COMPUTING INSOLATION BY EMPIRICAL METHODS

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ABSTRACT

An empirical method described by Klein is applied to calculate insolation for the Lake Hefner, Okla., area for 1 year of the Lake Hefner Studies. For some of the months, the computed values show unsatisfactory agreement with the observed amounts, but the computed annual total is in close accord with the observed annual total.

It is suggested that agreement between the monthly values may be improved by introducing a curvilinear regression in the formula whereby account is taken of the depletion of insolation by sky coverage.

In connection with the recent Lake Hefner Studies, Anderson [1] has tested the usefulness of some empirical methods for the computation of insolation. He compares the results with the values determined by means of a pyrheliometer and finds that, *e. g.*, Mosby's formula

$$Q_s=k\ (1-0.071c)h, \tag{1}$$

where Q_s is the amount of insolation on the horizontal in gcal. cm.$^{-2}$ min.$^{-1}$, k, a constant depending on geographical latitude, c, the average cloud cover in tenths of the sky covered, and h the average altitude of the sun in degrees, yields results which are too low if one retains Mosby's values of the constants. The insolation calculated for the period September 1, 1950, to August 31, 1951, is approximately 15 percent less than the observed amount.

As for a number of climatological applications it is desirable to have an indirect method which will give results of greater accuracy, we have tested the method described by Klein [2] for the Lake Hefner area for the period examined by Anderson. In this, monthly averages of the following basic meteorological and geophysical data were used, enumerated in the order as they appear in the calculations:

1. *Surface vapor pressure e:* The agencies cooperating in the Lake Hefner Studies have measured e over the lake itself but we have deliberately chosen the data of an ordinary meteorological station. Anderson has used in equation (1) the cloud cover data of the Weather Bureau station at Will Rogers Airport, which is situated about 20 km. south of the lake. As we also shall use the cloud cover data of the same station [3], the most consistent procedure would have been to take the vapor pressure data of the Airport. These, however, were not available in published form and, therefore, we have resorted to the vapor pressure data of the station at the Weather Bureau Office, Oklahoma City, Okla., the station being located some 13 km. southeast from the lake. The data in table 1 are the averages of the published 6-hourly observations [4].

2. *Precipitable water W in the atmosphere:* This was computed from a Hann-type empirical equation $W=\alpha e$, α being a suitable constant and e the surface vapor pressure. To reduce errors in these estimates, we have taken, as recommended by Klein ([2], p. 120), $\alpha=2.5$ for winter (December through February), $\alpha=2.1$ for summer (June through August); for spring and fall, the original Hann value of $\alpha=2.3$ was applied. With these constants, e is in cm. Hg. and W in cm.

3. *Barometric pressure p:* This is required to correct for station level the values of mean solar air mass published for sea level pressures. The pressure values in table 1 are those of Will Rogers Airport [3], increased by 3 mb. to correct for difference in elevation (the elevation of the airport is about 27 m. greater than that of the lake surface). This correction is purely nominal as it does not affect the first decimal of the figures representing mean solar air mass. There is no point in computing the mean solar air mass beyond the first decimal.

4. *Mean solar air mass m*: Obtained by interpolation from a table by Kennedy [5] and then corrected for station-level pressure.

5. *Fraction d of insolation depleted by atmospheric dust*: It will be assumed that $d=0$. This assumption will be reconsidered below.

6. *Daily amount of solar radiation I_0 reaching the top of the atmosphere*: This quantity can be computed from a theoretical formula (e. g., Humphreys, [6], p. 88). It is also available in tables, as for instance in Kennedy's [5] paper.

7. *Cloud cover c*: Data of Will Rogers Airport [3]. As mentioned above, the cloud cover data of this station were used by Anderson in connection with Mosby's equation.

With the help of the data 1 to 5 inclusive, one can compute the transmission coefficient of insolation $(a+\frac{1}{2}s)$ where a is the transmission coefficient of insolation for a cloudless sky and s the fraction representing total depletion by atmospheric scattering and diffuse reflection, dust absorption having been assumed to be negligible. The factor ½ of s in the coefficient above is connected with the assumption, due to Kimball (Klein, [2], p. 122), that about half of the incoming radiation scattered and diffusely reflected by the atmosphere is received at the ground as diffuse radiation from the sky. The daily amount of insolation Q_c on the horizontal reaching the surface from a cloudless sky is obtained from the formula

$$Q_c = I_0 \left(a + \tfrac{1}{2} s\right) \tag{2}$$

while the depletion of insolation by a cloud cover of c tenths is approximated with the aid of the formula

$$Q_s = Q_c \left(1 - 0.071\, c\right). \tag{3}$$

It is seen from table 1 that the computed monthly values of daily insolation diverge, in some cases considerably, from the observed values. Particularly poor is the result for August 1951. The yearly averages, however, are in rather good agreement, the computed value being about 1 percent higher than the observed value. Had we considered the estimated effect of depletion by dust and introduced such values of d as mentioned *e. g.* by Klein, the resulting annual average would have come out 1 or 2 percent lower than the observed average, so that the result would have hardly been less good.

It is of course very probable that the good agreement between the annual figures is partly fortuitous. But it is worth noting in table 1 that the signs of deviations of the monthly figures are not as uniform as in the case examined by Anderson, a fact which is conducive to producing the agreement in the annual figures.

The method of computation does not take account of the type and thickness of clouds, nor the height above surface of the cloud cover. Indeed, it would be rather difficult to consider these factors as they are seldom observed in a reliable manner. However, the table does suggest that the agreement between the computed and the observed monthly values might be improved by the use of a curvilinear regression in lieu of the straight-line regression $(1-0.071\, c)$ whereby allowance is made for the depletion of insolation by the amount of cloud. It is seen from the table that, for this small sample at least, the straight-line regression tends to underestimate insolation at the greater cloud amounts and overestimate it at the lesser cloud amounts.

TABLE 1.—*Computation of insolation on the horizontal with the aid of method described by Klein [2] and comparison of results with observed amounts for the Lake Hefner, Okla., area: September 1950 to August 1951*

[Symbols defined and sources of data [3, 4] stated in the text]

Month	e	W	p	m	$(a+\frac{1}{2}s)$	I_0	Q_c	c	Q_s		Percent error
	cm. Hg.	cm.	mb.			gcal/cm.² day	gcal/cm.² day	tenths	gcal/cm.² day		
									comp.	obs.	
1950											
Sept	1.41	3.2	972	2.9	0.89	805	716	5.8	422	417	+1
Oct	.98	2.3	973	3.3	.89	598	532	3.0	420	390	+8
Nov	.39	.9	975	3.7	.91	460	419	3.8	306	287	+7
Dec	.31	.8	974	4.2	.90	392	353	3.8	258	233	+11
1951											
Jan	.30	.7	974	3.9	.92	428	394	4.8	260	275	−5
Feb	.43	1.1	974	3.5	.91	547	498	6.0	284	322	−12
Mar	.40	.9	971	3.1	.92	701	645	5.2	406	424	−4
Apr	.61	1.4	969	2.8	.92	847	779	5.0	506	516	−2
May	1.12	2.6	971	2.7	.90	946	851	5.3	528	541	−3
June	1.63	3.4	969	2.7	.90	981	883	5.6	530	591	−10
July	1.84	3.9	971	2.7	.89	959	854	3.7	632	611	+3
Aug	1.65	3.5	971	2.8	.90	946	851	2.8	681	576	+18
							Average*		437	433	

*Obtained from monthly values weighted for length of month.

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