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## DETERMINATION OF ALTITUDES FROM THE ADIABATIC CHART AND THE REFSDAL AEROGRAM

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[U. S. Navy Postgraduate School, Annapolis, Md., March 1936]

The recently-developed Refsdal "aerogram" [1] possesses the interesting property of providing an exceedingly simple, direct and exact means of evaluating geopotential, and hence altitude, from the graph of an aerological sounding. The aerogram includes, in fact, a geopotential scale on the right margin for this purpose.

A somewhat parallel, and almost equally simple and exact, method of altitude determination with the familiar adiabatic chart does not appear to have had the attention that it seems to merit.[2] This method does not involve any inherent approximations; and it avoids the rather awkward constructions, and the moderate approximations, that are involved in the usual method of using the chart for this purpose.

The procedure depends on the familiar static relation for the atmosphere,

$$dp=-\rho g dz=-\frac{p}{RT}\, g\, dz; \qquad (1)$$

or, integrating,

$$z=-\int\frac{R}{g}\,T\,\frac{dp}{p}=-\int\frac{R}{g}\,T\,d(\log_e p) \qquad (2)$$

$$=\frac{R}{g}\int T\,d(\log_e p),$$

if $R$ and $g$ may be regarded as practically constant. Here the integral is evidently the area between the line that represents zero temperature and the line that represents simultaneously-observed temperatures and pressures at various altitudes, plotted in the coordinates of the adiabatic chart, viz, temperature and logarithm of pressure. Thus, the area under consideration is *abcd* in figure 1, where the heavier solid lines represent the boundary of the usual adiabatic chart.

As this area will equal the area *amnd* if a temperature line *mn* is so drawn that area *cno* equals area *obm*, it follows that the absolute temperature corresponding to line *mn* may properly be regarded as a "mean effective" (constant) temperature which would be the equivalent of the actual temperatures insofar as the actual temperatures compositely affect the altitude computations.

For a mean effective absolute temperature, $T_e$, so determined, equation (2) would reduce to,

$$z-z_o=\frac{R}{g}\,T_e\,(\log_e p_o-\log_e p) \qquad (3)$$

$$=29.3\,(t_e+273)\,(\log_e p_o-\log_e p) \qquad (4)$$

$$=67.3\,(t_e+273)\,(\log_{10} p_o-\log_{10} p), \qquad (5)$$

where $t_e$=the mean effective centigrade temperature ascertained by so drawing a temperature line (intersecting

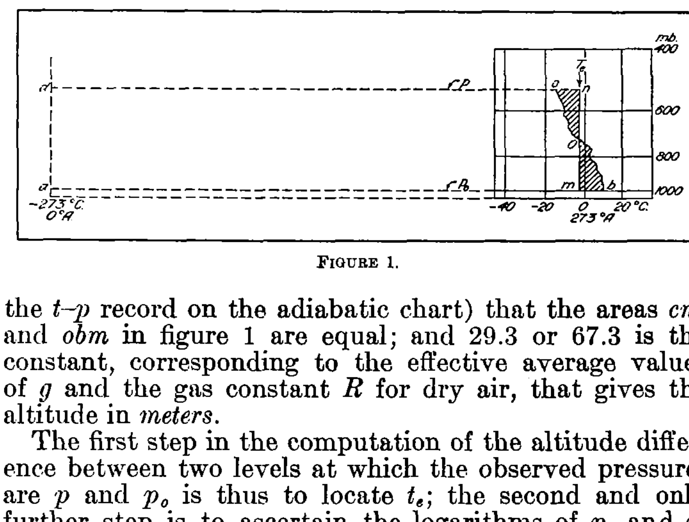


FIGURE 1.

the $t$–$p$ record on the adiabatic chart) that the areas *cno* and *obm* in figure 1 are equal; and 29.3 or 67.3 is the constant, corresponding to the effective average values of $g$ and the gas constant $R$ for dry air, that gives the altitude in *meters*.

The first step in the computation of the altitude difference between two levels at which the observed pressures are $p$ and $p_o$ is thus to locate $t_e$; the second and only further step is to ascertain the logarithms of $p_o$ and $p$, and perform the multiplication in equation (4) or (5).

