ERRATUM IN THE SMITHSONIAN METEOROLOGICAL TABLES: TEMPERATURE CORRECTION FACTOR IN THE HYPSOMETRIC FORMULA (ENGLISH MEASURES)

By V. E. JAKL

[Weather Bureau Airport Station, Chicago, Ill.]

The introduction to the Smithsonian Meteorological Tables, in dealing with the determination of heights by the barometer, using English measures, contains an erroneous mathematical transformation of the air-temperature term in the hypsometric formula, and hence the values for the temperature correction which are computed on the basis of this transformation are correspondingly discrepant to a slight degree. This error requires one of the tables, viz table 52 of the fourth and fifth editions, to be modified. To be specific, the tables in question contain the following statement (e. g., see p. xlii of the fourth edition, 1918, or p. xlvii of the fifth dition, 1931):

> In order to make the temperature correction as small as possible for average air temperatures, 50° F. will be taken as the temperature at which the correction factor is zero. This is accomplished by the following transformation:
>
> $1+0.002039(\theta-32°)=[1+0.002039(\theta-50°)][1+0.0010195\times 36°]$.

As may be easily verified, this is not an identity except when the mean temperature of the air column, θ, is equal to 50° F. The correct transformation is:

$$1+0.002039(\theta-32°)=[1+0.001967(\theta-50°)][1+0.0010195\times 36°].$$

This identity may be established by the following procedure:

Let $\alpha=0.002039$; then

$$1+\alpha(\theta-32)=1+\alpha(\theta-50)+18\alpha=(1+18\alpha)+\alpha(\theta-50).$$

Let a be a number such that

$$\alpha=\frac{a}{1-18a};$$

that is

$$\alpha=(1+18\alpha)a.$$

Substitute this value of α in the coefficient of $(\theta-50)$, whence we obtain

$$1+\alpha(\theta-32)=(1+18\alpha)+(1+18\alpha)a(\theta-50)=[1+18\alpha][1+a(\theta-50)].$$

Substituting numerical values, we get

$$a=\frac{\alpha}{1+18\alpha}=\frac{0.002039}{1+0.0367}=0.001967,$$

$$18\alpha=\frac{\alpha}{2}\cdot 36=0.0010195\times 36;$$

and these, when inserted in the expression last obtained, give the correct transformation as set forth above.

We may arrive at the same result by considering the steps leading to the hypsometric formula. Taking note of two facts: (1) That according to the gas laws, the thermal coefficient of expansion of air is equal to the reciprocal of the absolute temperature at which density (or volume) is regarded as standard, and (2) that the thermal coefficient of expansion of air which must be used in the temperature correction factor of the hypsometric formula should have a value appropriate to the temperature adopted for the (standard) density employed in the "barometric constant", it follows that if the (standard) densities employed in the "barometric constant" are based on the temperatures 32° F. or 50° F., respectively, the appropriate corresponding thermal coefficients of expansion of air are

$$\alpha=\frac{1}{491}=0.002039, \text{ very nearly, or}$$

$$a=\frac{1}{509}=0.001967, \text{ very nearly,}$$

respectively, since 491° and 509° are the absolute temperatures on the Fahrenheit scale equivalent to 32° and 50° F., respectively.

In the Smithsonian Meteorological Tables, the "barometric constant" for English measures is based on a temperature of 50° F. (see table 51 of the fourth and fifth editions), hence the latter coefficient is the appropriate one for the temperature correction factor.

In view of the above considerations, table 52 of the fourth and fifth editions of the tables in question, viz, Determination of Heights By The Barometer, English Measures, Term for Temperature: $0.002039(\theta-50°)Z$, should be modified to conform to the correct temperature term for these tables, viz $0.001967(\theta-50°)Z$. The following table, which was computed in the Aerological Division at the Central Office of the Weather Bureau, represents the result of applying the latter form of the correction:

TABLE 52.—*Determination of heights by the barometer, English measures*

Term for Temperature: 0.001967 (θ −50°) Z.

