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STRUCTURE AND MAINTENANCE OF DRY-TYPE MOISTURE DISCONTINUITIES NOT DEVELOPED BY SUBSIDENCE

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INTRODUCTION

In a previous paper (1) the author discussed in some detail the development and structure of subsidence inversions. It was pointed out that subsidence is not necessarily responsible for all inversions in which there is an abrupt fall of humidity; and that dry, warm air masses are frequently observed flowing over moister and colder currents in cases where no subsidence is indicated. For long periods of time, this warm, dry air layer can be found aloft over large sections of the country, particularly over the southern United States, and for this reason its life history presents an exceedingly important problem of synoptic meteorology. The source of this air is a controversial topic (2, 3). Indeed, the present state of the question is such that even the general classification Polar or Tropical cannot be applied with certainty; and in this report no attempt will be made to label this air mass so as to imply a certain source region or to ascribe to it any definite history. I shall be concerned merely with its existence above moister air; and for the sake of conformity with customary synoptic usage, shall refer to it as Ts air.[1]

The extremely sharp transitions in moisture observed in passing from underlying moist air masses into the *Ts* currents provide an opportunity to study several interesting questions, one of which is the thermal balance accompanying these transitions. In the case of subsidence inversions there can be little doubt that thermodynamic factors, particularly at upper levels, play the dominant role in governing the thermal conditions within the inversion layer. On the other hand, the discontinuities wherein *Ts* is the superimposed air mass seem to be governed in large part by radiative factors. This naturally implies that throughout these *Ts* currents vertical motions are comparatively small and slow acting. If this is true we may look upon the *Ts* flows as rather inert aerological entities which, because of their conditionally unstable structure, become at times catalysts in the process of rainmaking.

In this report I shall attempt to show from certain theoretical principles and observed data that there is in these discontinuities a radiative governing mechanism. The complexity of this entire problem is such that a rough approximation to a quantitative solution is as much as can be hoped for at present. It is the purpose of this paper to suggest such an approximation, or at least to suggest the more logical lines of attack, in order to obtain a solution.

[1] U. S. Weather Bureau meteorologists are now using for this air mass the designation "S", standing for superior or subsiding. At present, perhaps the most definite fact concerning this air is that it has its origin at upper levels, in contrast to other air masses which are known to have a surface source region.

STABILITY

Whatever the original cause of a humidity discontinuity, the presence or absence of condensation in the moist layer below the discontinuity will have considerable bearing on the treatment of the problem as one of radiational balance. This follows from the well-known fact that a cloud of any thickness greater than a few meters acts as a black body. The experimental investigations of the absorption spectrum of liquid water by Rubens and Ladenburg (4) form the basis for this supposition. Mal, Basu, and Desai (5) studied dry inversions on the assumption that the top of the underlying moist layer may be treated as a black body whether or not it be saturated. Their argument for this assumption is "that a haze layer is always associated with a humidity discontinuity, the maximum humidity coinciding with the top of the haze layer." Their procedure, then, treats clouds and haze layers alike with respect to radiating power. Aerological soundings in the United States show that humidity discontinuities are not always associated with pronounced haze discontinuities. Furthermore, although we have no observational data on the subject, there appears to be no sound justification for assuming that an unsaturated haze layer radiates and absorbs as a black body. The case of actual droplets of condensed water is quite different from that of a concentration of small solid haze particles, even though they be hygroscopic in nature. Though definite proof on this point is lacking, a clue is furnished by observations of inversions with and without clouds; almost invariably, those without condensation are less pronounced than those with clouds (for the same general moisture and temperature distribution). This indicates a deviation from black-body action in the moist layer underlying the inversion when clouds are absent.

Thus the quantitative treatment of dry inversions will be discussed first for an unsaturated moist base with an overlying relatively dry stratum; and later we shall deal with inversions where clouds are present.

Applicable to all cases, however, is the following consideration of the static equilibrium in a given moisture and temperature stratification: It is reasonable to suppose that the density cannot increase with height in the free air. If we assume a moisture discontinuity of the first order (mathematically abrupt) it is easy to show, as pointed out by Margules in 1906, that a temperature inversion is a static necessity, because a lapse of temperature would represent a highly unstable state which certainly could not exist aloft, for the slightest turbulence would upset it. Let the discontinuity in moisture be at the level defined by pressure p; and let the vapor pressure in the moist air be e_1, and in the overlying dry air e_2. Let ρ_1, T_1, and R_{a1} be respectively the density, temperature, and gas constant of the air immediately below the discontinuity, and ρ_2, T_2, and R_{a2} the corresponding values in the overlying dry air. Then we have

$$\rho_1=\frac{p}{R_{a1}T_1}, \tag{1}$$

$$\rho_2=\frac{p}{R_{a2}T_2}. \tag{2}$$

For static equilibrium the following inequality must hold:

$$\rho_1>\rho_2,$$

or, from (1) and (2)

$$R_{a1}T_1<R_{a2}T_2. \tag{3}$$

The limiting case of stability is therefore

$$R_{a1}T_1=R_{a2}T_2. \tag{4}$$

But, the gas constant for damp air (R_a) is related to that for perfectly dry air (R_d) by (very nearly)

