mine it, there was only a slight indication of a "Brückner cycle" of 37 to 40 years.

In the second [29] paper he applied the method of correlation to rainfall data for the British Isles from 1834 to 1924 and found a number of periodicities of which the most important were 24½, 41, and 51 years, which are in the ratio ¾, 1, and 5/4. Others were slightly over 43 years and the Brückner cycle of 34 years. All these are submultiples of a major period of 613 years. Alter calculated the 6-monthly rainfall from the periodicities, and extended the calculated curve to 1940, and though he emphasized that "the future values are given for test purposes only," it is interesting to note that his test prediction indicates a remarkably low rainfall in the first half of 1929, followed by a recovery in the second half—and incidently a slight excess of rain in the first half of 1930, which is repeated here "for test purposes only."

LONG-RANGE FORECASTING BY WEATHER CYCLES IN SUBTROPICAL LATITUDES

It is in subtropical latitudes that rainfall cycles of a few years in length seem to offer the greatest prospect of practical utility. In Java they have been investigated by C. Braak [30] and later by H. P. Berlage [31] and tentative forecasts have been issued for some years. The dominant periodicity here is one of three years, but this breaks down from time to time and begins again at a different phase, so that the statistical result is a periodicity of between three and three and three-fourths years. It is probable that there is a local natural period of oscillation between the East Indies and Australia to which the succession of the seasons gives a length of exactly three years, but this oscillation is alternately set in motion and stopped by some more general cause which recurs at intervals of about three and one-half years or of some multiple of that. Berlage has discovered a cycle in the atmospheric and oceanic circulations of the Pacific which has a length of seven years, and this may be the more general cause which modifies the local 3-year oscillation. A picture of these swings may be given by supposing that a pendulum has a natural period of three seconds, but that the bob is struck from right to left every seventh second

In southern Rhodesia Mr. C. L. Robertson [32] has been making tentative rainfall forecasts by a method of correlation similar to that employed successfully in India, and so highly developed by Sir Gilbert Walker. It is interesting to notice, however, that one of the terms of Robertson's equation is the rainfall of Rhodesia four years before, which has a correlation of —0.33 with the rainfall of the season to be forecast. This negative correlation points to a periodicty having a length of about 8 years or possibly of 2 to 3 years, or of both combined, so that the Rhodesian forecasts are in part based on cycles of rainfall.

In the rainfall, of Algeria [33] L. Petitjean has discovered periodicities of 6, 15, and 35 years, as a result of which the rainfall curve in its broad lines is symmetrical about 1903, that is, the rainfall of 1898–1902 resembles that of 1904–1908, 1894–1898 resembles 1909–1913, and so on. On the strength of this resemblance the author boldly prophesies the general course of rainfall in Algeria up to 1975, and foretells severe famines between 1940 and 1945 and between 1970 and 1975.

The most recent application of weather cylces to long-range forecasting in subtropical regions was made in Australia by Mr. H. A. Hunt,[34] who found evidence for a 4-year cycle in rainfall and temperature, which he attributed to a purely local chain of causes and effects. Some striking results were advanced, but the series of general rainfall values available in Australia appears to be too short as yet for them to be used with confidence in making predictions.

This seems, indeed, to be the chief conclusion to which a consideration of various weather cycles leads. They are either indefinite, or if they are expressed precisely they usually break down when tested over long periods. When a cycle has been found which is of real practical value in forecasting, it will be welcomed by meteorologists even though science may be unable to furnish any clue as to its origin.

[29] A group or correlation periodogram, with application to the British Isles. *Washington*, MONTHLY WEATHER REVIEW, 55, 1927, p. 263.

[30] *Batavia, K. Mag. en Meteor. Observatorium. Verh.* no. 5. Atmospheric variations of short and long duration in the Malay Archipelago, and the possibility to forecast them. By C. Braak. Batavia, 1919.

[31] *Batavia, K. Mag. en Meteor. Observatorium. Verh.* no. 20. East Monsoon forecasting in Java. By H. P. Berlage. Batavia, 1927. (See *Meteor. Mag.* 62, 1927, p. 268).

