SURFACE FRICTION IN A HURRICANE

VANCE A. MYERS

Hydrometeorological Section, U.S. Weather Bureau, Washington, D.C.

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ABSTRACT

In an extension of earlier work by Johnson, it is found that the apparent friction of the wind over Lake Okeechobee, Fla., in the 1949 hurricane is related to the speed by $F_s=0.022V^2$ and $F_n=0.020V^2$. F_s and F_n are the frictional accelerations in mi. hr.$^{-2}$ tangential and normal to the wind, and V is the anemometer-level wind speed in m.p.h. Frictional accelerations over land are about three times the over-water values at the same wind speed. At a given storm radius the land and water tangential frictional accelerations are nearly equal.

LIST OF SYMBOLS

V_s = wind speed, always positive.
t = time.
s = distance along streamline; also direction tangent to streamline, positive in direction of wind.
n = direction normal to s, positive to right of the wind.
θ = angle between tangent to streamline and tangent to circular isobar, positive for inward incurvature.
α = specific volume.
p = pressure.
r = distance between the center of concentric isobars and any point, positive outward.
R_t = radius of curvature of trajectory.
f = Coriolis parameter.
F_s = frictional force* tangent to trajectory, positive opposite the wind.
F_n = frictional force* normal to trajectory, positive to right of the wind.
V_H = speed of forward motion of storm.
ψ = azimuth of any point in storm from the storm center, measured from direction of motion of storm with clockwise rotation being positive.

1. INTRODUCTION

This paper is a sequel to an earlier paper by Johnson [1]. He computed the apparent frictional retardation of the wind in the hurricane that passed over Lake Okeechobee, Fla. on August 26–27, 1949. Unusually detailed wind and pressure observations were obtained by the Corps of Engineers network at the Lake and were analyzed by the U.S. Weather Bureau [2]. An important feature of the data is that wind speeds were measured over the water, by anemometers installed on navigation light pylons, as well as from shore stations.

Johnson's method was to compute values from the data for all terms in the equations of horizontal motion except the friction terms and thereby calculate these as residuals. His friction values are mean values for the storm, partly over land and partly over the Lake. This derives from his use of mean radial wind profiles based on all the wind observations, some over water, and some at the shore, with both off-water and off-land wind directions.

*Throughout this paper the term "force" means force per unit mass.

In the present study the Lake Okeechobee hurricane data have been reworked by Johnson's method, separately for over-water winds and off-land winds. It was found that the over-water friction components are about proportional to the square of the wind speed. The over-water values are probably the best available estimates of the low-level frictional forces in a hurricane over a water surface.

2. EQUATIONS OF MOTION

Stationary storm.—The equation of motion along any horizontal streamline in a stationary, circularly symmetrical storm is:

$$\frac{dV_s}{dt}=-V_s\frac{\partial V_s}{\partial r}\sin\theta=\alpha\frac{\partial p}{\partial r}\sin\theta-F_s. \qquad (1)$$

The respective terms give the total acceleration along the streamline or trajectory, the acceleration in terms of the wind speed and wind speed gradient, the component of pressure gradient force along the streamline, and the frictional force along the streamline. The directions involved in this equation and the next equation below are illustrated in figure 1.

The corresponding equation for acceleration normal to the streamline or trajectory is:

$$\frac{V_s^2}{R_t}=\frac{V_s^2}{r}\cos\theta-V_s^2\frac{\partial\theta}{\partial r}\sin\theta=\alpha\frac{\partial p}{\partial r}\cos\theta-fV_s-F_n. \qquad (2)$$

Here the first term on the left is the centrifugal force. The next two terms express the centrifugal force in two parts, that which must be overcome to maintain a constant θ along the streamline and that which must be overcome to increase θ. The final three terms are the component of the pressure gradient force normal to the streamline, the Coriolis force, and the frictional force normal to the streamline, respectively.

Equations (1) and (2) are the equations that were used by Johnson [1]. The next subsection will justify the application of equations (1) and (2) for a stationary storm to the storm of August 26–27, 1949 which was moving about 16 m.p.h.

