CLIMATOLOGICAL ANALYSIS OF FREEZE DATA FOR IOWA

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[Manuscript received March 14, 1958; revised June 5, 1958.]

ABSTRACT

A set of freeze thresholds is defined which replaces the loosely defined element "killing frost." Freeze series are tested and found to be random. This makes it possible to fit frequency distributions. It is hypothesized that these distributions are normal and a test of significance verifies this. A hypothesis that the variance parameter is constant over Iowa is tested and found to be true. It is shown that spring and fall freeze are independent and this results in a simple form of freeze-free period distribution. The calculation of probabilities of freeze and freeze-free season are explained.

1. INTRODUCTION

There seems to have been little statistical treatment of frost or freeze data prior to the excellent works of Reed and Tolley [7, 8] in 1916. Previous writers on this subject for the most part were content to give means with little explanation of their significance. Few seemed to recognize the role of the random variable in computing means or in climatology in general, and randomness was usually looked on only as giving rise to errors in physical measurements which were to be avoided if possible.

About the time of the publication of the Reed and Tolley papers, the technique of linear correlation was being introduced rapidly in applied climatology. Here was a "magic" tool which depicted relationships and seemingly avoided the randomness that was so difficult to treat. Correlation immediately absorbed the energies of many able climatologists in attempts to solve problems for which the use of the technique was largely inappropriate. Many did not recognize the random character of the sample correlation coefficient and frequently interpreted correlations erroneously. Probably more in climatology than in other fields, the correlation technique tended to displace more objective statistical methods and to delay progress.

In contrast Reed and Tolley's papers show a remarkable understanding of the statistical method and its application in climatology. Indeed, one is amazed at their surprisingly modern slant in this discussion of over 40 years ago. Yet these papers seem to have drawn little attention, for work carried out 20 and more years later had not yet taken advantage of Reed and Tolley's contributions. Even much of the work being done today is not up to the high standard set by these able climatologists.

2. KILLING FROST VERSUS FREEZE

While the occurrence of the temperature at a point in time and space can be measured and expressed as a definite value with particular interest, there are also critical values below which effects of interest are the same. Thus, if we are interested in the freezing of water as a simple physical event, we might say that water freezes at 32° F. or any temperature lower than 32°, therefore 32° is a critical value for water. The interpretation of frost or freeze in meteorology is similar in that an effect produced by a critical value is also produced by any temperature lower than the critical value. Hence we define a freeze as the occurrence of a minimum temperature of specified value or lower, which can produce some special freezing effect. A 32° freeze is therefore the occurrence of a minimum temperature of 32° or lower and more generally a t-degree freeze is the occurrence of minimum temperature of t° or lower.

It is to be noted that our definition of freeze is a numerical one; i. e., it is defined in terms of a thermometer measurement. This is in contrast to "killing frost" which is defined non-numerically as the frost or freezing condition which kills the staple vegetation in the vicinity of the observing station. As has been pointed out many times previously this element has many obvious faults not the least of which are its looseness of definition and the difficulties of observation. Early criticisms of "killing frost" and "growing season" were made by Ward [12] and Landsberg [6]. In 1948 the Weather Bureau discontinued the use of "killing frost" as a climatic element and substituted the freeze thresholds 32°, 28°, 24°, 20°, and 16° F. These were arrived at in consultation with agronomists and horticulturists.

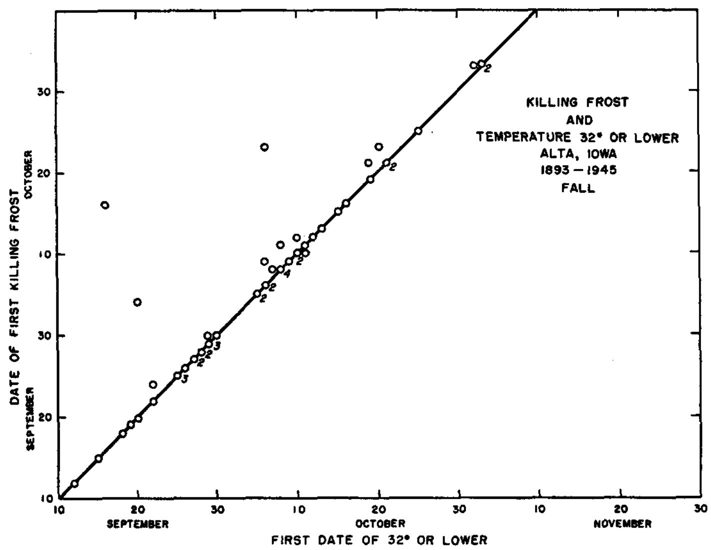

FIGURE 1.—Killing frost dates plotted against dates of 32° F. threshold in fall, Alta, Iowa.

