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THE OCCURRENCE OF HAIL

EDITOR'S NOTE.—In response to the demand for more specific data on the occurrence of hail in the United States, the Weather Bureau began in the April number of this REVIEW the publication of reports on the occurrence of hailstorms as observed by its regular and cooperative observers, numbering approximately 5,200. Cooperative observers report directly to the Weather Bureau officials in charge of the several section centers and these officials in turn transmit the reports to the Central Office in Washington, D. C. The reports are incorporated in the table which hitherto has borne the title "Severe Local Storms." That table will be found on pages 282–284 of this REVIEW, and it will appear in approximately the same position hereafter.

ASCENSIONAL RATE OF PILOT BALLOONS[1]

629.132.1 : 551.55

By WILLIAM C. HAINES, Meteorologist

(Weather Bureau, Washington, May 6, 1924)

Pilot balloons furnish us with an efficient and economical as well as a fairly accurate means of determining the direction and velocity of the wind in the free air. The two-theodolite method, when used in connection with a base line of 2,500 meters or more in length and well chosen with respect to the direction of the wind, will give results as accurate as the readings of the theodolites. However, in exceptionally long observations, an hour or more in length, or when the balloon is moving in the vicinity of the direction of the base line, the results are not so satisfactory. In either case the angles of the triangle become so small that a slight error in the reading of the angles makes a considerable error in the computed distance and altitude of the balloon, and therefore an error in the resulting wind velocity and direction.

In general, the single-theodolite method is better adapted for the procurement of free-air data than is the double-theodolite method, but the accuracy of its results is dependent upon the accuracy with which the altitude of the balloon is known. The Meteorological Section, Signal Corps, carried on during the war an extended investigation in order to develop a formula that would give the ascensional rate of balloons.[2] As a result of these studies, the following empirical formula which is a modification of the Dines' formula was developed and adopted as the one giving the best results:

$$V=71\left(\frac{l^3}{L^2}\right)^{.208} \qquad (1)$$

in which V is the rate of ascent in m./min., l is the free lift or ascensional force in grams, and L is the free lift plus the weight of the balloon. This formula was based on about 1,000 two-theodolite observations taken in all seasons of the year and at all times of the day. After the war a slight revision was made as the result of further study and the inclusion of additional data secured by the Weather Bureau and the Signal Corps. The revision consisted of a change in the constant from 71 to 72 and of the introduction of small additive corrections for the first five minutes of ascent.[3] The Weather Bureau has used this revised formula since April, 1921.

The two-theodolite work has been continued by the Weather Bureau at the various aerological stations in order to verify the ascensional rate formula in use; also to determine to what extent the ascensional rate is affected by convection, and to study the behavior of balloons at high altitudes. The first step taken toward this end was to standardize the ascensional rate of balloons. Since the latter part of 1921, the balloons have been inflated by an automatic weighing device to give an ascensional rate of 180 m./min in both single and double theodolite observations. The balloons used are 6 inch rubber weighing from 25 to 35 grams, and when inflated are approximately 60 centimeters in diameter. The a. m. observations are ordinarily taken between 7 and 8 and the p. m. between 3 and 4, 75th meridian time.

This paper is based on the study of all two-theodolite observations taken by the Weather Bureau since the standardized ascensional rate was adopted, or on more than 800 observations of 10 minutes or more in length. The following method was employed to determine the actual rate of ascent of the balloons at successive altitudes: In order to show to what height convection influences the ascensional rate, the average rate of ascent for each minute for the first 10 minutes was obtained. From altitudes of 2,000 to 11,000 meters, the average rates were obtained for four-minute periods immediately above each thousand-meter level, and above 11,000 meters the average rates for five-minute periods were taken. The data were treated in this manner to get the ascensional rate through the various strata of air from the surface to the highest altitude, independent of convection which might have affected the ascensional rate in the lower levels. The a. m. data, p. m. data, and a. m. and p. m. data combined, were considered separately. The means were plotted as ordinates and the altitudes as abscissae, and empirical curves were fitted to the points by the method of least squares. It was found that the points were best fitted by two equations of the form of

$$R=ah^2+bh+c \qquad (2)$$

in which R is the rate of ascent per minute, h is the altitude in meters and a, b and c are constants. The original data to which the curves were fitted are given in Table 1.

