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SOME EMPIRICAL CHARACTERISTICS OF LONG WAVES ON MONTHLY MEAN CHARTS[1]

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ABSTRACT

The geographical frequency of ridges and troughs appearing on 30-day mean 700-mb. charts in the North American area during the past 20 winters is described as a useful supplement to the normal chart. Statistics pertaining to the dimensions, symmetry, and motion of planetary waves on these charts are summarized by use of graphical correlation and synoptic models. The results throw light on the structure of the centers of action in the atmosphere and indicate their interdependence.

CONTENTS

INTRODUCTION

The concept of planetary or long waves in the upper westerlies has played an increasingly important role in meteorology since the pioneering work of Bjerknes [2] and Rossby [35]. It is now generally recognized that these long waves to a large extent control the various weather elements, such as temperature and precipitation. They are also closely related to the semipermanent centers of action at sea level, as well as the motion and development of individual cyclones and anticyclones.

Considerable work of a theoretical nature has been done on the motion of long waves. Rossby [35] expressed wave speed in terms of the zonal wind, wave length, and latitudinal variation of the Coriolis parameter. Subsequent investigations have included the effect of additional variables, such as lateral width (Haurwitz [16], Petterssen [34])-spherical shape of the earth (Haurwitz [17]), horizontal wind shear (Garstens [15], Thompson [40]), horizontal temperature gradient (Jaw [19], Yeh [46]), vertical wind shear (Charney [6], Fleagle [14]), and horizontal trough tilt (Machta [26], Petterssen [34]). Empirical studies relating the motion of long waves to these theoretical treatments have lagged behind. The simple Rossby formula, with some modification, has been successfully applied to both 5-day mean troughs (Namias and Clapp [28]) and daily troughs (Cressman [10], Sumner [38]); Petterssen's formula has yielded good results with short or minor daily waves (Johannessen and Cressman [20]); but few of the other formulas have been checked extensively. None of these theoretical equations will be explicity tested in this investigation since they were not designed for monthly mean maps. Instead the theoretical studies, together with synoptic experience, have been used to suggest important variables whose effect on wave motion will be tested empirically.

Other properties of long waves have not been studied as extensively as their motion. A large part of this report will therefore be devoted to the relationships which exist between simultaneous values of various wave characteristics. Three of these, wave length, amplitude, and wind speed, were found to be positively intercorrelated on

[1] Part of this paper was presented at the 114th National Meeting of the American Meteorological Society, New York City, January 29, 1952.

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5-day mean maps in North America by Bortman [4]. His study will be extended in this report to monthly mean maps and to additional features of the wave pattern. Some properties of long waves appearing on monthly mean maps have been discussed previously (Namias [29], Klein [21], Namias [31]). This report will describe these properties in greater detail and in a more quantitative fashion, but will not dwell on the physical mechanisms involved. It is hoped that this material will contribute to understanding of the general circulation and hence to extended forecasting.

The charts used in this project are 30-day means at the 700-mb. level (10,000 ft. before May 1945) obtained from the files of the Extended Forecast Section of the U. S. Weather Bureau dating back to October 1932.[2] The maps prior to November 1942 are drawn for calendar months only and are based in part upon judicious extrapolation of surface observations. Subsequent maps are based on superior upper air data and are analyzed for 30-day periods from mid-month to mid-month as well as for calendar months. Despite the partly overlapping nature of these later maps, each was considered as an independent case and treated in the same fashion as the non-overlapping maps for the earlier period. In this way the greater reliability of the later maps was used to advantage, and all available data were utilized. Use of the maximum possible number of maps was effected by including maps (after November 1942) for the periods mid-November to mid-December and mid-February to mid-March within the definition of winter season (December, January, and February) to which this study was restricted. Thus all winter monthly mean maps from December 1932 to February–March 1951, comprising a total of 93 cases, were used. As in previous studies, troughs and ridges were defined as lines connecting the points of minimum or maximum latitude reached by the isobars or contours. Axes of maximum contour curvature were not considered because they are difficult to locate and frequently coincide with ridge or trough lines defined in the objective manner given above.

GEOGRAPHICAL FREQUENCY OF RIDGES AND TROUGHS

The preliminary phase of this project consisted of a tabulation of the location of all mean ridges and troughs (as defined above) observed during the winter months of the past 20 years. This study was limited to the Atlantic, United States, and eastern Pacific at latitudes 30° N., 40° N., and 50° N. because of scarcity of data, particularly during the early war years, in other regions. At each of these latitudes the total number of ridges observed within each 5° longitude band was plotted on a map. These numbers were then combined in overlapping fashion to give the frequency of ridges within boxes of dimensions 10° of latitude by 10° of longitude, and isopleths were drawn, as shown in figure 1a. Isopleths of trough frequency were obtained in a similar manner, as illustrated in figure 1b.[3]

Many of the features of figure 1 can be clarified by reference to the long-period normal 700-mb. chart for the winter season, figure 2. This chart was prepared by averaging the normal 700-mb. height at standard intersections for the months of December, January, and February, as recently revised in the Extended Forecast Section, U. S. Weather Bureau [41]. As one might expect, monthly mean ridges are most frequent in regions occupied by normal 700-mb. ridges, in the northwestern United States, eastern Atlantic, and southeastern Pacific, while troughs occur infrequently in these areas. Likewise the regions of eastern North America at high latitudes and the west coast of North America at low latitudes are characterized by normal troughs, maximum monthly trough frequency, and small monthly ridge frequency.

Figure 1 contains additional information, however, which could not be readily anticipated from the normal chart. Most striking, perhaps, is the virtually complete absence of ridges in the northern and central Great Plains of the United States. This is a region where lee troughs frequently form as the prevailing westerlies blow across the Rocky Mountains. Because of this process a secondary maximum of trough frequency is located in Kansas and Nebraska (fig. 1b). Farther north the westerlies are normally stronger and the distance to the normal ridge shorter so that lee troughs (once formed) usually move rapidly eastward. As a consequence not a single mean trough or ridge was observed along the border between North Dakota and Saskatchewan during any winter month of the 20-year period comprising this study! It is also noteworthy that troughs are less frequent near the southern and central portions of the Rocky Mountains than they are on either side, where troughs tend to accumulate. Many features of figure 2 have also been noted for 5-day mean data (Wilkins [44], Myers [27]) and can be inferred from the theoretical pressure profiles computed by Colson [8] for air flow across the Rocky Mountains.

The effect of the Appalachian Mountains on ridge and trough frequency is somewhat similar to that of the Rockies but less marked. Downstream from the Appalachians, off the east coast of the United States, ridges are virtually absent but troughs are abundant. In addition monthly mean troughs are more frequent both east and west of the Appalachians than they are right along the mountain chain. Similar results have been obtained for the frequency of both 5-day mean and daily troughs at 700 mb. in this locality. A corresponding effect is well-

[2] Monthly mean charts of this sort have been published regularly in the Monthly Weather Review since January 1944.

[3] In analyzing these charts the degree of longitude was assumed to be a constant unit of distance. In order to make the frequencies at 50° N. and 30° N. strictly comparable to those at 40° N. the former should be multiplied by the ratio: sin 50°/sin 40° or 1.19, and the latter by the ratio: sin 30°/sin 40° or 0.884. This refinement was neglected, however, because of the many experimental errors inherent in the data.

