ON THE INVESTIGATION OF CYCLES AND THE RELATION OF THE BRÜCKNER AND SOLAR CYCLE

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The laws of sequence of the weather conditions and the correlation of solar and terrestrial phenomena manifest their importance in constantly arising new problems of far-reaching interest. For instance, a sharp controversy has raged for several years between the States bordering on the Great Lakes and the city of Chicago on account of the diversion of lake water into the Chicago drainage canal, supposedly the cause of the abnormally low lake levels prevailing at present. The question arises however whether or not this is merely another of the periodically recurring variations in lake level in accordance with the climatic cycle established by Brückner. Another field for which these laws are of importance is found in the widespread utilization of water power. The variations of river flow and their possible anticipation are of preeminent interest in the operation of hydroelectric plants.

Study of problems such as these have led to the results here presented. At the outset it was found that a certain confusion appears to prevail in the field of cycle investigation. So many periodicities are found as to cast doubt on their reality. Different parts of a record yield different sets of elements. These, if continued into the future, do not furnish the actual continuation of the record. Correlations, if found, are fragmentary, discontinuous, irregular.

Marvin (*1*) in an investigation of rainfall, finds 24 possible elements. Baur (*2*) investigating temperature, finds 20 possible elements. Investigating the sunpot cycle, Kimura, Schuster, and Michelson each find a different set of elements, the last named as many as 33. (*3*) Dinsmore Alter (*4*) investigating long rainfall records, finds apparently little relation between various localities.

Many more investigations could be cited, the results invariably being so incoherent as to cast considerable doubt on their reality. Adverse opinion is therefore voiced by many investigators regarding their value. Abbott states (*1*) "The speaker is by no means an enthusiast in cycles." Marvin confirms (*1*) "Harmonic analysis, of course, is not adequate to prove the reality of cycles." Clements believes (*1*) "There may be the gravest doubt of the reality of periodicities beyond that of a year." Humphreys (*1*) also insists that the mere analysis of a curve of temperature, rainfall, or other data into a series of harmonics is no proof that such harmonics exist in nature.

The probable cause of these unsatisfactory results has often been discussed. Many investigators ascribe them to the methods of analysis employed. Berget (*5*) states: "Les météorologistes, statisticiens surtout, ont cherché uniquement a compiler des moyennes générales dans la superposition desquelles disparaissent les ondes cherchées." Bigelow (*6*) claims: "The former method (harmonic and periodogram analysis) always leads to zero results in dealing with solar and terrestrial phenomena." Brunt concludes (*7*) "it is clear that the periodogram analysis is not in itself sufficient to deal with temperature variations."

Still others are more specific and point out the reasons why the methods employed are unsatisfactory. Thus Michelson (*3*) speaks of the probability that the cycles are variable, and considers the many homogeneous elements usually found, to be illusory. This theory has been supported by Clough in an important study (*8*), establishing the fact that the variability is systematic. His conclusions are further supported by theoretical considerations given by Marvin (*8*).

The methods hitherto employed in cycle investigations are based on the presence of homogeneous elements. They are (*9*):

(1) Periodic curves, homogeneous elements. These are the methods of harmonic analysis; the 6, 12, 24 point method of Runge, Fisher-Hinnen, Thompson, Turner; the graphical methods of Perry, Wedmore, Ashworth; the harmonic analysers of Michelson and Stratton, Henrici and many others.

(2) Nonperiodic curves, homogeneous elements. These are the methods commonly called periodogram analysis, and include the methods of Lagrange, Dale, Oppenheim, Hopfner, Schuster, Chrystal, Vercelli, Wallén.

Of the last-named group, the methods of Vercelli, Chrystal (*10*) (*11*) and Wallén, though based on homogeneous cycles, can also be used by approximation on variable cycles.

In attempting to resolve a curve into variable cycles, the problem leaves the field of rigorous mathematics, and is only solvable in an approximate way. It becomes difficult, if not impossible, to give a proper mathematical interpretation of the very indefinite premises. The limits of variability are not given. Not only are the amplitudes variable, but the periods as well. Only those cycles which clearly are caused by the earth's motions—the diurnal and annual cycles—are of equal period, but have by no means a homogeneous amplitude. Horace Lamb (*12*) states: "The problem is indeed indefinite, for the solution is possible in an infinite number of ways."

Variable cycles furthermore restrict the resolving power of any method of analysis, for only in case the limits of variability of one element do not too closely approach the limits of another can these elements be distinctly separated.

In spite of these difficulties it may nevertheless be possible to resolve different sets of weather data into variable cycles which constantly recur and which will permit cross identification where the original data will not. Marvin states (*13*) "Only those features which consistently survive and emerge from every analysis can be regarded as real periodic features in any body of data."

The probable limits of variability may be determined empirically and used to determine the probable extension of the curve. In this manner variable cycles may also be used in prognostication, as will be shown in the examples given.

It is impossible to derive mathematically an expression which will not only represent the curve of observations, which would be simple enough, but the future extension as well. There is no way open but to arrive by some general considerations at a hypothetical expression which, though lacking absolute proof, nevertheless seems reasonable. Thus it may be stated that all values are real, and for each value of x only one value of y exists. The curve is univalent, and finite. The curve oscillates repeatedly between rather narrow limits, and we may start by writing the expression

$$y = c_0 + f(t)$$

regarding the ordinate values as oscillating around a constant mean.

Taken over a very long period of time this mean undoubtedly changes; however, these changes are of such a small amplitude compared with the short term variations that they may be disregarded as a first approximation. Enough evidence may be found in the longest available records, such as Douglass' and Huntington's Sequoia record and de Geer's measurements of varvae, as well as the shorter weather records to support adoption of a constant plus a variable portion. From the accompanying Table 1 it may be seen that the variations in the average yearly rainfall at Lund (*4*) do not exceed 4.6 per cent while the individual years vary as much as 40 per cent.

The variable portion oscillates repeatedly between certain limits above and below the mean value. The curve can therefore not be represented by any but some Abelian function having one or more real periods.

Taking only a small part of the curve between narrow abscissae limits $\triangle t$ it is undoubtedly permitted to use the simplest forms, namely the trigonometric functions. From the solutions obtained it appears that the elements it contains are either homogeneous or else damped or increasing oscillations. The constant can of course only be obtained by taking a mean of the longest records available, and in the absence of these no reliable determination of the constant is possible.