[1] Presented before American Meteorological Society at Washington, April 30, 1924.
[2] Sherry, B. J. and Waterman, A. T., The military Mereorological Service in the United States during the War. MO. WEATHER REV. April, 1919. 47: 218.
[3] Sherry, B. J. The Rate of Ascent of Pilot Balloons. MO. WEATHER REV. Dec. 1920, 48: 692-694.

TABLE 1.—*Number of observations and average rate of ascent at various altitudes. Ascensional rate-altitude curves based on these data*

[Sections of this table show, respectively, one-minute, four-minute, and five-minute averages, as explained in the text]

a. m.			p. m.			a. m. and p. m. combined		
No.	Altitude	Rate	No.	Altitude	Rate	No.	Altitude	Rate
292	102	203.7	513	111	222.3	805	108	215.6
292	296	184.6	513	325	205.8	805	324	197.1
292	478	180.9	513	530	203.6	805	511	196.4
292	660	181.1	513	725	196.8	805	704	190.4
292	841	181.5	513	923	190.6	805	894	187.3
292	1,023	181.2	513	1,110	184.7	805	1,079	183.5
292	1,203	179.5	513	1,295	183.9	805	1,261	182.2
292	1,384	183.0	513	1,478	181.8	805	1,443	182.3
292	1,566	180.1	513	1,660	183.2	805	1,626	182.1
292	1,747	181.4	513	1,843	182.9	805	1,808	181.3
238	2,451	181.1	434	2,455	183.5	672	2,453	182.6
176	3,458	184.0	331	3,470	184.0	507	3,460	184.0
127	4,458	184.0	248	4,450	182.8	375	4,456	184.8
91	5,461	183.8	187	5,458	185.8	278	5,458	184.8
72	6,455	181.4	135	6,473	187.6	207	6,467	185.4
57	7,456	184.2	106	7,475	189.3	163	7,469	187.5
44	8,469	190.4	85	8,470	190.6	129	8,470	190.5
29	9,475	189.8	59	9,478	192.9	88	9,490	191.8
18	10,508	188.2	35	10,498	192.8	53	10,498	191.2
4	11,437	180.0	18	11,656	200.9	22	11,586	195.1
2	12,445	223.7	9	12,640	204.7	11	12,597	208.9
1	13,503	199.0	5	13,591	176.0	6	13,572	181.2
			2	14,695	221.3	2	14,693	222.5

The first four minutes of the a. m. data were fitted by the equation,

$$R = .00013755h^2 - .14505h + 217.353 \qquad (3)$$

and the remainder of the data by the equation,

$$R = .0000002227h^2 - .001358h + 182.884; \qquad (4)$$

the first eight minutes of the p. m. data by the equation,

$$R = .00001714h^2 - .05578h + 226.453 \qquad (5)$$

and the remainder of the data by the equation,

$$R = .0000001214h^2 - .0002299h + 182.973; \qquad (6)$$

and the first ten minutes of the a. m. and p. m. data combined were fitted by the equation,

$$R = .00001682h^2 - .04914h + 216.828 \qquad (7)$$

and the remainder of the data by the equation,

$$R = .0000001737h^2 - .0008857h + 183.827. \qquad (8)$$

Figure 1 represents these equations fitted to the data. Inspection shows the resulting ascensional rate-altitude curves to be decidedly similar, excepting in the lower levels where convection is an important factor. All three curves lie slightly above the 180 m./min. line between the point where convection ceases and the 5 or 6 thousand meter level, thereafter diverging at first slowly, and later more rapidly until the 200 m./min. line is reached by all three curves at an altitude of 14,000 meters, the limit of the a. m. curve. It is evident that in the morning hours before convection sets in no additive corrections are necessary, except to the first and second minutes, whereas in the afternoon somewhat larger corrections are needed than those that have been adopted. Also the assumed altitudes in the higher levels should be modified somewhat. Owing to insufficient data, and the liability of error in computation, the results above 12,000 or 13,000 meters should be held in abeyance until more data are collected at high altitudes.

