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LOCAL FORECAST STUDIES—WINTER PRECIPITATION

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[Weather Bureau, Honolulu, Hawaii, January 5, 1924]

In a previous paper [1] occurrences of precipitation during the summer months, May to August, inclusive, at Dubuque, Iowa, were tabulated with reference to the meteorological conditions obtaining at the previous regular morning observation at 7 a. m., 90th meridian time, and the percentage of days with rain under varying conditions shown in a series of curves. In the present paper a similar study is made of the relation between local meteorological conditions and the subsequent precipitation during the winter months, also at Dubuque, Iowa; latitude 42° 30′, north; longitude 90°, 44′, west; elevation 698 feet.

The data used are for the winter months, November, December, January, and February, for the 33-year period, 1889 to 1921, inclusive, with some omissions due to interrupted record, making a total of 3,606 observations. The method employed is that of the previous paper, namely, the construction of a series of curves, "showing the relation, expressed as a percentage, which the number of observations followed by rain in 12 or 24 hours bears to the total number of observations" within the various groups. To facilitate comparison with summer conditions, Figures 1, 2, and 3, have been made directly comparable with Figures 1, 2, and 6 of the earlier paper and include traces of precipitation and days when precipitation was occurring at observation.

The total probability of rain under all conditions is 0.34 within 12 hours and 0.45 within 24 hours. For the summer months the corresponding probabilities were 0.31 and 0.46, respectively.[2] Decreasing probability with increased barometric height is well shown in the "all observations" curves of Figures 1 and 2, but even with very high pressures there is considerable probability of rain, provided the pressure is decreasing. The downward tendency of the curves for pressures above 30.15 inches is much less marked in winter than in summer. With cloudy weather and falling pressure the percentages are continuously above 50 and rise to 80 for the 12-hour period and 84 for the 24-hour period. A distinct secondary maximum occurs with falling barometer at pressures between 30.25 and 30.34, at a point where, in summer, the chances are rapidly and steadily decreasing to negligible values. With cloudy weather and falling pressure the percentage within 12 hours rises to 78, which is within 2 per cent of that at the lowest pressures. As will be shown later this maximum is much less marked when traces are omitted from consideration. These characteristics of the curves describe the fact, which I think is generally recognized, of the high probability of light snowfall within 12 hours when the barometer is high but has begun to fall.

With clear weather and rising pressure the probability varies between 0.22 and 0.06 for 12 hours, and between 0.34 and 0.17 for 24 hours. It is least when the temperature is also rising. Consistently, at the other extreme, the probability is greatest with falling temperature, combined with falling pressure and cloudy skies. This latter is true only for pressures below 29.84 inches. Above that point the temperature relation seems doubtful, the curves crossing each other three times. The corresponding curves for the summer data were less consistent, both the highest and the lowest probabilities occurring with falling temperature.

Figure 3 is comparable with Figure 6 of the previous paper, and similarly shows that temperature changes alone afford no basis for rain prediction. It will be noted that a rapid fall in pressure is a much less reliable indicator of rain in winter than in summer. In summer a barometric decline of 0.25 inch within 12 hours is practically certain to result in rain in the following 12 hours, but in winter the same pressure change is followed by rain within 24 hours less than two-thirds of the time.

There is no forecasting value, however, in including days when rain is falling at the time of observation, as has been done in the above discussion, since they must all be recorded as having rain within the following 12 hours, and traces of precipitation have little practical significance. Omitting the former observations entirely, and counting as days with rain only those which have 0.01 inch or more, the total probability of rain within 12 and 24 hours, respectively, is reduced from 0.34 and 0.45 to 0.11 and 0.22. The contrast is clearly seen by comparing Figures 1 and 2 with Figures 5 and 6. In order to bring out more clearly the frequent occurrence of insignificant amounts of precipitation, Figure 4 was prepared, showing that during the winter months in fully one-half of the cases in which precipitation is recorded the amount of moisture received is too small to measure.

Under none of the combinations shown in Figures 5 and 6 are there as much as even chances of rain, the highest percentage found being 48. As previously mentioned, the marked secondary maximum of Figures 1 and 2 at pressures between 30.25 and 30.34 becomes much less pronounced when traces are omitted, but in the latter case another secondary maximum develops at pressures between 29.95 and 30.04, which, indeed, with cloudy weather and falling barometer, becomes the primary maximum for the 24-hour period. With rising pressure the primary maximum is at the next lower level of pressure, 29.85 to 29.94. It appears, then, that a measurable amount of snow is more apt to follow median pressures, whether the tendency be upward or downward, than very low ones, while traces are more frequent with distinctly low or distinctly high pressures.

The downward slope of the curves with rising pressure is even less in Figures 5 and 6 than in Figures 1 and 2. Figure 6 plainly says that the actual height of the barometer in winter affords little indication of the chances of rain, little at least as compared with its indications in summer. Neither does change in pressure, as shown in Figures 7 and 8, give a very accurate forecast. The slope of the lines is more consistent, but the range of probabilities is not great. A rapid fall of the barometer is followed by measurable precipitation in winter less than one-third of the time, but by flurries or traces about two-thirds of the time.

We find more definite indications when we consider the relations of wind direction to precipitation, Figure 9, and for the first time discover a condition under which the odds in favor of a measurable amount of rain are better than one to one. With an east wind and falling pressure, it has rained within 24 hours in 62 per cent of the cases. With falling pressure the probability of rain is distinctly greatest with wind from the east, i. e., with a center of low pressure slightly south of west and a center of high pressure slightly north of east. In the previous article

[1] Blair, T. A., Local Forecast Studies-Summer Rainfall, Mo. WEA. REV., April, 1921, 49: 183-190.

[2] "Rain" and 'rainfall" are used to include all precipitation, much of which is snow during the months under consideration. "Probability" is used as equivalent to percentage, not calculated by the more exact formula, $p=\frac{m+1}{n+2}$.

