

phases of such periods. It is believed, therefore, that beyond the possible demonstration of their existence, little can be done to secure accurate ephemerides until at least another half century of data are secured.

The writer's studies are primarily concerned with probabilities of observed departures from the error law, not with physical explanations of results found. However, he wishes to suggest very tentatively an explanation. Various writers have, during the past 30 years, urged the planetary tides as an explanation of sun spots. Most noteworthy of these is an early paper by E. W. Brown (2), using the tides of Jupiter and Saturn. With these he gave an excellent representation of the past epochs of maxima and minima. An ephemeris computed for the 30 years since his paper was published is almost perfect in locating epochs.

The tides have always seemed an impossible explanation on account of their feebleness in comparison with the sun's gravitational field. However, the recent study of radiation pressure and of the solar spectrum have proved an almost perfect balance of forces in the solar atmosphere. This being the case small tidal effects may possibly produce large results.

To test this possibility, an examination was made to see whether there is any unique relationship of the planetary periods to the assumed primary impulse period of 252 years. Exact multiples and half multiples of their periods must be considered, since, except for eccentricity effects, the latter give the same tidal effects as the former. So far as probabilities are concerned the deviations of the nearest multiples from a common multiple may be as great as 25 per cent of a planet's period. If there is much less average deviation than half this amount, there is a straw of evidence in favor of the hypothesis.

THE PERIODS OF SOLAR AND TERRESTRIAL PHENOMENA¹

By Prof. H. FRITZ

[Translated by W. W. Reed]

In the past decades there have appeared numerous papers on the periodic phenomena whose changes show more or less marked agreement with the periodic change in solar activity as it is most readily traceable in the changing frequency of sun spots. A similar change together with apparent relation, unexplainable or not directly explainable, between processes on the earth and processes on the sun can not be astonishing since the manifestation of energy, all animate or inanimate nature on the earth, is subject to energy radiated from the sun to the planet. The earth provides the matter, the sun supplies the energy. In contrast to the supply of energy from the sun, that from the interior of the earth, that radiated to the earth from the stars, and that reflected to the earth from the moon fade into nothingness. The moon acts most effectively through the attraction exerted on the earth and its constituent parts.

Inevitably every variation in solar activity must be reflected in terrestrial processes, although because of the

¹ Astronomers and meteorologists appear to be but little aware of the general intrinsic value of the work done by Prof. H. Fritz which culminated in *Die Perioden solarer und terrestrischer Erscheinungen*, published in *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zurich*, Heft 1, 1893.

As this publication is not accessible to many American students its translation and republication in the MONTHLY WEATHER REVIEW seems to be particularly appropriate at the present time when various authorities are seeking to establish indirect correlations between solar activity and features of weather sequence on the earth. It is a pleasure, therefore, to commend to the careful attention of the readers of the REVIEW Mr. Reed's translation of Professor Fritz's paper, more particularly with reference to the data and final tables of the epochs of the maxima of the 11-year cycle, which, with the modern tables of Wolf, cover a total range of 17 and 34 years. The author's discussion of long periods and the possible causes of sun spots must stand on its own merits.—C. F. Marvin.

Planet	Period	Multiple factor	Product	Per cent deviation in terms of planet's period
Jupiter.....	11.862	21	249.10	4.8
Saturn.....	29.458	8½	250.39	2.4
Uranus.....	84.015	3	252.04	2.8
Neptune.....	164.788	1½	247.16	1.5
Mean.....			249.67	

In every case the percentage deviation is found to be very small. This 250-year multiple is the only one to be found for these planets. The uncertainty of our 252-year period is greater than the difference from this mean. Though one would not wish to claim anything for the coincidence, it certainly is one to be borne in mind.

The writer wishes to acknowledge the aid of a grant from the research committee of the University of Kansas, under which he engaged Mr. James Edson to do the majority of the computing.

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general constitution of the earth, the variability frequently remains more or less out of sight, becomes not directly observable, or, it may be, escapes all observation due to compensation by other forces or effects. If, for example, with increasing solar radiation more water evaporates from the sea and, with the condensation of water vapor in the higher strata of the atmosphere, the liberated latent heat rises immediately to higher regions and into space, and at the same time insolation undergoes greater depletion on account of the greater amount of water vapor in the atmosphere, then even rather large differences in radiation of heat by the sun will be without influence on our measuring instruments and will no longer be shown by them. This single example will serve as well as many.

If there exist in the different phenomena of the earth periodic changes dependent on solar activity, then they can not be limited to a few decades; they must be present, on the contrary, in the oldest observations available at the present. In the contrary case one would be permitted to apply not entirely without reason the conventional word "accident."

Unfortunately, research extending far into the past is possible only in a restricted way; most of the useful data relative to the different kinds of phenomena are very recent.

Because of its definitely decided periodicity, its uniqueness, and its awesomeness, one terrestrial phenomenon can

be followed in observations going far backward through the years. This phenomenon is the aurora. On account of the decidedly periodic change in frequency, extent, and grandeur shown by the aurora and the fact that everyone's attention was drawn to the "northern lights," records were not to be omitted in the earliest works on history and especially could they not be overlooked in the chronicles. (For "southern lights" we have no record of phenomena earlier than those observed in Chile in the spring of the year 1640.) Even if not a few of the old records leave the reader in doubt whether the reference is to aurora, comet, or other fire phenomenon, there still remains such a large number of certain records of auroras that the epochs of the periods are very definitely manifest, especially in the regions in which the phenomenon sometimes fails to appear for a decade and only the most significant instances become noticeable.

