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THE DIURNAL VARIATION OF FREE-AIR TEMPERATURE AND OF THE TEMPERATURE LAPSE RATE [1]

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I. THEORETICAL DISCUSSION OF THE SUBJECT

A. FACTORS PERTAINING TO TEMPERATURE CHANGES AT THE SURFACE AND IN THE LOWER ATMOSPHERE

1. *Solar radiation.*—It is well known that on the average the temperature at the surface begins to increase shortly after sunrise, reaches a maximum, and then falls until about sunrise the next day. A large part of the increase is caused by the absorption of solar energy, and its magnitude depends mainly upon the following factors:

(*i*) Amount, intensity, and kind of radiation reaching the surface, which in turn depends upon—

(*a*) The quality of the radiation reaching the outer limits of the atmosphere, which quality changes at least with the period of sun spots,

(*b*) The normal intensity of the radiation at the outer limits of the atmosphere. At a given place, this intensity changes with the time of day, with the season and with any variations there may be in the solar constant,

(*c*) The absorption of the incoming radiation by carbon dioxide, ozone, water vapor, dry air, etc., in the atmosphere,

(*d*) The depletion of the incoming radiation by reflection, and scattering by dust, haze, air molecules, clouds, etc.,

(*e*) The length of the day,

(*f*) If on land, the height of the surface above sea level, since it determines to some extent the amount and composition of the overlying atmosphere.

(*ii*) Albedo of the surface, which depends upon—

(*a*) The nature of the surface, such as water, type of soil, snow covering, vegetation, etc.,

(*b*) The angle of inclination of the incident radiation.

(*iii*) Specific heat of the surface material which differs widely between water and soil.

(*iv*) Rate of loss of heat, depending upon—

(*a*) The temperature of the radiator,

(*b*) The nature of the radiating surface,

(*c*) Conduction by the surface material and between the surface and adjacent air,

(*d*) Thermal and eddy convection,

(*e*) Change of phase of water,

(*f*) Advection,

(*g*) Rate of return of heat from the air, reflection by under surfaces of the clouds, etc.

In the lower atmosphere (sea level to 3,000 meters) the percentage of absorption of the incoming solar radiation is, to a large extent, a function of the water-vapor and carbon-dioxide content. Pure dry air in this region absorbs but little incoming radiation of any kind. That which it would absorb has already been taken out by the air above. The carbon-dioxide content of the atmosphere is fairly constant at all times and places. The water-vapor content, however, varies widely with time of day, season, and geographical location. Since it has been quite definitely established that the power of absorption of air for solar radiation increases with increase in water-vapor content, it is obvious that this absorption must be greatest, other things being equal, where and when the absolute humidity is greatest. Likewise, water vapor is a good radiator. Therefore, so far as atmospheric absorption and radiation alone are concerned, we should expect to find the greatest diurnal temperature ranges during seasons and at stations where the water vapor content is relatively large.

2. *Reradiation between the ground, clouds, and atmosphere.*—The radiation involved here is a long-wavelength type of which the air as a whole is more absorptive than it is of solar radiation. The rate at which any particular sample (of air or surface) radiates depends mainly on its absolute temperature and its nature, that is, kind of substance, state (gaseous, liquid or solid), conductivity, etc.

The absolute temperature of the sample depends upon its heat capacity and the amount of heat it absorbs whether supplied by radiation, convection or otherwise, and the amount of heat which, in the meantime, it similarly loses.

Reradiation keeps the temperature of the radiating body lower than it otherwise would be. Evidently, absorption and reradiation jointly produce a diurnal range of temperature both in the free air and at the surface. Since the temperature difference between the ground and the air next to it usually is much greater than that between two adjacent air layers, the greatest exchange of heat per unit area and the greatest diurnal range of temperature takes place near the surface.

Since exchange of heat by radiation between two like objects results in a net loss by the warmer and gain by the cooler, it follows that this process tends always to decrease the numerical value of the average temperature lapse rate in the free air. That is, it tends to bring the air of different levels to the same temperature.

This subject has been discussed by, among others, D. Brunt (5), who obtains the following relationship between change of lapse-rate with height and temperature change with time:

$$\frac{\delta T}{\delta t}=K_r\frac{\delta^2 T}{\delta Z^2} \qquad (1)$$

where T=temperature, t=time, Z=height, and K_r, which is analogous to the conductivity coefficient in the

[1] A good historical sketch of studies made up to 1912 of the diurnal variation of temperature is given by Josef Reger (1). A later study and one which seems to be better supplied with data than these previous studies was made by Hergesell (2) in 1922. The important early work done on this subject in the United States was carried out at Blue Hill (3), and at Mount Weather (4).

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similar well-known differential equation of heat conduction in a solid, is defined by the expression

$$\frac{\delta E}{\delta T}\frac{T}{p_w}\frac{b}{60\rho C_p}.$$

In the latter expression E is the amount of radiation of "wave-lengths within which water-vapor amounting to 0.3 mm of precipitable water radiates like a black body", T and p_w are, respectively, the mean absolute temperature and mean vapor-pressure in millibars within a layer containing this quantity of precipitable water, b a constant, ρ the density, and C_p the specific heat at constant pressure, of the air. Equation 1 is obtained in this relatively simple form only if K_r is considered constant with altitude.

3. *Vertical convection.*—There are two types of vertical convection, (*a*) thermal and (*b*) mechanical turbulence. Both cause mixing of the air and thus transfer heat. In addition, both produce temperature changes by dynamical processes and by helping to change water vapor to liquid or solid water and the reverse. In so far, then, as convection has a diurnal period, to that extent it influences the diurnal temperature march both at the surface and aloft.

Thermal convection on the large scale that leads to the development of cumulus clouds is more or less irregular and will not be further considered here. Mechanical turbulence is more generally present, however, and plays an important role in the diurnal heating of the air up to certain levels.

Due to its flow over irregular surfaces there always is considerable eddy motion and turbulence in the air and, consequently, an absence of persistent instability in the layers so disturbed. Furthermore, some air which was stable in its initial position is constantly being carried upward or downward mechanically by turbulence. If the lapse rate exceeds the adiabatic for the air in question, that which is carried upward will always be at a higher temperature than the immediately surrounding air, therefore, lose heat to it and decrease the lapse rate. Conversely, when the lapse rate is less than the adiabatic, air carried upward always is cooler than the surrounding air, from which it takes heat and thereby increases the lapse rate. In each case the lapse rate is rendered more nearly adiabatic.

Evidently, heating of the air aloft by transfer of heat from the ground by turbulence normally can occur during only a limited time through the day, and extend up only to moderate heights, because for this process to occur the average lapse rate between the level heated and the surface must be superadiabatic. An inversion immediately off the ground prevents convection because any turbulence then starting at the ground is almost immediately damped-out. It should be emphasized, however, that the temperature at any level can be raised due to the vertical transfer of heat by turbulence even though the lapse rate is not as great as the adiabatic. For example, consider a level such that the lapse rate above the level is less than the lapse rate below it. The motion downward serves to heat the air at the level in question and the motion upward serves to cool it. If equal masses are moved upward and downward across the level, it is evident that the effect of the heating will exceed the effect of the cooling, and the temperature at the level will rise.

Taylor (**6**), Brunt (**5**), Richardson (**7**), Schmidt (**8**), and others have treated the subject of heat transfer by turbulence and arrived at various quantitative expressions. According to Brunt (**5**), the heating produced by turbulence can be approximately expressed by the equation,

$$\frac{\delta T}{\delta t}=K\frac{\delta^2 T}{\delta Z^2} \qquad (2)$$

where T, t, and Z are the same as in equation 1, and K is a measure of the capacity of the air for being heated by turbulence. Humphreys (**9**) has shown how this coefficient may be related to wind and rate of evaporation changes.

4. *Advection.*—The largest and most rapid temperature changes in the free air over a given place are caused by the horizontal transportation of heat in the passage of cyclonic disturbances, etc. These changes, however, are so irregular that with sufficient data they should cancel out. In cases where the observations are insufficient in number, or where a preponderance of them was obtained in some particular type of changing weather conditions, the effects of this method of heat transfer appear in the results.

Diurnal variations of wind, such as occur on a sea coast, a mountain slope, or in a valley, produce important effects on the diurnal variation of temperature.

Besides transporting heat mechanically, winds serve in at least two other important ways to affect the diurnal variation of temperature. One is by changing the degree of turbulence, the effects of which have already been discussed, and the other by increasing the rate of evaporation. The latter is a cooling process of considerable effect over moist surfaces.

5. *Changes of phase of water.*—The heat content of water differs so greatly with the phase that any change thereof is an important factor in heat transference. This difference is greatest between the solid and vapor and the liquid and vapor phases. That these changes may be important in the present problem, they must have a diurnal variation, and such diurnal tendencies do exist in the formation of clouds, rain, dew, frost, and in evaporation.

The diurnal tendency to the formation of clouds and rain is most pronounced at certain average levels and in certain seasons, as illustrated by the development of cumulus clouds on summer afternoons. These processes produce a direct effect on the diurnal temperature march in the region of formation and an indirect effect at all levels below it and for some distance above.

The formation of dew and frost, occurring mainly at night, is accompanied by the liberation of considerable heat, and thus has a tendency to reduce the fall in temperature. The evaporation of this dew and frost, as well as the evaporation of surface moisture, during the day has a tendency to reduce the temperature rise of the surface in two ways: (1) Much heat is absorbed in the actual evaporation, and (2) the increased water vapor content of the air serves to increase the depletion of solar energy by the air.

The processes just described, no doubt, have a very important part in maintaining the temperature balance in the lower atmosphere, serving specifically to decrease the diurnal range near the surface and to increase it in the free air.

6. *Conduction of heat between the ground and adjacent air.*—The amount of heat transferred from the surface to the adjacent air by conduction is a function of the rate of flow of heat, which depends mainly upon the temperatures, thermal conductivities, vertical temperature gradients, and specific heats of the surface material and of the adjacent air and indirectly upon the roughness of the

surface. This amount is considerable when the ground is very hot and the wind is strong enough to maintain, by mixing, a large temperature difference between the surface and the adjacent air.

B. THE DIURNAL RANGE OF TEMPERATURE

It is obvious that the diurnal range of temperature at any level depends upon the combined effect of all the factors discussed in I, A, and others not specifically mentioned. The relation between these factors and the magnitude of the range can be more clearly seen, however, if a coordinated discussion is given under the headings indicated in the following paragraphs.

1. *Water vapor.*—Water vapor in the air, as is well known, decreases the diurnal range at the surface by decreasing the maximum and increasing the minimum temperatures. The maximum temperature is decreased largely (1) by the depletion of solar energy in passing through the water vapor, and (2) by the evaporation of surface moisture. The minimum is increased (1) by the absorption of ground radiation, and (2) by the condensation of water vapor to form dew or frost.

At levels well above the surface in an atmosphere which is fairly uniformly humid throughout, water vapor presumably should serve to increase the diurnal range because moist air is a much better absorber and radiator than dry air.

The total effect of water vapor, then, presumably, is to decrease the diurnal temperature range at or near the surface and to increase it beyond certain levels above the surface. This should be true quantitatively as well as qualitatively; hence, other things being equal, the diurnal range at the surface should be least at stations where the amount of water vapor in the atmosphere is greatest and at these stations the rate of decrease of range with altitude would be least. This presumption appears to be supported by the data presented later.

2. *Nature of the surface.*—The nature of the surface largely influences the diurnal temperature range to a considerable height. Thus, the range is much less over snow, a good reflector, than over bare soil, a much better absorber. Hence in mid latitudes the range generally averages less during winter than summer.

Also, since wet surface material has a greater specific heat than dry its temperature is raised less for the same amount of energy absorbed, and due to its lower temperature it cools more slowly. The effect of this factor therefore, most pronounced over a body of water, is to decrease the diurnal range.

Evidently too, the temperature at a ground surface, and its diurnal range, are materially affected by the conductivity of the soil, in such manner that the higher the conductivity the less the range.

3. *Mean air displacement.*—The observational data for a diurnal study often can be so treated that the warming or cooling that accompanies the passage of cyclones and anticyclones does not appreciably affect the *direction* of the average diurnal change at any hour. The magnitude of the average diurnal range, however, is altered appreciably, if not materially, by such disturbances when frequent. Thus, frequent periods of decreasing or low temperature, resulting in a large number of days during which the temperature rises only a relatively small amount, reduce the average diurnal range. Conversely, it is increased by frequent warming or warm periods.

The conditions just described and their attending effects are more or less irregular in frequency and magnitude. The average direction, velocity, and origin of the wind also have an important bearing on the magnitude of the diurnal range. Air coming from a relatively warm region, as over land on clear summer days, serves to augment the effect of solar radiation during the day, and when this wind decreases in velocity or stops at night, the diurnal range is increased. A system of land and sea breezes, however, decreases the diurnal range.

At stations where upslope winds or downslope winds are prevalent the effects of dynamical cooling or warming are important. Nightly air drainage at mountain and valley stations also is important, as it causes a greater diurnal range at the surface in the valleys than on the slopes.

4. *Number of hours of sunshine.*—The number of hours of sunshine varies greatly with the season in high latitudes, and, generally, with latitude, on even the same day. The relation between the diurnal range and the number of hours of sunshine is quite obvious, but attention must be called to the effect of this relationship in the two seasons, spring and autumn. In the northern hemisphere the average duration of sunshine in the spring months—March, April, and May—is longer than in the autumn months, September, October, and November. This difference increases with latitude. Hence, other things being equal, the diurnal range should be greater in spring than in autumn, and the difference in range between the two seasons should be greater, the greater the latitude. The effect of this factor upon the diurnal range is in the same direction at all levels.