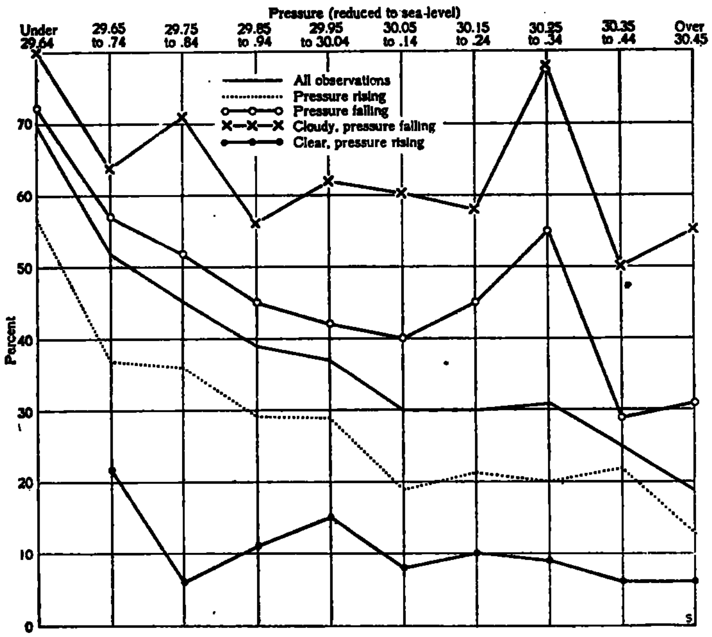

FIG. 1.—Percentage of observations followed by rain within 12 hours, as related to height of barometer and other factors. Days with trace and with rain falling at observation included. Based on 3,606 observations at Dubuque, Iowa

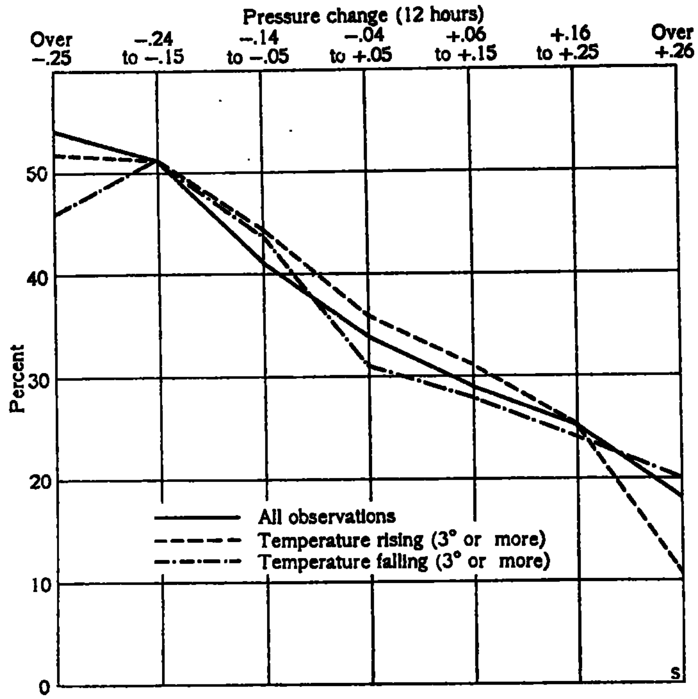

FIG. 3.—Percentage of observations followed by rain within 24 hours, as related to changes in pressure and temperature. Days with trace and with rain falling at observation included; 3,603 observations

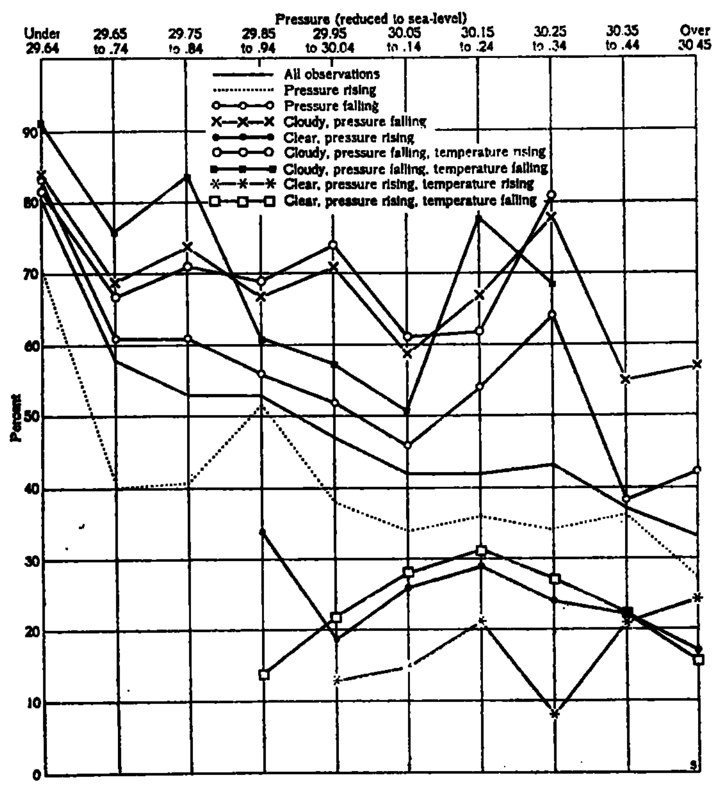

FIG. 2.—Percentage of observations followed by rain within 24 hours, as related to height of barometer and other factors. Days with trace and with rain falling at observation included. Based on 3,606 observations at Dubuque, Iowa