Moving storm.—Equation (1) is a special case of the following more general equation for the central part of a moving hurricane in which an unvarying circularly symmetrical pressure field moves forward in a straight line at a fixed speed. The wind field is not necessarily symmetrical but is fixed with respect to the center; that is, the pattern of isogons and isotachs moves forward with the speed of the storm but otherwise remains unchanged.

$$\frac{dV_s}{dt}=-\frac{\partial V_s}{\partial r}(V_s\sin\theta+V_H\cos\psi)+\frac{1}{r}\frac{\partial V_s}{\partial\psi}(V_H\sin\theta-V_s\cos\psi)=\alpha\frac{\partial p}{\partial r}\sin\theta-F_s. \qquad (3)$$

Equation (3) is derived by expansion of the derivative dV_s/dt. As wind speed under the stated restrictions is a function of r and ψ only, by the rules for expansion of derivatives,

$$\frac{dV_s}{dt}=\frac{\partial V_s}{\partial r}\frac{dr}{dt}+\frac{\partial V_s}{\partial\psi}\frac{d\psi}{dt}. \qquad (4)$$

Here dr/dt is the total time rate of change of the distance from the storm center to an air parcel. But

$$\frac{dr}{dt}=\left(\frac{dr}{dt}\right)_1+\left(\frac{dr}{dt}\right)_2 \qquad (5)$$

where $(dr/dt)_1$ is the rate of change of length r due to the motion of the air parcel and $(dr/dt)_2$ is the change in r due to the motion of the storm center.

From the geometry of the model,

$$\left(\frac{dr}{dt}\right)_1=-V_s\sin\theta;\quad \left(\frac{dr}{dt}\right)_2=-V_H\cos\psi \qquad (6)$$

or

$$\frac{dr}{dt}=-V_s\sin\theta-V_H\cos\psi. \qquad (7)$$

By similar reasoning,

$$\frac{d\psi}{dt}=\left(\frac{d\psi}{dt}\right)_1+\left(\frac{d\psi}{dt}\right)_2=-\frac{V_s}{r}\cos\theta+\frac{V_H}{r}\sin\psi. \qquad (8)$$

Substituting (7) and (8) in (4) gives:

$$\frac{dV_s}{dt}=-\frac{\partial V_s}{\partial r}(V_s\sin\theta+V_H\cos\psi)+\frac{1}{r}\frac{\partial V_s}{\partial\psi}(V_H\sin\psi-V_s\cos\theta). \qquad (9)$$

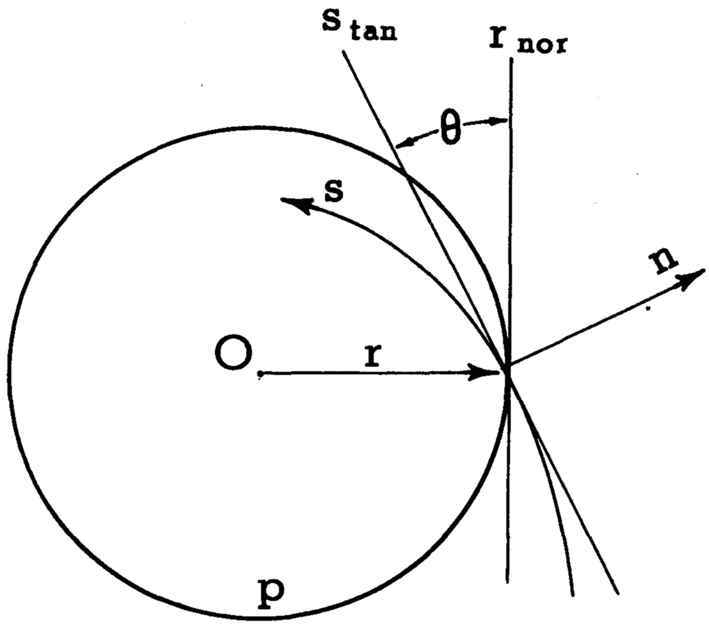

FIGURE 1.—Model hurricane. O=center of concentric circular isobars; p=isobar; s=streamline; s_{tan}=tangent to s; r=radius from O to point on s; r_{nor}=normal to r; n=normal to s; θ=deflection angle between r_{nor} and s_{tan}.