5. *Miscellaneous.*—While the factors already named have the greatest controlling influence upon the diurnal range, sometimes certain others, such as cloudiness, smoke, haze, dust, height of station above sea level, and physiographic environment, also are important.

Thus, near industrial centers the average amount of smoke is considerably greater than in the open country, resulting in a great decrease in the amount of solar radiation reaching the surface. The effect on the diurnal temperature range is obvious. This condition generally is most pronounced in winter.

As to height, the higher a station the more intense, in general, the sunshine, and the less the sky radiation, and therefore the greater the diurnal range.

C. THE DIURNAL VARIATION OF TEMPERATURE LAPSE RATE

Since reradiation, as previously explained, tends to remove all "kinks" from the temperature-height curve, while turbulence tends to produce an adiabatic lapse rate, therefore each helps to determine the temperature change at any particular level. Hence, in so far as these two phenomena themselves have a diurnal variation they tend to produce a like variation in the temperature lapse rate.

If the temperature changes occur substantially independently of the lapse rate, any change in the latter with time is a function of the variation of the magnitude of the diurnal range with altitude and of the change of phase of the diurnal temperature period with altitude. Thus, if there is no variation of these 2 controlling factors with altitude between any 2 levels, there will be no diurnal variation of the average lapse rate between these 2 levels. On the other hand, when conditions are such that the temperatures at any two levels, between which there is a positive lapse rate, are rising or falling at unequal rates, it is evident that the lapse rate between the levels will correspondingly change. When the temperature at the lower level is rising faster than the temperature at the upper level, the average lapse rate between the two will be increasing; and when the reverse conditions prevail, the lapse rate will be decreasing with time.

During the morning and early afternoon we should expect to find increasing lapse rates, and during the remainder of the day we should expect decreasing lapse rates to occur. The effect of the variable diurnal march of temperature at different levels is thus equivalent to a diurnal march of the lapse rate.

II. THE OBSERVATIONAL DATA AND METHOD OF SMOOTHING

A. THE DATA

The data upon which the present study is based were obtained by means of numerous series of kite flights made over a number of years at six aerological stations in the United States. Pertinent facts regarding these stations and the period of years covered will be found in table 1. A series of kite flights consists of a set of flights made as frequently as possible. Kite flights are usually of such length that about seven to nine flights can be made in a 24-hour period.

In making these flights the Marvin-Hargrave kite was used and the free-air data were recorded by the Marvin kite-meteorograph. The procedure employed in making and computing a kite flight is described in **(10)**.

It was intended that two series of flights each continuing over a period of at least 24 hours be made each month. This schedule could not always be carried out, however, due largely to the relatively small number of days when there was sufficient wind over a 24-hour period to fly kites. As a result there was much irregularity as to elapsed time between series, between flights in any one series, and in the heights reached by the individual flights in any given series. Furthermore, the different series were neither started at the same time of day nor did they all continue for a minimum period of 24 hours. These irregularities caused many difficulties in the smoothing of the data.

B. SMOOTHING OF DATA: DIURNAL VARIATION OF TEMPERATURE

In the first step toward smoothing the original data the series were treated as units. For each series the measured temperatures at each of the standard levels up to and including 3,000 m, m.s.l., were plotted against time. A smooth curve was then drawn through each of the sets of points (for each level) and the temperatures at the exact hours read from the curves. Both the ascent and descent records obtained from each individual kite flight were used so that each flight provided two measurements of the temperature at each standard level. Thus, the plotted points were usually never more than 2 hours apart for the levels near the ground. For the higher levels the plotted points were often as far as 3 hours apart (between flights) because at these levels there was less elapsed time between the ascent and descent measurements. It will be evident that this procedure provided hourly temperatures at each standard level for the duration of each series.

Several methods of summarizing similar data have been used in diurnal studies but none appears to be entirely satisfactory. Since the temperatures obtained in the first step did not all cover the same period of hours, the diurnal trend could not be determined from the averages of the hourly temperatures. Any method employing the differences for each hour from some reference hour or from the mean for the day has the especially objectionable feature of being adaptable only with great difficulty to data which do not cover a uniform period. Thus, if one is using the hourly differences from the daily mean, only those observations which cover a 24-hour period can be used. This necessitates discarding much otherwise good data and this is very objectionable when the data are already meager.

The method of smoothing used by Hergesell **(2)** appears to be quite satisfactory, but necessitates a great amount of work, since the computation involves a least-squares solution of many equations.

The fundamental thing which it is desired to know is how the temperature normally behaves from hour to hour. The method of determining this by averaging *observed temperature changes from hour to hour* at once suggests itself. This is the fundamental principle of the method of smoothing used in this investigation, which was as follows.

When the observed temperatures for any level are plotted as ordinates against time as abscissae and a smooth curve is drawn for each series, each such curve discloses a more or less regular diurnal period. The true nature of this period is not immediately disclosed, however, because combined in the curve as drawn are (*a*) the average annual trend for a calendar day, (*b*) the average trend resulting from the accumulation of systematic errors, and (*c*) the average trend resulting from the accumulation of accidental errors. Therefore, segments of these curves taken for 1 calendar day (24 hours) are not identical. By superimposing a number of such segments obtained from a limited period, such as a season, and then drawing a smooth curve to represent their average trend we obtain a curve which shows the average diurnal trend over the period considered, viz, the season, together with the average annual trend, any systematic errors present, and all accidental errors which have not cancelled out.

The most important causes of the systematic errors just referred to are the limitations inherent in the making of kite observations. Since the wind must be fairly strong, say 8 to 12 miles per hour, before kites can be flown, all kite flights are made in more or less particular conditions. When a series is to be made, the conditions must be of a still more special nature, because in this case the wind must continue to be sufficiently strong for at least 24 hours. It was found by experience that such conditions were usually most likely to occur as a low pressure area was moving toward the station. This meant that usually the temperature in general was rising during a series, which, in fact, is borne out by the nature of the residuals, V, discussed later.

Owing to the effects just considered, the initial and final temperatures of the average diurnal curve obtained are not identical—i.e., the curve is not periodic. The normal curve, however, which it is desired to find, is free from these effects. This normal curve can be approximated by removing the difference between the initial and final temperatures on the average curve by a prorating process.

Let T_i = temperature at the i^{th} hour of the day as would be given by the normal curve for any given season,

$\Delta T_i \equiv T_i - T_{i-1}$ = difference in temperature between the i^{th} and $(i-1)^{th}$ hours on the normal curve,

$d_{ji} \equiv T_{ji} - T_{j(i-1)}$ = observed difference in temperature between the i^{th} and $(i-1)^{th}$ hours on the day j, and $D_i \equiv \frac{1}{n_i}\sum_{j=1}^{n_i} d_{ji}$ = average observed difference in temperature between the i^{th} and $(i-1)^{th}$ hours for n_i days during the given season, where n_i = number of days during the given season when temperature observations were available for both the i^{th} and $(i-1)^{th}$ hours.

Since ΔT_i is nearly equal to D_i, we may write

$$\epsilon_i \equiv \Delta T_i - D_i \tag{1}$$

where ϵ_i represents a small correction which, when added to the average temperature difference D_i gives the normal temperature difference ΔT_i.

From equation (1) we have

$$\sum_{i=1}^{24} \epsilon_i = \sum_{i=1}^{24} \Delta T_i - \sum_{i=1}^{24} D_i \tag{2}$$

By hypothesis $T_i = T_{(i+24)}$, or $T_{24} - T_0 = 0$. Hence,

$$\sum_{i=1}^{24} \Delta T_i = \sum_{i=1}^{24} T_i - \sum_{i=1}^{24} T_{(i-1)} = T_{24} - T_0 = 0 \tag{3}$$

Substituting equation (3) in (2) we obtain

$$\sum_{i=1}^{24} \epsilon_i = -\sum_{i=1}^{24} D_i \equiv V, \text{ say.} \tag{4}$$

If the number of observations is reasonably large, we may assume that as a close approximation the average value of ϵ_i obtainable from equation 4 may be substituted for the individual hourly values, hence,

$$\epsilon_i = \frac{1}{24} V \equiv -\frac{1}{24}\sum_{i=1}^{24} D_i \text{ (approximately)} \tag{5}$$

or by equation (1)

$$\Delta T_i = D_i + \frac{1}{24} V \tag{6}$$

In a few cases V was found to be relatively large owing to the accumulation of systematic and accidental errors. In these cases, inspection of the observed differences, d_{ji}, usually disclosed some values which were so large that they were obviously erroneous. These values could then be discarded on the basis of their deviations from the mean value. Otherwise, equations 5 and 6 were considered valid and were used throughout.

The values of ΔT_i, as well as the values of T_i, clearly form a periodic function of time. Therefore, the values of ΔT_i obtained in the manner just described were represented by means of a Fourier series of four terms, the arithmetical values already computed being smoothed in this manner. The equation used in this step may be written,

$$\Delta T_i = f(t) = a_1 \cos \Theta + a_2 \cos 2\Theta + b_1 \sin \Theta + b_2 \sin 2\Theta \tag{7}$$

where $\Theta = \frac{2\pi i}{24}$. In this computation i was considered to be zero (or 24) at 1 a.m., and 23 at midnight.

It will be evident that the constant term a_0, usually required in such a series, is zero in this case because the values of ΔT_i are relative numbers. The constants a_1, a_2, b_1, and b_2 were computed by the method of least squares. New smoothed normal temperature differences were then computed from equation 7 and regarded as the final values. Table 2 shows the values of the constants a_1, a_2, b_1, and b_2 for each level, season, and station.

The reliability of the ΔT_i's obtained as described, depends largely upon the number of observed differences in temperature, which itself differed for the various hours of the day. Ordinarily, a series was begun between 8 a.m. and noon. In most cases the series continued for about 30 hours, so that in a great many instances a series furnished two observations of the temperature difference for these morning hours. In many cases the series was not continued for 24 hours, and so lacked observations for the early morning hours (2 a.m. to 8 a.m.). It was thus found that as a general rule the greatest number of observations occurred at about 11 a.m. and the least at about 6 a.m. Table 3 shows the total number of observations for the four seasons for these two hours arranged according to stations and levels. The number of observations in a single season may be found roughly by dividing the tabular value by 4. The number of series is roughly equal to the number of observations at 6 a.m.

The method of smoothing described above is believed to possess the following important advantages:

(1) Temperature differences are used throughout and the results thus obtained are more reliable than those gotten by the use of actual temperatures.

(2) All data obtained during a series can be used regardless of whether or not they are continuous over a 24-hour period from some reference hour.

(3) A convenient and justifiable basis is provided for discarding data which obviously are either erroneous or so abnormal that they would injure the quality of the entire work. Discarding can be done on the basis of least squares. Any observational value can be rejected without disturbing any other value. The observed differences usually are so small and uniform that any large differences can be immediately detected and investigated.

(4) Small numbers are used throughout, thus reducing the work.

(5) The values of the ΔT form a comparatively smooth curve without further smoothing by graphical means or by means of a series representation.

C. COMPUTATION OF SMOOTHED VALUES FOR THE DIURNAL VARIATION OF LAPSE RATE

The diurnal variation of the lapse rate can easily be computed if the diurnal variation of the temperature is known, as follows:

Let T_i = the normal temperature at any standard level, at hour i,

t_i = the normal temperature at the next higher level, at hour i

L_i = the average lapse rate between the levels at hour i,

$= -\frac{t_i - T_i}{\Delta h}$, where Δh = the difference in altitude of the two levels in hundreds of meters.

$L_i - L_{(i-1)}$ = the change in the lapse rate from the hour $(i-1)$ to the hour i.

Then,

$$L_i - L_{(i-1)} = \frac{-(t_i - t_{(i-1)}) + (T_i - T_{(i-1)})}{\Delta h} \tag{8}$$

Since $(t_i - t_{(i-1)}) \equiv \Delta t_i$ and $(T_i - T_{(i-1)}) \equiv \Delta T_i$, we get

$$L_i - L_{(i-1)} = \frac{\Delta T_i - \Delta t_i}{\Delta h} \tag{9}$$

The smoothed temperature differences computed by means of equation 7 thus can be used in equation 9 for the computation of smoothed hourly lapse-rate changes.

III. COMPARATIVE DISCUSSION OF THE SMOOTHED DATA

A. THE DIURNAL TEMPERATURE TREND

Hourly values of ΔT_t for each level up to 3,000 m, m.s.l., for each season and station were computed by means of equation 7 (sec. II) and accumulated from hour to hour. The accumulated values were plotted against time of day as shown for all seasons and stations in figures 1 to 6, inclusive. The times against which the accumulated values are plotted are those of the meridian in local use.

The curves represent the direction and magnitude of the temperature changes during the day—no actual temperatures being known. Once the normal temperature has been determined for any level at any particular hour, however, the values indicated by the curves (or computed by means of equation 7) are reliable enough to give a close approximation to the normal temperature at any other hour.

The curves that show the diurnal trend at the surface are similar to others previously published, the greatest difference being in the diurnal range. On superimposing the various surface curves given in figures 1 to 6 for any given season, one finds that the range varies considerably and that the maxima and minima at the different stations do not occur at the same standard time but at substantially the same local solar time. All the stations but one, Due West, use 90th meridian time, though they vary considerably in longitude.