$$R_a=\frac{R_d}{\left(1-\frac{3}{8}\frac{e}{p}\right)}.$$

Substituting in (4) and rearranging, we have

$$T_2=\frac{\left(1-\frac{3e_2}{8p}\right)}{\left(1-\frac{3e_1}{8p}\right)}T_1$$

or

$$=\left(\frac{8p-3e_2}{8p-3e_1}\right)T_1$$

$$T_2=\left[1+\frac{3(e_1-e_2)}{8p-3e_1}\right]T_1. \tag{5}$$

Neglecting $3e_1$ because it is small compared to $8p$, calling $(e_1-e_2)=\Delta e$, and $T_2-T_1=\Delta T$ we have

$$\Delta T=T_2-T_1=\frac{3}{8}-\frac{\Delta e}{p}T_1. \tag{6}$$

Taking values that are frequently observed in the case of *Ts-Tg* discontinuities: $T_1=290°$A, $\Delta\ e=10$ mb, $p=800$ mb, we find that $\Delta T=1.36°$ C.

It must be remembered that in the above development we have treated the discontinuity as being mathematically abrupt. Thus the resulting ΔT will be a discontinuous change at the level of p from T_1 to T_2. Even if such a discontinuity existed in the atmosphere it would be impossible to detect with our present means of securing temperature-altitude records, for the lag of the thermometric element of the thermograph would cause a continuous rather than a discontinuous record of temperature at the level p. There would, however, be a rapid increase of temperature with elevation shown by the thermogram at this point. The fact that no such rapid increase in temperature usually occurs at the *Tg-Ts* boundary surface, but that most observed cases are isothermal layers, indicates that these transitions of moisture are not extremely abrupt. From (6) it is also clear that these so called minimal inversions are not capable of accounting for the pronounced inversions actually observed at lower temperatures and humidities. For example, let $T_1=275°$A, $e_1=6$ mb, $e_2=2$ mb, and $p=750$ mb. Then $\Delta T=0.55°$ C. With these values of T, e, and p, the observed inversions are generally of the order of 2° or 3°.

We may consider the same problem in the case of a discontinuity of second order (i. e., assuming a continuous though rapid change of vapor pressure with height). Brunt (*6*) has given such a development; for stability, Brunt finds that the following inequality must hold:

$$-\frac{\partial T}{\partial z}<0.0001+\frac{3}{8}\frac{T}{p}\frac{\partial e}{\partial z}, \tag{7}$$

where $\frac{\partial T}{\partial z}$ is the lapse rate in the transition zone; z, of course, represents elevation. (The value 0.0001 is really a close approximation to the dry adiabatic lapse rate in °C. cm).

In the limiting case, if we are to have an isothermal layer

$$\frac{\partial T}{\partial z}=0$$

which is stable, and as before, $T=290$, $p=800$, the moisture distribution is given by

$$\frac{\partial e}{\partial z}=-\frac{8\times0.0001\times800}{3\times290}=-.000735 \text{ mb/cm}= -7.35 \text{ mb/100m}$$

Now it is difficult to say what is actually the nature of the variation in vapor pressure with elevation in these discontinuities. Assuming e to be a linear function of z, which seems reasonable, and obtaining $\frac{\partial e}{\partial z}$ for some 40 *Ts-Tg* boundary layers chosen at random, the writer finds an average far less than 7.35 mb/100 m. In fact, the average of these random 40 cases was less than 2 mb/100 m, and the individual values rarely exceeded 2 mb/100 m. Computing the steepest possible lapse rate which would be stable for $\frac{\partial e}{\partial z}=2$ mb/100m we have

$$-\frac{\partial T}{\partial z}=0.0001.+\frac{3}{8}\times\frac{290}{800}(-0.0002)=0.728°\text{C./100m.}$$

From the above analysis it becomes manifest that some factor other than the maintenance of static equilibrium must be at work to produce the pronounced stability of observed moisture discontinuities. It is interesting to note that moisture discontinuities of the *Ts-Tg* type simply cannot possess very steep lapse rates—a fact which (7) clearly shows. The effect of mechanical turbulence acts in the same general direction—that is, as the lapse rate becomes steeper the upward transport of moisture increases, thereby destroying the abruptness of the humidity transition.

RADIATIVE TRANSFER ACROSS A SURFACE SEPARATING DRY AIR FROM UNDERLYING MOIST AIR

Objections to the treatment of an unsaturated moist layer as a black body have already been stated. The following development will be based on several assumptions which, though rough approximations, are believed to hold with sufficient accuracy to be justified.

The work of Simpson (7) and its extension by Brunt (8, 9) have opened a way to the solution of many problems of radiation which previously seemed hopelessly complex; and the success of Simpson and Brunt in applying their methods is encouraging.