Since the vintage is decidedly periodic in amount and since from olden times special mention has been made of it (and of hail which is damaging to this and all other produce of the fields), the related information, although it is scant, may be used with the auroral phenomena of earlier times to establish agreement in their periodicity. For good reasons this can not be done at all, or at best only very imperfectly, with other phenomena for times so far removed in the past.

Table 1 contains the old sun spot observations from the year 188 up to the exact detection of the spots by Fabricius in the year 1610. The data are taken from the Chinese observations, supplemented in part by records from Europe. The sources of data are given in

detail in Wolf's *Mitteilungen über die Sonnenflecken* and his *Astronomische Mitteilungen* including 80 numbers to date (1893).

The second and third columns of Table 1 show the frequency of the months and days with visible spots; the fourth column gives the dates of the probable maxima of the short period.

Table 2 gives the year and the annual number of the old auroral observations between the years 194 and 1635 according to the phenomena recorded in the *Polarlichtkatrol* and its supplements.² Since these relate altogether to middle and southern Europe, the observations of Tycho de Brahe on the island of Hven (1582-1591) are added in brackets. The last column contains the dates of the probable maxima of the short period.

Tables 3 and 4 give the favorable vintage years mentioned in the old writings relating to Germany, Austria, and Switzerland, and the years that have become noted on account of heavy fall of hail. In both cases the years that were most conspicuous are printed in bold-faced type and the dates of the rather probable maxima of the short period are added in the last column.

In Table 5 there are placed side by side the maxima of the short period for the four different phenomena as given in the preceding tables, also the mean date of the maximum as determined from the preceding columns, the interval in periods of about 11 years, and the epoch of the period of 55.3 years, calculated from the table. Table 5a³ is a continuation of Table 5.

² See *Verzeichnis beobachteter Polarlichter*. H. Fritz. Wien. 1873.
³ Given separately on account of different form.—Translator.

TABLE 1.—Sun spot phenomena¹

Year	Number of months	Number of days	Year of probable maximum	Year	Number of months	Number of days	Year of probable maximum	Year	Number of months	Number of days	Year of probable maximum	Year	Number of months	Number of days	Year of probable maximum	Year	Number of months	Number of days	Year of probable maximum
188	1	1	188	389	1	1		842	1	1		1136	2	2		1376	1	1	
299	1	1		395	1	1		864	1		864	1337	2	2		1381	1	5	
300	1	1		396	1	1	398	865	1		874	1138	2	2	1138	1382	1	1	1382
301	2	3	301	400	1	2		874	1		874	1139	2			1383	1	1	
302	1	1		409	1	1	409				974	1145	1	1	1145				
304	1	1						1005	2	2	974	1160	1		1160	1511			1511
307	1	1		499	1	1		1005	1	1	1005	1185	3	16	1185	1529			1529
311	1	1	311	501	1		501				1077	1186	2	2	1193	1547	1		1649
321	2	2	321	502	2	2					1078	1200	3	12	1193	1588	7	1	
322	2	2		509	1			1079	2	12	1078	1201	2	32		1590	8	1	1591
342	2	2	342	510	1		511	1089	1	1	1089	1202	1	1	1202	1593	8	1	
354	1	2	354	513	2			1096	1	1	1096	1204	1	1		1596	10	1	
355	1	2		535	14		535	1096	1	1		1205	2	14		1607	11	1	1603
359	2	2		577	1	1	577	1103	1	1		1238	1	1	1238	1609	12	1	
360	1		360	580	1			1104	1	1	1104	1276	1	1	1276	1616	12	1	1617
361	1			626	8		626	1105	1							1617	1		
369	1	1		778	1		778	1112	1	2	1112	1370	3	19		1618	12	1	3
370	1	2		807	7		807	1118	2	2		1371	3	46		1624	12	2	
372	1	1	372	826	1	1	826	1120	3	3	1120	1372	4	4	1372	1626			
373	3	3		832	1	2		1122	1	1		1373	1	1		1638	12	1	
374	2	3		837	1	2		1129	1	3		1374	1	5					
375	1	1		840	4	90	840	1129	3	3	840	1375	2	2					
388	1	1	388	841	1	1		1131	1	3	1130								

¹ According to Wolf the first exactly determined spot maximum was that of 1626. The most conspicuous maxima were those of 1727.5, 1778.4, 1788.1, 1837.2, and 1870.1.
² 535-536, diminished brightness of the sun.
³ Sun partially darkened.
⁴ After Lycosthenes.
⁵ After Eginhardt.
⁶ (Humboldt, Kosmos.)

⁷ After Secchi.
⁸ Reported by the ship Richard of Arundell.
⁹ After Henneberg.
¹⁰ After Fausten.
¹¹ Observed by Keppler.
¹² After Henry Hudson.
¹³ Observed in China.