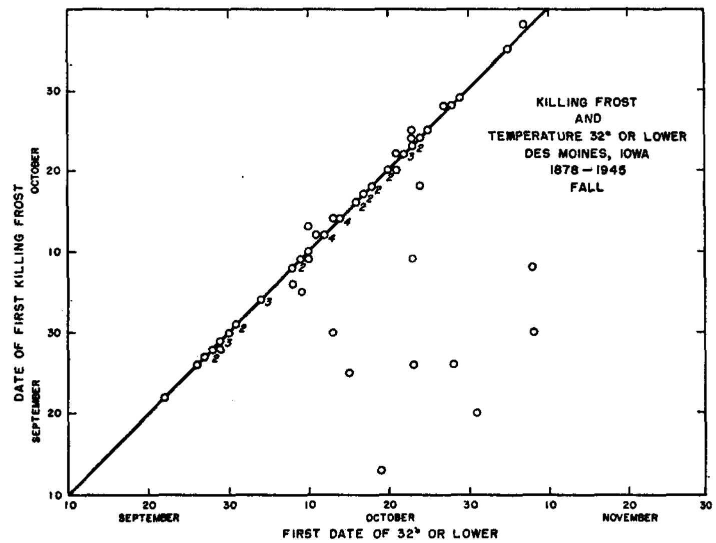

FIGURE 2.—Killing frost dates plotted against dates of 32° F. threshold in fall, Des Moines, Iowa.

Figures 1 and 2 show killing frost dates plotted against 32° F. threshold dates for Alta, a cooperative station, and Des Moines, a first order station. Note that for Alta the first killing frost date in fall comes later than the first 32° date whereas at Des Moines the 32° date comes later. An examination of several other examples showed similar inconsistencies, thus giving evidence of the shortcomings of the killing frost observation.

3. THE FREEZE SERIES

By a freeze series we mean the sequence of dates of annual occurrence of last spring or first fall freeze observed with stable instrument exposure. To apply statistical methods [10] to such a series it is necessary that an element of randomness be present. Although it never happens that this element is completely absent in a sequence of observations whatever their character, homogeneous freeze series are unconditionally random; i. e., there is no significant condition which can be imposed other than that of belonging to the class of freeze dates.

Reed and Tolley [8] evidently noted this randomness in "killing frost" series, for they compiled frequency distributions and treated frost dates as if they formed a random series. Although Reed and Tolley did not test the randomness of their frost series, the self-ordering of climatological data in time and the various extraneous effects which could introduce breaks and trends in climatological series make it good practice to apply some test for randomness. Many tests both parametric and non-parametric are available for such series, but since little is known of the power of such tests, the choice is based mostly on convenience of application. There is however, in this case the choice between a parametric and a non-parametric test, since as will be shown later, the freeze series treated here are normally distributed. In this instance then we have applied the autocorrelation test, the necessary distribution for which was derived by Anderson [1] for a normally distributed series. Grouping tests based on the tables of Swed and Eisenhart [9] have also been applied because of ease of application. In making the autocorrelation test it was assumed that the correlogram decreases from maximum values at short lags and hence only coefficients for lags one and three were computed.

The nature of the freeze series would naturally make the suspicion of non-randomness less than in other climatological series; in fact, all series tested were random on both tests. It seems, therefore, unnecessary to present results in detail although those for one station, Pocahontas, may be of interest. These are given in table 1.

