A CRITICAL TEST OF THE PLANETARY HYPOTHESIS OF SUN SPOTS

By DINSMORE ALTER

[University of Kansas, Lawrence, Kans., February 23, 1929]

With Schwabe's discovery that there is a cycle in sun-spot numbers, nearly equal to the period of the planet Jupiter, it is obvious that a study of the possibility of the tides produced by the planet Jupiter, on the sun, as a cause of the phenomenon would follow. The first paper on such a hypothesis seems to be that by Fritz in 1866 (**1**). This was followed by a series of papers on the same and related subjects by him and by Wolf, throughout the remainder of the century. It is a great pity that this most important of all series of sun-spot studies is not more generally available. Fortunately synopses and résumés are common. The MONTHLY WEATHER REVIEW has recently published a translation by Mr. Reed of one of Fritz's more important papers, a proceeding in which the editor is to be commended and urged to continue (**2**).

However, the most important single analytical study of the possibilities of the hypothesis was that by E. W. Brown (**3**). In this paper Professor Brown compares the observed epochs of maxima and minima of all available data with the curve formed by a rough combination of the tides of Jupiter and Saturn. The agreement is so excellent that usually it would be accepted as sufficient evidence of the truth of an hypothesis. In this case, however, the fact that the tidal forces are negligible in comparison with the sun's own gravitational field made even such strong evidence seem insufficient and the hypothesis found very few followers. Many other hypotheses have been advanced but uniformly have met with unsurmountable objections. For a number of years the feeling has been growing that the variations can not be accounted for by any fairly constant periodicities and that investigations were a waste of time.

Recently the writer made another analysis (**4**) of the sun-spot numbers, applying the new correlation periodogram. This periodogram, which compares the actual shape of two sections of the curve, and not merely one Fourier term at a time, is more sensitive than other methods in determining, and weighing the probability of actual physical existence of irregularly shaped repetitions of data. In that paper the conclusion was reached that if a series of periodicities do exist which account for the main variations in sun-spot numbers they must be harmonics of approximately 252 years.[1]

About this time a conversation with Professor Brown led to the computation of an ephemeris based on Brown's early paper. The ephemeris fitted the data during the 30 years since the paper had appeared as accurately as during the years used in the original paper. It is given here as Figure 1. Such a fulfillment of predictions was unique in the history of the subject.

Following this, the writer examined the 252-year period to find whether it bore any relationship to the planetary tidal periods. It was found that 250 years, well within the limits of error of his work, was an almost perfect least common multiple of the tidal periods of all the planets from Jupiter outward.

During the past few years modern physical theory and the spectroscopic study of the solar atmosphere have removed the great objection which has existed with respect to the tida hypothesis, by showing that the reversing layer is in almost perfect equilibrium between the inward force of gravity and the outward force of light pressure. Under such a condition, long thought to exist only when the rate of rotation was rapid enough, very small tidal forces might conceivably produce very marked disturbances. If so, they would be a common feature for all stars surrounded by planets.

In all this work the tides of the inner planets, equal in force to those of the outer, were neglected. Sun spots develop slowly and often last for many months. A period of two or three months could not be expected to show a variation comparable with that which would be produced by an equal force acting through a longer period. However, there should be some such variation present, its magnitude being a function of the force and of the length of period through which it acted. An examination of the data for such periods might form a critical test of the hypothesis.

