the sciences and may be expected in the future to emulate those sciences which are at present ahead of it. In its infancy a science is not able to stand the strong meat of mathematics; and mathematical ideas initially find an application in it by slow processes, although even in the preliminary stages mathematics makes an essential contribution in that it furnishes the only language in which exact information can be expressed in a form such that the mind can readily grasp the import of the data, viz., through elementary statistical methods. Political science, economics, biology, psychology, and geology are now at this stage of development. In the period of vigorous youth, where chemistry may be said to be, a science is not reduced to a mathematical form, yet its laws are so stated as to be subject to the sharpest experimental verification, and it often employs the elements of the differential calculus. The most mature sciences are those which make the most extensive use of mathematics; physics is perhaps the most mature natural science at present, and the one whose recent progress has been the most rapid and whose practical applications the most remarkable; it continually taxes to the utmost the resources of pure mathematics.[15]

The elucidation of the reason for the fundamental and powerful role which pure mathematics plays in physical science forms a fascinating and extensive philosophical inquiry. One aspect of the matter is, however, apparent upon a little thought: Mathematics is the only language adequate for use in a science which is to attain to the standard of exactness and perfection which, in order to yield the maximum practical usefulness and mental satisfaction, every science strives to attain. Language of some kind is a condition of all progress in knowledge beyond the merest rudiments—the mind can not go far without a coherent system of symbols for the representation of its concepts and constructs, and as an aid to reason. Language introduces immense economy of mental effort; it enables communication between individuals; it is a fundamental element in accumulating and retaining the heritage of the past. The ordinary language of daily life, however, while adequate for the purposes which it is designed to serve, has many shortcomings and defects which render it completely inadequate for use where precise and rigorous thinking in intricate and abstract matters is involved; and what the language of daily life does for the thought of usual intercourse the language of mathematics does for exact scientific thought. Mathematics furnishes the essential means for the expression of the latter; aside from its rigorous logic, its certainty, and its universality, mathematics owes its superior power to its symbolic language, the most powerful language known, which enables it to completely outstrip every other science and every other method of reasoning; it supplies the support without which the mind would be unable to carry through the processes necessary to attain the desired results. "There is a certain storing, as it were, of intellectual force in the mathematical symbols, from which it can be released with almost explosive power. These become mighty engines through the aid of which we can rear intellectual structures quite inaccessible to our unsupported power" or to the power of ordinary language.[16] Theoretically, the use of mathematical symbols is an irrelevant convenience only; the content of any mathematical expression could be translated into ordinary language, but the result would be so lengthy, involved, and cumbersome that no meaning could be seen in it and no deductions performed upon it.

Although there is probably nothing which more directly concerns each and every person than the weather, and there are few things of more vital importance to many lines of activity than a knowledge of its future course, meteorology has been a comparatively neglected science, especially on its physical and mathematical side. It is rarely taught in any adequate or extensive way at universities; comparatively few people have even an elementary knowledge of the subject; it has not been endowed with great observatories and institutions to the extent that astronomy has; and it has not attracted the attention of a very great number of able physicists and mathematicians. Furthermore, practical meteorologists do not always look with favor upon the attempts of the mathematical physicists to advance the science of meteorology; but it must be recognized that behind every high art there stands a higher science, that complete success in weather forecasting demands an equally complete knowledge of the sciences involved in the motions of the atmosphere, and that satisfactory progress in the improvement of daily predictions can only be based on corresponding progress in our knowledge of theoretical meteorology obtained with the help of the best mathematicians and physicists.[17] The method of forecasting by empirical rules and past experience is only a stage in our advance; it leads, as we know, to excellent and valuable results in the hands of the experienced, but its capacity is limited and the limit is very soon reached. To improve it, to carry it further, or to make out the true inwardness of its application in special cases, we must, as Shaw, Abbe, and others have insisted, depend on advancing our knowledge of the dynamics and physics of the atmosphere. It would be utterly absurd at the present time to entertain seriously the idea of forecasting weather, now or probably for many generations to come, by mathematical equations as the movements of the heavenly bodies are forecast; nevertheless we are even now introducing *scientific* considerations into the making of forecasts, and every small advance in our understanding of atmospheric phenomena will, sooner or later, be found to be of practical use in improving predictions, and will contribute in some measure toward making forecasting less of an empirical art and more of an exact science, whether or not we are ever enabled to reach or even remotely approach the ultimate ideal goal of exact mathematical calculation of the coming weather. Those who are impatient with mathematical studies on the atmosphere ask, "How can this be of any use? The