TABLE 2.—Aurora borealis ¹

Year	Times observed	Year of probable maximum	Year	Times observed	Year of probable maximum	Year	Times observed	Year of probable maximum	Year	Times observed	Year of probable maximum	Year	Times observed	Year of probable maximum	Year	Times observed	Year of probable maximum
194	(?)	194	673			940		940	1153		1151	1403		1401	1574	3	
394			676		676	944		944	1157		(?)	1432		1432	1575	2	
397		397	677	10		945		945	1166			1437		1435	1576	3	
400			710		710	957		957	1173			1460		1460	1580	15	1580
434		434	727		727	970		970	1174			1461		1462	1581	13	
450			740			971		971	1175			1465			1582	17	6(2)
453		453	741			978		978	1177	2	1177	1517			1583	9	18
454			742		742	979		979	1179			1518		1518	1584	4	14
479			743			992		992	1187		1187	1521			1585	3	4
480			745			993		993	1189			1524			1586	3	5
488			765		765	999		999	1192			1526			1587	4	3
502	(?)	502	773			1000		1000	1193		3	1193			1588	5	5
504			776		776	1002		1002	1194		3	1194		1529	1589	5	5
512		512	778			1003		1003	1195			1531		1529	1590	4	15
538		540	786		786	1014		1014	1200			1532		1532	1591	4	4
541	(?)		800			1069		1069	1203		3	1203			1592	3	
551			803			1084		1084	1204		(?)	1536			1593	8	
555		555	806		806	1093		1093	1226			1538		1538	1599	3	
556			807	(?)		1094		1094	1243			1539			1603	4	
560			808	(?)		1095		1095	1245			1540			1604	6	1604
563			827		827	1096		1096	1251			1541			1605	4	
564			828			1097		1097	1262			1542			1606	3	
566		70	839	(10)		1098		1098	1263			1544		(12)	1607	2	
567			840	(16)	840	1099		1099	1269			1545		2	1608	2	
570			842	3		1101		1101	1271			1548		2	1609	3	
577		577	848	2	848	1102		1102	1280			1549		3	1610	1	
580			855			1104		1104	1304			1550			1612	1	
582	3		859	11	859	1105		1105	1307			1551		4	1613	1	
583			861			1106		1106	1309			1553			1615	2	
584			870		870	1107		1107	1323			1554			1621	19	4
585	3	585	871			1114		1114	1325			1555		3	1622	3	
586			879			1116		1116	1332			1556		3	1623	10	
587			887			1117		1117	1336			1557		3	1624	3	
590			890		890	1118		1118	1348		4	1560		12	1625	10	
595		595	905	(?)	906	1119		1119	1352		2	1350		2	1626	6	
599			911			1120		1120	1353			1562		2	1627	4	
600			912			1121		1121	1354			1563		3	1628	5	
601			917			1122		1122	1361		12	1361		8	1629	18	1629
603		603	918		918	1130		1130	1375			1567		2	1630	10	
616	(?)	616	919			1131		1131	1379			1568		4	1631	1	
624		624	926			1132		1132	1381			1569		4	1632	2	
654			927		928	1138		1138	1388		3	1388		9	1633	3	
660		657	930			1139		1139	1389			1572		10	1634	1	
670			937			1150		1150	1399			1573		8	1635	2	

¹ Phenomena characterized by decidedness, frequency, or prolonged extent toward the south occurred in or near the following years: 194, 535, 807, 993, 1097, 1117, 1203, 1308, 1361, 1529, 1572, 1590, 1716, 1789, and 1848.

² Often and great.
³ Visible as far as Syria.
⁴ Often.
⁵ 70 days.
⁶ Great, middle Europe.
⁷ In low latitudes of China.
⁸ 10 days.
⁹ Intense.

¹⁰ Several times, intense.

¹¹ Very intense.
¹² Great.
¹³ Very great.
¹⁴ Visible as far as Palestine.
¹⁵ Great, visible as far as Portugal.
¹⁶ Several times.

¹⁷ The numbers in parentheses refer to observations by Tycho Brahe on the island of Hven.

¹⁸ Very great, visible in Syria.

The aurora australis showed well-defined phenomena in Chile in 1640, in Siam in 1730, and in South America from 1737 to 1745; it was observed in the Pacific Ocean in 1773 and 1774 and at Rio de Janeiro in 1783. For the greater part these appearances correspond closely with the maxima of the aurora borealis. The greater frequency of the phenomena in southern latitudes in 1831, 1840, 1848, 1860, and 1870 is closely related to the maxima for the Northern Hemisphere.

TABLE 3.—Years characterized by abundant and fine wine harvests ¹

Year	Year of probable maximum
976, 77	976
1070	
1122, 37, 53, 85	
1201, 28, 36, 40, 56, 59, 63, 70, 76, 77, 78, 84	1277
90, 91, 93, 94	1292
1303, 13, 23, 33, 36, 37, 39, 55, 63, 72, 73, 84	1337, 1372
86, 87, 94	1386
1420, 27, 31, 32, 42, 43, 47, 57, 63	1431, 1442
72, 73, 79, 82, 83, 84, 99	1472, 1483
1504, 18, 35, 39, 40, 52, 59, 84, 93, 99	1518, 1540

¹ In this and some of the subsequent tables the author prints in bold-face type certain years, evidently with the intention of marking them as outstanding years in the occurrence of the phenomena described.—*Trans.*

TABLE 4.—Years noted for damage by hail ¹

Year	Year of probable maximum
325, 66, 77	325, 377
407	407
579, 86	579
823, 24, 32, 55, 72, 82, 89	823
906	906
1011, 57	1011, 1057
1104, 20, 62, 67, 79, 83, 84, 86	1104, 1120, 1181
90, 94	1190
1202, 22, 23, 24, 29, 37, 49, 52, 54, 55, 56	1223, 1237, 1255
57, 62, 67, 75, 79, 80, 81, 88, 89	
90, 91	1260
1312, 43, 45, 48, 55, 60	1345, 1360
1415, 37, 43, 49, 51, 60, 74, 78, 90, 91, 92	1491
1501, 2, 7, 8, 9, 13, 15, 16, 17, 22, 24	1516
25, 28, 33, 37, 38, 49, 51, 53, 55, 56	1525, 1549
57, 58, 59, 62, 63, 64, 65, 67, 68, 71	1563
72, 73, 74, 75, 76, 77, 78, 80, 82, 83	
84, 86, 88, 89, 90, 91, 93, 97	1591
1601, 8, 16, 17, 20, 21, 22, 23, 24, 26, 27	
28, 30, 33, 35, 36, 37, 40, 42, 43, 45	1637
46, 48, 49, 50, 51, 52, 53, 56, 61, 62	1649
64, 66, 68, 69, 73, 74, 75, 76, 79, 80	1676
83, 85, 86, 87, 88, 89, 93, 95, 97	1688

¹ Widespread damage in the years 1186, 1360, 1593, and 1688.