The acceptance regions for ${}_LR_N$ where L is lag for number of observations N=42 and 43 may be taken directly from Anderson's table 6 for lag 1. For lag 3 the same table may be used since N/L=4 and N=4L, a criterion given by Anderson [1] for the suitability of the lag 1 table. Since the 95 percent acceptance region for N=42, 43 is approximately $(-.327<{}_1R_{42}<.278)$, it is seen that the coefficients of table 1 lie on this interval so we accept the hypothesis of randomness for both spring and fall series.

For the grouping test we again employ a two-tailed test since our alternatives to randomness are linear trend

TABLE 1.—*32° freeze autocorrelation coefficients, ${}_LR_N$, for Pocahontas, Iowa*

	Spring	Fall
Lag 1	0.224	−0.006
Lag 3	−0.032	−0.229
N	42	43
	Total Runs	
u	18	25

and location shift as well as oscillatory movement, the former giving rise to too many runs and the latter to either too many or too few. Swed and Eisenhart's [9] tables give the 95 percent acceptance region as (15, 28) approximately, for runs above and below the median with N=42, 43. The total numbers of such runs, u, for spring and fall as shown in table 1 are well within this region, and we conclude again that the series are random.

4. THE FREEZE DISTRIBUTION FUNCTION

Reed and Tolley [8] concluded that their "killing frost" series were normally distributed and fitted these series with this distribution; however, the means by which they reached this conclusion was not very powerful. The χ^2-test of goodness of fit was available at that time, but they did not report having employed it. This was perhaps just as well as this test is non-parametric and is not particularly sensitive in detecting divergence from normality. Geary and Pearson [5] have presented tests which are more sensitive in testing for non-normality. In fact Geary [4] states that these seem to be efficient tests for skewness and kurtosis for a wide range of alternatives to normality. Geary and Pearson's tests employ the statistics

$$a=\frac{\Sigma|x-\bar{x}|}{n\Sigma(x-\bar{x})^2}$$

for kurtosis, and

$$b_1=\frac{m_3}{(m_2)^{3/2}}$$

for skewness, where m_2 and m_3 are the second and third moments respectively. These were computed for a random sample of stations for 32°, 24°, and 16° freeze in spring and fall and are tabulated in tables 2 to 7 for the stations studied. The underlined values of $\sqrt{b_1}$ and a are those which fall outside of the 95 percent acceptance region. The relative infrequency of these when the reliability of the original data is considered leads us to the conclusion that the normal distribution is quite adequate for depicting freeze probability.

Since the sample mean and variance are jointly sufficient for estimating the parameters of the normal distribution, i. e., they extract all information from the sample available for fitting the normal distribution, it is only necessary to estimate these to obtain the freeze distribution function, or as we have called it, the "freeze hazard function." Values of the mean, $\bar{x}$, and variance, s^2, estimated from the various freeze series are also shown in tables 2 to 7.

Either one of two procedures may now be followed to obtain estimated probabilities for individual stations: (a) Employ $\bar{x}$ and s with any normal probability integral table, or (b) use normal probability coordinate paper for plotting the mean at 50 percent and $\bar{x}\pm 2s$ at 2.275 percent and 97.725 percent respectively. Method (a) may also be conveniently applied using the abbreviated normal probability table given in table 8. The results of following procedure (b) are shown in figure 3. A similar procedure may be applied to the 20° and 28° thresholds.

It is noted that the original data are not plotted. This is justified by the fact that plotting would involve first estimating probabilities, a procedure which is known to be much less efficient than the method of fitting described above; hence, the points would add nothing to the graphs and might actually detract from them if they tended to change the position of the lines. It may be further noted that spring freeze hazard is read from the upper scale and fall freeze hazard from the lower scale. These give the "best" estimates in the statistical sense of the

TABLE 2.—*Spring, 32° freeze. Underlined values are those which fall outside the 95 percent acceptance region*