From the experimental results of Hettner (10) on the absorption spectrum of water vapor, Simpson computed the absorption coefficients for individual wave lengths for a layer containing 0.3 mm of precipitable water per square centimeter cross section in the form of vapor. He constructed from these computations a graph showing the absorption by water vapor as a function of wave length. He then plotted on the same graph the absorption curve for carbon dioxide, reduced to the probable amount which would be present in such a layer of air. The source of his data was the experiments of Rubens and Aschkinass (11). From this superimposition of curves he was able to make the following generalizations:

A column of air which contains 0.3 mm of percipitable water in the form of water vapor and 0.06 gram of carbon dioxide (1) behaves like a black body for wavelengths between $5\frac{1}{2}\mu$ and 7μ and for all wave lengths greater than 14μ; (2) absorbs part, and transmits part, of the radiation it receives between 4μ and $5\frac{1}{2}\mu$, between 7μ and $8\frac{1}{2}\mu$, and between 11μ and 14μ; and (3) is completely transparent to radiation between $8\frac{1}{2}\mu$ and 11μ.

Since terrestrial radiation is all beyond 4μ, wave lengths shorter than this were not considered.

The existence of a transparent band between $8\frac{1}{2}\mu$ and 11μ, and the semitransparent bands between 4μ and $5\frac{1}{2}\mu$, between 7μ and $8\frac{1}{2}\mu$, and between 11μ and 14μ, have been found to be extremely important in the heat balance of the atmosphere as a whole.

Some objection has been raised to Simpson's application of Hettner's results on the ground that Hettner's observations were made under one pressure, and that other experimental results suggest that the absorption coefficient depends upon pressure. As far as the author knows, however, no fundamental revision of Simpson's results has yet appeared; and the success of other investigators in working with them seems to justify their use here.

Brunt (8) has suggested that the term "W-radiation" be used to denote that radiation composed of all the wavelengths wherein water vapor radiates and absorbs like a black body. This terminology will be used here. He has shown that the mean downward radiation R of the atmosphere on clear nights can be very well represented by a formula of the form

$$R=\sigma T^4(a+b\sqrt{e}),$$

where T is the absolute temperature and e the vapor pressure of the surface layers; σ, the Stefan-Boltzmann constant; and a and b are constants. R is composed chiefly of W-radiation, although some downward radiation comes from the semitransparent bands. An explanation of the dependence of R upon the square root of e, based upon Dennison's theory of the shape of the infrared absorption lines, has been offered by Pekeris (12).

The values of the constants a and b seem to differ widely for different places, and the reason for this variation probably lies chiefly in the instruments with which the observations are made. There can, however, be no doubt that the distribution of water vapor and of temperature with elevation have some effect upon a and b. Brunt's empirical formula shows that part of the downward radiation comes from some source other than water vapor. Suggestions that this source may be carbon dioxide or ozone have been made, although it has not been shown that Brunt's formula holds under low vapor pressures.

Let us now proceed to the problem of the *Ts–TG* boundary layer. It may be stated in this way: Given a dry stratum of *Ts* air immediately overlying a moist and relatively cooler current of *TG* air. Assuming that a radiative balance is established after a finite interval of time, what will be the general nature of this equilibrium?

The following assumptions will be made: (1) Simpson's conclusions concerning the absorption by a layer of air containing 0.3 mm of precipitable water and 0.06 gram CO_2 per square centimeter cross section are valid. (2) Brunt's formula for the downward radiation of the atmosphere holds at all levels, and the constants a and b remain the same throughout the small interval of height from the base to the top of the transition layer.

Consider, then, an abrupt transition from the *TG* to the overlying *Ts* layer, figure 1. Divide the air above and below the discontinuity into layers each of which contains in the vertical 0.3 mm of precipitable water per square centimeter cross section in the form of water vapor; the length l of a unit column can readily be found in terms of

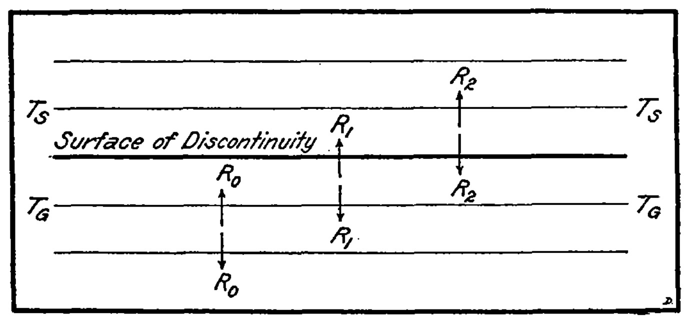

FIGURE 1.

the absolute temperature T and the vapor pressure e. If the density of the water vapor be ρ_w, its vapor pressure e, and the gas constant for water vapor R_w, then

$$e=\rho_w R_w T,$$

or

$$\rho_w=\frac{e}{R_w T}. \tag{8}$$

But $\rho_w l$ is to equal 0.3 g/cm². Thus

$$l=\frac{0.3}{\rho_w}=0.3\frac{R_w T}{e}=1.39\frac{T}{e}\text{ meters.} \tag{9}$$