[32] Robertson, C. L. The possibility of seasonal forecasting and prospects for rainfall season 1922–23. *Salisbury, Rhodesian Agric. J.*, 1922, pp. 648–655.

[33] Petitjean, L. Sur une périodicité et une symmétrie de la courbe des pluies à Alger. Application à la prévision des périodes sèches et pluvieuses en Algérie. *C. R.*, 185, 1927, pp. 472–473.

[34] Quart. Jour. Roy. Met. Soc. 55:323-330

ARE METEOROLOGICAL SEQUENCES FORTUITOUS?

By C. F. MARVIN

[Weather Bureau, Washington, January, 1931]

Any reader who expects this will carry an answer to the question raised in the title is destined to disappointment, but he may profit possibly by the information presented, which is the outcome of a file of correspondence with Mr. S. L. Moyer, civil engineer, interested in flood control and other hydrological problems.

Mr. Moyer says:

> The dilemma of contradictory weather cycles as shown by well authenticated data from two adjoining counties in the States of Washington and Idaho, may perhaps be explained by the examination of the premise which assumes periodicity.
>
> The existence of apparent rhythmic fluctuations in weather phenomena naturally suggests an assumption of periodicity; but to regard such periodicity as proven by the apparent rhythmic flow of the phenomena, is really quite unwarranted. It may possibly be true that haphazard contributions [i. e., departures from normal due to fortuitous influences C. F. M.], considered as resulting in accumulated departures from normal, may also display an apparent periodicity.

Without quoting his reasoning, he, briefly, assumed precipitation could be likened to the haphazard casts of four dice scored by the products of the points, and he adds:

> With this multiple haphazard premise in mind, 600 throws of four ordinary dice were made, the numbers exposed being multiplied together to arrive at a score. Theoretically, the mean or normal score for four dice in multiplication is 150.06, when the full range of opportunities is exhausted. For convenience, the normal was taken to be 150, and the excess above or shortage below this value was carried forward for each throw into a subtotal of accumulated departures. The annexed graph (fig. 1) gives a picture of the results of this process.
>
> This diagram suggests the unsoundness of assuming that an apparent rhythm proves periodicity; but, on the other hand, neither can it be said that the haphazard premise is demonstrated thereby. Each of these views has its value, and when the smoke of controversy clears away there may be reason to believe that the fluctuations in the weather arise out of both periodic and fortuitous sources.

On various occasions the writer has urged that perhaps the best way to examine the question of physical control upon the weather, as contrasted to fortuity, in the order of succession of meteorological data, consists in placing numbers which represent the meteorological data under examination on uniform balls (marbles or steel ball bearings). These are then thoroughly mixed by tumbling for 74 years from three relatively nearby stations, namely, Peoria, Ill., St. Louis and Oregon, Mo., beginning 1856.

The heavy full line represents the accumulated sums of departures from the normal, 36.6 inches, based on the natural order of succession. The remaining lines represent the accumulated sums of the same departures in two fortuitous orders of succession.