Station	n	$\bar{x}$	s^2	s	$\sqrt{b_1}$	a	n'
Albia	30	4/28	158.8	12.6	.1197	$\underline{.2065}$	48
Ames	30	5/1	95.3	9.8	$\underline{.6182}$	.8327	53
Corning	29	5/2	138.5	11.8	.3681	.8163	55
Davenport	30	4/12	95.1	9.8	−.0891	.9775	73
Decorah	29	5/16	218.0	14.8	.3678	.8391	50
Denison	30	5/5	132.8	11.5	−.0614	.7926	49
Des Moines	30	4/20	134.4	11.6	.2400	.8282	68
Dubuque	30	4/19	86.1	9.3	.3177	.7993	73
Fairfield	30	4/29	121.0	11.0	.2499	.7791	46
Iowa City	30	4/28	132.6	11.5	−.0155	.7983	54
Iowa Falls	30	5/4	138.6	11.8	.1936	$\underline{.8479}$	54
Northwood	30	5/6	130.3	11.4	.3293	.7838	51
Olin	21	5/6	146.7	12.1	.4032	.8321	44
Pocahontas	30	5/7	153.2	12.4	−.2843	.7941	42
$g=.1257$							

TABLE 3.—*Spring, 24° freeze. Underlined values are those which fall outside the 95 percent acceptance region*

Station	n	$\bar{x}$	s^2	s	$\sqrt{b_1}$	a	n'
Albia	30	3/31	139.6	11.8	−.1756	.7962	48
Ames	30	4/6	151.8	12.3	−.0372	.7964	50
Corning	29	4/7	146.7	12.1	.2048	.8083	54
Davenport	30	3/23	174.5	13.2	−.0797	.8243	53
Decorah	30	4/20	141.4	11.9	−.0075	.8438	53
Denison	30	4/10	103.6	10.2	.0541	$\underline{.9596}$	49
Des Moines	30	3/26	183.9	13.6	$\underline{.5351}$	.8132	53
Dubuque	30	3/27	160.7	12.7	−.3543	.8219	47
Fairfield	30	4/2	183.9	13.6	−.3154	.7594	45
Iowa City	30	4/2	166.0	12.9	−.1393	.7825	51
Iowa Falls	30	4/10	155.9	12.5	.0903	.8218	51
Northwood	29	4/10	115.6	10.8	−.5383	.8126	51
Olin	21	4/9	99.0	9.9	.1351	.8316	33
Pocahontas	30	4/9	117.1	10.8	.2220	.8340	42
$g=.0948$							

TABLE 4.—*Spring, 16° freeze. Underlined values are those which fall outside the 95 percent acceptance region*

Station	n	$\bar{x}$	s^2	s	$\sqrt{b_1}$	a	n'
Albia	30	3/15	114.4	10.7	.0768	.8120	49
Ames	30	3/18	106.2	10.3	−.0311	.8419	51
Corning	30	3/23	173.7	13.2	.3025	.8007	54
Davenport	30	3/9	168.0	13.0	−.4094	.7974	53
Decorah	30	3/28	175.9	13.3	$\underline{.5328}$	.7892	54
Denison	30	3/24	129.2	11.4	.1693	.8403	49
Des Moines	30	3/13	160.6	12.7	−.1709	.7894	53
Dubuque	30	3/13	135.2	11.6	−.4098	.8312	47
Fairfield	30	3/13	161.5	12.7	.1543	.8194	46
Iowa City	30	3/17	113.4	10.6	−.0536	.8371	52
Iowa Falls	30	3/21	143.3	12.0	.0367	$\underline{.8608}$	49
Northwood	30	3/23	159.6	12.6	−.1402	.8448	51
Olin	21	3/20	181.9	13.5	.4641	.8591	33
Pocahontas	29	3/25	174.2	13.2	−.2464	.8247	42
$g=.0918$							