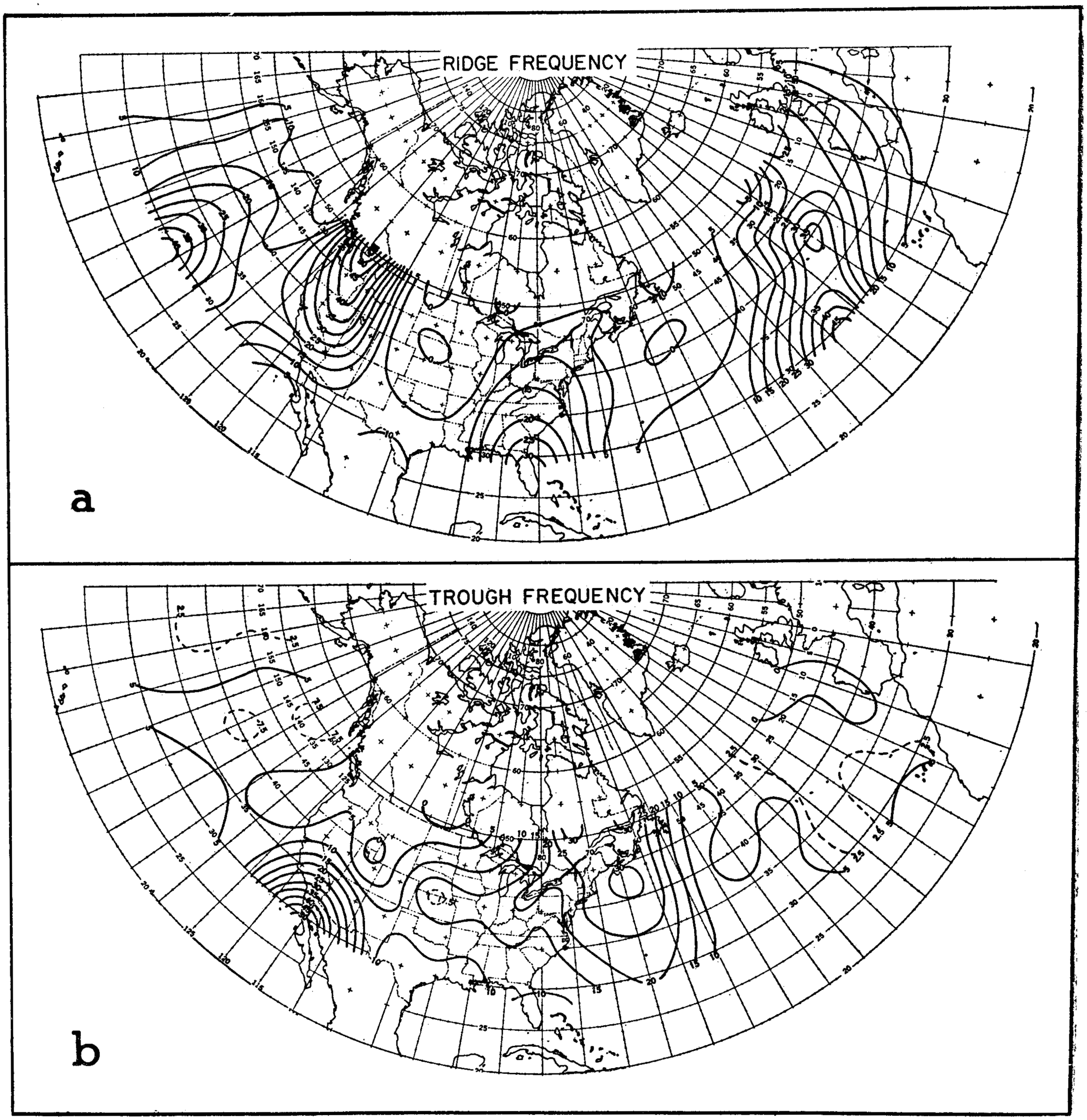

FIGURE 1.—Frequency by geographical location of ridges and troughs observed on 93 30-day mean 700-mb. charts during all winter months from December 1932 to March 1951 between 30° and 50° N. and 0° and 160° W. The isopleths give the number of ridges (a) and troughs (b) within approximately 10° squares and are drawn at intervals of 5 with intermediate isopleths dashed.

known at sea level where storms generally move northeastward along either the Atlantic Coast or the Ohio Valley, but rarely along the mountain chain. In this area, of course, the mountain influence is enhanced by the normal solenoidal field along the Atlantic coast.

Another surprising feature of figure 1 is the large frequency of ridges just inside the east coast of the United States. Comparable results were found on daily maps during the winters of 1946–48, except that the axis of maximum ridge frequency was displaced a few degrees to

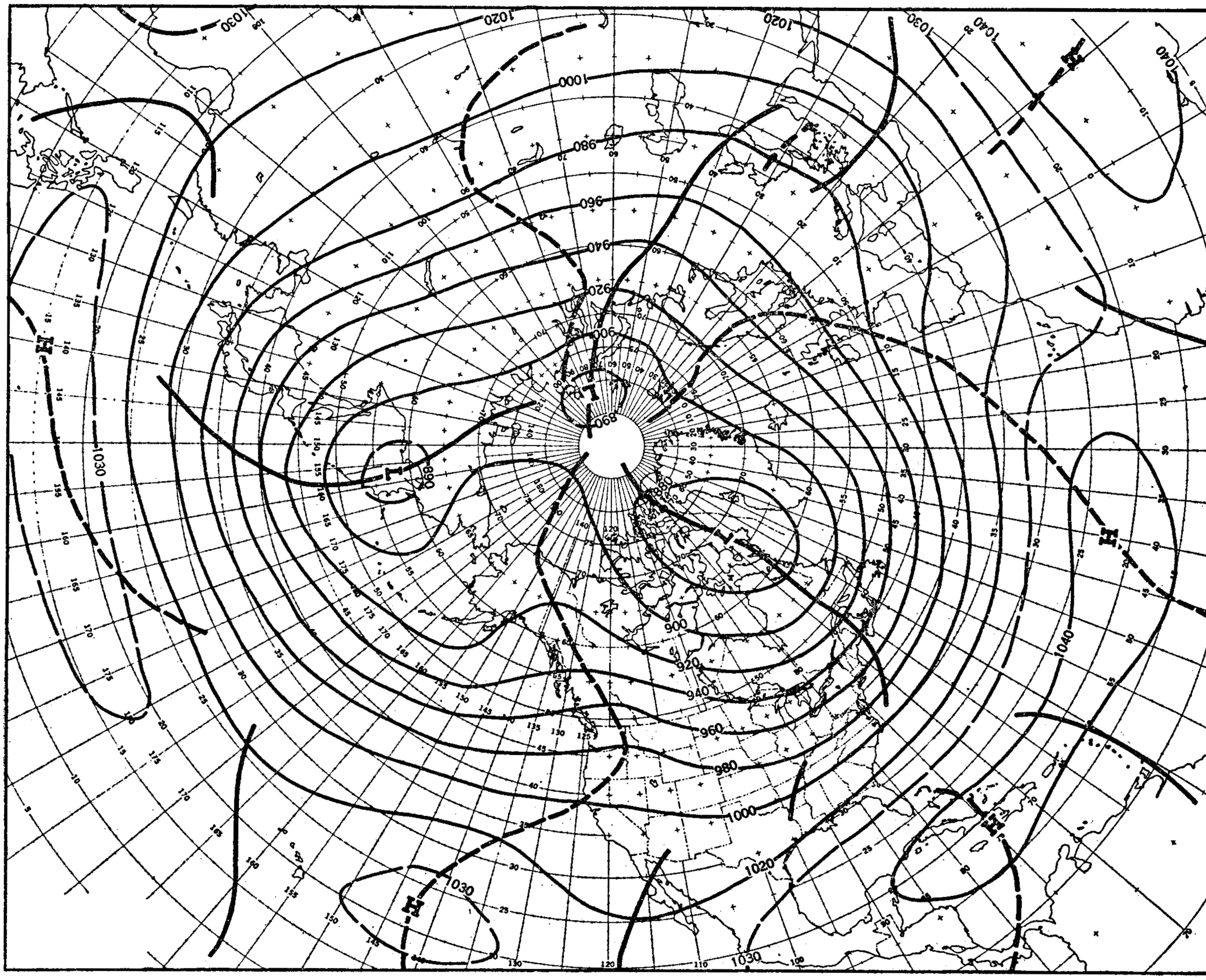

FIGURE 2.—Normal 700-mb. chart for the winter season (Dec., Jan., Feb.). Contours are labeled in tens of feet and shown by solid lines with intermediate contours dashed. Thicker lines show minimum-latitude trough location (solid) and maximum-latitude ridge location (dashed).

the east. There is only a moderate frequency of troughs in this area but these troughs are generally quite intense, as indicated by the proximity of the trough line on the normal map (fig. 2). The East Coast therefore appears to be a region of large pressure variability, where deep troughs alternate with frequent ridges. The Far West is also a region of great pressure variability, where an axis of maximum ridge frequency extending southward overlaps the northward projection of an axis of maximum trough frequency. By contrast the central portion of the United States, where daily pressure variability aloft is normally less than on either coast (Klein [22]), has only a moderate number of troughs and very few ridges. Thus the preferred wave pattern in the United States in winter appears to be of two main types; either a trough in the East and a ridge in the West, or a ridge in the East and a trough in the West. The first pattern is most common in the North, the second in the South. Because of this out-of-phase character of the wave pattern in different latitudes, a band of confluence is normally found in the eastern United States (fig. 2). A third basic type, consisting of a trough in the center of the country and a ridge along or off each coast, occurs less frequently than the first two. A similar conclusion was reached by Wulf, Hodge, and Obloy [45], who found that troughs and ridges around the 200-mb. level (on both daily and monthly mean maps) are usually located in either the eastern or the western third of the United States, but rarely in the center of the country, where straight flow generally prevails.[4]

Over the oceans, where the data are uncertain, the isopleths of ridge and trough frequency are not as informa-

[4] It is well known, of course, that ridges occur frequently in the central part of the United States during the summer months.

tive as they are over the United States. In the eastern Pacific, however, the well-known "Gulf of Alaska Low" development is indicated by a weak axis of maximum trough frequency between the 140° W. and 150° W. meridians (fig. 1b). These longitudes also bound an axis of maximum ridge frequency (fig. 1a). This suggests that this is a third "key" area of great pressure variability, where either a trough or a ridge tends to occur in harmony with the preferred wave pattern over the United States.

INTERRELATIONSHIP BETWEEN VARIOUS WAVE PROPERTIES

AVERAGE CHARACTERISTICS

The remainder of this report will be limited to study of the 700-mb. wave pattern which occurs most frequently in winter in the northern half of the United States and adjacent oceans. This consists of a ridge in or west of the Rocky Mountains and a trough to the east, as described in the preceding section. Only the first ridge to the west of the mountains and the first trough to the east of the mountains at latitudes 40° N. and 50° N. were considered. All the ridges were located between 107° W. and 160° W., while the troughs were found between 60° W. and 106° W. at 40° N. and between 40° W. and 95° W. at 50° N.

The half-wave length between trough and ridge upstream was computed in two ways, as illustrated in figure 3. The latitude half-wave length, $\frac{1}{2}L_l$, was measured along latitude parallels in the standard manner by taking the difference between the longitude of the ridge and the longitude of the trough at the same latitude. The contour half-wave length, $\frac{1}{2}L_c$, and also the double-amplitude, 2A, were measured by following the contour from its intersection with the trough at 40° N. or 50° N. upstream to its maximum latitude at the ridge. The difference between longitudes of the contour at the ridge and trough was then designated as $\frac{1}{2}L_c$, while the difference in latitude was called 2A. Thus, for each trough location, two upstream ridge locations were available, one along the latitude, from which $\frac{1}{2}L_l$ was obtained, and one along the contour, from which $\frac{1}{2}L_c$ was obtained. These two measures of wave length were equal when the ridge extended along the same meridian at all latitudes, but in many cases considerable horizontal tilt was evident. In fact most ridges and troughs at middle latitudes in the Northern Hemisphere are normally oriented from northeast to southwest (fig. 2), as required for maintenance of the angular momentum balance (Starr [37]). Geostrophic wind speeds were obtained from the spacing of the contours and intensities from the 700-mb. height at the ridge and trough locations. Geostrophic rather than gradient winds were used because the former are easier to compute and are better approximations to monthly resultant winds (Aubert and Winston [1]).