Taken over a small period of time the curve can therefore be represented by

$$y = c_0 + \Sigma_1^m c_1 \sin \frac{2\pi}{\theta_1}(t+\phi_1) + \Sigma_1^n c_2 e^{\alpha t} \sin \frac{2\pi}{\theta_2}(t+\phi_2)$$

wherein $C_0 C_1 C_2 \theta_1 \theta_2 \phi_1 \phi_2 \alpha \ldots$ may be regarded as constant values.

In this function the various periodic elements can be separated by repeated integration. Taking the function in three parts, the constant plus a homogeneous cycle plus a damped or increasing oscillation, the integration gives for the constant:

1 integration, $c_0' + c_0 t$

2 integrations, $c_0'' + c_0' t + c_0 \frac{t^2}{2}$

3 integrations, $c_0''' + c_0'' t + c_0' \frac{t^2}{2} = C_0 \frac{t^3}{6}$

It may be seen that the result is a potential series. For the first integration the series is a straight line, but multiple integrations will change this into a curve which will distort the result. It will be necessary to minimize this effect. The second term furnishes—

1 integration, $-\frac{c_1}{\frac{2\pi}{\theta_1}} \cos \frac{2\pi}{\theta_1}(t+\phi_1) - \frac{c_1'}{\frac{2\pi}{\theta_1'}} \cos \frac{2\pi}{\theta_1'}(t+\phi_1') - \ldots$

2 integrations, $-\frac{c_1}{\left(\frac{2\pi}{\theta_1}\right)^2} \sin \frac{2\pi}{\theta_1}(t+\phi_1) - \frac{c_1'}{\left(\frac{2\pi}{\theta_1'}\right)^2} \sin \frac{2\pi}{\theta_1'}(t+\phi_1') - \ldots$

3 integrations, $+\frac{c_1}{\left(\frac{2\pi}{\theta_1}\right)^3} \cos \frac{2\pi}{\theta_1}(t+\phi_1) + \frac{c_1'}{\left(\frac{2\pi}{\theta_1'}\right)^3} \cos \frac{2\pi}{\theta_1'}(t+\phi_1') + \ldots$

It may be seen that the integral is composed of elements which change in phase 90° for each integration, while the amplitudes diminish the faster, the smaller the period. Continued integration therefore tends to eliminate all the elements in the order of their length of period, leaving the longest periods remaining. *It is therefore a selective means of separating the elements.*[1]

The third term furnishes, taking one term of the sum only:

1 integration,

$$c_2\left[\frac{\alpha}{\alpha^2+\left(\frac{2\pi}{\theta_2}\right)^2} e^{\alpha t} \sin \frac{2\pi}{\theta_2}(t+\phi_2) - \frac{\frac{2\pi}{\theta_2}}{\alpha^2+\left(\frac{2\pi}{\theta_2}\right)^2} e^{\alpha t} \cos \frac{2\pi}{\theta_2}(t+\phi_2)\right]$$

2 integrations,

$$c_2\left[\frac{\alpha^2-\left(\frac{2\pi}{\theta_2}\right)^2}{\left\{\alpha^2+\left(\frac{2\pi}{\theta_2}\right)^2\right\}^2} e^{\alpha t} \sin \frac{2\pi}{\theta_2}(t+\phi_2) - \frac{2\alpha\left(\frac{2\pi}{\theta_2}\right)}{\left\{\alpha^2+\left(\frac{2\pi}{\theta_2}\right)^2\right\}^2} e^{\alpha t} \cos \frac{2\pi}{\theta_2}(t+\phi_2)\right]$$

3 integrations,

$$c_2\left[\frac{\alpha^3-3\alpha\left(\frac{2\pi}{\theta_2}\right)^2}{\left\{\alpha^2+\left(\frac{2\pi}{\theta_2}\right)^2\right\}^3} e^{\alpha t} \sin \frac{2\pi}{\theta_2}(t+\phi_2) + \frac{\alpha^2\frac{2\pi}{\theta_2}+\left(\frac{2\pi}{\theta_2}\right)^3}{\left\{\alpha^2+\left(\frac{2\pi}{\theta_2}\right)^2\right\}^3} e^{\alpha t} \cos \frac{2\pi}{\theta_2}(t+\phi_2)\right]$$

It may be seen that the integral is again a damped or increasing oscillation, (α negative or positive) and that the elements again vanish in the order of the length of their periods, the longest period remaining.

For a periodic function the constants may be eliminated by shifting the X-axis over a distance

$$c_0 = \frac{\int_0^{2\pi} y\,dt}{2\pi}$$

before each integration, and repeated integration becomes another method of harmonic analysis, and therefore subject to the same errors if applied to nonperiodic curves with variable elements, in the above manner. For a periodic function the integraph is therefore also a harmonic analyzer.

Applied on nonperiodic functions with variable elements, elimination of the constant in the above manner is no longer permissible as soon as the smaller periods have vanished, for the mean is then largely a mean value

[1] G. G. Stokes, in Proceedings of the Royal Society of London, vol. 29, 1879, pp. 122–23, pointed out the possibilities of an integration method in arriving at the solution of trigonometric functions, but his conception appears to have been essentially that of trial periods, afterwards more fully developed by Shuster. Otherwise, apparently, there is no earlier application than the one outlined herein to the problem of hidden periodicities.—C. F. M

of the ordinates of the element with long period, which may only partly be contained in the graph. The error may then be considerable. In the beginning the amplitude of the shorter elements usually predominates, and an error in the mean caused by the long cycle is usually of small influence.

A further approximation in the case of variable cycles would be found in the use of a record of finite length, for which the above function would not be the correct representation, but would only approximately apply.

For application on nonperiodic curves the integration is therefore extended only over a small part of the curve, as described below. It is of interest, however, to first regard the properties of a single integration, the so-called mass curve.

The mass curve has a distinct physical meaning, for its ordinates represent the total quantity of the subject investigated during a given time interval. Comparing the mass curve with the original curve, it may be seen that the irregularities have largely vanished, the mass curve presenting a much smoother appearance than the original curve. This is evident, as the irregularities, whether real or due to fortuitous errors, are in fact but discontinuous elements of perhaps large amplitude but small period, and a single integration makes these vanish.

