ATMOSPHERIC WAVES ON ISENTROPIC SURFACES AS EVIDENCED BY INTERFRONTAL CEILING OSCILLATIONS

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One cannot long be a student of dynamic oceanography without coming to realize the importance of wave motion in any fluid body. A natural tendency for a meteorologist who had undertaken such a study would be to attempt an application of the principles of wave motion to that greatest of all terrestrial fluid bodies, the atmosphere. The formation, characteristics, and effects of ocean waves are apparent to all who have viewed the sea, hence it is only natural that they should have received the attention of oceanographers at an early date. Atmospheric waves, on the other hand, are not nearly so apparent; and while they may be far greater in magnitude than those in even the wildest sea, they are noted by comparatively few casual observers, and probably for this reason, among others, they have not been given proportionate attention by meteorologists. However, the formation of smoke waves and the high altitude billow cloud has been attributed to their effects; and their existence, as evidenced by pressure, temperature, wind and precipitation fluctuations, has been studied and described by various investigators.

Von Helmholtz (1), writing during the latter part of the nineteenth century, showed that whenever two fluids of different densities flow one over the other with unequal velocities, wave motion is induced at the surface of juxtaposition of the two fluids. It seems logical to assume that any wave, propagated on the surface of discontinuity between two air masses with different entropies, would be evidenced by a corresponding wave effect in any cloud stratum which might form at such a surface through either adiabatic or radiative processes of cooling.

CEILING OSCILLATIONS OBSERVED AT SAN DIEGO

This reasoning appears to be validated by two interesting cases of interfrontal ceiling oscillation, showing this wave effect, which were recently observed by the writer at the Weather Bureau Airport Station, San Diego, Calif., and which occurred at times when careful measurement of the period and amplitude of fluctuation was possible. Ceilings in the San Diego area normally remain fairly constant, any change being a slow lowering or a more rapid raising coincident with frontal movements, diurnal fluctuations, or occasionally, especially with very low ceilings, a rapid variation of cloud height with sudden changes in wind direction or velocity.

Shortly after sundown on January 15, 1936, however, it was noted that the cloud height appeared to be fluctuating a great deal; this fact was a matter for some concern, because ceilings were originally very low, and it was a question of how long flying conditions would remain hazardous at Lindbergh Field. Accordingly, at 7:05 p. m., observations of ceiling height were begun, and measurements taken at intervals of 5 minutes until 8:05 p. m. From that time, since it was apparent that the frequent oscillatory motion was slowing down, the measurements were made at 10-minute intervals until 11:05 p. m. The 5 p. m. synoptic chart showed the presence of a weak cold front a short distance to the northwest of San Diego; while at 11 p. m., airway weather reports indicated that the front had passed all stations on the coast, and was advancing rapidly eastward, a fresh NPP (transitional polar Pacific) air mass then occupying the entire region. Previous to 7 p. m. the ceiling had been lowering slowly, but no oscillatory motion was noted until shortly before the comparative observations were initiated. The rapid rise in ceiling which took place at 8:45 p. m. (fig. 1) was concomitant with the passage of the cold front, and marked the cessation of the oscillations. The cloud type at this time changed from the characteristic prefrontal stratus to stratocumulus which, at 11 p. m., began to break somewhat, necessitating the termination of the interesting series of observations.

An examination of figure 1 reveals that the fluctuations before 9 p. m. (previous to the passage of the cold front) tend to occur at more or less regular intervals. Whatever irregularity may be in evidence can be accounted for, in part, by the method of observation; a continuous record of the ceiling oscillation was not possible. In each case it may be observed that the lag or advancement of the wave crest is a whole number of the observational intervals. Between 7:10 p. m. and 8:45 p. m. there were six complete oscillations, giving an average period of 15.8 minutes. The amplitudes of the waves, on this occasion, were somewhat variable, which would be expected under the influence of such rapidly changing physical conditions; the average was 22.8 meters with an extreme amplitude of 53 meters. The average height of the ceiling during the period of oscillation was 185 meters, which for the purpose of this study is taken to be the altitude of the discontinuity surface, the assumption being that the stratus clouds formed directly above this surface.

On February 22, 1936, a similar situation presented itself and observations of ceiling height were made at approximately 10-minute intervals from 6:15 p. m. until 11:55 p. m. On this occasion the oscillatory movement was even more pronounced than in the preceding case, the wave effect being unmistakably present; and a longer series of observations was possible, as no front passed the station during the period. The cloud layer remained uniform and unbroken throughout the entire 6 hours. It appears that in this case the discontinuity surface existed between a shallow wedge of NPP (transitional polar Pacific) air at the surface and a slowly overrunning mass of TP (tropical Pacific) air above. The cold front, followed by PP (polar Pacific) air, did not pass the station until shortly after 5 a. m. the next morning, 5 hours after this series of observations had been terminated.