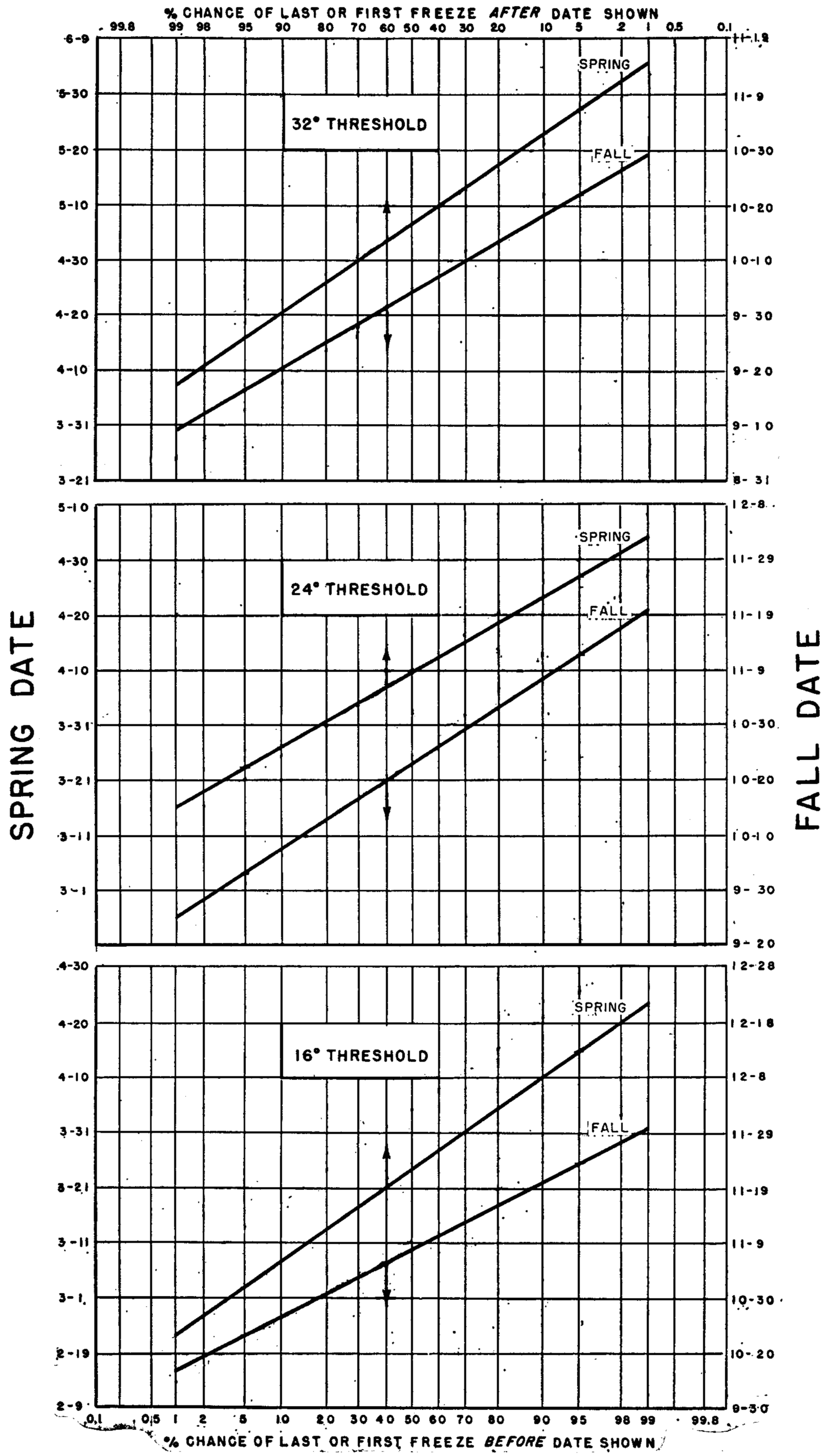

FIGURE 3.—Freeze hazard at Pocahontas, Iowa, 32°, 24°, and 16° thresholds.

TABLE 5.—*Fall, 32° freeze. Underlined values are those which fall outside the 95 percent acceptance region*

Station	n	$\bar{x}$	s^2	s	$\sqrt{b_1}$	a	n'
Albia	30	10/15	149.6	12.2	.0971	.8196	48
Ames	30	10/9	194.3	13.9	.6214	.7863	54
Corning	30	10/12	168.3	13.0	.2832	.8226	54
Davenport	30	10/24	133.6	11.6	−.0558	.8178	73
Decorah	30	9/25	170.8	13.1	.2447	.7873	52
Denison	30	10/4	125.8	11.2	.3972	.8405	51
Des Moines	30	10/19	146.4	12.1	.1611	.8031	68
Dubuque	30	10/19	143.0	12.0	−.0037	.8272	73
Fairfield	30	10/10	208.5	14.4	.1579	.8470	45
Iowa City	30	10/11	191.9	13.9	.1427	.7960	53
Iowa Falls	30	10/2	79.6	8.9	.3891	.7887	54
Northwood	30	10/5	98.2	9.9	.1801	.8588	50
Olin	20	10/1	80.1	8.9	.2226	.8215	42
Pocahontas	30	10/4	119.7	10.9	.3429	.8320	43