The integration is therefore an automatic smoothing process.

The mass curve has furthermore the property that the potential series of the constants is a straight line, and therefore does not distort the curve. Distortion, if any, is due to irregularity or asymmetry of the elements of the original curve, and this is empirically found to be unimportant.

STRATO ANALYSIS

It is important, therefore, to reduce the potential series to a constant if one applies repeated integration. This can be done by integrating in parts, reducing the limits of each integration to a minimum. The trigonometric function is then also a closer approximation. The minimum value between the integration limits, if done numerically, is the interval between two ordinates. The integration is therefore confined to the addition of two successive ordinates, having the abscissae t and $t+\Delta t$.

The addition of two successive ordinates now furnishes

$$y=2c_0+\Sigma_1^m c_1 \sin\frac{2\pi}{\theta_1}(t+\phi_1)+\Sigma_1^m c_1 \sin\frac{2\pi}{\theta_1}(t+\Delta t+\phi_1)+$$

$$\Sigma_1^n c_2\, e^{\alpha t}\sin\frac{2\pi}{\theta_2}(t+\phi_2)+\Sigma_1^n c_2\; e^{\alpha(t+\Delta t)}\sin\frac{2\pi}{\theta_2}(t+\Delta t+\phi_2),$$

or

$$(1)\ y=2\ c_0+2\ \Sigma_1^m\ c_1\cos\frac{\pi\Delta t}{\theta_1}\sin\frac{2\pi}{\theta_1}\left(t+\phi_1+\frac{\Delta t}{2}\right)+$$

$$2\ \Sigma_1^n\ c_2\, e^{\alpha t}\cos\frac{\pi\Delta t}{\theta_2}\sin\frac{2\pi}{\theta_2}\left(t+\phi_2+\frac{\Delta t}{2}\right)$$

It may be seen that repeated addition again *separates the elements* as the multiplier $\cos\frac{\pi\Delta t}{\theta}$ is larger for a larger period θ.

Inversely, the subtraction of two successive ordinates t and $t+\Delta t$ gives

$$y=-2\ \Sigma_1^m\ c_1\sin\frac{\pi\Delta t}{\theta_1}\cos\frac{2\pi}{\theta_1}\left(t+\phi_1+\frac{\Delta t}{2}\right)-$$

$$2\ \Sigma_1^n\ c_2\, e^{\alpha t}\sin\frac{\pi\Delta t}{\theta_2}\cos\frac{2\pi}{\theta_2}\left(t+\phi_2+\frac{\Delta t}{2}\right),$$

taking $e^{\alpha\Delta t}\cong 1$.

Repeated subtraction therefore has the opposite effect, as the elements with longer period tend to vanish relative to the elements with shorter period. This is in accordance with the effect of differentiation.

The method, besides being related to Chrystal's and Vercelli's, is also related to the method of Lagrange. In the latter the difference between observations (notation α) are added (notation $E\alpha$) and the following relations are derived

Ordinate, $y=c_0+c\sin(\alpha+r\theta)$

First difference, $\alpha_r=y_{r+1}-y_r=2c\sin\frac{\theta}{2}\cos(\alpha+(2r+1))\frac{\theta}{2}$

1° add. $E\,\alpha_r=\alpha_{r-1}+\alpha_{r+1}=2\ \alpha_r\cos\theta$

2° add. $E^2\alpha_r=E\alpha_{r-1}+E\alpha_{r+1}=4\alpha_r\cos^2\theta$

which necessarily are the same proportions as in formula (1).

In the case of variable cycles it is necessary that the interval Δt be considerably smaller than the period of the element to be isolated. The smallest periods obtained are therefore more or less fictitious, if, as is usually the case, the small periods are variable. After the small periods have vanished it becomes permissible to increase the interval between ordinates as an approximation to abridge the work involved.

As before, the phase is to be restored by a shift equal to $m\frac{\Delta t}{2}$, wherein m is the number of additions used. The restoration to scale requires multiplication with $\left(\frac{1}{2}\right)^m$ (m = number of additions).

In order to find the separate elements, the following method is used. After several additions the resultant curve, restored to scale and phase, is subtracted from the original. The difference is called the *stratum*, the remainder the *residual*.

The stratum is given by the formula

$$\Sigma_1^m c_1\left(1-\cos\frac{\pi\Delta t}{\theta_1}\right)\sin\frac{2\pi}{\theta_1}\left(t+\phi_1\right)+$$

$$\Sigma_1^n c_2 e^{\alpha t}\left(1-\cos\frac{\pi\Delta t}{\theta_2}\right)\sin\frac{2\pi}{\theta_2}\left(t+\phi_2\right).$$

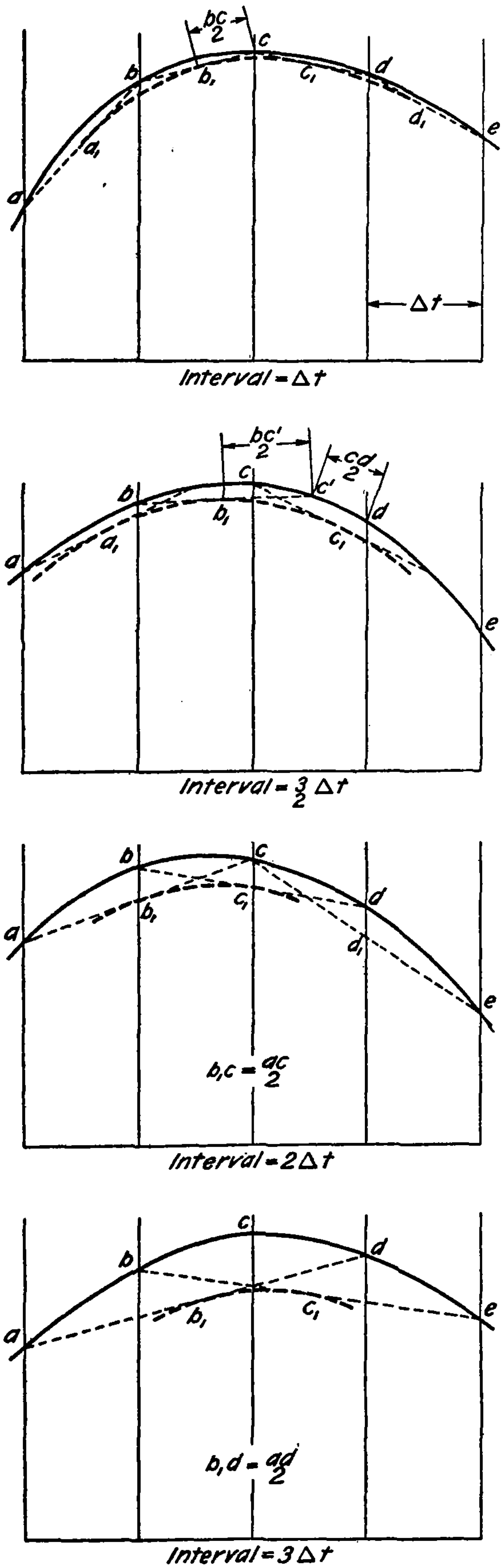
and has important characteristics. It may be seen that the stratum has properties exactly opposite to those of the residual. In the residual the elements with long period predominate, whereas in the stratum the element with short period predominates.