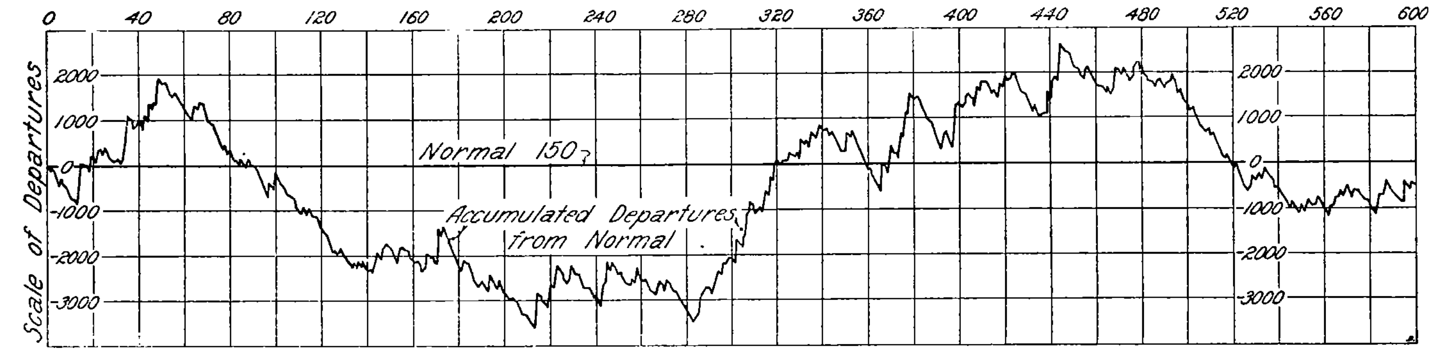

FIGURE 1.—Ebb and flow in luck displayed by throws of four dice scored by multiplication. S. L. Moyer, C. E., Montevideo, Minn.

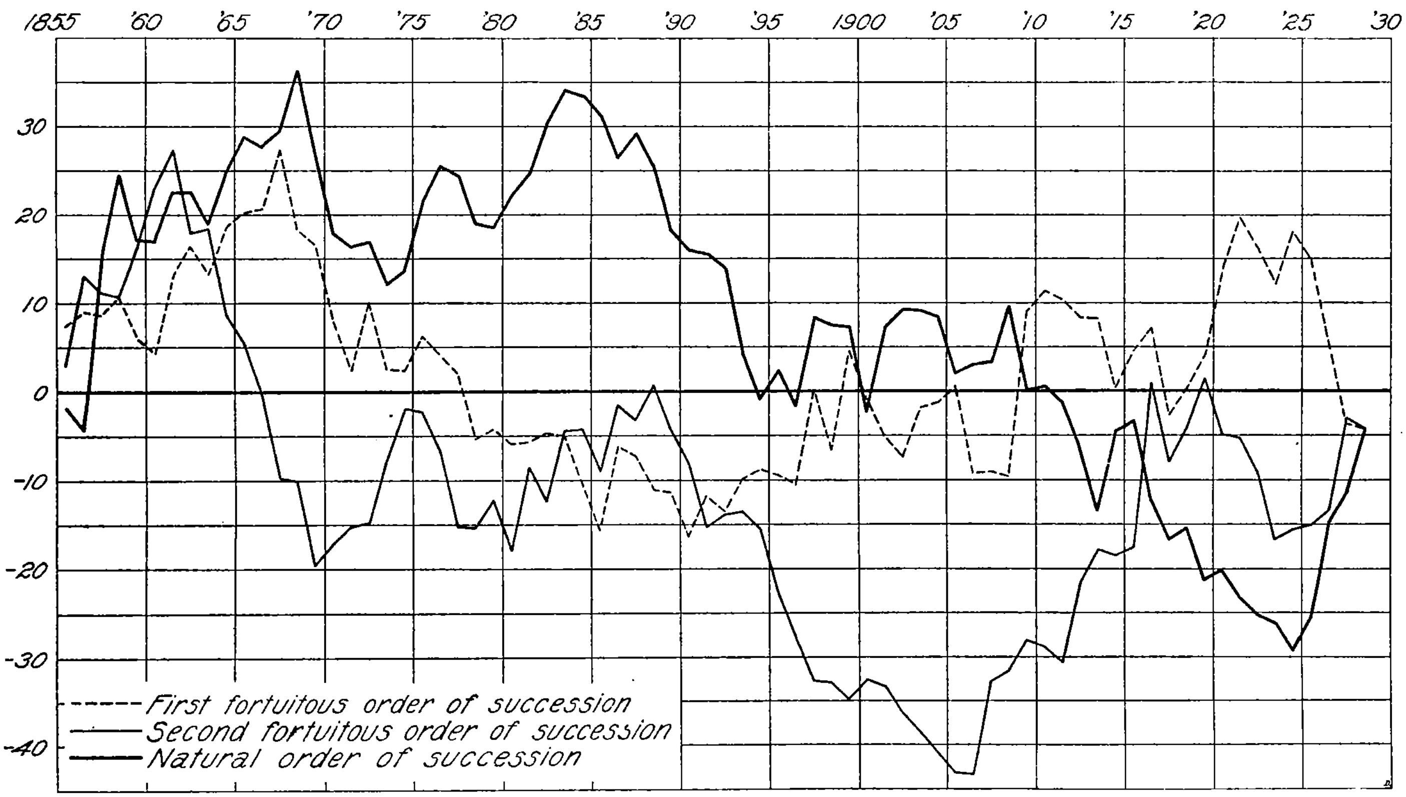

FIGURE 2.—Graph of accumulated sums of departure from mean of 74 years of rainfall record for stations at Peoria, Ill., St. Louis and Oregon, Mo.