The $t_e$ line may be located with sufficient accuracy by mere inspection, in conjunction with a transparent straight edge; a more exact equalization of the two areas is ordinarily quite unnecessary: Inspection will easily locate the line to within 0.5° C. on the usual adiabatic chart; the error in $T_e$ would, therefore, not exceed, say 0.18 percent. This gives the absolute temperature to the fourth significant figure, well within the range of certainty that attaches both to instrumental observations of temperature and pressure and to the value of $R$. A more exact equalization may, of course, be made by the use of a planimeter.

[1] A. Refsdal. Das Aerogramm. Met. Zeit., 52:1–5. 1935.

[2] On the method of using the tephigram for this purpose, see Sir Napier Shaw. Geopotential and Height in a Sounding with a Registering Balloon. Mem. Roy. Met. Soc., Vol. I, No. 8. 1927.

For humid air, either an appropriate average value of $R$ for the mixture may be employed, or else the *virtual* temperatures may be plotted.

The advantages of the preceding method are: ($a$) Absence of any inherent approximations; ($b$) ease with which accuracy may be obtained; and ($c$) general rapidity and simplicity of the operations.

*These features are in contrast to the inherent* approximations and troublesome nature of other methods. Those who are familiar with the latter are aware of the necessity for ($a$) determining the arithmetical mean temperature for each series of segments into which the sounding record is divided, ($b$) finding the slope of the dry-adiabatic line through each of these points, ($c$) delineating and progressively joining successive lines with these slopes,

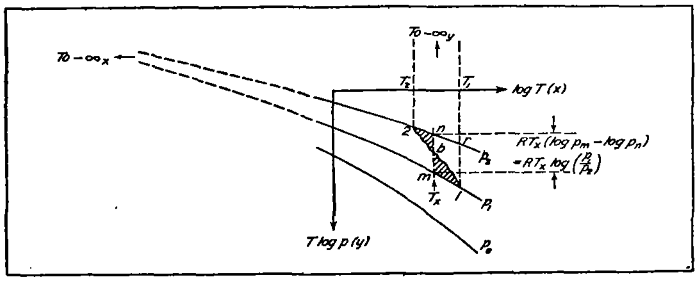


FIGURE 2.

($d$) reading off the subtended distances on the temperature axis, ($e$) correcting these distances, etc.[3] The method outlined above would seem to offer very real and distinct advantages.

### THE COORDINATE SYSTEM OF THE REFSDAL DIAGRAM

The mathematical considerations underlying Refsdal's selection of coordinates, and connected with altitude determinations by the aerogram, have not as yet had very wide dissemination; hence this opportunity will be taken to outline the basis of the system of coordinates and the dry-stage lines of the aerogram.

To appreciate the character of the coordinates, and the presumable reasons for their selection, the following analysis is offered; the analysis proceeds again from the familiar static relation, equation (1):

From equation (2),

$$-g\,dz=R\,Td\,(\log p) \qquad (2)$$

$$=R[d(T\log p)-(\log p)dT] \qquad (6)$$

$$=R\left[d(T\log p)-(T\log p)\frac{dT}{T}\right]$$

$$=R[d(T\log p)-(T\log p)d(\log T)]. \qquad (7)$$

Refsdal has in effect selected as ordinate ($T\log p$), and as abscissa ($\log T$). These are the coordinates indicated in figure 2. The absolute zero of temperature would be located at an infinite distance to the left, since $\log 0=-\infty$; similarly (for any finite temperature) the zero of pressure would be at an infinite distance *upward*, since the ordinate increases downward and $T\log 0=-\infty$.

This coordinate system has the following unique properties:

1. At *constant temperature*, from equation (2) and with subscripts that correspond to the representative points in figure 2, we have

$$g(z_n-z_m)=RT_x\log(p_1/p_2)=R\,[(T_x\log p_m)-(T_x\log p_n)]. \qquad (8)$$

Since the bracketed term on the right of this equation is the linear separation of the intersections of a constant-temperature line with the several pressure lines, therefore this linear distance is a direct index to the difference of geopotential, and thus of altitude, along an isothermal line.

Furthermore, considering the equivalence of the bracketed term and the term $T_x\log(p_1/p_2)$, it appears that the separation of two pressure lines, measured along a temperature line or ordinate, is directly proportional to the absolute temperature that corresponds to this ordinate. This fact accounts for the progressive divergence of the pressure lines in the figure with increasing temperature; it also shows that along any particular temperature ordinate, the distances from some one line of constant pressure $p_o$ to other lines for pressures $p_1$ and $p_2$ are in the ratio

$$\frac{\log(p_o/p_1)}{\log(p_o/p_2)}.$$

The above considerations are, in fact, the only ones that need be or are taken into account in the construction of pressure lines on the actual working diagram as issued and copyrighted by Refsdal. On that diagram the pressure line for 1,050 *mb*, the effective $p_o$ line, is *arbitrarily* made rectilinear, and the other pressure lines are then merely drawn relatively to the 1,050 line in conformity with the foregoing. This modification in no way invalidates the attribute of the diagram that was developed from equation (8).