The stratum plainly shows the shorter period, if present, superimposed on a residual, which is similar in outline to, but proportionately smaller than, the second stratum.

A curve, drawn free-hand, following in outline the second stratum, but of smaller amplitude, isolates the small element from the first stratum. The residual is then added to the next stratum and the process repeated. Finally the various elements are corrected against each other, and their sum necessarily must be equal to the original curve.

Drawing the empirical curve can be replaced by a subtraction process applied to the first stratum, which makes the longer elements vanish. In practice nothing is gained however, as the personal equation in drawing the curve is minimized if it is aimed to obtain the element to be isolated as homogeneous as possible.

A preliminary trial, with increasing intervals, will disclose quickly the lengths of periods present and the final



number of additions and the interval finally used as one proceeds derived therefrom. It should be remembered that for too large values of the interval fictitious values of the variable elements will result.

GRAPHICAL METHOD

Residual and stratum can also be obtained graphically in correct phase by a simple construction. Let a, b, c, d, e, be the points of the original curve. Then a_1, b_1, c_1, d_1, are points of the residual for the interval as shown. The difference between the dotted and the full line is the stratum in correct phase. Calling the interval between two ordinates Δt we have for different values of the interval between ordinates to be added I the following constructions:

ON THE RELATION OF THE BRÜCKNER AND THE SOLAR CYCLE

By applying the foregoing methods a relation between the solar cycle and terrestrial phenomena and with the Brückner cycle is easily demonstrated.

Figure 1 is a graph of the mass curve of the Wolf numbers. It may be seen that a single integration has a smoothing effect, and the long periods in the sun-spot curve become plainly visible. The major elements of the sun-spot curve have hereby been isolated to such an extent that these may be isolated by mere inspection.

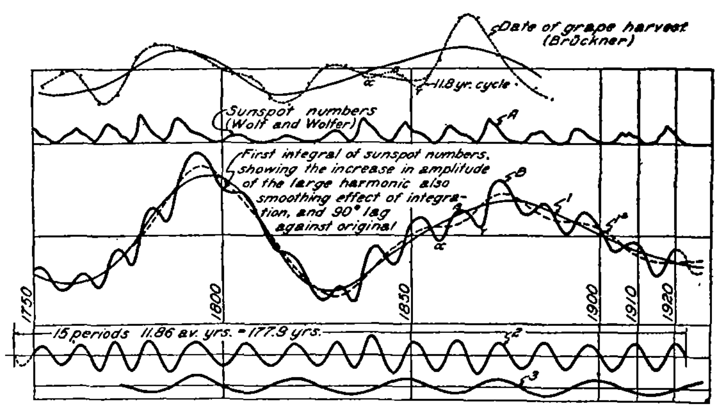


FIG. 1.—Example of integration of Fourier series, sun spot numbers, and their first integral

Drawing the center line through the 11-year cycle, it may be seen that this center line consists of a longer cycle superimposed on the still longer secular cycle.

If now the sun-spot curve be analyzed by means of strato analysis, the same elements necessarily emerge. This is given in Figures 2 and 3. Figure 2 shows the graphical process applied to the sun-spot numbers, using increasingly larger intervals between ordinates. This abridges the work but at the same time introduces a further approximation, since the cycles are variable. The approximation resulting is in this case unimportant. The last residual is transferred to Figure 3 and a curve drawn through the individual points. By continuation of the same process the dotted line in Figure 3 is finally obtained, the 11-year cycle having vanished.

Comparing Figures 1 and 3 and taking into consideration that in Figure 1 the elements are leading by one quarter period, and also considering the difference in amplitude, it may be seen that the results are identical. There appear to be three major variable cycles in the sun spot curve which for identification purposes may be called the Jupiter, Saturn and Secular cycle.

Returning to terrestrial phenomena, a comparison with the levels of the Great Salt Lake and Lake Ontario is shown in Figure 3. The curve for the variation of