TABLE 5.—Short and long periods of sun spots, auroras, wine harvests, and hailstorms; their mean maximum epochs, intervals between epochs, and the epochs of the 55.3-year period

Sun spots	Auro- ras	Wine har- vests	Hail- storms	Mean maxi- mum epoch (1-4)	Years be- tween epochs	Epoch of 55.3- year period	Sun spots	Auro- ras	Wine har- vests	Hail- storms	Mean maxi- mum epoch (1-4)	Years be- tween epochs	Epoch of 55.3- year period	Sun spots	Auro- ras	Wine har- vests	Hail- storms	Mean maxi- mum epoch (1-4)	Years be- tween epochs	Epoch of 55.3- year period	
(1)	(2)	(3)	(4)	(1-4)			(1)	(2)	(3)	(4)	(1-4)			(1)	(2)	(3)	(4)	(1-4)			
188				190		190	840	840			840	13			1226			1223	1225	2x11	
301	194			301		301	848	848			848	12		1238	1226			1237	1238	13	
311				311	10		864	859			859	11	853		1247				1247	9	
321				323	12		872	870			872	13						1255			
342			325	342	2x10		890	890		906	906	16	908		1262				1260	13	
354				354	12	356	918	918			918	12		1276	1270	1277			1270	10	
360				360	6		928	928			928	10							1278	8	
372				374	14		940	940			940	12				1292	1290		1291	13	1295
398	397			397	2x11		957	957			957	2x9	963		1308			1308	1308	2x9	
409			407	408	11	411	974	979		976	979	7			1324			1324	1324	16	
	434			434	2x13			993			993	14			1334	1337		1345	1348	14	1350
	453			453	2x10	466		1002			1003	10		1372	1372	1372		1360	1360	12	
501				501	4x12		1005			1011	1003	10			1378	1372		1372	1372	12	
511	512			512	11	521		1014			1013	10		1382	1386	1386		1380	1380	8	
535				538	2x13			1057			1057	4x11	1018		1388			1388	1388	8	
	540			555	2x9			1069			1069	12			1401			1401	1401	13	
	555			566	11		1078	1084			1081	12	1074		1435	1431		1435	1435	3x11	1405
	577		579	577	11	577	1089	1084			1081	12			1462	1442			1462	2x13	1460
	585			585	8		1096	1097			1096	15				1472			1472	10	
	595			595	10		1104	1104		1104	1104	8				1483			1483	11	
	603			603	8		1112				1112	13		1511			1516	1511	2x14	1516	
	616			616	13			1117			1117	13			1518	1518		1518	1518	7	
626	624			625	9	632	1120	1130		1120	1130	13	1129	1529	1529			1525	1529	11	
	657			657	3x11		1130	1131			1130	13			1538			1540	1538	9	
	676			676	2x10	687	1138	1138			1138	8		1549	1548			1549	1549	11	
	710			710	3x11		1145				1148	10			1560			1563	1560	11	
	727			727	2x9			1151			1151	13			1572			1572	1572	12	1571
	742			742	15	742	1160			1162	1161	13			1580			1580	1580	8	
	765			765	2x11			1177			1177	16			1591	1590		1591	1591	11	
778	776			776	11					1181	1185	8	1184	1617	1604	1605	1591	1603	1603	12	
	787			787	11		1185	1187			1185	8			1616	1616	1617	1618	1618	15	
807	806			806	2x10	797	1193	1193		1190	1193	8			1620			1620	1620	9	1626
828	827		823	827	2x10		1202	1203			1203	10		1626	1629	1624	1630	1627	9		

¹ 1626.0 is the second spot maximum determined by Wolf. After that date the data are rather sufficient for exact determination of the maxima and become more and more sufficient up to the present.

TABLE 5a.—Eleven-year periods of sun spots, auroras, etc., determined more accurately ¹

Sun spots	Auroras	Wine harvests	Hailstorms	Epoch of 55.3-year period	Sun spots	Auroras	Wine harvests	Hailstorms	Epoch of 55.3-year period
1626.0	1629	1624	1630	1626	1761.3	1760.9	1762	1762	-----
1639.5	1640	1637	1640	-----	1769.7	1772.8	1775	1770	-----
1649.0	1647	1648	1649	-----	1778.6	1778.0	1782	1780	-----
1660.0	1661	1657	-----	-----	1788.3	1788.2	1790	1788	1792
1675.0	1677	1678	1676	-----	1804.2	1801.5	1804	1804	-----
1685.0	1683	1686	-----	1681	1816.4	1818.5	1819	1819	-----
1695.0	1690	1686	-----	-----	1829.9	1829.9	1829	1829	-----
1705.5	1707	1704	-----	-----	1837.2	1840.2	1837	1839	-----
1718.2	1719	1718	1720	-----	1848.1	1850.1	1848	1848	1848
1727.3	1730.5	1727	1731	-----	1860.1	1860.6	1860	1859	-----
1738.7	1739.8	1737	1740	1737	1870.1	1870.9	1870	1869	-----
1750.3	1748.7	1748	-----	-----	1883.9	1883	1883	1883	-----

¹ This table without number appears as part of Table 5 in the original paper.
² According to Kircher's testimony (Frick: *Philosophische und theologische Bedenken von den Cometen*, "Ulm 1681, 4") (see Wolf: *Sonnenfleckenliteratur*, Nr. 3) the number of spots visible in 1639 was equaled only three or four times in a century.