In order to compare the results of the ascensional rate-altitude curves with our assumed altitudes for the various minutes, it becomes necessary to obtain the altitude as a function of the time. From the Calculus, we have,

$$R = \frac{dh}{dt} = ah^2 + bh + c \qquad (9)$$

Integrating this expression and evaluating the constant of integration, we obtain the equation,

$$t = \frac{2}{\sqrt{4ac - b^2}} \tan^{-1} \frac{2ah + b}{\sqrt{4ac - b^2}} - \frac{2}{\sqrt{4ac - b^2}} \tan^{-1} \frac{b}{\sqrt{4ac - b^2}} \qquad (10)$$

which is the general equation of the time-altitude curves. Plotting this equation by substituting the values of a, b, and c in equations (3)–(8) of our ascensional rate-altitude curves we obtain for various values of h the three time-altitude curves as shown in Figure 2. It will be noted that these three curves lie close together with the same

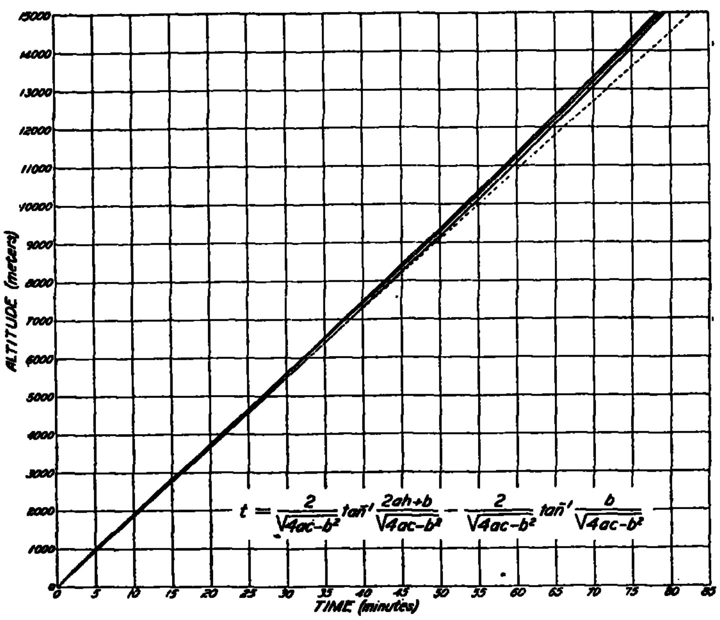

FIG. 1.—Ascensional rate-altitude curves for the a. m. data, p. m. data, and a. m. and p. m. data combined.

general variation from the assumed curve. In the lower levels the a. m. curve lies below the assumed and the p. m. curve above, whereas the curve of the combined a. m. land p. m. data practically coincides with the assumed curve.[4] In the higher levels all three curves lie above the assumed. Table 2 gives the altitudes shown by the curves, and the altitudes assumed for the first five minutes and at ten minute intervals thereafter.

TABLE 2.—*Altitudes shown by time-altitude curves and assumed heights for the first five minutes and at ten-minute intervals thereafter*

Minute	a. m.	p. m.	a. m. and p. m. combined	Assumed
1	202	223	210	216
2	398	429	413	414
3	571	631	609	612
4	752	826	800	801
5	930	1,012	989	990
10	1,830	1,930	1,900	1,890
20	3,640	3,740	3,720	3,690
30	5,450	5,580	5,550	5,490
40	7,290	7,460	7,410	7,290
50	9,160	9,330	9,280	9,090
60	11,070	11,290	11,220	10,890
70	13,060	13,270	13,200	12,690
80	15,140	15,320	15,260	14,490

[4] Cf. Figure 4 in Pilot-balloon observations at San Juan, Porto Rico, MO. WEATHER REV. January, 1924, 52: 22–26. (Discussion by W. R. Gregg and W. C. Haines.)