about in an adequate container and poured out into a long V-shaped trough or groove, where the balls assume a perfectly fortuitous order of succession, which may be read from either end.[1]

The results of two runs of the balls are shown in Figure 2. The data are the mean values of annual precipitation

The merit of the method of balls in introducing the element of chance in the order of succession is that none of the characteristic features of the numerical data, such as frequency distribution, standard deviation, normal, etc., are changed in the slightest. All remain unchanged except the order of succession.

If not designated on the diagram, who could pick out the natural from the chance order of succession? In the chance order of succession we imagine that the plus and minus departures are equally probable, and that the magnitude of the departures are questions of chance. The line of accumulated sums must therefore tend to cross and recross the base or zero line with frequency.

[1] Some 200 steel balls about seven-sixteenths inch in diameter have been numbered consecutively for this purpose from 1 to 99. A black or blank ball serves as 100 when needed. A second series of 99 balls bear the same consecutive numbers, but these balls are all colored brown or blue by heating. The color scheme is resorted to in order to facilitate separating the balls when less than 100 are desired. The purpose of this is to have individual balls that can represent any sequence of observational data from 1 to nearly 200 items, that is, the observations themselves are numbered consecutively from 1 up to the last observation. The necessary number of balls are chosen to represent the exact number of observations, and after pouring out in a chance order of succession the data are identified by choosing the departures from normal corresponding to the item of record represented by the particular ball in question.

It seems reasonable to expect that some useful information could be gathered from a further application of the methods described. Such studies should of course include analytical discussions of the effects of the so-called "laws of chance," which are words we use to comprise what we know about events which result from the operation of a large number of independent causes.

Unquestionably a comparatively large number of factors conspire in causing any known succession of values of weather conditions. This in itself tends to make sequences more or less fortuitous and involves difficulties in our efforts to segregate the controlled from the wholly uncontrolled or chance happenings.

In order to properly understand the curves from chance throws of dice and the flow of the balls, it is essential to have in mind the frequency characteristics of the dice and ball problems. These are shown in Figure 3. The similarity between the irregular distribution of the small number of unevenly spaced rainfall departures and the theoretically complete set of products of dice is rather striking.

It seems worth while to present here, briefly, how the various sums, products, frequencies, etc., for the throws of four dice can be worked out.

It is well known that the total number of possible combinations of four dice is $6^4 = 1296$. These of course fall in many duplicate products and sums. There are 21 different sums ranging by unit intervals from 4 to 24. As shown, the frequency distribution is exactly symmetrical. The throws yield a possible total of 75 different products, whose frequency distribution is not only strikingly unsymmetrical but discontinuous. The individual frequencies range from 1 to 60. The full details are given in Table 1, which includes every possible type-throw, 126, listed according to the products below and above 150, which is practically the mean product. Each type throw has its own product, which is sometimes the same for as many as four different throws, and for each of which there is a certain frequency which falls in one of five distinct classes, depending upon the likeness or unlikeness of the four digits which appear in each throw, also upon the number of permutations of which each-type-throw is capable. For example, any throw of four dice with the same digit can form but one possible combination, hence it has a frequency of 1 with reference to the total possible cases of 1296.

Class 2 consists of throws in which three digits of the same kind appear, with a fourth different one. Obviously, a throw like 1112 has four permutations, that is, 1112, 1121, 1211, 2111, relative frequency 4.

Class 3 comprises two different pairs of digits, as 2244, which has six possible permutations, frequency 6.