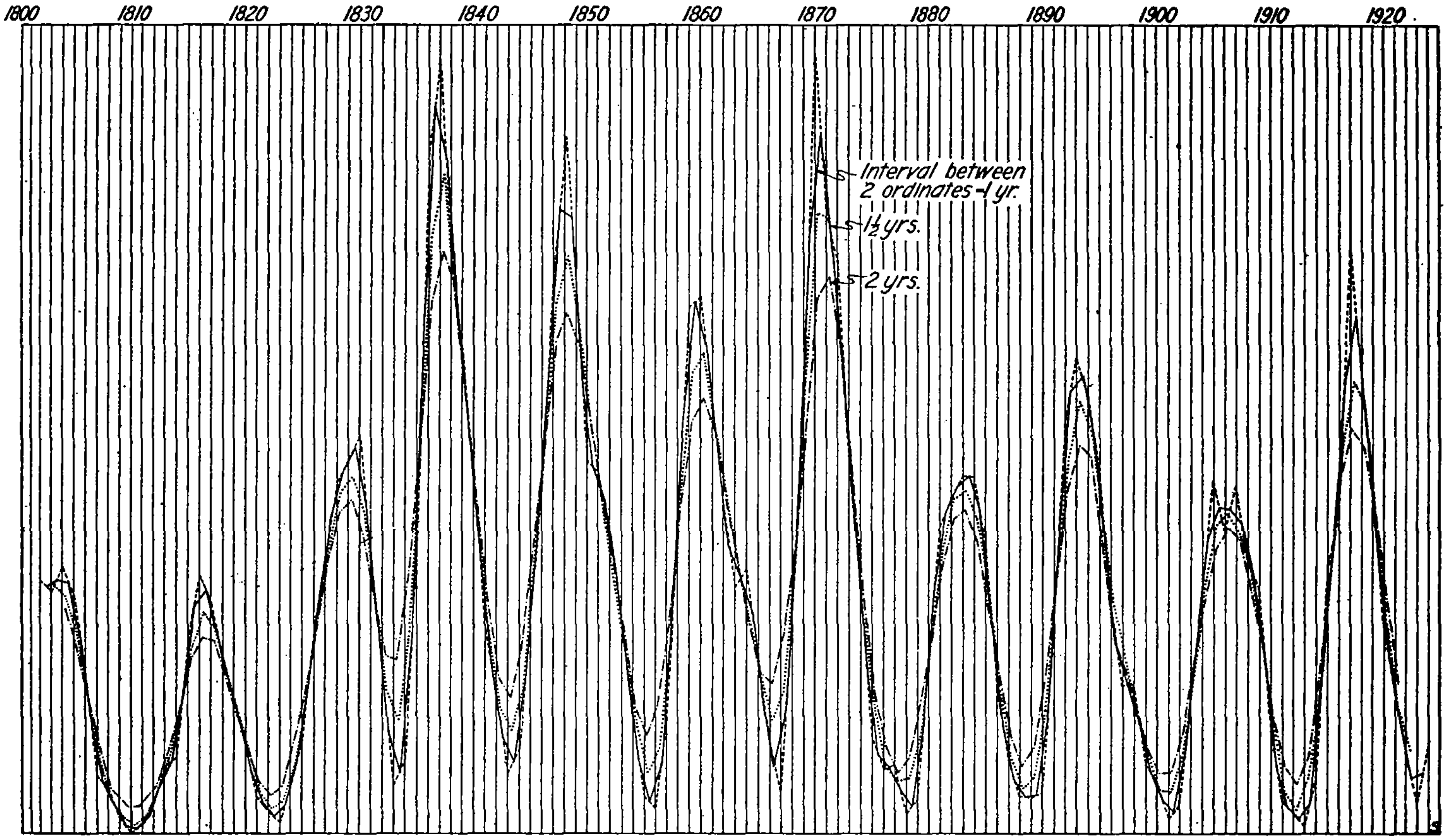


FIG. 2.—Strato analysis of the Wolf numbers

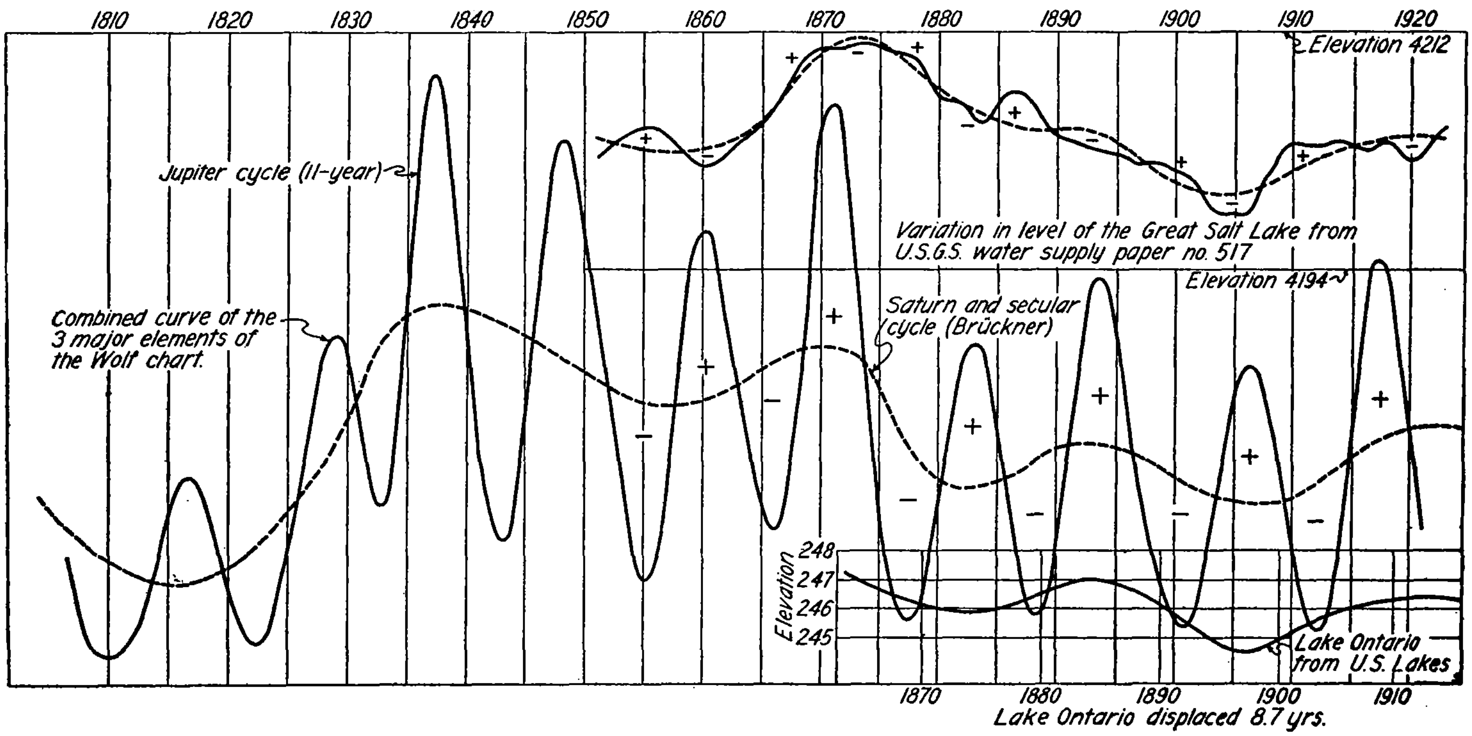


FIG. 3.—Comparison of lake levels and Wolf numbers

Lake Ontario was obtained by repeated addition of the values given by the United States Lake Survey. It may be seen that this curve follows the Saturn and Secular cycle with a change in phase.