The terms $\alpha(\partial p/\partial r)\sin\theta-F_s$ are the same in equations (1) and (3).

The data most readily available for this study were average values of pressure, wind speed, deflection angle, and radial gradients of these meteorological elements along respective radii. Averaging each term of equation (3) over azimuth, at any one radius, yields:

$$\overline{\frac{dV_s}{dt}}=-\overline{\frac{\partial V_s}{\partial r}V_s\sin\theta}-\overline{\frac{\partial V_s}{\partial r}V_H\cos\psi}+\overline{\frac{1}{r}\frac{\partial V_s}{\partial\psi}V_H\sin\theta}-\overline{\frac{1}{r}\frac{\partial V_s}{\partial\psi}V_s\cos\psi}=\overline{\alpha\frac{\partial p}{\partial r}\sin\theta}-\overline{F_s}. \qquad (10)$$

As the fluctuation of the various variables with azimuth is relatively modest, equation (10) can be approximated by:

$$\frac{\overline{dV_s}}{dt}=-\frac{\partial\overline{V_s}}{\partial r}(\overline{V_s}\sin\bar\theta+V_H\,\overline{\cos\psi})+\frac{1}{r}\frac{\overline{\partial V_s}}{\partial\psi}(V_H\sin\bar\theta-\overline{V_s}\,\overline{\cos\psi})=\alpha\frac{\partial p}{\partial r}\sin\bar\theta-\overline{F_s}. \qquad (11)$$

If the average is over 360° rather than only one sector,

$$\overline{\cos\psi}=0;\quad \frac{\overline{\partial V_s}}{\partial\psi}=0. \qquad (12)$$

Equation (11) then reduces to:

$$\overline{\frac{dV_s}{dt}}=-\overline{V_s\frac{\partial V_s}{\partial r}\sin\theta}=\alpha\overline{\frac{\partial p}{\partial r}\sin\theta}-\overline{F_s} \qquad (13)$$

which is in the same form as equation (1). The stationary storm equation, (1), if we accept the approximation of product of means equal to mean of products, is applicable, then, to a moving storm, for mean values taken over the entire 360° of azimuth. The last requirement is one reason that no attempt was made to derive separate frictional values for front and rear halves of storm in this study, as did Johnson [1].

The equation of motion normal to the trajectory for a moving storm reduces to equation (2) in the same fashion and under the same restrictions that equation (3) reduces to equation (1).

3. FRICTIONAL FORCES

Observed parameters.—The smoothed mean values of observed pressure and wind used to compute the friction are shown in figure 2. The pressure and wind speed profiles are from [3], the deflection angle from [2], also reproduced in [3]. This interpretation of the deflection angle gives more weight to a few points near the center than does Johnson [1] in his figure 6.

Computation of friction.—Values of the frictional forces, F_s and F_n, were computed for water and for land surfaces at various storm radii by substituting data from figure 2 into equations (1) and (2). Density was fixed at 1.15×10^{-3} gm. cm.$^{-3}$*. The F_s and F_n values are plotted against wind speed on logarithmic scale in figure 3. Most of the points outside the radius of maximum wind speed (23.5 miles) fall approximately in straight lines, suggesting a relation of the form $F=kV^m$. The computed points pertaining to the region inside the radius of maximum winds should be expected to depart from a relation, because the assumption of horizontal trajectories implicit in the method of computing friction probably does not hold across the boundary of the eye.

Relation of friction to wind speed.—Straight lines were fitted by eye to the portion of each set of data outside the radius of maximum winds. The equations of the lines are shown in the upper part of table 1.

The power of V_s for F_s over water was so close to the theoretical value of 2 that it was set equal to this value and k recomputed (lower part of table 1). There is no a priori knowledge of the exponent of V_s in the F_n relation. However, great convenience results in some applications if the exponent is the same for F_s and F_n, as the ratio F_s/F_n then remains constant. For F_n over-water m was also set equal to 2 and the line of best fit with this restriction drawn.