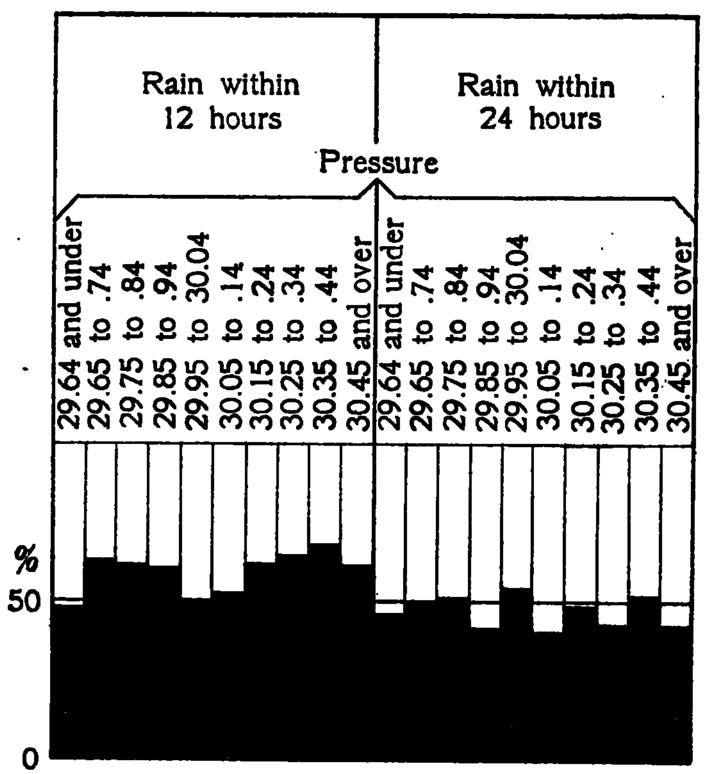

FIG. 4.—Percentages of traces in total number of occurrences of precipitation at Dubuque, Iowa, during the winter months. Number of observations, 856 for 12-hour period; 1,274 for 24-hour period

the same condition was found to obtain for the summer months. With rising pressure rain is most probable with a northeast wind, i. e., the LOW is moving eastward south of the station and pressure is rising in the north and northeast. In summer this pressure distribution is distinctly dry and the reverse conditions are those most favorable for rain with rising pressure. Instead of northeast it is southwest winds that give the highest summer frequency, the station being in the southwest quadrant of a baro-

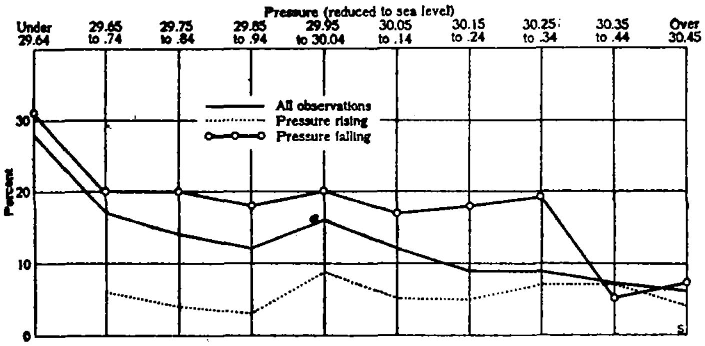

FIG. 5.—Percentage of observations followed by 0.01-inch or more of precipitation within 12 hours, as related to pressure and pressure change. Days with rain falling at observation omitted. Based on 3,246 observations at Dubuque, Iowa

metric depression and the rainfall being mostly due to thunderstorms. In winter, winds with a westerly component are very dry under all conditions, and the contrast between the probabilities with easterly and those with westerly winds is much sharper than in summer.

The relation between kind of clouds and subsequent precipitation is shown in Table 1. The probability under most conditions is highest with dense fog and with stratus clouds, and lowest with no clouds, but is also distinctly low on the average with cumulus and alto-cumulus clouds.

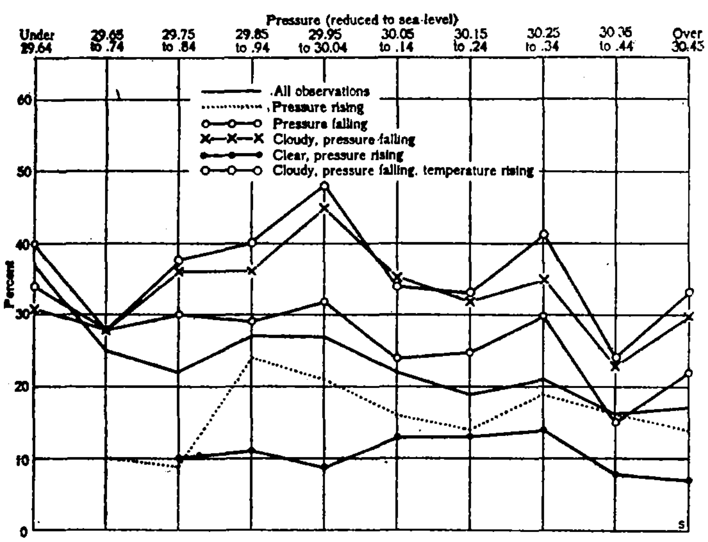

FIG. 6.—Percentage of observations followed by 0.01-inch or more of precipitation within 24 hours, as related to pressure, pressure change, and cloudiness. Days with rain falling at observation omitted. Based on 3,246 observations at Dubuque, Iowa

However, with alto-cumulus accompanying a rising barometer the chances of rain within 12 hours are not only relatively good but are absolutely higher than when the barometer is falling. With falling pressure cirro-cumulus and strato-cumulus clouds are frequent precursors of rain. These results are consistent with those described for France by Schereschewsky and Wehrlé [3] in their recently

[3] Schereschewsky and Wehrlé, Les Systèmes Nuageux, Office National Météorologique de France, 1923.

published and illuminating discussion of cloud systems. In their typical cloud system associated with a center of falling pressure, cirro-cumuli directly precede the center of the system, and are therefore in an area of falling pressure generally followed by rain, while alto-cumuli with falling pressure are found on the extreme margin

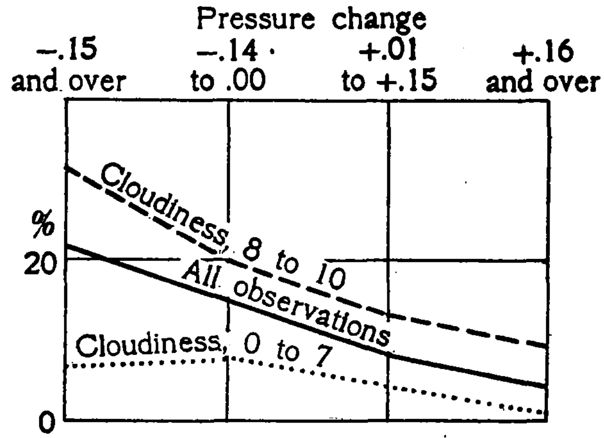

FIG. 7.—Percentage of observations followed by 0.01-inch or more of precipitation within 12 hours, as related to pressure change and cloudiness; 3,132 observations

of the system, where rain is infrequent, but alto-cumuli with rising pressure occur in the train of the system, where showers or patches of rain are frequent.