The curve of the variations of the Great Salt Lake is taken from Water Supply Paper No. 517 of the United States Geological Survey, by merely averaging the yearly

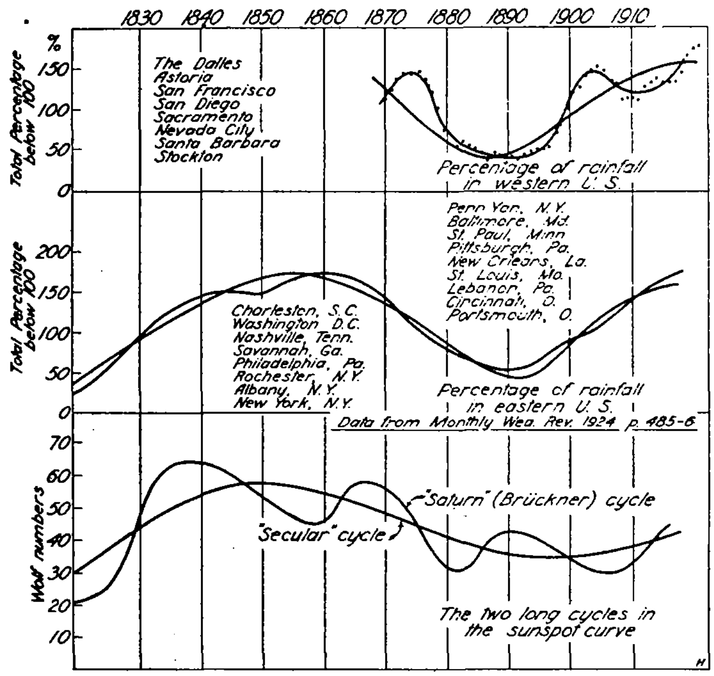


FIG. 4.—Percentage of rainfall in the eastern and western United States and the two long cycles in the sun spot curve

fluctuations. This could be reduced by repeated addition, but since the curve is simple in appearance the results are the same if merely a median curve is drawn.

The lake level fluctuates above and below this median line and by comparison with the solar cycles it may be seen that again identification with the major sun-spot

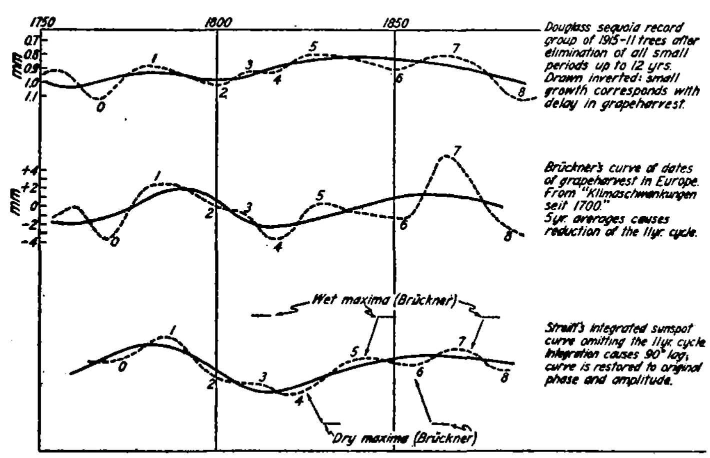


FIG. 5.—Comparison of Douglass' sequoia record, Brückner's European grape harvest record, and Streiff's integrated sun spot curve omitting the 11-year cycle

cycle is possible. A remarkable coincidence is here visible. It appears that the lake level seems to follow also the 11-year cycle, but with a change in phase of about 180°.

It appears therefore that the lake levels change in accordance with the major sun-spot cycles. This being the case, the conclusion may be drawn that rainfall must necessarily show the same correlation. In order to investigate this point, the long rainfall records given by Dinsmore Alter (*4*) for eastern and western United States were used. (Fig. 4.)

The mass curve for both these rainfall records was first computed and this mass curve subjected to repeated addition as described. The elements with short periods hereby vanished; the long cycles remained. It may be noted that the "Saturn" cycle is plainly visible in rainfall, and remarkably enough changes 180° in phase from east to west coast, whereas the "Secular" cycle retains practically the same phase. As before, a few integrations are sufficient to demonstrate the relation of rainfall and sun-spot numbers.

Since rainfall shows a distinct correlation with sun spots, it is to be expected that tree growth will likewise

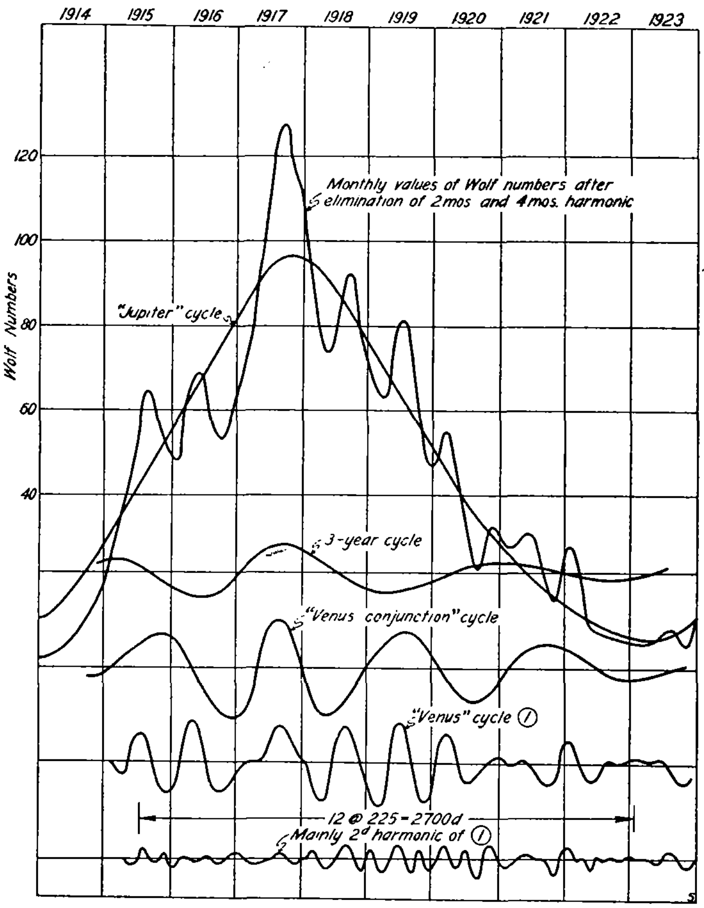


FIG. 6.—Strato analysis of monthly sun spot numbers and comparison with various cycles