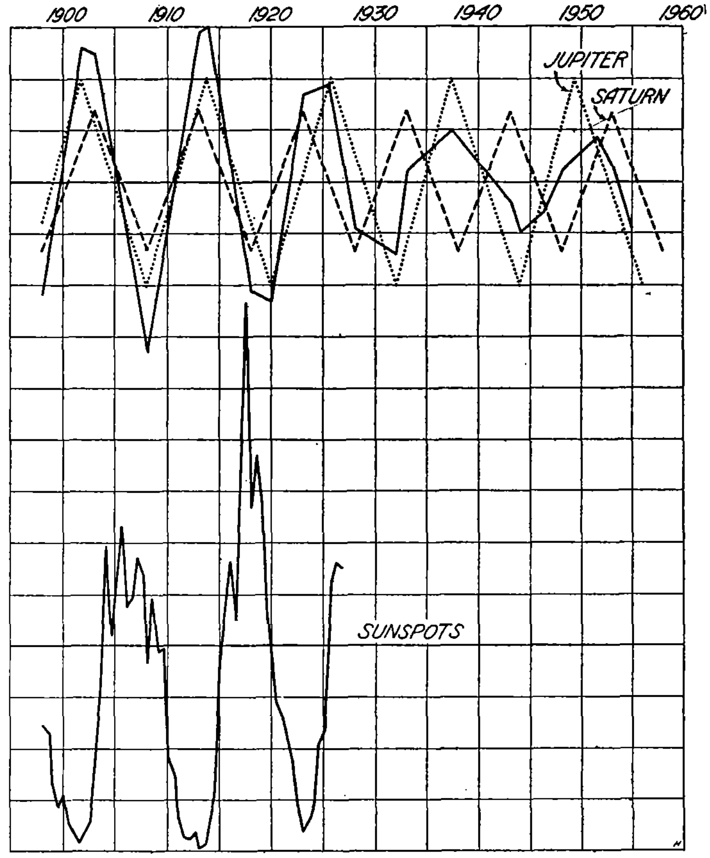

FIGURE 1.—A continuance of Prof. E. W. Brown's curves as published in Monthly Notices of Royal Astronomical Society, volume 60, 1899-1900. The dashed line marked "Jupiter" represents the tidal force due to the eccentricity of that planet and the dashed line marked "Saturn" is the expression of the tidal force due to Saturn's position in the sky with respect to Jupiter and the full curve is their sum. The minima are at the top, the maxima at the bottom

The calculation of the periods and magnitudes of such tidal forces is standard and need not be repeated here. The periods are the synodic periods of the three planets and in addition the period of revolution of Mercury, due to its large eccentricity. The eccentricity of Mercury will also cause a variable amplitude to its combinations with Venus and the earth. The period due to the combination of Venus and the earth may be considered as a

[1] Cf. Alter Dinsmore, A new analysis of sun-spot numbers; Mo. Wea. Rev. 56:401.

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constant one. *We can calculate in advance exactly where such periodicities should show peaks in the periodogram. If they are found there regularly, the truth of the hypothesis is established. If they are not found, or if they are often missing, the hypothesis is not entirely voided, but the evidence against it becomes serious.*

In the calculations it is assumed that the density of the solar atmosphere is small enough that each molecule may be assumed as independent of the others. In this case the forces due to each of the planets may be projected to get a resultant force in considering the tides raised. Forces do not appreciably affect the heights of tides along lines perpendicular to them, in such a case.

The square of the resultant force of the three planets at the point on the sun, where it is at any instant a maximum, is given by:

$$R^2 = f_v^2 + f_m^2 + f_e^2 + 2f_m f_v \cos m + 2f_e f_m \cos (e-m) + 2f_e f_v \cos e.$$

In this equation f_v and f_e are the practically constant tidal forces of Venus and of the earth, the former taken as unity; f_m is the tidal force of Mercury, varying from 0.756 to 0.215, depending on its true anomaly; e is the angle between the earth and Venus and m is the angle between Mercury and Venus. The expression for the angle between Venus and the vector R is not needed in this work. It is stated merely for the sake of completeness.

$$\tan R = \frac{f_m \sin m + f_e \sin e}{f_v + f_m \cos m + f_e \cos e}$$

To express accurately the effects on it of each of the terms involved in R would require a series expansion, similar to that of general perturbations. However, we can very easily consider the magnitudes of the contributions to the square of R. Such are entirely sufficient for our purpose, since we need only the order of importance of the different terms. The numbers given doubly in parenthesis are the maximum and the minimum values of f_m.

$$R^2 = 1 + \begin{Bmatrix} 0.756 \\ 0.215 \end{Bmatrix}^2 + 0.451^2 + 2\begin{Bmatrix} 0.756 \\ 0.215 \end{Bmatrix} \cos m + 0.902 \begin{Bmatrix} 0.756 \\ 0.215 \end{Bmatrix} \cos (e-m) + 0.902 \cos e$$

In the table below where the terms, or periods, are segregated the magnitudes stated are the contributions of the terms to the square of R.

Cause of term	Designation	Length of period	Magnitude
Mercury-Venus	A	72 d. 28	1.512 to 0.430.
Venus-earth	B	291.96	0.902.
Mercury-earth	C	57.94	0.451 times mag. of A.
Eccentricity of Mercury	D	87.97	0.541.

The magnitudes of the contributions to R^2 show that only the first two terms, designated as A and B need be considered. The forces from these terms will average about the same, but the term B is of a period almost exactly four times as long as term A. For this reason we will expect its effect on the number of spots to be considerably greater than that of any of the other terms.