It has never rained within 12 hours when cumulus clouds and rising pressure were observed at the morning observation, fully justifying the expression, "fair weather cumulus;" but for falling pressure or for a 24-hour

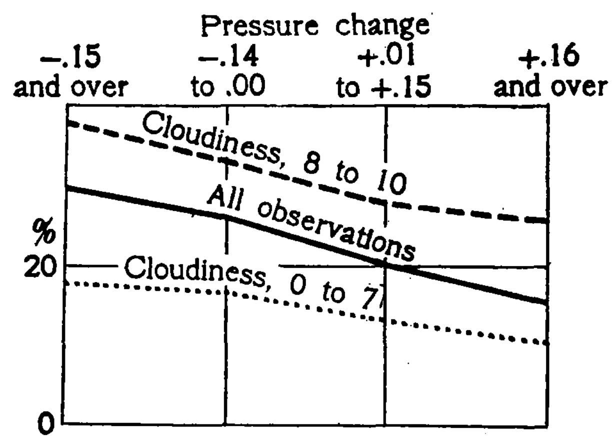

FIG. 8.—Percentage of observations followed by 0.01-inch or more of precipitation within 24 hours, as related to pressure change and cloudiness; 3,132 observations

period with rising pressure the frequency of rain following cumulus clouds is about the average of all observations. A contrast appears between summer and winter fogs. In summer the probability of rain following dense fog was found to be less than the average probability, but in winter dense fogs surpass any other cloud conditions as indicators of rain in three of the four columns of Table 1. The different nature of summer and winter fogs in this region was pointed out by Martin.[4] He classed summer fogs as local radiation phenomena and winter fogs as advective, due to air movement. In the Dubuque data 65 per cent of the summer fogs occur with rising pressure, and 54 per cent of the winter fogs with falling pressure.

[4] Martin, H. H., Fog in central Ohio and its relation to subsequent weather changes, MO. WEATHER REV., July, 1919, 47: 471-472.

In the study of the probabilities of rain resulting from the simultaneous consideration of two meteorological elements certain further relationships are brought out by Tables 1, 2, and 3, and Figures 10 and 11. Both tables and figures are in the form used by Rolf [5] in his study of Swedish data along these lines. The upper set of curves in Figure 10 brings out the fact that with all pressures the chances of rain are better with northeast to east winds than with winds from any other direction, but that the difference becomes very marked only when pressures are not far from normal. With other winds, the probability of rain shows no striking change with change in height of the barometer. The tendency to increased rain probability at pressures between 29.95 and 30.04, pointed out in connection with Figure 6, is shown in the lower part of Figure 10, in the curve for relative humidities above 91 per cent.

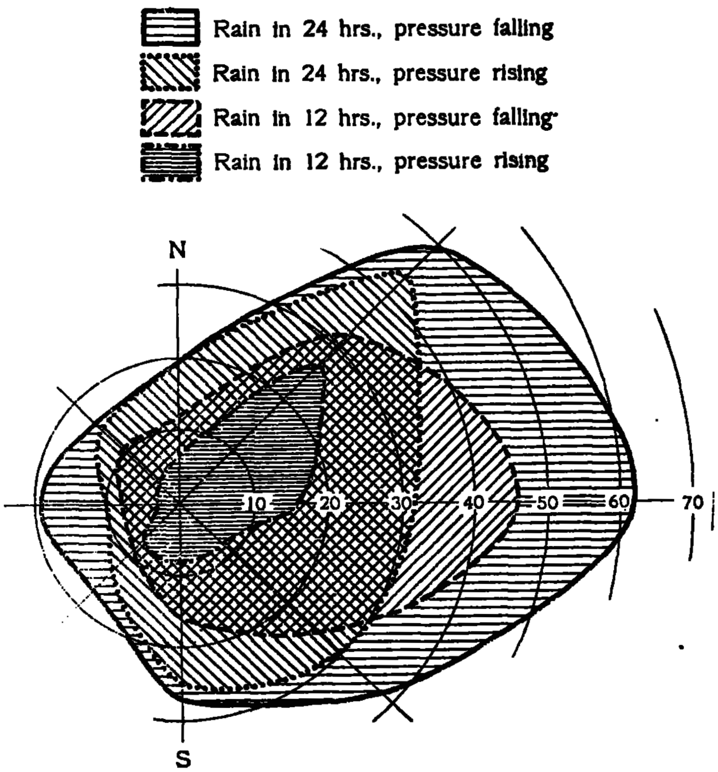

FIG. 9.—Percentage of observations followed by 0.01-inch or more of precipitation within 12 and 24 hours, as related to wind direction. Days with rain falling at observation omitted; 3,132 observations

The second set of curves of Figure 10, and the data in Table 2, on which they are based, yield a quite unexpected relation between wind velocity and rainfall, for which no adequate explanation is evident. For the greater part of their length the four curves are arranged in the order of the velocities represented, the probability of rain *decreasing* consistently with *increasing* velocity. Especially distinct is the increased probability following conditions of practical calm, winds of zero to 4 miles per hour. The general condition holds, though somewhat less consistently, when the data are classified by pressure change, as in Table 3.