An inspection of the data plotted in figure 1 reveals that the fluctuations were definitely isochronal. Between 6:15 p. m. and 9:15 p. m. the period between successive crests was exactly 40 minutes; this interval shortened somewhat during the next hour and one-half, which a later mathematical consideration will show would be expected of waves formed on a steadily lowering discontinuity surface. Between 6:35 p. m. and 10:55 p. m. there were seven complete oscillations giving an average period of 37.1 minutes. The average amplitude was 34.6 meters with an extreme of 70 meters. The average height of the ceiling from 6:15 p. m. until 10:55 p. m. was 158 meters which, again, is assumed to be the altitude of the discontinuity surface.

In both of these cases the characteristic weather phenomena were those which would be expected to precede rather than follow the passage of the warm front; such a

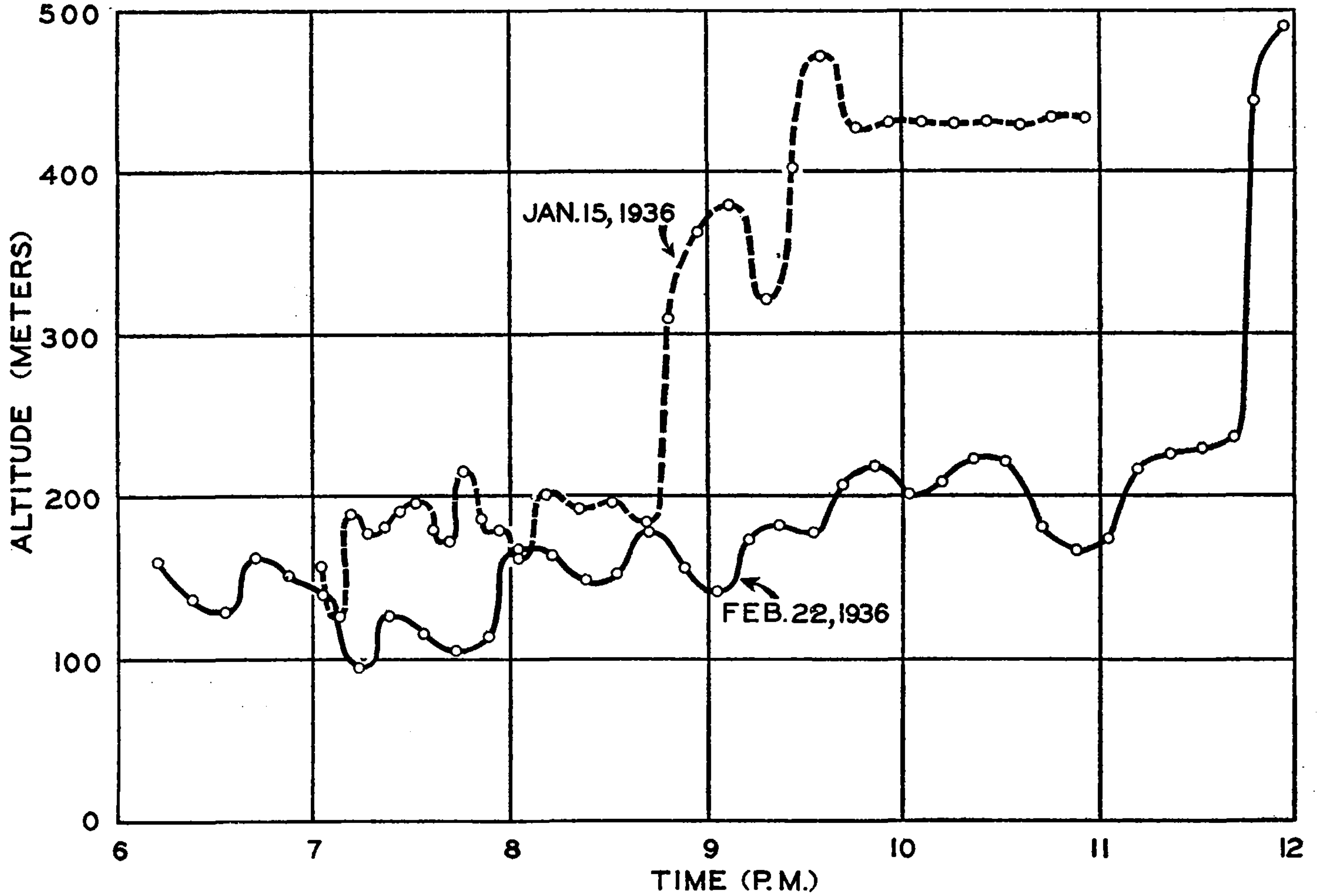

FIGURE 1.—Ceiling oscillation at San Diego, Calif.