*The horizontal density variation in a hurricane is proportional to the pressure variation. However, speeds were measured with Dines (pitot tube) anemometers, presumably calibrated at normal sea level density, and were not corrected for density. Adjustments to density and wind speed tend to be opposite and compensating and are therefore not required in equations such as (1) and (2), involving specific forces or accelerations.

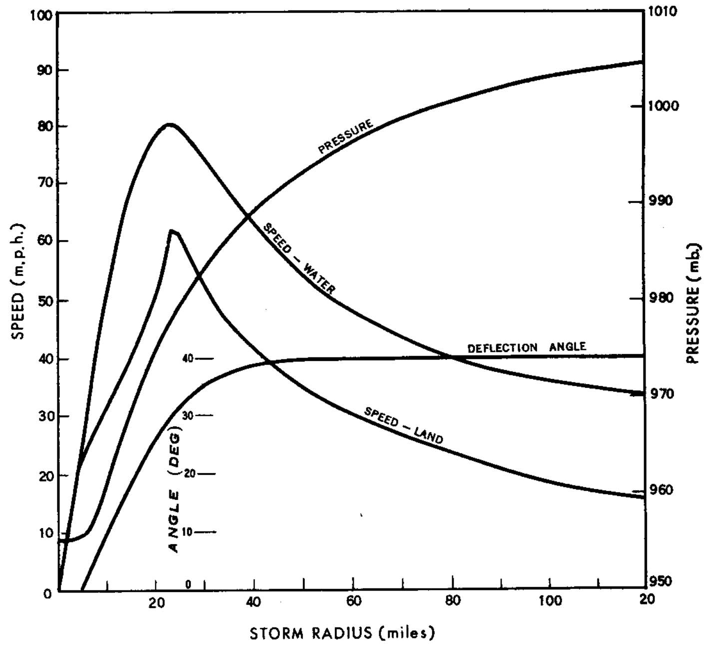

FIGURE 2.—Wind and pressure profiles, hurricane of August 26–27, 1949, at Lake Okeechobee, Fla.

The off-land friction was proportional to about the 1.7 power of the speed. The eye-fitted and adjusted relations are shown in table 1.

Comparison of friction over water and land.—The land values of friction are about three times the water values at the same surface wind speeds (fig. 3). But at the same storm radius *the land and water tangential friction is nearly the same* (fig. 4). This interesting result implies that, at least in this circumstance of a storm partly over land and partly over water, beneath some upper-level wind velocity that is essentially the same over land and water the surface speed adjusts itself to the roughness of the surface such that some requisite frictional retardation is attained.

TABLE 1.—*Relations of friction F_s and F_n to wind speed V_s (see fig. 3)*

Line of best fit by eye	
	Line shown in fig. 3
Over water	
$F_s=0.0195V_s^{2.04}$	No
$F_n=0.0053V_s^{1.87}$	Yes
Over land	
$F_s=0.29V_s^{1.6}$	Yes
$F_n=0.15V_s^{1.8}$	Yes
Smoothed to common exponent	
Over water	
$F_s=0.022V_s^{2}$	Yes
$F_n=0.020V_s^{2}$	Yes
Over land	
$F_s=0.20V_s^{1.7}$	No
$F_n=0.21V_s^{1.7}$	No

F_s=frictional acceleration, mi. hr.$^{-2}$, opposite wind vector. F_n=frictional acceleration, mi. hr.$^{-2}$, to right of wind vector. V_s=wind speed, m.p.h.