[15] Carmichael, *l. c.*

[16] R. D. Carmichael: The larger human worth of mathematics. *Scientific Monthly* 14, 447–468, 1922. Also, E. D. Roe, Jr.: A survey of mathematics and astronomy, *ibid.*, 17, 248, 1923.

[17] See Cleveland Abbe: The progress of science as illustrated by the development of meteorology. *Ann. Rept. of the Smithsonian Institution for 1907*, pp. 287–309. Washington, 1908.

calculations must require a preposteriously long time. Under the most favorable conditions it will take the learned gentlemen perhaps three months to calculate the weather that Nature will bring about in three hours. What satisfaction is there in being able to calculate tomorrow's weather if it takes us a year to do it?" Bjerknes has answered quite clearly: "If only the calculation shall agree with the facts, the scientific victory will be won. Meteorology would then have become an exact science, a true physics of the atmosphere. When that point is reached, *then* the practical results will soon develop. It may require years to bore a tunnel through a mountain. Many a laborer may not live to see the cut finished. Nevertheless this will not prevent later comers from riding through the tunnel at express train speed."[18]

The fourth and present period of meteorological history opens with the first attempts at constructing mathematical theories of the phenomena of the earth's atmosphere, beginning with the remarkable pioneer work of the American William Ferrel,[19] which gave a strong impulse to similar theoretical research by Helmholtz, Guldberg and Mohn, Hertz, von Bezold, Oberbeck, Sprung, Margules, and others. Most of this literature has been collected and translated into English by Abbe,[20] and in addition we have a comparative study of the different theories by Bigelow,[21] and an excellent textbook by Exner.[22] Shortly after 1850 the various governments began establishing weather bureaus, synoptic charts were prepared, and with the advent of the telegraph weather forecasting was initiated; in the latter part of the century the exploration of the upper air by kites and sounding balloons began, and in recent years has increased greatly, especially through the use of the pilot balloon. Much of the observational information upon which the construction of theories must rest has been gathered together by Hildebrandsson and Teisserenc de Bort,[23] but meteorologists are still greatly handicapped by the lack of adequate observational information. Whether one is studying meteorology from the standpoint of practical day-to-day forecasting (by whatever method) or from the standpoint of the mathematical theory of meteorological phenomena, satisfactory progress can not result until we have daily synoptic charts and aerological data covering in detail at least an entire hemisphere. Meteorology is peculiarly insatiable among the sciences in its demands for data. In recent years, much has been done toward making possible a fairly satisfactory daily surface synoptic chart for a large part of both the land and water of the Northern Hemisphere, but information concerning the upper air still remains extremely fragmentary.

In order to make our problem tractable at all, particularly at the start, simplifying assumptions have to be made—the natural reality must be replaced by a reasonable, workable ideal. Much of the work just mentioned deals with more or less *ideal* cases,[24] in some of which assumptions are introduced which carry us right away from actual conditions into hypothetical states which never exist, and in others of which there is considered only one of the great fundamental factors operating. We must, in order to reach a complete and satisfactory solution, simultaneously consider the actual conditions existing in the atmosphere under the influence of all the great factors—the rotation of the earth, the gravitational field, the distribution of temperature, friction and viscosity, the condensation and evaporation of moisture, thermodynamics of ascending and descending air, radiation and absorption, etc.—as expressed in one systematic set of mathematical equations. Needless to say, this calls for mathematical ability of a high order. At the present time there are several mathematicians and physicists who are working at the foundations of weather forecasting along strictly scientific lines, and are conducting important researches in theoretical meteorology, more and more along the lines of treating the *actual* conditions of nature by mathematico-physical methods.