In the relation of wind direction and cloudiness to rainfall, the spreading of the curves at northeast and east in Figure 11 indicates that in most cases of rain with these winds cloudiness has already developed at the morning observation to give additional indication. The curves showing the relation between pressure change and wind direction emphasize in fuller detail the fact already indicated in Figure 9 that with falling pressure the most frequent rain-bearing winds are from the east, and with rising pressure from the northeast. With a rise of 0.16 inch or more, this becomes very marked, northeast rising to 59 per cent, and east falling to 17 per cent, which is below southeast, south, and north. When it has been raining with northeast wind and rising barometer and

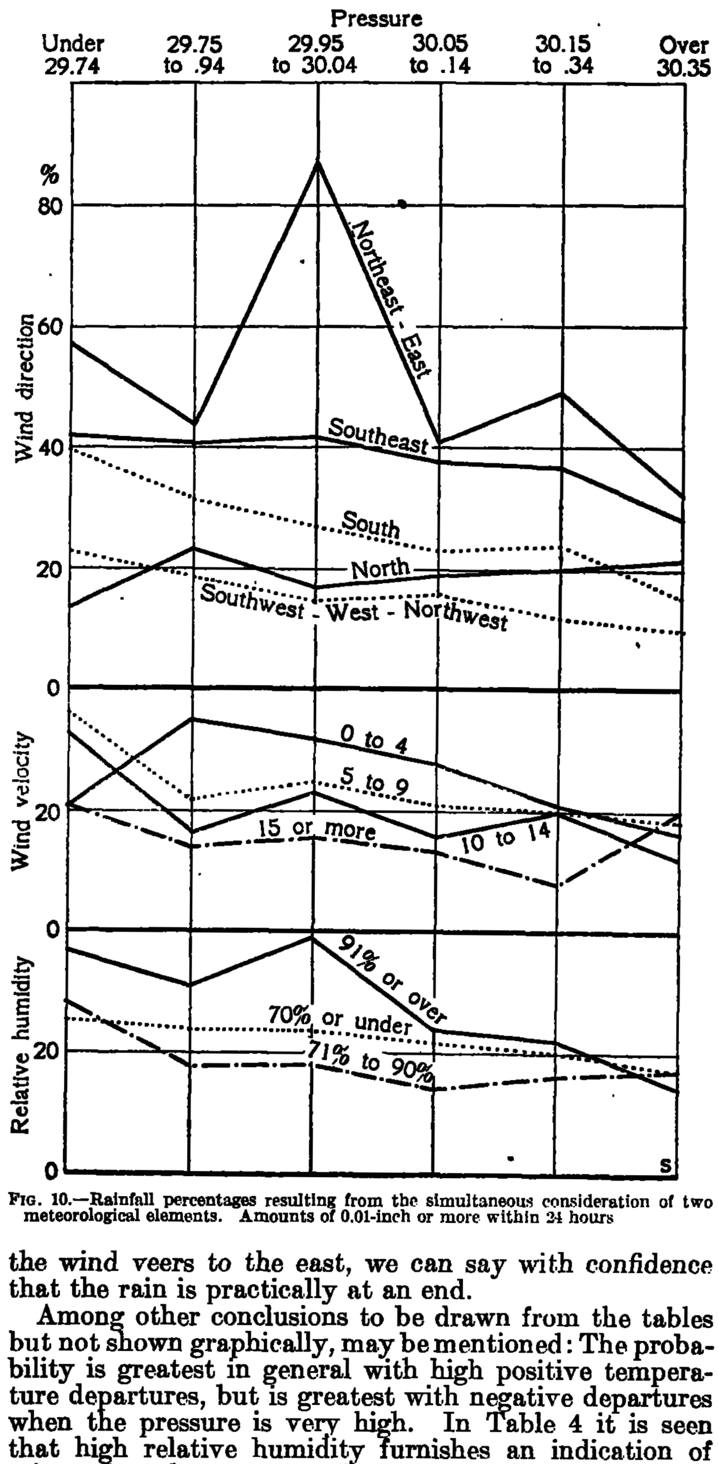

FIG. 10.—Rainfall percentages resulting from the simultaneous consideration of two meteorological elements. Amounts of 0.01-inch or more within 24 hours

the wind veers to the east, we can say with confidence that the rain is practically at an end.

Among other conclusions to be drawn from the tables but not shown graphically, may be mentioned: The probability is greatest in general with high positive temperature departures, but is greatest with negative departures when the pressure is very high. In Table 4 it is seen that high relative humidity furnishes an indication of rain only when combined with positive temperature departures or cloudy weather. With negative departures

[5] Rolf, Bruno, Probabilité et Pronostics des Pluies D'Été, Upsala, 1917.

it even seems more likely to rain with average or low humidities; with clear weather humidity is without significance as to coming rain. That pressure change, cloudiness, relative humidity, temperature change, and wind direction are not independent variables must be borne in mind, however. The closely interrelated character of the data with which we are dealing is made very evident by these tables and curves.

In the actual application to forecasting of such results as have been obtained in this discussion, Besson [6] has used a simple method which deserves to be more widely