$g=.1081$

TABLE 6.—*Fall, 24° freeze*

Station	n	$\bar{x}$	s^2	s	$\sqrt{b_1}$	a	n'
Albia	30	11/1	201.1	14.2	−.2286	.7610	49
Ames	30	10/27	127.9	11.3	−.2479	.8119	50
Corning	30	10/28	212.8	14.6	.1176	.8220	51
Davenport	30	11/13	126.7	11.3	−.1191	.7882	52
Decorah	30	10/17	211.4	14.5	−.2338	.8459	51
Denison	30	10/26	151.2	12.3	−.4701	.8326	50
Des Moines	30	11/7	121.9	11.0	−.0949	.7805	52
Dubuque	30	11/11	132.9	11.5	−.2058	.7791	52
Fairfield	30	10/28	166.7	12.9	.0482	.7985	42
Iowa City	30	10/30	160.1	12.7	.0086	.7665	52
Iowa Falls	30	10/25	139.6	11.8	−.1227	.8439	52
Northwood	29	10/27	158.2	12.6	−.2875	.8240	46
Olin	20	10/27	160.1	12.7	.2330	.7962	37
Pocahontas	30	10/23	151.3	12.3	−.1375	.8054	43

$g=.1032$

TABLE 7.—*Fall, 16° freeze*

Station	n	$\bar{x}$	s^2	s	$\sqrt{b_1}$	a	n'
Albia	30	11/23	196.5	14.0	.3515	.7517	49
Ames	30	11/15	110.9	10.5	−.2245	.8049	50
Corning	30	11/13	122.7	11.1	−.4692	.7960	51
Davenport	30	11/27	197.3	14.0	.5853	.7900	51
Decorah	30	11/9	155.5	12.5	−.3924	.7798	52
Denison	30	11/7	154.3	12.4	−.1285	.7745	51
Des Moines	30	11/25	192.4	13.9	.5127	.7735	52
Dubuque	30	11/25	176.3	13.3	.3899	.7826	52
Fairfield	30	11/22	182.2	13.5	.1082	.7882	44
Iowa City	30	11/20	125.1	11.2	−.2005	.7979	52
Iowa Falls	30	11/12	117.3	10.8	−.3577	.7948	52
Northwood	29	11/11	135.4	11.6	.1079	.8005	47
Olin	20	11/14	178.1	13.3	.1258	.8251	34
Pocahontas	30	11/8	93.8	9.7	−.1927	.7811	43

$g=.1007$

TABLE 8.—*Abbreviated table of normal probability distribution*

For probability less than ($\bar{x}\pm ts$) read down		
$P[x<(\bar{x}-ts)]$	t	$P[x<(\bar{x}+ts)]$
.01	2.33	.99
.02	2.05	.98
.05	1.64	.95
.10	1.28	.90
.15	1.04	.85
.20	.84	.80
.25	.67	.75
.30	.52	.70
.35	.39	.65
.40	.25	.60
.45	.13	.55
.50	0	.50
$P[x>(\bar{x}+ts)]$	t	$P[x>(\bar{x}-ts)]$
For probability greater than ($\bar{x}\mp ts$) read up		

probability of freeze occurring after a particular date in spring and before a particular date in fall.

In the above analysis n years were used to estimate the mean, variance, and standard deviation whereas the full length of record n' was used in estimating $\sqrt{b_1}$, and a. The n-year statistics came from State tabulations of freeze statistics issued to Weather Bureau Offices. These were designed according to the general principles set forth here and treated insofar as possible as a homogeneous 30-year record at cooperative stations.

5. CLIMATIC VARIATION OF THE PARAMETERS

We have seen that the two parameters of the freeze hazard distribution are the mean and standard deviation. It is quite clear from physical principles that the mean freeze date must vary with latitude and other factors. This makes it possible to draw isoline maps of mean frost (or freeze) and there are, of course, familiar examples of such maps in the literature.