TABLE 6.—Chief maximum epochs, number and length of eleven-yr. and 55.3-yr. periods ¹

Periods between chief maximum epochs ²	Interval in years	Number and length of 11-year periods	Number and length of 55.3 year periods
190 to 301.....	111	10x11. 10	2x55. 5
301 to 585.....	284	26x10. 92	5x56. 8
585 to 806.....	221	20x11. 05	4x55. 3
806 to 993.....	187	17x11. 00	3x62. 3
993 to 1096.....	103	9x11. 44	2x50. 2
1096 to 1360.....	264	24x11. 00	5x52. 8
1360 to 1529.....	169	15x11. 33	3x56. 3
1529 to 1627.....	98	9x10. 88	2x49. 0
1627 to 1848.....	221	20x11. 05	4x55. 3
301 to 1848.....	1,547	140x11. 05	28x55. 25
190 to 1848.....	1,658	150x11. 05	30x55. 26

¹ Heading of table supplied by translator.
² See dates printed in bold-faced type under "Mean maximum epoch" in Table 5.

Tables 5 and 5a show not only the well-known agreement between the times when auroras were well marked and visible far toward the Equator and the times when sun spots were visible to the unaided eye but also the times of several successive good vintages and years with very destructive hail nearly coincident with the maxima of solar activity and auroral phenomena. The coincident periods for all four of the phenomena could easily be increased without doing violence to the records, as a comparison of data in the first four tables will show.

The turning points of the maxima of the several periods, derived with a sure probability from the four series, are given in Table 6. The years printed in bold-faced type indicate maxima especially conspicuous because of outstanding phenomena. In all cases of the selection of the chief maximum and of the fixing of its date the chief weight had to be given to the aurora since its phenomena can be most certainly judged from what has been transmitted to the present time.

A grouping by years of the most conspicuous auroral maxima regardless of other phenomena resulted, by combining period lengths of from 54.8 to 55.8 years, in a mean of 55.6 years, or almost exactly the total length of five periods having a uniform length of 11½ years. (*Polarlicht*, 1881. Leipzig.)

From the old observations of sun spots Wolf selected (*Astronomische Mitteilungen*, Nr. LXXIV) as the most conspicuous maxima those of the years 372, 840, 1078, 1133, and 1372. These are separated by 18 periods (9+4+1+4) having an average length of 55.5 years. The 90 short periods give a mean length of 11.11 years.⁴

⁴ Wolf counted 42+21+5+22 short periods although the interval between 840 and 1078 on the one hand and 1133 and 1372 on the other differ by only one year (238 and 239 years, respectively). By the assumption of 43 instead of 42 periods in the first interval and the assumption of only 21 periods in the last interval the result would remain the same.

If, in view of the incompleteness of the data, one is not denied a certain license in the division of the long intervals into shorter and longer periods, then there appears in the different schemes arranged at different times—in which arbitrary treatment was avoided as much as possible—the striking agreement that the longer periods were somewhat longer than 55 years and thus include five times 11.11 years, or five of Wolf's periods. It is by no means necessary that the long intervals be multiples of the shorter periods; indeed, it appears not even advantageous on account of the greater complications arising therefrom, and it stands in opposition to the change in solar activity as a whole, which change follows a rule that is not, at any rate, simple.

As the tables, especially Table 5, show, there occur so many periods of 11 years, nearly 11 years, or intervals of years nearly divisible by 11 years between the dates at which the phenomena were observed in the earliest times that it must appear unquestionable that the 11-year period does not belong to modern times.

Previous to the year 190 the data became far more scanty. The material is limited almost entirely to a number of northern lights—of which those of 465 B. C., when the sky was lighted up for 75 days, and those of 443 B. C., when there were similar phenomena for 60 days, remind us very much of the twilight phenomena following the eruption of Krakatoa in 1883—the year of the dark sun in 45 B. C., and the Chinese observations of sun spots in the years 28 and 20 B. C. and 188 A. D. Pliny mentions 121 B. C. as the best vintage year at that period.

The sun spot observations just mentioned and a part of the phenomena to be considered as northern lights, especially those observed in southern China in 208 B. C. and at Carthage in 202 B. C., arrange themselves well in the period system previously set forth.

The fourth, sixth, eighth, twelfth, fourteenth, and sixteenth centuries were characterized by special frequency of the phenomena, auroras especially; the fifth, seventh, tenth, and fifteenth centuries, by rareness of the same. To be more exact, the dates of the chief maxima were 397, 585, 785, 1107, 1361, 1580, and 1775, with intervals, in years, as follows: 188, 200, 322, 254, 219, and 195. Since there can be inserted a somewhat less decided maximum in 990, the times with conspicuous maxima have probably recurred on an average after nearly 200 years (more exactly, 197 years).

On closer examination of the dates mentioned and in so far as the maxima of the phenomena can be determined with definiteness or sure probability, the periods of 55.25 years correspond to the maxima of the phenomena as follows: Exactly, 6 times; within 2 years, 11 times; and within 3 to 5 years, 11 times. Thus in 61 per cent of the cases the crests fall near the dates assumed as probable for the chief maxima of the phenomena.