Class 4 comprises one pair of like digits, with two unlike numbers, which arrangement has 12 possible permutations, and therefore a relative frequency of 12.

Finally, class 5 comprises four unlike digits, with 24 possible permutations, and relative frequency 24.

TABLE 1.—*Data on throws of four dice. Products, type throws, and relative frequencies when all possibilites are exhausted*

[Mean or normal product, 150.06]

Products	Products 150 and below					Products	Products above 150			
	Type throws				Combined frequencies		Type throws			Combined frequencies
1	1111				1	1296	6666			1
2	1112				4	1080	5666			4
3	1113				4	900	5566			6
4	1114	1122			10	864	4666			4
5	1115				4	750	5556			4
6	1116	1123			16	720	4566			12
8	1124	1222			16	648	3666			4
9	1133				6	625	5555			1
10	1125				12	600	4556			12
12	1126	1134	1223		36	576	4466			6
15	1153				12	540	3566			12
16	1144	1224	2222		19	500	4555			4
18	1136	1233			24	480	4456			12
20	1145	1225			24	450	3556			12
24	1146	1226	1234	2223	52	432	2666	3466		16
25	1155				6	400	4455			6
27	1333				4	384	4446			4
30	1156	1235			36	375	3555			4
32	1244	2224			16	360	2566	3456		36
36	1166	1236	1334	2233	48	324	3366			6
40	1245	2225			28	320	4445			4
45	1335				12	300	2556	3455		24
48	1246	1344	2226	2234	52	288	3446	2466		24
50	1255				12	270	3356			12
54	1336	2333			16	256	4444			1
60	1256	1345	2235		60	250	2555			4
64	1444	2244			10	240	3445	2456		36
72	1266	1346	2236	2334	60	225	3355			6
75	1355				12	216	1666	2366	3346	28
80	1445	2245			24	200	2455			12
81	3333				1	192	2446	3444		16
90	1356	2335			36	180	1566	2356	3345	48
96	1446	2246	2344		36	162	3336			4
100	1455	2255			18	160	2445			12
108	1366	2336	3334		28					
120	1456	2256	2345		60					
125	1555				4					
128	2444				4					
135	3335				4					
144	1466	2266	2346	3344	48					
150	1556	2355			24					

Number of cases 150 and below	41
Number of cases above 150	34
Total cases	75
Sum of frequencies 150 and below	899
Sum of frequencies above 150	397
Total frequencies	1,296

It will be noted that the gaps in distribution of products can never be filled, however large the number of throws of dice become. Similarly, notable gaps appear in the departures of even long records of rainfall. These considerations tend to support Mr. Moyer's assumption of likeness between rainfall and the casts of 4 or 5 dice scored by products.

Accuracy of the dice.—Theoretically, if any side of any or each of the four dice is a bit heavy, we may feel quite certain that in a large number of throws this heavy side will tend to go down too frequently, with a corresponding effect on the ultimate score.

The writer has found a very delicate test that shows any imperfections of this character. It consists in preparing a liquid solution in which the dice will almost but not quite sink. For celluloid dice a solution of potassium iodide does the trick. The dice must be completely submerged, which requires that the lowermost layers of the solution be a little more dense than those above. The heavy side takes an undermost position, while the dice seems to poise suspended in the solution. Four high-grade commercial dice, nearly 10 millimeters cube, invariably floated with the 4, 5, and 6 sides uppermost at some oblique angle. It is not difficult to understand why this should come about, because it is easy to see that an appreciable amount more celluloid is cut out in forming the small dimples for the dots on the 4, 5, and 6 faces than on the others, making them a trifle lighter than the other faces. The amount of the imperfection of balance could not be easily measured, and it is not known whether it is sufficient to influence the throws. I was prompted to develop and apply the test, however, when I noticed that after some 600 throws of these dice the normal ran appreciably higher than the theoretical normal, 150.06.