Obviously it is impossible to look for a small amplitude term of period 72.28 days with data which are monthly averages. Fortunately five-day averages, or more accurately, averages over one seventy-third of a year are available for the years 1876–1911. Doctor Elsa Frenkel (now Dagobert) in 1913 investigated, by means of Schuster's periodogram, the possibilities of short periods (5). She arrived at the conclusion that two highly variable periods exist, one of length averaging 68.5 days, the other in the neighborhood of 200 days. The method used, apparently showed the shorter to vary in length from 50 to 100 days. Ten-day averages are about the proper length for our search, accordingly means were made of each consecutive pair of her data. Since her paper is available, the data will not be reprinted here.

We are looking for terms of small amplitude, therefore we must choose the lags for the periodogram, such that there will be many products for each point computed. It was decided to compute a periodogram from lags, varying by two seventy-thirds of a year between 609 and 688 of such intervals. In this stretch we can compute the position of 11 crests of the A term and 2 crests of the B term. The number of products of data formed for each point varies from 705 for the 609 interval lag to 626 for the 688 interval lag. Even such a large number of products will leave accidental errors of somewhat troublesome proportions.

An interesting feature of any sharp peaks to be found in the periodogram, is that they must be followed at an interval of one rotation of the sun by a satellite. If to-day we were to observe an unusual number of sun spots, since many of them would persist, we would have another maximum in about 26 days, due to the adopted method of counting. Such a rotational pseudo period could easily be differentiated from a true one by the fact that after one or more rotations of the sun it would die out, and when such a period would show again in another part of the periodogram, it would bear no phase relationship to the former set, unless its primary were due to a real periodicity. On the other hand, peaks due to a real periodicity would follow regularly at intervals equal to the period length throughout the whole stretch of the periodogram. This last feature is one of the strongest advantages of the correlation periodogram in a search for periodicities.

The 292-day period is long enough that the variation due to the 11-year cycle will mask it, unless the latter is eliminated. The 72.28 day period, however, should show sharp peaks, displaced in the direction of ascent of the 11-year cycle. Each of these should be followed by a satellite. Since our smallest interval for the periodogram is 10 days, the satellites must follow sometimes at 20-day intervals and at others at 30-day intervals. If we find at approximately the computed places, the crests of the A term and find them followed at the proper intervals by their satellites, and if we find few or no peaks other than those computed in advance, the point will be established that the planetary tides are a major factor in production of sun spots. The divergence from such ideal conditions will mark the uncertainty of the conclusion. Afterward it will be necessary to eliminate the 11-year cycle from the results and see whether the 292-day period appears.

It would be natural to expect that if tides do affect the number of sun spots, the part of the sun undergoing the maximum tidal agitation would be that which would show the most spots. There are two such nearly equal areas opposite each other on the sun. If we had used 5-day means we would expect the peaks of the shorter terms to be fish-tailed with a separation of one-half a solar rotation. The 10-day means are nearly enough the half rotation period that we will get merely a broadening and sometimes a displacement either way. This half rotation period can not extend to the interval at which the satellites follow, for they merely, in a lesser degree, retrace what has already occurred in the primary peak one rotation period earlier.

In this paper we are not interested in the actual correlation coefficients. It is sufficient to take terms which are proportional to them. The means of the products formed for each periodogram point are practically so proportional and can be computed in much less time than the coefficients themselves. The amount of work to be done in forming and adding more than 50,000 products is tremendous and any saving of time becomes worth while, so long as it can in no way invalidate the results.

The periodogram as computed, and without elimination of the 11-year term is given as Figure 2. The abscissæ are in units of two seventy-thirds of a year and the ordinates in means of the products used in forming the points. Since the lags vary from a little less than one and a half of the 11-year cycles to a little more, the means of the products are all very large negatives. An arrow, with the letter A over it, points to each place where a crest of the 11-year term has been computed and one with an S over it follows each A by 26 days, to indicate the computed position of its satellite.

Three misses are seen at a glance. The ones at 612 and 648 are extra peaks, the only ones in the whole periodogram. The one at 634 is, of course, a badly displaced A peak. The other 10 A peaks all fall sufficiently close to the computed positions to be called fits. Eight of these peaks come within one periodogram point of the computed positions and are, therefore, exact. Each of the 11 A peaks is followed by its satellite, in only one case at an interval different from the computed 20 to 30 days. The 612 and 648 peaks seem rather large to be accidental but no explanation of them has been found.