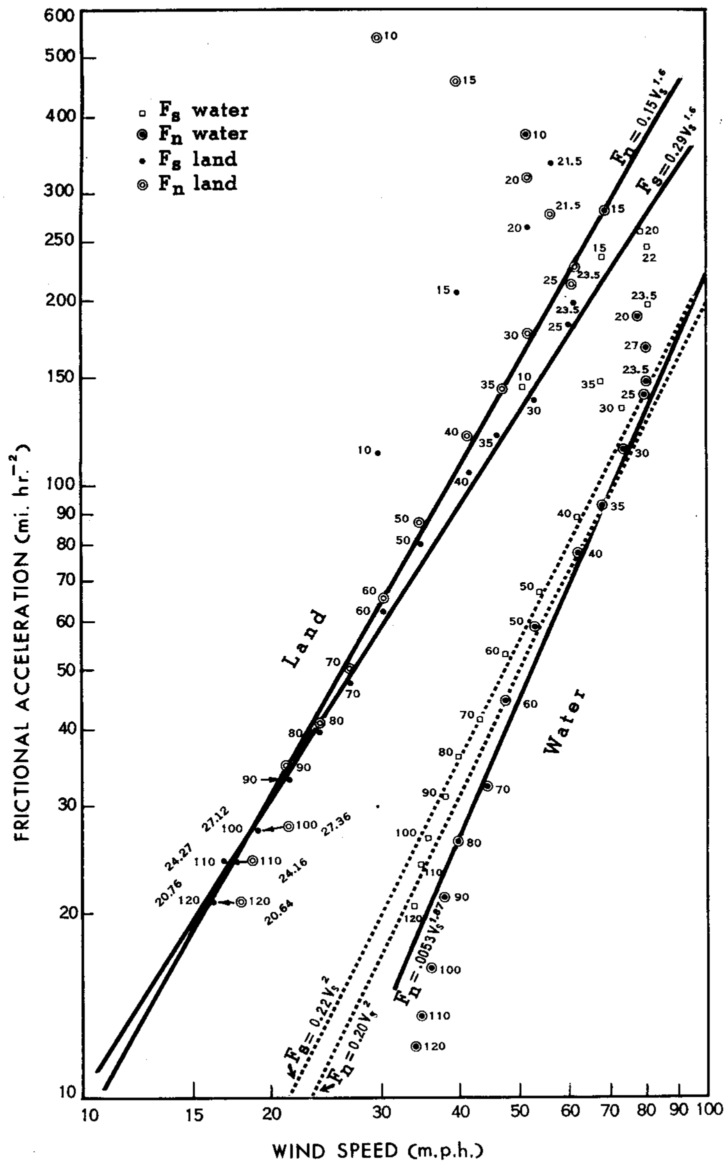

FIGURE 3.—Frictional acceleration vs. wind speed at anemometer level. Hurricane of August 26–27, 1949, at Lake Okeechobee, Fla. Numbers beside points are storm radius in miles.

The land normal friction is about 50 percent greater than the over-water friction at the same distance from the storm center.

Comparison with other authors' results.—Johnson's [1] values, derived from the same storm but with mixed frictional category, naturally lie between the values derived in this study (fig. 5). In addition to over-water and off-land winds, Johnson includes the "off-water" category of winds (measured at shore stations, wind direction off the lake) which were not used at all in the present study. Hubert [4] has measured friction in other recent hurricanes by similar techniques. The higher wind speeds of his investigation overlap the lower speeds of this study. Comparisons with some of his values are shown in figure

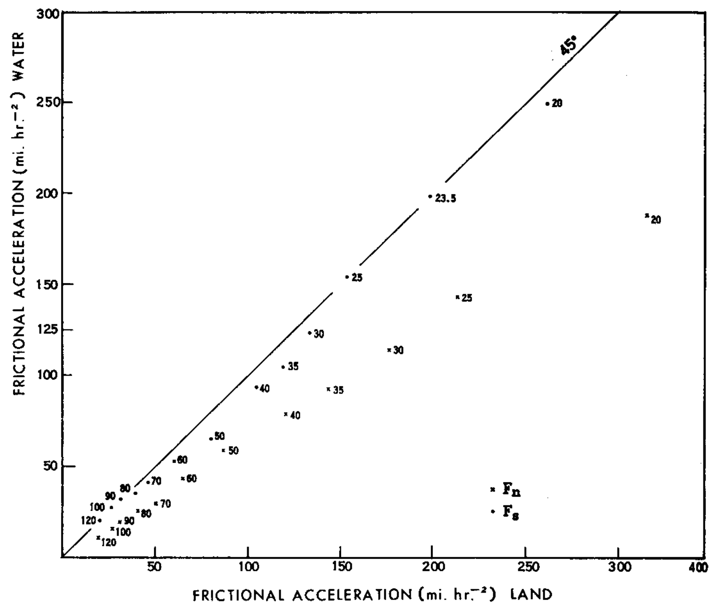

FIGURE 4.—Comparison of frictional accelerations over water and land at same storm radius. Hurricane of August 26–27, 1949, at Lake Okeechobee, Fla. Numbers beside points are storm radius in miles.