The appearance of the actual diagram is thus that of figure 3. Values of the function $T\log p$ do not appear on the chart, although we have seen that it is the fundamental ordinate of the coordinate system.[4]

2. *At constant pressure*, from equation (1) with $dp$ or $d\,(\log p)$ equal to zero, we have

$$-g\,dz=dp/\rho=0,$$

so that, from equation (7),

$$d\,(T\log p)_p=(T\log p)\,d\,(\log T)_p, \qquad (9)$$

where the subscript denotes the constancy of pressure.

By integration from $T_2$ to $T_1$, we obtain

$$-(T_2-T_1)\log p=-(T_2\log p-T_1\log p)$$

$$=\int_{T_2,\,p}^{T_1,\,p}(T\log p)\,d\,(\log T). \qquad (10)$$

Here the left side of the equation is represented graphically on figure 2 by the linear vertical separation (along the

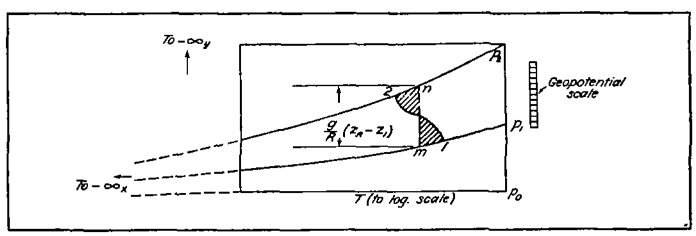


FIGURE 3.

$T\log p$ ordinate) of two points such as (2) and ($r$) located at the pressure line for $p_2$. The right side of the equation is represented graphically by the area (between the $T_2$ and $T_1$ lines) that would extend from the $p_2$ line upward to $-\infty$. The indicated equality thus has some significance on figure 2; any significance it may have on figure 3 is not readily apparent nor particularly important.

[3] *See* L. P. Harrison: Mathematical Theory of the Graphical Evaluation of Meteorograph Soundings by Means of the Adiabatic Chart. Mo. WEA. REV., vol. 63, pp. 123-135. 1935.

[4] This method of determining pressure lines with respect to a rectilinear contour for some $p_0$ amounts, in effect, to placing the axis of $T\log p$ at an angle of less than 90° with the axis of $\log T$, and employing a nonuniform scale for $T\log p$.

By integration from 0 to, say, $T_2$ (or from $-\infty$ to $\log T_2$) at constant pressure $p_2$, we obtain

$$T_2 \log p_2 = \int_{-\infty}^{\log T_2} (T \log p)\, d\, (\log T); \tag{11}$$

on figure 2 the integral would be represented by the total area (between the $T_2$ line and $-\infty$ *to the left*) that would extend from the $p_2$ line *upward* to $-\infty$. Such areas will hereinafter be denoted by symbols of the type $(-\infty_x)$ $(p_2)$ $(-\infty_y)$. Although this area is of infinite extent, it does possess a direct utility that becomes evident in the following paragraph.

3. *For any joint variation of temperature and pressure,* as with altitude, we have from equation (7),

$$g\,(z_2-z_1)=R\left[\int_{(2)}^{(1)} d\,(T\log p)-\int_{(2)}^{(1)}(T\log p)\,d\,(\log T)\right] \tag{7}$$

$$=R[(T\log p)_1-(T\log p)_2 - \int_{(2)}^{(1)}(T\log p)\,d\,(\log T)] \tag{12}$$

On figure 2 the three terms on the right of equation (12) would be represented graphically by the area

$$(-\infty_x)\ (p_1)\ (-\infty_y)-(-\infty_x)\ (p_2)\ (-\infty_y) - (-\infty_y)\ (p_1)\ (p_2)\ (-\infty_y) = (-\infty_x)\ (p_1)\ (p_2)\ (-\infty_x).$$

Thus,

$$g\,(z_2-z_1)=R\cdot \text{area}\ [(-\infty_x)\ (p_1)\ (p_2)\ (-\infty_x). \tag{12a}$$