The epochs of the chief maxima of the longer period of 59.6 years—a period derived by the writer in a theoretical manner (*Die Sonnenfleckenperioden und die Planetenstellen. Vierteljahrsschrift der Naturforschenden Gesellschaft in Zurich*, 1883)—correspond with the known or probable culmination points of the causative phenomena as follows: Exactly, 9 times; with differences of 1 to 2 years, 7 times; and with differences of 3 to 5 years, 12 times—thus in 58 per cent of the cases in a manner rather close. In five cases the previously accepted epochs of the 55.6-year period coincide with the epochs of chief maxima as shown in bold-faced type in Table 5, and in five cases there are differences as follows: 13, 9, 25, 22, and 10 years. The assumed epochs of the 59.6-year period coincide with the epochs of the above chief maxima

in four instances and approach them in four others, the difference in time being 21, 22, 11, and 21 years, respectively. These differences in time are noticeably near 11 years or twice 11 years. According to this the second somewhat longer period of 59.6 years is not to be disregarded.

If the above-mentioned dates of the earliest phenomena could be considered conclusive one might decide as to that one of the longer periods most favored by the data prior to the Christian era. In view of the two sun spot observations given us by the Chinese and some auroral phenomena this might prove correct, but on the basis of other data it is questionable.

The 55.6-year period makes the best showing when the above-mentioned chief maxima as taken from *Polarlicht* are made the basis of the study. In four out of the nine instances the difference does not amount to as much as 2 years and the average difference for all instances is 3.4 years. The longer period [59.6 years] gives more considerable differences and here again in a striking manner do the differences correspond closely with the 11-year periods.

The decision as to systematic change in solar activity over long periods requires continuous observation of the sun for a long time. The existing series of observations, including almost 200 years and resting on a statistical basis, would not be sufficient for investigation even if all of the data presented had equal weight. The following table illustrates this.

TABLE 7.—Sun spots: Epochs and relative numbers¹

Epochs of the minima	Mean relative number for period ²	Epochs of the maxima	Relative number at maximum ²	Epochs of the minima	Mean relative number for period ²	Epochs of the maxima	Relative number at maximum ²
1700-1712	17	1706	49	1799-1810	30	1804	73
1713-1723	27	1718	50	1811-1823	19	1816	46
1724-1733	43	1727	90	1824-1833	40	1830	71
1734-1744	41	1739	85	1834-1843	65	1837	138
1745-1755	33	1750	83	1844-1856	52	1848	124
1756-1766	32	1761	86	1857-1867	50	1860	96
1767-1775	63	1770	106	1868-1878	57	1870	139
1776-1784	69	1778	151	1879-1889	32	1884	64
1785-1798	50	1788	132				

¹ Heading supplied by translator.

² Monthly values.

The table shows agreement of the average relative number for the periods, counting from minimum to minimum, and the maximum values of the yearly mean at the time of spot maximum. The lowest values, those for 1706 and 1816, lie 110 years apart; the succeeding maxima, those for 1727 and 1837, are likewise 110 years apart, as are also the rather small values for 1750 and 1860. Should the trend in change in spots show the same behavior in the future then of necessity there would now (after 1891) follow a high maximum. However, while in 1770 the spottedness was high, in 1880, 110 years later, it fell away. Thus a conclusion can not be drawn.

If we reckon the mean epochs of the minima and the maxima from the dates given and under the assumption that the epochs need not coincide with the respective turning points for the shorter periods, then there are obtained as turning points for the minima: 1706, 1745, 1812, 1857, and 1884, with intervals, in years, of 39, 67, 45, and 27 and as turning points for the maxima: 1732,⁵ 1775,⁵ 1842,⁵ and 1868, with intervals, in years, of 43, 67, and 26.

The minima of 1745 and 1857 are separated by 112 years, the maxima of 1732 and 1842 (as also the observed maxima of 1727 and 1837), by 110 years; on an average there lies between them 55 plus 56 years. Thus the

⁵ It is striking that the auroral maxima are actually shifted to 1731, 1774, and 1840, or then there are still many phenomena in those years.

intervals are again symmetrically divided into parts of nearly equal length. The epochs of minima and maxima alternate at intervals of 26, 13, 30, 37, 30, 15, 11, and 16 years; in this only the third, fourth, and fifth, and then the sixth, seventh, and eighth intervals are symmetrical, the lengths, however, being very unequal. Up to this point 11 and 55-56 remain the best divisors, although the latter determine the main grouping only for the period 1700-1891.⁶

If the main groups 1745 to 1812 and 1812 to 1884 are taken together the average length of the interval is 69 years, while their chief maxima (obtained by averaging) are 79 years apart. The minima give 69.5 years. Counting back by one-half of this period we come to the year 1710, a time when there were low sun spot maxima and still fewer auroras than in 1745.

Let the investigation be carried out as it may, one arrives at similar varying or indefinite results since on one hand the numerical series is too short and on the other hand the course of the change in solar activity is determined not by a few simple waves, but in all probability by a series of waves having very different lengths and amplitudes. In a short series of observations even with only a moderate number of waves combining into the observed waves, the resolution becomes difficult if, as appears probable to the writer, the cause thereof is to be sought in the constantly changing positions of the planets with reference to the sun and to one another, in which the combination becomes so very complicated that only after clear knowledge of the influence of every one of the disturbing bodies will short series of observations be sufficient. If this is not the case then only very long series of observations can bring about a solution of the problem. This holds in still greater measure for the still far more complicated processes in the earth's atmosphere.

In *Mitteilungen über die Sonnenflecken*, Nr. X (p. 281), Professor Wolf comments on a period of 7.65 months or 0.638 year which may appear along with a yearly period with maxima in spring and autumn near the dates of the equinoxes.