condition, however, is not unusual in the warm sector of the cyclone in this region, since orographic influences are of prime importance. There is little doubt that these oscillations were Helmholtz waves induced along the surface of discontinuity between two masses of air of different entropy; such a surface might be formed between a shallow wedge of colder maritime air entrapped at the surface against the low hills or mountain ranges which parallel the coast in this region, and an overrunning layer of warmer maritime air above—the whole process probably an entirely local phenomenon.[1]

METHOD OF DETERMINING CEILING HEIGHTS

The ceiling heights were determined by the use of the ceiling-light projector and Marvin clinometer; and care was exercised to be certain that each observation was representative and included several distinct measurements each time. It is realized that this method of cloud height determination is subject to some error, especially with the higher ceilings over 600 meters, but it is felt that any error of observation would be a negligible part of the fluctuations actually observed and should tend to be constant, either slightly too high or too low.

[1] According to Byers a similar discontinuity surface might be produced in a tropical maritime air mass which has been cooled in the lower levels because of its long trajectory over colder waters, and has been rendered extremely stable by the time it has reached San Diego. Inasmuch as the amplitude of such waves decreases rapidly with increasing distance from the surface of origin, it is doubtful that the wave effect could be evidenced by ceiling oscillations in a cloud stratum formed beneath such a surface unless the cloud layer were thin or the amplitude of the waves great.

TABLE 1.—*Amplitudes of half-waves*

Wave no. X2	Amplitude of ½ wave [1]	
	Jan. 15, 1936	Feb. 22, 1936
	Meters	*Meters*
1	53	32
2	13	70
3	17	31
4	24	23
5	43	61
6	36	21
7	2	28
8	19	38
9	39	38
10	10	4
11	3	39
12	15	18
13		21
14		60
Average	22.8	34.6

[1] The difference in height between 1 crest and the following trough, etc.

It is true that a similar appearance of ceiling oscillation might be produced by a series of measurements of the heights of the base of a cloud stratum which was irregular or partially broken in some sequential manner. On January 15, however, the clouds presented the uniform appearance characteristic of the stratiform type until after the passage of the cold front; on February 22 the clouds remained uniform throughout the entire 6-hour period covered by the observations. In both cases surface winds were light and constant in direction, and no scud or breaks in the overcast were observed.

BAROMETRIC OSCILLATIONS IN CONJUNCTION WITH THE WAVES

An examination of the barogram for January 15 revealed a slight wavy appearance of the trace from 6 p. m. until 9 p. m. in what was otherwise the record of a slowly rising barometer. A pressure oscillation was hardly noticeable on February 22. In both cases the fluctuation was less than 0.3 mb. Unfortunately the record of a microbarograph for these periods was not available; and a more thorough examination of the pressure fluctuations on the ordinary barogram obviously was impossible.

Fluctuations of other meteorological elements, such as are commonly regarded as indicators of atmospheric waves, were absent. Since no rain fell during either of the two periods under consideration, it was impossible to determine whether waves of these amplitudes would produce corresponding fluctuations in precipitation. Simple computations show, however, that any adiabatic cooling or heating due to the waves would be very slight and, therefore, any variation in precipitation from these causes would, of necessity, be exceedingly small. No periodic fluctuations in wind or temperature were observed, but such effects would not be expected unless the point of observation were in such a position relative to the discontinuity that the wave surface would be first above and then below this point.