The length l will in general be quite small, so that as a first approximation the temperature within one of these layers may be considered constant. For example, if $T=290$ and $e=10$ mb, l comes out about 40 meters; l will, of course, increase as T increases, and as e decreases. For radiative equilibrium in the uppermost layer of the *TG* air, the total incoming radiation must equal the total outgoing. We need only deal with the vertical streams of radiation, assuming that the layer is of infinite horizontal extent and that other streams cancel out. The emission from this layer, then, is R_1 both upward and downward. The layer receives radiation R_2 from the overlying layer containing 0.3 mm of precipitable water vapor, and also radiation R_0 from the adjacent lower layer. For the equilibrium of this uppermost *TG* layer, we need not be concerned with any other layers, for we are dealing with W-radiation. Strictly, the radiation entering any layer consists not only of W-radiation but also of some semitransparent radiation, a small part of which is absorbed; it is assumed that this is negligibly small compared to the total W-radiation. Thus for equilibrium:

$$R_1+R_1=R_2+R_0. \tag{10}$$

But in the moist T_G, R_0 is practically equal to R_1, for l is small and hence the temperatures of the two uppermost T_G layers are practically the same. Then from (10)

$$R_1=R_2; \tag{11}$$

but

$$R_1=\sigma T_1^4(a+b\sqrt{e_1}) \text{ and } R_2=\sigma T_2^4(a+b\sqrt{e_2})\cdot$$

Substituting in (11) and rearranging,

$$\left(\frac{T_1}{T_2}\right)^4=\frac{a+b\sqrt{e_2}}{a+b\sqrt{e_1}}, \tag{12}$$

or

$$\frac{T_1}{T_2}=\left(\frac{a+b\sqrt{e_2}}{a+b\sqrt{e_1}}\right)^{\frac{1}{4}}=\left[1+\frac{b(\sqrt{e_2}-\sqrt{e_1})}{a+b\sqrt{e_1}}\right]^{\frac{1}{4}}.$$

Expanding and neglecting terms of higher order,

$$\frac{T_1}{T_2}=1+\frac{1}{4}b\left(\frac{\sqrt{e_2}-\sqrt{e_1}}{a+b\sqrt{e_1}}\right). \tag{13}$$

(The series converges rapidly, for $\frac{b(\sqrt{e_2}-\sqrt{e_1})}{a+b\sqrt{e_1}}$ may be shown from any of the observed sets of a and b values to be small compared to one.) Thus

$$\frac{T_1}{T_2}=1-\frac{0.25(\sqrt{e_1}-\sqrt{e_2})}{\left(\frac{a}{b}+\sqrt{e_1}\right)},$$

or

$$T_1=T_2-\frac{0.25(\sqrt{e_1}-\sqrt{e_2})T_2}{\left(\frac{a}{b}+\sqrt{e_1}\right)} \tag{14}$$

If we set $\beta=\frac{0.25T_2}{\frac{a}{b}+\sqrt{e_1}}$ and call $T_2-T_1=\Delta T$, (14) becomes

$$\Delta T=\beta(\sqrt{e_1}-\sqrt{e_2}), \tag{15}$$

which, then, should give the magnitude of the inversion. While T_2 and e_1 are easily obtained from upper air soundings through T_G–T_S boundaries, a and b values are not available. However, from the sets of values given by Brunt (**8**), a is always much larger than b; in the average of his data for six places of observation, a comes out about six times as large as b. The relatively small variations of e_1 in the denominator of the fraction representing β will, therefore, be of small consequence in the variation of β. Similarly, the percent deviation in T_2 in the numerator is quite small. It is probable then that the greatest variations in β are caused by the variation of the constants a and b. It seems safe to assume that a and b change chiefly because of the variation of the moisture and temperature distribution with elevation. If, then, we choose data from T_G–T_S discontinuities where T and e are similarly distributed, we should be able to determine the appropriate value of β.

A complication arises from the fact that abrupt discontinuities do not exist in Nature, but are transformed into transition zones through the action of turbulence. If, then, we assume that a particle in the T_S air, after having reached radiative equilibrium immediately above the discontinuity, is carried adiabatically to the top of the transition zone, we may replace ΔT in (15) by $\Delta\theta$, where $\Delta\theta$ is the difference in potential temperature between the base and top of the transition zone. Thus

$$\Delta\theta=\beta(\sqrt{e_1}-\sqrt{e_2}),$$

or

$$\beta=\frac{\Delta\theta}{\sqrt{e_1}-\sqrt{e_2}}. \tag{16}$$

In order to test the validity of the above reasoning it is necessary to have appropriate aerological soundings in which the records of temperature and particularly humidity are reasonably accurate. Ascents in relatively warm air are peculiarly well adapted for this study, for it is well known that the hair hygrograph reacts more quickly and registers more reliably under an environment of higher temperatures. The errors in the recorded humidity during aerological soundings appear to be due in large part to the lag of the hair used in the hygrograph. Spilhaus (**13**) has recently made a fundamental study of the transition rate of the human hair, suggesting methods by which the lag constants appropriate to any particular hair hygrograph may be determined. It is to be hoped that in the future these corrections will be applied. Until this procedure is adopted generally, investigators must place upon the hygrograph records an interpretation of their own. Indeed, synoptic meteorologists (**14, 1**) have been doing this very thing for some time. It is likewise necessary in the preset discussion to "interpret" the records. This does not mean that particular records were modified to fit the theory; all humidity values were used *as recorded*. It is in explaining deviations from the theory that interpretation is helpful.