In *Astronomische Mitteilungen*, Nr. LVI (p. 197), Wolf employed periods of 10, 11, and 81 years with the view of deriving a formula for the representation of the sun-spot situation. In the next volume of that publication (LVII) the first two values were fixed more accurately at 9.917 and 11.33 years.

In *Astronomische Mitteilungen*, Nr. LXXIII, Wolf announces that he has been led to take a period of 66.67 instead of 55.56 years, and this was followed by taking (in Nr. LXXIV) 83.33 instead of the 81 years mentioned above.

In addition there may be mentioned the depression in 1863, occurring at the time of Jupiter's aphelion, and the following reaction (mentioned by Wolf in *Astronomische Mitteilungen*, Nr. XXV) and the fact that similar anomalies appeared at the time of earlier aphelia while at several perihelia there probably occurred the opposite case. "It is not improbable that there is in this a certain conformity to law and not accident."

The preceding remarks, all based on detailed studies, support lengths of period that are often rather variable and certainly support what has been said before on the complicated course of the change of the phenomena on the sun.

When considered more closely, the period of 66.67 and 83.33 years must approximate the older observations, which conform to the period of 55.56 years since—if one does not wish to deny the 11-year period in earlier ages—the 66.7-year period, including one more 11-year period than the 55.6-year period, must frequently conform well if the 55.6-year period is generally satisfactory. It is the same with the 83.33-year period containing exactly 1.5 times 55.56 years. A period of 111 years will conform still better to epochs separated by twice 55.56 years.⁷

As to the smaller periods, that of 11.1 years is to be adhered to for the present. There is no denying that it represents the mean length of the shorter period of solar activity, terrestrial magnetism, auroras, etc., not only for the last centuries, but as far back as the observations extend or as far as they can be held sufficiently accurate. These waves of phenomena with their maxima and minima which compose the secular periods or which divide the latter into smaller divisions are in turn made up of short waves whose amplitudes increase at the time of maxima and decrease at the time of minima, but always remain perceptible.

Wolf first found (*Mitteilungen über die Sonnenflecken*, Nr. X), as already noted, a period conforming to the year on the earth in the manner that the spots were observed to be somewhat more numerous at the times of the equinoxes than in other seasons. After the use of a longer series of observations this period lost in definiteness. In its place there plainly appeared a period of 0.638 year, which is somewhat longer than the sidereal period for Venus, 0.615 year. The period corresponds exactly to the mean of the synodic periods of Venus relative to Jupiter and Saturn $(0.649 + 0.628) \div 2$ equals 0.638. Also this interval approaches one-half the synodic period of Venus relative to the earth, $1.598 \div 2$ equals 0.799, and is nearly twice as great as the synodic period of the earth relative to Mercury, 0.317×2 equals 0.634.

Wolf further found, as mentioned above, a period of 9.917 years, which in combination with that of 11.33 years could approximately produce the 11.1-year period with its deviations from the mean. This period of nearly 10 years corresponds almost exactly to one-half the synodic period of Jupiter relative to Saturn, $19.858 \div 2$ equals 9.929 years.

Now if Wolf, in *Astronomische Mitteilungen*, Nr. XXV, points out the possible influence of the planets, especially Jupiter at perihelion or aphelion, then with the above values have we approached farther to the point from which the writer proceeded when, in 1866, he first made public the hypothesis: "The changes in sun spots (now better denominated solar activity) may originate in the influences of the planets on the central body." While the manner of manifestation is not the same, the nature of the action is to be viewed as in agreement with the laws that relate to the attraction of the sun and moon as evidenced in the ebb and flow of the ocean. Referring to the *Programm des eidgenössischen Polytechnikums* von 1866; Wolf's *Astronomische Mitteilungen*, Nr. XXV; *Die Sonnenflecken-Periode und die Planetenstellung in Vierteljahrschrift der Naturforschenden Gesellschaft in Zurich*, Jahrgang XXVII; *Die wichtigsten periodischen Erschein-*

⁶ Through the investigation of a series of phenomena Brückner arrives at a period of 32 years. The figures presented by him (in G8a, B. 27) result from durations of wet and dry, cold and warm years extending over periods of 32 to 34 years, and from the period of advance in glaciers, 35 years (that is, 32, 34, or 37 years), or then in general periods of 3 times 11 years or periods that are one-half of the 69-year period.

By Dr. Früh (*Vortrag auf dem Rathhause Zurich im Frühjahr 1892*) and also by Brückner the vintage is said to show the 32-year period. On the basis of a rather abundant collection of yields for 15 years, a compilation that is hardly excelled to-day by any other wine statistics, the writer was able to give (Nr. XXXVI, *Vierteljahrschrift der naturforschenden Gesellschaft*) the following 5-year totals: 1825-1829, 8.7; 1830-1834, 5.2; 1835-1839, 6.4; 1840-1844, 4.6; 1845-1849, 5.5; 1850-1854, 4.2; 1855-1859, 4.7; 1860-1864, 4.4; 1865-1869, 6.2; 1870-1874, 4.9; 1875-1879, 4.6; 1880-1884, 4.5; and 1885-1889, 3.6.

According to these values and those of the original table the chief maxima (1826 and 1870) are separated by 44 years. The interval between the chief minima can not be determined even approximately.

⁷ The value of 69.5 or 70 years mentioned above corresponds to 1.25 times the length of the 55.6-year period.

ungen (Leipzig, 1889) by the writer; etc., it may be remarked briefly that when the true time of the sun's rotation is taken as 25.234 days (after Spörrer) and 25.74 days (after Buys-Ballot) the respective synodic times are: For Mercury, 35.37 and 36.38 days; for Venus, 28.42 and 29.06 days; for the earth, 27.10 and 27.687 days; and for Jupiter, 25.38 and 25.89 days.