Investigations are being conducted in a particularly exemplary manner by the Norwegian meteorologists under the leadership of V. Bjerknes: (1) In the first place, the Bergen meteorologists have, with an extremely close network of observing stations, brought to light details not known before, and have sought to show that their synoptic maps can be simulated by wave motion on either side of a surface of discontinuity which separates a cap of polar air from the warm equatorial air. The successive cyclones of the temperate zones are Helmholtz waves on this boundary surface, and the corresponding boundary line—the "polar front"—at the ground traverses the centers of depressions all around the polar region; the examination of depressions individually from their formation to their disappearance shows that young cyclones correspond to slight waves with small excursions from the equilibrium state; later the amplitudes increase simultaneously with a deepening of the depression. The life cycle of a cyclone has been worked out in great detail from actual observations interpreted in the light of this theory; the Norwegian weather maps are now drawn in accordance with the principles of the theory, and forecasts made accordingly. Marked success has attended the Norwegian methods of scientific forecasting. The germs of the polar front theory are contained in the work of Dove, and many of Bjerknes's ideas are an extension of those of Helmholtz, Brillouin, and Margules; similar ideas have been arrived at empirically by Humphreys and by Shaw; but never before has the theory been so elaborately developed and scientifically applied to daily forecasting. The generalizations established for Norway have not been tested in other regions of the world to any extent, however, because of the lack of sufficiently detailed data; the theory itself has been criticised from both the observational and the theoretical standpoints, and can not be regarded as final and proved, particularly as regards the details of the structure of cyclones in other countries than Norway.[25] (2) Then, in the second place, these meteorologists have at the same time, as a matter of *research*, tried to work out the course of the weather in a certain limited region by mathematical calculations based directly on physical laws applied to meteorological data as complete as obtainable, mainly by employing graphical methods to solve the differential equations involved.[26]

[18] See V. Bjerknes: Meteorology as an exact science. *Monthly Weather Review*, 42, 11-14, 1914.

[19] Wm. Ferrel: The motions of fluids and solids on the earth's surface. *Prof. Papers of the Signal Service*, No. VIII. Washington, 1882. Meteorological researches for the use of the Coast Pilot, *Rept. Supt. U. S. Coast Survey*, 1875, 1878, and 1881. Recent advances in meteorology, *Rept. Chief Signal Officer for 1885*, App. 71, Washington, 1886. *A popular treatise on the winds*, New York, 1889.

[20] Short memoirs on meteorological subjects. *Ann. Rept. Smith. Inst. for 1877*, pp. 376-478. Washington, 1878. The mechanics of the earth's atmosphere, second collection, *Smith. Misc. Coll.* 843, Washington, 1891; third collection, *Smith. Misc. Coll.*, vol. 51, No. 4, Washington, 1910. See also Marcel Brillouin: *Mémoires originaux sur la circulation générale de l'atmosphère*, Paris, 1900.

[21] F. H. Bigelow: Report on the International cloud observations. *Rept. of the Chief of the Weather Bureau for 1898-99*, vol. II. Washington, 1900.

[22] F. M. Exner. *Dynamische Meteorologie*. Leipzig, 1917.

[23] Hildebrand Hildebrandsson and L. Teisserenc de Bort: *Les bases de la météorologie dynamique*. Paris, 1898-1905. 2 vols. See also the British *Réseau Mondial*.

[24] Of this nature are the very recent kinematical studies of T. Kobayasi, "On the mechanism of cyclones and anticyclones," *Quar. Jour. Roy. Met. Soc.*, 49, 177-189, 1923; and V. H. Ryd, "Travelling cyclones," *Pub. fra det Danske Meteorologiske Institut, Meddelelser Nr. 5*, Kjöbenhavn, 1923.