The mean and standard deviation of each variable described in the preceding paragraph were computed for

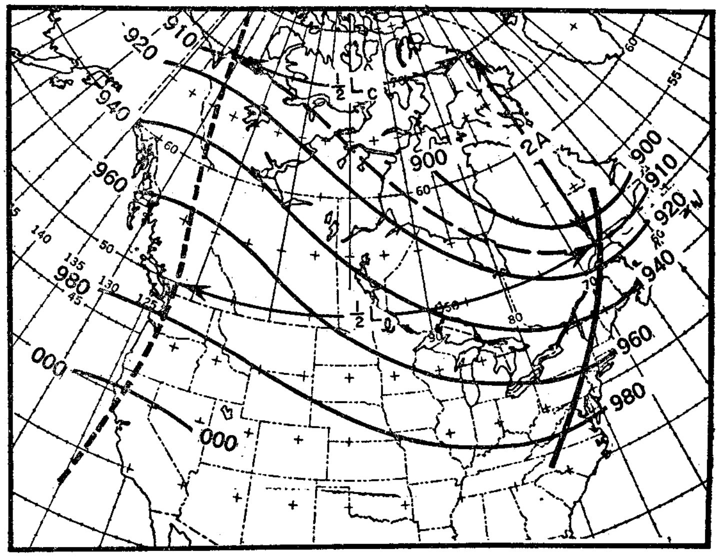

Figure 3.—Schematic monthly mean 700-mb. chart for North America during winter based on average values listed in table 1. Values of wave length and amplitude for the trough at 50° N. are illustrated by lines with arrowheads; where 2A is the double-amplitude, $\frac{1}{2}L_l$ the latitude half-wave length, and $\frac{1}{2}L_c$ the contour half-wave length.

all winter months from December 1932 to March 1948. Some of these statistics are summarized in table 1 and incorporated schematically in figure 3. The mean ridge and trough are located along the west and east coasts of North America, near their normal positions (fig. 2) but they are slightly more intense than normal. The mean trough position at 40° N., however, is about 7° west of the normal location, probably because trough frequency in eastern North America at 40° N. has a tri-modal rather than a normal distribution (fig. 1b). The wave with minimum latitude at 50° N. has larger average amplitude but smaller average wind speed than the wave with minimum latitude at 40° N. The half-wave lengths at 50° N. average numerically larger than those at 40° N. when expressed in degrees of longitude, but they are actually about equal in absolute units. Many of these features are also evident on the normal 700-mb. chart.

It is interesting to compare the monthly mean figures given in the first four columns of table 1 with the corre-

Table 1.—*Means and standard deviations of selected features of monthly mean 700-mb. wave in North America during winter. Corresponding values obtained previously for 5-day mean data are also given when available. (All units in degrees of longitude except double-amplitude which is in degrees of latitude and wind speed which is in meters per second)*

	Monthly mean at 40° N		Monthly mean at 50° N		5-day mean at 45° N	
	Mean	σ	Mean	σ	Mean	σ
1. Trough location	77° W.	13	66° W.	10	72 W.	14
2. Ridge location along latitude	123° W.	11	122° W.	9		
3. Ridge location along contour	124° W.	10	132° W.	14		
4. Latitude half-wave length ½ L_l	46	12	56	11	42	26
5. Contour half-wave length ½ L_c	47	13	66	16	49	15
6. Double-amplitude 2A	12	6	19	6	21	10
7. Mean wind speed along contour from trough to ridge	13	2	10	2	14	3

sponding statistics derived from 5-day mean charts by Namias and Clapp [28], Clapp [7], and Bortman [4, 5] given in the last two columns of table 1(and subsequent tables). The average 5-day mean wave lengths at 45° N. are generally smaller than the average monthly mean wave lengths at 40° N. and 50° N. Unlike wave length, the amplitude and wind speed of the 5-day mean wave average considerably larger than that of the monthly mean wave. The variability of the 5-day mean wave pattern is uniformly greater than that of the monthly mean, as shown by comparing the standard deviations of all elements listed in table 1. This result would be anticipated merely from statistical considerations since, for completely independent data, the standard deviation of the 30-day mean would be obtained by dividing the standard deviation of the 5-day mean by the square root of 6. It is noteworthy, however, that the monthly pattern, because of serial correlation and persistent recurrence of the daily circulations of which it is composed, has greater variability that it would have if it consisted of randomly distributed data. As a result each month's circulation pattern has its own distinctive anomalous character.

SIMULTANEOUS INTERRELATIONSHIPS

Table 2 summarizes the results obtained by correlating simultaneous values of wave length and amplitude (for the period 1932 to 1948) with each other and with several additional variables. Perhaps the most striking feature of this table is the high positive correlation between contour half-wave length, $\frac{1}{2}L_c$, and double-amplitude, 2A, as previously noted by Bortman [4] for 5-day mean data. The tendency for large amplitude to be accompanied by long wave length and small amplitude by short wave length is clearly indicated in the scatter diagram, figure 4. This relationship applies only to the trough in eastern North America and vicinity in the principal band of westerlies and to the ridge immediately upstream. It has not been tested in any other area on 5-day mean maps, and it did not hold when applied to the monthly mean wave pattern at 40° N. in either the Pacific (correlation .16) or the Atlantic (correlation .07).

This interrelation can be further clarified by considering that both $\frac{1}{2}L_c$ and 2A are positively correlated with wind speed at the trough but negatively correlated with wind speed at the ridge (table 2). A similar relation holds for $\frac{1}{2}L_l$, most markedly for 5-day mean data given by Clapp [7]. These correlations indicate that systems of large amplitude and long wave length in North America tend to have strong winds at the trough and weak winds at the ridge, while waves of small dimensions tend to have faster winds at the ridge and slower winds at the trough. This means that large amplitudes and long wave lengths are accompanied by confluence or convergence of the contours as they proceed from ridge to trough, while small amplitudes and short wave lengths are concomitants of difluence or divergence of the contours. The positive

TABLE 2.—*Simple linear correlation coefficients between selected features of monthly mean 700-mb. waves in North America during winter and simultaneous values of latitude half-wave length, contour half-wave length, and double-amplitude. Corresponding correlations obtained previously for 5-day mean data are also given, where available*

Independent variable	Monthly mean at 40° N			Monthly mean at 50° N			5-day mean at 45° N		
	$\frac{1}{2}L_l$	$\frac{1}{2}L_c$	2A	$\frac{1}{2}L_l$	$\frac{1}{2}L_c$	2A	$\frac{1}{2}L_l$	$\frac{1}{2}L_c$	2A
Latitude half-wave length, $\frac{1}{2}L_l$	1	0.70	0.31	1	0.55	0.36	1	------	0.09
Contour half-wave length $\frac{1}{2}L_c$	.70	1	.60	.55	1	.72	------	1	.82
Double-amplitude, 2A	.31	.60	1	.36	.72	1	.09	.82	1
Wind speed at the trough	.33	.38	.15	.19	.27	.22	.66	------	------
Wind speed at the ridge	−.08	−.37	−.66	−.04	−.08	−.13	−.44	------	------
Mean wind speed from trough to ridge	.13	−.13	−.48	.02	.14	.07	.17	.38	.22
Wind speed difference, trough minus ridge	.26	.50	.56	.18	.30	.34	------	------	------
Height at the trough	−.35	−.47	−.75	−.38	−.43	−.59	------	------	------
Height anomaly at ridge	.04	.48	.65	.32	.19	.29	------	------	------
Longitude of the trough	−.62	−.71	−.32	−.65	−.43	−.21	−.58	------	------
Longitude of the ridge	.38	.47	.36	.53	.83	.80	------	------	------
Horizontal trough tilt	−.50	−.55	−.17	−.01	.08	−.06	------	------	------

correlation between wave length, amplitude, and wind speed at the trough is in good agreement with Rossby's [36] constant absolute vorticity trajectories and Bortman's [4] empirical findings. The negative correlations obtained with wind speed at the ridge appear to conflict with theory, but they are probably a regional peculiarity caused by the fact that the mountains of western North America disrupt the westerly flow at the 700-mb. level.

The interrelationship of 2A, $\frac{1}{2}L_c$, and the difference between wind speeds at trough and ridge, is portrayed graphically in figure 5 for troughs at 40° N. Each observed value of 2A has been plotted as the dependent variable opposite its value of $\frac{1}{2}L_c$ as abscissa and wind speed difference as ordinate. The mean amplitude in each of about a dozen boxes of approximately equal size and number of observations was computed and plotted in the center of each box. Lines of equal amplitude were then

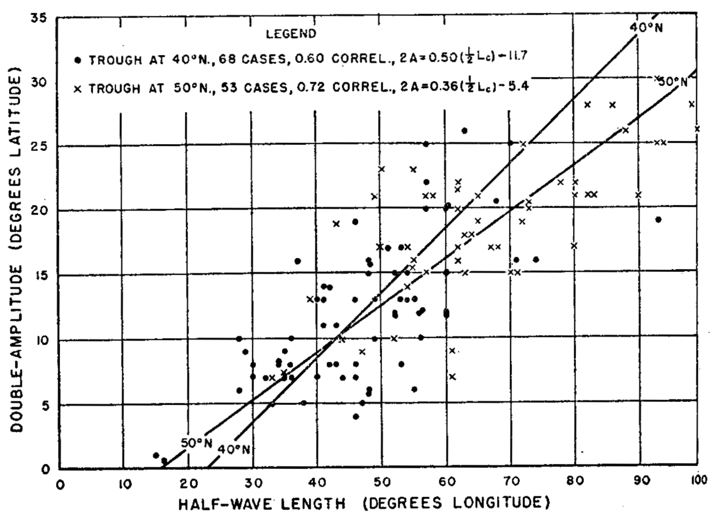

FIGURE 4.—Scatter diagram for monthly mean 700-mb. troughs at 40° N. (dots) and 50° N. (crosses) in North American area during winter. The interrelation between simultaneous values of double-amplitude (2A) and contour half-wave length ($\frac{1}{2}L_c$) is shown by the "best-fit" lines, regression equations, and correlation coefficients.

drawn smoothly free hand with approximately equal spacing to fit these means as closely as possible. In order to measure the accuracy of figure 5, a suitable value of 2A was estimated from the family of curves by entering the graph with each observed value of wind difference and $\frac{1}{2}L_0$. The correlation coefficient between these estimates and the observed amplitude is plotted in the lower right hand corner of the figure (r=0.76). It indicates that amplitude can be estimated with greater accuracy from the graph combining wave length and wind speed than it can from the simple regression with wave length alone (figure 4, r=0.60). It should be remembered, however, that these correlations are based on the dependent data from which the graphs were originally derived.