If there were available perfect soundings which penetrate moisture discontinuities of the type under discussion, and if the balance suggested above were always effective, we might plot on a graph the appropriate values of $\Delta\theta$ (the difference in potential temperature through the transition zone) as ordinate and the corresponding values of $\sqrt{e_1}-\sqrt{e_2}$) as abscissa. If β were constant, we should expect in accordance with (16) to obtain a straight line passing through the origin, the slope of which would give the value of β. The writer has taken at random 41 cases of abrupt moisture discontinuities in which a dry stratum overlay a moist and potentially colder layer. For reasons given above, the selection was restricted to soundings made through relatively warm air masses. Cases in which the base of the discontinuity was below 1,000 meters were not included, because of the complexity of effects observed within the surface layers. Cases in which a cloud layer covered more than five-tenths of the sky were also eliminated, for here it seems logical to assume that a different type of equilibrium (discussed in the next section) obtains. Of the 41 cases, 13 contained clouds covering five-tenths or less of the sky; the remaining 28 cases were entirely free of clouds.

In figure 2, $\Delta\theta$ is plotted as ordinate against $(\sqrt{e_1}-\sqrt{e_2})$ for the 41 cases. The straight line, drawn from inspection, appears to fit the observed points fairly well, corresponding to a constant slope $\beta=4$; open circles designate those points which deviate most widely from the straight line. To offer an explanation of these deviations, it is necessary to inspect the original records from which the values were obtained. The particular soundings are shown in figure 3; those in the upper half of the figure are the flights corresponding to the open circles above the line in figure 2 and those in the lower half to the open circles below the line. Numbers beside the soundings are the specific humidities at the respective levels. Arrows indicate the points chosen as the upper and lower boundaries of the transition zone. The outstanding distinction between the soundings in the upper and in the lower half of figure 3 is that the upper soundings indicate that the recorded values of specific humidity at the upper boundary of the transition zone are much too high. It is reasonable to assume that above the transition zone the variation of the moisture content with elevation is very small compared to the variation within the transition zone; this is clearly demonstrated by the older kite soundings wherein a much slower rate of ascent of the meteorograph made possible more reliable humidity measurements. Yet in the soundings referred to, the specific humidities at the first significant points above the upper arrows (the tops of the transition zones) are decidedly lower than those at the tops of the corresponding transition zones. Take, for example, the sounding at Oklahoma (OL) City on June 17, 1935. Here the humidity apparently falls from 6.6 g/kg at 2,990 m to 3.2 g/kg at 4,300 m, in spite of the adiabatic lapse rate within the intervening layer; this seems highly improbable. Again, take the case of OL on August 1, 1935. The specific humidity is recorded as falling off from 8.6 g/kg to 4.5 g/kg from 2,100 m to 3,190 m; it is much more probable that in passing through a transition zone of only 280 m, as in this case, the hygrograph did not have sufficient time to record true values. In view of the probability that e_2, the vapor pressure at the top of the transition zone, is much too large and hence that $(\sqrt{e_1}-\sqrt{e_2})$ is actually larger than recorded, it seems likely that all the crosses above the straight line in figure 2 should be displaced to the right. The ascents in the lower half of figure 3 indicate errors in the same direction, but here the probable errors are comparatively small. In fact, all these lower ascents indicate an unusual sensitivity of the hygrograph. Take for example the sounding at San Antonio (ZN) on June 17, 1935. The recorded specific humidity falls off from 13.4 g/kg to 7.3 g/kg from 1,590 m to 1,850 m. The fact that at 2,900 m the specific humidity is only 0.5 g/kg lower than it is at 1,850 m indicates that the value at 1,850 m is approximately correct. It is probable, then, that the crosses well above and those well below the line represent two extremes: the former corresponding to unusual sluggishness of the hygrograph, the latter to unusual sensitivity. The points represented as crosses would assumedly fall more nearly along a straight line if more reliable values of e_2 were available. The other points in figure 2 may thus be considered to represent cases in which the hygrograph behaved normally and similarly. It is these cases from which we must ascertain the most probable value of β. It is obvious that the true value will be somewhat below that given by the slope of the straight line in figure 2.