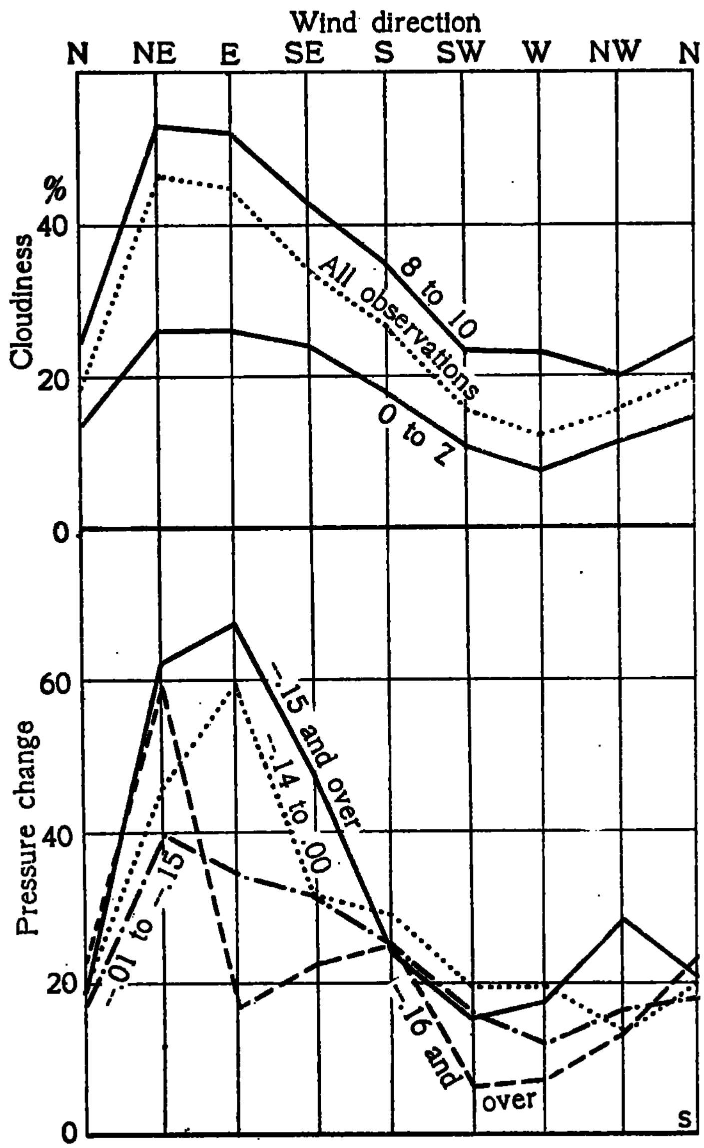

FIG 11.—Rainfall percentages resulting from the simultaneous consideration of two meteorological elements. Amounts of 0.01-inch or more within 24 hours

known and widely applied. He has used simple tables giving essentially the same facts as are given by the curves discussed in the beginning of this article, but has gone further by making actual forecasts from the data and computing the percentage of verification. Table 5 presents after the manner of Besson the data shown graphically in the "all observations" curves of Figures 1 and 2, and Table 6 a portion of the data used in Figure 9.

In order to consider the probability of rain as a function of two elements, Besson constructs tables of rectangles, and draws thereon probability isograms. The general method is illustrated in Figure 12, in which the probability of rain in 24 hours, including traces, is expressed as a function of pressure and pressure change. The values are smoothed by calculating the percentage

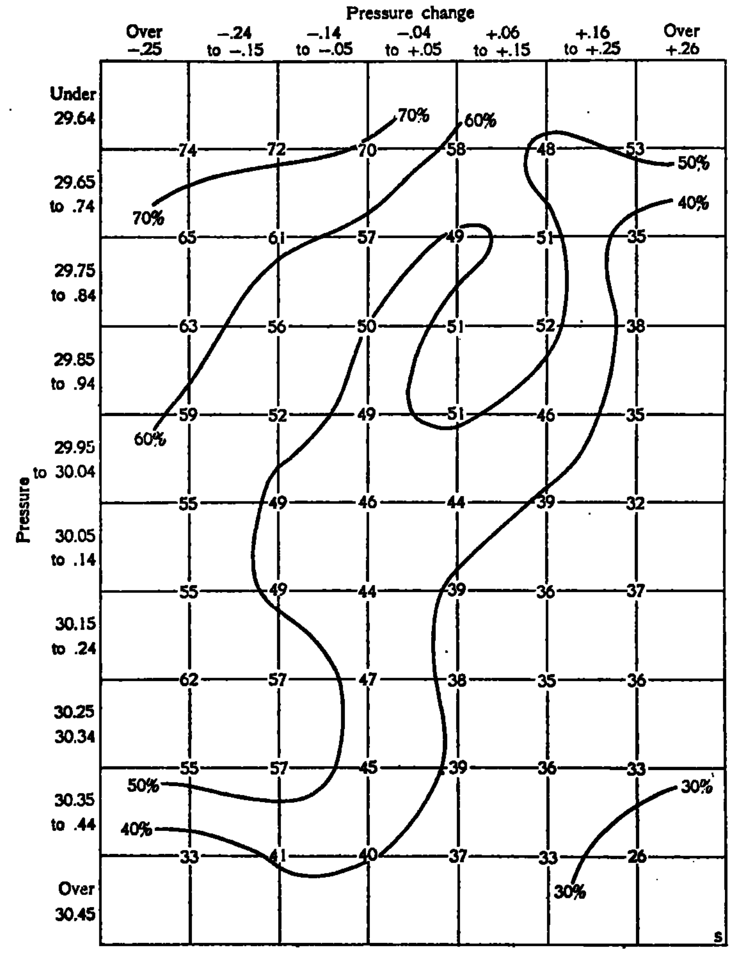

FIG. 12.—Rain probability as a function of pressure and pressure change (after Besson). The numbers entered on the figure are the mean percentages of rain within 24 hours, including traces, for the four adjoining squares. Data of 3,606 observations at Dubuque, Iowa

for the four squares at the center of which the percentage is entered. The results are fairly consistent, and a diagonal from the lower left-hand corner approximately represents the 50 per cent line. By forecasting rain for all cases represented here when the percentage is over

TABLE 1.—*Percentage of observations followed by 0. 01 inch or more of precipitation within 12 and 24 hours, as related to kind of clouds*

[Days with rain falling at observation omitted; 3,455 observations]

Pressure falling				Pressure rising			
12 hours		24 hours		12 hours		24 hours	
Clouds.	Per cent	Clouds	Per cent	Clouds	Per cent	Clouds	Per cent
Fog	30	Fog	45	A. cu	12	Fog	24
St	23	St	33	St	11	St	23
St. Cu	22	Ci. Cu	33	St. Cu	11	Ci	22
Ci. Cu	20	St. Cu	28	Ci	8	A. St	19
A. St	15	A. St	26	A. St	7	A. Cu	19
Ci. St	11	Ci. St	26	Ci. Cu	5	Ci. St	18
Cu	10	Cu	20	Fog	4	St. Cu	17
Ci	6	Ci	18	Ci. St	4	Cu	17
A. Cu	4	A. Cu	13	0	2	Ci. Cu	11
0	3	0	12	Cu	0	0	10

[6] Besson, Louis, Essai de prévision méthodique du de l'emps, Annales Observatoire Municipal (Montsouris), vol. 6, 1905; 473-496.