While geographical variation of mean freeze date can be explained physically, there is no cogent physical basis for hypothesizing the geographical variation of the freeze date standard deviation. If, however, we think of the standard deviation as a scaling parameter, then the standard deviation of temperature is a climatological scale of temperature, and the standard deviation is a scale of freeze date. Now it is known that the climatological scale of temperature varies slowly with geographical factors, and since the freeze threshold date is a kind of inverse time function of temperature for a fixed temperature range, we might suspect that the scale of freeze threshold also changes slowly in a similar fashion. This, as we shall see, seems to be the situation.

Reed and Tolley [8] prepared isoline maps of frost threshold standard deviation which give the impression of being drawn to small islands of homogeneity rather than to some underlying geographical functions. Although they do not remark on the difficulties in drawing these isolines, they must have wondered some about drawing the same isoline value for northern North Dakota as for northern Florida. One hypothesis that could be made is that the islands are due to correlations in the random residuals. Such correlations decrease with increasing distance between stations and this, together with small climatic heterogeneities, could lead to such islands.

Although the Reed and Tolley maps give no reason to question the constancy of the freeze scale parameter over quite large areas, we did examine one possible causal factor; this was station elevation. The variances of 57 Iowa stations were correlated with station elevation. For spring this gave a correlation coefficient of only 0.1054 and for fall of only −0.0173. Neither of these is significant at the 5 percent level, leading us to the conclusion that the variance is probably not related to the station elevation.

The next logical step in our analysis of scale variation seems to be to hypothesize that the scale is essentially homogeneous over quite large areas or specifically over the

State of Iowa. To test such a hypothesis it is convenient to deal with the variance rather than the standard deviation. This is no more complicated than to consider that the variance is our population parameter rather than the standard deviation. Although there have been many tests derived for testing the homogeneity of variances, there is no test for homogeneity when there is a given correlation between the populations. We shall, therefore, use a simple test due to Cochran [2] with consideration given to the effect of dependence.

Cochran has given the distribution of g which is the ratio of the largest variance of k sample variances to the sum of the k sample variances when the k populations are independent. Extended tables for the 5 percent and 1 percent significance limits are given in reference [3]. To adjust the test for dependence we proceed as follows: Since we have only the hypothesis on the invariance of the scale, we wish to accept this hypothesis although, of course, we will not let this wish influence our decision. We note that Cochran's tables provide for k populations (stations) and sample size n (length of record in years). With Olin omitted from consideration it is seen in tables 2–7 that n is about 30 and k would be 13 if the stations were independent of each other. However, the stations are not independent and hence, k has some effective value less than 13. Now, if we examine the 5 percent limits of Cochran's table [3] we see that the g values increase with decreasing k. Consequently, if a g based on k dependent stations is not significant according to the significance limit based on k independent stations, it is all the more not significant on some smaller k resulting from the existing dependence.

The values of g for each threshold and season are entered at the bottoms of tables 2–7. Using Cochran's tables and the above reasoning with respect to dependence one can see roughly that none of these g's is significant at the 5 percent level. Again on the assumption of independence a more exact probability of exceeding $g=0.12567$ for spring 32° feeeze was computed using a normal approximation for the beta distribution. This gave a value of 0.15 which is at least this large for the true situation of dependence among the stations. Hence, the probability of exceeding $g=0.12567$ is greater than 0.15 and the probability of exceeding the g's for other thresholds is still greater. The threshold series are highly correlated (see [10]); therefore, the g's add little information to each other about heterogeneity of the variance in general. As a consequence the individual tests of the g's must be accepted as a verification of the physical hypothesis that the variances are homogeneous.

If the variances do not differ significantly over the State of Iowa, then the standard deviations cannot differ significantly, and it is a considerable simplification to assume one value of the standard deviation for the whole State. This has already been done in [11]. On the assumption of homogeneity of the variance over a large area such as a State, a much improved estimate of the variance may be obtained by taking the weighted average of the station variances. The square root of this provides a good estimate of the standard deviation. Using the standard tabulations of freeze data referred to above, table 9 was prepared. These average standard deviations may be used with any individual station mean to obtain the freeze hazard distribution for that station.