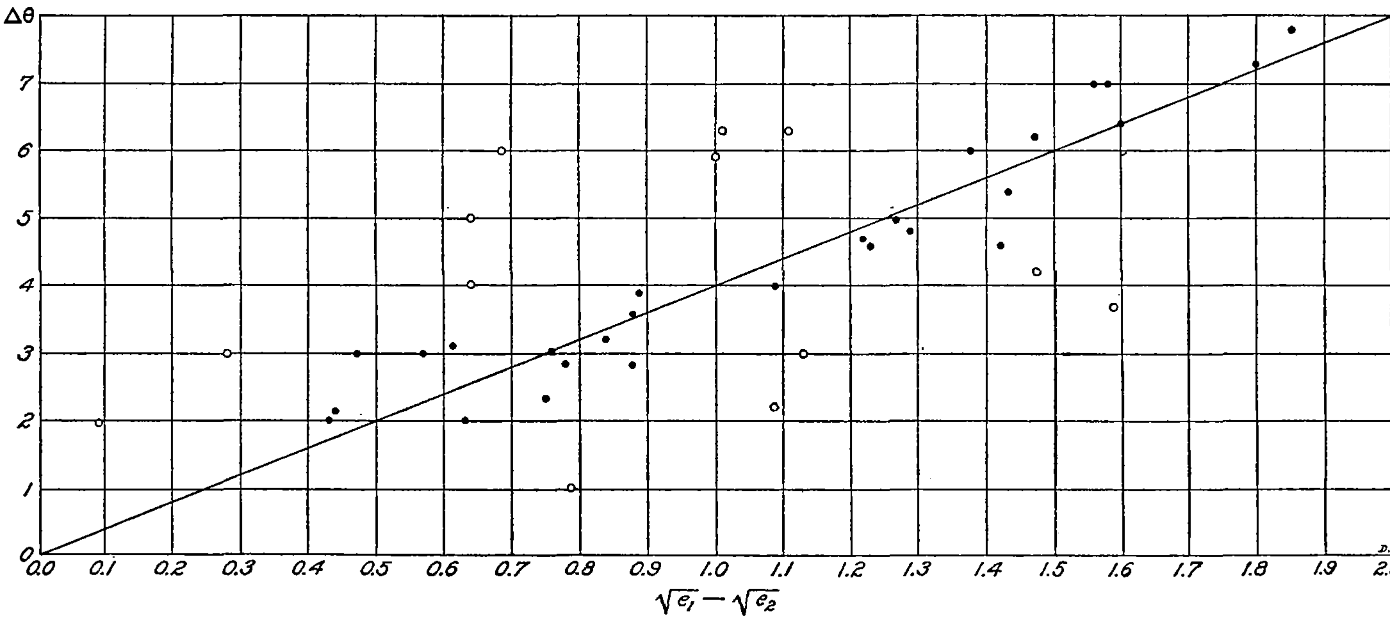

FIGURE 2.

While $\Delta\theta$ enables one to compute the difference in entropy through the transition zone, it is impossible from the above analysis to find its thickness. Clearly, this should depend upon the degree of turbulence. Qualitatively we should expect the thicker zones of transition to be formed when the overlying dry air is not much warmer than the underlying moist stratum, when the lapse rates in the air masses are steepest, and when the wind shear at the discontinuity surface is most pronounced. In the limited set of data used, some evidence of these factors appears, although it is not conclusive. This phase of the problem appears to be extremely complicated, and here no attempt to treat it will be made.

Because clouds act as black bodies, the currents of radiation emanating from their upper surfaces are appreciably greater than the W-radiation of unsaturated air with the same temperature and moisture content. In general, then, we should expect a greater loss of heat from the base of the moisture discontinuity when cloud forms are present. Therefore, $\Delta\theta$ and consequently β will be larger with clouds at the inversion base. The previous analysis obviously

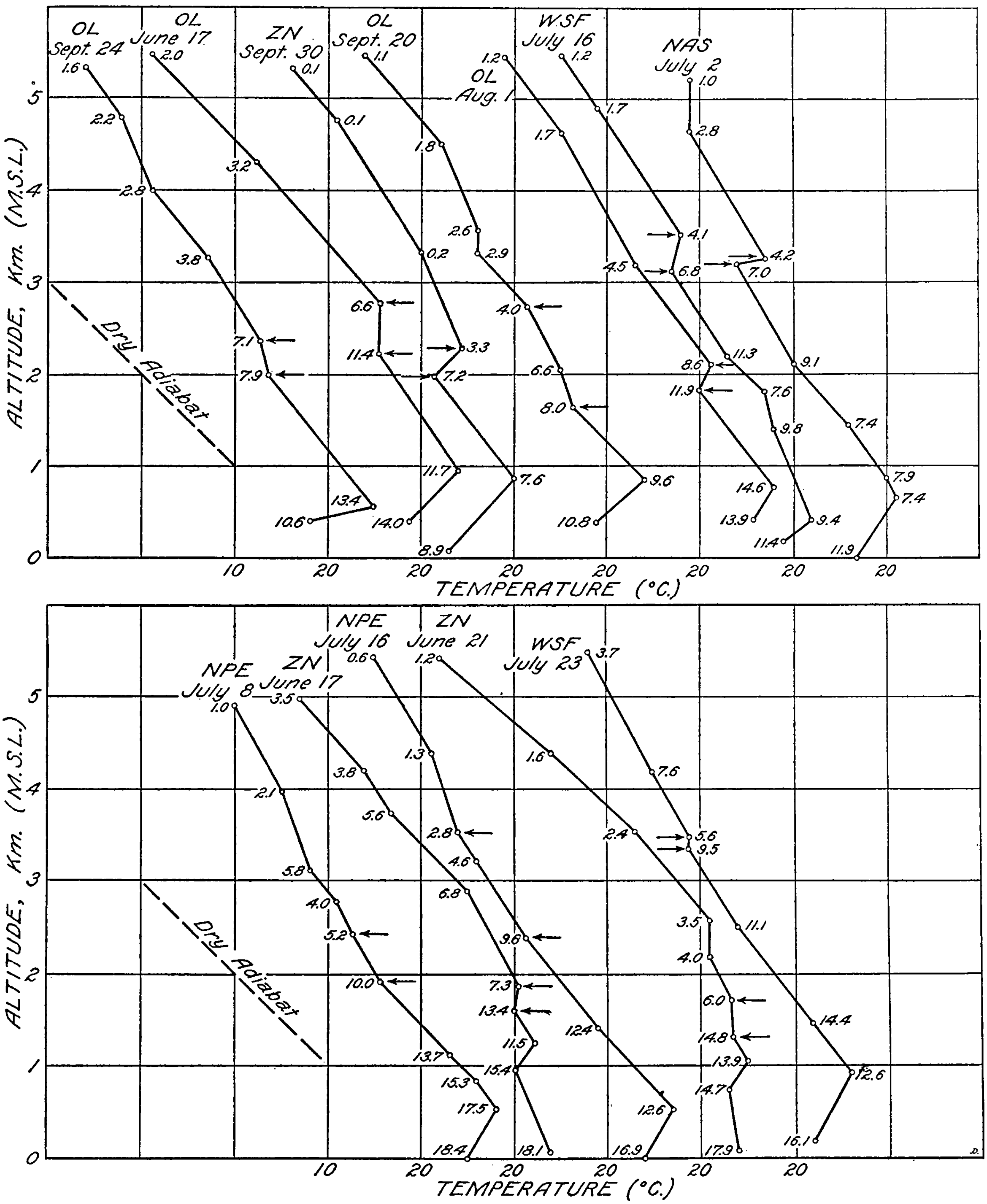
OL Sept. 24
OL June 17
ZN Sept. 30
OL Sept. 20
OL Aug. 1
WSF July 16
NAS July 2
Dry Adiabat
ALTITUDE, Km. (M.S.L.)
TEMPERATURE (°C.)
NPE July 8
ZN June 17
NPE July 16
ZN June 21
WSF July 23
Dry Adiabat
ALTITUDE, Km. (M.S.L.)
TEMPERATURE (°C.)