The curves in Figure 2 show averages. It is desirable also to know how much, or how little, the individual observations depart from these averages. In order to determine the departures in a more or less general manner, the average altitude at the end of the tenth minute was considered, and departures from this determined. This particular altitude was selected as representative, because it is great enough to show the maximum effects of convection, and still include the majority of observations. The number of individual observations that departed 5, 10, 15 per cent etc., above and below this average was determined. This was done for the a. m., p. m., and combined a. m. and p. m. observations. These departures are shown by means of frequency histograms in Figure 3. It will be noted that in the morning hours when convection is of little consequence, 77 per cent of the observations lie within 5 per cent of the average, and 94 per cent within 10 per cent, whereas in the mid-afternoon when convection is an important factor, 37 per cent of the observations lie within 5 per cent, and 61 per cent within 10 per cent of the average. Of the combined a. m. and p. m. observations, approximately 75 per cent lie within 10 per cent of the average.[5] As a usual thing, when convection is most active, the wind speed is low and the resulting error is of little consequence. Nevertheless it is obvious that the accuracy of single-theodolite work will be greatly increased if the observations are taken at the time when convection is least pronounced.

A careful examination has been made of approximately 700 time-altitude curves of two-theodolite observations to determine the effects and extent of convection on individual observations. In general we find convection appreciably affects the ascensional rate only in the lower levels—below 1,500 or 2,000 meters. However, during the summer months and especially at southern stations, the effects are noticeable occasionally to much higher altitudes. The rate of ascent may be either increased or decreased by convection. Table 3 shows the a. m. and p. m. observations classified as to season, no convection, upward convection and downward convection.

TABLE 3.—*Number of observations that showed no convection, upward convection, and downward convection*

Season	a. m.				p. m.			
	No convection	Upward	Downward	Total	No convection	Upward	Downward	Total
Spring	35	1	0	36	24	38	21	83
Summer	68	5	2	75	56	79	44	179
Autumn	94	5	1	100	85	69	22	176
Winter	2	0	0	2	27	7	4	38