For temperatures {above 50° F. / below 50° F.} the values are to be {added / subtracted.}

Mean temperature θ (° F.)		Approximate difference of height obtained from table 51												
		20	40	60	80	100	200	300	400	500	600	700	800	900
		Feet	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*
49	51	0	0	0	0	0	0	1	1	1	1	1	2	2
48	52	0	0	0	0	0	1	1	2	2	2	3	3	4
47	53	0	0	0	0	1	1	2	2	3	4	4	5	5
46	54	0	0	0	1	1	2	2	3	4	5	6	6	7
45	55	0	0	1	1	1	2	3	4	5	6	7	8	9
44	56	0	0	1	1	1	2	4	5	6	7	8	9	11
43	57	0	1	1	1	1	3	4	6	7	8	10	11	12
42	58	0	1	1	1	2	3	5	6	8	9	11	13	14
41	59	0	1	1	1	2	4	5	7	9	11	12	14	16
40	60	0	1	1	2	2	4	6	8	10	12	14	16	18
39	61	0	1	1	2	2	4	6	9	11	13	15	17	19
38	62	0	1	1	2	2	5	7	9	12	14	17	19	21
37	63	1	1	2	2	3	5	8	10	13	15	18	20	23
36	64	1	1	2	2	3	6	8	11	14	17	19	22	25
35	65	1	1	2	2	3	6	9	12	15	18	21	24	27
34	66	1	1	2	3	3	6	9	13	16	19	22	25	28
33	67	1	1	2	3	3	7	10	13	17	20	23	27	30
32	68	1	1	2	3	4	7	11	14	18	21	25	28	32
31	69	1	1	2	3	4	7	11	15	19	22	26	30	34
30	70	1	2	2	3	4	8	12	16	20	24	28	31	35
29	71	1	2	2	3	4	8	12	17	21	25	29	33	37
28	72	1	2	3	3	4	9	13	17	22	26	30	35	39
27	73	1	2	3	4	5	9	14	18	23	27	32	36	41
26	74	1	2	3	4	5	9	14	19	24	28	33	38	42
25	75	1	2	3	4	5	10	15	20	25	30	34	39	44
24	76	1	2	3	4	5	10	15	20	26	31	36	41	46
23	77	1	2	3	4	5	11	16	21	27	32	37	42	48
22	78	1	2	3	4	6	11	17	22	28	33	39	44	50
21	79	1	2	3	5	6	11	17	23	29	34	40	46	51
20	80	1	2	4	5	6	12	18	24	30	35	41	47	53
19	81	1	2	4	5	6	12	18	24	30	37	43	49	55
18	82	1	3	4	5	6	13	19	25	31	38	44	50	57
17	83	1	3	4	5	6	13	19	26	32	39	45	52	58
16	84	1	3	4	5	7	13	20	27	33	40	47	54	60
15	85	1	3	4	6	7	14	21	28	34	41	48	55	62
14	86	1	3	4	6	7	14	21	28	35	42	50	57	64
13	87	1	3	4	6	7	15	22	29	36	44	51	58	66
12	88	1	3	4	6	7	15	22	30	37	45	52	60	67
11	89	2	3	5	6	8	15	23	31	38	46	54	61	69
10	90	2	3	5	6	8	16	24	31	39	47	55	63	71
9	91	2	3	5	6	8	16	24	32	40	48	56	65	73
8	92	2	3	5	7	8	17	25	33	41	50	58	66	74
7	93	2	3	5	7	8	17	25	34	42	51	59	68	76
6	94	2	3	5	7	9	17	26	35	43	52	61	69	78
5	95	2	4	5	7	9	18	27	35	44	53	62	71	80
4	96	2	4	5	7	9	18	27	36	45	54	63	72	81
3	97	2	4	6	7	9	18	28	37	46	55	65	74	83
2	98	2	4	6	8	9	19	28	38	47	57	66	76	85
1	99	2	4	6	8	10	19	29	39	48	58	67	77	87
0	100	2	4	6	8	10	20	30	39	49	59	69	79	89

TABLE 52.—*Determination of heights by the barometer, English measures*—Continued

Term for Temperature: 0.001967 (θ −50°) Z.