Perhaps the outstanding feature of the curves for free-air temperature trend, by comparison with most of the curves heretofore published, is the absence of sufficient visual evidence (the results were not subjected to harmonic analysis), to indicate definitely the existence of any other than the 24-hourly temperature period. Such periods are not plainly indicated by inspection for the levels up to and including 1,500 m, m.s.l., and for levels beyond this no great reliance can be placed in the present values. The greatest difficulty in obtaining reliable values at or above 1,500 m is the fact that at about this height the periodic changes have become so much smaller than the aperiodic that irregularities occur unless the number of observations is very large. For example, the failure of one large aperiodic temperature change to cancel in the data for the 2,000-m level could easily produce a change in the entire appearance of the curve for this level, while with the same number of observations the same error at the surface might produce no noticeable result. In actual practice the greater the height the fewer the observations, and therefore the percentage error in the results for the upper levels is greater for two reasons.

The curves for the standard levels between the surface and 1,500 m, m.s.l., indicate, with only two important exceptions, a very regular diurnal variation of temperature. The magnitude of the period decreases with height and the time of the maxima and minima occur later in the day as the altitude increases (up to certain limits). Evidently, however, curves for any standard height above sea level for different stations are not intercomparable, because the diurnal variation occurs largely with respect to height above ground, and not sea level, and no two of the stations have the same sea-level height.

To compare the displacement of the time of maximum with altitude, the times have been plotted against height above ground for the surface and the standard levels. The results for levels up to 1,500 m, m.s.l., are shown in figure 7. At greater heights there is little or no apparent consistency, the times being spread throughout the day and, seemingly, just as likely to occur at one hour as another when all seasons and stations are considered. This fact appears to deny the existence of a definite diurnal period at these higher levels. It may be, however, (*a*) that a period does exist but is masked by the relatively large errors present, or (*b*) that the diurnal periods at these high levels are actually very different in phase for the various stations, seasons, and levels.

The latter alternative conclusion is possible but does not seem probable.

The two exceptions previously mentioned to a regular diurnal trend of temperature in the lower levels are at Ellendale in winter. Here the winter curves for the 750 and 1,000 m levels indicate a decided tendency toward a secondary maximum between 1 and 3 a.m. The data for these levels were examined carefully for large errors but none was found. The numbers of observations in the various hours were no smaller than for many other curves. Therefore, even though this condition did not appear to be present at the other stations, it seems likely that it is one which actually exists. Since it amounted to only a few tenths of a degree its cause possibly may be dynamical heating due to the sinking of large masses of stable air.

It has been said that a definite 24-hour period exists at 1,000 m, and that the existence of this period at 2,000 m is, in many cases, not clearly indicated by the present data. The 1,500-m level, then, occupies a unique position. We might expect this level to lie either above or below the upper limit of a pronounced diurnal period. But this limit probably changes considerably with season, according as the upper limit of turbulence, the average cloud heights, etc., change with season, and it certainly varies between stations because of their differences in altitude above sea level and the differences in their surrounding topography. Therefore, we should expect to find, as we do, little pronounced regularity in the 1,500-m, m.s.l., curves. In many instances the curve for the 1,500-m level differs from the curves for the levels both above and below, though usually, it is similar to one or the other.

As shown later, there is a region between 1,000 m and 2,000 m above sea level at all the stations under consideration in which the diurnal range is very small and above which the range decreases very slowly with altitude. If any period or periods of diurnal temperature variation exist above this region, they are very small in amplitude, and commonly lost in irregular changes.

In some cases it is necessary to know actual temperatures. To this end, the temperatures read at the morning observations at the six stations over a number of years were averaged, and the values thus found listed in table 4. The average time of the readings was about 6:45 a.m., local time, for all the stations except Due West where it was about 7:45 a.m.

The average lapse rate at 11 a.m. was found for each station, season, and level interval from the data furnished by the series flights; these are shown in table 5. The hour 11 a.m. was chosen because (*a*) the greatest number of observations occurred at about this time, and (*b*) the observed lapse rates were more nearly uniform at this hour than either earlier or later.

Approximate average temperatures can be found for all hours of the day at all levels by means of tables 4 and 5 and equation 7.

B. THE DIURNAL TEMPERATURE RANGE

The diurnal range for each station, season, and level was computed from the absolute maximum and minimum for the day.

To facilitate the study of the variation of range with season, the ranges for the various levels were plotted against altitude and smooth curves drawn through the points. Figures 8 to 13, inclusive, show these curves for all seasons grouped according to station.

To compare the ranges at different stations for any level we must know their ranges at the same height above ground. To this end the heights of the various standard levels above ground were computed for each station and the ranges for the various levels plotted against these heights. Smooth curves were drawn through the points. Figures 14a to 14d show these curves for all stations grouped by seasons.

Figures 8 to 13 show that at the surface, with one exception, the diurnal range was greater in spring than in summer at all stations for which data for the two seasons were available. The exception occurred at Royal Center, where little difference was found in the ranges for the two seasons. The greatest difference between the ranges for the two seasons was found at Ellendale and amounted to about 0.6° C. No such regularity was found, however, for the other seasons. In autumn, the ranges at Ellendale and Royal Center were less than those for spring and summer but greater than those for winter. At the remaining stations the range was greater in autumn than in any other season. The smallest range occurred in winter at Ellendale, Drexel, Royal Center, and Due West, while at Broken Arrow the range in winter was less than in autumn but greater than in spring and summer.

The present discussion of the results of this investigation is not intended to be explanatory, but it might be mentioned that the albedo and evaporation of the surface snow cover, the relatively small number of hours of sunshine, and the low average temperature are all probably quite instrumental in decreasing the diurnal ranges at Ellendale, Drexel, and Royal Center in winter to a value lower than that for the other stations.

At levels from a few meters above the surface to 700 to 1,200 m above the surface, the diurnal ranges at all stations show a regular seasonal march. At these levels for two of the stations, viz, Royal Center and Drexel, the range is greater in summer than in spring; at Groesbeck and Broken Arrow it is greater in spring than in summer; and at Ellendale the ranges for these two seasons are nearly equal at most levels. The winter curves for Royal Center and Ellendale are apparent exceptions to this rule at levels greater than about 500 and 600 m above ground, respectively. At and above these levels at these stations in winter the rate of decrease of range with altitude is very small, and the range is greater than in the other seasons.

At levels higher than those just considered the diurnal range remains nearly constant with increasing altitude, insofar as can be determined from the present data, and there is little apparent regularity of variation in the range with respect to season.

Figures 14a to 14d show an especially outstanding feature—the position of the curves for Groesbeck for the respective seasons. In all seasons at Groesbeck the ranges in the intermediate levels are greater than at any other station considered. This difference is most marked in spring and summer, the latter season not being represented for Due West. In autumn the ranges for Groesbeck and Due West are nearly equal at these levels, but both are decidedly greater than for any other station. In winter there is the least difference in the ranges, but the range at Groesbeck is still the largest. This outstandingly large range is neither shown for the surface nor above levels varying from about 900 m above ground in winter to about 1,600 m above ground in spring.

There are two exceptions to the statements just made. Notably, the range at Due West is greater than that at Groesbeck at altitudes above, roughly, 700 m above ground in autumn and 1,100 m in spring. It is to be regretted that data for summer for Due West are not available.

The relatively large ranges at Due West and Groesbeck are probably brought about mainly by the relatively great amounts of water vapor at these two stations.

There appears to be a tendency in the lower levels for the range to increase as one proceeds southward during all seasons. Conditions at Ellendale, where the range is, for most seasons, greater than we should expect from the geographical position of the station, seem to depart most widely from this general rule.

C. THE DIURNAL VARIATION OF LAPSE RATE

The normal changes in lapse rate from hour to hour were computed by means of equation 9. The differences were then accumulated in a manner similar to that employed in accumulating the temperature differences and the accumulated values plotted against time of day. The curves are not shown herewith, but the change of lapse rate can be traced graphically by noting the variation of the distance between the curves in figures 1 to 6. The distances themselves are not proportional to the actual lapse, but the changes in distance between any two curves are proportional to the changes in the average lapse rate between the respective levels.

Approximate hourly average lapse rates between the various levels can be found by means of table 5 and equation 9.

The manner in which the lapse-rate changes during the day due to the change of the diurnal range with altitude is of interest, since an inversion during part of the day is a necessary consequence if the decrease of range with altitude is great enough, or if there is a large change of phase with altitude. Thus, near the surface where the decrease of temperature range with altitude is comparatively great, the lapse-rate range is also large and an inversion must occur during the night unless very large lapse-rates exist during the day.

As a general rule, inversions exist at all stations and all seasons near the surface during the night hours, and superadiabatic lapse rates during some hours of the afternoon except for the winter season at northern stations.

IV. EVALUATION OF COEFFICIENTS RELATING TEMPERATURE CHANGES TO RADIATION AND TURBULENCE

As stated above, according to Brunt the time rate of change of temperature due to radiation between superposed air layers is given approximately by the expression $K_r\frac{\delta^2 T}{\delta Z^2}$ and that due to turbulence is given approximately by $K\frac{\delta^2 T}{\delta Z^2}$. Brunt then concludes that the combined effects of radiation and turbulence are given roughly by

$$\frac{\delta T}{\delta t}=(K+K_r)\frac{\delta^2 T}{\delta Z^2} \quad (1)$$

The value of K can be shown to be so much larger than that of K_r that $(K+K_r)$ may be roughly considered as representing K alone when its value is much larger than the largest limit of K_r (about 10^3), (Brunt (5)). This usually is true when there is even a small amount of turbulence. If $(K+K_r)$ could be evaluated, then, we could gain a valuable insight into the degree and extent of turbulence.

$(K+K_r)$ could evidently be evaluated for all times of the day if the distribution of the temperature were known in sufficient detail; that is, if the hourly temperatures were known at all elevations. It does not appear, however, that the term $\frac{\delta T}{\delta t}$ in equation 1 takes into account heating or cooling from any source other than radiation between layers and turbulence, while any observed $\frac{\delta T}{\delta t}$ depends upon the heating or cooling from all sources. Therefore, if $(K+K_r)$ is evaluated for the daylight hours from temperature data, one must assume either that the temperature change caused by all factors other than reradiation and turbulence is negligible or that the values obtained for $(K+K_r)$ are unreliable to a certain degree.

An attempt was made to evaluate this quantity at various times of the day, at different levels, and for different seasons from the data at hand. It was found, however, that the values obtained varied over a wide range, and that many large negative values were obtained. These negative values usually were found at times near the minimum or maximum and at higher levels where either $\frac{\delta T}{\delta t}$ or $\frac{\delta^2 T}{\delta Z^2}$ or both were relatively small. It is most probable that the discrepancies found were caused by errors in the lapse rates used. This seems likely when it is considered that an error of ± 0.05 in each of the lapse rates in two successive air layers may change the sign of $(K+K_r)$ at the times and levels just mentioned. Since the errors in the lapse rates used can easily be much larger than this, it is useless to attempt to get any values for $(K+K_r)$ at times or for levels where such errors in the lapse rates could produce such relatively large errors in the result. The investigation was therefore limited to conditions in which $\frac{\delta^2 T}{\delta Z^2}$ had a relatively large numerical value, either positive or negative; that is, it was limited to times well removed from the maximum or minimum and to the first or second standard levels nearest the ground.

In view of the foregoing, equation 1 was used to compute values of $(K+K_r)$ only for the 750 m, m.s.l., level for all seasons for various hours of the day at Drexel, Nebr. The results are shown in table 6. These values are seen to be ten to a hundred times as large as K_r, which alone should not exceed about 10^3, hence they indicate considerable turbulence.

$(K+K_r)$ was much larger during the middle of the day than during the night, indicating more turbulence during the daytime.

However, the values during the hours of sunshine are probably too large, so that the degree of turbulence probably is somewhat less than is indicated by the values of $(K+K_r)$ shown. The values during these two periods of the day (daytime and night) are roughly constant, within the range of accuracy of the observations, in each period.

$(K+K_r)$ is largest, as would be expected, in summer. The smallest values appear to have a tendency to occur in winter, but here again, even at the 750 m level, due to the steep winter inversion, relatively large errors are likely to occur because of the relatively small value of $\frac{\delta^2 T}{\delta Z^2}$. At this level in winter we should expect to find little turbulence, so that K_r is probably of the same order of magnitude as K.

G. I. Taylor (**6**) has given a solution of equation 1 and has shown how the value of the coefficient may be computed from the diurnal temperature ranges at two different altitudes. The value of $(K+K_r)$ obtained by using this solution is strictly valid only if the temperature varies in a manner similar to the sine function. This is approximately true of the temperature variation near the surface. According to this solution

$$b^2 = \frac{\Pi}{T(K+K_r)}, \text{ where} \qquad (2)$$

$$b = \frac{\log_e R_1 - \log_e R_2}{\Delta Z} \qquad (3)$$

T is the periodic time (1 day = 86,400 sec.); R_1 and R_2 are the diurnal ranges at levels 1 and 2, respectively, which differ in altitude by ΔZ centimeters.