All that remains now is to eliminate the 11-year variation from the periodogram to see whether the 292-day period shows and whether its amplitude is greater than the 72-day, as it should be; and whether its crests fall at the proper places. Four times the A period is 289.12 days, almost exactly equal to the long 291.96-day period. If, therefore, we form averages of 29 consecutive points, both these periods are eliminated. Curvature of the 11-year cycle during this interval will cause the means not to fall exactly on the 11-year curve, but the errors caused are not large and can bear no possible relationship to the periods for which we are searching. These means can be subtracted from the periodogram points and will leave us any short variations. Changes in the curvature of the 11-year variation, during the stretch of the periodogram, will somewhat warp our resultant modified periodogram, but can neither introduce nor hide any significant features. Figure 3 is this modified periodogram. In it the A period and its satellites are marked as in the first figure. The computed crests for the B term are shown by arrows under that letter. Both fall in the proper places and the amplitude is found to be around two and a half times that of the 72-day term. The 648 peak now resembles a displaced A peak, rather than an extra one, as we considered it from the original periodogram.

The agreements between hypothesis and observation in this paper and in Brown's early paper are so excellent that there seems to be no question that planetary action, presumably tidal, is the principal controlling factor in the number of sunspots observed. This leads to several interesting speculations regarding the effects of tides of stars on the sun and of the light changes in certain variable stars. The periods considered in this paper have forces equal to those of the outer planets, yet they produce much smaller disturbances. Evidently the variation must continue through a few years if it is to show its maximum effect. Apparently, from the approximate agreement of magnitude ratios in Brown's calculations, periods of a length such as those of Jupiter and Saturn have time to get in their full effect. A star of mass equal to that of the sun would have to approach to within

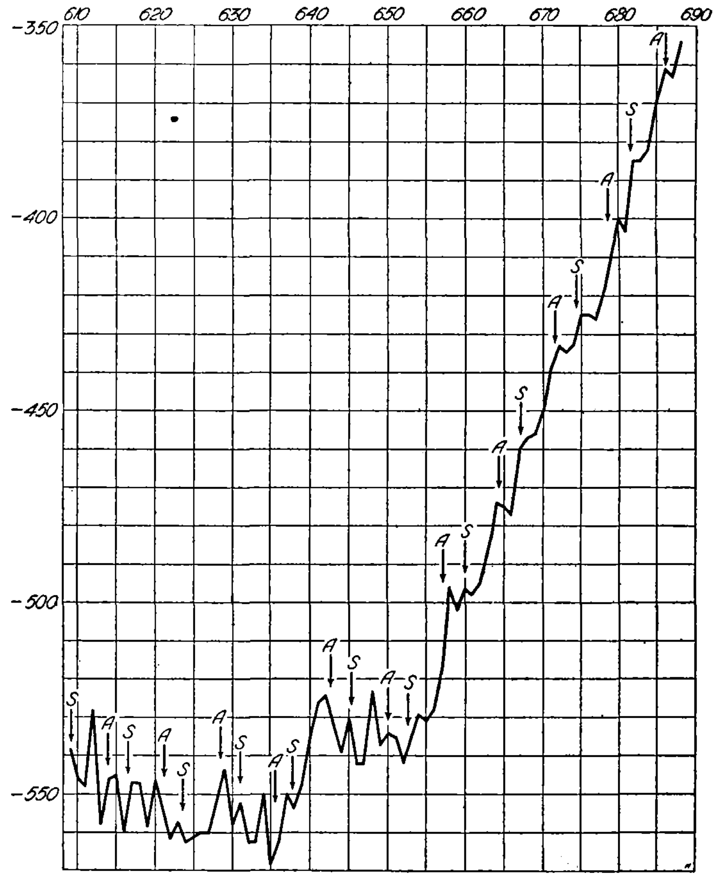

FIGURE 2.—Correlation periodogram of 10-day sun-spot means. The "A" arrows point to the positions where the Venus-Mercury tidal force should bring peaks according to theory. The "S" arrows point to the positions where the satellites of "A" should fall

53 astronomical units to have an equal tidal effect. This is about four thousand times as close as any star known at present. If the sun were distended to one thousand times its present radius, far larger than any known giant, the distance would still have to be four hundred times as close as Alpha Cantauri. Perhaps the writer is in error, for he is no authority on the subject, but he can not see that there would be any tidal difference in the cases of equilibrium due on the one hand to light pressure and gravitation and on the other hand to rotational speed and gravitation. It would seem to him that a star to have had an effect in the evolution of planets would have had to approach within an unreasonably small distance. Material left behind by a pulsating giant during its return from its maximum diameter would seem a more likely source of planetary material. Once the material for one such planet had gathered, it would begin to produce serious tidal disturbances on the giant and probably, for a time at least, cause more material to be lost and larger planets to be formed. Later, as the star contracted,

especially if its oscillations in diameter decreased, planets formed would be smaller. The subject matter of this paragraph is offered very tentatively, and the writer has seriously considered whether its possibilities are great enough to include it in a paper whose body is so much more definite.