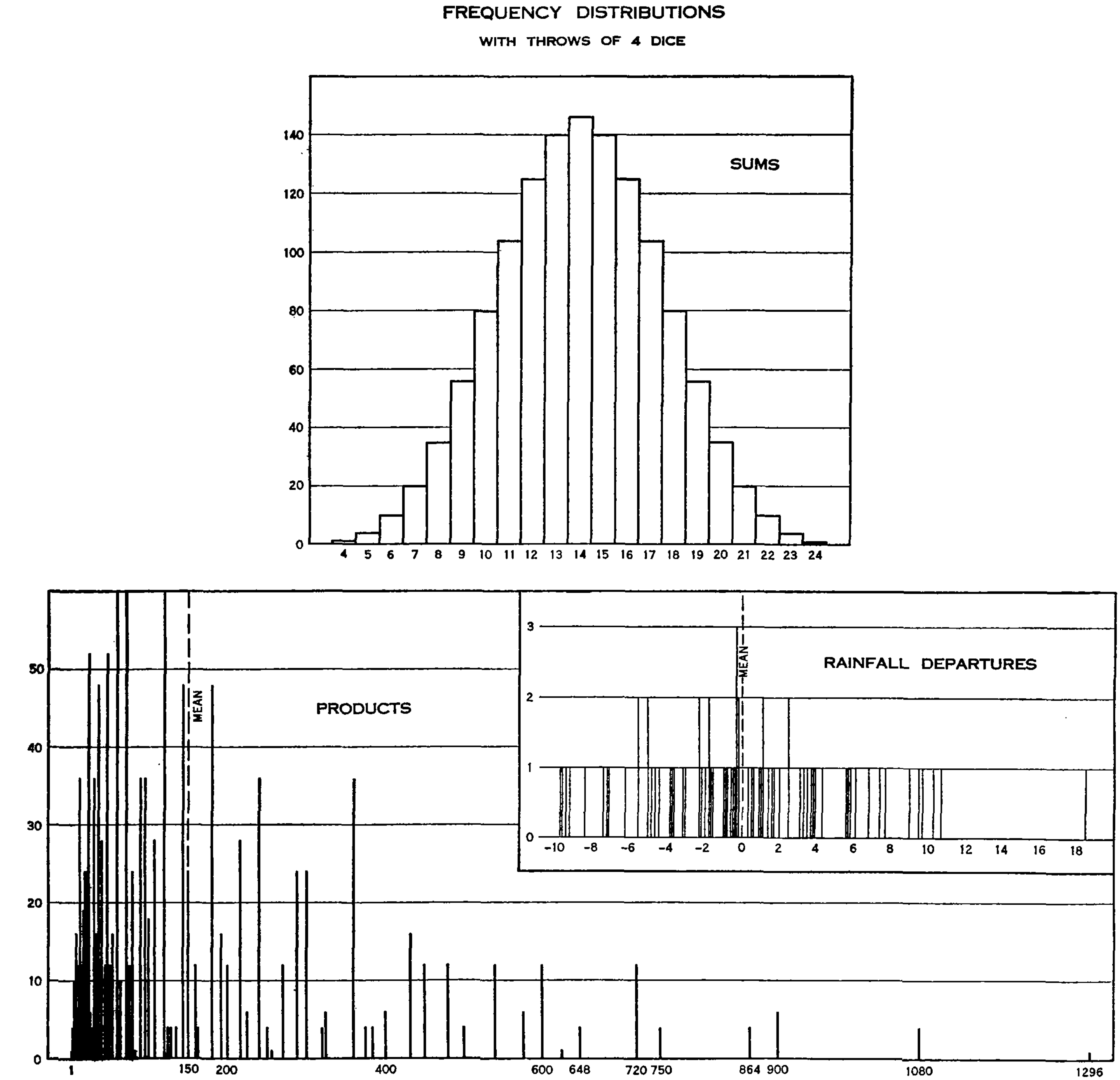

FIGURE 3.—Frequency distribution of throws of four dice scored by sums and products. Inset, distribution of rainfall departures in Figure 2

The average of the first 500 throws was slightly over 157, which gives a decided upward trend to the plot of accumulated sums. No evidence of this effect appears in Mr. Moyer's data, and the discordance just mentioned may still be only a long-sustained dominance of high values by the vagaries of pure chance. The subject is a fascinating and alluring one to pursue.