Taking Z = 20,000 cm and solving for $(K+K_r)$, we get

$$(K+K_r) = \frac{27.43 \times 10^2}{\left(\log_{10} \frac{R_1}{R_2}\right)^2} \qquad (4)$$

In order to get some idea of the degree of turbulence at different stations, equation 4 was used to compute $(K+K_r)$ for successive 200-m intervals above the surface for the various seasons at four stations. The magnitudes of the ranges at the different heights were read from curves plotted as shown in figure 14. The values of $(K+K_r)$ thus obtained are shown in table 7.

These values show a wide variation with season and station. Some variation is to be expected because of the differences in the mean wind velocity and direction and surface characteristics at the different stations. In a general way, however, the values of $(K+K_r)$ appear to be consistent with many known temperature tendencies. For the present considerations it is permissible to regard the values as representative of K alone.

At Ellendale in all seasons $(K+K_r)$ was found to be much smaller near the surface than at altitudes a little higher. This is to be expected, since, owing to the large surface inversions at this station we should expect to find little turbulence near the ground. Near the level of about 400 m above ground, however, the steepest part, if not all, of the inversion has been passed, especially in the seasons other than winter, and we find that the values of $(K+K_r)$ indicate considerable turbulence above this level. Between the surface and 200 m little turbulence is indicated, which is probably true, since an inversion exists most of the day in this region in winter.

At Drexel and Broken Arrow the values of $(K+K_r)$ average smaller and vary less with height than at Ellendale, and are roughly of the same magnitude at both stations. This is an indication of less turbulence, on the average, at the former two stations than at the latter. The values at Drexel for the layers near the surface for all seasons except summer appear to be comparatively small, an indication of the surface inversion.

The values for Groesbeck for spring and summer are consistently larger than those for the other stations for any season. Observations show that there is a very frequent inversion at Groesbeck in summer in the neighborhood of 600 m or 800 m above ground. This fact is indicated by the present data in the sharp decrease of $(K+K_r)$ for this season and station in the 600–800 m level, which indicates that there is a decided decrease in the degree of turbulence above the 600-m level. At this station again, the value of $(K+K_r)$ is decidedly smaller in winter near the surface.

Comparatively large errors are expected to be present in the values of $(K+K_r)$ derived as described above, the main sources of which are believed to be

(1) Inapplicability of the method of computation to the data, owing to, (*a*) inconstancy of $(K+K_r)$ with altitude and (*b*) material deviation of many of the temperature curves from the simple sine curve.

(2) Disregard of that part of the heating or cooling which might have been produced by direct absorption of solar energy, formation and evaporation of clouds, and other factors operating.

V. SUMMARY

The principal factors operating to produce temperature changes at the surface and in the free air have been introduced and a brief explanation given of their mode of operation. The factors discussed are (1) solar radiation, (2) reradiation between the ground, clouds, and atmosphere, (3) vertical convection, (4) changes of phase of water, (5) advection, (6) conduction.

A qualitative discussion of the magnitude of the diurnal range to be expected under various conditions is given under the headings (1) water vapor, (2) nature of surface, (3) mean air displacement, (4) number of hours of sunshine, (5) miscellaneous.

A diurnal variation in the lapse rate is shown to be a necessary consequence of differential changes of temperature with altitude.

Because of its large relative importance in a study such as this in which there is a very limited amount of data, the method of smoothing has been treated in some detail.

Various tables and figures are introduced showing the trend and magnitude of the diurnal temperature change; the variation of the time of diurnal maximum and minimum with altitude; the variation of the magnitude of the diurnal range with altitude; and the variation of the latter with season and station. Tables also are given showing various values of $(K+K_r)$, where $(K+K_r)$ is defined by equation 1 (sec. IV).

These figures and tables are discussed qualitatively and in a general way rather than in very great detail because this is a preliminary study only. It is necessary to know the magnitudes of several factors before any attempt is made to explain many of the characteristics of the temperature curves and even before too much emphasis is laid upon the actual existence of these characteristics. Similar studies of the diurnal variation of relative humidity, vapor pressure, and wind, based on this same set of material, will be made in the future. It is to be expected that relationships will be found between many of the various factors.

At present, however, it appears to be justifiable to draw the following conclusions:

(1) A definite single period of diurnal temperature variation exists up to an altitude roughly in the neighborhood of 1,000 m above ground.

(*a*) The magnitude of the diurnal range decreases rapidly with height immediately above the ground and then somewhat less rapidly up to this level.

(*b*) Beyond this level the magnitude of the diurnal range is, in general, less than 1° Centigrade.

(*c*) Above this level, if a definite diurnal period exists, the amplitude is quite small.

(2) In view of the number of observations upon which they are based and the size of the aperiodic changes present, the data at hand cannot be expected to indicate clearly in all cases the true diurnal period at levels higher than about 1,200 m above ground.

(3) The times of diurnal maximum and minimum occur later in the day with increasing altitude for some distance above the surface. At heights greater than about 800 m above ground, however, the rate of lag of time of maximum or minimum with altitude in many cases becomes irregular, and at heights greater than about 1,000 m above ground there is little apparent consistency in these times.

(4) The lapse rate has a regular diurnal period over the range in altitude in which a definite temperature period exists.

(5) More observations are necessary for the higher levels where the periodic changes are relatively small, than are necessary for the levels near the ground to obtain the same degree of accuracy in the final results.

(6) No immediate conclusions can be drawn regarding the change of the diurnal temperature march with latitude or longitude because certain other factors which cannot at present be evaluated appear to produce effects of the same order of magnitude as those of geographic position.

(7) A graph showing range plotted against altitude is not as good a criterion, in general, for the existence of a definite period as one showing time of maximum or minimum plotted against altitude. The latter, however, does not present proof.

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TABLE 1.—*Stations at which series were made*

Station	Altitude, m.s.l.	Latitude north		Longitude west		Meridian time used	Period covered by series
		°	′	°	′		
Ellendale, N.Dak	444	45	59	98	34	90th	1918–25
Drexel, Nebr	396	41	20	96	16	do	1916–25
Broken Arrow, Okla	233	36	02	95	49	do	1920–24
Groesbeck, Tex	141	31	30	96	28	do	1918–25
Royal Center, Ind	225	40	53	86	29	do	1918–25
Due West, S.C	217	34	21	82	22	75th	1921–29

TABLE 2.—*Fourier Series Constants*

Station	Level, m., m.s.l.	Spring				Summer				Autumn				Winter			
		A_1	A_2	B_1	B_2	A_1	A_2	B_1	B_2	A_1	A_2	B_1	B_2	A_1	A_2	B_1	B_2
Ellendale, N.Dak.	Surface (444)	−1.09	0.27	1.20	−0.33	−0.98	0.19	1.23	−0.16	−0.78	0.46	1.00	−0.60	−0.68	0.54	0.63	−0.47
	750	−.58	.11	.07	−.18	−.50	.24	.39	−.03	−.48	.25	.17	−.04	−.11	.26	.06	−.12
	1,000	−.41	.07	−.06	−.03	−.41	.17	.18	−.05	−.18	.09	−.06	.04	−.07	.21	−.13	−.10
	1,500	−.19	−.03	−.10	.01	−.15	.04	.03	.03	−.03	.00	.05	.00	−.14	.14	−.05	−.15
	2,000	−.11	−.03	−.06	.05	−.02	−.07	−.01	.07	.00	.06	.06	−.08	−.10	.12	.07	.07
	2,500	−.07	−.02	−.02	.04	−.08	−.10	−.04	.00	.01	.03	.04	−.09	−.11	.10	−.05	.08
	3,000	.00	−.01	.00	.02	−.11	−.07	.03	−.03	−.01	.06	.07	−.06	−.15	.01	.10	.04
Drexel, Nebr.	Surface (396)	−.98	.37	.97	−.30	−.94	.26	.98	−.31	−.96	.44	1.01	−.69	−.81	.58	.55	−.41
	750	−.41	.27	.07	−.03	−.57	.21	.06	−.01	−.37	.15	.00	.01	−.27	.15	−.06	−.08
	1,000	−.15	.15	.05	−.03	−.29	.12	−.08	.12	−.17	.08	−.03	.00	−.10	.10	−.03	−.08
	1,500	.01	.07	−.03	−.01	−.09	−.01	−.06	.01	−.08	.03	.05	−.05	−.08	.03	.02	−.03
	2,000	.00	−.08	.04	.00	−.09	−.01	.01	−.04	−.08	.04	−.01	−.08	−.06	.03	.12	−.04
	2,500	−.02	−.03	.03	−.05	−.07	.00	.02	.01	−.13	.05	.04	−.07	−.08	.05	.11	−.05
	3,000	.02	.00	.05	−.06	−.07	.02	.02	.08	−.07	−.06	.02	−.06	−.04	−.02	.12	−.02
Broken Arrow, Okla.	Surface (233)	−.82	.22	.96	−.28	−.80	.22	.97	−.32	−.75	.37	1.05	−.65	−.89	.60	.84	−.60
	500	−.67	.20	.36	−.13	−.59	.21	.35	−.14	−.45	.14	.28	−.28	−.50	.26	.26	−.13
	750	−.47	.24	.12	.05	−.39	.22	.05	.00	−.32	.12	.05	−.03	−.19	.04	.03	.06
	1,000	−.24	.19	.03	.09	−.23	.13	.03	.05	−.17	.00	.02	−.07	−.03	.01	.04	.12
	1,500	−.06	.10	.01	.04	−.03	.05	−.04	.05	.01	.00	.00	−.02	.06	.03	.10	.06
	2,000	−.10	−.03	.03	.06	.01	.04	−.07	.01	.08	.00	−.03	.12	.00	−.02	.03	.01
	2,500	.06	.00	.07	.15	.01	−.03	−.05	.04	−.02	.03	−.08	.03	.02	−.10	.04	−.02
	3,000					−.02	.06	−.02	.00	−.02	−.01	−.08	.08				
Groesbeck, Tex.	Surface (141)	−.99	.37	.84	−.16	−.89	.17	.91	−.16	−.94	.25	.98	−.57	−.98	.44	.80	−.44
	500	−.79	.30	.40	.15	−.78	.24	.19	.00	−.59	.32	.34	.07	−.43	.18	.07	.15
	750	−.61	.32	.17	.14	−.59	.29	.22	.04	−.42	.24	.20	.08	−.39	.05	.01	.09
	1,000	−.32	.41	−.01	.10	−.36	.30	.08	.03	−.29	.18	.03	.00	−.19	−.02	−.11	−.03
	1,500	−.04	.15	−.14	−.06	−.11	.10	−.05	.07	−.08	.01	−.01	−.06	.06	.02	−.09	−.04
	2,000	.03	−.01	−.10	−.04	−.02	−.08	−.06	−.08	−.01	−.04	.03	−.05	−.07	.07	−.09	−.02
	2,500	−.02	.03	.01	.01	−.09	−.04	−.02	−.07	−.02	−.05	.04	−.07	−.09	.00	−.08	−.09
	3,000	−.04	.01	−.03	.05	−.06	−.06	.02	−.03	−.09	−.07	−.03	−.03	−.09	−.02	.07	.08
Royal Center, Ind.	Surface (225)	−.75	.18	1.14	−.49	−.65	−.04	1.26	−.30	−.60	.12	1.02	−.72	−.49	.31	.49	−.56
	500	−.51	.21	.31	−.21	−.55	.15	.58	−.30	−.32	.23	.28	−.27	−.34	.17	.02	−.17
	750	−.31	.06	.13	−.03	−.33	.12	.26	−.09	−.19	.17	.10	−.12	−.29	.10	−.08	−.05
	1,000	−.17	.09	.08	.00	−.26	.18	.06	.02	−.10	.08	.10	−.06	−.20	.06	−.07	.02
	1,500	−.03	.02	.08	.09	−.06	.02	.11	.01	.00	−.01	.14	−.04	−.19	−.06	.01	−.07
	2,000	−.01	−.05	.01	.16	−.07	−.09	.05	.07	.03	.04	.15	−.03	−.18	−.07	.09	.04
	2,500	−.03	−.07	.02	.01	−.06	−.05	.03	.06	.09	.05	.10	.08	−.22	−.02	.07	.01
	3,000	−.07	−.08	.02	−.09	−.11	−.05	.02	.08	−.03	.11	.11	.06				
Due West, S.C.	Surface (217)	−.84	.14	.68	−.32					−.88	.27	.88	−.54	−.71	.43	.67	−.40
	500	−.65	.27	.30	−.08					−.60	.30	.33	−.14	−.44	.26	.33	−.10
	750	−.50	.18	.15	.08					−.48	.20	.16	.01	−.32	.08	.14	−.03
	1,000	−.38	.16	.15	.02					−.38	.06	.09	.06	−.20	.00	.07	.02
	1,500	−.14	.04	.25	.10					−.17	−.14	.09	.08	−.04	.00	.13	.04
	2,000	−.05	.08	.22	.11					−.08	.03	.10	.08	.06	.13	−.01	−.09
	2,500	.01	.13	.12	.00					−.11	.01	.05	.00	.01	.07	−.07	.15
	3,000									−.10	.04	.01	.10				

TABLE 3.—*Total number of observations for the 4 seasons*

Level, m., m.s.l.	Ellendale, N.Dak.		Drexel, Nebr.		Broken Arrow, Okla.		Groesbeck, Tex.		Royal Center, Ind.		Due West, S.C.	
	6 a.m.	11 a.m.	6 a.m.	11 a.m.	6 a.m.	11 a.m.	6 a.m.	11 a.m.	6 a.m.	11 a.m.	6 a.m.	11 a.m.
Surface	58	116	116	204	50	91	48	88	59	109	[1] 35	[1] 58
500					50	91	47	88	57	109	[1] 35	[1] 58
750	58	114	112	201	50	90	48	88	55	108	[1] 34	[1] 55
1,000	55	108	108	193	50	89	48	85	49	108	[1] 32	[1] 53
1,500	50	106	109	192	47	86	46	81	42	104	[1] 28	[1] 48
2,000	43	106	106	188	43	82	45	76	38	96	[1] 23	[1] 45
2,500	30	83	97	173	28	62	37	66	27	71	[1] 14	[1] 34
3,000	22	57	82	150	[1] 16	[1] 33	20	41	13	30	[2] 5	[2] 9

[1] 3 seasons only.