TABLE 2.—*Rainfall percentages resulting from the simultaneous consideration of two meteorological elements, based on 3,245 observations at Dubuque Iowa,*

[n=number of cases. p=percentage with rain of 0.01 inch or more within 24 hours]

Other element	Pressure											
	29.74 and under		29.75 to 29.94		29.95 to 30.04		30.05 to 30.14		30.15 to 30.34		30.35 and over	
Wind direction:	n	p	n	p	n	p	n	p	n	p	n	p
N	14	14	31	23	24	17	43	19	98	20	112	21
NE	8	75	16	50	12	92	15	40	34	50	50	30
E	4	25	4	25	11	82	17	41	27	48	26	35
SE	31	42	54	41	50	42	64	38	108	37	75	28
S	55	40	109	32	82	27	91	23	137	24	93	15
SW	32	34	70	20	40	5	47	15	81	17	50	4
W	29	17	66	17	53	17	46	17	102	11	75	5
NW	64	20	157	19	115	17	126	16	260	11	224	13
Wind velocity (m.p.h.):												
0-4	66	21	192	35	152	32	225	28	435	21	385	16
5-9	92	37	193	22	158	25	162	21	315	20	270	18
10-14	60	33	103	17	70	23	62	16	95	20	65	12
15+	24	21	37	14	19	16	14	14	37	8	15	20
Temperature Dept.:												
−11+	8	—	29	21	28	18	64	25	234	19	381	19
−4 to −10	16	19	45	11	55	20	93	17	184	20	162	18
−3 to +3	30	23	100	25	87	29	101	22	201	19	103	11
+4 to +10	56	23	163	20	111	25	104	17	138	18	58	12
+10+	132	35	187	34	118	33	86	29	94	30	19	11
Relative humidity:												
91 per cent+	67	37	131	31	105	39	107	24	185	22	187	14
71 per cent to 90 per cent	141	26	320	24	250	24	297	22	572	20	470	17
70 per cent −	34	29	73	18	44	18	44	14	94	16	66	17

TABLE 3.—*Rainfall percentages resulting from the simultaneous consideration of two meteorological elements, based on 3,245 observations at Dubuque, Iowa*

[n=number of cases. p=percentage with rain of 0.01 inch or more within 24 hours]

Other element	Pressure change									
	−.15 and over		−.14 to −.05		−.04 to +.05		+.06 to +.15		+.16 and over	
Wind velocity:	n	p	n	p	n	p	n	p	n	p
0-4	147	36	279	26	470	24	352	19	207	15
5-9	228	27	216	28	248	21	231	19	267	16
10-14	107	34	54	17	63	26	88	9	143	15
15 & +	26	19	8	—	21	19	32	12	59	10

	−.15 and over		−.14 to .00		+.01 to +.15		+.16 and over	
Wind direction:	n	p	n	p	n	p	n	p
N	10	20	49	20	163	18	100	23
NE	16	62	38	45	59	39	22	59
E	12	67	27	59	44	34	6	—
SE	128	47	162	31	83	31	9	—
S	174	24	269	29	116	25	8	—
SW	82	15	123	19	82	16	33	6
W	36	17	88	19	148	12	99	7
NW	39	28	135	13	390	16	382	13
Cloudiness:								
0-7	193	18	404	17	606	13	431	10
8-10	304	38	487	33	479	28	228	26

TABLE 4.—*Rainfall percentages resulting from the simultaneous consideration of two meteorological elements, based on 3,245 observations at Dubuque, Iowa*

[n=number of cases. p=percentage with rain of 0.01 inch or more within 24 hours]

Cloudiness	Wind direction															
	N		NE		E		SE		S		SW		W		NW	
	n	p	n	p	n	p	n	p	n	p	n	p	n	p	n	p
0-7	157	14	34	26	23	46	132	24	282	18	193	11	253	8	560	11
8-10	165	25	101	53	66	52	250	43	285	35	127	23	118	23	386	20

Relative humidity	Temperature departure									
	−11 and over		−4 to −10		−3 to +3		+4 to +10		+11 and over	
	n	p	n	p	n	p	n	p	n	p
91 per cent and over	202	17	121	18	122	22	141	24	196	42
71-90 per cent	468	21	381	17	453	21	395	19	353	29
70 per cent and under	74	16	53	28	46	11	94	15	87	19

TABLE 4.—*Rainfall percentages resulting from the simultaneous consideration of two meteorological elements, based on 3,245 observations at Dubuque, Iowa*—Continued

Relative humidity	Cloudiness					
	0-3		4-7		8-10	
	n	p	n	p	n	p
91 per cent and over	283	13	65	25	432	34
71-90 per cent	870	11	269	21	911	31
70 per cent and under	165	12	53	11	137	28

TABLE 5.—*Probability of rain with different pressures and different changes of pressure; includes days with trace and with rain falling at observation*

[R_{12}=number of times rain recorded in 12 hours. R_{24}=number of times rain recorded in 24 hours. P_{12} and P_{24} are the corresponding probabilities]

Pressure	R_{12}	R_{24}	Total	P_{12}	P_{24}
Pressure change:					
29.64 and under	112	130	161	0.70	0.81
29.65–74	86	94	163	0.53	0.58
29.75–84	108	127	239	0.45	0.53
29.85–94	143	197	370	0.39	0.53
29.95–30.04	160	203	436	0.37	0.47
30.05–14	152	208	501	0.30	0.42
30.15–24	154	218	517	0.30	0.42
30.25–34	137	192	446	0.31	0.43
30.35–44	81	119	321	0.25	0.37
30.45 and over	84	147	452	0.19	0.33
Total	1,217	1,635	3,606	.34	0.45
Verification				0.68	0.59
Pressure change:					
−.25 or more	150	167	260	0.58	0.64
−.24 to −.15	205	236	389	0.53	0.61
−.14 to −.05	259	326	639	0.41	0.51
−.04 to +.05	281	383	862	0.33	0.44
+.06 to +.15	197	295	753	0.26	0.39
+.16 to +.25	81	147	417	0.19	0.35
+.26 or more	44	81	286	0.15	0.28
Total	1,217	1,635	3,606	0.34	0.45
Verification				0.65	0.59

TABLE 6.—*Probability of rain with falling pressure and with wind from different directions; days with trace and with rain falling at observation not included*

Direction	R_{24}	Total	P_{24}
N	12	59	0.20
NE	27	54	.50
E	24	39	.62
SE	111	290	.38
S	120	443	.27
SW	35	205	.17
W	23	124	.19
NW	28	174	.16
Total	380	1,388	.27
Verification			.73

50, we get a verification of 62 per cent. This is only slightly better than the 59 per cent obtained for the corresponding period in Table 5, and corresponds to the experience of Besson, who says that contrary to expectations the two-element combinations do not offer much superiority over the results with one element. However, he constructs eight such figures, giving eight combinations of two elements each, and according as the arithmetical mean of the eight probabilities shown is above or below 0.50, forecasts rain or no rain, and obtains a verification of 73 per cent for 943 cases.