[25] See J. Bjerknes: On the structure of moving cyclones, MONTHLY WEATHER REVIEW, 47, 95-99, 1919; V Bjerknes: The meteorology of the temperate zone and the general atmospheric circulation, *Nature*, 105; 522-524, 1920; J. Bjerknes and H. Solberg: Life cycle of cyclones and the polar front theory of atmospheric circulation, *Geofysiske Publikationer*, Vol. III, No. 1, Kristiania, 1922.

[26] V. Bjerknes, *et al:* Dynamic meteorology and hydrography. 2 vols. *Carnegie Institution of Washington, Pub. 88.* Washington, 1910-1911.

In the system of fundamental partial differential equations expressing the state of the atmosphere, there are four independent variables—time, elevation above mean sea level, latitude, and longitude—and seven dependent variables—the three components of wind velocity, pressure, temperature, density, and joint mass of solid, liquid, and gaseous water per mass of atmosphere. The equations are the three hydrodynamical equations of motion of a fluid covering a rotating globe, the equation of continuity, the characteristic equation of state for gases, and two thermodynamical equations relating to the conveyance of heat and water.[27]

Special difficulties are at once encountered in connection with the hydrodynamical equations.[28] In the first place, classical hydrodynamics assumes the density to be a function of the pressure only,[29] whereas in the atmosphere many other independent variables, such as temperature and humidity, affect the density. The hydrodynamics of *baroclinic* fluids, i. e., fluids in which the density is a function of other variables besides the pressure—has been worked out by Bjerknes, but strangely enough has been incorporated in only one treatise on hydrodynamics.[30] In such fluids surfaces of equal density are not always surfaces of equal pressure and the formation and annihilation of vortices are possible.[31] In the second place, the resistance of the surface of the earth and the disturbances caused by the irregularities thereof give rise to a very turbulent condition of the lower atmosphere, the effect of which must be taken into account. The ordinary viscosity of the air is so small that it may be neglected, but the effect of turbulence is to give the air a virtual or pseudo viscosity many thousand times greater than the ordinary viscosity.[32] This turbulence is an extremely important factor in the transmission of heat and water vapor through the atmosphere.

The only forces acting on any mass of air are gravity, hydrostatic pressure, and friction; the acceleration of the mass is composed of two parts—acceleration relative to the surface of the earth, which is observable, and acceleration common to this surface itself; it is the acceleration of the surface which gives rise to the so-called "deflecting force of the earth's rotation." There are three cases:[33] (1) Eulerian winds, in which the rotational and frictional terms in the equations of motion are so small in comparison with the accelerational terms that they may be neglected—the observable acceleration corresponds to the horizontal pressure gradient, as in ordinary elementary hydrodynamics; (2) Geostrophic winds,[34] in which the accelerational and frictional terms are negligible in comparison with the terms arising from the rotation of the earth—the velocity is at *right angles* to the pressure gradient, giving rise to the interesting paradox that, in the Northern Hemisphere, air pushed north blows east;[35] and (3) Antitriptic winds, in which the accelerational and rotational terms are negligible in comparison with the frictional term—the wind blows in the direction of the pressure gradient, but without acceleration. Examples of these three classes are, respectively, tropical cyclones, slow-moving extratropical cyclones, land-and-sea breezes.

Eastern mathematical genius has recently made its entry into our field, in the work of Fujiwhara.[36] In ordinary fluids, vortices of the same sense of rotation have a tendency to approach, and those of opposite senses to separate, just the opposite of what happens in an ideal fluid; this may explain Okada's Law concerning the behaviour of cyclones and anticyclones. Employing a well known equation, due to Hesselberg and Friedmann, for the determination of the rate of change of the vorticity of horizontal motion in the atmosphere, Fujiwhara shows that the most important source of energy for a cyclone is in the vorticity of the surrounding air, and undertakes a new theory of the production and growth of cyclones along the polar front, in which a number of known facts come together in harmony.