Now, if on figure 2 a temperature line $mn$ is drawn similar to the one used in the adiabatic chart, figure 1, so that area *2nb* equals area *bim*, we have

$$g\,(z_2-z_1)=R\cdot\text{area}\ [(-\infty_x)\ (n)\ (m)\ (-\infty_x)] = R\cdot\text{area}\ [(-\infty_x)\ (m)\ (-\infty_y) - (-\infty_x)\ (n)\ (-\infty_y)], \tag{13}$$

or, by the remarks following equation (11),

$$g\,(z_2-z_1)=R[(T\log p)_m-(T\log p)_n] = R\,T_{mn}\,(\log p_1-\log p_2). \tag{15}$$

The significance of equation (14) is that, if the temperature-pressure record of a sounding be depicted on the coordinates $\log T$ and $T \log p$, and if we delineate a "mean effective" temperature line $mn$ in the way specified, then the difference of geopotential, and thus of altitude between points *1* and *2*, is equal to $R$ times the linear separation of points $m$ and $n$; this conclusion is valid so long as the construction of the pressure lines conforms to the requirements indicated above, which are fulfilled in the Refsdal chart. As previously mentioned, that chart provides a scale on the right margin to which linear distance $mn$ may be transferred and from which the difference of geopotential may be read directly.

The identity of equations (3) and (15) will be evident, as well as the essential identity of the constructions for determining the required mean effective temperature on the two charts. The only essential difference between the two procedures for the determination of geopotential and altitude in a static atmosphere is that with the adiabatic chart, $R$, $T_e$ and $\log (p_o/p_1)$ must actually be multiplied, whereas by a rather circuitous but very ingenious line of reasoning Refsdal has evolved a chart and scale that provide the product directly.

The Refsdal chart also provides the usual dry and wet adiabatics, the mixing ratio at saturation, etc. As on the tephigram, areas on the Refsdal chart have the dimension and significance of energy; but, unlike the tephigram, distances along the ordinates have also the above significance of geopotential energy.[5]

The writer ventures to suggest that meteorology is much in need of a diagram that will represent other energy data for the atmosphere, in addition to geopotential, as linear distances rather than as areas. Areas are too troublesome of accurate evaluation and frequently too difficult of proper interpretation. It is exactly for this reason that the engineer has abandoned the temperature-entropy diagram in practical work, and has found the Mollier (enthalpy-entropy) chart to be so much more useful.

[5] Cf. footnote 2. p. 69.

# VARIABILITY ISOCRYMAL MAPS FOR THE GREAT PLAINS

By EARL E. LACKEY

[University of Nebraska, Lincoln, Nebr., March 1936]

The series of variability isocrymal maps of the Great Plains described in this paper is a continuation of a corresponding study recently made by the author for Nebraska only.[1]

In constructing a series of frost maps, it was considered more desirable to use median and percentile dates than dates calculated from means and standard deviations. Not only is it easier to calculate percentile dates from the median, but also such dates register exactly what has occurred according to the record, whereas dates calculated from the means and standard deviations, although they may present a symmetrical picture, do not indicate what actually has occurred.

Means have not been computed for all the 500 stations in the 10 States represented by this study; but means and corresponding percentile deviations have been computed for Nebraska.[2] It was found that both the mean date and median date of the first killing frost of autumn at Beatrice, Nebr., according to a 43-year record, was October 10. The mean dates and median dates for the 71 Nebraska stations were in agreement only 30 percent of the time. In normal distributions medians and means are always in agreement.

The mean date of the first killing frost of autumn at Madison, Nebr., according to a 37-year record, was October 5—3 days *earlier* in the year than the median date, October 8. Expressed in another way, the first killing frost of autumn at Madison occurred earlier than the mean date 15 times, and later 20 times. At 25 percent of the 71 Nebraska stations, the mean date of the first killing frost of autumn was *earlier* than the median date.

The mean date of the first killing frost of autumn at Broken Bow, Nebr., according to a 39-year record, was October 2—7 days *later* in the year than the median date, September 25. In fact, the first killing frost of autumn occurred 23 times before, and 16 times after, the mean date. Of course the first killing frost of autumn occurred

[1] The Geographical Review, Vol. XXVI, No. 1, January 1936.
[2] Climatic Summary of the United States (U. S. Weather Bureau Bulletin W), Sections 38 and 39. Washington 1930.