show a distinct correlation. In order to test this, Brückner's curve of the dates of the grape harvest in Europe was taken unaltered from the work, "Klimaschwankungen seit 1700." From Douglass's work, "Climatic cycles and tree growth," the Sequoia record was taken. The latter was subjected to repeated addition until the 11-year cycle had vanished.

Figure 5 shows the relation of these two curves with the integrated sun-spot curve. It may be seen that the relation between the Bruckner cycle and the major sun-spot cycles is plainly evident, although the phase changes. The change in phase with geographic location has been emphasized by many investigators, as Baur (*2*) and Huntington (*14*).

As is well known, Professor Brückner investigated the existence of a long-term climatic cycle by tabulating five-year means of a great amount of various weather data. Since the relation between this long climatic cycle and the long solar cycles is unmistakable, the question arises whether a relation of the smaller elements of the sun-spot curve and weather data could be demonstrated. Since the variations in the sun-spot curve are accompanied by a variation of the solar radiation the influence on terrestrial phenomena should not necessarily be confined to the long solar cycles only, but the short cycles as well may be expected to cast their shadow on earth.

In order to demonstrate this, the monthly values of the sun-spot numbers, which were obtained by the writer through the courtesy of Doctor Abbot, were subjected to strato analysis. (Fig. 6.) On this chart, the smallest cycle (2d harmonic of 1) is largely fictitious, as the interval of the ordinates was too large for this variable cycle. It may be seen that the Wolf numbers contain four main variable elements. The nomenclature is merely used for easy identification and has no reference to origin. An interesting fact is here visible. Near the sun-spot maximum, all elements appear to be in phase.

In order to demonstrate the relation of these short solar cycles with rainfall, an indirect route was selected. Instead of rainfall, comparison is made with the periodic variations in river flow.

River flow, as an index of precipitation, has an important advantage over the direct measurement by means of a rain gauge. The drainage area may be many thousands of square miles and fortuitous variations may therefore be considered eliminated. The large drainage area acts as an integrator of rainfall; although it can not be used for absolute measurements of the amount of precipitation it is very well suited to demonstrate the periodic variations in this weather element.

This is immediately visible if it is attempted to find periodic variations in rainfall and river flow. The cycles in river flow are clear, regular, and distinct, while the cycles in rainfall are erratic in character.

Figure 7 shows an analysis of the mass curve of the Penobscot River, Me., obtained by means of strato analysis. The elements shown are mass curves and therefore in order to be in the correct phase should be shifted to the left over one-quarter of their period. Their amplitude should likewise be changed inversely proportional to the length of their period, in accordance with the effect of a differentiation, in order to obtain the amplitudes of the elements of the original curve in the correct proportion. Thus, taking the amplitudes of 4 as a standard, the amplitudes of 5 should be multiplied by 2, of 3 divided by 1.63, of 2 divided by 3, and of 1 divided by about 7. The amplitudes are then in the correct proportion as they occur in the run-off curve.