MATHEMATICAL ANALYSIS OF DATA FOR FEBRUARY 22

A theoretical discussion of the wave lengths and velocities requires a knowledge of upper air conditions. Unfortunately on January 15 the Naval Air Station at North Island (2½ miles southwest of Lindbergh Field) did not make a morning aerographic airplane flight, nor did the Weather Bureau undertake the usual afternoon pilot balloon sounding. On February 22, however, both these observations were made; and although they preceded the time of ceiling measurements by a considerable period they may be utilized to give a fairly representative picture of the general conditions which prevailed aloft at the time. The aerographic sounding made at 6 a. m. on this date showed a temperature of 14° C. at the surface, and 17° C. at an elevation of 590 meters, a temperature inversion of 3° C. This inversion level is, of course, considerably above the height assumed 12 hours later, but it would be expected that some lowering of the discontinuity surface would have occurred during the intervening period. The pilot balloon flight made at 1:46 p. m. showed a marked wind discontinuity at 500 meters; but the practice of observing the balloon position at 180 meter intervals and computing by 2-minute periods tends to smooth out any sudden wind discontinuity, and the actual level may have been as much as 360 meters below that indicated. The upper air winds below 1,000 meters at the time were as follows:

	Meters per second
Surface, west	1. 8
250 meters, northwest	3. 8
500 meters, northwest	2. 0
750 meters, south-southwest	0. 8
1,000 meters, southwest	3. 2

Above this level the wind continued from the wouthwest and increased rapidly in velocity with elevation; it was southwest, 23.6 miles per second, at 5,300 meters, the maximum altitude reached.

The wave length (λ) of any simple transverse wave may be expressed as

$$\lambda=\frac{U}{\mu}\equiv U\tau \qquad (1)$$

where U is commonly assumed, as a first approximation, to be one-half the difference in speed between the two layers, μ is the frequency of oscillation, and τ the period. Taking on the basis of the above data the arbitrary but reasonable value $2U=4.0$ meters per second, the wave length is found to be 4,452 meters. A similar value of U for January 15 gives a wave length of 1,896 meters on that date. Both these values compare favorably with those derived by Haurwitz (2) for pressure oscillations produced under similar conditions at Blue Hill Observatory in December 1933.

The wave length may also be expressed in terms of temperature and difference in velocity (**2**):

$$\lambda=\frac{\left(\frac{2\pi}{g}\right)U^2T_2}{T_1\left(1-\frac{U^2}{gh}\right)-T_2} \qquad (2)$$

where g is gravity acceleration, T_1 the temperature in the upper layer, T_2 the temperature in the lower, and h the thickness of the lower layer or the height of the discontinuity surface. Inasmuch as there is a direct linear relation between temperature and density, T in these equations may be replaced by ρ without appreciable error. This formula should be applied when $\lambda/20>h$ which, from (1), is found to be true for February 22. The upper air data for 6:00 a. m. of that date give $U=2.0$ meters per second, $T_1=290°$ A, $T_2=287°$ A, and $h=158$ m, but the inadmissible figure of 241 m is then found for λ. Examination of (2) shows that λ increases with either increasing U or decreasing ΔT.[2] Evidently then, either $2U>4.0$ meters per second or $T_1-T_2<3°$. The value for U seems reasonable under the conditions observed, but a smaller value for ΔT appears proabable. Now, equation (2) can readily be transformed into the following (**2**):

$$T_1-T_2=\frac{U^2}{g}\left(\frac{T_1}{h}+\frac{2\pi T_2}{\lambda}\right) \qquad (3)$$

Substituting the same value for U, taking the values for T_1 and T_2 on the right as equal to 287° (the observed surface temperature throughout the 6-hour period), and the value of λ derived from (1), T_1-T_2 is found to be 0.9°, which is an entirely reasonable figure.[3] On the other hand, from (2) or (3),

$$U=\left\{\frac{g\Delta T}{\frac{T_1}{h}+2\pi\frac{T_2}{\lambda}}\right\}^{1/2} \qquad (4)$$

and substituting in this equation the values for h, T_1 and T_2 which were observed, namely, 158 m, 290° and 287° respectively, 2U is found to be 7.4 meters per second which is, however, too large a value under the observed conditions.

[2] Decreasing ΔT to a certain limiting value would give negative figures in the denominator of equation (2), rendering values for λ meaningless. Decreasing it still farther would bring $T_2>T_1$ in which case there would no longer be an inversion but a condition approaching instability, where the formulae obviously could not apply.

[3] Actually the temperature difference would be slightly smaller, because the effect of compressibility has been disregarded, and the oscillations assumed to be simple transverse waves.