The wave length-amplitude interrelation is reflected in some additional correlation coefficients given in table 2. For instance each measure of wave length and amplitude tested was positively correlated with both longitude and height of the ridge but negatively correlated with longitude and height of the trough. This indicates a distinct tendency for long wave lengths and large amplitudes to occur with abnormally deep troughs, located well east of the mean position, and abnormally intense ridges, displaced west of the normal location. This relation can also be attributed to the frequent development of lee troughs just east of the Rocky Mountains. These troughs are characteristically weak aloft when they originate, but they usually deepen rapidly as they move eastward. Since a quasi-stationary ridge is frequently found in western North America, both wave length and amplitude generally increase as the trough moves eastward and deepens.

The final variable listed in table 2 is horizontal trough tilt. This was measured by simply subtracting the longitude of the trough 10° north of a given trough location from the longitude of the trough 10° to the south. Most of the correlations show that troughs which tilt more sharply than normal from northeast to southwest tend to have shorter wave lengths and smaller amplitude than those with little or reverse tilt. This may be merely a reflection of the fact that troughs with large tilts are usually located farther west than troughs with small tilt, as shown by a correlation coefficient of 0.65 between location and tilt of troughs at 40°N. This correlation is in accord with experience since troughs near the Atlantic and Pacific coasts may tilt in either direction, but troughs in the Great Plains characteristically have large tilt from northeast to southwest, as, for example the typical lee trough described by Myers [27]. (See fig. 8.) It may appear paradoxical that the lee trough actually intersects the Rocky Mountains at a large angle. The explanation may lie in the greater speed of the zonal current around 50°N., where the trough is well east of the mountains, than around 25°N., where the trough is usually west of the mountains in the normal trough off Lower California.

The preceding discussion has been limited to the wave in North America and vicinity. The amplitude and wave length of this wave are positively correlated with the corresponding feature of adjacent waves, both upstream in the Pacific, and downstream in the Atlantic. The most striking relationship of this type, between component parts of the circulation separated by thousands of miles, is the simple linear correlation of 0.85 obtained between simultaneous values of amplitude in North America and the Pacific. This interrelationship is one of the features illustrated in figure 6, prepared by the method of graphical correlation previously described. This graph indicates

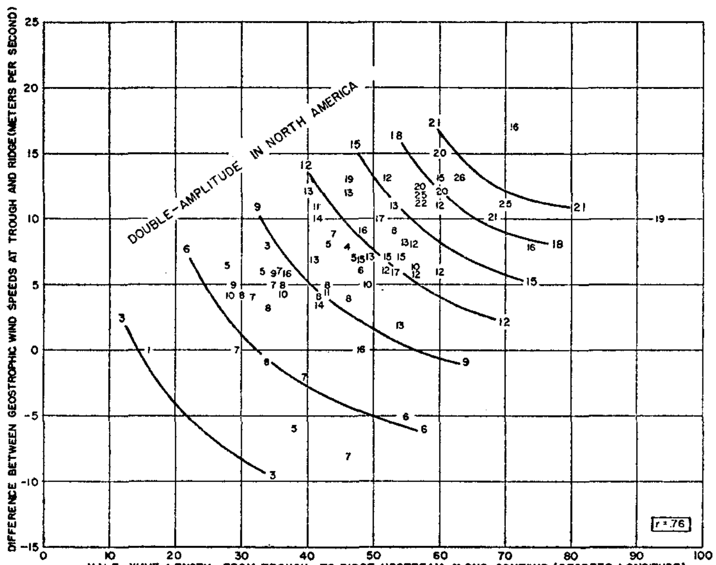

FIGURE 5.—Joint relationship of contour half-wave length ($\frac{1}{2}L_0$) and difference between geostrophic wind speeds at trough and ridge with contemporary double-amplitude (2A) for monthly mean 700-mb. troughs at 40° N. in North American area during winter. The linear correlation coefficient between observed values of 2A plotted on the graph and estimates based on the isopleths is shown in the lower right hand corner.

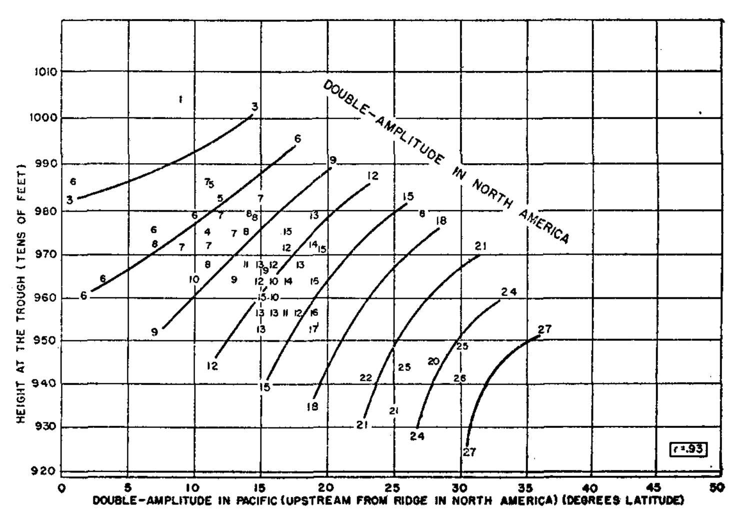

FIGURE 6.—Graphical interrelation between double-amplitude (2A) of monthly mean 700-mb. wave in North American area during winter for troughs at 40° N., contemporary height at the trough, and double-amplitude from the ridge in North America upstream along the contour to the trough in the Pacific. The linear correlation coefficient between observed values of 2A plotted on the graph and estimates based on the isopleths is shown in the lower right hand corner.

that when the amplitude is large between the trough at 40°N. in eastern North America and the ridge upstream along the contour, then the amplitude between this ridge and the next trough upstream in the Pacific is also large, while heights at the trough in North America tend to be low. There was close connection between these three variables for the dependent data used here, as indicated by the multiple correlation of 0.93 between observed values of 2A and those estimated from the graph. In part these relations are geometric necessities, but they also indicate that large scale waves tend to be symmetrical in harmony with the principle of conservation of vorticity, as suggested by Rossby [36]. This symmetry is not perfect, however, since for troughs at 40°N. 2A is correlated less highly with amplitude in the Atlantic (0.45) than in the Pacific (0.85). This difference can probably be attributed to the fact that ridges in the Atlantic are frequently prevented from becoming as strong as expected on the basis of vorticity transfer from deep troughs in eastern North America because of the effects of the Greenland ice cap, the confluence associated with large amplitude in eastern North America, and the orientation of the Atlantic coast of North America.

SYNOPTIC REPRESENTATION

In order to illustrate synoptically the nature of the interrelationships described previously, two composite maps were prepared, figure 7. The upper chart is the mean of the 10 observed 700-mb. monthly mean maps with the largest difference between the wind speed at the trough at 40° N. and at the ridge upstream along the contour. Naturally the contours are closely spaced at the trough and far apart at the ridge. But the map also contains a wave of large amplitude and long wave length in North America. Large amplitude is also suggested in the Pacific but the data are incomplete. The trough off the Atlantic coast is located east of its mean position in a region where cyclonic developments are favored by strong thermal contrast between land and sea. As a result it is about 200 feet deeper than normal at middle latitudes. Most of these features are also indicated by correlation coefficients based on all available data (table 2), not merely the 10 selected cases comprising this map. This chart can therefore be regarded as a model for a wave of large dimensions in North America.

The lower map stands in sharp contrast as a model small amplitude-short wave length map. It was prepared by averaging the 10 observed cases with the smallest difference in wind speed measured at the trough at 40° N. and the ridge upstream along the contour. The outstanding feature of this map is the large horizontal tilt of the trough stretching from the northeastern to the southwestern part of the United States. This trough is formed by a merger of the troughs normally located in eastern Canada and lower California. It crosses the 40th parallel at 95° W., where troughs to the lee of the Rockies are most frequent according to both a theoretical study by Colson [8] and an empirical study by Myers [27]. (See also fig. 1b.) In fact this map has many features in common with the composite map for lee troughs presented by Myers and reproduced in figure 8. This suggests that the Rocky Mountains are instrumental in producing short flat waves at 40° N. The composite map reveals some additional characteristics of these waves. They are usually strongly tilted, less intense than normal, displaced west of the normal position, and associated with a stream of fast flat westerly flow in the eastern Pacific at middle latitudes north of a well-developed eastern Pacific High. This flow undergoes considerable difluence as it enters North America so that wind speeds at the trough are considerably weaker than those at the ridge. In the upper map, on the other hand, the flow strikes the west coast of North America at a greater angle and with less speed, so that the direct effect of the mountains of the western United States is lessened.