[2] 2 seasons only.

TABLE 4.—*Mean temperature at the morning observation*

Station	Spring	Summer	Autumn	Winter	Period averaged
	° C.	° C.	° C.	° C.	
Ellendale, N.Dak.	1.1	15.2	2.3	−12.5	13 years.
Drexel, Nebr.	5.4	19.1	6.8	−7.6	10 years.
Broken Arrow, Okla.	10.8	22.0	11.5	.4	12 years.
Groesbeck, Tex.	14.6	23.4	15.2	6.0	Do.
Royal Center, Ind.	6.6	19.1	8.5	−4.5	Do.
Due West, S.C.	12.8	22.7	13.6	3.8	10 years.

TABLE 5.—*Average lapse rates at 11 a.m.*

Level interval (levels in m., m.s.l.)	Ellendale, N.Dak.				Drexel, Nebr.				Broken Arrow, Okla.				Groesbeck, Tex.				Royal Center, Ind.				Due West, S.C.			
	Spring	Summer	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring	Summer	Autumn	Winter
Surface—500									1.17	1.36	1.18	0.55	1.17	1.31	1.16	0.97	1.31	1.46	1.29	0.75	1.18		1.38	0.86
Surface—750	1.06	1.20	0.77	−0.33	1.04	1.20	0.81	−0.11																
500–750									.91	.98	.86	.29	.81	.95	.78	.34	.94	1.02	.82	.15	.86		.92	.54
750–1,000	.76	.81	.28	−.38	.48	.69	.24	−.26	.65	.71	.50	.01	.68	.78	.47	.10	.56	.90	.45	.06	.69		.62	.44
1,000–1,500	.71	.53	.25	.06	.38	.47	.31	.08	.53	.68	.39	.12	.29	.48	.46	.14	.52	.61	.34	.13	.57		.39	.40
1,500–2,000	.61	.66	.42	.34	.49	.70	.51	.31	.50	.66	.36	.43	.33	.39	.38	.36	.43	.58	.47	.34	.50		.28	.30
2,000–2,500	.58	.61	.48	.52	.61	.73	.59	.49	.57	.59	.44	.52	.49	.57	.48	.46	.50	.56	.49	.39	.57		.42	.40
2,500–3,000	.63	.55	.60	.51	.67	.80	.56	.47		.60	.43		.58	.54	.48	.45	.52	.58	.45				.51	

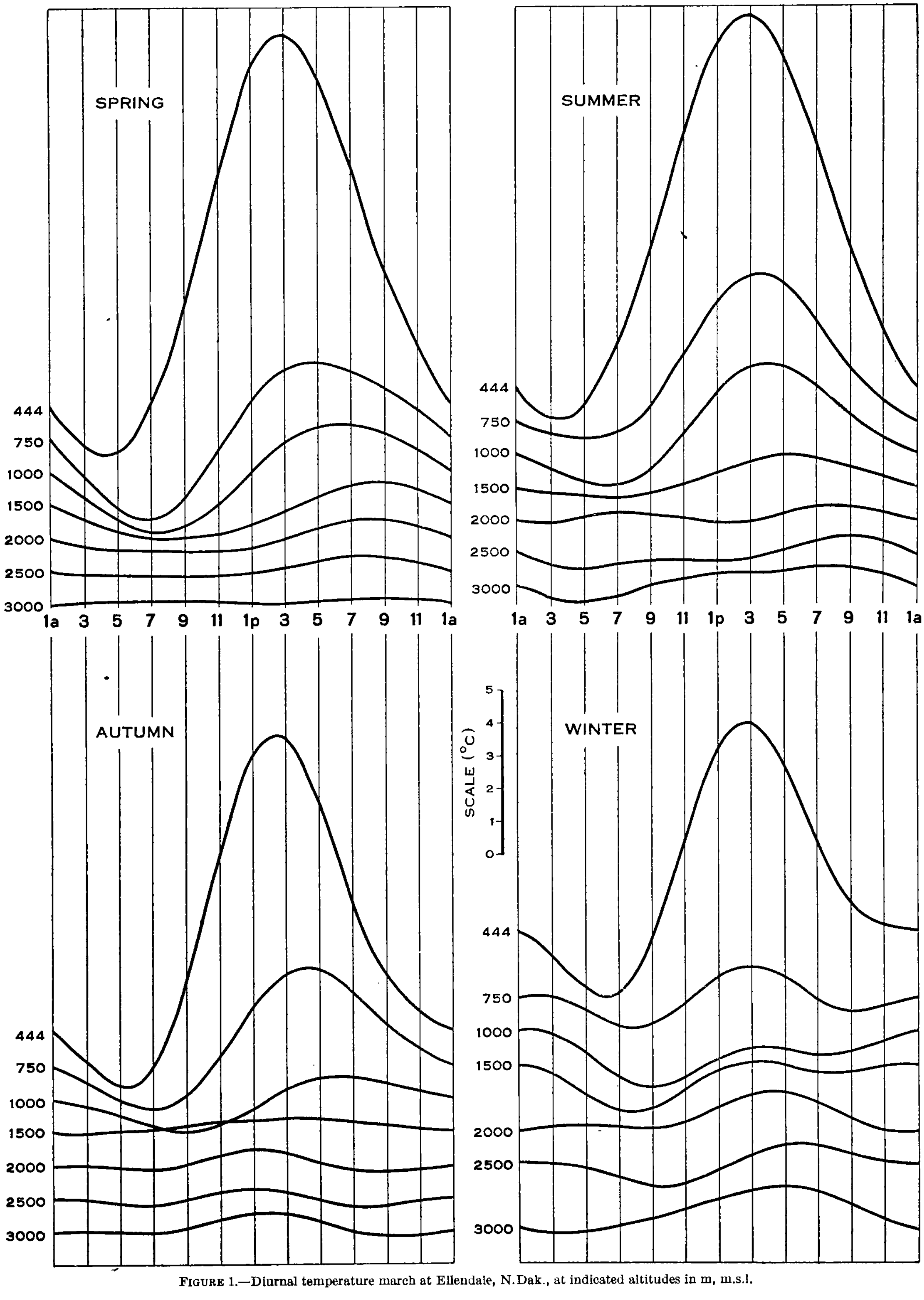

FIGURE 1.—Diurnal temperature march at Ellendale, N.Dak., at indicated altitudes in m, m.s.l.

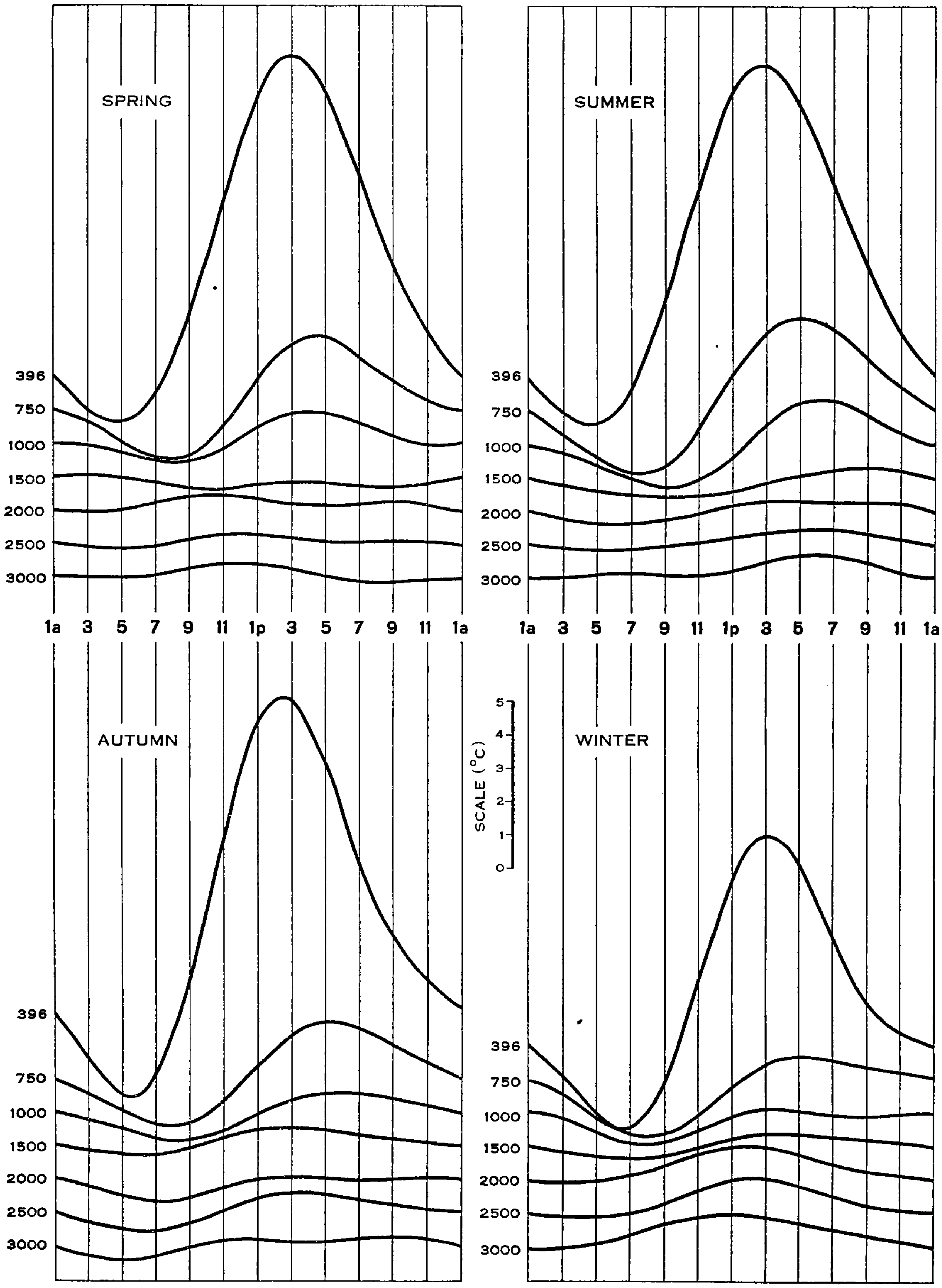

FIGURE 2.—Diurnal temperature march at Drexel, Nebr., at indicated altitudes in m, m.s.l.

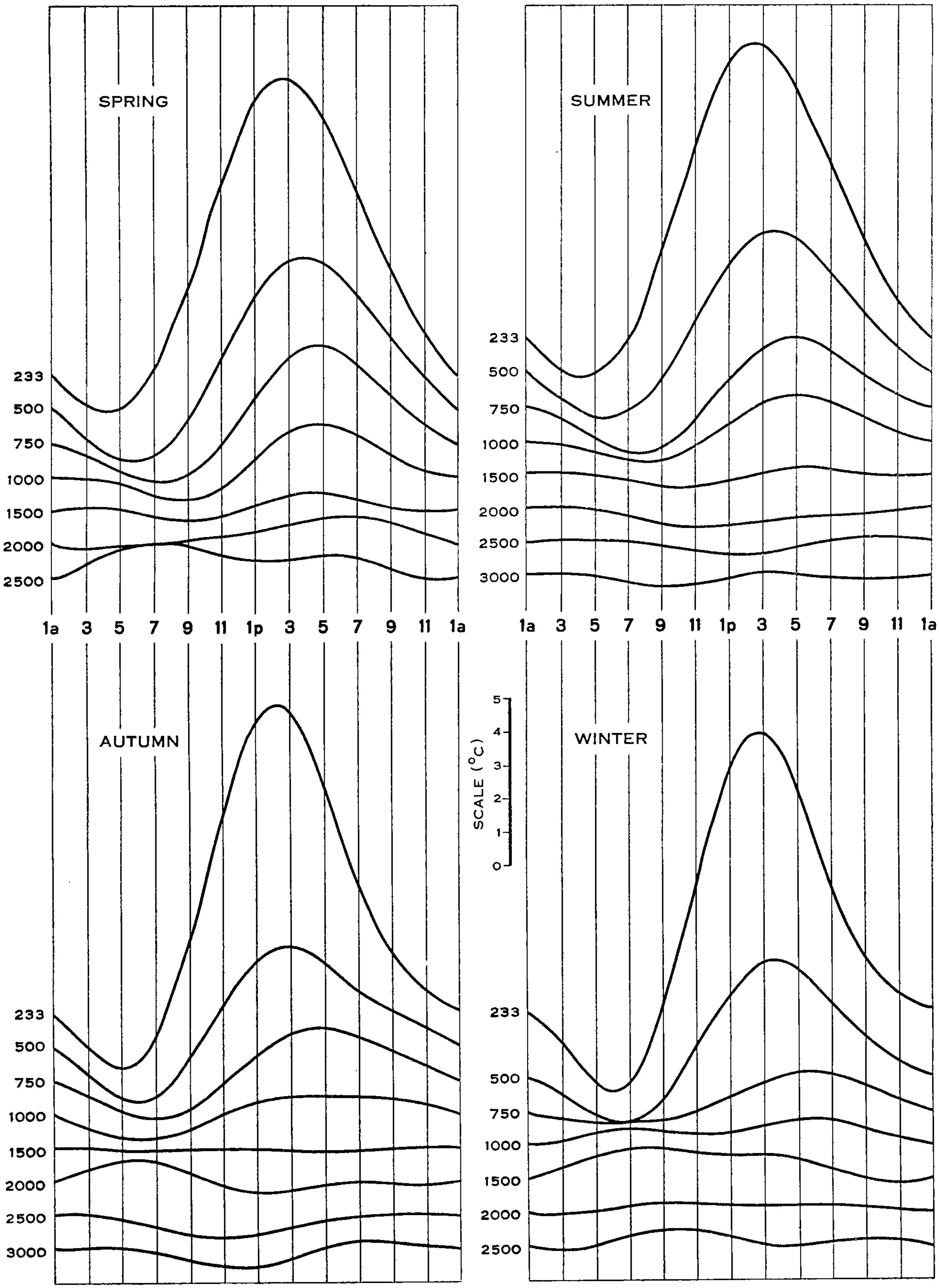

FIGURE 3.—Diurnal temperature march at Broken Arrow, Okla., at indicated altitudes in m, m.s.l.