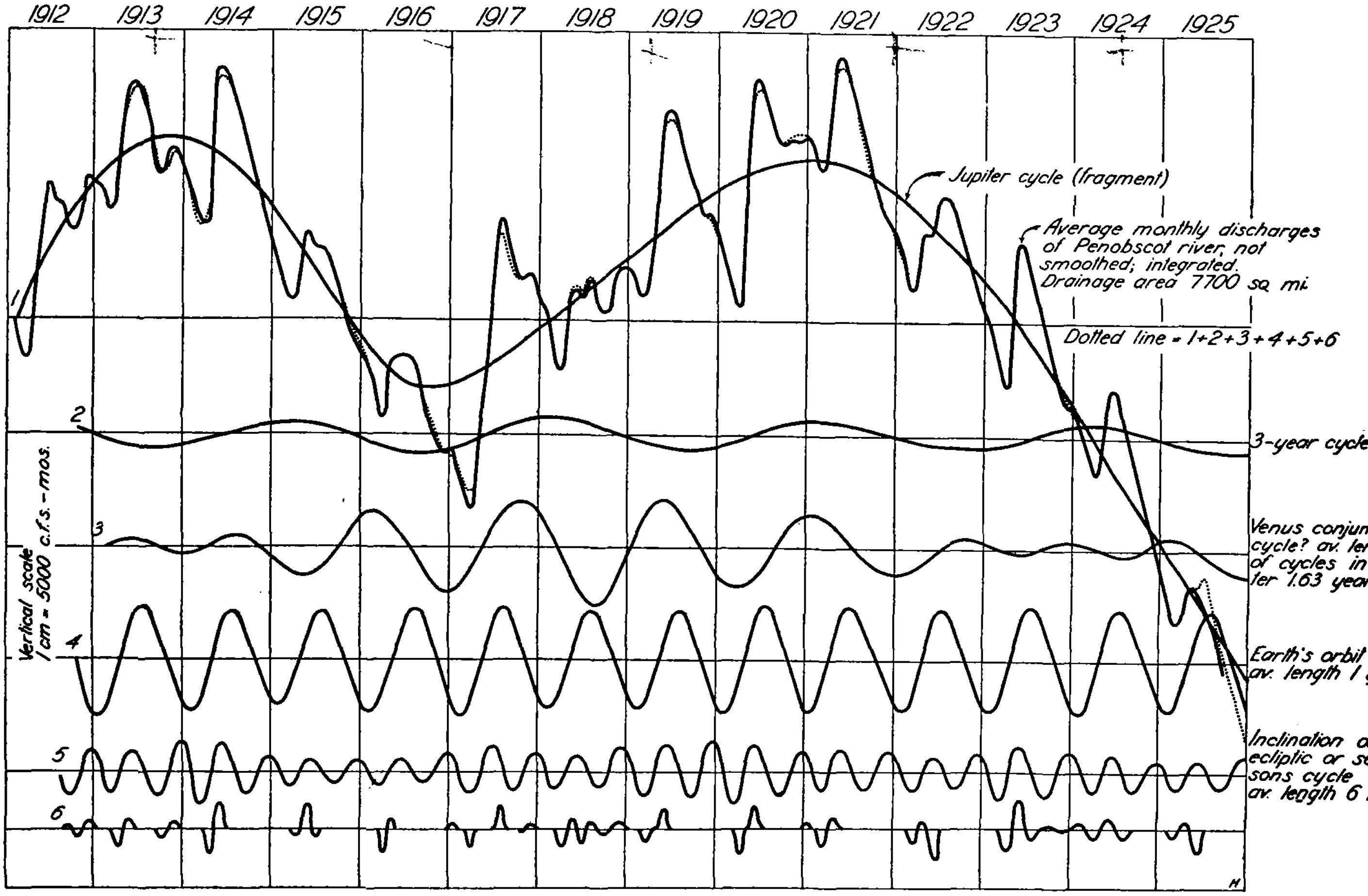


FIG. 7.—Flow analysis of Penobscot River, Me.

Applying strato analysis to river flow, an interesting fact is observed. The sum of the ordinates of 4, 5, and 6 taken over one year is practically zero. These cycles are plainly "terrestrial," due to the earth's motions, and the relative wetness or dryness of the year is independent of these. The yearly cycle, though largest in amplitude of all flow cycles, is practically homogeneous from year to year, and does not influence the relative wetness or dryness of the year.

In order to estimate the relative wetness of the following year, it is therefore only necessary to estimate the continuation of 1, 2, and 3 which in the run-off curve are rather flat curves, and permit an estimate with a fair degree of accuracy. The river, in this manner, becomes a useful meteorological instrument for the estimate of future storage quantities in irrigation projects.

If now Figures 6 and 7 are compared, an interesting correlation is observed. The annual and semiannual cycles (4 and 5, fig. 7) are absent in the Wolf numbers, which is to be expected. Its place is occupied by a smaller cycle (1, fig. 6) which on the other hand is not in evidence in the river chart. The periods larger than one year appear to be identical for the river and the Wolf numbers.

If the largest cycle (1, fig. 7) is shifted to the left one quarter period, the maximum, which is the true maximum of the run-off curve comes in 1918 and the difference in phase for the Jupiter cycle for river and Wolf numbers is therefore about one year.

The smaller cycles (2 and 3, fig. 7) have a smaller phase difference. It may be seen that in the year 1917 the elements were all in phase on sun and river. Elements 2 and 3 (fig. 7) are opposite in phase both on sun and in river in 1919.

It may further be seen that numerous sun spots concur for this location with increased flow. Thus far it has been found by the writer that the same elements can be identified in rivers 900 miles apart, the phase changing gradually with geographical location.

In the foregoing example the precipitation is therefore related with the smaller changes in the Wolf numbers as well as with the larger variations, and the Wolf chart may therefore be expected to be in fact the extension of the Brückner cycle.

Once the investigations are freed from the requirement of homogeneity, progress in the determination of correlations between solar and terrestrial phenomena is rapidly made. The variable cycles obtained by the above method constantly recur in different sets of data and may therefore well be regarded as real.

TABLE 1.—*Variations of mean value of annual rainfall at Lund (Sweden)*

Average of—	Percentage	Average of—	Percentage
10 years	101.8	90 years	102.2
20 years	101.2	100 years	101.5
30 years	102.2	110 years	100.7
40 years	103.4	120 years	100.8
50 years	104.2	130 years	100.7
60 years	104.6	140 years	99.9
70 years	103.6	150 years	100.4
80 years	103.4	157 years	100

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THE NASSAU HURRICANE, JULY 25–26, 1926

Not since many a day has there been written a more vivid account by an eye witness of a West India hurricane than that printed below. It is taken from a copy of the *Nassau Guardian* of July 28, received by the Weather Bureau from the State Department.

The hurricane which the Bahamas has just experienced is more fearful and devastating than any most people can remember. Nassau is ravaged; from every district come stories of ruin and havoc, and the tale is not yet fully told, for there are the reports from the out islands still to come.

The radio messages on Friday and Saturday morning found most people optimistic. * * * Saturday afternoon brought more serious news. The indication was that the storm was headed straight for these islands; still people talked of sudden curves to the gulf. There seemed nothing abnormal in the weather, only fresh breezes, and the glass was steady. On Sunday morning came worse news, a direct warning of the hurricane's approach. The wind rose higher. People looked to see that their shutters were in good order, and nailed and battened down windows. Still the glass was steady; ships of all sizes began to make for a safer anchorage in the waterfront near Potter's Cay and at the Eastern Creek. All the time the wind rose; the sky was heavy with clouds, and the sea began to lash angrily. As evening came on all the storm signs grew more intense. The barometer made a sudden drop, and weather-wise people shook their heads. As the sparse congregations came out of the churches after evening service it was plain that the storm was near. The wind rushed along Bay Street, swirling leaves and scraps of paper and stray sponges into doorways. The shopkeepers had almost without exception taken the precaution to board up their windows and Bay Street looked ready for a barricade, as indeed it needed to be. The sea came dashing up Rawson Square, throwing spray far over, and the harbor at this part was strangely dark any deserted. Here and there electric lights began to fuse, and street after street was plunged into darkness. Hardly anyone was to be seen, save unfamiliar policemen in long dark coats and storm helmets. People were all at home trying to make their houses additionally secure.

The wind was now blowing a heavy gale, dashing through the trees, and shaking the houses, growing steadily more terrible. Everything that could be shaken loose it began tearing down—shutters, signboards, gates. There was little sleep that night for anyone on the island, just listening to this merciless crashing and