In general, the lower map is a typical "high index" map with the principal belt of westerlies north of normal, while the upper map has much greater meridional flow with the westerlies displaced to the south. In this respect, it is noteworthy that the upper composite map is composed of six Februarys, one January, and three Decembers; whereas the lower composite comprises six Januarys, three Decembers, and only one February. This is strongly reminiscent of the type of singularity described by Namias [30] when he showed that the zonal westerlies are normally displaced farthest south in late February.

WAVE MOTION

AVERAGE PROPERTIES

Trough displacement was defined as the current longitude of the trough at 40° or 50° N. minus its longitude on the 30-day mean observed a month later at the same latitude. In this way positive values indicate progressive or eastward motion and negative values retrogression or westward motion. A similar system was used for ridges. Since most monthly mean maps contain only one long wave in the North American area, the identification of troughs and ridges from one month to the next was a fairly straightforward process. Cases where new troughs or ridges appeared or old ones disappeared, resulting in doubtful continuity and discontinuous changes in wave length, were excluded. Because of this restriction 23 months were eliminated at 40° N., but only 5 months at 50° N., from the total of 93 monthly mean maps considered. These numbers indicate that the formation of new troughs or ridges on monthly mean 700-mb. charts in North America in winter occurs almost five times as frequently at 40° N. as at 50° N. These cases generally involve new trough formation on the east (lee) side of the

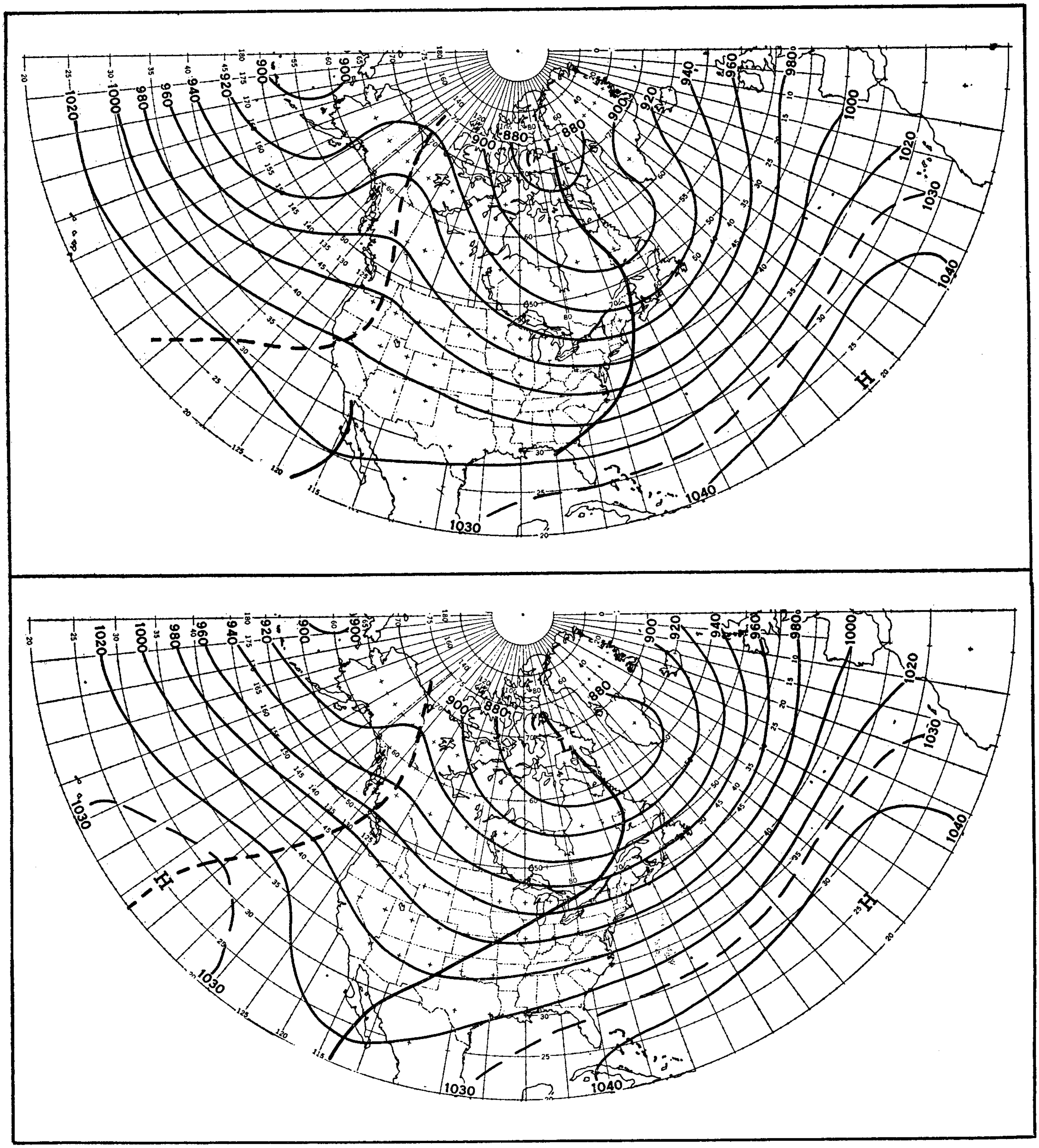

FIGURE 7.—Composite 30-day mean 700-mb. charts during winter: Upper, average of the ten months with the largest wind speed difference between trough at 40° N. and ridge up stream along the contour (Feb. 1934, Dec. 1934, Feb. 1936, Dec. 1939, Jan. 1940, Feb. 1940, Dec. 1943, Feb. 1944, Feb. 1945, and Feb. 1947); lower, average of the ten months with the smallest wind speed difference (Dec. 1932, Jan. 1933, Jan. 1934, Dec. 1936, Dec. 1937, Jan. 1939, Jan. 1946, Jan. 1947, Jan. 1948, Feb. 1948).

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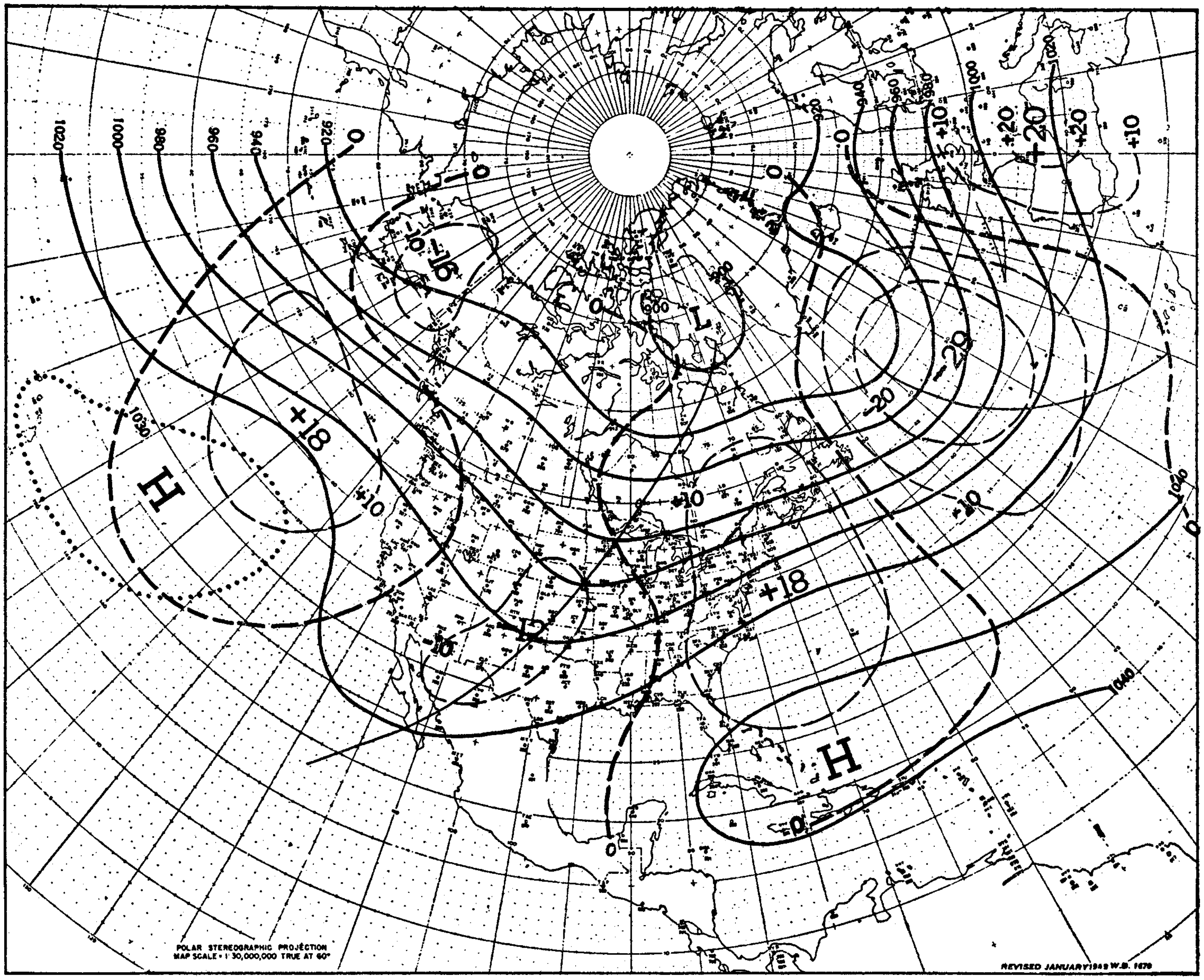

FIGURE 8.—Composite 700-mb. chart averaged from ten 5-day means, each one-half week subsequent to the development of a deep lee trough in the Great Plains of the United States (*after Myers* [27]). Contours are shown by solid lines, intermediate contours by dotted lines, and 700-mb. height departures from normal by dashed lines. Contours, anomaly centers, and anomaly isopleths are labeled in tens of feet. Minimum latitude trough locations are shown by thin solid lines.