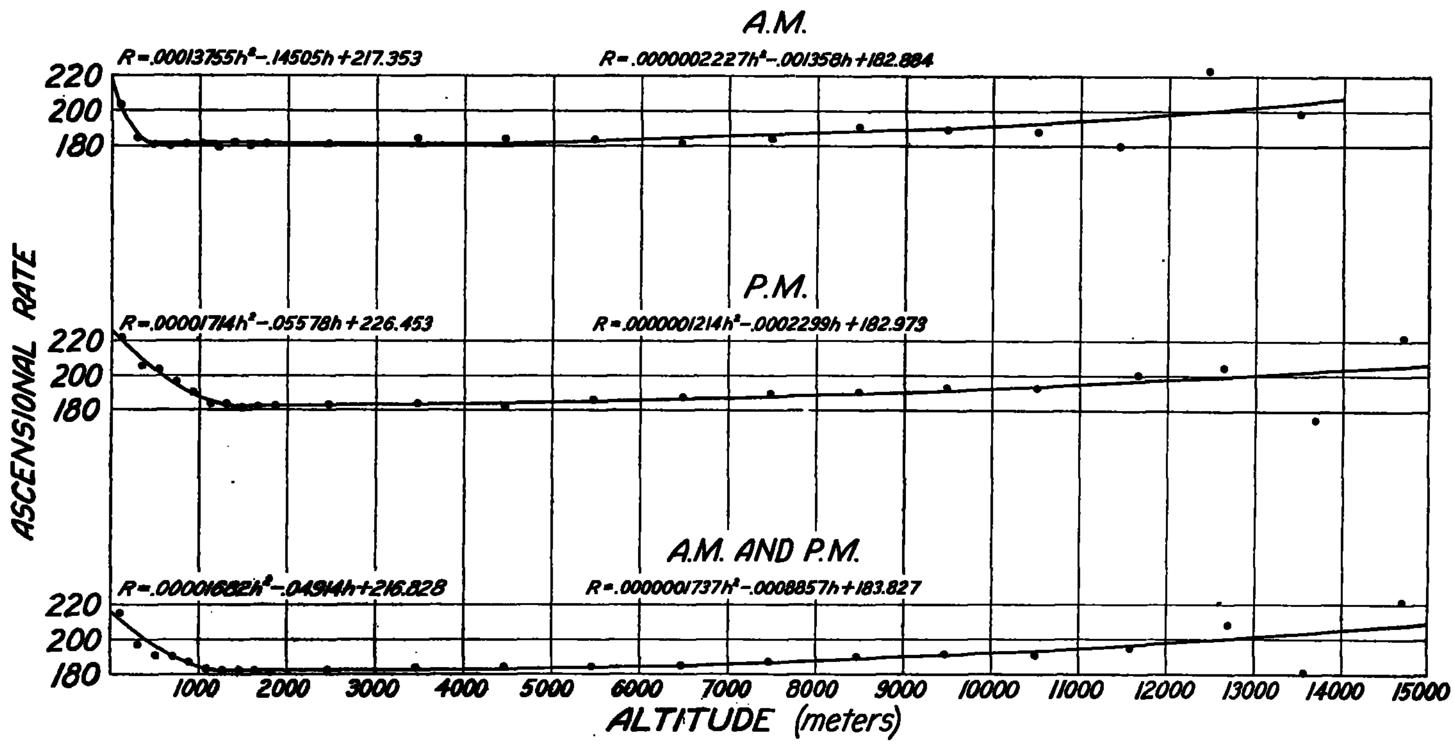

FIG. 2.—Time-altitude curves for the a. m. data (lower solid curve), p. m. data (upper solid curve), and the a. m. and p. m. data combined (intermediate solid curve). The broken curve shows the assumed altitudes.

It will be observed from the table that less than 7 per cent of the morning observations show effects of convection, whereas approximately 60 per cent of the afternoon observations were affected to a greater or less extent. In many cases the effects were not so pronounced as to impair the results materially. The a. m. observations that showed convection to any extent were without exception taken after 8 o'clock. In this connection we mention the fact that the percentage of an error that is caused by convection in the lower levels, becomes less and less with increasing altitude; thus an observation that is in error 25 per cent, or 500 meters in altitude, at the 2,000-meter level, is in error only 10 per cent at the 5,000-meter level, and 5 per cent at the 10,000-meter level, providing the rate of ascent above 2,000 meters is near the assumed.

The percentage of error in the wind velocities of single-theodolite observations caused by convection in

[5] These figures differ slightly from those given on page 25 of the discussion of Dr. Fassig's paper, loc. cit.[4] In that case, however, the variations were from the *assumed* rate; in the present case, they are from the *average* rate. The difference is made clear by reference to Figure 3.