For temperatures {above 50° F. / below 50° F.} the values are to be {added / subtracted.}

Mean temperature θ (° F.)		Approximate difference of height obtained from table 51										
		1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000	10,000	20,000
		Feet	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*
49	51	2	4	6	8	10	12	14	16	18	20	39
48	52	4	8	12	16	20	24	28	31	35	39	79
47	53	6	12	18	24	30	35	41	47	53	59	118
46	54	8	16	24	31	39	47	55	63	71	79	157
45	55	10	20	30	39	49	59	69	79	89	98	197
44	56	12	24	35	47	59	71	83	94	106	118	236
43	57	14	28	41	55	69	83	96	110	124	138	275
42	58	16	31	47	63	79	94	110	126	142	157	315
41	59	18	35	53	71	89	106	124	142	159	177	354
40	60	20	39	59	79	98	118	138	157	177	197	393
39	61	22	43	65	87	108	130	151	173	195	216	433
38	62	24	47	71	94	118	142	165	189	212	236	472
37	63	26	51	77	102	128	153	179	205	230	256	511
36	64	28	55	83	110	138	165	193	220	248	275	551
35	65	30	59	89	118	148	177	207	236	266	295	590
34	66	31	63	94	126	157	189	220	252	283	315	629
33	67	33	67	100	134	167	201	234	268	301	334	669
32	68	35	71	106	142	177	212	248	283	319	354	708
31	69	37	75	112	149	187	224	262	299	336	374	747
30	70	39	79	118	157	197	236	275	315	354	393	787
29	71	41	83	124	165	207	248	289	330	372	413	826
28	72	43	87	130	173	216	260	303	346	389	433	865
27	73	45	90	136	181	226	271	317	362	407	452	905
26	74	47	94	142	189	236	283	330	378	425	472	944
25	75	49	98	148	197	246	295	344	393	443	492	984
24	76	51	102	153	205	256	307	358	409	460	511	1,023
23	77	53	106	159	212	266	319	372	425	478	531	1,062
22	78	55	110	165	220	275	330	386	441	496	551	1,102
21	79	57	114	171	228	285	342	399	456	513	570	1,141
20	80	59	118	177	236	295	354	413	472	531	590	1,180
19	81	61	122	183	244	305	366	427	488	549	610	1,220
18	82	63	126	189	252	315	378	441	504	566	629	1,259
17	83	65	130	195	260	325	389	454	519	584	649	1,298
16	84	67	134	201	268	334	401	468	535	602	669	1,338
15	85	69	138	207	275	344	413	482	551	620	688	1,377
14	86	71	142	212	283	354	425	496	566	637	708	1,416
13	87	73	146	218	291	364	437	509	582	655	728	1,456
12	88	75	149	224	299	374	448	523	598	673	747	1,495
11	89	77	153	230	307	384	460	537	614	690	767	1,534
10	90	79	157	236	315	393	472	551	629	708	787	1,574
9	91	81	161	242	323	403	484	565	645	726	806	1,613
8	92	83	165	248	330	413	496	578	661	744	826	1,652
7	93	85	169	254	338	423	507	592	677	761	846	1,692
6	94	87	173	260	346	433	519	606	692	779	865	1,731
5	95	89	177	266	354	443	531	620	708	797	885	1,770
4	96	90	181	271	362	452	543	633	724	814	905	1,810
3	97	92	185	277	370	462	555	647	740	832	924	1,849
2	98	94	189	283	378	472	566	661	755	850	944	1,888
1	99	96	193	289	386	482	578	675	771	867	964	1,928
0	100	98	197	295	393	492	590	688	787	885	984	1,967

The differences between the original table 52 and the modified table herein are exemplified by the following tabulation for Z=5,000 feet:

θ (° F.)	0.002039(θ−50°)Z	0.001967(θ−50°)Z
50	0	0
60	102	98
70	204	197
80	306	295
90	408	393

The differences are thus obviously not very great under average conditions.