Without considering the remaining planetary actions, which appear but little effective according to the hypothesis mentioned, there results for the four planets alone an important combination of smaller influences for shorter periods.

If we assume the period of 27.6868 days, first derived by Buys-Ballot from the temperature observations of European stations and reflected in solar activity, auroras, terrestrial temperature changes, and other phenomena,⁸ to be produced by the synodic revolutions of Venus and Jupiter, which are most influential on the sun, then, reckoning from Venus the rotation period of the sun would be 24.65 days. This value agrees closely with periods determined as 24.12 days from meteorological phenomena, 24.33 days from magnetic phenomena, and 24.7 days from spot observations.

Since Venus and Jupiter must have nearly the same influence on variability of solar activity—if the hypothesis on planetary influence based on the laws of attraction corresponds to fact—there must be in the observations two almost equal waves, one of 25.9 and the other of 29.1 days, combining into the mean wave of 27.67 days.

Another derivation of this short period would be possible through recourse to an intramercurial planet with a period of revolution amounting to 50.577 days. (See *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zurich*, Nr. XXVII.) If it is desired to explain this short period as being a mean period from the synodic revolutions of Venus and Jupiter relative to the sun, then the above values for it must change to 26.07 and 29.29 days. The latter value now agrees closely with that of 29.39 days determined by J. Unterweger (*Die kleinen Perioden der Sonnenflecken*. Wien. 1891) from Tacchini's and Wolf's observations from 1880 to 1887. If investigation is made of the series of observations presented by Unterweger in statistics and graphs, corresponding increases are actually found 98 times in 112 periods of 26.07 days and 76 times in 99 periods of 29.29 days. If this result is well founded, then after every 0.3245 year, or 118.54 days (one-half of the synodic period of Venus relative to Jupiter), the waves must alternately increase and decrease. In fact, in 13 cases (especially striking on June 30 and October 15, 1883) the higher values were well marked and in other cases still noticeable in variable degree, so that in the 2,685 days taken there fall 23 periods.

If this compilation is investigated relative to the 27.687-day period then one finds—as in the earlier contributions of the writer (*Beziehungen der Sonnenflecken zu den magnetischen und meteorologischen Erscheinungen* in Wolf's *Astronomische Mitteilungen*, Nr. XXVII) also in the series of spot observations studied by Unterweger—that it represents that spots occur more abundantly in 80 out of 105 cases (76 per cent) of the 27.687-day period. Increased spottedness frequently appears at the midway point in the length of the period, 13.84 days after the maximum, and 77 cases (76 per cent of 105 cases) of secondary maximum can be pointed out. Over 60 per

cent of the number of prominences do not depart more than four days from the theoretical mean. In 1880, 1882 to 1883, 1885, and 1887 the times of more abundant spots correspond in marked manner to the epochs of the 27.687-day period; in the remaining period they correspond more to the intermediate halfway points, and only from March, 1881, to March, 1882, does there take place a reversal in the two epochs. In the meteorological phenomena the 14-day epochs generally occur rather regularly, while on the sun many of these fail to appear or are not observed. The reversal after every two or three years occurs in such way that it can be thought to be caused by two waves varying slightly in intensity so that little by little the secondary waves gain upon the primary until they attain the ascendancy.

A survey of the numerical values of the series of observations, and especially their graphic representation, allows hardly any other impression than that the resultant curves are made up of individual primary curves of unequal length and amplitude. The difference between the heights of the wave crests and the depths of the troughs increases at the times of the maxima and decreases at the times of the minima. The process reminds one very much of the flood curves of the ocean. If in these the influences of the sun and the moon as well as the local conditions and the effects due to the nature of the coast, depth and position of the sea can be examined with relative ease, it becomes more difficult with the spot curve. This is composed of a rather large number of waves. The most natural assumption as to the cause of the production of the individual waves leads more and more back to the planets. On account of mass and distance Jupiter, Venus, Mercury, and the earth must be viewed as most disturbing; Saturn and the other planets less effective in this way. The inner planets come into consideration chiefly in relation to the shorter waves, the outer planets in relation to the longer waves. The probable influence of comets, meteor swarms, and the like, or even the movement of the sun and its system, in space can be eliminated for the present even in considerable degree.

What has been said indicates the difficulty that presents itself when definitive investigations are undertaken while there are not available for each phenomenon to be studied far longer and, especially, more accurate series of observations than those now at hand. For periods of 50 to 75 years the observations must extend over 100 years since only in few cases do the periods always show the same length, oscillating for the most part about a mean length. For temperature and rainfall some series extend back to the year 1700, for river stages, ice conditions, thunderstorms, winds, etc., only to 1750. Air pressure observations came in later. Only rare reports on glacier changes are available for the preceding century. Crop statistics began in very recent times. Vintage statistics for a rather large region with data on the yield for definite surface extend (in Prussia) only as far back as 1820. If we add incompleteness to rareness of older series we can judge at once how great the accuracy and credibility of the same can be in by far the greatest number of cases when the epochs are separated by 50 or more years. For the present we must be satisfied with the determination of shorter periods. The determination of longer periods can be attempted only in rare cases. In view of the complex mechanism of meteorological phenomena individual series of observations are but little sufficient for discovery and confirmation of laws. If there is satisfaction therewith there should be no surprise at contradictions that arise. From individual series there can be found and maintained more or less complete contraries.

⁸ Among other things this agrees well with the magnetic observations by the Austrian Polar Expedition to Jan Mayen in 1882-83 and observations up to the present in middle Europe.