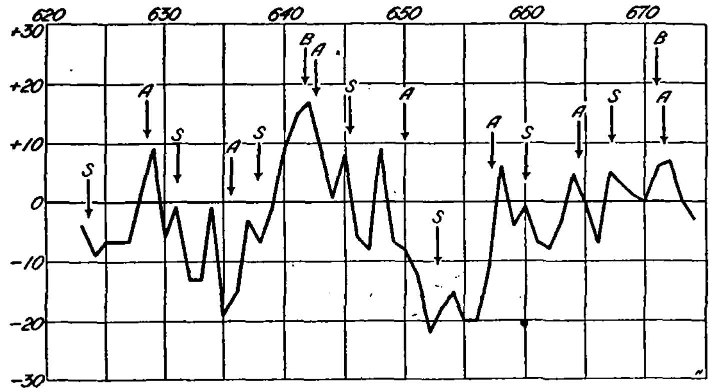

FIGURE 3.—Correlation periodogram of 10-day means of sunspots with 11-year variation eliminated. The arrows "A" and "S" are identical with those in Figure 2. The "B" arrows point where the Venus-Earth term should show maxima

A by-product of this investigation is to show that monthly or semimonthly means, used to compute a correlation periodogram, will in this manner locate the dates of sun-spot maxima and minima more accurately than do the methods now used. There is no single point to stand out and catch the eye, and the result depends upon a consideration of every variation in shape during the cycle.

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LOW RELATIVE HUMIDITY IN OREGON

551.571 (795)

By CHARLES I. DAGUE

[Fire-Weather Warning Service, Portland, Oreg., April 1, 1929]

Relative humidity, in recent years, has become most generally recognized on the Pacific slope, and over many other forested regions as well, as the best and most important index of the inflammability of forest materials. It has provided a simple fundamental basis for determining when fires will burn most readily, when they may be brought under control most easily, or when they will not burn at all. There are other important factors for rating forest-fire hazards such as strong winds, steep slopes, and intense heat from burning materials, but fires will soon die down and remain in a smoldering stage, or go out altogether if the relative humidity becomes high, regardless of wind, slope, or heat.

Past history of forest fires shows that the largest and most destructive fires have occurred, almost without exception, within a group of one to three days during a time when the relative humidity was low, regardless of whether or not the season as a whole was wet or dry. Present-day experiences with forest fires merely lend support to the fact that extreme forest-fire hazards are always the result of low relative humidity. Now it is these days, or groups of days, with low relative humidity that forest-fire protective organizations wish to anticipate so that they may more fully prepare their organizations to cope with any fire situation which may present itself on these days.

The importance of relative humidity as a controlling factor of forest-fire hazard is also indicated in the fact that the Logging Underwriting & Inspection Association writes a special policy containing a humidity warranty for logging risks whereby a logging operator is granted substantially reduced rates on the premium of his policy by agreeing to suspend all logging operations for any period that the relative humidity is 30 per cent or lower.

Since the special fire-weather warning service of the Weather Bureau was established in Oregon late in the season of 1924, fire-weather forecasts and warnings, embodying forecasts of relative humidity, winds, temperature, and thunderstorms, when thunderstorms are probable, have been issued twice daily, both morning and evening, throughout the greater portion of each season. The Weather Bureau, however, as long ago as 1913, began its fire-weather warning service on the Pacific coast by issuing forecasts that were known as fire-wind forecasts. Fire-weather forecasts, as they are known to-day, are available daily to the various forest-fire protective organizations, the logging operators, and to the public, by radio, the newspapers, and the printed weather map. Special fire-weather warnings of approaching spells of moderately low, low, or extremely low relative humidity are telegraphed to the State forester, district fire wardens, forest supervisors, and others whenever they are imminent. These special warnings are usually sent out by telegraph in the evening, the time of day when the protective organizations are most closely in touch with the personnel of their organizations, to provide them with ample opportunity for preparing beforehand for any possible emergencies that may arise. Only in an emergency, due to sudden changes in the weather, are the special fire-weather warnings ever sent out by telegraph in the morning.

The special fire-weather warnings, sent out by telegraph in Oregon, are received by 16 forest fire patrol associations, 5 State patrolled counties, 14 national forests, 2 Indian reservations, and 1 national park. They are also sent, upon request, to several logging operators who bear the cost of having these messages telegraphed. All of these organizations look after the dissemination, in their respective districts, of all fire-weather warnings telegraphed them by the Weather Bureau.

The length of the fire-weather season in the Pacific Northwest has been arbitrarily fixed, for convenience, to