TABLE 9.—*Variances and standard deviations*

	Threshold				
	32°	28°	24°	20°	16°
	Spring				
s^2	133.7	142.0	155.4	171.9	154.1
s	11.6	11.9	12.5	13.1	12.4
	Fall				
s^2	141.2	164.4	167.0	157.1	163.1
s	11.9	12.8	12.9	12.5	12.8

6. THE DISTRIBUTION OF FREEZE-FREE PERIOD

Since we have shown above that the freeze hazard distribution is normal, it follows from a well-known theorem in statistics that the difference between fall and spring threshold dates or freeze-free period will also be distributed in a normal distribution. This distribution will have a mean equal to the difference between the fall and spring mean dates. Its variance, however, will depend on the correlation coefficient between spring and fall freeze dates. This cannot be obtained from the statistics given above, and so must be given separate consideration.

The long time interval between spring and fall freeze dates naturally leads one to the conclusion that the correlation between them must be small. Reed [7] found

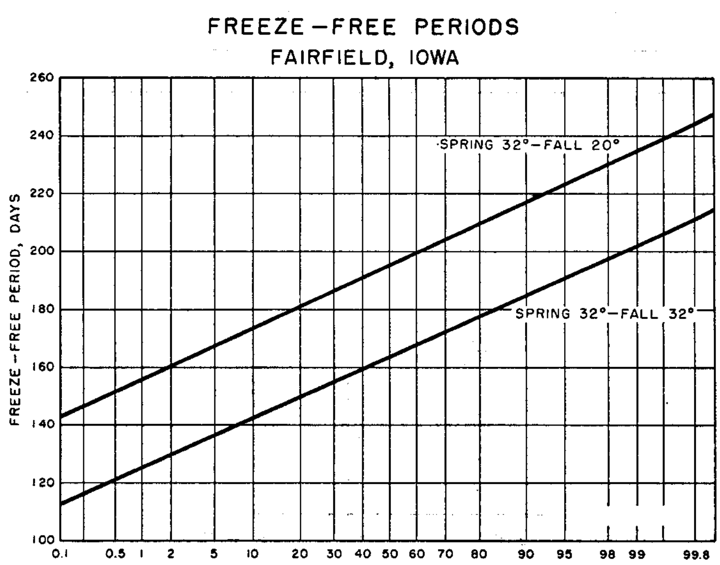

FIGURE 4.—Distribution of freeze-free period, Fairfield, Iowa.

the spring-fall correlation coefficient for one station, Keokuk, Iowa, to be —.179. Using a test which is now obsolete, he concluded correctly that this was not significantly different from zero. He made no further analysis of this correlation, and even though this single value was not significant, he included it in estimating the standard deviation of the growing season distribution. The effect of the correlation is to increase the standard deviation a little more than one day.

We have computed a number of spring-fall correlation coefficients for Iowa and found none to be significant. Two typical examples are Fairfield —.103 and Northfield —.080. Although further study of this correlation may be of interest, its small effect on the standard deviation of the freeze-free distribution makes the fruitfulness of such an effort questionable. With close approximation it may be assumed that the freeze-free period standard deviation is the square root of the sum of the spring and fall freeze variances and the mean is, of course, the difference between the fall and spring means. This gives the same statistics we had for the freeze distributions and hence, the freeze-free period distribution may be plotted on probability paper in the same manner. From the means of tables 2 and 5 and the variances of table 9, we find the spring 32°-fall 32° freeze-free period mean and standard deviation for Fairfield to be 163.9 and 16.6, respectively. The distribution is shown in figure 4.

It will be clear that the same threshold need not be used at both ends of the freeze-free period. The need for using different thresholds in spring and fall could come from either a different sensitivity of a single crop in the spring and fall or one crop being subject to the spring hazard and another to the fall hazard. Figure 4 also shows the distribution for the spring 32°-fall 20° freeze-free period which is an illustration of an application to seed corn growing. Here, the spring hazard is that of killing the tender plant, whereas the fall hazard is reduction of germination percentage. The spring and fall variances were obtained from table 9 and the mean 32°–20° freeze-free period, 195.5, from manuscript tabulations.

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