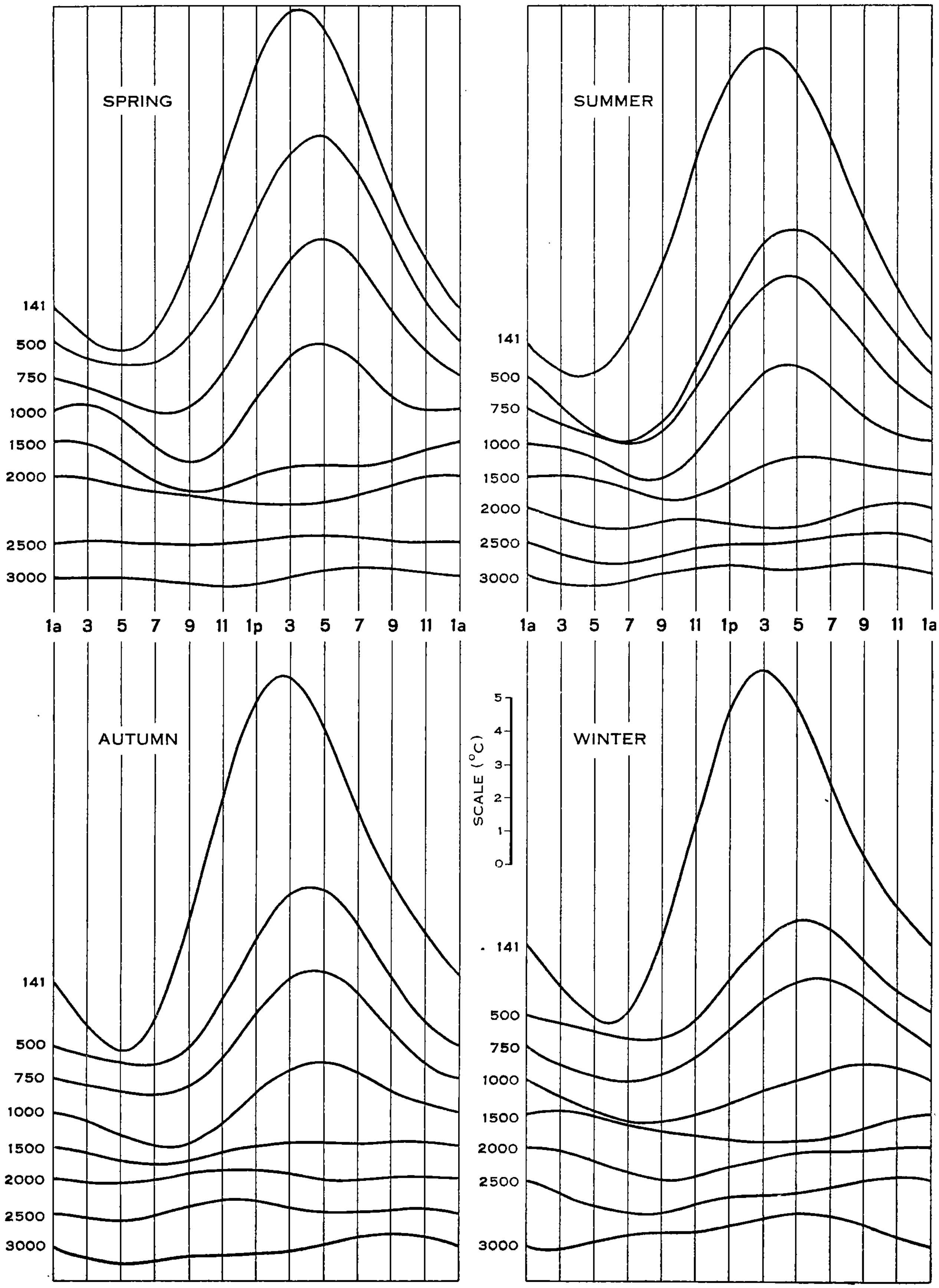

FIGURE 4.—Diurnal temperature march at Groesbeck, Tex., at indicated altitudes in m, m.s.l.

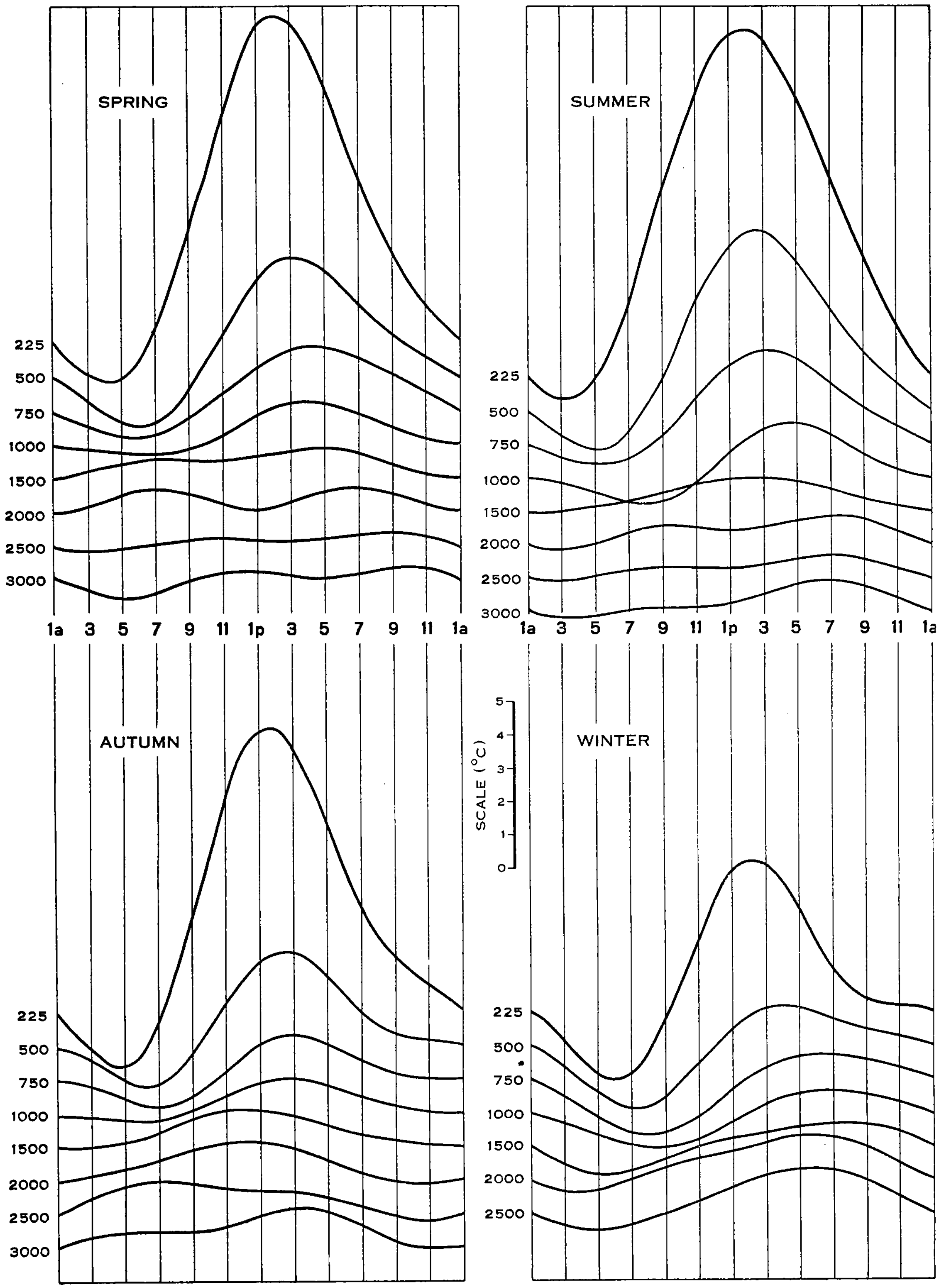

FIGURE 5.—Diurnal temperature march at Royal Center, Ind., at indicated altitudes in m, m.s.l.

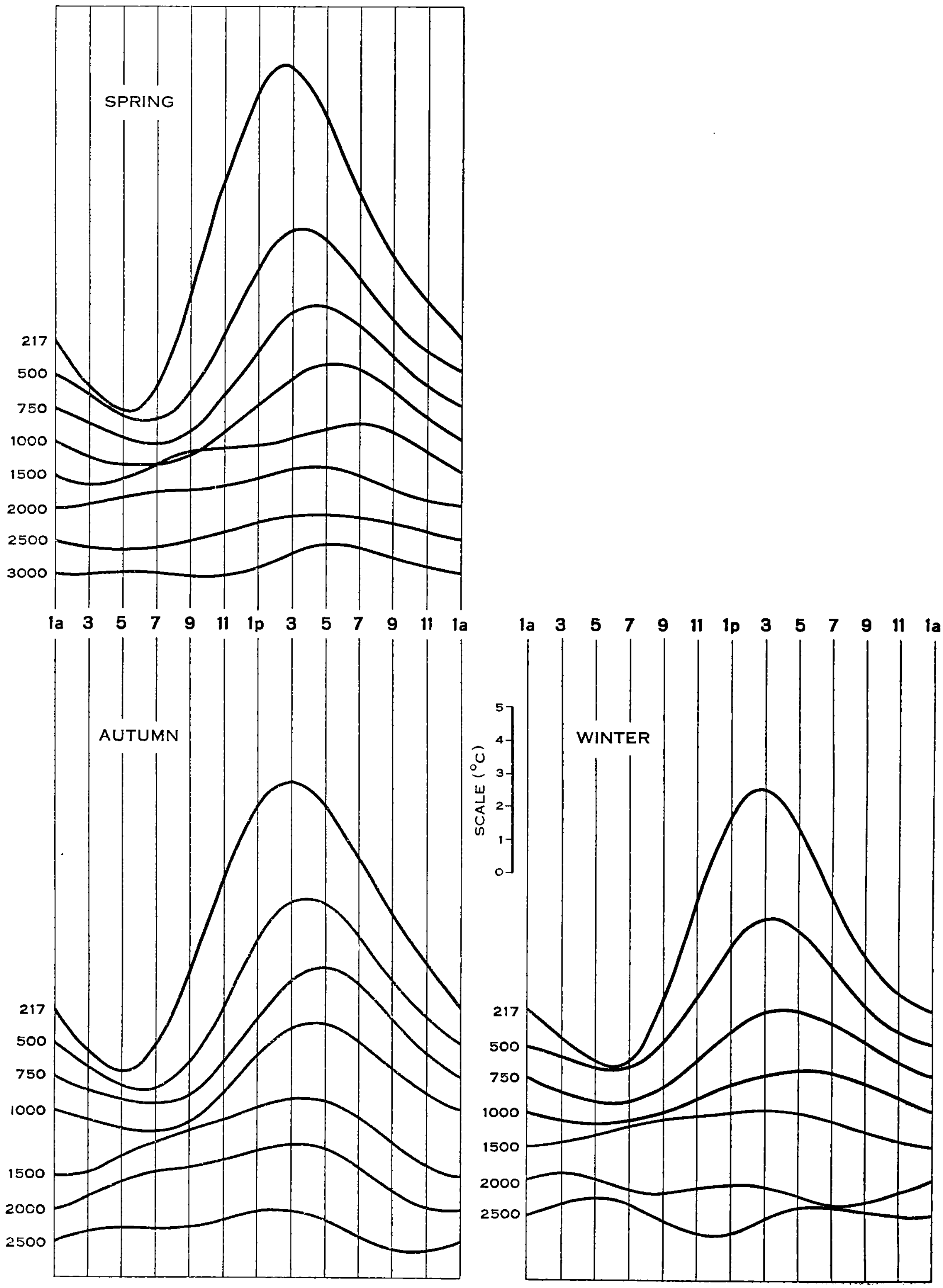

FIGURE 6.—Diurnal temperature march at Due West, S.C., at indicated altitudes in m, m.s.l.