5. Hubert's values are lower, his over-land values being approximately the same as the over-water values of this study for given wind speeds.

Analyses of the friction of the surface wind elsewhere than in hurricanes have also shown large values of normal component of friction [4, 5, 6, 7].

4. SUMMARY

Some detailed observations from a hurricane passing over a lake have been analyzed to determine the apparent frictional force on the anemometer-level wind flow. The component of friction to the right of the wind was about equal to the component opposite the wind over land, and almost as large over water. Outside the eye of the storm and in over-water flow both components were nearly proportional to the square of the wind speed. The winds appeared to adjust themselves in such a way that the total friction of the anemometer-level wind was about the same at any storm radius over water and over land.

The relations found can be used for estimates of the friction of the low-level wind in hurricanes at sea, though the questions remain of the relative roughness of the ocean as compared with Lake Okeechobee and of variations due to differences in structures of individual storms.

ACKNOWLEDGMENT

The over-water friction value computations were taken, in part, from a related study by Herman Lake [8].

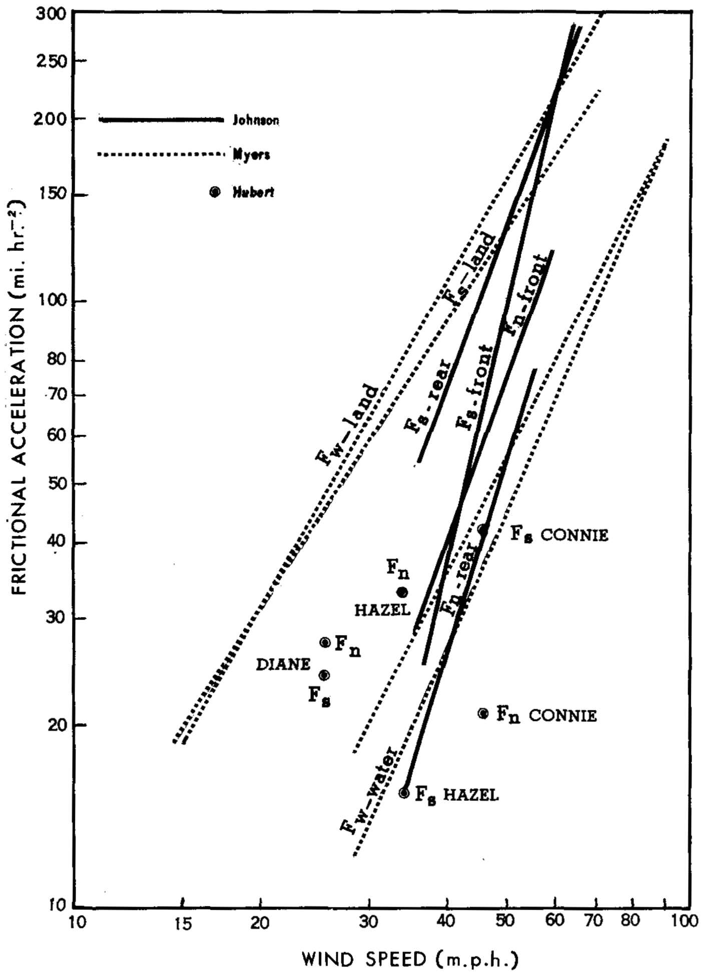

FIGURE 5.—Comparison of computed frictional accelerations. Johnson [1]: hurricane of August 26–27, 1949, at Lake Okeechobee, Fla. Values stratified into front and rear of storm but mixed as to category of underlying surface. Myers: same hurricane, from figure 3. Values stratified into over-water and off-land but mixed as to azimuth. Hubert [4]: selected hurricanes 1954–55, over land, mixed azimuth.

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