Besson forecasts rain when the tables show a probability above 0.50, and no rain when below 0.50. In Table 5 only when the barometer is below 29.75 is the probability of rain within 12 hours above 0.50. Given only this element, then, he would forecast rain whenever the barometer is below 29.75, and fair weather under all other conditions. To get the percentage of verification which such forecasts yield, add the number of cases of rain when the pressure is below 29.75 to the number of cases it did not rain when the pressure is above this figure, and divide by the total number of observations.

In this case we get $198 + 2,263 \div 3,606 = 0.68$. By this simple rule the forecasts are correct 68 per cent of the time for rain in 12 hours, and by a similar calculation 59 per cent for rain in 24 hours, forecasting rain in the latter case when the pressure is below 29.95 inches. Besson uses this method for the observations at Montsouris with six elements, pressure, direction and velocity of wind, temperature, cloudiness, and pressure change, and obtains percentages of verification ranging from 58 to 67. Traces and days when rain was falling at observation are included in Table 5. Excluding these the winter probability at Dubuque never rises to 0.50 except with northeast and east winds in combination with falling pressure as shown in Table 6. Under these conditions the verification is 73 per cent.

Owing to the low probability of precipitation of measurable amount, the complete application of the above method to the data of this paper is not practicable, and we must revert to the graphical representation of the facts for the essential relationships. The results obtained by Besson and those obtainable from the graphs fall short of those obtained by the experienced forecaster using the synoptic chart, and it is not to be assumed that conditions observed at a single station can give a complete basis for formulating a forecast, as, indeed, the chart and the local observations combined can not do. On the other hand, many weather proverbs and many of the predictions of mariners, farmers, and other outdoor men have a certain validity, due to the fact that locally observable phenomena do precede and announce coming weather changes. A statistical study of local data, by classifying these phenomena in relation to subsequent weather, furnishes valuable supplementary information and suggestion to the forecaster.

551.501

A NEW PRINCIPLE IN THE ANALYSIS OF PERIODICITIES

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[Weather Bureau, Washington, March 24, 1924]

What appears to be an important new principle has come out in the course of a recent application of the Fourier analyses made by the writer to evaluate a supposed sun-spot period in terrestrial temperatures. This has to do with what happens in the use of the analyses or other schemes for investigating periodicities when the data under examination have within them certain known or unknown periodicities the length of which is shorter than the last term of the Fourier series.

The demonstration which follows is built up around the Fourier series merely as a convenience contributing clearness and comprehensiveness of presentation. The principle itself is concerned only with the body of data investigated and the significance of certain features thereof, and the Fourier theorem is nothing whatever but a symbolism employed to conveniently exhibit the facts.

The present consideration presupposes, of course, that the whole number of phase values of data available is finite and limited, and consequently a finite and limited, although possibly a large number of terms of the series will nevertheless completely reproduce the original observations.

The point brought out in this note may be old and well known, but the writer has not as yet seen it discussed,[1] and experience indicates that it is neither avoided nor adequately discounted in many serious studies of alleged periodicities in meteorological data.

The principle may be stated in the following form:

The several terms of a limited and finite Fourier series do not necessarily represent single or unique harmonic components of the data analyzed, but each term may theoretically at least represent two or even several wholly independent components with widely different periods.

The analysis of this question does not appear to present material difficulties.

Suppose we have a long record, say of temperature for a period of 50 to 100 years or more, and that the individual phase values are weekly or monthly means. Now we know that this entire series of values can be exactly reproduced by a Fourier series with a finite number of terms.

Let K = the large number of phase values in the original record.

Let p = the relatively small time interval between phase values.

Let l = the length of period in the same time units.

Now we learn from the Fourier theorem that there will be a term in the series whenever the fraction $\frac{Kp}{l}$ equals an integer 1, 2, 3, etc.; that is, all the features of the original long record will be expressed as if they were equivalent to a long series of periodicities of designated wave lengths. It is of no significance whatever in this study whether the component periods are real or false. Some may be real and others false, or all may be real or false.

Assuming simply for convenience that K is exactly divisible by 4 we find that the last term in the Fourier series corresponds to the wave length $\frac{Kp}{l=2p} = \frac{K}{2}$ and the whole series appears thus:

$$\frac{Kp}{l} = 1,\ 2,\ 3,\ 4,\ 5,\ \ldots\ldots\ldots\ \frac{K}{2} \quad (1)$$

Now we are never able to make practical use of the theory when p is a relatively small interval of time and K is a very large number running into the hundreds or thousands, as assumed above. In practice we content ourselves with making p a relatively large number, P, and K a correspondingly smaller number, k so that the product $kP = Kp$.

It is perfectly obvious that these simplifying assumptions can not in the slightest way affect any of the characteristic features of the original data or the real existence of all of the terms of the series (1) as representing the original data.

We assume in the foregoing that the k phase values at large intervals P are actual observations at the times in question, and we shall continue this assumption for a moment. In actual problems of this kind it is a customary although a faulty practice to assemble a greater or less number of observations contiguous to the desired phase date into a representative mean *as of that date.*

[1] While this note was in process of publication the March issue of the Proceedings of the Royal Society, series A, vol. 105, No. A 731, was received at our library, and the article, A Difference Periodogram, by C. E. P. Brooks, p. 346, brought to my attention. While Dr. Brooks does not discuss in its generality the principle presented by the writer, nevertheless he makes use of it to evaluate short periodicities in the disguise of periods of much greater length.