Rocky Mountains in a region where trough frequency is much greater at 40° N. than at 50° N. (fig. 1b). A corresponding difference can be noted on the normal chart (fig. 2), where a single ridge-trough system of large amplitude is found at 50° N., while the wave at 40° N. is weaker and more poorly defined.

After cases of trough development or disappearance had been eliminated and the winters of 1949–51 set aside for later testing, the remaining data, totaling 54 cases at 40° N. and 70 cases at 50° N., were used to study wave motion. Table 3 shows that the mean displacement of both troughs and ridges on monthly mean charts was close to zero. This results because retrogression was about as large and frequent as forward motion, as illustrated by the frequency distributions of figure 9. These distributions are generally platykurtic (broad-shouldered) for trough displacement and leptokurtic (peaked) for ridge displacement, thus indicating that ridges tend to be quasi-stationary more often than troughs. Table 3 reveals no appreciable difference, however, between average trough and ridge displacement, taken both with and without regard to sign.

The last two lines of table 3 give statistics on trough motion on 5-day mean maps (Clapp [7]) and daily maps (Namias and Clapp [28]). Comparison with the numbers for monthly means reveals that, as the period of averaging increases, from 1 to 5 to 30 days, there is a marked decline in wave speed and variability, provided that all the data are expressed in the same units (°long./day, given in parentheses). This decrease is considerably smaller when

TABLE 3.—*Average wave displacement at 700 mb. in North America during winter. All units are in degrees of longitude; per month for monthly mean troughs, per week for 5-day mean troughs, and per day for daily troughs. Values in parentheses give conversions to degrees of longitude per day*

	Arithmetic mean	Absolute average	Standard deviation
Monthly mean ridges at 40° N	1 (0.03)	10 (0.33)	13 (0.43)
Monthly mean ridges at 50° N	0 (0)	10 (0.33)	13 (0.43)
Monthly mean troughs at 40° N	2 (0.07)	10 (0.33)	12 (0.40)
Monthly mean troughs at 50° N	1 (0.03)	9 (0.30)	12 (0.40)
Five-day mean troughs at 45° N	6 (0.86)	12 (1.7)	15 (2.1)
Daily troughs at 40° N (approximate)	12 (12)	13 (13)	16 (16)

the motions are expressed in units approximately equal to the time intervals of the maps (i. e., °long. per month for 30-day mean troughs, per week for 5-day mean troughs, and per day for daily troughs). In these units the declines of absolute average wave speed (col. 2) and standard deviation (col. 3) with increasing averaging interval are much slower than the decline of the arithmetic mean speed (col. 1). This is a consequence of the fact that retrogression occurs frequently on monthly means, occasionally on 5-day means, and rarely on daily maps. It also indicates a tendency for the increase of the averaging interval (from 1 to 30 days) to balance the decrease of the wave speed (in °long./day); so that the product of these two quantities, the absolute wave displacement (in °long.), remains approximately constant.

TROUGH DISPLACEMENT

Table 4 presents some of the results obtained by correlating trough displacement with a large number of independent variables, each measured one month prior to the

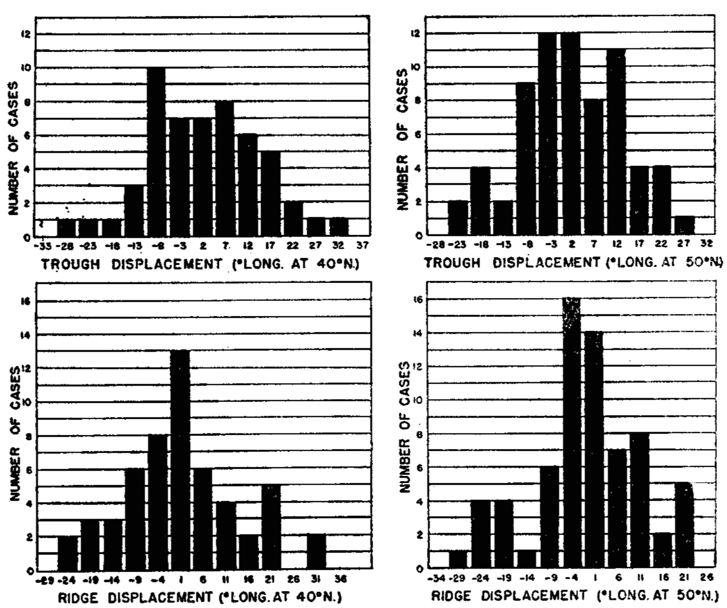

FIGURE 9.—Frequency distributions of one-month displacement of 30-day mean 700-mb. troughs (above) and ridges (below) at 40° N. (left) and 50° N. (right) in North American area during all winter months from December 1932 to March 1948. Displacements are grouped by 5 degrees of longitude.

observed trough motion. At both 40° and 50° N. subsequent trough displacement was more highly correlated with initial trough location than with any other variable tested. This implies that monthly mean troughs do not generally continue to move at a constant rate. Instead they tend to oscillate about a mean or preferred location which is determined by many complex factors. Trough location was also found to be important on 5-day mean charts (Clapp [7]), but examination of the statistics given in table 4 shows that monthly mean troughs are more likely than 5-day mean troughs to move toward the preferred trough location.

The second most highly correlated variable was wave length. Of eight different measures of wave length tested, the latitude half-wave length, $\frac{1}{2}L_l$, was best correlated with trough displacement. The correlations were negative as expected from theory, and of about the same magnitude (−0.5) at both 40° N. and 50° N., for both monthly and 5-day means, and for both the first and second powers of the wave length. A comparable correlation was obtained for daily 3-km. troughs in this area during the winter of 1936 by excluding minor troughs and sudden wave length changes (Haurwitz and Craig [18]). The correlations were higher than comparable ones obtained by measuring half-wave length on the 700-mb. mean height anomaly chart (Dickson [12]). This may indicate that the real flow is more suitable than its anomaly for application of physical principles to mean charts.

Table 4. Simple linear correlation coefficients between selected features of monthly mean 700-mb. charts and subsequent one-month displacement of trough in eastern North America during winter. Corresponding correlations obtained previously for one-week displacement of 5-day mean troughs are also given, where available

Independent variable	Monthly mean at 40° N	Monthly mean at 50° N	5-day mean at 45° N
Longitude of the trough	0.54	0.64	0.53
Latitude half-wave length, $\frac{1}{2}L_l$	−.50	−.53	−.52
Contour half-wave length, $\frac{1}{2}L_c$	−.43	−.24	−.30
Half-wave length west of ridge (Pacific)	−.12	−.23	----------
Wind speed at the trough	.08	.01	−.02
Wind speed at the ridge	.11	.08	.21
Mean wind speed from trough to ridge	.17	.13	.11
Zonal index from 35° to 55° N. and 0° to 180° W	.26	−.25	----------
Wind speed at the hemispheric jet	−.12	−.16	----------
Wind speed difference, trough-ridge	.03	−.05	----------
Double-amplitude, 2A	−.11	.02	−.40
Height at the trough	.15	.12	----------
Height at the ridge	.14	.00	----------
Lateral width of the westerlies	−.36	.23	----------
Hemispheric wind shear, 40° N.−50° N	−.12	.14	----------
Horizontal trough tilt	.42	−.03	----------

It is interesting to note that trough displacement was more highly correlated with $\frac{1}{2}L_l$ than with $\frac{1}{2}L_c$, although the latter wave length was better for the simultaneous interrelationships discussed in the preceding section. The wave length taken immediately upstream from the trough gave better results than the comparable wave length taken either downstream from the trough or upstream from the ridge. The latter wave length was negatively correlated with trough displacement, however, thus offering some support for Cressman's [11] contention that trough motion is influenced by the wave length distribution upstream.

Use of the full-wave length, measured from the trough in North America to the trough in the Pacific, was not as effective as use of the half-wave length from trough to ridge in North America. A similar result was found for 5-day mean troughs in this area (Namias and Clapp [28]). It is probably an effect of regional topographic features since the full-wave length was found to be more highly correlated than the half-wave length with 5-day mean trough displacement in the Atlantic and Pacific (Bortman [5]). It may also be a result of the arbitrary method used to define a trough, since axes of maximum vorticity or curvature in the eastern Pacific frequently affect the wave spacing as if they were minimum-latitude troughs (Klein [24]).

The graphical relationship between trough displacement, trough location, and $\frac{1}{2}L_1$ is illustrated in figure 10 for troughs at 40° N. and in figure 11 for troughs at 50° N. The figure clearly indicates that both independent variables contribute to displacement, despite the fact that they are correlated with each other. Troughs with short wave lengths to the west of the mean trough location tend to move eastward; troughs retrograde when they are located east of the mean position and the wave length is long. The stationary wave length coincides with the line of zero trough displacement and is therefore a function of trough location. Estimates of trough displacement were obtained for each case by entering the graphs with the observed value of trough location and wave length. These estimates were correlated more highly with the observed displacement than the simple correlations obtained by use of either independent variable alone.

Of the remaining variables listed in table 4, the wind speed was tested most intensively. It was measured in a dozen different ways, but in all cases the correlation with trough displacement was very low, despite the fact that all theoretical equations for trough motion except one derived by Craig [9] ascribe a primary role to the zonal wind. It thus appears that the relation between wind speed and trough displacement is just as poor on monthly mean maps as previously noted on both daily charts (Haurwitz and Craig [18]) and 5-day mean charts (Clapp [7]).