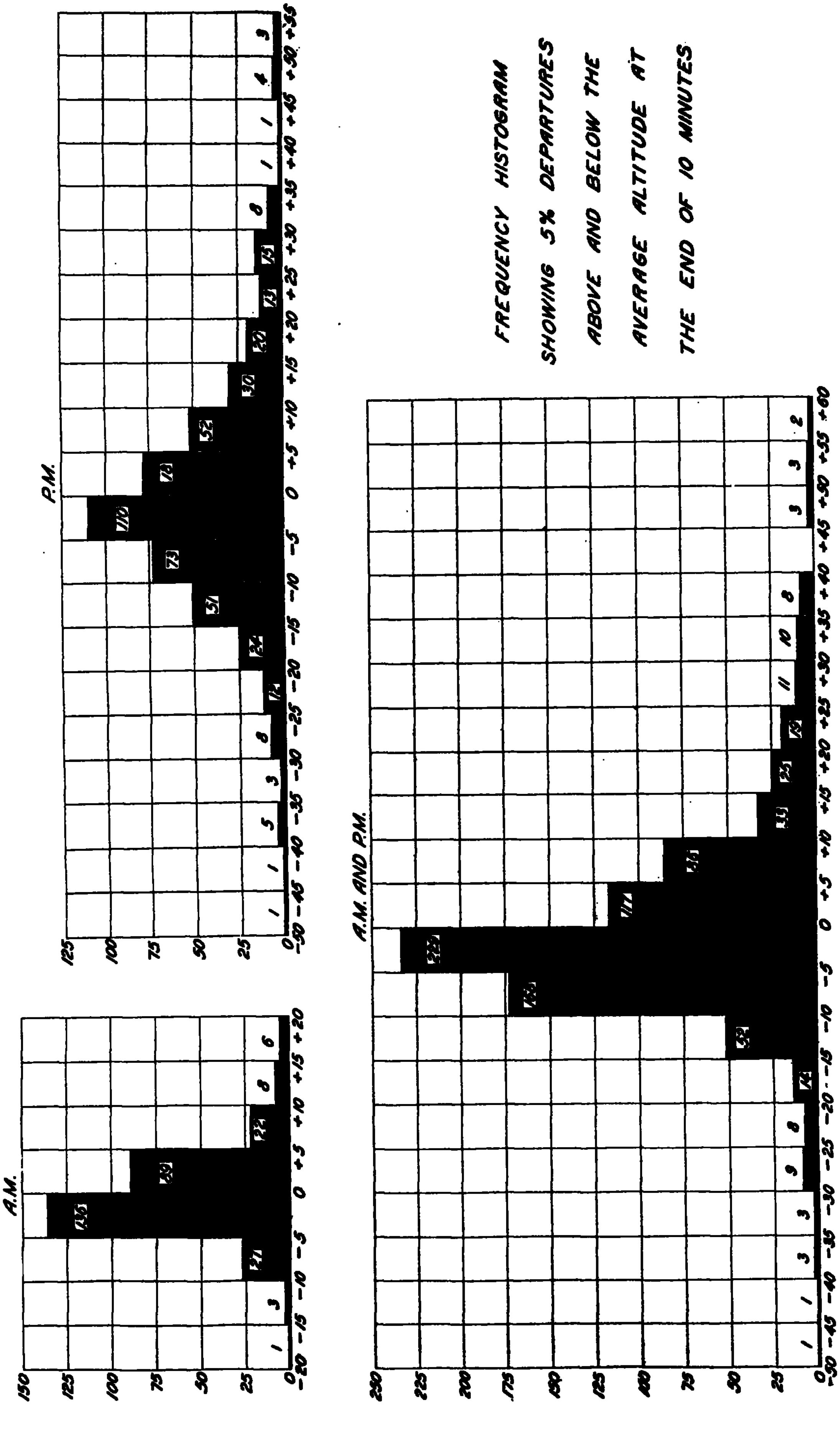
FREQUENCY HISTOGRAM
SHOWING 5% DEPARTURES
ABOVE AND BELOW THE
AVERAGE ALTITUDE AT
THE END OF 10 MINUTES
A.M.
P.M.
A.M. AND P.M.

FIG. 3.

the lower levels is not accumulative, but instead becomes less and less with increasing altitude. In general the percentage of error in the computed velocities is practically the same as that in the assumed altitudes. This is true, however, only when the percentage of error in the assumed altitudes does not change materially from minute to minute as is the case in the higher levels.

Special efforts have been made to determine to what altitude our balloons will ascend. As a rule the observations that reached considerable altitudes ended because of the bursting of balloons, or disappearance on account of distance, haziness or clouds. No cases were found where there was convincing evidence that the balloon reached a state of equilibrium and floated. This is not in accord with the results obtained by British investigators.[6] Our conclusions are based on more than 800 two-theodolite observations, of which more than 50 reached above 10,000 meters in altitude and 1 to an altitude of approximately 15,500 meters.