FIGURE 3.

does not hold when a cloud deck is present; it is interesting, however, to note the extreme divergence of β values when clouds cover more than five-tenths of the sky. A plot of $\Delta\theta$ against $\sqrt{e_1}-\sqrt{e_2}$ in 20 such cases shows no systematic arrangement of points. Furthermore, the average β for these 20 cases is 16.9, a value which is greater than any individual value of the noncloudy cases shown in figure 2. The cases in which clouds covered five-tenths or less of the sky have an average β of 6.2 compared to 4.15 for the noncloudy cases, showing the expected trend.

INFLUENCE OF CLOUDS

In the foregoing section it has been pointed out that by virtue of its black body behavior a cloud at the base of a moisture discontinuity modifies the radiative transfer. The upward current of radiation R_1 from the top of the cloud may thus be represented by $R_1=\sigma\ T_1^4$. The downward current R_2 which must equal this in radiative equilibrium may be expressed by Brunt's formula $R_2=\sigma\ T^4(a+b\sqrt{e_2})$; this downward radiation is composed mainly of W-radiation, although there is some radiation which comes from the semitransparent bands. Equating the two streams, we have

$$T_1^4=T_2^4(a+b\sqrt{e_2}),$$

from which, if T_2, a, b, and e_2 are known, we may compute T_1, the equilibrium temperature of the cloud top. In hundreds of observed inversions above clouds, there appears to be little or no relation between the magnitude of the inversion and the moisture distribution. That is, at different times nearly identical moisture discontinuities have widely different temperature or potential temperature distribution through the transition zone. This fact in itself suggests the conclusion which might *a priori* be expected, namely, that the time element is exceedingly important in establishing radiative equilibrium above a cloud. Furthermore, if one constructs, on one graph, curves of black body radiation and of W-radiation for the same temperature range as Wexler has done in his study of the structure of *Pc* air masses (**15**), it becomes clear that in order for the upward and downward streams of radiation to be equal there must be a tremendous inversion. The excessive surface inversions found in *Pc* air source regions bear this out. Because no such inversions are ever observed in the upper atmosphere in migratory air masses, it must be concluded that conditions do not maintain their status quo sufficiently long for equilibrium to become established. Probably the chief reason for the slowness of the cooling process lies in the fact that the cooling of the cloud readily extends downward because of the necessary upward transfer of heat through convection occasioned by the steepened lapse rate. In this manner the loss of heat from the cloud is supplied by a fairly thick layer, and the cooling process goes on at a slow rate. Early morning ground inversions, on the other hand, generally exceed dry inversions in the free air because here the chief source of heat compensation is the surface of the ground, which can communicate heat to the overlying air only as fast as it is supplied by molecular conduction from the layers of earth below the surface and the warmer air above. The latter process is clearly much slower than the convective transport operative below the cloud. There are many conceivable reasons for the upset of conditions which leads to the dissipation of the cloud or perhaps to the establishment of a different moisture distribution, and here we need mention only such factors as turbulent exchange, vertical motions, and air mass modification.

Thus it is of no use to attempt to find values of a and b through observations.

An interesting theoretical treatment of this problem has been offered by Mal, Basu, and Desai (**5**), who consider the change in lapse rate with time brought about by the presence of a cloud sheet at the base of a moisture discontinuity. These investigators dealt solely with W-radiation, considering that the effect of radiation in the semitransparent bands was relatively inconsequential. Perhaps the most uncertain part of their analysis lies in the evaluation of the coefficients of radiative diffusivity and eddy conductivity. The writer has previously pointed out (**1**) that their treatment of subsidence as a possible factor in the development of dry inversions was incomplete, since they merely considered that form of subsidence brought about by the frictional outflow from the lowest layers of an anticyclone, neglecting the more important effects of horizontal divergence. Furthermore, if their analysis held, one would expect an approximate continuity in lapse rate between the air above the inversion and that some distance below, for they assume that the original condition within the air mass is a linear lapse of temperature and that the presence of a cloud layer, through its black body behavior, changes this distribution of temperature so that an inversion is formed. In all the subsidence inversions which the writer has studied, not a *single* instance in which any such continuity is to be found has yet been encountered.