The velocity of propagation of surface waves, such as those between air and water, is given by

$$V=\sqrt{gh} \tag{5}$$

when the depth h is small compared to the wave length; here, however, $\Delta\rho/\rho_1$ is taken as unity; in a consideration of internal waves formed between two similar fluid bodies, a correction for small differences in density must be applied, and according to Ekman (**3**), equation (5) becomes

$$V=\sqrt{\frac{gh\Delta\rho}{\rho_1}}\cong\sqrt{\frac{gh\Delta T}{T_1}}, \tag{6}$$

in which ρ_1 is the density of the upper layer. Assuming ΔT to be 3° and T_1 to be 287°, a velocity of propagation of 4.0 meters per second is found. By taking ΔT as 0.9° and T_1 as 287.9° a velocity of propagation of 2.1 meters per second is found, which checks with the computed velocity of 2.0 meters per second for the wave length of 4,452 meters and period of 37.1 minutes.

WAVES ON AIR AND WATER SURFACES COMPARED

Waves in the atmosphere and in the sea have previously been compared, in a general way, as to magnitude; a similar comparison of the energy of wave forms in the two media is also possible. In any wave, two forms of energy occur; namely, the potential energy of the deformation and the kinetic energy of the motion. If, then, the energies of the waves in the two media are to be compared, a comparison of the densities of the two fluids is all that is necessary, assuming that the velocities are equal, and neglecting, for the time being, the effect of the different compressibilities of air and water. If the surface of the sea be at 14° C. (salinity 35.00) and the atmosphere (dry air) at 14.9° C. (pressure 1,013.3 mb), the ratio of densities is 1:0.001. It follows that waves formed on a level sea surface, corresponding to the atmospheric waves observed on January 15 and February 22, would be insignificant ripples of amplitudes 22.8 millimeters and 34.6 millimeters, respectively.

Helmholtz (**1**) concludes from the principle of mechanical similarity that if waves between two air masses (with a temperature discontinuity of 10° C.) and between air and water (both at 0° C.) are to be similar, the quantities $\frac{\sigma}{1-\sigma}\cdot\frac{b_1^2}{n}$ and $\frac{1}{1-\sigma}\cdot\frac{b_2^2}{n}$ must remain unchanged, where σ denotes the ratio of densities on either side of the discontinuity, b_1 and b_2 the velocities parallel to the surface of discontinuity, and n the linear dimension.

He finds that for the waves formed on these two surfaces, with the same wind velocity, to be geometrically similar, the wave length of the air wave must be increased in the ratio of 1 to 2630.3. With the same ratios, sea waves corresponding to the atmospheric waves on January 15 and February 22 would have wave lengths of 0.7 m and 1.7 m, respectively.[4]

Helmholtz' comparison of the internal waves fo the atmosphere with surface waves of the sea is, however, misleading inasmuch as two entirely different wave forms are being considered. According to McEwen and Chambers, of the Scripps Institution, if internal waves in the sea, such as boundary waves, be considered instead of surface forms, the wave lengths and the amplitudes would be more nearly comparable to those of atmospheric waves.

CONCLUSION

Whether manifested by ceiling oscillations or by turbulent conditions aloft, the existence of atmospheric waves is of importance to the aerographer, the airplane pilot, and the airway weather forecaster. The study of atmospheric waves under different meteorological conditions and as influenced by various topographical features, with the end in view of forecasting their occurrence and effects, should prove to be of vital importance to these groups, and of interest to the meteorologist. Additional studies of ceiling oscillations, similar to the pressure oscillation studies by Haurwitz, Stone, and Brooks (**2**), Clayton (**4**), Lamb (**5**), Namekawa (**6**), and Murase (**7**), might reveal interesting facts regarding the formation and effects of atmospheric waves on isentropic surfaces.

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[4] The "breaking" effect commonly observed in water waves at shallow depths was not evident. It appears that the slopes of the curves in figs. 1 and 2 are as often steeper in the posterior portions as otherwise.

WEATHER OF 1936 IN THE UNITED STATES

By J. P. KOHLER

[Weather Bureau, Washington, D. C., February 1937]

The weather during the year 1936 was characterized by marked extremes in temperature and precipitation. Unparalleled prolonged periods of subzero temperatures obtained in many Western States in the early months of the year followed by unprecedented drought conditions during the summer months.

January and February 1936 brought the most severe weather ever experienced to several States in the north and middle sections of the Mississippi and Missouri Valleys; also locally in parts of the Ohio Valley. In the month of January only six States, namely, California, Colorado, Nevada, Oregon, Utah, and Washington had average temperatures above normal. The greatest negative departures were centered in the northern portions of the Missouri and Mississippi valleys. The mean temperature for North Dakota was −5.8°, or 12.1° below normal; likewise in Minnesota the departure from normal was −10.8°; South Dakota, −10.2°; Iowa, −9.0°; and