The most thoroughgoing and comprehensive attempt to reduce the treatment of the phenomena of the atmosphere, as actually presented to us in Nature, to a systematic branch of mathematical physics is that of the British physicist L. F. Richardson.[37] In his study, practically every physical and dynamical process in the atmosphere is analyzed in detail, all meteorological phenomena are coordinated into a systematic whole and expressed by a set of equations which aid greatly in obtaining an insight into how the meteorological elements act and react on each other. Whatever direction practical forecasting may take in the future, Richardson's work can not fail to be of great practical benefit, aside from its purely scientific interest and importance.

At present it is perhaps best to separate such highly theoretical investigations from any suggestion of immediate direct application to forecasting; it is extremely interesting to know, however, that Richardson obtains a solution of his set of equations by means of the calculus of Finite Differences, and applies it to the computation of coming weather, using a very complete set of surface and free-air data which happens to be available for middle Europe at 1910, May 20, 7h., G. M. T. The resulting "forecast" is rather disappointingly in error, however, particularly in respect to the pressure changes, because, apparently, of inaccuracies in the observational data; and of course the calculations are so prohibitively laborious as to preclude the possibility of ever actually making forecasts in this way; besides, much of the really interesting detail in the weather is "smoothed out" by the method of solution.

An adequate comprehensive treatise on dynamical meteorology, for the reader with extensive mathematical training, has not yet been written, although greatly needed. The advancement of meteorology from this standpoint will long offer a fertile field for research by the mathematical physicist, and it is desirable that as many as possible be attracted to it. Purely mathematical difficulties, no matter how great, must eventually be overcome, for the Philosophy of Mathematics teaches us[38] that the power of pure mathematics is unlimited—its advance can not be stayed—because it meets difficulties by a creative act of the mind which leaps over them.

[27] See Bjerknes: MONTHLY WEATHER REVIEW, 42, 11–14, 1914; Abbe, The weather map on the polar projection, MONTHLY WEATHER REVIEW, 42, 36–38, 1914; and L. F. Richardson: *Weather prediction by numerical process*, chap. iv, Cambridge Press, 1922.

[28] For the derivation of these equations see: J. H. Jeans, *Dynamical theory of gases*, 3 ed., Cambridge Press, 1921, pp. 165–175; P. Appell, *Traité de mécanique rationelle*, Tome III, 3 ed., Paris, 1921; H. Lamb, *Hydrodynamics*, 4 ed., Cambridge, 1916, p. 318.

[29] Lamb, *op. cit.*, Art. 8; Appell, *op. cit.*, Art. 627.

[30] Appell, *op. cit.*, chap xxxii, pp. 562–605.

[31] See V. Bjerknes: On the dynamics of the circular vortex, with applications to the atmosphere and atmospheric vortex and wave motions. *Geopysiske Publikationer*, Vol. II, No. 4, Kristiania, 1921.

[32] See G. I. Taylor: Eddy motion in the atmosphere, *Phil. Trans.*, A215, 1–26, 1914, and later papers cited in the bibliography on p. 92 of Richardson, *op. cit.*

[33] Harold Jeffreys: On the dynamics of wind. *Quar. Jour. Roy. Met. Soc.*, 48, 29–47' 1922.

[34] Sir Napier Shaw: *Manual of meteorology*: Part IV, The Relation of the wind to the distribution of barometric pressure. Cambridge Press, 1919.

[35] W. J. Humphreys: *Weather proverbs and paradoxes*, p. 88. Baltimore, 1923.

[36] S. Fujiwhara: On the growth and decay of vortical systems and the mechanism of extratropical cyclones. *Bull. Central Met. Obs. of Japan*, 3, 151–208, 1923.

[37] L. F. Richardson. *Weather prediction by numerical process*. Cambridge Press, 1922.

[38] See J. B. Shaw. *Lectures on the philosophy of mathematics*. Chicago, 1918.