CONCLUSIONS

1. Although the ascensional rate of balloons is not constant, as has been assumed, there is striking agreement between the assumed and actual heights up to moderate altitudes. In the higher levels the agreement is not so close, however. At an altitude of 15,000 meters the assumed height is approximately 5 per cent below the actual altitude.
2. In individual observations the actual heights at about the 2,000-meter level are within 10 per cent of the assumed in approximately three-fourths of the cases. Observations taken in the early morning hours before convection sets in are within 10 per cent of correct in practically all cases.
3. At moderate heights from 2,000 to 10,000 meters the accuracy is still greater, since convection is absent and the balloons ascend at essentially a constant rate very close, as a rule, to the assumed rate.
4. Convection appreciably affects the ascensional rate only in the lower levels—below 1,500 or 2,000 meters. The effects are negligible in the early morning hours.
5. The balloons continue to ascend at the ordinary rate until they either burst, or disappear on account of distance, cloudiness, etc. No cases were found in which the balloon reached a state of equilibrium and floated. It is worthy of note also that only two or three balloons showed evidence of a slow leak caused by pinholes.
6. As to what takes place above 16,000 meters, we have at present no information. It seems likely, though, that the best balloons continue to rise to perhaps 20,000 meters or more, and that the rate of ascent continues to increase.

Acknowledgment is due to Mr. W. R. Gregg for valuable suggestions and to Mr. Edgar W. Woolard for aid in the mathematical portion of the paper.

551.574 (794)

THE PROBABILITY OF CERTAIN MINIMUM TEMPERATURES IN THE SANTA CLARA VALLEY, CALIFORNIA IN SPRING

By Esek S. Nichols, Meteorologist

[Weather Bureau Office, San Jose, California, March 15, 1924]

The average daily minimum temperature, based on 17-years' records at the Weather Bureau office in San Jose, Calif., is 42.1° for the month of March, 43.7° for April, and 45.9° for May. The lowest recorded in March during the entire period is 30°; in April, 33°; and in May, 35°. On the average, less than 1 day per year in March has a minimum of 32° or lower, tenths of a degree considered; and that minimum has not been reached on any day in April or May during the 17-year period. The average date of the last killing, or very severe frost in spring is February 11; and the latest is March 31. However, frosts of less severity, sometimes sufficient to damage tender vegetation considerably, occur in April and May. Some frost occurs in May in nearly one-half of the years.

The above paragraph contains such minimum temperature and frost data as are usually given in climatological articles. For many purposes, notably in connection with studies of damage to fruit by low temperatures and protection from such damage, more detailed knowledge of the occurrence of low temperatures of different degrees is desired; not only for San Jose but also for other places in the Santa Clara Valley, particularly in the colder sections thereof. This article is written for the purpose of supplying such additional information. Also, a method of increasing the usefulness of a short temperature record taken near a station having a long record is illustrated.

FREQUENCIES OF CERTAIN TEMPERATURES AT SAN JOSE

The first step taken in this study was to determine the number of days in the first half of March (1st to 15th, inclusive) throughout the 17 years of record at San Jose having each minimum temperature from the lowest recorded up to 45°, in whole degree intervals. Then beginning with the lowest, progressive sums were taken, so as to show the total number of days on which the thermometer fell to or below each temperature considered. Dividing these sums by 17, carrying the divisions to tenths, gives the numbers in the second column of Table 1, the average numbers of days during the first half of March with each minimum temperature or lower. Thus, there is an average of 0.1 day per year, or 1 day in 10 years, with a minimum of 30° or lower in the first half of March; with 32° or lower the number is 0.8, or 8 days in 10 years. Similar data were obtained for the second half of March (16th to 31st, inclusive), the two halves of April, and the first 15 days of May, all of which figures are entered in the third and subsequent columns of Table 1.

Figure No. 1 is a line diagram on which have been plotted the data from Table 1, minimum temperatures in degrees as abscissas and average numbers of occurrences as ordinates. For each of the five semimonthly periods considered the resulting curve is roughly a parabola with axis vertical; showing that, for the lowest temperatures in each case, especially the first three or four, increase of frequency with increase of temperature is slower than it is for higher temperatures. Thus, for the fore part of April, increase of frequency from 37° to 38° is much greater than the increase from 33° to 34°. Also, the lower portions of the curves for April, and to a lesser degree that for the latter part of March, are crowded together; showing a comparatively slow decrease in the frequency of very cold days as the season advances. Thus, temperatures of 35° or lower are nearly as numerous in the latter part of April as they are in the first half of that month; and 33° or lower occurs nearly as often in the first half of April as in the second half of March.

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[6] Johnson, N. K., Quar. Jour. Roy. Met'l Soc., Vol. XLVII, p. 49.