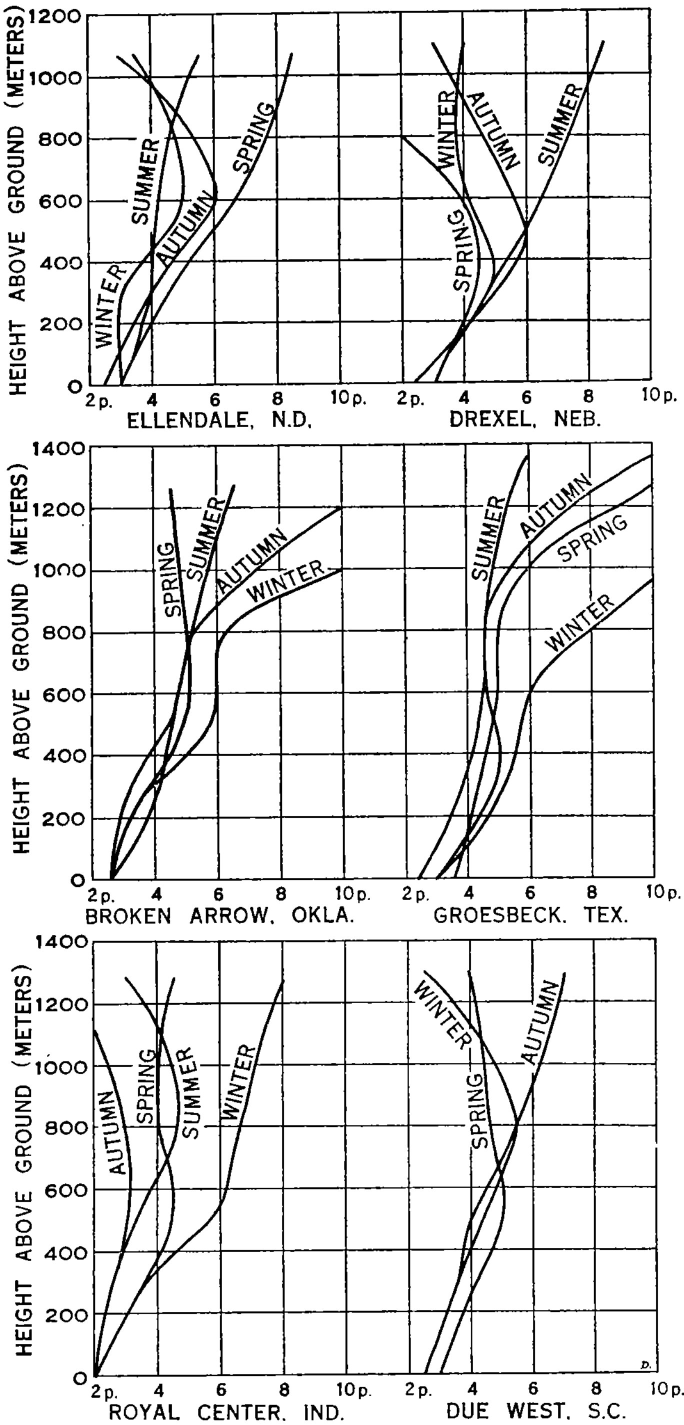

FIGURE 7.—Variation of time of maximum temperature with altitude.

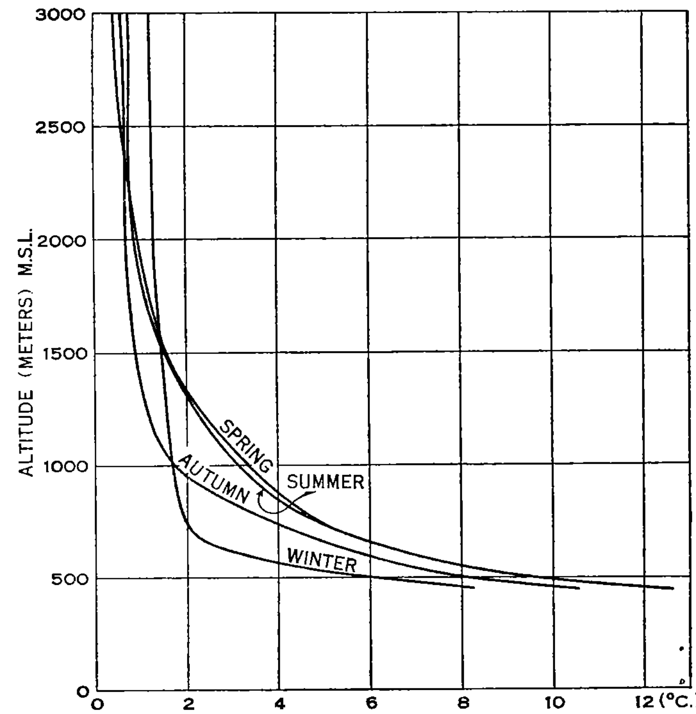

FIGURE 8.—Variation of the diurnal temperature range with altitude at Ellendale, N.Dak.

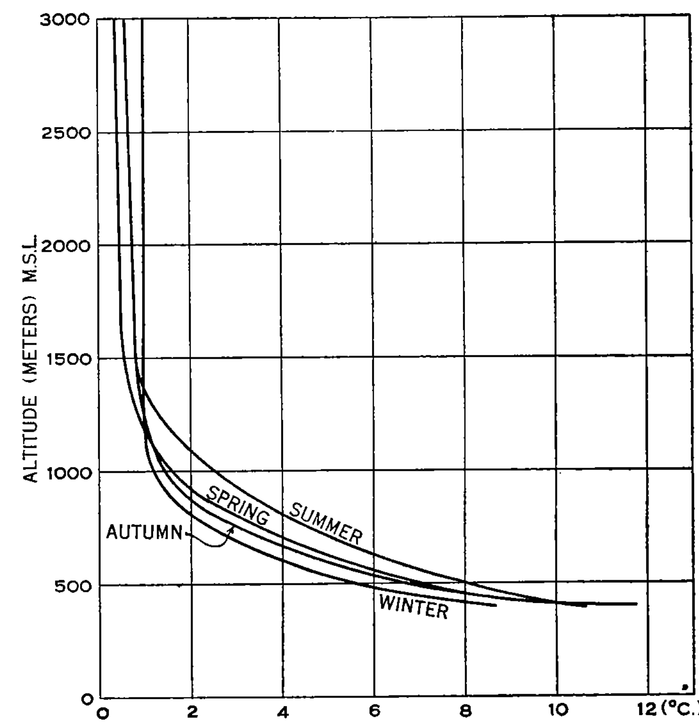

FIGURE 9.—Variation of the diurnal temperature range with altitude at Drexel, Nebr.

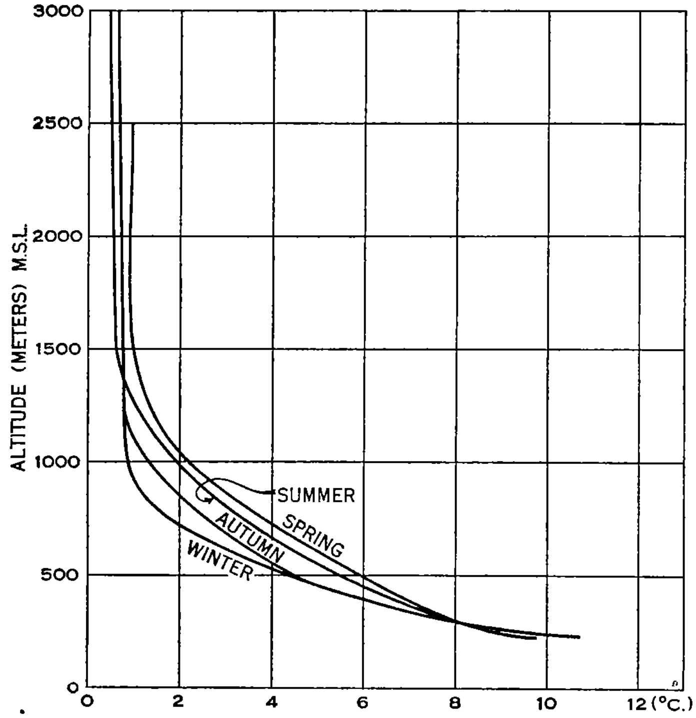

FIGURE 10.—Variation of the diurnal temperature range with altitude at Broken Arrow, Okla.

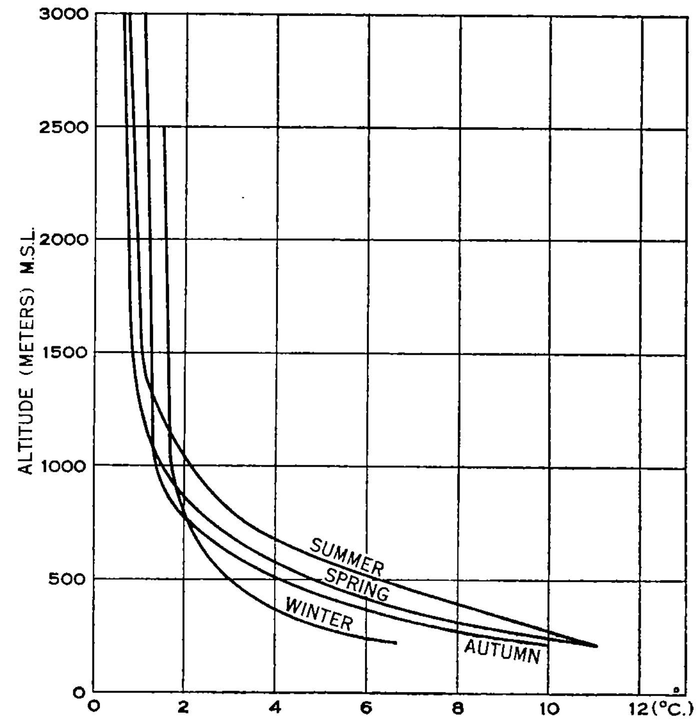

FIGURE 12.—Variation of the diurnal temperature range with altitude at Royal Center, Ind.

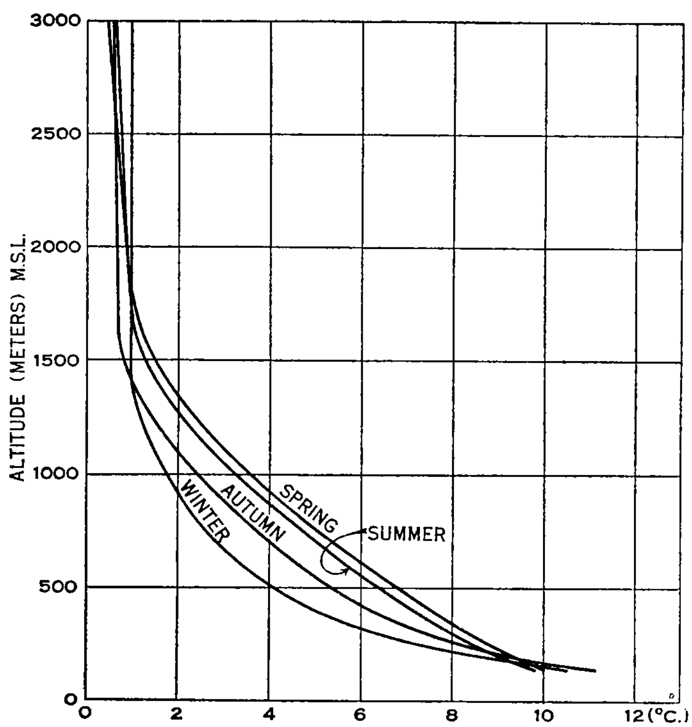

FIGURE 11.—Variation of the diurnal temperature range with altitude at Groesbeck, Tex.

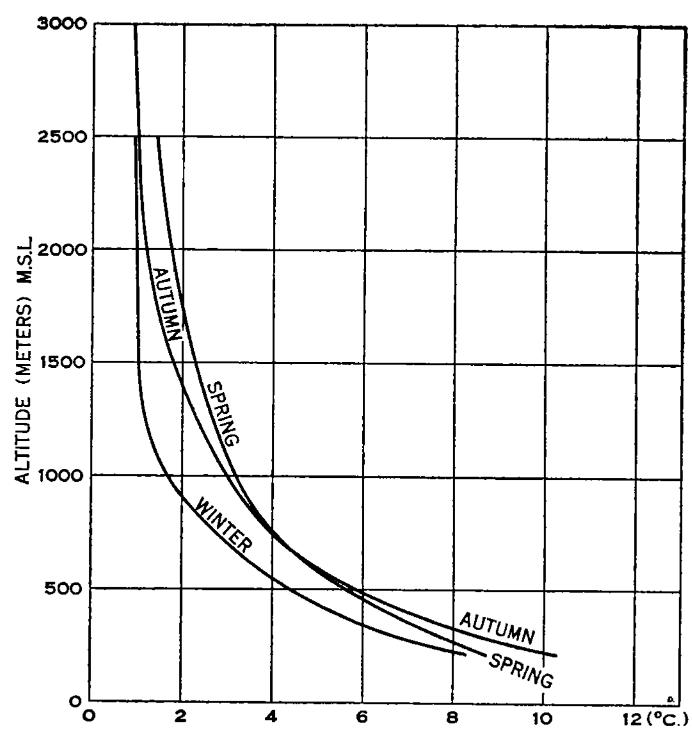

FIGURE 13.—Variation of the diurnal temperature range with altitude at Due West, S.C.