The effect of amplitude was also tested rather thoroughly. This factor does not appear in the simple Rossby formula for waves of infinitesimal amplitude nor in Neamtan's [32] equation for waves of finite amplitude. How-

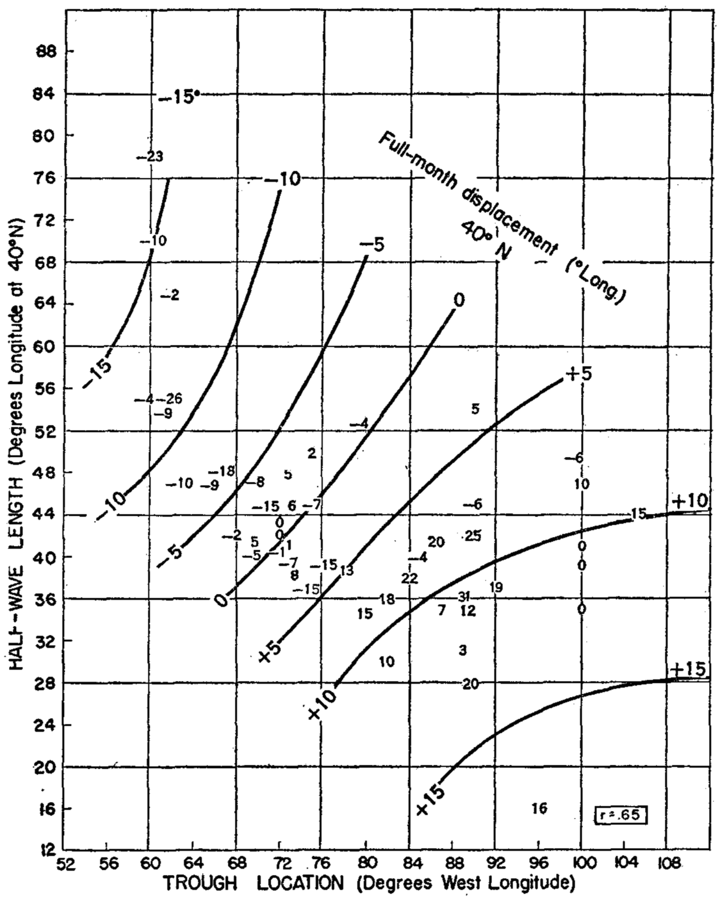

Figure 10.—Joint relationship of trough location and latitude half-wave length ($\frac{1}{2}L_1$) with subsequent one-month displacement of 30-day mean 700-mb. trough at 40° N. in North American area during winter. The linear correlation coefficient between observed values of displacement plotted on the graph and estimates based on the isopleths is shown in the lower right hand corner.

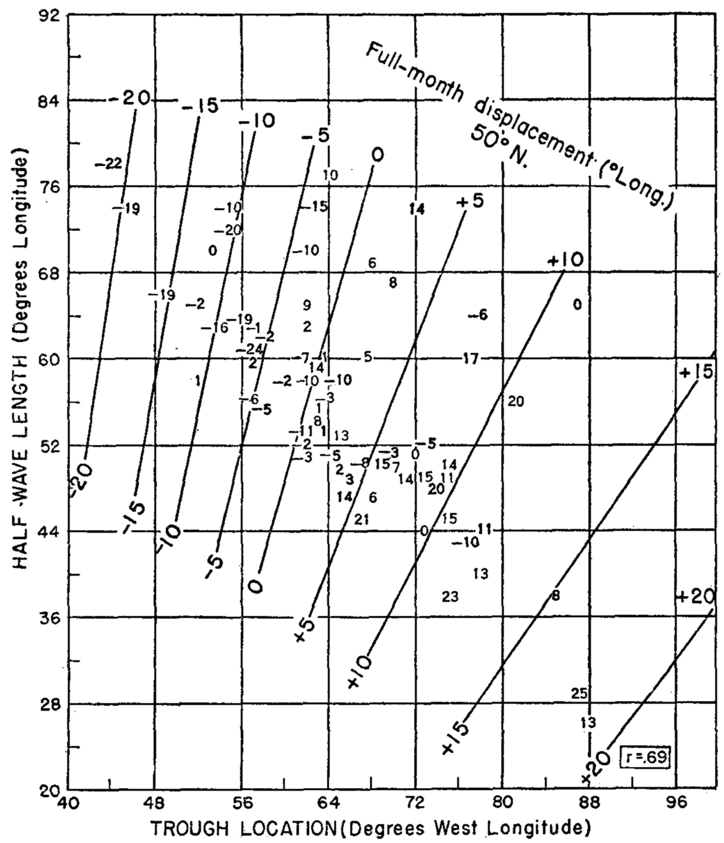

Figure 11.—Joint relationship of trough location and latitude half-wave length ($\frac{1}{2}L_1$) with subsequent one-month displacement of 30-day mean 700-mb. trough at 50° N. in North American area during winter. The linear correlation coefficient between observed values of displacement plotted on the graph and estimates based on the isopleths is shown in the lower right hand corner.

ever, some evidence has been presented that amplitude is important for trough motion, theoretically by Walsh [42] and empirically by Bortman [4, 5]. Five measures of amplitude or intensity were therefore correlated with full-month trough displacement, but all the correlation coefficients were very low, and in some cases the signs were conflicting. Thus, the effect of amplitude on trough motion appears to be slight, in conformance with the empirical findings of Clapp [7] on 5-day mean charts and Sumner [38] on daily maps.

Poor results were also obtained by use of several crude measures of lateral width, horizontal wind shear, and horizontal trough tilt. Since these variables (and also wind speed) appear in many theoretical equations for wave motion it is interesting to speculate on the reasons for failure to obtain better results. Of course the theory is not strictly applicable to mean charts especially when taken over long time periods. Furthermore, many of the variables are difficult to define precisely and may require more refined methods of measurement and testing. Better results might be obtained by using some of the more complex measures which have recently been suggested such as the half-width of the current (Petterssen [34]), the thermal zonal wind (Sutcliffe [39]), the shear of the absolute vorticity (Platzman [33]), and the second derivative of wind shear (Rossby [35] and Thompson [40]). Likewise the zonal wind might be measured differently, for example, at the level of nondivergence (Bjerknes and Holmboe [3], Charney [6]), in the core of the current (Cressman [10]), at the latitude of maximum absolute vorticity (Kuo [25]), or at the upper level jet stream (Estoque [13]). However, it was not feasible to consider these factors in this study.

RESIDUAL TROUGH DISPLACEMENT

It is possible that trough displacement was poorly correlated with some of the independent variables listed in table 4 because mutual relationships among these variables masked their true effect. Each of these variables (and many others not listed) was therefore correlated with the residual trough displacement, defined as the difference between the observed trough displacement and its estimate obtained from the graph combining wave length and trough location (figs. 10 and 11). However, the results were largely negative.

Of all variables correlated with residual trough displacement best results were obtained by use of the subsequent one-month displacement of the ridge measured upstream along the latitude circle. This variable indicates how the wave length would change with time if the trough remained stationary, a factor originally emphasized by Wexler [43]. In figures 12 and 13 this ridge displacement is plotted as ordinate, while the trough displacement estimated from the graph of wave length and trough location (figures 10 or 11) is plotted as abscissa. The family of curves has been analyzed for the observed trough displacement, plotted as dependent variable, by the smoothing process described previously and is intended to give an improved estimate of full-month trough displacement. The graphs show that the trough does not move as far eastward as forecast from its initial location and wave length when the ridge upstream subsequently retrogrades and increases the wave length, but its eastward motion exceeds expectations when the ridge also moves eastward. In other words both trough and ridge tend to move in harmony to conserve the wave length. Estimates

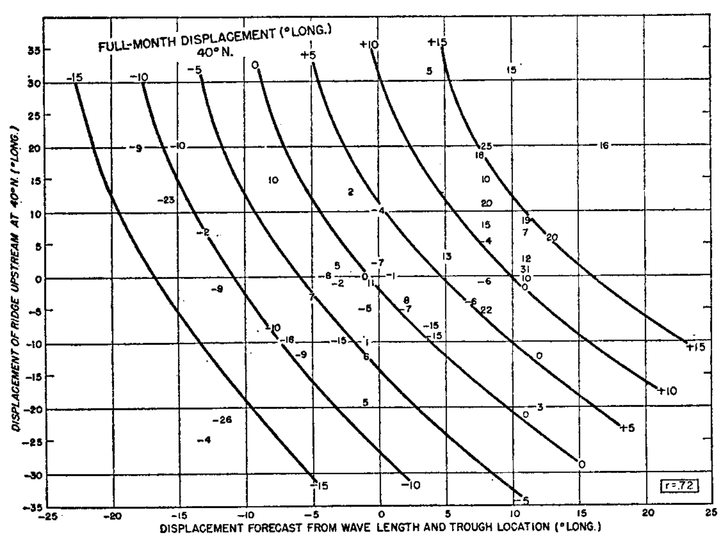

FIGURE 12.—One-month displacement of 30-day mean 700-mb. trough at 40° N. in North American area during winter in relation to contemporary one-month displacement of ridge to the west and initial estimates of trough displacement based on wave length and trough location (fig. 10). The linear correlation coefficient between observed values of trough displacement plotted on the graph and estimates based on the isopleths is shown in the lower right hand corner.