There appears, however, to be no serious objection to Mal's treatment in the event no appreciable dynamic forces are in operation.

It is generally supposed that insolation, because of its short wave character, is almost completely reflected back to space from the highly reflecting cloud surfaces. It would seem that a certain amount of absorption through diffuse radiation might take place at the top of the cloud, although it has not yet been possible to determine the magnitude of this effect. Until appropriate observations are at hand it seems most logical to suppose that the effects of insolation upon clouds is, compared with other effects, negligibly small.

SYNOPTIC IMPORTANCE OF EQUILIBRIUM

If the two types of equilibrium outlined in the preceding sections actually tend to prevail at moisture discontinuities, some light is thrown upon several questions important to synoptic meteorology. It has long been observed that clouds are generally colder than the clear spaces making up the environment at the same level. This phenomenon was for a time embarassing to the proponents of the theory of penetrative convection, but now it is quite generally accepted that the cooling is due to the inertial movement of the saturated mass of air beyond its equilibrium level. It seems possible, however, that these horizontal gradients of temperature may in part be due to the difference in radiative balance discussed in the foregoing sections; this difference in balance, and evaporation as well, would naturally make the clouds colder at their tops than the surrounding dry air.

It is also conceivable that the difference of radiative effects may play a part in the development of convectional thunderstorms. Although an energy diagram based on a sounding made in the morning hours may indicate conditions favorable for the formation of thunderstorms, yet later in the day upper air indications may be unfavorable After cumulus clouds have formed, and built up to the base of the mositure discontinuity, they must affect appreciably the temperature distribution both below and above the discontinuity, for a black body surface has been substituted for one emitting only W-radiation. Assuming that the effects of insolation are unimportant, there must therefore be a net loss of heat from the cloud top and a smaller gain of heat in the unsaturated air above the discontinuity, for much of the black body radiation of the cloud passes unabsorbed through the transparent bands of the unsaturated air. This heat is probably supplied mainly by the internal energy of the air at the cloud level. It should be noted, however, that the formation and development of the cloud is a process wherein heat is transferred through convection from the lower insolation-heated levels to the upper levels. If there is an appreciable loss of heat at the level of the cloud top, the stability of the transition zone above it must be increased. In this manner more work must be done by the rising cloud mass in order to penetrate the more stable layer and develop to thunderstorm proportions.

Surfaces of subsidence may also be modified appreciably by radiation influences. Some of these effects were mentioned in a previous paper (**1**). With limited data on subsidence inversions, the analysis discussed above would not be applied with reasonable success; that is, there appeared to be no general agreement in the computed values of β. In many cases it was obvious that the humidity values were at fault, for subsidence inversions appear to be restricted to the colder air masses where the hydrograph is least reliable. Aside from the errors in the measurement of humidity, it seems probable that for the most part the effect of radiation upon noncloudy subsidence inversions is small compared with thermodynamic effects. In order to obtain a value of β agreeing with the above value of 4, subsidence inversions where $\Delta\theta$ is 6° C. would have to have a moisture discontinuity wherein $\sqrt{e_1}-\sqrt{e_2}=1.5$. If e_1 is 6 mb, a not uncommon value, then e_2 would have to be 0.9 mb. A discontinuity having these values has probably never been measured in a layer of a few hundred meters thickness, and it is hard to imagine its presence in one and the same air mass. It should be pointed out, though, that β may be a function of the temperature and moisture distribution, or that Brunt's downward radiation formula may not hold at low moisture contents. The very fact that subsidence inversions show markedly varying values of $\Delta\theta$ for similar moisture discontinuities seems to indicate that thermodynamic factors are more significant in their maintenance than radiation.

It is interesting to note that some 50 subsidence inversions showed in the mean a remarkable constancy of equivalent temperature through the inversion. In fact, the *mean* equivalent temperature at the base of these inversions was 274.1° A, while that at the top of the inversion was also 274.1° A! This agreement strongly indicates the presence of a convection mechanism pictured by Rossby (**16**); in his treatment Rossby arrived at the final distribution of temperature and moisture in and just above a layer stirred so that convective equilibrium is reached. The relevant conclusion as stated by Rossby is: "Thus, in the case of convection with condensation the theory indicates that there must be a temperature inversion at the upper boundary of the convective layer and this inversion must have such a value that the equivalent potential temperature and therefore also the equivalent temperature remain constant."[3] It is true that condensation is not always present at the base of a subsidence inversion, but the relative humidity at this level is practically always very close to 100 percent, and it is probable that condensation sets in from time to time. The alternate appearance and disappearance of ACu clouds at subsidence inversions seems to indicate that this is true.

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[3] Strictly speaking, the constancy of equivalent potential temperature through an inversion does not imply the constancy of equivalent temperature. For conditions observed within the atmosphere, however, the variation is negligibly small.