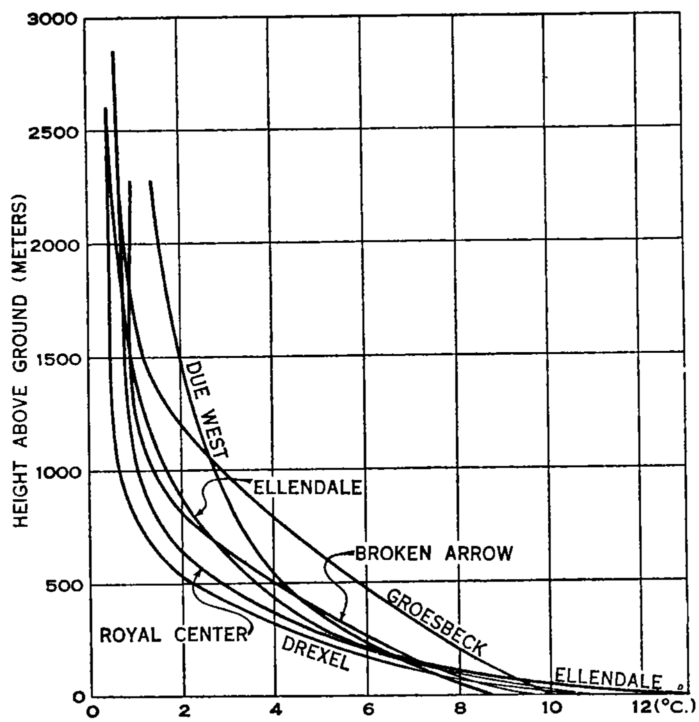

FIGURE 14a.—Variation of the diurnal temperature range with altitude in spring, at various stations.

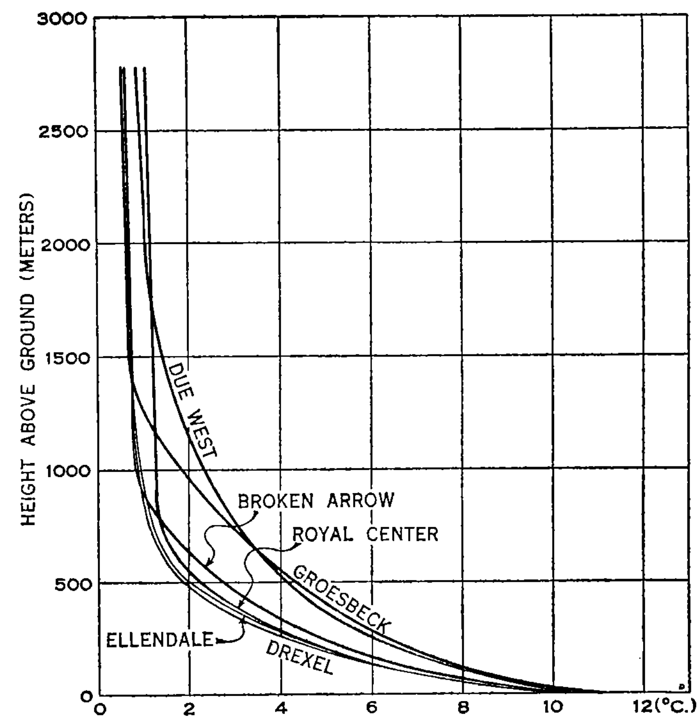

FIGURE 14c.—Variation of the diurnal temperature range with altitude in autumn at various stations.

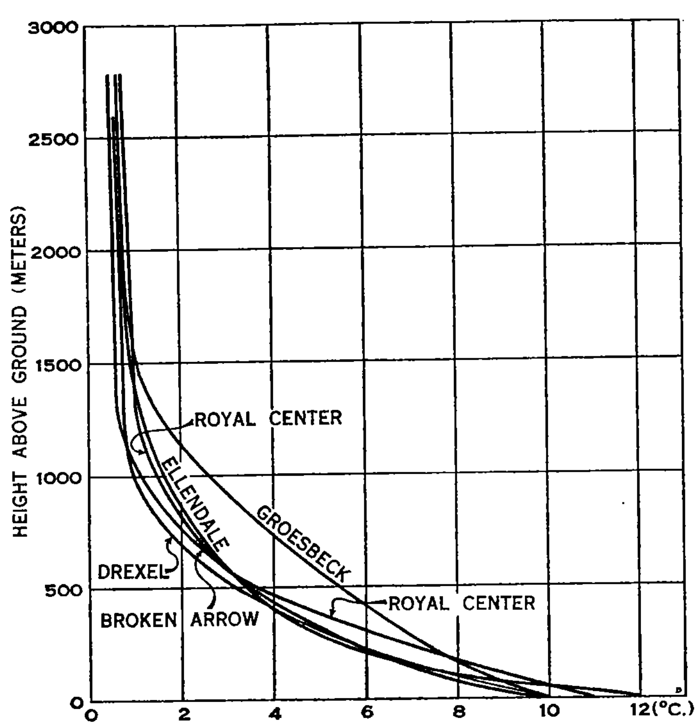

FIGURE 14b.—Variation of the diurnal temperature range with altitude in summer at various stations.

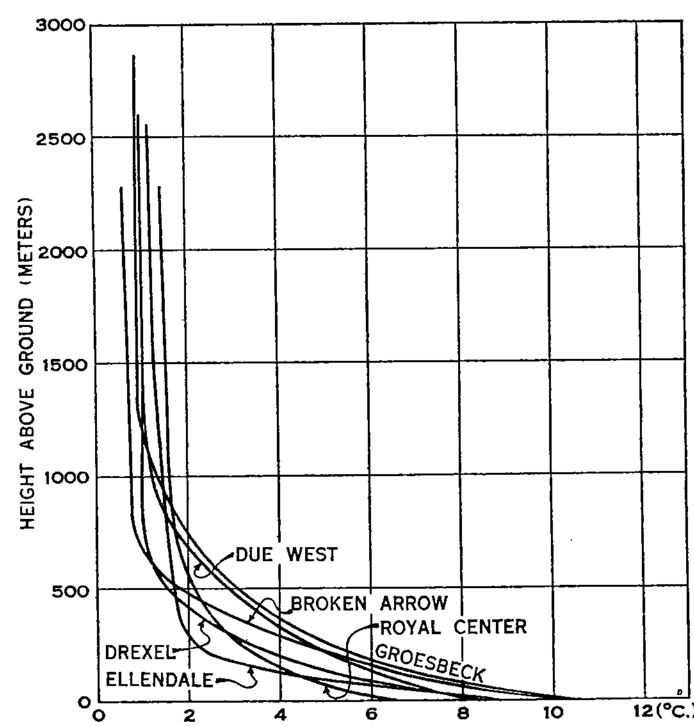

FIGURE 14d.—Variation of the diurnal temperature range with altitude in winter at various stations.

TABLE 6.—*Hourly values of* $(K+K_r)\times10^{-4}$ *for Drexel, Nebr., for the 750 m, m.s.l., level*

Time	Spring	Summer	Autumn	Winter
1 a.m.	1.1	3.2	1.6	1.5
2 a.m.	1.3	3.2	1.8	2.1
3 a.m.	1.6	3.3	1.6	2.6
4 a.m.	2.1	3.7	1.5	3.0
5 a.m.	2.4	3.8	1.5	2.9
6 a.m.	2.4	3.9	1.2	2.3
7 a.m.	1.7	---	0.7	1.4
8 a.m.	---	---	---	---
9 a.m.	---	13.0	---	---
10 a.m.	10.9	12.0	---	---
11 a.m.	9.1	11.3	6.4	---
12 m.	8.2	11.1	4.8	8.4
1 p.m.	7.4	11.4	4.1	5.6
2 p.m.	6.5	11.7	4.0	3.9
3 p.m.	5.2	12.5	3.8	2.8
4 p.m.	2.5	---	3.8	1.8
5 p.m.	---	---	---	---
6 p.m.	---	---	2.1	---
7 p.m.	7.6	7.7	2.3	2.3
8 p.m.	5.3	5.6	2.3	1.4
9 p.m.	3.8	4.7	2.1	0.9
10 p.m.	2.5	4.0	2.0	0.6
11 p.m.	1.6	3.5	1.8	0.6
12 p.m.	1.2	3.2	1.6	0.9

TABLE 7.—*Values of* $(K+K_r)\times10^{-4}$ *computed from the diurnal temperature range*

	Height above surface (meters)	Ellendale, N. Dak.	Drexel, Nebr.	Broken Arrow, Okla.	Groesbeck, Tex.
Spring	0–200	3.0	3.3	9.5	29.2
	200–400	4.7	4.1	16.2	29.2
	400–600	19.9	3.4	9.5	29.2
	600–800	13.4	4.4	7.1	21.1
	800–1,000	10.9	8.8	8.8	14.0
Summer	0–200	3.4	5.7	6.9	25.5
	200–400	7.8	7.4	9.5	27.2
	400–600	14.7	5.9	8.8	31.5
	600–800	15.4	5.5	7.4	15.4
	800–1,000	14.0	5.5	5.9	14.0
Autumn	0–200	3.0	2.1	3.2	6.2
	200–400	4.0	3.2	7.1	14.0
	400–600	3.4	4.8	6.8	18.9
	600–800	29.2	15.4	4.0	12.3
	800–1,000	17.9	36.7	8.8	8.8
Winter	0–200	1.2	2.7	3.3	4.1
	200–400	8.8	3.0	3.5	7.1
	400–600	42.9	4.6	2.3	14.7
	600–800	42.9	43.7	6.1	16.2
	800–1,000	31.7	---	---	11.4

THE USE OF GLASS COLOR SCREENS IN THE STUDY OF ATMOSPHERIC DEPLETION OF SOLAR RADIATION

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[Apr. 10, 1933]

There are two kinds of atmospheric depletion of solar radiation to be considered, namely, (1) scattering by the gas molecules and dust of the atmosphere, and (2) selective absorption by atmospheric gases, principally water vapor. The scattering by pure dry air may be computed by the use of equations developed by Lord Rayleigh and modified by King (1). Fowle (2) has shown the relation between the amount of water vapor in the atmosphere and the depletion of solar radiation in the great infrared water-vapor absorption bands of the solar spectrum. There remains, therefore, the absorption by gases other than water vapor, for which Fowle (3) estimates that by ozone to be from 0.2 to 0.4 percent, and that by the remaining permanent gases to be less than 1 percent of the solar constant of radiation. But the water-vapor content of the atmosphere can be obtained through measurements, other than by the spectroscope, only approximately. If, therefore, the reduction of solar radiation through scattering, in addition to the scattering by pure dry air, can be determined, the absorption by water vapor becomes the only unknown factor in atmospheric depletion, and since pyrheliometric measurements give the total reduction, we may determine the amount due to water vapor with a degree of accuracy that depends upon the accuracy with which the other factors are known. Then from this value and the relation between water-vapor content and water-vapor absorption of solar radiation in the atmosphere, as developed by Fowle (2), it is possible to determine the water-vapor content of the atmosphere, a matter of considerable importance to meteorologists.

Ångström (4) has shown that the depletion of solar radiation through scattering by dry air containing ordinary atmospheric dust (and he includes in this the scattering that Fowle (2) found associated with water vapor in the atmosphere) may be expressed by $(e^{-\beta/\lambda^{1.3}})^m$. Hence the intensity of solar radiation after depletion by scattering may be expressed by the equation

$$I_m=\int_{\lambda=o}^{\lambda=\infty} e_{o\lambda}\,(a_{a\lambda})^m\ (e^{-\beta/\lambda^{1.3}})^m\,d\lambda \qquad 1$$

in which

$e_{0\lambda}$ = the intensity of radiation of wave length λ before depletion by the atmosphere,

$a_{a\lambda}$ = the atmospheric transmission coefficient for radiation of this same wave length,

m = the air mass, approximately the secant of the sun's zenith distance,

e = the base of the Naperian system of logarithms, and

β = the coefficient of atmospheric turbidity as defined by Ångström.

Equation 1 has been solved for the 38 different values of λ given in table 111, Smithsonian Meteorological Tables, fifth revised edition, hereafter referred to as table 111, and corrected for the ultraviolet and the infrared radiation not measured. It also has been solved for the values of $e_{0\lambda}\ a_{a\lambda}$ given in the same table, for values of m = 0.0, 0.526, 1.0, 2.0, 3.0, and 4.0; and for β = 0.0, 0.025, 0.050, 0.075, 0.100, 0.150, and 0.200. The integration of equation 1 has been effected graphically by summing up values of I_m equally spaced with reference to the U.V. glass deviation from ω_1. (See table 111.) It will be noted that some of the intensity values near the extremes of the spectrum are twice as far apart on the deviation scale as those nearer the center of the spectrum. Such values were given double weight in the summation.

Figure 1 is a reproduction of a spectrobologram of solar radiation obtained by the Astrophysical Observatory of the Smithsonian Institution. It will be noted that while the wave lengths change relatively faster at the ultraviolet than at the infra-red end of the spectrum, the prismatic deviation is uniform throughout. It is for this reason that deviations instead of wave lengths are used in connection with computations of radiation intensities in this paper. The results obtained from equation 1 have been plotted on figure 2 and connected by curved lines.

In the integrations no attention has been paid to the water-vapor absorption bands in the infra-red. (See figure 1.) Therefore, the curved lines of figure 2 show