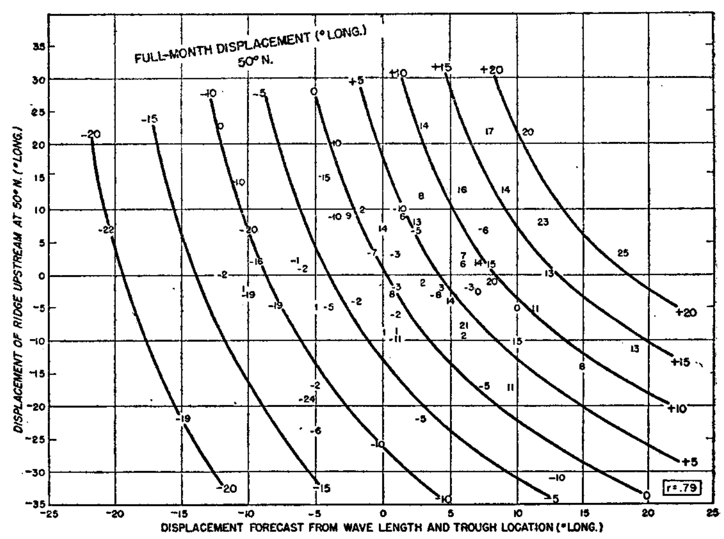

FIGURE 13.—One-month displacement of 30-day mean 700-mb. trough at 50° N. in North American area during winter in relation to contemporary one-month displacement of ridge to the west and initial estimate of trough displacement based on wave length and trough location (fig. 11). The linear correlation coefficient between observed values of trough displacement plotted on the graph and estimates based on the isopleths is shown in the lower right hand corner.

of trough displacement made from figures 10, 11, 12, and 13 were substantially more accurate than estimates made from figures 10 and 11 alone, as indicated by comparing the correlation coefficients given in the lower right corner of each figure. In practice the subsequent ridge displacement will not be known on forecast day since it refers to motion (during the next month) which is contemporary with the motion of the trough under consideration. It can be estimated fairly accurately and objectively, however, by judicious use of trend and kinematic techniques.

TEST ON INDEPENDENT DATA

The outstanding relationships derived in the preceding sections with dependent data for the winters from 1932 to 1948 were tested on independent data for the winter months from mid-November 1948 to mid-March 1951. Of the 21 months in the test period, it was necessary to eliminate 3 at 50° N. and 6 at 40° N. because of new trough development. Some statistics obtained by use of the remaining cases are given in tables 5, 6, and 7.

Table 5 shows that the relation between trough displacement and latitude half-wave length $\frac{1}{2}L_l$ is a rather stable one since correlation coefficients of about −0.5 were obtained for both the dependent and independent data at both 40° N. and 50° N. At both latitudes, however, the correlation coefficients between trough displacement and trough location were lower on the independent than on the dependent data. As a result, estimates of trough displacement based on figure 10 or 11 were less accurate for the independent than the dependent data. These estimates were improved by considering the effect of subsequent observed ridge displacement as expressed in figure 12 or 13. At 50° N. the magnitude of the improvement obtained by using two forecast graphs instead of one was about the same for both dependent and independent data. At 40° N. the improvement was more marked for the independent data, perhaps because the estimates based on the first graph alone were quite poor (correlation of 0.42). As a result of this compensation process, the independent objective estimates based on both sets of graphs were about equal in accuracy at both 40° and 50° N. (correlation of 0.64 between forecast and observed displacement).

The accuracy of these estimates has also been expressed in terms of the standard error of estimate (table 6). Since this statistic is smaller than the standard deviation of observed trough displacement for both dependent and independent data at both 40° and 50° N., a reasonable degree of accuracy is to be expected in using the two-graph estimates of trough displacement. Considerable reliance can be placed on the sign, at least, of the estimated displacement when it is large in magnitude. For example out of the 19 test cases with estimated trough displacement in excess of ±7° in only one did the trough actually move in a direction opposite to that expected! However, estimates of trough displacement made on the independent data suffered from appreciable negative bias, as indicated by comparing the mean errors listed in table 6. In other words, the trough failed to move as far eastward as expected during the three winters comprising the test data. At 40° N. the mean trough displacement was actually negative (retrograde) during this period, contrasted to slight mean forward motion observed in the past on dependent data.

Table 7 shows that some of the simultaneous interrelationships noted on the dependent data at 40° N. were also weaker on the independent data. For example, simple and graphical correlations between amplitude, wave length, and wind speed difference were all reduced. Some of these differences are attributable to the fact that the mean trough in the United States at 40° N. was displaced an average of 15° west of its normal position during the three winters comprising the test period. The unusual nature of these winters has been discussed elsewhere (Klein [21], Namias [31], Klein [23], and the Monthly

TABLE 5.—*Simple linear correlation coefficients between selected features of monthly mean 700-mb. charts and subsequent one-month displacement of trough in eastern North America during winter*

Independent variable	Dependent data		Independent data	
	40° N	50° N	40° N	50° N
Half-wave length along latitude $\frac{1}{2}L_l$	−0.50	−0.53	−0.48	−0.53
Trough location	.54	.64	.42	.58
Subsequent ridge displacement	.38	.26	.34	.14
Estimates based on $\frac{1}{2}L_l$ and trough location	.65	.69	.42	.53
Estimates based on $\frac{1}{2}L_l$, trough location, and ridge displacement	.72	.79	.64	.64

TABLE 6.—*Observed one-month trough displacements on monthly mean 700-mb. charts in North America during winter compared to estimates based on wave length, trough location, and ridge displacement (units in degrees of longitude)*

	Dependent data		Independent data	
	40° N	50° N	40° N	50° N
Standard error of estimated displacements	9	7	9	10
Standard deviation of observed displacements	12	11	12	16
Mean error (observed − estimated displacement)	0	0	−6	−3
Mean trough displacement	2	1	−2	0

TABLE 7.—*Simple linear correlation coefficients between contemporary double-amplitude, 2A, of monthly mean 700-mb. wave in North America during winter and selected variables*

Independent variable	Dependent		Independent	
	40° N	50° N	40° N	50° N
Contour half-wave length $\frac{1}{2}L_c$	0.60	0.72	0.46	0.81
Trough-ridge wind speed difference along contour	.56	.34	.02	.62
Double-amplitude upstream from ridge (Pacific)	.85	-------	.81	.86
Height at the trough	−.75	−.59	−.59	−.55
Estimate based on $\frac{1}{2}L_c$ and wind speed difference	.76	-------	.54	-------
Estimates based on trough height and Pacific amplitude	.93	-------	.88	-------

Weather Review series of articles on the weather and circulation of each month since January 1950). At 50° N., on the other hand, the correlations between amplitude, wave length, and wind speed difference were actually higher on the independent than the dependent data. Although the mean trough position at 50° N. was 8° west of normal during the test period, it was still 15° east of the trough position at 40° N. Apparently then most of these simultaneous interrelationships operate as long as the trough is located within a broad zone but they tend to break down when mean trough location exceeds certain critical limits. This supports the previous conclusion that these interrelationships are closely connected to the normal topographic and solenoidal factors of the North American area.

Failure to obtain better results also can be attributed to paucity of upper air data and errors of instrumentation, analysis, and chart reading. Because of these experimental errors it is impossible to accurately locate a trough or ridge to the nearest degree, as has been attempted here, or to obtain a perfect correlation between any two variables. More important, perhaps, is the great complexity of the atmospheric circulation. Because the atmosphere reacts to a multitude of forces it is obviously impossible for any simple empirical study of this type to completely account for all phases of long wave structure and behavior. Attainment of this goal will require further studies, of both theoretical and empirical nature, in order to arrive at a fuller understanding of the mechanism of the general circulation.

SUMMARY AND CONCLUSIONS

It should be realized that this study is quite limited in scope. It applies only to the winter season in the North American region and contributes little to the important problems of deepening and new trough formation. The basic cause of large-scale anomalies of the circulation pattern are not considered. Furthermore the results depict only the average behavior of the circulation. Few of the correlation coefficients account for much more than half of the observed variability and none exclude the possible existence of more complicated curvilinear or joint relationships. Hence large deviations from the expected relations may occur in individual cases or future years. Nevertheless the following conclusions are believed to be generally applicable to monthly mean maps at the 700-mb. level during the winter season in the North American area:

1. The preferred wave pattern in the United States consists of two main types; either a trough in the East and a ridge in the West, or a ridge in the East and a trough in the West.

2. Although the monthly mean flow pattern is generally flatter, weaker, and less variable than the 5-day mean pattern, the former exhibits more variability than would be expected from random data.

3. When there is a long wave length between the trough in eastern North America at middle latitudes and the ridge immediately upstream, the amplitude is usually large; while short wave lengths tend to be accompanied by small amplitude.

4. Systems of large amplitude and long wave length generally have strong winds at the trough and weak winds at the ridge, while waves of small dimensions tend to have faster winds at the ridge and slower winds at the trough. Troughs of the first type are frequently located along the Atlantic coast and produced during periods of low index; small amplitude waves are frequently strongly tilted from northeast to southwest and formed as lee troughs to the east of the Rocky Mountains.

5. The amplitude of the wave in North America is closely related to the amplitude of adjoining waves, particularly in the Pacific, since planetary waves tend to be symmetrical, in harmony with the principle of conservation of vorticity.

6. The formation of new troughs generally occurs to the lee of the Rocky Mountains and much more often at 40° N. than at 50° N.

7. As the period of averaging increases the wave speed decreases, but the absolute wave displacement from one chart to the next tends to remain constant, regardless of the time interval over which the map is averaged.

8. Retrogression is about as large and frequent as forward motion on monthly mean charts, but 5-day mean troughs retrograde only occasionally and daily troughs rarely.

9. Troughs with short wave length located west of the mean position tend to move eastward during the next month, while troughs retrograde when they are located east of the mean position and the wave length is long. This trough motion is modified by the simultaneous displacement of the ridge upstream since both trough and ridge tend to move in harmony to conserve the wave length.

10. Wind speed, amplitude, lateral width, wind shear, and trough tilt appear to have only a slight effect upon trough motion.

These findings may be further summarized in the following two conclusions, which have been emphasized repeatedly in the past century, namely:

1. There are definite interrelationships between the component parts of the general circulation.

2. The initial circulation pattern contains within itself at least some of the factors determining its subsequent evolution.

Many of the findings are also in reasonable agreement with the theory of long waves in the westerlies which has been developed during the past 15 years, despite the fact that the theory was designed for short period phenomena. Thus the monthly mean map may be considered as a dynamic entity to which certain physical principles can be applied. It is therefore hoped that this study will encourage theoreticians to devote more attention to the problem of weather processes involving time periods longer than a few days.

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